A Bouncing Cosmology from VECROs

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March, 2023

A thesis submitted to McGill University in partial fulfillment of the requirements of the degree of

Master of Science

Abstract

There are many different cosmological models that aim to resolve the problems encountered in Standard Big Bang cosmology. One such problem is the singularity problem that occurs because the universe is compressed into an infinitely dense point at the time of the Big Bang. We propose that we must use string theory to describe the physics of the early universe because it has enough degrees of freedom to prevent the cosmological singularity from occurring. When applied to black holes, it has been argued that string theory prevents the formation of horizons and leads to the formation of horizon sized objects called VECROs (Virtual Extended Compression Resistant Objects). First, we review how VECROs have been used to resolve the information paradox and the unbounded entropy problem encountered in the classical description of black holes. Then, we briefly discuss cosmology in a contracting universe and summarize the previous results for black hole formation in a contracting universe. Finally, we propose an application of VECROs to cosmology. We argue that it is possible that a background composed of a gas of VECROs can lead to a non-singular bouncing cosmology.

Abrégé

Il existe plusieurs modèles cosmologiques visant à résoudre les problèmes rencontrés dans le Modèle Standard de la cosmologie. Un de ces problèmes est le problème de singularité qui se produit lorsque l'univers est compressé en un point à densité immense au moment du Big Bang. Nous suggérons d'utiliser la théorie des cordes pour décrire la physique de l'univers primitif, car elle a assez de degrés de liberté pour prévenir l'apparition d'une singularité cosmologique. Lorsqu'appliquée dans le cas des trous noirs, il a été suggéré que la théorie des cordes prévient la formation d'horizons et mène à la formation d'objets de la taille d'horizons nommés VECROs (Virtual Extended Compression Resistant Objects). En premier lieu, nous faisons un résumé de comment les VECROs peuvent être utilisés pour résoudre le paradoxe d'information et le problème de croissance incontrôlée d'entropie présente dans la description classique des trous noirs. Ensuite, nous discutons brièvement l'évolution cosmologique d'un univers en contraction et résumons certains procédées connus menant à la formation de trous noir dans un univers en contraction. Finalement, nous proposons une application possible des VECROs dans le domaine de la cosmologie. Nous amenons l'hypothèse qu'il est possible qu'un gaz de VECROs mène à un rebond de l'univers, évitant ainsi le problème de singularité.

Acknowledgements

This endeavor would not have been possible without the guidance, patience, and extensive knowledge of my supervisor, Robert Brandenberger. I would also like to acknowledge support from an NSERC CGSM scholarship and the Lorne Trottier Science Accelerator Fellowship. Lastly, I'd like to recognize Samir Mathur for developing the fuzzball conjectures for black holes.

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List of Acronyms

AdS/CFT Anti-de Sitter/Conformal field theory.

CMB Cosmic Microwave Background.

EOM Equation of Motion.

FLRW Friedmann–Lemaître–Robertson–Walker.

LIGO Laser Interferometer Gravitational-Wave Observatory.

PBB Pre-Big-Bang.

Chapter 1

Introduction

General relativity tells us that the gravitational force arises due to distortions of spacetime, and it is the most successful theory of gravity. Previously, Newton's more basic framework was useful for describing the motion of massive objects, even though the origin of the attractive force was unknown. However, experiments showed that Newton's model did not line up with certain observations. One such observation was made in the 1800s, and it was the precession of the perihelion of Mercury. Rather than the elliptical orbit predicted by Newtonian gravity, general relativity predicted that the perihelia of the planets would rotate around the Sun. Another important observation was that light does not travel along straight lines in the presence of massive objects. Although Newtonian gravity did predict that starlight would bend as it passed the Sun, it wasn't until Einstein's formulation of general relativity that a reliable value that could be replicated in experiments was calculated. The gravitational redshifting of light and the related gravitational time dilation were also predicted by general relativity, and subsequently tested via light escaping a white dwarf star, and through the use of atomic clocks respectively.

A more recent consequence of Einstein's theory of general relativity was the detection of gravitational waves from binary black hole mergers by LIGO and Virgo. Gravitational waves, which are ripples in spacetime that travel at the speed of light, also could not have been discovered with Newton's developments alone. They are especially important in studying the very early universe because gravitational waves are generated during inflation, which is one of the popular theories for the early universe, and have an effect on the cosmic microwave background (CMB) [28].

Despite the many successes of general relativity, there are still a few major problems. One of those problems is the spacetime singularities that are present in the classical description of black holes and the Big Bang, as shown by the Hawking-Penrose singularity theorems [20]. In general relativity, singularities occur when the spacetime curvature becomes infinite, which is a sign that the theory is incomplete due to the fact that spacetime is ill defined. Black holes form when an object collapses to a sufficient degree after which it becomes an infinitely dense point of spacetime surrounded by a hypersurface called an event horizon. Once a particle crosses the event horizon, it can never escape. On the other hand, the cosmological singularity is what happens when we use general relativity to extrapolate backwards in time to deduce what happened in the far past. For instance, the equations for a model of spacetime called the Friedmann–Lemaître–Robertson–Walker (FLRW) metric can be followed back in time. From this, it was found that the universe began in an infinitely hot and dense state, which is known as the traditional Big Bang. The singularity arising from the FRLW metric tells us that general relativity is insufficient for describing the laws of physics of the early universe.

Although general relativity is the best relativistic theory of gravity, it still only a classical theory. This is concerning because the beginning of the universe involves very high energies, which means that quantum effects are expected to be significant. For example, the information paradox is a contradiction that arises when both general relativity and quantum mechanics are applied to the study of black holes (see e.g. [23] for an introduction to the information paradox). Hawking applied the laws of quantum mechanics to the general relativity definition of black holes and found that black holes emit radiation, currently known as Hawking radiation [19]. It was also argued that the radiation would only depend on the mass, charge, and angular momentum of the initial black hole (see e.g. recent review [2]). However, many different states can be identical in these three properties. This becomes problematic when we consider the formation of a black hole that eventually evaporates away leaving only radiation in its wake. Many different initial states could have evolved into this final state, which results in a permanent loss of information about the initial state. This is in direct violation of quantum unitarity, which implies that the quantum state at any time can be used to determine what the state looked like in the past or future [2]. We know that the time evolution of a quantum state is a unitary process, therefore the problem must lie in the general relativity description of black holes.

For a long time, it was thought that the information paradox could be solved by considering the effects of small quantum gravity corrections (see e.g. the discussion in [26]). For example, if the small corrections introduced correlations between the radiated quanta, then at the end of the evaporation process, the radiation is not entangled with the remaining hole and we have no paradox [2]. However, it was argued that small modifications are not sufficient to provide a solution to the information paradox; instead, we require order-unity corrections [26]. Another possible way around the paradox was to conclude that the evaporation of a black hole stops when it reaches the Planck mass, and leaves behind a Planck-sized remnant [2]. Starting with a black hole with an arbitrarily large mass, results in an arbitrarily large entanglement with the remnant [2]. This means that the remnant has an infinite number of internal states [2]. However, the AdS/CFT duality within string theory [22] does not allow for the existence of remnants because there cannot be an infinite number of state for remnants at the center of AdS [26]. As we will see in the body of the thesis, we must deviate from the classical description of a black hole in order to avoid the information paradox. Instead of a spacetime singularity surrounded by a vacuum horizon, black hole microstates should instead be horizon sized quantum objects with no horizon, and no vacuum region, as was argued in [26]. As will be discussed later in this thesis, black holes in the fuzzball picture can release information easily since there is no microscopical horizon. In fact, the final stages of the evaporation are easier to describe since there is no singularity. The initial stages of the evaporation will be the same as in the semiclassical picture. These quantum objects are called fuzzballs, and they are intrinsically stringy [2]. The vacuum virtual counterpart of a fuzzball is called a VECRO [26], and will be the focus of this thesis.

While there are no complete quantum theories of gravity, one promising candidate is superstring theory [16]. As we have seen, general relativity coupled to matter as point particles is not the correct way to describe nature. In string theory, rather than point particles, the building blocks are instead fundamental strings. Strings have three types of states: momentum modes, oscillatory modes, and winding modes. In comparison with point particles, we see that the new degrees of freedom are the oscillatory modes and the winding modes [4] (see [10] for a review). Furthermore, the T-duality symmetry from string theory also plays a very important role. T-duality provides a relationship between winding modes in one compact space, with momentum modes in a dual compact space [4]. Because strings allow for new degrees of freedom and introduce new symmetries, they lead to a very different interpretation of the early universe [4, 8]. One such interpretation is string gas cosmology [4], which addresses the issues of an initial spacetime singularity, as well as the infinite temperature that arises as we approach the Big Bang. String gas cosmology makes use of T-duality, string winding modes, and string oscillatory modes. This model comes from coupling a gas of closed strings to a classical background, with all spatial dimensions compactified [4, 10]. When the spatial dimensions are taken to be toroidal, the T-duality symmetry tells us that the energy spectrum of the string states is the same if the winding quantum numbers and the momentum quantum numbers are switched [4, 10].

The key realization comes from considering the thermodynamics of strings. As was first discovered in [18], when a gas of closed strings is in thermal equilibrium, its temperature cannot exceed a particular finite value known as the Hagedorn temperature. Moreover, as seen in [4], for a gas of closed strings in a box, the temperature of the gas decreases as the radius of the box falls to a value much smaller than the string length. If the gas of strings has a large entropy, then there is a large range of possible box radii for which the temperature of the gas is near the Hagedorn temperature [4]. This is known as the Hagedorn phase of string cosmology. The universe smoothly exits the Hagedorn phase and transitions directly into the radiation dominated era of Standard Big Bang cosmology [10]. The discovery of the Hagedorn temperature provided a solution to the infinite temperature problem, and also leads to the absence of a curvature singularity [4]. However, despite this, the background dynamics of string gas cosmology are not yet understood, which means that there is no quantitative model for the Hagedorn phase [10]. Note that it is assumed in String Gas Cosmology that the dilaton is fixed. Hence, the Einstein and string frames coincide.

Rather than have the universe start in an hot, dense state with high curvature, we can consider a model that starts in a contracting phase, then “bounces" into the typical expanding phase. In bouncing cosmology, the universe starts out as cold and large; vacuum perturbations on sub-Hubble scales in this initial phase generate the primordial perturbations that become the structures we see today (for a review of bouncing cosmologies see e.g. [6]). However, if general relativity and matter satisfy the usual energy conditions, we end up with a singular bounce, that is, the energy density becomes infinite [10]. As we discussed previously, general relativity is incomplete, so we do not expect it to be able to correctly describe what is happening near the bounce. In order to achieve a nonsingular bounce, we must use matter that violates the weak energy condition, or go beyond Einstein gravity [10]. However, we need new Planck-scale physics in order to get the necessary background cosmology that is not yet well established [10]. This is the main challenge when trying to realize the bouncing scenario.

Aside from string gas cosmology, there have been a number of other attempts to use string theory to resolve the Big Bang singularity. Two such approaches are the Pre-Big-Bang (PBB) model and the Ekpyrotic scenario. Both of these models contain a cosmological bounce, however, their effective actions break down at the bounce point [38]. This makes it impossible to properly study how the background evolves as well as the resulting perturbations. In order to determine the resulting cosmology after the bounce, the challenge in both models is to match the contracting universe to the expanding universe along a space-like hypersurface that represents the bounce region [38].

In the original Ekpyrotic scenario, the universe was taken to be a five-dimensional spacetime that starts out cold and nearly vacuous [21]. The spacetime is bounded by two (3+1)-dimensional surfaces: the visible brane, and the hidden brane [21]. The visible brane is the usual four-dimensional universe with particles and radiation [21]. The bulk volume that is bounded by the boundary branes also contains a brane that is attracted to the visible brane [21]. The two branes approaching one another corresponds to the contracting phase, which is driven by scalar field matter with a negative exponential potential [7]. When the bulk brane and the visible brane collide, there is a singular bounce from the low energy effective field theory perspective [6]. Then, some of the kinetic energy from the bulk brane is converted into a hot bath of matter and radiation on the visible brane [21]. Rather than starting from an infinite temperature as in Standard Big Bang cosmology, the hot expanding phase in this scenario starts evolving from a finite temperature before entering the FLRW radiation dominated phase [21]. The main challenges in the Ekpyrotic scenario are figuring out how to obtain a nonsingular bounce, as well as a mostly scale-invariant spectrum of curvature fluctuations [7].

String theory predicts the existence of the dilaton, which has an associated scalar field, and plays an important role in the PBB scenario [13]. The PBB phase begins in a flat, empty state, composed of incoming gravitational and dilatonic waves [13]. This string vacuum is gravitationally unstable, and therefore leads to the evolution towards higher and higher curvature and coupling constants [15]. Therefore, unlike the Ekpyrotic scenario, the PBB scenario's contraction period is deflationary and driven by the kinetic energyy of the dilaton [13]. The PBB phase ends with global singularity, where the Hubble constant is infinite [21]. The challenge is to match across the singularity to the post-Big-Bang FLRW universe [21].

The goal of this thesis is to show that a gas of VECROs in a contracting universe will lead to a nonsingular cosmological bounce. In chapter 2 we define VECROs and see how they are used to solve problems in the current black hole formalism. We motivate the use of bouncing cosmologies in Chapter 3, and describe the background and cosmological perturbations in a contracting universe. Chapter 4 gives an overview of previous work done in [35] that determines the conditions under which density perturbations grow and collapse into black holes in a contracting universe. In chapter 5 we argue that VECROs naturally result in a nonsingular bounce, and produce a scale-invariant spectrum of curvature perturbations. The appendix includes a paper published by Robert Brandenberger and myself about shape moduli stabilization in the context of string gas cosmology.

Chapter 2

Black Holes and VECROs

2.1 Classical Black Holes

Black holes are regions of spacetime that are separated from infinity by a hypersurface called an event horizon. Once particles cross an event horizon, they can never escape to infinity. Though black holes have horizons, the external geometry is the same as it is for more typical bodies, such as stars or planets. In general relativity, the unique static spherically symmetric vacuum solution is the Schwarzschild metric, which is given by:

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \tag{2.1.1}$$

We see that the Schwarzschild event horizon is located at $R_S = 2GM$, which means that all future timelike paths cannot cross past this radius once they enter it. All stationary, asymptotically flat black hole solutions that are nonsingular outside the event horizon are fully characterized by mass, electric and magnetic charge, and angular momentum [14]. This is called the 'no-hair theorem', and it applies when electromagnetism is the only long-range nongravitational field [14]. The solutions for charged and rotating black holes, are given by the Kerr and Reissner Nordström metrics, respectively [14]. In the context of this paper, only Schwarzschild black holes are relevant.

2.2 Black Hole Puzzles

2.2.1 The Problem of Unbounded Entropy

Bekenstein's expression for the entropy of a black hole is given by:

$$S_{Bek} = \frac{A}{4G} \tag{2.2.1}$$

where A is the area of the black hole [1]. This is considered to be the maximum possible entropy that can be in the region $r \leq 2GM$. Therefore, for a system of mass M, fixed volume V, and energy $E = M$, that is confined to R_S, we expect:

$$S(E,V) \leq S_{Bek}(M)$$

However, in the usual black hole metric where there is an event horizon, it was seen in [26] that it is possible to construct a slice of spacetime with an arbitrarily large entropy inside R_S, while keeping the energy at M. Referring back to the metric in equation 2.1.1, we will see that we can construct a spacelike slice in this geometry with unbounded entropy.

For a constant $\bar{t}$ and $\bar{r}$, a spacelike slice can be taken as $t = \bar{t}$ outside the horizon, however when r is inside R_S, the spacelike slice will be $r = \bar{r}$ [26]. This is because space and time interchange roles when you cross over the horizon. To demonstrate the problem of unbounded entropy, we can choose to set $\bar{r} = GM$ so that we are not close to the horizon of the singularity. We have the freedom to construct a slice like this for any black hole, and using the generalized metric:

$$ds^2 = -f(r)du^2 + 2dudr + r^2 d\Omega^2 \tag{2.2.2}$$

where $u = t + r^*$, $r^* = \int \frac{dr}{f}$, and $f(R_S) = 0$ [26].

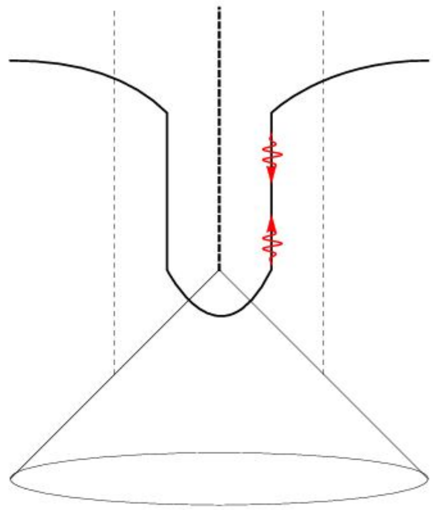

Figure 2.1: This figure shows a null shell (the cone) collapsing into a black hole [26]. The thin dashed lines are the horizon, and the thick dashed line is the singularity. The solid black line is the hypersurface $r = \bar{r}$ (inside the horizon) that we are adding quanta to, which are depicted by the red arrows.

We can choose to add entropy to this slice by adding quanta to the portion $r = \bar{r}$ inside the horizon. For massless quanta moving in the radial direction, the momenta are the following null vectors:

$$\vec{k}_1 = (0, k_1^r) \tag{2.2.3}$$

$$\vec{k}_2 = (k_2^u, \frac{1}{2}\bar{f}k_2^u) \tag{2.2.4}$$

where $f(\bar{r}) \equiv \bar{f}$, and $k_1^r < 0$ and $k_2^u > 0$ [26]. To ensure the momenta lie in the forward light cone, we have required that $\vec{k} \cdot \hat{n} < 0$, where $\hat{n}$ is the unit normal to the slice pointing in the direction of decreasing r. The energies of the quanta are given by $E = -\vec{K} \cdot \vec{k}_i$ where $\vec{K}$ is the Killing vector ∂_t. Therefore, we have [26]:

$$E_1 = -k_1^r > 0 \tag{2.2.5}$$

$$E_2 = \frac{1}{2}\bar{f}k_2^u < 0 \tag{2.2.6}$$

This means that a quantum with momentum k_1 provides a positive contribution to the mass of the black hole, whereas a quantum with momentum k_2 will provide a negative contribution. In order to make a state with unbounded entropy, we can take our previously discussed spacelike slice and add photons to the $r = \bar{r} = GM$ portion that carries the energies E_1 and E_2 above [26]. Moving in the direction of increasing u, we can place photons along the slice, and alternate between $\bar{k_1}$ and $\bar{k_2}$ momenta. Each photon's energy is very small, hence the mass inside the horizon is $\approx M \pm \delta$, where δ is very small [26].

Each photon will have two spin states, and carry an entropy of ln2. Then, if there are N photons corresponding to $\bar{k_1}$ and N corresponding to $\bar{k_2}$, the total entropy will be [26]:

$$S = 2N \ln 2 > S_{Bek} \tag{2.2.7}$$

The absence of an upper bound on the entropy comes from the fact that the slice we have chosen has an infinite proper length, meaning that N can be arbitrarily large [26]. In specific, [26] shows that if the length, L, of the $r = \bar{r}$ part of the slice inside the horizon satisfies:

$$L \gtrsim \left(\frac{R_S}{l_{Pl}} \right)^{\frac{d-1}{d+1}} R_S \tag{2.2.8}$$

then we can get an entropy that exceeds S_{Bek} without a significant disturbance of the semiclassical black hole metric. Here, d is the spacetime dimension, and l_{Pl} is the Planck length. The only way we can get around this problem, is if we find something that prevents the allowance of such long slices in the black hole geometry [26].

2.2.2 The Information Paradox

In the case of a collapsing shell, when the shell reaches the interior of its horizon, new physics on it can no longer affect the behaviour of fields at the horizon due to causality [26]. However, around the horizon, entangled particle-antiparticle pairs are constantly being created [25]. One quantum in the pair falls into the hole, and the other quantum escapes to infinity as Hawking radiation [25]. When the black hole fully evaporates, the ending product is radiation that is entangled, but there will be nothing that it is entangled with [25]. This results in a permanent loss of information, known as the 'information paradox'.

To demonstrate this more thoroughly, we can consider a spherically symmetric shell of massless particles moving in the radial direction. Let M be the mass of the shell, and let its proper radius be $R(\lambda)$, where we use λ to parametrize null trajectories. Causality tells us that two particles p and p' on the shell cannot send signals to each other if they are not infinitesimally close together. Furthermore, we can say that all fluctuations of importance can be confined to a length scale $\lesssim l_{Pl}$ [26]. The equivalence principle tells us that the inward path of particle p is identical to what it would be in a section of spacetime that is approximately flat, because the classical gravity theory and quantum fluctuations are not affected by the rest of the shell [26]. This would also be the case as p falls through its horizon at $R(\lambda) = 2GM$.

Light cones in the region $R(\lambda) < r < 2GM$ point inwards, so assuming causality, the behaviour of the shell as $R(\lambda) \to 0$ cannot have any significant effects on the low energy fields by the horizon [26]. Therefore, we can only expect $O(\varepsilon)$ modifications to the Hawking pairs at the horizon, where $\varepsilon \ll 1$ [26]. Unfortunately, the small corrections theorem prevents the continued rise in entanglement entropy between Hawking pairs from being resolved through small corrections to the emission process [26].

2.3 VECROs

Horizons, and consequently singularities, only arise when we use a theory that doesn't have enough degrees of freedom. String theory, however, has a sufficient amount, and it leads to a different microstructure of black holes than we what we find in the semi-classical theory [2]. That is, when we try to construct black holes via string theory, we get horizon sized objects with no horizon, and those are the previously mentioned fuzzballs [2].

We must first make two assumptions. Firstly, semiclassical gravity will hold to leading order if there are no horizons, and curvatures are low ($\mathscr{R} \ll l_P^{-2}$) [26]. Secondly, causality will hold to leading order if curvatures are low [26]. Then, we will see that there must be a new component of the vacuum quantum gravity wavefunctional, that is not normally considered in the quantum field theory vacuum [26]. This novel addition is composed of quantum fluctuations called VECROs, and we will argue that they are required to solve the problems discussed in sections 2.2.1 and 2.2.2, whilst also preserving our above two assumptions.

2.3.1 Spacetime Topology

Before getting into a more in-depth description of VECROs, we will first briefly discuss non-trivial spacetime manifolds and why they are of interest to us. A good example to consider, is the situation where a scalar field arises from a compactified extra dimension. For example, if we compactify a dimension into a circle and parametrize it by ω, then the scalar field, ϕ, is defined by [26]:

$$g_{\omega\omega} = e^{\phi} \tag{2.3.1}$$

Fluctuations of the radius in the compactified direction correspond to excitations of scalar fields on the noncompactified spacetime [26]. This is depicted in figure 2.2(a). In this case, the physics is what we would expect from quantum fields on curved spacetime [26].

Figure 2.2: (a) Fluctuations of the compact radial direction are included in the semiclassical approximation [26]. (b) Fluctuations where the circle pinches off leads to a different type of topology that is not included in the semiclassical approximation [26].

However, other possibilities are fluctuations where the compact directions pinch off, as in figure 2.2(b). In this case, the compact spacetime is not a direct product with the noncompact spacetime

[26]. Here, the dynamics of the scalar field are not described by the semiclassical approximation like figure 2.2(a) [26]. These types of nontrivial spacetime manifolds are important to consider, because they appear in the fuzzball solutions for black hole microstates in string theory [26]. Without some of the semiclassical limitations, we get horizon sized objects with no horizon that will not collapse. These objects are known as fuzzballs, and are excitations of the theory with various masses [26].

Presently, we will be focused on the vacuum wavefunctional rather than the fuzzball solutions, i.e. the excitations, themselves. The scalar fields, ϕ, that exist on the spacetime manifold will fluctuate in the vacuum. As we saw previously, it is possible for fluctuations in the radius of the compactified directions to yield deformations to the vacuum wavefunctional like in figure 2.2(b). Furthermore, it follows that the vacuum wavefunctional should have support over nontrivial manifolds like figure 2.2(b) [26]. The part of the vacuum wavefunctional with this support is called a VECRO, which is an acronym that stands for Virtual Extended Compression-Resistant Object. The wavefunctional amplitudes will be lower in this case because it takes more action to reach these configurations [26]. Black hole microstates have support over these topologically nontrivial configurations (figure 2.2(b)), and have a very large number of states [26]. Therefore, even though the wavefunctional's amplitude will be smaller on the nontrivial configurations, the large number of states ensures that the VECRO part of the vacuum wavefunctional is significant [26]. We will see that there are situations where the classical theory would result in horizon formation, whereas with VECROs there is no horizon.

2.3.2 When VECROs Become Important

Let us describe the curvature of a spacetime region by: L, the length scale over which the curvature persists, and R_c, the curvature length scale with $R_c \sim \mathscr{R}^{\frac{-1}{2}}$. If

$$L \ll R_c \tag{2.3.2}$$

is true everywhere, then the dynamics are governed by the semiclassical rules of quantum field theory on a curved spacetime. The VECRO portion of the quantum gravity wavefunctional will adjust to the changes in curvature adiabatically [26].

However, if

$$L \gtrsim R_c \tag{2.3.3}$$

The VECRO configurations are squeezed and modified by order unity, causing an increase of the VECRO energy that is on par with the classical energy that gives rise to R_c [26]. We will discuss why this happens in more detail in section 2.3.5. Instead of the vacuum, we now have the creation of on-shell fuzzballs, which are excitations analogous to particle creation. We encounter this situation for black holes, and the new physics allows us to avoid the problems in sections 2.2.1 and 2.2.2.

2.3.3 Virtual

The quantum electrodynamics vacuum has fluctuations that occur in particle-antiparticle pairs such as electron-positron pairs, or quark-antiquark pairs, but it is also possible to have fluctuations of more complicated bound states [26]. Similarly, vacuum fluctuations for large or extended objects, like atoms and benzene rings, should also exist virtually [26]. We do not normally consider these types of fluctuations because they have a low probability of happening. We can approximate the probability for a state with energy E, existing for a time T, as [26]:

$$P \sim e^{-\tilde{S}} \sim e^{-ET} \tag{2.3.4}$$

It follows from our reasoning above that the vacuum should also contain virtual black hole fluctuations for all masses M. In a $d+1$ dimensional spacetime, with the horizon radius denoted as R_0, the action is:

$$\tilde{S} \sim MR_0 \sim \left(\frac{R_0}{l_p}\right)^{d-1} \tag{2.3.5}$$

where we approximate T, the duration of the fluctuation, as R_0 [26]. This means that the probability is small for $R_0 \gg l_p$. However, black holes have a very large degeneracy, $\mathscr{N} \sim Exp[S_{Bek}]$ [27]. This larger number of degrees of freedom makes it possible for the total probability to be of order unity [26]. What we take from this, is that the vacuum virtual fluctuations of black holes are significant for all M.

2.3.4 Extended

Fuzzballs are extended objects that are bound states in string theory [26]. Although fuzzball states are quantum, their size can be approximated by looking at specific subsets of states that have classical descriptions [27]. In [27], a few simple cases were studied and it was found that the bound state is horizon sized, which implies that the area of the boundary of the state gives the degeneracy via a Bekenstein relation. This was proven for the brane bound states of Strominger and Vafa, as well as the BFSS matrix model [2]. The full construction of states for all 2-charges also showed that individual microstates do not have a horizon or singularity [27].

In semiclassical gravity, black holes are essentially empty space with a singularity at the centre. As mentioned before, string theory shows that black hole microstates are fuzzballs, which are the excited states of VECROs. Fuzzball solutions can arise due to characteristics of string theory such as extended objects and a larger number of dimensions, whereas $3+1$ dimensional quantum gravity prohibits these solutions [26]. We can see this in the following example:

$$ds^2 = -dt^2 + \left(1 - \frac{r_0}{r}\right) d\tau^2 + \frac{dr^2}{1 - \frac{r_0}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{2.3.6}$$

where the Euclidean time, τ, is compact, and $0 \leq \tau \leq 4\pi r_0$ [24]. This is essentially adding a time direction that is trivial to the $3+1$ dimensional Schwarzschild solution. Dimensionally reducing on the circle τ, allows us to view the solution as a $3+1$ dimensional metric in (t, r, θ, ϕ) coupled to the scalar field that describes the radius of the compact direction [24]. We would usually discard this solution because pressures diverge as $r \to r_0$. However, the problem isn't the solution, but rather the dimensional reduction map itself when the radius of the compact circle goes to zero [24]. Equation 2.3.6 is still an acceptable solution of the $4+1$ dimensional vacuum Einstein equations.

Many fuzzball microstates have been constructed in string theory, and most have the structure of the example in figure 2.3. Figure 2.3 is an example of a configuration of the gravity theory that fuzzball wavefunctionals are supported on. We will not be concerned with the details of the constructions of fuzzballs; what is relevant for this paper is the fact that the field configurations that support fuzzballs are extended. That is, energy is not squeezed into a point-like neighbourhood they way it is in the semiclassical situation.

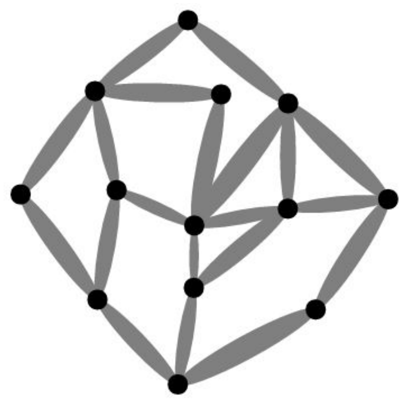

Figure 2.3: A depiction of a fuzzball [26]. The black dots are KK monoploes and anitmonopoles, and the grey ellipses are S^2 spheres that carry gauge field fluxes that maintain the rigidity of the structure [26].

2.3.5 Compression Resistant

The gravitational force grows with Newton's constant, which means that matter gets more and more compressed as G increases. However, we also know that the size of the black hole horizon also increases with G. Therefore, we expect that the index states would collapse behind a horizon when the gravitational attraction becomes too strong for the matter to sustain the pressure necessary to prevent core collapse [2]. String theory prevents this outcome through fractionation of its spectrum on extended objects like VECROs. Fractionation occurs because string theory contains solitonic branes. If N is a large number that represents the amount of branes in a set, and a string or brane is bound to that set, then the tension of that brane becomes $1/N$ of its original tension [2]. As we mentioned before, VECROs are bound states in string theory, so this yields the third key property: compression resistance.

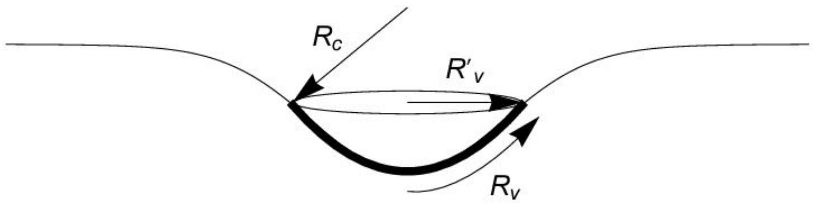

Figure 2.4: A compressed VECRO configuration [26]. See text for explanation.

Because VECROs are compression resistant objects, their potential energy will rise quickly if we try to squeeze or expand them. Suppose we have a VECRO configuration in flat space with radius R_V. If the spacetime is then deformed into a section of a sphere with a curvature radius of R_c, then the newly compressed radius of the VECRO is [26]:

$$R'_V = R_c \sin \frac{R_V}{R_c} \tag{2.3.7}$$

This is depicted in figure 2.4. We define δ such that the surface of the VECRO is compressed by a factor of $1 - \delta$. There are two cases: either $\delta \ll 1$, or $\delta \sim 1$. In the first case, the compression is small, so [26]:

$$\delta \approx \frac{1}{6} \left(\frac{R_V}{R_c} \right)^2 \tag{2.3.8}$$

Furthermore, the potential energy change is quadratic in the compression, which means that $U \sim \delta^2$ [26]. In the second case, when $\delta \sim 1$, the VECRO breaks, and its potential energy function levels out at this critical value even upon further compression or expansion [26]. Now, we model the potential energy as:

$$U = k \sin^2 \left(\frac{\pi}{2} \frac{\delta}{\delta_c} \right), \qquad -\delta_c < \delta < \delta_c \tag{2.3.9}$$

where δ_c is an order unity constant [26].

To determine the energy scale, k, we assume that it corresponds to the mass of a black hole that has a horizon radius R_ν [26]:

$$M \sim \frac{R_\nu^{d-2}}{G} \tag{2.3.10}$$

Therefore,

$$k = \beta \frac{R_\nu^{d-2}}{G} \tag{2.3.11}$$

where β is an order unity constant.

We will consider an example using an ordinary object like a star. Let R_{star} be its radius, and let the constant ρ_0 be the uniform density of the star in $d+1$ dimensions. The curvature radius in the region of the star is then [26]:

$$R_c \sim \frac{1}{\sqrt{G\rho_0}} \tag{2.3.12}$$

We define a μ such that:

$$R_{star} = \mu R_c \tag{2.3.13}$$

For a star, $\mu \ll 1$, but for an object whose radius is similar to its Schwarzschild radius, $\mu \sim 1$ [26]. Furthermore, we know that the energy of the star is as follows:

$$E_{star} \sim \rho_0 R_{star}^d \tag{2.3.14}$$

As we discussed previously, the curvature caused by the star will compress the VECROs and therefore lead to an increase in energy. The most significant changes in energy will come from the largest VECROs, which have $R_\nu \sim R_{star}$. Since μ is small, δ follows equation 2.3.8. For VECROs with $R_\nu = R_{star}$, the potential energy change is [26]:

$$U \sim (\rho_0 R_{star}^d) \left(\frac{R_{star}}{R_c}\right)^2 \tag{2.3.15}$$

However, because

$$\frac{U}{E_{star}} \sim \mu^2 \ll 1 \tag{2.3.16}$$

the compression energy coming from VECROs will not significantly impact this star. In contrast, when $\mu \sim 1$, we will have

$$\frac{U}{E} \sim 1 \tag{2.3.17}$$

which means that VECROs will have a significant impact.

2.4 The VECRO Picture for Black Holes

2.4.1 Resolving the Unbounded Entropy Problem

In section 2.2.1, we saw that it was possible for there to be no upper bound on the entropy in the horizon, while still keeping the mass in the horizon at $\approx M$, if we keep the traditional metric [26]. Now we will show that the VECRO hypothesis disallows the particular slices that are required to get states with unbounded entropy.

We can first consider a hypersurface that is not stretched in the way required to get unbounded entropy, and focus on the VECROs with radius $R_V \gtrsim R_S$. After the slice is evolved for a time $t \sim R_S$ the VECRO will have to expand to a larger volume [26]. The energy of the VECRO configuration must then increase. For $\delta \sim 1$, the deformation of the VECRO wavefunctional gives the following energy increase [26]:

$$\Delta E_{vecro} \sim U \sim \frac{R_S^{d-2}}{G} \sim M \tag{2.4.1}$$

According to [26], $\Delta E_{vecro} > M$ so the total energy on the slice satisfies:

$$E_{total} = M + \Delta E_{vecro} > 2M \tag{2.4.2}$$

Placing negative energy quanta on the slice would lower the energy, but because $E_{negative} > -M$, it is not possible to bring E_{total} back down to M [26].

$$E_{total} = M + \Delta E_{vecro} + E_{negative} > M \tag{2.4.3}$$

This is because the the horizon radius would vanish if the energy enclosed on the slice reached zero, and consequently a spacelike slice wouldn't be able to exist within the horizon [26]. Therefore, we cannot have this type of slice in the black hole geometry because it does not allow energy to be conserved.

2.4.2 Resolving the Information Paradox

In section 2.2.2, we described why black holes have an information loss problem, and now we will argue how it can be avoided. Far outside of a collapsing shell, that is, $R(\lambda) \gg R_S$, the gravitational pull outside the shell is weak, and VECROs are compressed to a slightly smaller radius [26]. The distortion of the vacuum is small, and is already included in quantum field theory on curved spacetime [26]. For the case of a VECRO with radius R_v, with $R(\lambda) < R_v < R_S$, the VECRO part of the vacuum wavefunctional will be significantly deformed due to the inward pointing light cones inside the Schwarzchild radius. The compression reaches $\delta \sim 1$, which results in a linear superposition of on-shell fuzzball states in the spacetime region between R_S and the shell radius. In particular, a collapsing shell tunnels into a linear combination of VECRO and fuzzball states such that no microphysical state has a horizon [2]. This is similar to how the distortion of the semiclassical wavefunctional creates on-shell particles [26]. Because fuzzballs radiate from their surfaces like normal bodies, rather than by pair creation from the vacuum; we are able to avoid the information paradox [2].

2.5 VECRO Equation of State

We will now argue that the energy density of a VECRO must be negative in order to eliminate the metric singularity present in classical black holes. First, we start with the metric of an uncharged, non-rotating, vacuum black hole

$$ds^2 = -(1-\frac{2GM}{r})dt^2 + (1-\frac{2GM}{r})^{-1}dr^2 \tag{2.5.1}$$

where, t is the time of an observer far from the black hole's centre, and r is the radial distance such that $4\pi r^2$ is the area of a sphere surrounding the black hole at a radius r [14]. For simplicity, we have dropped the angular variables. A static spherically symmetric spacetime in the presence of matter will have the general form:

$$ds^2 = -A(r)dt^2 + B(r)dr^2 \tag{2.5.2}$$

where the functions $A(r)$ and $B(r)$ depend on the matter distribution [3]. For asymptotically flat spacetime, $A(r), B(r) \to 1$, and we recover the Minkowski metric. For the Schwarzschild metric $A(r) = 1 + \frac{2GM}{r}$, and $B(r) = \frac{1}{A(r)}$. In both of these cases we have no matter source, but in our case, we have non-zero components of the stress-energy tensor, $T_{\mu\nu}$. If we introduce the functions $f(r)$ and $h(r)$ to parametrize $A(r)$ and $B(r)$ as follows:

$$A(r) \equiv e^{2h(r)} f(r) \qquad B(r) \equiv f(r)^{-1} \tag{2.5.3}$$

then, we can use the Einstein equations,

$$R^{\mu}_{\nu} = 8\pi G(T^{\mu}_{\nu} - \frac{1}{2}\delta^{\mu}_{\nu} T^{\sigma}_{\sigma}) \tag{2.5.4}$$

to give us the following equations of motion [3]:

$$h'(r) = 4\pi G r f(r)^{-1}(T^r_r - T^t_t) \tag{2.5.5}$$

$$[r(f(r)-1)]' = 8\pi G r^2 T^t_t \tag{2.5.6}$$

These equations determine $A(r)$ and $B(r)$ in terms of the components of $T_{\mu\nu}$. As in [3], we can redefine $r(f(r)-1)$ as

$$r(f(r)-1) \equiv -2m(r) \tag{2.5.7}$$

so that

$$f(r) = 1 - \frac{2m(r)}{r} \tag{2.5.8}$$

where $m(r)$ can be interpreted as the effective mass of the solution at radius r. Using this expression for $f(r)$, equation 2.5.6 becomes:

$$m'(r) = 4\pi G r^2(-T_t^t) = 4\pi G r^2 \rho(r) \tag{2.5.9}$$

where we have assumed that matter is a perfect fluid [3]. In particular,

$$T_{\mu\nu} = (p+\rho)U_\mu U_\nu + p\eta_{\mu\nu} \tag{2.5.10}$$

$$U^\mu = (1,0,0,0) \tag{2.5.11}$$

where U^μ is the four-velocity and $\eta_{\mu\nu}$ is the Minkowski metric. Therefore, we can solve the differential equation to find the effective mass at r.

$$m(r) = 4\pi G \int_0^r dr' r'^2 \rho(r') \tag{2.5.12}$$

If ρ is a constant, we can see that $f(r) = 1 - \frac{8\pi G\rho}{3}r^2$, so there is no singularity at $r = 0$. For VECROs, we don't know the form of the energy density, but we can make an argument for its sign. If the energy comes from a VECRO component with energy density ρ_v, and an infalling shell with mass m_0, then the effective mass becomes

$$m(r) = m_0 + 4\pi G \int_0^r dr' r'^2 \rho_v(r') \tag{2.5.13}$$

Without the VECROs, we would have $f(r) = 1 - \frac{2m_0}{r}$, which clearly has a singularity at $r = 0$. In order for the VECROs to remove the singularity produced by m_0, we conclude that ρ_v must be negative.

The equation of state shows how the pressure is related to energy density as: $p = w\rho$, where w is the equation of state parameter. We now argue that the equation of state parameter for a gas of VECROs is $w = 1$. That is,

$$p_V = \rho_V \tag{2.5.14}$$

To justify this, we will use the same logic in [36] that was used for a stringy black hole gas. The key input in the analysis of [36], is that the entropy of a single VECRO is given by its area, just as it is for a black hole. Therefore, the entropy of a gas of VECROs is

$$S = NM_V^2 G, \tag{2.5.15}$$

where M_V is the mass of a VECRO, and N is the number of VECROs in a fixed comoving volume V. The energy of a gas of VECROs is given by

$$E = NM_V. \tag{2.5.16}$$

We can obtain the equation of state from equation 2.5.14 by using the following thermodynamic identities:

$$p = T(\partial S/\partial V)_E \tag{2.5.17}$$

$$T^{-1} = (\partial S/\partial E)_V \tag{2.5.18}$$

Chapter 3

Cosmology Overview

3.1 Bouncing Cosmology

There are many different models for the very early universe, but the most reliable ones should explain our observations of the CMB. That is, they should predict a nearly scale-invariant power spectrum of curvature perturbations with a small red tilt, a small tensor-to-scalar ratio, and small non-Gaussianities [35]. One of the first models to accomplish this is inflationary cosmology [17], which is characterized by a very early phase of near exponential expansion of the universe. This model provides a causal mechanism for structure formation, and also solves some of the problems encountered in Standard Big Bang Cosmology [9]. Despite the successes of inflationary cosmology, there are still a number of conceptual problems.

One important challenge for the inflationary scenario is the singularity problem. Because the dynamics of inflation come from scalar field matter coupled to the Einstein-Hilbert action, the Hawking-Penrose singularity theorems imply that the must be a singularity before the inflationary period [6]. This means that inflation cannot tell us the complete history of the very early universe. Another challenge is the trans-Planckian problem for fluctuations. If the inflationary period lasts only slightly longer than the minimal period required to solve the problems of Standard Big Bang theory, then the wavelengths of modes we see today originated at sub-Planckian values [6]. The trans-Planckian regime is where general relativity and quantum field theory break down, therefore the computations in inflation make assumptions about physics beyond the Planck scale [6].

Although inflation is self-consistent as an effective field theory, it does not emerge from an ultraviolet complete theory. In particular, inflation does not naturally emerge from superstring theory [5]. Superstring theory is the best available theory for describing physics at very high energies, and is very restrictive [5]. The physics emerging from superstring theory should be able to be described by an effective field theory at low energies [5]. However, there is a vast landscape of effective field theories, and only a small subset of them are consistent with superstring theory [5]. The compatible theories are constrained by the swampland criteria, and the rest lie in the "swampland" [5]. The slow-roll inflationary model does not satisfy the swampland criteria, hence it is not consistent with superstring theory [5]. On the other hand, string gas cosmology comes directly from the fundamental principles of string theory, and the Ekpyrotic scenario can be easily embedded in string theory [6]. Therefore, it is a good idea to consider alternatives to inflation.

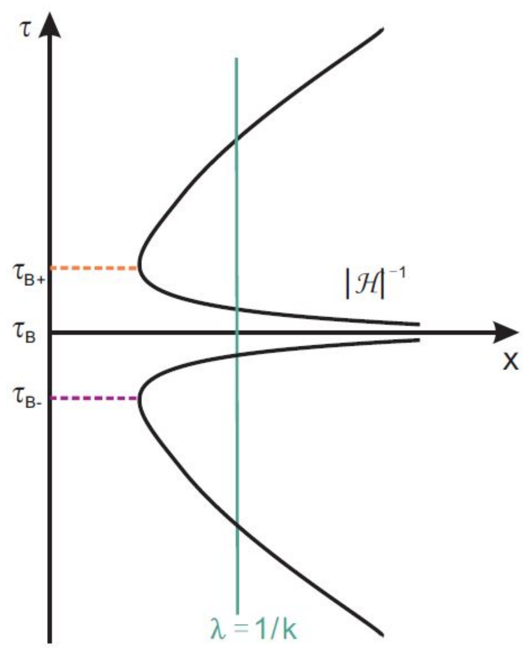

Figure 3.1: A spacetime sketch of a bouncing cosmology: the vertical axis is the conformal time, the horizontal axis is the comoving spatial coordinate, $\mathscr{H}^{-1}$ is the comoving Hubble radius, and λ is the physical wavelength of a cosmological fluctuation mode [10].

One such alternative is the cosmological bounce scenario, which will be our focus for the duration of this paper. By introducing new physics to obtain the bounce, this scenario naturally avoids the singularity problem. Figure 3.1 shows a spacetime sketch of a bounce at a time $\tau = \tau_B$. There are three main phases of a nonsingular bounce [10]. Initially, the universe existed forever in a contracting phase where the Hubble radius decreased linearly with the magnitude of time. Then, there is the bounce phase, where we transition from a contracting universe to an expanding one. And thirdly, we end up in the usual expanding phase of Standard Cosmology. As we can see in figure 3.1, the scales we observe today started out early in the contracting phase at sub-Hubble lengths, which means that there is also a causal mechanism for structure formation [10]. We can see that the trans-Planckian problem is avoided here because the length scales we observe are much larger than the Planck scale [6].

One of the difficulties with bouncing cosmology is the growth of anisotropies in a contracting universe [35]. In later sections, we will discuss previous work from [35], where the conditions under which the growth of density perturbations in a contracting universe forms large inhomogeneities that collapse into black holes was studied. Then, we will do a similar analysis that accounts for the presence of VECROs during the contracting phase.

3.2 Gravitational Potential In a Contracting Universe

3.2.1 The Background

In order to have nonsingular bouncing cosmology, we must have a background where the scale factor, $a(t)$, bounces in a nonsingular manner [10]. First, we define the reduced Planck mass by $M_{Pl} \equiv (8\pi G_N)^{-1/2}$, where G_N is Newton's gravitational constant [35]. Then, in order to eventually find the evolution of perturbations in a contracting universe, we start with the following generic action:

$$S = -\frac{1}{16\pi G_N}\int \mathrm{d}^4x\sqrt{-g}R + S_m \tag{3.2.1}$$

where $g_{\mu\nu}$ is the metric tensor, $g \equiv \det(g_{\mu\nu})$, R is the Ricci scalar, and S_m is the matter action [14]. We will work in a flat Friedmann-Lemaître-Robertson-Walker(FLRW) metric, so the background metric is:

$$ds^2 = g^{(0)}_{\mu\nu}dx^\mu dx^\nu = a(\eta)^2(d\eta^2 - \delta_{ij}dx^i dx^j) \tag{3.2.2}$$

where a is the scale factor, η is the conformal time defined as $d\eta \equiv a^{-1}dt$, with t being the physical time, and x_i are the Cartesian comoving coordinates [28]. The energy-momentum tensor is defined as [14]:

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta_g S_m}{\delta g^{\mu\nu}} \tag{3.2.3}$$

We assume the energy-momentum tensor takes the form $T^\nu_\mu = \mathrm{diag}(\rho, -p\delta^j_i)$, where p is the pressure and ρ is the energy density. Using this, the background equations of motion (EOMs) are then:

$$\mathscr{H}^2 = \frac{8\pi G_N}{3}a^2\rho \tag{3.2.4}$$

$$\mathscr{H}' = -\frac{4\pi G_N}{3}a^2\rho(1+3w) \tag{3.2.5}$$

where the prime is a conformal time derivative, and $\mathscr{H} \equiv a'/a$ is the conformal Hubble parameter [35].

We will be working in a hydrodynamical fluid setup, so we will assume the matter action has the following form:

$$S_m = -\int \mathrm{d}^4x\sqrt{-g}\rho \tag{3.2.6}$$

The fluid's sound speed is defined by the variation of the pressure, p, with respect to the energy density, ρ, at constant entropy density, s [28]. That is,

$$c_s^2 \equiv \left(\frac{\partial p}{\partial \rho}\right)_s \tag{3.2.7}$$

We will assume that the fluid only has adiabatic fluctuations, which means that we will be ignoring entropy perturbations throughout the analysis.

3.2.2 Cosmological Perturbations

Now, we perturb the background introduced above with linear scalar perturbations. The new metric in the longitudinal gauge is [12]:

$$ds^2 = a(\eta)^2\{[1+2\Phi(\eta,\mathbf{x})]d\eta^2 - [1-2\Phi(\eta,\mathbf{x})]\delta_{ij}dx^i dx^j\} \tag{3.2.8}$$

where we have no anisotropic stress, that is, ($\delta T_{ij} = 0$ for $i \neq j$). The metric fluctuation variable, Φ, is the Newtonian gravitational potential. Then, using the perturbed Einstein equations, $\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu}$, we can get the EOM [28]:

$$\Phi'' + 3\mathscr{H}(1+c_s^2)\Phi' + [2\mathscr{H}' + (1+3c_s^2)\mathscr{H}^2]\Phi - c_s^2\nabla^2\Phi = 0 \tag{3.2.9}$$

If we use the Friedmann equations and transform to Fourier space, we can rewrite the EOM as:

$$\Phi_k'' + 3\mathscr{H}(1+c_s^2)\Phi_k' + 3(c_s^2 - w)\mathscr{H}^2\Phi_k + c_s^2 k^2 \Phi_k = 0 \tag{3.2.10}$$

where k is the magnitude of the comoving wavenumber associated with the perturbations [35].

We will consider a universe whose evolution can be separated into different periods of constant EoS parameter and constant speed of sound. Hence, for $w =$ constant, we can find the following solution to the background EOMs [35]:

$$a \propto (-\eta)^{\frac{2}{1+3w}} \tag{3.2.11}$$

which yields:

$$\mathscr{H} = -\frac{2}{1+3w}(-\eta)^{-1} \tag{3.2.12}$$

and

$$\mathscr{H}' = -\frac{2}{1+3w}(-\eta)^{-2} \tag{3.2.13}$$

We can use the two equations above to rewrite the EOM as:

$$\Phi_k'' - \frac{6(1+c_s^2)}{1+3w}\frac{1}{(-\eta)}\Phi_k' + \left(c_s^2 k^2 + \frac{12(c_s^2-w)}{(1+3w)^2}\frac{1}{(-\eta)^2}\right)\Phi_k = 0 \tag{3.2.14}$$

The most general solution to this ordinary differential equation is a combination of Bessel functions of the first and second kind, $J_\nu(x)$ and $Y_\nu(x)$:

$$\Phi_k(\eta) = [2(1+3w)(-\eta)]^{\nu_1}[C_{1,k}J_{\nu_2}(-c_s k\eta) + C_{2,k}Y_{\nu_2}(-c_s k\eta)] \tag{3.2.15}$$

where $C_{1,k}$ and $C_{2,k}$ are the constants of integration, and are set by the initial conditions [35].

$$\nu_1 \equiv -\frac{5+6c_s^2-3w}{2(1+3w)} \quad \text{and} \quad \nu_2 \equiv \frac{\sqrt{25+12c_s^2+36c_s^4+18w-36c_s^2w+9w^2}}{2(1+3w)} \tag{3.2.16}$$

Furthermore, [35] shows that having a constant equation of state parameter tells us that $w = c_s^2$. Therefore, the indices can be simplified to:

$$\nu_1 = -\frac{5+3w}{2(1+3w)} \quad \textit{and} \quad \nu_2 = \frac{5+3w}{2(1+3w)} \tag{3.2.17}$$

which allows us to define $\nu \equiv \nu_2 = -\nu_1$.

We define two more useful quantities for a generic flat contracting universe using the results above. The first is the gauge-invariant density contrast (in Fourier space):

$$\delta_k \equiv \frac{\delta\rho_k^{(gi)}}{\rho^{(0)}} = -\frac{2}{3}\left(\frac{k^2}{\mathscr{H}^2}\Phi_k + \frac{3}{\mathscr{H}}\Phi_k' + 3\Phi_k\right) \tag{3.2.18}$$

where $\delta\rho(\eta,\mathbf{x})$ is the energy density fluctuation, and $\rho^{(0)}(\eta)$ is the background energy density [35]. Using equation 3.2.12 and the general solution for Φ_k, the density contrast can be written more specifically as:

$$\delta_k(\eta) = -\frac{1}{3\cdot 2^{\nu+1}(1+3w)^{\nu}(-\eta)^{\nu}}\{6(1+3w)x[C_{1,k}J_{\nu-1}(x)+C_{2,k}Y_{\nu-1}(x)] \\ + \left[12-6(5+3w)+\frac{(1+3w)^2x^2}{c_s^2}\right]\left[C_{1,k}J_{\nu}(x)+C_{2,k}Y_{\nu}(x)\right]\} \tag{3.2.19}$$

where $x \equiv c_s k(-\eta)$ [35]. The second quantity is the power spectrum of the density contrast. In dimensionless form, this is defined as [35]:

$$\mathscr{P}_{\delta}(k,\eta) = \frac{k^3}{2\pi^2}|\delta_k(\eta)|^2 \tag{3.2.20}$$

For adiabatic fluctuations, the equations of motion for the perturbations show us that there is a critical wavelength, called the Jeans length λ_J. We will look at the density contrast and power spectrum in relation to the Jeans scale and the Hubble scale. The sub-Jeans scales correspond to the limit $\lambda \ll \lambda_J$, and the super-Jeans scales corresponds to the limit $\lambda \gg \lambda_J$, where

$$k_J = \sqrt{\frac{3}{2}}\frac{|\mathscr{H}|}{c_s} = \frac{\sqrt{6}}{1+3w}\frac{1}{c_s|\eta|} \tag{3.2.21}$$

and $\lambda_J \equiv 2\pi/k_J$ [35]. The Hubble scale is given by

$$k_H \equiv |\mathscr{H}| = \frac{2}{(1+3w)|\eta|} \tag{3.2.22}$$

so $\lambda_J < \lambda_H$ [35].

Chapter 4

Black Hole Formation in a Contracting Universe

In [35], the adiabatic perturbations of a hydrodynamical fluid with constant EoS parameter and sound speed were analyzed in a flat contracting universe. The evolution of the density contrast also determined on sub-Jeans, super-Jeans/sub-Hubble, and super-Hubble scales. It was found that the amplitude of the density contrast on sub-Jeans scales does not change over time during the radiation-dominated regime. However, during the matter-dominated phase, it was found that the amplitude of the density contrast grows with time on super-Jeans/sub-Hubble scales.

The generalized density contrast power spectrum was also determined in [35] for two different initial conditions. For quantum vacuum initial conditions, the power spectrum for a matter-dominated contracting universe was seen to be scale invariant on super-Jeans/sub-Hubble scales, and grows as $H^2(t)/M_{Pl}^2$. In contrast, for thermal initial conditions, was not scale invariant because $P_\delta \sim k^3$ for the radiation-dominated phase (sub-Jeans) scales, and $P_\delta \sim k^{-1}$ for the matter dominated phase (super-Jeans/sub-Hubble scales).

Next, the analysis in [35] discussed the requirements for the overdensities from the growth of perturbations to collapse into black holes. Assuming that the hoop conjecture holds, a black hole will form at the point $\mathbf{q}_\star$ and time $t_\star$ if

$$\int_{\mathscr{B}(R_S,\mathbf{q}_\star)} \mathrm{d}^3\mathbf{q}\,\delta\rho(t_\star,\mathbf{q}) \geq \frac{R_S}{2G_N} \tag{4.0.1}$$

where $\mathscr{B}(R_S,\mathbf{q}_\star) \equiv \{\mathbf{q}\in\mathbb{R}^3 \big| |\mathbf{q}-\mathbf{q}_\star| \leq R_S\}$ [35]. After smoothing out the perturbations and utilizing the Friedmann equations, equation 4.0.1 could be reduced to

$$\delta(\eta,\mathbf{x};\mathscr{R}_S) \geq \delta_c(\mathscr{R}_S,\eta) \tag{4.0.2}$$

at conformal time η and comoving position x. $\mathscr{R}_S$ is the comoving Schwarzschild radius, and δ_c is the critical density contrast where

$$\delta_c(\mathscr{R}_S,\eta) = \left(\frac{\mathscr{H}^{-1}(\eta)}{\mathscr{R}_S}\right)^2 \tag{4.0.3}$$

The Press-Schechter formalism gives the Gaussian probability for the fraction of mass in spheres of radius $\mathscr{R}$ with overdensity $\delta > \delta_c$, as

$$\mathscr{F}(\mathscr{R},\eta) = \frac{2}{\sqrt{2\pi}\sigma(\mathscr{R},\eta)}\int_{\delta_c(\mathscr{R},\eta)}^{\infty} \mathrm{d}\delta\exp\left[-\frac{\delta^2}{2\sigma^2(\mathscr{R},\eta)}\right] = \mathrm{erfc}\left[\frac{\delta_c(\mathscr{R},\eta)}{\sqrt{2}\sigma(\mathscr{R},\eta)}\right] \tag{4.0.4}$$

where σ is the variance [35]. Using the Press-Schechter formalism and the requirement in 4.0.2, it was shown that Hubble-sized black holes form first during the matter dominated phase when the Hubble parameter reaches $|H| \sim c_s^{5/2} M_{Pl}$ for quantum vacuum initial conditions. The results were similar for thermal initial conditions, except $|H| \sim c_s^{18/5}(M_{Pl}/H_{ini})^{1/5} M_{Pl}$ was the energy scale for black hole formation. For the radiation-dominated period, black holes could only form when $|H| \sim M_{Pl}$. It was concluded in [35] that a nonsingular bounce is robust against the black hole formation when the sound speed is sufficiently large. Once the Hubble-sized black holes form, the perturbation analysis can no longer be used, so further evolution of the universe was not provided.

Possible outcomes, such as holographic cosmology, and the evolution of black holes into “string holes" were briefly discussed in [35].

Chapter 5

VECROs in a Contracting Universe

5.1 Obtaining a Nonsingular Bounce

In section 2.5, we saw that VECROs can remove the singularity present in black holes due to their negative energy density. We will now argue that their negative energy density and stiff equation of state will also result in a nonsingular bouncing cosmology.

We start with a flat, homogeneous, isotropic, contracting universe that contains cold matter, radiation, and VECROs. Their equation of state parameters are: $w_m = 0$, $w_r = 1/3$, and $w_v = 1$, respectively. The continuity equation

$$\rho \propto a^{-3(1+w)} \tag{5.1.1}$$

tells us that the energy density scales as $\rho_m \sim a^{-3}$ in cold matter, as $\rho_r \sim a^{-4}$ in radiation, and as $|\rho_v| \sim a^{-6}$ in VECROs [28]. In the following discussion, we will disregard cold matter because it is dominated by radiation and VECROs as we approach the bounce point at $a(t) = 0$.

Early in the contracting phase, we assume that the energy density in VECROs is

$$\rho_v(t) = -A_v \left(\frac{a(t)}{a(t_i)}\right)^{-6} \tag{5.1.2}$$

where t_i is the initial time, and A_ν is a positive amplitude representing the initial VECRO density. Similarly, the energy density in radiation is

$$\rho_r(t) = A_r \left(\frac{a(t)}{a(t_i)} \right)^{-4} \tag{5.1.3}$$

where A_r is the initial radiation density amplitude. We can now express the total energy density as

$$\rho(t) = \rho_m \left[\left(\frac{a(t)}{a(t_i)} \right)^{-4} - \left(\frac{a(t)}{a(t_i)} \right)^{-6} \right] \tag{5.1.4}$$

where ρ_m is a constant with units of energy density that is close to the maximum density during contraction. This is the energy density when VECROs start to prevent further contraction of space, and we expect this scale to be given by the string scale.

We recall the Friedmann equations

$$H^2 = \frac{8\pi G}{3}(\rho) \tag{5.1.5}$$

and

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \tag{5.1.6}$$

The magnitude of the VECRO energy density starts out smaller than the radiation energy density. However, as the universe contracts and the scale factor decreases, the magnitude of the VECRO density will catch up to the radiation energy density, becoming equal at some time $t_\star$. Therefore, from equation 5.1.5, we will have

$$H(t_\star) = 0 \tag{5.1.7}$$

Furthermore, because VECROs have a stiffer equation of state, that is $w_\nu > w_r$, at time $t_\star$ the magnitude of the VECRO pressure will be larger than the radiation pressure. Thus, by equation 5.1.6, we see that

$$\dot{H}(t_\star) > 0 \tag{5.1.8}$$

Equations 5.1.7 and 5.1.8 tell us that the addition of the VECRO component yields a smooth cosmological bounce at $t = t_{\star}$.

5.2 Evolution of the Scale Factor

Previously, we established that the bounce facilitated by the VECROs is nonsingular. Now we will determine the evolution of the scale factor around the bounce point. We can switch to the conformal time, η, since $t = \eta$ at the bounce point, and $\rho(t_i) = 0$. We linearize the scale factor so that $\tilde{a}$ is a small perturbation from the initial scale factor, and we choose $\eta = 0$ to be the bounce time.

$$a(\eta) = a_0 + \tilde{a}(\eta) \qquad \tilde{a}(0) = 0 \tag{5.2.1}$$

Using equation 5.2.1, we can approximate the total energy density as:

$$\rho(\eta) \simeq 2\rho_m \frac{\tilde{a}}{a_0} \tag{5.2.2}$$

The equation of state implies that the pressure can be written as:

$$p(t) = \rho_m \left[\frac{1}{3} \left(\frac{a}{a_0} \right)^{-4} - \left(\frac{a}{a_0} \right)^{-6} \right] \tag{5.2.3}$$

Near the the initial time, this can be approximated as $p(t) \simeq -\frac{2}{3}\rho_m$. Now, using the linearization of the scale factor, we can rewrite the first Friedmann equation, i.e. equation 3.2.4, as:

$$\mathscr{H} \simeq \frac{\tilde{a}'}{a_0} \tag{5.2.4}$$

so that

$$\mathscr{H}' \simeq \frac{\tilde{a}''}{a_0} \tag{5.2.5}$$

We can use the second Friedmann equation, i.e. equation 3.2.5, and equation 5.2.3 to write:

$$\tilde{a}'' \simeq \frac{8\pi G}{3}\rho_m \tag{5.2.6}$$

Then, it follows immediately that the scale factor is:

$$a(\eta) \simeq \frac{8\pi G}{3}\rho_m \eta^2 + a_0 \tag{5.2.7}$$

5.3 Scale-Invariance of Cosmological Fluctuations

Starting from vacuum initial conditions, we will now show that a gas of VECROs produces a scale-invariant spectrum of curvature fluctuations at late times, and that the fluctuations remain in the linear region as long as the energy density at the bounce (which is given by the string scale) is lower than the Planck density. We first introduce the canonical Mukhanov-Sasaki variable [30, 37]:

$$v = a\left(\delta\varphi + \frac{z}{a}\Phi\right) \tag{5.3.1}$$

where

$$z = \frac{a\varphi_0'}{\mathscr{H}} \tag{5.3.2}$$

and Φ is the scalar gravitational potential, and φ is the scalar matter field that is linearized about its background as

$$\varphi(\mathbf{x},t) = \varphi_0(t) + \delta\varphi(\mathbf{x},t). \tag{5.3.3}$$

The Mukhanov-Sasaki variable is proportional to the curvature fluctuation, ζ, in the comoving gauge where $\delta\varphi = 0$ [29]:

$$\zeta \equiv \frac{v}{z}. \tag{5.3.4}$$

The variable ζ is what determines the curvature fluctuations at late times, therefore we need to compute its power spectrum:

$$P_\zeta(k) = k^3|\zeta(k)|^2 \tag{5.3.5}$$

Note that we have used the Fourier transform of ζ.

In Fourier space, the EOM of the Mukhanov-Sasaki variable is as follows [29]:

$$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0 \tag{5.3.6}$$

with vacuum initial conditions that impose:

$$v_k(t_i) = \frac{1}{\sqrt{2k}}. \tag{5.3.7}$$

Looking back at equation 5.2.7, we consider the time interval when a_0 is negligible, so that $a(\eta) \sim \eta^2$. This means that the VECROs result in a matter dominated phase of contraction. In this case, the solution of equation 5.3.6 on super-Hubble scales is

$$v_k(\eta) = c_1\eta^2 + c_2\eta^{-1} \tag{5.3.8}$$

where c_1 and c_2 are constants that are determined by the initial conditions [6]. In a contracting universe, the dominant mode of this solution is the second one, which means that the canonical variable scales as $v_k(\eta) \sim \eta^{-1}$. We can now rewrite the power spectrum in terms of v_k:

$$P_\zeta(k,\eta) = k^3 z^{-2}(\eta)|v_k(\eta)|^2. \tag{5.3.9}$$

Assuming that we have hydrodynamical matter where the kinetic energy dominates, we see that $z(\eta) = a(\eta)M_{Pl}$ [29]. Now, making use of this, as well as the initial conditions and the dominant scaling of v_k, we find that:

$$P_\zeta(k,\eta) = k^3 z^{-2}(\eta)\left(\frac{v_k(\eta)}{v_k(\eta_i(k))}\right)^2 |v_k(\eta_i(k))|^2 \tag{5.3.10}$$

$$\sim k^2 z^{-2}(\eta)\left(\frac{\eta_i(k)}{\eta}\right)^2 \tag{5.3.11}$$

$$\sim k^2 a^{-2}(\eta) M_{Pl}^{-2} \eta^{-2} \eta_i(k)^2 \tag{5.3.12}$$

The time of Hubble radius crossing is [6]

$$k^{-1} a(t_i(k)) = t_i(k), \tag{5.3.13}$$

and since $a(t) \sim t^{2/3}$ and $H(\eta_i(k))^{-1} = t_i(k)$, we can see that

$$\eta_i(k) \sim k^{-1}. \tag{5.3.14}$$

Using the above equation, and the fact that $a(\eta) \sim \eta^2$, we see that the power spcetrum has no k dependence, and is thus scale-invariant:

$$P_\zeta(k,\eta) = a^{-2}(\eta) M_{Pl}^{-2} \eta^{-2} \tag{5.3.15}$$

$$\sim \eta^{-4} M_{Pl}^{-2} \eta^{-2} \tag{5.3.16}$$

$$\sim H^2 M_{Pl}^{-2}. \tag{5.3.17}$$

Furthermore, the Friedmann equation states that $H^2 = \rho M_{Pl}^{-2}$, so

$$P_\zeta(k,\eta) \sim \frac{\rho}{M_{Pl}^4} \tag{5.3.18}$$

The fluctuations remain linear when $P_\zeta(k,\eta) < 1$, which means that $\rho < M_{Pl}^4$, and subsequently, $\rho_m < M_{Pl}^4$. This linear regime is important because when fluctuations become non-linear, our perturbative analysis breaks down.

Chapter 6

Discussion

The VECRO hypothesis [26] was first applied to black holes as a way to solve the information paradox. VECROs are virtual fluctuations of extended compression resistant objects that are intrinsically stringy and have no horizon. They, along with fuzzballs [2], make up the horizon sized microstates of a black hole. When a shell of matter is collapsing and approaches its Schwarzchild radius, it will tunnel into a linear combination of VECROs and fuzzballs. The absence of horizon formation in black holes is what allows us to avoid the information loss problem. It was also argued that VECROs would eliminate the metric singularity at the centre of classical black holes. We proposed that in order to do so, they must have a negative effective energy density. Furthermore, motivated by previous work done with string holes [36], we argued that the equation of state should be that of a stiff fluid. That is, the equation of state parameter should be $w = 1$, such that $p = \rho$.

The inflationary scenario [17] is one of the most widely regarded theories of early universe cosmology because it made a number of correct predictions about the CMB, and provided a causal mechanism for structure formation. However, we stressed the importance of considering alternatives to inflation, especially those that belong to the class of bouncing cosmologies, as they aim to resolve the initial singularity. For example, the PBB scenario [13] is motivated by string theory and thus has both the graviton and dilaton as massless modes. The Pre-Big-Bang phase is contracting, and its solution has a growing string coupling, whereas the Post-Big-Bang phase is the typical radiation dominated expanding one. New physics is required for the Pre-Big-Bang and Post-Big-Bang phases to be connected smoothly. We also discussed the original Ekpyrotic scenario [21], where a negative exponential potential causes a “bulk brane" and a “visible brane" to approach each other in what corresponds to a contracting phase. When the branes collide, there is a singular bounce into the usual expanding phase. Another alternative is string gas cosmology [4], which makes use of the new degrees of freedom and symmetries that come from superstring theory. This model compactifies all spatial dimensions, and is based on coupling a classical background to a gas of closed strings [4]. One of the major benefits of string gas cosmology is that there is no temperature singularity because a gas of strings cannot exceed the Hagedorn temperature [18].

The theory of cosmological perturbations [29] is one of the most important parts of modern cosmology because it allows us to make a connection between early universe theories and observational data. Nonlinear structures, such as the galaxies and clusters we see today, initially started out as small perturbations at early times. Describing the growth of these primordial perturbations mathematically requires solving the linearized Einstein equations in an expanding background. However, because we are interested in a bouncing cosmology, we referred to the analysis in [35], which determines the evolution of perturbations in a contracting universe. The goal of [35] was to determine when a contracting universe is robust against the formation of black holes. The density contrast and power spectra were computed for both thermal and quantum vacuum initial conditions, and the Press-Schechter formalism [34] was used to derive the general requirement for black hole formation. It was found that black holes will not form in a nonsingular bounce when the sound speed is sufficiently large, and that bouncing cosmologies drievn by matter satisfy the conditions for black hole formation. However, we did not consider black hole formation in our study of VECROs in cosmology, and instead focused on the perturbative analysis in [35].

Just as fuzzballs and VECROs emerge to prevent Schwarzchild singularities from forming in black holes, VECROs also have a similar effect in the case of the Big Bang metric singularity. Hence, we can view VECROs as a component of the quantum vacuum that works to counteract the formation of singularities. Using this logic, we have given a proposal for a new nonsingular bouncing cosmology using VECROs. We were able to realize a nonsingular bounce by using the two key arguments we developed. Namely, that the equation of state of VECROs is $p = \rho$, and that they have a negative energy density. We then determined the behaviour of the scale factor near the bounce, which allowed us to do an analysis of the cosmological perturbations under vacuum initial conditions. We found that a gas of VECROs produces a scale-invariant spectrum of curvature fluctuations at late times. Furthermore, we also showed that the fluctuations remain linear if the energy density at the bounce is less than the Planck density.

As we mentioned previously, bouncing cosmologies can be obtained by modifying matter, or modifying the effective gravitational action. However, it would be ideal if a bounce were obtained via an ultraviolet complete theory. Superstring theory [16] is is a theory that unifies all four forces of nature, and is the best candidate. There are many constraints imposed by fundamental physics that are in tension with inflation, which is why alternatives must be considered despite its many successes. We have discussed the Ekpyrotic [21] and PBB [13] scenarios, which both use stringy effects to obtain a nonsingular bounce, but neither come from a complete quantum gravity theory. We also outlined string gas cosmology, which is based on the principles of superstring theory, but has no rigorous non-perturbative formalism [6]. Our proposal based on VECROs in cosmology faces a similar problem in the sense that we cannot yet describe it in terms of an effective action. In fact, the specific structure of the VECROs themselves are not well understood. In our arguments, we have only made use of their basic qualitative properties to propose a mechanism to yield a bounce. Our hypothesis is still in the very early stages, but is still relevant because it addresses one of the main challenges in bouncing cosmologies.

The swampland criteria [11] are composed of a list of constraints that tell us if an effective field theory can be embedded in superstring theory. One such constraint is the distance conjecture [32], which says that the canonically normalized scalar field of an effective field theory must have a field range that is smaller than an order one constant multiple of the Planck mass. Another one of the swampland criteria is the de Sitter conjecture [31], which imposes a constraint on the steepness of the field's potential. In string theory, the scalar fields in the low energy effective action are called moduli fields, and they represent the radii and shapes of the extra compactified dimensions [10]. Showing that the radion and shape moduli fields are stabilized is one of the major difficulties in field theory models that are motivated by string theory. Our starting point in the work done in Appendix A was string gas cosmology, where matter is a gas of strings with momentum and winding modes coupled to the background. It is well known that the radion, R, is stabilized by the winding and momentum modes [33], therefore, we only studied the shape moduli, θ. We derived the effective potential for the shape modulus field on an internal two dimensional torus, and showed that it is consistent with the distance and de Sitter conjectures. The shape modulus was found to be stabilized at the value $\theta = 0$, and the radion was stabilized at the self dual radius $R = 1$. Our work provided further support for the validity of the swampland conjectures.

Chapter 7

Conclusions

We proposed a new nonsingular bouncing cosmology based on VECROs. We took inspiration from the VECRO hypothesis for black holes, which resolved the information paradox and eliminated the metric singularity at the centre of classical black holes. An original contribution was our argument that VECROs must have a negative energy density and a stiff equation of state in order to remove the black hole singularity. By combining these two new VECRO properties, we were able to show that a gas of VECROs in a contracting universe naturally results in a nonsingular cosmological bounce. We then analyzed the curvature fluctuations and showed that they produce a scale-invariant spectrum. What we have done thus far is just a first step in developing the full theory of a VECRO bounce. New physics is required to realize the bounce, and we expect that it will be string or Planck scale physics. Furthermore, since VECROs have negative energy densities, they violate the null energy condition. However, because VECROs are quantum objects, it is possible that we have a quantum violation of the null energy condition that is consistent with the swampland criteria. This is an open issue, and more development is required to be able to embed this model into superstring theory.

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Appendix A

Note on shape moduli stabilization, string gas cosmology and the swampland criteria

Eur. Phys. J. C (2021) 81:39
https://doi.org/10.1140/epjc/s10052-021-08843-0

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Note on shape moduli stabilization, string gas cosmology and the swampland criteria

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Received: 3 November 2020 / Accepted: 6 January 2021 / Published online: 18 January 2021

Abstract In String Gas Cosmology, the simplest shape modulus fields are naturally stabilized by taking into account the presence of string winding and momentum modes. We determine the resulting effective potential for these fields and show that it obeys the *de Sitter* conjecture, one of the *swampland criteria* for effective field theories to be consistent with superstring theory.

1 Introduction

In recent years there has been a lot of interest in constraints on low energy effective field theories which can emerge from superstring theory. These constraints pick out a small subspace (the *string landscape*) of the huge space of possible effective field theories. Effective field theories which are not embeddable in string theory are said to lie in the *swampland* (see e.g. [1,2] for reviews). Two key criteria are the *distance conjecture* and the *de Sitter* constraint. The *distance conjecture* [3] states that an effective field theory of a canonically normalized scalar field ϕ is only consistent with string theory if the field range $\Delta\phi$ is smaller than $c_1 m_{pl}$, where m_{pl} is the four space-time dimensional Planck mass, and c_1 is a constant of order one. The *de Sitter condition* [4] states that the potential $V(\phi)$ of such a field has to be sufficiently steep, i.e.

$$|\frac{V'}{V}| > \frac{c_2}{m_{pl}}, \tag{1}$$

where c_2 is another constant of the order one, and a prime denotes the derivative with respect to the field. In the case that the potential has a local extremum, then the condition (1) is may not be met, but in that case an extended version of the criterion applies which states that [5,6]

$$\frac{V''}{V} < -\frac{c_3}{m_{pl}^2}, \tag{2}$$

where c_3 is a positive constant of the order one.

The swampland conjectures have been formulated based on well-motivated ideas from string theory, but they have not yet been rigorously established. Assuming string theory, all scalar fields appearing in a low energy effective action are *moduli fields* of the string compactification, e.g. the radii and shape parameters of extra dimensions. Hence, their potentials are determined by string theory. In a recent paper [7] the effective potential of the scalar field corresponding to the radius of an extra dimension was studied. The starting point was the *String Gas Cosmology* model [8] (see also [9] for earlier work, and [10] for reviews), in which matter is described by a gas of strings including both momentum and winding modes, and is coupled to a background space-time. It is known [11–14] that the radion modulus is stabilized by the presence of both winding and momentum modes. The momentum modes prevent the radion from decreasing to zero while the winding modes prevent it from expanding without limits. The resulting effective potential for the radion has vanishing potential energy at its minimum. We found that the effective potential is quadratic about the minimum and hence satisfies the de Sitter criteria (1, 2).

In this paper, we will focus on the shape moduli. We follow [15] and consider two internal toroidal dimensions with radius R and angle θ. From the work of [15] it is known that string effects stabilize the shape modulus field. Here we will consider the effective potential for θ and show that it is also consistent with the conditions (1, 2).

In the following section we give a brief review of SGC. In Sect. 3 we derive the effective potential for our shape modulus field θ and show that it obeys the swampland criteria. We conclude with a discussion of our results. We work in natural

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units in which the speed of light and Planck's constant are set to 1. We also set the string length equal to 1 in our units.

2 Brief review of string gas cosmology

String gas cosmology is based upon coupling a classical background (including the graviton and dilaton fields) to a gas of strings. Strings have three types of states: momentum modes, oscillatory modes, and winding modes. These string states, as well as the T-duality symmetry, are the key features of string theory that are used to develop string gas cosmology. String theory requires six extra spatial dimensions, which we take to be compactified and toroidal. Namely, the space-time metric is:

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu + \gamma_{ab}dx^a dx^b \tag{3}$$

where the Latin indices label the compactified dimensions, and the Greek indices label the four dimensional FRW metric. The torus is parametrized by shape and size moduli.

A major challenge in string cosmology is stabilizing the string moduli, that is, stabilizing the sizes and shapes of the extra dimensions, as well as the dilaton. Note that we are here considering moduli stabilization at late times when the background can be described by dilaton gravity.[1] The basic principle of size modulus stabilization is that the winding modes prevent expansion because their energies increase with R, whereas the momentum modes prevent contraction because their energies increase with $\frac{1}{R}$ [11–14]. As we will review here, the coupling of the string gas to the background also provides a stabilization mechanism for a simple shape modulus field [15]. It can be shown [18] that nonperturbative effects due to gaugino condensation can stabilize the dilaton without interfering with size and shape modulus stabilization, and the same mechanism leads to high scale supersymmetry breaking [19].

3 Shape modulus potential

In the following we consider our space-time dimensions to be non-compact, and add a number of compact dimensions which we take to be toroidal. Strings have momentum and winding numbers about the extra dimensions.

The matter action for a mas of strings at temperature β^{-1} in D space-time dimensions is given by

[1] In the initial high temperature Hagedorn phase of strings, dilaton gravity is inapplicable since it is not consistent with the basic symmetries of string theory. For a recent attempt to construct a background which is consistent with the T-duality symmetry of string theory see e.g. [16,17].

$$S = \int d^D x\sqrt{-G} \times \sum_{n^a, w_a, N, \tilde{N}, \mathbf{p}_{nc}} \mu_{n^a,w_a,N,\tilde{N},\mathbf{p}_{nc}} \epsilon_{n^a,w_a,N,\tilde{N},\mathbf{p}_{nc}}, \tag{4}$$

where G is the determinant of the full metric, and the sum runs over all free string states. These states are labelled by their momentum numbers n^a, winding numbers w_a, numbers N and $\tilde{N}$ of right- and left-moving oscillatory modes, and momentum $\mathbf{p}_{nc}$ in the non-compact directions. The quantity $\mu_{n^a,w_a,N,\tilde{N},\mathbf{p}_{nc}}$ is the number density of a state with the specified quantum numbers, and $\epsilon_{n^a,w_a,N,\tilde{N},\mathbf{p}_{nc}}$ is its energy, given by

$$\epsilon_{n^a,w_a,N,\tilde{N},\mathbf{p}_{nc}} = \sqrt{\mathbf{p}_{nc}^2 + M^2_{n^a,w_a,N,\tilde{N}}}, \tag{5}$$

where $M_{n^a,w_a,N,\tilde{N}}$ is the mass of such a state. In the absence of string interactions, the number density of states is constant in comoving coordinates. We denote the comoving number density by $n_{n^a,w_a,N,\tilde{N}}$. In thermal equilibrium, the number density of excited states is suppressed by the thermal Boltzmann factor. Hence, we can restrict the summation to run over the lowest mass states which are the massless states (the tachyon which appears in bosonic string theory does not appear in the spectrum of superstring theory). Since the determinant of the metric reduced to our expanding spatial dimensions cancels out between $\sqrt{G}$ and μ, the matter action becomes

$$S = \int d^D x\sqrt{-G_{00}} \sum_{\text{restricted}} n_{n^a,w_a,N,\tilde{N},\mathbf{p}_{nc}} \epsilon_{n^a,w_a,N,\tilde{N},\mathbf{p}_{nc}} \tag{6}$$

where the sum is restricted to the lowest mass states, and G_{00} is the time component of the metric (the scale factor dependence in the spatial dimensions has been cancelled against the corresponding factor appearing in the number densities μ).

In [11–15], the equations of motion for the size and shape moduli were derived by deriving the energy-momentum tensor of the higher dimensional theory with matter action (6), and inserting the resulting expressions into the higher dimensional Einstein action. Here, following [7], we obtain the equations of motion for the moduli fields by inserting the ansatz for the metric into the matter action (6) (more specifically, by inserting the ansatz into the mass formula), viewing the resulting action as an action for the moduli fields, and taking the resulting variational equations.

The string mass for a toroidal compactification depends on the three sets of string quantum numbers, and is given by:

$$M^2_{\mathbf{n},\mathbf{n},\mathbf{N}} = \frac{1}{R^2}\gamma^{ab}n_a n_b + \frac{R^2}{\alpha'^2}\gamma_{ab}w^a w^b + \frac{2}{\alpha'}\left(2N + n^a w_a - 2\right), \tag{7}$$

where n^a, w^a, and N are the momentum, winding, and oscillatory quantum numbers, respectively,[2] and γ_{ab} is the space-time metric. In the string units which we are using, $\alpha' = 1$. The indices a and b run over all compact dimensions. Later in this note we will be considering a two dimensional torus representing two of the internal dimensions.

In SGC, the universe begins in a thermal state of strings[3] The string partition function is then dominated by the string states with lowest mass. These states satisfy $N = 1$ and $n^a = w_a = \pm 1$. Note that for each such state, there are momenta and windings in only one of the directions.

If matter is treated as a gas of strings, then the dynamics is governed by the following d space-time dimensional full low energy effective action (the action for matter and geometry):

$$S = \frac{1}{2\kappa_0^2}\int d^d X\sqrt{-G}e^{-2\Phi_d} \times \left[\hat{R}^d + 4\partial_\mu\Phi_d\partial^\mu\Phi_d - \frac{1}{4}\partial_\mu\gamma_{ac}\partial^\mu\gamma^{ab} - 2\kappa_0^2 e^{-2\Phi_d} n\langle E_1\rangle\right], \tag{8}$$

where κ_0^{-1} is the reduced gravitational constant, n is the number density of the strings, G is the determinant of the metric, Φ is the dimensionless dilaton field, $\hat{R}$ is the Ricci scalar, and $\langle E_1\rangle$ is the thermal average of the energy of a single string. We consider this as the action for the moduli fields.

The final term in (8) contains no derivatives of the moduli fields and hence acts as a potential for these fields, i.e.

$$V(\phi) = e^{2\Phi_d} n\langle E(\phi)\rangle. \tag{9}$$

Since the partition function is dominated by the lowest mass string states, the potential energy can be expressed in terms of the mass of these states, i.e.

$$V(\phi) \sim e^{2\Phi_d} n\sqrt{\mathbf{p}_{nc}^2 + M_{1,-1,1}^2}. \tag{10}$$

We will now review how this potential stabilizes the moduli fields, and extract the shape of the potential for a canonical shape modulus field.

4 Moduli stabilization on a 2-dimensional torus

To be specific, we shall here consider the case of an internal two dimensional torus. Thus, the metric we will be using is that of Eq. (3) where γ_{ab} is the metric for the torus:

$$\gamma_{ab} = \begin{bmatrix} R^2 & R^2 sin\theta \\ R^2 sin\theta & R^2, \end{bmatrix} \tag{11}$$

where R gives the radius of the torus, and θ is the shape parameter. For $\theta = 0$, it is easy to see that the square mass function (7) has a minimum at the self-dual radius $R = 1$, and hence leads to stablization of the radial modulus field R, as has been considered in [11–14]. The resulting modulus potential was shown [7] to be consistent with the de Sitter conjecture. Here, we focus on the shape modulus and its potential.

The metric γ_{ab} generally involves scalar fields ϕ^I called moduli fields. The kinetic terms of the moduli fields are:

$$-\frac{1}{4}\partial_\mu\gamma_{ac}\partial^\mu\gamma^{ab} = -g_{IJ}\partial_\mu\phi^I\partial_\mu\phi^J \tag{12}$$

In our case, we fix R and hence consider only one moduli field. It corresponds to the shape modulus θ. The corresponding canonically normalized field is

$$\phi \equiv \frac{M_{pl}}{R}\theta, \tag{13}$$

which leads to the internal space metric

$$g_{IJ} = \begin{bmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{4} \end{bmatrix} \tag{14}$$

in the limit of small ϕ. Note that $\phi = 0$ corresponds to a rectangular torus, the enhanced symmetry point.

The stable fixed point is the rectangular torus where the complex structure modulus θ is zero, and R is set to unity. We can explicitly find the string mass for the metric γ_{ab} defined in (11). Expanding equation (7) about small field values gives:

$$M_{1,-1,1}^2 \sim \left(\phi^2 + \mathcal{O}(1)\frac{\phi^4}{M_{pl}^2}\right). \tag{15}$$

Using (10), we see that the resulting potential for the modulus field ϕ is (dropping the quartic term in ϕ)

$$V(\phi) = e^{-2\Phi_d} n\sqrt{\mathbf{p}_{nc}^2 + \phi^2} \tag{16}$$

and that hence

$$\frac{V'}{V} = \frac{1}{\sqrt{2}}\frac{\phi}{\mathbf{p}_{nc}^2 + \phi^2} \sim \frac{1}{\sqrt{2}\phi}, \tag{17}$$

where in the last step we have evaluated the result at late times when the momentum $\mathbf{p}_{nc}$ in the non-compact directions is negligible (since these dimensions are expanding and the momentum hence redshifts).

[2] More precisely, N is the number of right-moving modes, and we have made use of the level matching condition to take into account the left-moving modes.

[3] Thermal fluctuations of strings in the early phase provide the origin for the cosmological fluctuations [20] and gravitational waves [21] observed today.

Let us now make contact with the swampland criteria. First, we note from (13) that, since $|\theta| < \pi/2$, the range of the modulus field does not exceed the Planck scale, in agreement with the distance conjecture [3]. Next, we see from (17) that, since the field is confined to values $|\phi| < (\pi/2)M_{pl}$, the de Sitter conjecture (1) is automatically satisfied, with a constant c_2 which (making use of the maximal value of $|\phi|$), is given by $c_2 = \sqrt{2}/\pi$.

There is a caveat to our conclusions regarding the de Sitter conjecture. As originally formulated [4], the de Sitter conjecture applies to a bare potential in the four-dimensional effective field theory. Our potential, however, is an effective potential which in fact changes its amplitude as our three dimensional spatial sections expand.[4] On the other hand, the derivation of the de Sitter conjecture in [6] based on the covariant entropy bound of Bousso [22] concerns the potential function which enters the Friedmann equations, which is the effective potential. Hence, when considering applications of the de Sitter conjecture to cosmology it is reasonable to consider the effective potential, as we are doing.

5 Conclusions

We have studied shape modulus stabilization using in the context of string gas cosmology, and have seen that the effective potential for this modulus field is consistent with the swampland constraints (specifically, the distance and the de Sitter constraint). Note that the string gas yields a potential that stabilizes both the radial and the shape modulus fields, the radial field being stablized at the self dual radius $R = 1$ of the extra dimensions, and the shape modulus θ at the value $\theta = 0$ which corresponds to a square torus. Our analysis showed that the de Sitter conjecture is satisfied, and the value of the constant c was found to be $\frac{\pi}{4}$.

Our analysis was done in the context of the simplest model for the extra dimensions, but the physics which yields the effective potentials which are consistent with the swampland constraints should be generalizable to more complicated compactifications. It is also important to point out the the origin of the potentials which we are considering are stringy. In a pure effective point particle field theory approach the winding modes which are responsible for modulus stabilization and the shape of the potentials are not present, and the effects we have discussed could not be seen.

In summary, our study provides further support for the swampland conjectures.

[4] As shown in [12], such a time-dependent effective potential is consistent with constraints from late time cosmology. The analysis in that paper was performed for the volume modulus field, but the conclusions will carry over to the shape modulus field which we are considering in the present paper.

Acknowledgements GM acknowledges support from a NSERC CGSM scholarship. RB thanks the Pauli Center and the Institutes of Theoretical Physics and of Particle- and Astrophysics of the ETH for hospitality. The research at McGill is supported, in part, by funds from NSERC and from the Canada Research Chair program.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: Data sharing not applicable to this article as no datasets were generated or analysed during the current study.].

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