Shifting Pre-Service Teachers' Beliefs About Mathematics Teaching: The Contextual Situation of a Mathematics Methods Course

Abstract

For pre-service teachers (PSTs) who have been exposed to traditional approaches, teacher education courses can be a revelatory experience in their development as educators. This study explores if Canadian upper elementary/lower secondary (grades 4-10) PSTs change their beliefs about mathematics teaching as a result of taking a mathematics methods course and how the course influenced these beliefs. Surveys were used to measure participants' mathematics beliefs and results show that PSTs' beliefs moved to favour reform-based approaches. Qualitative data complemented the survey results, suggesting that experiencing new approaches and having the opportunity to apply them into practice is important to their development as mathematics teachers.

Keywords: methods courses; pre-service teachers; situated learning; teacher beliefs; teacher education programs

Introduction

Teachers who recognize that their pedagogical practices have a positive impact on student learning and achievement tend to be more willing to implement new teaching strategies, take risks with their instructional practices, and work to achieve student goals (Bruce & Ross, 2008). Research has shown that this type of teacher contributes to an increase in student achievement (e.g., Bandura, 1997; Henson, 2002). Pre-service teachers (PSTs) enter their teacher education programs with strong beliefs rooted in their experiences as students and their own interactions with teachers (Lortie, 1975; Pajares, 1992). In mathematics, where PSTs often have traditional experiences as students, they may enter teacher education

programs with a narrow vision of mathematics education (Swars, Hart, Smith, Smith, & Tolar, 2007). Thus, their teacher education program is of great importance as in order to successfully create a reform-based environment as teachers, traditionally-minded PSTs need to change their beliefs about mathematics education (Richardson & Placier, 2001).

While there has been much research that has indicated that teacher education programs have had minimal impact on teacher practice (e.g., Burkhardt & Schoenfeld, 2003; Cochran-Smith & Zeichner, 2005), others have more optimistic perspectives (e.g., Brouwer & Korthagen, 2005; Day, 1999). Researchers have written of the need to go beyond broad research on the impact of teacher education and to instead focus on specific aspects of teacher education programs that positively affect PST development (Bransford, Darling-Hammond, & LePage, 2005; Grossman & Schoenfeld, 2005; Shulman, 2004). Responding to this call, this study specifically explores the impact of one component of a teacher education program, a mathematics methods course, on PSTs' beliefs about mathematics teaching and learning.

Methods courses have the potential to change PSTs' beliefs (Evans, Leonard, Krier, & Ryan, 2013; Wilkins & Brand, 2004). This study complements previous research by investigating course factors that contribute to change in PSTs' beliefs. For this two-part study, the research question guiding Phase I was: How have the beliefs of Canadian upper elementary/lower secondary PSTs changed as a result of a mathematics methods course? Phase II of the study investigated the question: How did the course change their beliefs?

Approaches to Mathematics Education

Many mathematics PSTs start their teacher education program having only experienced learning mathematics using traditional approaches. In a traditional approach to education, teachers "transmit facts, skills, and values to students" (Miller, 2010, p. 15). This approach

often uses a textbook to support student learning and follows a step-by-step, procedural format for skill development. The learning context is teacher-centered with the teacher as a knowledge provider. Advocates of the traditional approach to teaching assert that it is the most efficient way of teaching (Miller, 2010). In mathematics, this approach emphasizes a fluency in procedures (Ball, 1990) rather than conceptual understanding of mathematical ideas and follows a behaviourist learning model (Battista, 1994).

By contrast, a reform-based approach is currently advocated for and values student-centered and exploratory learning (Hiebert, 1990). Teaching strategies that align with a reform-based approach include problem-based learning (e.g., Lambros, 2002; Savery, 2006) and inquiry-based learning (e.g., Li, Moorman, & Dyjur, 2010; Towers, 2010). While each strategy has unique characteristics, all share the following similarities: the learning context has a degree of ambiguity and open-endedness; teachers take on the role of facilitator, guiding student learning by supporting and guiding students' discoveries; and students are at the fore of the learning context constructing their learning through discussion and collaboration (Lampert & Ball, 1998). It follows that principles of constructivism form the underpinnings of many strategies in reform-based classrooms (Steffe & Cobb, 1988). Other strategies that align with reform-based approaches include the use of manipulatives, multiple forms of representation (oral, visual, written), and real-world connections can encourage connections between ideas (Lesh, Post, & Behr, 1987). Reform-based approaches also support current international trends in mathematics education (e.g., Australia – Board of Studies, NSW, 2012; China – Liu & Li, 2010; Netherlands – Van den Heuvel-Pauhauizen & Drijvers, 2014; United Kingdom – Department for Education, 2014).

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Although research has discussed the benefits of and promoted a reform-based approach, many teachers continue to use traditional approaches. Mathematics teachers tend to teach the way that they were taught (Ball, 1988) and this may be attributed to their beliefs as teachers. PSTs' previous experiences as students contribute to their beliefs about education (Lortie, 1975; Pajares, 1992) and these in turn guide their teaching practices (Feiman-Nemser & Floden, 1986; Richardson, 1996). For PSTs, their teacher education program is the last opportunity to experience teaching and learning mathematics in a reform-based way before entering the workplace. Thus, teacher education programs should provide opportunities for PSTs to change their existing traditional beliefs.

Teacher Education Programs – An Opportunity to Re-examine PSTs' Beliefs

A belief is a proposition of content accepted as true by the individual holding it (Evans et al., 2013). Many mathematics PSTs come into their teacher education with certain beliefs and traditionally, teacher education programs have not challenged PSTs to re-examine these beliefs (Leaman & Flanagan, 2013). Although these beliefs may be hard to change (Pajares, 1992) mathematics teachers whose beliefs align with reform-based ideals are more likely to use reform-based practices.

Typically, teacher education programs are comprised of coursework and practice teaching blocks (practica). The inclusion of practica attempts to allow the opportunity for PSTs to experience teaching and learning in a context similar to what they will experience in the future, yet some research has shown that these practica may inadvertently reinforce undesirable practices (Maynard, 2001). Thus a thoughtful effort is needed to ensure that coursework provides optimal opportunities for PSTs' development.

Teacher education courses are challenged with the task of bridging the gap between teaching theories and teaching practice and re-examining PSTs' beliefs about education (Feiman-Nemser & Norman, 2000). It is the role of these courses to move the PSTs forward from their limited insights of the intricacies of teaching developed through their *apprenticeship of observation*. This term, coined by Lortie (1975) describes the conceptions about teaching developed by PSTs through their time spent as students observing and evaluating their own teachers. Throughout this apprenticeship of observation, PSTs do not consider the teacher's intentions and reflections on classroom events. Thus, teacher education courses should foster the PSTs' transition to an active member of the classroom who recognizes and reflects upon the decision-making process that their teachers undergo before, during and after the lesson (Leaman & Flanagan, 2013).

PSTs hold closely to their own mathematics learning experiences and related beliefs; this is of particular concern for mathematics PSTs who experienced only traditional approaches as students. These experiences result in two different, yet related challenges. The first, concerns PSTs' views that students are passive members of the mathematics classroom being the recipients of the teacher's actions. The PSTs experience of learning mathematics in traditional contexts only intensifies the perils of their apprenticeship of observation. Secondly, many PSTs have no experience with reform-based practices and teaching theories discussed in current literature. So, in addition to the previously discussed need for PSTs to become active and reflective members of their learning, teacher education courses should provide opportunities to experience mathematics learning in a context similar to those which they will create as teachers themselves. In doing so, PSTs will be better prepared to face real classroom situations (Biza, Nardi, & Zacharidades, 2007).

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Situated Learning

Situated learning theory asserts that learning takes place in the context in which it occurs (Bell, Maeng, & Binns, 2013). Situated learning theory assumes the following: understanding of a concept is constantly developing, knowledge is learned in an authentic context, and learning is a social activity (Lave & Wenger, 1991). Thus, for the context of teacher education, PSTs' learning context should reflect the context in which they shall teach in the future (Ethell & McMeniman, 2000).

McLellan (1996) proposed a model of instruction based on situated learning theory with the following four components: 1) cognitive apprenticeship and coaching, 2) opportunities for multiple practice, 3) collaboration, and 4) reflection. A *cognitive apprenticeship* occurs when teachers provide their students with increasingly complex problems to solve and foster opportunities for the students to apply their learning in new, yet relevant, situations (Brown & Duguid, 1989). In teacher education courses, an example of cognitive apprenticeship may occur if PSTs are asked to apply a teaching strategy that was modelled for them by the teacher educator. The teacher's role in the cognitive apprenticeship model is crucial to student development. Rather than directly telling students what they need to do or know to solve the presented problem, teachers assume the role of *coaches* and scaffold and guide student learning (Huang, Lubin, & Ge, 2011). The multi-layered scaffolding and opportunities for application of student learning provide *opportunities for multiple practice*. C*ollaboration* is encouraged through discussion with peers and the teacher. Teachers may ask students to *reflect* on their learning as part of the discussion process or provide other opportunities for students to make observations, predictions and suggest theories about the material that they are learning. Each of these components should occur within an authentic

context thus, teacher educators should create suitable experiences and realistic situations in which PSTs can encounter and grapple with reform-based approaches of mathematics education.

Teaching methods courses provide a natural venue for PSTs to experience situated learning. Generalists teachers are required to teach all areas of the curriculum. Thus, mathematics methods courses are a subject-specific venue to support PSTs' development in a certain domain of their teaching and provide a rich opportunity for PSTs to experience mathematics in a situated way. More specifically, in this course, the PSTs should have the opportunity to engage with mathematics as both learners and teachers that reflect reform-based ideals.

Context of the Study – The Mathematics Methods Course

A mathematics methods course provides an introduction to exemplary mathematics education techniques during which PSTs learn the role of the teacher in implementing, evaluating and designing mathematics curricula (Dunn, 2005). The context of this study was a 39 hour (thirteen, three-hour classes) mathematics method course taken by upper elementary/lower secondary PSTs enrolled in their first year of a two-year teacher education program in a faculty of education in Central Canada[1]. This course is the only mathematics methods course taken by PSTs in this program. In this region of Canada, upper elementary/lower secondary comprises grades six through ten (students ages 11-15) and graduates of this program typically seek employment as generalist teachers.

As the instructor of this course, I aimed to create a situated context (Lave & Wenger, 1991) in which PSTs could experience learning in an environment that modelled reform-

[1] The program consists of coursework (15 courses in total), a major research paper, and four, four-week practica.

based ideals. A variety of techniques such as discussions, presentations, and active participation incorporating small and large group learning were used in the course. Although primarily dealing with pedagogy, mathematics content was embedded through in-class activities used to highlight specific teaching practices. PSTs were assigned to read research-based articles for each class and classes were assigned a group of guiding pedagogy- and content-based themes. The themes were selected to match reform-based ideals and areas of focus for mathematics education (McDougall, 2004) as highlighted by the National Council of Teachers of Mathematics (2000). Examples of themes included: algebra, fractions, program planning, problem solving, and technology.

Each class centered around two or three mathematics reform-based activities that dealt with both the mathematics content and pedagogical themes. As the facilitator of these activities, I modelled teacher behaviours that I hoped the students would develop for themselves. During the activity, I would fluctuate between the role of mathematics teacher (facilitating the activity) and teacher educator (describing my pedagogical decisions). As described by Swars et al. (2007), "thoughtful reflection and examination of teaching and learning" is needed for PSTs "to align their pedagogical beliefs with current thinking" (p.325). Thus, when the activity was complete, I continued a discussion of the pedagogy of the activity and encouraged PSTs to contribute to the discussion by asking them to reflect on the activity. Specifically, I asked PSTs to expand upon their experiences as a learner, and suggest modifications and adaptations to improve my activity and meet the needs of various learners and learning contexts. Furthermore, throughout our discussions I urged PSTs to make connections to the assigned readings.

Assignments for the course were purposefully created to provide opportunities for PSTs to be reflective practitioners and build upon their experiences as students in the course. Many of the assignments were scaffolded to provide multiple opportunities for PSTs to practice specific skills and deepen their understanding of reform-based mathematics teaching practices. For example, an earlier assignment in the course asked PSTs to create a lesson plan for a 40-minute lesson on a topic of their choosing. PSTs were given the rubric used to assess their lesson plan, and descriptors on the rubric indicated that I would be looking for reform-based ideals to be incorporated into their lesson. In a study by Swan et al. (2002), PSTs expressed the value of seeing exemplary teaching practices modelled within a context of a lesson prior to having to implement similar practices into their own lessons. For this reason, I had modelled two different 40-minute lessons so that the PSTs experienced learning mathematics in a reform-based way. Accompanying these lessons was a written lesson plan using the same format that the PSTs would be using for their own assignments. When we debriefed each of these lessons, I had PSTs discuss their experiences, observations, and critiques of my lesson (and lesson plan) and its delivery. As described by Loughran (2006), it was just as important for me to share my pedagogical reasoning for the different components in my lesson as it is for the PSTs to be able to do the same.

A later assignment in the course asked students to work in small groups to create a series of lessons that could constitute a unit of study covering a group of curriculum expectations in a grade of their choosing. Each group gave a presentation about their unit, during which, each presenter was responsible for facilitating a short activity. Similar to the individual lesson plan assignment, assessment criteria for the unit plan indicated that it should include reform-based ideals. A final assignment was an essay in which PSTs were asked to reflect upon their

journey as a developing mathematics educator taking into account their experiences including those as a mathematics student, a student in my course, and in practica.

Methods

This study explored whether PSTs' beliefs about mathematics education changed after completing a mathematics methods course and how this course influenced these beliefs. Informed by the exploratory case method (Yin, 2009), data for this study was comprised of surveys, PST essays and interviews. This mixed methods (Creswell & Plano Clark, 2007) approach was chosen to "obtain different but complementary data on the same topic" (Morse, 1991, p.122). Qualitative data were used to validate and expand quantitative results. Participants were PSTs enrolled in the mathematics methods course described earlier and formed the case for this study. Twice throughout the course, classes paused for a practice teaching block. PSTs may or may not have had the opportunity to teach mathematics during these practica.

In Phase I of the study, data was gathered via a survey about their beliefs about mathematics. This survey had three sections. The first section contained questions to classify the participant and uncover their mathematics background. The second section contained 20 Likert-type questions (Appendix A) about the participant's beliefs about mathematics education. The survey questions were taken from McDougall's (2004) Attitudes and Practices to Teaching Math Survey, a research-based and validated survey created to provide feedback regarding the degree to which the teacher's attitudes and teaching practices align with current mathematics education thinking. The final section had open-ended questions that allowed participants to elaborate on their current beliefs. The results from the quantitative data and open-ended questions provided a framework for components of Phase II of the

study. Participants completed the survey twice: once in the first class and then again in the second last class of the course.

Phase II consisted of an essay, which was the final assignment for the course, and PST interviews. As described in the context of the study, the essay asked PSTs to reflect on their journey as developing mathematics educators. Participants had access to their completed surveys and were encouraged to refer to them in their essays. PSTs were advised of this final assignment in the first class of the course when reviewing course objectives and assignments. The guidelines of the assignment stated that the essay was to be between four to six pages double-spaced in length and submitted within one week following the final class of the course. One-on-one PST interviews were completed after the course and followed-up on themes that arose from reviewing participants' surveys and essays. Interviews ranged in length from 45 minutes to 120 minutes based on the availability of the PSTs. Interviews were audio-taped and transcribed in preparation for analysis using nVivo9. Qualitative data was coded through a constant comparison analysis method (Miles & Huberman, 1994). The initial inductive coding scheme was based on themes from literature (McLellan, 1996) and was deductively elaborated on based on the emerging themes from the data.

All PSTs completed the surveys and essay as part of the course. Prior to completing the first survey (in the first class of the course), PSTs were reassured that their surveys were kept confidential and that I would be the only one reading their answers. I encouraged the PSTs to be honest and open about their views and experiences as I would be using their responses to shape course themes and activities and wanted an authentic starting point from which to work off of. The PSTs were also reminded that their completed two surveys would be returned to them to aid the writing of their final essay assignment. I told the PSTs that I would be looking

for them to synthesize their personal experiences and reflect upon how these experiences shape their developing philosophy of mathematics education. I reminded the PSTs that each may be at a different stage in their development as mathematics educators and that they each may have a different philosophy of mathematics education.

PSTs taking the course were invited to participate in any or all components of the study. As my position as course instructor may have shaped the data that was collected, PSTs were assured that participation in the study would not affect their performance in the course. Additionally, PST interviews and data analysis occurred after final grades had been submitted. Names of participants have been replaced with pseudonyms. From a class of 28 PSTs, 19 consented to the use of their mathematics beliefs survey and essay for the research study and 12 agreed to be interviewed for Phase II of the study. While my dual position of researcher and course instructor may have coloured PSTs' survey responses, essays, and interview responses, I hoped that my attempt to create an open learning (and research) environment allowed the PSTs to voice their honest opinions and shape their own beliefs about mathematics teaching and learning. My approach seemed to be embraced by the PSTs, as Gretchen noted in her interview, "The whole year, you were kind of like, 'Make it your own. And don't do what I'm going to do because it's not necessarily going to work for you or your students.'" Kim also indicated that she felt comfortable being honest in sharing her journey as a developing mathematics educator in spite of her negative views towards mathematics. When prompted to discuss a passage of her choosing of her essay, Kim replied:

> "The only thing worse than being a student of math is being a teacher of math."
>
> That's how I felt seven months ago. I remember reading this [as a] rough draft, and I

thought, "That's going to go before I hand this in." But it stayed because I feel like if you're not going to do a reflection honestly, then what's the point?

While this feedback may be indication that PSTs felt comfortable articulating their true beliefs about mathematics teaching and learning, I acknowledge that PSTs possibly conveyed beliefs that they thought I would want to hear.

Results and Discussion

In this section, I first respond to the research question posed in Phase I (How have the PSTs beliefs changed?), then conclude by addressing the question for Phase II (How did the course change PSTs' beliefs?).

Changes in PSTs' Beliefs

Due to the small sample size, I used a Wilcoxon Signed Rank Test to determine whether PSTs moved significantly from traditional views to reform-based views throughout the course. I calculated each PST's average responses on the questionnaire and found that this average increased from 4.22 to 4.51 (Wilcoxon $w = 28$, $p=0.013$) thus indicating that PSTs shifted their mathematics beliefs towards a more reform-based approach.

PSTs' initial beliefs. Qualitative data suggest that PSTs entered the mathematics methods course with beliefs shaped by their previous experiences with mathematics. Catherine described her mindset when completing the first survey and how her experiences as a mathematics student had shaped her initial beliefs: "I was basing…my responses on my experiences in math; my experiences were very teacher directed…I had paper and pencil tests and…loved copying down notes off of the board from the teacher." Catherine enjoyed and did well in her mathematics courses. At the beginning of the methods course, Catherine's beliefs about mathematics education paralleled the approaches to teaching and learning

mathematics that she enjoyed as a student and allowed her to be successful in her learning. River spoke of other factors that contributed to his beliefs entering the methods course, "After many years of work (as an engineer), I was expecting to come to university and start to learn (more) about what I knew about mathematics education based on all of my experiences." In addition to his experiences as a mathematics student, River mentioned that his use of mathematics in a professional context impacted his beliefs. Most PSTs described the majority of their experiences with mathematics as being in a traditional learning context. It was these experiences that shaped their beliefs and were reflected in their responses in the first mathematics beliefs survey.

The mathematics methods course: An opportunity for change. The mathematics methods course provided an opportunity for PSTs to (re)consider their beliefs. Warren shared that right from the first activity, he realised that his beliefs about mathematics education may need to be reconsidered. In this activity, I engaged the PSTs in a paper-cutting activity during which I asked PSTs to problem solve, predict outcomes of various cuts, and propose alternative explorations. Warren spoke of the introductory activity as a learning experience that was in contrast to his beliefs about mathematics education, "[This activity] was something that I did not expect. It made me really think about what mathematics instruction could be." Warren reinforced that the methods course provided him with experiences and opportunities to re-shape his belief. Other PSTs echoed that the course was pivotal in shaping their evolving beliefs. As Flora said in her interview, "The key shifts [in my beliefs] that happened were as a result of the (methods) course. This idea that math could be collaborative and fun. That was all you."

Components of the Mathematics Methods Course Leading to Changes in PSTs' Beliefs

Each of the questions on the mathematics beliefs survey (Appendix A) focus on a component of mathematics education. PSTs' responses suggest the degree to which their beliefs align with the component highlighted in the question. Table 1 contains the changes observed on a per-question basis.

Table 1
Changes Observed on a Per-question Basis

Question	Pre-test question average (out of 7)	Post-test question average (out of 7)	Change from pre- to post-test	Wilcoxon *w*	P-value (* if <0.05)
1	4.74	5.11	0.37	25.0	0.28
2	5.68	5.79	0.11	3.5	0.32
3	5.21	4.68	-0.53	74.5	0.04*
4	5.21	5.21	0.00	18.0	1.00
5	4.84	4.79	-0.05	44.0	0.94
6	5.58	5.37	-0.21	17.5	0.16
7	2.89	3.32	0.42	14.0	0.09
8	3.68	4.21	0.53	42.0	0.31
9	3.16	3.95	0.79	16.5	0.01*
10	3.74	4.53	0.79	23.5	0.02*
11	3.11	3.58	0.47	28.0	0.06
12	3.68	4.37	0.68	8.0	0.02*
13	4.74	5.11	0.37	22.0	0.35
14	1.89	2.26	0.37	43.0	0.34
15	5.32	5.53	0.21	37.5	0.33
16	3.05	3.47	0.42	36.5	0.18
17	5.47	5.47	0.00	17.5	0.59
18	5.11	4.79	-0.32	41.0	0.49
19	4.47	5.37	0.89	13.0	0.01*
20	2.89	3.37	0.47	52.0	0.25

Positive changes in PSTs' responses suggest changing beliefs favouring reform-based approaches, while negative changes in PSTs' responses suggest changing beliefs favouring traditional approaches. PSTs' responses to five questions from the mathematics beliefs survey showed significant change. Four questions highlighted PSTs' changing beliefs favouring reform-based approaches (Q9, Q10, Q12, and Q19) and one question highlighted

change favouring traditional approaches (Q3). While is it interesting to note the four questions suggesting changes favouring reform-based approaches were phrased negatively, seven other questions on the survey were also phrased negatively (Q5). Four of these questions resulted in slight positive changes (Q7, Q13, Q16, Q20) and two with no changes (Q4, Q17).

In the next sections, I present findings as suggested by the qualitative data exploring how the mathematics methods course supported changes in PSTs' beliefs, first by exploring facets of the course that led PSTs to favour reform-based and traditional approaches respectively. I then describe beliefs resulting in no changes and conclude by describing other elements of the teacher education program that had an impact on PSTs' changing beliefs.

Changes to Favour Reform-based Approaches. Four questions from the mathematics beliefs survey were more effective than others in illuminating components of PSTs' changing beliefs to favour reform-based approaches. Each question focused on a particular component of mathematics teaching and was an area of focus in the course: use of student-centered learning (Q9), non-traditional assessment strategies (Q10), classroom discussions (Q12), and open-ended tasks (Q19).

Providing a new experience. As most of the PSTs had only experienced traditional learning contexts previous to the mathematics methods course, the course provided the PSTs with opportunities to experience a reform-based approach. As described by Catherine, "[The class] was really different from my own math education.…[in this course], we were figuring it out rather than you just telling us what was right and how to do things." Catherine continued by describing that these new experiences with student-centred learning (Q9) encouraged her to consider the benefits of this approach. Other PSTs echoed that

experiencing reform-based approaches as a mathematics learner influenced their developing beliefs. Katie provided an experience that shaped her beliefs about student-centered learning (Q9) and classroom discussions (Q12):

> In the class with concept attainment, you let the students direct the learning…you let us sort it out. And we sort of had this big class discussion, just the students in trying to understand the concepts.

PSTs spoke of the importance of experiential learning in shaping their beliefs. To reiterate that he had not experienced reform-based approaches previously as a mathematics student, Manoel said,

> The hands-on stuff was really good for me because of my relationship with math. In literacy, I feel the opposite. Reading really theoretical stuff in literacy was really helpful because I have an ease. I know what to do. But in math, I need those things.

Manoel's words underline his differences in experiences as a learner. He had never experienced learning mathematics that mirror current educational theories and therefore valued the opportunity to participate in reform-based learning activities. In other courses in the teacher education program (e.g., the literacy methods course) Manoel could rely on his previous experiences as a literacy student to make connections to the theories discussed in class. Although PSTs described experiencing reform-based approaches in the course as fundamental to their shift in beliefs, findings indicate that there were a variety of causes for PSTs to change their beliefs as a result of these experiences.

Many PSTs were encountering reform-based approaches for the first time in the mathematics methods course. Research indicates that doing mathematics supports change in PSTs' beliefs (Anderson & Piazza, 1996). The course provided opportunities for PSTs to

experience reform-based approaches first as mathematics learners before considering their use in their practice.

Negating a previously negative experience. For some PSTs who previously had negative experiences with mathematics as a student, experiencing an approach in the course that was in contrast to the (traditional) approach that they had experienced as a student encouraged a shift in their beliefs. In her essay, Wendy described the impact of experiencing student-centred learning (Q9) in the course: "I believe [student centred] math is engaging as students are discovering for themselves rather than following teacher instructions and just doing." When asked to elaborate on this statement in her interview, Wendy said,

> [T]hat was a really big shift for me. [At the beginning of the course] I had no concept that I could be [a teacher] so different from what I originally thought and what I experienced. I am so happy that [there is] something different because that makes [mathematics] a lot more exciting and a lot less scary to me.

This PST directly attributed her change in beliefs to experiencing a different approach to mathematics as compared to her own schooling. Initial beliefs about mathematics education were shaped by her previous, albeit negative, experiences as a mathematics student but a different and more positive experience in the course led to a change in beliefs.

Generalist PSTs often enter their teacher education program with mathematics anxiety and lacking content knowledge. Ashcraft (2002) described mathematics anxiety as the negative attitude towards, or lack of confidence in, mathematics. For many PSTs, learning mathematics in a traditional learning context contributed to their mathematics anxiety (Stuart & Thurlow, 2000; Trujillo & Hadfield, 1999). Conversely, learning through reform-based approaches has been found to reduce mathematics anxiety as these approaches provide

diverse ways for students to engage in the mathematics and are more accessible to a greater variety of learners (Furner & Berman, 2003; Vinson, 2001). Experiencing reform-based approaches in the course benefitted these mathematics anxious learners.

Creating an enhanced learning experience. Experiencing an approach and seeing that it enhanced their own mathematical understanding resulted in a shift in PSTs' beliefs. As described by Brittany, "When we would do the activities in class, that was something really valuable and I could see the benefits for myself. That meant that I could also see benefits for future students." For Brittany, realizing that an approach supported her own learning meant that it could more than likely support the learning of students in her own classroom.

To explain her shift in beliefs towards a more reform-based approach, Gretchen described a student-centred activity (Q9) that the PSTs' engaged in during the course. In this activity, students investigated the concept of Venn Diagrams by creating a human Venn Diagram in response to a variety of prompts that I provided. Gretchen shared that at the beginning of the activity, she could not remember what a Venn Diagram was and how to create one. She shared, "I probably would have remembered what a Venn Diagram was if I had learned it as a kid by moving around or by doing it in a different way instead of just drawing circles." This student-centred activity allowed Gretchen to engage in learning about Venn Diagrams. For PSTs, (re-) learning mathematics using reform-based approaches allowed them to address misconceptions and strengthened content knowledge (Ball, Lubienski, & Mewborn, 2001; Green, Piel & Flowers, 2008). PSTs developed positive feelings towards this reform-based approach and feeling empowered because they understood mathematics concepts for which they previously had a fragile understanding. This enhanced learning experience contributed to shifting her beliefs about mathematics education.

Fostering an enjoyable learning experience. To explain his belief that open-ended tasks (Q19) are effective in supporting students' learning, Martin named the Math Trail, during which students worked in groups on a series of tasks in the community, as a moment in the course that stood out to him. "[The Math Trail] was an opportunity for me to shine in the class. I was able to show my expertise in math, so I really enjoyed that." Martin's beliefs were shaped by a positive experience in the course and more specifically, an opportunity for the PST to take on a leadership role and show off his strengths in front of his peers.

For many of the PSTs, the opportunity to experience these approaches as a mathematics learner and seeing the positive benefits of the approach (either for their own learning or their peers) had an impact on their beliefs. Yet, negative moments in the course also shaped PSTs' beliefs. Gretchen described in instance in the course that did not support her learning and specifically shaped her beliefs about the benefits of student discussion (Q12).

> Any time it was something individual and I could not consult someone else for help or reassurance, I immediately felt anxiety and it was just an overall negative experience. But I realized that helped me understand how not to teach my class.

While these experiences in the course were not positive, the PSTs used them to form their beliefs by considering these experiences as examples of approaches that they would not use in their own teaching. Findings suggest that PSTs formed beliefs though their experiences in the course (positive and negative). PSTs beliefs about mathematics education were in parallel to approaches that support their own learning as mathematics students.

Having an opportunity to reflect. PSTs appreciated the opportunity to reflect on their experiences with reform-based approaches. As Kim described in her interview:

> I feel like we did a lot of reflecting in the course. Individual time to write down whatever…I really appreciate the process of reflection. Especially after the final reflection assignment where I didn't even realize what I'm thinking until I'm writing it down. It's a really important part of the process because you really don't figure out a lot (about your beliefs) until you [reflect].

In her essay, Kim wrote about her change in beliefs regarding student centred learning (Q9), and encouraging student discussion (Q12).

PSTs shared that reflection allowed them to more deeply consider their beliefs and how they could translate their developing beliefs into practice. Brittany described this when discussing her beliefs about alternative forms of assessment (Q3): "I was questioning myself, 'How could I bring this into my math class…what am I going to do with it…and how are [my students] going to be assessed on it?'"

PSTs commented on the value of dedicated moments in the course for reflection. Without these moments, the PSTs said that they would not have as deep an understanding of their experiences and the additional insights that they gained from critiquing and reflecting upon their teaching and learning. As Martin said,

> [These opportunities] allowed me to sit back and actually reflect on what went on which I don't think I had much of an opportunity to do that before. Just in general…day to day. Yeah, you sit back and you think about your [experiences], but you don't actually jot down notes or reflect on how [they] actually went which I should have probably done. But just to sit back and to grapple with the issues that I was dealing with, it was good to do that.

Class discussions and self-reflection tasks provided targeted opportunities for the PSTs to draw upon their new experiences learning through reform-based approaches in order to extend PSTs' thinking about mathematics education (Calderhead, 1989; Jay & Johnson, 2002).

Changes to Favour Traditional Approaches

Not all beliefs shifted to favour reform-based approaches. One question from the mathematics beliefs survey highlighted a significant shift in PST beliefs, however, to favour traditional approaches. This question was about the use of manipulatives in the mathematics classroom (Q3). It appears that after taking the mathematics methods course, the PSTs were now more resistant to the use of manipulatives. This result is interesting as the use of manipulatives was also a focus of the course and my approach towards introducing these learning tools to the PSTs was no different than other teaching strategies used.

The PSTs' change in beliefs may have been affected by their own confusions of the efficacy of manipulatives as they themselves successfully learned mathematics without them. Kramer wrote in his essay, "I find manipulatives challenging since I was taught mathematics in a traditional paper and pencil approach." Many PSTs commented that manipulatives made exploring the mathematics concept more complicated than necessary. Wendy eluded to this in her interview: "I just did not get it and it bothered me because I just did not want to use them. I just was not used to it so I was learning from the beginning again."

As described earlier, many PSTs' beliefs were shaped by their opportunities to experience the teaching approach and more specifically, experiences in which the approach had a positive affect on the learner. Said Martin, "I still haven't seen a situation where students really understand something because of manipulatives." Katie spoke of not having

experienced the positive benefits of manipulatives and how this shaped her beliefs regarding their use. “At the beginning of the course, I really thought that I would like manipulatives, but then [throughout the course] I realized that…they did not do anything (for my learning). They seemed useless.” As with many of her peers, Katie did not find that manipulatives enhanced her personal learning experience. This directly impacted her beliefs.

PSTs shared that they may consider using manipulatives in their teaching but were unsure of their efficacy. Manoel said, “I really like the idea…[but] it is a confusing one.” He went on to say that as a novice teacher, he would require “someone to show me how [manipulatives] would apply to a specific teaching point and how [to] apply them effectively”. In his reflective paper, Warren described his curiosity with manipulatives having encountered them for the first time in the mathematics methods course. Warren wrote that he, along with many of his colleagues were intrigued by this learning tool, however, by virtue of their “newness” he was not able to critically consider their efficacy in a mathematics classroom. Within the span of the course, it seemed that the PSTs were not able to experience manipulatives in a way so that they were convinced of their benefits in mathematics education.

Perhaps particular learning approaches and the context in which they were presented required the PSTs to revisit their preconceptions in a different way and the challenge of this task proved to deter the PSTs from seeing the benefits of the learning tool (Vinson, 2001). This may have been the case with manipulatives. For these PSTs who learned mathematics in a largely traditional way, it is expected that there is a learning curve for reform-based approaches (Huang et al., 2011; Swars et al., 2007) and ample time is required for PSTs to explore new approaches (Wubbels, Korthagen, & Broekman, 1997).

As the course instructor, further introspection into my own teaching practices is necessary to ensure that I am adequately supporting PSTs' learning, especially with unfamiliar approaches. As a novice teacher educator, as conscientious as I may be, I am still developing my own awareness and aptitude of navigating the complexities of being a teacher educator (Lunenberg & Hamilton, 2008). Collins et al. (1987) suggest that modelling should be accompanied by a running meta-commentary of the reasoning and cognitive process of the teacher. While I attempted to share my pedagogical reasoning with my PSTs in my classes, some activities were not as conducive to this meta-commentary. In an introductory lesson on manipulatives, although I described some benefits and challenges of the use of these learning tools, I left the PSTs to discover the manipulatives individually and in small groups. In retrospect, my debrief may have been too vague about the learning tools as a genre rather than honing in on the specifics each of the manipulatives that they encountered in my course (e.g., algebra-tiles, Cuisinaire Rods). As their instructor, without explicitly articulating why I did what I did, the PSTs were unable to access "the pedagogical reasoning, uncertainties, and dilemmas of practice that are inherent in understanding teaching as being problematic" (Loughran, 2006, p.6). PSTs do not naturally reflect upon the nuances of our practices (de Freitas, 2008). This is a skill that teacher educators need to foster and encourage.

No Changes in Beliefs

Some questions from the survey did not indicate a change in PSTs' beliefs: multiple approaches to solving problems (Q4) and cooperative group work (Q5). This lack of change could be a result of an already reform-based viewpoint, thus having limited room to shift towards stronger reform-based beliefs (Q4). It is important to note that all themes addressed in the mathematics beliefs survey were integrated into the mathematics pedagogy course, thus

themes that did not result in favourable shifts in beliefs may indicate a need for adjustments and reflection regarding content and approaches of the course itself. As previously mentioned, my intention was to spend a relatively equal amount of time on each theme, yet this may not yield the best results. For example, if students come into the course with traditional perspectives towards inquiry-based learning, greater opportunities for students to engage in an inquiry-based approach should be taken (Q14). Similarly, as a potentially challenging theme for PSTs to take up, more time during the course should spent engaging with and debriefing manipulatives (Q3) to help consolidate the PSTs' experiences.

Other Contributors to Changing Beliefs

While PSTs spoke of the mathematics methods course as having an impact on their changing beliefs, the PSTs also acknowledged other components of their teacher education program. Most commonly, PSTs cited that their practica were also significant in their development as educators. PSTs described these blocks as: "the most useful", "where I learned in the most rapid fashion", and "where I [learned] a lot about actual teaching". These comments indicate that while the methods course can impact PSTs beliefs, the opportunity to put their new beliefs into practice is important to the development of PSTs. Manoel said, "The practicum stands out because you really feel it. You feel what it is like. You try things out and they do not work or they work and you learn from that." Eldwin wrote about his change in beliefs about open-ended tasks (Q19): "In my practicums I have had the chance to use open ended tasks for introducing concepts to students, and for summatively assessing them. My change in beliefs is clearly evidenced by my increased perception of their effectiveness (from practicum)." While PSTs' beliefs may be shaped by their own positive experiences learning mathematics using reform-based approaches, so too does seeing the

benefits for students. Therefore, it is important to provide PSTs with opportunities to experience reform-based practices in teaching blocks (Swars et al., 2007).

Although PSTs commented on appreciating the opportunity to be mathematics learners in a reform-based context, resulting in a shift in beliefs, many shared that they were still grappling with how to implement these strategies into their own practice. Martin reflected on his evolving beliefs about student-centred learning (Q9), "I think I value the inquiry approach. I think it's much more obvious…(but) I'm still trying to figure out how we can make (the mathematics classroom) as student-centered as it should be." Katie shared a similar sentiment, "I'm trying to make [reform-based methods] fit for me. That's what I'm trying to discover." To elaborate on his belief that students should be assessed in a variety of ways (Q10), Alvin said, "This was a rather novel concept to me this year and something I'm still trying to master". For many PSTs, their end of the course beliefs were based on new and still limited experiences from their first year in the teacher education program. Although shifted towards a more reform-based approach, the PSTs' beliefs are fragile and still in development. The PSTs' beliefs will continue to evolve as they continue to grapple with, and question the direct connection of their experiences with the reform-based approaches from the course to their practice (Beswick, 2006).

Conclusion

Fundamental to the Canadian mathematics education landscape, the NCTM (2000) advocates for mathematics educators to move away from traditional approaches to teaching. International trends in mathematics education also value the benefits of reform-based approaches. Findings from this exploratory study suggest that Canadian upper elementary/lower secondary PSTs, through taking a mathematics methods course, can begin

to shift their beliefs about mathematics education. This course provided opportunities for new, and positive mathematics learning using reform-based approaches, which deepened PSTs' mathematical understanding, and opportunities to reflect on these learning experiences. Research shows that teachers who have experienced learning using reform-based practices are more likely to use these practices in their teaching (Stofflett & Stoddart, 1994). The small sample size of this study warrants the need for additional studies to support current findings. A closer investigation of content and approaches used in mathematics methods courses will be of particular interest to better explore PSTs' beliefs with themes that did not result in favourable shifts in beliefs.

Some PSTs shared that they were uncertain as to how to apply their new beliefs into their teaching practice. The PSTs felt that they were still lacking tangible resources to implement into their teaching. For others, the course provided exposure to reform-based approaches and the PSTs were open to further consideration of these approaches but were not yet convinced of their benefits. PSTs changed their beliefs related to some practices (student-centered learning, non-traditional assessment strategies, classroom discussions and open-ended tasks), other themes (manipulatives) did not yield similar results. While many of the experiences from the methods class were new to the PSTs, the discrepancy of changing beliefs in some areas but not others requires additional investigation.

Teacher education programs are comprised of a variety of components: content courses, methods courses, general education courses, and practica. While each component has a unique contribution and focus for PSTs' development, all should be working in collaboration towards and shared vision (Darling-Hammond, 2000). This study used the context of a mathematics methods course to explore its impact on PSTs' beliefs about mathematics

teaching and learning. Findings from this study strengthened the idea that mathematics methods courses can change PSTs' beliefs, yet the course was not the only contributor. Consistent with previous research (Charalambous, Philippou, & Kyriakides, 2008; Ng, Nicholas, & Williams, 2010), PSTs also spoke of practica as significant to their evolving beliefs. Further research is necessary to continue to explore how other components of teacher education programs impact PSTs' beliefs. Perhaps more crucial is an investigation to explore how components may work in cooperation to support development of reform-based beliefs.

This study spanned the first year of PSTs' time in a two-year program. Although PSTs' beliefs shifted to favour reform-based approaches, it is important to acknowledge that the PSTs are still continuing on their journey towards becoming a teacher of mathematics and that their beliefs are fragile and fluid (Pajares, 1992). Future studies to investigate PSTs' changing beliefs as they continue through their teacher education program are needed to support these future mathematics teachers. Similarly, teachers' beliefs continue to evolve once they enter the profession. Research shows that teachers' beliefs do not necessarily translate into their practice (Cooney, 1985; Herbel-Eisenmann, Lubienski, & Id-Deen, 2006). Research on the nature of change of teachers' beliefs, factors that shape these changes, and mechanisms to support teachers' transfer of reform-based beliefs into their practice is needed. An analysis of early career mentoring, professional development, and the implications of teaching context can contribute to the knowledge base about teachers' beliefs and supports for these mathematics educators to not just "talk the talk" but also "walk the walk".

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Appendix A: Questions from mathematics beliefs survey (taken from McDougall, 2004)

1. The use of technology supports student learning in math.
2. Instead of giving answers, a math teacher should ask questions to get students thinking and to puzzle things out for themselves.
3. Math should be taught through using tactile objects that allow students to experience math in a hands-on way.
4. There is only one correct way to solve a math problem.
5. It's often not very productive for students to work together during math class.
6. It is important to teach math embedded into real-life problems.
7. Assigning questions from a textbook is a good way for students to learn math.
8. Teachers should incorporate all subject areas (math, science, language, etc.) within the same lesson.
9. Teachers should be the ones to explain concepts to students.
10. Paper and pencil tests are the best way to assess student mathematical knowledge.
11. The content of a math unit should be driven by what students want to learn and if of interest to them.
12. It is important for the math classroom to be a quiet space.
13. Not all students are capable of being successful in math.
14. Students need not master basic operations before they tackle complex problems.
15. Students should have a chance to discuss their understanding of concepts with their peers.
16. A lot of math may not be fun or interesting, but it's important to learn anyway.
17. Math learning only happens in the classroom.
18. When working on math problems, the process is more important than the final answer.
19. Open-ended tasks or explorations are not effective in the math classroom.
20. If a student doesn't know the answer to a problem, the teacher should teach them how to solve it.

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