# Cosmological Instabilities

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# Abstract

Though historically the word "tachyon" has been used to describe hypothetical particles which propagate faster than the speed of light, in a more modern context the term has been recycled to refer to certain unstable states in field theory. This thesis explores the role of tachyonic instabilities in cosmology considering tachyons which arise in string theory and also more conventional, field theoretic instabilities. Our study of such instabilities is, in part, motivated by attempts to embed inflation into string theory. We will argue that the study of string theory models of inflation is well-motivated and may provide a rare potential observational window into string physics.

After reviewing the necessary background material concerning inflation, cosmological perturbation theory and tachyonic instabilities we study in detail the dynamics of the tachyonic instability which marks the end of a particular string theory model of inflation, focusing on the processes of reheating and cosmic string production. We show that the peculiar dynamics of the open string tachyon leads to various novelties in these processes and consider also potential observational consequences.

We consider tachyonic preheating at the end of hybrid inflation in a conventional field theory setting and show that the preheating process can leave an observable imprint on the Cosmic Microwave Background, either through $n = 4$ contamination of the power spectrum or else through large nongaussian signatures. The possibility of large nongaussianity is particularly interesting since it demonstrates that hybrid inflation provides one of the few well-motivated models which can generate an observable nongaussian signature.

Finally, we study a novel string theoretic model of inflation, $p$-adic inflation. This model is nonlocal, however, it is free of the usual problems (such as ghosts) which plague nonlocal theories. Furthermore, the nonlocal structure of the theory leads to a variety of unexpected dynamics including the possiblity of a slowly rolling inflaton, despite an extremely steep potential.

# Résumé

Bien qu'historiquement le mot "tachyon" fut introduit pour désigner d'hypothétiques particules se déplaçant plus rapidement que la vitesse de la lumière, de nos jours il désigne aussi certaines configurations instables en théorie des champs. Cette thèse explore le rôle des instabilités tachyoniques en cosmologie, particulièrement celui des tachyons apparaissant en théorie des cordes, mais aussi celui de tachyons plus conventionnels de la théorie des champs. Notre étude de ces instabilités est, en partie, motivée par une tentative d'inscrire l'inflation à l'intérieur de la théorie des cordes. Nous démontrerons que l'étude de modèles d'inflation en théorie de cordes est bien justifiée, et pourrait constituer une fenêtre d'accès privilégiée pour l'observation expérimentale de la théorie des cordes.

Après avoir revu les notions nécessaire en matière d'inflation, de théorie cosmologique des perturbations ainsi que d'instabilités tachyoniques, nous examinons en détails la dynamique de l'instabilité tachyonique qui marque la fin d'un modèle particulier d'inflation en théorie des cordes, en mettant l'accent sur les processus de réchauffement de la matière et de production de cordes cosmiques. Nous montrons que les particularités de la dynamique du tachyon de la corde ouverte apportent des éléments nouveaux à ces processus, et discutons des conséquences possibles pour les observations.

Dans le cadre conventionnel de la théorie des champs, nous étudions le préréchauffement par le tachyon qui survient à la fin de l'inflation hybride et montrons que ce processus peut laisser des empreintes sur le fond diffus cosmologique (CMB), soit par une contamination $n = 4$ du spectre de puissance, soit par une importante signature non-gaussienne. La possibilité d'une importante non-gaussianité est particulièrement intéressante, puisque parmi les modèles d'inflation qui soit bien motivés l'inflation hybride est l'un des rares possédant une telle signature.

Finalement, nous considérons un nouveau modèle d'inflation s'inscrivant à l'intérieur de la théorie des cordes: l'inflation $p$-adique. Il s'agit d'un modèle non-local, mais néanmoins exempt des difficultés affectant habituellement ce genre de théories (tels les

fantômes). Cette non-localité confère au modèle des propriétés remarquables, notamment la possibilité d'une inflation en roulement lent malgré un potentiel extrêmement abrupt.

# Acknowledgements

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# Statement of Original Contributions

This thesis contains material previously published in [1]-[7].

Chaper 2 is based on [1] which was written in collaboration with J. Cline. I performed most of the calculation and wrote a large fraction of the body of the text.

Chapter 3 is based on [2] which was a single author paper and hence I have performed all calculations and wrote the entirety of the text.

Chapter 4 is based on [3] which was written in collaboration with C. Burgess and J. Cline. I studied the mode functions and helped to set up the two throat model described in section 2. I independently verified the calculations in the paper to ensure accuracy.

Chapter 5 is based on [4] which was written in collaboration with A. Berndsen, J. Cline and H. Stoica. I performed all the analytical calculations of defect formation in sections 3 and 4, and studied the comological consequences in collaboration with my co-authors in section 6. I also wrote a large fraction of the body of the text and contributed most of the appendix.

Chapter 6 is based on [5] and [6], both written in collaboration with J. Cline. In [5] I performed all of the cosmological perturbation theory calculations, both at first and second order, including the construction of the tachyon curvature perturbation and the construction of the Green function for the master equation. I performed some (though not all) of the calculations involving the tachyon mode functions section III. I independently verified both the numerical and analytical analysis in sections V and VI and performed the reduction of the KKLMMT model in section VII. I contributed appendices A-E and wrote a large fraction of the body of the text. In [6] I independently verified the adiabatic approximation computation in section III and derived the result demonstrating that it is possible to obtain large nongaussianity without saturating the observational limit on the linearity parameter in this case. I performed the reductions of KKLMMT and P-term inflation in sections IV and V and performed all of the calculations in section VI (second order cosmological perturbation theory for a multi-component tachyon). I contributed both appendices. I also wrote a large fraction of the body of the text.

Chapter 7 is based on [7] written in collaboration with T. Biswas and J. Cline. I contributed section 3 and was responsible for the initial observation that the inflaton can roll slowly despite the steepness of the potential. I independently verified all of the calculations in subsections 4.1, 4.2 and contributed subsection 4.3 on the friction-dominated approximation. I constructed the mode function solutions and the curvature perturbation in section 5 and contributed much of the discussion of initial conditions. I contributed all of the appendices and wrote a large fraction of the body of the text.

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# Chapter 1

# Introduction

## 1.1 Modern Cosmology

Inflation [8] plays a key role in modern cosmology. Inflation has become the dominant paradigm for explaining the high degree of homogeneity and isotropy of the universe. In addition to explaining why the universe is so homogeneous, inflation also provides a mechanism for explaining the existence (and scale invariant nature) of the small perturbations which are present in the Cosmic Microwave Background (CMB). As we will see, these are generated by the quantum fluctuations of the inflaton field during inflation. The literature on this subject contains a number of excellent review papers on the theory of inflation and cosmological perturbations, for example [9, 10, 11].

We begin with a very brief review of some of the salient features of modern cosmology. Modern cosmology is based on Einstein's theory of General Relativity (GR). One of the essential assumptions of modern cosmology (confirmed by observation) is that, to a first approximation, the universe is homogeneous and isotropic. This assumption turns out to be quite restrictive and the only metric compatible with this assumption is the Friedmann-Robertson-Walker (FRW) metric[1]

$$ds^2 \equiv g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a(t)^2\left[\frac{dr^2}{1-Kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right] \tag{1.1}$$

where $K = 0, \pm 1$ defines the geometry on spatial hypersurfaces and $a(t)$ is the scale factor of the universe. We will typically set $K = 0$ which is in good agreement with observation and is motivated by the assumption of a period of primordial inflation. The stress-energy tensor for matter content which is consistent with the assumption of homogeneity and isotropy is of the form

$$T^\mu_\nu = \mathrm{diag}\left(-\rho(t), P(t), P(t), P(t)\right) \tag{1.2}$$

where $\rho$, $P$ are the energy density and pressure respectively. It is conventional to define the equation of state $\omega$ of the matter by

$$P = \omega\rho \tag{1.3}$$

[1]We employ the "mostly plus" convention for the metric signature throughout so that $\eta_{\mu\nu} = \mathrm{diag}(-1, +1, \cdots, +1)$.

In many cases of interest $\omega$ is constant. In particular, for a gas of nonrelativistic particles (pressureless dust) one has $\omega = 0$ while for a gas of relativistic particles (radiation) one has $\omega = 1/3$. A pure cosmological constant ($T_{\mu\nu} \propto g_{\mu\nu}$) corresponds to $\omega = -1$.

The dynamics of the system is described by the Einstein equations

$$\begin{aligned} G_{\mu\nu} &= \frac{1}{M_p^2} T_{\mu\nu} \\ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} &= \frac{1}{M_p^2} T_{\mu\nu} \end{aligned} \tag{1.4}$$

as well as the conservation equations

$$\nabla^\mu T_{\mu\nu} = 0 \tag{1.5}$$

Throughout Greek indices $\mu, \nu = 0, 1, 2, 3, 4$ run over the full spacetime while Latin indices only run over the spatial directions $i, j = 1, 2, 3$. In (1.4) $G_{\mu\nu}$ is the Einstein tensor, $R_{\mu\nu}$ is the Riemann tensor, $R$ is the Ricci scalar and $M_p = (8\pi G_N)^{-1/2} \cong 2.43 \times 10^{18}$GeV is the reduced Planck mass. Equations (1.4, 1.5), along with the ansatz (1.1), imply the following system

$$H^2 = \frac{\rho}{3M_p^2} - \frac{K}{a^2} \tag{1.6}$$

$$0 = \dot{\rho} + 3H(\rho + P) \tag{1.7}$$

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_p^2}(\rho + 3P) \tag{1.8}$$

In the above the dot denotes differentiation with respect to cosmic time $\dot{f} = \partial_t f$ and we have defined $H \equiv \dot{a}/a$. Only two of these equations are independent, indeed (1.8) may be derived by differentiating (1.6) with respect to time and using (1.7). With (1.3) the two equations (1.6, 1.7) are sufficient to close the system.

For constant $\omega \neq -1$ and assuming $K = 0$ one has $\rho \propto a^{-3(1+\omega)}$ and $a(t) \propto t^{\frac{2}{3}\frac{1}{(1+\omega)}}$. For $\omega = -1$ we would have had $\rho, H = \text{const}$ and $a \propto e^{Ht}$. Notice that for both radiation and matter dominated cosmologies one has a *decelerating* universe, $\ddot{a} < 0$, while a universe dominated by a cosmological constant will *accelerate*, $\ddot{a} > 0$.

Historically, the development of inflation was motivated by several conceptual problems associated with big bang cosmology (none of which represented an actual conflict with observation). Though the big bang model enjoyed immense phenomenological success in explaining Hubble's law, the existence (and backbody nature) of the Cosmic Microwave Background (CMB) radiation and the abundance of light elements (nucleosynthesis), it nevertheless seemed to require unnaturally fine tuned initial conditions in order to explain the observed flatness and homogeneity of the universe. Perhaps the most serious conceptual problem with the simple big bang model was the difficulty in constructing a causal theory of structure formation. The resolution to these puzzles, which has now become a widely accepted part of the standard cosmological model, is to posit a phase of accelerated expansion known as *inflation* preceeding the big bang. Because the inflation is a *predictive* scenario (and these predictions are in excellent agreement with observation) we will not linger on its historical motivation and will rather focus on the predictions.

### 1.1.1 Inflation

How can one obtain a prolonged phase of accelerated expansion? It is clear from our previous discussion that a universe dominated by dust or by radiation will not lead to acceleration. A pure cosmological constant would do the trick; however in that case inflation would never end and there could be no smooth transition to the big bang. The most popular (and perhaps simplest) implementation of this idea is inflation driven by a single scalar field, $\varphi$, which we refer to as the *inflaton*. The action for the inflaton field is

$$S_\varphi = -\int d^4x\sqrt{-g}\left[\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + V(\varphi)\right] \tag{1.9}$$

where $V(\varphi)$ is the potential energy of the field. For a homogeneous field configuration $\varphi = \varphi(t)$ the stress tensor associated with the inflaton takes the form (1.2) with

$$\rho = \frac{1}{2}\dot{\varphi}^2 + V(\varphi) \tag{1.10}$$

$$P = \frac{1}{2}\dot{\varphi}^2 - V(\varphi) \tag{1.11}$$

To close the system of dynamical equations describing the inflaton coupled to gravity we need only consider the Friedmann equation (1.6) and the Klein-Gordon equation for the inflaton field

$$H^2 = \frac{1}{3M_p^2}\left[\frac{1}{2}\dot{\varphi}^2 + V(\varphi)\right] \tag{1.12}$$

$$0 = \ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) \tag{1.13}$$

where $V'(\varphi) = \partial V/\partial\varphi$. Equation (1.13) can be derived by minimizing (1.9) or equivalently by inserting (1.10, 1.11) into (1.7). If we would like to use the inflaton field to drive a period of inflation we want $\rho \cong -P$ so that the field (temporarily) mimics a cosmological constant. From (1.10, 1.11) it is clear that this requires $\dot{\varphi}^2/2 \ll V$. This will ensure that $H \cong$ const, however, if $\ddot{\varphi}$ is too large large then this inflationary phase will not last very long. In order to ensure a long period of inflation we must also therefore demand that $\ddot{\varphi} \ll H\dot{\varphi}$. It is conventional to quantify this by defining the *slow roll parameters*. There are two logically distinct definitions of slow roll parameters which appear in the literature. The first definition, the Hubble slow roll parameters, is in terms of the dynamics of the fields

$$\epsilon_H \equiv \frac{1}{2M_p^2}\frac{\dot{\varphi}^2}{H^2} = -\frac{\dot{H}}{H^2} \tag{1.14}$$

$$\epsilon_H - \eta_H \equiv \frac{\ddot{\varphi}}{H\dot{\varphi}} \tag{1.15}$$

In the second equality in (1.14) we have used the fact that $\dot{H} = -(\rho + P)/(2M_p^2) = -\dot{\varphi}^2/(2M_p^2)$ and thus $\epsilon = -\dot{H}/H^2$, which follows from (1.6) and (1.8). It is clear that if $\epsilon_H \ll 1$, $|\eta_H| \ll 1$ then the time evolution of $\varphi(t)$ and $H(t)$ will be slow compare to the Hubble scale. An alternative definition is the potential slow roll parameters

$$\epsilon_V \equiv \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2 \tag{1.16}$$

$$\eta_V \equiv M_p^2\frac{V''}{V} \tag{1.17}$$

so that for $\epsilon_V \ll 1$, $|\eta_V| \ll 1$ the scalar potential is flat. For a flat potential one can show that $\epsilon_H \cong \epsilon_V$, $\eta_H \cong \eta_V$ so in this case there is no need to distinguish between these two types of slow roll parameter. We will assume this to be the case throughout this chapter and hence we drop the subscipts and refer to the slow roll parameters simply as $\epsilon$, $\eta$. In chapter 7 we will consider a model of inflation where the distinction between Hubble and potential slow roll parameters is crucial.

The challenge of inflationary model building is to find potentials which are sufficiently flat but are nevertheless well motivated from a particle physics perspective. We will return to this issue in a subsequent subsection. Assuming that we have found a potential for which $\epsilon, |\eta| \ll 1$, then the dynamics is described by the slow roll equations

$$0 \cong 3H\dot{\varphi} + V' \tag{1.18}$$

$$H^2 \cong \frac{1}{3M_p^2} V(\varphi) \tag{1.19}$$

Notice that the time derivatives of the slow roll parameters are second order in the slow roll expansion[2]

$$|\dot{\epsilon}| = 2\epsilon\, |\eta - 2\epsilon|\, H \ll \epsilon H$$

$$|\dot{\eta}| = 2\epsilon |\eta| H \ll |\eta| H$$

which means that we are justified in treating $\epsilon, \eta$ as constant if we work only to leading order in a slow roll expansion. This fact will be quite useful when we solve the perturbation equations.

During inflation the Hubble scale $H$ remains approximately constant so that expansion of the universe is approximately that of de Sitter space $a(t) \cong e^{Ht}$. The acceleration of the scale factor can then be written as

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = (1 - \epsilon) H^2$$

so that inflation will come to an end once $\epsilon(\varphi_f) \cong 1$ where $\varphi_f \equiv \varphi(t_f)$ is the value of the inflaton field at the end of inflation. It is common to define the total number of e-foldings of inflation between some reference time $t$ and the end of inflation $t_f$ as

$$N_e \equiv \int_t^{t_f} H dt \cong -\frac{1}{M_p^2} \int_{\varphi}^{\varphi_f} \frac{V}{V'} d\varphi \tag{1.20}$$

so that typically $N_e \cong H(t_f - t)$.

[2]The equality for $\dot{\eta}$ only strictly holds for $V''' = 0$ though this contribution will in general be small.

### 1.1.2 Cosmological Perturbation Theory

In addition to explaining why the universe is so homogeneous and isotropic, inflation also provides a mechanism for generating the small inhomogeneities which lead to the temperature fluctuations of the CMB. In this subsection we will show how quantum fluctutations during the inflationary epoch lead to a nearly scale-invariant large scale power spectrum.

#### Quantum Fields in de Sitter Space

During inflation the expansion of the universe is approximately that of pure de Sitter space. Noting that the $\eta$ slow roll parameter can be written as

$$\eta = \frac{m^2}{3H^2}$$

the slow roll condition $|\eta| \ll 1$ therefore implies that the inflaton field must be light compared to the Hubble scale $m \ll H$. Hence the inflaton field fluctuation is expected to behave very much like a light quantum field in de Sitter space.

As a warm-up exercise for the full calculation we therefore consider the dynamics of some quantum field, $\chi$, (not necessarily the inflaton) in pure de Sitter space (see, for example, [12]). For the sake of simplicity we assume a potential of the form $V = m_\chi^2 \chi^2/2$. The $\chi$-field is expanded in terms of annihilation/creation operators as

$$\chi(t, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} \left[a_\mathbf{k} \chi_\mathbf{k}(t) e^{i\mathbf{k}\cdot\mathbf{x}} + \text{h.c.}\right] \tag{1.21}$$

where h.c. denotes the Hermitian conjugate of the preceeding term. The annihilation/creation operators satisfy

$$\left[a_\mathbf{k}, a^\dagger_{\mathbf{k}'}\right] = \delta^{(3)}\left(\mathbf{k} - \mathbf{k}'\right)$$

The c-number valued mode functions $\chi_\mathbf{k}(t)$ satisfy an equation of the form

$$\ddot{\chi}_k + 3H\dot{\chi}_k + \left[\frac{k^2}{a^2} + m_\chi^2\right] \chi_k = 0 \tag{1.22}$$

The calculation is simplest in terms of conformal time, $\tau$, which is related to cosmic time as $a d\tau = dt$. For pure de Sitter space $a(\tau) = -1/(H\tau)$ and it is convenient to define the conformal time Hubble scale as

$$\mathcal{H} \equiv \frac{a'}{a} = -\frac{1}{\tau}$$

In terms of conformal time (1.22) becomes

$$\chi_k'' + 2\mathcal{H}\chi_k' + \left[k^2 + a^2 m_\chi^2\right]\chi_k = 0$$

and, defining $V_k = a\chi_k$, this equation simplifies to

$$V_k'' + \left[k^2 + \frac{1}{\tau^2}\left(\frac{m_\chi^2}{H^2} - 2\right)\right] V_k = 0. \tag{1.23}$$

This is the variable in terms of which the action is canonically normalized

$$S \cong -\frac{1}{2}\int d\tau d^3x \left[-(V')^2 + \partial_i V \partial^i V + m_{\text{eff}}^2(\tau) V^2\right]$$

so that the field $V$ behaves like a quantum field in Minkowski space with a time varying mass, $m_{\text{eff}}(\tau)$, which encodes information about the expansion of the universe. The general solutions to (1.23) are

$$V_k(\tau) = \sqrt{-\tau}\left[c_1(k) H_\nu^{(1)}(-k\tau) + c_2(k) H_\nu^{(2)}(-k\tau)\right] \tag{1.24}$$

where the order of the Hankel functions is $\nu = \sqrt{9/4 - m_\chi^2/H^2}$. This analysis makes no assumptions (yet) about the size of $m_\chi/H$ and all the formulae we derive are valid for arbitrary complex $\nu$ unless otherwise stated.[3] Notice that $-\tau \geq 0$ for all $t$ so that the arguments of the Hankel functions are always positive definite.

In the solution (1.24) there are two unknown coefficients $c_i(k)$, $i = 1, 2$ corresponding to the two initial data required to uniquely specify the solution of a second order differential equation. How should one fix the initial data? Inspection of (1.21) suggests that different choices of normalization for the mode functions $\chi_k$ will lead to alternative definitions of the annihilation/creation operators. Since the vacuum is defined by $a_{\mathbf{k}}|0\rangle = 0$ it follows that different initial data correspond to different

[3]Of course, for $\chi$ corresponding to the inflaton perturbation we are interested in $m_\chi \ll H$. However, for the time being we leave $m_\chi$ unspecified. The behaviour of heavy fields in de Sitter space will be relevant for the analysis of chapter 6.

vacuum choices. This is a standard issue is curved space quantum field theory: there is no unique prescription for how to choose the vacuum state.[4] There are many interesting physical consequences associated with this ambiguity, however, a complete discussion would be out of place here. (Indeed, the above discussion can be put on rigorous footing using the tools of curved space quantum field theory.) For our purposes it is sufficient to note that on small scales $k/(aH) = -k\tau \gg 1$ de Sitter space is indistinguishable from Minkowski space. Thus, a natural presciption for normalizing the modes $\chi_k$ will one be which reproduces the well known Minkowski space result $V_k = e^{-ik\tau}/\sqrt{2k}$ on small scales. This choice is refered to as the Bunch-Davies vacuum prescription. Thus, we fix $c_i(k)$ by demanding that

$$V_k(\tau) \approx e^{-ik\tau}/\sqrt{2k} \tag{1.25}$$

for $-k\tau \gg 1$. The solution (1.24) reproduces the desired asymptotics (1.25) with the choice

$$c_1(k) = \frac{\sqrt{\pi}}{2}\exp\left[\frac{i\pi}{2}\left(\nu+\frac{1}{2}\right)\right], \quad c_2(k) = 0.$$

The solution for the Mukhanov variable $V_k$, for all $k$, becomes

$$V_k(\tau) = \frac{\sqrt{-\pi\tau}}{2}\exp\left[\frac{i\pi}{2}\left(\nu+\frac{1}{2}\right)\right]H_\nu^{(1)}(-k\tau) \tag{1.26}$$

In terms of cosmic time and the original field variable (1.26) becomes

$$\chi_k(t) = \frac{1}{2}\sqrt{\frac{\pi}{a^3H}}\exp\left[\frac{i\pi}{2}\left(\nu+\frac{1}{2}\right)\right]H_\nu^{(1)}\left(\frac{k}{aH}\right). \tag{1.27}$$

Ultimately we will be interested in the large scale behaviour of this solution. Taking the limit $-k\tau \to 0$ of (1.26) gives

$$\begin{aligned} V_k(\tau) \;\; \to \;\; & \frac{\sqrt{-\pi\tau}}{2}\exp\left[\frac{i\pi}{2}\left(\nu+\frac{1}{2}\right)\right] \\ \times \;\; & \left[\frac{1}{\Gamma(\nu+1)}\left(-\frac{k\tau}{2}\right)^{\nu} - i\frac{\Gamma(\nu)}{\pi}\left(-\frac{k\tau}{2}\right)^{-\nu}\right] \end{aligned} \tag{1.28}$$

[4]Indeed, this ambiguity is already present in flat-space quantum field theory. However, since Minkowski space enjoys a high degree of symmetry, once the zero-particle state has been defined then all inertial observers will agree on this vacuum.

To simplify further we should focus on either $m_\chi/H > 3/2$ or $m_\chi/H < 3/2$.

We first consider a field which is heavy compared to the Hubble scale $m_\chi \gg H$ (not the inflaton) so that $\nu \approx im_\chi/H$ is pure imaginary. How do the two terms in the square braces of (1.28) compare? The functions $(-k\tau)^{\pm im_\chi/H}$ are oscillatory so the relative magnitude of these two terms depends only on the gamma function prefactors. Using the results (for $\beta$ real and arbitrary complex $z$)

$$\Gamma(1+z) = z\Gamma(z), \quad |\Gamma(1+i\beta)| = \sqrt{\frac{\pi\beta}{\sinh(\pi\beta)}}$$

from the theory of gamma functions we find that for $m_\chi/H \gg 1$

$$\left|\frac{1}{\Gamma(1+im_\chi/H)}\right| \sim \sqrt{\frac{H}{2\pi m_\chi}} \exp\left(\frac{\pi m_\chi}{2H}\right),$$
$$|\Gamma(im_\chi/H)| \sim \sqrt{\frac{2\pi H}{m_\chi}} \exp\left(-\frac{\pi m_\chi}{2H}\right)$$

so that the first term in the square braces on the second line of (1.28) dominates at large $m_\chi/H$. Going back to the original variable we find

$$|\chi_k| \sim a^{-3/2}\frac{1}{2^{3/2}\sqrt{m_\chi}} \quad \text{for } m_\chi \gg H \tag{1.29}$$

on large scales $-k\tau \ll 1$.

We consider now a field (such as the inflaton) which is light compared to the Hubble scale $m_\chi \ll H$. In this case $\nu \approx 3/2$ is pure real and the first term in the square braces in (1.28) goes to zero and we have

$$|\chi_k| \sim \frac{H}{\sqrt{2k^3}} \quad \text{for } m_\chi \ll H \tag{1.30}$$

on large scales $-k\tau \ll 1$.

Notice that for any value of $m_\chi/H$ we have, by construction, the asymptotics

$$|\chi_k| \sim a^{-1}\frac{1}{\sqrt{2k}} \tag{1.31}$$

on small scales $-k\tau \gg 1$.

We now define the power spectrum of the field in terms of the two-point function as

$$\begin{aligned} \langle 0|\chi^2(t,\mathbf{x})|0\rangle &= \int \frac{d^3k}{(2\pi)^3} \left|\chi_k(t)\right|^2 \\ &= \int \frac{dk}{k} \mathcal{P}_\chi(k) \end{aligned} \tag{1.32}$$

where

$$\mathcal{P}_\chi(k) = \frac{k^3}{2\pi^2} \left|\chi_k(t)\right|^2 \cong \frac{H^2}{(2\pi)^2} \left(\frac{k}{aH}\right)^{n-1} \tag{1.33}$$

The quantity $n$ in (1.33) is the *spectral index.* The spectrum is called scale-invariant if $n = 1$, blue-tilted if $n > 1$ and red-tilted if $n < 1$.

From equation (1.29) we see that any heavy field in de Sitter space will have a large-scale spectrum with a spectral index $n = 4$ and which is damped as $a(t)^{-3/2} = e^{-3Ht/2}$. On the other hand, from equation (1.30) we see that any light field in de Sitter space will have a large-scale spectrum which is almost scale invariant

$$n = 1 + 3 - 2\nu \cong 1 + \mathcal{O}\left(\frac{m^2}{H^2}\right) \tag{1.34}$$

and which suffers no exponential damping. We expect, then, that inflation should generate a nearly scale invariant spectrum of perturbations on large scales. In the next subsubsection we will confirm this intuition.

### The Full Calculation

The previous calculation shed some light on the dynamics of quantum fields during inflation, however, this calculation neglected departures from pure de Sitter expansion. Furthermore, this calculation neglected the possibility of metric perturbations. Because Einstein's equations (1.4) couple the geometry to the matter, it is clear that quantum fluctuations in the inflaton $\delta\varphi$ will induce fluctuations of the geometry $\delta g_{\mu\nu}$. A consistent calculation must include both.

We expand the inflaton field in perturbation theory as

$$\varphi(\tau,\mathbf{x}) = \varphi_0(\tau) + \delta\varphi(\tau,\mathbf{x}) \tag{1.35}$$

The most general expansion of the metric is

$$\begin{aligned} g_{00} &= -a(\tau)^2\left[1+2\phi\right] && (1.36)\\ g_{0i} &= a(\tau)^2\left[\partial_i\omega+\omega_i\right] && (1.37)\\ g_{ij} &= a(\tau)^2\left[(1-2\psi)\delta_{ij}+D_{ij}\chi\right.\\ &+ \left.(\partial_i\chi_j+\partial_j\chi_i+\chi_{ij})\right] && (1.38)\end{aligned}$$

where $D_{ij} = \partial_i\partial_j - \frac{1}{3}\delta_{ij}\partial^k\partial_k$ is a trace-free operator. It is conventional to decompose the metric perturbations according to their behaviour under three-dimensional rotations. The fluctuations are therefore decomposed such that the vector perturbations are transverse $\partial^i\omega_i = \partial^i\chi_i = 0$ while the tensor perturbations are transverse, traceless and symmetric: $\partial^i\chi_{ij} = 0$, $\chi^i_i = 0$, $\chi_{ij} = \chi_{ji}$. This scalar/vector/tensor decomposition is an inherently nonlocal procedure, however, it is both conventional and convenient.[5] Clearly at linear order the scalar, vector and tensor modes will decouple and we can study each type of perturbation independently. Our calculation will focus on the scalar perturbations. It can be shown that vector perturbations decay in an expanding universe while tensor perturbations are suppressed with respect to the scalar perturbations by slow roll parameters.

There is one further subtlety to this perturbative expansion. As we have mentioned, quantum fluctations in the matter sector $\delta\varphi$ will induce metric perturbations $\delta g_{\mu\nu}$. However, it is easy to convince oneself that metric perturbations can also be induced by performing coordinate transformations of the form

$$x^\mu \rightarrow x^\mu + a^\mu \tag{1.39}$$

where we assume that $a^\mu$ is a small perturbation. Clearly fluctuations associated with (1.39) have no physical meaning. This problem is completely analogous to what occurs in ordinary electromagnetism: the Maxwell equations $\partial_\mu F^{\mu\nu} = 0$ are insufficient to completely determine the vector potential $A_\mu$. The ambiguity is related

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[5] At linear order and focusing on inflationary perturbations this nonlocality is not manifest, however, if one considers second order perturbations or perturbations associated with field configurations having compact support (such as cosmic strings) then this nonlocality becomes quite evident [13].

to the freedom to perform gauge transformations in electromagnetism $A_\mu \to A_\mu + \partial_\mu a$. Of course, this ambiguity does not appear in the *physical* quantities; for example the field strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is invariant under $A_\mu \to A_\mu + \partial_\mu a$ . Similarly, the freedom to perform coordinate transformations (1.39) represents a gauge freedom in cosmological perturbation theory. Just as in other gauge theories, there are two ways to proceed:

1. Rewrite the system in terms of quantities which are gauge invariant and work only with those quantities.

2. Fix the gauge by placing some constraints on the metric perturbations. One must still be careful to ensure that whatever physical quantities are computed in the end are independent of the gauge fixing.

We will adopt the gauge-fixed approach which is typically simpler. We employ the longitudinal gauge, defined by $\omega = \chi = 0$, so that the metric, ignoring vector and tensor modes, takes the simple form

$$ds^2 = a(\tau)^2\left[-(1+2\phi)d\tau^2 + (1-2\psi)\delta_{ij}dx^i dx^j\right]$$

The perturbed Einstein equations $\delta G^\mu_\nu = M_p^{-2}\delta T^\mu_\nu$ can be found in elsewhere (see [9] for example) and we do not reproduce them here. The $\delta G^i_j = M_p^{-2}\delta T^i_j$ Einstein equation for $i \neq j$ implies that $\phi = \psi$ which is a well known result. We apply this simplification in the following. The $\delta G^0_i = M_p^{-2}\delta T^0_i$ Einstein equation is a constraint

$$\phi' + \mathcal{H}\phi = \frac{1}{2M_p^2}\varphi_0'\,\delta\varphi \tag{1.40}$$

which means that the first order metric perturbation $\phi$ and the first order inflaton perturbation $\delta\varphi$ are not independent. Once either $\phi$ or $\delta\varphi$ is known, the other may be computed from (1.40), though it is simplest to solve for $\phi$ and use (1.40) to compute $\delta\varphi$. One obtains a dynamical equation for the metric perturbation by applying (1.40) to the sum of $\delta G^0_0 = M_p^{-2}\delta T^0_0$ and $\delta G^i_i = M_p^{-2}\delta T^i_i$ Einstein equations. The result is

$$\phi_k'' + 2\left(\mathcal{H} - \frac{\varphi_0''}{\varphi_0'}\right)\phi_k' + \left[2\left(\mathcal{H}' - \mathcal{H}\frac{\varphi_0''}{\varphi_0'}\right) + k^2\right]\phi_k = 0 \tag{1.41}$$

Notice that the perturbed Klein-Gordon equation for the inflaton is not needed to close the system of equations. We now discuss the slow roll solutions of (1.41).

The conformal time slow roll parameters are

$$\epsilon = 1-\frac{\mathcal{H}'}{\mathcal{H}^2}=\frac{\kappa^2}{2}\frac{\varphi_0'^2}{\mathcal{H}^2}, \tag{1.42}$$

$$\epsilon-\eta = \frac{\varphi_0''}{\mathcal{H}\varphi_0'}-1. \tag{1.43}$$

During a pure deSitter phase $\epsilon = \eta = 0$ and the scale factor evolves as $a(\tau) = -1/(H\tau)$ with $H$ constant. During inflation, however, the Hubble scale evolves slowly as (1.42) so that for small $\epsilon$ one has [9]

$$a(\tau) = -\frac{1}{H\tau}\frac{1}{1-\epsilon}$$

and

$$\mathcal{H} = \frac{a'}{a} = \dot{a} = aH = \frac{-1}{\tau(1-\epsilon)}. \tag{1.44}$$

The dynamical equation for $\phi^{(1)}$ (1.41) can be rewritten in terms of the slow roll parameters as

$$\phi_k'' - \frac{2}{\tau}(\eta-\epsilon)\phi_k' + \left[\frac{2}{\tau^2}(\eta-2\epsilon)+k^2\right]\phi_k = 0 \tag{1.45}$$

where we used (1.44) and dropped higher order terms in $\epsilon$, $\eta$. Treating the slow roll parameters as constant, the equation (1.45) has an exact solution

$$\phi_k(\tau) = (-\tau)^{1/2+\eta-\epsilon}\left[c_1(k)H_\nu^{(1)}(-k\tau)+c_2(k)H_\nu^{(2)}(-k\tau)\right] \tag{1.46}$$

where $\nu \cong 1/2+(3\epsilon-\eta)$ to lowest order in slow roll parameters.

It remains to fix the coefficients $c_1(k)$, $c_2(k)$ in (1.46). The variable in terms of which the action is canonically normalized is the Mukhanov variable [10]

$$V_k = a\left[\delta\varphi_k + \frac{\varphi_0'}{\mathcal{H}}\phi_k\right]$$

where the inflaton fluctuation is solved for using the constraint equation (1.40). We fix $c_1(k)$, $c_2(k)$ by requiring that $V_k \cong e^{-ik\tau}/\sqrt{2k}$ on small scales $-k\tau \gg 1$ which corresponds to the usual Bunch-Davies vacuum choice. This leads to

$$c_1(k) = i\frac{H}{M_p}\sqrt{\frac{\pi\epsilon}{2}}\frac{\exp\left[\frac{i\pi}{2}(\nu+1/2)\right]}{2k}, \quad c_2(k)=0.$$

The solution for the metric perturbation, then, is

$$\phi_k(\tau) = i\sqrt{\frac{\pi\epsilon}{2}}\frac{H}{M_p}\frac{\exp\left[\frac{i\pi}{2}(\nu+1/2)\right]}{2k} \times (-\tau)^{1/2+\eta-\epsilon}H_\nu^{(1)}(-k\tau). \quad (1.47)$$

It is straightforward to construct the inflaton fluctuation using (1.40). The quantity relevant for observation is the *curvature perturbation*

$$\zeta = -\frac{\mathcal{H}}{a\varphi_0'}V = -\phi - \frac{\mathcal{H}}{\varphi_0'}\delta\varphi$$

This quantity is gauge invariant and is approximately constant on large scales (in the absence of nonadiabatic pressure). The resulting large scale power spectrum (defined as in (1.32)) is

$$\mathcal{P}_\zeta = A_\zeta^2\left(\frac{k}{aH}\right)^{n-1} \quad (1.48)$$

where the amplitude of the spectrum is

$$A_\zeta^2 = \frac{1}{2M_p^2\epsilon}\frac{H^2}{(2\pi)^2} \quad (1.49)$$

and the spectral index is

$$n-1 = 2\eta - 6\epsilon \quad (1.50)$$

so that $|n-1| \ll 1$. The spectrum (1.48) is meant to be evaluated when scales relevant for CMB observations exited the horizon, roughly 60 e-foldings before the end of inflation. It turns out that $N_e \cong 60$ also coincides with the minimal amount of inflation necessary to resolve the horizon problem of big bang cosmology.

Quantum fluctuations which are “born” on small scales $-k\tau \gg 1$ are redshifted by the expansion of the universe and become classical as they cross the horizon $-k\tau = 1$. The quantum to classical transition is most easily seen by writing the hamiltonian in the form $H_k = \omega_k(n_k+1/2)$ where $\omega_k$ is the energy eigenvalue and $n_k$ is the occupation number. It is straightforward to show that

$$n_k \sim (-k\tau)^{-3}$$

so that once $-k\tau \ll 1$ we have $n_k \gg 1$, the "quantum" factor of 1/2 in $H_k$ can be neglected and the fluctuations may indeed be considered classical.[6]

### The CMB

During inflation quantum fluctuations of the inflaton field with comoving wavenumber $k$ are generated by the usual Minkowski space vacuum fluctuations on small scales $k \gg aH$, as in (1.31). The physical wavenumbers $k_{\rm phys} = k/a \sim e^{-Ht}k$ are redshifted by the expansion of the universe (while $H(t)$ remains approximately constant) until *horizon crossing* $k = aH$ (or $-k\tau = 1$). Once a mode exits the horizon it becomes "frozen in" as a classical perturbation, as in (1.30).

After inflation the energy density in the inflaton field must be transferred to visible radiation in order to make contact with the successes of the big bang model. This process is called *reheating* (see [14] for a review). Immediately after reheating the universe is radiation dominated so that $a(t) \sim t^{1/2}$, $H(t) \sim t^{-1}$. However, because $\rho_{\rm rad} \sim a^{-4}$ while $\rho_{\rm dust} \sim a^{-3}$ it follows that eventually pressureless dust will come to dominate. During the matter dominated The crucial point is that after inflation physical wavenumbers red-shift as $k/a \sim kt^{-\alpha}$ with some $\alpha < 1$ while $H(t) \sim t^{-1}$. Thus modes which crossed the horizon during inflation will subsequently re-enter the horizon at some point after reheating.

As perturbation modes $\zeta_k$ re-enter the causal horizon during radiation- or matter-domination they create density perturbations $\delta\rho_k$ by the gravitational attraction of potential wells and ultimately induce the observed temperature fluctuations $\delta T$ in the CMB. The amplitude of the spectrum $A_\zeta$ (equation (1.49)) fixes the amplitude of the temperature fluctuations $\delta T/T$ in the CMB. Consistency with observation requires $\delta T/T \sim 10^{-5}$ on large angular scales [15] which translates into

$$\frac{V}{\epsilon M_p^4} = 6\times 10^{-7} \quad \Rightarrow \quad \left(\frac{V}{\epsilon}\right)^{1/4} = 6.7\times 10^{16}\,\mathrm{GeV} \tag{1.51}$$

We refer to (1.51) as the COBE normalization of the inflaton potential.

[6]Another way to see the quantum-to-classical transition is by considering the commutator $[\delta\varphi, \delta\dot{\varphi}]$. It is straightforward to show that this quantity damps to zero exponentially as $-k\tau \to 0$.

The prediction $|n-1| \ll 1$ (equation (1.50)) is in excellent agreement with observation. Indeed, recent WMAP analysis [16] suggests that $n \cong 0.95$. Inflation also predicts a nearly scale-invariant spectrum of tensor perturbations (gravitational waves) with an amplitude that is suppressed by a factor of $\epsilon$ as compared to $A_\zeta$, though these have yet to be observed.

### 1.1.3 Gaussianity of the Perturbations

In simple single-field models of inflation (which, so far, we have limited our discussion to describing) the perturbations are almost gaussian. Working strictly to linear order in perturbation theory we find that

$$\begin{aligned}
\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle &= 0 \\
\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \zeta_{k_4} \rangle &= \langle \zeta_{k_1} \zeta_{k_2} \rangle \langle \zeta_{k_3} \zeta_{k_4} \rangle + \text{perms} \\
&\cdots
\end{aligned}$$

where "perms" denotes various permutations of the momenta $k_i$. Similarly all higher odd $n$-point functions vanish while all even $n$-point functions can be reduced to products of two-point functions. This result is simply a consequence of Wick's theorem and the fact that $\zeta$ contains only one annihilation/creation operator. Thus, working strictly to linear order in perturbation theory, the two-point function is the only independent statistic. This is precisely the definition of *gaussian* perturbations. Since the inflationary perturbations are small (they are indeed suppressed as compared to the homogeneous background by $\delta T/T \sim 10^{-5}$) it follows that linear theory should be applicable and that the perturbations must be approximately gaussian.

However, Einstein's equations are nonlinear and one generically expects *some* nongaussianity to be generated during inflation, though one must work beyond linear order in perturbation theory in order to see this effect (for a review see [17]). From the above discussion it is clear that the three-point function (the bispectrum) is the lowest order statistic which will allow us to discriminate between gaussianity and nongaussianity. Defining $P$, $B$ by

$$\begin{aligned}\langle\zeta_{k_1}\zeta_{k_2}\rangle &= P(k)\,\delta(k_1+k_2)\\ \langle\zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle &= (2\pi)^{-3/2}\,B(k)\,\delta(k_1+k_2+k_3)\end{aligned}$$

it is conventional to define the *nonlinearity* parameter $f_{NL}$ by

$$B(k_i) = -\frac{6}{5}f_{NL}\,[P(k_1)P(k_2) + \text{perms}]$$

The nonlinearity parameter provides a dimensionless measure of the nongaussianity in a particular model of inflation.

We will return to the issue of calculating $f_{NL}$ using second order cosmological perturbation theory in chapter 6. For the time being we simply cite the result of a full calculation. In a simple, single field inflationary model (such as we have discussed in the previous subsection) one finds that [18]

$$f_{NL} \sim \frac{5}{12}(n-1)$$

so that $|f_{NL}| \ll 1$. On the other hand, observation provides the rather weak constraint $|f_{NL}| \lesssim 100$ [16]. Even though the simplest models predict negligible nongaussianity there is still considerable interest in computing $f_{NL}$ in various scenarios because this quantity is extremely model dependent and can (at least in principle) be used as a powerful tool to discriminate between various realizations of inflation. Furthermore, an observation of $|f_{NL}| \gtrsim \mathcal{O}(10)$ would be strongly indicative of some novelty in the dynamics during inflation or shortly afterwards.

### 1.1.4 Inflation from String Theory

Despite the great phenomenological success of inflation, it remains a paradigm in search of a theory. There are a number of unresolved problems concerning this scenario including:

1. *What is the inflaton?* The above construction gives no information about how the field $\varphi$ should be interpreted from a particle physics perspective.

2. *How does reheating after inflation take place?* A complete understanding of reheating after inflation will require that we understand how the inflaton field couples to standard model degrees of freedom. We will discuss this issue in much greater detail in chapters 2 and 4.

3. *The issue of fine tunings.* The requirements $\epsilon, |\eta| \ll 1$ and (1.51) seem to require considerable fine tuning of the inflaton potential. For example, the *chaotic inflation* model [19] with $V = m^2\varphi^2/2$ requires $\varphi \gg M_p$ in order to achieve slow roll. At such large field values one might expect nonrenormalizable terms in the potential to become important and generically such corrections would spoil the flatness of the potential. Furthermore, the COBE normalization in this model demands $m \sim 10^{13}$ GeV which means that we have introduced a hierarchy into the particle physics model building. It remains an outstanding problem to explain convincingly why the inflaton potential should be so flat and how this flatness is protected against quantum corrections.

4. *The trans-Planckian problem.* If inflation lasts much longer than the minimal $\sim 60$ e-foldings then physical scales of current interest will have started out smaller than the Planck scale. However, on scales comparable to the Planck scale one expects quantum gravity effects to become important and the semi-classical analysis detailed above to be invalidated.

This list could be extended (see, for example, [10]). These, and other, outstanding problems provide a strong motivation to try to embed inflation into some complete theory of particle physics which incorporates quantum gravity. Indeed, it is difficult to imagine a convincing resolution to the above problems which does not involve identifying the inflaton as a physical degree of freedom in some ultra-violet (UV) complete theory of particle physics. To date there is only one viable candidate for such a theory: *string theory.*

The starting point of string theory is the assumption that elementary particles are not point-like but are, in fact, one-dimensional extended objects (or strings). The characteristic string length is taken to be extremely small so that these objects will appear as point-like at low energies. These strings may appear as closed loops or they may be open. In this picture the various species of particles which we observe in nature arise as distinct vibrational modes of the string. Remarkably, one of these excitations describes a massless spin-2 particle - the graviton - which mediates the gravitational interaction in the same way that the photon mediates electromagnetic interactions. Hence string theory is a quantum theory of gravity which has the potential to unify all the observed forces in nature. (In fact, despite considerable effort, string theory seems to be the only such theory.) It is well known that string theory predicts the existence of compact extra spatial dimensions (in addition to the three large spatial dimensions which we observe). The theory also predicts the existence of extended higher dimensional objects known as "branes" (short for "membrane") on which open strings can end. The difficulty of finding direct laboratory probes of string theory is well known. We will discuss below the possibility of gaining an observational window into string theory through cosmology.

Why can string theory help with the aforementioned problems of inflation? String theory provides inflationary model builders with a vast number of scalar fields, some of which may provide candidates for the inflaton. Since string theory is a unified theory of particle physics, it is in principle possible to identity how some candidate inflaton couples to standard model degrees of freedom and hence it should be possible to address the reheating issue quantitatively. Furthermore, it seems to be possible to generate large hierarchies in string theory in a natural way by taking advantage of the properties of warped compactifications [20, 21] so there may be hope to resolve the naturalness issues which plague inflation. Finally, string theory is a complete theory of quantum gravity, hence it should be able to describe dynamics all the way down to the Planck scale and it is at least in principle possible to resolve the trans-Planckian problem.

We have argued that the search for inflation in string theory is of practical interest for a cosmologist because it is probably necessary in order to resolve the "big" open questions associated with inflation. Embedding inflation into string theory is also interesting from the perspective of a string theorist because it might provide a rare observational window into the theory.

For the reasons described above there has been intense effort on the part of both cosmologists and string theorists to develop inflationary models in string theory (there are far too many proposal to cite here, see [22] for a review and see subsequent chapters for more comprehensive referencing of the literature). Though none of the existing proposals is completely satisfactory, there has been considerable progress recently. In particular, there has been progress in developing fully string theoretic brane inflation models for which the moduli (scalar fields describing the size and shape of the compact dimensions) are stabilized[7] by the addition of fluxes [24]. Furthermore, in this construction it is possible to identify where the standard model degrees of freedom should reside [25]. In such a construction it is possible to address the issue of reheating after inflation in a quantitative manner [3] (see also [26] for subsequent work). This constuction is not without problems [27]; however, we feel that it is encouraging that realistic string theory constructions can begin to address (at least some of) the aforementioned problems of inflation.

In chapter 7 we will describe a novel string inflation model for which slow roll inflation can proceed even with an extremely steep potential. This provides a particularly interesting possibility for circumventing the issue of fine tuning of the inflaton potential.

It is worth spending some time thinking about why string theory and cosmology can usefully interact with one another [28]. It seems surprising that this should be possible since cosmology deals with the properties of the large scale universe while string theory (ostensibly) describes physics on the smallest scales. Shouldn't the principle of decoupling[8] forbid string theory from playing any interesting role in cosmology? It is

[7]This stabilization is required for consistency with observation. Time variation of the extra dimensions will lead to a time variation in the four-dimensional effective Newton's constant. Such time variation is tightly constrained by observation (see, for example, [23]).

[8]For example, we know that atoms are made up of smaller constituent electrons, protons and neutrons and futhermore that the protons and neutrons are made up of quarks and gluons. However, a detailed understanding of atomic properties (like chemistry) does *not* depend on the complicated

quite remarkable that string theory and cosmology *can* usefully inform one another. Indeed the fact that this is possible was already evident from the trans-Planckian problem (above). The possibility that inflation could be sensitive to quantum gravity effects means that, from the perspective of a string theorist, the trans-Planckian problem is really the trans-Planckian *window of opportunity* [10]. There appear to be two main reasons why inflation and string theory can interact [28]:

1. *Access to high energy scales.* Inflation probably took place at an extremely high energy scale, close to $10^{16}$ GeV, for which stringy degrees of freedom would be relevant. (From eq. (1.51) it is clear that the scale of inflation must be $V^{1/4} \lesssim 10^{16}$ GeV because $\epsilon < 1$. A priori there is no reason why we should be close to the upper bound since almost any value $\epsilon \lesssim 10^{-2}$ is compatible with observation. However, one might argue that because $\epsilon \ll 1$ requires fine tuning it follows that the most natural value of $\epsilon$ is the largest value which is compatible with observation. Furthermore, recent reconstructions of the inflaton potential from the CMB data seem to suggest a large value of $\epsilon$ [30].)

2. *Dependence on UV sensitive properties.* The phenomenological success of inflationary model building relies on properties of the underlying particle physics model (like the flatness of the potential) which are UV sensitive. Inflation requires light scalar masses $m < H$ (to keep $|\eta| < 1$), but scalar masses are notoriously difficult to keep small when integrating out high energy degrees of freedom.

## 1.2 Tachyonic Instabilities

Historically the term "tachyon" has been used to refer to hypothetical particles which travel faster than light. Such particles would carry negative mass-squared ($m^2$) since the relativistic relation $v = p/\sqrt{p^2 + m^2}$ between velocity $v$ and momentum $p$ clearly

details of the interactions between quarks. In retrospect, the decoupling of small scales from large scale physics was a prerequisite for the success of physics as a discipline.

gives $v \leq 1$ unless $m^2 < 0$. From a more modern perspective the idea of faster-than-light propagation is abandoned and the term "tachyon" is recycled to refer to a quantum field with $m^2 \equiv V'' < 0$. Clearly $V''$ can be negative about a maximum of the potential. Fluctuations about such a point will be unstable (since the dynamics of the system will tend to minimize the energy) and hence tachyons are associated with the presence of some physical instability. Tachyons play an important role in symmetry breaking and the formation of cosmic defects. We will illustrate this below using an extremely simple example: a single real scalar field with a double well potential. The results below generalize readily to more complicated theories.

### 1.2.1 Tachyons in Field Theory

Consider the field theory

$$S = -\int d^4x \left[\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{\lambda}{4}\left(\sigma^2 - v^2\right)^2\right] \tag{1.52}$$

The potential is $V(\sigma) = \lambda\left(\sigma^2 - v^2\right)^2/4$ and the Klein-Gordon equation

$$\ddot{\sigma} - \partial^i\partial_i\sigma + V'(\sigma) = 0$$

has static solutions at the critical points where $V' = 0$. This potential has critical points $\sigma = 0$ and $\sigma = \pm v$.

Let us first consider linearizing the theory about the point $\sigma = 0$ which has $V''(0) < 0$ and hence represents a tachyonic maximum. Writing $\sigma = 0 + \delta\sigma$ for some small fluctuation $\delta\sigma$ the mode functions obey

$$\delta\ddot{\sigma}_k + k^2\delta\sigma_k - \lambda v^2\delta\sigma_k \cong 0 \tag{1.53}$$

so that

$$\delta\sigma_k \sim \frac{1}{\sqrt{2k}}e^{\sqrt{\lambda v^2 - k^2}t}$$

and the modes with $k < \sqrt{\lambda}v$ grow exponentially. Quickly the fluctuations become large and the linearized approximation breaks down once $\langle(\delta\sigma)^2\rangle^{1/2} \sim v$. We see that if the field is initially localized near $\sigma = 0$[9] then it will quickly roll towards the critical points $\sigma = \pm v$.

One the other hand, one might imagine linearizing about the critical points $\pm v$ as $\sigma = \pm v + \delta\sigma$. In this case the mode functions obey

$$\delta\ddot{\sigma}_k + k^2\delta\sigma_k + 2\lambda v^2\delta\sigma_k \cong 0$$

so that

$$\delta\sigma_k \sim \frac{1}{\sqrt{2k}} e^{i\sqrt{2\lambda v^2 + k^2}t}$$

which describes small, stable oscillations about the vacuum. The critical points with $V'' > 0$ therefore correspond to the true vacua of the theory while the point $\sigma = 0$ is the false (unstable) vacuum.

To see the connection to symmetry breaking, notice that the original field theory (1.52) enjoys a reflection symmetry associated with $\sigma \to -\sigma$. However, writing $\sigma = +v + \chi$ (we could equally well have chosen $-v + \chi$, the point is that we have to choose *some* vacuum) we get an action for the $\chi$ field of the form

$$S = -\int d^4x \left[\partial_\mu\chi\partial^\mu\chi + \lambda v^2\chi^2 + \lambda v\chi^3 + \frac{\lambda}{4}\chi^4\right]$$

which contains a term proportional to $\chi^3$. We see that the reflection symmetry of the original action is spontaneously broken by the vacuum.

It is interesting to note that the action (1.52) admits a classical soliton solution depending on one spatial coordinate $x^1 \equiv x$

$$\sigma(x) = v \tanh\left(\sqrt{\lambda}\, v\, x\right) \tag{1.54}$$

called the "kink". This solution interpolates between the vacua $\sigma = -v$ at $x = -\infty$ and $\sigma = +v$ at $x = +\infty$ while the field remains trapped in the false vacuum at $x = 0$ (the core of the kink). Embedded into the full $3+1$-dimensional spacetime the kink describes a two-dimensional membrane called a domain wall, with thickness

[9]One could imagine trapping the field near the unstable maximum by thermal effects or else by interactions with other fields. We will give an explicit example below.

$\Delta x \sim \lambda^{-1/2} v^{-1}$, which is most easily seen by constructing the stress-energy tensor associated with this topological defect

$$\begin{aligned} T^{00} &= -T^{22} = -T^{33} = \frac{\lambda v^4}{2} \mathrm{sech}\left(\sqrt{\lambda}\, v\, x\right) \\ T^{11} &= 0 \end{aligned} \tag{1.55}$$

These results generalize in a straightforward manner to more complicated tachyon actions. Of particular interest is the generalization of (1.52) to a complex tachyon. In that case the symmetry which is spontaneously broken is $U(1)$. The theory admits a soliton solution called the vortex which appears as a one-dimensional topological defect called the cosmic string, when embedded into the full spacetime.

### 1.2.2 Hybrid Inflation and Tachyonic Preheating

The discussion of the previous subsection can be made more concrete by considering an explicit cosmological application. We will consider hybrid inflation [31], which is described by the action

$$\begin{aligned} S &= \frac{M_p^2}{2} \int d^4x \sqrt{-g}\, R \\ &- \int d^4x \sqrt{-g} \left[\frac{1}{2}(\partial\varphi)^2 + \frac{1}{2}(\partial\sigma)^2 + V(\sigma,\varphi)\right] \end{aligned} \tag{1.56}$$

with

$$\begin{aligned} V(\sigma,\varphi) &= \frac{\lambda}{4}\left(\sigma^2 - v^2\right)^2 + \frac{m^2}{2}\varphi^2 + \frac{g^2}{2}\varphi^2\sigma^2 \\ &= \frac{\lambda v^4}{4} + \frac{1}{2}\left(-\lambda v^2 + g^2\varphi^2\right)\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{m^2}{2}\varphi^2 \end{aligned} \tag{1.57}$$

Initially we take $g^2\varphi^2 > \lambda v^2$ so that $\sigma$ has a positive mass-squared and remains trapped in the false vacuum $\sigma = 0$. This is the inflationary phase. The effective potential driving inflation is then

$$V = \frac{\lambda v^4}{4} + \frac{m^2}{2}\varphi^2 \cong \frac{\lambda v^4}{4}$$

where we assume that $\lambda v^4 \ll 2m^2\varphi^2$ throughout inflation.[10] The slow roll parameters along the inflationary trajectory are

$$\eta \cong 4\frac{M_p^2 m^2}{\lambda v^4}, \qquad \epsilon \cong 8\left(\frac{M_p m^2 \varphi}{\lambda v^4}\right)^2$$

so that if $\lambda v^4 \ll 2m^2\varphi^2$ then $\epsilon \ll \eta$ and $n - 1 \cong 2\eta$. During the slow roll phase the inflaton evolves as

$$\varphi(t) = \frac{\lambda^{1/2} v}{g} \exp\left(-\frac{m^2(t-t_c)}{3H}\right) = \frac{\lambda^{1/2} v}{g}\left(\frac{a(t)}{a(t_c)}\right)^{-\eta}$$

where $t_c$ is the time at which $\varphi = \varphi_c \equiv \lambda^{1/2} v/g$ and $H \cong \lambda^{1/2} v^2/(2\sqrt{3}M_p)$. One can show that the COBE normalization in this model fixes the inflaton mass as $m^2 \cong 230 g\lambda v^5/M_p^3$ while the constraint $|n-1| \lesssim 10^{-1}$ requires $gv/M_p \lesssim 5 \times 10^{-5}$. Notice that $\eta > 0$ so that this model always gives a blue-tilted spectrum which is disfavoured by the recent WMAP data.

Once $t > t_c$ the field $\sigma$ becomes tachyonic. The subsequent evolution is called tachyonic preheating [32, 33]. If the dimensionless couplings $g, \lambda$ are not too small then the system experiences a very strong force along the unstable direction in field space and the tachyonic instability will bring inflation to an end very quickly (this corresponds to satisfying the so-called "waterfall conditions" of hybrid inflation). In this regime the tachyonic preheating completes in a time which is very short compared to $H^{-1}$ and we are justified in neglecting the expansion of the universe to describe this phase. It is also common to make the "instantaneous quench" approximation in which one neglects also the time variation of the mass term $m_\sigma^2 \equiv -\lambda v^2 + g^2\varphi^2$. In this approximation one takes the $\sigma$ field to be massless at $t = t_c$ while quickly at $t > t_c$ a tachyonic mass $m_\sigma^2 = -\lambda v^2$ is turned on. The large scale mode functions grow exponentially as in (1.53) and the variance of the fluctuations grows as [34]

$$\langle(\delta\sigma)^2\rangle = \frac{1}{8\pi}\int_0^{\lambda v^2} dk^2 \frac{\lambda v^2}{\lambda v^2 - k^2} \sinh^2\left[(t-t_c)\sqrt{\lambda v^2 - k^2}\right]$$

[10] It is not necessary to demand that the false vacuum energy $\lambda v^4/4$ dominate to achieve successful inflation, however, if the mass term dominates then this model is not different from chaotic inflation.

where we have neglected the divergent vacuum contribution. This tachyonic growth persists until $\langle(\delta\sigma)^2\rangle \sim v^2$ at which point the dynamics is dominated by oscillations about the true minima $\pm v$ and the symmetry breaking process has completed.

We will discuss some cosmological implications of tachyonic preheating after hybrid inflation in chapter 6, showing that preheating can lead to observable consequences in the CMB. In particular, we will show that it is possible to generate significant nongaussianity during tachyonic preheating.

We see that once $t > t_c$ the tachyon field rolls down the unstable direction towards the true minima $\pm v$. Causality forbids the tachyon field $\sigma$ from rolling to the same vacuum in two spatial regions which are causally disconnected. The universe therefore fragments into domains of $\sigma = +v$ and $\sigma = -v$. Continuity of the field $\sigma$ means that there must be some region in between these domains where the field remains trapped at $\sigma = 0$: this is exactly the kink solution (1.54). This mean that tachyonic preheating will produce a network of domain wall topological defects in the early universe. Since during radiation/dust domination the causal horizon is $\sim H^{-1}$ it follows that we must expect *at least* one defect per Hubble volume. However, this is merely a lower bound on the produced number of defects and typically the microscopic dynamics will produce a network with characteristic defect separation $\sim \lambda^{-1/2}v^{-1}$ [32, 33, 34]. (The argument above is a caricature of the Kibble mechanism [35] which ensures that topological defects will be formed at cosmological phase transitions. See [36] for a more modern review.) After formation the defect network will undergo some highly nontrivial nonlinear dynamics which is thought to lead to a *scaling solution* so that at late times there is roughly one defect per Hubble volume, regardless of the initial density.[11] We will discuss the formation of topological defects at the end of brane inflation and the cosmological consequences of the defect network in chapter 5.

We should mention that a cosmological network of domain walls is not compatible with late-time cosmology since these defects will overclose the universe unless their

[11] In the case of a cosmic string network, the existence of such a scaling solution is fairly well understood. For a domain wall network this remains to be demonstrated conclusively [37, 38], however, a scaling solution is expected.

tension is fine tuned to be unreasonably small. The above discussion also holds with only minor modifications for preheating with a complex tachyon which leads to the formation of a network of cosmic strings. Cosmic string networks are not ruled out and lead to a variety of interesting observational consequences.

### 1.2.3 Tachyons in String Theory

Certain D-brane configurations in string theory carry no conserved charges and hence are unstable to decay (see [39, 40] for reviews). In particular, the following systems are unstable:

1. *The brane-antibrane system.* The lightest stretched string mode between a parallel brane and antibrane separated by a distance $y$ has a mass-squared given by
$$m^2 = m_s^2 \left[ -\frac{1}{2} + \frac{(m_s y)^2}{(2\pi)^2} \right]$$
where $m_s$ is the string mass. This state become tachyonic once the branes are sufficiently close, signaling that the system has become unstable to decay.

2. *Wrong dimensional branes.* In type IIA/IIB string theory odd/even dimensional branes contain a tachyonic excitation in the spectrum of open strings ending on the brane.

3. *The brane-brane system with fluxes.* Certain brane-brane systems with fluxes turned on have tachyonic instabilities. One example in the D3/D7 system [209] in which case the presence of non-self-dual D7 fluxes can generate a negative contribution to the mass-squared of the stretched string modes between the branes so that a tachyon develops at sufficiently small inter-brane separation.

The decay of unstable objects in string theory may lead to interesting cosmological consequences. In order to explore these consequences we must first derive an effective field theoretic decription of the open string tachyon. To do so we recall that the D$p$-brane in string theory is described by the Dirac-Born-Infeld (DBI) action (see, for example, [41])

$$S = -T_p \int d^{p+1}\sigma e^{-\phi} \sqrt{-\det\left(G_{AB} + B_{AB} + 2\pi\alpha' F_{AB}\right)} \tag{1.58}$$

The brane tension is

$$T_p = \frac{m_s^{p+1}}{(2\pi)^p}$$

while the dilaton $\phi$ is related to the string coupling as $e^{-\phi} = g_s^{-1}$ and $\alpha' \equiv m_s^{-2}$. We define indices $A, B = 0, \cdots, p$ which run over the directions parallel to the brane while $\mu, \nu = 0, \cdots, D-1$ run over the full D-dimensional spacetime and $a, b = p+1, \cdots, D-1$ run over the directions perpendicular to the brane. The coordinates $\sigma^A$ parameterize the embedding of the brane into the full spacetime. In (1.58) $G_{AB}$ is the pull-back of the spacetime metric $g_{\mu\nu}$ to the hypersurface of the brane

$$G^{AB} = \frac{\partial \sigma^A}{\partial x^\mu} \frac{\partial \sigma^B}{\partial x^\nu} g^{\mu\nu} \tag{1.59}$$

Similarly $B_{AB}$ and $F_{AB}$ are the pull-backs of the antisymmetric tensor and the open string $U(1)$ gauge field strength respectively. Setting $F_{AB} = B_{AB} = 0$ and assuming a constant dilaton field the action becomes

$$S = -\tau_p \int d^{p+1}\sigma \sqrt{-\det\left(G_{AB}\right)} \tag{1.60}$$

where

$$\tau_p = \frac{m_s^{p+1}}{(2\pi)^p} \frac{1}{g_s}$$

Intuitively this action makes sense because $\delta S = 0$ minimizes the surface area of the D-brane. We can choose the coordinates along the brane such that

$$\begin{aligned} \sigma^A &= x^A \\ \sigma^a &= y^a(x) \end{aligned}$$

(this choice is referred to as the static gauge) so that (1.60) becomes

$$S = -\tau_p \int d^{p+1}x \sqrt{-g}\, \sqrt{1 + \delta^{ab} g^{\mu\nu} \partial_\mu y_a \partial_\nu y_b} \tag{1.61}$$

where the fields $y^a(x)$ are the transverse scalars which describe the fluctuations of the brane surface. Notice that in the absence of transverse fluctuations the stress tensor associated with the D$p$-brane is of the form $T_{AB} = -g_{AB}\,\tau_p\,\delta^{D-p-1}(\mathbf{x}-\mathbf{x_0})$ while $T_{ab} = T_{Ab} = 0$ (here $\mathbf{x_0}$ gives the position of the brane in the space spanned by $\{x^a\}$).

We would like to generalize the D$p$-brane action (1.61) to incorporate the tachyon field which appears on the world-volume of a wrong dimensional brane. A natural generalization, usually called the Sen action, is [42, 43, 44]

$$S = -\int d^{p+1}x\sqrt{-g}\,V(T)\,\sqrt{1+g^{\mu\nu}\partial_\mu y^a \partial_\nu y_a + g^{\mu\nu}\partial_\mu T\partial_\nu T} \tag{1.62}$$

where $T$ is a real scalar field describing the open string tachyon and $V(T)$ is the tachyon potential which satisfies

$$\begin{aligned} V(0) &= \tau_p, \quad V''(0) < 0, \\ V(T_{\rm min}) &= 0, \quad V''(T_{\rm min}) > 0 \end{aligned}$$

and $T_{\rm min}$ is the value of $T$ which minimizes the potential (we will eventually be interested in the limit $T_{\rm min} \to \infty$). The configuration $T = 0$ is an unstable maximum of the potential which describes a system consisting of a single wrong-dimensional D$p$-brane. One expects the tachyon to roll to $T_{\rm min}$, the true minimum, which describes the vacuum without any D$p$-brane (since $V(T)$ plays the role of a time-varying brane tension). In a full string theory calculation the brane will decay to a gas of closed strings [45].

Though it has not been derived from first principles, the simple action (1.62) does a remarkably good job of describing the dynamics. In particular:

1. This action can be obtained from string theory in some limit [46] with the potential
$$V(T) = \frac{\tau_p}{\cosh\left[\frac{T}{\sqrt{2}\,l_s}\right]} \tag{1.63}$$
where $l_s \equiv m_s^{-1}$ is the string length. (For the remainder of the chapter we assume this potential.)

2. This action reproduces the correct tachyonic mass in the unstable vacuum [45].

3. It yields the correct stress tensor for homogeneous tachyon condensation [45, 47, 49].

4. The action admits soliton solutions corresponding to lower dimensional branes. The tension of the defect reproduces the expected D-brane tension [50].

5. The action for the fluctuations about the defect solutions reproduces the expected DBI action for a brane [50].

6. In studies of dynamical defect formation using this action, the gradient of the tachyon field blows up in a finite time near the core of the defect [4, 51], leading to a delta function divergence in the stress tensor [1]. The same result was observed in a fully string theoretic calculation in [47, 48].

7. The late time solutions close to the tachyon vacuum give a coarse-grained description of the gas of closed strings which are produced during brane decay [52].

In chapter 3 we will describe in detail the dynamics of the tachyon field close to the true vacua $T \to \pm\infty$. In particular, we will show that small initial inhomogeneities in the field profile will lead to the formation of *caustics* where second (and higher) spatial derivatives of the field blow up, invalidating the effective description (1.62).

Let us now try to construct a kink solution the action (1.62) neglecting the transverse scalars $y^a = 0$. The following discussion follows closely [50]. We are interested in a profile which depends on only one spatial coordinate $x^1 \equiv x$. The $(x, x)$ component of the stress-tensor is

$$T_{xx} = \frac{V(T)}{\sqrt{1 + (\partial_x T)^2}}$$

and energy-momentum conservation gives $\partial_x T_{xx} = 0$ so that $T_{xx}$ is constant. Far from the core of the kink (which we take to be at $x = 0$) the tachyon should be in the true vacuum $T = \pm\infty$ (recall eq. (1.54)) so that $T_{xx} = 0$. At the core of the kink $T = 0$ so that $T_{xx} = 0$ requires $\partial_x T = \infty$. Clearly the solution appears singular, however, the stress tensor is well-defined and is independent of how we regularize the kink solution. Consider the profile

$$T(x) = f\left(\frac{x}{\varepsilon}\right) \tag{1.64}$$

where $f(u)$ satisfies

$$f(u) = -f(-u), \quad f'(u) > 0 \ \forall \ u, \quad f(\pm\infty) = \pm\infty$$

but is otherwise arbitrary. This profile is meant to be understood in the limit $\varepsilon \to 0$. Such a field profile satisfies energy-momentum conservation and is a solution of the full equation of motion for $T$. The resulting stress tensor gives the expected result for an infinitely thin defect (compare to eq. (1.55))

$$\begin{aligned} T_{00} &= -T_{22} = \cdots = \tau_{p-1}\delta(x) \\ T_{xx} &= T_{0x} = \cdots = 0 \end{aligned}$$

(For finite $\varepsilon$ one obtains a strongly peaked function which tends to a delta function in the limit $\varepsilon \to 0$.) The fact that the kink solution is singular no longer seems surprising when one recalls that the D-brane should be infinitely thin. Since the D-brane stress tensor has a delta function profile, it is not surprising that tachyon kink solution must also be a distribution. When we consider the dynamical formation of such a tachyon defect in chapters 2 and 5 we will see that the defect forms within a finite time $t_c$ and that the role of the parameter $\varepsilon$ which regulates the kink is played by $t_c - t$.

As we have mentioned previously, the action describing small fluctuations about this kink solution exactly reproduces the expected DBI action for a stable D$(p-1)$-brane. We see that the tachyon kink, which would be expected to form during brane decay, *is* a lower dimensional brane. The above discussion also generalizes to the brane-antibrane system in which case the tachyon is a complex field and the resulting vortex-like singular soliton solution describes a codimension-two brane, which results as a decay product from brane-antibrane annihilation.

# Chapter 2

# Creating the Universe from Brane-Antibrane Annihilation

# Abstract

When $p$-dimensional branes annihilate with antibranes in the early universe, as in brane-antibrane inflation, stable $(p-2)$-dimensional branes can appear in the final state. We reexamine the possibility that one of these $(p-2)$-branes could be our universe. In the low energy effective theory, the final state branes are cosmic string defects of the complex tachyon field which describes the instability of the initial state. We quantify the dynamics of formation of these vortices. This information is then used to estimate the production of massless gauge bosons on the final branes, due to their coupling to the time-dependent tachyon background, which would provide a mechanism for reheating after inflation. We improve upon previous estimates indicating that this can be an efficient reheating mechanism for observers on the brane.


## 2.1 Introduction

In the last few years significant progress has been made in constructing string theoretic cosmological models where inflation is driven by the naturally occuring potentials between D-branes and their antibranes [24, 25], [53]-[55]. The formation of lower-dimensional branes at the end of inflation can lead to interesting signals: cosmic-string-like or higher dimensional defects could be observable remnants [56, 57, 58], possibly providing a rare clue to the stringy origin of inflation. A more radical idea was explored in [59]: perhaps our own observable universe is such a defect in the higher-dimensional spacetime predicted by string theory. Since the stable branes in Type IIB string theory have spatial dimensionalities which are odd, a 3-brane would have descended from annihilation of 5-branes in this picture, and thus they would be codimension-two defects in of the effective 6D theory. Codimension-two braneworlds have attracted interest lately because of their novel features, which might have some bearing on the cosmological constant problem [60, 61].

Our interest in this scenario is motivated by questions about the efficiency of reheating in brane-antibrane inflation [62]. It is possible that the energy density liberated from the brane collisions will be converted mostly into closed string states, ultimately gravitons, and not necessarily into visible radiation [63]. A generic mechanism which could avoid this problem was proposed in [59], wherein the reheating in D-brane driven inflation is due to the coupling of massless gauge fields to a time-dependent tachyon condensate, which describes the annihilation process. However, ref. [59] considered only the formation of a tachyon kink instead of the more realistic case of a vortex, and it used a somewhat crude ansatz for the background tachyon field. The problem of finding the actual tachyon background predicted by string theory was studied numerically in [51] but no attempt was made to improve on the reheating computation. In this chapter we aim to analytically determine the dynamics of formation of lower dimensional branes described as tachyon defects—both kinks and vortices—and to improve on the reheating calculation of [59].

Let us begin by describing the scenario we have in mind. In the simplest version of D-brane inflation a parallel brane and antibrane begin with some separation between them in one of the extra dimensions. Although parallel branes are supersymmetric and have no force between them, the brane-antibrane system breaks supersymmetry so that there is an attractive force and hence a nonvanishing potential energy. It is the latter which drives inflation. Once the branes have reached a critical separation one of the stretched string modes between the branes, $T$, becomes tachyonic and the branes become unstable to annihilation. The tachyon field starts from the unstable maximum $T = 0$ and rolls towards the vacuum $T \to \pm\infty$. However, topological defects may form through the Kibble mechanism [56, 57, 58] so that $T = 0$ stays fixed at the core of the defect. These defects are known to be consistent descriptions of branes whose dimension is lower than that of the original branes [50, 64, 65]. For example, the brane-antibrane system has a complex tachyon field, leading to vortices which represent codimension-two branes. On the other hand, an unstable brane has a real tachyon which leads to kinks representing codimension-one branes.

The formation of tachyon defects at the endpoint of D-brane inflation is a dynamical process where the tachyon couples to gauge fields which will be localised on the descendant brane. It is thus expected that some radiation will be produced by the rolling of the tachyon and the problem of reheating becomes quantitative: can this effect be efficient enough to strongly deplete the energy density of the tachyon fluid so the the universe starts out being dominated by radiation rather than cold dark matter? It is important to stress that though the situation is somewhat analogous to that of hybrid inflation (where the tachyon plays the role of the unstable direction in field space which allows for inflation to end quickly) the mechanism for reheating is qualitatively different. The difference is that in the low energy effective field theory which describes the tachyon $T$, the potential is minimized at $T = \pm\infty$ and there are no oscillations about the minimum of the potential. In a normal hybrid inflation model, $T$ would have a minimum at some finite value and the oscillations of $T$ around its minimum would give rise to reheating in the usual way. In the present case, the time dependence of the background is monotonic, not oscillatory. Reheating thus might seem to resemble gravitational particle production [66] rather than the standard picture in which the inflaton decays. However, in this work we highlight an important difference between reheating through tachyon condensation and gravitational particle production, which can make the former much more efficient: there is a divergence in the stress-energy tensor of the tachyon field within a finite time, which corresponds to the formation of the lower-dimension D-brane.

In this chapter we study analytically the dynamical formation of the tachyon vortex and improve the reheating calculation, using a slightly simplified model of particle production by tachyon condensation, which captures the essential physics revealed by the analysis of vortex formation. In section 2.2 we review the formation of tachyon kinks which describes the condensation of a brane to a brane of codimension-one. In sections 2.3-2.5 we study analytically the formation of a tachyon vortex on the brane-antibrane pair. Section 2.3 introduces Sen's action for the complex tachyon field describing this situation. Section 2.4 presents new analytic results for the time-dependent, complex tachyon field representing vortex formation, both near to and far from the vortex core. In section 2.5 we show that the stress-energy tensor for the system splits into a localized, singular piece describing the descendant branes, plus a bulk contribution that describes the rolling tachyon condensate. Section 2.6

introduces the effective action for U(1) gauge bosons which become localized on the final-state 3-brane, in the rolling tachyon background. This provides a model for the visible radiation produced during reheating. In section 2.7 we calculate the energy density of this produced radiation on the 3-brane, using some reasonable simplifying assumptions. Section 2.8 gives our conclusions, including speculation about how the final brane-antibrane system could be stabilized.

## 2.2 Dynamical Tachyon Kink Formation on Unstable Dp-branes

In this section we review the dynamical formation of a D$(p-1)$-brane through tachyon condensation on an unstable D$p$-brane, and derive a few new results. The equations of motion in this case are simpler than in the case of the vortex and we will use the analysis of this section to reinforce our conclusions when we analyze the vortex since many of the results are quite analogous.

### 2.2.1 Effective Field Theory and Equations of Motion

We will work with the effective action for the tachyon on an unstable Dp-brane [42, 44][1]

$$S = -\int V(T)\sqrt{-\det|\eta_{MN} + \partial_M T \partial_N T|}\, d^{p+1}x \tag{2.1}$$

where we have set the gauge fields and transverse scalars to zero. We use the potential $V(T) = \tau_p \exp(-T^2/a^2)$ where $\tau_p$ is the tension of a D$p$-brane and $a = 2\sqrt{\pi\alpha'}$. The value of the constant $a$ is chosen so that the potential satisfies the normalization condition

$$\int_{-\infty}^{+\infty} V(y)\, dy = 2\pi\sqrt{\alpha'}\tau_p = \tau_{p-1} \tag{2.2}$$

[1]The convention for indices is that upper case roman indices $\{M, N\}$ run over the full space-time coordinates $\{0, 1, \cdots, p\}$, greek indices $\{\mu, \nu\}$ run over the defect coordinates $\{0, 1\}$ and "hatted" greek indices $\{\hat{\mu}, \hat{\nu}\}$ run over the remaining spatial coordinates $\{2, 3, \cdots, p\}$. We use metric signature $\mathrm{diag}(-1, 1, 1, \cdots)$.

proposed in [50]. This normalization was used in [50] to fix the tension of the singular static kink solution of the action (2.1) to correspond to the tension of a D($p-1$)-brane. For a time-dependent kink solution we take $T$ to be a function of $x^\mu = (t, x)$ so that the action (2.1) becomes

$$S = -\int V(T)\sqrt{1+\partial_\mu T \partial^\mu T}\, d^{p+1}x. \tag{2.3}$$

Static solutions of the theory (2.3) are well studied in the literature [50, 67]. Inhomogeneous solutions have also been studied in some detail [59, 51, 68, 69, 70]. The energy momentum tensor for (2.3) is

$$T_{\mu\nu} = \frac{V(T)}{\sqrt{1+\partial_\rho T \partial^\rho T}}\partial_\mu T \partial_\nu T - \eta_{\mu\nu} V(T)\sqrt{1+\partial_\rho T \partial^\rho T}, \tag{2.4}$$

and the Euler-Lagrange equation of motion is

$$\partial^\mu \partial_\mu T - \frac{\partial_\mu \partial_\nu T \partial^\mu T \partial^\nu T}{1+\partial^\rho T \partial_\rho T} - \frac{V'(T)}{V(T)} = 0 \tag{2.5}$$

where $V'(T) = \frac{\partial V(T)}{\partial T}$. It is worth noting, as in [71], that the equation of motion (2.5) is equivalent to conservation of energy $\partial_\mu T^{\mu\nu} = 0$ for nonconstant $T$ since $\partial_\mu T^{\mu\nu} = \partial^\nu T\left[\partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu T)}\right) - \frac{\partial \mathcal{L}}{\partial T}\right]$. It will be useful in the ensuing analysis to define

$$\Sigma = \frac{V(T)}{\sqrt{1+\partial_\mu T \partial^\mu T}}. \tag{2.6}$$

### 2.2.2 Solutions Near the Core of the Defect

At the core of the kink we expect the field to stay pinned at $T = 0$. Consider initial data $T(t=0, x) = T_i(x)$ and $\dot{T}(t=0,x) = \dot{T}_i(x) = 0$ [2]. One expects the field to start to roll where $T_i(x) \neq 0$ due to the small displacement from the unstable maximum $V'(T) = 0$. At $t = 0$ the equation of motion (2.5) is

$$\ddot{T}_i(x)(1+T_i'(x)^2) = T_i''(x) + 2a^{-2}T_i(x)(1+T_i'(x)^2).$$

[2]For the remainder of this section the dot denotes differentiation with respect to time while the prime denotes differentiation with respect to the $x$ coordinate.

Clearly any point $x_0$ where $T_i(x_0) = T_i''(x_0) = 0$ will be a fixed point where $\ddot{T}(t, x_0) = 0 = \dot{T}(t, x_0)$ throughout the evolution. We restrict ourselves only to considering intial data such that $\text{sgn}(\ddot{T}_i(x)) = \text{sgn}(T_i(x))$ for all $x$ to ensure that the solutions are increasing.

At the site of the kink (which we take to be $x_0 = 0$) we have $T = 0$; hence there should always be some neighbourhood of the point $x = 0$ where we can take $V'(T) \cong 0$ so that (2.5) yields

$$\ddot{T}(1 + T'^2) = (1 - \dot{T}^2)T'' + 2\dot{T}T'\dot{T}'. \tag{2.7}$$

This has an increasing solution with $T'' = 0$

$$T(x,t) = x \tan\left[\frac{\omega}{a}(t - t_c) + \frac{\pi}{2}\right]. \tag{2.8}$$

Near the site of the kink the slope of the tachyon field diverges as $(t_c - t)^{-1}$ as $t$ approaches the critical time $t_c$, similar to the solutions in [51].

The finite-time slope divergence was observed both numerically and analytically in [51] and leads to the formation of a singularity in the energy density at $t = t_c$. This effect was also found in an exact string theoretic calculation in [48]. As $t \longrightarrow t_c$ we have $\Sigma(t \longrightarrow t_c, x \cong 0) \longrightarrow 0$. The case $\Sigma = const$ arises as a first integral of the motion in the static case and the limit $\Sigma \longrightarrow 0$ corresponds to the singular soliton solution of Sen [67]. It is natural to expect, then, that as $t \longrightarrow t_c$, near $x = 0$, the tachyon field $T(t = t_c, x = 0)$ coincides with the stable kink solution of Sen and the time evolution in this neighbourhood stops. We will argue that at this point a codimension-one brane has formed.

### 2.2.3 Vacuum Solutions

Away from the site of the kink the field is expected to roll towards the vacuum $T \longrightarrow \pm\infty$ so that $V(T) \longrightarrow 0$ at late times for $x \neq 0$. To analytically study the dynamics near the vacuum it is easiest to work in the Hamiltonian formalism [72, 73] since the Lagrangian vanishes in the limit $V(T) \longrightarrow 0$, whereas the Hamiltonian remains well-defined. Defining the momentum conjugate to $T$ as $\Pi = \delta S/\delta\dot{T}$ the Hamiltonian is given by $\mathcal{H} = \sqrt{\Pi^2 + V(T)^2}\sqrt{1 + \partial_i T \partial_i T}$. It is useful to rewrite Hamilton's equations of motion, $\dot{\Pi} = -\frac{\partial \mathcal{H}}{\partial T}$ and $\dot{T} = \frac{\partial \mathcal{H}}{\partial \Pi}$, in a manifestly covariant way as

$$\partial^\mu T \partial_\mu T + 1 = \frac{V(T)^2}{\Sigma^2} \tag{2.9}$$

and

$$\Sigma\, \partial_\mu [\Sigma \partial^\mu T] = V(T)V'(T), \tag{2.10}$$

where $\Sigma = \Pi/\dot{T}$ is defined in (2.6). In the limit $V(T) \rightarrow 0$ equations (2.9), (2.10) yield

$$\partial^\mu T \partial_\mu T + 1 = 0 \tag{2.11}$$

and

$$\partial_\mu [\Sigma \partial^\mu T] = 0. \tag{2.12}$$

The solutions of (2.11) were found for arbitrary Cauchy data in [68] using the method of characteristics. The generic solutions exhibit the formation of caustics where second and higher order derivatives become singular. Caustics are known to form in systems with a pressureless fluid, which is a good description of the tachyon field as it approaches its ground state $T \rightarrow \infty$. It is not known whether the caustics are a genuine prediction of string theory or just an artifact of the derivative truncation which leads to the Born-Infeld Lagrangian for the tachyon.

In any case, caustics are not present in the simplest solution of eq. (2.11), $T = \pm t$, which is the asymptotic form for the homogeneously rolling tachyon. For $\dot{T}^2 = 1$, eq. (2.12), which is equivalent to energy conservation, implies that $\Sigma(t, x) = \Sigma(x)$ is an arbitrary function of $x$. In this regime the energy momentum tensor (2.4) is identical to that of pressureless dust $T_{\mu\nu} = \Sigma(x) u_\mu u_\nu$ where $u_\mu = \partial_\mu T$ is interpreted as the local velocity vector and $\Sigma$ is interpreted as a Lorentz-invariant matter density [44, 72, 73].

### 2.2.4 Stress-Energy Tensor

We are interested in the behavior of $T_{MN}$ as $t \rightarrow t_c$. First we consider the neighbourhood near $x = 0$ where $T(t, x) \cong kx/(t_c - t)$ as $t \rightarrow t_c$ (see 2.8). Near $x = 0$, the

Hamiltonian is

$$T_{00} \cong \frac{\tau_p k}{t_c - t} \exp\left(-\frac{k^2 x^2}{a^2 (t_c - t)^2}\right)$$

and

$$\lim_{t \to t_c^-} T_{00} = \sqrt{\pi} a \tau_p \delta(x) = \tau_{p-1} \delta(x) \tag{2.13}$$

using the normalization (2.2) for the potential. Similarly

$$T_{\hat{\mu}\hat{\nu}} \rightarrow -\tau_{p-1}\, \delta(x)\, \delta_{\hat{\mu}\hat{\nu}}$$

and $T_{11} \rightarrow 0$ as $x \rightarrow 0$.

Consider now the late-time behavior of $T_{MN}$ away from the site of the kink. Using the solutions of section 2.2.3 we find $T_{00} \rightarrow \Sigma(x)$ while $T_{11}$, $T_{01}$ and $T_{\hat{\mu}\hat{\nu}}$ tend to zero for $x \neq 0$.

To summarize, we find that in the limit of condensation the energy momentum tensor is identical to that of a D$(p-1)$-brane:

$$\begin{aligned} T_{00} &= \tau_{p-1}\, \delta(x) + \Sigma(x) \\ T_{11} &= T_{01} = 0 \\ T_{\hat{\mu}\hat{\nu}} &= -\tau_{p-1}\, \delta(x)\, \delta_{\hat{\mu}\hat{\nu}}. \end{aligned}$$

The extra bulk energy density $\Sigma(x)$ is similar to the result in [71] and corresponds to what has been dubbed tachyon matter.

## 2.3 Effective Tachyon Field Theory on the Brane-Antibrane Pair

We would like to generalize the results of the previous section to study the dynamical formation of a tachyon vortex and hence a codimension-two brane. This is the more realistic situation, since the stable D-branes of a given string theory are those whose dimensions differ by multiples of two. We will work with an effective action proposed by Sen [50] for the tachyon on a brane-antibrane pair. The field content for this system is a complex tachyon field T, massless gauge fields $A_\mu^{(1)}$, $A_\mu^{(2)}$ and scalar fields $Y^I_{(1)}$, $Y^I_{(2)}$ corresponding to the transverse fluctuations of the branes. The index $(i) = (1), (2)$, which we call the brane index, labels which of the original branes (actually the brane or the antibrane) the field is associated with. The effective action is:

$$\begin{aligned} S = -\int V(T, Y^I_{(1)} - Y^I_{(2)}) \Big(\sqrt{-\det M^{(1)}} \\ + \sqrt{-\det M^{(2)}}\Big)\, d^{p+1}x \end{aligned} \tag{2.14}$$

where [3]

$$\begin{aligned} M^{(i)}_{MN} &= g_{MN} + \alpha' F^{(i)}_{MN} + \partial_M Y^I_{(i)} \partial_N Y^I_{(i)} \\ &+ \frac{1}{2} D_M T D_N T^* + \frac{1}{2} D_M T^* D_N T, \end{aligned} \tag{2.15}$$

$$F^{(i)}_{MN} = \partial_M A^{(i)}_N - \partial_N A^{(i)}_M, \quad D_M = \partial_M - iA^{(1)}_M + iA^{(2)}_M. \tag{2.16}$$

For the remainder of this chapter we will ignore the transverse scalars and choose $V(T,0) = V(T) = \tau_p \exp(-|T|^2/a^2)$ where $a$ is chosen so that the static singular vortex solutions of the theory (2.14) have the correct tension to be interpreted as codimension 2 D-branes according the the normalization proposed in [50]. We will discuss the normalization of the potential proposed in [50] in more detail when we calculate the energy momentum tensor for the theory (2.14).

Though the action (2.14) was not derived from first principles it obeys several necessary consistency conditions which are discussed in [50]. There have been various other proposals for the tachyon effective action and vortex solutions on the brane-antibrane pair [74, 75, 76]. See [77] for a discussion of various models including (2.14).

[3]Greek indices $\{\mu, \nu\}$ are now understood to run over the coordinates $\{0, 1, 2\}$ on which the vortex solutions depend, and hatted greek indices $\{\hat{\mu}, \hat{\nu}\}$ run over the spatial coordinates parallel to the vortex $\{2, 3, \cdots, p\}$, where $p = 6$ for a vortex which describes a 3-brane. Upper case roman indices $\{M, N\}$ still run over the full space-time coordinates $\{0, 1, \cdots, p\}$. Finally it will be convenient later on to refer to lower case roman indices $\{m, n\}$ which run over only the time and radial coordinates $\{0, 1\}$.

## 2.4 Vortex Solutions on the Brane-Antibrane Pair

To construct vortex solutions we ignore the transverse scalars and take the remaining fields to depend only on the polar coordinates $x^\mu = (x^0, x^1, x^2) = (t, r, \theta)$ with metric $g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + dr^2 + r^2 d\theta^2$. Since the vortex solution should have azimuthal symmetry we make the ansatz:

$$T(t,r,\theta) = e^{i\theta} f(t,r), \quad A_\theta^{(1)} = -A_\theta^{(2)} = \frac{1}{2} g(t,r) \tag{2.17}$$

with all other components of $A_\mu^{(i)}$ vanishing. This generalizes the ansatz used in [50] to include time-dependence in the fields $f$ and $g$. For (2.17) one has

$$D_t T = e^{i\theta} \dot{f}(t,r), \quad D_r T = e^{i\theta} f'(t,r),$$

$$D_\theta T = e^{i\theta} i(1 - g(t,r)) f(t,r)$$

and

$$F_{t\theta}^{(1)} = \frac{1}{2}\dot{g}(t,r) = -F_{t\theta}^{(2)}, \quad F_{r\theta}^{(1)} = \frac{1}{2} g'(t,r) = -F_{r\theta}^{(2)}$$

where the dot denotes differentiation with respect to time and the prime now denotes differentiation with respect to the radial coordinate. The matrices $M_{MN}^{(i)}$ are

$$\begin{aligned} \left[M_{MN}^{(1)}\right] &= \begin{bmatrix} -1+\dot{f}^2 & \dot{f}f' & \alpha'\dot{g}/2 & 0 \\ \dot{f}f' & 1+f'^2 & \alpha' g'/2 & 0 \\ -\alpha'\dot{g}/2 & -\alpha' g'/2 & r^2 + (1-g)^2 f^2 & 0 \\ 0 & 0 & 0 & \delta_{\hat{\mu}\hat{\nu}} \end{bmatrix}, \\ \left[M_{MN}^{(2)}\right] &= \left[M_{MN}^{(1)}\right]^T . \end{aligned} \tag{2.18}$$

We also have $\det(M^{(1)}) = \det(M^{(2)})$ since $M_{MN}^{(1)} = M_{NM}^{(2)}$ and so we omit the brane index $(i)$ on $\det(M^{(i)})$ in subsequent calculations.

The action for this ansatz simplifies to:

$$\begin{aligned} S &= -2\int V(f) \Bigg( (1 + \partial_m f \partial^m f) \left[r^2 + f^2(1-g)^2\right] \\ &+ \frac{\alpha'^2}{4}\partial_m g \partial^m g - \frac{\alpha'^2}{4}\left(\epsilon^{mn}\partial_m f \partial_n g\right)^2 \Bigg)^{1/2} d^{p+1}x \end{aligned} \tag{2.19}$$

where $x^m = (x^0, x^1) = (t, r)$ and $g_{mn}dx^m dx^n = -dt^2 + dr^2$. For notational convenience we define the scalar quantity

$$\Sigma(t,r) = \frac{V(f)}{\sqrt{-\det(M)}} \tag{2.20}$$

in analogy with (2.6). The equation of motion for the tachyon is

$$\begin{aligned} &\partial_m \left[ \Sigma \left[ r^2 + (1-g)^2 f^2 \right] \partial^m f - \frac{\alpha'^2 \Sigma}{4} (\epsilon^{ab} \partial_a f \partial_b g) \epsilon^{mn} \partial_n g \right] \\ &= \Sigma(1 + \partial^m f \partial_m f)(1-g)^2 f + \frac{V'(f) V(f)}{\Sigma}, \end{aligned} \tag{2.21}$$

and the nontrivial component of the equation of motion for the gauge field is

$$\begin{aligned} &\frac{\alpha'^2}{4} \partial_m \left[ \Sigma \partial^m g \;-\; \Sigma(\epsilon^{ab} \partial_a f \partial_b g) \epsilon^{mn} \partial_n f \right] \\ &\qquad = \Sigma(1 + \partial^m f \partial_m f) f^2 (g-1). \end{aligned} \tag{2.22}$$

Alhough these equations are somewhat cumbersome, inspection of (2.22) tells us that there should exist a solution $g(t,r)$ such that at $g = 1$, the vacuum, we have $\partial_m g = 0$. This is the asymptotic behavior which corresponds to a vortex solution; it is already known from [50] that the static solution $g(r)$ is a monotonically increasing function which varies between 0 and 1. Thus we shall only consider solutions with the asymptotic behavior $\partial_m g(t,r) \to 0$ as $g(t,r) \to 1$. We will take initial data $g(0,r) = g_i(r)$ such that $g_i(0) = 0$, $0 \leq g_i(r) \leq 1$ for all $r$ and $g_i'(r) \geq 0$ for all $r$. In addition we will focus on initial tachyon profiles $f(0,r) = f_i(r)$ such that $f_i(0) = 0$ and $f_i'(r) > 0$ for all $r$. For these initial conditions the tachyon must start rolling for $r \neq 0$ due to its displacement from the unstable vacuum $V'(f) = 0$. Since the asymptotic $g_i(r \to \infty) \to 1$ is an exact solution of (2.22) and $g(t, r = 0) = 0$ by construction, we therefore expect $g(r,t)$ to increase towards unity for finite $r \neq 0$.

### 2.4.1 Solutions Near the Core of the Defect

As in [51], to analytically study the dynamics near the core of the vortex, $r = 0$, we make the ansatz:

$$f(t,r) \cong p(t)r, \quad g(t,r) \cong q(t)r \tag{2.23}$$

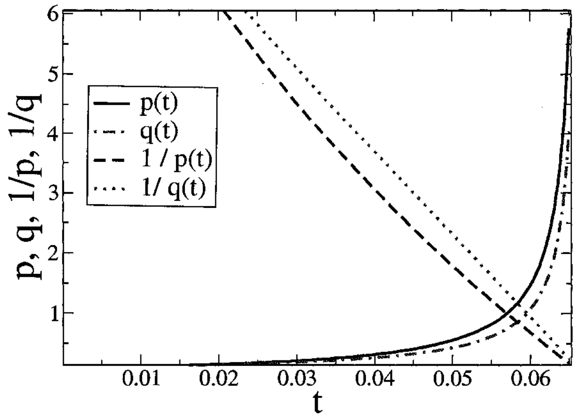


Figure 2.1: Numerical solution for $p(t)$ and $q(t)$ of (2.24-2.25) showing the finite-time slope divergence. $1/p$ and $1/q$ are also shown, demonstrating the linearity of these functions near the critical time.

for small $r$. Dropping terms which are subleading in $r$ yields a set of coupled ODEs for $p(t)$ and $q(t)$

$$pq^3\ddot{q} - \ddot{p}q^4 + 2\dot{p}q^3\dot{q} - 2pq^2\dot{q}^2 + \frac{4}{\alpha'^2}p^3q^2 + \frac{4}{\alpha'^2}pq^2 + \frac{2}{a^2}pq^4 = 0 \qquad (2.24)$$

from (2.21) and

$$\begin{aligned} -p\ddot{p}q^3 &+ p^2q^2\ddot{q} + q^2\ddot{q} - 2q\dot{q}^2 + 2p\dot{p}q^2\dot{q} - 2p^2q\dot{q}^2 + \frac{4}{\alpha'^2}p^4q \\ &+ \frac{8}{\alpha'^2}p^2q + \frac{4}{\alpha'^2}q + \frac{2}{a^2}p^2q^3 = 0 \qquad (2.25) \end{aligned}$$

from (2.22). Although (2.24,2.25) are difficult to solve analytically it is straightforward to verify that in the regime where $\dot{p}$, $\dot{q}$ and higher derivatives are large compared to $p$, $q$ there exists an approximate solution to (2.24-2.25) where both $p(t)$ and $q(t)$ are divergent in finite time $t_c$:

$$p(t) = \frac{p_0}{t_c - t}, \qquad q(t) = \frac{q_0}{t_c - t} \qquad (2.26)$$

in analogy with the kink solution of [51]. Numerical solutions to (2.24-2.25) agree with this prediction, as shown in figure 2.1.

### 2.4.2 Solutions Away From the Core of the Defect

We are interested in solutions where $\partial_m g \to 0$ as $g \to 1$ and where $f(t,r) \to \infty$ as $t \to \infty$. Since, as we have seen above, $g'(t, r = 0)$ is diverging in finite time, therefore $g$ must be increasing to unity for $r \neq 0$ so that at late times we expect $g(t,r)$ to resemble a step function. Thus to study the dynamics away from the core of the defect we begin with the ansatz:

$$g(t,r) = 1 - \varepsilon\sigma(t,r) \tag{2.27}$$

and work only to leading order in $\varepsilon$. The leading order contribution to (2.21) decouples completely from $\sigma(t,r)$

$$\begin{aligned} &\left[\ddot{f}(1+f'^2) - f''(1-\dot{f}^2) - 2\dot{f}f'\dot{f}' - \frac{2f}{a^2}(1-\dot{f}^2+f'^2)\right] r \\ &- \left[1-\dot{f}^2+f'^2\right] f' = 0. \end{aligned} \tag{2.28}$$

We can consistently find solutions of (2.28) by taking $f$ to be a solution of the eikonal equation $1 - \dot{f}^2 + f'^2 = 0$. Subject to this constraint the second term in the square braces in (2.28) vanishes trivially. The constraint that the first term in the square braces in (2.28) vanishes is exactly the same as the equation of motion one would derive from $\mathcal{L} = -V(f)\sqrt{1+\partial_m f \partial^m f}$, the Born-Infeld Lagrangian. The eikonal equation yields the Born-Infeld equation as a differential consequence, which is not surprising since this amounts to minimizing the action by setting $\mathcal{L} = 0$. Thus the PDE (2.28) is automatically satisfied when $f$ is a solution of the eikonal equation. We find, then, that when $g(t,r) \cong 1$ the tachyon field must obey

$$\partial_m f \partial^m f + 1 = 0 \tag{2.29}$$

as in section 2.2.

The ansatz (2.27) yields no simplification of (2.22); however, we may solve for $\sigma(t,r)$ given (2.29) based on more fundamental constraints. The argument of the square root in (2.19) must be nonnegative to ensure reality of the Lagrangian. Thus for $f(t,r)$ given by (2.29) the requirement that the Lagrangian be real translates to

$$\partial_m \sigma \partial^m \sigma - (\epsilon^{mn}\partial_m f \partial_n \sigma)^2 \geq 0.$$

or, with $f$ given by (2.29),

$$-\left(\dot{\sigma}\sqrt{1+f'^2}-\sigma' f'\right)^2 \geq 0. \tag{2.30}$$

For real fields it is clear that (2.30) can only be solved when the equality is taken. It is worth noting that since the Lagrangian vanishes when the equality is taken in (2.30), this constraint ensures that the full equations of motion are satisfied.

As in section 2.2 we can avoid the difficulties of caustic formation in the general solutions of (2.29) found in [68] by taking the Cauchy data to be linear and using the one-parameter family of solutions

$$f(t,r) = \alpha t + \sqrt{\alpha^2 - 1}\, r. \tag{2.31}$$

Reality of the Lagrangian requires

$$\alpha\dot{\sigma} - \sqrt{\alpha^2-1}\,\sigma' = 0.$$

This PDE is separable and we find the solution

$$g(t,r) = 1 - \varepsilon \exp\left(-\frac{r}{R}\right) \exp\left(-\frac{t}{R}\sqrt{\frac{\alpha^2-1}{\alpha^2}}\right) \tag{2.32}$$

where $R$ is a separation constant. Note that the solution (2.32) becomes static in the limit $\alpha^2 \to 1$, the homogeneous rolling tachyon. In fact, for $\alpha^2 = 1$ any function $\sigma(t,r) = \sigma(r)$ satisfying the necessary boundary condition $\sigma(r \to \infty) \to 0$ will generate a solution. We will ultimately be interested in this limit.[4]

It is noteworthy that taking $g(t,r)$ close to unity (2.27) ultimately translates into the requirement that $\det(M)$ must vanish. The solutions (2.31-2.32) should be thought of as late-time asymptotics where $V(f) \to 0$ since $f \to \infty$. In this limit $\Sigma(t,r)$ defined in (2.20) has the indeterminant form $\frac{0}{0}$ as in section 2.2. The quantity $\Sigma$ is of some interest for two reasons. First, it parametrizes the manner in which we take the limits $V(f) \to 0$ and $\det(A) \to 0$ as we approach the vacuum state. Second,

[4]The exact functional form of $\sigma$ at large $r$ and late times turns out to be of little importance to the ensuing analysis.

the form of $\Sigma$ for $r \neq 0$ will determine the form of the energy-momentum tensor in that regime, as in the case of the kink. Following the discussion in section 2.2 we will use energy-momentum conservation to place constraints on the aymptotic form of $\Sigma$.

## 2.5 Stress-Energy Tensor

In this section we demonstrate that the vortex solutions found above give rise to the formation of a singularity in the stress-energy tensor of the tachyon field, which corresponds exactly with that of a codimension-two D-brane in the final state, whose tension has the value expected from string theory. We also derive the bulk stress-energy tensor for the leftover tachyon matter, which continues to roll even after the formation of the D-brane.

The stress-energy tensor for the action (2.14) is

$$\begin{aligned} T^{MN} &= -\frac{V(T, Y^I_{(1)} - Y^I_{(2)})}{r} \Big[\sqrt{-\det M^{(1)}} \big(M^{-1}_{(1)}\big)^{MN}_S \\ &+ \sqrt{-\det M^{(2)}} \big(M^{-1}_{(2)}\big)^{MN}_S\Big] \end{aligned} \tag{2.33}$$

where the subscript $S$ denotes the symmetric part of the matrix, *i.e.*, $(M^{-1}_{(i)})^{MN}_S = \frac{1}{2}[(M^{-1}_{(i)})^{MN} + (M^{-1}_{(i)})^{MN}]$. The components of $T^{MN}$ parallel to the vortex simplify to

$$T^{\hat{\mu}\hat{\nu}} = -\frac{2}{r} V(f) \sqrt{-\det(M)} \delta^{\hat{\mu}\hat{\nu}}. \tag{2.34}$$

For the components involving $t, r, \theta$, it is useful to rewrite $T^{\mu\nu}$ in terms of $\Sigma$ and the symmetrized cofactor matrix of $M^{(i)}_{\mu\nu}$, which we define as $C^{\mu\nu}_{(i)}$.[5] Since $C^{\mu\nu}_{(1)} = C^{\mu\nu}_{(2)}$ we drop the brane index on the cofactor matrices. In the $t, r, \theta$ directions, then, we have

$$T^{\mu\nu} = \frac{2\Sigma}{r} C^{\mu\nu}. \tag{2.35}$$

[5] That is to say $C^{\mu\nu}_{(i)} = \det(M)(M^{-1}_{(i)})^{\mu\nu}_S$.

The nonzero components of the cofactor matrix for the Lagrangian (2.19) are

$$\begin{aligned}
C^{tt} &= \left[r^2 + f^2(1-g)^2\right](1+f'^2) + \frac{\alpha'^2}{4} g'^2, \\
C^{tr} &= -\left[r^2 + f^2(1-g)^2\right]\dot{f}f' - \frac{\alpha'^2}{4}\dot{g}g', \\
C^{rr} &= -\left[r^2 + f^2(1-g)^2\right](1-\dot{f}^2) + \frac{\alpha'^2}{4}\dot{g}^2, \\
C^{\theta\theta} &= -(1-\dot{f}^2 + f'^2).
\end{aligned}$$

### 2.5.1 Normalization of the Potential

In [50] Sen finds that the action (2.14) provides a good effective description of the tachyon on the brane-antibrane system provided the potential $V(T)$ is chosen to satisfy the normalization constraint

$$\tau_{p-2} = 4\pi \int_0^\infty V(z)\sqrt{z^2(1-\hat{G}(z))^2 + \frac{\alpha'^2}{4}(\hat{G}'(z))^2}\, dz \tag{2.36}$$

where $\tau_{p-2} = (2\pi)^2\alpha'\tau_p$ is the tension of a $(p-2)$-brane, $\hat{G}(z) = G(F^{-1}(z))$, and $\{f(r) = F(br),\ g(r) = G(br)\}$ are the static soliton solutions to be understood in the limit that $b \to \infty$. This constraint is necessary to ensure that the vortex solution has the correct tension to be interpreted as a D$(p-2)$-brane. In the time-dependent case there is some ambiguity as to how to interpret (2.36) since this statement appears to depend on the functional form of the solutions, which would make the right-hand-side apparently time-dependent.[6]

But physically, it makes sense to impose (2.36) at $t \geq t_c$ since $t_c$ is the time by which the brane has actually formed, and in the limit $t \to t_c$ the time-dependent solutions should coincide with the soliton solutions in the neighbourhood of $r = 0$. In the time-dependent case, it is $(t_c - t)^{-1}$ which tends to infinity (as $t \to t_c$) and plays the role of $b$. Although the exact functional forms of $G(z)$, $F(z)$ are not known for all $z$, we can infer from (2.26) that $G(z) \cong q_0 z$ and $F(z) \cong p_0 z$ near $z = 0$.

[6] Arguments are presented in [50] for why (2.36) is in fact only a constraint on $V(T)$ and not on the solutions $T(x)$ themselves.

Furthermore, we know that $G(z) \to 1$ for sufficiently large $z$ by construction. We also have $F(z=0)=0$ and $F(z\neq 0)\neq 0$ so that $F^{-1}(0)=0$ and $F^{-1}(z\neq 0)\neq 0$.

Let us consider the two terms under the square root in (2.36). The first term, $z^2(1-\hat{G}(z))^2$, is small near $z=0$ due to the overall multiplicative factor of $z^2$. On the other hand, at large $z$ this term is also small since $F^{-1}(z)$ is large and hence $\hat{G}(z)=G(F^{-1}(z))\cong 1$. We conclude that the derivative term under the root in (2.36) dominates. At small $z$ we have $F^{-1}(z)\cong p_0^{-1}z$ and thus $\hat{G}(z)\cong q_0F^{-1}(z)\cong \frac{q_0}{p_0}z$ so that $\hat{G}'(z)\cong \frac{q_0}{p_0}$. For simplicity we take $\hat{G}'(z)\cong\frac{q_0}{p_0}$ for all $z$ since this expression is multiplied by $V(z)$ which tends to zero quickly for large $z$. We find then that

$$\frac{q_0}{p_0}\int_{-\infty}^{\infty} e^{-z^2/a^2}\,dz = 4\pi. \tag{2.37}$$

The normalization (2.37) is equivalent to $a=4\sqrt{\pi}\frac{p_0}{q_0}$. We shall see later on that the relation $a=4\sqrt{\pi}\frac{p_0}{q_0}$ may be equivalently viewed as a constraint on the arbitrary function $\Sigma(t,r)$.

### 2.5.2 Stress-Energy Tensor at $r=0$

At $r\cong 0$ and $t\to t_c$ the solutions (2.26) are valid and the Hamiltonian is

$$\begin{aligned} T^{00} &= \frac{2\Sigma}{r}\left(\left[r^2+f^2(1-g)^2\right](1+f'^2)+\frac{\alpha'^2}{4}g'^2\right)\\ &\cong \frac{\tau_p}{r}\frac{q_0\alpha'}{t_c-t}\exp\left(-\frac{p_0^2r^2}{a^2(t_c-t)^2}\right)\end{aligned}$$

to leading order in $r$. Then, using $a=4\sqrt{\pi}\frac{p_0}{q_0}$,

$$\begin{aligned}\lim_{t\to t_c} T^{00} &= 4\pi\tau_{p-2}\frac{\delta(r)}{r}\\ &= \tau_{p-2}\,\delta(r\cos\theta)\,\delta(r\sin\theta)\end{aligned}$$

and the components parallel to the vortex are

$$\begin{aligned} T^{\hat{\mu}\hat{\nu}} &= -\frac{2}{r}\delta^{\hat{\mu}\hat{\nu}}V(f)\sqrt{-\det(M)}\\ &\to -\tau_{p-2}\,\delta^{\hat{\mu}\hat{\nu}}\delta(r\cos\theta)\,\delta(r\sin\theta).\end{aligned}$$

The remaining components, $T^{11}$ and $T^{01}$, are vanishing at $r=0$. The angular component $T^{22}=T^{\theta\theta}$ contains a delta function at $r=0$; however this is an artifact of $\theta$

being a bad coordinate at $r = 0$; it can be seen that $T_{\theta\theta} = 0$, and going to Cartesian coordinates confirms that the $T_{\mu\nu} = 0$ for the transverse coordinates, as should be the case for a D-brane. This result is the same as in the static case [50].

### 2.5.3 Stress-Energy Tensor at $r > 0$

For $r > 0$ at late times the solutions (2.31,2.32) are valid. For simplicity we take $\alpha^2 = 1$ and work only to leading order in $\varepsilon$. The Hamiltonian is

$$T^{00} \cong 2r\Sigma(t,r)$$

and the remaining components of $T^{MN}$ vanish at late times for $r > 0$. Notice that conservation of energy $\partial_M T^{MN} = 0$ at large $r$ forces

$$\Sigma(t,r) = \Sigma(r).$$

That is, $\Sigma$ is an arbitrary function of $r$ as in section 2.2.

To summarize, we find that in the limit of condensation the energy momentum tensor is identical to that of a D$(p-2)$-brane

$$\begin{aligned} T^{00} &= \tau_{p-2}\,\delta(r\cos\theta)\,\delta(r\sin\theta) + 2r\Sigma(r) \\ T^{11} &= T^{22} = 0 \\ T^{\hat{\mu}\hat{\nu}} &= -\delta^{\hat{\mu}\hat{\nu}}\tau_{p-2}\delta(r\cos\theta)\,\delta(r\sin\theta) \end{aligned} \tag{2.38}$$

with all off-diagonal components vanishing. The extra bulk energy density $2r\Sigma(r)$ is similar to the result in [71] and section 2.2 and corresponds to tachyon matter rolling toward $T \to \infty$ in the bulk.

### 2.5.4 Conservation of Energy

We can constrain $\Sigma(r)$ using conservation of energy. Initially the system consists of two D$p$-branes with energy density $2\tau_p V_2$ in the $(p-2)$-dimensional subspace spanned by $\{x^{\hat{\mu}}\}$, where $V_2$ is the volume of the 2-dimensional subspace spanned by $\{r, \theta\}$. At late times, after the codimension 2 brane and its antibrane have formed, the energy density in the $(p-2)$-dimensional space is given by the sum of the D$(p-2)$-brane tensions, $2\tau_{p-2}$, and the energy density due to tachyon matter $4\pi \int r^2\Sigma(r)\,dr$. Conservation of energy thus implies

$$\begin{aligned} 2\tau_p V_2 &= 2\tau_{p-2} + 4\pi \int dr\, r^2 \Sigma(r) \\ \left(2V_2 - 2(2\pi)^2\alpha'\right)\tau_p &= 4\pi \int dr\, r^2\Sigma(r). \end{aligned}$$

Since $\Sigma(r)$ is arbitrary we can take

$$\Sigma(r) = \frac{\tau_p}{r} + \frac{\tilde{\Sigma}(r)}{4\pi r^2} \tag{2.39}$$

where $\tilde{\Sigma}(r)$ satisfies the constraint

$$\int dr\, \tilde{\Sigma}(r) = -8\pi^2\alpha'\tau_p. \tag{2.40}$$

The conditions (2.39),(2.40) are equivalent to (2.37) and may be thought of as an alternative to the normalization (2.36).

## 2.6 Inclusion of Massless Gauge Fields

We will now restrict ourselves to a (5+1)-dimensional spacetime with $\{M,N\}=\{0,1,\cdots,5\}$, $\{\mu,\nu\}=\{0,1,2\}$ and $\{\hat{\mu},\hat{\nu}\}=\{3,4,5\}$. There are two gauge fields in the problem: $A^M_{(1)}$ and $A^M_{(2)}$, or equivalently $A^M_+ = A^M_{(1)} + A^M_{(2)}$ and $A^M_- = A^M_{(1)} - A^M_{(2)}$, which have different couplings to the tachyon. We have already shown that $A^\mu_-$ is the field which condenses in the vortex, hence its associated gauge symmetry is spontaneously broken. For reheating it is thus $A^{\hat{\mu}}_+$ which most closely resembles the Standard Model photon. We will ignore fluctuations of the heavy fields $A^{\hat{\mu}}_-$, $A^\mu_+$, and $A^\mu_-$, keeping only the background solution for $A^\mu_-$ (which was given in section 2.4), and the fluctuations of the photon $A^{\hat{\mu}}_+$. This leads to considerable simplification since it ensures that $D_{\hat{\mu}}T = 0$. To compute the production of photons in the time-dependent background, we want to expand the action (2.14) to quadratic order in $A^{\hat{\mu}}_+$.

The matrix $M^{(i)}_{MN}$ of eq. (2.15) can be written in block diagonal form as

$$\left[M^{(i)}_{MN}\right] = \begin{bmatrix} \left[\mathcal{V}^{(i)}_{\mu\nu}\right] & \left[S^{(i)}_{\mu\hat{\nu}}\right] \\ -\left[S^{(i)}_{\mu\hat{\nu}}\right]^T & \left[\delta_{\hat{\mu}\hat{\nu}} + \alpha' F^{(i)}_{\hat{\mu}\hat{\nu}}\right] \end{bmatrix} \tag{2.41}$$

where $\mathcal{V}^{(i)}_{\mu\nu} = M^{(i)}_{\mu\nu}$ is the contribution from the vortex background given in (2.18), $S^{(i)}_{\mu\hat{\nu}} = \alpha'\partial_\mu A^{(i)}_{\hat{\nu}}$ is the contribution from $\{t, r, \theta\}$ derivatives of $A^{(i)}_{\hat{\mu}}$, and $F^{(i)}_{\hat{\mu}\hat{\nu}}$ is the field strength tensor for $A^{(i)}_{\hat{\mu}}$. Using a well-known identity for determinants we can write

$$\det(M^{(i)}) = \det \mathcal{V}^{(i)} \det\left(\mathbf{1} + \alpha' F^{(i)} + S^T_{(i)}\mathcal{V}^{-1}_{(i)}S_{(i)}\right). \tag{2.42}$$

Expanding $\det(\mathbf{1} + \alpha' F^{(i)} + S^T_{(i)}\mathcal{V}^{-1}_{(i)}S_{(i)})$ to quadratic order in $A^{(i)}_{\hat{\mu}}$, the action (2.14) becomes

$$\begin{aligned} S \ &\cong\ -\alpha'^2\frac{1}{4}\int d^{p+1}x V(f)\sqrt{-\det\mathcal{V}}\left((\mathcal{V}^{-1})^{\mu\nu}_{(S)}\partial_\mu A^+_{\hat{\mu}}\partial_\nu A^{\hat{\mu}}_+\right. \\ &+\ \left.\partial_{\hat{\mu}}A^+_{\hat{\nu}}\partial^{\hat{\mu}}A^{\hat{\nu}}_+\right) \end{aligned} \tag{2.43}$$

where we have chosen the gauge $\partial_{\hat{\mu}}A^{\hat{\mu}}_+ = 0$ and disregarded the piece which does not depend on $A^{\hat{\mu}}_+$. We omit the brane index on $\mathcal{V}_{(i)}$ since the determinant and the symmetric part of $\mathcal{V}^{-1}_{(i)}$ are equal for both $i = \{1, 2\}$. Defining an effective metric $G_{MN}$ by

$$[G_{MN}] = \begin{bmatrix} \left[\mathcal{V}^{(S)}_{\mu\nu}\right] & [0] \\ [0] & [\delta_{\hat{\mu}\hat{\nu}}] \end{bmatrix} \tag{2.44}$$

the action (2.43) may be written as

$$S = -\frac{\alpha'^2}{4}\int V(f)\sqrt{-G}G^{MN}\delta^{\hat{\mu}\hat{\nu}}\partial_M A^+_{\hat{\mu}}\partial_N A^+_{\hat{\nu}}\, d^{p+1}x. \tag{2.45}$$

From (2.45) one sees that the fluctuations of the photon behave like a collection of massless scalar fields propagating in a nonflat spacetime described by the metric $G_{MN}$, with a position- and time-dependent gauge coupling given by $g^2 = 1/V(f(t,r))$.

To get an intuitive sense for the behavior of the action (2.45) we note that the stress-energy tensor derived in section 2.5 can be written as

$$T^{MN} = -\frac{2}{r}V(f)\sqrt{-G}G^{MN}.$$

In the limit of condensation, $T^{MN}$ is given by (2.38), so that once the brane has formed the action (2.45) reduces to a description of gauge fields propagating in a (3+1)-dimensional Minkowski space, with an additional component which couples the gauge fields to the tachyon matter density in the bulk. In other words, the effective metric $G_{MN}$ starts off being smooth throughout the bulk, but within the time $t_c$, its support collapses to become a delta function $\delta^{(2)}(\vec{x})$ in the relevant extra dimensions $\{r, \theta\}$.

The equations of motion resulting from the effective action (2.45) are difficult to solve analytically since the effective metric $G_{MN}$ depends nontrivially on both $r$ and $t$ and is nondiagonal in the subspace of $\{t, r\}$. For this reason we would like to propose a simplified model of the condensation which captures the essential features of the action (2.45). We have derived solutions for the vortex background valid at small $r$, $r \lesssim (t_c - t)$, and at large $r$, $r \gtrsim (t_c - t)$. Similarly, the energy momentum tensor we have derived corresponding to these solutions has very different behavior in the $r \leq (t_c - t)$ and $r > (t_c - t)$ regions of the spacetime.

For $r \leq (t_c - t)$ the energy momentum tensor contracts to a delta function centered at $r = 0$, with $(t_c - t)$ playing the role of the small parameter which regularizes the delta function. That is to say,

$$rT^{MN} \cong \frac{1}{t_c - t} \exp\left(-\frac{p_0^2 r^2}{a^2 (t_c - t)^2}\right) H^{MN}$$

at small $r$, where the matrix entries $H^{00}$, $H^{\hat{\mu}\hat{\nu}}$ are finite as $t \longrightarrow t_c$ and the remaining components of $H^{MN}$ tend to zero (near $r = 0$) as $t \longrightarrow t_c$.

In the $r > (t_c - t)$ region, the energy momentum tensor has quite different behavior. After condensation of the defect has completed at $t = t_c$, the energy density in the bulk ($r > 0$) is due entirely to tachyon matter, while the part of the stress-energy which is going into the tension of the defect vanishes in this region. Hence

$$rT^{MN} \longrightarrow 2\delta_0^M \delta_0^N r^2 \Sigma(r)$$

as $t \longrightarrow t_c$, for $r > (t_c - t)$.

In our simplified model of the particle production due to the tachyon condensation we therefore split the energy momentum tensor into brane and bulk pieces $T^{MN} = T^{MN}_{\text{brane}} + T^{MN}_{\text{bulk}}$ where $T^{MN}_{\text{brane}}$ contracts to a delta function as $t \longrightarrow t_c$ and $T^{MN}_{\text{bulk}} \longrightarrow 2\delta_0^M \delta_0^N r \Sigma(r)$ in the same limit. The action (2.45) then splits into two components $S = S_{\text{bulk}} + S_{\text{brane}}$. We expect most of the particle production to occur near the end of the condensation, when the background tachyon field is becoming singular near the vortex, so the best approximations for the simplified gauge field action are those which describe the exact expression most accurately near $t = t_c$:

$$S_{brane} \propto -\int \frac{1}{t_c - t} \exp\left(-\frac{p_0^2 r^2}{a^2(t_c - t)^2}\right) H^{MN} \delta^{\hat{\mu}\hat{\nu}} \partial_M A^+_{\hat{\mu}} \partial_N A^+_{\hat{\nu}} \, dt dr d\theta dx^{\hat{\alpha}}$$

and

$$S_{bulk} \propto -\int r^2 \Sigma(r) \left(-\delta^{\hat{\mu}\hat{\nu}} \dot{A}^+_{\hat{\mu}} \dot{A}^+_{\hat{\nu}}\right) dt dr d\theta dx^{\hat{\alpha}}.$$

At earlier times the coefficient of the bulk part of the Lagrangian would have time dependence, and the bulk Lagrangian would contain contributions from all the derivative of the gauge field, but this form is valid close to $t_c$.

Finally we argue that the bulk part of the action can be ignored. To this end, let us change to coordinates which are comoving with the contraction of the vortex core:

$$\tilde{r} = \frac{r}{t_c - t}, \qquad \tilde{t} = t_c - t. \tag{2.46}$$

In terms of these coordinates the "small $r$" solutions are valid for $\tilde{r} \leq 1$ and the "large $r$" solutions are valid for $\tilde{r} > 1$. The Jacobian of this transformation is $-\tilde{t}$ so that

$$S_{\text{brane}} \propto -\int \exp\left(-\frac{p_0^2 \tilde{r}^2}{a^2}\right) H^{MN} \delta^{\hat{\mu}\hat{\nu}} \partial_M A^+_{\hat{\mu}} \partial_N A^+_{\hat{\nu}} \, d\tilde{t} d\tilde{r} d\theta dx^{\hat{\alpha}}.$$

To lowest order in $\tilde{r}^2$ the matrix entries $H^{MN}$ in terms of these new coordinates are all constant. Consider now the piece of the action which couples to the tachyon matter density in the bulk, written in terms of these new coordinates:

$$S_{\text{bulk}} \propto -\int \tilde{r}^2 \tilde{t}^2 \Sigma(\tilde{r}\tilde{t}) \left(-\delta^{\hat{\mu}\hat{\nu}} \dot{A}^+_{\hat{\mu}} \dot{A}^+_{\hat{\nu}}\right) d\tilde{t} d\tilde{r} d\theta dx^{\hat{\alpha}}.$$

Since $\Sigma(z)$ is not singular at $z = 0$, the bulk piece of the action become negligible near the end of the contraction $\tilde{t} \to 0$. This is a consequence of the fact that as the condensation proceeds the gauge field is confined to the descendant brane.

## 2.7 Simplified Model of Reheating

In the previous section we argued that the gauge field couples most strongly to the part of the tachyon background which is collapsing to form the defect. This closely resembles a gauge theory defined on a manifold in which a two-dimensional subspace which is shrinking with time. As a simplified model of the interaction we thus consider a massless spin-1 field

$$S = -\frac{1}{4}\int \sqrt{-g} g^{MA} g^{NB} F_{MN} F_{AB}\, d^{p+1}x$$

propagating in a FRW-like background

$$g_{MN} dx^M dx^N = -dt^2 + R(t)^2 \left(d\tilde{r}^2 + \tilde{r}^2 d\theta^2\right) + \delta_{\hat{\mu}\hat{\nu}} dx^{\hat{\mu}} dx^{\hat{\nu}}. \tag{2.47}$$

The coordinate $\tilde{r}$ in (2.47) is fixed with the expansion and is thus corresponds to $\tilde{r}$ defined in (2.46). However for simplicity of notation we will drop the tilde and write $r$ instead of $\tilde{r}$ in the remainder of the chapter. We take $r$ to be dimensionless while $t$, $x^{\hat{\mu}}$ and $R$ have dimensions of length.

If we restrict ourselves to configurations with $A^{\mu} = 0$, $A^{\hat{\mu}} \neq 0$[7] and impose the gauge condition $\partial_{\hat{\mu}} A^{\hat{\mu}} = 0$ then the action simplifies to

$$S = -\frac{1}{2}\int \sqrt{-g} g^{MN} \delta^{\hat{\mu}\hat{\nu}} \partial_M A_{\hat{\mu}} \partial_N A_{\hat{\nu}}\, d^{p+1}x \tag{2.48}$$

Notice that for the metric (2.47) and the gauge field configuration $A^{\mu} = 0$ we have chosen, one has $\nabla_M A_{\hat{\mu}} = \partial_M A_{\hat{\mu}}$ and $R_{MN} A^M A^N = R_{\hat{\mu}\hat{\nu}} A^{\hat{\mu}} A^{\hat{\nu}} = 0$.

We will impose homogeneous boundary conditions at $r = 1$ and take the scale factor in (2.47) to be

$$R(t) = \begin{cases} R_0 & \text{if } t < 0; \\ R_0 - \eta t & \text{if } 0 \leq t \leq t_c; \\ R_0 - \eta t_c = \epsilon & \text{if } t > t_c. \end{cases} \tag{2.49}$$

[7]This restriction will only underestimate the reheating. Since we want to show that the reheating can be efficient, this approximation will not weaken the ensuing argument.

where $R_0$ represents the initial radial size of the extra dimensions. This approximation for the time-dependence of the vortex core is the simplest form which has the same qualitative behavior as the true background, while still allowing us to solve analytically for the gauge field wave functions in the background. A shortcoming of this approximation is that $\dot{R}$ is discontinuous at the interfaces, which leads to an ultraviolet divergence in the production of gauge bosons. The behavior of the actual $R(t)$ is smooth, and must yield a finite amount of particle production [59]. In addition, we will find a separate UV divergence in the particle production in the limit as the vortex core thickness goes to zero. This is presumably an artifact of the effective field theory which is not present in the full string theory, and we deal with it by introducing the cutoff $\epsilon$ on the final radius of the defect core, which we will take to be of order the string length $l_s$.

### 2.7.1 Gauge Field Solutions

The first step in computing the production of photons in the time-dependent background is to solve their equation of motion following from (2.48):

$$-2\frac{\dot{R}}{R}\dot{A}^{\hat{\rho}} - \ddot{A}^{\hat{\rho}} + \frac{A^{\hat{\rho}''}}{R^2} + \frac{A^{\hat{\rho}'}}{rR^2} + \frac{\partial_\theta^2 A^{\hat{\rho}}}{r^2R^2} + \partial^{\hat{\mu}}\partial_{\hat{\mu}}A^{\hat{\rho}} = 0 \tag{2.50}$$

where the dot and prime denote differentiation with respect to $t$ and $r$, respectively.

Equation (2.50) separates as

$$A^{\hat{\rho}}(t,r,\theta,x^{\hat{\mu}}) = \phi(t)\varphi(r)\Theta(\theta)\chi^{\hat{\rho}}(x^{\hat{\mu}}) \tag{2.51}$$

where

$$\partial^{\hat{\mu}}\partial_{\hat{\mu}}\chi^{\hat{\rho}} = -k^2\chi^{\hat{\rho}},\ \partial_{\hat{\nu}}\chi^{\hat{\nu}} = 0,\ k^2 = k^{\hat{\mu}}k_{\hat{\mu}} = \vec{k}\cdot\vec{k} \tag{2.52}$$

$$\partial_\theta^2\Theta = -m^2\Theta \tag{2.53}$$

$$\varphi'' + \frac{1}{r}\varphi' + \left(c^2 - \frac{m^2}{r^2}\right)\varphi = 0 \tag{2.54}$$

$$\ddot{\phi} + 2\frac{\dot{R}}{R}\dot{\phi} + \left(\frac{c^2}{R^2} + k^2\right)\phi = 0 \tag{2.55}$$

and $c$ is a separation constant.

The particular solutions of (2.52) are labeled by the momenta $\vec{k}$ in the 3 large dimensions ($\{x^4, x^5, x^6\}$) and we take them to be normalized according to

$$\int \chi^{\hat{\mu}}_{\vec{k}} \chi^{\hat{\mu}}_{\vec{k}'} dx^4 dx^5 dx^6 = \delta_{\vec{k}\vec{k}'}.$$

The particular solution of (2.53) is a sum of sines and cosines. The odd parity and even parity modes (under $\theta \to -\theta$) do not mix with the even ones, and for simplicity, we restrict our attention to the even modes, which include the massless one. This can underestimate the effciency of the reheating by a factor of 2 at most. The solution of (2.53) is thus

$$\Theta_m(\theta) = \frac{1}{\sqrt{\pi}} \cos(m\theta).$$

These are orthogonal for different values of $m$, and requiring that the solution be single-valued restricts $m$ to be an integer.

So far we have not been specific about the geometry of the two extra dimensions around which the original annihilating branes were wrapped. One simple possibility is a 2-sphere, where the descendant brane and antibrane (vortices) form at antipodal points. Since all the singular behavior of the tachyon background is localized near these points, the curvature and topology in the bulk should have little effect on particle production near the defects. To simplify the mathematics, we therefore replace either of the two hemispheres of the sphere with a flat disk, in the coordinate region $r \leq 1$. The correct boundary condition on radial eigenfunctions of the bulk Laplacian is that their derivatives vanish at $r = 1$, so that they are smooth at the interface where the two halves of the space are glued together.

The solution of (2.54), subject to the boundary condition $\varphi'(r = 1) = 0$ and the requirement that $\varphi(r)$ be regular at the origin, is

$$\varphi_{mn}(r) = \frac{\sqrt{2}}{J_{m+1}(c_{mn})} J_m(c_{mn} r) \tag{2.56}$$

where $c_{mn}$ is the $n$th zero of $J'_m(r)$. The solutions (2.56) are orthogonal for different values of $n$. The zero mode $n = 1$, $m = 0$ must be treated separately; it is the constant solution, where $c_{01} = 0$ and $\varphi_{01}(r) = 1$.

The solution of (2.55) depends on the scale factor. For $t < 0$ and $t > t_c$ the solutions are trivial and are given by

$$\phi_{mn}(t) = \frac{1}{R_0\sqrt{2\omega_{mn}}}\left(a_{mn}e^{-i\omega_{mn}t} + a^\dagger_{mn}e^{i\omega_{mn}t}\right) \tag{2.57}$$

and

$$\phi_{mn}(t) = \frac{1}{\epsilon\sqrt{2\bar{\omega}_{mn}}}\left(d_{mn}e^{-i\bar{\omega}_{mn}t} + d^\dagger_{mn}e^{i\bar{\omega}_{mn}t}\right) \tag{2.58}$$

respectively. We have defined $\omega^2_{mn} = \frac{c^2_{mn}}{R_0^2} + k^2$ and $\bar{\omega}^2_{mn} = \frac{c^2_{mn}}{\epsilon^2} + k^2$. The mutiplicative factors in (2.57) and (2.58) are introduced for later convenience and ensure that $a_{mn}$ and $d_{mn}$ will be properly normalized annihilation operators in the appropriate spacetime region when the gauge field is quantized. It will be convenient for what follows to introduce phase-shifted annihilation and creation operators in the region $t > t_c$: $\bar{d}_{mn} = e^{-i\bar{\omega}_{mn}t_c}d_{mn}$ and $\bar{d}^\dagger_{mn} = e^{i\bar{\omega}_{mn}t_c}d^\dagger_{mn}$. In terms of these operators (2.58) becomes

$$\phi_{mn}(t) = \frac{1}{\epsilon\sqrt{2\bar{\omega}_{mn}}}\left(\bar{d}_{mn}e^{-i\bar{\omega}_{mn}(t-t_c)} + \bar{d}^\dagger_{mn}e^{i\bar{\omega}_{mn}(t-t_c)}\right).$$

We are suppressing the dependence of the annihilation/creation operators on the 3-momenta $\vec{k}$.

In the region $0 < t < t_c$ where the scale factor depends nontrivially on time, the solution of (2.55) is

$$\begin{aligned}\phi_{mn}(t) \quad &= \quad \frac{1}{\sqrt{R_0 - \eta t}}\left(B_{mn}J_{p_{mn}}\left[\frac{k}{\eta}(R_0 - \eta t)\right]\right.\\ &\qquad\left. + C_{mn}J_{-p_{mn}}\left[\frac{k}{\eta}(R_0 - \eta t)\right]\right)\end{aligned} \tag{2.59}$$

where $p_{mn} = \frac{1}{2\eta}\sqrt{\eta^2 - 4c^2_{mn}}$. Some comments are in order concerning this solution, which has different behavior for massive and massless modes.

In the massless case $c_{mn} = 0$, $p_{mn} = 1/2$ and $J_p$, $J_{-p}$ are linearly independent, since $p$ is noninteger. The constants $B_{mn}$ and $C_{mn}$ in (2.59) should be interpreted as independent, real-valued constants so that $\phi_{mn}(t)$ is real.

In the massive case $c_{mn} \neq 0$, $\eta^2 \leq 1$,[8] and the order of the Bessel functions in (2.59), $p_{mn}$, is pure imaginary. Such Bessel functions are complex-valued and there are several options for constructing real solutions. Since $(J_\nu(x))^\star = J_{\nu^\star}(x)$ for real

$x$, we can impose $C_{mn} = B^{\dagger}_{mn}$. Calculating the Wronskian of $J_{is}$, $J_{-is}$ verifies that for real $s \neq 0$ these two solutions are linearly independent. Since we do not need to explicitly quantize the field in the region $0 < t < t_c$, the dagger can be thought of simply as complex conjugation. The interpretation of $B_{mn}$ and $B^{\dagger}_{mn}$ as annihilation and creation operators is unnecessary, since there are no asymptotic states in this region.

We summarize this subsection by putting these results together to write the general solution of (2.50) as

$$A^{\hat{\rho}}(x^M) = \sum_{m,n} \sum_{\vec{k}} \chi^{\hat{\rho}}_{\vec{k}}(x^{\hat{\mu}}) \Theta_m(\theta) \phi_{mn}(t) \varphi_{mn}(r).$$

where

$$\phi_{mn}(t) = \begin{cases} \frac{1}{R_0\sqrt{2\omega_{mn}}} \left( a_{mn} e^{-i\omega_{mn} t} + a^{\dagger}_{mn} e^{i\omega_{mn} t} \right) & \text{if } t < 0; \\ \frac{1}{\sqrt{R_0 - \eta t}} \left( B_{mn} J_{p_{mn}}[\frac{k}{\eta}(R_0 - \eta t)] + C_{mn} J_{-p_{mn}}[\frac{k}{\eta}(R_0 - \eta t)] \right) & \text{if } 0 \leq t \leq t_c; \\ \frac{1}{\epsilon\sqrt{2\bar{\omega}_{mn}}} \left( \bar{d}_{mn} e^{-i\bar{\omega}_{mn}(t - t_c)} + \bar{d}^{\dagger}_{mn} e^{i\bar{\omega}_{mn}(t - t_c)} \right) & \text{if } t > t_c. \end{cases}$$

### 2.7.2 Spectra of Produced Particles

The next step is to impose continuity of $A^{\hat{\mu}}$ and $\partial_M A^{\hat{\mu}}$ at $t = 0$ and $t = t_c$ in order to compute the Bogoliubov coefficients, which relate the annihilation and creation operators for $t > t_c$ ($\bar{d}_{mn}$ and $\bar{d}^{\dagger}_{mn}$) to those in the region $t < 0$ ($a_{mn}$ and $a^{\dagger}_{mn}$).[9] Smoothness of the solutions at the interfaces is ensured by the continuity of $\phi_{mn}$ and $\dot{\phi}_{mn}$, since $\phi_{mn}$ is the only part of $A^{\hat{\rho}}$ which changes between the different spacetime regions.

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[8]One might expect $\eta^2 \leq 1$ on physical grounds. In fact, for the ensuing arguments to hold one need only restrict $\eta \leq 3.7$ to ensure that $p_{mn}$ is pure imaginary and nonzero for all massive modes.

[9]We are ultimately interested in calculating the number of $d_{mn}$ quanta in the vacuum annihilated by $a_{mn}$. For this purpose it is just as good to use $\bar{d}_{mn}$, $\bar{d}^{\dagger}_{mn}$ as $d_{mn}$, $d^{\dagger}_{mn}$ since the phase shift cancels out of the number operator. $d^{\dagger}_{mn} d_{mn} = \bar{d}^{\dagger}_{mn} \bar{d}_{mn}$.

**Massive Modes**

For the massive modes continuity of $\phi_{mn}(t)$ and $\dot{\phi}_{mn}(t)$ at $t = 0$ implies

$$\begin{pmatrix} a_{mn} \\ a^{\dagger}_{mn} \end{pmatrix} = U \begin{pmatrix} B_{mn} \\ B^{\dagger}_{mn} \end{pmatrix} = \begin{bmatrix} u_1 & u_2 \\ u_2^{\dagger} & u_1^{\dagger} \end{bmatrix} \begin{pmatrix} B_{mn} \\ B^{\dagger}_{mn} \end{pmatrix}. \tag{2.60}$$

The entries of the matrix $U$ are given by

$$\begin{aligned} u_1 &= \left(\sqrt{\frac{R_0\omega_{mn}}{2}} + \frac{i\eta}{2\sqrt{2R_0\omega_{mn}}}\right) J_{p_{mn}}(kR_0/\eta) \\ &- ik\sqrt{\frac{R_0}{2\omega_{mn}}} J'_{p_{mn}}(kR_0/\eta) \\ u_2 &= \left(\sqrt{\frac{R_0\omega_{mn}}{2}} + \frac{i\eta}{2\sqrt{2R_0\omega_{mn}}}\right) J_{-p_{mn}}(kR_0/\eta) \\ &- ik\sqrt{\frac{R_0}{2\omega_{mn}}} J'_{-p_{mn}}(kR_0/\eta). \end{aligned}$$

Continuity at $t = t_c$ similarly gives

$$\begin{pmatrix} \bar{d}_{mn} \\ \bar{d}^{\dagger}_{mn} \end{pmatrix} = V \begin{pmatrix} B_{mn} \\ B^{\dagger}_{mn} \end{pmatrix} = \begin{bmatrix} v_1 & v_2 \\ v_2^{\dagger} & v_1^{\dagger} \end{bmatrix} \begin{pmatrix} B_{mn} \\ B^{\dagger}_{mn} \end{pmatrix}. \tag{2.61}$$

The matrix entries $v_i$ are obtained from $u_i$ by replacing $R_0$ with $\epsilon$ and $\omega_{mn}$ with $\bar{\omega}_{mn}$.

From (2.60), (2.61) we can write

$$\begin{pmatrix} \bar{d}_{mn} \\ \bar{d}^{\dagger}_{mn} \end{pmatrix} = VU^{-1} \begin{pmatrix} a_{mn} \\ a^{\dagger}_{mn} \end{pmatrix} = \begin{bmatrix} \alpha_{mn} & \beta_{mn} \\ \beta^{*}_{mn} & \alpha^{*}_{mn} \end{bmatrix} \begin{pmatrix} a_{mn} \\ a^{\dagger}_{mn} \end{pmatrix}.$$

The last equality defines the Bogoliubov coefficients. Note that there is *no summation* implied over any of the indices in the above expression. The indices $m$ and $n$ label the modes of the in- and out-states, and the summation which appears in the general definition of the Bogoliubov coefficients is not present here.

We can now determine the spectrum of Kaluza-Klein (KK) excitations of the photon which is produced in the tachyon vortex background. Observers in the future see a spectrum of massive particles in the final state given by

$$N^{mn}_{M>0}(k) = |\beta_{mn}|^2 = \frac{1}{|\det U|^2}|u_1 v_2 - u_2 v_1|^2.$$

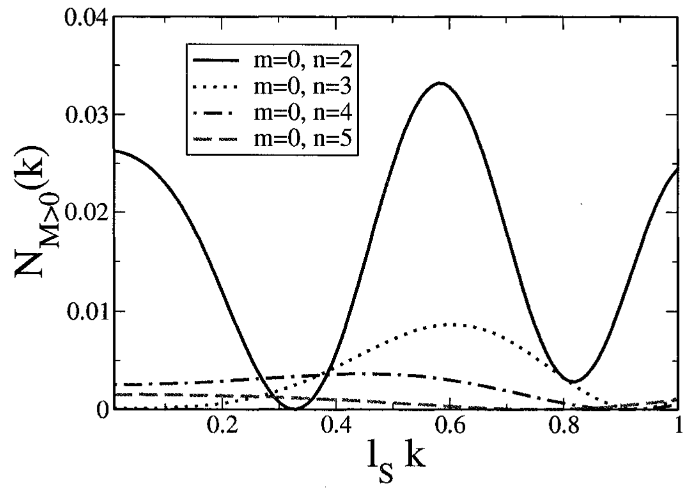


Figure 2.2: $N^{mn}_{M>0}(k)$ versus $k$ for increasing values of the radial quantum number $n$ showing decreased production of heavier modes.

The determinant is

$$\begin{aligned} \det U &= \det V = -\frac{2i\eta}{\pi}\sin\left(\pi p_{mn}\right) \\ &= \frac{2\eta}{\pi}\sinh\left(\frac{\pi}{2\eta}\sqrt{4c_{mn}^2-\eta^2}\right) \end{aligned} \tag{2.62}$$

which may be obtained by using the Wronskian of $J_p$ and $J_{-p}$. The fact that the two determinants are equal ensures the appropriate normalization $|\alpha_{mn}|^2 - |\beta_{mn}|^2 = 1$ of the Bogoliubov coefficients. The explicit expression for $N^{mn}_{M>0}(k)$ can be obtained analytically, but it is complicated and we do not write it out here; instead we will give numerical results.

The mass of a KK mode with quantum numbers $m, n$ is $c_{mn}/\epsilon$, which increases with $m$ and $n$. Figures 2.2-2.3 illustrate the dependence of $N^{mn}_{M>0}(k)$ on the 3-momentum $k$ for the lightest few massive modes. The parameters of the vortex background are taken to be $R_0 = 10\, l_s$, $\epsilon = l_s$ and $\eta = 1$. (The dependence of $N^{mn}_{M>0}(k)$ on $R_0$, $\epsilon$ and $\eta$ is essentially the same as that of the spectrum for massless modes, $N_{M=0}(k)$, which we will discuss in the next section.)

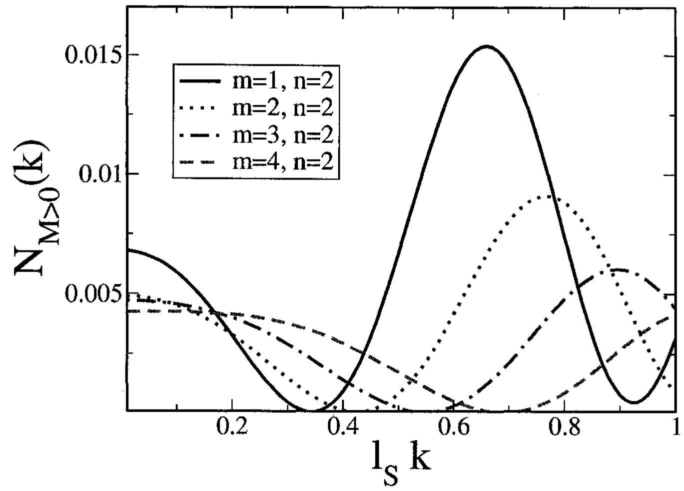


Figure 2.3: Same as fig. 2.2, but varying the angular quantum number $m$.

**Massless Modes**

The analysis for the massless modes proceeds similarly to the calculations in the preceding section, but more simply since in this case $p_{mn} = 1/2$ and $\omega_{mn} = \bar{\omega}_{mn} = k$. The Bogoliubov coefficients do not depend on the mode indices:

$$\begin{pmatrix} \bar{d}_{mn} \\ \bar{d}^{\dagger}_{mn} \end{pmatrix} = \begin{bmatrix} \alpha & \beta \\ \beta^* & \alpha^* \end{bmatrix} \begin{pmatrix} a_{mn} \\ a^{\dagger}_{mn} \end{pmatrix}.$$

The observed spectrum of particles in the final state

$$\begin{aligned} N_{M=0}(k) &= |\beta|^2 \\ &\cong \frac{1}{\epsilon^2} \frac{\eta\pi R_0}{8k} \left( J_{\frac{1}{2}} \left(kR_0/\eta\right)^2 + J_{\frac{3}{2}}(kR_0/\eta)^2 \right) \end{aligned} \tag{2.63}$$

where the second line shows the leading small-$\epsilon$ behavior. The exact result can also be obtained in closed form. Using explicit representations of $J_{n/2}$, we will be able to integrate this expression exactly to obtain the energy density of produced radiation. Figures 2.4-2.6 show plots of $N_{M=0}(k)$ as a function of $k$ for various values of the parameters $R_0$, $\epsilon$ and $\eta$.

## 2.7.3 Energy Density

To find the total energy density of radiation produced by the massive modes we should integrate over all $k$ and sum over the mode indices $m$ and $n$ and the number

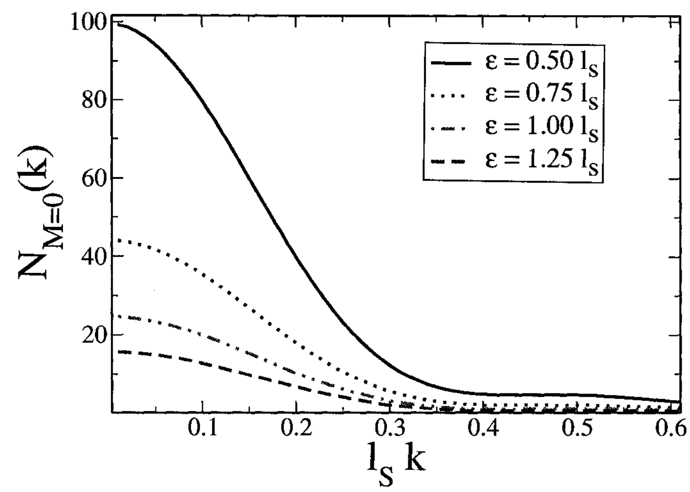


Figure 2.4: $N_{M=0}(k)$ versus $k$ for different values of $\epsilon$. $R_0 = 10\, l_s$ and $\eta = 1$.

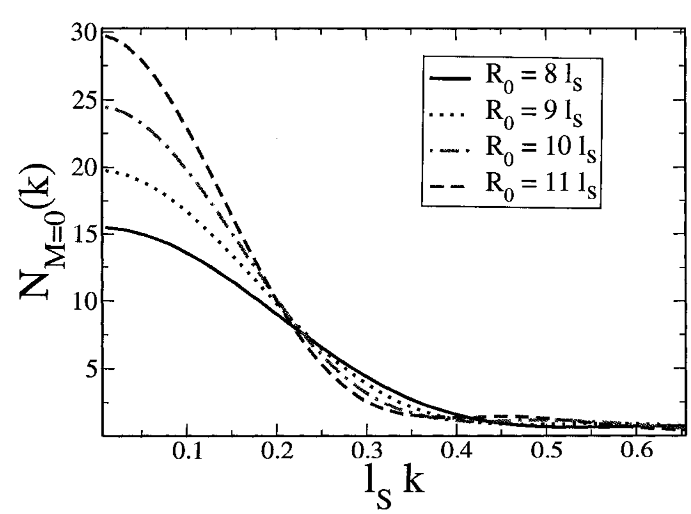


Figure 2.5: $N_{M=0}(k)$ versus $k$ for different values of $R_0$. $\epsilon = l_s$ and $\eta = 1$.

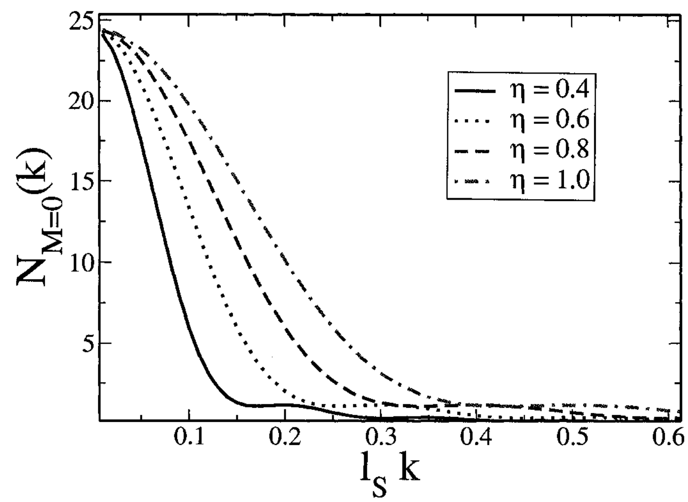


Figure 2.6: $N_{M=0}(k)$ versus $k$ for different values of $\eta$. $R_0 = 10\, l_s$ and $\epsilon = l_s$.

of polarizations (3 for massive vector bosons):

$$\rho_{M>0} = 3 \sum_{m,n} \int \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + \frac{c_{mn}^2}{\epsilon^2}} N_{M=0}^{mn}(k). \tag{2.64}$$

For the massless modes we only integrate over $k$ and sum over the 2 polarizations, since $N_{M>0}(k)$ has no dependence on the mode indices:

$$\rho_{M=0} = 2 \int \frac{d^3k}{(2\pi)^3} k N_{M=0}(k). \tag{2.65}$$

As anticipated above, the integrals in (2.64), (2.65) are not convergent due to the fact that the simplified tachyon background (2.49) has discontinuous time derivatives at $t = 0$ and $t = t_c$. This has the consequence that the spectrum $N(k)$ decreases like $k^{-2}$ for large $k$, which is too slow for convergence. In reality $n$th derivatives of $R(t)$ do not exceed $O(1/l_s^n)$, so the production of modes with $k > l_s^{-1}$ should be exponentially suppressed. We therefore introduce a UV cutoff, $k_{\max} = \Lambda \sim l_s^{-1}$. Moreover $N(k)$ is divergent as the vortex radius $\epsilon \longrightarrow 0$, so $\epsilon$ is also presumably limited by the string scale, $\epsilon \sim l_s$.

In the final state, the heavy KK modes have mass given by $c_{mn}/\epsilon$, where the $c_{mn}$'s are order unity and larger. These are near the cutoff, so their contributions are of the same order as other UV contributions which we are omitting. For consistency we should thus neglect the massive states' contribution to the total energy density on the vortex. Again, this underestimates the efficiency of particle production and makes our estimates conservative. Henceforth we will refer to $\rho_{M=0}$ as simply $\rho$.

Although the integral (2.65) can be performed analytically, the resulting expression for $\rho$ is cumbersome. Rather than write it out explicitly we will discuss some noteworthy features, plot $\rho$ with respect to the parameters of the model and present some useful simplifications of the complete expression in various limits.

Figures 2.7-2.8 show the dependence of $\rho$ on the initial size of the brane. For $R_0$ greater than a few times $l_s$, the energy density is relatively insensitive to changes in $R_0$. We can therefore reduce the dimensionality of the parameter space by simply assuming that the extra dimensions are somewhat larger than $l_s$. In fact in the limit of large $R_0$, the energy density takes the very simple form

$$\lim_{R_0 \to \infty} \rho = \frac{\eta^2}{8\pi^2} \frac{\Lambda^2}{\epsilon^2} \tag{2.66}$$

which is the main result of this section. We recall that $\eta$, which parametrizes the speed at which the vortex forms, is predicted from eq. (2.46) to be $\eta = 1$.

As a check on our calculations, we have also considered the limit as $R_0 \to \epsilon$, which corresponds to a static background, with no vortex condensation. As expected, the energy density of produced particles goes to zero,

$$\rho \cong \frac{\Lambda^4}{4\pi^2\epsilon^2}(R_0 - \epsilon)^2 - \frac{\Lambda^4}{4\pi^2\epsilon^3}(R_0 - \epsilon)^3 + \dots$$

as $R_0 \to \epsilon$.

### 2.7.4 Efficiency of Reheating

To quantify the efficiency of the reheating we need to determine how much energy is available to produce the photons on the final-state 3-brane. Initially the system consisted of a D5-brane plus antibrane, whose 3D energy density was given by $2\tau_5 V_2$, where $\tau_5$ is the tension of a D5-brane and $V_2$ is the volume of the compact 2-space $\{r, \theta\}$ wrapped by the branes. The final state consists of a D3-brane/antibrane with total tension $2\tau_3$. Until now we considered just half of the 2-sphere and focused on a single vortex located at $r = 0$. Conservation of Ramond-Ramond charge requires the

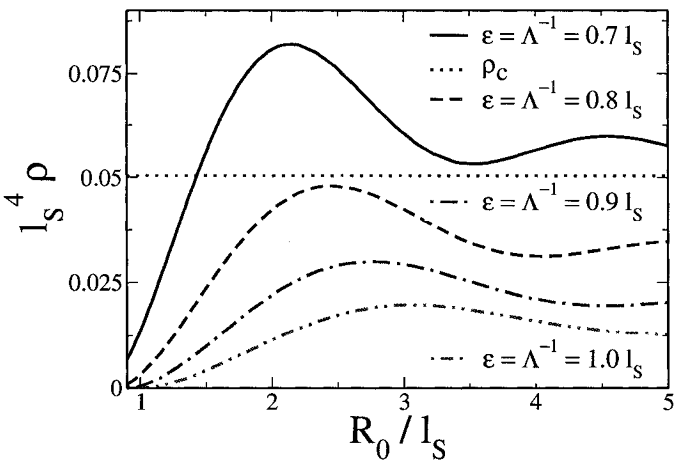


Figure 2.7: $\rho$ versus compactification radius $R_0$ for different values of $\epsilon = \Lambda^{-1}$ with $\eta = 1$. Dotted line is the critical density for efficient reheating $\rho_c$, (2.67).

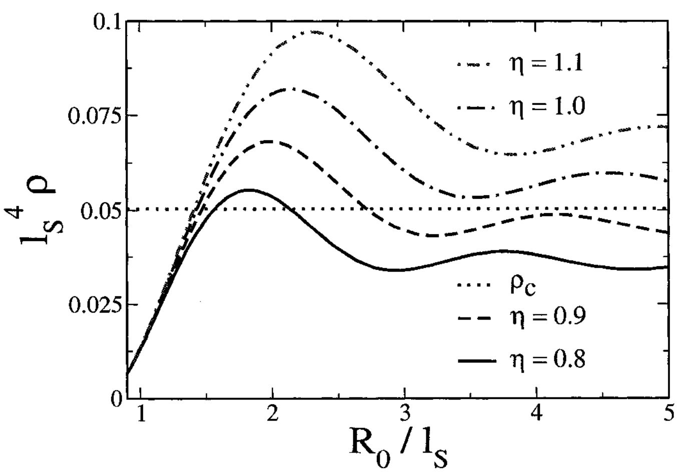


Figure 2.8: Same as figure 2.7 but for different values of vortex collapse rate $\eta$ and with $\epsilon = \Lambda^{-1} = 0.7\, l_s$.

second vortex, which we place at the south pole of the sphere to preserve azimuthal symmetry. These two defects are identical and are matched at the equator of the sphere. The vortex at the south pole represents the D3-antibrane.

Since reheating on each final-state brane should be equally efficient, the 3D energy density available for reheating on one them, which we call the critical energy density $\rho_c$, is half the difference between the initial and final tensions of the branes and antibranes:

$$\rho_c \equiv \tau_5 V_2 - \tau_3 = \tau_3 \left( \frac{V_2}{4\pi^2\alpha'} - 1 \right)$$

where we have used the recursion relation $2\pi\sqrt{\alpha'}\tau_p = \tau_{p-1}$. The tension of a D3-brane is given by [78]

$$\tau_3 = \frac{1}{g_s} \frac{1}{(2\pi)^3 \alpha'^2}.$$

The string coupling $g_s$, in 5+1 dimensions of which two are compact, is determined by the gauge coupling evaluated at the string scale, $\alpha(M_s)$ [54]:

$$g_s = \frac{V_2}{2\pi^2\alpha'} \alpha(M_s).$$

Thus we have

$$l_s^4 \rho_c = \frac{1}{16\pi^3 \alpha(M_s)} \left( 1 - \frac{4\pi^2 l_s^2}{V_2} \right). \tag{2.67}$$

In the regime where $V_2 \sim 2\pi R_0^2 \gg l_s^2$, which was where we could most easily quantify the particle production, the second term in parentheses can be neglected, and in any case it would be unimportant for a rough estimate unless it accidentally canceled the first term (1) to high accuracy. Hence we drop this term and take $l_s^4 \rho_c \cong (16\pi^3 \alpha(M_s))^{-1}$.

The energy density $\rho_c$ is the critical value at which the conversion into radiation would be 100% efficient. In our analysis we take $\alpha(M_s) \cong \frac{1}{25}$ [54] which gives $\rho_c l_s^4 = 0.05$. The critical energy density is shown as a dashed horizontal line in figures 2.7 and 2.8. We see that the criterion for efficient reheating can be achieved for moderate values of the parameters. We only need for the length-scale cutoffs $1/\Lambda$ and $\epsilon$ to be somewhat smaller than the string scale, while the size of the extra dimensions should exceed a few times $l_s$. Using (2.66), we can write the criterion for efficient reheating as

$$\sqrt{\frac{\epsilon}{\eta\Lambda}} \lesssim (2\pi\alpha(M_s))^{1/4}\, l_s \cong 0.7\, l_s \tag{2.68}$$

We have not taken into account the back-reaction of the particle production on the tachyon background, which is why our calculation allows for more reheating than is energetically possible. The back reaction will suppress somewhat the actual efficiency of reheating, but we don't expect a dramatic reduction. Given that we have been conservative in our estimates, such as ignoring the contributions from produced KK photons which will decay into massless photons, our result makes it plausible that a large fraction of the original energy can be converted into visible radiation.

## 2.8 Conclusions

We have argued for the possibility that our visible universe might be a codimension-two brane left over from annihilation of a D5-brane/antibrane pair at the end of inflation. In this picture, reheating is due to production of standard model particles (*e.g.* photons) on the final branes, driven by their couplings to the tachyon field which encodes the instability of the initial state as well as the vortex which represents the final brane. We find that reheating can be efficient, in the sense that a sizable fraction of the energy available from the unstable vacuum can be converted into visible radiation, and not just gravitons.

The efficiency of reheating is greatest if the radius of compactification of the extra dimensions is larger than 2-3 times the string length $l_s$. The efficiency also depends on phenomenological parameters we had to introduce by hand in order to cut off ultraviolet divergences in the calculated particle production rate: namely $\epsilon$, a nonvanishing radius for the final brane, and $\Lambda$, an explicit cutoff on the momentum of the photons produced. The latter must be introduced to correct for discontinous time derivatives in our simplified model of the background tachyon condensate; the actual behavior of the condensate corresponds to a cutoff of order $\Lambda \sim 1/l_s$. It is less obvious why the effective field theory treatment should give divergent results as the thickness of the final brane ($2\epsilon$) goes to zero, but it seems clear that a fully string-theoretic computation would give no such divergence, and therefore it is reasonable to cut off the field-theory divergence at $\epsilon \sim l_s$. Given only these mild assumptions, our estimates (2.66,2.67) predict that the fraction of the available energy which is converted into visible radiation is $\rho/\rho_c \cong \pi\alpha(M_s) \cong 0.25$. This simple estimate counts only photons; in a more realistic calculation, it would be enhanced by the number of light degrees of freedom which couple to the tachyon, which could be much greater 1. Moreover it could also be enhanced by the production of massive KK modes, which correspond to string excitations when the vortex has formed [76, 79].

Our analysis makes significant improvements to the previous work of [59]. We considered the formation of a vortex in the tachyon field rather than a kink, in line with the descent relations for stable Dp-branes. We found analytic solutions for the tachyon field which give the time dependence in the vicinity of the defect while it is forming, for both the vortex and the kink solutions. In the latter case we verified that this solution reproduces the known dynamics of kink formation which was determined numerically in [51], giving us more confidence in the vortex solutions, which are quite analogous. Our explicit solution for the tachyon background is nevertheless too complicated for computing the production of particles on the defect. We therefore approximated it by a simpler ansatz with the same qualitative behavior, which allows for analytic solutions of the gauge fields in the background. This ansatz resembles a gauge field in a 6D spacetime with two compact spatial dimensions which are contracting with time, and leads to simple analytic results for the energy density of photons produced during the contraction, in the regime where the extra dimensions are large compared to the string scale.

There are still some outstanding questions to be addressed concerning this scenario. First, we have made reference to the Kibble mechanism for the creation of the final state defects. If we assume the causal bound of one defect per Hubble volume then this would imply that the size of the extra dimensions must exceed the inverse Hubble rate; otherwise there would be enough time for the fields to straighten themselves out and the putative vortex-antivortex pair would immediately annihilate. For example if we take the string scale $M_s$ to be $10^{16}$ GeV then $H \sim M_s^2/M_p$ and we would need the compactification scale to be of order $R_0 \sim (M_p/M_s)l_s$. Our results indicate that efficient reheating is compatible with a large compactification scale. However, taking the remaining four extra dimensions (which the initial 5-brane/antibrane pair do not wrap) to be string scale is not consistent with getting inflation since the initial state 5-brane and antibrane cannot be sufficiently separated to satisfy the slow roll conditions.

One the other hand, our results indicate that the reheating *can* be efficient for $R_0$ only a few times the string length, though in this scenario a naive application of the Kibble mechanism does not favor having the final state defects span the three large dimensions. These requirements may not be prohibitive since the question of how these defects form dynamically at the end of inflation is a quantitative one which merits further investigation. In principle the correlation length for the initial fluctuations of the tachyon field could be as small as the string length. We point out also that it is possible that the dynamics of the formation of tachyon defects is qualitatively different from defect formation in a conventional scalar field theory. For example, the numerical investigation of [51], it was found that small kinks in the initial configuration which are in causal contact with each other *do not* dynamically straighten themselves out as they would in a conventional, nontachyonic field theory. Instead, every place where the field crosses zero in the initial state develops a full-blown kink, so long as there was enough energy in the bulk to produce the required number of kinks. Another indication that the dynamics of the tachyon field may be qualitatively different from an ordinary scalar field theory comes from the [80] in which the causal structure of the tachyon Dirac-Born-Infeld action was studied. The authors of [80] found that small fluctuations of the tachyon field propagate according to an effective metric which depends on the tachyon background. In the case of a homogeneous rolling background it was found that as the condensation proceeds the effective metric contracts to the Carroll limit of the Lorentz group so that the tachyon light cone collapses into a timelike half line and the tachyon fields at different spatial points are decoupled. We feel that quantitatively determining the dynamics of the

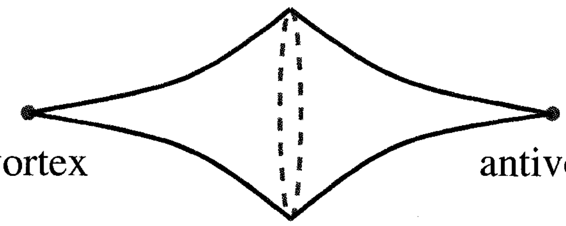


Figure 2.9: Warped compactification with branes localized in throats on opposite sides of a stack of branes.

formation of tachyon defects at the endpoint of D-brane inflation is a question which deserves further investigation.

In the present analysis we have not included the gravitational or Ramond-Ramond forces between the vortex and antivortex which would attract them toward each other and lead to their eventual annihilation. How do we insure that the braneworld on which we are supposed to live is safe from annihilation with an antibrane in the bulk? One possibility is to have warping caused by a stack of branes which wraps only the equator of the extra dimensions, as illustrated in fig. 2.9. Such a braneworld scenario using the AdS soliton solution for the bulk has been considered in ref. [81, 82, 61]. The advantage for our scenario is that the warping can provide a barrier to the annihilation of the brane-antibrane pair, since it is energetically favorable for them to remain within their respective throats. In chapter 4 we will consider a model of reheating in brane-antibrane inflation which does not suffer from this difficultly.

In solving for the tachyon background we have also ignored the possibility of caustic formation in the bulk [68] by taking initial profiles without too much curvature. It is possible that caustic formation may be an artifact of the derivative truncation which leads to the Born-Infeld type of Lagranian for the tachyon. We will study this issue in greater detail in chapter 3.

Brane-antibrane inflation and braneworld cosmology are two of the most important applications of string theoretic ideas to cosmology. We find it intriguing that these two ideas might be combined in the way we have described. An outstanding challenge is to find some observable signatures that would be able to test our scenario, for example through the gravitational wave component which is expected to be a major component of the radiation produced during reheating.

## Acknowledgements

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# Chapter 3

# Caustic Formation in Tachyon Effective Field Theories

# Abstract

Certain configurations of D-branes, for example wrong dimensional branes or the brane-antibrane system, are unstable to decay. This instability is described by the appearance of a tachyonic mode in the spectrum of open strings ending on the brane(s). The decay of these unstable systems is described by the rolling of the tachyon field from the unstable maximum to the minimum of its potential. We analytically study the dynamics of the inhomogeneous tachyon field as it rolls towards the true vacuum of the theory in the context of several different tachyon effective actions. We find that the vacuum dynamics of these theories is remarkably similar and in particular we show that in all cases the tachyon field forms caustics where second and higher derivatives of the field blow up. The formation of caustics signals a pathology in the evolution since each of the effective actions considered is not reliable in the vicinity of a caustic. We speculate that the formation of caustics is an artifact of truncating the tachyon action, which should contain all orders of derivatives acting on the field, to a finite number of derivatives. Finally, we consider inhomogeneous solutions in $p$-adic string theory, a toy model of the bosonic tachyon which contains derivatives of all orders acting on the field. For a large class of initial conditions we conclusively show that the evolution is well behaved in this case. It is unclear if these caustics are a genuine prediction of string theory or not.


## 3.1 Introduction

Recently effective actions describing the open string tachyon have recieved considerable attention in the literature. These effective actions are interesting to study since they permit a relatively simple formulation of the complicated open string dynamics in terms of classical field theories. In particular, the use of effective actions has triggered significant interest in the possible role of the tachyon in cosmology. For example, there have been numerous attempts to make use of the tachyon as either the inflaton or as quintessence [83] (see [84] for reviews of tachyon cosmology). Constraints and shortcomings of these scenarios have been discussed in [85].

Perhaps the most promising application of the tachyon to cosmology comes from D-brane inflation [24, 25], [53]-[55] In the context of D-brane inflation, the annihilation of a brane-antibrane pair at the endpoint of inflation is described by the rolling of the tachyon, $T$, from the unstable maximum $T = 0$ to the degenerate vacua of the theory $T = \pm\infty$. The tachyon field may roll to the same vacuum everywhere in the space or topological defects may form through the Kibble mechanism [56, 57, 58]. The latter possibility corresponds to the formation of lower dimensional branes at points where $T = 0$. In the regions between the descendant branes the tachyon field asymptotically rolls towards the true vacuum of the theory. The formation of lower dimensional branes is of some phenomenological importance since the resulting cosmic-string-like or higher dimensional defects could be observable remnants of inflation [56, 57, 58]. In [86, 1] a more radical idea was explored: it was proposed that our own observable universe could be such a defect in the higher dimensional spacetime predicted by string theory. In this scenario the coupling of the time dependent tachyon condensate to gauge particles which will be localized on the descendant brane can provide an efficient mechanism for reheating at the end of brane-antibrane inflation.

Applications of the tachyon to cosmology generically involve the tachyon field rolling from the unstable maximum of its potential to the true vacuum of the theory, at least in some region of the space, so it is of some interest to study the dynamics in this regime. Often only homogeneous tachyon profiles are considered, though a consistent study of the tachyon field in cosmology must include spatial inhomogeneities. In the context of the formation of lower dimensional branes at the endpoint of D-brane inflation spatial inhomogeneity is unavoidable since the field must cross $T = 0$ at one or more points in the space. However, even in the absence of topological defects one expects that spatial inhomogeneities will be generated starting from a homogeneous profile by vacuum fluctuations of the tachyon field.

The full tachyon action contains an infinite number of derivatives acting on the field, though the action is not known explicitly to all orders in derivatives. Rather, numerous effective actions describing the tachyon have been proposed in the literature on different grounds and have been shown to reproduce various nontrivial aspects of the full dynamics expected from string theory. Most studies of the tachyon dynamics in terms of effective actions have considered actions involving only first derivatives of the field and it is not clear how higher derivative corrections may alter the dynamics. Indeed, studies of $p$-adic string theory suggest that there may be fundamental differences between dynamical equations with an infinite number of derivatives and those with a large finite number of derivatives. In [87] it was shown that the initial value problem is qualitatively different in these two cases.

In this chapter we study analytically the dynamics of the inhomogeneous tachyon field as it rolls towards the vacuum in the context of several different effective field theories. Our interest is motivated by [68, 69] in which inhomogeneous vacuum solutions were discussed using the popular tachyon Dirac-Born-Infeld (DBI) action. The authors of [68, 69] found that in general the inhomogeneous solutions of the DBI action formed caustics where the second and higher derivatives of the tachyon field blow up at some point. In the vicinity of these caustics the tachyon DBI action is not reliable since the higher derivative corrections which have been ignored will become important. It is thus not known if the formation of caustics is a genuine prediction of string theory or an artifact of the DBI action for the tachyon. To shed some light on this question it is interesting to study inhomogeneous vacuum solutions of different tachyon effective actions and look for caustics or similar singularities.

The finite time formation of caustics found in [68, 69] must be distinguished from the finite time divergences in the first derivatives of the tachyon field at points where $T = 0$ [1, 51] which corresponds to the formation of topological defects. In both cases the divergences in the derivatives of the tachyon field lead to divergences in the energy density. In the case of topological defect formation, however, this finite time divergence in the energy density has the form of a delta function and hence leads to finite total energy [1]. We point out that similar behaviour was found in [88] where the decaying tachyon on the supergravity background corresponding to space-like branes in string theory was studied. In this case the first derivative of the tachyon field and the energy density of the tachyon matter both divergence in finite time producing a space-like curvature singularity in the metric.

The organization of this chapter is as follows. In section 3.2 we consider three tachyon effective actions, all of which contain only first derivatives of the field and show that all three display surprisingly similar dynamics near the true vacuum of the theory. In section 3.3 we briefly review how this universal vacuum structure leads to the formation of caustics. In section 3.4 we consider the vacuum dynamics of the tachyon in the context of yet another effective action which contains terms with second derivatives of the field and show that the problem of caustic formation is not ameliorated. In section 3.5 we briefly review the bosonic tachyon action of $p$-adic string theory and study small spatial inhomogeneities about a time dependent solution which rolls towards the minimum of the potential (which is unbounded from below). We find that for a large class of initial data our perturbative expansion is reliable throughout the evolution and the solutions are well behaved. Finally we conclude and discuss the difficulty of interpreting caustic formation physically.

## 3.2 First Derivative Tachyon Effective Actions and the Eikonal Equation

For simplicity we restrict ourselves to studying real tachyon fields in $1+1$-dimensional Minkowski space with metric $\eta_{\mu\nu}dx^\mu dx^\nu = -dt^2 + dx^2$. In the context of brane annihilation the rolling of the tachyon in $1+1$ dimensions from the unstable maximum to the ground state describes the decay of an unstable D$p$-brane to either the vacuum (in the case that the tachyon rolls to the same vacuum everywhere) or to a brane of codimension one (in the case that kinks form). We work in units where $\alpha' = 2$.

### 3.2.1 The Tachyon Dirac-Born-Infeld Action

We first consider the tachyon Dirac-Born-Infeld action [42, 43, 50]

$$\mathcal{L} = -U(T)\sqrt{1 + \partial_\mu T \partial^\mu T} \tag{3.1}$$

with potential

$$U(T) = \frac{\tau_p}{\cosh(T/2)} \tag{3.2}$$

where $\tau_p$ is the D$p$-brane tension. Solutions of this theory have been widely studied in the literature [86, 1, 68, 69, 50, 89, 90]. This action can be obtained from string theory in some limit [46] and has been shown to reproduce numerous nontrivial aspects of the full string theory dynamic, as discussed in chapter 1.

The action (3.1) should be thought of as describing tachyon profiles which are "close" [1] to the homogeneous rolling tachyon solution

$$T(t) = 2\sinh^{-1}\left(T_+ e^{t/2} + T_- e^{-t/2}\right). \tag{3.3}$$

We are interested in the dynamics of (3.1) close to the vacuum $T \to \pm\infty$, $U(T) \to 0$. The vacuum structure of this theory has been studied analytically in [43, 90] which we briefly review. The vacuum structure is most easily described in the Hamiltonian formalism since the Lagrangian does not survive the limit $U(T) \to 0$, but the Hamiltonian does. Defining the momentum conjugate to $T$ as $\Pi = \partial\mathcal{L}/\partial\dot{T}$ [2] the Hamiltonian is given by

$$H = \int d^2x\, \mathcal{H}, \quad \mathcal{H} = \sqrt{\Pi^2 + U(T)^2}\sqrt{1 + T'^2}.$$

In the limit as $U(T) \to 0$ one finds

$$\mathcal{H} = \Pi\sqrt{1 + T'^2}. \tag{3.4}$$

In the vacuum Hamilton's equations of motion are

$$\dot{T} = \sqrt{1 + T'^2} \tag{3.5}$$

and

$$\dot{\Pi} = \partial_x\left(\Pi \frac{T'}{1 + T'^2}\right) \tag{3.6}$$

[1] That is to say taking $T_+$, $T_-$ to be slowly varying functions of $x^\mu$.

[2] Here and throughout this chapter $\dot{T} = \partial_0 T = \partial_t T$ and $T' = \partial_1 T = \partial_x T$.

for $\Pi > 0$. Equation (3.5) is precisely the eikonal equation

$$\partial_\mu T \partial^\mu T + 1 = 0. \tag{3.7}$$

It was observed numerically in [68, 69] that the general solutions of (3.1) tend towards the first order equation (3.7).

### 3.2.2 The Boundary String Field Theory Action

The action

$$\mathcal{L} = -V(T) F\left(\partial_\mu T \partial^\mu T\right) \tag{3.8}$$

where the potential is

$$V(T) = \sqrt{2}\tau_p \exp\left(-\frac{T^2}{4}\right) \tag{3.9}$$

and the kinetic term is

$$F(z) = \frac{4^z z \Gamma(z)^2}{2\Gamma(2z)} \tag{3.10}$$

can be derived from boundary string field theory (BSFT) assuming a linear profile $T = a + u_\mu x^\mu$, and summed to all orders in $u_\mu$ [91]. This action should therefore be considered reliable only when describing profiles where second and higher derivatives of the tachyon field are small.

The dynamics of the action (3.8) have been discussed in [71] which we briefly review. Before attempting to describe the vacuum dynamics of the theory (3.8) we discuss some limiting behaviour of the function $F(z)$.

At small $z$ the function $F(z)$ has a Taylor expansion

$$F(z) \cong 1 + \ln(4) z + \cdots$$

and at large positive $z$ the function $F(z)$ has the behaviour

$$F(z) \rightarrow \sqrt{\pi z} \quad \text{as } z \rightarrow \infty. \tag{3.11}$$

It is also noteworthy that $F(z)$ is singular at $z = -n$ ($n = 1, 2, \cdots$). Of particular interest to the ensuing analysis will be the limiting behaviour of $F(z)$ near the singular point $z = -1$

$$F(z) \cong \frac{-1}{2(z+1)}, \qquad F'(z) \cong \frac{1}{2(z+1)^2}, \qquad F''(z) \cong \frac{-1}{(z+1)^3}. \tag{3.12}$$

We now proceed to study the dynamics of the action (3.8). The equation of motion which follows from (3.8) is

$$\partial^\mu T \partial_\mu z \, F''(z) + \partial^\mu \partial_\mu T \, F'(z) - \frac{1}{2} T \, zF'(z) + \frac{1}{4} T \, F(z) = 0 \tag{3.13}$$

where $z = \partial_\mu T \partial^\mu T$. We will assume initial conditions such that $z \approx 0$ initially. Furthermore, we assume that z does not cross the singularity at $z = -1$ [3].

The homogeneous solutions of (3.13) have the asymptotic behaviour $T \to \pm t$ at late times [92] so we expect that the tachyon will roll towards the vacuum $T \to \pm\infty$ as $t \to \infty$. We consider first the case where $z$ does not approach $-1$ as $T \to \infty$. In this case $F(z)$, $F'(z)$ and $F''(z)$ are well-behaved and the leading contribution to (3.13) is

$$F(z) - 2zF'(z) = 0.$$

It is easy to show that the quantity $F(z) - 2zF'(z)$ is greater than or equal to 0 for all $z > -1$ and that $F(z) - 2zF'(z) \to 0$ as $z \to +\infty$ (see equation (3.11)). We conclude then that for solutions where $z$ is increasing the equation of motion requires $z \to +\infty$ as $T \to \pm\infty$.

We now consider the possibility that $z \to -1$ at late times. Near $z = -1$ the function $F(z)$ and its derivatives are given by (3.12) and hence the leading contribution to (3.13) is

$$\partial_\mu T \partial^\mu z \, F''(z) - \frac{1}{2} T \, zF'(z) = 0. \tag{3.14}$$

[3] This is not a particularly restrictive assumption since if $z$ did cross the singularity it would lead to infinite action. Note that this does not exclude the possibility that $z \to -1$ as $V(T) \to 0$.

This equation can be satisfied for profiles where $z \rightarrow -1$. Since $z+1 \rightarrow 0$ as $T \rightarrow \pm\infty$ it is reasonable to assume that we can write $z+1 = f(T)$. This is consistent with (3.14) which implies

$$z + 1 \sim V(T)^{1/2}.$$

It is important to stress that, barring any asumptions on the form of the solution, the equation of motion (3.13) requires that $\partial_\mu T \partial^\mu T$ tends to either $+\infty$ or $-1$ as $T \rightarrow \pm\infty$. The former case describes tachyon kinks of the form $T(x) = x/\epsilon$ to be understood in the limit that $\epsilon \rightarrow 0$. The latter case describes the rolling tachyon and corresponds to the vacuum structure we are interested in. We conclude then that close to the vacuum the tachyon field obeys the eikonal equation

$$\partial_\mu T \partial^\mu T + 1 = 0.$$

### 3.2.3 The Lambert and Sachs Action

The action

$$\mathcal{L} = -V(T)K(\partial_\mu T \partial^\mu T) \tag{3.15}$$

where the potential is given by (3.9) and the kinetic term is

$$K(z) = e^{-z} + \sqrt{\pi z}\,\text{erf}\left(\sqrt{z}\right) \tag{3.16}$$

was proposed in [93]. This action is derived by requiring that the profile

$$T(x) = \chi \sin\left(\frac{x - x_0}{2}\right)$$

be an exact solution to the equations of motion. The residual freedom is fixed by demanding that $\mathcal{L} = \sqrt{2}\tau_p \exp(-T^2/4)$ for constant profiles for agreement with (3.8).

Before proceeding to study the dynamics of (3.15) we consider the asymptotic behaviour of the kinetic term (3.16). For $z \geq 0$ the function $K(z)$ is remarkably similar to the BSFT function $F(z)$ given by (3.10). Near $z = 0$ the function $K(z)$ has a Taylor expansion

$$K(z) \cong 1 + z + \cdots .$$

For large positive values of $z$ the function $K(z)$ has the aymptotic behaviour

$$K(z) \to \sqrt{\pi z},\ K'(z) \to \frac{1}{2}\sqrt{\frac{\pi}{z}},\ K''(z) \to -\frac{\sqrt{\pi}}{4z^{3/2}} \tag{3.17}$$

so that $K(z)$ tends to infinity slowly as $z \to +\infty$ while $K'(z)$, $K''(z)$ tend to zero. Finally we consider the behaviour of $K(z)$ for large negative $z$. Using the formula

$$\sqrt{\pi}\,\mathrm{erf}(iy) \to ie^{y^2}\left(\frac{1}{y} + \frac{1}{2y^3} + \mathcal{O}(y^{-5})\right)$$

for $y \to \infty$ it is easy to show that

$$K(z) \to -\frac{e^{|z|}}{2|z|},\quad K'(z) \to +\frac{e^{|z|}}{2|z|},\quad K''(z) \to -\frac{e^{|z|}}{2|z|} \tag{3.18}$$

for large negative $z$. Note that though $K(z)$ has very similar behaviour to $F(z)$ and $\sqrt{1+z}$ for large positive $z$, these three kinetic terms have very different behaviour on $z < 0$.

The equation of motion which follows from (3.15) is

$$\partial^\mu T \partial_\mu z\, K''(z) + \partial^\mu \partial_\mu T\, K'(z) - \frac{1}{2} T\, zK'(z) + \frac{1}{4} T\, K(z) = 0 \tag{3.19}$$

where $z = \partial_\mu T \partial^\mu T$. By construction equation (3.19) has the homogeneous solution

$$T(t) = T_+ e^{t/2} + T_- e^{-t/2}.$$

Comparing this to the homogeneous solution (3.3) demonstrates that a field redefinition is necessary to make accurate comparisons between the two theories. Notice that in the case of the homogeneous solution the tachyon field rolls towards the vacuum $T \to \pm\infty$ at late times with $z \to -\infty$.

We proceed now to study the vacuum structure of the theory (3.15). First we consider the possibility that $z$ tends to some finite positive value as $T \to \pm\infty$. In this case the leading contribution to (3.19) is

$$TzK'(z) = \frac{1}{2}TK(z) \tag{3.20}$$

or

$$0 = e^{-z}.$$

This suggests that we require $z \to +\infty$ if $z > 0$ and indeed, using the limiting behaviour (3.17) the leading contribution to (3.19) for $T \to \pm\infty$ with $z \to +\infty$ is satisfied.

We consider now possibility that $z$ approaches some finite negative value as $T \to \pm\infty$. Assuming $z \neq -\infty$ then $K(z)$, $K'(z)$ and $K''(z)$ are all well behaved and the leading contribution to (3.19) is still (3.20) or equivalently $0 = e^{|z|}$ which has no solution.

Finally we consider the possibility that $z \to -\infty$ as $T \to \pm\infty$. In this case the limiting behaviour (3.18) is applicable and $K(z) \sim -K'(z) \sim K''(z)$ so that the leading contribution to (3.19) is

$$2\partial_\mu T\partial^\mu z + Tz = 0. \tag{3.21}$$

Since $z \to -\infty$ as $T \to \pm\infty$ it is reasonable to assume $z = f(T)$ in this regime. This assumption is consistent with (3.21) and we find $f(T) = -T^2$ up to an arbitrary additive constant which is irrelevant as $T^2 \to \infty$. One may verify that given these assumptions the leading behaviour (3.21) is order $T^3$ while the terms we have disregarded in (3.19) are order $T$ so that this series of approximations is self-consistent. We conclude that for $z < 0$ the equation of motion requires that the tachyon field obeys

$$\partial_\mu T\partial^\mu T + T^2 = 0.$$

Defining $T = \exp(\tilde{T})$ we find that $\tilde{T}$ obeys the eikonal equation

$$\partial_\mu \tilde{T}\partial^\mu \tilde{T} + 1 = 0.$$

We stress that the equation of motion (3.19) requires either $z \to +\infty$ or $z \to -\infty$ as $T \to \pm\infty$. The former case describes tachyon kink solutions while the latter case describes the rolling tachyon and corresponds to the vacuum structure we are interested in.

## 3.3 Caustic Formation

We have found that all three actions (3.1,3.8,3.15) predict that the tachyon field is described by the eikonal equation (3.7) close to the vacuum. Inhomogeneous solutions of (3.7) were found in [68, 69] for arbitrary Cauchy data by the method of characteristics. The solution is given along a set of characteristic curves which, in $1+1$ dimensions, are defined by

$$x(q,t) = q - \frac{T_i'(q)}{\sqrt{1+T_i'(q)^2}}t \tag{3.22}$$

where the parameter $q$ defines the initial position of the curve on the $x$ axis such that $x(q, t=0) = q$ and $T_i(x)$ is the Cauchy data at time $t = 0$. The parameter $q$ should be thought of as labelling the curves. The value of the tachyon field along a given characteristic curve is

$$T(q,t) = T_i(q) + \frac{t}{\sqrt{1+T_i'(q)^2}}. \tag{3.23}$$

It is also worth noting that the derivatives of the field are constant along the curves $T'(q,t) = T_i'(q)$ and $\dot{T}(q,t) = \sqrt{1+T_i'(q)^2}$. From (3.22) it is clear that in general for initial data where $T_i'' \neq 0$ there will be curves originating from different initial points on the $x$ axis which will cross in some finite time. At points where the characteristic curves intersect the field becomes multi-valued since evolving (3.23) along two different curves leads to two different values of $T$ at the point of intersection. This corresponds to the formation of caustics and signals a pathology in the evolution. To illustrate the problem, consider $T''$ along a characteristic curve

$$T''(q,t) = \frac{T_i''(q)}{1 - \frac{T_i''(q)}{(1+T_i'(q)^2)^{3/2}}t}.$$

At some time $t_c$ and point $q_c$ where $T_i''(q_c) > 0$ the denominator vanishes and the second derivative blows up. For a given $q_c$ the caustics form at time

$$t_c = \frac{(1+T_i'(q_c)^2)^{3/2}}{T_i''(q_c)}.$$

In each of the three effective actions considered above (3.1,3.8,3.15) this caustic formation signals a breakdown of the theory since terms in the action involving second and higher derivatives of the field which have been neglected will become important.

## 3.4 Vacuum Dynamics in a Higher Derivative Action

In light of the analysis of the preceeding section it is tempting to speculate that the caustic formation described above is an artifact of the derivative truncation which leads to (3.1,3.8,3.15) since these effective descriptions break down in the vicinity of a caustic. We would like to reconsider the effective description of the tachyon near the vacuum in the context of an action which does contain terms with second derivatives of the field. One might consider attempting to generalize the action (3.8) to a profile of the form $T = a + u_\mu x^\mu + v_{\mu\nu}x^\mu x^\nu$ to obtain an action which is valid for profiles where $\partial_\mu T \neq 0$ and $\partial_\mu\partial_\nu T \neq 0$ without constraint on the size of the first and second derivatives. However, deriving such an action (following [91]) would involve performing a path integral which is no longer Gaussian. On the other hand, the superstring tachyon effective action has been calculated in an expansion in small momenta (derivatives) around a constant profile up to six orders in derivatives ($\mathcal{O}(\partial^6)$) in [94]. The action, in units where $\alpha' = 2$ and truncated to fourth order in derivatives, is

$$\mathcal{L} = -V(T)\,\Big( \;1 + \ln(4)\partial_\mu T\partial^\mu T + \left(\tfrac{1}{2}(\ln(4))^4 - \zeta(2)\right)(\partial_\mu T\partial^\mu T)^2 + \zeta(2)T\partial_\mu\partial_\nu T\partial^\mu T\partial^\nu T + \left[(2\zeta(2) - 8(\ln(2))^2) + \tfrac{\zeta(2)}{2}T^2\right]\partial_\mu\partial_\nu T\partial^\mu\partial^\nu T \;\Big) \qquad (3.24)$$

where $\zeta(z)$ is the Riemann zeta function. The action (3.24) is valid to all orders in $T$ but for small derivatives and hence naively does not seem applicable to studying the vacuum dynamics where, the analysis of the preceeding sections would suggest, $\partial_\mu T\partial^\mu T$ is order unity or larger. However, one might expect the vacuum structure of the theory to be significantly modified due to the presence of terms like $T^2\partial_\mu\partial_\nu T\partial^\mu\partial^\nu T$ in the action (3.24) which are vanishing on linear profiles but are large in the vacuum for nonlinear profiles. One might hope that the vacuum structure of the theory (3.24) is such that when one considers Cauchy data where the derivatives of $T$ are small but nonzero, then the derivatives stay small throughout the evolution and the approximations which lead to (3.24) remain self consistent. This turns out not to be the case, as we will show.

The equation of motion which follows from (3.24) is cumbersome and we do not write it out here. In the vacuum, as $T \to \pm\infty$, assuming that the derivatives of the field are well behaved [4] the leading contribution to the equation of motion is

$$\partial^\mu\partial^\nu T\partial_\mu T\partial_\nu T = 0 \qquad (3.25)$$

where the subleading terms are of order $T^{-1}$ and smaller.

We note that (3.25) may be re-written as

$$\partial_\mu T\partial^\mu\left(\partial_\nu T\partial^\nu T\right) = 0$$

so that the solution set of (3.25) contains the solutions of the first order equation

$$\partial_\nu T\partial^\nu T = \kappa$$

where $\kappa$ is a constant which may be set to $+1$, $-1$ or $0$ by rescaling $T$. The case $\kappa = -1$ corresponds to the eikonal equation and we conclude that (3.25) does admit solutions

[4] If we drop this assumption then (3.24) ceases to be a reasonable description of the dynamics.

with caustic formation in the case of Cauchy data such that $\dot{T}_i(x)^2 = 1 + T_i'(x)^2$. However, equation (3.25) is second order and of course it is not necessary to constrain the Cauchy data in this manner. For completeness we discuss the construction of more general solutions of (3.25) in appendix A.

It is beyond the scope of this chapter to perform a comprehensive analysis of the derivative singularities admitted by (3.25). We have shown an explicit reduction of (3.25) to the eikonal equation, which exhibits caustics. As discussed in the appendix, we believe more general solutions of (3.25) exhibit similar pathologies. For our purposes, this is sufficient to demonstrate that the higher derivative action (3.24) does not ameliorate the problem of caustic formation. In light of these results, it is tempting to speculate that it is necessary to use an action which contains all orders of derivatives acting on the field to obtain a fully consistent description of the dynamics of the tachyon near the vacuum.

As an illustrative example we will discuss inhomogeneous solutions in a $p$-adic string theory, a toy theory of the bosonic string tachyon which contains an infinite number of derivatives acting on the field which is known explicitly to all orders in derivatives.

## 3.5 Inhomogeneous Solutions in $p$-adic String Theory

The action of $p$-adic string theory is [95]

$$S = \frac{p^2}{g^2(p-1)} \int d^D x \left[ -\frac{1}{2} \phi \, p^{-\frac{1}{2}\partial_\mu \partial^\mu} \phi + \frac{1}{p+1} \phi^{p+1} \right] \tag{3.26}$$

where $\phi$ is the open string tachyon, $g$ is the open string coupling constant and, though the action was derived for $p$ a prime number, it appears that $p$ can be continued to any positive integer (the action makes sense even in the limit $p \to 1$ [96]). The differential operator $p^{-\frac{1}{2}\partial_\mu \partial^\mu}$ is to be understood as the series expansion

$$p^{-\frac{1}{2}\partial_\mu \partial^\mu} = \exp\left( -\frac{1}{2} \ln(p) \, \partial_\mu \partial^\mu \right) = \sum_{n=0}^{\infty} \left( -\frac{1}{2} \ln(p) \right)^n \frac{1}{n!} \left( \partial_\mu \partial^\mu \right)^n .$$

The equation of motion which follows from (3.26) is

$$p^{-\frac{1}{2}\partial_\mu \partial^\mu} \phi = \phi^p \tag{3.27}$$

and the potential is

$$V(\phi) = \frac{p^2}{g^2(p-1)} \left( \frac{1}{2}\phi^2 - \frac{1}{p+1}\phi^{p+1} \right). \tag{3.28}$$

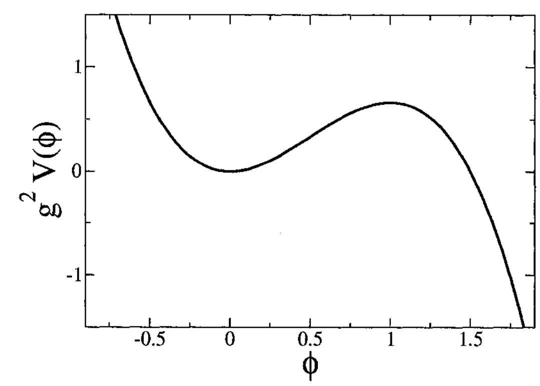


Figure 3.1: Plot of the potential $V(\phi)$ for even $p$.

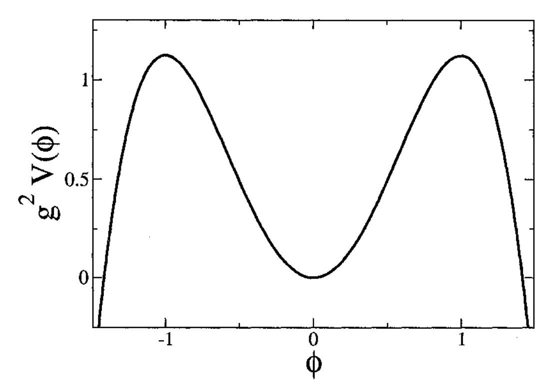


Figure 3.2: Plot of the potential $V(\phi)$ for odd $p$.

The cases of odd and even $p$ are qualitatively different. For odd $p$ the potential is an even function of $\phi$ and for even $p$ the potential is an odd function of $\phi$. In both cases the potential is unbounded from below. In the case of even $p$ the perturbative vacuum is at $\phi = 1$ and in the case of odd $p$ there is an equivalent false vacuum at

$\phi = -1$. In both cases the true vacuum of the theory is at $\phi = 0$. Figures 3.1 and 3.2 show plots of the potential (3.28) for the cases of even and odd $p$ respectively.

We note that the action (3.26) is intended to describe the bosonic string tachyon, rather than the superstring tachyon which we have considered in our previous analysis (3.1,3.8,3.15,3.24). Furthermore, (3.26) is a toy model which is only expected to qualitatively reproduce some aspects of a more realistic theory. These points should be kept in mind when comparing analysis using (3.26) to the previous analysis using (3.1,3.8,3.15,3.24). That being said we note that there are several nontrivial qualitative similarities between $p$-adic string theory and tachyon matter. For example, near the true vacuum of the theory $\phi = 0$ the field naively has no dynamics since its mass squared goes to infinity [5]. This is the $p$-adic version of the statement that there are no open string excitations at the tachyon vacuum. A second similarity between $p$-adic string theory and tachyon matter is the existence of lump-like soliton solutions representing $p$-adic D-branes [65]. The theory of small fluctuations about these lump solutions has a spectrum of equally spaced masses squared for the modes [97], as in the case of normal bosonic string theory. On the other hand, there are some important differences between the theory (3.26) and (3.1,3.8,3.15,3.24). In the case of tachyon matter the vacuum is at infinity and the tachyon never reaches this point, whereas in the case of the $p$-adic string the vacuum is at a finite point in the field configuration space and homogeneous solutions rolling towards the vacuum typically pass this point without difficulty [87]. In fact, the numerical studies of [87] found *no* homogeneous solutions which appeared to correspond to tachyon matter (vanishing pressure at late times).

Keeping in mind that the connection between (3.26) and (3.1,3.8,3.15,3.24) is not entirely clear we proceed to study inhomogeneous solutions of (3.26) as an example of a theory with an infinite number of derivatives which may have some qualitative similarities to the string theory tachyon. At first glance it is not immediately clear

[5] Reference [87] found anharmonic oscillations around the vacuum by numerically solving the full nonlinear equation of motion. However, these solutions do not correspond to conventional physical states.

how to proceed since (3.27) is difficult to solve for profiles with nontrivial dependence on more than one variable. The simplest solution is to study small fluctuations about some known time-dependent solution of (3.27). We are interested in the dynamics near the vacuum and [87] found anharmonic homogenous oscillations near $\phi = 0$ so one might consider small spatial inhomogeneities about these solutions. However, these anharmonic oscillations cannot be found by solving the linearized equation of motion and hence we should not expect to be able to study small inhomogeneities near the vacuum by linearizing about these oscillators. One might consider the closest analogy to the solutions found in the preceeding sections to be small fluctuations about a time dependent solution which interpolates between the unstable vacuum $\phi = 1$ (or also $\phi = -1$ in the case of odd $p$) and the stable vacuum $\phi = 0$. However, it was shown in [87, 98] that no such time dependent solution to (3.27) exists [6]. We therefore will consider inhomogeneous fluctuations about the rapidly increasing solution

$$\phi_0(t) = p^{\frac{1}{2(p-1)}} \exp\left(\frac{1}{2}\frac{p-1}{p\ln p}t^2\right). \tag{3.29}$$

We stress that this solution does not roll from the unstable maximum of the potential to the true vacuum of the theory (indeed no such solution appears to exist) and thus the ensuing analysis may be of limited relevance since the connection between $p$-adic string theory and tachyon matter is unclear. The homogeneous solution (3.29) does bear some qualitative similarity to the solutions of (3.1,3.8,3.15,3.24) in the sense that this solution represents the tachyon rolling down the potential though in the case of (3.29) the tachyon rolls towards $V \to -\infty$ and not $V = 0$.

Writing $\phi(t,x) = \phi_0(t) + \delta\phi(t,x)$ with $\phi_0(t)$ given by (3.29) the equation of motion (3.27) is

$$p^{-\frac{1}{2}\partial^\mu\partial_\mu}\delta\phi(t,x) = p\phi_0(t)^{p-1}\delta\phi(t,x) \tag{3.30}$$

to linear order in $\delta\phi/\phi_0$. The particular solutions of (3.30) may be written as

$$\delta\phi_\lambda(t,x) = \phi_0(t)K_\lambda(i\alpha t)\chi_\lambda(x) \tag{3.31}$$

[6]More precisely, it was shown that there exists no homogeneous nonnegative bounded continuous solution of (3.27) with $\phi(t \to -\infty) = 1$ and $\phi(t \to +\infty) = 0$.

where

$$\alpha = \sqrt{\frac{1}{2}\frac{p^2-1}{p\ln p}}$$

and $K_\lambda(z)$ is any solution of the Hermite equation

$$\frac{\partial^2 K_\lambda(z)}{\partial z^2} - 2z\frac{\partial K_\lambda(z)}{\partial z} + 2\lambda K_\lambda(z) = 0. \tag{3.32}$$

Plugging (3.31) into (3.30) one finds that the spatial modes $\chi_\lambda(x)$ are determined by the equation

$$p^{-\frac{1}{2}\partial_x^2}\chi_\lambda(x) = p^{1-\lambda}\chi_\lambda(x). \tag{3.33}$$

Before attempting to solve (3.33) for the spatial dependence of the particular solutions of (3.30) some comments are in order concerning the separation of variables (3.31). It is straightforward to show that the solutions of (3.30) separate as (3.31) using the identity [97]

$$\begin{aligned} e^{a\partial_t^2}\left[K_\lambda(i\alpha t)e^{bt^2}\right] &= (1-4ab)^{-1/2}\left(1+\frac{4a\alpha^2}{1-4ab}\right)^{\lambda/2} \\ &\times\; K_\lambda\left(\frac{i\alpha t}{\sqrt{(1-4ab)(1-4ab+4a\alpha^2)}}\right)\exp\left(\frac{bt^2}{1-4ab}\right) \end{aligned} \tag{3.34}$$

This identity was considered in [97] for $\lambda = n = 0,1,2,\cdots$ and $K_n(z) = H_n(z)$, the Hermite polynomials, though it holds for arbitrary $\lambda$ which is most easily shown by noting that the two solutions of (3.32), $K_\lambda(z) = P_\lambda(z)$ and $K_\lambda(z) = Q_\lambda(z)$, have contour integral representations [99]

$$P_\lambda(z) = \int_{C_1}\frac{d\xi}{2\pi i}\,\xi^{-\lambda-1}e^{-\xi^2+2\xi z} \tag{3.35}$$

and

$$Q_\lambda(z) = \int_{C_2}\frac{d\xi}{2\pi i}\,\xi^{-\lambda-1}e^{-\xi^2+2\xi z} \tag{3.36}$$

where the curves $C_1$ and $C_2$ are given in figure 3.3. The standard definition of the Hermite polynomials for $\lambda = n = 0,1,2,\cdots$ is given by

$$\begin{aligned} H_n(z) &= n!\,(P_n(z)+Q_n(z)) \\ &= n!\int_C\frac{d\xi}{2\pi i}\,\xi^{-n-1}e^{-\xi^2+2\xi z} \end{aligned} \tag{3.37}$$

where $C$ is the contour given in figure 3.4. For subsequent calculations we will be particularly interested in defining the Hermite functions $H_\lambda(z)$ on $\lambda \leq 1$. For $\mathrm{Re}(\lambda) < 0$ we can contract the path of integration $C$ to the origin and write

$$H_\lambda(z) = \frac{1}{\Gamma(-\lambda)} \int_0^\infty d\xi \, \xi^{-\lambda-1} e^{-\xi^2-2\xi z} \tag{3.38}$$

where the normalization has been chosen to agree with [100]. Note that when $\mathrm{Re}(\lambda) > -1$ we can also represent the Hermite functions by a real integral [100]

$$H_\lambda(z) = \frac{2^\lambda e^{z^2}}{\sqrt{\pi}} \int_0^\infty e^{-\xi^2} \xi^\lambda \cos\left(2z\xi - \frac{\lambda\pi}{2}\right) d\xi \tag{3.39}$$

which coincides with the standard definition of the Hermite polynomials when $\lambda = n = 0, 1, 2, \cdots$. With these conventions the Hermite functions are normalized so that

$$H_\lambda(0) = \frac{2^\lambda \Gamma(1/2)}{\Gamma\left(\frac{1-\lambda}{2}\right)}, \quad H'_\lambda(0) = \frac{2^\lambda \Gamma(-1/2)}{\Gamma(-\lambda/2)}.$$

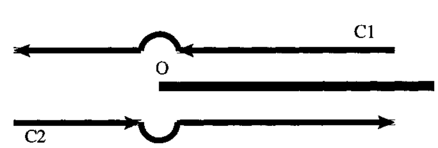


Figure 3.3: The contours of integration $C_1$ and $C_2$.

We now proceed to determine the spatial dependence of the modes (3.31). Equation (3.33) has solutions

$$\chi_\lambda(x) = a_\lambda \cos(\omega_\lambda x) + b_\lambda \sin(\omega_\lambda x) \tag{3.40}$$

where $\omega_\lambda = \sqrt{2(1-\lambda)}$. For initial data which are periodic on some interval $[-L, +L]$ one takes $\omega_\lambda = \pi m / L$ where $m = 0, 1, 2, \cdots$. With this choice the degree of the Hermite function is

$$\lambda = 1 - \frac{\pi^2 m^2}{2L^2}.$$

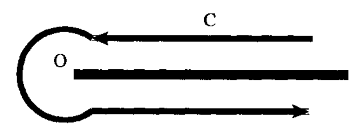


Figure 3.4: The contour of integration $C$.

For the zero mode $m = 0$ we have $\lambda = 1$ and for all other $m$ we have $\lambda \neq 0, 1, 2, \cdots$ unless $L$ is chosen so that $\sqrt{2}\, L/\pi$ is an integer.

The Hermite equation (3.32) is second order and we expect to be able to find two linearly independent solutions for each $\lambda$. It is most convenient to choose one of these to be the Hermite functions as defined by (3.37,3.38,3.39). To obtain a second solution we note the Wronskian formula [100]

$$W\left[H_\lambda(iz), H_\lambda(-iz)\right] = \frac{2^{\lambda+1}\sqrt{\pi}}{\Gamma(-\lambda)} e^{-z^2}. \tag{3.41}$$

so that

$$\begin{aligned} \delta\phi_\lambda^{(+)}(t,x) &= \phi_0(t) H_\lambda(+i\alpha t)\, \chi_\lambda^{(+)}(x), \\ \delta\phi_\lambda^{(-)}(t,x) &= \phi_0(t) H_\lambda(-i\alpha t)\, \chi_\lambda^{(-)}(x) \end{aligned}$$

are linearly independent for $\lambda \neq 0, 1, 2, \cdots$ (the spatial eigenmodes $\chi_\lambda^{(\pm)}$ are any linearly independent conbinations of $\cos(\omega_\lambda x)$ and $\sin(\omega_\lambda x)$). The linear independence of these two solutions fails in the exceptional case $\lambda = 1$ for all $L$. The linear independence of these two solutions may also fail on other values of $m$ if $\sqrt{2}\, L/\pi$ is an integer. For simplicity we exclude the case of integer $\sqrt{2}\, L/\pi$ from the present analysis and take $\lambda = 1$ to be the only special case. (We note, however, that is is straighforward to extend our analysis to include integer values of $\sqrt{2}\, L/\pi$.) In the case $\lambda = 1$ ($m = 0$) we can take, as one solution of (3.32), the Hermite polynomial (3.37)

$$H_1(z) = 2z.$$

The second solution may be written formally as

$$P_1(z) - Q_1(z)$$

where the functions $P_\lambda$ and $Q_\lambda$ are given by (3.35,3.36). The function obtained in this manner does not have a simple closed form expression as does $H_1(z)$.

Putting the results of this section together we write the general solutions of (3.30) as

$$\begin{aligned}\delta\phi(t,x) = \quad & \phi_0(t)\,\mathrm{Re}\,[P_1(i\alpha t) - Q_1(i\alpha t)]\,\tfrac{1}{2}\,a_0^{(1)} + \phi_0(t)\,\tfrac{t}{2}\,a_0^{(2)} \\ & \phi_0(t)\textstyle\sum_{m=1}^{\infty}\tilde{H}^{(1)}_{1-\pi^2 m^2/2L^2}(t)\,\chi_m^{(1)}(x) + \phi_0(t)\sum_{m=1}^{\infty}\tilde{H}^{(2)}_{1-\pi^2 m^2/2L^2}(t)\,\chi_m^{(2)}(x)\end{aligned} \tag{3.42}$$

where we have defined the functions

$$\tilde{H}^{(1)}_\lambda(t) = \frac{\Gamma(\frac{1-\lambda}{2})}{2^\lambda\Gamma(1/2)}\frac{1}{2}\left(H_\lambda(i\alpha t) + H_\lambda(-i\alpha t)\right)$$

and

$$\tilde{H}^{(2)}_\lambda(t) = \frac{\Gamma(-\lambda/2)}{2^\lambda\Gamma(-1/2)}\frac{1}{2\alpha i}\left(H_\lambda(i\alpha t) - H_\lambda(-i\alpha t)\right).$$

The normalizations have been chosen so that $\tilde{H}^{(1)}_\lambda(0) = 1$ and $\partial_t\tilde{H}^{(2)}_\lambda(t)|_{t=0} = 1$. The spatial modes $\{\chi_m^{(i)}\}$ are

$$\chi_m^{(i)}(x) = a_m^{(i)}\cos\left(\frac{\pi m x}{L}\right) + b_m^{(i)}\sin\left(\frac{\pi m x}{L}\right)$$

for $i = 1, 2$. The coefficients $\{a_m^{(1)}, b_m^{(1)}\}$ determine the fourier expansion of $\delta\phi/\phi_0$ at $t = 0$ and the coefficents $\{a_m^{(2)}, b_m^{(2)}\}$ determine the fourier expansion of $\partial_t(\delta\phi/\phi_0)$ at $t = 0$.

For practical numerical computations it is simplest to consider initial data where $a_0^{(1)} = 0$ to eliminate the function $P_1(i\alpha t) - Q_1(i\alpha t)$ from the expression (3.42). For example, this will be true for any initial data such that $\delta\phi/\phi_0$ at $t = 0$ is an odd function of $x$.

Recall that the linearized equation (3.30) is valid for $|\delta\phi(t,x)/\phi_0(t)| \ll 1$. The stability of the solution (3.42) therefore depends on the asymptotic behaviour of the functions $\tilde{H}^{(1)}_\lambda(t)$ and $\tilde{H}^{(2)}_\lambda(t)$ at late times. It is straightforward to show that $\tilde{H}^{(1)}_\lambda(t)$ and $\tilde{H}^{(2)}_\lambda(t)$ have the asymptotic behaviour

$$\tilde{H}^{(1)}_\lambda(t) \sim \tilde{H}^{(2)}_\lambda(t) \sim t^\lambda$$

for large $t$. That these two solutions become linearly dependent as $t \to \infty$ was to be anticipated from (3.41). We find, then, that for modes with $\lambda = 1 - \frac{\pi^2 m^2}{2L^2} < 0$ one has $\delta\phi_m^{(i)}(t,x)/\phi_0(t) \to 0$ and the linearized approximation is stable. On the other hand, modes with $\lambda = 1 - \frac{\pi^2 m^2}{2L^2} > 0$ [7] are increasing functions of time and hence the linearized approximation will eventually break down. In the case that $L < \pi/\sqrt{2}$ and $a_0^{(1)} = a_0^{(2)} = 0$ then all of the modes are decreasing and the perturbative expansion is reliable throughout the evolution. In this case we can make definitive statements about the absence of caustics or similar singularities in the theory (3.26). In fact, for initial data which satisfy these restrictions, the profile $\phi(t,x)$ becomes homogeneous at late times. One might argue that (3.42) implies the absence of caustic formation even for more general initial data (for example when $L$ is large and many of the modes $\delta\phi_m^{(i)}(t,x)/\phi_0(t)$ are increasing) since in this case the homogeneous zero mode of $\delta\phi/\phi_0$ increases faster than all other modes.

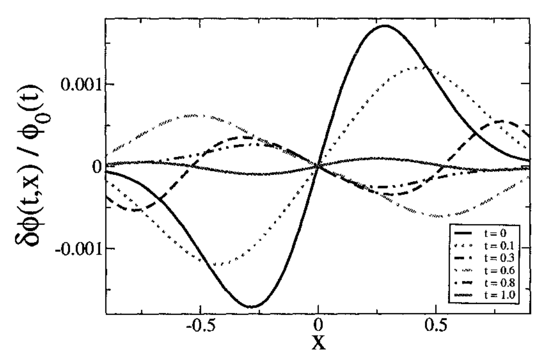


Figure 3.5: Plot of the spatial profiles $\delta\phi/\phi_0$ for a series of increasing time steps. The initial data is such that the linearized approximation is valid throughout the evolution.

Figure 3.5 shows a plot of the spatial profiles $\delta\phi/\phi_0$ given by (3.42) for a series of increasing time steps for the case $p = 2$, $L = 1$. The initial data are $\delta\phi(0,x)/\phi_0(0) = 0.01\, x\, e^{-(x/0.4)^2}$ and $\partial_t(\delta\phi(t,x)/\phi_0(t))|_{t=0} = 0.001\, \sin(\pi x/L)$ on $-L \le x \le +L$. For

[7] Recall that we are excluding the case $\lambda = 0$.

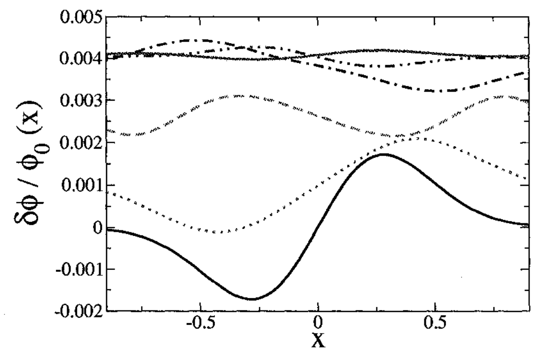


Figure 3.6: Plot of the spatial profiles $\delta\phi/\phi_0$ for a series of increasing time steps. The initial data is such that the linearized approximation breaks down at late times.

this choice of initial data $a_0^{(1)} = a_0^{(2)} = 0$ and $\lambda < 0$ for all $m$. The perturbative expansion is stable for this example and the field tends towards a homogeneous profile at late times. Figure 3.6 shows the same plot for the initial data $\delta\phi(0,x)/\phi_0(0) = 0.01\, x\, e^{-(x/0.4)^2}$ and $\partial_t(\delta\phi(t,x)/\phi_0(t))|_{t=0} = 0.01$ on $-L \leq x \leq +L$. With this choice of initial data $\delta\phi/\phi_0 \sim t$ at late times since $a_0^{(2)} \neq 0$ and the linearized approximation will eventually break down.

Finally we comment on the interpretation of the Cauchy problem for the linearized differential equation (3.30). We have found a general solution of (3.30) for which we are free to specify the initial field $\delta\phi/\phi_0$ at $t = 0$ and the time derivative $\partial_t(\delta\phi/\phi_0)$ at $t = 0$. This may seem surprising since (3.30) contains an infinite number of time derivatives and one might naively expect to have the freedom to specify an infinite number of initial data $\partial_t^{(n)}(\delta\phi/\phi_0)|_{t=0}$ for $n = 0, 1, 2, \cdots$. However, the initial value problem for homogeneous solutions of (3.27) was studied in [87] and it was found that the equation of motion itself imposes an infinite number of consistency conditions on the initial data which one can consider. In fact, it was speculated in [87] that the space of allowable initial conditions for (3.27) may be finite. This conjecture seems consistent with our results. It is interesting to note that (3.30) seems to be an example of an equation containing an infinite number of derivatives whose solution space is surprisingly similar to that of equations containing only two time derivatives.

## 3.6 Conclusions

We have studied several different effective actions describing the open superstring tachyon. In the case of actions containing only first derivatives of the tachyon field we have studied three different theories proposed on different grounds. These three actions are expected to be valid in different limits of string theory, however, we find that that vacuum structure of all three theories is remarkably similar. In particular, we have shown that all three theories lead to the formation of caustics where the field becomes multi-valued and where second and higher derivatives of the field blow up. Each of these three actions cannot be trusted in the vicinity of a caustic since higher derivative corrections which have been neglected become important. We considered also an effective action containing second order derivatives of the field and found a similar structure of derivative singularities. Finally, in the context of $p$-adic string theory, we studied small inhomogeneities about a time dependent solution which rolls from the unstable vacuum to infinity in field configuration space. In this case we found that for a broad class of initial data the linearized approximation is reliable throughout the evolution and we could conclusively show the absence of caustics or similar pathologies. This result seems suggestive that the formation of caustics is an artifact of truncating a theory with an infinite number of derivatives, however, the connection between $p$-adic string theory and tachyon matter is still unclear.

Since we have restricted our analysis to the study of effective actions we cannot conclude if the phenomenon of caustic formation is a genuine prediction of string theory or not. If this prediction is borne out by string theory it will be necessary to find some physical interpretation of the caustics, though none is obvious to us. As pointed out in [68, 69] such an interpretation could depend on the dimensionality of the theory. One avenue for future study would be to try to interpret caustic formation in terms of the gas of massive closed strings which is described by the tachyon fluid [52].

If these caustics are in fact a real prediction of string theory it will also be necessary to find some way to predict the field value in the vicinity of a caustic. The situation is qualitatively similar to the formation of shock waves in nonlinear gas dynamics. In that case the multi-valued continuous solution in the vicinity of a shock is replaced with a single valued discontinuous solution using the Rankine-Hugoniot jump condition to quantify the discontinuity in the field. It is possible that some similar auxiliary condition could be used to predict the value of the tachyon field in the vicinity of a caustic.

## Acknowledgments

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# Chapter 4

# Warped Reheating in Brane-Antibrane Inflation

## Abstract

We examine how reheating occurs after brane-antibrane inflation in warped geometries, such as those which have recently been considered for Type IIB string vacua. We adopt the standard picture that the energy released by brane annihilation is dominantly dumped into massive bulk (closed-string) modes which eventually cascade down into massless particles, but argue that the this need not mean that the result is mostly gravitons with negligible visible radiation on the Standard Model brane. We show that if the inflationary throat is not too strongly warped, and if the string coupling is sufficiently weak, then a significant fraction of the energy density from annihilation will be deposited on the Standard Model brane, even if it is separated from the inflationary throat by being in some more deeply warped throat. This is due to the exponential growth of the massive Kaluza-Klein wave functions toward the infrared ends of the throats. We argue that the possibility of this process removes a conceptual obstacle to the construction of multi-throat models, wherein inflation occurs in a different throat than the one in which the Standard Model brane resides. Such multi-throat models are desirable because they can help to reconcile the scale of inflation with the supersymmetry breaking scale on the Standard Model brane, and because they can allow cosmic strings to be sufficiently long-lived to be observable during the present epoch.


## 4.1 Introduction

There has been significant progress over the past years towards the construction of *bona fide* string-theoretic models of inflation. The main progress over early string-inspired supergravity [102] and BPS-brane based [103] models has come due to the recognition that brane-antibrane [104, 105] and related [106, 107] systems can provide *calculable* mechanisms for identifying potentially inflationary potentials. Even better, they can suggest new observable signatures, such as the natural generation of cosmic strings by the brane-antibrane mechanism [104, 56, 57, 58]. The central problem to emerge from these early studies was to understand how the many string moduli get fixed, since such an understanding is a prerequisite for a complete inflationary scenario.

Recent developments are based on current progress in modulus stabilization within warped geometries with background fluxes for Type IIB vacua [20, 108, 21]. Both brane-antibrane inflation [24] and modulus inflation [109] have been embedded into this context, with an important role being played in each case by branes living in strongly-warped 'throat-like' regions within the extra dimensions. These inflationary scenarios have generated considerable activity [110, 111] because they open up the possibility of asking in a more focused way how string theory might address the many issues which arise when building inflationary models. For instance, one can more fully compute the abundance and properties of any residual cosmic strings which might survive into the present epoch [58]. Similarly, the possibility of having quasi-realistic massless particle spectra in warped, fluxed Type IIB vacua [113] opens up the possibility of locating where the known elementary particles fit into the post-inflationary world [25], a prerequisite for any understanding of reheating and the subsequent emergence of the Hot Big Bang.

Even at the present preliminary level of understanding, a consistent phenomenological picture seems to require more complicated models involving more than a single throat (in addition to the orientifold images).[1] This is mainly because for the single-throat models the success of inflation and particle-physics phenomenology place contradictory demands on the throat's warping. They do so because the energy scale in the throat is typically required to be of order $M_i \sim 10^{15}$ GeV to obtain acceptably large temperature fluctuations in the CMB. But as was found in ref. [25], this scale tends to give too large a supersymmetry breaking scale for ordinary particles if the Standard Model (SM) brane resides in the same throat. This problem appears to be reasonably generic to the KKLT-type models discussed to date, because these models tend to have supersymmetric anti-de Sitter vacua until some sort of supersymmetry-

[1]Two-throat models are also considered for reasons different than those given here in ref. [112].

breaking physics is added to lift the vacuum energy to zero. The problem is that the amount of supersymmetry-breaking required to zero the vacuum energy also implies so large a gravitino mass that it threatens to ruin the supersymmetric understanding of the low-energy electroweak hierarchy problem.

No general no-go theorem exists, however, and there does appear to be considerable room to try to address this issue through more clever model-building. Ref. [114] provides a first step in this direction within the the framework of 'racetrack' inflation [109]. Another possibility is a picture having two (or more) throats, with inflation arising because of brane-antibrane motion in one throat but with the Standard Model situated in the other (more about this proposal below). By separating the scales associated with the SM and inflationary branes in this way, it may be possible to reconcile the inflationary and supersymmetry-breaking scales with one another.

Besides possibly helping to resolve this problem of scales, multi-throat models could also help ensure that string defects formed at the end of inflation in the inflationary throat have a chance of surviving into the present epoch and giving rise to new observable effects [58]. They are able to do so because if the Standard Model were on a brane within the same throat as the inflationary branes, these defects typically break up and disappear by intersecting with the SM brane.

At first sight, however, any multi-throat scenario seems likely to immediately founder on the rock of reheating.[2] Given the absence of direct couplings between the SM and inflationary branes, and the energy barrier produced by the warping of the bulk separating the two throats, one might expect the likely endpoint of brane-antibrane annihilation to be dump energy only into closed-string, bulk modes, such as gravitons, rather than visible degrees of freedom on our brane. In such a universe the energy which drove inflation could be converted almost entirely into gravitons, leaving our observable universe out in the cold.

It is the purpose of the present work to argue that this picture is too pessimistic, because strongly-warped geometries provide a generic mechanism for channelling

[2]See ref. [115] for a discussion of issues concerning brane-related reheating within other contexts.

the post-inflationary energy into massless modes localized on the throat having the strongest warping. They can do so because the massive bulk Kaluza-Klein (KK) modes produced by brane-antibrane annihilation prefer to decay into massless particles which are localized on branes within strongly-warped throats rather than to decay to massless bulk modes. As such, they open a window for obtaining acceptable reheating from brane-antibrane inflation, even if the inflationary and SM branes are well separated on different throats within the extra dimensions.

The remainder of the chapter is organized as follows. In §2 we introduce a simple generalization of the Randall-Sundrum (RS) model [116] containing two $AdS_5$ throats with different warp factors, as a tractable model for the KLMT inflationary scenario [24] with two throats. Here we recall the form of the KK graviton wave functions in the extra dimension. This is followed in §3 by an account of how the tachyonic fluid describing the unstable brane-antibrane decays into excited closed-string states, which quickly decay into KK gravitons. §4 Discusses the tunneling of the KK modes through the energy barrier which exists between the two throats because of the warped geometry. §5 Gives an estimate of the reheating temperature on the SM brane which results from the preferential decay of the KK gravitons into SM particles. Our conclusions are given in §6.

## 4.2 Tale of Two Throats

We wish to describe reheating in a situation where brane-antibrane inflation occurs within an inflationary throat having an energy scale of $M_i$, due to the warp factor $a_i = M_i/M_p$, where $M_p$ is the 4D Planck mass. This throat is assumed to be separated from other, more strongly warped, throats by a weakly warped Giddings-Kachru-Polchinski (GKP) manifold [20] whose volume is only moderately larger than the string scale, so $M_s \lesssim M_p$. In the simplest situation there are only two throats (plus their orientifold images), with the non-inflationary (Standard Model) throat having warp factor $a_{sm} \ll a_i$.

There are two natural choices for the SM warp factor, depending on whether or not the SM brane strongly breaks 4D supersymmetry. For instance, if the SM resides on an anti-D3 brane then supersymmetry is badly broken and the SM warp factor must describe the electroweak hierarchy *à la* Randall and Sundrum [116], with $a_{sm} \sim M_W/M_p \sim 10^{-16}$. Alternatively, if the SM resides on a D3 or D7 brane which preserves the bulk's $N = 1$ supersymmetry in 4D, then SUSY breaking on the SM brane is naturally suppressed by powers of $1/M_p$ because it is only mediated by virtual effects involving other SUSY-breaking anti-D3 branes. In this case the electroweak hierarchy might instead be described by an intermediate-scale scenario [117], where $a_{sm} \sim M_{int}/M_p \sim (M_W/M_p)^{1/2} \sim 10^{-8}$.

A potential problem arises with the low-energy field theory approximation if $a_{sm} < a_i^2$, because in this case the string scale in the SM throat, $M_{sm} \sim a_{sm}M_p$, is smaller than the inflationary Hubble scale $H_i \sim M_i^2/M_p \sim a_i^2 M_p$ [118]. In this case string physics is expected to become important in the SM throat, and stringy corrections may change the low-energy description. The intermediate scale, where $a_{sm} \gtrsim a_i^2$, is more attractive from this point of view, since for it the field-theory approximation may be justified.

To proceed we use the fact that within the GKP compactification the geometry within the throat is well approximated by

$$ds^2 = a^2(y)(-dt^2 + dx^2) + dy^2 + y^2\, d\Omega_5^2\,, \tag{4.1}$$

where $y$ represents the proper distance along the throat, $a(y) = e^{-k|y|}$ is the throat's warp factor and $d\Omega_5^2$ is the metric on the base space of the corresponding conifold singularity of the underlying Calabi-Yau space [119]. Of most interest is the 5D metric built from the observable 4 dimensions and $y$, which is well approximated by the metric of 5-dimensional anti-de Sitter space.

A simple model of the two-throat situation then consists of placing inflationary brane-antibranes in a throat at $y = -y_i$ and putting the Standard Model brane at $y = +y_{sm}$, as is illustrated in Fig. 4.1. Our analysis of this geometry follows the spirit of ref. [120]. Since most of the interest is in the throats, we simplify the description of the intervening bulk geometry by replacing them with a Planck brane at $y = 0$, with the resulting discontinuity in the derivative of $a(y)$ chosen to reproduce the smoother (but otherwise similar) change due to the weakly-warped bulk. This approximation is illustrated in Fig. 4.2, with the smooth dashed curve representing the warp factor in the real bulk geometry and the solid spiked curve representing the result using an intervening Planck brane instead.

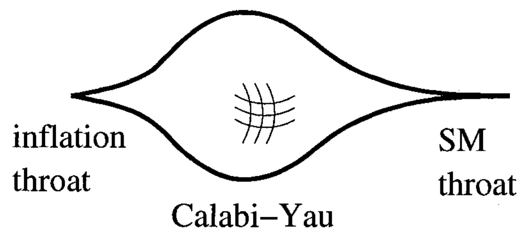


Figure 4.1: A Type IIB vacuum with a mildly warped inflationary throat and a strongly warped Standard Model throat. This diagram suppresses any image throats arising due to any orientifolds which appear in the compactification.

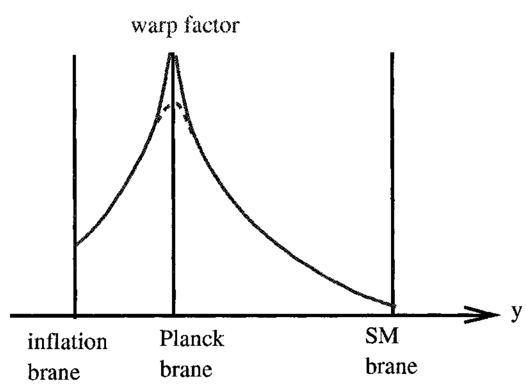


Figure 4.2: The warp factor as function of a bulk radial coordinate in a simplified model of two asymmetric throats. As shown in the figure, the part of the internal space outside of the throats can be regarded as a regularization of a 'Planck' brane of a Randall-Sundrum geometry.

Of particular interest in what follows are the massive Kaluza-Klein modes in the bulk, since these are arguably the most abundantly-produced modes after brane-antibrane annihilation. For instance, focussing on the 5 dimensions which resemble

AdS space in the throat, a representative set of metric fluctuations can be parameterized as $h(x,y)$ in the line element,

$$ds^2 = a^2(y)(-dt^2 + dx^2 + h_{\mu\nu}dx^\mu dx^\nu) + dy^2 \,. \tag{4.2}$$

In the static AdS background, the KK modes have spatial wavefunctions of the form

$$h(x,y) = \sum_n \phi_n(y)\, e^{ip\cdot x} \tag{4.3}$$

with $p \cdot x = -E_n t + \mathbf{p}\cdot\mathbf{x}$, and $\phi_n(y)$ satisfying the equation of motion

$$-\frac{d}{dy}\left(e^{-4k|y|}\,\frac{d\phi_n}{dy}\right) = m_n^2 e^{-2k|y|}\phi_n \,. \tag{4.4}$$

Here $m_n^2 = p \cdot p$ is the mode's 4D mass as viewed by brane-bound observers.

Exact solutions for $\phi_n(y)$ are possible in the Planck-brane approximation [116, 120, 121], and are linear combinations of Bessel functions times an exponential

$$\phi_n(y) = N_n\, e^{2k|y|}\left[J_2\left(\frac{m_n}{k}e^{k|y|}\right) + b_n\, Y_2\left(\frac{m_n}{k}e^{k|y|}\right)\right] \tag{4.5}$$

where, for low lying KK modes ($m_n \ll k$) one has

$$b_n \cong \frac{\pi m_n^2}{4k^2} \tag{4.6}$$

while for heavy KK modes ($m_n/k \cong 1$) one has

$$b_n \cong -0.47 + 1.04\left(\frac{m_n}{k}\right). \tag{4.7}$$

$N_n$ is determined by the orthonormality condition, which ensures that the kinetic terms of the KK modes are independent of $a_{sm}$:

$$\int_{-y_i}^{y_{sm}} dy\, e^{-2k|y|}\phi_n\phi_m = \delta_{nm} \,. \tag{4.8}$$

These wavefunctions are graphed schematically in Fig. 4.3.

For strongly-warped throats it is the exponential dependence which is most important for the KK modes. Because of the exponential arguments of the Bessel functions in eq. (4.5), the presence of the Bessel functions modifies the large-$y$ behaviour slightly. Due to the asymptotic forms $J_2(z) \propto z^{-1/2}$ for large $|z|$, and similarly for

$Y_2(z)$, we see that $\phi_n(y) \sim e^{3k|y|/2}$ for $m_n e^{k|y|} \gg k$. It is only this behaviour which we follow from here on. Taking the most warping to occur in the SM throat we find that $m_n$ is approximately quantized in units of $M_{sm} \equiv a_{sm} M_p$, which is either of order $M_W \sim 10^3$ GeV or $M_{int} \sim 10^{10}$ GeV depending on whether or not supersymmetry breaks on the SM brane. Keeping only the exponentials we find that orthonormality requires $N_n^{-2} \sim \int^{y_{sm}} dy \, e^{-2ky} \left(e^{3k|y|/2}\right)^2 \sim (k \, a_{sm})^{-1}$, and so

$$\phi_n(y) \sim (a_{sm} \, k)^{1/2} \, e^{3k|y|/2} \,, \tag{4.9}$$

showing that these modes are strongly peaked deep within the throat. This is intuitively easy to understand, since being localized near the most highly-warped region allows them to minimize their energy most effectively.

Thus, even the most energetic KK modes still have exponentially larger wave functions on the TeV brane, with the more energetic modes reaching the asymptotic region for smaller $y$. This is illustrated in Figure 4.4, which shows $\ln|\phi_n(y)|$ versus $y$ in the representative case of a throat having warp factor $a = e^{-10}$, for a series of KK states with masses going as high as $M_p$ ($n = 20,000$). As the figure shows, the wave functions grow exponentially toward the TeV brane, with the onset of the asymptotic exponential form setting in earlier for larger mode number.[3] This behaviour is central to the estimates which follow.

Among the KK modes it is the zero modes which are the exceptional case because their wavefunction is constant, $\phi_0 \sim \sqrt{k}$, and so they are not exponentially peaked inside the throat. It is the strong exponential peaking of the lightest massive KK modes relative to the massless modes which is central to the reheating arguments which follow.

In our simplified model, the presence of two throats is not much more complicated than the original RS model. Mathematically it is the same, except that RS identified the two sides $y \leftrightarrow -y$ through orbifolding. Instead we interpret them as two separate

[3]For the lowest-lying modes having the smallest nonzero masses it can happen that the asymptotic form of the Bessel functions is not yet reached even when $y = y_{sm}$, in which case the exponential peaking is slightly stronger than discussed above.

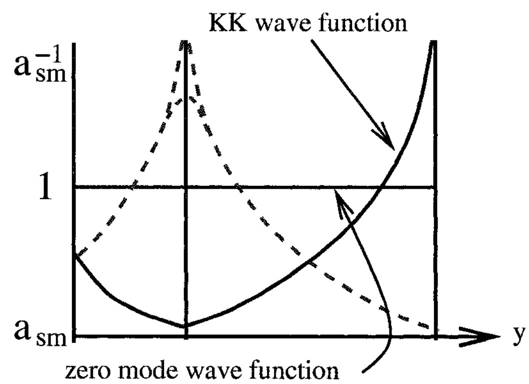


Figure 4.3: Wave functions of KK gravitons on the internal space.

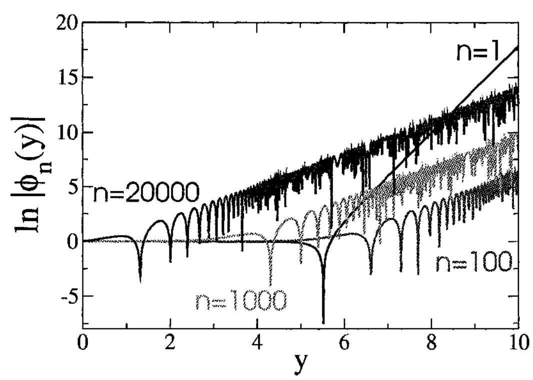


Figure 4.4: Unnormalized wave functions for highly excited KK gravitons with KK numbers $n = 1$, 100, 1000 and 20,000.

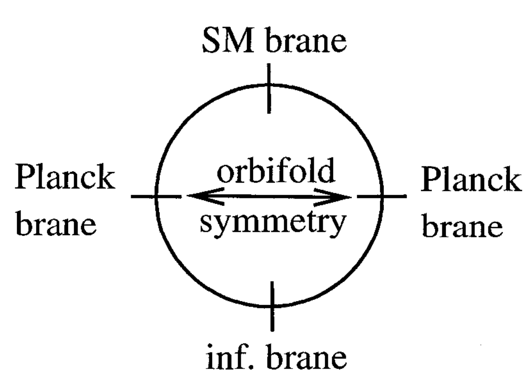


Figure 4.5: How to place two throats on an $S_1$ with $Z_2$ orbifold symmetry.

throats with different depths defined by the brane locations $-y_i$ and $+y_{sm}$. One can imagine doubling this entire system on $S_1$ and orbifolding as shown in Fig. 4.5, to define the boundary conditions on the metric and its perturbations at the infrared branes. In this figure, the orbifold identification acts horizontally so that the inflation and SM branes are distinct fixed points.

## 4.3 Brane-Antibrane Annihilation

In brane-antibrane inflation the energy released during reheating is provided by the tensions of the annihilating branes. Although this annihilation process is not yet completely understood, present understanding indicates that the energy released passes through an intermediate stage involving very highly-excited string states, before generically being transferred into massless closed-string modes. The time frame for this process is expected to be the local string scale.

For instance many of the features of brane-antibrane annihilation are believed to be captured by the dynamics of the open-string tachyon which emerges for small separations for those strings that stretch between the annihilating branes.[4] In flat space and at zero string coupling ($g_s = 0$), the annihilation instability has been argued to be described by the following tachyon Lagrangian [43]

$$\mathcal{L}_T = -2\tau_0 \, e^{-|T|^2/l_s^2} \sqrt{1 + |\partial_\mu T|^2} \tag{4.10}$$

where $T$ is the complex tachyon field, $\tau_0$ is the tension of either of the branes, and $l_s$ is the string length scale. During inflation, when the brane and anti-brane are well separated, $\dot{T} = 0$ and the pressure of the system $p_i$ is simply the negative of the tension of the two branes, $p_i = -\rho_i$, while afterward $\dot{T} \to 1$ and $p_i \to 0$. In this description the pressureless tachyonic fluid would dominate the energy density of the universe and lead to no reheating whatsoever.

[4]See, however, ref. [122] for a discussion of an alternative mechanism for which the relevant highly-excited strings are open strings, but for which the annihilation energy nonetheless eventually ends up in massless closed string modes.

However, for nonvanishing $g_s$, the time evolution of the tachyon fluid instead very quickly generates highly excited closed-string states [45, 40]. For D$p$ systems with $p > 2$ the rate of closed string production in this process is formally finite, whereas it diverges for $p \leq 2$ (and so passes beyond the domain of validity of the calculation). This divergence is interpreted to mean that for branes with $p \leq 2$ all of the energy liberated from the initial brane tensions goes very efficiently into closed string modes. For spatially homogeneous branes with $p > 2$ the conversion is less efficient and so can be dominated by other, faster processes. In particular, it is believed that these higher-dimensional branes will decay more efficiently inhomogeneously, since they can then take advantage of the more efficient channels which are available to the lower-dimensional branes. For example a D3 brane could be regarded as a collection of densely packed but smeared-out D0 branes, each of which decays very efficiently into closed strings. Since the decay time is of order the local string scale, $l_s = 1/M_s$, the causally-connected regions in this kind of decay are only of order $l_s$ in size, and so have a total energy of order the brane tension times the string volume, $\tau_0 l_s^3 \sim M_s/g_s$.

These flat-space calculations also provide the distribution of closed-string states as a function of their energy. The energy density deposited by annihilating D3-branes into any given string level is of order $M_s^4$, and so due to the exponentially large density of excited string states the total energy density produced is dominated by the most highly-excited states into which decays are possible. Since the available energy density goes like $1/g_s$ the typical closed-string state produced turns out to have a mass of order $M_s/g_s$, corresponding to string mode numbers of order $N \sim 1/g_s$. On the other hand, the momentum transverse to the decaying branes for these states turns out to be relatively small, $p_T \sim M_s/\sqrt{g_s}$ [45, 40], and so the most abundantly produced closed-string states are nonrelativistic.

How do these flat-space conclusions generalize to the warped Type IIB geometries which arise in string inflationary models? If the annihilating 3-branes are localized in the inflationary throat, then the tension of the annihilating branes is of order $\tau_0 \sim (a_i M_s)^4/e^{\phi_i}$, where $\phi_i$ denotes the value taken by the dilaton field at the throat's tip. The highly-excited closed-string states that are produced in this way live in the bulk, with the energy density produced being dominated by those whose masses are of order $a_i M_s/e^{\phi_i}$. Once produced, these closed-string bulk modes decay down to lower energies and, as might be expected from phase space arguments, most of them typically drop down to massless string states very quickly. An important exception to this would arise for those states carrying the most angular momentum at any given string mass level, since these must cascade more slowly down to lower energies in order to lose their angular momentum [123]. However these seem unlikely to be produced in appreciable numbers by brane-antibrane annihilation.

We are led in this way to expect that the annihilation energy is distributed relatively quickly amongst massless string states, or equivalently to KK modes of the higher-dimensional supergravity which describes these states. Although the initial massive string modes would be nonrelativistic, with $M \sim a_i M_s/e^{\phi_i}$ and $p_T \sim a_i M_s/e^{\phi_i/2}$, the same need not be true for the secondary string states produced by their decay, whose masses are now of order the KK mass scales. Consequently these states may be expected not to remain localized in the inflationary throat, and so if the extra dimensions are not too large compared with the string scale these modes would have time to move to the vicinity of the SM throat before decaying further. Once there, they would be free to fall into the potential wells formed by the throats as their energy is lost by subsequent decays into lower-energy levels.

This physical picture is supported by the exponential peaking of the KK-mode wave-functions in the most deeply-warped throats. In order to estimate the efficiency with which energy can be transferred amongst KK modes, we can use the approximate behavior of the wave functions given in the previous section to keep track of powers of the throat's warp factor, $a_{sm}$. For instance, consider the trilinear vertex among 3 KK states having mode numbers $n_1$, $n_2$ and $n_3$ which is obtained by dimensionally reducing the higher-dimensional Einstein-Hilbert action, $\sqrt{g}R$. Keeping in mind that $\sqrt{g}g^{\mu\nu} \propto a^2$ and that $\psi_n \propto a_{sm}^{1/2}/a^{3/2}$ for the nonzero modes (eq. (4.9)), we find that the trilinear vertex involving $0 \leq r \leq 3$ massive KK modes (and $3-r$ massless KK modes) has the following representative estimate

$$
\begin{aligned}
\mathcal{L}_{\rm int} &\sim \int_{-y_i}^{y_{sm}} dy\, \sqrt{|g|}\, g^{\mu\nu} g^{\alpha\beta} g^{\kappa\sigma} g^{\rho\delta} h_{\alpha\kappa} \partial_\mu h_{\sigma\rho} \partial_\nu h_{\beta\delta} \\
&\sim \int_{-y_i}^{y_{sm}} dy\, e^{-2k|y|}\, \eta^{\mu\nu} \psi_{n_1}(x,y)\, \partial_\mu \psi_{n_2}(x,y)\, \partial_\nu \psi_{n_3}(x,y) \\
&\sim \psi_{n_1}(x)\, \psi_{n_2}(x)\, \psi_{n_3}(x)\, p_2 \cdot p_3\, a_{sm}^{r/2} \int_{-y_i}^{y_{sm}} dy\, e^{-2k|y|} (e^{3k|y|/2})^r \\
&\sim \psi_{n_1}(x)\, \psi_{n_2}(x)\, \psi_{n_3}(x) \left( \frac{p_2 \cdot p_3}{k} \right) a_{sm}^{\eta} \qquad (4.11)
\end{aligned}
$$

where

$$\eta = 2 - r \quad \text{if} \quad r \geq 2, \qquad \text{and} \qquad \eta = \frac{r}{2} \quad \text{if} \quad r = 0, 1\,. \qquad (4.12)$$

Notice for this estimate that since derivatives in the compactified directions are proportional to $g^{mn}$ rather than $g^{\mu\nu}$, they suffer from additional suppression by powers of $a = e^{-ky}$ within the throat. Here $m$, $n$ label the internal directions perpendicular to the large $3+1$-dimensional Minkowski space.

Thus a trilinear interaction amongst generic KK modes ($r = 3$), even those with very large $n$, is proportional to $1/(a_{sm}k) \sim 1/M_{sm}$, and so is only suppressed by inverse powers of the low scale. Similarly, $r = 2$ processes involving two massive KK modes $B$ and $B'$, and one massless bulk mode $ZM$ — such as the reaction $B \to B' + ZM$ — are $\propto 1/k \sim 1/M_p$ and so have the strength of 4D gravity inasmuch as they are Planck suppressed. The same is also true of the $r = 0$ couplings which purely couple the zero modes amongst themselves.[5] Finally, those couplings involving only a single low-lying massive mode and two zero modes ($r = 1$) — such as for $B \to ZM + ZM'$ — are proportional to $a_{sm}^{1/2}/k \sim (M_{sm}/M_p^3)^{1/2}$ and so are even weaker than Planck-suppressed.

Similar estimates may also be made for the couplings of the generic and the massless KK modes to degrees of freedom on a brane sitting deep within the most strongly-warped throat. Using the expressions $\phi_0(y_{sm}) \sim 1$ and $\phi_n(y_{sm}) \sim 1/a_{sm}$ for

[5] The appendix shows that this agrees with the size of the couplings found in the effective 4D supergravity lagrangian which describes the zero-mode and brane couplings.

massless and massive KK modes respectively, this gives:

$$\begin{aligned} \mathcal{L}_i &= M_p^{-1} \left( h_{\mu\nu}^{(0)} \phi_0(y_{sm}) + \sum_n h_{\mu\nu}^{(n)} \phi_n(y_{sm}) \right) T_{sm}^{\mu\nu} \\ &\sim \left( \frac{h_{\mu\nu}^{(0)}}{M_p} + \sum_n \frac{h_{\mu\nu}^{(n)}}{M_{sm}} \right) T_{sm}^{\mu\nu} . \end{aligned} \tag{4.13}$$

We see here the standard Planck-suppressed couplings of the massless modes (such as the graviton) as compared with the $O(1/M_{sm})$ couplings of the massive KK modes.

The picture which emerges is one for which the energy released by brane-antibrane annihilation ends up distributed among the massive KK modes of the massless string states. Because the wavefunctions of these modes tend to pile up at the tip of the most warped (SM) throat, their couplings amongst themselves — and their couplings with states localized on branes in this throat — are set by the low scale $M_{sm}$ rather than by $M_p$. Furthermore, because the $\mathcal{O}(1/M_{sm})$ couplings to the massless modes on the SM branes are much stronger than the Planck-suppressed couplings to the massless bulk modes, we see that the ultimate decay of these massive KK modes is likely to be into brane states. If it were not for the issue of tunneling, which we consider below, the final production of *massless* KK zero modes would be highly suppressed. Although we make the argument here for gravitons, the same warp-counting applies equally well to the other fields describing the massless closed-string sector.

In summary, we see that strong warping can provide a mechanism for dumping much of the energy released by the decay of the unstable brane-antibrane system into massless modes localized on branes localized at the most strongly-warped throat, regardless of whether the initial brane-antibrane annihilation is located in this throat. It does so because the primary daughter states produced by the decaying brane-antibrane system are expected to be very energetic closed strings, which in turn rapidly decay into massive KK modes of the massless string levels. The strong warping then generically channels the decay energy into massless modes which are localized within the most strongly-warped throats, rather than into massless bulk modes.

## 4.4 Tunneling

However the above arguments are too naive, since they ignore the fact that there is an energy barrier which the initial KK gravitons must tunnel through in order to reach the Standard Model throat. The efficiency of reheating on the SM brane will be suppressed by the tunneling probability.

The tunneling amplitude for a KK mode with energy $E_n$, in a Randall-Sundrum-like two-throat model just like ours, has been computed exactly in [120]:

$$\mathcal{A} \sim a_{\rm inf}^2 \left(\frac{E_n}{M_i}\right)^2 \tag{4.14}$$

where $M_i$ is the characteristic energy scale at the bottom of the inflation throat, out of which the particle is tunneling. Intuitively, this can be understood in the following way. For a mode with minimum (but nonzero) energy, the tunneling amplitude is given by the ratio of its wave function at the bottom of the throat to that at the top:

$$\mathcal{A} \sim \frac{\phi_n(y_{\rm inf})}{\phi_n(0)} \sim a_{\rm inf}^2 \tag{4.15}$$

Since energies in the throat scale linearly with the warp factor, a high-energy mode, with energy $E_n$, should have the larger tunneling amplitude given by (4.14). In the present case, the highest KK modes have energies determined by the tension of a D0-brane (as argued above); but we must remember that it is the warped tension which counts, so the maximum energy scale is given by $E_n \sim a_{\rm inf}\, M_s/g_s$ whereas the characteristic energy scale in the throat is $M_i \sim a_{\rm inf}\, M_s$. The tunneling probability is therefore

$$P = \mathcal{A}^2 = \left(\frac{a_{\rm inf}}{g_s}\right)^4 \tag{4.16}$$

To maximize this, we need a high scale of inflation (so that the inflationary warp factor is not too small) and a small string coupling. Optimistically, we could imagine that inflation is taking place near the GUT scale, $10^{16}$ GeV, which saturates the bound on the inflation scale coming from gravitational waves contributing to the CMB anisotropy, and $g_s = 0.01$. Then $a_{\rm inf} = 10^{-3}$ and the tunneling probability is $P = 10^{-4}$.

With a small tunneling probability $P$, the universe immediately after reheating would be dominated by massless gravitons, the final decay product of KK gravitons confined to the inflation throat. Only the small fraction $P$ of the original false vacuum energy density which tunneled into the SM throat would efficiently decay into ultimately visible matter on the SM brane. Such a distribution of energy density would be strongly ruled out by big bang nucleosynthesis were it to persist down to low temperatures. There are several natural ways in which this outcome can be avoided however. Since reheating occurs at a high scale (given that $P$ is not *too* small, as we shall quantify in the next section), the number of effectively massless degrees of freedom $N(T_{\rm rh})$ could be quite large at the temperature of reheating. As the heavier of these species go out of equilibrium, they transfer their entropy into the lighter visible sector particles, resulting in a relative enhancement factor $N(T_{\rm rh})/N(T_{\rm nuc})$ of the entropy in visible radiation at the nucleosynthesis temperature $T_{\rm nuc}$. On the other hand the entropy density in gravitons remains fixed because they were already thermally decoupled from the moment they were produced. If this is the only mechanism for diluting gravitons, we would require $P\,N(T_{\rm rh})/N(T_{\rm nuc}) \gtrsim 10$, so that gravitons make up no more than 10% of the total energy density at BBN.

Additionally, gravitons can be efficiently diluted if any heavy particles decay out of equilibrium at a temperature $T_{\rm dec}$ before BBN, so that they come to dominate the energy density during a significant interval.[6] In this case the gravitons are diluted by an additional factor of $T_{\rm dec}/M$ by decaying particles of mass $M$. Similarly, a period of domination by coherent oscillations of a scalar field (for example a flat direction with a large initial VEV, that gets lifted during a phase transition) will behave as though matter-dominated, and give the same kind of dilution.

There is one more criterion which must be satisfied in order for tunneling to be significant: the lifetime of the heaviest KK states should be of the same order as or longer than the typical tunneling time. The typical momentum of the KK modes is of order $M_i/\sqrt{g_s}$, hence the velocity transverse to the decaying brane is of order $\sqrt{g_s}$

[6]We thank Andrei Linde for pointing out this possibility.

[45], and the length of the throat is of order $R = cM_s^{-1}$ where $c \gtrsim 10$ in order to have a reliable low-energy description of the inflationary dynamics. Thus the tunneling time is

$$\tau_t \gtrsim 10\frac{M_s^{-1}}{\sqrt{g_s}}P^{-1} \tag{4.17}$$

while the lifetime of a KK mode is estimated to be

$$\tau_l \sim \left(g_s^2\,\frac{M_i}{g_s}\right)^{-1} \tag{4.18}$$

where the factor of $g_s^2$ comes from the squared amplitude for the two-body decay and $M_i/g_s$ is the phase space. This translates into the requirement $10\,g_s^{9/2} < a_{\rm inf}^3$. To satisfy this, we need to take a string coupling which is somewhat smaller than 0.01, say $g_s = 0.006$. The bound is then saturated for an inflationary warp factor of $a_{\rm inf} = 10^{-3}$. In this case the tunneling probability is $8 \times 10^{-4}$.

It would be interesting if there exist warped compactifications in which the background dilaton field is varying between the two throats. In this case it may be possible to have a smaller string coupling in the inflation throat, as would be desirable for the tunneling problem, while keeping the string coupling in the SM throat at a phenomenologically preferred value.

It is worth emphasizing that even when the tunneling probability for energetic KK modes is large enough for reheating, the lifetime of cosmic strings in the inflation throat is still cosmologically large. Copeland *et al.* [58] estimate the barrier penetration amplitude for a string to be

$$e^{-1/a_{\rm inf}^2} \sim e^{-10^6} \tag{4.19}$$

which makes the strings stable on cosmological time scales.

## 4.5 Warped Reheating

From the previous sections we see that the endpoint of brane-antibrane inflation can be considered as a gas of nonrelativistic closed strings with mass $M_i/g_s$, density $M_i^3$ and decay rate $\Gamma \sim g_s\,M_i$, localized in the inflationary throat. These heavy states cascade down to massless gravitons through a sequence of KK gravitons, a fraction $P$ (eq. 4.16) of which tunnel to the SM brane and decay into visible sector particle.

Initially there will be two relevant reheat temperatures: one for the massless gravitons, $T_{\rm grav}$ and one for the visible sector, $T_{\rm vis}$. By the standard reheating estimate [125] we see that

$$T_{\rm grav} \sim 0.1\,(\Gamma M_p)^{1/2} \sim 0.1\,a_i^{1/2}\,\sqrt{M_s\,M_p} \sim 10^{-3}\,M_p\,, \tag{4.20}$$

where the last estimate uses $a_i \sim 10^{-3}$. and $M_s \sim M_p/10$. On the other hand, since a fraction $P$ of the false vacuum energy was converted to visible sector particles, we deduce that

$$T_{\rm vis} = P^{1/4}\,T_{\rm grav} \sim T_{\rm grav}/10 \tag{4.21}$$

using the optimistic estimate of the previous section for $P$. This estimate is high enough to avoid potential problems to which a low reheat temperature can give rise. One should take this result with a grain of salt since it is marginally larger than both $M_i$ and $M_{sm}$, and because it is larger than the string scale in the throats it invalidates the 4D field-theoretic calculation on which it is based. A more careful calculation must instead be based on a higher-dimensional, string-theoretic estimate of the energy loss, which goes beyond the scope of this article.

In conventional inflation models, such a high reheating temperature would be in conflict with the gravitino bound (overproduction of gravitinos, whose late decays disrupt big bang nucleosythesis). It is interesting in this regard that the KKLT scenario gives a very large gravitino mass, around $m_{3/2} = 6 \times 10^{10}$ GeV [114], which is so large that there is effectively no upper limit on the reheat temperature (see for instance ref. [133]). The disadvantage of such a large gravitino mass is that supersymmetry is broken at too high a scale to explain the weak scale of the SM. If SUSY is this badly broken, one possibility for explaining the weak hierarchy is that the large landscape of string vacua provides a finely-tuned Higgs mass, as well as cosmological constant, as has been suggested in ref. [134]. If this is the case, then the degree of warping in the SM model brane would not be crucial for determining the

TeV scale, and the existence of an extra throat to contain the SM model brane would be unnecessary. However, given the large number of 3-cycles in a typical Calabi-Yau manifold, each of which can carry nontrivial fluxes, the existence of many throats should be quite generic, and it would not be surprising to find the SM brane in a different throat from the inflationary one.

## 4.6 Conclusions

We have argued that for brane-antibrane inflation in strongly-warped extra-dimensional vacua — such as have been considered in detail for Type IIB string models — there is a natural mechanism which channels a fraction of the released energy into reheating the Standard Model degrees of freedom. This is because a nonnegligible fraction of the false vacuum energy of the brane-antibrane system naturally ends up being deposited into massless modes on branes which are localized inside the most strongly-warped throats, rather than being dumped completely into massless bulk-state modes.

This process relies on what is known about brane-antibrane annihilation in flat space, where it is believed that the annihilation energy dominantly produces very massive closed-string states, which then quickly themselves decay to produce massive KK modes for massless string states. What is important for our purposes is that the wave functions for all of the massive KK modes of this type are typically exponentially enhanced at the bottom of warped throats, while those for the massless KK bulk modes are not. This enhancement arises because the energies of these states are minimized if their probabilities are greatest in the most highly warped regions. This peaking is crucial because it acts to suppress the couplings of the massive KK modes to the massless bulk states, while enhancing their couplings to brane modes in the most warped throats.

Although the couplings of the KK modes to SM degrees of freedom are enhanced, the KK modes must first tunnel from the inflation throat to the SM throat. This results in most of the energy density of the brane-antibrane system ending up as massless gravitons, and only a small fraction $P$ going into visible matter. Nevertheless, for reasonable values of the string coupling and the warp factor of the inflationary throat, $P$ can be as large as $10^{-3} - 10^{-4}$. Since the initial reheat temperature is high, there are many decades of evolution in temperature during which the decoupled gravitons can be diluted by events which increase the entropy of the thermalized visible sector particles relative to the gravitons. In this way it is quite plausible that big bang nucleosynthesis bounds on the energy density of gravitons can be satisfied.

From this point of view, it is possible to efficiently reheat the SM brane after brane-antibrane inflation, so long as there are no other hidden branes lying in even deeper throats than the SM, which would have a larger branching ratio for visible sector decay than the SM. This observation is all the more interesting given the attention which multiple-throat inflationary models are now receiving, both due to the better understanding which they permit for the relation between the inflationary scale and those of low-energy particle physics, and to the prospects they raise for producing long-lived cosmic string networks with potentially observable consequences. We will discuss the formation and evolution of such cosmic string networks in chapter 5.

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# Chapter 5

# Overproduction of Cosmic Superstrings

# Abstract

We show that the naive application of the Kibble mechanism seriously underestimates the initial density of cosmic superstrings that can be formed during the annihilation of D-branes in the early universe, as in models of brane-antibrane inflation. We study the formation of defects in effective field theories of the string theory tachyon both analytically, by solving the equation of motion of the tachyon field near the core of the defect, and numerically, by evolving the tachyon field on a lattice. We find that defects generically form with correlation lengths of order $M_s^{-1}$ rather than $H^{-1}$. Hence, defects localized in extra dimensions may be formed at the end of inflation. This implies that brane-antibrane inflation models where inflation is driven by branes which wrap the compact manifold may have problems with overclosure by cosmological relics, such as domain walls and monopoles.


## 5.1 Introduction

Although it is notoriously difficult to test string theory in the laboratory, exciting progress has been made on the cosmological front through the realization that superstrings could appear as cosmic strings within recent popular scenarios for brane-antibrane inflation [56, 58]. Gravity wave detectors and pulsar timing measurements could thus give the first positive experimental signals coming from superstrings [135] (see, however, [136]). If seen, it will be challenging to distinguish cosmic superstrings from conventional cosmic strings. One distinguishing feature is the possibility that superstrings have a smaller intercommutation probability than ordinary cosmic strings [137]. In this chapter we consider whether the mechanism of formation of cosmic superstrings might provide another source of distinction, by studying in detail the formation of string and brane defects, emphasizing differences with the standard picture of defect formation.

To set the stage, let us briefly recall how defect formation works in a standard theory [35] (see chapter 1 for a review). Consider a scalar field theory with potential

$\frac{\lambda}{4}(|\phi|^2 - \eta^2)^2$. In the standard picture as the universe cools below some critical temperature $T_c$ the $U(1)$ symmetry is broken and $\phi$ rolls to the degenerate vacua of the theory $|\phi| = \eta$. Causality implies that the field cannot be correlated throughout the space and hence the field rolls to different vacua in different spatial regions leading to the formation of topological defects. The defect separation is set by the correlation length, $\xi$. For a universe expanding with Hubble rate $H$ one expects $\xi < H^{-1}$ and hence a minimum defect density of about one per Hubble volume. However, this is only an upper bound on $\xi$. A more careful estimate can be made using condensed matter physics methods: equating the free energy gained by symmetry breaking with the gradient energy lost. Very close to $T_c$ thermal fluctuations which can restore the symmetry are probable; however, once the universe cools below the Ginsburg temperature, $T_G$, there is insufficient thermal energy to excite a correlation volume into the state $\phi = 0$ and the defects "freeze out." For the scalar field theory described above this estimate yields a correlation length of order the microscopic scale: $\xi \sim \lambda^{-1}\eta^{-1}$.

The physics of tachyon condensation on the unstable D$p$ brane-antibrane system is quite different however, due to the peculiar form of the action [43]

$$S = -2\tau_p \int d^{p+1}x \, e^{-|T|^2/b^2} \sqrt{1 + |\partial T|^2}. \tag{5.1}$$

The tachyon potential has a "runaway" form and there are no oscillations of the field near the true vacuum which can restore the symmetry. The decaying exponential potential of the complex tachyon field multiplies the kinetic term. Once condensation starts, $T$ rolls quickly to large value, and damps the gradient energy exponentially. This essentially eliminates the restoring force which would tend to erase gradients within a causal volume in an ordinary field theory.

In this chapter we perform a quantitative analysis of the formation of string defects starting from the unstable tachyonic condensate that describes unstable brane-antibrane systems. Having established that defects form with an energy density comparable to that which is available from the condensate, we then examine the possible cosmological consequences of this larger-than-expected initial density.

The organization of this chapter is as follows. In section 5.2 we briefly discuss brane-antibrane inflation and the formation of defects at the end of brane-antibrane inflation by tachyon condensation. In section 5.3 we study the formation of codimension-one branes in the decay of a nonBPS brane both analytically as well as through lattice simulations. In section 5.4 we repeat the study, this time for the formation of codimension-two branes in the decay of the brane-antibrane system. We contrast these results with the formation of conventional cosmic strings in section 5.5. The reader who is interested in the cosmological implications may wish to skip directly to section 5.6 where we consider new constraints on brane inflation models which may arise due to the large initial density of defects. Conclusions are given in section 5.7. In an appendix we justify the assumed initial conditions for the tachyon fluctuations which lead to defects.

## 5.2 Brane Inflation

Relic cosmic superstrings can form at the end of inflation driven by the attractive interaction of branes and antibranes [24, 25], [53]. In this picture, inflation ends when the brane and antibrane (or a pair of branes oriented at angles) become sufficiently close that one of the open string modes stretching between the branes becomes tachyonic. The subsequent rolling of this tachyon field describes the decay of the brane-antibrane pair. Quantum fluctuations produce small inhomogeneities in the tachyon field, which will cause it to roll toward different vacua in different spatial regions, leading to the formation of topological defects which are known to be consistent descriptions of lower dimensional branes.

The branes which drive inflation must span the three noncompact dimensions and may wrap some of the compact dimensions. The defects which form are lower dimensional branes whose world-volume is within the world-volume of the parent branes. The argument was made in [56] (illustrated in figure 5.1) that by applying the reasoning of the Kibble mechanism to the formation of the lower-dimensional branes one concludes that the branes which are copiously produced as topological defects will always wrap the same compact dimensions that the parent branes wrap and hence these branes appear as cosmic strings to the 3-dimensional observer.[1] This argument is based on the fact that the size of the compact dimensions is orders of magnitude smaller than the Hubble distance during (and at the end of) inflation and hence there are no causally disconnected regions along the compact dimensions (a reasonable estimate is $H^{-1} \sim 6 \times 10^3 M_s^{-1}$ while typically the extra dimensions are compactified at a scale $R$ closer to $M_s^{-1}$). Therefore along these directions the tachyon field would always roll toward the same vacuum and no topological defects would form. The causally disconnected regions occur only along the extended directions, so the defects are localized along the extended directions. An identical argument implies that cosmic strings should form with a density of about one string per Hubble volume.

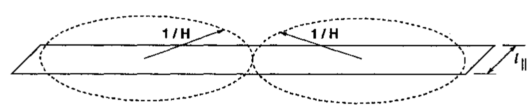


Figure 5.1: Illustration of the Kibble argument in the case of branes spanning extended dimensions and wrapping compact ones. The compact dimension is much smaller in size than the Hubble distance, $1/H \gg \ell_{||}$, so there are no causally disconnected regions along this dimension. The branes that will form as topological defects will wrap the same compact dimensions as the parent branes, and will be localized on the extended dimensions.

Reference [56] uses the analogue of the condensed matter physics argument paraphrased in section 5.1 to estimate the correlation length of the tachyon field at the end of inflation. Since the universe has zero temperature at the end of inflation the thermal fluctuations are replaced by the quantum fluctuations of de Sitter space: $H/(2\pi)$. In that analysis the potential depends on the brane separation and as a result of the relative motion of the brane and the antibrane, the curvature of the potential at $T = 0$ changes sign. It is also assumed that the tachyon potential, $V(T)$,

[1]The production of phenomenologically dangerous monopole-like and domain wall-like defects is suppressed.

has a minimum for some finite value of $T$. In this case there is an interplay between the correlation length, $\xi_C$ given by the curvature of the potential at $T = 0$, and the Ginsburg length, $\xi_G$, which is the largest scale on which quantum fluctuations of the field can restore the symmetry. As the brane separation decreases and the shape of the potential changes, the correlation length and the Ginsburg length change as well. The density of defects freezes when $\xi_C = \xi_G$. This analysis yields a correlation length which is not much different from the Hubble scale: $\xi \sim 1.6 \times 10^4 M_s^{-1} \sim 2.7 H^{-1}$.

There are several reasons to consider a more quantitative study of defect formation at the end of brane inflation. Firstly, the estimate of one defect per Hubble volume provides only a lower bound on the defect density via causality. The actual network of defects produced is determined by the complicated tachyon dynamics as it rolls from the unstable maximum of its potential to the degenerate vacua of the theory. Second, the effective field theory which describes the dynamics of the tachyon [43] has a rather unusual causal structure [138] and the usual reasoning of the Kibble mechanism may not be applicable. In [138] it was found that in the case of the homogeneous rolling tachyon the small fluctuations of the field propagate according to a "tachyon effective metric" which depends on the rolling tachyon background. As the tachyon rolls towards the vacuum the tachyon effective metric degenerates, the tachyon light cone collapses onto a time-like half line and the tachyon fields at different spatial points are causally decoupled.

Finally we comment on the validity of the effective field theory description. At some point the effective field theory description of the decaying brane will become inappropriate and the correct degrees of freedom will be the decay products of the brane annihilation. Since the topological defects of the tachyon field form in a short time of order $M_s^{-1}$, [48, 1], the effective field theory description should be applicable during the period of defect formation. Furthermore, there are reasons to believe that the effective field theory is valid even at late times when the tachyon is close to the vacuum, since in this regime the tachyon effective field theory may give a dual description of the closed strings which are produced during brane decay [52].

## 5.3 Tachyon Kink Formation in Compact Spaces

In this section we consider the formation of codimension-one branes from tachyon condensation on a nonBPS brane. We study defect formation both by solving the full nonlinear equations of motion on a lattice, as well as providing analytical approximations to the behavior of the tachyon field in different regimes of interest. We study the evolution of the tachyon field starting from a profile which is close to the unstable vacuum $T = 0$ (in this respect our analysis is similar to [32, 33, 34]). In our analysis there is no parameter which causes continuous variation of the potential, we consider the brane-antibrane system to be coincident at $t = 0$ when the initial conditions are imposed (this potentially important difference should be kept in mind when comparing our results to [56]).

### 5.3.1 Action and Equation of Motion

We will work with the tachyon Dirac-Born-Infeld effective action [43, 42, 50]

$$S = -\int V(T)\sqrt{1+\partial_\mu T\partial^\mu T}\,\sqrt{-g}\,d^{3+1+1}x. \tag{5.2}$$

The action (5.2), with $V(T) = V_0\,/\cosh(T/\sqrt{2\alpha'})$, can be derived from string theory in some limit [46] and has been shown to reproduce various nontrivial aspects of the full string theory dynamics [50, 45]. Here we take the potential to be

$$V(T) = \tau_p \exp\left(-T^2/\,b^2\right) \tag{5.3}$$

where $\tau_p$ is the D$p$-brane tension. The constant $b$ determines the tachyon mass in the perturbative vacuum as $M_T = \sqrt{2}b^{-1} \sim 1/\sqrt{\alpha'}$. Qualitatively the results will depend very little on the specific functional form of $V(T)$. In fact, for much of the analytical analysis which follows we will not even need to make reference to the specific functional form of $V(T)$.[2] We consider real tachyon fields in a spacetime with three expanding, noncompact dimensions $\{x^i\}$ (where $i = 2, 3, 4$) and one compact

[2]Provided of course that $V'(T=0) = 0$ and $V(T \to \pm\infty) \to 0$.

dimension $x^1 = x$ which we take to be static. The metric is

$$g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + dx^2 + a(t)^2\delta_{ij}\, dx^i\, dx^j. \tag{5.4}$$

For simplicity we take the tachyon field to depend only on time, $x^0 = t$, and the compact spatial coordinate $x^1 = x$. The equation of motion which follows from (5.2) with the metric (5.4) is

$$\left(1+T'^2\right)\ddot{T} = T''\left(1-\dot{T}^2\right) - \left(3H\dot{T} + \frac{V'(T)}{V(T)}\right)\left(1-\dot{T}^2+T'^2\right) + 2\dot{T}T'\dot{T}' \tag{5.5}$$

where $\dot{T} = \partial_0 T = \partial_t T$, $T' = \partial_1 T = \partial_x T$, $V'(T) = \partial V/\partial T$ and $H = \dot{a}/a$.

### 5.3.2 Lattice Simulation of Kink Formation

We have solved (5.5) on a lattice for different values of the compactification radius and the Hubble parameter $H$ (which we take to be constant for simplicity). We consider vanishing initial velocity $\dot{T}(t=0,x) = \dot{T}_i(x) = 0$. The initial profile $T(t=0,x) = T_i(x)$ is a Fourier series truncated such that the minimum wavelength is comparable to the lattice spacing.[3] The amplitudes of the Fourier coefficients are given by a random Gaussian distribution and the overall amplitude of the initial profile is chosen to be small compared to $b$.

As in [51] we find that the gradient of the tachyon field near the core of the kink becomes infinite in a finite time, forcing us to halt our evolution. After this point the codimension-one branes have formed and if one wants to follow the evolution beyond this time it is necessary to consider the dynamics between branes and antibranes in a compact space; the field theory description is no longer adequate.

Typically the initial profile crosses $T = 0$ at many points. In the early stages of the evolution the field begins to grow due to the small displacement from the unstable vacuum. During this phase of the evolution many of the small fluctuations of the field will straighten themselves out. Large Hubble damping tends to kill off the high

[3]See the appendix for a discussion of the validity of these initial conditions.

frequency fluctuations faster. At the end of this initial stage there may be several locations where the field stays pinned at $T = 0$, depending most crucially on the radius of compactification. The evolution very quickly enters a nonlinear regime in which the field begins to roll quickly towards $T \to +\infty$ where $T > 0$ and $T \to -\infty$ where $T < 0$.

The most important parameter for determining the density of the defect network once the daughter branes have formed is the radius of compactification, $R$. Perhaps surprisingly, we find that for a compactification radius as small as a few times $b$, tachyon kinks will form. Once the field enters the nonlinear regime the defect formation depends only on the local physics near the core of the kink. In figures 5.2, 5.3 we plot the tachyon field versus $t$ and $x$, showing the formation of codimension-one branes from the decay of a nonBPS brane for radii of compactification $R = 8M_T^{-1}$ and $R = 15M_T^{-1}$. The Hubble constant is taken to be vanishing, $H = 0$, in these figures. We have also considered $H \neq 0$ and find that the Hubble damping has little effect on the final kink/anti-kink network. Hence we find that tachyon kinks *do* form in the compact directions even when the field is initially in causal contact throughout the extra dimension.

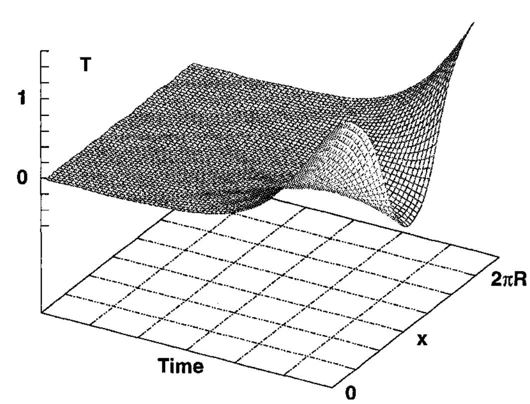


Figure 5.2: Formation of tachyon kinks for $R = 8M_T^{-1}$. The time axis is measured in units of $M_T^{-1}$.

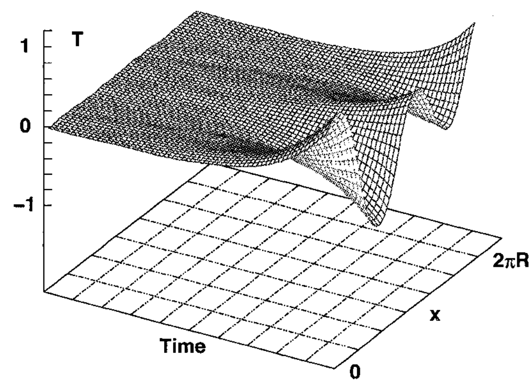


Figure 5.3: Formation of tachyon kinks for $R = 15M_T^{-1}$. The time axis is measured in units of $M_T^{-1}$.

### 5.3.3 Analytical Study of Tachyon Kink Formation

Here we describe analytically the formation of tachyon kinks. Since the full equation of motion (5.5) is difficult to solve analytically for arbitrary initial data, we study the dynamics of (5.5) in different regimes: near the core of the defect, where the field stays pinned at $T = 0$, and away from the core, where $T$ rolls towards the vacuum. To simplify our analysis we neglect the compact topology of the extra dimension $x$, which should be a good approximation since the defect solutions are infinitely thin and therefore highly localized.

#### Solutions Near the Core of the Defect

Here we briefly review the analytical studies of kink formation near the core of the defect presented in [1]. Consider initial data $T(t = 0, x) = T_i(x)$ and $\dot{T}(t = 0, x) = \dot{T}_i(x) = 0$. The field should start to roll where $T_i(x) \neq 0$ due to the small displacement from the unstable maximum $V'(T) = 0$, and furthermore it must stay pinned at $T = 0$ at the core of the kink. At $t = 0$ the equation of motion (5.5) is

$$\ddot{T}_i(x)\left(1 + T_i'(x)^2\right) = T_i''(x) + \frac{2}{b^2}T_i(x)\left(1 + T_i'(x)^2\right).$$

Clearly any point $x_0$ where $T_i(x_0) = T_i''(x_0) = 0$ will be a fixed point where $\ddot{T}(t, x_0) = 0 = \dot{T}(t, x_0)$ throughout the evolution. We restrict ourselves to considering only initial data such that $\mathrm{sgn}(\ddot{T}_i(x)) = \mathrm{sgn}(T_i(x))$ for all $x$ to ensure that the solutions are increasing. To analytically study the dynamics near $x_0$ it is therefore reasonable to make the ansatz

$$T(t,x) \cong u(t)(x - x_0). \tag{5.6}$$

For $u(t) = \mathrm{const} \to \infty$ this solution corresponds to the singular static soliton solution of Sen [50]. Inserting the ansatz (5.6) into (5.5) and working only to linear order in $(x - x_0)$ one obtains an ordinary differential equation for the slope

$$\ddot{u} = \frac{2}{b^2}u + 2\frac{u\dot{u}^2}{1+u^2} - 3H\dot{u}. \tag{5.7}$$

We have solved (5.7) numerically for both constant $H$ and $H \sim 1/(t+t_0)$. We find that generically the solutions of (5.7) become singular in a finite time $t = t_c$. Larger $H$ tends to delay the onset of the singularity and soften the singular behaviour. To understand this finite time slope divergence analytically it is simplest to consider $H = 0$. In this case if we assume that initially $\dot{u}$ is close to zero then the second term on the right hand side of (5.7) can be neglected and therefore

$$u(t) \cong u_+ e^{\sqrt{2}t/b} + u_- e^{-\sqrt{2}t/b}$$

at early times. Clearly $\dot{u}$ grows quickly and the second term on the right hand side of (5.7) very quickly becomes important. In the regime where $u$ and its derivatives become large the solution has the behaviour

$$u(t) \sim \frac{k}{t_c - t}$$

where the critical time $t_c$ depends on the initial conditions. We find that within finite time the slope becomes singular and the time dependent tachyon field near the core of the kink coincides with the singular soliton solution of Sen [50]. Hence we conclude that the codimension-one brane forms in a finite time.

This finite-time slope divergence was observed both numerically and analytically in [51] and leads to a finite-time divergence in the stress-energy tensor. This divergence

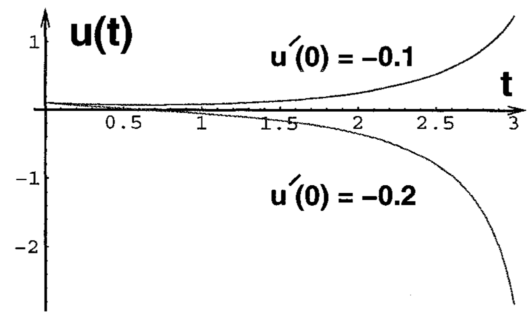


Figure 5.4: Behavior of the slope at the core of the kink for different initial conditions. The time axis is measured in units of $b$.

was also found in an exact string theory calculation in [48]. For the potential (5.3) the stress-energy tensor near $x = x_0$ and $t = t_c$, with $H = 0$, is [1]

$$T_{00} \cong \frac{\tau_p k}{t_c - t} \exp\left(-\frac{k^2(x-x_0)^2}{b^2(t_c-t)^2}\right).$$

Hence, with the normalization $b = 2\sqrt{\pi\alpha'}$ proposed in [50], as $t \rightarrow t_c$ we have $T_{00} \rightarrow \tau_{p-1}\delta(x - x_0)$. Similarly one can show that in this regime $T_{01} = T_{11} = 0$ and that $T_{ij} = -\tau_{p-1}\delta_{ij}\delta(x - x_0)$. In the limit of condensation, then, the stress-energy near $x = x_0$ is identical to that of a D$(p-1)$-brane. If we take into account the rolling of the tachyon for $x \neq x_0$ then there will be an additional component to $T_{\mu\nu}$ corresponding to tachyon matter, as in [71].

### Solutions Near the True Vacuum

Away from the site of the kink the field will roll towards the true vacuum $T \rightarrow \pm\infty$ so that $V(T) \rightarrow 0$ at late times for $x \neq x_0$. Solutions in this regime were discussed in detail in chapter 3.

## 5.4 Tachyon Vortex Formation in Compact Spaces

In this section we generalize the results of section 5.3 to consider the formation of codimension-two branes from tachyon condensation on the brane-antibrane pair. This situation is of more direct interest to brane-antibrane inflation since inflation ends when a brane-antibrane pair annihilate. This situation is also more realistic since the stable branes in a given string theory are those whose dimension differs by a multiple of two.

### 5.4.1 Action and Equations of Motion

We wish to generalize the results of the previous section to consider complex tachyon fields which depend on time and two spatial coordinates which we compactify on a square torus. We generalize the action (5.2) to

$$S = -2\tau_p \int e^{-|T|^2/b^2} \sqrt{1 + \partial_\mu T \partial^\mu \bar{T}} \sqrt{-g}\, d^{3+2+1}x \tag{5.8}$$

with metric

$$g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + dx^2 + dy^2 + a(t)^2 \delta_{ij} dx^i dx^j$$

where $x^1 = x$ and $x^2 = y$ are Cartesian coordinates on the torus and $x^i$ denotes the three noncompact dimensions, as in section 5.3. We consider tachyon fields which depend on $x^0 = t$, $x^1 = x$ and $x^3 = y$. If we separate the tachyon field into into real and imaginary components as

$$T(t, x, y) = T_1(t, x, y) + i\, T_2(t, x, y)$$

then (5.8) may be rewritten in terms of components as

$$S = -2\tau_p \int \exp\left(-T_J T_J / b^2\right) \sqrt{1 + \partial_\mu T_I \partial^\mu T_I} \sqrt{-g}\, d^{3+2+1}x$$

where the upper-case Roman indices label the real and imaginary components of the tachyon field ($I = 1, 2$). The equation of motion is (no sum over $I$, sum over $J$ and $K$):

$$\begin{aligned} &V(|T|)\, \partial_\alpha \left[ \frac{\sqrt{-g} g^{\alpha\beta} \partial_\beta T_I}{\sqrt{1 + \partial_\alpha T_J \partial^\alpha T_J}} \right] + \frac{\partial V(|T|)}{\partial T_K} \left[ \frac{\sqrt{-g} \partial_\alpha T_K \partial^\alpha T_I}{\sqrt{1 + \partial_\alpha T_J \partial^\alpha T_J}} \right] \\ &- \frac{\partial V(|T|)}{\partial T_I} \sqrt{-g} \sqrt{1 + \partial_\alpha T_J \partial^\alpha T_J} = 0. \end{aligned} \tag{5.9}$$

We note that the action (5.8) has not been derived from first principles and has several drawbacks from a theoretical perspective. For other proposals of the effective field theory on the brane-antibrane pair see [50],[139]-[141]. One theoretical difficulty is that the action (5.8) does not evade Derrick's theorem [67]. This is of no practical concern to us since we study time dependent solutions and since our interest is in defect formation in a compact space where charge conservation precludes the possibility of isolated defect solutions. More seriously, for the action (5.8) a static profile of the form $T = u(x + iy)$ with $u \to \infty$ does not correctly reproduce the stress tensor for a codimension-two brane. We will attempt to address this difficulty by considering an alternative description of the complex tachyon dynamics in a subsequent subsection.

### 5.4.2 Lattice Simulations of Vortex Formation

We solve the system of two coupled partial differential equations (5.9). As in subsection 5.3.2 we find that the gradient of tachyon field becomes singular near the core of the defect in a finite time, forcing us to halt our lattice evolution. As in the case of the kink we choose as initial data $\dot{T}(t = 0, x, y) = \dot{T}_i(x, y) = 0$ and $T(t = 0, x, y) = T_i(x, y)$ given by a truncated Fourier series with random Gaussian coefficients with the overall amplitude small compared to $b$. We take $H = 0$ for our examples, since the Hubble damping plays little qualitative role in the dynamics. In figures 5.5, 5.6, 5.7 and 5.8 we plot $-|T| = -\sqrt{T_1^2 + T_2^2}$ against $\{x, y\}$ with the $T$ axis measured in units of $b$. Figure 5.5 shows typical initial conditions used for our numerical analysis. In figures 5.6, 5.7 and 5.8 we plot the final configurations close to $t = t_c$ when the gradients become infinite for various radii of compactification. (Note that because we are plotting $-|T|$ rather than $+|T|$ the vortices appear as spikes in the final configuration.) Again we find that vortices do form for radii of compactification as small as a few times $M_T^{-1}$, even though the field is initially in causal contact throughout the compact space.

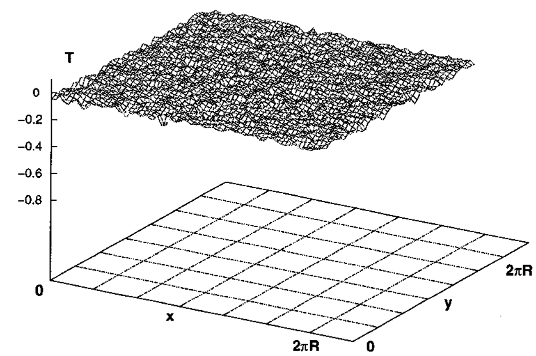


Figure 5.5: Typical initial configuration for numerical studies of vortex formation.

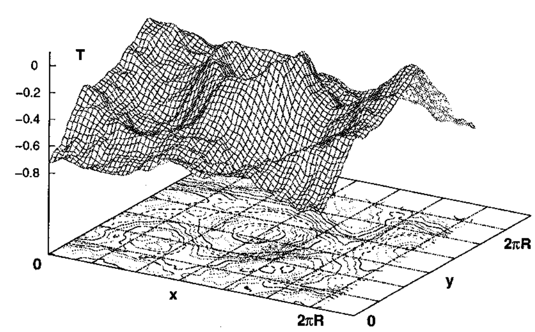


Figure 5.6: The final configuration of the complex tachyon field for $R = 7M_T^{-1}$.

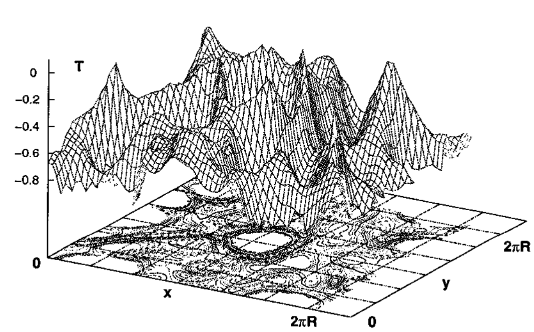


Figure 5.7: The final configuration of the complex tachyon field for $R = 15M_T^{-1}$.

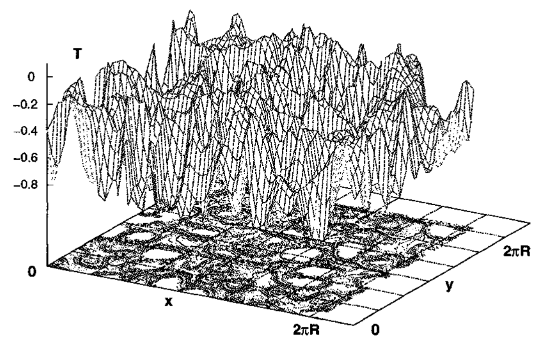


Figure 5.8: The final configuration of the complex tachyon field for $R = 25M_T^{-1}$.

### 5.4.3 Analytical Study of Vortex Formation

As in subsection 5.3.3 we expect that in the case of the vortex there will exist fixed points where the field stays pinned at $T = 0$ throughout the evolution. To see this we consider the equation of motion for $T_1$ at $t = 0$ for initial data such that $\dot{T}(t = 0, x, y) = 0$. To simplify the expression we write the equation evaluated at a point $(x_0, y_0)$ such that $T(t = 0, x_0, y_0) = 0$:

$$\left[-\ddot{T}_1 + \partial_x^2 T_1 + \partial_y^2 T_1\right]\left[1 + (\partial_x T_1)^2 + (\partial_y T_1)^2 + (\partial_x T_2)^2 + (\partial_y T_2)^2\right]$$
$$-\Big[(\partial_x T_1)^2\,\partial_x^2 T_1 + (\partial_y T_1)^2\,\partial_y^2 T_1 + 2\,\partial_x T_1\,\partial_y T_1\,\partial_x\partial_y T_1$$
$$\partial_x T_1\,\partial_x T_2\,\partial_x^2 T_2 + \partial_y T_1\,\partial_y T_2\,\partial_y^2 T_2 + (\partial_x T_1\,\partial_y T_2 + \partial_y T_1\,\partial_x T_2)\,\partial_x\partial_y T_2\Big] = 0.$$

The equation for $T_2$ is identical with $1 \leftrightarrow 2$. It is clear, then, that any point $(x_0, y_0)$ where $T_I(0, x_0, y_0) = \partial_x^2 T_I|_{(0,x_0,y_0)} = \partial_y^2 T_I|_{(0,x_0,y_0)} = \partial_x\partial_y T_I|_{(0,x_0,y_0)} = 0$ (for $I = 1, 2$) will be a fixed point of the evolution where $\ddot{T}_I(t, x_0, y_0) = \dot{T}_I(t, x_0, y_0) = 0$ for all $t$ and hence the field $T$ stays pinned at zero throughout the evolution. In the neighborhood of the point $(x_0, y_0)$ we thus should be able to write the field in the form $T \cong u(t)(x - x_0) + v(t)(y - y_0)$ with $u$ and $v$ complex. Therefore to study analytically the dynamics of the field near the core of the vortex we make an ansatz of the type:

$$T_1\,(t, x, y) = u\,(t)\,(x - x_0), \qquad T_2\,(t, x, y) = u\,(t)\,(y - y_0). \tag{5.10}$$

We have chosen $v(t) = i\,u(t)$ and $u(t)$ real since the vortex solution is expected to take the form

$$T\,(t,z,\overline{z}) = u\,(t)\prod_i\,(z-z_i)\prod_j\,(\overline{z}-\overline{z}_i) \tag{5.11}$$

where we have defined complex coordinates $z = x+iy$, $\overline{z} = x-iy$. The profile (5.11), with $u(t) = \text{const} \to \infty$ was used for the multi-vortex solutions of [139] where it was shown that each holomorphic zero of (5.11) corresponds to a brane and each anti-holomorphic zero of (5.11) corresponds to an antibrane. We insert now the ansatz (5.10) into the equations of motion (5.9) which corresponds to studying the dynamics close to the core of a single vortex located at $z_0 = x_0 + iy_0$. In this regime the equations of motion for the real and imaginary parts of the field, $T_1$ and $T_2$, give the same equation for the slope near $z = z_0$:

$$\ddot{u} = \frac{2}{b^2}\,u\,(1+u^2) + \frac{3\,u\,\dot{u}^2}{1+2u^2} \tag{5.12}$$

where we have taken $H = 0$ for simplicity. We have verified numerically that this equation yields solutions which diverge in a finite time for generic initial data. In the regime where $u(t)$ and its derivatives are large the dominant contribution to (5.12) is

$$\ddot{u} \cong \frac{2u^3}{b^2} + \frac{3}{2}\frac{\dot{u}^2}{u} \tag{5.13}$$

which has the solution

$$u(t) = \frac{b}{2(t_c - t)}. \tag{5.14}$$

where the critical time when the slope diverges, $t_c$, is fixed by the initial data. This singular behavior corresponds to the finite-time formation of a codimension-two brane in the annihilation of a brane-antibrane pair.

### 5.4.4 Alternative Complex Tachyon Action

The action (5.8) used above has the advantage of making the analysis tractable, and the resulting dynamics is analogous to kink formation. However, as discussed in subsection 5.4.1 this action has theoretical drawbacks. Here we consider an alternative description of the complex tachyon dynamics.

The tachyon action has been calculation in boundary string field theory (BSFT) in [140] by assuming a linear tachyon profile. For a linear profile, gauge and space-time transformations allow one to write $T = u_1 x + i\, u_2 y$, and the action one derives is

$$S = -2\tau_p \int d^{p+1}x \, \exp\left[-2\pi\alpha' \left((u_1 x)^2 + (u_2 y)^2\right)\right] F(4\pi\alpha'^2 u_1^2)\, F(4\pi\alpha'^2 u_2^2) \qquad (5.15)$$

where the function $F(z)$ is given by

$$F(z) = \frac{4^z z \Gamma(z)^2}{2\Gamma(2z)}.$$

To make our analysis tractable we generalize the action (5.15) to nonlinear profiles using two simplifications. The first is to consider a generalization which reduces to (5.15) for linear tachyon profiles only when $u_1 = u_2 = u$. We feel this is a reasonable simplification since for the profile $T = u_1 x + i\, u_2 y$ to minimize the action one requires both $u_1$ and $u_2$ to be infinite.

Our next simplification is to replace the complicated function $F(z)$ by $\sqrt{1+\pi z}$. As justification we note that these two functions have identical behavior at large $z$ since $F(z) \to \sqrt{\pi z}$ for $z \to \infty$.

We consider, then, the action

$$S = -2\tau_p \int \sqrt{-g}\, d^{p+1}x \exp\left(-|T|^2/\, b^2\right) \left(1 + \partial_\mu T \partial^\mu \bar{T}\right) \qquad (5.16)$$

where we have performed a rescaling of $T$ and, for consistency with (5.15), $b = \sqrt{\pi\alpha'}$. This action has also been studied in connection with tachyon condensation in [64]. Writing the tachyon field in real and imaginary components as $T(t,x,y) = T_1(t,x,y) + i\, T_2(t,x,y)$ the equation of motion one derives from (5.16) is

$$g^{\mu\nu}\nabla_\mu\nabla_\nu T_I + \frac{1}{b^2}\left[T_I\left(1 + g^{\mu\nu}\nabla_\mu T_K \nabla_\nu T_K\right) - 2T_K g^{\mu\nu}\nabla_\mu T_K \nabla_\nu T_I\right] = 0. \qquad (5.17)$$

We have solved the system of two coupled partial differential equations (5.17) on a lattice, as in subsection 5.4.2. The results are similar to those following from the action (5.8), though in this case the nonlinear effects are less dramatic. One still has defect formation, though on a longer time scale and defects begin to form for somewhat larger radius of compactification ($R \sim 30 M_T^{-1}$ is sufficient). The qualitative result that cosmic strings can form even for compactification scales well below the Hubble scale is unchanged. We consider now the dynamics near the core of the defect by plugging the ansatz

$$T_1(t,x,y) = u(t)(x-x_0), \quad T_2(t,x,y) = u(t)(y-y_0)$$

into (5.17) and working only to leading order in $(x-x_0)$, $(y-y_0)$. For $H=0$ the equation for the slope of the field at the core of the kink is

$$\ddot{u} - \frac{1}{b^2}u = 0 \tag{5.18}$$

leading to an exponentially increasing slope. We see that in this case the slope does not become singular in finite time, which can result in a different final density of defects.

## 5.5 Comparison with Ordinary Cosmic String Formation

If the potential for the tachyon has the usual runaway form, $\exp(-T^2/b^2)$, then once the field starts rolling, it continues rolling towards $T=\pm\infty$. The gradient force is insufficient to stop or reverse the rolling. In this section we remind the reader how this differs from the mechanism for formation of defects in conventional field theories, where their production is much more strongly suppressed. The conventional case corresponds to a theory with a global $U(1)$ symmetry, the potential $\frac{\lambda}{4}\left(|\phi|^2-\sigma^2\right)^2$ and a standard kinetic term.

In the case of $\phi^4$ theory, the evolution is well-behaved and one can follow the formation and the annihilation of kinks and anti-kinks indefinitely into the future. Comparing the two potentials $\exp(-T^2/b^2)$ and $\frac{\lambda}{4}\left(|\phi|^2-\sigma^2\right)^2$ (see figure 5.9), we see that in the $\phi^4$ theory one expects oscillations of the field which can restore symmetry and wipe out the defects.

In that case the final density of defects formed depends strongly on how fast the oscillations are damped, either through Hubble expansion, or through the coupling of

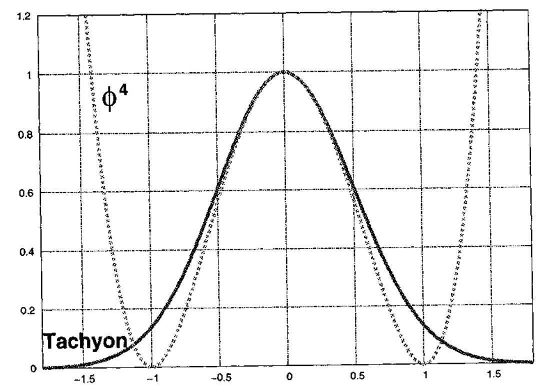


Figure 5.9: Comparison of the tachyon and $\phi^4$ potentials. In the case of the $\phi^4$ theory, the finiteness of the slope of the kink, as well as large oscillations of the field, strongly suppress the formation of defects.

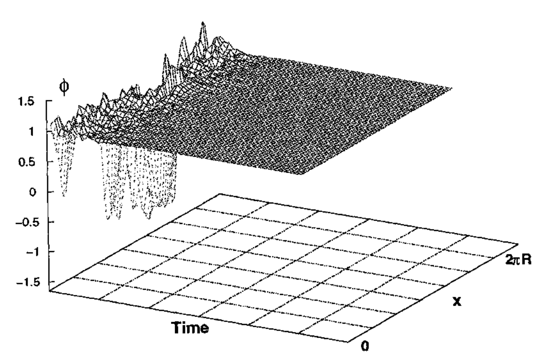


Figure 5.10: The effect of damping on the formation of vortices in the $\phi^4$ theory. Small damping results in a large number of oscillations of the field, and effective homogenization.

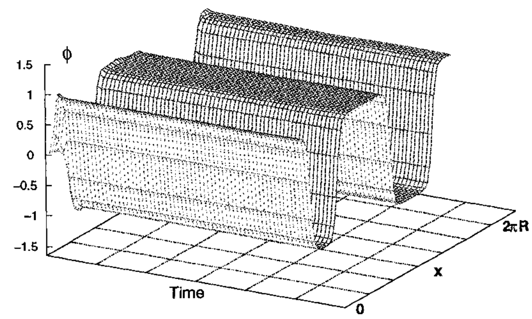


Figure 5.11: The effect of damping on the formation of vortices in the $\phi^4$ theory. An unphysically large damping is used in order to show that the density of defects which survive is larger.

the field $\phi$ to other fields. In figures 5.10 and 5.11 we show the effect of the damping in the $\phi^4$ theory for radius of compactification $R = 60M_s^{-1}$. A large value of $H$ results not only in a higher density of defects, but also slows down the motion of the defects that have formed, effectively reducing the rate at which they annihilate. This is in contrast to the tachyon field theory, in which case the Hubble damping plays essentially no role in determining the final density of defects.

## 5.6 Cosmological Consequences

### 5.6.1 String defects

Our studies of defect formation in compact space can easily be extrapolated to imply a density of roughly one defect per string volume in the non-compact directions aswell. We now turn to the cosmological implications of having a very large initial density of strings. In normal cosmic string networks, the details of the initial conditions are not important because the network quickly reaches a scaling solution. This can be demonstrated quite simply, using the "one-scale" model for the energy density $\rho$ in long strings whose characteristic length is $L$ [142]:

$$\dot{\rho} \cong -2H\rho - f(P)\frac{\rho}{L}. \tag{5.19}$$

The terms on the r.h.s. represent respectively the effects of dilution through expansion and the loss due to breaking off small loops, which eventually disappear by shrinking and emitting gravity waves. $f(P)$ is a function of the intercommutation probability, which is believed to go like $f \sim \sqrt{P}$ [143, 144]. Taking $\rho = \mu/L^2$ (where $\mu$ is the string tension) and $H = \beta/t$, one can verify that this has a stable attractor solution

$$L = \gamma(t)\, t \equiv \frac{f(P)\, t}{2(1-\beta)} \tag{5.20}$$

known as the scaling solution, since the string length becomes a constant fraction of the horizon size $H^{-1}$, and the energy density in long strings also tracks that of the dominant component,

$$\rho = \frac{\mu}{\gamma^2 t^2}. \tag{5.21}$$

If the initial energy density was much greater than the scaling value, we can find the time scale for reaching scaling by solving (5.19) in the approximation that the Hubble expansion term is negligible compared to the loop-emission term, giving the solution $\gamma(t) = \gamma_0 + \frac{1}{2} f(P)\, \ln(t/t_0)$. Inverting, we find that the time required to reach a value $\gamma = f/(2-2\beta)$, starting from high densities where $\gamma_0 \ll 1$, is

$$t \cong t_0\, e^{1/(1-\beta)} \tag{5.22}$$

which is not much greater than $t_0$. For a radiation dominated universe, with $\beta = 1/2$, the scaling solution is reached in $e^2/2$ Hubble times in the usual case, and subsequent evolution is quite insensitive to the initial conditions. This conclusion is unchanged even for very small intercommutation probabilities.

We have investigated the approach to the scaling solution using a more detailed model of network evolution, which takes into account the possibility that loops may reconnect to long strings when the initial density is very high, and thereby possibly delay the onset of scaling. Suppose that the density of small loops with characteristic size $l$ is $1/x^3$, defining the average distance between loops at a given time. One can estimate that the rate per unit volume for loops to recombine with long strings is

$$\frac{dn_s}{dt} = \tilde{f}\, x^{-3}\, L^{-3}\, l\, L^2\, \frac{v_{\rm rel}}{\min(x, L)}. \tag{5.23}$$

Here $\tilde{f}$ is the probability of a reconnection, $x^{-3}$ and $L^{-3}$ are the number densities of loops and long strings, respectively, $v_{\rm rel}$ is the relative velocity between loops and strings, which we take to be $O(1)$, and $\min(x, L)$ is the distance a loop typically travels before reaching a string. The probability of a collision must be proportional to $l$, not $l^2$, since the size of the loop in the direction parallel to the long string does not affect the cross section.

In this model, long strings and small loops are treated as two separate components, $\rho_s = \mu/L^2$ and $\rho_l = \mu l/x^3$, whose energy densities are governed by

$$\begin{aligned} \dot{\rho}_s &= -2\,H\rho_s - f\frac{\rho}{L} + \mu\, l\,\frac{dn_r}{dt}, \\ \dot{\rho}_l &= -3\,H\rho_l + f\frac{\rho}{L} - \mu\, l\,\frac{dn_r}{dt} - \Gamma G\mu^2\frac{1}{x^3}. \end{aligned} \tag{5.24}$$

The last term represents the power emitted by loops due to gravitational radiation, $\Gamma G\mu^2$ [146], where $\Gamma \cong 50$ and $10^{-11} \lesssim G\mu \lesssim 10^{-6}$ [56, 58]. The loop size is taken to always be a fixed fraction of the long string correlation length: $l/L = \Gamma G\mu$, so long as this is not smaller than the fundamental string length scale $l_s$. We have integrated these equations numerically, together with the Friedmann equation and the evolution equations for energy density in visible and gravitational radiation, keeping the short distance cutoff $l_s$ on the size of the loops (in fact the results do not change noticeably if we assume the loops remain as small as this cutoff). This more detailed study confirms that the scaling solution is attained in only a few Hubble times, as can be seen from the time evolution of the fractions of the critical density for each component, shown in figure 5.12. We note, however, that in the above analysis we assumed that string velocities remain of order unity; if there is significant freezing out of the relative string motions in the large in the large dimensions this could have significant impact on the approach to scaling [142]. We note also that the effect of friction due to particle scattering may also significantly alter this picture [145].[4]

The initial density of strings is many orders of magnitude greater than the Kibble estimate, which gave the initial correlation length $L$ as $\sim H^{-1}$ at the moment of formation of the network; instead, the initial energy density of the network is comparable

[4]We thank J.J. Blanco-Pillado and Carlos Martins for bringing this to our attention.

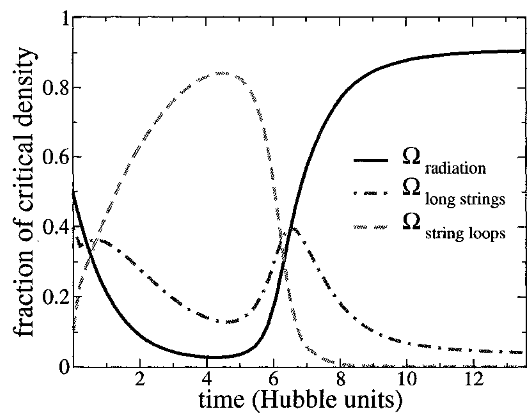


Figure 5.12: Fraction of critical density versus time for long strings, loops, and visible radiation starting from initial values $\Omega_s = 0.4$, $\Omega_l = 0.1$ and $\Omega_{rad} = 0.5$. Not shown is the fraction of energy density in gravity waves produced by decay of the loops.

to the total energy density available, $\sim \mu^2$, so that $L \sim \mu^{-1/2}$, smaller by a factor of $\mu^{-1/2}/M_p$ than the Kibble value. Since $\rho = \mu/L^2$, the initial density is greater by a factor of $M_p^2/\mu$ than the Kibble value. However, this huge excess is so shortlived that it has no observable effects. For instance, the contribution from the early nonscaling regime to the gravity wave background is negligible due to the small size of the loops which are formed and subsequently radiate during this era. Following ref. [146], one can estimate the amplitude of these gravity waves at frequency $\omega$ as being of order

$$h \sim \frac{(G\mu)^{7/6}\rho_0^{7/12}}{M_p^2\omega^{1/3}} \sim 10^{-66} \tag{5.25}$$

where $\rho_0$ is the present energy density of the universe. The estimate (5.25) is some 40 orders of magnitude below the sensitivity of LIGO at the frequencies of interest, $\omega \sim 100$ Hz. As for the cosmic microwave background, the wavelength of density perturbations created during the nonscaling regime of the network is too short to be relevant: initially $\lambda \sim \mu^{-1/2}$, which gets stretched to the scale of the present energy density $\lambda \sim \rho_0^{-1/4} \sim 0.1$ mm. This length scale is also too small to be relevant for the formation of primordial black holes (PBH's) since the mass contained in volume $\lambda^3$ is far below that needed for cosmologically long-lived PBH's.

### 5.6.2 Domain Walls

Thus the fast approach to the scaling solution for three-dimensional string networks erases all sensitivity to the initial conditions, even though the initial density is orders of magnitude greater than for conventional cosmic strings, and this is true regardless of the reduced intercommutation probability. However, there are situations where our modified understanding of the network's initial conditions may make a dramatic difference: namely, in the case of higher dimensional defects. Let us illustrate with the example of D5-$\overline{\text{D5}}$ annihilation, where two of the dimensions are wrapped on an internal manifold with coordinates $(y_1, y_2)$ and the remaining three dimensions span the euclidean space $(x^1, x^2, x^3)$. The codimension-two defects which form from the annihilation are D3 branes, and these can have various orientations with respect to the world-volume of the parent branes. The choices are exemplified by the three situations:

1. extended in $x_3, y_1, y_2$ directions, localized in $x_1, x_2$: looks like D1 in 3D.
2. extended in $x_2, x_3, y_2$ directions, localized in $x_1, y_1$: looks like D2 in 3D.
3. extended in $x_1, x_2, x_3$ directions, localized in $y_1, y_2$: looks like D3 in 3D.

The first case looks like ordinary cosmic strings to the 3D observer since the defects have only one long direction among the three large ones. Their effects have already been discussed. Case 3 is a network of 3-branes, all of whose dimensions are large. Their tension will contribute to the effective 3D cosmological constant.[5] Case 2, illustrated in fig. 5.13, is the interesting one because these appear as domain walls to the 3D observer, and their energy density redshifts too slowly: $\rho \sim 1/a^2$ in terms of the scale factor of the large dimensions. A single domain wall of tension $\tau_2 = \eta^3$ within our horizon would dominate the present energy density unless $\eta \lesssim 1$ MeV. The effective tension of a 3-brane wrapping one compact dimension of size $R$ is $\tau_2 = R\tau_3 \sim \mu^{3/2}$; hence the string scale would have to be $\lesssim 1$ MeV, absurdly small.

[5]These can be safely assumed to annihilate quickly since they are not separated in the expanding space $(x^1, x^2, x^3)$.

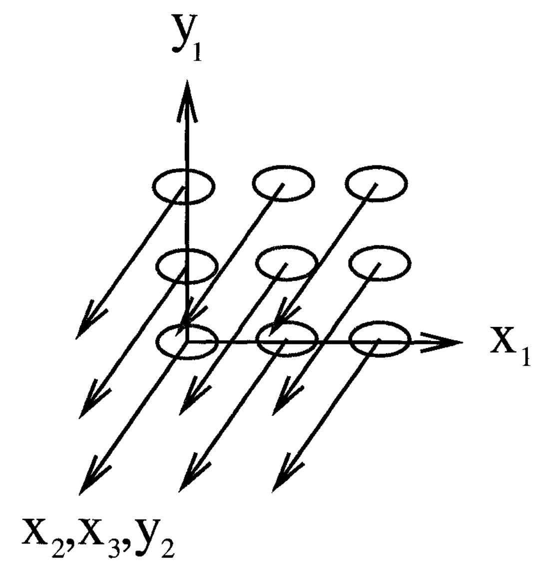


Figure 5.13: An array of codimension-two D3-branes from D5-anti-D5 annihilation, partially localized in the compact dimensions, which look like strings in the $x_1$-$y_1$ plane, and domain walls in the large $x_i$ dimensions.

Our conclusion differs from that of ref. [57], which speculated that no defects partially localized in compact directions can form. The argument was based on the Kibble mechanism, and thus assumed the correlation length could not be smaller than $H^{-1} \sim M_p/\mu$, whereas the size of the compact dimensions must be much smaller, of order $\mu^{-1/2}$. We have seen that in fact the initial correlation length is of the same order as the string scale, so that this argument is invalidated.

Still, one might be skeptical as to whether defects partially localized in the compact directions can survive until today, since intuitively they might be able to find each other and annihilate very quickly in the compact space. Ref. [147] attempts to address this question, and concludes that domain wall defects like those in case 2 *cannot* efficiently annihilate, since they are not completely localized in the compact (nonexpanding) space. The analysis of [147] assumes that the number density of D$p$-branes satisfies a rate equation which is nearly the same as that governing monopoles:

$$\dot{n} = -(3 - p_{||})Hn - \frac{\Gamma}{T^{D-2}}n^2 \tag{5.26}$$

where $p_{||}$ is the number of dimensions of the brane spanning the large dimensions, and $D$ is the total number of spacetime dimensions. This ignores the effect of self-intersections for reducing the density of long defects, which is known to be the dominant means for string networks to reach scaling (cf. eq. (5.19)). Further, it unrealistically assumes that the defects are parallel, so that they will annihilate rather than intercommute when they meet. It is therefore not immediately clear how far we can trust their conclusions [147].

On the other hand, numerical evolution of domain wall networks shows that self-intersections are not generic, and it is suggested that the dominant energy-loss mechanism is direct gravitational radiation rather than through collisions [148]. Furthermore it is observed that the approach to the scaling solution is slower for domain walls than for cosmic strings [149].

### 5.6.3 Full String Network Simulations

To attempt to address the issue of whether domain walls disappear or not, we have considered the dynamics of D3-branes in $(5+1)$ dimensions in the approximation of projecting out the dynamics in one compact direction, $y^2$, and one noncompact direction, $x^3$, to give an effective $3+1$-dimensional system. In this case the D3-branes appear as one-dimensional objects (see figure 5.13) and the dynamics can be modeled by considering string evolution in an anisotropic space with two large and one small dimension. In this setup those "strings"—string-like from the $(x^1, x^2, y^1)$ point of view—which span the large dimensions $x^1$ and $x^2$ appear as domain walls to the 3D observer while the "strings" which span the compact dimension $y^1$ appear as cosmic strings in 3D.

Following the setup of Smith and Vilenkin [150] we performed numerical simulations of the defect evolution in this approximation, keeping track of the extent to which defects preferentially spanned the compact direction $y^1$—thus appearing as strings in the $x^1, x^2, x^3$ subspace, relative to spanning the large directions, which appear as domain walls. Figure 5.14 shows the fractional energy density in wound strings wrapped about the three directions $x^1, x^2, y^1$ as a function of time. This particular run started with equal energy in each direction so that the correlation lengths, and hence the interaction rates, were roughly the same. We observe that the branes wrapping the large directions lose their winding energy more quickly than those wrapping the compact direction: this can be attributed to several factors, including the smaller cross-sectional area of the long strings, and the greater energy radiated away when two long strings annihilate. The implication is that defects which look like strings to the 3D observer tend to survive preferentially over those that appear as domain walls. However since this is a toy model for the actual higher-dimensional defects, such a conclusion awaits validation from actual domain wall simulations.

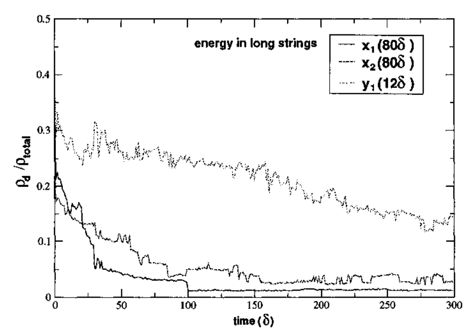


Figure 5.14: Fraction of wound string density about the three anisotropic directions (two large and one small) starting from equal energy density in strings wrapping each direction. Top curve is for strings winding in the small direction.

As further evidence that defect evolution after formation tends to favor the defects which are localized in the large dimensions (and hence tends to favor the survival of cosmic string-like defects) we consider a field theory simulation of domain wall evolution for a real scalar field with potential $\frac{\lambda}{4}\left(\phi^2-\sigma^2\right)^2$ and standard kinetic term [6] We studied the formation and evolution of domain walls in this theory in an anisotropic $(3+1)$-dimensional space with two large and one compact dimension.

[6]Though it is of more direct interest to consider the dynamics of the tachyon field theory with action (5.2) the finite time slope divergence prevents us from following the dynamics after the defects have formed.

Starting from a random initial profile close to the false vacuum (as in subsection 5.3.2) we find that the evolution of the defects after formation tends to annihilate domain walls which are localized in the compact direction and to favor the survival of domain walls which are localized in the large directions. Figure 5.15 shows a plot of the domain wall network late in the evolution after the domain walls localized in the compact direction have disappeared.

We also explored the effect of increasing the anisotropy, varying the initial distribution of winding modes, and varying the intercommutation probability; however, the system consistently evolved to favor winding about the compact direction. These results seem to corroborate the claim that domain wall-like defects are suppressed.

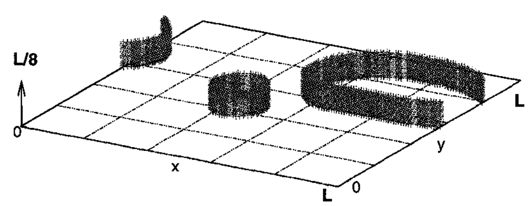


Figure 5.15: Plot of domain wall network in an anisotropic space with two large and one small dimensions with $L = 80M_s^{-1}$. This snapshot is taken late in the evolution and shows that domain walls which are localized in the large dimensions are preferred by the defect evolution.

We note that there are several factors which we have not taken into account which may radically alter the picture. For example, in figure 5.14 we have unrealistically neglected the dynamics in two dimensions to facilitate numerical studies. Furthermore, in both of the examples above we have neglected the expansion of the large dimensions—an effect which could play an important role in the dynamics. It is therefore unclear whether domain walls will pose a cosmological problem in models of brane-antibrane inflation where the branes driving inflation wrap the compact space. We feel that this is a problem which merits further investigation and it is likely that a complete resolution of this issue will require full simulations of 3-brane dynamics in an anisotropic $(5+1)$-dimensional spacetime with 3 expanding dimensions. While such simulations would be of interest, they are beyond the scope of the present work.

### 5.6.4 Monopoles

Finally consider an example of how monopole-like defects may be formed through a cascade of annihilations in D5-$\overline{\text{D5}}$ inflation. The initial state D5 and $\overline{\text{D5}}$ span the three large dimensions and wrap two compact dimensions. These may produce D3 and $\overline{\text{D3}}$ which wrap the two compact dimensions and are extended in one large dimension; hence these defects appear string-like to the 3-dimensional observer. A parellel D3-$\overline{\text{D3}}$ pair may then annihilate to produce D1 branes and antibranes which can span the same large dimension as the parent D3-$\overline{\text{D3}}$, or alternatively could wrap the compact dimensions. Those D1-branes which span the large dimension appear as cosmic string defects to the 3-dimensional observer while those which wrap the compact dimensions will appear as point-like (monopole) defects. If the compact dimensions admit nontrivial 1-cycles (like $T^2$ for example) then these monopoles will be stable. Our results indicate that in general both string-like and monopole-like defects should be produced in this cascade. To understand the subsequent evolution of such defects we consider numerically the dynamics of D1-branes in a 3D space with one large dimension and two small dimensions. Figure 5.16 shows the fractional energy density in wound strings wrapped about the three directions $x^1, y^1, y^2$ as a function of time. Strings wrapping the large direction $x^1$ appear as genuine cosmic strings to the 3D observer while strings wrapping small directions $y^1, y^2$ appear as monopole-like defects in 3D. As in subsection 5.6.2 we start with equal energy in each direction so that the correlation lengths are roughly equal. Again we find that strings wrapping the large direction lose their winding energy quicker than those wrapping the compact directions. Physically, this suggests that monopoles are preferentially produced over cosmic strings in this particular cascade of annihilations.

The question of whether such monopole-like defects will pose a cosmological problem depends crucially on the long-range forces between these defects and is, we feel, an issue which merits further investigation (see [151] for a solution of the monopole problem which is independent of inflation).

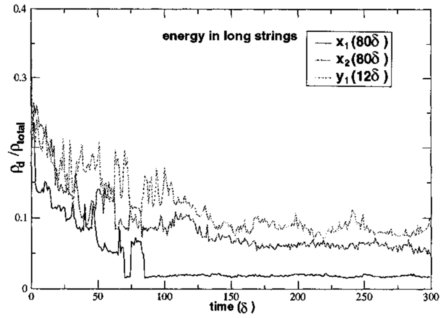


Figure 5.16: Fraction of wound string density about the three anisotropic directions (one large and two small) starting from equal energy density in strings wrapping each direction.

## 5.7 Summary and Conclusions

We have studied the formation of topological defects during the decay of a nonBPS brane or a coincident brane-antibrane pair. The problem was treated both analytically, by solving the equation of motion for the tachyon field at the core of the defect, as well as numerically, by evolving the tachyon field on a lattice. We showed that defects form with a correlation length proportional to the string length rather than the horizon. Defects localized within compact dimensions can form even if the compactification radius is as small as $7M_S^{-1}$ and the tachyon dynamics is insentitive the the Hubble damping. Depending on the exact form of the action (*e.g.*, Sen's version, or the boundary string field theory version) the slope of the field at the defect could either increase exponentially in time, or else diverge within a finite time, potentially changing the initial density of defects. We compared the evolution of the tachyon field to that of a scalar field in the $\phi^4$ theory and noted that the most efficient way to suppress the formation of defects is through symmetry restoration, caused by large oscillations of the scalar field. This is not possible if the potential has a runaway form, as for the string tachyon, which inevitably leads to the formation of a higher density of defects in the string case. Once the defects form the field theory description is no longer adequate, so in order to analyze the annihilation of the defects formed one has to use a description in terms of branes and antibranes interacting in a compact space.

As a result, the initial density of string defects is much greater than previous estimates. For strings which are genuine 1D objects, we showed that the string network nevertheless attains scaling behavior within just a few Hubble times, so that there are no observable consequences of the initial high string density. Of course this assumes that the network is not frustrated [152], so that scaling can indeed be achieved. Whether this is the case for cosmic superstrings which are bound states of fundamental and D-strings is an interesting open question [153].

On the other hand, we argue that in models where inflation is driven by branes which wrap the compact manifold (for example D5-$\overline{\text{D5}}$), domain wall-like and monopole-like defects are inevitably produced. The stability and subsequent evolution of such defects is complicated and may depend crucially on the details of the compactification, for example, on whether or not the compact manifold which the parent branes wrap admits nontrivial 1-cycles. We leave detailed studies of whether such models are phenomenologically viable to future investigations; however, we have shown that the formation of defects at the end of brane-antibrane inflation is much more complicated and model-dependent than one might have expected.

## Acknowledgments

We would like to thank Robert Brandenberger, J.J. Blanco-Pillado, Guy Moore and Henry Tye for discussions. This work was supported in part by NSERC and FQRNT.

# Chapter 6

# Nongaussianity from Tachyonic Preheating in Hybrid Inflation

# Abstract

We show that in hybrid inflation it is possible to generate large second-order perturbations in the cosmic microwave background due to the instability of the tachyonic field during preheating. We carefully calculate this effect from the tachyon contribution to the gauge-invariant curvature perturbation. We clarify some confusion in the literature concerning nonlocal terms in the tachyon curvature perturbation; we show explicitly that such terms are absent. We find that large nongaussianity occurs only when the tachyon remains light throughout inflation, whereas $n = 4$ contamination to the spectrum is the dominant effect when the tachyon is heavy. We compute quantitatively the amount of nongaussianity or spectral distortion generated during preheating and use this calculation to place constraints on the parameter space of hybrid inflation. We apply our results also to popular brane-antibrane, F-term and D-term models of inflation. In the case of warped-throat brane-antibrane inflation we find interesting constraints from nongaussianity. For F-term and D-term inflation models from supergravity, we obtain nontrivial constraints from the spectral distortion effect. Finally, we also establish that our analysis applies to complex tachyon fields.


## 6.1 Introduction

In the simplest models of inflation, the primordial density perturbations have a negligible degree of nongaussianity. The parameter $f_{NL}$ which characterizes nongaussianity is of the order of $|n-1| \ll 1$ (where $n$ is the spectral index) in conventional inflation models [156]-[158], [18] whereas the current experimental limit is $|f_{NL}| \lesssim 100$ [16]; one can additionally characterize the nongaussianity using the trispectrum, which is also small in conventional models [159]. Nevertheless, there have been intense theoretical efforts to find models which predict observably large levels [160] (see chapter 1 for a review). These efforts are motivated, in part, by the fact that $f_{NL}$ can potentially be used as a powerful tool to discriminate between various theoretical realizations of inflation.

It has been difficult to find examples which can give large $f_{NL}$. In single field inflation models a small inflaton sound speed is necessary to achieve large nongaussianity [161], as in the models of [162, 163] (unless the inflaton potential has a sharp feature [164]). It is also possible to generate significant nongaussianity using the curvaton mechanism [165] for the generation of cosmological perturbations. Indeed, the curvaton scenario may be ruled out by future non-observation of nongaussianity [166] (see, however, [167]). The simplest multi-field models do not seem to give large nongaussianity [168], though it is not clear if this is true also of more complicated models.

We will show that one of the most prevalent classes of models, hybrid inflation [31, 169], is able to yield large nongaussianity for certain ranges of parameters (see [170] for constraints on the parameter space of hybrid inflation coming from the WMAP data). The effect is due to the growth of the waterfall field—tachyonic preheating—which contributes to the curvature perturbation, and hence the temperature anisotropy, only starting at second order in cosmological perturbation theory; see also [171]-[177].

The scenario is summarized as follows. During hybrid inflation the inflaton field $\varphi$ is displaced from its preferred value, and inflation is driven by the false vacuum energy of the "waterfall" or tachyon field $\sigma$ which is trapped in its false vacuum by interactions with the inflaton. When the inflaton reaches some critical value the tachyon effective mass squared becomes negative and its fluctuations grow exponentially due to the spinodal instability. The exponential growth continues until the fluctuations of the tachyon start to oscillate about the true vaccuum. At this stage the tachyon fluctuations become nonperturbative and their back-reaction brings inflation to an end. Here we study the evolution of the second-order curvature perturbation due to the tachyonic instability and find that it can lead to a large level of nongaussianity, if the tachyon was lighter than the Hubble scale $m_\sigma < 3H/2$, during a sufficient part of the inflationary epoch. If $m_\sigma \gg H$, the fluctuations of $\sigma$ are exponentially suppressed during inflation (see section 1.1.1), however, in this case it is still possible for preheating to impact the Cosmic Microwave Background (CMB) in a nontrivial way (if the exponential growth of the tachyonic fluctutations during preheating is so large that it overcomes the exponential damping during inflation). In the case of a heavy tachyon the preheating phase generates a nonscale-invariant ($n = 4$) contribution to the power spectrum. For certain parameter values this nonscale-invariant contribution can be made so large that it dominates over the scale invariant contribution coming from the inflaton.

The possibility that preheating may impact the large scale curvature perturbation has also been considered in [178]-[181] (though these studies were restricted to first order in perturbation theory). In particular, [179, 181] showed that, contrary to previous claims, there is no violation of causality necessary for preheating to amplify the large scale curvature perturbation. This is so because inflation has already set up correlations on large scales, the preheating dynamic simply amplifies those existing large scale fluctuations.

We apply our results for nongaussianity and spectral distortion to popular models of hybrid inflation including brane inflation [24] and P-term inflation [182], which is a synthesis of supergravity inflationary models interpolating between F-term and D-term inflation. In the case of brane inflation we find that large nongaussianity is possible, while for P-term inflation spectral distortion is the dominant effect.

Finally we demonstrate that our analysis applies not only to real tachyon fields but also to the case of complex tachyon fields. The latter situation is more realistic since in that case cosmic strings, rather than domain walls, are formed at the end of inflation.

We begin in section 6.2 by defining the perturbations up to second order in the metric and matter fields. In section 6.3 the hybrid inflations model is reviewed, starting with the dynamics of the fields at zeroth order, and then their first order perturbations, with emphasis on the dynamics of the tachyon field fluctuations toward the end of inflation. In section 6.4 we solve for the second order curvature perturbation which is induced by the tachyonic fluctuations. This is used in section 6.5 to compute the bispectrum (three-point function) and the nonlinearity parameter $f_{NL}$ as well as contributions to the spectrum itself. In section 6.6 we incorporate experimental constraints on nongaussianity, as well as on the spectrum, to derive excluded regions in the parameter space of the hybrid inflation model. We then adapt these constraints to the cases of brane-antibrane inflation in section 6.7 and P-term inflation in section 6.8. We further extend our analysis to the more realistic case of a complex tachyon field in section 6.9, showing that the extra components of the tachyon add in a simple way and modify the real-field results by factors of order unity. Conclusions are given in section 6.10. Appendix D-1 gives details about the matching between early- and late-time WKB solutions of the tachyon fluctuation mode functions. Appendix D-2 gives details about the second order perturbed Einstein equations. Appendix D-3 deals with the construction of the inflaton curvature perturbation. Appendix D-4 discusses the Fourier transforms of convolutions. Appendix D-5 provides technical details concerning the constructions of the tachyon curvature perturbation. Finally, appendix D-6 gives details about the source term of the curvature perturbation for complex tachyons.

## 6.2 Metric and Matter Perturbations

In this section we write down the perturbations of the metric and matter fields about a spatially flat Robertson-Walker background following [157]. Greek indices run over the full spacetime $\mu, \nu = 0, 1, 2, 3$ while latin indices run only over the spatial directions $i, j = 1, 2, 3$. We will frequently employ conformal time $\tau$, related to cosmic time $t$ by $dt = ad\tau$. Differentiation with respect to conformal time will be denoted by $f' = \partial_\tau f$ and with respect to cosmic time by $\dot{f} = \partial_t f$.

The metric is expanded up to second order in fluctuations as

$$g_{00} = -a(\tau)^2 \left[1 + 2\phi^{(1)} + \phi^{(2)}\right] \tag{6.1}$$

$$g_{0i} = a(\tau)^2 \left[\partial_i \omega^{(1)} + \frac{1}{2}\partial_i \omega^{(2)} + \omega_i^{(2)}\right] \tag{6.2}$$

$$\begin{aligned} g_{ij} = \; & a(\tau)^2 \left[(1 - 2\psi^{(1)} - \psi^{(2)})\delta_{ij} + D_{ij}(\chi^{(1)} + \frac{1}{2}\chi^{(2)}) \right. \\ & \left. + \frac{1}{2}(\partial_i \chi_j^{(2)} + \partial_j \chi_i^{(2)} + \chi_{ij}^{(2)})\right] \end{aligned} \tag{6.3}$$

where $D_{ij} = \partial_i\partial_j - \frac{1}{3}\delta_{ij}\partial^k\partial_k$ is a trace-free operator. The fluctuations are decomposed such that the vector perturbations are transverse $\partial^i\omega_i^{(2)} = \partial^i\chi_i^{(2)} = 0$ while the tensor perturbations are transverse, traceless and symmetric: $\partial^i\chi_{ij}^{(2)} = 0$, $\chi_i^{i(2)} = 0$, $\chi_{ij}^{(2)} = \chi_{ji}^{(2)}$. In the above we have neglected the vector and tensor perturbations at first order, which are small, since vector perturbations decay with time, while tensors are suppressed by the slow roll parameter $\epsilon$. The same is not true at second order, however, since the second order tensors and vectors are sourced by the first order scalar perturbations. We adopt the generalized longitudinal gauge defined by $\omega^{(1)} = \omega^{(2)} = \omega_i^{(2)} = 0$ and $\chi^{(1)} = \chi^{(2)} = 0$. The metric in this gauge becomes

$$g_{00} = -a(\tau)^2\left[1 + 2\phi^{(1)} + \phi^{(2)}\right] \tag{6.4}$$

$$g_{0i} = 0 \tag{6.5}$$

$$\begin{aligned} g_{ij} &= a(\tau)^2\Big[\left(1 - 2\psi^{(1)} - \psi^{(2)}\right)\delta_{ij} \\ &+ \frac{1}{2}\left(\partial_i\chi_j^{(2)} + \partial_j\chi_i^{(2)} + \chi_{ij}^{(2)}\right)\Big]. \end{aligned} \tag{6.6}$$

In hybrid inflation the matter content consists of two scalar fields which are expanded in perturbation theory as

$$\varphi(\tau, \vec{x}) = \varphi_0(\tau) + \delta^{(1)}\varphi(\tau, \vec{x}) + \frac{1}{2}\delta^{(2)}\varphi(\tau, \vec{x}) \tag{6.7}$$

$$\sigma(\tau, \vec{x}) = \sigma_0(\tau) + \delta^{(1)}\sigma(\tau, \vec{x}) + \frac{1}{2}\delta^{(2)}\sigma(\tau, \vec{x}). \tag{6.8}$$

where $\varphi$ is the inflaton and $\sigma$ the tachyon (or "waterfall" field). In hybrid inflation the time-dependent vacuum expectation value (VEV) of the tachyon field is set to zero $\sigma_0(\tau) = 0$, about which we will say more later.

The perturbations are defined so that $\langle\delta^{(i)}\varphi\rangle = 0$, hence $\langle\varphi(\tau, \vec{x})\rangle = \varphi_0(\tau)$. At first order in perturbation theory this is automatic since $\delta^{(1)}\varphi$ contains only one annihilation/creation operator. However, at higher order in perturbation theory the homogeneous $k = 0$ mode of the fluctuation must be subtracted by hand in order to ensure that all of the zero mode of the field is described by the nonperturbative background.

The Einstein tensor and stress-energy tensor expanded up to second order in perturbation theory can be found in [183]. We do not reproduce these results here, but we have carefully checked all the results from [183] which are relevant for our analysis.

## 6.3 Hybrid Inflation

We consider hybrid inflation in which both the inflaton and the tachyon are real fields with the potential

$$V(\varphi,\sigma)=\frac{\lambda}{4}\left(\sigma^2-v^2\right)^2+\frac{m_\varphi^2}{2}\varphi^2+\frac{g^2}{2}\varphi^2\sigma^2. \tag{6.9}$$

This potential will give rise to topological defects at the end of inflation—domain walls in the $\sigma$ field—which could produce large nongaussianities apart from the ones which we consider. To avoid overclosure of the universe by domain walls we must either render the domain walls unstable (for example through the addition of a small term like $\mu\sigma^3$ to the potential) or else consider the generalization to complex tachyon field. In the case of a complex tachyon the symmetry breaking process will lead to the formation of cosmic strings, which are much more phenomenologically viable than domain walls. We will return to this possibility in a subsequent section, showing that our analysis generalizes to complex fields in a straightforward manner.

### 6.3.1 Background Dynamics

At the homogeneous level, the usual Friedmann and Klein-Gordon equations for the scale factor and the matter fields are

$$3H^2 = \frac{\kappa^2}{2}\left(\dot\varphi_0^2+\dot\sigma_0^2\right)+\kappa^2 V, \tag{6.10}$$

$$0 = \ddot\varphi_0+3H\dot\varphi_0+\frac{\partial V}{\partial\varphi} \tag{6.11}$$

where $\kappa^2 = M_p^{-2} = 8\pi G_N$. Here and elsewhere the potential and its derivatives are understood to be evaluated on the background values of the fields so that $V = V(\varphi_0,\sigma_0)$ and $\partial V/\partial\varphi = \partial V/\partial\varphi|_{\{\varphi=\varphi_0,\sigma=\sigma_0\}}$, for example. For the potential (6.9) we have $\partial V/\partial\sigma = \partial^2 V/\partial\sigma\partial\varphi = 0$, provided that $\sigma_0 = 0$. We will apply this simplification to all subsequent results.

To see why one should set $\sigma_0(\tau) = 0$, notice that initially the tachyon effective mass is $m_\sigma^2 = g^2\varphi_0^2 - \lambda v^2 > 0$. That is, the tachyon mass squared starts out being positive during inflation. Provided that there was a long enough prior period of inflation, any initial departure of $\sigma$ from zero would be exponentially damped. At some point, $m_\sigma^2$ becomes negative, and the tachyonic instability begins. However, it is still true that $\sigma_0(\tau)$ remains zero even then, since the universe will consist of equal numbers of domains with $\sigma > 0$ and $\sigma < 0$. On average, these give zero, which is the definition of the zeroth order field $\sigma_0$. The departures of $\sigma$ from zero between domain walls which form are taken account in the fluctuations of the field. Thus it is consistent to set $\sigma_0(\tau) = 0$ for all times in our analysis.

Notice that we do not need to replace the average of the fluctuations $\left\langle(\delta^{(1)}\sigma)^2\right\rangle^{1/2}$ with an effective homogeneous background $\sigma_0(\tau)$ because we work to second order in perturbation theory and the effect of these fluctuations enters into the calculation through the second order perturbed energy momentum tensor $\left\langle\delta^{(2)}T^\mu_\nu\right\rangle$. When $\left\langle\delta^{(2)}T^\mu_\nu\right\rangle$ becomes sufficiently large the backreaction will stop inflation. We take $\langle(\delta^{(1)}\sigma)^2\rangle^{1/2} = v/2$ (below) as our criterion for the end of inflation. We have checked numerically that this is a somewhat more stringent constraint than demanding that the energy density in the fluctuations $\delta^{(1)}\sigma$ does not dominate over the false vacuum energy which drives inflation $\lambda v^4/4$.

We will make extensive use of the slow roll parameters, defined by

$$\begin{aligned} \epsilon &= \frac{\kappa^2}{2}\frac{\dot{\varphi}_0^2}{H^2} = -\frac{\dot{H}}{H^2} \cong \frac{M_p^2}{2V^2}\left(\frac{\partial V}{\partial\varphi}\right)^2, \\ \epsilon - \eta &= \frac{\ddot{\varphi}_0}{H\dot{\varphi}_0} \cong \epsilon - \frac{M_p^2}{V}\left(\frac{\partial^2 V}{\partial\varphi^2}\right) \end{aligned}$$

so that, during inflation

$$\eta \cong 4\,\frac{M_p^2\, m_\varphi^2}{\lambda\, v^4}, \qquad \epsilon \cong 8\left(\frac{M_p\, m_\varphi^2\, \varphi_0}{\lambda\, v^4}\right)^2 \tag{6.12}$$

Notice that if $m_\varphi^2\varphi_0^2 \ll \lambda v^4$, then $\epsilon \ll \eta$. This is equivalent to demanding that the false vacuum energy of the tachyon dominates during inflation, which is the assumption usually made for hybrid inflation:

$$V(\varphi_0, \sigma_0 = 0) = \frac{\lambda v^4}{4} + \frac{m_\varphi^2}{2}\varphi_0^2 \cong \frac{\lambda v^4}{4}.$$

During the slow roll phase the inflaton equation of motion $3H\dot{\varphi}_0 + m_\varphi^2\varphi_0 \cong 0$ has solution

$$\varphi_0(t) = \varphi_s \exp\left(-\frac{m_\varphi^2(t-t_s)}{3H}\right) = \varphi_s\left(\frac{a(t)}{a_s}\right)^{-\eta} \tag{6.13}$$

where we used $a(t) = a_s e^{H(t-t_s)}$, with $t_s$ an arbitrary time. The Hubble scale remains approximately constant, $3H^2 \cong \lambda v^4/(4M_p^2)$. Since $\varphi_0$ is decreasing and $\sigma_0 = 0$, the slow roll parameter $\epsilon$ actually decreases slowly during inflation while $\eta$ remains constant.

For $\sigma_0 = 0$ the first order metric and inflaton perturbations obey exactly the same equations as in single field inflation. Hence, the analysis presented in chapter 1 all applies without modification. Indeed, the results of section 1.1.1 will come in handy since similar equations will arise when we study the second order metric perturbations.[1]

### 6.3.2 Conditions for a slowly varying tachyon mass

Now we come to an important point for this chapter, that if $\eta$ is sufficiently small, then the tachyon is a light field during some part of the observable period of inflation. The tachyon mass is given by $m_\sigma^2 = -\lambda v^2 + g^2\varphi_0^2(t)$. If we choose the arbitrary time $t_s$ in (6.13) to be when $m_\sigma^2 = 0$, then $g^2\varphi_s^2 = \lambda v^2$, and

$$\begin{aligned} m_\sigma^2 &= -\lambda v^2\left(1 - \left(\frac{a(t)}{a_s}\right)^{-2\eta}\right) \cong -2\eta\lambda v^2 H(t-t_s) \\ &= -2\eta\lambda v^2 N \end{aligned} \tag{6.14}$$

where $N$ is the number of e-foldings of inflation occurring after the tachyonic instability begins. At some maximum value $N = N_*$, inflation will end. If the inflaton rolls slowly enough, then the tachyon mass remains close to zero for a significant number

[1] As will be shown, the second order metric fluctuation $\phi^{(2)}$ obeys an inhomogeneous equation where the differential operator is identical to the one which determines the dynamics of $\phi^{(1)}$; thus an understanding of the first order solutions simplifies the construction of the Green function for the second order fluctuations.

of e-foldings. In general, we will have $-N_i \equiv N_e - N_*$ e-foldings of inflation before the spinodal time, followed by $N_*$ e-foldings during the preheating phase.

The approximation in (6.14), that the tachyonic mass changes slowly enough for its time dependence to be approximated as linear, is true so long as $|2\eta N| \ll 1$. This can be rephrased using the definition of $\eta$ in (6.12), and eliminating $m_\varphi^2$ using the COBE normalization of the inflationary power spectrum: $V/(M_p^4\epsilon) \cong 150\pi^2(2\times 10^{-5})^2 = 6\times 10^{-7}$ (see chapter 1). Using $V = \frac{1}{4}\lambda v^4$ and eq. (6.12) for $\epsilon$, the COBE normalization gives

$$m_\varphi^2 \cong 230\, g\lambda \frac{v^5}{M_p^3} \tag{6.15}$$

Then with $N \sim 60$, the requirement $|2\eta N| < 1$ becomes

$$g < 10^{-5}\frac{M_p}{v} \tag{6.16}$$

Interestingly, the bound (6.16) turns out to be a requirement that must often be satisfied for different reasons, namely the experimental limit on the spectral index of the first-order inflaton fluctuations. In terms of the slow-roll parameters, the deviation of the spectral index from unity is given by

$$n - 1 = 2\eta - 6\epsilon \cong 2\eta$$

where we have used the fact that $\epsilon \ll \eta$ in hybrid inflation (this is equivalent to the requirement that the energy density which drives inflation is dominated by $\lambda v^4/4$). The experimental constraint on the spectral index is roughly $|n-1| \lesssim 10^{-1}$. Writing $\eta$ in terms of model parameters this translates into the constraint

$$g\frac{v}{M_p} \lesssim 5\times 10^{-5} \tag{6.17}$$

This is just five times weaker than the technical assumption (6.16).

In passing, we note that there is also a lower bound on $g$ from the assumption that the false vacuum energy density is dominated by $\lambda v^4/4 > m_\varphi^2\varphi_0^2/2$. Using $g^2\varphi_s^2 = \lambda v^2$ and (6.15), one finds

$$g > 460\,\lambda\frac{v^3}{M_p^3} \tag{6.18}$$

### 6.3.3 Tachyonic Instability

To quantify the evolution of the tachyonic instability at the end of inflation, we consider the equation of motion for the tachyon field fluctuation in Fourier space,

$$\delta^{(1)}\ddot{\sigma}_k + 3H\delta^{(1)}\dot{\sigma}_k + \left[\frac{k^2}{a^2} + \left(g^2\varphi_0^2 - \lambda v^2\right)\right]\delta^{(1)}\sigma_k = 0. \tag{6.19}$$

Once $\varphi_0 < \lambda^{1/2}v/g$ the tachyon effective mass parameter $\partial^2 V/\partial\sigma^2$ becomes negative and the fluctuations $\delta^{(1)}\sigma$ are amplified due to the spinodal instability. The efficient transfer of energy from the false vacuum energy $\lambda v^4/4$ to the fluctuations $\delta^{(1)}\sigma$ is refered to as *tachyonic preheating* in the literature [32]-[34].

#### Tachyonic Preheating in the Instantaneous Quench Approximation

The initial studies of tachyonic preheating focused on the flat space dynamics of $\delta^{(1)}\sigma_k$ in the instantaneous quench approximation. In this approximation the field $\delta^{(1)}\sigma$ is initially assumed to have zero mass and at $t = 0$ a negative mass squared term $-|m_\sigma^2|(\delta^{(1)}\sigma)^2/2$ is turned on. We briefly review these dynamics here following closely [32]. Initially the tachyon has the usual Minkowski space mode functions $e^{-ikt+i\vec{k}\cdot\vec{x}}/\sqrt{2k}$ [2]. Once the negative mass squared term is turned on the modes with $k = |\vec{k}| < |m_\sigma|$ grow exponentially with a dispersion

$$\begin{aligned}\left\langle(\delta^{(1)}\sigma)^2\right\rangle &= \frac{1}{4\pi^2}\int_0^{|m_\sigma|} dk\, k\, e^{2t\sqrt{|m_\sigma^2|-k^2}} \\ &= \frac{|m_\sigma^2|}{4\pi^2\alpha^2}\left(e^{\alpha}(\alpha-1)+1\right); \quad \alpha \equiv 2|m_\sigma|t\end{aligned} \tag{6.20}$$

which produces a spectrum with an effective cutoff $k_{\max} = |m_\sigma|$. The tachyonic growth persists until the dispersion saturates at the value

$$\left\langle(\delta^{(1)}\sigma)^2\right\rangle^{1/2} \cong \frac{v}{2} \tag{6.21}$$

at which point the curvature of the effective potential vanishes and the tachyonic growth is replaced by oscillations about the true vacuum. This process completes

[2] Even in deSitter space the Bunch-Davies vacuum choice will ensure that this behaviour is respected on small scales $k \gg aH$ regardless of the tachyon mass during inflation. See chapter 1 for a review.

within a time

$$t_s \sim \frac{1}{2|m_\sigma|} \ln\left(\frac{\pi^2}{\lambda}\right) \tag{6.22}$$

which we call the spinodal time. At this point a large fraction of the vacuum energy $\lambda v^4/4$ has been converted into gradient energy of the field $\delta^{(1)}\sigma$ so that the universe is divided into domains with $\sigma \pm v$ of typical size $l \sim |m_\sigma|^{-1}$ and on average one still has $\langle\sigma\rangle = 0$ so that $\sigma_0(t) = 0$. These analytical argument are backed up by semi-classical lattice field theory simulations in [32, 33].

### Dynamics of Tachyonic Preheating Beyond the Instantaneous Quench Approximation

The discussion of the dynamics of tachyonic preheating above apply strictly only in Minkowski space. The dynamics of tachyonic preheating including the dynamics of the inflaton but neglecting the expansion of the universe were considered in [34]. The dynamics of tachyonic preheating including both the dynamics of the inflaton and the expansion of the universe were considered in [184] wherein the authors reach conclusions identical to those discussed above. The authors of [184] also find that the spinodal time is somewhat modified from (6.22) due to the background dynamics (see also [185, 186]).

In the present work, we are interested in a situation which is different from the instantaneous quench, where the instability may turn on slowly compared to the Hubble expansion, rather than suddenly. Of course, all of our analysis will also apply to the case where the instability turns on quickly. We are approximating the time dependence of the tachyon mass as being linear around the time when it vanishes, eq. (6.14), so the mode equation can be written in the form

$$\frac{d^2}{dN^2}\delta^{(1)}\sigma_k + 3\frac{d}{dN}\delta^{(1)}\sigma_k + \left[\hat{k}^2 e^{-2N} - cN\right]\delta^{(1)}\sigma_k = 0. \tag{6.23}$$

where $N = H(t - t_s)$, $\hat{k} \equiv k/H$ and, incorporating the COBE normalization as in (6.15),

$$c \cong 22000\, g\, M_p/v\,. \tag{6.24}$$

From eq. (6.16), $c$ is limited to values

$$c \ll \left(\frac{M_p}{v}\right)^2 \tag{6.25}$$

for the validity of the approximation that the tachyon mass squared varies linearly with time. The quantum mechanical solution in terms of annihilation and creation operators $a_k$, $a_k^\dagger$ has the usual form [3]

$$\delta^{(1)}\sigma(x) = \int \frac{d^3k}{(2\pi)^{3/2}}\, a_k\, \xi_k(N)\, e^{ikx} + \text{h.c.} \tag{6.26}$$

but the mode functions $\xi_k$ will be complicated by the time-dependence of the tachyon mass. We normalize the mode functions $\xi_k$ according to the usual Bunch-Davies prescription which is discussed in 1.1.1.

Since (6.23) has no closed-form solution, we approximate it in two regions. First, when $\hat{k}^2 e^{-2N} > c|N|$, we ignore the mass term and use the massless solutions, $\xi_k \sim a^{-3/2} H_{3/2}^{(1)}(\hat{k}e^{-N})$. We match this onto the solution in the region where $\hat{k}^2 e^{-2N} < c|N|$, where the term $\hat{k}^2 e^{-2N}$ is ignored in the equation of motion. The transition between the two regions occurs at different times $N_k$ for different wavelengths, given implicitly by

$$N_k = \ln \frac{\hat{k}}{\sqrt{c}} - \ln \sqrt{|N_k|} \tag{6.27}$$

This is a multivalued function of $x \equiv \hat{k}/\sqrt{c}$, because for $x < (2e)^{-1/2}$, the $\hat{k}^2$ term in the differential equation comes to dominate again for a short period around $N = 0$, the moment when the tachyon is massless. To deal with this, we are going to assume that the solution is still well-approximated by the massive one during this short period. This amounts to replacing the multivalued function with the single-valued one shown in figure 6.1. We checked this approximation in conjunction with other approximations we will make for the mode functions, as described below eq. (6.30). Appendix D-1 provides some technical details about the matching time $N_k$.

In the second region, with $N > N_k$, the solutions are approximated by Airy functions, but it is more convenient to use the WKB approximation to obtain an

[3]Our conventions for fourier transforms and mode functions are discussed in detail in appendix D-4.

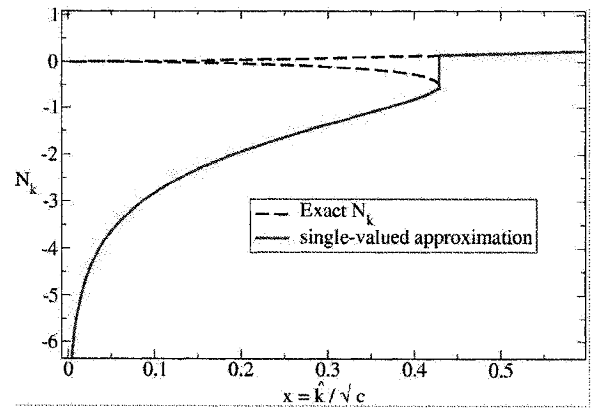


Figure 6.1: Exact solution and our approximation for the function $N_k$ in eq. (6.27).

expression in terms of elementary functions. Ignoring overall phases (which will cancel out of the final result), in this way we obtain

$$\xi_k \cong \begin{cases} \left(2H\hat{k}^3\right)^{-1/2}\left(1+i\hat{k}e^{-N}\right), & N < N_k \\ b_k\, e^{-\frac{3}{2}N+\frac{9}{4c}z^{3/2}}(1+|z|)^{-1/4}, & N > N_k \end{cases} \tag{6.28}$$

with

$$b_k = \frac{1-i\sqrt{c|N_k|}}{\sqrt{2H}(c|N_k|)^{3/4}}\frac{(1+|z_k|)^{1/4}}{\exp\left(\frac{9}{4c}z_k^{3/2}\right)} \tag{6.29}$$

and

$$z \equiv \left(1+\frac{4}{9}cN\right); \quad z_k \equiv \left(1+\frac{4}{9}cN_k\right) \tag{6.30}$$

In the above expressions, we have for simplicity matched the amplitudes but not derivatives of the solutions at $N = N_k$. This will not affect the estimates we make below. We used (6.27) to reexpress exponential dependence on $N_k$ as power law dependence. Notice that exponent in (6.29) becomes purely imaginary when $\frac{4}{9}cN_k < -1$. We also replaced $|z|^{1/4} \to (1+|z|)^{1/4}$ to correct the spurious singularity at $z = 0$ where the WKB approximation breaks down. We numerically verified that this gives a good approximation to the exact Airy function solutions.

Moreover, we have checked the approximate solution by numerically integrating the mode equations, starting from the small-$N$ region $k^2e^{-2N} \gg c|N|$, where the massless solutions with known amplitude tell us the initial conditions, and integrating into the large-$N$ region where the exponential growth due to the tachyonic instability becomes important. We did this for two orthogonal solutions to the mode equations, $\xi_{1,2}$, whose behavior in the small-$N$ region is

$$\begin{aligned}\xi_1 &= (2Hk^3)^{-1}e^{-N}\left(k\cos(ke^{-N}) - \sin(ke^{-N})\right)\\ \xi_2 &= (2Hk^3)^{-1}e^{-N}\left(k\sin(ke^{-N}) + \cos(ke^{-N})\right)\end{aligned}$$

Evolving these initial conditions to large $N$, the envelope of these functions, which is also the modulus of the complex solutions, is $\xi = \sqrt{\xi_1^2 + \xi_2^2}$. We compared this numerical solution to the modulus $|\xi_k|$ of our approximation (6.28) for a large range of $c$ and $k$ values. In the large $N$-region, $|\xi_k|$ agrees with $\xi$ up to a numerical factor of order unity. This factor, the ratio of the actual solution to the approximation, is shown in figure 6.2. Because the exponential growth of the mode function is a very steep function of $N$, these small errors have an imperceptible effect on the exclusion plots we will present in section 6.6. Furthermore, we have checked which values of $c$ and $k$ actually give constraints in the parameter space of the hybrid inflation model below, and found that in the regions where $c$ is large, $k$ is exponentially small. Extrapolating the results of figure 6.2 indicates that the error becomes quite small as $k \to 0$. Therefore our approximations for the mode functions are quite good for the purposes of this chapter.

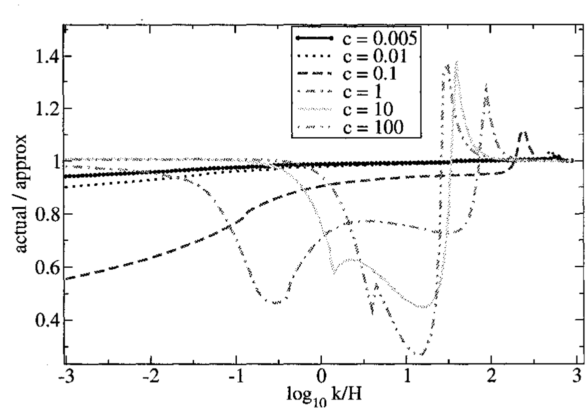


Figure 6.2: Ratio of the exact mode functions to the approximation (6.28), at large times.

With the above approximate solution, we are in a position to recompute the dispersion of the tachyon fluctuations taking into account both the dynamics of the tachyon mass and also the background evolution of the scale factor, $\langle(\delta^{(1)}\sigma)^2\rangle = (2\pi)^{-3}\int d^3k|\xi_k|^2$. Following the discussion of subsection 6.3.3, we set this equal to $v^2/4$ at the end of inflation, $N = N_*$:

$$\int \frac{d^3k}{(2\pi)^3}|\xi_k|^2\bigg|_{N=N_*} = \frac{v^2}{4} \tag{6.31}$$

which implicitly determines $N_*$ in terms of parameters of the hybrid inflation model,

$$N_* = N_*\left(g, \lambda, v/M_p\right) \tag{6.32}$$

The main contribution to the integral at $N = N_*$ comes from wave numbers for which the exponentially growing solution in (6.28) applies. These modes satisfy $k < k_{\rm max} \equiv He^{N_*}\sqrt{cN_*}$. We have numerically performed the integral for a wide range of values of $c$ and $N_*$. The result is displayed in figure 6.3, where contours of $\ln M_p^2/\lambda v^2$ are shown in the plane of $N_*$ and $\ln c$. Recall that $c = 22000\, g\,(M_p/v)$, eq. (6.24).

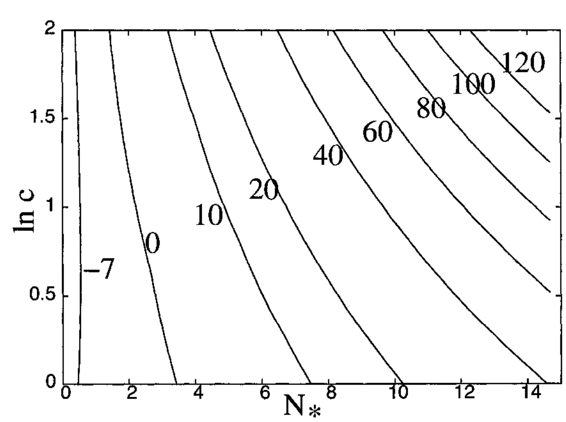


Figure 6.3: Contours of $\ln M_p^2/\lambda v^2$ in the plane of $N_*$ and $\ln c$.

## 6.4 Second Order Fluctuations in the Long Wavelength Approximation

### 6.4.1 The Master Equation

The authors of [183] have derived a "master equation" for the second order potential $\phi^{(2)}$ which can be written as

$$\begin{aligned} \phi''^{(2)} + 2\mathcal{H}(\eta - \epsilon)\phi'^{(2)} &+ \left[2\mathcal{H}^2(\eta - 2\epsilon) - \partial^k\partial_k\right]\phi^{(2)} \\ &= J(\tau, \vec{x}) \end{aligned} \tag{6.33}$$

where the source terms are constructed entirely from first order quantities and can be split into inflaton and tachyon contributions

$$J(\tau, \vec{x}) = J^{\sigma}(\tau, \vec{x}) + J^{\varphi}(\tau, \vec{x}). \tag{6.34}$$

Although we have explicitly inserted the slow roll parameters, equation (6.33) is quite general and we have not yet assumed that $\eta, \epsilon$ are small (although recall that we have set $\sigma_0 = 0$). We have verified both the second order Einstein equations and the master equation presented in [183] and these results are discussed in appendix D-2.

Because the equation (6.33) is linear we can split the solutions $\phi^{(2)}$ into three parts: the solution to the homogeneous equation, the particular solution due to the inflaton souce and the particular solution due to the tachyon source. The solution to the homogeneous equation will be proportional to $\phi^{(1)}$ since the differential operator on the left hand side of (6.33) is identical to the operator which determines $\phi^{(1)}$ (see section 1.1.1). In cosmological perturbation theory the split between background quantities and fluctuations is unambigiuous, since background quantities depend only on time while fluctuations depend on both position and time. However, the split between first order and second order fluctuations is arbitrary and the freedom to include in the solution for $\phi^{(2)}$ a contribution which is proportional to $\phi^{(1)}$ reflects this. We fix this ambiguity by including only the particular solutions for $\phi^{(2)}$ which is due to the source, $J(\tau, \vec{x})$.

During the preheating phase the tachyon fluctuations are amplified by a factor $v/H$ and so $J^\sigma$ will come to dominate. This implies that the particular solution for $\phi^{(2)}$ which is due to $J^\sigma$ will come to dominate over the particular solution which is due to $J^\varphi$. Although our analysis will focus on this part of the solution, we will also consider the inflaton source in appendix D-3 in order to verify that our formalism reproduces previous results.

### 6.4.2 The Green Function

In this subsection we construct the Green function for the master equation so that $\phi^{(2)}$ may be determined in terms of first order quantities. As discussed previously we consider only the particular solution for $\phi^{(2)}$ due to the source $J$ and neglect the solution to the homogeneous equation, which is equivalent to assuming that the second order fluctuations are zero before the source is turned on. During a quasi-deSitter phase the master equation can be written as

$$\begin{aligned} \partial_\tau\left[(-\tau)^{2(\epsilon-\eta)}\partial_\tau\phi_k^{(2)}\right] &+ (-\tau)^{2(\epsilon-\eta)}\left[\frac{2}{\tau^2}(\eta-2\epsilon)+k^2\right]\phi_k^{(2)} \\ &= (-\tau)^{2(\epsilon-\eta)}J_k(\tau) \end{aligned} \tag{6.35}$$

where it is assumed that $\epsilon, \eta \ll 1$, and the differential operator on the left hand side of the master equation is written in a manifestly self-adjoint form. In deriving (6.35) we used (1.44) and treated the slow roll parameter as constant, which is consistent at first order in the slow roll expansion. The causal Green function for this operator is

$$\begin{aligned} G_k(\tau,\tau') &= \frac{\pi}{2}\Theta(\tau-\tau')\left(\tau\tau'\right)^{1/2+\eta-\epsilon} \\ &\times \left[J_\nu(-k\tau)Y_\nu(-k\tau')-J_\nu(-k\tau')Y_\nu(-k\tau)\right] \end{aligned} \tag{6.36}$$

where the order of the Bessel functions is $\nu \cong 1/2+3\epsilon-\eta$ as in section 1.1.1.

The solution for the metric perturbation $\phi^{(2)}$ can then be written as

$$\phi_k^{(2)}(\tau) = \int_{-(1+\epsilon)/a_i H}^{0} d\tau' G_k(\tau,\tau')\left(-\tau'\right)^{2(\epsilon-\eta)} J_k(\tau') \tag{6.37}$$

where $a_i = a(t_i)$ is the scale factor at the some initial time, well before the tachyonic instability has set in. This solution is quite general; it applies during any slow roll phase, including during the tachyonic instability.

There are several interesting limiting cases of (6.36). In the long wavelength limit $k \to 0$ the Green function reduces to

$$\begin{aligned} G_k(\tau, \tau') &= \Theta(\tau - \tau')(1 + 2\eta - 6\epsilon) \\ &\left[-(-\tau)^{1+2\epsilon}(-\tau')^{2(\eta-2\epsilon)} + (-\tau')^{1+2\epsilon}(-\tau)^{2(\eta-2\epsilon)}\right]. \end{aligned}$$

In the case of pure deSitter expansion $\epsilon = \eta = 0$, but for all $k$, the Green function reduces to

$$G_k(\tau, \tau') = \Theta(\tau - \tau')\frac{1}{k}\sin\left[k\left(\tau - \tau'\right)\right].$$

Finally, the form of the Green function in the case of $\epsilon = \eta = 0$ and $k \to 0$ may be of some interest

$$G_k(\tau, \tau') = \Theta(\tau - \tau')(\tau - \tau'). \tag{6.38}$$

### 6.4.3 The Tachyon Source

We now study the tachyon contribution to the source (6.34). In position space $J^\sigma$ takes the form (see equation (D-21))

$$\begin{aligned} J^\sigma(\tau, \vec{x}) &= a^2\kappa^2 m_\sigma^2 \left(\delta^{(1)}\sigma\right)^2 - 2\kappa^2\left(\delta^{(1)}\sigma'\right)^2 \\ &+ 2\kappa^2\mathcal{H}(1 + \eta - \epsilon)\Delta^{-1}\partial_i\left(\delta^{(1)}\sigma'\partial^i\delta^{(1)}\sigma\right) \\ &+ 4\kappa^2\Delta^{-1}\partial_\tau\partial_i\left(\delta^{(1)}\sigma'\partial^i\delta^{(1)}\sigma\right) \\ &- \mathcal{H}(1 + 2\epsilon - 2\eta)\Delta^{-1}\gamma_\sigma' + \Delta^{-1}\gamma_\sigma''. \end{aligned} \tag{6.39}$$

The quantity $\gamma_\sigma$ can be written in the form (see equation (66) of [183], or equivalently (D-20))

$$\begin{aligned} \gamma_\sigma &= -\kappa^2\Delta^{-1}\left[3\partial_i\left(\partial^k\partial_k\delta^{(1)}\sigma\partial^i\delta^{(1)}\sigma\right)\right. \\ &+ \left.\frac{1}{2}\partial^k\partial_k\left(\partial_i\delta^{(1)}\sigma\partial^i\delta^{(1)}\sigma\right)\right] \\ &= -3\kappa^2\Delta^{-1}\partial_i\left(\partial^k\partial_k\delta^{(1)}\sigma\partial^i\delta^{(1)}\sigma\right) \\ &- \frac{\kappa^2}{2}\left(\partial_i\delta^{(1)}\sigma\partial^i\delta^{(1)}\sigma\right). \end{aligned}$$

Notice that the terms in the first line of the (6.39) are local, the terms in the second and third line are non-local (containing an inverse laplacian $\Delta^{-1}$) and the fourth line contains terms which are both local and doubly non-local (containing $\triangle^{-2}$). The Fourier transforms of the source terms are computed in appendix D-4 wherein we also discuss our conventions for the inverse laplacian operators.

In the following, we will need the Fourier transform of terms like $\triangle^{-1}\gamma_\sigma$,

$$\begin{aligned}\mathcal{F}\left[\triangle^{-1}\gamma_\sigma\right] &= -3\frac{\kappa^2}{k^4}\int\frac{d^3k'}{(2\pi)^{3/2}}k'^2 k\cdot(k-k')\delta^{(1)}\tilde{\sigma}_{k'}\delta^{(1)}\tilde{\sigma}_{k-k'} \\ &- \frac{\kappa^2}{2k^2}\int\frac{d^3k'}{(2\pi)^{3/2}}k'\cdot(k-k')\delta^{(1)}\tilde{\sigma}_{k'}\delta^{(1)}\tilde{\sigma}_{k-k'}\end{aligned}$$

This expression is operator-valued and can be written in terms of annihilation/creation operators and mode functions as $\delta^{(1)}\tilde{\sigma}_k = a_k\xi_k(t) + a^\dagger_{-k}\xi_{-k}(t)$ (see appendix D-4 for more details). In the Fourier transformed expression for $\triangle^{-1}\gamma_\sigma$, the scale dependence of the mode functions $\delta^{(1)}\tilde{\sigma}_k$ is integrated over so that $\triangle^{-1}\gamma_\sigma$ gets contributions from the tachyon fluctuations on all scales. The large scale limit of terms like $\triangle^{-1}\gamma_\sigma$ is not transparent and therefore we do not neglect any terms in the tachyon source which contain inverse Laplacians.

In order to consistently compute $\phi^{(2)}_k$ (6.37) in the large scale limit it is necessarly to keep the next-to-leading order terms in the small $(k/\mathcal{H})^2$ expansion of the Green function (6.36). To see this, notice that powers of $k^2$ cancel inverse Laplacians in the source:

$$k^2 J^\sigma_k = -\gamma''_{\sigma,k} + \mathcal{H}\gamma'_{\sigma,k} + \cdots \tag{6.40}$$

where $\cdots$ denotes gradient terms which are small on large scales. In appendix D-2 it is shown that $\gamma_\sigma$ can be written on large scales as (D-19)

$$\begin{aligned}\gamma_\sigma &\cong \frac{3\kappa^2}{2}\left[\left(\delta^{(1)}\sigma'\right)^2 - a^2 m_\sigma^2\left(\delta^{(1)}\sigma\right)^2\right] \\ &- 3\kappa^2\triangle^{-1}\partial_\tau\partial_i\left(\delta^{(1)}\sigma'\partial^i\delta^{(1)}\sigma\right) \\ &- 6\mathcal{H}\kappa^2\triangle^{-1}\partial_i\left(\delta^{(1)}\sigma'\partial^i\delta^{(1)}\sigma\right).\end{aligned}$$

Plugging this result into (6.40) and comparing to (6.39) we see that $k^2 J^\sigma_k/\mathcal{H}^2$ contains terms which are of the same form as those which appear in $J^\sigma_k$ (6.39). Hence these terms must be included to consistently study $\phi^{(2)}_k$ (and hence the curvature perturbation) on large scales. Will we see shortly that the inclusion of such terms is crucial for the cancellation of nonlocal terms in the curvature perturbation.

### 6.4.4 The Gauge Invariant Curvature Perturbation

The curvature perturbation is expanded to second order as

$$\zeta = \zeta^{(1)} + \frac{1}{2}\zeta^{(2)}$$

In hybrid inflation, where $\sigma_0 = 0$, the first order contribution comes entirely from the inflaton sector

$$\zeta^{(1)} = -\phi^{(1)} - \mathcal{H}\frac{\delta^{(1)}\rho}{\rho_0'}$$

where $\rho_0 = -(T_0^0)_{(0)}$, $\delta^{(1)}\rho = -\delta^{(1)}T_0^0$ are the unperturbed and first order stress tensor respectively. It is also conventional to define the comoving curvature perturbation at first order

$$\mathcal{R}^{(1)} = \phi^{(1)} + \mathcal{H}\frac{\delta^{(1)}\varphi}{\varphi_0'}$$

which, on large scales, is related to $\zeta^{(1)}$ as

$$\mathcal{R}^{(1)} + \zeta^{(1)} \cong 0.$$

Because $\sigma_0 = 0$ the inflationary trajectory is straight the case of hybrid inflation and $\zeta^{(1)}$ is conserved on large scales [190]-[193]. However, this is not the case at second order [187] and one expects that $\zeta^{(2)}$ will be amplified due to the tachyonic instability. This amplification of $\zeta^{(2)}$ will continue until $N = N_\star$ at which point the backreaction sets in and stops inflation. For $N > N_\star$ the inflaton is no longer dynamical since all of its energy has been converted into tachyon fluctuations. Thus for $N > N_\star$ the large scale curvature perturbation is conserved at all orders in perturbation theory since only one field (the tachyon) is dynamical and there are no non-adiabatic pressures.

The definition of the first order curvature perturbation $\zeta^{(1)}$ is generally agreed upon in the literature (up to a sign). At second order, however, there are several definitions of the curvature perturbation in the literature (see [187] for a comprehensive discussion). The definition we adopt follows [188] and generalizes the definition of Malik and Wands [189] (valid on large scales) to multiple scalar fields. (Our definition differs from [183]. The definition of [183] generalizes [157] to two scalar fields and applies only during inflatoin [193, 187] since it is conserved only in the slow roll limit.)

In [188] the second order large scale curvature perturbation is written in terms of the first and second order Sasaki-Mukhanov variables as

$$\begin{aligned}
\zeta^{(2)} &= \frac{1}{3-\epsilon}\frac{1}{(\varphi_0')^2}\left[\varphi_0' Q_\varphi'^{(2)} + a^2\frac{\partial V}{\partial\varphi}Q_\varphi^{(2)}\right] \\
&+ \frac{1}{3-\epsilon}\frac{1}{(\varphi_0')^2}\left[\left(Q_\sigma'^{(1)}\right)^2 + a^2 m_\sigma^2\left(Q_\sigma^{(1)}\right)^2\right] \\
&+ \frac{1}{3-\epsilon}\frac{1}{(\varphi_0')^2}\left[\left(Q_\varphi'^{(1)}\right)^2 + a^2 m_\varphi^2\left(Q_\varphi^{(1)}\right)^2\right] \\
&+ 4(3+\epsilon-\eta)\left(\frac{3-2\epsilon}{3-\epsilon}\right)\left(\frac{\mathcal{H}}{\varphi_0'}Q_\varphi^{(1)}\right)^2 \\
&+ (-10+2\epsilon+2\eta)\left(\frac{\mathcal{H}}{\varphi_0'}Q_\varphi^{(1)}\right)^2 \qquad (6.41)
\end{aligned}$$

where the first order Sasaki-Mukhanov variables [4] are

$$Q_\varphi^{(1)} = \frac{\varphi_0'}{\mathcal{H}}\mathcal{R}^{(1)} \qquad (6.42)$$

$$Q_\sigma^{(1)} = \delta^{(1)}\sigma \qquad (6.43)$$

and the second order Sasaki-Mukhanov variable is

$$\begin{aligned}
Q_\varphi^{(2)} &= \delta^{(2)}\varphi + \frac{\varphi_0'}{\mathcal{H}}\psi^{(2)} \\
&+ (2+2\epsilon-\eta)\frac{\varphi_0'}{\mathcal{H}}\left(\phi^{(1)}\right)^2 \\
&+ 2\frac{\varphi_0'}{\mathcal{H}^2}\phi^{(1)}\phi'^{(1)} + \frac{2}{\mathcal{H}}\phi^{(1)}\delta^{(1)}\varphi'. \qquad (6.44)
\end{aligned}$$

In writing (6.41-6.44) we have restricted ourselves to hybrid inflation, made use of the background equations and inserted $\epsilon, \eta$ (but not assuming slow roll).

We split $\zeta^{(2)}$ into contributions coming from the inflaton and the tachyon as

$$\zeta^{(2)} = \zeta_\varphi^{(2)} + \zeta_\sigma^{(2)}$$

and study each piece separately. This splitting is different from the one discussed in [188], where the curvature perturbation is defined for each fluid in such a way that the total curvature perturbation is a weighted sum of the individual contributions.

[4] Notice that $Q_\varphi^{(1)}$ is related to the variable $V^{(1)}$ discussed chapter 1 by $V^{(1)} = a\, Q_\varphi^{(1)}$.

Instead, we simply divide $\zeta^{(2)}$ into terms which depend respectively on the tachyon and inflaton fluctuations, $\delta^{(1)}\sigma$ and $\delta^{(1)}\varphi$, $\phi^{(1)}$, which is only possible because $\sigma_0 = 0$.

The tachyon part of the curvature perturbation $\zeta^{(2)}_\sigma$ gets contributions from the first and second line of (6.41), both explicitly through $Q^{(1)}_\sigma$ and implicitly through $\delta^{(2)}\varphi, \psi^{(2)}$. The inflaton part of the curvature perturbation $\zeta^{(2)}_\varphi$ contains contributions from the last three lines of (6.41) as well as from the first line of (6.41), both implicitly through $\delta^{(2)}\varphi$, $\psi^{(2)}$ and explicitly through the definition of $Q^{(2)}_\varphi$.

The inflaton part of the curvature perturvation, $\zeta^{(2)}_\varphi$ coincides with the $\zeta^{(2)}$of single field inflation and has been derived previously [18, 158]. We have considered the construction of $\zeta^{(2)}_\varphi$ using our formalism and these results are presented in appendix D-3.

### 6.4.5 The Tachyon Curvature Perturbation

We now compute $\zeta^{(2)}_\sigma$. Our focus is on the leading order contribution to $\zeta^{(2)}$ in the slow roll and large scale limit. If we work only to leading order in the slow roll parameters it is sufficient to use the Green function (6.36) in the limit $\epsilon = \eta = 0$ and keep only the terms in the tachyon source (6.39) which are not slow-roll suppressed. However, to consistently compute $\zeta^{(2)}_\sigma$ we must keep the next-to-leading order terms in the small $(k/\mathcal{H})^2$ expansion of the Green function, as we have discussed in subsection 6.4.3.

In appendix D-5 we use the second order Einstein equations and the previously derived expressions for the Green function and tachyon source to write $\zeta^{(2)}_\sigma$ (equation (6.41)) in terms of first order fluctuations. The result is

$$
\begin{aligned}
\zeta^{(2)}_\sigma \;\cong\; & \frac{\kappa^2}{\epsilon}\int_{-1/a_i H}^{\tau} d\tau' \left[ \frac{\left(\delta^{(1)}\sigma'\right)^2}{\mathcal{H}(\tau')} \right. \\
- \; & \left. \frac{\mathcal{H}(\tau')^2}{\mathcal{H}(\tau)^3}\left( \left(\delta^{(1)}\sigma'\right)^2 - a^2 m_\sigma^2 \left(\delta^{(1)}\sigma\right)^2 \right) \right] \qquad (6.45)
\end{aligned}
$$

where the tachyon fluctuations $\delta^{(1)}\sigma$ are functions of the integration variable $\tau'$. The corrections to (6.45) are either subleading in the slow roll expansion or are total gradients which can be neglected on large scales. Equation (6.45) is the main result of this section.

Several comments are in order concerning (6.45). First, note that the integration by parts which we have performed gives rise to terms which are evaluated between the initial and final time of the form $[\cdots]_{\tau'=-1/a_i H}^{\tau}$. Since we are interested in the preheating phase during which the fluctuations are amplified exponentially we can safely drop the contribution at $\tau' = -1/a_i H$ relative to the contribution at $\tau' = \tau$. Consistency of this approximation requires that the tachyonic growth of the perturbations overcomes any damping which may have occured during inflation. (If this is not the case then we can safely assume that no significant nongaussianity is produced.) We will return to this issue.

The expression (6.45) satisfies an important consistency check, namely that it is a local expression. Nonlocal operators $\triangle^{-n}$ appeared in many of the intermediate steps in the calculation. These nonlocal terms arise due to the decomposition of the metric perturbations into scalar, vector and tensor components. One expects that the nonlocal terms should cancel out of physical quantities, similarly to electrodynamics in Coulomb gauge. The second order curvature perturbation is related to the observable CMB temperature fluctuations in a nontrivial way, so this by itself does not prove that $\zeta^{(2)}$ must be local. However [194] has recently shown that under the conditions present in our model, $\zeta^{(2)}$ should indeed by local.

To clarify, we have shown that at leading order in slow roll parameters the nonlocal contributions to $\zeta_\sigma^{(2)}$ cancel. We have not checked that such terms cancel at higher order in the slow roll expansion, though we believe that they do. This cancellation was not observed in previous studies because the subleading (in $k^2$) corrections to the Green function were not included and thus the large scale expansion was inconsistent.

A comment is in order concerning the long wavelength approximation which we have used in deriving (6.45). In writing the expression for $k^2 J_k^\sigma$ we have dropped terms which are total gradients even though such terms will be integrated over time in computing $\zeta_\sigma^{(2)}$, due to the time integral in (6.37). Strictly speaking, one should only apply the large scale limit $k\tau \ll 1$ *after* the time integral has been performed. The reason for this is that the time integral extends from when the modes are well within the horizon, where they oscillate, to when the modes are outside the horizon, including horizon crossing. Mode-mode coupling near the epoch of horizon crossing can contribute momentum-dependent terms which are not suppressed on large scales [18, 195]. However in our application, we are justified in dropping the contributions which are generated near horizon crossing since they are exponentially suppressed compared those which are produced during the preheating phase.

### 6.4.6 Time-integrated tachyon perturbation

We can make our main result (6.45) more explicit by substituting in the solutions (6.26,6.28). The result is simplified by taking the Fourier transform of $\zeta^{(2)}$ and evaluating it at vanishing external wave number, and at the final time corresponding to the end of inflation, $N = N_*$:

$$\begin{aligned} \zeta^{(2)}_{k=0}(N_*) &= \frac{\kappa^2}{\epsilon}\int \frac{d^3p}{(2\pi)^{3/2}}\left(a_p\, b_p\, a^\dagger_p\, b_{-p} + \text{perms}\right) \\ &\quad \times \int_{\max(N_p,N_i)}^{N_*} f(c,N,N_*)\, dN \end{aligned} \tag{6.46}$$

where $N_i$ denotes the value of $N$ at the beginning of inflation, $f(c,N,N_*)$ is given by

$$\begin{aligned} f(c,N,N_*) = e^{-3N+\frac{9}{2c}z^{3/2}}\left(1+|z|\right)^{-1/2} \times \\ \left[\frac{9}{4}\left(1-e^{3(N-N_*)}\right)\left|\sqrt{z}-1-\frac{2c\ \text{sign}(z)}{27(1+|z|)}\right|^2 - \right. \\ \left. cNe^{3(N-N_*)}\right] \end{aligned} \tag{6.47}$$

where $z \equiv (1+\frac{4}{9}cN)$, and "perms" in (6.46) indicates the three other combinations of $a_p\, b_p$ and $a^\dagger_p\, b_{-p}$.

For illustration, we show the behavior of the function $f(c,N,N_*)$ for sample parameter values $N_* = 22.5$ and $\ln c = 1$. Figure 6.4 plots $(\text{sign}(f)\ln(1+|f|)$ as a function of $N$. The function is exponentially peaked at the initial value $N = N_i$, and at the final value $N = N_*$. Moreover it always becomes negative at $N_*$ because the negative mass squared term in (6.47) comes to dominate. Although it is not obvious in the figure, the negative value is orders of magnitude larger than the positive maximum just preceding it, so the negative extremum dominates in the integral in (6.47).

Whether the extremum of $f$ at $N_*$ or at $N_i$ dominates overall depends on how $|3N_i|$ compares to $9/(2c)z^{3/2}$ evaluated at $N_*$. To see this, note that the dominant time-dependence of $f$ is determined by the combination $-3N+\frac{9}{2c}z^{3/2}$ in the exponent. The $e^{-3N}$ decay factor is typical of massive modes, which redshift as $\delta\sigma_k \sim a^{-3/2}$. The $e^{\frac{9}{2c}z^{3/2}}$ growth factor is a result of the tachyonic instability. As we noted in the discussion following eq. (6.45), (6.46) is valid only when the late-time behavior dominates (since otherwise terms which we have discarded are no longer subdominant to the terms which appear in (6.45)). Therefore an important consistency condition for all of our analysis is that

$$\frac{9}{2c}z_*^{3/2} \equiv \frac{9}{2c}\left(1+\frac{4}{9}cN_*\right)^{3/2} > 3|N_i|\,. \tag{6.48}$$

If this condition is violated then the preheating is not playing any significant role in the dynamics and we can safely assume that no significant nongaussianity is produced.

Because of the exponential growth of $f$ at its extrema, it is a very good approximation to the integral to write $f = e^g$ and expand $g = g_m + g'_m(N - N_m)$ in the vicinity of the maximum value, whether it is at $N_i$ or $N_*$. Since the integral is so strongly peaked near the extremum, there is an exponentially small error in extending the range of integration to the half-line. In this way one obtains

$$\int dN\, f \cong \frac{e^{g_m}}{|g'_m|} \tag{6.49}$$

We will use this approximation below to numerically evaluate the integral.

## 6.5 Bispectrum and spectrum of second order metric perturbation

### 6.5.1 The tachyon contribution to the bispectrum

Here we calculate the leading contribution to the three-point function (bispectrum) of the second-order curvature perturbation due to the tachyon,

$$\left\langle \zeta^{(2)}_{k_1}\,\zeta^{(2)}_{k_2}\,\zeta^{(2)}_{k_3}\right\rangle \equiv (2\pi)^{-3/2}\,\delta^{(3)}(\vec{k}_1+\vec{k}_2+\vec{k}_3)\,B(\vec{k}_1,\vec{k}_2,\vec{k}_3) \tag{6.50}$$

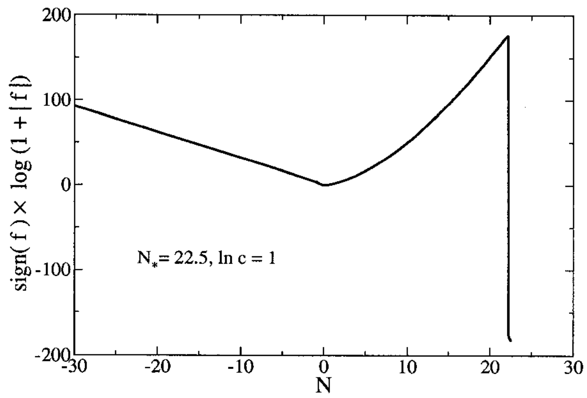


Figure 6.4: sign($f$) $\ln(1+|f|)$ versus $N$, showing behavior of the the function $f$ defined in (6.47), for $\ln c = 1$, $N_* = 22.5$

It is understood that only the connected part of the correlation function is computed, which is equivalent to subtracting the expectation value of $\zeta^{(2)}$ from the quantum operator. This three-point function is straightforward to compute, using the free-field two-point functions

$$\langle \delta^{(1)}\tilde{\sigma}_{p_i}\delta^{(1)}\tilde{\sigma}_{q_i}\rangle = \xi_{p_i}\xi_{q_i}\,\delta^{(3)}(p_i+q_i) \tag{6.51}$$

Carrying out the contractions of pairs of fields which contribute to the connected part of the bispectrum, one finds eight terms, which in the limit of vanishing external wave-numbers, are all equal. The result is

$$B = 8\frac{\kappa^6}{\epsilon^3}\int \frac{d^3p}{(2\pi)^3}|b_p|^6\left[\int_{\max(N_p,N_i)}^{N_*} dN\, f(c,N,N_*)\right]^3 \tag{6.52}$$

The integrand of the $p$-integral is exponentially strongly peaked, either near $\ln(p/\sqrt{c}H) \cong N_i$ if $f$ has its global maximum near $N = N_i$, or else near $\ln(p/\sqrt{c}H) \cong 0$ if $f$ is dominated by its behavior near $N_*$. The logarithm of the integrand for a typical case is shown in figure 6.5. If the cusp-like peak is dominating, we can use the same approximation as in (6.49) to evaluate the $p$ integral, on both sides of the maximum. If the other local maximum at larger values of $p$ is the global maximum, as sometimes happens, then we should treat the integral as a gaussian, since the derivative of the integrand vanishes at the maximum. In this case we similarly write the integrand of the $p$ integral as $f = e^g$ and expand $g = g_m + \frac{1}{2}g''_m(p - p_m)^2$, where $g''_m < 0$; then (6.49) is replaced by

$$\int dN\, f \cong \sqrt{\frac{2\pi}{|g''_m|}}e^{g_m} \tag{6.53}$$

We have carried out the evaluations numerically over a range of values of $\ln c$ and $N_*$.

In figure 6.6 we plot contours of $\ln|\tilde{B}|$, where $\tilde{B}$ is defined by

$$B = \frac{e^{-6N_i}}{2\pi^2}\left(\frac{\kappa^2}{c\epsilon}\right)^3 \tilde{B} \tag{6.54}$$

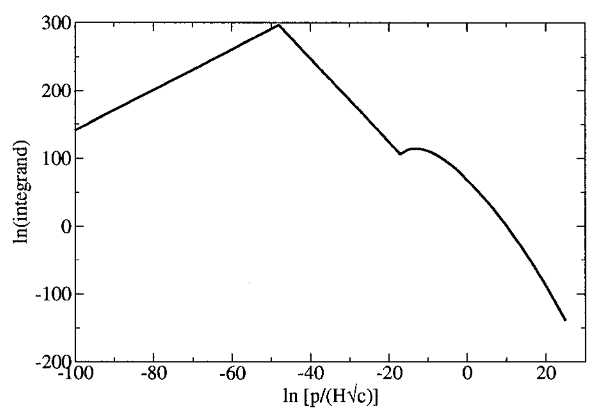


Figure 6.5: Log of the integrand of the $p$ integral in eq. (6.52), for a case where the maximum occurs near $\ln(p/\sqrt{c}H) = N_i$ .

The boundary between the two regions in fig. 6.6 is described analytically by the relation

$$3|N_i| = \frac{9}{2c}z_*^{3/2} \tag{6.55}$$

with $z_* = 1 + \frac{4}{9}cN_*$. Only the region to the right, where $\frac{9}{2c}z_*^{3/2} > 3|N_i|$, is relevent for our analysis (see eq. (6.48)).

To make contact with experimental constraints, we want to compare the predicted bispectrum with that of single-field inflation, where nongaussianity is conventionally expressed via a nonlinearity parameter $f_{NL}$, defined through

$$B(k_i) = -\frac{6}{5}f_{NL}\left(P_\phi(k_1)\,P_\phi(k_2) + \text{ permutations}\right) \tag{6.56}$$

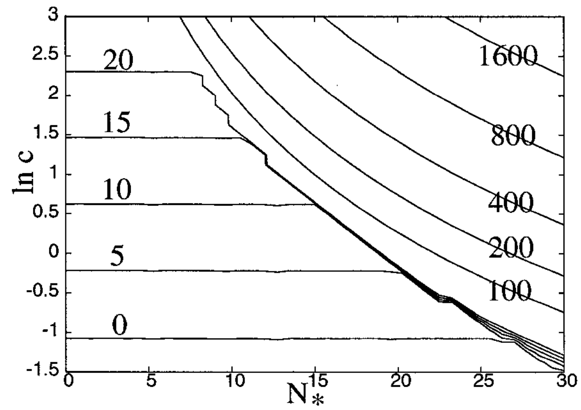


Figure 6.6: Contours of $\ln|\tilde{B}|$, defined in (6.54), in the plane of $N_*$ and $\ln c$. $N_i = -30$ in this example.

where $P_\phi(k)$ is the usual inflationary power spectrum, $P_\phi(k)^{1/2} \sim 10^{-5}\,(2\pi)/k^{3/2}$. If we assume that all $k_i$ are the same, $k_i \cong k$, then

$$f_{NL} = -\frac{5}{18}(2\pi)^{-4}10^{20}\,B(k)\,k^6 \tag{6.57}$$

We can thus convert our predicted bispectrum into an effective $f_{NL}$, for which the present experimental constraint is roughly $|f_{NL}| < 100$.

Using (6.54) in (6.57) and recalling that $N_e = |N_i| + N_*$ we find that

$$f_{NL} = -e^{17.5-6N_*}c^{-3}\tilde{B} \tag{6.58}$$

where we used $H^2 = V/(3M_p^2)$ as well as the COBE normalization $V/(\epsilon M_p^4) = 6 \times 10^{-7}$. The contours of $\tilde{B}$ can thereby be converted into contours of $f_{NL}$, and demanding that $|f_{NL}| < 100$ gives a constraint in the parameter space $c, N_*, N_i$.

### 6.5.2 The tachyon contribution to spectrum

The analogous calculation can also be carried out for the tachyon contribution to the two-point function of the curvature perturbation.

Closely following the preceding calculation, it is straightforward to show that, in the limit of vanishing external wave numbers,

$$\left\langle \zeta^{(2)}_{k_1} \zeta^{(2)}_{k_2} \right\rangle_{\rm con} = \delta(\vec{k}_1 + \vec{k}_2) \times$$
$$2\frac{\kappa^4}{\epsilon^2} \int \frac{d^3p}{(2\pi)^3} |b_p|^4 \left[ \int_{N_p}^{N_*} dN\, f(c, N, N_*) \right]^2$$
$$\equiv \delta(\vec{k}_1 + \vec{k}_2)\, S(k_i) \tag{6.59}$$

In analogy to (6.54), we define[5]

$$S = \frac{e^{-3N_i}}{4\pi^2} \frac{H}{c^{3/2}} \left( \frac{\kappa^2}{\epsilon} \right)^2 \tilde{S} \tag{6.60}$$

and we display contours of $\tilde{S}$ in fig. 6.7. Analogously, only the right-hand region is relevent for our analysis (since this is the region where 6.48 is satisfied).

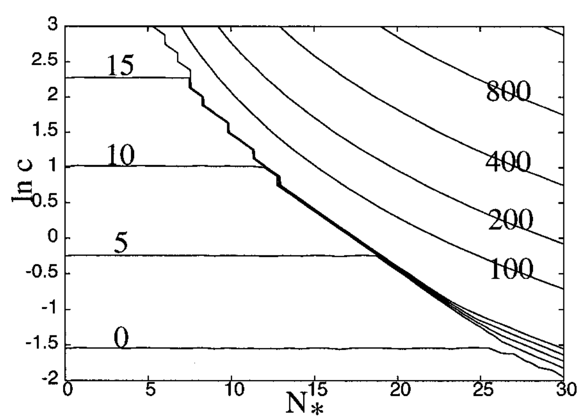


Figure 6.7: Contours of $\ln|\tilde{S}|$ in the plane of $N_*$ and $\ln c$, at $N_i = -30$.

Demanding that the tachyon contribution to the spectrum is sub-dominant to the inflaton contribution gives

$$\left\langle \zeta^{(2)}_{k_1} \zeta^{(2)}_{k_2} \right\rangle_{\rm con} \leq \frac{(2\pi)^2}{k^3} \delta(\vec{k}_1 + \vec{k}_2)\, \mathcal{P}_\zeta \tag{6.61}$$

with $\mathcal{P}_\zeta^{1/2} \cong 10^{-5}$. Combining this with (6.59) and (6.60), we define the *linearity parameter*

$$f_L \equiv \frac{S}{P_\zeta} \equiv \frac{10^{10}}{(2\pi)^4} \frac{H}{c^{3/2}} \left( \frac{\kappa^2}{\epsilon} \right)^2 \tilde{S}\, e^{-3N_i}\, k^3 \tag{6.62}$$

[5]The powers of $H$ and $c$ can be understood as follows: $b_p \sim H^{-1/2} c^{-3/4}$, whereas $p \sim \sqrt{c}H$.

The linearity parameter quantifies the tachyon contribution to the power spectrum. Since we would like the spectrum to be generated by the inflaton, we will demand that $|f_L| < 1$. One might consider being more conservative and imposing, say, $|f_L| < 0.01$ rather than $|f_L| < 1$, as we have done. Our exclusion plots are actually quite insensitive to the value assumed for $f_L$ (this is also true for $f_{NL}$) since the effect turns on exponentially fast.

## 6.6 Constraints on hybrid inflation parameter space

We have shown that contours of $\tilde{B}$, $\tilde{S}$ can be converted into contours of $f_{NL}$, $f_L$. Demanding that $|f_{NL}| < 100$, $|f_L| < 1$ gives constraints in the parameter space of $c$, $N_*$, $N_i$. The only obstacle to working directly in the model parameter space $g$, $\lambda$, $v$ is the implicit dependence of $N_*$ on $(g, \lambda, v)$. We have numerically inverted the relation depicted in fig. 6.3 to determine $N_*(g, \lambda, v)$. It then becomes straightforward to scan the model parameters, recomputing $f_{NL}$ and $f_L$. In the next section we will display our constraints in the space of $\log_{10} \lambda$, $\log_{10} g$ for a range of values of $\log_{10}(v/M_p)$.

Note that once a set of values for $\lambda$, $g$ and $v/M_p$ are chosen and $N_*$ has been calculated, one must still determine $N_i$ in order to calculate the parameters $f_L$ and $f_{NL}$. We do so by first computing the total number of e-foldings using the standard result

$$N_e = 62 - \ln\left(\frac{10^{16}\ \text{GeV}}{V^{1/4}}\right) - \frac{1}{3}\ln\left(\frac{V^{1/4}}{\rho_{\text{r.h.}}^{1/4}}\right) \tag{6.63}$$

where $V \cong \lambda v^4/4$ is the energy density during inflation, and $\rho_{\text{r.h.}}$ is the energy density at reheating. In the following, we will ignore the gravitino bound $\rho_{\text{r.h.}}^{1/4} \lesssim 10^{10}$ GeV and assume instant reheating, $\rho_{\text{r.h.}} = V$. The value of $N_i$ then follows from $N_i = N_* - N_e$. We have checked that incorporation of the gravitino bound does not create a noticeable change in the excluded regions.

### 6.6.1 The issue of scale invariance

In evaluating the tachyonic contributions to the spectrum or bispectrum we find that these are either nearly scale invariant ($S \sim 1/k^3$, $B \sim 1/k^6$), or else to scale invariance is badly violated ($S$, $B \sim k^0$). In the latter case, the spectral index for the tachyon contribution to the two-point function is $n = 4$ (this is characteristic of massive fields and respects the Traschen integral constraints on causality in general relativity [196]).

The two regimes, scale-invariant and nonscale-invariant, can be understood in reference to the condition (6.48) which must be satisfied in order for tachyonic preheating to play any significant role. There are two ways to satisfy (6.48). One is to take $cN_\star \gg 1$, which usually requires $c > 1$. This is the regime in which the tachyon mass is not small compared to $H$ during most of inflation, and so it corresponds to nonscale-invariant fluctuations of $\delta\sigma$. The tachyon fluctuations are Hubble damped as $\delta\sigma \sim a(t)^{-3/2}$ prior to inflation, but this suppression can be overcome on large scales if the amplification during the preheating phase is sufficiently large, which typically requires very small values of the self-coupling $\lambda \ll 1$. This nonscale-invariant regime corresponds to a region of the parameter space where the waterfall condition of hybrid inflation is satisfied (and one typically has $N_\star \ll 1$).

The second way to satisfy (6.48) is to take $c|N| < 1$ for all $N \in [N_i, N_\star]$. This gives a scale-invariant spectrum for the tachyon and also for $\zeta_\sigma^{(2)}$, which is most easily seen by writing the tachyon mass-squared as $|m_\sigma^2|/H^2 = c|N|$. It is clear that if $c|N| < 1$ for all $N \in [N_i, N_\star]$ then the tachyon field will have been light compared to the Hubble scale for all $\sim 60$ e-foldings of inflation which ensures a nearly scale-invariant spectrum for the tachyon. Also, in this case the instability will typically take several e-foldings to complete so that $N_\star > 1$. This scale-invariant regime corresponds to a region of the parameter space where the usual waterfall condition of hybrid inflation is violated (which is equivalent to saying $N_\star \gtrsim 1$).

### 6.6.2 Nonscale-invariant case

In the nonscale-invariant case $f_L$ and $f_{NL}$ depend on $k$. Because the tachyon spectrum is blue in this case the strongest constraint comes from evaluating $f_L$, $f_{NL}$ at the largest values of $k$ which are measured by the CMB. In deriving our constraints we conservatively take this to be $k = e^6 H e^{-N_e}$ where $N_e$ is the total number of e-foldings of inflation (6.63). The resulting constraints in the plane of $\log_{10} g$ and $\log_{10} \lambda$ are shown in the right-hand region of figure 6.8, for several values of $\log_{10} v/M_p$. We find that in this region the most stringent constraints come from $f_L$ rather than $f_{NL}$, so we expect to see distortions of the spectrum rather than nongaussianity at the left-hand boundaries of the excluded regions. (The other boundaries are excluded for different reasons described in the next paragraph.)

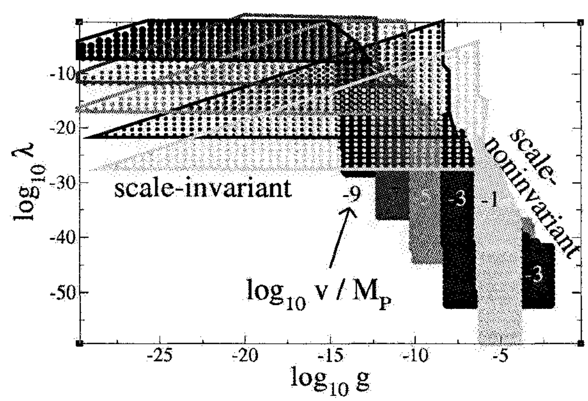


Figure 6.8: Excluded regions of the hybrid inflation parameter space, for $\log_{10} v/M_p = -1, -3, -5, -7, -9$.

In computing $f_L$, $f_{NL}$ over a wide range of $g, \lambda, v/M_p$ we also checked that several additional assumptions were respected: the tachyon mass-squared $m_\sigma^2$ varies linearly with the number of e-foldings, which was shown previously to require $gv/M_p < 10^{-5}$; the false vacuum energy density $\lambda v^4/4$ dominates during inflation, leading to the bound $g > 460\lambda\,(v/M_p)^3$; the reheat temperature exceeds 100 GeV, so that baryogenesis can occur at least during the electroweak phase transition, leading to the lower bounds on $\lambda$.

### 6.6.3 Scale-invariant case; the adiabatic approximation

On the left-hand side of fig. 6.8, we display new constraints for which the spectrum and bispectrum are scale-invariant. In contrast to the right-hand side, $f_{NL}$ provides the dominant constraint here, so that nongaussianity is playing the important role. To obtain these results, we employ a different approximation for the tachyon mode functions, namely the adiabatic approximation (described in some detail in appendix F of [5]). Because the tachyon has a small mass during the entirety of inflation (subsequent to horizon crossing), its mass is changing slowly, and we can use the standard mode functions for light fields, but with a time-dependent mass:

$$\delta\sigma(x) = \int \frac{d^3k}{(2\pi)^{3/2}} \frac{H}{\sqrt{2k^3}} (-k\tau)^{\eta_\sigma(\tau)} e^{ikx}\, a_k + \text{h.c.} \tag{6.64}$$

Here $\eta_\sigma = M_p^2 V_{,\sigma\sigma}/V$ is the slow-roll parameter for the tachyon, given by

$$\eta_\sigma(\tau) = 8\eta \frac{M_p^2}{v^2} \ln|H\tau| \tag{6.65}$$

where $\eta = M_p^2 V_{,\varphi\varphi}/V$. We have also verified that the solution (6.64) can be reproduced by (6.28) in the appropriate limit.

Since the mode functions have a simple form in the adiabatic approximation, it is possible to go farther analytically in this case. Notably, we can find an implicit equation for $N_*$ after evaluating the integral (6.31):

$$N_* \cong \frac{v/M_p}{15000\, N_e\, g} \ln\left[1 + 2\times 10^6\, N_* g \left(\frac{M_p}{v}\right)^3\right] \tag{6.66}$$

This expression for $N_*$ is much easier to evaluate than the one which arises in the WKB approximation since the latter leads to a numerical integral $F(N; \lambda, g, v)$ which must be inverted to find the $N_*$ which satisfies $F(N_*; \lambda, g, v) = v^2/4$.

Moreover, the time ($N$) integral in (6.45) can be evaluated explicitly using the saddle point method, since it is dominated by the exponential growth near $N = N_*$.

[6] Defining $\eta_f$ to be $|\eta_\sigma|$ at $N = N_*$, this results in the expression

$$\begin{aligned} B(\vec{k}_i) &= 4H^{6\eta_f} d_*^3 \int \frac{d^3p}{(2\pi)^3} |p|^{-3-2\eta_f} |p-k_3|^{-3-2\eta_f} \\ & \left(|p+k_2|^{-3-2\eta_f} + |p+k_1|^{-3-2\eta_f}\right) \end{aligned} \quad (6.67)$$

for the bispectrum,[7] where $d_* = H^2\kappa^2\eta_f e^{2\eta_f N_*}/(2\epsilon)$. In the limit of small $c \sim \eta_f$, this is manifestly nearly scale invariant, $B \sim 1/k_i^{6(1+\eta_f)}$, by power-counting the divergent behavior of the integral in the infrared. The divergence must be cut off in the usual way, by ignoring modes with $p$ smaller than the horizon. Numerically evaluating the remaining $p$ integral for $k_i \sim k$, and using $k\tau_* \sim e^{N_i}$ for modes near the horizon,[8] we find

$$B(k) \cong 45\, k_i^{-6(1+\eta_f)} \left(\frac{\kappa^2 H^2 \eta_f e^{2\eta_f N_e}}{2\pi\epsilon}\right)^3 \quad (6.68)$$

Further using the COBE normalization to write $\kappa^2 H^2/\epsilon = 2 \times 10^{-7}$, we find that the nonlinearity parameter is

$$f_{NL} = -2.6 \times 10^{-5} \left(\eta_f e^{2\eta_f N_e}\right)^3. \quad (6.69)$$

Moreover the COBE normalization implies $\eta_f = 7360 N_* g M_p/v$. Demanding that $|f_{NL}| < 100$ gives the new excluded regions on the left-hand side of fig. 6.8.

We have claimed that in the scale-invariant regime the dominant constraint is coming from $f_{NL}$ and not from $f_L$, contrarily to the nonscale-invariant regime. We now justify this claim. Repeating the steps above for the tachyon spectrum, $S$, one obtains

$$|f_L| \sim 10^{-6} \left(\eta_f e^{2\eta_f N_e}\right)^2 \quad (6.70)$$

Thus, in the scale-invariant regime, the linearity and nonlinearity parameters are related as

$$|f_{NL}| \sim 10^6 |f_L|^{3/2} \quad (6.71)$$

---

[6]As discussed previously, an integral of the form $\int dN e^g$ is approximated by $e^{g_*}/\sqrt{|g_*'|}$ where $g_*$ is the maximum value (at $N = N_*$) and $g_*'$ is the derivative evaluated at the same point.

[7]In [5], the conformal time when the instability starts is (perhaps confusingly named) $\tau_* = -1/H$, due to our choice of $N = 0$ for the beginning of the instability.

[8]The horizon-crossing condition is $k\tau_i = 1$, and $\tau_*/\tau_i = e^{N_i}/e^0$.

so that $|f_{NL}| > |f_L|$ except when $|f_L|$ is extremely small. This demonstrates that it is indeed possible to obtain significant nongaussianity in this region of the parameter space. We have verified that the result (6.71) can also be derived using the mode function solutions (6.28).

## 6.7 Implications for Brane-Antibrane Inflation

We now apply our results to a popular model of inflation from string theory, brane-antibrane inflation [24]. This can be done by mapping the low-energy effective action for the brane-antibrane system onto the hybrid inflation model. We focus on the popular KKLMMT scenario [24, 25] which reconciles brane inflation with modulus stabilization using warped geometries with background fluxes for type IIB string theory vacua [21]. In this model, the antibrane is at the bottom of a Klebanov-Strassler (KS) throat [197], with warp factor $a_i \ll 1$, and the brane moves down the throat.[9] Within the KS throat the geometry is well approximated by

$$ds^2 = a(y)^2 g_{\mu\nu} dx^\mu dx^\nu + dy^2 + y^2 d\Omega_5^2$$

where $y$ is the distance along the throat, $a(y) \cong e^{ky}$ is the warp factor and $d\Omega_5^2$ is the metric on the base space of the corresponding conifold sigularity of the underlying Calabi-Yau space. In the subsequent analysis we ignore the base space and treat the geometry as $AdS_5$.

In the following we compute only the nongaussianity which is due to the preheating dynamics and ignore the possible effects of the inflaton sound speed [162, 163, 199]; hence our results may be thought of as a lower bound on the nongaussianity from brane inflation.

In string theory the open string tachyon $T$ between a D3-brane and antibrane,[10]

[9]See [198] for other discussions on nongaussianity in string theory models of inflation.

[10]We restrict ourselves to inflation models driven by D3-branes since inflation driven by higher dimensional branes have problems with overclosure of the universe by defect formation [4].

separated by a distance $y$, is described by the action [43]

$$\begin{aligned} S_{\rm tac} &= -\int d^4x\,\sqrt{-g}\,\mathcal{L} \\ \mathcal{L} &= V(T,y)\,\sqrt{1+(a_i M_s)^{-2} g^{\mu\nu}\partial_\mu T^* \partial_\nu T} \end{aligned} \tag{6.72}$$

Here the small-$|T|$ expansion of the potential is

$$\begin{aligned} V(T,y) &= 2\,a_i^4\,\tau_3\left[1+\frac{1}{2}\left(\left(\frac{M_s y}{2\pi}\right)^2-\frac{1}{2}\right)|T|^2\right. \\ &\qquad\qquad \left. + \,\mathcal{O}(|T|^4)+\cdots\right]. \end{aligned} \tag{6.73}$$

where $M_s$ is the string mass scale, $\tau_3 = g_s^{-1}M_s^4/(2\pi)^3$ is the D3-brane tension, and $g_s$ is the string coupling. Notice that in the warped throat scenario the instability does not set in until the branes are separated by the unwarped string length,[11] $M_s^{-1}$. An interesting difference between the string tachyon and that of ordinary hybrid inflation is that (at $y=0$) the tachyon potential $V(|T|)$ in the string case does not have a local minimum; rather

$$V(T,0) \sim \tau_3\, e^{-|T|^2/4} \tag{6.74}$$

The potential is minimized as $T \to \infty$. Therefore $T$ does not have a VEV. Nevertheless, the unstable brane-antibrane system decays into closed strings soon after the instability begins, and the large-$T$ part of the potential is not meaningful for determining the actual evolution of the tachyon. In hybrid inflation, it is also true that the end of inflation occurs somewhat before the fluctuations of the tachyon become as large as the VEV. We will see that even in the absence of a $T^4$ coupling, we can still define the equivalent of $\lambda$ and $v$ for the brane-antibrane system, by equating $\frac{1}{4}\lambda v^4$ to the false vacuum energy, and $\lambda v^2$ to the tachyon mass scale. This amounts to replacing the condition for the end of inflation (6.31) by

$$\left.\int \frac{d^3k}{(2\pi)^3}|\xi_k|^2\right|_{N=N_*} = \frac{\text{false vacuum energy}}{|\text{tachyon mass}|^2} \tag{6.75}$$

[11] There is some confusion on the literature on this point, with some papers having stated that the instability is determined by the warped string length, but this is not the case [163]. We thank L. Leblond for pointing this out to us.

Despite that fact that the tachyon potential is only minimized at $|T| \to \infty$ the condition (6.75) is quite reasonable. Detailed numerical simulations of the symmetry breaking in the theory (6.72) were performed in [4]. Comparing to the analysis of [4] one finds that $N_\star$ as defined in (6.75) roughly corresponds to the time at which singularities in the spatial gradients of the tachyon field form [51]. The appearance of singularities within a finite time corresponds to the formation of lower dimensional branes [1] and hence by $N = N_\star$ the inflaton field ceases to exist as a physical degree of freedom. This means that, as in our previous analysis, for $N > N_\star$ there no longer exists any nonadiabatic pressure (since only one field, the tachyon, is dynamical) and the large scale curvature perturbation becomes conserved to all orders in perturbation theory [192, 194].

The effective values of the couplings can be found by rewriting the action in terms of the canonically normalized fields $\sigma = a_i\sqrt{\tau_3}T/M_s$ and $\varphi = \sqrt{\tau_3}y$ (see equations 3.6, 3.10 or C.1 in [24]), and then matching to the hybrid inflation potential (6.9). This gives the correspondence

$$v = \sqrt{\frac{2}{\pi^3}}\frac{a_i M_s}{\sqrt{g_s}} \tag{6.76}$$

$$\lambda = \frac{\pi^3}{4}g_s \tag{6.77}$$

$$g = \sqrt{2\pi g_s}\, a_i \tag{6.78}$$

For the analysis of the preceding sections to be valid, the inflaton potential must be well-described by $V_0 + \frac{1}{2}m_\varphi^2\varphi^2$ during the relevant e-foldings of inflation. The full potential can be written as

$$V_{\text{inf}} = \frac{m_\varphi^2}{2}\varphi^2 + V_0\left(1 - \frac{\nu}{4\pi^2}\frac{V_0}{\varphi^4}\right) \tag{6.79}$$

where $V_0 = 2a_i^4\tau_3$ and $\nu$ is a geometrical factor which is given $\nu = 27/16$ for the KS throat. It is typical to parameterize the inflaton mass in terms of the dimensionless quantity $\beta$ as $m_\varphi^2 = \beta H_0^2$ where $H_0^2 = V_0/(3M_p^2)$. Using the COBE normalization, we find that

$$\beta = 10^{7/2}\, a_i^3\left(\frac{M_s}{M_p}\right)^3. \tag{6.80}$$

Demanding that the mass term in (6.79) dominate over the Coulomb term even when the branes are separated by the local string length yields a lower bound on $\beta$:

$$\beta > 324\pi^4 g_s^2 a_i^{10} \left(\frac{M_p}{M_s}\right)^2$$

The parameter $\beta$ is also bounded from above by the requirement that $|n-1| \lesssim 10^{-1}$ which corresponds to $g_s^2 a_i^{10}(M_p/M_s)^2 \ll 5 \times 10^{-6}$.

Our results apply only in the case that $\beta > 0$; moreover the case where $\beta < 0$ does not make sense from the string theory point of view, since $\varphi = 0$ denotes the bottom of the throat, and the brane must roll toward that point, not away from it [27].

As in hybrid inflation, we still have three parameters even after normalizing the spectrum, which we can take to be $g_s$, $a_i$ and the ratio of the warped string scale to the Planck scale, $a_i M_s/M_p$. Taking into account the additional restrictions on $\beta$, we find that the scale-noninvariant exclusions (right-hand side of fig. 6.8) do not survive at all in the KKLMMT model; however all the scale-invariant ones do. Therefore this model has the potential for producing large nongaussianity, and is even constrained by producing too high levels of nongaussianity.

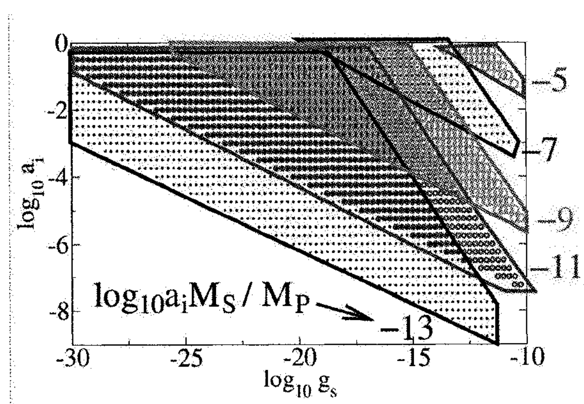


Figure 6.9: Excluded regions of the KKLMMT brane-antibrane inflation parameter space, in the plane of $\log_{10} a_i$ versus $\log_{10} g_s$ for $\log_{10} a_i M_s/M_p = -13, -11, \ldots, -5$ from nongaussianity.

The constraints in the string parameter space are shown in figure 6.9. The excluded regions shown here correspond to very small values of $g_s \lesssim 10^{-10}$. In the simplest way of connecting type IIB string theory to low-energy phenomenology, the gauge couplings of the Standard Model are related to $g_s$ by running down from the string scale, which would render such small values of $g_s$ incompatible with observations. However, type IIB strings are dual to themselves under SL(2,Z) transformations which take $g_s \to 1/g_s$. In the dual picture, the string coupling is very large, and the gauge dynamics at the string scale would be confining. It is conceivable that the Standard Model arises as a remnant of a strongly coupled gauge theory at the high scale, similar to technicolor models. In this case, the small values of $g_s$ which give rise to large nongaussianity could still be compatible with particle physics constraints.

## 6.8 Implications for P-term Inflation

In realistic models of hybrid inflation from supergravity, the potential is generated by F- or D-terms. P-term inflation is a class of models which combines the two kinds of terms and can interpolate between them [182]. The potential for P-term inflation, along the inflationary trajectory, is

$$V_{\rm inf} = \frac{g^2\xi^2}{2}\left(1 + \frac{g^2}{8\pi^2}\ln\frac{\varphi^2}{\varphi_c^2} + \frac{f}{8}\frac{\varphi^4}{M_p^4} + \cdots\right) \tag{6.81}$$

where $\cdots$ denotes terms of order $\varphi^6/M_p^6$ and higher. Here $\varphi_c = \xi = \sqrt{|\xi_+|^2 + \xi_3^2}$ is defined in terms of two Fayet-Iliopoulos parameters $\xi_+$ and $\xi_3$, and in (6.81) $f$ must lie in the interval $0 \leq f \leq 1$, since it is defined as $f = |\xi_+|^2/\xi^2$. The limits $f = 0$ and 1 correspond to D-term [200] and F-term [201] models, respectively. We will consider each of these limits separately. We do not consider the complications which arise when these models are coupled to moduli fields [202].

As in the models previously considered the false vacuum energy dominates during inflation and the Hubble scale is given by

$$H \cong \frac{g\,\xi}{\sqrt{6}M_p} \tag{6.82}$$

The inflaton couples to two scalar fields $\Phi_\pm$, of which one linear combination $\sigma$ is tachyonic. Its mass-squared is given by

$$m_\sigma^2 = g^2\left(\varphi^2 - \xi\right) \tag{6.83}$$

By comparing (6.82) and (6.83) to the hybrid inflation potential (6.9) we can determine the hybrid inflation model parameters as

$$\lambda = \frac{g^2}{2} \tag{6.84}$$

$$v = \sqrt{2\xi} \tag{6.85}$$

The coupling $g$ retains its original meaning in P-term inflation.

### 6.8.1 D-term Inflation

D-term inflation corresponds to taking $f = 0$ in (6.81). During a slow roll phase the inflaton field evolves as

$$\varphi_0(t)^2 = \varphi_c^2 - \frac{g^3\xi}{2\sqrt{6}\pi^2}(t - t_c)$$

which implies that $m_\sigma^2$ varies linearly with the number of e-foldings. Scales relevant for the CMB left the horizon when $\varphi = \varphi_N$ where

$$\varphi_N^2 = \varphi_c^2 + \frac{g^2 N}{2\pi^2} M_p^2 = \xi + \frac{g^2 N}{2\pi^2} M_p^2$$

Two distinct regimes are possible depending on the value of the coupling $g$. It is often assumed that $g$ is relatively large so that $g^2 N/(2\pi^2) \gg \xi/M_p^2$ [203] which gives the correct amplitude of density perturbations with $\xi \cong 10^{-5} M_p^2$ and requires $g \gtrsim 2 \times 10^{-3}$ for consistency. In this regime $\varphi_N \gg \varphi_c$ so that slow roll at the onset of the instability is not guaranteed and our previous analysis of the tachyon mode functions does not apply. However, in this regime it is also difficult to satisfy the constraints coming from the cosmic string tension, to avoid overproduction of cosmic strings, $\xi \lesssim 4 \times 10^{-7} M_p^2$ [204]. (Ref. [205] has pointed out that the constraints on the cosmic string tension can be weakened by incorporating the effect which strings have on the observed spectral index.)

We are therefore driven to consider D-term inflation in the regime of small coupling $g^2$ so that $g^2 N/(2\pi^2) \ll \xi/M_p^2$ and $\varphi_N \cong \varphi_c$. In this case we are guaranteed that the universe will still be in a slow roll phase at the onset of the instability and our previous analysis holds without modification. This corresponds to very small couplings $g \ll 2 \times 10^{-3}$, however, there is no obstruction to taking such a small coupling since $g^2$ is not necessarily related to the gauge coupling constant in a GUT [182]. In this regime the COBE normalization fixes $\xi \cong 7 \times 10^{-4} g^{2/3} M_p^2$ so that $g$ is the only independent model parameter. The cosmic string constraint $\xi \lesssim 4 \times 10^{-7} M_p^2$ then restricts the coupling $g$ to be smaller than $g \lesssim 1.3 \times 10^{-5}$.

Applying our previous analysis of hybrid inflation to D-term inflation,[12] including the additional constraints mentioned above, we find that there is a range of couplings,

$$-10.0 < \log_{10} g \lesssim -8.7 \tag{6.86}$$

which is ruled out because of the spectral distortion constraint, in the scale-noninvariant region of figure 6.8. On the other hand there is no constraint coming from nongaussianity for this model.

### 6.8.2 F-term Inflation

F-term inflation [201, 207] corresponds to taking $f = 1$ in (6.81). In this case the dynamics are somewhat more complicated than the D-term model. Again there are two possible regimes: a large coupling regime where $\varphi_N \gg \varphi_c$ and our previous analysis does not apply and also a small coupling regime where $\varphi_N \cong \varphi_c$ and our previous analysis does apply. The large coupling regime corresponds to $g \gtrsim 2 \times 10^{-3}$ and again the cosmic string tension constraints are difficult to satisfy (see, however, [208]). We are therefore driven to consider only the small coupling regime, $g \lesssim 2 \times 10^{-3}$. For couplings $3 \times 10^{-7} \ll g \lesssim 2 \times 10^{-3}$ it can be shown that the quadratic term $\varphi^4/M_p^4$ in the potential (6.81) can be neglected and the dynamics is identical to D-term inflation, which we have already considered. Thus we consider only the F-term model for $g \ll 3 \times 10^{-7}$ since this is the only region of parameter space for which the model differs significantly from the D-term model. [13]

[12]See [206] for further discussion of preheating in D-term inflation.

[13]We have neglected the intermediate regime $0.06 \lesssim g \lesssim 0.15$ which will not yield significant nongaussianity or spectral distortion.

For $g \lesssim 3\times10^{-7}$, so the $f$-term is dominating the potential, the slow roll parameter $\epsilon = \xi^3/(8M_p^6)$, and the COBE normalization fixes $\xi \cong 6.7 \times 10^6 g^2 M_p^2$ and the cosmic string tension is within observational bounds for $g \lesssim 2 \times 10^{-7}$. Again applying the general hybrid inflation constraints, we find the excluded region

$$-13.0 \lesssim \log_{10} g \lesssim -9.5 \tag{6.87}$$

which, as in the case of D-term inflation, comes from the $k^3$ spectral distortion effect rather than nongaussianity.

For more general P-term models with $0 < f < 1$ we expect the excluded regions to interpolate between (6.86) and (6.87). In deriving our constraints we have been driven to the small coupling regime by the requirement that the cosmic sting tension be within observational bounds. Our analysis does not give any significant constraint on the string theoretic D3/D7 model [209] since in this case the cosmic strings are not stable and there is no motivation to consider the small values of the coupling $g$ in (6.86) and (6.87). Indeed, such small couplings are difficult to motivate from string theory [210].

## 6.9 The Case of a Complex Tachyon

In the preceeding sections we have applied results which were derived under the assumption that $\sigma$ is a real field, to models in which the tachyon is actually complex. In doing so we have assumed that the generalization of the analysis of [5] to the case of a complex tachyon does not significantly modify the exclusion plot, figure 6.8. Here we verify this claim.

### 6.9.1 Cosmological Perturbation Theory for an $O(M)$ Multiplet

Before restricting to the case of a complex tachyon we consider the somewhat more general case of an $O(M)$ symmetric multiplet of tachyon fields $\sigma_A$ with $A = 1, \ldots, M$.

The matter sector is expanded in perturbation theory as

$$\varphi(\tau,\vec{x}) = \varphi^{(0)}(\tau)+\delta^{(1)}\varphi(\tau,\vec{x})+\frac{1}{2}\delta^{(2)}\varphi(\tau,\vec{x}) \quad (6.88)$$

$$\sigma_A(\tau,\vec{x}) = \delta^{(1)}\sigma_A(\tau,\vec{x})+\frac{1}{2}\delta^{(2)}\sigma_A(\tau,\vec{x}). \quad (6.89)$$

As before the time-dependent vacuum expectation value (VEV) of the tachyon fields are set to zero $\langle\sigma_A\rangle \equiv \sigma^{(0)} = 0$ which is a consequence of the $O(M)$ symmetry of the theory. Notice, however, that the tachyon field *does* develop an effective VEV for the radial component

$$\langle|\sigma|\rangle \equiv \langle\sqrt{\sigma_A\sigma^A}\rangle \neq 0$$

We also assume that

$$\frac{\partial V}{\partial\sigma_A} = \frac{\partial^2 V}{\partial\sigma_A\partial\varphi} = 0$$

but $V$ is, for the time being, otherwise arbitrary. Here and elsewhere the potential and its derivatives are understood to be evaluated on background values of the fields so that $V = V(\varphi^{(0)},\sigma^{(0)})$, for example.

We consider only the $\delta^{(2)}G^0_0 = \kappa^2\delta^{(2)}T^0_0$, $\partial_i\delta^{(2)}G^i_0 = \kappa^2\partial_i\delta^{(2)}T^i_0$ and $\delta^i_j\delta^{(2)}G^j_i = \kappa^2\delta^i_j\delta^{(2)}T^j_i$ equations since the second order vector and tensor fluctuations decouple from this system. In the case that $\sigma^{(0)}_A = 0$ the second order tachyon fluctuations $\delta^{(2)}\sigma_A$ decouple from the inflaton and gravitational fluctuations up to second order and hence we do not need to solve for $\delta^{(2)}\sigma_A$. Note also that the Klein-Gordon equation for the inflaton fluctations is not necessary to close the system. In this section we sometimes insert the slow roll parameters $\epsilon$ and $\eta$ explicitly though we do not yet assume that they are small. We also introduce the shorthand notation $m^2_\varphi \equiv \partial^2 V/\partial\varphi^2$.

The second order $(0,0)$ equation is

$$\begin{aligned} &3\mathcal{H}\psi'^{(2)} + (3-\epsilon)\mathcal{H}^2\phi^{(2)} - \partial^k\partial_k\psi^{(2)} \\ &= -\frac{\kappa^2}{2}\left[\varphi_0'\delta^{(2)}\varphi' + a^2\frac{\partial V}{\partial\varphi}\delta^{(2)}\varphi\right] + \Upsilon_1 \end{aligned} \quad (6.90)$$

where $\Upsilon_1$ is constructed entirely from first order quantities. Dividing $\Upsilon_1$ into inflaton and tachyon contributions we have

$$\Upsilon_1 = \Upsilon_1^\varphi + \Upsilon_1^\sigma$$

where

$$\begin{aligned}\Upsilon_1^{\varphi} &= 4(3-\epsilon)\mathcal{H}^2\left(\phi^{(1)}\right)^2 + 2\kappa^2\varphi_0'\phi^{(1)}\delta^{(1)}\varphi' \\ &- \frac{\kappa^2}{2}\left(\delta^{(1)}\varphi'\right)^2 - \frac{\kappa^2}{2}a^2m_\varphi^2\left(\delta^{(1)}\varphi\right)^2 - \frac{\kappa^2}{2}\left(\vec{\nabla}\delta^{(1)}\varphi\right)^2 \\ &+ 8\phi^{(1)}\partial^k\partial_k\phi^{(1)} + 3\left(\phi'^{(1)}\right)^2 + 3\left(\vec{\nabla}\phi^{(1)}\right)^2 \end{aligned} \tag{6.91}$$

and

$$\begin{aligned}\Upsilon_1^{\sigma} &= -\frac{\kappa^2}{2}\left[\delta^{(1)}\sigma_A'\delta^{(1)}\sigma'^A + \partial_i\delta^{(1)}\sigma_A\partial^i\delta^{(1)}\sigma^A\right. \\ &+ \left. a^2\frac{\partial^2 V}{\partial\sigma_A\partial\sigma_B}\delta^{(1)}\sigma_A\delta^{(1)}\sigma_B\right]. \end{aligned} \tag{6.92}$$

The divergence of the second order $(0,i)$ equation is

$$\partial^k\partial_k\left[\psi'^{(2)} + \mathcal{H}\phi^{(2)}\right] = \frac{\kappa^2}{2}\varphi_0'\partial^k\partial_k\delta^{(2)}\varphi + \Upsilon_2 \tag{6.93}$$

where $\Upsilon_2 = \Upsilon_2^{\varphi} + \Upsilon_2^{\sigma}$ is constructed entirely from first order quantities. The inflaton part is

$$\begin{aligned}\Upsilon_2^{\varphi} &= 2\kappa^2\varphi_0'\partial_i\left(\phi^{(1)}\partial^i\delta^{(1)}\varphi\right) + \kappa^2\partial_i\left(\delta^{(1)}\varphi'\partial^i\delta^{(1)}\varphi\right) \\ &- 8\partial_i\left(\phi^{(1)}\partial^i\phi'^{(1)}\right) - 2\partial_i\left(\phi'^{(1)}\partial^i\phi^{(1)}\right) \end{aligned} \tag{6.94}$$

and the tachyon part is

$$\Upsilon_2^{\sigma} = \kappa^2\partial_i\left(\delta^{(1)}\sigma_A'\partial^i\delta^{(1)}\sigma^A\right). \tag{6.95}$$

The trace of the second order $(i,j)$ equation is

$$\begin{aligned} & 3\psi''^{(2)} + \partial^k\partial_k\left[\phi^{(2)} - \psi^{(2)}\right] + 6\mathcal{H}\psi'^{(2)} \\ + & \; 3\mathcal{H}\phi'^{(2)} + 3(3-\epsilon)\mathcal{H}^2\phi^{(2)} \\ = & \; \frac{3\kappa^2}{2}\left[\varphi_0'\delta^{(2)}\varphi' - a^2\frac{\partial V}{\partial\varphi}\delta^{(2)}\varphi\right] + \Upsilon_3 \end{aligned} \tag{6.96}$$

where $\Upsilon_3 = \Upsilon_3^{\varphi} + \Upsilon_3^{\sigma}$ is constructed entirely from first order quantities. The inflaton part is

$$\begin{aligned}\Upsilon_3^{\varphi} &= 12(3-\epsilon)\mathcal{H}^2\left(\phi^{(1)}\right)^2 - 6\kappa^2\varphi_0'\phi^{(1)}\delta^{(1)}\varphi' \\ &+ \frac{3\kappa^2}{2}\left(\delta^{(1)}\varphi'\right)^2 - \frac{3\kappa^2}{2}a^2m_\varphi^2\left(\delta^{(1)}\varphi\right)^2 - \frac{\kappa^2}{2}\left(\vec{\nabla}\delta^{(1)}\varphi\right)^2 \\ &+ 3\left(\phi'^{(1)}\right)^2 + 8\phi^{(1)}\partial^k\partial_k\phi^{(1)} + 24\mathcal{H}\phi^{(1)}\phi'^{(1)} \\ &+ 7\left(\vec{\nabla}\phi^{(1)}\right)^2 \end{aligned} \tag{6.97}$$

and the tachyon part is

$$\Upsilon_3^\sigma = \kappa^2 \left[\frac{3}{2}\delta^{(1)}\sigma'_A\delta^{(1)}\sigma'^A - \frac{1}{2}\partial_i\delta^{(1)}\sigma_A\partial^i\delta^{(1)}\sigma^A \right. \\ \left. - \frac{3}{2}a^2\frac{\partial^2 V}{\partial\sigma_A\partial\sigma_B}\delta^{(1)}\sigma_A\delta^{(1)}\sigma_B\right] \tag{6.98}$$

The derivation of the master equation which was presented in appendix D-2 follows here unmodified except for the new definitions of $\Upsilon_1^\sigma$, $\Upsilon_2^\sigma$ and $\Upsilon_3^\sigma$. The master equation is

$$\phi''^{(2)} + 2(\eta-\epsilon)\mathcal{H}\phi'^{(2)} + \left[2(\eta-2\epsilon)\mathcal{H}^2 - \partial^k\partial_k\right]\phi^{(2)} = J \tag{6.99}$$

where the source is

$$J = \Upsilon_1 - \Upsilon_3 + 4\Delta^{-1}\Upsilon_2' + 2(1-\epsilon+\eta)\mathcal{H}\Delta^{-1}\Upsilon_2 \\ + \Delta^{-1}\gamma'' - (1+2\epsilon-2\eta)\mathcal{H}\Delta^{-1}\gamma'. \tag{6.100}$$

and the quantity $\gamma$ is defined as

$$\gamma = \Upsilon_3 - 3\Delta^{-1}\Upsilon_2' - 6\mathcal{H}\Delta^{-1}\Upsilon_2 \tag{6.101}$$

We can split the source into tachyon and inflaton contributions $J = J^\varphi + J^\sigma$ in the obvious manner, by taking the tachyon and inflaton parts of $\Upsilon_1, \Upsilon_2, \Upsilon_3, \gamma$.

In appendix D-6 we prove the identity (see eqn. D-41)

$$\gamma_\sigma = -\frac{\kappa^2}{2}\left(\partial_i\delta^{(1)}\sigma_A\partial^i\delta^{(1)}\sigma^A\right) \\ - 3\kappa^2\Delta^{-1}\partial_i\left(\partial^k\partial_k\delta^{(1)}\sigma_A\partial^i\delta^{(1)}\sigma^A\right)$$

which is analogous to the result for a real tachyon field.

We now proceed to derive the tachyon curvature perturbation. The derivation of $\zeta_\sigma^{(2)}$ presented previously follows unmodified except, of course, for the change in the definitions of $\Upsilon_1^\sigma$, $\Upsilon_2^\sigma$, $\Upsilon_3^\sigma$ and $\gamma_\sigma$. From this point onwards we assume that $\epsilon, |\eta| \ll 1$. The leading contribution to the tachyon curvature perturbation is

$$\zeta_\sigma^{(2)} \cong \frac{1}{\epsilon}\int_{\tau_i}^{\tau} d\tau' \left[-\frac{\Upsilon_1^\sigma}{\mathcal{H}(\tau')} + \frac{1}{3}\frac{\Upsilon_3^\sigma}{\mathcal{H}(\tau')} \right. \\ \left. - \frac{2}{3}\frac{\mathcal{H}(\tau')^2}{\mathcal{H}(\tau)^3}\Upsilon_3^\sigma\right]$$

Now, using equations (D-8) and (D-14) we can write this in terms of the tachyon fluctuation $\delta^{(1)}\sigma$ as

$$\begin{aligned}\zeta_\sigma^{(2)} &\cong \frac{\kappa^2}{\epsilon}\int_{-1/a_i H}^{\tau} d\tau' \left[\frac{\delta^{(1)}\sigma_A'\delta^{(1)}\sigma'^A}{\mathcal{H}(\tau')}\right. \\ &- \frac{\mathcal{H}(\tau')^2}{\mathcal{H}(\tau)^3}\left(\delta^{(1)}\sigma_A'\delta^{(1)}\sigma'^A\right. \\ &- \left.\left. a^2\frac{\partial^2 V}{\partial\sigma_A\partial\sigma_B}\delta^{(1)}\sigma_A\delta^{(1)}\sigma_B\right)\right] \end{aligned} \tag{6.102}$$

The corrections to (6.102) are either total gradients or are subleading in the slow roll expansion. In deriving (6.102) we have restricted ourselves to the preheating phase during which the fluctuations $\delta^{(1)}\sigma_A$ grow exponentially.

Using (6.102) the second order tachyon curvature perturbation can be computed once the fluctuations $\delta^{(1)}\sigma_A$ are determined. The first order tachyon fluctuations are described by the perturbed Klein-Gordon equation

$$\delta^{(1)}\sigma_A'' + 2\mathcal{H}\delta^{(1)}\sigma_A - \partial^k\partial_k\delta^{(1)}\sigma_A + a^2\frac{\partial^2 V}{\partial\sigma_A\partial\sigma_B}\delta^{(1)}\sigma_B = 0 \tag{6.103}$$

### 6.9.2 Complex Tachyon Mode Functions

At this point we restrict our attention to the case with $M = 2$ and the potential

$$\begin{aligned} V &= \frac{\lambda}{4}\left(\sigma_A\sigma^A - v^2\right)^2 + \frac{g^2}{2}\varphi^2\sigma_A\sigma^A \\ &+\frac{m_\varphi^2}{2}\varphi^2 \end{aligned} \tag{6.104}$$

For $\sigma_A^{(0)} = 0$ the mass matrix is diagonal

$$\begin{aligned}\frac{\partial^2 V}{\partial\sigma_A\partial\sigma_B} &= \left(-\lambda v^2 + g^2\varphi_0^2\right)\delta_{AB} \\ &\equiv m_\sigma^2\delta_{AB}\end{aligned}$$

so that the tachyon fluctuations with $A = 1$ and $A = 2$ evolve independently (see eqn. 6.103).

As previously the quantum mechanical solutions $\delta^{(1)}\sigma_A$ are written in terms of annihilation and creation operators $a_k^A$, $a_k^{A\dagger}$ in the usual way

$$\delta^{(1)}\sigma_A(x) = \int \frac{d^3k}{(2\pi)^{3/2}}\, a_k^A\, \xi_k(t)\, e^{ikx} + \text{h.c.} \tag{6.105}$$

Both components $A = 1$ and $A = 2$ have the same time dependence owing to the fact that the mass matrix is diagonal. The $\xi_k$ in (6.105) are thus identical to the solutions (6.28), which we have already studied.

### 6.9.3 The End of Symmetry Breaking

For a multi-component tachyon the condition defining $N_\star$ must be modified as $\langle \delta^{(1)}\sigma_A \delta^{(1)}\sigma^A \rangle (N = N_\star) = v^2/4$ which, for the case $M = 2$, changes (6.31) to

$$\int \frac{d^3k}{(2\pi)^3} |\xi_k|^2 \bigg|_{N=N_\star} = \frac{v^2}{8}$$

### 6.9.4 Tachyon Curvature Perturbation

For the potential (6.104) the tachyon curvature perturbation $\zeta_\sigma^{(2)}$ decomposes into a sum of term

$$\zeta_\sigma^{(2)} = \sum_{A=1,2} \zeta_A^{(2)}$$

where $\zeta_A^{(2)}$ is the contribution to $\zeta_\sigma^{(2)}$ coming from $\sigma_A$. Consider, as an example, the spectrum of the tachyon curvature perturbation

$$\begin{aligned} \langle \zeta_{\sigma,k_1}^{(2)} \zeta_{\sigma,k_2}^{(2)} \rangle &= \langle \zeta_{1,k_1}^{(2)} \zeta_{1,k_2}^{(2)} \rangle + \langle \zeta_{2,k_1}^{(2)} \zeta_{2,k_2}^{(2)} \rangle \\ &+ \langle \zeta_{1,k_1}^{(2)} \zeta_{2,k_2}^{(2)} \rangle + \langle \zeta_{1,k_2}^{(2)} \zeta_{2,k_1}^{(2)} \rangle \end{aligned}$$

Because the annihilation/creation operators $a_k^1$ and $a_k^2$ are independent the cross-terms on the last line do not contribute to the connected part of the correlation function. This means that

$$\langle \zeta_{\sigma,k_1}^{(2)} \zeta_{\sigma,k_2}^{(2)} \rangle = 2 \, \langle \zeta_{1,k_1}^{(2)} \zeta_{1,k_2}^{(2)} \rangle$$

The quantity $\langle \zeta_{1,k_1}^{(2)} \zeta_{1,k_2}^{(2)} \rangle = \langle \zeta_{2,k_1}^{(2)} \zeta_{2,k_2}^{(2)} \rangle$ will be identical to the $\langle \zeta_{\sigma,k_1}^{(2)} \zeta_{\sigma,k_2}^{(2)} \rangle$ which we have already computed for the real tachyon field. We see, then, that the effect of having a complex tachyon field (as opposed to a real field) is to multiply $f_L$ and $f_{NL}$ by a factor of 2 and also to slightly reduce $N_\star$. The net change in $f_L$, $f_{NL}$ is order unity and the new exclusion plot is difficult to visually distinguish from figure 6.8. This justifies our previous claims that our constraints do not change significantly when one considers a complex tachyon field.

## 6.10 Conclusions

In this chapter we have studied the evolution of the second order curvature perturbation during tachyonic preheating at the end of hybrid inflation. We have found that, depending on the values of certain model parameters, two interesting effects are possible:

- Preheating generates a scale-invariant contribution to the curvature perturbation. In this case significant nongaussianity can be generated during preheating and the model is even constrained by producing too high a level of nongaussianity.

- Preheating generates a nonscale-invariant contribution to the curvature perturbation with spectral index $n = 4$. In this case the strongest constraint comes from the distortion of the power spectrum and no significant nongaussianity can be produced.

In both cases one typically requires fairly small values of the dimensionless couplings $g, \lambda$ in order to obtain a strong effect. Note that a small coupling $g$ does not require fine tuning in the technical sense, since $g^2$ is only multiplicatively renormalized: $\beta(g^2) \sim \mathcal{O}(g^2\lambda, g^4)/(16\pi^2)$. That is, if $g$ is small at tree level then loop corrections do not change its effective value significantly.

We have applied our constraints on hybrid inflation to several popular models: brane inflation, D-term inflation and F-term inflation. In the case of brane inflation we have found that significant nongaussianity from preheating is possible for sufficiently small values of the warp factor. For both D- and F-term inflation we have shown that no nongaussianity is produced during preheating, however, we still put interesting constraints on the model due to the distortion of the spectrum by nonscale-invariant fluctuations.

We have also generalized our results to the case of a complex tachyon field, confirming the claim that this modification does not significantly alter our exclusion plots.

We should note that the model of hybrid inflation considered here always gives a small blue tilt to the spectral index, $n > 1$, which is disfavoured by recent data [16]. One avenue for future study [211] is to generalize our results to the case of inverted hybrid inflation [212] which always gives $n < 1$.

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# Chapter 7

# $p$-adic Inflation

# Abstract

We construct approximate inflationary solutions rolling away from the unstable maximum of $p$-adic string theory, a nonlocal theory with derivatives of all orders. Novel features include the existence of slow-roll solutions even when the slow-roll parameters, as usually defined, are much greater than unity, as well as the need for the Hubble parameter to exceed the string mass scale $m_s$. We show that the theory can be compatible with CMB observations if $g_s/\sqrt{p} \sim 10^{-7}$, where $g_s$ is the string coupling, and if $m_s < 10^{-6}M_p$. A red-tilted spectrum is predicted, and the scalar-to-tensor ratio is bounded from above as $r < 0.006$. The $p$-adic theory is shown to have identical inflationary predictions to a local theory with superPlanckian parameter values, but with the advantage that the $p$-adic theory is ultraviolet complete.


## 7.1 Introduction

Many string theorists and cosmologists have turned their attention to building and testing stringy models of inflation in recent years. The goals have been to find natural realizations of inflation within string theory, and novel features which would help to distinguish the string-based models from their more conventional field theory counterparts. The more popular categories include brane-antibrane [24, 25, 53], D3/D7 [209], modular [213], DBI [162] and tachyon-driven [214] inflation (see [22] for a review).

In most examples to date, string theory has been used to derive an effective 4D field theory operating at energies below the string scale. Since string theory provides a complete description of dynamics also at higher energies, it may be interesting to consider a model which takes advantage of this distinctive feature. This is usually daunting since the field theory description should be supplemented by an infinite number of higher dimensional operators at energies above the string scale, whose detailed form is not known. In the present work, we propose to take a small step in the direction of overcoming this barrier, by considering a simplified model of string theory invented in 1987 [95], in which the world-sheet coordinates of the string are restricted to the field of $p$-adic numbers. Scattering amplitudes of open string theory can be related to those of the $p$-adic strings. A great advantage in $p$-adic string theory is that it is possible to compute *all* amplitudes of its lowest state and to determine a simple field-theoretic Lagrangian which exactly reproduces them. The result is a nonlocal field theory which is nevertheless sensible in the far ultraviolet.

The $p$-adic string resembles the bosonic string in that its ground state is a tachyon, whose unstable maximum presumably indicates the presence of a decaying brane, analogous to the unstable D25-brane of the open bosonic string theory [215]. Similarly to the bosonic string, the potential is asymmetric around the maximum, with one direction leading to a zero-energy vacuum, while in the other direction the potential is unbounded from below. We will consider whether it is possible to get successful inflation from rolling toward the bounded direction. This has been tried before in the context of the open string tachyon, and is difficult [216] because the tachyon potential is not flat enough to give a significant period of inflation, and there are no parameters within the theory which can tune the potential to be more flat. In contrast, we will show that the $p$-adic string tachyon *can* roll slowly enough to give many e-foldings of inflation. There are two distinct regions of parameter space which allow for successful inflation. There is a region with $p = \mathcal{O}(1)$ for which the $p$-adic field potential is flat and slow roll inflation proceeds in the usual manner. However, there is also a region of parameter space with $p \gg 1$ for which the potential is *extremely* steep ($|\eta| = M_p^2|V''/V|$ may be as large as $10^{11}$) but the $p$-adic scalar field nevertheless rolls slowly. This remarkable behaviour relies on the nonlocal nature of the theory: the effect of the higher derivative terms in the action is to slow down the field sufficiently, despite its steep potential. This new effect manifests itself only in the regime where the higher derivative interactions cannot be ignored. It is also interesting that in our model the kinetic energy is responsible for driving inflation for a significant number of e-foldings, unlike in conventional models of inflation.

One may worry about the presence of these higher derivative terms, because they are usually known to introduce ghosts[1] into the theory. In fact, it is easy to check that for a scalar field theory if one introduces only a finite number of higher derivatives, then the model invariably contains ghost degrees of freedom. The reason why the $p$-adic string action can evade this problem is because it is intrinsically nonperturbative in nature, the propagator being modified in such a way as to not contain any poles. In other words there are no physical states, ghosts or otherwise, around the true vacuum. This novel way of curing the problem of ghosts in higher derivative theories, while retaining some nice properties such as improved UV behaviour, was already pointed out in [218] in the context of gravity. It was also pointed out in [218] that such theories can exibit interesting new cosmological features. For instance, one can obtain nonsingular bouncing solutions by making gravity weak at short distances. More recently, such models have also been shown to possess inflationary solutions [219]. However, the models of [218, 219] are phenomenological, while the $p$-adic action that we consider is an actual (albeit exotic) string theory and reproduces many nontrivial features of conventional string theories.

We start by reviewing the salient features of $p$-adic string theory in section 7.2. In section 7.3 we show that this theory does not give inflation if the higher-derivative terms in the action are ignored. However, near the top of the potential, the energy can be large enough to justify keeping all higher derivative terms. In section 7.4 we

[1]One may also worry about classical instabilities which usually plague higher derivative theories; generically they go by the name of Ostrogradski instabilities (see [217] for a review). These are the classical manifestations of having ghosts in the theory: they can have arbitrarily large negative energy, which leads to classical instability. Since the nonlocal theory under consideration does not contain any ghosts we also do not expect to find such instabilities. One way to see how such theories may avoid the Ostrogradski instability argument, valid for finite higher derivative theories, is by noting that one cannot construct the usual Ostrogradski Hamiltonian because there is no highest derivative in such nonlocal actions. Also, in arriving at the Ostrogradski Hamiltonian, one assumes that all the derivatives of the field (except the maximal one) are independent canonical variables. This is no longer true for theories with derivatives of infinite order. For instance, it is not possible to independently choose an initial condition with arbitrarily specified values of all the derivatives of the field. See ref. [87] for a discussion of this point.

show how to resum their contributions and we construct approximate inflationary solutions valid near the top of the potential, by solving the coupled equations for the tachyon and scale factor of the universe. We give two different approximate methods in this section. In section 7.5 we solve for the fluctuations around this background to determine the power spectrum of scalar and tensor perturbations which can be probed by the cosmic microwave background (CMB). There we show that it is possible to choose parameters which are compatible with the measured amplitude and spectral index, and that the scalar-to-tensor ratio is bounded from above as $r < 0.006$ in this model. We also argue that it can be natural to have initial conditions compatible with inflation in the $p$-adic theory. We give conclusions in section 7.6. Appendix E-1 gives details about the $p$-adic stress tensor and the approximate inflationary solution of the Friedmann equation. Appendix E-2 gives mathematical details about the incomplete cylindrical functions of the Sonine-Schaefli form. Appendix E-3 explains a formal equivalence between the dynamics of the $p$-adic tachyon and those of a local field theory with a super-Planckian vacuum expectation value (VEV).

## 7.2 Review of $p$-adic string theory

The action of $p$-adic string theory is given by [95]

$$\begin{aligned} S &= \frac{m_s^4}{g_p^2}\int d^4x\left(-\frac{1}{2}\phi\, p^{-\frac{\Box}{2m_s^2}}\phi+\frac{1}{p+1}\phi^{p+1}\right) \\ &\equiv \frac{m_s^4}{g_p^2}\int d^4x\left(-\frac{1}{2}\phi\, e^{-\frac{\Box}{m_p^2}}\phi+\frac{1}{p+1}\phi^{p+1}\right) \end{aligned} \tag{7.1}$$

where $\Box = -\partial_t^2 + \nabla^2$ in flat space and we have defined

$$\frac{1}{g_p^2} \equiv \frac{1}{g_s^2}\frac{p^2}{p-1} \text{ and } m_p^2 \equiv \frac{2m_s^2}{\ln p} \tag{7.2}$$

The dimensionless scalar field $\phi(x)$ describes the open string tachyon, $m_s$ is the string mass scale and $g_s$ is the open string coupling constant. Though the action (7.1) was originally derived for $p$ a prime number, it appears that it can be continued to any postive integer and even makes sense in the limit $p \to 1$ [96]. Setting $\Box = 0$ in the action, the resulting potential takes the form $V = (m_s^4/g_p^2)(\frac{1}{2}\phi^2 - \frac{1}{p+1}\phi^{p+1})$. Its shape is shown in figure 7.1.

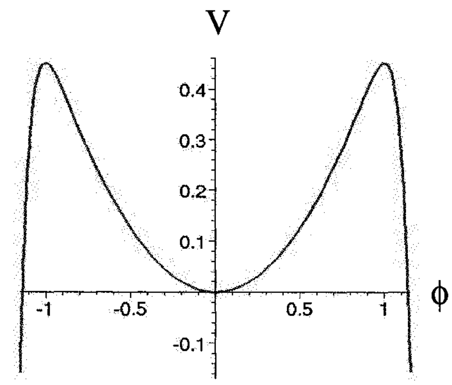


Figure 7.1: The potential of the $p$-adic tachyon for $p = 19$.

The action (7.1) is a simplified model of the bosonic string which only qualitatively reproduces some aspects of a more realistic theory. That being said, there are several nontrivial similarities between $p$-adic string theory and the full string theory. For example, near the true vacuum of the theory $\phi = 0$ the field naively has no particle-like excitations since its mass squared goes to infinity.[2] This is the $p$-adic version of the statement that there are no open string excitations at the tachyon vacuum. A second similarity is the existence of lump-like soliton solutions representing $p$-adic D-branes [65]. The theory of small fluctuations about these lump solutions has a spectrum of equally spaced masses squared for the modes [65],[97], as in the case of normal bosonic string theory.

It should also be noted that the connection between (7.1) and the DBI-type tachyon actions, which have been widely studied in the literature in the context of tachyon matter [43], is not entirely clear (see [220] for a discussion of the relation between $p$-adic and ordinary strings). In the case of tachyon matter, solutions which roll towards the vacuum $T \to \infty$ have late time asymptotics $T \sim t$ and hence the

[2]Reference [87] found anharmonic oscillations around the vacuum by numerically solving the full nonlinear equation of motion. However, these solutions do not correspond to conventional physical states.

tachyon never reaches this point [47], whereas in the case of the $p$-adic string the vacuum is at a finite point in the field configuration space and homogeneous solutions rolling towards the vacuum typically pass this point without difficulty [87] (at least in flat space). In fact, the numerical studies of [87] found *no* homogeneous solutions which appeared to correspond to tachyon matter (vanishing pressure at late times). This issue has been considered in the case of cubic string field theory in [221]. Related rolling tachyon solutions in cubic string field theory have been discussed in [222].

It is worth pointing out that one obtains very similar actions to (7.1) with exponential kinetic operators (and usually assumed to have a cubic or quartic potential) when quantizing strings on random lattice [223]. These field theories are also known to reproduce several features, such as the Regge behaviour [224], of their stringy duals. Although our analysis focuses on the specific $p$-adic action, it can easily be applied to such theories as well. The connection between $p$-adic string theory and ordinary string theory on a discrete lattice was explored in [225].

The field equation that results from (7.1) is

$$e^{-\Box/m_p^2}\,\phi = \phi^p \tag{7.3}$$

We are interested in perturbing around the solution $\phi = 1$, which is a critical point of the potential, representing the unstable tachyonic maximum. For odd $p$ one also has another unstable point at $\phi = -1$, but we will restrict our attention to solutions that start to evolve from the omnipresent $\phi = 1$ maximum. In passing we also note that there is also the stable vacuum of the tachyon, at $\phi = 0$. For both even and odd $p$ the potential is unbounded from below.

It is also worth commenting on the physical interpretation of the fact that the potential is unbounded from below. The instability associated with the decay of the "closed string vacuum" $\phi = 0$ to the "true vacuum" $\phi = \infty$ is thought to be associated with the closed string tachyon instability [215].

Notice that in the limit $p \to 1$ the equation of motion (7.3) becomes a local equation

$$\Box\phi = 2\phi\ln\phi$$

Therefore $p \to 1$ is the limit of local field theory. In the present work we will also consider a very different limit, $p \gg 1$, in which the nonlocal structure of (7.1) is playing an important role in the dynamics.

## 7.3 Absence of Naive Slow Roll Dynamics

One may wonder whether the field theory (7.1) naively allows for slow roll inflation in the conventional sense. Naively one might expect that for a slowly rolling field the higher powers of $\Box$ in the kinetic term are irrelevant and one may approximate (7.1) by a local field theory. The action (7.1) can be rewritten as

$$S = \int d^4x \left[\frac{1}{2}\chi\Box\chi - V(\chi) + \cdots\right] \tag{7.4}$$

where we have defined the field $\chi$ as

$$\chi = \chi_0 \phi \tag{7.5}$$

$$\chi_0 = \frac{p}{g_s}\sqrt{\frac{\ln p}{2(p-1)}}\, m_s \tag{7.6}$$

and the potential is

$$V(\chi) = \frac{m_s^2}{\ln p}\chi^2 - \frac{m_s^4}{g_s^2}\frac{p^2}{p^2-1}\left(\frac{\chi}{\chi_0}\right)^{p+1} \tag{7.7}$$

In (7.4) the $\cdots$ denotes terms with higher powers of $\Box$. The truncation (7.4) is quite analogous to what is usually performed in the literature when one studies inflation from the string theory tachyon [214]. In the case of the usual string theory tachyon corrections involving $\Box^2$ and higher are expected, but the full infinite series of higher derivative terms is not known explicitly (see, however, [94] for a calculation of the string theory tachyon action up to order $\partial^6$). Thus, the standard approach is to simply neglect such terms. We will show, however, that under certain circumstances the higher derivative corrections may play an extremely important role in the inflationary dynamics.

Working in the context of the action (7.4) let us consider the slow roll parameters describing the flatness of the potential (7.7) about the unstable maximum $\chi = \chi_0$. It is straightforward to show that

$$\frac{M_p^2}{2}\frac{1}{V(\chi_0)^2}\left(\frac{\partial V(\chi)}{\partial\chi}\right)^2\Bigg|_{\chi=\chi_0} = 0 \tag{7.8}$$

$$M_p^2\frac{1}{V(\chi_0)}\frac{\partial^2 V(\chi)}{\partial\chi^2}\Bigg|_{\chi=\chi_0} = -\frac{4g_s^2}{\ln p}\frac{p^2-1}{p^2}\left(\frac{M_p}{m_s}\right)^2 \tag{7.9}$$

Thus one would naively expect that inflation is only possible in the theory (7.1) for $g_s M_p/(\sqrt{\ln p}\, m_s) \ll 1$. For $p = \mathcal{O}(1)$ we will see that this expectation is correct, however, for $p \gg 1$ this intuition is incorrect and that successful inflation can occur even with $M_p^2 V^{-1}\partial^2 V/\partial\chi^2 \sim -10^{16}$! The reason for this surprising result is that the dynamics of the $p$-adic tachyon field is set by the mass scale which appears in the kinetic term, $m_s$, rather than the mass scale which is naively implied by the potential

$$m_\chi^2(\chi_0) \equiv \frac{\partial^2 V}{\partial\chi^2}\Bigg|_{\chi=\chi_0} = -\frac{2(p-1)}{\ln p}m_s^2 \tag{7.10}$$

Clearly for $p \gg 1$ we have $|m_\chi^2| \gg m_s^2$.

## 7.4 Approximate Solutions: Analytical techniques

In this section we construct the approximate solutions for the scalar field and the quasi-de Sitter expansion of the universe, in which $\phi$ starts near the unstable maximum ($\phi = 1$) of its potential and rolls slowly toward the minimum ($\phi = 0$). Cosmological solutions of similar nonlocal theories have also been considered in [226].

We use two different formalisms to construct inflationary solutions. We first devise a perturbative expansion in $e^{\lambda t}$ similar to what was carried out in [87] to study rolling solutions in flat spacetime. Our second formalism is the analogue of the usual slow roll approximation: we assume that the friction term in the $\Box$ operator dominates over the acceleration term and also neglect the time variation of $H$.

We first discuss the perturbative expansion in powers of $e^{\lambda t}$. Our starting point is the ansatz

$$\phi(t) = 1 - \sum_{r=1}^{\infty}\phi_r e^{r\lambda t} \tag{7.11}$$

and

$$H(t) = H_0 - \sum_{r=1}^{\infty} H_r e^{r\lambda t} \qquad (7.12)$$

We have chosen the parameterisation such that at $t \to -\infty$, $\phi$ starts from the top of the hill where the universe is undergoing de Sitter expansion with Hubble constant $H_0$. As $t \to -\infty$, $e^{\lambda t} \to 0$ and all the correction terms, which quantify the departure from the pure de Sitter phase, vanish. As $t$ increases, the field rolls toward the true vacuum $\phi = 0$, in fact reaching it at a finite time. Classically, the model admits infinitely many e-foldings of inflation, although only the last 60 e-foldings before the end of inflation are relevant for observation. This idealized behaviour is an artifact of neglecting quantum fluctuations; quantum mechanically the field cannot sit at $\phi = 1$ for an infinite amount of time. We will return to this issue later and show that the inclusion of quantum fluctuations does not spoil inflation, which it would in ordinary local field theory if the $\eta$ parameter is large.

Given the ansatz (7.11,7.12), we can expand the field equation for the $p$-adic scalar (7.3) and the Friedmann equation as a series in $e^{\lambda t}$ and then determine the coefficients $\{\phi_r, H_r\}$ systematically, order by order. We will show that this can be done consistently. The zeroth order Klein-Gordon equation is trivially satisfied, by virtue of the fact that we start from a maximum of the potential.

### 7.4.1 $p$-adic Scalar Field Evolution

Let us first find an approximate solution for the scalar field equation of motion (7.3). We note that to compute quantities such as $\Box^n e^{\lambda t}$ to the order of interest, it is sufficient to use a truncation of (7.11) and (7.12):

$$\phi = 1 - u - \phi_2 u^2, \qquad H = H_0 - H_1 u - H_2 u^2 \qquad (7.13)$$

where we have used the freedom to choose the origin of time to set $\phi_1 \equiv 1$, and for convenience we have defined the new variable

$$u \equiv e^{\lambda t} \qquad (7.14)$$

in terms of which the $\Box$ operator takes the form

$$\Box = -\lambda^2 \left[ u^2 \frac{\partial^2}{\partial u^2} + \left(1 + \frac{3H}{\lambda}\right) u \frac{\partial}{\partial u} \right] \tag{7.15}$$

We wish to compute quantities such a $\Box^n e^{\lambda t}$ up to $\mathcal{O}(u^2)$. This can be done recursively. Writing $\Box^n \phi$ (where $n \geq 1$) as

$$(-\Box)^n \phi = A_n u + B_n u^2 + \ldots \tag{7.16}$$

and applying another $\Box$ operator one finds the following recursion relations for the coefficients $A_n$ and $B_n$:

$$A_{n+1} = A_n(\lambda^2 + 3H_0\lambda) \tag{7.17}$$

and

$$B_{n+1} = B_n(4\lambda^2 + 6H_0\lambda) - 3H_1\lambda A_n \tag{7.18}$$

Equation (7.17) has the solution

$$A_n = a_1 \left(\lambda^2 + 3H_0\lambda\right)^n \tag{7.19}$$

with $a_1 = -1$ (from examination of $\Box\phi$) while a suitable ansatz for $B_n$ is given by

$$B_n = b_1(\lambda^2 + 3H_0\lambda)^n + b_2(4\lambda^2 + 6H_0\lambda)^n \tag{7.20}$$

The coefficients $b_1, b_2$ can be deduced from (7.18) using the initial values

$$b_1 = -\frac{H_1}{H_0 + \lambda} \quad \text{and} \quad b_2 = \frac{H_1}{H_0 + \lambda} - \phi_2 \tag{7.21}$$

which follow from explicitly computing $\Box\phi$ and $\Box^2\phi$. Putting everything together we now have

$$\begin{aligned}(-\Box)^n \phi &= \delta_{n,0} - (\lambda^2 + 3H_0\lambda)^n u + \left[ \left( \frac{H_1}{H_0 + \lambda} - \phi_2 \right) (4\lambda^2 + 6H_0\lambda)^n \right. \\ &\quad \left. - \frac{H_1}{H_0 + \lambda} (\lambda^2 + 3H_0\lambda)^n \right] u^2 + \mathcal{O}(u^3)\end{aligned} \tag{7.22}$$

which works also for the case $n = 0$. Using (7.22) one can resum the contributions coming from all the powers of $\Box$ in the exponential operator $e^{-\Box/m_p^2}$ to give

$$e^{-\Box/m_p^2} \phi = 1 - e^{\mu_1} u + \left[ (\sigma - \phi_2)\, e^{\mu_2} - \sigma e^{\mu_1} \right] u^2 + \cdots \tag{7.23}$$

where we have introduced

$$\mu_1 \equiv \frac{\lambda^2 + 3H_0\lambda}{m_p^2} \quad , \quad \mu_2 \equiv \frac{4\lambda^2 + 6H_0\lambda}{m_p^2} \quad \text{and} \quad \sigma \equiv \frac{H_1}{H_0 + \lambda} \tag{7.24}$$

We conjecture that such resummations are possible for higher order terms as well. Notice that (7.23) reduces to $\phi$ in the limit $m_p \to \infty$, as it should.

To solve the equation of motion for the scalar field, we must equate (7.23) to the right-hand-side of (7.3):

$$\phi^p = [1 - u - \phi_2 u^2]^p = 1 - pu - p\left(\phi_2 - \frac{p-1}{2}\right)u^2 + \cdots \tag{7.25}$$

matching coefficients for each order in $u$. The zeroth order equation is identically satisfied, as promised earlier, while matching at first order gives

$$e^{\mu_1} = e^{(\lambda^2 + 3H_0)/m_p^2} = p \tag{7.26}$$

which, using (7.2) can be rewritten in the form

$$\lambda^2 + 3H_0\lambda = 2m_s^2 \tag{7.27}$$

which is independent of $p$. Later we will see that $H_0 \gg m_s$ is necessary for getting inflation, so the solution of (7.27) is approximately

$$\lambda \cong \frac{2m_s^2}{3H_0} \tag{7.28}$$

Finally, matching coefficients at second order gives

$$\sigma e^{\mu_1} + (\phi_2 - \sigma)\, e^{\mu_2} = p\left(\phi_2 - \frac{p-1}{2}\right) \tag{7.29}$$

In summary, by solving the scalar field equation of motion (7.3) to second order in the expansion in powers of $u = e^{\lambda t}$, we have obtained two relations, (7.27) and (7.29). The former determines the parameter $\lambda$ while the latter determines $\phi_2$.

### 7.4.2 The Stress Energy Tensor and the Friedmann Equation

To complete our approximate solution for the classical background, we must solve the Friedmann equation

$$H^2 = \frac{1}{3M_p^2}\,\rho_\phi \tag{7.30}$$

to second order in $u$. To find the energy density $\rho_\phi$, we turn to the stress energy tensor for the $p$-adic scalar field. A convenient expression for $T_{\mu\nu}$ was derived in [227] (see also [228])

$$T_{\mu\nu} = \frac{m_s^4}{2g_p^2} g_{\mu\nu} \left[ \phi e^{-\frac{\Box}{m_p^2}} \phi - \frac{2}{p+1} \phi^{p+1} + \frac{1}{m_p^2} \int_0^1 d\tau \left( \Box e^{-\frac{\tau\Box}{m_p^2}} \phi \right) \left( e^{-\frac{(1-\tau)\Box}{m_p^2}} \phi \right) \right. \tag{7.31}$$
$$\left. + \frac{1}{m_p^2} \int_0^1 d\tau \left( \partial_\alpha e^{-\frac{\tau\Box}{m_p^2}} \phi \right) \left( \partial^\alpha e^{-\frac{(1-\tau)\Box}{m_p^2}} \phi \right) \right] - \frac{m_s^4}{m_p^2 g_p^2} \int_0^1 d\tau \left( \partial_\mu e^{-\frac{\tau\Box}{m_p^2}} \phi \right) \left( \partial_\nu e^{-\frac{(1-\tau)\Box}{m_p^2}} \phi \right)$$

One may verify that the $T_{\mu\nu}$ is symmetric by changing the dummy integration variable $\tau \to 1-\tau$ in the last term. For homogeneous $\phi(t)$ the above expression simplifies, and for $T_{00}$ we find

$$\rho_\phi = -T_{00} = \frac{m_s^4}{2g_p^2} \left[ \phi e^{-\frac{\Box}{m_p^2}} \phi - \frac{2}{p+1} \phi^{p+1} + \frac{1}{m_p^2} \int_0^1 d\tau \left( \Box e^{-\frac{\tau\Box}{m_p^2}} \phi \right) \left( e^{-\frac{(1-\tau)\Box}{m_p^2}} \phi \right) \right.$$
$$\left. + \frac{1}{m_p^2} \int_0^1 d\tau\, \partial_t \left( e^{-\frac{\tau\Box}{m_p^2}} \phi \right) \partial_t \left( e^{-\frac{(1-\tau)\Box}{m_p^2}} \phi \right) \right] \tag{7.32}$$

One can evaluate the above expression term by term, keeping up to $\mathcal{O}(e^{2\lambda t}) \sim u^2$. The final result reads

$$\begin{aligned} T_{00} &= \frac{m_s^4}{2g_p^2} \left[ 1 - u(1+e^{\mu_1}) - \frac{2[1-(p+1)u]}{p+1} + u(e^{\mu_1}-1) \right] + \mathcal{O}(u^2) \\ &= \frac{m_s^4(p-1)}{2g_p^2(p+1)} + \mathcal{O}(u^2) \end{aligned} \tag{7.33}$$

The $\mathcal{O}(u)$ terms cancel out and matching the coefficients in the Friedmann equation gives us the simple results

$$H_0^2 = \frac{m_s^4}{6M_p^2} \frac{p-1}{g_p^2(p+1)} \tag{7.34}$$

and

$$H_1 = 0 \tag{7.35}$$

for zeroth and first order respectively.

The $\mathcal{O}(u^2)$ contribution to $T_{00}$ is quite complicated (see appendix E-1) but once we use (7.35) it simplifies greatly. Matching coefficient at order $\mathcal{O}(u^2)$ in the Friedmann equation gives

$$H_2 = \frac{\lambda m_s^4}{4g_p^2 m_p^2 M_p^2} e^{\mu_1} = \frac{1}{8g_s^2} \frac{p^3 \ln p}{p-1} \left( \frac{m_s}{M_p} \right)^2 \lambda \tag{7.36}$$

Because of our sign convention for $H_r$, the fact that $H_2 > 0$ means that the expansion is slowing as $\phi$ rolls from the unstable maximum, as one would expect in a conventional inflationary model.

Using (7.28), (7.34) and (7.36) to compute $H_2/H_0 \sim p \ln p$ it is clear that the perturbative expansion in $u$ breaks down once $u \sim (p \ln p)^{-1/2}$ (recall that $H \cong H_0 - H_2 u^2$). Thus one expects that once $u \sim p^{-1/2}$ then inflation ends. We verify this claim using an alternative formalism in the next subsection.

To summarize, we have determined the five parameters $\lambda, \phi_2, H_0, H_1$ and $H_2$ which appear in the solutions for $\phi(t)$ and $H(t)$ up to $\mathcal{O}(u^2)$ through the equations (7.27-7.29), (7.34-7.36). As a check of our result, we can take $H_0 \ll m_s$ and compare it to the the Minkowski background solution that was found in [87]. For $H_0 \ll m_s$ we see from (7.27) that

$$\lambda \cong \sqrt{2} m_s \left(1 - \frac{3H_0}{2\sqrt{2} m_s}\right) \tag{7.37}$$

the first term corresponding to the known Minkowski result. We can also compute the coefficient $\phi_2$ in this limit from (7.29),

$$\phi_2 \cong -\frac{1}{2(p^2 + p + 1)} \tag{7.38}$$

This too coincides with the coefficient that was determined in [87] for $p = 2$.

### 7.4.3 The Friction-Dominated Approximation

In the previous subsections we have constructed an approximate solution for the $p$-adic scalar rolling down its potential by performing an expansion of $\phi$, $H$ in a power series in $u$. Furthermore, we have shown that once $u \sim p^{-1/2}$ then this solution breaks down (because the $\mathcal{O}(u^2)$ term in $H(t)$ become larger than the zeroth order term, $H_0$). At this point inflation has ended. Because the equations of motion are complicated, we now verify this behaviour using an alternative formalism which does not rely on small $u$.

The method is the same as the slow-roll approximation in ordinary inflation, which assumes that $\dot{\phi} \ll H\phi$. To justify it within the $p$-adic theory, we will provisionally assume that

$$\lambda^2 \ll 3H_0\lambda$$

so that the evolution is friction-dominated in the usual sense. The consistency of this approximation will be established when we match the theory to the observables from the inflationary power spectrum later, in eqs. (7.70), (7.71). It follows that it is a good approximation to take

$$-\Box \cong 3H_0\partial_t$$

Then the $p$-adic scalar field equation becomes

$$e^{\alpha\partial_t}\phi = \phi^p \tag{7.39}$$

where we have defined

$$\alpha \equiv \frac{3H_0}{m_p^2} \tag{7.40}$$

Our procedure is to treat $H_0$ as exactly constant, solve for $\phi(t)$ and compute the energy density $\rho_\phi$. If $\rho_\phi$ is approximately constant then this series of approximations is self-consistent and the solution is reliable. Once $\rho_\phi$ begins to deviate significantly from a constant value then the solution breaks down and we conclude that inflation has ended.

We now proceed to solve (7.39) for constant $H_0$. To this end we expand $\phi(t)$ in Fourier modes as

$$\phi(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} dke^{-ikt}\phi_k \tag{7.41}$$

so that

$$\begin{aligned} e^{\alpha\partial_t}\phi(t) &= \frac{e^{\alpha\partial_t}}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} dke^{-ikt}\phi_k & (7.42)\\ &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} dke^{-ik(t+\alpha)}\phi_k & (7.43)\\ &= \phi(t+\alpha) & (7.44) \end{aligned}$$

The $p$-adic scalar equation of motion (7.39) then takes the simple form

$$\phi(t+\alpha) = \phi(t)^p \tag{7.45}$$

It is quite remarkable that this equation admits dynamical solutions. It is straightforward to check that (7.45) has the solution

$$\phi(t) = e^{-u(t)} \tag{7.46}$$

where $u(t) = e^{\lambda t}$ and $\lambda = \alpha^{-1}\ln p = 2m_s^2/(3H_0)$, as before. One can easily check by acting on (7.46) with the full operator $-\Box = \partial_t^2 + 3H_0\partial_t$ that indeed the friction term dominates as long as $u \lesssim 1$.

Performing a Taylor expansion of (7.46) about $u = 0$ gives

$$\phi \cong 1 - u + \frac{1}{2}u^2 + \cdots$$

which reproduces our solution in the small-$u$ expansion in the limit that $m_s \ll H_0$[3] (see equations 7.13 and 7.72). We also have

$$e^{-\Box/m_p^2}\phi = e^{-up} \cong 1 - pu + \frac{1}{2}p^2u^2$$

which again reproduces our previous results (which can be verified by inserting the solutions for $\mu_1$, $\mu_2$, $\sigma$ and $\phi_2$ into equation 7.23).

We now proceed to construct the energy density for $\phi(t)$ in this approximation. It is straightforward to show that

$$\begin{aligned} e^{-\tau\Box/m_p^2}\phi &= \exp\left(-up^{\tau}\right) \\ e^{-(1-\tau)\Box/m_p^2}\phi &= \exp\left(-up^{1-\tau}\right) \end{aligned} \tag{7.47}$$

We write $\rho_\phi$ (7.32) as

$$\rho_\phi \equiv \frac{m_s^4}{2g_p^2}\left[T_1 + T_2 + T_3 + T_4\right] \tag{7.48}$$

where

$$T_1 \equiv \phi e^{-\Box/m_p^2}\phi \tag{7.49}$$

$$T_2 \equiv -\frac{2}{p+1}\phi^{p+1} \tag{7.50}$$

$$T_3 \equiv \frac{1}{m_p^2}\int_0^1 d\tau \left(\Box e^{-\frac{\tau\Box}{m_p^2}}\phi\right)\left(e^{-\frac{(1-\tau)\Box}{m_p^2}}\phi\right) \tag{7.51}$$

$$T_4 \equiv \frac{1}{m_p^2}\int_0^1 d\tau \partial_t\left(e^{-\frac{\tau\Box}{m_p^2}}\phi\right)\partial_t\left(e^{-\frac{(1-\tau)\Box}{m_p^2}}\phi\right) \tag{7.52}$$

---

[3] We will see later that this is the same as taking the spectral index equal to unity $n_s \to 1$.

Using the scalar field equation (7.3) and (7.46) the first two terms, (7.49) and (7.50), are trivial

$$T_1 + T_2 = \frac{p-1}{p+1} e^{-up-u} \tag{7.53}$$

Since this term is proportional to $\phi^{p+1}$ we identify it with the potential energy of the $p$-adic scalar.

The next simplest term to evaluate is $T_4$ (7.52) which gives

$$T_4 = \frac{\lambda^2 u^2}{2m_s^2} p \ln p \int_0^1 d\tau \exp\left[-u\left(p^\tau + p^{1-\tau}\right)\right]$$

It is useful to change the variable of integration to $\omega = up^\tau$ and cast this result in the form

$$T_4 = \frac{\lambda^2}{2m_s^2}(pu^2) \int_u^{up} \frac{d\omega}{\omega} \exp\left[-\omega - \frac{pu^2}{\omega}\right] \tag{7.54}$$

The $d\omega$ integral can be performed exactly (though not in closed form) in terms of special functions. Since $T_4$ is subdominant to a contribution coming from $T_3$ we will not investigate the behaviour of (7.54) any further.

We now study $T_3$ (7.51). This can be written in the form

$$\begin{aligned} T_3 &= -\frac{\lambda^2}{2m_s^2} \int_u^{up} d\omega\omega \exp\left[-\omega - \frac{pu^2}{\omega}\right] \\ &+ \frac{\lambda^2}{2m_s^2}\left(1 + \frac{3H_0}{\lambda}\right) \int_u^{up} d\omega \exp\left[-\omega - \frac{pu^2}{\omega}\right] \end{aligned} \tag{7.55}$$

The dominant contribution to $T_3$ is the one proportional to $H_0$ since the evolution is friction-dominated. This term is also larger than $T_4$. The leading contribution to $T_3 + T_4$ is then

$$T_3 + T_4 \cong \int_u^{up} d\omega \exp\left[-\omega - \frac{pu^2}{\omega}\right] \tag{7.56}$$

where we have used the fact that $3H_0\lambda/(2m_s^2) \cong 1$ which follows from (7.27) when $\lambda^2 \ll 3H_0\lambda$. Since $T_3$ and $T_4$ contain time derivatives acting on $\phi$ it is natural to identify (7.56) with the kinetic energy of the $p$-adic scalar.

Let us study the behaviour of the kinetic energy, equation (7.56), as a function of $u$. The integral in (7.56) can be performed in terms of the incomplete cylindrical functions of the Sonine-Schlaefli form [230] (see appendix E-2 for a review).

$$\int_u^{up} d\omega \exp\left[-\omega - \frac{pu^2}{\omega}\right] = 2\pi u p^{1/2} S_{-1}(-u, -up; 2iup^{1/2})$$

We now study the behaviour of this integral in various limits. We assume that $p \gg 1$ throughout since the previous method of expanding in $u$ is valid in the case where $p \sim 1$. At very early times, $u < 1/p$, this integral goes to zero as

$$\int_u^{up} d\omega \exp\left[-\omega - \frac{pu^2}{\omega}\right] \cong up \quad \text{for} \quad u < 1/p$$

For intermediate times, $1/p \ll u \lesssim 1/p^{1/2}$, it is a good approximation to treat the upper and lower limits of integration as 0 and $+\infty$ respectively. In this approximation we can write the incomplete cylinder function in terms of a Hankel function of order $\nu = -1$ (see appendix E-2 for details). The small argument asymptotics of $H_{-1}^{(1)}$ gives

$$\int_u^{up} d\omega \exp\left[-\omega - \frac{pu^2}{\omega}\right] \cong 1 + 2u^2 p \ln(2up^{1/2}) \quad \text{for} \quad 1/p \ll u < 1/p^{1/2}$$

Finally we consider late times, $u > 1/p^{1/2}$. It is still reasonable to extend the integral as $\int_u^{up} d\omega \cong \int_0^{\infty} d\omega$ and hence the integral can still be written in terms of $H_{-1}^{(1)}$. This time the large-argument asymptotics of the Hankel function are appropriate and one has

$$\int_u^{up} d\omega \exp\left[-\omega - \frac{pu^2}{\omega}\right] \cong \sqrt{\pi u p^{1/2}}\, e^{-2up^{1/2}} \quad \text{for} \quad u > 1/p^{1/2}$$

We have verified these asymptotic expressions numerically.

We can now write the dominant contribution to $\rho_\phi$ in the friction-dominated approximation,

$$\rho_\phi = \frac{m_s^4}{2g_s^2}\frac{p^2}{p-1}\left(\frac{p-1}{p+1}e^{-up-u} + \int_u^{up} d\omega \exp\left[-\omega - \frac{pu^2}{\omega}\right]\right) \tag{7.57}$$

The first term, proportional to $e^{-up}$, represents the potential energy and the second term represents the kinetic energy. Using our previous analysis of the kinetic energy the behaviour of $\rho_\phi$ as a function of $u$ (assuming $p \gg 1$) is clear. At early times $u < 1/p$ the potential energy dominates and we have $\rho_\phi \cong m_s^4/(2g_p^2)$. At intermediate times $1/p \ll u \lesssim 1/p^{1/2}$ the potential energy goes to zero and the kinetic energy dominates and we have $\rho \cong m_s^4/(2g_p^2)$. At late times $u > 1/p^{1/2}$ and $\rho_\phi$ damps to zero as $e^{-2up^{1/2}}$. We have verified this behaviour numerically. Figure 7.2 shows the behaviour of $\rho_\phi$ as a function of $t$, verifying that $\rho_\phi$ is approximately constant for $u < p^{-1/2}$. The time evolution of both the potential energy and the kinetic energy contributions to $\rho_\phi$ are shown, demonstrating that the latter dominates in the interval $p^{-1} < u < p^{-1/2}$. In this figure we have taken $p = 10^5$ for illustrative purposes.

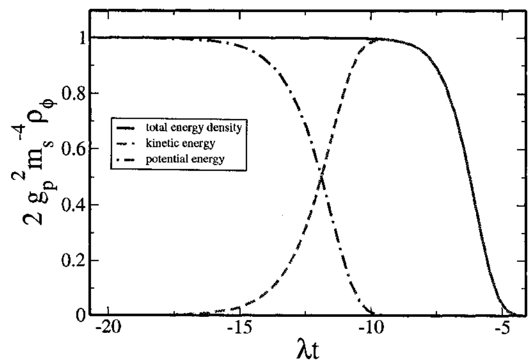


Figure 7.2: Plot of the energy density of the $p$-adic scalar as a function of $t$, for $p = 10^5$. This figure shows that $\rho_\phi$ is approximately constant for $u < p^{-1/2}$. The individual contributions from the potential and kinetic energy are also shown.

We conclude that for $u \lesssim 1/p^{1/2}$ the scalar field energy density $\rho_\phi$ is approximately constant and inflation proceeds. At $u = 1/p^{1/2}$ the energy density of the $p$-adic scalar begins to decrease quickly and the analysis of this subsection is no longer applicable. We conclude that inflation ends, roughly, when $u = 1/p^{1/2}$.

It is quite interesting that during the intermediate phase $p^{-1} < u < p^{-1/2}$ it is in fact the *kinetic* energy which is driving inflation, rather than the potential energy. This is quite different from what occurs in a local field theory.

## 7.5 Fluctuations and Inflationary Predictions

In this section we consider the spectrum of cosmological fluctuations produced during $p$-adic inflation. The full cosmological perturbation theory for the $p$-adic string model (7.1), which should include metric perturbations and also take into account the departure of the background expansion from pure de Sitter, is complicated and beyond the scope of the present chapter. We leave a detailed study of these matters to future investigation [235]. To simplify the analysis we will neglect scalar metric perturbations as well as deviations of the background metric from pure de Sitter space during horizon crossing. In standard cosmological perturbation theory these approximations reproduce the more exact results to reasonable accuracy and we therefore assume that the situation is similar for the action (7.1).

### 7.5.1 $p$-adic Tachyon Fluctuations

We are approximating the background dynamics as de Sitter which amounts to working in the limit $u \longrightarrow 0$ so that

$$H^2 \equiv H_0^2 = \frac{m_s^4}{6M_p^2} \frac{p-1}{g_p^2(p+1)} \tag{7.58}$$

$$\phi \equiv \phi_0 \equiv 1 \tag{7.59}$$

We expand the $p$-adic tachyon field in perturbation theory as

$$\begin{aligned} \phi(t,\vec{x}) &= \phi^{(0)}(t) + \delta\phi(t,\vec{x}) \\ &= 1 + \delta\phi(t,\vec{x}) \end{aligned}$$

The perturbed Klein-Gordon equation (7.3) takes the form

$$e^{-\Box/m_p^2}\,\delta\phi = p\,\delta\phi \tag{7.60}$$

(Inhomogeneous solutions in $p$-adic string theory have also been considered in [2].) One can construct solutions by taking $\delta\phi$ to be an eigenfunction of the $\Box$ operator. If we choose $\delta\phi$ to satisfy

$$-\Box\delta\phi = +B\,\delta\phi \tag{7.61}$$

then this is also a solution to (7.60) if

$$B = m_p^2 \ln p = 2m_s^2 \tag{7.62}$$

where in the second equality we have used (7.2).

The solutions of (7.61) are well known. However, in order to make contact with the usual treatment of cosmological perturbations we need to define a field in terms of which the action appears canonical. This presents a serious difficulty because, in general, there is no local field redefinition which will bring the kinetic term

$\phi\left(1-e^{-\Box/m_p^2}\right)\phi$ into the canonical form $\phi\Box\phi$. (One might imagine simply truncating the expansion in powers of $\Box$ as we have described in section 7.3, however, the higher order terms are not negligible in general.) Fortunately, for fields which are on-shell (that is, when (7.61) is solved) the field obeys

$$\begin{aligned}
\left(1-e^{-\Box/m_p^2}\right)\delta\phi &= \left(1-e^{B/m_p^2}\right)\delta\phi \\
&= \left(1-e^{B/m_p^2}\right)\frac{1}{(-B)}(-B)\delta\phi \\
&= \left(1-e^{B/m_p^2}\right)\frac{1}{(-B)}\Box\delta\phi \\
&= \frac{p-1}{2m_s^2}\Box\delta\phi
\end{aligned}$$

Thus, for on-shell fields the kinetic term in the Lagragian can be written as

$$\begin{aligned}
\mathcal{L}_{\text{on-shell}} &= \frac{m_s^4}{g_p^2}\frac{1}{2}\phi(1-e^{-\Box/m_p^2})\phi+\cdots \\
&= \frac{m_s^4}{g_p^2}\frac{p-1}{2m_s^2}\frac{1}{2}\phi\Box\phi+\cdots \\
&= \frac{1}{2}\varphi\Box\varphi+\cdots
\end{aligned} \tag{7.63}$$

In (7.63) we have defined the "canonical" field

$$\varphi \equiv A\phi \tag{7.64}$$

where

$$A \equiv \frac{m_s p}{\sqrt{2}\,g_s} \tag{7.65}$$

The field $\varphi$ has a canonical kinetic term in the action, at least while (7.61) is satisfied. Notice that $\varphi$ is distinct from the field $\chi$ (see (7.5)) which we introduced in section 7.3. The field $\chi$ corresponds to the canonical field which one would naively define when neglecting terms $\mathcal{O}(\Box^2)$ and higher in the action (as is typical in studies of tachyonic inflation) while $\varphi$ is the appropriate definition of the canonically normalized field when taking into account the infinite series of higher derivative corrections.

Now, let us return to the task of solving (7.61), bearing in mind that $\delta\varphi = A\delta\phi$ is the appropriate canonically normalized field. We write the quantum mechanical solution in term of annihilation/creation operators as

$$\delta\varphi(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} \left[a_k \varphi_k(t) e^{ikx} + \text{h.c.}\right]$$

and the mode functions $\varphi_k(t)$ are given by

$$\varphi_k(t) = \frac{1}{2}\sqrt{\frac{\pi}{a^3 H_0}}\, e^{\frac{i\pi}{2}(\nu+1/2)}\, H_\nu^{(1)}\left(\frac{k}{aH_0}\right) \tag{7.66}$$

where the order of the Hankel functions is

$$\nu = \sqrt{\frac{9}{4} + \frac{B}{H_0^2}} = \sqrt{\frac{9}{4} + \frac{2m_s^2}{H_0^2}} \tag{7.67}$$

and of course $a = e^{H_0 t}$. In the second equality in (7.67) we have used (7.62) and (7.2). In writing (7.66) we have used the usual Bunch-Davies vacuum normalization so that on small scales, $k \gg aH_0$, one has

$$|\varphi_k| \cong \frac{a^{-1}}{\sqrt{2k}}$$

which reproduces the standard Minkowski space fluctuations. This is the usual procedure in cosmological perturbation theory. However, we note that the quantization of the theory (7.1) is not transparent and it might turn out that the usual prescription is incorrect in the present context. We defer this and other subtleties to future investigation.[4] On large scales, $k \ll aH_0$, the solutions (7.66) behave as

$$|\varphi_k| \cong \frac{H}{\sqrt{2k^3}} \left(\frac{k}{aH_0}\right)^{3/2-\nu}$$

which gives a large-scale power spectrum for the fluctuations

$$P_{\delta\varphi} = \left(\frac{H_0}{2\pi}\right)^2 \left(\frac{k}{aH_0}\right)^{n_s-1}$$

with spectral index

$$n_s - 1 = 3 - 2\nu$$

[4]Our prescription for choosing the vacuum has the property that in the local limit $p \to 1$ the cosmological fluctuations are identical to the well-known solutions in local field theory.

From (7.67) it is clear that to get an almost scale-invariant spectrum we require $m_s \ll H_0$. In this limit we have

$$n_s - 1 \cong -\frac{4}{3}\left(\frac{m_s}{H_0}\right)^2 \tag{7.68}$$

which gives a red tilt to the spectrum, in agreement with the latest WMAP data [16]. For $n_s \cong 0.95$ one has $m_s \cong 0.2 H_0$. Comparing (7.66) to the corresponding solution in a local field theory we see that the $p$-adic tachyon field fluctuations evolve as though the mass-squared of the field was $-2m_s^2$ which may be quite different from the mass scale which one would infer by truncating the infinite series of derivatives: $\partial^2 V/\partial\chi^2(\chi = \chi_0)$ (see eq. (7.10)). The fact that $H_0 > m_s$ is an unusual feature; we will comment on it below.

It is worth pointing out that we have constructed solutions of a partial differential equation with infinitely many derivatives, for which we are free to specify two initial data; to obtain eq. 7.66 we fixed these using the Bunch-Davies prescription. Precisely the same result was obtained when inhomogeneous solutions were studied in [2]. Partial differential equations with infinitely many derivatives constitute a new class of equations in mathematical physics about which little is presently known. In particular, it is not clear how to pose the intial value problem for such equations. We conjecture that the most general solutions of (7.3) are specified by two initial data (for example $\phi(0,\vec{x})$ and $\dot{\phi}(0,\vec{x})$), just like equations containing only one power of $\Box$. See [229] for mathematical work on constructing solutions of equations with infinitely many derivatives.

### 7.5.2 Determining Parameters

We now want to fix the parameters of the model by comparing to the observed features of the CMB perturbation spectrum. There are three dimensionless parameters, $g_s$, $p$ and the ratio $m_s/M_p$. The important question is whether there is a sensible parameter range which can account for CMB observations, *i.e.* , the spectral tilt and the amplitude of fluctuations. Using (7.34) in (7.68), we can relate the tilt to the model parameters via

$$|n_s - 1| = \frac{8(p+1)}{p^2}\left(\frac{M_p}{m_s}\right)^2 g_s^2 \quad \Longleftrightarrow \quad \left(\frac{m_s}{M_p}\right)^2 = \frac{8(p+1)}{p^2}\frac{g_s^2}{|n_s - 1|} \tag{7.69}$$

Thus one can have a small tilt while ensuring that the string scale is smaller than the Planck scale, provided that $g_s^2/p \ll 1$. Henceforth we will use (7.69) to determine $m_s/M_p$ in terms of $p$, $g_s$, and $|n_s - 1| \cong 0.05$. All the dimensionless parameters in our solution, $m_s/H_0$, $\lambda/m_s$, $\phi_2$ and $H_2/m_s$, are likewise functions of $n_s - 1, p$ and $g_s$. From (7.34) and (7.69) we see that for $p \gg 1$,

$$\frac{m_s}{H_0} \cong \sqrt{6}\, g_p \frac{M_p}{m_s} \cong g_s\sqrt{\frac{6}{p}}\frac{M_p}{m_s} \cong \frac{1}{2}\sqrt{3|n_s - 1|} \tag{7.70}$$

It may seem strange to have $H$ exceeding $m_s$ since that means the energy density exceeds the fundamental scale, but this is an inevitable property of the $p$-adic tachyon at its maximum, as shown in eq. (7.33). This is similar to other attempts to get tachyonic or brane-antibrane inflation from string theory, since the false vacuum energy is just the brane tension which goes like $m_s^4/g_s$.

Next we determine $\lambda/m_s$, where $\lambda$ is the mass scale appearing in the power series in $e^{\lambda t}$ which provides the ansatz for the background solutions. Consider eq. (7.26) for $\lambda$ in the $H_0 \gg m_s$ limit. The positive root for $\lambda$ gives

$$\frac{\lambda}{m_s} \cong \sqrt{\frac{|n_s - 1|}{3}} \tag{7.71}$$

From (7.70) and (7.71) we find that $3H_0\lambda \gg \lambda^2$ which means that the evolution is friction-dominated in the usual sense. As for $\phi_2$, from eq. (7.29) it follows that

$$\phi_2 \cong -\frac{1}{2}\frac{p-1}{p^{1+\beta} - 1} \tag{7.72}$$

with

$$\beta \cong |n_s - 1|/3 + \mathcal{O}(|n_s - 1|^2)$$

so that $\beta \ll 1$. Notice that for $p \gg 1$, we have $\phi_2 \cong -0.5p^{-\beta}$. Finally we have $H_2$ which from eq. (7.36) is given by

$$\frac{H_2}{m_s} = \frac{p}{\sqrt{3}}\frac{p-1}{p+1}\frac{\ln p}{|n_s - 1|^{1/2}} \tag{7.73}$$

To go further, we must impose the COBE normalization on the amplitude of the density perturbations, to show that it is possible to satisfy all the experimental constraints while keeping $m_s/M_p < 1$. The latter requirement is usually needed for the validity of any 4D effective description of string theory. In a compactification of the $d$ extra dimensions whose volume is of order $V_d \sim m_s^{-d}$, we would have $M_p \sim m_s$ whereas more generally $M_p^2 = m_s^{2+d}V_d$. The 4D effective theory would normally need to be supplemented by higher dimension operators if $V_d$ was small compared to $m_s^{-d}$.

### 7.5.3 Curvature Perturbation and COBE normalization

In order to fix the amplitude of the density perturbations we consider the curvature perturbation $\zeta$. We assume that

$$\zeta \sim -\frac{H}{\dot{\varphi}}\delta\varphi$$

as in conventional inflation models. To evaluate the prefactor $H/\dot{\varphi}$ we must work beyond zeroth order in the small $u$ expansion. We take $\phi = 1 - u$ to evaluate the prefactor, even though the perturbation $\delta\chi$ is computed in the limit that $\phi = 1$. This should reproduce the full answer up to $\mathcal{O}(u)$ corrections. The prefactor is

$$\begin{aligned} -\frac{H}{\dot{\varphi}} &\cong \frac{H_0}{A\lambda u} \\ &\cong \frac{2^{3/2} g_s}{p} \frac{1}{|n_s - 1|} \frac{1}{u} m_s^{-1} \end{aligned}$$

We should evaluate $u$ at the time of horizon crossing, $t_\star$, defined to be approximately 60 e-foldings before the end of inflation $t_{\rm end}$, assuming that the energy scale of inflation is high (near the GUT scale). We must therefore estimate $t_{\rm end}$. We have shown in the last subsection that inflation ends when $u \sim 1/p^{1/2}$. From eqs. (7.70-7.71) we see that $H_0/\lambda = 2/|n_s - 1|$; therefore we can write the scale factor $a(t) \cong e^{H_0 t}$ in the form

$$a(t) \cong u(t)^{2/|n_s - 1|} \tag{7.74}$$

so that $a_\star = e^{-60} a_{\rm end}$ corresponds to

$$u_\star = e^{-30|n_s - 1|} u_{\rm end} \equiv e^{-30|n_s-1|} \frac{1}{p^{1/2}} \tag{7.75}$$

The power spectrum of the curvature perturbation is given by

$$P_\zeta = \left|\frac{H}{\dot{\varphi}}\right|^2 P_{\delta\varphi} \equiv A_\zeta^2 \left(\frac{k}{aH_0}\right)^{n_s-1} \tag{7.76}$$

where the amplitude of fluctuations $A_\zeta$ can now be read off as

$$A_\zeta^2 = \frac{8}{3\pi^2}\frac{g_s^2}{p}\frac{e^{60|n_s-1|}}{|n_s-1|^3} \tag{7.77}$$

As an example, taking $n_s \cong 0.95$ one can fix the amplitude of the density perturbations $A_\zeta^2 \cong 10^{-10}$ by choosing

$$\frac{g_s}{\sqrt{p}} \cong 0.48 \times 10^{-7} \tag{7.78}$$

To get a more general idea of how the inflationary observables constrain the parameters of the model, we will allow $n_s$ to vary away from the value 0.95, which is a fit to the WMAP data under the assumption that the tensor contribution to the spectrum is negligible. Setting $A_\zeta^2 = 10^{-10}$ and using (7.77) gives and expression for $g_s$ in terms of $p$ and $|n_s - 1|$

$$g_s = \sqrt{\frac{3\pi^2}{8}}\,\sqrt{p}\,e^{-30|n_s-1|}|n_s-1|^{3/2} \times 10^{-5} \tag{7.79}$$

Combining (7.79) with (7.69), we also obtain

$$\frac{m_s}{M_p} = \sqrt{3\pi^2}\sqrt{\frac{p+1}{p}}\,e^{-30|n_s-1|}|n_s-1| \times 10^{-5} \tag{7.80}$$

We graph the dependence of $g_s$ and $m_s/M_p$ on $p$ for several values of the spectral index in figures 7.3 and 7.4. We see that the string scale is bounded from above as $m_s/M_p \lesssim 0.94 \times 10^{-6}$ and that for typical values of $p, n_s$ it is close to $m_s/M_p \cong 0.61 \times 10^{-6}$. We also see from (7.79) that $g_s$ is unconstrained and that $g_s$, $p$ are not independent parameters. If we wish to take $g_s \sim \mathcal{O}(1)$ then we must choose extremely large values $p \sim 10^{14}$. If we restrict ourselves to the perturbative regime $g_s < 1$ then this places an upper bound on $p$:

$$p < 4.3 \times 10^{14}$$

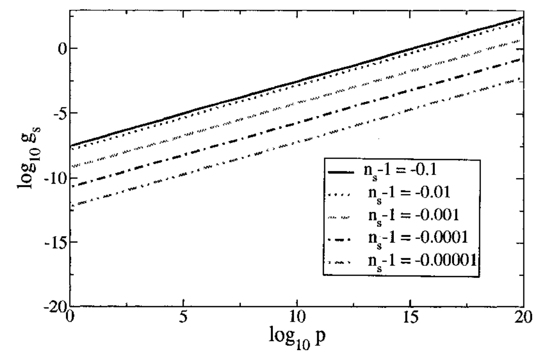


Figure 7.3: Log of the string coupling $g_s$ as a function of $\log_{10} p$ for several values of $n_s$.

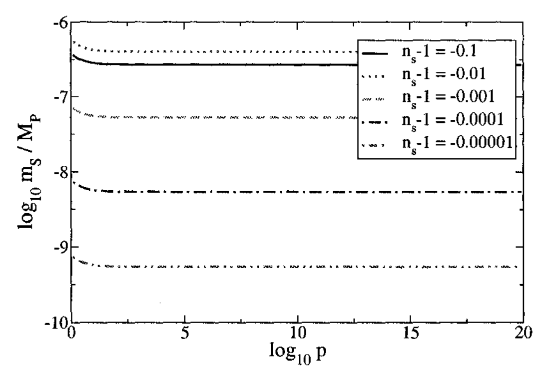


Figure 7.4: Log of the ratio of the string mass scale to the Planck mass as a function of $\log_{10} p$ for several values of $n_s$.

### 7.5.4 Comments on Slow Roll and the Relation to Local Field Theory

It is quite remarkable that our predictions for inflationary observables and also the solutions for $\chi$, $H$ are essentially identical to the results from *local* field theory with potential $V(\psi) = g_s(\psi^2 - v^2)^2/4$ where $g_s v^2 \equiv 2m_s^2$ (see appendix E-3 for a detailed comparison). Indeed, the dynamics of the $p$-adic tachyon are fixed by the mass scale in the kinetic term, $m_s$, rather than by the naive mass scale $\sqrt{-\partial^2 V/\partial\chi^2}$ (see section 7.3), which may be much larger than $m_s$.

It is an interesting feature of this theory that the canonical $p$-adic tachyon, $\phi$, can roll slowly despite the fact that, working in a derivative truncatation (as in section 7.3), one would conclude that the tachyon has an extremely steep potential. To see this we first define the Hubble slow roll parameters $\epsilon_{\mathcal{H}}$, $\eta_{\mathcal{H}}$ by

$$\epsilon_{\mathcal{H}} \equiv \frac{1}{2M_p^2}\frac{\dot{\varphi}^2}{H^2} \tag{7.81}$$

$$\epsilon_{\mathcal{H}} - \eta_{\mathcal{H}} \equiv \frac{\ddot{\varphi}}{H\dot{\varphi}} \tag{7.82}$$

These are the appropriate parameters to describe the rate of time variation of the inflaton as compared to the Hubble scale. Using the solution $\phi \cong 1 - u$ (recall that $\varphi = A\phi$, $A = m_s p/(\sqrt{2}\, g_s)$) we find that

$$\epsilon_{\mathcal{H}} \cong \frac{1}{2}\frac{p+1}{p}e^{-60|n-1|}|n_s - 1| \tag{7.83}$$

$$\eta_{\mathcal{H}} \cong -\frac{|n_s - 1|}{2} \tag{7.84}$$

We see that the Hubble slow-roll parameters, as defined above, are small. This means that the $p$-adic tachyon field rolls slowly in the conventional sense. One reaches the same conclusion if one defines the potential slow roll parameters *using the correct canonical field*, which is $\varphi$ (7.64):

$$\frac{M_p^2}{2}\left(\frac{1}{V}\frac{\partial V}{\partial \varphi}\right)^2\bigg|_{\varphi=A} = 0 \tag{7.85}$$

$$M_p^2\,\frac{1}{V}\frac{\partial^2 V}{\partial \varphi^2}\bigg|_{\varphi=A} = -\frac{1}{2}|n_s - 1| \tag{7.86}$$

On the other hand, consider the potential slow roll parameter which one would naively define using the the derivative truncated action (7.4):

$$\frac{M_p^2}{2}\left(\frac{1}{V}\frac{\partial V}{\partial \chi}\right)^2\Bigg|_{\chi=\chi_0} = 0 \tag{7.87}$$

$$M_p^2\,\frac{1}{V}\frac{\partial^2 V}{\partial \chi^2}\Bigg|_{\chi=\chi_0} = -\left(\frac{p-1}{\ln p}\right)\frac{1}{2}|n_s-1| \tag{7.88}$$

where in (7.88) we have used equations (7.9) and (7.69). We see that (7.88) can be enormous, though the tachyon field rolls slowly. Taking the largest allowed value of $p$, $p \sim 10^{14}$, and $n_s \cong 0.95$ we have $M_p^2 V^{-1}|\partial^2 V/\partial\chi^2| \sim 10^{11}$!

Since large values of $p$ are required if one wants to obtain $g_s \sim 1$, it follows that it is somewhat natural for $p$-adic inflation to operate in the regime where the higher derivative corrections play an important role in the dyanimcs. However, in the regime where $p \sim 1$ (corresponding to very small coupling $g_s$) this novel features is not present. For example, with $p = 3$, $n_s = 0.95$ one has $g_s \sim 10^{-7}$ and $M_p^2 V^{-1}|\partial^2 V/\partial\chi^2| \sim 0.05$ and so the slow-roll dynamics are not surprising.

### 7.5.5 Tensor Modes

Since the $p$-adic stress tensor (7.32) does not contribute any anisotropic stresses up to first order in perturbation theory it follows that the first order tensor perturbations of the metric do not couple to the first order tachyon perturbation. In fact, the action for the tensor perturbations is given by

$$S_{\text{grav}} = \frac{M_p^2}{2}\int dt\, d^3x\, a(t)^3\, \frac{1}{2}\partial_\mu h_{ij}\,\partial^\mu h^{ij}$$

The standard procedure gives a power spectrum for the gravity waves

$$P_T = A_T^2\left(\frac{k}{aH_0}\right)^{n_T}$$

with amplitude

$$A_T^2 = \frac{8}{M_p^2}\left(\frac{H_0}{2\pi}\right)^2 \tag{7.89}$$

We would like to compare this to the power in scalar modes (7.77). Defining the tensor-to-scalar ratio in the usual way

$$r \equiv \frac{\frac{1}{100}A_T^2}{\frac{4}{25}A_\zeta^2}$$

we find that

$$r = \frac{1}{2M_p^2}\frac{\dot{\varphi}^2}{H_0^2} = \epsilon_{\mathcal{H}} \tag{7.90}$$

which reproduces the usual result from local field theory. Using (7.83), we can evaluate $r$ as a function of $p$ and $n_s - 1$

$$r = \frac{1}{2}\frac{p+1}{p}e^{-60|n_s-1|}|n_s - 1| \tag{7.91}$$

It is easy to see that $r$ is maximal when $p = 2$, $n_s = 1 - 1/60$ and hence it follows that the scalar-tensor ratio is bounded from above as $r < 0.006$, which is very small.

### 7.5.6 Comments on Initial Conditions

As we have previously noted, our classical solution $\phi(t)$ sits at the unstable maximum of the potential for an infinite amount of time, thus this model would seem to admit infinitely many e-foldings of inflation. Of course, this cannot be the case quantum mechanically and one expects quantum fluctuations to displace $\phi$ from the false vacuum and cause it to roll down the potential (we have assumed that this rolling takes place towards $\phi = 0$, rather than down the unbounded side of the potential). As a consistency check we note that

$$\frac{\langle(\delta\phi)^2\rangle^{1/2}}{\phi_0} = \langle(\delta\phi)^2\rangle^{1/2} \sim \frac{H_0}{A} \tag{7.92}$$

We should compare this to $u_\star$ (see equation 7.75), the distance the field has rolled classically at horizon crossing. It is straightforward to show that

$$\frac{\langle(\delta\phi)^2\rangle^{1/2}}{u_\star} \sim 10^{-5}\,|n_s - 1|$$

where we have used (7.78). Since $\langle(\delta\phi)^2\rangle^{1/2} \ll u_\star$ for all parameter values it follows that the de Sitter space fluctuations (which are present as $u \longrightarrow 0$) will not displace the field far enough from the maximum of the potential to have any significant effect on the number of observable e-foldings of inflation, although they prevent inflation from being past-eternal. We note, however, that if one incorporates thermal fluctuations (and initial momentum) then this model may suffer from problems related to fine tuning the intial conditions as in small field inflationary models [231]. However, it is not clear if these objections apply to our model for several reasons. The first reason is that the dynamics of this theory is peculiar and it is not clear how (or if) the phase space arguments of [231] apply. The second reason is that it is not clear what initial conditions for the field $\phi$ are most natural from a string theory perspective. Finally we note that since the field $\varphi$ rolls a distance $A > M_p$ in field space, our model is not a "small field" model in the conventional sense.[5]

If $p$-adic superstrings exist, it might also be possible to justify the initial conditions for inflation by having topological inflation [232], if the tachyon potential is symmetric about the unstable maximum. This distinction exists between the tachyon of the open bosonic string [215] (describing the instability of D25 branes), and the tachyon of unstable branes in superstring theory [233]. Any realistic extension of the model should have a potential which is bounded from below, and if it is supersymmetric, the minima should be at zero, hence the additional minima will be degenerate with the one at $\phi = 0$. The existence of domains of the universe in the different minima ensures that there will be regions in between where inflation from the maximum of the potential is taking place, so long as the minima are discrete and not connected to each other by a continuous symmetry.

As we have noted previously the fact that the potential $V(\phi)$ is unbounded from below is thought to be a reflection of the closed string tachyonic instability of bosonic string theory. If this conjecture is correct then the addition of supersymmetry should indeed lead to a symmetric potential for the $p$-adic tachyon which is bounded from below, as we have suggested above.

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[5]The skeptical reader might have reservations about the validity of our analysis since $A > M_p$. We note that the action (7.1) is not a low energy effective field theory and hence we believe that we are justified in using this action even for super-Planckian symmetry breaking scale.

## 7.6 Conclusions

In this chapter we have constructed for the first time approximate solutions of the fully nonlocal $p$-adic string theory coupled to gravity, in which the $p$-adic tachyon drives a sufficiently long period of inflation while rolling away from the maximum of its potential. In our solution, the nonlocal nature of the theory played an essential role in obtaining slow-roll, since with a conventional kinetic term the potential would have been too steep to give inflation. One of the novel features of this construction is that the Hubble parameter is larger than the string scale during inflation, a condition which would usually invalidate an effective field theory description, but which is consistent in the present context because of the ultraviolet-complete nature of the theory.

We found that the experimental constraints on the amplitude of the spectrum of scalar perturbations produced by inflation require a small value of the string coupling $g_s$, and can be consistent with a large range of values of the parameter $p$, $1 \lesssim p \lesssim 10^{14}$. The regime $p \gg 1$ is interesting because it exhibits qualitatively different behavior relative to conventional inflationary models: slow roll despite the potential being steep, and inflation being driven by the kinetic as well as potential energy of the field. This regime is also interesting because it corresponds to $g_s = \mathcal{O}(1)$ and hence appears more natural from a string theory perspective. Since the $p$-adic string is not construed to be a realistic model by itself, it may not be very meaningful to question how natural such values might be. However, it may not be unreasonable to think of real strings as being composed of constituent $p$-adic strings because the Veneziano amplitude of the $p$-adic theory is related to that of the full bosonic string by $\mathcal{A}^{-1} = \prod_p \mathcal{A}_p$ where the product is over all prime numbers $p$. Thus it may not be unreasonable to expect similar behavior to the large-$p$ results from a more realistic model.

The model predicts a red spectrum, in agreement with the latest WMAP data, whose tilt is related to the ratio of the string scale to the Hubble rate during inflation via $H/m_s = 2/\sqrt{3|n_s - 1|}$. For $n_s = 0.95$ this gives $H/m_s \cong 5$. This is in contrast to most stringy models of inflation which require $H < m_s$ in order for the effective field theory to be valid. We find the bound $r < 0.006$ on the tensor modes. It has been estimated that future experiments could eventually have a sensitivity of $r \sim 6 \times 10^{-5}$ [234] and hence the tensor components may in fact be observable.

We noted that the $p$-adic model succeeds with inflation where the real string theory tachyon fails. But our analysis makes it clear that this could be due to the failure to keep terms with arbitrary numbers of derivatives in the action. The effective tachyon action of Sen [43] is a truncation which keeps arbitrary powers of first derivatives but ignores higher order derivatives, which were essential for obtaining our solution. Thus the new features for inflation which we find in $p$-adic string theory could also be present in realistic string theories, if we knew how to include the whole tower of higher dimensional kinetic terms.

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## Note Added

Upon completing this analysis a related work appeared [236] in which an alternative normalization for the fluctuations of the $p$-adic scalar was proposed. Motivated by this work, we reconsidered our choice of normalization and concluded that (7.64,7.65) is the most appropriate definition of a "canonical" field. Notice, however, that our field $\varphi$ differs from the definition of a canonical field which was proposed in [236]. In [236] a field redefinition is advocated which puts the stress tensor $T_{\mu\nu}$ into canonical form, though this definition does not have canonical kinetic term in the action. Though we believe that our definition is more natural, we stress that in the case of current interest, $p$-adic inflation, the distinction does not generate any significant *quantitative* difference because our definition differs from that of [236] by a factor proportional to $\sqrt{\ln p}$ (which is less than an order of magnitude for the values of $p$ which we consider). We are grateful to J. Lidsey for sending us a draft of his manuscript prior to publication and also for interesting and enlightening discussions.

# Chapter 8

# Conclusions and Discussion

*All I know is what the words know, and dead things, and that makes a handsome little sum, with a beginning and a middle and an end, as in the well-built phrase and the long sonata of the dead.* -Samuel Beckett

Inflation offers an elegant resolution to the conceptual problems associated with the simple big bang model, however, this resolution brings with it a new list of conceptual problems to be addressed. Lest the reader fear that we have simply traded one set of conceptual problems for another (neither representing an actual conflict with observation) we remind the reader that inflation is a predictive framework and that, to date, these predictions have recieved spectacular confirmation by observation. (Indeed, the predictions of inflation are so well confirmed by observation that it is difficult, at this point, to imagine any measurement which could falsify inflation.) We have argued that a proper resolution to the outstanding conceptual problems of inflation will require embedding into (or at least input from) a complete theory of particle physics which incorporates quantum gravity. Since the only viable candidate is string theory, we have argued that the construction of string theoretic inflationary models is well motivated and provides a rare (perhaps the only) potential window into stringy physics. It is quite a remarkable feature of the interaction between cosmology and particle physics that it may be possible to glean information about extremely high energy physics (as high as $10^{16}$ GeV) through observations which will be made within our lifetimes.

In this thesis we have explored the cosmological consequences of tachyonic instabilities, both within the context of string theory and also in more conventional field theory models. Motivated by the KKLMMT model of brane inflation, we have considered the cosmological consequences of D-brane decay in chapters 2, 3, 4 and 5. We have studied in detail post-inflationary dynamics like reheating and cosmic string production in such models. We have shown that the dynamics of these processes may be quite different from the dynamics of analogous field theory models. We have also examined the cosmological consequences of these peculiar dynamics.

In chapter 6 we have studied tachyonic preheating at the end of hybrid inflation, showing that this phase of violent, nonperturbative particle production can have highly nontrivial implications for the observed spectrum of cosmological perturbations. We have shown that, depending on the values of certain model parameters, preheating may leave its imprint on the CMB either through $n = 4$ distortions of the spectrum or else through large nongaussian signatures. If large nongaussianity is observed in future missions it will certainly demonstrate the existence of some novel dynamics during (or shortly after) the inflationary epoch. There are very few inflationary models which can give rise to observably large nongaussianity and many of the models which do are somewhat contrived or are difficult to motivate from a particle physics perspective. Thus it is quite significant that large nongaussianity can be obtained in hybrid inflation, a simple and well-motivated model of inflation which appears to arise naturally in string theory and supergravity. We have used our analysis of nongaussianity from tachyonic preheating to show that current CMB observations constrain several popular string theory and supergravity models of inflation.

Finally, in chapter 7 we have explored a novel string theory model of inflation based on the tachyonic mode of the fully nonlocal $p$-adic string theory. The nonlocal structure of this theory leads to extremely peculiar dynamics, including the possibility of slow roll with a steep inflaton potential. This result opens up the possibility of addressing the issue of fine tuning of the inflaton potential in a new and novel way.

We feel that the work presented in this thesis represents some small progress in the direction of producing a realistic and natural string theory model of inflation. Though we (as a community) are still far from this goal, it is encouraging that significant progress has been made in recent years. Furthermore, the interface between string theory and cosmology is one which has been mutually beneficial. Indeed, even if string theory turns out not to be the theory of quantum gravity which nature has chosen, we feel that the exercise has still been a fruitful one since it has revealed a wealth of previously unexpected possibilities for inflationary model building. We look forward to future surprises (from both theory and observation) and the exciting possibility that future cosmological measurements may provide a window into new physics at extraordinarily high energy scales.

# Chapter A

# Appendix to Chapter 3

# A-1 The Legendre Transformation

In $1+1$ dimensions equation (3.25) takes the form

$$\dot{T}^2\ddot{T} + T'^2T'' - 2\dot{T}T'\dot{T}' = 0 \tag{A-1}$$

To construct general solutions of (A-1) we preform the Legendre transformation (see [101] for example) defined by the relations

$$\xi = T', \quad \eta = \dot{T}, \quad T(t,x) + \omega(\eta,\xi) = x\xi + t\eta \tag{A-2}$$

where $\omega(\eta,\xi)$ is a new dependent variable and $\{\eta,\xi\}$ are the new independent variables. It follows from (A-2) that

$$x = \frac{\partial\omega}{\partial\xi}, \quad t = \frac{\partial\omega}{\partial\eta}. \tag{A-3}$$

It is straightforward to verify the relations

$$T'' = J\frac{\partial^2\omega}{\partial\eta^2}, \quad \ddot{T} = J\frac{\partial^2\omega}{\partial\xi^2}, \quad \dot{T}' = -J\frac{\partial^2\omega}{\partial\xi\partial\eta} \tag{A-4}$$

where the Jacobian of the transformation is

$$\begin{aligned} J &= T''\ddot{T} - \left(\dot{T}'\right)^2 \\ &= \left[\frac{\partial^2\omega}{\partial\xi^2}\frac{\partial^2\omega}{\partial\eta^2} - \left(\frac{\partial^2\omega}{\partial\xi\partial\eta}\right)^2\right]^{-1} \end{aligned} \tag{A-5}$$

which we assume is nonzero. Note that this analysis excludes any solutions where $J=0$. The nonlinear partial differential equation (A-1) is transformed to a linear partial differential equation in terms of the new variables

$$\xi^2\frac{\partial^2\omega}{\partial\eta^2} + \eta^2\frac{\partial^2\omega}{\partial\xi^2} + 2\eta\xi\frac{\partial^2\omega}{\partial\xi\partial\eta} = 0. \tag{A-6}$$

Given a solution $\omega(\eta,\xi)$ of (A-6) the relations (A-3) along with

$$T = \xi\frac{\partial\omega}{\partial\xi} + \eta\frac{\partial\omega}{\partial\eta} - \omega(\eta,\xi) \tag{A-7}$$

define the solution of (A-1) parametrically.

We consider now solutions of the linear partial differential equation (A-6). To simplify the ensuing analysis we will restrict ourselves to solutions where $\partial_\mu T \partial^\mu T$ does not change sign. This is expected to be a reasonable restriction for the vacuum structure of the theory (3.24) since previous analysis of the actions (3.1,3.8,3.15) suggests that $\partial_\mu T \partial^\mu T$ is either increasing or decreasing as $T \to \pm\infty$. We only consider the case $\partial_\mu T \partial^\mu T \leq 0$, which we expect to most closely resemble the vacuum structure of the actions (3.1,3.8,3.15). In this regime we can define new coordinates $\{\rho, \sigma\}$ by

$$\eta = \rho \cosh \sigma, \qquad \xi = \rho \sinh \sigma \tag{A-8}$$

such that

$$\rho^2 = -\xi^2 + \eta^2 = -\partial_\mu T \partial^\mu T \geq 0, \qquad \tanh \sigma = \frac{\xi}{\eta}.$$

In terms of these new variables (A-6) takes the remarkably simple form

$$-\rho \frac{\partial \omega}{\partial \rho} + \frac{\partial^2 \omega}{\partial \sigma^2} = 0. \tag{A-9}$$

with particular solution

$$\omega_m(\eta, \xi) = \rho^{m^2} \left[ \alpha_m e^{m\sigma} + \beta_m e^{-m\sigma} \right]$$

for arbitrary $m$, $\alpha_m$, $\beta_m$. The most convenient way to fix the Cauchy data is to specify $\omega$ and $\partial\omega/\partial\rho$ at $\sigma = 0$. In this case we write the general solution of (A-9) as

$$\omega(\rho, \sigma) = \sum_{n=0}^{\infty} \rho^n \left( a_n \cosh\left(\sqrt{n}\,\sigma\right) + b_n \sinh\left(\sqrt{n}\,\sigma\right)\right) \tag{A-10}$$

where $\{a_n\}$ determine the coefficients in the taylor expansion of $\omega$ at $\sigma = 0$ and $\{b_n\}$ determine the coefficients in the taylor expansion of $\partial\omega/\partial\rho$ at $\sigma = 0$.

It is straightforward to compute $x(\eta, \xi)$, $t(\eta, \xi)$ from (A-3) and

$$T(\rho, \sigma) = \sum_{n=0}^{\infty} \rho^n (n-1) \left( a_n \cosh\left(\sqrt{n}\,\sigma\right) + b_n \sinh\left(\sqrt{n}\,\sigma\right)\right)$$

from (A-7). Because this solution is not in closed form and defined parametrically, it is difficult to get an intuitive feeling for the behaviour of $T$ as a function of $t$ and $x$. However, we believe that these general solutions do admit derivative singularities and have, for several choices of $\{a_n, b_n\}$, found points $(\rho_c, \sigma_c)$ corresponding to finite $x$ and $t > 0$ at which $T''$ is singular though $T$, $T'$ and $\dot{T}$ are regular. In this case the second derivative blows up because the Jacobian of the Legendre transformation (A-5) is singular.

# Chapter B

# Appendix to Chapter 4

# B-1 The 4D View

In this appendix we compute the low-energy couplings amongst the bulk zero modes and brane modes in the effective 4D supergravity obtained after modulus stabilization *à la* KKLT [21]. Besides checking the scaling of the kinetic terms obtained by dimensionally reducing the Einstein-Hilbert action, this also allows the study of the couplings in the scalar potential which arise from modulus stabilization and so are more difficult to analyze from a semiclassical, higher-dimensional point of view.

To this end imagine integrating out all of the extra-dimensional physics to obtain the low-energy effective 4D supergravity for a Type IIB GKP vacuum having only the mandatory volume modulus (and its supersymmetric friends) plus various low-energy brane modes (such as those describing the motion of various D3 branes). The terms in this supergravity involving up to two derivatives are completely described once the Kähler function, $K$, superpotential, $W$, and gauge kinetic function, $f_{ab}$, are specified.

Denoting the bulk-modulus supermultiplet by $T$ and the brane multiplets by $\phi^I$, we use the Kähler potential [126, 21, 24, 127]

$$K = -3\log\left[r\right] , \tag{B-1}$$

where $r = T + T^* + k(\phi, \phi^*)$. For instance, if $\phi^I$ denotes the position of single brane, then $k$ is the Kähler potential for the underlying 6D manifold. This implies the scalar kinetic terms are governed by the following Kähler metric in field space

$$K_{TT^*} = \frac{3}{r^2}, \qquad K_{IT^*} = \frac{3\,k_I}{r^2} \qquad \text{and} \qquad K_{IJ^*} = \frac{3}{r^2}\left[k_I\,k_{J^*} - r\,k_{IJ^*}\right] , \tag{B-2}$$

with inverse

$$K^{T^*T} = \frac{r}{3}\left[r - k^{L^*N}k_{L^*}k_N\right] , \quad K^{J^*T} = \frac{r\,k^{J^*L}k_L}{3} \quad \text{and} \quad K^{J^*I} = -\frac{r\,k^{J^*I}}{3} . \tag{B-3}$$

In the absence of modulus stabilization the superpotential of the effective theory is a constant [128], $W = w_0$, and the supergravity takes the usual no-scale form [129], with vanishing scalar potential. If, however, there are low-energy gauge multiplets associated with any of the D7 branes of the model then their gauge kinetic function is $f_{ab} = T\,\delta_{ab}$. For nonabelian multiplets of this type gaugino condensation [130, 131] can generate a nontrivial superpotential, of the form

$$W = w_0 + A\,\exp\left[-a\,T\right]\,, \tag{B-4}$$

where $A$ and $a$ are calculable constants.

With these choices the Kähler derivatives of the superpotential become

$$D_T W = W_T - \frac{3W}{r}\,, \qquad \text{and} \qquad D_I W = -\frac{3k_I\,W}{r}\,, \tag{B-5}$$

and so the supersymmetric scalar potential [132] becomes

$$V = \frac{1}{3r^2}\,\left[\left(r - k^{I^*J}k_{I*}k_J\right)|W_T|^2 - 3(W^*W_T + WW_T^*)\right]\,. \tag{B-6}$$

Notice that use of these expression implicitly requires that we work in the 4D Einstein frame, and so are using 4D Planck units for which $M_p = \mathcal{O}(1)$.

If we specialize to the case of several branes, for which $\{\phi^I\} = \{\phi^i_n\}$, with $i$ labelling the fields on a given brane and $n = 1, \dots, N$ labelling which brane is involved, then we typically have

$$k(\phi^I, \phi^{I*}) = \sum_n k^{(n)}(\phi^i_n, \phi^{i*}_n)\,. \tag{B-7}$$

In this case the Kähler metric built from $k$ is block diagonal, with $k_{i_n j_m} = k^{(n)}_{ij}\,\delta_{mn}$, and so $k^{I^*J}k_{I*}k_J = \sum_n k^{i^*j}_{(n)}k^{(n)}_{i^*}k^{(n)}_j$ and so on.

We may now see how strongly the bulk KK zero modes, $g_{\mu\nu}$ and $T$, couple to one another and to the brane modes. Setting $k = 0$ in the above shows that the couplings of $T$ and $g_{\mu\nu}$ to one another are order unity, and since our use of the standard 4D supergravity formalism requires us to be in the Einstein frame, this implies these are all of 4D Planck strength (in agreement with our higher-dimensional estimates).

Couplings to the branes are obtained by keeping $k$ nonzero, and in the event that the branes are located in highly warped regions, we must take $k^{(n)} = \mathcal{O}(a_n^2)$ with $a_n \ll 1$ denoting the warp factor at the position of brane $n$.[1] In this case the combination $k^{i^*j}_{(n)}k^{(n)}_{i^*}k^{(n)}_j$ is also $\mathcal{O}(a_n^2)$.

[1]For instance, this power of $a_n$ reproduces the $a_n$-dependence of the factor $\sqrt{g}g^{\mu\nu}$ obtained by dimensionally reducing the higher-dimensional kinetic terms.

Suppose we now expand the functions $k^{(n)}$ in powers of $\phi$ and keep only the leading powers:

$$k^{(n)} \approx a_n^2 \sum_i \phi_n^{i*} \phi_n^i \,. \tag{B-8}$$

Then, since the $\phi_n$ kinetic terms are $\mathcal{O}(a_n^2)$, we see that the canonically-normalized fields are $\chi_n^i = a_n \phi_n^i$. Once this is done the leading couplings to $T$ and $g_{\mu\nu}$ are those which involve those parts of $k^{(n)}$ that are quadratic in $\chi_n^i$, and since these are also order unity, these couplings are also of Planck strength (again in agreement with our earlier estimates).

Alternatively, consider now those couplings which only involve the brane modes. Working to leading order in $a_n^2$, we see that a term in $k^{(n)}$ of the form $(\chi_n^i)^k$ has a strength which is of order $a_n^{2-k}$. For instance the case $k = 3$ generates cubic couplings from the kinetic lagrangian of order $a_n^{-1} \chi \partial \chi \partial \chi$, whose coefficient is of order $(a_n M_p)^{-1} = M_{sm}^{-1}$. These are larger than Planck suppressed ones, as expected.

# Chapter C

# Appendix to Chapter 5

## C-1 Initial Conditions for Defect Formation

Here we briefly discuss the initial conditions for defect formation at the end of brane inflation. During the slow roll inflationary phase the tachyon behaves as an ordinary massive Klein-Gordon scalar field (provided $T \ll l_s^{-1}$). We consider here for simplicity a standard field theory of a scalar $\chi$ in 3+1-dimensions whose mass squared parameter abruptly becomes negative. This type of theory has been considered in detail in [32, 33, 34].

During the de Sitter phase (before the mass parameter becomes tachyonic) vacuum fluctuations yield a blackbody spectrum of produced particles $\langle 0|N_k|0\rangle = (e^{\omega_k/T}-1)^{-1}$ with temperature $T = H/(2\pi)$ (see, for example, [154] for a review). However, in any region of de Sitter space which is small compared to the Hubble scale, the space is locally Minkowski, and even in the vacuum state there are quantum fluctuations quantified by the two point functions of the fields

$$\begin{aligned}\langle \chi^\star(\vec{k})\chi(\vec{k}')\rangle &= \frac{1}{2|\vec{k}|}(2\pi)^3\delta^3(\vec{k}-\vec{k}') \\ \langle \Pi^\star(\vec{k})\Pi(\vec{k}')\rangle &= \frac{|\vec{k}|}{2}(2\pi)^3\delta^3(\vec{k}-\vec{k}')\end{aligned}$$

where $\Pi = \dot{\chi}$.[1] The initial stages of the string tachyon condensation are identical to the tachyonic preheating scenario described in [32, 33]. The tachyonic instability amplifies exponentially those modes with $|\vec{k}| < m$ where the $\chi$ field has mass squared parameter $-m^2$ and the variance of the fluctuations grows as [34]

$$\langle \chi^2(t)\rangle = \langle \chi^2(0)\rangle + \frac{1}{8\pi}\int_0^{m^2} dk^2 \frac{m^2}{m^2-k^2}\sinh^2\left(t\sqrt{m^2-k^2}\right)$$

where $\langle \chi^2(0)\rangle$ is a divergent vacuum contribution. The above result was derived in (3+1) dimensions, but the generalization to higher dimensions must have the form

$$\langle \chi^2(t)\rangle = \langle \chi^2(0)\rangle + c\sum_i \int_0^{m^2} dk^2 \frac{m^2}{m^2-k^2-m_i^2}\sinh^2\left(t\sqrt{m^2-k^2-m_i^2}\right)$$

where $m_i$ are the masses of the Kaluza-Klein excitations. These fluctuations grow to be of order the classical VEV, $\langle \chi^2(t)\rangle - \langle \chi^2(0)\rangle \sim m^2/\lambda$ before the linear treatment

[1] Similar initial conditions are taken for defect formation at the end of inflation in [155].

breaks down. Notice that this growth occurs on a microscopic time scale. Since in the case of the string theory tachyon the potential has no minimum, we can conservatively take the initial exponential growth to be over a shorter time scale, $\langle\chi^2\rangle \sim m^2 \sim M_s^2$. These fluctuations are much larger than the de Sitter fluctuations $\langle\chi^2\rangle \sim H^2$ and are thus the dominant seeds for defect formation. Furthermore, these fluctuations have a minimum wavelength comparable to the string scale which is sufficient to initiate the formation of defects localized in the compact dimensions. This justifies our choice of random initial conditions for the tachyon field in the numerical studies of defect formation.

# Chapter D

# Appendices to Chapter 6

# D-1 The Matching Time $N_k$

The matching time $N_k$ which determines the boundary between large- and small-scale behaviour of the mode functions (6.28) is determined by the transcendental equation

$$|N_k|e^{2N_k} = \frac{\hat{k}^2}{c} \tag{D-1}$$

The solutions may be written exactly in terms of the branches of the Lambert-W functions. In the region $\hat{k} < \sqrt{c/(2e)}$ the solution is triple-valued and may be written as

$$N_k = \begin{cases} \frac{1}{2}W_{-1}\left(-\frac{2\hat{k}^2}{c}\right) & \text{for the branch with } N_k < -1; \\ \frac{1}{2}W_0\left(-\frac{2\hat{k}^2}{c}\right) & \text{for the branch with } -1 < N_k < 0; \\ \frac{1}{2}W_0\left(+\frac{2\hat{k}^2}{c}\right) & \text{for the branch with } N_k > 0. \end{cases} \tag{D-2}$$

In the region $\hat{k} > \sqrt{c/(2e)}$ the solution is single valued and can be written as

$$N_k = \frac{1}{2}W_0\left(+\frac{2\hat{k}^2}{c}\right) \tag{D-3}$$

One may derive some asymptotic expressions for $N_k$ in various regions of interest. When $|N_k| \gg 1$ we have

$$N_k \cong \ln\left(\frac{\hat{k}}{\sqrt{c}}\right) \tag{D-4}$$

which describes $N_k$ at $\hat{k} \gg \sqrt{c/(2e)}$ and also the lower branch of $N_k$ at $\hat{k} \ll \sqrt{c/(2e)}$. For $\hat{k} \lesssim \sqrt{c/(2e)}$ there are two more branches of the solution with approximate behaviour

$$N_k \cong \pm\frac{\hat{k}^2}{c} \tag{D-5}$$

In our analysis we have used the approximation that $N_k$ is a single-valued function, described by

$$\begin{aligned} N_k^{\text{s.v.}} &= \frac{1}{2}\Theta\left(\sqrt{c/(2e)} - \hat{k}\right) W_{-1}\left(-\frac{2\hat{k}^2}{c}\right) \\ &+ \frac{1}{2}\Theta\left(\hat{k} - \sqrt{c/(2e)}\right) W_0\left(+\frac{2\hat{k}^2}{c}\right) \end{aligned}$$

where $\Theta(x)$ is the Heaviside step function. We have verified both numerically and analytically that this approximation does not alter our result

## D-2 Perturbed Einstein Equations and the Master Equation

Using Maple we have carefully verified the results of [183] for the perturbed Einstein equations and the master equation. Here we briefly review those results relevent for the computation of $\zeta^{(2)}$. We present only the $\delta^{(2)}G^0_0 = \kappa^2\delta^{(2)}T^0_0$, $\partial_i\delta^{(2)}G^i_0 = \kappa^2\partial_i\delta^{(2)}T^i_0$ and $\delta^i_j\delta^{(2)}G^j_i = \kappa^2\delta^i_j\delta^{(2)}T^j_i$ equations since the second order vector and tensor fluctuations decouple from this system. In the case that $\sigma_0 = 0$ the second order tachyon fluctation $\delta^{(2)}\sigma$ decouples from the inflaton and gravitational fluctuations. Analogously to the first order fluctuations, the Klein-Gordon equation for the inflaton fluctation at second order $\delta^{(2)}\varphi$ is not necessary to close the system of equations. In the subsequent text we sometimes insert the slow roll parameters $\epsilon$ and $\eta$ explicitly though we make no assumption that they are small. We also introduce the shorthand notation $m^2_\varphi = \partial^2 V/\partial\varphi^2$ and $m^2_\sigma = \partial^2 V/\partial\sigma^2$ and assume that $\partial^2 V/\partial\sigma\partial\varphi = 0$.

The second order $(0,0)$ equation is

$$\begin{aligned}
& 3\mathcal{H}\psi'^{(2)} + (3-\epsilon)\mathcal{H}^2\phi^{(2)} - \partial^k\partial_k\psi^{(2)} \\
= & -\frac{\kappa^2}{2}\left[\varphi_0'\delta^{(2)}\varphi' + a^2\frac{\partial V}{\partial\varphi}\delta^{(2)}\varphi\right] + \Upsilon_1
\end{aligned} \tag{D-6}$$

where $\Upsilon_1$ is constructed entirely from first order quantities. Dividing $\Upsilon_1$ into inflaton and tachyon contributions we have

$$\Upsilon_1 = \Upsilon_1^\varphi + \Upsilon_1^\sigma$$

where

$$\begin{aligned}
\Upsilon_1^\varphi = & \; 4(3-\epsilon)\mathcal{H}^2\left(\phi^{(1)}\right)^2 + 2\kappa^2\varphi_0'\phi^{(1)}\delta^{(1)}\varphi' \\
- & \; \frac{\kappa^2}{2}\left(\delta^{(1)}\varphi'\right)^2 - \frac{\kappa^2}{2}a^2 m^2_\varphi\left(\delta^{(1)}\varphi\right)^2 - \frac{\kappa^2}{2}\left(\vec{\nabla}\delta^{(1)}\varphi\right)^2 \\
+ & \; 8\phi^{(1)}\partial^k\partial_k\phi^{(1)} + 3\left(\phi'^{(1)}\right)^2 + 3\left(\vec{\nabla}\phi^{(1)}\right)^2
\end{aligned} \tag{D-7}$$

and

$$\begin{aligned}
\Upsilon_1^\sigma = & -\frac{\kappa^2}{2}\left[\left(\delta^{(1)}\sigma'\right)^2 + \left(\vec{\nabla}\delta^{(1)}\sigma\right)^2\right. \\
+ & \left. a^2 m^2_\sigma\left(\delta^{(1)}\sigma\right)^2\right].
\end{aligned} \tag{D-8}$$

The divergence of the second order $(0,i)$ equation is

$$\partial^k\partial_k\left[\psi'^{(2)}+\mathcal{H}\phi^{(2)}\right]=\frac{\kappa^2}{2}\varphi_0'\partial^k\partial_k\delta^{(2)}\varphi+\Upsilon_2 \tag{D-9}$$

where $\Upsilon_2=\Upsilon_2^{\varphi}+\Upsilon_2^{\sigma}$ is constructed entirely from first order quantities. The inflaton part is

$$\begin{aligned}\Upsilon_2^{\varphi} &= 2\kappa^2\varphi_0'\partial_i\left(\phi^{(1)}\partial^i\delta^{(1)}\varphi\right)+\kappa^2\partial_i\left(\delta^{(1)}\varphi'\partial^i\delta^{(1)}\varphi\right)\\ &- 8\partial_i\left(\phi^{(1)}\partial^i\phi'^{(1)}\right)-2\partial_i\left(\phi'^{(1)}\partial^i\phi^{(1)}\right)\end{aligned} \tag{D-10}$$

and the tachyon part is

$$\Upsilon_2^{\sigma}=\kappa^2\partial_i\left(\delta^{(1)}\sigma'\partial^i\delta^{(1)}\sigma\right). \tag{D-11}$$

The trace of the second order $(i,j)$ equation is

$$\begin{aligned}&3\psi''^{(2)}+\partial^k\partial_k\left[\phi^{(2)}-\psi^{(2)}\right]+6\mathcal{H}\psi'^{(2)}\\ +\ &3\mathcal{H}\phi'^{(2)}+3(3-\epsilon)\mathcal{H}^2\phi^{(2)}\\ =\ &\frac{3\kappa^2}{2}\left[\varphi_0'\delta^{(2)}\varphi'-a^2\frac{\partial V}{\partial\varphi}\delta^{(2)}\varphi\right]+\Upsilon_3\end{aligned} \tag{D-12}$$

where $\Upsilon_3=\Upsilon_3^{\varphi}+\Upsilon_3^{\sigma}$ is constructed entirely from first order quantities. The inflaton part is

$$\begin{aligned}\Upsilon_3^{\varphi} &= 12(3-\epsilon)\mathcal{H}^2\left(\phi^{(1)}\right)^2-6\kappa^2\varphi_0'\phi^{(1)}\delta^{(1)}\varphi'\\ &+ \frac{3\kappa^2}{2}\left(\delta^{(1)}\varphi'\right)^2-\frac{3\kappa^2}{2}a^2m_\varphi^2\left(\delta^{(1)}\varphi\right)^2-\frac{\kappa^2}{2}\left(\vec{\nabla}\delta^{(1)}\varphi\right)^2\\ &+ 3\left(\phi'^{(1)}\right)^2+8\phi^{(1)}\partial^k\partial_k\phi^{(1)}+24\mathcal{H}\phi^{(1)}\phi'^{(1)}\\ &+ 7\left(\vec{\nabla}\phi^{(1)}\right)^2\end{aligned} \tag{D-13}$$

and the tachyon part is

$$\begin{aligned}\Upsilon_3^{\sigma} &= \kappa^2\left[\frac{3}{2}\left(\delta^{(1)}\sigma'\right)^2-\frac{1}{2}\left(\vec{\nabla}\delta^{(1)}\sigma\right)^2\right.\\ &- \left.\frac{3}{2}a^2m_\sigma^2\left(\delta^{(1)}\sigma\right)^2\right]\end{aligned} \tag{D-14}$$

We now proceed to derive the master equation. Adding (D-6) to the inverse laplacian of the time derivative of (D-9) and then using (D-9) to eliminate $\delta^{(2)}\varphi$

yields

$$\begin{aligned} &\psi''^{(2)} - (1+2\epsilon-2\eta)\mathcal{H}\psi'^{(2)} + \mathcal{H}\phi'^{(2)} \\ - \;& 2(2\epsilon-\eta)\mathcal{H}^2\phi^{(2)} - \partial^k\partial_k\psi^{(2)} = \Upsilon_1 + \triangle^{-1}\Upsilon_2' \\ - \;& 2(2+\epsilon-\eta)\mathcal{H}\triangle^{-1}\Upsilon_2. \end{aligned} \tag{D-15}$$

Notice that we have decoupled $\psi^{(2)}$ and $\phi^{(2)}$ from the inflaton perturbation $\delta^{(2)}\varphi$. It remains now to express $\psi^{(2)}$ in terms of $\phi^{(2)}$. To this end we subtract the inverse laplacian of (D-9) from (D-12) and again use (D-9) to eliminate $\delta^{(2)}\varphi$ which gives

$$\partial^k\partial_k\left[\phi^{(2)} - \psi^{(2)}\right] = \Upsilon_3 - 3\triangle^{-1}\Upsilon_2' - 6\mathcal{H}\triangle^{-1}\Upsilon_2$$

or, equivalently

$$\psi^{(2)} = \phi^{(2)} - \triangle^{-1}\gamma. \tag{D-16}$$

Following the notation of [183] we have defined

$$\gamma = \Upsilon_3 - 3\triangle^{-1}\Upsilon_2' - 6\mathcal{H}\triangle^{-1}\Upsilon_2 \tag{D-17}$$

which can be split into inflaton and tachyon components $\gamma = \gamma_\varphi + \gamma_\sigma$ in an obvious fashion. Our $\Upsilon_2$ is related to the quantities $\alpha, \beta$ defined in [183] by

$$\Upsilon_2 = \kappa^2\beta - \alpha.$$

Now, using (D-16) to eliminate $\psi^{(2)}$ from (D-15) gives the master equation

$$\begin{aligned} &\phi''^{(2)} + 2(\eta-\epsilon)\mathcal{H}\phi'^{(2)} + \left[2(\eta-2\epsilon)\mathcal{H}^2 - \partial^k\partial_k\right]\phi^{(2)} \\ = \;& \Upsilon_1 + \triangle^{-1}\Upsilon_2' - 2(2+\epsilon-\eta)\mathcal{H}\triangle^{-1}\Upsilon_2 - \gamma \\ - \;& (1+2\epsilon-2\eta)\mathcal{H}\triangle^{-1}\gamma' + \triangle^{-1}\gamma'' \end{aligned}$$

Inserting explicitly the expression for $\gamma$ (D-17) this can be written as

$$\phi''^{(2)} + 2(\eta-\epsilon)\mathcal{H}\phi'^{(2)} + \left[2(\eta-2\epsilon)\mathcal{H}^2 - \partial^k\partial_k\right]\phi^{(2)} = J$$

where the source is

$$\begin{aligned} J \;=\;& \Upsilon_1 - \Upsilon_3 + 4\triangle^{-1}\Upsilon_2' + 2(1-\epsilon+\eta)\mathcal{H}\triangle^{-1}\Upsilon_2 \\ +\;& \triangle^{-1}\gamma'' - (1+2\epsilon-2\eta)\mathcal{H}\triangle^{-1}\gamma'. \end{aligned} \tag{D-18}$$

We can split the source into tachyon and inflaton contributions $J = J^{\varphi} + J^{\sigma}$ in the obvious manner, by taking the tachyon and inflaton parts of $\Upsilon_1, \Upsilon_2, \Upsilon_3, \gamma$.

We now derive some results concering the tachyon source terms which will be useful in the text. First we consider the tachyon contribution to $\gamma$ (D-17):

$$\gamma_\sigma = \Upsilon_3^\sigma - 3\Delta^{-1}\partial_\tau \Upsilon_2^\sigma - 6\mathcal{H}\Delta^{-1}\Upsilon_2^\sigma.$$

Using equations (D-11) and (D-14) we can write this as

$$\begin{aligned}\gamma_\sigma &= \kappa^2 \left[\frac{3}{2}(\delta^{(1)}\sigma')^2 - \frac{1}{2}\partial_i\delta^{(1)}\sigma\partial^i\delta^{(1)}\sigma - \frac{3}{2}a^2 m_\sigma^2(\delta^{(1)}\sigma)^2\right] \\ &- 3\kappa^2\Delta^{-1}\partial_\tau\partial_i(\delta^{(1)}\sigma'\partial^i\delta^{(1)}\sigma) \\ &- 6\kappa^2\mathcal{H}\Delta^{-1}\partial_i(\delta^{(1)}\sigma'\partial^i\delta^{(1)}\sigma) \qquad \text{(D-19)}\end{aligned}$$

We now write $\gamma_\sigma$ as

$$\begin{aligned}\gamma_\sigma &= \kappa^2\Delta^{-1}\left[\frac{3}{2}\partial^k\partial_k(\delta^{(1)}\sigma')^2 - \frac{1}{2}\partial^k\partial_k(\partial_i\delta^{(1)}\sigma\partial^i\delta^{(1)}\sigma)\right. \\ &- \frac{3}{2}a^2m_\sigma^2\partial^k\partial_k(\delta^{(1)}\sigma)^2 - 3\partial_\tau\partial_i(\delta^{(1)}\sigma'\partial^i\delta^{(1)}\sigma) \\ &- \left. 6\mathcal{H}\partial_i(\delta^{(1)}\sigma'\partial^i\delta^{(1)}\sigma)\right]\end{aligned}$$

and, after some algebra, we have

$$\begin{aligned}\gamma_\sigma &= \kappa^2\Delta^{-1}\left[-\frac{1}{2}\partial^k\partial_k(\partial_i\delta^{(1)}\sigma\partial^i\delta^{(1)}\sigma)\right. \\ &- 3\partial_i\left(\delta^{(1)}\sigma'' + 2\mathcal{H}\delta^{(1)}\sigma' + a^2m_\sigma^2\delta^{(1)}\sigma\right)\partial^i\delta^{(1)}\sigma \\ &- \left. 3\left(\delta^{(1)}\sigma'' + 2\mathcal{H}\delta^{(1)}\sigma' + a^2m_\sigma^2\delta^{(1)}\sigma\right)\partial^k\partial_k\delta^{(1)}\sigma\right]\end{aligned}$$

The last two lines can be simplified using the equation of motion for the tachyon fluctuation

$$\delta^{(1)}\sigma'' + 2\mathcal{H}\delta^{(1)}\sigma + a^2m_\sigma^2\delta^{(1)}\sigma = \partial^k\partial_k\delta^{(1)}\sigma$$

which gives

$$\begin{aligned}\gamma_\sigma &= \kappa^2\Delta^{-1}\left[-\frac{1}{2}\partial^k\partial_k\left(\partial_i\delta^{(1)}\sigma\partial^i\delta^{(1)}\sigma\right)\right. \\ &- \left. 3\partial_i(\partial^k\partial_k\delta^{(1)}\sigma)\partial^i\delta^{(1)}\sigma - 3\partial_i\partial^i\delta^{(1)}\sigma\partial^k\partial_k\delta^{(1)}\sigma\right] \\ &= \kappa^2\Delta^{-1}\left[-\frac{1}{2}\partial^k\partial_k\left(\partial_i\delta^{(1)}\sigma\partial^i\delta^{(1)}\sigma\right)\right. \\ &- \left. 3\partial_i\left(\partial^k\partial_k\delta^{(1)}\sigma\partial^i\delta^{(1)}\sigma\right)\right] \qquad \text{(D-20)}\end{aligned}$$

This result has also been derived in [183]. We now consider the tachyon contribution to the source. We take

$$\begin{aligned} J^\sigma &= \Upsilon_1^\sigma - \Upsilon_3^\sigma + 4\Delta^{-1}\partial_\tau\Upsilon_2^\sigma + 2(1-\epsilon+\eta)\mathcal{H}\Delta^{-1}\Upsilon_2^\sigma \\ &+ \Delta^{-1}\gamma_\sigma'' - (1+2\epsilon-2\eta)\mathcal{H}\Delta^{-1}\gamma_\sigma' \end{aligned}$$

and, using (D-8), (D-11) and (D-14), we have

$$\begin{aligned} J^\sigma(\tau,\vec{x}) &= a^2\kappa^2 m_\sigma^2 \left(\delta^{(1)}\sigma\right)^2 - 2\kappa^2\left(\delta^{(1)}\sigma'\right)^2 \\ &+ 2\kappa^2\mathcal{H}(1+\eta-\epsilon)\Delta^{-1}\partial_i\left(\delta^{(1)}\sigma'\partial^i\delta^{(1)}\sigma\right) \\ &+ 4\kappa^2\Delta^{-1}\partial_\tau\partial_i\left(\delta^{(1)}\sigma'\partial^i\delta^{(1)}\sigma\right) \\ &- \mathcal{H}(1+2\epsilon-2\eta)\Delta^{-1}\gamma_\sigma' + \Delta^{-1}\gamma_\sigma''. \end{aligned} \qquad \text{(D-21)}$$

# D-3 The Inflaton Contribution to $\zeta^{(2)}$

In this appendix we consider the calculation of the inflaton part of the second order curvature perturbation $\zeta_\varphi^{(2)}$ using results from appendix D-2 and following closely the calculation of the tachyon part of the second order curvature perturbation.

The last four lines of (6.41) are relatively simple to evaluate in the large scale and slow roll limit. On the other hand, the first line of (6.41) is somewhat more complicated and its evaluation requires the second order Einstein equations. We first focus first on the following contribution to $\zeta^{(2)}$

$$\zeta^{(2)} \ni \frac{1}{3-\epsilon}\frac{1}{(\varphi_0')^2}\left[\varphi_0' Q_\varphi'^{(2)} + a^2\frac{\partial V}{\partial\varphi}Q_\varphi^{(2)}\right]. \qquad \text{(D-22)}$$

From the definition of $Q_\varphi^{(2)}$ it is clear that this contains a contribution of the form $\varphi_0'\delta^{(2)}\varphi' + a^2\partial V/\partial\varphi\,\delta^{(2)}\varphi$ which also appears in the second order $(0,0)$ Einstein equation (D-6). We therefore use (D-6) to eliminate $\delta^{(2)}\varphi$ from (D-22). We also eliminate $\psi^{(2)}$ in favour of $\phi^{(2)}$ using (D-16). The result is that (D-22) takes the form

$$\begin{aligned} \zeta^{(2)} &\ni -\frac{\phi'^{(2)}}{\epsilon\mathcal{H}} - \left(\frac{1}{\epsilon}+1\right)\phi^{(2)} + \frac{1}{3-\epsilon}\frac{\partial^k\partial_k\phi^{(2)}}{\epsilon\mathcal{H}^2} \\ &+ \frac{1}{\epsilon\mathcal{H}}\Delta^{-1}\gamma' + \Delta^{-1}\gamma - \frac{1}{3-\epsilon}\frac{1}{\epsilon\mathcal{H}^2}\gamma \\ &+ \frac{1}{3-\epsilon}\frac{\Upsilon_1}{\epsilon\mathcal{H}^2} + \frac{1}{3-\epsilon}\left(\frac{\Upsilon_4'}{\varphi_0'} + \frac{a^2}{(\varphi_0')^2}\frac{\partial V}{\partial\varphi}\Upsilon_4\right) \end{aligned} \qquad \text{(D-23)}$$

where the quantities $\Upsilon_1, \gamma$ are constructed entirely from first order fluctuations and are defined explicitly in appendix D-2 in equations (D-7, D-8, D-10, D-11, D-13, D-14). The quantity $\Upsilon_4$ is also contructed from first order fluctuations and is defined as the last two lines of (6.44). That is,

$$\begin{aligned}\Upsilon_4 &= (2+2\epsilon-\eta)\frac{\varphi_0'}{\mathcal{H}}\left(\phi^{(1)}\right)^2 \\ &+ 2\frac{\varphi_0'}{\mathcal{H}^2}\phi^{(1)}\phi'^{(1)} + \frac{2}{\mathcal{H}}\phi^{(1)}\delta^{(1)}\varphi'. \end{aligned} \tag{D-24}$$

The first line of (D-23) can be solved for using the Green function for the master equation as:

$$\begin{aligned}&- \frac{\phi_k'^{(2)}}{\epsilon\mathcal{H}} - \frac{\phi_k^{(2)}}{\epsilon} - \frac{k^2\phi_k^{(2)}}{3\epsilon\mathcal{H}^2} = \\ &\frac{1}{\epsilon}\int_{\tau_i}^{0} d\tau'\Theta(\tau-\tau')J_k^{\varphi}(\tau') \\ \times &\left[\tau' + \left(-\frac{1}{6}\tau'^3 + \frac{5}{6}\tau^2\tau' - \frac{2}{3}\tau^3\right)k^2\right]. \end{aligned} \tag{D-25}$$

This equation allows us to remove all of the explicit dependence of $\zeta^{(2)}$ on the second order metric perturbation $\phi^{(2)}$.

Having removed the explicit dependence of $\zeta^{(2)}$ on the second order metric perturbation $\phi^{(2)}$ we next eliminate $\phi^{(1)}, \delta^{(1)}\varphi$ in favour of $\zeta^{(1)}$. Using the solution (1.47) and the first order constraint equation (1.40) one may verify that on large scales

$$\zeta^{(1)} \cong -\phi^{(1)} - \frac{\varphi_0'}{\mathcal{H}}\delta^{(1)}\varphi \cong -\frac{1}{\epsilon}\phi^{(1)}$$

and

$$\delta^{(1)}\varphi \cong \frac{\varphi_0'}{\mathcal{H}}\frac{1}{\epsilon}\phi^{(1)} \cong -\frac{\varphi_0'}{\mathcal{H}}\zeta^{(1)}.$$

On large scales the first order curvature perturbation is approximately constant since [190]

$$\zeta'^{(1)} = \frac{\partial^k\partial_k}{\epsilon\mathcal{H}}\phi^{(1)}$$

using the fact that $\sigma_0 = 0$ (so that there are no anisotropic stresses, whose absence guarantees the conservation of $\zeta^{(1)}$ on super-Hubble scales). Thus on large scales we have

$$\delta^{(1)}\varphi' \cong -(2\epsilon-\eta)\varphi_0'\zeta^{(1)}.$$

It is straightforward to compute the last three lines of (6.41) using the fact that $Q_\varphi^{(1)} \approx -\varphi_0'\zeta^{(1)}/\mathcal{H}$ and $Q_\varphi'^{(1)} \approx -(2\epsilon-\eta)\varphi_0'\zeta^{(1)}$ on large scales. The result is

$$\zeta^{(2)} \ni \left[\frac{1}{3}\left(\frac{a\,m_\varphi}{\mathcal{H}}\right)^2 + 2 + 2\epsilon - 2\eta\right](\zeta^{(1)})^2 \qquad \text{(D-26)}$$

where we have dropped terms which are higher order in slow roll parameters or which contain gradients. Notice that the quantity $(a\,m_\varphi/\mathcal{H})^2$ is first order in the slow roll expansion because

$$\eta \approx \frac{1}{\kappa^2}\frac{1}{V}\frac{\partial^2 V}{\partial\varphi^2} = \frac{1}{\kappa^2}\frac{m_\varphi^2}{V} \approx \frac{1}{3}\left(\frac{a\,m_\varphi}{\mathcal{H}}\right)^2.$$

To (D-26) we must add the contribution coming from the fist line of (6.41) which can be written explicitly in terms of first order quantities using the Green function for the master equation. Here we consider only the particular solution for $\phi^{(2)}$ due to the inflaton source $J^\varphi$.

In order to compute $J^\varphi$ we first study the quantities $\Upsilon_1, \Upsilon_2, \Upsilon_3, \Upsilon_4, \gamma$ which are defined in appendix D-2. On large scales and to leading order in slow roll we have

$$\Upsilon_1^\varphi \approx \left[12\epsilon^2 - \epsilon\left(\frac{a\,m_\varphi}{\mathcal{H}}\right)^2\right]\mathcal{H}^2(\zeta^{(1)})^2.$$

The quantity $\triangle^{-1}\Upsilon_2^\varphi$ can be written as (see equation 39 of [183])

$$\begin{aligned}\triangle^{-1}\Upsilon_2^\varphi &= \frac{\kappa^2}{2}(\epsilon-\eta)\mathcal{H}(\delta^{(1)}\varphi)^2 + 3\mathcal{H}(\phi^{(1)})^2 - 2\phi^{(1)}\phi'^{(1)}\\ &+ \frac{2}{\varphi_0'}\triangle^{-1}\left(\partial_i\partial^k\partial_k\phi^{(1)}\partial^i\delta^{(1)}\varphi\right.\\ &+ \left.\partial^k\partial_k\phi^{(1)}\partial_i\partial^i\delta^{(1)}\varphi\right).\end{aligned}$$

The inverse laplacians on the last two lines contribute only to the momentum dependence of $f_{NL}^\varphi$ which we neglect. On large scales and in the slow roll limit we have

$$\triangle^{-1}\Upsilon_2^\varphi \approx (4\epsilon^2 - \epsilon\eta)\mathcal{H}(\zeta^{(1)})^2 + \cdots$$

where the $\cdots$ denotes momentum dependent terms. On large scales and in the slow roll limit we also have

$$\Upsilon_3^\varphi \approx \left[36\epsilon^2 - 3\epsilon\left(\frac{a\,m_\varphi}{\mathcal{H}}\right)^2\right]\mathcal{H}^2(\zeta^{(1)})^2.$$

The quantity $\Upsilon_4$ defined in (D-24) depends only on the inflaton fluctuation and is given by

$$\Upsilon_4 = \Upsilon_4^{\varphi} \approx (6\epsilon^2 - 2\epsilon\eta)\frac{\varphi_0'}{\mathcal{H}}(\zeta^{(1)})^2$$

on large scales and in the slow roll limit. Using these results one may readily verify that $\gamma_\varphi/\mathcal{H}^2$ is third order in slow roll parameters

$$\gamma_\varphi \approx \mathcal{O}(\epsilon^3)\mathcal{H}^2(\zeta^{(1)})^2.$$

Using these results one may verify that the only term on the first line of (6.41) which contributes at lowest order in slow roll parameters is the term proportional to $\Upsilon_1^{\varphi}/\epsilon\mathcal{H}^2$. Thus, the contribution to the second order curvature perturbation due to the first line of (6.41) is

$$\zeta_\varphi^{(2)} \ni \left[4\epsilon - \frac{1}{3}\left(\frac{a\, m_\varphi}{\mathcal{H}}\right)^2\right](\zeta^{(1)})^2.$$

Adding all the contributions together we find

$$\zeta_\varphi^{(2)} \approx (2 - 2\eta + 6\epsilon)(\zeta^{(1)})^2.$$

The contribution $2(\zeta^{(1)})^2$ stems from using the Malik and Wands [189] definition of the second order curvature perturbation. It can be related to the definition of Lyth and Rodriguez [187] (which also agrees with Maldacena [18]) using

$$\zeta^{(2)} = \zeta_{LR}^{(2)} + 2(\zeta^{(1)})^2.$$

The Lyth-Rodriguez curvature perturbation, due to the inflaton up to second order, can thus be written as

$$\zeta_{LR}^{\varphi} = \zeta^{(1)} - \frac{3}{5}f_{NL}^{\varphi}(\zeta^{(1)})^2$$

where

$$f_{NL}^{\varphi} = \frac{5}{6}(2\eta - 6\epsilon).$$

In writing $\zeta_{LR}^{\varphi}$ above we have suppressed the homogeneous $k = 0$ mode of $\zeta$ which should be subtracted to ensure that $\langle\zeta\rangle = 0$. This result differs from previous studies [18, 158] by a factor of two. The calculation of [18, 158] takes into account the effect of nonlinear evolution aswell as the effect of computing the bispectrum in the vacuum of the interacting theory, as opposed to the vacuum of the free theory. Our calculation does not consider the effect of the change in vacuum which is the same order of magnitude. Thus we should not expect to reproduce exactly the results of [18, 158]. The change in vacuum will not change the calculation of the tachyon part of the curvature perturbation $\zeta_\sigma^{(2)}$ since $\zeta^{(1)}$ does not depend on $\delta^{(1)}\sigma$ and hence contributions to $\zeta_\sigma$ due to the change in vacuum will be higher than second order in perturbation theory.

## D-4 Fourier Transforms, Mode Functions and Inverse Laplacians

We define the Fourier transform of some first order quantity $\delta f(t,\vec{x})$ by

$$\begin{aligned}\delta f(t,\vec{x}) &= \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\vec{k}\cdot\vec{x}} \delta\tilde{f}_k(t) \\ \delta\tilde{f}_{\vec{k}}(t) &= \int \frac{d^3x}{(2\pi)^{3/2}} e^{-i\vec{k}\cdot\vec{x}} \delta f(t,\vec{x})\end{aligned} \tag{D-27}$$

where the Hermiticity of $\delta f(t,\vec{x})$ implies that $\delta\tilde{f}_k(t) = \delta\tilde{f}_{-k}(t)^\dagger$ so that we can define

$$\delta\tilde{f}_k(t) = a_k\xi_k(t) + a^\dagger_{-k}\xi_{-k}(t)^\star \tag{D-28}$$

where $a_k$ is an operator and $\delta f_k(t)$ is a c-number valued mode function. We then re-write the Fourier transform as

$$\begin{aligned}\delta f(t,x) &= \int \frac{d^3k}{(2\pi)^{3/2}} \left[a_k\xi_k(t)e^{ikx} + a^\dagger_{-k}\xi_{-k}(t)^\star e^{ikx}\right] \\ &= \int \frac{d^3k}{(2\pi)^{3/2}} \left[a_k\xi_k(t)e^{ikx} + a^\dagger_k\xi_k(t)^\star e^{-ikx}\right].\end{aligned}$$

In this form it is clear that $a_k$ is the usual creation operator satisfying

$$\left[a_k, a^\dagger_{k'}\right] = \delta^{(3)}(\vec{k}-\vec{k}')$$

and one should expect

$$\xi_k(\tau) \approx \frac{1}{\sqrt{2k}} e^{-ik\tau}$$

on small scales, which corresponds to the Bunch-Davies vacuum $a_k|0\rangle = 0$. This is consistent with the usual definition of the power spectrum in terms of the two-point function

$$\begin{aligned}\langle 0|\delta f^2(t,\vec{x})|0\rangle &= \int \frac{d^3k}{(2\pi)^3}|\xi_k(t)|^2 \\ &= \int \frac{dk}{k}\mathcal{P}_f(k)\end{aligned}$$

so that

$$\mathcal{P}_f(k) = \frac{k^3}{2\pi^2}|\xi_k(t)|^2.$$

We now discuss the Fourier transform of the tachyon source. Typical terms in the source have the form

$$\begin{aligned}J^{(1)}(t,\vec{x}) &= b(t)\delta f(t,\vec{x})\delta g(t,\vec{x}), \\ J^{(2)}(t,\vec{x}) &= b(t)\triangle^{-1}\left[\delta f(t,\vec{x})\delta g(t,\vec{x})\right], \\ J^{(3)}(t,\vec{x}) &= b(t)\triangle^{-2}\left[\delta f(t,\vec{x})\delta g(t,\vec{x})\right]\end{aligned}$$

where $\delta f$, $\delta g$ are some first order quantities and $b$ is constructed from zeroth order quantities. The Green's function for the laplacian is defined as appropriate in the absence of boundary surfaces

$$(\triangle^{-1}f)(t,\vec{x}) = \int d^3x' G(\vec{x}-\vec{x}')f(t,\vec{x}')$$

where

$$G(\vec{x}-\vec{x}') = -\frac{1}{4\pi}\frac{1}{|\vec{x}-\vec{x}'|}.$$

In Fourier space we have

$$\begin{aligned}G(x,x') &= \int \frac{d^3k}{(2\pi)^{3/2}}e^{ikx}G_k(x') \\ G_k(x') &= -\frac{1}{(2\pi)^{3/2}}\frac{e^{-ikx'}}{k^2}.\end{aligned}$$

In Fourier space the souce terms contain convolutions

$$\begin{aligned}J_k^{(1)}(t) &= b(t)(\delta\tilde{f}\star\delta\tilde{g})_k(t), \\ J_k^{(2)}(t) &= -b(t)\frac{1}{k^2}(\delta\tilde{f}\star\delta\tilde{g})_k(t), \\ J_k^{(3)}(t) &= b(t)\frac{1}{k^4}(\delta\tilde{f}\star\delta\tilde{g})_k(t).\end{aligned}$$

where $\tilde{f}_k$, $\tilde{g}_k$ are operator valued Fourier transforms defined as in (D-27). These may be related to the mode functions as (D-28). Finally, we have defined convolution by

$$(\delta\tilde{f}\star\delta\tilde{g})_{\vec{k}}(t) = \int \frac{d^3k'}{(2\pi)^{3/2}}\delta\tilde{f}_{\vec{k}'}(t)\delta\tilde{g}_{\vec{k}-\vec{k}'}(t)$$

# D-5 Construction of The Tachyon Curvature Perturbation

In this appendix we include technical details about the contruction of $\zeta_\sigma^{(2)}$ using the Green function for the master equation. For clarity we repeat some details which are included in the text. We consider only contributions to the equations which depend on the tachyon field and we remind the reader that $\zeta^{(1)}$ is independent of $\sigma$. The second order curvature perturbation is (see (6.41-6.43))

$$\begin{aligned}\zeta^{(2)} &= \frac{1}{3-\epsilon}\frac{1}{(\varphi_0')^2}\left[\varphi_0' Q_\varphi'^{(2)} + a^2\frac{\partial V}{\partial\varphi}Q_\varphi^{(2)}\right] \\ &+ \frac{1}{3-\epsilon}\frac{1}{(\varphi_0')^2}\left[\left(\delta^{(1)}\sigma'\right)^2 + a^2 m_\sigma^2\left(\delta^{(1)}\sigma\right)^2\right] \\ &+ \text{ inflaton contributions}\end{aligned} \tag{D-29}$$

where the second order Sasaki-Mukhanov variable is

$$\begin{aligned}Q_\varphi^{(2)} &= \delta^{(2)}\varphi + \frac{\varphi_0'}{\mathcal{H}}\psi^{(2)} \\ &+ \text{ inflaton contributions}\end{aligned} \tag{D-30}$$

Inserting (D-30) into (D-29) and using the $(0,0)$ Einstein equation (D-6) to eliminate the contribution $\varphi_0'\delta^{(2)}\varphi' + a^2\partial V/\partial\varphi\delta^{(2)}\varphi$ gives

$$\begin{aligned}\zeta_\sigma^{(2)} &= -\frac{\phi'^{(2)}}{\epsilon\mathcal{H}} - \left(\frac{1}{\epsilon}+1\right)\phi^{(2)} + \frac{1}{3-\epsilon}\frac{\partial^k\partial_k\phi^{(2)}}{\epsilon\mathcal{H}^2} \\ &+ \frac{1}{\epsilon\mathcal{H}}\Delta^{-1}\gamma_\sigma' + \Delta^{-1}\gamma_\sigma - \frac{1}{3-\epsilon}\frac{1}{\epsilon\mathcal{H}^2}\gamma_\sigma\end{aligned} \tag{D-31}$$

where we have also eliminated $\psi^{(2)}$ in favour of $\phi^{(2)}$ using (D-16). Notice that using (D-6) introduces a term proportional to $\Upsilon_1^\sigma$ to the curvature perturbation which cancels the contribution proportional to $(\delta^{(1)}\sigma')^2 + a^2 m_\sigma^2(\delta^{(1)}\sigma)^2$ on the second line of (D-29) up to a gradient term $(\vec{\nabla}\delta^{(1)}\sigma)^2$ which can be neglected on large scales. The result (D-31) is valid only on large scales but we have not yet assumed slow roll. In (D-31) it is understood that $\phi^{(2)}$ denotes only the particular solution due to the tachyon source, $J^\sigma$. We now use the Green function (6.36) to solve for $\phi^{(2)}$ in terms of the tachyon source (D-21). We work only to leading order in slow roll parameters. We also work in the large scale limit. To lowest order in $\epsilon$, $\eta$ and up to order $k^2$ we can write

$$\begin{aligned} & - \frac{\phi_k^{\prime(2)}}{\epsilon\mathcal{H}} - \frac{\phi_k^{(2)}}{\epsilon} - \frac{k^2\phi_k^{(2)}}{3\epsilon\mathcal{H}^2} = \\ & \frac{1}{\epsilon}\int_{\tau_i}^{0} d\tau'\Theta(\tau-\tau')J_k^\sigma(\tau') \\ \times & \left[\tau' + \left(-\frac{1}{6}\tau'^3 + \frac{5}{6}\tau^2\tau' - \frac{2}{3}\tau^3\right)k^2\right] \end{aligned} \tag{D-32}$$

using the Green function (6.36) and the relation (6.37). Equation (D-32) allows us to eliminate the dependence of $\zeta_\sigma^{(2)}$ on $\phi^{(2)}$. At leading order in slow roll parameters the tachyon source is (see (D-18))

$$\begin{aligned} J^\sigma &= \Upsilon_1^\sigma - \Upsilon_3^\sigma + 4\Delta^{-1}\partial_\tau\Upsilon_2^\sigma + 2\mathcal{H}\Delta^{-1}\Upsilon_2^\sigma \\ &+ \Delta^{-1}\partial_\tau^2\gamma_\sigma - \mathcal{H}\Delta^{-1}\partial_\tau\gamma_\sigma. \end{aligned} \tag{D-33}$$

We must now insert (D-33) into (D-32), perform numerous integrations by parts, and then insert this result into (D-31). We evaluate term-by-term the last line of (D-32). The first contribution (the term proportional to $k^0$) is

$$-\frac{\phi_k^{\prime(2)}}{\epsilon\mathcal{H}} - \frac{\phi_k^{(2)}}{\epsilon} - \frac{k^2\phi_k^{(2)}}{3\epsilon\mathcal{H}^2} \ni -\frac{1}{\epsilon}\int_{\tau_i}^{\tau} d\tau'\frac{J^\sigma(\tau')}{\mathcal{H}(\tau')}.$$

Inserting (D-33), noting that $\mathcal{H}(\tau') = -1/\tau'$ at leading order in slow roll and integrating by parts gives

$$\begin{aligned} & - \frac{\phi_k^{\prime(2)}}{\epsilon\mathcal{H}} - \frac{\phi_k^{(2)}}{\epsilon} - \frac{k^2\phi_k^{(2)}}{3\epsilon\mathcal{H}^2} \ni -\frac{1}{\epsilon}\int_{\tau_i}^{\tau} d\tau'\frac{J^\sigma(\tau')}{\mathcal{H}(\tau')} \\ \cong & \frac{1}{\epsilon}\int_{\tau_i}^{\tau} d\tau'\left[-\frac{\Upsilon_1^\sigma}{\mathcal{H}(\tau')} + \frac{\Upsilon_3^\sigma}{\mathcal{H}(\tau')} - 6\Delta^{-1}\Upsilon_2^\sigma\right] \\ + & \frac{1}{\epsilon}\left[-4\frac{\Delta^{-1}\Upsilon_2^\sigma}{\mathcal{H}} - \frac{\Delta^{-1}\gamma_\sigma'}{\mathcal{H}}\right] + \cdots \end{aligned} \tag{D-34}$$

where the terms under the $d\tau'$ integral are evaluated at $\tau'$ while the terms in the square braces on the second line are evaluated at $\tau$. The $\cdots$ denotes constant terms evaluated at $\tau = \tau_i$ which arise from the integration by parts. Since our interest is in the preheating phase during which the fluctuations $\delta^{(1)}\sigma$ are amplified exponentially we can safely drop these constant terms.

The second contribution to the last line of (D-32) has the form

$$-\frac{\phi_k'^{(2)}}{\epsilon\mathcal{H}} - \frac{\phi_k^{(2)}}{\epsilon} - \frac{k^2\phi_k^{(2)}}{3\epsilon\mathcal{H}^2} \ni \frac{1}{6\epsilon}\int_{\tau_i}^{\tau} d\tau' \frac{k^2 J_k^\sigma}{\mathcal{H}(\tau')^3}.$$

In evaluating this we need only consider terms in $k^2 J_k^\sigma$ which are not suppressed on large scales. Using (D-33) one may verify that

$$k^2 J_k^\sigma \cong -\gamma_{\sigma,k}'' + \mathcal{H}\gamma_{\sigma,k}' \tag{D-35}$$

on large scales (recall that $\Upsilon_2^\sigma$ is a gradient, see (D-11)) so that we have

$$\begin{aligned} - \quad & \frac{\phi_k'^{(2)}}{\epsilon\mathcal{H}} - \frac{\phi_k^{(2)}}{\epsilon} - \frac{k^2\phi_k^{(2)}}{3\epsilon\mathcal{H}^2} \ni \frac{1}{6\epsilon}\int_{\tau_i}^{\tau} d\tau' \frac{k^2 J_k^\sigma}{\mathcal{H}(\tau')^3} \\ \cong \quad & \frac{1}{\epsilon}\int_{\tau_i}^{\tau} d\tau' \left[-\frac{2}{3}\frac{\Upsilon_3^\sigma}{\mathcal{H}} + 6\Delta^{-1}\Upsilon_2^\sigma\right] \\ + \quad & \left[-\frac{1}{6\epsilon}\frac{\gamma_\sigma'}{\mathcal{H}^3} - \frac{1}{3\epsilon}\frac{\gamma_\sigma}{\mathcal{H}^2} + 2\frac{\Delta^{-1}\Upsilon_2^\sigma}{\mathcal{H}}\right] + \cdots \end{aligned} \tag{D-36}$$

Equation (D-35) shows why it was necessary to include the $k^2$ terms in the large scale expansion of the Green function (D-32), noting that $\gamma_\sigma$ may be written as (D-17) and comparing to (D-33) we see that $k^2 J_k$ contains terms which are of the same size as those in $J_k$, on large scales. Thus a consistent large scale expansion of $\zeta_\sigma^{(2)}$ requires that we work up to order $k^2$ in the expansion of the Green function.

The third contribution to the last line of (D-32) is

$$\begin{aligned} - \quad & \frac{\phi_k'^{(2)}}{\epsilon\mathcal{H}} - \frac{\phi_k^{(2)}}{\epsilon} - \frac{k^2\phi_k^{(2)}}{3\epsilon\mathcal{H}^2} \ni -\frac{5}{6\epsilon}\int_{\tau_i}^{\tau} d\tau' \frac{k^2 J_k^\sigma}{\mathcal{H}(\tau')\mathcal{H}(\tau)^2} \\ \cong \quad & \frac{5}{6\epsilon}\left[\frac{\gamma_\sigma'}{\mathcal{H}^3}\right] + \cdots \end{aligned} \tag{D-37}$$

using the same procedure as above.

Finally, the fourth contribution to the last line of (D-32) is

$$\begin{aligned}
& - \frac{\phi_k'^{(2)}}{\epsilon\mathcal{H}} - \frac{\phi_k^{(2)}}{\epsilon} - \frac{k^2\phi_k^{(2)}}{3\epsilon\mathcal{H}^2} \ni \frac{2}{3\epsilon}\int_{\tau_i}^{\tau} d\tau' \frac{k^2 J_k^\sigma}{\mathcal{H}(\tau)^3} \\
\cong & -\frac{2}{3\epsilon}\frac{1}{\mathcal{H}(\tau)^3}\int_{\tau_i}^{\tau} d\tau' \mathcal{H}(\tau')^2 \Upsilon_3^\sigma \\
+ & \frac{1}{\epsilon}\left[2\frac{\triangle^{-1}\Upsilon_2^\sigma}{\mathcal{H}} + \frac{2}{3}\frac{\gamma_\sigma}{\mathcal{H}^2} - \frac{2}{3}\frac{\gamma_\sigma'}{\mathcal{H}^3}\right] + \cdots
\end{aligned} \tag{D-38}$$

Summing up (D-34), (D-36), (D-37) and (D-38) and inserting the result into (D-31) gives

$$\begin{aligned}
\zeta_\sigma^{(2)} \cong & \frac{1}{\epsilon}\int_{\tau_i}^{\tau} d\tau' \left[-\frac{\Upsilon_1^\sigma}{\mathcal{H}(\tau')} + \frac{1}{3}\frac{\Upsilon_3^\sigma}{\mathcal{H}(\tau')}\right. \\
- & \left.\frac{2}{3}\frac{\mathcal{H}(\tau')^2}{\mathcal{H}(\tau)^3}\Upsilon_3^\sigma\right]
\end{aligned}$$

Now, using equations (D-8) and (D-14) we can write this in terms of the tachyon fluctuation $\delta^{(1)}\sigma$ as

$$\begin{aligned}
\zeta_\sigma^{(2)} \cong & \frac{\kappa^2}{\epsilon}\int_{-1/a_i H}^{\tau} d\tau' \left[\frac{\left(\delta^{(1)}\sigma'\right)^2}{\mathcal{H}(\tau')} - \frac{\mathcal{H}(\tau')^2}{\mathcal{H}(\tau)^3}\left(\left(\delta^{(1)}\sigma'\right)^2\right.\right. \\
- & \left.\left. a^2 m_\sigma^2 \left(\delta^{(1)}\sigma\right)^2\right)\right]
\end{aligned} \tag{D-39}$$

## D-6 An Identity Concerning $\gamma_\sigma$

In this appendix we derive an identity concering the tachyon source term $\gamma_\sigma$ (6.101):

$$\gamma_\sigma = \Upsilon_3^\sigma - 3\triangle^{-1}\partial_\tau\Upsilon_2^\sigma - 6\mathcal{H}\triangle^{-1}\Upsilon_2^\sigma.$$

Using equations (6.95) and (6.98) we can write this

$$\begin{aligned}
\gamma_\sigma = & \kappa^2\triangle^{-1}\left[\frac{3}{2}\partial^k\partial_k(\delta^{(1)}\sigma_A'\delta^{(1)}\sigma'^A)\right. \\
- & \frac{1}{2}\partial^k\partial_k(\partial_i\delta^{(1)}\sigma_A\partial^i\delta^{(1)}\sigma^A) \\
- & \frac{3}{2}a^2\frac{\partial^2 V}{\partial\sigma_A\partial\sigma_B}\partial^k\partial_k(\delta^{(1)}\sigma_A\delta^{(1)}\sigma_B) \\
- & 3\partial_\tau\partial_i(\delta^{(1)}\sigma_A'\partial^i\delta^{(1)}\sigma^A) \\
- & \left.6\mathcal{H}\partial_i(\delta^{(1)}\sigma_A'\partial^i\delta^{(1)}\sigma^A)\right]
\end{aligned}$$

and, after some algebra, we have

$$\gamma_\sigma = \kappa^2 \Delta^{-1} \left[ -\frac{1}{2} \partial^k \partial_k (\partial_i \delta^{(1)} \sigma_A \partial^i \delta^{(1)} \sigma^A) \right. \tag{D-40}$$
$$- \; 3\partial_i \left( \delta^{(1)} \sigma''_A + 2\mathcal{H} \delta^{(1)} \sigma'_A + a^2 \frac{\partial^2 V}{\partial \sigma_A \partial \sigma_B} \delta^{(1)} \sigma_B \right) \partial^i \delta^{(1)} \sigma^A$$
$$\left. - \; 3 \left( \delta^{(1)} \sigma''_A + 2\mathcal{H} \delta^{(1)} \sigma'_A + a^2 \frac{\partial^2 V}{\partial \sigma_A \partial \sigma_B} \delta^{(1)} \sigma_B \right) \partial^k \partial_k \delta^{(1)} \sigma^A \right]$$

In deriving this equation we have used that fact that

$$\frac{\partial^2 V}{\partial \sigma_A \partial \sigma_B} = \frac{\partial^2 V}{\partial \sigma_B \partial \sigma_A}$$

which follows from the $O(M)$ symmetry of the theory. The last two lines of (D-40) can be simplified using the equation of motion for the tachyon fluctuation (6.103) which gives

$$\gamma_\sigma = -\frac{\kappa^2}{2} \left( \partial_i \delta^{(1)} \sigma_A \partial^i \delta^{(1)} \sigma^A \right)$$
$$- \; 3\kappa^2 \Delta^{-1} \partial_i \left( \partial^k \partial_k \delta^{(1)} \sigma_A \partial^i \delta^{(1)} \sigma^A \right) \tag{D-41}$$

# Chapter E

# Appendices to Chapter 7

# E-1 The Stress Energy Tensor and the Friedmann Equation

Here we compute the $O(u^2)$ term of the approximate solutions

$$\begin{aligned} \phi &= 1 - u - \phi_2 u^2 \\ H &= H_0 - H_2 u^2 \end{aligned}$$

We write the different terms appearing in the energy density as

$$\rho_\phi = -T_{00} = \frac{m_s^4}{2g_p^2}\left[\phi e^{-\frac{\Box}{m_p^2}}\phi - \frac{2}{p+1}\phi^{p+1} + \frac{1}{m_p^2}\int_0^1 d\tau\,\left(\Box e^{-\frac{\tau\Box}{m_p^2}}\phi\right)\left(e^{-\frac{(1-\tau)\Box}{m_p^2}}\phi\right)\right.$$
$$\left. + \frac{1}{m_p^2}\int_0^1 d\tau\;\partial_t\left(e^{-\frac{\tau\Box}{m_p^2}}\phi\right)\partial_t\left(e^{-\frac{(1-\tau)\Box}{m_p^2}}\phi\right)\right] \equiv \frac{m_s^4}{2g_p^2}(T_1+T_2+T_3+T_4) \qquad \text{(E-1)}$$

where $T_1$, $T_2$, $T_3$ and $T_4$ are defined as in (7.49-7.52). We now evaluate this expression term-by-term.

We first consider $T_1 + T_2$, which can be written as

$$T_1 + T_2 = \frac{p-1}{p+1}\phi^{p+1}$$

using (7.3). Using (E-1) we have

$$T_1 + T_2 = \frac{p-1}{p+1} + (-p+1)u + (p-1)\left[-\phi_2 + \frac{p}{2}\right]u^2 + \mathcal{O}(u^3) \qquad \text{(E-2)}$$

To evaluate $T_3$, $T_4$ we note that

$$\begin{aligned} e^{-\tau\Box/m_p^2}\phi &= 1 - e^{\tau\mu_1}u - \phi_2 e^{\tau\mu_2}u^2 + \cdots \\ e^{-(1-\tau)\Box/m_p^2}\phi &= 1 - e^{(1-\tau)\mu_1}u - \phi_2 e^{(1-\tau)\tau\mu_2}u^2 + \cdots \end{aligned} \qquad \text{(E-3)}$$

(see equation 7.23 with $\sigma = 0$). It is straightforward to show that

$$\Box e^{-\tau\Box/m_p^2}\phi = e^{\tau\mu_1}(\lambda^2 + 3H_0\lambda)u + \phi_2 e^{\tau\mu_2}(4\lambda^2 + 3H_0\lambda)u^2 + \cdots \qquad \text{(E-4)}$$

We are now in a position to compute $T_3$. Using (E-3) and (E-4) we have

$$\begin{aligned} T_3 &= \frac{1}{m_p^2}\int_0^1 d\tau\,\left[(\lambda^2 + 3H_0\lambda)e^{\tau\mu_1}u\right. \\ &+ \left.\left[-(\lambda^2 + 3H_0\lambda)e^{\mu_1} + \phi_2(4\lambda^2 + 6H_0\lambda)e^{\tau\mu_2}\right]u^2 + \cdots\right] \end{aligned}$$

The $d\tau$ integrals are trivial to perform using the identity

$$\int_0^1 d\tau e^{\alpha\tau} = \frac{e^{\alpha} - 1}{\alpha} \tag{E-5}$$

We find that

$$T_3 = (p-1)u + \left[-p\mu_1 + (e^{\mu_2} - 1)\phi_2\right] u^2 + \mathcal{O}(u^3) \tag{E-6}$$

We now consider the integrand of $T_4$:

$$\partial_t\left(e^{-\frac{\tau\Box}{m_p^2}}\phi\right)\partial_t\left(e^{-\frac{(1-\tau)\Box}{m_p^2}}\phi\right) \cong \lambda^2 u^2 \partial_u\left(1 - e^{\tau\mu_1}u\right)\partial_u\left(1 - e^{(1-\tau)\mu_1}u\right) = \lambda^2 u^2 e^{\mu_1} \tag{E-7}$$

The $d\tau$ integral is trivial and gives

$$T_4 = u^2 \frac{\lambda^2}{m_p^2} e^{\mu_1} \tag{E-8}$$

It is straightforward to sum up the various contributions to $T_{00}$. We find that

$$\begin{aligned} \rho_\phi &= \frac{m_s^4}{2g_p^2}\left[\frac{p-1}{p+1}\right. \\ &+ \left.\left[(-p + e^{\mu_2})\phi_2 + \frac{p}{2}(p-1) - p\ln p + p\frac{\lambda^2}{m_p^2}\right] u^2 + \cdots\right] \end{aligned} \tag{E-9}$$

The fact that the coefficient of the $\mathcal{O}(u)$ term is zero verifies that $H_1 = 0$. We now solve the Friedmann equation

$$3H^2 = \frac{1}{M_p^2}\rho_\phi$$

noting that

$$H^2 = H_0^2 - 2H_0 H_2 u^2 + \mathcal{O}(u^3)$$

Matching the coefficients at order $u^0$ gives

$$H_0^2 = \frac{1}{6g_p^2}\frac{m_s^4}{M_p^2}\frac{p-1}{p+1}$$

as before. Matching the coefficients at order $u^2$ gives

$$\begin{aligned} -6H_0 H_2 &= \frac{m_s^4}{2g_p^2 M_p^2}\left[(e^{\mu_2} - p)\phi_2 + \frac{p}{2}(p-1) - p\ln p + p\frac{\lambda^2}{m_p^2}\right] \\ &= \frac{m_s^4}{M_p^2}\frac{p}{2g_p^2}\left[\frac{\lambda^2}{m_p^2} - \ln p\right] \\ &= \frac{m_s^4}{M_p^2}\frac{p}{2g_p^2}\left[\frac{-3H_0\lambda}{m_p^2}\right] \end{aligned}$$

In the first equality we have used that fact that

$$\phi_2 e^{\mu_2} = p\phi_2 - \frac{p}{2}(p-1)$$

which follows from the second order Klein-Gordon equation and in the second equality we have used

$$\frac{\lambda^2}{m_p^2} + 3\frac{H_0\lambda}{m_p^2} = \mu_1 = \ln p$$

which follows from the first order Klein-Gordon equation. Finally we arive at the result for $H_2$:

$$H_2 = \frac{m_s^4}{4g_p^2 M_p^2}\frac{p\lambda}{m_p^2} = \frac{1}{8g_s^2}\frac{p^3\ln p}{p-1}\left(\frac{m_s}{M_p}\right)^2\lambda \tag{E-10}$$

## E-2 Incomplete Cylindrical Functions of the Sonine-Schaefli Form

The Bessel function can be represented by a contour integral of the Sonine-Schaefli form

$$J_\nu(z) = \frac{1}{2\pi i}\left(\frac{z}{2}\right)^\nu \int_{c-i\infty}^{c+i\infty} \omega^{-\nu-1}\exp\left[\omega - \frac{z^2}{4\omega}\right]d\omega \tag{E-11}$$

as long as $\mathrm{Re}(\nu) > -1$. In (E-11) $c$ is an arbitrary positive constant. The incomplete cylindrical function, $S_\nu(r,s;z)$ generalizes (E-11) to arbitrary limits of integration

$$S_\nu(r,s;z) = \frac{1}{2\pi i}\left(\frac{z}{2}\right)^\nu \int_r^s \omega^{-\nu-1}\exp\left[\omega - \frac{z^2}{4\omega}\right]d\omega \tag{E-12}$$

It follows that

$$\int_r^s \omega^{-\nu-1}\exp\left[-\omega - \frac{z^2}{4\omega}\right]d\omega = 2\pi i e^{i\nu\pi/2}\left(\frac{z}{2}\right)^{-\nu} S(-r,-s;iz) \tag{E-13}$$

Taking the limits $r \to 0$ and $s \to +\infty$ this integral can be written in terms of Hankel functions of imaginary argument, $K_\nu(z)$, as

$$\int_0^\infty \omega^{-\nu-1}\exp\left[-\omega - \frac{z^2}{4\omega}\right]d\omega = 2\left(\frac{z}{2}\right)^{-\nu} K_\nu(z) \tag{E-14}$$

The function $K_\nu(z)$ is real-valued for real $z$ and is related to the usual Hankel function as $K_\nu(z) = (i\pi/2)e^{i\pi\nu/2}H_\nu^{(1)}(iz)$. Using the well known large-argument asymptotics of the Hankel functions one may show that

$$\int_0^\infty \omega^{-\nu-1} \exp\left[-\omega - \frac{z^2}{4\omega}\right] d\omega \cong \sqrt{\frac{2\pi}{z}} \left(\frac{z}{2}\right)^{-\nu} e^{-z} \tag{E-15}$$

for $z \gg 1$.

# E-3 Comparison to Local Field Theory

In this appendix we perform a detailed comparison of our results for the action (7.1) to the theory

$$S = -\int d^4x \left[\frac{1}{2}\partial_\mu\psi\partial^\mu\psi + V(\psi)\right] \tag{E-16}$$

with potential

$$\begin{aligned} V(\psi) &= \frac{g_s}{4}\left(\psi^2 - v^2\right)^2 \\ &\equiv \frac{gv^4}{4} - m_s^2\psi^2 + \frac{g}{4}\psi^4 \end{aligned} \tag{E-17}$$

where we have defined $m_s^2 \equiv 2g_s v^2$. We are interested in obtaining inflation near the unstable maximum $\psi = 0$. The flatness of the potential is parameterized by the dimensionless slow roll parameters

$$\epsilon = \frac{M_p^2}{2}\left(\frac{V''(\psi)}{V(\psi)}\right)^2 = \frac{8\psi^2 M_p^2}{(\psi^2 - v^2)^2} \tag{E-18}$$

$$\eta = M_p^2 \frac{V''(\psi)}{V(\psi)} = \frac{4M_p^2}{(\psi^2 - v^2)^2}\left[3\psi^2 - v^2\right] \tag{E-19}$$

Unlike in the $p$-adic theory we do not need to distinguish between $\epsilon_{\mathcal{H}}, \eta_{\mathcal{H}}$ and $\epsilon_V, \eta_V$ (see equations 7.81-7.88) and hence we drop the subscripts on the slow roll parameters. For $\psi \ll v$ the $\epsilon$ parameter (E-18) is automatically small while

$$|\eta| \cong 8\left(\frac{M_p}{v}\right)^2$$

which is small compared to unity for $v \gg M_p$. The fact that the symmetry breaking scale is large compared to the Planck scale may be reason to doubt the validity of the field theory (E-16). However, for our purposes this is irrelevant.

It is instructive to consider solving the equations of motion for this theory using the formalism of section 7.4. We begin by speculating solutions of the form

$$\psi = ve^{\lambda t} + \psi_2 v e^{2\lambda t} + \psi_3 v e^{3\lambda t} + \mathcal{O}(e^{3\lambda t}) \tag{E-20}$$

$$H = H_0 - H_1 e^{\lambda t} - H_2 e^{2\lambda t} + \mathcal{O}(e^{3\lambda t}) \tag{E-21}$$

and solving order by order in $e^{\lambda t}$. The ansatz (E-20,E-21) is analogous to (7.11,7.12) since in both cases the (classical) field spends an infinite amount of time at the unstable maximum, driving a past-eternal de Sitter phase, before rolling towards the true minimum of the potential. We suppose that $e^{\lambda t} \ll 1$ so that $\psi \ll v$ initially.

The Klein-Gordon equation is

$$\frac{\ddot{\psi}}{v} + 3H\frac{\dot{\psi}}{v} = 2m_s^2\frac{\psi}{v} + 2m_s^2\left(\frac{\psi}{v}\right)^3$$

Plugging in the ansatz (E-20) we obtain

$$\lambda^2 + 3H_0\lambda = 2m_s^2 \tag{E-22}$$

at first order in $e^{\lambda t}$. This result is identical to (7.27). At second and third order respectively we find

$$\psi_2 = 0$$

$$\psi_3 = \frac{2m_s^2 + 3H_2\lambda}{9\lambda^2 - 2m_s^2}$$

The Friedmann equation is

$$3H^2 = \frac{1}{M_p^2}\left[\frac{gv^4}{4} - m_s^2 v^2\left(\frac{\psi}{v}\right)^2 + \frac{gv^4}{4}\left(\frac{\psi}{v}\right)^4 + \frac{v^2}{2}\left(\frac{\dot{\psi}}{v}\right)^2\right]$$

At zeroth order in $e^{\lambda t}$ we obtain the familiar result

$$H_0 = \frac{g^{1/2}}{2\sqrt{3}}\frac{v^2}{M_p} \tag{E-23}$$

At first and second order we find that

$$H_1 = 0 \tag{E-24}$$

$$H_2 = \frac{2}{\sqrt{3g}}\frac{2m_s^2 - \lambda^2}{M_p} = \frac{1}{4}\left(\frac{v}{M_p}\right)^2\lambda \tag{E-25}$$

In writing the second equality in (E-25) we have used $\lambda^2 + 3H_0\lambda = 2m_s^2$.

Though a complete treatment of inhomogeneities including metric perturbations and nonzero slow roll parameters is straightforward in this context we choose to analyze this theory using the same approximations as we used in section 7.5 in order to make more explicit the comparison between the two theories. The perturbed Klein-Gordon equation (neglecting metric fluctuations and in the limit $e^{\lambda t} \to 0$) is

$$\left(\partial_t^2 + 3H_0\partial_t - \frac{\partial_i\partial^i}{a^2} - 2m_s^2\right)\delta\psi = 0$$

where $a = e^{H_0 t}$. The large scale solution is

$$|\delta\psi_k| \cong \frac{H}{\sqrt{2k^3}}\left(\frac{k}{aH_0}\right)^{3/2-\nu}$$

where

$$\nu = \sqrt{\frac{9}{4} + \frac{2m_s^2}{H_0^2}}$$

Near scale-invariance of the spectrum requires $m_s \ll H_0$. In this limit we have

$$n_s - 1 \cong -\frac{4}{3}\frac{m_s^2}{H_0^2} = -8\left(\frac{M_p}{v}\right)^2 \tag{E-26}$$

which is identical to (7.68). This result also reproduces the full calculation incorporating metric perturbations: $n_s - 1 \cong 2\eta - 6\epsilon \cong -8M_p^2/v^2$.

We can use (E-26) to write the dimensionless quantities $H_0/m_s$, $H_2/m_s$ and $\lambda/m_s$ in terms of $|n_s - 1|$. The solution of (E-22) can be written as

$$\frac{\lambda}{m_s} \cong \sqrt{\frac{|n_s - 1|}{3}}$$

which is identical to (7.71). For this solution we of course have $3\lambda H_0 \gg \lambda^2$ so that the evolution is friction dominated. It is also straightforward to show that

$$\begin{aligned} \frac{H_0}{m_s} &= \frac{2}{\sqrt{3|n_s - 1|}} \\ H_2 &= H_0 \end{aligned}$$

We see that the solutions $H$, $\chi$ for the theory (E-16) are identical to those of the theory (7.1) up to order $e^{\lambda t}$. At order $e^{2\lambda t}$ and higher, however, the dynamics of the two theories differs.

From (E-18) one can check that $\epsilon \cong 1$ at $\psi \lesssim v$ so that it is a good approximation to suppose that inflation ends at $e^{\lambda t} \cong 1$. It is straightforward to impose the COBE normalization $V/(\epsilon M_p^4) = 6 \times 10^{-7}$ for this model (which will impose $g_s \ll |n_s - 1|^2$), however, this is unnecessary for our purposes. It is clear that the inflationary dynamics and predictions predictions of the theory (E-16) are identical to those of the theory (7.1).

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