SPECTROSCOPIC ABSOLUTE MAGNITUDES & PARALLAXES OF 200 A-TYPE STARS

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ABSTRACT

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The spectroscopic absolute magnitudes and parallaxes of 200 A stars have been determined.

The material used was the large collection of 1-prism slit spectrograms of the Yerkes Observatory. Eighty stars having reliable trigonometric or cluster parallaxes were made the basis of calibration and seven criteria have been found giving correlations with absolute magnitude, namely - width of K, H_δ, $\lambda4481$, and intensity ratios $\lambda4215/\lambda4227$, $\lambda4233/\lambda4227$, $\lambda4535/\lambda4481$, $\lambda4549/\lambda4481$.

Relative to the standard stars, the systematic error of the magnitudes is -0.04 and the probable error ±0.5 . Relative to 108 Mt.Wilson spectroscopic magnitudes the systematic error is +0.09 and the probable error ±0.3 .

The Shajn Double Star test applied to 12 pairs gives satisfactory evidence in favour of the accuracy of the magnitudes, but an attempt to correlate magnitude with reduced proper motion gave a null result.

A comparison is given between Mt. Wilson , Arcetri and the writer's average magnitudes for each spectral subdivision and evidence is given in support of the claim that the magnitudes herein determined have greater individual accuracy than can be obtained by adopting the mean magnitude method.

Various problems involved in the interpretation of the spectra of A stars are discussed.

PART I

Introduction.

The young science of astrophysics is the direct and natural outcome of three of the oldest of all the sciences - astronomy, mathematics and physics. Arising as it does from this triple alliance it is endowed with the astronomical wisdom of the ages, it wields in its hand the powerful weapon of mathematical reasoning, it adapts to its use the most exquisite instruments of precise measurement that physics can devise and thus equipped it strides boldly forward into new and ever wider realms of thought and realms of space with all the courage and enthusiasm of youth.

In the days of ancient Greece, Aristotle taught, and Ptolemy of Alexandria perpetuated the fallacy, that whereas terrestrial things were composed of four "elements", earth, air, fire, water, the celestial bodies - sun, moon, planets, and stars - were composed of a fifth element, a perfect substance, unchanging and unchangeable, the "quintessence". As long as this idea held sway in the minds of men, a science of astrophysics was impossible. For physics deals essentially with <u>change</u> - change of position, change of configuration, change of state, change of energy distribution - change of some kind or there is no physics involved, save only in one limited subdivision of physics, namely, statics;

and even in statics the idea of change cannot be wholly avoided for many of its problems are solved by recourse to the principle of Virtual Work, a principle the basic idea of which is the effect of change.

When Galileo turned his hand-made telescope upon the Sun and found upon its surface strange, dark markings which altered both in shape and position from day to day, he threw down and trampled under foot forever the dogma of the immutable fifth substance. There were changes on the face of the Sun, great convulsions which darkened portions of its dazzling surface; there were small satellites which moved round and round Jupiter; there were stars which blazed forth a temporary brilliance - all testified to a great law of change ruling in the heavens as well as upon the earth; and from the moment this was realized a science of astrophysics became theoretically possible.

When Isaac Newton passed a beam of sunlight through a glass prism and obtained thus a separation of its rays, the beautiful orderly array of spectrum colours, he achieved the first step towards making a science of astrophysics practically possible. For since there is no means of gaining information about the stars save that which can be read in the beam of starlight itself, the first essential to the problem of disentangling the riddle of the stars is to be able to analyse star light.

In the judgment of F. J. M. Stratton[1], the science of astrophysics may be said to have had its actual beginning when in 1802 Wollaston observed that the spectrum of sunlight did not consist merely of a coloured band of light, but also of certain sharp narrow dark lines crossing the spectrum at right angles to the direction in which the colour changed from red to violet. He noted seven such lines.

In 1814 Fraunhofer made a more detailed study of the solar spectrum and mapped 574 of these dark lines. He compared also the lines in the spectrum of direct sunlight and of sunlight reflected from Venus. For his pioneer work in spectroscopy the scientific world honours him by referring to the solar lines as "Fraunhofer lines".

Forty-five years elapsed before an explanation of these dark lines in the solar spectrum was forthcoming and then it was Kirchhoff whose insight enabled him to associate each dark line with a definite type of atom in the outer atmosphere of the Sun. He realized that cool gases will absorb just those radiations which they can themselves emit if raised to sufficiently high temperatures and hence they rob the out-streaming solar radiation of certain definite frequencies. Thus the resulting absorption lines give an indication of the elements composing the solar atmosphere.

The pioneer in the application of spectroscopy to starlight and the first to design a spectroscope by which he saw and studied the individual stellar spectra, and hence in a very real sense deserving of the title of Father of Astrophysics, was Sir William Huggins.

The step from the visual study of star spectra to the obtaining of a photographic record of their spectra was taken in 1872 by Henry Draper. Shortly afterwards at the Harvard College Observatory, a line of research was initiated which has progressed unceasingly, and has led to several critical attempts to classify the stars according to their spectra, culminating recently in the completion of the Henry Draper Catalogue[2] of over 200,000 stars, the spectra of which are classified by Miss A. J. Cannon.

Hartmann of Potsdam achieved the accurate measurement of the wavelengths of the absorption lines in a spectrogram by means of a comparison spectrum whose lines are known and which is photographed on the same plate as the star light, both above and below it. Thus it became possible by the precise measurement of line positions to deduce their shift towards the red or towards the violet, if such shift existed, and in this way by the application of Doppler's principle to calculate the velocity in the line of sight.

With the beginning of the twentieth century there came an almost startling advance in the realm of pure physics. The work of J. J. Thomson and others on the nature of the electron, and the investigations of Rutherford and his associates into radioactivity and the nuclear structure of the atom led to the theoretical work of Bohr upon the hydrogen atom, and this gave for the first time a reasonable explanation of the empirical facts of spectroscopy. The elaboration of his theory by Bohr and also by Sommerfeld[3] has greatly amplified and strengthened this physical basis for the interpretation of spectra, and astrophysicists were not slow to recognize that herein lay the path to the understanding of the physical conditions prevailing in stellar atmospheres.

Absolute Magnitude and Parallax: One of the great problems of the astronomer is to determine the absolute magnitude of a star. Apparent magnitudes were placed upon a logical, though of necessity a quite arbitrary, basis by Sir John Herschel about 1859, but apparent magnitude is a function of the star's distance from the solar system as well as of the actual physical properties of the star, whereas absolute magnitude on any arbitrary scale is a function only of the physical condition of the star. By almost universal agreement the absolute magnitude of a star is taken as equal to the apparent magnitude which it would have if viewed from a distance of 10 parsecs -- a parsec being the distance corresponding to a parallax of one second of arc, that is to say, the angle at the star subtended by the Sun-earth distance is one second if the distance to the star be one parsec.

If apparent magnitude be denoted by m, absolute magnitude by M and parallax by p, a very simple relation can readily be shown to exist, as follows:-

$$M = m + 5 + 5 \log. p.$$

It is thus evident that given m and p, it is possible to calculate M and this has been the road of approach of the astronomer. He measured by trigonometric survey of the sky the distance of the star, or in a few particular cases deduced its distance from its proper motion, and then he calculated its absolute magnitude. The approach to this problem by the astrophysicist has been from the opposite direction. He attempts to interpret the star spectrogram in such a way as to deduce its absolute magnitude and then from this and its known apparent magnitude he calculates the parallax. Values of luminosity and distance thus obtained are called Spectroscopic Absolute Magnitude and Spectroscopic Parallax.

The first step toward the spectroscopic determination of absolute magnitudes was taken independently about the year

1914 by Hertzsprung[5] and Kohlschütter.[4] They found that two spectra might be so similar in general appearance as to fall naturally into the same class and yet in a few particular details exhibit such marked difference that each could be readily identified. Thus the two stars α Tauri and 61' Cygni are both placed in spectral classification K (α Tauri K5, 61' Cygni K7) and yet close examination shows important differences in certain lines in the two spectra, for example, the calcium line (λ 4455) is weak in the former and strong in the latter, whereas the strontium line (λ 4215) is strong in the first case and weak in the second. These facts of observation acquire a significance as soon as two things are realized: (1) α Tauri is a very luminous giant star of absolute magnitude 0.4 whereas 61' Cygni is intrinsically faint, a dwarf star of absolute magnitude 8.0; (2) the calcium line (λ 4455) is associated with an energy change occurring in a normal calcium atom, whereas the strontium line (λ 4215) can only be produced when strontium atoms are under such physical conditions that ionization of the atoms has taken place, or in other words, that the outermost electron of each atomic system has been removed leaving each strontium atom no longer neutral but with an excess unit positive charge. Thus since it is known that high ionization implies a low gas pressure in that portion of the stellar atmosphere in which the "enhanced" line originates, it follows from the above mentioned facts, that high stellar luminosity is to be associated with low pressures, and a consequent increase in ionization, whereas low luminosity is implied when the spectrum shows a relatively weak ionization and a strengthening of the lines of the neutral atoms.

These were the ideas developed by the first and chief worker in the field of spectroscopic parallaxes, W. S. Adams,[5] who by 1916 had found more than a dozen lines sensitive to changes in physical conditions in the stellar atmospheres and therefore suitable as the basis of a systematic search for a quantitative relation between the relative intensities of selected lines and absolute magnitude. Making use at first of 125 known trigonometrical parallaxes he found that satisfactory relations did exist between these factors, and in 1917 he published a preliminary list of absolute magnitudes and parallaxes thus determined.[6] By 1920 Adams and three of his associates at Mt. Wilson had determined these factors for 1646 stars,[7] and two years later 544 more were added.[8]

Since that time several others have been drawn into this work either as contributors to the theoretical basis or as partakers in the actual task of interpreting spectra with this end in view. Of the former Pannekoek,[9] Stewart[10,11] and Eddington[12] are the chief. Pannekoek has pointed out that the difference in the relative intensity of selected spark and arc lines (due respectively to an ionized and a neutral atom) in two stars indicates the difference in luminosity only indirectly -- it is in reality a measure of the difference in the value of surface gravity on the two stars and surface gravity is proportional to the ratio of mass to luminosity.

$$g = \sigma \, M/_L$$

Where g - gravity, M = mass, L = luminosity and σ = surface brightness, a constant for a given spectral class. Hence if a star's mass differed greatly from the average mass of stars of its class, the spectroscopic absolute magnitude would be very much in error.

The important result obtained by Eddington connecting mass and luminosity in the form

$$L = f(M) + \text{Const.}$$

has a direct bearing in this connection for it shows that luminosity may be interpreted as a function of gravity only, within a spectral class. Thus Pannekoek's main ground of criticism of the accuracy of spectroscopic parallaxes is rendered less serious.

Stewart has discussed the effects of different pressures upon radiating and absorbing atoms and has drawn attention to an important point in connection with the width of lines in the stellar spectra, namely that in giant stars, in contrast to dwarf stars, the pressure is low and the intensity of an absorption line will be greater on account of the greater depth of atmosphere; also the line will be narrower and sharper because even at this greater depth the pressure is less, there being both a lower surface gravity and a smaller pressure gradient than for a dwarf.

Following upon the work at Mount Wilson several other observatories equipped for taking slit spectrograms have undertaken the examination of their plates with the same purpose in view. At the Dominion Astrophysical Observatory, Victoria, B.C. this has been accomplished with marked success by Young and Harper.[13] They based their determinations on ratios of fourteen pairs of spectral lines, some of which hadalready been shown suitable by Adams, others of which they found for themselves and demonstrated to be reliable. Their list of 1105 spectroscopic parallaxss showed such a good agreement with the Mt. Wilson values that confidence in the method has been considerably strengthened.

At the Observatory of Upsala, careful determinations have been made by Lindblad[14,15] and at the Norman Lockyer Observatory, Devon, valuable contributions have been made by Edwards[16,17,18] and Rimmer.[19,20] Each worker evolves his own individual method applicable to the class of spectra he is examining, but the fact that diverse minds following diverse methods produce results which are confirmatory and harmonious is sufficient to encourage others to continue the quest and to inspire them with an assurance that the quest will not prove fruitless.

PART II

During the summer of 1925 the writer held an appointment as Volunteer Research Assistant for four months at the Yerkes Observatory, Williams Bay, Wis. (University of Chicago) The Director, Dr. E. B. Frost offered the writer, as a research problem, the determination of the Spectroscopic Absolute Magnitudes and Parallaxes of A-type stars. For this purpose there were available several thousand spectrograms of about 500 stars of spectral classification A0 to A9, or taking the next sub-class above and below, B9 to F0. A selection from these was to be made including as many stars as possible for which reliable group or trigonometic parallaxes were available, these to form the basis of calibration.

All these spectrograms were taken at the Yerkes Observatory with the 40-inch refractor and Bruce Spectrograph attachment. They are 1-prism spectrograms having a dispersion of 30Å to the millimetre at λ4500, while from H_β to H_ϵ approximately 691Å, is 38 mm.

This problem presented many features of interest and of difficulty, for though considerable work has been done on spectroscopic magnitudes of the later type stars (F,G,K,M), comparatively little has been done upon spectra of class A. In later type spectra (viz. F to M) there are many metallic lines; in spectra earlier in type than A (viz. B-stars) there are helium lines, hydrogen lines and spark lines of silicon and of a few other elements; but in the spectra of A-stars, especially Ao, there are very few lines well defined with the exception of strong hydrogen lines and the H and K lines of ionized calcium.

The following Table summarizes the criteria upon which other investigators have depended in their determinations of spectroscopic absolute magnitudes, and for the sake of completeness the criteria used in the present investigation are included:

(INSERT TABLE I HERE)

Examination of the Yerkes spectrograms made it evident that new criteria would have to be sought. The lines λλ 4326, 4384 employed by Adams do not appear at all on the Yerkes plates of Ao stars, and even in A1 and A2 stars if these lines are present they may be so faint as to make relative intensity estimates impossible.

The comparison of the density of the regions used with such success by Lindblad is ruled out in the present case because the range of the Yerkes 1-prism plates is from just to the red of H to just to the violet of K, approximately λ 4900 to λ 3920. Thus the bands at λλ 3884, 3907 are beyond the range of these spectrograms.

The widths of the hydrogen lines should, theoretically, show a correlation, but that this extends throughout the range of A spectra is doubted by Harvard investigators.[25,26]

TABLE I

Spec. Class	Investigator	Ref.	Criteria
B	Adams Joy	21	Correlation with spectral class based chiefly on helium and hydrogen lines as in Harvard system, but with "nebulous" and "sharp" subdivisions.
B	Edwards	16 17 18	λλ 4471, 4388 (helium) with H_γ. λ4144 (helium) with H_δ
B8- -A3	Lindblad	15	Comparison of regions λλ3884 - 3907 and λλ3907 - 3935
B9- -A9	Adams Joy	8	Correlation with spectral class based chiefly on λλ 4026, 4471 (helium) and λλ 4326, 4384 (Fe)
A	Struve	22	Width of λ4481 (Mg^+)
A	Douglas	-	Width of H_δ, K, λ4481; intensity ratios from λλ 4215, 4227, 4233, 4481, 4535, 4549.
A- -F5	Abetti	24	Correlation with spectral class involving trigonometrical, cluster, and all available spectroscopic dats from Mt. Wilson, Victoria, Sidmouth and Arcetri.
F- -M	Adams Joy etc.	7	Selected ratios from λλ4072, 4077, 4215, 4250, 4271, 4290, 4455, 4462.
F- -M	Lindblad	15	Arc line λ3900; Cyanogen bands λλ4144-4184, 4227-4272, 3993.
F- -M	Rimmer	19	Selected ratios from 4072, 4077, 4215, 4227, 4250, 4271, 4290. 4444, 4455, 4462.
F- -M	Young Harper	13	Selected ratios from λλ4072, 4077, 4162, 4168, 4215, 4247, 4250, 4258, 4271, 4290, 4455, 4482, 4489, 4494, 4496, etc.
F- -M	Macklin	23	Selected ratios from λλ 4174, 4227, 4290, 4326, 4387, 4444.

For the early A-stars, however, the writer felt that this line of attack should not be ignored. Likewise the width of λ 4481 correlated with luminosity by Struve[22] for 56 stars of types Ao to Fo appeared to warrant careful consideration.

In order to gain familiarity with spectra of this type and to find if possible new criteria upon which to work, the writer made a random selection of plates covering a wide range of absolute magnitudes as determined by Adams. These were studied in pairs on the Hartmann Spectrocomparator which allows of the simultaneous view, under magnification and equal illumination of the two spectra in exact contiguity throughout their entire range. The result of this preliminary study was threefold:

(a) It was thought that there might be a relation between absolute magnitude and the difference of intensity between H_β and its adjacent continuous background, and likewise H_γ, H_δ, K and the region of λ4535 each with its adjacent background.

(b) The widths of the broad hydrogen lines (β, γ, δ) and of K and the blend $H_\epsilon H$ seemed to show a progression for Ao and A1 stars at least.

(c) It seemed likely that a relation might be established between the intensities of the arc line λ 4227 (calcium) and the spark lines λ 4215 (Sr), λ 4233 (Fe), and between the arc lines λ 4535 (Fi), λ 4549 (Ti) and the spark line λ4481 (Mg).

In order to test out the value of (a), measurements were made of the photographic density of the regions by means of the Hartmann Wedge Photometer, in which a Lummer-Brodhun cube effects the isolation of the region under observation and its apparent juxtaposition with the adjustable background of the neutral-tinted wedge. The results obtained, though not wholly lacking in interest, did not lead to the simple relation anticipated and the method was eventually abandoned.

In the case of (b) and (c) however, preliminary measurements made with a Gaertner Measuring Machine strengthened the conviction that these measurements could be correlated with absolute magnitude, and the following criteria were finally adopted:

(1) Widths of $\lambda 4481$, H_δ, H_ε H, K

(2) Intensity ratios: $\frac{4218}{4227}$, $\frac{4233}{4227}$, $\frac{4535}{4481}$, $\frac{4549}{4481}$.

METHODS

In order to avoid any conscious or unconscious bias in the measurements and intensity estimates, the writer decided to make no reductions whatever until all the measurements had been made. The required data for a sufficiently large number of stars were obtained before leaving Yerkes Observatory in September, 1925, and the compilation and reduction of this data was commenced subsequently. Thus the correlations which have been obtained are felt to be free from prejudice and subject only to the personal factor which is inevitable in all such work.

The number of spectrograms of any one star varied from one in comparatively few cases to several score in certain cases of spectroscopic binaries where orbit determination had necessitated many plates being taken. Usually there were several plates and of these the two best were selected, special effort being made to get good definition at the extreme left - the H and K region. After preliminary examination under low power magnification to establish the essential similarity of the two, one was selected for final measurement. Where there were distinct differences between the two, both were measured and other plates of the same star examined. Cases of uncertainty were generally attributable to spectroscopic binary effects where considerable care was necessary lest apparent breadth of line was a result of Doppler displacement rather than an effect of the kind sought. The final measurement, in general, of one plate for each star has obvious advantages and, apart from the writer's own tests, it appears justified by the consistency of records of the widths of H_γ and H_δ made from different plates of the same star by means of a Moll microphotometer.[26]

The measurement of line widths involves a large uncertainty where the lines are wide and nebulous; in particular, in the A stars, the "wings" of the hydrogen lines are a very prominent feature of these spectra and it is a matter of individual opinion as to where the "line" begins and ends. All that can be hoped is that the measures made by one individual with magnification and illumination kept approximately uniform, will be consistent within themselves.

The measurement or estimate of line intensity presents difficulties which different investigators have sought to overcome in various ways. The writer depended solely on eye estimates endeavouring always to integrate mentally the total intensity from wings to line centre. It is this integrated intensity that is really sought and not merely the central maximum intensity obtained by wedge extinction methods. Eye estimates are relied upon in much of the work of Harvard investigators, even where the comparisons are made between lines on different plates. In the present case, however, comparisons were made only between closely adjacent lines on the same plate, and for this purpose an eye-piece giving magnification 3 was used.

The widths of lines were measured on a Gaertner machine under magnification 15. In order to eliminate temperature or other effects causing lack of complete uniformity in dispersion a factor was recorded for every plate measured, namely the distance between the titanium-iron spark comparison lines $\lambda\lambda$ 3963-3969, or when these were indistinct, $\lambda\lambda$ 3981-3998. All measured widths were subsequently reduced to the standard given by the average plate factor 0.370 mm in the former and 0.950 mm in the latter case.

Each plate was weighted at the time of measurement on the basis:- Good 3, Fair 2, Poor 1. Where the plate was not equally good throughout, separate weights were recorded for the different regions as required.

KNOWN PARALLAXES

The stars for which data had been obtained included 31 belonging to the Taurns and Ursa Major Clusters for which reliable parallaxes were available from the work of Rasmuson;[27] also 49 for which trigonometrical parallaxes were given in Schlesinger's General Catalogue of Parallaxes.[28] The Cluster parallaxes being absolute were used as they stand,[29] the trigonometric parallaxes however are differential, but 35 were made use of by W. S. Adams[8] and his value for the corresponding absolute parallax was taken in each case, the remaining 11 were reduced to absolute by the addition of an average factor $+0''.004$.[30] Of these 46, only one had a negative parallax and as it had the value $-0''.003$ the reduction to absolute brought it up to $+0''.001$ thus obviating any necessity of having to introduce the effect of negative parallaxes into the reduction of the data.

These 80 stars formed the basis upon which the absolute magnitude relations were determined.

<u>Absolute Magnitudes:</u> The absolute magnitude of each of these stars was calculated from the relation

$$M = m + 5 + 5 \log p$$

where M and m are the absolute and apparent magnitudes respectively and p the absolute parallax.

The values of m were taken in every case from the Henry Draper Catalogue -- photometric magnitude.

<u>Spectral Classification:</u> In assigning the spectral class to each star studied, the writer was definitely influenced and guided by the Mt. Wilson classification rather than the Harvard H. D. classification.

<u>Sharp and Nebulous Spectra:</u> Following the lead of the Mt. Wilson investigators, the writer classified all the spectrograms studied as s or n according as the absorption lines were sharp, narrow and clean cut or nebulous, wide and hazy. It seemed evident from the outset that there were many spectra which could not be said to be either definitely sharp or decidedly nebulous; these were labelled ns or sn and put with the group which they most resembled.

The writer decided to carry out separate reductions for the s and n stars and having done these quite independently the resulting curves should provide unbiased evidence as to whether this grouping has a physical reality or whether the stars form a homogeneous class.

Reduction Procedure: With absolute magnitude as abscissae and either line width or intensity ratio as ordinates, plots were made for each of the eight criteria in the case of the n- stars and likewise for the s- stars, sixteen plots in all. Cluster stars were differentiated from trigonometric parallax stars so that any systematic difference would become apparent. As was hoped, no such effect was evident.

The spectral class was also indicated beside each plotted point to aid in the grouping of the individual points when finding means. In three cases it was at once apparent that no correlation existed - the width of $H_\epsilon H$, both n and s, (as is not very surprising considering the complexity of the blend) and H_δ, n.

In the other thirteen cases mean points were computed and the lines of closest fit drawn in. In every case where more than one curve was required to represent the data on either the s or n graph or on both, it was found by superposition of the graphs afterwards that an s and n curve practically coincided. Hence the final curves take the forms shown in Figures 1 - 7.

Systematic Errors: The absolute magnitudes of the stars used in establishing the curves were redetermined from the curves. Weights were assigned to the estimate from each of the seven criteria based upon its apparent reliability and then the mean magnitude calculated. Comparison was then made for each criterion separately with the standard value (cluster or trigonometric), and wherever possible with Adams' value.

In the Tables the symbols used have the following significance:-

S = the standard value of absolute magnitude whether dependent upon cluster or trigonometric parallax.

A = the spectroscopic absolute magnitude as determined at Mt. Wilson.

D = ditto, as determined by the writer, from all the criteria applicable.

1 = ditto, as determined by the writer from width of K

2 = ditto, from width of H_δ

3 = Ditto, from width of $\lambda 4481$.

4 = Ditto, from Ratio $\lambda 4215 / \lambda 4227$

5 = ditto, from Ratio $\lambda 4233 / \lambda 4227$

6 = ditto, from Ratio $\lambda 4535 / \lambda 4481$

7 = ditto, from Ratio $\lambda 4549 / \lambda 4481$.

N = Weight (number of stars)

e = systematic error.

r = probable error.

TABLE II

SYSTEMATIC ERRORS

Comparison	e	N	Comparison	e	N
S - 1	-0.01	65	A - 1	0.33	56
S - 2	0.05	30	A - 2	0.45	26
S - 3	-0.10	50	A - 3	0.25	40
S - 4	0.30	26	A - 4	0.54	19
S - 5	-0.01	50	A - 5	0.26	42
S - 6	-0.34	43	A - 6	0.10	34
S - 7	-0.28	50	A - 7	0.00	42

This information, together with that given in the section on Probable Errors, led to an adjustment of some of the curves, separate n and s corrections being applied as indicated in Table III.

TABLE III

SYSTEMATIC CORRECTIONS

Criterion	s	n
1	0.2	-0.1
2	0.0	0.0
3	0.0	0.0
4	0.4	-0.2
5	-0.2	0.0
6	-0.2	-0.2
7	-0.2	-0.2

After applying these corrections the weighted mean magnitude was calculated for each star and the systematic error determined. It was noted that one star was largely responsible for the error and as this star (Boss 606) is the one for which the differential parallax is negative, it seemed fair to omit it from the total and consider that the resulting low value of the systematic error indicated that the criteria were satisfactory from this point of view. In Table IV the value of e is given both with and without Boss 606. It may be remarked that for this star the absolute magnitude derived from the absolute parallax +0".001 and apparent magnitude 5.4 is -4.6, whereas the writer's determination is +0.5. It is not included in the Mt. Wilson list.

TABLE IV.

Comparison of Systematic Errors

	e	N	Remarks
S-D	-0.11	74	Boss 606 included
S-D	-0.04	73	Boss 606 excluded
S-A	-0.16	69	Boss 606 excluded
A-D	+0.15	66	Boss 606 excluded

PROBABLE ERRORS The average error, or mean deviation, was found for each star with respect to each estimate individually and to the weighted mean. In Table V. the Probable Errors (31) corresponding to these average errors are set forth. Boss 606 is not included; if it were the errors would all be increased by 5% to 10%.

TABLE V.

Summary of Probable Errors

	r	N	Remarks
S-1	± 0.75	64	-
S-2	0.59	30	
S-3	0.67	49	
S-4	0.74	26	
S-5	0.57	49	
S-6	0.67	42	
S-7	0.76	49	
S-D	0.52	73	
S-A	0.60	69	
A-D	0.45	65	
S-D	0.48	71	Boss 3960,6031 omitted
S-A	0.57	67	Boss 3960,6031 omitted

It is gratifying to find that the probable error of the luminisities determined in the present investigation is somewhat less than that of the Mt. Wilson results. In their determination of luminosities of later type stars the probable errors vary for various types from ±0.64 to ±0.00, the average probable error being given as 0.^{m}40.[32] That the corresponding figure in the case of the more difficult A-type stars should be as low as ±0^{m}.5 seems to be all that can be hoped for at the present stage of our knowledge of the complex conditions which evidently exist in atmospheres of stars of this interesting class.

THEORETICAL CONSIDERATIONS: While it must be explicitly understood that the relationships that have been obtained are empirical, it is also to be remembered that there is a theoretical basis for relationships of the kind found. Considering first the question of the width of an absorption line, there are at least four points to be taken into account:

(a) Minimum width

(b) Doppler widening

(c) Rayleigh scattering

(d) Stark effect.

(a) For a given instrument and a given slit width, there is a minimum width to be expected for an absorption line of any given wave length. For the Bruce Spectrograph with slit width

0.05 mm. and the ratio $f/D = 19$, the minimum width of λ4481 on a 1-prism plate has been calculated by O. Struve[22] to be 0.68 Å. This is based upon the theories of Schuster[33] and Newall[34] who find that two lines cannot be resolved into separate lines if the difference in the wave length, $d\lambda$ is less than λ/p or at most, $\lambda/2P$, where P, the purity of the spectrum, is some fraction of the resolving power, R. This $P = pR$ where $0 < p < 1$. If the slit were of infinitely narrow width then $p = 1$ and $P = R$. For the case under consideration, the value of p is believed to lie between 1/6 and 1/7[22].

For K a similar calculation indicates that the minimum width would be approximately 0.79 Å. In the case of H_δ (λ4341) the limit would lie intermediate between the values given for λ 4481 and λ 3933.

A glance at the graphs (Figs. 1-3) shows that the lowest measured values are just about at these calculated minima, the great majority lying well above.

(b) A line may be considerably widened as a result of Doppler shifts due to the translatory motion of the radiating and absorbing atoms. That this is not the main cause of widening is evident from the observed fact that line width tends to increase as temperature decreases and pressure increases in the stellar atmosphere. If the widening were chiefly dependent upon thermal agitation the reverse woulf be true. Stewart, following the treatment of this question by Lorentz, believes the Doppler effect to be negligible except in the case of hydrogen where it may become appreciable though not dominant.

(c) An important factor in producing width is undoubtedly <u>Rayleigh scattering</u>, "the intrinsic lack of sharpness in the 'tuning' of the active (scattering) molecules". This does not involve "absorption" in the restricted use of the word advocated by Stewart[10] who limits absorption to cases involving transformation of radiant to thermal energy. Scattering does not involve such transformation. The radiant energy taken up momentarily by an atom is re-radiated without important change in frequency and the resultant opacity is due to diffusion in direction of the incident beam of radiation.

Stewart deduces the relation for line width Δ

$$\Delta = 5.8\ 10^{-13}\ \lambda / \sqrt{n}$$

Where N is the number of atoms in a column of 1 cm^2 cross-section in the line of sight above the reversing layer. Thus it is evident that the greater the density of the stellar atmosphere the greater the width of the absorption line, a relation borne out by the graphs in Fig.1-3.

(d) Merton[36] was perhaps the first to draw attention to the broadening effect which will be produced in star spectra by the natural influences of their own electric and magnetic fields upon the radiating and absorbing particles. Conditions in the stellar atmosphere, producing closer packing and frequent collisions, will thus be accompanied by widening of the absorption lines due to Stark effect and to abnormal electron orbit distortions.

Hulbert[38] has discussed the breadth pf the Balmer lines of hydrogen in the stellar spectra by combining the Stark theory with the Saha theory of high temperature ionization. He finds that the observed width in A-type stars far exceeds the theoretical width unless either the pressure is equivalent to several atmospheres or there are a very large number of free electrons present. The former assumption is ruled out on many astrophysical grounds, the latter is quite admissable.

Other causes of broadening are mentioned by Rayleigh[37,39] as, for example, the possible rotation of the radiating particles, but those already discussed appear to indicate that there is a theoretical justification for the relationships that have been established. The effect of stellar rotation, which may well be a disturbing factor, will be referred to later.

When we come to investigate the theoretical basis for correlations such as those in Figs. 4-7, the ratio is found, in every case, to be that of an arc to a spark line or vice versa. Indeed these lines were selected with this in mind, because it is an obvious fact that conditions which weaken an arc line may enhance a line arising from an ionized atom. In the early type stars where effective temperature is high and pressure low there is a high degree of ionization, and the proportion of neutral atoms is low. As pressure increases and temperature falls, which corresponds roughly to lower luminosity, the amount of ionization falls off and the intensity of the arc lines is increased. This increase will, of course, not continue indefinitely, there being a definite set of conditions corresponding to maximum intensity for any line. Fowler and Milne,[40, 41,42] have evaluated this in terms of the ionization potential, the energy of the given excited state, the partial electron pressure and various constants.

The lines involved in this investigation are the following:

λ 4215.5 is due to once ionized strontium and has the series relation[43] $1s^2 - 1p^2$ and excitation potential 0.0 .

λ 4226.7 from the normal calcium atom has the series relation $1S - 1P$ and E.P. 0.0. It grows gradually stronger through the A,F,G,M stars. It is unblended and a very satisfactory basis for comparison.

λ 4233 is less simple. It is a blend of ionized iron (Fe^{+} λ 4233.16) $2p^4 - 1d'^4$, E.P. 2.68 and normal iron (Fe λ 4233.6) $1d'^7 - md^7$, E P. 2.46. In general its behaviour is that of a spark line in the A stars, but an occasional anomalous intensity may be due to complications arising from the blend. Near it are two manganese lines λ4235.2, λ4235.1 of type $1d^4 - 1p'^4$, but blending with these could only occur in extreme cases of stars having line characteristic n.

λ4481 arising from ionized magnesium with series relation $2d^2 - 3f^2$, E.P.8.83. It is a close doublet λλ4481.33, 4481.13. It is an admirable line to study being visible on every spectrogram and well situated in the region of best definition and intensity.

It is said to reach maximum [44] at A2, its subsequent rise being attributed by C.H.Payne to blending with iron which predominates in the cooler stars. As the present writer is not interested in relative intensities of any one line at different spectral classes but in ratios of two lines, it is sufficient for the present purpose that the intensity of λ 4481 increases less rapidly than the intensity of λ 4535 and λ4549.

λ 4535 is due to neutral titanium, $1f^5 - 2f'^5$.

Fout Ti lines seem to fall close together here:

$\lambda\lambda$ 4536.00 4535.92, 4535.58, 4534.78, the last two being the strongest according to laboratory intensity estimates. E.P. of all four 0.82.

λ 4549 appears to be a blend of lines of different kinds; neutral titanium gives rise to λ4548.77 ($1f^5$-$2f'^5$) of laboratory intensity (35), while ionized titanium has a line at λ4549.64 ($1h^2$ - $1g'^2$) intensity (25) and ionized iron gives λ4549.48 ($2f^2$ - $1d'^4$) intensity (4). Adams describes this line as Fe^+ Ti^+ but the writer's data go to prove that the blend as a whole behaves as an unionized line, the ratio λ4549/ λ4481 growing greater as luminosity diminishes. The excitation potential of the Ti line is 0.82, of the Ti^+ line 1.57, of the Fe^+ line 2.82.

Thus it is to be expected that the ratios λ4215/λ4227 and λ4233/λ4227 will have a negative slope, whereas the ratios λ4535/λ4481 and λ4549/λ4481 will have a positive slope. Figures 4-7 exhibit these characteristics. There is certainly no theoretical basis for straight line relations, but the data at the writer's disposal warranted no other more complicated representation.

CORRELATION CURVES:

The writer believes this to be the first time that definite relations have been obtained for the width of K and the width of H_δ with absolute nagnitude. That these are real relations seems undoubted.

In the case of Figure 1, the Bravais-Pearson correlation coefficient has been evaluated for each curve. Let x be the deviation of the individual star magnitudes from the weighted mean and y the corresponding deviation of line width, then the correlation coefficient is given by

$$\frac{\Sigma\, x\, y}{\sqrt{\Sigma\, x^2 \cdot \Sigma\, y^2}}$$

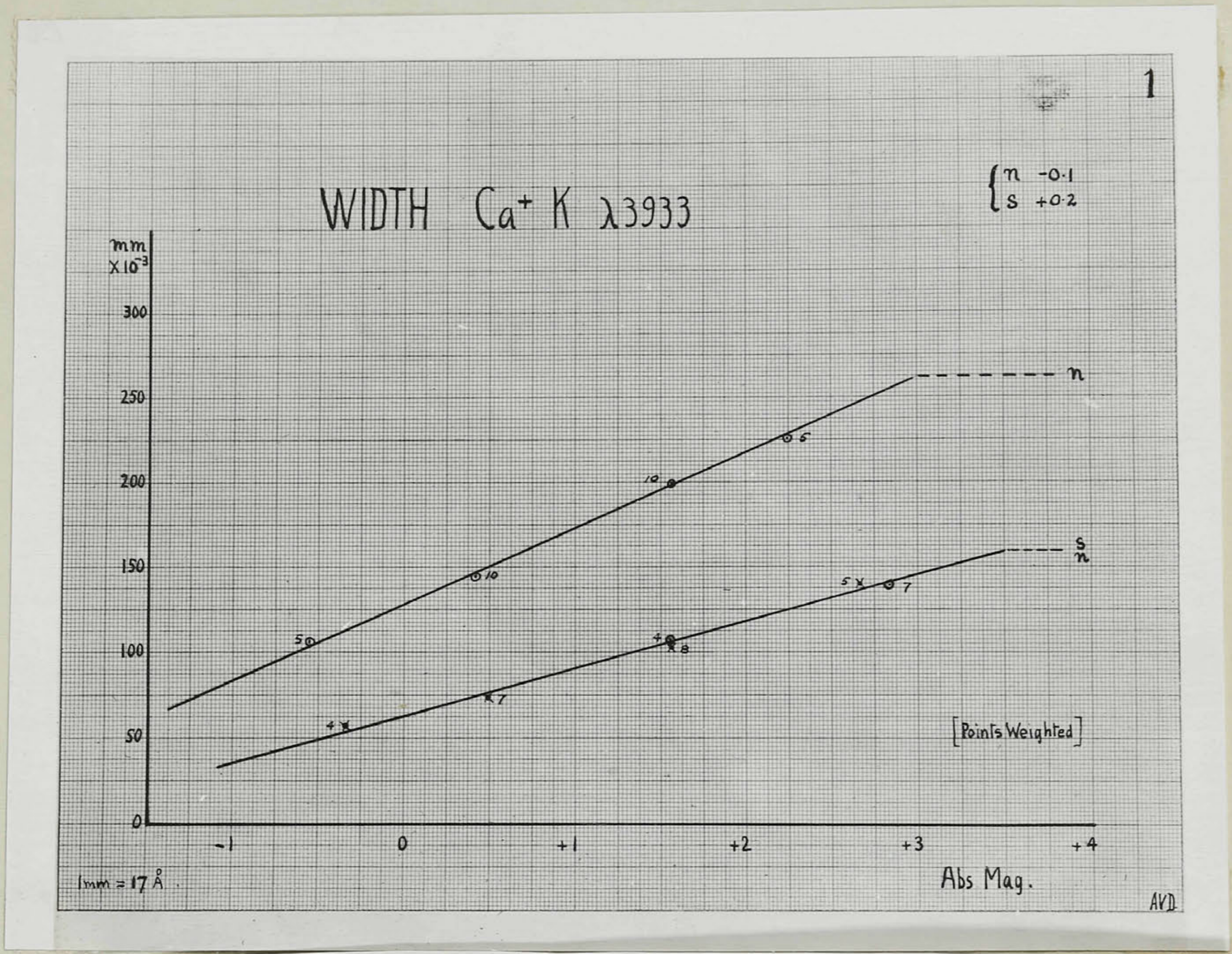
In the case of Figure 2, it has been evaluated for the main curve (Ao-Fo). The results are as follows:

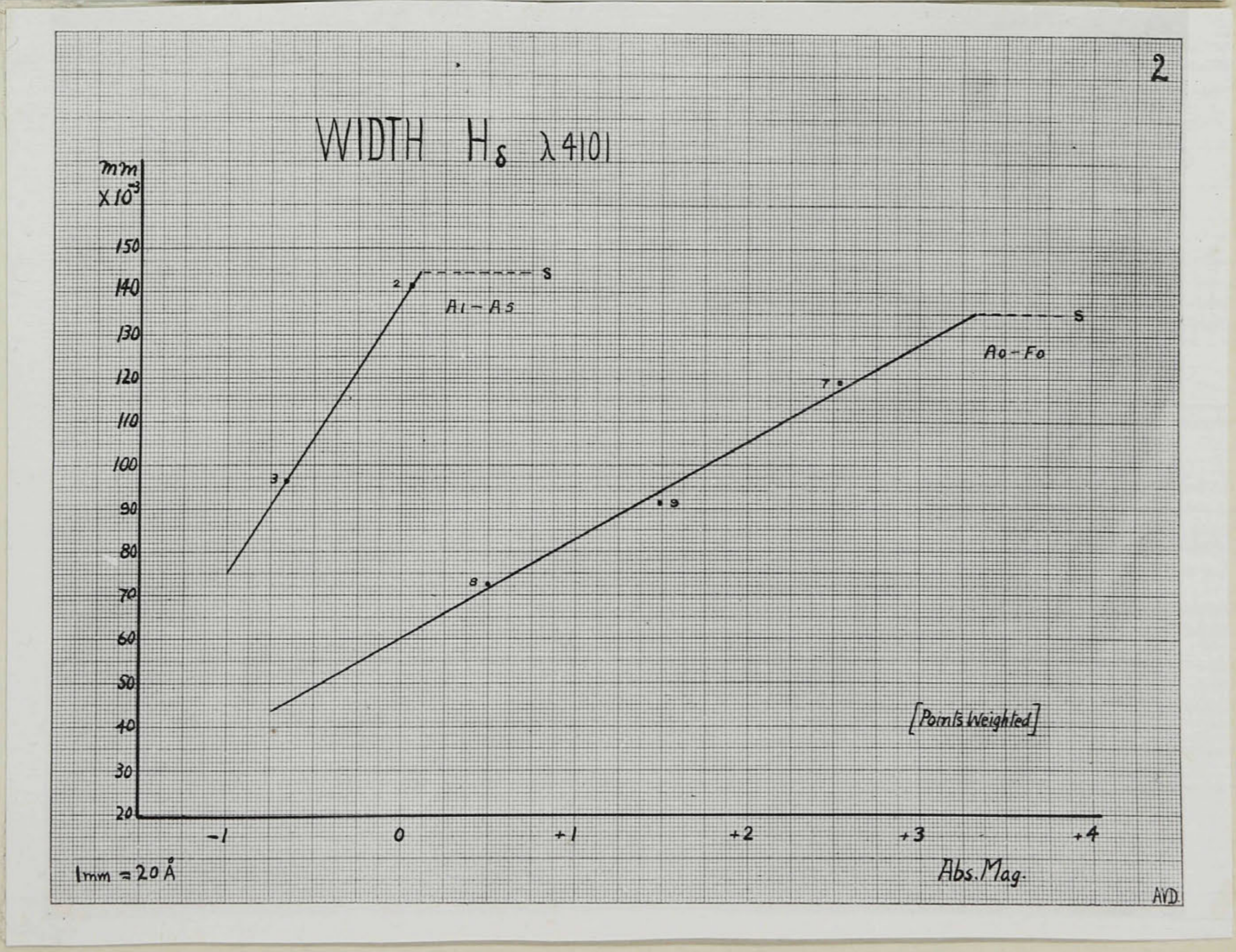
Width K (n)	0.67	Prob.error	± 0.067
" K (sn)	0.67	" "	± 0.063
" H_δ (Ao-Fo)	0.68	" "	± 0.074

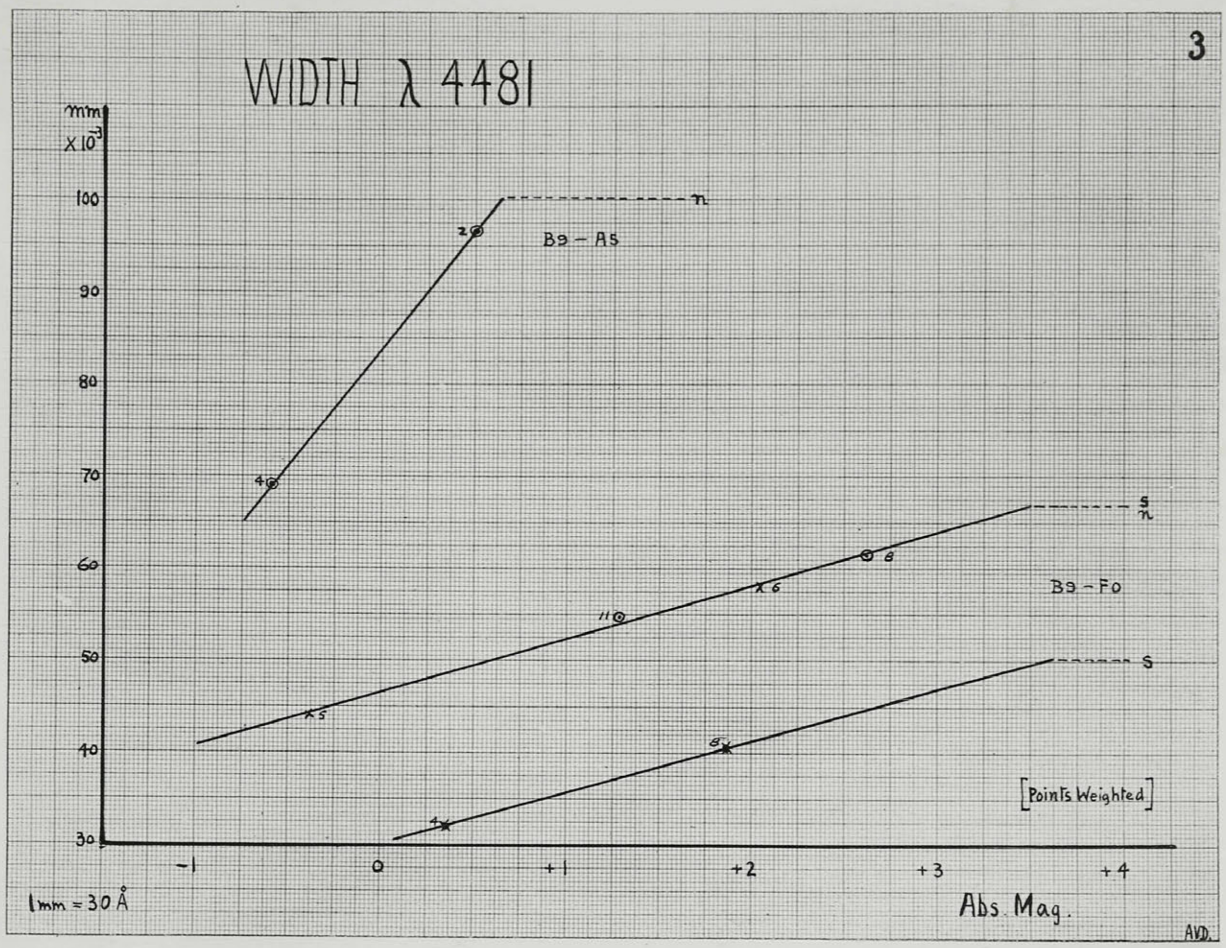
These figures are very satisfactory, for the chance that the correlations they represent are unreal, that is, due merely to "sampling", is very small, of the order of 10^{-10} or less.[45]

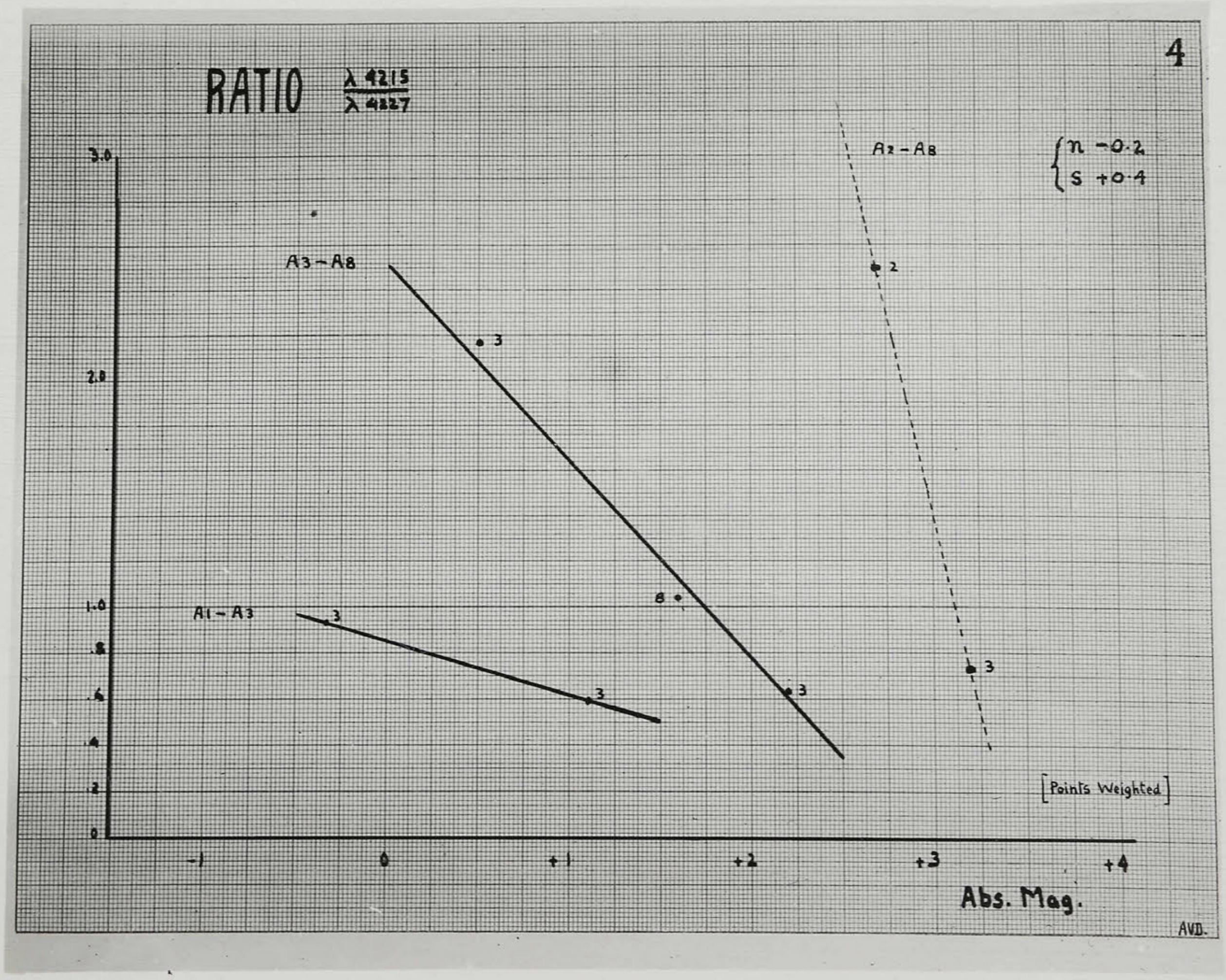
In the application of all these correlation curves, it is evident that uncertainty may easily arise as a result of their multiple character. In general, it has been found that a glance at the spectrogram, or the brief description of it on the writer's records, was sufficient to indicate the general brightness and hence to determine which curve was most applicable in the case of each criterion. Where uncertainty remained, and either one of two curves seemed equally applicable, the usual procedure adopted was to take the mean, and in general to weight this lower than unique values from other criteria.

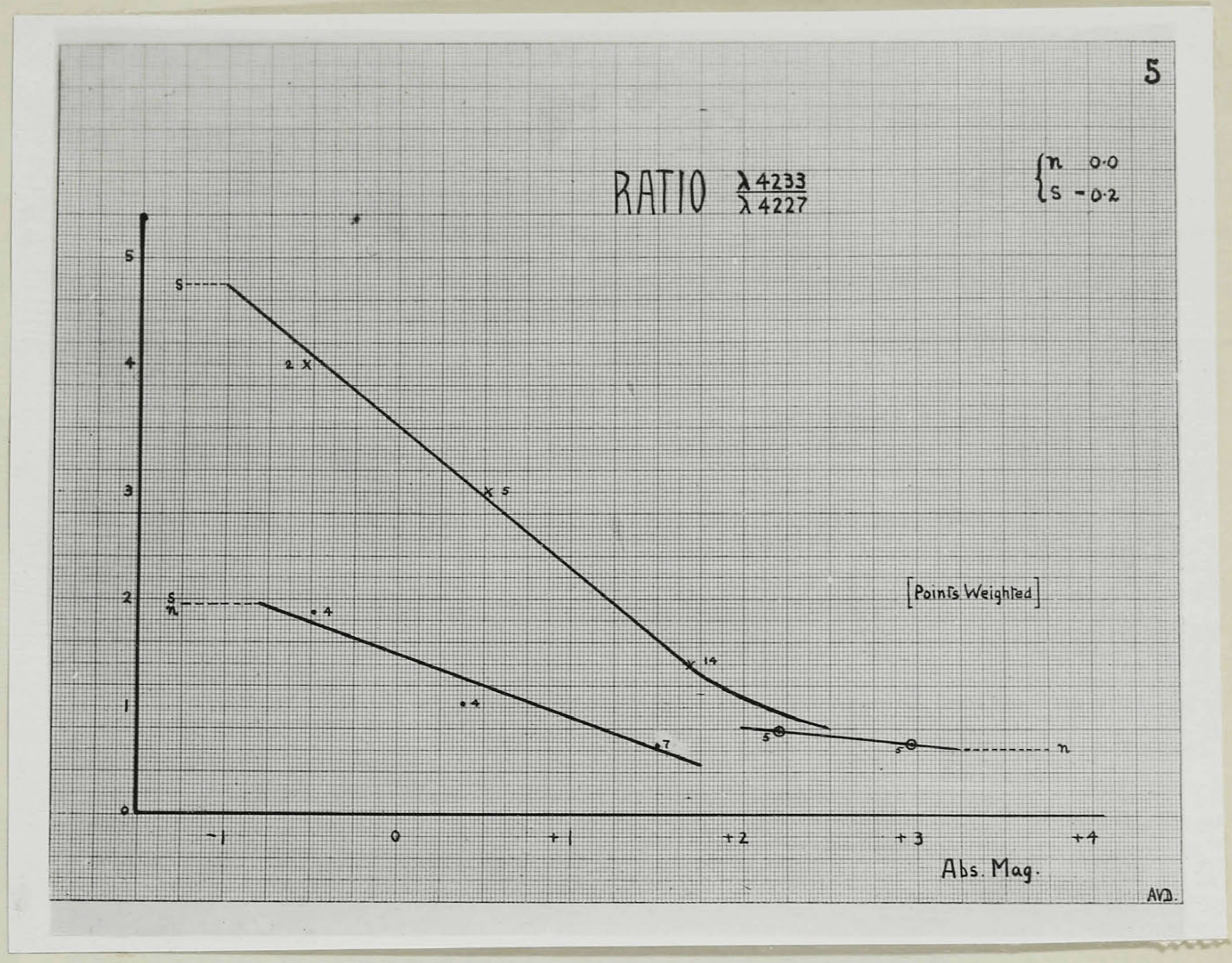
These curves are definitely <u>not</u> built up upon spectral classification, and therefore in so far as the writer has been successful in eliminating the influence of spectral class from the mind in interpreting the curves, the resulting absolute magnitudes should be something more than just averages for spectral class as are the Mt. Wilson[8] and Arcetri[24] determinations of magnitudes of A-Stars.

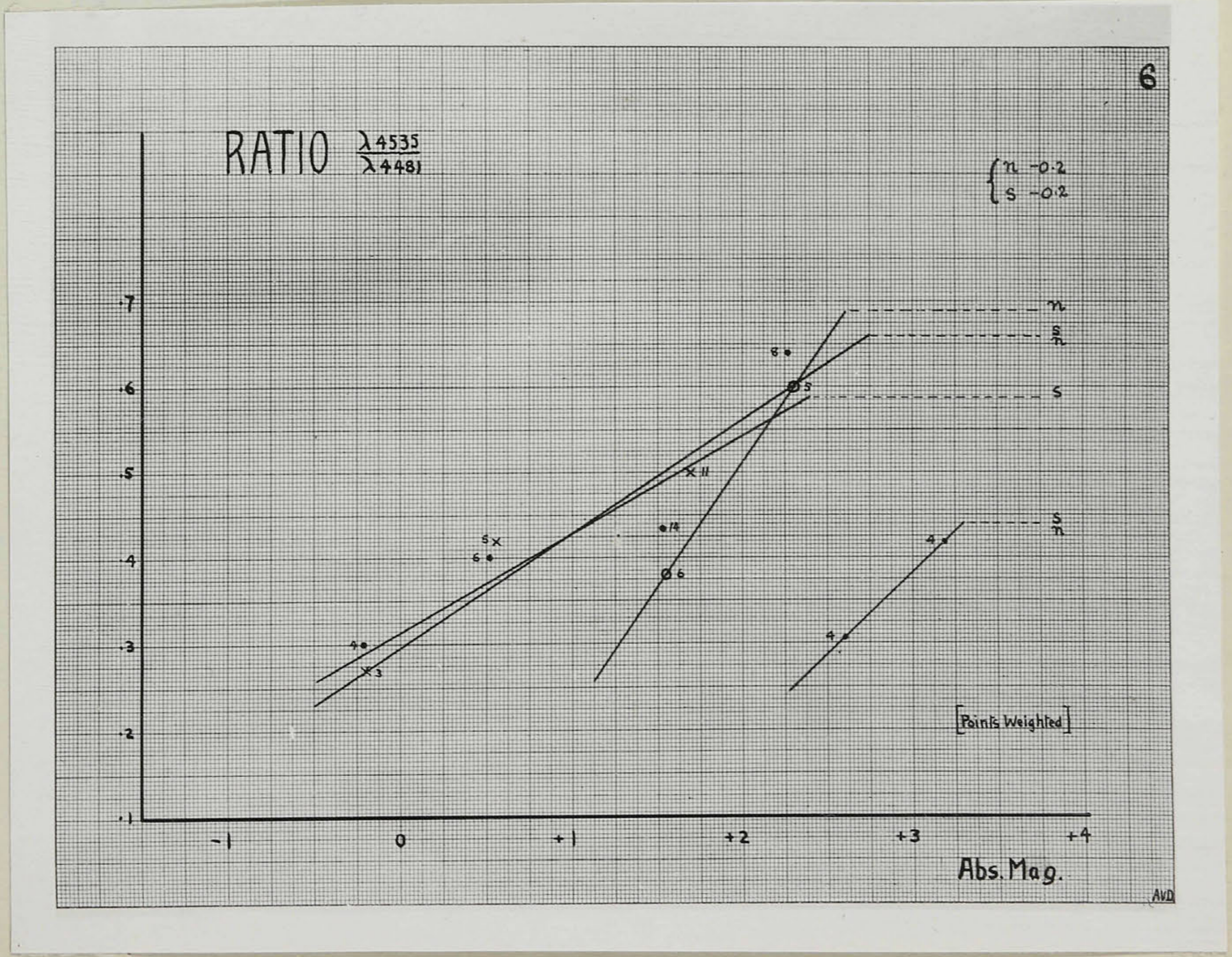

1
WIDTH Ca+ K λ3933
n -0·1
s +0·2
mm
X10-3
300
250
200
150
100
50
0
-1
0
+1
+2
+3
+4
n
s
n
[Points Weighted]
1mm = 17 Å
Abs Mag.
AVD

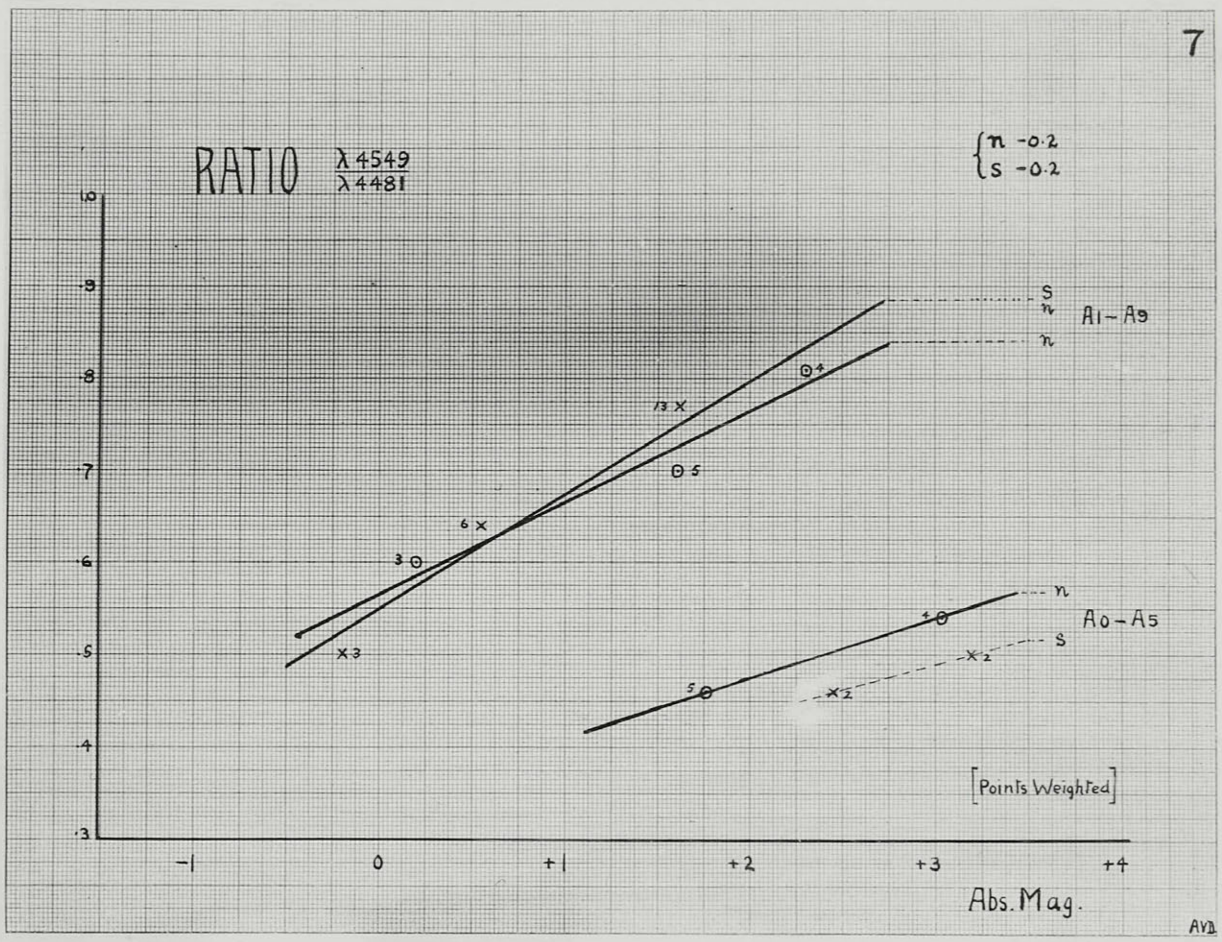

2
WIDTH Hδ λ4101
mm
X10-3
150
140
130
120
110
100
90
80
70
60
50
40
30
20
-1
0
+1
+2
+3
+4
S
A1 - A5
S
A0 - F0
[Points Weighted]
1mm = 20 Å
Abs. Mag.
AVD


3
WIDTH λ 4481
mm
x 10⁻³
100
90
80
70
60
50
40
30
n
B9 – A5
s
n
B9 – F0
s
[Points Weighted]
-1
0
+1
+2
+3
+4
1mm = 30 Å
Abs. Mag.
AVD.


4
RATIO λ 4215 / λ 4227
3.0
2.0
1.0
.8
.6
.4
.2
0
A2 – A8
n -0.2
s +0.4
A3 – A8
A1 – A3
[Points Weighted]
-1
0
+1
+2
+3
+4
Abs. Mag.
AVD.


5
RATIO λ4233 / λ4227
n 0·0
s -0·2
[Points Weighted]
Abs. Mag.
AVD.


6
RATIO λ4535 / λ4481
n -0·2
s -0·2
[Points Weighted]
Abs. Mag.
AVD


7
RATIO λ4549 / λ4481
n -0.2
s -0.2
1.0
.9
.8
.7
.6
.5
.4
.3
-1
0
+1
+2
+3
+4
Abs. Mag.
A1 – A9
A0 – A5
s
n
[Points Weighted]
AVD

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PART III

The correlation curves obtained and tested as described in Part II, have been employed to give values of the absolute magnitudes of sone 170 stars for which trigonometric or cluster parallaxes are not available.

The weighting of the values from each criterion was done as already described and the weighted mean value alone is recorded in Table Vl. In some cases where the data were very meagre it was thought best to exclude these stars pending further study; they belong in most instances to the Group Aon which are by far the most difficult to handle in the light of the present investigation,

Table VI. contains the spectroscopic absolute magnitudes and parallaxes of two hundred A stars as determined by the writer. In addition to these, six stars are included which belong to the set of standard stars upon which the correlation curves were based, but for these six reliable determinations from the present criteria were impossible.

Successive columns of Table VI give (1) the Boss Number, (Preliminary General Catalogue); (2) the visual apparent magnitude; (Henry Draper Catalogue); (3) the reduced proper motion, H; (4) (5) the absolute magnitude, M, and parallax, p, as determined trigonometrically (T) or by group motion (G); (6) (7) (8) the spectral class, spectroscopic absolute magnitude and parallax as determined at Mt. Wilson; (9) (10) (11) the class and spectroscopic M and p as determined by the writer from the Yerkes Observatory spectrograms. The designation "Yerkes" cannot be given to these three columns as might at first glance seem more appropriate, because to do so would be to set the official stamp of approval of that institution upon these results. For defects in this work the Yerkes Observatory is in no way responsible and criticisms of these results must be borne solely by the writer.

TABLE VI (i)

Spectroscopic Magnitudes and Parallaxes of 200 A Stars

Boss	H.D. m	H	Trig. or Gr M	Trig. or Gr p	Mt. Wilson Class	Mt. Wilson M	Mt. Wilson p	Douglas Class	Douglas M	Douglas p
10	2.2	3.8	-	-	A1n	1.1	0".060	A1n	1.0	0".059
43	4.4	3.0	-	-	A1n	1.5	.026	A3n	1.7	.029
50	4.5	-	0.6	0".017T	A0n	1.1	.021	A0n	0.2	.014
145	5.0	2.8	-	-	-	-	-	A1n	1.3	.018
154	5.5	2.7	-	-	A2s	1.1	.014	A1s	0.9	.012
203	3.9	4.9	1.5	.033T	-	-	-	A6n	2.0	.042
246	5.2	6.4	-	-	-	-	-	A4n	2.2	.025
269	4.9	2.1	-	-	-	-	-	A4n	1.6	.022
295	5.3	4.0	-	-	A1n	1.5	.018	A2n	1.4	.017
300	4.7	-	0.9	.017T	-	-	-	A2n	2.3	.033
314	2.8	5.2	-	-	A3n	2.0	.068	A5n	2.5	.087
368	5.5	6.2	-	-	-	-	-	A5n	2.5	.025
370	5.5	4.2	-	-	-	-	-	A2s	0.9	.012
422	4.8	5.5	-	-	-	-	-	A0n	0.7	.015
423	4.8	5.5	-	-	-	-	-	A2s	0.6	.014
428	2.7	-	1.9	.068T	-	-	-	A5s	1.9	.069
441	4.8	-	2.1	.029T	-	-	-	A7n	2.1	.029
446	4.7	-	1.2	.020T	A4s	2.0	.029	A3n	2.0	.029
449	4.1	2.3	-	-	A1n	1.1	.025	A1s	1.2	.026
452	5.4	6.2	-	-	-	-	-	A1n	1.4	.016

TABLE VI (ii)

Boss	H.D. m	H	Trig. or Gr. M	p	Mt. Wilson Class	M	p	Douglas Class	M	p
463Ft	5.2	-	-	-	A3n	1.8	0".022	A3s	1.5	0".018
463Br	4.3	2.1	-	-	-	-	-	A3s	0.4	.017
466	5.4	2.2	-	-	A2n	1.5	.016	A1n	2.3	.024
476	5.1	3.2	-	-	-	-	-	A5s	1.9	.023
480	4.8	5.8	-	-	-	-	-	A1n	1.8	.025
482	3.1	-	-0.7	0".017T	-	-	-	A3s	1.3	.044
522	5.1	3.9	-	-	A1n	0.9	.014	A0n	0.3	.011
550	4.6	0.5	-	-	A3s	1.4	.021	A3s	2.0	.030
560	4.3	2.3	-	-	-	-	-	A0n	0.8	.020
597	5.8	2.3	-	-	-	-	-	A2n	1.6	.014
606	5.4	-	-4.6	.001T	-	-	-	A1n	0.5	.010
628	5.2	6.0	-	-	-	-	-	A6s	2.4	.028
629	4.4	-	2.2	.036T	-	-	-	A7n	1.9	.032
666	5.3	-	-	-	A0n	1.1	.014	A0n	1.2	.015
674Br	5.3	1.4	1.6	.018T	A3s	1.0	.014	A2s	1.2	.015
677	5.2	4.4	-	-	-	-	-	A2n	1.1	.015
730	5.0	4.4	-	-	-	-	-	A1n	0.9	.015
791	5.0	2.6	-	-	A0n	0.9	.015	A0n	1.1	.016
850	5.4	4.6	-	-	-	-	-	A4s	1.1	.014
883	5.4	5.7	-	-	-	-	-	A3n	2.5	.026
923	5.1	-0.5	-	-	-	-	-	A1s	0.5	.012

TABLE VI (iii)

Boss	H.D. m	H	Trig. or Gr.		Mt. Wilson			Douglas		
			M	p	Class	M	p	Class	M	p
932	3.9	-1.8	-	-	-	-	-	A4s	1.0	0".026
971	5.4	-1.1	-	-	-	-	-	A8s	2.5	.026
974	5.1	-	-	-	A1n	1.1	0".015	A0n	1.2	.017
986	4.6	4.0	-	-	-	-	-	A0s	0.6	.016
998	5.3	4.2	-	-	-	-	-	A1s	0.3	.010
1007	5.3	-	2.4	0".027G	A9n	2.8	.031	A6n	2.0	.022
1022	4.8	-	2.1	.029G	A6n	2.4	.034	A5s	2.0	.028
1023	5.1	2.4	-	-	A3n	2.0	.017	A2n	2.1	.025
1026	4.4	4.6	1.3	.025G	A3s	1.8	.030	A2n	1.5	.026
1027	5.4	6.0	2.8	.030G	A3n	2.1	.022	A1n	1.7	.018
1029	4.2	-	1.3	.026G	A3s	1.4	.028	A3s	1.7	.032
1033	4.4	-	1.6	.028G	A2n	1.8	.030	A2n	-	-
1034	4.6	-	1.8	.028G	A0n	1.3	,022	A2n	-	-
1046	3.6	-	0.7	.026G	A3s	1.6	.042	A5s	1.7	.042
1047	5.1	-	2.3	.028G	A2n	2.0	.024	A4n	2.1	.025
1051	5.7	-	2.8	.026G	A3n	2.1	.019	A3n	2.6	.024
1054	4.8	-	2.1	.029G	A5s	2.0	.028	A5s	2.1	.029
1067	4.8	-	1.9	.027G	A2n	2.0	.028	A2n	1.2	.019
1087	4.3	-	1.6	.028G	A3n	2.1	.036	A3n	1.8	.032

TABLE VI (iv)

Boss	H.D. m	H	Trig. or Gr.		Mt. Wilson			Douglas		
			M	p	Class	M	p	Class	M	p
1088	5.3	4.4	-	0." -	A4s	2.0	0".022	A6s	2.3	0".025
1090	4.9	-	1.6	0.023G	A3n	2.2	.028	Q3n	2.2	.029
1092	5.6	-	2.3	.023G	A5s	2.2	.020	A5s	2.5	.024
1095	5.0	3.3	-	-	A4s	1.7	.022	A3s	1.8	.023
1114	5.4	-	2.6	.029G	A8s	2.6	.027	A5n	2.1	.022
1117	5.4	-	1.4	.016T	-	-	-	A7s	1.9	.020
1122	5.4	-	1.9	.020G	A7s	2.2	.024	A6s	1.8	.019
1143	5.1	-	2.2	.026G	A2n	1.7	.021	A1n	1.7	.021
1153	4.5	1.7	-	-	-	-	-	A0n	1.9	.030
1194	4.7	-	1.6	.024G	A3n	2.1	.030	A4n	1.4	.022
1220	2.9	-	0.5	.033G	A1n	1.5	.052	A0n	1.6	.055
1244	5.1	1.1	-	-	-	-	-	A7s	2.6	.032
1268	5.2	1.2	-	-	-	-	-	A6s	2.3	.026
1352	5.3	3.0	-	-	-	-	-	A0n	1.3	.016
1392	4.9	3.4	-	-	-	-	-	A0n	0.9	.016
1452	5.3	2.3	-	-	-	-	-	A1s	0.7	.012
1453	4.9	0.6	-	-	A2s	1.4	.020	A1s	0.6	.014
1482	2.7	-	-0.9	.019T	A1s	0.6	.037	A1s	0.1	.030
1488	5.3	0.6	-	-	-	-	-	A0n	0.1	.009

TABLE VI (v)

Boss	H.D. m	H	Trig. or Gr.		Mt. Wilson			Douglas		
			M	p	Class	M	p	Class	M	p
1492	5.1	4.0	-	-	-	-	-	A7s	1.8	0".022
1516	5.0	-2.6	-	-	-	-	-	A6s	3.2	.044
1575	4.4	-	1.5	0".026T	A2s	1.2	0".024	A1n	0.7	.018
1690	1.9	-	0.5	.053T	A2s	0.9	.063	A2s	0.3	.048
1714	5.1	5.1	-	-	-	-	-	A1n	1.4	.018
1716	4.9	1.8	-	-	A1n	1.1	.014	A1n	1.1	.017
1759	5.2	3.3	-	-	-	-	-	A0s	0.4	.011
1763	3.6	-	0.4	.023T	-	-	-	A0n	0.5	.024
1782	5.3	0.7	-	-	-	-	-	A4n	1.7	.019
1853	4.1	-0.7	-	-	-	-	-	Aon	0.5	.019
1886	3.6	-	1.5	.038T	A2n	1.5	.038	A2n	1.8	.044
1928	4.5	2.9	-	-	-	-	-	A1s	0.7	.017
1968	4.8	0.0	-	-	-	-	-	A8s	2.0	.028
1974	5.3	0.9	-	-	-	-	-	A5n	2.3	.025
2051	5.1	1.3	-	-	A7n	2.6	.031	A5n	2.0	.024
2078	5.0	3.4	-	-	A0n	1.1	.016	A1n	1.1	.017
2088	5.3	4.5	-	-	-	-	-	A0s	0.7	.012
2091	5.4	3.8	-	-	-	-	-	A1s	1.0	.013
2120	4.6	2.9	-	-	-	-	-	A0n	0.7	.017
2138	5.1	4.8	-	-	-	-	-	A0n	0.6	.013

TABLE VI.(vi)

Boss	H D. m	H	Trig. or Gr. M	p	Mt.Wilson Class	M	p	Douglas Class	M	p
XXXXX	X.X	-	-X.XX	X".XXXX	XXX	X.X	0".XXX	XXX	X.X	0".XXX
2185	5.5	-1.0	-	-	A3n	2.0	.019	A3n	1.9	.019
2237	4.0	-	-0.6	.012T	A0n	0.9	.024	A0n	0.1	.017
2264	5.4	5.0	-	-	-	-	-	A6s	2.2	.023
2327	4.7	-	-0.3	.010T	-	-	-	A1n	0.4	.014
2339	5.6	3.0	-	-	-	-	-	A5s	1.2	.013
2398	5.5	5.5	-	-	-	-	-	A2n	2.0	.020
2404	3.1	-	2.9	.090T	A4n	2.2	.066	A4n	1.9	.058
2407	4.3	2.9	-	-	A4s	1.7	.030	A4s	1.8	.032
2479	3.8	-	-0.1	.017T	A1n	1.3	.032	A0n	-	-
2495	4.0	-	1.8	.037T	B9n	0.6	.021	A0n	1.9	.038
2559	4.5	1.6	-	-	-	-	-	A3n	1.9	.030
2584	5.2	1.7	-	-	A5n	2.4	.026	A5n	2.1	.024
2637	4.5	-0.7	-	-	A1s	0.9	.015	A2s	1.2	.022
2642	5.3	-	3.2	.039T	A5n	2.2	.024	A4n	2.3	.025
2655	5.3	1.6	-	-	-	-	-	A1n	0.0	.009
2692	4.5	3.1	-	-	A2n	1.8	.028	A2n	1.3	.023
2697	4.5	1.9	-	-	-	-	-	A0s	0.0	.013
2729	3.5	4.6	-	-	-	-	-	A4s	1.2	.035
2735	5.4	6.4	-	-	-	-	-	A7s	2.6	.028
2754	4.9	1.9	- -	-	-	-	-	A3s	1.1	.017
2900	4.8	3.6	-	-	-	-	-	A0s	0.0	.011
2930	2.4	-	0.8	.046G	A3s	1.0	.054	A3s	0.3	.038

TABLE VI (vii)

Boss	H.D. m	H	Trig. or Gr. M	p	Mt.Wilson Class	M	p	Douglas Class	M	p
2932	4.4	-	0.1	0".014T	-	-	0".-	A4s	0.2	0".014
2972	2.6	-	2.2	.085G	A2n	1.7	.066	A1n	1.0	.048
2974	3.4	-	0.0	.021T	A2s	0.9	.032	A3s	0.2	.023
2987	4.8	4.8	-	-	A0n	0.7	.016	A1s	0.2	.012
2990	4.1	4.0	-	-	B9s	-0.2	.014	B9s	0.0	.015
3023	5.3	3.9	-	-	B9n	0.8	.012	B9n	0.4	.010
3063	5.5	6.4	-	-	A4n	2.0	.020	A6n	1.9	.019
3088	5.1	4.3	-	-	A1n	1.5	.019	A1n	1.2	.017
3097	5.2	3.7	-	-	A1n	0.8	.015	A1s	0.6	.012
3101	2.2	-	2.5	.114T	A2n	1.7	.079	A5s	2.4	.110
3117	2.5	-	0.6	.041G	A0n	0.9	.048	A0n	0.7	.044
3126	5.2	3.8	-	-	-	-	-	A0s	0.7	.013
3139	4.6	2.2	-	-	-	-	-	A3n	1.4	.023
3182	5.1	1.7	-	-	-	-	-	F0n	2.9	.036
3190	3.4	-	1.7	.045G	A0n	0.9	.032	A0n	-	-
3210	4.0	3.1	-	-	-	-	-	A2s	1.2	.028
3240	5.2	2.5	-	-	-	-	-	A6n	3.0	.036
3244	5.0	0.6	-	-	-	-	-	A4s	2.6	.033
3266	5.4	1.9	-	-	A3s	1.0	.014	A3s	1.0	.013
3277	5.4	5.0	-	-	B9n	0.6	.010	B9n	0.2	.009
3283	4.8	4.1	-	-	-	-	-	A0s	0.3	.013
3309	5.0	5.6	-	-	B9n	0.6	.014	B9n	0.4	.012
3310	5.5	5.0	-	-	B9n	0.6	.010	A0n	0.7	.011

TABLE VI (viii)

Boss	H.D. m	H	Trig. Or Gr. M	Trig. Or Gr. p	Mt.Wilson Class	Mt.Wilson M	Mt.Wilson p	Douglas Class	Douglas M	Douglas p
3323	5.2	5.5	-	-	A6n	2.4	0".027	A7s	2.4	0".028
3354	5.8	3.0	-	-	-	-	-	A0s	0.4	.008
3356	5.3	2.8	-	-	-	-	-	A0n	0.2	.009
3370	5.4	7.3	-	-	-	-	-	A8s	3.3	.028
3371	2.9	-	1.1	0".044T	A1s	0.6	.034	A1s	1.1	.044
3409	4.4	-	-0.4	.011T	A2s	0.9	.020	A2s	-0.3	.011
3450	5.1	2.3	-	-	-	-	-	B9n	0.3	.011
3474	2.4	-	0.6	.044G	A2s	1.1	.054	A2s	0.6	.044
3475	4.0	-	2.3	.046G	A8s	2.0	.052	A5s	1.8	.036
3480	4.0	-	2.1	.042G	A1n	1.1	.026	A1n	2.2	.044
3506	4.9	-	0.6	.014G	A3s	1.5	.021	A5s	0.7	.014
3508	3.4	5.7	-	-	-	-	-	A0n	0.9	.032
3509	5.5	1.7	-	-	-	-	-	A0n	0.8	.011
3512	4.6	5.6	-	-	-	-	-	A0n	0.7	.017
3518	4.9	-	3.5	.052T	A1n	1.5	.021	A1n	1.8	.024
3526	5.5	-	1.4	.015T	A7n	2.5	.018	A7n	2.5	.025
3530	5.3	6.2	-	-	-	-	-	A1n	1.0	.014
3561	5.5	6.4	-	-	-	-	-	A1s	0.6	.010
3612	4.3	1.9	-	-	A1n	1.1	.023	A0n	1.4	.026
3654	4.6	-	1.6	.025T	A4n	2.2	.034	A5n	1.8	.028

TABLE VI (ix)

Boss	H.D.	H	Trig. or Gr.		Mt.Wilson			Douglas		
	m		M	p	Class	M	p	Class	M	p
3666	4.3	-	2.4	0".041T	A1n	1.1	0".023	A0n	1.3	0".025
3692	5.1	-	1.2	.017T	B9n	0.8	.014	B9n	0.9	.014
3722	3.0	4.3	-	-	A3n	2.0	.062	A5n	2.3	.072
3749	4.9	0.6	-	-	-	-	-	A2s	-0.3	.009
3752	4.4	-	0.4	.016G	A0n	0.9	.020	A0n	0.5	.017
3787	2.9	3.5	-	-	F1n	2.9	.100	A6n	2.2	.072
3911	5.5	1.7	-	-	A2n	1.5	.016	A2n	1.8	.018
3928	3.1	-0.7	-	-	-	-	-	A2s	1.5	.048
3939	5.1	5.0	-	-	-	-	-	A6s	2.5	.030
3960Br	4.2	-	-0.2	.013 T	A4n	2.4	.044	A7n	2.7	.050
3961	2.3	-	0.3	,041G	A0n	0.9	.052	A1n	0.8	.050
3998	3.9	-	0.5	.021T	A0n	1.1	.026	A0n	0.8	.024
4004	5.5	4.5	-	-	-	-	-	A1s	0.5	.010
4009	3.7	-	0.6	.023G	A1n	1.1	.030	A0n	0.0	.018
4016	3.6	3.4	-	-	A0s	0.4	.022	A0s	1.0	.030
4022	5.8	0.8	-	-	A6s	2.3	.020	A8s	1.9	.017
4026	3.8	-	1.5	.035T	A6s	1.8	.042	A4s	1.7	.038
4028	5.2	5.2	-	-	A0n	0.9	.014	A0n	1.1	.015
4072	5.0	6.4	-	-	A5n	2.5	.032	A4n	1.8	.023
4081	4.8	1.0	-	-	-	-	-	A3n	1.5	.022

TABLE VI (x)

Boss	H.D. m	H	Trig. Or Gr. M	p	Mt. Wilson Class	M	p	Douglas Class	M	p
4229	5.6	2.5	-	-	B9s	0.4	0".009	B9s	0.4	0".009
4232	5.6	2.3	-	-	-	-	-	B9n	0.2	.008
4376	3.2	-	0.4	0".028T	A0n	0.9	.035	A0n	1.0	.036
4581	3.7	-	1.5	.037T	A5s	1.8	.042	A5s	2.1	.048
4747	5.1	3.7	-	-	-	-	-	A1n	1.1	.016
4749	5.1	4.1	-	-	-	-	-	A1n	1.3	.017
4752	4.3	1.5	1.4	.026T	A5s	2.2	.037	A4s	1.5	.028
4754	5.9	2.6	3.3	.030T	A1n	1.5	.013	A1n	1.9	.016
4761	4.4	-	3.1	.056T	A2n	2.0	.034	A4n	2.6	.044
4802	4.5	2.7	-	-	-	-	-	A1n	1.3	.023
4803	5.4	3.8	-	-	-	-	-	A1n	1.8	.019
4824	3.3	-	-1.3	.012T	B9s	-0.2	.020	B9n	-0.2	.020
4858	3.0	-	0.8	.037T	B9n	0.8	.036	B9n	-	-
4988	3.9	-	0.2	.018T	A1n	1.1	.028	A1n	1.0	.026
5048	3.0	-	1.7	.055T	A1n	1.1	.042	B9n	-	-
5062	0.9	-	2.5	.203T	A2n	1.7	.145	A1n	2.0	.166
5186	5.0	1.1	-	-	-	-	-	A1n	1.6	.021
5187	4.0	-4.6	-	-	-	-	-	B9s	0.6	.021
5337	3.8	-	0.6	.023T	A1n	1.6	.037	A1s	1.0	.028
5480	2.6	-	2.2	.084T	A2n	1.3	.056	A2n	2.5	.096
5600	3.0	-	3.3	.115T	A3m	2.2	.068	A6s	2.2	.069
6031	4.9	-	3.1	.043T	A3s	1.2	.018	A3s	0.7	.014

COMPARISON WITH MT. WILSON MAGNITUDES: Of the 200 stars in Table Vl, 108 are also contained in the list of Adams and Joy.[8] Comparing the two classifications a close agreement is found as is to be expected; only occasionally is a star classed n or s by Mt. Wilson given the reverse class by the writer and the decimal sub-class is seen to be rarely very different.

Taking the writer's n and s groups and comparing the Mt.Wilson values of absolute magnitude the systematic and probable errors were computed as follows:

Table Vll

	Systematic error	Probable error	No.of Stars
	σ	r	N
n	+ 0.07	± 0.31	68
S	+ 0.14	± 0.30	40
All	+ 0.09	± 0.31	108

Thus it is seen that there is a fair agreement, the present magnitudes being systematically smaller by about $0^m.1$ or in other words, the stars are here given brighter than at Mt. Wilson by $0^m.1$.

COMPARISON WITH ARCETRI MAGNITUDES: Of the 81 stars in Table VI for which neither trigonometrical, group nor Mt. Wilson spectroscopic values exist, 22 were found amongst the 275 stars of early type determined by Abetti at Arcetri, Florence[24]. His values are given in Table V111 for comparison.

TABLE V111

Spectroscopic Absolute Magnitudes

Boss	Abetti	Douglas	Remarks
597	1.7	1.6	
730	0.4	0.9	
850	0.9	1.1	
932	1.2	1.0	
971	2.4	2.5	Systematic error
1352	1.4	1.3	
1452	1.4	0.7	-0.027
1488	1.1	0.1	
1759	1.1	0.4	
1974	2.1	2.3	
2088	0.6	0.7	Probable error
2754	0.6	1.1	
3139	1.3	1.4	±0.30
3182	1.8	2.9	
3283	0.4	0.3	
3354-	0.4	0.4	
3356	0.4	0.2	
3749	0.8	-0.3	
3928	1.2	1.5	
3339	2.1	2.5	
4802	1.2	1.3	
4803	1.8	1.8	

Though the comparison is limited in extent, it is, as far as it goes, highly satisfactory.

REDUCED PROPER MOTION AND MAGNITUDE: Reduced proper motion is defined as the same function of apparent magnitude and proper motion as is absolute magnitude of apparent magnitude and parallax.

$$M = m + 5 + 5 \log p$$

$$H = m + 5 + 5 \log \mu$$

Where μ is the total proper motion and H is the reduced proper motion. Since μ varies with distance as well as with space velocity, it is logical to suppose that considered statistically M and H should exhibit a strong correlation. With this in mind the correlation coefficient was worked out rigorously from the data in Table Vl, where values of H are recorded for 129 stars including all those for which no trigonometrical or cluster parallaxes are given.

Instead of finding a reasonable correlation the coefficient came out to be 0.030. This low value is not due to a few exceptional stars, but is thoroughhky representative of the data, over half the stars having x and y deviations of opposite sign. Why this should be the case is not at all clear, especially as Adams records a very close correlation between parallax and proper motion.

SHAJN DOUBLE STAR TEST: As a means of testing the accuracy of spectroscopic parallaxes, Shajn [46] has made use of the relation which should exist between the apparent and absolute magnitudes of components of multiple star systems. Since the parallax should be the same for each component it follows that

$$\Delta M - \Delta m = o$$

where ΔM is the difference in spectroscopic absolute magnitude of the two components and Δm is their difference in apparent magnitude. Applying this test to the Mt. Wilson determinations Shajn finds that for 55 double stars of late type

$$\Delta M - \Delta m = \pm 0.28 \text{ to } \pm 0.48$$

which means a 14% to 25% error in the parallaxes. For 139 early type (B,A) doubles, however,

$\Delta M - \Delta m = \pm 1.08$ to ± 1.24 corresponding to 65% to 75% errors in parallax.

Attention having been called to Shajn's work by Dr. O. Struve, the writer compiled a list of A-type double stars of sufficient apparent magnitude and angular separation to allow of separate spectrograms being obtainable. By the courtesy of the Director, the writer was given permission to use the Yerkes 40-inch telescope throughout the week Sept.6th-13th,1925, and every effort was made to obtain these spectrograms. Unfortunately, the nights were unpropitious and only seven plates were secured, but the Yerkes observers have very kindly taken spectrograms of several of these stars and sent them to the writer for examination. Thus it has been possible to apply Shajn's test to the following star systems:

Table IX

Boss	Name	Class	Spec p	Spec M	H.D. m	ΔM - Δm.
422	5 γ Arietis N	Aon	0".015	0.7	4.8	
423	" S	A2s	.014	0.6	4.8	+0.1
463	α¹ Piscium Ft	A3s	.018	1.5	5.2	
463	α² " Br	A3s	.017	0.4	4.3	+0.2
1026	65 K Tauri Br	A2n	.026	1.5	4.4	
1027	67 K² " Ft	Aln	.018	1.7	5.4	-0.8
3354	32¹ H Camel-opardi Ft	Aos	.008	0.4	5.8	
3356	32² " Br	Aon	.009	0.2	5.3	-0.3
3370	12 α¹ Can.Ven.Ft	A8s	.028	3.3	5.4	
3371	12 α² " Br	Als	.044	1.1	2.9	-0.3
3474	79 ζ¹ Urs. Maj. (Mizar)	A2s	.044	0.6	2.4	
3475	79 ζ² "	A5s	.036	1.8	4.0	-0.4
3480	80 g " (Alcor	Aln	.044	2.2	4.0	0.0
4229	16 Draconis Ft	B9s	.009	0.4	5.6	
4232	17 " Br	B9n	.008	0.2	5.6	+0.2
4747	ε¹ Lyrae Br	Aln	.016	1.1	5.1	
4749	5 ε² Lyrae	Aln	.017	1.3	5.1	+0.2
4752	6 ζ¹ Lyrae Br	A4s	.028	1.5	4.3	
4754	6 ζ² Lyrae Ft	Aln	.016	1.9	5.9	-1.2
4802	63 θ¹ Seppentis Br	Aln	.023	1.3	4.5	
4803	63 θ² " Ft	Aln	,019	1.8	5.4	-0.4
5186	30 o¹ Cygni Ft	Aln	.021	1.6	5.0	
5187	31 o² " Br	B9s	.021	0.6	4.0	0.0

In two cases the deviation from zero is somewhat large. For the K Tauri pair Mt. Wilson shows a departure from zero of -0.7, and Rasmuson's group parallaxes give magnitudes having a deviation +0.5. In the case of the ζ Lyrae pair the trigonometrical values are in good accord, deviation +0.3, but the Mt. Wilson magnitudes give a deviation -2.3.

For the eleven systems, however, the average deviation

$$\Delta M - \Delta m = \pm 0.34$$

is as low as can be expected considering the probable error involved in each determination.

THE PROBLEM OF A-Stars. (1) It is disappointing that not one of the criteria is single-valued. In the stars of Class A we are face to face with a serious problem. Some as yet unknown or unrecognized factor is playing an important part in determining the character of the spectrum. Stars of this type are evidently at a critical stage of development, the transition from the giant stage to the dwarf stage. On the older theory of Russell[47.48] this might be thought of as the transition from the state of a perfect gas to a denser state where the gas laws begin to break down, but since Eddington[12] has shown that the dwarf stars of the main sequence are probably to be regarded as also conforming to the perfect gas laws, on account of the high degree of ionization produced by their great central temperatures, it now becomes necessary to look elsewhere for the cause of complexity. Why do stars turn down the main sequence? Jeans[49] explains it in terms of the automatic reduction in the rate of production of radiant energy at the centre of a star when its central temperature exceeds about 30 million degrees. Fowler and Guggenheim [50] have given quantitative evidence in favour of the assumptions of Eddington and Jeans that at these temperatures there would be 99% ionization. As a star approaches complete ionization, its radiation will be unable to increase further. Though its central temperature will be maintained, its density will continue to increase, accompanied by an increase in the absorption coefficient, and thus luminosity will gradually fall off.

There is in this theory, however, no direct clue to the interpretation of the spectra of stars at the transition stage. The writer's material provides independent evidence that the Mt. Wilson sub-divisions, n and s (according as the absorption lines are nebulous or sharp), are of the utmost importance in forming magnitude correlations but do <u>not</u> represent two distinct classes of stars, there being all gradations in line character from the extremely hazy and ill-defined to the extremely sharp, clean cut, narrow line.

(2) If stellar rotation were the dominating factor in line width, the relative numbers of stars of n, sn, s character should be proportional to the probable orientation of stars in space, n corresponding to stars viewed along their equatorial planes, s to stars seen along their polar axes and sn to stars viewed obliquely. But if stellar rotation govern line width such correlations as found in figures 1-3 are meaningless. There may be some evidence, however, against this possibility. The orientations of the polar axes of stars of a binary or multiple system should be approximately the same, and therefore it occurred to the writer that this might serve as the basis for a test of the influence on the spectrum of stellar rotation. The spectra of both components should be either s, sn or n. While this is often the case, a glance at Table IX shows that there are six cases of difference or generalizing from admittedly meagre data, 50% of the binary systems considered give evidence that stellar rotation is not the main cause of line width.

This argument tacitly assumes that the two components of a binary system would have much the same rate of rotation. Where the ratio of masses differed widely from unity, this might not be a legitimate assumption, and even if the mass and the angular velocity were the same for the two components, but their density very dissimilar, the argument breaks down, since it is linear velocity at opposite extremities of a diameter that affects line width. To give some idea of the velocities required to influence line width appreciably, consider λ4481 from a star having a diameter of 10^6 miles and density 1.0, probably a tyoical star like Sirius. Figure 3 shows that a line width of 2Å is not excessive; this would mean a widening in either direction of 1Å. Hence the Doppler equation $\frac{d\lambda}{\lambda} = \frac{v}{c}$

gives $\frac{1}{4481} = \frac{v}{3 \times 10^{10}}$

or $v = 6.7 \times 10^6$ cm per sec.

Since $v = \omega r$ we havefor the angular velocity of the star whose radius is $r = 5 \times 10^5 \times 1.6 \times 10^5$ cm $= 8 \times 10^{10}$ cm

$$\omega = \frac{6.7 \times 10^6}{8 \times 10^{10}} = 8 \times 10^{-5} \text{ radians per sec.}$$

Now, the angular velocity of solar rotation is approximately 3×10^{-6} radians per sec., so that to give this observed width of λ 4481 our typical Sirian star would have to rotate ten or twenty times faster than does the sun.

This speculation is of such interest that it may be worth keeping in mind for future consideration, but no weight whatever is laid upon its interpretation in the light of the absolutely inadequate data here considered.

(3) That there is some as yet unrecognized factor in the atmospheres of A stars seems certain. Harvard investigators have stressed this, pointing out that a one dimensional classification of A stars is inadequate.[51] The Henry Draper classification is based primarily upon the intensity of H and K and is consistent on this basis. Mt. Wilson investigators have adopted a different basis. If the helium lines λλ4026, 4471, 4686 are showing in a Draper Ao star, they call it Bq and they classify the A stars chiefly by the number and intensity of the metallic lines without reference to the intensity of H and K. The writer followed Mt. Wilson fairly closely until gradually a personal classification was felt to be shaping itself. The presence of the helium "raies ultimes" was considered sufficient to warrant the designation Bq, unless there were just a trace of these lines accompanied by well developed K. An Ao star usually showed only λλ4481, 4227, 4233, 4215, 4549, 4535 with barest traces of anything else (except of course H, K, and the Balmer lines of hydrogen, and even these lines too weak in general for relative intensity estimates to be made. Growing intensity of these lines and the appearance of other metallic lines marked the A_1 to A_5 stars, but no star was classed by

the writer later than A_5 no matter how many lines were up unless the hydrogen lines were beginning to stand out less conspicuously. In an A_8 star the hydrogen lines are sinking into comparability with the stronger metallic lines and in an Fo star there is equality between the outstanding metallic lines and the diminishing hydrogen lines. In addition to the suffix n or s to spectral class, the writer is determined on all subsequent work on A stars to adopt the sn suffix, regularly employed by Abetti, to distinguish those stars which one investigator might call n and another s, so intermediate are its lines between either extreme.

(4) Shapley and Fairfield [26] found no correlation to exist between the width of hydrogen lines and absolute magnitude, but they found slight correlation between width and reduced proper motion for late B and early A stars, indicating that there is a tendency for narrow line stars to have low space velocities. No explanation has been hazarded. The present material throws some light on the question. The n and s stars must be dealt with separately - there is no apparent correlation between widths of n-lines with absolute magnitude, but a strong correlation in the case of the s-line stars (Fig.2). This being so, one would naturally anticipate a correlation between

widths of S-lines and reduced proper motion. In the light of the winter's failure to find a general correlation between M and H, however, this point evidently calls for considerable further attention with less limited material; but it indicates the possibility that the relationship with space velocity is illusory.

(5) A perplexing problem is presented by a small class of A stars whose spectra exhibit lines so narrow and sharp as to resolve the usual $H_\epsilon H$ blend into two distinct lines of independently measurable widths. Of the 250 stars studied, 10 stars fall into this class. Three of these stars have well-established parallaxes and in each case the writer's M was too low. That the present criteria failed badly in the case of Sirius shook faith in the applicability of these criteria to stars of this extreme class.

Table X shows in successive columns the Boss number and the name of the star, the writer's spectral classification and absolute magnitude, the trigonometrical or cluster magnitude, the Mt. Wilson magnitude, the measured width of H_ϵ , H , K and finally the reduced proper motion.

TABLE X

Stars of exceptional s-Characteristic.

Boss	Name	Type (D)	Abs. Mag. (D)	Abs. Mag. (S)	Abs. Mag. (A)	Width ($\times 10^{-3}$ mm) H_ϵ	Width H	Width K	H
82	+43°92 Androm.	A5	0.6	-	-	56	48	59	5.2
364	21 Persei	A2	?	-	-	100	40	35	3.1
1241	5μ Leporis	A0	-0.2	-	-	48	39	28	1.8
1657	13 Monoc.	A1	-0.2	-	-	44	65	50	-0.5
1732	9α Can. Maj.	A2	-0.3	$1.3_{G.T.}$	0.9	70	39	44	4.0
2035	τ Puppis	A5	0.9	-	-	68	53	52	-0.0
3363	77ε U. Maj.	A2	0.0	$-0.2_{G.}$	0.9	61	-	39	2.0
3626	11α Drac.	A1	0.3	-	-	33	37	47	2.3
4722	α Lyrae	A0	-0.3	$0.2_{G.}$ $0.3_{T.}$	0.6	-	-	53	2.8.
5320	α Cygni	A2	-0.1	-	-	59	50	63	-

None of the above data seems to exhibit suggestive relationships and, pending further study of these and similar spectra, the above stars have been omitted from Table VI.

(6) Certain stars have individual peculiarities which offer problems upon which as yet it is premature to attempt explanations. A few of these may be noted.

Boss 370. 43 ω Cassiopaiae A2s. It is very unusual to find λ4535 equal to and λ4549 greater than λ4481. While simultaneously λλ4215, 4233 are more than twice as intense as λ4227.

Boss 1516. 17 Leporis A6s. λ4481 is very faint and a line at about λ4546 is very strong. This star has the H.D. class Ao which the writer finds difficult to understand for though the K line is certainly faint there are many metallic lines up very sharply and intense and H_β, H_γ, H_δ are waning.

Boss 3749. 29 π Boötis (Br) A2s. The line λ3984 is strong. It is probably the line of unknown origin recorded by Belopolsky[52] in α Canum Venaticorum and by Lockyer and Baxandall[53] in α Andromedae. Another unusual line to be outstanding is λ4137, a very weak Fe^+ line undoubtedly blended with some line of unknown origin.[54]

Boss 300. 90 ν Piscium A2n

" 1968 97 G Puppis A8s

" 2974 71 θ Leonis A3s

In these and a few others a line is present at about λ4586 so intense as to equal or approximate the intensity of λ 4481. It may be Ca λ4585.87 ($1d^3$-$3f^3$) .Stratton[54] records a line unknown origin at λ4586 in the Corona. Why this line comes up so prominently in just a few cases, α Cygni, for example, is not understood, but it seems unlikely that calcium alone is responsible.

Boss 476	12	K Arietis		A5s
" 3409	51	θ Virginis		$A2_s$ -
" 3506	78	o	"	$A5_s$
3561	84		Urs.Maj.	Als

In these stars a pair of chromium lines, are present with unusual intensity; they are Cr^+ λλ4558.89, 4588.43.

Twenty-four strontium stars are amongst those in Table Vl, remarkable for the intensity of the Sr^+ line λ4215. The Boss numbers and writer's classification and magnitude, also the average magnitude for that class are given in Table Xl.

TABLE XI.

Strontium Stars.

Boss	Class D	M (D)	$\overline{M}$ ($\overline{D}$)	Remarks
370	A2s	0.9	0.7	
423	A2s	0.6	0.7	
463	A3s	0.4	1.1	Si^+ $\lambda\lambda$4128,31 also strong
476	A5s	1.9	1.9	
550	A3s	2.0	1.1	
674	A2s	1.2	0.7	
677	A2s	1.1	0.7	Sr^+ λ 4078 also strong
850	A4s	1.1	1.4	
923	A1s	0.5	0.7	Si^+ $\lambda\lambda$4128,31 also stro
1117	A7s	1.9	2.3	Sr^+ λ 4078 " "
1122	A6s	1.8	2.3	ditto
1268	A6s	2.3	2.3	
1453	A1s	0.6	0.7	
1492	A7s	1.8	2.3	ditto
1968	A8s	2.0	2.3	
2339	A5s	1.2	1.9	Si^+ $\lambda\lambda$4128,31 also stro
2932	A4s	0.2	1.4	ditto
3266	A3s	1.0	1.1	Sr^+ λ 4078 " "
3475	A5s	1.8	1.9	
3506	A5s	0.7	1.9	
3561	A1s	0.6	0.7	
3749	A2s	-0.3	0.7	very abnormal
4022	A8s	1.9	2.3	Sr^+ λ4078 also stro.
4026	A4s	1.7	1.4	
6031	A3s	0.7	1.1	

The comparison between M and $\bar{M}$ indicates that on the whole the strontium stars are $0.^m24$ brighter than the average stars of the same spectral type, but here again the scantiness of the data makes generalisations dangerous. C.H.Payne[55], arguing from proper motion relations and a few individual cases of dwarf strontium stars, concludes that there is no sufficient justification for the statement[56] that these stars are "distinctly brighter than the average". The writer has made as the criterion for the inclusion of a star in this class not any arbitrary scale of absolute line intensity but the one condition that $\lambda 4215 > \lambda 4227$. As indicated in the Table, the other member of the Sr^+ doublet λ4077.7 is sometimes also of outstanding intensity.

The silicon stars are represented by fifteen given in Table XII, one or both of the pair of Si^+ lines λ4128.1 λ4131.1 being unusually prominent.

A comparison between the magnitudes of these stars and the mean magnitude for their respective classes indicates that they are brighter on the average by $0^m.5$.

It is worth noticing that frequently, though by no means always, do the lines of Si^+, Cr^+ and Sr^+ occur with unusual intensity in the same star. Evidently conditions favouring one, favour also the others, and the absence of any one or two given the third is a matter of the abundance of the element in the stallar atmosphere.

TABLE XII

Silicon Stars

Boss	Name	Class (D)	M (D)	$\bar{M}$ (D)	Remarks
463	α^2 Pisc Br	A3s	0.4	1.1	4128 > 31, both strong
923	36 τ^9 Erid.	A1s	0.5	0.7	4128,31 both > 4481
998	56 Tauri	A1s	0.3	0.7	ditto
1046	78 θ^2 Tauri	A5s	1.7	1.9	4128 strong
1117	4 Camel	A7s	1.9	2.3	ditto
1482	37 θ Aurigae	A1s	0.1	0.7	4128,31 strong
2088	+79° 265 Camel	A0s	0.7	0.5	ditto
2339	49b Cancri	A5s	1.2	1.9	ditto
2754	30 H Urs.Maj.	A3s	1.1	1.1	ditto
2900	45 ω Urs.Maj.	A0s	0.0	0.5	4128,31 prominent
2932	60b Leonis	A4s	0.2	1.4	4128,31 strong
3409	51 θ Virginis	A2s	-0.3	0.7	4128,31 fairly strong
3474	79 ζ Urs.Maj.	A 2s	0.6	0.7	ditto
3506	78 o Virginis	A5s	0.7	1.9	4128,31 strong
3749	29 π Boötis Br.	A2s	-0.3	0.7	ditto

MEAN MAGNITUDE AND SPECTRAL CLASS. One of the main problems of the A stars is the question as to whether it is possible to interpret the spectra with individual accuracy or whether the Mt. Wilson and Arcetri method of merely adopting the mean magnitude for spectral class is all that can be done at present.

This investigation has been a definite attempt to maintain the former position. How far it has been successful it is difficult to say. The following Table, together with Tables IV., V., presents the evidence for and against the writer's claim that the magnitudes herein determined have a greater individual accuracy than can be obtained by following the Mt. Wilson and Arcetri methods. What is the true interpretation of this evidence, the writer is not in a position to say, the decision must rest with the critical astronomer.

In Table XIII a comparison is given separately for the n and s stars, between the average deviations of the present magnitudes (D) from the Mt. Wilson magnitudes (A) and from the mean magnitude ($\bar{D}$) per spectral class. As previously, N indicates the number of stars.

TABLE X111

Average Deviations

Class	A — D				D̄ — D			
	n	N	s	N	n	N	s	N
B9	±0.22	5	±0.10	2	±0.21	7	±0.23	3
A0	.41	17	.60	1	.38	31	.27	9
A1	.34	15	.42	8	.43	29	.26	15
A2	.45	8	.56	5	.37	12	.51	10
A3	.18	6	.40	8	.31	10	.52	11
A4	.42	6	.30	3	.31	9	.51	8
A5	.43	6	.29	8	.20	8	.35	11
A6- -Fo	.38	5	.22	5	.36	10	.32	17
All	0.37	68	0.36	40	0.37	116	0.37	84

In Table XlV the mean absolute magnitudes for each spectral class are given. The Mt. Wilson figures are averages for the individual means of Adams and Joy [57]. The Arcetri figures are taken from Abetti's diagram[58], upper full curves, the s and sn curves being averaged with weight given to the s curve.

The general agreement is good. The Arcetri values are based on all the material available from every source, trigonometrical, group and spectroscopic data being all included, and are to be given greatest weight. The Mt. Wilson values are smoothed by graphical means from the original data of Adams and Joy[59] on 101 n-stars and 48 s-stars. The writer's values are here given unsmoothed. The group of A6-Fo includes only 5 stars that are later than A7.

TABLE XIV.

Average Absolute Magnitude

Class	Mt. Wilson		Arcetri		Douglas			
	n	s	n	s	n	N	s	N
B9	0.6	-0.2	0.6	0.3	0.3	7	0.3	3
Ao	0.9	0.2	0.8	0.5	0.8	31	0.5	9
A1	1.3	0.6	1.1	0.7	1.3	29	0.7	15
A2	1.7	1.0	1.3	1.0	1.7	12	0.7	10
A3	2.0	1.3	1.5	1.1	2.0	10	1.1	11
A4	2.2	1.6	1.7	1.3	2.0	9	1.4	8
A5	2.3	1.8	1.8	1.5	2.2	8	1.9	11
A6 -Fo	2.7	2.4	2.4	1.9	2.3	10	2.3	17
						116		84

FURTHER INVESTIGATIONS. This work is the preliminary to a more extensive investigation which the writer hopes to carry out in the near future, involving the whole of the A stars of which the Yerkes Observatory has spectrograms. With the permission of the Director of the Yerkes Observatory, many of the spectrograms studied in the course of this work will be re-studied and many others not yet measured will be examined. As Dr. J.S.Plaskett has remarked, the determination of spectroscopic magnitudes is a matter of closer and closer approximations towards the truth. In the light of a greatly enlarged material, the criteria used in the present work will probably require modification and readjustment. It is hoped that new criteria may be found especially in regard to the Ao stars, so many of which had to be omitted from the present list. Dr Plaskett has advised the close study of the line Cr λ4352. Its behaviour is said to be that of an ionized line. Its identification is with the neutral atom ($1d^5$ - $1f^5$) λ 4351.8 and with the ionized atom, series relation unknown, λ 4351.9. Dr Shapley advises the continuation of width measurements with efforts to find unique correlations with something—possibly with some ionization criteria – in the hope of finding the clue to some of the problems already referred to in regard to the interpretation of the spectra of Class A. stars.

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