Bridging the Gender Gap:

Embodiment of Spatial Reasoning in Young Girls

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Abstract

Spatial reasoning is essential to science, technology, engineering, and mathematics (STEM) fields. Spatial reasoning is body-centered, or embodied, and involves *how* one solves spatial tasks. While studies have investigated children's *accuracy* in solving spatial tasks, few have aimed to understand how children *reason* about solving spatial tasks. Moreover, despite women and young girls' underperformance in spatial tasks compared to men and boys and the deficit perspective placed on young girls and women in spatial research, few studies have aimed to investigate how young girls reason about spatial tasks to address this issue. This study investigated how young girls embodied spatial reasoning as they solved six geometric spatial scaling tasks.

When solving spatial tasks, children draw from their learnt experiences. This study used situative learning theory and embodied cognition theory to provide context of prior mathematical learning and center reasoning in an embodied way. I employed case study methodology to understand how each student reasoned about solving spatial scaling tasks in an embodied way. I interviewed 17 girls in Grade 3 from an all-girls private school. I conducted individual video-recorded, semi-structured task-based interviews to capture each student's embodied reasoning. The spatial scaling tasks involved matching an object on one picture to a picture of a different scale. Using thematic analysis, I analyzed the interviews to capture the embodied reasoning of each student. Student reasoning was identified by linguistic cues and gestural cues.

Overall, the findings revealed seven forms of embodied reasoning when solving spatial scaling tasks: (1) comparing positions, (2) comparing amounts, (3) comparing shapes, (4) partitioning space to aid comparison, (5) two or more coordinated lines to locate a point, (6) zooming, and (7) I just saw it. The forms of reasoning *comparing positions, comparing amounts,*

and *partitioning space to aid comparison* were divided into four, eight, and two sub-forms of reasoning, respectively. Moreover, the findings revealed that intersections of forms of reasoning occurred for over half the total number of episodes. The results also showed trends by episodes, spatial scaling tasks, and class groups. Finally, the results of six student cases revealed complex and multiple forms of reasoning where students used objects in their surroundings and past experiences to solve the spatial scaling tasks.

This study delivers insights into how young children go about solving spatial scaling tasks and the role of embodiment in spatial reasoning. Importantly, I expand on the current discourse surrounding gender and spatial reasoning and counter the deficit perspective that has been placed on women and young girls by demonstrating the complex and embodied reasoning of young girls as they solve spatial tasks. This study concludes by suggesting teacher practices of teacher integrating student reasoning with gestures in their classroom and recognizing implicit bias towards gender in mathematics.

Résumé

Le raisonnement spatial est essential pour les disciplines des sciences, de la technologie, de l'ingénierie et des mathématiques (STIM). Le raisonnement spatial est centré sur le corps, en d'autres mots, la personnification et implique *la manière* dont une personne résout des tâches spatiales. Bien que des études ont aient déjà exploré la *précision* avec laquelle les enfants résolvent des tâches spatiales, peu d'entre elles ont cherché à comprendre comment les enfants *raisonnent* lorsqu'ils les abordent. Bien que les femmes et les jeunes filles obtiennent des résultats moins élevés que les hommes et les garçons dans les tâches spatiales, et malgré la perspective déficitaire souvent attribuée aux jeunes filles et aux femmes dans la recherche sur le raisonnement spatial, peu d'études ont cherché à examiner comment les jeunes filles raisonnent face à ces tâches pour aborder cette problématique. Cette étude a examiné comment des jeunes filles personnifient le raisonnement spatial en résolvant six tâches de mise à l'échelle spatial géométrique.

Lorsque les enfants résolvent des tâches spatiales, ils s'appuient de leurs expériences acquises. Cette étude a utilisé la théorie de l'apprentissage situé et la théorie de la cognition personnifiée pour contextualiser les apprentissages mathématiques antérieurs et recentrer le raisonnement d'une perspective personnifiée. J'ai utilisé la méthodologie de l'étude de cas pour comprendre comment chaque élève raisonnait lors de la résolution des tâches de mise à l'échelle spatiale de manière personnifiée. J'ai interviewé dix-sept filles de troisième année d'une école privée de filles. J'ai mené des entretiens individuels semi-structurés, enregistrés sur vidéo et basés sur des tâches, pour capturer le raisonnement personnifié de chaque élève pendant qu'elle résolvait six tâches de mise à l'échelle spatial géométrique. Les tâches consistaient à faire correspondre un objet d'une image à un objet d'une autre échelle. En utilisant l'analyse

thématique, j'ai analysé les entretiens pour identifier le raisonnement personnifié de chaque élève. Ce raisonnement a été identifié à travers des indices linguistiques et gestuels.

Globalement, les résultats ont révélé sept formes de raisonnement personnifié lors de la résolution de tâches de mise à l'échelle spatiale : (1) comparaison des positions, (2) comparaison des montants, (3) comparaison des formes, (4) partitionnement de l'espace pour faciliter la comparaison, (5) deux ou plusieurs lignes coordonnées pour localiser un point, (6) zoomer, et (7) je l'ai simplement vu. Les formes de raisonnement *comparaison des positions, comparaison des montants,* et *partitionnement de l'espace pour faciliter la comparaison* ont été respectivement subdivisées en quatre, huit, et deux sous-formes de raisonnement. Les résultats ont révèlé que les croisements de formes de raisonnement se produisaient pour plus de la moitié des épisodes. Les résultats ont également montré des tendances par épisodes, tâches d'échelles spatiales, et groupes de classe. Enfin, les résultats de six cas d'élèves ont révélé des formes de raisonnement complexes et multiples dans lesquelles les élèves ont utilisé des objets de leur environnement et des expériences passées pour résoudre les tâches de mise à l'échelle spatiale.

Cette étude aide à comprendre comment les jeunes enfants abordent les tâches de mise à l'échelle spatiale et le rôle de personnification dans le raisonnement spatial. L'étude élargit le discours actuel concernant le genre et le raisonnement spatial en réfutant la perspective déficitaire placée sur les femmes et les jeunes femmes. Ceci est accompli en démontrant la complexité de la personnification du raisonnement des jeunes femmes lorsqu'elles résolvent des tâches spatiales. Enfin, l'étude propose des pratiques pédagogiques qui intègrent le raisonnement des élèves avec des gestes en classe et en reconnaissant les biais implicites liés au genre dans l'enseignement des mathématiques.

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Contribution of Authors

This thesis was written by me, Sabrina Turrin. As the principal investigator of this research project, I developed the interview protocols, recruited participants, conducted the interviews, and analyzed the data. I also created the figures and tables for this study. My supervisor, Dr. Marta Kobiela, provided support, recommendations, and suggestions throughout the research project. Dr. Marta Kobiela collaborated with me on the development of the coding scheme. Notably, she inspired me to pursue a research project on spatial reasoning.

Chapter One: Introduction

Despite efforts to mitigate the gender gap in the *science, technology, engineering, and mathematics* (STEM) workforce, there persists significant disparities in engagement. In 2021, only 23.5% of Canadian women held STEM occupations (Statistics Canada, 2022; Wall, 2019). Moreover, at the university level, low engagement varies across STEM disciplines; fewer women participate in engineering and physics than in biology and chemistry (Cheryan et al., 2017; Wall, 2019). Notably, such participation is not caused by women *leaving* STEM in university as most graduate (Wall, 2019). Rather, girls are not pursuing STEM in university to begin with, where the STEM engagement gap emerges *before* adulthood (Card & Payne, 2021). Therefore, it is crucial to investigate how young girls engage in STEM fields to better understand the underpinnings of the STEM participation gap. In this study, I address this gender gap by investigating a field *foundational* to STEM that emerges in early childhood: *spatial reasoning*.

Spatial reasoning involves sensemaking about the orientation, movement, and changes (e.g., in size, shape, etc.) of objects in 2- and 3-dimensional space (Davis & Spatial Reasoning Group (SRG), 2015). Spatial reasoning is essential to solve spatial tasks. Indicated as early as childhood, achievement in spatial tasks is *significantly* predictive of STEM engagement and academic achievement (Tian et al., 2023; see also Berkowitz & Stern, 2018; Khine, 2017; Sorby et al., 2018; Uttal & Cohen, 2012). Specifically, Tian et al. (2023) conducted a longitudinal study investigating the predictive nature of fourth graders' accuracy of spatial tasks to their later choice in STEM discipline in university. Overall, they determined that a child's accuracy of spatial tasks is *directly* and *significantly indirectly* predictive of their future choice of STEM major. Moreover, they identified that girls' underperformance in spatial tasks contributed to decreased STEM engagement in university. While girls' accuracy of response in spatial tasks has been addressed by previous studies, it remains unclear how girls *reason* (i.e., go about solving) spatial tasks. This study aims to understand the nature of spatial reasoning in young girls to better leverage their reasoning in the classroom.

In this chapter, I detail how investigating spatial performance in terms of gender differences is problematic and necessitates understanding spatial *reasoning* in children, particularly young girls. I then introduce and define spatial reasoning. Further, I conceptually frame spatial reasoning as both a logical and body-driven process, or an *embodied* process, to justify my focus on examining girls' gestures as they reason. I finally specify how and why I focus on *spatial scaling*, a component of spatial reasoning which involves resizing, or *scaling*, 2- and 3-dimensional shapes.

1.1 The Socially Constructed Gender "Gap" in Spatial Research

This study positions gender as "the socially constructed roles, behaviours, expressions, and identities" of people, where gender is *not* determined by biological attributes (e.g., biological sex, hormones) (Canadian Institutes of Health Research, 2023, para. 3). Moreover, I position the gender gap in STEM and the accuracy of spatial tasks as *socially constructed* and *not* biologically constructed (e.g., caused by hormonal differences) (Bartlett & Camba, 2023; Butler, 1999). I argue that identified gender differences in the performance of spatial tasks vary depending on the task and are caused by "differential treatment and socialization" (Bartlett & Camba, 2023, p. 23). In children, differential treatment can appear as categorizing specific toys, games, sports, and clothing as masculine or feminine. Socialization is contextually and culturally dependent and includes engaging in specific behaviours and practices based on socially accepted norms as to *who* should carry out those practices (Carter, 2014). Regarding gender, socialization includes associating toys, colours, household roles, and school subjects with specific genders

(Carter, 2014). Overall, social factors, including upbringing and education, can influence a child's approach to solving spatial tasks.

As argued by Bartlett and Camba (2023), spatial reasoning research has been "co-constructed with gender" (p. 1), contributing to socially constructed gender narratives in spatial reasoning and STEM. One area where this co-construction has occurred is the selection of spatial tasks. Namely, selecting *specific* spatial tasks has resulted in more drastic differences between men and women (Bartlett & Camba, 2023; Lauer et al., 2019). Task-specific factors involve the *design* of spatial tasks that have been used in spatial studies and the *assessment* of participant accuracy in spatial tasks. In the following paragraphs, I explain how the *design* and *assessment* of spatial tasks can exaggerate differences in accuracy between men and women. I then argue that measuring the accuracy of an individual's responses to spatial tasks does not measure *reasoning*. Reasoning can only be understood by asking participants about their sensemaking of the task.

Bartlett and Camba (2023) argued that the *design* of the spatial task chosen can perpetuate gender differences. They examined a component of spatial reasoning, spatial rotation, which involves the rotation of objects. They critiqued how the design of a specific spatial rotation test, called the "mental rotation test (MRT)" (p. 1), resulted in more exaggerated gender differences compared to other spatial rotation tests. The MRT involves participants answering multiple-choice questions about what an object would look like if it were rotated by a certain amount. However, Bartlett and Camba noted that the type of perspectives used for the images in the multiple-choice questions and the amount of visibility of the images was different from other rotation tests, which resulted in more considerable differences between men and women (see also Linn & Petersen, 1985; Voyer et al., 1995). Similarly, Lauer et al. (2019) reviewed spatial rotation literature and identified that the type of rotation task was predictive of participants' *performance* on the task. In addition to performance, in their study, Lehrer et al. (1998) suggested that the children's *reasoning* differed depending on the spatial task. Therefore, the design of a spatial task can impact a student's performance *and* reasoning (Bartlett & Camba, 2023; Lauer et al., 2019).

Gender differences in spatial task performance can also be exacerbated by the *assessment* of the task (Bartlett & Camba, 2023). Spatial tasks often use assessment models that measure participants' accuracy of the task to determine their overall performance (e.g., Frick & Newcombe, 2012; Gilligan et al., 2019; Möhring & Szubielska, 2023). Specifically, spatial research has repeatedly compared results in accuracy of spatial tasks between young girls and boys or women and men (e.g., Jansen & Heil, 2009; Scholnick et al., 1990). However, such a comparison of results requires consideration of the assessment models that researchers used, as the participants' performance can be measured differently for the same task (Bartlett & Camba, 2023). Specifically, they highlighted the work of Voyer et al. (2004) and how the MRT was designed to have two correct answers for each question. Specifically, Bartlett and Camba (2023) described how Voyer et al. (2004) scored 40 rotation questions out of every two correct responses rather than out of every single correct response to reduce the impact of guessing to obtain the correct answer. When introducing their hypotheses on this choice, Voyer et al. (2004) hypothesized that women would be more likely to obtain one or two incorrect answers rather than leaving blank answers because "their limited level of spatial ability would compel them to guess" (p. 74). While Bartlett and Camba (2023) highlighted that Voyer et al. (2004) disproved that hypothesis, I argue that assessment models should *not* be developed based on generalized assumptions of behaviours of participants as it can lead to unrepresentative results lacking in empirical grounding.

Finally, even when the *same* spatial task is used, results on gender differences can vary for different studies, suggesting other possible sources of variation (Lauer et al., 2019). For example, Gilligan et al. (2018a) investigated spatial scaling in children (N = 386) between ages 5 and 10. Their methods involved showing the children a tiled picture containing all orange tiles except one that was black. They then asked the children to match the original picture with one of four options, all proportionally scaled by a certain amount. Focusing on the accuracy of the children's results, they identified that boys significantly outperformed girls in the task. However, Gilligan et al. (2018b) conducted the same spatial scaling task with different participants (N = 155) ages 6 to 10 and found no significant differences in performance between boys and girls. While both studies used the same task and took place in London, United Kingdom, it is possible that factors such as age, the number of participants, or the classroom context resulted in such differences rather than gender. Inconsistencies of gender differences in spatial tasks similarly occur with spatial rotation tasks (Bartlett & Camba, 2023; Lauer et al., 2019). Indeed, the performance of men and women varying even across studies using the *same* spatial task has resulted in debate as to what the *cause* is of the "gap" in the accuracy of spatial tasks and whether that "gap" is task-specific (Newcombe, 2020). While such discourse is ongoing, in the following section, I argue *against* the proposed suggestion that spatial task performance differences are caused by biology.

Proposing biological (e.g., hormonal) differences as the cause of differences in spatial accuracy is harmful as it places a deficit model on women and young girls, which perpetuates the socialized narratives of the capabilities of women in STEM. As mentioned previously, there is extensive debate as to whether the *cause* of differences between men and women in spatial accuracy is socially constructed, biologically constructed, or both (Bartlett & Camba, 2023; Lauer et al., 2019). I agree with the overwhelming evidence that spatial reasoning is socially constructed. First, training people on spatial tasks can significantly reduce and even eliminate gender differences in the performance of spatial tasks, suggesting that differences in performance are task-specific and temporary when an intervention is used (e.g., Kass et al., 1998; Lauer et al., 2019; Spence et al., 2009; Tzuriel & Egozi, 2010). Moreover, as described in the previous section, inconsistencies of results between studies using the same tasks suggest the overwhelming evidence that accuracy is socially constructed (i.e., based on past experiences of related tasks, education, culture, etc.) and not biologically constructed (Lauer et al., 2019). Finally, works like Lauer et al. (2019) have presented counterevidence from studies that describe how women not only perform equally to men on spatial tasks but have *outperformed* men in spatial tasks. Overall, promoting biological differences as a potential cause of gender differences in spatial tasks is harmful as it wrongfully positions women and young girls as inherently less capable than men and young boys.

Overall, there is a need to move beyond the discussions of whether a gap in performance exists, as assessing the accuracy of spatial tasks does not assess how children are *reasoning* about spatial tasks. *Only* by understanding how children reason about spatial tasks, recommendations and suggestions can arise to leverage their learning in the classroom. While extensive research has investigated gender differences in terms of children's *accuracy* of spatial tasks, there is a need to investigate *how* children reason about spatial tasks.

1.2 Spatial Reasoning as an Embodied Sensemaking Process

Spatial reasoning is foundational to STEM fields, such as organic chemistry (Stieff et al., 2012), robotics (Landsiedel et al., 2017), and geometry (Mulligan, 2015). In the following paragraphs, I define spatial reasoning by describing *spatial* and *reasoning* separately.

I define *spatial* as objects within a defined space (Mulligan, 2015). For example, if one were working on a spatial task involving shrinking a shape by a specific amount on a sheet of paper, the object is the shape, and the defined space is the paper. In another example, if a child were playing with toys in a sandbox, the objects would be the toys and the sand, and the defined space would be the sandbox they are playing in. Spatial reasoning involves interacting with or manipulating objects to obtain different results, such as *scaling, changing dimensions, decomposing,* or *orienting* (Whiteley et al., 2015, p. 5).

In defining *reasoning*, it must also be distinguished from “spatial thinking” (Bednarz & Lee, 2011, p. 103), “spatial visualization” (Titus & Horseman, 2009, p. 242), and especially “spatial ability” (Pellegrino et al., 1984, p. 239). In contrast to thinking defined by Bednarz and Lee (2011), I argue that reasoning is *not* a set of defined “skills” (p. 103). Moreover, in contrast to visualization and ability, *reasoning* is mediated through the body and necessitates “logic” (Whiteley et al., 2015, p. 10). Extending beyond visualization and awareness, reasoning involves the underlying logical meaning-making processes behind spatial tasks. Revisiting the former example of shrinking a shape, each person will use an intentional process to create the shrunken shape. For instance, identifying the mathematical operational relations of the lengths of each side to shrink the shape, executing such operations, and using tools to illustrate the shrunken shape on a piece of paper. Moreover, each person will have an explanation for the way they created the shrunken shape. To illustrate the idea of reasoning further, Whiteley et al. (2015) explain:

> Spatial reasoning is a specific area of non-verbal reasoning, with a suite of powerful tools. Therefore, when we speak of spatial reasoning, we are speaking of explanations of some sort (of why or when or how), and so we first need to understand the various non-

verbal ways in which reasoning might (and should) occur (including through gestures, diagrams and mental imagery) and also need to value, in the classroom, these forms of reasoning (p. 10).

Spatial reasoning concerns the underlying processes occurring as an individual solves spatial tasks. Overall, the body acts as a mediator to surrounding social and environmental cues (Whiteley et al., 2015). This study engages with *verbal* and *embodied* reasoning occurring during spatial reasoning tasks. Verbal reasoning involves what ideas are being vocalized or said while solving spatial scaling tasks. *Embodiment* or the types of *gestures and bodily movement* one makes throughout sensemaking, can provide insight as to how a child describes complex mathematical situations like spatial scaling (Alibali & Nathan, 2011; Davis & SRG, 2015).

To address the socially constructed gender gap in spatial reasoning research, this study investigates the way in which young girls embody a type of spatial reasoning task called *spatial scaling*. Understanding the bodily processes associated with spatial reasoning can allow for better support in children's spatial reasoning in the classroom as it can provide a fuller portrait of how children make sense of spatial tasks. However, since spatial reasoning consists of many elements, this study focuses specifically on embodied reasoning when solving spatial scaling tasks.

1.3 Spatial Scaling

In this study, I define spatial scaling as the change of a shape's dimensions by a single multiplicative factor to preserve proportions between the shape's dimensions (Barenblatt, 2003; Lehrer et al., 2002). For example, a rectangle's dimensions can be scaled *up* by a factor of two by applying a multiplicative factor of two to all the sides. Spatial scaling research has often used tasks that involve scaling 2-dimensional objects (Frick & Newcombe, 2012; Gilligan et al., 2019). Spatial scaling has also been found to be a significant predictor of mathematical performance and is associated with number line estimation, proportional reasoning, and fractions (Gilligan et al., 2019; Jirout & Newcombe, 2018). Overall, spatial scaling tasks involve transforming objects by a multiplicative factor.

Former spatial studies have largely measured spatial scaling through participant's accuracy in tasks involving using maps to identify the location of objects (e.g. Frick & Newcombe, 2012; Huttenlocher et al., 1999; Jirout et al., 2018; Rosetti et al., 2017; Uttal, 1996). However, the nature of reasoning about spatial scaling tasks has yet to be investigated. To address the deficit model placed on young women and girls in spatial research, this study illuminates the ideas and voices of young girls as they make sense of solving spatial scaling tasks. In understanding the processes that are engaged when young girls solve spatial scaling tasks, there can be better support for children, especially young girls, in the mathematics classroom.

1.4 Research Gap and Questions

As previously identified, while many studies have engaged in the accuracy of children's spatial scaling performance, studies have yet to investigate how children reason when solving spatial scaling tasks. To address this gap, this study will investigate two elements of reasoning: *verbal* and *embodied* reasoning in solving spatial scaling tasks. While the embodiment of other spatial reasoning tasks like rotation has been established, studies have yet to identify how children embody reasoning when solving spatial scaling tasks (Lee-Cultura & Giannakos, 2020). Considering the discourse on gender in spatial reasoning, understanding how young girls embody spatial reasoning can provide insight for teachers to better leverage their reasoning in the classroom.

This study investigates how young girls solve spatial scaling tasks within the context of geometry. Spatial reasoning is necessary for solving geometric tasks (Mulligan, 2015). Geometry is a foundational component of the mathematics curriculum in Canada and is an aspect of the elementary education curriculum. Specifically, Ontario's mathematics curriculum describes geometry and spatial reasoning as *one* domain in mathematics (Ontario Ministry of Education, 2020). Therefore, this study will investigate young girls' embodied reasoning in solving geometric spatial scaling tasks.

To address the research gaps in spatial scaling research, this study is guided by two research questions:

(RQ1) How do young girls reason when solving spatial scaling tasks within geometry?

(RQ2) In what ways do young girls use their bodies when reasoning about spatial tasks?

In this study, I use *case study methodology* and individually interview young girls at the elementary level to address these research questions. Moreover, as I describe in Chapter Three, I focus on each student's individual reasoning to better understand how each student goes about solving spatial tasks with their bodies.

1.5 Researcher Positionality

As a STEM educator, I am passionate about uplifting and supporting students in their math and science learning. In my bachelor's degree, I engaged in science and technology education as well as engineering education. Currently, as a teacher of science and mathematics, I am interested in investigating areas which impact *all* STEM fields. I believe that supporting and uplifting students' spatial reasoning in mathematics and science is important to build their self-confidence in these fields so they consider themselves fit for careers in STEM as they progress through their educational journey. As a result, I developed an interest in spatial reasoning, as it is relevant to all STEM fields. Importantly, my perspectives on spatial reasoning greatly shape the way in which this study is conducted. Specifically, my perspectives of spatial reasoning are that it is an embodied and social process. As a result, the design of the study, such as the methods, were task-based interviews to capture the embodied and social nature of spatial reasoning. Moreover, the analysis of these interviews were further designed to capture the embodied reasoning of students as they solve the task, rather than their results of the task. Approaching solving spatial scaling tasks through an embodied framework, this study will contribute to embodied spatial reasoning research and provide resources to support teachers as they engage in spatial reasoning in their classrooms.

Moreover, as a woman who is an education researcher and former STEM student, I am particularly interested in deconstructing discourse in research that negatively places women and young girls in STEM. With the privilege of meeting successful women in STEM and being cognizant of the complex phenomenon to measure a student's learning, I am skeptical when encountering research that places women and young girls as less capable than men and young boys in STEM fields. Through my education and experience in becoming an educator, I believe in and support students in achieving their academic goals. Positioning learning as influenced by social and cultural norms, it is crucial to position *all* students as competent in their reasoning. Specifically, I chose to interview young girls and highlight their reasoning of spatial tasks due to the deficit perspective that has been placed on girls and women in spatial reasoning research. In all, the purpose of this study is to forefront the participants' voices and their reasoning.

1.6 Study Overview

This study consists of six chapters: *Introduction, Theory and Literature Review, Methodology, Results Part I, Results Part II,* and *Discussion and Conclusion.*

The chapter *Introduction* set the stage for the importance of investigating embodiment of spatial scaling in young girls. It also provided context to the associations between spatial reasoning and STEM engagement. The second chapter, *Theory and Literature Review,* establishes the theoretical frameworks centered in this study, *situated learning theory* and *embodiment theory.* The chapter also provides a more extensive overview of the existing literature on spatial reasoning in relation to spatial reasoning and spatial scaling. *Methodology,* the third chapter, establishes the relevance of *case study methodology* in relation to this study. It also presents a thorough overview of the methods employed throughout the study. The fourth chapter, *Results Part I,* describes the embodied forms of reasoning students used when solving the spatial scaling tasks. The fifth chapter, *Results Part II*, describes the trends of embodied forms of reasoning in terms of overall trends, trends by spatial scaling tasks, and trends by class group. I also present several case cases to show how students coordinated multiple forms of reasoning simultaneously to reason about the tasks. The sixth and final chapter, *Discussion and Conclusion,* discusses elements of the results, the implications and limitations of the study, as well as contributions of the study.

Chapter Two: Theory and Literature Review

The beginning of this chapter describes the two theories guiding my study: *situated learning theory* and *embodied cognition*. After highlighting the ways in which embodiment has shaped spatial reasoning research, I present a brief overview of the spatial reasoning literature. Afterwards, I define spatial scaling as a type of spatial task and provide a review of current spatial scaling research.

2.1 Cognition as Embodied and Situated

2.1.1 Situated Learning Theory

The first theoretical framework of this study is *situated learning theory.* Building from Lave and Wenger (1991), I define learning and reasoning as shaped by social and environmental interactions *inside* and *outside* the classroom. Moreover, all reasoning, including spatial reasoning, is influenced by interactions that occur and *have occurred* both inside and outside the classroom as well as the larger social context in which they are situated (Greeno, 1997; Lave & Wenger, 1991). For example, *inside the classroom*, students are influenced by the interactions that take place with their peers, their teacher, the class content, and the classroom environment (Langer, 2009). In class, students are also influenced by the interactions that have taken place with their friends and family outside of class. Alternatively, *outside the classroom*, students are shaped by interactions with their friends and their family and the interactions that took place when they were in class (Greeno, 1997; Roth & Jornet, 2013). Importantly, these social interactions and the larger social context can include the different treatment of students based on gender (Bartlett & Camba, 2023). In this study, I consider the influences from both inside and outside the school as students solve the spatial scaling tasks.

2.1.2 Embodied Cognition

The second theoretical framework of this study is *embodied cognition theory.* In defining *embodied cognition,* I answer the epistemological question: *how does one come to know and reason?* I argue that *cognition,* or reasoning and the production of knowledge, is not restricted to the mind and is *embodied*, or also extends to the person's body (Bayne et al., 2019; Wilson, 2002). In the following sections, I discuss the historical foundations of embodiment and describe the ways in which embodied cognition has been investigated.

Indicated to emerge as early as the 1700s, embodiment as a concept became popularized by philosopher Maurice Merleau-Ponty in the 1950s (Davis & SRG, 2015; Shapiro, 2014; Wilson, 2002). Embodiment emerged as oppositional to Western Cartesian dualism, or the ideology that mind and body are separated, where learning occurs only within the mind (Davis & SRG, 2015; Wilde, 1999). Cartesian dualism was popularized by Descartes and Plato, whose arguments propelled traditional cognitive science research (Davis & SRG, 2015; Gerofsky, 2015; Shapiro, 2014; Wilde, 1999; Wilson, 2002). While cognitive sciences continue to play a substantial role in current spatial reasoning research, I approach cognition and spatial reasoning as embodied. Agreeing with previous works on embodiment, I propose that learning is mediated and facilitated by the body (Clark, 1997; Roth & Jornet, 2013; Shapiro, 2014). As described by Clark (1997), solving a puzzle requires "physical manipulation" of the puzzle pieces in order to solve it (p. 36). Studies have yet to investigate how young girls embody cognition when solving spatial tasks. In doing so, I argue that this can disrupt the deficit narrative placed on young girls as it will bring in different perspectives for how girls reason about spatial tasks.

Overall, I approach learning and cognition as being *both* embodied and situated in the social context (e.g. Clark, 1999; Rohrer, 2007; Roth & Jornet, 2013; Wilson, 2002). Moreover, I argue that how a person embodies cognition is socially influenced, where embodiment is influenced by current and past experiences (Núñez et al., 1999; Rohrer, 2007). In this study, I proceed to describe embodiment in terms of spatial reasoning and how interactions with objects and surroundings play a role in embodiment. Moreover, I present my framework of embodied cognition about spatial tasks, or the way in which reasoning occurs through interactions about spatial tasks. My framework is divided into three parts: *visible embodied cognition and environment (VEC), invisible embodied cognition (IEC),* and *spoken language*. In the following, I describe these components and how these components appear and connect when solving a spatial task.

2.1.2.1 Visible Embodied Cognition (VEC) and Environment. The first aspect of this framework is *visible embodied cognition* (VEC) which concerns the ways in which an individual *visibly* moves their body and physically interacts with the environment (Wilson, 2002; Ziemke, 2003). This study aims to understand the ways that students *visibly embody* their reasoning about spatial scaling tasks. Visible embodied cognition plays a role in spatial reasoning within mathematics in two ways: as *moves* (Abrahamson & Lindgren, 2014; Hostetter & Alibali, 2008; Hostetter & Alibali, 2019; Lakoff & Núñez, 2000; Parsons, 1994) and as *interactions* with objects in one's environment (Wilson, 2002; Ziemke, 2003).

I define *moves* as the ways in which an individual moves their body and physically interacts with objects in their surroundings when reasoning about spatial tasks (Alibali, 2005; Wilson, 2002). Building on Francis et al. (2016), I use the word *moves* to illustrate their conceptualization of the "situated movement of a body" when problem-solving (p. 4). *Moves* can include head movements (e.g., nodding, turning one's head), body movements (Nathan & Walkington, 2017), and gesturing (Abrahamson et al., 2020; Nemirovsky & Ferrara, 2009; Nemirovsky et al., 2011). This study will particularly focus on students' gestures as hand movements (e.g., Goldin-Meadow et al., 1993; Hostetter & Alibali, 2008). Importantly, gestures can facilitate the communication of ideas surrounding spatial mathematics tasks (Alibali et al., 2000; Alibali, 2005; Alibali & Nathan, 2011). For example, Alibali (2005) described how gestures can help organize and partition complex spatial ideas into simple components of the complex spatial idea. Moreover, Alibali and Nathan (2011) built on McNeill (1992) and described different types of gestures that took place when individuals explained concepts in mathematics and the roles that different gestures can play. One gesture Alibali and Nathan (2011) focused on was *pointing gestures,* which help show objects or concepts that are being described (e.g., pointing at properties of a cube when describing a cube) has also been investigated by other scholars (Kobiela & Lehrer, 2019). Another type of gesture, *representational gestures,* can demonstrate motion about an object (e.g., showing the rotation of an object with a gesture) (Alibali & Nathan, 2011; de Freitas & Sinclair, 2012). Finally, *metaphoric gestures* represent abstract ideas and metaphoric language (e.g., discussing mathematical operations with gestures) (Alibali & Nathan, 2011; Nathan & Walkington, 2017).

While gestures can *align* with ideas expressed aloud (i.e., they *match*), sometimes gestures can also *misalign* with ideas said aloud (i.e., they *mismatch*) (Goldin-Meadow et al., 1993; Goldin-Meadow, 2011). For instance, Goldin-Meadow (2011) identified that some young children, when explaining their reasoning about a spatial task comparing amounts of water between two different cups, made gestures that did not match their explanation. Goldin-Meadow (2011) proposed that misalignment in gesture and speech indicated "that the speaker is in a transitional state and ready to learn" (p. 605). In this study, I consider that gestures may not always match ideas which are said aloud, where the visible embodiment of ideas, or gestures, may misalign what is said aloud.

Regarding gestures and gender, Ehrlich et al. (2006) compared how boys and girls performed and gestured in spatial rotation training interventions. Overall, they identified that girls outperformed girls in spatial tasks. Moreover, they identified that boys gestured more frequently than girls and proposed a correlative relationship between the performance of spatial tasks and frequency of gestures. They highlighted how students gestured on average in just over half of the spatial tasks. In this study, I investigate how young girls embody spatial scaling tasks. Differently from Ehrlich et al.'s work, this study is *not* an intervention and does *not* aim to train students and is strictly observational to understand how young girls gesture about spatial tasks without outside influence.

2.1.2.2 Invisible Embodied Cognition (IEC). The second component, *invisible embodied cognition*, or IEC is the embodied retention of knowledge and memories grounded in prior interactions with tools and the environment (Pouw et al., 2014). In other words, memory is not only retained by the mind, but by the body. While I propose that IEC is always occurring as it is *memory,* I propose that IEC can appear in two ways. The first, in the context of this study, is visible to an outsider watching and includes the person solving a spatial task relying on prior bodily actions to execute a specific task (Pouw et al., 2014). For example, if someone were to solve a task without the tools they would typically require, the way in which they would solve the task would nonetheless rely on the "sensorimotor routines" that were established when they used those tools (Pouw et al., 2014, p. 61). In the mathematics contexts, these routines would be established prior in the classroom or elsewhere, where even if the individual were not using tools or gesturing, the way in which they reason about mathematical tasks is still embodied. Through long-term routines, I argue that making *memories* is not isolated to the mind and is also occurring in the body, or in other words, enacted (Sutton & Williamson, 2014). Similarly, Abrahamson et al. (2021) described "knowing is doing" (p. 164), where the storage of knowledge, or memories, resides within the body and not only the mind. In the context of spatial scaling tasks, an individual may be using body-based memories of sizes of objects to visualize the transformation of an object increasing in size prior to explaining their answer out loud. Importantly, without the communication of ideas through *spoken language*, *gesture,* or *both,* such embodied visualization is *invisible* to an outsider.

The second way IEC occurs is when the individual is solving a task but is not visibly moving to an outsider watching. Wilson (2002) described the phenomenon of solving tasks outside of the working environment (e.g. in one's mind) as "offline cognition" (p. 626). Specifically, Wilson (2002) explained how, while not visible to the outsider, the functions of imagining visual scenarios and problem-solving rely on "mechanisms of sensory processing and motor control" (Wilson, 2002, p. 626). Therefore, even when not in physical motion, the individual is still solving tasks in an embodied way. For example, Wilson (2002) provided an example of counting with one's fingers and then reducing such motion so that there is none (p. 633). The sensation on the fingers can still be felt, but there is no movement. Overall, IEC can be approached as embodied memory that can be described or physically demonstrated out loud by the individual solving the spatial task (Pouw et al., 2014).

2.1.2.3 Spoken Language. The final component of the embodied cognition framework in solving spatial tasks involves *spoken language.* Language in embodied cognition can carry different roles depending on the context. While the way language is used in spatial scaling tasks has yet to be determined, it can be either accompanied with gestures and physical interactions towards a spatial scaling task or can be used to make explicit the ideas that would not be indicated by the body if left unsaid (Abrahamson et al., 2020; Glenberg & Kaschak, 2002). In the context of motion, Talmy (2000) describes the spatial language of real situations as “factive” language, whereas “fictive” language is used to describe imaginary or metaphoric situations (p. 168). For example, factive language may be to describe distance literally. However, as described by Lakoff and Núñez (1997), fictive language in arithmetic may be used to describe a small number being “far way” from a large number, even though there is no literal distance occurring (p. 37). Moreover, Krauss (1998) described the way in which gestures affect speech related to “spatial content” (p. 58). Specifically, Krauss identified that when an individual’s gestures are hindered when they are discussing spatial concepts, the individual’s flow of speech is also disrupted. On the other hand, hindering an individual’s gestures when they are *not* discussing spatial concepts did *not* disrupt their flow of speech (Krauss, 1998). Overall, spoken language plays a critical role in accompanying both bodily motion and externalizing thoughts.

2.1.2.4 Mediational Means in Embodied Cognition. As previously discussed, I approach learning as shaped by social and cultural context. Shaped by cultural and social context, Wertsch (1994) builds off Vygotsky’s work on mediation, which Wertsch described as “how tools and signs mediate human action” (p. 204). For example, a tool such as a pencil will be used over a marker to facilitate or mediate the act of writing. The tools used to facilitate human action are what Wertsch (1994) described as mediational means. Regarding spatial tasks, I approach mediational means as including, but are not limited to, objects, spatial language, gestures, and images to facilitate solving the task. Mediational means can also consist of physical objects or abstract concepts. However, the way in which we use these mediational means is not permanent. Wertsch (1994) specified that since mediational means are shaped by cultural and social context, the use of these mediational means can lead to the creation of new means or changing existing means of facilitating human action.

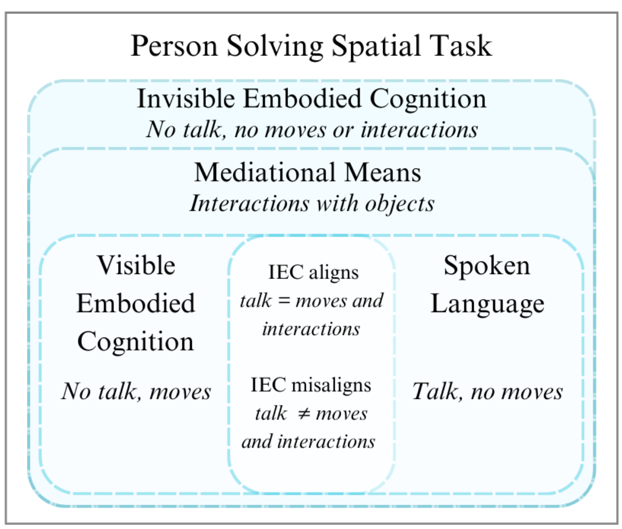
In expanding on Wertsch's (1994) *mediational means*, I argue that the use of mediational means is embodied. Specifically, individuals can "off-load" tasks with the use of mediational means (Wilson, 2002, p. 626). However, it is essential to mention that objects only play a role in embodied cognition when the individual is *using* the objects or has used the objects in the past.

In spatial mathematics, mediational means have been investigated by scholars as well (Kobiela & Lehrer, 2019; Nemirovsky et al., 1998). I highlight the work of Kobiela and Lehrer (2019), who investigated students' conceptions of measurement and area and had students use various objects to reason about changing length and area (e.g., using a squeegee on paint to wipe the space to conceptualize length and area). Moreover, they used the terminology "sweeping" as mediational mean and fictive language to conceptualize this idea of making a length longer or an area larger (Kobiela & Lehrer, 2019, p. 180). Importantly, their work highlighted examples of how physical objects and language can serve as mediational means.

2.1.2.5 Embodied Cognition of Spatial Tasks Framework. Drawing upon VEC, IEC, and spoken language, I introduce the embodied cognition of spatial tasks framework (Figure 1). In developing this framework, I aim to answer: How would VEC, IEC, and spoken language occur as an individual is solving a spatial task in real life? As described in Section 2.1.1., I first proposed that the way in which one solves a spatial task is shaped by past interactions and experiences. Moreover, I previously described that IEC is embodied memory, where I propose it occurs throughout the entire course of solving a spatial task. In isolation, IEC appears to an outsider as the individual not moving and not speaking (Figure 1). However, when IEC intersects with VEC, I propose that the individual solving the spatial task appears to be moving and is not speaking out loud. On the other hand, when IEC intersects with spoken language, the individual is explaining their reasoning and is not moving. In the following, I propose that when all three combine, two possible options occur based on Goldin-Meadow et al.'s (1993) work on matching and mismatching gestures with what is said out loud. The first possibility is that when the individual speaks out loud and their moves match what the person said out loud, the IEC, or memory and understanding of concepts, aligns. Moreover, the way in which the person is embodying their reasoning aligns with the way in which they remembered and understood them. The second possibility is that when the individual speaks aloud, and their moves do not align with what they said, IEC is misaligned. In other words, the way in which they remembered and understood prior concepts does not fully align with the way in which they explained their ideas out loud, hence the lack of alignment in moves (Goldin-Meadow et al., 1993). Overall, embodied cognition can include how the individual uses objects in their surroundings to further leverage their reasoning. In all, this study is guided by situated learning theory and embodied cognition theory.

Figure 1

Embodied Cognition of Spatial Tasks Framework



Note. All embodied cognition is situated in context, where this is first person perspective. Talk refers to speaking aloud, moves refers to gestures, and interactions refer to physical interactions with objects and surroundings.

2.2 Spatial Reasoning

Spatial reasoning is the embodied and situated sensemaking of spatial tasks. There are various types of spatial reasoning tasks, such as spatial scaling, folding, rotation, and orientation (Whiteley et al., 2015). This study investigates how students reason about a type of spatial reasoning task, spatial scaling. Prior to detailing spatial scaling, I briefly summarize the research in spatial reasoning and related concepts within spatial research. Spatial research has addressed various topics, where I highlight two: spatial research as a measurement of spatial *skills* and spatial research as understanding spatial *reasoning* about spatial tasks. The first, spatial reasoning as a set of skills, has primarily been influenced by a framework developed by Uttal et al. (2013). The framework describes spatial skills as trainable concepts, where spatial skills can be categorized depending on the properties of the spatial task at hand. The framework considered two aspects. The first aspect, *intrinsic* or *extrinsic*, referred to either a singular object or many objects (Uttal et al., 2013). The second aspect, *static* or *dynamic*, concerned whether the objects were not in motion or in motion (Uttal et al., 2013). Under their framework of categorization of spatial reasoning skills, spatial scaling would be categorized as *extrinsic static* (Davis & SRG, 2015; Gilligan et al., 2018b; Uttal et al., 2013). Many studies that focused on spatial skills investigated these skills by assessing performance accuracy when solving spatial tasks. However, this study does not follow this framework as the spatial skills within Uttal et al.'s (2013) framework may "shift to another category depending on how tasks are interpreted or performed" (Davis & SRG, 2015, p. 144). As such, considering that different reasoning approaches are possible for spatial tasks (Lehrer et al., 1998) and may not necessarily align with Uttal et al.'s (2013) framework, this study considers the possibility that students may reason in ways beyond what has been established in Uttal et al.'s (2013) framework. Moreover, while the categorization of spatial components is helpful in addressing the ways spatial tasks present themselves, I argue that researching spatial *skills* does not address spatial *reasoning.*

I build from Lithner (2000) to define spatial *reasoning.* For this study, I define *reasoning* as an *embodied* "line of thought, the way of thinking, adopted to produce assertions and reach conclusions" about spatial tasks (p. 166). Specifically, the author described reasoning about tasks as a form of "plausible reasoning" and as "based on established experiences from the learning environment" or based on the situative perspective (p. 165). Moreover, Lithner (2000) emphasizes that an essential component of reasoning is argumentation, "the part of reasoning that aims at convincing oneself" (p. 166). While spatial reasoning does involve making sense of spatial tasks, spatial reasoning is an embodied process in how people make sense of spatial tasks which is mediated by the body and has been most researched in the form of gestures (e.g., Alibali, 2005; Elia & Evangelou, 2014; Kim et al., 2011; Nathan & Martinez, 2015; Ng & Sinclair, 2013; Thom, 2018). Overall, I argue that reasoning is not limited to the mind and is embodied. In short, reasoning can be considered as *how* or *why* an individual comes about reaching an answer for a spatial task. This study focuses on how young girls reason about spatial scaling tasks.

Within spatial mathematics research, there is limited research related to student *reasoning*. As mentioned by Lowrie & Logan (2018), geometry is often the focus of spatial reasoning in mathematics education. However, in mathematics education research, reasoning about spatial mathematics focusing on geometric tasks is limited (e.g., Battista, 1999; Clements et al., 2012; Lehrer et al., 1998; Lehrer et al., 2002; Ng & Sinclair, 2015). Together, scholars have investigated reasoning in multiple domains. For example, Lehrer et al. (1998) investigated students' conceptions of various domains of spatial mathematics, such as angles, spatial visualization, identifying shapes, length of shapes, and area of shapes. In a later study, Lehrer et al. (2002) investigated how students understand the multiplicative similarity of shapes and particularly focused on students' expression of mathematical language and concepts. Similarly, Battista (1999) and Clements et al. (2012) also investigated student reasoning related to geometry and spatial mathematics. Recently, Ng & Sinclair (2015) investigated how young children reason about symmetry and how their reasoning developed over two lessons, specifically through the development of their language, gestures, and use of symbols to represent symmetry. Overall, they identified that these three aspects developed throughout the course of the two lessons. In all, these researchers aimed to understand how young students reasoned about specific areas related to spatial mathematics. Similarly, this study aims to understand how young students reason about spatial scaling tasks based on geometry.

2.3 Spatial Scaling as a Type of Spatial Task

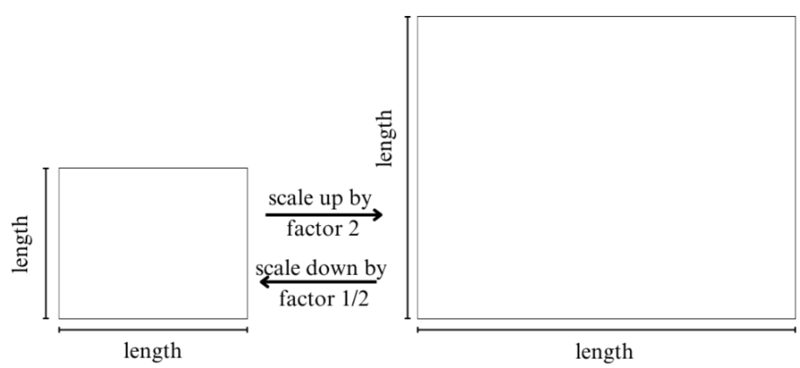
This study focuses on individuals' reasoning about spatial scaling tasks. From the literature, spatial scaling is considered a component of spatial reasoning (Whiteley et al., 2015). Since this study investigates how young girls reason about spatial scaling tasks, I define spatial scaling as a type of spatial task concerning the resizing of one or more objects (Frick & Newcombe, 2012). In the following, I propose that *reasoning* about spatial scaling tasks is embodied and describe the foundations of spatial scaling tasks. I will then provide an overview of spatial scaling as embedded in mathematics. I will give a brief summary of the prior literature on spatial scaling related to map reading and establish the relations of spatial scaling to other domains of mathematics, specifically proportional reasoning and number line estimation.

2.3.1 Foundations of Spatial Scaling Tasks

In regard to 2-dimensional objects, spatial scaling concerns how an object's *lengths*, or measured properties of a defined space, change with a multiplicative numerical factor or *scale* (Barenblatt, 2003; Frick & Newcombe, 2012; Piaget et al., 196). Depending on the situation, an object's lengths, area, and volume can either *scale up* or *scale down* depending on the multiplicative factor applied (Frick & Newcombe, 2012). Scaling up occurs when an object's lengths are transformed by a scale factor greater than the number one, while scaling down occurs when the scale factor applied is less than the number one. For example, the lengths of an object can become longer when the multiplicative numerical factor of the number two is applied (Figure 2). On the other hand, an object's length can become shorter when a multiplicative numerical factor of less than the number one is applied (Figure 2). A change in spatial scale can be reverted to its original form by applying an inverse operation (e.g., the inverse of the scale factor of the number two is one-half). Overall, in this study, spatial scaling of a 2-dimensional object involves changing the dimensions of an object by a multiplicative factor.

Figure 2

Spatial Scaling of a Single Object

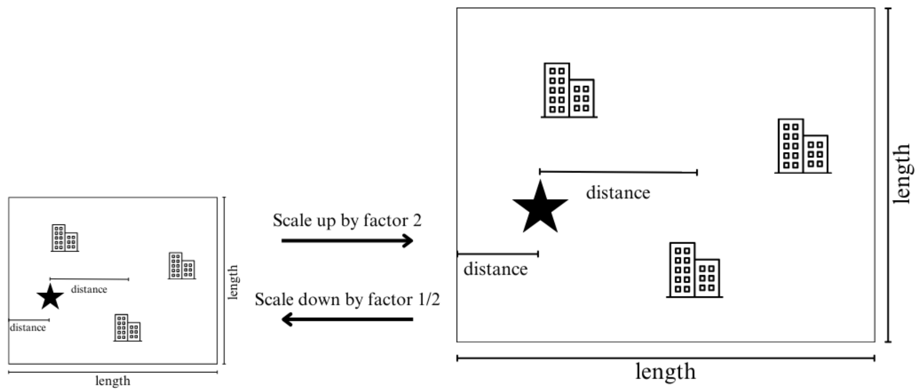


Note. Scaling up (left to right) or scaling down (right to left)

Spatial scaling tasks can involve more than one object. Spatial scaling tasks can also concern the distance *between* objects. Building off Piaget et al. (1960), *distance* is defined as the measured space between two objects. Advancing the first illustration, inserting additional objects inside an empty rectangle (Figure 2) transforms the shape into a usable *map* (Figure 3).

Figure 3

Spatial Scaling of Multiple Objects



Note. Scaling up (left to right) or scaling down (right to left). For both images, the map is the larger rectangle holding the contents inside such as the buildings and the star.

When spatial scaling tasks concern multiple objects, the relations of the distances between the objects must be preserved by the scale factor, which has been referred to as "distance coding" (Huttenlocher et al., 1994, p. 116). Map reading involves distance coding. As shown in Figure 2, enlarging a picture by a specific scale factor will also transform the lengths between the objects by the same scaling factor (Frick & Newcombe, 2012). Depending on the type of distance coding that has occurred, it has been proposed that the strategy occurring has either been a *relative* change in distance or an *absolute* change in distance. In relation to mathematics, *relative change* refers to proportional (or multiplicative) change in distance, while *absolute change* refers to an additive change in distance (Degrande et al., 2018; Frick & Newcombe, 2012; Hund & Plumert, 2007). Specifically, if the relation between the same distance between two pictures of different scales is proportional, that relation can otherwise be considered *relative* (Frick & Newcombe, 2012; Hund & Plumert, 2007). For example, consider the position of the star on the left-hand side of Figure 3. The smaller map is 6 cm to the right of the left edge of the map, whereas the distance of the star in the right-hand, larger map is 12 cm from the lefthand edge. In other words, the relative (i.e., proportional) change in distance from the smaller picture to the larger picture is changed by a multiplicative factor of the number two (Frick & Newcombe, 2012). In terms of *absolute* distance, the star in the right-hand map is 6 cm more to the right than the star in the smaller map. (Figure 3). Therefore, spatial scaling tasks can involve matching the location of objects between two different objects of different scales. In the following, I expand on spatial scaling literature which almost always involved map reading tasks.

2.3.2 Overview of Spatial Scaling Literature Involving Map Reading

Most literature encompassing spatial scaling have used navigational tasks, specifically, that of map reading (e.g., Frick & Newcombe, 2012; Gilligan et al., 2019; Jirout & Newcombe, 2014; Plumert et al., 2019; Rosetti et al., 2017; Möhring et al., 2018; Szubielska & Möhring, 2019; Uttal, 1996). As previously mentioned, map reading involves not only scaling the objects but also making sense of how the distance between objects changes when a scale factor is applied to the entire map and the contents within the map. This study focuses on how young girls reason about spatial scaling tasks involving map reading. Given this study's focus on spatial scaling tasks related to map reading, I review the existing literature related to spatial scaling of map reading. Map reading requires proportionally translating distances as presented in a map (i.e., the *representation*/graphic model) onto maps of different sizes or real-life distances (i.e., the *referent space*) (Frick & Newcombe, 2012; Liben & Downs, 1994, p. 551). This study aims to investigate how young children can transfer their understanding from one representation to another.

Overall, spatial scaling literature on map reading has touched on the following themes: accuracy of task by scale factor, accuracy between age groups, accuracy between boys and girls, and strategies for solving spatial scaling tasks. Spatial scaling tasks involve locating where objects should go with a map onto another map of a different scale. Researchers have investigated the accuracy of participant's answers (e.g., Frick & Newcombe, 2012; Gilligan et al., 2019; Jirout et al., 2018; Möhring et al., 2018; Szubielska et al., 2022). Overall, these studies similarly present results in student accuracy of task by a scale factor, where the accuracy of spatial scaling tasks tends to decrease as the scale factor increases, Similarly, Boyer and Levine (2012) investigated the scaling of fractions and also found this accuracy relation. Moreover, in studies investigating differences in the results of the tasks between age groups, children generally improved in accuracy with age (Frick & Newcombe, 2012; Gilligan et al., 2019). However, while Frick & Newcombe (2012) investigated up to 6 years old and compared to adults, Gilligan et al., (2019) investigated up to 10 years old, where they identified that performance in spatial scaling tasks did not increase, where they identified a potential "ceiling" in performance with age (p. 13). While most spatial scaling studies generally find that girls and boys perform similarly, (e.g., Rosetti et al., 2017; Möhring et al., 2014; Gilligan et al., 2019), one study has identified that boys outperform girls significantly in spatial scaling (Gilligan et al., 2018a).

As previously mentioned, spatial scaling research has suggested that there are three general strategies to solving spatial scaling tasks: a) scaling by absolute distances, b) scaling by relative distances, and c) scaling by transformation or zooming (Newcombe et al., 2018). While I have previously described absolute distances and relative distances in the previous section of this chapter, zooming is described as generally increasing and decreasing the overall image at the same time (Vasilyeva & Huttenlocher, 2004). Moreover, Vasilyeva & Huttenlocher (2004) discuss the possible combination of these three strategies. While zooming can be done independently of the other two strategies, they describe how a relative distances strategy does not require transformation, or zooming, while the combination of absolute distances and then transformation would lead to more mapping errors. Overall, the conclusions of using absolute distances as a strategy was determined by the accuracy of the student's response in mapping as well as the differences in accuracy between tasks of different scales. Specifically, Frick & Newcombe (2012) compared students' accuracy of tasks between scales of 1:2 and 1:4. They describe how if the distance of an object on one map is identically applied on another map of a different scale, the similarities in distances would indicate an absolute distance strategy. However, since the accuracy between these two tasks were similar, they described how students could possibly use relative distances as a strategy or zooming/transformation as a strategy as they would both yield similar results. To determine whether zooming/transformation was used, studies have described that a faster response time in answering the tasks would indicate that zooming/transformation was used rather than relative distances (Möhring et al., 2016; Newcombe et al., 2018).

Scaling, as a phenomenon, has been heavily investigated in various fields (e.g., engineering, forestry, mathematics, specifically geometry) where the commonality between all fields is the retention of "self-similarity" of the objects (Barenblatt, 2003, p. 1). Importantly, middle school students' conceptions of scaling have been found to be significantly correlated with both science and mathematics achievement (Chesnutt et al., 2019). In the geometry context, Foerster (2017), investigated the application of Minecraft in Grade 5 and 6 student learning of geometry. One area they investigated was having students scale what they made by a factor of two. Foerster (2017) described how almost all students had a challenging time completing this task due to improper scaling of a 3-dimensional structure as well as time constraints of the task. Moreover, Luneta (2015) investigated Grade 12 students' accuracy of geometry tasks, which included scaling. Overall, there were two questions related to scaling. In the first scaling question, which involved scaling a polygon by a certain scale factor, over half of students responded accurately, while there was a set of responses that were unclear in student accuracy, which Luneta (2015) proposed to be a conceptual misunderstandings of scaling. The second question required comparing the areas of polygons and required scaling to respond to the question. Luneta (2015) described how this was the "most difficult question amongst the majority of the students samples, such that even in [the most advanced group] only 44% got it right" (pp. 7-8). Studies have identified the importance and underperformance of spatial scaling tasks related to geometry. However, there is a limited focus on how children reason about scaling tasks.

Accuracy in solving spatial scaling tasks has been identified to be a substantial role in predicting performance in mathematics in early childhood (Frick, 2019). Additionally, spatial scaling has been identified to be related to understanding numbers (Newcombe et al., 2015), significantly correlative with number line estimation (Möhring et al., 2018) and related to proportional reasoning as it involves numerical scaling (e.g. Boyer & Levine, 2012; Möhring et al., 2015; Möhring et al., 2018). Importantly, spatial scaling tasks require reasoning about geometric shapes, where Cox (2013) and Lehrer et al. (2002) investigated the role of scale and changing the measurement of shapes. Considering that spatial scaling and learning are socially situated, I aim to also understand the context of prior mathematics learning in the children's classroom, which may potentially impact student communication of reasoning related to spatial scaling tasks.

2.3.3 Research on the Body's Role of Solving Spatial Scaling Tasks

While there have yet to be discussions on how an individual solves spatial scaling tasks through an embodied framework, I highlight papers that focus on the role of the body in spatial scaling and related tasks. Past research has considered spatial scaling as a component of spatial reasoning (Whiteley et al., 2015). Klatzky (1998) describes spatial scaling, as a phenomenon, as understanding distances either from an “allocentric” or “egocentric” perspective. *Allocentric* refers to understanding the distance between two items exterior of one’s body, whereas *egocentric* refers to understanding the distance between two items *in relation* to where one’s body is positioned (Klatzky, 1998). Frick (2019) investigated spatial scaling with maps of different scales and argued that spatial scaling aligns with the egocentric reference frame. From an embodied framework, the body plays a role in how individuals engage with both allocentric and egocentric perspectives. While the egocentric perspective is embodied as it refers to the perspective from the individual, I also view the allocentric view as embodied. Furthermore, I regard the allocentric view as embodied and requires the individual to *imagine*, which is an embodied phenomenon of visualization, to “observe” objects from a different perspective (Klatzky, 1998). For example, if someone imagines what an object looks like from the other side, the phenomenon of imagined observation is mediated by the person’s conceptualization of *seeing* altogether. That conceptualization of *seeing* is constructed by the person’s experience of seeing from their own eyes. In other words, visualizing scaling, or the expansion or shrinkage of an object, is an embodied process and relies on embodied experiences with perception and sight.

Moreover, Proffitt (2013), argued that the *perception* of objects is embodied. Perception is defined as the way an individual sees their surroundings (Proffitt, 2013). Specifically, Proffitt (2013) argued that the way a person understands the size of an object “derive[s] from the body” (p. 475). As spatial scaling requires one to observe tasks from a certain distance as well as a certain perspective angle, I propose that the embodied act of perspective is essential of solving spatial scaling tasks. Therefore, considering spatial scaling in the same approach, the body itself plays a role in how an individual transforms the scale of objects.

Finally, one study examined the role of touch in spatial scaling. Recently, Möhring & Szubielska (2023) have analyzed spatial scaling by comparing spatial scaling accuracies and response times when students were relying on *only vision*, *touch and vision*, and *only touch*. Overall, they identified that students performed similarly in the only vision and touch and vision categories. Specifically, students were most accurate in the vision categories (*only visual* and *touch and vision*) and less accurate in the *only touch* category.

Only a few spatial scaling studies have focused on the role of the body. However, to my knowledge, there have yet to be studies examining spatial scaling through an embodied reasoning perspective. Again, I argue that spatial scaling is a form of spatial reasoning, where reasoning is embodied and situated. This study aims to understand how young girls reason about spatial scaling tasks and the ways the body moves when describing reasoning about spatial tasks.

2.4 Summary of Chapter Two

Overall, research has yet to ask students what they are thinking *as* they are solving spatial scaling tasks. Moreover, studies have yet to investigate how children *reason* when solving spatial scaling tasks. Overall, my study aims to understand the ways in which young children reason about 1-dimensional and 2-dimensional spatial scaling tasks involving one and multiple objects.

While prior literature has described the differences in student accuracy of spatial scaling tasks, spatial scaling research has yet to *ask* students how they solved spatial scaling tasks. This study aims to understand how young girls reason about spatial scaling tasks of multiple objects. I also aim to understand how young girls' reasoning changes along spatial scaling tasks focusing on 1-dimension and 2-dimensions.

This chapter commenced by introducing the theoretical frameworks guiding my research. Specifically, my research is guided by situated learning theory and embodied cognition theory, where I introduced the embodied cognition framework. Subsequently, I defined spatial scaling and described the correlation of spatial scaling to mathematics education. Finally, I presented existing literature discussing gender differences in spatial scaling literature and identified what is to be explored in this study based on past research. In the following chapter, I detail the methodological approaches of this study.

Chapter Three: Methodology and Methods

In the following chapter, I detail the methodological approach employed for this study. I describe the participants and setting of the study, the interview protocol, and how data was collected. I then explain how the data was analyzed and how trustworthiness and validity was considered in the study.

3.1 Case Study Research Methodology

This study employed case study methodology. A form of qualitative research and at times, mixed methods research, Yin et al. (2018) framed case study research as: (1) focused on the "how" and "why" questions, (2) centered on a "contemporary event," and (3) the researcher had "little to no control" of events or participants' responses (p. 13). In revisiting the research goals of this study, this study focuses on *how* young girls reason about spatial scaling tasks and centered on the phenomenon of *embodiment*. I position embodiment as a contemporary event in the context of spatial scaling research as it has yet to be well established. Lastly, the methods were developed so that I had *no control* over how participants responded to solving spatial scaling tasks. Importantly, this study uses a situative theoretical lens and considers that learning is situated in the social context. Moreover, the purpose of a case study is to "understand a real-world case and assume that such an understanding is likely to involve important contextual conditions" (Yin et al., 2018, p. 15). Therefore, case study methodology aligns with my situative theoretical framework as both my theory and methodology consider the larger situated context.

While case study research adheres to the formerly mentioned criteria, there are choices in methodological design which distinguish case studies from each other. The first and most essential choice is to define what the *case* is. In case study research, a *case* can be, but is not limited to, a single person, a group of people, or even a large community (Yin et al., 2018). Since

I aim to individually capture the reasoning of each student for this study, each student is a case for this study. Additionally, there are different types of case studies. While Baxter & Jack (2008) described the ways in which case study methodology can be particularly designed, I follow the framework established by Yin et al. (2018). Specifically, Yin (2018) described how a case study can be a *single case study* (i.e., has only one participant) or a *multiple case study* (i.e., has more than one participant). Considering the research questions of the study ask how girls reason about spatial scaling tasks, and I, therefore, have multiple participants for this study, my study will follow *multiple case study methodology*. Moreover, I chose to conduct a *multiple case study* rather than a *single case study* to obtain more evidence of the embodied reasoning of young girls and, in turn, render results that are more indicative of *how* young girls, in general, reason about spatial tasks. Also, in a case study, there is further categorization in terms of *holistic* and *embedded* approaches (Yin et al., 2018). While holistic case study looks at each individual case generally and focuses on the overall case, embedded approaches look at specific aspects for each case. In this case study methodology, I follow *multiple holistic case study methodology* as I am looking at each participant generally for how they reason about the spatial tasks.

3.2 Participants and Setting

This study took place in an all-girls private school in eastern Canada from May to early June of the end of the school year. The participants included 17 grade three students and their mathematics teacher. While not all the students were in the same class, they all shared the same mathematics teacher and received the same mathematics education. As later described in the following section, the mathematics teacher was provided a survey to complete to describe what content related to the study she had already taught.

The grade level was determined based on consultation with the school principal and aligns with the purpose of the study to understand how *young* girls embody reasoning about spatial scaling tasks. In recruiting students, I visited the grade and introduced myself and my research project. All the girls in the grade were given a consent form to bring home to their parents. Students who returned with a signed consent form by their parent or legal guardian were eligible to participate. The participating mathematics teacher was given a consent form to complete a questionnaire related to the spatial math content the students learned. All participation was entirely voluntary, and participants could withdraw consent at any time.

Among the 17 student participants, seven students were from what I call Class A and 10 students were from Class B. In future chapters, I shall refer to these students by class and number by order in which I interviewed them (e.g., student A1 was the first student I interviewed in Class A). All the students identified as girls as they self-selected, with their guardians, to attend an all-girls school. Six students were later selected for additional fine-grained analysis. Each student was chosen based on their varying ways of reasoning as well as their level of embodiment, where some students visibly gestured more than others. Specific details of selection for fine-grained analysis are discussed later in this chapter in Section 3.3.3.

Importantly, it has been proposed that the "ideal number of participants ranges from eight to 12" for case study methodology, where single case study methodology can have as few as one participant (Baškarada, 2014, p. 14). In this case study, I had 17 participants, which was chosen to reduce the skewing of results that would occur if there were substantially fewer participants. Moreover, the fine-grained analysis of this study was limited to six participants to ensure an in-depth description of how students reason on a case-by-case basis. Moreover, Yin et al. (2018) do not propose a specific number of participants for multiple case study methodology and describe how cases can be very different depending on the study. As well, having 17 participants will facilitate the practice of "replication logic" or demonstrate a reliable pattern of findings, in this study, forms of reasoning across multiple participants (Yin et al., 2018, p. 55). I argue that in this study, as it is the first to investigate the embodied reasoning of spatial scaling in young girls, having a high number of cases is essential to establish replication of findings.

3.3 Data Collection

I collected data from two sources: a questionnaire completed by the teacher and video-recorded student interviews. In the following, I describe the methods of data collection.

3.3.1 Teacher Questionnaire

The mathematics teacher of the class was sent an online questionnaire asking about the mathematics content they taught previously in the school year (Appendix A). Specifically, the teacher was asked about spatial or geometry-related content that they taught their students. When I asked the teacher in the questionnaire about their teaching on spatial content, I provided a brief definition of both spatial reasoning and spatial scaling. There was also space to describe how they taught the content and add any other information if they wished. The questionnaire was distributed by Microsoft Forms, where the participating teacher completed the questions and submitted them through the platform.

The purpose of the teacher questionnaire, as previously stated, was to contextualize and situate the student learning and reasoning since students are shaped by the content that takes place within their classroom. The content the students were taught is summarized in Table 1. Overall, the students were taught geometry for a week, how to locate objects with a Cartesian plane, the metric system, spatial rotation, and scaling. However, the extent to which the teacher taught spatial reasoning and scaling was not specified. In the following section, I detail the methods of the individual student interviews.

Table 1

Class Context of Student Learning of Geometry

Geometry timeline	1 week
Locating objects topics	Cartesian plane
Metric system topics	Meters (m), decimeters (dm), centimeters (cm), millimetres (mm)
Spatial reasoning & scaling topics	2-dimensional and 3-dimensional shapes, nets, solids, rotating shapes. Taught spatial scaling, multiplicative comparison, and additive comparison of shapes.

Note. This is an exhaustive and detailed summary of the teacher's questionnaire responses.

3.3.2 Individual Interview

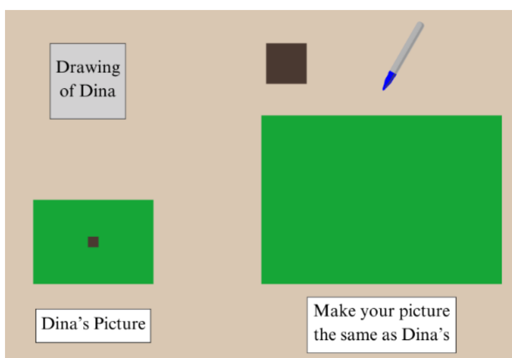
In this section, I elaborate on how I drew upon literature to design the interview, the materials and procedure of the interview, and the types of questions students were asked in the interview. Importantly, the purpose of the interview was to understand how each student reasoned about the spatial scaling tasks. As my study focuses on how young girls reason about spatial scaling tasks, the interviews were individual and semi-structured (Adams, 2015). I video-recorded the interviews to capture their talk and gestures while reasoning about spatial scaling tasks. As previously mentioned, the interviews were conducted over a five-week period the middle of May to early June. The interviews averaged 20 minutes long and took place in an unoccupied room during the school day.

The interview centered on an activity called *The Farmer's House.* This activity was inspired by the methods of Frick & Newcombe (2012). This activity was suitable for my study as many spatial scaling studies have replicated this activity or versions of this activity across elementary grades and age groups (e.g., Gilligan et al., 2019; Jirout & Newcombe, 2014; Möhring et al., 2014; Möhring et al., 2018; Szubielska et al., 2022; Vasilyeva & Huttenlocher, 2004). The task requires students to map locations between pictures of different scales. While previous studies have used the task to establish participant accuracy on a quantitative level, this study addresses the gap in how students reason about this particular spatial scaling task.

Overall, the activity aimed to understand the participants' reasoning when scaling on a 2-dimensional space. The activity, *The Farmer's House*, was presented to students as a situation where they must help a farmer named Dina, a fictional character. When students would enter the room, they would see the materials as depicted in Figure 4. All materials for the activity were placed on a corkboard surface, which is represented by the beige background in Figure 4. The materials included farmland represented by green paper, farmhouses represented by brown blocks, and a blue marker for students to use for the activity. Students were also shown a drawing of Dina's face to further engage them in the activity.

Figure 4

Layout of Materials for The Farmer's House Task



Note. Dina's picture was on the left-hand side with the location of the house indicated. On the right-hand side was the student's picture, which was missing the house's location. Above the student's picture was their house (represented by a brown block) as well as a marker to place a dot where they thought the house should go.

To start the activity, I told the student a story about Dina (see Appendix B for full story). I explained that Dina bought land and was looking to build a farmhouse. I pointed to *Dina's picture* that showed the location of Dina's house, which was represented by a brown block on top of green paper (Figure 4). I then told the student that Dina wants to make additional pictures to show her friends where she was thinking of building her farmhouse. I showed the student an additional picture, labelled in Figure 4 as “Make your picture the same as Dina’s,” which I will refer to as the *student’s picture*. The *student’s picture* was of a different scale and was missing the location of Dina’s house, where the house was placed outside of the picture. I then told the student that Dina needed their help in matching the *student’s picture* with *Dina’s picture*. The student was instructed to first use a blue marker to place a dot where they thought the house should go and then instructed to place the house on top of the dot they made with the marker. The reason for using the marker was to encourage students to take their time in solving the task as well as to recall where the student placed the house in the case that it would be moved to ask about alternative locations where the house might be placed.

The interview was structured in two parts (Table 2). The first part of the interview involved scaling *up* and the second part involved scaling *down* (Frick & Newcombe, 2012). Both tasks had the dimensions of Dina’s pictures scaled up or down by a factor of four. Specifically, Dina’s pictures were one-fourth the length of the student’s pictures in the scaling up part and four times longer for the scaling down part. The order of these parts was chosen for both consistency with previous literature (e.g. Frick & Newcombe, 2012) and to reduce the possibility of scaffolding student learning between tasks as previous works have discussed that scaling up may be more challenging than scaling down (e.g., Möhring and Szubielska, 2023). The choice to scale up first as a task was to ensure consistency with spatial scaling studies that *only* investigated spatial scaling *up* (e.g., Frick & Newcombe, 2012) as well as follow the order of scaling up and scaling down that was done previously by Möhring and Szubielska (2023). Moreover, while prior research discussed that scaling up might be more challenging than scaling down, I propose that the order to scale up or down first would not have made a difference in difficulty, as Plumert et al. (2019) found that there was no difference in difficulty between the tasks. The choice of scaling by a factor of four was to maintain consistency with past studies such as Frick and Newcombe (2012) who also had scaling by this factor in their research. In their work, when assessing a child's scaling, they started with scaling up by a factor of four as they argued the task would be easier to comprehend as well as that "for 1:4 trials it is easier to differentiate systematic variance in scaling accuracy from unsystematic error variance, and thus 1:4 trials are more informative than 1:2 trials" (Frick & Newcombe, 2012, p. 275).

There were six tasks in all, or six different locations that students were shown (Table 2). In the first part, students were shown three locations for Dina's farmhouse for scaling up: centered, the top left corner, and the bottom left corner. Figure 4 shows the scaling up, centered task. In the second part, students were shown three locations as well for scaling down: centered, top right corner, and bottom right corner. The locations were chosen in this order to reduce student influence between the parts of the task as well as ensure that all quadrants of the map were used for the task including the origin. Reducing student influence was to ensure that no two locations shown were exactly the same. For instance, if the first task was *scaling up, centered* and the second task was *scaling down, centered,* that would highly likely influence student reasoning as both locations would be in the center. For all tasks, the location of the corner was always one quarter distance of the dimensions from the corner that was described. For instance, the top left corner location was one-quarter of the length from the top left corner and one-quarter of the width from the top left corner.

Table 2

Description of Six Spatial Scaling Tasks

Parts of Activity	Type of picture	Dimensions of map, length x width (cm)	Proportion (length/width)	Location of Dina's house
Scaling up	Dina's picture	7.5 x 6	1.25	1. Centered 2. Top left corner 3. Bottom left corner
	Student's picture	30 x 24	1.25	
Scaling down	Dina's picture	5 x 4	1.25	1. Centered 2. Top right corner 3. Bottom right corner
	Student's picture	20 x 16	1.25	

Note. This table describes the type of task, type of photograph, dimensions, proportions between the lengths and width, and location of Dina's house for all six tasks.

The dimensions, or lengths and widths, of Dina's pictures and the student's pictures were different between scaling up and scaling down tasks to reduce students being influenced by their responses in the previous part. Nonetheless, the proportions between the length and width for all pictures were maintained for consistency.

As mentioned, students were asked to place the house where they thought it should go so that it matched Dina's picture. Afterwards, they were asked a variety of questions to further understand their reasoning, such as how their picture matched Dina's, how they knew their spot was correct, whether alternative house locations matched Dina's picture, why alternative locations did or did not match, and how they would explain where to place the house to their classmate (for full protocol, see Appendix B). Overall, the probes were used to further understand the student's reasoning as well as clarify potential areas of misunderstanding.

While this case study took place within an academic setting, the purpose of the study was *not* to evaluate students on their understanding of spatial scaling so as not to perpetuate the deficit discourse surrounding young girls in spatial tasks. Throughout the interviews, I placed the students as competent and approached their reasoning in a non-evaluative way. Placing the students competently involved responding and asking questions to students in ways that used their vocabulary as well as their responses. For example, if a student described the house as located in the *top left corner,* I used the same language when I probed on their reasoning so as not to influence them in their response. Moreover, in approaching their reasoning in a non-evaluative way, students were never provided the correct response as to where the house should go. To specify, I probed on their reasoning and treated their ideas as *correct* regardless of their responses so as not to evaluate them. Never at any point during or after the interview were students provided the correct locations of where Dina's house should go.

3.4 Analysis Methods

In the following, I describe the process of data analysis. Overall, the data analysis procedure consisted of the following four stages: (1) organization and video noticing, (2) episode creation, (3) coding scheme development, and (4) coding sessions to finalize coding scheme, re-coding of data, and analysis of trends and student cases.

3.4.1 Organization and Video Noticing

As soon as interviews were completed, the videos were labeled and documented in an excel by student participant ID and by class group. Upon collecting videos of student interviews, I started conducting video noticing sessions of the interviews, both alone and with my supervisor, Dr. Marta Kobiela. The video noticing sessions were essential in the starting phases of developing the coding scheme as the observations developed helped identify general commonalities between students' reasoning and gestures. Overall, we had three collective video noticing sessions. During these sessions, we would first watch the video and write notes silently and then pause the video to share what we noticed together. Observations included what the student said, the way each student moved (e.g., gesture, standing, moving closer to the picture), any probes that I asked that were notable, differences in student's responses, and changes or inconsistencies in what the student said throughout the interview. We focused these collective sessions on shorter segments of student interviews, specifically one task of the student interview. The moments of pausing were typically at the end of each task.

3.4.2 Episode Creation

After the three video noticing sessions, I developed the episode rules. I define an episode as containing an interviewer's question and the student's response. In most cases, most questions I asked the students during the interview resulted in a new episode. However, a new episode was not created when I asked questions that required previous context. This was done to ensure that episodes could be watched on their own and did not require the context of episodes that came before. For instance, if I asked the student a question to clarify their previous response, this clarification question was included in the episode if the context of their last response was needed to understand their later response. For example, I often asked clarification questions about terms the student used (e.g., what they meant by the *middle*).

To create the episode rules, I first created a preliminary draft set of rules and then proceeded with iterations of feedback and revision. Overall, there were four iterations of episode rules to develop the final episodes. The modifications conducted were determined by creating the episode rules, applying the rules to a sample of student episodes, and taking notes on inconsistencies or areas in the rules lacking clarity. Overall, there were 452 episodes in total, where the lengths of the episodes were as short as 5.2 seconds and as long as 4 minutes and 1.2 seconds. There were as few as 17 episodes per student and as many as 37 episodes per student. The average number of episodes per student was 26.59 episodes. The number of episodes per student is listed in Table 3 below. These episodes served as the unit of analysis for the coding, which I describe next.

Table 3

Total Number of Episodes by Student Profile

Class	Student	Number of episodes (total)
Group A	A1	17
	A2	25
	A3	26
	A4	28
	A5	31
	A6	31
	A7	20
Group B	B1	24
	B2	31
	B3	26
	B4	37
	B5	32
	B6	31
	B7	34
	B8	21
	B9	21
	B10	17

Note. Students are listed by class group (A or B) and then ordered by the order in which they were interviewed.

3.4.3 Coding Scheme Development

My supervisor and I collaborated on the development of the coding scheme. Coding was conducted by observing the videos rather than reading transcripts. Considering the purpose of the study is to understand *how* young girls gesture as they solve spatial scaling tasks, it was essential to observe videos for coding as reasoning is captured through *both* talk and gesture. The coding scheme focused on forms of student reasoning as this study is interested in investigating *how* students reason about solving spatial scaling tasks.

After the three video noticing sessions, I developed the first draft of the coding scheme on my own. My supervisor and I then revised this draft to finalize our first coding scheme which we used for our first coding session.

Each coding session involved the coding of at least 10 episodes. After coding these episodes individually, we gathered our codes in a consensus file and discussed each episode and our codes for each episode. Overall, there were 10 coding sessions to achieve over 70% iter-rater reliability. I detail how we coded in the following paragraph and detail the iter-rater process in Section 3.5. A sample of the coding consensus of one student is included in Table 4.

Table 4

Consensus of Codes Example

Student	**Episode**	**ST Codes**	**MK Codes**	**Consensus codes**	**Notes**
STUDENT	11:14 – 11:57	Comparing amounts – 2 other spaces	Comparing positions – Using map positions Comparing Amounts – Unclear	Comparing amounts – 2 other spaces Comparing positions – Using map positions	Comparing amounts - student gestures between maps, which according to our coding scheme can be two spaces.

Note. This is an example of what one row appears as for the consensus coding sessions.

As mentioned in Chapter Two, reasoning in the context of this study is *how* or *why* the student reaches an answer about the spatial scaling task. Moreover, we operationalized or determined that a student was reasoning when the student responded by describing *how* or *why* they came to conclusions surrounding the task. We called the ways that students reasoned *forms of reasoning*. Almost all episodes were coded with a form of reasoning as the questions I asked during the interview targeted *how* students solved the tasks and *why* they knew that their solution for the task was correct. In addition, when relevant, we coded multiple codes per episode to capture the different forms of reasoning that students used simultaneously. Throughout the coding sessions, both my supervisor and I identified the form of reasoning through evidence of what the student was *saying* and what *gestures* the student was making. Alternatively, this can be described as *linguistic cues* and *gestural cues*. Linguistic cues were indications of how the students reasoned by what they said aloud (e.g., "space"). Gestural cues were indications of a form of reasoning based on the student's movements or gestures. Sometimes, depending on the form of reasoning, linguistic cues alone were sufficient to determine the reasoning used (I describe examples of this in the following sections of this chapter). However, gestural cues required linguistic cues as some forms of reasoning had identical gestural cues. As later explained in Section 3.5, we reflected on whether alternative forms or "disconfirming evidence" of reasoning were possible when coding (Creswell & Miller, 2000, p. 127). For example, we would ask each other whether the gestures or linguistic cues observed could also align with a different form of reasoning. In cases where there was insufficient evidence to determine what form of reasoning the student was reasoning about *and* it was unclear what the student was reasoning about, the episode was coded as *unclear* forms of reasoning. When a form of reasoning was clear but did not fit our existing codes, we coded this as *other*.

Overall, the coding scheme resulted in seven forms of reasoning excluding *other* and *unclear* forms of reasoning. Three of the seven forms of reasoning were divided into sub-forms of reasoning to further specify each form of reasoning. It was possible to specify forms of reasoning as gestures and language further indicated aspects such as directionality and amounts of items being described. The forms of reasoning will be further described in Chapter Four.

3.4.4 Re-Coding of Data and Analysis of Trends and Cases

Once the coding scheme was finalized, my supervisor and I coded four students together to determine our interrater reliability. Once it was established that the interrater reliability was above 70%, I then re-coded all of the remaining data on my own. Overall, among the 452 episodes that were identified, 22 episodes were coded as unclear forms of reasoning. The episodes coded as *unclear* forms of reasoning were excluded from analysis as it was unclear what the student was expressing to the point that it was not possible to code the episode to a form of reasoning in the coding scheme. Therefore, I did not consider these episodes for analysis as there was insufficient content from the episode to analyze. This left 430 episodes for analysis.

I then conducted two rounds of additional analysis. For the first, I identified trends within the coded forms of reasoning *by episode, spatial task, and by class.* To identify trends by episode, among the 430 episodes, I investigated trends such as the frequency of codes and the intersection or co-occurrence of codes in the same episode. I chose to analyze the trends by episode to understand how the codes may co-occur in the same episode and how many times these codes co-occurred. Regarding analysis by spatial task, there were six spatial tasks in all (see Table 2 in Section 3.3.2). I analyzed the frequency and trends of intersecting forms of reasoning by spatial task to better understand the situated nature of student reasoning. I chose to analyze trends by task to investigate how forms of reasoning varied between the tasks and understand whether co-occurrence trends occurred throughout the tasks as well. Finally, since the participants consisted of two separate classes, I analyzed the frequency and trends of forms of reasoning by class group to understand how their reasoning might be situated within their classroom contexts.

Second, to understand how students coordinated multiple forms of reasoning simultaneously, I conducted a fine-grain analysis of select student cases. I selected six students who had a high rate of intersecting forms of reasoning per episode (i.e., over 70% of intersections occurred in the total number of student episodes). The students and further details on the selection of students are described in Chapter Five. When conducting the fine-grained analysis, I looked to capture *how* multiple forms of reasoning were expressed in the same episode. In doing so, I looked to capture the student's language, gestures, and interaction with surroundings as these multiple forms of reasoning coordinated.

To better understand students' coordination of forms of reasoning, I also took note of how mediational means appeared as they reasoned (Wertsch, 1994). Mediational means such as language and gestures were analyzed throughout the coding. However, we did not code for types of mediational means, such as interaction with objects or language specific to experiences, for two reasons. First, students were limited to interacting with two objects throughout the task: the marker when they placed the dot where they thought the house should be placed and their model of the house. Moreover, once students solved the task, they were not expected to interact with the model or the marker afterwards, where there would be limited to no opportunity for mediational means with objects. The second reason was that personal experiences are specific to each student. Therefore, associating a code with personal experiences would generalize unique experiences that would result in an abundance of codes. Each case will be illustrated in Chapter Five.

3.5 Validity of Study

In this section, I describe the ways in which validity, or trustworthiness, was assessed throughout the study. Validity is a set of measurable research practices which demonstrate the trustworthiness of the researcher and their study (Butler-Kisber, 2010). While I use validity as the term to describe this practice, I do not adhere to the positivist perspective of validity, where I view validity as *trustworthiness* (Butler-Kisber, 2010). Importantly, the spatial scaling tasks were specifically inspired by previous spatial scaling research to supplement the preexisting findings on student accuracy by focusing on their reasoning about these tasks (Frick & Newcombe, 2012). Moreover, the interview protocol and order of the spatial scaling tasks were reviewed by my supervisor to ensure that both the tasks and the interview questions would not sway students towards a specific response. Finally, the interview protocol was practiced in a pilot interview to ensure as much consistency between the participants as possible. Moreover, my supervisor and I watched the video recording of the pilot to refine the interview protocol and develop follow-up questions from the main interview questions. Validity, or trustworthiness, was further determined through Creswell and Millers' (2000) description of the "Lens of the Researcher": *triangulation, disconfirming evidence,* and *researcher reflexivity* (p. 126).

In this study, *triangulation* was achieved by having the seventeen participants conduct the same tasks to determine the forms of reasoning (Creswell & Miller, 2000). Moreover, the same coding scheme was used for all participants. Finally, the coding scheme involved both evidence from gesture and language, where evidence from two sources from the participants was almost always necessary to determine the form of reasoning. Triangulation was conducted *both* during the development of the coding scheme and *during* the final coding process.

R*esearcher reflexivity* was conducted throughout the course of the coding scheme development. Specifically, *disconfirming evidence* in this study was a form of researcher reflexivity to ensure that our assumptions and bias were impacting the coding process and to ensure that we were coding based only on the coding scheme to the most accurate degree possible. Moreover, *researcher reflexivity* was essential during the meetings on coding consensus as we would take time to reflect on our biases. For each coding session, I selected student episodes and created coding templates. The coding templates included the student episode video time stamps, a space to write the associating code, and a space for notes. For each coding session, there were approximately 10 episodes that we observed, coming from different students to provide a more representative sample of episodes. I used a separate file to keep track of all the episodes we coded to ensure episodes from all students were observed. For each session, my supervisor and I coded separately on our own copy of the coding template using the coding scheme. Importantly, a section of the coding template included *notes*, which we would use to further justify *why* we chose a code with video evidence as well as list concerns or ideas about the coding scheme. During this process we individually considered *disconfirming evidence* prior to the meeting on the codes together (Creswell & Miller, 2000). *Disconfirming evidence* involved investigating alternative forms of reasoning that the student could be presenting (Creswell & Miller, 2000). For instance, if it was determined the students were reasoning by a specific form of reasoning, we reviewed the forms of reasoning to determine whether other forms of reasoning were possible. However, since a student could use more than one form of reasoning in an episode, it was essential to review the coding scheme to ensure that alternative options were reviewed and considered. The section dedicated to notes also served as a place for researcher reflexivity, where we also used this space to reflect on our choices for the codes and in what ways our biases and assumptions could influence the codes we chose (Creswell & Miller, 2000; Pillow, 2010). We also determined that each episode would be treated as its own clip and not be influenced by previous episodes. The reason for this was to prevent miscoding and bias from previous episodes. One way we reduced the impacts of bias was to write down evidence for *both* linguistic and gestural cues to confirm the choice of form of reasoning.

During the meetings, we would share our individual codes and notes to reach a consensus on the codes as well as further clarify or change the coding scheme as needed. When we had disagreements about codes for episodes or any questions about any episodes even if we agreed on them, we would rewatch the episodes together and pause at moments we wished to discuss. For example, disagreements included one of us coding for a form of reasoning while the other did not code for it, or each of us coding different codes. We first reviewed the evidence we wrote down and discussed how we reflected on disconfirming evidence during our individual coding process (Creswell & Miller, 2000). We then also reflected on the coding scheme and ensured that there were no additional codes we did not consider that could fit with the student's reasoning. It was essential that there was enough evidence through linguistic and gestural cues to determine that the form of reasoning was being used. Once we reached sufficient evidence for the chosen codes, we wrote it in the consensus codes section of the template (shown in Table 4). In our notes section, we took note of how we agreed on the change and described the changes we would make to either the coding process or the coding scheme itself.

Using the process mentioned above, it was sometimes possible that we would keep both codes if we coded for different forms of reasoning depending on if there was enough evidence. However, most times, only one form of reasoning would be chosen as there was almost always more gestural and linguistic evidence to support one form of reasoning over the other.

As previously mentioned, the validity of the coding scheme was determined by coding four student interviews with my supervisor to attain over 70% iter-rater reliability. Iter-rater reliability was determined by dividing the total number of common codes over the maximum total number of codes identified in one session. For example, if I had identified 18 codes and Marta identified 17 codes and we had 15 codes in common by appearance, the iter-rater reliability would be 15/18. Once 70% was achieved, there were at least two more coding sessions to ensure consistency over 70% of the iter-rater reliability and stability of the coding scheme.

Once we completed coding these four complete student interviews together, I then coded the rest of the student interviews until completion. As I continued coding, I maintained the practice of notetaking to continue practicing reflexivity to ensure that the code I chose was backed by evidence which aligned with the coding scheme, as well as to ensure that there were no alternative options possible. All notes were preserved in the final coding software file for the coding software MAXQDA.

3.6 Summary of Chapter Three

In this chapter, I described how I employed case study methodology for this study. I then described the participants and setting of the study and described the questionnaire protocol and interview protocol. Finally, I described the method of analysis employed for this study and discussed how trustworthiness was verified throughout the course of data collection and analysis. In the following chapter, I discuss the embodied forms of reasoning that students used as they solved the spatial scaling tasks.

Chapter Four: Results Part I

This study focused on the students' embodied reasoning about six spatial scaling tasks. I divide the results into two parts. This first part describes the forms of reasoning students expressed during the interview. The second part, Chapter Five, describes the frequency and trends of the forms of reasoning in relation to episodes, tasks, and class group. I also show how six cases of students coordinated forms of reasoning and drew upon mediational means as they did so. Both chapters focused on the students' embodied reasoning about six spatial scaling to answer the two research questions regarding the overall interviews: (RQ1) How do young girls reason when solving spatial scaling tasks within geometry? (RQ2) In what ways do young girls use their bodies when reasoning about spatial tasks?

As described in Chapter Three, the students solved six spatial scaling tasks of different house locations, they were: *scaling up* (1) center, (2) top left, (3) bottom left, and *scaling down* (4) center, (5) top right, and (6) bottom right. I consider student reasoning as *the process* of how they placed the house in the location they placed it or the *justification* of why their location was correct. Throughout, I refer to the three gestures described by Alibali and Nathan (2011), *pointing, representational,* and *metaphoric gestures.* For all student reasoning, I argue that there was alignment between the student's gestures and speech (Goldin-Meadow, 2011).

Following analysis of the interviews, *seven* forms of student reasoning about the spatial scaling tasks were identified. The forms of reasoning were: (1) *comparing positions,* (2) *comparing amounts,* (3) *comparing shapes,* (4) *partitioning space to aid comparison,* (5) *two or more coordinated lines to locate a point,* (6) *zooming,* and (7) *I just saw it.* Three forms of reasoning, *comparing positions, comparing amounts,* and *partitioning space to aid comparison,* were composed of sub-forms of reasoning, which further specified the way in which students reasoned about the tasks. All forms and sub-forms of reasoning identified are listed in Table 5. In the cases when a student was clearly reasoning and their reasoning did *not* align with any of the forms of reasoning, their reasoning was categorized as *other* forms of reasoning.

Table 5

Forms of Reasoning

Form of reasoning	**Sub-forms of reasoning**	**Student quotes**
Comparing positions	Using map positions	“It’s in the **middle** like her house over here [points to middle].”
	Not case	“I just look at it and think – that’s **not** [points to incorrect spot].”
	Symmetry	“I placed it a bit like where I put it before, well just like, just under, ‘cause it’s like the **opposite** [points to symmetrical spot].”
	Aligning with original	“When it’s close, I can see more better [**places pictures side-by-side**].”
Comparing amounts	Horizontal lengths	“There’s a bit of a **gap** [claw-like gesture between house and right edge of picture, thumb is start-point and index finger is end point].”
	Vertical lengths	“You have a tiny bit **more space** than Dina’s [claw-like gesture vertically where thumb is at house and finger is at top edge of picture].”
	Diagonal lengths	“It’s a bit **too long** here [traces diagonal distance from corner to house].”
	Two Horizontal spaces	“This is way **too little space** [points to space above house in horizontal direction].”
	Two Vertical spaces	“This **space** would have **way more** than this [traces vertically space with hand].”
	Two Other spaces	“There’s a little bit of **space** here, and there’s a little bit of **space** there [circles small space near top left corner and large space near bottom left corner.”
	More than two spaces	“Because they’re all equal parts for each other [**points to four spaces**].”
	Unclear amounts	“It has more space here than it does here [with unclear gestures].”
Comparing shapes	Not applicable (N/A)	“Here I see a **square** there [draws a square] but here I see more of a **rectangle** [draws a rectangle].”
Partitioning space to aid comparison	Cutting into pieces	“**Split** into four [**draws partitions** with hand cutting picture into four pieces].”
	Creating a new map	“That’s like a square [gestured square] and you put it in the middle of the square [points to center of square].”
Two or more coordinated lines to locate a point	N/A	“No, because, it still hits it (i.e., the house), but not all of the lines hit it. Only…only some of the diagonal lines [traces lines with finger to intersect at the house].”
Zooming	N/A	“Well, I acted as if I **zoomed** in on it [stretches out hand].”
I just saw it	N/A	“I just see where it goes and I put it there [no gesture].”
Other	N/A	“If you bend it like this [pretends to fold paper].”

Note. The student quotes contain gestures that are shown in [brackets] and N/A = not applicable.

In the following sections, I will describe each form of reasoning mentioned above. For each form of reasoning, I specify the linguistic and gestural cues that were determining factors of the form of reasoning and provide student examples of how it was presented in the interviews. All gestures have been replicated and drawn to protect participants' anonymity and better see the gestures.

4.1 Comparing Positions

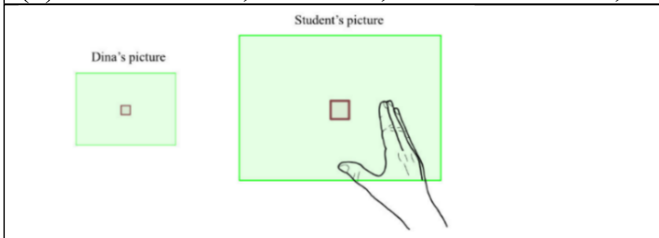
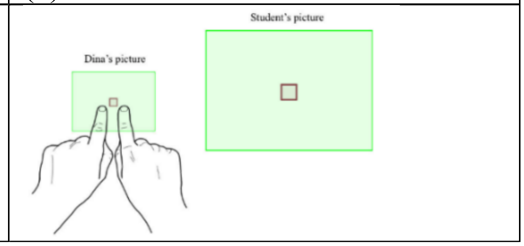
The form of reasoning *comparing positions* was determined when students would reason about the location of Dina's house by comparing positions between Dina's picture and the student's picture. There were four sub-forms of reasoning for *comparing positions*: *using map positions, not case, symmetry,* and *aligning with original.* These sub-forms of reasoning will be discussed in the following paragraphs.

4.1.1 Using Map Positions

A student reasoned by *comparing positions – using map positions* when the student compared specific named locations of the house between Dina's picture and their student's picture (e.g., "center," "middle," "left corner"). To solve the task, students used positional terminology to justify where the house should be located. For example, I asked student A1 how her picture matched Dina's in the first task (i.e., scaling up, center). As illustrated in Figure 5, she pointed to her student's picture and responded, "Because, well, the house, it's in the middle also," and then used a *pointing gesture* to show Dina's house with both her index fingers and continued, "Like her house over here." Linguistic cues included using positional terminology such as: "middle," "center," "top corner," "bottom corner," and equivalent. Gestural cues involved pointing or gesturing to the location the student described.

Figure 5

Comparing Positions – Using Map Positions Example

(1) “because well, the house, it’s in the middle,”	(2) “like her house over here”.
	

Note. From left to right in numerical order, student A1’s gestures and quotes are represented.

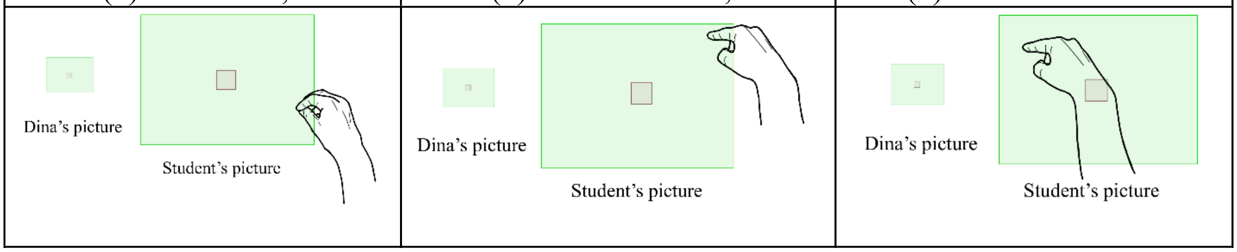
4.1.2 Not Case

Students reasoned using *comparing positions – not case* when they justified their house was in the correct location by identifying *incorrect* alternative locations for the house. That is, students identified positions on the pictures that would *not* be correct. For example, students reasoned that their house was in the correct location by showing an incorrect location on the picture and describing that the house does *not* go there, either by using linguistic cues “not” or equivalent. As illustrated in Figure 6, when student B8 explained for the first task (i.e., scaling up, center) how she knew that her house was in the middle, she responded, “I just look at it [referred to her student picture] and think, that’s not, and that’s not, and that’s not,” and used *pointing gestures* to indicate different locations on the map that were *not* in the center. Gestural and interactive cues included pointing to the incorrect location or moving the house to the incorrect location they were describing. This sub-form of reasoning was not considered when the interviewer presented an incorrect location. Students needed to introduce the incorrect location themselves.

Figure 6

Comparing Positions – Not Case Example

(1) "I just look at it [referring to student picture] and think,"		
(2) "that's not,"	(3) "and that's not,"	(4) "and that's not".



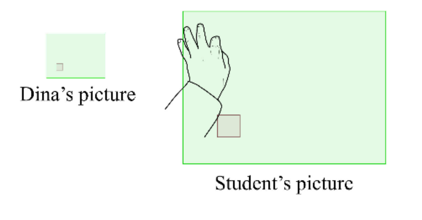
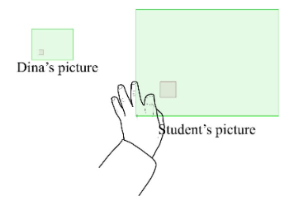
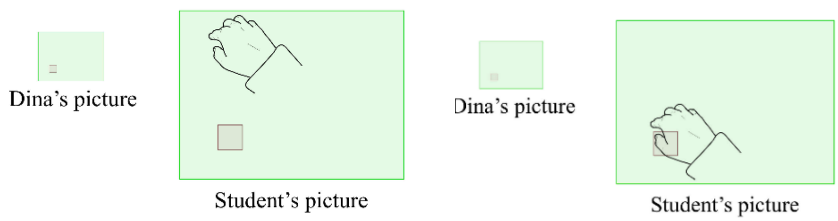
Note. With the gesture below the quotes, student B8's quotes are in numerical order.

4.1.3 Symmetry

A student reasoned with *comparing positions – symmetry* when they reasoned their house was in the correct location by relating it to another location having symmetrical properties. Specifically, students reasoned where Dina's house should be placed by identifying a symmetrical location and describing that their house was in the *same* location as the symmetrical position if *either* the entire picture was changed (e.g., rotated, flipped) or the house was moved (e.g., moved to the other side of the picture). For example, referring to Figure 7, when I asked student B2 how she would explain how to place the house for the third task (i.e., scaling up, bottom left corner), she responded: "Because, I think it's like a bit like over here [pointed to the top left corner]. I placed it a bit like where I put it before, well just like, just under," she said and used a *pointing gesture* to the student house at the bottom left corner, "'cause it's like the opposite." As illustrated in this example, students did not need to explicitly use the word "symmetry." Instead, linguistic cues included stating that the location of the house was similar to a different location, but just on the "opposite side," "other side," or "rotated." Gestural cues included *pointing gestures* or moving to the block to the location they described. However, if a student presented a symmetrical location but only expressed that it was *not* where the house goes and did not express symmetry using the indicated linguistic cues, then this sub-form of reasoning was not considered.

Figure 7

Comparing Positions – Symmetry Example

(1) "Because, I think it's like a bit like over here [pointed to top left corner of picture]	(2) I placed it a bit like where I put it before, well just like, just under," [pointed to bottom left corner and her hand was not touching the paper in this instance]
	
(3) "'cause it's like the opposite," [touching the paper, she dragged her finger down].	
	

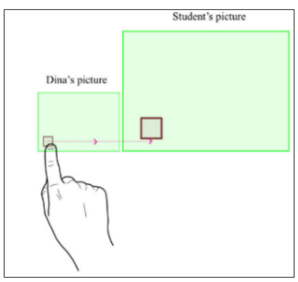
Note. From left to right chronologically, student B2 described the similarity of the location to the last task, gesturing by pointing to the previous location and showing the symmetrical location with gesture.

4.1.4 Aligning with Original

When *comparing positions – aligning with the original*, students reasoned their house was in the correct location by aligning the student picture with the original picture (i.e., Dina's picture) at the edges and using this alignment to determine where their house should be placed. Specifically, *aligning with original* helped students compare the two locations or positions of the house between the two pictures more easily. For example, in the third task, student A1 aligned both Dina's and the student's picture at the bottom edge and placed them side-by-side to assist her in placing Dina's house (Figure 8). From Dina's house, she traced an imaginary horizontal line to the right into her student picture until she reached a point on her picture to help her match the pictures, as seen in Figure 8. Although she later adjusted the vertical distance of her house to account for the scaling up, the alignment provided her with an initial starting place for horizontal distance. For the same task, student A4 similarly aligned the pictures, yet her justification was different. She explained: “When it’s like this [she moved the pictures far apart from each other] it’s kind of hard for me to see like a little bit that so, so sometimes I estimate… I do it here,” and she pointed to an incorrect location. As she aligned the papers together, she said, “and when it’s close, I can see more better.” Gestural and interactive cues that were essential for this form of reasoning included moving the pictures towards each other in a specific manner.

Figure 8

Comparing Positions – Aligning with Original Example



Note. Student A1 used the alignment to locate where the house should be placed and created a horizontal line and dragged it along into the student picture.

4.2 Comparing Amounts

Students reasoned about the house’s location by creating and *comparing amounts* that were either 1-dimensional lengths or 2-dimensional spaces. Overall, this form of reasoning was divided into eight sub-forms of reasoning. Three sub-forms of reasoning involved *comparing 1-dimensional lengths* and were specified by directionality: *horizontal lengths, vertical lengths, and diagonal lengths.* Four sub-forms of reasoning were dedicated to *comparing 2-dimensional spaces* and were distinguished by directionality and number of spaces imagined: *two vertical spaces, two horizontal spaces, two other spaces, and more than two spaces*. The last sub-form of reasoning *unclear amounts* was determined for students when they were clearly comparing amounts, yet it could not be confirmed which amounts of space or length were being described due to insufficient evidence. I describe the sub-forms of reasoning in the following paragraphs.

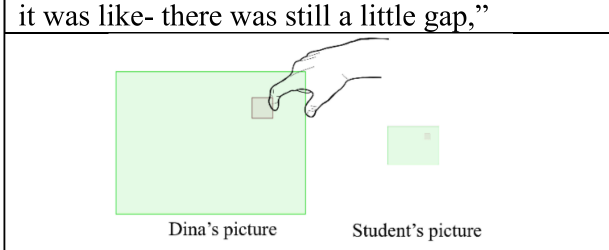
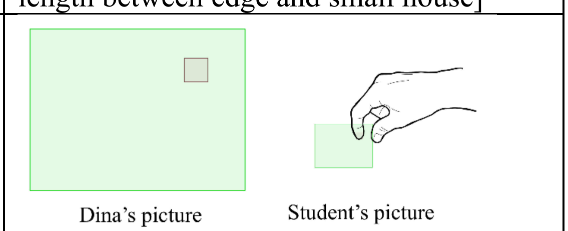
4.2.1 Comparing Lengths

The sub-forms of reasoning *comparing lengths* were determined when students created and compared two similar lengths that were either *horizontal*, *vertical*, or *diagonal*. Students determined where their house should be placed by measuring or estimating linear distances on Dina’s house (e.g., the distance between the house and the right edge of the picture) and replicating these distances on their student picture to scale. Since the lines were distinguished by directionality, the lengths were straight and not curved. In the following, I provide examples of how horizontal, vertical, and diagonal lengths presented themselves in interviews.

Regarding horizontal lengths, when I asked student B3 in task 5 what she was thinking as she matched Dina’s picture, she responded, “So when I looked at it over here, I saw that it was like- there was still a little gap,” and then created a *representational gesture* to show that “there was still a little bit” of horizontal distance from the cube to the right edge of the picture, as depicted in Figure 9.

Figure 9

Comparing Amounts – Horizontal Lengths Example

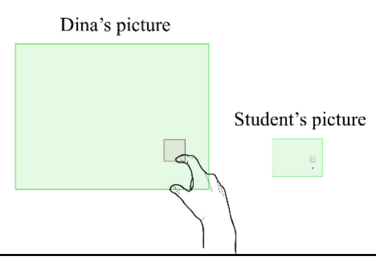
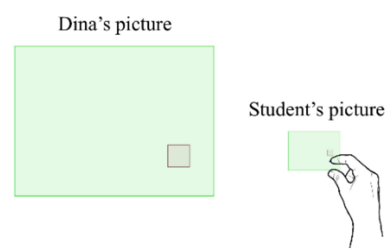
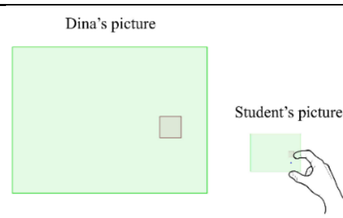
(1) “So when I looked at it over here, I saw that it was like- there was still a little gap,”	(2) “there was still a little bit,” [shows length between edge and small house]
	

Note. In numerical order, the student B3’s gestures are represented below the relevant quote.

For vertical distances, I asked student A6 for the sixth task (i.e., scaling down, bottom right corner) whether an alternative location (above where she placed the house) still matched Dina's picture. She responded, "Yes and no, because it's not touching the side (Figure 10). But, you can tell that Dina's barn has this much space," and made a *representational gesture* to show the vertical distance between the house and the bottom edge of Dina's picture. She created a similar gesture on her student picture and explained, "and then… it needs this much space here." She finally made a gesture from the edge of Dina's picture to the alternative location and said, "but you have a tiny bit more space than Dina's."

Figure 10

Comparing Amounts – Vertical Lengths Example

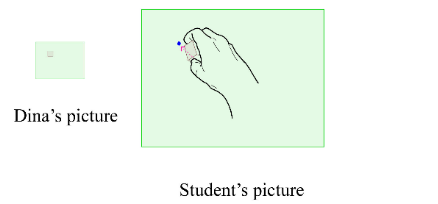
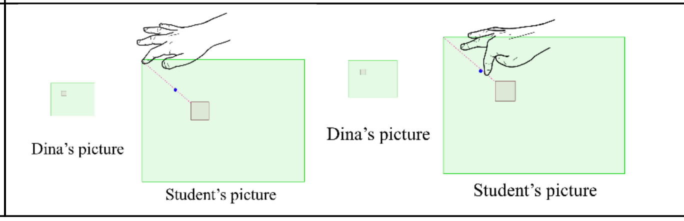
<table>
<tr><td>(1) "Yes and no, because it's not touching the side. But, you can tell that Dina's barn has this much space," [length between house and edge]</td><td>(2) "and then… it needs this much space here," [length between dot and edge]</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td colspan="2">(3) "but you have a tiny bit more space than Dina's." [length between the alternative location of house and the edge]</td></tr>
<tr><td colspan="2">
</td></tr>
</table>

Note. Student A6's gestures are represented below the student's quote in numerical order.

Finally, for diagonal distances, I showed student B9 an alternative location for the second task (i.e., scaling up, top left corner) and asked if it was correct. She responded it would not be correct because, "it's a bit too long here," and created a diagonal line with a *representational* *gesture* from the corner to where I had moved the house (Figure 11).

Figure 11

Comparing Amounts – Diagonal Lengths Example

(1) "I would suggest to put it a bit like here [moved block to her location, on top of the blue dot] because like,"	(2) [she quickly moved block back to alternative location shown below]"it's a bit too long here".
	

Note. Student B9's quotes and gestures are presented in numerical order. The blue dot was created by the student with her marker to indicate where she thought the house should go.

Linguistic cues when comparing lengths included "distance," "length," "space," or equivalent terminology. Additionally, linguistic cues that indicated relative magnitude were necessary, such as "more," "less," "closer," "farther," "shorter," "longer," or similar terminology. Gestural cues included making representational gestures by either tracing the length with their fingertip, dragging the house by a certain amount, or by showing the starting point of the line with their thumb and the end point with their index finger. In cases where I provided alternate locations in which I moved the house by a certain direction and amount, linguistic cues included describing that the house moved "too much to the left" or "too right" for *horizontal lengths*, "more down" or "less up" for *vertical lengths*, or "too diagonal" for *diagonal lengths.* Overall, for all gestural or interactive cues, the lengths had to have clear starting and stopping points.

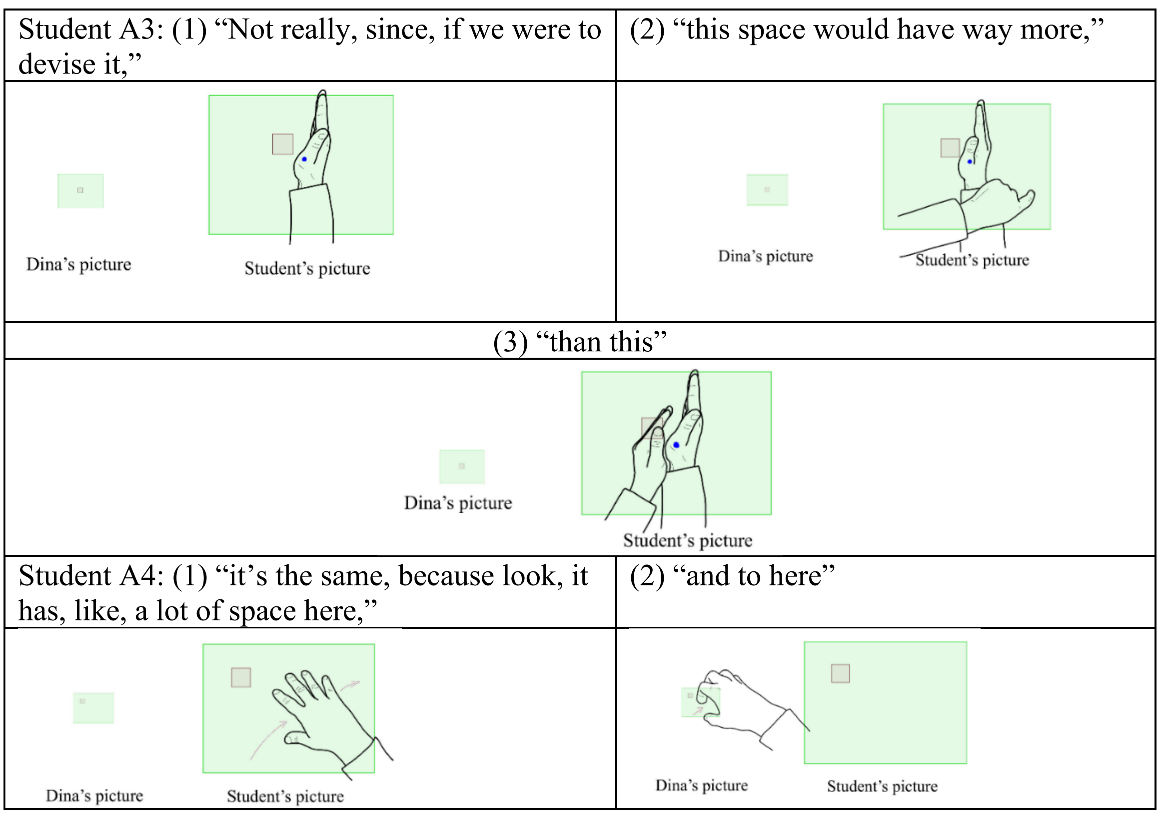
4.2.2 Comparing Spaces

The second component of *comparing amounts* was when students reasoned about the house's location by describing and comparing amounts of *space*. As previously mentioned, there were four categories of *comparing spaces*: *two vertical spaces, two horizontal spaces, two other spaces, and more than two spaces*. In contrast with comparing *lengths*, comparing *spaces* involved students reasoning about the location of Dina's house by creating and comparing 2-dimensional *spaces*. Specifically, students determined the location of their house on their student picture by identifying and determining amounts of 2-dimensional space on Dina's picture, often between the house and the edges of the picture. The student matched those amounts on the student's picture to help them locate where the house should be placed. Notably, those amounts on their student picture were larger or smaller depending on the task. In the following, I present examples of student reasoning for each sub-form of comparing spaces.

We determined when students reasoned by *two vertical spaces, two horizontal spaces,* and *two other spaces* when they determined the house's location by comparing two 2-dimensional spaces that they created. *Two vertical spaces* and *two horizontal spaces* were indicated by the directionality of the spaces they were observing. For example, in the first task (i.e., scaling up, center), I asked student A3 whether an alternative location would work for the first task. She responded by *comparing two vertical spaces*: "Not really, since, if we were to devise it [placed her right arm to *represent* a vertical division of the picture (Figure 12)], this space would have way more than this [made *pointing gesture* to the right side of the division and then to the left side of the division]." In the second task (i.e., scaling up, top left corner), when I asked whether an alternative location was correct on the student picture, she responded by reasoning about *two horizontal spaces* with a *representational gesture* as she dragged her finger side-to-side above the house on the student picture. She said, "No, since the top… maybe if this were a tiny bit more here… but no since this is way too little space, like this is smaller, so," as she dragged her finger horizontally on the space over Dina's house to indicate the horizontal space. We interpreted her horizontal gesture above the house to indicate space since there was no obvious start or stopping point, as would have been the case with a length gesture. On the other hand, *two other spaces* did not have to have clear directionality and involved a comparison of two spaces in two nonuniform areas of the picture. For example, when student A4 was asked how her picture matched Dina's picture in the second task (i.e., scaling up, top left corner), she responded by *comparing two other spaces*: "It's the same, because look, it has, like, a lot of space here, and to here," and gestured a space on the student's picture and then on Dina's picture (Figure 12). Additional examples of comparing *other* spaces included comparing a vertical space to a horizontal space and comparing the space in one corner to another corner.

Figure 12

Comparing Amounts – Two Vertical Spaces and Two Other Spaces Examples



Note. Student A3 and A4s' quotes and gestures are indicated in numerical order. For student A4, the student waved her hand in the direction of the pink arrow indicated.

We determined when students reasoned by *two vertical spaces, two horizontal spaces,* and *two other spaces* when students noted the house's location by comparing two 2-dimensional spaces that they created. In contrast to comparing two spaces, comparing *more than two spaces* occurred when students compared *more* than two spaces on the pictures to determine the location of the house on their picture. On the other hand, *two other spaces* did not have to have clear directionality and could even be a comparison of two spaces in two different areas of the picture. *More than two spaces* did not require specific directionality but required at least three spaces created on one picture to determine that the student was reasoning about more than two spaces. The spaces compared could be within one picture or between Dina's and the student's picture. For example, when student B5 was asked how she knew that her house was in the center, she responded and compared *more than two spaces*: "Because they're all equal parts for each other." When I asked what she meant by "parts," she elaborated: "Parts, so it's… because it's a rectangle, so we split into four, and each corner gets one part," she said as she showed how the pieces split (i.e., halfway along the vertical direction and halfway along the horizontal direction) to demonstrate where the four spaces are. In another example, when student A7 was asked for Task 1 how she knew her house was centered, she at one point described how she saw nine squares on the picture (i.e., appearing as a three-by-three grid). In this case, she compared multiple spaces within the picture to determine where the house is centered.

Linguistic cues for reasoning about *comparing spaces* included "space," "land," "area," or equivalent. Moreover, students need to indicate the relative magnitude of 2-dimensional space through gesture and linguistic cues. For example, linguistic cues of magnitude included "more," "less," or "equal." Gestural cues of magnitude indicated students waving hands over a space, cupping hands over a space, making circular motions over the spaces, showing where the spaces are, or placing their entire hand over the spaces as if to cover the space with their hand. Almost always, spaces were rectangular, except for *two other spaces* as gestural cues. An exception for rectangular spaces was when students reasoned about the *entire* space by circling their hand over the entire picture as previously described.

4.3.4 Unclear Amounts

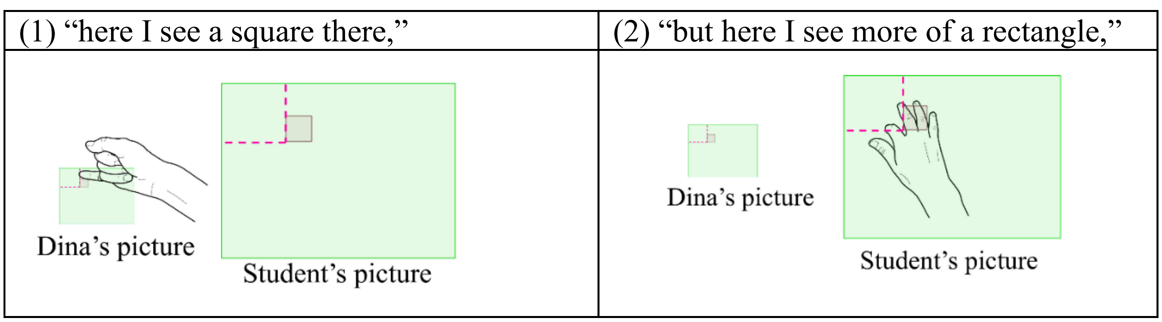
The sub-form of reasoning *unclear amounts* was identified when students reasoned by comparing amounts, yet it was unclear what kinds of amounts they were reasoning by. For instance, in some cases, it was unclear if the student was reasoning about space or length. In other instances, it was clear that they were referring to space or length, but the student did not specify the directionality of the space or length they described. For example, when student B4 was asked how her picture matched Dina's picture for the second task, she responded, "It has more space here than it does here" and gestured by pointing toward two areas of the picture. However, she pointed too far away from the pictures, so it was unclear what spaces she pointed at. Therefore, it could have been possible that she referred to *two vertical spaces* or *two other spaces*. As a result, it was determined to be *unclear amounts* as it could not be confirmed how she reasoned. As demonstrated, linguistic cues include students describing "space" but not gesturing or elaborating on what the spaces were. Gestural cues include limited or no gestures to the point it could not be determined where on the pictures they were reasoning about.

4.3 Comparing Shapes

Another form of reasoning that students used in solving the spatial scaling tasks was *comparing shapes.* When reasoning about where to place their house, students first saw a shape in Dina's picture. Then, students would create a similar shape on their student picture and use that shape to identify the location of the house on their picture. Importantly, the shape on the student’s picture was at the same location as the shape they saw in Dina’s picture. However, the size of the picture was different from Dina’s picture as it was smaller or larger depending on the scaling task. All students who compared shapes saw rectangles. For example, for the second task, I asked student A7 whether an alternative location of the house matched Dina’s picture. She responded that the alternative location was not correct and justified her response, “here I see a square there.” She then drew a square in Dina’s picture with her index finger. She continued, “but here I see more of a rectangle,” and drew a rectangle on the student picture with her index finger (Figure 13). As a result, she described that for the location of the house to be correct, she would need to see squares on both pictures, not a square and a rectangle. Importantly, these shapes she *saw* were imagined. Linguistic cues for this form of reasoning included naming shapes such as “square,” “thin rectangles,” or similar terminology. Gestural cues included *representational gestures* such as tracing the shapes they saw with their finger or with their hand.

Figure 13

Comparing Shapes Example



Note. From left to right, student A7 showed how, after I moved her house in the student picture, she now saw two different shapes as she gestured along the pink lines above. In Dina’s picture, she saw a square, while in her picture, she saw a rectangle. The house is sitting on top of the dot.

4.4 Partitioning Space to Aid Comparison

Partitioning space to aid comparison was a form of reasoning that students used to aid comparisons they made, whether the comparisons were spaces, lengths, shapes, or positions.

When partitioning, students divided the pictures into two or more different spaces. For example, the students might describe an aspect about lengths, space, or positions and then partition the picture to reinforce their reasoning further. There were two sub-forms of partitioning space: *cutting into pieces* and *creating a new map,* which I describe in the following sections.

4.4.1 Cutting into Pieces

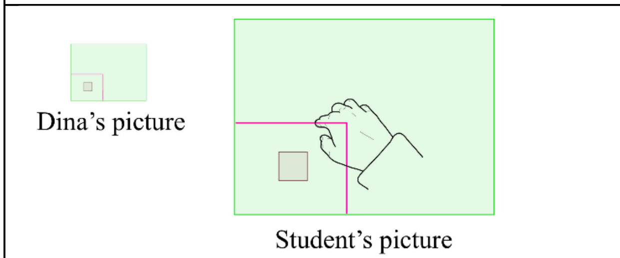
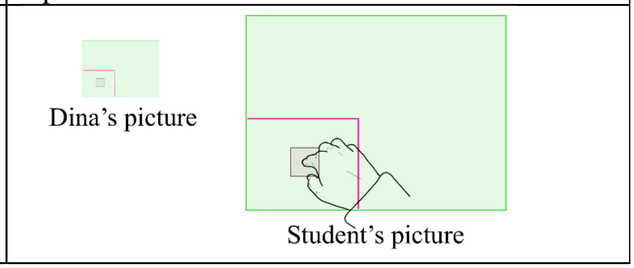
The sub-form of reasoning, *cutting into pieces,* was determined when students reasoned about the house's location with the aid of partitioning the picture into two or more pieces in various directions. As described in the section on *comparing spaces*, students like student B5 described how they "split [the picture] into four," to aid their comparison of four equal spaces. Moreover, students like student A7 partitioned the picture into as many as nine parts to justify their reasoning about the house's position. For the first task (i.e., scaling up, centered), she divided the picture into three rows and three columns to aid comparison as to what the "middle" means for her picture. Linguistic cues included words that describe partitioning, such as "split," "cut," "divide," or equivalent. Gestural cues were necessary to determine how the student partitioned the pictures and showed the partitions by tracing them with a finger or placing their hand or arm over the partition as if to cut the paper. In all instances, students partitioned linearly, where all the pieces created were rectangular.

4.4.2 Creating a New Map

The sub-form of reasoning *creating a new map* was determined when students created a partition and reasoned about a single partitioned space within the picture. In contrast to *cutting into pieces*, *creating a new map* involved students focusing on the partition they made as if it were a new picture, or map. Students' partitions were linear and of varying size. For example, some students chunked the map into a smaller version of the original map by tracing out a small rectangle within a corner of the map. For instance, in the third task (i.e., scaling up, bottom left corner) student B8 gestured a square in the bottom corner and explained, “that’s like a square,” and pointed to the center of the square and said, “And you put it in the middle of the square,” as shown in Figure 14. Here, B8 also used *comparing map positions* (“middle”).

Figure 14

Partitioning Space to Aid Comparison, Creating a New Map Example

(1) “that’s like a square,”	(2) “And you put it in the middle of the square”
	

Note. Student B8 gestured the new map on both pictures and pointed to the center of the new map.

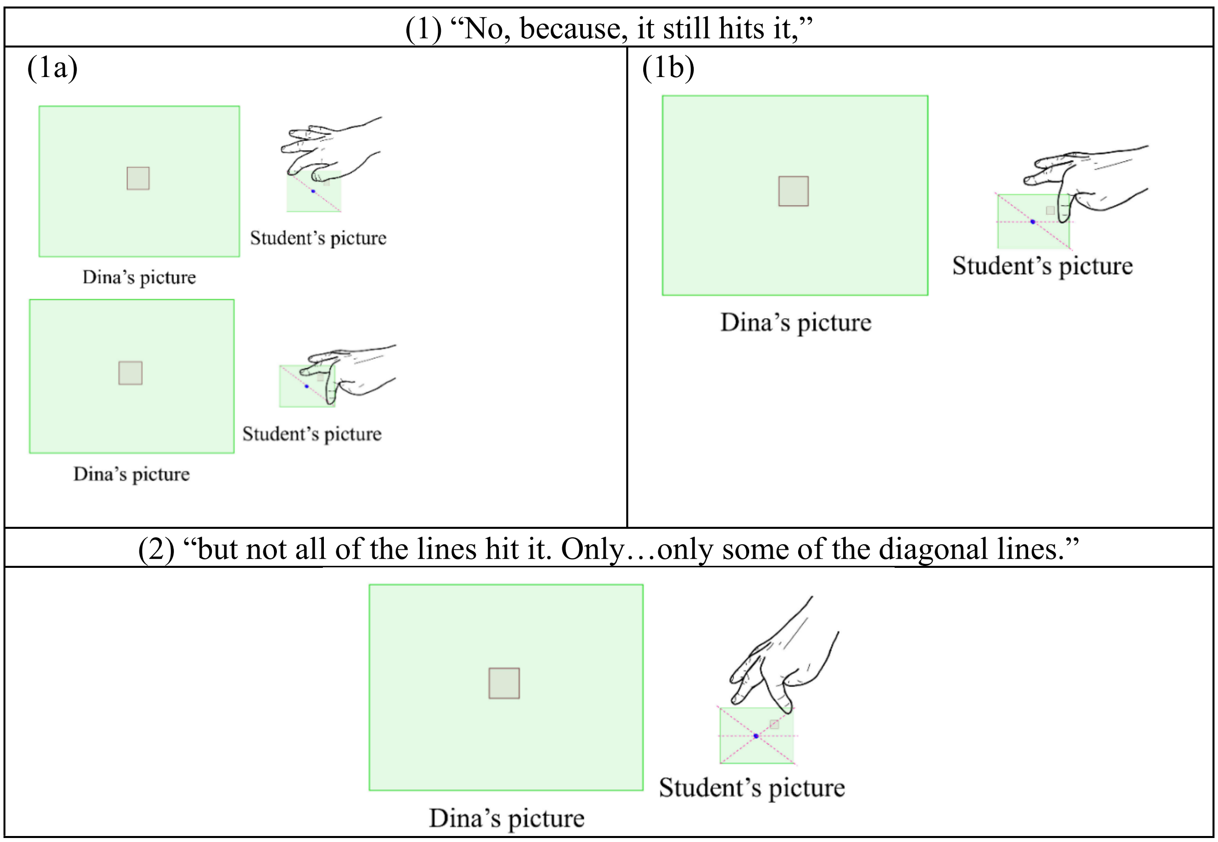
However, rather than finding the middle of the entire map, she found the middle of her new partitioned square, illustrating how she used this partition to aid comparison. Linguistic cues included describing the partition they made by naming it, such as “box,” “rectangle,” “square,” or equivalent. Unlike *comparing shapes*, linguistic cues required interaction with only *one* piece that was defined by their partition. Gestural cues included tracing the partition they made with their finger or hand. In all, the student described the new space in which they were working with and interacted with properties regarding that space they made.

4.5 Two or More Coordinated Lines to Locate a Point

Two or more coordinated lines to locate a point was determined when students reasoned about the house’s location by means of intersecting two or more coordinated lines to a single point. Specifically, students created lines as a means to join them at a specific converging *point* on the picture that was where they believed the house should go. These lines started from the edges, from the corners, or from two outer areas of the picture. For example, when student B1 was asked for the fourth task (i.e., scaling down, center) whether an alternative location was correct, she said, "No, because, it still hits it, but not all of the lines hit it. Only…only some of the diagonal lines." She gestured these lines, as demonstrated in Figure 15.

Figure 15

Two or More Coordinated Lines to Locate a Point Example



Note. In numerical order, student B1 described how the coordinated lines she made determined that the alternative location I moved the house was incorrect as only some of the coordinated lines hit the house. She repeatedly made the same gestures along the line as she spoke, where repeated gestures have been omitted for redundancy.

Linguistic cues involved students describing they were creating "lines," were visualizing the space as a "graph," were "connecting" lines, and/or showed lines that "meet at" or "hit" a "point." Representational gestural cues included students creating and showing lines with their fingers or hands.

4.6 Zooming

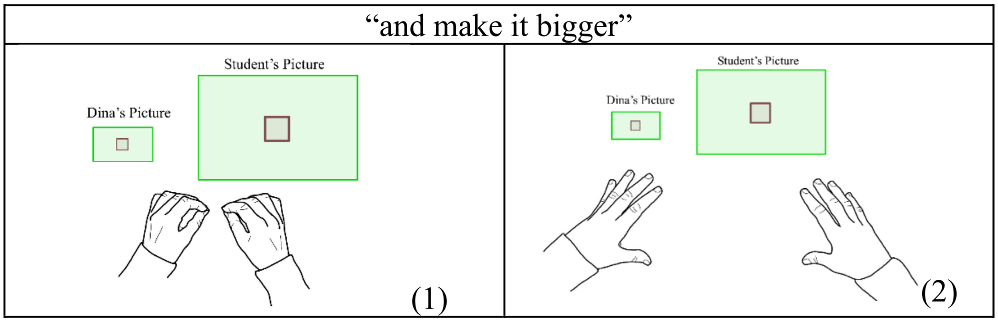
The form of reasoning *zooming* occurred when students described their understanding of the house's location by discussing the entire space or various aspects of the picture collectively getting larger or smaller, functioning similarly to the zooming strategy suggested by previous scholars (e.g., Newcombe et al., 2018). Students determined where their house should be placed by imagining what Dina's picture would look like if it were larger or smaller depending on the task. For example, when I asked student A5 how she matched the pictures for second task (i.e., scaling up, top left corner), she explained, "Well, I was first looking at this version here [held Dina's picture], and, and I was trying to imagine it very big with my photographic memory. So, I basically took a picture of it in my head and then after, I said, well what would it look like bigger?" After I asked what she did with the picture, she responded:

> Well, I acted as if I zoomed in on it a lot [made a *metaphoric gesture* stretched out her right hand], and if I zoomed in, it would be, like, a bigger space, than this one [she held the small Dina picture] because here it's so much tinier, and here [pointed to the larger student picture] there's a bigger space because it's a bigger picture.

In another interview, when I asked student 8B in the first task (i.e., scaling up, center) how she would explain to a peer how to match the picture, she described, using *pointing* and *metaphoric gestures*, "You need to take that [made a *pointing gesture* to Dina's picture] and make it bigger [made a *metaphoric gesture* as shown in Figure 16], and take that [pointed to Dina's house], and make it bigger." Overall, linguistic cues includes when students described how the picture and house were becoming "larger," "bigger," "smaller," or "zooming," or equivalent. Gestural cues included when students appeared to stretch out or pinch their hands.

Figure 16

Zooming Example



Note: From left to right, student B8 started by pinching her hands towards the small picture, and then moved her hands outwards and rested her hands over the larger picture.

4.7 I Just Saw It

The form of reasoning, *I just saw it*, was used when students reasoned about the house's location by stating that they just saw the house goes where they placed it. The students did not elaborate as to *how* they saw it, but were adamant that they knew the house should go where they placed it because they just saw it should go there. For example, when I asked student B6 for the fifth task (i.e., scaling down, top right corner) how she knew her house should be placed where she put it, she said, "you can see that it looks like the same." I asked her how she knew they were both the same, and she responded, "because, I just came up with it sometimes, because I just see where it goes and I put it there." There was little to no elaboration as well as no other forms of reasoning present when students used *I just saw it*. To elaborate, if a student said they *just saw it* and later elaborated on their reasoning within the same episode, then the form of reasoning *I just saw it* would be removed. In all, there were only seven instances of this form of reasoning, which indicates that students almost always had other forms of reasoning when describing why and how they placed Dina's house on their picture.

4.8 Other

This form of reasoning was considered when the student's reasoning did not align with any of the other forms of reasoning. However, in contrast to *unclear,* this form of reasoning was used when it was certain that there *was* reasoning, but it did not align with any other forms of reasoning. For example, in the first task, I asked student B10 how she knew her house was in the center. She responded and gestured about her student picture, "If you bend it like this [she made a *representational gesture* to fold it in half where the fold would be vertical] and this [she gestured to fold it so the fold is horizontal], and then the middle of this is here [she did a *pointing gesture* to the middle]."Although she seemed to be partitioning space to create intersecting lines, the way she did so was distinct from all other instances of these forms of reasoning. We thus categorized her response as *other* forms of reasoning in order to distinguish it from other examples. Overall, for this form of reasoning, linguistic cues and gestural cues did not align with any of the forms of reasoning.

4.9 Summary of Chapter Four

This chapter described the forms of reasoning that were determined throughout all the student interviews. Overall, there were seven forms of reasoning and one additional form of reasoning that was used to capture any other forms of reasoning. Moreover, some forms of reasoning were subdivided into further forms of reasoning. The following chapter discusses the trends of the results by episode and by the type of task and presents cases of students who coordinated multiple forms of reasoning throughout the interviews.

Chapter Five: Results Part II

The previous chapter described the forms of reasoning students engaged with throughout the spatial scaling tasks. In this chapter, I describe the trends of the forms of reasoning and how the forms of reasoning co-occurred or intersected. Specifically, I first describe the trends of student reasoning by episode. I then detail trends of reasoning by spatial scaling task. I conclude by describing the trends of student thinking by class group and focus on six student cases who had various forms of reasoning intersect throughout the tasks.

5.1 Trends of Forms of Reasoning by Episode

As mentioned in Chapter Three, there were 452 episodes (or short interview clips) in all. Among the 452 episodes, 22 episodes (or less than 5% of the total number of episodes) were determined to have *unclear/no* forms of reasoning. Since I am focused on student *reasoning,* all results exclude the 22 episodes containing unclear/no forms of reasoning so as only to capture student reasoning. Therefore, all results were drawn from the 430 episodes of student reasoning.

Among the 430 episodes, 208 episodes contained only *one* form of reasoning. The remaining 222 episodes contained two or more sub-forms of reasoning. Among these 222 episodes, 136 episodes, 64 episodes, 17 episodes, 4 episodes, and 1 episode contained two, three, four, five, and six sub-forms of reasoning, respectively. Therefore, more than half of the time, students were using more than one form of reasoning to reason about the spatial scaling task.

Table 6 shows the overall trends, by episode, of forms and sub-forms of reasoning. As mentioned in Chapter Three, each sub-form of reasoning could only be coded once per episode. For example, if a student reasoned by zooming *twice* in an episode, it would only be listed *once* when coding. Therefore, the frequency of sub-forms of reasoning was the number of episodes each sub-form of reasoning was in. As shown in Table 6 *comparing positions, using map* *positions* appeared in the most episodes: 223 episodes out of the 430 episodes. Meanwhile, the form of reasoning *comparing amounts – two horizontal spaces* appeared in 5 episodes out of 430 episodes. Notably, the 766 total forms of reasoning are greater than the 430 total episodes because the 430 total episodes does not account for intersecting reasoning, as I elaborate on later.

Table 6

Frequency of Forms of Reasoning

Form of reasoning	Sub-form of reasoning	Frequency of reasoning	Frequency and percentage of students using the form or sub-form of reasoning
Comparing Positions	*Using map positions*	223	17 [100%]
	Not case	87	14 [82%]
	Symmetry	18	11 [65%]
	Aligning with original	11	6 [35%]
Comparing Amounts	*Horizontal lengths*	80	17 [100%]
	Vertical lengths	72	17 [100%]
	Diagonal lengths	40	12 [71%]
	Two Horizontal spaces	5	1 [6%]
	Two Vertical spaces	26	7 [41%]
	Two Other spaces	15	8 [47%]
	More than two spaces	16	6 [35%]
	Unclear amounts	60	16 [94%]
Comparing Shapes	*Not applicable (N/A)*	12	3 [18%]
Partitioning Space to Aid Comparison	*Cutting into pieces*	34	9 [53%]
	Creating a new map	25	7 [41%]
Two or More Coordinated Lines to Locate a Point	*N/A*	20	9 [53%]
Zooming	*N/A*	10	5 [29%]
I just saw it	*N/A*	7	4 [24%]
Other	*N/A*	5	3 [18%]
	Total	766	N/A

Note. N/A means not applicable, where there are no sub-forms of forms of reasoning for the rows indicated. Frequency of reasoning is number of episodes that the reasoning appeared in.

Addressing the intersection of forms of reasoning by episode, the most common intersection of *two* sub-forms of reasoning within the same episode was *comparing positions – not case* and by *using map positions,* which occurred 37 times. When including episodes where these two sub-forms of reasoning intersected with additional forms of reasoning, the total episode occurrence was 67 times. Following, *comparing lengths* by *horizontal* and *vertical lengths* intersected 11 times. When including episodes containing additional intersecting sub-forms of reasoning, there were 33 episodes containing the intersection of these two forms of reasoning. For *three* sub-forms of reasoning, the most frequent intersection was *using map positions, horizontal lengths, and vertical lengths,* occurring 7 times. For *four* or more sub-forms of reasoning, there were no more than two episodes containing the same combination of particular sub-forms of reasoning.

In sum, the most frequent co-occurrence of sub-forms of reasoning, with additional intersections of reasoning, comprised less than 16% of the total number of episodes. Therefore, throughout the tasks, students reasoned differently from one another, where intersections of forms of reasoning varied by the type of task and by individual student. In the following, I discuss trends of reasoning by task.

5.2 Co-occurrences of Types of Reasoning by Task

As previously mentioned in Chapters Three and Four, each student solved six spatial scaling tasks during the interview varying by the type of scaling and location of Dina's house: (1) scaling up, center; (2) scaling up, top left corner; (3) scaling up, bottom left corner; (4) scaling down, center; (5) scaling down, top right corner; (6) scaling down, bottom right corner. In the following section, I describe the trends of reasoning in terms of the six spatial scaling tasks. The occurrences of each type of form of reasoning per task are indicated in Table 7.

Table 7

Frequency of Forms of Reasoning by Task

Form of reasoning	Sub-form of reasoning	T1	T2	T3	T4	T5	T6	SUM
Comparing positions	Using map positions	41	42	26	41	38	35	223
	Not case	10	15	15	14	20	13	87
	Symmetry	0	2	8	0	2	6	18
	Aligning with original	1	1	6	2	1	0	11
Comparing amounts	Horizontal lengths	14	14	17	10	16	9	80
	Vertical lengths	5	16	21	6	9	15	72
	Diagonal lengths	1	21	7	1	4	6	40
	2 Horizontal spaces	0	2	0	0	0	3	5
	2 Vertical spaces	7	5	4	6	4	0	26
	2 Other spaces	1	6	5	1	0	2	15
	More than 2 spaces	6	2	1	1	3	3	16
	Unclear amounts	7	14	12	9	8	10	60
Comparing shapes	Not applicable (N/A)	0	5	4	1	2	0	12
Partitioning space to aid comparison	Cutting into pieces	10	5	4	6	6	3	34
	Creating a new map	0	10	7	0	5	3	25
2 or more coordinated lines to locate a point	N/A	10	0	2	8	0	0	20
Zooming	N/A	1	2	3	1	2	1	10
I just saw it	N/A	0	0	2	0	3	2	7
Other	N/A	3	0	0	1	0	1	5
SUM		117	162	144	108	123	112	766

Note. The frequency of each form of reasoning by task (e.g., task 1 = T1, task 2 = T2) is indicated on the righthand side. For each row, based on the range of the values per row, the colours of the values shift from light orange (10^{th} percentile) to darker orange (50^{th} percentile) and darkest orange (90^{th} percentile). The shades vary by row, where the shades of colours fall on a gradient between the lowest to highest value within each row.

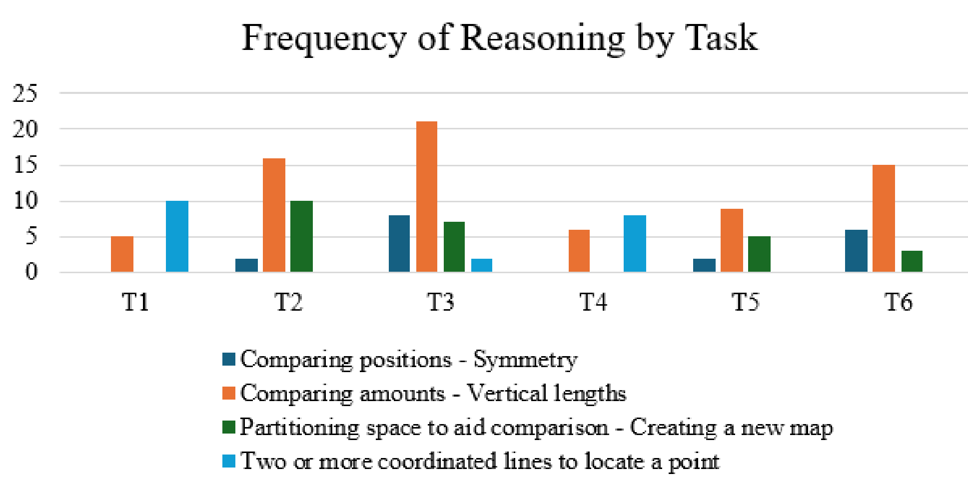
Regarding frequency by spatial task, the most used form of reasoning overall, *comparing positions – using map positions,* was also the most used form of reasoning for all the spatial tasks. However, this pattern does not hold true for all forms of reasoning. For instance, the second most used form of reasoning overall, *comparing positions – not case,* was the second most commonly used only for tasks four and five. Instead, *comparing amounts* was the second most frequent form of reasoning for other tasks, specifically *vertical lengths* for tasks three and six, *horizontal lengths* for task one, and diagonal lengths for task two. Moreover, *partitioning space to aid comparison* and *two or more coordinated lines to locate a point* were used more or the same amount as *not case* depending on the task. Therefore, my results show that frequency of form of reasoning overall is *not* indicative of frequency of form of reasoning by *task.*

In focusing on the spatial scaling tasks where the house was located at the *center* (i.e., tasks one and four), the following forms of reasoning were more present in these tasks compared to the other tasks: *comparing amounts – two vertical spaces, partitioning space to aid* and *two or more coordinated lines to locate a point.* The sub-form of reasoning, *comparison – cutting into pieces,* appeared most in task one and second most in task four (tied with task five). On the other hand, the following forms of reasoning were *least* present or *not* present in tasks at the center: *comparing positions – symmetry, comparing amounts – vertical lengths, diagonal lengths, and two horizontal spaces, partitioning space to aid comparison – creating a new map,* and *I just saw it.* For these trends generally, one possible explanation is that the spatial task itself elicited some forms of reasoning more than other forms of reasoning.

To highlight trends of reasoning by task, I focus on four forms of reasoning: (1) *symmetry,* (2) *vertical lengths,* (3) *two or more coordinated lines to locate a point,* and (4) *creating a new map* as shown in Figure 17.

Figure 17

Trends of Four Forms of Reasoning by Task



Note. The forms of reasoning are indicated in the legend above. The type of task is numbered at the bottom (T1-T6) and the frequency of the form of reasoning per task is indicated on the left.

To highlight an example of reasoning by task, *creating a new map* appeared only in tasks where the location of Dina's house was at the *corners* (i.e., tasks two, three, five and six). Therefore, *creating a new map* did not serve the students' purpose during the tasks located at the center compared to other forms of reasoning such as *two or more coordinated lines to locate a point* and additional forms of reasoning listed in Table 7 (e.g., *cutting into pieces, two vertical spaces*). One possible explanation for this trend is that the nature of these two forms of reasoning both consist of creating lines, which, in most cases, intersected perpendicularly. Moreover, the lines in both these forms of reasoning were used to help students locate the house.

In the following, I highlight how trends of forms of reasoning were present when examining the tasks in order. Reasoning by *symmetry* occurred only at the tasks with locations at the corners (i.e., tasks two, three, five, and six), with increases in occurrence between tasks two and three and between tasks five and six (Figure 16). Similarly, reasoning by *comparing amounts – vertical lengths* increased from tasks one to three and then increased again between tasks four to six. Overall, reasoning by *symmetry* was identified 18 times. In 12 of those instances, students were also found to reason by *vertical lengths*. Out of those 12 instances, one occurred in the second task, six occurred in the third task, and five occurred in the sixth task. Overall, reasoning by *symmetry* and *vertical lengths* both had similar trends in frequency throughout the task and, importantly, intersected. For example, students like B2 would describe how their house is similar to the previous location and describe it as being *lower* than the last location by showing a change in vertical length. Therefore, some students used the information from earlier tasks to inform them of the later tasks they solved. A possible reason for this trend includes that the nature of the tasks two and three and tasks five and six *are* symmetrical to each other *and* have a difference of only vertical distance. Importantly, reasoning by *comparing amounts – diagonal lengths* had a high frequency in task two (i.e., scaling up, top left corner) yet quickly dropped in the third task. One possible explanation is that other forms of reasoning such as *comparing positions – symmetry* and *comparing amounts – vertical lengths* were more useful for students as there was relations between the second and third tasks. The second possible explanation is that the *order* of tasks elicited different forms of reasoning as well as different frequencies of forms of reasoning. Therefore, there may be influence from the task itself that led to this trend of form of reasoning.

5.3 Cases of Students Who Coordinated Multiple Forms of Reasoning

In this section, I will compare frequencies and co-occurrences of forms of reasoning by sharing six student cases who most frequently coordinated forms of reasoning. To situate the cases, I first start by comparing the trends between both classes, Group A and Group B.

5.3.1 Reasoning by Group A and Group B

There were seventeen participants in this study. Seven students were from one class, Group A, and ten students were from another class, Group B. The total frequencies of the forms and types of reasoning of each student are indicated in Table 8 for their entire interview.

Table 8

Frequency of Forms of Reasoning of Student by Value and Percentage

Form of reasoning	Sub-form of reasoning	A1		A2		A3		A4		A5		A6		A7		B1		B2		B3		B4		B5		B6		B7		B8		B9		B10		Sum A	Sum B
Comparing positions	Using map positions	8	47%	7	29%	10	38%	8	33%	15	52%	24	80%	10	50%	8	35%	13	43%	20	77%	14	39%	18	58%	15	52%	16	55%	18	86%	13	68%	6	38%	82	141
	Not case	4	24%	6	25%	11	42%	7	29%	6	21%	12	40%	7	35%	2	9%	6	20%	1	4%	0	0%	5	16%	0	0%	7	24%	6	29%	7	37%	0	0%	53	34
	Symmetry	3	18%	1	4%	2	8%	1	4%	0	0%	0	0%	2	10%	1	4%	2	7%	0	0%	0	0%	1	3%	1	3%	0	0%	0	0%	3	16%	1	6%	9	9
	Aligning with original	2	12%	0	0%	1	4%	2	8%	1	3%	0	0%	1	5%	0	0%	4	13%	0	0%	0	0%	0	0%	0	0%	0	0%	0	0%	0	0%	0	0%	7	4
Comparing amounts	Horizontal lengths	10	59%	13	54%	1	4%	2	8%	10	34%	2	7%	6	30%	5	22%	4	13%	5	19%	4	11%	7	23%	1	3%	3	10%	1	5%	2	11%	4	25%	44	36
	Vertical lengths	10	59%	12	50%	2	8%	3	13%	6	21%	8	27%	2	10%	4	17%	3	10%	1	4%	2	6%	8	26%	2	7%	1	3%	1	5%	4	21%	3	19%	43	29
	Diagonal lengths	0	0%	3	13%	1	4%	6	25%	10	34%	0	0%	0	0%	2	9%	3	10%	3	12%	2	6%	2	6%	0	0%	1	3%	4	19%	3	16%	0	0%	20	20
	2 Horizontal spaces	0	0%	0	0%	5	19%	0	0%	0	0%	0	0%	0	0%	0	0%	0	0%	0	0%	0	0%	0	0%	0	0%	0	0%	0	0%	0	0%	0	0%	5	0
	2 Vertical spaces	0	0%	2	8%	15	58%	2	8%	0	0%	0	0%	0	0%	1	4%	1	3%	0	0%	2	6%	3	10%	0	0%	0	0%	0	0%	0	0%	0	0%	19	7
	2 Other spaces	0	0%	0	0%	1	4%	2	8%	0	0%	0	0%	1	5%	1	4%	0	0%	0	0%	2	6%	2	6%	0	0%	5	17%	1	5%	0	0%	0	0%	4	11
	More than 2 spaces	1	6%	0	0%	2	8%	0	0%	0	0%	0	0%	3	15%	0	0%	0	0%	1	4%	0	0%	3	10%	0	0%	0	0%	0	0%	0	0%	6	38%	6	10
	Unclear amounts	2	12%	1	4%	1	4%	2	8%	12	41%	1	3%	1	5%	6	26%	4	13%	2	8%	18	50%	1	3%	1	3%	4	14%	2	10%	2	11%	0	0%	20	40
Comparing shapes	Not applicable (N/A)	0	0%	0	0%	0	0%	0	0%	0	0%	0	0%	5	25%	1	4%	0	0%	6	23%	0	0%	0	0%	0	0%	0	0%	0	0%	0	0%	0	0%	5	7
Partitioning space to aid comparison	Cutting into pieces	0	0%	0	0%	6	23%	1	4%	0	0%	4	13%	6	30%	0	0%	1	3%	1	4%	0	0%	4	13%	0	0%	0	0%	4	19%	0	0%	7	44%	17	17
	Creating a new map	0	0%	0	0%	0	0%	0	0%	0	0%	0	0%	2	10%	1	4%	0	0%	3	12%	0	0%	0	0%	5	17%	0	0%	6	29%	1	5%	7	44%	2	23
2 or more coordinated lines to locate a point	N/A	1	6%	0	0%	0	0%	0	0%	0	0%	0	0%	1	5%	4	17%	0	0%	3	12%	0	0%	1	3%	5	17%	1	3%	3	14%	0	0%	1	6%	2	18
Zooming	N/A	0	0%	1	4%	0	0%	0	0%	5	17%	0	0%	1	5%	0	0%	0	0%	0	0%	0	0%	0	0%	1	3%	0	0%	2	10%	0	0%	0	0%	7	3
I just saw it	N/A	0	0%	0	0%	0	0%	0	0%	1	3%	1	3%	0	0%	0	0%	0	0%	0	0%	0	0%	0	0%	4	14%	0	0%	0	0%	1	5%	0	0%	2	5
Other	N/A	0	0%	0	0%	1	4%	0	0%	0	0%	0	0%	0	0%	0	0%	2	7%	0	0%	0	0%	0	0%	0	0%	0	0%	0	0%	0	0%	2	13%	1	4
Sum of sub-forms of reasoning		41	N/A	46	N/A	59	N/A	36	N/A	66	N/A	52	N/A	48	N/A	36	N/A	43	N/A	46	N/A	44	N/A	55	N/A	35	N/A	38	N/A	48	N/A	36	N/A	37	N/A	**348**	**418**
Sum of episodes by student		17	N/A	24	N/A	26	N/A	24	N/A	29	N/A	30	N/A	20	N/A	23	N/A	30	N/A	26	N/A	36	N/A	31	N/A	29	N/A	29	N/A	21	N/A	19	N/A	16	N/A	**430**	

Note. Forms of frequency are separated by students (e.g., A1 = student A1). The green shading represents the percentile weight of reasoning by student profile (e.g., light green = 10th percentile, dark green = 90th percentile). Zero (0) occurrence is left blue for visibility. The percentages represent form of reasoning/total student episodes (e.g., percentage of 8/17 for student A1 is 47%).

Overall, the following sub-forms of reasoning were used by all the students: *using map positions, horizontal lengths,* and *vertical lengths* (n = 17/17 students). The following sub-forms of reasoning were used by more than half the students, *not case, symmetry, unclear amounts,* and *diagonal lengths* (each used by n ≥ 10/17 students). The following sub-forms of reasoning were used by approximately half the total number of students: *two other spaces, cutting into pieces* and *two or more coordinated lines to locate a point* (each used by 8 ≤ n ≤ 9 students). The remaining forms of reasoning were used by less than half the students: *aligning with original, two horizontal spaces, two vertical spaces, more than two spaces, comparing shapes, creating a new map, zooming, I just saw it,* and *other* (each used by n ≤ 7 students). Importantly, one form of reasoning was used by only one student: reasoning by *two horizontal spaces*.

Table 8 also shows class-specific differences between these two class groups. For instance, Group A almost exclusively used the form of reasoning *aligning with the original*. Whereas 5/7 of the students in Group A used this form of reasoning, only 1/10 students from Group B did. Moreover, Group B almost exclusively used the form of reasonings *creating a new map* (6/10 students in Group B compared to 1/7 in Group A) and *two or more coordinated lines to locate a point* (7/10 students in Group B compared to 2/7 in Group A). These trends highlight the possibility of differences between the classes, whether by the way they were taught mathematics material, by the compositions of the class themselves, or by student-specific differences. In the remainder of this section, I focus on these student-specific differences.

In chapter four, I addressed the ways in which students engaged with *one* form of reasoning at a time. In the following section, I describe the cases of students who frequently used more than one form of reasoning at a time. I selected six students who used more than one sub-form of reasoning per episode for over 70% of the episodes (Table 9): students A1, A3, A5, A7, B8, and B10. In the following sections of this chapter, I describe these students based on the ways in which they presented more than one form of reasoning throughout the tasks. I also highlight the mediational means they drew from when solving the tasks (Wertsch, 1994).

Table 9

Selection of Student Profile Cases

Student	Episodes with more than one form of reasoning	Total number of episodes (excluding unclear/no code)	Percentage of episodes with more than one form of reasoning (rounded up)	Selected (Yes/No)
A1	14	17	82%	Yes
A2	13	24	54%	No
A3	20	26	77%	Yes
A4	9	24	38%	No
A5	22	29	76%	Yes
A6	19	30	63%	No
A7	14	20	70%	Yes
B1	11	23	48%	No
B2	11	30	36%	No
B3	14	26	54%	No
B4	8	20	20%	No
B5	17	31	55%	No
B6	4	29	14%	No
B7	7	29	24%	No
B8	17	21	81%	Yes
B9	11	19	58%	No
B10	13	16	81%	Yes

Note. Students selected were highlighted in green. Sub-forms and forms of reasoning were also considered as more than one form of reasoning.

5.3.2 The Lengths and Positional Reasoner

Student A1, or the lengths or positional reasoner, reasoned by *comparing lengths* and *comparing positions* throughout almost all the scaling tasks. This student mainly reasoned by comparing different lengths and/or different aspects of positions to determine and justify the location of their house so that it matched Dina's picture. In sum, over 75% of the total episodes consisted of *only* comparing lengths and/or *only* comparing positions. Out of the 17 episodes, one contained only *comparing positions,* three episodes contained only *comparing lengths,* and nine episodes contained only *comparing positions* and *comparing lengths.* The remaining episodes in which she reasoned by lengths and positions contained intersections with other forms of reasoning. In the following, I present examples of how she reasoned about comparing lengths and positions, first individually and then together.

In the instance where she *compared positions*, she reasoned about the position of her house by comparing positions as described in Chapter Four, Section 4.1.1. When she was only *comparing lengths*, she used horizontal and vertical lengths. For example, in the second task (i.e., scaling up, top left corner), when I asked her how her picture was the same as Dina's, she said, "Well, because there was a teensy bit of space over here," and gestured the distance from the edge of the picture to Dina's house on Dina's picture. She then said, while gesturing the same distance on the student picture, "So after I left a little amount of space, but except I put it a bit more, because since the landscape is bigger, I needed to leave a bit more space than what would actually be over here." She then compared the vertical distance from the edge similarly.

While she compared different distances at the same time, she also compared distances and positions at the same time. When comparing positions and lengths, she tended to reason about the position first and then describe the lengths she saw. For instance, when asked about how her picture matched Dina's picture in the fifth task (i.e., scaling down, top right corner), she compared both positional aspects and distances between Dina's picture and her student picture:

> Well, because, Dina's house is at the right, and here it's also at the right [reasoning by *using map positions*]. Then also there must be a little bit of space over here [she places her fingers between the edge of Dina's house and the edge of the picture and then does the same on her student picture to indicate *comparing amounts*]. Here, it's about three fingers so here, since it's a bit smaller, it would be about a half of one finger [she then

placed one finger between the edge of her student house and picture] [reasoning about *horizontal distance*].

She then repeated this process but using vertical distances between the house and the closest edge of the picture. Uniquely, this student often referred to mathematical graphs when using distances. For example, when asked whether an alternative location was correct for the second task, she said: "It would be too much space, because if you used the graph again [she traced the y-axis and x-axis of a graph on the left and bottom edge of Dina's picture and the student's picture], after you would have your dots with your lines [she made dots as if they were markers on the x-axis and y-axis]. But after you would see that over here it's a vertical line [reasoned about vertical distances]." Despite her repeated references to graphs, she only reasoned once by *coordinating two or more lines to locate a point*. Instead, it appeared her graph helped her reason about horizonal and vertical distances from the edges rather than using the lines of the graph to locate points. She also used mathematical language when solving the tasks. For example, she used the concept of "proportions" when solving the fifth (i.e., scaling down, top right) and sixth tasks (i.e., scaling down, bottom right corner). For example, in the fifth task, she described how, since Dina's picture is larger, "you need to make the proportions that are over here [Dina's picture] smaller to adjust to this landscape [her student picture]." When asked to describe the proportions she created, she described them as horizontal and vertical distances of the house from the edge. In the following, I describe student A3 and how she reasoned by comparing horizontal and vertical spaces as well as positions when determining the location of Dina's house on her student picture.

5.3.3 The Comparing Two Spaces, Comparing Positions, and Partitioning Reasoner

Throughout the tasks, student A3 reasoned predominantly by *comparing two spaces*, *partitioning by cutting into pieces*, and *comparing positions*. Specifically, out of the 26 episodes in her interview, in 15 of those instances, she reasoned by comparing *two vertical spaces* and was the student with the highest percentage of occurrence of this reasoning. For six of of 15 instances, she supported her reasoning by *partitioning* her picture into two pieces. Moreover, for five of the 15 instances, she also reasoned by *comparing positions.* For one of these 15 instances, she reasoned by both *comparing positions* and *partitioning* her picture.[1] Often, she partitioned the paper into two vertical pieces with her hand or with her arm and compared the amount of vertical spaces on either side of the partition. For example, in the second task (i.e., scaling up, top left corner), when I asked her how her picture matched Dina's, she created a vertical partition on each picture and compared each side of the partition for each picture: "It's also the same, since there's a tiny stretch of land here [she places her hand on top of the partition on Dina's picture to show the vertical space], and a giant one here [she places her arm on the same partition line on the student picture and then points to the larger space on the right-hand side of the partition], like in Dina's picture." In all, she often partitioned the space vertically and then compared the vertical spaces on either side.

When reasoning by comparing two vertical spaces, she also supported her reasoning by comparing positions. For example, she reasoned with *not case* and *two vertical spaces* in the fifth task (i.e., scaling down, top right corner). When I asked her how she would explain to me where the house should be placed if I were one of her classmates, she responded by comparing the space between the house and the right edge of the map on both her picture and Dina's picture:

> I think it should go here [pointed to her house], because as I can see there's only a little bit of space [she showed *vertical space* between Dina's house on Dina's picture and the

[1] For the five episodes containing comparing positions and comparing two spaces, there were three episodes that also involved comparing other amounts. I focus only on comparing positions to highlight main co-occurrences.

right edge of the map] and it's wider than here [showed a *not case* by moving her house on her picture closer to the right edge], but it's not, but it's not as wide as here [now moved her house to another *not case* much farther from the right edge]. So, I think this would be the perfect, 'cause this looks like a big ribbon [showed the space on Dina's house from before] and this too [showed the same space on student picture]. Well, a bit bigger than a ribbon I don't know how to describe it.

Unique from the other students, she related her understanding of space to familiar objects, the theatre, and the story that the task was based on. As shown in the example above, she often referred to the vertical spaces between the house and the edge of the picture for the corner locations as "ribbons," "strings," or "cut piece of paper." For example, she would often describe slim vertical pieces as "ribbons" or "strings" while the "cut piece of paper" would be used to refer to larger vertical pieces, where she leveraged her understanding of sizes of everyday objects to justify her reasoning on where the house should be placed. Additionally, she drew from her understanding of the theatre. For example, in the first task, when she said her house was in the middle, and I asked what she meant by middle, she responded, "Center… center, if in theatre, center stage, the center of all the green," as she made a circular gesture around her house to show the green space around it. Since the task was centered on helping Dina who is a farmer, she also referred to providing space for the animals. For example, in the second task (i.e., scaling up, top left corner), when I asked her how to make sure the picture is the same as Dina's picture, she placed her arm to show a vertical partition and said, "You might want to know how many spaces there might be," while she gestured to the spaces on either side of the partition. When I asked her for clarification as to what spaces we were looking for, she responded, "space for animals" and showed the vertical spaces that it would be for the animals. Moreover, in the third task, she commented that the location of Dina's house as "impractical" as there would not be enough space for animals. Through the storytelling, she not only reasoned with her understanding of the story, but she also established her preferences of the locations based on the practicality of the use of the land. Overall, student A3 reasoned by comparing two vertical spaces and used partitioning spaces and comparing positions to support her reasoning about these imagined spaces.

5.3.4 The Unique Distances and Zooming Reasoner

Student A5, while also similarly reasoning by comparing lengths and positions like student A1, was unique in that she reasoned by also often comparing unique distances which were determined to be unclear amounts. Overall, she reasoned by unclear amounts in 12 out of 29 episodes. Importantly, these episodes were not categorized as another form of reasoning as her reasoning was clearly about *amounts* and *distances* based on what she said aloud. However, it remained unclear exactly *what* lines she was imagining as her gestures were minimal or she did not gesture. Moreover, to categorize her reasoning as *other* forms of reasoning would fail to capture that she was reasoning by amounts. Therefore, the form of reasoning *unclear amounts* was most suitable for her reasoning as it was unclear how many distances she was creating, where the distances started and stopped, and what exactly the indirect distances looked like. Finally, it was not suitable to create new forms of reasoning in the coding scheme due to these areas of uncertainty.

Specifically, A5 reasoned by comparing positions and either *indirect* distances or *multiple* distances. Reasoning by *indirect* distances involved distances that did not consist of only a straight line. The student described how for the second task (i.e., scaling up, top left corner) when she was asked how her picture matched Dina's, she responded, referring to the house on the student picture, "First, because, it's off-centered by a lot- also, it can't go straight-away to the corner diagonally, it would have to go in more of a squiggly line." Moreover, in terms of *multiple* distances, she reasoned about equidistance from all the edges of the pictures when matching the locations when the house was centered. For example, in the first task, when the student was asked how her picture matched Dina's, she reasoned about equidistance from all sides, "the perimeter, has the same distance, so this has the same distance from the perimeter." Moreover, when I asked her how she would explain where the house would go in this task, she responded, "her house, should be centered so that the farm around it is all equal and in the middle so that it's the same distance to go to everything." Here, she used the map position of "center" along with equidistance to determine the house's location. Although her language specified distance, her gestures did not show where those distances were.

Unique from other students as well is that student A5 reasoned by zooming and reasoned by zooming more than any of the other students. Throughout all the tasks, the student reasoned by zooming five times. Out of those five times, four times she also reasoned by comparing lengths. While I will highlight this common co-occurrence, in one of the four instances she reasoned by comparing positions – using map positions, and in the remaining instance of those five she reasoned by unclear amounts. In highlighting zooming and comparing lengths, for example, for the fifth task (i.e., scaling down, top right corner), I asked her what was going on in her head as she placed the house. She said, "Well, what happened in my head, was that, well, ok, that this is a much bigger distance [she gestured with a claw like gesture the horizontal distance from the right edge to the house on Dina's picture], so if you just slowly shrink it [she brings her hands together to show shrinking], it would be smaller of everything. So smaller block, smaller paper, and smaller distance."

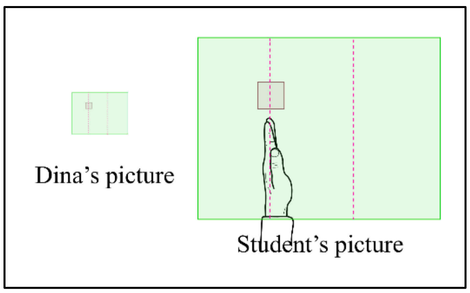
Regarding mediational means, this student drew from her understanding of mathematical concepts of measurement, such as centimetres and inches and perimeter. She also drew upon photography to describe zooming. For example, she described how she would take a photo of something and zoom in or out depending on the task.

5.3.5 The Comparing Shapes and Partitioning Spaces Reasoner

Student A7 reasoned in a variety of ways throughout her tasks. Overall, the student reasoned by comparing positions, comparing amounts, comparing shapes, creating partitions, creating two or more coordinated lines to locate a point, and zooming. While she reasoned in a variety of ways that have been previously described, the way she reasoned about two forms of reasoning presented itself as unique from all the other students. Notably, student A7 often reasoned by comparing multiple rectangular spaces and shapes on one picture, partitioning the picture in ways that were unique from the other students. For example, when the student reasoned about where the house should be placed, she often described a position she saw (i.e., *comparing positions – using map positions*) and reasoned that the position was correct by partitioning her picture by either cutting it into pieces or creating a new map. When cutting the picture into pieces, she often created a three-by-three grid (i.e., three rows and three columns, totalling 9 spaces) on her student picture and determine which space the house should be placed. For example, in the second task (i.e., scaling up, top left corner), she described how her house was in the map position of "left side corner." She then showed how her house was in the top left space of her three-by-three grid, by quickly gesturing vertical lines (i.e., partitions) to signal the partition. When I asked her what she meant by lines, showed how she made two vertical partitions as if to separate the paper into thirds vertically, and two horizontal partitions, partitioning the line into thirds horizontally. These lines resulted in a grid consisting of three rows and three columns. Student A7 then focused on the vertical partition that was closer to the left side of the picture and described how Dina's house falls "on the side of the line, or in the line and like, here [placed her hand as shown in Figure 18], it's kind of in or out of the line, depends, and, that's how I, I see it." Therefore, partitions did not only serve to compare spaces but also served to help her locate where the house should be placed.

Figure 18

Partitioning Space to Aid Comparison - Cutting Into Pieces Example



Note. Focusing only on the vertical partitions, student A7 split the picture into thirds.

When reasoning, she also described shapes and defined them by partitioning the picture into pieces. Therefore, student A7 reasoned by intersecting *comparing shapes* and *partitioning space to aid comparison.* Student A7 was one of only 3 students who compared shapes. Throughout the activity, she often described how she saw "rectangles" or "squares" based on the partitions she made, as previously described in Chapter Four in Section 4.3. When partitioning the space by either cutting it into pieces or creating a new map, she described these spaces as squares or rectangles and sometimes use Dina's picture as a mediational means to help demonstrate the size of the partitioned space. For example, in task three (i.e., scaling up, bottom left corner), she described how she saw rectangles. When I asked her how many she saw, she used Dina's picture and referred to the previous picture: "Earlier, it was, I saw squares, because it would be equal amount here, here, here, here [placing Dina's picture at the top left corner and moving it from left to right and then going downwards once she reached the top right corner]."

Unique from the other students, at the start of the activity, she described how Dina's picture was “plastified” as it was laminated for reusability. She later used the plastified part as a mediational means. When discussing her ideas on partitioning, unprompted, she described why she partitioned the paper in the second task. She said, “When there’s a book like that book [pointed to the books in a bookshelf] sometimes there’s like a little piece that’s plastified [she made vertical gestures up and down with her hand] and that makes like one section, so I think of it [she appeared to cut her student map into multiple vertical partition by chopping it with her hand] like a book [she made horizontal gestures to indicate partitioning] that would be plastified just a little bit.” Here, Dina’s picture being laminated prompted her to consider the nearby bookshelf. Since a bookshelf has books sorted vertically next to each other, she described how she used this to help her vertically partition the pictures to determine the location of Dina’s house.

Overall, while this student also reasoned by comparing amounts and positions, she often reasoned by comparing shapes and partitioning to aid comparison. Moreover, she leveraged her reasoning by using familiar objects around her and the aspects about the objects in the activity (i.e., Dina’s picture being laminated). Finally, she used various gestures while talking and when not talking when reasoning about the spatial tasks. For example, when she described the spaces she saw on her picture, she counted the number of spaces. Specifically, she used her hands to show the spaces as she was counting them.

5.3.6 The Partitioning Space and Using Map Positions Reasoner

Similarly to student A7, student B8 also used a variety of reasoning and reasoned by comparing positions, amounts, partitioning space to aid comparison, coordinating 2 or more lines to locate a point, and zooming. In terms of intersecting forms of reasoning, B8 often reasoned by *comparing positions* and *partitioning space to aid comparison*. Including coordination of sub-forms of reasoning, B8 appeared in comparing positions in 90% of episodes (19 episodes), partitioning space in 38% of episodes (8 episodes), and both forms of reasoning in 33% of episodes (7 episodes). Moreover, she used these forms of reasoning for five of the six spatial scaling tasks and used them the most for the locations that were in the corner. For example, in the fifth task, when asked how her picture matched Dina's, she responded, "They're both not in the middle" and continued by describing places that were *not cases* by first pointing to a location in Dina's picture and then pointing to the matching location in the student picture, "If it was here, that would be here, and then there would be there, and that would be that, and that would be that." She then aided her comparison by partitioning the space into four quadrants and then created a map by tracing the quadrant in the top right, and describing how the house is in the "middle" of that quadrant. She often used this method of coordinating *partitioning space to create a new map* with *map positions* to identify the location of Dina's house on her picture. Moreover, she used her understanding of partitioning space, particularly into fourths, to determine where to create her new partitioned map. For example, in task three (i.e., scaling up, bottom left corner), when I asked her why she created her map to the size she made rather than a larger size, she said, "cause that's, like, one fourth of the whole page."

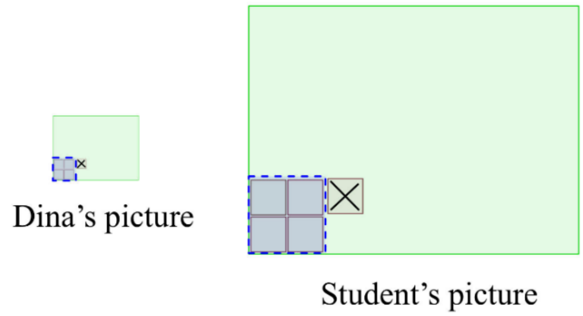
As these examples show, student B8 drew on mediational means that included her understanding of locations, such as using the language of "middle" to confirm that her house was placed in the center of her partitioned rectangle. As previously described, she drew from her knowledge on fractions, such as fourths, when partitioning the space. Compared to other students, student B8 did not explicitly refer to outside knowledge from other classes or materials when solving the task. However, she did refer to the story of the task, describing the green picture as "grass" and describing the block as a "house."

5.3.7 The Creating a New Map and Cutting the Map into Pieces Reasoner

Student B10 had 16 episodes that contained forms of reasoning. Among the 16 episodes, six of them contained the intersection of three forms of reasoning, all appearing in tasks where the house was located at the corner of the picture: *creating a new map, cutting into pieces,* and *comparing more than two spaces.* Moreover, the way in which B10 combined these forms of reasoning was unique from the other students. First, she focused in on a corner of the space, being a new map, she created. Then, in that new map she created, she partitioned her map into four pieces (Figure 19). She ensured these pieces were adequately sized by comparing the spaces of the pieces so that they were the same size as the house. In other words, she used the house as a measuring unit to determine where the house should be placed. Then, she placed her house just next to one of the spaces to determine the location of Dina's house. For example, in the third task (i.e., scaling up, bottom left), when I asked her how she would explain it to me if I was one of her classmates, she showed how she made spaces as shown in Figure 19: "I think you put it here because this is like, using this house I make four boxes [she pointed to Dina's house on Dina's picture and then pointed where she placed the houses in the four spaces as seen in Figure 19]." She proceeded to describe how she "just put it on the top [she pointed to the house on the top right box]." She then described how she did the same thing for her student picture. Specifically, she took her house and placed it on top of the imaginary four spaces as indicated in Figure 19 and then placed her house as to what appeared to be just next to the edge of the top right space.

Figure 19

Intersection of Reasoning Example



Note. The student first focused on the corner of Dina’s picture (indicated by the blue dotted line). She then partitioned that blue space with the house by placing Dina’s house in the four spaces as shown by the brown lines inside the blue space. She compared more than two spaces by using Dina’s house as a measuring unit. She then placed the house just outside of this space on the top right of the space.

However, for the tasks located at the center (tasks one and four), B10 reasoned mostly by comparing vertical or horizontal lengths and using map positions. When reasoning about lengths for the centered location, she sometimes used the length of the house as a unit of measurement. For example, in the fourth task, she counted how many blocks fit along the horizontal length and then placed the house halfway along that total length.

As these examples illustrate, student B10 used the house as a mediational means by using it as a measurement tool throughout all the tasks. Depending on the task, she used the house as a measurement tool to measure length or area. In the first and fourth tasks, where the houses were located at the center, she often used linear measurement. However, in the second, third, fifth, and sixth tasks, she used the house to measure the area of the space to determine the location of Dina’s house. Moreover, she almost always referred to Dina’s house as a “box.” Her language use of “box” may have helped her associate the object with something she was familiar with and may have facilitated reasoning about the box as a measurement tool. In other words, reasoning about the house as a “box” rather than a “house” may have been a mediational mean to solve the task.

5.4 Summary of Chapter Five

In this chapter, I described the forms of reasoning by episode and task of the activity and presented student cases of intersecting forms of reasoning. In all, over half of the episodes contained more than one form of reasoning, where students were using more than one form of reasoning at a time. Among the students, six of them used more than one form of reasoning throughout the tasks for over 70% of the episodes. In the following chapter, I conclude this thesis by discussing the contributions of the study, describing implications for teaching practice, and discussing the limitations and future directions of this study.

Chapter Six: Discussion and Conclusion

This study explored young girls' embodied reasoning when solving six spatial scaling tasks, where I addressed two research questions:

(RQ1) How do young girls reason when solving spatial scaling tasks in geometry?

(RQ2) In what ways do young girls use their bodies when reasoning about spatial tasks?

I addressed both research questions in Chapter Four, where I described the forms and sub-forms of embodied reasoning that appeared when students solved spatial scaling tasks. We determined the form of reasoning being used when the students demonstrated that they were describing *how* or *why* the house was placed at the location through linguistic and gestural evidence. Overall, the results indicated that students collectively used seven forms of reasoning, where three forms of reasoning contained sub-forms of reasoning. In Chapter Five, I further addressed both research questions by examining trends of these forms and sub-forms of reasoning. In all, the most used sub-form of reasoning was *comparing positions – using map positions*, despite no indications of positional vocabulary from the interviewer during the activity. Over half of the episodes in the interviews contained more than one sub-form of reasoning, indicating that this sample of students often used more than one form of reasoning simultaneously when solving spatial tasks. Moreover, certain sub-forms of reasoning, as well as gestures, were almost exclusively used by certain class groups and specific students, suggesting class-specific and individually situated differences. Focusing on the cases of six students who used more than one form of reasoning for at least 70% of their episodes, I detailed the ways in which the forms of reasoning intersected. Overall, the combinations of forms of reasoning and the ways in which they combined differed greatly from student to student, indicating that reasoning occurs beyond strategies described in the previously established literature (e.g., Frick

& Newcombe, 2012). In answering the second research question, students used their bodies by gesturing and interacting with objects in their general surroundings and within the activity. In highlighting the individually situated student cases, students used a variety of different mediational means (Wertsch, 1994) when reasoning about spatial tasks, namely interacting with objects in their surroundings and using specific gestures to determine where the house should go in each task. As mentioned in Chapters Four and Five, I addressed the ways students used their bodies by identifying gestures for different forms of reasoning as well as addressing how students used their bodies to interact with objects in their surroundings.

In this chapter, I first describe the contributions of the study. Specifically, I explain how this study builds on spatial scaling research and spatial reasoning research. I highlight how I contribute to the conversation of mediational means, embodiment, and gender regarding spatial scaling and spatial reasoning research. I then describe the implications for teacher practice. I conclude by describing the limitations of this study and identifying future directions of spatial research from this study.

6.1 Contributions of the Study

6.1.1 Expanding Notions of Spatial Scaling as Reasoning

In the following section, I describe how this study contributes to spatial scaling research in terms of students' approaches to solving spatial scaling tasks. Specifically, I describe how this study extends research on *how* students solve spatial scaling tasks.

Past research studies have proposed that there are, generally, three ways that students solve spatial scaling tasks: using absolute distances, using relative distances, and zooming (e.g., Frick & Newcombe, 2012; Möhring et al., 2014; Vasilyeva & Huttenlocher, 2004). Overall, the results of the students' responses aligned with Frick & Newcombes' (2012) spatial scaling by *relative* distances, where all students understood that the distances must be changed in accordance with the size of the image. While this remained consistent, this study provides new forms of reasoning students used when solving spatial scaling tasks beyond strategies involving distances. Specifically, only three sub-forms of reasoning were related to distances, whereas the remaining forms of reasoning identified extend beyond what has been previously established. Moreover, my results differ from past research in what forms of reasoning were prevalent. For example, one of the common strategies for solving spatial scaling tasks that was suggested by previous works was *zooming* (Möhring et al., 2014; Vasilyeva & Huttenlocher, 2004). While this study did establish zooming as a form of reasoning for solving the tasks, it occurred in only 10 episodes, which is 2.3% of the total episodes containing forms of reasoning. In other words, zooming was one of the *least* used forms of reasoning. Considering this, future scaling research should consider other possible forms of reasoning as possible to solving spatial scaling tasks. Importantly, while their reasoning aloud and gestures indicated an understanding of different distances being required, the actual locations of where they placed the house did not always align with their reasoning spoken aloud. For example, in the second task (i.e., scaling up, top left corner), student 7A placed the house very close to the corner even though her reasoning was very complex and aligned with the task. Furthermore, it is essential to consider that student response alone is insufficient to gather student *reasoning.* Future scholars interested in student spatial reasoning can consider a qualitative or mixed-methods research approach, specifically including interview-based methods which target *how* the student reasoned about spatial tasks.

Previous research by Gilligan et al. (2019) indicated that there is a potential ceiling in the performance of spatial scaling tasks at age 10, where students at this age perform similarly to adults. Considering the participants of this study were in Grade 3, which is approximately 9 years old, their understanding of the task was consistent with Gilligan et al.'s (2019) findings. Overall, students understood that the distances would change according to the picture.

Finally, previous literature has yet to suggest that students could be using multiple forms of reasoning in solving spatial scaling tasks. This study identified that over half of the time; students used at least two sub-forms of reasoning when solving spatial scaling tasks. While previous literature has suggested the use of a single strategy at a time or no more than two strategies being combined (i.e., zooming and relative distances) (Vasilyeva & Huttenlocher, 2004), this study identified that students can use *more* than two forms of reasoning at a time to solve spatial scaling tasks. Therefore, future studies should consider multiple forms of reasoning are possible when solving spatial scaling tasks.

6.1.2 Mediational Means in Solving Spatial Scaling Tasks

This study also investigated the mediational means that were used by students when they solved spatial tasks (Wertsch, 1994). As mentioned in Chapter Two, mediational means consist of the objects, language, and gestures that a student can use when solving a spatial scaling task. Previous spatial scaling studies using interviews had yet to investigate how mediational means play a role in solving spatial scaling tasks. Overall, students used mediational means such as conceptual material from their classes, gestures, and materials within the task and interview room to support their reasoning. Regarding class material, students drew from their experiences and understanding of concepts they learned in mathematics, theatre, and art class. For example, students brought in their understanding of concepts and language from mathematics, Cartesian planes, and area to solve the spatial scaling tasks. Students also used language, specifically positional terminology, such as *middle* and *top left corner*, despite no indication of positional language being used by the interviewer or the spatial activity. They also used language to describe and compare quantities, whether that be 1-dimensional or 2-dimensional quantities. Regarding mediational means that were material items, students used materials in the room and in the activity to further support their reasoning. However, when physical materials were being used, students often used their conceptual understanding of measurement when applying it to the tasks. For instance, as mentioned in Chapter Five, student B10 used the house as a measurement tool to measure the horizontal distance of the picture. Finally, students used various gestures to facilitate solving the spatial task, as described in Chapter Four and Five. In all, this study contributes to new findings in how students use mediational means, such as choice of language, gestures, and interactions with the environment to solve spatial scaling tasks.

While most papers focused on accuracy of participant's spatial scaling, Plumert et al. (2019) investigated whether scaling up was more challenging than scaling down in spatial scaling tasks in adults and children. In the end, Plumert et al. (2019) determined that neither type of task is more difficult. Rather, they determined that the task itself and the materials made the scaling tasks more challenging. Specifically, they noted how the setup of the task itself created challenges for participants as they had trouble seeing the entire picture when solving the scaling up task. This was similarly identified in this study. As mentioned in Chapter Four, student A4 reasoned by *comparing positions – aligning with original* and explicitly explained how it helped her see both pictures more easily. Moreover, almost half of the students, specifically students A2, A4, A7, B2, B4, B7, B8, and B9, stood up at one point during the activity when reasoning about the tasks, where I propose they did so to change their perspective when solving the task, relating to Proffitt's (2013) work on perception. With consideration of this, future research can consider how the use of mediational means can change depending on the challenges that might arise in the spatial tasks.

As mentioned in Chapter Two, I approach mediational means as a form of embodiment, where gestures, language, and objects are used in a way that is body-based. Overall, students used a variety of mediational means to facilitate their solving of the spatial scaling task. In the following section, I describe how this study built on embodied spatial reasoning research.

6.1.3 Expanding on Embodied Spatial Reasoning Research

This study investigated how young girls *embody* spatial scaling tasks. As previously mentioned, this study is framed by embodied cognition, which considers both what an individual says and what gestures are occurring in solving spatial tasks. While research on embodiment of spatial reasoning and mathematics (e.g., Alibali & Nathan, 2011; Nathan et al., 2021; Ng & Sinclair, 2013) has started to be investigated by a few scholars, studies have yet to explore gestures about spatial scaling tasks. Spatial scaling research has only examined participants' accuracy in solving spatial scaling tasks (e.g., Frick & Newcombe, 2012; Möhring et al., 2018; Vasilyeva & Huttenlocher, 2004). To my knowledge, this is the first study to investigate the *embodied reasoning* of spatial scaling in children.

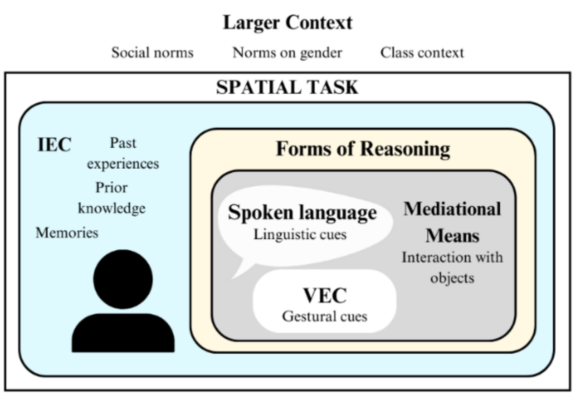
In all, various gestures played a role during students' reasoning of spatial tasks, such as pointing, making a claw-like gesture, waving hands, and cutting up paper with one's hand. These gestures have the potential to be categorized within McNeill's (1992) categories of gestures. They also align with the gestures described by Alibali and Nathan (2011) regarding mathematical concepts. This study adds to this work by presenting specific ways that students use pointing, representational, and metaphoric gestures.

This study adds to spatial research discussing the role of language and gesture. Newcome and Stieff (2011) addressed *myths* related to discussions in research about spatial *thinking*. One myth was "spatial language determines spatial thought," where they argue that language is not fully reliable for describing visual representations (Newcombe & Stieff, 2011, p. 964). My results bolster their argument that language is insufficient in capturing what an individual is thinking and, to add to this argument, reasoning as language may not sufficiently capture spatial aspects of the task (Newcombe & Stieff, 2011). For example, when students described “space,” the spatial meaning of that word could be either describing 1-dimensional distance or 2-dimensional space, where a gesture was essential in identifying the amount being described. On the other hand, this study identified that gestures did not always match the reasoning that students were describing aloud (Goldin-Meadow, 2011; Goldin-Meadow & Singer, 2003). For example, reasoning by *two or more coordinated lines to locate a point* and *partitioning space to aid comparison – cutting into pieces* had similar gestures of tracing lines despite the linguistic cues being different. This study addressed the role of visible embodiment, specifically student’s spoken language *and* gesture as they solved solving spatial scaling tasks. Therefore, future studies can consider the role of gestures *and* language in student reasoning.

In revising the embodied cognition framework in Chapter Two (Figure 1), I introduce the reasoning of spatial tasks framework, which is presented in Figure 20 below, about how students solve spatial tasks. Previously in Chapter Two, I considered the alignment, or *matching,* of language and gesture (Goldin-Meadow, 2011). However, as mentioned in the previous paragraph, the results of this study determined the combined nature of language and gesture and all reasoning being aligned. Considering how this study described embodied reasoning as combining language and gesture, which was mentioned in the previous paragraph, I omit the alignment aspect of gestures and speech aspect from Figure 1. In all, the revised embodied forms of reasoning consider *forms of reasoning* as the combination of *spoken language, visible embodied cognitions (VEC),* and *mediational means.* The way the forms of reasoning present themselves are specified in Figure 20. Importantly, when a student solves a spatial task, *IEC* (invisible embodied cognition) is present in the form of memories and past experiences. Moreover, a student's forms of reasoning are visible to teachers in the form of *spoken language*, *VEC* (visible embodied cognition), and now, *interactions with objects* in the surroundings occur. Finally, the larger context, such as social norms, norms about gender, and class context, influence the student's IEC and the way in which they solve spatial tasks.

Figure 20

Embodied Reasoning About Spatial Tasks Framework



Note. Forms of reasoning are visible to outside observers. IEC is still involved with reasoning yet is not visible to outside viewers.

Overall, this study supplements current spatial scaling literature and proposes additional and alternative ways in which students go about solving spatial scaling tasks. Moreover, this study contributes to the literature which informs research on how students may go about solving spatial tasks (Seccia & Goldin-Meadow, 2024). Future studies can expand on the current framework of embodied reasoning to identify additional nuances and forms of reasoning related to spatial tasks.

6.1.4 Gender and Spatial Reasoning

This study addresses the larger gender gap related to participants' accuracy of spatial tasks that has been repeatedly placed on young girls and women in spatial research. As previously discussed, past research has highlighted inconsistencies (e.g., Bartlett & Camba, 2023; Lauer et al., 2019; Newcombe, 2020) in spatial research on the accuracy of specific spatial tasks regarding gender. In the following paragraphs, I first extend the discussion that spatial reasoning is co-constructed with gender (Bartlett & Camba, 2023). I then describe how this study contributes to countering the discussion in spatial research that places women and young girls in a deficit model.

Considering that spatial reasoning is co-constructed with gender, I argue that the idea of success in spatial reasoning is *not* gender neutral. With reference to Due's (2012) work on students' ideas of a successful physicist, Gonsalves (2014) presented the *paradox of gender neutrality.* She described that Due's "participants discuss that successful physicists are gender neutral, but on the other hand, when they think about physicists, they think about men" (p. 463). In short, the *paradox* is that gender neutrality is linked to socially constructed masculine attributes (Gonsalves, 2014). Considering that spatial reasoning is a foundational aspect of STEM, I argue that spatial reasoning is a contributing factor to this paradox in STEM fields as there also exists this paradox in spatial reasoning research. Specifically, in spatial reasoning, the performance of tasks has been similarly approached as gender neutral, yet the design and assessment of spatial tasks have positively impacted men (Bartlett & Camba, 2023). Also, past literature has discussed research that investigated performance of spatial tasks as associated with biological differences (Lauer et al., 2019). Considering that gender is constructed and *performative,* I question: *What is the goal of investigating gender as a statistical variable, if not to obtain perception of gender neutrality, yet which is constructed with masculinity?* (Butler, 1999; Gonsalves, 2014; Salzinger, 2004). Therefore, spatial tasks and, as a result, spatial research have co-constructed with gender to the extent that performing and being successful in spatial tasks are not gender neutral but are rather associated with socially constructed masculine traits (Bartlett & Camba, 2023; Gonsalves 2010; Gonsalves, 2014). I argue that reframing the gender paradox in spatial reasoning requires consideration of what it means to investigate *reasoning*. Moreover, there is a need for future research to push *against* the harmful gender discourse that negatively impacts women and young girls.

Current spatial research has led to circulating discourse that women generally perform worse in spatial tasks compared to men, which is a contributing factor to how spatial reasoning has been co-constructed with gender and moreover, encourages discussion that does not consider the socially constructed nature of spatial reasoning altogether (Bartlett & Camba, 2023; Butler, 1999). This study addresses how spatial reasoning is the sensemaking process as one solves spatial tasks. Specifically, it was identified that each participant's reasoning was multifaceted, embodied, and individually situated to their personal life experiences. Considering that the spatial *reasoning* of each student had differences, I argue that spatial research must move away from the generalizing language of gender-related performance altogether, as it does not account for the socially constructed nature of learning, communication, and reasoning (Butler, 1999). Through evidence of results of the student's embodied reasoning, I found not only *no* evidence for the deficit model placed on women and young girls, but also found overwhelming evidence *against* the deficit model.

In this study, I purposefully investigated *only* how young girls reasoned about spatial tasks to avoid positioning girls in opposition to those of other genders. The results of the study, specifically in the six student cases in Chapter Five, showed that there are differences in reasoning even among girls of the same age in the same class. Importantly, even though I identified seven different forms of reasoning, these forms of reasoning appeared uniquely in various combinations depending on the student. In doing so, the results demonstrate direct counterevidence to the practice of *gender essentialism* in spatial research, as each student's reasoning was unique. Gelman and Taylor (2000) defined essentialism as "the view that categories have an underlying reality or true nature that one cannot observe directly but that gives an object its identity" (p. 169). In the context of gender, gender essentialism is the belief that there are innate or inherent aspects about girls "that distinguishes them from boys" (p. 169). In spatial research, I propose that the "artificially imposed" idea that biology is the cause of gender differences has contributed to the "overgeneralization" of discourse surrounding gender performance that negatively positions women (p. 185). Such concerns of "overgeneralization" (p. 185) were the precise reason why I did not analyze girls' *accuracy* of spatial tasks, as it would directly eliminate the individually situated reasoning identified in this study and not highlight the capabilities of the girls in this study. In contrasting the discourse in spatial research, I specifically respond to Newcombe and Stieff (2011), who proposed the argued that differences in sex performance are not "pervasive" (p. 959). In contrast, I argue that such an examination of performance is unproductive and pervasive as it can reinforce discourse generalizing individuals based on their gender despite these spatial tests being task specific. Moreover, these reinforced generalized discussions can lead to the stereotyping of performance based on gender, as Newcombe and Stieff (2011) highlighted themselves. With consideration of the literature, previous works such as Ceci and Williams (2009), who examined spatial research and STEM, determined "that the patterns across studies of hormones and cognitive ability between women and men is not strong and consistent enough to justify that hormones are the *primary* cause of sex differences in STEM careers" (p. 75). While 15 years have passed and it has been repeatedly established that biological factors are insufficient as evidence (e.g., Bartlett & Camba, 2023; Lauer et al., 2019), I ask the question: *why is there continual examination of biology if not to reaffirm gender essentialism?* Moreover, in listing a few examples from Chapters One and Two, I argue that the long-term generalization of women as being less capable than men in spatial reasoning, the continual examination of biological and hormonal causes of these capabilities, and the artificial exaggeration of the gap through spatial task design and assessment is sufficient evidence that such examination is pervasive as it continually targets women (Bartlett & Camba, 2023; Gelman & Taylor, 2000; Lauer et al., 2019; Newcombe & Stieff, 2011). It is critical that future spatial research reconsiders that spatial reasoning is individually situated.

Importantly, at the start of this study, I aimed to understand how young girls reasoned about spatial tasks to address the larger gender gap in STEM participation in both university and the workforce. It was determined that young girls were not only capable of solving spatial scaling tasks, but they expressed multiple forms of embodied reasoning at the same time to solve the tasks. Moreover, I argue, from the evidence, that the negative discourse surrounding the performance in spatial reasoning of women and young girls is socially generated. I propose that the choice to pursue STEM is socially influenced and *not* correlated with accuracy in spatial tasks. Rather than solely focusing on accuracy, future studies should examine the socially constructed nature of spatial reasoning and its impact on an individual's choice to pursue STEM.

6.2 Implications for Mathematics Teaching Practice

This study aimed to understand how young girls *reason*, particularly with a focus on gestures, about spatial scaling tasks. In Chapters Four and Five, I described the ways that girls gestured when reasoning about spatial scaling tasks. Moreover, I described in Chapter Five how students interacted with objects in their surroundings and how gestures were presented when forms of reasoning intersected in the same episode. Focusing on gestures provided insight into the students' reasoning, where linguistic cues were not always sufficient in identifying how the student was making sense of the task. In the classroom, teachers might particularly focus on the ways in which their students' gesture and interact with classroom materials as they reason about spatial mathematics. Such an emphasis on various ways of learning, or *modes*, is otherwise known as multimodal teaching approaches (Abrahamson et al., 2020). Multimodal teaching includes focusing on teaching practices beyond *modes* that are verbal and written and can include teaching with gestures and additional materials. Moreover, the forms of reasoning determined in this study can be used as a framework to support teachers' multimodal noticing (Walkoe et al., 2023) of students' reasoning when their students solve spatial tasks. Specifically, mathematics teachers can take note of and consider gestures when students reason about mathematics tasks, particularly those that involve geometry. Importantly, such noticing of gestures can be accompanied by students' verbal forms of reasoning (Jarry-Shore & Borko, 2023). In all, by noticing students' spatial reasoning through language, gestures, and interaction with surroundings, teachers can better leverage student reasoning about mathematic tasks. Specifically, in teacher practice, teachers can ask students about their gestures as they solve spatial mathematics tasks to better understand how their students are reasoning.

Finally, this study examines the role of gender in spatial reasoning and counters generalizing discourse that positions women and young girls as underperforming in spatial reasoning (Butler, 1999). Moreover, I described in previous sections of this chapter how such discourse has been socially constructed and, as a result, can influence behaviours towards young girls as they learn spatial mathematics (Butler, 1999). In teacher practice, teachers can continually reflect on their implicit bias about gender in mathematics. For example, Mendick

(2005) examined students' identity in mathematics and how gender plays a role in mathematics self-esteem. In this work, Mendick (2005) described how socialized aspects of high mathematics performance have been attributed to masculinity, and highlighted a few, importantly, being able to "reason" (p. 212). Moreover, Mendick (2005) described various social factors that play a role in reinforcing this narrative, such as stories that are highlighted in television and film. In teaching, at the same time that they promote the *reasoning* of spatial mathematics, teachers must also be mindful of these social biases of mathematics being masculine and reflect on ways in which they may be unintentionally reproducing these narratives.

6.3 Limitations and Future Directions

This study investigated how students reason about specific spatial scaling tasks. As illustrated by my results, students' reasoning can change depending on the task even when the same spatial reasoning domain is considered. Future directions may include understanding how students reason about *different* spatial scaling tasks. Specifically, while this study aimed to understand how students reason about scaling with 2-dimensional shapes, future directions could include having tasks where students reason about scaling with 3-dimensional shapes.

Moreover, this study investigated students' reasoning based on spatial scaling tasks that involved matching locations. In this study, it was identified that certain forms of reasoning were elicited depending on the location of the house. For example, as mentioned in Chapter Five, reasoning about *comparing positions – symmetry* and *comparing amounts – vertical lengths* emerged most in tasks three and six which indicated that students reasoned about previous locations that were symmetrical to assist them in their response for the current task. Not only did the task elicit forms of reasoning, but the alternative locations that I presented to students elicited forms of reasoning. For example, in Chapter Four section 4.2.2., I described how I presented an alternative location to student A3 and it elicited reasoning about *comparing amounts – two horizontal spaces.* Such findings align with my previous arguments in Chapter Two about the situated nature of reasoning and how the *design* of the task can influence the participant's response (Bartlett & Camba, 2023). Future scholars can consider how student reasoning changes depending on the type of spatial task and other aspects of the context. For example, future studies can investigate how student reasoning differs between the spatial scaling task of this study with a spatial scaling task that presents the locations in a different order.

This study investigated the forms of reasoning of individual students as they solved a spatial scaling task related to map reading. Accordingly, this study did not capture how students reason within group settings. Understanding how students reason in group settings can expand understanding of students' reasoning through a situative perspective as learning and reasoning are socially occurring. Future directions include understanding how students reason about spatial scaling tasks in group settings. Moreover, as this study took place within an unoccupied classroom in a school setting, this study did not capture how students reason about spatial scaling tasks when in their mathematics classroom during class hours. Specifically, students were not provided any resources beyond those provided in the interview. Future directions could include how students reason about spatial tasks within their mathematics classroom with access to resources that they would have in their classroom such as measurement tools. Moreover, future studies can focus on the role of mediational means for individual student learning in the classroom setting. Alternatively, future studies can focus on the role of these means and construction of knowledge with consideration of the social dynamics of the classroom, also called *social semiotics* (Lemke, 2003).

Finally, this study examined how young girls reason about spatial tasks. In this study, I have defined gender as socially constructed and emphasized the need to move beyond gender essentialism in spatial research. However, countering spatial research that has targeted women involved describing and responding to research that has continually placed gender in a binary model. While this study gives voice to young girls in spatial research, future scholars can aim to move discourse of spatial reasoning beyond the gender binary.

6.4 Conclusion

This study investigated how young girls reasoned about spatial scaling tasks through an embodied framework. This study disrupted the narrative that there are only simple and singular strategies at play to solve spatial tasks. Seven embodied forms of reasoning were identified and the students used multiple forms of reasoning at the same time when solving spatial tasks. Specifically, students used various embodied forms of reasoning and mediational means *at the same time* when solving spatial tasks. Students drew from their personal experiences and prior knowledge to solve the tasks, where reasoning varied from student to student. It is essential to re-evaluate *spatial reasoning* as this study demonstrated the embodied nature of spatial reasoning.

Finally, I investigated how young girls reason to counter negative discourse towards women and young girls in spatial reasoning. It is crucial to consider reasoning as *an embodied sensemaking practice* to move away from accuracy-based discourse that negatively positions women and young girls in spatial research. By moving away from discussions of capabilities based on accuracy and focusing on understanding *reasoning*, researchers and teachers can leverage student learning on spatial reasoning in ways that are more authentic for students. In approaching learning and reasoning as embodied, teachers can better understand, communicate with, and support students' spatial reasoning. To conclude, this study demonstrated that young girls are clearly capable of solving spatial tasks, and their reasoning extends far beyond what has been established in the literature. It is essential that future studies recognize implicit biases about gender in spatial mathematics research and education and aim to counter discourse that generalizes women's and young girls' capabilities in spatial reasoning and STEM.

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Appendices

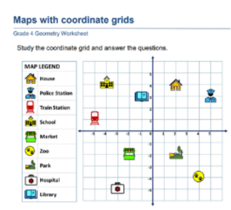
Appendix A: Teacher Questionnaire

The questionnaire was completed on Microsoft forms. Questions without selection based required written responses.

1. Select the grades you are teaching **this school year** below. You can select more than one response:
   - ☐ Grade 1
   - ☐ Grade 2
   - ☐ Grade 3
   - ☐ Grade 4
   - ☐ Grade 5
   - ☐ Grade 6

2. How many days or weeks have you taught **geometry** so far this school year? <u>Please specify the grade level in your response.</u>
   Sample answer: "Grade 1 - I have taught geometry for 2 weeks. Grade 3: This year, I taught 1 week of geometry..."
   *If you did **not** teach geometry yet this year to any grade, you can respond: not applicable (NA).*

3. Mathematics involves learning about **locating objects in space (e.g. in front of, behind, next to), on an axis such as a Cartesian plane, or a map**. Did you teach your students anything about locating objects this school year?

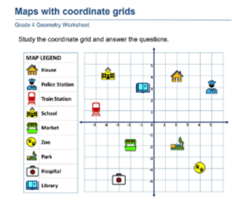


   If yes, select the grades this applies to (you can select more than one option):
   - ☐ Grade 1
   - ☐ Grade 2
   - ☐ Grade 3
   - ☐ Grade 4
   - ☐ Grade 5
   - ☐ Grade 6

   Image source:

Maps with coordinated grids. (n.d.). *K5 Learning.* https://www.k5learning.com/free-math-worksheets/fourth-grade-4/geometry/coordinate-grid-map

4. Please specify **what topics related to locating objects in space** you taught your students. <u>Please specify the grade level in your response.</u>



   Sample answer: "Grade 1: We covered positional vocabulary (e.g. behind, in front of, etc.). Grade 4: I taught locating objects in Cartesian plane".

   *If you did **not** teach locating objects, you can respond: not applicable (NA).*

   Image source:
   Maps with coordinated grids. (n.d.). *K5 Learning. https://www.k5learning.com/free-math-worksheets/fourth-grade-4/geometry/coordinate-grid-map*

5. Geometry often requires knowledge of **measurement**, specifically the **lengths** of objects.
   This school year, did you teach your students about **measuring lengths of objects or shapes**? If yes, please select the grades you taught measurement to.

   *If you did **not** teach measurement, you can respond: not applicable (NA).*

6. Comparing lengths of shapes can involve *additive comparison* or *multiplicative comparison*. See the image below for the difference.

   Did you teach **additive comparison** this school year? If yes, select the grade levels you taught additive comparison to.

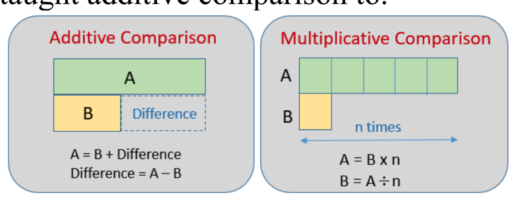


   Image source:
   Multiplication or division word problems (grade 4). (n.d.). *OnlineMathLearning.com.* https://www.onlinemathlearning.com/multiply-divide-word-problems-4oa2.html#google_vignette

   - ☐ Grade 1
   - ☐ Grade 2
   - ☐ Grade 3
   - ☐ Grade 4
   - ☐ Grade 5
   - ☐ Grade 6

7. Comparing lengths of shapes can involve *additive comparison* or *multiplicative comparison*.

   Did you teach **multiplicative comparison** this school year? If yes, select the grade levels you taught multiplicative comparison to.

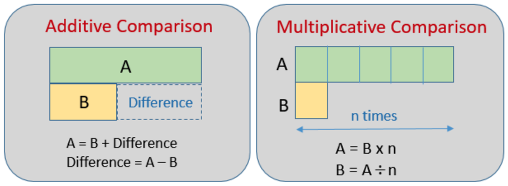
   

   Image source:
   Multiplication or division word problems (grade 4). (n.d.). *OnlineMathLearning.com.* https://www.onlinemathlearning.com/multiply-divide-word-problems-4oa2.html#google_vignette

8. Understanding numerical relationships can involve **proportional reasoning (example: 5/10 = 20/40)** This school year, did you teach **proportional reasoning?** If yes, select the grades below:
   - ☐ Grade 1
   - ☐ Grade 2
   - ☐ Grade 3
   - ☐ Grade 4
   - ☐ Grade 5
   - ☐ Grade 6

9. **Spatial reasoning** in mathematics involves asking students on their ideas of 2-dimensional and 3-dimensional shapes. These questions can involve rotating shapes, shrinking and enlarging shapes, locating objects, composite shapes, folding and unfolding shapes (nets).

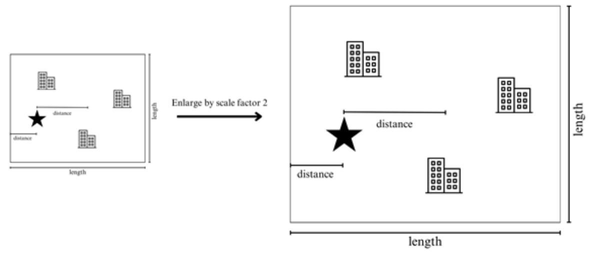
   Have you taught spatial reasoning this year to your students? If **yes**, please select the grades this applies to.

   - ☐ Grade 1
   - ☐ Grade 2
   - ☐ Grade 3
   - ☐ Grade 4
   - ☐ Grade 5
   - ☐ Grade 6

10. Please elaborate on how you have taught **spatial reasoning** in your classes in as much detail as possible. Please indicate the grade level in your answer.

    If you have not taught spatial reasoning this year, you can respond: not applicable (NA).

11. Spatial scaling in mathematics involves asking students on scaling shapes. For example, shapes can be scaled to be 2x larger or 0.5x the size. Another example of spatial scaling involves using the same diagrams of different scales (see the image attached).



    Have you taught spatial scaling this year to your students? If **yes**, please select the grades this applies to.

    - ☐ Grade 1
    - ☐ Grade 2
    - ☐ Grade 3
    - ☐ Grade 4
    - ☐ Grade 5
    - ☐ Grade 6

12. Please elaborate on how you have taught **spatial scaling** in your classes in as much detail as possible if you have taught it. Please indicate the grade level in your answer.

    If you have not taught spatial scaling this year, you can respond: not applicable (NA).

Appendix B: Student Interview Protocol

The Farmer's House

Interviewer: "So we are going to start our first activity. I'm going to read a short story to you, about a farmer named Dina, and then we will get started, okay?" [wait for student response]

"There is a farmer named Dina. Dina bought land and is going to build a farmhouse. Dina has a few ideas of where to place her house, but she wants to show her friends some places she is thinking of building the house to ask them what they think! Dina has made a few pictures to show her friends, and she made one that looks like this. *Reveal model to student.*

"What do you notice?" [wait for student response]

"Dina wants to make pictures for her friends and made pictures of different sizes. Here's one that she made, what do you notice about this picture?" [wait for student response]

Introduce problem:
"Dina has a problem; she does not know where to place the house so that it matches the other picture! Dina needs your help in putting the house so that it matches her picture. How does that sound?" [wait for student response]

"So, I want you to use this marker and make a dot where you think the house should go. I'll give you the house so that you can put it right on top of your dot."
"Can you show me where we should place the house?"

Change Location:
"I'm going to show you another picture she made." "What do you notice about this picture?"

Prompts:
- *Why did you place it there?*
- *What [was going on in your head/ were you thinking] as you were placing the house on that spot?*
- *Imagine I am one of your classmates. How would you explain to me how to find out where to place the house if I was having trouble?*
- *Can you show me?*
- *If I placed it here (absolute distance answer), would this be right?*