PREDICTION OF UNPLANNED DILUTION IN UNDERGROUND MINES THROUGH MACHINE LEARNING TECHNIQUES

Carlos Tapia Cadenillas

A thesis submitted to McGill University in partial fulfillment of the requirements of the degree of Master of Science

Department of Mining and Materials Engineering

McGill University

March 2023

Abstract

Mining companies must be in continuous improvement of their process and control of their costs in order to improve their profitability. Given that dilution is a significant problem that reduces the profit of underground operations, its monitoring, prediction, and reduction enhance the operation economy. Several methods to predict dilution have been addressed over the last few decades. However, the recent progress in machine learning has opened new avenues to predict unplanned dilution.

The contribution of this thesis is divided into two folds. The first is the development of a prediction tool for uncontrollable dilution based on regression techniques, Artificial Neural Networks (ANN), Random Forest (RF), and Recurrent Neural Networks (RNN). Using these techniques, different predictive models were developed based on a data set of 99 stopes from an underground mine in North America. The primary mining method applied in this mine is longitudinal retreat long-hole with variations in specific stopes. An accurate predictive dilution model will help improve the economic assessment of each stope. The second contribution of this thesis is identifying the most significant variables in the predictive process of the models. The variables involved in developing the models are related to the geometry of the stope, the location, the drilling and blasting process, and the rock support. The research outcomes showed that unplanned dilution is highly impacted by the powder factor and the rock support in each stope, in this study case. Also, unplanned dilution can successfully be predicted through machine learning models, and they can reach a higher accuracy than traditional methods.

Résumé

Les entreprises minières doivent être en amélioration continue de leurs processus et en contrôle de leurs coûts afin d'améliorer leur rentabilité. Étant donné que la dilution est un problème important qui réduit le profit des opérations souterraines, sa surveillance, sa prévision et sa réduction améliorent l'économie d'exploitation. Plusieurs méthodes pour prédire la dilution ont été abordées au cours des dernières décennies. Cependant, les progrès récents de l'apprentissage automatique ont ouvert de nouvelles voies pour prédire la dilution non planifiée.

La contribution de cette thèse se divise en deux volets. Le premier est le développement d'un outil de prédiction de la dilution incontrôlable basé sur des techniques de régression, les réseaux de neurones artificiels (ANN), les forêts aléatoires (RF) et les réseaux de neurones récurrents (RNN). À l'aide de ces techniques, différents modèles prédictifs ont été développés à partir d'un ensemble de données de 99 chantiers d'une mine souterraine en Amérique du Nord. La principale méthode d'extraction appliquée dans cette mine est le retrait longitudinal par longs trous avec des variations dans des chantiers spécifiques. Un modèle de dilution prédictif précis aidera à améliorer l'évaluation économique de chaque chantier. La deuxième contribution de cette thèse est d'identifier les variables les plus significatives dans le processus prédictif des modèles. Les variables impliquées dans le développement des modèles sont liées à la géométrie du chantier, à l'emplacement, au processus de forage et de dynamitage et au support rocheux. Les résultats de la recherche ont montré que la dilution non planifiée est fortement impactée par le facteur poudre et le support rocheux dans chaque chantier, dans ce cas d'étude. De plus, la dilution non

planifiée peut être prédite avec succès grâce à des modèles d'apprentissage automatique, et ils peuvent atteindre une précision plus élevée que les méthodes traditionnelles.

Acknowledgments

I want to acknowledge and give my thanks to Prof. Mustafa Kumral and DT Solutions Services for their tremendous support and constant guidance. This thesis is the result of the knowledge and advice provided by them.

I would like to give special thanks to my family for their support and their words of encouragement when undertaking my research.

Special thanks to the Peruvian Institute of Mining Engineers (IIMP) for funding my studies and supporting this research.

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List of Abbreviations

ANN: Artificial Neural Network

ELOS: Equivalent Linear Overbreak/Slough

FW: Foot Wall

HW: Hanging Wall

LSTM: Long-Short Term Memory

MAE: Mean Absolute Error

MLR: Multiple Linear Regression

MLRA: Multiple Linear Regression Analysis

MNLR: Multiple Non-Linear Regression

RF: Random Forest

RNN: Recurrent Neural Network

VCR: Vertical Crater Retreat

1 Introduction

1.1 Problem Statement

Within the mining industry, extracted material is categorized into either ore or waste. The classification of rock into these two groups is solely based on economic factors. Ore is rock that contains a specific concentration of minerals that can be mined and sold at a profitable margin given the existing economic conditions (Hustrulid & Bullock, 2001). As such, accurate identification of rock classification requires a deep understanding of the geological features of the orebody and the economic environment of the mine. It is worth noting that only a tiny proportion of the "run-of-mine" ore contains valuable metals, necessitating further processing to concentrate or extract the metal content.

Due to the intricate shapes of orebodies and operational constraints, the surrounding rock waste is often mined and added to the ore stream. This practice leads to the contamination of the ore with additional rock, commonly known as dilution. Achieving perfect selectivity of ore is challenging, which makes dilution an inherent problem in mining operations. Initially defined by Wright (1983) as the contamination of ore by waste material during the mining process, dilution was later classified into *planned and unplanned dilution* during stope design by Scoble and Moss (1994). Planned dilution refers to the waste rock located within the stope boundaries that cannot be avoided due to the mining method's selectivity. On the other hand, unplanned dilution is the additional waste rock from outside the stope boundaries caused by blasting or weak rock stability of the walls. The inclusion of rock waste in the ore stream has a negative impact on a company's revenue by decreasing the grade of the run-of-mine ore and the amount

of material extracted to obtain the same equivalent metal content (Lyndon Michael, 1998). Dilution amount is a financial parameter required for the economic assessment of a mine, but its prediction remains highly complex and not entirely controlled. Unplanned dilution significantly adds to a mine operator's costs, leading to unnecessary haulage and mineral processing expenses. If left unaddressed, this problem can jeopardize the operation's economy. Although considerable efforts have been made in the past, the problem remains unsolved. However, emerging machine learning techniques offer new opportunities for addressing the issue more effectively.

In the last five decades, various methods have been developed to predict unplanned dilution in underground mining. The stability graph method is a standard method in the open stope design and the prediction of dilution (Madenova & Suorineni, 2020). The stability graph's validity remains because it has been modified several times with database expansions, redefinition of the zone boundaries, and modification of the adjustment factors during the last few decades (Madenova & Suorineni, 2020).

Clark and Palkanis (1997) introduced ELOS (Equivalent Linear Overbreak/Slough) as a variant of the stability graph method, transforming it from a qualitative to a quantitative method. It has become one of the most prevalent predictive dilution models since then. ELOS uses rock quality, stope dimensions, and rock-joint features to predict unplanned dilution; however, it ignores essential factors such as blasting parameters, wall exposure time, and stope mining sequence (Jang, Topal, & Kawamura, 2015). After that, other approaches to address dilution prediction were proposed, such as "dilution density" (Henning and Mitri, 2007) and the simple linear regression method (Germain and Hadjigeorgiou, 1997).

Henning and Mitri (2007) used 3D elastic-plastic numerical models to quantify unplanned dilution through mining depth, rock properties, stope geometry, extraction sequence, and rock stresses. They introduced the term "dilution density" to define the contour of the relaxation zone ($\sigma_3 = \sigma_t$) created by the void after the blasting process, and the volume within this contour is the potential overbreak in the stope.

Germain and Hadjigeorgiou (1997) used regression techniques to define the relationship between different variables and the stope dilution. Their research aimed to prove that more variables are involved in the dilution occurrence phenomenon than just the stope dimension and the quality of the rock. Germain et al. (1997) concluded that the unplanned dilution is highly correlated with the stope geometry (r=0.746), and the powder factor has no or little impact on the stope performance (r=-0.083 or r=0.282) according to the performance at Louvicourt Mine.

This research aims to propose new prediction approaches to reduce unplanned dilution in underground mining operations. The problem sentence of this thesis is how can a statistical or machine learning model can improve the dilution prediction with higher accuracy than the traditional methods.

1.2 Research Objectives

- Develop predictive models for unplanned dilution based on statistics and machine learning methods that can achieve an accuracy that is the same or higher than the traditional methods.

- Explore the impact of each variable and rank them according to their importance in the predictive process of dilution.
- Deliver a tool to evaluate the different parameters of a stope during the planning process and help minimize unplanned dilution in the mine design phase.
- Provide comparative performance measurement analysis of the proposed approaches.

1.3 Economic and Environmental Benefits

Economic Benefits

- Improve the accuracy of the economic assessments of mines or stopes as a result of a better prediction of the unplanned dilution.
- Capacity to improve the design of the stopes to reduce unplanned dilution and its impact on profitability.

Environmental Benefits

- Reduce haulage cost and gas emissions emitted by haulage equipment
- Use less energy and generate fewer tailing quantities in mineral processing through dilution reduction.

1.4 Originality and Success

The originality of this work rests on developing statistical and machine learning models to predict unplanned dilution. This study focuses on three main objectives: first, using multiple methods to determine the most suitable according to the features of the data in this case study;

second, comparing the accuracy of every method against the traditional techniques to predict dilution; and finally, ranking the variables involved in the process according to their importance in the prediction of dilution.

1.5 Thesis organization

This thesis is organized into the following sections: Chapter 1 describes the problem statement, the research objectives, and the originality of the work. Chapter 2 explores the literature related to the case study's mining method, the previous predictive dilution models, and the statistical and machine learning techniques that will be applied in the developed models. Chapter 3 describes the methodology used to create, train, and test the predictive models. Chapter 4 introduces the case study and describes the variables involved in the prediction process. Chapter 5 introduces the results of the models applied to a test data set and discusses each model's different restrictions and features. It also compares the accuracy of the developed models to the traditional techniques. Chapter 6 concludes the thesis with a discussion of the methodology, results, and additional steps that can be considered in the future to improve the work done in this thesis.

2 Literature Review

2.1 Underground Mining Methods and Unplanned Dilution

The selection of mining method is essential to conducting feasibility studies and economic assessments of the mining project. Once enough information is collected from the orebody, it is possible to outline and quantify the ore that can be extracted. The combination of the layout and the manner of ore extraction is known as the mining method.

The choice of the mining method serves as the basis for a project's design and its feasibility study (Hustrulid & Bullock, 2001). The selection of the most suitable mining method relies on the shape of the orebody, dimensions, inclination, equipment selection, rock quality, and a set of economic, geological, or operational constraints. Furthermore, the project's complexity could lead to selecting more than one mining method in different areas.

This section aims to briefly show the features of the major mining methods and describe the potential dilution impact in each case.

2.1.1 Room and Pillar

The room-and-pillar method was designed to be used in low-dip-angle or flat-bedded deposits. The extraction is done through open stopes created along the deposit, leaving behind pillars supporting the roof (Figure 2-1). Minimizing the area and number of pillars will increase the ore recovery, but this is limited according to the rock support required, which is dependent on the rock quality and the dimensions of the stopes (Hustrulid & Bullock, 2001).

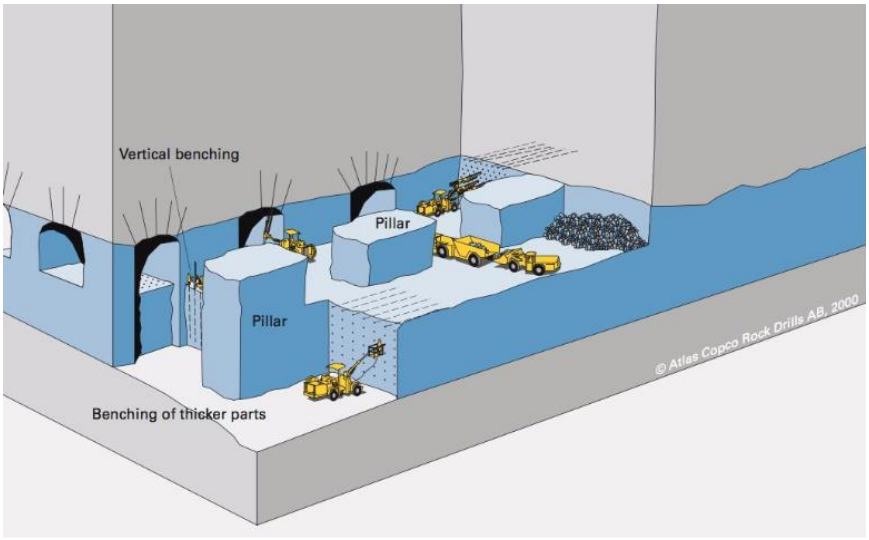

Figure 2-1. Classic Room and Pillar Scheme (Hustrulid & Bullock, 2001)

There are two significant advantages of this method: First, simple development work is required to open the stope, and the roadway excavations are combined with the production of ore within the stopes. Second, multiple phases of production can occur in the stope at the same time, so several production areas can be established inside the same stope, avoiding dependency on the production activities (drilling, blasting, mucking).

As in all the underground mining methods, the unplanned dilution level depends greatly on the amount of slough rock that falls into the stope or the haulage area. Thus, the level of unplanned dilution potentially depends on the area of the unconfined wall in the boundary between the host rock and the orebody. Usually, the boundaries of the orebody in the room-and-pillar method are located in the floor and the roof of the stope. Therefore, the level of dilution is highly correlated with the area of roof unsupported in each cut, which is relatively small. However, the big challenge in the room-and-pillar method is the mining recovery that depends on the transversal area of the pillar left in the stope.

2.1.2 Shrinkage stoping

Shrinkage stoping was designed to be used in steep deposits where the rock can dump directly into the rail car from the chutes in the crosscut located at the bottom of the stope. The extraction of the ore is conducted through horizontal slices from the bottom up. Part of the broken ore is mined from the chutes in the draw point located in the loading crosscuts (Figure 2-2), and part of it is left to be used as the floor for the subsequent drilling and blasting process. The total ore is recovered when all the stope has been blasted. The development structure required in this mining method is the set of crosscuts that are used as draw points of the ore, a transport drift, and a raise from the bottom of the stope that provides access and ventilation to each horizontal slide that will be mined (Hustrulid & Bullock, 2001).

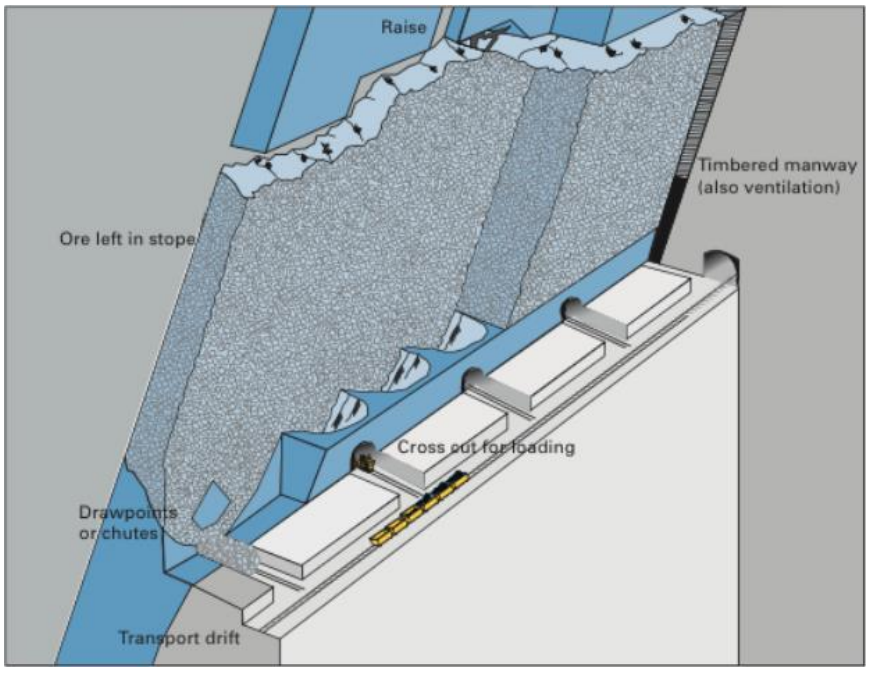

Figure 2-2. Shrinkage stoping (Hustrulid & Bullock, 2001)

Shrinkage stoping is a non-mechanized method that was very popular when jumbo drillers were not widespread in underground mining. The ore pile in the stope is used as a floor for drilling and blasting, and its lack of firmness prevents mechanized equipment from being used. Due to its labor intensity, limited capacity, and ore retention for an extended period, this method has been abandoned for other massive mining methods.

The non-mechanized feature of this method allows a high selectivity in mining. The horizontal slices blasted are usually between 1.5-4.0 meters high, which creates a small-scale unconfined wall. The ore pile used as the floor plays the role of rock support preventing extra unplanned dilution within the stope.

2.1.3 Cut and Fill stoping

The cut-and-fill method is preferred in mines where the orebody has an irregular shape because of the method's adaptability in the mining layout. The ore is extracted in horizontal slides from the bottom undercut up or from the uppercut down (preferred when the rock quality is poor, and the paste backfill can be used as roof). The height of the mining slices depends on the selectivity wanted and is restricted by the maximum height the driller can reach. After the material has been mined, the void is backfilled with tailing sand or hard rock in combination with cement in the last pour to add support and create a hard fill to form a firm platform for machinery in the next slice (the content of cement depends on the rock quality or the direction of the cut-and-fill method).

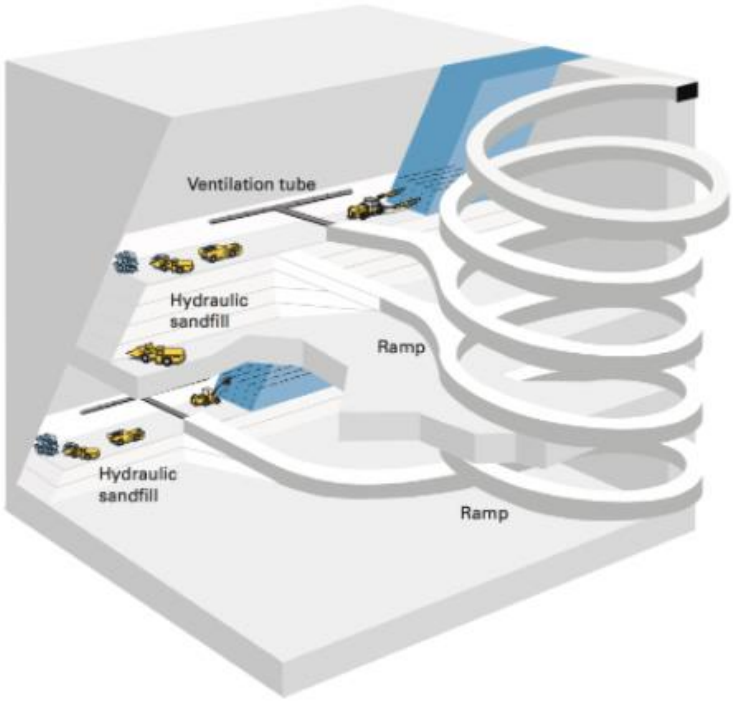

Figure 2-3. Cut and fill method (Hustrulid & Bullock, 2001)

The main development infrastructure is the haulage drift along the orebody at each level. A ramp with access drives the orebody with raising connections that allow the ore extraction and fill the stopes. Additional development is required as water drainage in the stope undercuts, and ore passes to connect the stopes with the draw points.

The versatility of the mining method allows for following the variations in ore boundaries, creating sections of low-grade and high-grade material to be mucked out separately. The drill pattern can be modified several times in the same horizontal slice of ore (Figure 2-4), improving the rock's fragmentation and increasing the efficiency of LHD cycles (Hustrulid & Bullock, 2001).

The dilution level in the cut-and-fill method is relatively low compared to massive methods. The unconfined wall in the boundaries between the orebody and the host rock has a small area that is related to the size of the cut (usually no more than 5 meters).

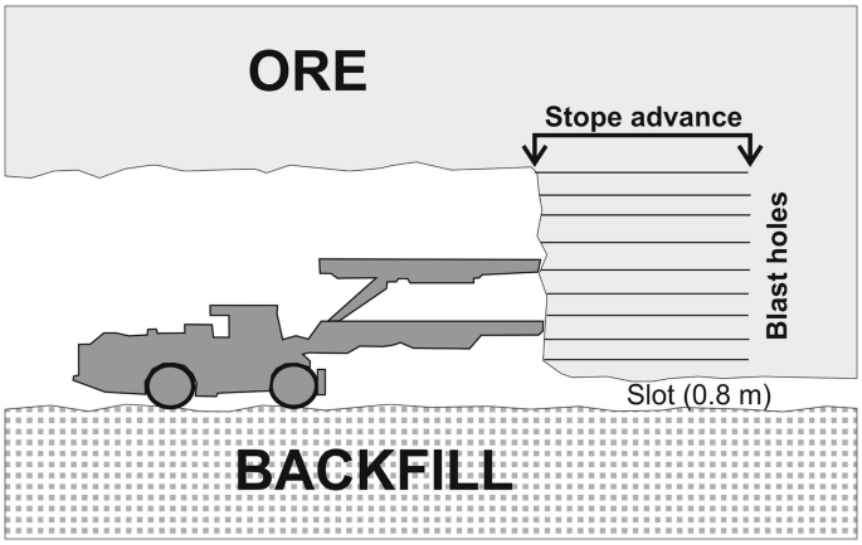

Figure 2-4. Face drilling in the cut-and-fill method (Abzalov, 2016)

2.1.4 Sub-level open stoping

This method divides the ore body into several open stopes, which are mined using sublevels. In each sublevel, ore sections are set aside as pillars to support the hanging wall, which can be recovered at the end when all the stopes in that sublevel are already mined. Also, horizontal sections of ore are left as crown pillars between sublevels. After all the ore is mined in one stope, it is backfilled to allow mining in the following stope (Abzalov, 2016).

The enlargement of the stopes increases the efficiency of the mining process. However, the rock quality is the most limiting constraint, and the stope must be stable enough to auto-support itself until all the material is extracted (Hustrulid & Bullock, 2001). Additionally, higher dilution values are usually related to larger dimensions in stopes; consequently, the economic parameters become an additional constraint to the enlargement of the stopes.

Drifts are excavated in each sublevel as accesses for machinery to drill the stope. The drilling pattern covers the entire stope and requires high precision in the angle and depth of the drill holes to avoid blasting rock outside the boundaries and producing unplanned dilution. After blasting occurs, the ore is extracted from the draw points located in the stope bottom (Figure 2-5). Different layouts can be done in the developing drifts and crosscuts underneath the stope. The most popular is where the loading level is combined with the undercut as only one drift, which makes the development investment cheaper for each stope.

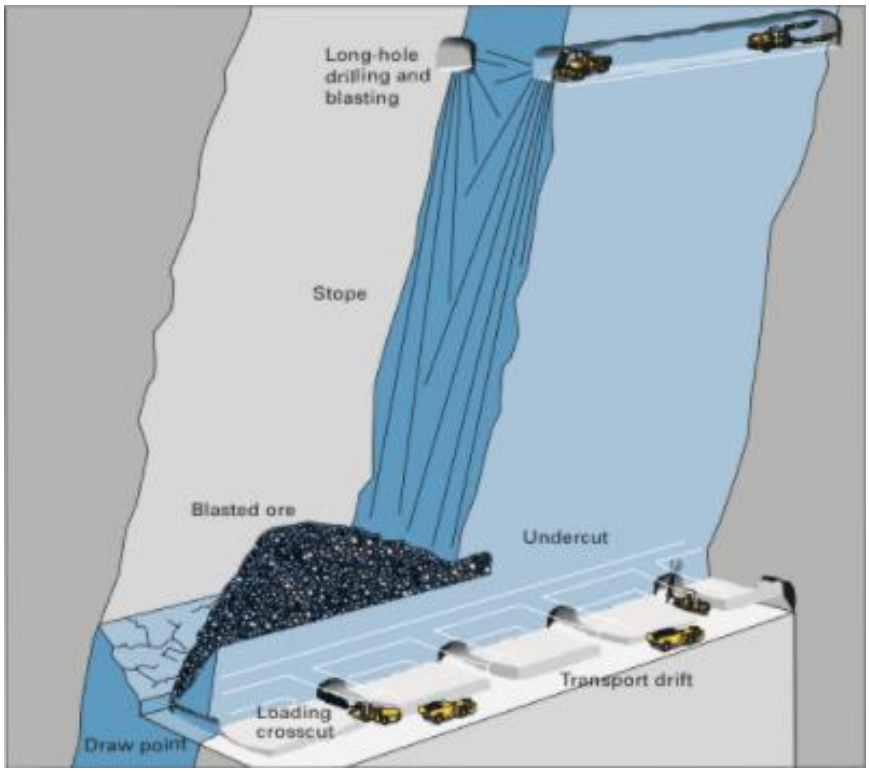

Figure 2-5. Sub-level stoping method (Hustrulid & Bullock, 2001)

In contrast with the cut-and-fill method, sub-level stoping requires well-defined boundaries and regular shapes in the orebody. The selectivity is diminished at the expense of production; as a result, if some waste material is inside the stope, it will be included in the ore

analysis, and the grade will be diluted during the design process (planned dilution). Additionally, extra material from the walls can slough into the ore stream that was not considered in the stope design, which also increases the dilution in the process (unplanned dilution). The unconfined walls around the stopes have a larger area than in the previous methods.

2.1.5 Vertical crater retreat (VCR)

The vertical-crater-retreat method (VCR) was initially used in Canadian mines and then spread worldwide as an established and proven mining method. This method shares many similarities with sub-level stoping in the development structure, which also requires an overcut drift for drilling machinery and an undercut as a haulage drift (Hustrulid & Bullock, 2001).

The drilling pattern consists of large diameter blast holes drilled downward from the overcut to the undercut. VCR is based on "crater blasting," in which a powerful charge of explosive is located and fired at the same distance from the bottom of the blast hole (Abzalov, 2016) (Figure 2-6). As a result, a slice of the ore is fragmented, drops into the undercut, and can be extracted from there. When part of the ore is mucked, and there is enough space to allocate more broken ore, then a subsequent slice of ore can be blasted following the same procedure. This repetitive process ends when all the stope has been fired and mucked. Finally, the stope can be backfilled and the next can be mined.

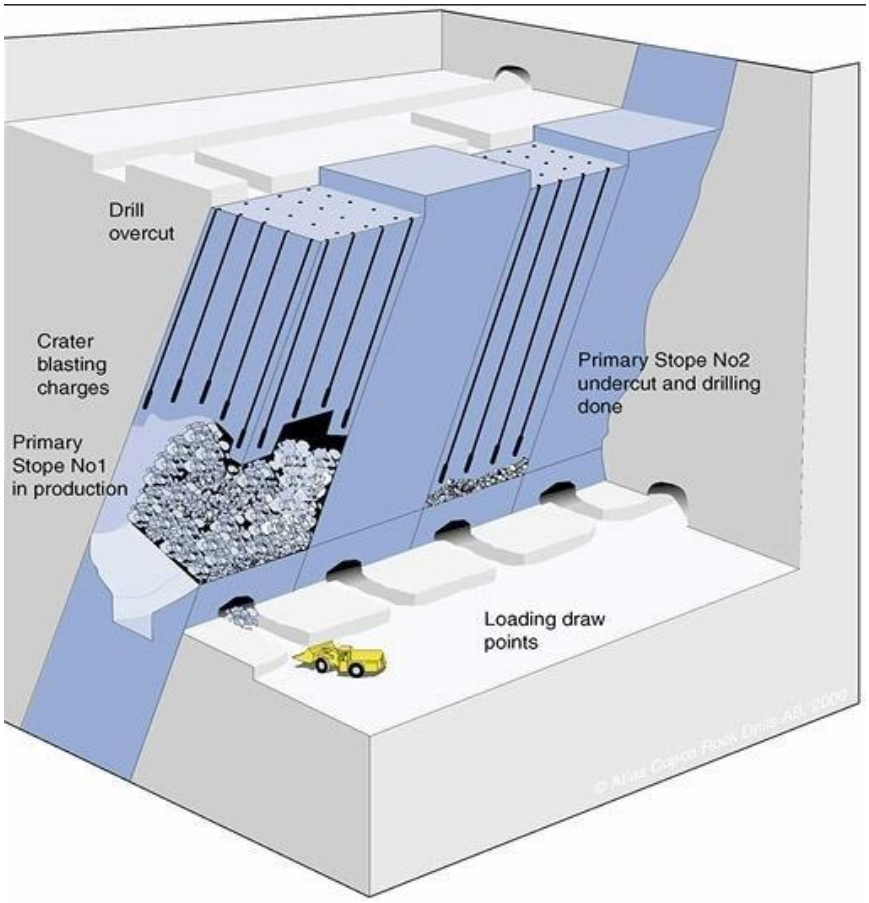

Figure 2-6. Vertical-crater-retreat method (Hustrulid & Bullock, 2001)

2.1.6 Sub-level caving

Sub-level caving is used in steep and large orebodies. The ore is extracted through parallel drifts (transversal to the strike of the orebody) developed at the bottom of each level, which will also allow the drilling and blasting process (Figure 2-7). The main goal of the blasting process is to fracture the rock, and the unsupported void created results in a caving process around it. Caving requires that the rock mass has a rock quality that allows it to fracture and collapse in

controlled conditions. This continuous caving in the rock mass avoids the creation of cavities that would lead to a sudden collapse or rock falls into the installations (Hustrulid & Bullock, 2001).

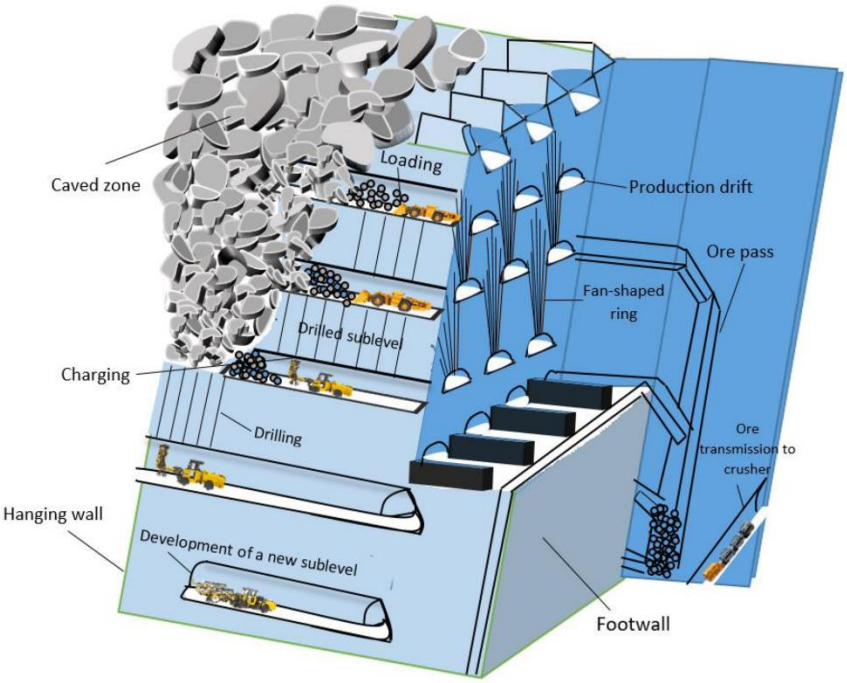

Figure 2-7. Sub-level caving method (Hustrulid & Bullock, 2001)

Compared with other mining methods, the development infrastructure required is extensive and costly. However, caving methods are highly productive, leading to the lowest unit prices. The low mining cost required allows for mining ore bodies with low grades. Due to its massive capacity, this method is applied to large orebodies where selectivity is not the primary purpose.

2.1.7 Block caving

The ore extraction in block caving is similar to sub-level caving, which is based on the controlled collapse of the rock mass. However, in this method, the stope is much larger, called a

"block," and it can reach thousands of square meters. In tonnage production, block caving is the leader among all mining methods. The development infrastructure consists of a group of drifts exacted underneath the block, drawbells beneath the undercut that collect the fragmented ore, and a lowermost level where the ore is hauled (Figure 2-8).

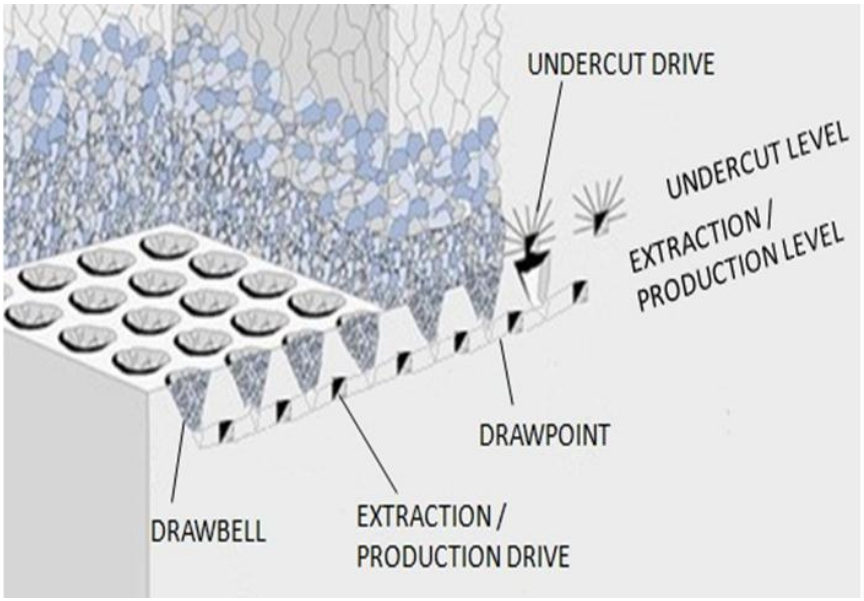

Figure 2-8. Block caving method (Hustrulid & Bullock, 2001)

Openings under the block are under high internal stresses, which leads to the use of heavy rock support to secure the integrity of the machinery, infrastructure, and process. The extraction process relies totally on gravity, in which the ore sloughs into the chutes and then is hauled to the surface (Hustrulid & Bullock, 2001). Supposedly, no blasting is required to fracture the rock, but in practice, long-hole drilling and blasting are needed to assist in the fragmentation of boulders that interrupt the continuous flow of the method (Abzalov, 2016).

Both caving methods are the least selective methods from the list presented in this section. Due to the low selectivity intrinsic to this kind of mining method, dilution tends to be

higher than in the previous methods. However, these massive mining methods can manage the dilution because of their low operational costs.

2.2 Dilution in underground mining

Underground mine design relies on key factors such as cost criteria, production, safety, and dilution (Villaescusa, 1998). Therefore, the correct definition of these factors would allow the development of an accurate design and select the most suitable mining method for the project. As with all the other factors, dilution results from many parameters around the stopes, making it complicated to predict. Over the last few decades, many methods to predict dilution have been developed. Some are summarized in this chapter.

2.3 The stability graph method

Mathews (1981) developed the stability graph method as a guide to design stopes in underground mining. The graph method qualitatively assesses the stability of the stope through a plot of the "Stability Number" (N) against the hydraulic radius. This empirical method was initially constructed from 26 historical cases from 3 mines and then expanded to 175 cases from 34 mines by Potvin (1988).

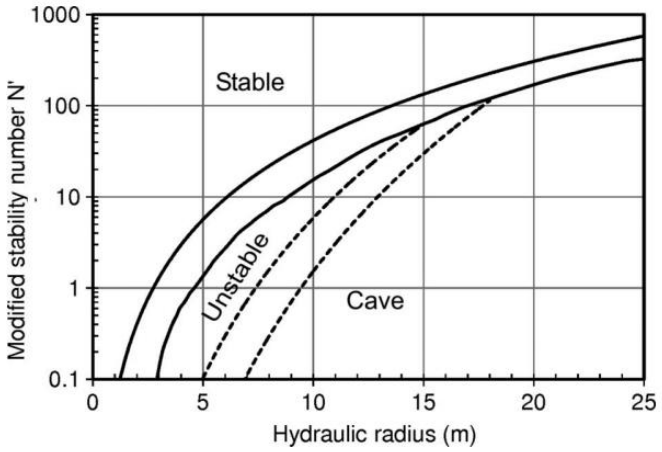

Figure 2-9. The standard stability graph, Mathews (1981)

The hydraulic radius is defined by the ratio between the hanging wall (or foot wall) area and its perimeter. Equation 2-1

$$Hydraulic\ Radius = \frac{Area\ of\ the\ Hanging\ Wall}{Perimeter\ of\ the\ Haging\ Wall} \qquad (2\text{-}1)$$

The "Stability Number" (N) is a product of four different factors: The first is the Q' developed by Barton (1974), which defines the rock-mass quality through its different features. The second is the stress factor A, which accounts for the ratio between stress and rock strength. Next is the B factor, which accounts for the difference in strike and dip between the stope and the joints. Finally, the gravity factor C accounts for the mode of failure of the wedges formed by the joints and the angle of the stope (gravity fall, sliding, or slabbing).

$$N' = (Q')(A)(B)(C) \qquad (2\text{-}2)$$

$$Q' = \left(\frac{RQD}{J_n}\right)\left(\frac{J_r}{J_a}\right) \qquad (2\text{-}3)$$

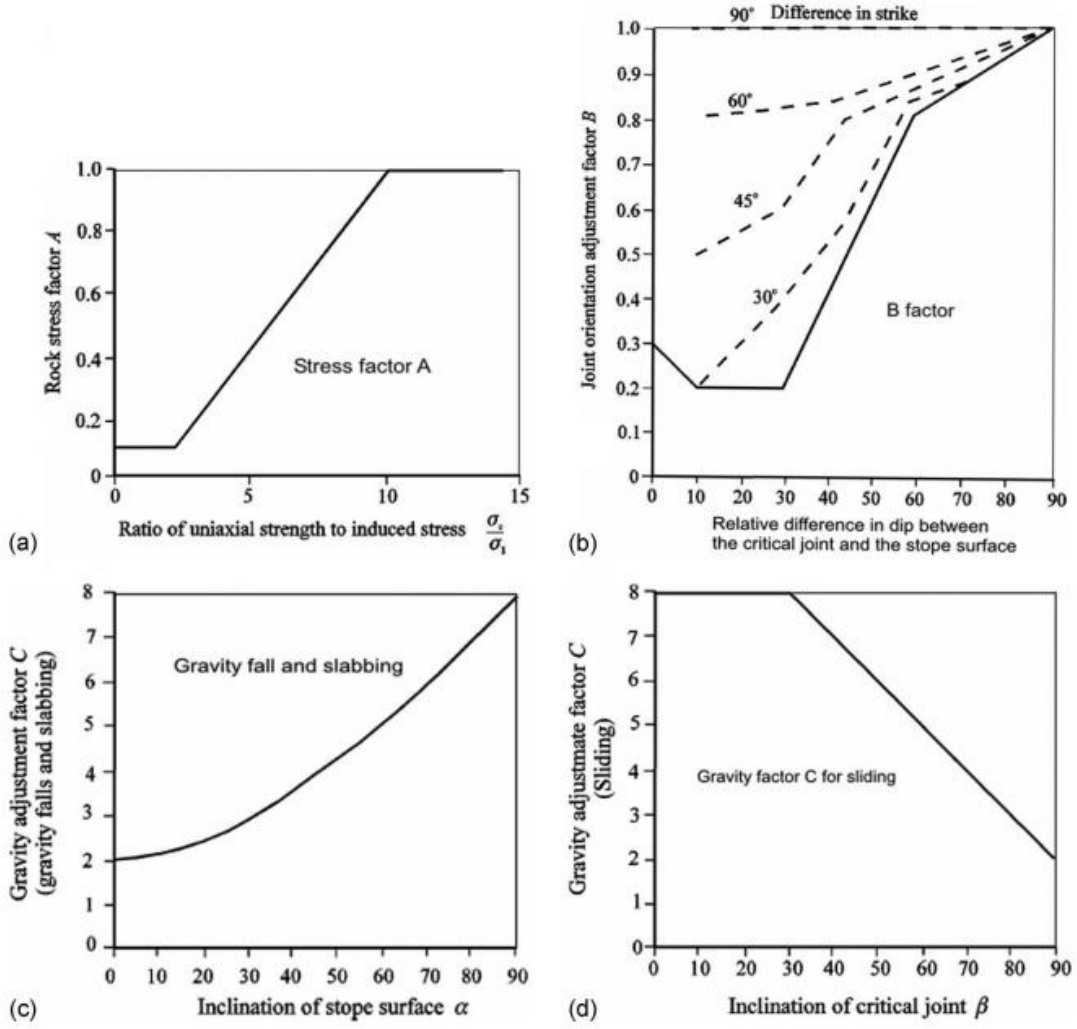

Figure 2-10. Chart for the definition of the stability graph factors, Mathews (1981)

The conventional graph method can only designate a stope as stable or unstable. However, this method cannot quantify the dilution in the stopes. Scoble and Moss were the first to propose dilution lines for a Stability Graph, and later Clark and Palkanis (1997) introduced the

concept of Equivalent Linear Overbreak/Slough (ELOS). ELOS within the Stability Graph allows for predicting the unplanned dilution.

2.4 Equivalent Linear Overbreak/Slough (ELOS)

Clark and Palkanis (1997) introduced the concept of ELOS to quantify the dilution in open stopes. ELOS (in meters) is the ratio between the volume of overbreak and the hanging wall or foot wall area.

$$ELOS = \frac{Volume\ of\ overbreak}{Stope\ height\ \times wall\ strike\ length} \quad (2\text{-}4)$$

$$Dilution\ (\%) = \frac{ELOS}{Orebody\ Width} \quad (2\text{-}5)$$

Using Canadian mine databases, Palkanis et al. (1995) and Gauthier (2001) recognized that the classic formulae to determine unplanned dilution are highly sensitive to wall sloughing in narrow deposits. Therefore, ELOS became famous for introduce a standard dilution metric for different-size stopes, which was complicated with the standard dilution metric (in percentage). However, the lack of consideration of the underbreak in the ELOS method is a significant limitation for correctly assessing dilution in stopes.

2.5 O'Hara's quick guide

O'Hara (1980) collected data from different mines in 1978 regarding ore body characteristics, mine size, capital costs, and operating costs. Through this information, he created a rule of thumb for each factor involved in the economic evaluation of a deposit at the pre-

feasibility stage. O'Hara's (1980) guide is quite comprehensive, considering capital and operating costs for each part of the mining and plant process. Dilution was pointed out as an adverse factor on revenue, and predicting it became necessary to accomplish the economic evaluation. Thus, he developed what could perhaps be the first deterministic dilution model.

$$\%Dilution = \frac{100}{W^{0.5}} Sin(A°) \tag{2-6}$$

Where:

W: Stope width (ft)

A°: Orebody dip angle

Also, two more parameters were identified in the causality of the dilution: the mining method and the quality of the rock walls. O'Hara developed an empirical graph method to predict the dilution percentage in each mining method. The resulting dilution from the graph must be adjusted according to the rock mass quality of the walls. When the rock quality of the stope walls is competent, the dilution must be reduced by 0.7 times from the calculated with the graph in Figure 3. Ergo if it is of poor quality, the dilution should be increased up to 1.5 times that shown in the graph. This final assessment is purely subjective as O'Hara did not give any hint of how to evaluate the rock mass as competent or weak.

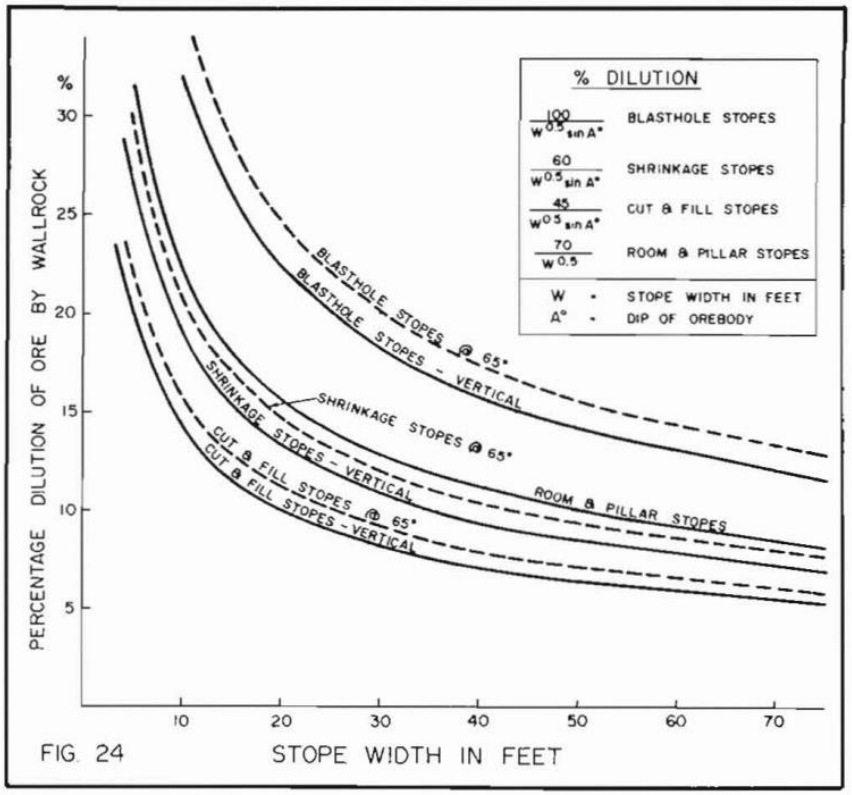

Figure 2-11. O'Hara's plot of stope width against dilution as a function of the mining method and dip angle.

2.6 Parameters influencing dilution

Villaescusa (1998) identified the most common parameter influencing underground mining dilution. These parameters were grouped into five stages during the mining process.

2.6.1 Orebody Delimitation

Orebody delimitation depends on the accuracy of the geological information available. This process is not limited to defining the boundaries of the orebody but also establishes geological parameters such as the grade, shape, and rock mass quality. Without detailed geological information, dilution control would be weak and ineffective.

The reliability of geological information depends on the number of samples taken from the orebody and the sampling process quality. Diamond drilling and geophysical logging are the most common methods to obtain geological information. According to assumptions based on geostatistical tools, the rock mass properties are extrapolated from hole to hole (Villaescusa, 1998).

These parameters can impact the orebody delimitation and mislead dilution control:

- Under-sampling of orebody boundaries.
- Errors in decisions regarding cut-off grades.
- Downhole survey errors.
- Lack of geotechnical characterization.

2.6.2 Design and Sequencing

At this stage, engineering, geological, and operating inputs are combined to form a mine plan containing the most suitable mining method, stope geometry, and sequencing. The most economical mining method selected depends on the orebody's geotechnical properties and geometric configuration. Massive mining methods usually have lower operational costs and higher dilution than those with more selectivity; finding the balance to maximize the revenue is the mine planner's duty. The stope size selection must ensure optimizing the revenue by counting the dilution and ore losses.

Economic evaluation and stability analysis are performed to evaluate the sequencing of the stopes, the backfill material, and the ore recovery. However, Villaescusa (1998) identified

some parameters that can negatively influence the process and mislead the optimization of unplanned dilution:

- Poorly designed infrastructure.
- Poor stope design (dimensions).
- Lack of proper stope sequencing.
- Lack of economic assessment.

2.6.3 Stope Development

Villaescusa (1998) stated that infrastructure location is a factor that can impact dilution in stopes if these openings are nearby. Along with Villaescusa, Henning et al. (2007) recognized undercutting as a factor contributing to stope instability and dilution. Crosscuts can work as an undercut for the stope, and incorrect positioning can increase dilution. Joint sets parallel to the stope over the undercut cause instability in the stope walls, so geology mapping is critical to avoid this issue. Studies conducted by Yao et al. (Yao, Allen, & Willett, 1999) and Dunne et al. (Dunne, Palkanis, Mah, & Vongpaisa, 1996) proved undercutting in stopes could increase unplanned dilution up to 5%.

Villaescusa stated some parameters that negatively influence the mining process at this stage:

- Non-alignment of sill horizons.
- Poor geological control during mining.
- Mining not following geological markups.
- Inappropriate reinforcement schemes.

2.6.4 Drilling and Blasting

The blasting in mining aims to break the rock to a required fragmentation, avoiding damage to the stope walls. "The blasting process involves the interaction of the rock mass, the explosives, the initiation sequences and the drill hole pattern" (Villaescusa, 1998). However, some factors can lead to an increase in overbreak and underbreak:

- Poor initial markup of holes.
- Set-up, collaring, and deviation of blast holes.
- Incorrect choice of blasting patterns, sequences, and explosive types.

2.6.5 Production Stage

During the mucking process, ore contamination can occur due to digging the floor by the LHDs. Similarly, if the mucking equipment ramps up and leaves broken ore on the floor, the production suffers losses. If the mucking time extends, the probability of slough rock in the stope increases, resulting in more unplanned dilution. Villaescusa defined the key factors during this stage as the following:

- Mucking of backfill floors.
- Mucking of fall-offs and stope wall failures.
- Contamination of broken ore by backfill.
- Leaving broken ore inside the stopes.
- Poor management of waste rock (tipped into the ore stream).

2.6.6 Mine Management

Ashcroft (1991) and Villaescusa (1998) determined that management issues can be as critical for dilution as technical issues. Usually, the dilution is not assessed correctly and is inferred from the mucking tonnage. Also, the blasting pattern is based only on the experience of the blasting designer, who only focuses on the production tonnage. Moreover, the lack of quantifiable data avoids dilution prediction and control.

2.7 Mine Planning for Ore Dilution through Numerical Modeling

Any underground opening produces the re-distribution of the rock mass stresses near the excavation. Stresses normal to the stope faces dissipate due to the absence of rock mass in this direction. Consequently, a zone of stress relaxation occurs around the opening, causing loss of the clamping effect that holds the rock pieces together. The stress relaxation process in the stope walls and a fractured rock mass increase the rock slough and dilution (Henning & Mitri, 2007).

Henning and Mitri (2007) evaluated the potential overbreak volume caused by stress relaxation using 3D numerical modelling software. In their research, they replicated the Canadian Shield Environment at different depths. The conditions of the rock mass stresses are shown inTable 2-1. Hoek-Brown criterion was used to model the rock mass, and the parameters are shown in Table 2. This numerical modelling allowed Henning et al. to describe the stress around stopes and examine its variation due to parameter changes such as depth, stope geometry, and type of stope.

Table 2-1. Stress parameter used in the 3D numeric modelling. Henning et al. (2007)

Depth Category	Depth below Surface	K	σH	σh	σv

Shallow	750m	1.9	37.3 MPa	25.2 MPa	19.5 MPa
Moderate	1500m	1.6	64.2 MPa	47.1 MPa	39.0 MPa
Deep	2250m	1.5	89.3 MPa	68.4 MPa	58.5 MPa

Table 2-2.Hoek-Bown criterion parameters used in the 3D numeric modelling. Henning et al. (2007)

Material Values - Host and orezone Rock		**Rockmass Values calculated with GSI values**	
GSI	65	Rockmass elastic Modulus (Erm)	23,713 MPa
Uniaxial compressive Strenght (σc)	175 MPa	Hoek-Brown m,s	7.16 / 0.021
		Uniaxial tensile Strenght (σt)	0.50 MPa

Through stress relaxation, Henning (2007) introduced dilution density as a new way to quantify and predict unplanned dilution. Dilution density is the ratio between the volume limited by the iso-line where σ3=0 (or σ3=tensile strength) and the hanging wall area. Two terminologies come up in this concept, No-tension Overbreak (σ3=0) and Confinement Overbreak (σ3=tensile strength). In the first, the iso-line envelops the overbreak volume that may happen. In contrast, Confinement Overbreak is the ratio where the overbreak volume will slough. Figure 4 shows the overbreak regimes on hanging walls at different depths.

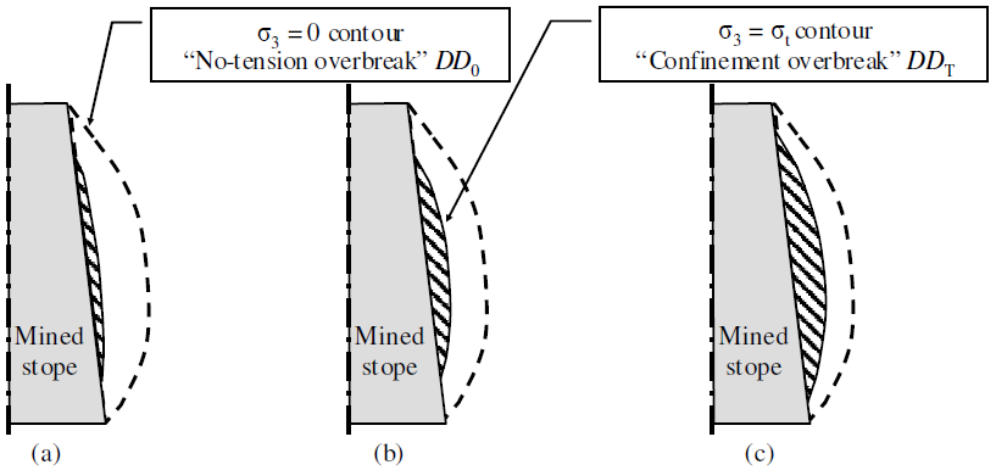

Figure 2-12. Schematic illustration of the influence of mining depth on hanging wall overbreak regimes: (a) shallow depth, (b) moderate depth, and (c) deep. Henning et al. (2007)

Henning et al. (2007) defined a "typical" stope to evaluate the effects of depth in dilution density based on the standard geometry of Canadian long-hole stopes. The dimensions selected were 30 m high, 10 m thick, a strike length range of 10 m to 40 m, and the chosen dip angle was 80 degrees.

Depth was the first parameter assessed in the 3D numerical model, and the results were shown in Figure 5. The dilution density increases when the stope deepens, but the increasing ratio varies in each stope size. 15m-strike-length stopes or smaller are less sensitive to depth than stopes with larger strike lengths. Hence, the interaction of more than one parameter (size and depth) is the leading cause of the unplanned dilution increase.

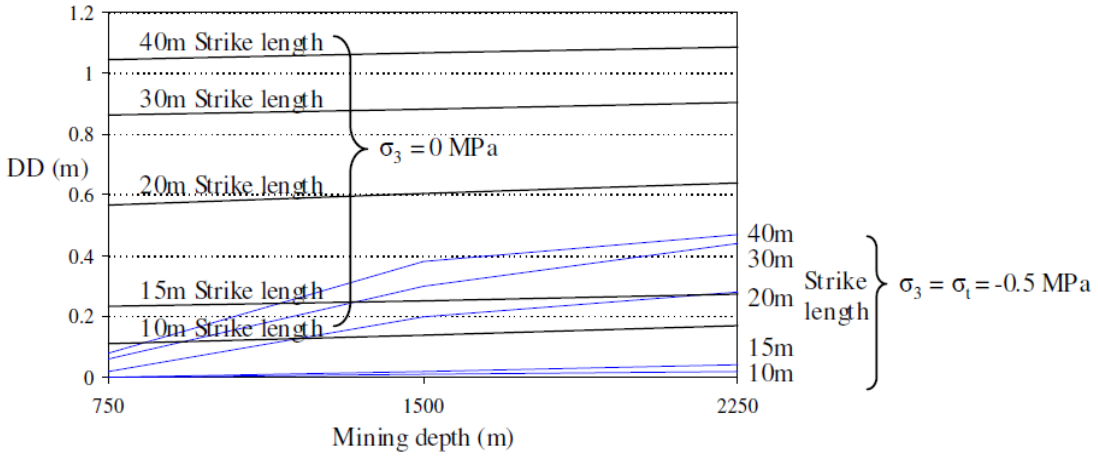

Figure 2-13. Dilution density model according to depth and strike length. Henning et al. (2007)

In agreement with Mathews (1981) and Potvin (1988), Henning (2007) pointed out that the area of the hanging wall is a crucial parameter causing unplanned dilution. The ratio between the strike length and the stope's height was tested at a constant depth according to the dilution density. Figure 2-14 shows that dilution density improves in small stopes and increases when both dimensions enlarge together. Although dilution is not desirable in stopes, small sizes increase cost and time during the mining process. Consequently, it is essential to determine the acceptable overbreak to keep an economic mining ratio.

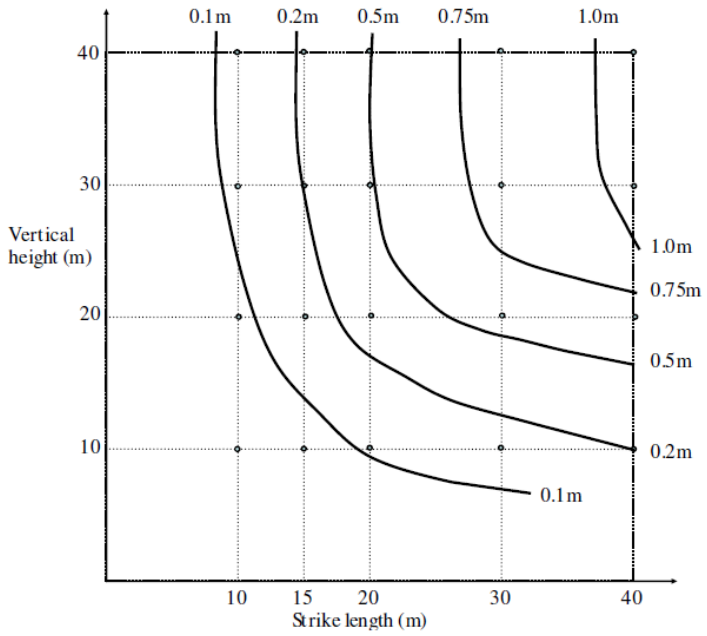

Figure 2-14. Dilution density as a function of hanging wall dimensions. Henning (2008)

Stope type in local mine sequencing is a parameter affecting unplanned dilution. Depending on the mining sequence, the stope can be bounded by intact rock on both walls (a primary stope) or surrounded by backfill stopes (secondary stopes). Henning knew that a stope next to a backfilled one would respond differently to rock slough. To test the effect of the stope type on dilution density, Henning divided the primary stopes into three sub-groups (P1, P2, P3) and the secondary ones in two (S1, S2) (Figure 7). P1 is a primary stope surrounded only by intact rock, P2 is over a backfilled P1 stope, and P3 is over more than one backfilled primary stope. S1 shares a wall with a backfilled stope, and S2 has both walls shared with a backfilled one.

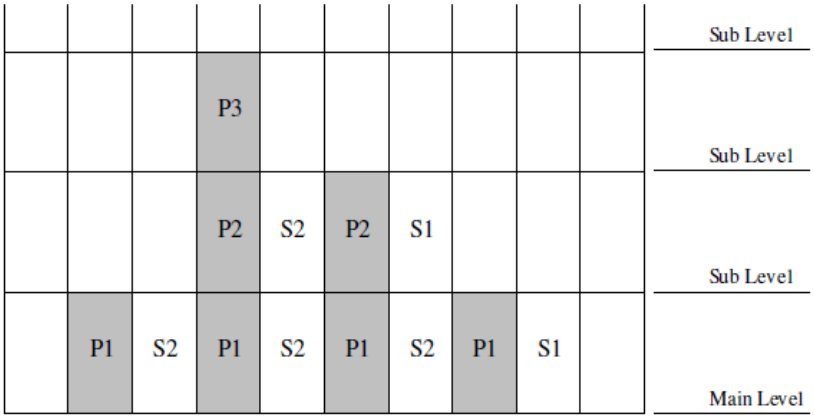

Figure 2-15. Stope type in mining sequence. Hemming (2008)

The dilution density was tested by the type of stope in different strike lengths. Figure 8 shows that secondary stopes are associated with greater values of dilution density, and the increase in S2 from S1 is more significant than any increase in the primary stopes. Indeed, backfill walls tend to be much more sensitive to rock slough into the stope.

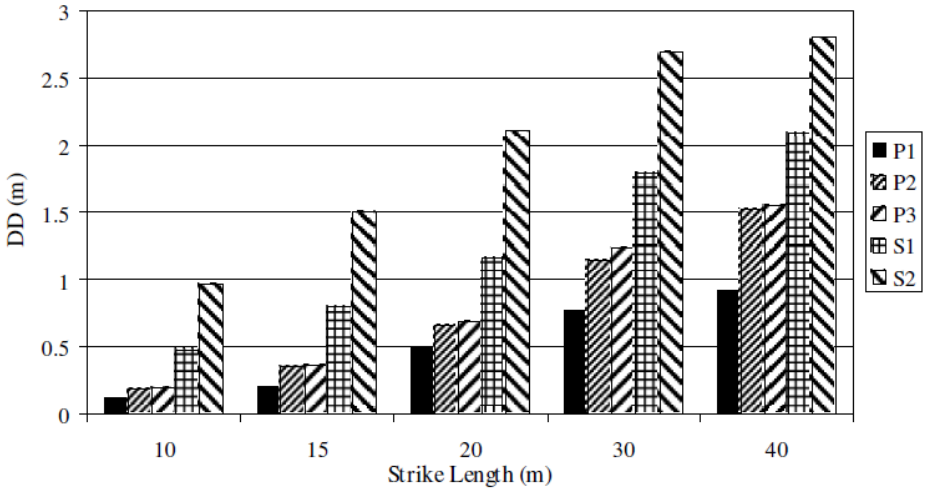

Figure 2-16. Influence of stope type on dilution density. Henning (2008)

2.8 Dilution Metrics

Defining the metrics of dilution can be complicated due to the wide range of approaches to calculating it. Even with multiple mining disclosure codes worldwide, the way to determine unplanned dilution is still unstandardized. Palkanis (1986) designed a survey to assess the state of knowledge of stope design among 38 underground mines (22 mines replied) throughout Canada. The questionnaire investigated several areas, one of which was the assessment of dilution into the stopes. The following formulae are the most common ways to calculate dilution identified by the survey.

$$Dilution = \frac{Tons\ waste\ mined}{(Tons\ ore\ mined)} \tag{2-7}$$

$$Dilution = \frac{(Tons\ waste\ mined)}{(Tons\ ore\ mined + tons\ waste\ mined)} \tag{2-8}$$

$$Dilution = \frac{Undiluted\ in\ situ\ grade\ as\ derived\ from\ drill\ holes}{(Sample\ assay\ grade\ at\ the\ drawpoint)} \tag{2-9}$$

$$Dilution = \frac{Undiluted\ in\ situ\ grade\ as\ derived\ from\ drill\ holes}{(Mill\ head\ grades\ obtained\ from\ same\ tonnage)} \tag{2-10}$$

$$Dilution = \frac{(Tonnage\ mucked\ -\ tonnage\ blasted)}{(tonnage\ blasted)} \quad (2\text{-}11)$$

$$Dilution \quad (2\text{-}12)$$
$$= Difference\ between\ backfill\ tonnage\ actually\ placed\ and\ theorical\ required\ to\ fill\ void$$

$$Dilution = Dilution\ visually\ observed\ and\ assessed \quad (2\text{-}13)$$

$$Dilution = \frac{(\text{x}\ amount\ of\ meter\ of\ footwall\ slough + y\ amount\ of\ hanging\ wall\ slough)}{(Ore\ width)} \quad (2\text{-}14)$$

$$Dilution = \frac{Tons\ drawn\ from\ stopes}{(Calculated\ reserve\ tonnage)} \quad (2\text{-}15)$$

Besides the material that sloughs into the stope, dilution can consider the ore losses in its calculation. Ore losses refer to any amount of material left in place or not adequately blasted that was not sent to the processing plant due to different operation limitations (Ibarra-Gutierrez & LaFlamme, 2021). In this study case, dilution is defined as the overbreak and slough of material from outside the boundaries of the stope compared to the planned ore minus the ore losses (*2-16*).

$$Operational\ Dilution\ (\%) = \frac{Material\ from\ out\ of\ the\ boundaries\ of\ the\ stope}{Estimated\ ore\ inside\ the\ stope\ limits - Operational\ Ore\ Losses} \quad (2\text{-}16)$$

3 Methodology

Statistical methods have been used to predict unplanned dilution before, such as the cases of Germain and Hadjigeourgiou (1997) using simple linear regression or Jang et al. (2015) using a neural network. This study will go through linear and non-linear regression analysis and neural network techniques, including ANN, RNN, and RF. In the following sections, the term dilution will denotate only the unplanned dilution resulted after the blasting and hauling processes

3.1 Multiple Linear Regression Analysis (MLRA)

Regression analysis is a statistical tool that determines relationships between a collection of independent variables to a single dependent variable (Uyanik & Guler, 2013). Linear regression has three objectives: The first is to determine a linear function that represents the linear relationship between the independent variables and the dependent variable better than any other function. The second is to investigate the magnitude of the relationship between dependent and independent variables by determining the coefficient of variation. Finally, the third is to examine whether the relationship between the dependent and the independent variables can be generalized to the population (significance) (Tacq, 1997).

The general formula of the MLRA is assumed to be:

$$Y = b_0 + b_1 x_1 + b_2 x_2 \dots b_n x_n \quad (3\text{-}1)$$

Without regression, the best prediction of the dependent variable would be its mean value ($\bar{y}$), and the error of this prediction can be represented by the sum of squares in the dependent variable:

$$Sum\ of\ squares\ (mean) = \sum (y - \bar{y})^2 \tag{3-2}$$

If a regression model exists, the error in the prediction values ($\hat{y}$) is represented by:

$$Sum\ of\ squared\ errors\ (Regression) = \sum (y - \hat{y})^2 \tag{3-3}$$

If the error in regression is smaller than the error with the mean, then the regression model improves the prediction. Then the difference $\sum(y - \bar{y})^2 - \sum(y - \hat{y})^2$ is the reduction of error, dividing this subtraction by the original probability results in proportional reduction, also known as R^2.

$$R^2 = \frac{\sum(y - \bar{y})^2 - \sum(y - \hat{y})^2}{\sum(y - \bar{y})^2} \tag{3-4}$$

Some assumptions must be met to ensure the existence of a linear relationship between the independent and dependent variables:

- Variables are significant and can be generalized: The significance of a variable determines if its impact on the predictive model is due to chance or if there is enough statistical

evidence to establish so. Stating a "null hypothesis" is the starting point to evaluate significance. The significance level is measured through the possibilities that the null hypothesis is true. This possibility is expressed usually as the "p-value," and if it is below the confidence level desired, the hypothesis is rejected, concluding that the variable is significant.

- The residual values are independent: MLR analysis assumes that each observation in the database is independent; thus, the residual according to the regression fit should be independent too. One of the most widely used ways of proving independence in the observation is the Durbin-Watson Test (*3-5*) (Tillman, 1975):

$$d = \frac{\sum_{t=2}^{T}(e_t - e_{t-1})^2}{\sum_{t=1}^{T} {e_t}^2} \tag{3-5}$$

Where:

  - T: The total number of observations
  - e_t: The t^{th} residual from the regression model
  - d: The Durbin-Watson variable

The lack of independence between observations is considered as an autocorrelation problem. Autocorrelation denotes that there is a relationship between the observations and the dependent variable not only depends on the independent variables but also on more variables that are not considered. If d=2 indicates there is no autocorrelation, and if it is greater than 2.5 or less than 1.5, then there is a potential autocorrelation problem.

- The variance of the residual is constant: MLR assumes that the variance of the residual along the linear model has constant variance. If this is not the case, the linear model is facing a heteroscedasticity problem. The presence of heteroscedasticity makes the model unreliable. The lack of constant variance in the residual denotes that the regression model declares a variable as significant when actually it is not.

  The simplest way to determine if this assumption is met is with a fitted value vs residuals plot (Figure 5-1). It can be graphically determined if the variance is increasing or decreasing along the linear model. Heteroscedasticity can be determined using student tests (Koenker, 1981) or estimator formulae (Muller & Stadtmuller, 1987); however, it can turn out to be highly complex.

- The values of the residuals are normally distributed: Multiple linear regression assumes the existence of a linear relationship between the independent variables and the dependent variable. If this assumption is met, then the residuals are normally distributed along the linear model. The easiest way to determine the linearity between the variables is to create a Normal Probability Plot from the residuals, which is a graphical technique to assess whether the residual is normally distributed.

- Multicollinearity: The predictor variables should be independent of each other, which means that there is no correlation between the independent variables. The existence of multicollinearity in the regression analysis makes the determined coefficients imprecise due to the dependency between two or more predictor variables. Also, the accuracy of the significance test is reduced, which makes it difficult to determine the actual statistically significant variables. The two most common ways to detect multicollinearity

are the creation of a matrix table of correlation coefficients between all the variables and the use of VIF (variance inflation factor). The correlation matrix helps us to understand the relationship between the variables much better and to determine how to manage them. Dealing with multicollinearity can be problematic, since it is important to determine how to resolve it: remove variables, combine them, or perform the analysis with the correlated variables because both are needed for further analysis.

Additionally, the dependent variables to be used in the model should be statistically significant so they can be generalized to the population. In order to determine which variables are significant, a step-by-step iterative process must be performed to select the independent variable to be used in the linear model. This iterative process is called Stepwise Regression.

The goal of the Stepwise Regression is to select the set of significant variables to be used in the regression analysis; this is done by adding or removing potential explanatory variables and testing their significance (p-value) in each iteration. There are three approaches for Stepwise Regression:

- Forward Selection: Starts with no variables in the analysis. The significance of each variable is analyzed as it is added, and the most significant ones are kept (user should select a confidence level).
- Backward Selection: Starts with all the independent variables, and the ones with lowest significance (highest p-value) are deleted one at a time. The iteration stops when all the variables are into the confidence level.

- Bidirectional elimination: Combines both previous methods. This method adds and eliminates variables looking for the best set of significant variables.

3.2 Multiple Non-Linear Regression (MNLR)

Non-linear regression is a regression model that uses a non-linear function between the dependent variables. MNLR determines the coefficient and parameter of the model minimizing the residual sum of squares (the distance of the datapoints to the curve). However, unlike multiple linear regression, MNLR cannot be solved in one step in a deterministic way; instead, the model has to be created iteratively. During the process, the iteration keeps adjusting the model parameters to improve the fit of the curve (Motulsky & Ransnas, 1987).

One of the most common algorithms used to determine a MNLR model is the Gauss-Newton method which uses steepest descent moves along the direction of the steepest descent curve with small steps (Motulsky & Ransnas, 1987). The initial iterations approach the goal quickly, while later iterations usually take much more time to minimize the residual sum of squares.

Fitting a curve demands a large amount of computational resources due to the number of iterations; this demand drastically increases when the number of variables also increases. To simplify the process and reduce the time to determine the non-linear model, a transformation of the data must be done. This method will transform the curved relationship into a linear one to perform a linear regression. One of the most common linear transformations is applying logarithm in each variable:

The general shape of a Multiple Non-Linear Regression function is assumed to be:

$$Y = b_0(x_1)^{b1}(x_2)^{b2} \dots (x_n)^{bn} \quad (3\text{-}6)$$

The estimation of the parameters b_1-b_n in a non-linear relationship can be performed by an alteration to a linear domain using a log transformation. The new shape of the MNLR is changed to:

$$\log(Y) = \log(b_0) + b_1 \log(x_1) + b_2 \log(x_2) \dots b_n \log(x_n) \quad (3\text{-}7)$$

After the log transformation, the parameter b_i can be determined by the traditional linear regression method. In this case, the regression is applied to $\log(Y)$ on $\log(x_1) + \log(x_2) \dots \log(x_n)$.

After the transformation, the data is analyzed with a Multiple Linear Regression, and the assumptions required have to be met. Transforming the data to perform linear regression has several advantages, and the most important one is the simplicity of the process. However, the linear regression model enhances errors when it is used to predict y values that are out of the range used to create the model; in other words, this MNLR model has to be used carefully when predicting data out of the training data set.

3.3 Artificial Neural Network

Wang (2003) defined an Artificial Neural Network as a group of neurons arranged in layers that mimic the sophisticated interconnection of the human brain neurons in order to learn (Figure 3-1). The main structure of an ANN is the array of "neurons" in input, hidden, and output layers (Figure 3-1). The "neurons" represent each input variable in the first layer (input layer), but the neurons are independent from the initial variables in the following layers. Each neuron is fully

interconnected to the others in the next layer, and these neuron connections represent weights (coefficients). The optimum weights lead to the model's best accuracy through a backward process with a learning algorithm during the training stage (Jang, Topal, & Kawamura, 2015). The optimum weights can be determined using the Adam (Adaptative Moment Estimation) Algorithm (Kingma & Lei Ba, 2015), which is a stochastic gradient-based optimization proposed by Kingma and Lei Ba (2015).

The Adam Algorithm has replaced the classic stochastic gradient descent procedures in determining the network weights. The previous methods used to take hours to perform several iterations to find the optimum (or close) results. In contrast, the Adam Algorithm computes adaptive learning rates (steps) to move forward in the gradient descent estimation. The algorithm keeps updating the gradient of the function assessed in steps determining minimums and maximums along the loss function by analyzing its gradient. The weights that lead to the minimum loss function will be the optimum ones for the predictive model (the neural network).

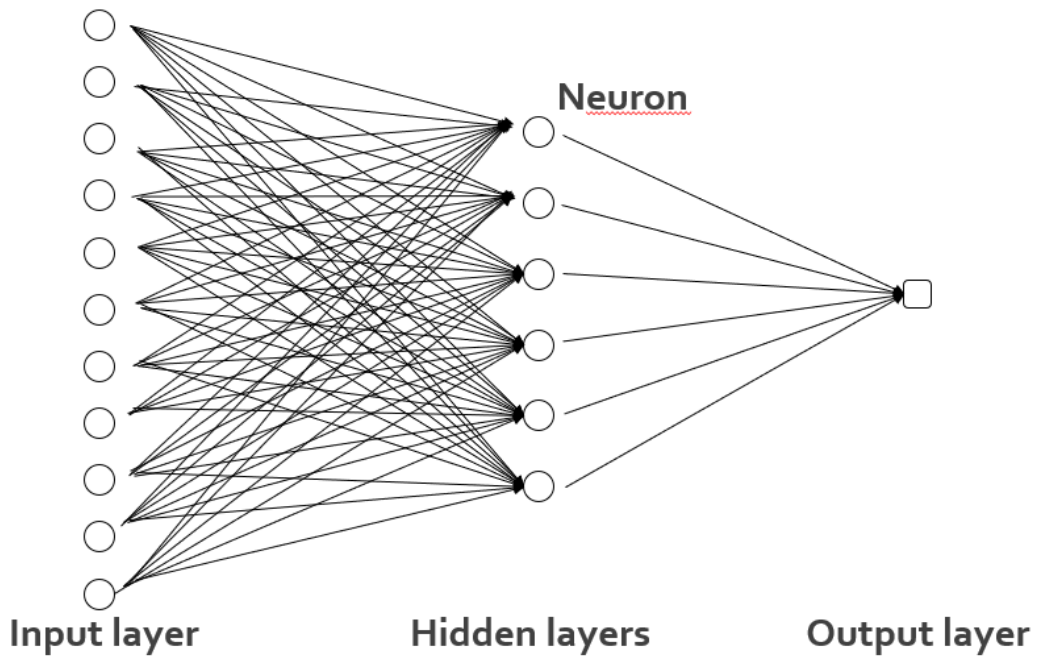

Figure 3-1. Neural Network Layout

The value (h_i) of a neuron i in any hidden or output layer is defined by the following formula:

$$h_i = \sigma\left(\sum_{j=1}^{N} V_{ij}x_j + B_i\right) \tag{3-8}$$

Where σ is the activation function, N is the number of neurons in the previous layer, V_{ij} is the weight, x_j is the value of the neuron in the previous layer, and B_i is the bias (a constant determined for each neuron).

The primary purpose of the activation function is to introduce nonlinearity into the neural network (Wang, 2003). Many activation functions can be applied to neural networks, and choosing the most suitable can be challenging. The Rectified Linear Unit function is the most

widely used in predictive models due to its simplicity. The following function is applied to every neuron in a neural network model:

$$\sigma(h_i) = \begin{cases} \sigma(h_i) = 0 & h_i < 0 \\ \sigma(h_i) = \sum_{j=1}^{N} V_{ij} x_j + B_i & h_i \geq 0 \end{cases} \quad (3\text{-}9)$$

When the architecture of a neural network is set, the training process occurs. The neural network model is trained by an input data set called the training data set. The weights and biases are adjusted to minimize the error function in the training process, which usually compares the model and actual output. The most used metrics in error functions are mean square error and mean absolute error (Wang, 2003).

The training data set must be large enough for the model to identify the trends embedded in the data set. During training, the fitting process can lead to overfitting the data; hence, a validation data set is applied to avoid it. Also, the validation set is independent of the training set and allows for testing the model's accuracy and picking the best according to its performance.

3.4 Random Forest

Breiman (2001) introduced the Random Forest algorithm as a combination of predictor trees in which each tree depends on a group of predictor variables sorted in a *p*-dimensional vector $X = (x_1, x_2, x_3 \dots x_p)^T$ and a result or response variable $Y = (y_1, y_2, y_3 \dots y_T)$. The main objective of this algorithm is to determine a prediction function $f(X)$ to forecast Y, and the prediction of this function is given by the minimization of a loss function $L(Y, f(X))$. Generally,

the metrics used of L are square error loss $L\big(Y, f(X)\big) = (Y - f(X))^2$ or absolute error loss $L\big(Y, f(X)\big) = |Y - f(X)|$. In this case study, absolute error will be used to standardize the metrics with the machine learning techniques.

The prediction function $f(X)$ is made up of an ensemble of classifiers or "base learners," $h_1(x), h_2(x), h_3(x) \ldots h_j(x)$. The *j*th classifier is typically detonated as $h_j\big(X, \ominus_j\big)$, where $\ominus_j$ is an independent random variable which describes the randomness in the node splitting in the predictor trees. A splitting node is defined by a binary partition which is the base unit for each tree. Each node splits the data set into two descendant nodes, called "terminal nodes." In a continuous variable, the split is determined by a specific value, and all the entries smaller than the split value go to one terminal node, and the rest go to the second terminal node. Testing every predictor's possible splits leads to finding the best possible split according to the "goodness of fit" criteria.

A typical criterion used in a split node is the mean squared residual at each terminal node:

$$Q = \frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})^2 \tag{3-10}$$

Where n is the sample size of the terminal node and $y_i \, \epsilon \, (y_1, y_2 \ldots y_n)$ is a subgroup of Y that was split by the splitting node. Therefore, the splitting criteria for the two possible descendants (left and right) determine $Q_L \; and \; Q_R$ and their sample size as $n_L \; and \; n_R$. The best possible split will be determined by the "goodness of fit" criteria which require minimizing $Q_{split} = Q_L n_L + Q_R n_R$.

When the best split has been chosen, the data partitioned into two goes through the same process as the original node and the data keeps being split. This process continues until the

sample size of the descendants meets a stopping criterion previously defined by the user. When the process of splitting ends, the last descendants take the name of "terminal nodes." All the split and terminal nodes define an individual tree, which is ready to predict a new data set $P(X, Y)$.

In a group of predictor trees, which is the main structure of the Random Forest algorithm, the prediction is the unweighted average of the predictions made by each individual tree. If there are J predictor trees, the prediction of the forest is defined as:

$$\hat{y} = f_{(X)} = \frac{1}{J}\sum_{j=1}^{J} h(X) \tag{3-11}$$

The combination of the randomness in the selection of the split values and a large number of predictor trees prevent overfitting in the algorithm.

3.5 Recurrent Neural Network (RNN)

The recurrent neural network contains one hidden layer or layers that are recurrent. The term recurrent denotes neurons in this layer determined by both the input variables and the past states of the same neurons (previous state). The standard recurrent layer is the vectorial sum of the layer's inputs and the recurrent information (the prior state of the neurons). Because the vector sum is a repetitive procedure in training and predicting processes, the sigmoid or hyperbolic tangent transformation functions are required to avoid the values in neurons diverging out of their ranges (Figure 3-1).

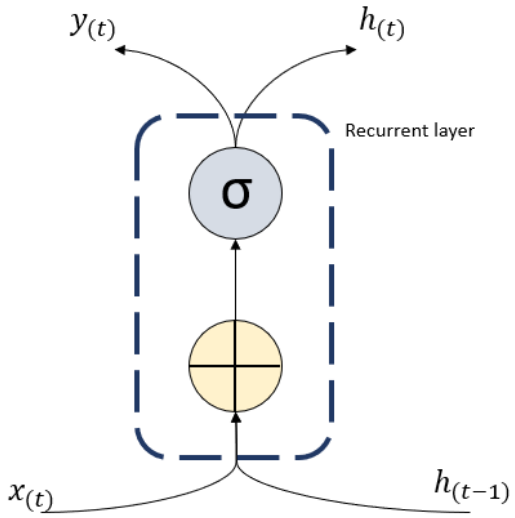

Figure 3-2. Recurrent layer

The recurrent neural networks are successful when the output depends on the input variables and the previous results or states. However, the standard recurrent neural layer is incapable of handling long-term dependencies. Hochreiter & Schmidhuber (1997) introduced the long short-term memory layer. They increase the capacity to remember beyond the previous state by adding "gates" to the layer. The "input gate" uses the input variables and the recurrent information to select which variables will be stored in the cell state while the "output gate" decides the output of the layer state with the information provided by the input gate (Figure 3-2). The gates are vectors populated with 0 and 1. Hence, the vector multiplication of the gates will allow which information will pass (when multiplied by 1) and which will be filtered (when multiplied by 0). The hyperbolic tangent function in both gates allows the creation of the gate vector (combination of vectors populated with 0 and 1s).

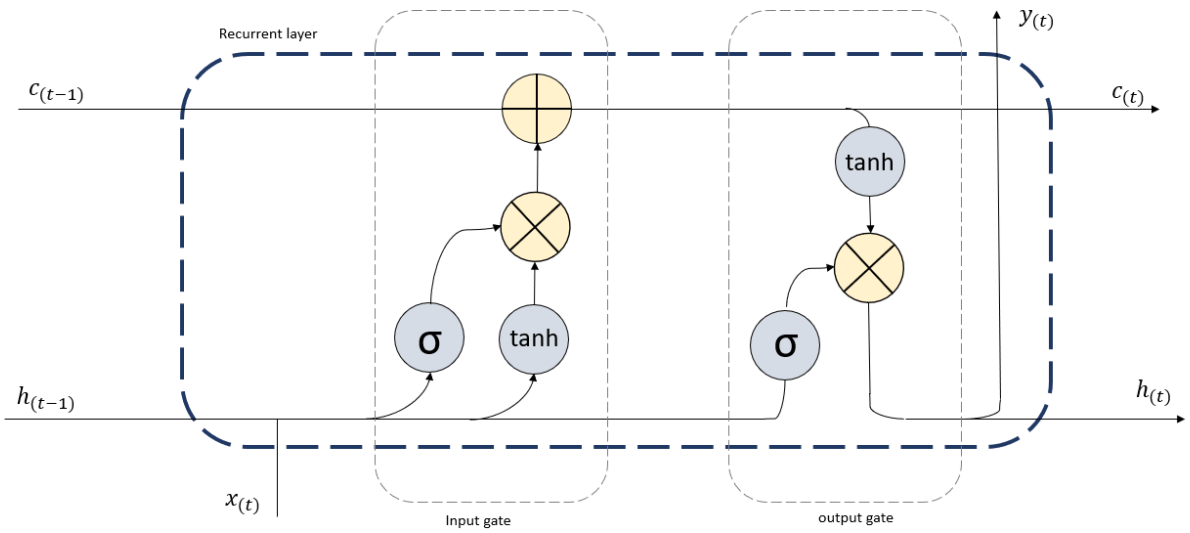

Figure 3-3. LSTM recurrent layer

Several variations of the LSTM layer have been developed in the last few years. The most famous one is the LSTM layer with Forget Gate, which adds an initial layer, previous to the input gate, that allows forgetting information with a vectorial multiplication. In this case study, the standard LSTM layer will be used.

Similar to neural networks, in RNN, the hidden and recurrent layers are based on weights and biases, which are optimized using the Adam Algorithm (Kingma & Lei Ba, 2015). Optimization in RNN is extended to determine the optimum combination of 0 and 1 in the gates inside the recurrent layer.

Sorting the data is the most crucial part of RNN. The model's prediction accuracy depends on the recurrent information stored on the layers, which will be used in the subsequent prediction. If the data has no order or is sorted incorrectly, RNN is not applicable.

3.6 Workflow

The workflow of this research is summarized in Figure 3-4.

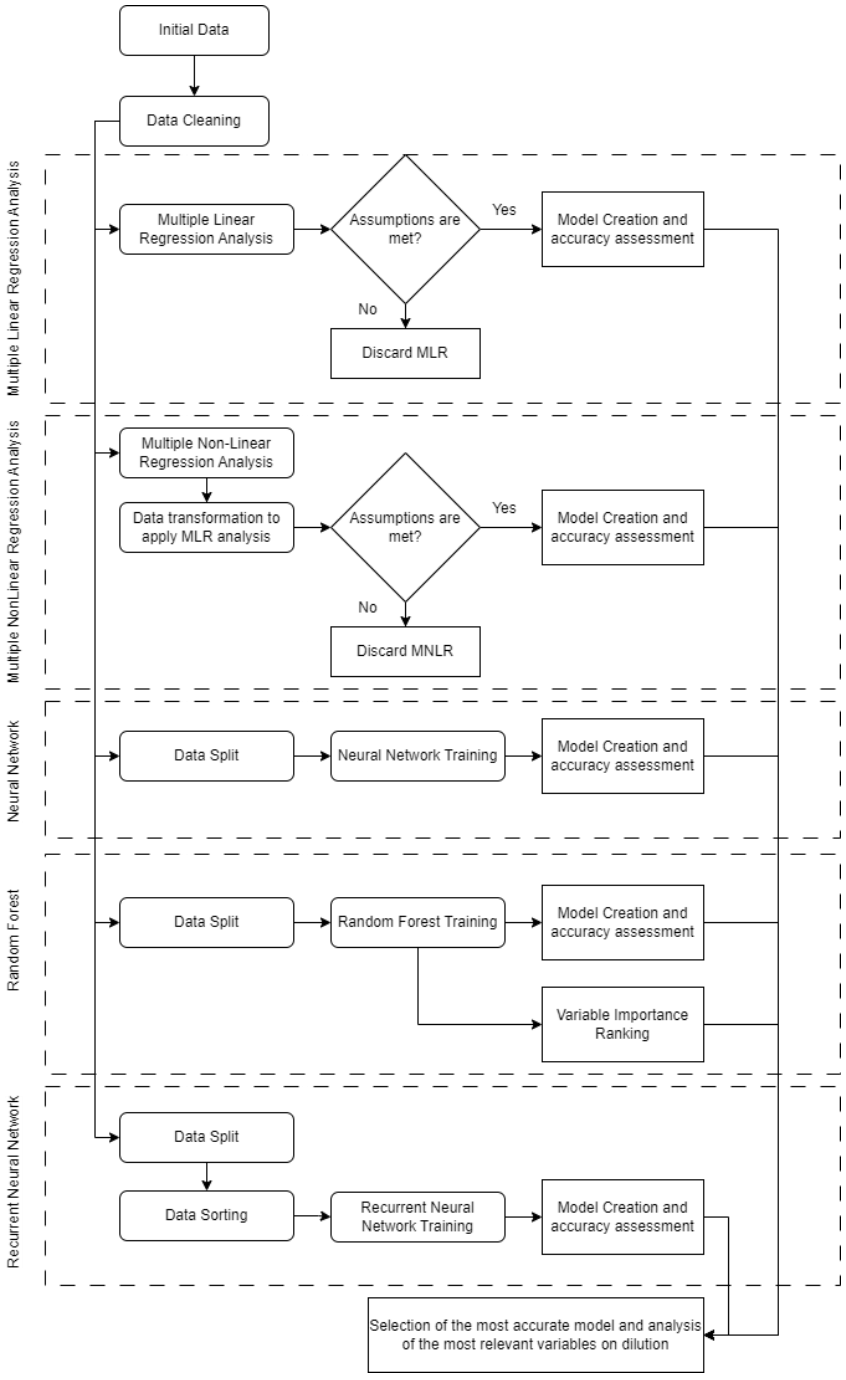

Figure 3-4. Summary of research workflow

4 Case Study

The predictive capability of any statistical model depends on the data quality gathered. In this analysis, the data set consists of 99 entries with 68 parameters from an underground mine in North America. The primary mining method applied is longitudinal retreat long-hole. The stopes are partially backfilled (one-third to half of the volume) with cemented rockfill, and the rest is filled with regular rockfill.

The independent variables were grouped into the six categories listed below.

4.1.1 Location

- Zone: The mine was divided into two significant zones (1 & 2). Each zone was defined according to the two orebodies that have been mined (Figure 4-1).

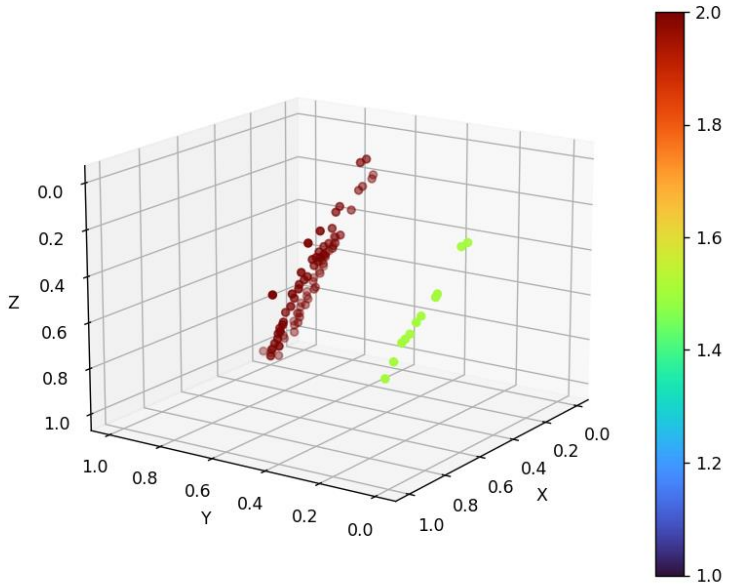

Figure 4-1. 3D Plot of the zones in the mine.

- X, Y, and Z coordinates: The three coordinates of the stopes are used as independent variables in the analysis.
- Level and depth: These two variables are related to the Z coordinate of the stopes. Both were introduced in the analysis but discarded during the backward process of the regression analysis.
- Location panel: This variable represents the location of the stope in the longitudinal axis according to the orebody (Figure 4-2).

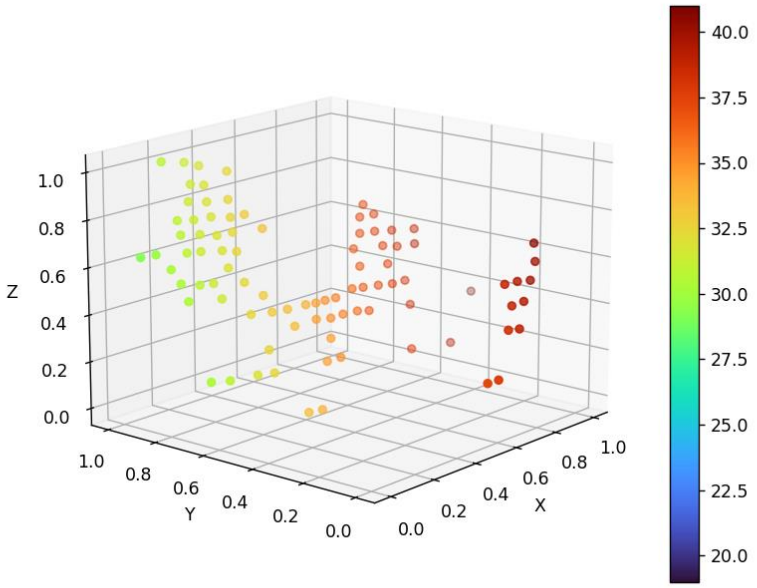

Figure 4-2. 3D plot of the Panels in the mine

4.1.2 Stope Geometry

- Angle: This is the angle generated between the hanging wall and the horizontal reference.
- Stope height, width, and length: The three dimensions of the stopes during the planning process.

- Number of hanging walls: The hanging wall can change its angle along itself. The number of hanging walls is the number of angle changes plus one (Figure 4-3).

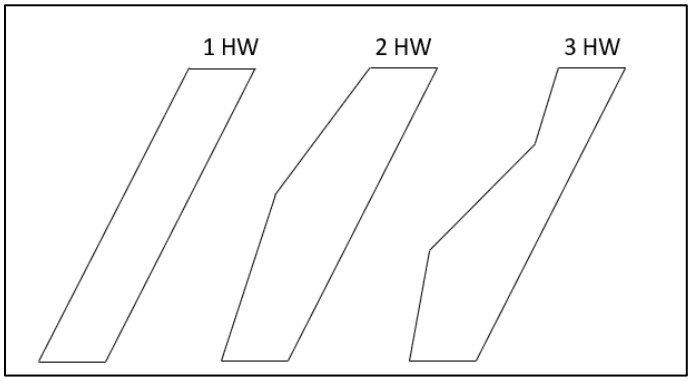

Figure 4-3. Number of Hanging Walls

- Infrastructure intersection (lower part of the stope): This is a binary variable to define if there is any other infrastructure (tunnel) in the intersection of the stope in the lower level.
- Infrastructure intersection (upper part of the stope): This is a binary variable to define if there is any other infrastructure (tunnel) in the intersection of the stope in the upper level.
- Height with angle: This height with angle is calculated using the angle of the stope. $Height\ with\ angle = \frac{Height}{Sin(angle\ of\ stope)}$
- Hydraulic radius (roof): The hydraulic radius is defined as the area of the surface (in this case, the roof of the stope) divided by the perimeter.
- Hydraulic radius (hanging wall and foot wall): The hydraulic radius is defined as the area of the surface (in this case, the hanging wall of the stope) divided by the perimeter.
- Planned tonnage: This is the volume of the stope calculated using its dimensions (height, width, length, and angle) times the rock density.

4.1.3 Drilling and Blasting

- Burden and spacing: Spacing is the distance between two blast holes in the same row, and the burden is the distance between rows in the drilling design.
- Mining method: This is a nominal variable. The methods used in this mine are Longitudinal (the most used method), Longitudinal-sill, Avoca, primary, and secondary.
- Powder factor: The relationship between the amount of rock blasted and how much explosive is used in the blasting process. This variable is calculated with the "planned tonnage" and the "quantity of explosive" variables.
- Number of shots: Number of shots that the blasting process takes to break all the stope.
- Explosive (type): This variable was not used. The blasting process of all the stopes used the same type of explosive: emulsion subtek.
- Drilling direction: This is a binary variable that defines the drilling process's direction. This stope could be drilled from the lower level to the upper level (up) or vice versa (down).
- Blasting cap (type): This variable was not used. The blasting process of all the stopes used the same type of blasting cap: electronic.
- Driller model: This nominal variable refers to the five different models of drillers that are used in the mine.
- Explosive quantity: The amount of explosive used in the blasting process.
- Drill type (mode): This nominal variable refers to the type of drill.
- Drilling meters planned: The total meters drilled in the drilling process.

- Drilling diameter: The size of the drill (diameter) used in the driller.

4.1.4 Rock Mass Stability

- N' Number (hanging wall and back wall): These two variables were not used in the analysis due to their invariance along all the stopes in the database.
- Lithology (foot wall and hanging wall): Two binary variables that define the lithology of the foot wall or hanging wall (Tuff or Diorite).
- Shear zone (foot wall and hanging wall): Two nominal variables that describe the presence and size of shear zones in the foot wall and hanging wall.
- Overbreak in development: This set of variables defines the length of the overbreak in the stopes due to the upper and lower development infrastructures. The overbreak is produced when the levels used to drill the stope are wider than the stope, producing an overbreak in the top and the bottom of the stope.

4.1.5 Rock Support

- Wall and roof support density: Two variables that define the support density of the walls or roof in the stope. This is a specific metric used in the mining company.
- Cable 10 m or 6 m: The number of cables used in support of the rock in different parts of the stope (lower and upper part).
- Cable length: The total length of cable used in each part of the stope (lower and upper part).
- Height and length not supported: These two variables refer to the distance in each direction of the stope that was not supported in the mining process.

4.1.6 Mining Sequence

- Span time before mucking (ore in place), during mucking, and backfilling: Three variables measure the time in days that it takes to start mucking after the blasting, mucking the ore, and backfilling the stope.
- Type of backfill: There are three types of backfill used in this mine (rockfill, cemented 3.5%, and cemented 6%).
- Neighbour backfilled: This set of variables describes whether the neighbour stopes were backfilled before the blasting or are intact rock.

Table 4-1. List of Variables

Location • Zone • Level • X coordinate • Y coordinate • Z coordinate • Depth • Location panel	**Stope Geometry** • Angle • Stope height • Stope width • Stope length • Number of hanging walls • Infrastructure intersection (lower part of the stope) • Infrastructure intersection (upper part of the stope) • Height with angle • Hydraulic radius (roof) • Hydraulic radius (hanging wall and foot wall) • Planned tonnage
Drilling and Blasting • Burden • Spacing • Mining method • Powder factor • Number of shots • Explosive (type) • Drilling direction • Blasting cap (type) • Driller model • Explosive quantity • Drill type (mode) • Drilling meters planned • Drilling diameter (inches)	**Rock mass stability** • N` (Stability number - hanging wall) • N` (Stability number - back wall) • Lithology foot wall • Lithology hanging wall • Shear zone (foot wall) • Shear zone (hanging wall) • Overbreak in development - FW down • Overbreak in development - HW down • Overbreak in development - FW up • Overbreak in development - HW up
Rock Support • Wall support density • Roof support density • Cable 10 m stope lower part • Cable 6 m stope lower part • Cable 10 m stope upper part • Cable 6 m stope upper part • Cable length lower part • Cable length upper part • Height not supported • Length not supported	**Mining Sequence** • Span time of mucking • Time for drilling • Type of backfill • Down neighbour is backfilled • East neighbour is backfilled • Up neighbour is backfilled • West neighbour is backfilled • Span time ore in place • Span time backfilling

5 Results

5.1 Analysis of Multiple Linear Regression (MLR) Results

MLR assumptions were scrutinized to determine the existence of a linear relationship between the independent and the dependent variables. A lack of significance in several variables was detected in the first attempt using the MLR model. Therefore, stepwise regression methods removed the variables without the required statistical significance level (95% confidence interval), and the output was an MLR model with 31 variables (Table 5-1). However, the analysis of the standardized errors shows heteroscedasticity, which means that the error variance increases when the forecasted dilution value increases (Figure 5-1).

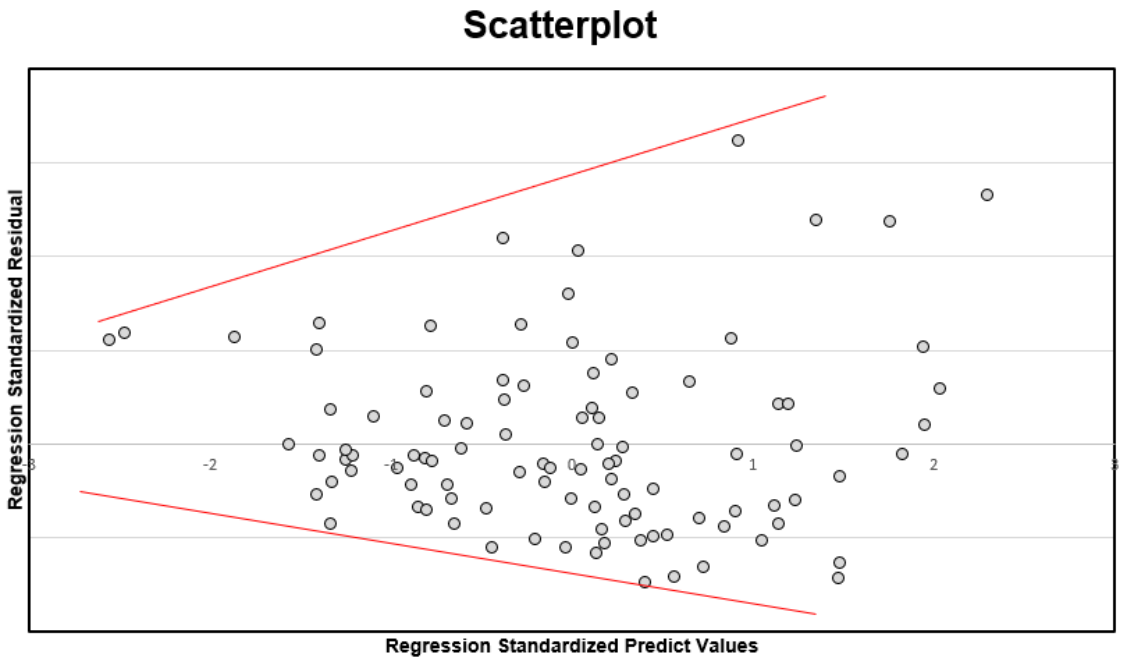

Figure 5-1. Scatterplot Standardized Residuals - MLR

Heteroscedasticity rejects the existence of a linear relationship between variables and dilution. Reducing the number of variables can help to find a linear relationship between dilution and the significant variables; however, this will reduce the yield, $R^2 = 0.790$.

Table 5-1. Significant Variables in the MLR model

Variables	Non-standardized coefficients		Standardized coefficients	Sig.
	B	Error	Beta	
(Constant)	109.920	171.119		0.523
Zone	-2.259	0.418	-4.628	0.000
Angle	-0.018	0.003	-0.862	0.000
X coordinate	-0.005	0.002	-4.320	0.007
Y coordinate	0.014	0.002	8.399	0.000
Z coordinate	-0.006	0.001	-2.681	0.000
Burden	-2.915	0.665	-5.650	0.000
Stope height	0.049	0.015	1.261	0.002
Location panel	0.216	0.047	8.542	0.000
Stope width	0.033	0.008	1.064	0.000
Spacing	3.081	0.658	5.994	0.000
Wall support density	0.904	0.167	0.626	0.000
Roof support density	0.230	0.064	0.322	0.001
Cable 10 m stope upper part	-0.015	0.004	-0.305	0.000
Cable 6 m stope upper part	-0.002	0.001	-0.215	0.043
Time for drilling (days)	0.011	0.002	0.403	0.000
Cable length lower part	0.000	0.000	0.252	0.020
Height not supported	-0.011	0.002	-0.429	0.000

Table 5 1. Significant Variables in the MLR model

Hydraulic radius (roof)	-0.178	0.030	-0.691	0.000
Hydraulic radius (hanging wall and foot wall)	-0.414	0.087	-2.268	0.000
Overbreak in development - HW down	-0.087	0.016	-0.468	0.000
Overbreak in development - HW up	0.098	0.027	0.386	0.001
Method-Avoca	0.219	0.073	0.389	0.004
Lithology FW - Diorite	-0.106	0.036	-0.286	0.005
Lithology HW - Diorite	0.229	0.051	0.383	0.000
Type of backfill – cement 6%	-0.244	0.039	-0.743	0.000
Direction of drilling - up	-0.273	0.072	-0.351	0.000
Infrastructure intersection (lower part of the stope)	-0.212	0.036	-0.593	0.000
Down neighbour is backfilled – intact rock	0.357	0.065	1.142	0.000
Down neighbour is backfilled – cement 6%	0.135	0.062	0.331	0.035
Down neighbour is backfilled – cement 3.5%	0.249	0.062	0.598	0.000
East neighbour is backfilled – intact rock	0.374	0.099	1.061	0.000
East neighbour is backfilled – cement 3.5%	0.334	0.103	0.780	0.002
East neighbour is backfilled – cement 6%	0.230	0.101	0.358	0.026
East neighbour is backfilled – rockfill	0.473	0.124	0.434	0.000
Up neighbour is backfilled – cement 6%	-0.199	0.056	-0.284	0.001

Table 5 1. Significant Variables in the MLR model

Shear zone HW - enclosed	-0.167	0.062	-0.342	0.009
Driller model - CUBEX01	0.229	0.039	0.703	0.000
Driller model - FLXLH707	0.158	0.067	0.177	0.022
West neighbour is backfilled – cement 6%	0.123	0.040	0.270	0.003

5.2 Analysis of Multiple Non-Linear Regression (MNLR) Results

Because the required assumptions were not fulfilled to fit a linear regression model to the data set, an MNLR analysis was performed to obtain a suitable predictive model. In this case, a non-linear relationship between the variable was assumed, which led to the model in (*5-1*):

Working with a power model is much more complex than with an MLR; therefore, modifying it through a log transformation is a feasible solution. This modification allows calculating the power coefficients using an MLR analysis.

$$\ln(\hat{Y}) = \ln(b_0) + b_1 \ln(x_1) + b_2 \ln(x_2) + \cdots b_n \ln(x_n) \quad (5\text{-}1)$$

The power model established by MNLR proved suitable for the data set because all the required assumptions were met. The significant variables were selected through backward stepwise regression analysis, shown in Table 5-2. The variables with the highest impact (standardized coefficients) on dilution belong to the *Stope Geometry* and *Drilling and Blasting* categories.

Table 5-2. Significant Variables in the MNLR model

Variables	Non-standardized coefficients		Standardized coefficients	Sig.
	B	Error	Beta	
(Constant)	-1613.657	545.737		0.004
Zone	-1.019	0.313	-2.087	0.002
Method - Primary	-0.108	0.044	-0.202	0.018
Lithology HW - Tuff	-0.164	0.052	-0.274	0.003
Drilling direction - Up	-0.216	0.082	-0.277	0.011
Infrastructure intersection (lower part of the stope)	-0.116	0.034	-0.324	0.001
Down neighbour is backfilled – intact rock	0.122	0.032	0.390	0.000
Down neighbour is backfilled – rock fill	-0.132	0.063	-0.205	0.040
East neighbour is backfilled – intact rock	0.087	0.028	0.245	0.003
Up neighbour is backfilled – intact rock	0.195	0.055	0.279	0.001
Driller model - CUBEX01	0.172	0.038	0.528	0.000
Angle	-1.041	0.266	-0.804	0.000
Level	0.501	0.183	1.040	0.008
Y coordinate	156.953	53.029	3.421	0.004
Burden	-4.613	1.676	-2.946	0.008
Stope height	3.200	0.857	4.516	0.000
Stope width	1.874	0.910	3.343	0.043
Location panel	1.080	0.417	1.418	0.012
Stope length	2.729	0.787	3.403	0.001

Table 5 2. Significant Variables in the MNLR model

Spacing	5.140	1.660	3.295	0.003
Wall support density	0.739	0.175	0.454	0.000
Roof support density	0.197	0.091	0.198	0.033
Cable 10 m stope upper part	-0.034	0.015	-0.184	0.022
Powder factor	3.590	1.497	2.380	0.019
Time for drilling	0.073	0.024	0.269	0.003
Explosive quantity	-1.841	0.776	-4.916	0.020
Length not supported	-0.046	0.013	-0.324	0.001
Hydraulic radius - HW	-2.727	0.878	-2.393	0.003
Overbreak in development - HW down	-0.121	0.039	-0.249	0.003
Overbreak in development - HW up	0.130	0.042	0.306	0.003

ANOVA table (Table 5-3) of the MNLR model exhibits a $R^2 = 0.727$. The 29 significant variables in Table 5-2 can explain only 72.7% of the variation in dilution.

Table 5-3. ANOVA Table of MNLR model

	Sum of Squares	**Degrees of freedom**	**Mean Squares**	**F**	**Sig.**
Regression	1.693	31	0.055	5.75	<.001
Residual	0.637	67	0.010		
Total	2.330	98			

5.3 Analysis of Artificial Neural Network Results

Defining the neural network's optimum structure can be challenging because there is no theory on the number of hidden layers or neurons a model should have. The optimum numbers depend on experimentation with iterations to determine the best architecture. The data set was tested for 1 to 5 hidden layers with 2 to 200 neurons each. According to the accuracy of the forecasting of the validation set, the optimum architecture was the following:

- Number of hidden layers: 4
- Number of neurons: 87, 110, 79, 111

Special attention was paid to choosing this model's activation function and the optimizer algorithm. Many activation functions were proved during the process; however, ReLU (Rectified Linear Units) had the best performance according to our data set; in many cases, the other functions led the model to a "dead" neural network (where only one neuron is activated in all of the process, getting the same results beside the inputs). Also, the Adam Algorithm (Kingma & Lei Ba, 2015) was selected due to its fast and straightforward convergence analysis that estimates the optimum weights of the neurons. Important parameters required in the Adam Algorithm are the learning rate (steps) and the batch size of data (random subsamples). If the batches are too small, the algorithm will not converge on the optimum weights, and if the batches are too big, there will be overfitting issues. Something similar occurs with the size of the learning rate. Large steps cannot detect the optimum weights in the iterations, and tiny steps can trap the algorithm in a local optimum.

The model was assessed through iteration to determine the optimum learning rate and the batch size:

- Learning rate: 0.001
- Batch Size: 45

The training process used two sub-datasets: the training data set with 76 stopes and a validation data set to avoid overfitting with 13. Part of the initial data set was taken aside (10 stopes) from the learning process. The test data set's correlation coefficient (r) was 0.842 ($R^2 = 0.710$), and the MEA was 3.9% in dilution forecasting.

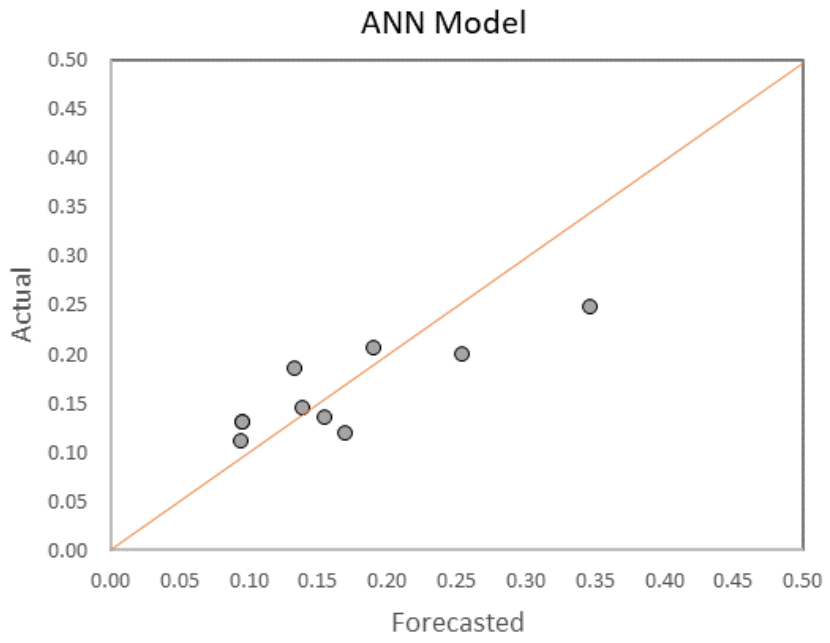

Figure 5-2. Scatterplot - Forecast values and actual values (ANN)

5.4 Anlaysis of Random Forest Results

Random Forest models do not require a validation data set; their classification algorithm based on creating decision trees prevents this model from falling into overfitting issues. As with ANN models, defining optimum hyperparameters is required, specifically the number of estimators and the minimum number of samples to splits. In this case, both were determined by random search through iterations. The number of estimators or trees was 349, and the minimum number of samples split was 7 in this model.

The same test data set used with the ANN model was assessed with the RF model to compare both accuracies.

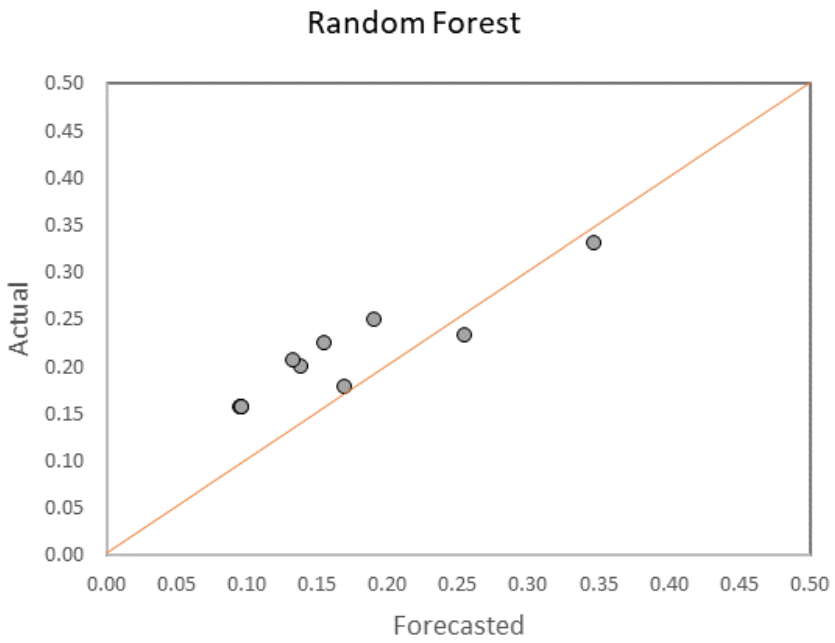

Figure 5-3. Scatterplot - Forecast values and actual values (RF)

The test data set's correlation coefficient (r) was 0.925 ($R^2 = 0.855$), and the MEA was 5.00% in dilution forecasting. The Random Forest model not only obtained better results than the ANN

model, but RF also allowed the calculation of the importance of each variable. The complexity of the interaction of the weights in a neural network makes it impossible to follow the significance of each variable. The ten variables with the highest impact on dilution according to the RF model are shown in Table 5-4.

Table 5-4. Variable ranking according to the RF model

Rank	Variable	Impact
1	Length not supported	9.47%
2	Powder factor	8.75%
3	Height not supported	8.00%
4	Span time ore in place	4.20%
5	Hydraulic radius (roof)	3.92%
6	Cable length upper part	3.18%
7	Stope width	3.14%
8	Overbreak in development - HW up	3.00%
9	Y coordinate	2.81%
10	Planned tonnage	2.74%

5.5 Analysis of Recurrent Neural Network Results

The most critical step for the recurrent neural network method is to define how to sort the data. The recurrent information required in the algorithm is based on the entry's order; sorting the data requires a variable to follow. Multiple ways of sorting were tried during the process, and the first ones were by the coordinates. Sorting the stopes following coordinates (X,

Y, or Z) created dependencies between nearby stopes. However, these dependencies were useless when trying to predict new data sets where the mined stopes were not close to each other and usually belonged to different zones. Finally, the data were sorted according to the blasting date of each stope. The dependency on time was suitable for predicting the new data set where the model would work because it was sorted according to the planned blasting date.

After sorting the data sets, defining the optimal model structure was required before training the model. Random search methodology was used to optimize the model's architecture: to determine the number of neurons in the hidden layers, the model was tested with from 2 to 150 neurons in each layer. The learning ratio of the Adam Algorithm was optimized using a range from 0.1 to 0.01. The hyperparameters selected are the following:

- Learning ratio: 0.01
- Number of neurons in the recurrent layer: 125
- Number of neurons in the additional hidden layer: 76

The training process used two sub-datasets, the training data set with 76 stopes and a validation data set to avoid overfitting with 13 stopes, and part of the initial data set was taken aside (10 stopes) from the learning process. The test data set's correlation coefficient (r) was 0.637 ($R^2 = 0.406$), and the MEA was 11.7% in dilution prediction.

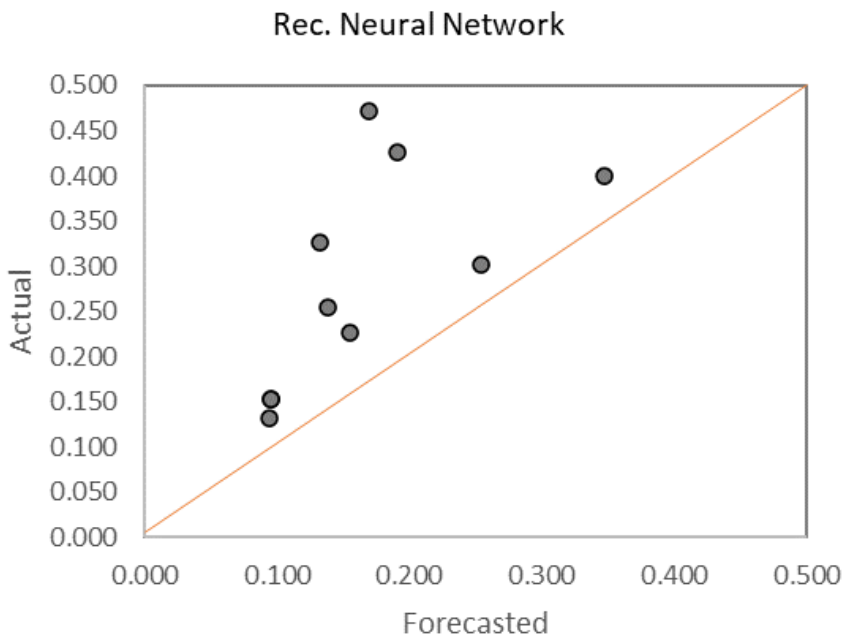

Figure 5-4. Scatterplot - Forecast values and actual values (RNN)

5.6 Comparison with a traditional method

The primary objective of this research is to enhance the accuracy of unplanned dilution prediction by utilizing both conventional methods and emerging machine learning techniques. To assess the effectiveness of the newly developed predictive models, the accuracy of each model was evaluated using a test dataset and compared with the ELOS method. A summary of the results is presented in Table 5-5. Notably, five models developed in this study have demonstrated a similar or higher level of accuracy compared to the traditional ELOS method, as depicted in Figure 5-5.

It is worth highlighting that ELOS is a static method, and its performance will not improve over time, whereas the accuracy of machine learning models will consistently enhance.

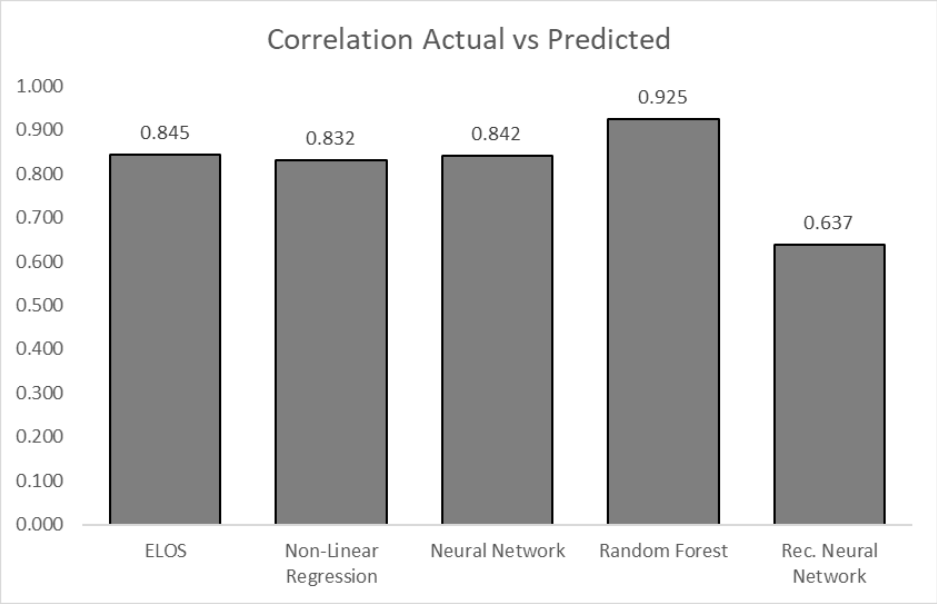

Figure 5-5. Accuracy comparison of each model

Table 5-5. Results table - Comparison with ELOS

Actual Values	ELOS		Non-Linear Regression		Neural Network		Random Forest		Rec. Neural Network	
	Predicted	Error	Predicted	Error	Predicted	Error	Predicted	Error	Predicted	Error
0.095	0.094	0.001	0.190	-0.095	0.111	-0.016	0.157	-0.062	0.133	-0.038
0.347	0.405	-0.058	0.378	-0.031	0.247	0.100	0.330	0.017	0.400	-0.053
0.190	0.186	0.005	0.221	-0.031	0.207	-0.016	0.249	-0.059	0.426	-0.236
0.155	0.141	0.014	0.084	0.071	0.135	0.020	0.224	-0.069	0.227	-0.072
0.096	0.105	-0.010	0.032	0.064	0.131	-0.035	0.157	-0.062	0.152	-0.057
0.254	0.139	0.115	0.238	0.016	0.199	0.056	0.233	0.022	0.301	-0.047
0.096	0.130	-0.034	0.032	0.064	0.131	-0.035	0.157	-0.062	0.152	-0.057
0.139	0.095	0.044	0.075	0.064	0.145	-0.006	0.200	-0.061	0.255	-0.117
0.169	0.097	0.072	0.227	-0.058	0.118	0.051	0.179	-0.009	0.471	-0.302
0.133	0.107	0.025	0.186	-0.054	0.185	-0.053	0.206	-0.074	0.326	-0.193
	Correlation	MAE	Correlation	MAE	Correlation	MAE	Correlation	MAE	Correlation	MAE
	0.845	0.038	0.832	0.055	0.842	0.039	0.925	0.050	0.637	0.117

5.7 Variable Significance and Ranking

In addition to prediction of dilution, ranking the variables affecting unplanned dilution is required to develop a dilution management strategy. Machine learning techniques can be highly complex, and tracing the weights of each variable can turn out to not be possible. However, the Random Forest and Regression models are able to show the importance of each variable. Figure 5-6 shows the significant variables according to Multiple Non-Linear Regression Analysis, where variables related to blasting (explosive quantity, powder factor, burden, and spacing), stope geometry (length, width, height, and hydraulic radius), and location (Y coordinate and zone) are the variables with higher importance to the prediction of dilution. Nonetheless, the most critical significant variables according to Random Forest are related to rock support (length and height not supported and cable length), blasting (powder factor), stope geometry (hydraulic radius, stope width, and planned tonnage), the span of time the ore is placed in the stope, and the infrastructure around the stope (Figure 5-7).

Most of the most significant variables in both methods are similar, such as blasting and stope geometry, which denotes the importance of the design of each stope and the blasting process. However, the Random Forest models give extra attention to the rock support within the stope and the time the ore stays in the stope.

The "Y coordinate" in both cases shows high importance in the prediction of dilution. The presence of this variable can indirectly show that there is a hidden variable that is not considered in the analysis and changes along the Y-axis. According to Figure 4-1, the gap that separates the two orebodies is along the Y-axis. The importance of the Y-axis variable in both models reveals the importance of the difference between these two orebodies and the different features that

they have, such as rock quality, the width of the vein, orebody dip, and several variables that were not measured in this analysis.

Besides the similarities between the two lists of variables of each model, many discrepancies were also detected. On the one hand, rock support is a main feature of Random Forest, which is not the case in MNLR analysis. On the other hand, stope dimension is highly important in MNLR analysis, but its importance decays considerably in Random Forest. Consequently, due to the difference between them, it is difficult to select the most accurate list. Both lists have enough statistical evidence to support them; thus, it is a fact that they are reliable. However, their reliability depends directly on the accuracy of their models. While the MNLR model reached a correlation of 0.832, the Random Forest model overperformed it with a correlation of 0.925. According to these results, the list provided by the RF model is recommended.

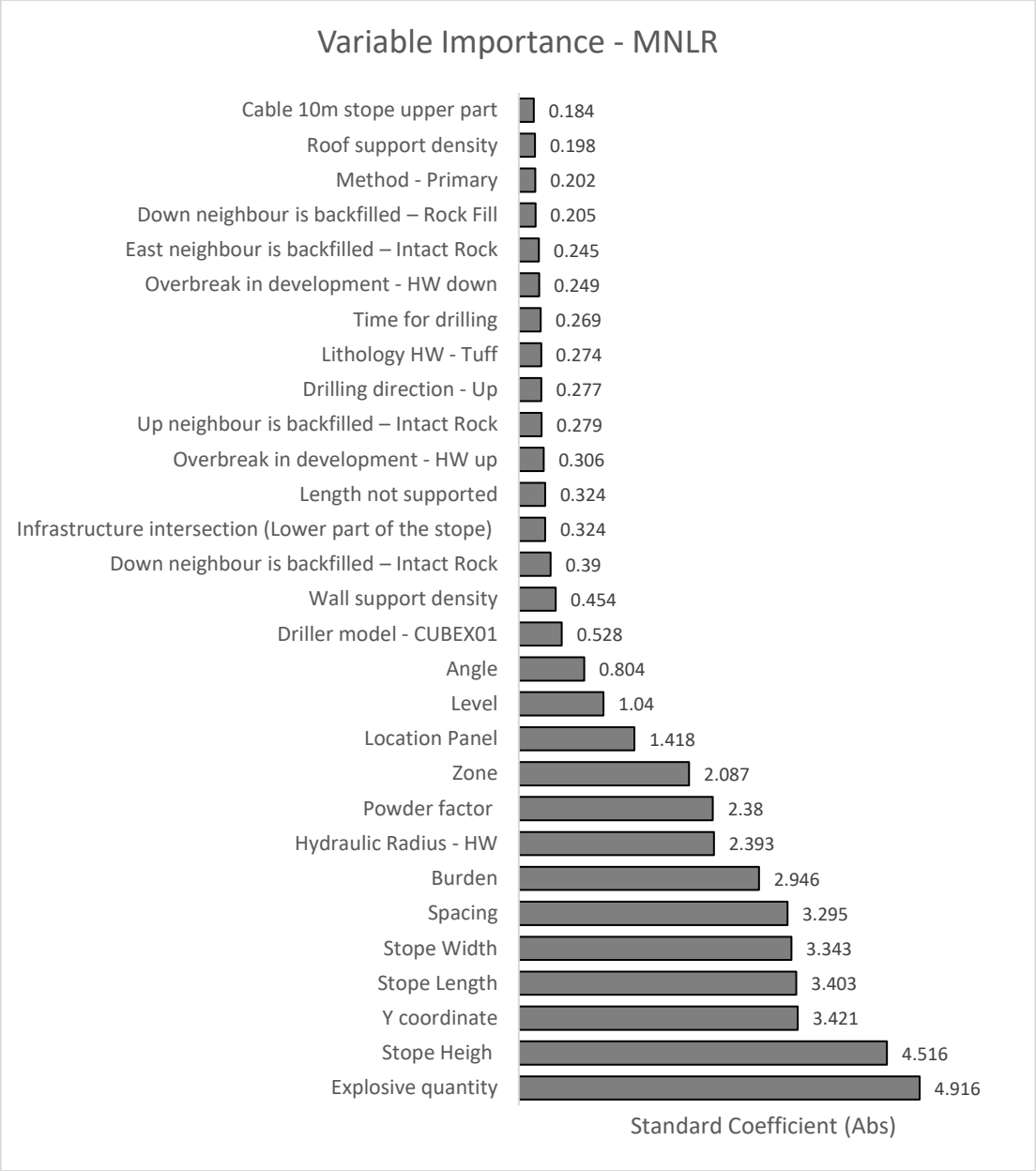

Figure 5-6. Variable Importance - MNLR

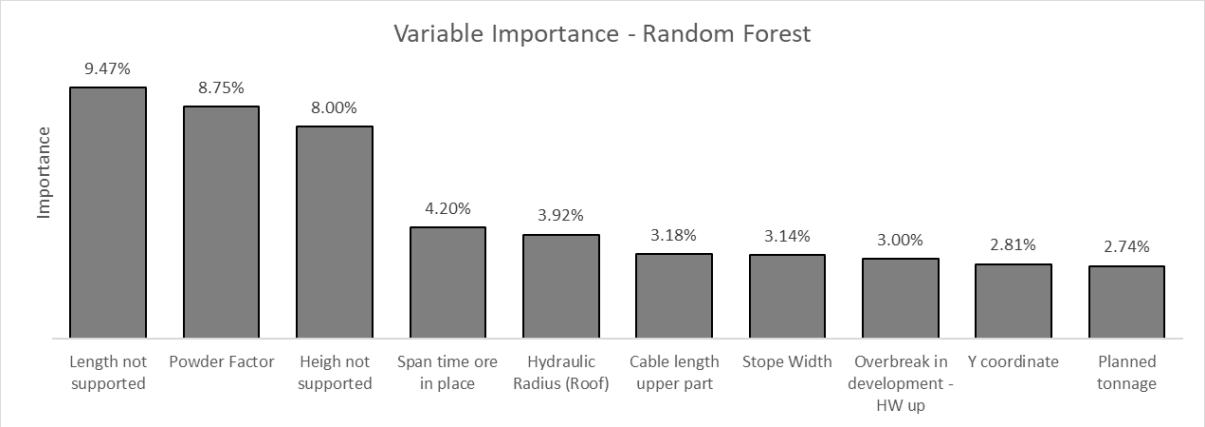

Figure 5-7. Variable Importance - Random Forest

6 Conclusions and Future Work

As technology, equipment, and mining methods have evolved over time, the problem of dilution has remained a persistent challenge in the industry. With the increasing complexity of deposits, it has become imperative to adopt better practices for mine design in order to address dilution effectively. Traditional methods for predicting dilution are limited in their scope and fail to consider a comprehensive range of variables. However, advancements in computer science techniques and the increasing computing power have now enabled the mining industry to employ a range of tools to determine trends and relationships between a greater number of variables and dilution. By harnessing the power of these techniques, mining companies can gain a deeper understanding of the causes of dilution, increase the accuracy of their predictions, and reduce dilution wherever possible. This enhanced understanding of dilution is key to achieving a more consistent and reliable economic assessment of mining projects.

This thesis presents a comprehensive overview of the development, improvement, and practical use of dilution-predictive models based on advanced statistics and machine learning

techniques. This research has identified that regression analysis, neural networks, and random forest techniques are particularly well-suited for creating accurate and reliable dilution-prediction models, given their capacity to effectively manage multiple variables. Some other machine learning methods applied to the dilution problem were evaluated, but some exhibited poor accuracy, while others were not suitable for the type of data we were working with. As a result, the methodology presented in this thesis focuses specifically on the most effective techniques, ensuring that our predictive models are robust, efficient, and highly accurate.

The predictive models developed in this study have demonstrated a similar or even higher level of accuracy than traditional methods. However, the key feature of these models is their ability to learn and improve over time as more data is inputted into the model. In particular, the Random Forest model achieved a remarkable correlation coefficient of r=0.925 on the test data set, with an average error of only 5% on the dilution value. This model outperformed other models, including the traditional model, due to its unique variable-classifying feature, which allows it to excel in small databases when compared to neural networks. Instead of applying weights or coefficients to each variable, the Random Forest model creates classifiers or "base learners" that split the data set into different subsets. Tuning the split ranges requires less data than tuning coefficients, making the model more efficient and accurate. Overall, the results of this study demonstrate the potential of machine learning techniques, particularly Random Forest models, to greatly enhance the accuracy and effectiveness of dilution-prediction models in the mining industry.

The analysis conducted in this study incorporated a wide range of variables in order to identify the most essential factors for predicting dilution. However, it is important to note that

the goal of this analysis is not to generalize the importance of specific variables, but rather to demonstrate that the most critical variables can vary significantly in each case, due to the inherent complexity of each mine. By considering a broad range of variables, we can identify the specific factors that are most relevant for each mining operation and optimize our predictive models accordingly. Ultimately, this approach enables us to better understand the unique factors that contribute to dilution in different mining scenarios and develop tailored solutions that are optimized for each specific context. By leveraging these insights, we can work to mitigate dilution more effectively, reducing costs and improving the overall economic viability of mining operations.

Although the models developed in this study do not track the importance of variables in the same way as neural network techniques, the Random Forest and Multiple Non-Linear Regression models are capable of providing a list of the most significant variables and their respective rankings in the predictive process. Interestingly, the lists of significant variables differ between the models, raising questions about which factors truly define dilution. Upon further analysis of the important variables highlighted by each method, it was found that blasting variables, such as powder factor, were consistently present in the group of variables with the highest impact on dilution, suggesting that blasting is the most critical process for unplanned dilution in both models. In the case of the Random Forest model, rock support emerged as a crucial factor, while its importance decayed in the Multiple Non-Linear Regression model. Similarly, stope dimensions were deemed essential in the Multiple Non-Linear Regression model, but their importance diminished in the Random Forest model. Nevertheless, it is worth noting that in both models, these variables are among the most significant factors, albeit with different

magnitudes. By understanding the specific variables that contribute most to dilution in each model, mining companies can develop more effective strategies to mitigate dilution and optimize their operations, ultimately leading to improved efficiency, profitability, and sustainability.

The "Y coordinate" variable is an essential factor that deserves particular attention, as it is considered significant in both the Random Forest and Multiple Non-Linear Regression models. The presence of the Y coordinate may indirectly suggest the existence of another variable that is not currently being considered but has an impact on dilution and changes along the north axis. One possible explanation for this variable is the discrimination of the two orebodies identified in the analysis, which are separated along the Y-axis. However, additional research is required to confirm this hypothesis. Understanding the importance of the Y coordinate variable could potentially provide valuable insights into the underlying causes of dilution and aid in the development of more accurate predictive models.

Future work on this topic could involve introducing more information to the data set to follow the improvement in the accuracy of the models. Regarding the contrast in the significant variables according to different methods, future studies should be considered to understand each variable's impact thoroughly. According to many authors, the quality of the rock mass should be a variable of vital importance for dilution. In this case study, no rock mass variables were included due to the lack of measurement of these parameters. Incorporating this kind of information could also result in a considerable improvement in the prediction of dilution. The "Y coordinate" variable in the significant variables can indirectly suggest a change in the rock mass quality in different zones.

7 References

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