MATHEMATICS TEACHING AND MATHEMATICS ACHIEVEMENT IN HONG KONG - AN EXTENSION OF THE INTERNATIONAL PROJECT FOR THE EVALUATION OF EDUCATIONAL ACHIEVEMENT

BY

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ABSTRACT

Mathematics teaching and mathematics achievement were studied in Hong Kong, at a time when the world wide movement for the modernization of mathematics education had begun to affect that city. The International Project for the Evaluation of Educational Achievement (IEA) had listed a number of environmental variables believed to be related to educational achievement, and studied the effect of these upon achievement in mathematics in twelve countries. Three parallel achievement tests of one and a half hour duration were constructed, using all the items of the IEA test for thirteen year old children, and applied to a sample of schools in Hong Kong. The interrelationships of the societal factors in this study were compared with those of the twelve nations of the IEA study.

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CHAPTER I

INTRODUCTION

"Hong Kong is a barren island with hardly a house upon it",[1] wrote Lord Palmerston in 1841 when Hong Kong was ceded to Britain after the Opium War. This once barren island has grown in slightly more than a century into a densely populated city which has additionally transformed its economic basis from an entrepot to an industrial center in the last two decades. These changes have been so rapid that difficulties are encountered in providing sufficient social services and educational facilities.

In terms of population, Hong Kong has four million people. In terms of international trade, Hong Kong in its import and export trade outrates many Asian nations. Yet geographically speaking Hong Kong is extremely small. The island has only 28 square miles and in most parts is too hilly to be inhabited. Across the Victoria Harbour, there is the Kowloon Peninsula which was leased by China

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1. Hong Kong Government, Hong Kong Annual Report, 1968 (Hong Kong, 1969), p.268.

to Britain for 99 years from 1898. On the one hand this lease has added three hundred and sixty square miles of valuable land to Hong Kong and without it the growth of the city would not have been possible. On the other hand the expiration of this lease in the year 1997 poses a formidable threat to the future of this Colony, and is responsible for a feeling of insecurity among its citizens.

Hong Kong's geographical position is a favourable one. Hong Kong lies 22 degrees north of the Equator and is just within the tropics. To the North is the China mainland with its border running approximately along the Shum Chun River. Through the Pearl River and its upper courses, (the East Kong, the West Kong and the North Kong,) there is easy communication with Canton and other parts of South China. Within a circular arc one finds Shanghai, Manila and Saigon, with Singapore, North Borneo, the Philippines and Japan slightly more distant. The harbour provides an excellent anchorage for the large ocean-going vessels and because the gradient of its shore is steep, ships of great tonnage and even modern giant liners can come close to the land, thus facilitating easy loading.

The climate is good, Temperature ranges from 40 to 85 degrees, there is summer rainfall and cool dry winters.

The numerous islets and the bays along the coast provide ideal fishing places for the Colony's fishery, in which 50,000 people are directly or indirectly engaged.

The inhabitants are mainly Chinese, with minorities of British, Portuguese, Indian, American and a few thousand transients of all nationalities. Most of the Chinese came from the area near Canton, the others being Shanghainese, Fokienese and the indigenous Hoklo and Tangka. Although they have their own dialects, there is no great language difficulty as all of them speak Cantonese. Gradually the differences between one group and the others have become less, due to assimilation and education.

Industrial development in Hong Kong was first initiated by the political changes in China and the subsequent Korean War. These changes made it impossible for Hong Kong to continue as an entrepot between China and the rest of the world. The arrival of refugees from the Mainland brought in additional manpower and technical knowledge as well as capital. As a result, while the entrepot trade declined, manufactures increased. These were mainly light industries with cotton textiles as the most important. The rate of growth has been great, and the rapid industrialization has brought with it an increasing need to improve the utilization of all resources, particularly technical manpower. However, Hong Kong has abundant low-cost labour, efficient banking services and easy access to the world by sea. Above all there is a comparatively stable political environment. These are attractive to the foreign investors, although most of the capital resources are self-generated by the local Chinese industrialists. This economic growth has had a great impact on the people of Hong Kong and has provided the financial resources for the rapid expansion in education which has taken place during the same period.

CHAPTER II

THE HISTORICAL DEVELOPMENT OF EDUCATION IN HONG KONG

The Establishment of Modern Schools

Soon after the British came, western type schools appeared largely through the efforts of such missionary bodies as the Morrison Education Society, the London Missionary Society and the Roman Catholic Church. Schools were established for the training of Chinese clergymen and to provide some education for these children whose parents could not afford to send them to Canton. As in Britain education was left to voluntary efforts. Yet grants were made available to the philanthropic and religious schools, and villagers were encouraged to build their own schools with the help of government money. However, when the government attempted to restrict religious teaching in schools, the church schools refused to accept the grants. Finally, the churches won concessions from the government which left catechistic teaching absolutely in the hand of the church. This struggle between the government and the church made it clear that the government would have to provide schools of its own if it meant to take education into its own hands.

Soon, a significant number of government schools appeared, and other independent types of schools also emerged in large numbers. They were the Chinese Kaifong (neighbourhood) Welfare Societies' Schools. These schools differed from the government and church schools by conforming to the pattern of education provided in China.

With the increase in the numbers of schools, more attention was turned to the improvement of the quality of education provided. School inspections began to be both more frequent and more serious. Teachers were to be praised or punished, given extra salary or dismissed. Inquiry was made into curriculum, and, as a result, the scope of study was expanded by the addition of more modern subjects. Standards were further raised by the introduction of an English university examination, the Cambridge Local Examination. All these measures, together with an increase of educational expenditure which amounted to 2.8% of the total budget of the government, provided the base of public education at the end of the nineteenth century in Hong Kong,[1] whilst ideas on higher education for Hong Kong received an impetus in 1910 when the foundation stone was laid for the University of Hong Kong which embraced the existing Hong Kong College of Medicine.

Thus before the out-break of the First World War

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1. Hong Kong Government, Triennial Survey-Education Department, 1964-67 (Hong Kong, 1968), p.5.

an education system operating at all levels was to be found in Hong Kong. The focus of attention until this time had been upon education for children of the upper and the upper-middle classes. Even the government concentrated its effort on providing few but good schools. The education of the masses was neglected and was left to philanthropic bodies and other organisations. In the words of the then Director of Education, "Our policy was to encourage upper grade schools, both government and grant rather than lower grade schools."[1] This attitude truly reflected both the British view on educating the elite and the Chinese view that education was only for the gifted.

Development Between The Two World Wars

This period saw a rapid expansion in the provision of education. In less than thirty years, the number of pupils increased four fold from 25,000 in 1914 to 118,000 in 1941.[2] The expansion of numbers was largely due to the increase of population, particularly during the period when the Japanese invaded China and caused a great influx

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1. Hong Kong Education Department, 1913 Education Ordinance (Hong Kong, 1914).
2. Hong Kong Education Department, Triennial Survey 1964-67 (Hong Kong, 1968), p.5.

of refugees into Hong Kong. The nature of this influx forced the government to pay more attention to education of the masses. There was also the enactment of a new Education Ordinance which gave the government the responsibility for control of primary and secondary education. Their efforts showed progress in the amount and nature of education provided, particularly in the provision of vernacular education. A new normal school was opened in 1925 and an in-service training course for teachers was also in operation.

Progress was also made in the education of girls. By 1939 there were 37,370 girls out of a school population of 118,193[1] or 33% of the total. This was due to the changing attitudes of the Chinese toward education for girls which had always, in the past, been considered as a luxury. In well-to-do families girls received the same education as boys, and some even proceeded to university. It is rather surprising to find that in 1929 the number of secondary pupils was 4,398 males and 3,150 females.[2] The numbers were so similar that it might suggest that

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1. Hong Kong Education Department, Annual Report 1947 (Hong Kong, 1948).
2. Ibid.

both sexes had an equal chance to go to school. Actually the development of girls' school exceeded the demand, for many of the secondary girl schools had less than 20 pupils in the higher classes.

Vocational education began when the Salesian Order provided elementary education in conjunction with vocational training for orphans and poor children. Pupils divided their time between ordinary school subjects and practical works. Later the government opened a junior technical school and a technical institute, each providing more formal technical education. A great variety of subjects were offered including surveying, building engineering, wireless telegraphy and nautical courses. By the end of 1939 there were 20 vocational schools, one of which, though small in size, specialized in agricultural science, and trained farmers to provide high quality fruits and vegetables for the local markets.

Hong Kong had passed her centenary in 1941 when the Japanese invasion came. In this hundred years, a foundation of education had been laid and different types of education at all levels had been provided, although still somewhat poor in quality and limited in quantity. The outstanding feature of education in this hundred years had been the growth of two separate systems of schools side by side -- the English school and the Chinese school. The former followed the British ideals and the latter developed in line with the modern schools of China. There were many differences in organisation, syllabuses and aims of these two types of schools. The Chinese schools, had a 6-3-3 system and prepared students for the universities on the mainland. They were more concerned with the teaching of Chinese culture and moral discipline. The English schools for their part offered a type of secondary education similar to that of the grammar schools in England. The more intensive study of the English language in these schools opened up to students greater opportunity of employment in government and foreign commercial firms.

Post Second World War Development

The period of Japanese occupation from 1941 to 1945 saw the destruction of practically all that had been built up. Due to the destruction of most of the school buildings and equipment and the loss of qualified teachers, the work of rehabilitation was more difficult than expected, and many years had to be spent in recovery. But this process was soon made much harder by the great influx of refugees from China as the communists began to take over mainland China. Temporary measures were taken to absorb more students within the existing educational facilities; the government ran two-shift schools and allowed private schools to be housed in whatever premises could be found. Attention was given to long term development. It was decided that fifty government vernacular primary schools should be established over a period of ten or less years. But the plan was soon made out-of-date, such was the increase of population. In 1954 a new and more ambitious 7-years development plan was launched which aimed at universal education for all children of primary school age from 6 to 11. This called for the creation of 215,000 places in school. To cope with such a development two new teacher training colleges were opened to produce at least 700 primary school teachers per year. The original target was met within five years and by the scheduled time, 1961, the actual increase was 313,000, some 98,000 more than forecast.

While development of primary education was in full swing, secondary and higher education were comparatively neglected. From 1954 to 1961, there was not even one secondary school of any kind built by the government. The gap was only partly filled by the opening of many private secondary schools, and most of these were run purely for profit making purposes.

Expansion in higher education was achieved partly by the growth of the University of Hong Kong and the founding of a new Chinese University in 1963 and partly by the increase of post-secondary colleges which moved to Hong Kong from China during the time when communists took over that country.

Looking back over the past, there is some satisfaction to see so much done in so short a time. In only two decades, the net increase of student enrolment in Hong Kong was more than one million, for while the school population in 1948 was still under 100,000 in 1968 it was already 1,133,041.[1]

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1. Hong Kong Government, Hong Kong Annual Report, 1968 (Hong Kong, 1969), p.337.

CHAPTER III

THE EDUCATION SYSTEM

The education system in Hong Kong appears somewhat similar to that of England, for it has been modelled on the British system. Thus we find in Hong Kong six year primary schools which lead to a Secondary School Entrance Examination at the age of 12, parallel to the British junior schools which held the eleven-plus examination at the end of the junior school period to select pupils for different types of secondary education. We also find three types of secondary schools in Hong Kong -- the grammar schools, the technical schools and the secondary modern schools as in England. In higher education, Hong Kong University takes in students with passes in G.C.E. advanced level or its equivalent, like the matriculation examination of the University of Hong Kong. Curricula, length of course and course requirement closely follow those of British universities. Yet, in spite of the similarity there are many differences. In the first place, Hong Kong differs from England in having a centralized system. Schools have to follow the same syllabus, have the same structure and are geared to the same examinations. In the second place, education in Hong Kong is neither free nor compulsory. In the third place, the educational concepts and practices are a compromise between those of the East and the West. In particular the concept of a scholar, the function of a school, the teacher-pupil relationship and discipline are strongly influenced by the Chinese heritage. These factors make education in Hong Kong unique -- an apparently western orientation superimposed upon a Chinese foundation.

Not only is Hong Kong a meeting place of East and West, bringing together people from many parts of the world, its Chinese, indigenous population is far from homogeneous, and contributes a wide variety of cultural traits. Educationally, the large number of alien or expatriate professors, administrators and sponsors occupying key posts have great influence on educational policy. They propagate ideas and formulate policy, and are largely responsible for the western framework within which Hong Kong education is administered. Yet within this framework aspects of both Chinese and European civilizations are nurtured and transmitted.

There is a lack of long term planning in Hong Kong. In this it seems to reflect very much the British empiricism in dealing with social political issues. Since Hong Kong's problems are so unique and its social and economic changes are so unpredictable the use of a series of adjustments and adaptations, expansions and diversions rather than a theoretically based, pre-conceived plan has been more profitable. In Hong Kong the enactment or the repeal of a law depends on the immediate need of the environment. Opposing view points and antagonisms are resolved by compromise. Education like every thing here grows on a delicate equilibrium. Without an understanding of this fact, many educational features in Hong Kong could not be explained.

Hong Kong like many other Asian countries has a high proportion of people under the age of 20 (50.00% of the total population).[1] This means that a comparatively small number of productively active people have to support a larger number of students. In 1966 the student population was 27.3% of the total population where the working population was 37.7% of the total.[2] Every student in Hong Kong is thus supported by 1.4 producers while in many countries a student is usually supported by 3-4 persons in work. Hong Kong like many other Asian countries has a high illiteracy rate. The 1966 By-census showed that 25.24% of the total population of age 15 and over have never been in school. Figure 3.1 illustrates the age

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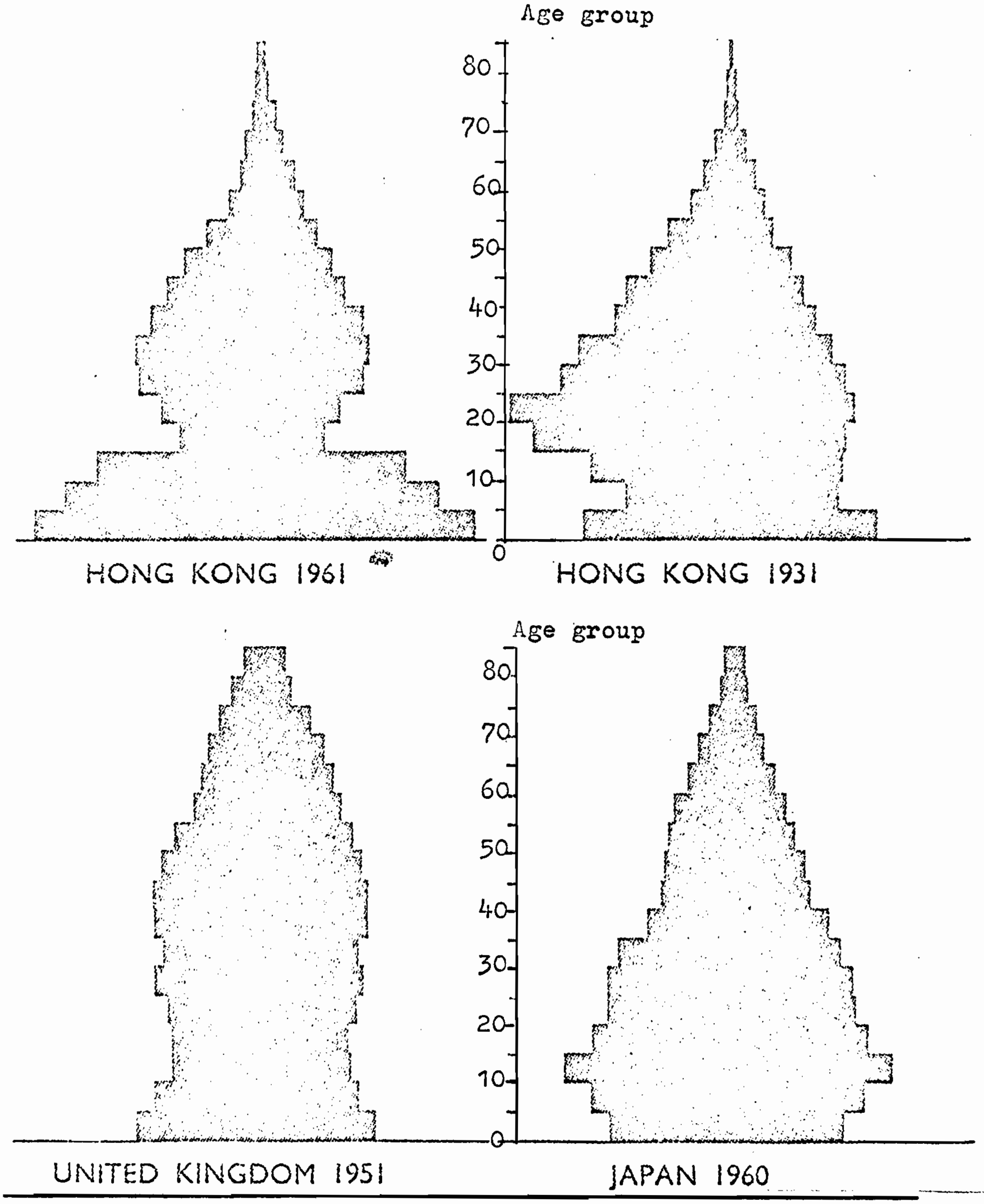
1,2. Hong Kong Census Department, <u>Hong Kong Statistics 1947-67</u> (Hong Kong, 1968).

composition in comparison to that of United Kingdom and Japan. The broad base of the 1961 Hong Kong pyramid indicates the large number of school age compared with the size of the adult population who will have to pay for the schooling. The position in the United Kingdom and in Japan is much more favourable. Figure 3.2 shows the proportion of population which has been in school. This graph does not show the length of schooling. It indicates however, that the problem of female illiteracy is being rapidly eliminated and that about 90% of all the children receive some schooling even if the start of it is somewhat delayed.

Organisation in Education

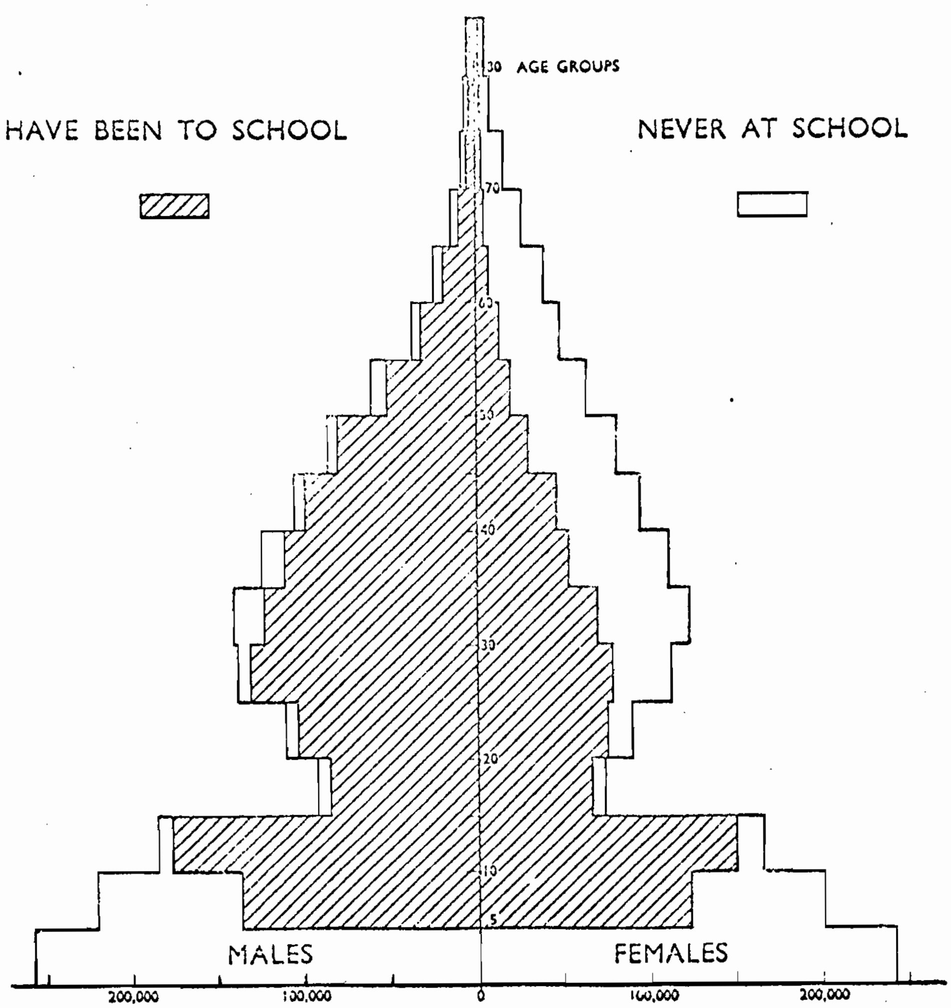
The highly centralized nature of education in Hong Kong places all educational matters directly under government control. This required a large bureaucratic organisation. The Education Department is headed by a Director and a Deputy Director and has five sub-departments, each under an Assistant Deputy Director. Three of them are in charge of the three levels of education respectively, the other two deal with school inspection and education development. Outside of this administrative organisation is the Board of Education which

Figure 3.1 Comparative Population Pyramids[1]



1. Census and Statistics Department, The Census and You (Government Printer, Hong Kong, 1961).

Figure 3.2 Educational Attainments in Hong Kong



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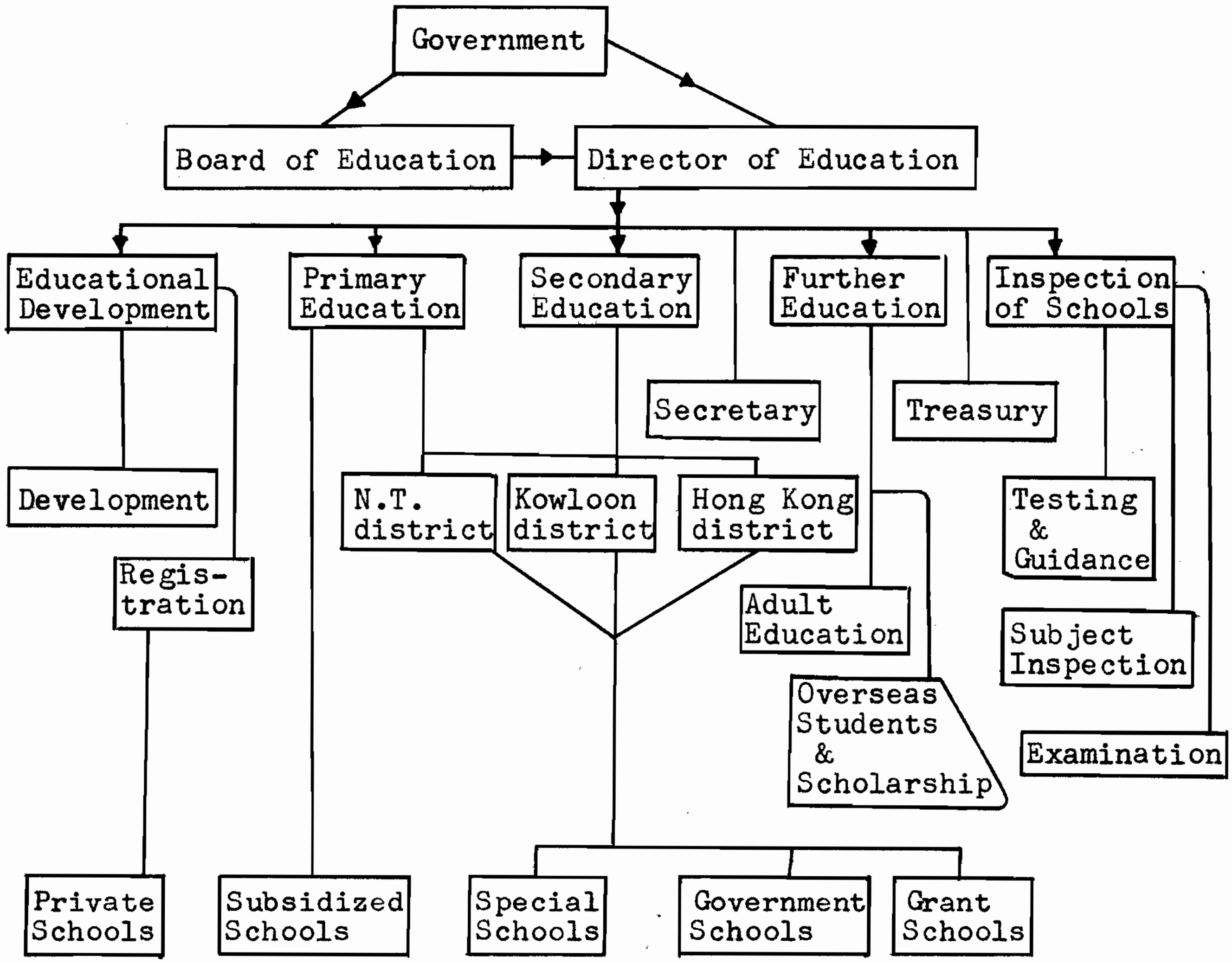
1. Ibid.

gives advice and exercises general supervision of all educational provisions. The schematic diagram below shows this administrative organisation. It will be noted that the Government, and the Board of Education intervene only through the Director of Education, who, with his administrative staff is responsible for the day to day supervision of the schools.

Figure 1.1:

The Administrative Organisation of Education in Hong Kong



School System

In the year 1968 there were 2,594 schools in Hong Kong educating 1,133,041 pupils.[1] These schools can be classified into three types according to their languages of instruction. They are Chinese schools, English schools and Anglo-Chinese schools. The English schools are for the English speaking population (largely expatriate) and do not study the Chinese language at all. Their curriculum is closely allied to that of an English Grammar School. The Anglo-Chinese schools use English as the language of instruction but offer Chinese as a second language. The Chinese schools, originally modelled on the schools of mainland China are now undergoing a transformation to relate more clearly to higher educational opportunities in Hong Kong, and elsewhere than mainland China.

Schools may also be classified according to the nature of their financial support. Several overlapping types are found. Thus there are government schools -- schools provided and maintained by the government, grant-in-aid schools -- schools receiving deficiency payments but providing a quality education as well as religion and

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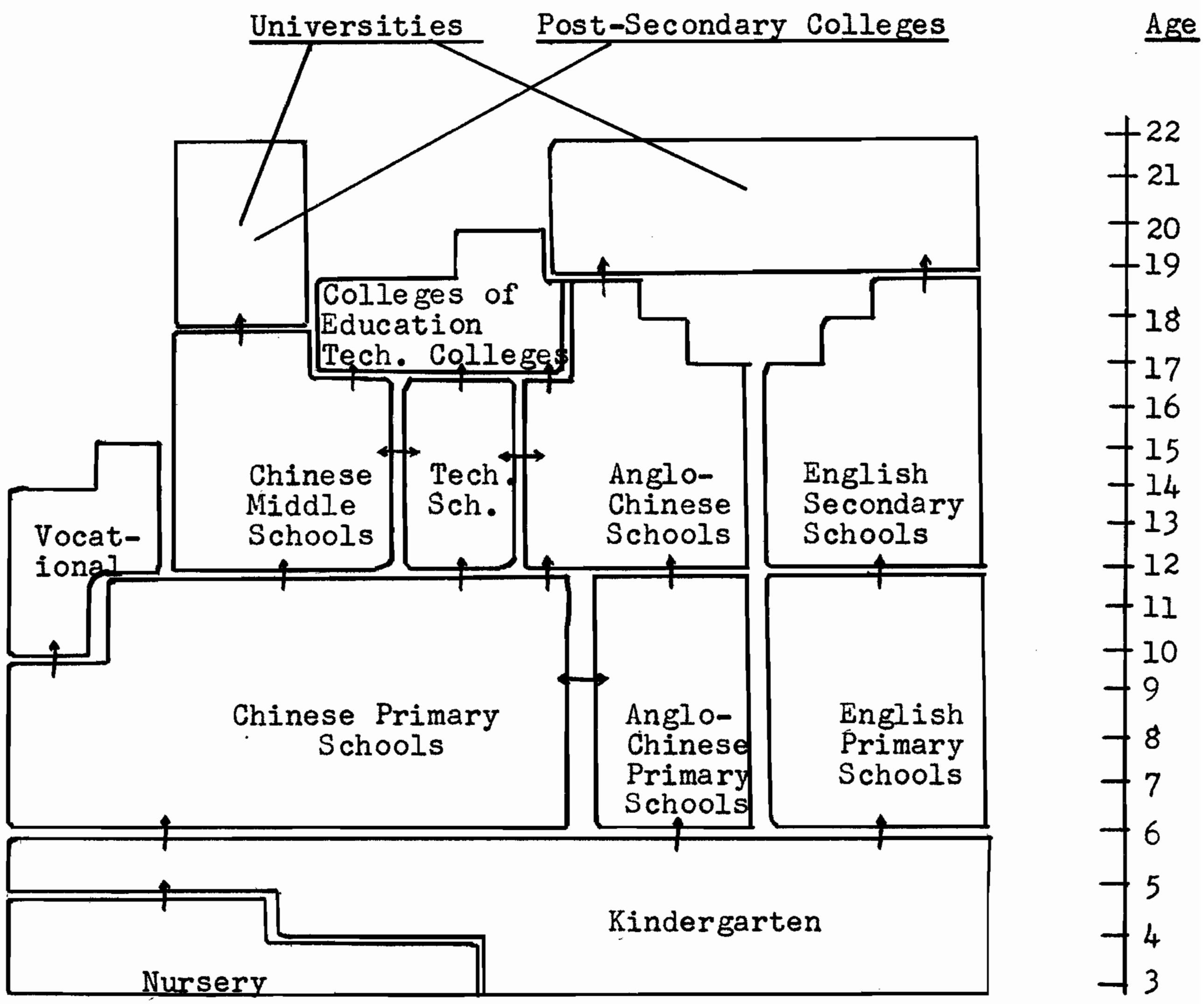
1. Hong Kong Government, Hong Kong Annual Report, 1968 (Hong Kong, 1969), p.337.

catechistic teaching, and subsidized schools -- schools at the primary or secondary level receiving a fixed annual subsidy, but not necessarily meeting a deficit. Again some private, non-profit schools may from time to time receive aid, whilst schools run for profit are unaided. Sometimes a school may be designated by words from both classifications, thus, an English private unaided school, a Chinese subsidized school, and so on.

At the age of 6 a pupil enters a six year primary school. He will remain there to take the Secondary School Entrance Examination at the end of primary school. If he enters an academic secondary school, he will follow a five year course leading to the Hong Kong School Leaving Certificate Examination. To proceed to higher education he must stay for two years (English or Anglo-Chinese) to prepare for the Advanced Level of the General Certificate of Education, or for the Hong Kong University Matriculation Examination. The Chinese student may attend a Chinese Middle School for one year in preparation for the Matriculation Examination of the Chinese University of Hong Kong. If admitted there he will have a four year university education, as compared with three years for the English or Anglo-Chinese School graduate. This system is represented diagrammatically below.

Figure 1.2: School System In Hong Kong



(Flow from one unit to another when indicated by arrows).

Primary Education

There is no compulsory education in Hong Kong, not even at the primary level. But such is the enthusiasm for education in Hong Kong that most families are eager to send their children to school. In fact, in 1968 the number of children in primary day schools, 666,834, was 104% of the estimated number of children in the 6-11 age group.[1] This remarkable number is due to the number of over age children retained in the various grades. It indicates, however, that the goal of compulsory primary education would be realisable, should the government seek to require this. The schools themselves are almost entirely Chinese schools, with the exception of a few English and Portuguese schools for minority groups. The government run schools are occupied by two separate schools during the day, a morning school and an afternoon school, whilst in the evening the schools are used for adult education. Schools are multi-storey, and are built on as little land as possible, and to accommodate as many as possible.

Private primary schools exist, and provide about one half of the total number of places, but they are to be

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1. Hong Kong Education Department, Annual Summary 1967-68 (Hong Kong, 1969), p.18.

found in ordinary dwelling houses, badly housed, and short of qualified staff. In some resettlement areas roof top schools also exist. These partly covered roof tops were planned to be recreational areas, but have been converted to school use by bricking up walls and cutting wooden frames for windows. Though subsidized by the government they are under the control of philantrophic agencies. If the population continues to grow, and it is estimated to reach five and one half millions in 1981, and if unsatisfactory school buildings are to be replaced by more suitable ones, then Hong Kong's building force would be kept fully employed for the next two decades on this task alone.

The primary school lasts for six years, and sets out to provide a general education. Almost half of the 35 weekly lesson periods are devoted to the study of Chinese. In the second or third year English is introduced and may receive from three to five periods per week. Arithmetic is taught in each year of the course. These three subjects form the core of the examination which terminates the period of primary education.

In 1949 the Joint Primary Six Examination was established to regulate entry into the government operated and controlled secondary schools. In 1962 this became the

Secondary Schools Entrance Examination. Being successful in this examination is a highly prized result, on the part of the children, the parents and the schools. Only one in five of the entrants pass from the primary school into a government secondary school. There is therefore great pressure, from both parents and schools, upon primary school children. Many adverse criticisms have been made of this state of affairs; only beliefs that education should be reserved for the gifted are urged in its defence.

Secondary Education

At present about four fifths of Hong Kong children who enter primary schools successfully complete the six year course, and of these perhaps four fifths enter a secondary school of one kind or another. A recent figure placed the number at 69.4% of whom 47% entered Anglo-Chinese schools and the other 20% entered the Chinese Middle Schools[1], a fact which emphasizes the demand for some form of English education with increased opportunity for a job in some commercial enterprise at the end of secondary school. The Government Secondary Schools provide only 20% of the total places for secondary school children.

As noted above all types of Anglo-Chinese schools prepare students for admission to the English speaking University of Hong Kong. In a like manner the Chinese Middle Schools prepare their students for admission to the Chinese University of Hong Kong, to the Post-Secondary Colleges, and to universities in Taiwan. It is now impossible for Hong Kong students to go to universities on the mainland. Students in each school may follow a five year course leading to the Hong Kong School Certificate Examination, which may be taken in English or Chinese.

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1. Hong Kong Education Department, Annual Summary, 1967-68 (Hong Kong, 1969), p.2.

A Credit pass in this examination ranks as a Pass at Ordinary Level in the General Certificate of Education Examinations of English university examining boards.

The Chinese Middle Schools originally had a six year course, and were often divided into Junior Middle School and Senior Middle School, on the assumption that many may not be able to afford to stay beyond three years. With the lack of entry into Chinese universities on the mainland this distinction has been dropped and the Chinese Middle Schools have become increasingly like the other Anglo-Chinese Secondary schools with a five year course. Rather than stay for a sixth year to enter the Chinese University of Hong Kong many students seek positions in business offices at the end of five years. The position of these schools has become increasingly acute. Although there are nearly four million Chinese in Hong Kong there are less than 50,000 in attendance at these Chinese Middle Schools. This lack of enthusiasm for Chinese education has been deplored by many. One such example is taken from a speech by Bishop Hall, who said,

> "Those who abandon their own language and culture to pursue English for reasons of economic security are in serious danger of making the pursuit of wealth

the main aim of their life."[1]

Many of the Chinese students are to be found in the Anglo-Chinese schools, attempting to learn in a foreign language. If, in addition, the standard of English of some of the teachers is inadequate to explain difficult concepts with any degree of clarity, then students resort to rote learning of facts from text books, for the students then lack the ability to express an idea clearly in their own words. This sacrifice of understanding and sacrifice of interest in learning because of the high economic value of English remains one of the continuing problems of education in Hong Kong.

Problems exist also in the private Anglo-Chinese Secondary Schools. These have increased rapidly over the past few years, and attracted a great many Chinese students. They now offer one half of all secondary school places. But many of them are poorly equipped and badly staffed, offering an education of somewhat low standard. They remain profitable to their proprietors by charging high fees and employing poorly paid and unqualified teachers. The Government Secondary Schools offer an

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1. Bishop Hall, Address appeared in the South China Morning Post, December, 11, 1965.

education of a fairly high standard, with the study of English taking up about one third of the usual forty period school week. Grouping of subjects for the purpose of the Hong Kong School Certificate Examination takes place only after three years of general education. Only about one fifth of the students who complete the fifth year enter into the sixth form for further study leading to A-level examinations and possible admission to the University of Hong Kong.

Technical Education

In many countries there is an imbalance between academic and technical education; in the case of Hong Kong this imbalance is quite extreme. Ninety four percent of secondary education is devoted to a grammar school type, and only four percent to some form of technical education[1]. Even this is something of an exaggeration since some of the technical schools ape the grammar schools in aiming for the School Leaving Certificate Examination in many of the subjects of the humanities. One or two schools offer a vocational training with no pretence of academic work. There is one post-secondary technical college, the

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1. R.F. Simpson, Technical Education and Economic Development (Hong Kong University, 1966), p.2.

Hong Kong Technical College with full and part-time instruction and day and evening classes. Many of the courses offered are linked to the City and Guilds of London Institute examinations. Nine tenths of the students attend on a part time basis, but all told it offers more than one hundred separate courses to some sixteen thousand students grouped into eight separate departments[1]. Building and textile instruction are the most popular options. Even at the technical level there is the same extreme competition for admission, and the unsuccessful candidates often take up some other form of adult education offering instruction in language and mathematics. Certain industries, particularly those which have many small firms prefer to rely upon on the job training, sometimes through apprenticeship training. The larger industries have well organized training schemes.

Expenditure on technical education takes up thirteen million out of a total of two hundred and ninety four million Hong Kong dollars[2]. This must be regarded as meagre for a

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1. Hong Kong Education Department, Triennial Survey, 1964-67 (Hong Kong, 1968), p.39.
2. Hong Kong Education Department, Annual Summary, 1967-68 (Hong Kong, 1969), Table V.

city which boasts of its industrial base, for nearly forty per cent of all employed persons are engaged in manufacturing industries[1]. The shortage of trained technical personnel has been recognized as an increasingly acute problem, and has led to the setting up of the Industrial Training Advisory Committee. Its main functions are,

> "to make recommendations on industrial training problems as they emerge and to plan to establish a comprehensive system of training to meet the needs of Hong Kong industry as they develop."[2]

This step can scarcely be considered to match the urgency of the problems of technical and vocational training.

Teacher Training

There is evident need for a scheme for the training of the teachers necessary for the more than one million students currently in the schools. To fulfill this task are the three Colleges of Education. The first one, Northcote College of Education was established in 1939,

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1. Hong Kong Government, Annual Report, 1967-68 (Hong Kong, 1969), Appendix III.
2. Hong Kong Education Department, Triennial Survey, 1964-67 (Hong Kong, 1968), p.38.

conducts its courses in English and has residential facilities. The other two, Grantham and its offshoot, the Sir Robert Black College offer their courses in Chinese and are non-residential. The majority of the four hundred students at Northcote College follow a two year course leading to employment in the primary schools and the lower classes of the secondary schools. They will be recipients of the Hong Kong School Leaving Examination with five passes, of which two must carry the mark of Credit. Holders of diplomas from the post-secondary colleges in Hong Kong are admitted to a special one year course at Northcote College, and train for positions in the upper classes of both the Chinese Middle Schools and the Anglo-Chinese secondary schools.

The Grantham College of Education was established in 1951 to help staff the expanding primary schools. During the seven year plan it offered only a one-year course to prepare teachers for the Chinese primary schools. The course has been lengthened subsequently into two years. Of recent years it has offered instruction in English because of the demand from the schools for English instruction. The third of the government operated training colleges, the Sir Robert Black College, was opened in 1960 as a branch college but soon became independent of its parent institution.

The colleges follow an English model, except in the length of course. There are courses for personal development, through the study of one or two subjects in depth, as well as curriculum studies, professional education and supervised practice teaching. Despite these aims, for a number of years their graduates remained ill equipped for their profession, being provided more with a set of rules and precepts than possessing cultivated minds, able to judge, to reason and to make independent assessments. More recently a new third year course for specialists has begun. Students may specialize in one of five subjects - mathematics, music, art, domestic science and physical education - and are recruited on the basis of outstanding performance in the two year course. One half of the places in this course are reserved for promising teachers in government and grant aided schools who may be seconded on full pay for the one year or full time study.

At present there are about two thousand teachers under training in the three colleges, about half of them in evening in-service courses. The aim is to produce a thousand qualified teachers each year. Not all of them can find employment in the government schools or subsidized or grant aided schools which can afford to pay the approved salary scale. For those unable to do so, life is hard since they are unlikely to be employed in the private enterprise schools which prefer unqualified teachers at extremely low rates of pay. One solution which has been canvassed is the payment of the difference in salary to the private proprietors for each qualified teacher they employ. But this at best is a poor solution. The only lasting solution is for the provision of government schools for the whole primary population. Some satisfaction can be claimed here since they now provide nearly two thirds of the total as against little more than half a few years ago.

University Education

There are two universities in Hong Kong, the University of Hong Kong established in 1911 and the Chinese University of Hong Kong which resulted from the recommendations of the Chinese University Commission of 1962 under the chairmanship of J.S. Fulton, Vice Chancellor of the University of Sussex. This is a federal university incorporating a number of Chinese colleges which were set up after the Communist take over of power on mainland China. Chung Chi College, New Asia College and the United College became foundation colleges of the new university which was formally inaugurated in 1963. The government has provided some 273 acres of land at Shatin for the erection of the Central University Building.

The Hong Kong University follows the English pattern, has about three thousand students with the majority in the Faculty of Arts, and is recognized as one of the Commonwealth Universities with accepted standards of education. It has five faculties, Arts, Science, Medicine, Engineering and Architecture, and Social Sciences. Some students attend the Chinese Language School and a number take education courses. Nine tenths of the students are undergraduates. Graduate and post-graduate work is of recent origin, and much of it is confined to the Centre of Asian Studies established in 1968 by the combination of the Institute of Oriental Studies and the Institute of Modern Asian studies. The new institute recruits its students from a variety of Asian countries other than Hong Kong. English professional bodies recognize the degrees of the university, a fact particularly important to the faculties of medicine and engineering.

The Chinese University has about two thousand students, though the numbers are growing and are likely to increase further with the completion of new buildings. It already possesses a Graduate School, a School of Education, offering a one year diploma course to graduates wishing to become teachers, an Institute of Business

Administration and a Computer Centre. Its original Chinese flavour is being replaced by a more clearly American one, though the language of instruction continues to be Chinese. Both universities have Extra-Mural Departments which offer a wide range of courses and attract a large enrolment, almost twice that of the full time students. Courses of a professional nature are given as well as the more usual courses of general educational interest. Both universities are autonomous, and free from direct government control, operating under a University Grants Committee, set up for the first time in 1965. Ordered growth is seen for both.

As in other areas of education the striking feature of Hong Kong universities is the paucity of admissions compared with the large number of potential applicants. In the United States one child in three has a chance of admission to a university, in Britain it is estimated to be one in twelve, if one includes all forms of tertiary education, sandwich courses included. Hong Kong universities offer five thousand places to a community of four million people. Since admission follows upon matriculation by examination, the level of success in the respective examinations is set at a high standard. The University of Hong Kong, for example, requires three good passes at the Advanced level of the G.C.E. examination. In spite of the ordered expansion which has been forseen, this is likely to be insignificant in comparison with the growth of population.

Post Secondary Colleges

There are a number of so called "Post-Secondary" colleges in Hong Kong, the main impetus behind their establishment being the influx of students and teachers from universities in China during the period of the Chinese Civil War in 1946-49. These colleges are of varying standards and offer day and evening courses of great variety. Among them are the Chu Hoi College, the Baptist College, the Tsing Hwa College, the Columbia College, the Sun yet Sen College, the Kwong Tai College and the Overseas Chinese Engineering and Business Administration College. A Post-Secondary College Ordinance was enacted in 1960 with the object of giving statuatory recognition to those colleges whose status approaches but not yet attains that of a university institution. To date no college has been given such recognition, which should follow when an institution reaches a certain standard in a wide variety of matters relating to academic standards, governing body, legal status, finances and facilities.

With the exception of the Hong Kong Baptist College which has recently moved into a newly built premises all the others are housed in unsuitable tenant flats or share a small school building with a middle school. The teaching staff are poorly paid and usually of low quality. Facilities are nearly non-existent.

Some day colleges offer full-time four-year courses of study in arts, science, engineering and commerce; evening courses are usually in a limited specialized field. A group of colleges has organized joint entry examination to select candidates for admission. In 1966, 1,432 sat the examination for admission to the eight colleges and 771 passed.[1]

Nevertheless, these private post-secondary colleges in Hong Kong are fulfilling a useful function in providing education capable of approaching university standards for those who are unable to gain a place in the two universities. With an ever increasing demand for higher education, there is no reason why these colleges should not grow one day into institutes deserving the name of universities. As Simpson said: "Had these colleges

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1. Hong Kong Education Department, Triennial Survey 1964-67 (Hong Kong, 1968), p.52.

been in the Philippines and a number of other Asian countries, they might already be awarding their own degrees. Even if the standards are low, should they not be encouraged?"[1]

Hong Kong needs to increase greatly her tertiary education. In 1960 the enrolment ratio in full-time course in this level was only 1.8%(even with all post-secondary colleges and Upper 6th form included) while the Philippines, Taiwan, Republic of Korea and even Thailand with 5.7%, 6.1%, 5.6% and 2.5% respectively[2] have much higher enrolment ratios than Hong Kong. By contrast Japan had 12.1%. Economically speaking, Hong Kong is much closer to Japan in its higher GNP per capita than the other Asian countries and therefore should be able to cater for more higher education. Perhaps the sluggishness in making more provision and in giving encouragement to the low quality colleges is still due

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1. R.F. Simpson, Perspectives in University Development (Hong Kong Council for Educational Research, 1966), p.2 and p.30.
2. R.F. Simpson, Comparative Statistics on Economic and Educational Development in Asian Countries (Hong Kong Council for Educational Research, 1966), Appendix CA13.

to the inheritance of the British prejudice against non-residential colleges and part-time degrees which do not accord with the British notion of a university where full time residential "university life" is considered to be as important as academic studies. Perhaps as Britain changes, Hong Kong may also drop her prejudices against any but the traditional forms of education, particularly university education.

PART II

TEACHING OF MATHEMATICS IN HONG KONG

CHAPTER IV

MATHEMATICS IN CHINA

China is one of the oldest nations in the world, and as such has had mathematical knowledge from early times. Their early arithmetical arts developed from the regular observance of religious rites and ceremonies. According to old documents the Huang-ti or the Yellow Emperor, who reigned over China in the 27th. century B.C., made Tai-nao construct the chia-tsŭ of the sexagesimal system, Li-shou invent the art of numbers, and Yung-cheng employ all these to regulate calendars.[1]

The chia-tsŭ that has been employed from the time of the Yellow Emperor down to the present day is formed by combining the Ten Heavenly Stems (T'ien-kan)[2] with the Twelve Earthly Branches (Ti-chih)[3] to obtain a cycle of 60. The Ten Heavenly Stems are chia, yi, ping, ting, wu, chi, kêng, hsin, jên, kuei,[4] and the Twelve Earthly

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1. Yoshio, Mikami, Mathematics in China and Japan (Chelsea Publishing Co., 1913), p.2.
2. 天干.
3. 地支.
4. 甲,乙,丙,丁,戊,己,庚,辛,壬,癸.

Branches are *tzŭ*, *ch'ou*, *yin*, *mao*, *ch'ên*, *ssŭ*, *wu*, *wei*, *shên*, *yu*, *hsü*, *hai*.[1]

To explain how these form a numeration cycle, let us denote the Ten Heavenly Stems by $x_1, x_2, \ldots x_{10}$ and the Twelve Earthly Branches by $y_1, y_2, \ldots y_{12}$. A numeration system is obtained by matching the x's and the y's in the forms as $x_i y_j$, where i 1,2, ..., 10, j 1,2, ..., 12 are chosen in order. Thus we have

$$\begin{array}{lll}
x_1y_1, \; x_2y_2, & \cdots & x_{10}y_{10}, \\
x_1y_{11}, \; x_2y_{12} & \cdots & x_{10}y_8, \\
x_1y_9, \; x_2y_{10} & \cdots & x_{10}y_6, \\
x_1y_7, \; x_2y_8 & \cdots & x_{10}y_4, \\
x_1y_5, \; x_2y_6 & \cdots & x_{10}y_2, \\
x_1y_3, \; x_2y_4 & \cdots & x_{10}y_{12}.
\end{array}$$

These are all the 60 members in the *chia-tsu* cycle.

In the sacred book of I-king[2] (11th century B.C.) symbols of *ying* -- and *yang* ——[3] were used to form *sz'siang* and *pa-kua*[4] which were then extended into the

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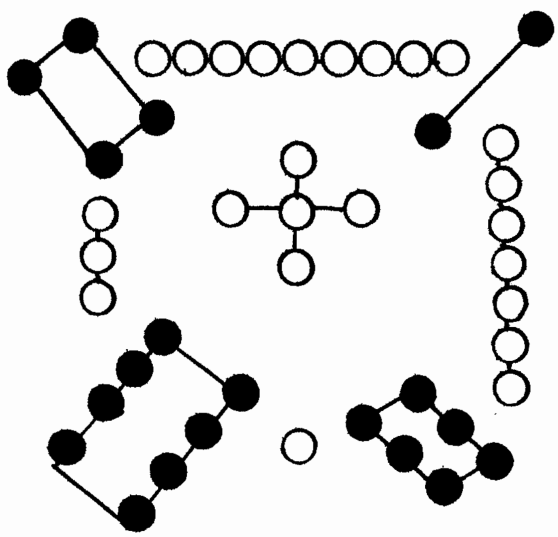
1. 子,丑,寅,卯,辰,巳,午,未,申,酉,戌,亥.
   Tung Tso-pin, *Chronological Tables of Chinese History* (University of Hong Kong, 1960), p.vi.
2. 易經.
3. 陰陽.
4. 四象,八卦.

sixty-four hexagrams. The principle of this is based upon the use of a scale of two, with <u>ying</u> -- for zero and <u>yang</u> —— for one. So we have the eight trigrams[1] as:-

pa-kua	☷	☶	☵	☴	☳	☲	☱	☰
binary number	000	001	010	011	100	101	110	111
tenary number	0	1	2	3	4	5	6	7

However, I-king though it is not a book on mathematics shows the first evidence of an interest in the permutation of <u>ying-yang</u> and the mathematics of magic squares. The <u>lo-shu</u>[2] in I-king takes the following form:-



4	9	2
3	5	7
8	1	6

Rod numerals were also developed in the middle of Chou. The system consists of the following numerals.

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1. D.E. Smith, <u>History of Mathematics</u> (Ginn and Company, Boston, 1951), p.26.
2. 洛書, <u>Ibid</u>., p.28.

	1	2	3	4	5	6	7	8	9
Units Hundreds Ten thousands	𝍠	𝍡	𝍢	𝍣	𝍤	𝍥	𝍦	𝍧	𝍨
Tens Thousands	𝍩	𝍪	𝍫	𝍬	𝍭	𝍮	𝍯	𝍰	𝍱

Each rod in a vertical position represents one and the horizontal rod represents five, while the rods in the columns of tens and thousands are arranged in the opposite way.[1] Thus, for example, the number 7836 will appear as 𝍯𝍧𝍫𝍥. These written forms of numbers coincide with the arrangements of calculating rods which were used in calculation. These rods were made of small bamboo or wooden pieces of two colours, red or black, representing the positive and negative.

The Chinese numerals which are still in use have derived from the above forms since 16th century. They are

〡	〢	〣	〤	〥	〦	〧	〨	〩	十	〇	千	万
1	2	3	4	5	6	7	8	9	10	0	1,000	10,000

As calculating techniques developed, a number of arithmetic books came to be written. The oldest important works are the Chou-pei Suan-ching[2]. Kiu-ch'ang Suan-ching

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1. Joseph Needham, Science and Civilization in China (Cambridge University Press, 1959), Vol. III, p.9.
2. Suan-ching (算經) means arithmetic classic.

(Arithmetic in Nine Chapters), _Sun-tsǔ Suan-ching_ and the _Hai-tao Suan-ching._[1] In _Ch'ou-pei_, the Pythagorean theorem was discussed. The _Kiu-ch'ang Suan-ching_ deals with calculations grouped in nine chapters under _Fung-tien_ (Squaring the farm), relating to surveying, _Su-mi_ (calculating the cereals), relating to percentage and proportion, _Shuai-fen_ (Calculating the shares), relating to partnership, _Shao-kuang_ (Finding length) relating to the finding of the sides of figures, square and cube roots, _Shang-kung_ (Finding volumes), _Chün-shu_, relating to motion problems, _Ying-pu-tsu_ (Excess and deficiency), _Fang-ch'eng_ (Equation) relating to simultaneous linear equations and determinants, and _Kon-ku_ relating to Pythagorean Triangles.[2]

Mathematical knowledge continued to grow in China as time passed. In Tang and some other dynasties the governments included mathematics in the vast national examinations for the election of civil officers. So the learning of mathematics had a place in education even in the early days in China. But owing to the lack of a public education system, mathematics learning could only be obtained by private coaching and was treated

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1. 周髀算經. 九章算經, 孫子算經, 海島算經.

2. D.E. Smith, _History of Mathematics_, Vol I., p.32.

as a part of the Chinese classical heritage rather than an independent science.

When modern education developed under the influence of the Western countries at the turn of the twentieth century, mathematics then became an integral part of the school syllabus. Mathematics study was more or less similar to that in the western schools in other countries except that there was always some Chinese flavour as in the use of problems which originated from the classical Suan-chings.

CHAPTER V

MATHEMATICS EDUCATION IN HONG KONG BEFORE 1960

Generally speaking, language and mathematics are the two subjects which have received most attention in Hong Kong schools. Chinese has a Mandarin based non-phonetic structure, with mastery of a great many characters required before continuous written work can be produced. It is read orally in a dialect, Cantonese in Hong Kong. Mathematics came to Hong Kong with the traditional mainland trappings of a heavy, involved syllabus. In Hong Kong this has been increased by the use of the British system of weights and measures, the increasing metrication of measurement, the adoption of the New Standard Chinese System (dating from the time of Sun Yat Sen) and confounded by the persistence of the original Chinese system, which, even though it is no longer taught in schools is in daily use in the markets.

Mathematics begins in kindergarten, where children are taught both to count and to write up to 100. Addition is also taught for adding numbers from 1 to 9. More formal written works are given in primary I. The work is heavy even at this stage. The following are some of the topics in the syllabus of primary 1:-

Subtraction of numbers up to two digits.

Relation between addition and subtraction.

Meaning of zero.

Understanding of Long measures (Both Chinese and British measures commonly used in Hong Kong).

Understanding of comparative lengths, weights, thickness, quantities and sizes.

Recognition of shapes (circles, triangles, squares and rectangles).

Counting of odd and even numbers up to 20.

Counting of 5's and 10's up to 100.

Counting backwards from 20 down to 1.

Composition and decomposition of numbers: 6,7,8,9 and 10.

Addition of three numbers, the sum being under 100.

Simple oral problems of addition and subtraction.[1]

Upon this base, calculation is soon extended to include multiplication, division and the manipulation of fractions. The numbers involved are usually large and problems are not always straight-forward. The function of mathematics learning is viewed both as a training of calculating skill and as a mental discipline. Therefore harder problems are given for the presumed purpose of training pupils to think. Throughout the whole primary

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1. C.Y. Ho et al., Primary Arithmetic, Book VII (Tin Li Co., Hong Kong, 1958), p.51.

stage, six to seven lessons on arithmetic are given a week with a lot of home work requiring about the same amount of time. Most teachers believe that practice perfects the skill. The hardship of school mathematics is mainly due to the following eight reasons:-

(1) Heavy mechanical calculations.

Calculation is not treated as a means to illuminate manipulation and to explain concepts so much as a means to acquire skill. The result is that rigid rules are given without much explanation and are followed by a large number of exercises aiming to drill the skill desired.

To support this criticism let me quote an example from a widely used text book[1]:-

Topic: Bracket and its application.

Rule: The numbers inside a bracket must be considered as one number, and should be calculated first; all other numbers outside the brackets should be dealt with only after all brackets have been removed, therefore they (the numbers outside the brackets) should all be brought down to the next step, without missing one.

This lengthy and not explicit rule is then followed by five illustrative examples which end up with another

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1. C.Y. Ho et al., Primary Arithmetic, Book VII (Tin Li Co., Hong Kong, 1958), p.51.

rule: In the case that addition and subtraction in an expression require calculation preceding multiplication and division, they (the numbers) must be put inside brackets.

To conclude the lesson a set of twenty-two exercises is given:-

1. 51001 + (785 + 3458) =

2. 72 - (24 + 6x5) + 9 + 10 =

. . .

. . .

21. (1440 + 3 + 24x5) + (7x3-15) + 50 =

22. (147 + 24) ÷ (15x6-29x3) + 5 =

From this example one can get the impression that this text book is nothing more than a collection of arithmetic rules and exercises. In fact most of the text books in use in Hong Kong have much the same outlook as this.

(2) Too many systems of measurements and currency are taught.

In Hong Kong a pupil has to learn three systems in every measurement -- the British system, the Metric system and the Chinese system[1]. The learning of these

1. The Chinese measurements are sometimes referred to as the "Standard systems" (標準制) which were created in 1919 to replace old systems. The "Standard systems" are closely related to the Metric systems with equivalent ratios 1, 1/2 and 1/3 in capacity, weight and length. But the introduction is only partly successful.

not only causes confusion but also lacks the support of daily experience. For the latter two systems do not exist in Hong Kong. In the Chinese market the traditional measurements are still in use. Yet this widely used system is not taught in school. Therefore a pupil who has learned three systems in school still does not know the system used in daily life.

(3) Problems are too difficult.

Chinese texts frequently contain problems which owe their existence to the classical Suan-chings and are beyond the pupils' ability to solve. Consequently a model solution is given to classified problems. Pupils are expected to memorize the method of solving each type of problem. Obviously the inclusion of these kinds of problems in text books has neither educational value nor practical purpose. For instance, in a Primary Six arithmetic text the following problem is found.[1]

There are certain things whose number is unknown. Repeatedly divided by 3, the remainder is 1; by 4 the remainder is 2; and by 5 the remainder is 3. What will be the number?

The problem first looks like the indeterminate

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1. C.Y. Ho et al., Primary Arithmetic, Book XI.

equations:

$3p+1 = 4q+2 = 5r+3$, where p,q,r are all natural numbers.

But the particular solution meant for this exercise is that when 2 is added to this number it will be divisible by 3,4,5 and hence the L.C.M. of 3,4,5 leads to the solution.

A well known classical arithmetic problem from Sun-tsu Suan-Ching resembles this. It is :-

There are certain things whose number is unknown. Repeatedly divided by 3; the remainder is 2; by 5 the remainder is 3; and by 7 the remainder is 2. What will be the number?[1]

Another type of typical exercise in the Chinese mathematics text books is the problem of "Chicken and rabbits". For example:-

There are some chickens and rabbits kept in the same cage. Taking the chickens and the rabbits as a whole, there are 18 heads and 48 legs. How many chickens and rabbits are there?[2]

A model solution is given in the book and "problems" of exactly the same structure follow, even to the extent of using the same words. In "solving" them, pupils need

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1. Mikami, loc. eit., p.32.
2. C.Y. Ho et al., Primary Arithmetic (Tin Li Co., Hong Kong, 1960), Book X, p.41.

no more than a substitution of numbers into the model supplied and can then carry out the mechanical calculation involved. The chance for reasoning is therefore denied, while memorizing the rigid method is encouraged.

All the other types of arithmetic problems are also classified. Among them are problems using the unitary method, Sum and difference problems, excess and deficiency[1] problems, planting trees problems, speed of boat sailing against or with the current problems, distance and time problems etc.[2]

(4) Heavy mechanical calculation.

As mechanical skill has been unduly emphasized, large numbers and complicated compound quantities, using many units at the same time are used in the arithmetic text books published in Hong Kong. The following are selected from a number of text books for primary III.

Examples: 1. 9850-59x60-4009 = ?[3]

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1. The Kiu-ch'ang Suan-ching has a chapter on Excess and Deficiency. 盈不足.
2. 和差.植樹.流水,行程問題.
3. Yan Dai Education Association, Arithmetic, Book V (Hong Kong, 1958), p.21.

2. 24000 taels = ? tans [1]

3. 88363 + 209 = ?

4. 7654 x 321 = ?

5. 1601921 - 1518493 = ?

6. 78469 chuen = ? Li ? chuen ? chek[2]

7. 43 tans 21 catties and 2 taels ÷ 12 = ?

8.

Yuen	Kok	Fen[3]
98	7	6
-67	8	9

(5) Subject matters bear little relevance to pupils' experience.

As mentioned above, the systems that the pupils learn are seldom used in daily life in Hong Kong. These compound quantities like pound, shilling, pence, stone, hundredweight, tan, meter and kilometer do not exist in their experience at all. Neither do the problems have real meaning nor practical use to a child. They are at most a sort of training for "mental discipline".

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1. *Ibid.*, p.45.

   Tael catty and tan are Chinese weights, where 16 taels = 1 catty; 100 catties = 1 tan.

2. Chinese length: Li 里, chern 丈, chek 尺, chuen 寸.

3. Chinese money; Yuen 元 (dollar), Kok 角 (coin = 10 cents) and Fen 分 (cent).

(6) Environmental restriction

The importance of having real experience in learning is most important. The lack of much chance to use the currency in shopping or to measure the length of a meter could reduce learning to meaningless rote. Having no chance to handle and play with concrete objects, or to observe through activities, pupils have to rely very much on bookish learning. Due to limited financial resources, and perhaps the lack of realization of the importance of the "concrete" approach, schools are seldom equipped for mathematics teaching. Visual aids are seldom found in primary schools and models, charts and film projectors are lacking in most secondary schools.

(7) The influence of examinations

Primary schools have to prepare pupils for the Secondary School Entrance Examination at the end of the primary stage. As the examination is highly competitive, those who can work fast have a great advantage. In this examination there are only four subjects, and therefore arithmetic weighs heavily on the result, particularly as the other subjects like Chinese and English have less variation in mark distribution. 60 questions are given in the arithmetic paper for 45 minutes time. Great speed and accuracy in mechanical calculation are therefore essential in this examination, so the primary schools vigorously drill their pupils for acquiring skills before the examination. Understanding of mathematics has to be sacrificed; discovery through activities is considered to be a waste of time.

(8) Heavy syllabus

The primary arithmetic syllabus in Hong Kong is heavy, for it includes all the rudiments in calculation, topics in percentage, interest, currency, graphs and the study of three measurement systems. The use of the abacus is sometime included too. Cramming of undigested knowledge has become a common phenomenon in Hong Kong schools.

Secondary Mathematics

Until recently mathematics teaching in Hong Kong at the secondary level was also somewhat unsatisfactory. The teaching tended to pay more attention to the logical arrangement and development of mathematical ideas in the abstract than to the utility of these ideas in actual life. Mathematics was, therefore, not treated as a science where an experimental and inductive approach was used. It was also not treated as an art where beauty of harmony and order are appreciated. It was looked upon as a set of rigid rules which have no other meaning except as tools to solve problems.

In the teaching of geometry, right from the beginning, the axiomatic approach prevailed. There was no stage preceding the logical deduction such as the "boy-scout geometry" for observing geometrical properties. The use of instruments and other simple apparatus was rare. Instead, pupils were asked to memorize the definitions and axioms and to proceed to formal proofs and difficult riders. The importance of having inductive experiment before deductive logic had not been realized by most of the teachers. Analysis of the links between hypothesis and conclusion was not pursued. Instead, the teachers only showed the synthesis of the proofs. Class teaching was formal. There was no oral discussion and no activity for discovery.

Adding to these difficulties, the text books in geometry, especially the Chinese texts, were written in a florid style, as mathematical treatises rather than books for children. They included large number of classical riders which even experienced teachers would be afraid to attempt. Failing to tackle the problems by their own efforts both teachers and students had to rely on key books. The content of the subject was unduly heavy, to the extent of including the complicated problems of the nine-points circle and Simpson's line. This kind of material was not quite suitable for most secondary pupils except, perhaps, a few gifted ones.

The teaching of algebra at that time was not much better than the teaching of geometry. Algebra was treated as rigid manipulations with which the finding of unknowns seemed to be the only goal. It was not regarded as a language where clarity of expression was achieved by employing symbols for quantities in operations. Concepts of relations and functions and laws of algebraic operations were ignored. The domain and range in which lies the choice of the use of integers, rationals or reals had not been made clear. The subject also suffered from the lack of an adequate introduction. The gap between arithmetic dealing with numbers and algebra dealing more with symbols had not been bridged. The change over from primary arithmetic to secondary mathematics was so abrupt that most students felt lost and needed years to recover their interest and confidence, if they ever recovered at all.

In trigonometry, too much time was spent on the identities. The properties of the circular functions were not discussed. Many texts emphasized too much the practical side, as if its main concern was surveying, and failed to present trigonometry as pure mathematics.

These three branches of mathematics were treated as three separate subjects put into water-tight compartments as if there was not and should not be any connection among them. They were then taught separately and their problems were dealt with by using mathematical methods restricted to the one branch of the subject according to where the problems appeared. In short, the possible unification of the various branches of mathematics was not foreseen in these days.

In the English schools the teaching of mathematics was further hindered by the language. Pupils in lower secondary classes, coming from Chinese primary schools failed to understand the explanations given by teachers and lacked the ability to read the texts themselves. Most pupils then found their relief by following the worked examples given in the text. The change of teaching medium together with the emergence of new mathematical terms was a source of great difficulty.

In the Chinese Middle Schools, the syllabuses followed those schools in China where mathematics study used to be more emphasized. The course there went beyond the G.C.E. Advanced level. The following are some items found in the syllabus for Senior Middle III:-[1]

Algebra:

Harder examples of permutation and combination

Probability

Binomial theorem extended to a negative index

Complex numbers up to De Moivre's theorem

Analytic Geometry:

Relation and transformation between Cartesian and Polar Coordinates

Parabola, ellipse, hyperbola and their parametric equations.

Asymptotes

Rotation of Axes

Central conics and their properties

Calculus:

Higher derivates

Maximum and minimum

Volumes of solids of revolution

Integration by parts

Approximate integration

Centre of mass

Moment of inertia

Length of arc

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1. Education Department of Hong Kong, Chinese Middle Schools Syllabus (Hong Kong, 1950).

Exponential functions and their differentiation and integration

The binomial, exponential and logarithmic series and their ranges of validity

Volumes of prisms, cylinder, cone, frustrum and sectors of spheres.

The study of all the above, together with some other topics, in one short year for every student sounds more like a dream than a practical aim. For so much had to be learned in so short of time, that the learning, needless to say, tended to be superficial and ill digested.

CHAPTER VI

THE REVOLUTION IN MATHEMATICS

The situation in Hong Kong, however, was not greatly different from that existing in other parts of the world. Very similar criticisms were made about the teaching of mathematics in England, even though attempts had been made in the immediate post war years to bring about a more unified approach to mathematics. Certain university examining boards offered single papers in mathematics, replacing the traditional papers of arithmetic, algebra, geometry and trigonometry. For Hong Kong, the added complications of language, confusion of measurement systems, inadequate buildings, equipment and staff, and the great pressure imposed upon students, just made the situation that much worse.

> "The changes in Mathematics in progress at the present time are so extensive, so far-reaching in their implications and so profound that they can be described only as

a revolution".[1]

These are the words of G. Baley Price in the Report of Regional Orientation Conferences in Mathematics of the National Council of Teachers of Mathematics in 1961. In the past one and half decades, a movement towards modernizing school mathematics has been occurring on a world wide scale. The impetus behind this move is the rapid development of modern mathematics and its application in various technological fields. Mathematics to-day is in many respects entirely different from what it was at the turn of the century. New developments have been extensive and new concepts have been revolutionary. Yet the mathematics taught to students in schools until just a few years ago was no less than 200 years' old. Mathematics has come to be regarded as a living language. It has been enriching itself so rapidly all the time by extensive researches and discoveries that it has become a new language in the last fifty years. Therefore the question is whether the schools should reform the

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1. G.B. Price, Progress in Mathematics and Its Implications For the Schools, The Revolution in School Mathematics (National Council of Teachers of Mathematics, Washington, 1961), p.1.

teaching of mathematics in order to cope with the changes or whether they should teach the subject as a dead language. In his introductory address to the Organization for European Economic Co-operation (O.E.E.C.) Marshall H. Stone of the University of Chicago said:

> "There are two major factors which require us to examine with fresh eyes the mathematics we propose to teach to young pupils in the secondary schools and in the first year at the university. One is the extraordinary growth of pure mathematics in modern times. The other is the increasing dependence of scientific thought upon mathematical methods, coinciding in time with a more and more urgent social demand for the services of scientists of every description.
>
> The forces exerted by these two factors on our educational system are quite clearly on the point of shattering the traditional

> framework of mathematical instruction and thus preparing the way for an overdue modernization and improvement of our teaching of mathematics."[1]

Many new branches of modern mathematics have been included in university mathematics courses. Thus a student who has completed the traditional mathematical education in a secondary school may very soon find the university course very difficult. There is a gap between the university and the secondary school. To bridge the gap, either the undergraduate schools of the universities or the secondary schools must reform, as Jean Dieudonne said:

> "In the last 50 years, mathematicians have been led to introduce not only new concepts but a new language, a language which grew empirically from the needs of mathematical research and whose ability to express mathematical statements

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1. H.S. Marshall, Reform in School Mathematics, <u>New Thinking in Mathematics</u> (O.E.E.C.,1961), p.15.

> concisely and precisely has repeatedly been tested and has won universal approval. But until now the introduction of this new terminology has been steadfastly resisted by the secondary schools, which desperately cling to an obsolete and inadequate language. And so when a student enters the university, he will most probably never have heard such common mathematics words as set, mapping, group, vector space, etc. No wonder he is baffled and discouraged by his contact with higher mathematics."[1]

The desirability of modernising the schools syllabus was expressed in a number of events. In 1957 J.M. Hammersley organized at Oxford a highly successful triangular conference of mathematics teachers from schools and universities and representatives of Industry. It was made aware that there was such a wide range of

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1. Ibid., p.34.

mathematics, modern as well as traditional, employed in industrial activities. In the United States of America there had been much discussion on curriculum reform. In 1958 Max Beberman's Inglis Lecture on "An Emerging Program of Secondary School Mathematics" outlined the feasible steps for the modernization.

Following this there emerged several schemes in the U.S.A., the one best known being the School Mathematics Study Group (SMSG). It was originally under the direction of E.G. Begle, and represents the largest united effort for improvement in the history of mathematics education. It is national in scope. The material produced for the texts is unique in that it represents the combined thinking of many people - psychologists, test makers, mathematicians from colleges and industry, and school teachers.

Sample textbooks and teacher manuals were written and tried out in 45 states by more than 400 teachers and 42,000 pupils[1] post 1959. Suggestions and criticisms were studied and analyzed and then shaped the revised new texts. This series of books contains new topics as well as changes in the organization and presentation

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1. The Revolution of School Mathematics, p.17.

of older topics. Attention is focused on important mathematical facts and skills and on basic principles that provide a logical framework for them.

The University of Illinois Curriculum Study in Mathematics (UICSM) began work for the change as early as 1952. Under the direction of Max Beberman experiments were conducted in 25 states involving 10,000 pupils from 1959. The scheme puts more emphasis on consistency, precision of language, structure of mathematics and understanding of basic principles through pupils discovery of generalization by the students in the basic technique.

In England the movement began as a result of the Southampton Mathematical Conference in April 1961. The conference was an exceedingly strenuous one which divided its time between studying the shortage of mathematicians and planning a school syllabus. As is well known the outcome of this conference has been the establishment of the School Mathematics Project under the direction of Bryan Thwaites. In its first sub-committee report on a new general mathematical course, for schools, it said,

> "In view of the great changes which have taken place during recent years in mathematics at university level,

> including those changes which are often considered collectively under the term "Modern Mathematics", there is a prima facie case for a critical look at the content of the present school syllabi . . .
>
> During the time that this research is taking place, there is a clear need for examining the way in which the traditional content of the syllabus is taught, with a view to inspiring in children something of the modern attitude towards the structure, pattern and beauty of mathematics."[1]

The committee then came to the conclusion that the unification of the course should be encouraged and examination with separate papers in arithmetic, algebra and geometry should be discontinued; that Set Theory, since it provides a common language for many branches of mathematics, should be utilized whenever appropriate,

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1. Bryan Thwaites, Southampton Mathematics Conference, On Teaching Mathematics (Southampton University Press, 1961), p.34.

even from the earliest years, and that clarity of thought and precise use of the mathematical language is vital.

In order to make room for modern mathematics such as vector algebra, matrices, number theory, set theory, permutations and groups, the committee suggested a substantial reduction in the geometrical content and the complexity of manipulation required.

The School Mathematics Project (SMP) was first carried out by eight grammar schools. Material was tried and then revised for the production of a series of text-books, which are now widely used all over England. Other projects in England are the Midland Mathematics Experiment and the Project of the St. Dunstan's College.

In the United States there are large number of Mathematics Teaching Projects. Among them are the University of Maryland Mathematics Project, the Ball State Teachers College Experimental Program, the Southern Illinois University Developmental Project in Secondary Mathematics, the Commission on Mathematics of the College Entrance Examination Board and the Secondary School Curriculum Committee of the National Council of Teachers of Mathematics.

Numerous mathematics modernizing programs have sprung up all around the world in the recent years. It would be difficult to find a place where mathematics teaching has not been influenced in some or other way by this movement.

The focus of emphasis and level of attention may differ from one project to another; some changes are more drastic in the exclusion of all plane geometry using an axiomatic approach, some retain all the traditional mathematics but present them within a new approach. In spite of all these differences, there are features common to all. One of the most important similarities is the existence of a unifying theme in mathematics. All the programs attempt to avoid the presentation of new materials as a string of unrelated topics. Such unifying themes are stressed through the use of set language, logical deduction, number systems, operations and their inverses, statistical inference and valid generalizations.

Not all the changes have taken place in schools. Undergraduate curricula have been changed, to accommodate the demands of the university syllabus to the entering abilities, and backgrounds, of freshman students. A notable early contribution in this field was instituted by the American Mathematical Association, and led to the highly popular first year university text of Kemeney et al.. The reform of mathematics education has been going on everywhere, and at all levels, primary, secondary, and tertiary. It has obviously not yet reached a final form. It is still subject to experiment, still in the search for the best presentation of the most desired aspects of mathematics.

In the next chapter the effects of these movements and changes upon the teaching of mathematics in Hong Kong schools will be examined in some detail.

CHAPTER VII

MODERNIZATION OF MATHEMATICS

TEACHING IN HONG KONG

It has already been noted that the movement for the modernization and improvement in mathematics teaching has become almost world wide. In the process one or two countries have seemed to reach pre-eminence, and one or two individuals, such as Beberman and Thwaites, have been acclaimed as world leaders. But several factors are at work, and the same combination of factors is not necessarily operative in each country. In general, progress in science and technology has created a demand for more individuals capable of making even faster changes. This in turn has placed emphasis upon recent graduates of the universities, particularly those who have reached some degree of research sophistication whilst still retaining energy and ideas. But the input into the universities, from secondary schools, has been insufficient in both quantity and quality to meet all the demands for expansion. Even more the out of phase nature of the two operations, school and university, has promoted unrest in each sector. Hence attention has been directed in both areas to the effect upon the subject matter of mathematics, at what is suitable to be taught, and at what level, but somewhat less upon the effect of the teaching upon the individual as an individual. Some discontent has remained local, other discontent has managed to gain support from the national and international level. In so doing it has been exposed to new thought concerning content, expressed as syllabi or aims, in methods of instruction appropriate to the new skills and to the individuals who are to learn them, and finally to the over all implications of the new mathematics for society. Consequently an attempt is made below to outline the changes which took place in Hong Kong, their nature, effects and directions as well as the participants and agents of change. The tale is far from clear and far from being complete.

The world wide movement of modernization of mathematics teaching soon influenced Hong Kong. The impact of the reform was brought in by visiting educators, students returning from abroad and from university staff who had spent sabbatical study leave in Europe and America. As early as 1957 the mathematics department of the University of Hong Kong under the professorship of Y.C. Wong, began to revise its syllabus in line with the modernization. Topics like set theory, topology and abstract algebra were included in the undergraduate mathematics syllabus. After 1960 elementary concept of set and logic, relations and mapping and the idea of algebraic structure and binary operations have been taught in form VI. The Matriculation Examination Handbook points out that the emphasis of the examination would henceforth be on the understanding of basic mathematical concepts and their application.[1] During the period Dr. K.F. Leung and Dr. D. Chen published their two-volume textbooks on elementary set theory for the sixth formers preparing for the advanced level examination, and opened a new era in the teaching of "new mathematics" in Hong Kong.

In 1961 professor Y.C. Wong attended the well known Southampton Mathematics Conference. After returning to Hong Kong he became more active in shaping the reform. The need to change the content in mathematics teaching in secondary schools to correspond to the change in the matriculation examination and in the university course

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1. University of Hong Kong, Handbook of the Advanced Level Examination (University of Hong Kong, 1968), p.16.

was discussed in the Secondary School Syllabus Committee. Representatives from the University brought with them the necessary knowledge, the intention and the methods of change to the committee. Through these contacts among the university, secondary schools and the Education Department, the stage for the reform was set. In 1962 the Hong Kong Secondary Mathematics Project Committee was formed from representatives of all three sides -- the university, the schools and the education department. The Committee under the chairmanship of Dr. S.T. Tsou, the Head of Mathematics Department of the United College of the Chinese University of Hong Kong, began to work rapidly. With the cooperation of some of the best mathematics educators, including the chief inspector of schools, the lecturers of the colleges of education, principals of secondary schools and some secondary school teachers, the committee soon produced a new syllabus and decided to test its suitability for teaching in secondary schools by means of an experimental project.

In September 1962, the experiment began in the Queen Elizabeth School in Kowloon with the cooperation of its energetic head mathematics teacher S.M. Tsui. Four Form I classes took part in the experiment. The general response both from students and their parents was encouraging. Material was written mainly by the teachers in the school, with suggestions from the Project Committee. Other secondary schools showed great interest and concern in this experiment, too. A year later two more secondary schools joined in the scheme. One of them was the largest secondary school in Hong Kong, the New Method College. It brought 3,000 participants to the experiment.

Meanwhile some education officers and government school teachers were sent to England and North America to learn from other countries' experience. Upon returning, they helped to promote the reform, in spreading the new idea, writing material and training teachers. In 1962 the present writer visited the centre of the School Mathematics Project in Southampton University and had the opportunity of discussing problems with the chief project coordinator Dr. A. G. Howson.

At the same time a group of enthusiastic teachers in Hong Kong gathered together to exchange views on teaching of "modern mathematics". Their regular meetings led to the formation of the "Mathematics Study Monoid". This group of teachers took up part of the work left undone by the Secondary School Mathematics Project Committee. Under the directorship of Dr. S. T. Tsou and members T. McC. Chamberlain, S. L. Ho, S.M. Tsui, H.H. Tsou, C.P.Poon, W. T. Poon and the writer, the Monoid produced two series of textbooks for both the secondary schools and primary schools of Hong Kong. Other textbooks published by projects all over the world are mainly for their particular national need and are not totally suitable for Hong Kong, so it is necessary to have our own local textbooks, incorporating the use of local examples and data for illustration in order to make mathematics more practical in life.

The new syllabus of mathematics produced by the Hong Kong School Mathematics Project Committee was approved by the Board of the Hong Kong School Leaving Certificate Examination as an alternate to the existing syllabus in 1964. Candidates then had the option of writing the examination in either syllabus. This approval was a green light to those schools which had decided earlier to take a wait and see attitude towards the reform. The battle for reform was finally won in 1964 when all the catholic secondary schools, regarded as the better organized and more prestigious schools, changed to teaching the "new mathematics".

The reforms carried out so far in the modernization of mathematics teaching in Hong Kong have been only a part of the world wide movement but have their own particular characteristics. As the chairman of the Hong Kong School Mathematics Project Committee, S.T. Tsou, said, the reform in Hong Kong was meant to be more radical than many similar projects in other places. Being sure that the trend of mathematics development is in this direction and having advantages of learning from the experience of other pioneer schemes, we could be more confident in pushing ahead with more drastic and ambitious plans.

As a rule, in the modernization movement, the major change in the new syllabus has been the increase in algebraic content, and, in particular, the introduction of the idea of structures. Set notation and language makes an early appearance and is used as the unifying element throughout the whole course. A major innovation is the study of Euclidean space by means of the geometrical transformations of rotation, reflection and translation. Transformation in space also introduces the use of vectors. The algebraic treatment of transformation leads to the study of matrices and gives an approach to groups. Euclidean systematic geometry is nearly all omitted, for the study of geometrical properties in Euclidean geometry can be achieved by the early study of analytical geometry and motion geometry, while the training of deductive reasoning by systematic geometry can be replaced by logic and algebraic deduction. Elementary statistics and probability have been included because of their wide use in the everday world. The reduction in the amount of arithmetical computation and algebraic manipulation serves to make more time available for the teaching of new topics.

The changes can be more clearly seen if one examines the "alternate syllabus" of the Hong Kong School Leaving Examination which introduces the following main topics to the traditional syllabus:-

1) Approximation and estimation, significant figures.
2) Scales of numeration, other than the denary.
3) Inequalities.
4) Calculation devices, including the use of slide rule.
5) Elementary concept and operations of sets.
   -- Union, intersection, complementary, subset, null and universal sets, Venn diagram with symbols.
6) 2 x 2 matrices -- the unit matrix, the formation of the inverse of a non-singlar matrix and applications to simultaneous equations and linear transformation.

7) Vectors.

8) Relationships, mapping, function, functional relation. One-to-one correspondence. Inverse function.

9) Linear programming.

10) Motion geometry - operations of reflection, rotation, translation and enlargement. Symmetry about planes, lines and points. Combination of transformations.

11) Simple probability - sum and product law.

12) Graphical representation of numerical data, calculation of the mean, median and quartiles.

Time for teaching the above is gained by omitting the following traditional topics:

Euclidean geometry.

Harder fractions and fractional equation.

Harder loci.

Geometrical illustrations of identities.

Extensions of Pythagoras theorem.

Apollonius theorem.

Harder identities.

Harder constructions.

Ceva's theorem and its converse.

Nine-point circle.

Homothetic polygons.

Partial fractions.

Binomial theorem.

The difference between the new and the traditional syllabuses is so great in contents that the reformed mathematics may be more suitably regarded as a new subject. Yet the emphasis of the reform is placed even more on the approach than the contents. The criterion for choosing a topic is the consideration of the ideas brought about by the topic rather than the immediate use of the topic. For example, through the teaching of sets, the concept of one-to-one correspondence, numbers and operations of numbers can be best explained. The subtle idea of variable can also be explained as an un-named element of a set. The use of letters to stand for variables gives the convenience of describing the general properties of known sets and can be extended to algebraic operations. The treatment of the material is indeed more important than the material itself in bringing out fundamental mathematics. The multiplication of numbers can be learned as a mechanical manipulation or can be used to illuminate a basic property. Consider the example 217 x 34 :-

$$\begin{array}{r} 217 \\ \times \quad 34 \\ \hline 6510 \\ 868 \\ \hline 7378 \end{array}$$

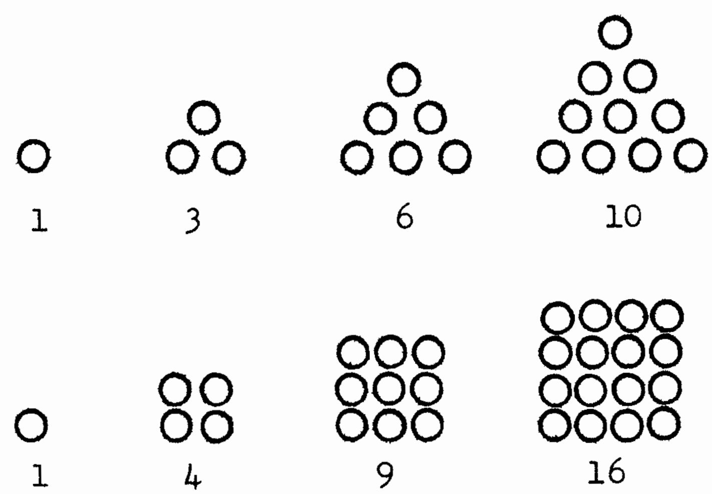
The string of operations involved may be used to show the correctness of the distributive and associative property. In this calculation students have chance to see that

$$
\begin{aligned}
217 \times 34 &= 217 \times (30 + 4) \\
&= 217 \times 30 + 217 \times 4 && \text{Distributive property} \\
&= 217 \times (3 \times 10) + 217 \times 4 \\
&= (217 \times 3) \times 10 + 217 \times 4 && \text{Associative property}
\end{aligned}
$$

We do not need to look for new topics to find examples of "modern mathematics". The basic structure of mathematics still remains but its exposition depends on the approach taken.

It is believed that the synthesis of conceptual learning and technical proficiency can only be achieved through the proper treatment of content learning, and practice will lead to technical proficiency in later applications. If however, we begin the other way about, with rote-learning of rules followed by practice unintelligently following them, it will be more difficult to acquire understanding, for once a learner knows how to work out an answer correctly, his desire to understand the calculation will lessen. Thus understanding must come first before technique.

In general the newly published textbooks on "modern mathematics" in Hong Kong do not lack this spirit. In most cases chances are provided for children to experiment before drawing a conclusion. Texts are written so as to lead to discovery and conceptual forming. In the Mathematics Study Monoid's book, for example, paper folding is used to introduce a number of geometrical elements -- lines, angles, parallelism and perpendicularity.[1] The arrangement of marbles in patterns leads to the study of figurative numbers -- triangular number, square number and rectangular number.[2]



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1. S.T. Tsou, Modern Mathematics for Secondary Schools (United Press, Hong Kong, 1965), Book I, Part I, pp. 1-3.
2. *Ibid.*, pp. 14-15.

There are, of course, other new textbooks in Hong Kong contributing to the improvement of mathematics teaching. These include Modern Mathematics for Middle Schools by C. Y. Wong[1], Sets and Logic by C. L. Tang and Modern Mathematics by Vincent Lo.

Some schools produced their own teaching material for their students in the form of notes. Some use several sets of textbooks including the local published books and the S.M.P. texts of England. Four years after first publication the Mathematics Study Monoid revised their texts, basing them on the revised syllabus and profiting from the criticism from schools where the earlier books had been used.

As more and more schools changed to teaching the new syllabus the demand for teachers able to teach the new topics become acute. To augment the supply of teachers qualified to teach modern mathematics, courses are organized by the education department, the colleges of education and the universities. Since 1961 the Northcote College of Education has begun to teach mathematics students a number of new topics which cover

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1. 中學新數學, 黃卓然.

a large part of the secondary school new syllabus. These topics include the following[1]:

Topics	1st year	2nd year
Set	4 weeks	2 weeks
Number systems	4	2
Coordinate geometry	6	
Higher algebra and Trigonometry	3	
Symbolic logic	4	
Statistics	3	2
Differential and integral Calculus	4	3
Matrices		4
Algebraic structures		3
Geometrical transformation		3
Vector and vector space		2
Linear programming		2

Through the two year course mathematics lectures are given twice a week together with tutorials supplementing

1. Two-Year Mathematics Course Syllabus, Northcote College of Education, Hong Kong, 1969.

the lectures. The rest of the time is devoted to lectures and discussion on mathematics teaching methods (about five weeks in each year) and teaching practice (10 weeks each year). The other two colleges of education -- the Grantham College and the Sir Robert Black College have similar courses for the training of mathematics teachers to cope with the reform. In 1963, the mathematics section of the Hong Kong Education Department started a course for in-service training of mathematics teachers preparing for the change. This course meets two evenings a week for a period of one year. Since then many teachers have received some elementary training in this respect.

Meanwhile, the two universities have been offering courses on these modern topics of mathematics designed specially for teachers, in their extra-mural departments. A diploma course on the teaching of modern mathematics in secondary schools, designed for practising teachers, was offered in 1967-68 and 68-69 by the Chinese University of Hong Kong. This course, taught by S. T. Tsou, S. M. Tsui and the writer, dealt with main new topics in more detail and depth in its 130 hours of lectures. As most of the teachers in this course are university graduates and are teaching in secondary schools, it has received more credit and prestige in providing better qualified teachers specially for the higher forms.

In 1968, a third year specialist mathematics course was added at the Northcote College of Education. Entrants to this course are graduates of the two-year course in one of the colleges of education. Half of them are recent graduates selected directly from the mathematics students in the final year of their two year training. The other half are selected from the serving mathematics teachers who are recommended as worthy of further training at government expense so that they would be more prepared to teach the higher level mathematics in secondary schools. These serving teachers are released from their teaching posts with full salary to attend the course. The student teachers directly promoted from the two-year course are given annual allowance of $2,400 for the course and also a $1,200 interest free loan if required. As a reward, they will enter the salary scale two steps higher on return to employment.

In this third-year specialist course, students devote all the time to the academic study of mathematics.

The topics studied in the two-year course will be extended to higher level.

The following topics are included:-

Symbolic logic, Boolean algebra.

Sets, algebra of sets; countable and non-countable sets.

Relations and mappings; equivalence classes, partitioning of sets.

Systems of numbers, elementary number theory, modulo arithmetic.

Polynomials.

Complex numbers. De Moivres theorem.

Roots of Complex numbers, the form $r(\cos\theta + i\sin\theta)$.

Circular functions.

Coordinate geometry 2 and 3 dimensions, conic sections.

Geometrical transformations; reflection, rotation, translation, shearing and enlargement. Symmetry.

Vector and Vector algebra.

Matrices. Matrix algebra. Elementary transformations of matrix.

Inverse matrices. Reduction to echelean form. Application to system of equations.

Sequences and series, Convergence. Continuity.

Differential and integral calculus.

Algebraic structures. Properties of group, ring and field.

Kinds of geometries including intuitive topology.

Statistics: measure of spread. Normal distribution. Test for significance and simple correlations. Probability;

addition and multiplication law.[1]

This intensive study of mathematics should produce teachers capable of taking the specialist posts in secondary schools and, in a way, it may be taken as the second road to degree study. It is hoped that the best students from this course may succeed in the external degree examinations, in part or whole, in mathematics subjects. One third of the 1969 graduates have registered for the external degree examination of the University of London, and another third have shown their intention of doing so.

In recent years it has been realized that to make a good teacher it is more important for one to pursue academic study as high as possible in order to ensure complete understanding of the subject rather than to receive a set of cook book type rules in teaching. The reform of the mathematics syllabus makes this truth more clear. Without the thorough understanding of mathematics one could never be a good mathematics teacher. Signs show that the colleges of education in Hong Kong are making changes in accordance with this realization.

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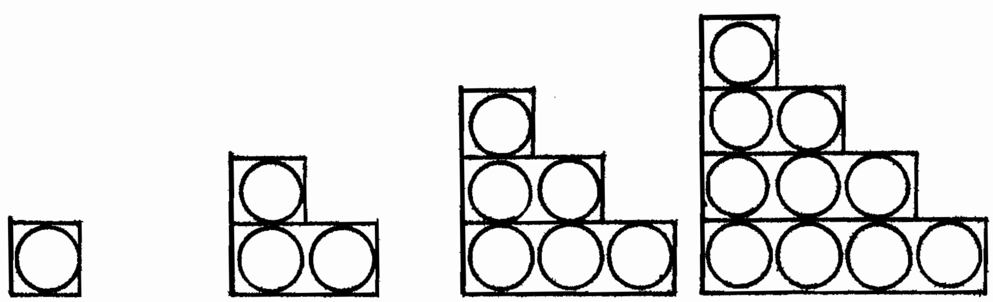
1. Mathematics Syllabus for the Third Year Specialist Course, Northcote College of Education, Hong Kong, 1969.

The modernization movement of mathematics teaching in Hong Kong has been well propagated in a number of conferences organized by the mathematics school inspectors of the education department, with the assistance from colleges of education and other areas. In the summer of 1966 a four-day conference on the teaching of mathematics was held in Grantham College of Education. Four hundred teachers and school administrators attended this conference. A wide range of topics was treated in lecture and discussion on the reform of mathematics teaching -- from the aim of teaching mathematics, the recent development of mathematics education, to textbooks and class teaching. Due to the success of this conference and the desire to improve mathematics teaching, similar conferences have been organized every succeeding summer.

In 1969 the renowned mathematics educator J.B. Biggs visited Hong Kong. She was invited by the education department to give a series of lectures and to conduct a project for training teachers to use the experimental approach in teaching primary mathematics. With the assistance of many mathematics inspectors, and the students in the mathematics specialist course in Northcote College of Education, hundreds of teachers from all the large primary schools took part in the one-week training experience. The main theme of the project was to show how simple experiments using measurement, counting and observation with simple everyday life objects could lead to the discovery of mathematics and to reveal the relationship of variables.

Efforts to improve primary school mathematics teaching first came in 1965 when the education department decided to experiment in teaching with structural apparatus. Four government primary schools and one subsidized school started the project with about 500 primary I pupils. The Hong Kong education department entered into a contract with Seton Pollock to translate and to rewrite the textbooks on Colour-factor mathematics in Chinese for the use of Hong Kong schools. An officer in the mathematics section of the education department was assigned for the work. He also supervised the project. Although it is too early to conclude whether the use of structural apparatus in Hong Kong can bring better results in the learning of primary mathematics, there are signs showing that pupils have adopted a more favourable attitude towards the learning of arithmetic. Cuisenaire rods have been in use in several primary schools too. However the lack of a Chinese text and the high cost of the apparatus prevent a wider adoption beyond the scale of a sponsored experiment.

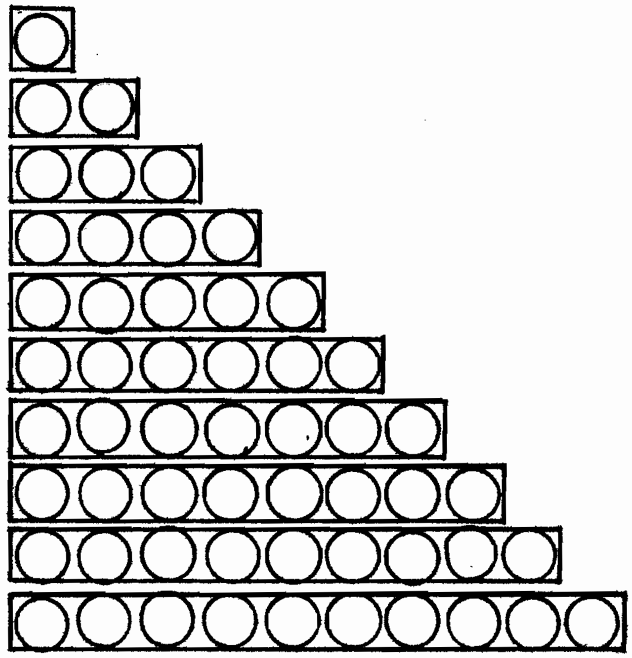
In 1966, the writer and his colleague S. L. Ho began to conceive a multi-model method for teaching primary mathematics. A year later the publication of a series of textbooks for primary schools was begun, using a combination of rods, blocks and a number of geometrical shapes refered to as the Groupoid apparatus. It is believed that such a model will be more versatile in demonstrating the structure of mathematics. For understanding numbers and their composition (like $5 = 1+1+1+1+1 = 1+4 = 2+3 = 3+2 = 4+1$) rods are used. For the learning of numeration, number bases and volume blocks are used. Playing with rods leads to the recognition of the figurative numbers. For instance the triangular numbers are obtained by arranging the rods in the following way:



1 3 6 10

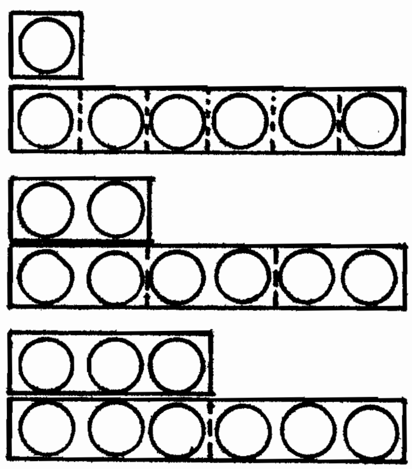
The Groupoid colour rods consist of the following 10 pieces:

紅	red
橙	orange
黄	yellow
綠	green
青	light-blue
藍	blue
紫	purple
褐	tan
灰	grey
黑	black



The lengths are from 1 centimeter to 10 centimeters. The colour assigned to each rod is in the Chinese order of naming colours. No mathematical implication attached to the use of colour as it is doubted whether children would see the weak relation between the composition of primary colours and the composition of a number with primes as claimed by Seton Pollock. Nor is any advantage perceived in classifying the numbers in families distinguished by colours as in Cuisenaire. In fact, the advantage for colouring the rods only lies in the convenience of abstraction, when there are names, and hence symbols, to refer to quantities.

The adding of small circles in the rods will provide a chance to see the patterns of numbers. The number of circles in a rod should not destroy its "oneness" and hinder the representation of an algebraic quantity other than the number indicated by the circles. For instance, the blue rod has 6 circles and is 6 when red is used as unit, but will be 3 when it is measured by the orange rod and is 2 when yellow rod is the unit. Thus children will see:-



Unit

6

Unit

3

Unit

2

This will bring out the relationship of numbers and will explain operations more easily than the rods without a number indication. In fact, there is a proposal to return to the idea of using "Tellich Bricks". It could be that the colour rods apparatus developed from the "Tellich Bricks" have sacrificed the basic advantage of showing number relation for the claimed advantage in algebraic abstraction. Such sacrifice may not be justified, and the gain in algebraic skill is doubtful. Again, the learning of numbers and relations should be the first concern at this stage when apparatus is used.

The books seek to emphasize conceptual learning by introducing the idea of sets, one-to-one correspondence and matching. The use of apparatus is mainly for concept forming at the early stage and does not provide a complete arithmetic programme. Once the concepts appropriate to each stage have been attained, it will be necessary to drill them and apply them.

The emergence of these textbooks happily coincide with the publication by the education department of a new primary mathematics syllabus. For the teaching of "modern mathematics" in secondary schools called for a corresponding change in primary mathematics. The new syllabus drafted by the Syllabus Committee has changes in two respects; the shift of emphasis from arithmetic skills to concept learning and the shift from teacher's domination to pupils learning through their own activities. In the introduction to the syllabus, the Committee wrote,

> "New concepts should be introduced, when possible, through practical activities where children have the opportunity of discovering the concepts and relationships. Whilst some interesting facts could be used as illustrations in developing a new topic others should be left to be discovered by the pupils through questioning and not merely enumerated by the teacher or in the text."[1]

It further emphasized the importance of the role of activities for understanding. It said,

> "Particular emphasis should be laid on learning through practical activities. In no case should any emphasis be placed on computation with large numbers."[2]

1,2. Education Department, Suggested Syllabuses For Primary Schools (Hong Kong, 1967), p.2.

Other suggestions are that mental drill should be regularly given, that lessons should be planned to maintain interest of pupils and, in particular, that the use of structural materials for teaching, especially in the lower classes, should be encouraged.

With the same spirit, the committee further suggested the inclusion of the following items for primary school mathematics:

Pre-number concepts.

Relationship between operations.

Study of simple geometrical shapes.

Introducing algebraic symbols.

Statistical charts, means.

Other numerals.

Number patterns.

Simple Venn diagrams.

Intuitive geometry through paper folding.

Curve stitching.[1]

Thus the stage for improving mathematics teaching in Hong Kong at all levels has been set. Although the process of change could be a long one, as there are problems related to teacher's retraining, textbooks'

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1. Ibid., pp. 3-16.

writing and contents selecting, the movement is well started. To help to evaluate the general attitude and progress and to collect information, the education department has been investigating the case of reform since 1966. Questionnaire forms are sent out to all secondary schools every year. The main items for investigation are the intention of adoption of the new syllabus; the preparation for the change with respect to teachers and materials; and the difficulties encountered if such change has been taking place in school.

The returned statistics have been analyzed. In 1968 it showed that there were 76 Anglo-Chinese secondary schools teaching the alternate syllabus out of about 200 total of such schools in Hong Kong.[1] As most of these 76 schools were the larger schools, the number of students learning the new material was already well over half of the student population in the age group 12-13. Of the schools teaching the alternate syllabus three out of four are using the textbooks published by the Mathematics Study Monoid. Again about one half of those schools teaching traditional mathematics indicated that

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1. A report on education news, Wah Kiu Yet Po, Hong Kong, 1968.

they would change to teach "new mathematics" within one or two years, when preparation for the change was completed. The rest of them were more conservative, their responses ranged from suspicion to open hostility to the change.

From personal interviews with members of the Inspectorate it is possible to place in some order of importance the kinds of difficulties cited by those schools engaged in teaching the new mathematics. In a roughly descending order of frequency these are as follows:-

1. Shortage of qualified mathematics teachers able to teach the new mathematics.
2. Shortage of reference books, particularly books in Chinese.
3. Exercises in the newly published textbooks are too few for drilling to acquire skills.
4. Schools lack of facility for the teaching of some of the new topics, in particular, in teaching calculating devices.
5. The preparation of teachers in teaching "new mathematics" requires too much time.
6. Language difficulty poses a barrier for students to read the text by themselves.

7. Difficulty of students on transferring to schools or classes where the mathematics programs are different.
8. Heavy syllabus.
9. Parents' opposition.

In the early years of change such difficulties are understandable. It is hoped that many of them will disappear in time. However, there are four major obstacles to the modernization and improvement of the teaching of mathematics in Hong Kong which are probably endemic. These are:-

(1) The Chinese tradition of the great authority of teachers would prevent a real use of child-centered discovery method, which is the gist of the modernization. Pupils are so used to looking up to their teachers for information that they fear to venture to draw their own conclusion. There is evidence that the "new" material is taught by the "old" method.

(2) Schools in Hong Kong are in general not well equipped, nor are funds available to spend on materials for teaching. The lack of facilities makes the teaching of some modern topics difficult. For instance, in the

teaching of calculating devices, there is no desk calculator, slide rules. There is even no special room for activities. Classes are usually large, crowded with 40-45 students sitting in rigid alignment. Desks are fixed and students have no room to move around. Without improving the environment the talk of experimental approach and teaching by activities is nothing more than wishful thinking. Worst of all, this situation will remain unchanged in the near future as Hong Kong will be preoccupied by other needs, considered more urgent.

(3) There is misunderstanding of the aims of teaching modern mathematics. Many think that it is solely to serve the needs of modern technology in this space age. A typical example concerns the learning of binary numeration. Because such knowledge has been useful in the circuitry of computers it is concluded that binary numbers and binary numeration are taught solely for this use. In fact, input into computers is still denary, the conversion to binary form being performed within the machine. Such misunderstanding has been used as a means of resisting change, particularly as Hong Kong has not yet reached an advanced stage of technological

application of mathematical knowledge.

(4). The official body to direct the reform is the Hong Kong School Mathematics Project Committee. The work of this committee is mainly concerned with conceiving plans and making recommendation for reform rather than putting ideas into action, and there is no other working party to execute the ideas. Reform in schools is not well coordinated; there is no center of guidance to direct the changes. When difficulty occurs, the teacher has no one to look to for advice. The mathematics inspectors in the education department are mainly concerned with routine administrative work, and advice regarding academic study is not usually given. Hence the reform in individual schools is left to the initiative of the teachers.

In spite of all the difficulties mentioned, the reform in Hong Kong will go ahead as it has done in other parts of the world. There is no reason to doubt that the reform will not finally produce the expected fruit. The modernization of the teaching of one subject is indeed only a part of the modernization of education as a whole which is being continuously guided by evolving educational theories and practices.

PART III

MATHEMATICS ACHIEVEMENT IN HONG KONG

CHAPTER VIII

THE INTERNATIONAL PROJECT FOR THE EVALUATION OF EDUCATIONAL ACHIEVEMENT

Only two decades ago comparative education consisted mainly of separate descriptions of educational systems in various countries. The change from a descriptive to a more analytical emphasis began when attempts were made by certain comparative educators to juxtapose systems for closer comparative study. At about the same time the postwar rapid development of technology had aroused great concern in educational planning in many countries, particularly the United States which was perturbed over the achievements of Russian technology, which led to the launching of Sputnik. The need of a society to plan ahead to cope with its future development became quite clear. Its acceptance, in some degree, followed. The study of educational systems could more profitably be related to educational planning in the countries involved.

The change in the fifties was marked by the publication of many encyclopaedic works of world surveys of education, by a number of institutions, including U.N.E.S.C.O. and the International Bureau of Education. More qualitative analyses, supported by statistical data, comments and interpretation were seen. This phase of change was described by Bereday as the juxtapositioning phase.

Later the Unesco Institute for Education in Hamburg which has been a meeting center for the world educationalists became the fermenting ground for international studies. Through the contacts of educationalists from many countries came attempts to establish evaluation techniques which could be applied to the measurement of cross-national educational aspects. Such kinds of large scale measures have required co-operation from many individuals from many countries. The idea of setting up an international organization for the cross-national study of educational development was conceived.

At the same time the founding of the center for Comparative Education at the University of Chicago added a force in international educational research in this direction. Anderson[1] argued in favour of giving high priority to research on cross-national achievement in education. He pointed out that trying to discuss the

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1. C.A. Anderson, "Methodology of Comparative Education", International Review of Education, Vol. VII (1961), no.1.

influence of various hypothetical independent variables would be futile until measures of dependent variables had been obtained. This requires quantitative analyses which can partly compensate for the scarcity of information about the outcomes of educational systems. To further progress in our understanding of these matters, some attempts at model building are necessary, and it has been realized that certain basic problems in education cannot be successfully tackled unless one tries to utilize cross-system variability such as differences in input, process and the product.

The feasibility of setting up a permanent administration so as to carry out an international survey was discussed in the Hamburg meetings. It led to the formation of the council of the International Project for the Evaluation of Educational Achievement (IEA) which was incorporated as an international scientific association in 1967.

The IEA decided to carry out a detailed study on mathematics achievement as its first pilot project. The choice of mathematics as the first study coincided with a growing concern with improving mathematics teaching to cope with the technological development, at the time when many countries were involved in re-examining the curricula, methods of teaching and the effects of the introduction of the "new mathematics". Furthermore, the relative large common denominator in the content of school mathematics in different countries would make the survey more comparable.

The aim of this venture was the study of the effects of the differences among the school systems of the world and the differences in the achievement, interests and attitudes of the students. The diversity of educational practices in different countries may make it possible to reveal the relationships between educational achievement and societal factors. Such a finding would then throw more light on a number of hypotheses relating education to socio-economic background, and provide guidance in educational policy-making.

In 1962 the work of the survey of mathematics achievement in twelve nations began. These participating nations were Australia, Belgium, England, Federal Republic of Germany, Finland, France, Israel, Japan, The Netherlands, Scotland, Sweden and the United States. Canada did not participate, although invited. The administrative and technical coordination services were mainly provided by the Unesco Institute for Education.

Professor Husén of University of Stockholm was elected as the Chairman of the IEA council to direct the survey. H.H. Stern and T.N. Postlethwaite were in turn the co-ordinators of the project.

In order to ensure representation of the target population in the test, great caution was taken in the method of sampling. Each national research center appointed a sampling expert for its own country. The target populations were the 13 year old students and the pre-university grade, since they represent the two major terminal points of education in each country, namely, the last point where approximately 100% of an age group was still in full-time schooling and the termination of formal schooling where the product of secondary education may be assessed. These populations were further subdivided as follows:-

Population 1a:- All students who were aged between 13:0-13:11 year at the date of testing.

Population 1b:- All students at the grade level where the majority of students of age 13:0-13:11 were found.

Population 3a:- Those studying mathematics as an integral part of their course for their future training at the pre-university level.

Population 3b:- Those studying as a complementary part of their studies in the pre-university level.

The test was constructed after detailed analyses of school mathematics contents in the countries for each target population. Items were selected and organized into nine one-hour test units - A,B,C,4,5,6,7,8,9. The following table shows the composition of these tests:-

Table 8.1: Summary of Content of Tests for Different Populations[1]

Topic	Popn 1 (A,B,C)	Popn 3a (5,7,8,9)	Popn 3b (4,5,6)
Basic arithmetic	13		3
Advanced arithmetic	18	3	9
Elementary algebra	12	1	5
Intermediate algebra	4	19	13
Euclidean geometry	13	5	13
Analytic geometry	1	8	5
Sets	4	4	4
Trigonometric and circular functions		3	3
Analysis		8	1
Calculus		9	
Probability		1	1
Logic		8	1
Affine geometry	3		

1. T.N. Postlethwaite, "IEA Mathemtics Study", International Review of Education (Hamburg, 1969), Vol XV, No.2, p.139.

The overall objective of the test was to arrive at internationally valid measuring instruments covering a wide range of content and objectives, within which each country could find its own emphases. Apart from measuring mathematical achievement, descriptive information of the teaching, of the school organization and of the student was also collected. To collect this relevant information about these environmental factors questionnaires were constructed separately for students, teachers and schools.

The students questionnaires include questions on students' background yielding such information as grade, sex, age, size of mathematics class, amount of mathematics instruction and home work, father's and mother's occupation and education, aspirations, and expectations for further mathematics, further schooling and future occupation, best-and least-liked subjects, examination(s) taken and extracurricular mathematics activities. The teacher's questionnaires collected information about professional training, experience, refreshed mathematics courses and the degree of freedom in teaching. The information requested from schools was mainly organizational, such as the type of school, enrolment, size of staff, number of trained mathematics teachers and school expenditure. These case-study questionnaires have collected valuable quantitative and qualitative data for determining the role of each environmental factor in the learning of mathematics.

The test was well administered and completed in 1964. All together 132,775 students from 5,348 schools were tested and questionnaires were filled in by 13,364 teachers and 5,348 headmasters.[1] The result of the major findings were analyzed and published in the two-volume book edited by T. Husén, the Chairman of the project, with the title International Study of Achievement in Mathematics -- A Comparison of Twelve Countries.

The results of the test show that the mean achievements of the twelve countries differ a great deal. The range among the participating countries for the 13-year-old population, for example, is greater than one standard deviation of the combined distribution, which indicates that those who are much above the average in one country might be regarded as mathematically rather backward in another country. In the la population Japan has the highest mean (31.2) while Sweden has the lowest mean

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1. Torsten Husén, (ed) International Study of Achievement in Mathematics. (Stockholm, 1967), Vol I, p.53.

(15.7). However the productivity of an educational system of a country and the effect of the instruction cannot be assessed solely from national means. There are too many variables contributing to the achievement, such as the age of school entry, the difference in selectivity and the contents of the subject. Variability among student achievement is also marked. In the la population the standard deviation is almost twice as large in England as in Finland.

Mathematics scores were correlated with 45 independent variables characterizing the school, teacher and student. It was found that the length of teacher training, parents' education and occupation were strongly and positively related to achievement, whilst students' interest in mathematics and aspiration for further education were also highly correlated with their scores. The amount of information collected in this survey is enormous. The bank of data will provide material for educationalists to analyze for a long time to come.

CHAPTER IX

THE IEA MATHEMATICS TEST IN HONG KONG

In order to evaluate the effect of a number of environmental variables on the learning of mathematics in Hong Kong, the IEA test was adopted as a pilot test in an attempt to collect information for both an internal analysis and an international comparison. It is obvious, of course, that a survey on a scale as large as that undertaken by the IEA could not be done by individual effort. It was decided to concentrate the enquiry on one student population rather than four, and to restrict the investigation to fewer but, hopefully, more important aspects of the learning of mathematics in Hong Kong. The problem therefore became one of constructing a new test, retaining the essence of the IEA test for a given population, making it possible to compare Hong Kong with the twelve nations of the IEA enquiry, and at the same time within the competence of a single investigator, the present author.

Population 1b was chosen, and the corresponding IEA test. This was a test of 70 questions divided into three separate papers and applied to a class of students, the majority of whom were expected to be in the thirteen year old age range. To reduce the time required of the co-operating schools in Hong Kong it was further decided to present one paper of thirty five questions and to have a single time limit of one and a half hours instead of three. At the same time it was necessary to use all 70 questions so that some information, in terms of each question, would be available for comparative purposes.

The original test has three papers labelled A,B,C. Three new papers X,Y,Z were constructed. Some of the original 70 questions were allocated to each of X,Y and Z. Some were common to papers X and Y, some common to X and Z and some common to Y and Z. In the allocation of questions a study of their "uniqueness" was first made. A question classified as unique, in the sense of the knowledge required and the technique necessary for correct solution, should be included in all three papers. In this way every student in the Hong Kong sample would have a chance to be exposed to this item. Again, there may be an area of knowledge in which more than one question exists. Such a case occurs with the addition of fractions, where three questions B1, C3 and A4 are available. One of these is allocated to each of the papers X,Y,Z. B20, on analytical geometry is judged unique, the only one in any of A,B and C, and so is to be found in the three papers X,Y and Z. Continuing this process, seven questions were common to all three papers, twenty one questions were available to be distributed between two papers, whilst the remaining forty two of the original questions were allocated among the three papers X,Y and Z. In terms of topics and questions we have the following:-

1. Questions common to all three papers.

   B21 Binary number.

   B5 Closure of addition and multiplication of integers.

   C19 Set.

   B20 Analytic geometry (Equation of line).

   B7 Approximation.

   C17 Set of integers.

   A18 Problem where knowledge of set may be applied.

2. Questions common to two papers:-

Questions	Knowledge	X paper	Y paper	Z paper
B 6	Percentage	X	X	
C 16			X	X

Questions	Knowledge	X paper	Y paper	Z paper
A 6	Square root	X		X
A 9			X	X
C 4	Ratio	X		X
A 20	Simultaneous equation	X	X	
C 9			X	X
B 11	Simplification on Algebraic expression	X		X
B 13		X	X	
B 15		X		X
C 21	Algebraic problem		X	X
A 21	Elementary geometry	X	X	
B 18			X	X
A 16		X		X
C 22		X	X	
C 11			X	X
B 12	Directed number	X	X	
A 3	Use of blacket	X		X
C 1	Large number		X	X
C 10	L.C.M.	X		X
B 22	Fraction	X	X	
	Total	14	14	14

3. Questions used in only one paper:-

Of the remaining forty two questions, fourteen appear in each of the three papers. Thus the structure of a paper is therefore 7 items common to each of three papers, fourteen items which are found in one other paper, and fourteen items to be found only in that particular paper.

The final arrangement was as follows:-

Table 9.1: Arrangement of test questions

No.	X paper	Y paper	Z paper
1	A2	C1	A2
2	A3	B2	B1
3	B3	C2	C1
4	C3	A4	A3
5	C4	B4	C4
6	A5	B5	B5
7	B5	B6	C5
8	A6	C6	A6
9	B6	A7	B7
10	B7	B7	B8
11	A8	C7	A9
12	B10	C8	B9
13	C10	A9	C9
14	A11	C9	C10
15	B11	A10	B11
16	B12	C11	C11
17	B13	B12	A12
18	C13	B13	C12
19	A14	B15	A13
20	B15	A15	B14
21	A16	C16	C14
22	B16	A17	C15
23	B17	C17	A16
24	C17	A18	C16
25	A18	B18	C17
26	A19	C18	A18
27	C19	C19	B19
28	A20	A20	B18
29	B20	B20	C19
30	A21	A21	B20
31	B21	B21	C20
32	A22	C21	B21
33	B22	B22	C21
34	C22	C22	B23
35	C23	A23	B24
Average difficulty	.398	.399	.399

The papers finally selected should, of course, be of the same order of difficulty as that of the original enquiry. In the allocation of questions to papers this fact was noted, and the average level of difficulty of each paper was calculated. They are 0.398, 0.399 and 0.399, in comparison with an average difficulty of 0.43 of the original A,B,C papers. (This figure is based upon averaging of normalized proportions expressed as deviations before averaging. The Direct averaging of the difficulty indices gave the figures for papers X,Y and Z). The papers presented to Hong Kong children are no easier than those presented in the twelve nations.

The questionnaires designed for students and teachers were simplified and modified to suit a Hong Kong environment. Information about the students concentrated on a few more important points such as age, sex, homework, fathers' occupation and education, economic background, aspiration of education, interest in mathematics and the length of time spent learning "modern mathematics".

The teacher questionnaires also collected information about the school. Questions were asked on the type of school enrolment, size of class, time for mathematics lessons per week, number of periods taught by teachers, training of teachers - in particular the training for teaching new mathematics topics - experience, textbook and extra-curriculum activities.

Test papers were also translated into Chinese to guarantee thorough understanding. As most of the schools in the test were Anglo-Chinese schools where English was used as the teaching medium, all schools were given test papers in English but with a Chinese version attached. It was reported later that only in Chinese schools were such translated versions needed.

After this preparation, the test papers and questionnaires were printed and mailed to Hong Kong in February 1970. The test was applied in the sample of Hong Kong Schools in March 1970 under the direction of the mathematics department of the Northcote College of Education in Hong Kong. The administration of the test, the invigilation, the marking and the preparation of the data for item analysis was carried out by selected students from the third year specialist mathematics course. These students were trained and directed in this task by the head of the Mathematics department of the College, Mr. S.L. Ho.

Sampling

The fourteen participating schools were selected on the basis of their enrolment, and known characteristics. The selection was made by the author, but the composition was confirmed by his colleagues, and it is believed that it would be confirmed by other competent authorities among the inspectorate, for example. There are urban and rural components, English and Chinese, and grammar and technical schools within the sample. The composition of these schools is given in table 9.2.

At the level of the individual school the attempt was made to secure a sample of approximately 30 students from each school. The selection of this sample within a school was made by the mathematics teachers of the school in consultation with the mathematics department of Northcote College of Education. At this distance it is not possible to say with certainty that each within-school sample was fully representative of the school; it can only be said that this point was emphasized in correspondence with colleagues in Hong Kong who carried out the field testing. In this respect the "absentee" individual investigator is at a great disadvantage when compared with those in a more prestigious international enquiry. However, the compactness of Hong Kong, the strictly observed common syllabus, the homogeneity of the various areas in terms of status and population characteristics all increase the likelihood that the sample of fourteen schools, and the within sample populations may be treated as representative of the target population of the classes where one may expect to find the majority of thirteen year old children.

Table 9.2: Classification of school samples

School	Classifications											
	Teaching medium		Financial Support			Type of education		Location		Sex		
										Single Sex		Coeducat-ional
	English	Chinese	Government	Subsidized	Private	Grammar	Technical	Urban	Rural	Boy	Girl	
Belliois School	X		X			X		X			X	
Choi Hung School	X			X		X		X		X		
Confucian School		X		X		X		X				X
Ho Tung Tech. School	X		X				X	X			X	
King's College	X		X			X		X		X		
Kowloon Technical School	X		X				X	X		X		
Maryknoll School	X			X		X		X			X	
Queen Elizabeth's School	X		X			X		X				X
Shaukiwan School	X		X				X	X				X
Tai Shing Middle School					X	X		X				X
True Light Middle School		X			X	X		X			X	
Tsuen Wan Technical School	X		X				X		X	X		
Wellington School	X				X	X		X				X
Yuen Long Middle School		X	X			X			X			X
Total	10	4	8	3	3	10	4	12	2	4	4	6
	14		14			14		14		14		

CHAPTER X

DISTRIBUTION OF TEST SCORES

Eventually the test was found to have been applied to 415 Hong Kong Students 215 male, 200 female in the fourteen schools. The major distribution scores are given in tables below. Where possible, comparison statistics from the international enquiry are provided in parallel. Discussion of the results, and a consideration of the implications is given in a separate section. In a subsequent chapter the results are analyzed in terms of the mathematics teaching given in Hong Kong.

Distribution of scores.

Table 10.1: Distribution of scores in percentage

Score	Hong Kong	Twelve nations total
0		1.0
1-5		10.0
6-10	0.5	11.0
11-15	0.2	12.0
16-20	0.7	12.0
21-25	2.5	12.0
26-30	3.4	10.0
31-35	7.7	10.0
36-40	12.0	7.0
41-45	18.3	6.0
46-50	20.7	4.0
51-55	16.9	3.0
56-60	10.4	1.0
61-65	5.1	0.5
66-70	1.4	0.1
Mean	44.2	23.0
S.D.	9.9	15.0

Figure 10.1: Distribution of Test Score in Hong Kong

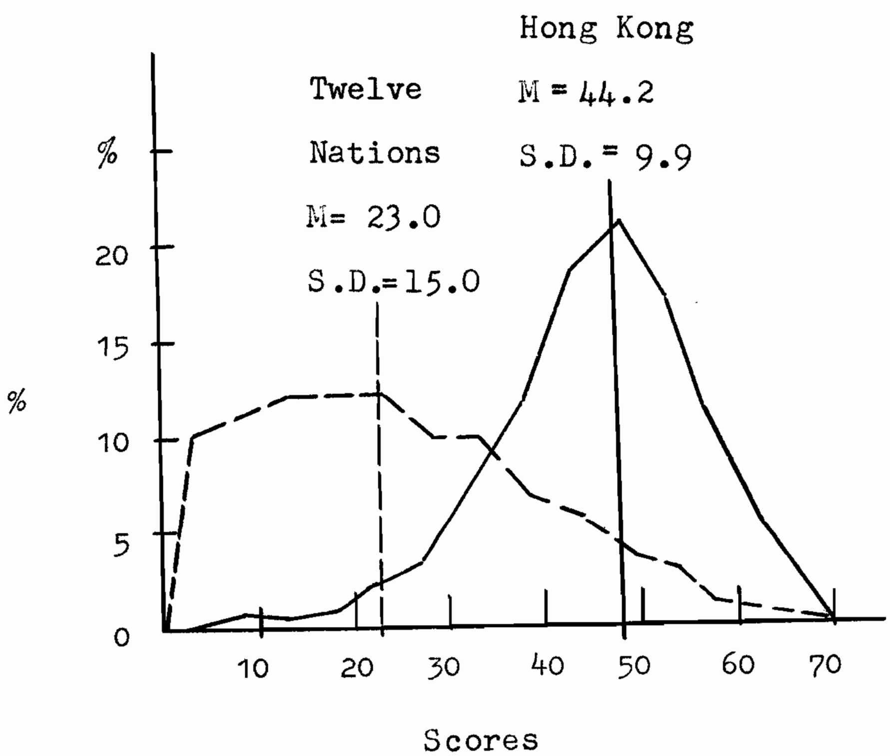
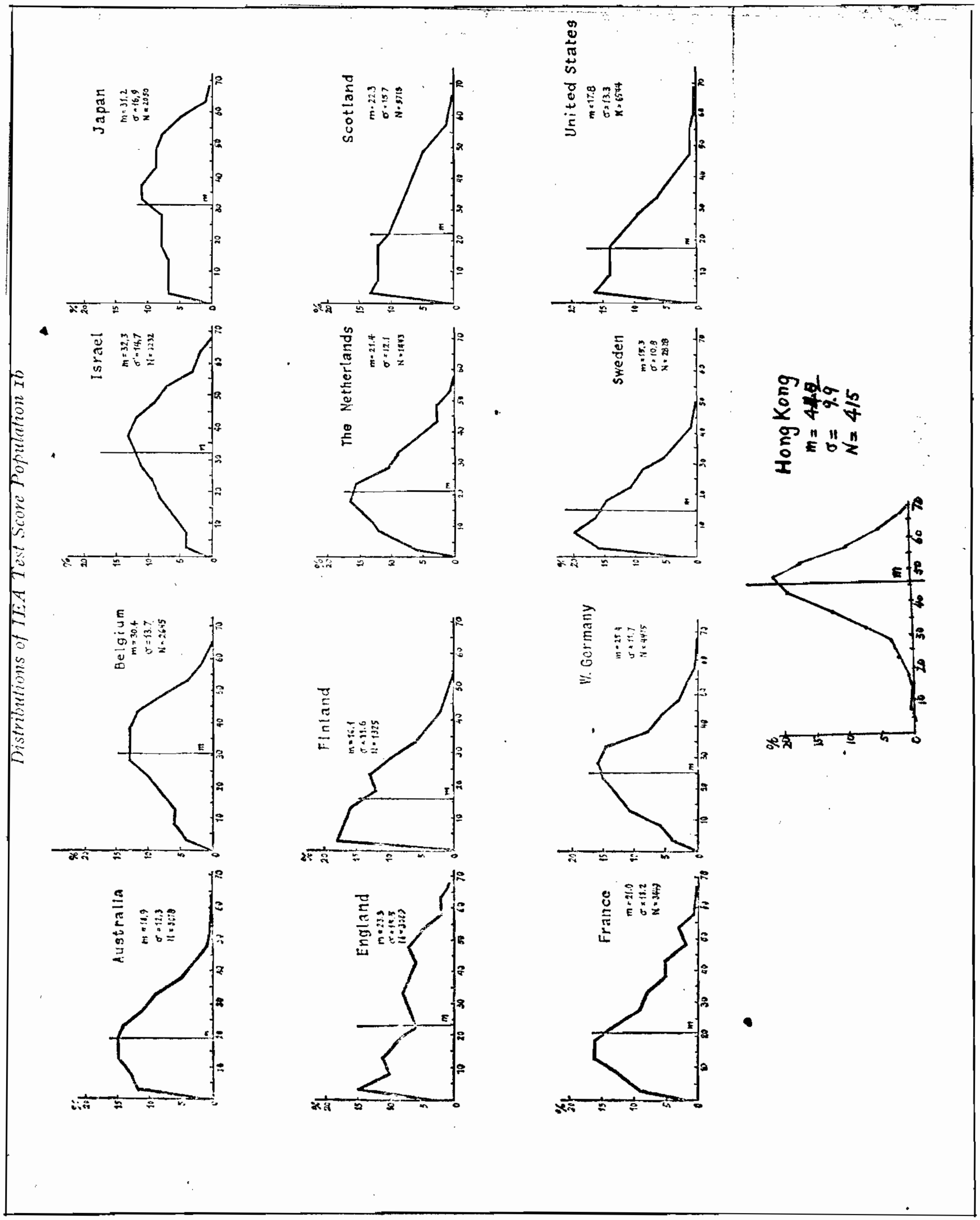


Figure 10.2: Distributions of IEA Test Score[1]



1. Shigeru Shimada, National Analysis: Japan, International Review of Education (Hamburg, 1969). Vol. XV, No. 2, p.170.

Distribution of scores by schools

Table 10.2: Mean score of Schools in Hong Kong

School	Mean Score	S.D.	N
A	40.9	8.7	28
B	48.9	8.4	40
C	45.4	8.7	43
D	40.8	12.3	36
E	56.6	5.6	28
F	48.3	12.1	36
G	40.9	7.5	27
H	48.8	7.4	38
I	45.2	10.6	30
J	24.5	9.8	6
K	39.0	7.8	20
L	43.0	8.5	15
M	38.7	10.6	44
N	46.0	7.7	25
Total	44.2	9.9	415

<u>Distribution of scores by type of school</u> (form of financial support)

Table 10.3: <u>Comparison of Achievement of Grammar Schools in Hong Kong</u> (in percentage)

Score	Government	Subsidized	Private
0			
1-5			
6-10			2.0
11-15			2.5
16-20		0.1	4.0
21-25	2.1	1.5	6.0
26-30	1.1	4.6	10.0
31-35	3.2	9.2	14.0
36-40	9.5	11.3	16.0
41-45	17.0	17.0	22.0
46-50	13.8	26.1	12.0
51-55	24.5	19.2	6.0
56-60	17.0	3.8	4.5
61-65	8.5	5.4	1.0
66-70	3.2	0.6	
Mean	48.3	44.5	37.0

Distribution of scores by type of school (type of course provided)

If the schools are classified into grammar technical and rural schools, the distribution of scores will be:

Table 10.4: Distribution of score in grammar, technical and rural schools

	Grammar		Technical		Rural (All purpose)	
Scores	boy	girl	boy	girl	boy	girl
0						
1-5						
6-10	0.7			1.9		
11-15	0.0	0.7		0.0		
16-20	0.7	0.7		1.9		3.6
21-25	0.7	4.4	3.1	1.9	2.4	3.6
26-30	2.2	6.6	1.5	1.9	2.4	3.5
31-35	4.4	11.8	3.1	15.4	2.4	7.1
36-40	8.7	15.5	10.9	15.4	11.9	3.6
41-45	14.5	21.3	12.6	25.1	19.0	28.6
46-50	18.0	20.6	26.6	13.5	28.5	28.6
51-55	24.6	12.5	9.4	15.4	14.3	14.3
56-60	11.6	5.2	25.0	5.7	14.3	7.1
61-65	10.8	0.7	4.7	1.9	2.4	
66-70	2.9		3.1		2.4	
%	100.0	100.0	100.0	100.0	100.0	100.0
Mean	48.2	41.0	47.9	41.3	46.4	42.9

Distribution of scores by language of instruction

When the schools are grouped by language of instruction, the following comparison is obtained.

Table 10.5

Score	English School boy %	English School girl %	Chinese Middle School boy %	Chinese Middle School girl %
0				
1-5				
6-10			2.7	
11-15			0.0	
16-20			0.0	1.7
21-25		2.7	0.0	5.3
26-30		5.5	5.3	5.3
31-35	2.3	8.2	5.3	10.5
36-40	5.7	17.8	5.3	14.0
41-45	10.5	23.3	19.0	19.3
46-50	18.2	22.0	24.3	26.3
51-55	28.5	13.6	30.0	12.2
56-60	17.0	5.5	0.0	5.3
61-65	13.6	1.4	8.1	
66-70	4.5			
Mean	51.7	42.5	44.9	41.3

Distribution of scores by control of school (religious/secular)

Table 10.6: Comparison of scores in Religious Schools and in Secular Schools.

(in cumulative percentages)

Score	Secular school boy	Secular school girl	Religious school boy	Religious school girl
0-5				
6-10	1.1			
11-15	1.1	1.2		
16-20	2.2	3.7		
21-25	3.3	8.5		2.9
26-30	6.5	14.6	1.7	10.0
31-35	9.8	25.6	6.7	22.9
36-40	20.7	36.6	13.3	40.0
41-45	32.6	63.4	28.3	58.6
46-50	47.8	76.8	51.7	87.1
51-55	64.1	91.5	81.7	97.1
56-60	82.6	98.8	86.7	100.
61-65	91.3	100.	98.3	
66-70	100.		100.	
Mean	49.1	41.0	48.6	41.0

Fig.10.3: Comparison of scores by three types of grammar schools

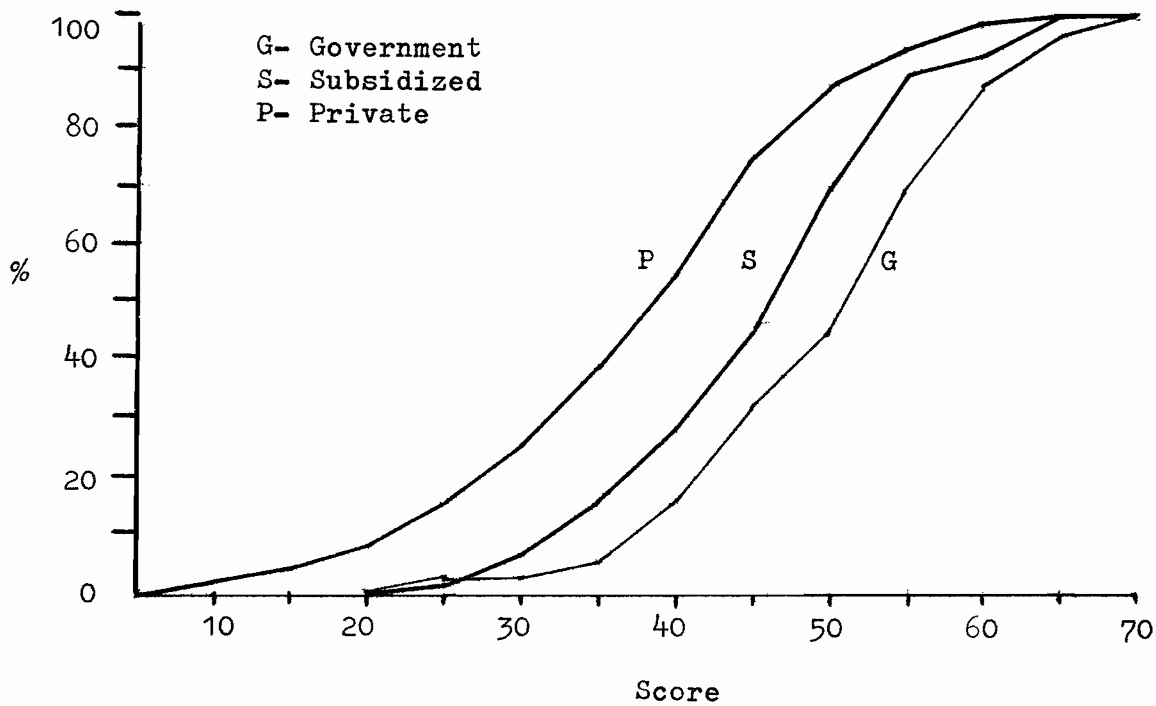


Fig.10.4: Comparison of scores made by Religious and Secular schools

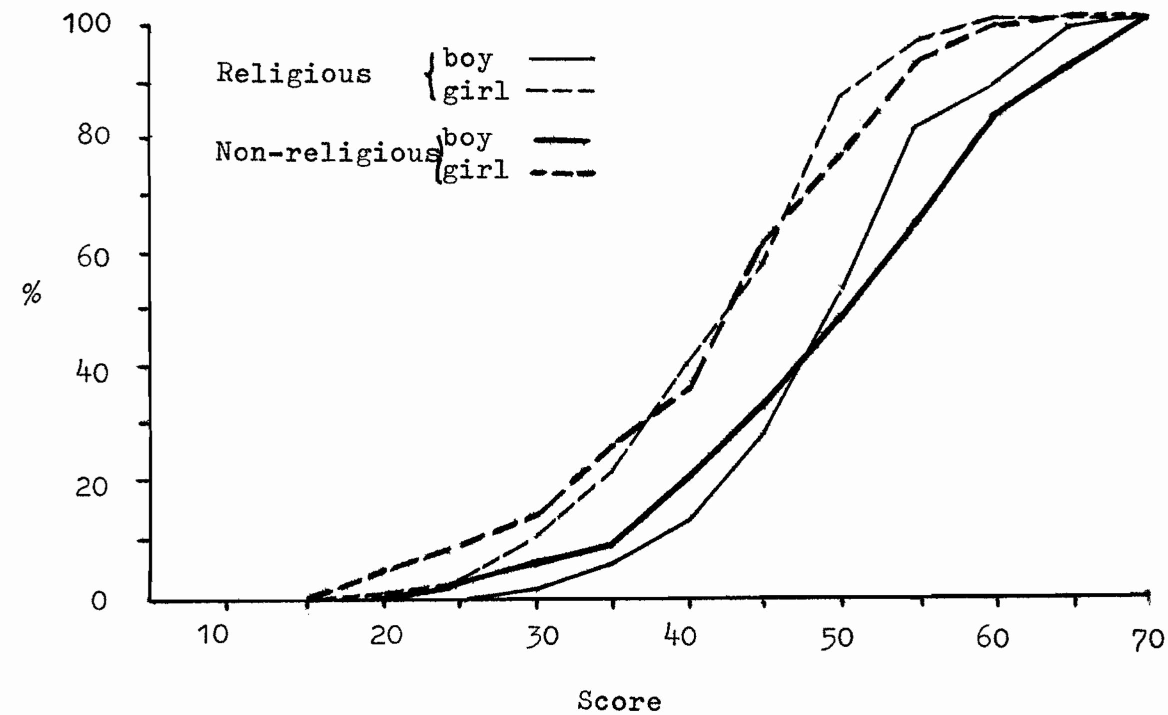


Table 10.7: Comparison of Hong Kong Mathematics Achievement with the Upper 50% of Scores in 12 Nations.

Population 1b.

Score	Aus	Bel	Eng	Fin	France	Ger	Isr	Jap	Net	Sco	Swe	USA	Total	Hong Kong
0														
1-5														
6-10														0.5
11-15	1.0										11.0			0.2
16-20	12.0		1.2		8.6				1.6	1.2	30.0	21.0		0.7
21-25	28.0		10.0		26.0				32.0	20.0	22.0	24.0	16.8	2.5
26-30	22.0		14.0	37.0	18.0	31.4			22.0	18.0	18.0	20.0	20.0	3.4
31-35	18.0	21.0	16.0	28.0	16.0	30.0	8.0	13.2	18.0	16.0	10.0	14.0	20.0	7.7
36-40	10.0	26.0	14.0	20.0	10.0	16.0	26.0	22.0	12.0	14.0	6.0	10.0	14.0	12.0
41-45	6.0	24.0	12.0	8.0	10.0	12.0	24.0	18.0	6.0	12.0	2.0	6.0	12.0	18.3
46-50	2.0	16.0	14.0	6.0	4.0	6.0	18.0	18.0	6.0	10.0	0.8	2.0	8.0	20.7
51-55	0.6	8.0	10.0	1.0	6.0	4.0	14.0	16.0	2.0	6.0	0.2	2.0	6.0	16.9
56-60	0.2	4.0	4.0	0.0	1.0	0.6	6.0	10.0	0.2	2.0		0.8	2.0	10.4
61-65	0.2	1.0	4.0		0.4		4.0	2.0	0.2	0.8		0.2	1.0	5.1
66-70	0.0	0.0	0.8		0.0			0.8					0.2	1.4
Mean	28.73	42.1	39.6	34.05	31.84	35.28	44.9	45.18	31.29	35.26	23.86	28.51	34.49	44.2

Table 10.8

The distribution of age of the Hong Kong students in the test
(in months)

	Boys		Girls		Total
Age	%	Cum.%	%	Cum.%	
149-151	.5	.5			.2
152-154	.9	1.4	.5	.5	.7
155-157	1.4	2.8	1.5	2.0	1.4
158-160	4.7	7.4	9.0	11.0	6.7
161-163	7.9	15.3	11.0	22.0	9.4
164-166	7.4	22.8	9.0	31.0	8.2
167-169	9.3	32.1	16.5	47.5	12.8
170-172	19.1	51.2	15.5	63.0	17.4
173-175	16.3	67.4	14.0	77.0	15.2
176-178	6.0	73.5	4.0	81.0	5.1
179-181	5.1	78.6	4.0	85.0	4.6
182-184	9.3	87.9	4.0	89.0	6.7
185-187	5.1	93.0	1.0	90.0	3.1
over 187	7.0	100.0	10.0	100.0	8.4
	Mean 172.2 S.D. 8.4		Mean 170.0 S.D. 8.9		Mean 171.0 S.D. 8.8

The following table shows the comparison of the mean ages of the students by nations in the IEA test.

Table 10.9: Mean age of students - population 1b.[1]

Country	Mean	S.D.
Australia	159	7.7
Belgium	168	8.8
England	172	4.2
Finland	167	7.3
France	163	7.8
Germany	164	6.6
Israel	167	5.6
Japan	161	3.4
The Netherlands	157	11.6
Scotland	168	5.4
Sweden	164	4.9
United States	164	6.8
All countries	166	7.7
Hong Kong	171	8.8

1. Torsten Husén (ed.), International Study of Achievement in Mathematics (Stockholm, 1967) Vol. II, p.69.

Discussion of the results obtained

The Hong Kong students made a mean score of 44.2 with a standard deviation of 9.9. This mean score is high in comparison with that obtained by the twelve nations of the I.E.A. study. The students themselves felt that the test was easy and that many finished within a one hour period. From table 10.1 it can be seen that two thirds of the Hong Kong students scored between 40 and 60 out of a total possible mark of 70, with more than 6% gaining a mark above 60. This would indicate a low ceiling for the test. The overall mean is 21.2 marks higher than the twelve nations, or 1.4 standard deviations higher.

This high achievement is accounted for by the fact that only 50% of the age group remains in school, for education is not compulsory, and is insufficiently provided for even those who are anxious to continue. Because of the selective entry into government school, and, to a less extent, into the subsidized schools, this may well be the upper 50% of the age group. It would be unfair to make direct comparison with nations in the twelve nations study. Certain assumptions may be made to facilitate such a comparison. If we assumed that the remaining 50% (not in school) had taken the test and scored zero, we should have had a skewed distribution with a mean of 22.1, slightly less than the twelve nation mean of 23.0. If we assumed the same kind of rectangular distribution of scores below the mean as in the total population, the Hong Kong mean would be 28.6, whereas if we had assumed a normal distribution of this missing 50% within a range from zero to 23.0, then the Hong Kong mean would have become 27.8. On any plausible assumption, therefore, the mean score in Hong Kong would be not less than the mean score of the twelve nations. The various assumptions would, of course, increase the standard deviation of the distribution, calculated to be 9.9 - the smallest of any country apart from Finland. (This low spread is attributable to the homogeneity of the Hong Kong population in comparison with a population distributed amongst a whole nation, and also to some extent to the common syllabus which the schools follow, or are required to follow.)

If we did not wish to make assumptions based on hypothetical results of a missing Hong Kong population we could truncate the distribution of the other nations to the upper 50%. This would alter the form of their distribution as well as the value of the mean. Means for the twelve nations calculated on this assumption are given in Table 10.7. Then only the scores of Japan and Israel would be higher than those for Hong Kong.

When we make an inter school comparison the results are shown in Table 10.2. This shows the difference between the best school and the poorest one to be as great as 32.1. Even if we ignore the small sample in the lowest scoring school, the range would be 17.6 -- that is, from 39.0 to 56.6. Thus there is a great difference in academic standard between schools in Hong Kong. This difference in standard is closely related to the form of support given to the schools. This is made clear in Table 10.3. Among the government supported schools there is some difference in homogeneity - the standard deviation being lowest in the government grammar schools, higher in the government technical schools. The standard deviation of the subsidized schools is lower than that of the private schools, though for reasons of confidentiality these can not be too clearly demonstrated in the tables other than by naming the schools concerned. As befits the better staffed, the better equipped schools, which take the pick of the Hong Kong children by selective entrance examination, their mean score is significantly higher than that of the other schools. Entrants into the private schools have failed to gain admission to government schools or subsidized schools.

Distribution of scores between groups in grammar, technical or rural (all purpose) schools show no significant differences (Table 10.4), but do reveal significant differences between the sexes in each type of school, particularly in the grammar schools. This table should be read in conjunction with Table 10.2, which stresses inter-school differences. On the other hand the language of instruction has less effect upon the performance of girls, who obtain means of 42.5 in English and 41.3 in Chinese schools. Per contra for the boys the language of instruction does appear to be important. This seems to be related to Hong Kong patterns of male dominance in opportunities for education and for career prospects, tied to a demand for English language instruction as the means of entry into commercial and professional life. The final comparison upon the secular or religious control of the schools once more demonstrates the sex difference and negates the influence of this form of control of schools upon the performance of boys (Table 10.6). In both forms of schools boys have significantly higher mean scores than the girls, and non-significant inter school (by control) differences.

The clearest relation seems to have been demonstrated in the case of form of financial control of school upon mathematics performance of students. It may be conjectured how far this is related to the form of development, or stage of development reached in Hong Kong, where the expenditure allowed a school directly affects its output. In Hong Kong there is little difference among the schools, when these are classified as grammar, technical or vocational, though the most homogeneous and the highest scoring are found to be in the government controlled grammar schools. This may be explained by the emphasis upon academic studies no matter what the designation of the school. All the technical schools offer a five year academic programme to prepare students for the School Leaving Certificate Examination if they desire it, and so many do. Since the technical schools are restricted in equipment, and in the qualification of their teachers, and there is a marked lack of enthusiasm on the parents and students for technical education it becomes a moot point whether direct expenditure on more grammar schools would not give a better academic yield than continuing, insufficient expenditure upon technical schools.

The IEA lb test is aimed at the population of students in classes where the majority is of age 13-14. It was believed that most of the form II students in Hong Kong were in this age group. The findings of the test unexpectedly reveals a much higher age at this grade. Table

10.8 shows the distribution of age of the students in the test. The overall mean was 171.0 with standard deviation 8.8 months. This is 5 months higher than the twelve nations' mean (166 months) and apart from England's 172 months Hong Kong is the highest. The variability is also greater in Hong Kong as compared in table 10.9.

The correlation of age and score is found to be only 0.07 in Hong Kong. This is not significant. The hypothesis that increased age is associated with increased score as confirmed in the IEA analysis is not supported by Hong Kong's finding. For an older student in Hong Kong may be a repeater and is more backward. Coincidently, the school with the highest mean age and the school with the lowest mean age scored exactly the same mark 40.9. The mean age also varies greatly from school to school. This may be a reflection of the different standards required for entrance in different types of schools.

Table 10.8 also shows that the boys were older. The 2.2 months is a significant difference. This may be due to the fact that educational opportunity is not quite equal for both sexes. Sex discrimination may discourage the less able girls from continuing their schooling and subsequently the girl student body has less over aged repeaters.

In general, the higher age of students in Hong Kong is partly due to the shortage of educational provision which delays the enrolment of children of school age and partly due to the class system which requires many to repeat a class upon failing to pass a promotion examination.

CHAPTER XI

ITEM ANALYSIS

The three one-hour IEA mathematics test were re-arranged into three 1½ hour test papers X,Y,Z for use in Hong Kong, though each student in the test answered only one paper. In each participating school the students were randomly assigned one of the papers, so that number of students writing each paper was approximately the same. In the analysis of the results, one concern is to determine whether the arrangement of the papers affected the performance of the students.

In the test 138 students attempted X paper giving 3403 correct answers; 145 attempted Y paper giving 3549 correct answers; and 132 attempted Z paper giving 3238 correct answers. The mean number of correct answers given in these papers are 24.66 for X, 24.48 for Y and 24.53 for Z. This shows that the three papers are well balanced as far as difficulty is concerned. On the other hand, the students assigned to do X, Y or Z paper do not show any difference in ability, for the analysis of seven common items shows that the students scored nearly the same marks in this set of common questions. The average degree of difficulty of these seven questions for students who answered X paper was 0.65, for students answering Y paper was 0.67 and for students answering Z paper was 0.70. These may be regarded as the same. We could then assume that the students are from the same population and that there has been no effect on performance arising from the difference of papers, X,Y,Z.

Degree of difficulty

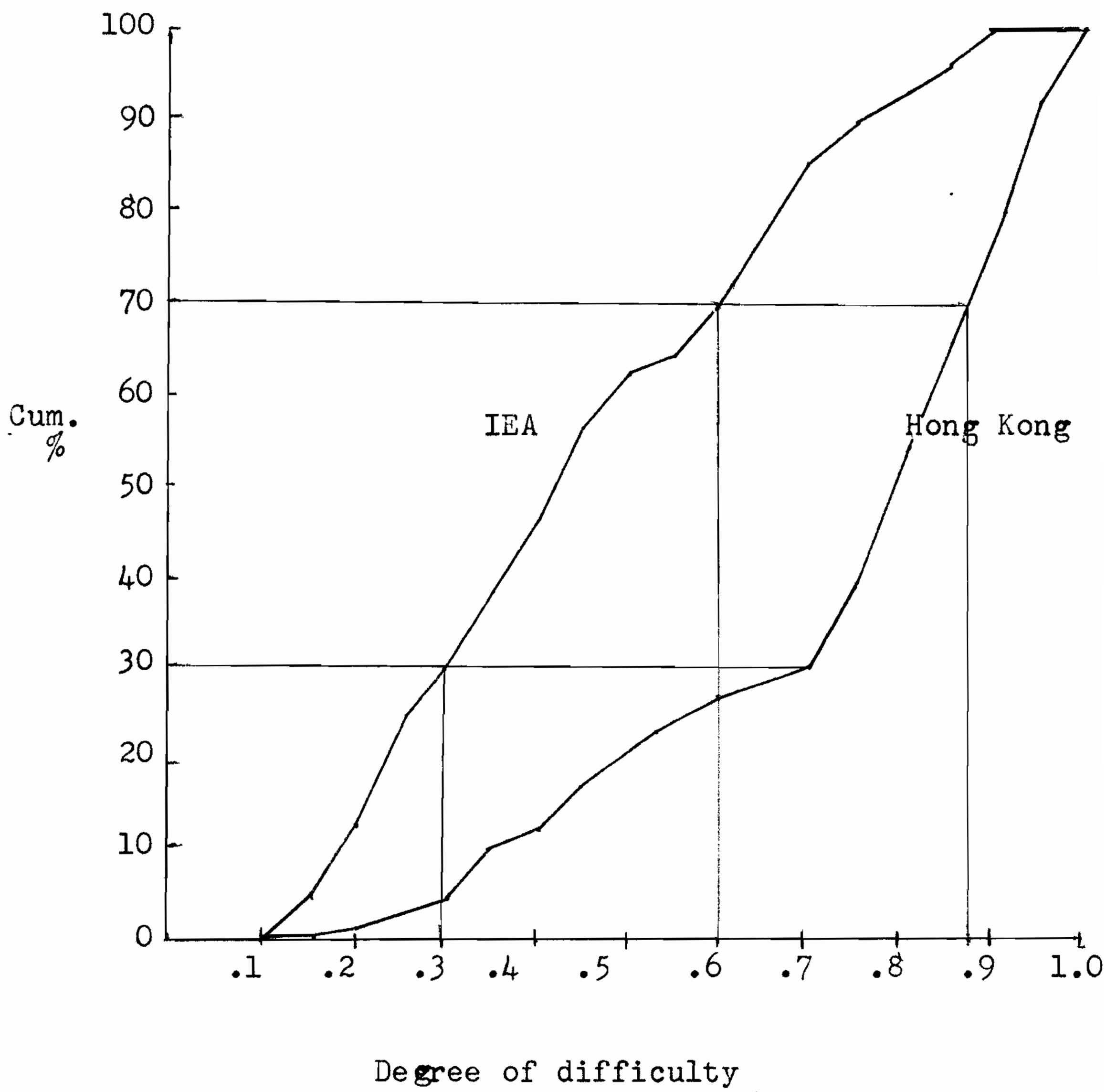
In the IEA test the degree of difficulty of a question is defined as the ratio of number of correct answers obtained to the total number of participants in the test, so that a higher ratio denotes an easier question. Table 11.1 presents a comparison of the degree of difficulty in the IEA and in Hong Kong.

The mean difficulty for IEA was 0.446 which is much lower than Hong Kong's 0.723. To show the difference of the distribution of the degree of difficulty of question, the cumulative percentages are calculated and shown in the figure 11.1. The horizontal distance between the two curves shows the great difference between Hong Kong and the IEA. At 30 percentile the difference is the greatest. One is 0.3 and the other is 0.7. That means 30% of the questions in IEA had difficulty equal to or less than 0.3, while in

Table 11.1: Degree of difficulty

	Paper A		Paper B		Paper C	
question	IEA	HK	IEA	HK	IEA	HK
1	.85	.99	.74	.98	.81	.84
2	.77	.98	.81	.97	.71	.85
3	.87	.95	.59	.91	.69	.92
4	.66	.95	.62	.81	.42	.87
5	.49	.78	.62	.82	.66	.93
6	.46	.85	.20	.70	.69	.88
7	.24	.44	.40	.69	.51	.90
8	.44	.85	.88	.94	.44	.86
9	.30	.76	.40	.92	.42	.80
10	.22	.64	.63	.97	.20	.82
11	.72	.94	.41	.95	.58	.80
12	.34	.70	.39	.81	.33	.34
13	.41	.46	.61	.78	.24	.29
14	.30	.79	.32	.71	.38	.31
15	.29	.78	.28	.90	.55	.50
16	.20	.41	.57	.73	.17	.46
17	.23	.40	.48	.74	.13	.33
18	.67	.77	.18	.23	.18	.54
19	.18	.55	.27	.19	.19	.74
20	.10	.42	.11	.56	.43	.79
21	.19	.81	.63	.78	.33	.85
22	.26	.39	.19	.28	.49	.78
23	.66	.92	.39	.71	.36	.63
24			.76	.91		

Figure 11.1: <u>Comparison of Degree of Difficulty</u>



Hong Kong only 30% of the questions gave a difficulty less than 0.7. If we divide the questions into three groups: easy questions which are of difficulty 0.7 or up; average questions which are of difficulty between 0.3 to 0.7 and difficult questions which are of degree of difficulty below 0.3, the following comparison is obtained.

	Easy questions	Average questions	Difficult questions
IEA	14%	56%	30%
Hong Kong	70%	25%	5%

Analysis of subscores

The average degree of difficulty of each type of questions for IEA and for Hong Kong can be compared by the classification below:-

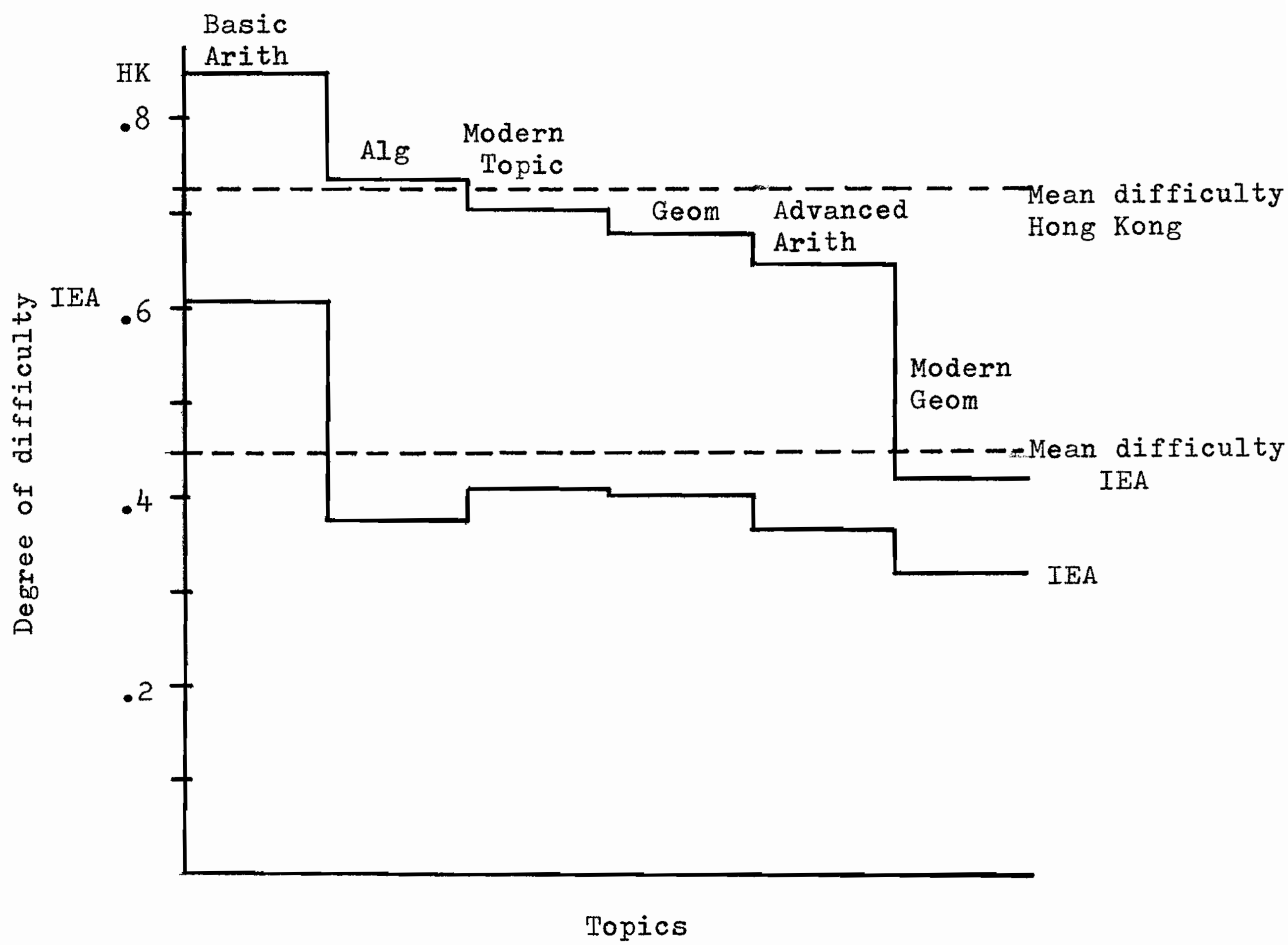
Table 11.2

	Degree of difficulty	
Type of questions	IEA	Hong Kong
Basic arithmetic	0.61	0.85
Advanced arithmetic	0.37	0.64
Modern topics	0.41	0.70
Euclidean geometry	0.40	0.67
New geometry	0.32	0.42
Elementary algebra	0.38	0.73

Figure 11.2 presents the results in a striking manner. In spite of the great difference in value of the degree of difficulty, the patterns of the difficulties in these types of questions for Hong Kong and for IEA are very much alike. Both made high scores in basic arithmetic and low in advanced arithmetic and in new geometry. The difference in respect to other topics is less marked. As the students in the twelve nations generally felt the papers difficult, they gained scores mainly in the comparatively easy topics like basic arithmetic, in which 48% of their total score was made. In the more difficult items like algebra and geometry, the gains were only 21% and 20% respectively of the total. The Hong Kong scores in these three categories were: basic arithmetic 34%, algebra 26% and geometry 22% of the total. It seems that the Hong Kong students have wider knowledge in mathematics at this stage. This is most probably due to the heavy syllabus in both primary and secondary schools in Hong Kong, for all topics in the test have been taught at this level including the equation of line and simultaneous equations.

In figure 11.2 the difference in degree of difficulty between Hong Kong and IEA mean is largest in algebra questions. In Hong Kong, algebra questions scored slightly

Figure 11.2: Degree of difficulty by topics



higher than their mean mark, while the IEA scored lower than their mean. In this figure the topics are arranged from the easiest to most difficult according to Hong Kong's performance.

Besides doing well in basic arithmetic, Hong Kong students also did well in algebra and modern topics. This may be the result of the teaching "modern mathematics" in most of the Hong Kong schools. The new syllabus for the first two years in secondary schools has included topics on set, elementary analytic geometry and binary numbers. On the other hand Euclidean geometry has been greatly reduced. The large quantity of harder arithmetic problems which used to be taught in first year of secondary school has been replaced by the new material. The degree of difficulty of the topics seems to coincide naturally with the emphasis of the mathematics teaching in Hong Kong.

For the subscores for the national groups in different types of questions, shown in table 11.3 the 70 questions were first grouped into "lower process" and "higher process" questions. This classification was made by judgement of several reviewers of the IEA who decided whether a higher mental process was required in solution. The second classification was made according to whether a question was in verbal form or not, whilst the remaining classification of questions was made according to the nature of the questions. In this table it can be seen that Hong Kong's scores were always greater than the IEA by at least one standard deviation. In this comparison the highest achievement of Hong Kong was made in algebra by a standard score of 227 (in IEA terminology).

Further item analysis shows that performance in Hong Kong is particularly weak in several questions, where the scores made are even lower than the IEA average. They are B19, A22, A7, and A13. In B19, the degree of difficulty for Hong Kong is 0.19 against 0.27 for the IEA. Surprisingly, all these questions involve metric system. Two of them refer to map scales in kilometers, the other two on volume in cubic centimeters. The reason is obvious. Hong Kong students have to learn three systems of measurements -- the English system, the Chinese system and the metric system, and since the metric system is not in use in daily life in Hong Kong, it is the least familiar system.

There are also two other questions in which Hong Kong scored lower than the IEA, C14 and C15 both on motion geometry. These are long verbal questions. From information received, most of the students did not bother to look at the Chinese translation, and the meaning of these questions might not be clear to them. Language difficulty in these questions could be held to account for the low score.

As noted above, Hong Kong students did well in the items on modern topics. For example, Hong Kong scored 0.74 for C19, a question on set in which the IEA difficulty is 0.19, and 0.79 on the binary number question. Further analysis of the modern topics including questions on associative, distributive and commutative laws, binary numbers, integers, equations of lines and sets shows that the schools teaching "modern mathematics" are superior in answering this type of questions. Yet it is not certain whether this superiority is due to the teaching of modern mathematics or due to other variables, for not all Chinese schools have yet changed to teaching the new syllabus and they are in general less advanced than the English schools. However, it was found that the schools teaching "modern mathematics" scored much better in modern topics than in Euclidean geometry, but in the schools where traditional mathematics is taught, the opposite was the case.

The effect of the recent reform of mathematics teaching in Hong Kong on mathematics achievement, understanding and interest and intent to learn is still unknown. In this test, the schools teaching the two different syllabuses are different types of achools and are therefore not comparable. To answer this question a further survey, on a larger scale, with more questions designed to test the difference of these two approaches on mathematics understanding, skill, interest and the ability to solve problem is still needed.

Table 11.3: Subscores for National Groups (in Standard Scores)

	No. of Items	Aus	Bel	Eng	Fin	France	Ger	Isr	Jap	Net	Sco	Swe	U.S.	H.K.
Total Score	70	-28	49	5	22	-14	16	62	54	-11	-5	-52	-35	141
Lower Process	49	-26	52	1	16	-12	15	58	57	-13	-5	-53	-31	135
Higher Process	21	-27	34	13	36	-17	17	63	38	-2	-2	-41	-37	145
Verbal	41	-24	44	7	26	-16	25	58	53	-14	-6	-46	-39	147
Computational	29	-30	51	1	15	-9	2	60	49	-5	-3	-54	-25	104
New Maths.	13	-15	42	10	54	-29	10	35	21	-16	-6	-28	-16	112
Basic Arith.	18	-31	32	-19	45	-12	28	43	32	12	-7	-40	-19	116
Advanced Arith.	14	-30	45	2	26	-16	28	62	56	-2	-12	-56	-32	201
Algebra	17	-9	47	29	30	-25	-27	40	52	-29	14	-46	-24	227
Geometry	17	-24	53	8	-29	6	23	75	50	-22	-13	-43	-48	142

CHAPTER XII

SOCIAL FACTORS AND MATHEMATICS ACHIEVEMENT

During the past decade there have been a large number of studies attempting to portray the link between educational attainment and home background. The IEA is only one of such attempts made on cross-national studies. It has been found that the relationship between socio-economic level and learning is generally strong, yet it varies greatly from one country to another. It was of interest, therefore, to see whether such a relationship, confirmed by the IEA test, also exists in Hong Kong, and if it exists, to what degree it affects education.

(I) Achievement and Father's Occupation

The IEA found that there is a strong correlation between a student's achievement and his father's occupational status. The correlation for the lb population was as high as 0.25. In most of the IEA countries the higher the father's occupational status, the higher is the achievement. But this relation becomes weaker at a higher level of schooling where the student groups are more selective. This selectivity makes the lower socio-economic classes less represented, but represented by the more able students. The relationship under these circumstances may become negative. At the pre-university level, the 3a population, this negative correlation exists in several countries, particularly in those countries where educational opportunity is limited.

The 1b population is already a selective group in Hong Kong, so that, the relation, if it exists, may be hidden by the fact that the lower class children are not well represented. It was found that the correlation of achievement to father's occupational status was only 0.01.

According to the IEA classification, fathers' occupations are grouped into the following nine categories:[1]

Group 1. Higher professional and technical.

Group 2. Administrators, executives and proprietors of large and medium scale establishment.

Group 3. Subprofessional and technical.

Group 4. Small working proprietors in retail trade and personal services.

Group 5. Proprietors and managers in agriculture, forestry and fishing.

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1. Ibid., Vol I, p.144.

Group 6. Clerical and sales workers.

Group 7. Manual workers skilled and semiskilled.

Group 8. Labourers in agriculture, forestry, fishing.

Group 9. Unskilled manual workers.

Group 0. Unclassified, no answer.

To calculate the correlation between occupational status and achievement, the above groups were pooled into four categories:

Highest occupations (groups 1 and 2),

Second highest occupations (groups 3,4 and 6),

Second lowest occupations (groups 5 and 8),

Lowest occupations (group 7 and 9).

The mean score of the students in each of the above four categories were calculated. Table 12.1 presents the necessary data. Table 12.2 and Figure 12.1 give comparative data.

Table 12.1 : Students scores as grouped by fathers' occupational status

Fathers' occupational status	Students' mean score	Standard deviation
Highest	46.50	8.64
Second highest	42.95	10.70
Second lowest	47.67	8.10
Lowest	44.73	10.55

If any nonzero order correlation were to exist between students' scores and parental occupation the data in the two columns would need to be sequentially ordered. This is not so. The highest score is made by the second lowest occupation group, the lowest score by the second highest occupational group. The difference between the means is significant. In contrast to this, the IEA shows a positive correlation between the two variables. Only France shows a trend which is parallel to that of Hong Kong.

The analysis of the home background presents interesting data, the composition of socio-economic home background in Hong Kong being quite different from the other countries. The percentage of the highest three categories is very low in Hong Kong. As shown in table 12.3, it is only 9% compared with 19% in all the other nations. But the percentages in group 4 and 6 are large. Group 4 is for small working proprietors in retail trades, personal services and artisan proprietors. In Hong Kong there is a large number of small shops in the retail trade and personal services. This category accounts for 17% of the total as compared with only 8% in other countries. The large percentage in group 6 for clerical and sales workers can also be well explained by Hong Kong's socio-economic background. As a commercial

Table 12.2:

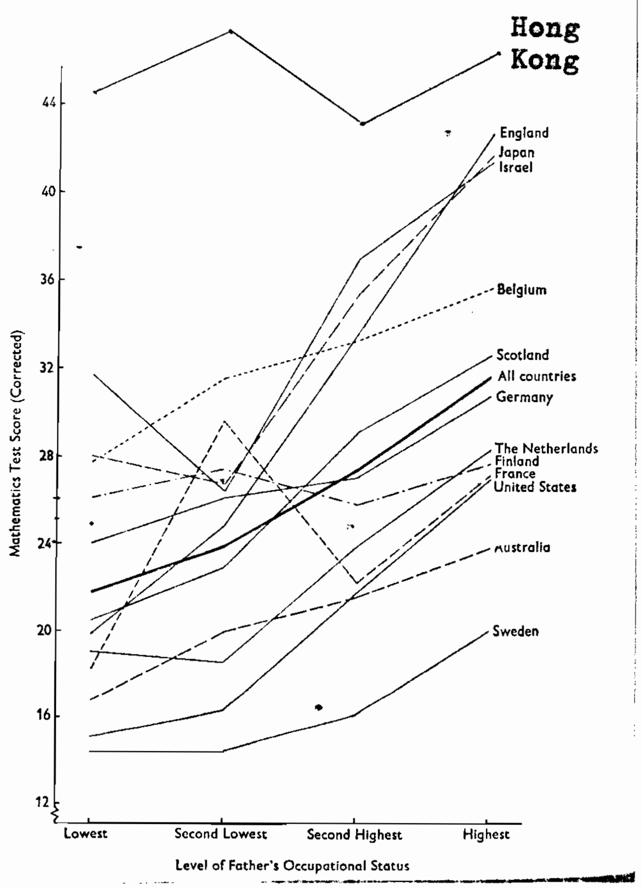
Total Mathematics Test Score by Level of Father's Occupational Status

Population 1b[1]

Country	Lowest Occupations M.	S.D.	Second Lowest Occupations M.	S.D.	Second Highest Occupations M.	S.D.	Highest Occupations M.	S.D.	Difference Highest -- Lowest
Australia	16.8	11.79	19.78	11.78	21.53	11.95	22.55	12.1	6.9
Belgium	27.44	13.94	31.16	13.01	32.96	12.85	35.22	12.6	7.8
England	19.7	16.6	24.6	17.9	33.2	17.3	42.2	14.0	22.5
Finland	25.9	9.5	27.1	9.5	25.5	9.2	27.3	11.8	1.5
France	18.2	11.2	29.2	13.8	22.0	12.9	26.9	11.1	8.9
Germany	23.8	11.6	25.8	12.1	26.7	11.4	30.6	10.5	6.8
Israel	31.2	14.2	26.1	13.8	36.5	13.1	40.8	11.0	9.6
Japan	27.7	16.4	26.5	15.6	35.0	16.0	41.3	15.3	13.6
The Netherland	18.9	11.4	18.4	9.6	23.5	12.1	27.9	13.1	9.1
Scotland	20.2	14.9	22.6	14.9	28.7	15.4	32.2	15.1	12.0
Sweden	14.3	10.1	14.3	10.7	16.9	10.9	19.9	11.9	5.6
U.S.A.	15.0	11.9	16.3	12.6	21.6	12.6	26.8	13.8	11.9
All Countries	21.6	12.8	23.5	12.9	27.0	13.0	31.2	12.7	9.7
Hong Kong	44.7	10.6	47.7	8.1	43.0	10.7	46.5	8.6	4.7

1. *Ibid.*, Vol. II, p.207.

Figure 12.1: <u>Distribution of score by fathers' occupational status</u>[1]



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1. <u>Ibid.</u> , Vol. II, p.208.

Table 12.3: Percentage of Students by Country and occupational category of father

Country	Occupational category 1	2	3	4	5	6	7	8	9	0	N
Belgium	4	4	9	12	5	18	40	0	7	1	1656
England	4	1	8	9	2	8	62	2	1	3	2899
Finland	5	4	14	5	25	2	40	1	4	0	743
France	2	2	11	9	7	7	53	1	4	4	2292
Germany	4	5	8	9	9	14	42	1	5	3	4318
Japan	3	10	6	16	24	10	24	2	2	3	1969
Netherland	4	7	8	6	11	13	43	5	2	1	423
Scotland	5	2	8	4	3	6	61	2	4	5	4972
Sweden	3	5	13	7	16	4	43	4	2	3	2458
U.S.	8	7	10	6	5	12	43	2	1	6	5806
All countries	4	5	10	8	11	9	45	2	3	3	
Hong Kong	2	1	6	17	4	18	29	4	12	7	400

Table 12.4 Percentages of the Three Highest Occupational Status Groups in Different Countries

Country	%
Australia	20
Belgium	20
England	13
Finland	22
France	16
Germany	17
Israel	14
Japan	19
Netherland	23
Scotland	17
Sweden	22
U.S.	24
Hong Kong	9

port, Hong Kong may have as much as 18% of her work force so employed. The combined percentage of skilled and unskilled workers is 41 which is close to the IEA countries' average. But the proportion of unskilled workers to skilled workers differs greatly. In Hong Kong it is 4:10, while in the other countries it is only 1:15. The large number of unskilled labourers is due to long years of neglect of technical training and to historical factors related to Hong Kong's dependent status.

(II) Achievement and Father's Educational Level

The IEA mathematics project had demonstrated that in all the participating countries, at the lb level, there was a significant relation between scores and the level of education of the father. As selectivity increases, and the nearer one approaches a university level the greater the decrease in the size of the correlation, until a point is reached at which the correlation is negative. This negative correlation is attributable to the bias introduced by the selection process, where only the higher socio economic groups are represented in any large numbers. There is thus greater homogeneity on one variable and heterogeneity on the other variable.

In Hong Kong the first selection at eleven plus selects approximately 50% by means of the Secondary School Entrance Examination, not all of whom are under the age of twelve, or even thirteen. Thus the lb population of Hong Kong is already more homogeneous on attainment at this age than would be the case in the other countries. In many respects Hong Kong would represent for its lb population conditions seen elsewhere only at the immediate pre-university level. For the 244 students who responded to this question the correlation between mathematics score and parent's education level is -.145, significant at the 0.05 level.

(III) Achievement and the Economic Background of the Home

It has been noted above that there is a positive relationship between achievement in mathematics and home background. To examine this proposition for Hong Kong, the student samples were subdivided into five groups, ranging from poor to wealthy, and mean scores calculated. Table 12.5 illustrates this.

Table 12.5: Mean Achievement of Socio-economic Groups

Class		Mean	N
I.	Poor	41.25	14
II.	Less poor	45.25	108
III.	Average	44.17	120
IV.	Less wealthy	44.22	99
V.	Wealthy	43.53	34
		44.20	375

For a positive correlation the order amongst the classes would need to be paralleled by the order amongst mean achievement. There are two striking exceptions - the highest mean score is obtained by a relatively poor group, the second lowest mean score by the children from the wealthy group. Additionally it should be noted that the poorest group has the lowest mean score. This may be over determined by poor home conditions, poor educational level of parents, and the school attended.

In an attempt to get some index of the effect of the school in relationship to socio-economic background of its students, a tentative index was calculated. This starts with the assumption that in Hong Kong the living area of the home, in a city where space of all kinds is at a premium, is one crude measure of socio-economic status. For the other crude measure, parental occupation was dichotomized between the two upper groups and all others. On the basis of these two measures a simple weighted measure was made by taking floor area of the home (in multiples of ten square feet) and the percentage in the upper two occupational groups. Table 12.6 shows how this crude, simple weighted composite measure applies to Hong Kong schools. Class distinctions are present in Hong Kong schools. Girls' Grammar schools stand highest in this respect, technical schools the lowest.

Table 12.6

Socio-Economic Background of Schools

School	Type of School	Index I	Index II	Combined Index (1/10 I + II)
Maryknoll	Grammar, girl	890	73	162
True Light	Grammar, girl	525	85	138
B.P.S.	Grammar, girl	562	60	116
King's College	Grammar, boy	443	63	107
Tai Tung	Grammar, boy	450	60	105
Yuen Long	Rural, Coed.	592	37.5	97
Choi Hung	Grammar, boy	502	40.5	91
Wellington	Grammar, boy	352	54	89
Ho Tung	Tech., girl	431	43	86
K.T.S.	Tech., boy	414	44	85
Shaukiwan	Tech., coed.	450	38	83
Tsuen Wan	Tech., boy	400	42	82
Q.E.S.	Grammar, coed.	466	34	80
Confucian	Grammar, coed.	329	22	55

(IV) Achievement and Time Variable

In Hong Kong the student in the second year of secondary school spends an average of 3.8 hours per week in mathematics instruction. That is 5.5 periods of 40 minutes out of a total of 40 periods of instruction per week. No correlation between instruction time for mathematics and achievement has been reported. In fact the time for mathematics instruction in Hong Kong is nearly the least in all the countries where the IEA test has been conducted. A comparison of the time spent on mathematics is presented below:-

Table 12.7 Mathematics Instruction For Population 1b[1]

(hours per week)

Country	Instruction	S.D.
Australia	5.2	0.6
Belgium	4.7	1.0
England	4.0	0.8
Finland	3.0	0.2
France	4.4	0.8
Germany	3.9	0.6
Netherland	4.6	1.5
Israel	4.1	0.5
Japan	4.5	0.5
Scotland	4.6	1.0
Sweden	3.8	0.9
United States	4.6	1.3
Hong Kong	3.8	0.6

1. *Ibid.*, Vol. I, p.278.

The standard deviation-or homogeneity of school practice - is interestingly revealed in the above table.

Table 12.8 represents the distribution of means and standard deviations of the time spent on mathematics homework by students in the twelve countries, to which has been added the data from Hong Kong.

Table 12.8 Means and Standard Deviation of Hours per week for Mathematics Homework (MH) and all Homework (AH)[1]

	MH		AH		MH/AH
Country	Mean	S.D.	Mean	S.D.	%
Australia	2.5	1.6	6.1	3.8	39
Belgium	3.7	2.5	11.4	7.3	32
England	1.8	0.9	5.4	3.1	31
Finland	2.9	2.2	11.0	6.9	26
France	3.4	1.9	9.1	5.7	37
Germany	3.4	1.9	9.0	4.4	38
Israel	4.4	2.6	14.0	7.0	31
Japan	3.0	1.8	8.3	5.5	36
Netherland	2.6	1.9	9.0	6.9	29
Scotland	2.2	1.6	4.8	3.7	48
Sweden	1.9	1.3	6.1	3.9	31
United States	3.1	2.3	6.9	5.5	45
All countries	2.7	2.0	7.1	5.4	38
Hong Kong	4.9	3.4	11.5	6.2	43

1. Ibid., Vol. I, p.278.

It will be noted that the average Hong Kong figure is 4.9 hours per week, higher than any of the twelve countries. Only Finland and Belgium devoted about the same time for total homework, and only Israel exceeded it. Hong Kong stands high in the ranks of those who spend a high proportion of total homework time on mathematics. Its standard deviation of the mean time spent on mathematics homework is also the highest, so that schools and individuals must differ greatly in what is required by the term "mathematics homework".

Figure 12.2 suggests the existence of some slight relationship between mathematics achievement and the amount of mathematics home work.

As a crude measure of correlation a line of best fit would join Sweden and Belgium, in which the average of the deviations approximates zero. It would intersect the x axis at an angle $\tan^{-1}.53$. This in turn would suggest that about one quarter of the variance in scores on the mathematics test is due to the influence of varying amounts of time spent on homework.

Table 12.9 presents separate data for Hong Kong on the mean time spent on homework for each of the Hong Kong schools, subdivided by sex.

Figure 12.2: Relation of Mathematics home work and Score

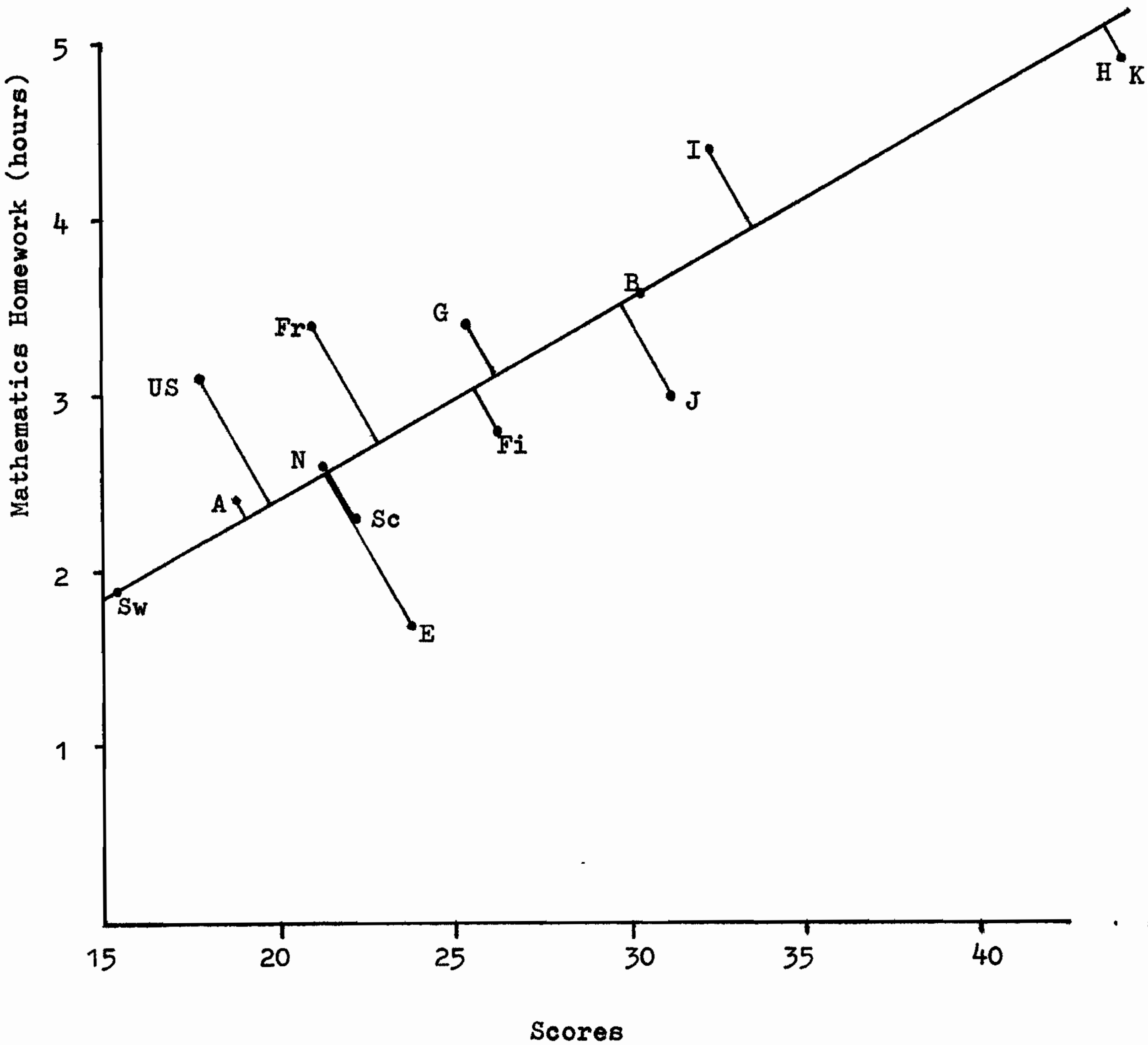

Table 12.9: Mean time per school spent on mathematics homework

Hours of homework

School	boy	girl	All boys and girls
A		3.57	3.57
B	7.34		7.34
C	3.85	7.39	5.74
D		3.74	3.74
E	1.27		1.27
F	3.21		3.21
G		6.33	6.33
H	4.28	2.94	3.64
I	5.46	4.53	4.97
J	7.75	7.50	7.67
K		4.00	4.00
L	4.00		4.00
M	7.52	7.77	7.63
N	5.61	5.17	5.40
Average	4.88	4.99	4.94

From the above means, and a knowledge of the mean mathematics achievement in the schools, it was possible to obtain a correlation between the two of -0.51, significant at the 5% level. The negative value indicates that in the poorer schools, with lower mathematics achievement more time is spent on mathematics homework. Using all the data for all students, the correlation between mathematics score and time spent on mathematics homework is zero order.

(V) Achievement and Aspiration in Education

Question 11 in the student questionnaire for Hong Kong student asked for the number of year of education the student expected to complete. Answers were grouped into five: 1-2 years, 3-4 years, 5-6 years, 7-8 years and over 8 years. The mean of the expectation of education is 7 years. Analysis of the scores and the answers given to this question shows that the correlation is 0.196. This is significant at 0.01 level for a sample of 412 respondents.

Question 13 asked them to give their best liked subjects in order of preference. The result confirmed the hypothesis that interest and achievement in mathematics are related. The correlation is found to be 0.37, also significant at the 1% level. It is interesting to note that mathematics is the best liked subject in Hong Kong, according to the list of best liked subjects.

Table 12.10 Best-liked subjects in Hong Kong

Order	Subject	% given as best-liked
1	Mathematics	27.0
2	English	18.8
3	Science subjects	18.2
4	Chinese	12.8
5	History	6.2
6	Geography	4.0
7	Others	13.0
		100.0

This data may be treated with some reserve, since the very fact that it was elicited by a questionnaire sampling opinion about mathematics may have created a response bias in favour of mathematics. There is inter-school variation, however, so that girls' schools gave higher percentages in favour of history and English, while boys' schools gave higher percentages in favour of mathematics and science subjects. The Hong Kong figures should be interpreted as supporting the hypothesis that student achievement is related to interest in the subject, and to a desire for more education, presumably of the same kind.

CHAPTER XIII

THE TEACHERS' VARIABLES

For Hong Kong, the questionnaire returned by the teachers also gave the basic information about the school, its name, size, type and time spent on mathematics. The teachers responded directly to questions seeking information as to sex, age, length and type of education, number of subjects taught, number of mathematics lessons per week, length of teaching experience, participation in in-service mathematics courses, availability of reference books and the possibility of extra curricular activities of a mathematical nature. To render the data comparable, table 13.1 adds the Hong Kong data to that of the twelve nations.

From further analysis it was found that the average size of schools in the Hong Kong sample was 912, but there is no correlation between school size and mathematics achievement. The teachers of mathematics are more likely to be male (64%), they are likely to be called upon to teach more than one subject (79% teach at least one other subject), so that the average number of subjects taught by these teachers is 2.14. Each teacher, on the average, teaches

23 mathematics lessons per week (there are forty lessons per week). Their experience ranges from zero up to twenty years with a mean of 6.57 years. Only three of the fourteen schools continue to teach only the tradition mathematics curriculum, two teach both old and "new", whilst nine teach only "new" mathematics.

The first item in table 13.1 shows that Hong Kong has the highest percentage of young teachers, under age of thirty, in great contrast to Germany's 12%. The rapid expansion of education in Hong Kong in the recent years accounts for this high percentage. For percentage of male mathematics teachers and the length of training, Hong Kong roughly stands in the middle of the array; but in professional training Hong Kong is far behind the other nations, for 36% of the mathematics teachers in the survey are untrained. They compensate by taking in-service courses. This helps to explain why there are 71% of teachers who have been in some mathematics training course concerned either with learning mathematics or the method of teaching mathematics. This high percentage may also explain the fact that the Hong Kong mathematics teachers have the greatest opportunity to teach "new mathematics", for 78% of

Table 13.1: Teachers variables (percentage)

Age under	Aus	Bel	Eng	Fin	Fra	Ger	Net	Isr	Jap	Sco	Swe	US	HK
30	55	47	31	29	33	12	19	24	26	26	31	31	57
Male	72	64	65	71	60	88	93	59	77	57	76	51	64
Year of post-secondary education	2.8	2.3	3.0	3.2	2.1	3.2	4.3	2.8	3.3	4.0	4.6	4.4	3.1
Type of professional training													
Teacher training college	45	85	46	58	53	60	88	46	63	21	75	18	50
University	45	11	47	30	16	25	1	32	20	74	8	82	14
Other	6	4	5	2	11	15	8	17	17	2	5	0	0
None	2	0	2	10	20	0	3	4	0	3	12	0	36
Inservice maths. course	23	4	20	11	34	18	12	59	28	13	23	60	71
Teaching "new math"	40	10	23	0	25	9	5	16	5	15	3	66	78

the Hong Kong mathematics teachers have to teach "new mathematics".

The variation between countries in the provision of in-service courses is great, varying from zero in France, 3% in Sweden to 66% in United States with the highest, 78% in Hong Kong. It seems that the opportunity to teach "new mathematics" may demand more in-service courses. In fact these two items are related, the correlation being 0.58 which is just significant at 0.5 level.

It will have been noted among the teacher variables that the Hong Kong teachers seem to lie at the extremes of the continua on which they are surveyed. They are the youngest, they possess the greatest number of untrained members, they rank with the countries low on the variable "number of years of post secondary education", but on the other hand they show the highest frequency of those attending courses, and they appear to have the greatest opportunity to teach "new mathematics".

In the original IEA project some 54 variables were selected for study in relation to the major hypotheses of the project. Using mathematics score as the criterion, some 26 variables were used in a correlation study followed by a regression analysis to determine the contribution of several of these variables to the total variance of the mathematics score. Six of these related to the teacher and were described as "teacher variables"[1]. Five of these were studied in relation to the population of Hong Kong teachers - the degree of freedom given to the teacher being omitted, since teachers are required to follow the prescribed syllabus. For all of the teacher variables combined in the IEA study, the best estimate of the variance explained by them is of the order of 5.5%. Only in England and Scotland do values of the order of 20% appear. These two countries have the highest standardized regression coefficients for the student's opportunities for learning, which appear to weigh heavily in the overall percentage of variance explained. It is believed that teachers' opinions and attitudes may contribute greatly to this. In Hong Kong this is less so. The student's ability to pass examinations would, prima facie, appear to have the greatest effect upon subsequent opportunity. This would act to reduce the teacher's effect, except

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1. Ibid., Vol. II, p.260 et seq.

in so far the teacher uses this threat as a motive-incentive condition for learning. On the other hand, the traditional Chinese position of a strong relation or bond between teacher and student, should considerably increase the effect. When the shortage of good text books reinforces the recourse of the student to the authority of the teacher as a major source of knowledge, the effect may be at least as great as in England and Scotland, but for entirely different reasons.

CHAPTER XIV

SUMMARY AND CONCLUSIONS

Impressed with the major aim of the Project for the International Study of Achievement in Mathematics (IEA), with the aid of psychometric techniques to compare outcomes in different educational systems,[1] and the necessity for further work of this kind, a study of similar intent was carried out on a sample of Hong Kong schools. The IEA report suggested that among the further research needed case studies of particular countries would be undertaken.

> "Such studies would have to imply through analyses of curricula, text-books, and other learning materials. In some instances, special attention would have to be directed to incentives used in instruction, the relation between learning at school and at home, to what extent the parents are supporting the school" etc.[2]

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1. Ibid., Vol I, p.30.
2. Ibid., Vol I, p.309.

The present study cannot be regarded as a case study in that sense, though some attention was directed to some of the aspects raised there. Rather it was an attempt to show what the individual researcher can accomplish in this field, given only the support he can muster from personal contacts and official position. It is necessarily limited in the scope of its findings, which may not be valid for any Hong Kong schools outside the sample used. Nevertheless it is hoped that the analysis may reflect many aspects of education, especially mathematics education, in Hong Kong.

In Part I, the educational system in Hong Kong has been described in considerable detail. Within this setting the teaching of mathematics and its reform was discussed. The second part describes the attempt to measure achievement against this background. It is still not known how the recent reform of mathematics teaching affects achievement in the subject. Has the reform raised the mathematics standard in Hong Kong or lowered it instead? To answer this question investigation is needed of more schools of equal academic standing but teaching two different syllabuses. In this application as only three schools are still teaching traditional mathematics, the small size of the sample could not be sufficient to draw valid conclusion. In addition the difference in type of schools would make such comparison meaningless.

One may be reasonably sure that the teaching of modern topics in most of the schools in Hong Kong has contributed to the high scores made on some of these modern items in the test. Many of these items may possess little intrinsic difficulty but owe their present attributed difficulty to the small number of children able to solve them because they have not had the necessary exposure and experience. But whether the reform of teaching has improved mathematics performance as a whole, or just the part on the new topics added is still unknown. The test itself was not designed to test for such an effect, but was designed to test attainment in general and to relate it to certain social factors.

However the high achievement in Hong Kong in the test -- 1.41 standard deviation higher than the all IEA mean -- needs more explanation. After the analyses of the factors related to achievement, the followings were suspected as the main causes:

(1) Selective system.

The exact figure for the percentage of the target population, the age group 13:0 - 13:11, in school was not available. It was estimated as follows. In 1968, the number of children in primary school was 104.8% of the age group 6-11. Excluding 20.2% of over aged children, there were 84.6% of the age group children in primary schools. Of all those who had completed primary schooling 69.4% were admitted into the secondary school certificate course.[1] This means that the first year in secondary school would have 58.7% of the age group of children in Hong Kong. With a drop out rate of 24% over five years, the second year of secondary education would have an estimated student population around 55% of the whole of age group. As admission to secondary school is largely determined by examinations, the 55% of students in school could be the upper 55% in ability. It was therefore more comparable to regard Hong Kong's achievement as though made by the upper 50% of students. Under this comparison Hong Kong's achievement would be the third-highest in the list of the test's users.

(2) Heavy mathematics syllabus.

Hong Kong is perhaps the only area in the IEA test

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1. Hong Kong Education Department, <u>Annual Summary 1967-68</u> (Hong Kong, 1968), pp.18-19.

which has taught at this level all the topics tested in the 1b papers. In fact, many teachers in Hong Kong who saw this test paper unanimously commented on the easiness of the paper. They were of the opinion that the questions could be answered without much difficulty by the primary six pupils in Hong Kong. It was reported that the majority of students completed the paper in half of the time assigned to these papers.

(3) Mathematics home work.

The average time spent on mathematics home work is highest in Hong Kong. It is 4.9 hours per week. Even at the primary level, the students have to do mathematics home work regularly. It may come to as much as half an hour every week day or two and half hours a week, being particularly heavy at the primary six and five level when pupils prepare themselves for the important secondary School Entrance Examination. Few amongst the IEA countries assign home work for primary students. The constant home work could have given Hong Kong students more chance to drill the basic skills and to solve problems.

(4) The Chinese language.

The Chinese characters are symbols. Mathematics is also a language of symbols. The similarity of these two may have some effect on the learning. For one used to writing symbols (character) to express himself must find it easier to learn to use symbols for mathematical expressions. In the comparison, it was found that the Hong Kong students were particularly good in the algebraic items.

(5) Strict discipline.

Mathematics has always been regarded as a subject which conferred special powers on the minds of those who studied it. The belief persists, although the form of the explanation may differ. It requires the observation of rules, formulae and methods of manipulation. Learning mathematics may discipline one's mind and a disciplined person may be more ready to learn mathematics. In Hong Kong the highly authoritarian position traditionally ascribed to teachers forces some discipline upon students, who are used to following instructions from a visible authority. They accept the given rules, and learn to apply methods according to set patterns. Mathematically, algorithms are a very part of their existence. This characteristic may make mathematics easier to learn at this stage, although the strict discipline may be harmful later when more imagination and creativeness become more important at the higher level of learning. Can algorithms become heuristic?

It was not the main concern of this survey to compare mathematics achievement in Hong Kong with other nations, for the comparison of different countries with different educational systems is not yet meaningful. The main concern of this survey was to measure the effect of possible variables contributing to mathematics learning in Hong Kong. For this, a number of factors were analyzed. Some obvious relations were seen to exist, some were not so clear.

Finally, to conclude this report, a few further points are noted:-

1) The omission of Euclidean geometry in the new syllabus needs to be reviewed. For the test revealed the comparative weakness of Hong Kong students on this type of question. Perhaps some of the elementary geometrical properties should be included at this level as informal study before the teaching of analytical geometry.

2) The findings confirmed the relation between achievement and expenditure on schools. For the further improvement

of education in Hong Kong it will be necessary to increase the investment particularly in terms of equipment and qualified staff.

3) The high proportion of untrained teachers in Hong Kong shows that there is a need to expand teacher education. As most of those untrained teachers are university graduates the education departments of the two universities in Hong Kong should be expanded.

There are a great many questions unanswered by this analysis, in particular, the effect of the modernization of mathematics teaching in recent years. Further large scale researches would be needed to determine the causes and effects of educational changes and to guide the further development of education in Hong Kong.

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Appendix

A. Hong Kong English School Leaving Certificate Examination Mathematics B Syllabus.

The Hong Kong Mathematics Test:

B. Students questionnaire

C. Teachers questionnaire

D. Item analysis forms

E. Test papers - X paper, Y paper, Z paper.

F. Model Answers

(FOR INTRODUCTION IN 1969)

MATHEMATICS

SYLLABUS B

GENERAL

1. There will be two papers of 2 hours each.

2. The aim of the examination is to test ability to understand and to apply mathematical concepts rather than to test ability to perform lengthy manipulations.

3. The syllabus sets out only the final achievements expected of a candidate on completion of a five-year secondary course, and for teaching purposes some items may have to be dealt with repeatedly with different approaches at different stages before reaching the requirements of the syllabus. The order of the various topics in the syllabus does not suggest the sequence in which they should be taught.

4. Candidates will be expected to do some deductive thinking and to do some simple proofs. Credit will be given to a clear and systematic presentation of an argument. Symbolic expressions are often helpful in making statements concise and precise, and candidates will be expected to be familiar with the use of approved symbols which are listed in the syllabus. However, a strict axiomatic treatment of the topics will not be required; and some sections, where specified, will be treated entirely by intuitive methods.

5. Slide Rules, flexible and French curves may be brought into the examination. Four-figure mathematical tables will be provided.

EXAMINATION SYLLABUS

Knowledge of Primary School Mathematics is assumed.

Rough estimates, approximations, significant figures and limits of accuracy.

Primes and factorisation of natural numbers. Principles of simple divisibility tests for 2, 3, 4, 5, 8, 9, 11. L.C.M. and H.C.F. including the general principles of finding the H.C.F. (Euclidean algorithm).

Simple statements, the negation of a statement ($\sim$), compound statements using connectives "and" ($\wedge$), "or" ($\vee$), "if . . . then" ($\Rightarrow$), "if and only if" ($\Leftrightarrow$). Truth values and truth tables. The use of the above in presenting arguments. (The emphasis will be on the understanding and presentation of logical arguments rather than on formal manipulations).

APPENDIX A

Principle of mathematical induction.

Sets, member (or element) of a set, subset, union, intersection, difference, complement, universal set and empty set. Venn diagrams and their use in illustrating set operations and in solving problems. (Approved symbols: $\{\ldots\}$, $\{:\}$, $\in$, $\notin$, $\subset$, $\not\subset$, $\cup$, $\cap$, $-$, $'$, $\emptyset$). The use of composition laws for sets including $A \cup A = A$, $A \cap A = A$, $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$ (formal proofs will not be required). Ordered pairs, simple ideas of mappings (or functions).

Representation of integers by means of different bases, including base 2, (The number a to base b will be expressed as a_b with b always in denary). Simple flow charts.

Informal discussion of integers, rational numbers, the real number system and the complex number system. (The concept of ordered pairs applied to the above where appropriate).

Binary operations. Informal discussion of associativity, commutativity, distributivity, neutral element and inverse. Modulo arithmetic treated intuitively. The elementary idea of a group and its sub-groups as illustrated by examples such as modulo arithmetic, integers and addition, non-zero rational numbers and multiplication, rotation and reflection of triangles, quadrilaterals and regular n-sided polygons, matrices and matrix-multiplication.

$m \times n$ matrices with $1 \leq m, n \leq 3$, and operations on them: addition, multiplication and scalar-multiplication. System of m linear equations in n unknowns with $1 \leq m, n \leq 3$. Square matrices and determinants of order ≤ 3. Unit matrices, non-singular matrices and simple numerical cases of their inverses. Vectors and simple applications in geometry.

Knowledge of the following will be assumed but no formal proofs of any kind will be required: Parallel lines and their tests. Angle sums of triangles and polygons, base angles of an isosceles triangle, equiangular triangles have their corresponding sides proportional, Pythagoras' relation, angle in the alternate segment, angles in the same segment, angles in a cyclic quadrilateral. Lengths, areas and volumes; mensuration of common plane and solid figures, including polygons, polyhedra, circle, cylinder, cone and sphere. Plans and maps, areas and volumes of similar figures.

Simple transformations of the plane; reflection, rotation and translation, combination of the above transformations; identity and inverse transformation. The transformations connecting directly or oppositely congruent figures. The ideas of shearing and stretching. Transformations in terms of co-ordinates: reflection in the lines $x = 0$, $y = 0$, $x = \pm y$; rotation through multiples of 90°. Expression, by 2×2 matrices, of reflection, rotation, enlargement and shearing.

Symmetry about a point, a line and a plane.

Informal discussion of simple networks: odd nodes, even nodes, unicursal (one-stroke) network, matrix description of a network.

Three-dimensional figures: angle between a straight line and a plane and between two planes, nets of solids, Euler's relation for convex polyhedra. Plans and elevations (ability to produce technically correct figures will not be required).

Polynomials in one variable and their fundamental operations (questions will not be set on lengthy multiplication and division). Simple algebraic fractions. Factorisation of $ax + bx$, $a^2 - b^2$, $a^2 \pm 2ab + b^2$. Easy identities, easy equations including the general quadratic equation.

Functional relations and graphs. Simple curves including the curve $y=e^x$. Gradients of curves by drawing, estimation of area under curves by square counting or trapesium rule. The idea of rate of change.

Laws of indices. Slide rule.

Rectangular Cartesian co-ordinates in 2 and 3-dimensional spaces. Equations of lines in 2-dimensional space and planes in 3-dimensional space. Simple loci in 2 and 3-dimensional spaces.

Measurement of angles (degrees and radians). The functions sine, cosine, tangent and their graphs. The relations $\tan A = \sin A/\cos A$, $\sin^2 A + \cos^2 A = 1$. Solution of triangles in cases reducible to right-angled triangles, Simple 3-dimensional problems. Courses and bearings. Latitude, longitude, great and small circles, nautical miles, distances along parallels of latitudes and along meridians.

Simple probability theory including application of the sum and product laws.

Collection and organisation of numerical data, and their graphical representation by bar chart, pie chart, frequency polygon and cumulative frequency polygon, histogram. Calculation of the mean. Estimation of the median and quartiles, inter-quartile range.

Appendix B

THE NCE MATHEMATICS TEST

I STUDENT QUESTIONNAIRE

Please answer all the following questions.

1. Name of School ______________________________

2. Name of student ______________________________

3. Age: ____ years and ____ months. 4. Class _____ 5. Sex: boy ☐ girl ☐

(Check the appropriate answer box with ✓)

6. How many hours a week do you usually spend on home work? Mathematics: _____ hours Other subjects: _____ hours

7. Do you receive private coaching? (including those from family members) Yes ☐ No ☐ if yes, say how many hours. ______ hours.

8. Estimate the living area of your home.
   - Under 100 sq ft ☐
   - 100-200 . . ☐
   - 201-400 . . ☐
   - 401-700 . . ☐
   - over700 . . ☐

9. What is your father's occupation? __________

10. For how many years did your father receive full-time education?
    - 0 year ☐
    - 1-3 ☐
    - 4-6 ☐
    - 7-9 ☐
    - 10-12 ☐
    - 13-14 ☐
    - over14 ☐

11. How many more years of full time education do you expect to complete?
    - 1-2years ☐
    - 3-4 ☐
    - 5-6 ☐
    - 7-8 ☐
    - 9or more ☐

12. How many years have you learnt "modern maths."?
    - Oyear ☐
    - 1 ☐
    - 2 ☐
    - 3 ☐
    - 4or more ☐

13. Give 3 subjects you like most in order of preference. (1)__________ (2)__________ (3)__________

WAIT for instruction to turn this page.

Appendix C

TEACHER QUESTIONNAIRE

(Invigilator please write down the following answers by interviewing the mathematics teacher).

The School

1. Name of the School: ______________________________

2. Type of the School: ____________ 3. Size of the School: ________

4. Number of students in test: Boys ________ Girls ________ Total ________

5. Time of mathematics teaching of the class in test per week: ________ periods of ________ minutes each.

The Teacher

6. Sex ________

7. About what age? ________

8. Education: University graduate
   Trained university graduate
   Trained teacher from college of education, ______ year-course
   Post-secondary college graduate
   Trained Post-sec. college grad.
   Others
   ____

9. How many subjects does he(she) teach? ________

10. How many mathematics lessons does he(she) teach per week? ________

11. How many year teaching experience has he(she)? ________

12. Has he (she) attended any course dealing with modern maths? yes no
    If yes, give the length of the course. ______ hours.

13. What mathematics text book(s) is(are) in use?

____

14. What other mathematics books are used for reference?

____

15. Is there any extra-curriculum mathematics activity in the school? ______________

16. Other comments on syllabus, textbooks, teaching methods, difficulties etc:

Appendix D

Item Analysis I

School:____. Boys/Girls

(Use separate sheets for boys and girls)   No. of students:_____

Student	Paper X	Y	Z	Age in months	Score	Math home work	Private lesson	Living cond.	Father occup.

Item Analysis II

Distribution of Scores and Age for EACH school.

School:_______________________________________.

I Score

	Boys	Girls	Total	Cum.Freq
0				
1-2				
3-4				
5-6				
7-8				
9-10				
11-12				
13-14				
15-16				
17-18				
19-20				
21-22				
23-24				
25-26				
27-28				
29-30				
31-32				
33 and over				
Total				
	Mean=	Mean=		
	S.D.=	S.D.=		

II Age

months	Boys	Girls	Total
Under 140			
140-142			
143-145			
146-148			
149-151			
152-154			
155-157			
158-160			
161-163			
164-166			
167-169			
170-172			
173-175			
176-178			
179-181			
182-184			
185-187			
over187			

Over all mean=

S.D.=

Item Analysis III

Degree of difficulty of each question for a School.

School.________________________________ Class.____

Number of students in the test. ____________. Paper X ______
(total) Y ______
Z ______

Degree of difficulty of a question in Paper X = $\frac{\text{No. of students gave CORRECT answer to it}}{\text{Total No. of students in the test doing Paper X}}$

For example: In school ABC, 15 students did Paper Y and 10 students gave oorrect answer to question no.31, then $\frac{10}{15}$ is the degree of difficulty of this question.

$\therefore$ Degree of difficulty of question no.31 in Paper Y = $\frac{10}{15}$ = .67 (to two decimals)

Question	Paper X	Paper Y	Paper Z
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			

Question	Paper X	Paper Y	Paper Z
19			
20			
21			
22			
23			
24			
25			
26			
27			
28			
29			
30			
31			
32			
33			
34			
35			

1. How many seven-man teams can you make out of 7 nine-man team?

   A. 7 B.8 C.9 D.16 E.63 Ans._____

2. (22 x 18)-(47+59) is equal to

   A.290 B.300 C.384 D.480 E.502 Ans._____

3. In the division on the right, the correct answer is

   $.004\overline{)24.56}$

   A. .614
   B. 6.14
   C. 61.4
   D. 614
   E. 6140 Ans._____

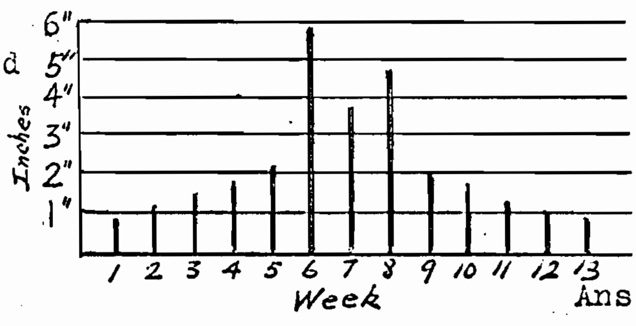
4. The sum of $9\frac{4}{5}$ and $13\frac{1}{4}$ is equal to

   A. $22\frac{5}{9}$ B. $22\frac{9}{20}$ C. 23 D. $23\frac{1}{20}$ E. $23\ \frac{1}{5}$ Ans._____

5. The ratio of 2 to 5 equals the ratio of what number to 100? Ans._____

6. In the graph on the right, rainfall in inches is plotted for 13 weeks. The average weekly rainfall during the period is approximately

   A. 1inch B. 2 inches
   C. 3 inches D. 4 inches
   E. 5 inches



Ans._____

7. Which of the following operations with whole numbers will <u>always</u> give a whole number?
   I.Addition II.Multiplication III.Division

   A. I only B.II only C.III only
   D. I and II only E. II and III only Ans._____

8. The value of $2^3 \times 3^2$ is

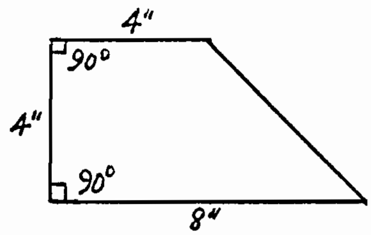
A. 30 B. 36 C. 64 D. 72 E. none of these Ans._____

9. If the selling price of an article was \$55 and a profit of 10% was made on the cost Price, what was the cost price in dollars? Ans._____

10. The value of 0.2131×0.02958 is approximately

A. 0.6 B. 0.06 C. 0.006 D. 0.0006 E. 0.00006 Ans._____

11. There is a brass plate of the shape and dimensions shown in the adjoining figure. What is its area in square inches?



A. 16 B. 24 C. 32 D. 64 E. 96 Ans._____

12. If P=LW and if P=12 and L=3, then W is equal to

A. $\frac{3}{4}$ B. 3 C. 4 D. 12 E. 36 Ans._____

13. One bell rings every 8 minutes, while another bell rings every 12 minutes. They have rung together once at the same moment. After how many minutes will they ring together again

A. for the first time? Ans._____

B. for the second time? Ans._____

C. for the tenth time? Ans._____

14. A shopkeeper has x lb of tea in stock. He sells 15 lb and then receives a new lot weighing 2y lb. What weight of tea does he now have?

A. x-15-2y B. x+15+2y C. x-15+2y

D. x+15-2y E. none of the above Ans._____

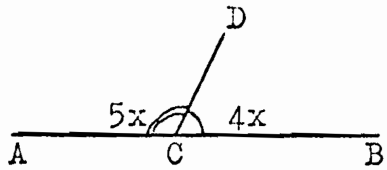
15. Simplify: 5x+3y+2x-4y

A. 7x+7y B. 8x-2y C. 6xy

D. 7x-y E. 7x+y Ans._____

16. What is the value of (-6)-(-8)? Ans._____

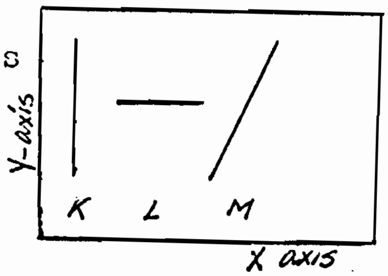
17. If AB is a straight line, what is the measure in degrees of angle BCD in the figure on the right?



A. 20 B. 40 C. 50
D. 80 E. 100

Ans._____

18. Imagine that the geometrical figures K,L,M,N and O have been drawn on a rubber sheet. The lines are assumed to have no width. The rubber sheet is stretched parallel to the X-axis while leaving all the distances measured parallel to the Y-axis unchanged. The stretching is uniform, that is, the same for every part of the sheet.
For which of the segments K,L,M will the length remain unchanged?



A. only K B. only L C. only M D. K and L
E. K and M

Ans._____

19. If $\frac{4x}{12} = 0$, then x is equal to

A. 0 B. 3 C. 8 D. 12 E.16

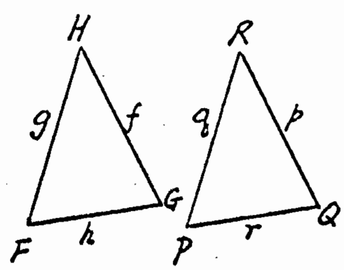
Ans._____

20. If x=-3, the value of -3x is

A. -9 B. -6 C. -1 D. 1 E. 9

Ans._____

21. Which of the following sets of conditions is not sufficient for the congruence of $\triangle$ FGH and $\triangle$ PQR when f is less than g?



A. $\angle$F=$\angle$P g=q f=p

B. $\angle$F=$\angle$P h=r $\angle$G=$\angle$Q

C. g=q $\angle$F=$\angle$P h=r

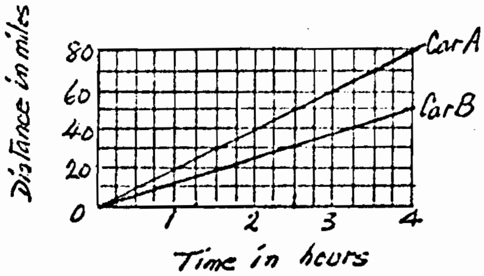
D. h=r g=q f=p

E. f=p $\angle$G=$\angle$Q h=r

Ans._____

22. Use the graph on the right. Three hours after starting, car A is how many miles ahead of car B?

A. 2 B. 10 C. 15 D. 22.5 E. 25



Ans._____

23. Use the same graph in the above question. How much longer does it take car B to go 50 miles than it does for car A to go 50 miles?

A. 1 hour 15 minutes B. 1 hour 30 minutes
C. 2 hours D. 2 hours 30 minutes
E. 2 hours 35 minutes

Ans._____

24. Let the symbol $\overline{a,b}$ denote the set of integers between a and b. For example, $\overline{3,7}$ consists of the integers 4,5 and 6. Which of the following pairs of sets has a larger number of integers in common than any of the other pairs?

A. $\overline{0,15}$ and $\overline{7,20}$ B. $\overline{5,15}$ and $\overline{16,30}$
C. $\overline{5,14}$ and $\overline{5,17}$ D. $\overline{4,18}$ and $\overline{8,20}$
E. $\overline{0,12}$ and $\overline{6,12}$

Ans._____

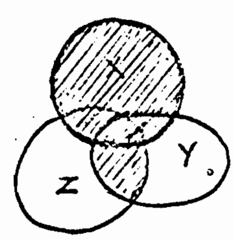
25. There are 227 boys in a school. Every boy in the school belongs to either the music club or the sports club, and some boys belong to both clubs. The music club has 120 members, and 36 of these are also members of the sports club. What is the total membership of the sports club?

Ans._____

26. The lengths of the sides of a triangle XYZ are 4,7 and 10. If a similar triangle has a perimeter of 147, what is the length of its shortest side?

Ans._____

27. The symbol $P \cap Q$ represents the intersection of sets P and Q and the symbol $P \cup Q$ represents the union of sets P and Q. Which of the following represents the shaded portion of the diagram?



A. $(X \cap Y) \cup Z$ B. $X \cup (Y \cap Z)$ C. $X \cap (Y \cup Z)$
D. $(X \cap Y) \cap Z$ E. $(X \cup Y) \cap Z$

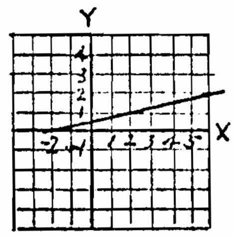
Ans._____

28. In the solution of the following system of equations,

$$\begin{cases} 2x+y=7 \\ x-4y=4 \end{cases}$$ the value of y equals to

A. $-\frac{5}{3}$ B. -9 C. $\frac{1}{9}$ D. $-\frac{1}{9}$ E. $\frac{5}{3}$ Ans.____

29. The equation of the line shown in the graph is



A. $x+4y=4$
B. $2x-y=4$
C. $2x=y-2$
D. $x-4y+2=0$
E. $4x-y=2$

Ans.____

30. Which of the following is true for any parallelogram ABCD which has an acute angle at B and diagonals AC and BD?

A. $AB < BC$ B. $AB=BC$ C. $AB > BC$
D. $AC < BD$ E. None of them Ans.____

31. Which of the following numbers in base two is (are) even?
I. 110011 II. 110010
III. 110101 IV. 100100

A. I only B. III only C. I and III only
D. II and IV only E. I, III and IV Ans.____

32. The distance between two towns A and B, is 150 kilometers. This distance is represented on a certain map by a length of 30 centimeters. The scale of this map is

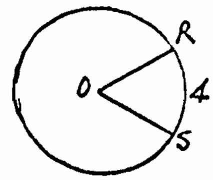
A. 1/500,000 B. 30/150 C. 1/20,000
D. 1/5,000 E. 1/200,000 Ans.____

33. The expression $\frac{a}{b-c} + \frac{a}{c-b}$ where $a \neq 0$, is equal to

A. 0 B. $\frac{2a}{b-c}$ C. $\frac{a}{b^2-c^2}$

D. $\frac{a}{2b}$ E. 2a Ans.____

34. The length of the circumference of the circle on the right with center at O is 24 and the length of arc RS is 4. What is the measure in degrees of the central angle ROS?



A. 24 B. 30 C. 45 D. 60 E. 90

Ans._____

35. Given any fraction whose numerator is less than the denominator, if you then add 2 to both the numerator and the denominator, the new fraction is

A. Equal to the original fraction
B. larger than the original fraction
C. twice the original fraction
D. smaller than the original fraction
E. 1 more than the original fraction

Ans._____

II Mathematics Test   Time: $1\frac{1}{2}$ hours.   PAPER Y

1. Which of the following is the same as a quarter of a million?

   A. 25,250   B. 40,000   C. $\frac{1}{4,000,000}$ D. 250,000   E. 2,500,000   Ans._____

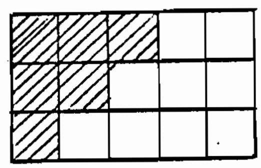
2. Peter and Paul decided to start saving money. Peter can save 3 dollars each month and Paul can save 5 dollars. At this rate, after how many months will Paul have exactly 10 dollars more than Peter?

   A. 2
   B. 3
   C. 4
   D. 5
   E. 8   Ans._____

3. 0.40 x 6.38 is equal to

   A. .2552   B. 2.452   C. 2.552   D. 24.52   E. 25.52   Ans._____

4. In the figure on the right the little squares are all the same size and the area of the whole rectangle is equal to 1. The area of the shaded part is equal to



   A. $\frac{2}{15}$ B. $\frac{1}{3}$ C. $\frac{2}{5}$ D. $\frac{3}{8}$ E. $\frac{1}{2}$ Ans._____

5. The arithmetic mean (average) of 1.50, 2.40, 3.75 is equal to

   A. 2.40   B. 2.55   C. 3.75   D. 7.65   E. none of these   Ans._____

6. Which of the following operations with whole numbers will <u>always</u> give a whole number?
   I. Addition   II. Multiplication
   III. Division

   A. I only
   B. II only
   C. III only
   D. I and II only
   E. II and III only   Ans._____

7. If the selling price of an article was $55 and a profit of 10% was made on the Cost Price, what was the cost price in dollars?   Ans._____

8. On level ground, a boy 5 feet tall cast a shadow 3 feet long. At the same time a nearby telephone pole 45 feet high casts a shadow the length of which, in feet, is

   A. 24   B. 27   C. 30   D. 60   E. 75   Ans._____

9. A box has a volume of 100 cc. Another box is twice as long, twice as wide and twice as high. How many cc is the volume of the second box? Ans._____

10. The value of 0.2131x0.02958 is approximately

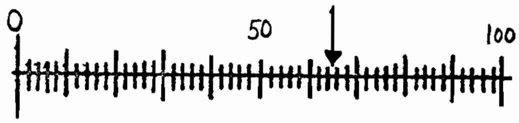
A. 0.6
B. 0.06
C. 0.006
D. 0.0006
E. 0.00006

Ans._____

11. A runner ran 3,000 meters in exactly 8 minutes. What was his average speed, in meters per second?

A. 3.75 B. 6.25 C. 16.0 D. 37.5 E. 62.5 Ans._____

12. On the scale to the right, the reading indicated by the arrow is between



A. 51 and 52 B. 57 and 58 C. 60 and 62
D. 62 and 64 E. 64 and 66

Ans._____

13. What is the square root of 12x75?

A. 6.25 D. 625
B. 30 E. 900
C. 87

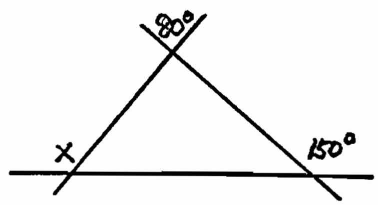
Ans._____

14. If x+y = 4 and x-y =2, then x is equal to

A. 0 C. 2 E. 6
B. 1 D. 3

Ans._____

15. Three straight lines intersect as shown in the figure on the right. What is x equal to in degrees?



A. 30 B. 50 C. 60 D. 110 E. 150

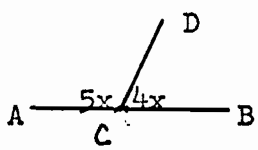
Ans._____

16. At 4 o'clock, the measure of the angle between the minute hand and the hour hand of a clock, in degree, is

A. 30 B. 45 C. 60 D. 90 E. 120 Ans._____

17. What is the value of(-6)-(-8)? Ans._____

18. If AB is a straight line, what is the measure in degrees of angle BCD in the figure on the right?



A.20
B.40
C.50
D.80
E.100

Ans._____

19. If x= -3, the value of -3x is

A. -9 B. -6 C. -1 D. 1 E. 9 Ans._____

20. The floor of a room is covered with wooden rectangular blocks. When blocks measuring a inches by b inches are used, M blocks are needed. If blocks fit exactly, how many blocks will be needed if each block measures x inches by y inches?

A. $\frac{Mab}{xy}$ B. $\frac{ab}{Mxy}$ C. $\frac{(a+b)M}{x+y}$ D. $\frac{ab \cdot xy}{M}$ E. $\frac{Mxy}{ab}$ Ans._____

21. A factory produces m units per week. How many units per week will it produce after production is increased p percent?

A. $100p+m$ B. $100m+mp$ C. $\frac{m+mp}{100}$ D. $m+\frac{mp}{100}$ E. $\frac{p}{100}+m$ Ans._____

22. Which of the following is(are) true?

I. $(53 \times 73) \times 17 = 53 \times (73 \times 17)$
II. $133 \times (78+89) = (133 \times 78)+89$
III. $133 \times (78+89) = (133 \times 78)+(133 \times 89)$

A. I only
B. II only
C. III only
D. I and II only
E. I and III only

Ans._____

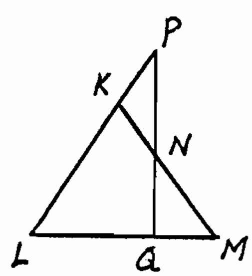
23. Let the symbol $\overline{a,b}$ denote the set of integers between a and b. For example, $\overline{3,7}$ consists of the integers 4, 5 and 6. Which of the following pairs of sets has a larger number of integers in common than any of the other pairs?

A. $\overline{0,15}$ and $\overline{7,20}$
B. $\overline{5,15}$ and $\overline{16,30}$
C. $\overline{5,14}$ and $\overline{5,17}$
D. $\overline{4,18}$ and $\overline{8,20}$
E. $\overline{0,12}$ and $\overline{6,12}$

Ans._____

24. There are 227 boys in a school. Every boy in the school belongs to either the music club or the sports club, and some boys belong to both clubs. The music club has 120 members, and 36 of these are also members of the sports club. What is the total mambership of the sports club? Ans._____

25. In $\triangle$KLM on the right, KL=KM, PQ$\perp$LM, and LP is a straight line. Then $\triangle$ NKP is isosceles because



A. $\angle$P=$\angle$KNP, since both are complements of the equal angles L and M.

B. NK = PK, since $\angle$P=$\angle$M.

C. its sides are parallel to the sides of $\triangle$KLM.

D. its sides are perpendicular to the sides of $\triangle$KLM.

E. $\angle$P=$\angle$KNP, since both are half the supplement of angle M.

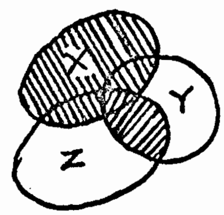
Ans._____

26. What are all values of x for which

the inequality $5x+\frac{5}{3} \leqslant -2x-\frac{2}{3}$ is true?

A. $x \leqslant -\frac{7}{9}$ B. $x \leqslant -\frac{1}{3}$ C. $x \geqslant 0$ D. $x \geqslant \frac{7}{3}$ E. $x \geqslant \frac{9}{3}$ Ans._____

27. The symbol $P \cap Q$ represents the intersection of sets P and Q and the symbol $P \cup Q$ represents the union of sets P and Q. Which of the following represents the shaded portion of the diagram?



A. $(X \cap Y) \cup Z$ B. $X \cup (Y \cap Z)$ C. $X \cap (Y \cup Z)$
D. $(X \cap Y) \cap Z$ E. $(X \cup Y) \cap Z$ Ans._____

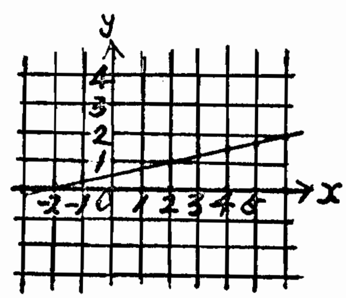
28. In the solution of the system of equations;- $\begin{cases} 2x+y=7 \\ x-4y=4 \end{cases}$

the value of y is equal to

A. $-\frac{5}{3}$ B. -9 C. $\frac{1}{9}$ D. $-\frac{1}{9}$ E. $\frac{5}{3}$ Ans._____

29. The equation of the line shown in the graph is

A. $x+4y=4$
B. $2x-y=4$
C. $2x=y-2$
D. $x-4y+2=0$
E. $4x-y=2$



Ans._____

30. Which of the following is true for any parallelogram ABCD which has an acute angle at B and diagonals AC and BD?

A. $AB < BC$ B. $AB=BC$ C. $AB > BC$ D. $AC < BD$
E. none of them Ans.____

31. Which of the following numbers in base two is (are) even?

I. 110011 II. 110010
III. 110101 IV. 100100

A. I only B. III only C. I and III only
D. II and IV only E. I, III and IV Ans._____

32. Each of 9 boys had t marbles. In order to play a game, they divided the marbles among 12 boys in such a way that each had the same number. How many marbles did each of the 12 have?

A. $\frac{3t}{4}$ B. t-3 C. $\frac{4t}{3}$

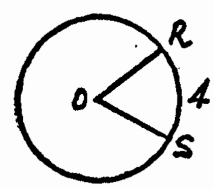
D. 9t-12 E. 12t-9 Ans._____

33. The expression $\frac{a}{b-c} + \frac{a}{c-b}$ where $a \neq 0$, is equal to

A. 0 B. $\frac{2a}{b-c}$ C. $\frac{a}{b^2-c^2}$

D. $\frac{a}{2b}$ E. 2a Ans._____

34. The length of the circumference of the circle on the right with center at O is 24 and the length of arc RS is 4. What is the measure in degrees of the central angle ROS?



A. 24 B. 30 C. 45

D. 60 E. 90 Ans._____

35. Which of the following equals 7x(3+9)?

A. (7x3)+(7x9)

B. (7x9)+(3x9)

C. (7x3)+(3x9)

D. 7x27

E. 21+ 9 Ans._____

II. Mathematics Test   Time: $1\frac{1}{2}$ hours   PAPER Z

1. 43.0 - 17.6   Ans._____

2. $\frac{2}{5} + \frac{3}{8}$ is equal to A. $\frac{5}{13}$ B. $\frac{5}{40}$ C. $\frac{6}{40}$
   D. $\frac{16}{15}$ E. $\frac{31}{40}$ Ans._____

3. Which of the following is the same as a quarter of a million?
   A. 25,250 B. 40,000 C. $\frac{1}{4,000,000}$
   D. 250,000 E. 2,500,000   Ans._____

4. (22 x 18) - (47 + 59) is equal to
   A. 290 B. 300 C. 384 D. 408 E. 502   Ans._____

5. The ratio of 2 to 5 equals the ratio of what number to 100 ?   Ans._____

6. Which of the following operations with whole numbers will always give a whole number?
   I. Addition II. Multiplication III. Division
   A. I only B. II only C. III only
   D. I and II only E. II and III only   Ans._____

7. In a given triangle the measures of two angles in degrees are 60 and 70. What is the measure of the third angle in degrees?   Ans._____

8. The value of $2^3 \times 3^2$ is
   A. 30 B. 36 C. 64 D. 72 E. none of these   Ans._____

9. The value of 0.2131 x 0.02958 is approximately

A. 0.6 B. 0.06 C. 0.006 D. 0.0006 E. 0.00006 Ans._____

10. Joe had three test scores of 78,76 and 74, while Mary had scores of 72,82 and 74. How did Joe's average compare with Mary's?

A. Joe's was 1 point higher
B. Joe's was 1 point lower
C. Both averages were the same
D. Joe's was 2 points higher
E. Joe's was 2 points lower

Ans._____

11. What is the square root of 12 x 75 ?

A. 6.25 B. 30 C. 87 D. 625 E.900 Ans._____

12. Which of the following is <u>false</u> when a and b are different real numbers:

A. (a + b) + c = a + (b + c)
B. ab = ba
C. a+b=b+a
D. (ab)c = a(bc)
E. a - b = b - a

Ans._____

13. If x+y=4 and x-y=2, then x is equal to

A. 0 B. 1 C.2 D. 3 E. 6 Ans._____

14. One bell rings every 8 minutes, while another bell rings every 12 minutes. They have rung together once at the same moment. After how many minutes will they ring together again?

Ans. For the first time:_____
second time:_____
tenth time:_____

15. Simplify 5x+3y+2x-4y

A. 7x+7y B. 8x-2y C. 6xy
D. 7x-y E. 7x+y

Ans._____

16. At 4 o'clock, the measure of the angle between the minute hand and the hour hand of a clock, in degrees, is

A. 30   B. 45   C. 60   D. 90   E. 120   Ans._____

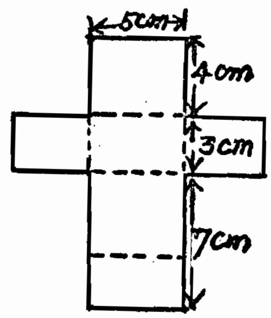
17. If $\frac{x}{2} < 7$, then

A. $x < \frac{7}{2}$ B. $x < 5$ C. $x < 14$ D. $x > 5$ E. $x > 14$ Ans._____

18. Any two regular polygons with the same number of sides are

A. congruent   B. noncongruent
C. Similar   D. not similar
E. equal in area   Ans._____

19. A piece of tin with dimensions as shown is to be folded along the dotted lines to make a box. What is the volume, in cubic centimeters, enclosed in the box?

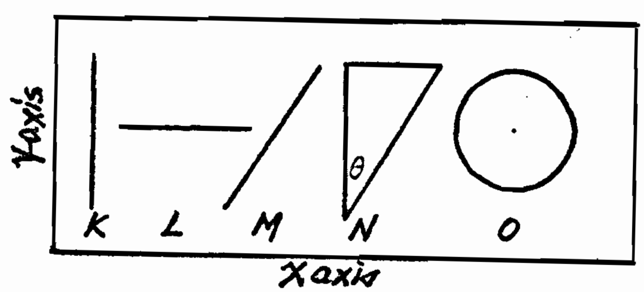


Ans._____

20. If x=y=z=1, then $\frac{x-z}{x+y}$ is equal to

A. -2   B. -1   C. 0   D. $\frac{1}{2}$ E. 1   Ans._____

21. Imagine that the geometrical figures K,L,M,N and O have been drawn on a rubber sheet. The lines are assumed to have no width. The rubber sheet is stretched parallel to the X-axis while leaving all the distances measured parallel to the Y-axis unchanged. The stretching is uniform, that is, the same for every part of the sheet.



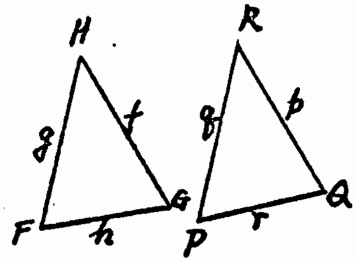
What will happen to the measure of angle θ of triangle N?

A. It will remain the same
B. It will become larger
C. It will become smaller
D. One cannot tell from the data whether A,B or C is correct   Ans._____

22. In the above question, what will happen to circle O?

A. It will still be a circle.
B. It will no longer be a circle.
C. One cannot tell the data whether A or B is correct. Ans.____

23. Which of the following sets of conditions is not sufficient for the congruence of $\triangle$ FGH and $\triangle$ PQR on the right when f is less than g?



A. $\angle F=\angle P$, $g=q$, $f=p$

B. $\angle F=\angle P$, $h=r$, $\angle G=\angle Q$

C. $g=q$, $\angle F=\angle P$, $h=r$

D. $h=r$, $g=q$, $f=p$

E. $f=p$, $\angle G=\angle Q$, $h=r$

Ans.____

24. A factory produces m units per week. How many units per week will it produce after production is increased p percent?

A. $100p+m$ B. $100m+mp$ C. $\frac{m+mp}{100}$
D. $m+\frac{mp}{100}$ E. $\frac{p}{100}+m$ Ans.____

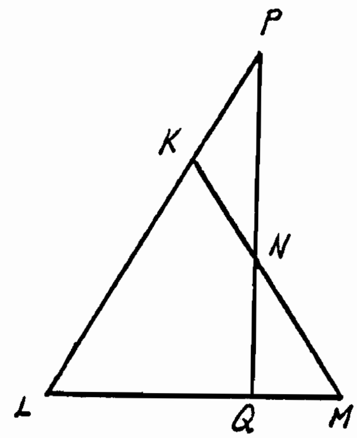
25. Let the symbol $\overline{a,b}$ denote the set of integers between a and b. For example, $\overline{3,7}$ consists of the integers 4, 5 and 6. Which of the following pairs of sets has a larger number of integers in common than any of the other pairs?

A. $\overline{0,15}$ and $\overline{7,20}$ B. $\overline{5,14}$ and $\overline{16,30}$
C. $\overline{5,14}$ and $\overline{5,17}$ D. $\overline{4,18}$ and $\overline{8,20}$
E. $\overline{0,12}$ and $\overline{6,12}$ Ans.____

26. There are 227 boys in a school. Every boy in the school belongs to either the music club or the sports club, and some boys belong to both clubs. The music club has 120 members, and 36 of these are also members of the sports club. What is the total membership of the sports club? Ans.____

27. The distance between two schools on a map with a scale of 1 : 10,000 is 20 cm. What is the actual distance in kilometers between the two schools? Ans.____

28. In $\triangle$KLM on the right, KL=KM, PQ$\perp$LM, and LP is a straight line. Then $\triangle$NKP is isosceles because



A. $\angle$P=$\angle$KNP, since both are complements of the equal angles L and M.

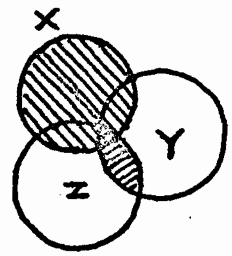
B. NK=PK, since $\angle$P=$\angle$M.

C. its sides are parallel to the sides of $\triangle$KLM.

D. its sides are perpendicular to the sides of $\triangle$KLM.

E. $\angle$P=$\angle$KNP, since both are half the complement of angle M.

Ans._____

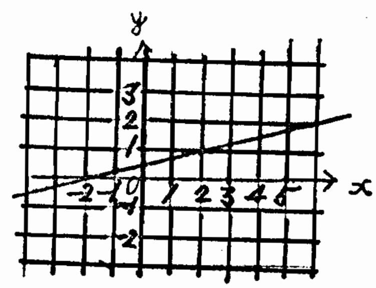


29. The symbol P$\cap$Q represents the intersection of sets P and Q and the symbol P U Q represents the union of sets P and Q. Which of the following represents the shaded area of the diagram?

A. (X$\cap$Y) U Z   B. X U (Y$\cap$Z)   C. X$\cap$(Y U Z)
D. (X$\cap$Y)$\cap$Z   E. (X U Y)$\cap$Z

Ans._____

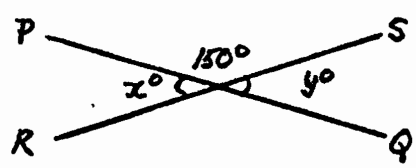
30. The equation of the line shown in the graph is



A. x+4y=4
B. 2x-y=4
C. 2x=y-2
D. x-4y+2=0
E. 4x-y=2

Ans._____

31. If , in the given figure, PQ and RS are intersecting straight lines, then x+y is equal to



A. 15   B. 30   C. 60   D. 180   E. 300

Ans._____

32. Which of the following numbers in base two is(are) even?

I. 110011
II. 110010
III. 110101
IV. 100100

A. I only
B. III only
C. I and III only
D. II and IV only
E. I,III and IV

Ans._____

33. Each of 9 boys had t marbles. In order to play a game, they divided the marbles among 12 boys in such a way that each had the same number. How many marbles did each of the 12 have?

A. $\frac{3t}{4}$ B. t-3 C. $\frac{4t}{3}$ D. 9t-12 E. 12t-9

Ans._____

34. Soda costs a cents for each bottle but there is a refund of b cents on each empty bottle. How much will Henry have to pay for x bottles if he brings back y empties?

A. ax+by
B. ax-by
C. (a-b)x
D.(a+x)-(b+y)
E. none of these.

Ans._____

35. From a long stick of wood a man cut 6 short sticks each 2 feet long. He then found he had 1 foot left over. Which of the following would tell him the length of the original stick of wood?

A. 6 x (2+1)
B. (6 x2) +1
C. (6+2)-1
D. (6 x 2) -1
E. (6+2) +1

Ans._____

Appendix F

MODEL ANSWERS

PAPER-X

X-1		X-2		X-3		X-4		X-5		X-6	
1	C	8	D	16	+2	22	D	28	D	34	D
2	A	9	50	17	D	23	B	29	D	35	B
3	E	10	C	18	A	24	D	30	D		
4	D	11	B	19	A	25	143	31	D		
5	40	12	C	20	E	26	28	32	A		
6	B	13	24,48,240*	21	A	27	B	33	A		
7	D	14	C								
		15	D								

PAPER-Y

Y-1		Y-2		Y-3		Y-4		Y-5	
1	D	9	800	20	A	26	B	32	A
2	D	10	C	21	D	27	B	33	A
3	c	11	B	22	E	28	D	34	D
4	C	12	E	23	D	29	D	35	A
5	B	13	B	24	143	30	D		
6	D	14	D	25	A	31	D		
7	50	15	D						
8	B	16	E						
		17	+2						
		18	D						
		19	E						

PAPER-Z

Z-1		Z-2		Z-3		Z-4		Z-5		Z-6	
1	25.4	9	C	16	E	22	B	28	A	32	D
2	E	10	C	17	C	23	A	29	B	33	A
3	D	11	B	18	C	24	D	30	D	34	B
4	A	12	E	19	60	25	D	31	C	35	B
5	40	13	D	20	C	26	143				
6	D	14	24,48,240*	21	B	27	2				
7	50	15	D								
8	D										

* 1 mark for all three correct answers in this question, 0 mark for any mistake