Aspects of Cosmology from Particle Physics Beyond the Standard Model

Natalia Shuhmaher

Doctor of Philosophy

Department of Physics

McGill University
Montréal, Québec, Canada
July, 2007

A thesis submitted to McGill University
in partial fulfillment of the requirements of the degree of Doctor of Science

Abstract

The interface of Cosmology and High Energy physics is a forefront area of research which is constantly undergoing development. This thesis makes various contributions to this endeavor. String-inspired cosmology is the subject of the first part of the thesis, where we propose both a new inflationary and a new alternative cosmological model. The second part of the thesis concentrates on the problems of integrating cosmology with particle physics beyond the Standard Model.

Inspired by new opportunities due to stringy degrees of freedom, we propose a non-inflationary resolution of the entropy and horizon problems. In this string-inspired scenario, 'our' dimensions expand while the extra dimensions first expand and then contract, before eventually stabilizing. The equation of state of the bulk matter (which consists of branes) is negative. Hence, there is a net gain in the total energy of the universe during the pre-stabilization phase. At the end of this phase, the energy stored in the branes is converted into radiation. The result is a large and dense 3-dimensional universe.

Making use of similar ideas, we propose a not-fine-tuned model of brane inflation. In this scenario the brane separation, playing the role of the inflaton, is the same as the overall volume modulus. The bulk matter provides an initial expansion phase which drives the inflaton up its potential, so that the conditions for inflation are realized. The specific choice of the inflationary potential nicely fits the cosmological observations.

Another aspect of this research concentrates on the cosmological moduli problem: namely, the existence of weakly coupled particles those decay is late enough to interfere with Big Bang Nucleosynthesis. As a solution, we suggest parametric and tachyonic resonances to shorten the decay time. Even heavy moduli are dangerous for cosmology if they cause the overproduction of gravitinos. We find that tachyonic decay channels help to transfer most of the energy of these dangerous moduli into a scalar sector, preventing the excess gravitino abundance.

Résumé

L'interface entre la Cosmologie et la Physique des hautes énergies est un sujet de recherche d'avant-plan en constant développement.

La cosmologie inspirée par la théorie des cordes est le sujet de la première partie de cette thèse, dans laquelle nous proposons d'une part un nouveau mécanisme pour l'inflation et d'autre part une nouvelle alternative de modèle cosmologique.

Dans la seconde partie nous nous concentrons sur les problèmes reliés à l'intégration de la cosmologie dans un modèle de physique des particules au-delà du Modèle Standard.

Motivés par les nouvelles possibilits venant des degrés de liberté de la théorie des cordes, nous proposons une résolution non-inflationiste aux problèmes d'entropie et d'horizon. Selon notre scenario fondé sur la théorie des cordes, les trois dimensions spatiales habituelles ainsi que les dimensions supplémentaires s'étendent, mais ces dernières se contractent eventuellement avant de se stabiliser. L'équation d'état de la matière du bulk, qui consiste de branes, est négative. Il y a donc un net gain dans l'énégie totale de l'univers durant la phase de pré-stabilisation. A la fin de cette phase, l'énergie stockée dans les branes est convertie en radiation. Le résultat est un univers tri-dimensionel large et dense.

En utilisant des idées similaires, nous proposons un modèle d'inflation qui ne requiert pas d'ajustements fins. Dans ce scénario, la séparation entre les branes, qui joue le rôle de l'inflaton, est la même que le module du volume global. La matière du bulk fournit une phase d'expansion initiale qui pousse l'inflaton vers le haut de son p`otentiel, réalisant ainsi les conditions pour l'inflation. Le choix spécifique du potentiel de l'inflaton est en accord avec observations cosmologiques.

Un autre aspect de ma these adresse le problme cosmologique des champs de module: c'est-à-dire l'existence de particules faiblement couples dont la désintégration a lieu suffisamment tard pour interférer avec la Nucléosynthèse primordiale. Comme solution nous suggérons une résonance paramétrique et tachyonique pour réduire le temps de désintégration. Même les champs de module lourds sont dangereux pour la cosmologie s'ils causent une surproduction de gravitinos. Nous obtenons que le canal de désintégration tachyonique aide le transfert de la plus grande partie de l'énergie de ces champs de module dangereux dans un secteur scalaire, empêchant ainsi la surproduction de gravitinos.

Acknowledgements

It is a pleasure to thank Thorsten Battefeld for his collaboration. I wish also thank professors of High Energy group, Jim Cline, Keshav Dasgupta and Guy Moore, for advises and instructions. Many thanks to Diana Battefeld, Tirthabir Biswas, Dick Bond, Cliff Burgess, Sean Carroll, Jean Dufaux, Masahiro Ibe, Subodh Patil, Michael Peskin, Erich Poppitz, Andrey Katz, Justin Khoury, Ryuichiro Kitano, Lev Kofman, Andrei Linde, Jerome Martin, Anupam Mazumdar, Rabindra Mohapatra, Alessio Notari, Yael Shadmi, Tomer Volansky and Scott Watson for useful discussions, comments and suggestions. It is also a great pleasure to acknowledge the advise of Robert Brandenberger, my thesis supervisor. I wish to thank him for taking a roll in all of the above during the last three years, in addition to providing guidance, support and important feedback.

I also thank Aaron Berndsen, William Witczack-Krempa, Larissa Lorenz and Patrick Martineau for language related corrections in the introduction, and William Witczack-Krempa, Larissa Lorenz and Claudia de Rham for helping with the French translation of the abstract.

Finally, thanks for Loison Hoi for the LaTeX template.

Contributions of Authors

This is a manuscript [1, 2, 3, 4, 5] based thesis with footnotes added in response to questions received from the thesis committee. The first Chapter introduces the reader to the cosmological problems and difficulties to integrate cosmology with particle physics. In particular I discuss puzzles of Big Bang Cosmology, Inflation as a resolution of these puzzles, nonperturbative techniques to reheat the universe, and the cosmological moduli problem. Chapters 2,3 and 5 are based on the work I did with Robert Brandenberger [1, 2, 4], chapter 4 is based on the work with Thorsten Battefeld [3], and chapter 6 is based on my single author paper [5].

I initiated and contributed main ideas of [1]. In particular, I suggested the mechanism to overcome the entropy and horizon problems avoiding an inflationary period. The main idea of [2] originated in the process of discussions with Robert Brandenberger and is heavily based on the ingredients developed in [1]. In both papers I contributed to all calculations. In the followup paper [3], I wrote more than a half of the manuscript. I mainly worked on the first part, namely I calculated the cosmological parameters and made an estimate of the fundamental string length. I was also involved in detailed discussions of the second part.

In the paper [4] I suggested the specific interaction model based on the particle physics nature of the cosmological moduli problem and independently performed all the calculations. I also noticed that triliniar interactions might result in tachyonic resonance which may help to resolve the problem. This observation led to the main idea of my single author paper [5] where I performed all the calculations and wrote all the text.

Contents

List of Tables

List of Figures

Chapter 1

Introduction

Early universe cosmology is a vastly developing area that provides a natural framework to test models of High Energy Physics. The canon of the early universe cosmology is the theory of Big Bang Cosmology (BBC), which establishes the thermal history of the universe. According to BBC, the beginning of time is associated with an infinitely hot universe. As time progresses the universe adiabatically cools down and grows. At a temperature of around 1 MeV the nucleosynthesis processes form the nuclei of light elements. As the temperature reaches 1 eV (the recombination epoch) electrons no longer have enough energy to overcome the attractive force of atomic nuclei; hence, atoms form. The universe becomes transparent to photons. Later on, the tiniest fluctuations of density in the early universe cause structure formation. The universe continues to expand and cool until it reaches today's 1 meV temperature.

The evolution of the universe is governed by the Einstein equations

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{1.1}$$

where $G_{\mu\nu}$ is the Einstein tensor, μ, $\nu = 0, .., 4$, $T_{\mu\nu}$ is the energy-momentum tensor and G is the Newtonian gravitational constant. Experiments reveal that our universe is homogeneous and isotropic to a high degree of precision. In the homogeneous and isotropic limit the line element is unique and we obtain the so called Friedmann-Robertson-Walker (FRW) universe

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right] \tag{1.2}$$

where $a(t)$ is the scale factor, t, r, θ, ϕ are time and spherical spatial coordinates, and $k = 1, 0, -1$ is the curvature signature. If the matter is in the state of a perfect fluid

$$T^\mu_\nu = \text{diag}(-\rho, p, p, p)\,, \tag{1.3}$$

where ρ and p are the energy density and pressure, the Einstein equations reduce to the set of equations:

$$H^2 = \frac{1}{3m_{pl}^2}\rho - \frac{k}{a^2}\,; \tag{1.4}$$

$$\dot{H} = -\frac{1}{2m_{pl}^2}(\rho + p) - \frac{k}{a^2}\,, \tag{1.5}$$

where $m_{pl}^2 = \frac{1}{8\pi G}$ and $H = \frac{\dot{a}}{a}$. The equations (1.4,1.5) are known as the Friedmann equations.

Different aspects of BBC are tested to a high level of precision. One of them is the theory of nucleosynthesis, which correctly predicts the primordial abundance of the very light elements. Another is the presence of the cosmic microwave background (CMB) which, according to BBC, has formed once the universe became transparent to photons.

Despite tremendous success, BBC does not explain why we observe homogeneous, isotropic and spatially flat universe, nor what is the source of the fluctuations responsible for structure formation as well as other problems (see e.g. [6, 7] for a review). Inflation – a short period of accelerated expansion – manages to complete the picture. However, the Standard Model (SM) of particle physics does not incorporate the necessary degrees of freedom to describe inflation. Other cosmological observations, for example the origin of dark matter and dark energy, are unexplained within the SM as well. The success of models beyond the SM to explain the above phenomena does not ensure the absence of new obstacles in a way of successfully integrating modern cosmology and particle physics. Specifically, various extensions of the SM generically predict new forces and particles. It may happen that new degrees of freedom help to explain old puzzles in new, unconventional ways. In particular, what are the consequences of the dynamics of the extra dimensions in the phase preceding their stabilization on the evolution of the very early universe?

As a part of this thesis, a new string-inspired scenario of the evolution of the universe is presented. The new model makes use of extra dimensions, a gas of p-branes in the bulk to drive an initial isotropic but non-accelerated expansion of the universe, as well as orbifold fixed planes responsible for the eventual contraction of the extra dimensions while our three dimensions continue to expand. Depending on the details, the toy model either explains the entropy and horizon problems of BBC without invoking the paradigm of inflation or provides a not fine-tuned emerging brane inflation model.

While theories beyond the SM present unexplored opportunities, their ingredients are not always compatible with BBC (e.g. moduli and gravitinos). In the second part of the thesis, the possibility to overcome one of the cosmological problems associated with physics beyond the SM, the cosmological moduli problem, is investigated. The cosmological moduli problem is the problem of the over-abundance of weakly interacting particles present in supersymmetric models, particularly, in string-inspired models. To overcome this, nonperturbative decays of moduli into SM degrees of freedom are considered. While the final decision regarding the success of the solution depends upon the details of the model, we find ranges of parameters that avoid the cosmological moduli problem.

The following part of this chapter is dedicated to a review of the main points and basic ideas which serve as background material to the thesis.

1.1 Problems of Big Bang Cosmology

One of the cosmological puzzles is the horizon problem. It arises as a result of the inconsistency of the isotropy of the CMB and predictions of Big Bang Cosmology. The photons which are observed in the CMB [8, 9], last scattered when Universe was 1200 time smaller ($a_{CMB} = 1/1200$) at $t = t_{rec}$. The co-moving horizon size at the surface of last scattering is

$$l_p(t_{rec}) = \int_{t_i}^{t_{rec}} \frac{dt}{a} = 2H_0^{-1}\sqrt{a_{CMB}} \approx 6 \text{ x } 10^{-2} H_0^{-1} \,, \tag{1.6}$$

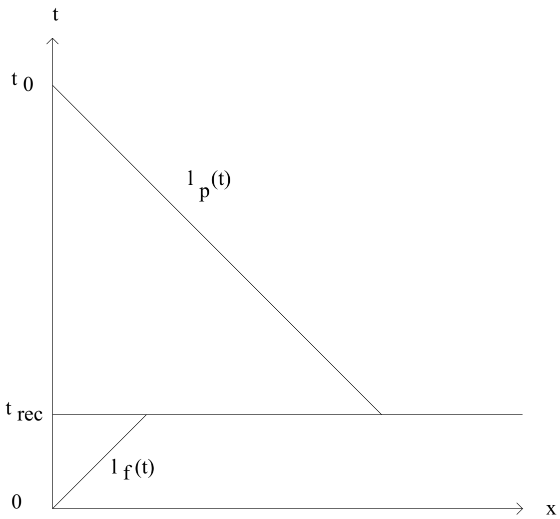

Figure 1.1: The sketch of time (t) versus co-moving distance (x). The line at t_{rec} corresponds to the surface of last scattering. The backward light cone (l_p) is substantially larger than the forward light cone (l_f). This constitutes the horizon problem. Fig from Ref [6]

where t_i stands for the beginning of expansion. The co-moving distance photons travel from a point on the CMB surface to an observer on the Earth is

$$l_f(t_{rec}) = \int_{t_{rec}}^{t_0} \frac{dt}{a} \approx 2H_0^{-1} \tag{1.7}$$

where t_0 stands for the current time. Comparing (1.6) to (1.7) (see Fig. 1.1) leads to the conclusion that the CMB map should consist of a vast number of uncorrelated regions. Instead, the observations shows that the surface of last scattering is isotropic to 1 part in 10^4 [10, 11, 12, 13]. To explain this phenomenon, one is required to modify the causal structure of Standard Big Bang Cosmology in a way such that

$$\int_{t_i}^{t_{rec}} \frac{dt}{a} \gg \int_{t_{rec}}^{t_0} \frac{dt}{a} \,. \tag{1.8}$$

Another issue is the following, anisotropies observed in the form of galaxies and clusters require a causal mechanism which generates density perturbations. In Standard Cosmology horizon grows faster than the distance between objects. Perturbations starts to grow once the matter energy density exceeds the radiation energy density, at t_{eq}. The scales above 50Mpc where not in causal contact at t_{eq}. Hence,

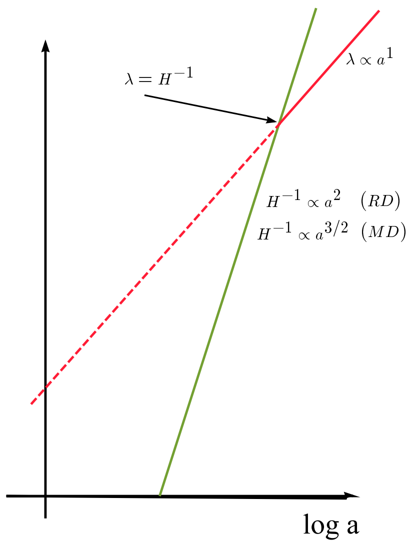

Figure 1.2: The sketch of logarithm of any physical length scale λ and the Hubble radius H^{-1} versus logarithm of the scale factor. The physical scale crosses the Hubble radius only once. In the radiation and matter dominated universes, H^{-1} coincides with horizon. Hence, correlations on physical scales which at t_{eq} are outside H^{-1} cannot have causal origin. The dotted line indicates periods when λ is larger than H^{-1}. Fig. from Ref [14]

the correlations between galaxies and clusters of galaxies observed on scales above 50Mpc [15, 16, 17] are unexplained if the perturbations are generated before t_{eq} (see Fig. 1.2). This is the structure formation problem.

Another puzzle of Big Bang Cosmology is the entropy problem. During the adiabatic expansion, the entropy per co-moving volume (S) in the Universe is constant, and

$$S \propto g_* a^3 T^3 \tag{1.9}$$

where g_* is the number of ultra-relativistic degrees of freedom. g_* doesn't change by more than a couple orders of magnitude during the history of the Universe and we neglect its time dependence in the following analysis. The energy of the co-moving volume corresponding to the current Hubble patch which is stored in the relativistic degrees of freedom is

$$E = \rho H_0^{-3} \approx g_* H_0^{-3} T_0^4 \approx S_U T_0 \simeq 10^{90} T_0 \,. \tag{1.10}$$

S_U, the entropy in the current Hubble volume, is a conserved quantity in an adiabatically expanding universe. Hence, naively extrapolating back to Planck times, one finds that the energy of the universe is 90 orders of magnitude above the expected value.

The energy problem can be rephrased in terms of the size problem. The value of the Hubble radius today is $H_0^{-1} \approx 10^{42}\, GeV^{-1}$. Since the Planck epoch, the universe grew up by a factor of

$$\frac{a_0}{a_p} = \frac{T_p}{T_0} = 10^{32}\,. \tag{1.11}$$

The co-moving size at the Planck epoch corresponding to the current Hubble radius is $l_{H_0} = 10^{10}\, GeV^{-1}$. In a power law expanding Universe the Hubble radius is roughly equal to the maximal causally connected region (horizon). However, at the Planck epoch the size of the universe corresponding to the Hubble radius at that time is $l_p = 10^{-19}\, GeV^{-1}$. Hence the size of the universe at the Planck epoch is 29 orders of magnitude above the expected value. To solve the entropy and the size problems one requires either to explain the large amount of entropy or to blow the initial patch (l_p) to the size l_{H_0} without significant loss in energy density.

Today the universe is very close to being flat ($k = 0$). To quantify this statement, it is very useful to define the ratio:

$$\Omega = \frac{\rho}{3H^2 m_p^2}\,. \tag{1.12}$$

In terms of Ω, the Friedmann equation (1.4) is

$$\Omega - 1 = \frac{k}{a^2 H^2}\,. \tag{1.13}$$

During the period of radiation domination $H \propto a^{-2}$. Hence, extrapolating Ω back to the Planck era and comparing to today's value $\Omega \approx 1$ leads to extreme fine-tuning in the initial value of Ω

$$\frac{|\Omega - 1|_{T=T_{PL}}}{|\Omega - 1|_{T=T_0}} \approx \left(\frac{a_{PL}^2}{a_0^2}\right) \approx \left(\frac{T_0^2}{T_{PL}^2}\right) \approx \mathcal{O}(10^{-64}) \tag{1.14}$$

This fine-tuning problem is called the flatness problem.

1.2 Inflation

The puzzles discussed in the previous section as well as other problems of BBC (see e.g. [6, 7] for a review) are possible to resolve if one invents a new period of cosmological expansion. The most successful one is the period of accelerated expansion of the universe - inflation [18]. A toy model approach to obtain inflation consists of introducing one or more scalar fields (inflatons), which evolve slowly due to some appropriately tuned potential. If the energy density of the universe is dominated by a spatially homogeneous scalar field φ

$$\rho = \frac{1}{2}\dot{\varphi}^2 - V(\varphi)\,, \tag{1.15}$$

and if this scalar field is slowly rolling, i.e

$$\dot{\varphi}^2 << V(\varphi), \tag{1.16}$$

then the Hubble parameter is constant:

$$H^2 \approx \frac{1}{3m_{pl}^2}\left[\frac{1}{2}V(\varphi)\right]\,. \tag{1.17}$$

During the period of applicability of (1.16), the scale factor grows exponentially ($a \propto e^{Ht}$) while the energy density remains constant (which means inflation). This is precisely the requirement to solve the entropy problem. Note that $V(\varphi)$ must be chosen such that (1.16) is satisfied for a sufficient long period. This requires the condition

$$\ddot{\varphi} \ll 3H\dot{\varphi} \tag{1.18}$$

to be satisfied as well.

Inflation easily solves the horizon and structure formation problems. During the period of inflation the Hubble radius (H^{-1}) is almost unchanged while the horizon continues to grow as

$$l_f(t_R) = \int_{t_R}^{t_i} \frac{dt}{a}\,, \tag{1.19}$$

where i stands for the beginning of inflation and R for its end. Thus, a long enough period of inflation (in fact fractions of a second) allows the co-moving horizon size to grow much larger than the size of the past light cone at recombination, which explains the horizon problem (Fig. 1.3). The same argument allows a resolution of the structure formation problem. Since all scales inside the present Hubble radius could be fully inside the horizon (Fig. 1.4), a causal microphysical mechanism to generate perturbations is possible. In particular, quantum fluctuations during inflation are redshifted and might be a source of the perturbations responsible for structure formation. The amplitude of quantum fluctuations is independent of time (for an order H^{-1}). Due to this fact, the inflationary scenario implies an almost scale-invariant spectrum of adiabatic cosmological fluctuations [19, 20], a prediction which was made more than a decade before the cosmic microwave background anisotropies were mapped out [10, 11, 12, 13]. The conditions for successful slow roll inflation are (1.16) and (1.18). They lead to the conditions

$$\varepsilon = \frac{m_{pl}^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1 \tag{1.20}$$

$$\eta = m_{pl}^2\frac{V''}{V} \ll 1\,. \tag{1.21}$$

In this approximation, one can compute the scalar spectral index (n_s), the scalar to tensor ratio (r) and the tensor spectral index (n_T) to [21]

$$n_s \approx 1 - 6\varepsilon + 2\eta\,, \tag{1.22}$$

$$r \approx 16\epsilon\,, \tag{1.23}$$

$$n_T \approx -r/8\,, \tag{1.24}$$

where ϵ and η have to be evaluated at the time when the relevant scales leave the Hubble radius during inflation.

Another glance at the Friedman equations

$$\Omega - 1 = \frac{k}{a^2H^2} \tag{1.25}$$

shows that the flatness problem is resolved as well. During inflation $\ddot{a} = \dot{H}a > 0$, therefore, the function $\Omega - 1$ decreases as time progresses and the universe becomes closer to the flat one.

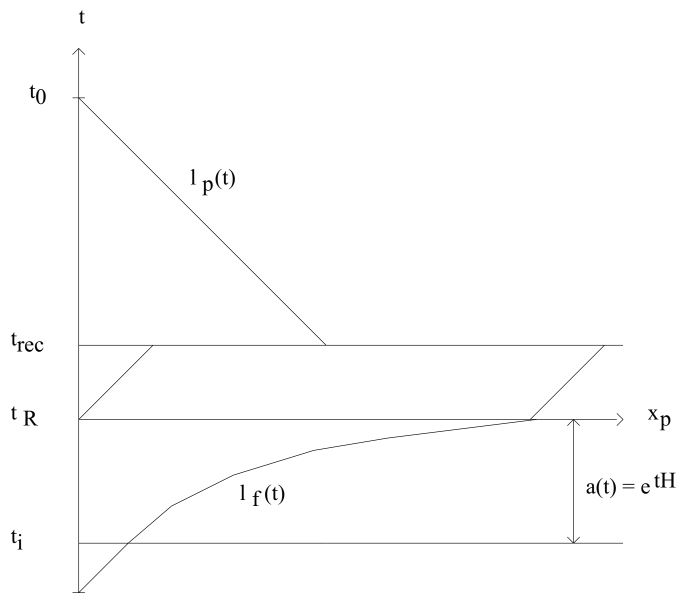

Figure 1.3: In this sketch, the evolution of the horizon is compared with the past light cone at recombination. During inflation ($a(t) \approx e^{Ht}$), the co-moving horizon size grows exponentially in time. Fig. from Ref [6]

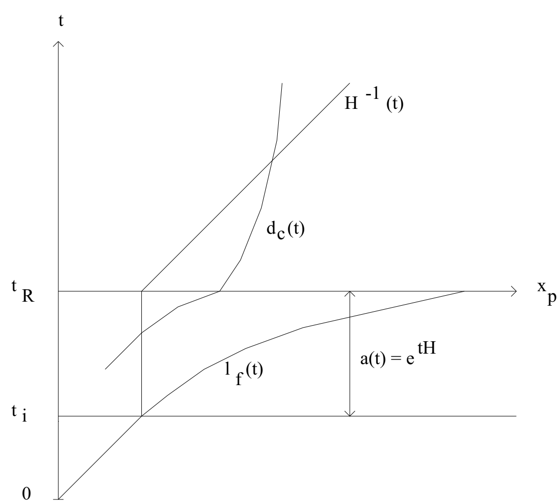

Figure 1.4: The sketch of an evolution of physical scales as a function of time. If sufficiently long inflation took a place, the physical separation between two clusters (d_c) is inside the forward light cone at all times. Note that d_c crosses the Hubble radius (H^{-1}) during inflation and re-enters only after the end of inflation. Hence, during inflation the causal mechanism of structure formation is possible. Fig. from Ref [6]

There are many toy model potentials which result in inflation (see e.g. [22]). However, there is no fully established and accepted inflationary theory. One of the reasons is the fine-tuning problems involved, another is that there is no well established fundamental theory to embed the inflationary scenario in. The last reason is a partial reason for the 'not fine-tuning' problems of inflationary cosmology, trans-planckian and singularity problems [6].

Alternative explanations of the puzzles of the BBC have been proposed alongside inflationary models. Famous examples of these are the Pre-Big-Bang [23] and Ekpyrotic scenarios [24]. However, none of these are able to explain the large initial size of the universe, and thus the entropy problem remains unsolved. In Chapter 2, we demonstrate a string theory inspired solution of the entropy and horizon problems. Note that currently there is no alternative scenario able to overcome all the puzzles of the BBC, in particular, the flatness problem remains unsolved. Hence, inflation remains the most attractive paradigm.

1.3 Brane Inflation

Encouraged by the great success of the inflationary paradigm, one is motivated to find a successful realization of inflation within more fundamental theories, such as string theory. Branes are fundamental objects of string theory on which strings ends. One of such constructions with 3 spatial dimensions (3-brane) can represent our world. Interactions between branes can lead both to attractive and repulsive potentials. Examples of sources of inter-brane potentials are exchange of massless and massive bulk modes, and strings stretched between branes. The slow relative-motion of brane and (anti)brane can be interpreted as inflation on the worldsheet of the brane [25, 26].

In 1998, Dvali and Tye [25] proposed a D-brane inflationary scenario which makes use of slow motion of branes towards each other. They noticed that the joint force due to gravitational attraction, exchange of dilaton and Ramond-Ramond fields cancels while supersymmetry is preserved. However, once the supersymmetry breaking corrections are included, a potential of the form

$$V = A + \frac{B}{r^n}\,, \tag{1.26}$$

where A, B and n are constants, results. The basic (and well known [27]) drawback of this and similar brane-antibrane proposals [26] is the 'η problem': smallness of the slow roll parameter η (1.21) requires the inter-brane separation to be larger than the size of the manifold.

The phenomenological constructions discussed above were the first attempts to obtain inflation in string theory. These constructions did not take into account the issue of stabilization of the internal dimensions. Without stabilization of the degrees of freedom controlling compactification, a typical 4 dimensional effective potential has a typical form (see e.g. [28])

$$V(\varphi, \rho, \phi) \sim e^{a\varphi - b\rho}\tilde{V}(\phi) \tag{1.27}$$

where a and b are model dependant positive constants, φ and ρ are canonically normalized fields representing the dilaton field and the volume moduli. In order for the potential $\tilde{V}(\phi)$ to drive inflation, the dilaton field has to be stabilized in order not to run to minus infinity and the volume not to decompactify. After the discovery that flux constructions can lead to a stabilization mechanism for most moduli fields of string theory [29, 30], a lot of attention (beginning with [31, 32]) was focused on how to obtain inflationary models in the context of flux compactifications (see [33] for reviews and comprehensive lists of references). These constructions are, once again, in the context of static bulk configurations, and have to assume very special configurations (special configurations of branes and special flux choices).

On the other hand, one of the compactification moduli can serve as the inflaton at the last stages of stabilization. An example of modulus inflation, based on the KKLT construction [31], is Racetrack Inflation [34]. In Chapters 3 and 4, we propose a scenario with the overall volume modulus playing the roll of the inflaton. The origin of the potential for the overall modulus is assumed to be in the brane-(anti)brane interactions. The new ingredient of the scenario is the preceding bulk expansion phase which naturally leads to the required pre-inflationary conditions. The Kahler modulus inflation model suggested in [35] has similarities with our scenario. In particular the inflaton is one of the volume components (4-cycle volume) and the potential has the form

$$V = V_0 \left(1 - \xi e^{-a\phi^n}\right) \tag{1.28}$$

where V_0, ξ, a and n are model-dependent constants, and ϕ is the canonically normalized inflaton. In [35], this form of the potential is obtained in the following way. The Kahler moduli appear only non-perturbatively in the superpotential and result in terms in the potential of the form $e^{-a\phi^n}$. The uplifting of the potential is achieved through α' corrections [36] and provides the potential with a constant piece.

1.4 Framework

The scenario of the multidimensional universe which is developed in Chapters 2,3 and 4 has similar initial conditions to those assumed in the hot Big Bang, the only difference being the number of dimensions. The universe was born small with a typical scale of string size, l_s. We assume the manifold to be

$$\mathcal{M} = \mathcal{R} \times T^3 \times T^d/Z_2\,, \tag{1.29}$$

so that our three dimensions have the topology of a torus T^3, and the d extra dimensions are compactified on the orbifold T^d/Z_2. The $d+3$ dimensional universe was born dense with all stringy degrees of freedom present. To ensure the success of our scenario, we need to assume a weak attractive force between the orbifold fixed planes which is generated via some potential V. While the specific form of the potential is less important for the success of the scenario, the ultimate requirement on V is to prevent decompactification of the volume. The potential can be generated due to branes pinned to orbifold fixed planes.

The evolution of the universe has three stages. In the fist stage, the bulk matter leads to isotropic expansion. Bulk matter consists of stringy degrees of freedom and specifically p-branes. This pre-inflationary expansion [1, 2] is responsible for a large inter brane separation and volume of the internal space. As the universe expands, the energy density stored in the gas of p-branes gets diluted until a weak attractive force generated by the potential V comes into play and changes the overall dynamics. The second stage is the process of contraction of the extra dimensions while our dimensions continue to expand (and even inflate). In our scenario, the volume modulus is identified with the interbrane separation. Thus, the contraction potential V plays an important role in the overall volume stabilization. Once the extra dimensions shrink down to a small scale, moduli trapping [37, 38, 39, 40] and pre-heating [41, 42, 43, 44, 45, 46] occurs. This is the third stage of the process which is followed by the epoch of BBC.

The evolution of the $d+4$ dimensional Universe is governed by the Einstein equations. Let G_{ab} be the metric for the full space-time with coordinates X^a. The line element of spatially flat but anisotropic universe is

$$ds^2 = G_{ab} dX^a dX^b \;=\; dt^2 - a(t)^2 d\mathbf{x}^2 - b(t)^2 d\mathbf{y}^2\,, \tag{1.30}$$

where $\mathbf{x}$ denotes the three coordinates parallel to the orbifold fixed planes and $\mathbf{y}$ denotes the coordinates of the d perpendicular directions. The action of the Universe is described by

$$S = \int d^{d+4}X\sqrt{-\det G_{ab}}\left\{\frac{1}{16\pi G_{d+4}}R_{d+4} + \hat{\mathcal{L}}_M\right\}\,, \tag{1.31}$$

where R_{d+4} is the $d+4$ dimensional Ricci scalar and $\hat{\mathcal{L}}_M$ is the matter Lagrangian density with the metric determinant factored out. In the first stage the dominant component of the Lagrangian is the bulk matter perfect fluid with equation of state

$$P = w\rho \tag{1.32}$$

where P is the pressure and ρ is the energy density. For p-branes,

$$w = -\frac{p}{3+d}\,. \tag{1.33}$$

The universe is isotropic,

$$a(t) = b(t) \propto t^{2/(3+d-p)}\,. \tag{1.34}$$

The bulk energy density gets diluted as

$$\rho(t) \sim b(t)^{-d-3+p}\,. \tag{1.35}$$

and the expansion lasts until the potential V begins to dominate.

To follow the evolutions of 'our' three spatial dimensions from the effective four dimensional point of view, we need to replace the $b(t)$ by a canonically normalized scalar field $\varphi(t)$ which is related to $b(t)$ through

$$\varphi = \beta^{-1} m_{pl} \ln(b)\,, \tag{1.36}$$

where we have defined

$$\beta^{-1} \equiv \sqrt{\frac{d(d+2)}{2}}\,. \tag{1.37}$$

In terms of φ the effective reduced four dimensional action after performing a conformal transformation to arrive at the Einstein frame is

$$\begin{aligned} S = \int d^4x \sqrt{-\tilde{g}_{\mu\nu}} \left\{ \frac{1}{2} m_{pl} R_4 \right. &- \frac{1}{2}(\partial\varphi)^2 \\ &+ \left. \mathcal{V} e^{-d\varphi/m_{pl}\beta} \hat{\mathcal{L}}_M \right\}\,, \end{aligned} \tag{1.38}$$

where

$$\mathcal{V} = \int d^d\mathbf{y} = l_s^d \tag{1.39}$$

is the coordinate volume of the extra dimensions, and $\tilde{g}_{\mu\nu}$ is the metric in the Einstein frame. To simplify the notation, let us define:

$$\tilde{\mathcal{L}} = \mathcal{V} e^{-d\varphi/m_{pl}\beta} \hat{\mathcal{L}}_M\,. \tag{1.40}$$

In terms of $\tilde{\mathcal{L}}$, the reduced energy-momentum tensor takes the form

$$\tilde{T}_{\mu,\nu} = \tilde{\nabla}_\mu\varphi\tilde{\nabla}_\nu\varphi - \frac{1}{2}\tilde{g}_{\mu,\nu}\tilde{g}^{\rho\sigma}(\tilde{\nabla}_\rho\varphi)(\tilde{\nabla}_\sigma\varphi) - 2\frac{\delta\tilde{\mathcal{L}}}{\delta\tilde{g}^{\mu,\nu}} + \tilde{g}_{\mu,\nu}\tilde{\mathcal{L}}\,. \tag{1.41}$$

Then, the FRW equations read:

$$\begin{aligned} H_E^2 &= \tilde{T}_{00} & (1.42)\\ \dot{H}_E &= \frac{1}{2m_{pl}}(\tilde{T}_{00} + \tilde{T}_{11}/\alpha^2) & (1.43) \end{aligned}$$

where α is the scale factor of our dimensions in the Einstein frame and $H_E = \dot{\alpha}/\alpha$. The equation of motion for the field φ is

$$\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\tilde{\nabla}_\nu\varphi + \frac{\partial\tilde{\mathcal{L}}}{\partial\varphi} = 0 \tag{1.44}$$

The three equations (1.42,1.43,1.44) fully determine the evolution of our three dimensions once $\tilde{\mathcal{L}}$ is known.

The scenario [1, 2] adopts the mechanism of stabilization of the shape and volume moduli through trapping at enhanced symmetry points [37, 38, 39, 40, 47]. In string gas cosmology (see e.g. [48, 49, 28] for an introduction) the self dual radius serves as a point at which new degrees of freedom become light (see [40] for a toy model). As the size of the extra dimensions shrink to the string scale, new massless degrees of freedom get produced and trap the volume moduli [40]. To introduce the main idea of moduli trapping at enhanced symmetry point, consider the Lagrangian

$$L = -\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{2}m^2\varphi^2 - \frac{1}{2}g^2\varphi^2\chi^2 \tag{1.45}$$

where χ stands for a light field, φ for a modulus, m for the mass of the modulus, and g for the coupling. Assuming homogeneity, the equations of motion for φ and χ are

$$\ddot{\varphi} + m^2\varphi + g^2\varphi\chi^2 = 0 \tag{1.46}$$

$$\ddot{\chi} + g^2\varphi^2\chi = 0 \tag{1.47}$$

As χ gets produced, the energy density of created particles grows

$$\rho_\chi \approx m_\chi^2\langle\chi^2\rangle \approx g^2\varphi^2\langle\chi^2\rangle \tag{1.48}$$

where m_χ is the effective mass of the χ particles. The number density of the created particles can be defined as

$$n_\chi = \frac{\rho_\chi}{m_\chi} \approx g|\varphi|\langle\chi^2\rangle \tag{1.49}$$

Plugging the expression for $\langle\chi^2\rangle$ into the equations of motion for φ,

$$\ddot{\varphi} + m^2\varphi = -g^2\varphi\frac{n_\chi}{g|\varphi|} \tag{1.50}$$

we see that the backreaction of the created particles generates an attractive force for the modulus towards the point of enhanced symmetry. Since the force remains attractive while the modulus oscillates around the point, trapping occurs.

1.5 Preheating Techniques

The process of reheating precedes the radiation epoch. During reheating the pre-existing form of energy is converted non-adiabatically into radiation. The reheating process is model dependent. In inflationary models, as soon as the expansion rate is smaller than the inflaton decay rate (Γ), the inflaton decays within a time $1/\Gamma$. However, under certain conditions, which will be reviewed below, inflationary models admit nonperturbative regimes of inflaton decay, e.g narrow and broad parametric resonance. We will consider a specific, namely the chaotic inflationary scenario.

In the chaotic inflationary scenario the Lagrangian can take the following form

$$L = -\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{2}m_\varphi^2\phi^2 - \frac{1}{2}m_\chi^2\chi^2 - \frac{1}{2}h\phi\chi^2 - \frac{1}{2}g^2\phi^2\chi^2\,, \tag{1.51}$$

where φ is the inflaton and m_φ is its mass, χ is the matter field inflaton couples to and m_χ is its mass, g and h are coupling constants.

The equations of motion for φ and χ are

$$\ddot{\varphi} \;+\; \left(-\nabla^2 + m_\varphi^2 + g^2\chi^2\right)\varphi + \frac{1}{2}h\chi^2 \;=\; 0\,, \tag{1.52}$$

$$\ddot{\chi} \;+\; \left(-\nabla^2 + m_\chi^2 + h\varphi + g^2\varphi^2\right)\chi \;=\; 0\,. \tag{1.53}$$

If φ is a homogeneous field, i.e. only the zero mode of φ is excited, and the initial excitations of χ are negligible, then φ performs oscillations with frequency m_φ^{-1}:

$$\varphi = \Phi\sin(m_\varphi t)\,, \tag{1.54}$$

where Φ is the amplitude of φ. The situation described above is common for chaotic type inflationary models where at the end of inflation $\Phi \sim M_p$.

The nonperturbative process we discuss is the excitation process of the k'th mode of the quantum field χ. The Heisenberg representation of χ in terms of the creation ($\hat{a}_k$) and annihilation ($\hat{a}_k^\dagger$) operators of the k'th mode is

$$\chi(\mathbf{x},t) = \frac{1}{(2\pi)^(3/2)}\int d^3k\left(\hat{a}_k\chi_k(t)e^{-i\mathbf{kx}} + \hat{a}_k^\dagger\chi_k^*(t)e^{i\mathbf{kx}}\right) \tag{1.55}$$

We plug the above representation for φ into the equation of motion for the k'th mode of χ to obtain

$$\ddot{\chi}_k + \omega_k^2\chi_k \;=\; 0\,. \tag{1.56}$$

where

$$\omega_k^2 = k^2 + m_\chi^2 + h\Phi sin(m_\varphi t) + g^2\Phi^2 sin^2(m_\varphi t) \tag{1.57}$$

In the adiabatic approximation the solution of (1.56) can be written in term of the Bogoliubov coefficients $\alpha_k(t)$ and $\beta_k(t)$

$$\chi_k(t) = \frac{\alpha_k(t)}{\sqrt{2\omega_k}} e^{-i\int \omega_k dt} + \frac{\beta_k(t)}{\sqrt{2\omega_k}} e^{i\int \omega_k dt} \tag{1.58}$$

If one chooses as initial conditions the positive-frequency solution, then $\alpha_k(0) = 1$ and $\beta_k(0) = 0$, and the number density of the created particles is

$$n(t) = \int \frac{d^3k}{(2\pi)^3} n_k(t) \tag{1.59}$$

with

$$n_k(t) = |\beta_k|^2(t) = \frac{\omega_k}{2}\left(|\chi_k|^2 + \frac{|\dot{\chi}_k|^2}{\omega_k}\right) - \frac{1}{2}\,. \tag{1.60}$$

In order to find explicit solutions for χ_k, we would like to put the equation (1.72) into the form of the well known Mathieu equation [50]. For the time being we assume $h\Phi \gg g^2\Phi^2$ and introduce a dimensionless variables via

$$z = \frac{1}{2} m_\phi t + \frac{\pi}{4}\,. \tag{1.61}$$

The differentiation with respect to z will be denoted by a prime. In this case, the above equation (1.72) takes the form

$$\chi_k'' + (A_k - 2q\cos 2z)\chi_k = 0\,, \tag{1.62}$$

where

$$\begin{aligned} A_k &= 4\frac{k^2 + m_\chi^2}{m_\phi^2} \\ q &= 2\frac{h\Phi}{m_\phi^2}\,. \end{aligned} \tag{1.63}$$

The behavior of (1.62) is well known [50] and is illustrated in Fig. 1.5. The analytic investigation is about to follow.

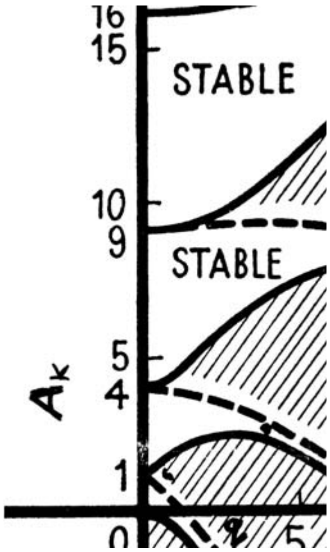

Figure 1.5: The instability chart of the Mathieu equation. The shaded regions correspond to unstable solutions. From Ref. [50]

1.5.1 Narrow Parametric Resonance

Small values of $q \ll 1$ fall in the domain of narrow resonance. In this case the solution can be found analytically [51, 52]. The most important contribution for χ_k comes from the 1'st instability band $A_k \in [1-q, 1+q]$. The resonance band is centered at

$$k_m = \frac{m_\phi}{2}(1 - \frac{4m_\chi^2}{m_\phi^2})^{1/2} . \quad (1.64)$$

with the width

$$\Delta k \approx \frac{m_\phi q}{2} . \quad (1.65)$$

All modes which fall into the band get excited with the amplitude

$$\chi_k(t) \propto e^{2\mu_k z} \quad (1.66)$$

where μ_k is the Floquet index

$$\mu_k = \sqrt{\left(\frac{q}{2}\right)^2 - \left(\frac{2\sqrt{k^2+m_\chi^2}}{m_\varphi} - 1\right)^2} . \quad (1.67)$$

The number density of the created particles with momenta k grows exponentially,

$$n_k \approx e^{8\mu_k m_\varphi t} . \quad (1.68)$$

Hence, the total number density is

$$n(t) = \int \frac{d^3k}{(2\pi)^3} e^{8\mu_k m_\varphi t} \approx \frac{\Delta k}{(2\pi)^3} k_m^2 e^{4qm_\varphi t} \approx \frac{h\Phi m_\varphi}{(2\pi)^3} e^{8h\Phi t/m_\varphi} \,. \tag{1.69}$$

Since the energy density is conserved the number density of the φ field decreases.

If the quartic term in (1.57) is more important then the cubic one, we have

$$\begin{aligned} z &= m_\varphi t \\ A_k &= \frac{k^2 + m_\chi^2}{m_\phi^2} + 2q \\ q &= 2\frac{g^2\Phi^2}{4m_\phi^2} \,. \end{aligned} \tag{1.70}$$

and the calculation (1.69) is adjusted accordingly. The process described above is equivalent to the decay of a φ particle with mass m_φ into two particles with energy $m_\varphi/2$ which is exactly what happens in perturbative decay. The main difference is that the narrow parametric resonance is a nonperturbative process and only particles with certain momenta get excited.

The above discussion does not take into account the expansion of the universe. In the expanding universe, the energy density of φ red-shifts as matter

$$\rho_\varphi = \frac{1}{2} m_\varphi^2 \Phi^2 \propto a^{-3} \,. \tag{1.71}$$

Therefore, the value of q reduces with time and eventually the perturbative decay rate Γ dominates. In addition, the equation of motion for χ_k acquires an additional frictional term

$$\ddot{\chi} + 3H\dot{\chi} + \omega_k^2 \chi \, = \, 0 \,. \tag{1.72}$$

The friction dilutes the k'th mode

$$\chi_k \propto e^{-3Ht} \,. \tag{1.73}$$

Hence, one has to compare the dilution of the k'th mode due to expansion with its growth due to narrow parametric resonance. We can establish the condition for efficiency of narrow parametric resonance in the expanding universe for

$$qm_\varphi > \max{(H, \Gamma)} \tag{1.74}$$

In the expanding universe the physical momentum p red-shifts as a^{-1}, namely $p = k/a$, where k is the co-moving momentum. In a time interval Δt, assuming $m_\chi < m_\phi/4$, the change in the physical momentum corresponding to the middle of the lowest resonance band (k_m) is

$$\Delta p = pH\Delta t \simeq \frac{m_\phi}{2} H\Delta t\,. \tag{1.75}$$

Comparing the expression to the width of the resonance band (1.65) we infer that p remains in the resonance band during the time interval

$$\Delta t \simeq qH^{-1}\,. \tag{1.76}$$

To justify neglecting the expansion of space, we must require that the exponent the growth factor (1.66) is at least 1 during this time interval. This leads to a more severe constraint on q comparing to (1.74):

$$q^2 m_\phi > H\,. \tag{1.77}$$

For cubic interactions the condition (1.77) translates to

$$\frac{h^2 m_p}{m_\varphi^4} > \frac{1}{\Phi} \propto t\,. \tag{1.78}$$

Therefore, the narrow resonance decay channel eventually shuts off. The same condition for quartic interactions reads

$$\frac{g^4 m_p}{m_\varphi^4} > \frac{1}{\Phi^3} \propto t^3\,, \tag{1.79}$$

and the resonance shuts off even faster.

Along with excitations of the modes in the 1'st resonance band $A_k \in [1-q, 1+q]$, modes with $A_k < 2q$ are excited as well via the process of tachyonic resonance (see instability chart in Fig. 1.5). Tachyonic resonance is named after the fact that, during part of the oscillation period, χ_k has a negative squared mass. Another glance at the definitions (1.63, 1.70) reveals that tachyonic resonance occurs only if cubic terms are dominant. During tachyonic resonance, excitations of χ_k roughly grow as

$$\chi_k \propto e^{\sqrt{2q-A_k}z}\,. \tag{1.80}$$

All modes with

$$k < \sqrt{h\Phi} \tag{1.81}$$

are excited. Hence, the physical momentum cannot redshift out of the tachyonic resonance band and the efficiency condition in the expanding universe reads

$$\sqrt{q}m_\varphi > H \tag{1.82}$$

or equivalently

$$\frac{\sqrt{h}m_p}{m_\varphi} > \sqrt{\Phi} \propto t^{-1/2}\,. \tag{1.83}$$

As can be seen from the instability chart of the Mathieu equation, tachyonic resonance shuts off once $q = 1/2$.

1.5.2 Broad Parametric Resonance

For large q values, particles are created in the broad resonance regime. The analytic theory of the broad resonance was first proposed in [43, 44]. At the onset of oscillations the amplitude of the inflaton can be large, e.g. as in the chaotic inflation model where $\varphi \sim M_p/10\,,\ m_\varphi \sim 10^{-6}M_p$ and

$$q = \frac{g^2\Phi^2}{4m_\varphi^2} \sim g^2 10^{10} \gg 1\,. \tag{1.84}$$

As φ crosses zero, the adiabatic condition

$$\left|\frac{\dot{\omega}_k}{\omega_k^2}\right| < 1 \tag{1.85}$$

is violated, allowing the nonperturbative particle production. Notice that for small values of φ, $\dot{\varphi} = m_\varphi\Phi$. Thus, for the quartic interaction, the condition (1.85) is satisfied for a wide range of momenta

$$k^2 \leq (g^2\varphi m_\varphi\Phi)^{2/3} - g^2\varphi\,. \tag{1.86}$$

This window opens up as soon as φ drops below $\sqrt{m\Phi/g}$ and particles with typical momentum $k = \sqrt{gm_\varphi\Phi}$ is produced.

The field φ crosses zero at $t_j = \frac{j\pi}{m_\varphi}$. The t_j are points of time at which creation of particles is concentrated. While t is far from these points the adiabatic condition holds. To study equation of motion (1.56) near the moments of particle production, let us expand the frequency ω near the points t_j as

$$\omega_k^2(t) = \omega_k^2(t_j) + \frac{1}{2}\ddot{\omega}^2{}_k(t_j)(t-t_j)^2 \tag{1.87}$$

and make the change of variables

$$\tau \equiv [2\ddot{\omega}^2(t_j)]^{1/4}(t-t_j)\,, \tag{1.88}$$
$$\kappa^2 \equiv \frac{\omega_k^2(t_j)}{\sqrt{2\ddot{\omega}^2(t_j)}} = \frac{k^2+m_\chi^2}{2gm_\varphi\Phi}\,. \tag{1.89}$$

In the new variables the equation (1.72) becomes

$$\frac{d^2\chi_k}{d\tau^2} + \left(\kappa^2 + \frac{\tau^2}{4}\right)\chi_k = 0 \tag{1.90}$$

which can be viewed as a Schrödinger equation for a wave function scattering in an inverted parabolic potential. The solutions of (1.90) are well known [53]: they are parabolic cylinder functions, $W(-\kappa^2, \pm\tau)$. In particular, after the first scattering (no previous particles are present)

$$n_k = e^{-2\pi\kappa^2}\,. \tag{1.91}$$

Later on, the number density is growing exponentially, $n_k \sim e^{2\mu_k z}$ with typical Floquet exponent $\mu \approx 0.175$ [44].

1.6 The Cosmological Moduli Problem

Nowadays, the predictions of the SM are tested to a high degree of precision. However, some phenomena remain unexplained. The origin of dark matter, neutrino masses and dark energy demands physics beyond the SM. Components of models beyond the SM may not only provide the necessary explanation of the phenomena they are invited to explain but they also pose new problems. The cosmological moduli problem [54, 55, 56, 57] is one of them. Moduli are weakly coupled scalar fields. The cosmological moduli problem arises if the moduli are overproduced in the early universe and threaten to spoil the success of the BBC scenario.

Consider a modulus ϕ with mass m_ϕ and a potential of the form

$$V(\phi) = \frac{1}{2} m_\phi^2 \phi^2 \,. \tag{1.92}$$

In the expanding universe, the moduli slowly roll until the Hubble rate, H, drops below the mass, m_ϕ and, thus, they start to oscillate. During oscillations the energy density of the moduli redshifts as a matter field $\rho_\phi \propto a^{-3}$ while the energy density in radiation redshifts as $\rho_r \propto a^{-4}$. Hence, an initially radiation dominated universe might become overclosed by the oscillations of the moduli. The typical decay rate Γ_{all} of a scalar ϕ with only gravitational strength couplings is

$$\Gamma_{all} \;\sim\; \frac{1}{4\pi} \frac{m_\phi^3}{m_p^2} \,. \tag{1.93}$$

The reheating temperature of the universe $T_R \approx \sqrt{m_p \Gamma_{all}}$ is below the scale when Nucleosynthesis takes place unless the moduli are heavier than $\sim 100 TeV$. Even if the initial energy density of moduli is low enough not to overclose the universe upon their decay, the decay products of moduli threaten to spoil the success of BBN or overproduce dark matter components. In particular, hadronic or radiative decay of moduli can significantly affect the primordial abundances of the light elements. There are stringent constraints on the abundance of moduli coming from the non-thermal production of D, ^3He and ^6Li [58, 59]. The range of moduli masses which are dangerous for cosmology is model dependent but typically is between 10 eV and 100 TeV. The long lived fermions have similar constraints from cosmology as the moduli fields discussed above. The best known example of a long lived fermion is the gravitino, a component of the supergravity multiplet.

Supersymmetric or supergravity extensions of the SM are considered to be among the most promising candidates to explain new physics. They are also a major source of moduli, fields which have flat potentials in the supersymetric limit and are only weakly coupled to the SM particles. Another source of moduli are compactifactions in string theory which yield volume and shape moduli. Moduli obtain masses during the process of supersymmetry breaking. There are many scenarios of supersymmetry breaking, among them gravity and gauge mediation scenarios are a natural source of weakly coupled moduli with masses in the dangerous range.

Consider for example the Polonyi model [60] which is one of the classical examples of gravity mediated supersymmetry breaking. The description of the Polonyi model presented here closely follows [61]. There are hidden and visible sectors. The hidden sector proposed by Polonyi contains only one chiral multiplet (ϕ_p, χ_p), which has the following superpotential

$$W_p = \mu^2(\phi_p + \omega)\,, \tag{1.94}$$

where μ and ω are parameters which will be determined by phenomenological requirements later on. In additional, we adopt the simplest Kähler potential

$$K = \phi_p^*\phi_p\,. \tag{1.95}$$

Hence, the combined superpotential of the theory is

$$W = W_P + W_{obs} \tag{1.96}$$

where W_{obs} stands for the superpotential of the observable sector and doesn't depend on ϕ_p. Calculations of the interaction potential between ϕ_p and fields in the observable sectors show that all mutual interactions are Planck suppressed [61].

The auxiliary field F_p is given by

$$\begin{aligned} F_p &\equiv e^{K/(2m_p^2)^2} D_{\phi_p^*} W_p^* && (1.97)\\ &\equiv e^{K/(2m_p^2)^2} \left\{ \frac{\partial W_p^*}{\partial \phi_p^*} + \frac{1}{m_p^2}\frac{\partial K}{\partial \phi_p^*} W_p^* \right\} && (1.98)\\ &= \mu^2 \left\{ \frac{\phi_p}{m_p^2}(\phi_p + \omega) + 1 \right\} e^{\phi_p\phi_p^*/2m_p^2} && (1.99) \end{aligned}$$

F_p is the supersymmetry breaking parameter. Since no solution to the equation $F_p = 0$ exists, supersymmetry is spontaneously broken. Making the requirement of zero cosmological constant at the minimum of the potential ($\langle V \rangle = 0$) for ϕ_p

$$V = e^{K/m_p^2} \left\{ |D_{\phi_p} W_p|^2 - \frac{3}{m_p^2}|W_p|^2 \right\}\,, \tag{1.100}$$

we determine the following set of parameters

$$
\begin{aligned}
\omega &= (2-\sqrt{3})\, m_p && (1.101)\\
\langle\phi\rangle &= (\sqrt{3}-1)\, m_p && (1.102)\\
\langle W_p\rangle &= \mu\, m_p && (1.103)\\
\langle F_p\rangle &= \sqrt{3}\, e^{2-\sqrt{3}}\mu^2\,. && (1.104)
\end{aligned}
$$

The Super-Higgs mechanism provides the gravitino with a mass

$$m_{3/2} = \langle e^{K/(2m_p^2)^2}\frac{W_p}{m_p^2}\rangle\,. \tag{1.105}$$

Hence,

$$m_{3/2} = e^{2-\sqrt{3}}\frac{\mu^2}{m_p}\,. \tag{1.106}$$

Further, plugging $\phi_p = (\sqrt{3}-1)\, m_p + \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$ into the potential (1.100), we can determine the masses of the excitations ϕ_1 and ϕ_2

$$m_{\phi_1} = 2\sqrt{3}\, m_{3/2}^2 \text{ and } m_{\phi_1} = (4-2\sqrt{3})\, m_{3/2}^2\,. \tag{1.107}$$

The masses of squarks and sleptons are related to the mass of the gravitino. For example, in models with minimal kinetic term, the following (tree level) super-trace formula among the mass matrixes M_j^2's holds:

$$StrM^2 \equiv \sum_{spinJ}(-1)^{2J}(2J+1)trM_j^2 \approx 2(n-1)m_{3/2}^2 \tag{1.108}$$

where n stands for the number of the chiral multiplets in the spontaneously broken local SUSY model. In this case, all the SUSY breaking masses of squarks and sleptons are of order of the gravitino mass. Therefore, in order to obtain an interesting phenomenology, one struggles to keep the gravitino mass around TeV scale. The relation between the supersymmetry breaking parameter F and gravitino mass is determined by the requirement of zero cosmological constant. Roughly speaking, $\langle V\rangle = 0$ leads to

$$\langle F\rangle < \langle\frac{W}{m_p}e^{K/2m_p^2}\rangle \sim O(m_{3/2}m_p)\,. \tag{1.109}$$

This is the relation we obtained in the Polonyi model. The phenomenological requirement to keep gravitino mass around TeV scale determines the scale $\sqrt{F_p}$ or the μ parameter around $\sim 10^{11}$ GeV.

The Polonyi model is an example of a model which has weakly coupled scalars with the mass of order the gravitino mass. In fact, this is the situation in a variety of supergravity mediation models [62]. The "Polonyi field", ϕ_p is an example of a modulus. During inflation, quantum fluctuations of a scalar field ϕ with the effective mass $m_{eff} << H$ yield a variance of ϕ growing as [63]

$$\langle\phi^2\rangle \approx \frac{3H^4}{8\pi^2 m_\phi^2}(1 - e^{-(2m^2/3H^2)t}), \tag{1.110}$$

while if $m_{eff} >> H$ the variance goes as

$$\langle\phi^2\rangle \approx \frac{H^3}{12\pi^2 m}. \tag{1.111}$$

Hence, we see that if the effective mass of the moduli during inflation is substantially lower that the inflation scale, the vacuum expectation value of the moduli can be of the order of the Planck scale. Even in the case of large effective mass during inflation, the offset of the high and low temperature minima usually is of the order of the Planck scale and one generically expects moduli to acquire large expectation values at the end of inflation. The large expectation values of moduli fields give rise to the cosmological moduli problem as discussed above.

While moduli with mass above 100 TeV decay before BBN, their decay products still may be dangerous for cosmology. In particular, the large branching ratio of the modulus decay into gravitinos may result in the overproduction of gravitinos [64, 65]. Consider the part of the Lagrangian describing the gravitino-modulus couplings in the unitary gauge in the Einstein frame

$$\begin{aligned} e^{-1}\mathcal{L} \;\;=\;\; & -\frac{1}{8}\epsilon^{\mu\nu\rho\sigma}(G_\phi \partial_\rho \phi + G_{\phi^\dagger}\partial_\rho \phi^\dagger)\bar{\psi}_\mu \gamma_\nu \psi_\sigma && (1.112) \\ & -\frac{1}{8}e^{G/2}(G_\phi \phi + G_{\phi^\dagger}\phi^\dagger)\bar{\psi}_\mu[\gamma^\mu, \gamma^\nu]\psi_\nu && (1.113) \end{aligned}$$

where ψ_μ stands for the gravitino and G_i is a derivative with respect to the field i of the

$$G = K/M_p^2 + \ln(|W|^2/M_p^6). \tag{1.114}$$

K and W are the Kähler potential and superpotential, respectively. Based on these couplings, the perturbative decay rate of the real and imaginary components of the moduli ϕ, ϕ_R and ϕ_I, into gravitinos is

$$\Gamma_{3/2} \equiv \Gamma(\phi_{R,I} \to 2\psi_{3/2}) \approx \frac{|G_\phi|^2}{288\pi K_{\phi\phi^\dagger}} \frac{m_\phi^5}{m_{3/2}^2 M_p^2} \,. \tag{1.115}$$

where $K_{\phi\phi^\dagger}$ is the Kähler metric and the calculations are done in the limit $m_\phi \gg m_{3/2}$ after the canonical normalization $\hat{\phi} = \sqrt{K_{\phi\phi^\dagger}}\phi$.

The dimensionless auxiliary field of the modulus ϕ, G_ϕ, in general, can be small to suppress $\Gamma_{3/2}$ relative to the total decay rate Γ_{all} (6.1). However, suppressed G_ϕ is not the typical case. For example, in the framework of $4D$ supergravity, G_ϕ obtains a minimal value $\sim m_{3/2}/m_\phi$. The statement follows from the requirement that the potential (1.100) at the minimum should vanish. In terms of G, the potential (1.100) takes the form

$$V = m_p^4 e^G (G^i G_i - 3) \tag{1.116}$$

where $G^i = K^{-1}_{ij^*} G_{j^*}$. The constraint of having zero cosmological constant requires that at least one of the $G_i \sim 0(1)$. For value of G_ϕ to be much less than 1, we need to introduce a hidden sector field Z for which $G_z \sim 0(1)$. To derive the lower bound on G_ϕ, we minimize the potential in the ϕ direction, $V_\phi = \partial V/\partial\phi = 0$ which leads to

$$G^i \left(G_{i\phi} - K_{ij^*\phi} K^{-1}_{ij^*} G_i \right) + G_\phi = 0 \,. \tag{1.117}$$

Carefully estimating the contributions to (1.117), we arrive at [64] $G^\phi G_{\phi\phi} \sim \mathcal{O}(1)$. In the limit $m_\phi \gg m_{3/2}$, $G_{\phi\phi} \sim W_{\phi\phi}/W \sim m_\phi/m_{3/2}$ where $m_\phi = \langle e^{K/2} K^{-1}_{ij^*} W_{\phi\phi} \rangle$, and, hence, $G_\phi \geq m_{3/2}/m_\phi$. Since the typical value of $K_{\phi\phi^*}$ is of the order one, the branching ratio $Br_{3/2} = Br(\phi_{R,I} \to 2\psi_{3/2}) \sim \mathcal{O}(0.01 - 1)$.

On the other hand, the abundance of the unstable gravitino is severely constrained not to jeopardize Nucleosynthesis or overproduce the lightest supersymmetric particles. The stable gravitino is constrained by the dark matter abundance. For example, the constraint from the overproduction of ^3He [58, 59] yields

$$m_{3/2} Y_{3/2} < O(10^{-14} \sim 10^{-11}) \text{ GeV} \,. \tag{1.118}$$

where $Y_{3/2}$ is the gravitino yield. The gravitino yield is defined as following

$$Y_{3/2} \equiv \frac{n_{3/2}}{s} = \frac{n_{3/2}}{n_\phi}\frac{3}{4}\frac{T_R}{m_\phi} \tag{1.119}$$

where $n_{3/2}$ and n_ϕ are, respectively, the number densities of gravitino and moduli particles,

$$T_R = \left(\frac{\pi^2 g_*}{90}\right)^{-1/4} \sqrt{m_p \Gamma_{all}} \approx 5.5 * 10^{-6} GeV \left(\frac{m_\phi}{10^3 GeV}\right)^{3/2} . \tag{1.120}$$

Each moduli particle can decay to two gravitino particles. Hence, $n_{3/2}/n_\phi \approx 2Br_{3/2}$. In terms of $Br_{3/2}$, the relative gravitino abundance is

$$m_{3/2}Y_{3/2} = \frac{3}{2}B_{3/2}\frac{T_R}{m_\phi}m_{3/2} . \tag{1.121}$$

In order to satisfy the constraint (1.118), the branching ratio of moduli into gravitinos cannot exceed

$$Br_{3/2} < \mathcal{O}(10^{-6} \sim 10^{-3}) \left(\frac{1GeV}{m_{3/2}}\right) . \tag{1.122}$$

In Chapters 5 and 6 of this thesis, a partial resolution of the moduli problem is investigated. In Chapter 5 we consider nonperturbative decay (see Chapter 1.5) of the moduli to prevent moduli from dominating the energy density of the universe. Specificallly, we consider trilinear couplings of moduli to another scalar field χ which is strongly coupled to SM degrees of freedom. Hence, the transfer of energy into χ is equivalent to the transfer of energy into radiation. In Chapter 6, we investigate the problem of large branching ratios of heavy moduli into gravitinos. We again use triliniar couplings of moduli to an additional scalar field χ in order to show that large $Br_{3/2}$ does not pose a problem if the moduli undergo nonperturbative decay into χ. [1]

[1]Further research is required for a successful implementation of the proposal into an explicit particle model. In particular, one of the expected problems in this direction is the mass of the χ in Chapter 5, which is required to be much smaller than the expected mass of the new yet undiscovered particles.

Chapter 2

The Confining Heterotic Brane Gas: A Non-Inflationary Solution of the Entropy and Horizon Problems of Standard Cosmology

We propose a mechanism for solving the horizon and entropy problems of standard cosmology which does not make use of cosmological inflation. Crucial ingredients of our scenario are brane gases, extra dimensions, and a confining potential due to string gas effects which becomes dominant at string-scale brane separations. The initial conditions are taken to be a statistically homogeneous and isotropic hot brane gas in a space in which all spatial dimensions are of string scale. The extra dimensions which end up as the internal ones are orbifolded. The hot brane gas leads to an initial phase (Phase 1) of isotropic expansion. Once the bulk energy density has decreased sufficiently, a weak confining potential between the two orbifold fixed planes begins to dominate, leading to a contraction of the extra spatial dimensions (Phase 2). String modes which contain momentum about the dimensions perpendicular to the orbifold fixed planes provide a repulsive potential which prevents the two orbifold fixed planes from colliding. The radii of the extra dimensions stabilize, and thereafter our three spatial dimensions expand as in standard cosmology. The energy density after the stabilization of the extra dimensions is of string scale, whereas the spatial volume has greatly increased during Phases 1 and 2, thus leading to a non-inflationary solution of the horizon and entropy problems. [1]

2.1 Introduction

The Inflationary Universe scenario [18] (see also [66, 67, 68]) has been extremely successful phenomenologically. It has provided a solution to some of the key problems of standard cosmology, namely the horizon and flatness problems, and yielded a mechanism for producing primordial cosmological perturbations using causal physics, a mechanism which predicted [20, 19] (see also [69, 66]) an almost scale-invariant spectrum of adiabatic cosmological fluctuations, a prediction confirmed more than a decade later to high precision by cosmic microwave background anisotropy experiments [10, 11, 12, 13].

In this Chapter, we will pay special attention to the "entropy problem" of standard cosmology [18]. The problem consists of the fact that without accelerated expansion of space, it is not possible to explain the large entropy, size and age of our current universe without assuming that at very early times the universe was many orders of magnitude larger than would be expected on dimensional arguments.

In the inflationary scenario, the entropy problem is solved by postulating a sufficiently long period of accelerated expansion, after which the universe reheats to a temperature comparable to that prior to the onset of the period of acceleration. In most models of inflation, the accelerated expansion of space is sourced by the potential energy of a slowly rolling scalar field. Such models, however, are subject to serious conceptual problems (see e.g. [6, 70] for recent overviews of these problems). Most importantly, the source of the acceleration is very closely related to the source of the

[1] The homogeneity and isotropy problems is addressed as well in the chapter. A long period of expansion allows the region corresponding to our current Hubble radius to be in a causal contact and, hence, to solve the problems.

cosmological constant in field theory, a constant which is between 60 and 120 orders of magnitude larger than the maximal value of the cosmological constant allowed by current observations. Because of the existence of these conceptual problems, it is of great importance to look for possible alternatives to scalar field-driven inflationary cosmology.

There have been various suggestions for alternative cosmologies. In varying speed of light models [71, 72], postulating the existence of a period in the early universe during which the speed of light decreased very fast leads to a solution of the horizon problem. In the "Pre-Big-Bang scenario" [23], the Universe is born cold, flat and large, undergoes a period of super-exponential contraction before emerging into the period of radiation-dominated expansion of standard cosmology. The contracting phase and the expanding phase are related via a duality of string theory, namely "scale-factor duality". In a more recent cosmological scenario motivated by heterotic M-theory [73], namely the "Ekpyrotic scenario" [24], the collision of a bulk brane onto our boundary orbifold fixed plane generates a non-singular expansion of our brane. However, neither the Pre-Big-Bang nor the original Ekpyrotic scenario can explain why our Universe is so large and old (without assuming that the Universe is already much larger than would be expected by dimensional arguments at the end of the phase of contraction (see e.g. [74, 75]) (this problem is avoided in the "cyclic scenario" [76], a further development of ideas underlying the Ekpyrotic scenario, but this is achived at the cost of additional ad hoc assumptions about the cosmological bounce). The size problem has so far also prevented the "string gas cosmology" scenario [77, 78] (see e.g. [79, 28] for recent reviews) from making contact with late time cosmology, although a stringy mechanism for producing a scale-invariant spectrum of cosmological perturbations does exist in this context [80].

In this Chapter, we present a potential solution of the entropy problem which does not make use of a period of accelerated expansion. Our solution makes use of several ingredients from string theory: extra spatial dimensions, the existence of branes and orbifold fixed planes as fundamental extended objects in the theory, and a stringy mechanism for stabilizing the shape and volume moduli of string theory via the production of massless string states at enhanced symmetry points in moduli space. Thus, it is possible that our mechanism will find a natural realization in string theory.

2.2 Overview of the Model

Our starting point is a topology of space in which all but three spatial dimensions are orbifolded, and the three dimensions corresponding to our presently observed space are toroidal. Specifically, the space-time manifold is

$$\mathcal{M} = \mathcal{R} \times T^3 \times T^d / Z_2 \,, \tag{2.1}$$

where T^3 stands for the three-dimensional torus, and d is the number of extra spatial dimensions, which we will take to be either $d = 6$ in the case of models coming from superstring theory, or $d = 7$ in the case of models motivated by M-theory. We will assume that there is a weak confining force between the orbifold fixed planes [2].

As our initial conditions, we take the bulk to be filled with an isotropic [3] gas of branes, as in the studies of [83, 84, 85]. These studies show that, in the context of Type IIB superstring theory, the bulk of the energy density will end up in three and possibly seven branes. However, if the initial Hubble radius is large relative to the size of space, there will be no residual seven branes. In the case of heterotic string theory or taking the starting point to be M-theory, we would be dealing with Neveu-Schwarz 5-branes.

Assuming that the universe starts out small and hot, it is reasonable to assume that the energy density in the brane gas will initially be many orders of magnitude

[2]It may be necessary to have branes pinned to the orbifold fixed planes in order to induce such a potential. Our approach, at this stage, is purely phenomenological, and we simply postulate the existence of a potential with the required properties

[3]Note that the orbifolding will prohibit the existence of certain branes along certain of the dimensions and will thus lead to a breaking of the condition of isotropy. The details are fairly model-specific and will be discussed in a followup paper. The bottom line, however, is that the noninflationary bulk expansion of the first phase in all directions remains a valid conclusion.

larger than the potential energy density generated by the force between the orbifold fixed planes. Thus, initially our universe will be expanding isotropically. We denote this as Phase 1. Our key observation is that in this phase, the energy density projected onto the orbifold fixed planes does not decrease. The reason is that the tension energy of the p-branes increases as $a(t)^p$, where here $a(t)$ is the bulk scale factor. The volume parallel to the orbifold fixed planes is increasing as $a(t)^3$, and hence the projected energy density does not decrease (it is in fact constant in the case of 3-branes).

During Phase 1, the bulk energy density will decrease. Hence, eventually the inter-orbifold potential will begin to dominate. At this point, the cosmological evolution will cease to be isotropic: the directions parallel to the orbifold fixed planes will continue to expand while the perpendicular dimensions begin to contract. We denote this phase as Phase 2.

Once the orbifold fixed planes reach a microscopic separation, a repulsive potential due to string momentum modes becomes important (one example is the production of massless states at enhanced symmetry points [40, 39]). The interplay between this repulsive potential which dominates at small separations and the attractive potential which dominates at large distances, coupled to the expansion of the three dimensions parallel to the orbifold fixed planes, will lead to the stabilization of the radion modes at a specific radius (presumably related to the string scale). In the context of heterotic string theory, we could use the string states which are massless at the self-dual radius to obtain stabilization of the radion modes at the self-dual radius [38, 47] (see also [37]). These modes would also ensure dynamical shape moduli stabilization [89]. We denote the time of radion stabilization by t_R since this time plays a similar role to the time of reheating in inflationary cosmology. The branes decay into radiation either during or at the end of Phase 2. This brane decay is the main source of reheating of our three dimensional space.

After the radion degrees of freedom have stabilized at a microscopic value which presumably is set by the string scale, the three spatial dimensions parallel to the orbifold fixed planes will continue to expand. The energy density which determines the three-dimensional Hubble expansion rate is the projected energy density ρ_p, i.e.

the bulk energy density integrated over the transverse directions. The key point is that during Phase 1, ρ_p does not decrease. If the bulk is dominated by 3-branes, ρ_p is constant, if it is dominated by 5-branes, ρ_p in fact increases. In Phase 2, the projected energy density ρ_p also remains constant if the bulk is dominated by 3-branes, modulo the conversion of brane tension energy into radiation as the bulk branes decay or are absorbed by the fixed planes. If we approximate the evolution by assuming that all of the brane tension energy converts to radiation at the time of radion stabilization, then the value of ρ_p at t_R, which is the energy density which determines the evolution of the scale factor of our three spatial dimensions after t_R, is equal to the projected energy density at the initial time, which we take to be given by the string scale [4]. If the branes decay during Phase 2, then the projected energy density at t_R is larger than the initial value, in which case we may be driven to a Hagedorn phase of string theory. The main point, however, it that since the volume of our three spatial dimensions has been expanding throughout Phases 1 and 2, the horizon and entropy problems of standard cosmology can easily be solved by simply assuming that the phase of bulk expansion lasted sufficiently long (numbers will be given later).

Note that we are assuming in this paper that the dilaton has been stabilized by some as yet unknown mechanism. In this case, the equations of motion of the bulk are those of homogeneous but anisotropic general relativity. The metric is in this case given by

$$ds^2 \,=\, dt^2 - a(t)^2 d\mathbf{x}^2 - b(t)^2 d\mathbf{y}^2 \,, \tag{2.2}$$

where $\mathbf{x}$ denote the three coordinates parallel to the boundary planes and $\mathbf{y}$ denote the coordinates of the perpendicular directions. In the case of d extra spatial directions

[4]We assume that initial radii and densities are all set by the string scale, i.e. we introduce no unnaturally small or large numbers.

the equations of anisotropic cosmology are [5]

$$\begin{aligned} \ddot{a} &+ \dot{a}\big(2H + d\mathcal{H}\big) \qquad (2.3)\\ &= 8\pi G a\big[P - \frac{1}{3+d-1}(3P + d\tilde{P}) + \frac{1}{3+d-1}\rho\big]\,, \end{aligned}$$

$$\begin{aligned} \ddot{b} &+ \dot{b}\big(3H + (d-1)\mathcal{H}\big) \qquad (2.4)\\ &= 8\pi G b\big[\tilde{P} - \frac{1}{3+d-1}(3P + d\tilde{P}) + \frac{1}{3+d-1}\rho\big]\,, \end{aligned}$$

and

$$(3H + d\mathcal{H})^2 - 3H^2 - d\mathcal{H}^2 \;=\; 16\pi G\rho\,, \qquad (2.5)$$

where $H \equiv \dot{a}/a$, $\mathcal{H} \equiv \dot{b}/b$ are the expansion rates of the parallel and perpendicular dimensions, respectively, ρ is the bulk energy density and P and $\tilde{P}$ are the parallel and perpendicular pressures, respectively.

2.3 The Phase of Bulk Expansion

During the phase of bulk expansion, the two scale factors coincide, $P = \tilde{P}$, and both equations (2.3) and (2.4) reduce to

$$\frac{\ddot{a}}{a} + (2+d)\big(\frac{\dot{a}}{a}\big)^2 \;=\; \frac{8\pi G}{3+d-1}\big[\rho - P\big]\,. \qquad (2.6)$$

Making use of the equation of state $P = w\rho$, and inserting (2.5), the dynamical equation (2.6) becomes

$$\frac{\ddot{a}}{a} + (2+d)\big(\frac{\dot{a}}{a}\big)^2 \;=\; \frac{1}{2}(3+d)(1-w)\big(\frac{\dot{a}}{a}\big)^2\,, \qquad (2.7)$$

which leads to power law expansion

$$a(t) \;\sim t^{\alpha} \qquad (2.8)$$

[5]In the following treatment we omit inter-brane potential contribution during the phase of expansion and the contribution of the bulk matter during the stages of contraction. The full treatment should include both contributions simultaneously (See, for example, [86, 87]) The work in this direction is in progress. [88]

where the value of α depends on the equation of state:

$$\alpha = \frac{2}{(3+d)(1+w)} . \qquad (2.9)$$

If the bulk energy is dominated by the tension of p-branes, then we have

$$w = -\frac{p}{3+d} . \qquad (2.10)$$

In the example motivated by perturbative Type IIB superstring theory, namely $d = 6$ and $p = 3$ we obtain

$$a(t) \sim t^{2/(3+d-p)} = t^{1/3} . \qquad (2.11)$$

What is important for us is that this is not accelerated expansion. Starting with heterotic string theory, we would have $d = 6$ and $p = 5$ and for M-theory we would take $d = 7$ and $p = 5$. These two cases lead to faster expansion rates, namely $\alpha = 1/2$ in the former case and $\alpha = 2/5$ in the latter.

2.4 The Phase of Orbifold Contraction

If we want the expansion which takes place in this initial phase to solve the size and horizon problems of standard cosmology independent of any further expansion during Phase 2, then the effective four dimensions scale factor, which is defined by $ab^{d/2}$ needs to increase by a factor $\mathcal{F}$ of at least

$$\mathcal{F} \sim 10^{30} . \qquad (2.12)$$

This result comes about by demanding that the predicted radius of the universe evaluated at the present temperature be greater than the presently observed Hubble radius, i.e. greater than $10^{42}\mathrm{GeV}^{-1}$, by taking the density at the time t_R to be given by the string scale which we take to be 10^{17}GeV, and taking into account that the scale factor in standard cosmology increases by a factor of about 10^{29} between when the temperature is of string scale and today. Correspondingly, the radiation temperature of the bulk will decrease by the same factor $\mathcal{F}$.

We now assume the existence of a confining potential V between the orbifold fixed planes. In order to generate such a non-vanishing potential, we will need to assume that branes are stuck to the orbifold fixed planes. In terms of the distance $r = l_s b$ between these planes (l_s being the string length), a typical confining potential is

$$V(r) \, = \, \mu r^n \, = \, \mu(l_s b)^n, \tag{2.13}$$

where n is an integer, $\mu \equiv \Lambda^{d+n+4}$, and Λ is the typical energy scale of the potential. As we will show below, a value $n \, \geq \, \sqrt{d(d+2)} + d$ is required for our scenario to work.

The presence of this potential will lead to a transition between the phase of isotropic expansion to a phase in which the extra dimensions contract while the dimensions parallel to the fixed planes keep on expanding (and we will verify below that the expansion is not inflationary). The transition between Phase 1 and Phase 2 takes place when the bulk energy density and the inter-brane potential become comparable. The bulk energy density in Phase 1 scales as

$$\rho_b(t) \, \sim \, b(t)^{-d-3+p} \tag{2.14}$$

(recall that in this phase $a(t) = b(t)$). Assuming that the initial bulk energy density is set by the string scale, and using the result (2.11), it follows that in order for the bulk to have expanded by the factor of (2.12), the upper bound on Λ should satisfy:

$$\Lambda \, \sim \, l_s^{-1} 10^{-60 \frac{d-p+3+n}{(d+4+n)(d+2)}} \, . \tag{2.15}$$

For example, in the case $d = 6$, $p = 3$, and $n = 14$ we obtain

$$\Lambda \, \sim \, l_s^{-1} 10^{-5.31} \, \sim 10^{11} \mathrm{GeV} \, . \tag{2.16}$$

For $d = 6$ and $p = 5$ the result is $\Lambda \sim l_s^{-1} 10^{-45/8} \sim 10^{10}\mathrm{GeV}$.

We will analyse the evolution during Phase 2 using a four-dimensional effective field theory, where we replace the radion $b(t)$ by a scalar field $\varphi(t)$. In order that φ be canonically normalized when starting from the higher dimensional action of General Relativity, φ and b must be related via (see e.g. [28], Appendix A)

$$\varphi \, = \, m_{pl} \sqrt{d(d+2)/2} \log(b) \, , \tag{2.17}$$

where m_{pl} is the four-dimensional Planck mass. If the bulk size starts out at the string scale, then $b(t_i) = 1$, where t_i is the initial time. With these normalizations, $\varphi = 0$ corresponds to string separation between the branes. In terms of φ, the potential (2.13) then induces an effective potential for φ:

$$V_{eff}(\varphi) \;=\; \mu l_s^{(n+d)} e^{\tilde{n}\varphi/m_{pl}} \,, \tag{2.18}$$

where $\tilde{n} = (n-d)\sqrt{2/(d(d+2))}$. Note that the original bulk potential needs to be multiplied by the area of the orbifold fixed plane in order to obtain the effective potential for φ, $V_{eff}(\varphi)$. There is also a factor of b^{-2d} coming from converting to the Einstein frame (see e.g. [28], Appendix A). The equation of motion for φ then becomes

$$\ddot{\varphi} + 3H\dot{\varphi} \;=\; -\tilde{n}\frac{\mu l_s^{(n+d)}}{m_{pl}} e^{\tilde{n}\varphi/m_{pl}} \tag{2.19}$$

with

$$H^2 \;=\; \frac{1}{3m_{pl}^2}\Big[\frac{\dot{\varphi}^2}{2} + \frac{\mu l_s^{(n+d)}}{m_{pl}} e^{\tilde{n}\varphi/m_{pl}}\Big] \tag{2.20}$$

During Phase 2, the scale factor $a(t)$ of the three spatial dimensions parallel to the orbifold fixed planes will expand according to the usual four space-time dimensional cosmological equations, where matter is dominated by the scalar field φ. The solution of the equations of motion (2.19 and 2.20) in the cases $\tilde{n} = 1$ and $\tilde{n} = 2$ is given by

$$\varphi \;=\; \frac{m_{pl}}{\tilde{n}} \ln \frac{2m_{pl}^2(6-\tilde{n}^2)}{\tilde{n}^4 \mu {l_s}^{(n+d)} t^2} \tag{2.21}$$

The corresponding values of the equation of state parameter are

$$\tilde{w} \;=\; \frac{\tilde{n}^2 - 3}{3} \,. \tag{2.22}$$

For $\tilde{n} = 1$ this equation of state corresponds to an accelerating background, but for $\tilde{n}^2 = 2$ the background evolution is non-accelerating. In fact, as $\tilde{n}$ grows one can easily show that the usual inflationary slow-roll conditions are grossly violated. Thus, for a value of $\tilde{n}^2 \geq 2$ or equivalently $n \geq \sqrt{d(d+2)} + d$ the evolution of $a(t)$ during this phase will be non-inflationary.

Taking into account the bulk expansion during Phase 1 of (2.12), it follows that for $d = 6$ and $p = 3$ the initial value of φ is about $69 m_{pl}$. The exponential form of the potential will lead to a rapid collapse of the extra dimensions. To estimate the time scale of the decrease, we replace the source of the right hand side of (2.19) by its initial value and estimate the time interval Δt for φ to decrease by an amount m_{pl}. We find that this time interval equals the initial Hubble time. Thus, a rough estimate of the duration of Period 2 is $10^2 H^{-1}$.

2.5 Modulus Stabilization and Late Time Cosmology

The next crucial step in our scenario is to invoke a mechanism to stabilize the radius of the extra dimensions at a fixed radius. Such modulus stabilization mechanisms have recently been extensively studied both in the context of string theory models of inflation (see e.g. [33] for recent reviews) and in string gas cosmology [90]. We will make use of the mechanism developed in the latter approach.

String modes which carry momentum about the extra dimensions will generate an effective potential for the radion which is repulsive. These repulsive effects will dominate for values of the radion smaller than the self-dual radius. Since these modes are very light at large values of the radion, it is likely that they will be present in great abundance. Even if they are not, the subset of such modes which are massless at enhanced symmetry points will be copiously produced when the value of the radion approaches such points [40, 39] [6]. The induced potential will lead to a source term in the equation of motion for the scale factor $b(t)$ which is of the form [38, 47]

$$\ddot{b} + 3H\dot{b} \, = \, 8\pi G n(t) \big[\big(\frac{1}{b}\big)^2 - b^2 \big] + \dots , \tag{2.23}$$

where the dots indicate extra source terms from other string modes, as well as terms quadratic in $\dot{b}$. Note that $n(t)$ is given by the number density of the modes. Translating to the scalar field φ, and neglecting terms quadratic in $\dot{\varphi}$, the above equation

[6] As discussed in [47], stabilization via string modes which are massless at the self-dual radius leads to a consistent late time cosmology.

becomes

$$
\begin{aligned}
&\ddot{\varphi} + 3H\dot{\varphi} \; = \qquad (2.24)\\
&8\pi\, G\, n(t)\, e^{-\sqrt{2/(d(d+2))}\varphi/m_{pl}} \sqrt{d(d+2)/2}\, m_{pl} \big(e^{-\sqrt{8/(d(d+2))}\varphi/m_{pl}} - e^{\sqrt{8/(d(d+2))}\varphi/m_{pl}}\big)\,.
\end{aligned}
$$

Thus, it follows that after approaching the self-dual radius, $b(t)$ will perform damped oscillations about $b(t) = 1$, or, in other words, $\varphi(t)$ will undergo damped oscillations about and get trapped at $\varphi = 0$ (which corresponds to string scale separation between the orbifold fixed planes). At this separation, the four dimensional effective potential V_{eff} becomes

$$
V_{eff} \; = \; \Lambda^{d+4+n} l_s^{n+d}\,, \qquad (2.25)
$$

and, taking upper limit on Λ from (2.15), this becomes

$$
V_{eff} \; = \; l_s^{-4} 10^{-60(d-p+3+n)/(d+2)}\,. \qquad (2.26)
$$

Thus, starting with vanishing cosmological constant in the bare bulk Lagrangian, our scenario accidently generates a cosmological constant energy density in our present universe which is suppressed by $60 \times (d-p+3+n)/(d+2)$ orders of magnitude. This will provide the correct order of the cosmological constant to account for the current acceleration if $d = n - p - 1$.

Either at some point during the phase of contraction, or else when the distance between the orbifold fixed planes has decreased to the string scale, all of the bulk branes will decay, presumably predominantly into radiation along the fixed plane directions. The three unconfined spatial dimensions will thus emerge in the expanding radiation-dominated phase of standard cosmology. The energy density which at late time governs the dynamics of our scale factor $a(t)$ is the bulk energy density integrated over the transverse dimensions. Since the bulk energy in Phase 1 is dominated by the $p = 3$ branes, the integrated energy density is constant. Thus, at the beginning of the radiation-dominated phase the effective energy density is of the same order of magnitude as the initial bulk energy density, namely given by the string scale.

From the point of view of late time cosmology, what has been achieved during Phase 1 is to increase the size of our spatial sections without decreasing the effective energy density. Without extra spatial dimensions, the energy density can only remain constant if the expansion of space is inflationary, but making use of the dynamics of extra spatial dimensions, constant effective energy density can be achieved using non-accelerated expansion of all dimensions.

Note that in the case of $p > 3$, specifically in the cases where we use Neveu-Schwarz 5-branes in the bulk, the projected energy density actually increases in Phase 1. If it decreases less during Phase 2 than it increased during Phase 1 (which will be the case e.g. if the branes convert to radiation during Phase 2), then the possibility emerges that we are driven to a Hagedorn phase of string theory towards the end of Phase 2 [77, 91]. In this case, a very nice mechanism for the generation of a scale-invariant spectrum of fluctuations [80] can be realized. This possibility will be briefly discussed in the next section.

There is another key prediction of our model which is closely related to the chosen topology of space. No odd-dimensional cycles exist on the inner space T^6/Z_2, thus prohibiting certain stable configurations of p-branes. Given that we are using odd-dimensional branes in our examples, only 1 or 3 brane dimensions can wrap our three-dimensional toroidal space T^3, because no odd-dimensional stable p-branes can have an odd number of their brane dimensions wrapped about the inner space. This prevents the creation of stable "stringy" domain walls and monopoles in our universe, but it may predict the existence and future detection of cosmic strings.

2.6 Discussion and Conclusions

By making use of some tools coming from string theory, we have proposed a mechanism to solve the entropy (size) problem of standard cosmology without inflation. According to our proposal, the universe begins hot, small and dense. We assume that the six extra spatial dimensions of perturbative superstring theory are orbifolded, the three dimensions we see today are not (they are toroidal). The universe emerges with a gas of bulk branes (e.g. three branes if we have the perturbative limit of Type IIB superstring theory in mind or 5-branes if we start from heterotic string theory or

M-theory) which drives an initial phase of isotropic bulk expansion of all nine spatial dimensions. During this phase, the energy density projected onto the orbifold fixed planes does not decrease, even though the scale factor is expanding (as $t^{1/3}$ in the case of 3-branes in six extra dimensions). We assume the presence of a weak confining potential between the orbifold fixed planes (the cosmological scenario which emerges when considering a more conventional type of potential will be discussed in the next chapter). Such a potential will eventually dominate over the bulk energy density and will lead to a second phase in which the extra spatial dimensions rapidly contract while our three spatial dimensions continue to expand. Once the orbifold fixed planes approach each other to within the string scale, stringy effects previously studied in the context of string gas cosmology will stabilize the radion degrees of freedom. The bulk branes decay, and the universe emerges into the radiation-dominated phase of standard cosmology, with a temperature which is of string scale, but a size which is many orders of magnitude larger than what would be expected on dimensional arguments [7].

Since the initial spatial section is in thermal contact, the horizon problem of standard cosmology is explained, as well. Our scenario, however, does not solve the flatness problem of standard cosmology. If the initial spatial sections are curved, then the curvature will lead to a re-collapse of the universe. One way to address the flatness problem is to invoke a special symmetry such as the BPS symmetry (see e.g. [93] for a textbook discussion) which prohibits spatial curvature.

In order to provide an alternative to inflation in terms of solving all of the cosmological problems of standard cosmology which inflation addresses, we need to find a mechanism for generating fluctuations. Work on this topic is in progress. Since the universe is initially in causal contact, there are no causality arguments which prevent the generation of adiabatic fluctuations. It is possible that bulk fluctuations similar to

[7] Note that our proposal has certain similarities with the approach of [92], in which - in the context of brane world cosmology - it was proposed that the decay of Kaluza-Klein bulk modes will lead to an entropy flow from the bulk to the brane which can solve the entropy and homogeneity problem of standard cosmology without requiring a phase of inflationary expansion.

the ones proposed in the Ekpyrotic scenario could play this role. Provided there are scale-invariant fluctuations in bulk metric variables during the contracting phase, the work of [94] (see also [95]) shows that such fluctuations will induce a scale-invariant spectrum of four dimensional metric fluctuations in the radiation-dominated phase. Another possibility, in particular in the context of branes with spatial dimension larger than three, is that the post-collapse phase will lead to such high densities that a quasi-static Hagedorn phase will result. The Hagedorn phase makes a smooth transition to the radiation-dominated phase of standard cosmology. In this case, string thermodynamics automatically generates a scale-invariant spectrum of adiabatic fluctuations on all scales smaller than the Hubble radius during the quasi-static phase [80]. (See [81, 82] for problematic points of this scenario.)

Chapter 3

Brane Gas-Driven Bulk Expansion as a Precursor Stage to Brane Inflation

We propose a new way of obtaining slow-roll inflation in the context of higher dimensional models motivated by string and M theory. In our model, all extra spatial dimensions are orbifolded. The initial conditions are taken to be a hot dense bulk brane gas which drives an initial phase of isotropic bulk expansion. This phase ends when a weak potential between the orbifold fixed planes begins to dominate. For a wide class of potentials, a period during which the bulk dimensions decrease sufficiently slowly to lead to slow-roll inflation of the three dimensions parallel to the orbifold fixed planes will result. Once the separation between the orbifold fixed planes becomes of the string scale, a repulsive potential due to string effects takes over and leads to a stabilization of the radion modes. The conversion of bulk branes into radiation during the phase of bulk contraction leads to reheating.

3.1 Introduction

The Inflationary Universe scenario [18] (see also [66, 67, 68]) has been extremely successful phenomenologically. It has provided a solution to some of the key problems of standard cosmology, namely the horizon and flatness problems, and yielded a mechanism for producing primordial cosmological perturbations using causal physics, a mechanism which predicted [20, 19] (see also [69, 66]) an almost scale-invariant spectrum of adiabatic cosmological fluctuations, a prediction confirmed more than a decade later to high precision by cosmic microwave background anisotropy experiments [10, 11, 12, 13].

However, it has proven difficult to find convincing realizations of inflation in the context of quantum field theory models of matter in four space-time dimensions. It is usually assumed that the quasi-constant potential energy of a slowly rolling scalar field (the so-called "inflaton") leads to the accelerated expansion which inflation requires. The Standard Model of particle physics, however, does not contain a scalar field whose dynamics leads to slow-rolling. In single field models with a renormalizable potential, field values larger than m_{pl} (the four-dimensional Planck mass) are required in order to obtain slow-rolling as a local attractor in the phase space of homogeneous solutions to the scalar field equations of motion [96].

Superstring theory and M-theory, on the other hand, contain a lot of degrees of freedom which at the level of the four space-time dimensional effective field theory are described by scalar fields. Supersymmetry ensures that some of these fields (the so-called "moduli fields" are sufficiently weakly coupled to provide potential candidates to be an inflaton.

In the context of brane world cosmology [97, 98], an appealing possibility is that the separation between a brane and an antibrane [25, 26] can serve as the inflaton. A problem with the proposed constructions, which were all in the context of a static bulk, was that the bulk size was generically too small to allow for the large values of the inflaton field required to generate inflation. This problem was addressed in [99, 100]. Another possibility is to have topological brane inflation [101], but this construction also requires special parameters in order to obtain a wide enough brane.

The constructions mentioned in the previous paragraph were all done in the context of phenomenological field theoretical models inspired by string theory. After the discovery that flux constructions can lead to a stabilization mechanism for most moduli fields of string theory [29, 30], a lot of attention (beginning with [31, 32]) was focused on how to obtain inflationary models in the context of flux compactifications (see [33] for reviews and comprehensive lists of references). These constructions are, once again, in the context of static bulk configurations, and have to assume very special configurations (special configurations of branes and special flux choices).

In this chapter, we present a new model of brane inflation. In contrast to previous constructions, the dynamics of the bulk dimensions is essential to our model. Also, in contrast to previous constructions, we start with initial conditions which we consider to be very natural, namely a hot brane gas in the context of an initial universe in which all spatial dimensions are democratically small (of the string scale), similar to what is assumed in "string gas cosmology" [77] and its brane generalizations [78, 83, 84, 85]). The hot brane gas leads to an initial phase of isotropic bulk expansion (Phase 1 of our cosmology). During this phase, the bulk energy density decreases.

The isotropy of space is explicitly broken by our assumption that the extra spatial dimensions are orbifolded. This leads to the existence of orbifold fixed planes. We assume the existence of a weak attractive potential between the orbifold fixed planes [1]. Eventually, the associated potential energy will begin to dominate the dynamics and will lead to a contraction of the dimensions perpendicular to the orbifold fixed planes (Phase 2). We will consider a potential of the form

$$V(r) \,=\, -\mu\frac{1}{r^n}\,, \tag{3.1}$$

where r is the separation of the orbifolds, and n is an exponent which we will fix later. Such a potential could emerge from charges on branes pinned to the orbifold fixed planes. We will show that such a potential can lead to slow-roll inflation. The inflationary slow-roll parameters are set by the coefficient $\mu \equiv \Lambda^{4+d-n}$ (where Λ has dimensions of energy) which characterizes the strength of the potential. The requirement of a sufficient number of e-foldings to solve the cosmological problems of standard cosmology [18] sets an upper bound on Λ.

[1]Note that in terms of having inflation driven by the potential between orbifold fixed planes, our setup is similar to that of [100].

Once r decreases to the string scale, a repulsive potential created by stringy effects will take over. The competition of the repulsive short range force and the attractive long range force, together with the continued expansion of space parallel to the orbifold fixed planes, will lead to a stabilization of r. This stabilization scenario is an application of the mechanism of radion stabilization which has recently been studied extensively in the context of string gas cosmology [38, 47, 37] (see [90] for a short review). Either during Phase 2, or once the separation of the orbifold fixed planes has decreased to the string scale, the bulk branes annihilate and decay into radiation. This leads to a smooth transition into the radiation phase of standard cosmology.

Note that a very similar setup was used to construct a non-inflationary solution to the entropy and horizon problems of standard cosmology. In Chapter 2, we assumed that the inter-brane potential was confining, a potential of the type that could be generated by non-perturbative effects. Here, we take the potential (3.1) which could come from string exchange between branes [102].

3.2 The Model

Our starting point is a topology of space in which all but three spatial dimensions are orbifolded, and the three dimensions corresponding to our presently observed space are toroidal. Specifically, the space-time manifold is

$$\mathcal{M} \,=\, \mathcal{R} \times T^3 \times T^d/Z_2\,, \tag{3.2}$$

where T^3 stands for the three-dimensional torus, and d is the number of extra spatial dimensions, which we will take to be either $d = 6$ in the case of models coming from superstring theory, or $d = 7$ in the case of models motivated by M-theory. We will assume that there is a weak force between the orbifold fixed planes given by the potential (3.1) [2].

[2]It may be necessary to have branes pinned to the orbifold fixed planes in order to induce such a potential. Our approach, at this stage, is purely phenomenological, and we simply postulate the existence of a potential with the required properties

As our initial conditions, we take the bulk to be filled with a gas of p-branes, as in Chapter 2. In the context of Type IIB superstring theory we will have D-branes with $p = 3$, in the case of heterotic string theory or taking the starting point to be M-theory, we have Neveu-Schwarz 5-branes ($p = 5$).

Assuming that the universe starts out small and hot, it is reasonable to assume that the energy density in the brane gas will initially be many orders of magnitude larger than the potential energy density generated by the force between the orbifold fixed planes. Thus, initially our universe will be expanding isotropically. As shown in Chapter 2, this expansion is non-inflationary. During this phase (Phase 1), the bulk energy density will decrease. Hence, eventually the inter-orbifold potential will begin to dominate. At this point, the cosmological evolution will cease to be isotropic: the directions parallel to the orbifold fixed planes will continue to expand while the perpendicular dimensions begin to contract: this marks the beginning of Phase 2. In the following we will show that for a wide class of potentials, the expansion of our dimensions will be inflationary.

The metric in the non-isotropic phase (and in the absence of spatial curvature) is given by

$$ds^2 \;=\; dt^2 - a(t)^2 d\mathbf{x}^2 - b(t)^2 d\mathbf{y}^2 \,, \tag{3.3}$$

where $\mathbf{x}$ denote the three coordinates parallel to the boundary planes and $\mathbf{y}$ denote the coordinates of the perpendicular directions. Hence, the radius r of the dimensions perpendicular to the orbifold fixed planes is given by $r(t) \;=\; l_s b(t)\,.$

We will analyse the evolution during Phase 2 using a four-dimensional effective field theory, where we replace the radion $b(t)$ by a scalar field $\varphi(t)$. In order that φ be canonically normalized when starting from the higher dimensional action of General Relativity, φ and b must be related via (see e.g. [28], Appendix A for a review)

$$\varphi \;=\; \sqrt{\frac{d(d+2)}{2}}\, m_{pl} \log(b)\,. \tag{3.4}$$

If the bulk size starts out at the string scale, then $b(t_b) \;=\; 1$, where t_b is the initial time. With these normalizations, $\varphi = 0$ corresponds to string separation between the branes. The dimensional reduction of the higher dimensional gravitational action to the four space-time dimensional Einstein frame action yields the following effective potential for φ [28]

$$V_{eff}(\varphi) \,=\, l_s^d b(\varphi)^{-d} V(r(\varphi))\,. \tag{3.5}$$

Note that the dilaton is assumed to be fixed, and the dilaton-dependence of the potential is neglected. From the potential (3.1) and inserting the relation (3.4) we obtain

$$\begin{aligned} V_{eff}(\varphi) &= -\Lambda^{4+d-n} l_s^{d-n} e^{-\frac{\sqrt{2}(d+n)}{\sqrt{d(d+2)}}\frac{\varphi}{m_{pl}}} \\ &= -\Lambda^{4+d-n} l_s^{d-n} e^{-\tilde{\alpha}\frac{\varphi}{m_{pl}}}\,, \end{aligned} \tag{3.6}$$

where we have defined a constant $\tilde{\alpha} \equiv \sqrt{2}(d+n)/\sqrt{d(d+2)}$.

To ensure vanishing of the four-dimensional cosmological constant today, we must add a positive constant V_0 to the effective potential (3.6). If the stabilization radius of the extra dimensions is the string scale l_s, then V_0 is given by

$$V_0 \,=\, \Lambda^{4+d-n} l_s^{d-n}\,. \tag{3.7}$$

From the form of the potential, it should be expected that a period of slow-roll inflation is possible as long as the initial value of φ at the beginning of Phase 2 is larger than m_{pl}. The special feature of our scenario (and the major advantage compared to previous versions of brane inflation), is that such large values of φ dynamically emerge and do not have to be inserted as ad hoc initial conditions.

In our scenario, inflation has a graceful exit. Once the orbifold fixed planes reach a microscopic separation, Kaluza-Klein momentum modes of strings (e.g. the momenta of the massless states produced at enhanced symmetry points) produce a repulsive potential which scales as b^{-2} [40, 39] and hence on short distances overwhelms the large-distance attractive potential (provided $n < 2$). The interplay between this repulsive potential which dominates at small separations and the attractive potential which dominates at large distances, coupled to the expansion of the three dimensions parallel to the orbifold fixed planes, will lead to the stabilization of the radion modes at a specific radius (presumably related to the string scale). In the context of heterotic string theory, we could use the string states which are massless at the self-dual radius to obtain stabilization of the radion modes at the self-dual radius [38, 47] (see also [37]). These modes would also ensure dynamical shape moduli stabilization [89]. The branes decay into radiation either during or at the end of Phase 2. This brane decay is the main source of reheating of our three dimensional space. We denote the time of radion stabilization and reheating by t_R. After reheating, our three spatial dimensions emerge in the radiation phase of standard cosmology.

3.3 The Phase of Bulk Expansion

The phase of isotropic bulk expansion ($a(t) = b(t)$) proceeds as discussed in Chapter 2. The equation of motion for $a(t)$ is

$$\frac{\ddot{a}}{a} + (2+d)\Big(\frac{\dot{a}}{a}\Big)^2 \;=\; \frac{8\pi G}{3+d-1}\big[\rho - P\big]\,, \tag{3.8}$$

where ρ is the energy density and P denotes the pressure. Making use of the equation of state $P = w\rho$, and inserting the Einstein constraint equation

$$\big((3+d)^2 - 3 - d\big)H^2 \;=\; 16\pi G\rho\,, \tag{3.9}$$

where $H \equiv \dot{a}/a$, we obtain power law expansion

$$a(t) \;\sim t^{\alpha} \;\text{ with }\; \alpha \;=\; \frac{2}{(3+d)(1+w)}\,. \tag{3.10}$$

In the case of bulk energy dominated by the tension of p-branes, we have

$$w \;=\; -\frac{p}{3+d}\,. \tag{3.11}$$

Thus, in the example motivated by perturbative Type IIB superstring theory, ($d = 6$ and $p = 3$) we obtain $\alpha = 1/3$. Starting with heterotic string theory ($d = 6$ and $p = 5$) we obtain $\alpha = 1/2$, and for M-theory ($d = 7$ and $p = 5$) we get $\alpha = 2/5$. Thus, the phase of bulk expansion is non-inflationary.

3.4 The Period of Inflation

The period of bulk expansion ends when the bulk potential energy becomes equal to the bulk brane energy density. The bulk energy density in Phase 1 scales as

$$\rho_b(t) \sim a(t)^{-d-3+p} \tag{3.12}$$

Assuming that the initial bulk energy density is given by the string scale, i.e.

$$\rho_b(t_b) \sim l_s^{-4-d} \tag{3.13}$$

where t_b denotes the initial time, then the transition between Phase 1 and Phase 2 takes place at a time t_i given by

$$b(t_i)^{-(d+3-p-n)} = \left(\Lambda l_s\right)^{d+4-n} . \tag{3.14}$$

The value of the radion φ at this time is given by

$$\varphi(t_i) = \sqrt{\frac{d(d+2)}{2}} m_{pl} \log(b(t_i)) . \tag{3.15}$$

Since φ is canonically normalized, its equation of motion is given by (see the form of the effective potential of (3.6))

$$\ddot{\varphi} + 3H\dot{\varphi} = -\Lambda^{4+d-n} l_s^{d-n} \frac{\tilde{\alpha}}{m_{pl}} e^{-\tilde{\alpha}\varphi/m_{pl}} . \tag{3.16}$$

The slow-roll conditions are satisfied provided:

$$\varphi \gg \frac{m_{pl}}{\tilde{\alpha}} \log\left\{1 + \tilde{\alpha}^2\right\} . \tag{3.17}$$

Thus, to get N efolding of inflation, the initial value of φ should exceed the bound

$$\varphi(t_i) > \frac{m_{pl}}{\tilde{\alpha}} \log\left\{\tilde{\alpha}^2(N+1) + 1\right\} \tag{3.18}$$

leading to the condition

$$l_s\Lambda < \left[\tilde{\alpha}^2(N+1) + 1\right]^{-\frac{d+3-p-n}{(d+n)(d+4-n)}} \tag{3.19}$$

which allows Λ of order of the string scale.

We conclude that, provided the bound (3.19) on the energy scale Λ is satisfied, Phase 2 will provide a sufficient length of inflation of our three spatial dimensions, inflation driven by the slow rolling of the modulus field.

3.5 Discussion and Conclusions

In this paper we have proposed a new way of obtaining inflation in the context of theories with extra dimensions and branes. We assume that our three spatial dimensions are singled out by the orbifold construction of (3.2), and that there is a weak potential between branes pinned to the orbifold fixed planes. We assume attractive potentials such as could emerge if opposite charges were localized on the two branes.

In our scenario, the universe begins small and hot, filled with an isotropic gas of branes. This brane gas drives a period of isotropic bulk inflation which continues until the potential between the branes localized on the orbifold fixed planes becomes dominant. We have shown that the potential of the radion supports a period of slow-roll inflation. The new feature of our model compared to other models of brane inflation is that the large values of the radion required to obtain sufficient inflation are dynamically generated during the phase of bulk expansion.

Inflation ends when the radion shrinks to string-scale values, the bulk branes annihilate into radiation, and the radion becomes stabilized by string gas effects.

An interesting lesson obtained by comparing our present results with those of the preceding Chapter is that the details of the potential between the orbifold fixed planes is very important in determining the evolution of our three spatial dimensions. For a sufficiently confining potential, our three spatial dimensions never undergo a period of accelerated expansion - but the period of bulk expansion still enables us to solve the horizon and entropy problems because the energy density of the brane gas projected onto the orbifold fixed planes does not decrease. The condition on the power n appearing in the potential (3.1) in order to obtain inflation is $n < d + 3 - p$. This limit mirrors the requirement that orbifold fixed plane potential is diluted slower than energy density of p-branes in the bulk.

Chapter 4

Predictions of Dynamically Emerging Brane Inflation Models

We confront the inflationary proposal of Chapter3 with WMAP3+SDSS, finding a scalar spectral index of $n_s = 0.9659^{+0.0049}_{-0.0052}$ in excellent agreement with observations. The proposal incorporates a preceding phase of isotropic, non accelerated expansion in all dimensions, providing suitable initial conditions for inflation. Additional observational constraints on the parameters of the model provide an estimate of the string scale.

A graceful exit to inflation and stabilization of extra dimensions is achieved via a string gas. The resulting pre-heating phase shows some novel features due to a redshifting potential, comparable to effects due to the expansion of the universe itself. However, the model at hand suffers from either a potential over-production of relics after inflation or insufficient stabilization at late times.

4.1 Introduction

Inflation provides a natural explanation for major problems of standard cosmology such as the homogeneity, horizon and flatness problems [18]. An almost scale-invariant spectrum of adiabatic cosmological fluctuations was predicted [19, 20] more than a decade before the cosmic microwave background anisotropies were analyzed [13, 12, 11, 10]. Encouraged by the great success of the inflationary paradigm, one is urged to find a successful realization of inflation within more fundamental theories, such as string theory.

The heuristic approach of obtaining inflation consists of introducing one or more scalar fields (inflatons), which evolve slowly due to some appropriately tuned potential. Simple single field models require the inflaton to start out at a value larger than the Planck mass. However, at these values radiative corrections to the inflaton mass threaten to spoil slow roll dynamics. Therefore, unless some underlying symmetry protects the inflaton mass, it is hard to implement an inflationary scenario within gauge theories. Moreover, in the framework of four dimensional inflationary models the physical interpretation of the inflaton is unclear. Generally, it is taken to be a singlet of the Standard Model and typically of all of the visible sector too. (See [103] for a recently proposed exception.)

The advent of extra dimensions [97, 98] opened up a venue for new inflationary scenarios where the inflaton has a physical meaning; for example, in brane-antibrane inflationary models the inter-brane separation serves as the inflaton [25, 26]. However, to provide a sufficient amount of inflation in brane-antibrane models, one requires fine tuning of initial conditions [99, 100], e.g. large inter brane separation, special configurations or very weak couplings.

In this chapter, we would like to continue discussions of the proposal of emerging brane inflation suggested in Chapter 3. The proposal makes use of extra dimensions, a gas of p-branes in the bulk to drive an initial isotropic but non accelerated expansion of the universe, as well as orbifold fixed planes responsible for contraction of the extra dimensions and inflation of our three dimensions. The presence of a string gas at the end of inflation provides a graceful exit, pre-heating and stabilization of extra dimensions. The model at hand does not need any fine tuning of initial conditions and will turn out to be in good agreement with observations.

In this scenario the multidimensional universe starts out small and hot, with our three dimensions compactified on a torus and the extra dimensions on an orbifold of the same size. The pre-inflationary expansion is governed by topological defects (p-branes) in the bulk and responsible for a large inter brane separation. As the universe expands isotropically due to the gas of p-branes, the energy density stored in the gas gets diluted until additional weak forces come into play, changing the overall dynamics. For example, branes pinned to orbifold fixed planes, which exhibit an attractive force, may eventually cause a contraction of the extra dimensions while our dimensions inflate. From the four dimensional point of view, the inflaton is identified with the radion and consequently, the pre-inflationary bulk expansion explains the large initial value of the inflaton. Inflation comes to an end when the extra dimensions shrink down to a small scale where moduli trapping [37, 38, 39, 40] and pre-heating [41, 42, 43, 44, 45, 46] can occur.

Our main goal in this chapter is to examine the viability of the emerging brane inflation model outlined above and to make contact with observations.

The outline of this chapter is as follows: In section 4.2 we review the details of the model, followed by a computation (section 4.3) within the slow roll approximation of the spectral index n_s, the running of the index $d\,n_s/(d\ln k)$, the scalar to tensor ratio r and the tensor index n_T. We confront our predictions with the observation of the cosmic microwave background radiation measured by the Wilkinson Microwave Anisotropy Probe (WMAP3) [104, 105, 106, 107] and the Sloan Digital Sky Survey (SDSS) [108], resulting in good agreement. In order to get sufficient initial expansion one requires the inter brane potential to remain subdominant for a long time compared to the energy density stored in the bulk p-branes. This requirement imposes constraints on the scale of interactions between branes pinned to the orbifold fixed points. This, together with constraints from observational data, will be sufficient to provide an estimate of the string scale. In section 4.4, we study the viability of pre-heating after inflation and stabilization at late times. While pre-heating can occur in the standard manner, albeit some novel effects are present, we do find potential problems associated with either late time stabilization or relics: if the branes pinned to the orbifold fixed planes annihilate after inflation they could produce an over-abundance of relics such as cosmic strings, and if they do not annihilate they will destabilize the extra dimensions at late times. We conclude with a comment on open issues within the framework of emerging brane inflation.

4.2 The Model

Following Chapter 3, we assume a spacetime

$$\mathcal{M} = \mathcal{R} \times T^3 \times T^d/Z_2\,, \tag{4.1}$$

so that our three dimensions have the topology of a torus T^3, and the d extra dimensions are compactified on the orbifold T^d/Z_2. Note, that this specific choice of the manifold is not crucial for the model – many other manifolds distinguishing our three dimensions could be chosen instead. Next, we assume that pairs of branes are pinned to the different orbifold fixed planes a distance r apart. Furthermore, we also assume inter brane interactions such that a weak attractive force is generated via some potential V. It should be noted that we take a phenomenological approach and postulate the existence of a potential with the desired properties. A discussion of possible origins of inter-brane potentials can be found in [25].

The special feature of the underlying scenario is the pre-inflationary dynamics which explains the large size of the extra dimensions just before inflation. Initially, the universe starts out small and hot with all spatial dimensions of the same size. The bulk is filled with a gas of p-branes. In this phase, the energy density in the brane gas is assumed to be many orders of magnitude larger than the potential energy density, which provides the force between the orbifold fixed planes. Thus, the universe expands isotropically but not inflationary, as shown in Chapter 2. During the expansion phase the bulk energy density of the gas decreases and eventually the potential V begins to dominate, causing inflation of the directions parallel to the orbifold fixed planes, and contraction of the extra dimensions. It should be noted that whether or not inflation occurs is sensitive to the form of the inter-brane potential V.

Following Chapter 3, we consider a potential of the form

$$V(r) = -\mu \frac{1}{r^n}\,, \tag{4.2}$$

where r is the inter brane separation and $n > 0$ is a free parameter, which could in principle be computed from the underlying fundamental theory. As we shall see below, this form of the potential results in inflation.

We shall first compute how dynamics can be described in a four dimensional effective theory. Let G_{ab} be the metric for the full space-time with coordinates X^a. In the absence of spatial curvature, the metric of a maximally symmetric space which distinguishes 'our' three dimensions is given by

$$ds^2 = G_{ab}dX^a dX^b \; = \; dt'^2 - \alpha(t')^2 d\mathbf{x}^2 - b(t')^2 d\mathbf{y}^2 \,, \tag{4.3}$$

where $\mathbf{x}$ denotes the three coordinates parallel to the orbifold fixed planes and $\mathbf{y}$ denotes the coordinates of the d perpendicular directions.

Our goal is to find a four-dimensional effective potential which governs the inflationary phase. We start out with the higher dimensional action

$$S = \int d^{d+4}X \sqrt{-\det G_{ab}} \left\{ \frac{1}{16\pi G_{d+4}} R_{d+4} + \hat{\mathcal{L}}_M \right\} \,, \tag{4.4}$$

where R_{d+4} is the $d+4$ dimensional Ricci scalar and $\hat{\mathcal{L}}_M$ is the matter Lagrangian density with the metric determinant factored out. Note that the dilaton is assumed to be fixed already, e.g. via the proposal of [109]. In the effective four-dimensional action, the radion $b(t)$ is replaced by a canonically normalized scalar field $\varphi(t)$ which is related to $b(t)$ through [28, 110]

$$\varphi = \beta^{-1} m_{pl} \ln(b) \,, \tag{4.5}$$

where

$$m_{pl}^2 = \frac{1}{8\pi G_4} \tag{4.6}$$

is the reduced four dimensional Planck mass and we defined

$$\beta^{-1} := \sqrt{\frac{d(d+2)}{2}} \,. \tag{4.7}$$

After performing a dimensional reduction and a conformal transformation to arrive at the Einstein frame [28, 110] we are left with

$$\begin{aligned} S = \int d^4x a^3 \left\{ \frac{1}{2} m_{pl} R_4 \right. & - \; \frac{1}{2}(\partial\varphi)^2 \\ & \left. + \; \mathcal{V} e^{-d\varphi/m_{pl}\beta} \hat{\mathcal{L}}_M \right\} \,, \end{aligned} \tag{4.8}$$

where

$$\mathcal{V} = \int d^d\mathbf{y} = l_s^d \tag{4.9}$$

is the volume of the extra dimensions, and

$$\begin{aligned} ds_E^2 &= b^d(dt'^2 - \alpha^2 d\mathbf{x}^2) \\ &= dt^2 - a(t)^2 d\mathbf{x}^2 \end{aligned} \tag{4.10}$$

is the effective four dimension metric. Further, assuming that the initial separation between the orbifold fixed planes is of string length l_s one can compute the distance between the orbifold fixed planes to

$$r(t') = l_s b(t') \,. \tag{4.11}$$

Note that $b = 1$ corresponds to the string scale. Setting $\hat{\mathcal{L}}_M = V$ yields

$$\begin{aligned} V_{4d}(\varphi) &= g_s^4 l_s^d b(\varphi)^{-d} V(r(\varphi)) \\ &= -\mu g_s^4 l_s^{d-n} e^{-\tilde{\alpha}\frac{\varphi}{m_{pl}}} \,, \end{aligned} \tag{4.12}$$

where we used (4.2), restored the string coupling dependence and defined

$$\tilde{\alpha} := (n+d)\beta \,. \tag{4.13}$$

To account for the brane tension/zero cosmological constant today we add a positive constant V_0 to the effective potential (4.12) and arrive at the effective four dimensional potential

$$\begin{aligned} V_{eff}(\varphi) &= V_0 - \mu g_s^4 l_s^{d-n} e^{-\tilde{\alpha}\frac{\varphi}{m_{pl}}} \\ &= V_0(1 - \zeta e^{-\tilde{\alpha}/m_p \varphi}) \end{aligned} \tag{4.14}$$

where we defined

$$\zeta := \frac{\mu g_s^4 l_s^{d-n}}{V_0} \,. \tag{4.15}$$

This potential yields inflation for large enough values of the radion/inflaton φ. Similar potentials have been considered before, see e.g. [35] in the context of brane inflation or [111] in the context of supergravity. These proposals differ from ours in the form of the graceful exit, the details of pre-heating as well as the the pre-inflationary dynamics of our model, which pull φ far from its minimum such as to provide suitable initial conditions for inflation without fine tuning.

4.3 Predictions

Inflation gives rise to a viable mechanism of structure formation: quantum vacuum fluctuations, present during inflation on microscopic scales, exit the Hubble radius and are subsequently squeezed, resulting in classical perturbations at late times, see e.g. [112]. Moreover, current observations are precise enough to distinguish between different inflationary models [104, 105, 106, 107, 108, 21, 113].

In the following, we derive observable quantities within the slow roll approximation and make contact with observations. Thereafter, we show how one can estimate the string scale in the model at hand.

4.3.1 Cosmological Parameters

The equation of motion for a scalar field in an expanding universe is given by

$$\ddot{\varphi} + 3H\dot{\varphi} + V_I' = 0\,, \tag{4.16}$$

where $H = \dot{a}/a$ is the Hubble parameter, $V_I := V_{eff}$ from (4.14) is the inflaton potential and $V_I' := dV_I/d\phi$. If the scalar field φ governs the evolution of the Universe, the Friedmann Robertson Walker equations become

$$H^2 = \frac{1}{3m_{pl}^2}\left[\frac{1}{2}\dot{\varphi}^2 + V_I(\varphi)\right]\,, \tag{4.17}$$

$$\left(\frac{\ddot{a}}{a}\right) = \frac{1}{3m_{pl}^2}\left[V_I(\varphi) - \dot{\varphi}^2\right]\,. \tag{4.18}$$

If the potential energy of the inflaton dominates over the kinetic energy, accelerated expansion of the universe results. In other words, if the potential is flat enough to allow for slow roll of the inflaton field, inflation occurs. In this case (4.16) and (4.17) become

$$3H\dot{\varphi} = -V_I'\,, \tag{4.19}$$

$$H^2 = \frac{V_I}{3m_{pl}^2}\,, \tag{4.20}$$

where we assumed $\dot{\varphi}^2 \ll V_I$ and $\ddot{\varphi} \ll 3H\dot{\varphi}$.

This approximation is valid if both the slope and the curvature of the potential are small, that is if the slow roll parameters

$$\varepsilon = \frac{m_{pl}^2}{2}\left(\frac{V_I'}{V_I}\right)^2, \tag{4.21}$$

$$\eta = m_{pl}^2\frac{V_I''}{V}. \tag{4.22}$$

satisfy $\varepsilon \ll 1$ and $\eta \ll 1$. Let φ_i denote the value of the inflaton field N e-folds before the end of inflation. This value can be determined from

$$\begin{aligned} N &= \int_{t_i}^{t_f} H(t)dt \\ &\approx \frac{1}{m_{pl}^2}\int_{\varphi_f}^{\varphi_i}\frac{V}{V'}d\varphi\,, \end{aligned} \tag{4.23}$$

where φ_f is the value of the inflaton field at which the slow roll approximation breaks down.

Within the slow roll regime one can then compute the scalar spectral index, the scalar to tensor ratio and the tensor spectral index to [21]

$$n_s \approx 1 - 6\varepsilon + 2\eta\,, \tag{4.24}$$

$$r \approx 16\epsilon\,, \tag{4.25}$$

$$n_T \approx -r/8\,, \tag{4.26}$$

where ϵ and η have to be evaluated at φ_i. For the potential (4.14) the slow roll parameters become

$$\varepsilon = \frac{\tilde{\alpha}^2}{2}\frac{1}{(\zeta^{-1}e^{\tilde{\alpha}/m_{pl}\,\varphi} - 1)^2}, \tag{4.27}$$

$$\eta = -\tilde{\alpha}^2\frac{1}{\zeta^{-1}e^{\tilde{\alpha}/m_p\,\varphi} - 1}. \tag{4.28}$$

Since $|\eta| > \varepsilon$ in our case, inflation ends once $|\eta| = \mathcal{O}(1)$, that is once φ approaches

$$\varphi_f = \frac{m_{pl}}{\tilde{\alpha}}\ln\left((\tilde{\alpha}^2+1)\zeta\right). \tag{4.29}$$

By using V_I from (4.14) in (4.23) we can compute the required initial value of the inflaton by solving

$$N = \frac{e^{\tilde{\alpha}/m_{pl}\,\varphi_i} - e^{\tilde{\alpha}/m_{pl}\,\varphi_f}}{\tilde{\alpha}^2\,\zeta} + \frac{(\varphi_f - \varphi_i)}{\tilde{\alpha}\,m_{pl}}, \tag{4.30}$$

for φ_i, which can be done analytically. Neglecting $\mathcal{O}(1)$ terms in (4.29) gives $\varphi_f \approx \frac{m_{pl}}{\tilde{\alpha}} \ln(\tilde{\alpha}^2\zeta)$ which in turn leads to

$$N \approx \frac{1}{\tilde{\alpha}^2}\left(e^{\tilde{\alpha}/m_{pl}\,\varphi} - \tilde{\alpha}^2\right), \tag{4.31}$$

after neglecting the second term in (4.30). This expression can be solved to

$$\varphi_i \approx \frac{m_{pl}}{\tilde{\alpha}} \ln(\tilde{\alpha}^2\zeta N + \tilde{\alpha}^2\zeta). \tag{4.32}$$

The slow roll parameters (4.21) and (4.22) evaluated at φ_i can now be approximated by

$$\epsilon \approx \frac{1}{2\tilde{\alpha}^2 (N+1)^2} \sim \mathcal{O}\left(\frac{1}{(N\tilde{\alpha})^2}\right), \tag{4.33}$$

$$\eta \approx -\frac{1}{N+1} \sim \mathcal{O}\left(\frac{1}{N}\right). \tag{4.34}$$

Henceforth, the scalar spectral index becomes

$$n_s \approx 1 - \frac{2}{N+1} \tag{4.35}$$

$$\approx 1 - \frac{2}{N}, \tag{4.36}$$

whereas the scalar to tensor ratio and the tensor spectral index read

$$n_T = -\frac{r}{8} \approx -\frac{1}{\tilde{\alpha}^2 (N+1)^2} \tag{4.37}$$

$$\approx -\frac{1}{\tilde{\alpha}^2 N^2}. \tag{4.38}$$

In addition, the running of the scalar spectral index can be evaluated to

$$\frac{d\, n_s}{d\,\ln(k)} = -m_{pl}\frac{V_I'}{V_I}\frac{d\, n_s}{d\,\varphi} \tag{4.39}$$

$$\approx -\frac{2}{N^2}, \tag{4.40}$$

which is negligible.

Now, we can evaluate (4.24)-(4.26) after specifying some parameters: first, since our model is motivated by string theory and the dilaton is fixed, we have $d = 6$ extra dimensions. Second, as we shall see in section 4.4, stabilization of the extra dimensions after inflation at the string scale is possible. Lastly, we have to specify the exponent n in (4.14), which in turn determines $\tilde{\alpha}$. This last parameter will barely influence the scalar spectral index but has some effect on the scalar to tensor ratio and the tensor spectral index. If we take $n = 4$ [1] and $N = 54 \pm 7$ we get

$$n_s = 0.9659^{+0.0049}_{-0.0052}\,, \tag{4.41}$$

$$r = \left(6.1^{+1.9}_{-1.3}\right) \times 10^{-4}\,, \tag{4.42}$$

$$n_T = \left(-7.6^{+2.4}_{-1.6}\right) \times 10^{-5}\,, \tag{4.43}$$

where we used the more cumbersome exact analytic expressions within the slow roll regime. These predictions [2] can now be compared with observational data. To be specific, the combined observational data of WMAP3 [104, 105, 106, 107] and SDSS [108] was used by Kinney et.al. in [21]: the above predictions for n_s and r lie in the middle of the 1σ region in the case of negligible running (see Fig. 1 in [21]).

Hence, the model of emerging brane inflation presented in Chapter 3 passes this first observational test.

4.3.2 Estimate of the Fundamental String Length

In the proposed scenario, brane inflation emerges after the inflaton got pushed up its potential in the preceding bulk expansion phase. The inflaton is related to the scale factor of extra dimensions, b, through (4.5). Therefore, the requirement to obtain N e-foldings of inflation (4.32) leads to a constraint on the minimal value of b at the beginning of inflation,

$$b_i \geq (\tilde{\alpha}^2 N\zeta)^{\beta/\tilde{\alpha}}\,. \tag{4.44}$$

[1] We have $n = 4$ in our specific setup, since $n = d + 3 - p - 2$ is expected from the form of inter-brane attraction potential, $p = 3$ corresponds to a 3-brane on the orbifold fixed-planes, and $d = 6$ is number of dimensions in some types of string theories.

[2] The limit $p \to -\infty$ of [114] corresponds to an exponential potential like the one discussed in this article; however, no estimate of r and n_T, which depend on the exponent $\tilde{\alpha}$, were given, and the WMAP3 data set alone was used for comparison. This led Alabidi and Lyth to conclude that an exponential potential would be allowed at the 2σ level.

On the other hand, the preceding expansion phase sets an upper limit on the scale factor b [2]. The end of bulk expansion and the beginning of inflation is indicated by $V = \rho_b$ where ρ_b is the energy density of the brane gas. This yields the condition

$$\mu l_s^{-n} b^{-n} \;=\; l_s^{-d-4} b^{-d-3+p} \,, \tag{4.45}$$

where we assumed that the energy density stored in p-branes at the beginning of the bulk expansion phase is of the order of the string scale. Rearranging parameters leads to

$$b_i \leq (\mu l_s^{d+4-n})^{-1/(d+3-p-n)} \,. \tag{4.46}$$

The bound (4.46) relates the scale of the inter-brane potential $\Lambda = \mu^{1/(d+4-n)}$ to the scale factor of the extra dimensions. Therefore, the requirement of N e-folds set an upper bound on Λ

$$(\Lambda l_s)^{d+4-n} \leq (\tilde{\alpha}^2 N \zeta)^{-\beta/\tilde{\alpha}(d+3-p-n)} \,. \tag{4.47}$$

Next, we can use observational data to constrain the effective inflationary potential. To be specific, COBE data implies [22] for the scale of inflation

$$\left(\frac{V}{\varepsilon}\right)^{1/4} = 0.027 m_{pl} \,. \tag{4.48}$$

Evaluating this expression N e-folds before the end of inflation leads to

$$0.027 m_{pl} \;\simeq\; V_0(\sqrt{2}\tilde{\alpha}N)^{1/2} \tag{4.49}$$

$$=\; \frac{\mu l_s^{d-n} g_s^4 (\sqrt{2}\tilde{\alpha}N)^{1/2}}{\zeta} \tag{4.50}$$

$$\leq\; \frac{g_s^4 (\sqrt{2}\tilde{\alpha}N)^{1/2}}{\zeta\,(\tilde{\alpha}^2 N\zeta)^{(d+3-p-n)/(d+n)}}\, l_s^{-4} \,, \tag{4.51}$$

where we used (4.47) in the last expression. Substituting l_s^{-2} with $g_s^2 m_{pl}^2$, one eventually arrives at a constraint for the string coupling

$$g_s^8 \geq 0.027\, \frac{\zeta\,(\tilde{\alpha}^2 N\zeta)^{(d+3-p-n)/(d+n)}}{(\sqrt{2}\tilde{\alpha}N)^{1/2}} \,. \tag{4.52}$$

For $(\zeta, d, p, n, N) = (1, 6, 3, 4, 54)$ this expression reduces to

$$g_s \geq 0.53 \,. \tag{4.53}$$

In conclusion, we found in the model at hand that inflation of about 60 e-folds requires the string scale to be slightly below the Planck scale.

A word of caution might be in order here: we assumed the dilaton to be fixed throughout bulk expansion and inflation; but if the dilaton is rolling during these early stages, it will modify the above estimate. Hence, a better understanding of the dilatons stabilization mechanism is of great interest.

4.4 Stabilization and Pre-heating

We saw in the previous sections how brane inflation can emerge in a higher dimensional setup. The specific inflaton potential in the effective four dimensional description was given by (4.14)

$$V_I = V_0 \left(1 - e^{-\tilde{\alpha}\varphi}\right) , \tag{4.54}$$

where we set $m_p \equiv 1$ and fine tuned $\zeta = 1$. The inflaton is related to the radion via (4.5) where $\beta^{-2} = d(d+2)/2$ was introduced. Furthermore, we assumed an already stabilized dilaton, e.g. via the proposal of [109]. It should be noted that a free dilaton could potentially invalidate the predictions of the model at hand.

In the following we would like to address three questions: How does inflation end, how does the universe reheat and can the radion/inflaton be stabilized at late times?

4.4.1 Stabilization

Based on the idea of moduli stabilization at points of enhanced symmetry [37, 38, 39, 40, 47] it was advocated in [94, 28] that an inflationary phase driven by the radion could be terminated by the production of nearly massless states if the radion comes close to such a point [3]. To be specific, if we work within heterotic string theory ($d = 6$)

[3]We focus on the overall volume modulus here – all other moduli (e.g. complex structure moduli and Kähler moduli) are assumed to be stabilized already. Since it is not always possible to find points of enhanced symmetry, one can not use the notion of quantum moduli trapping [28] for all of them.

such a point of enhanced symmetry could be the self dual radius corresponding to $\varphi = 0$. This was already anticipated by setting $\zeta = 1$ so that the potential V_I in (4.54) vanishes at $\varphi = 0$ (the self dual radius) [4].

The mechanism for stabilizing moduli at points of enhanced symmetry was illustrated in detail in [40, 39], and can be implemented in string gas cosmology. In the specific toy model of [40] it was shown that new massless states, gauge vectors and scalars, appear at the self dual radius. These states have to be included in the effective four dimensional action, leading to trapping of the volume modulus: as the radius shrinks down to the string size, the evolution becomes non-adiabatic and light states are produced via parametric resonance. Since the coupling of moduli to vector states is a gauge coupling, one expects parametric resonance to be efficient. The produced vectors stop to be massless as the radius shrinks further, generating an effective potential for the volume modulus. As a consequence, the size of extra dimensions ceases to shrink. The mechanism of moduli trapping at enhanced symmetry points (ESP) was discussed more generally in [39]: the trapping force is proportional to the number of states that becomes massless at the ESP, since enlarging the amount of new light degrees of freedom effectively causes an enhanced coupling of the moduli. As a consequence of the larger coupling, the effectiveness of parametric resonance and the trapping effect are enhanced. Therefore, points with greater symmetry are dynamically preferred.

We refer the interested reader to [48, 49] for a basic introduction and to [28] for a technical review of string gas cosmology, and jump into the discussion right after the string gas got produced.

As mentioned above, the string gas leads to an effective potential from a four dimensional point of view which is given by [115, 28]

$$V_S = \frac{\tilde{N}}{a^3} e^{-\frac{d}{2}\beta\varphi} \sqrt{\frac{q^2}{a^2} + \sinh^2(\beta\varphi)}\,, \tag{4.55}$$

where q parameterizes the momentum of the string gas along the three large dimensions and $\tilde{N}$ is proportional to the number density of strings. We shall treat both

[4] By choosing $\zeta = 1$ we effectively set the cosmological constant to zero.

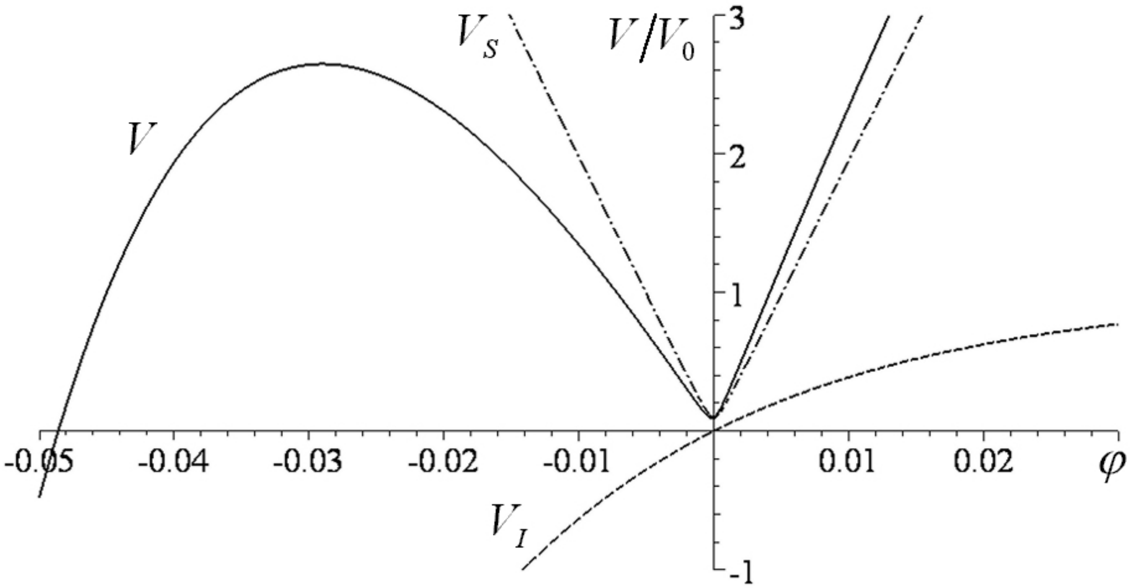

Figure 4.1: The inter-brane potential V_S of (4.54), string gas potential V_I of (4.55) and total potential $V = V_I + V_S$ with $d = 6$, $n = 4$. The height of V at the minimum $\varphi = \sigma \approx 0$ is given by the momentum of the string gas along the three large dimensions q. We chose $l := q/a = 10^{-6}$ and $\tilde{N}/(V_0 a^3) = 4\tilde{\alpha}/\beta$ for instructive reasons only, such that V_I, V_S and V are clearly discernable. Note that V_S is only viable around $\varphi = \sigma$. In order for a minimum to exist and moduli trapping to occur, conditions (4.58) and (4.62) need to be satisfied.

parameters as free ones [5]. Note the novel feature that the potential redshifts like matter, unlike potentials usually encountered for scalar fields.

This redshifting leads to a problem if we insist that the present day radion be stabilized by V_S, which can be seen as follows: let us for simplicity set $q = 0$ for the time being and ask whether the total potential

$$V = V_I + V_S \,, \tag{4.56}$$

[5]Both $\tilde{N}$ and q could in principle be computed via a study of the production mechanism of the sting gas. This process shares similarities to pre-heating and in fact overlaps with the early stages of pre-heating. Consequently, pre-heating might be influenced (see section 4.4.2).

which is plotted in Fig. 4.1, exhibits a minimum. Expanding V around $\varphi = 0$ yields

$$V \approx V_0 \tilde{\alpha} \varphi + \frac{\tilde{N}}{a^3} |\beta \varphi| \,. \tag{4.57}$$

In order to stabilize the radion at $\varphi = 0$ we need

$$\frac{\tilde{N}}{a^3 V_0} \gg \frac{\tilde{\alpha}}{\beta} = \mathcal{O}(1) \,, \tag{4.58}$$

so that the stabilizing potential V_S is able to prevent the collapse of the internal dimensions due to the inter brane potential V_I. Since the universe expanded roughly another 60 e-foldings after inflation until today, we would need

$$\frac{\tilde{N}}{V_0} \gg e^{180} \tag{4.59}$$

if we want a stable radion at late times, which is clearly an unreasonable condition. This problem is a simple reflection of the fact that the inter brane potential does not redshift, whereas the string gas redshifts like matter. Henceforth, it is not surprising that the attractive force between the branes wins in the long run. Notice that the same reason makes this type of stabilization incompatible with the presence of a cosmological constant [116].

If one insists on achieving stabilization via a string gas, there must be a mechanism present that cancels out V_I; luckily, such a mechanism seems possible in our scenario: once the branes associated with the orbifold fixed planes approach each other within the string scale they could annihilate via tachyon decay [117, 118, 119, 120, 121, 122, 123]. It should be noted that the universe itself does not go through a singularity: the radion gets stabilized at the self dual radius so that there is no big crunch.

What is more, one can imagine that this decay contributes to pre-heating, similar to the mechanism employed in the cyclic/ekpyrotic scenario [6] or in more recent realizations of brane inflation as in the KKLMMT proposal [31, 124, 125]. However, this mechanism comes with a price: the potential over-production of relics like cosmic

[6]Note that the cyclic scenario includes a singular collision of branes pinned to orbifold fixed planes, which is not what we are dealing with here: the branes in the scenario at hand come close to each other (within string length), but do not actually collide.

strings. If too many of these un-observed relics are produced, the model at hand would be ruled out [126] [7]. Hence, we shall assume that a mechanism to cancel the inter brane potential exists without producing too many relics.

Since the redshifting of the string gas potential can potentially spoil stabilization at late times, it is a concern whether this redshifting will also spoil standard pre-heating methods or leave them unaffected. Thus, we will address this question in the next subsection.

4.4.2 Pre-heating

Assuming that the inter brane potential cancels via some unspecified mechanism near the self dual radius, the complete potential for the radion is provided by the string gas alone, that is

$$V = \frac{N}{a^3} e^{-\frac{d}{2}\beta\varphi} \sqrt{\frac{q^2}{a^2} + \sinh^2(\beta\varphi)}\,. \tag{4.60}$$

We would now like to address the question whether the standard theory of pre-heating after inflation can be applied. The novel feature in our model is the dependence of the potential on the scale factor a. If one could neglect this feature, pre-heating would progress as usual, see e.g. [41, 42, 43, 44, 45, 46] for a sample of the extensive literature on the subject.

As a first estimate we can compare the rate at which the potential changes with the Hubble factor. As we shall see below in (4.66), both quantities are of the same order. Hence we expect any effects due to the redshifting of the potential to be of the same magnitude as those directly caused by the expansion of the universe. As a consequence, whenever the Hubble expansion needs to be included, e.g. in the case of stochastic pre-heating [44] (broad parametric resonance in an expanding universe), one should also include the time dependence of the potential.

---

[7] One way to avoid the defect overabundance problem is to enhance the symmetry which is broken during the annihilation; this can be achieved by having several overlapping branes instead of just one [131, 132].

In order to examine more carefully whether the redshifting of an inflaton potential can be neglected under the assumption that the expansion of the universe itself is unimportant, we will focus on a specific toy model for pre-heating [44, 45]: narrow parametric resonance [41, 42]. It should be noted that narrow or broad parametric resonances will not be viable reheating mechanisms if the inflaton is identified with the radion (as in our case), since the couplings between the radion and other matter-fields are heavily suppressed [8]. Nevertheless, we will focus on narrow resonance as an instructive example, since the mechanism is quite simple and very sensitive to changes in the shape of V: any change in the potential during the time-scale of pre-heating will cause the center of the resonance band to shift. If this shift is larger than the width of the resonance band, modes would not stay within the band long enough to get reasonably amplified. But if the shift is small compared to the width, narrow resonance will commence in the usual way. As we shall see below, the latter is the case so that there are no new effects and/or constraints due to the redshifting potential.

To study pre-heating, let us first expand the potential around the minimum of the potential at $\varphi_{min} =: \sigma$ and thereafter couple the radion to a scalar matter field χ. At this first stage we neglect the expansion of the universe so that $n := N/a^3 \approx const$ and $l^2 := q^2/a^2 \approx const$. The minimum of (4.60) can be found at

$$\sigma(l) \;=\; \frac{1}{2\beta} \ln\left(-3l^2 + \frac{3}{2} - \frac{\sqrt{36(l^4 - l^2) + 1}}{2}\right), \tag{4.61}$$

Where we used $d = 6$. Note that a minimum only exists for

$$0 \leq l < \tilde{l} \tag{4.62}$$

with $\tilde{l} := (\sqrt{12} - \sqrt{6})/6$, leading to $0 \leq \sigma < \ln(2)/(4\beta)$. Expanding the potential around σ leads to

$$V \;\approx\; \tilde{V}_0 + \frac{m^2}{2}\phi^2\,, \tag{4.63}$$

[8] Nevertheless, there are possibilities to enhance suppressed reheating channels by considering large vacuum expectation values of scalar matter fields after inflation, see e.g. [127].

where we used a shifted inflaton $\phi := \varphi - \sigma$ and

$$\tilde{V}_0 = n e^{-3\beta\sigma} \sqrt{l^2 + \sinh^2(\beta\sigma)}\,, \tag{4.64}$$

as well as

$$m \approx \beta \sqrt{\frac{n}{l}} - \beta \frac{63}{4} \sqrt{n} l^{3/2}\,, \tag{4.65}$$

where we expanded m around $l = 0$. Note that $m(l = \tilde{l}) \equiv 0$ exactly, so that any value of m can be achieved by appropriately tuning l. We will not need the cumbersome exact expression for m in the following, hence we shall omit it.

At this point we should step back for a second and estimate the rate of change of the potential. Using $l \propto 1/a$ and $q \propto 1/a^3$ we arrive at

$$\frac{\dot{m}}{m} \approx -H\,, \tag{4.66}$$

where we only kept the leading order term in (4.65). Hence, we naively expect that the expansion of the universe and the redshifting of the potential lead to comparable effects. This estimate can be made more concrete at the level of the toy model of narrow parametric resonance: if we couple the radion to a scalar matter field via $V_{int} = -g^2\phi^2\chi^2$, the system will be in the regime of narrow resonance if $g\Phi \ll \sigma \ll m$ holds, where $\Phi(t)$ is the amplitude of the oscillating inflaton [44]. This condition can be satisfied if we are free to tune g and m appropriately. Following the analysis of [44] closely, we find the first resonance band of the resulting Mathieu-equation for χ_k at the wave-number $k \approx m/2$ with a width of $\Delta k \approx \tilde{q}m/2$ where $\tilde{q} := 4g^2\sigma\Phi/m^2 \ll 1$.

Since parametric resonance usually commences during the first few oscillations of ϕ around its minimum, the characteristic time-scale is given by the period of these oscillations $T = 2\pi/m$.

Turning on the expansion of the universe yields the requirement

$$H \ll \tilde{q}^2 m\,, \tag{4.67}$$

in order for narrow resonance to take place [9], otherwise modes would leave the resonance band too fast [44]. Given that inequality, we can give an upper bound on

[9]Notice that g is expected to be small in our model. As a consequence, condition (4.67) is not satisfied and pre-heating will not progress in the regime of narrow parametric resonance.

the change of the scale factor a within the period T: because the scale factor makes a transition from an inflating one to a solution for a radiation dominated universe during pre-heating, we can use the inflationary solution as an upper bound for the change in a, that is

$$\frac{a(t_0+T)}{a(t_0)} \approx e^{2\epsilon\pi\tilde{q}^2} \tag{4.68}$$

where we used $H \approx \epsilon\tilde{q}^2 m$ with $\epsilon \ll 1$. This results in a change of the potential's shape via a change in the inflaton mass

$$\begin{aligned} m(t_0+T) &\approx \beta\sqrt{\frac{n}{l}} && (4.69)\\ &\approx m(t_0)e^{-\epsilon 2\pi\tilde{q}^2} && (4.70)\\ &\approx m(t_0)(1-\epsilon 2\pi\tilde{q}^2)\,, && (4.71) \end{aligned}$$

where we only kept the leading order term in l from (4.65), plugged in $l \propto 1/a$ as well as $n \propto 1/a^3$ and expanded around $\tilde{q} = 0$. Since the position of the first resonance band is located at $k = m/2$, we see that the shift of its position is given by $D_k = \epsilon\pi\tilde{q}^2 m(t_0)$. This shift has to be compared with the width of the band $\Delta k \approx \tilde{q}m(t_0)/2$. We immediately see that $D_k \ll \Delta k$ and henceforth, we can safely ignore the slight change of the radion potential.

4.4.3 Discussion

We saw in the previous section that the time dependence of the inflaton potential does not interfere much with the process of pre-heating. We estimated the effect on the toy model of narrow parametric resonance, because this pre-heating mechanism is most sensitive to changes in the mass of the inflaton. We found that new effects due to the redshifting potential are comparable to the ones already present due to the expanding universe.

Hence, we expect no novel features during pre-heating if a string gas supplies the stabilizing potential for the inflaton, and the standard theory of pre-heating can be applied (we refer the reader to [41, 42, 43, 44, 45] and follow up papers for the relevant literature). However, whenever the expansion of the universe itself is crucial, one should also consider the redshifting of the potential; for example, in the case of stochastic resonance [44] the Hubble expansion causes a mode to scan many resonance bands during a single oscillation of the inflaton. Naturally, including the redshifting of the potential will add to this effect, since the resonance bands themselves shift, just as in the case of narrow resonance we examined in the previous section.

There is another issue worth stressing again: since the inflaton is identified with the radion in our setup, its couplings to matter fields are heavily suppressed. As a consequence, we do not expect parametric resonance to be the leading pre-heating channel (see however [127] for the possibility of *enhanced pre-heating*), but instead tachyonic pre-heating (see e.g. [46, 128, 129] and references therein), which occurs in case of a negative effective mass term for the matter field. This effect was used to address the moduli problem in [4] and warrants further study [130].

Yet another possibility to reheat the universe could be provided by the annihilation of the boundary branes via tachyon decay [117, 118, 119, 120, 121, 122, 123] once the branes come close to each other. A potential hinderance could be an over-production of relics such as cosmic strings. It seems possible to avoid this problem in certain circumstances [131, 132], but we postpone a study of this interesting possibility to a future publication, since it is beyond the scope of this article.

Last but not least, since the production of the stabilizing string gas will overlap with the early stages of pre-heating, one should discuss both processes in a unified treatment.

4.5 Conclusions

In this chapter, we examined observational consequences of the recently proposed emerging brane inflation model. After reviewing the aforementioned model, observational parameters were computed within the slow roll regime, once and foremost the scalar spectral index $n_s = 0.9659^{+0.0049}_{-0.0052}$. This index is a generic prediction of emerging brane inflation, independent of model specific details and in excellent agreement with recent constraints of WMAP3 and SDSS. Furthermore, based one the COBE normalization we were able derive a bound onto the fundamental string scale, (4.52).

Thereafter, we examined the consequences of a redshifting string gas potential, which arises at the end of inflation. Even though the radion/inflaton can initially be stabilized, the mechanism fails at late times as long as there is a contribution to the effective potential that does not redshift, like a cosmological constant or a remaining interbrane potential. Consequently, a mechanism to cancel out all such contributions needs to be found in order for the model to work.

Related to this mechanism, we encountered another potential problem: since the interaction of boundary branes is responsible for inflation, but branes have to be absent at late times in order to keep extra dimensions stable, we concluded that they had to annihilate after inflation. During this annihilation, which could in principle be responsible for pre-heating, relics like cosmic strings are expected to be produced. Mechanisms to avoid an overproduction of said relicts are conceivable, but warrant further study.

Concerned that pre-heating after inflation might also get disrupted via the time dependence of the potential, we focused on narrow parametric resonance as a toy model for pre-heating to estimate the magnitude of new effects: we find that new effects are comparable to those originating directly from the expansion of the universe. Henceforth, we concluded that the standard machinery of pre-heating can be applied to the model at hand, but the time dependence of the potential needs to be incorporated if expansion effects are crucial for pre-heating, as is the case in e.g. stochastic resonance. Since the annihilation of boundary branes and the production of the stabilizing string gas occurs during the early stages of pre-heating, one should incorporate these effects in a detailed study of pre-heating.

To summarize, the proposal of emerging brane inflation is a viable realization of inflation, if the potential problems associated with the annihilation of branes after inflation can be overcome.

Chapter 5

Nonperturbative Instability as a Solution of the Cosmological Moduli Problem

It is widely accepted that moduli in the mass range 10eV - 10^4GeV which start to oscillate with an amplitude of the order of the Planck scale either jeopardize successful predictions of nucleosynthesis or overclose the Universe. It is shown that the moduli problem can be relaxed by making use of parametric resonance. A new nonperturbative decay channel for moduli oscillations is discussed. This channel becomes effective when the oscillating field results in a net negative mass term for the decay products. This scenario allows for the decay of the moduli much before nucleosynthesis and, therefore, leads to a complete solution of the cosmological moduli problem.

5.1 Introduction

In many theories beyond the Standard Model of particle physics, in particular in supergravity and string theories, there are many scalar and fermionic fields with masses smaller or equal to the electroweak scale and gravitational strength couplings to ordinary matter. Such fields are called *moduli fields* and behave as non-relativistic matter at late time. Since they decay late because of their weak interactions, they lead to the so-called cosmological moduli problem [54, 55, 56, 57]. Below, independently of their origin, fields having only Planck scale couplings and weak scale mass are denoted collectively as moduli fields.

In the case of moduli in supersymmetric theories, the mass m_ϕ of these fields ϕ is generated during supersymmetry breaking. A wide variety of these scenarios predict masses of the moduli in the dangerous range, $m_\phi \sim 10eV - 10^4 GeV$. According to our ideas of early Universe cosmology, we expect moduli to be produced in great abundance in the early universe. In the context of Big Bang cosmology, both scalar and fermionic moduli particles will be part of the initial thermal bath of particles of the very early universe. Even assuming that the moduli particles are not part of the initial thermal bath (for example in the context of inflationary cosmology) it is hard to avoid the presence of excited moduli fields at late times. For example, in the case of scalar moduli, since the moduli are massless before supersymmetry breaking, there is no reason that the moduli field values before supersymmetry breaking coincide with the values which turn into the minima of the potential after supersymmetry breaking [133]. The offset will lead to moduli fields which oscillate about their potential minima. An offset of a scalar modulus can also be produced by quantum fluctuations in the early phases of inflation, as follows from the computation of the coincident point two point function of a low-mass scalar field during inflation [134, 135]. The excessive production of moduli is predicted during the preheating stage of inflationary cosmology in a wide variety of models [136]. A further source of moduli particles is gravitational particle production between the end of inflation and the time of nucleosynthesis [137, 138].

Due to their weak interactions, the decay of the moduli fields is slow. Widely used estimates based on dimensional analysis for the perturbative decay rate Γ give

$$\Gamma \sim \frac{m_\phi^3}{M_p^2} \, . \tag{5.1}$$

The presence of excited moduli fields at late times is dangerous since the presence of the extra moduli field energy during the time of nucleosynthesis could spoil the success of the standard Big Bang nucleosynthesis scenario [139]. This danger is acute in particular for the heavier moduli fields. Both lighter moduli fields and heavier fields which do not decay before the time of equal matter and radiation threaten to overclose the Universe at that time (and thus also today if they do not decay between the time of equal matter and radiation and the present time).

To be more specific, it is the fact that the interactions of moduli fields come from non-renormalizable terms in the supersymmetry (SUSY) potential that leads to the moduli problem. For example, the following term in the Kähler potential

$$\delta K = \frac{1}{M_p^2}\phi_I^\dagger \phi_I \phi^\dagger \phi \,, \tag{5.2}$$

results in a contribution to the square mass for ϕ of the form ρ/M_p^2 [140]. By ϕ we denote a canonically normalized modulus field (with bare mass m_ϕ), by M_p - the Planck mass, by ϕ_I - a field which dominates the energy density of the universe, the inflaton, and by ρ - the energy density contained in the inflaton. Since $\rho = 3H^2M_p^2$, the effective mass m_{eff} becomes

$$m_{eff}^2 = cH^2 + m_\phi^2 \,, \tag{5.3}$$

where c is a constant. If $c \gg 1$, then inflation drives the moduli to the minima of their high temperature effective potential. However, typically the high temperature minima are offset from the zero temperature minima of the moduli potentials by a value which has Planck order of magnitude. If $c \ll 1$ then $H > m_{eff}$, and then quantum fluctuations during inflation will also excite the field ϕ to a value of the order of the Planck mass. During reheating the energy density of the inflaton stops dominating the Universe and the effective mass of ϕ relaxes to m_ϕ. After reheating, the Hubble constant decreases in the radiation dominated phase. Once $H \sim m_\phi$ the condition for slow rolling of ϕ ($V''(\phi) < H^2$) is no longer satisfied, and at that point the field ϕ starts to oscillate around its low temperature minimum which we take to be zero. The energy density of ϕ decreases like that of non-relativistic particles (i.e. proportional to $a(t)^{-3}$, where $a(t)$ is the cosmological scale factor), whereas the energy density of the radiation dominated universe falls as $a(t)^{-4}$. Thus, the modulus field ϕ may come to dominate the energy density of the universe, or at least contribute too much during the period of nucleosynthesis, unless the field decays early. The problem is that the modulus fields are coupled only gravitationally to themselves and other particles and thus have very small perturbative decay rates.

There has been some previous work to try to mitigate the cosmological moduli problem. Entropy production at late times will dilute the moduli density. For example, weak scale inflation [141] or thermal inflation [142] could sufficiently dilute unwanted moduli (see e.g. [143] for a detailed study of the potential of thermal inflation to solve the moduli problem for masses in the above-mentioned dangerous range predicted in models with hidden sector and gauge-mediated supersymmetry breaking). For certain ranges of parameters of a gauge-mediated supersymmetry breaking model, the oscillations of the modulus field itself might sufficiently dilute the string moduli density [144]. The decay of an unstable domain wall network [145] is another way to generate entropy and dilute the moduli density at late times. A common danger of these approaches is that the baryon density might also be diluted to an unacceptably low value. Another approach is to invoke effects which give the moduli fields a contribution to the square mass of the order of H^2 which would allow them to roll down their potential during inflation [146, 147] and prevent them from acquiring a large expectation value during inflation (see also [148]). However, this solution does not work if the low temperature minimum of the moduli potential does not coincide [133] with its high-temperature ground state (new symmetries which could force the two states to be the same were analyzed in [140]). A recent proposal to solve the moduli problem is moduli trapping at enhanced symmetry points [39, 40]. In terms of the use of parametric resonance instabilities, our work has similarities with that of [39, 40]. However, in contrast to these works, in our study the focus is on the traditional moduli problem as formulated in [54, 55, 56, 57].

In this chapter, we propose a way to solve the cosmological problems of scalar moduli fields which requires no external mechanism for the dilution of moduli. Instead, it makes use of non-perturbative decay channels. Non-perturbative decays have been shown to completely change the scenario of reheating in inflationary cosmology. In particular, the decay of the inflaton field by a parametric resonance instability has been shown to be very important [149, 43, 150, 44]. In certain models, a tachyonic instability renders the inflaton decay even more efficient [151].

We consider a toy model which potentially gives rise to a cosmological moduli problem. In the framework of this model we investigate non-perturbative decay channels (decays into particles whose lifetime is much shorter than that of the moduli fields). We study the decay of the oscillating modulus field via parametric resonance, and propose a new tachyonic decay channel. We quantify the conditions on the parameters of the model for which the decay channels are effective. Note that the decay channels work for an initial field amplitude up to the order of the Planck scale. No external mechanism for diluting the moduli density is required.

5.2 The Model

If the moduli problem arises as a consequence of supersymmetry breaking, the moduli potential takes the form (see e.g. [141])

$$\mathcal{V}(\Phi) \;=\; m_{3/2}^2 M_p^2 \mathcal{G}(|\Phi|/M_p)\,, \tag{5.4}$$

where $m_{3/2}$ is the gravitino mass and $\mathcal{G}$ is some function. We will assume that the modulus field couples to some matter field χ which we treat as a scalar field (following the analyses in the study of fermionic preheating [152] it could also be taken to be a fermionic field). Making use of (5.4), and of the fact that the dimension five and six operators are suppressed by M_p and by M_p^2, respectively, we can construct a typical potential for the modulus ϕ and the matter scalar field χ

$$\begin{aligned} V \;=\; & \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_\chi^2\chi^2 + \frac{1}{2}\frac{m_I^2}{M_p}\phi\chi^2 \\ & +\lambda_1\phi^2\chi^2 + \lambda_2\chi^4 + \lambda_3\phi^4\,, \end{aligned} \tag{5.5}$$

where the coupling constants $\lambda_1,\ \lambda_2,\ \lambda_3$ are small enough such that for $|\phi|,\ |\chi| < M_p$ the low energy effective potential for ϕ takes form

$$V_l \;=\; \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_\chi^2\chi^2 + \frac{1}{2}\frac{m_I^2}{M_p}\phi\chi^2\,. \tag{5.6}$$

Thus, the toy model we consider is described by the following Lagrangian:

$$\begin{aligned} L = \quad & \partial^\mu \phi \partial_\mu \phi + \partial^\mu \chi \partial_\mu \chi \\ & -\frac{1}{2} m_\phi^2 \phi^2 - \frac{1}{2} m_\chi^2 \chi^2 - \frac{1}{2} \frac{m_I^2}{M_p} \phi \chi^2 \, . \end{aligned} \tag{5.7}$$

In the above, the mass m_I sets the scale of the interaction between ϕ and χ. It is reasonable to assume that this scale is much less than M_p.

In analogy to the situation encountered in the study of the decay of the inflaton, if we want to study the decay of the modulus field, it is crucial to focus on the equation of motion for matter fields χ which the modulus field couples to. In the presence of the oscillating modulus field, this equation (in an expanding space-time) is

$$\begin{aligned} \ddot{\chi}_k \quad + & \quad 3H\dot{\chi}_k + \left(\frac{k^2}{a^2} + m_\chi^2 + \frac{m_I^2}{M_p} \Phi(t) \sin(m_\phi t) \right) \chi_k \\ = & \quad 0 \, , \end{aligned} \tag{5.8}$$

where $\Phi(t)$ is the amplitude of ϕ. The amplitude Φ decreases as a consequence of the expansion of space. In the above, k denotes the comoving wavenumber and $a(t)$ is the cosmological scale factor.

In a first step, we will put the above equation into the form of the well-known Mathieu equation. To absorb the expansion of space, we define a rescaled field via

$$\eta_k \, = \, a^{3/2} \chi_k \, . \tag{5.9}$$

Then, the equation of motion for η_k becomes:

$$\ddot{\eta}_k + \left(\frac{k^2}{a^2} + m_\chi^2 + \frac{m_I^2}{M_p} \Phi(t) \sin(m_\phi t) - \Delta \right) \eta_k \, = \, 0 \, , \tag{5.10}$$

where

$$\Delta \, = \, \frac{3}{4} H^2 + \frac{3}{2} \frac{\ddot{a}}{a} \, = \, \frac{3}{2} (1/2 + (p-1)/p) H^2 \, , \tag{5.11}$$

where for the second equality we have assumed that $a(t) \propto t^p$.

It is convenient to introduce a dimensionless time variable via

$$z \, = \frac{1}{2} m_\phi t + \frac{\pi}{4} \, . \tag{5.12}$$

The differentiation with respect to z will be denoted by a prime. In this case, the above equation (5.10) takes the form

$$\eta_k'' + (A_k - 2q\cos 2z)\eta_k \; = \; 0\,, \tag{5.13}$$

where

$$A_k \;\; = \;\; 4\frac{k^2}{m_\phi^2 a^2} + 4\frac{m_\chi^2}{m_\phi^2} - 4m_\phi^{-2}\Delta \tag{5.14}$$

$$q \;\; = \;\; 2\frac{m_I^2 \Phi}{m_\phi^2 M_p}\,. \tag{5.15}$$

If it is justified to neglect the expansion of space compared to the rate of the processes which will be discussed in the following, then Δ vanishes, and the equation (5.13) takes the form of the Mathieu equation. Note that k/a is the physical wavenumber.

5.3 Parametric Resonance Instability

The modulus field ϕ is frozen until the Hubble parameter H drops to a value comparable to m_ϕ. Then, ϕ begins to oscillate about $\phi = 0$ with a frequency m_ϕ, its amplitude $\Phi(t)$ being damped by the expansion of the spatial background and by the energy loss of ϕ to other fields. The second effect is a back-reaction effect which we will neglect. The condition required that the oscillation of ϕ begins before the time of nucleosynthesis is that the Hubble damping term in the equation of motion for ϕ becomes smaller than the force term $V'(\phi)$ driving the oscillations. It yields

$$H(T_{NS}) \; < \; m_\phi\,, \tag{5.16}$$

(where T_{NS} is the temperature at which nucleosynthesis takes place) a condition which is satisfied for all masses in the dangerous range. Once the modulus field starts to oscillate, resonant excitation of all fields coupled to ϕ is possible, in particular the excitation of η.

The first instability we will study is the parametric resonance instability, first applied to the decay of the inflaton field in [149]. There are two types of resonance

[43], *broad parametric resonance* and *narrow parametric resonance*. The condition for broad resonance is $q > 1$. where q is the parameter appearing in equation (5.13). This condition is satisfied for large values of the amplitude Φ, namely for

$$\Phi \, > \, \Phi_b \equiv M_p \big(\frac{m_\phi}{m_I}\big)^2 \, . \tag{5.17}$$

Evaluating this condition at the time when perturbative decay of the modulus field sets in, i.e. when $\Gamma = H$, and using for formula for Γ applicable to our toy model (given below in (5.29)), we find that unless

$$m_\phi \, \ll \, m_I \big(\frac{m_I}{10 M_p}\big)^{1/2} \, , \tag{5.18}$$

broad parametric resonance can relax but not solve the moduli problem without an additional decay channel being present. Both narrow resonance (discussed below) and the tachyonic decay discussed in the following section can provide the additional channel.

For smaller values of Φ, we are in the domain of narrow parametric resonance. In this phase, the growth of η_k is known [51, 52]:

$$\eta_k \, \sim \, e^{qz} \, \sim \, e^{q m_\phi t/2} \, . \tag{5.19}$$

Only modes in narrow resonance bands are amplified, and the first such band is centered at a value of $k = k_m$ given by [51, 52]

$$A_k \, = \, 1 \, , \;\; k_m \, = \, \frac{m_\phi}{2}(1 - \frac{4 m_\chi^2}{m_\phi^2})^{1/2} \tag{5.20}$$

from which it follows that the band does not exist unless $m_\chi < m_\phi/2$. Other resonance bands occur for larger values of A_k but are of higher order in perturbation theory and hence have a negligible effect.

The first condition for resonance to be effective is that the typical time scale for the growth of η_k is shorter than the Hubble time, i.e.

$$q m_\phi \, > \, H \, . \tag{5.21}$$

In addition, one should take into account the change of the momentum as a result of the background expansion [44]. In a time interval δt, assuming $m_\chi < m_\phi/4$, the change in the physical momentum $p = k/a(t)$ corresponding to the middle of the lowest resonance band is

$$\Delta p \;=\; pH\Delta t \;\simeq\; \frac{m_\phi}{2} H\Delta t \,. \tag{5.22}$$

The width of the resonance band is

$$\Delta p \;=\; \frac{q m_\phi}{2} \,. \tag{5.23}$$

Thus, p remains in the resonance band during a time interval

$$\Delta t \;\simeq\; q H^{-1} \,. \tag{5.24}$$

To justify neglecting the expansion of space, we must require that the exponent in the growth factor (5.19) is at least 1 during this time interval. This leads to a more severe constraint on q:

$$q^2 m_\phi \;>\; H \,. \tag{5.25}$$

Inserting the value of q from (5.15), we find that narrow resonance is efficient provided

$$\Phi(t) \;>\; \Phi_c(t) \equiv \frac{\sqrt{H} M_p m_\phi^{3/2}}{m_I^2} \,. \tag{5.26}$$

Since $\sqrt{H}$ decreases as $t^{-1/2}$ whereas in a radiation-dominated phase $\Phi(t)$ decreases only as $t^{-3/4}$, the narrow resonance decay channel eventually shuts off. In order that the moduli field does not dominate the energy density at the time of the shutoff at temperature T, the following condition needs to be satisfied:

$$m_\phi^2 \Phi_c^2 \;\ll\; T^4 \,, \tag{5.27}$$

which, inserting the expression (5.27) for Φ_c, turns into the condition

$$m_\phi \;\ll\; m_I \Big(\frac{T^2}{m_I M_p}\Big)^{1/5} \,. \tag{5.28}$$

For narrow parametric resonance to solve the cosmological moduli problem, one needs to check that at the time of moduli decay (which occurs when $\Gamma \sim H$), the condition (5.28) still holds. The decay rate of ϕ for our Lagrangian (5.7) is given by

$$\Gamma \;=\; \frac{m_I^4}{32\pi M_p^2 m_\phi} \,. \tag{5.29}$$

Thus, the solution of the cosmological moduli problem requires:

$$m_\phi \ll m_I \Big(\frac{m_I}{10M_p}\Big)^{1/3} . \tag{5.30}$$

Inserting the temperature of nucleosynthesis to get lower bound from (5.29) on the potentially dangerous mass range, and using the value $m_I = 10^2$ GeV, the problem is solved for values of m_ϕ satisfying (more general results are shown in Fig. 5.1)

$$10^{-7}GeV < m_\phi < 10^{-4}GeV . \tag{5.31}$$

In the context of our model (for $m_I = 10^2$GeV), moduli with masses smaller than 10^{-7}GeV decay before time of nucleosynthesis and thus do not cause the problem. Note that this scaling of the decay rate with the mass of the decaying particle is going against the intuition that lighter moduli should decay later than heavier one. This curious aspect of our toy model decreases the potentially dangerous mass range, and this realization might be useful in some concrete models suffering from a moduli problem.

It appears at this point of our study that the period of narrow parametric resonance has the potential of solving the moduli problem for values of m_ϕ and m_I which satisfy the relation given by (5.30). One issue which we have not taken into account is the fact that late moduli decay may provide a large source of non-thermal photons which could distort the black-body nature of the CMB. The constraints resulting from this effect must be studied in any concrete model with late-decaying moduli fields.

The previous analysis has missed a second important condition for the efficiency of narrow parametric resonance. It is not sufficient that the modes η_k increase with a rate faster than H. Since the resonance occurs only in narrow bands [51, 52], it is important to check that the rate of energy increase integrated over all modes of η be larger than the decrease in the energy density of ϕ taking into account the expansion of space alone. Otherwise, the energy stored in the moduli field would still scale as matter in spite of the exponential increase in the occupation number of certain field modes. This condition reads

$$\dot{\rho_\eta} > H\rho_\phi . \tag{5.32}$$

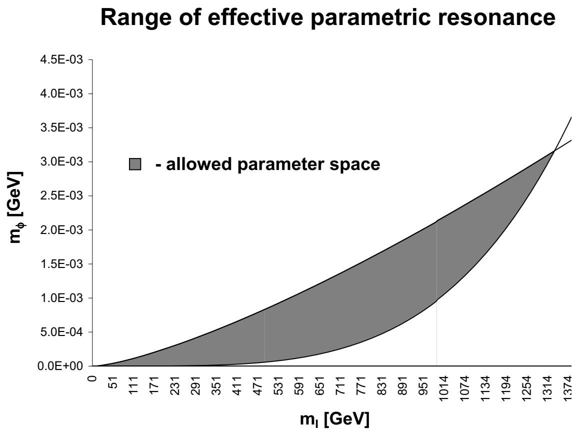

Figure 5.1: The shaded region represents the parameter space where parametric resonance alone completely solves the cosmological moduli problem. The upper bound is obtained from a combination of two conditions: the moduli never dominate the universe and parametric resonance is effective up to the moment when the moduli decay perturbatively. The lower bound limits the shaded region to moduli masses which are potentially dangerous.

The rate of increase $\dot{\rho}_\eta$ in the energy density of η can be estimated by considering the increase in the amplitude of all modes of η in the lowest instability band. This band is located at $k \sim m_\phi$ and its width is given by qm_ϕ. Since the rate of increase (from (5.19)) is qm_ϕ and since the initial mode (vacuum) energy is about k, we obtain

$$\dot{\rho}_\eta \sim m_\phi^5 q^2 \,. \tag{5.33}$$

Hence, the condition (5.32) for efficiency of the resonance process becomes

$$m_\phi < m_I \frac{m_I^2}{T^2} \frac{m_I}{M_p} \,, \tag{5.34}$$

which is to be evaluated at the temperatures when the presence of moduli fields are dangerous for cosmology. Using, as before, the value $m_I = 10^2$GeV, and evaluating the above condition at the temperature of nucleosynthesis, we find that the condition is satisfied as long as the mass m_ϕ is smaller than about 10^{-5}GeV. The condition (5.34) becomes increasingly well satisfied at lower temperatures, and is no longer a concern at the time of recombination. Note that the condition (5.32) for the efficiency of narrow resonance is a very conservative one. As long as the first condition (5.25) is satisfied, the mode amplitude will grow, and hence the vacuum mode amplitude used in the above estimate should be replaced by the excited amplitude, thus relaxing the constraint by an exponential factor.

The problem, however, is that the condition (5.34) conflicts up to coefficients of order unity with the condition that the perturbative decay rate is negligible. Thus, in our toy model, and using the very conservative form of our conditions for the effectiveness of the resonance process, the narrow resonance decay channel is only effective near the time when the perturbative decay is also become important. This result, however, is a consequence of the particular scaling of Γ with m_ϕ, $\Gamma \propto m_\phi^{-1}$. In models where moduli decay to fermions, $\Gamma \propto m_\phi$, and, therefore, we expect those models do not suffer from this specific problem.

The above discussions neglected the expansion of the universe. Taking into account this expansion changes the Mathieu equation into a more general equation of Floquet type, and leads to a stochastic nature of the resonance process [44]. However, the property that the number of particles is growing exponentially at a rate given by (5.19) is preserved.

5.4 Tachyonic Decay of the Oscillating Modulus Field

In the case of the decay of the inflaton field at the end of the period of inflation, it is known [151] that for certain models there is a tachyonic instability channel which is more efficient than parametric resonance. In this section, we will study a similar process for moduli decay.

Let us return to the basic equation (5.13), with the values of the parameters A_k and q given by (5.14) and (5.15), respectively. We immediately see that for large values of Φ, the effective m^2 term in the equation will be negative for part of the oscillation period of ϕ. This tachyonic instability occurs provided

$$\frac{m_I^2 \Phi}{m_\phi^2 M_p} > \frac{m_\chi^2}{m_\phi^2} \,. \tag{5.35}$$

The minimal value for which the tachyonic decay channel is open is given by setting the two sides in (5.35) equal and will be denoted by Φ_m.

The condition under which the tachyonic decay channel can solve the moduli problem is then given by

$$m_\phi^2 \Phi_m^2 \ll T^4 \,, \tag{5.36}$$

where T is the temperature corresponding to the period one is interested in [1]. Evaluating (5.36) at the time of nucleosynthesis, we obtain

$$m_\phi \ll T_{NS} \frac{T_{NS}}{M_p} \frac{m_I^2}{m_\chi^2} \,. \tag{5.37}$$

The upper mass bound on m_ϕ for which the above tachyonic decay is effective thus depends sensitively on the ratio of m_I and m_χ. Unless the latter mass is much smaller than the former, the tachyonic decay channel cannot reduce the amplitude of moduli oscillations to a level consistent with the observational constraints. Further constraints on m_χ come from the requirement that the tachyonic channel be more efficient than pertubative decay (see Fig. 5.2).

The range of values of m_ϕ for which the two decay channels - narrow parametric resonance (neglecting for a moment the issue that in our model it starts to be efficient together with the perturbative decay) and tachyonic decay - are open depends on the values of the masses. While if $m_\chi > m_\phi$, the only allowed channel is the tachyonic one, the narrow parametric resonance works for a wider range of masses m_ϕ when $m_\chi \sim m_\phi$. The latter can be seen by inserting $m_\chi = m_\phi$ into (5.37) and comparing

[1]While the energy density of χ is built up there is no reduction in Φ amplitude. In the case that the space expansion can be neglected, conservation of the total energy density eventually causes reduction in the moduli amplitude, Φ decreases. Thus, potentially, Φ can reach even lower values than Φ_m, however, the non-negligible fraction of the energy density might still remain in the moduli. All we ask for is to lower the energy density of the moduli below the total energy density 5.36 at the moment the perturbative decay takes place, namely, below $\Gamma_{all}^2 m_p^2$. See Fig. 5.2

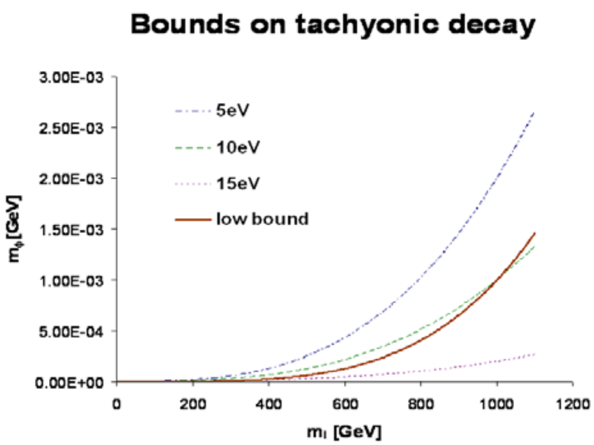

Figure 5.2: The combination of two conditions - moduli never dominate the universe and tachyonic resonance is effective up to the moment the moduli decay perturbatively - sensetively depends on the mass of the decay products (m_χ). As an example, we take $m_\chi = 5$, 10 and 15 eV. The upper bound set by the above conditions seriously reduces the range of applicability of the tachyonic resonance for decay products with large masses. The situation can be fixed by dropping the condition that the moduli never dominate, which will remove the bound and still allow the method to work. The lower bound limits the parameter space to moduli masses which are potentially dangerous.

with (5.28). However, as the value of m_χ is reduced, the range of masses for which the tachyonic decay channel is open grows and begins to dominate over that of narrow parametric resonance. Tachyonic decay can be made to work even for $m_I \sim m_\phi$.

For values of m_ϕ for which both decay channels are open, the tachyonic decay is more efficient for two reasons. First, it leads to the excitation of all long wavelength modes, and not just to modes in a narrow resonance band. Modes with

$$k_p^2 \, < \, m_I^2 \frac{\Phi}{M_p} \, \equiv \, k_{crit}^2 \tag{5.38}$$

are excited. For such modes, the value of η_k increases with a maximal rate given by

$$\eta_k \, \sim \, exp(\sqrt{q} z) \, , \tag{5.39}$$

which for $q < 1$ is a larger rate than that which occurs for modes in the resonance band during narrow resonance (see (5.19)). This is the second reason for the larger efficiency of the tachyonic decay channel.

Let us estimate the energy density ρ_η stored in the quanta produced during the tachyonic decay process. The phase space of modes which are excited tachyonically is of the order k_{crit}^3. Each mode grows with a rate which varies in time, the maximal rate being given by (5.39), and the growth occurs for approximately half the oscillation period (the period during which the effective square mass is negative). The mean growth rate is given by $1/\sqrt{2}$ of the maximal rate. Thus

$$\rho_\eta(t) \, \sim \, m_I^4 \frac{\Phi^2}{{M_p}^2} exp(\frac{1}{\sqrt{8}} m_I \sqrt{\Phi/M_P} t) \, . \tag{5.40}$$

The prefactor in front of the exponential factor is much larger than the corresponding factor in the case of the narrow resonance decay process.

5.5 Discussion and Conclusions

In this chapter we have studied two non-perturbative decay processes which can substantially dilute the density of dangerous moduli fields. Both processes occur during the phase in which the moduli fields oscillate about their ground states. The first non-perturbative process is parametric resonant excitation of fields coupled to the modulus field. In this paper we modeled such fields as a scalar field χ coupled to the modulus field ϕ via dimension five operators suppressed by the Planck mass. The second decay process is tachyonic decay which makes use of the fact that the effective square mass of χ in the presence of the oscillating ϕ field is negative for part of the oscillation period. These decay processes are analogous to the parametric resonant decay of the inflaton during reheating [149], and to tachyonic preheating [151], respectively.

We have established the conditions under which either of the two decay processes can solve the moduli problem, i.e. reduce the energy density of the modulus field to values consistent with big bang nucleosynthesis. It appears that narrow parametric resonance has the potential to solve the modulus problem but, for the toy model considered here, and using very conservative conditions for the effectiveness of the resonance process, it happens only at the time when the pertubative decay rate also becomes important. Undoubtedly, tachyonic decay successfully solves the problem given masses of decay products which are much smaller than the scale of interaction - m_I. Moreover, the tachyonic decay channel allows for excitation of particles with masses heavier than that of the decaying particle. Depending on the values of the other masses in the Lagrangian, either of the two decay processes can be open for a wider range of masses m_ϕ. For values of m_ϕ for which both decay channels are open, the tachyonic decay is much more efficient, as is true in the case of the decay of the inflaton. Note that the presence of an interaction term in the Lagrangian linear in ϕ was important in order to obtain the tachyonic decay. For example, it can be generated as a result of a nonrenormalizable term in the Kähler potential after integrating out the field I responsible for F-type SUSY breaking:

$$\int d\theta^4 I^\dagger I \frac{\phi\chi^2}{M_p^3} = \frac{F^2}{M_p^3}\phi\chi^2 \approx m_{3/2}^2 \frac{\phi}{M_p}\chi^2 \,, \tag{5.41}$$

where $m_{3/2}$ is the gravitino mass.

It will be of great interest to study the applicability of these decay channels to concrete models with moduli fields. This work is left for future research.

The form of the potential in our toy model Lagrangian inevitably suggests inflation at low scales. This natural source of inflation does not only dilute heavier relics but could also mitigate the flatness, horizon and entropy problems. It needs to be studied whether this type of models can provide a successful reheating mechanism, if the inflaton has only gravitationally suppressed interactions. Once again, non-perturbative instabilities like those used in preheating [149, 43, 150, 44, 151] are likely to be successful. If the modulus field comes to dominate the energy density of the universe for some period (without necessarily leading to inflation), it can provide a candidate for the curvaton (see e.g. [153] for an extensive discussion of moduli fields as candidates for the curvaton).

Chapter 6

A Note on the Moduli-Induced Gravitino Problem

The cosmological moduli problem has been recently reconsidered. Papers [64, 65] show that even heavy moduli ($m_\phi > 10^5$ GeV) can be a problem for cosmology if a branching ratio of the modulus into gravitini is large. In this paper, we discuss the tachyonic decay of moduli into the Standard Model's degrees of freedom, e.g. Higgs particles, as a resolution to the moduli-induced gravitino problem. Rough estimates on model dependent parameters set a lower bound on the allowed moduli at around $10^8 \sim 10^9$ GeV.

6.1 Introduction

The cosmological moduli problem is a disease of many supersymmetry/supergravity theories [54, 55, 56, 57]. Many supersymmetry/supergravity theories contain fields which have flat potentials in the supersymetric limit and only Planck suppressed couplings to Standard Model (SM) particles. We generically call them moduli. The cosmological moduli problem arises whenever the decays of moduli are in conflict with cosmological observations. Masses of moduli depend on the type of supersymmetry breaking. Moduli much lighter than the Hubble scale during inflation acquire a vacuum expectation value (VEV) of order the Planck scale [134, 135]. Later on, a large abundance of moduli threatens to overclose the Universe or jeopardize the processes of nucleosynthesis. Several solutions of the moduli problem have been suggested, see e.g. [154, 155, 140, 4].

The cosmological moduli problem is automatically avoided in heavy moduli scenarios. A widely used estimate for the perturbative decay rate Γ_{all} of moduli is

$$\Gamma_{all} \sim \frac{1}{4\pi}\frac{m_\phi^3}{M_p^2}\,. \tag{6.1}$$

where ϕ is the modulus field and m_ϕ is the modulus mass. Moduli decay once the Hubble rate is of the order of Γ_{all}. Therefore, moduli of mass below 100 TeV decay near or after the time of nucleosynthesis, when the universe is nearly 1 second old. If the mass is above 100 TeV then the moduli decay before the time of Big Bang Nucleosynthesis (BBN). Examples of scenarios with heavy moduli exist [156, 141, 157, 158, 159].

The heavy moduli scenario as a solution of the cosmological moduli problem has recently been reconsidered starting with the papers [64, 65]. It was shown that the decay of moduli into gravitinos is unsuppressed (for an opposite example see [160]). The part of the Lagrangian describing the gravitino-modulus couplings is

$$e^{-1}\mathcal{L} = -\frac{1}{8}\epsilon^{\mu\nu\rho\sigma}(G_\phi\partial_\rho\phi + G_{\phi^\dagger}\partial_\rho\phi^\dagger)\bar{\psi}_\mu\gamma_\nu\psi_\sigma \tag{6.2}$$

$$-\frac{1}{8}e^{G/2}(G_\phi\phi + G_{\phi^\dagger}\phi^\dagger)\bar{\psi}_\mu[\gamma^\mu,\gamma^\nu]\psi_\nu \tag{6.3}$$

where ψ_μ stands for the gravitino and G_ϕ is a non vanishing dimensionless auxiliary field with $G = K/M_p^2 + \ln(|W|^2/M_p^6)$. The subscript i denotes the derivative with respect to the field i. K and W are Kähler potential and superpotential respectively. Based on these coupling, the perturbative decay rate of moduli into gravitinos is

$$\Gamma_{3/2} \equiv \Gamma(\phi \to 2\psi_{3/2}) \approx \frac{|G_\phi|^2}{288\pi}\frac{m_\phi^5}{m_{3/2}^2 M_p^2}\,. \tag{6.4}$$

The auxiliary field of the modulus, G_ϕ, in general, can be small to suppress $\Gamma_{3/2}$ to the total decay rate Γ_{all} (6.1). However, suppressed G_ϕ is not a typical case, e.g. in the framework of the $4D$ supergravity $G_\phi \geq m_{3/2}/m_\phi$. Performing elaborate calculations, authors of [64, 65] have shown that the typical branching ratio $Br(\phi \to$

$2\psi_{3/2}) \sim \mathcal{O}(0.01-1)$. The large branching ratio of heavy moduli into gravitinos causes gravitino overproduction. Hence, even having a modulus mass above 100 TeV does not resolve the cosmological moduli problem. A detailed re-analysis of the cosmological moduli problem taking into account constraints on gravitino overproduction pushes up the gravitino mass above $10^5 - 10^6$ GeV [65]. This is the moduli-induced gravitino problem.

The previously published literature on the moduli-induced gravitino problem does not include nonperturbative decay channels. We propose a solution of the moduli-induced gravitino problem by having most of the moduli energy decay into the SM degrees of freedom through a tachyonic decay into a boson pair, e.g. Higgs. The decay process *moduli* $->$ *bosons* is rapid and occurs before moduli start to perturbatively decay into gravitinos. The scheme allows to find a range of moduli masses ($m_\phi > 10^8 \sim 10^9$ GeV) which does not suffer from the moduli-induced gravitino problem. Making use of conservative approximations, we find a range of masses with no overproduction of gravitinos.

6.2 Basic Idea

The general idea can be introduced in the following way. As was mentioned previously, moduli have only Planck suppressed couplings to other fields and during inflation obtain a VEV of the order of the Planck scale. After inflation, the modulus field slowly rolls preserving its energy. When the Hubble parameter reaches the value of m_ϕ, the modulus field starts to oscillate. In the following, we assume that moduli have a trilinear coupling to a scalar field χ,

$$\phi\chi^2 \,. \tag{6.5}$$

The effective potential, $V(\phi,\chi)$ is

$$V(\phi,\chi) = \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_\chi^2\chi^2 + \frac{1}{2}\frac{\alpha}{M_{Pl}}m_\phi^2\phi\chi^2 + \frac{1}{4}\lambda\chi^4 \,. \tag{6.6}$$

The equation of motion for χ field with switched off the expansion of space is

$$\ddot{\chi}_k + \left(k^2 + m_{eff}^2\right)\chi_k \;=\; 0 \,. \tag{6.7}$$

where

$$m_{eff}^2 = m_\chi^2 + \lambda\chi^2 + \frac{\alpha}{M_p} m_\phi^2 \phi\,, \tag{6.8}$$

The oscillations of the field ϕ induce a negative mass for the field χ. The modes of the field χ with $k < \sqrt{-m_{eff}^2}$ are excited,

$$\chi_k \propto e^{\sqrt{-m_{eff}^2 - k^2}\,t} \tag{6.9}$$

and the energy is transferred from the oscillating ϕ into excitations of χ in a preheating-like process. The process has a name of tachyonic resonance and is widely discussed in the literature starting with [149, 129, 46], in particular, the implementation of tachyonic resonance in the context of the resolution of the moduli problem is discussed in [4]. Thus, we see that for a certain range of parameters, the energy density stored in the moduli nonperturbatively transfers into excitations of χ field much before moduli perturbatively decay into gravitinos. The couplings of χ to Standard Model particles are assumed to be unsuppressed and, as a result, the decay rate of χ is much larger than 1 sec^{-1}. Thus, the modulus energy is converted into radiation much before the time of BBN.

To study the stability of the potential (6.6), we find the minimum of the $V(\phi,\chi)$ in the ϕ direction which occurs for

$$\phi = -\frac{1}{2}\frac{\alpha}{M_p}\chi^2\,. \tag{6.10}$$

Substituting (6.10) into $V(\phi,\chi)$ leads to

$$V(\phi,\chi) = -\frac{1}{4}\left(\frac{1}{2}\frac{\alpha^2}{M_p^2}m_\phi^2 - \lambda\right)\chi^4 + \frac{1}{2}m_\chi^2\chi^2\,, \tag{6.11}$$

and, we see that the effective potential is unstable for

$$\frac{1}{2}\frac{\alpha^2}{M_p^2}m_\phi^2 > \lambda\,. \tag{6.12}$$

Thus, the presence of additional terms with Planck suppressed couplings is important to stabilize the potential (6.6) at large values of the fields.

The efficiency of the tachyonic resonance must be carefully checked against the effects of dilution due to the expansion of space. For the tachyonic resonance to be effective, the growth of the mode k (6.9) shall dominate the dilution due to the expansion of space. The appropriate condition would be

$$\sqrt{-m_{eff}^2 - k^2} > H \tag{6.13}$$

or

$$\frac{\alpha}{M_p} m_\phi^2 \Phi > \frac{m_\phi^2 \Phi^2}{M_p^2} \tag{6.14}$$

where Φ is the amplitude of the ϕ field. The above condition is fulfilled once

$$\alpha M_p > \Phi\,. \tag{6.15}$$

At the onset of oscillations $\Phi < M_p$, thus for $\alpha \geq 1$ we can neglect the expansion of space in our analysis.

In addition to the growing mode (6.9), there is also the decaying mode

$$\chi_k \propto e^{-\sqrt{-m_{eff}^2 - k^2}t}\,. \tag{6.16}$$

The decaying mode causes inference terms and may put further restrictions on the region of applicability of the tachyonic resonance. The equation 6.7 can take the form of the well known Mathieu equation (see e.g. [50]). In fact as it can be seen from the instability chart of the Mathieu equation, the resonant production is terminated as soon as $q \equiv \alpha\Phi/m_p \leq 1/2$; hence $\alpha \gg 1$. In the context of gauge supersymmetry breaking and anomaly mediation scenarios the interaction couplings are expected to be larger than Planck suppressed which corresponds to $\alpha \gg 1$ in our parametrization. In gravity mediation supersymmetry breaking scenarios relatively large couplings α can also be obtained if moduli couples to Bose Condensate [127]. Another way to increase α is to consider many trilinear interactions which effectively causes an enhanced coupling of the moduli to the scalar sector.

Tachyonic preheating in the parameter range corresponding to large α was extensively studied in [128]. The authors have shown that trilinear terms lead to faster re-scattering and thermalization. As a bonus, trilinear terms allow complete decay of the moduli. In addition to positive effects, enhanced resonance and fast subsequent thermalization may enlarge the reheating temperature beyond the allowed region which threatens to overproduce gravitinos through re-scattering processes [161].

The trilinear interaction term (6.5) may arise, for example, from the non-renormilizable term in the Kähler potential [1]

$$\mathcal{L}_H = \int d^4\theta \frac{\lambda_H}{M_{pl}} \phi H_u^* H_d^* + h.c. \tag{6.17}$$

where H_u and H_d are up-type and down-type Higgs supermultiplets or corresponding scalar fields, respectively. The ϕ field is the moduli supermultiplet and, in the following, its scalar part. After integrating out the superspace coordinates, we obtain

$$\begin{aligned} \mathcal{L}_H &= \frac{\lambda}{M_p} \left(D_\mu D^\mu \phi H_u^* H_d^* \right. \\ &\quad \left. + F_\phi H_u^* F_d^* + F_\phi H_d^* F_u^* + c.c. + \cdots \right) \end{aligned} \tag{6.18}$$

where $F_i = -M_p^2 e^{G/2} (G^{-1})^i_j G_j$ is the auxiliary field of the i'th supermultiplet, D_μ is the covariant derivative. The process of energy transfer described above makes use of on-shell degrees of freedom. Hence, we make use of the equation of motion for the ϕ field to replace $D_\mu D^\mu \phi$ with $m_\phi^2 \phi$. As a result, the following interaction term is part of the Lagrangian:

$$\mathcal{L}_H \supset \frac{\lambda_H}{M_{pl}} m_\phi^2 \phi H_u^* H_d^* + h.c. \tag{6.19}$$

In the low energy effective Lagrangian, the term (6.19) is responsible for the interaction (6.5), where χ is the neutral scalar component of the lightest Higgs field in the mass basis.

6.3 Estimates

In the following we would like to estimate the region of moduli mass for which the moduli-induced gravitino problem is resolved. Another glance at the equation of motion of the χ field

$$\ddot{\chi}_k + \left(k^2 + m_\chi^2 + \lambda \chi^2 + \frac{\alpha}{M_p} m_\phi^2 \phi \right) \chi_k \; = \; 0 \, ,$$

[1] Here we provide only one example of the origin of trilinear terms. Large α might require other interactions.

reveals that the tachyonic process is more effective for larger masses of the moduli. We assume that the tachyonic resonance works as long as m_{eff}^2 can obtain negative values,

$$\frac{m_\chi^2}{m_\phi^2} < \alpha \frac{\Phi}{M_p} \,. \tag{6.20}$$

All the energy converted into excitations of the χ field afterwards is transferred to SM degrees of freedom. Further, since $Br_{3/2} = \mathcal{O}(0.01 \sim 1)$ we assume that once the bound (6.20) is violated all the energy is transferred to gravitinos. The above assumptions allow us to estimate the gravitino abundance neglecting the effect of the expansion of space. At the end, we insert the known bounds on the gravitino abundance and derive the lower bound on the gravitino mass.

We distinguish between two cases at the onset of moduli field oscillations: in the first case, the universe is supercooled and $\langle\chi^2\rangle \sim 0$; or, in the second case, the universe is dominated by radiation and $\langle\chi^2\rangle \sim T^2 = \sqrt{m_\phi M_p}$. The universe is supercooled if oscillations of the moduli were preceded by an inflationary period, and the energy is still stored in the oscillations of an inflaton, or if the modulus itself is the inflaton (see [162, 163] for discussions on the moduli-induced gravitino problem in this case). In this paper, we primary concentrate on the first case. In this case, we omit the self interaction term to obtain order of magnitude estimates for the bound on the allowed moduli mass.

While the tachyonic resonance is in effect, the energy density in ϕ is transferred to χ particles and then to radiation. Neglecting the expansion of space,

$$\rho_{rad} = m_\phi^2 M_p^2 \tag{6.21}$$

The tachyonic resonance ends as soon as Φ reaches the value [2]

$$\Phi_{min} = \frac{m_\chi^2}{m_\phi^2}\frac{M_p}{\alpha} \,. \tag{6.22}$$

At this point, the remaining energy density in the moduli is

$$m_\phi^2 \Phi_{min}^2 = \frac{m_\chi^4 M_p^2}{\alpha^2 m_\phi^2} \equiv \rho_{3/2} \,. \tag{6.23}$$

[2] We have assumed that tachyonic resonance ends before the perturbative decay takes place. This assumption is equivalent to $m_\chi > \Gamma_{all}$.

The energy density stored in the gravitino, $\rho_{3/2}$, allows us to determine the gravitino abundance.

$$\begin{aligned} m_{3/2}Y_{3/2} &\equiv m_{3/2}\frac{n_{3/2}}{s} && (6.24)\\ &= \frac{\rho_{3/2}}{s} && (6.25)\\ &= \frac{m_\chi^4 M_p^2}{\alpha^2 m_\phi^2 s} && (6.26) \end{aligned}$$

where $Y_{3/2}$ is the gravitino yield, $n_{3/2}$ is the number density of gravitino particles and s is the entropy of the ultra-relativistic particles.

$$s = \frac{\rho + p}{T_R} = \frac{4}{3}\frac{\rho_{rad}}{T_R} \approx (m_\phi M_p)^{3/2}\,, \qquad (6.27)$$

where T_R is the reheating temperature (temperature of ultra-relativistic plasma at the moment it reaches thermal equilibrium). While the actual reheating temperature depends on the thermalization processes, the upper bound is

$$T_R < \sqrt{m_\phi \Phi_{in}} \leq \sqrt{m_\phi M_p} \qquad (6.28)$$

where Φ_{in} is the amplitude of the field ϕ at the onset of oscillations. Since we have neglected the expansion of space throughout the calculations, we have plugged $T_R = \sqrt{m_\phi M_p}$ to obtain the last equality in (6.27).

The gravitino abundance is severely constrained in order not to jeopardize the success of BBN or from the danger of overproducing of lightest supersymmetric particles. The most stringent constraint comes from the overproduction of ^3He [58, 59] which yields

$$m_{3/2}Y_{3/2} < O(10^{-14} \sim 10^{-11})\ \text{GeV}\,. \qquad (6.29)$$

The limit (6.29) is equivalent to

$$\begin{aligned} m_{3/2}Y_{3/2} &= \frac{m_\chi^4}{\alpha^2 m_\phi^4} T_R && (6.30)\\ &= \frac{3}{4}\frac{m_\chi^4}{\alpha^2 m_\phi^4}\sqrt{m_\phi M_p} \\ &< O(10^{-14} \sim 10^{-11})\ \text{GeV} \end{aligned}$$

where we have inserted the expression for s (6.27). Making further assumptions: $\alpha \sim O(1)$, $m_\chi \approx 100$ GeV, the moduli is safe from the overproduction of gravitinos in direct decay if

$$10^8 \sim 10^9 \text{ GeV} \leq m_\phi \,. \tag{6.31}$$

The lower bound (6.31) is the main result of the paper.

In the second case, when the field χ is a part of the thermal bath and the contribution of the self interaction term to the effective mass can be large, we have

$$\begin{aligned} m_{eff}^2 &= m_\chi^2 + \lambda\langle\chi^2\rangle + \frac{\alpha}{M_p} m_\phi^2 \phi \\ &= m_\chi^2 + \lambda T^2 + \frac{\alpha}{M_p} m_\phi^2 \phi \,, \end{aligned} \tag{6.32}$$

where we have used the Hartree approximation to go from the first to the second line. The large λT^2 term threatens to prevent the tachyonic resonance from occurring. Particulary, if, at the onset of oscillations, the condition

$$1 < \frac{\alpha}{\lambda} \frac{m_\phi}{M_p} \tag{6.33}$$

is not satisfied, the effective mass (6.32) is positive. In an expanding moduli-dominated universe, the temperature redshifts as

$$T^2 = m_\phi M_p \left(\frac{\Phi}{M_p}\right)^{4/3} \tag{6.34}$$

Hence, m_{eff}^2 remains positive during oscillations of the ϕ if

$$1 > \frac{\alpha^3}{\lambda^3} \frac{m_\phi}{M_p} \tag{6.35}$$

where we have inserted $\Phi_f = \frac{m_\phi^2}{M_p}$ - the value of Φ at the time of perturbative decay (6.1). In the case $m_\chi^2 > \lambda T^2$, the estimates on moduli mass reduce to (6.21-6.31).

The decay of moduli dilutes the pre-existing abundance of gravitinos. Let us denote the initial gravitino yield by $Y_{3/2}$. The entropy produced in the decay of moduli into radiation $s_n \propto T_n^3$, hence, the new gravitino yield is

$$\begin{aligned} Y_{3/2}^n &= \frac{n_{3/2}}{s_f + s_n} Y_{3/2}^n \approx \frac{Y_{3/2} s_f}{s_n} = \frac{Y_{3/2} s_f}{s_n} \\ &= \frac{T_f^3}{T_n^3} Y_{3/2} \,. \end{aligned} \tag{6.36}$$

where s_f and T_f stands for the values of the preexisting entropy and temperature of radiation at $\Gamma_{all} = H$. Making use of (6.34), we deduce

$$Y_{3/2}^n = \frac{m_\phi}{M_p} Y_{3/2} \tag{6.37}$$

6.4 Conclusions

In this chapter, we have discussed the influence of the tachyonic resonance on the moduli-induced gravitino problem. We primarily have discussed the case when χ is not a part of the thermal bath at the onset of oscillations of the modulus field. In this case, the rough estimates shows that moduli masses above $10^8 \sim 10^9$ are free from overproduction of gravitinos in direct decay of moduli. The estimates omit several model dependent points which may either enhance or diminish the influence of the resonance. In particular, in the process of calculations we did not take into account the expansion of space. In the case when χ is a part of the thermal bath at the onset of the oscillations of ϕ, we have found that the tachyonic resonance is less likely to work. In any case, even if the tachyonic resonance is inefficient, the decay of moduli dilutes the initial abundance of gravitinos.

Chapter 7

Conclusions

Integration of Cosmology with High Energy Physics is one further step towards the theory of everything. The interface of early universe cosmology and string theory is not yet possible on a rigorous basis since string theory is not yet fully developed theoretically. Nevertheless it can be mutually beneficial to combine ideas from string theory and cosmology. From one side, new degrees of freedom open up new venues for string-inspired scenarios - both inflationary and alternative cosmologies. On the other side, the main drawback of string theory, the lack of predictability, may be put aside if connections to the cosmological observations are found. Later time cosmology constraints put further restrictions on models beyond the Standard Model. This thesis contributes to this interface area.

Inflationary cosmology is a robust and predictive paradigm. Because of its robustness the chances are high that inflation is indeed a part of the history of the universe. However, there are many ways to obtain inflationary expansion of the universe which makes it difficult to agree on the unique model of inflation. Past decades gave rise to many models of inflation which cannot be distinguished on the basis of current observations. A successful alternative model which predicts yet untested deviations from inflation could open up a further way to test the inflationary paradigm. Finding an alternative scenario is a difficult task and tackling only part of the problems of BBC remains useful. Note that no currently known alternative scenario solves all the puzzles of BBC which are solved by inflation.

In Chapter 2, we propose an alternative string-inspired scenario which solves the entropy and horizon problems of BBC. Inspired by the idea of a hot big bang, we propose that the $4+d$ dimensional universe emerges with all stringy degrees of freedom present, and being compact. The topological difference between 3 and d spatial dimensions leads to a difference in the evolution of the extra dimensions compared to the evolution of our three currently observed spatial dimensions. During the phase of bulk expansion, the total energy density of the matter is growing. The following phase in which the extra dimensions contract is necessary to obtain later on am effectively 4 dimensional cosmology. Once the size of the extra dimensions is stabilized and the energy of the bulk transfers into radiation, the radiation epoch proceeds as in the BBC scenario. Note, that the expansion is non-accelerated in all stages of evolution.

A weak point the inflationary models is the fine-tunings involved in order to obtain inflation. Special initial conditions are not naturally expected to emerge in most of the models. In the inflationary string inspired model presented in Chapter 3, this weak point is ameliorated by the existence of a period of preceding expansion which results in the correct initial conditions. The setup of the scenario is very similar to the one used in Chapter 2. The difference is in the way the extra dimensions contract. In Chapter 3, the contraction is inflationary.

Chapter 4 follows up on Chapter 3. The observational consequences of the model presented in Chapter 3 are investigated. In particular, spectral index is found to be in excellent agreement with observations. Furthermore, we derive a bound on the fundamental string scale, examine compatibility with late time cosmology and discuss preheating.

Compatibility of late time cosmology with particle physics models is a subject under investigation. In particular, predictions of Nucleoshynthesis are challenged if the particle physics model at hand contains moduli fields. In Chapters 5 and 6 we consider decay channels for moduli which have been so far neglected, namely nonperturbative decay channels. In Chapter 5 we build a toy model Lagrangian which is inspired by the nature of the problem. Coupling of moduli to another scalar field through trilinear terms allows transfer of energy by virtue of parametric and tachyonic resonances. We find a parameter range which does not exhibit the cosmological moduli problem. In Chapter 6, the nonperturbative decay of moduli prevents domination of the moduli energy-density upon its perturbative decay, and as a byproduct it avoids the overproduction of gravitinos. We estimate the range of moduli masses for which the theory is free from the moduli-induced gravitino problem. While the final resolution of the cosmological moduli problem is model-dependent, our investigations open a new window of opportunity to solve the problem.

Bibliography

[1] R. Brandenberger and N. Shuhmaher, "The confining heterotic brane gas: A non-inflationary solution to the entropy and horizon problems of standard cosmology," JHEP **0601**, 074 (2006) [arXiv:hep-th/0511299].

[2] N. Shuhmaher and R. Brandenberger, "Heterotic brane gas inflation," Phys. Rev. Lett. **96**, 161301 (2006) [arXiv:hep-th/0512056].

[3] T. Battefeld and N. Shuhmaher, "Predictions of dynamically emerging brane inflation models," Phys. Rev. D **74**, 123501 (2006) [arXiv:hep-th/0607061].

[4] N. Shuhmaher and R. Brandenberger, "Non-perturbative instabilities as a solution of the cosmological moduli problem," Phys. Rev. D **73**, 043519 (2006) [arXiv:hep-th/0507103].

[5] N. Shuhmaher, "A note on the moduli-induced gravitino problem," arXiv:hep-ph/0703319.

[6] R. H. Brandenberger, "Inflationary cosmology: Progress and problems," arXiv:hep-ph/9910410.

[7] A. Riotto, "Inflation and the theory of cosmological perturbations," arXiv:hep-ph/0210162.

[8] A. A. Penzias and R. W. Wilson, "A Measurement of excess antenna temperature at 4080-Mc/s," Astrophys. J. **142**, 419 (1965).

[9] P. J. E. Peebles and R. H. Dicke, "Origin of the Globular Star Clusters," Astrophys. J. **154**, 891 (1968).

[10] G. F. Smoot *et al.*, "Structure in the COBE DMR first year maps," Astrophys. J. **396**, L1 (1992).

[11] C. B. Netterfield *et al.* [Boomerang Collaboration], "A measurement by BOOMERANG of multiple peaks in the angular power spectrum of the cosmic microwave background," Astrophys. J. **571**, 604 (2002) [arXiv:astro-ph/0104460].

[12] S. Hanany *et al.*, "MAXIMA-1: A Measurement of the Cosmic Microwave Background Anisotropy on angular scales of 10 arcminutes to 5 degrees," Astrophys. J. **545**, L5 (2000) [arXiv:astro-ph/0005123].

[13] C. L. Bennett *et al.*, "First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Preliminary Maps and Basic Results," Astrophys. J. Suppl. **148**, 1 (2003) [arXiv:astro-ph/0302207].

[14] E. W. Kolb, "Dynamics of the inflationary era," arXiv:hep-ph/9910311.

[15] V. de Lapparent, M. J. Geller and J. P. Huchra, "A Slice of the universe," Astrophys. J. **302**, L1 (1986).

[16] H. Lin, R. P. Kirshner, S. A. Shectman, S. D. Landy, A. Oemler, D. L. Tucker and P. L. Schechter, "The Luminosity Function of Galaxies in the Las Campanas Redshift Survey," Astrophys. J. **464**, 60 (1996) [arXiv:astro-ph/9602064].

[17] S. A. Shectman, S. D. Landy, A. Oemler, D. L. Tucker, H. Lin, R. P. Kirshner and P. L. Schechter, "The Las Campanas Redshift Survey," Astrophys. J. **470**, 172 (1996) [arXiv:astro-ph/9604167].

[18] A. H. Guth, "The Inflationary Universe: A Possible Solution To The Horizon And Flatness Problems," Phys. Rev. D **23**, 347 (1981).

[19] V. N. Lukash, "Production Of Phonons In An Isotropic Universe," Sov. Phys. JETP **52**, 807 (1980) [Zh. Eksp. Teor. Fiz. **79**, (1980)].

[20] V. F. Mukhanov and G. V. Chibisov, "Quantum Fluctuation And 'Nonsingular' Universe. (In Russian)," JETP Lett. **33**, 532 (1981) [Pisma Zh. Eksp. Teor. Fiz. **33**, 549 (1981)].

[21] W. H. Kinney, E. W. Kolb, A. Melchiorri and A. Riotto, "Inflation model constraints from the Wilkinson microwave anisotropy probe three-year data," Phys. Rev. D **74**, 023502 (2006) [arXiv:astro-ph/0605338].

[22] A. Liddle and D. Lyth, *Cosmological Inflation and Large-Scale Structure* (Cambridge Univ. Press, 2000).

[23] M. Gasperini and G. Veneziano, "Pre - big bang in string cosmology," Astropart. Phys. **1**, 317 (1993) [arXiv:hep-th/9211021].

[24] J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, "The ekpyrotic universe: Colliding branes and the origin of the hot big bang," Phys. Rev. D **64**, 123522 (2001) [arXiv:hep-th/0103239].

[25] G. R. Dvali and S. H. H. Tye, "Brane inflation," Phys. Lett. B **450**, 72 (1999) [arXiv:hep-ph/9812483].

[26] G. R. Dvali, Q. Shafi and S. Solganik, "D-brane inflation," arXiv:hep-th/0105203.

[27] F. Quevedo, "Lectures on string/brane cosmology," Class. Quant. Grav. **19**, 5721 (2002) [arXiv:hep-th/0210292].

[28] T. Battefeld and S. Watson, "String gas cosmology," Rev. Mod. Phys. **78**, 435 (2006) [arXiv:hep-th/0510022].

[29] K. Dasgupta, G. Rajesh and S. Sethi, "M theory, orientifolds and G-flux," JHEP **9908**, 023 (1999) [arXiv:hep-th/9908088];
S. Gukov, C. Vafa and E. Witten, "CFT's from Calabi-Yau four-folds," Nucl. Phys. B **584**, 69 (2000) [Erratum-ibid. B **608**, 477 (2001)] [arXiv:hep-th/9906070].

[30] S. B. Giddings, S. Kachru and J. Polchinski, "Hierarchies from fluxes in string compactifications," Phys. Rev. D **66**, 106006 (2002) [arXiv:hep-th/0105097].

[31] S. Kachru, R. Kallosh, A. Linde, J. M. Maldacena, L. McAllister and S. P. Trivedi, "Towards inflation in string theory," JCAP **0310**, 013 (2003) [arXiv:hep-th/0308055].

[32] K. Dasgupta, C. Herdeiro, S. Hirano and R. Kallosh, "D3/D7 inflationary model and M-theory," Phys. Rev. D **65**, 126002 (2002) [arXiv:hep-th/0203019].

[33] C. P. Burgess, "Inflatable string theory?," Pramana **63**, 1269 (2004) [arXiv:hep-th/0408037];
A. Linde, "Inflation and string cosmology," eConf **C040802**, L024 (2004) [arXiv:hep-th/0503195];
J. M. Cline, "Inflation from string theory," arXiv:hep-th/0501179.

[34] J. J. Blanco-Pillado *et al.*, "Racetrack inflation," JHEP **0411**, 063 (2004) [arXiv:hep-th/0406230].

[35] J. P. Conlon and F. Quevedo, "Kaehler moduli inflation," JHEP **0601**, 146 (2006) [arXiv:hep-th/0509012].

[36] K. Becker, M. Becker, M. Haack and J. Louis, "Supersymmetry breaking and alpha'-corrections to flux induced potentials," JHEP **0206**, 060 (2002) [arXiv:hep-th/0204254].

[37] S. Watson and R. Brandenberger, "Stabilization of extra dimensions at tree level," JCAP **0311**, 008 (2003) [arXiv:hep-th/0307044].

[38] S. P. Patil and R. Brandenberger, "Radion stabilization by stringy effects in general relativity and dilaton gravity," Phys. Rev. D **71**, 103522 (2005) [arXiv:hep-th/0401037].

[39] L. Kofman, A. Linde, X. Liu, A. Maloney, L. McAllister and E. Silverstein, "Beauty is attractive: Moduli trapping at enhanced symmetry points," JHEP **0405**, 030 (2004) [arXiv:hep-th/0403001].

[40] S. Watson, "Moduli stabilization with the string Higgs effect," Phys. Rev. D **70**, 066005 (2004) [arXiv:hep-th/0404177].

[41] J. H. Traschen and R. H. Brandenberger, "Particle Production During Out-Of-Equilibrium Phase Transitions," Phys. Rev. D **42**, 2491 (1990).

[42] Y. Shtanov, J. H. Traschen and R. H. Brandenberger, "Universe reheating after inflation," Phys. Rev. D **51**, 5438 (1995) [arXiv:hep-ph/9407247].

[43] L. Kofman, A. D. Linde and A. A. Starobinsky, "Reheating after inflation," Phys. Rev. Lett. **73**, 3195 (1994) [arXiv:hep-th/9405187].

[44] L. Kofman, A. D. Linde and A. A. Starobinsky, "Towards the theory of reheating after inflation," Phys. Rev. D **56**, 3258 (1997) [arXiv:hep-ph/9704452].

[45] P. B. Greene, L. Kofman, A. D. Linde and A. A. Starobinsky, "Structure of resonance in preheating after inflation," Phys. Rev. D **56**, 6175 (1997) [arXiv:hep-ph/9705347].

[46] G. N. Felder, J. Garcia-Bellido, P. B. Greene, L. Kofman, A. D. Linde and I. Tkachev, "Dynamics of symmetry breaking and tachyonic preheating," Phys. Rev. Lett. **87**, 011601 (2001) [arXiv:hep-ph/0012142].

[47] S. P. Patil and R. H. Brandenberger, "The cosmology of massless string modes," JCAP **0601**, 005 (2006) [arXiv:hep-th/0502069].

[48] R. H. Brandenberger, "Moduli stabilization in string gas cosmology," Prog. Theor. Phys. Suppl. **163**, 358 (2006) [arXiv:hep-th/0509159].

[49] R. H. Brandenberger, "Challenges for string gas cosmology," arXiv:hep-th/0509099.

[50] N. W. McLachlan, *Theory and application of Mathieu functions* (Clarendon, Oxford, 1947).

[51] L. Landau and I. Lifshitz, *Mechanics* (Pergamon, Oxford, 1960).

[52] V. Arnold, *Mathematical Methods of Classical Mechanics* (Springer, New York, 1974).

[53] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions* (Dover, New York, 1972).

[54] G. D. Coughlan, W. Fischler, E. W. Kolb, S. Raby and G. G. Ross, "Cosmological Problems For The Polonyi Potential," Phys. Lett. B **131**, 59 (1983).

[55] J. R. Ellis, D. V. Nanopoulos and M. Quiros, "On The Axion, Dilaton, Polonyi, Gravitino And Shadow Matter Problems In Supergravity And Superstring Models," Phys. Lett. B **174**, 176 (1986).

[56] B. de Carlos, J. A. Casas, F. Quevedo and E. Roulet, "Model independent properties and cosmological implications of the dilaton and moduli sectors of 4-d strings," Phys. Lett. B **318**, 447 (1993) [arXiv:hep-ph/9308325].

[57] T. Banks, D. B. Kaplan and A. E. Nelson, "Cosmological implications of dynamical supersymmetry breaking," Phys. Rev. D **49**, 779 (1994) [arXiv:hep-ph/9308292].

[58] M. Kawasaki, K. Kohri and T. Moroi, "Hadronic decay of late-decaying particles and big-bang nucleosynthesis," Phys. Lett. B **625**, 7 (2005) [arXiv:astro-ph/0402490].

[59] M. Kawasaki, K. Kohri and T. Moroi, "Big-bang nucleosynthesis and hadronic decay of long-lived massive particles," Phys. Rev. D **71**, 083502 (2005) [arXiv:astro-ph/0408426].

[60] J. Polonyi, Hungary Central Research Institute Report KFKI-77-93 (Budapest, 1977) (unpublished).

[61] T. Moroi, "Effects of the gravitino on the inflationary universe," arXiv:hep-ph/9503210.

[62] I. Joichi, Y. Kawamura and M. Yamaguchi, "Stability of gauge hierarchy in Einstein supergravity," arXiv:hep-ph/9407385.

[63] A. Vilenkin and L. H. Ford, "Gravitational Effects Upon Cosmological Phase Transitions," Phys. Rev. D **26**, 1231 (1982); A. D. Linde, "Scalar Field Fluctuations In Expanding Universe And The New Inflationary Phys. Lett. B **116**, 335 (1982); K. Enqvist, K. W. Ng and K. A. Olive, "Scalar Field Fluctuations In The Early Universe," Nucl. Phys. B **303**, 713 (1988); A. Vilenkin, "Quantum Fluctuations In The New Inflationary Universe," Nucl. Phys. B **226**, 527 (1983).

[64] M. Endo, K. Hamaguchi and F. Takahashi, "Moduli-induced gravitino problem," Phys. Rev. Lett. **96**, 211301 (2006) [arXiv:hep-ph/0602061].

[65] S. Nakamura and M. Yamaguchi, "Gravitino production from heavy moduli decay and cosmological moduli problem revived," Phys. Lett. B **638**, 389 (2006) [arXiv:hep-ph/0602081].

[66] K. Sato, "First Order Phase Transition Of A Vacuum And Expansion Of The Universe," Mon. Not. Roy. Astron. Soc. **195**, 467 (1981).

[67] R. Brout, F. Englert and E. Gunzig, "The Creation Of The Universe As A Quantum Phenomenon," Annals Phys. **115**, 78 (1978).

[68] A. A. Starobinsky, "A New Type Of Isotropic Cosmological Models Without Singularity," Phys. Lett. B **91**, 99 (1980).

[69] W. Press, Phys. Scr. **21**, 702 (1980).

[70] R. H. Brandenberger, "Looking beyond inflationary cosmology," arXiv:hep-th/0509076.

[71] J. W. Moffat, "Superluminary universe: A Possible solution to the initial value problem in cosmology," Int. J. Mod. Phys. D **2**, 351 (1993) [arXiv:gr-qc/9211020].

[72] A. Albrecht and J. Magueijo, "A time varying speed of light as a solution to cosmological puzzles," Phys. Rev. D **59**, 043516 (1999) [arXiv:astro-ph/9811018].

[73] P. Horava and E. Witten, "Heterotic and type I string dynamics from eleven dimensions," Nucl. Phys. B **460**, 506 (1996) [arXiv:hep-th/9510209];
P. Horava and E. Witten, "Eleven-Dimensional Supergravity on a Manifold with Boundary," Nucl. Phys. B **475**, 94 (1996) [arXiv:hep-th/9603142].

[74] M. S. Turner and E. J. Weinberg, "Pre-big-bang inflation requires fine tuning," Phys. Rev. D **56**, 4604 (1997) [arXiv:hep-th/9705035];
N. Kaloper, A. D. Linde and R. Bousso, "Pre-big-bang requires the universe to be exponentially large from the very beginning," Phys. Rev. D **59**, 043508 (1999) [arXiv:hep-th/9801073].

[75] R. Kallosh, L. Kofman and A. D. Linde, "Pyrotechnic universe," Phys. Rev. D **64**, 123523 (2001) [arXiv:hep-th/0104073].

[76] P. J. Steinhardt and N. Turok, "Cosmic evolution in a cyclic universe," Phys. Rev. D **65**, 126003 (2002) [arXiv:hep-th/0111098].

[77] R. H. Brandenberger and C. Vafa, "Superstrings In The Early Universe," Nucl. Phys. B **316**, 391 (1989).

[78] S. Alexander, R. H. Brandenberger and D. Easson, "Brane gases in the early universe," Phys. Rev. D **62**, 103509 (2000) [arXiv:hep-th/0005212].

[79] R. Brandenberger, *Challenges for String Gas Cosmology*, arXiv:hep-th/0509099, to appear in the proceedings of the 59th Yamada Conference "Inflating Horizon of Particle Astrophysics and Cosmology" (University of Tokyo, Tokyo, Japan, June 20 - June 24, 2005).

[80] A. Nayeri, R. Brandenberger and C. Vafa, "Producing a Scale-Invariant Spectrum of Perturbations in a Hagedorn Phase of String Cosmology", arXiv:hep-th/0511140

[81] R. H. Brandenberger *et al.*, "More on the spectrum of perturbations in string gas cosmology," JCAP **0611**, 009 (2006) [arXiv:hep-th/0608186].

[82] N. Kaloper, L. Kofman, A. Linde and V. Mukhanov, “On the new string theory inspired mechanism of generation of cosmological JCAP **0610**, 006 (2006) [arXiv:hep-th/0608200].

[83] M. Majumdar and A.-C. Davis, “Cosmological creation of D-branes and anti-D-branes,” JHEP **0203**, 056 (2002) [arXiv:hep-th/0202148].

[84] R. Durrer, M. Kunz and M. Sakellariadou, “Why do we live in 3+1 dimensions?,” Phys. Lett. B **614**, 125 (2005) [arXiv:hep-th/0501163].

[85] A. Karch and L. Randall, “Relaxing to three dimensions,” arXiv:hep-th/0506053.

[86] J. M. Cline, “Inflation from extra dimensions,” Phys. Rev. D **61**, 023513 (2000) [arXiv:hep-ph/9904495].

[87] N. Arkani-Hamed, S. Dimopoulos, N. Kaloper and J. March-Russell, “Rapid asymmetric inflation and early cosmology in theories with sub-millimeter dimensions,” Nucl. Phys. B **567**, 189 (2000) [arXiv:hep-ph/9903224].

[88] N. Shuhmaher, in preparation.

[89] R. Brandenberger, Y. K. Cheung and S. Watson, “Moduli stabilization with string gases and fluxes,” arXiv:hep-th/0501032.

[90] R. Brandenberger, *Moduli Stabilization in String Gas Cosmology* to appear in the proceedings of YKIS 2005 (Yukawa Institute for Theoretical Physics, Kyoto, Japan, June 27 - July 1, 2005). arXiv:hep-th/0509159.

[91] A. A. Tseytlin and C. Vafa, “Elements of string cosmology,” Nucl. Phys. B **372**, 443 (1992) [arXiv:hep-th/9109048].

[92] G. D. Starkman, D. Stojkovic and M. Trodden, “Large extra dimensions and cosmological problems,” Phys. Rev. D **63**, 103511 (2001) [arXiv:hep-th/0012226]; G. D. Starkman, D. Stojkovic and M. Trodden, “Homogeneity, flatness and ’large’ extra dimensions,” Phys. Rev. Lett. **87**, 231303 (2001) [arXiv:hep-th/0106143].

[93] J. Polchinski, *String Theory, Vol. 2*, (Cambridge Univ. Press, Cambridge, 1998).

[94] T. J. Battefeld, S. P. Patil and R. H. Brandenberger, "On the transfer of metric fluctuations when extra dimensions bounce or stabilize," Phys. Rev. D **73**, 086002 (2006) [arXiv:hep-th/0509043].

[95] A. J. Tolley, N. Turok and P. J. Steinhardt, "Cosmological perturbations in a big crunch / big bang space-time," Phys. Rev. D **69**, 106005 (2004) [arXiv:hep-th/0306109].

[96] R. H. Brandenberger and J. H. Kung, "Chaotic Inflation As An Attractor In Initial Condition Space," Phys. Rev. D **42**, 1008 (1990).

[97] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, "The hierarchy problem and new dimensions at a millimeter," Phys. Lett. B **429**, 263 (1998) [arXiv:hep-ph/9803315];
I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, "New dimensions at a millimeter to a Fermi and superstrings at a TeV," Phys. Lett. B **436**, 257 (1998) [arXiv:hep-ph/9804398].

[98] L. Randall and R. Sundrum, "An alternative to compactification," Phys. Rev. Lett. **83**, 4690 (1999) [arXiv:hep-th/9906064].

[99] C. P. Burgess, M. Majumdar, D. Nolte, F. Quevedo, G. Rajesh and R. J. Zhang, "The inflationary brane-antibrane universe," JHEP **0107**, 047 (2001) [arXiv:hep-th/0105204].

[100] C. P. Burgess, P. Martineau, F. Quevedo, G. Rajesh and R. J. Zhang, "Brane antibrane inflation in orbifold and orientifold models," JHEP **0203**, 052 (2002) [arXiv:hep-th/0111025].

[101] S. H. S. Alexander, "Inflation from D - anti-D brane annihilation," Phys. Rev. D **65**, 023507 (2002) [arXiv:hep-th/0105032].

[102] M. Fabinger and P. Horava, "Casimir effect between world-branes in heterotic M-theory," Nucl. Phys. B **580**, 243 (2000) [arXiv:hep-th/0002073];
M. R. Douglas and G. W. Moore, "D-branes, Quivers, and ALE Instantons," arXiv:hep-th/9603167;
J. Polchinski, S. Chaudhuri and C. V. Johnson, "Notes on D-Branes," arXiv:hep-th/9602052;
J. Polchinski, "Lectures on D-branes," arXiv:hep-th/9611050.

[103] R. Allahverdi, K. Enqvist, J. Garcia-Bellido and A. Mazumdar, "Gauge invariant MSSM inflaton," Phys. Rev. Lett. **97**, 191304 (2006) [arXiv:hep-ph/0605035].

[104] D. N. Spergel *et al.*, "Wilkinson Microwave Anisotropy Probe (WMAP) three year results: Implications for cosmology," arXiv:astro-ph/0603449.

[105] L. Page *et al.*, "Three year Wilkinson Microwave Anisotropy Probe (WMAP) observations: Polarization analysis," arXiv:astro-ph/0603450.

[106] G. Hinshaw *et al.*, "Three-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: Temperature analysis," arXiv:astro-ph/0603451.

[107] N. Jarosik *et al.*, "Three-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: Beam profiles, data processing, radiometer characterization and systematic error limits," arXiv:astro-ph/0603452.

[108] M. Tegmark, A. J. S. Hamilton and Y. Z. S. Xu, "The power spectrum of galaxies in the 2dF 100k redshift survey," Mon. Not. Roy. Astron. Soc. **335**, 887 (2002) [arXiv:astro-ph/0111575].

[109] S. Cremonini and S. Watson, "Dilaton dynamics from production of tensionless membranes," arXiv:hep-th/0601082.

[110] S. M. Carroll, J. Geddes, M. B. Hoffman and R. M. Wald, "Classical stabilization of homogeneous extra dimensions," Phys. Rev. D **66**, 024036 (2002) [arXiv:hep-th/0110149].

[111] E. D. Stewart, "Inflation, supergravity and superstrings," Phys. Rev. D **51**, 6847 (1995) [arXiv:hep-ph/9405389].

[112] V. F. Mukhanov, H. A. Feldman and R. H. Brandenberger, "Theory Of Cosmological Perturbations. Part 1. Classical Perturbations. Part 2. Quantum Theory Of Perturbations. Part 3. Extensions," Phys. Rept. **215**, 203 (1992).

[113] J. Martin and C. Ringeval, "Inflation after WMAP3: Confronting the slow-roll and exact power spectra to CMB data," arXiv:astro-ph/0605367.

[114] L. Alabidi and D. H. Lyth, "Inflation models after WMAP year three," arXiv:astro-ph/0603539.

[115] T. Battefeld and S. Watson, "Effective field theory approach to string gas cosmology," JCAP **0406**, 001 (2004) [arXiv:hep-th/0403075].

[116] F. Ferrer and S. Rasanen, "Dark energy and decompactification in string gas cosmology," JHEP **0602**, 016 (2006) [arXiv:hep-th/0509225].

[117] A. Sen, "Rolling tachyon," JHEP **0204**, 048 (2002) [arXiv:hep-th/0203211].

[118] A. Sen, "Tachyon matter," JHEP **0207**, 065 (2002) [arXiv:hep-th/0203265].

[119] P. Mukhopadhyay and A. Sen, "Decay of unstable D-branes with electric field," JHEP **0211**, 047 (2002) [arXiv:hep-th/0208142].

[120] S. J. Rey and S. Sugimoto, "Rolling tachyon with electric and magnetic fields: T-duality approach," Phys. Rev. D **67**, 086008 (2003) [arXiv:hep-th/0301049].

[121] B. Chen, M. Li and F. L. Lin, "Gravitational radiation of rolling tachyon," JHEP **0211**, 050 (2002) [arXiv:hep-th/0209222].

[122] N. Lambert, H. Liu and J. Maldacena, "Closed strings from decaying D-branes," arXiv:hep-th/0303139.

[123] M. Gutperle and P. Yi, "Winding strings and decay of D-branes with flux," JHEP **0501**, 015 (2005) [arXiv:hep-th/0409050].

[124] X. Chen and S. H. Tye, "Heating in brane inflation and hidden dark matter," arXiv:hep-th/0602136.

[125] L. Kofman and P. Yi, "Reheating the universe after string theory inflation," Phys. Rev. D **72**, 106001 (2005) [arXiv:hep-th/0507257].

[126] H. Firouzjahi and S. Watson, private communication.

[127] R. Allahverdi, R. Brandenberger and A. Mazumdar, "Enhanced reheating via bose condensates," Phys. Rev. D **70**, 083535 (2004) [arXiv:hep-ph/0407230].

[128] J. F. Dufaux, G. Felder, L. Kofman, M. Peloso and D. Podolsky, "Preheating with trilinear interactions: Tachyonic resonance," JCAP **0607**, 006 (2006) [arXiv:hep-ph/0602144].

[129] B. R. Greene, T. Prokopec and T. G. Roos, "Inflaton decay and heavy particle production with negative coupling," Phys. Rev. D **56**, 6484 (1997) [arXiv:hep-ph/9705357].

[130] N. Shuhmaher and R. Brandenberger, in preparation.

[131] K. Dasgupta, J. P. Hsu, R. Kallosh, A. Linde and M. Zagermann, "D3/D7 brane inflation and semilocal strings," JHEP **0408**, 030 (2004) [arXiv:hep-th/0405247].

[132] P. Chen, K. Dasgupta, K. Narayan, M. Shmakova and M. Zagermann, "Brane inflation, solitons and cosmological solutions: I," JHEP **0509**, 009 (2005) [arXiv:hep-th/0501185].

[133] A. S. Goncharov, A. D. Linde and M. I. Vysotsky, "Cosmological Problems For Spontaneously Broken Supergravity," Phys. Lett. B **147**, 279 (1984).

[134] A. D. Linde, "Scalar Field Fluctuations In Expanding Universe And The New Inflationary Universe Scenario," Phys. Lett. B **116**, 335 (1982).

[135] A. Vilenkin and L. H. Ford, "Gravitational Effects Upon Cosmological Phase Transitions," Phys. Rev. D **26**, 1231 (1982).

[136] G. F. Giudice, A. Riotto and I. I. Tkachev, "The cosmological moduli problem and preheating," JHEP **0106**, 020 (2001) [arXiv:hep-ph/0103248].

[137] G. F. Giudice, I. Tkachev and A. Riotto, "Non-thermal production of dangerous relics in the early universe," JHEP **9908**, 009 (1999) [arXiv:hep-ph/9907510].

[138] G. N. Felder, L. Kofman and A. D. Linde, "Gravitational particle production and the moduli problem," JHEP **0002**, 027 (2000) [arXiv:hep-ph/9909508].

[139] J. R. Ellis, A. D. Linde and D. V. Nanopoulos, "Inflation Can Save The Gravitino," Phys. Lett. B **118**, 59 (1982).

[140] M. Dine, L. Randall and S. D. Thomas, "Supersymmetry breaking in the early universe," Phys. Rev. Lett. **75**, 398 (1995) [arXiv:hep-ph/9503303].

[141] L. Randall and S. D. Thomas, "Solving the cosmological moduli problem with weak scale inflation," Nucl. Phys. B **449**, 229 (1995) [arXiv:hep-ph/9407248].

[142] D. H. Lyth and E. D. Stewart, "Thermal inflation and the moduli problem," Phys. Rev. D **53**, 1784 (1996) [arXiv:hep-ph/9510204].

[143] T. Asaka and M. Kawasaki, "Cosmological moduli problem and thermal inflation models," Phys. Rev. D **60**, 123509 (1999) [arXiv:hep-ph/9905467].

[144] T. Asaka, M. Kawasaki and M. Yamaguchi, "Cosmological moduli problem and oscillating inflation in gauge-mediated supersymmetry breaking," Phys. Lett. B **451**, 317 (1999) [arXiv:hep-ph/9810334].

[145] M. Kawasaki and F. Takahashi, "Late-time entropy production due to the decay of domain walls," Phys. Lett. B **618**, 1 (2005) [arXiv:hep-ph/0410158].

[146] M. Dine, W. Fischler and D. Nemeschansky, "Solution Of The Entropy Crisis Of Supersymmetric Theories," Phys. Lett. B **136**, 169 (1984).

[147] G. D. Coughlan, R. Holman, P. Ramond and G. G. Ross, "Supersymmetry And The Entropy Crisis," Phys. Lett. B **140**, 44 (1984).

[148] A. D. Linde, "Relaxing the Cosmological Moduli Problem," Phys. Rev. D **53**, 4129 (1996) [arXiv:hep-th/9601083].

[149] J. H. Traschen and R. H. Brandenberger, "Particle Production During Out-Of-Equilibrium Phase Transitions," Phys. Rev. D **42**, 2491 (1990).

[150] Y. Shtanov, J. H. Traschen and R. H. Brandenberger, "Universe reheating after inflation," Phys. Rev. D **51**, 5438 (1995) [arXiv:hep-ph/9407247].

[151] G. N. Felder, J. Garcia-Bellido, P. B. Greene, L. Kofman, A. D. Linde and I. Tkachev, "Dynamics of symmetry breaking and tachyonic preheating," Phys. Rev. Lett. **87**, 011601 (2001) [arXiv:hep-ph/0012142].

[152] P. B. Greene and L. Kofman, "Preheating of fermions," Phys. Lett. B **448**, 6 (1999) [arXiv:hep-ph/9807339].

[153] K. Enqvist and A. Mazumdar, "Cosmological consequences of MSSM flat directions," Phys. Rept. **380**, 99 (2003) [arXiv:hep-ph/0209244].

[154] G. Lazarides, C. Panagiotakopoulos and Q. Shafi, "Phenomenology And Cosmology With Superstrings," Phys. Rev. Lett. **56**, 432 (1986).

[155] G. Lazarides, C. Panagiotakopoulos and Q. Shafi, "Baryogenesis And The Gravitino Problem In Superstring Models," Phys. Rev. Lett. **56**, 557 (1986).

[156] T. Moroi, M. Yamaguchi and T. Yanagida, "On the solution to the Polonyi problem with 0 (10-TeV) gravitino mass in supergravity," Phys. Lett. B **342**, 105 (1995) [arXiv:hep-ph/9409367].

[157] M. Kawasaki, T. Moroi and T. Yanagida, "Constraint on the Reheating Temperature from the Decay of the Polonyi Field," Phys. Lett. B **370**, 52 (1996) [arXiv:hep-ph/9509399].

[158] T. Moroi and L. Randall, "Wino cold dark matter from anomaly-mediated SUSY breaking," Nucl. Phys. B **570**, 455 (2000) [arXiv:hep-ph/9906527].

[159] S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, "De Sitter vacua in string theory," Phys. Rev. D **68**, 046005 (2003) [arXiv:hep-th/0301240].

[160] M. Dine, R. Kitano, A. Morisse and Y. Shirman, "Moduli decays and gravitinos," Phys. Rev. D **73**, 123518 (2006) [arXiv:hep-ph/0604140].

[161] D. Lindley, "Cosmological Constraints On The Lifetime Of Massive Particles," Astrophys. J. **294**, 1 (1985); M. Y. Khlopov and A. D. Linde, "Is It Easy To Save The Gravitino?," Phys. Lett. B **138** (1984) 265; F.Balestra et all., Sov. J. Nucl. Phys. **39**, 626 (1984); M. Y. Khlopov, Yu. L. Levitan, E. V. Sedelnikov and I. M. Sobol, "Nonequilibrium cosmological nucleosynthesis of light elements: Calculations by the Monte Carlo method," Phys. Atom. Nucl. **57**, 1393 (1994) [Yad. Fiz. **57**, 1466 (1994)]; J. R. Ellis, J. E. Kim and D. V. Nanopoulos, "Cosmological Gravitino Regeneration And Decay," Phys. Lett. B **145**, 181 (1984); R. Juszkiewicz, J. Silk and A. Stebbins, "Constraints On Cosmologically Regenerated Gravitinos," Phys. Lett. B **158**, 463 (1985); J. R. Ellis, D. V. Nanopoulos and S. Sarkar, Nucl. Phys. B **259**, 175 (1985); J. Audouze, D. Lindley and J. Silk, "Big Bang Photosynthesis And Pregalactic Nucleon Synthesis Of Light Elements," Astrophys. J. **293**, L53 (1985); M. Kawasaki and K. Sato, "Decay Of Gravitinos And Photodestruction Of Light Elements," Phys. Lett. B **189**, 23 (1987).

[162] M. Kawasaki, F. Takahashi and T. T. Yanagida, "Gravitino overproduction in inflaton decay," Phys. Lett. B **638**, 8 (2006) [arXiv:hep-ph/0603265].

[163] M. Kawasaki, F. Takahashi and T. T. Yanagida, "The gravitino overproduction problem in inflationary universe," Phys. Rev. D **74**, 043519 (2006) [arXiv:hep-ph/0605297].