CORRELATIONS BETWEEN HIGH FREQUENCY AND LOW FREQUENCY NOISE IN A HIGH FREQUENCY OSCILLATOR

by

Hugh A. Hamilton

A thesis submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy in the Faculty of Graduate Studies and Research

Eaton Electronics Research Laboratory
McGill University, Montreal
April, 1953

TABLE OF CONTENTS

		Page
	ACKNOWLEDGEMENTS	I
	ABSTRACT	II
I	INTRODUCTION	1
II	A REVIEW OF NOISE THEORY AND EXPERIMENTAL KNOWLEDGE APPLICATIONS OF THE PRESENT STUDY	
	1. General Considerations	6
	2. Thermal Noise	6
	3. Shot Noise	8
	4. Partition Noise	9
	5. Induced Grid Noise	10
	6. Flicker Noise and Current Noise	11
	7. Applications of the Present Study of Noise in Oscillators	13
III	EXPERIMENTAL PROCEDURE AND APPARATUS	
	1. General Considerations	16
	2. Noise Power Spectra	16
	3. Correlation Measurements	39
IV	THE RADIO FREQUENCY OSCILLATOR AND DETECTOR	44
V	EXPERIMENTAL RESULTS	
	1. Foreword	50
	2. Summary of Experiments	51
	3. Noise Spectra	58
	4. Correlation Measurements	65
VI	DISCUSSION AND CONCLUSIONS	76
	APPENDIX	80
	BIBLIOGRAPHY	83

ACKNOWLEDGMENTS

The author wishes to acknowledge the guidance and assistance of Professor G.A. Woonton and Dr. J.R. Whitehead, who were the joint directors of this research problem. The advice and encouragement of Professor Woonton, together with the many detailed discussions of the problem which were held with him, contributed greatly to the success of the project. The guidance of Dr. Whitehead was of great assistance in the study of noise and in its particular application to this problem. The design of the equipment was greatly facilitated by his many suggestions and his detailed knowledge of the problems involved.

Mr. V. Avarlaid contributed technical advice on a number of occasions. Part of the equipment was constructed by Mr. B. Meunier and Mr. B. Szymkowiak.

The work was carried out with the aid of a National Research Council Studentship (1951-52), a summer extension of that Studentship (summer 1952), and a National Research Council Fellowship (1952-53).

ABSTRACT

The noise appearing as a modulation on the radio frequency energy generated by oscillators has been examined. These measurements have revealed a new effect: flicker noise and shot noise, generated within a tube at very low frequencies, modulates the radio frequency oscillation and in doing it produces radio frequency noise. At frequencies close to the carrier this noise greatly exceeds the noise from all previously recognized sources.

Spectrum and correlation measurements have been made on both the audio frequency and the radio frequency noise generated simultaneously by an oscillator tube. The spectra of the radio frequency and the audio frequency noises corresponded. Other measurements showed that the noise trains were as much as seventy per cent correlated up to the limit of measurement, namely 10 kilocycles per second different from the carrier frequency.

A method is provided for the calculation of the magnitude of the noise arising from this source.

I INTRODUCTION

There are a number of applications, for example in radio frequency bridge measurements and in doppler radar, in which the magnitude of the noise produced by a radio frequency (rf) oscillator or amplifier is of importance. In calculating the noise in the output of an rf oscillator, it is customary to consider only the noise generated in the tube at the frequency and within the pass band defined by the resonant circuit of the oscillator. There are several accepted sources of noise within such a circuit. This thesis deals with a new and hitherto unrecognized source of noise, namely, noise appearing as a modulation on the rf output of the oscillator due to sources commonly supposed to affect only the audio frequency performance of the tube.

One of the sources of noise in the rf output which is commonly recognized is shot noise, due to the random arrival at the anode of the electrons comprising the plate current of the tube. It has uniform magnitude over the frequency spectrum up to frequencies comparable to the transit time of the electrons in the tube. Its magnitude is directly proportional to the D.C. current flowing in the tube.

Partition noise, which has similar characteristics, will be present if there is more than one electrode which draws current in the tube. It is caused by the further disarrangement of the electron stream by the separation of the two or more currents.

Thermal noise, which is due to the random motion of conduction electrons in the resistive portion of any impedance, and whose magnitude is also independent of frequency, will occur in components in the circuit.

Finally, if the frequency considered is high enough, induced grid noise must be included. This type of noise, which arises due to charges being induced on the control grid of the tube by passing electrons, becomes important at a frequency comparable to the transit time of an electron in the tube, and is relatively constant above that frequency.

A further source of noise within the tube, flicker noise, is customarily ignored, since it varies in magnitude approximately as 1/f (where f is the frequency) and therefore is only important at very low audio frequencies. This type of noise is due to random changes in the emissive properties of the cathode surface of the tube.

This thesis will show that noise at audio frequencies is of great importance in an rf oscillator. Although the flicker noise does not have significant components at the oscillation frequency, it has been found that the low frequency fluctuation of this noise does appear as an amplitude modulation on the rf output. For example, in a typical case, it was found that at a modulation frequency of 25 cps, the magnitude of this noise was ten times that of the shot, thermal and partition noise together.

Since the effect of noise generated at audio frequencies on the output of an rf oscillator has not hitherto been considered, the study of this phenomenon is of great practical and theoretical importance. In practical applications, any process which attempts to detect a low audio frequency modulation upon an rf signal must take into account the effect of this source of noise.

A Doppler radar system, of the continuous wave (cw) type is an example of this. In this case, a frequency change which may be as low as 1 cps, or lower, must be detected. A signal of relatively large magnitude will leak from the transmitting to the receiving antenna. The noise in this signal, which is largely the noise generated in the rf oscillator of the transmitter, may be an important factor in determining the sensitivity of the receiver to small radar signals. If this noise has a particularly large low frequency component, then the sensitivity of the receiver may be reduced at these frequencies, thus reducing the effectiveness of the radar set.

There are several steps in the experimental proof of the existence of this source of noise.

First, it must be shown that the noise modulation on the rf output of an oscillator has a power spectrum similar to that of the audio noise output of the oscillator tube, which means that it follows a 1/f law spectrum due to the presence of flicker noise. This was done by measuring the detected rf noise of a specially constructed electron coupled oscillator operating at 5.06 Mc with a precision noise measuring apparatus. This system was so arranged that it would measure the noise power within a narrow band of frequencies ($\delta f \doteq 0.01 f_o$ where δf is the band width of a tuned filter centered on the frequency of measurement f_o) at any frequency from 20 cps to 10 kc. A plot of mean square noise voltage versus frequency showed that the noise in the rf output of the oscillator did indeed have the same variation with frequency as the low frequency audio noise generated in the tube.

In addition, if the noise modulation of the rf oscillations is due, in part, to the low frequency flicker noise in the tube, the audio frequency noise and the noise modulation on the rf signal should be at least partially correlated, which means that they should have the same time variation of amplitude.

In order to demonstrate this fact the audio noise generated in the oscillator tube, which was made to appear across an audio load resistor at the plate of the tube, and the detected rf noise were separately amplified. They were then compared by selecting narrow bands of frequencies by means of identical filters and photographing the resulting wave trains as they appeared simultaneously on a double beam oscilloscope. At all frequencies where this process was feasible, the two traces appeared to be very similar in shape. However comparison was only possible at low frequencies (less than 50 cps) by this means with the equipment available.

A further comparison was obtained by applying the separate wave trains, or selected frequency bands from these trains, to the vertical and horizontal deflection plates of an oscilloscope. In this case perfect correlation in phase and wave shape would be indicated by a straight line at an angle of 45^{o} across the tube. Complete lack of correlation would be indicated by a fuzzy circle in the middle of the screen. This experiment indicated correlations ranging from approximately 80% at low frequencies to 50% at 10 kc. Two different types of oscillators gave different degrees of correlation.

Further extensive experiments were carried out in order to show that this correlation was not due to any external causes, such as unwanted coupling between the two noise trains, or plate modulation of the rf oscillator due to external effects. It was conclusively shown that the correlation was due to some form of internal modulation of the rf output by the low frequency noise within the tube.

This thesis consists of a section dealing briefly with the theory and the state of the experimental knowledge of each of the sources of electrical noise, with particular emphasis on flicker noise. In addition, an examination of some of the practical applications of this study of noise in rf oscillators is included. Section III deals in detail with the experimental techniques and the apparatus involved in the measurements mentioned above. The design of the rf oscillator used for the measurements is explained in Section IV. Section V deals with the results of the experiments carried out, while the last section summarizes the results and conclusions of the experiments and discusses a possible physical mechanism for the phenomenon.

II A REVIEW OF NOISE THEORY AND EXPERIMENTAL KNOWLEDGE. APPLICATIONS OF THE PRESENT STUDY

1. General Considerations

The purpose of the experiments described in this thesis was to study the sources of noise in a high frequency oscillator, with particular emphasis on any effect that the flicker noise and other audio frequency noise in the oscillator tubes might have. It has already been pointed out that there are a number of sources of electrical noise. For each of these sources, one is interested in the magnitude of the noise measured as the mean square voltage fluctuations, $\overline{e_n^2}$ or the current fluctuation $\overline{i_n^2}$, within a given band of frequencies, δf. This may readily be transformed into the noise power per unit bandwidth, or the noise power density. This quantity is often a function of frequency, and conditions in the circuit such as temperature and D.C. current flowing.

Theoretical studies have been made to determine the mechanisms which lead to the noise being produced. In this way, insight into conditions within the material may be gained, and the study of noise becomes an important investigation of solid state physics.

2. Thermal Noise

Johnson[1] first showed experimentally that the minute currents caused by the random thermal motion of conduction electrons in a resistor can be detected as noise fluctuations. This is now known as thermal or Johnson noise.

At the same time, Nyquist[2], by applying the statistical theory of

---

1. Johnson, J.B. Phys. Rev., 29, 367 (1927)
   Phys. Rev., 32, 97 (1928)
2. Nyquist, H., Phys. Rev., 32, 110, (1928)

thermodynamics to the problem showed that, for this type of noise,

$$\overline{e_n^2} = 4kTR\delta f, \qquad \text{II, 1.}$$

where R = the resistance, in ohms,

D = Boltzmann's constant,

= 1.377×10^{-23} Joules/degree Kelvin,

and T = absolute temperature.

It will be noted that the magnitude of this type of noise is dependent only on the resistance and the temperature, and is independent of the frequency.

The Nyquist derivation is independent of the mechanism of production of the noise. Several other derivations of this equation have been presented, based on various models for electric conduction through a metal[3,4,5]. However, a completely satisfactory derivation based on the modern theory of metals is still lacking.

The predictions of the thermal noise equation have been exhaustively verified by experiments[6,7,8]. In all cases the theory was found to hold with great accuracy.

It has also been predicted, from theoretical and practical considerations, that at some very high frequency, the noise power density must decrease.[9] If this were not so, an infinite amount of energy would be contained in the whole noise spectrum. However, different theoretical

---

3. Bernamont, J., Ann. Phys. 7, 71, (1937)
4. Bakker, C.J., and G. Heller, Physica, 6, 262, (1939)
5. Spenke, E., Wiss Veroffentl Siemens Werken, 18, 54, (1939)
6. Moullin, E.B., and H.D.M. Ellis, J.I.E.E., 74, 331, (1934)
7. Wilbur, D.A., "Thermal Agitation of Electricity in Conductors", Dissertation, U. of Mich. (1932)
8. Willians, F.C., J.I.E.E., 81, 751, (1937)
9. For a detailed discussion, see Lawson, J.L., and G.E. Uhlenbeck, "Threshold Signals" (Rad. Lab. Ser. Vol. 24), McGraw-Hill, 1950, p. 77.

approaches yield different critical frequencies for this phenomenon. Experimental investigation of this point is difficult, since the frequencies involved are too high for accurate measurement with present day techniques.

3. Shot Noise

The electric current emitted from a hot cathode consists of the combined effects of a large number of independently emitted electrons, each of which has its own individual effect at the anode of the tube. Therefore, the anode current of the tube is never steady, but has small random fluctuations.

Schottky[10] was the first to predict this effect, and to verify it experimentally. He showed that for a diode, operating in the temperature limited region,

$$\overline{i_n^2} = 2eI\delta f, \qquad \text{II-2}$$

where e = the electronic charge,

and I = D.C. current flowing in the tube.

It will be noted that the power density of this type of noise, too, is independent of frequency. This holds up to frequencies which are of the order of the reciprocal of the transit time of the electron in the tube.

Equation II-2 is so well verified[11] that diodes operated in this manner, are used as precision calibrated, "white" noise sources at frequencies from 100 cps to 3000 mc/s.

If the tube is operated in the space charge limited region of its characteristics, the presence of the space charge cloud will tend to smooth out fluctuations in the plate current. For if a large group of electrons is emitted from the cathode, some of the extra electrons will

---

10. Schottky, W., Ann. Physik., 57, 541 (1918)

11. Best results. Williams, H., and W.S. Huyford, Phys. Rev. 73, 773, (1929)

be used up temporarily to increase the space charge, thus smoothing somewhat the fluctuation in the current. A similar smoothing action takes place if the emitted current fluctuates below its mean value. In this case, some of the electrons in the space charge are given up temporarily to smooth out the fluctuation.

In order to allow for this effect, a space charge reduction factor, Γ^2, whose value is less than 1, and which is a function of tube geometry and operating conditions, is inserted into the Schottky formula.

$$\overline{i_n^2} = \Gamma^2 \, 2eI\delta f, \qquad \text{II-3.}$$

Many theoretical and experimental investigations have been carried out to determine the value of Γ^2 that should be used.[12] For simple electrode geometry, and frequencies below the critical frequency set by the transit time of electrons in the tube, it has been possible to predict the value of Γ theoretically.[13,14] The value of Γ is thought to be independent of frequency up to the region of the spectrum where transit time effects begin to occur. Above this point, the value of Γ may be expected to increase. Further experimental and theoretical work is required on this point.

4. Partition Noise

In tubes in which more than one electrode draws current, such as pentodes or tetrodes, the random nature of the division of the electron stream between the collecting electrodes causes additional fluctuations in the current. This additional noise is known as partition noise.

---

12. A short review of this subject, with references is given in Lawson and Uhlenbeck. Op. Cit. Page 93.
13. North, D.O., R.C.A. Review. 4, 441 (1940)
14. Schottky,, W. and E. Spenke, Wiss Veroffentl Siemens Werken 16 (No. 2), 1, 19, (1937)

Lawson and Uhlenbeck[15] give an expression for the noise in a pentode including space charge suppressed shot noise and partition noise,

$$\overline{i_n^2} = 2eI_2 \frac{I_1 + \Gamma^2 I_2}{I_1 + I_2} \delta f \qquad \text{II-4}$$

where I_2 is the anode current

and I_1 is the screen current.

This expression has been experimentally confirmed by North.[16]

5. <u>Induced Grid Noise</u>

When an electron passes through a grid in the tube, a current pulse is induced in the grid. As the electron approaches the grid the current rises rapidly; when the electron passes through the grid, the current drops to a negative value. The net current is, of course, zero, since no electron capture occurs. However the tube has had a pair of pulses of very short duration (less than the transit time) applied to its control grid and hence a noise signal appears. These pulses, because of their short duration, will generate high frequency noise; a detailed analysis leads to the expression[17]

$$\overline{i_n^2} \propto f^2. \qquad \text{II-5.}$$

This effect has been investigated theoretically and experimentally.[18,19] Good agreement between theory and experiment has been obtained. However, it is apparent that the noise cannot continue to increase with f^2, since the noise would become infinite. The theory is limited by the assumption that the frequency considered is lower than a critical frequency determined by the cathode grid transit time of the electron.

15. Lawson and Uhlenbeck, Op. Cit. Page 93
16. North, D.O., R.C.A. Rev., <u>5</u>, 244, (1940)
17. Goldman, S. "Frequency Analysis, Modulation and Noise", McGraw-Hill 1948, p. 376.
18. North, D.O., and W.R. Ferris, Proc. I.R.E. <u>29</u>, 49, (1941)
19. Bakker, C.J., Physica, <u>8</u>, 23, (1941)

6. Flicker Noise and Current Noise

Current or flicker noise, which occurs when current is passed through a vacuum tube or through a semi-conducting material, has the general properties that its magnitude varies directly as the square of the D.C. current and increases with decreasing frequency. Flicker noise is particularly important in vacuum tubes with coated cathodes. Current noise occurs in all kinds of semi-conducting materials, such as crystal diodes, germanium and silicon transistors, photo-conductive cells, and carbon resistors.

This type of noise was first observed by Johnson[20] in 1925, and since that time, numerous investigators have made further observations[21]. These investigations show that the correct expression for the noise power density of this type of noise is

$$\overline{i_n^2} \propto \frac{I^2}{f^x} \, \delta f \qquad \text{II-6}$$

where x may vary with frequency. From practical considerations, it may be seen that as the frequency approaches zero, x must have a value less than 1, in order for the energy contained in the whole spectrum to remain finite. Different observers have reported various values of x, but it is generally accepted today that x has a value of approximately 1, falling slightly below 1 as the frequency approaches zero, and sometimes rising as high as 1.5 or even 2 at very high frequencies.

Extreme difficulties are encountered in measuring the mean square value of this noise fluctuation at very low frequencies, which explains the scarcity of reliable measurements below about 20 cps. As a result no comprehensive picture of the variations of x with frequency, or the factors which affect its value has been built up as yet.

20. Johnson, J.B., Phys. Rev. 26, 71 (1925)
21. For bibliography, see MacDonald, D.K.C., Reports in Progress in Physics 12, 56 (1948-49)

A detailed discussion of the techniques and difficulties of noise measurements of this type will be found in section III, 2.a, and in appendix 1.

Several attempts have been made to derive an expression for the magnitude of this noise which will fit experimental data. A review of these theories has recently been presented by Van Wyngaarden and Van Vliet[22].

It is assumed in all of these theories that some event occurs at random which increases emission or conduction, and which decays exponentially with a time constant T. If one assumes that there is but a single time constant involved, a $1/f^2$ spectral distribution results[23]. However, as several investigators have pointed out,[24,25,26] if a distribution of time constants is employed, it is possible to achieve a result which more nearly fits experimental evidence. Indeed, as Van der Ziel[25], has stated, by the proper choice of a distribution of T's, it is possible to match any experimentally determined law whatsoever. There must, therefore, be some good physical reason for choosing the distribution of T which is employed. This is the basis of most of the modern theories.

Perhaps the most successful theory for the origin of flicker noise thus far has been that of Macfarlane.[27] Although this theory has some mathematical errors[28], they do not qualitatively affect the overall result.

---

22. Van Wyngaarden, J.S. and K.M. Van Vliet, Physica, 18, 10, 683 (1952)
23. Schottky, W., Phys. Rev. 28, 74 (1926)
24. Campbell, R.H. and R.A. Chipman, Proc. I.R.E. 37, 938, (1949)
25. Van der Ziel, A., Physica 16, 359, (1950)
26. Macfarlane, G.G., Proc. Phys. Soc. 59, 366, (1947)
27. Macfarlane, G.G., Proc. Phys. Soc. 63, 807, (1950)
28. Petritz, Catholic Univ. Wash., D.C., Oct. 1952 (Report to Wash. Meeting of Am. Phys. Soc.)

Macfarlane considers an emitting surface, which has 'hot' and 'cold' patches, patches of high and low emission due to the orientation of crystals in the surface. This 'patch effect' is well known in studies of thermionic emission.[29] Macfarlane next considers the modulation of the Schottky potential barrier at the emitting patches due to random variations of concentration of mobile adsorbed impurity ions at these patches.

A distribution of time constants, which is dependent upon the shape of the patch, is obtained by calculating the probability that an impurity ion somewhere in the patch may have emerged from it in a time T. This distribution is then employed to calculate the auto correlation function of the noise, and hence, by Fourier transform theory, its power spectrum. The resulting law is

$$\overline{i_n^2} \; \alpha \; \frac{I^2}{f^x} \qquad \text{II-7}$$

where x is a known function of frequency, shape of the patches and temperature.

More accurate data, especially at very low frequencies, is necessary to enable further theoretical investigations of this type of noise to be made. In addition, a comprehensive study of the dependence of flicker noise upon temperature and other physical conditions is still required.

7. Applications of the Present Study of Noise in Oscillators

There are many cases where it is important that the noise included in the output of a high frequency oscillator, particularly noise lying close to the oscillation frequency, be kept as low as possible. In these cases, a knowledge of the sources of noise within an oscillator, and the distribution with frequency of this noise is important.

---

29. Herring, C. and M.H. Nichols, Rev. Mod. Phys. 21, 185 (1949)

As an example, the techniques of high frequency bridge measurements may be considered. The usual procedure in this case is to beat the high frequency output signal of the bridge down to an audible frequency by means of a local oscillator and a mixer. If the high frequency signal applied to the bridge contains an appreciable quantity of noise, this noise may not be balanced out in balancing the bridge, and will contribute to the noise level of the equipment; the noise then determines the ultimate sensitivity of the bridge. Also, noise in a frequency band, lying on either side of the local oscillator frequency will beat with the local oscillator signal and appear as audio noise in the output.

A further example is the case of a Doppler radar set of the continuous wave type described by Barlow[30]. In this type of radar, a cw signal of fixed frequency is transmitted by means of a directional antenna. Any object possessing a radial velocity, v, which is in the beam will reflect a radio signal, changed in frequency due to the Doppler effect, by $2v/\lambda$ where λ is the wavelength of the transmitted signal. This reflected signal is mixed in the receiver with a signal of the same frequency as the transmitted signal, resulting in an audio signal output of frequency $2v/\lambda$. It can be seen that for some target velocities and some signal frequencies, the frequency of this detected signal may be very low.

It can be shown[30] that the maximum range, r, of this radar set is given by

$$r = \left[\frac{1}{128\pi^3 kT} \quad \frac{P_t\, \sigma\, G_o^2\, \lambda^2}{F_1\, N} \right]^{1/4} \qquad \text{II-8}$$

30. Barlow, E.J., Proc. I.R.E., 37, 340, (1949)

where P_t = power transmitted

σ = scattering cross section of the target

G_o = antenna gain relative to an isotropic radiator

F_1 = audio bandwidth of the receiver

N = effective noise figure of the receiver and all other symbols are as previously defined.

Consider the quantity N. The noise figure of a receiver is actually the number of times greater the noise output of the receiver is than it would be if the only source of noise in the circuit were that injected at the input.

With this system, a relatively large leakage signal from the transmitting antenna will reach the receiving antenna. The magnitude of this signal can be controlled, to some extent, by careful placing of the antennas; it may, however, be very much greater than the minimum received signal.

Any noise on this leakage signal may be considered as extra noise increasing the noise figure N, in equation II-8. For example, in a case where the leakage signal is 100 db above the desired minimum received signal, the noise on the transmitted signal must be more than 100 db below the transmitted signal level in order not to obscure the received signal.

Since the object of this type of radar is to detect a low frequency audio signal, the noise at low audio frequencies in the receiver is particularly important. This thesis will show that the noise in an rf signal increases at frequencies close to the signal frequency and therefore assumes increased importance in this application.

III EXPERIMENTAL PROCEDURE AND APPARATUS

1. General Considerations

The purpose of this investigation was to show that noise occurring in a radio frequency (rf) oscillator circuit at audio frequencies appears as a modulation on the rf output of the tube. If this is true, the audio noise and the noise detected as a modulation at the rf frequency should have the same time variation, or wave shape, and the same power spectrum.

The power spectrum of the audio noise generated within the tube should follow a 1/f law at low frequencies due to the presence of flicker noise. Therefore the power spectrum of the noise modulation of the rf signal should follow the same law at low modulation frequencies. For this reason power spectra were measured for both the audio noise and the noise on the rf output of an rf oscillator.

The second major experiment carried out was a comparison of the audio noise train with the train of audio noise occurring as a modulation on the rf signal. For this purpose, the degree of correlation of these two noise signals was measured; the correlation coefficient serves as a measure of the extent to which the noise trains are alike in phase and wave shape.

2. Noise Power Spectra

(a) General Considerations.

Fig. 1 is a block diagram of a conventional measuring apparatus for audio noise. It will be seen that such a system consists of an amplifier, flat over the range of frequencies considered, a precision attenuator, a variable-frequency tuned amplifier of known bandwidth to select the band of frequencies measured, and a square law detector, incorporating some sort of smoothing circuit, to measure the average noise power.

It can be shown (appendix 1) that the mean fluctuation of the output of the smoothing circuit, expressed as a fraction, e, of the mean noise power, is given by

$$e = \frac{A}{\sqrt{BT}} \qquad \text{III-1}$$

where B is the noise bandwidth,

T is the smoothing time constant,

and A is a numerical factor depending upon the shape of the filter.

It will be seen from this equation, that as the frequency of measurement becomes lower, and thus the permissible bandwidth, B, becomes smaller, the time constant T must become very long in order to achieve accuracy. This agrees with the practical consideration that a large number of cycles must be considered in order to obtain an accurate average. These limitations clearly make it impractical to make accurate measurements at very low frequencies (< 5 cps) by this method.

There are several procedures for taking a reading in the frequency range above five cycles per second.

The output meter could be calibrated in terms of input power. This requires that the overall gain of the system be extremely stable, and be independent of temperature, of impedance of the source of noise, and of small changes in supply voltages.

Instead of this, it is better to use a substitution method. In this case a reading of the noise is taken on the output meter and is compared with the reading of a signal of known magnitude applied under the same conditions. This may be accomplished in two ways.

A signal sufficient to double the original reading may be added to the noise at the input. The reference signal power is then equal to

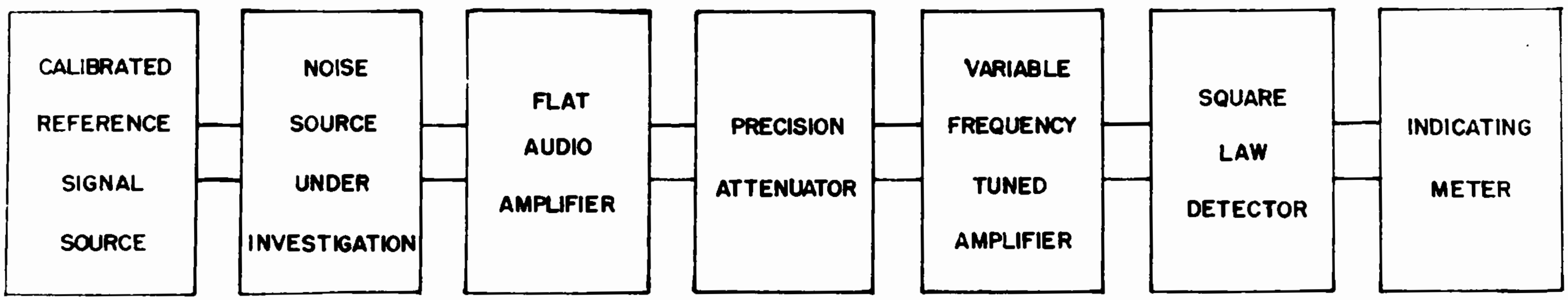


FIG. 1 BLOCK DIAGRAM OF A MEASURING APPARATUS FOR AUDIO FREQUENCY NOISE

the noise power but this requires that the power response of the measuring system be strictly linear. In addition, even if the reference signal is a pure sine wave, large fluctuations in the output will occur due to the noise which is still present.

These objections may be overcome in the following manner: the original noise reading is noted, 20 db of attenuation is inserted into the system with the precision attenuator, and then reference signal is added, until the original reading is duplicated. The reference signal input is now 100 times the original noise power, or 10 times the original noise voltage. A small error is introduced by neglecting the noise still present. However, the linearity of that part of the system which follows the attenuator is no longer important, since the same reading is used in each case. In addition, since the signal being measured in the second measurement may be practically a pure sine wave, little or no smoothing is required for the second reading. This type of measurement was employed in the present work.

Fig. 2 is a block diagram of the noise measurement apparatus assembled for these experiments.

It was desired to measure noise signals of the order of 1 microvolt with the greatest possible accuracy. Therefore, it was necessary that the effect of all extraneous sources of noise, such as hum, microphony and electrical interference be less than this value.

The most critical units of the system are those where the noise signal under investigation is smallest. For this reason, particular care was taken in the design and construction of the oscillator which was being studied, the reference signal injection circuit, and the pre-amplifier. Each of these units was built in a doubly shielded chassis.

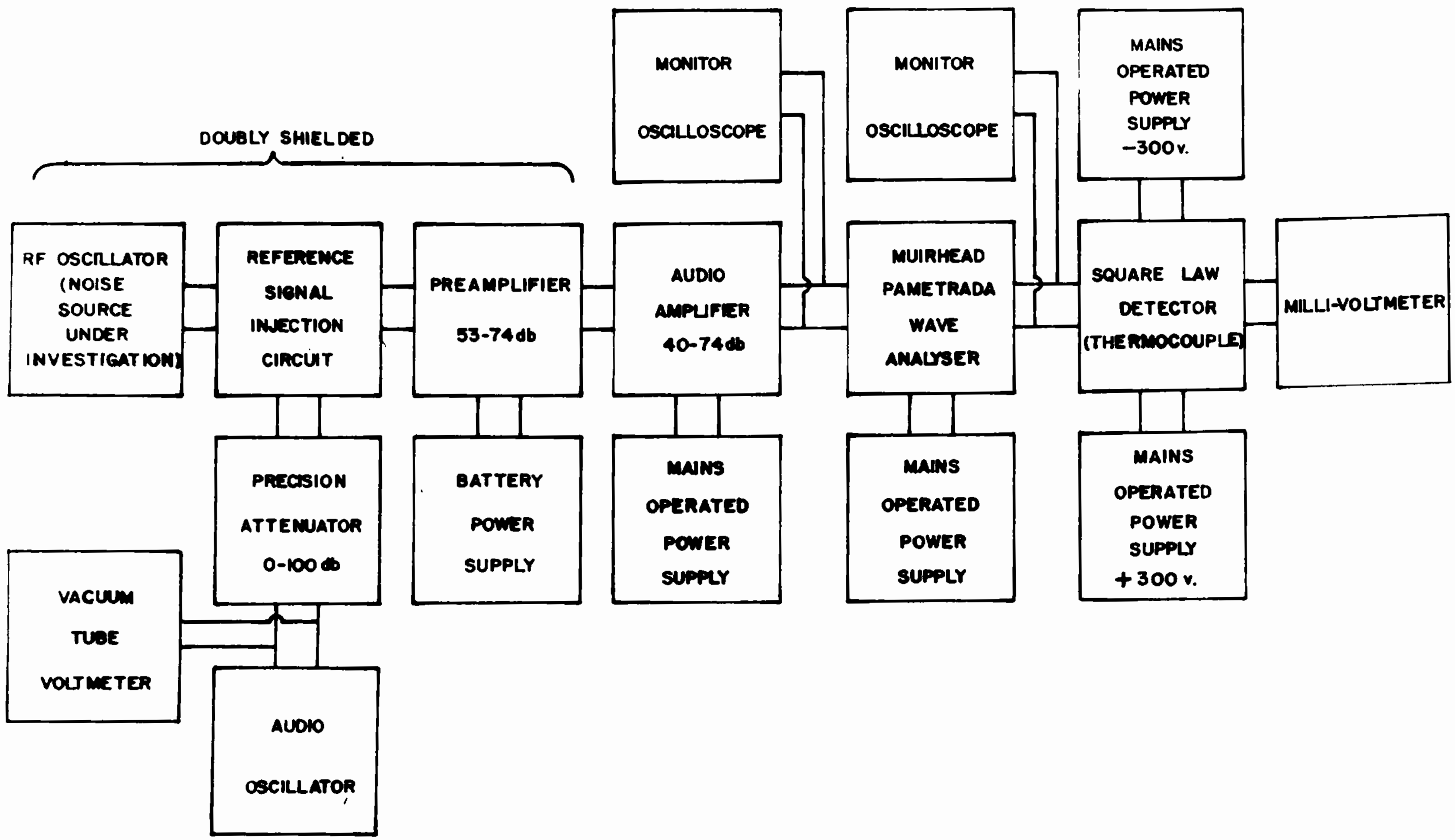


FIG. 2 BLOCK DIAGRAM OF THE APPARATUS USED TO MEASURE THE SPECTRA OF AUDIO NOISE

The circuits were carefully constructed, with special attention to layout and rigidity. Each unit was shock mounted within its own chassis. The doubly shielded chassis were designed to permit quick and convenient access to the circuits. Fig. 3 is a cross sectional view of the preamplifier, showing the special chassis construction.

A study was made of grounding techniques. Baxandall[31] has pointed out that since a relatively large hum signal will be generated in any small conducting loop that lies in a small magnetic hum field, it is extremely important to avoid such loops. Strict attention was paid to this fact in the design of this system. The whole system was grounded at only one point, at the input of the preamplifier, for if the system were grounded in two separate places, a very large ground loop would be formed, which might cause a hum signal to be generated which could completely obscure any noise signal being measured.

The circuit ground and the shield of the circuits ahead of the preamplifier in the system were carried separately to the common ground at the preamplifier. If these circuits were grounded to their own shields away from the common ground point, a loop would be formed in which hum signals might be generated.

Care was taken in the layout of the input lead, ground return lead and grid leak resistor of the amplifier so that the loop they form was as small as possible.

Whenever possible leads and their ground returns were twisted together, as is usually done with filament leads. Shielded cables were used throughout. The low-level units were completely battery operated and were kept at least eight feet from all mains transformers.

---

31. Baxandall, P.J. Wireless World, 53, 57, (1947)

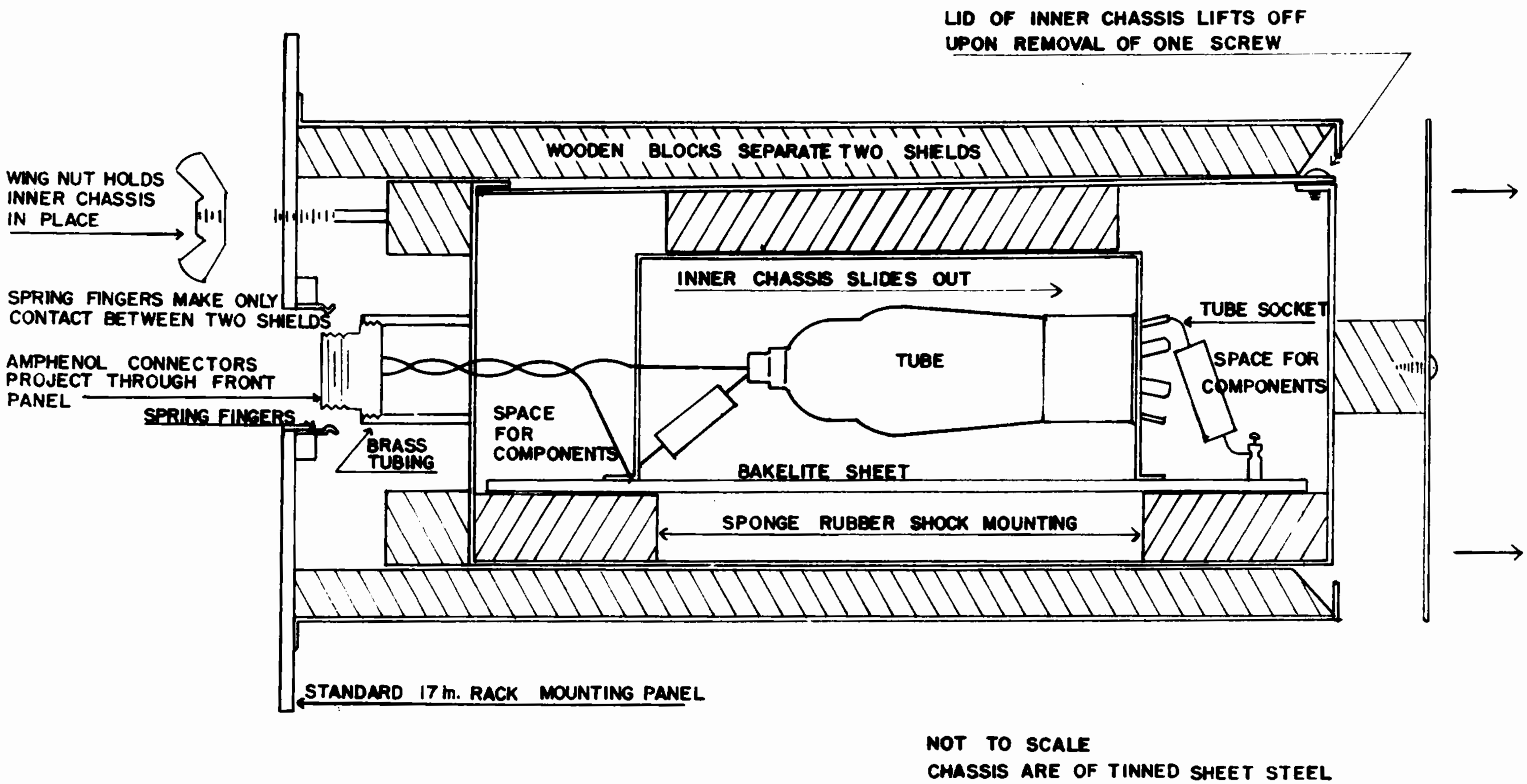


FIG. 3 CROSS SECTIONAL VIEW OF THE PREAMPLIFIER SHOWING THE CONSTRUCTION OF THE DOUBLY SHIELDED CHASSIS

By observing these precautions, it was possible to take low-level noise measurements at any time, and with any apparatus in the laboratory in operation.

(b) The Reference Signal Source and Injection Circuit.

It is possible to use any signal source which generates a known signal of fixed amplitude at the frequency of measurement as the reference signal source.

The reference signal is actually being compared with a narrow frequency band of noise. Therefore, if a sine wave is used as the reference signal, it is necessary to know the bandwidth of the system in order to calculate the noise power density. However, a source may be used which generates a calculable quantity of noise power, whose density is independent of frequency. Specially constructed diodes, which generate pure shot noise, are often used for this purpose. In this case it is not necessary to know the system bandwidth, for

$$NP \cdot \delta f = 2eIR\delta f$$

where NP = Noise Power density

R = noise diode load resistor

and the δf's cancel out.

Unfortunately, most diodes generate flicker noise at low audio frequencies, and hence cannot be used as reference sources for audio noise. Recently, it has been shown[32] that vacuum photo-tubes generate pure shot noise down to D.C., and therefore may be used as 'white' noise sources at audio frequencies.

A commercial audio oscillator was used as the reference signal source. For the purpose of the measurement it was necessary that this

---

32. Brown, D.A.H., T.R.E. Memo, No. L3-34 (1947)

unit furnish a signal of accurately known magnitude which was stable in amplitude and frequency. The oscillator used was manufactured by the Southwestern Industrial Electronics Co. Its frequency was variable from 1 cps to 120 kc and it had excellent frequency stability. The output of this oscillator was set at 0.1 volts by means of a Hewlett Packard Vacuum Tube Voltmeter, and was applied to a precision audio attenuator, manufactured by Radiometer of Copenhagen. The output of this attenuator was matched into the injection circuit, and could be read directly in microvolts. The calibration of the vacuum tube voltmeter was checked with a 0.1% accuracy A.C. meter and was adjusted to be within 1% of the correct reading at the point used.

Fig. 4 is a schematic of the reference signal injection circuit. The reference signal is applied across the 600 ohm matching resistor, which is in series with the 280 kilohm load resistor across which the noise signal to be measured is applied. Since some of the reference signal appears across the 280 kilohm load resistor, a simple calculation must be made to determine the fraction of the applied reference signal which is actually applied to the input of the measuring system by the network.

In a case where the internal impedance of the source of the noise being measured is low in comparison with the 280 kilohm load resistor, nearly all of the applied reference signal voltage would appear across the 280 kilohm load resistor, and very little would be applied to the input of the measuring system. To overcome this difficulty, the injection circuit was designed with a separate input for low impedance sources. When this input was used, a 280 kilohm resistor was placed in series with the noise source, thus increasing its apparent impedance. This device reduced the noise voltage measured to approximately one half the noise

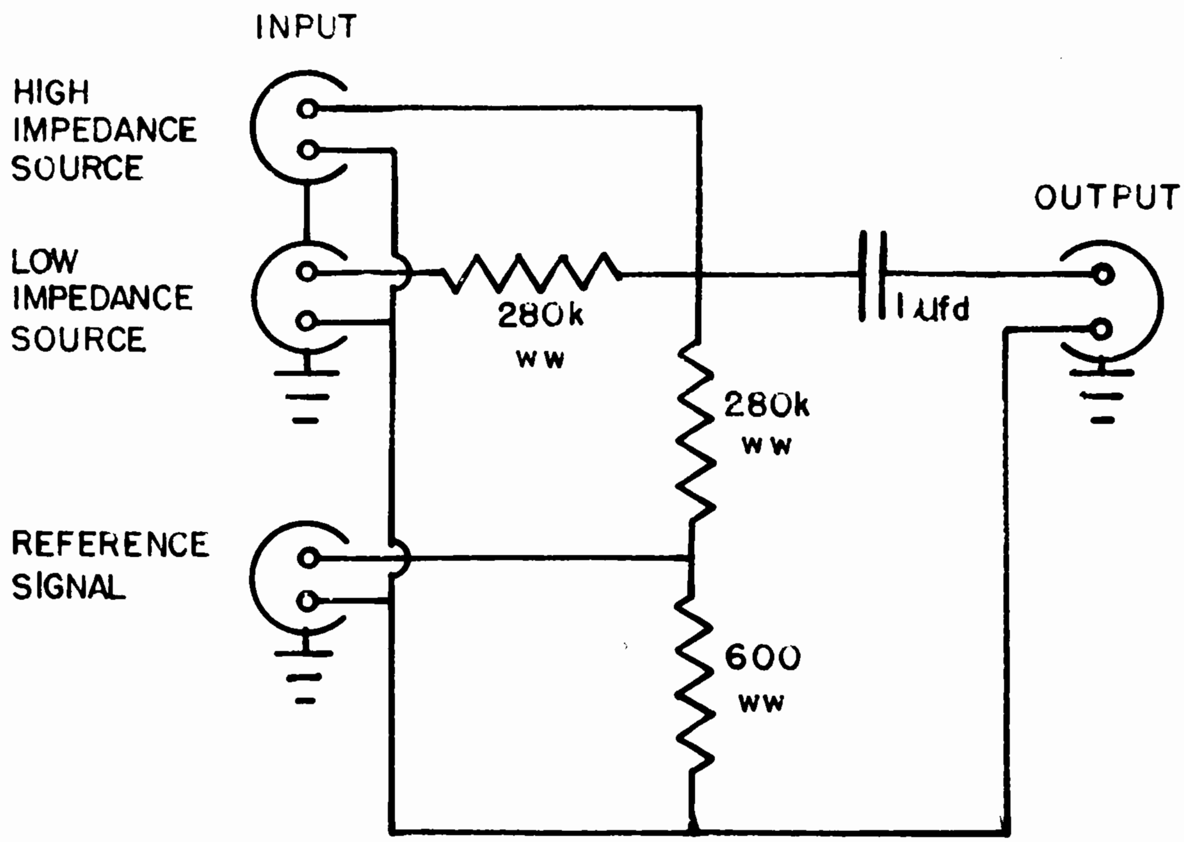


FIG. 4 SCHEMATIC DIAGRAM OF THE REFERENCE SIGNAL INJECTION CIRCUIT

generated in the source.

(c) The Preamplifier.

This unit was required to amplify signals of the order of one microvolt, and of frequencies from 5 cps to 20 kc/s. A highly stabilized high gain amplifier with a wide frequency response and with low internal noise characteristics was required. Further, unwanted external noise could not be allowed to enter the system. Fig. 5 is a schematic of the preamplifier circuit. This is a copy of the circuit published by D. Barber[33] for the same purpose.

The gain of this amplifier was controlled by feedback, and was variable in 10 db steps from 50 to 70 db. At the highest gain position, 30 db of negative feedback remained to provide stabilization and flat response. This amplifier had a flat response from 1 cps to above 20 kc.

Several tubes, including the 12AY7, 12AU7, 12AT7, and the EF37A were tested for low noise, low microphony properties. The EF37A, which is manufactured by Mullard of England, was found to be the most satisfactory tube for the purpose. It is designed to have particularly low noise, low hum and low microphony characteristics.

The use of this tube, together with double shielding, a battery power supply and other precautions detailed in section III, 2,(a) resulted in an extremely low noise level in this amplifier. Measurements showed that this amplifier generated noise equivalent to 1.26×10^{-8} volts/cycle at the input at frequencies above 200 cps. This value increased at lower frequencies due to the presence of flicker noise. At 100 cps, the equivalent input noise signal was 2.24×10^{-8} volts/cycle.

---

33. Barber, D., T.R.E. Tech. Note., No.135 (1952)

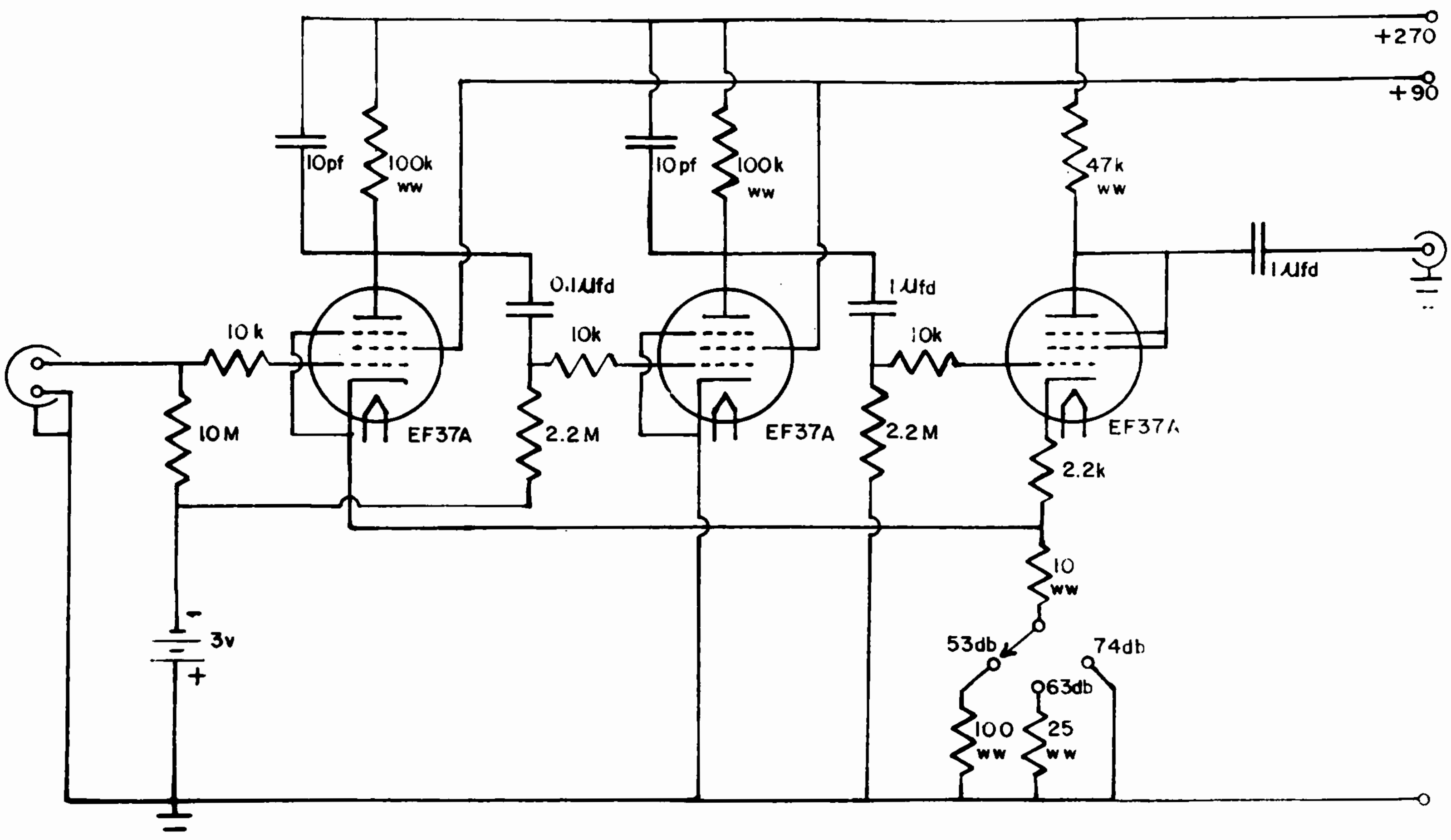


FIG. 5 SCHEMATIC DIAGRAM OF THE PREAMPLIFIER

(d) The Audio Amplifier

Although the design of the second audio amplifier was not as critical as that of the preamplifier, it was required to amplify signals of the order of one millivolt. Therefore, low hum and low noise, along with stabilization of gain and frequency response, were still of primary importance.

Fig. 6 is a schematic of the circuit employed. It is a variation of the preamplifier circuit designed to operate with a mains operated regulated power supply. The EF37A input tube of this amplifier provides an extremely low hum level at the output.

These two audio amplifiers could be operated side by side in the same rack in cascade without any difficulty. The large coupling condensers, which were necessary to attain the required low frequency response, resulted in large transients lasting about a minute when the amplifiers were switched on or when the input conditions were changed. However, these did not affect the operation of the units under steady conditions.

(e) The Tuned Amplifier.

The tuned amplifier was required to select the noise signal in the desired band of frequencies, and to attenuate signals outside the band to a level where they did not interfere with the measurement. The nature of the measurement made it necessary that the frequency of measurement be variable, and that the bandwidth be narrow enough to provide accuracy of frequency measurement. The design was made more difficult by the fact that appreciable drifts of gain or bandwidth could not be tolerated.

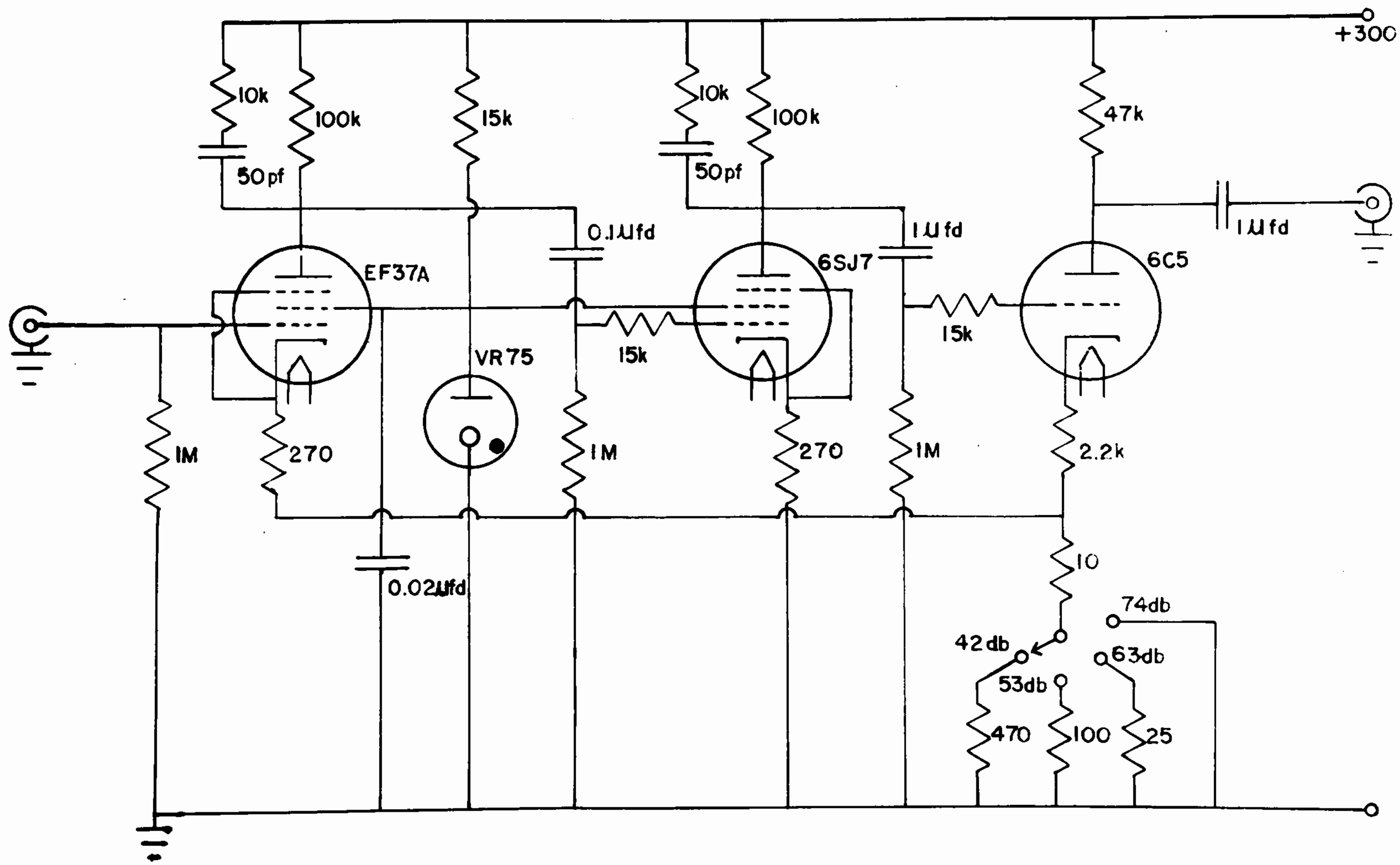


FIG. 6 SCHEMATIC DIAGRAM OF THE AUDIO AMPLIFIER

A Muirhead Pamedrada Wave Analyser was used for this purpose. This instrument consists of a precision attenuator, followed by two stabilized Wien Bridge variable frequency tuned amplifiers in cascade. The frequency is continuously variable from 19 cps to 21 kc, and the effective Q of the circuit is variable, in steps, from 1 to 100. The input attenuator provides full scale output for inputs from 1 millivolt to 300 volts. The instrument is run from a separate mains operated power supply, arranged so that only D.C. power is fed to the instrument. No trace of hum could be detected in the output of the instrument even in its most sensitive gain position. Fig. 7 is a plot of the variation of the noise bandwidth of this instrument with frequency in the high Q position. The instrument was used at this setting throughout the experiments.

(f) The Square Law Detector.

In noise measurements, the mean value of the voltage fluctuation about the D.C. voltage is zero. Therefore, it is necessary to measure the mean square value of the voltage fluctuations, or the mean power. For this reason a detector which responds to the square of the applied voltage must be used.

There are a number of devices which act in this way, such as thermistors, thermocouples and electronic square law detectors.

The simplest and most stable of these devices is a thermocouple, where the junction is heated by the electrical noise power. The thermocouple delivers a D.C. voltage, usually of the order of a few millivolts, which is directly proportional to the temperature of the junction, and is therefore proportional to the noise power, or the square of the noise voltage, that is being used to heat the element.

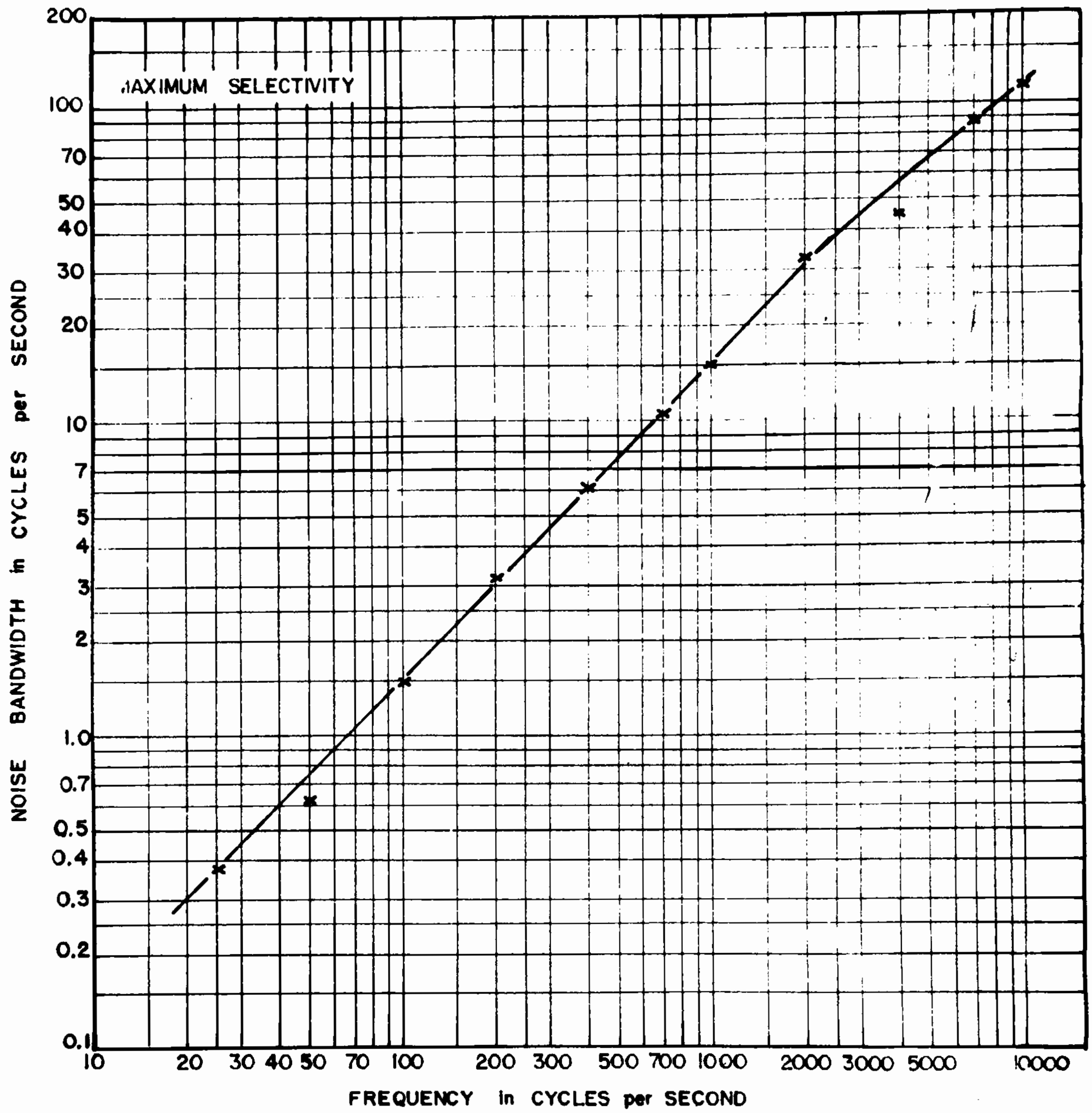


FIG. 7 EXPERIMENTALLY DETERMINED VARIATION WITH FREQUENCY OF THE NOISE BANDWIDTH OF THE MUIRHEAD PAMETRADA WAVE ANALYSER

Since the thermocouple is a low impedance device, some difficulty was encountered in designing a circuit with a sufficiently low output impedance and sufficiently high power handling capabilities to drive it. Fig. 8 is a schematic of the thermocouple unit, which has a low output impedance due to its large negative feedback. A 6V6 power tube was used as the output tube in order to give sufficient output current for the thermocouple.

The thermocouple itself was mounted in a separate compartment in this unit, and was thermally insulated from the rest of the circuit. This prevented the occurrence of extraneous thermal emf's in the thermocouple loop. A 500 ohm dummy load resistor could be substituted for the thermocouple when the unit was not in use.

Fig. 9 is a graph showing the linear relation of the output of this unit with applied power. Fig. 10 is a plot showing the linearity of the whole noise measuring system. As a precaution against burnout, the power applied to the thermocouple was always limited to such a value that the output meter reading was always in the lower half of its range.

As indicated in section III, 3,(a), it is necessary to smooth out the fluctuations in the output of the thermocouple. An RC circuit with a sufficiently long time constant has a very high impedance, and when used in conjunction with a thermocouple would result in a very small output. With conventional thermocouples, the difficulty could be overcome only by the use of an expensive direct current amplifier. Another approach was used in the design of the apparatus, made possible by the fact that a thermocouple itself possesses a time constant, since it takes

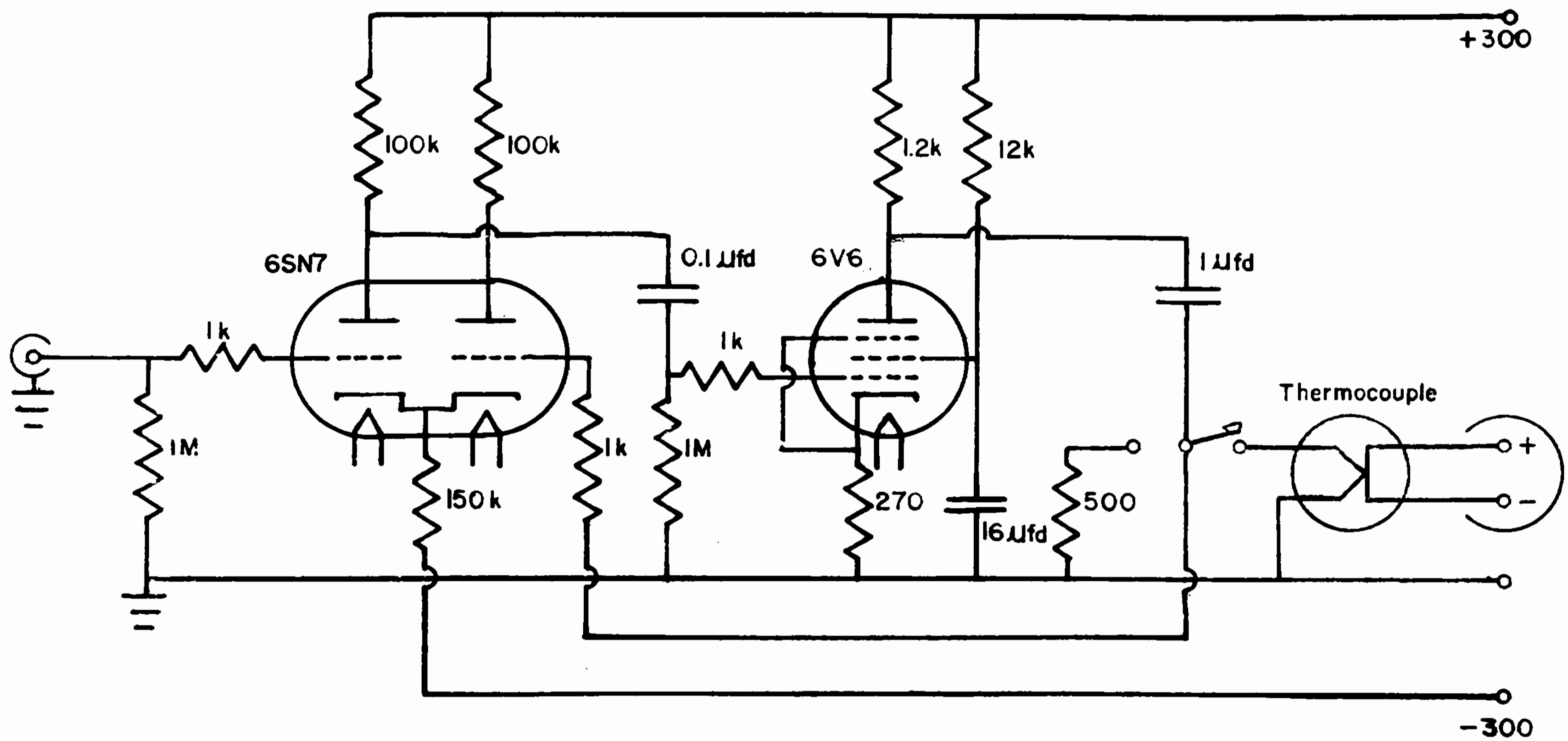


FIG. 8 SCHEMATIC DIAGRAM OF THE THERMOCOUPLE CIRCUIT

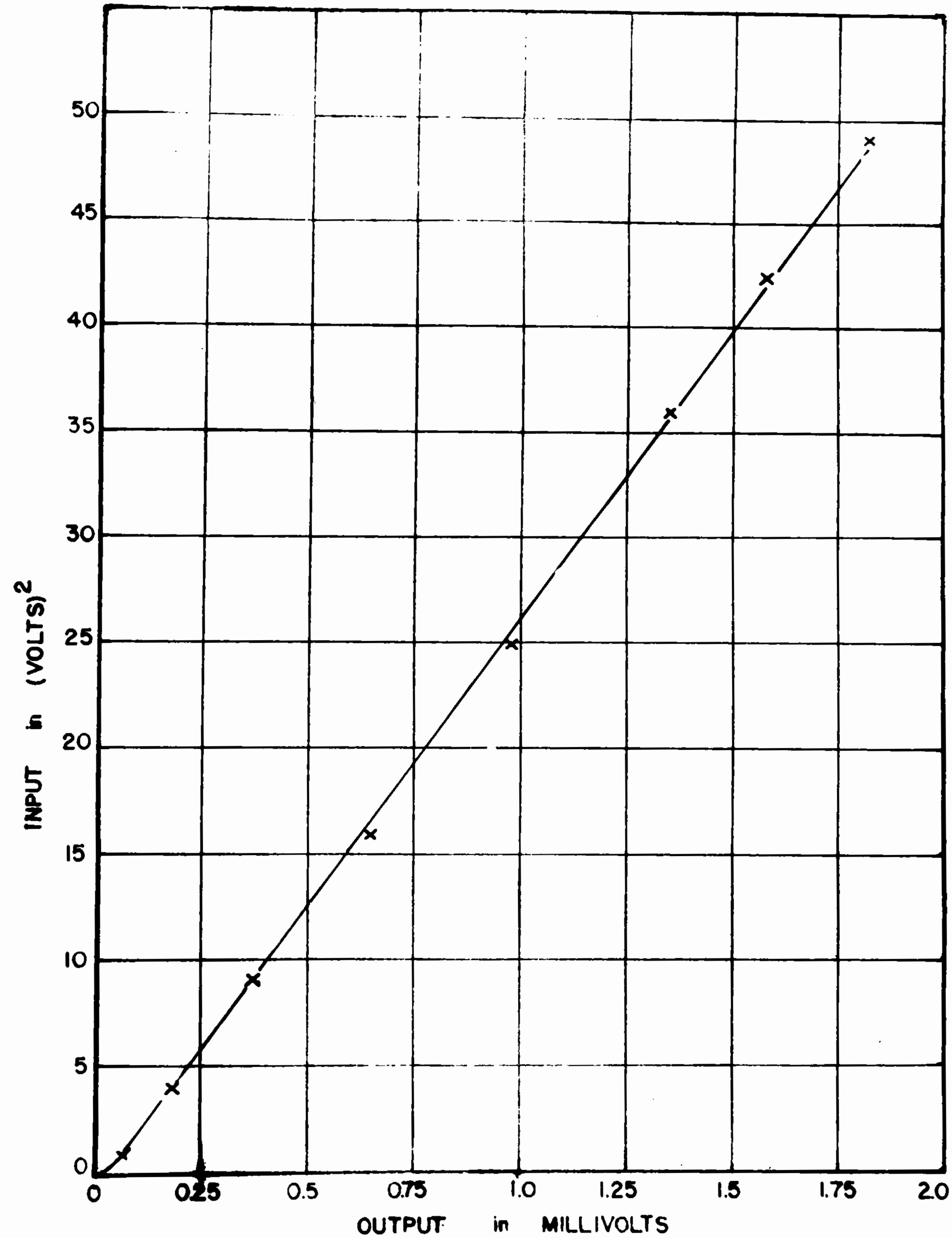


FIG. 9 A GRAPH SHOWING THE LINEARITY OF THE RESPONSE OF THE THERMOCOUPLE CIRCUIT TO INPUT POWER

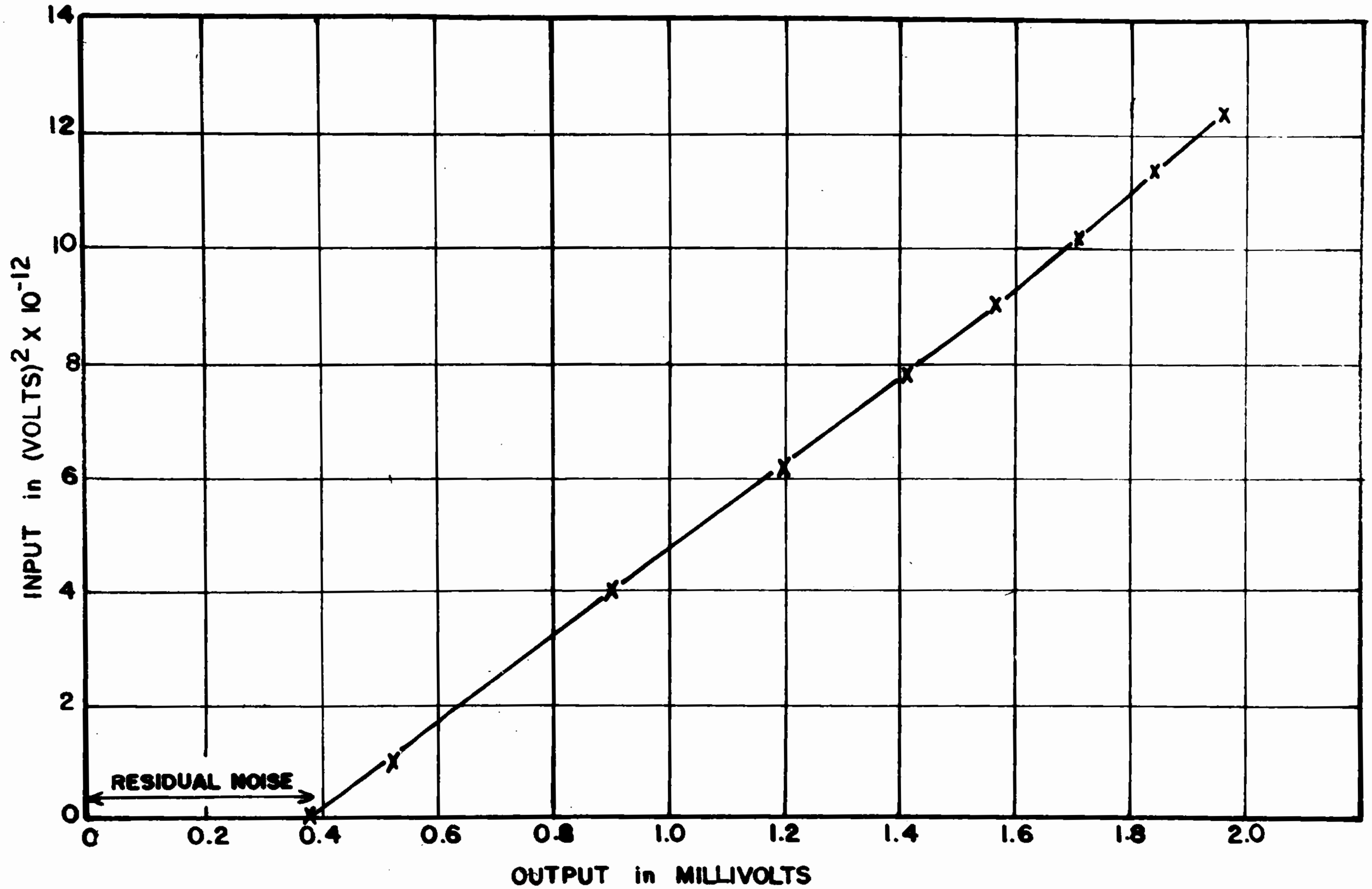


FIG. 10 A GRAPH SHOWING THE LINEARITY OF THE RESPONSE OF THE ENTIRE MEASURING SYSTEM TO INPUT POWER

a finite time for the element to heat up and cool down. Commercial thermocouples are designed to respond rapidly, and the longest time constant found in commercial units which were tested was five seconds. A vacuum thermocouple of iron and constantin was therefore designed and built which had a time constant of thirty seconds. Heavier wires were used in this unit to increase its thermal capacity and thus achieve a longer thermal time constant. This thermocouple was used in all measurements.

Fig. 11 is the schematic diagram of an electronic square law detector. In this device, a phase splitting circuit applies the noise signal, 180° out of phase, to the grids and screens of two amplifiers, which have the same plate load. The output of a vacuum tube amplifier may be represented by the equation

$$i_p = av + bv^2 + \text{——} \qquad \text{III-4}$$

where i_p = plate current

v = applied signal voltage

and a, b = constants of the tube.

For the two tubes, operated in antiphase, the output is therefore[34,35]

$$i_p = 2bv^2 + \text{-----}$$

This signal, which is an A.C. signal, is amplified, passed through a linear detector, and applied to a smoothing netowrk and vacuum tube voltmeter circuit of a type used by Barber.[33]

Fig. 12 is a curve showing that the output of this unit is directly proportional to the square of the input voltage up to 5 volts input. However, this instrument would operate satisfactorily only over the

33. Barber, D., T.R.E., Tech Note., No. 135 (1952)
34. Ross, H.McG. and A.L. Schuffrey, J. Sci. Inst., 25, 200, (1948)
35. Berger, F.B. and D. MacRae, "Waveforms", McGraw-Hill, (1949), p. 179 (M.I.T. Rad. Lab. Ser. Vol. 19)

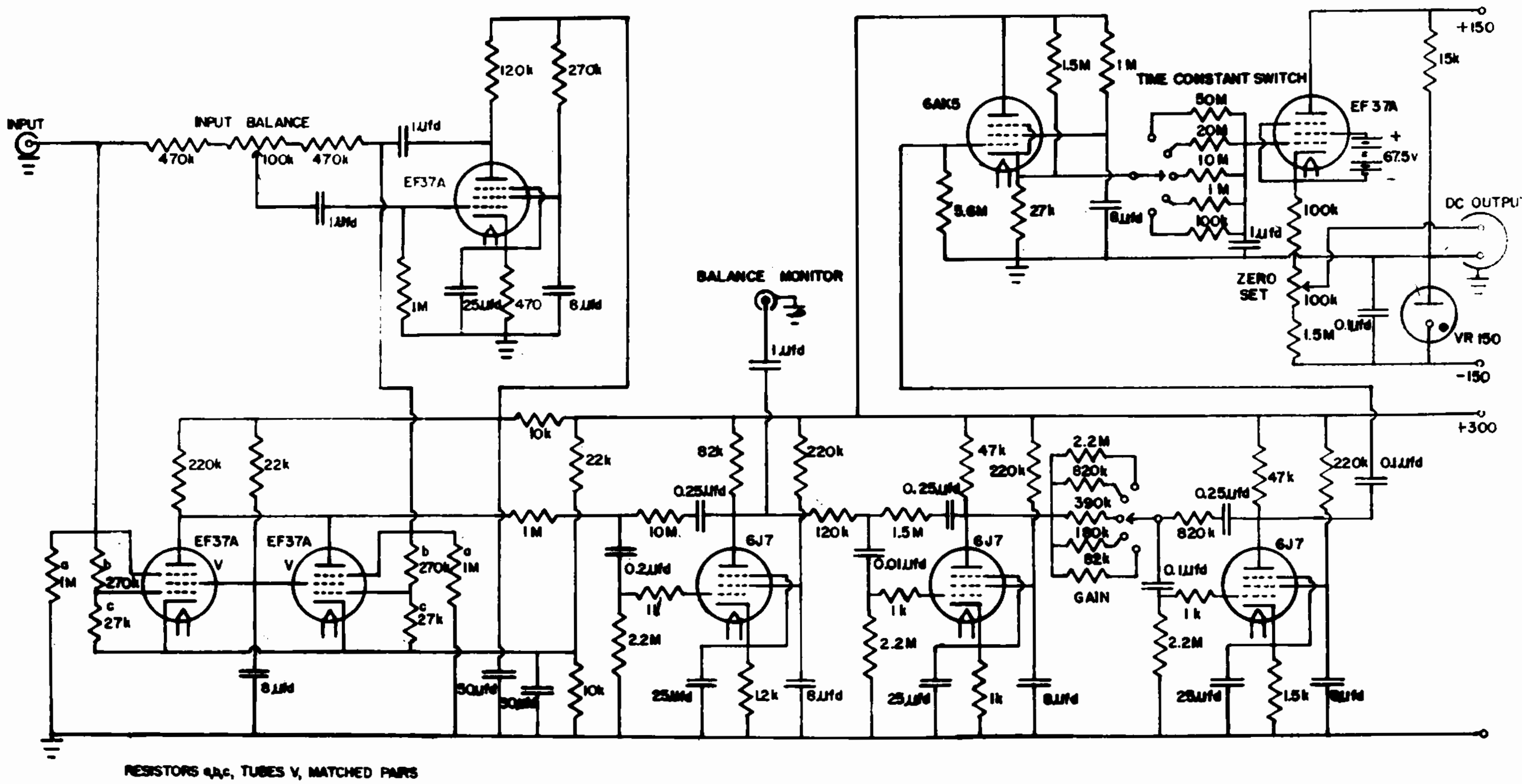


FIG. 11 SCHEMATIC DIAGRAM OF AN ELECTRONIC SQUARE LAW DETECTOR

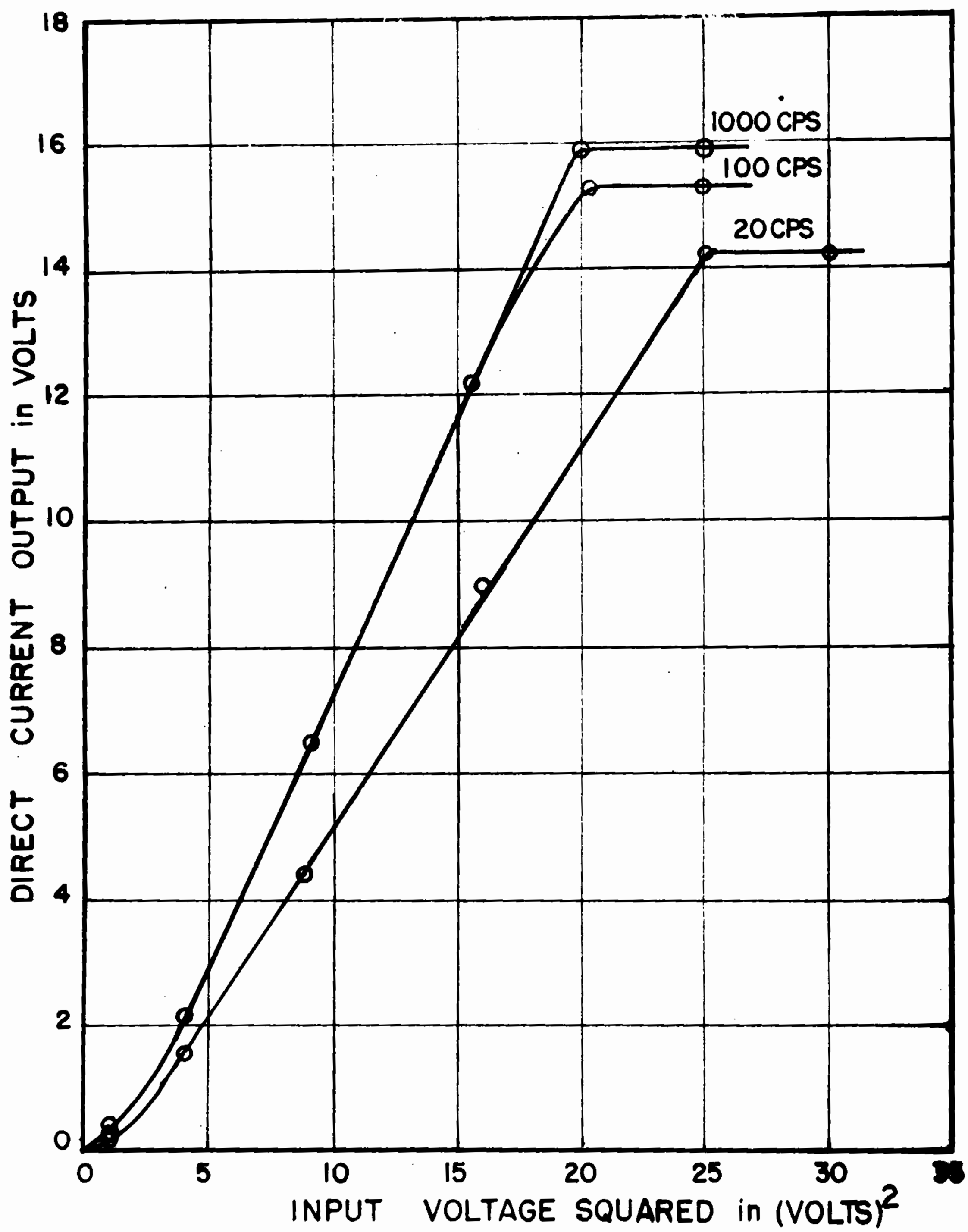


FIG. 12 RESPONSE CURVES OF THE ELECTRONIC SQUARE LAW DETECTOR

frequency range from 20 cps to 1 kc. This was due primarily to adverse phase shifts caused by poor electrical layout.

Further improvement in the frequency response of this instrument would make it suitable for use in the noise measuring system.

3. Correlation Measurements

(a) General Considerations

A comparison was required of the noise occurring at audio frequencies in the tube, and that appearing as an audio noise modulation on the rf output of the oscillator.

The shape of two noise trains may be compared by amplifying each of them, and by recording them simultaneously by means of a pen recorder or by photographic means. The latter course was chosen. The two noise traces in question were displayed simultaneously on a double beam oscilloscope, and a continuous record was kept by means of a 35 mm oscilloscope camera.

Visual observation of the resulting record will detect any strong correlation. However, considerable mathematical analysis is necessary to obtain the actual percentage correlation.

The percentage of correlation may be obtained by multiplying the two noise signals together by electronic or mechanical means. If the two signals are completely uncorrelated, there is an equal probability of their product having a positive and a negative sign. The mean result is therefore zero. If the two signals are exactly alike, their product will always be positive. The mean result will be a maximum in this case. Other percentages of correlation will yield results lying between these two extremes. Because this method requires the construction of a complex and costly electronic multiplier, other methods of comparison were employed.

Correlation can be detected by applying the separate signals to the vertical and horizontal deflecting plates of a cathode-ray oscilloscope. Perfect correlation would be indicated by a straight line, lying at an angle across the screen. Complete lack of correlation would result in a circular blur in the centre of the tube. Partial correlation would result in a filled in elliptical pattern, lying at an angle across the screen. The percentage of correlation could be determined approximately by comparison of these patterns with those resulting from known correlations.

Fig. 13 is a block diagram of the apparatus assembled for the correlation measurements. The audio noise originating in the oscillator and the detected audio frequency noise modulation on the rf signal were each amplified by means of separate, identical, amplifier chains. The amplifiers were the same as those described for use in measuring noise power spectra. A band of frequencies was then selected from each noise train by a pair of identical tuned audio amplifiers. The resulting signals were then tested for correlation by either one or the other of the methods already described.

(b) The Tuned Amplifiers.

Fig. 14 is a schematic diagram of the circuit of a pair of identical audio band pass filters, designed by Dr. J.R. Whitehead of this laboratory. For this circuit, the tuned frequency is given by $2\pi fCR = 1$ and the effective Q of the circuit is approximately $k/2$.

The circuit was so designed that separate frequency determining units, containing the condensers Ck and C/k could be plugged in at will. A number of these units ranging in frequency from 25 cps to 10 kc and having Q's of approximately 6 were constructed and tested. These resulted in bandwidths of about 1/6 the centre frequency of the filter.

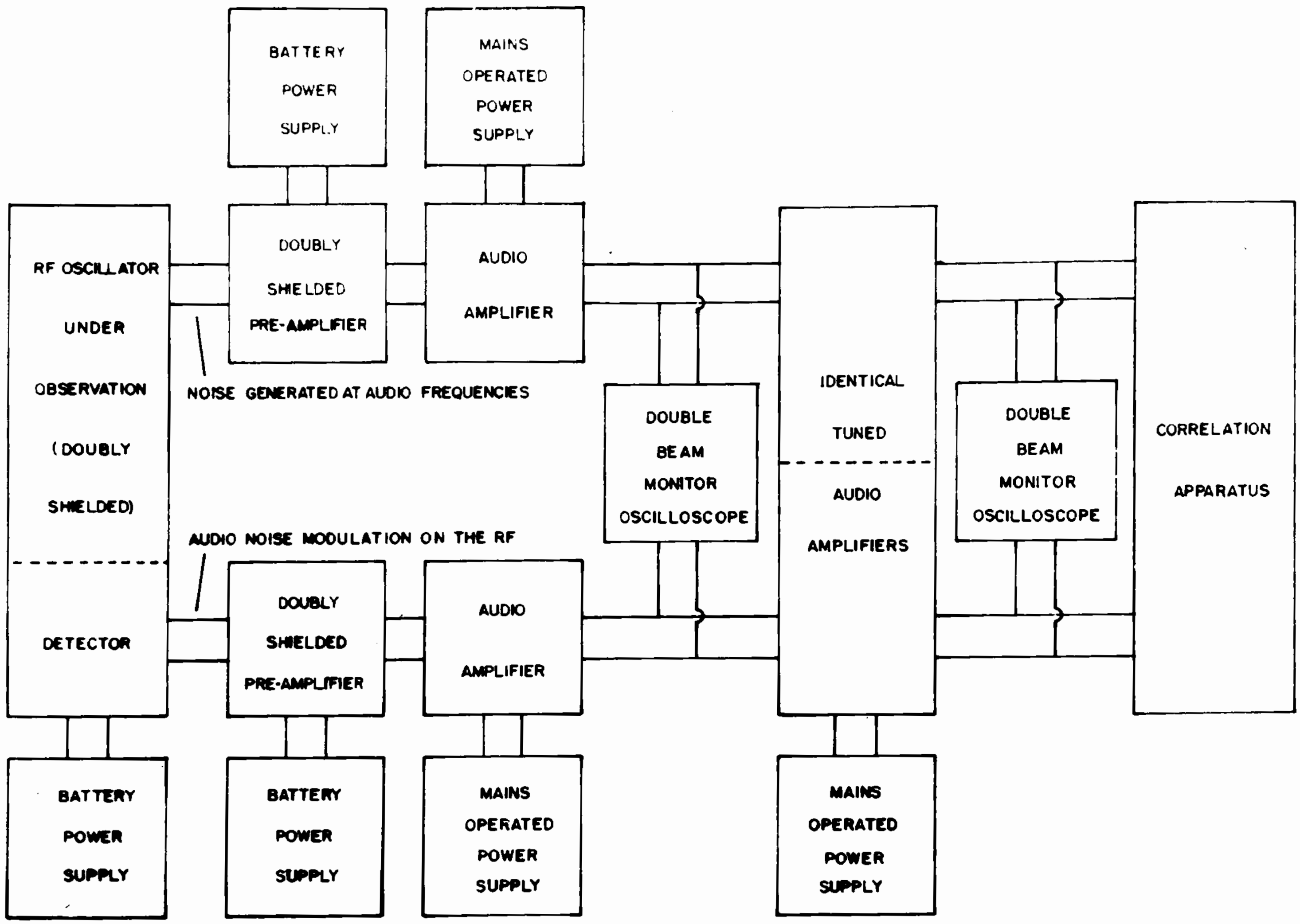


FIG. 13 BLOCK DIAGRAM OF THE APPARATUS USED TO MEASURE THE CORRELATION BETWEEN THE NOISE GENERATED AT AUDIO FREQUENCIES AND THE AUDIO NOISE MODULATION ON THE OUTPUT OF A RADIO FREQUENCY OSCILLATOR

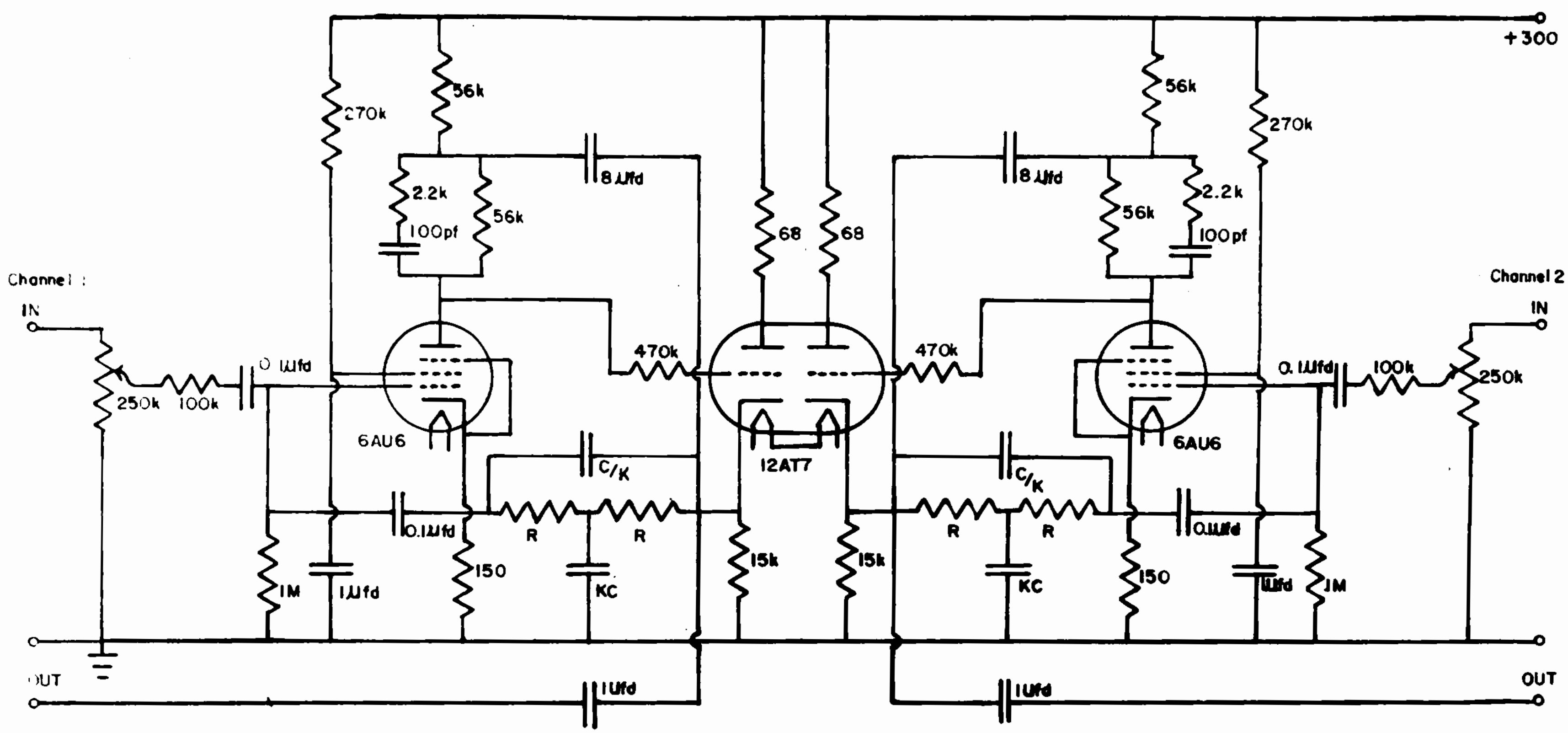


FIG. 14 SCHEMATIC DIAGRAM OF A PAIR OF IDENTICAL TUNED AUDIO AMPLIFIERS

Careful tests showed that no appreciable interaction occurred between the two filters, in spite of the fact that they had a common power supply and were built on the same chassis.

IV THE RADIO FREQUENCY OSCILLATOR AND DETECTOR

It was desired to study the noise arising from shot, partition and flicker effects in the tube of an rf oscillator; it was necessary, therefore, that noise arising from other effects in the circuit be negligible compared to the noise from these sources. To carry out this project two separate noise signals had to be investigated; the noise occurring at audio frequencies within the tube, and the noise occurring as a noise modulation of the rf output.

The noise at the rf frequency could be studied by applying the rf output to a detector. The noise on the rf oscillation will then appear as an audio noise signal at the detector output. However, any change in the output level of the oscillator will also appear as a noise fluctuation at the output of the detector. Since the noise signal being observed may be as small as one microvolt, a very small change in rf output level would obscure the desired signal.

The audio noise generated in an rf oscillator tube usually cannot be studied directly, since the anode circuit of the oscillator normally presents no impedance at audio frequencies. An audio load resistor was therefore placed in series with the anode tank circuit. This resistor was decoupled at radio frequencies, so that only the audio noise signal appeared across it.

In the design of rf oscillators it is difficult to avoid low frequency fluctuations of amplitude of oscillation, due to slight changes in the frequency of oscillation, changes in external loading conditions, and small fluctuations in the tube operating conditions.

Frequency stability may be achieved by the use of a crystal to control the oscillation frequency. All changes in loading, oscillation frequency and tube operating conditions may be minimized by the use of a buffer amplifier between the oscillator and the output load impedance. However, a separate buffer amplifier in the circuit in this case would have obscured the desired noise signal by adding its own noise to the output.

A number of circuits were tried in an effort to build an oscillator which satisfied the requirements for noise study. An electron-coupled oscillator, whose circuit is shown in Fig. 15, was found to be suitable for noise measurements. This circuit oscillated at 5.06 Mc.

In this circuit, the cathode, control grid and screen grid act as a triode oscillator, while the suppressor and anode act as a buffer amplifier. This circuit provided sufficient stability for all the measurements which were done.

It was found that most rf oscillators are particularly susceptible to microphony, both in the tube and the components. An EF37A Mullard tube was used as the oscillator tube. As has already been stated in section III, 2, (c), this tube was found to have an extremely low level of noise due to microphony.

Particular care was taken in the layout and construction of the circuit to ensure the greatest possible rigidity of the circuit components. Early trials indicated that contact noise at the tube socket was troublesome. All connections to the tube were soldered directly to the base pins in order to avoid this difficulty.

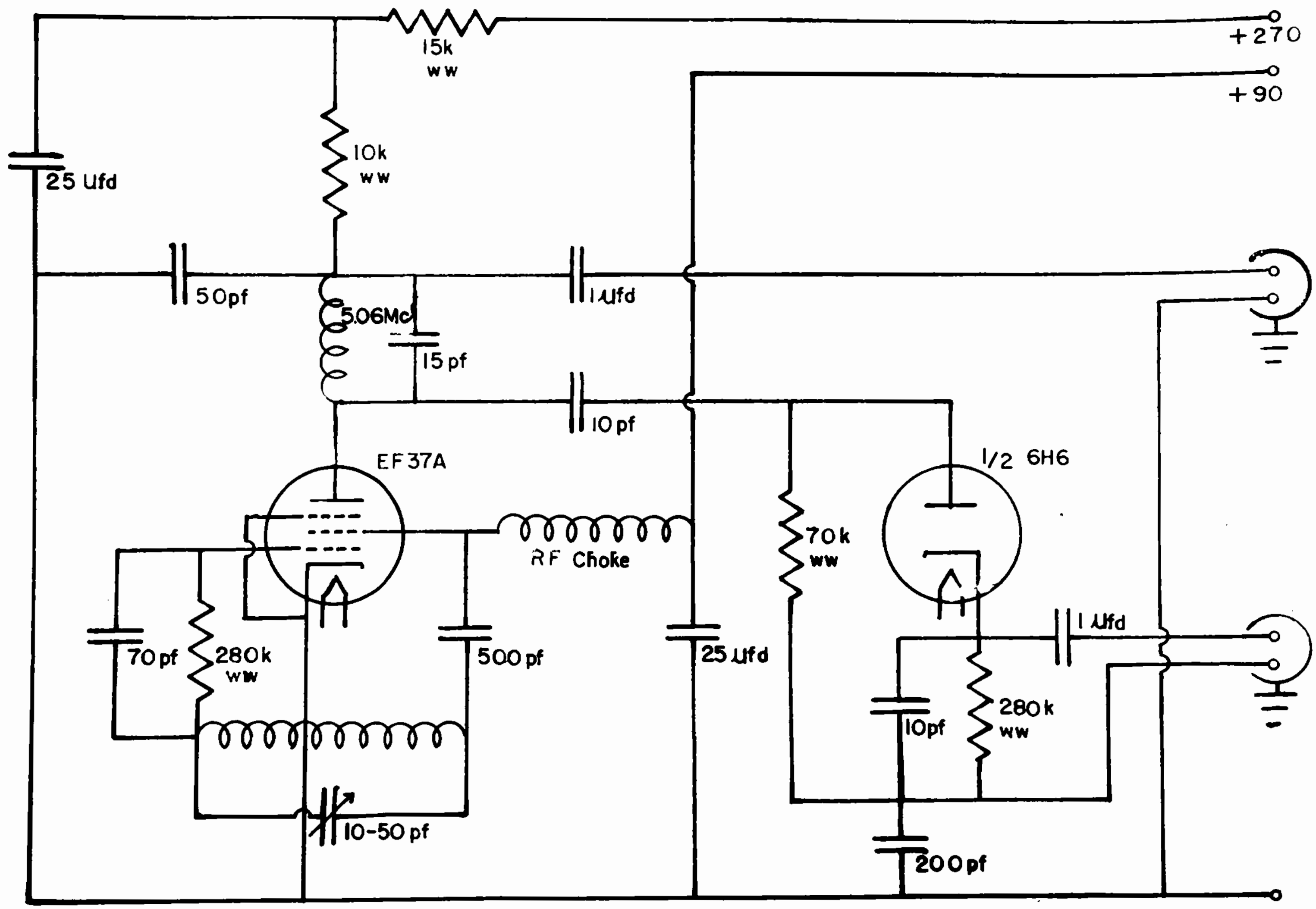


FIG. 15 SCHEMATIC DIAGRAM OF THE ELECTRON COUPLED OSCILLATOR

The circuit was mounted on foam rubber within a doubly shielded chassis of the same type as that used for the preamplifier, and detailed in Fig. 3. The chassis was then mounted on a sponge rubber pad, placed on a solid shelf, and the shelf was rigidly attached to one on the concrete support pillars of the laboratory. The frequency of any mechanical oscillations which did occur was reduced by a heavy weight placed on the chassis. The cables leading to the chassis were as flexible as possible, and were clamped solidly to the supporting shelf. Fig. 16 is a photograph of the oscillator and the amplifiers in operating position.

It was found that if the detector was located in a separate chassis, great difficulties arose with contact noise, variation of coupling, and microphony in the coupling cable and connections. Therefore, a detector of a very simple type was built in the same chassis as the oscillator using the same precautions in construction as the oscillator circuit. Care was taken to choose the coupling and bypass condensers of the circuit so that the separate noise signals could not interact due to leakage from one part of the circuit to another.

Extreme care was taken to avoid the occurrence of hum signals. It will be seen from Fig. 13, the block diagram of the correlation measurement system, that the oscillator and detector, the two amplifier chains and the correlation apparatus could have formed a very large ground loop. This loop was avoided by isolating the audio ground of the detector circuit from the oscillator audio ground, by means of a 200 pf rf bypass condenser. These grounds were carried separately to the inputs of their respective preamplifiers. By this means high hum currents were prevented from occurring in the low level portions of the circuit.

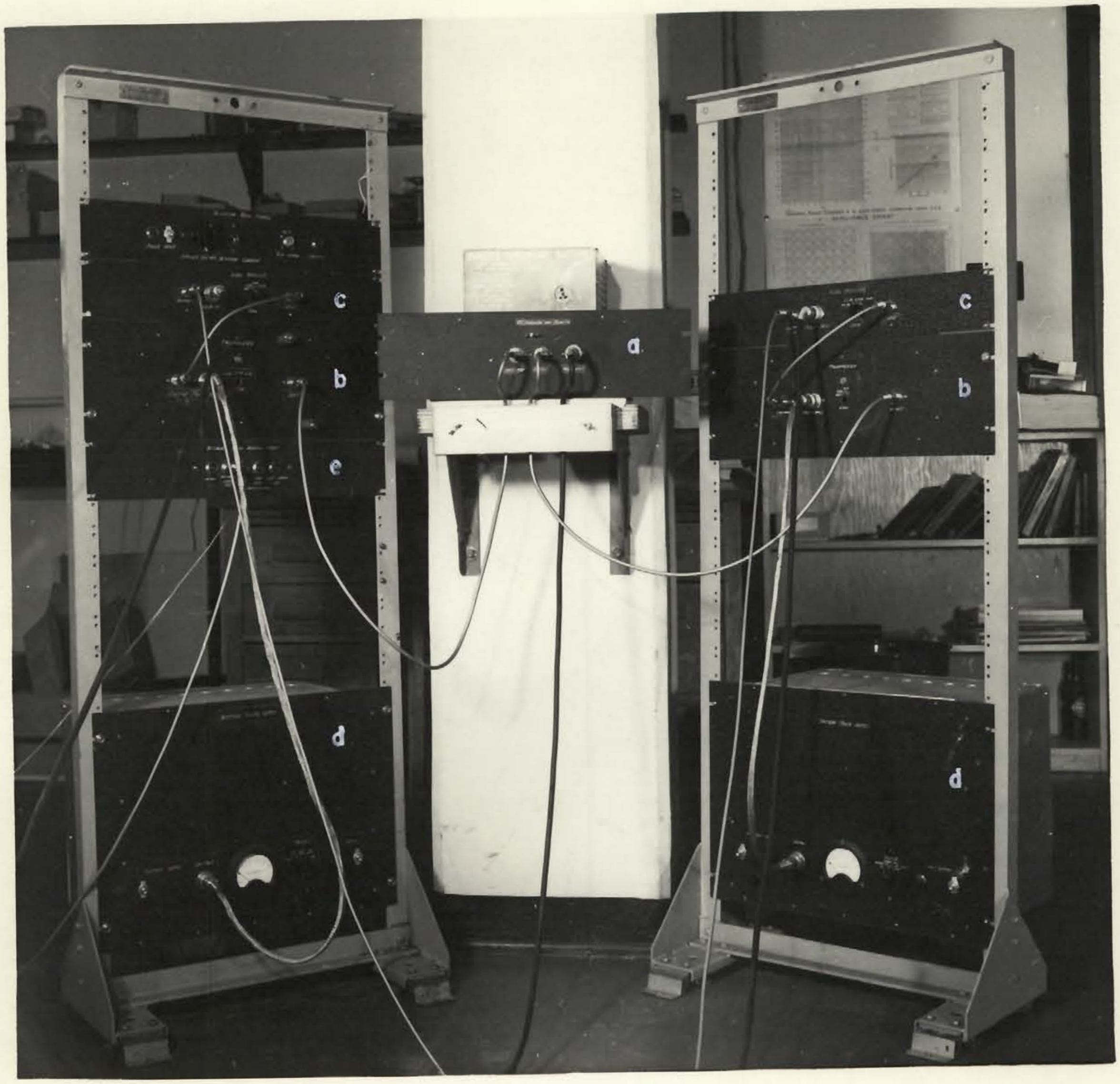


FIG. 16 A PHOTOGRAPH SHOWING THE RF OSCILLATOR AND THE TWO AMPLIFIER CHAINS IN OPERATING POSITION

a) Electron coupled oscillator, shock mounted on a rigid shelf
b) Preamplifiers
c) Audio amplifiers
d) Batteries for preamplifiers
e) Reference signal injection chassis

An 8.66 Mc crystal controlled oscillator, whose circuit is shown in Fig. 17, was also built. Although the same precautions were employed in its construction, it was much more susceptible to extraneous noise than the electron coupled oscillator, and only a few measurements were made on it.

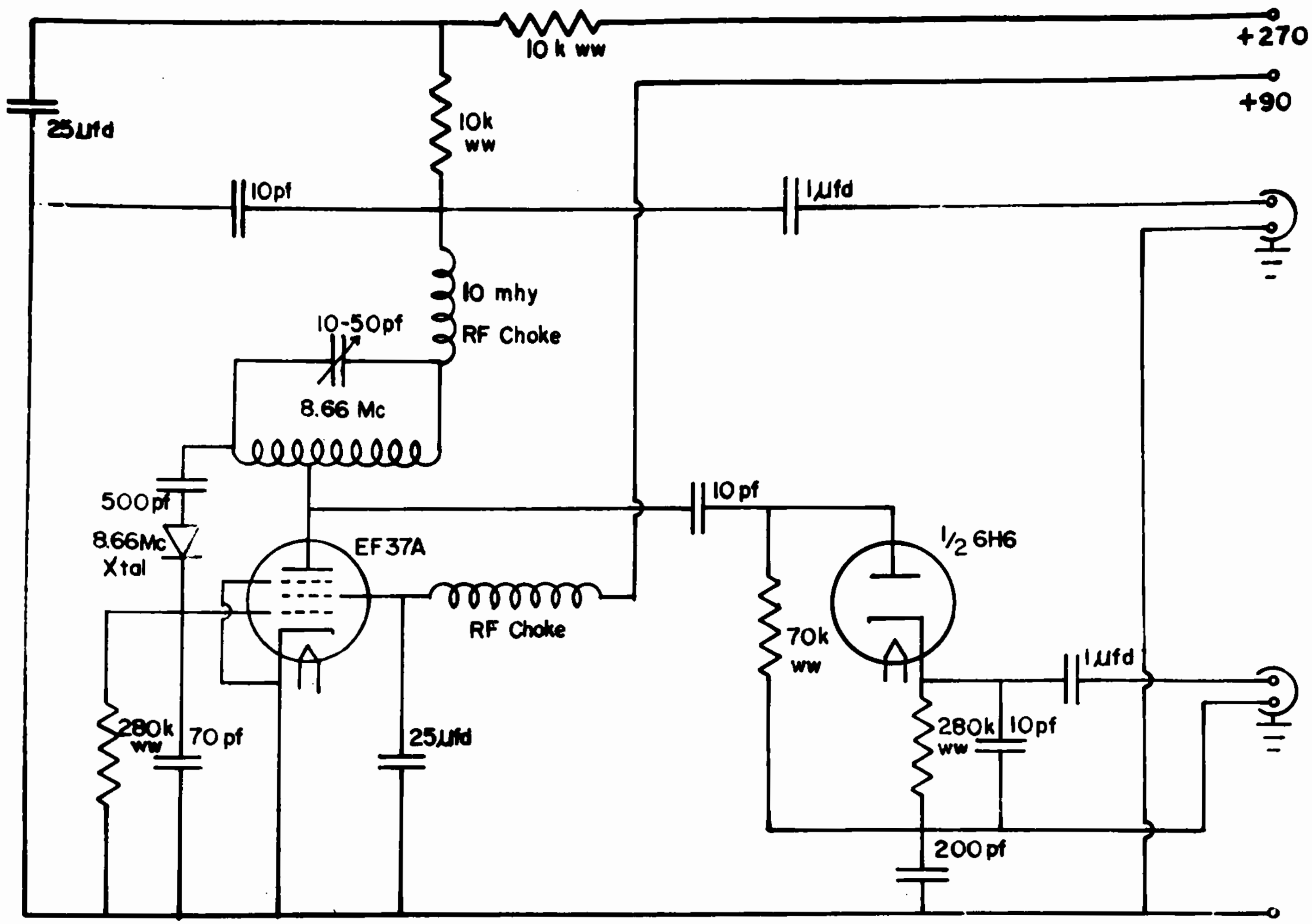


FIG. 17 SCHEMATIC DIAGRAM OF THE CRYSTAL CONTROLLED OSCILLATOR

V EXPERIMENTAL RESULTS

1 Foreword

The experimental proof that noise which is generated at audio frequencies within an rf oscillator tube also appears as an audio noise modulation on the rf output of the oscillator required a number of experiments. These fell into two main classes, as explained in section III, 1.

Measurements were made to determine the magnitude and frequency distribution of the noise occurring at audio frequencies within the oscillator tube, and of the audio noise detected as a modulation of the rf output of the oscillator. Comparisons were made of the two noise spectra obtained in order to show that the noise which was present as a modulation on the rf output had the same characteristics as that occurring at audio frequencies. These measurements are described at length in part 3 of this section.

Comparisons were made of the wave-form, or time-variation of the two noise signals described above in order to show by their similarity that they had the same origin. These measurements are described in detail, under the heading of correlation measurements, in part 4 of this section.

Part 2 of this section is a summary of all the experiments carried out, together with a brief notation of their results.

Most of the measurements described in this section were made on the electron-coupled oscillator described in section IV. The plate current of the oscillator was 2.1 milliamps, and the screen current was 650 micro-amps. The rf output was such that the steady potential difference across the output of the detector was twelve volts and the steady current in that circuit was 43 microamps.

2. <u>Summary of Experiments</u>

1) Nature of measurement: Noise power spectrum.

Purpose: to test the accuracy of the noise measuring apparatus as a function of frequency.

Noise source: a parallel combination of resistors = 585 kilohms.

Refer to: page 59 , Fig. 18.

Result: The experimental measurement was higher than theory by 0.2 db from 200 cps to 10 kc. Below 100 cps, flicker noise in the amplifier was evident.

Conclusion: The apparatus was adequate to measure frequency spectra for which (noise voltage)^2 density was in excess of 2×10^{-15} (volts)^2 per cycle over the range of 100 cps to 10 kc, and in excess of 10^{-14} (volts)^2 per cycle over the range from 10 cps to 100 cps.

2) Nature of measurement: noise power spectrum.

Purpose: to measure noise introduced directly by the linear detector which was used to demodulate the radio frequency signal.

Noise source: one half of a 6H6 diode, operated with 12 volts applied between anode and ground to simulate the conditions when it was connected to the oscillator.

Refer to: page 59, Fig. 18.

Result: The spectrum followed a 1/f law in the low frequency range, and fell to shot noise level at 1000cps. The detector noise was 10 db below the noise measured on the rf signal.

Conclusion: The noise measured at the detector output when excited by the rf signal could be attributed to the rf oscillator to within 0.5 db.

3) Nature of Measurements: noise power spectrum.

Purpose: to measure the noise power spectrum of the noise generated at audio frequencies in the rf oscillator.

Noise source: audio noise generated across 50 kilohm audio load resistor in the plate circuit of oscillator.

Refer to: page 60, Fig. 18.

Result: The spectrum followed a 1/f law in the low frequency range, and fell to the calculated shot noise level at 1 kc.

Conclusion: Flicker noise was generated which was 10 db. above shot noise at 25 cps.

4) Nature of experiment: noise power spectrum.

Purpose: to measure the noise power spectrum of noise modulation upon the rf output of the oscillator.

Noise source: audio output of the detector, which was used to demodulate the rf signal.

Refer to: page 61, Fig. 18.

Result: The spectrum followed a 1/f law in the low frequency region, and fell to the calculated shot noise level at 2 kc. The curve was approximately the same shape as the audio noise spectrum (3).

Conclusion: Flicker noise, which was 13 db above shot noise at 25 cps, was present as a modulation of the rf output of the oscillator.

5) Nature of Experiment: calculation of noise current spectra of the two noise signals measured in 3 and 4.

Purpose: to discover the relationship between the audio frequency noise current, and the noise current which would generate the noise modulation measured on the rf output.

Refer to: page 62, Fig. 19.

Result: Spectral plots of the two noise currents designated above almost coincided.

Conclusion: The noise current occurring within the oscillator at audio frequencies also acts as the generator of the noise modulation on the rf signal.

6) Nature of experiment: noise power spectrum.

Purpose: to discover any change in the noise power spectrum of the noise modulation of the rf signal which occurred when the audio load resistor was removed.

Noise source: the audio output of the detector. This represented the noise modulation on the rf signal. The audio load resistor in the oscillator was short circuited.

Refer to: page 63, Fig. 18.

Result: The spectrum followed a 1/f law in the low frequency region, and fell to the shot noise level at 700 cps. The spectrum was 3 db below the spectrum measured with the audio load resistor in the circuit.

Conclusion: Flicker noise in the noise modulation on the rf signal was not due to direct modulation caused by the audio noise signal appearing across the audio load resistor.

7) Nature of experiment: noise power measurement,

Purpose: to measure the variation of the magnitude of the noise modulation on the rf signal as a function of plate current at a low frequency.

Noise source: audio output of the detector, which represented the noise modulation on the rf signal.

Refer to: page 64 , Fig. 20.

Result: The magnitude of the square of the noise voltage varied directly as the square of the plate current at a modulation frequency of 100 cps.

Conclusion: The noise modulation on the rf signal at low modulation frequencies was due primarily to flicker noise, which was the only noise being generated in the circuit which varied as I^2.

8) Nature of experiment: Lissajous correlation experiment.

Purpose: to provide Lissajous patterns representing known percentage correlations of two noise signals for comparison purposes.

Noise sources: (1) thermal and circuit noise generated in an amplifier chain. (2) Thermal and circuit noise generated in this and a second amplifier chain, mixed in known ratios.

Refer to: page 65, Figs 21, 22.

Result: Patterns were obtained for zero, 25%, 50%, 75%, and 100% correlation for comparison with experimental measurements.

9) Nature of experiment: Lissajous correlation experiment.

Purpose: to discover any coupling between the two noise amplifier trains.

Noise sources: No power was supplied to oscillator.

(1) thermal noise in the oscillator audio load resistor.

(2) thermal noise in the detector load resistor.

Refer to: page 66 .

Result: No correlation was detected.

Conclusion: There was no interaction between the two amplifiers, due to their proximity.

10) Nature of experiment: Lissajous correlation experiment.

Purpose: to discover any correlation due to generation in the same circuit of the two signals under investigation.

a) Noise sources: The oscillator was not permitted to oscillate.

1) noise generated at audio frequencies in the tube. This noise signal appeared across audio load resistor.

2) thermal noise in the detector load resistor.

Refer to: page 66.

Result: No correlation was detected.

b) Noise sources: The oscillator was operating, with the audio load resistor short circuited.

1) Noise generated by the amplifier chain.

2) Noise detected as a modulation on the rf signal.

Refer to: page 67.

Result: No correlation was detected.

Conclusion: (a and b) There was no interaction between the two noise sources due to the fact that they were in the same circuit.

11) Nature of experiment: Lissajous correlation experiment.

Purpose: to determine whether the audio noise signal appearing across the audio load resistor in the plate circuit of the oscillator was modulating the rf signal directly.

Signal sources: 1) audio signal applied to audio load resistor.

2) detected audio noise modulation and signal modulation on the rf oscillation.

Refer to: page 67.

Result: One millivolt of audio signal was required to modulate the rf signal to a detectable degree.

Conclusion: Noise signals appearing at this point were 40 db below 1 millivolt. Therefore, modulation of the rf by the audio noise did not occur in this manner.

12) Nature of Experiment: Lissajous correlation experiment.

Purpose: to discover any correlation due to hum signals, or external audio signals.

a) Signal sources: 1) 60 cps mains signal. 2) either the noise signal generated at audio frequencies within the oscillator tube, or the audio noise modulation on the rf signal, and selected frequency bands from these signals.

Refer to: page 67 .

Result: No correlation was detected.

Conclusion: No correlation was caused by the occurrence of hum signals anywhere in the system.

b) Signal sources: 1) variable frequency audio oscillator.

2) either noise signal originating in the rf oscillator, or selected frequency bands from these signals.

Refer to: page 68 .

Result: No correlation was detected at any frequency.

Conclusion: Any observed correlation was not caused by an external audio signal.

13) Nature of experiment: photographic recording of noise traces.

Purpose: to detect a correlation between audio frequency noise generated in the rf oscillator and audio noise modulation occurring on its rf output, by a comparison of photographic records of the noise traces.

Noise sources: low frequency bands of noise selected from the two noise signals listed above, which were generated in the electron-coupled rf oscillator.

Refer to: page 68, Fig. 23.

Result: Frequency bands from 25 cps to 80 cps were tested. In all cases an easily detectable correlation was observed.

Conclusion: The two noise signals had the same origin within the oscillator.

14) Nature of experiment: Lissajous correlation measurement.

Purpose: to detect a correlation between the two noise signals tested above by means of the Lissajous figure method.

a) Noise sources: 1) noise at audio frequencies and 2) noise modulation on the rf signal, generated in the electron-coupled oscillator.

Refer to: page 69, Fig. 24, 25.

Result: about 50% correlation was indicated at all frequencies tested from 25 cps to 10 kc.

b) Noise sources: 1) audio noise and 2) the noise modulation on the rf signal, generated in the crystal controlled oscillator.

Refer to: page 74, Fig. 26.

Result: About 25% correlation was indicated in this case.

Conclusion: Noise occurring at audio frequencies with an oscillator tube also appears as an audio noise modulation on the rf output of the oscillator.

3. Noise Spectra

(a) General Considerations

Noise is measured, either as a mean square voltage, $\overline{e_n^2}$, or a mean square noise current, $\overline{i_n^2}$. Either of these quantities is related to noise power through the resistance of the noise source. Therefore, measurements of $\overline{e_n^2}$, or $\overline{i_n^2}$ are commonly referred to as noise power measurements, since such measurements can easily be changed into noise power data. It should be recognized that a vacuum tube is a noise current generator, and the noise power generated depends upon the resistance through which the noise current passes.

Since noise occurs throughout the frequency band, and does not possess a finite amount of power at a specific frequency, a noise density must be calculated, either as a power density, in watts per cycle, a mean square noise voltage density, in volts squared per cycle, or a mean square noise current density, in amperes squared per cycle.

It was desired to detect the presence of flicker noise by measuring the noise power spectra. From section II, it will be seen that there are two types of noise present in the system; flicker noise, whose magnitude varies as 1/f, and shot, thermal and partition noise, whose magnitude is independent of frequency. This may be represented by

$$\overline{e_n^2} = A + \frac{B}{f} \qquad \text{V - 1}$$

where A and B are constants of proportionality. Taking the logarithms of both sides,

$$\log \overline{e_n^2} = \log \left(A + \frac{B}{f}\right)$$

$$\doteq \log B - \log f$$

where $\frac{B}{f} \gg A$.

Therefore, if the spectra are plotted on a logarithmic scale, the presence of flicker noise will be indicated by a rise at low frequencies with a slope of minus one.

(b) Measurements on Thermal Noise.

The purpose of this experiment was to test as a function of frequency the accuracy of the measuring system which was described in section III, 2. For this purpose, measurements were made on a 600 kilohm wire wound resistor, which, in parallel with the input grid leak resistor of the amplifier formed a resistance of 585 kilohms.

The results of this measurement are plotted in Fig. 18. The correct noise density, which is shown as a dashed line, was 9.7×10^{-15} volts^2/cycle. The measured value was 1.01×10^{-14} volts^2/cycle, over the range from 200 cps to 10 kc. This represented an error of 0.2 db. Below 200 cps the flicker noise in the amplifier can be detected as a further error.

These measurements showed that the system was adequate for measurements of mean square noise voltage density as small as 2×10^{-15} volts^2/cycle without any correction for amplifier noise down to 100 cps. Between 100 cps and 10 cps, noise densities less than 10^{-14} volts^2/cycle required corrections for amplifier noise.

(c) Measurement of the Detector Noise Spectrum.

The purpose of this experiment was to determine the magnitude of the noise introduced into the signal by the detector which was used to detect the noise modulation on the rf signal. For this purpose, the oscillator was disconnected from the detector, and a 12 volt direct current voltage was applied between its plate and ground in order to simulate operating conditions.

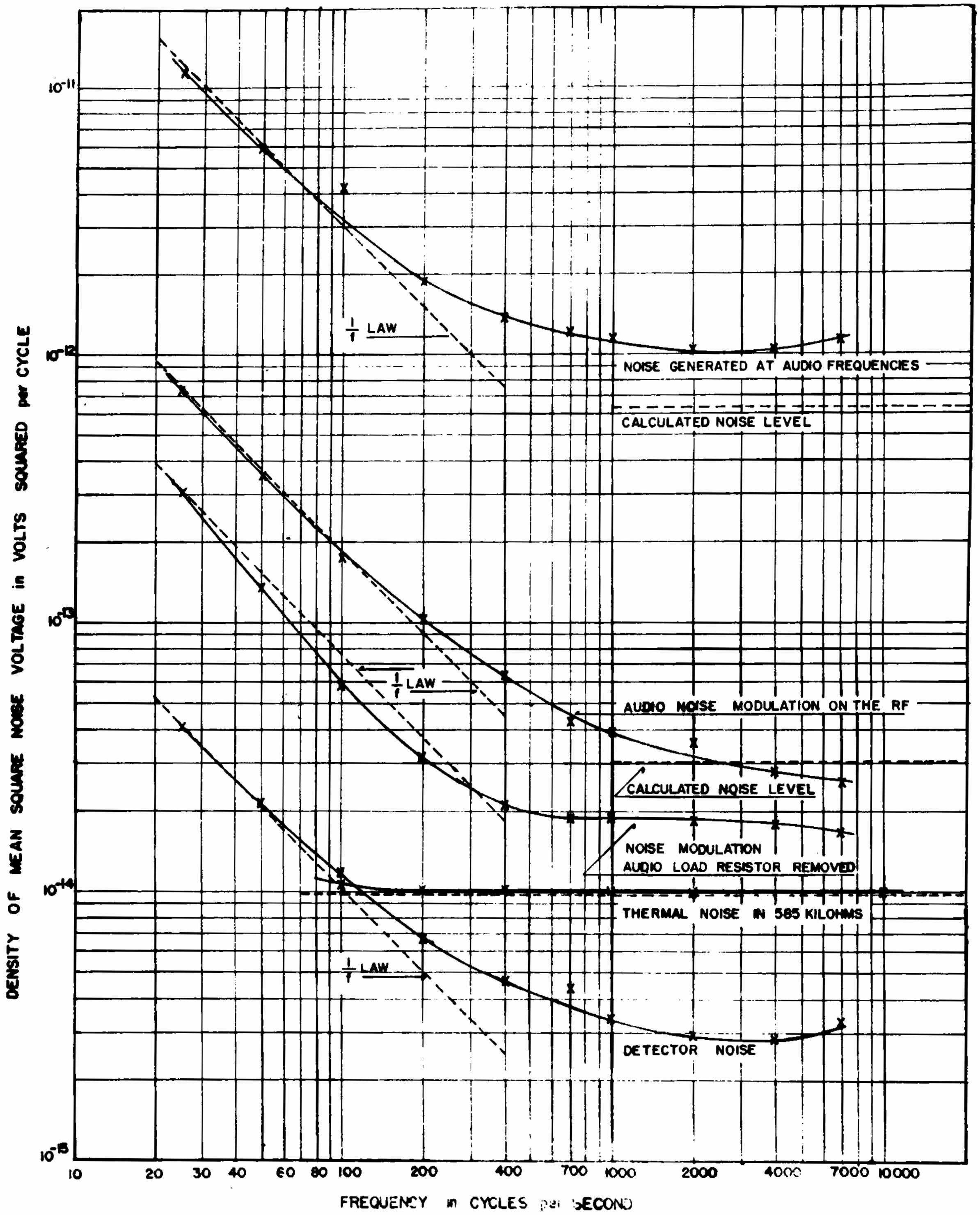


FIG. 12 A PLOT OF THE MEASURED NOISE SPECTRA OF THE ELECTRON COUPLED OSCILLATOR

The noise spectrum measured under these conditions is plotted in Fig. 18. It will be seen that at frequencies below 200 cps this spectrum follows a 1/f law, indicated by the dashed line with a slope of -1. This indicates the presence of flicker noise.

The rise at the high frequency end of this spectrum indicates that at frequencies above 5 kc the effect of capacitive loading on the amplifier input circuit made measurements unreliable.

It will be seen that this noise spectrum lies about 10 db lower than the noise spectrum which was measured for the audio noise modulation on the rf signal. Therefore, the detector contributed very little noise to the noise signal being investigated.

(d) Spectrum of the Noise Generated at Audio Frequencies in the Oscillator Tube.

The spectrum of the noise signal which appeared across the audio load resistor in the plate circuit of the oscillator was measured. The audio load resistor in this case was 50 kilohms. This signal represented the noise generated at audio frequencies in the tube.

This spectrum is plotted in Fig. 18. It will be seen that it follows a 1/f law at low frequencies, which indicates the presence of flicker noise in the signal. Above 1 kc, the spectrum falls to a level of 1.02×10^{-12} volts^2 per cycle.

In calculating the shot noise level in a tube, it is customary to assume a value of 1/5 for the space charge smoothing factor, Γ^2, if more accurate information is not available. This value of Γ^2 will almost certainly be within a factor of 2 of the correct value.

If a value of 0.2 is assumed for the space charge smoothing constant, Γ^2, the shot noise level may be calculated[1], from the formula

$$\overline{e_n^2} = 2eI_p \frac{(I_s + \Gamma^2 I_p)}{I_s + I_p} R_L^2 \qquad \text{V - 2}$$

where I_p = plate current

I_s = screen current

and R_L = audio load resistance in this case. This calculation yields a shot noise level of 6.25×10^{-13} volts^2/cycle which is in fair agreement with the experimental result of 1.02×10^{-12} volts^2 per cycle.

(e) Spectrum of the Audio Noise Modulation on the rf Signal.

The purpose of this experiment was to measure the spectrum of the noise modulation on the rf signal and to compare it with the spectrum of the noise occurring at audio frequencies.

This spectrum is plotted in Fig. 18. It will be seen that it had almost the same shape as the spectrum of the noise at audio frequencies. It follows a 1/f law very closely below 200 cps, which indicates the presence of flicker noise.

The curve falls to a shot noise level of about 2.5×10^{-14} volts^2/cycle. One may estimate the Q of the tank circuit loaded by the detector, at 10, since it was deliberately designed to have at low Q in order to minimize the effect of any frequency drifts upon the amplitude of oscillation. Allowing for wiring capacitance, the tank circuit inductance may be calculated as 33 microhenries. The resonant impedance of the tank circuit is therefore 10.8 kilohms. Substituting this value into equation V-2 and again using a value of 0.2 for Γ^2 yields a shot noise level of 3.05×10^{-14} volts^2/cycle. The almost exact agreement between theory

1. See section II, 4.

and measurement is gratifying, but it is, of course, slightly fortuitous in view of the fact that both the value of Γ^2, the space charge smoothing factor, and the Q of the tank circuit could only be assumed. However, the assumption of the value of 0.2 for Γ^2 is partly justified by the agreement obtained with experiment in calculating the shot noise level in the case of the noise generated at audio frequencies in part d. The results obtained in the calculation of the noise current which would have generated the noise modulation on the rf signal, in part f of this section, together with the agreement obtained in this section between theory and experiment tended to support the assumption made of the value of Q. A higher value of Q would have resulted in much too high a value for the calculated shot noise level while a lower Q would have meant that more noise current was generated as a modulation on the rf signal than was present as audio noise on the tube.

The measurements in this experiment clearly indicated the presence of flicker noise as a modulation of the rf output of an oscillator. It was concluded from these measurements that noise generated at audio frequencies in an oscillator tube appears as a noise modulation on the rf oscillations.

(f) A Comparison of the Spectra of the Noise Generated at Audio Frequencies in the Tube, and the Audio Noise Modulation of the rf Signal.

A vacuum tube may be considered as a noise current generator. The noise voltage which is measured is the product of the noise current generated and the impedance across which the noise is measured.

A comparison was made of the mean square noise current spectra of the noise currents which gave rise to the noise voltages of the noise generated at audio frequencies and the noise modulation on the rf signal, measured in parts d and e of this section. The purpose of this calculation was to discover what relationship existed between the magnitudes of the two noise signals.

The noise current for the noise occurring at audio frequencies was calculated from Ohm's law using the known 50 kilohm audio load resistance.

The rf signal is generated by an rf current modulated by an audio noise signal acting on the resonant impedance of the tank circuit. This impedance has already been calculated as 10.8 kilohms. The detector, which is a voltage measuring device, measures the rf amplitude as a steady voltage (12 volts) and the noise modulation as a voltage fluctuation at an audio frequency. The measured noise voltage is therefore determined by the impedance of the tank circuit at the rf oscillation frequency. The noise current for this noise was therefore calculated from Ohm's law, using the measured noise modulation voltage and the calculated resonant impedance of the tank circuit.

Fig. 19 is a plot of the resulting mean square noise current spectra. It will be seen that the two spectra almost coincide below 500 cps, where the majority of the noise modulation on the rf signal is due to modulation by the audio noise signal; and that their widest separation, at 7000 cps, is only 3 db.

It is apparent therefore that the noise current fluctuation generating the noise occurring at audio frequencies may also be considered to generate the audio noise modulation on the rf signal which occurs due to this source.

(g) Spectrum of the Audio Noise Modulation of the rf Signal with the Audio Load Impedance Removed.

The purpose of this experiment was to discover any change in the

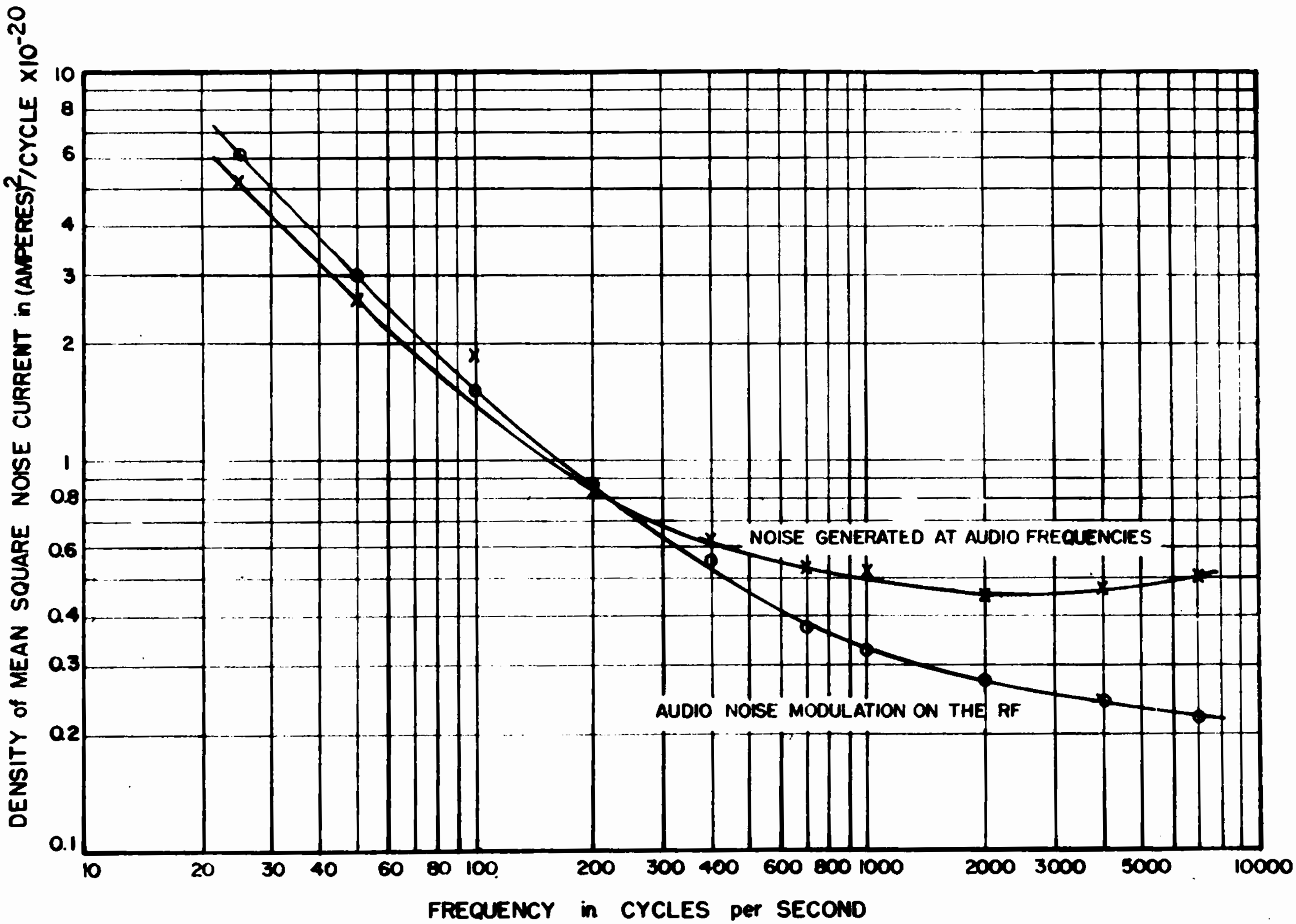


FIG. 19 A PLOT OF THE SPECTRA OF THE MEAN SQUARE NOISE CURRENTS

spectrum of the noise modulation of the rf which occurred when the audio load resistance was removed from the plate circuit of the oscillator.

The spectrum of the noise modulation with the audio load resistor removed was measured and is plotted in Fig. 18. It will be seen that it lies about 3 db below the original noise modulation spectrum, and is slightly changed in shape. However it still indicates a large content of flicker noise, which shows that a modulation depending upon the noise generated at audio frequencies did appear on the rf signal, independent of the audio load resistor.

It might be concluded from the differences in the two spectra that some direct modulation of the rf signal was occurring due to the audio noise signal appearing across the audio load resistor. Later experiments showed that this definitely was not the case. It can only be suggested, therefore, that the differences between these two spectra were caused by some change in the operating conditions of the tube due to the removal of the load resistor.

(h) Variation of the Noise Modulation with Plate Current.

A series of measurements was made of the noise modulation on the rf signal at a modulation frequency of 100 cps, for different plate currents. The screen voltage was varied in steps of 22 1/2 volts from 45 volts to 112 1/2 volts, which resulted in plate currents ranging from 0.65 ma to 2 ma.

Fig. 20 is a plot of the variation of the mean square noise voltage as a function of the square of the plate current. It will be seen from this curve that the magnitude of the mean square noise voltage varies linearly with the square of the plate current over the greater part of the range of measurement. The departure from this law at low currents is difficult to explain, and it can only be suggested that at these low

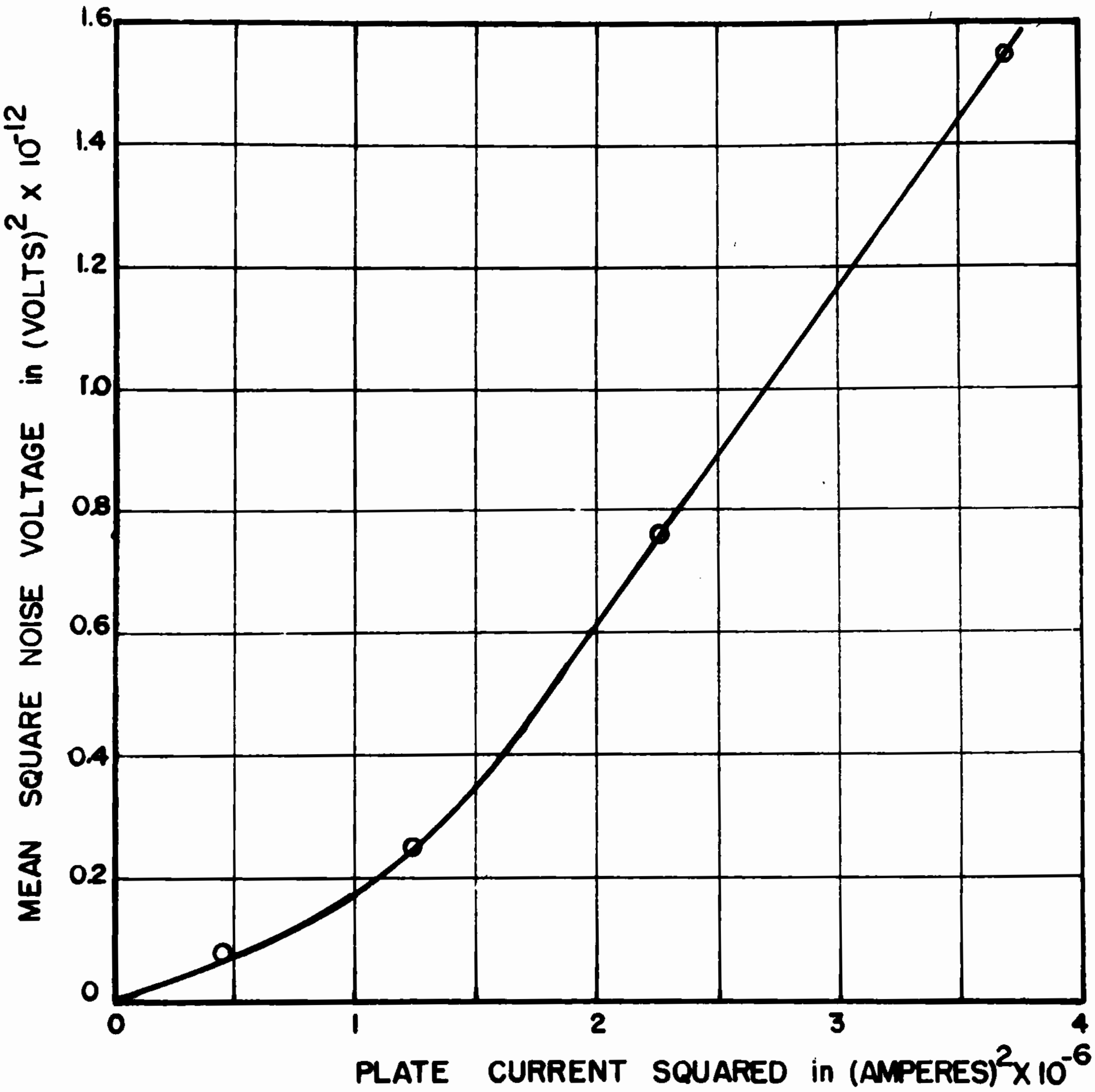


FIG. 20 A GRAPH SHOWING THE VARIATION WITH THE SQUARE OF THE PLATE CURRENT OF THE MEAN SQUARE VOLTAGE OF THE NOISE MODULATION ON THE RF SIGNAL GENERATED BY THE ELECTRON COUPLED OSCILLATOR AT A MODULATION FREQUENCY OF 100 CPS

currents, some change in the efficiency of the modulation process may have occurred.

It was concluded from this experiment that nearly all of the noise occurring as a modulation on the rf signal at low audio frequencies was due to flicker noise, since this is the only type of noise being generated in the circuit which varied as the square of the plate current.

4. Correlation Measurements

(a) General Considerations

The purpose of the correlation measurements was to show that the noise signal occurring as a modulation on the rf signal had the same fluctuations, or time variation as the noise signal occurring in the tube at audio frequencies. The approximate degree to which this is true, which may be stated as a percentage of correlation, was also of interest.

(b) Reference Patterns for Lissajous Correlation Measurements.

The percentage of correlation of experimental Lissajous correlation patterns could be determined by comparison with patterns of a known percentage of correlation. A circuit, whose schematic diagram is given in Fig. 21, was built to provide such reference patterns.

The two amplifier chains were arranged to deliver completely uncorrelated noise, originating as thermal noise in the input resistors of the preamplifiers. One of these noise signals was used as the first signal forming the Lissajous figure. The two signals were then mixed in a known ratio in the circuit in Fig. 21 to form the second signal used in generating the Lissajous pattern. The resulting patterns for various correlations are shown in Fig. 22. The percentages of correlation,

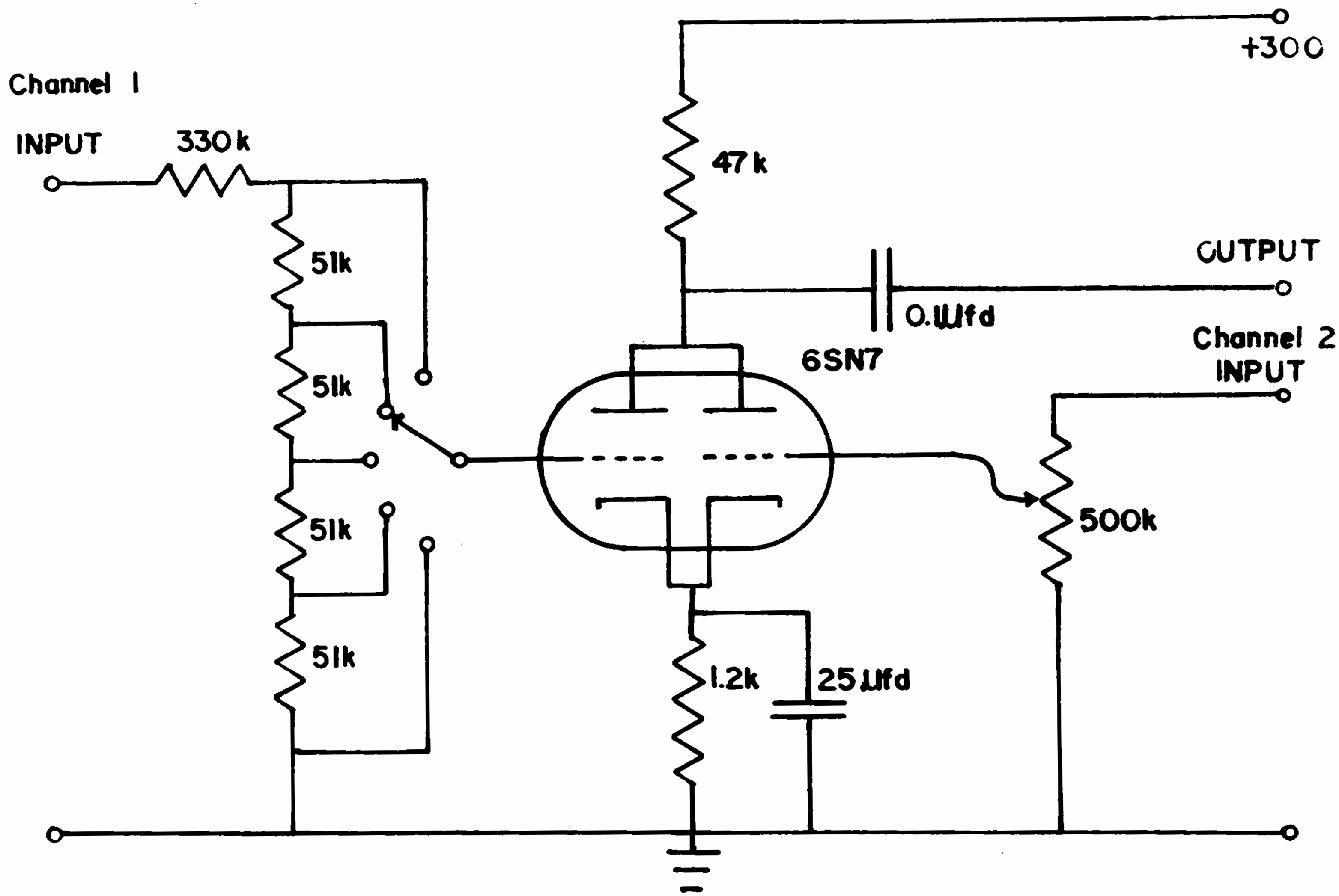
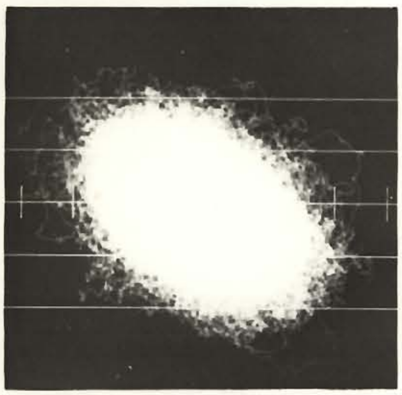


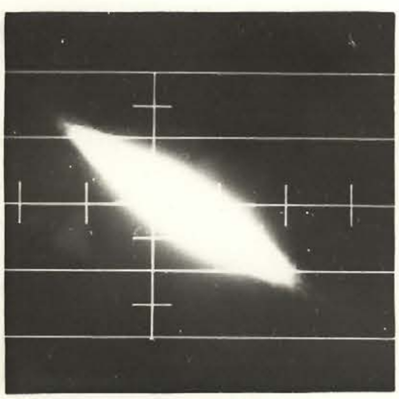
FIG. 21 SCHEMATIC DIAGRAM OF THE CIRCUIT USED TO MIX TWO UNCORRELATED NOISE SIGNALS IN A KNOWN RATIO

a) Zero correlation



b) 25% correlation

c) 50% correlation



d) 75% correlation

e) 100% correlation

FIG. 22 LISSAJOUS CORRELATION PATTERNS REPRESENTING KNOWN PERCENTAGES OF CORRELATION FOR REFERENCE PURPOSES in this case, are known to within 5%. The shape of these patterns was found to be independent of the frequency of the band of noise selected to form them. All patterns are 10 second exposures, and therefore represent integrated patterns. Comparison of patterns obtained experimentally with these patterns enables the percentage correlation of the experimental patterns to be determined approximately.

(c) Control Experiments.

A number of experiments were carried out to show that no correlation occurred in the measuring system described in section III, 3 due to spurious or external effects. In each experiment, the Lissajous figure method, which is a very sensitive method, was used to detect any correlation which was occurring.

In order to discover any interaction which was occurring between the two amplifier chains due to their proximity or because the signals they amplified were generated in the same circuit, three experiments were carried out. In all cases both amplifier chains were connected to their respective signal outputs of the oscillator.

In the first case, no power was supplied to the oscillator. The signals amplified were therefore the thermal noise signals generated in the audio load resistor and the detector load resistor in the oscillator.

No correlation was detected. It was therefore concluded that there was no interaction between the two amplifier chains due to their proximity.

In the second experiment, power was supplied to the oscillator, but oscillations were stopped by grounding the oscillator grid. In this case, audio noise was generated in the tube, and appeared as a signal across the audio load resistor. This formed one output signal. Since no rf signal was developed, the noise output from the detector circuit was the thermal noise generated in the detector load resistor. No correlation was detected in this case, showing that no interaction resulted from the generation of the two noise signals in the same circuit.

In the third case, the oscillator was permitted to oscillate, but the audio load resistor was short circuited. In this case, an audio noise modulation was detected upon the rf signal and formed the output of the detector. The other noise signal was the noise generated in the amplifier chain.

No correlation was detected, showing again that no interaction resulted from the two noise signals being generated in the same circuit.

Measurements were made to determine whether the audio noise signal appearing across the audio load resistor in the oscillator plate circuit could modulate the rf signal directly.

With the circuit operating in its normal condition, an audio signal at the frequency of the filters was applied across the audio load resistor. It was found that 1 millivolt of signal was necessary at this point in order to produce a detectable modulation of the rf signal. The maximum audio noise signal measured at this point was less than 10 microvolts, which is 40 db below the signal required to produce a detectable modulation. It was therefore concluded that no modulation of the rf signal took place in this manner. Further, this experiment showed that no audio signal appearing at the output of the detector could leak back through the circuit to the audio noise circuit.

Experiments were carried out to show that no extraneous audio signal, such as a hum signal, was occurring in the system and causing correlation to appear.

The Muirhead Pametrada Wave Analyser, set at its highest Q position, was used to search for hum in the noise occurring at audio frequencies in the oscillator, and the audio noise modulation on the rf signal. No trace of hum could be found.

Selected frequency bands from each of the two noise signals in turn were used as one of the Lissajous signals, with a 60 cps mains signal forming the other. No correlation was found.

The same bands of noise from the same two signals were separately tested for correlation with the output of a variable frequency audio oscillator. No correlation could be found at any frequency.

It was therefore concluded that no correlation occurred due to external audio signals either at hum frequency or any other audio frequency.

It was felt that these control experiments showed that any correlation detected between two noise signals applied to the input of the amplifier systems under the conditions of this experiment was real, and did not occur due to any spurious effects.

(d) Correlation Measurements on the Electron-Coupled Oscillator.

The degree of correlation of the noise generated at audio frequencies within the oscillator tube and the audio noise modulation on the rf output of the electron coupled oscillator was investigated, using the methods described in section III, 3.

Two noise signals, obtained by selecting similar bands of frequencies from the noise generated at audio frequencies in the tube, and the audio noise modulation on the rf signal were displayed simultaneously on a double beam cathode ray oscilloscope. A high degree of correlation could be detected visually at frequencies up to 200 cps in this manner. Above this frequency the eye cannot follow the noise fluctuations fast enough to detect correlation.

Fig. 23 a, c, and e, are photographs of these noise traces for frequencies of 25 cps, 50 cps and 80 cps. Fig. 23 b and d are similar photographs of uncorrelated noise at 25 and 50 cps for comparison purposes.

It will be seen that the correlated noise traces are 180° out of phase. This is due to the 180° phase shift introduced by the audio load resistor of the oscillator. Apart from this, the correlated noise traces are alike in wave shape, phase and amplitude. The uncorrelated noise traces, on the other hand, bear no constant relation to each other in any of these ways.

It will be seen from Fig. 23 e that the speed of the camera mechanism limited useful recording by this means to frequencies below 80 cps.

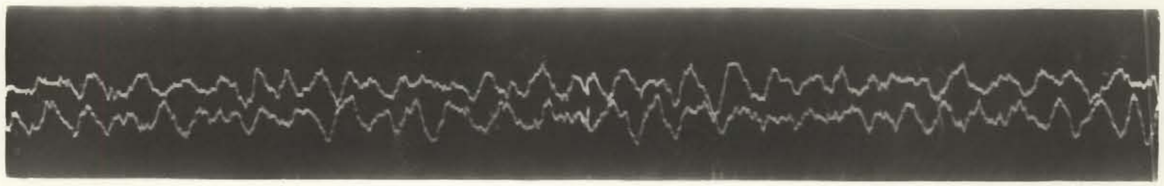
The percentage of correlation of the two signals cannot be estimated with any accuracy from these photographs. A lengthy mathematical analysis is necessary to obtain this information from this type of record.

Correlation measurements were made of noise from the electron-coupled oscillator by the Lissajous method at frequencies from 25 cps to 10 kc. Correlation was observed at all frequencies tested.

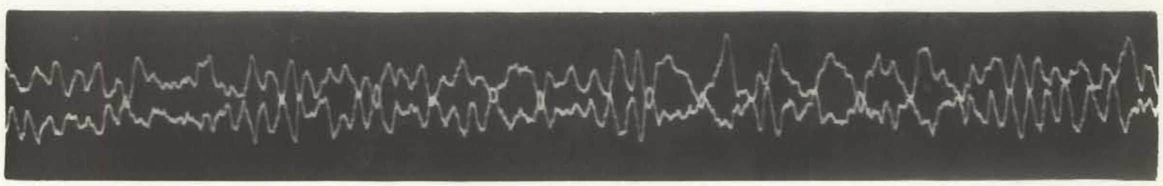
Figs 24 and 25 are a series of photographs of the resulting patterns. On each page a known 50% correlation pattern from Fig. 22 is reprinted for comparison purposes.

All the patterns shown for the oscillator noise except Fig. 24b are 10 second exposures, representing integrated pictures. It will be seen that the degree of correlation appeared to be about 50% for all frequencies

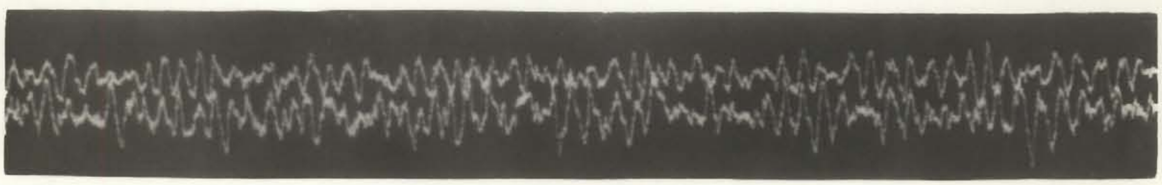
a) Correlated noise in a 4 cps band about 25 cps



b) Uncorrelated noise in a 4 cps band about 25 cps



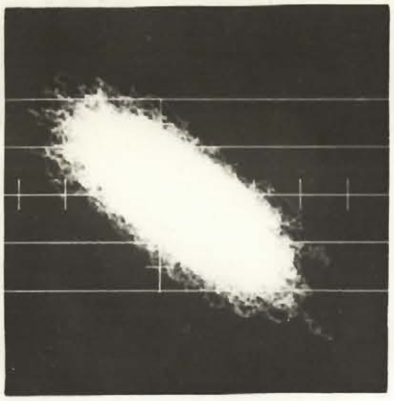
c) Correlated noise in an 8 cps band about 50 cps



d) Uncorrelated noise in an 8 cps band about 50 cps

e) Correlated noise in a 12 cps band about 80 cps

FIG. 23 SEGMENTS OF PHOTOGRAPHIC RECORDS
OF NARROW FREQUENCY BANDS OF NOISE



a) Known 50% Correlation Pattern for Reference Purposes

b) Correlation in a 4 cps band about 25 cps
1 second exposure
Estimated Correlation, 80%

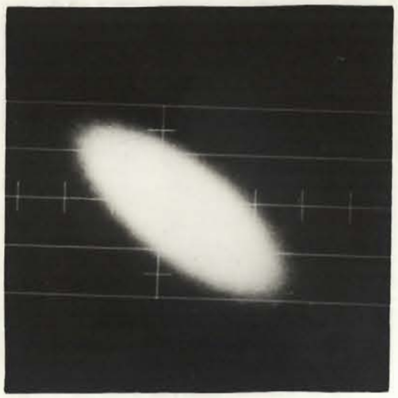
c) Correlation in a 4 cps band about 25 cps
10 second exposure
Estimated Correlation, 40%

d) Correlation in a 12 cps band about 80 cps
10 second exposure
Estimated Correlation, 40%

e) Correlation in a 35 cps band about 200 cps
10 second exposure
Estimated Correlation, 50%

f) Correlation in a 70 cps band about 400 cps
10 second exposure
Estimated Correlation, 50%

FIG. 24 LISSAJOUS CORRELATION PATTERNS
FOR THE ELECTRON COUPLED OSCILLATOR



a) Known 50% Correlation
Pattern for Comparison

b) Correlation in a 130 cps
band about 800 cps
Estimated Correlation, 50%

c) Correlation in a 200 cps
band about 1250 cps
Estimated Correlation, 60%

d) Correlation in a 400 cps
band about 2500 cps
Estimated Correlation, 60%

e) Correlation in an 800 cps
band about 5000 cps
Estimated Correlation, 60%

f) Correlation in a 1500 cps
band about 10000 cps
Estimated Correlation, 60%

FIG. 25 LISSAJOUS CORRELATION PATTERNS
FOR THE ELECTRON COUPLED OSCILLATOR
All exposures are 10 second exposures.

with a slightly higher correlation at higher frequencies. This was contrary to expectations, for it was expected that the greatest correlation would exist at the lower audio frequencies, where the majority of the noise modulation on the rf signal was due to the large quantity of flicker noise occurring in the tube at audio frequencies. At higher audio frequencies, the noise modulation on the rf signal due to audio noise occurring in the tube would be expected to be smaller in relation to the noise actually occurring in the tube at the rf frequency.

This can be explained, at least in part, by the fact that in spite of all the precautions taken, the oscillator-detector circuit was still somewhat microphonic. This microphony chiefly affected the rf parts of the circuit and the detector, and would thus be expected to affect the noise modulation on the rf signal more strongly than the noise being generated at audio frequencies. This effect was actually observed on the monitor oscilloscope.

The noise added by small microphonic vibrations would be expected to be low frequency noise. For this reason, the percentage correlation might appear to be less at low frequencies than at high frequencies for an integrated picture. Fig. 24b is a one second exposure of the pattern occurring for 25 cps noise, and clearly shows that the percentage correlation of the noise at this frequency would actually be about 75% were it not for microphonic interference.

The correlation observed was not itself the product of microphony, since it was observed at all frequencies. This would be extremely unlikely if it were caused by microphonic vibrations.

These correlation measurements on the electron coupled oscillator showed that the noise occurring at audio frequencies within an oscillator tube appears as a modulation upon the rf output of the oscillator.

(e) Measurements on the Crystal Controlled Oscillator.

A series of correlation measurements, using the Lissajous method, were performed on the crystal controlled oscillator. Correlations of about 25% were detected in this case.

This oscillator was much more susceptible to extraneous noise than the electron coupled oscillator, and the fact that this resulted in a lower percentage of correlation again showed that microphony and extraneous noise will reduce the degree of correlation.

Fig. 26 is a series of photographs showing Lissajous patterns obtained for this oscillator.

These observations on the crystal controlled oscillator served to confirm the conclusion that the noise generated in an oscillator tube at audio frequencies appears as a modulation on its rf output.

a) Known 25% Correlation
Pattern for Reference Purposes

b) Correlation in a 120 cps
band about 800 cps
1 second exposure
Estimated Correlation, 15%

c) Correlation in a 120 cps
band about 800 cps
1/25 second exposure
Estimated Correlation, 25%

d) Correlation in a 50 cps
band about 320 cps
10 second exposure
Estimated Correlation, 20%

e) Correlation in a 50 cps
band about 320 cps
1/2 second exposure
Estimated Correlation, 25%

FIG. 26 LISSAJOUS CORRELATION PATTERNS
FOR THE CRYSTAL CONTROLLED OSCILLATOR

VI DISCUSSION AND CONCLUSIONS

A series of experiments was carried out which showed that noise generated at audio frequencies in a radio frequency (rf) oscillator tube also appears as an audio noise modulation upon the rf output of the oscillator. This is a new and hitherto unrecognized source of noise in an oscillator. It may well be the major source of noise at low modulation frequencies, since flicker noise, at very low audio frequencies, is the largest source of noise in the tube.

This result was established by comparing the noise generated at audio frequencies with the audio frequency noise modulation on the rf signal in two ways. The frequency spectra of the two noise signals were measured and compared (Figs 18, 19) and the correlation, or degree to which the two noise wave trains were alike was determined (Figs 23, 24, 25, and 26).

The measurements of the noise spectra of an electron-coupled oscillator operating at 5.06 Mc showed that the noise generated at audio frequencies and the noise occurring as a modulation on the rf signal had approximately the same spectral distribution, each spectrum showing the presence of flicker noise. Further, it was found that both these noise signals could be considered to be generated by the same noise current (Fig. 19). In the case of the noise at audio frequencies, this noise current, passing through the audio load resistor in the plate circuit of the oscillator generates the measured noise voltage. The same noise current modulates the rf current, which generates the rf signal across the resonant impedance of the plate tank circuit. This means that the noise modulation voltage may be calculated by multiplying the audio noise current by the resonant impedance of the tank circuit.

It was further shown that the noise modulation on the rf signal at a low audio frequency varied as the square of the plate current, as would be expected if this noise were caused by flicker noise.

The degree of correlation of the two noise signals was investigated in two ways.

Signals representing bands of low frequency noise selected by filters from each of the noise signals, were displayed simultaneously on a double beam oscilloscope and photographed. A visual comparison of the photographic records revealed the correlation between the two noise trains. (Fig. 23.)

Frequency bands were selected at frequencies from 25 cps to 10 kc from both the noise generated at audio frequencies and the audio noise modulation on the rf signal, and were applied to the vertical and horizontal deflection plates of an oscilloscope. An elliptical Lissajous figure resulted. From the theory of Lissajous figures, this pattern would have been a straight line lying at an angle across the screen for perfect correlation, and would have become circular at zero correlation (Fig. 22). For the electron-coupled oscillator, about 50% correlation was found between the two noise signals at all frequencies; for a crystal controlled oscillator, whose output contained much extraneous noise, 25% correlation was observed.

These correlation experiments demonstrated the fact that the noise generated at audio frequencies in the tube appears, in exactly the same wave shape, as a modulation on the rf signal.

A number of experiments were performed to show that the results obtained were not due to spurious effects such as interaction of the amplifier chains, direct modulation of the rf by the audio noise signal appearing at the plate, or interference by external signals.

Section V, 2, is a complete summary of the experiments performed.

The process whereby the audio noise modulates the rf signal may be understood by considering the process of modulation. In that case, an external signal is applied to one of the controlling elements of the tube in such a way that the operating conditions of the tube are changed according to the instantaneous value of the applied modulation signal. This results, in turn, in a variation of the mean plate current in the tube according to the modulation signal. The magnitude of the rf signal, which depends upon the mean plate current, is therefore varied by the applied modulating signal.

When a noise signal is generated within the tube, it appears as a modulation on the plate current, as if it had been applied as an external signal. The noise signal will therefore appear as a modulation on the rf signal. The magnitude of the modulation will be governed solely by the magnitude of the noise modulation on the mean current.

It is important to be able to calculate the magnitude of the noise modulation on an rf signal at any modulation frequency. This may be done quite simply from one measurement of the mean noise current generated within the tube at a low audio frequency. It should be recognized, however, that the calculation does not take into account the large quantity of microphonic noise which is present in most rf oscillators.

For example, from Fig. 19, the mean square noise current for the noise generated at a frequency of 25 cps within the oscillator was 6×10^{-21} amperes^2/cycle. This current, passing through the calculated resonant tank circuit impedance, of 10.8 kilohms, would result in a mean square noise voltage of

$$\overline{e_n^2} = \overline{i_n^2} \, R_L^2$$
$$= 6 \times 10^{-21} \times (10.8)^2 \times 10^6$$
$$= 6.75 \times 10^{-13} \text{ volts}^2/\text{cycle at 25 cps.}$$

The measured value of the mean square noise modulation voltage at 25 cps was 7.40×10^{-13} volts^2/cycle. The error was therefore 0.5 db.

It has been shown that within the flicker noise range, the mean square voltage of the noise modulation on the rf varies according to a 1/f law. Therefore, from the same single measurement, the noise at any frequency within the flicker noise range may be calculated.

For example, using the same figures to find the noise at 100 cps,

$$\overline{e_n^2 \text{ (100 cps)}} = \overline{e_n^2 \text{ (25 cps)}} \times 25/100$$
$$= 6.75/4 \times 10^{-13}$$
$$= 1.69 \times 10^{-13} \text{ volts}^2/\text{cycle at 100 cps.}$$

The measured value (Fig. 18) was 1.65×10^{-13} volts^2/cycle. In this case, the error is 0.1 db.

Similarly, it has been shown, (Fig. 20) that the mean square noise modulation voltage varies as the square of the plate current. Therefore, the noise at any other plate current could be calculated from the formula

$$\overline{e_n^2 \, (I_1)} = \overline{e_n^2 \, (I_2)} \quad \frac{(I_1)^2}{(I_2)^2}$$

These calculations provide criteria for the design of oscillators for specific applications. The phenomenon described in this thesis makes it imperative that this source of noise be considered in all applications where noise modulation on an rf signal is of importance.

APPENDIX.

A derivation is given of the expression relating the mean fluctuation of a noise measurement to the mean noise power, in terms of bandwidth and circuit time constant.

Let w(f) be the power spectrum of the noise output of the filter before squaring.

Let g(f) be the time response of the smoothing circuit, with Fourier transform S(f).

The squared noise has a mean D.C. amplitude, equal to

$$\int_0^\infty w(f)\,df.$$

The mean D.C. level of the output of the square law detector and smoothing circuit is

$$\int_0^\infty w(f)\,df \int_0^\infty S(o)\,df$$

$$= \int_0^\infty w(f)\,df \int_0^\infty g(f)\,dt$$ from Fourier transform theory.

The RMS value of the fluctuation of this level may be calculated as follows.

The power spectrum after squaring is

$$W(f) = \int_{-\infty}^{\infty} w(x) \quad w(f - x)\,dx,$$

or the convolution of the noise spectrum with itself.

If a sufficiently long time constant, corresponding to a narrow acceptance band about zero frequency, is used, the high frequency components of this power may be ignored.

The spectral density near f = 0 is

$$W(o) = \int_{-\infty}^{\infty} w(x)\,w(-x)\,dx$$

$$= 2\int_0^\infty w^2\,(f)\,df.$$

Since we may consider the squared noise to extend over positive and negative frequencies, this becomes

$$W'(o) = \int_0^{\infty} w^2(f)\, df.$$

The mean square fluctuation of the output is therefore

$$W'(o) \int_{-\infty}^{\infty} |S(f)|^2 df = \int_0^{\infty} w^2(f)\, df \int_0^{\infty} g^2(f)\, dt$$

from Parseval's theorem.

The ratio, e, of the RMS value of the output fluctuations, and the mean D.C. output level, is therefore

$$e = \frac{\sqrt{\int_0^{\infty} w^2(f)\, df \int_0^{\infty} g^2(f)\, dt}}{\int_0^{\infty} w(f)\, df \int_0^{\infty} g(f)\, dt.}$$

As the simplest example, the noise is passed through an ideal rectangular filter of bandwidth B before squaring, and the output is smoothed by an RC network of time constant T.

For a rectangular filter

$$\int_0^{\infty} w(f)\, df = B$$

$$\int_0^{\infty} w^2(f)\, df = B$$

when the noise spectrum is normalized.

For an RC circuit

$$\int_0^{\infty} g(t)\, dt = \int_0^{\infty} e^{-t/T}\, dt = T$$

$$\int_0^{\infty} g^2(t)\, dt = \int_0^{\infty} e^{-2t/T}\, dt = \frac{T}{2}$$

Therefore, applying equation 1

$$e = \frac{\sqrt{B \cdot T/2}}{BT} = \frac{1}{\sqrt{2BT}}$$

For other forms of filter, by the same process, it can be shown that

$$e = \frac{A}{\sqrt{BT}}$$

where A is a numerical factor, depending on the filter.

BIBLIOGRAPHY

Books

Goldman, S. Frequency Analysis, Modulation and Noise, McGraw-Hill, 1948.

Lawson, J.L., and G.E. Uhlenbeck, Threshold Signals. (M.I.T. Rad. Lab. Ser., Vol. 24), McGraw-Hill, 1950.

Moullin, E.B., Spontaneous Fluctuations of Voltage, Clarendon Press, 1938.

Review Articles

Bakker, C.J., and B. van der Pol. Report on Spontaneous Fluctuations of Current and Potential, U.R.S.I. Document. AG1938, No. 67 Comm. V.

Department of Scientific and Industrial Research (UK), Special Report No. 20. "Valve and Circuit Noise", H.M. Stationary Office, London 1951.

MacDonald, D.K.C., Spontaneous Fluctuations, Reports on Progress in Physics, 12, 56, 1949.

Pierce, J.R., Noise in Resistances and Electron Streams, Bell Syst. Tech. J. 27, 158, 1948

Schottky, W, Spontaneous Current Fluctuations in Various Conductors, Ann. Physik, 57, 541, 1918.

Flicker Noise and Current Noise

Bakker, C.J., Fluctuations and Electron Inertia, Physica, 8, 23, 1941

Barber, D. An Apparatus for the Measurement of Current Noise in Semi Conductors at Audio Frequencies, T.R.E. Tech. Note No. 135 (1952) DRB ref 52/3783.

Bernamont, J., Fluctuations in the Resistance of Thin Films., Proc. Phys. Soc. 49 (extra part), 138, 1947.

Bernamont, J., Potential Fluctuations at the Ends of a Metallic Conductor of Small Volume Traversed by a Current, Ann. de Physique 7, 71, 1937

Boardman, F.D., A Spectrum Analyser for Low Frequency Noise, T.R.E. Tech. Note No 141, 1951. DRB ref No. 51/12996.

Brown, D.H.H., Noise Measurements on Photo Electric Cells, T.R.E. Memo No. L3-34, 1947.

Brillouin, L., Fluctuations of Current in a Conductor, Helv. Phys. Acta. Supp. 7, 47, 1934.

Campbell, R.H., and R.A. Chipman., Noise from Current Carrying Resistors 20 - 500 kc. Proc. Inst. Radio Engrs, 37, 938, 1949.

Christensen, C.J. and G.L. Pearson, Spontaneous Resistance Fluctuations in Carbon Microphones and other Granular Resistances, Bell Syst. Tech. J. 15, 197, 1936.

Davydov, B.J. and B.K. Gurevich., Voltage Fluctuations in Semiconductors, Jour. of Phys. 7, 138, 1943

Furth, R. and D.K.C. MacDonald, Spontaneous Fluctuations of Current in a Photo Electric Cell, Nature 159, 608, 1947.

Harris, E.J., Circuit and Current Noise, Elect. Engng. 20, 145, 1948

Harris, E.J. and P.O. Bishop, Low Frequency Noise from Thermionic Valves, Working under Amplifying Conditions, Nature 161, 971, 1948.

Harris, E.J., W. Abson and W.L. Roberts, Semi Conductor Noise at Low Frequencies, T.R.E. Report No. T2051, 1949.

Johnson, J.B., The Schottky Effect in Low Frequency Circuits, Phys. Rev. 26, 21, 1925.

Macfarlane, G.G., A Theory of Flicker Noise in Valves and Impurity Semi-conductors, Proc. Phys. Soc. 59, 366, 1947

Macfarlane, G.G., A Theory of Contact Noise in Semi-conductors, Proc. Phys. Soc. B, 63, 807, 1950

Miller, P.H., Jr., The Noise Spectrum of Crystal Rectifiers, Proc. Inst. Radio Engrs, 35, 252, 1947.

Montgomery, H.C., Electrical Noise in Semi-Conductors, Bell. Syst. Tech. J. 31, 950, 1952

Pennsylvania University (Moore School of Electrical Engineering), Investigation of the Measurement of Noise, Report No. NE-120803, DRB ref 52/1843, 1952

Petritz, R.L., On the Theory of Noise in P-N Junctions and Related Devices Proc. Inst. Radio Engrs, 40, 1441, 1952.

Richardson, J.M., The Linear Theory of Fluctuations Arising from Diffusional Mechanisms - an Attempt at a Theory of Contact Noise, Bell Syst. Tech. J. 29, 117, 1950.

Schottky, W., Small Shot and Flicker Effect, Phys. Rev. 28, 75, 1926.

Schottky, W., Weakening Action of the Space Charge or the Shot and the Flicker Effects, Physica, 4, 175, 1937.

Shockley, Electrons and Holes in Semi Conductors, p. 343, D. Van Nostrand Co., 1950

Wyngaarden, J.G. van, K.M. van Vleet, C.J. van Leeuwen, and E.F. de Haan, Low Frequency Noise in Electron Tubes.
A) Space Charge Reduction of the Flicker Effect.
B) Measurements on Triodes under Normal Operating Conditions
C) Electrometer Tubes, Physica, 18, 683, 1952

Ziel, A van der, A Study of the Cause and Effect of Flicker Noise in Vacuum Tubes., U.S. Sig. Corps. Contract No. DA 36-039-sc. 15355, Quarterly Reports - Minnesota University, Inst. of Technology, Oct., 1951, Jan., 1952, April 1952.

Ziel, A van der., On the Noise Spectra of Semi-Conductor Noise and of Flicker Effect, Physica 16, 359, 1950.

General Noise Theory

Bakker, C.J., Fluctuations and Electron Inertia, Physica 8, 23, 1941

Burgess, R.E., Noise in Receiving Aerial Systems, Proc. Phys. Soc. 53, 293, 1941

Friis, H.T., Noise Figures of Radio Receivers, Proc. Inst. Radio Engrs, 32, 419, 1944.

Furth, R. and D.K.C. MacDonald, Shot Effect in Diodes under Retarding Field Conditions, Nature 157, 841, 1946.

Furth, R., and D.K.C. MacDonald, Statistical Analysis of Spontaneous Electrical Fluctuations, Proc. Phys. Soc., 59, 388, 1947

Johnson, J.B., Thermal Agitation of Electricity in Conductors, Phys. Rev. 29, 36, 1927., Phys. Rev. 32, 97, 1928.

MacDonald, D.K.C. and Furth, Spontaneous Fluctuations of Electricity in Thermionic Valves under Retarding Field Conditions., Proc. Phys. Soc. 59, 375, 1947.

MacDonald, D.K.C., Transit Time Deterioration of Space Charge Reduction of Shot Effect., Phil. Mag. 40, 561, 1949.

North, D.O., Fluctuations in Space-Charge - Limited Currents at Moderately High Frequencies, Part II. Diodes and Negative Grid Triodes, R.C.A. Review 4, 441; 5, 106; 1940.

North, D.O., Fluctuations in Space Charge Limited Currents at Moderately High Frequencies., Part III, Multi Collectors. R.C.A. Review 5, 244, 1940.

North, D.O., The Absolute Sensitivity of Radio Receivers, R.C.A. Review 6, 332, 1942.

North, D.O., and W.R. Ferris, Fluctuations Induced in Vacuum Tube Grids at High Frequencies, Proc. Inst. Radio Engrs, 29, 49, 1941.

Nyquist, H., Thermal Agitation of Electric Charge in Conductors, Phys. Rev., 29, 614, 1927, Phys. Rev. 32, 110, 1928

Peterson, L.C., Space-Charge, Signal and Noise in Microwave Tetrodes, Proc. Inst. Radio Engrs, 35, 1264, 1947

Rice, S.O., Mathematical Analysis of Random Noise, Bell Syst. Tech J. 23, 282, 1944, 24, 46, 1945.

Riche, F.F., Microwave Magnetrons (Chap. 9 Noise) (M.I.T. Rad. Lab. Ser. Vol. 6), McGraw-Hill, 1948.

Schremp, E.J., Amplifier Sensitivity, Chap. 12 of Vacuum Tube Amplifiers (M.I.T. Rad. Lab. Ser., Vol. 18), McGraw-Hill, 1948

Spenke, E., On the Corpuscular Treatment of the Thermal Noise of Electrical Resistances, Wiss. Ver. Siemens, 18, 54, 1939.

Thompson, B.J. and D.O. North, Fluctuations in Space-Charge-Limited Currents at Moderately High Frequencies. Part IV. Fluctuations Caused by Collision ionization., R.C.A. Review 5, 371, 1941.

Torrey, H.C. and C.A. Whitmer, Crystal Rectifiers (Chap. 6, Noise Generation) (M.I.T. Rad. Lab. Ser. Vol. 15), McGraw-Hill, 1948.

Wang, M.C. and G.E. Uhlenbeck, On the Theory of Brownian Motion, Rev. Modern Physics, 17, 323, 1945.

Williams, F.C., Thermal Fluctuations in Complex Networks, Journ. Inst. Elect. Engrs, 81, 751, 1937.

Williams, F.C., The Fluctuations of Space-Charge-Limited Currents in Diodes, Journ. Inst. Elect. Engrs, 88, Part III, 219, 1941.

Woodward, S., The Mathematical Description of Random Noise, Proc. Camb. Phil. Soc. 44, 404, 1948.