Distributed Deployment Strategies for Prioritized Coverage of a Field under Measurement Error and Limited Communication Capabilities

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Abstract—In this work, a novel diagram called guaranteed power diagram with limited communication range is introduced. The proposed diagram is used to develop distributed deployment algorithms for a network of nonidentical mobile sensors with limited communication ranges, where the coverage priority of different points in the field is specified by a priority function, and the information of sensors’ locations is inaccurate due to measurement errors. The proposed algorithms are iterative and in each iteration, the sensors find their new locations and move towards them. Simulation results confirm the effectiveness of the proposed algorithms.

Index Terms—Mobile sensor network, coverage, distributed deployment algorithm, priority function, measurement error, limited communication.

I. INTRODUCTION

Wireless mobile sensor networks have attracted considerable attention in recent years. A network of mobile sensors could have several applications such as environmental monitoring [1], target tracking [2] and health monitoring [3], to name only a few. A mobile sensor network (MSN) is often comprised of wireless mobile nodes that can communicate with each other and move in different directions. In developing the control algorithms for mobile sensor networks, several practical constraints may be existed. For example, in many real-world applications, the initial positions of sensors are not known a priori. Hence, the developed algorithms should be independent of the initial locations of sensors. Also, since the nature of system is often decentralized, it is often desired to use distributed decision-making algorithms.

Voronoi diagram is a basic tool for developing the deployment algorithms in mobile sensor networks. Two Voronoi-based algorithms, namely VOR and Minimax, are developed in [4] to determine the final location of the sensors for efficient network coverage. Several cost-effective resource management strategies are developed in [5] to prolong the network lifetime. The authors in [6] propose an ant colony optimization algorithm to solve the energy-efficient coverage problem in wireless sensor networks. A virtual force algorithm (VFA) is proposed in [7] in which a force-directed approach is used for increasing the coverage. It is assumed that a cluster head executes the VFA and finds the appropriate sensors’ locations, and all sensors can communicate with it. The authors in [8] introduce an algorithm to estimate the optimal location of sensors for maximizing the covered area. In this work it is assumed that the sensors can detect their neighbors’ locations and their local uniformity. An algorithm is proposed in [9] to increase the area covered by sensors. The proposed algorithm reduces the coverage overlap of sensors by properly aligning their directions. The above-mentioned algorithms assume that the sensors can communicate with each other and each sensor knows its exact location as well as the exact location of the other sensors. These assumptions are not realistic in many applications. Also, in many existing works in the literature (including the above-mentioned papers) it is assumed that the coverage importance for different points in the field is the same. Although this can be a realistic assumption, in many practical problems covering certain areas in the field is more important.

In this paper, a novel Voronoi-based diagram called Guaranteed Power Diagram with Limited Communication Range (GPD-LC) is introduced. The proposed diagram assigns a distinct region to each sensor such that the regions are mutually disjoint and they have the so-called Golden Property. Then, two algorithms are proposed to increase the prioritized coverage in a network of mobile sensors with limited communication ranges. Note that in this work, it is assumed that due to measurement errors, the exact location of sensors is not available. The proposed algorithms are iterative, and in each iteration each sensor finds its new location and moves towards it such that the weighted coverage area is improved.

The organization of the remainder of the paper is as follows. The problem is formally defined in Section II. Section III introduces a novel Voronoi-based diagram, and subsequently the deployment algorithms are proposed in Section IV. Simulation results are given in Section V to demonstrate the effectiveness of the proposed algorithms. Finally, the conclusions of the work are summarized in Section VI.

II. PROBLEM STATEMENT

Consider a group of n mobile sensors denoted by S := {S_1, S_2, . . . , S_n}, randomly deployed in the 2D sensing field F. It is assumed that the sensing ranges of the sensors are not necessarily the same for all sensors. Also, it is assumed that their communication ranges are limited and the sensors are not necessarily identical in terms of communication capabilities. Let the sensing and communication ranges of the sensors be circles centered at their positions and the i-th sensor be denoted by S_i = {p_i, r_s,i, r_c,i} where p_i is the position of the i-th sensor, r_s,i > 0 is its sensing range, and r_c,i > 0 denotes its communication range, for any i ∈ N := {1, 2, . . . , n}. Assume each sensor can measure the position of some of its neighboring sensors by using a localization technique or direct communication with them, and the upper bound of measurement error is available for the sensors. More precisely, if the obtained i-th sensor location by sensor S_j is p_{ij}, then the exact location of the i-th sensor is somewhere within a disk of radius ε_{ij}.
centered at $p_{ij}$. In addition to the inaccurate obtained measures from some of the neighboring sensors, it is assumed that each sensor measures its position with error. Let the upper bound of measurement error be available for each sensor. If the obtained position of the $i$-th sensor by $S_i$ is denoted by $p_{ii}$, then the exact location of that sensor is within a disk of radius $\epsilon_{ii}$ centered at $p_{ii}$. Let the coverage priority of different points in $F$ be specified by a priority function $\varphi(q)$. In other words the coverage importance of the point $q$ is more than that of the point $p$ if and only if $\varphi(q) > \varphi(p)$.

**Problem Definition:** It is desired to develop algorithms for moving the sensors and placing them in proper positions such that the more important points are covered as much as possible and the total weighted covered area in the field increases. More precisely, the objective is to compute the locations of sensors such that the following function is maximized:

$$H = \int_{\mathbb{R}^2} C(p, r_{s,i}) \varphi(q) dq$$  \hspace{1cm} (1)

where $C(p, r_{s,i})$ denotes a circle of radius $r_{s,i}$ centered at $p_i$.

**III. Guaranteed Power Diagram with Limited Communication Range (GPD-LC)**

One of the popular approaches used for solving the coverage problems is to assign a proper region to each sensor and relocate the sensors in such a way that each sensor covers the points of its corresponding region as much as possible. Let region $\Pi_i$ be assigned to the $i$-th sensor for any $i \in \mathbb{N}$. The region assignment $\Pi = \{\Pi_1, \Pi_2, \ldots, \Pi_n\}$ has the Golden Property if any point inside a region that cannot be covered by its corresponding sensor cannot be covered by any other sensors either.

First case: Assume all sensors can communicate with each other and their communication ranges are unlimited. Also, assume the sensors are identical in terms of sensing ranges. In addition, assume there is no measurement error such that sensors can obtain their positions and also other sensors’ locations accurately. In this case, the sensors can construct the Voronoi diagram accurately. The mathematical characterization of the $i$-th region in the Voronoi diagram is as follows:

$$\Pi_i = \{\text{point } q \in F | d(q, p_i) \leq d(q, p_j), \forall j \in \mathbb{N}\}$$  \hspace{1cm} (2)

where $d(q, p_j)$ denotes the Euclidean distance between the $j$-th sensor and point $q$. According to (2) it is straightforward to show that if point $q$ cannot be covered by it corresponding sensor no other sensor can cover it either. In other words the conventional Voronoi diagram has the Golden Property in this case.

Second case: This case is similar to the previous one but the sensing ranges of the sensors are not necessarily the same. Note that, the conventional Voronoi diagram does not have the Golden property in this case. In other words when the sensors have different sensing ranges, it can be shown that a point which is not covered by the sensor corresponding to the Voronoi region containing that point, may be covered by another sensor which has the greater sensing range. The power diagram proposed in the sequel remedy this shortcoming. The mathematically characterization of the $i$-th region in the power diagram is as follows:

$$\Pi_i = \{\text{point } q \in F | d^2(q, p_i) - r_{s,i}^2 \leq d^2(q, p_j) - r_{s,j}^2, \forall j \in \mathbb{N}\}$$  \hspace{1cm} (3)

**Proposition 1.** The power diagram has the Golden Property.

**Proof:** The proof is omitted due to space restrictions.

Third case: The difference between this case and the previous one is that in the third case it is assumed that there are measurement errors and each sensor measures its position and other sensors’ locations with error. In fact, as it was mentioned before, if the obtained position of the $i$-th sensor by $S_i$ denoted by $p_{ij}$, then the exact location of that sensor is within a disk of radius $\epsilon_{ij}$ centered at $p_{ij}$. Let the region of the $i$-th sensor be a set of all sensors which can send the required information to $S_i$, be denoted by $\Pi_i$.

**Assumption 1.** It is assumed that the minimum communication ranges of the sensors denoted by $r_{\text{min}}$ (i.e., $r_{\text{min}} = \min_{j \in \mathbb{N}}(r_{c,j})$) is known by each sensor a priori. Also, since the communication range of a mobile sensor is typically much larger than the sensing range, it can be shown that when there are measurement errors the power diagram constructed based on the measured locations does not have necessarily the Golden Property. In fact, a point inside the $i$-th region (constructed based on the inaccurate measured locations) which is not covered by $S_i$ can be covered by another sensor. The guaranteed power diagram (GPD) proposed in our previous work [11] does not have this shortcoming. The $i$-th region of the guaranteed power diagram is mathematically characterized as follows:

$$\Pi_i = \{\text{point } q \in F | d(q, p_i) + \epsilon_{ii} \leq d(q, p_j) + \epsilon_{ij}, \forall j \in \mathbb{N}\}$$  \hspace{1cm} (4)

**Proposition 2.** The guaranteed power diagram has the Golden Property.

**Proof:** The proof is omitted due to space restrictions.
larger than its sensing range \([12]\), it is assumed that the sensing ranges of the sensors are less than or equal to \(\frac{r_{\text{min}}}{2}\).

The Guaranteed Power Diagram with Limited Communication Range (GPD-LC) of \(S\) is the set of the regions \(\Pi(S) = \{\Pi_1, \Pi_2, \ldots, \Pi_n\}\), where region \(\Pi_i\) includes all points \(q \in F\) such that the following two inequalities are satisfied for all \(j \in Ix(i)\):

\[
\max_{p \in C(p_i,e_{ii})} d(q,p) \leq \frac{r_{\text{min}}}{2}
\]

\[
\max_{p_i \in C(p_i,e_{ii})} d(q,p_i)^2 - r_{s,i}^2 < \min_{p_j \in C(p_j,e_{jj})} d(q,p_j)^2 - r_{s,j}^2
\]

(6a)

(6b)

where \(C(x,y)\) denotes a disk with radius \(y\) centered at point \(x\).

Note that the minimum distance between \(q\) and a point in disk \(C(p_{ji},e_{ji})\) is equal to \(d(q,p_{ji}) - e_{ji}\). Also since the maximum distance between \(q\) and a point in disk \(C(p_{ii},e_{ii})\) is equal to \(d(q,p_i) + e_{ii}\), then the \(i\)-th GPD-LC region can also be characterized by the following equation:

\[
\Pi_i = \{q \in F \mid d(q,p_i) \leq \frac{r_{\text{min}}}{2} - e_{ii}, d(q,p_i) + e_{ii}, \forall j \in Ix(i)\}
\]

Proposition 3. For any \(i, j \in n\), \(d(p_i, p_j) \leq d(p_{ii}, p_{jj}) + e_{ii} + e_{jj}\)

\(d(q,p_i) \leq \frac{r_{\text{min}}}{2} - e_{ii}\), \(d(q,p_j) \leq \frac{r_{\text{min}}}{2} - e_{jj}\)

(8)

(9)

Also, according to the Proposition 3:

\[
\max_{p_i \in C(p_i,e_{ii})} d(q,p_i)^2 - r_{s,i}^2 < \min_{p_j \in C(p_j,e_{jj})} d(q,p_j)^2 - r_{s,j}^2
\]

(10)

Using (9), (10) and the triangle inequality, it can be deduced that:

\[
d(q,p_i) + d(q,p_j) + e_{ii} + e_{jj} \leq r_{\text{min}}
\]

(11)

From (11) and on noting that \(r_{\text{min}} \geq \min\{r_{c,i}, r_{c,j}\}\), one arrives at:

\[
i \in Ix(j), \ j \in Ix(i)
\]

(12)

Since \(q \in \Pi_i\) and \(j \in Ix(i)\), by setting \(p_1 = p_{ii}\) and \(p_2 = p_{jj}\) in (6b) it can be concluded that:

\[
d(q,p_{ii})^2 - r_{s,i}^2 < d(q,p_{jj})^2 - r_{s,j}^2
\]

Likewise, since \(q \in \Pi_j\) and \(i \in Ix(j)\), thus:

\[
d(q,p_{jj})^2 - r_{s,j}^2 < d(q,p_{ii})^2 - r_{s,i}^2
\]

(13)

(14)

which contradicts the inequality (13), and hence invalidates the initial assumption \(\Pi_i \cap \Pi_j \neq \emptyset\).

Proposition 5. The GPD-LC has the Golden Property.

\(\Pi\) does not have the Golden Property and there is sensor \(S_j\) such that point \(q\) can be covered by it. Thus:

\[
d(q,p_i) > r_{s,i}
\]

(15)

\[
d(q,p_j) \leq r_{s,j}
\]

(16)

Consider the following two cases:

i) First case: assume \(j \notin Ix(i)\). From (16) and according to Assumption 1 one arrives at:

\[
d(q,p_j) \leq \frac{r_{\text{min}}}{2}
\]

(17)

Also, since \(q \in \Pi_i\) and according to (6a), thus:

\[
d(q,p_i) \leq \frac{r_{\text{min}}}{2}
\]

(18)

Using (17), (18) and the triangle inequality, it can be concluded that:

\[
d(p_i, p_j) \leq r_{\text{min}}
\]

(19)

which means \(S_j \in L\) and contradicts the assumption \(j \notin Ix(i)\).

ii) Second case: assume \(j \in Ix(i)\). From (15) the inequality \(d^2(q,p_i) - r_{s,i}^2 > 0\) is concluded, and according to (6b) one arrives at:

\[
d^2(q,p_j) - r_{s,j}^2 > 0
\]

(20)

or equivalently \(d(q,p_j) > r_{s,j}\) which contradicts the initial assumption \(S_j\) can cover \(q\). Hence GPD-LC has the Golden Property.

Fig. [1] shows examples of power diagram, GPD and GPD-LC for a group of 4 sensors with the sensing ranges of 4.8m, 6m, 4.2m and 3m, and the communication ranges of 48m, 60m, 42m and 30m. The measurement errors \(\epsilon_{ii}\) and \(\epsilon_{jj}\) are assumed to be 1m and 2m, respectively.

IV. DEPLOYMENT PROTOCOLS

In this section, two distributed deployment algorithms are developed to increase the weighted coverage area in a network of nonidentical mobile sensors in the presence of measurement error and limited communication capabilities.

Definition 1. The integral of the priority function over the intersection of the \(i\)-th GPD-LC region and \(C(p_i,r_{s,i})\) will be referred to as the \(i\)-th local weighted coverage area for all \(i \in n\). The \(i\)-th local weighted coverage area is mathematically characterized as follows:

\[
\beta_i = \int_{\Pi_i \cap C(p_i,r_{s,i})} \phi(q) dq
\]

A. Deployment algorithms

The proposed algorithms in this work are iterative, and each iteration consists of three phases. The mentioned three phases of the \(k\)-th time interval \([T_i(k),T_j(k)]\) can be summarized as follows:

• First phase: In the subinterval \([T_i(k),T_i(k)]\) every sensor broadcasts its information to other sensors and constructs its GPD-LC region based on the received information. Note that, because of the limited communication ranges each sensor is not necessarily aware of the information of all other sensors.

• Second phase: In the subinterval \([T_i(k),T_j(k)]\) every sensor uses a proper strategy to find its new location. Note that,
B. Movement Strategies

The proposed algorithms differ only in the strategy used for finding the new sensors’ locations. These strategies will be proposed in the next subsection.

• Third phase: In the subinterval \([T_2(k), T_f(k)]\) every sensor moves to its new location if and only if its local weighted coverage area increases by moving to the new location. Otherwise, the sensor remains in its current location.

If the local weighted coverage area by no sensor is increased, or the number of iterations exceeds a predefined value the algorithms are terminated.

B. Movement Strategies

In the second phase of the above-mentioned algorithms, every sensor uses a proper strategy for finding its new location. The following strategies are borrowed from [10] to find the new locations of sensors.

• Maximum Weighted Point (MWP) Strategy: In this strategy each sensor selects a point with maximum value of priority function in its GPD-LC region as the new location. Although the MWP strategy is effective in many practical cases, it is not suitable when the priority function is smooth. For instance, when the priority function of all points are equal, sensors do not move under the MWP strategy. To remedy this shortcoming the following movement strategy will be proposed which is based on both distance and value of priority function.

• Maximum Distance Weight (MDW) Strategy: In this strategy, for any \(i \in \mathbf{n}\), the new location of the \(i\)-th sensor is a point inside the \(i\)-th GPD-LC region whose squared distance from \(S_i\) multiplied by its value of priority function is maximum.

Based on which of the above-mentioned movement strategies are used, the following two algorithms can be introduced:

i) MWPGPD-LC algorithm

ii) MDWGPD-LC algorithm

V. SIMULATION RESULTS

In this section, the effectiveness of the proposed algorithms is investigated by some examples.

Example 1. Consider a 50m×50m field containing 15 randomly deployed sensors: 5 sensors with a sensing range of 1.6m, 6 sensors with a sensing range of 2m, 3 sensors with a sensing range of 2.4m, and 1 sensor with a sensing range of 2.8m.

Moreover, the communication range of each sensor is assumed to be 20m. The priority function for the network coverage in this example is given by \(\varphi(q) = \exp(-0.1([x_q - 15]^2 + [y_q - 15]^2)) + \exp(-0.1([x_q - 37.5]^2 + [y_q - 40]^2))\), where \(x_q\) and \(y_q\) are the abscissa and ordinate of point \(q\), respectively. The final configuration of the sensors (denoted by small yellow disks), and their trajectories under the MWPGPD-LC algorithm are shown in Fig.2. As it can be observed from this figure, the sensors move towards the points with large value of priority function and they properly cover the most important areas.

Example 2. In this example, we investigate the performance of the MDWGPD-LC algorithm when the priority function is smooth. Assume 30 sensors with the communication range of 20m are randomly deployed in a 50m×50m field: 9 sensors with a sensing range of 2.4m, 12 sensors with a sensing range of 3m, 6 with a sensing range of 3.6m, and 3 sensors with a sensing range of 4.2m. The priority function for this example is equal to \(\varphi(q) = \exp(-0.001([x_q - 20]^2 + [y_q - 30]^2))\). Figs. 3(a) and 3(b) show the initial and final configurations of sensors, respectively. In these figures, the sensing area of the sensors is depicted by filled circles. As it can be observed from these figures, in the final round sensors are located in proper positions such that they do not have much overlap to each other which is very important when the sensing ranges of sensors are relatively large and the priority function is smooth.

In the next examples, both algorithms proposed in the previous section are applied to a flat space of size 50m×50m, and the results are all the average values obtained by using 100 random
initial deployments for the sensors.

**Example 3.** In this example, consider 10 sensors randomly deployed in the field: 3 sensors with a sensing range of 0.8m, 4 sensors with a sensing range of 1m, 2 sensors with a sensing range of 1.2m, and 1 sensor with a sensing range of 1.4m. The communication range of the sensors is assumed to be 10m. Let the priority function be equal to $\phi(q) = \exp(-0.01[(x_q - 20)^2 + (y_q - 30)^2])$. Fig. 4 shows the weighted coverage per round for both proposed algorithms. As it can be seen from this figure, the MWPGP-LC algorithm outperforms the MDWGP-LC algorithm in this example.

**Example 4.** In this example, the effect of the sharpness of the priority function is investigated. Consider 30 sensors with the communication range of 20m randomly deployed in the field: 9 sensors with a sensing range of 4.8m, 12 sensors with a sensing range of 6m, 6 sensors with a sensing range of 7.2m, and 3 sensors with a sensing range of 8.4m. Let the priority function be in the form of $\phi(q) = \exp(-k[(x_q - 15)^2 + (y_q - 15)^2]) + \exp(-k[(x_q - 37.5)^2 + (y_q - 40)^2])$, where $k$ reflects the sharpness of the priority function. Fig. 5 shows the final weighted coverage for different values of $k$. As it can be observed from this figure when there is a relatively large number of sensors with large sensing ranges in the field, the MDWGP-LC algorithm outperforms the MWPGP-LC algorithm and this superiority is significant when the priority function is smooth.

**VI. CONCLUSIONS**

Distributed deployment algorithms are developed in this work to improve the weighted coverage area in a network of nonidentical mobile sensors with limited communication ranges. The proposed algorithms use the guaranteed power diagram with limited communication ranges (GPD-LC) for finding the sensors’ new locations. GPD-LC is an appropriate diagram for developing sensor deployment strategies when the information of sensors’ locations is inaccurate and the communication ranges of sensors are limited. Using the proposed algorithms, the sensors move iteratively such that the weighted covered area is increased. Simulations confirm the efficacy of the proposed algorithms in improving the weighted coverage area.

**REFERENCES**


