THE NUCLEAR MAGNETIC MOMENT

OF

BORON OF MASS ELEVEN

A thesis submitted to the Faculty of Graduate Studies and Research of McGill University in part fulfilment of the requirements for the degree of Doctor of Philosophy.

by

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January,
1949.

ACKNOWLEDGMENTS

The work described, started in the Macdonald Physics Building, was largely carried out in the new Radiation Laboratory with its excellent facilities at hand.

The author is deeply grateful to Dr. J. S. Foster who suggested the work originally, for his continued encouragement and advice. The writer was joined in May, 1948 by Mr. D. Hunten whose efforts in building some of the measuring instruments and suggestions in general are greatly appreciated. He has to thank Messrs. Massé, Doig and the late Mr. Genereux of the Machine Shop staff who did most of the machine work, Misses Lyttle, Mulligan and Mrs. R. L. Clarke who did some of the wiring.

Grateful acknowledgment is due to the National Research Council for three successive studentships awarded during the course of the work.

SUMMARY

The extensive equipment which is described has been built up for the study of magnetic moments of nuclei, particularly for accurate comparison of their gyromagnetic ratios, and the relative spins of isotopes.

The nuclear induction and the nuclear absorption resonances of the proton have been observed. The induction resonance of B^{11} has been observed at around 10 mc. in an alkaline solution of KBO_2. Its frequency has been compared with that of the proton in the same field giving a measure of the ratio of the two gyromagnetic ratios:

$$\frac{\gamma_B}{\gamma_P} = 0.320904 \pm 4.3 \times 10^{-6}$$

Based on the best known value of $2.7928 \pm .0008$ for the magnetic moment of the proton, the magnetic moment of the B^{11} nucleus becomes

$$2.6887 \pm .0008 \quad n.m.$$

which is an improvement in precision by a factor of ten over the best known previous value.

A check on the amplitude of the B^{11} induction resonance is made and the relaxation times of B^{11} in KBO_2 solution estimated as:

$$T_1 \sim 0.01 \text{ sec.}$$

$$T_2 > 10^{-4} \text{ sec.}$$

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INTRODUCTION

Theoretical Background

The first evidence that the nuclei have magnetic properties was provided by the discovery that the fine structure lines of some atoms are further divided into energy levels whose quantized separations are roughly one two-thousandth that caused by orbital and spin interactions of the orbital electrons. Isotopic mixture of the nuclei in question accounted for only a part of this hyperfine structure; Pauli in 1924 brought forward the suggestion that the nuclei have angular momentum, and thus since they involve conglomerated circling charge, their magnetic fields may be described by assigning magnetic moments.

In strict analogy with the atomic picture, the nuclear spin momentum is observed in only integral or one-half integral multiples of $\frac{h}{2\pi}$, where h is Planck's constant. Wave mechanics gives the total angular momentum vector the value,

$$I(I + 1) \frac{h}{2\pi}$$

where I is the spin quantum number, and one component, m_I, where M_I and I are both integers or half-integers and $-I \leq M_I \leq I$ may be observed. From the classical analogue the magnetic moment is expected to be for the proton:

$$\frac{e}{2mc} \cdot p$$

where e = electronic charge, m = atomic mass unit, c = velocity of light, and p = the angular momentum. If we take p as one unit of $\frac{h}{2\pi}$, we have a natural unit for the measurement of nuclear magnetism:

$$\frac{eh}{4\pi mc} \equiv 1 \text{ nuclear magneton.}$$

However, the experimental values of the magnetic moments of nuclei are not integral values of this unit, and a factor, g, called the nuclear g-factor is used, defined as follows:-

$$\mu = gI$$

where μ = magnetic moment of the nucleus. The nuclear g-factors and spins of many nuclei have been measured and some correlations among their values are noticed[1,2]. The former range from that of the neutron, -3.82, to that of the triton which is 5.95.

1. Latham, Proc. of Phys. Soc. November 1947, p. 979.
2. Rarita and Schwinger, Phys. Rev., March 1, 1941, p. 436

Historical Outline of Previous Work.

The first estimates of magnetic moments were made from spectroscopic data on the hyperfine structure splitting. Both spin and magnetic moment can be obtained if the resolution of the hyperfine structure or flag pattern of a line is adequate. In a weak magnetic field, such as that of the atom itself, the nuclear spin is coupled to the atomic frame and quantization exists in discrete component values, M_F, of the total quantum number, F, of the atom. This quantum number, F, is the sum of I and J, the spin numbers of nucleus and orbital electrons respectively. If this structure can be resolved, the spin I is known immediately that J is known. In a strong field the nuclear magnetic moment gives rise to a strong nuclear Zeeman effect. The weak coupling between I and J is negligible and the levels of different I but the same J form a hypermultiplet which has 2I + 1 terms. If these are resolved well enough to be counted, the I is thus obtained, and if well enough to measure separation, an estimate of magnetic moment may be made. This method was first employed by Bach and Goudsmit for bismuth.[3]

The hyperfine structure splitting demands the utmost resolution of which optical instruments are capable. The most work on nuclear magnetic moments has been done by Rabi and his co-workers with a skilful

3. Goudsmit and Bach Z. f. Physik 43:321,1927. Goudsmit and Bach. ibid.. 47: 174, 1928.

adaption of the Stern-Gerlach technique in which the effective resolution is many times that obtainable optically. Rabi's most significant method, so far as this work is concerned, makes use of the fact that a nucleus, when placed in a field of a few thousand gauss will precess -- because of the interaction of spin and the couple produced by magnetic moment in the field -- at a Larmor frequency which is in the radio range. This frequency is given by

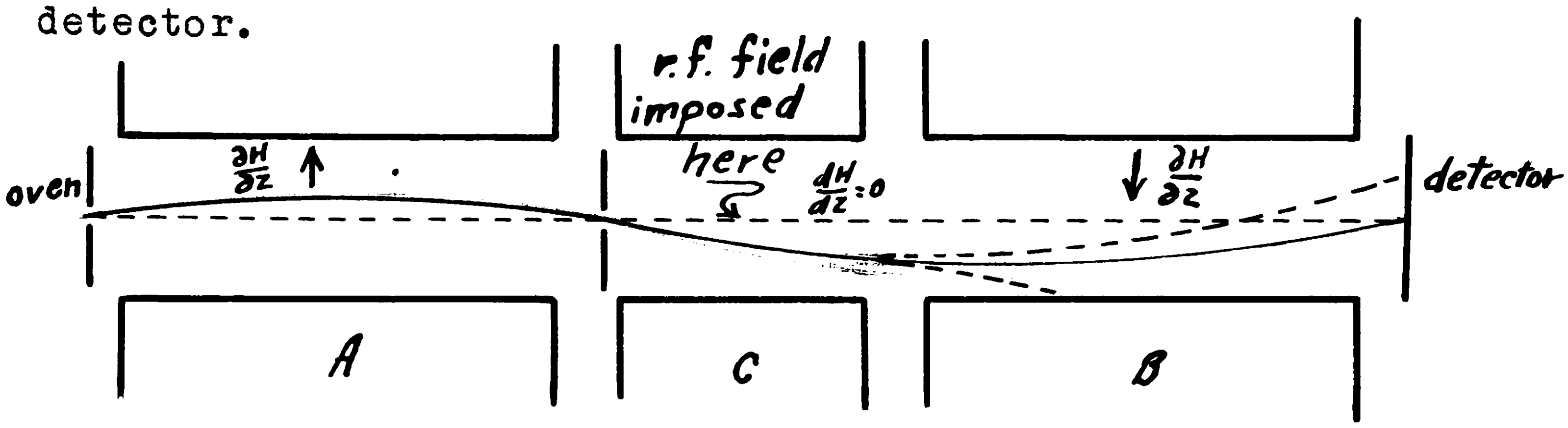
$$\nu = \frac{\mu H}{h I}$$

where μ is the total magnetic moment, H is the field, and h is Planck's constant.

In this method[4], the gaps of three magnets, A, B and C (as shown in the sketch) are placed carefully in line and form an evacuated system. The pole faces of A and B are shaped so as to give a highly inhomogeneous field, the first with a field whose $\frac{dH}{dz}$ is in one direction, the second with $\frac{dH}{dz}$ equal and opposite to it, z being the coordinate along the field gradient. When molecules of zero spin are projected thermally from an oven, and collimated so as to form a fine beam in the gap, they will be deflected along the changing gradient

4. Kellogg and Millman, Rev. of Modern Physics, July 1946, p. 323.

by a force $\frac{dH}{dz}$, in a direction dependent on the orientation of their nuclear magnetic moments in the field of A. If nothing happens to reorient them, the field of B will bring them back to a focus at the detector.



In order to upset these orientations, a third magnet C, having a homogeneous field is interposed and a radio frequency magnetic field introduced at right angles to the main one. When this frequency is tuned through the Larmor precession frequency, transitions occur and the detector shows a decrease in the number of molecules arriving. Values of the field and frequency at which this happens give the gyromagnetic ratio. This was the first successful application of radio methods to the measurements of nuclear moments. Using this technique values were obtained with a precision several times those obtained spectroscopically.

In 1936, two years previous to Rabi's discovery, Gorter made an attempt to observe the absorption of energy which results from these induced transitions. He used two solid alums at liquid hydrogen temperatures expecting a heating effect, but was unable to detect one. In his second unsuccessful attempt in 1942, he looked for a change in magnetic susceptibility.

After the war, with many new radio frequency techniques and new apparatus available, the direct observation of this effect was attempted with renewed vigour by two teams of workers: Purcell, Pound and Torrey who found an absorption effect[5], and Bloch, Hansen and Packard who observed nuclear induction[6]. An outstanding contribution of this work has been the precise comparison of the magnetic moments of proton, deuteron and neutron by the Stanford team, [7,8] and by Roberts and Arnold at Argonne[9]. Further measurements of the gyromagnetic ratios of other nuclei have been made by others [10,11,12,13]. Pound at Harvard[14,15] has built a recording spectrometer for this specific purpose and using the absorption resonance compared the gyromagnetic ratios of a number of nuclei with those of the proton and sodium.

5. Purcell, Torrey, Pound, Phys. Rev. Jan. 1946, p. 37.
6. Bloch, Hansen, Packard, Phys. Rev. Feb. 1946, p. 127
7. Bloch, Levinthal, Packard, Phys. Rev. Dec. 1, 1947, p. 1125
8. Bloch, Nicodemus, Staub, Phys. Rev. Nov. 1, 1948 p. 1025
9. Arnold and Roberts, Phys. Rev., June 15, 1947, p. 878
10. Poss, Phys. Rev., Oct. 1, 1947, p. 637
11. Anderson and Novick, Phys. Rev., March 15, 1947
12. Bloch, Graves, Packard, Spence, Phys. Rev. April 15/47 p.551
13. Anderson and Novick, Phys. Rev. April 15, 1948, p. 919
14. Pound, Phys. Rev. March 1, 1948 p. 523
15. Pound, Phys. Rev., May 1, 1948, p. 112.

Previous work on B^{11}.

At the time the measurement described here was made the only previously published values[16] for the magnetic moment of B^{11} was that of Millman, Kusch and Rabi at Columbia using the molecular beam apparatus[17]. Heating the alkali metaborates and tetraborates to 1100°K in their oven, they found resonance dips in the molecular beams arriving at the detector in the neighbourhood of 2000 gauss and with frequencies which gave the B^{10} and B^{11} nuclei magnetic moments values of 0.597 ± 0.003 and 2.682 ± 0.008 respectively. A little later they reported a value of $2.694 \pm .008$ for $\mu_{B^{11}}$, obtained by a comparison with Li^7 whose nuclear g-factor was assumed to be 2.167. Recent determinations[18] of the spin of B^{11} show their assumption of a spin of $\frac{3}{2}$ is justified.

Value of the Magnetic Moment of the Proton.

Since the value for the magnetic moment of B^{11} is based on that of the proton in this experiment, a note on the value of the latter is necessary. The value of $2.7896 \pm .0008$ was obtained in 1941 by Millman and Kusch[19] using the molecular beam apparatus. They calibrated their magnetic field by reference to electronic transitions between various (F,m) levels of the rubidium, sodium, and caesium atoms. The calibration of magnetic field arrived at involves

16. Millman, Kusch, Rabi; Phys. Rev., July 1939 p. 165
    Also abstract Kusch, Millman Phys. Rev. July 15, 1939 p. 213
17. Rabi, Millman, Kusch, Zacharias, Phys. Rev., 55, 1939. p.526
18. Gordy, Ring and Burg; Phys. Rev., Nov. 1, 1948 p. 1191
19. Millman and Kusch, Phys. Rev., July 15, 1941, p. 91

quantum mechanical expressions which give the energy of an alkali in a magnetic field and involves the assumption[20] that the electronic gyromagnetic ratio for an alkali atom in the ground state is exactly 2. Recent work [21,22,23] indicates that the electron spin gyromagnetic ratio is

$$g_s = 2(1.00119 \pm 0.00005)$$

bringing the magnetic moment of the proton to the value of

$$2.7928 \pm 0.0008 \text{ nuclear magnetons}$$

which is the value used here.

20. M. Phillips; Phys. Rev., July 15, 1941, p. 100
21. Kusch and Foley; Phys. Rev., August 1, 1948, p. 250
22. Nafe, Nelson, Rabi; Phys. Rev., June 15, 1948, p. 914.
23. Nagle, Julian, Zacharias, Phys. Rev., Nov. 1, 1947, p. 971

THE NUCLEAR INDUCTION EXPERIMENT

The nuclear induction experiment, as carried out by Bloch and collaborators[24], calls for a strong magnetic field into which the nuclei are placed. Another magnetic field of only a few gauss and alternating at a radio frequency is imposed at right angles to the first. When the strong magnetic field is varied so that the precession frequency coincides with the alternating field frequency, the nuclei are made to precess synchronously and will induce a voltage of the same frequency in a second coil wound around the sample so that its axis is perpendicular to both the above fields.

The theoretical mechanism of nuclear induction has been given by Bloch[25] with a later analysis of signal shapes by Jacobsohn and Wangsness[26]. A brief synopsis of the relevant features of the first article is given here to explain the experimental procedure.

Magnitude of the Effect

The nuclear paramagnetic susceptibility, χ, is defined by

$$M = \chi H$$

where M is the resultant nuclear polarization per c.c. in the direction of the field, H. The susceptibility, χ, is related to the nuclear g-factor of the "n" nuclei by

24. Bloch, Hansen, Packard; Phys. Rev., October,1946 p. 474
25. Bloch; Phys. Rev., October 1946, p. 460.
26. Jacobsohn and Wangsness; Phys. Rev., May 1, 1948 p. 942.

the Curie formula:

$$\chi = \frac{I(I + 1)\ ng^2}{3kT}$$

where I = spin number
k = Boltzmann's constant
T = absolute temperature.

It is the time variation of a component of M which gives rise to $B = 4\pi M$ lines of induction which is observed.

Let the strong field, H_z, be assumed parallel to the z-axis. In practice it is made as homogeneous as possible over the space occupied by the sample. The radio frequency field of amplitude $2H_1$ and frequency ω radians per second is assumed parallel to the x-axis so that the imposed field has components:

$$H_x = 2H_1 \cos \omega t$$

$$H_y = 0$$

$$H_z = H_o$$

Since the nuclei have spin momenta and magnetic moment vectors parallel to each other, the interaction of torque due to the strong field and their spin gives rise to the precession of each one around the field direction. The quantum mechanical expectation of the whole system may be solved for its time variation by the classical equations of motion. Considering only quantities involving the whole system of nuclei in the sample, and allowing the strong magnetic field to vary back and forth relatively slowly through its resonance value

$$H^* = \frac{\omega}{|\gamma|}$$

where γ is the gyromagnetic ratio $(= \frac{g}{\hbar})$ which may be negative, the amplitude of voltage induced in the receiver coil at resonance turns out to be:

$$a_r = \frac{10^{-8}}{\pi} N A n \frac{I(I+1)}{3kT} h^2 \gamma \omega^2 \quad \text{volts}$$

where n = number of nuclei/c.c.

A = area of receiving coil

N = number of turns

h = Planck's constant

ω = resonant radian frequency.

Effect of Relaxation Times

The above amplitude represents a maximum figure which is not reached in practice due to the influence of the local fields and thermal agitation on the nuclei. These are described by assigning two relaxation times:-

T_1 = time constant of an assumed exponential approach of the polarization M_z to its equilibrium value M_o given by the Curie formula.

T_2 = time constant of an assumed exponential approach of a transverse component, M_x or M_y, to its equilibrium. This is also called the line width parameter and plays a role in a resonance similar to that of the "Q" of a coil in an ordinary resonant circuit.

Two extreme cases with simplified conditions are treated mathematically giving signals of the types rapid and slow passage through resonance. Imposing the adiabatic condition

$$\frac{\partial \delta}{\partial t} << |\gamma| H_1$$

where $\delta = \dfrac{H_o - H^*}{H_1}$, the following solutions are obtained for the three components of the polarization **M**, assuming that the driving field H_1 is predominent over the relaxation effects described by T_1 and T_2 (i.e. $|\gamma| H_1 >> \frac{1}{T_1}$; $|\gamma| H_1 >> \frac{1}{T_2}$):

$$M_x = \frac{M}{(1+\delta^2)^{\frac{1}{2}}} \cos \omega t$$

$$M_y = \mp \frac{M}{(1+\delta^2)^{\frac{1}{2}}} \sin \omega t$$

$$M_z = \frac{M \delta}{(1+\delta^2)^{\frac{1}{2}}}$$

The optional sign for M_y allows for the opposite rotation due to a negative magnetic moment. M is a function of time, t, and is given by:

$$M(t) = \frac{1}{T_1} \int_{-\infty}^{t} \frac{\delta(t') \exp\{-[\theta(t) - \theta(t')]\} M_o(t')}{[1 + \delta^2(t')]^{\frac{1}{2}}} dt'$$

$$M_o(t') = \chi H_o(t')$$

$$\theta(t) - \theta(t') = \frac{1}{T_1} \int_{t'}^{t} \frac{\delta^2(t'') + T_1/T_2}{1 + \delta^2(t'')} dt''$$

If the relaxation times are long or if the passage through resonance occurs so rapidly that the exponential does not vary, the integral takes on a constant value accumulated during considerably previous times and the resonance shape is dominated by

$$M_y = \mp \frac{M}{(1+\delta^2)^{\frac{1}{2}}}$$

which gives a "rapid passage" resonance. This is a simple resonant curve whose maximum ($\delta = 0$) coincides with the actual resonant value of the field in time.

If the relaxation times are short, or passage of the magnetic field through the resonance interval very slow, the exponential part of the integrand approaching unity rapidly causes the polarization to reach a position of equilibrium at each point through the resonance interval, generating a curve represented by:

$$M_y = \mp \frac{M_0(t)\ \delta(t)}{\delta^2(t) + T_1/T_2}$$

which is zero when $\delta = 0$ and reverses sign as δ does. This is the typical "slow passage" resonance.

A particular case of the "rapid passage" resonance is of particular interest to us here. The optional sign for M_y is given to allow for the possibility of approaching resonance with a decreasing or an increasing magnetic field. In the extreme case of long T_1 the value of the integral is governed by the average value of H_z so that over a 60 c.p.s. sweep, for example, both fore and back sweeps give a resonance deflection of the same polarity. When T_1 is of the order of the half period of the sweep, the integral has time to change sign due to the changing of sign of the δ, and the fore and back sweeps may be of opposite polarity as is the case of the boron resonance shown in the photograph. To account for this type of signal exactly mathematically involves evaluating the integral which is very difficult and not necessary for an accurate interpretation of our result.

<u>Bloch's Theory Applied to B^{11} Resonance</u>

The amplitude of signal expected without regard to relaxation times is calculated from the equation given, using the values of the quantities known from our experimental apparatus.

Volume of sample	=	1.8 c.c.
Turns of coil	$N =$	6
Area	$A =$	1.27 cm^2
Concentration of nuclei	$n =$	0.58×10^{22}/c.c.
Gyromagnetic ratio of B^{11}	$\gamma_B =$	0.86×10^4 c.g.s. units
Frequency used	$\omega =$	$2\pi \times 9.6 \times 10^6$ radians/sec.
	$h =$	6.61×10^{-27} erg sec.
	$k =$	1.38×10^{-16} erg deg^{-1}
	$T =$	290° K
	$I =$	$\frac{3}{2}$

$$a_r = \frac{10^{-8}}{\pi} \, NAn \, \frac{I(I+1)}{3kT} \, h^2 \gamma \omega^2$$

$$= 6.0 \, \mu V.$$

DESCRIPTION OF APPARATUS

The apparatus which has been assembled and built up for the nuclear resonance experiment is shown schematically in figure 1. It may be classified under four headings:-

1. The Magnet and Associated Equipment.
2. Coils Assembly and Balance Circuit.
3. The Radio Frequency Oscillator and Supply.
4. The Receiving Circuits including Frequency Measuring Units.

1. MAGNET AND ASSOCIATED EQUIPMENT

Magnetic Field Control Circuits

The main magnetic field is supplied by a large water-cooled magnet with 4" pole faces, the current in which, provided by a 6 KVA generator, may be set at up to 10 amperes for long periods without excessive heating. This will provide the 1 5/8" gap with about 9000 gauss. The field of the larger generator is supplied by two 220 volt exciter generators in series whose fields in turn are excited by an electronically regulated current through the two parallel 807 beam tetrodes of the regulator circuit.

The regulator circuit consists of two parts, a standard type of d.c. amplifier using a "chopper", and a parallel anti-hunt circuit shown in figures 2 and 3 respectively.

The magnet current passes through a series 0.1 -ohm manganin resistor immersed in an oil bath through which also passes a copper tube carrying a fraction of the water circulating in the magnet. This develops a fraction of a volt (negative) which adds in series to a positive standard reference voltage taken from a potentiometer and storage battery circuit to give the error voltage for the regulator. This voltage is "chopped" at 60 c.p.s. by a Brown converter and applied to the a.c. amplifier through a filter tuned broadly for 60 c.p.s.

The a.c. part of the amplifier uses three 6SC7 tubes in push-pull. The first two tubes, the converter and standard reference circuit, are mounted in a small chassis on rubber mounts and on the larger chassis which contains the rest of the circuit. The push-pull circuit and the absence of a bias resistor in the first 6SC7 are features of this emplifier which were found to reduce spurious a.c. voltages coming mostly from filament-cathode sources. The final 6SC7 stage is tuned to 60 c.p.s. in the primary of its transformer. The secondary of this transformer drives a 6N7 phase-sensitive detector which reconverts the error voltage to d.c. and provides the bias for the two 807 exciter drivers. It was found that the whole magnet current could be varied manually (with the automatic gain control at zero) over practically its whole range by varying the bias on the 807's by a leak to the high tension supply through a 0.25-megohm potentiometer and a fixed 80-kilohm resistor in series. Most manual control of the field for anything but precision measurements was effected by varying the 100-ohm or the 6-ohm potentiometer in the standard reference circuit with the regulator gain up.

Since it was desired to make measurements with accuracy of the order of one part in 10^4, it was desired to make the magnet field remain constant about that degree during the short interval between the presentation of the two resonances, proton and boron, on the C.R.O. screen. Due to the cascaded delay of the correction voltage around the loop -- occuring mostly in the generators and the magnet -- the whole system tended to hunt at about 2 c.p.s. The anti-hunt circuit shown in figure 3 consisting of two 6AC7's, a cathode-coupled 6SN7-6SK7 stage; and twelve 807's in parallel makes a second closed feedback loop not including the generator. This has a frequency response which goes down to a fraction of a cycle per second and is fed by about -4 volts from a 0.5-ohm resistor carrying the magnet current. The 807's are in parallel with 7.5-ohms which together carry the magnet current, the 807's sharing about 20% of the whole current. It is possible to regulate the mean value of the current to between 1 and 2 parts in 10^4, and less than 1 part in 10^4 over several minutes.

The most troublesome feature of this system is a 20 c.p.s. variation which comes from a poor commutator on the generator giving rise to a jitter of about 1 gauss in the resonance display.

Magnetic Field Modulation

The two circuits for modulating the magnetic field about resonance are part of the circuits shown in figures 2 and 3.

Anticipating the possible need for a very slow variation of the magnetic field to suit long relaxation times, a 60 to 80 cycles/sec. phase shift oscillator was built into the main control circuit. The 6J5 cathode-follower driven by the 6AC7 oscillator provides a few millivolts of this frequency in series with the standard storage cell, resulting in a "beat" frequency of 0 to 20 c.p.s. in the output of the Brown converter. This oscillator is quite stable at close to 60 c.p.s. giving a slow cyclic variation of the field. The proton was first observed with this sweep but thereafter it was used very little because of distortion of the signal shape caused by spurious voltages from the generator when varied in this fashion.

The measurement on boron was taken using the 60 c.p.s. field modulation. This was provided by two coils of about 500 turns each, 4" inside diameter and about

2" apart axially, squeezed against the coil forms of the magnet windings with four simple jack screws. An insulating transformer and a variac supplies these with a current sufficient to modulate the field with an amplitude of up to 100 gauss. This appears to be a very satisfactory system for modulating the field in spite of the strong eddy currents which must circulate in the solid pole faces. No disturbance of the average value of the field by a.c. from this source getting into the "chopper", as was feared, has been noticed.

Magnetic Field Homogeneity

Disregarding the finer details of signal shapes, three main things contribute to the width of the simple rapid passage resonance:-

1. The natural width of the resonance which is proportional to the driving field and is $2H_1$ at $\frac{1}{2}$ of the maximum signal amplitude.

2. The local fields of the neighbouring atoms and nuclei contributing a width which is measured in terms of the time constant T_2 where $T_2 = \frac{1}{|\gamma| H_{local}}$

3. The inhomogeneity of the main magnetic field over the finite volume of the sample.

An effort was made to reduce the inhomogeneity of the field enough to make the desired precision possible. Pole pieces of cold rolled steel (0.15% carbon) were made

1.5" long and slightly smaller than the 4" diameter of the yoke piece in order to clear the modulating coil forms and the magnet winding. They are screwed into the yoke pieces with a slightly sloppy thread allowing a tilting adjustment by thin shims behind their shoulders to correct any wedge-shaped assymmetry of the field in the gap. Using paper shims, the gap was made uniform around the circumference to within 0.001".

Since the proton resonance had not yet been obtained, the homogeneity was checked using a specially designed search coil* with which it was possible to detect a field inhomogeneity of less than 1 gauss from the mean of several readings. The outside diameter of this coil is about 1 cm. It is mounted on a small carriage in a frame which allows it to be drawn rapidly away from the centre of the gap a predetermined distance operating a ballistic galvanometer.

A thickness of 0.042" was machined away from the centre of the pole pieces leaving a shim 0.344" wide, the values having been calculated from Rose.[27] When the field was checked with the search coil the drop-off of field at 0.5" radius was found to be still an average of 8.5 gauss.

*Designed and built by Mr. D. Hunten.

27. Rose, Phys. Rev. May 1, 1938, p. 718

Drop-off of Field at 0.5" Radius

Azimuth	0°	45°	90°	135°	180°	225°	270°	315°	Average
No Shims	15	18	16	14	12	11	14	14	14.2
"Rose" Shim	11.2	-	13.6	-	4.1	-	5.1	-	8.5
Double Shim	3.7	1.0	0	0.7	3.4	6.0	6.0	4.0	3.1

A second shim was added 0.688" wide and 0.019" thick, its dimensions estimated from the effect of the first shim using the following assumption*: The effect on the drop-off curve at the centre by a shim at the circumference is proportional to its cross-sectional area, and the curve is "squeezed" in towards the centre the same fractional amount as the centre of the shim is moved radially. When the field was tested after machining the double shim, the readings shown in the table were taken. They show that a fairly homogeneous field exists within the half-inch just above (90°) the centre of the gap. This was judged adequate for measurements. The detailed dimensions of the gap and pole pieces are shown in figure 4. That this spot is the best in the gap was confirmed later with the proton absorption resonance by moving the sample around in the gap.

The shape of the proton absorption signal in water was not noticeably different from that in the viscous borate solution used; it is seen in the photograph to resemble an isosceles triangle. The breadth of this signal is due almost

27. Rose, Phys. Rev. May 1, 1938, p. 718

* Suggested by Mr. D. Hunten.

completely to inhomogeneity in the field across the sample, since Purcell has shown the proton resonance in water to be very narrow[28] (< 0.015 gauss). The isosceles shape may be accounted for if the magnetic field strength is assumed to change uniformly across the diagonal of the cylindrical sample. The liquid sample is contained in a small bottle 0.5" high and 0.5" in diameter. Assuming the field to vary in this fashion, the intersections of the magnetic field equipotential surfaces with the sample are plane segments of an ellipse perpendicular to the diagonal parallel to the direction of field change. The nuclei in each plane resonate in turn as the resonance condition travels along this diagonal and give a signal shape shown in figure 5, which is a result of a calculation of the area of the segment of the ellipse as a function of the distance along the diagonal. The peak of this resonance is very sharp and gives a fairly satisfactory reference point for establishing the position of the proton resonance.

A further adjustment for homogeneity was carried out after the proton resonance had been obtained. It consisted of varying the gap spacing by a screw adjustment between yoke and winding cores to obtain the narrowest absorption resonance. The base of the proton resonance was 4 gauss wide at the time of the boron measurement

28. Bloembergen, Purcell, Pound, Phys. Rev. April 1,1948 P. 689.

indicating this total inhomogeneity along the diagonal. Since these various successive adjustments were carried out with considerable time intervals in between during which they may have been disturbed, it is most likely that a repeat of these, particularly checking parallelity of the pole faces, would yield a much more homogeneous field.

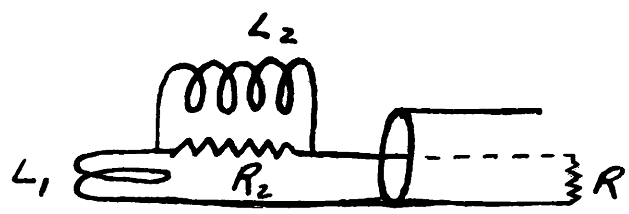
2. COILS ASSEMBLY AND BALANCE CIRCUIT

The parts associated with the induction and detector coils are mounted in a shield constructed of 10 cm. waveguide for rigidity and held by a heavy clamp fastened to the centre of the magnet yoke, the whole assembly designed for a minimum of mechanical vibrations which might induce microphonics in the receiver system. The end of the waveguide shield which goes between the pole faces has narrow slots intended to prevent eddy currents which might make difficult the 60 c.p.s. modulation of the magnetic field. The loose ends are held by screws to a piece of lucite machined to fit in the end of the guide. This construction is shown in one of the photographs. Later experiment showed these slots to be unnecessary -- very little attenuation of the 60 c.p.s. field variation being caused by such eddy currents. This shield is long enough to extend beyond the edge of the magnet windings; the receiver coaxial probe goes inside. The induction coil consists of 6 turns of number 16 gauge copper wire wound 1" diameter on the inside of a polystyrene form threaded 16 turns per inch. This form is fastened to either side of the shield, but between these flat surfaces, the outside of the form is spherical with centre coincident with the centre of the coil. A second piece of polystyrene fits and slides over the spherical section like part of a ball bearing carrying the coaxial probe on whose end the receiver coil is mounted. This part is shown in the photograph of the probe and regenerative receiver. The leakage between induction coil and receiver coil is minimized by varying the angle between the axes of these two coils slowly with a screw adjustment between the receiver probe and the shield. This adjustment may be expected to cancel out most of the common magnetic flux coupling. However, the capacitive coupling leakage, since it is through the high capacitive reactance between the coils, has a large component out of phase with the magnetic leakage and may not be cancelled out by orientation adjustment alone. Two devices are employed to reduce and control this leakage so that it can be set at the optimum value required by the detector used.

(a) Both coils are of a construction which gives electrical balance to ground. The induction coil is fed by a push-pull oscillator which means that the receiver coil should receive approximately equal and opposite capacitive coupling from either end of the induction coil. The receiver coil is wound in two parallel sections of 6 turns each.

The two sections are wound oppositely, their inner ends connected to the centre coaxial conductor and outer ends grounded, so that the more sensitive ends are shielded to some extent by the outer less sensitive ends. The coil is wound with number 24 gauge enamelled copper wire, closely spaced, 0.57" diameter, and has an inductance of 0.55μh. The coil form is a thin walled glass bottle which was blown in a mold from pyrex tubing, and has a capacity of 1.8 c.c. The shank, a short piece of pyrex tubing which is cemented to the bottle after the coil is wound on, has the coaxial connections inside it, and fits into the polystyrene "bearing" around the receiver probe. The Q of this coil is about 130 with the boron solution in the bottle. Under this condition it was possible to reduce the leakage by orientation to a fraction of a volt across the tuned circuit or a few millivolts of actually induced voltage.

(b) In order to reduce this leakage further and control it, a phase shifter was added to the power oscillator, capable of delivering about 1 volt at any phase relative to the oscillator phase into 50 ohms so that a voltage may be fed into the receiver to balance out undesirable leakage. This consists of two coils in series mounted on a polystyrene ball in the field of the tank coil of the oscillator. One coil consists of a single turn of heavy wire, the other with its axis at right angles to it consists of several turns and is almost completely shorted by 50 ohms across it. A knob on the panel of the oscillator cabinet rotates the ball via two bevel gears. The output is a 50-ohm cable supposed to be matched at the far end. The circuit is shown with that of the oscillator in figure 6 and again here, in which L_1 and L_2 are the coils of one and several turns respectively.



For the frequencies used,

$$\omega L_1 << R + R_2$$

$$\omega L_2 >> \frac{RR_2}{R + R_2}$$

Let θ be the angle of rotation of these coils in the field of the oscillator tank coil.

Output across R due to $L_1 = \dfrac{R}{R + R_2 + j\omega L_1} n_1 10^{-8} \dfrac{\partial \varphi}{\partial t} \cos \theta$

Output across R due to

$$L_2 = \frac{\frac{RR_2}{R + R_2}}{\frac{RR_2}{R + R_2} + j\omega L_2} n_2 10^{-8} \frac{\partial \varphi}{\partial t} \sin \theta$$

Neglecting the relatively small terms, the sum of these becomes:

$$E_R = \left(\frac{Rn_1}{R + R_2} \cos\theta - j \frac{RR_2 \quad n_2}{(R + R_2)\, \omega L_2} \sin \theta \right) 10^{-8} \frac{\partial \varphi}{\partial t}$$

The output of this phase shifter is a voltage with a manually variable phase and constant amplitude if

$$\frac{n_1}{n_2} = \frac{R_2}{\omega L_2}$$

At a fixed frequency this equivalence may be obtained and the phase of its output voltage varies as the rotation of the dial. Actually since R_2 increases with frequency increase somewhat, this property may hold fairly well over a considerable range. The variation of output amplitude for this phase shifter with θ is shown for 10 mc. and 30 mc. in figure 7. The values were adjusted to give a practically constant output over θ at around 10 mc. since this is where the B^{11} resonance was expected.

The balancing signal from the phase shifter is fed through an attenuator directly into the 50-ohm tap on the resonant circuit of the sample coil and the tuning condenser as shown in the receiver diagram of figure 8. The attenuator used was built as a general purpose piston attenuator designed according to Hartnett and Case[29]. A disadvantage in its use is its high insertion loss of 27.6 db. at 10 mc. Together the phase shifter and attenuator provide independent phase and amplitude control of the balancing voltage. The attenuator is intended to make sensitivity measurements for the receiver, and also to

29. Hartnett and Case, Proc. I.R.E. June 1935, p. 578

determine the amplitude of a resonance.

3. POWER OSCILLATOR AND SUPPLY.

A circuit diagram of the radio frequency power oscillator and supply is shown in figure 6.

The high voltage supply is an electronically regulated rectifier of conventional type getting its supply through a line regulator. The + 300 volts output is observed on an oscilloscope to have a variation of a few millivolts. The filaments of the oscillator tubes are supplied by a 6-volt storage battery.

The oscillator itself employs two 807's in push-pull, the tuned circuit across the plates being tapped on either side of the centre which is at ground potential, to provide feedback to the grids and voltage for the induction coil to which the taps are linked by a shielded pair. A meter reads the grid rectified current to indicate the peak r.f. voltage on the induction coil. The amplitude of oscillating r.f. field, 2 H_1, at the centre of the induction coil was measured as 5 gauss. The oscillator may be tuned from 9 mc. to about 80 mc. with the help of three plug-in tank coils. The oscillator is stable after it warms up and is unusually free from harmonics.

4. THE RECEIVERS

The circuit diagram of the system used for comparing two gyromagnetic ratios is shown in figure 8. With this system it is possible to go from displaying the proton to displaying the B^{11} resonance in a few seconds by throwing the single pole double throw switch connecting the sample coil.

The Regenerative Receiver

The proton signal may be displayed either by the induction method or by absorption. If the regenerative receiver is set just on the verge of oscillation with the power oscillator on, a strong proton induction resonance can be picked up. On switching the oscillator off, the regenerative receiver goes into oscillation producing the absorption signal[30].

The regenerative receiver consists of a single 6AK5 tube as a type of cathode-follower oscillator[31]. The feedback for this type occurs through the cathode-grid capacitance which is sufficient coupling in our circuit for any frequency above 14 mc. The regeneration is controlled by varying the cathode load resistance. There exists an optimum setting for this control. Too strong an oscillation causes the signal to decrease in amplitude; too weak an oscillation does not decrease the signal

30. Method suggested by Roberts, Rev.Sci.Instr.Nov.1947 p. 845.

31. Cathode-Follower Circuits, Schlesinger, Proc.I.R.E. Dec. 1945 p. 843.

amplitude, but increases the noise. If the oscillation becomes extremely weak, the simple proton resonance takes on a transient shape resembling the "slow passage" resonance, explained mathematically by Jacobsohn and Wangsness[26].

The regenerative receiver has a flexibility and a simplicity which commend it, but it is somewhat unstable in frequency. When it is used to receive a very weak induction signal it becomes very awkward to adjust with the balance circuit, owing to the interdependence of the various controls. The dependence of frequency on the regeneration control can be understood when it is realized that part of the input capacitance to the grid depends on the gain of the cathode-follower.

The Tuned-Radio-Frequency Receiver

Because of the aforementioned difficulties as well as the inferior sensitivity of the regenerative receiver, a new receiver was built. The amplifier described by Wallman, MacNee and Gadsden[32] was chosen for the following reasons:-

1. This receiver, having a very low-noise figure, is one of the best as a pre-amplifier for any type of induction or absorption resonance. It may be adapted to low-noise pre-amplifier use by adding an output tap to

32. Wallman, MacNee, Gadsden, A Low-Noise Amplifier, Proc. I.R.E. June, 1948, p. 700.

the third plug-in coil (across the 6AL5 diode), taking the output line to a conventional receiver.

2. This receiver alone was adequate for observation of the B^{11} induction resonance.

3. The T.R.F. receiver does not add the complication of image frequencies present when a strong signal is applied to the input, and there is no concern about local oscillator noise.

The condensers in the matching box of the probe match the sample coil to the 50-ohm line which in turn is made to appear as about 10 kilohms to the 6AK5. The plug-in tuning coils have high Q's so that the overall radio frequency bandwidth is the usual audio width. The tapping down of the input for each coil narrows the bandwidth, and in the case of the second coil, lowers the impedance seen looking into the cathode of the grounded-grid 6J6 preventing regeneration in the first 6AK5. The plug-in coils allow the possibility of simple adaption to any frequency at which it is desired to work, and it is possible to tune over a few megacycles with the trimmers, or with the iron-dust slugs of the slug-tuned coils.

The audio-amplifier consists of a 6SN7 mixer and a 6J5 output tube. By switching the cathodes of the 6SN7 it is possible to receive either resonance separately or both at once (as arranged for with an eye to future use when both signals might be viewed simultaneously). The audio response was taken down to a fraction of a cycle per second to allow for the possibility of observing a resonance while the field is being varied very slowly to allow for a long relaxation time; d.c. bias from No. 9 flashlight batteries being used for the same reason. The whole receiver is powered by a 6-volt storage battery and two 45-volt "B" batteries.

Frequency Indicators

Besides an oscilloscope (Dumont, model 208), other items of apparatus were two frequency indicators. The basic indicator* is a conventional circuit consisting of a precision quartz crystal (SMC-100) capable of oscillating at either 100 kc. p.s. or 1 mc. p.s. and a multivibrator working normally at the tenth subharmonic. Since the higher harmonics of the 100 kc.p.s. are not great enough to make a visible beat note in the regenerative circuit at frequencies above 20 mc., another type frequency indicator (Marconi, Type C2) which has a strong stable oscillator with five frequency ranges between 5 mc. and 10 mc. was used as an intermediate reference. When this oscillator is set at 10 mc. on the 100th harmonic of the crystal, its third harmonic is strong enough to pull

*Built by Mr. D. Hunten.

the frequency of the regenerative oscillator, working at 30 mc., into synchronism with it when coupled only loosely. The condenser of the Type C2-indicator oscillator is the straight line frequency type -- lending itself well to interpolation when it is calibrated against the multivibrator of the standard indicator. Interpolation in the intervals between multivibrator heterodynes was aided by audible reference to a calibrated audio oscillator. Another receiver which happened to be available facilitated observation of the heterodynes at 10 mc. and of the B^{11} resonance at around 9.62 mc. No reference to a standard such as WWV was deemed necessary since it is a ratio of frequencies only which was measured.

EXPERIMENTAL PROCEDURE AND RESULTS

The Samples Used.

Several attempts to observe the absorption and induction B^{11} resonances were made with the regenerative circuit in the same way as was successful for the proton. The liquid samples were placed in the "sample bottle" coil form described on page 26. A new coil was made for the solid samples which were tried on the vague chance that some might have T_1 short enough and T_2 long enough to allow a visible resonance. The coil consists of two oppositely wound sections of 15 turns each. The coil form and supporting shank are of polystyrene. The circuit was altered to suit the impedance of this coil while it was in use.

The frequency was set at about 10 mc., the magnetic field varied through the appropriate value, and the following samples tried:-

Pure amorphous boron

Boron carbide (B_4C)

Sodium tetraborate ($Na_2B_4O_7.10\ H_2O$) crystals, and saturated aqueous solution.

Trimethyl borate ($B(OCH_3)_3$), 40% pure in methyl alcohol.

Potassium metaborate (KBO_2) saturated aqueous solution.

The potassium metaborate was chosen because it is one of the most soluble* of the compounds of boron, and hence the solution contains a large number of boron atoms per unit volume.

The T.R.F. receiver was added to the equipment and a B^{11} resonance observed using an alkaline solution of potassium metaborate in water. The B^{11} resonance shown in the photograph was obtained with a freshly prepared solution of the borate made by mixing 8 grams of H_3BO_3 with 10 c.c. of a boiling solution of 7 grams of KOH, producing a system described by:

$$KOH + H_3BO_3 \rightleftharpoons KBO_2 + 2H_2O$$

This solution is about as viscous as concentrated sulphuric acid, and over a few hours what is presumably boric acid crystallizes out, the signal decreasing in amplitude as this happens.

Relaxation Times

Approximate information may be gleaned about the relaxation times in this solution. Inspection of the 60 c.p.s. display as the average magnetic field is varied slowly through the boron resonance shows that the signal just has time to reverse in half a sweep period. This means that T_1 is about 0.01 second.

* Suggestion by Dr. C. A. Winkler of the Department of Chemistry.

No accurate idea of T_2 may be obtained from this resonance, since the observed width can practically all be assigned to the inhomogeneity of H_0 and the natural width due to the strong r.f. induction field. The following readings describe conditions at the time the photograph was taken:-

Total Magnetic field (H_z) cyclic variation	= 16 gauss
Width of resonance (at .707 max. amplitude)	= 3.2 gauss
Voltage on 1.1 μh. induction coil	= 86 volts r.m.s.
Current in coil at 9.6 mc.	= 1.8 amperes peak
Turns in induction coil	= 6.3 turns/cm.

Calculating the field along the axis of the induction coil according to the formula,

$$H_z\,(\text{max.}) = \frac{2\pi n i}{10}\,(\cos\beta_2 - \cos\beta_1)$$

where β_1 and β_2 are the angles between the coil axis and the ends of the coil at the point on the axis, gives a field at the centre of 5.0 gauss. A rough estimation of the field off the axis[33] shows that:

average field over the sample = about that on the axis

= 4.3 gauss

i.e. $2H_1(\text{av.}) = 4.3$ gauss

33. Smythe, W.R., Static and Dynamic Electricity, McGraw-Hill p. 267.

When the known inhomogeneity of 0.9 gauss from the proton resonance is added to this, the expected width of B^{11} resonance becomes 5.2 gauss, about 60% greater than the observed width of 3.2 gauss. The conclusion is that the 86 volts is an overestimation of the coil voltage due to the inductance of leads between oscillator and induction coil. We may conclude that

$$T_2 > \frac{1}{|\gamma| \frac{3.2-0.9}{2}}$$

$$> 10^{-4} \text{ sec.}$$

Magnetic Moment of B^{11}

In making the comparison of the gyromagnetic ratios of the two nuclei, two features were desired in the display:

1. Any fluctuations in the magnetic field shall be random or cyclic so that they will average out over a number of readings. On simple inspection the movements of the resonance seemed to be of this type.

2. The readings must be taken in such a way that any transient phenomenon existing in the resonance or out-of-phase condition in the fore and back sweeps will not affect the average value obtained. The conditions in (2) do not matter in principle if the resonances are set in the centre of the sweep and the two adjusted together for symmetry.

The procedure followed to eliminate these errors was to adjust the phase of the sweep so that the two peaks of the proton resonances coincided, and then take as many readings of the boron fore-sweep resonance as of the back-sweep one. On account of the value of T_1, one or the other of these is larger depending upon which side of the centre of the sweep the boron resonance lies. In either case the larger of the two resonances was adjusted to the marker on the screen. Taking the resonance readings off-centre also improved the degree of discrimination between resonance and a slight amount of 120 c.p.s. spurious signal in the display.

The induction resonance of the B^{11} nucleus at about 9.6 mc. was compared to the absorption resonance of the proton at 30.0 mc. Between each reading of the B^{11} resonant frequency, the variable oscillator of the secondary frequency indicator was set on 10 mc. $\pm$ 30 c.p.s., the 100th harmonic of the SMC-100 crystal. The regenerative receiver frequency was adjusted to have a low or zero frequency difference with its third harmonic and simultaneously the magnetic field adjusted to make the average position of the proton resonance coincide with a mark on the screen. A switch to the B^{11}

resonance required only a few seconds. The power oscillator was then adjusted to make the B^{11} resonance coincide with the same mark, and its frequency read against the dial of the secondary indicator. A switch back again to the proton resonance found its average value on the mark and its frequency within an estimated $\pm$ 200 c.p.s. of 30 mc. The sweep field was 16 gauss. Before the readings were taken, the whole apparatus was allowed to warm up for over two hours, and the precision work was done in the evening so that there would be fewer surges in the a.c. lines due to the changing loads of power tools.

The dial readings taken for the B^{11} resonant frequency are as follows:-

Conditions	Secondary Standard Frequency Dial	Average	Residual (d_i)	d_i^2
H_0(av.) < resonance value	9.6249		+ 2 x 10^{-4}	4 x 10^{-8}
	9.6248		+ 1	1
	9.6241		- 6	36
H_0(av.) > resonance value	9.6247		+ 4	16
	9.6241		- 2	4
	9.6247		+ 4	16
	9.6233	9.62427	-10	100
	9.6243		0	0
	9.6245		+ 2	4
H_0(av.) < resonance value	9.6249		+ 2	4
	9.6248	9.62468	+ 1	1
	9.6246		- 1	1

Average of fore and back resonance = 9.62447 mc. d_i^2 = 187 x 10^{-8}

The calibration of the dial was effected by reference to the audio signal generator and the multivibrator of the primary frequency indicator. The latter, working at the ninth subharmonic, gives frequencies at

$$9.6000 \pm \frac{0.100}{9} n \qquad \text{mc. (n is an integer)}$$

Calibration yielded the following readings:

Dial Reading	Audio Note	Oscillatory Frequency
9.6220	0 c.p.s.	9.62222 mc.
9.6240	2080	9.62430
9.6250	3160	9.62538
9.6330	0	9.63333

By interpolation the boron resonance frequency becomes

9.62481 mc.

Assignment of Errors: The errors which may occur are of two kinds: random and systematic.

The chief causes of the spread in the above dial readings are the errors in judging exactly the average positions of the two resonances with respect to the mark on the screen, and the variation of the average value of the field between observations. The probable error computed from the readings is a fair measure of random error from these sources. This spread of readings also contains two cascaded errors involved in setting the regenerative receiver exactly on 30 mc. Estimated errors are:

$\pm$30 c.p.s. in setting the variable oscillator of the secondary standard on the 100th harmonic of the crystal, and $\pm$200 c.p.s. for adjusting the regenerative receiver to the variable oscillator. These two errors are not independent of the errors shown by the randomness of the readings, since if the frequency of the regenerative oscillator is set wrongly, the resulting setting for the boron resonance should be wrong by the same fractional amount. The probable error for the set of readings covers all random errors except the calibration error of the indicator dial. This calibration was carried out immediately after the readings were taken. There is no backlash in the dial; however, 0.1 division or 0.0001 mc. is thought to be a reasonable probable error. From the readings we have the probable error[34] for a single determination:

$$r = \pm 0.6745 \sqrt{\frac{\Sigma d_i^2}{n-1}}$$

where n is the number of readings.

$$= \pm 0.6745 \sqrt{\frac{187 \times 10^{-8}}{11}}$$

$$r = \pm 2.8 \times 10^{-4} \text{ mc.}$$

For the set the probable error is:

$$R = \pm \frac{r}{\sqrt{n}}$$

$$= \pm 0.8 \times 10^{-4} \text{mc.}$$

Adding the assumed dial error the total known probable error becomes:

$$P = \pm \sqrt{.8^2 + 1^2} \quad x\ 10^{-4}$$

$$= \pm\ 1.3 \times 10^{-4} \text{ mc.}$$

It is noticed in the readings that the averages of the fore and back resonances differ by 0.00041 mc. This could be due only to the presence of a transient component in one or both of the resonances, since the peaks of the proton resonance were adjusted together. The direction of the shift is such that the proton peak must occur just after $\delta = 0$, or the boron peak before $\delta = 0$. Inspection of the proton resonance shows a trace of the "wiggles" explained by Purcell[28] and by Jacobsohn and Wangsness[26] who show that the peak of the absorption is delayed a slight amount in this case. Because of the known small value of T_1 there is a slight mixture of slow passage type resonance in the boron signal. This mixture shifts the peak of the resonance so that it occurs slightly before $\delta = 0$ Both these shifts are in the same direction and combine to produce the above difference in the averages. In any case all but second order errors due to this shift are cancelled out by the method followed in taking the readings.

Systematic errors could arise from drift of the magnetic field or of the frequencies of the indicators consistently in one direction during the time of the experiment.

Because the return from the boron to the proton resonance after each measurement usually found the latter in good adjustment, it is believed that negligible drift in one direction of the magnetic field was present. Both frequency indicators are very stable after warming up, and could not have drifted more than a negligible amount over the time occupied by the readings. The drift during this time is the only one which could introduce an error since calibration was carried out immediately afterwards.

Assymmetry of the resonance due to an assymmetrical inhomogeneity over the sample or the presence of confusing spurious signals on the screen could also introduce systematic errors. Inspection of the photographs of the resonances shows them to be reasonably symmetrical, but that spurious signals which are confusing for the placing of the boron resonance are present. Systematic error from this source is difficult to assess, and is neglected because care was taken during the readings to place the resonance on flat rather than sloping parts of the display, and to read by areas between resonances and spurious signals rather than by the peaks of the former. There are no other reasons known for systematic errors.

Results: The boron resonant frequency may be stated as

$$9.62481 \pm 0.00013 \text{ mc.}$$

which gives a ratio for the two frequencies:

$$\frac{f_B}{f_P} = \frac{9.62481 \pm 0.00013}{30.0000} = 0.320827 \pm 0.0000043$$

<u>Corrections:</u> The Lamb[35] correction for diamagnetism in atoms is a factor of

$$(1 + 0.319 \times 10^{-4} (Z)^{4/3}) \text{ where } Z = \text{at. No.}$$

increasing the observed value of the gyromagnetic ratio because the field at the nucleus is weakened by the planetary electrons. Neglecting the effect of the molecular binding on the electronic wave functions the fraction to be applied to the above ratio is:

$$1 + 2.41 \times 10^{-4}$$

giving the result

$$\frac{\gamma_B}{\gamma_P} = 0.320904 + 0.0000043$$

Using the value 2.7928 $\pm$0.0008 for the proton, and a ratio of 3 for their spins,

$$\mu_B = \frac{\gamma_B}{\gamma_P} \cdot \frac{I_B}{I_P} \cdot \mu_P$$

$$= 2.6887 \pm 0.0008 \quad n.m.$$

<u>Agreement with Concurrent Work</u>

Since the above measurement was made, two determinations of the magnetic moment of the B^{11} nucleus have been made known. A recently published letter by Zimmerman and Williams[36] gives a value

$$= 2.700 \pm 0.008 \quad \text{nuclear magnetons}$$

35. Lamb, W.E., Phys. Rev., Dec. 1, 1941, p. 817.
36. Zimmerman and Williams, Phys. Rev. Dec. 15, 1948, p. 1885.

They detected the absorption resonances of B^{11} and Na^{23} in an aqueous solution of $Na_2B_2O_4$ using a super-regenerative receiver and referred to the value for the magnetic moment of sodium which was given as 2.217 ± 0.002 by Millman and Kusch[19]. In the same paper [19] they give the value of 2.687 ±0.008 for B^{11}, the error of which overlaps that of Zimmerman and Williams. The earlier paper[16] of Millman, Kusch and Rabi gives the lower answer, 2.682 ± 0.008 with which the value 2.700 is at variance.

The other determination is given in an internal report by Professor Bitter of Massachusetts Institute of Technology in which the result quoted is

2.6857 ± 0.02%

assuming the proton value to be 2.78960 and disregarding its error. Under the same premise the value obtained by the writer is

2.68558 ± 0.00004

which is in substantial agreement with the M.I.T. value but definitely at variance with the value of 2.700 ± 0.008.

Note on the Amplitude of the B^{11} Resonance

It was found experimentally that the minimum leakage from induction coil to receiving coil could be reduced considerably by paying careful attention to the position of the receiving coil along the axis of the former. The B^{11} signal became nicely visible without the need of the balance circuit and the attenuator was released for other use.

Using this attenuator, a known voltage was fed into the receiver 50-ohm line from a signal generator. A beat note whose half-period corresponded to the resonance width was produced on the display, caused by the leakage from the power oscillator. Its amplitude was adjusted to equal the resonance height. From this the B^{11} resonance amplitude may be estimated. If the sample coil and attenuator are matched to each other in this circuit, then, regardless of the receiver input impedance, the available signal powers from each are the same when the display voltages from each are equal. In the case of a mismatch the following formula may be derived:

$$a_r = \frac{\omega L \cdot E_0}{\sigma R_A}$$

where ωL = reactance of sample coil

E_0 = open circuit attenuator output

R_A = attenuator internal resistance

σ = voltage step-up ratio of the condensers matching sample coil to 50-ohm line.

In our case a mismatch exists. The attenuator internal resistor was made by putting aquadag on paper to get a radial conductor in the coaxial line which would have an r. f.

resistance approximating closely to its d.c. value. Its value increased to 69 ohms from its original value of about 50 ohms. Solution was spilled on the sample coil lowering its Q to 130 from a higher value. Both effects produce a mismatch in the same direction. At the time of the measurement the following were the pertinent values:-

$$\omega L = 33.2 \text{ ohms}$$

$$E_0 = 140 \mu v.$$

$$\sigma = 13$$

$$R_A = 69 \text{ ohms}$$

$$a_r = \frac{33.2 \times 140}{13 \times 69}$$

$$= 5.2 \mu v.$$

This must be increased by about 20% on account of field inhomogeneity, giving

$$a_r = 6.2 \mu v.$$

An error of ± 40% is estimated for this because of errors in comparing the small deflections on the screen, the length of coaxial lines with mismatches, and the errors in the various measured quantities involved. This value is to be compared with 6.0 μv. obtained for the amplitude from Bloch's theory disregarding the effect of relaxation times.

As explained by Bloch, the optimum condition for observation of a nuclear induction resonance occurs when

$$T_2 \sim T_1$$

causing the integral in the expression for M(t) to approach the value $T_1 \chi H_o$ and

$$M(t) \rightarrow \chi H_o$$

A suspicion that this condition impends in this experiment was gathered from the former conclusions that

$$T_1 \sim .01 \text{ sec.}$$
$$T_2 > 10^{-4} \text{sec.}$$

CONCLUSIONS

The Experiment.

Considerable accuracy has been attained in the comparison of the gyromagnetic ratios of the proton and B^{11} in spite of some adverse features in the apparatus. This has been accomplished by extreme care in the measurement, and the arrangement of factors tending to introduce errors so that as little as possible systematic error is introduced, the predominant error being that of pure random variations.

The viscous potassium metaborate used is a very good type of sample for this apparatus, having the two advantages of high density of nuclei and the correct relaxation times for maximum signal. In the investigation of other nuclei it is probably advantageous to look for a viscous sample like the one used here.

The Apparatus.

Noise in the signal comes mainly from two sources. It should be feasible to reduce it by some attention to these sources. Some noise coming from sparking in the commutator of the generator could be eliminated by overhauling the generator and also by building a new shield for r.f. components between the pole pieces. The slots in this shield which have been found to be unnecessary undoubtedly admit some spurious r.f. fields from the magnet and should be eliminated in any future design. There is modulation of a few millivolts in the high tension supply for the r.f. oscillator which adds to the noise. Detection of the signal in the 6AL5 diode occurs best when a few volts of leakage are present across it. This means that the modulation percentage of the r.f. oscillator should be small enough that it is negligible compared with the signal at this point. The scheme for reducing the leakage by using coils whose construction gives electrical balance to ground and orienting one coil mechanically to the other is very satisfactory in adjusting for the optimum leakage.

With the attenuator and phase shifter it is possible to adjust the leakage to so low a value that the 0 - 100 meter in series with the diode reads practically zero.

This apparatus does not lend itself to the measurement of the transverse relaxation time with induction, but line widths using the regenerative receiver absorption method can beestimated, although some repeated adjustments for homogeneity are necessary to give accurate results. It is not difficult to estimate the longitudinal relaxation time from the induction resonance when it is of the order of magnitude of the sweep period or less.

The feasibility of checking quite accurately the amplitudes of signals with this apparatus was established with the method outlined in the note on the amplitude of the B^{11} resonance (see page 46). The import of such a measurement is signified by the expression for a_r in which

$$a_r \propto nI(I+1)$$

If two isotopes whose relative abundance and gyromagnetic ratios are known exist in equivalent molecular states in the same sample, their relaxation times may be expected to be equal, or very closely so; and a determination of their relative spins may be made by measuring the amplitudes of their resonances. Bloch and his collaborators estimated the spin of the triton in this way[12]. Measurements would be made at the same frequency, changing the magnetic field from one resonance to the other, the precision piston attenuator being used to establish the ratio.

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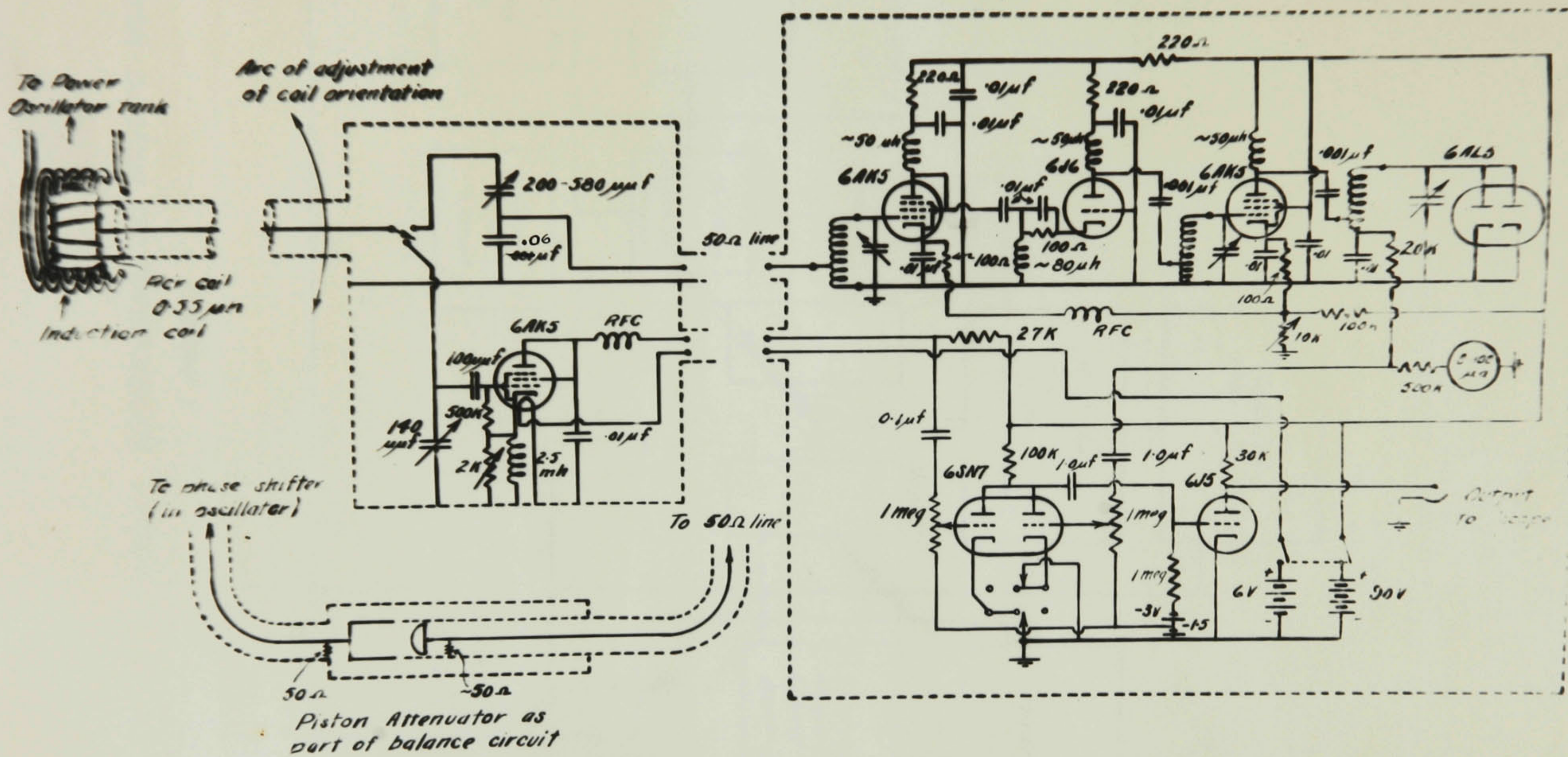


FIGURE 8

TRF RECEIVER, REGENERATIVE DETECTOR AND AMPLIFIER

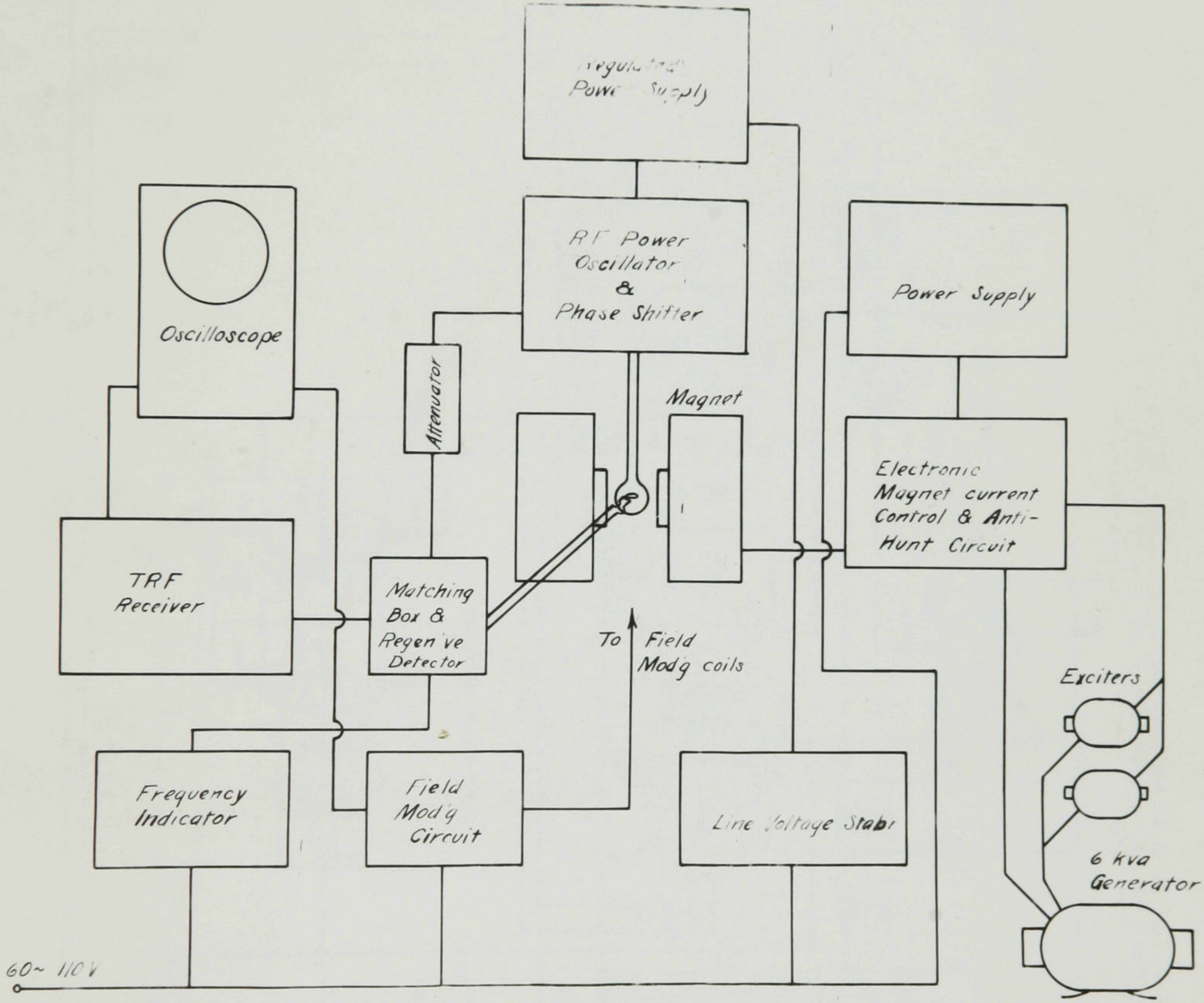


FIGURE 1

BLOCK DIAGRAM OF EXPERIMENTAL LAYOUT

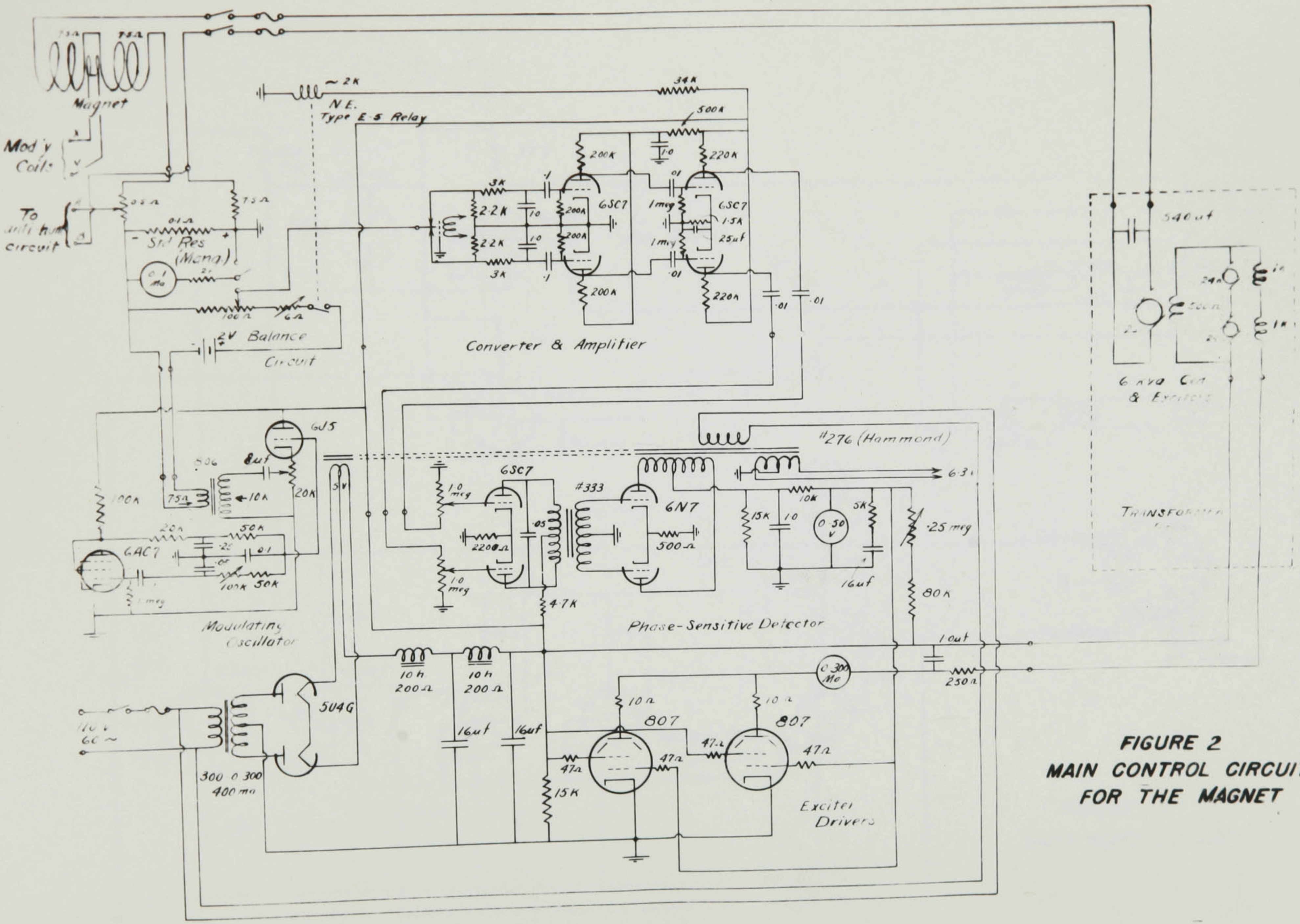


FIGURE 2
MAIN CONTROL CIRCUIT
FOR THE MAGNET

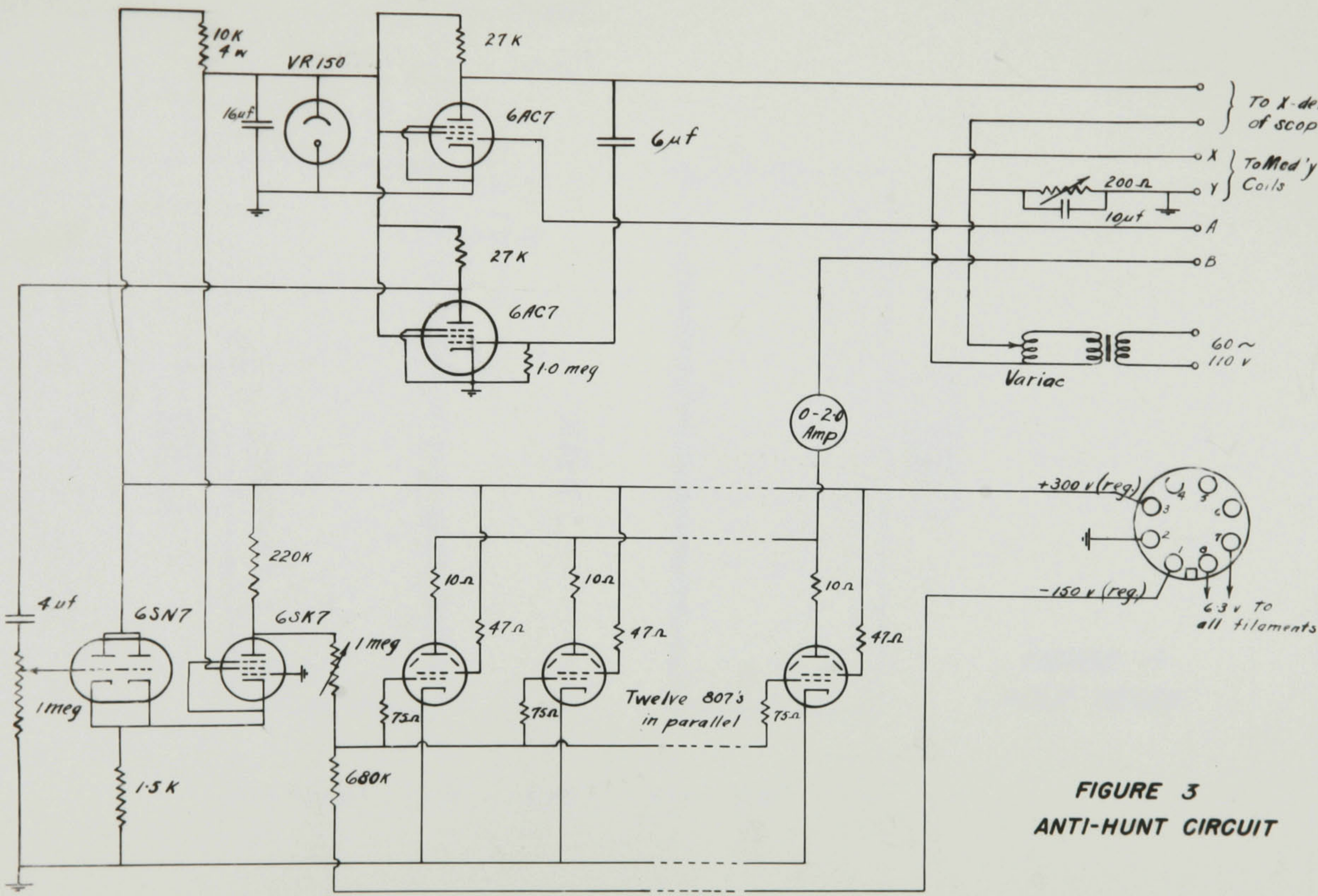


FIGURE 3
ANTI-HUNT CIRCUIT

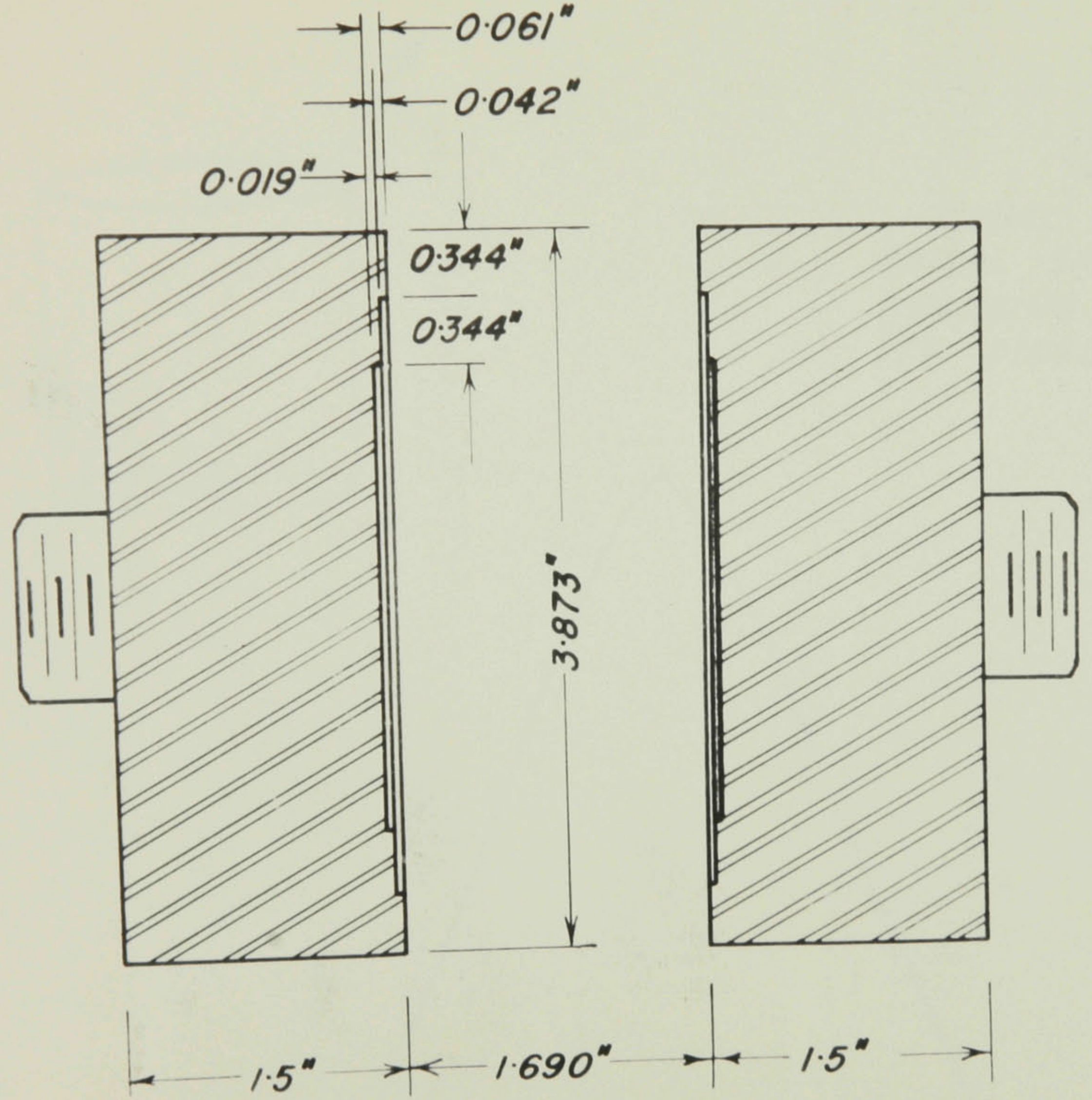


FIGURE 4
POLE PIECES

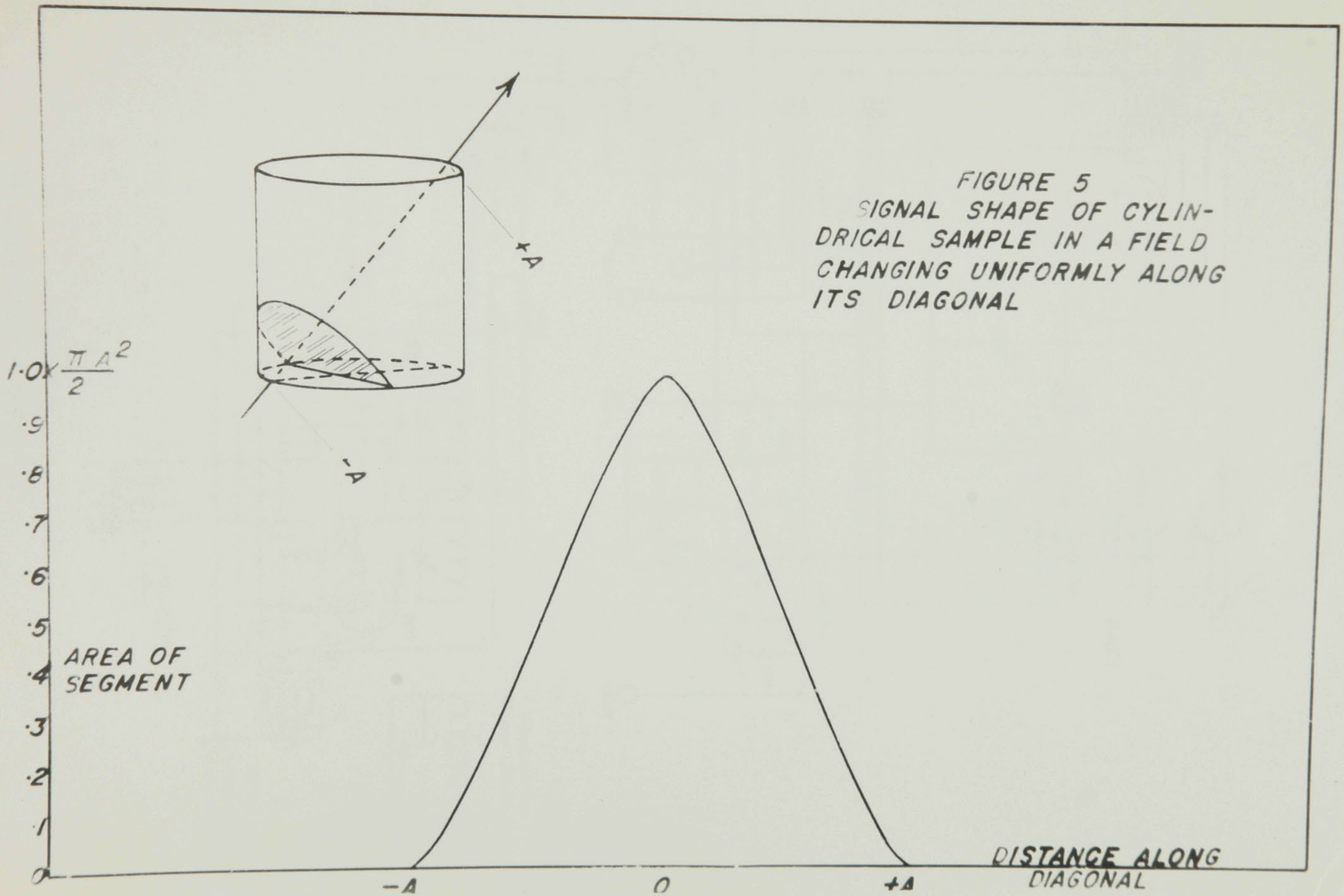


FIGURE 5
SIGNAL SHAPE OF CYLINDRICAL SAMPLE IN A FIELD CHANGING UNIFORMLY ALONG ITS DIAGONAL

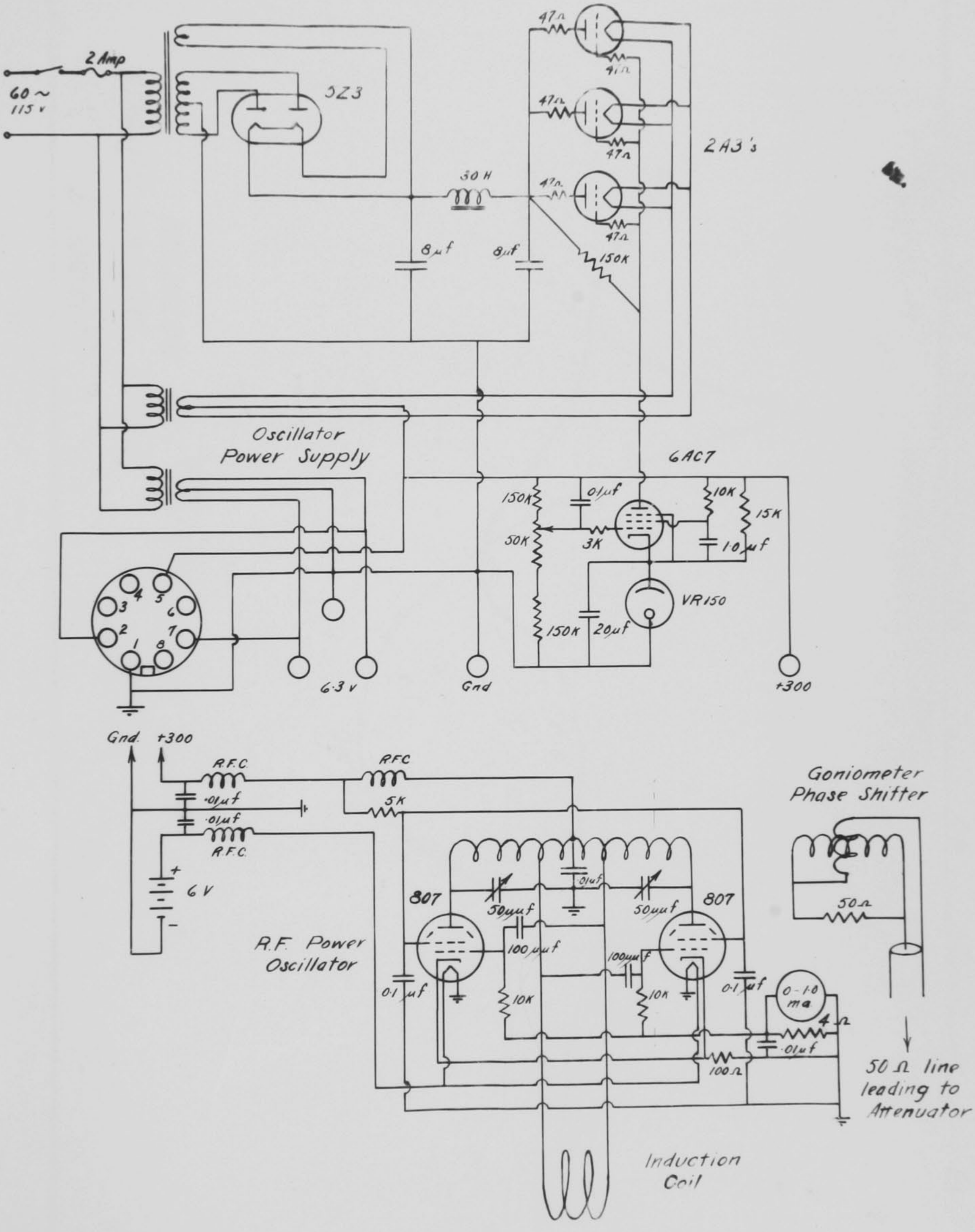


FIGURE 6

RF OSCILLATOR & SUPPLY

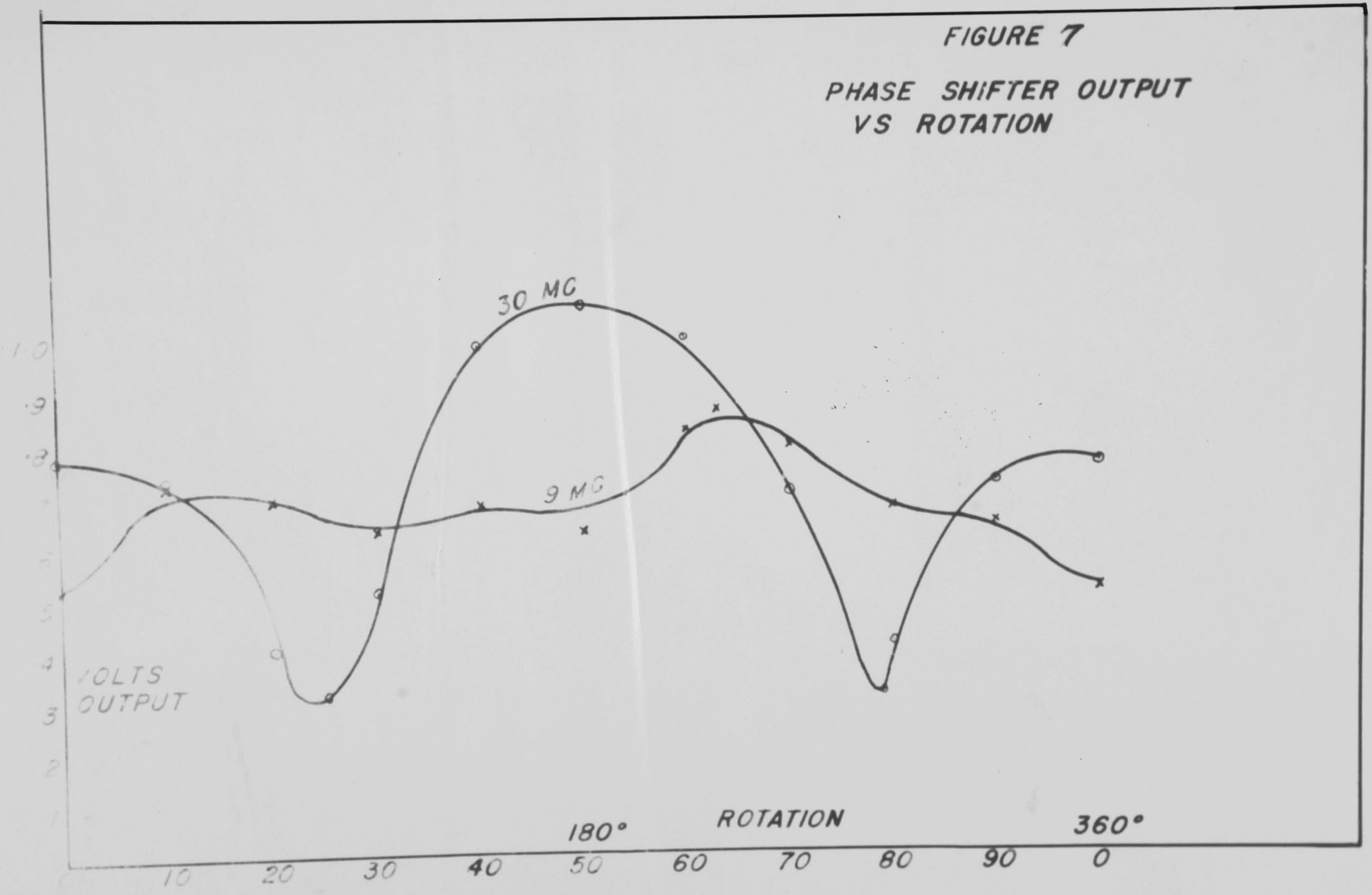
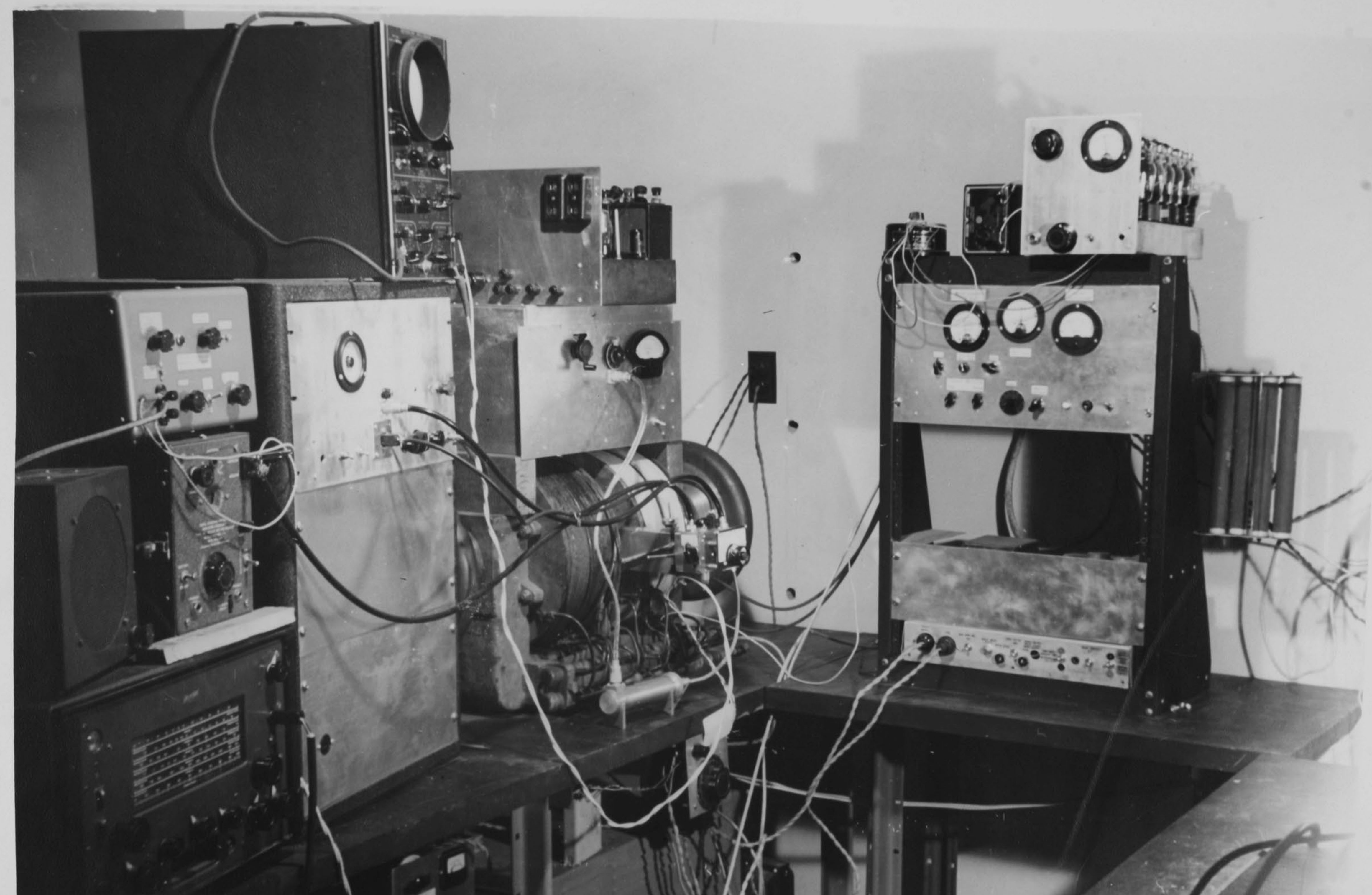
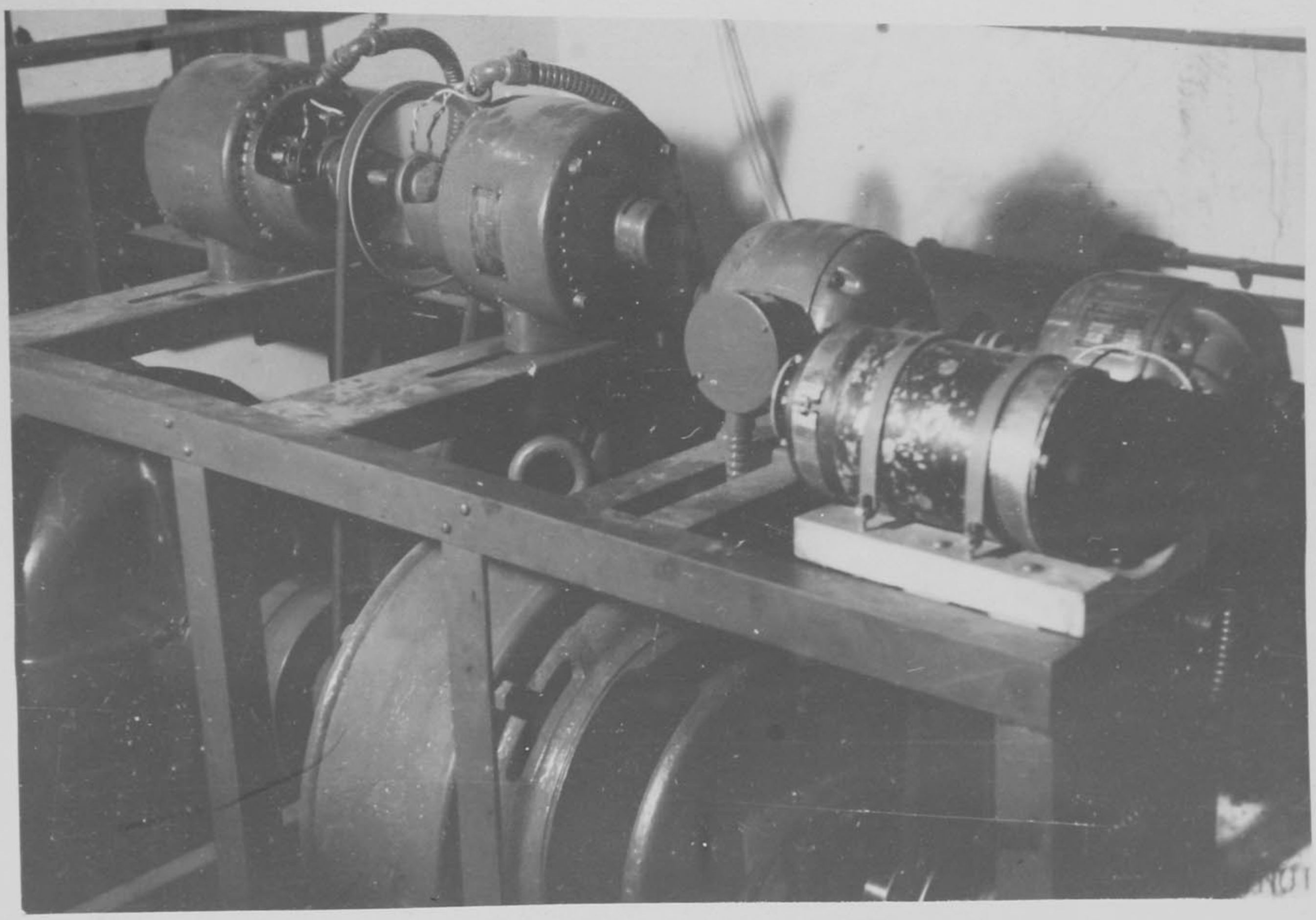


FIGURE 7

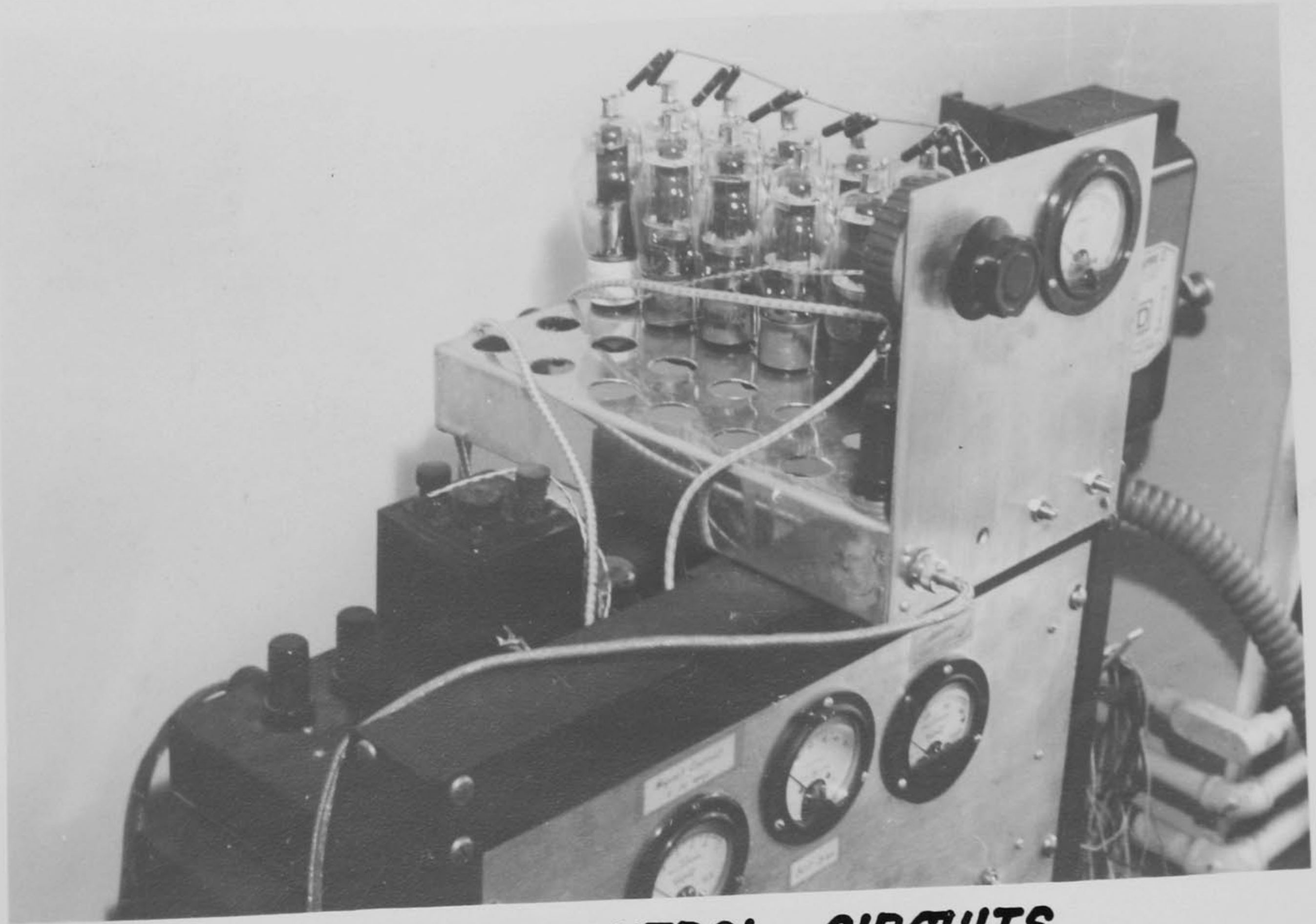
PHASE SHIFTER OUTPUT VS ROTATION



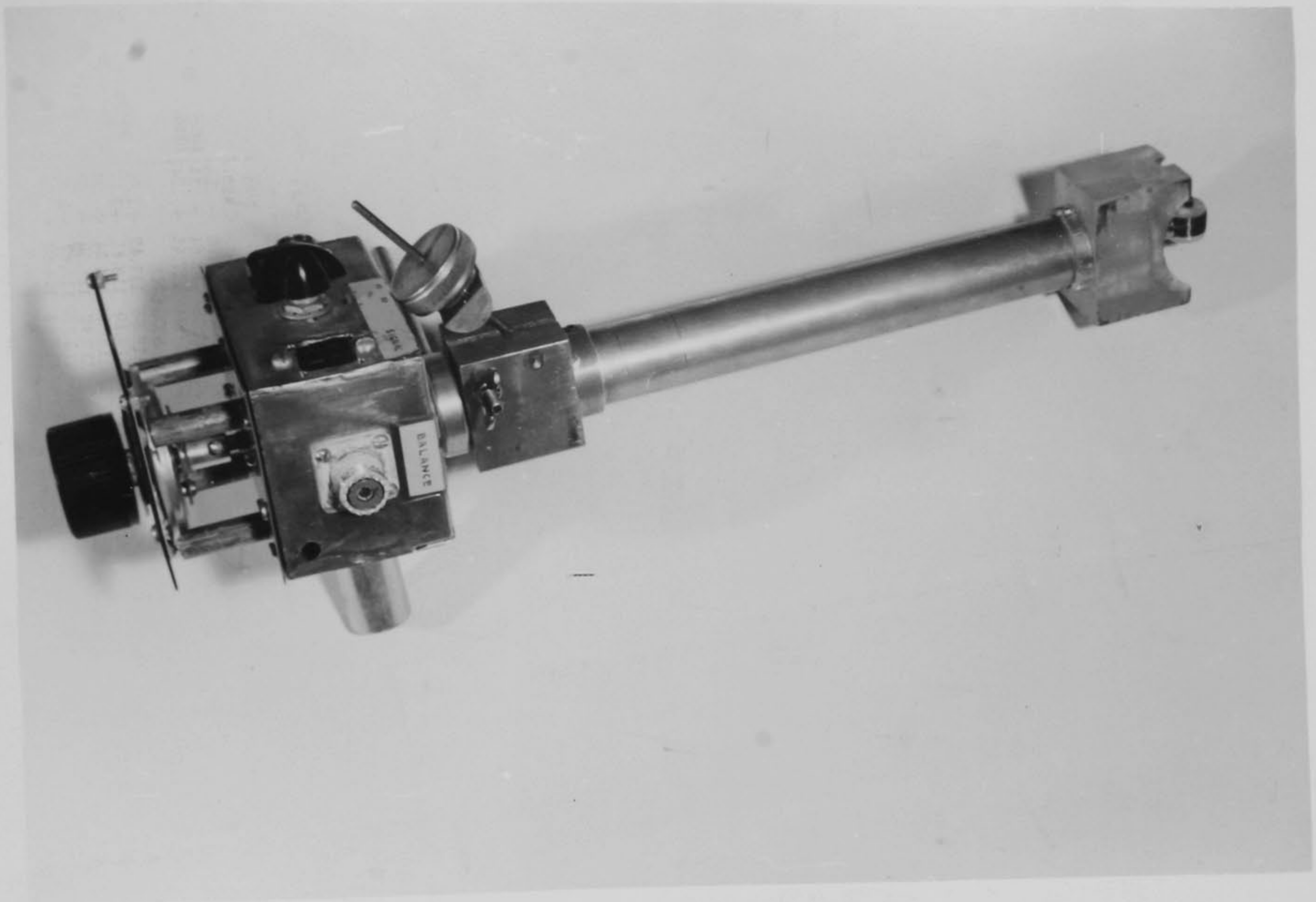
EXPERIMENTAL APPARATUS



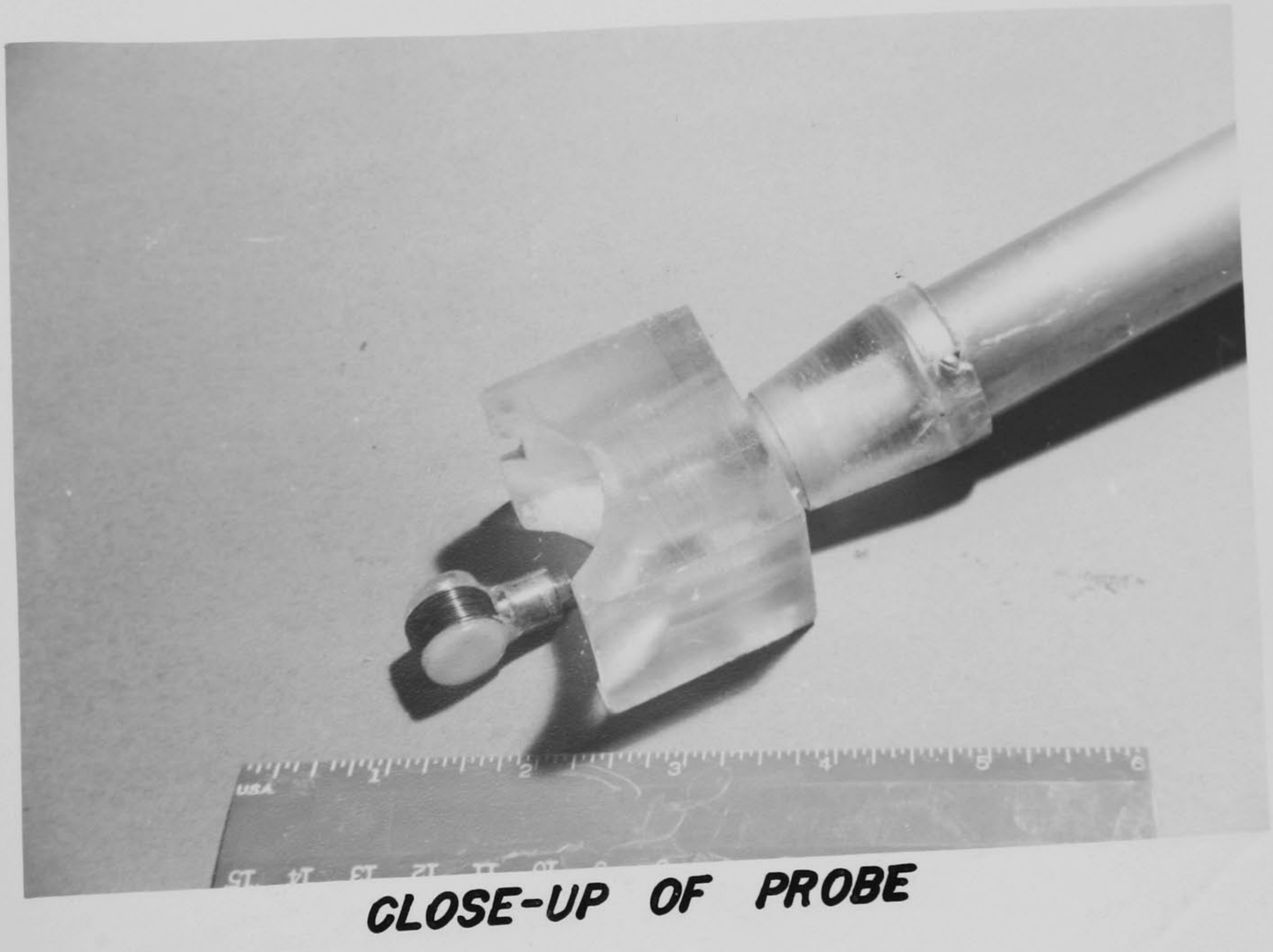
GENERATOR & EXCITERS



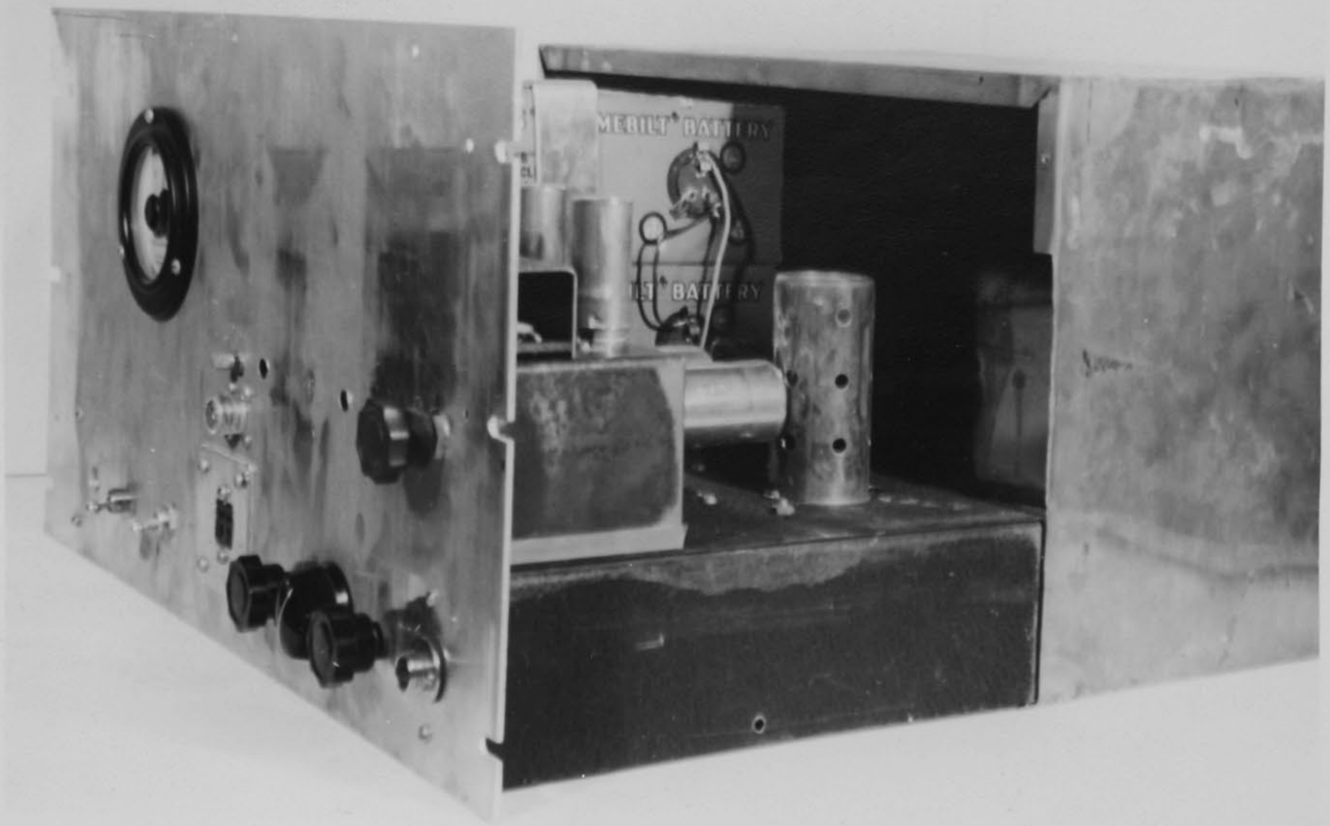
MAGNET CONTROL CIRCUITS



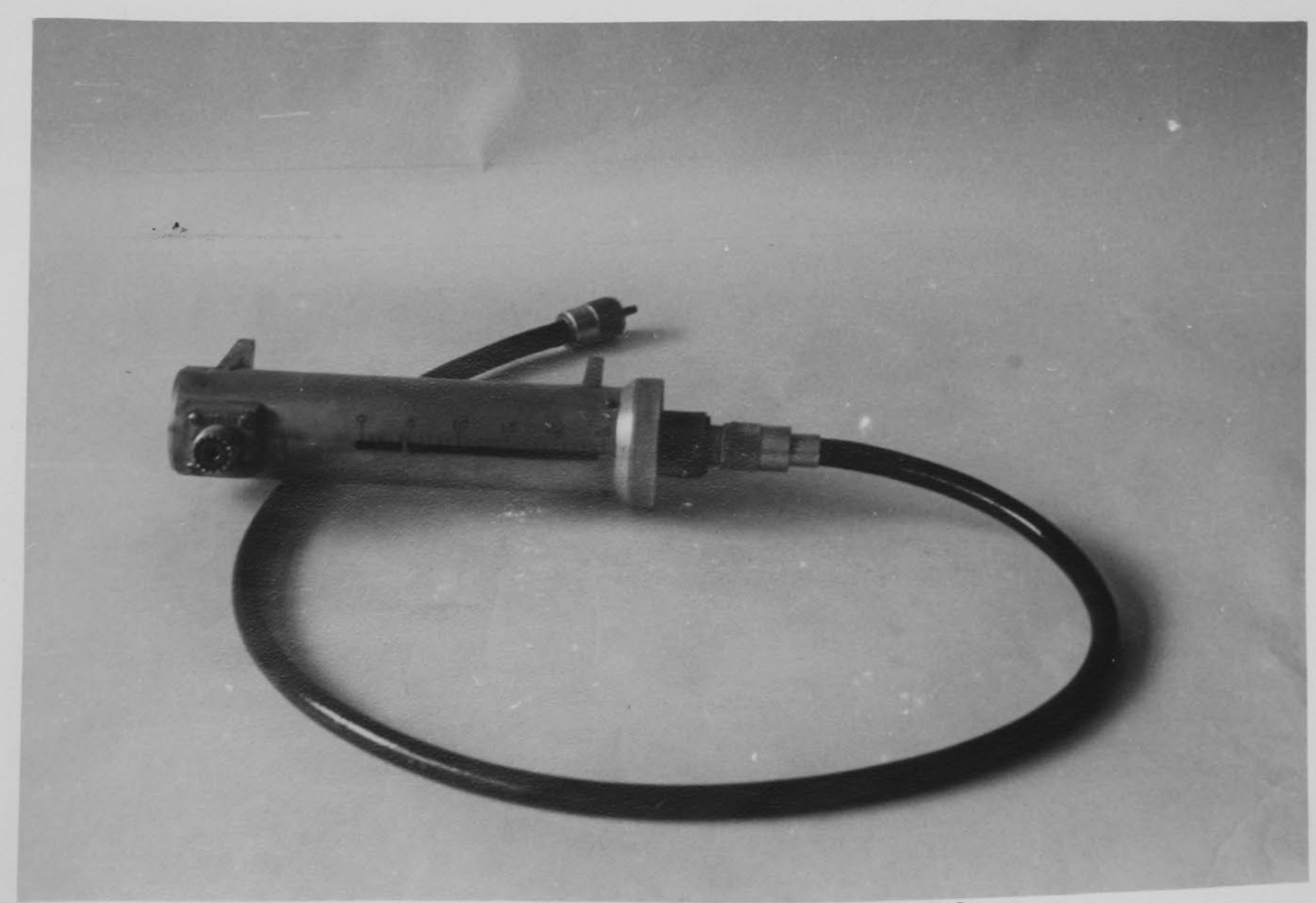
REGENERATIVE RECEIVER & PROBE



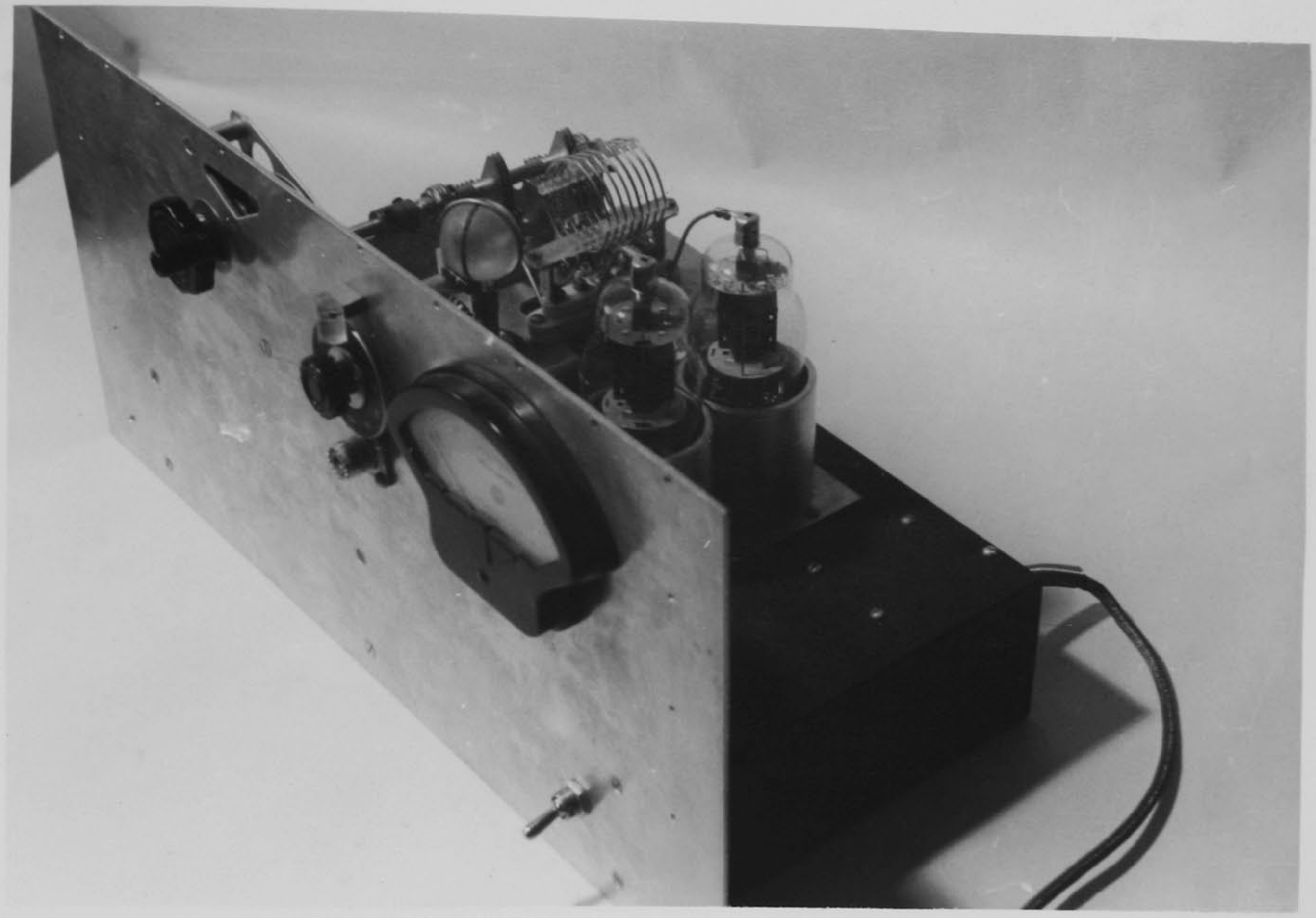
CLOSE-UP OF PROBE



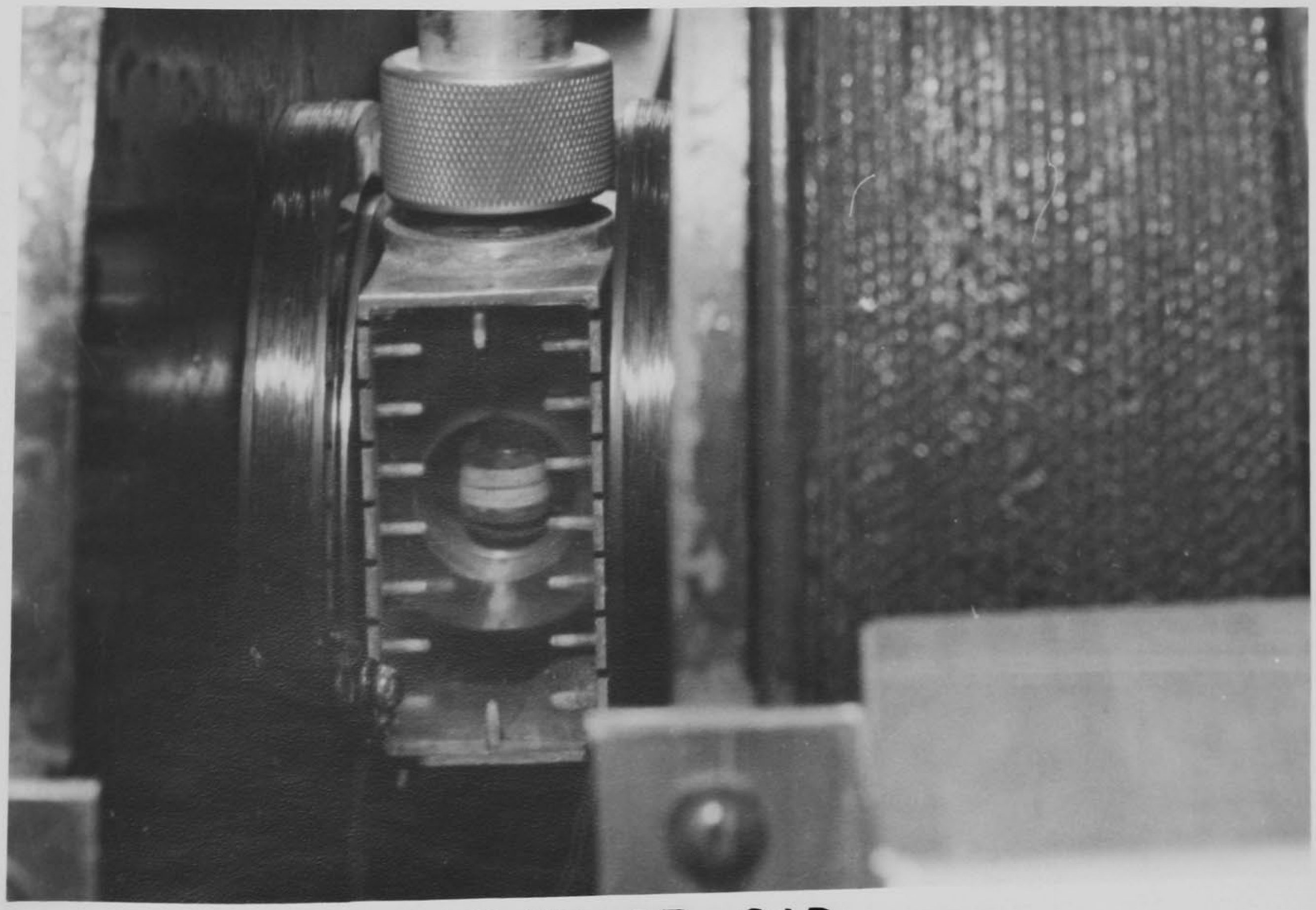
T. R. F. RECEIVER



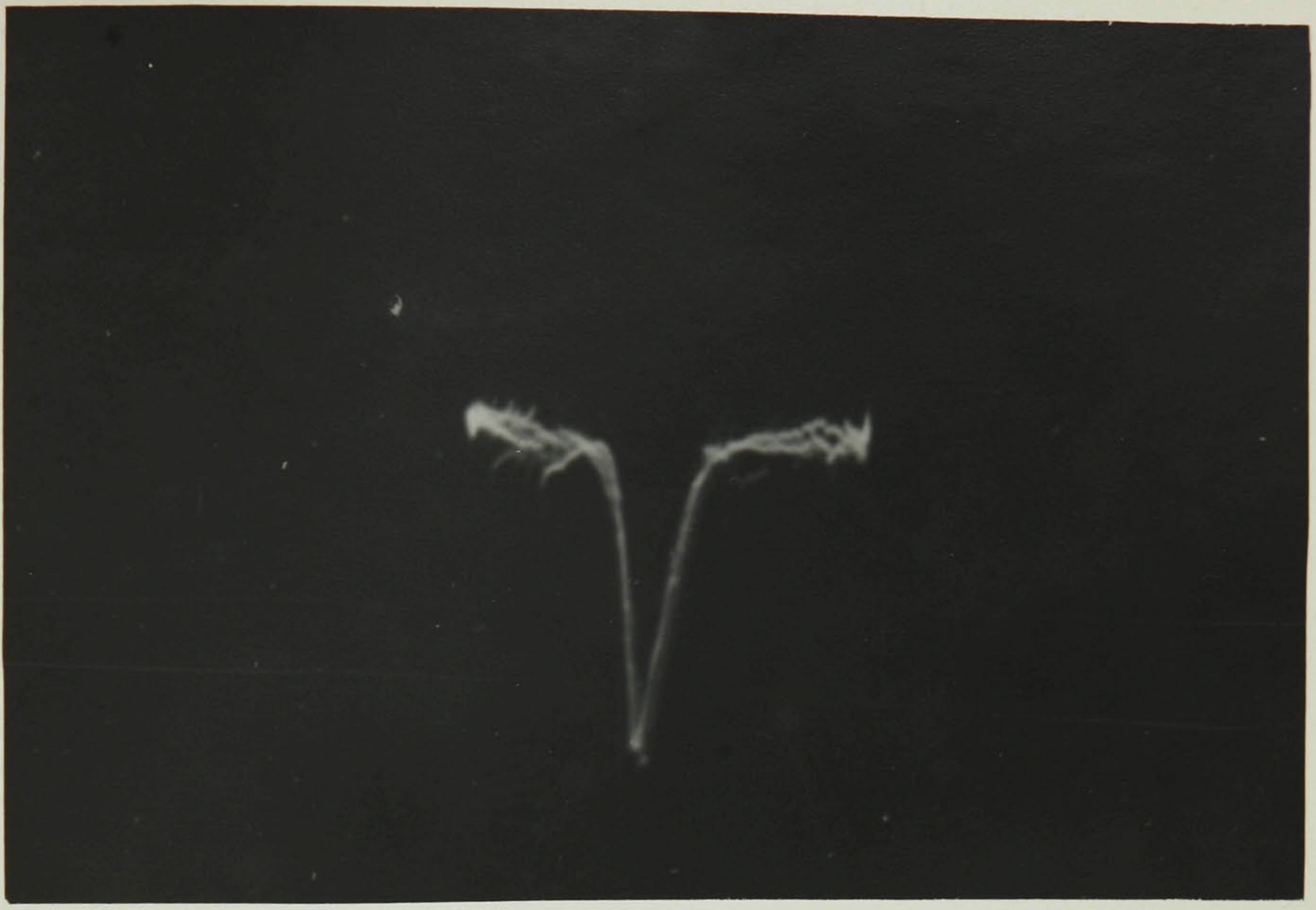
PRECISION ATTENUATOR



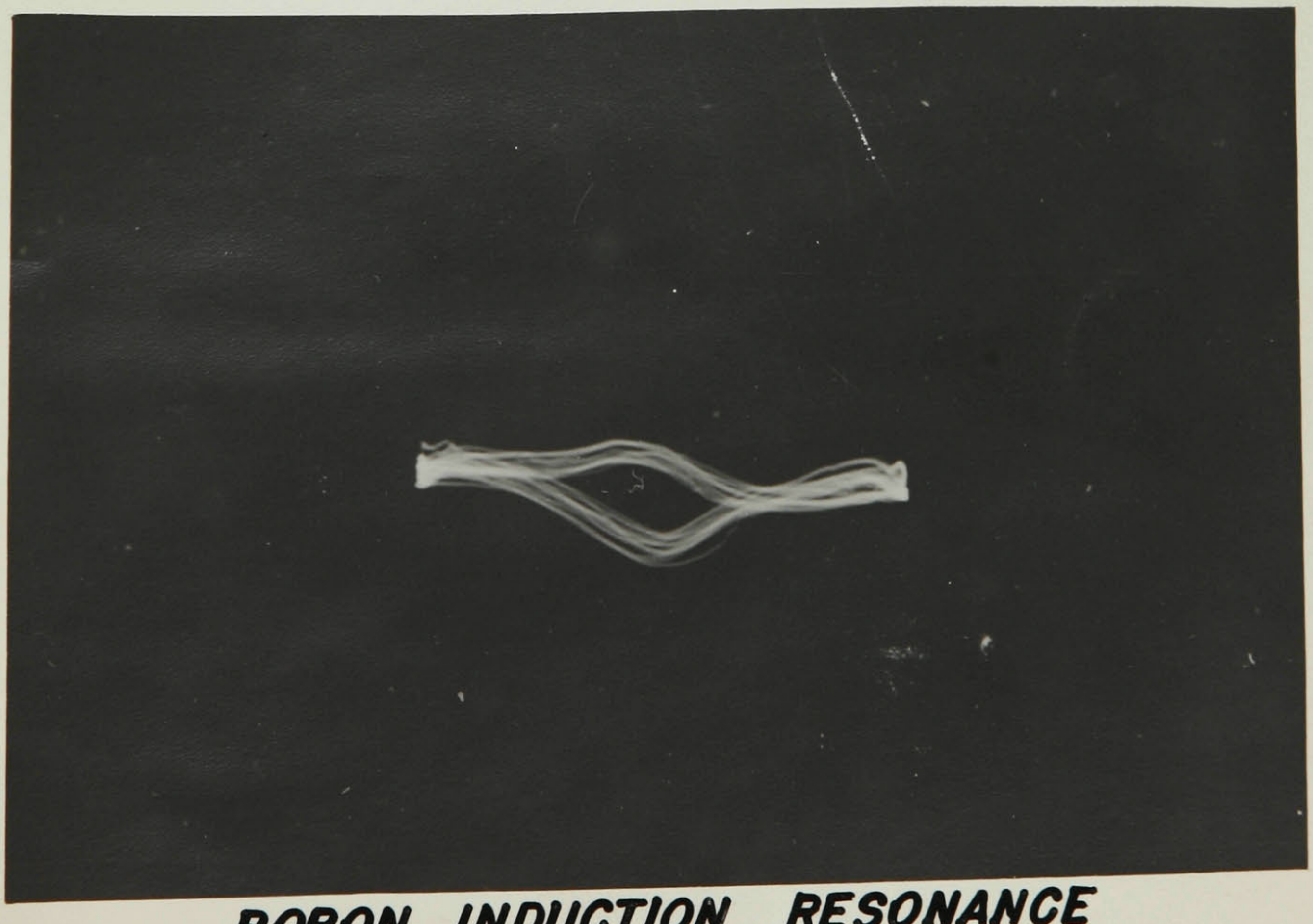
R.F. POWER OSCILLATOR



CLOSE-UP OF GAP



PROTON ABSORPTION RESONANCE



BORON INDUCTION RESONANCE