Essays on strategic trade policies, differentiated products, and exhaustible resources

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Abstract

This thesis consists of three essays. The firsts essay looks at the optimal export policy in the context of an vertically related industry with differentiated products, and analyze effects of the degree of product substitutability and market structure on the determination of such policy. It is shown that results obtained in a similar model with homogenous goods rivalry no longer hold when the goods are differentiated. Indeed, the degree of product substitutability plays an important role in the determination of export policies, it also determine whether a country can be better off under a trade policy war compared to free trade. The use of differentiated product setting also allows one to compare export policies and countries' welfare between Cournot and Bertrand competitions. It is found that the results of the comparison are also sensitive to the degree of product substitutability.

The second essay presents a model of tariff war involving n resource-importing countries and a coalition of resource-exporting countries that acts as a resource-cartel, coordinating the extraction paths of their resource-extracting firms. We deal with two different tariff war scenarios. In the first scenario, which we call the bilateral monopoly scenario, all resource-importing countries form a coalition that imposes a common tariff rate on the exhaustible resource. In the second scenario, the resource-importing countries act independently, each imposing its own tariff rate. We compare the outcomes under the two tariff war scenarios and with the free-trade outcome. In our model, it turns out that, given the rate of discount, there is a corresponding threshold level of the marginal cost parameter beyond which the resource-importing countries would prefer bilateral monopoly to world-wide free trade. The higher is the rate of discount, the greater is the corresponding threshold marginal cost level. We study the effects of asymmetry between resource-importing countries on the welfare of the exporting country, and of the importing countries. In the final section, we examine the welfare consequence of splitting the resource stock into two (possibly unequal) parts to serve the two import markets. It turns out that such market segregation is harmful to the exporting country.

The third essay analyzes the dynamic rivalry by two firms in the cultural goods market. Consumers are assumed to have homogeneous valuation of one good and heterogeneous valuation of the other good. One of the firm can choose the quality of its product from a continuum of quality levels. This firm is the far-sighted firm in the dynamic model. Consumer preferences evolve gradually over time. This evolution is driven by a network effect and a depreciation effect. The network effect is a function of the current market shares of the two firms. We consider the case where one firm is the dynamic optimizer and the other firm is myopic. The far-sighted firm solves its problem by manipulating its current market share, which in turn affects the evolution of consumer preferences. We show there exist two steady states, one of which is a stable in the saddle-point sense, while the other is unstable. Our comparative static analysis of the stable steady state shows that the steady state quality level of the far-sighted firm is increasing in the discount factor and decreasing in the cost and the speed of adjustment parameters. Moreover, the steady state quality level is lower than the equilibrium quality level of the static model.

Résumé

Cette thèse se compose de trois essais. Le premier essai examine la politique d'exportation optimale dans le contexte d'une industrie à l'aval qui s'approvisionne à l'amont, et qui produit des biens différenciés. On analyse des effets du degré de substituabilité des biens et la structure du marché sur les propriétés de la politique optimale. On montre que les résultats obtenus dans le cas des biens homogènes ne tiennent plus quand les biens sont différencies. En effet, le degré de substituabilité de produit joue un rôle important sur la détermination des politiques d'exportations. Il est un facteur important qui détermine si un pays peut améliorer son bien-être vis-à-vis le libre échange en s'engageant dans une guerre de politique commerciale. On compare les niveaux du bien-être social quand les firmes sont en concurrence à la Cournot et à la Bertrand. On constate que le résultat de la comparaison est également sensible au degré de substituabilité des biens.

Le deuxième essai présente un modèle de guerre de entre les pays exportateurs de ressources naturelles et les pays importateurs. Nous traitons deux scénarios différents de guerre de tarifs. Dans le premier scénario, que nous appelons le scénario de monopole bilatéral, les pays importateurs forment une coalition qui impose un taux commun de tarif sur l'importation de la ressource épuisable. Dans le deuxième scénario, les pays importateurs agissent indépendamment, chacun imposant son propre taux de tarif. Nous comparons les résultats sous les deux scénarios de guerre de tarifs et aux résultats sous le libre-échange. Dans notre modèle, il s'avère que, étant donné le taux d'escompte, il y a un niveau de seuil du paramètre de coût marginal au delà duquel les pays importateurs préféreraient le libre échange au monopole bilatéral. Le niveau de seuil du coût marginal est une fonction croissant du taux d'escompte. Nous étudions les effets de l'asymétrie entre les pays importateurs sur le bien-être du pays exportateur, et des pays importateurs. Dans la section finale, nous examinons la conséquence sur le bien-être social de diviser le stock de ressource en deux parties inégales pour servir deux marchés d'importation. Il s'avère qu'une telle ségrégation du marché est nocive au pays exportateur.

Le troisième essai analyses la rivalité dynamique entre deux firmes qui produisent des biens culturels. Les consommateurs sont homogènes dans leur évaluation d'un bien et hétérogènes dans leur évaluation de l'autre bien. Une firme choisit la qualité de son produit. Elle optimise son sentier de profits. La préférence des consommateurs s'évolue graduellement. Cette évolution est influence par un effet de réseaux et un effet de dépréciation. L'effet de réseaux est une fonction de la part de marche de la firme. On considère la case ou l'une des firmes optimise dans un contexte temporel tandis que l'autre est myope. La première firme solutionne son problème en manipulant sa part de marche, qui a un effet sur l'évolution de la préférence des consommateurs. On montre qu'il y a deux états stationnaires, dont un est stable dans le sens de point de selle, et l'autre est instable. A l'état stationnaire stable, la qualité du produit de la firme qui optimise est une fonction croissante du taux d'escompte et décroissante du cout et de la vitesse d'ajustement. La qualité de son produit dans l'état stationnaire est inferieur à celle qu'elle choisit dans le cas statique.

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Contributions of Authors

The second chapter of this thesis is coauthored with my supervisor, professor Long. He suggested the ideas and I analyzed the model and carried out the simulations. We both reviewed the literature on this topic.

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Introduction

After papers such as Brander and Spencer (1985) and Eaton and Grossman (1986) have been published, there has been an enormous wave of research on various extensions to study the robustness of the predictions on export policies under international oligopolistic (especially duopolistic) rivalry. As international outsourcing and fragmentation become more and more popular among firms, economists also become interested in trade policies involving vertically related industry. The main focus in this context has been on the optimal design of the government's trade policies such as import tariff and export subsidy to raise a country's welfare. Research along this line has generated many interesting findings and policy implications that have been practiced in the real world. In particular, Bernhofen (1997) directly extend the model of Brander and Spencer (1985) by incorporating an upstream monopolist who sells a key intermediate good to two downstream firms for export to a third country. The paper aims to study the effect of vertically related markets on the government's export policy, and it shows that, beside the horizontal rent-shifting effect of an export subsidy, there is also a vertical rent-shifting effect which suggests a tax instead of a subsidy as argued by Brander and Spencer (1985). Depending on the pricing schemes employed by the upstream monopolist, the vertical rent-shifting effect may dominates the horizontal one implying that the optimal policy is a tax. The majority of the papers in this area consider homogenous-good models; however, it is interesting to see if the policy prescription would be same in the case where goods are differentiated. After all, almost all kinds of goods in the real world are differentiated one way or the other. The model on differentiated product first developed by Dixit (1979), Singh and Vives, (1984), and Vives (1985) has been very popular due to the linear demand specified in the model which usually yields a relatively simple and computable solution. A more detailed introduction of the literature in this area will be presented in the Section 1.1. The first two essays in this thesis try to analyze the strategic trade policies in the context of differentiated products, vertically related industry, and natural resource while the third essay looks at firms' decision in a cultural good rivalry where the form of product differentiation is quite different than the one used in Vives (1985).

The first essay conducts an analysis on optimal export subsidy/tax, using a model similar to that of Bernhofen (1997) but with differentiated final goods. This generalization allows us to study not only the effect of the degree of product differentiation on the original results obtained in Bernhofen (1997) but also the Bertrand competition case. When the upstream firm is charging a uniform price on the intermediate good, Bernhofen (1997) concludes that the optimal policy is a still a subsidy which was first noted by Brander and Spencer (1985) because the horizontal rent-shifting effect (suggesting a subsidy) dominates the vertical rent-shifting effect (suggesting a tax); however, we find that this is only true if the two goods are sufficiently close to each other in our model. Indeed, the increase in the degree of product differentiation diminishes the horizontal effect which leads to a decrease in the government's incentive to impose a subsidy. If the goods become sufficiently differentiated, the vertical rent-shifting effect dominates, and it becomes optimal for the government to impose a tax. As in the price discrimination case, our finding is the same as Bernhofen (1997): the optimal export policy is a tax. We have also found that the familiar prisoners' dilemma where both exporting countries are worse off by subsidizing their own firms does not necessary hold in the context of differentiated-good duopoly. As long as the optimal export policy is a tax (subsidy), the country is better off intervening compared to free trade.

When the downstream firms compete in price, the governments impose taxes in equilibrium regardless of the pricing schemes. Although this finding is very similar to the one of Eaton and Grossman (1986), it still enables us to make some interesting findings by comparing the county's welfare in two competition modes. The main finding is that the country's welfare under Bertrand rivalry can be higher or lower than that under Cournot rivalry. In sum, given the pricing scheme employed by the upstream monopolist, the closer the two goods, the more likely that the country is better off under Cournot rivalry than under Bertrand rivalry.

The second essay also studies the optimal trade policy but in the context of a natural-resource-using economy. To our knowledge, the majority of the literature on resource economics has been focusing on the assumption that only one side of the market (demand/supply) exercises the market power while the other side is passive (price-taking behavior)[1]. Thus, this has motivated us to look at the case of a bilateral monopoly where both sides of the market exercise their market power at the same time. By constructing a model of extraction of exhaustible resource where the cost of extracting the resource is increasing with the accumulated extraction, we wish to find out the extraction path under bilateral monopoly setting and compare it to the one under free trade. We also generalize the model

[1] A more detailed discussion can be found in Section 2.2.

to allow for more than one resource-importing countries that can be potentially asymmetric in their market sizes and provide some numerical solutions in the case of two resource importing countries.

Comparing welfare under bilateral monopoly to the one under free trade, we found that, given the discount rate, there exists a corresponding threshold level of the marginal cost parameter beyond which the resource-importing country is better off under bilateral monopoly. Moreover, this threshold level becomes higher if the discount rate is higher. Another interesting question is what would be the division of gains from trade between two groups of countries (importing and exporting). Under free trade, we show that the gains from trade that accrue to the resource-exporting country depend on both the discount rate and the marginal cost parameter. For the bilateral monopoly case, a surprising result is that the resource-exporting country always shares two-third of the world-wide trade gains regardless the parameter values while the resource-importing country (countries) shares the remaining one-third. The numerical solutions for the two-importing-country case suggests that the countries' welfare can be sensitive to the total market size as well as the relative market sizes of the two importing countries.

We also consider a special case where the resource-exporting country is obligated to divide the deposit of its resource stock to serve two importing countries separately. Interestingly, it is optimal for the exporting country to split the deposit in such a way that matches two resource importing countries' relative market sizes in the first stage, and the tariff imposed by two importing countries and the tax/subsidy imposed by the exporting country in the following stage coincide with the ones under case where two resource importing countries form a custom union (cooperatively determining a common tariff rate). Thus, an importing country can potentially achieve the same goal of forming a custom union with the other importing country by signing a contract with resource exporting country on a portion of deposit that is to be sold solely to it.

The third essay studies the possibility of a cultural-good producer who may gain by driving out some cultural minorities. A classic example would be Hollywood movie producers versus some local (Japanese, French, or Canadian) movie producers who are relatively much smaller. Although (cultural) goods in this chapter are differentiated as well, the nature of demand for these goods is quite different than the one used in Chapter 1. Indeed, consumers in this model have homogenous valuation of one good and heterogenous valuation of the other good, and each consumer is assumed, due to some constraint such as time, to only consume either one of the two cultural goods each period of time. Therefore, if the price of one good is too high, the corner solution may arise leaving only the producer of the other good in the market. This is in sharp contrast to the model of differentiated-product duopoly based on Vives(1985), where both producers always coexist in the market at least in the case of Cournot rivalry. We analyze both static and dynamic competitions between two firms.

The static model is described as a two-stage game. In the first stage, the firm (a Hollywood movie maker) whose products are valued homogeneously by consumers can decide to improve the quality of its product from some pre-determined level through some costly investment. Both firms then choose prices of their products in the second stage. The static analysis is similar to the one of Francois and van Ypersele (2002) but instead of considering only two levels (high and low) of quality improvements, we generalize it into a continuum. We provide explicitly the conditions for the existences of both corner and interior solutions.

Introducing a network effect based on market shares of two goods on the evolution of consumers's preference into the static framework transforms it into a dynamic model where the current preference over French movie changes in a way that depends on a the market shares of two goods in the last period. By focusing on interior solution, we are to find one stable and one unstable steady state, and the property of the stable steady is reported through a local linearization at the neighborhood of that steady state. Analysis of comparative statics suggests that the steady-state level of consumers' valuation of French movie is increasing in the discount rate but decreasing in the speed of adjustment and the cost parameter.

Chapter 1
Differentiated Products, Vertically Related Markets, and Optimal Export Policies

1.1 Introduction

During the past two decades, many economists explore the possible strategic use of policy instruments in oligopolistic industries to increase the welfare of a country. However, the literature in this topic produce mixed results. Brander and Spencer (1985) consider a horizontal export-rivalry model where a "home" firm and a "foreign" firm engage in a Cournot competition to export the good to a third country. They show that both governments subsidize exports at the equilibrium. Eaton and Grossman (1986), on the other hand, show that the result is reversed in the sense that both governments impose taxes on exports if firms act as Bertrand competitors. Instead of considering an international duopoly, Dixit (1984) generalizes the Cournot export rivalry model to allow more than one firms in each country. In this oligopolistic setting, he shows that the optimal policy for the home country is still a subsidy given that the number of domestic firms is not too large. On the other hand, if the number of domestic firms is too large, then a tax is called for.

Emerging from the literature on optimal trade policy with horizontal competition, more and more papers start to analyze the optimal trade policy when the market structures are vertically related. Spencer and Jones (1992) consider a Cournot rivalry of the final good in the home country between a home firm and a foreign firm where the foreign firm is a vertically integrated producer of both intermediate and final good, and the home firm partially depends on the import of the intermediate good from the foreign firm. The main focus of their paper is not to solve for the optimal policy but to consider the reaction of the foreign vertically integrated firm on the exogenously change in trade policies. They show that a tariff on the import of the final good reduces the price of the intermediate input charged by the foreign vertically integrated firm. Spencer and Jones (1991), in a companion paper, analyze the effect of a foreign government's policies on both intermediate and final goods on the vertical supply decision by a foreign vertically integrated firm. They show that the optimal export policies by the foreign government may be a tax on both goods or a subsidy on both goods. Bernhofen (1997) consider a Cournot duopoly consisting of a foreign and a home firm that import intermediate input from a monopolist located in a country other than home and foreign and sell the good to a third country. The paper analyzes how an export subsidy provided by the home or foreign country can extract rents from the other countries both horizontally and vertically, and shows that the optimal government intervention is sensitive to the pricing scheme employed by the intermediate good monopolist. Ishikawa and Spencer (1999) examine a more general model where both upstream and downstream markets consist of many firms. They show that a subsidy on the final good by the domestic government intended to shift rents from foreign final good producers may instead shift the rent to the foreign intermediate good producers. Moreover, they show that the incentive for an export subsidy on the final good tends to be larger if the intermediate good market is purely domestic or purely foreign. Gaudet and Long (1996) also consider the case where there are many firms in both upstream and downstream industries; however, their focus is on the profitability of a vertical integration among these firms. Recently, Chang and Sugeta (2004) introduce conjectural variations and bargaining into a vertically related trade model where a foreign upstream firm supplies a commonly used input to two downstream firms that produce differentiated products for export. They show that the optimal export policies by the government depend crucially on the conjectural variations of the final good rivals, upstream firms' pricing scheme, firms' bargaining power, and the degree of product differentiation. Most of the papers mentioned above assume that goods are homogeneous. However, it is well noted that most goods are differentiated instead of homogenous. Therefore, it will be interesting to investigate the strategic trade policy in the context of differentiated good market.

The differentiated oligopoly model introduced by Dixit (1979), Singh and Vives, (1984), and Vives (1985) has been widely adopted by many people to study issues such as merger, R&D, dumping, and strategic trade policy. They assume a linear demand system. This yields a relatively easy and computable solution, and for this reason, our paper use a linear demand system as well. Clarke and Collie (2003) consider a model where in each of two countries there is a monopolist that produces a differentiated product. They show that a country always gains when it opens up the trade with the other country. A more recent paper, Clarke and Collie (2006), use a similar setting to compare the maximum-revenue tariff and optimum-welfare tariff. They show that both tariffs are lower under Bertrand duopoly than under Cournot duopoly. Also, the optimum-welfare tariff may exceed the maximum-revenue tariff under both Bertrand duopoly and Cournot duopoly depending on the degree of product differentiation.

This paper adopts the setting of Bernhofen (1997) where intermediate and final goods markets are vertically related. The model postulates an oligopoly consisting of two firms located in two different countries that buy and use intermediate good from a monopolist to produce differentiated final goods sold to a third country. We consider two competition modes, namely Cournot and Bertrand, for downstream market, and two pricing schemes employed by the upstream monopolist, price discrimination and uniform price. The governments of both countries use an export subsidy or tax to maximize welfare defined as profit net of subsidy or tax. The model is described as a three stage game where governments simultaneously decide their policies in the first stage. In the second stage, the upstream monopolist announces the price of the intermediate good sold to downstream firms. Both price discrimination and uniform pricing will be considered. In the final stage, the two downstream firms compete in a third market. The equilibrium concept is subgame perfection. We find that whether governments subsidize or tax its export depends not only on the market structure but also on the degree of product substitutability. Moreover, depending on how close the two final goods are, a country's welfare under Cournot competition can be lower or higher than that under Bertrand competition.

This paper is organized as follows. Section 2 provides an overview of the basic model. Section 3 analyzes the optimal trade policies and welfare under Cournot competition. Section 4 takes up Bertrand duopoly. Section 5 compares trade policies and welfare between the Cournot and the Bertrand modes of rivalry. Section 6 concludes the paper.

1.2 The Model

A monopolist produces and supplies the intermediate good to a downstream industry. The downstream market structure is a duopoly where each firm produces a differentiated final product. We adopt the setting of Bernhofen (1997) where the final good producers are located in two different countries namely, country H (Home) and F (Foreign), and the intermediated good monopolist is located in a country other than country H and F. The final good is sold to a third country. The production technology is linear: one unit of final good is produced using one unit of intermediate good.

Consider a three-stage game in which the governments of H and F choose their export policies in the first stage. In the second stage, the upstream monopolist commits to a price of the intermediated good sold to the downstream firms. We consider two pricing schemes of the monopolist: the price-discrimination scheme, and the uniform pricing scheme. In the third stage, the downstream firms, firm h, and firm f, take the price of the intermediate good and the governments' export policies as given and compete in the differentiated good market. The equilibrium concept adopted here is sub-game perfection.

Following Singh and Vives (1984) and Vives (1985), we assume the utility function of a representative consumer in the final good market takes the form

$$U(\mathbf{Q}, z) = a_h q_h + a_f q_f - \frac{1}{2}(b_h q_h^2 + b_f q_f^2) - \gamma q_h q_f + z \tag{1.1}$$

where $a_i, b_i, \gamma > 0$, $i = h, f$; $\qquad b_i \geq \gamma$.

The term $\frac{\gamma^2}{b_h b_f} \in [0, 1]$ measures the degree of product substitutability ranging from zero when the products are independent to one when the products are perfect substitutes. The variables q_h and q_f denote, respectively, the output level of firm h and firm f, and z is the consumption of a numeraire good where its price is normalized to unity. To simplify the model, assume that $a_h = a_f = a$ and $b_h = b_f = 1$. The degree of product substitutability is thus measured by γ ranging from zero to unity: in the limit as $\gamma \to 0$, the goods become independent, while in the limit as $\gamma \to 1$ they become perfect substitutes.

The representative consumer's inverse demand function for variety i can be obtained by solving the consumer's optimization problem and is thus given by

$$p_i = a - q_i - \gamma q_j, \qquad i, j = 1, 2, \qquad i \neq j, \tag{1.2}$$

The demand function is

$$q_i = \frac{1}{(1-\gamma^2)} [(1-\gamma)a - p_i + \gamma p_j] \qquad i, j = 1, 2, \qquad i \neq j, \tag{1.3}$$

It can be easily seen that the demand for good i is decreasing in its own price, p_i, but increasing in the price of good j, p_j. In the next two sections, we consider first the quantity-setting (Cournot) case and then the price-setting (Bertrand) case.

1.3 Cournot Equilibrium

When firms set quantities in the final stage sub-game, firm i chooses q_i to maximize its profit

$$\pi_i = (p_i - w_i + s_i)q_i = (a - q_i - \gamma q_j - w_i + s_i)q_i, \qquad i = h, f, \; j \neq i, \tag{1.4}$$

where w_i and s_i are the input price for firm i and the export subsidy applied on its export, respectively. Note that the input price is indexed by i which implies that the monopolist can potentially sell the intermediate good to two different firms at different prices. This possibility is further discussed in succeeding sections. In the Cournot-Nash equilibrium we obtain

$$q_i^C = \frac{(2-\gamma)a-2(w_i-s_i)+\gamma(w_j-s_j)}{(2-\gamma)(2+\gamma)}, \qquad p_i^C = \frac{(2-\gamma)a+\left(2-\gamma^2\right)(w_i-s_i)+\gamma(wj-s_j)}{(2-\gamma)(2+\gamma)},$$
$$i = 1, 2, \; j \neq i,$$

The equilibrium profit for firm i is thus

$$\pi_i = \frac{\left[(2-\gamma)\,a - 2\,(w_i - s_i) + \gamma\,(w_j - s_j)\right]^2}{\left(4-\gamma^2\right)^2} \qquad i = 1, 2, j \neq i,$$

It can be easily seen that an export subsidy by the home government increases the home firm's profit at the expense of the foreign firm. This rent shifting effect is characterized as "horizontal effect" of an export policy and is illustrated by Brander and Spencer (1985).

At the second stage, the upstream monopolist, denoted by M, and located in a country other than Home and Foreign announces the prices of the intermediate good sold to downstream firms. As in Bernhofen (1997), we consider two pricing schemes employed by the monopolist: (i) price discrimination, and (ii) uniform pricing. In the case (i), the monopolist is assumed to be able to segment the market for intermediate goods and charge different prices to firm h and f. In contrast, the monopolist in case (ii) is unable to segment the markets so is forced to charge a common price for the intermediate goods demanded by two downstream firms. We will first consider the decision of the upstream monopolist in the case of price discrimination.

The problem faced the monopolist M in this case is to choose input prices (w_h, w_f), taking both governments' trade policies as given, that maximize

$$\pi^M = (w_h - c)q_h^C + (w_f - c)q_f^C, \tag{1.5}$$

where c denotes the per-unit production cost of the intermediate good, and q_i^C, $i = h, f,$ is viewed as the derived demand for the intermediate good facing the monopolist. Differentiating this with respect to w_h and w_f yields two first order conditions,

$$\begin{aligned}
\frac{1}{\gamma^2 - 4}\left((\gamma - 2)(a + c) + 2(2w_h - s_h) - \gamma(2w_f - s_f)\right) &= 0 \\
\frac{1}{\gamma^2 - 4}\left((\gamma - 2)(a + c) + 2(2w_f - s_f) - \gamma(2w_h - s_h)\right) &= 0
\end{aligned}$$

The prices charged by the monopolist, w_h and w_f, must satisfy these two conditions simultaneously. Solving this system, we get

$$w_i = \frac{1}{2}a + \frac{1}{2}c + \frac{1}{2}s_i \qquad i = h, f. \tag{1.6}$$

Note that the input price that the monopolist charges to firm i, w_i does not depend on the export subsidy (or tax) of country j but only depends on s_i. Also, it does not depend on the degree of product differentiability, γ.

Under the uniform price scheme where $w_i = w_j = w$, the monopolist chooses a single price w that maximizes

$$\pi^M = (w-c)q = (w-c)\frac{1}{\gamma+2}(2a + s_f + s_h - 2w)$$

where $q \equiv q_h + q_f$ denotes the total quantity of final good produced by the downstream firms. The equilibrium input price under the uniform price scheme is

$$w = \frac{1}{2}(a+c) + \frac{1}{2}(\frac{s_i}{2} + \frac{s_j}{2}) \tag{1.7}$$

Unlike the price-discrimination scheme, the optimal input price is a function of an average of the two rates of subsidy. First, note that it is easy to check that the second order condition is satisfied for both pricing schemes. Also, an export subsidy (tax) increases (decreases) the input price charged to the downstream firms via the changes in downstream firms' demand elasticicities for the intermediate good. This suggests that the government should use an export tax to extract rent from the upstream monopolist. However, the effectiveness of the rent extraction is different under the two pricing schemes. A dollar increase in the export tax lowers the input price for its firm by $\frac{1}{2}$ dollars in the price discrimination case compared to $\frac{1}{4}$ dollars in the case of uniform pricing. Moreover, in the case of uniform pricing, an export tax by the home government extracts rent not only from the home firm but also from the foreign firm.

At the first stage, each government chooses the export subsidy (or tax) rate to maximize the national welfare which is defined as the firm's profit net of subsidy payment.

$$
\begin{aligned}
W_i &= \pi_i^C - s_i q_i^C \\
&= \frac{[(\gamma-2)a + 2(w_i - s_i) - \gamma(w_j - s_j)]}{(\gamma^2-4)^2} \\
&\quad \left[(\gamma-2)a + (2w_i + (2-\gamma^2)s_i) - \gamma(w_j - s_j)\right] \\
i &= h,\ f,\ i \neq j,
\end{aligned}
$$

Substituting input prices (both price discrimination and uniform price cases) from stage 2 into country i's welfare function, we get W_i^d and W_i^u, $i = h,\ f$, where they are the country i's welfare function for the case of discriminating and uniform input prices, respectively,

$$
\begin{aligned}
W_i^d &= \frac{[(\gamma-2)(a-c) + \gamma s_j - 2s_i]\,[(\gamma-2)(a-c) + \gamma s_j + (6-2\gamma^2)s_i]}{4(\gamma^2-4)^2} \qquad (1.8) \\
W_i^u &= \frac{[(2\gamma-4)(a-c) + (3\gamma+2)s_j + (-\gamma-6)s_i]}{16(\gamma^2-4)^2} \\
&\quad \left[(2\gamma-4)(a-c) + (3\gamma+2)s_j + \left(10-\gamma-4\gamma^2\right)s_i\right] \\
i &= h,\ f, \qquad i \neq j,
\end{aligned}
$$

Government i maximizes welfare by choosing the subsidy rate s_i taking as given the other government's policy s_j, and this yields its reaction function

$$s_i^d = \frac{(2-\gamma)(2-\gamma^2)}{4(\gamma^2-3)}(a-c) - \frac{\gamma(2-\gamma^2)}{4(\gamma^2-3)}s_j, i = h,\ f, \qquad i \neq j,$$

$$s_i^u = \frac{2(2-\gamma)(-2+\gamma+2\gamma^2)}{(\gamma+6)(10-\gamma-4\gamma^2)}(a-c) - \frac{(3\gamma+2)(-2+\gamma+2\gamma^2)}{(\gamma+6)(10-\gamma-4\gamma^2)}s_j,\ i = h,\ f, \qquad i \neq j,$$

where s_i^d and s_i^u denote the government i's export subsidy in the case of discriminating and uniform pricing schemes, respectively. It is easy to see that the two governments' export policies are strategic substitutes of each other in the discriminatory price scheme; however, in the uniform pricing scheme, whether policies are strategic substitutes or complements depends on the degree of product substitutability. Export subsidy/tax policies are strategic substitute (complement) if and only if $(-2+\gamma+2\gamma^2) > (<) \, 0$. Note that the expression $(-2+\gamma+2\gamma^2)$ is monotonic increasing in γ, and there exists some critical value of γ beyond which this expression is positive. Therefore, if γ is sufficiently large (the two goods are very close substitutes), governments' policies are strategic substitutes to each other.

Consider the case of unilateral intervention by government i where the other government sets its export policy, s_j to be zero. It can be easily checked that $s_i^d(s_j = 0) < 0$ suggesting that the government should use export tax if the monopolist practices price discrimination. As for the uniform price scheme, it, again, depends on the expression $(-2+\gamma+2\gamma^2)$, the larger is γ, the more likely that the government has an unilateral incentive to use export subsidy.

Proposition 1 *(i) The government have an unilateral incentive to impose an export tax under the price-discrimination case.*

(ii) In the case of uniform pricing, there exist a $\widetilde{\gamma} \in [0,1]$ *such that if* γ *is larger (smaller) than* $\widetilde{\gamma}$ *then the government has an unilateral incentive to impose an export subsidy (tax). In fact* $\widetilde{\gamma}$ *is the positive root of* $-2+\gamma+2\gamma^2=0$*, i.e.,* $\widetilde{\gamma} \simeq 0.78078$

One thing worth noting is that $s_i^d(s_j=0) < s_i^u(s_j=0) \; \forall \; \gamma \in [0,1]$ suggesting that the government should impose a higher tax in the case of price discrimination than in the case of uniform pricing. This is because a government's export tax intended to extract rent from the upstream monopolist, operating via a reduction in input price, benefits solely its own firm in the price discrimination case, while in the case of uniform pricing it also benefits the firm in the other country. Note that result (ii) is somewhat different from the result obtained by Bernhofen (1997) where he shows that the government has an unilateral incentive to impose a subsidy in the uniform price case. In our model, whether a tax or a subsidy is optimal depends on the product substitutability. The more (less) close the two goods, the larger the horizontal rent-shifting effect thus the more likely that the government imposes an export subsidy (tax) unilaterally. Indeed, in the limit where $\gamma = 1$ the two goods become homogeneous, the result is consistent with the one of Bernhofen (1997). In the other words, when the goods become less and less substitutable to each other, the horizontal rent-shifting effect becomes smaller and smaller.

Solving two governments reaction curves simultaneously, we obtain the symmetric Nash equilibrium policies for each of the two pricing schemes

$$\begin{aligned} s_C^d &= -\frac{(2-\gamma^2)}{(2\gamma-\gamma^2+6)}(a-c) \\ s_C^u &= -\frac{(2-\gamma-2\gamma^2)}{7\gamma-\gamma^2+14}(a-c) \end{aligned} \tag{1.9}$$

where the subscript C stands for Cournot competition in the downstream market. It can be seen that government imposes an export tax when the upstream monopolist practices price discrimination, and a subsidy (respectively, a tax) in the uniform pricing case if and only if $\gamma - \widetilde{\gamma} > 0$ (respectively, < 0).

The equilibrium welfare can be obtained by substituting (1.9) into (1.8)

$$\begin{aligned} W_C^d &= \frac{3-\gamma^2}{(-\gamma^2+2\gamma+6)^2}(a-c)^2 \\ W_C^u &= \frac{(\gamma+6)(10-\gamma-4\gamma^2)}{4(-7\gamma+\gamma^2-14)^2}(a-c)^2 \end{aligned}$$

It can be easily checked that $W_C^d > W_C^u$ which implies that the country's welfare is always higher when the upstream monopolist practices price discrimination.

Under free trade equilibrium, i.e., when the governments set $s_i = 0$, $i = h, f$, the welfare in the case of price discrimination and uniform price are identical and equal to

$$W_C^F = \frac{1}{4}\frac{(\gamma-2)^2}{(\gamma^2-4)^2}(a-c)^2$$

Comparing this with the bilateral intervention, one can show that both countries are better off intervening with price discrimination upstream, i.e., $W_C^d > W_C^F$. However, for the uniform price case, it again depends on the sign of $(\gamma + 2\gamma^2 - 2)$, if $(\gamma + 2\gamma^2 - 2) > (<) \, 0$,

$\left(W_C^u - W_C^F\right) < (>)\, 0$. In the other words, if γ is sufficiently large, so that $(\gamma + 2\gamma^2 - 2) > 0$, then both countries are worse off in a trade policy war as opposed to free trade when the upstream monopolist charges a uniform price for the intermediate good. We obtain the following proposition

Proposition 2 *In the Cournot competition case, if the upstream monopolist can price discriminates the downstream firms, both countries are better off intervening. However, if the upstream monopolist cannot price discriminate the downstream firms, both countries can be better off by intervening if and only if the degree of product substitutability is sufficiently low, i.e.,* $\gamma < \tilde{\gamma} \simeq 0.78078$. *Otherwise, both countries are better off in a trade war than under free trade.*

1.4 Bertrand Equilibrium

When the downstream firms set prices in the final stage sub-game, firm i chooses p_i to maximize its profit

$$\pi_i = (p_i - w_i + s_i)q_i = \frac{1}{1-\gamma^2}\left(p_i + s_i - w_i\right)\left(a - p_i - a\gamma + \gamma p_j\right), \qquad i = h,\ f,\ j \neq i,$$

In the Bertrand-Nash equilibrium we obtain

$$p_i^B = \frac{(2+\gamma)(1-\gamma)a + 2(w_i - s_i) + \gamma(w_j - s_j)}{4-\gamma^2}, \qquad q_i^B = \frac{(2+\gamma)(1-\gamma)a - (2-\gamma^2)(w_i - s_i) + \gamma(w_j - s_j)}{(4-\gamma^2)(1-\gamma^2)},$$
$$i = 1, 2,\ j \neq i,$$

The equilibrium profit for firm i is thus

$$\pi_i^B = \frac{\left((\gamma^2+\gamma-2)\,a+(2-\gamma^2)\,(w_i-s_i)-\gamma\,(w_j-s_j)\right)^2}{(1-\gamma^2)\,(\gamma^2-4)^2} \qquad i=1,2,j\neq i,$$

One can see that, ceteris paribus, a decrease in s_i, i.e. an increase in export tax or a decrease in export subsidy, increases firm i's profit. This suggests that the government should instead use an export tax as the instrument to extract the "horizontal rent". This is consistent with Eaton and Grossman (1986) when firms compete as Bertrand rivals rather than as Cournot rivals.

At the second stage, the upstream monopolist, M, announces the prices of intermediate good sold to downstream firms. In the case of price discrimination, using the Bertrand equilibrium quantities and (1.5), the problem facing the intermediate-good monopolist is

$$\max_{w_i,w_j}\left((w_i-c)q_i^B+(w_j-c)q_j^B\right)$$

The profit-maximizing pricing rule is

$$w_i=\frac{1}{2}a+\frac{1}{2}c+\frac{1}{2}s_i \qquad i=h,f.$$

Interestingly, the rule is the same as in the case where the downstream firms engage in the Cournot competition. The same is true for uniform pricing scheme where

$$w=\frac{1}{2}(a+c)+\frac{1}{2}(\frac{s_i}{2}+\frac{s_j}{2})$$

One should note that although the upstream monopolist's pricing rule is identical no matter what competition modes the downstream firms engage in, it is not necessary true for equilibrium input prices to be the same in two competition modes. It is because the governments' export policies are, in general, different under different competition modes.

At the first stage, the government chooses the export subsidy (tax) rate to maximize its national welfare, which is the firm's profit net of subsidy payment.

$$\begin{aligned} W_i &= \pi_i^B - s_i q_i^B \\ &= \frac{1}{(1-\gamma^2)(\gamma^2-4)^2}\left[\left(\gamma^2+\gamma-2\right)a + 2s_i + \left(2-\gamma^2\right)w_i - \gamma\left(w_j - s_j\right)\right] \\ &\quad \left[\left(\gamma^2+\gamma-2\right)a + \left(2-\gamma^2\right)\left(w_i - s_i\right) - \gamma\left(w_j - s_j\right)\right] \\ i &= h,\ f,\ i \neq j, \end{aligned}$$

Substituting input prices from stage 2 into country i's welfare function, we get W_i^d and W_i^u, $i = h, f$, where they are country i's welfare function for the case of discriminating and uniform input prices, respectively,

$$\begin{aligned} W_i^d &= \frac{1}{4(1-\gamma^2)(\gamma^2-4)^2}\left[\left(\gamma^2+\gamma-2\right)(a-c) - \left(2-\gamma^2\right)s_i + \gamma s_j\right] \\ &\quad \left[\left(\left(\gamma^2+\gamma-2\right)(a-c) + \left(6-\gamma^2\right)s_i + \gamma s_j\right)\right] \\ W_i^u &= \frac{1}{16(1-\gamma^2)(\gamma^2-4)^2}\left[\left(2\gamma^2+2\gamma-4\right)(a-c) + \left(3\gamma^2-\gamma-6\right)s_i + \left(3\gamma-\gamma^2+2\right)s_j\right] \\ &\quad \left[\left(2\gamma^2+2\gamma-4\right)(a-c) + \left(10-\gamma-\gamma^2\right)s_i + \left(3\gamma-\gamma^2+2\right)s_j\right] \\ i &= h,\ f,\ i \neq j, \end{aligned} \tag{1.10}$$

Government i maximizes welfare by choosing the subsidy rate s_i taking as given the other government's policy s_j, and this yields its reaction function

$$\begin{array}{l} s_i^d = \frac{\left(2\gamma^2+2\gamma-4\right)}{(\gamma^2-6)(\gamma^2-2)}\left(a-c\right)+\frac{2\gamma}{(\gamma^2-6)(\gamma^2-2)}s_j \\ s_i^u = \frac{2(\gamma-1)(\gamma+2)\left(-\gamma+\gamma^2+2\right)}{(\gamma+\gamma^2-10)(-\gamma+3\gamma^2-6)}\left(a-c\right)+\frac{\left(3\gamma-\gamma^2+2\right)\left(-\gamma+\gamma^2+2\right)}{(\gamma+\gamma^2-10)(-\gamma+3\gamma^2-6)}s_j \end{array}, \quad i=h,\ f,\ i\neq j,$$

where s_i^d and s_i^u denote the government i's export subsidy in the case of discriminating and uniform pricing schemes, respectively. It is easy to see that , unlike the case of Cournot competition, governments view each other's policy as strategic complements under both pricing schemes. Moreover, governments have unilateral incentives to impose taxes under both pricing schemes. The reason for this result is as follows: while the vertical rent extraction still suggests an export tax, the horizontal rent extract points also to an export tax instead of an export subsidy as in the case of Cournot competition. Therefore, both governments have incentive to impose tax in the equilibrium. The symmetric equilibrium policy is

$$\begin{aligned} s_B^d &= -\frac{2\left(1-\gamma\right)}{-4\gamma-2\gamma^2+\gamma^3+6}\left(a-c\right)<0 \\ s_B^u &= -\frac{\left(1-\gamma\right)\left(-\gamma+\gamma^2+2\right)}{2\gamma^3-5\gamma^2-7\gamma+14}\left(a-c\right)<0 \end{aligned} \tag{1.11}$$

where the subscript B stands for Bertrand competition in the downstream market. Comparing taxes in two pricing schemes, we find that

$$s_B^d - s_B^u = -\frac{\left(2-\gamma^3-2\gamma^2+\gamma\right)\left(2-\gamma\right)^3}{\left(-4\gamma-2\gamma^2+\gamma^3+6\right)\left(-7\gamma-5\gamma^2+2\gamma^3+14\right)}\left(a-c\right)$$

It is easy to verified that $s_B^d - s_B^u < 0$ indicating that governments impose higher taxes in the price discrimination case. This result is similar to the one under Cournot competition case because of the rent spill-over effect in the uniform pricing scheme.

The equilibrium welfare can be obtained by substituting (1.11) into (1.10)

$$\begin{aligned} W_C^d &= \frac{(1-\gamma)(6-\gamma^2)(2-\gamma^2)}{4(\gamma+1)(-4\gamma-2\gamma^2+\gamma^3+6)^2}(a-c)^2 \\ W_C^u &= \frac{(1-\gamma)(\gamma+\gamma^2-10)(-\gamma+3\gamma^2-6)}{4(\gamma+1)(-7\gamma-5\gamma^2+2\gamma^3+14)^2}(a-c)^2 \end{aligned}$$

It can be checked that $W_B^d > W_B^u$ because of the spill-over effect of vertical rent-shifting in the case of uniform price upstream.

Under the free trade equilibrium, where both governments set $s_i = 0$, $i = h, f$, the welfare in the case of price discrimination and uniform price are identical and equal to

$$W_B^F = \frac{1}{4}\frac{(\gamma^2+\gamma-2)^2}{(1-\gamma^2)(\gamma^2-4)^2}(a-c)^2$$

Comparing this with the bilateral intervention, one can show that both countries are better off intervening under both pricing schemes.

Proposition 3 *When the downstream firms compete as Bertrand rivals, both countries are better off in a trade policy war compared to free trade under both pricing schemes.*

1.5 Comparison

In this section, we compare the equilibrium values of quantities, final-good prices, firms' profits, input-prices, export subsidies (taxes), and welfare where these values are provided in the appendix. For all variables, the superscript describes the pricing schemes, d and u for price discrimination and uniform price, respectively. The subscript denotes the competition modes where C and B represents Cournot and Bertrand competitions respectively.

We first focus on the case where the upstream monopolist can segment the markets and charge different prices of inputs sold to downstream firms. All the equilibrium values of relevant variables are provided in the appendix. Comparing trade policies, we get

$$s_C^d - s_B^d = \frac{(4 - 4\gamma - 2\gamma^2 + \gamma^3)\,\gamma^2}{(6 + 2\gamma - \gamma^2)\,(-4\gamma - 2\gamma^2 + \gamma^3 + 6)}\,(a - c)$$

It is easy to see that the denominator is positive. Thus, the sign of $\left(s_C^d - s_B^d\right)$ depends on the expression $(4 - 4\gamma - 2\gamma^2 + \gamma^3)$ where it can take positive or negative values for $\gamma \in [0, 1]$. Moreover, $(4 - 2\gamma^2 - 4\gamma + \gamma^3)$ is monotonically decreasing in γ, therefore, for γ sufficiently large, i.e. $\gamma > \widehat{\gamma} \simeq 0.80606$, $(4 - 2\gamma^2 - 4\gamma + \gamma^3) < 0$ which implies $s_C^d - s_B^d < 0$. First, note that if the upstream monopolist price discriminates the downstream duopoly, the governments impose taxes in both Cournot and Bertrand equilibria. Thus we can conclude that if the two goods are sufficiently close to each other, then the export tax imposed by the government in the Cournot equilibrium is higher than that in the Bertrand equilibrium which gives $s_C^d - s_B^d < 0$. The comparison of the equilibrium input prices under the two competition modes is similar to that of the export policy. In fact, by inspecting the equation (1.6) we find there is a monotonic and one-to-one mapping from export tax (subsidy) to input price. It is easy to show that $q_C^d < q_B^d$ and $p_C^d > p_B^d$ which is consistent with the result of Vives (1985) that Bertrand competition yields a more competitive outcome than Cournot in a duopoly setting where goods are substitutes to each other. We also find out that firm's profit is higher under Cournot competition than Bertrand competition. As for welfare, we can show that

$$W_C^d - W_B^d = -\frac{\left(-22\gamma^2 - \gamma^3 + 3\gamma^4 + 36\right)\gamma^2\left(4 - 2\gamma^2 - 4\gamma + \gamma^3\right)}{4\left(\gamma + 1\right)\left(-2\gamma + \gamma^2 - 6\right)^2\left(-4\gamma - 2\gamma^2 + \gamma^3 + 6\right)^2}\left(a - c\right)^2$$

Whether welfare is higher under Cournot or Bertrand competition again depends on the expression $(4 - 2\gamma^2 - 4\gamma + \gamma^3)$. The welfare level is higher under Cournot competition, i.e., $W_C^d > W_B^d$, if and only if $(4 - 2\gamma^2 - 4\gamma + \gamma^3) > 0$. Thus, if γ is sufficiently large, then $(4 - 2\gamma^2 - 4\gamma + \gamma^3) < 0$ and $W_C^d > W_B^d$. We summarize these results in the following lemma

Lemma 4 *In the case where the upstream monopolist price discriminates the downstream firms, the export tax and welfare are lower under Bertrand competition than under Cournot competition if and only if* $\gamma \geq \widehat{\gamma} \simeq 0.80606$.

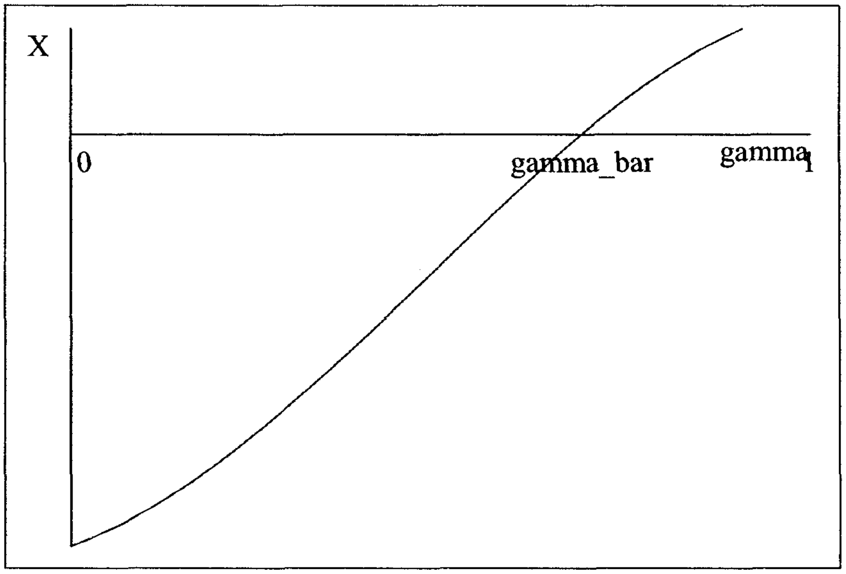
We now turn to the comparison between Cournot and Bertrand equilibria when the upstream firm has to charge a uniform price for intermediate goods. Unlike the price discrimination case, it is easy to show that $s_C^u > s_B^u$ for all values of $\gamma \in [0, 1]$. Recall that under Bertrand equilibrium, we have shown that the optimal policy is a tax no matter the monopolist can price discriminate or not; however, under Cournot competition, the optimal policy can be a tax or a subsidy depending on the substitutability of two goods. If it is such that $s_C^u > 0$, (i.e., an export subsidy in Cournot equilibrium), then it is obvious that $s_C^u > s_B^u$ where $s_B^u < 0$. If γ is sufficiently small and the governments impose a tax in the Cournot equilibrium, i.e. $s_C^u < 0$, they will impose an even higher tax rate in the Bertrand equilibrium. By looking at (1.7), it is easy to see that $w_C^u > w_B^u$. Again, the result of Vives (1985) holds in this case where $q_C^u < q_B^u$ and $p_C^u > p_B^u$. Similar to the case of price discrimination, firm's profit in the Cournot equilibrium is higher than in the Bertrand equilibrium. For welfare, we can show that

$$W_C^u - W_B^u = \frac{\gamma^2 X}{4(\gamma+1)(2\gamma^5 - 19\gamma^4 + 133\gamma^2 - 196)^2}(a-c)^2$$

where $X = (-16\gamma^8 - 33\gamma^7 + 465\gamma^6 + 297\gamma^5 - 3113\gamma^4 - 1560\gamma^3 + 6872\gamma^2 + 2576\gamma - 4368)$

It is clear that the sign of $(W_C^u - W_B^u)$ is the same as that of X. We represent X on figure 1.1

Figure 1.1



One can see that X is monotonically increasing in γ for $\gamma \in [0, 1]$ and intersects the horizontal axis at $\overline{\gamma} \simeq 0.76101$ where $X \geq (<) \ 0$ if and only if $\gamma \geq \overline{\gamma}$. From this, we obtain the following lemma

Lemma 5 *When the upstream monopolist charges an uniform price, a country's welfare is higher (lower) in the Cournot than in the Bertrand equilibrium if and only if* $\gamma > (<) \ \overline{\gamma}$ *where* $\overline{\gamma} \simeq 0.76101$.

Combining the two lemmas, we get the following proposition.

Proposition 6 *Given that* $\overline{\gamma} \simeq 0.76101$ *and* $\widehat{\gamma} \simeq 0.80606$, *if* γ *is sufficiently small where* $\gamma < \overline{\gamma}$, *the welfare is lower in the Cournot equilibrium than in the Bertrand equilibrium despite the pricing scheme. If* γ *has a intermediate value such that* $\overline{\gamma} < \gamma < \widehat{\gamma}$, *then* *under the price discrimination case (uniform price case), Cournot competition yields a higher (lower) welfare level compared to Bertrand competition. Given that the two goods are sufficiently close to each other such that $\gamma > \widehat{\gamma}$, Bertrand competition yields a higher welfare than Cournot competition under both pricing schemes.*

Finally, we can find out the consumers' surplus under various competition modes and pricing schemes. The consumers' surplus, denoted by CS, will be the utility less the expenditures on these two goods

$$CS = U(\mathbf{Q}, z) - \sum_{i=h,f} p_i q_i - z$$

Using (1.1) and due to symmetry, we get

$$CS = (2a - (1+\gamma)\, q - 2p)\, q \tag{1.12}$$

Substituting q_j^k and p_j^k, where $k = d, u$ and $j = C, B$, into (1.12), we get the consumers' surplus under the two competition modes and the two pricing schemes which is denoted by CS_j^k, where $k = d, u$ and $j = C, B$.

$CS_C^d = \frac{(\gamma+1)}{(-\gamma^2+2\gamma+6)^2}(a-c)^2$	$CS_B^d = \frac{(\gamma^2-2)^2}{4(\gamma+1)(-4\gamma-2\gamma^2+\gamma^3+6)^2}(a-c)^2$
$CS_C^u = \frac{(\gamma+1)(\gamma+6)^2}{4(-7\gamma+\gamma^2-14)^2}(a-c)^2$	$CS_B^u = \frac{(-\gamma+3\gamma^2-6)^2}{4(\gamma+1)(-7\gamma-5\gamma^2+2\gamma^3+14)^2}(a-c)^2$

Comparing consumers' surplus in two competition modes under the price discrimination case, we get

$$CS_C^d - CS_B^d = -(2-\gamma)\frac{\gamma^2(\gamma+2)(8\gamma - 20\gamma^2 - 4\gamma^3 + 3\gamma^4 + 24)}{4(\gamma+1)(-2\gamma+\gamma^2-6)^2(-4\gamma - 2\gamma^2 + \gamma^3 + 6)^2}(a-c)^2$$

One can see that $CS_C^d < CS_B^d$ for all $\gamma \leq 1$, thus under Cournot rivalry, the consumer is worse off when the downstream than under Bertrand rivalry.

We now turn to the uniform pricing case,

$$CS_C^u - CS_B^u = -(2-\gamma)\frac{\gamma^2(\gamma+3)(\gamma+2)(56\gamma - 53\gamma^2 - 26\gamma^3 + 6\gamma^4 + \gamma^5 + 84)}{(\gamma+1)(-7\gamma+\gamma^2-14)^2(-7\gamma - 5\gamma^2 + 2\gamma^3 + 14)^2}(a-c)^2$$

It is easy to see that $CS_C^u - CS_B^u < 0$ which implies that the consumer is worse off in the Cournot equilibrium than in the Bertrand equilibrium. Therefore, the standard result that consumers are better off when firms compete in the Bertrand fashion holds regardless of upstream monopolist's pricing schemes. This is consistent with the intuition that, when firms compete in prices, they compete more intensively which results in a higher quantity and a lower price.

It is also interesting to see how consumers' surplus is affected by different pricing schemes imposed by the upstream monopolist. In the case of Cournot competition, taking the difference of CS_C^d and CS_C^u, we get

$$CS_C^d - CS_C^u = -\frac{1}{4}\frac{(\gamma^5 + 5\gamma^4 - 40\gamma^3 - 44\gamma^2 + 128\gamma + 128)(\gamma+2)^2}{(\gamma^4 - 9\gamma^3 - 6\gamma^2 + 70\gamma + 84)^2}(a-c)^2$$

It is obvious that the above expression is negative for all $\gamma \in [0,1]$. Therefore, the consumer is better off if the upstream firm cannot price discriminates downstream firms given that downstream firms compete in the Cournot fashion. As for the Bertrand competition case, we get

$$CS_B^d - CS_B^u = \frac{(1-\gamma)(\gamma+2)(\gamma-2)^2(32\gamma+58\gamma^2-27\gamma^3-12\gamma^4+5\gamma^5-64)}{4(-2\gamma^6+9\gamma^5+5\gamma^4-60\gamma^3+30\gamma^2+98\gamma-84)^2}(a-c)^2$$

The sign of $\left(CS_B^d - CS_B^u\right)$ seems to depend on the sign of $(32\gamma+58\gamma^2-27\gamma^3-12\gamma^4+5\gamma^5-64)$ which may seems difficult to determine at first place. However, we can show that it is negative which implies that $CS_B^d - CS_B^u < 0$ or $CS_B^d < CS_B^u$. To see this, differentiating the expression with respect to γ, we get

$$\frac{d}{d\gamma}\left(32\gamma+58\gamma^2-27\gamma^3-12\gamma^4+5\gamma^5-64\right) = 25\gamma^4-48\gamma^3-81\gamma^2+116\gamma+32$$

One can see that the right-hand side is positive indicating that the expression is monotonic increasing in γ for all value of γ. One can also show that the expression yields a value of -8 when γ approaches 1. Therefore, for all $\gamma \in [0,1]$, the expression yields a negative value.

We thus conclude that the consumers are always better off in a Bertrand duopoly as compared to a Cournot duopoly. They are also better off when the upstream firm cannot practice price discrimination.

1.6 Concluding Remarks

We have taken a model of vertically related markets which consists of a upstream monopolist and a differentiated downstream duopoly to analyze the governments' export policy. We consider both Cournot and Bertrand rivalry between two downstream firms and also two pricing schemes, namely uniform pricing and price discrimination, practiced by the upstream monopolist. We have shown that whether the governments use a tax or a subsidy, and countries' welfare levels in equilibrium depend crucially on competition modes, pricing schemes, as well as the level of substitutability between the two goods.

We have shown that under downstream Cournot competition, the governments impose an export tax in the price discrimination case, and this yields a higher welfare compared to free trade. On the other hand, when the upstream monopolists uses the uniform price scheme, the governments only impose a tax provided the two goods are sufficiently independent to each other, otherwise, a subsidy is called for. Moreover, both countries are better off engaging in a trade policy war compared to free trade in the case of price discrimination, but under uniform pricing scheme, trade war is better for a country if and only if its government imposes a tax in equilibrium. In the case of Bertrand competition, governments impose taxes and are better off in a trade war regardless of the pricing schemes.

We also carry out the comparison between Cournot equilibrium and Bertrand equilibrium given the pricing scheme. In the price discrimination case, the export tax is higher when the downstream firms compete in the Bertrand fashion rather than in the Cournot one if and only if the two goods are sufficiently close to each other. Otherwise, Cournot competition yields a higher tax than Bertrand competition. If the upstream monopolist has to charge a uniform price, then the government always imposes a higher tax in Bertrand competition than in Cournot one. As for the analysis on welfare, the results of the comparison depends on the degree of product substitutability. If the degree of product substitutability is sufficiently small such that the demands for the two goods are relatively independent of each other, countries' welfare are lower in Cournot equilibrium as compared to the one in Bertrand equilibrium, regardless of the pricing schemes. On the other hand, if the degree of product substitutability has a intermediate value, the Cournot equilibrium welfare level is higher than the Bertrand one, given price discrimination by the upstream monopoly, but lower than Bertrand one given uniform pricing. Finally, if the degree of product substitutability is sufficiently large such that the two goods are close substitutes, Cournot competition yields a higher welfare level under both pricing schemes.

Lastly, we have confirmed the result of Vives (1985) that Bertrand competition yields a higher quantity consumed and a lower price of final good given that the two goods are substitutes to each other. In our model, the consumer is always better off when the downstream firms compete as Bertrand rivals, whether the upstream monopolist practice price discrimination or not.

Table 1.1: Equilibrium values in price discrimination case

$s_C^d = -\frac{\left(2-\gamma^2\right)}{2\gamma-\gamma^2+6}\,(a-c)$	$s_B^d = -\frac{2(1-\gamma)}{-4\gamma-2\gamma^2+\gamma^3+6}\,(a-c)$
$w_C^d = \frac{(\gamma+2)a+\left(\gamma-\gamma^2+4\right)c}{2\gamma-\gamma^2+6}$	$w_B^d = \frac{(2-\gamma)\left(2-\gamma^2\right)a+\left(\gamma^3-2\gamma^2-6\gamma+8\right)c}{2\gamma^3-4\gamma^2-8\gamma+12}$
$q_C^d = \frac{1}{-\gamma^2+2\gamma+6}\,(a-c)$	$q_B^d = \frac{2-\gamma^2}{2(\gamma+1)(-4\gamma-2\gamma^2+\gamma^3+6)}\,(a-c)$
$p_C^d = \frac{\left(\gamma-\gamma^2+5\right)a+(\gamma+1)c}{-\gamma^2+2\gamma+6}$	$p_B^d = \frac{\left(2\gamma^3-3\gamma^2-8\gamma+10\right)a+\left(2-\gamma^2\right)c}{2\gamma^3-4\gamma^2-8\gamma+12}$
$\pi_C^d = \frac{1}{(-\gamma^2+2\gamma+6)^2}\,(a-c)^2$	$\pi_B^d = \frac{(1-\gamma)\left(\gamma^2-2\right)^2}{4(\gamma+1)(-4\gamma-2\gamma^2+\gamma^3+6)^2}\,(a-c)^2$
$W_C^d = \frac{3-\gamma^2}{(-\gamma^2+2\gamma+6)^2}\,(a-c)^2$	$W_B^d = \frac{(1-\gamma)\left(6-\gamma^2\right)\left(2-\gamma^2\right)}{4(\gamma+1)(-4\gamma-2\gamma^2+\gamma^3+6)^2}\,(a-c)^2$

Table 1.2: Equilibrium values in uniform price case

$s_C^u = -\frac{\left(2-\gamma-2\gamma^2\right)}{7\gamma-\gamma^2+14}\left(a-c\right)$	$s_B^u = -\frac{(1-\gamma)\left(-\gamma+\gamma^2+2\right)}{2\gamma^3-5\gamma^2-7\gamma+14}\left(a-c\right)$
$w_C^u = \frac{\left(\gamma^2+8\gamma+12\right)a+\left(6\gamma-3\gamma^2+16\right)c}{-2\gamma^2+14\gamma+28}$	$w_B^u = \frac{(\gamma-2)\left(-\gamma+3\gamma^2-6\right)a+\left(\gamma^3-3\gamma^2-10\gamma+16\right)c}{4\gamma^3-10\gamma^2-14\gamma+28}$
$q_C^u = \frac{(\gamma+6)}{-2\gamma^2+14\gamma+28}\left(a-c\right)$	$q_B^u = \frac{-3\gamma^2+\gamma+6}{2(\gamma+1)(-7\gamma-5\gamma^2+2\gamma^3+14)}\left(a-c\right)$
$p_C^u = \frac{\left(7\gamma-3\gamma^2+22\right)a+\left(\gamma^2+7\gamma+6\right)c}{-2\gamma^2+14\gamma+28}$	$p_B^u = \frac{\left(4\gamma^3-7\gamma^2-15\gamma+22\right)a+\left(\gamma-3\gamma^2+6\right)c}{2(-7\gamma-5\gamma^2+2\gamma^3+14)}$
$\pi_C^u = \frac{(\gamma+6)^2}{(-2\gamma^2+14\gamma+28)^2}\left(a-c\right)^2$	$\pi_B^u = \frac{(1-\gamma)\left(-3\gamma^2+\gamma+6\right)^2}{4(\gamma+1)(-7\gamma-5\gamma^2+2\gamma^3+14)^2}\left(a-c\right)^2$
$W_C^u = \frac{(\gamma+6)\left(10-\gamma-4\gamma^2\right)}{4(-7\gamma+\gamma^2-14)^2}\left(a-c\right)^2$	$W_B^u = \frac{(1-\gamma)\left(\gamma+\gamma^2-10\right)\left(-\gamma+3\gamma^2-6\right)}{4(\gamma+1)(-7\gamma-5\gamma^2+2\gamma^3+14)^2}\left(a-c\right)^2$

Chapter 2
Dynamic Tariff War on Exhaustible Resource

2.1 Introduction

This paper presents a model of bilateral monopoly between a coalition resource-importing countries, denoted by M and a coalition of resource-exporting countries, denoted by X. Our main objective is to compare the time path of extraction under bilateral monopoly with that under free-trade with perfectly competitive (price-taking) extractive firms and price-taking consumers. For each group of countries, we also compare its welfare level under (world-wide) free trade and that under bilateral monopoly. We also provide some numerical solutions for the case with two importing countries and a resource exporting country where they play a dynamic game of export/import tariff setting.

Concerning welfare, it is well known that world welfare is maximized under free trade. An interesting question to ask is whether there exist parameter values such that one of the two groups of countries is better off under bilateral monopoly than under free trade. In our model, it turns out that, given the rate of discount, there is a corresponding threshold level of the marginal cost parameter beyond which the resource-importing countries would prefer bilateral monopoly to world-wide free trade. The higher is the rate of discount, the greater is the corresponding threshold marginal cost level.

Another related question is the division of gains from trade (whether free-trade, or tariff-ridden trade under bilateral monopoly) between the two groups of countries. Under free-trade, the resource-exporting countries' share of world-wide gains from trade is a function of the cost parameter and the (common) discount rate. This share can be very large, or very small. Under bilateral monopoly, however, two-third of the gains from trade accrues to the resource-exporting countries, regardless of the values of the marginal cost parameter and the discount rate. In the case of two importing countries, we show that asymmetries of market size between two countries also play a role in determining the countries' welfare under free trade or tariff war. We show that as the two importing countries become more asymmetric in terms of their sizes, the gains from trade are more likely to be higher under tariff war than under free trade.

To our knowledge, there has been no formal analysis of trade gains under bilateral monopoly in the market for natural resources in a truly dynamic setting. There are, on the other hand, quite a few models where one side of the market exercises market power while the other side of the market is passive. See the literature review section.

We begin by characterizing a world competitive equilibrium in which an exhaustible resource (say oil) is extracted and traded. We then compare this competitive scenario with the bilateral monopoly (or trade-war) scenario, in which the resource-exporting nation takes over the private deposit and acts as the sole supplier of the extracted resource, exploiting its monopoly power, while the collection of resource-importing countries imposes a tariff on imported oil in order to take advantage of its monopsony power. We compare the extraction paths, the price paths, and the welfare level of each group of countries in the two scenarios.

We also investigate the scenario where the exporting country can choose to commit on a division of resource to serve to two importing countries separately. We find that the optimal division corresponds to the one that divides up the resource deposit according to the two countries' market sizes. An interesting result arises where the exporting country is worse off in this case compared to the case where it has no option to divide up the resource and is required to supply the two importing countries from a common pool. Moreover, the tariff in this three-country case coincides with the tariff in the case where two importing countries form a custom union and charge the same tariff rate. This suggests that if forming a custom union that would give both importing countries higher gains from trade is not possible, an importing country may seek to ask for the commitment from the exporting country to serve it with a fixed portion of the resource deposit, and optimally, the exporting country will have to divide the resource up in a way that the tariff in the three-country case equals to the tariff in the case of cooperation between the two importing countries.

2.2 A Brief Review of the Literature on Market Power in Resources Market

The interest in the exercise of market power by the suppliers (facing price-taking demanders) of natural resource goods has given rise to the theory of resource cartels. Most papers in this area use the concepts of open-loop Nash equilibrium, or open-loop Stackelberg equilibrium. These equilibria are now known to suffer from lack of the (desirable) property of subgame perfection. See Salant (1976), Gilbert (1978), Pindyck (1987), Ulph and Folie (1980), Kemp and Long (1980), among others.

Concerning market power on the demand side (under the assumption that suppliers are price-takers), there is a literature on optimal tariff on exhaustible resources. Bersgstrom (1981) assumes that importing countries are committed to a constant tariff rate from the initial time until the resource-exhaustion time. Brander and Djajic (1983) use the same assumption. Newbery (1976) and Kemp and Long (1980) allow the tariff rate to vary over time, and show that, in the case of zero extraction cost, the optimal open-loop ad valorem tariff rate is a constant. They also point out that such an open-loop Stackelberg equilibrium is time-inconsistent: if the planner is released from his committed time path of tariff rate at some time in the future, he would want to choose a different tariff rate. Maskin and Newbery (1978, 1990) compute the time-consistent tariff rates. Karp and Newbery (1991, 1992) compute time-consistent tariff rates under the assumptions that importers and extractive firms do not move simultaneously.

There are a few papers that treat the case of bilateral monopoly. In a two-period model, Robson (1983) studies the extraction policy of the importing countries who also have their own resource stocks. Lewis, Lindsey and Ware (1986) consider a three-period model in which a coalition of consumers seeks a substitute for an exhaustible resource. In Harris and Vickers (1995), the resource-importing countries try to innovate to reduce reliance on the exhaustible resource. Liski and Tahvonen (2004) study optimal carbon taxes, which are similar in spirit to optimal tariffs. Rubio (2005) studies bilateral monopoly in exhaustible resources, under alternative assumptions about price setting and quantity setting.

2.3 Model

2.3.1 Consumers and firms

There are n resources-importing countries. The representative individual in importing country i has a utility function

$$U_i(q_i, y_i) = (Aq_i - \frac{1}{2\beta_i} q_i^2) + y_i$$

where q_i : consumption of oil, and y_i : consumption of the numeraire good. Also, assume that he is endowed with a flow of income $\overline{y_i}$. The consumer maximizes his utility subject to the budget constraint. This gives the demand for oil

$$\begin{array}{ll} q_i(t) = \beta_i(A - P(t)) & P(t) \in [0, A] \\ q_i(t) = 0 & P(t) > A \end{array}$$

The inverse demand function is:

$$P_i = A - \frac{1}{\beta_i} q_i$$

A is the "choke price" of oil for each importing country. If the price of oil is higher than the choke price, the demand drops to zero. β_i can be interpreted as the size of the market in country i: the larger the β_i, the greater is the demand for a given price of oil. The total demand for a given P and inverse demand facing the resource exporting countries are

$$Q \equiv \sum_{i=1}^{n} q_i = B(A - P(t))$$

$$P(t) = A - \frac{1}{B}Q$$

where $B \equiv \sum_{i=1}^{n} \beta_i$

The $(n+1)^{\text{th}}$ country is the oil exporting country. It consists of a continuum of identical resource-owners where owner j is endowed with a stock S_{j0} of oil, and a flow y_j of numeriare good. For simplicity, we assume that consumers in the resource exporting country do not consume oil so the utility is solely from consumption of the numeraire good:

$$U_{n+1}(y_{n+1}) = y_{n+1}$$

Denote by $E_j(t)$ owner j's extraction rate, we can define the accumulated extraction for j to be $Z_j(t)$:

$$Z_j(t) = \int_0^t E_j(\tau)d\tau \leq S_{j0}$$

We assume that the extraction cost is increasing in the accumulated extraction and is given by

$$C(E_j(t), Z_j(t)) = cZ_j(t)E_j(t)$$

where $c > 0$. The marginal cost of extraction in terms of the numeraire good at time t is then $cZ_j(t)$. This assumption reflects the fact it is more costly to pump up the remaining oil when the remaining stock is small. When $cZ_j(t)$ reaches A, the marginal cost equals to the choke price, and the stock will be abandoned.

Assumption 1: The initial stock is sufficiently large, in the sense that

$$cS_{j0} > A$$

Oil firm faces price path $P^F(t)$ which is not necessarily identical to $P_i(t)$, the oil price that consumers face, if there are import tariff or export tax.

Firms solve the following problem:

$$\max_{E(t)} \int_0^\infty e^{-\rho t}\{p^F(t)E(t) - cZ(t)E(t)\}dt$$

subject to

$$E(t) \geq 0$$

$$\dot{Z} = E(t)$$

with $Z(0) = 0$ and

$$\lim_{t\to\infty} Z(t) \leq S_0$$

Assume that $P^F(t)$ is such that extraction is positive for all t, then the firm's time path of $Z(t)$ must satisfy

$$\dot{P}^F = -\rho\psi = \rho(P^F - cZ)$$

To see this, formulate the Hamiltonian function

$$H = P^F E - cZE + \psi E$$

The necessary conditions are

$$\frac{\partial H}{\partial E} = P^F - cZ + \psi = 0 \tag{2.13}$$

$$\dot{\psi} = \rho\psi - \frac{\partial H}{\partial Z} = \rho\psi + cE$$

$$\dot{Z} = E(t)$$

Differentiating (2.13) with respect to time and using the necessary conditions, we get

$$\dot{P}^F = -\rho\psi = \rho(P^F - cZ)$$

This is the familiar Hotelling's rule: the rate of increase in price divided by "net price", equals the interest rate.

2.3.2 Competitive Equilibrium

The competitive equilibrium consists of a system of $(n+3)$ equations (with 2 boundary conditions)

$$\dot{P} = \rho(P - cZ)$$

$$\dot{Z} = E(t)$$

$$\sum_{i=1}^{n} q_i = E$$

$$q_i = (A - P)\beta_i \qquad i = 1, 2, \ldots, n$$

with $Z(0) = 0$, and $\lim_{t\to\infty} Z(t) \leq S_o$.

Combine the last three equations, we get

$$\dot{Z} = B(A - P)$$

Differentiate it with respect to time,

$$\ddot{Z} = -B\dot{P} = -B\rho(P - cZ) = -B\rho(A - cZ) + \rho\dot{Z}$$

Rearranging terms, we get,

$$\ddot{Z} - \rho\dot{Z} - cB\rho Z = -B\rho A$$

The solution of this second order differential equation is

$$Z(t) = K_1 e^{\lambda_1 t} + K_2 e^{\lambda_2 t} + \frac{A}{c}$$

where $\lambda_1 < 0$ and $\lambda_2 > 0$ are roots of

$$\lambda^2 - \rho\lambda - cB\rho = 0$$

To determine the constant of integration, we use the two boundary conditions $Z(0) = 0$ and $\lim_{t\to\infty} Z(t) \leq S_0$. Since $\lambda_2 > 0$ and $\lim_{t\to\infty} Z(t) \leq S_0$, we must have $K_2 = 0$. Using $Z(0) = 0$, we get $K_1 = \frac{-A}{c}$. Therefore, the solution is

$$Z(t) = \frac{A}{c}(1 - e^{-\kappa t}) \qquad \text{for } t \in [0, \infty) \tag{2.14}$$

where $\kappa \equiv -\lambda_1 > 0$ and is given by

$$\kappa = \frac{1}{2}[(\rho^2 + 4Bc\rho)^{\frac{1}{2}} - \rho]$$

Differentiating (2.14) with respect to time, we get

$$\dot{Z}(t) = \kappa\frac{A}{c}e^{-\kappa t} = \kappa(\frac{A}{c} - Z) > 0$$

The extraction path is

$$E(t) = \dot{Z}(t) = \kappa\frac{A}{c}e^{-\kappa t}$$

$$\frac{\dot{E}}{E} = -\kappa < 0$$

This implies that the extraction is decreasing over time at a constant percentage rate (equals to κ). Note that κ is increasing in B and in the discount rate ρ. Greater impatience implies higher extraction at the beginning. The price path will be $P(t) = A - \frac{1}{B}Q$ where $Q = E(t)$, so

$$P(t) = A\left(1 - \frac{\kappa}{cB}e^{-\kappa t}\right)$$

It is easy to show that $0 < \frac{\kappa}{cnB} < 1$, so $P(t) < A$ for $t \in [0, \infty)$ and $\lim_{t\to\infty} P(t) = A$. We can also express the price path in feedback form as follows:

$$P(Z) = A\left[1 - \frac{\kappa}{cB}(1 - \frac{cZ}{A})\right]$$

Welfare under Free Trade

The consumers' surplus for country i is denoted by S_i

$$S_i = U_i(q_i) - Pq_i = (Aq_i - \frac{1}{2\beta_i}q_i^2) - (A - \frac{1}{\beta_i}q_i)q_i = \frac{1}{2\beta_i}q_i(t)^2$$

Thus the gain from trade by resource-importing country i when it starts to import the oil at time zero is

$$V_i^M = \int_0^\infty e^{-\rho t}(\frac{1}{2\beta_i}q_i(t)^2)dt$$

where $q_i(t) = \frac{\beta_i}{B}Q(t) = \frac{\beta_i}{B}E(t)$. Evaluation of the integral yields

$$V_i^M = (\frac{1}{2})\beta_i(\frac{A}{cB})^2\frac{\kappa^2}{\rho + 2\kappa}$$

It can be shown that holding β_i constant, if B increases (other countries' shares increase relative to i's share), the gain from trade for country i's decreases. Conversely, if β_i increases and B is fixed (some β_j decreases) then the gain from trade for country i will be larger. The profit for the resource-exporting firms at time t is

$$\pi(t) = (P(t) - cZ(t))Q(t)$$

Since we assume the consumers in oil-exporting country do not consume oil, the gain from trade for the oil-exporting country is just the discounted profits

$$V^X = \int_0^\infty e^{-\rho t}\pi(t)dt = B(\frac{A}{cB})^2(\frac{\kappa^3}{\rho(\rho + 2\kappa)})$$

It can be shown that $\frac{\partial V^X}{\partial B} > 0$, so the exporting-country's profit is increasing in total market size.

Define R as the ratio of gain from trade for the exporting country over the aggregate gains for oil-importing countries:

$$R = \frac{V^X}{\sum_i^n V_i^M} = \frac{2\kappa}{\rho} = (1 + \frac{4Bc}{\rho})^{\frac{1}{2}} - 1$$

It can be easily seen that R is increasing in B. The resource-exporting country's share of gains is increasing in cost parameter. The intuition is that given the marginal cost is going to rise faster in the future, the firm will want to sell less in the beginning. The lower volume of sales and the resulting higher price reduce the consumers' welfare in the importing countries. Thus, the share of welfare of of the resource-importing countries decreases in the cost parameter.

2.3.3 Trade Policies Differential Game

Tariff and Export Taxes

Assumes that the government of each oil-importing country i ($i = 1, 2, \ldots, n$) imposes a tariff rate, $T_i(t)$, on each unit of oil imported, and the tariff revenue is distributed back to consumer in a lump sum fashion. Furthermore, we assume that the government of resource-exporting country takes over the resource deposits and behaves as the monopolist. This is equivalent to taxing the export of oil at some rate $T^X(t)$. We consider the following Markovian strategies employed by resource importing and exporting countries. The monopolist announces an export price strategy which is a function of the state variable, $Z(t)$.

$$P^X(t) = P^X(Z(t))$$

Furthermore, the monopolist is committed to satisfy the demand at the stated price. Because of the possibility of tariff imposed by importing countries, the price facing consumers in importing country i is

$$P_i^M(t) = P^X(t) + T_i(t)$$

The quantity demanded at $P_i^M(t)$ is

$$q_i(t) = \beta_i(A - P_i^M(t))$$

The monopolist's extraction in equilibrium is thus

$$E(t) = \sum_{i=1}^{n} \beta_i(A - P_i^M(t)) = B(A - P^X(t)) - \sum_{i=1}^{n} \beta_i T_i$$

The welfare of importing country i at t is the sum of consumers surplus $S_i(t)$ and tariff revenue $R_i(t)$:

$$W_i^M(t) = \frac{\beta_i^2(A - P_i^M(t))^2}{2\beta_i} + \beta_i(A - P_i^M(t))T_i(t)$$

Thus,

$$W_i^M(t) = \frac{1}{2}\beta_i[(A - P^X(t))^2 - T_i(t)^2]$$

Equilibrium Concept and Equilibrium Conditions

The equilibrium concept is Markov-perfect Nash equilibrium where each agent's strategy depends on the current level of the state variable, the total amount of extraction, $Z(t)$.

Importing country i announces a Markovian tariff strategy

$$T_i(t) = T_i(Z(t)) \qquad i = 1, 2, \ldots, n$$

It specifies tariff at t as a function of state variable $Z(t)$ at t. The exporting country also announces a Markovian price strategy

$$P^X(t) = P^X(Z(t))$$

An $n+1$ tuple of Markovian strategies $\{T_1(Z), T_2(Z), \ldots, T_n(Z), P^X(Z)\}$ is called a Markov-perfect Nash equilibrium if :

(i) Given strategies $T_1 = T_1(Z), T_2 = T_2(Z), \ldots T_{i-1} = T_{i-1}(Z), T_{i+1} = T_{i+1}(Z), \ldots,$ $T_n = T_n(Z)$ and $P^X = P^X(Z)$, the tariff strategy for country i, $T_i(Z)$, is a feedback representation of the time path $T_i(t)$ that maximizes the objective function of resource-importing country i:

$$J^i = \max_{T_1(\cdot)} \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \frac{1}{2} \beta_i [(A - P^X(Z))^2 - T_i(t)^2] dt$$

subject to

$$\dot{Z} = B(A - P^X(Z)) - \beta_i T_i(t) - \sum\nolimits_{j \neq i} \beta_j T_j(Z)$$

$$\lim_{t \to \infty} Z(t) \leq S(t_0)$$

(ii) Given tariff strategies $T_1 = T_1(Z)$, $T_2 = T_2(Z), \ldots, T_n = T_n(Z)$, the strategy $P^X(Z)$ is a feedback representation of time path $P^X(t)$ that maximizes **the** objective function of resource-exporting country

$$J^X = \max_{P^X(\cdot)} \int_{t_0}^{\infty} e^{-\rho(t-t_0)}[P^X(t) - cZ(t)][B(A - P^X(t)) - \sum_{i=1}^{n} \beta_i T_i(Z)]dt$$

subject to the same conditions.

To find the Markov-perfect Nash equilibrium, we solve the Hamilton-Jacobi-Bellman (HJB) equations. For importing country i:

$$\rho J^i(Z) = \max_{T_i} \left[\frac{1}{2}\beta_i[(A - P^X(Z))^2 - T_i(t)^2] + J^i_Z(Z)[B(A - P^X(Z)) - \beta_i T_i - \sum\nolimits_{j\neq i} \beta_j T_j(Z)]\right]$$

where

$$J^i_Z(Z) \equiv \frac{dJ^i(Z)}{dZ}$$

The first order condition for maximizing the right-hand side with respect to T_i yields

$$T_i = -J^i_Z(Z)$$

Using the first order condition, the HJB equation becomes

$$\rho J^i(Z) = \frac{1}{2}\beta_i(A-P^X(Z))^2+\frac{1}{2}\beta_i(J^i_Z(Z))^2+B(A-P^X(Z))J^i_Z(Z)+J^i_Z(Z)\sum\nolimits_{j\neq i}\beta_j J^j_Z(Z)$$

The boundary condition is

$$J^i(\frac{A}{c}) = 0$$

where $cZ = A$, i.e., when the marginal extraction cost is equals to choke price, the extraction falls to zero. So the value of the remaining stock is zero.

There will be n HJB equations like this one for n importing countries. The HJB equation for the resource-exporting country is

$$\rho J^X(Z) = \max_{P^X}\left[[P^X - cZ][B(A - P^X) - \sum_{i=1}^{n}\beta_i T_i(Z)] + J_Z^X[B(A - P^X) - \sum_{i=1}^{n}\beta_i T_i(Z)]\right]$$

The first order condition for maximizing the right-hand side with respect to P^X yields

$$P^X = \frac{1}{2B}\left[BA + BcZ - BJ_Z^X - \sum\nolimits_{i=1}^{n}\beta_i T_i(Z)\right] \equiv P^X(Z)$$

Substituting $P^X(Z)$ and $T_i(Z)$ into the monopolist's HJB equation, we get

$$\rho J^X(Z) = \frac{1}{4B}\left[BA - BcZ + BJ_Z^X + \sum\nolimits_{i=1}^{n}\beta_i J_Z^i(Z)\right]^2$$

with boundary condition

$$J^X(A/c) = 0$$

We use the technique of "undetermined coefficients" to solve for the close-loop solution. Assume the value functions, $J^i(Z)$ and $J^X(Z)$, are quadratic in Z,

$$\begin{aligned} J^i(Z) &= U_i + V_i Z + \frac{W_i}{2} Z^2 \qquad i = 1, 2, \ldots, n \\ J^i_Z(Z) &= V_i + W_i Z \end{aligned}$$

$$\begin{aligned} J^X(Z) &= U_X + V_X Z + \frac{W_X}{2} Z^2 \\ J^X_Z &= V_X + W_X Z \end{aligned}$$

We can then, in principle, determine the coefficients U_i, V_i, W_i, U_X, V_X, and W_X, where $i = 1, 2, \ldots, n$, by using the $(n+1)$ HJB equations. It is impossible to obtain analytical solution for the coefficients if there are more than one resource-importing countries even if they are symmetric in market size, so in general we will need to specify numerical values for the parameters. However, we can obtain the analytical solution for some special cases, such as bilateral monopolies and custom union.

2.3.4 Bilateral Monopolies

Suppose there is only one resource-importing country so that the differential game is characterized as a bilateral monopoly trade policies game. Also assume, for simplicity, that the slope of the inverse demand for the oil is unity. We get a pair of differential equations

$$\begin{aligned} \rho J^M(Z) &= \frac{1}{8}\left[A - cZ + J^X_Z(Z) + J^M_Z(Z)\right]^2 \\ \rho J^X(Z) &= \frac{1}{4}\left[A - cZ + J^X_Z(Z) + J^M_Z(Z)\right]^2 \end{aligned}$$

where $J^M(Z)$ is the value function for the resource-importing country. Note that $J^X(Z) = 2J^M(Z) \quad \forall Z$, therefore $J_Z^X(Z) = 2J_Z^M(Z)$. Substitute this into the pair of differential equations we get

$$\rho J^M(Z) = \frac{1}{8}\left[A - cZ + 3J_Z^M(Z)\right]^2$$

Assume that $J^M(Z)$ takes the form

$$J^M(Z) = U + VZ + \frac{W}{2}Z^2$$

where U, V, and W are to be determined. The solution is

$$J^M(Z) = \frac{\mu_1^2}{8\rho}Z^2 - \frac{A\mu_1}{4\rho c}Z + \frac{1}{2}(\frac{A}{c})^2(\frac{\mu_1}{2})^2\frac{1}{\rho}$$

where $\mu_1 = \frac{2}{3}((\rho^2 + 3c\rho)^{1/2} - \rho)$.

It can be verified that $J^M(A/c) = 0$ so the boundary condition is satisfied. Starting at time zero where $Z = 0$, the equilibrium welfare levels of the importing and exporting countries are respectively

$$J^M(0) = \frac{1}{2}(\frac{A}{c})^2(\frac{\mu_1}{2})^2\frac{1}{\rho}$$

$$J^X(0) = (\frac{A}{c})^2(\frac{\mu_1}{2})^2\frac{1}{\rho}$$

The equilibrium strategy pair is $T^M(Z) = (\frac{A}{c} - Z)\frac{\mu_1^2}{4\rho}$ and $P^X(Z) = \frac{A(\rho+\mu_1)\mu_1}{2\rho c} + \frac{2c+\mu_1}{6}Z$. Note that at $Z = \frac{A}{c}$, $T^M(\frac{A}{c}) = 0$ and $P^X(\frac{A}{c}) = A$. The equilibrium path of accumulated extraction is

$$Z(t) = \frac{A}{c}(1 - e^{-\mu_1 t/2})$$

The equilibrium price path and the price function in feedback form are

$$P^X(t) = A[1 - (\frac{2c + \mu_1}{6c})e^{-\mu_1 t/2}]$$

$$P^X(Z) = A[1 - (\frac{2c + \mu_1}{6c})(1 - \frac{cZ}{A})]$$

It is easy to show that $P^X(t) > 0$ for all t. The equilibrium tariff is

$$T^M(t) = \frac{\mu_1^2}{4\rho}(\frac{A}{c})e^{-\mu_1 t/2}$$

The extraction path is

$$E(t) = \frac{\mu_1}{2}(\frac{A}{c})e^{-\mu_1 t/2}$$

and

$$\frac{\dot{E}(t)}{E(t)} = -\frac{\mu_1}{2}$$

One can show that $\frac{\mu_1}{2} < \kappa$ thus initial extraction under bilateral monopoly is smaller than under the free trade scenario. This is consistent with the notion that the monopolist conserves the resources. We can now compare the welfare under policy game with that under free trade. For the resource importing country,

$$V^M(0) = (\frac{1}{2})(\frac{A}{c})^2 \frac{\kappa^2}{\rho + 2\kappa}$$

$$J^M(0) = \frac{1}{2}(\frac{A}{c})^2 (\frac{\mu_1}{2})^2 \frac{1}{\rho}$$

Thus, its welfare is higher under free trade if and only if

$$\frac{\kappa^2}{\rho + 2\kappa} > (\frac{\mu_1}{2})^2 \frac{1}{\rho}$$

This condition can be reduced to

$$LHS = \left(\frac{\sqrt{\rho^2 + 4c\rho} - \rho}{\sqrt{\rho^2 + 3c\rho} - \rho} \right)^2 > \frac{4}{9\rho}(\sqrt{\rho^2 + 4c\rho}) = RHS$$

As $c \longrightarrow 0$, the $LHS \longrightarrow 1$ and the $RHS \longrightarrow \frac{4}{9}$. So free trade is better for the resource-importing country if the extraction cost parameter, c, is small. As $c \longrightarrow \infty$, the $RHS \longrightarrow \frac{4}{3}$ and $LHS \longrightarrow \infty$. Thus bilateral monopoly is better for resource importing country if c is large. Indeed, $\left(\frac{\sqrt{\rho^2+4c\rho}-\rho}{\sqrt{\rho^2+3c\rho}-\rho} \right)^2$ is a decreasing function in c and $\frac{4}{9\rho}(\sqrt{\rho^2 + 4c\rho})$ is an increasing function in c. Therefore for a given value of $\rho \in (0, 1)$, there exists a cut-off value, $\widehat{c}$, such that if $c < \widehat{c}$, free trade is better, and if $c > \widehat{c}$, bilateral monopoly is better for the resource-importing country.

For the resource-exporting country, free trade is better if and only if

$$(\frac{A}{c})^2(\frac{\kappa^3}{\rho(\rho+2\kappa)}) > (\frac{A}{c})^2(\frac{\mu_1}{2})^2\frac{1}{\rho}$$

or

$$(\frac{2\kappa}{\mu_1})^2 > \frac{\rho+2\kappa}{\kappa}$$

$$LHS = \left(\frac{\sqrt{\rho^2+4c\rho}-\rho}{\sqrt{\rho^2+3c\rho}-\rho}\right)^2 > \frac{2}{9c}\left(4c+\rho+\sqrt{\rho^2+4c\rho}\right) = RHS$$

Note that both LHS and RHS are both decreasing functions of c. As $c \rightarrow 0$, $LHS \rightarrow 0$ and $RHS \rightarrow \infty$. Therefore, for a small c, bilateral monopoly benefits the exporting country. As c increases, RHS decreases at a faster rate than LHS so LHS will be larger than RHS for c greater than some critical value, given the discount factor, ρ. In this case, the bilateral monopoly is harmful for the exporting country compared to free trade. As $c \rightarrow \infty$, $LHS \rightarrow 4/3$ and $RHS \rightarrow 8/9$.

2.3.5 Custom Union

Assumes that the governments of all importing-importing countries ($i = 1, 2, \ldots, n$) form a joint government which is allowed to impose a single tariff rate, $T(t)$ for all importing countries, on each unit of oil imported, and the tariff revenue is distributed back to consumer in a lump sum fashion. The monopolist announces an export price strategy which is a function of the state variable, $Z(t)$.

$$P^X(t) = P^X(Z(t))$$

The price facing consumers in importing country i is

$$P^M(t) = P^X(t) + T(t)$$

Quantity demanded in each country i at $P^M(t)$ is

$$q_i(t) = \beta_i(A - P^M(t))$$

The monopolist's extraction in equilibrium is thus

$$E(t) = \dot{Z}(t) = \sum_{i=1}^{n} \beta_i(A - P^M(t)) = B(A - P^X(t) - T(t))$$

Welfare of importing country i at t is the sum of consumers surplus $S_i(t)$ and tariff revenue $R_i(t)$:

$$W_i^M(t) = \frac{\beta_i^2(A - P^M(t))^2}{2\beta_i} + \beta_i(A - P^M(t))T(t)$$

Thus,

$$W_i^M(t) = \frac{1}{2}\beta_i[(A - P^X(t))^2 - T(t)^2]$$

The trade block of importing countries announces a Markovian tariff strategy

$$T(t) = T(Z(t))$$

It specifies tariff at t as a function of state variable $Z(t)$ at t. The exporting countries also announces a Markovian price strategy

$$P^X(t) = P^X(Z(t))$$

The importing coalition's problem:

Given strategies $P^X = P^X(Z)$, the tariff strategy for trade block, $T(Z)$, is a feed back representation of the time path $T(t)$ that maximizes the objective function which is the sum of the joint welfare of importing countries:

$$J^B = \max_{T(\cdot)} \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \sum_{i=1}^{n} \frac{1}{2}\beta_i[(A - P^X(Z))^2 - T(t)^2]dt$$

subject to

$$\dot{Z} = B(A - P^X(Z) - T(t))$$

$$\lim_{t\to\infty} Z(t) \leq S(t_0)$$

The exporting country's problem:

Given the tariff strategy $T = T(Z)$, the exporting country's strategy $P^X(Z)$ is a feedback representation of time path $P^X(t)$ that maximizes objective function of resource-exporting country

$$J^X = \max_{P^X(\cdot)} \int_{t_0}^{\infty} e^{-\rho(t-t_0)}[P^X(t) - cZ(t)][B(A - P^X(t) - T(Z))]dt$$

subject to the same conditions.

To find the Markov-perfect Nash equilibrium, we solve the Hamilton-Jacobi-Bellman (HJB) equations. For the trade block:

$$\rho J^B(Z) = \max_T \left[\frac{1}{2}B[(A - P^X(Z))^2 - T(t)^2] + J_Z^B(Z)[B((A - P^X(Z)) - T(t))]\right]$$

where

$$J_Z^B(Z) \equiv \frac{dJ^B(Z)}{dZ}$$

The first order condition on the RHS yields

$$T = -J_Z^B(Z)$$

Using the first order condition, the HJB equation becomes

$$\rho J^B(Z) = \frac{1}{2}B(A - P^X(Z))^2 + \frac{1}{2}B(J_Z^B(Z))^2 + B(A - P^X(Z))J_Z^B(Z)$$

$$\rho J^B(Z) = \frac{1}{2}B[A - P^X(Z) + J_Z^B(Z)]^2$$

The boundary condition is

$$J^B(\frac{A}{c}) = 0$$

Thus when $cZ = A$, the marginal extraction cost is equal to the choke price, the extraction falls to zero.

The HJB equation of resource-exporting country

$$\rho J^X(Z) = \max_{P^X} \left[[P^X - cZ][B(A - P^X - T(Z))] + J_Z^X [B(A - P^X - T(Z))] \right]$$

The first order condition on the RHS yields

$$P^X = \frac{1}{2} \left[A + cZ - J_Z^X - T(Z) \right] \equiv P^X(Z)$$

Substitute $P^X(Z)$ into the monopolist's HJB equation

$$\rho J^X(Z) = \frac{1}{4} \left[A - cZ + J_Z^X - T(Z) \right]^2$$

with boundary condition

$$J^X(A/c) = 0$$

Substituting $(T(Z), P^X(Z))$ into the two HJB equations, we get a pair of differential equations

$$\rho J^B(Z) = \frac{1}{8} B \left(A + J_Z^B + J_Z^X - Zc \right)^2$$

$$\rho J^X(Z) = \frac{1}{4}B\left(A + J_Z^B + J_Z^X - Zc\right)^2$$

with the boundary conditions

$$J^X(\frac{A}{c}) = 0 = J^M(\frac{A}{c})$$

It follows that $J^X(Z) = 2J^M(Z)$ and thus $J_Z^X(Z) = 2J_Z^M(Z)$.

Substitution gives

$$\rho J^B(Z) = \frac{1}{8}B\left(A + 3J_Z^B - Zc\right)^2$$

We guess that $J^M(Z)$ takes the form

$$J^M(Z) = -U - VZ - \frac{1}{2}WZ^2$$

where U, V, and W are some constant to be determined. The solution are

$$W = -\frac{1}{9B\rho}\mu_1^2$$

$$V = \frac{1}{9}\frac{A}{Bc\rho}\mu_1^2$$

$$U = -\frac{1}{18}(\frac{A}{c})^2\frac{1}{B\rho}\mu_1^2$$

where

$$\mu_1 = \left((\rho^2 + 3Bc\rho)^{\frac{1}{2}} - \rho\right)$$

The solution is

$$J^M(Z) = \frac{\mu_1^2}{18B\rho}Z^2 - \frac{1}{9}\frac{A\mu_1^2}{Bc\rho}Z + \frac{\mu_1^2}{18}(\frac{A}{c})^2\frac{1}{B\rho}$$

We can verify that $J^M(\frac{A}{c}) = 0$.

Note that although every country in the trade bloc charges the same tariff rate, their welfare can be different due to different market size β_i.. Indeed, the welfare for the bloc and each importing country i at each level of Z are, respectively,

$$J^M(0) = \frac{\mu_1^2}{18}(\frac{A}{c})^2\frac{1}{B\rho}$$

$$J^i(0) = \frac{\beta_i}{B}J^M(0) = \frac{\beta_i}{B}\frac{\mu_1^2}{18}(\frac{A}{c})^2\frac{1}{B\rho}$$

For the resource-exporting country,

$$J^X(0) = \frac{\mu_1^2}{9}(\frac{A}{c})^2\frac{1}{B\rho}$$

It can be shown that both $\frac{\partial}{\partial B}J^M(0) > 0$ and $\frac{\partial}{\partial B}J^X(0) > 0$.

2.4 Numerical Solutions

As mentioned above, if there are more than two resource-importing countries setting tariff rate noncooperatively, we would not be able to obtain any analytical solution. To simplify the analysis, we consider the case where there are one resource exporting and two resource importing countries. The relationship between three countries' HJB equations are not trivial even for the symmetric importing countries case. We will obtain 9 equations with 9 unknowns (coefficients), U_i, V_i, and $W_i \quad i = 1, 2, X$, and these coefficients are not solvable analytically. Since the equilibrium strategy and thus welfare are functions of these coefficients so we cannot obtain an analytical solution. In order to obtain more insight we have calculated numerical solutions for these coefficients by setting $A = 100$, $c = 0.25$, and $\rho = 0.05$. We would like to see how asymmetries in the two importing countries' market size, and the difference in β_i holding total market size B constant, affect their welfare levels as well as exporting country's welfare in various competition modes namely, free trade, noncooperative tariff war, and cooperative tariff setting. We will also investigate the effect of larger total market size, B, on each country's welfare. The welfare for two importing countries under free trade, tariff war, and cooperation are given in Tables 1, 2, and 3.

Table 2.1 Welfare levels of importing countries under free trade

	$\beta_1 = \beta_2$	$\beta_1 = 1.5\beta_2$	$\beta_1 = 2\beta_2$	$\beta_1 = 3\beta_2$	$\beta_1 = 4\beta_2$
$B = 1$	$1400, 1400$	$1681, 1120$	$1867, 934$	$2101, 700$	$2241, 560$
$B = 2$	$1140, 1140$	$1368, 912$	$1520, 760$	$1710, 570$	$1824, 456$
$B = 3$	$990, 990$	$1188, 792$	$1320, 660$	$1485, 495$	$1584, 396$
$B = 4$	$889, 889$	$1067, 711$	$1185, 593$	$1333, 444$	$1422, 356$

Table 2.2 Welfare levels of importing countries under tariff war

	$\beta_1 = \beta_2$	$\beta_1 = 1.5\beta_2$	$\beta_1 = 2\beta_2$	$\beta_1 = 3\beta_2$	$\beta_1 = 4\beta_2$
$B = 1$	$1173, 1173$	$1409, 971$	$1597, 846$	$1888, 690$	$2109, 592$
$B = 2$	$1097, 1097$	$1315, 917$	$1496, 807$	$1788, 674$	$2020, 590$
$B = 3$	$1017, 1017$	$1218, 854$	$1388, 757$	$1669, 639$	$1896, 566$
$B = 4$	$951, 951$	$1139, 801$	$1299, 712$	$1567, 606$	$1787, 541$

Table 2.3 Welfare levels of importing countries under cooperation

	$\beta_1 = \beta_2$	$\beta_1 = 1.5\beta_2$	$\beta_1 = 2\beta_2$	$\beta_1 = 3\beta_2$	$\beta_1 = 4\beta_2$
$B = 1$	$2000, 2000$	$2400, 1600$	$2667, 1333$	$3000, 1000$	$3200, 800$
$B = 2$	$2318, 2318$	$2782, 1855$	$3091, 1546$	$3477, 1159$	$3709, 927$
$B = 3$	$2477, 2477$	$2972, 1981$	$3302, 1651$	$3715, 1238$	$3963, 991$
$B = 4$	$2577, 2577$	$3092, 2061$	$3436, 1718$	$3866, 1288$	$4123, 1031$

Note that the ratio of importing country i's welfare to the combined welfare is the same as the ratio of its market size, β_i, to the total market size, B. It is easy to see that the welfare is highest when the two countries cooperate in tariff setting for all values of β_1, β_2, and B. It can be seen that as the total market size is larger (total demand curve become flatter) the importing countries' welfare are smaller both under tariff war and under free trade. However, surprisingly, the two countries' welfare under cooperation increase as the total market size increase. It is interesting to compare the welfare under free trade and that under tariff war given different β_i. First notice that although each resource-importing country's welfare decreases in both scenarios when the total market size become larger, it decreases at a faster rate under free trade than that under tariff war. So given the constant rate of $\frac{\beta_1}{\beta_2}$, the welfare is higher under free trade given a small value of B, i.e. $B = 1$. As we increase B, the importing country i's welfare under tariff is larger than that under free trade, for example $B = 2$ and $\beta_1 = 3\beta_2$. Indeed, if $B = 4$, the welfare under tariff war will be higher than that under free trade for each value of $\frac{\beta_1}{\beta_2} = 1, \frac{3}{2}, 2, 3, 4$. So, as the total market size increases, the importing country is more likely to be better off under tariff war than under free trade.

As for the effect of asymmetry in importing countries' market size, one can see that as the two importing countries become more asymmetric, the welfare for the smaller country become smaller and smaller as its market size decreases. However, the large (small) country's welfare increases (decreases) at a faster (slower) rate under tariff war than that under free trade. So for small B ($B = 1, 2$), the welfare under free trade is higher starting from symmetric markét size, and the welfare under tariff war eventually becomes higher for both importing countries Therefore, we conclude that the welfare for importing countries are likely to be higher under tariff war than that under free trade if they are more asymmetric in their market size.

Table 2.4 Welfare of the exporting country

	free trade	Coop.	Tariff War				
			$\beta_1 = \beta_2$	$\beta_1 = 1.5\beta_2$	$\beta_1 = 2\beta_2$	$\beta_1 = 3\beta_2$	$\beta_1 = 4\beta_2$
$B = 1$	10034	8000	9919	9877	9800	9633	9484
$B = 2$	12317	9273	11981	11935	11849	11661	11487
$B = 3$	13480	9907	13078	13032	12947	12756	12578
$B = 4$	14222	10307	13796	13752	13668	13481	13304

Since the welfare of the exporting country under free trade and cooperation are the same for all values of $\frac{\beta_1}{\beta_2}$, we report its welfare under free trade, cooperation, and tariff war in Table 4. In contrast with the importing countries, the welfare of the exporting country increases (decreases) as the total market size, B increases under free trade and tariff war (cooperation). Also, different than importing countries, its welfare is the lowest if the importing countries form a custom union compared to free trade and noncooperative tariff war. Moreover, free trade is always better for the exporting country than tariff war. When the importing countries are more asymmetric in their market sizes, the exporting country's welfare decreases under tariff war, therefore, it would prefer the symmetric importing countries case.

Table 2.5 The ratio of welfare levels of two importing countries under tariff war

	$\beta_1 = \beta_2$	$\beta_1 = 1.5\beta_2$	$\beta_1 = 2\beta_2$	$\beta_1 = 3\beta_2$	$\beta_1 = 4\beta_2$
$B = 1$	1	1.45	1.89	2.74	3.56
$B = 2$	1	1.43	1.85	2.65	3.42
$B = 3$	1	1.43	1.83	2.61	3.35
$B = 4$	1	1.42	1.82	2.59	3.30

Table 5 gives the ratio of welfare levels of two importing countries under tariff war. Country 1 is the larger country. The ratio J_1/J_2 exceeds unity. As mentioned before, under free trade or custom union, the ratio of welfare levels is identical to the ratio of market sizes. This is not the case under non-cooperative tariff war. The figures in Table 5 supports the notion of free riding for the smaller country in the noncooperative tariff setting game.

2.5 Optimal Deposit Division

In this section, we consider a scenario where the resource exporting country is forced to divide its oil deposit to serve two importing countries separately. Think of a resource deposit that has a rectangular shape with the depth which is great enough to ensure abandonment before exhaustion. Normalize the length of the deposit to unity. The exporting country has to specify the length of $\alpha \in (0, 1)$, and cut the deposit into two parts to serve two importing countries separately. Without loss of generality, assume it uses the field with length equals to α to serve importing country 1 and the remaining field (with length $(1 - \alpha)$ to importing country 2. We have a two-stage game structure. This problem can be solved by backward induction. In the second stage, given α, the resource exporting country plays with each importing country separately in choosing the price of oil. As we showed earlier, the analytical solution can be obtained in the case of bilateral monopoly. We can thus find the equilibrium welfare for exporting country as a function of α. In the first stage, the exporting country maximizes its welfare by choosing the optimal division, α

$$\max_{\alpha}\left[\begin{array}{c} 2A^2\alpha\frac{\beta_1}{6c\beta_1+4\alpha\sqrt{\frac{1}{\alpha}(\alpha\rho^2+3c\beta_1\rho)}+4\alpha\rho} \\ +2A^2(1-\alpha)\frac{\beta_2}{6c\beta_2+4(1-\alpha)\sqrt{\frac{1}{(1-\alpha)}((1-\alpha)\rho^2+3c\beta_2\rho)}+4(1-\alpha)\rho} \end{array}\right]$$

We can show that the first order conditions are satisfied if the exporting country sets $\alpha = \frac{\beta_1}{B}$ and the second order conditions are also satisfied. This implies that the exporting country should divide the resource to serve two countries according to their relative market size. More surprisingly, if $\alpha = \frac{\beta_1}{B}$, the two importing countries will charge the same tariff rate and this rate is equal to the cooperative rate as mentioned in the previous section. We know that the cooperation make the importing countries better off and the exporting country worse off in the original differential game. It follows that if the exporting country is forced to commit right from the beginning to a fixed (though optimally chosen) division, its welfare will be lower than the non-commitment case. Another implication is that if forming a coalition of two importing countries is not possible, an importing country can gain by forcing the oil exporting country to commit on a fixed division of its deposit to serve it.

2.6 Concluding Remarks

By setting up a model of dynamic trade policy war between a country which extracts an exhaustible resource and one or more resource importing countries, we have been able to obtain a number of interesting results. In the case of one importing country, we are able to obtain the Markov perfect equilibrium tariff rate and export price. We then compare the tariff war equilibrium with the free trade equilibrium. We find that the initial extraction rate under bilateral monopoly is lower than that under free trade. This is consistent with the notion that the monopolist conserves the resources. Furthermore, we determine the effects of a higher extraction cost or higher discount rate on the welfare level of a resource importing country, and of a resource exporting country. The lower the extraction cost parameter, the more likely the resource exporting (importing) country is going to better (worse) off under tariff war compared to free trade. Indeed, there exists a threshold level of marginal cost parameter beyond which the oil importing country benefits from bilateral monopoly, and the higher the rate of discount, the greater is the corresponding threshold marginal cost level. We also obtain some result for the division of gains from trade between the two countries. The resource-exporting country's share of gains is increasing in the cost parameter and decreasing in the discount rate under free trade equilibrium. Under tariff war, two-third of gains from trade accrues to the resource-exporting countries regardless of parameter values.

We also generalize the model to the case of multiple resource-importing countries to analyze the effect of asymmetry between importing countries' market sizes on their welfare under various competition modes, namely, free trade, tariff war, and custom union. Due to the complexity of the HJB equations, the analytical solution is not obtainable, so we can only get the numerical solutions by specifying numerical values to model parameters. In the case of two importing countries and one exporting country, we find that the exporting (importing) country's welfare is increasing (decreasing) in the total market size under either free trade or tariff war but decreasing (increasing) under custom union formed by two importing countries. Another interesting result is that as the asymmetry between two importing countries' market sizes increases, each importing (exporting) country's welfare increases (decreases) in both free trade and tariff war cases. The exporting country's welfare is always higher in the free trade scenario compared to tariff war regardless of the degree of asymmetry. In contrast, whether free trade is better for an importing country as compared to tariff war depends on how asymmetric the market sizes are. Although the welfare levels of importing countries are increasing in the degree of asymmetry of market sizes in both cases, they increase at a faster rate under tariff war than free trade when the asymmetry becomes larger given that the total market size is big enough. Therefore, it is more likely for the importing countries' gain from trade to be larger under tariff war than free trade if the asymmetries in market sizes are large.

We discuss a special case where the exporting country has to commit on a division of its deposit of resource to serve to two countries separately. We argue that the optimal division is the one which splits the deposit according to importing countries' relative market size. The corresponding tariffs are the same as in the case where they form a custom union.

This implies that the exporting country is worse off compared to the case where it serves two importing countries with a common deposit.

Chapter 3
International Trade in Cultural Goods under Dynamic Adjustment of Consumer Preference

3.1 Introduction

Due to the globalization, more and more countries are concerned that their cultural goods are being threatened by international cultural hegemony. In particular they are afraid of being swamped by the US cultural goods. These goods include movies, television programs, newspapers and magazines. The focus of the literature on this topic has been to study the effect of the effort by local governments to protect their cultural heritage. To our knowledge, little has been said about the incentive of individual firms to drive out their rivals in order to achieve cultural hegemony. Thus we are motivated to analyze this particular issue.

We construct a model of competition between two producers from two different countries, each producing a cultural good. To fix ideas, assume that the two firms are a Hollywood movie producer and a French auteur movie producer, and the competition takes place in the French market only. The consumers are assumed to have a homogenous valuation of the Hollywood movie but a heterogenous one of the French movie. We analyze both static and dynamic versions of the model.

In the static model, we consider a two-stage game where in the first stage, the Hollywood firm can choose the quality of its movie, and in the second stage, the two firms engage in price competition to sell the DVD made from their movies. We begin by assuming that given the initial quality levels, when both firms compete as Bertrand rivals by setting their prices, the corresponding market shares are strictly inside the unit interval. In the other words, the equilibrium is interior, and both firms serve the market. We then proceed by assuming that the Hollywood firm can increase the quality of its movie through some costly operation (e.g. more special effects). This will increase consumers' valuation of the Hollywood DVD and thus change the equilibrium at the second stage. We therefore consider both interior and corner solutions. The corner solution represents the case where the level of additional quality of the Hollywood movie is such that the French firm is driven out of the DVD market in the second stage. Depending on consumers' relative valuations between the two goods, the Hollywood firm chooses one of the two strategies on the level of additional quality to put in its movie. One strategy results in an interior solution in the second stage, and the equilibrium market share of the Hollywood firm is increasing in the valuation of the Hollywood movie and decreasing in its cost parameter and in the valuation of French movie. The other strategy results in a corner solution, and the equilibrium price and quality level depend only on the cost parameter. Indeed, we show that there exists a cut-off level of the consumers' valuation of the French DVD such that if the actual valuation is larger (respectively, smaller) than this cut-off level, then the Hollywood firm would choose quality of its movie such that both firms coexist (respectively, the Hollywood firm becomes a monopolist) in the DVD market. The static version of our model is similar to that of Francois and van Ypersele (2002) but with some differences. They assume that the Hollywood producer can only choose between two levels of quality, high or low, and thus two associated fixed cost levels, high or low, in the first stage, and the corresponding equilibrium in the second stage is duopoly with low quality and monopoly with high quality. We, on the other hand, allow the Hollywood firm to endogenously choose the optimal level from a continuum.

We then extend the model using a dynamic setting where we assume that the consumers' valuation of the French auteur movie changes over time, and this is governed by some dynamic process. We postulate that the consumers' valuation of the French movie increases through a network effect and decreases through natural depreciation. The network effect is assumed to be the greatest when the market share of the French movie maker is a half. This is because each consumer who watches the movie can find a person who has not watched the movie and share the story of the movie with him. The depreciation effect implies a decrease in valuation of French movie over time, and it is assumed to be the same among all consumers. If the market shares of the two firms are relatively close to a half then each consumer's valuation of the French movie will increase in the next period because the network effect dominates the depreciation effect. However, if either firm captures the majority of the market, the depreciation effect will dominate. We also introduce a parameter which reflects the speed of the adjustment. We consider the case where only the Hollywood movie maker is a dynamic optimizer, and the French movie maker only cares about the current profit. We formulate an infinite horizon optimal control model, where the state variable is the consumers' valuation of the French movie. We solve for an interior solution where both firms exist in the market with the quality chosen by the Hollywood firm at each time period. There are two steady states. The steady state with a higher valuation of the French DVD is stable in the saddle-point sense and the lower steady state is not stable. Doing comparative statics analysis on the stable steady state, we find that the steady state is increasing in discount factor and decreasing in the cost and the speed of adjustment parameters. Moreover, the comparative statics results are in the same direction for the steady state values of the state variable (preference for French movies) and the control variable (the additional quality of the Hollywood movie). The comparative statics results imply that the more impatient the Hollywood firm is, the higher the quality of its movie. This in turn implies a higher level of consumers valuation of French movies at the steady state. We also find that at the steady state, the quality of Hollywood movie is lower than in the static equilibrium. This can be explained as follows. If the Hollywood firm chooses a slightly higher quality level than the steady-state quality level, its market share will increase in the current period, and its current profit will be higher. However, the consumer preference for the French movie will increase through the network effect which leads to a decrease in the Hollywood firm's future profit. We also discuss the possibility of the corner solution where the Hollywood firm might wish to drive out the French firm for a sufficient amount of time so that the consumer preference for the French movie depreciates to a value low enough such that it can enjoy the monopoly profit afterward. Thus, the Hollywood firm faces a conflict of short-term loss versus long-term gain.

3.2 Literature Review

Most of the formal models in this literature try to determine the conditions under which the national governments should protect their own cultural goods by using means such as cultural tariffs or quotas. These models include Francois and van Ypersele (2002), Rauch and Trindade (2005) and Richardson (2006). Francois and van Ypersele (2002) present a model in which cultural goods are produced by two different countries using increasing return to scale technologies and in which consumers have a homogenous valuation for one good but a heterogenous valuation for the other good. Francois and van Ypersele (2002) is closest to the static part of this paper. However they focus on the government's protection of the local cultural good whereas we focus on the dynamic optimization of the foreign cultural good producer. They show that a cultural tariff by the home government aimed at protecting the home cultural good can indeed raise welfare in both countries. On the other hand, Rauch and Trindade (2005) consider a dynamic model where cultural goods are characterized by some consumption externalities. Production of a new cultural good in one country is affected by the stock of ideas generated by previous cultural production in all countries. They show that the subsidy on cultural good production can be more beneficial than import restrictions on cultural goods from other countries. Richardson (2006) considers a model where two radio stations choosing the combinations of local and international contents to broadcast as well as the price for advertising time with their revenue being derived from sales of advertising time. The consumers are assumed to have some preference over the content of those combinations and get disutility from advertising time. The demand for advertising depends negatively on its price and positively on the station's market share. He shows that a local content requirement (cultural quota) by the government is welfare reducing in the absence of externality. He also finds that the policy of putting a cap on advertising can also be welfare reducing.

Instead of looking at protection of cultural goods by local governments, there are a few articles that focus on the consumption side. Brito and Barros (2005) models the consumption of a cultural good that exhibits habit formation. They solve for the optimal consumption path under a fixed price model and a flexible price model. In the fixed price model, they find that the long run demand for cultural goods is an increasing function of income and decreasing function of relative price. Bala and Long (2005) use the evolutionary approach to show that a preference type can become extinct if a small country is opened to trade in cultural goods with a large country.

3.3 Model

3.3.1 Static Analysis

We use a parable of two movies makers. We call them firm H (the Hollywood firm) and firm F (the French firm). We assume that in each period, each firm makes one movie, and markets the DVD of its movie. Firms compete in one market only (it could be the global market, or the French market). Consumers are uniformly distributed on the interval [0, 1]. Consumers are indexed by a parameter $\gamma \in [0, 1]$. High-γ consumers have stronger preferences for firm F's DVD than low-γ consumers. The enjoyment of a F DVD by consumer γ is $a + \gamma$, where $a > 0$. When time is explicitly introduced, we will use $a(t)$ as a state variable that changes over time according to some process to be specified.

The enjoyment of a H DVD is V, which is the same for all consumers, regardless of their γ values. A γ consumer's net surplus from buying a F DVD is $a + \gamma - q$, where q represents the price of a F DVD. Similarly, any consumer would enjoy a net surplus of $V - p$ from buying a H DVD at the price p. We assume that a consumer will not buy a firm's DVD if her net surplus is negative. Furthermore, we assume that due to time constraint, each consumer buys either zero or one DVD. We consider a two-stage game. In the first stage, firm H is allowed to increase the quality of its movie which leads to an increase in V. In the second stage, the two firms compete in the market given the decision made by firm H in the first stage.

In the second stage, firms use prices as decision variables, and we assume that no firm can practice price discrimination. Furthermore, assume that parameter values are such that any firm, if it is a monopolist, will find that profit maximization occurs at a price that would induce all consumers to participate (i.e., all consumers will buy the DVD). It follows that under duopoly, the whole market will be covered. The variable production cost of DVD is assumed to be zero for both firms. We consider the scenario where $a > 1$. This implies that if F is a "pure" monopolist (i.e., it has no potential rival), it will charge the price $q = a$ and the whole market will be served. We solve the game by backward induction. Let γ^* denote the "pivotal consumer" , i.e., the consumer who is indifferent between buying a Hollywood

DVD and a French DVD given the prices, p and q. The value is given by $V-p = a+\gamma^*-q$ or $\gamma^* = V+q-p-a$. For an interior solution, $\gamma^* \in (0,1)$, where

$$0 < (V-p+q-a) < 1$$

The first inequality is satisfied if and only if $p-q < V-a$. Else if $p-q \geq V-a$, then no consumer will buy a Hollywood DVD, i.e. $\gamma^* = 0$. The second inequality is satisfied iff $q-p < a+1-V$. If $q-p \geq a+1-V$, and then $\gamma^* = 1$, i.e., no one will buy a French DVD. Denote the total revenue function of the Hollywood firm to be π^i. It is given by

$$\pi^i(p,q) = p\gamma^*(p,q)$$

Maximizing π^i with respect to p gives the first order condition for an interior maximum: $(V+q-p-a)-p=0$. This yields the Hollywood firm's reaction function

$$p = \sigma^i(q) = \frac{1}{2}(V-a) + \frac{1}{2}q \tag{3.15}$$

Denote the total revenue (profit) function of the French firm by π:

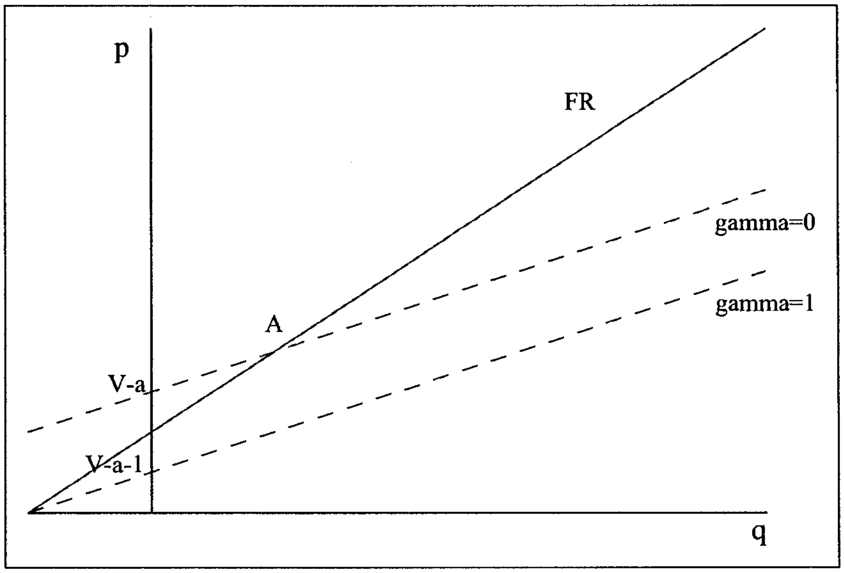
$$\pi(p,q) = q(1-\gamma^*(p,q))$$

Firm F maximizes π with respect to q. This gives the first order condition for an interior maximum: $a-V-2q+p+1=0$. Solving for q yields the French firm's reaction function

$$q = \sigma(p) = \frac{1}{2}(a + 1 - V) + \frac{1}{2}p \tag{3.16}$$

One can see that these reaction curves slope upward indicating that they are strategic complements. We now consider the case of corner solution. For $\gamma \leq 0$, we get $p \geq (V - a) + q$. This implies that if the Hollywood firm chooses any price level, p, such that this inequality is satisfied, its market share will be zero. For $\gamma \geq 1$, we get $p \leq (V - a - 1) + q$. If the price of firm F is so high such that this inequality is satisfied, then the market share of the French firm will become zero ($\gamma = 1$). Combining these two inequalities, we get a band with upper and lower boundaries as shown in Figure 3.1. Since we require that the market share of one firm has to be strictly between the interval [0, 1], the reaction functions of the two firms characterized by (3.15) and (3.16) have to lie inside the band or on its boundaries. Suppose firm F's reaction curve from (3.16) is represented by the curve FR in the figure 3.1. Then its effective reaction curve will be FR up to the point A and the band's upper boundary $\gamma = 0$ beyond point A. There is a kink at point A.

Figure 3.1



Depending on values of valuations of two movies, V and a, many cases can arise. The results are summarized in the following proposition.

Proposition 7 *The equilibrium prices of the Hollywood and the French firms are given by:*

$$p^* = \begin{cases} 0 & \text{if} \quad V-a < -1 \\ \frac{1}{3}(V-a+1) & \text{if} \quad -1 \leq V-a \leq 2 \\ V-a-1 & \text{if} \quad 2 < V-a \end{cases}$$

$$q^* = \begin{cases} a-V & \text{if} \quad V-a < -1 \\ \frac{1}{3}(2-(V-a)) & \text{if} \quad -1 \leq V-a \leq 2 \\ 0 & \text{if} \quad 2 < V-a \end{cases}$$

and resulting market share of firm H is

$$\gamma^*(p,q) = \begin{cases} 0 & \text{if} \quad V - a < -1 \\ \frac{1}{3}(V - a + 1) & \text{if} \quad -1 \leq V - a \leq 2 \\ 1 & \text{if} \quad 2 < V - a \end{cases} \tag{3.17}$$

Proof. See the appendix.

The corresponding equilibrium total revenue for firm H will be

$$R^H(p^*,q^*) = \begin{cases} 0 & \text{if} \quad V - a < -1 \\ \frac{1}{9}(V - a + 1)^2 & \text{if} \quad -1 \leq V - a \leq 2 \\ V - a - 1 & \text{if} \quad 2 < V - a \end{cases} \tag{3.18}$$

One can see that the revenue function for firm H is the square of its market share.

In the first stage, firm H can choose v to improve the quality of its movie where $V = V_0 + v$. We consider restrictions on V_0 such that

Condition 3.1

$$-1 < V_0 - a < 2$$

This condition implies that if the Hollywood firm wishes to leave unchanged the quality of its movie, i.e. $v = 0$, then $V = V_0$, and the interior solution will prevail in the second stage. We assume that the investment aimed at increasing the quality of firm H's movie entails a quadratic cost where $C(v) = \frac{1}{2}cv^2$. This implies that the marginal cost of additional quality, e.g., extra special effects, is increasing. Furthermore assume that c satisfies the following condition.

Condition 3.2

$$c > \frac{1}{3}$$

The profit of the Hollywood firm is its total revenue from DVD sales,namely expression (3.18) in the second stage, minus the cost of additional quality $C(v)$. Denote the profit function by π^H.It is given by

$$\pi^H(v) = R^H(p^*, q^*) - C(v)$$

The Hollywood firm will choose v to maximize its profit. One can see that from (3.18), the objective function will change if v is greater than some critical level. Denote this critical level by x. It is given by

$$x = 2 - (V_0 - a)$$

If the Hollywood firm chooses any v greater than x, then its revenue function will be $V_0 + v - a - 1$. Otherwise, the revenue function will be $\frac{1}{9}(V_0 + v - a + 1)^2$. One can check that at $v = x$, the values of the two revenue functions equal to each other: both equal unity. The firm H solves the following problem

$$\max_v \begin{cases} \pi^C(v) & \text{if} \quad 0 \leq v \leq x \\ \pi^M(v) & \text{if} \quad v < x \end{cases}$$

where

$$\pi^C(v) = \frac{1}{9}(V_0 + v - a + 1)^2 - \frac{1}{2}cv^2 \tag{3.19}$$

and

$$\pi^M(v) = V_0 + v - a - 1 - \frac{1}{2}cv^2 \tag{3.20}$$

It is easy to verify that both π^C and π^M are quadratic in v given Condition 3.2. Suppose we just maximize (3.19) with respect to v, then we can solve for the v which satisfies the first order condition. Call this the optimal interior additional quality and denote it v^C. It is given by

$$v^C = \frac{1}{9c-2}\left(-2a + 2V_0 + 2\right)$$

We can see that the optimal interior additional quality, v^C, is decreasing in a and c but increasing in V_0. Suppose we maximize (3.20) with respect to v and solve for the v which satisfies the first order condition, we get $v = v^M$ where

$$v^M = \frac{1}{c}$$

v^M is the optimal corner solution for the Hollywood firm's additional quality, and it depends only on the cost parameter. Moreover, v^M is decreasing in c as well. It is easy to verify that the second order conditions for both problems are satisfied with Condition 3.2. Given the cost parameter, c, and a, the optimal level of v for firm H is given by

$$v^* = \begin{cases} v^C & \text{if} \quad \pi^C(v^C) \in \arg\max\left(\pi^C(v^C), \pi^M(v^M), \pi(x)\right) \\ v^M & \text{if} \quad \pi^M(v^M) \in \arg\max\left(\pi^C(v^C), \pi^M(v^M), \pi(x)\right) \\ x & \text{if} \quad \pi(x) \in \arg\max\left(\pi^C(v^C), \pi^M(v^M), \pi(x)\right) \end{cases}$$

where

$$\pi^C(v^C) = \frac{c}{9c-2}(V_0 - a + 1)^2$$

and

$$\pi^M(v^M) = \frac{1}{2c}(-2c + 2cV_0 - 2ac + 1)$$

The results are summarized in the following proposition:

Proposition 8 *The equilibrium value of additional quality chosen by firm H is*

$$v^* = \begin{cases} v^M & \text{if } V_0 - 2 < a \leq a' \\ v^C & \text{if } a' < a < V_0 + 1 \end{cases} \tag{3.21}$$

where

$$v^M = \frac{1}{c} \tag{3.22}$$

$$v^C = \frac{1}{9c-2}(-2a + 2V_0 + 2) \tag{3.23}$$

and

$$a' = \frac{1}{2c^2}\left(2c + 2c^2V_0 + \sqrt{c^3(9c-2)} - 7c^2\right) \tag{3.24}$$

Proof. See the appendix.

Note that a' is the cut-off level of a, where firm H chooses between v^M and v^C. If a is greater than a', then firm H will choose v^C such that both firms coexist in the DVD market. This is because the cost to drive out the French firm, $c(2-(V_0-a))^2$, is too high given that the consumers' valuation of French movie, a, is high enough. In this case, if the Hollywood firm intends to drive out the French firm, the loss of doing so out-weights the gain from being a monopoly in the market. However, if a is less than a', then firm H will choose v^M such that it becomes a monopolist in the DVD market. If we take the limit of a' for c to hit its upper and lower bound, we obtain the following results:

$$\lim_{c\to\infty} a' = V_0 - 2$$

$$\lim_{c\to\frac{1}{3}} a' \simeq V_0 + 0.36603 < V_0 + 1$$

If c is infinitely large, then it follows that any v greater than zero is not sensible. In this case, firm H will not add any amount of quality to its movie, and both firms coexist in the market. We can show that a' is increasing in c

$$\frac{d}{dc}a' = -\frac{1}{2c^2\sqrt{c^3\left(9c-2\right)}}\left(2\sqrt{c^3\left(9c-2\right)}-c^2\right) < 0$$

The cut-off point of a is decreasing in c, and this implies that as the cost parameter becomes larger, firm H is more likely to choose v^C over V^M given a and V_0. So far, we have found the equilibrium prices charged by the two firms in the second stage. This includes both interior and corner solutions. We also solve for the equilibrium quality of the movie chosen by the Hollywood firm in the first stage with different values of parameters. We find that firm H will choose between v^C and V^M given by (3.23) and (3.22) depending on the relative valuations of two goods as well as the cost parameter. The quality level of v^C results in a coexistence of both firms in the market, but that of V^M results in the market being monopolized by firm H. Indeed, there exists a cut-off level of valuation of French movie, a, such that if a is greater (smaller) than this level, firm H chooses v^C (v^M). Moreover, the higher is the cost parameter, the lower is the value of this cut-off level of a. This implies that it is more likely that firm H chooses v^C, and both firms coexist in the market.

3.3.2 Dynamic Analysis

In this section, we assume that the consumers' preference toward the French DVD, a, evolves over time according to a dynamic process. We assume that there will be two factors affecting how a evolves through time. One of them is the network effect and the other is the depreciation effect. The network effect can be explained as follows. Consumers who watch the DVD would like to share it with the people who have not watched it. Thus people like the French movie more in the next period and the valuation of French DVD increases. The network effect is assumed to be at its maximum when the market share for the French movie is a half, so each consumer who watches the movie can find a partner to share the story of the movie. The depreciation effect takes a usual interpretation and it is assumed to be the same for all consumers. Moreover, we assume that the French firm is myopic in the sense that it only maximizes its current profit and does not take into account the change in consumers' preference over time. To fix the idea, we assume the dynamics of the valuation of French movie take the following form

$$\dot{a}(t) = \rho(\gamma^*(t) - \frac{1}{3})(\frac{2}{3} - \gamma^*(t)) \tag{3.25}$$

where ρ is a positive constant and represents the speed of adjustment, and γ^* is the equilibrium market share of the Hollywood DVD at time t.

In this setting, $a(t)$ increases as long as the market share of the Hollywood movie at time t is between $\frac{1}{3}$ and $\frac{2}{3}$. The rate of increase is the greatest when half of the consumers watch the Hollywood movie and the other half watch the French movie. If $\gamma^* < (>)\frac{1}{3}$, too many (few) people are watching DVD from firm F, and the depreciation effect dominates. Correspondingly, a decreases, i.e., $\dot{a}(t) < 0$. Note that the lowest value that $\dot{a}(t)$ can take is $-\frac{2}{9}\rho$ and this happens at $\gamma^*(t) \in \{0, 1\}$. Consider the case where $v \leq 2 - (V_0 - a)$. Then from (3.17), the equilibrium market share is $\gamma^* = \frac{1}{3}(V_0 + v - a + 1)$. Substituting this into (3.25), we get

$$\dot{a}(t) = -\frac{1}{9}\rho(a - V_0 - v)\,(a + 1 - V_0 - v) \tag{3.26}$$

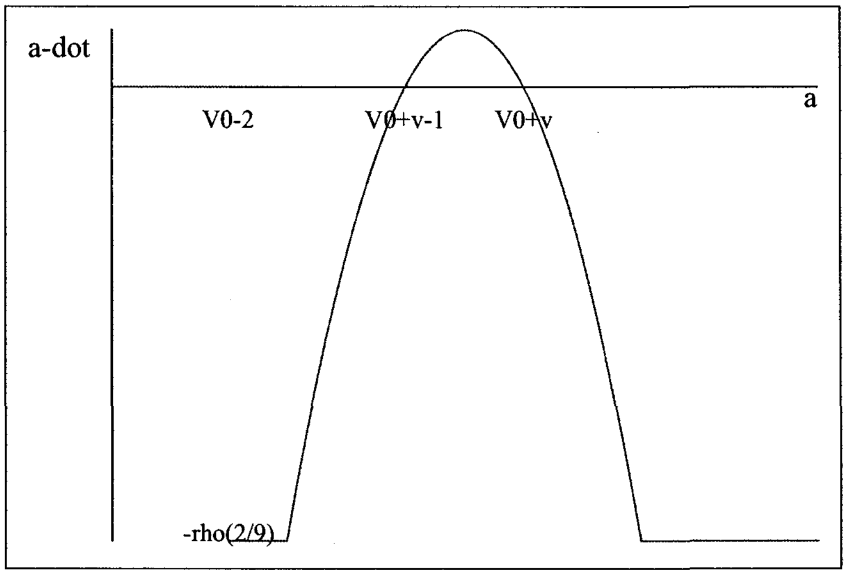
However, for any $v > 2 - (V_0 - a)$, the corresponding equilibrium market share will be unity. Thus $\dot{a}(t)$ is a constant and equals to $-\frac{2}{9}\rho$. Since we require the values of V_0 and a **to** satisfy Condition 1, we set a lower bound on a.

Assumption 3.1:

$$\dot{a} = 0 \quad \text{if} \quad a \leq V_0 - 2$$

This assumption guarantees that a will not be smaller than its lower bound, $V_0 - 2$. Combining the above analysis, we show $\dot{a}(t)$ as a function of a in Figure 3.2.

Figure 3.2:



The dynamic of the popularity of the French movie, a, is given by

$$\dot{a}(t) = \begin{cases} -\frac{1}{9}\rho\,(a - V_0 - v))\,(a + 1 - V_0 - v) & \text{if } \; 0 \leq v \leq a - (V_0 - 2) \\ -\frac{2}{9} & \text{if } \quad a - (V_0 - 2) < v \end{cases}$$

Given the current stock $a(t)$, the instaneous profit function for the Hollywood firm is

$$\pi(t) = \begin{cases} \pi^C = \frac{1}{9}((V_0 + v) - a + 1)^2 - \frac{1}{2}cv^2 & \text{if } \; 0 < v \leq a - (V_0 - 2) \\ \pi^M = V_0 + v - a - 1 - \frac{1}{2}cv^2 & \text{if } \quad a - (V_0 - 2) < v \end{cases}$$

One can see that depending on the control variable, v, there will be two different objective functions. We focus on the interior case where firm H has no incentive to choose any v greater than $a - (V_0 - 2)$. It is sufficient to assume that $v^M < x$, which is implied by Assumption 3.2 below.

Assumption 3.2:

$$a > \frac{1}{c} + V_0 - 2$$

Given Assumption 3.2., firm H has no incentive to further increase v beyond $a - (V_0 - 2)$ because that would necessarily decrease the current profit without bringing down a at any faster rate. The proof of the decrease in the current profit for $v > a - (V_0 - 2)$ can be found in the proof of proposition 2. So we can ignore the part where $v > a - (V_0 - 2)$. Thus, the optimization problem is to find $v(t)$ that maximizes the following:

$$U(a_0) = \int_0^\infty e^{-\delta t} \left(\frac{1}{9}((V_0 + v) - a + 1)^2 - \frac{1}{2}cv^2 \right) dt$$

subject to

$$v \leq a - (V_0 - 2)$$

$$\dot{a} = -\frac{1}{9}\rho\,(a - V_0 - v))\,(a + 1 - V_0 - v)$$

$$a(0) = a_0$$

The current-value Hamiltonian is

$$\widetilde{H} = \left(\frac{1}{9}((V_0+v)-a+1)^2-\frac{1}{2}cv^2\right) + \psi\left(-\frac{1}{9}\rho(a-V_0-v)\,(a+1-V_0-v)\right) \quad (3.27)$$

where ψ is the current valued shadow price.

The corresponding Lagrangian is

$$\mathcal{L} = \widetilde{H} + \lambda(a-(V_0-2)-v)$$

where $\lambda \geq 0$ is the Lagrangian multiplier.

The necessary conditions are

$$\begin{array}{l} \frac{1}{9}\left(1+2a-2v-2V_0\right)\rho\psi + \frac{2}{9}\left(-a+V_0-(\frac{9}{2}c-1)v+1\right)-\lambda = 0 \\ (a-(V_0-2)-v) \geq 0, \quad \lambda \geq 0, \quad \text{and} \quad \lambda(a-(V_0-2)-v)=0 \end{array} \quad (3.28)$$

$$\dot{\psi} = \delta\psi - \frac{\partial\widetilde{H}}{\partial a} = \frac{1}{9}\left(\left(9\frac{\delta}{\rho}-2v+1\right)+2\left(a-V_0\right)\right)\rho\psi + \frac{2}{9}\left(v-a+V_0+1\right) \quad (3.29)$$

$$\dot{a} = \frac{\partial\widetilde{H}}{\partial\psi} = (-\frac{1}{9}\rho\left(a-V_0-v\right))\left(a+1-V_0-v\right) \quad (3.30)$$

Assumption 3.3:

$$\delta > \rho$$

This assumption means that the speed of adjustment of the consumer's preference is not too fast relative to the discounted factor.

We shall divide the orthant in (a, ψ) into regions according to whether or not the constraints on the control is binding.

(a) Suppose that $\lambda > 0$, then $v = a - (V_0 - 2)$. Using (3.28), we get

$$cV_0 - 2c - \frac{1}{3}\psi\rho - ac + \frac{2}{3} = \lambda > 0$$

We can rewrite the above inequality as $\psi < -\frac{1}{\rho}\left(3c(2 - V_0 + a) - 2\right)$ and characterize this as region A in Figure 3.3. In this region, firm H chooses $v = a - (V_0 - 2)$ such that the corner solution prevails.

(b) Suppose that $\lambda = 0$, so $(a - (V_0 - 2) - v) \geq 0$. Using (3.28), we get

$$\psi \geq -\frac{1}{\rho}\left(3c(2 - V_0 + a) - 2\right)$$

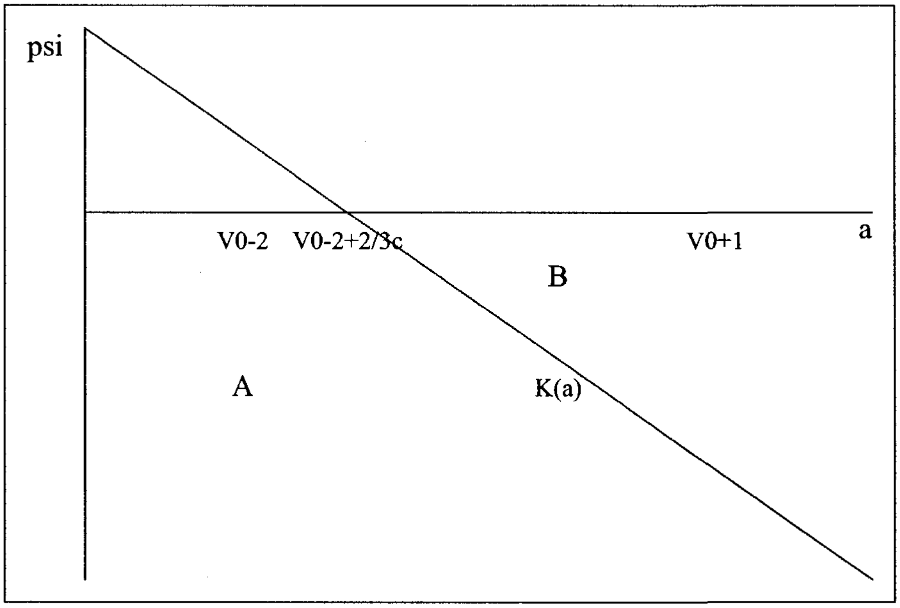
We characterize this as region B in Figure 3.3.

Denote the curve $K(a)$ to be the following

$$\psi = K(a) = -\frac{1}{\rho}\left(3c(2 - V_0 + a) - 2\right) \tag{3.31}$$

$K(a)$ represents the boundary between an interior solution and a corner one. The curve $K(a)$ intersects the horizontal axis at $a = V_0 - 2 + \frac{2}{3c}$ as shown in Figure 3.3. Note that as c increases, the curve $K(a)$ rotates clockwise at $(V_0 - 2, \frac{2}{\rho})$.

Figure 3.3:



From Assumption 3.2, we only analyze the case where $a > \frac{1}{c} + V_0 - 2$, so we only have to consider the area of region A where $a > \frac{1}{c} + V_0 - 2$. Furthermore, at $a = \frac{1}{c} + V_0 - 2$, $B(a) = -\frac{1}{\rho}$, and this implies that $\psi \leq -\frac{1}{\rho}$ in the relevant part of region A. Consider the $\dot{\psi} = 0$ locus and $\dot{a} = 0$ locus in region A. Substituting $v = a - (V_0 - 2)$ into $\dot{\psi}$ and setting it equal to zero, we get

$$\dot{\psi}(v = a - (V_0 - 2)) = \frac{1}{9}\psi\rho\left(9\frac{\delta}{\rho} - 3\right) + \frac{2}{3} = 0$$

The locus is

$$\psi = \frac{2}{-3\delta + \rho}$$

For $\psi > (<) \frac{2}{-3\delta+\rho}$, $\dot{\psi} < (>)0$. If $\frac{2}{-3\delta+\rho} \geq -\frac{1}{\rho}$, then the locus $\dot{\psi} = 0$ will lie outside the relevant area of region A, and $\dot{\psi} > 0$ for every point in region A. However, if $\frac{2}{-3\delta+\rho} < -\frac{1}{\rho}$, then the $\dot{\psi} = 0$ locus will be in relevant area of region A. Due to Assumption 3.3, we can show that this will be the case. Consider the locus for $\dot{a} = 0$ in region A. By substituting $v = a - (V_0 - 2)$ into(3.30) we get

$$\dot{a}(v = a - (V_0 - 2)) = -\frac{2}{9}\rho < 0$$

Firm H will be a monopoly if it sets $v = a - (V_0 - 2)$, and a will depreciate at its maximum rate, $-\frac{2}{9}\rho$.

Consider region B, where $\lambda = 0$. Using (3.28) we get

$$v = \frac{1}{9c + 2\psi\rho - 2}\left(2\left(\psi\rho - 1\right)a + 2(1 - \psi\rho)V_0 + 2 + \psi\rho\right) \tag{3.32}$$

Substituting (3.32) into (3.29) and (3.25) we get a pair of non-linear differential equations

$$\dot{\psi} = \frac{1}{9c+2\psi\rho-2}\left(2\delta\rho\psi^2 + \left(9c\delta - 2\delta + c\rho + 2ac\rho - 2c\rho V_0\right)\psi + \left(2c + 2cV_0 - 2ac\right)\right) \tag{3.33}$$

and

$$\dot{a} = \frac{1}{9}\frac{\rho}{(9c+2\psi\rho-2)^2}\left(9cV_0 + \psi\rho - 9ac + 2\right)\left(9c - 9cV_0 + \psi\rho + 9ac - 4\right) \tag{3.34}$$

To find the locus of $\dot{\psi} = 0$, set (3.33) equal to zero. This holds if and only if $g(a, \psi) = 0$ given that $9c + 2\psi\rho - 2 \neq 0$. The function $g(a, \psi)$ is given by

$$g(a, \psi) \equiv \left(2\delta\rho\psi^2 + (9c\delta - 2\delta + c\rho + 2ac\rho - 2c\rho V_0)\,\psi + (2c + 2cV_0 - 2ac)\right) = 0 \tag{3.35}$$

Setting (3.35) equal to zero and solving for ψ, we obtain two locus. They are given by

$$\psi = -\frac{1}{4\delta\rho}\left((9c-2)\delta + c\rho(2a - 2V_0 + 1) + \sqrt{R}\right) \tag{3.36}$$

and

$$\psi = -\frac{1}{4\delta\rho}\left((9c-2)\delta + c\rho(2a - 2V_0 + 1) - \sqrt{R}\right) \tag{3.37}$$

where

$$\begin{aligned} R \;=\; & 81c^2\delta^2 + c^2\rho^2 + 4\delta^2 - 36c\delta^2 + 4ac^2\rho^2 - 20c\delta\rho - 4c^2\rho^2V_0 + 4a^2c^2\rho^2 + 18c^2\delta\rho \\ & +4c^2\rho^2V_0^2 - 36c^2\delta\rho V_0 + 8ac\delta\rho - 8ac^2\rho^2V_0 - 8c\delta\rho V_0 + 36ac^2\delta\rho \end{aligned}$$

Note that $\frac{d}{da}R > 0$ given that $R > 0$. One can see that if $R = 0$ the two locus intersect, giving the same value of ψ. This value is given by

$$\psi = -\frac{1}{4\delta\rho}\left((9c-2)\delta + c\rho(2a - 2V_0 + 1)\right) \tag{3.38}$$

Combining the equations $R = 0$ and (3.38), we obtain a system two equations with two unknowns. Solving them, we obtain two sets of solutions

$$\begin{aligned} a_1 &= \frac{1}{2c\rho}\left(2\delta - 9c\delta - c\rho + 2\delta\left(2 - \sqrt{6}\sqrt{\frac{c}{\delta}(3\delta+\rho)}\right) + 2c\rho V_0\right) \\ \psi_1 &= \frac{1}{2\rho}\left(2 + \sqrt{6}\sqrt{\frac{c}{\delta}(3\delta+\rho)}\right) \end{aligned}$$

and

$$\begin{aligned} a_2 &= \frac{1}{2c\rho}\left(2\delta - 9c\delta - c\rho - 2\delta\left(2 - \sqrt{6}\sqrt{\frac{c}{\delta}(3\delta+\rho)}\right) + 2c\rho V_0\right) \\ \psi_2 &= \frac{1}{2\rho}\left(2 - \sqrt{6}\sqrt{\frac{c}{\delta}(3\delta+\rho)}\right) \end{aligned}$$

Since $\psi_1 > 0$, we reject the pair (a_1, ψ_1). It can be verified that $\psi_2 < 0$ given our assumptions on parameter values. The derivative of R with respect to a evaluated at a_2 and ψ_2 is

$$\frac{d}{da}R = 4c\rho\left(2\delta + 9c\delta + c\rho + 2ac\rho - 2c\rho V_0\right)$$

It can be shown that this will be greater than zero given the conditions and assumptions of the model. Therefore, the necessary condition for a real valued solution of $\dot{\psi} = 0$ is $a > a_2$. Also, as a increases, R increases so one of the locus is positively sloped and the other one is negatively sloped. The two locus are shown in figure 3.4.

Consider the locus of $\dot{a} = 0$ in region B. From(3.34), $\dot{a} = 0$ if and only if the following equation is satisfied

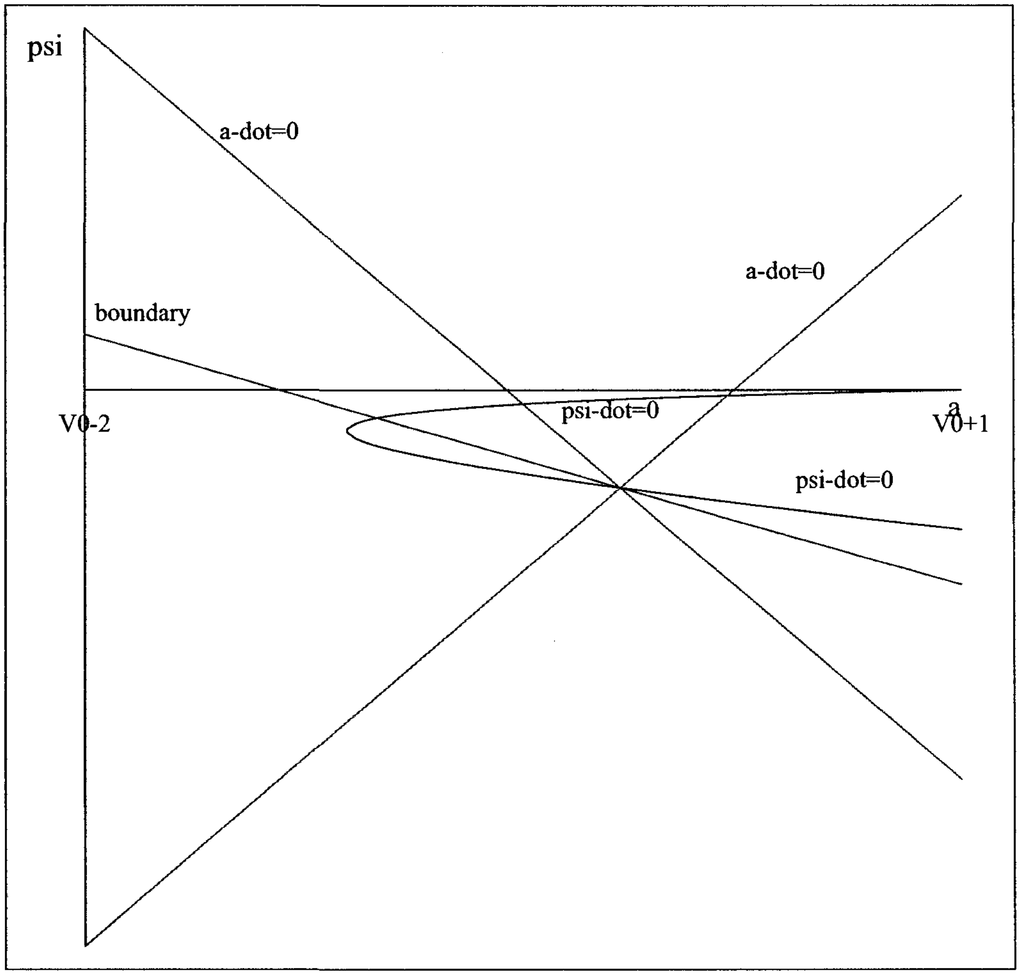
$$\left(9cV_0 + \psi\rho - 9ac + 2\right)\left(9c - 9cV_0 + \psi\rho + 9ac - 4\right) = 0$$

This gives us two locus, and they are given by

$$\psi = \begin{cases} -\frac{1}{\rho}\left(9cV_0 - 9ac + 2\right) \\ -\frac{1}{\rho}\left(9c - 9cV_0 + 9ac - 4\right) \end{cases} \tag{3.39}$$

Note that for (3.39) , one locus is positively sloped straight line and the other is a negatively sloped straight line. They intersect each other at the point where $a = \frac{1}{6c}\left(-3c + 6cV_0 + 2\right)$ and $\psi = -\frac{1}{\rho}\left(\frac{9}{2}c - 1\right)$. Also note that the boundary line that separates the corner and the interior solutions, $\psi = K(a)$, also intersects two $\dot{a} = 0$ locus at the same point. Moreover, the slope of the curve $K(a)$ is flatter than the negatively sloped $\dot{a} = 0$ locus. Therefore, there will always be two steady states. The intersection point is at $\left(a = \frac{1}{6c}\left(-3c + 6cV_0 + 2\right), \psi = -\frac{1}{\rho}\left(\frac{9}{2}c - 1\right)\right)$

Figure 3.4:



Solving $\dot{a} = 0$ and $\dot{\psi} = 0$ simultaneously, we obtain three steady states

$$\begin{array}{ll} a_1^{ss} = \frac{1}{6c}\left(-3c + 6cV_0 + 2\right) & \psi_1^{ss} = -\frac{1}{2\rho}\left(9c - 2\right) \\ a_2^{ss} = \frac{1}{c(9\delta+\rho)}\left(2\delta + cV_0(9\delta + \rho)\right) & \psi_2^{ss} = -\frac{2}{9\delta+\rho} \\ a_3^{ss} = \frac{1}{c(9\delta-\rho)}\left(4\delta + c(V_0 - 1)(9\delta - \rho)\right) & \psi_3^{ss} = -\frac{4}{9\delta-\rho} \end{array} \tag{3.40}$$

If we substitute (a_1^{ss}, ψ_1^{ss}) back to (3.34), we can see that both denominator and numerator are zero for $\dot{a}$. Therefore, we can conclude that (a_1^{ss}, ψ_1^{ss}) is indeed not a steady state. It can be shown that $\psi_3^{ss} < \psi_2^{ss} < 0$ given the assumption that $\delta > \rho$. Also, we can show that $a_2^{ss} > a_3^{ss}$.

Proof. We want to show that $a_2^{ss} - a_3^{ss} = \frac{1}{c(81\delta^2-\rho^2)}\left(81c\delta^2 - 18\delta^2 - 6\delta\rho - c\rho^2\right) > 0$. Since the numerator is positive, the sign of $(a_2^{ss} - a_3^{ss})$ depends on the denominator. It can be rewritten as $\left(c(\delta^2 - \rho^2) + (18c\delta - 6\rho)\delta + (62c - 18)\delta^2\right)$. Given $c > \frac{1}{3}$ and $\delta > \rho$, it can be easily checked that the term is positive. ■

We want to make sure that the smaller steady state, namely, a_3^{ss} still satisfies Assumption 3.2 so that both steady states are in the region of interior solutions. Using (3.40) and Assumption 3.2, and rearranging terms, we get

$$\frac{1}{c\,(9\delta - \rho)}\,(\rho - 5\delta + 9c\delta - c\rho) > 0$$

i.e.

$$c > \frac{5\delta - \rho}{(9\delta - \rho)}$$

It can be shown that $\frac{5\delta-\rho}{(9\delta-\rho)} > \frac{1}{3}$. This implies we have to make a more restrictive assumption on c.

Assumption 3.4:

$$c > \frac{5\delta - \rho}{(9\delta - \rho)}$$

To determine the stability properties of the two steady states, we first define

$$\dot{a} \equiv N(a, \psi)$$

$$\dot{\psi} \equiv M(a, \psi)$$

Although it is usually very difficult to solve the system of two nonlinear differential equations, we can linearize the differential equations within the neighborhood of each steady state. We obtain

$$\begin{bmatrix} \dot{a} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} N_a & N_\psi \\ M_a & M_\psi \end{bmatrix} \begin{bmatrix} a - a^{ss} \\ \psi - \psi^{ss} \end{bmatrix}$$

where

$$\begin{aligned} N_a &= -3c\tfrac{\rho}{(9c+2\psi\rho-2)^2}\left(3c - 6cV_0 + 6ac - 2\right) \\ N_\psi &= \tfrac{\rho^2}{(9c+2\psi\rho-2)^3}\left(3c - 6cV_0 + 6ac - 2\right)^2 \\ M_a &= -\tfrac{2c-2c\psi\rho}{9c+2\psi\rho-2} \\ M_\psi &= \tfrac{1}{(9c+2\psi\rho-2)^2}\left(4\delta - 36c\delta - 6c\rho + 81c^2\delta + 9c^2\rho + 4\delta\psi^2\rho^2 - 8\delta\psi\rho + 18ac^2\rho - 18c^2\rho V_0 + 36c\delta\psi\rho\right) \end{aligned} \tag{3.41}$$

Substituting (a_2^{ss}, ψ_2^{ss}) into (3.41), we get

$$\begin{aligned} N_a &= -c\rho\tfrac{9\delta+\rho}{3(-6\delta-2\rho+27c\delta+3c\rho)} \\ N_\psi &= -c\rho\tfrac{9\delta+\rho}{3(-6\delta-2\rho+27c\delta+3c\rho)} \\ M_a &= \tfrac{6c\delta+2c\rho}{6\delta+2\rho-27c\delta-3c\rho} \\ M_\psi &= \tfrac{1}{3(-6\delta-2\rho+27c\delta+3c\rho)}\left(3\delta\left(-6\delta - 2\rho + 27c\delta + 6c\rho\right) + c\rho^2\right) \end{aligned}$$

The trace of (a_2^{ss}, ψ_2^{ss}) is $N_a + M_\psi = \delta$, and the determinant is $N_a M_\psi - N_\psi M_a$ which is given by

$$D_2 = -\frac{1}{9}c\rho\frac{9\delta+\rho}{(6\delta+2\rho-27c\delta-3c\rho)^2}\left(18c\delta + 6c\rho - 18\delta^2 - 6\delta\rho + 81c\delta^2 + c\rho^2 + 18c\delta\rho\right)$$

It is easy to show that $D_2 < 0$ given Assumptions 3.3 and 3.4. This implies that the steady state (a_2^{ss}, ψ_2^{ss}) is a saddle point. To analyze the stability of the steady state (a_3^{ss}, ψ_3^{ss}), we substitute (a_3^{ss}, ψ_3^{ss}) into (3.41) to get

$$\begin{aligned} N_a &= c\rho\frac{\rho-9\delta}{-3(-6\delta-2\rho+27c\delta-3c\rho)} \\ N_\psi &= \rho^2\frac{\rho-9\delta}{-27(-6\delta-2\rho+27c\delta-3c\rho)} \\ M_a &= \frac{6c\delta+2c\rho}{6\delta+2\rho-27c\delta+3c\rho} \\ M_\psi &= \frac{1}{-3(-6\delta-2\rho+27c\delta-3c\rho)}\left(-3\delta\left(-6\delta-2\rho+27c\delta-6c\rho\right)-c\rho^2\right) \end{aligned}$$

The trace is δ, and the determinate is D_3 where

$$D_3 = -\frac{1}{27}c\rho\frac{(\rho-9\delta)^2}{6\delta+2\rho-27c\delta+3c\rho}$$

It is easy to show that $D_3 > 0$ given Assumptions 3.3 and 3.4. Therefore, we conclude that the steady state (a_3^{ss}, ψ_3^{ss}) is unstable. Starting from a_0 close enough to (a_2^{ss}, ψ_2^{ss}), the optimal policy for firm H is to choose an appropriate $\psi(0)$ so that $(a_0, \psi(0))$ is located on the stable branch of the saddle point. The comparative statics for the steady state with saddle point property are

$$\frac{\partial}{\partial\delta}a_2^{ss} = \frac{2}{c}\frac{\rho}{(9\delta+\rho)^2} > 0$$

$$\frac{\partial}{\partial\rho}a_2^{ss} = -\frac{2}{c}\frac{\delta}{(9\delta+\rho)^2} < 0$$

$$\frac{\partial}{\partial c}a_2^{ss} = -\frac{2}{c^2}\frac{\delta}{9\delta+\rho} < 0$$

So the stable steady state is increasing in the discount factor and decreasing in the speed of adjustment of dynamic and cost parameter. The more impatient the firm H is, the higher the consumers' preference toward French movies at the steady state will be. As the speed of adjustment of the dynamic, ρ, is faster or the cost of adding the quality to firm H's movie is higher, the steady state a will be lower. To see the intuition behind these results, consider the value of v at this steady state:

$$v^{ss} = 2\frac{\delta}{c\,(9\delta + \rho)}$$

First note that the steady-state level of v is less than $a_2^{ss} - (V_0 - 2)$ where it is equal to $\frac{2}{c(9\delta+\rho)}\,(\delta + 9c\delta + c\rho)$, so it is an interior solution. The comparative statics results for v^{ss} are

$$\frac{\partial}{\partial \delta} v^{ss} = \frac{2}{c}\frac{\rho}{(9\delta + \rho)^2} > 0$$

$$\frac{\partial}{\partial \rho} v^{ss} = -\frac{2}{c}\frac{\delta}{(9\delta + \rho)^2} < 0$$

$$\frac{\partial}{\partial c} v^{ss} = -\frac{2}{c^2}\frac{\delta}{9\delta + \rho} < 0$$

These are identical to the comparative statics results of the steady state consumer preference, a. The more impatient firm H is, the higher the level of additional quality it will put into its movie. The faster the speed of adjustment for the consumer preference or the more expensive it is to increase the quality of the movie, the lower the value of v

chosen by firm H at the steady state. The market share at this stable steady state a_2^{ss} is $\frac{1}{3}$. How does v^{ss} compare to the static model, if a is equal to a_2^{ss}? To answer this question, we compute

$$v^C \mid_{a=a_2^{ss}} = \frac{1}{c(9c-2)(9\delta+\rho)}(18c\delta - 4\delta + 2c\rho)$$

One can verify that $v^{ss} < v^C$. This implies that firm H will choose a lower v in the dynamic setting as compared to the static setting. The intuition is that if firm H chooses v higher than v^C, then its market share will be higher than $\frac{1}{3}$ which implies that the consumer preference toward the French movies is going to increase in the next period. This will hurt firm H, so it is optimal for firm H to choose $v = v^{ss}$.

In this section, we analyzed how firm H maximizes its discounted sum of profit stream taking into account the evolution of consumers' preference which is affected by the market shares of the two firms. We solved the optimal control problem in which firm H has no incentive to choose the quality of its movie such that it becomes monopolist in the DVD market. We found two steady states. One steady state has the saddle-point property and the other one is unstable. At the stable steady state, firm H chooses a greater (smaller) quality the larger (smaller) the discount factor (cost parameter and speed of adjustment). We also show that at this steady state, firm H chooses a lower level of quality compared to that found in the case where the firm maximizes its the static profit.

3.4 Discussion

The preceding results are based on the assumption that either c or a is large enough and Assumption 3.2 is satisfied. In this case firm H has no incentive to set v greater than x. However, it is interesting to look at the case where the above condition does not hold. In this case, If both firms are myopic and given some values of the parameters, firm H may choose some v where the steady state in which both firms coexist prevails (similar to our result). However, if firm H is a dynamic optimizer, it may wish to set a level of v high enough to drive out firm F at some time t. By doing this, it would make a loss in its current profit but the consumers' valuation of firm F's movie would decrease in the next period The popularity of French movies, a, would keep decreasing until it hits some critical value where firm H finds itself better off by choosing v^M, and it becomes the monopolist in the DVD market. Given v^M, the popularity of French movie will further decrease until it hits the lower bound ($V_0 - 2$), and firm H can enjoy the monopoly profit afterward. Thus, firm H faces a trade-off between short-term loss and long-term gain. The difficult part comes from the switching of two regimes given the level of control variables, and this serves as the next step of our research agenda. Another interesting line of extension is to include the possibility that the French government imposes some tariff on firm H's DVD. Thus, a differential game between the firm H and the government of the country where firm F locates would be analyzed. Potentially, we can see how the government protect its cultural good, and the impact of such policies on the welfare of the consumers.

3.5 Concluding Remarks

We constructed a dynamic model where two cultural good producers from two different countries compete in one of the markets. The consumers are assumed to have a homogeneous valuation of one good (Hollywood) but a heterogeneous one of the other good (French). The preference for one of the two goods is assumed to change over time by some process that is related to the market shares of the two goods. In the static analysis, the Hollywood firm (firm H) can choose to increase the quality of its product by incurring some cost. Given the quality of movie chosen by firm H, both firms engage in price competition in the DVD market. The conditions for the existence of both corner and interior solutions are established. More specifically, if either the cost of increasing the quality is low (respectively, high) or the preference for the French movies is low (respectively, high), the Hollywood firm will choose the additional quality of its movie such that monopoly prevails (respectively, both firms coexist) in the price competition stage. Furthermore, there exists a cut-off level of the preference for the French movies, at which the Hollywood firm is indifferent between driving out the French firm (by choosing a high quality level) or allowing it to remain in the market.

As for the dynamic analysis, we obtained the interior solution of the optimal control problem as well as the comparative statics results of the steady states. We found that there will be two steady states. One of these is stable in the saddle-point sense and the other is unstable. Due to the nonlinearity of the differential equations, the closed-form solution for the quality trajectory cannot be obtained. However, we provided the linearized properties at the neighborhood of the steady states. We found that the steady state with saddle-point property is increasing in the discount rate but decreasing in the speed of adjustment and the cost parameter. We also found that the quality chosen by the Hollywood firm at the steady state is lower than its static counterpart.

An interesting feature of the model is that it allows us to consider the case where the Hollywood firm might wish to corner the market by sacrificing the short term profit for the sake of long run monopoly profits. The analysis of such a situation is complicated because the firm's profit function changes its form when there is a regime change. Standard techniques cannot be used to find a solution. A more novel solution method is required.

3.A Appendix

3.A.1 Proof of Proposition 7 (Static Equilibrium Prices)

(a) If $V - a < -1$, and given $p = 0$, firm F's q must maximize

$$q\,[1 - \gamma^*(p,q)]$$

subject to $0 \leq \gamma^*(p,q) \leq 1$. Now $\gamma^*(p,q) = V + q - p - a = V + q - a$. The Lagrangian is

$$L = q\,[1 - V - q + a] + \lambda[V + q - a] + \mu\,[1 - V - q + a]$$

The first order condition yields

$$-2q + 1 - V - a + \lambda - \mu = 0$$

Try $\lambda = \mu = 0$, then $q = (1 - V + a)/2$, and the corresponding $\gamma = V - a + [(1 - V + a)/2] = (V - a - 1)/2 < 0$. So the constraint is violated. The solution must therefore be $\lambda > 0$ and thus $V + q - a = 0$. This proves that $q = a - V$ is the best reply to $p = 0$.

Given $q = a - V$, then firm H chooses $p \geq 0$ to maximize $p\gamma^*(p,q)$ subject to $0 \leq \gamma^*(p,q) \leq 1$. Now $\gamma^*(p,q) = V + q - p - a = -p$. So the Lagrangian is

$$p(-p) + \lambda(-p) + \mu[1 + p]$$

The first order conditions are

$$-2p - \lambda + \mu \leq 0, = 0 \text{ if } p > 0$$

$$\lambda \geq 0, \ -p \geq 0, \ \lambda(-p) = 0$$

$$\mu \geq 0, \ (1+p) \geq 0, \ \mu\,[1+p] = 0$$

These condition can be satisfied only if $p = 0$. Therefore we show that given $V - a < -1$, the equilibrium price pair $(p,\ q) = (0,\ a - V)$

(b) If $V - a > 2$, the proof of the equilibrium price pair $(p,\ q) = (V - a - 1,\ 0)$is similar to the one when $V - a < -1$.

(c) If $-1 \leq V - a \leq 2$, we show that the Bertrand equilibrium is the pair, $(p,\ q) = \left(\frac{V-a+1}{3},\ \frac{2-(V-a)}{3}\right)$. Suppose that the firm H's price is $p = \frac{V-a+1}{3}$, then the firm F must maximize

$$q\,[1 - \gamma^*(p,q)]$$

subject to $0 \leq \gamma^*(p,q) \leq 1$. Now $\gamma^*(p,q) = V + q - p - a = \frac{2}{3}V - \frac{2}{3}a + q - \frac{1}{3}$. The Lagrangian is

$$L = q\left[1 - (\frac{2}{3}V - \frac{2}{3}a + q - \frac{1}{3})\right] + \lambda[\frac{2}{3}V - \frac{2}{3}a + q - \frac{1}{3}] + \mu\left[1 - (\frac{2}{3}V - \frac{2}{3}a + q - \frac{1}{3})\right]$$

The first order condition yields

$$\frac{2}{3}a - \frac{2}{3}V - 2q + \lambda - \mu + \frac{4}{3} = 0$$

Try $\lambda = \mu = 0$, then $q = \frac{1}{3}(2 - (V - a))$. Note that under the condition on the value of $V - a$, $0 \leq q \leq 1$. The corresponding $\gamma = \frac{1}{3}(V - a + 1) \in [0, 1]$. Therefore, $q = \frac{1}{3}(2 - (V - a))$ is the best response to firm H setting $p = \frac{V-a+1}{3}$. Now consider if firm F sets $q = \frac{1}{3}(2 - (V - a))$, firm H must maximize

$$p\gamma^*(p, q)$$

subject to $0 \leq \gamma^*(p, q) \leq 1$. Now $\gamma^*(p, q) = V + p - p - a = \frac{1}{3}(2V - 2a - 3p + 2)$. The Lagrangian is

$$L = p\left(\frac{1}{3}(2V - 2a - 3p + 2)\right) + \lambda[\frac{1}{3}(2V - 2a - 3p + 2)] + \mu\left[1 - (\frac{1}{3}(2V - 2a - 3p + 2))\right]$$

The first order condition yields

$$\frac{2}{3}V - \frac{2}{3}a - 2p - \lambda + \mu + \frac{2}{3} = 0$$

Try $\lambda = \mu = 0$, then $p = \frac{1}{3}(V - a + 1) \in [0, 1]$. The corresponding $\gamma = \frac{1}{3}(V - a + 1)$ which can be easily verified that belongs to $[0, 1]$ under the initial condition on the value of $V - a$. The price pair, $(p,\ q) = \left(\frac{V-a+1}{3},\ \frac{2-(V-a)}{3}\right)$, is the best response to each other so it constitutes the Bertrand Equilibrium.

3.A.2 Proof of Proposition 8 (Static Equilibrium Additional Quality of

Firm H's Movie, v)

We can discuss this by dividing the problem into 3 cases.

First note that for $v^M \leq x$

$$2-(V_0-a)-\frac{1}{c}\geq 0 \Rightarrow a \geq V_0-2+\frac{1}{c}$$

Note that under this condition, if the firm H wishes to drive out the firm F, it will choose $v = x$ because setting any higher v will necessarily decrease its monopoly profit. lso, from Condition 3.1, the following inequality must be satisfied.

$$V_0-2+\frac{1}{c}<V_0+1$$

For $v^C \leq x$

$$\frac{1}{9c-2}(-2a+2V_0+2)-(2-(V_0-a))=-\frac{3}{9c-2}(6c-3cV_0+3ac-2)<0$$

i.e.

$$(6c-3cV_0+3ac-2)\geq 0$$

i.e.

$$a\geq V_0-2+\frac{2}{3c}$$

Therefore, $v^M \leq x \Rightarrow v^C < x$ because $V_0-2+\frac{1}{c}>V_0-2+\frac{2}{3c}$.

It can be easily seems that it is impossible for $v^M = v^C = x$.

Case 1. $\frac{1}{c} + V_0 - 2 \leq a < V_0 + 1$

In this case, $v^M \leq x$ and $v^C < x$. First note that $\frac{d}{dv}\pi^C \mid_{v=x} < 0$ and $\frac{d}{dv}\pi^M \mid_{v=x} < 0$ because both π^M and π^C are quadratic in v ,and $v^M \leq x$ and $v^C < x$. Moreover, we have shown that $\pi^M = \pi^C$ when $v = x$. Thus by choosing any $v > x$ in this case, firm H will definitely be worse off. Thus, firm H will choose $v \in [0,\ x]$ to maximize its profit. By definition of v^C, it will be the choice for firm H.

Case 2. $V_0 - 2 + \frac{2}{3c} \leq a < V_0 - 2 + \frac{1}{c}$

In this case, $v^C \leq x$, and $v^M > x$. There is a possibility that v^M gives a higher profit than v^C. Therefore, we have to compare $\pi^C(v^C)$ and $\pi^M(v^M)$. Define

$$D^{CM} \equiv \pi^C(v^C) - \pi^M(v^M) = \frac{1}{2c\,(9c-2)}\left(Ac^2 + Bc + 2\right)$$

where

$$A = 2\,(a - V_0 + 2)\,(a - V_0 + 5) > 0$$

$$B = 4(V_0 - a) - 13 < 0$$

Since the numerator is always positive, the sign of D^{CM} will depends on the denominator. Also, because of $A > 0$ and $B < 0$, for small c, the sign of D^{CM} will be negative. This indicates that firm H's profit is higher if it chooses v^M instead of v^C. However, if c is large enough, then D^{CM} starts to become positive where choosing v^C is better off for firm H than choosing v^M. The intuition is that the higher the c, the more costly to drive out firm

F by setting v^M where $v^M > x$. Differentiate D^{CM} with respect to a, we get

$$\frac{d}{da}D^{CM} = \frac{1}{9c-2}\left(7c - 2cV_0 + 2ac - 2\right)$$

This implies that $\frac{d}{da}D^{CM} > 0$ iff $(7c - 2cV_0 + 2ac - 2) > 0$. Rearrange terms, condition for $\frac{d}{da}D^{CM} > 0$ is

$$a > \frac{-7}{2} + V_0 + \frac{1}{c}$$

It can easily be showed that

$$\frac{-7}{2} + V_0 + \frac{1}{c} < V_0 - 2 + \frac{2}{3c}$$

Therefore, $\frac{d}{da}D^{CM} > 0$ in this case where

$$\frac{-7}{2} + V_0 + \frac{1}{c} < V_0 - 2 + \frac{2}{3c} \leq a < V_0 - 2 + \frac{1}{c}$$

Setting $D^{CM} = 0$ and solve for a, we get two solutions

$$a = \left\{ \begin{array}{lcl} a_1 & = & \frac{1}{2c^2}\left(2c + 2c^2V_0 - 7c^2 - \sqrt{c^3(9c-2)}\right) \\ a_2 & = & \frac{1}{2c^2}\left(2c + 2c^2V_0 - 7c^2 + \sqrt{c^3(9c-2)}\right) \end{array} \right.$$

where $a_2 > a_1$.

It can be showed that $a_1 < V_0 - 2 + \frac{2}{3c}$, i.e.

$$V_0 - 2 + \frac{2}{3c} - a_1 = \frac{1}{6c}\left(3\sqrt{c(9c-2)} - 2 + 9c\right) > 0$$

a_1 is not belong to the condition $V_0 - 2 + \frac{2}{3c} \leq a < V_0 - 2 + \frac{1}{c}$, therefore, we do not have to consider it. We want to show that $a_2 \in (V_0 - 2 + \frac{2}{3c}, V_0 - 2 + \frac{1}{c})$.

Suppose, $a_2 \leq V_0 - 2 + \frac{2}{3c}$, then it implies

$$a_2 - (V_0 - 2 + \frac{2}{3c}) = \frac{1}{6c}\left(2 + 3\sqrt{c\,(9c-2)} - 9c\right) \leq 0$$

i.e.

$$3\sqrt{c\,(9c-2)} \leq 9c - 2$$

i.e.

$$9c\,(9c-2) \leq (9c-2)^2$$

i.e.

$$9c \leq (9c-2)$$

Which is impossible for any real valued c.

Suppose $a_2 \geq V_0 - 2 + \frac{1}{c}$, then

$$a_2 - (V_0 - 2 + \frac{1}{c}) = \frac{1}{2c^2}\left(\sqrt{9c^4 - 2c^3} - 3c^2\right) \geq 0$$

i.e.

$$\sqrt{c^3 9(c-2)} \geq 3c^2$$

i.e.

$$c^3 9(c-2) \geq 9c^4$$

i.e.

$$9(c-2) \geq 9c$$

which again is impossible. Denote $a_2 \equiv a'$, and we can conclude that

$$V_0 - 2 + \frac{2}{3c} < a_2 \equiv a' < V_0 - 2 + \frac{1}{c}$$

Since we show that $\frac{d}{da}D^{CM} > 0$ and $D^{CM}(a = a') = 0$, thus

$$D^{CM}\begin{cases} < 0 \;\Rightarrow\; v = v^M & \text{if } V_0 - 2 + \frac{2}{3c} \leq a < a' < V_0 - 2 + \frac{1}{c} \\ > 0 \;\Rightarrow\; v = v^C & \text{if } V_0 - 2 + \frac{2}{3c} < a' < a < V_0 - 2 + \frac{1}{c} \end{cases}$$

Case 3. $V_0 - 2 < a < V_0 - 2 + \frac{2}{3c}$

Under this case, $v^M > x$ and $v^C > x$, therefore, the firm H will definitely set $v \geq x$ to maximize the profit. Since $v^M > x$ and by definition, v^M maximize the profit, so it will be the v chosen by the firm H.

Combine all three cases, we prove proposition 8.

Conclusion

The first essay studies the optimal export policy when markets are vertically related and goods are differentiated. We consider both Cournot and Bertrand rivalry between two downstream firms. We also consider two pricing schemes employed by the upstream monopolist, price discrimination and uniform pricing. Export policy and thus welfare depend not only on the competition mode but also on the pricing scheme. Moreover, the result is sensitive to the degree of product substitutability.

When the downstream market is a Cournot duopoly, an export tax is called for in the price discrimination case; however, when the upstream monopolist practices uniform pricing, there is a cut-off level of product substitutability beyond which the government imposes a subsidy. In the price discrimination case, a country is always better off under trade war as compared to free trade. For the uniform pricing case, a country is better off in trade war if and only if the optimal export policy is a tax. When downstream firms compete in the Bertrand fashion, the optimal policy is a tax regardless of the pricing schemes, and welfare is always higher under optimal policies equilibrium compared to free trade equilibrium.

Given that the upstream monopolist price discriminates downstream firms, the optimal export tax is higher under Bertrand duopoly than under Cournot duopoly if two goods are sufficiently close to each other. Otherwise, the reverse is true. If the upstream monopolist charges a uniform price, the export tax is always higher under Bertrand duopoly. The analysis also suggests that the degree of product substitutability plays an important role. If two goods are sufficiently independent to each other, then the Cournot equilibrium welfare level is higher than the Bertrand one under both pricing schemes. If the degree has an intermediate value, welfare level is higher (lower) in Cournot equilibrium than that in Bertrand equilibrium in the price discrimination case (uniform pricing case). If two goods are sufficiently close to each other, Bertrand competition between two downstream firms always yields a higher welfare regardless the pricing schemes.

The study on consumers' welfare reinforces the conventional result that a consumer is always better off in Bertrand competition than in Cournot one due to a higher quantity consumed and lower price. Moreover, this result is not sensitive to different pricing schemes employed by the upstream monopolist.

The second essay considers a model consisting of an exhaustible resource extracting country and one or more resource importing countries and analyzes the uses of export/import policies by these countries. In a bilateral monopoly case where there is only one importing country, we derive analytically the Markov perfect equilibrium tariff rate and export price. Comparing the bilateral monopoly equilibrium with free trade equilibrium, we show that bilateral monopoly yields a lower initial extraction rate compared to free trade which is consistent with the intuition that the monopolist conserves the resources. An importing finding is that if the extraction cost is lower than some threshold level, then the resource exporting (importing) country is better (worse) off in bilateral monopoly compared to free trade. Moreover, this threshold level is increasing in the rate of discount. Doing the analysis on division of gains from trade, we find that the resource exporting country's share of gains increases in the cost parameter and decreases in the discount rate under free trade. In bilateral monopoly, resource-exporting country share two-third of gains from trade regardless of parameter values.

In a more general setting where there are multiple resource importing countries that can be different in market sizes, a numerical simulation has been conducted. In the two-importing-countries case, exporting (importing) country's welfare is increasing (decreasing) in the total market size under either free trade or tariff war scenario but decreasing (increasing) under custom union (bilateral monopoly) scenario. Also, each resource importing (exporting) country's welfare increases (decreases) in both free trade and tariff war cases when the market size asymmetry of two importing countries increases. Given the values of parameters specified in the model, the resource exporting country's welfare is always higher in the free trade case compared to the tariff war regardless the degree of asymmetry. On the other hand, the more asymmetric the market sizes, the more likely it is for an importing country to be better off under tariff war compared to free trade.

Considering an interesting case where the resource exporting country is forced to commit on a (optimal) division of its stock of resource to serve two importing countries separately, we find that the optimal division is the one that split the stock according to importing countries' relative market sizes. Moreover, the corresponding tariffs are the same as in the case when two importing countries cooperatively choose their tariffs rate. A policy implication from this exercise is that a resource importing country may seek to sign a contract of exclusive right on some portion for oil stock with the exporting country to achieve the same outcome as under custom union by two importing countries.

The third essays studies the rivalry by two firms (Hollywood vs. French) in a cultural good (movie) market. Cultural goods are differentiated in the sense that the consumers have homogenous valuation of one good (Hollywood) and heterogeneous valuation of the other good. The model is characterized as a two-stage game. The firm which supplies the homogeneous-valuation good chooses the quality of its product in the first stage, and in the second stage, the two firms compete as Bertrand rivals. We first look at the static competition and then study the dynamic version where consumers' preference are affected by the market shares of two firms.

In the static analysis, we provide the conditions on parameter values for the existence of both corner and interior solutions. If either the cost of increasing the quality or the preference for the French is low, the Hollywood firm has an incentive to choose the quality of its movie such that monopoly prevails in the second stage of the game.

The dynamic analysis consists of an optimal control problem solved by the Hollywood firm as the French firm is assumed to be myopic in that it only cares about its current profit. We focus on the interior solution by considering those parameter values such that the Hollywood firm has no incentive to drive out the French firm by over-investing in the quality of its movie. There exists two steady states, one of which is a saddle point and the other one is unstable. The linearization at the neighborhood of the stable steady state suggests that the steady state value of preference of the French movie is increasing in the discount rate but decreasing in the speed of adjustment and the cost parameter. Moreover, steady state quality of Hollywood movie is lower than its static counter part.

References

Bala, V. and N.V. Long, (2005), "International Trade and Cultural Diversity with Preference Selection", *European Journal of Political Economy,* 21, 143-62.

Bergstrom, T.C., (1982), "On Capturing Oil Rents with National Excise Tax", *American Economic Review*, 71, 194-201.

Bernhofen, D.M., (1997), "Strategic Trade Policy in a Vertically Related Industry", *Review of International Economics*, 5, 429-33.

Brander, J. and S. Djajic (1983), "Rent Extracting Tariffs and the Management of Exhaustible Resources", *Canadian Journal of Economics*, 16, 288-98.

Brander, J.A. and B.J. Spencer, (1985), "Export subsidies and international market share rivalry"., *Journal of International Economics,* 18, 83–100.

Brito, P. and C. Barros, (2005), "Learning-by-Consuming and the Dynamics of the Demand and Prices of Cultural Goods", *Journal of Cultural Economics,* 29, 83-106.

Chang, W.W. and H. Sugeta, (2004), "Conjecture Variations , Market Power, and Optimal Trade Policy in a Vertically" Related Industry, *Review of International Economics*, 12, 12-26.

Clarke, R. and D.R. Collie, (2003), "Product differentiation and the gains from trade under Bertrand duopoly", *Canadian Journal of Economics*, 36, 658–73.

—, (2006), "Optimum-Welfare and Maximum-Revenue Tariffs under Bertrand Duopoly", *Scottish Journal of Political Economy*, 53, 398-408.

Dixit, A.K., (1979), "A Model of Duopoly Suggesting a Theory of Entry Barriers", *Bell Journal of Economics*, 10, 20–32.

Dixit, A.K., (1984), "International Trade Policy for Oligopolistic Industries," *Economic Journal Conference Papers,* 94, 1–16.

Dockner, E., S. Jorgensen, N. V. Long and G. Sorger, (2000), *Differential Games in Economics and Management Science*, Cambridge University Press.

Eaton, J. and G.M. Grossman, (1986), "Optimal Trade and Industrial Policy under Oligopoly", *The Quarterly Journal of Economics*, 101, 383-406.

Francois, P. and T. van Ypersele, (2002), "On the Protection of Cultural Goods", *Journal of International Economics,* 56, 359-69.

Gaudet, G. and N.V. Long, (1996), "Vertical Integration, Foreclosure, and profits in the Presence of Double Marginalization", *Journal of Economics and Management Strategy*, 5, 409-32.

Gilbert, R.J., (1978), "Dominant Firm Pricing Policy in a'Market for an Exhaustible Resource", *Bell Journal of Economics*, 9, ,385-95.

Gilbert, R.J. and S.N. Goldman, (1978), "Potential Competition and the Monopoly Price of an Exhaustible Resource", *Journal of Economic Theory*, 17, 319-31.

Harris, C. and J. Vickers, (1995), "Innovation and Natural Resources: A Dynamic Game with Uncertainty", *Rand Journal of Economics*, 26, 418-30.

Ishikawa, J. and B.J. Spencer, (1999), "Rent-Shifting Export Subsidies with an Imported Intermediate Product", *Journal of International Economics*, 48, 199-232.

Karp, L., (1984), "Optimality and Consistency in a Differential Game with Non-Renewable Resources", *Journal of Economic Dynamics and Control*, 8, 73-97.

Karp, L. and D.M. Newbery (1991), "Optimal Tariffs on Exhaustible Resources", *Journal of International Economics*, 30, 285-99.

—, (1992), "Dynamically Consistent Oil Import Tariffs", *Canadian Journal of Economics*, 25, 1-21.

Kemp, M.C. and N.V. Long, (1980), "Optimal Tariffs on Exhaustible Resources", in M.C. Kemp and N.V. Long (eds.), *Exhaustible Resources, Optimality, and Trade*, North Holland, Amsterdam.

Leonard, D. and N.V. Long, (1992), *Optimal Control Theory and Static Optimization in Economics*, Cambridge University Press.

Lewis, T., R. Lindsey, and R. Ware, (1986), "Long-Term Bilateral Monopoly: The Case of an Exhaustible Resources", *Rand Journal of Economics*, 17, 89-104.

Lewis, T.S., S.A. Matthews, and H.S. Burness, (1979), "Monopoly and the Rate of Extraction of Exhaustible Resources: Note", *American Economic Review*, 69, 227-30.

Liski, M. and O. Tahvonen, (2004), "Can Carbon Tax Eat OPEC's Rents?", *Journal of Environmental Economics and Management,* 47, 1-12.

Maskin E., and D.M. Newbery, (1978), "Rational Expectations and Market Power: The Paradox of the Disadvantageous Monopolist", *Warwick Economic Discussion Paper.*

—, (1990), "Disadvantageous oil Tariffs and Dynamic Consistency", *American Economic Review*, 80, 143-56.

Maskin E., and J. Tirole, (2001), "Markov-Perfect Equilibrium I: Observable Actions", *Journal of Economic Theory*, 100, 191-219.

Pyndick, R.S,. (1978), "Gains to Producers from the Cartelization of Exhaustible Resources", *Review of Economic and Statistics*, 60, 238-51.

Rauch, J.E., and V. Trindade, (2005), "Neckties in the Tropics: A Model of International Trade and Cultural Diversity", *NBER Working Papers*: 11890.

Richardson, M., (2006), "Commercial Broadcasting and Local Content Cultural Quotas, Advertising and Public Stations", *The Economic Journal*, 116, 605-25.

Robson, A.J., (1983), "OPEC versus the West: A Robust Equilibrium", *Journal of Environmental Economics and Management*, 10, 18-34.

Rubio, S.J., (2005), "Optimal Tariffs and Non-Renewable Resources International Monopolies", Fifth International Society of Dynamic Games Workshop, Sergovia, 21-25 September 2005.

Salant, S.W., (1976), "Exhaustible Resources and Industrial Structure: A Nash-Cournot Approach to the World Oil Market", *Journal of Political Economy*, 84, 1079-94.

Singh, N. and X. Vives, (1984), "Price and Quantity Competition in a Differentiated Duopoly", *Rand Journal of Economics*, 15, 546–54.

Spencer B.J. and R.W. Jones, (1991), "Vertical foreclosure and international trade policy", *Review of Economic Studies*, 58, 153-70.

—, (1992), "Trade and Protection in Vertically Related Markets", *Journal of International Economics*, 32, 31-55.

Ulph, A.M., and G.M. Folie, (1980), "Exhaustible Resources and Cartels: An Intertemporal Nash-Cournot Model", *Canadian Journal of Economics*, 13, 645-58.

Vives, X., (1985), "On the Efficiency of Bertrand and Cournot Equilibria with Product Differentiation", *Journal of Economic Theory*, 36, 166-75.