

Analysis of Planar EBG Structures Using Transmission Line Models

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ABSTRACT

The transmission line based analytical solutions have simplified engineering of complex microwave circuits like electromagnetic bandgap structures (EBGs). In this thesis, planar EBG structures are studied by derivation of lumped element and transmission line equivalent circuits followed by utilizing analytical formulations. Based on this approach, a code is developed that predicts the dispersion characteristics of these periodic structures in a matter of few seconds. Planar EBG structures containing meander sections are investigated and a method for development of an equivalent circuit for the meander line portion is presented. The analysis of the studied EBG structures begins from a simple 1D geometry and is extended to more complex 2D geometries. The analytical simulation results are evaluated against full-wave simulations. Inclusion of the meander sections reduces the beginning of the bandgap to below 1GHz resulting in a more attractive structure for low frequency omni-directional filtering.

Abrégé

Les solutions analytiques basées sur des lignes de transmission ont simplifié l'ingénierie de circuits micro-ondes complexes, tel que les EBG. La présente thèse étudie les structures coplanaires EBG à partir d'éléments discrets et de modèles de lignes de transmission, auxquels sont ensuite appliquées des formules analytiques. Grâce à cette approche, un logiciel a été développé permettant de prédire les caractéristiques de dispersion de ces structures périodiques en quelques secondes seulement. Les structures coplanaires EBG contenant des sections courbes sont étudiées et un modèle de circuit équivalent à la portion courbe est proposé. L'analyse des structures EBG commence par une simple géométrie 1D, puis est étendue à des géométries 2D plus complexes. Le résultat des simulations analytiques est évalué par rapport au résultat des simulations analogues. Lorsque les sections courbes sont incluses, le début de la bande interdite est porté en deçà de 1GHz, rendant la structure plus intéressante pour le filtrage basse fréquence omni- directionnel.

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TABLE OF CONTENTS

CHAPTER 1	Introduction	1
1.1	Motivations and Objectives.....	2
1.2	Methodology	3
1.3	Thesis organization	4
CHAPTER 2	Review of Transmission Lines.....	6
2.1	Transmission Line Circuits	6
2.2	Analysis of Two Conductor Transmission Lines.....	8
2.3	Analysis of Multi-conductor Transmission Lines (MTLs)	12
2.4	Conclusion.....	19
CHAPTER 3	Electromagnetic Bandgap Structures	20
3.1	Overview of EBG Geometries	21
3.2	Electromagnetic Waves Supported by EBG structures.....	22
3.2.1	Review of Maxwell's Equations	22
3.2.2	Floquet's Theorem	25
3.2.3	Forward, Backward and Evanescent Waves in a Periodic Array.....	27
3.2.4	Spatial Harmonics in Periodic Arrays.....	32
3.2.5	Surface Waves.....	34
3.3	Effective Model.....	35
3.4	Application of EBG Structures in Suppression of Power/Ground Noise.....	36
3.4.1	Use of Discrete Decoupling Capacitors	37
3.4.2	Use of Embedded Capacitors	37
3.4.3	Use of Shorting Vias and Islands	38
3.5	Other Applications of EBG Structures.....	38
3.6	Conclusion.....	39
CHAPTER 4	Modeling using Transmission Line Circuit	40
4.1	Analysis of a Single Microstrip Line	40

4.2 Analysis of 1D and 2D EBG Structures Made with Two Conductor Transmission Line Circuits	45
4.2.1 Derivation of Dispersion Equation: 1D Stepped Impedance Structure.....	46
4.2.2 Derivation of Dispersion Equation: 2D Stepped Impedance Structure.....	49
4.3 Dispersion Diagrams	53
4.4 Planar EBG Structures Based on Stepped Impedance Design.....	56
4.4.1 Studied 1D Stepped Impedance Structure.....	56
4.4.2 Predicting the Insertion Loss Information from the Dispersion Equation	58
4.4.3 Studied 2D Stepped Impedance Structure.....	60
4.4.4 Insertion Loss Simulation and Measurement.....	63
4.5 Conclusion.....	66
CHAPTER 5 Design and Modeling of Meander lines	67
5.1 Inductance of the Meander Line.....	67
5.1.1 Calculation of Self Inductance	67
5.1.2 Calculation of Mutual Inductance	69
5.1.3 Capacitance of a Meander Line.....	77
5.2 Conclusion.....	78
CHAPTER 6 Planar EBG Structures Containing Meander Line Sections	79
6.1 Unit cells with Meander Transmission Line Sections.....	79
6.2 1D EBG Design A.....	83
6.3 1D EBG Design B.....	89
6.4 2D EBG Design A.....	93
6.5 2D EBG Design B.....	98
6.6 Conclusion.....	103
CHAPTER 7 Conclusions	105
7.1 Concluding Remarks	105
7.2 Recommendations for Future Work.....	106
References	108

LIST OF FIGURES

Figure 1.1: (Left) Textured and (Right) planar EBG structures sandwiched between power planes for the suppression of noise in PCB designs.....	2
Figure 1.2: Analysis procedure using transmission line modeling method	5
Figure 2.1: Planar microwave transmission lines.....	8
Figure 2.2: Parallel plate two conductor system	8
Figure 2.3: Section of a transmission line [13]	9
Figure 2.4: Lumped component model of the section of transmission line [13]	9
Figure 2.5 Definition of contour for the derivation of first set of MTL equation [27]	13
Figure 2.6: Definition of contour for the derivation of second set of MTL equation [27]	17
Figure 2.7: Lumped component model of a three conductor MTL segment [27].....	19
Figure 3.1: Top view of unit cells of various types of EBG structures.....	23
Figure 3.2: One dimensional representation of a periodic structure	26
Figure 3.3: An <i>infinite</i> x <i>infinite</i> dipole array with inter-element spacings of D_x and D_z , and element length of $2l$ [50].	28
Figure 3.4: Array mutual impedance $Z^{2,1}$ between an array with element orientation $\hat{p}^{(1)}$ and an external element with orientation $\hat{p}^{(2)}$ is obtained from the plane wave expression, (3.19) [50].	30
Figure 3.5: Wave amplitudes in a periodic structure [42].....	32
Figure 3.6: Generation of Switching noise and its suppression using EBG structures.....	36
Figure 4.1: Cross section of a microstrip line configuration.....	40
Figure 4.2: Important parameters and port definition for the analysis of a microstrip line.....	41
Figure 4.3: (a) Simulation setup for the analysis of a microstrip line, and (b) field distribution corresponding to excitation of a microstrip line	44
Figure 4.4: Variation of (a) effective dielectric constant, and (b) characteristic impedance with frequency. The WebCalculator results are from [77]	44
Figure 4.5: (a) Return loss, and (b) insertion loss of the studied microstrip line.....	45

Figure 4.6: One dimensional representation of a periodic structure	46
Figure 4.7: Unit Cell of a 1D stepped impedance filter	46
Figure 4.8: Cascading of ABCD matrices of sections	47
Figure 4.9: A unit cell of a 2D EBG structure and its equivalent transmission line network.....	50
Figure 4.10: Four port network representation of a unit cell of a two dimensional periodic structure.....	51
Figure 4.11: Irreducible Brillouin zone of an isotropic square reciprocal lattice with lattice constant d.....	54
Figure 4.12: (a) Unit cell of a one dimensional stepped impedance filter, (b)Design parameters, and (c) The equivalent transmission line model.	57
Figure 4.13: (a) Dispersion diagram from TLM based code, (b) Dispersion diagram from full wave solver (c) Insertion loss for a finite structure consisting of three unit cells	60
Figure 4.14: Cross section of the TL model of the 2D stepped impedance EBG structure.....	62
Figure 4.15: Dispersion diagram of the 2D stepped impedance structure based on Design A predicted by the TLM code.....	62
Figure 4.16: Dispersion diagram of the 2D stepped impedance structure based on Design A obtained from HFSS full-wave solver	63
Figure 4.17: Illustration of the EBG structure constituted of several cascaded unit cells along with the port definitions for the calculation of the Insertion Loss	64
Figure 4.18: Magnitude of S_{21} for finite size stepped impedance structure for the arrangement shown in Figure 4.17 and calculated between 28 segments along x-direction	65
Figure 5.1: Various segments of a meander line for the calculation inductance	68
Figure 5.2: A conducting segment of a meander line	68
Figure 5.3: Middle segments used for the calculation of inductances	68
Figure 5.4: Mutual inductance of two equal parallel straight filaments.....	69
Figure 5.5: Two intersecting unequal filaments inclined at an angle θ [97].....	69

Figure 5.6: Two non-intersecting unequal filaments inclined at an angle θ [97]	70
Figure 5.7: Two parallel segments displaced by a finite distance.....	71
Figure 5.8: Two unequal overlapping parallel segments with one end along the same perpendicular.....	71
Figure 5.9: Two non-overlapping parallel segments with one end along the same perpendicular.....	71
Figure 5.10: Two non-overlapping segments sharing the same axial plane and separated by a finite distance	72
Figure 5.11: Two scenarios for Case I.....	72
Figure 5.12: Two scenarios for Case II.....	73
Figure 5.13: Segment alignment for Case III.....	73
Figure 5.14: Segment alignment for Case IV.....	74
Figure 5.15: Segment alignment for Case V	74
Figure 5.16: Segment alignment for Case VI.....	75
Figure 5.17: Segment alignment for Case VII	76
Figure 6.1: The unit cell design of an EBG structure, Design A, containing meander sections.....	79
Figure 6.2: Unit cell design of the EBG structure from [38] , called here as Design B, containing meander sections.....	80
Figure 6.3: Capacitor geometry to calculate the fringe capacitance due to two co- planar metal patches [28]	81
Figure 6.4: Basic model represent the meander loading	82
Figure 6.5: (a) 1D meander loaded transmission line Design A. (b) The equivalent transmission line model for the unit cell used in the TLM code.....	84
Figure 6.6: Dispersion diagram of 1D EBG structure Design A predicted by the TLM code based.....	85
Figure 6.7: Dispersion diagram of 1D EBG structure Design A predicted from full wave analysis.....	895
Figure 6.8: Insertion loss plot for Design A.....	88
Figure 6.9: Magnified image of Figure 6.8	88

Figure 6.10: (a) 1D meander loaded transmission line Design B. (b) The equivalent transmission line model for the unit cell used in the TLM code.....	89
Figure 6.11: Dispersion diagram of 1D EBG structure Design B predicted by the TLM code based.....	90
Figure 6.12: Dispersion diagram of 1D EBG structure Design B predicted from full wave analysis.....	90
Figure 6.13: Insertion loss plot of the structure shown in Figure 6.10 (a).....	91
Figure 6.14: Zoomed in version of the graph shown in Figure 6.13.....	92
Figure 6.15: Simulation setup for the unit cell of 2D EBG structure of Design A.	93
Figure 6.16: Dispersion diagram of the 2D EBG Design A obtained from the TLM code.	94
Figure 6.17: Dispersion diagram of the 2D EBG Design A obtained from full-wave analysis.	94
Figure 6.18: Unit cell of the 2D meander loaded EBG structure in ADS schematic window.....	95
Figure 6.19: Illustration of the EBG structure constituted of several cascaded unit cells along with the port definitions for the calculation of the Insertion Loss	96
Figure 6.20: (a) Magnitude of S_{21} for a finite size 2D meander loaded EBG structure Design A. (b) Magnified scale of the shaded region shown in (a).....	97
Figure 6.21: Unit cell of the 2D meander loaded EBG structure in ADS schematic window.....	98
Figure 6.22: Dispersion diagram of the 2D EBG Design B obtained from the TLM code.	99
Figure 6.23: Dispersion diagram of the 2D EBG Design B obtained from full-wave analysis.....	99
Figure 6.24: Illustration of the EBG structure constituted of several cascaded unit cells along with the port definitions for the calculation of the Insertion Loss	100
Figure 6.25: (a) Magnitude of S_{21} for a finite size 2D meander loaded EBG structure Design B. (b) Magnified scale of the shaded region shown in (a).....	102

LIST OF TABLES

Table 3.1: The differential form of Maxwell's equations [42]	23
Table 4.1: Microstrip line parameters	42
Table 4.2: Comparison between the stopbands found from TLM code and full wave solver (HFSS) results for 1D stepped impedance based EBG design.....	60
Table 4.3: Comparison between the TLM code and full wave solver results	65
Table 5.1: Inductance values for meander line designs with different number of turns	77
Table 6.1: Geometrical parameters of meander lines of Design A and B and calculated inductance	80
Table 6.2: Parameter Values for Capacitance calculation.....	82
Table 6.3: Comparison between the TLM code and fullwave solver results for 1D EBG designs.....	92
Table 6.4: Comparison between the TLM code and fullwave solver results for 2D EBG Design A and Design B from dispersion simulations	102
Table 6.5: Comparison between the TLM code and full wave solver results for 1D EBG Designs	103

LIST OF ACRONYMS

FSS	Frequency Selective Surface
PBG	Photonic Band-Gap
EBG	Electromagnetic Band-Gap
PDN	Power Distribution Network
AMC	Artificial Magnetic Conductor
EM	Electro-Magnetic
TLM	Transmission Line Modeling
TL	Transmission Line
IC	Integrated Circuit
TEM	Transverse Electro-Magnetic
TE	Transverse Electric
TM	Transverse Magnetic
PCB	Printed Circuit Board
KCL	Kirchhoff's Current Law
KVL	Kirchhoff's Voltage Law
RL	Return Loss
RHS	Right Hand Side
LHS	Left Hand Side
MTL	Multi-conductor Transmission Line
1D	One-Dimensional
2D	Two-Dimensional
3D	Three-Dimensional
SSN	Simultaneous Switching Noise
HIS	High Impedance Surface
PPW	Parallel Plate Waveguide
MoM	Method of Moments
TDSE	Time Dependent Schroedinger Equation
FEM	Finite Element Method

CHAPTER 1

Introduction

Advances in fabrication technology have enabled realization of many complicated microwave circuits using low cost printed circuit board technologies. Electromagnetic bandgap structures (EBGs) [1]-[3], metamaterials [4], [5] and substrate integrated waveguides (SIWs) [6], [7] are examples of such advanced structures. EBG structures are used in designing low profile antennas, microwave filtering and noise suppression in power delivery networks [3], [8]-[10].

Power/ground noise is an increasing problem in modern systems due to the ever rising clock frequencies and decreasing rise and fall times associated with the newer generations of electronic devices. EBG structures have been employed as filters for suppression of noise in power distribution networks (PDNs). This is achieved by replacing one of the power/ground planes by an EBG surface. The stopband induced by the EBG layer suppresses the noise in all planar directions within the PDN. The EBG layer is either a Textured (via based) or a Uniplanar (without vias) geometry. Figure 1.1 shows two parallel plate PDNs containing these two types of EBG structures. This thesis focuses on the analysis of Uniplanar or planar EBG structures and in particular the structures that can be modeled using lumped component and two-conductor transmission line circuits. Determination of the stopbands and the degree of isolation obtained in the forbidden bands of EBG structures in filtering and noise suppression applications are the two most important measures of their efficiency. These characteristics can be ascertained from the dispersion diagram and scattering parameter (s-parameter) plots.

Over the past few years, different methodologies have been employed to generate the dispersion and insertion loss plots and predict the modal characteristics of these geometries. These methods include full wave numerical analysis and analytical solutions. This thesis focuses on the analysis of planar EBG structures using transmission line modeling and analytical solutions.

1.1 Motivations and Objectives

As stated earlier, in order to predict the frequency selective behaviour of the EBG structures, the dispersion characteristics and the s-parameters should be extracted. The dispersion characteristic is the plot of propagation constant of every mode versus frequency, commonly known as $k - \beta$ diagram in waveguides. To obtain such plots, eigenvalues of the electromagnetic problem should be found. The most commonly used procedure to obtain these attributes is by utilizing commercial electromagnetic (EM) field solvers. These tools are very accurate in predicting the EBG characteristics, but they often require a lot of time to converge on the eigenvalues especially when a large number of modes are investigated. Hence, any modification to the layout for the purpose of design optimization results in hours of simulation time.

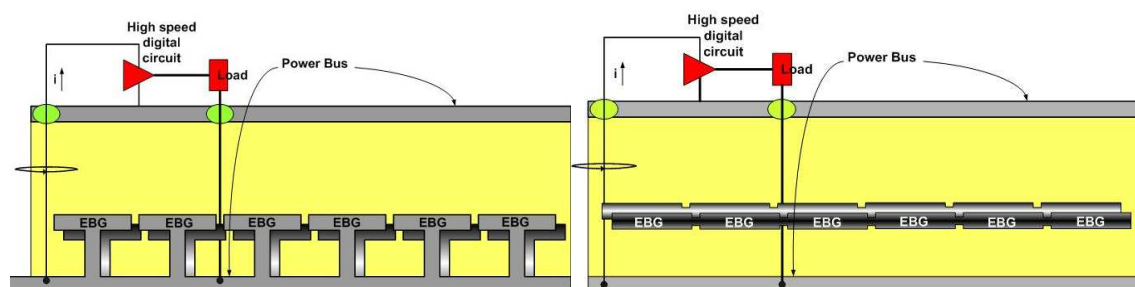


Figure 1.1: (Left) Textured and (Right) planar EBG structures sandwiched between power planes for the suppression of noise in PCB designs

Two most commonly used EM solvers are Ansoft's High Frequency Structure Simulator (HFSS) and Agilent's Advanced Design System (ADS). These tools are also used in the investigation of the studied structures. In this thesis, driven port simulations in HFSS and ADS Momentum are utilized for generating the s-parameters. Of the two solvers Eigenmode solution is only available in HFSS and is used for generating the dispersion diagrams. HFSS is a finite element method (FEM) based solver whereas ADS Momentum is a Method of Moments (MoM) solver. FEM solvers divide the domain into finite elements (triangles for 2D elements and tetrahedra for 3D elements). HFSS is an efficient tool for the extraction of modal information. It can be used for simulation of

antennas and includes definition of closed and open boundaries as well as perfectly matched layers (PML). ADS Momentum is a planar analysis tool which can be broadly used for the analysis of open and shielded 2.5D structures. The MoM also involves the subdivision of the domain into a mesh. Once it is achieved, the MoM code solves for the current on each small rectangular or triangular patch. Green's function is used to fill the moment matrix. Scattering parameters and radiation patterns are examples of output data generated by ADS Momentum analysis. Both HFSS and ADS give extremely accurate scattering parameter results but are expensive and time consuming. EBG structures have become very popular in microwave engineering and there is a need for an inexpensive way to analyze them. The goal of this thesis is to utilize an analytical method by which the frequency selective characteristics of EBG structures could be predicted with an acceptable degree of accuracy in a matter of few seconds, when compared to the simulation time required by EM solvers.

1.2 Methodology

The primary objective of this thesis is to develop an alternative approach to predict the dispersion and suppression characteristics of planar EBG structures in a cost effective way. This is achieved by employing the transmission line modeling (TLM) method to represent EBG structures. As stated earlier, Eigenmode analysis of EBG structures using commercial EM solvers is expensive and time consuming. Moreover, the size of the mesh associated with such structures is limited by the memory availability of the machine. Hence, the only way around such a problem is a reduction in the mesh size, which compromises the accuracy of the network and modal analysis. The method in this thesis mainly relies on the derivation of simplified models and obtaining Kirchhoff's Voltage Law (KVL) and Kirchhoff's Current Law (KCL) and multi-port network solutions. Since the recently developed EBG structures are based on printed circuit geometries, transmission line modeling (TLM) plays a significant role in their analytical investigation [5], [11], [12]. The equivalent circuit of an EBG structure can be composed of two or multi-conductor transmission line segments with some lumped loading components to represent the periodic discontinuity. Voltage and current variables, which could be

vectors in the case of multi-conductor structures, are used to derive the representative network parameters.

In order to find the eigenvalues of the system, Floquet's theorem is applied to voltage and current variables at the two ends of the unit cell of the EBG structure [13]. Solving the resultant matrix equation yields the dispersion (or characteristic) equation of the EM problem. The representative transfer matrix of the unit cell can be utilized to obtain insertion loss signature of the structure without including the effect of excitation ports and only by considering the number of cells between the two ports. A summary of the steps involved in each of the processes to obtain the desired output data is shown in Figure 1.2 . This diagram also serves as the basis for development of a TLM-code in MATLAB.

In this thesis, in order to demonstrate the capability of the TLM based analysis, few planar EBG geometries are analyzed. The results generated by the TLM-based code are compared with those obtained from full-wave analysis. Using this analytical methodology, the design of the EBG structures can be improved to achieve low frequency stopbands below 1GHz.

1.3 Thesis organization

After the brief introduction provided in this chapter, Chapter 2 presents a review of transmission line fundamentals, Chapter 3 surveys the history, fundamentals and applications of EBG structures. Chapter 4 discusses the methodology for analysis of EBG structures using transmission line modeling and provides examples of analysis of planar EBG structures. Chapter 5 describes an analytical technique to obtain the equivalent inductance of a meander line. Chapter 6 presents the analysis of a few planar EBG structures containing meander line sections. Dispersion diagrams are generated to study the modal characteristics of the geometries reviewed in Chapters 4 and 6. As well s-parameter plots are obtained to provide information about the isolation achieved in the investigated EBG geometries. In chapter 7, closing remarks and discussions about the scope of future work are presented.

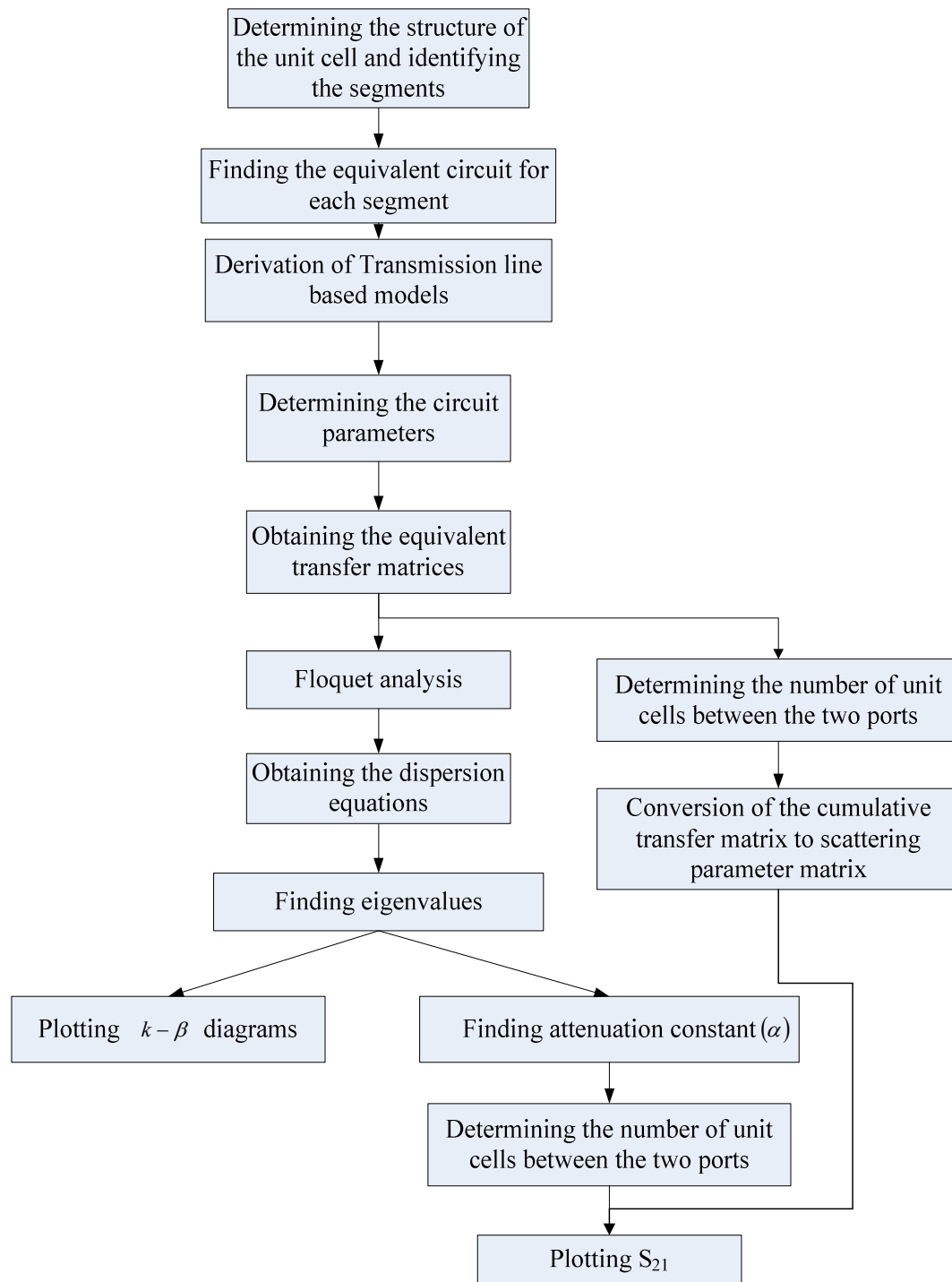


Figure 1.2: Analysis procedure using transmission line modeling method

CHAPTER 2

Review of Transmission Lines

This chapter reviews the analysis of various types of transmission lines that are commonly used in electronic systems and discusses the differences between them. The major part of the chapter deals with the derivation of the Telegraphers' equations for two conductor and multiconductor transmission line networks and describes the various circuit parameters involved in the analysis.

2.1 Transmission Line Circuits

When analyzing transmission line circuits, generally their lumped element model or distributed element model is referred. Maxwell's equations are applicable to all types of electromagnetic behaviour at all frequencies. When the electrical length of the circuit is smaller than the operating wavelength of the circuit, static field equations apply, and lumped components are used to represent the electrically small section of a transmission line. In such cases, the circuit components can be considered as frequency independent parameters. The lumped component model is also utilized when modeling transmission line discontinuities with parasitic components [14]. At higher frequencies, such as those in the microwave region (defined between 300 MHz to 30 GHz [13]), the electrical length of the circuit becomes relatively large due to which the time delay factor becomes prominent and wave equations should be used in the analysis of circuits. In this case, the circuit is known as a distributed circuit. In such cases, a distributed element model is preferred. The distributed model can be assumed to be composed of small lumped element sections. For circuits operating at microwave and millimetre wave frequencies and even in the analysis of micro-scale circuits such as ICs, distributed models are often used.

Transmission line circuits have been utilized as signal guiding structures for over 50 years [15]. Some of the most popular transmission lines that are used in printed circuits include striplines, microstrip lines, coplanar waveguides and twinstrips, slotlines and finlines [16]. Striplines most commonly employ homogenous dielectrics, thus supporting a dominant transverse electromagnetic (TEM) propagating mode. Their major

applications include the design of microwave filters [17], hybrids [18], directional couplers [19] and stepped impedance transformers [20]. Microstrip lines are preferred in low cost circuit design because of the convenience they provide in the mounting of the transistors or lumped components. The drawback associated with microstrip lines is the difficulty in accessing the ground plane that requires punching through the substrate. In addition to this, the substrate must be very thin in order to avoid higher order mode propagation at higher frequencies [16]. This would result in a very delicate framework. This was considered in the development of other types of transmission lines such as coplanar waveguides and twinstrip lines. These transmission lines were realized by moving the ground plane to the same side of the substrate as the signal land. The signal line could either be placed symmetrically between the two ground plane lands (coplanar waveguide) or adjacent to a single ground land (twinstrip). Slotlines have been used in conjunction with microstrip lines to design filters [21] and directional couplers [22]. Finlines (also referred to as E-planes [16], [23]) have been used to create filters [24], detectors [23] and mixers [25], [26]. Figure 2.1 shows the various commonly used microwave planar transmission lines.

Microstrip lines, owing to the non homogenous nature of the dielectric medium, support hybrid TE and TM modes. At lower frequencies, microstrips appear to have a guided wavelength and characteristic impedance very similar to that of a TEM line. As the frequency increases, this mode deviates from the TEM behaviour and hence such a mode is conceptually referred to as a quasi-TEM mode.

Another type of transmission line is a parallel plate two conductor system which, as the name suggests, consists of parallel conductor plates as shown in Figure 2.2. The power distribution network (PDN) in various printed circuit systems like PCBs, ICs, is used to provide biasing current to the circuit components constituting a system. In addition to this the power/ground planes also act as efficient shields.

In addition to routing signals between two or more locations, transmission lines are the basic structures used in creating filters, couplers, resonators and circuit components. Hence, analysis of a two conductor transmission line provides invaluable insight into the

propagation characteristics of electromagnetic waves in such structures. Derivations of voltage and current wave equations are discussed in the following section.

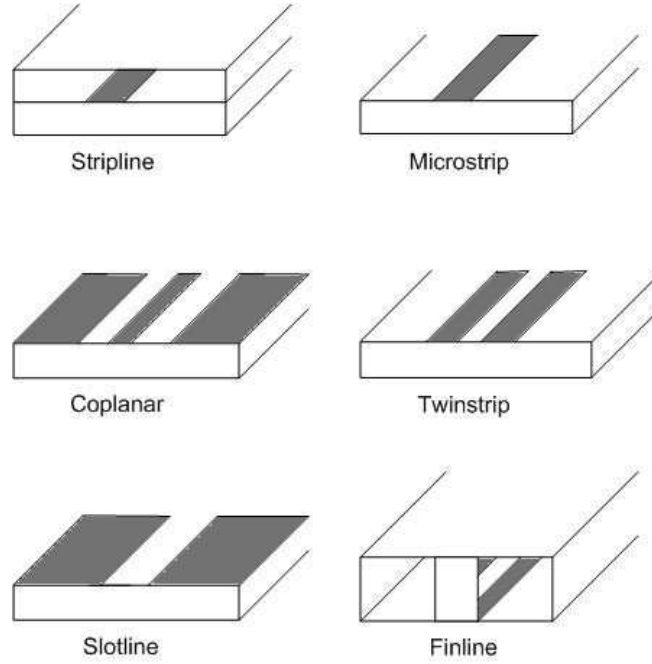


Figure 2.1: Planar microwave transmission lines

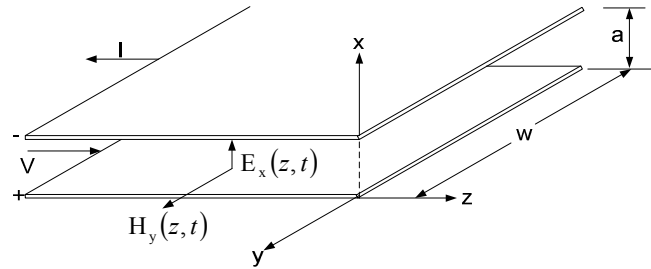


Figure 2.2: Parallel plate two conductor system

2.2 Analysis of Two Conductor Transmission Lines

As stated earlier, transmission line analysis is employed when the length of the transmission line is a comparable fraction of the signal wavelength. Circuit theory deals with only lumped component parameters whereas transmission line theory deals with

distributed component parameters. If we consider a small segment of a transmission line of length Δz shown in Figure 2.3, then the lumped element circuit model of that section of transmission line can be represented as shown in Figure 2.4 [13]. The lumped component model consists of series resistance, series inductance, shunt capacitance and shunt conductance, all of which have been normalized per unit length of the transmission line.

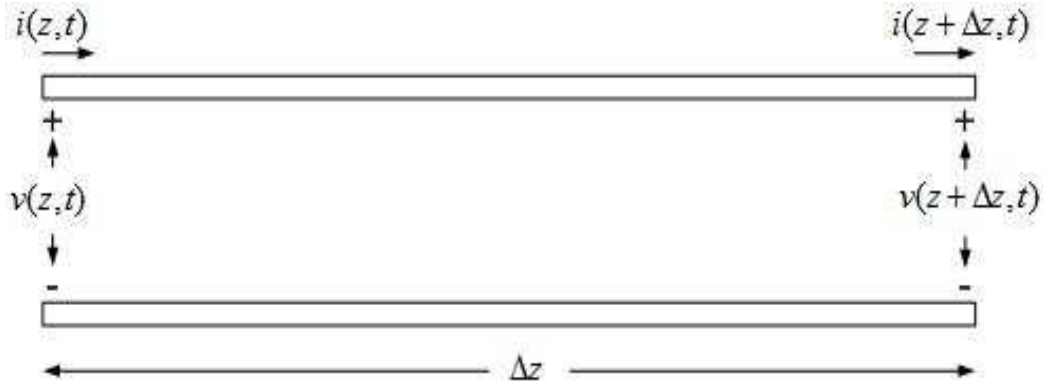


Figure 2.3: Section of a transmission line [13]

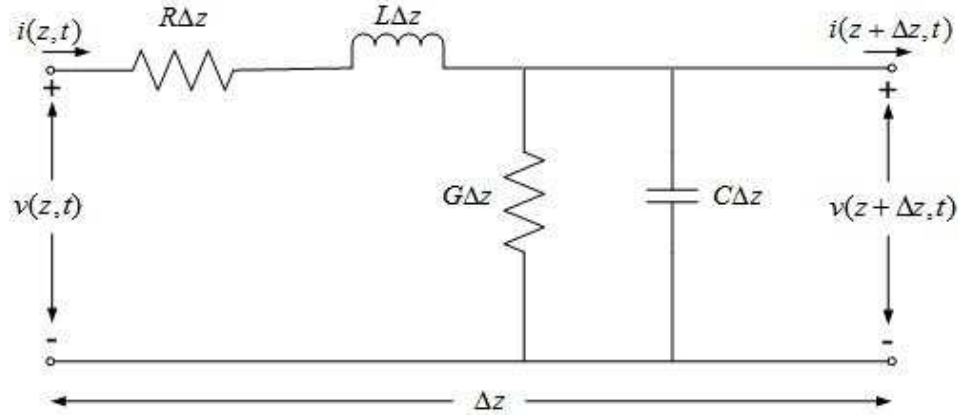


Figure 2.4: Lumped component model of the section of transmission line [13]

where R = Per unit length series resistance in Ω/m

L = Per unit length series inductance in H/m

G = Per unit length shunt conductance in S/m

C = Per unit length shunt capacitance in F/m

Applying Kirchhoff's Voltage Law to the circuit shown in Figure 2.4, we obtain,

$$v(z, t) - R\Delta z i(z, t) - L\Delta z \frac{\partial i(z, t)}{\partial t} - v(z + \Delta z, t) = 0 \quad (2.1)$$

Similarly, applying Kirchhoff's Current Law to the circuit shown in Figure 2.4, we obtain,

$$i(z, t) - G\Delta z v(z + \Delta z, t) - C\Delta z \frac{\partial v(z + \Delta z, t)}{\partial t} - i(z + \Delta z, t) = 0 \quad (2.2)$$

Dividing (2.1) and (2.2) by Δz and applying $\lim_{\Delta z \rightarrow 0}$, we obtain the following set of differential equations,

$$\frac{\partial v(z, t)}{\partial z} = -Ri(z, t) - L \frac{\partial i(z, t)}{\partial z} \quad (2.3)$$

$$\frac{\partial i(z, t)}{\partial z} = -Gv(z, t) - C \frac{\partial v(z, t)}{\partial z} \quad (2.4)$$

(2.3) and (2.4) are known as *Telegraphers' equations*.

The steady state equations for (2.3) and (2.4) are:

$$\frac{\partial V(z)}{\partial z} = -(R + j\omega L)I(z) \quad (2.5)$$

$$\frac{\partial I(z)}{\partial z} = -(G + j\omega C)V(z) \quad (2.6)$$

Partially differentiating (2.5) and (2.6) with respect to z

$$\frac{\partial^2 V(z)}{\partial z^2} = -(R + j\omega L) \frac{\partial I(z)}{\partial z} \quad (2.7)$$

$$\frac{\partial^2 I(z)}{\partial z^2} = -(G + j\omega C) \frac{\partial V(z)}{\partial z} \quad (2.8)$$

Substituting (2.5) and (2.6) in (2.8) and (2.7) respectively

$$\frac{\partial^2 V(z)}{\partial z^2} = (R + j\omega L)(G + j\omega C)V(z) \quad (2.9)$$

$$\frac{\partial^2 I(z)}{\partial z^2} = (G + j\omega C)(R + j\omega L)I(z) \quad (2.10)$$

Rearranging (2.9) and (2.10), we obtain the voltage and current wave equations as follows:

$$\frac{\partial^2 V(z)}{\partial z^2} - \gamma^2 V(z) = 0 \quad (2.11)$$

$$\frac{\partial^2 I(z)}{\partial z^2} - \gamma^2 I(z) = 0 \quad (2.12)$$

where
$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta \quad (2.13)$$

is the complex propagation constant of the transmission line and is a frequency dependent parameter. The propagation constant is a measure of the change in the amplitude and phase of a wave between the points of origin or reference, and a point at a distance z as it travels along the transmission line.

The propagation constant consists of two discernible parts: (a) A real part ‘ α ’ known as the attenuation constant which is a measure of the attenuation that a wave suffers per metre travel along the transmission line, and (b) an imaginary part ‘ β ’ known as the phase constant. β , along with the characteristic impedance Z_0 (refer to Equation (2.16)), is one of the two most important descriptive parameters of a transmission line and is a measure of the change in phase that a wave suffers per metre travel along the transmission line.

The solution to the set of differential equations (2.11) and (2.12) is given by

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z} \quad (2.14)$$

$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z} \quad (2.15)$$

where V_0^+ and I_0^+ are the amplitudes of the voltage and current of the incident wave, and similarly, V_0^- and I_0^- are amplitudes of the voltage and current of the reflected wave.

The characteristic impedance of a transmission line is defined by

$$Z_0 = \frac{V_0^+}{I_0^+} = \frac{V_0^-}{I_0^-} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \quad (2.16)$$

Whenever a transmission line is terminated in an arbitrary load, $Z_L \neq Z_0$, an important parameter, known as the reflection coefficient, comes into existence. The reflection coefficient is given by

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \quad (2.17)$$

The return loss (RL) of a transmission line is a measure of the amount of power delivered from the source to the load and is provided by the equation

$$\text{RL} = -20 \log|\Gamma| \text{ dB} \quad (2.18)$$

A matched load (which implies no part of the incident power is reflected) has RL of ∞ dB whereas total reflection (which implies all the incident power is reflected) has a RL of 0 dB.

2.3 Analysis of Multi-conductor Transmission Lines (MTLs)

Having investigated a two conductor transmission line in the previous section, we now investigate a multiconductor transmission line segment [27] as shown in Figure 2.5. The system is composed of 'n' conductors and the conductor numbered '0', which provides the return path for all the other conductors in the system, is defined as the reference conductor.

For the Voltage equation:

According to Faraday's law

$$\oint_c \vec{E} \cdot d\vec{l} = -\mu \frac{d}{dt} \int_s \vec{H} \cdot d\vec{s} \quad (2.19)$$

When we apply Faraday's law along the contours ab, bc, cd , and da which enclose the surface s_i and move in a clockwise direction, we obtain

$$\int_a^b \vec{E}_t \cdot d\vec{l} + \int_b^c \vec{E}_l \cdot d\vec{l} + \int_c^d \vec{E}_t \cdot d\vec{l} + \int_d^a \vec{E}_l \cdot d\vec{l} = -\mu \frac{d}{dt} \int_{s_i} \vec{H}_t \cdot (-\hat{a}_n) d\vec{s} \quad (2.20)$$

where $\vec{E}_t$ and $\vec{H}_t$ represent the transverse electric and magnetic fields (with the subscript 't' representing the transverse direction) and $\vec{E}_l$ represents the longitudinal electric field (with the subscript 'l' representing the longitudinal direction). $\hat{a}_n$ is the surface unit

vector $\hat{a}_n$ directed perpendicular to the surface s_i and is directed out of the plane. Since $\hat{a}_n$ and the magnetic field associated with the surface s_i are in opposite directions, hence the right hand side (RHS) term will become positive and hence (2.20) evolves to the following,

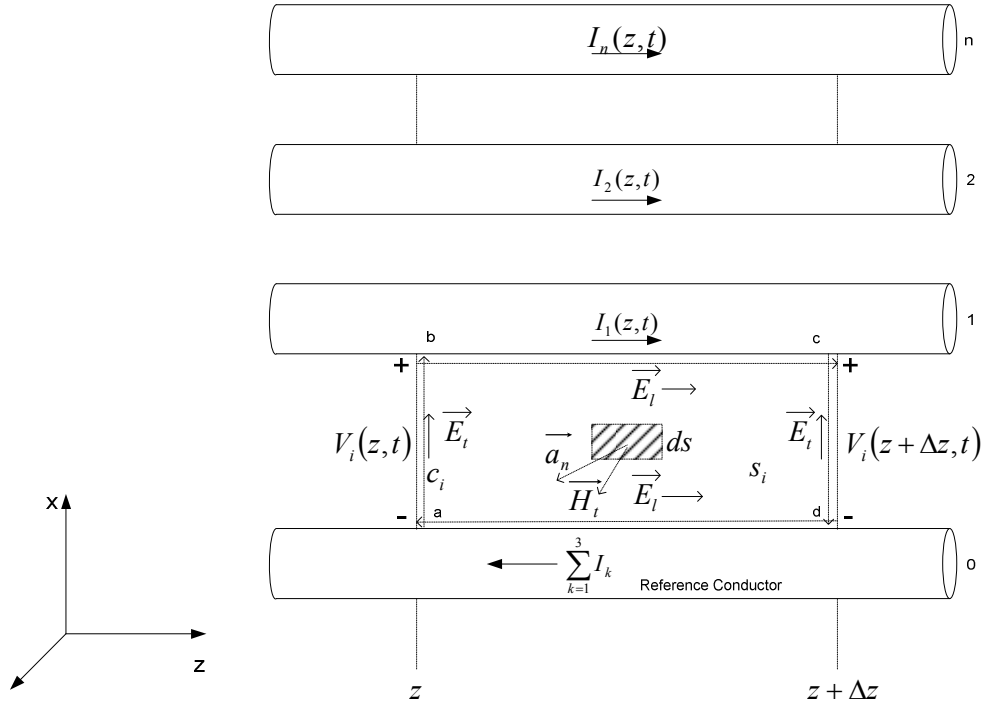


Figure 2.5 Definition of contour for the derivation of first set of MTL equation [27]

$$\int_a^b \vec{E}_t \cdot d\vec{l} + \int_b^c \vec{E}_t \cdot d\vec{l} + \int_c^d \vec{E}_t \cdot d\vec{l} + \int_d^a \vec{E}_t \cdot d\vec{l} = \mu \frac{d}{dt} \int_{s_i} \vec{H}_t \cdot (\hat{a}_n) d\vec{s} \quad (2.21)$$

We consider a TEM mode of propagation and in accordance with such a mode, the voltage relationships between the i^{th} conductor and the reference conductor at the beginning and at the end of the section are given by

$$V_i(z, t) = - \int_a^b \vec{E}_t(x, y, z, t) \cdot d\vec{l} \quad (2.22)$$

$$V_i(z + \Delta z, t) = - \int_c^d \vec{E}_t(x, y, z + \Delta z, t) \cdot d\vec{l} \quad (2.23)$$

Since a TEM mode of propagation is possible only in perfect conductors, in order to accommodate wave propagation in imperfect conductors in a *quasi*-TEM mode of propagation, we include a per-unit-length conductor resistance, $r \Omega/m$.

For the purpose of simplification of our analysis, let us consider an MTL segment consisting of three conductors and one reference conductor [27].

Voltage drops along the sections (assuming imperfect conductor):

$$\text{Along } bc = - \int_b^c \vec{E}_l \cdot d\vec{l} \quad (2.24)$$

$$\text{Along } da = - \int_d^a \vec{E}_l \cdot d\vec{l} \quad (2.25)$$

Or we can say that:

$$- \int_b^c \vec{E}_l \cdot d\vec{l} = -r_i \Delta z I_i(z, t) \quad (2.26)$$

$$- \int_d^a \vec{E}_l \cdot d\vec{l} = -r_0 \Delta z I_0(z, t) \quad (2.27)$$

where r_i & r_0 are the per unit conductor resistances of the i^{th} and reference conductors,

and $I_i(z, t)$ & $I_0(z, t) = \sum_{k=1}^3 I_k(z, t)$ (k varies from 1 to 3 because we are analysing a

specific case of three conductors for an n -conductor multiconductor transmission line system) are the currents on the i^{th} and reference conductors.

According to Ampere's law:

$$I_i(z, t) = \oint_{c_i} \vec{H}_t \cdot d\vec{l} \quad (2.28)$$

where c_i is a contour just off the surface of the i^{th} conductor and enclosing the i^{th} conductor.

Substituting (2.22) to (2.28) in (2.20) we get:

$$-V_i(z, t) + r_i \Delta z I_i(z, t) + r_0 \Delta z \sum_{k=1}^3 I_k(z, t) + V_i(z + \Delta z, t) = \mu \frac{d}{dt} \int_{s_i} \vec{H}_t \cdot (-\hat{a}_n) d\vec{s} \quad (2.29)$$

Rearranging the above equation

$$V_i(z + \Delta z, t) - V_i(z, t) = - \left(r_i \Delta z I_i(z, t) + r_0 \Delta z \sum_{k=1}^3 I_k(z, t) \right) + \mu \frac{d}{dt} \int_{s_i} \vec{H}_t \cdot (\hat{a}_n) d\vec{s} \quad (2.30)$$

Dividing the LHS and RHS by Δz

$$\frac{V_i(z + \Delta z, t) - V_i(z, t)}{\Delta z} = - \left(r_i I_i(z, t) + r_0 \sum_{k=1}^3 I_k(z, t) \right) + \mu \frac{1}{\Delta z} \frac{d}{dt} \int_{s_i} \vec{H}_t \cdot (\hat{a}_n) d\vec{s} \quad (2.31)$$

Hence,

$$\begin{aligned} \frac{V_i(z + \Delta z, t) - V_i(z, t)}{\Delta z} &= -r_i I_i(z, t) - r_0 [I_1(z, t) + I_2(z, t) + I_3(z, t)] \\ &\quad + \mu \frac{1}{\Delta z} \frac{d}{dt} \int_{s_i} \vec{H}_t \cdot (\hat{a}_n) d\vec{s} \end{aligned} \quad (2.32)$$

The total magnetic flux penetrating the surface is given by

$$\psi_i = -\mu \lim_{\Delta z \rightarrow 0} \frac{1}{\Delta z} \int_{s_i} \vec{H}_t \cdot \hat{a}_n d\vec{s} = \sum_{j=1}^{n=3} l_{ij} I_j \quad (2.33)$$

where l_{ij} refers to the per-unit-inductance in H/m

Applying $\lim_{\Delta z \rightarrow 0}$ in (2.32) and substituting (2.33) in it, we obtain

$$\begin{aligned} \frac{\partial V_i(z, t)}{\partial z} &= -r_0 I_1(z, t) \dots - (r_0 + r_i) I_i(z, t) \dots r_0 I_3(z, t) \\ &\quad - l_{i1} \frac{\partial I_1(z, t)}{\partial t} - l_{i2} \frac{\partial I_2(z, t)}{\partial t} - l_{i3} \frac{\partial I_3(z, t)}{\partial t} \end{aligned} \quad (2.34)$$

Using matrix notation, (2.34) can be rewritten as [27]

$$\frac{\partial V(z, t)}{\partial z} = -RI(z, t) - L \frac{\partial I(z, t)}{\partial t} \quad (2.35)$$

where,

$$V(z, t) = \begin{bmatrix} V_1(z, t) \\ V_2(z, t) \\ V_3(z, t) \end{bmatrix} \quad (2.36)$$

$$I(z, t) = \begin{bmatrix} I_1(z, t) \\ I_2(z, t) \\ I_3(z, t) \end{bmatrix} \quad (2.37)$$

The per-unit-inductance matrix is given by

$$L = \begin{bmatrix} l_{11} & l_{12} & l_{13} \\ l_{21} & l_{22} & l_{23} \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \quad (2.38)$$

and the per-unit-resistance matrix is denoted by

$$R = \begin{bmatrix} r_0 + r_1 & r_0 & r_0 \\ r_0 & r_0 + r_2 & r_0 \\ r_0 & r_0 & r_0 + r_3 \end{bmatrix} \quad (2.39)$$

We can clearly observe that (2.35) is identical to the two conductor transmission line equation (2.5) derived in the previous section.

For the current equation:

Referring to Figure 2.6, let us consider a surface $\hat{s}$ enclosing the i^{th} conductor as shown. It consists of two parts; the end areas represented by $\hat{s}_e$ and the enclosing section represented by $\hat{s}_o$. The continuity equation for the conservation of charge states

$$\oiint_{\hat{s}} \vec{J} \cdot d\vec{s} = -\frac{\partial}{\partial t} Q_{enc} \quad (2.40)$$

where Q_{enc} is the charge enclosed by the surface.

Applying the continuity equation to over the end caps we have,

$$\iint_{\hat{s}_e} \vec{J} \cdot d\vec{s} = I_i(z + \Delta z, t) - I_i(z, t) \quad (2.41)$$

Similarly, application of the continuity equation over the side surface yields,

$$\iint_{\hat{s}_o} \vec{J}_c \cdot d\vec{s} = \sigma \iint_{\hat{s}_o} \vec{E}_t \cdot d\vec{s} \quad (2.42)$$

where $\vec{J}_c = \sigma \vec{E}_t$ is the conduction current and (2.42) represents the transverse current flowing between the conductors.

Again, for the purpose of simplification of our analysis, let us consider an MTL segment consisting of three conductors and one reference conductor [27].

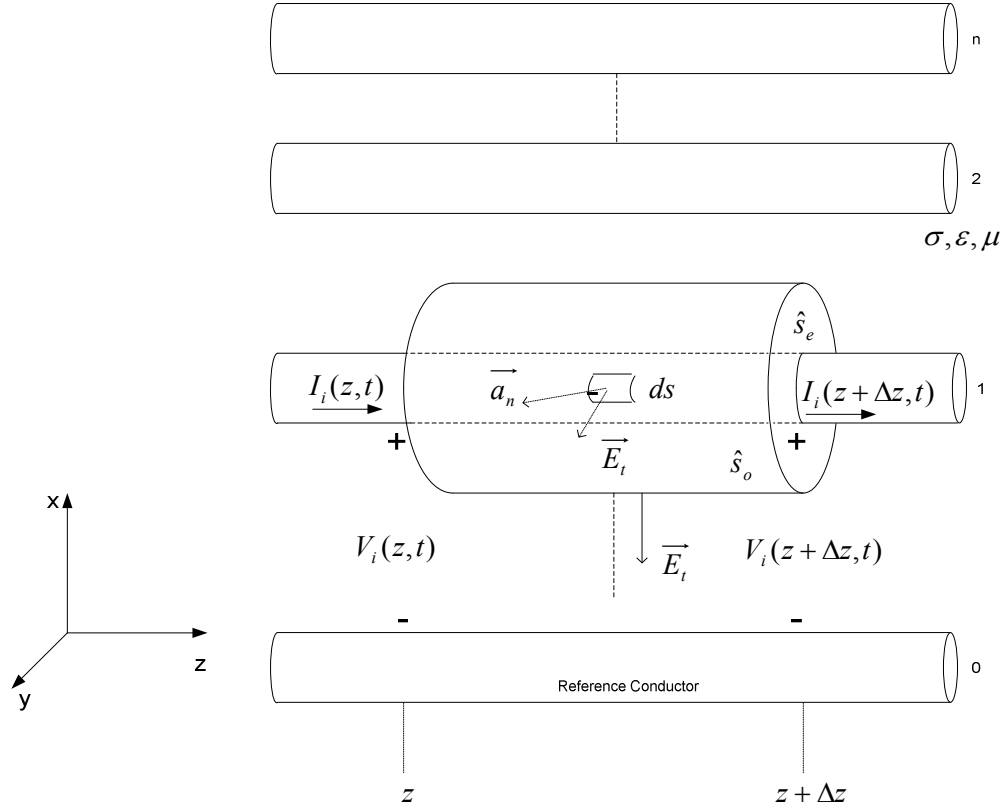


Figure 2.6: Definition of contour for the derivation of second set of MTL equation [27]

We know that,

$$\begin{aligned}
 \sigma \lim_{\Delta z \rightarrow 0} \frac{1}{\Delta z} \iint_{\hat{s}_0} \vec{E}_t \cdot d\vec{s} &= g_{i1}(V_i - V_1) + g_{i2}(V_i - V_2) + g_{i3}(V_i - V_3) \\
 &= -g_{i1}V_1(z, t) - g_{i2}V_2(z, t) - g_{i3}V_3(z, t) + \sum_{k=1}^3 g_{ik}V_k(z, t)
 \end{aligned} \quad (2.43)$$

where g_{ij} is the per-unit-conductance in S/m

By Gauss's law, the total charge enclosed by the surface encircling the conductor is given by,

$$Q_{enc} = \epsilon \iint_{\hat{s}_0} \vec{E}_t \cdot d\vec{s} \quad (2.44)$$

The charge per unit length of line is obtained by [27],

$$\begin{aligned} \varepsilon \lim_{\Delta z \rightarrow 0} \frac{1}{\Delta z} \iint_{\hat{s}_0} \vec{E}_t \cdot d\vec{s} &= c_{i1}(V_i - V_1) + c_{i2}(V_i - V_2) + c_{i3}(V_i - V_3) + c_{ii}V_i \\ &= -c_{i1}V_1(z, t) - c_{i2}V_2(z, t) - c_{i3}V_3(z, t) + \sum_{k=1}^3 c_{ik}V_k(z, t) \end{aligned} \quad (2.45)$$

where c_{ij} is the per-unit-capacitance in F/m

Substituting (2.41), (2.42) and (2.44) in equation (2.40), we get,

$$I_i(z + \Delta z, t) - I_i(z, t) + \sigma \iint_{\hat{s}_0} \vec{E}_t \cdot d\vec{s} = -\varepsilon \iint_{\hat{s}_0} \vec{E}_t \cdot d\vec{s} \quad (2.46)$$

Dividing both sides by Δz we get,

$$\frac{I_i(z + \Delta z, t) - I_i(z, t)}{\Delta z} + \frac{\sigma}{\Delta z} \iint_{\hat{s}_0} \vec{E}_t \cdot d\vec{s} = -\frac{\varepsilon}{\Delta z} \iint_{\hat{s}_0} \vec{E}_t \cdot d\vec{s} \quad (2.47)$$

Applying $\lim_{\Delta z \rightarrow 0}$ to (2.47) and substituting (2.43) and (2.45) yields,

$$\begin{aligned} \frac{\partial I_i(z, t)}{\partial z} &= g_{i1}V_1(z, t) + g_{i2}V_2(z, t) + g_{i3}V_3(z, t) - \sum_{k=1}^3 g_{ik}V_k(z, t) \\ &\quad + \frac{\partial}{\partial t} \left[c_{i1}V_1(z, t) + c_{i2}V_2(z, t) + c_{i3}V_3(z, t) - \sum_{k=1}^3 c_{ik}V_k(z, t) \right] \end{aligned} \quad (2.48)$$

$$\approx \frac{\partial I_i(z, t)}{\partial z} = GV(z, t) - C \frac{\partial}{\partial t} V(z, t) \quad (2.49)$$

where the per-unit-conductance matrix is represented as

$$G = \begin{bmatrix} g_{11} + g_{12} + g_{13} & -g_{12} & -g_{13} \\ -g_{21} & g_{21} + g_{22} + g_{23} & -g_{23} \\ -g_{31} & -g_{32} & g_{31} + g_{32} + g_{33} \end{bmatrix} \quad (2.50)$$

and the per-unit-capacitance matrix is represented as

$$C = \begin{bmatrix} c_{11} + c_{12} + c_{13} & -c_{12} & -c_{13} \\ -c_{21} & c_{21} + c_{22} + c_{23} & -c_{23} \\ -c_{31} & -c_{32} & c_{31} + c_{32} + c_{33} \end{bmatrix} \quad (2.51)$$

It can be observed that (2.49) resembles (2.6) derived in the previous section pertaining to two conductor transmission lines. Figure 2.7 shows a lumped component model of a four conductor MTL segment, one of which is a reference conductor.

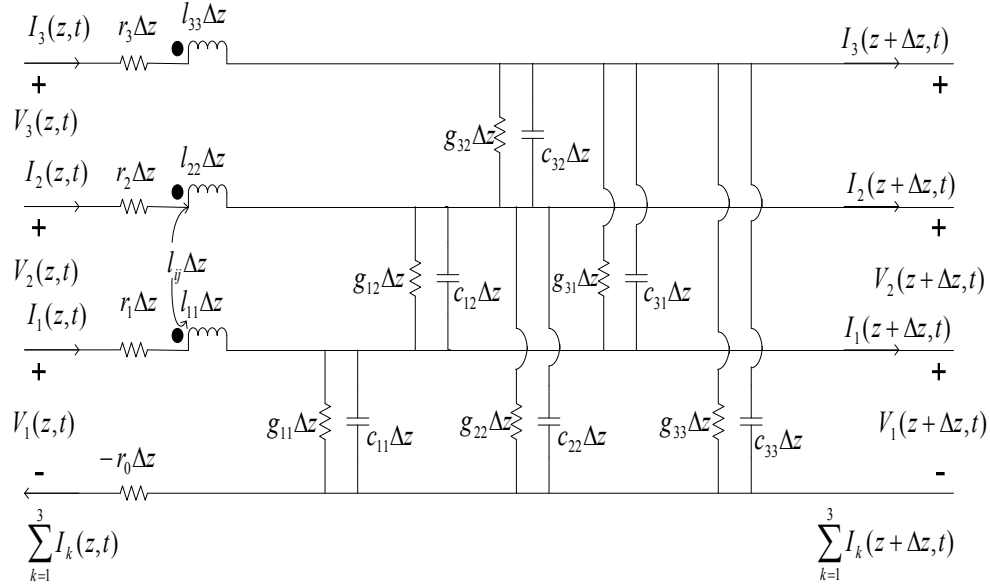


Figure 2.7: Lumped component model of a three conductor MTL segment [27]

2.4 Conclusion

This chapter presented a brief discussion about the types and applications of transmission lines. The fundamentals of transmission line networks discussed in this chapter are essential in analytical studies of a transmission line circuit. In the subsequent chapters, we will discuss equivalent models of various transmission line based geometries and utilize analytical derivations to find the network parameters of the studied structures. The important parameter that we intend to find in the analysis is the propagation constant, β , of the transmission line-based circuit.

CHAPTER 3

Electromagnetic Bandgap

Structures

Electromagnetic bandgap (EBG) structures are periodic geometries constructed by repetition of a unit cell or building block in one, two or three dimensions. Mostly they are used to ideally eliminate propagation of electromagnetic waves within certain frequency bands. Practically though, only partial elimination is possible. One dimensional (1D) periodic structures, i.e. structures in which repetition of the unit cell occurs in one dimension, are often utilized as microwave filters. Their properties have been studied in [13], [25]-[28]. Two dimensional (2D) periodic structures, i.e. repetition exists along two directions, are the most popular type of EBG structures. They have been utilized in the design of antennas, microwave filters and even for suppression of power/ground or simultaneous switching noise (SSN) in electronic circuits [29] and [30]. They have been dealt with in numerous publications due to the ease of fabrication especially when compared with 3D geometries. Three dimensional (3D) EBG structures, i.e. unit cell repeats along all three spatial dimensions, have been studied in [31]. Note that the 1D and 2D EBG structures may be 3D geometries as they are often developed on a dielectric substrate and may include a conductor plane for grounding or shielding. The most commonly used 2D EBG structures in microwave circuits contain conductor parts like an array of metal protrusions or metal patches arranged in one or two dimensions.

Terminologies like High impedance surfaces (HIS) [28] or Frequency Selective Surfaces (FSSs) are also used to refer to 2D EBG structures especially when they are used as reflector planes and for the purpose of spatial wave filtering or eliminating surface waves and currents.

This chapter provides a brief introduction about common 2D EBG structures, explains their modal characteristics and elaborates on some important applications.

3.1 Overview of EBG Geometries

EBG structures used in microwave and electronic engineering are mainly 2D periodic structures. These 2D geometries can be broadly classified into two categories: Textured (Mushroom or thumbtack) type and Uniplanar (or patterned) structures. Textured surfaces consist of an array of bumps and waves travelling over such an undulating surface, interact with these small bumps. The bumps basically behave as speed breakers and hence there is a change in the velocity of the bound waves when they travel over these bumps [28]. Due to the interference and change in velocity brought about by this association, the textured surfaces prevent the propagation of surface bound waves and this results in the production of a two directional bandgap [28]. Initial studies of this type of structure involved obtaining a high impedance surface by using small bumps patterned on a silver film [32], [33]. These bumpy structures were later developed into the mushroom structures, also known as Sievenpiper's structures [34]. These structures are inductive at low frequencies and hence support TM waves and are capacitive at high frequencies which enables them to support TE waves [28]. Within the forbidden gap though, these structures suppress both TE and TM waves from propagating through the structure.

Uniplanar or patterned EBG structures can be realized as a grid/meshed plane [35], a 2D stepped impedance structure [36], interconnected slotted patches [37], metal patches connected by meander lines [3], [38] or other parasitic sections. Some of the geometries used for developing planar patterned and mushroom EBG structures have been shown in Figure 3.1. Figure 3.1 (a) is the mushroom structure developed by Sievenpiper [28]. Figure 3.1 (c) is another uniplanar EBG used for suppression of ground/power noise [39]. Figure 3.1 (d) is also another one of Sievenpiper's EBG structures [26], [27], while Figure 3.1 (e) shows the structure without the via. Note that in Figures 3.1(a) and 3.1(e), a double sided substrate is used and the via stitches the patches on one side to the solid conductor on the other side. Figure 3.1 (f) is an L-bridge loaded transmission line structure [40]. Figure 3.1(g) is the stepped impedance structure and Figure 3.1 (h) is a meander loaded transmission line structure [41]. A modified geometry used to obtain

ultra wide bandgap, known as a super cell [3], is shown in Figure 3.1 (i). The super cell was designed by combining two different geometries into a super cell structure. The bandgap obtained due to the new structure is found to amalgamate the bandgaps arising due to individual geometries, in addition to the bandgap arising from the combined periodicity formed by the super cell topology. All of these geometries have shown bandgap properties and provide various degrees of isolation characteristics.

In this thesis, only patterned EBG geometries have been analyzed. The stepped impedance structure of Figure 3.1(g) and the structure of metal patches connected by meander lines shown in Figure 3.1(h) serve as the basis for the EBG geometries studied in this thesis.

The geometry of these EBG structures can be altered in order to obtain the bandgap in the desired frequency region. In simplified cases an EBG structure can be approximated with a surface impedance. This enables prediction of the bandgap using analytical approaches.

3.2 Electromagnetic Waves Supported by EBG structures

3.2.1 Review of Maxwell's Equations

Maxwell's equations hinge on the idea that light is an electromagnetic wave and they provide one of the most effective and succinct ways to correlate the electric and magnetic fields associated with the wave to their sources such as charge density and current density. Although deceptively simple to look at, these equations epitomize a high level of mathematical sophistication. Table 3.1 summarizes these equations.

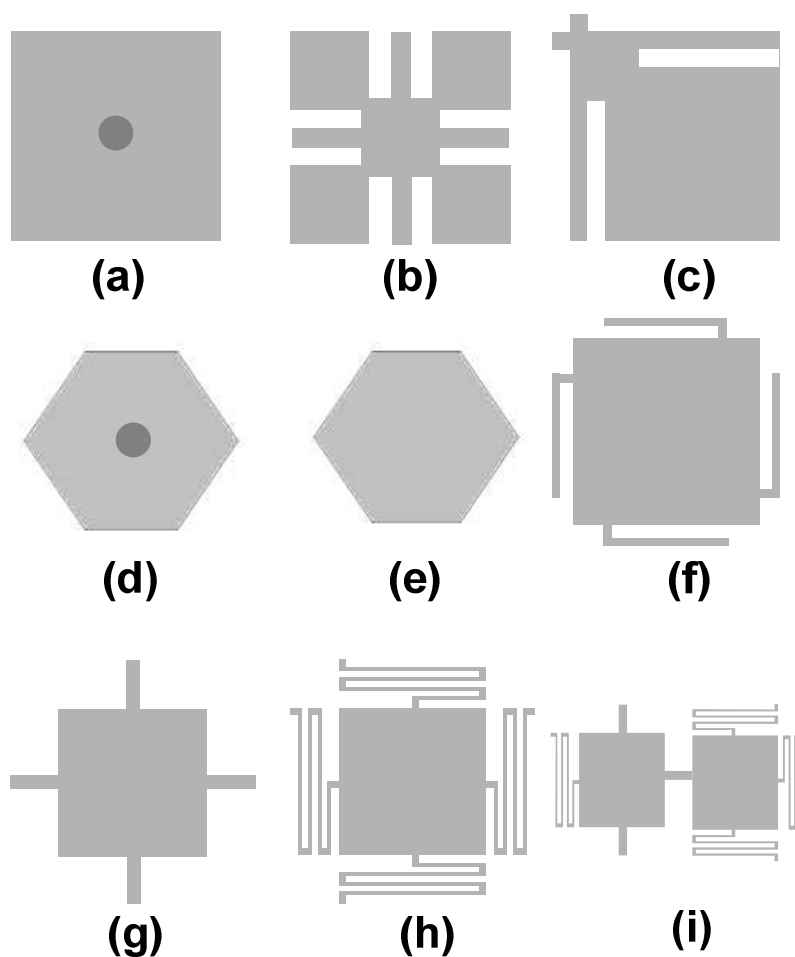


Figure 3.1: Top view of unit cells of various types of EBG structures

Table 3.1: The differential form of Maxwell's equations [42]

Name of the Law	Equation
Gauss' s Law	$\nabla \cdot \mathbf{D} = \rho$
Gauss's law for magnetism [43]*	$\nabla \cdot \mathbf{B} = 0$
Faraday's Law of Induction	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
Ampere's loop law	$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$

* Although the term is widely used, it is not a universal law. This law is also known as "Absence of free magnetic poles" [44]

where E is the electric field intensity in Volt/meter.

D is the electric flux density in Newton/Volt-meter.

ρ is the total charge density in Coulomb/m³

B is the magnetic field density in Tesla

H is the magnetic field intensity in Ampere/meter

J is the total current density in Ampere/m²

and,

$$E = \frac{1}{\epsilon} D \quad (3.1)$$

$$\text{Here} \quad \epsilon = \epsilon_r \epsilon_0 \quad (3.2)$$

ϵ_r is the relative permittivity of the medium, ϵ_0 is the free space permittivity in Farads/metre, the value of which equals $8.85 \times 10^{-12} \text{ m}^{-3} \text{ kg}^{-1} \text{ s}^4 \text{ A}^2$

Another important expression is the equation of continuity which relates the electric charge density to the electric current density.

$$\nabla \cdot J = -\frac{\partial \rho}{\partial t} \quad (3.3)$$

These are the basic equations which can be used individually, as well as in combination to describe the electromagnetic interactions taking place. The wave equation, also known as Helmholtz's equation, can be obtained by combining Faraday's law and Ampere's loop law and is given by,

$$\nabla \times \frac{1}{p} \nabla \times F - k_0^2 q F = 0 \quad (3.4)$$

where $k_0 = \omega \sqrt{\mu_0 \epsilon_0}$ is known as the free space wave number, and, $p = (\mu_r, \epsilon_r)$, $q = (\epsilon_r, \mu_r)$ for $F = (E, H)$.

(3.4) is the governing expression for electromagnetic wave propagation in any medium including periodic structure.

In a source-free region, (3.4) becomes

$$\begin{aligned}\nabla^2 E + k^2 E &= 0 \\ \nabla^2 H + k^2 H &= 0\end{aligned}\tag{3.5}$$

Where k is known as the wave number and is given by

$$k = \omega \sqrt{\mu \epsilon} = \frac{\omega}{u} \quad (\text{rad/m})\tag{3.6}$$

3.2.2 Floquet's Theorem

The fundamental expression underlying the analysis of periodic structures is known as Floquet theorem which was proposed by Gaston Floquet in [45]. The theorem is pertinent to quantum mechanics and an analysis dealing with time dependent Schroedinger equation (TDSE) has been described in [46]. Floquet theorem has also been discussed in detail in [47]. According to this theorem, if we have a periodic continuous function $Q(x)$ with a minimum period of π such that

$$Q(x + \pi) = Q(x)\tag{3.7}$$

Then the second order first degree differential equation

$$y'' + Q(x)y = 0\tag{3.8}$$

has two continuously differentiable solutions $y_1(x)$ and $y_2(x)$.

If we take a closer look at (3.5) and (3.8), we observe the correlation between the various parameters ($y \Leftrightarrow (E, H); Q \Leftrightarrow k$)

Consider a characteristic equation is given by,

$$\rho^2 - \{y_1(\pi) + y_2'(\pi)\}\rho + 1 = 0\tag{3.9}$$

The above equation has two Eigen values in $\rho_1 = e^{ia\pi}$ and $\rho_2 = e^{-ia\pi}$ respectively. Since both Eigen values are different from each other, then (3.9) has two linearly independent solutions,

$$f_1(x) = e^{iax} p_1(x)\tag{3.10}$$

$$f_2(x) = e^{-iax} p_2(x)\tag{3.11}$$

where $p_1(x)$ and $p_2(x)$ are periodic functions with period π .

In solid state physics, Felix Bloch applied the theorem to periodic boundary conditions and generalized it to three dimensions. This came to be known as Bloch's theorem. Bloch's work dealing with quantum mechanical structure of electrons in crystal lattices which are periodic geometries has been discussed in detail in [48], [49].

Hence in analysis of periodic structures in microwave engineering often the terminology Bloch-Floquet Theorem is used. According to this theorem, there exists a correlation between the fields at a point in an infinite periodic structure and the fields at a point period a away and they are found to differ from each other by a propagation factor $e^{-\gamma a}$, where γ is the propagation constant in the direction of propagation.

Based on this theorem, for the structure shown in Figure 3.2 depicting a one dimensional periodic structure with a period d , the voltage and current relationships are given by

$$V_{n+1} = V_n e^{-\gamma d} \quad (3.12)$$

$$I_{n+1} = I_n e^{-\gamma d} \quad (3.13)$$

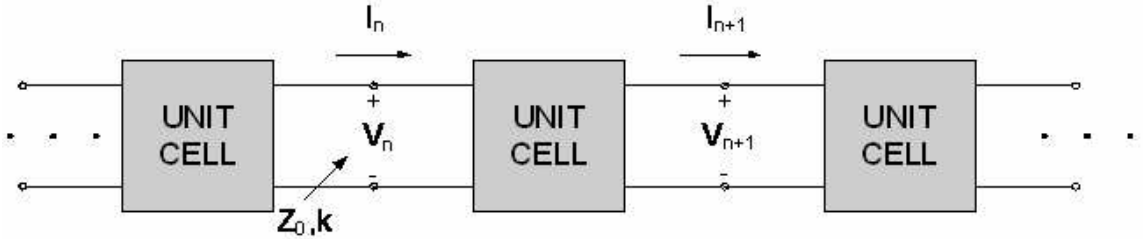


Figure 3.2: One dimensional representation of a periodic structure

The most important derivation from Floquet's theorem is the possibility of expressing fields in a periodic geometry by restricting the analysis to a unit cell of the periodic structure. Once the field solution F at a particular point is determined, it is possible to predict the field solutions at a period ma , away by the following relationship,

$$F(x, y, z + ma) = e^{-\gamma ma} F(x, y, z) \quad (3.14)$$

In this thesis, the analysis and prediction of the modal characteristics of the studied EBG structures has been performed by using the methodology mentioned above, which involves application of Bloch-Floquet theorem to a unit cell of the EBG structure.

3.2.3 Forward, Backward and Evanescent Waves in a Periodic Array

In order to examine the various modes supported by EBG structures, a review of propagating modes in a 2D infinite periodic structure from [50] is presented here. A 2D array of dipoles in Figure 3.3 is considered. The array is infinitely periodic along x- and z-directions with a period D_x and D_z , respectively. The array is excited by plane wave propagating in the direction described by

$$\hat{s} = \hat{x}s_x + \hat{y}s_y + \hat{z}s_z \quad (3.15)$$

where s_x, s_y, s_z are referred to as the direction cosines of the unit direction vector $\hat{s}$ of the plane wave.

Since it is an infinite periodic array, the current in each element obeys Floquet's theorem and is given by,

$$I_{qm} = I_{0,0} e^{-j\beta_q D_x s_x} e^{-j\beta_m D_z s_z} \quad (3.16)$$

where q and m refer to the column and row of an element, and

$I_{0,0}$ refers to the current in the reference element in the array.

In accordance with Ohm's law, the voltage in the reference element is,

$$V^{0,0} = [Z_L + Z^{0,0}] I_{0,0} \quad (3.17)$$

where

$$Z^{0,0} = \sum_{q=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} Z_{0,qm} e^{-j\beta_q D_x s_x} e^{-j\beta_m D_z s_z} \quad (3.18)$$

is known as the scan impedance [50] and is defined as the array mutual impedance of the reference element. It is composed of numerous elemental mutual impedances referred to as $Z_{0,qm}$. Z_L is the load impedance on the reference element.

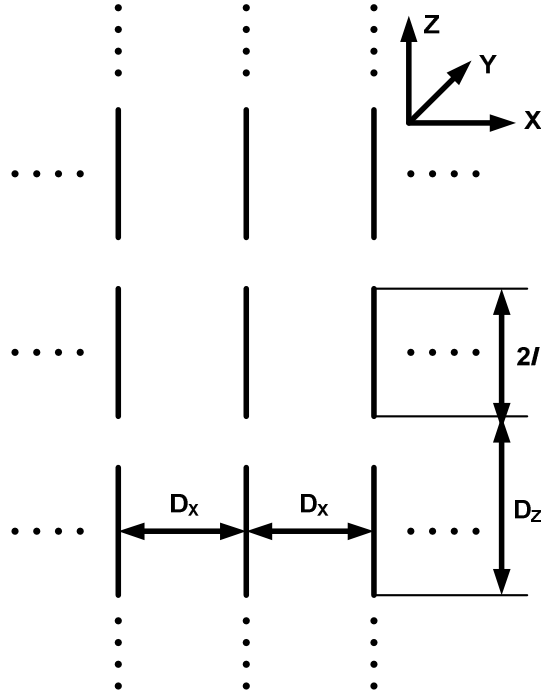


Figure 3.3: An *infinite* x *infinite* dipole array with inter-element spacings of D_x and D_z , and element length of $2l$ [50].

When we consider an array element arbitrarily oriented along $\hat{p}^{(1)}$ with the reference element of the array at $\bar{R}^{(1)}$ and an external element oriented along $\hat{p}^{(2)}$ and having a reference element at $\bar{R}^{(2)}$ (illustrated in Figure 3.4), then the mutual array impedance, based on (3.16) and (3.17) is given in [50] as,

$$Z^{2,1} = -\frac{V^{2,1}}{I_{0,0}} = \frac{Z_0}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{e^{-j\beta(\bar{R}^{(2)} - \bar{R}^{(1)}) \cdot \hat{r}}}{r_y} [\perp P^{(1)} \perp P^{(2)t} + \parallel P^{(1)} \parallel P^{(2)t}] \quad (3.19)$$

$$\text{where } \hat{r} = \hat{x} \left(s_x + k \frac{\lambda}{D_x} \right) \pm \hat{y} r_y + \hat{z} \left(s_z + n \frac{\lambda}{D_z} \right) \text{ for } y \neq 0 \quad (3.20)$$

$$r_y = \sqrt{1 - \left(s_x + k \frac{\lambda}{D_x} \right)^2 - \left(s_z + n \frac{\lambda}{D_z} \right)^2} \quad (3.21)$$

$$\perp P^{(1)} = \hat{p}^{(1)} \cdot \perp \hat{n} P^{(1)} \quad (3.22)$$

$$\underset{\parallel}{P}^{(2)t} = \underset{\parallel}{\hat{p}}^{(2)} \cdot \underset{\perp}{\hat{n}} P^{(2)t} \quad (3.23)$$

$$P^{(1)} = \frac{1}{I_0^{(1)}(\overline{R}^{(1)})} \int_{-l_1}^{l_1} I_0^{(1)}(l) e^{j\beta \hat{p}^{(1)} \cdot \hat{r}} dl \quad (3.24)$$

$$P^{(2)t} = \frac{1}{I_0^{(2)t}(\overline{R}^{(2)})} \int_{-l_2}^{l_2} I_0^{(2)t}(l) e^{-j\beta \hat{p}^{(2)} \cdot \hat{r}} dl \quad (3.25)$$

In the above expressions, $P^{(1)}$ and $P^{(2)t}$ are known as the pattern factors, $\underset{\perp}{\hat{n}}$ and $\underset{\parallel}{\hat{n}}$ are the unit vectors perpendicular and parallel to the directional vector $\hat{r}$. (3.20) and (3.21) are the governing expressions for determining the nature of the mode of propagation associated with the given array.

The exponential term in (3.19) is affiliated to a family of plane waves emanating from $\overline{R}^{(1)}$ and propagating in the direction $\hat{r}$. The direction and nature of these plane waves are strongly dependent upon the summation indices k and n . For the event when $k = n = 0$, the plane wave direction expression can be written as

$$\hat{r} = \hat{x}s_x + \hat{y}r_y + \hat{z}s_z \quad (3.26)$$

$$\text{and,} \quad \hat{r} = \hat{x}s_x - \hat{y}r_y + \hat{z}s_z \quad (3.27)$$

(3.26) refers to the fact that plane waves follow the direction of propagation of the incident wave, $\hat{s} = \hat{x}s_x + \hat{y}s_y + \hat{z}s_z$ and are hence termed *forward scattering waves*.

Similarly, (3.27) signify that the plane waves associated with them are in a direction opposite to the direction of the incident wave and are hence referred to as *bistatic reflected propagating waves*. The nature of the waves depends upon the period of the array too. If D_x and D_z are large, then (3.21) will result in real values of r_y , which implies that propagation is possible along this/these direction(s) as well. As D_x and D_z tend to ∞ , (3.21) yields imaginary values of r_y which results in the attenuation of the propagating waves.

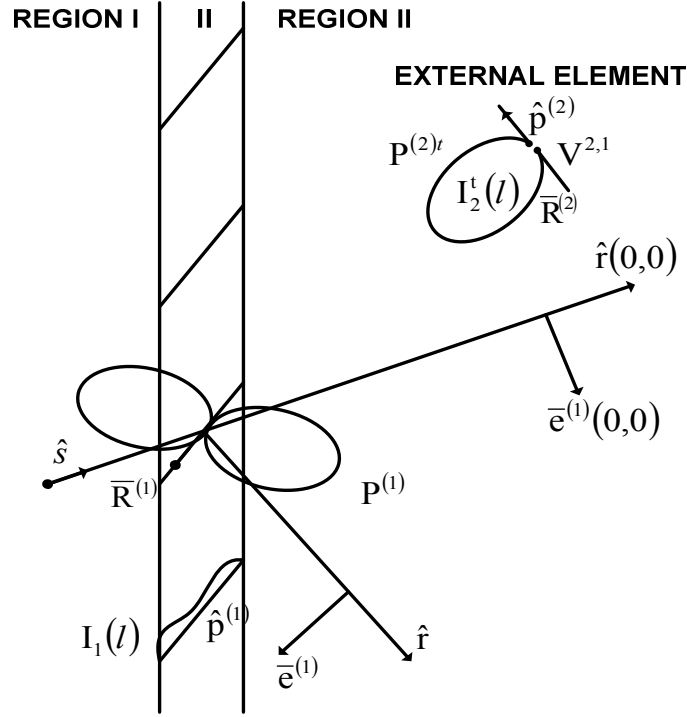


Figure 3.4: Array mutual impedance $Z^{2,1}$ between an array with element orientation $\hat{p}^{(1)}$ and an external element with orientation $\hat{p}^{(2)}$ is obtained from the plane wave expression, (3.19) [50].

These waves are known as *evanescent waves*. An interesting property of these waves is that their phase velocity remains real in the direction of periodicity [50], i.e. along r_x

and r_z , where $r_x = \left(s_x + k \frac{\lambda}{D_x} \right)$ and $r_z = \left(s_z + n \frac{\lambda}{D_z} \right)$, respectively.

However, the phase in the y -direction for imaginary r_y remains unchanged. Hence, the magnitudes of these waves are inconsequential when compared to those of the propagating modes as one tends to move further away from the array, but are extremely strong as the other one (evanescent) moves closer to the array. Another interesting phenomenon is the onset of grating lobes [50] which occurs when $r_y = 0$ and (3.21) modifies to,

$$\left(s_x + k \frac{\lambda}{D_x}\right)^2 - \left(s_z + n \frac{\lambda}{D_z}\right)^2 = 1 \quad (3.28)$$

This equation represents a family of circles with their centers at $k \frac{\lambda}{D_x}, n \frac{\lambda}{D_z}$.

The above explanation pertains to dipole arrays. In order to generalize the concept of forward and backward propagating waves on periodic structures, we consider an explanation provided in [42]. Referring to Figure 3.5, we consider the amplitudes of the forward and backward propagating waves at the n^{th} and $n+1^{\text{th}}$ section of a periodic structure as c_n^+ , c_n^- , c_{n+1}^+ and c_{n+1}^- . The amplitudes at the n^{th} and $n+1^{\text{th}}$ section are related to each other by the *wave amplitude transmission matrix* [42]. The wave amplitude transmission matrix relates the incident and reflected wave amplitudes on the input side of a junction to those on the output side of the junction. For sign convention, as seen from Figure 3.5, we consider waves propagating to the right with a positive notation and those propagating to the left with a negative notation. The equation below relates the input and output wave amplitudes as,

$$\begin{bmatrix} c_n^+ \\ c_n^- \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} c_{n+1}^+ \\ c_{n+1}^- \end{bmatrix} \quad (3.29)$$

According to Bloch-Floquet theorem,

$$\begin{bmatrix} c_{n+1}^+ \\ c_{n+1}^- \end{bmatrix} = e^{-\gamma d} \begin{bmatrix} c_n^+ \\ c_n^- \end{bmatrix} \quad (3.30)$$

Substituting (3.30) in (3.29) yields,

$$\begin{bmatrix} A_{11} - e^{-\gamma d} & A_{12} \\ A_{21} & A_{22} - e^{-\gamma d} \end{bmatrix} \begin{bmatrix} c_{n+1}^+ \\ c_{n+1}^- \end{bmatrix} = 0 \quad (3.31)$$

Since the voltage amplitudes cannot be zero, for a nontrivial solution the determinant should vanish. Hence,

$$A_{11}A_{22} - A_{12}A_{21} + e^{2\gamma d} - e^{\gamma d}(A_{11} + A_{22}) = 0 \quad (3.32)$$

If normalized amplitudes are used, (3.32) becomes,

$$\cosh(\gamma d) = \frac{A_{11} + A_{22}}{2} \quad (3.33)$$

The ratio of c_n^- to c_n^+ is called the characteristic reflection coefficient, Γ_B .

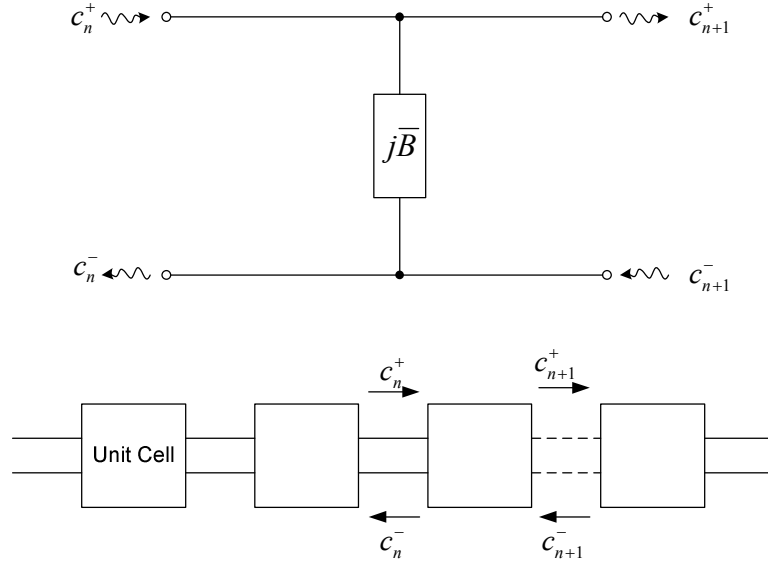


Figure 3.5: Wave amplitudes in a periodic structure [42]

3.2.4 Spatial Harmonics in Periodic Arrays

If we consider a unit cell with a length d , then for an infinite periodic structure, the Bloch modes repeat themselves at every terminal differing by a propagation factor $e^{-\gamma d}$.

The EM fields in a periodic structure which is periodic in the z -direction with a period d are defined by,

$$\begin{aligned} E(x, y, z) &= e^{-\gamma z} E_p(x, y, z) \\ H(x, y, z) &= e^{-\gamma z} H_p(x, y, z) \end{aligned} \quad (3.34)$$

where E_p and H_p are periodic functions given by,

$$F_p(x, y, z + nd) = F_p(x, y, z) \quad (3.35)$$

where $F_p = (E_p, H_p)$

From Bloch-Floquet theorem, the field at a point z_1 is related to the field at a point z_{1+d} by,

$$\begin{aligned} F(x, y, z_1 + d) &= e^{-\gamma(z_1+d)} F_p(x, y, z_1 + d) \\ &= e^{-\gamma(z_1+d)} F_p(x, y, z_1) \\ &= e^{-\gamma z_1} e^{-\gamma d} F_p(x, y, z_1) \\ &= e^{-\gamma d} F(x, y, z_1) \end{aligned} \quad (3.36)$$

which captures the repetitive properties of a Bloch wave as necessitated by (3.34).

Expanding $F_p(x, y, z)$ into its infinite Fourier series [51], the field solutions can then be represented by,

$$\begin{aligned} F(x, y, z) &= \sum_n F_{pn}(x, y) e^{-\gamma_n z} e^{-j \frac{2n\pi}{d} z} \\ &= \sum_n F_{pn}(x, y) e^{-\gamma_n z} \end{aligned} \quad (3.37)$$

where $\gamma_n = \alpha + j \left(\beta + \frac{2n\pi}{d} \right)$ and F_{pn} are the expansion coefficients that are vector functions of x and y given by,

$$F_{pn} = \frac{1}{d} \int_0^d F_p(x, y, z) e^{j \frac{2n\pi}{d} z} dz \quad (3.38)$$

Each term in the expansion (3.37) is called a spatial harmonic (or a Hartree harmonic).

The n th harmonic has a phase constant $\beta_n = \beta + \frac{2n\pi}{d}$, often referred to as Floquet's mode numbers. The phase velocity v_{pn} and the group velocity v_{gn} for an n^{th} harmonic is given by,

$$v_{pn} = \frac{\omega}{\beta_n} \quad (3.39)$$

$$v_{gn} = \frac{d\omega}{d\beta_n} \quad (3.40)$$

Some harmonics which have negative values of β_n , and hence negative phase and group velocities, are called backward travelling waves and the harmonics with the positive value of β_n are called forward travelling waves.

The exact solution for the one dimensional periodic problem involves finding the complex mode numbers γ_n and the Floquet periodic vector F_p . This method is termed as plane wave expansion method [52] and ordinarily results in an eigenvalue equation whose solution is obtained by equating the Fourier series coefficients on both sides of the equation. Finding expansion coefficients for two and three dimensional periodic

structures is a mathematically rigorous task and closed-form formulas are only available for specific unit shapes [53]. Therefore, alternative solution techniques, such as numerical methods should be used for general cases. Simple structures, like the one discussed in this thesis can be analyzed using transmission line modeling.

3.2.5 Surface Waves

Conductor surfaces, dielectric slabs and grounded dielectric slabs are capable of conducting surface waves from DC frequency onwards to higher frequencies [28].

Surface waves come into existence at the interface between two different media (for example an interface between two different dielectrics) and are referred to as surface waves due to their tendency to remain tightly bound to the interface [13]. The EBG surfaces are in fact such boundaries and hence, we expect the existence of surface waves on EBG structures.

When an infinite ground plane is used as reflector for an antenna, the ground plane will not only reflect the plane waves, but also allow the propagation of surface currents generated on the sheet due to the incident waves. These surface currents would result in only a minor reduction in the radiation efficiency. However, for all practical purposes, the ground plane is finite. Hence a surface wave propagating on a ground plane would encounter an edge (discontinuity). A current may be induced at the edge that can radiate and result in multipath interference or *speckle* [28] but some part of the surface wave may be reflected back (if the angle of incidence is greater than the critical angle [54] for the air-dielectric interface, the entire wave may be reflected back) and this may interfere with the incident surface wave and give rise to standing waves. Moreover, multiple components sharing the same ground plane may experience mutual coupling due these surface waves on the ground plane.

Surface waves are a major cause of concern in microwave engineering. EBG surfaces have been used for the suppression of surface waves and improvement of radiation patterns in antenna applications [55], [56]. In high-speed electronic circuits this characteristic of EBG structures in suppression of surface currents within certain

frequency bands is used to suppress unwanted parallel-plate waveguide mode in power distribution networks [29], [30]. This mode is behind the voltage fluctuations on the reference voltage planes and is excited by the switching current of electronic devices [25]. When a large number of electronic devices switch at the same time it gives rise to simultaneous switching noise (SSN). SSN is dependent upon the slew rate and switching time, thus needs significant attention in high edge rate devices [9], [57]. More discussion on this topic is provided in Section 3.4.

3.3 Effective Model

All high impedance structures can be studied on the basis of an effective model which consists of a resonant LC circuit as we will discuss in the following sections. In the case of mushroom structures, the capacitance arises due to the fringing fields between two adjacent metal plates whereas the inductance arises from current propagation through the small loop consisting of two adjacent vias and the ground plane [28]. Analytical formulas for sheet inductance and capacitance and approximate center frequency of the bandgap are provided in [28]. The center frequency of the bandgap is given by,

$$f_c = \frac{1}{2\pi\sqrt{LC}} \quad (3.41)$$

In the case of planar EBG structures, the sheet or effective capacitance arises not only because of the fringing fields between adjacent plates, but also due to the parasitic capacitance that comes into play due the substrate and the proximity between the patterned plane and the ground plane. The sheet or effective inductance results from the per unit length inductance of the transmission line sections as well as any meander structure that may be included in the planar design. Hence, often the EBG structure is crudely modeled as a resonant LC circuit. In [58] a planar EBG structure is investigated and the following formula is given to calculate the approximate bandwidth when the effective inductance and capacitance model is used.

$$BW = \frac{\Delta\omega}{\omega_0} = \frac{1}{\eta} \sqrt{\frac{L}{C}} \quad (3.42)$$

3.4 Application of EBG Structures in Suppression of Power/Ground Noise

Noise

Power delivery network (PDN) of modern electronic systems from chip level to PCBs and backplanes contain parallel conductor plate arrangements. The parallel-plate pair configures a parallel-plate waveguide which can be modeled by parallel-plate transmission lines. This parallel plate waveguide can be excited by the switching currents on the through vias used for power and ground connection. The dominant mode of the parallel-plate waveguide, which is TEM, results in fluctuations of the reference voltages that are supposed to be fixed DC values supplied throughout the system by the PDN. The cumulative effect of a large number of buffers switching at the same time creates appreciable power/ground noise or SSN. Such noise can be extremely detrimental to the performance of circuits as it may cause false switching and malfunctioning of devices. As well, when this unwanted mode is excited it propagates in the system substrate and reflects back and forth when reaching the edges of the board while also radiating to free space at the edges. Figure 3.6 depicts the generation of SSN in PCBs, propagation and suppression in an EBG structure (a Sievenpiper mushroom structure) and radiation from the edges of the PCBs [10].

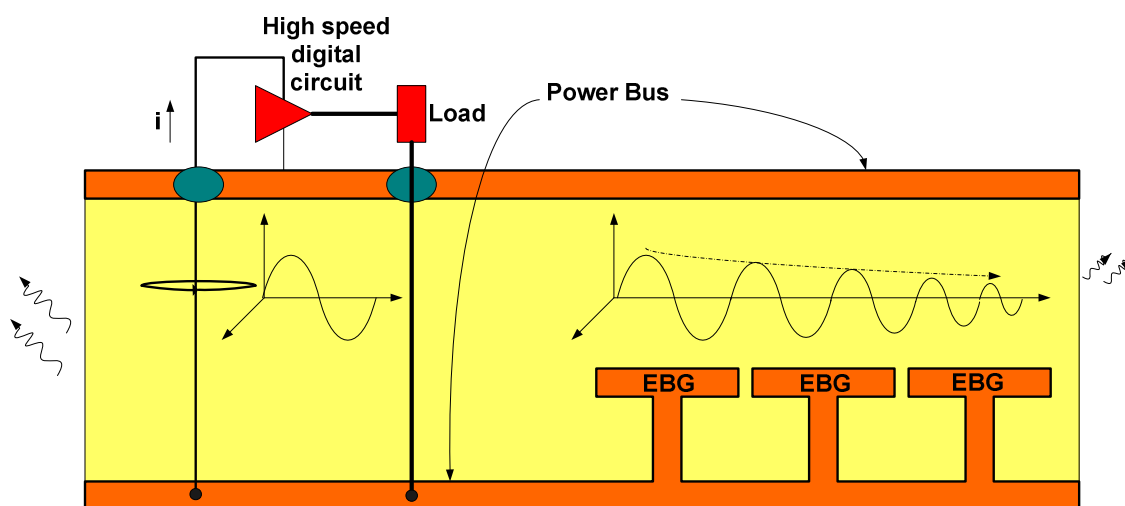


Figure 3.6 Generation of Switching noise and its suppression using EBG structures

In the following subsections, we discuss various methods that have been tried to mitigate the adverse effects of SSN, one of the most prevalent problems faced today by integrated circuits (ICs) operating at higher frequencies with lower power supply voltages.

3.4.1 Use of Discrete Decoupling Capacitors

One of the most popular noise suppression methods involves the use of decoupling capacitors [59]-[65]. Decoupling capacitors provide a shorting path for the SSN. The capacitor has connecting leads for attachment to the PDN. The inductance associated with the lead of the decoupling capacitors creates a self resonant frequency, thereby decreasing the effectiveness of the capacitor dramatically beyond this frequency [66]. The effectiveness of the capacitor can be increased by various means which include placing more decoupling capacitors closer to the chip [67], connecting the decoupling capacitors directly to the power and ground planes through vias rather than traces since the trace loop adds to the inductance. There are approaches to optimize the values and location of the decoupling capacitor [29], [66], which include placing the decoupling capacitors closer to the device power/ground pins [68].

However, the effectiveness of decoupling capacitors is basically in the low operating frequency regions (in the sub 600 MHz range).

3.4.2 Use of Embedded Capacitors

Another method involves the use of embedded capacitors to suppress power ground noise [69], [70]. This method has been found to very effective in the suppression of ground bounce noise up to 5GHz [71], [72]. The effectiveness of this method depends heavily upon the dielectric constant of the substrate and hence places a bottleneck on the performance of embedded capacitors. Very high dielectric constant substrates have been used to increase the capacitance between the layers [73] but the problem with such high permittivity dielectrics is that they are not as cost effective as discrete decoupling capacitors.

3.4.3 Use of Shorting Vias and Islands

This method uses shorting vias and virtual island- like patches to block SSN from reaching sensitive devices. The shorting vias provide a low impedance path to the returning current and the islands stop the propagation of PPW noises through the power/ground pair in the PCB's. In this method too, noise suppression is achievable only over a very narrow frequency range [74].

3.5 Other Applications of EBG Structures

As stated earlier, one of the main applications of EBG structures is as a reflective HIS. HISs are used in antenna applications and also in miniaturization of circuit components like printed inductors [75].

In these applications, the HISs behave as an artificial magnetic conductor (AMC). Whenever an antenna is placed in close proximity to an AMC surface, a part of the antenna signal is radiated and part of it is directed towards the AMC surface and gets reflected. When the antenna is placed close to a conductor reflector, it can technically be shorted as the reflection from the ground plane would interfere destructively with the radiated wave and result in a deterioration of the radiation pattern and cancellation of it in the extreme case. An AMC, surface on the other hand results in no phase change in the reflected wave and hence, this reflected wave interferes constructively with the radiated wave from the antenna to give an improved radiation pattern.

High impedance surfaces acting as AMC conductors have an additional advantage too. In addition to providing a scenario for constructive interference, these surfaces have forbidden bands associated with them. An important property of operation of these surfaces within the forbidden band is that they stop wave propagation within these bands. Once this occurs, the surface waves are eliminated within the forbidden band of operation or are almost negligible. Hence, in the absence of surface waves, the radiation pattern is further ameliorated [28]. This region of the bandgap where the reflected wave provides maximum constructive interference with the incident wave can be a subset of the

bandgap itself [76]. As discussed in [76], when a reflected signal is evaluated at a distance d from a reflective plane, the reflection phase introduced due to a regular and non patterned continuous metal ground plane is given by,

$$\phi = 180 - \left(\frac{d}{\lambda} \bullet 360 \right) \text{ (degrees)} \quad (3.43)$$

When the continuous metal plane is replaced by an EBG plane, the reflection phase for the reflected wave is given by,

$$\phi_{EBG} = \frac{\int_S \text{Phase}(E_{SCATTERED}) dS}{\int_S \hat{s} dS} \quad (3.44)$$

where $E_{SCATTERED}$ is the scattered E field data from an incident wave $\hat{s}$ and S is surface over which the phase needs to be evaluated.

3.6 Conclusion

In this chapter, an overview of EBG structures and the various geometries associated with them are presented. The concept of surface waves and the various modes that come into existence in periodic structures are discussed. The correlation of these modes with Maxwell's equations and the methodology behind capturing the modal characteristics are introduced. The concept of SSN and methods, particularly the use of EBG structures, to mitigate SSN are also discussed. The chapter also deals with other applications of EBG structures such as their use as AMC surfaces for antenna applications.

CHAPTER 4

Modeling using Transmission

Line Circuit

Most EBG structures used in microwave filtering applications consist of two conductors in which one is the reference conductor also acting as a shield. In PDNs and multilayer circuits, we generally encounter at least three conductor layers. The focus of this thesis is on the two metal layer structures as they can be modeled using conventional transmission line circuits. These structures are mainly composed of microstrip line sections. In order to better understand the characteristics of EBG structures realized using microstrip lines, we need to analyse the operation of a microstrip line. In this chapter, a review of a microstrip line is presented followed by the modeling of EBG structures. Then, a stepped impedance EBG structure is analyzed.

4.1 Analysis of a Single Microstrip Line

Microstrip lines are widely used as interconnects of choice due to their ease of fabrication. A microstrip line consists of a conductor placed over a grounded dielectric substrate. The important geometrical parameters in the design of the microstrip line are its width (W), length (L), distance from the ground plane (b) and the relative permittivity of the dielectric medium (ϵ_r). The fundamental characteristics of the microstrip line, Z_0 and β , depend upon the width of the microstrip line and the relative permittivity of the substrate. This can be observed from the expressions discussed later under this section.

Since the electromagnetic wave supported by a microstrip line is exposed to two different dielectric media, the microstrip line supports a quasi-TEM wave.

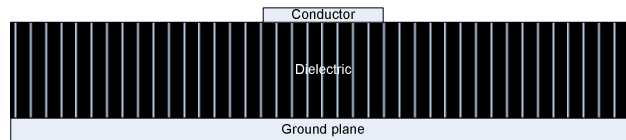


Figure 4.1: Cross section of a microstrip line configuration

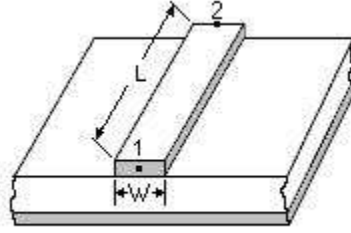


Figure 4.2: Important parameters and port definition for the analysis of a microstrip line

Based on [13], [77] references, the effective dielectric constant of a microstrip line is given by

$$\varepsilon_e = \begin{cases} \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \frac{1}{\sqrt{1 + 12 \frac{b}{W}}} & \text{for } \frac{W}{b} > 1 \\ \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \left[\frac{1}{\sqrt{1 + 12 \frac{b}{W}}} + 0.04 \left(1 - \frac{W}{b} \right)^2 \right] & \text{for } \frac{W}{b} \leq 1 \end{cases} \quad (4.1)$$

Same references provide the following formulas for the characteristic impedance of the microstrip line

$$Z_0 = \begin{cases} \frac{60}{\sqrt{\varepsilon_e}} \ln \left(8 \frac{b}{W} + \frac{1}{4} \frac{W}{b} \right) & \text{for } \frac{W}{b} \leq 1 \\ \frac{120\pi}{\sqrt{\varepsilon_e} \left[\frac{W}{b} + 1.393 + 0.667 \ln \left(\frac{W}{b} + 1.444 \right) \right]} & \text{for } \frac{W}{b} > 1 \end{cases} \quad (4.2)$$

To verify the closed-form formulas, the microstrip line is analyzed using ADS LineCalc. The dimensions of the simulated microstrip line being analyzed are provided in Table 4.1. FR4 is the dielectric substrate. For these design parameters, $\frac{W}{b} > 1$, and Equations (4.1) and (4.2) yield the DC effective dielectric constant of, $\varepsilon_{eff,0} = 3.3426$ F/m, and a characteristic impedance, $Z_0 = 48.89 \Omega$.

The frequency dependent characteristic impedance of a microstrip line as provided in [78] is given by

$$Z_{0,F} = Z_0 \left(\frac{\epsilon_{eff,0}}{\epsilon_{eff,F}} \right)^{0.5} \left(\frac{\epsilon_{eff,F} - 1}{\epsilon_{eff,0} - 1} \right) \quad (4.3)$$

where $\epsilon_{eff,F}$ is the frequency dependent effective dielectric constant.

Table 4.1: Microstrip line parameters

Parameter	Value	Units
Width of the Microstrip line W	10	Mil
Thickness of the Microstrip line t	1.3	Mil
Distance from the ground plane b	5	Mil
Permittivity of the medium ϵ_r	4.4	None
Length of the Mcrostrip line L	100	Mil

Modifying (4.3) in order to calculate $\epsilon_{eff,F}$ yields

$$\epsilon_{eff,F} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (4.4)$$

where $a = 1$

$$b = \left(\frac{-2\epsilon_{eff,0} - k^2 \epsilon_{eff,0}^2 - k^2 + 2\epsilon_{eff,0} k^2}{\epsilon_{eff,0}} \right)$$

$$c = 1$$

$$k = \frac{Z_{0,F}}{Z_0}$$

Choosing only the addition in the numerator of the RHS of Equation (4.4), the frequency dependent $\epsilon_{eff,F}$ can be calculated.

The most accurate way to find characteristic impedance and permittivity of a microstrip line is by conducting full-wave simulations which provide these parameters as functions of frequency. Hence, the microstrip described in Table 4.1 was simulated using two main

microwave circuit simulators; Ansoft's High Frequency Structure Simulator (HFSS) and Agilent's Advanced Design Systems (ADS). ADS is a 2.5-D solver based on the method of moments (MoM). By 2.5-D it is meant that only 2D planar layouts and particular vertical conductor structures like vias can be simulated. This allows for stacking up of one layer above another but not a structure with 3D complexities. ADS is widely used to analyze planar circuits like microstrip- or stripline-based geometries. HFSS is a 3D field solver operating based on the finite element method (FEM) for the numerical analysis. It is a robust and accurate software that enables the analysis of complex 3D geometries. FR4 was used as the dielectric substrate.

One of the most important steps in the design process using HFSS is the definition of the ports. A wave port is commonly chosen for the excitation. HFSS generates a solution by exciting each wave port individually. Each mode incident on a port contains one Watt of time-averaged power. Care has to be taken in defining the ports as wrong field excitation at the port can lead to incorrect results. In the conducted simulations, the height of the port is chosen to be approximately ten times the height of the substrate and the width was chosen to be approximately fifty percent of the dielectric width. This definition is in accordance with the recommended guidelines of the simulator. Another important feature is the definition of the radiation boundary. Since most of the designs discussed in this thesis are open boundary problems and hence radiation boundaries are used to emulate a wave radiating infinitely far into space. It does so by essentially absorbing the wave at the radiation boundary thereby distending the boundary to theoretical infinity.

Figure 4.3(a) shows the layout of a microstrip line in HFSS. Figure 4.3(b) shows the field distribution when a waveport is used for excitation. It clearly depicts the quasi-TEM nature of the wave supported by the microstrip line. Figure 4.4 (a) depicts the frequency dependent effective relative permittivity and Figure 4.4 (b) shows the plot of the characteristic impedance of the microstrip line and its variation with frequency. Three methods are used to derive the results of Figure 4.4; HFSS, ADS Momentum and the web calculator from [77].

Figure 4.5 shows the return loss and insertion loss of the studied microstrip line found from three methods, HFSS, ADS Momentum, ADS Schematic window simulations which uses library models for the transmission lines as opposed to full-wave simulations. It is observed at low frequencies, the microstrip line transmits almost all the power from the source to the load. This behaviour changes with the increase in frequency; the power at the load end is always lesser than the power at the source end. This is attributed to the radiation, conductor and dielectric losses.

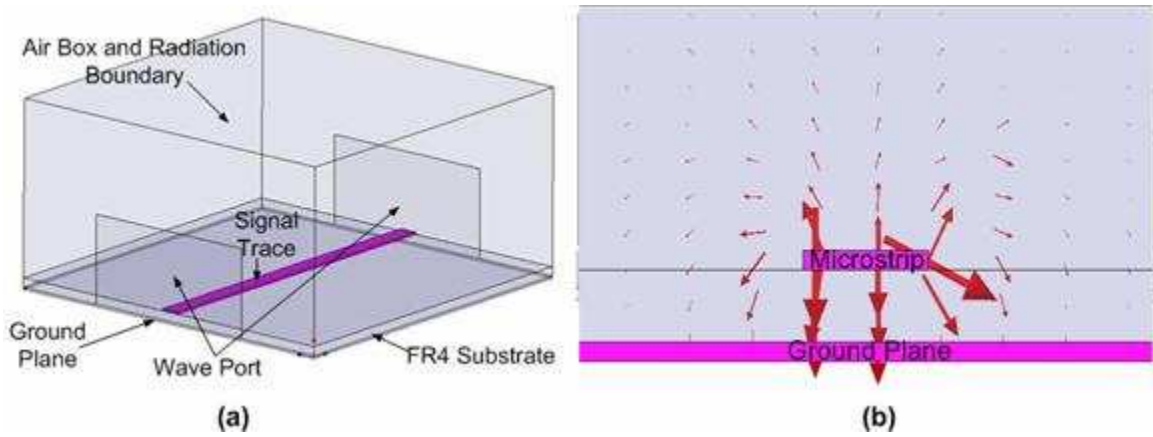


Figure 4.3: (a) Simulation setup for the analysis of a microstrip line, and (b) field distribution corresponding to excitation of a microstrip line

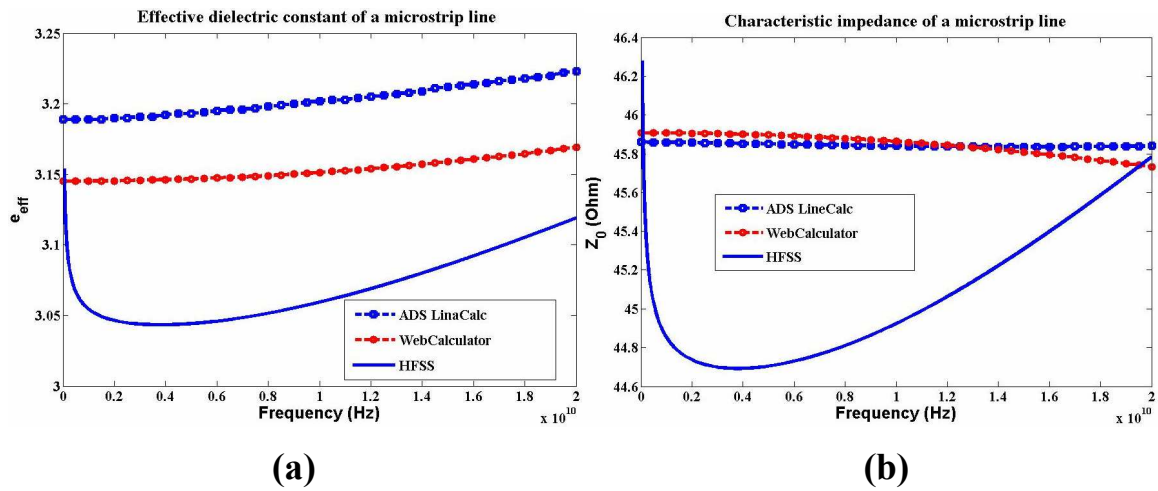


Figure 4.4: Variation of (a) effective dielectric constant, and (b) characteristic impedance with frequency. The WebCalculator results are from [77]

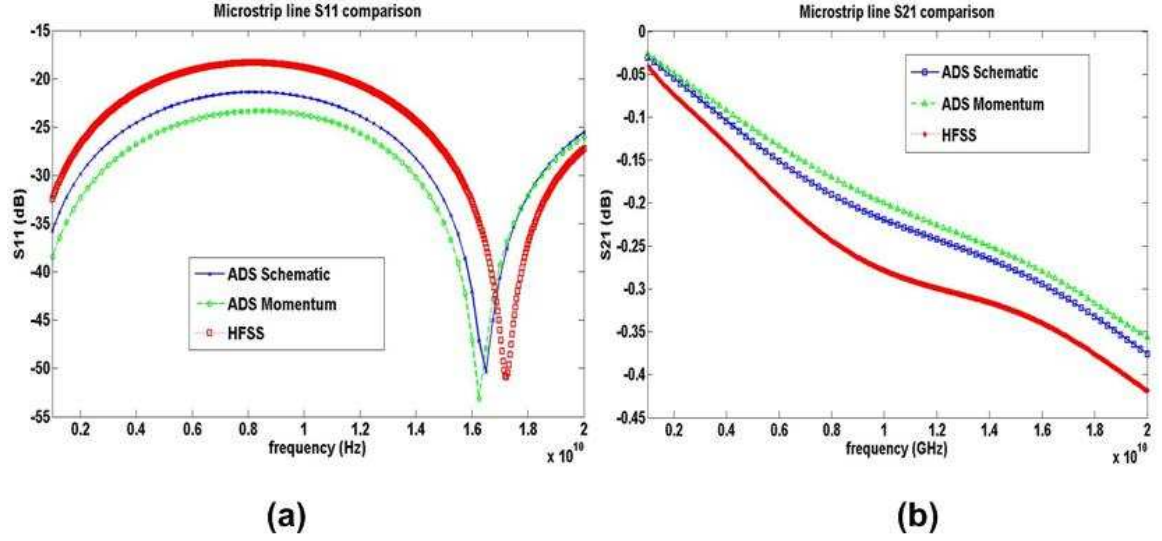


Figure 4.5: (a) Return loss, and (b) insertion loss of the studied microstrip line

Radiated power or electromagnetic interference at high frequencies can be a serious problem as it can affect the performance of the circuit as well as those of the adjacent circuits. This has been discussed in detail in [79].

4.2 Analysis of 1D and 2D EBG Structures Made with Two Conductor Transmission Line Circuits

As indicated in the introduction of this chapter, many of the EBG structures used in microwave and electronic engineering are designed by using transmission line circuits. The focus of study in this thesis is on the structures containing two conductor layers. In this chapter mainly the stepped impedance type EBG structures are considered for modeling and analysis using transmission line network theory and Floquet's Theorem. Using these analytical procedures, the characteristic equation or dispersion equation can be obtained which provides the information about the excited modes and their propagation constants.

A good understanding can be obtained by first analysing a 1D EBG structure. The two dimensional periodic structure can then be treated as a deduction of the one dimensional geometry.

4.2.1 Derivation of Dispersion Equation: 1D Stepped Impedance Structure

Figure 4.6 depicts a one dimensional periodic structure with the period. When this structure is based on transmission line structures, the voltage and current relationships according to Floquet's Theorem are given by

$$V_{n+1} = V_n e^{-\gamma d} \quad (4.5)$$

$$I_{n+1} = I_n e^{-\gamma d} \quad (4.6)$$

To showcase the derivation of the dispersion equation of a one dimensional periodic structure, a 1D stepped impedance filter also known as $Z_l Z_h$ (Z-low Z-high) filter is considered. The unit cell has been divided into three sections (refer to Figure 4.7). Then, the transfer matrix of each section is cascaded with the transfer matrix of the subsequent section (refer Figure 4.8) to obtain a single equivalent transfer matrix for the unit cell.

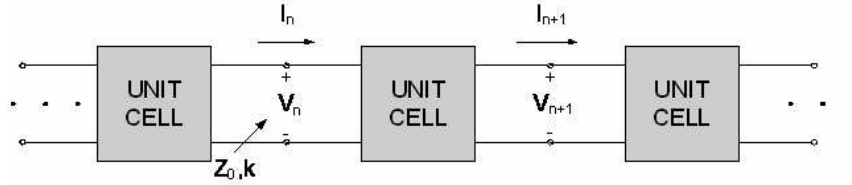


Figure 4.6: One dimensional representation of a periodic structure

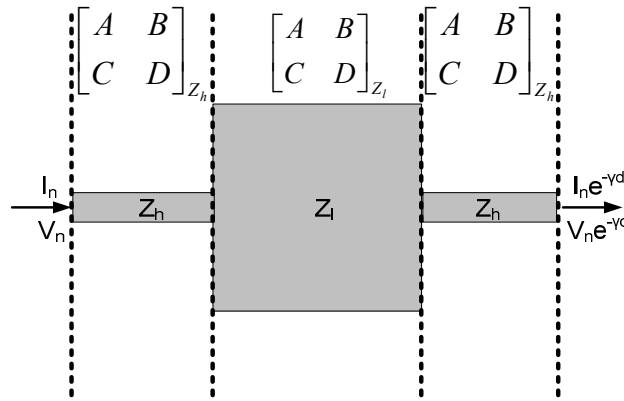


Figure 4.7: Unit Cell of a 1D stepped impedance filter

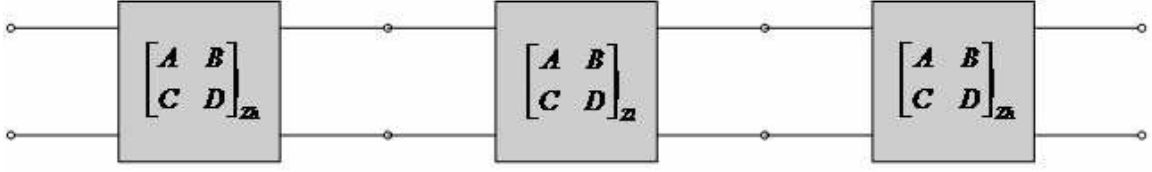


Figure 4.8: Cascading of ABCD matrices of sections

From the analysis of microstrip lines, we know that a narrow section has higher characteristic impedance than a wider section. Hence the parameters of the transmission line-based unit cell are as follows,

$$\text{Impedance of the broad section} = Z_L \quad \Omega$$

$$\text{Impedance of the narrow section} = Z_H \quad \Omega$$

$$\text{Admittance of the broad section} = Y_L = \frac{1}{Z_L} \quad \Omega^{-1}$$

$$\text{Admittance of the narrow section} = Y_H = \frac{1}{Z_H} \quad \Omega^{-1}$$

$$\text{Length of the sections 1 and 3 (the two narrow sections on either side)} = d_1$$

$$\text{Length of the section 2 (the broad section in the middle)} = d_2$$

$$\text{Relative permeability of the medium} = \mu_r$$

$$\text{Relative permittivity of the medium} = \varepsilon_r$$

$$\text{Permittivity of the medium } \varepsilon = \varepsilon_r \cdot \varepsilon_0$$

$$\text{Permeability of the medium } \mu = \mu_r \cdot \mu_0$$

$$\text{Where } \varepsilon_0 \text{ is the permittivity of free space} = 8.85 \times 10^{-12} \text{ m}^{-3} \text{kg}^{-1} \text{s}^4 \text{A}^2$$

$$\mu_0 \text{ is the permeability of free space} = 4\pi \times 10^{-7} \text{ N.A}^{-2}$$

$$\text{Intrinsic impedance of the medium } \eta = \sqrt{\mu \cdot \varepsilon}$$

$$\text{Wave Number } k = 2\pi \cdot f \cdot \eta$$

Where 'f' is the operational frequency

$$\text{Electrical length of the narrow sections} = \theta_1 = k \times d_1$$

$$\text{Electrical length of the broad section} = \theta_2 = k \times d_2$$

The transfer or ABCD matrices of the narrow and wide sections are

$$X = \begin{bmatrix} \cos(\theta_1) & jZ_H \sin(\theta_1) \\ jY_H \sin(\theta_1) & \cos(\theta_1) \end{bmatrix} \quad (4.7)$$

$$Y = \begin{bmatrix} \cos(\theta_2) & jZ_L \sin(\theta_2) \\ jY_L \sin(\theta_2) & \cos(\theta_2) \end{bmatrix} \quad (4.8)$$

where X and Y denote the transfer matrices of the high impedance and low impedance sections, respectively.

The ABCD matrix of the unit cell is, therefore $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = X*Y*X$ (4.9)

The Voltage and Current at the beginning and the end of a unit cell are defined using matrices, $\begin{bmatrix} V_n \\ I_n \end{bmatrix}$ & $\begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix}$ respectively

$$\therefore \begin{bmatrix} V_n \\ I_n \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} \quad (4.10)$$

Using Floquet's Theorem:

$$V_{n+1} = V_n e^{-\gamma d} \quad (4.11)$$

$$I_{n+1} = I_n e^{-\gamma d} \quad (4.12)$$

Hence the (4.10) can be rewritten as,

$$\begin{bmatrix} V_{n+1} e^{\gamma d} \\ I_{n+1} e^{\gamma d} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} \quad (4.13)$$

Rearranging the above equation yields,

$$\begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} \begin{bmatrix} A - e^{\gamma d} & B \\ C & D - e^{\gamma d} \end{bmatrix} = 0 \quad (4.14)$$

$$\begin{aligned} &\Rightarrow (A - e^{\gamma d})(D - e^{\gamma d}) - BC = 0 \\ &\Rightarrow AD - BC + e^{2\gamma d} - (A + D)e^{\gamma d} = 0 \end{aligned} \quad (4.15)$$

For a reciprocal network

$$\Rightarrow AD - BC = 1 \quad (4.16)$$

Therefore (4.15) becomes,

$$\begin{aligned} &\Rightarrow 1 + e^{2\gamma d} - (A + D)e^{\gamma d} = 0 \\ &\Rightarrow e^{-\gamma d} - e^{\gamma d} = (A + D) \end{aligned}$$

$$\Rightarrow 2\cosh(\gamma d) = (A + D)$$

$$\Rightarrow \cosh(\gamma d) = \frac{(A + D)}{2}$$

$$\Rightarrow \gamma d = \cosh^{-1}\left(\frac{A + D}{2}\right)$$

$$\Rightarrow (\alpha + j\beta)d = \cosh^{-1}\left(\frac{A + D}{2}\right) \quad (4.17)$$

$$\Rightarrow \alpha d = \text{real}(\gamma d) \quad (4.18)$$

$$\Rightarrow \beta d = \text{imaginary}(\gamma d) \quad (4.19)$$

This analytical derivation is clearly explained in [13]

4.2.2 Derivation of Dispersion Equation: 2D Stepped Impedance Structure

The transmission line model (TLM) developed in the one dimensional problem above can be extended to the two dimensional geometry as well. The two dimensional implementation of the transmission line model is discussed in detail in [80]-[84]. These ideas have also been extended to multilayered and multiconductor geometries [35], [85]-[87]. In references [36], [81], [88], these models are combined with Bloch-Floquet Theorem and derivation of 2D dispersion equation is presented [88].

To showcase these derivations again a sample 2D EBG structure composed of dissimilar transmission line sections shown in Figure 4.9 is analyzed. The length of the unit cell is a and the length of the broad section of microstrip line is d_2 . The length of the narrow transmission line sections is thus, $d_1 = \left(\frac{a - d_2}{2}\right)$.

The model consists of transmission line sections in two perpendicular directions as shown by overlapping circuit on the 2D stepped impedance structure depicted in Figure 4.9.

Each transmission line section is represented by its corresponding Transfer matrix as follows

$$T_h = \begin{bmatrix} \cos(\beta_d d_1) & jZ_h \sin(\beta_d d_1) \\ jY_h \sin(\beta_d d_1) & \cos(\beta_d d_1) \end{bmatrix} \quad (4.20)$$

where β_d is the phase constant of the transmission line section. This matrix pertains to the narrow transmission line segments, thus it is referred to as high impedance TL section represented by the subscript h in the equation.

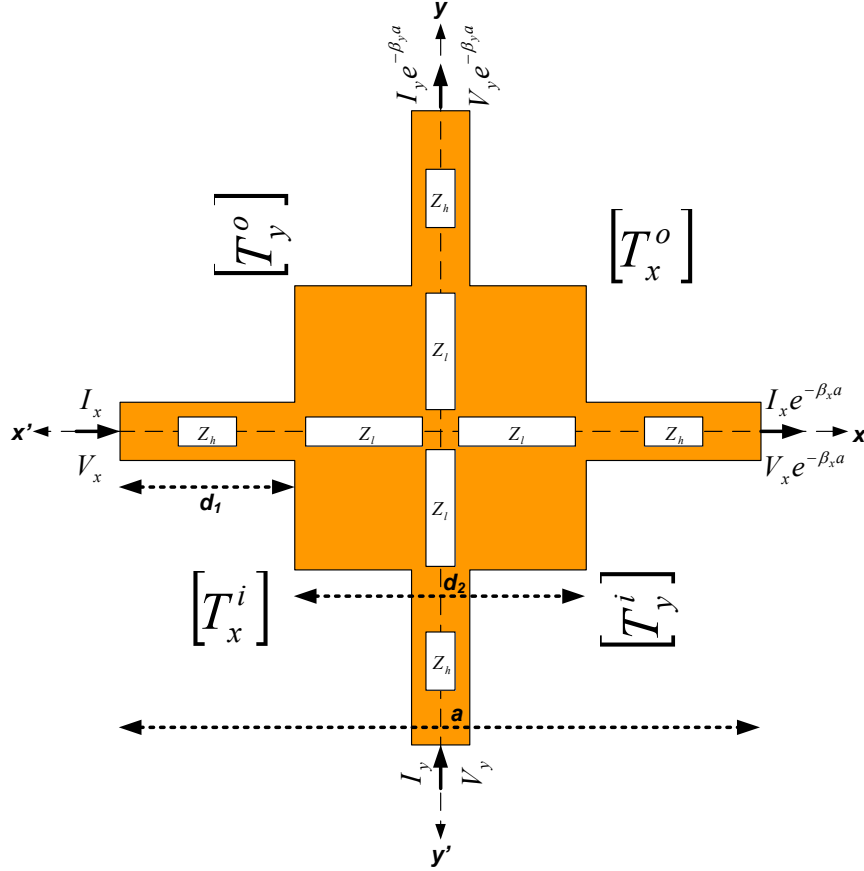


Figure 4.9: A unit cell of a 2D EBG structure and its equivalent transmission line network

Similarly, the transfer matrix of the wider section is represented by

$$T_l = \begin{bmatrix} \cos(\beta_d d_2) & jZ_l \sin(\beta_d d_2) \\ jY_l \sin(\beta_d d_2) & \cos(\beta_d d_2) \end{bmatrix} \quad (4.21)$$

where β_d is the phase constant of the transmission line section and is considered identical for both the transmission line sections as the substrate of all the transmission line sections is the same. The overall transmission matrix of each section composed of the high and low impedance transmission lines is found from

$$T_k^i = \begin{bmatrix} A_k^i & B_k^i \\ C_k^i & D_k^i \end{bmatrix} = T_h T_l \quad (4.22)$$

$$T_k^o = \begin{bmatrix} A_k^o & B_k^o \\ C_k^o & D_k^o \end{bmatrix} = T_l T_h \quad (4.23)$$

where the superscript i represents the input segment, o represents the output segment in case different transmission lines are used in the input and output sections as labelled in Figure 4.9. Note that there are two directions of propagation, x and y , which follow a similar derivation process to obtain the transfer matrices. Thus index k is included in the formulations where $k = x, y$ represents the direction along the axes under consideration.

The transmission line model constructed based on this analysis for structure of Figure 4.9 is depicted in Figure 4.10

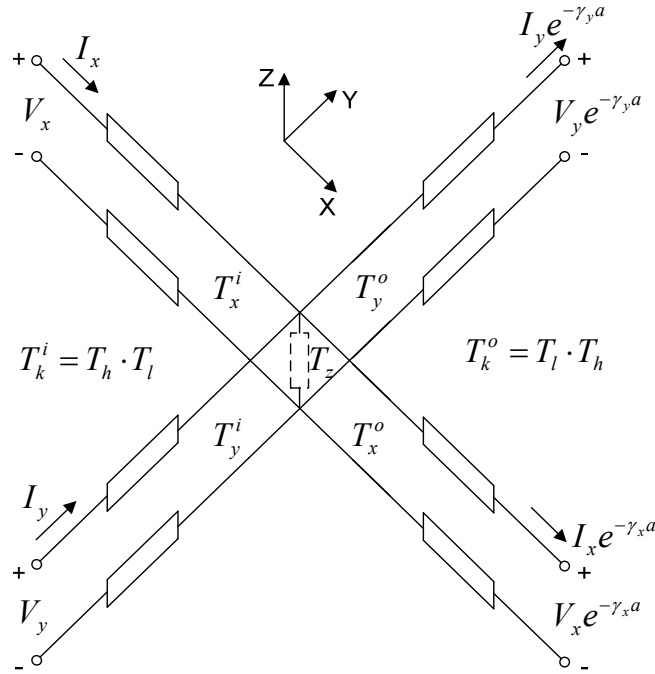


Figure 4.10: Four port network representation of a unit cell of a two dimensional periodic structure

An infinite periodic structure can be viewed as a repetitive array of one such unit cell in x and y directions. By applying the Bloch-Floquet theorem in both directions and thereby

relating the voltages and current at one port to the voltages and current at other port along the same axis, similar to what presented for the 1D structure, dispersion equation can be found. Note: here the Floquet's phase progression along the x and y directions is determined by $\beta_x a$ and $\beta_y b$ factors and denoted by $e^{-\beta_x a}$ and $e^{-\beta_y a}$ in the following equations. In addition to this, Kirchhoff's Voltage Law (KVL) and Kirchhoff's Current Law (KCL) are applied to establish the voltage and current relationships at the intersection of the x and y directed circuits. The following equations are the result of these calculations,

$$(D_x^i / \Delta_x^i - A_x^o e^{-\beta_x a}) V_x - (B_x^i / \Delta_x^i - B_x^o e^{-\beta_x a}) I_x = 0 \quad (4.24)$$

$$(D_y^i / \Delta_y^i - A_y^o e^{-\beta_y a}) V_y - (B_y^i / \Delta_y^i - B_y^o e^{-\beta_y a}) I_y = 0 \quad (4.25)$$

$$(A_x^o e^{-\beta_x a}) V_x + (B_x^o e^{-\beta_x a}) I_x - (A_y^o e^{-\beta_y a}) V_y - (B_y^o e^{-\beta_y a}) I_y = 0 \quad (4.26)$$

$$(C_x^i / \Delta_x^i - C_x^o e^{-\beta_x a}) V_x + (C_y^i / \Delta_y^i - C_y^o e^{-\beta_y a}) V_y - (A_x^i / \Delta_x^i - D_x^o e^{-\beta_x a}) I_x - (A_y^i / \Delta_y^i - D_y^o e^{-\beta_y a}) I_y = 0 \quad (4.27)$$

where $\Delta_k^i = A_k^i D_k^i - B_k^i C_k^i$, $k = x, y$ and β_x , β_y represent the phase constants along x and y axes, respectively.

Eliminating I_x , I_y from Equations (4.32) to (4.35) we obtain the simplified system of linear homogenous matrix equation

$$(G) \begin{pmatrix} V_x \\ V_y \end{pmatrix} = \begin{pmatrix} g_{1x} & g_{1y} \\ g_{2x} & g_{2y} \end{pmatrix} \begin{pmatrix} V_x \\ V_y \end{pmatrix} = 0 \quad (4.28)$$

where

$$g_{1k} = \frac{B_k^i A_k^o + B_k^o D_k^i}{B_k^i + B_k^o \Delta_k^i e^{-\beta_k a}} e^{-\beta_k a} \quad (4.29)$$

$$g_{2k} = \frac{1 + \Delta_k^o \Delta_k^i e^{-2\beta_k a} - (C_k^i B_k^o + C_k^o B_k^i + A_k^i A_k^o + D_k^o D_k^i) e^{-\beta_k a}}{B_k^i + B_k^o \Delta_k^i e^{-\beta_k a}} \quad (4.30)$$

where $\Delta_k^o = A_k^i D_k^o - B_k^o C_k^o$, $k = x, y$. These formulations are fully derived and explained in [88].

From Equation (4.28) it can be observed that the solution to the matrix equation is possible if either the determinant of the voltage matrix is zero, or if the determinant of the

coefficient G matrix is zero. For a non-trivial solution we conclude that $\det(G) = 0$. Solving for this condition, we obtain the following dispersion equation

$$\cosh(\beta_x a) + \cosh(\beta_y a) = 2 \cos(\beta_d a) + \left(2 - \frac{Z_l}{Z_h} - \frac{Z_h}{Z_l} \right) \sin(2\beta_d d_1) \sin(2\beta_d d_2) \quad (4.31)$$

where $a = 2d_1 + 2d_2$. Losses are ignored in this analysis; hence, the attenuation constant is neglected. The general dispersion equation including losses is:

$$\cosh(\gamma_x a) + \cosh(\gamma_y a) = 2 \cos(2\gamma_d d_1 + 2\gamma_d d_2) + \left(2 - \frac{Z_l}{Z_h} - \frac{Z_h}{Z_l} \right) \sin(2\gamma_d d_1) \sin(2\gamma_d d_2) \quad (4.32)$$

where $\gamma_x = \alpha_x + j\beta_x$, $\gamma_y = \alpha_y + j\beta_y$ represent the propagation constants along x and y axes, respectively.

4.3 Dispersion Diagrams

The dispersion diagram, also known as $k - \beta$ diagram, is obtained from dispersion equation. For a 1D periodic structure, this diagram is simply a $k - \beta$ diagram and β is the propagation constant along the direction of repetition, k is the wavenumber in the medium for propagation of light. When the structure is periodic along 2 or 3 directions plotting such a $k - \beta$ diagram is complicated. Hence, Brillouin Diagrams are used which were originally introduced by Leon Brillouin. Based on the directions of propagation, and propagation constants along those directions and lattice of periodic structures, Brillouin identified zones to cluster directions of propagations. These zones are studied in detail in [89]. The 2D periodic structures studied in this thesis are symmetric with respect to x and y axes and have a square lattice structure. According to Brillouin, all possible directions of propagation in x and y directions can be combined in an *irreducible Brillouin zone* [90]. The irreducible zone for the studied structures is a triangular wedge with $(1/8)^{\text{th}}$ the area of a square, i.e. the full Brillouin zone. The method of selection of the triangular irreducible Brillouin zone from the deducted square reciprocal lattice has been discussed in detail in [90]. Figure 4.11 shows a diagram of the irreducible Brillouin zone. This

figure shows all the possible values of phase constants along x and y directions. For plotting a 2D dispersion equation such as Equation (4.31), the possible values for β_x and β_y are found from the limits set by the irreducible Brillouin zone.

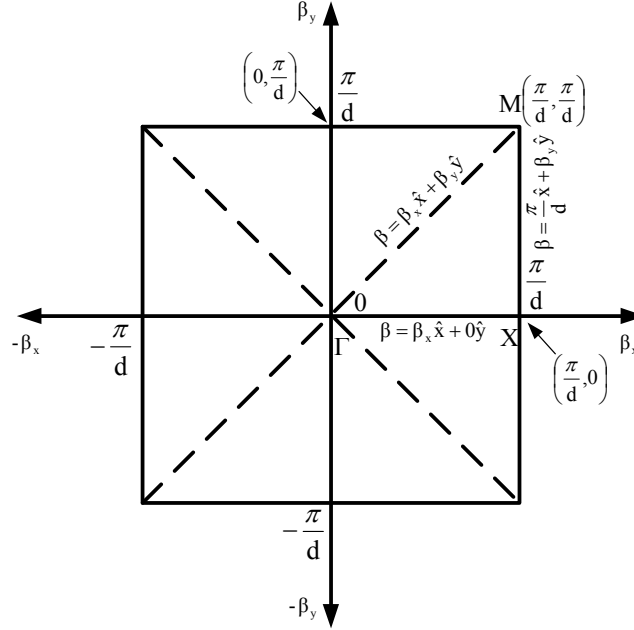


Figure 4.11: Irreducible Brillouin zone of an isotropic square reciprocal lattice with lattice constant d

Plotting the 2D dispersion diagram involves three major steps [91]

- The phase constant along the perpendicular direction y is fixed at 0 degrees. Correspondingly, the phase constant along x direction, the base of the Brillouin triangle, called $\Gamma - X$ direction, is varied from 0 to 180 degrees. Then, from numerical simulations or calculations of the dispersion equation the first set of Eigen mode frequencies are found. This generates the frequency dataset for the wave propagation in the $\Gamma - X$ segment of the Brillouin triangle.
- The phase constant along the $\Gamma - X$ segment of the Brillouin triangle, x direction, is then fixed at 180 degrees and the phase constant along the $X - M$ segment, y

direction, is varied from 0 to 180 degrees. Then, from simulations or calculation of dispersion equation the second set of Eigen mode frequencies are found

- The phase constant along both the segments, $\Gamma - X$ and $X - M$, are varied in synchronization with each other from 180 to 0 degrees. Hence, the third set of eigenmode frequencies are found for the $M - \Gamma$ segment of the Brillouin triangle.

Generally dispersion diagrams also contain the light line which indicates the propagation behaviour of light in $\Gamma - X$ and $M - \Gamma$ directions.

The dispersion diagram can be obtained from the numerical codes with eigen value solvers. The dispersion equations provided in this thesis are found from transmission line models thus predict the TM modes and lose accuracy as the frequency goes up. The numerical methods are more accurate as they provide eigen values for all possible modes, TE and TM. Ansoft HFSS or CST Microwave Studio Suite are the only popular CAD tools with such capability. CST Studio Suite seems to be more preferred over HFSS for closed structures as it is comparatively faster. However, the drawback associated with CST Microwave Studio Suite is its inability to simulate open structures which are very essential for antenna applications. Ansoft HFSS can predict the dispersion diagram for both open and closed structures, although it can be a bit tedious and requires a clear insight into the method of obtaining the dispersion diagram. This enables the user to optimize the results and obtain accurate results. A detailed approach for generating dispersion diagrams with Ansoft HFSS is discussed in [76]. Once the dispersion diagram is obtained, the filtering characteristics are predicted from the bandgap regions between the $k - \beta$ curves of various modes for all the phase combinations according to the irreducible Brillouin zone.

As indicated earlier in 2D dispersion diagrams the light lines are plotted in regions. They are very important in the sense that they are the boundaries of slow-wave (surface wave) region. The slope of these light lines marks the propagation velocity. For the waves bound by the substrate (surface waves), the speed is less than that of a light propagating in a homogeneous medium. Hence, in the 2D transmission line based structures studied in this thesis, the waves are bound to the transmission line.

4.4 Planar EBG Structures Based on Stepped Impedance Design

Planar EBG structures are via-less structures. They have the distinct advantage of the ease of fabrication over structures containing via-holes (Textured EBG structures). Planar EBG structures essentially consist of a patterned power/ground plane backed by a layer of dielectric substrate that may be grounded. A surface wave can easily propagate through such structures owing to their behaviour as a dielectric slab waveguide [92]. Planar EBG structures have been investigated in [1], [3], [93]-[95] .

In the following sections, the studied 1D and 2D EBG structures which are based on a stepped impedance structure are described and their design parameters are presented. Dispersion diagram of these structures are generated following the procedure explained earlier. The studied geometries has DC connectivity and can be considered to have a low pass configuration, hence, a passband exists at low frequencies.

4.4.1 Studied 1D Stepped Impedance Structure

A 1D stepped impedance filter is designed based on the lookup tables given in [13]. It can be realized by cascading alternate high and low impedance sections. This in turn can be achieved by either introducing a variation in the widths of the sections or a variation in the effective dielectric constant [96] or a combination of both. In the present studied case, a stepped impedance filter uses wide microstrip lines to approximate shunt capacitors and narrow lines to approximate series inductors in order to provide a lowpass frequency response. Two such stepped impedance filter designs are investigated in [36]. In this thesis Design A is considered for evaluations. Design A has the following parameters: $W_1 = 10$ mm, $W_2 = 0.5$ mm and $g = 6$ mm. Rogers RO3010 with a thickness h of 1.27 mm is used as the substrate of choice with thickness t of the copper cladding being 0.017 mm. Hence, the total length of the edges of the unit cell a is 16 mm. Figure 4.12 shows the layout of Design A with its equivalent transmission line circuit.

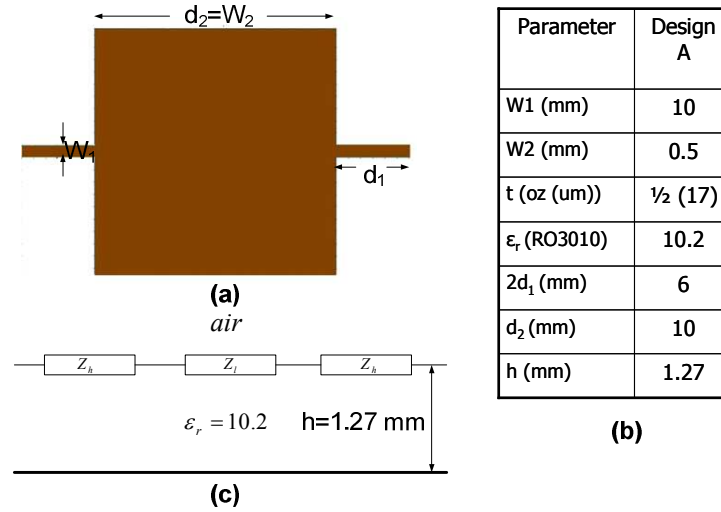


Figure 4.12: (a) Unit cell of a one dimensional stepped impedance filter, (b) Design parameters, and (c) The equivalent transmission line model.

Equation (4.17) is the dispersion equation for this planar 1D structure. Based on this, the dispersion diagram for the design is generated and compared with its corresponding dispersion diagram obtained using a full wave solver.

Since the structure is periodic in one dimension, propagation vector assumes the form $k = k_x \hat{x}$. Phase is varied along x-axis, $\Gamma - X$ direction, from 0 to 180 degrees and the corresponding eigenfrequencies are obtained. Figure 4.13 (a) and Figure 4.13 (b) depict the dispersion diagrams obtained using the code developed based on calculation of Equation (4.17) (let us refer to this as transmission line modeling method or TLM for brevity) and by using a full wave solver, respectively.

From the two figures it can be observed that the dispersion diagram obtained from the full wave solver predicts a higher number of modes. Plotting the light line divides this diagram into two distinct regions known as the fast wave region (lying above the light line) and the slow wave region (lying below the light line). The portion of the eigenmodes lying above the light line represents the portion of the eigenmodes that are not bound to the surface and are radiative in nature. These waves are known as *leaky*

waves [28]. The modes that lie above the light line can radiate energy away by coupling to the external plane waves (when the EBG surface is used as reflector in antenna applications). They are radiatively unstable and have an imaginary frequency component. Below the light line are the waves that are bound to the substrate and closer to what is predicted by the transmission line modeling method.

4.4.2 Predicting the Insertion Loss Information from the Dispersion

Equation

By modifying the TLM based code, the S_{21} -parameter characteristic of a finite size structure composed of n unit cells can be predicted. This can be achieved by the following relation

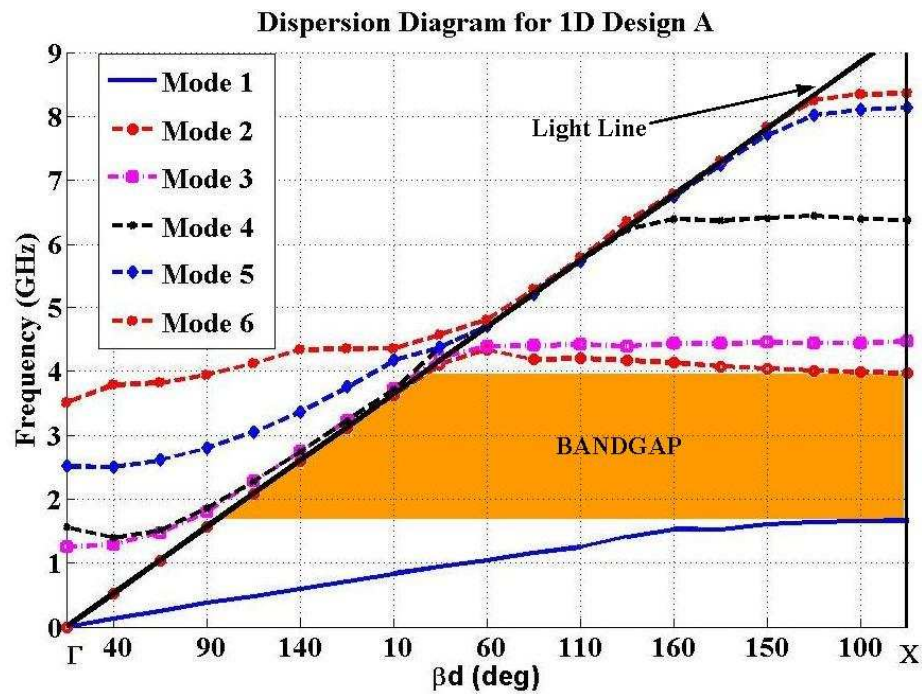
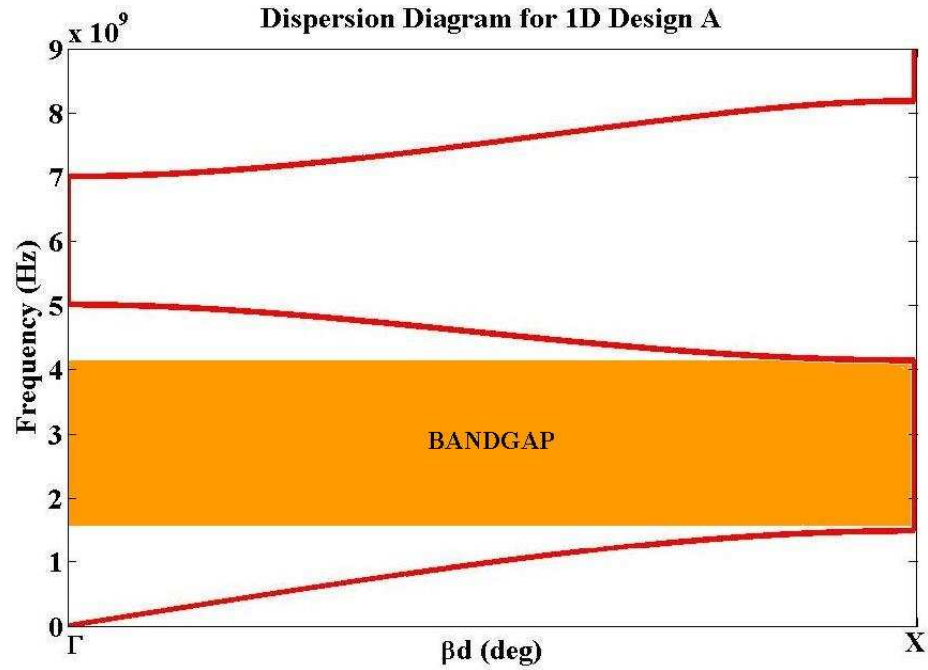
$$S_{21} = 20 \log \left(e^{-\alpha_{(f)} \hat{r} \, nd} \right) \quad (4.33)$$

where n is the number of unit cells included between the two measurement ports, $\alpha_{(f)}$ is the frequency dependent attenuation constant and d is the lattice constant and S_{21} is expressed in decibel. The attenuation constant for the different frequency points is in fact obtained from the complex propagation constant found from Equation (4.17). Basically it is β in the stopband.

To obtain other scattering parameters one can obtain the equivalent transfer matrix of the n unit cells by cascading the transfer matrices of all the cells between the ports converting them to scattering parameter matrix.

The insertion loss plots obtained by different methodologies, from full-wave HFSS, ADS Momentum and TLM method are depicted in Figure 4.13 (c). It can be observed that a very good match is found between the results obtained using the TLM based code and the full wave analysis.

Table 4.2 presents the numerical values of the stopband predicted from the results shown in Figure 4.13 (a) and (b).



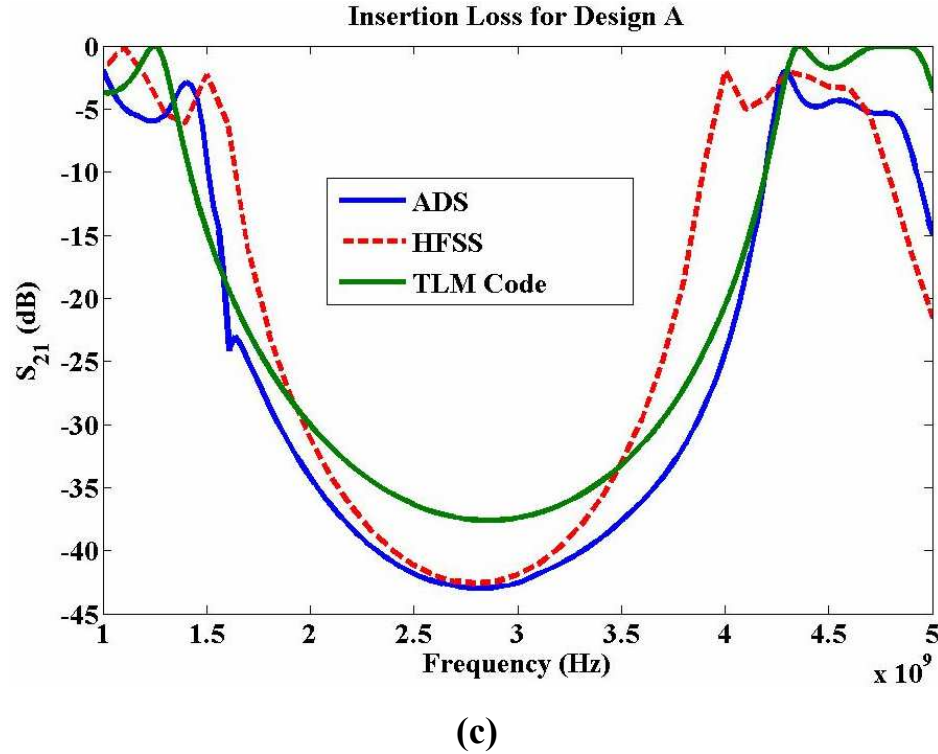


Figure 4.13: (a) Dispersion diagram from TLM based code, (b) Dispersion diagram from full wave solver (c) Insertion loss for a finite structure consisting of three unit cells

Table 4.2: Comparison between the stopbands found from TLM code and full wave solver (HFSS) results for 1D stepped impedance based EBG design

EBG geometry	Bandgap from TLM based code (BW_{code}) (GHz)	Bandgap from full wave solver (HFSS) ($BW_{full\ wave}$) (GHz)	Time to simulate the results using TLM based code	Time to simulate the results using HFSS	Percentage match in bandgap $\left(\frac{BW_{full\ wave}}{BW_{code}}\right) \times 100$
Design A	1.49~4.149	1.662~3.969	107 seconds	1 hour and 52 minutes	86.8

4.4.3 Studied 2D Stepped Impedance Structure

Having analysed the one dimensional stepped impedance filter, the same design specifications are extended to a two dimensional geometry. In fact the studied layout is

the structure shown in Figure 4.9. The analysis of the structure follows the same pattern as discussed for a one dimensional stepped impedance filter with the sectioning of the unit cell being the only exception. The one dimensional structure consisted of 3 sections for the analysis whereas the two dimensional structure, owing to its symmetry about the x- and y-axes, is split into four sections. The same method could have been applied to the one dimensional structure too, considering the unit cell is symmetrical about a transverse axis (in our case the y-axis) passing through the center of the unit cell. However, for ease of calculation, a three section method was followed. Figure 4.14 shows the cross section of a transmission line model for the unit cell of this 2D structure along one direction to show the segmentation used for the analysis. The four port network representation of the unit cell is shown in Figure 4.10.

Employing the methodology discussed in section 4.2.2, the dispersion equation for the planar 2D EBG structure shown in Figure 4.9 is given by

$$\cos(\beta_x a) + \cos(\beta_y a) = 2\cos(\beta_d a) + \left(2 - \frac{Z_l}{Z_h} - \frac{Z_h}{Z_l}\right) \sin(\beta_d W_1) \sin(\beta_d (a - W_1)) \quad (4.34)$$

where Z_l and Z_h are the characteristic impedances of the broad and narrow transmission line sections, respectively. The effective dielectric constant and characteristic impedance of a microstrip line is given by (4.1) and (4.2), respectively.

The stopband obtained by this EBG structure can be considered to be omni-directional as it occurs in all azimuthal directions of propagation.

Figure 4.15 and Figure 4.16 present a comparison between the dispersion diagrams obtained using the TLM method and a full wave solver (HFSS), respectively. Table presents the numerical values of the bandgap predicted from the results shown in Figure 4.15 and Figure 4.16

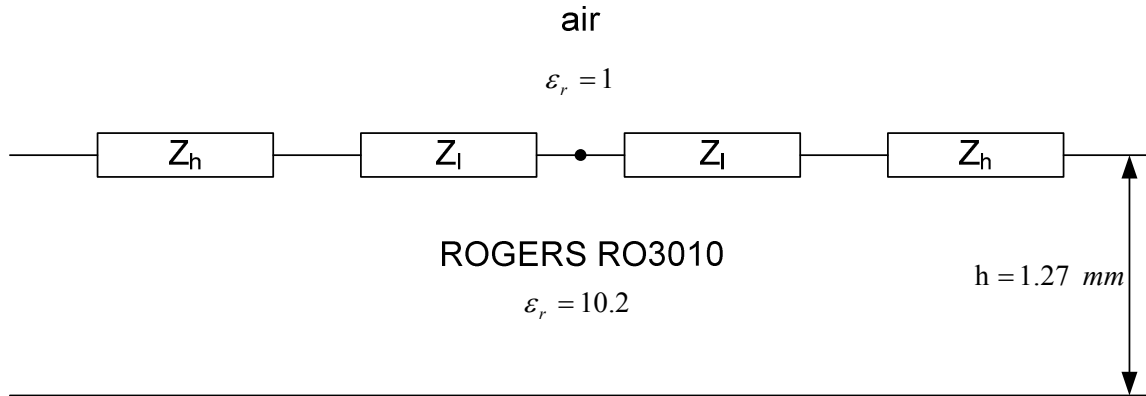


Figure 4.14: Cross section of the TL model of the 2D stepped impedance EBG structure

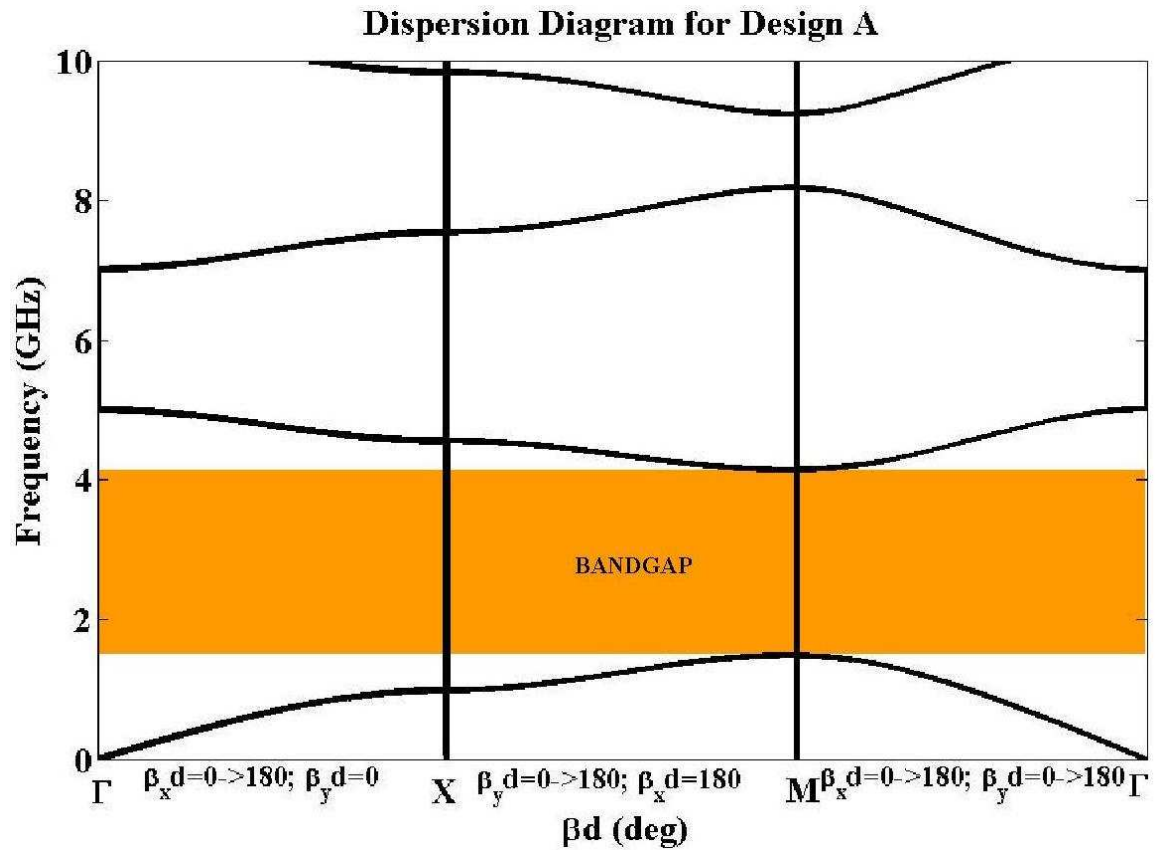


Figure 4.15: Dispersion diagram of the 2D stepped impedance structure based on Design A predicted by the TLM code

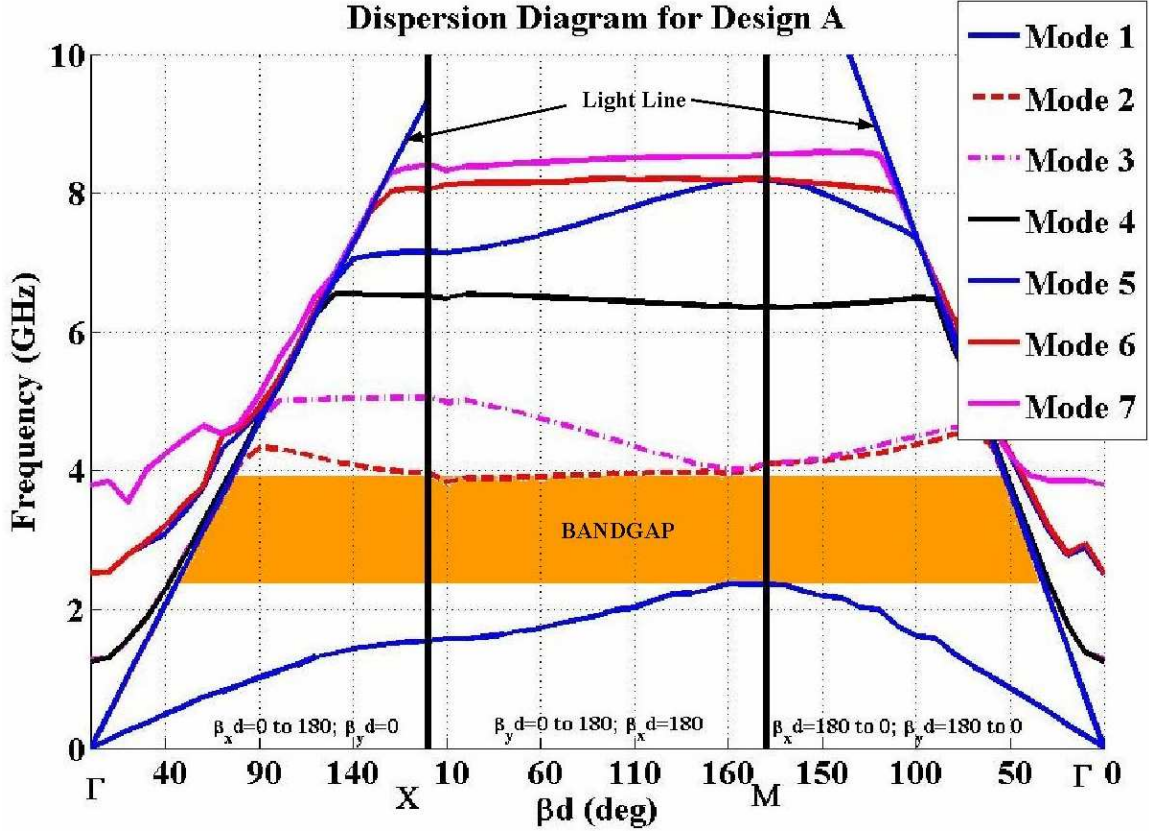


Figure 4.16: Dispersion diagram of the 2D stepped impedance structure based on Design A obtained from HFSS full-wave solver

4.4.4 Insertion Loss Simulation and Measurement

The insertion loss information is obtained from the scattering parameter S_{21} . In order to extract the values of S_{21} using the TLM method, the same approach that has been applied to the 1D EBG design is employed for the 2D EBG design. This involves generating the equivalent transfer matrix for n unit cells between two ports on the EBG structure and then carrying out the conversion from the transfer matrix to the scattering parameter matrix. Once the scattering parameter matrix is obtained, the S_{21} values are easily extracted from it.

Another method to obtain the scattering parameters is by conducting a measurement on the fabricated structure. The EBG structure used for insertion loss simulation and

measurement, is shown in Figure 4.17. The figure also denotes the location of the vertical SMA connectors used for excitation. Anritsu's 37397D Vector Network Analyzer (VNA) (Lightning, 40MHz to 65 GHz) is used for measuring the scattering parameters and from the recorded data, the S_{21} information is extracted and plotted.

The insertion loss plot in Figure 4.18 depicts the results obtained from using different methods, including full wave simulation results, along with the S_{21} results of a parallel-plate structure which is used as a reference. Port 1 is located at (40 mm,72 mm) and Port 2 is located at (120 mm,72 mm).

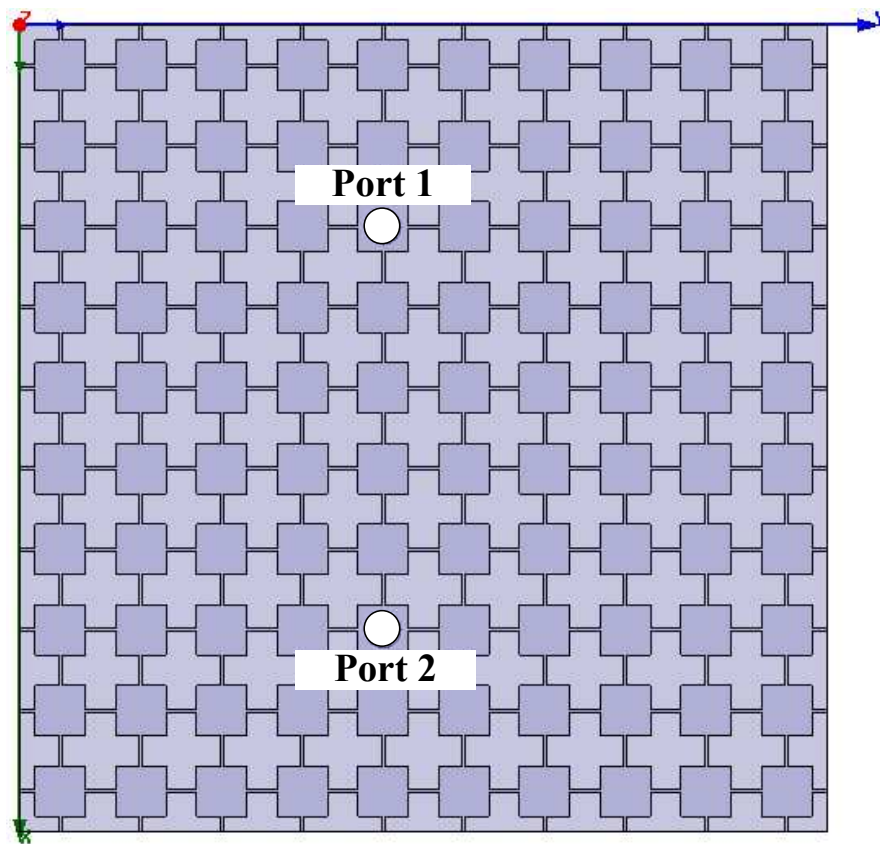


Figure 4.17: Illustration of the EBG structure constituted of several cascaded unit cells along with the port definitions for the calculation of the Insertion Loss

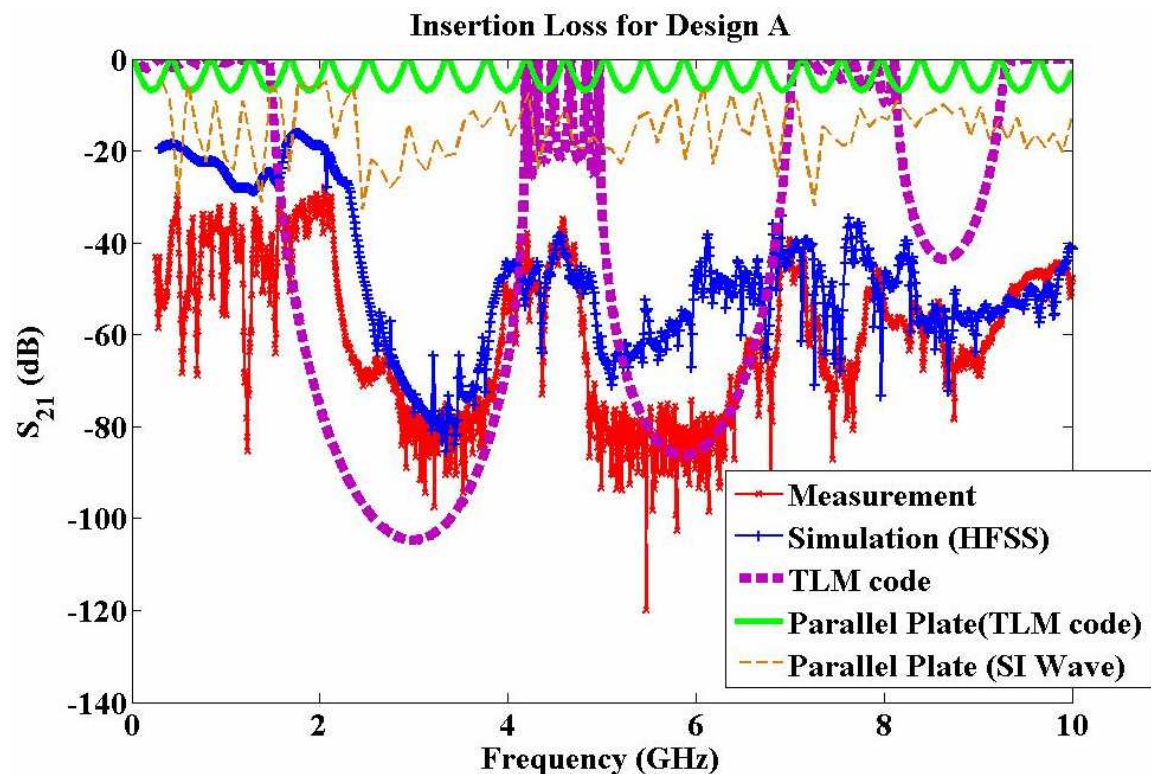


Figure 4.18: Magnitude of S_{21} for finite size stepped impedance structure for the arrangement shown in Figure 4.17 and calculated between 28 segments along x-direction

Table 4.3: Comparison between the TLM code and full wave solver results

EBG geometry	Bandgap from TLM based code (BW_{code}) (GHz)	Bandgap from full wave solver HFSS ($BW_{full\ wave}$) (GHz)	Time to simulate the results using TLM based code	Time to simulate the results using HFSS	Percentage match in bandgap $\left(\frac{BW_{full\ wave}}{BW_{code}}\right) \times 100$
Design A	1.49~4.151	2.368~3.836	115 seconds	3 hours and 36 minutes	55.1672

4.5 Conclusion

In this chapter, we analysed the behaviour of a microstrip line as this is the TL of our choice for realising the various EBG geometries discussed in this thesis. We also introduced the concept of using transmission line model (TLM) for the analysis of 1D and 2D two conductor EBG structures. The methodology behind the analysis was discussed in detail. We discussed the method of extraction of the modal characteristics of the EBG structures discussed in this thesis within a defined irreducible Brillouin zone. A stepped impedance based EBG structure was investigated in order to validate the results obtained using the TLM based code and verify its accuracy with respect to results obtained from full wave analysis.

The only drawback with the TL modeling is that only TM (Transverse Magnetic) modes of propagation are captured but not the TE (Transverse Electric) modes.

CHAPTER 5

Design and Modeling of Meander lines

As discussed in Section 3.3 of Chapter 3, an EBG structure can be approximated by an effective surface impedance and the centre frequency can be estimated by $\sqrt{LC}$ and the width of the bandgap is related to $\sqrt{L/C}$. Hence, it can be inferred that in order to achieve a bandgap at lower frequencies with a wide bandwidth, methods for increasing the effective inductance of the unit cell should be employed. In the planar EBG structures reported in [3], [38], [94], [95], a meander line is used to connect microstrip line sections, instead of the high impedance transmission line section shown in Figure 4.9 of Chapter 4. The meander line increases the inductance of the unit cell which in turn reduces the lower edge of the bandgap. Application of this method results in the achievement of ultra-wideband noise suppression as reported in [3].

In this chapter we review the methodology behind obtaining the lumped component model of a meander line so that it may be effectively employed in a TLM of an EBG structure consisting of metal patches connected by meander lines.

5.1 Inductance of the Meander Line

The layout of a typical meander line is shown in Figure 5.1. To extract the equivalent inductance of a meander line, its layout is decomposed into straight conductive segments. The total inductance is found from the sum of the self inductances of all the segments and the mutual inductances between all combinations of the straight segments.

5.1.1 Calculation of Self Inductance

The self inductance of a straight conductor with a rectangular cross section as shown in Figure 5.2 is given in [97], [98] by the following formula

$$L = 0.002 \cdot l \cdot \left\{ \ln \left[\frac{2l}{w+t} \right] + 0.50049 + \left[\frac{w+t}{3l} \right] \right\} \quad (5.1)$$

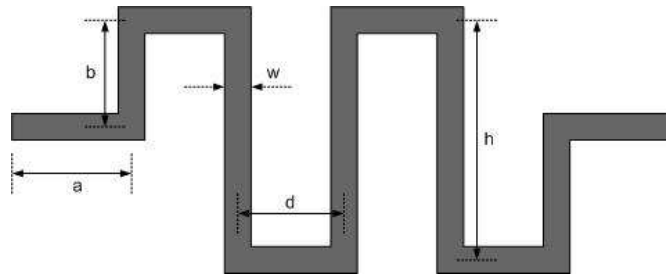


Figure 5.1: Various segments of a meander line for the calculation inductance

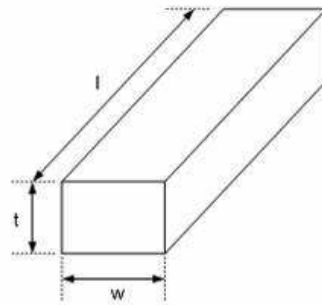


Figure 5.2: A conducting segment of a meander line

where L is the inductance in μH , l is the length, w is the width and t the thickness of the conducting segment in centimeters. The above equation is valid for materials with a magnetic permeability equal to unity and the inductance is assumed to be frequency independent. The closed form equation provides a cumulative self inductance due to both, the internal and the partial external inductance of an element.

As an example let us consider the meander inductor geometry shown in Figure 5.1

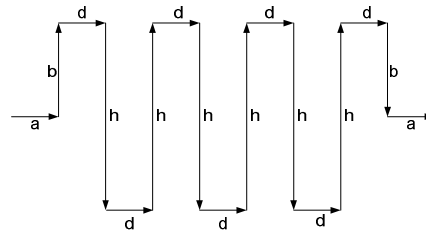


Figure 5.3: Middle segments used for the calculation of inductances

The total self inductance of all the segments is given by

$$L_{\text{selftot}} = 2L_a + 2L_b + NL_h + (N+1)L_d \quad (5.2)$$

where $L_{a,b,h,d}$ are self inductances of segments with lengths $l = a, b, h, d$ respectively.

N- Number of segments of greatest length h (which for the geometry shown in Figure 5.3 and Figure 5.3 is 6)

5.1.2 Calculation of Mutual Inductance

The general equation for the mutual inductance of the segments with equal length l and separated by a distance r is given by

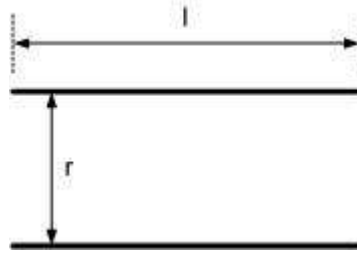


Figure 5.4: Mutual inductance of two equal parallel straight filaments

$$M_c(l,r) = \pm \frac{\mu_0}{2\pi} \cdot l \cdot \left[\ln \left(\frac{l}{r} + \sqrt{1 + \left(\frac{l}{r} \right)^2} \right) - \sqrt{1 + \left(\frac{r}{l} \right)^2} + \left(\frac{r}{l} \right) \right] \quad (5.3)$$

Mutual inductance between unequal filaments intersecting at a point at an angle θ is given by [97]

$$M = 0.002 \cos(\theta) \left[x \tanh^{-1} \left(\frac{y}{x+S} \right) + y \tanh^{-1} \left(\frac{x}{y+S} \right) \right] \mu H \quad (5.4)$$

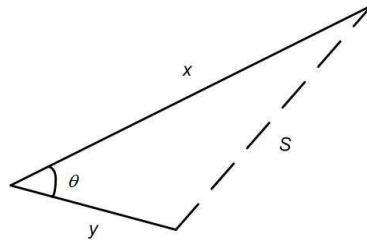


Figure 5.5: Two intersecting unequal filaments inclined at an angle θ [97]

Mutual inductance between two unequal filaments in the same plane not meeting at any point is given by [97]

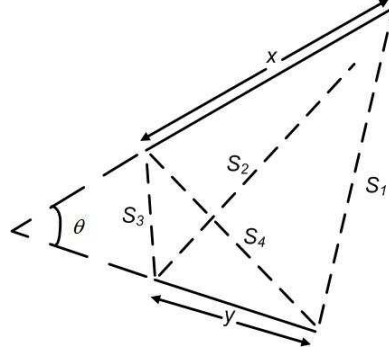


Figure 5.6: Two non-intersecting unequal filaments inclined at an angle θ [97]

$$M = 0.002 \cos(\theta) \left[(\mu + x) \tanh^{-1} \left(\frac{y}{S_1 + S_2} \right) + (\nu + y) \tanh^{-1} \left(\frac{x}{S_1 + S_4} \right) - (\mu) \tanh^{-1} \left(\frac{y}{S_3 + S_4} \right) - (\nu) \tanh^{-1} \left(\frac{x}{S_2 + S_3} \right) \right] \quad (5.5)$$

where

$$\mu = \frac{[2y^2(S_2^2 - S_3^2 - x^2) + \alpha^2(S_4^2 - S_3^2 - y^2)]x}{4x^2y^2 - \alpha^4} \quad (5.6)$$

$$\nu = \frac{[2x^2(S_4^2 - S_3^2 - y^2) + \alpha^2(S_2^2 - S_3^2 - x^2)]y}{4x^2y^2 - \alpha^4} \quad (5.7)$$

$$\alpha^2 = S_4^2 - S_3^2 + S_2^2 - S_1^2 \quad (5.8)$$

From (5.4) and (5.5), we conclude that mutually perpendicular segments exhibit no mutual inductance. Figure 5.1 shows the meander line geometry in addition to the various parameters that were used for the calculation of inductance.

Mutual inductance of other segments can be expressed using the above expressions. In [98] various scenarios are discussed. The expressions for calculating the mutual inductance in each case are presented herein.

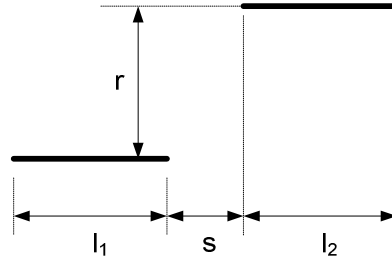


Figure 5.7: Two parallel segments displaced by a finite distance

For the arrangement shown in Figure 5.7, the mutual inductance between the two segments is given by

$$M_{a1}(l_1, l_2, r, s) = 0.5 \cdot [M_c(l_1 + l_2 + s, r) + M_c(s, r) - M_c(l_1 + s, r) - M_c(l_2 + s, r)] \quad (5.9)$$

For the arrangement shown in Figure 5.8, the mutual inductance between the two segments is given by

$$M_{a2}(l_1, l_2, r) = 0.5 \cdot [M_c(l_1, r) + M_c(l_2, r) - M_c(l_1 - l_2, r)] \quad (5.10)$$

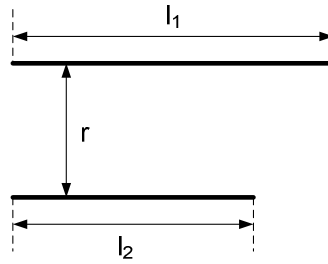


Figure 5.8: Two unequal overlapping parallel segments with one end along the same perpendicular

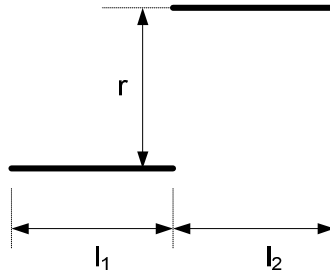


Figure 5.9: Two non-overlapping parallel segments with one end along the same perpendicular

For the arrangement shown in Figure 5.9, the mutual inductance between the two segments is given by

$$M_{a3}(l_1, l_2, r) = 0.5 \cdot [M_c(l_1 + l_2, r) - M_c(l_1, r) - M_c(l_2, r)] \quad (5.11)$$

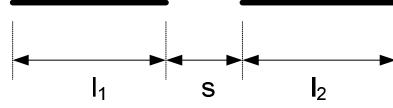


Figure 5.10: Two non-overlapping segments sharing the same axial plane and separated by a finite distance

For the arrangement shown in Figure 5.10, the mutual inductance between the two segments is given by

$$M_b(l_1, l_2, s) = \frac{\mu_0}{4\pi} \cdot [(l_1 + l_2 + s) \cdot \ln(l_1 + l_2 + s) - (l_1 + s) \cdot \ln(l_1 + s) - (l_2 + s) \cdot \ln(l_2 + s) + s \cdot \ln(s)] \quad (5.12)$$

Equations (5.9)-(5.12) provide the four basic expressions that are essential for the calculation of the various cases of mutual inductance in a meander line.

The following text provides more specific examples of these calculations for a meander line and their contribution to the ultimate equivalent inductance.

Case I

Mutual inductance between two opposite short sections joining the longer segments (see Figure 5.11)

$$M_1 = \sum_{i=1}^{N/2} (2N + 4 - 4i) M_{a1}(d, d, h, (2i - 2)d) ; \text{ If } N \text{ is even} \quad (5.13)$$

$$M_1 = \sum_{i=1}^{(N+1)/2} (2N + 4 - 4i) M_{a1}(d, d, h, (2i - 2)d) ; \text{ If } N \text{ is odd} \quad (5.14)$$

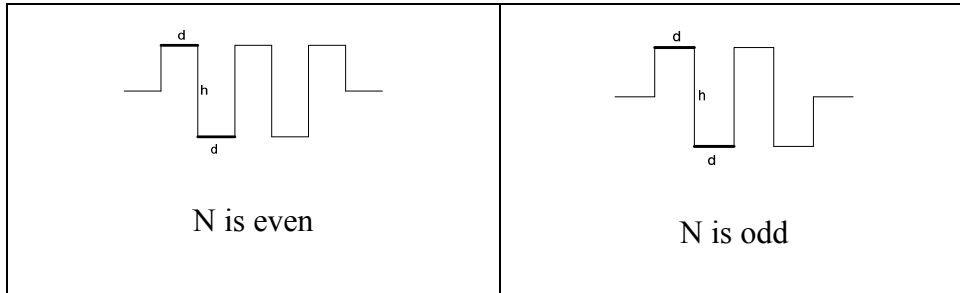


Figure 5.11: Two scenarios for Case I

For the case when $i=1$, the right hand side (RHS) of expression (6.13) becomes

$$2N \cdot M_{a1}(d, d, h, 0) \quad (5.15)$$

It can also be observed that,

$$M_{a1}(d, d, h, 0) \rightarrow M_{a3}(d, d, h) \quad (5.16)$$

Case II

Mutual inductance between two adjacent short sections joining the longer segments (see Figure 5.12)

$$M_2 = \sum_{i=1}^{N/2} (2N + 2 - 4i) M_b(d, d, (2i - 1)d) ; \text{ If } N \text{ is even} \quad (5.17)$$

$$M_2 = \sum_{i=1}^{(N-1)/2} (2N + 2 - 4i) M_b(d, d, (2i - 1)d) ; \text{ If } N \text{ is odd} \quad (5.18)$$

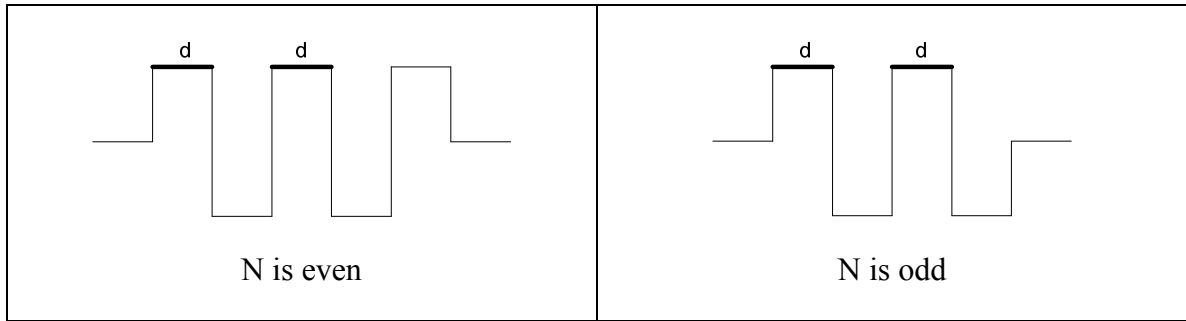


Figure 5.12: Two scenarios for Case II

Case III

Mutual inductance between the two end sections (refer Figure 5.13)

$$M_3 = 2 \cdot M_b(a, a, (N + 1)d) \quad (5.19)$$

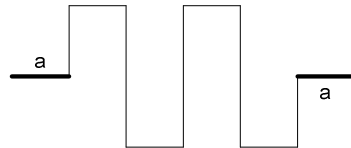


Figure 5.13: Segment alignment for Case III

Case IV

Mutual inductance between the end sections and the short sections joining the longer segments (see Figure 5.14)

$$M_4 = \sum_{i=0}^N 4 \cdot M_{a1}(a, d, b, id) \quad (5.20)$$

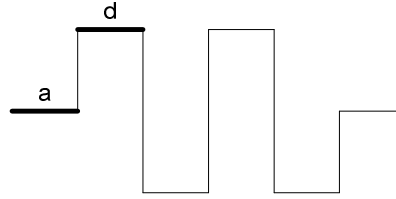


Figure 5.14: Segment alignment for Case IV

For the case when $i=1$, the RHS of expression (5.20) becomes

$$4 \cdot M_{a1}(a, d, b, 0) \quad (5.21)$$

It can also be observed that,

$$M_{a1}(a, d, b, 0) \rightarrow M_{a3}(a, d, b) \quad (5.22)$$

Case V

Mutual inductance between the longer segments (in Figure 5.15)

$$M_5 = \sum_{i=0}^{N-1} (-1)^i \cdot 2 \cdot (N - i) \cdot M_c(h, id) \quad (5.23)$$

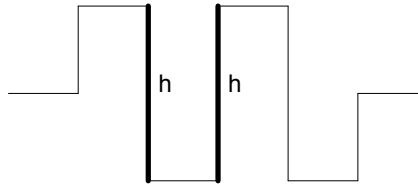


Figure 5.15: Segment alignment for Case V

For the case when $i=0$, the RHS of expression (5.23) becomes

$$(-1)^0 \cdot 2 \cdot (N - 0) \cdot M_c(h, 0) \quad (5.24)$$

If we observe closely we can notice that,

$$(-1)^0 \cdot 2 \cdot (N - 0) \cdot M_c(h, 0) \rightarrow \infty \quad (5.25)$$

The reason for this observation is the fact that if ‘r’ becomes zero, then the case becomes equivalent to the self inductance of the conducting segment.

Case VI

Mutual inductance between the two semi-long segments, i.e. segments with half the length of the long segments, at either end (refer Figure 5.16)

$$M_6 = -2 \cdot M_c(b, (N + 1)d) ; \text{ If } N \text{ is even} \quad (5.26)$$

$$M_6 = +2 \cdot M_{a3}(b, b, (N + 1)d) ; \text{ If } N \text{ is odd} \quad (5.27)$$

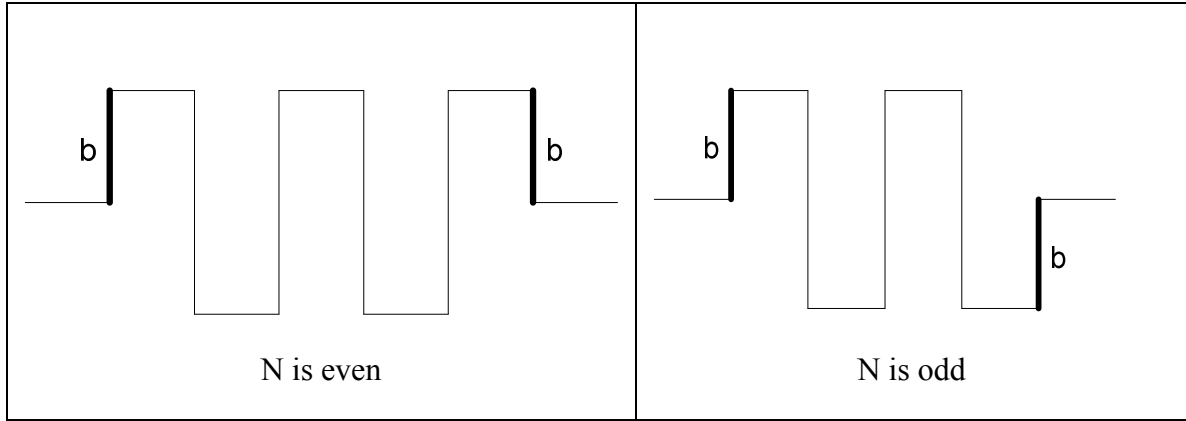


Figure 5.16: Segment alignment for Case VI

Case VII

Mutual inductance between a semi long segments and a long segment as shown in Figure 5.17 is

$$M_7 = \sum_{i=0}^N (-1)^i \cdot 4 \cdot M_{a2}(b, h, id) \quad (5.28)$$

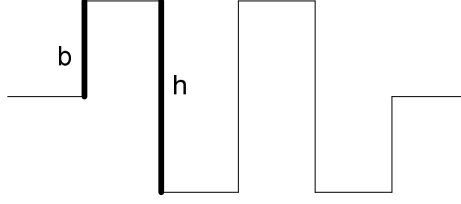


Figure 5.17: Segment alignment for Case VII

For the case when $i=0$, the RHS of equation (5.28) becomes

$$(-1)^0 \cdot 4 \cdot M_{a2}(b, h, 0) \quad (5.29)$$

A close observation of this expression reveals that

$$(-1)^0 \cdot 4 \cdot M_{a2}(b, h, 0) \rightarrow \infty \quad (5.30)$$

This is due to the fact that when distance 'r' becomes zero, the case is equivalent to the self inductance of a conducting segment.

The total mutual inductance due to all the segments is calculated by the following summation

$$M_{\text{tot}} = M_1 + M_2 + M_3 + M_4 + M_5 + M_6 + M_7 \quad (5.31)$$

The total inductance of the meander line geometry is found from,

$$L_{\text{tot}} = L_{\text{selftot}} + M_{\text{tot}} \quad (5.32)$$

where L_{tot} is the total inductance of the meander line in μH

L_{selftot} is the total self inductance of the meander line in μH

M_{tot} is the total mutual inductance due to all the segments in μH

It should be point out when current flows in the various segments it may have opposite directions in some parallel segments. This co-directional and contra directional currents affect the sign of the mutual inductance. This is already accounted for in the expressions. For example, for the meander design in Figure 6.1, M_5 corresponds to a negative value and for the meander design in Figure 6.2, both M_5 and M_7 have negative values.

In order to verify the analytical approach described herein for calculation of inductance, field simulations are conducted using Ansoft Q3D. Three meander lines with various number for segments with the layout parameters shown in Table 5.1 are simulated in Q3D. A MATLAB code is written that calculates the value total inductance using the aforementioned analytical formulas.

Table 5.1 presents the calculated and simulated results for the meander structures. It can be inferred that there is an excellent match between the inductance values obtained from the code and those obtained through field simulation.

Table 5.1: Inductance values for meander line designs with different number of turns

Design Number	Number of Segments	w (μm)	d (μm)	L_{measured} Extracted from Q3D (nH)	L_{Tot} Calculated (nH)
1	5	40	40	1.5	1.3
2	8	24	24	2.0	2.2
3	11	17	17	2.5	2.8

5.1.3 Capacitance of a Meander Line

In order to predict the behavior of a meander line with more accuracy, parasitic capacitances should also be accounted for in modeling. The parasitic capacitance effect arises due to the capacitance between the conducting segments of the meander line as well as due to the capacitance between the meander signal line and the bottom ground plane for which the substrate behaves as a dielectric [3], [5]. The total stray capacitance can be calculated from the equation presented in [99] and repeated here as follows:

$$C_p = l_{\text{tot}} \left[\epsilon_0 \epsilon_r \frac{2\pi}{\ln\left(1 + 2h/t + \sqrt{2h/t(2h/t + 2)}\right)} + \epsilon_0 \epsilon_r \frac{w - t/2}{h} \right] \quad (5.33)$$

where l_{tot} is the total length of the meander line, t is the thickness of the conductor, w is

the width of the conductor and h is the thickness of the substrate. It is suggested in [99], in order to model the meander line this capacitance comes in parallel with the L_{tot} component calculated in Section 5.1

5.2 Conclusion

In this chapter, a schema for improving the bandgap of the EBG structure analyzed in Chapter 4 is studied. It involves replacing the dc link in the design with a meander line. In order to analyze the EBG structure containing meander section using the TLM based code, the equivalent model for the meander line is needed. This chapter presents a methodology for derivation of the equivalent circuit using closed-form relations. This approach and the found parameters of the equivalent circuit in this chapter are used in the design and analysis of the EBG structures presented in the next chapter.

CHAPTER 6 Planar EBG Structures Containing Meander Line Sections

In the previous chapter it was shown that a meander section can be represented by an inductor. In this chapter 1D and 2D EBG structures that contain meander sections are studied using the general approach discussed in Chapter 4.

In this chapter, we will discuss two EBG structures consisting of metal patches linked by meander lines. The analysis of such structures was carried out using the TLM method after developing the proper models and by full wave analysis. The results have been tabulated at the end of the chapter.

6.1 Unit cells with Meander Transmission Line Sections

Two meander line designs (refer to Figure 6.1 and Figure 6.2) are analyzed using the approach of Chapter 5. Figure 6.1, called Design A, is designed based on the stepped impedance structure presented in Chapter 4 (Refer Figure 4.12 (a)). Figure 6.2, called Design B is the layout of a design presented in [38].

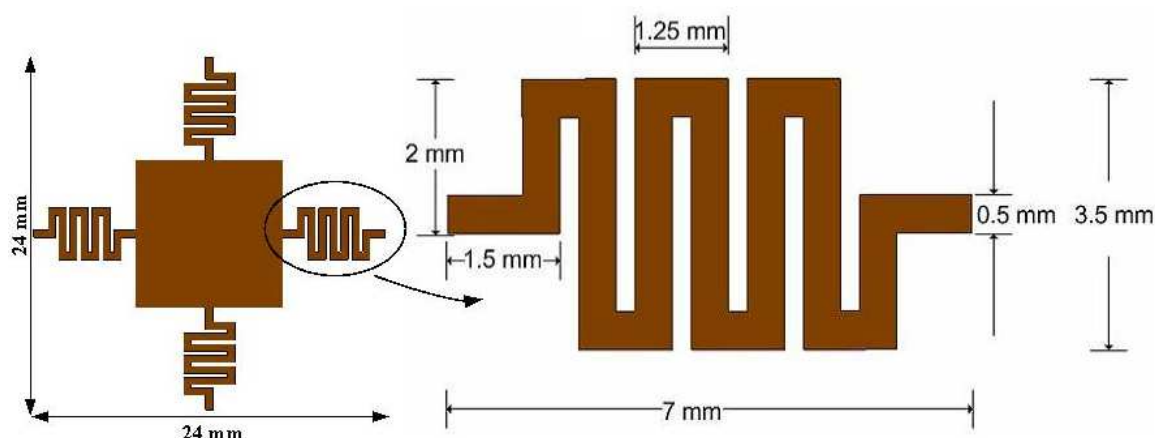


Figure 6.1: The unit cell design of an EBG structure, Design A, containing meander sections.

The first objective is the calculation of the equivalent inductance of the meander section Design A and Design B using the methodology of Chapter 5. Table 6.1 summarizes geometrical parameter values for each of the two meander line designs along with their calculated inductances

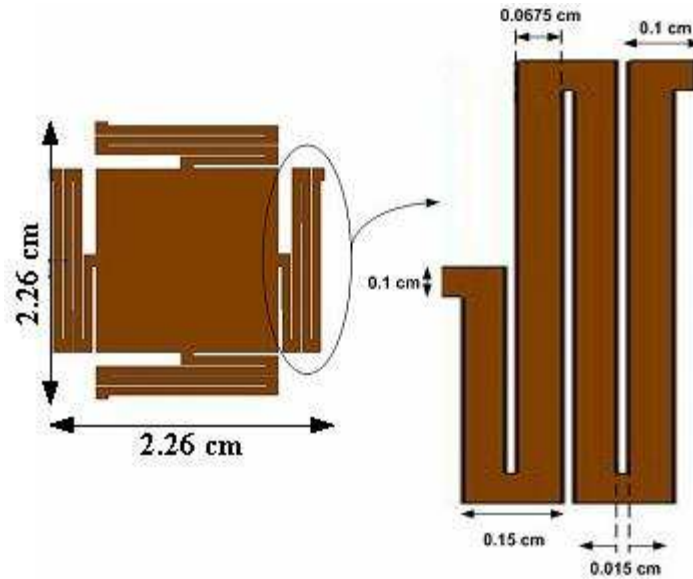


Figure 6.2: Unit cell design of the EBG structure from [38] , called here as Design B, containing meander sections.

Table 6.1: Geometrical parameters of meander lines of Design A and B and calculated inductance

Design Parameter	Design A	Design B
a (mm)	1.25	0.6625
b (mm)	1.75	7.5
d (mm)	0.75	0.825
h (mm)	3	15
t (mm)	0.017	0.017
L (nH)	13	20

When these meander sections interconnect conductor patches (the patches are seen in Figure 6.1 and Figure 6.2), a capacitive element comes in parallel with the effective inductance. In order to find this capacitance, two adjacent patches in the two consecutive unit cells should be considered. Please refer to Figure 6.3 to see the side view of adjacent patches.

The capacitance due to two adjacent co-planar metal patches results in a fringing capacitance which can be derived using conformal mapping technique as explained in [28]. If we consider two co-planar metal patches of width x surrounded by air on one side of the interface and by a dielectric substrate with relative permittivity ϵ_r on the other side, then the gap capacitance is given by the expression

$$C_g = \frac{2x\epsilon_0(1 + \epsilon_r)}{\pi} \cdot \cosh^{-1} \left[\frac{y}{2z} \right] \text{ Farad} \quad (6.1)$$

Where ‘ y ’ is the distance between the centers of the two adjacent metal patches and ‘ z ’ is the gap between the two patches as depicted in Figure 6.3.

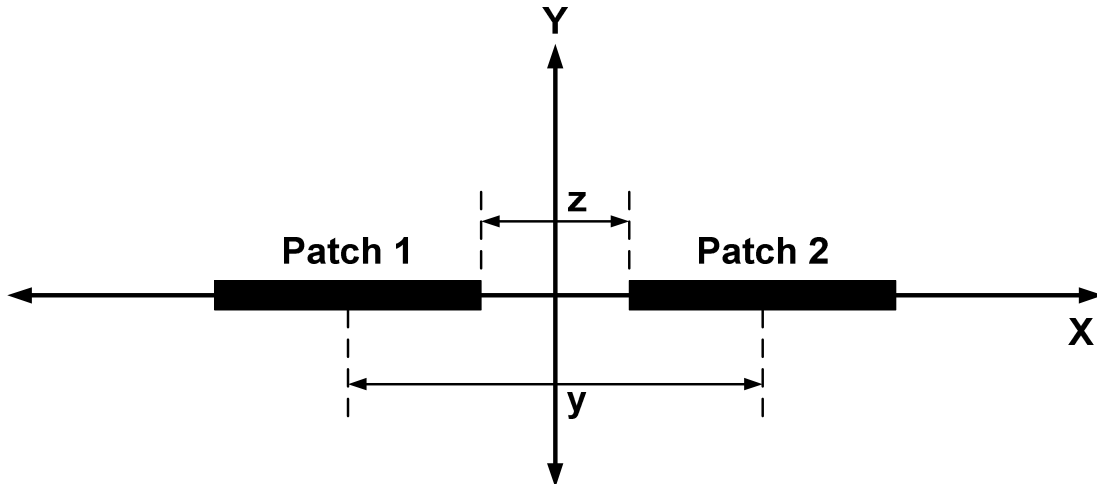


Figure 6.3: Capacitor geometry to calculate the fringe capacitance due to two co-planar metal patches [28]

Table 6.2: Parameter Values for Capacitance calculation

Design Parameter	Design A	Design B
y (mm)	17	18.8
x (mm)	10	15
z (mm)	7	3.8
C_g (pF)	0.40605	0.70735

For a preliminary and crude analysis of the EBG structure, the loading effect of the meander line can be modeled with an inductor in parallel with a capacitor. This approximate model is shown in Figure 6.4

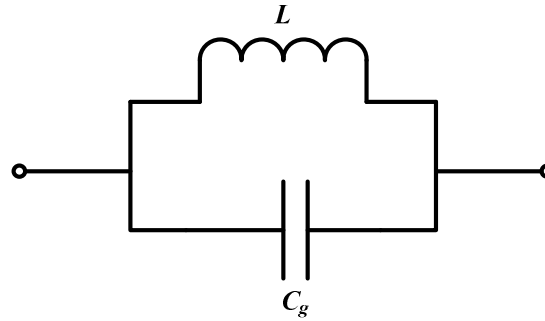


Figure 6.4: Basic model represent the meander loading.

Now we can proceed to calculate the resonant frequency of the meander line. This is of principal importance because the behaviour of an inductor changes with the resonant frequency. The resonant frequency of a meander line is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC_{gap}}} \quad (6.2)$$

For an inductance of 13 nH and parasitic capacitance of 0.40605 pF, the resonant frequency equates to 2.19 GHz.

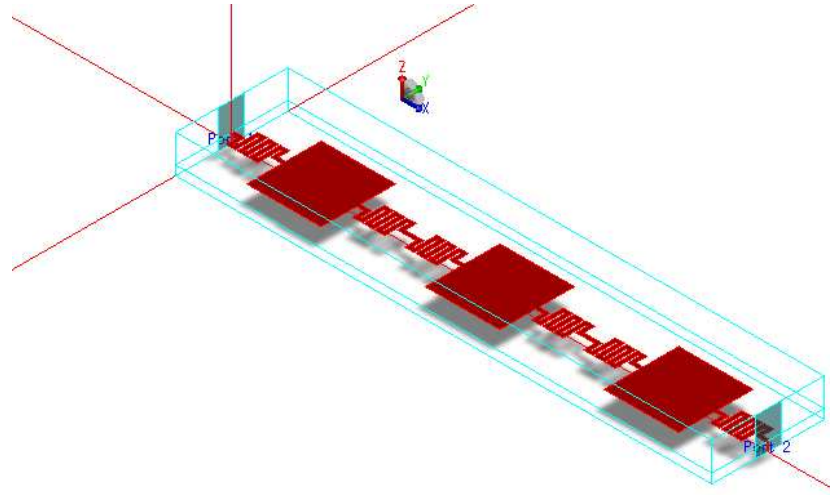
The unit cells depicted in Figure 6.1 and Figure 6.2 are used to create 1D and 2D EBG geometries in the following sections.

6.2 1D EBG Design A

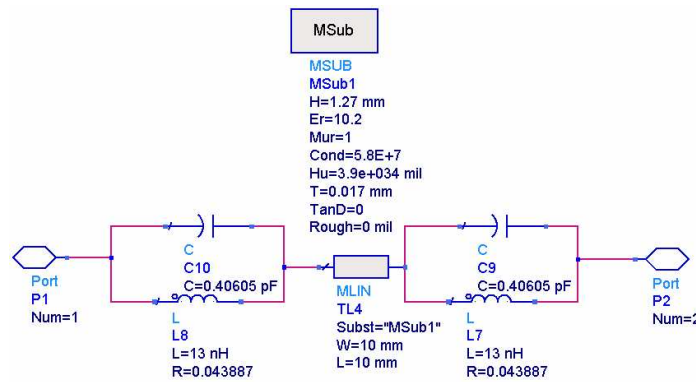
The unit cell shown in Figure 6.1 is now adjusted (only removing the meander loadings in the perpendicular direction) to create a 1D periodic structure. Two symmetric meander sections are inserted between wide microstrip line sections as depicted in Figure 6.5 (a). The meander sections load microstrip transmission lines having widths and lengths of 10 mm respectively. Figure 6.5 (b) depicts the equivalent circuit for the unit cell of this structure for use in the transmission line code.

The substrate and its thickness used for Design A is the same as the one used for the design analyzed in Chapter 4 (Rogers RO3010 which has $\epsilon_r = 10.2$ and a thickness of 1.27 mm).

A total of nine segments or 3 unit cells are used to create the 1D EBG structures. First the modified unit cell is simulated to generate the 1D dispersion diagrams from transmission line modeling and eigenvalue HFSS simulations. The results are shown in Figure 6.6 and Figure 6.7



(a)



(b)

Figure 6.5: (a) 1D meander loaded transmission line Design A. (b) The equivalent transmission line model for the unit cell used in the TLM code.

The dispersion diagram of this 1D periodic structure is created using the developed Code and HFSS Eigenvalue solver. From the dispersion diagrams we notice that the TLM

based code captures the $k - \beta$ diagrams of the first and second mode with a reasonable accuracy. HFSS result yields all the TE and TM modes dispersion results. TLM based code basically predicts only TM modes and does not provide the full modal characteristics. This causes the results from the TLM based code (Figure 6.6) to predict a stopband from 631 MHz to 4.39 GHz, while the HFSS result (Figure 6.7) shows a stopband only in the slow-wave region, below the light line, from 0.9 GHz to 1.97 GHz [28], [100], [101]. This again can be attributed to the fact that the TLM based code is not as accurate as a full-wave solver and the inclusion of the lumped element model of the meander line further compromises the accuracy of the code. However, for a fast estimation of the bandgaps, the TLM based code is a highly efficient tool.

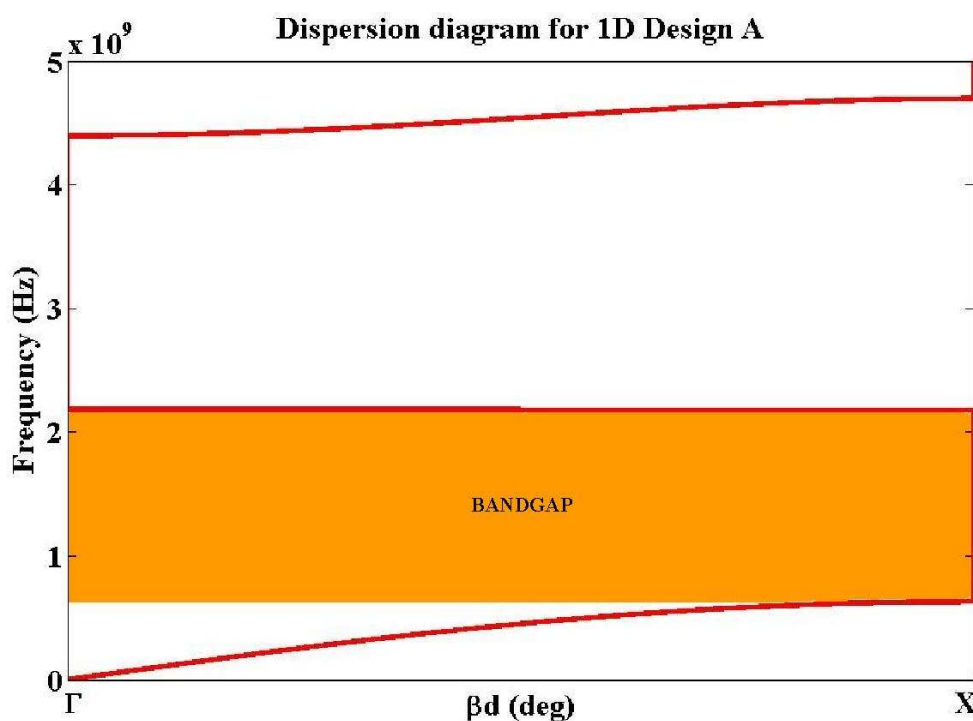


Figure 6.6: Dispersion diagram of 1D EBG structure Design A predicted by the TLM code based.

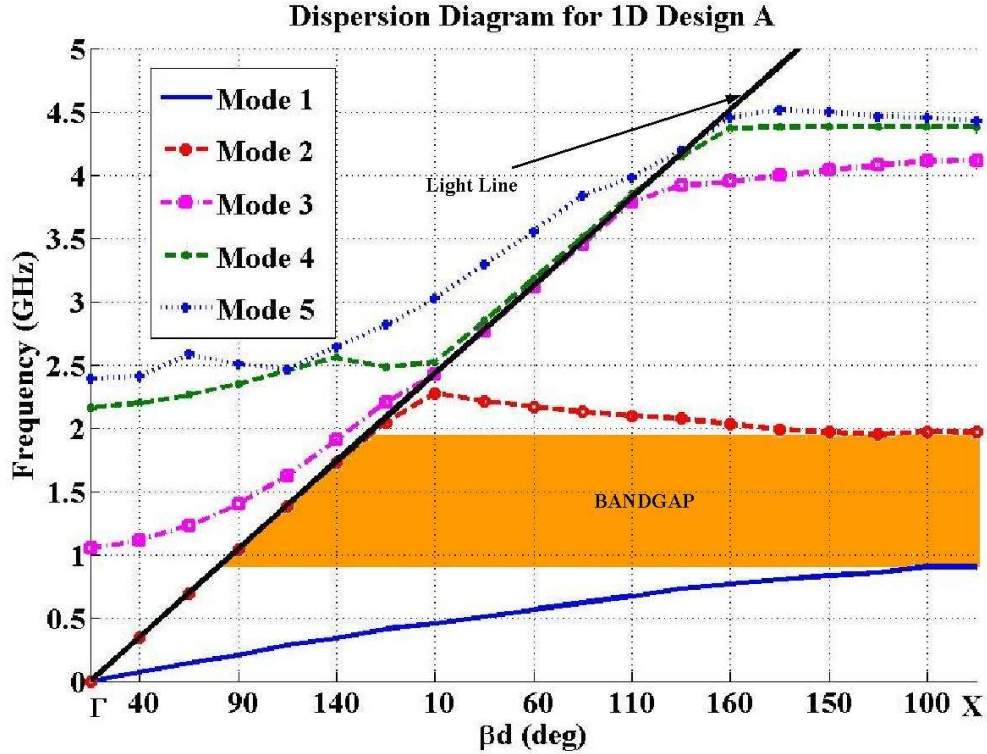


Figure 6.7: Dispersion diagram of 1D EBG structure Design A predicted from full wave analysis

To summarize the results, Table 6.3 is included at the end of Section 6.3 to compare the predicted stopband by various methods and state the simulation times. The computational platform used for these simulations is a dual Pentium Xeon 3.60 GHz system with a Windows XP (32-bit) platform which enables a total maximum memory usage of 4 GB (RAM). This summary table shows that the TLM based code predicts the eigen frequencies with acceptable accuracy and at a much faster speed than full wave solvers.

Next, the 1D structure composed of three unit cells (shown in Figure 6.5 (a)) is simulated to extract the insertion loss parameter. Again, ADS Momentum and HFSS driven simulations are used to evaluate the results obtained from transmission line code. From the insertion loss plot (refer to Figure 6.8 and Figure 6.9), it can be observed that the stopband predicted by ADS begins at 0.7 GHz whereas the one predicted by HFSS begins at 0.8 GHz.

If all the dispersion diagrams and S_{21} results are compared with the 1D stepped impedance EBG design (refer to Figure 4.13), it can be observed that inserting the meander line sections reduces the lower edge of the first bandgap by almost one-half the value obtained for the stepped impedance structure. Here, the inductance of the segment connecting the patches has increased from 1.5706 nH for the DC link (high-impedance transmission line section) in Chapter 4 to 13 nH for the meander line. The width of the bandgap has decreased from 2.659 GHz to 1.559 GHz (from the TLM code) and from 2.307 GHz to 1.061 GHz (from HFSS simulation result).

A closer look at the S_{21} results reveals that the TLM code fails to closely follow the insertion loss behaviour. Even though the start of stopband is very accurately predicted, (see Figure 6.8 and Figure 6.9) the width and shape of the stopband is not captured. This shows limitation of the code in full prediction of driven-port network simulations. However, the validity and efficiency of the approach to conduct eigenvalue solutions should not be undermined by this drawback which stems from not including the physical port and finite substrate geometry in the analysis.

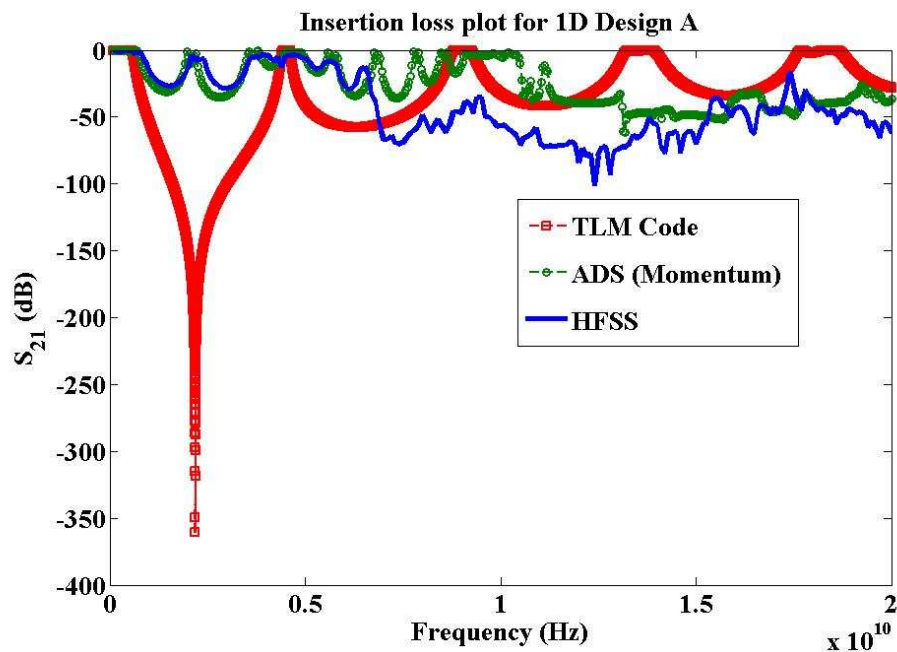


Figure 6.8: Insertion loss plot for Design A

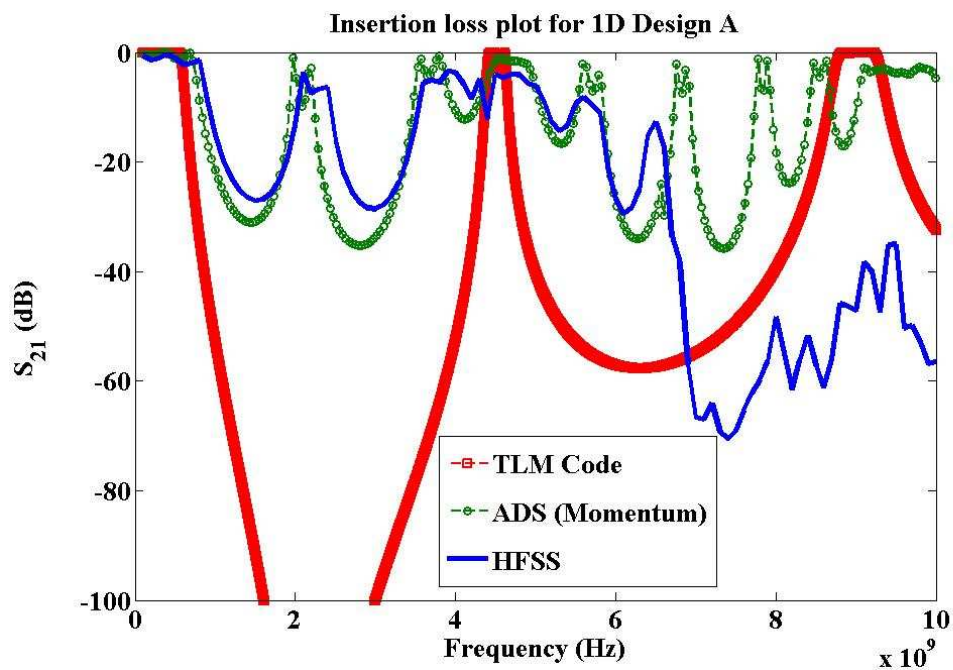
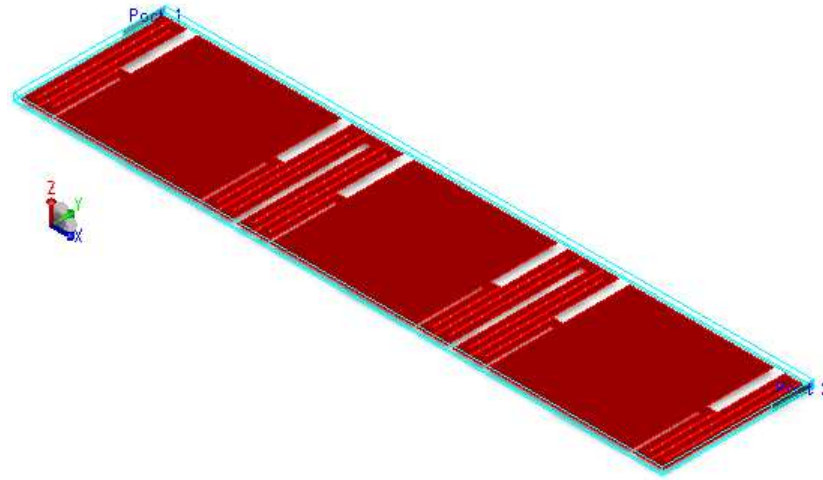


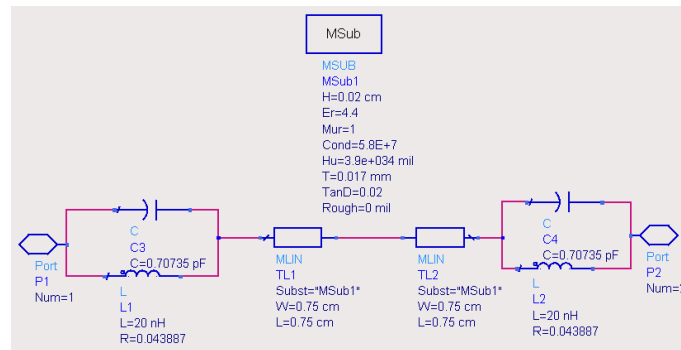
Figure 6.9: Magnified image of Figure 6.8

6.3 1D EBG Design B

Next, the unit cell shown in Figure 6.2 is modified by removing the meander loadings in the perpendicular direction to create another 1D periodic structure. The geometry of the 1D EBG structure based on Design B is shown in Figure 6.10 (a). Figure 6.10 (b) depicts the equivalent circuit of the unit cell of this structure for use in the transmission line code. The substrate and its thickness used for Design B is FR4 ($\epsilon_r = 4.4$) with a thickness of 0.02 cm as provided in [38] The microstrip patches have the length and width 1.5 cm.



(a)



(b)

Figure 6.10: (a) 1D meander loaded transmission line Design B. (b) The equivalent transmission line model for the unit cell used in the TLM code.

The modified unit cell is simulated to generate the 1D dispersion diagrams from transmission line modeling and eigenvalue HFSS simulations. These results are shown in Figure 6.11 and Figure 6.12

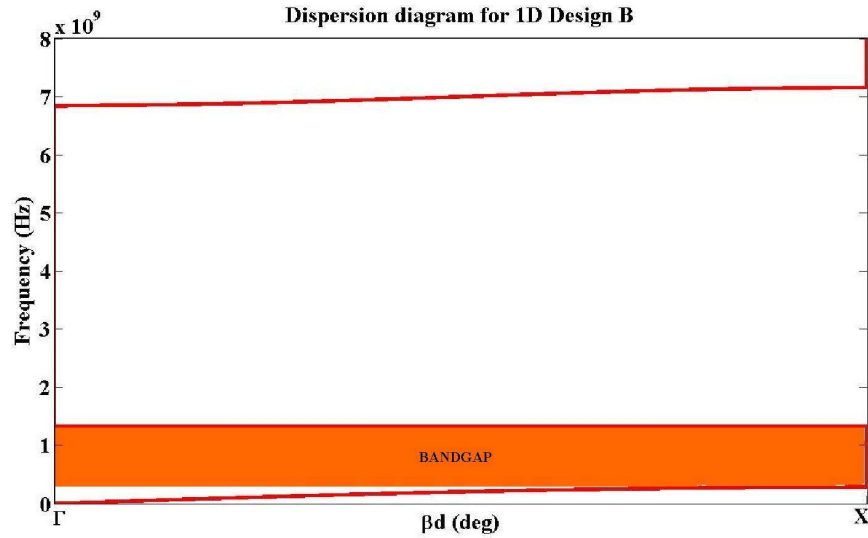


Figure 6.11: Dispersion diagram of 1D EBG structure Design B predicted by the TLM code based.

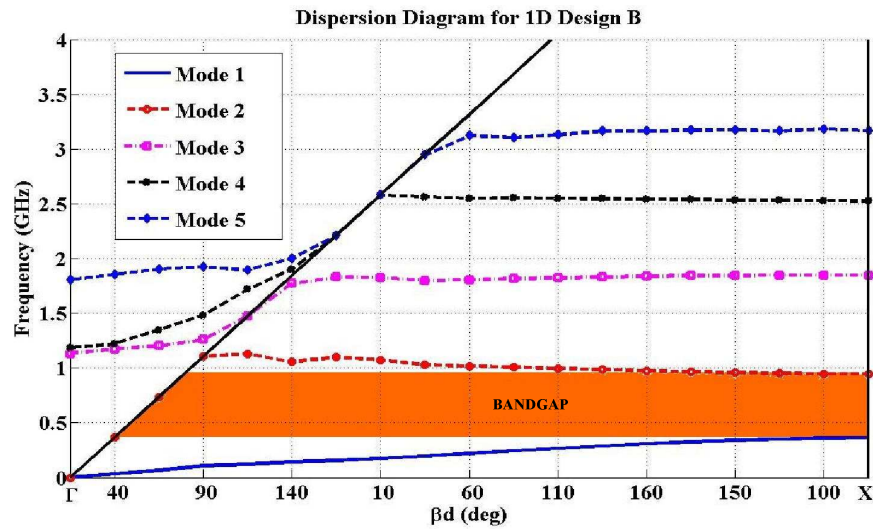


Figure 6.12: Dispersion diagram of 1D EBG structure Design B predicted from full wave analysis

From the dispersion diagrams it can be noticed that the TLM based code captures the $k - \beta$ diagrams of the first and second mode with a reasonable accuracy and does not predict the TE modes found from HFSS simulations. The TLM code predicts the

stopband from 0.282 MHz to 1.33 GHz (Figure 6.11) while the HFSS bandgap (Figure 6.12) is from 0.364 MHz to 0.947 MHz. A summary of these results are also listed in Table 6.3. The fast simulation time of the TLM code should be noted in Table 6.3.

Port driven simulation of the finite size structure (shown in Figure 6.10 (a)) are also conducted and the insertion loss results predicted by ADS Momentum, HFSS and TLM code are shown in Figure 6.13 and Figure 6.14. It can be observed that like Design A the beginning of the stopband is predicated by the TLM code with excellent accuracy while there are discrepancies between the full-wave simulations and code results in the shape and width of the stopband.

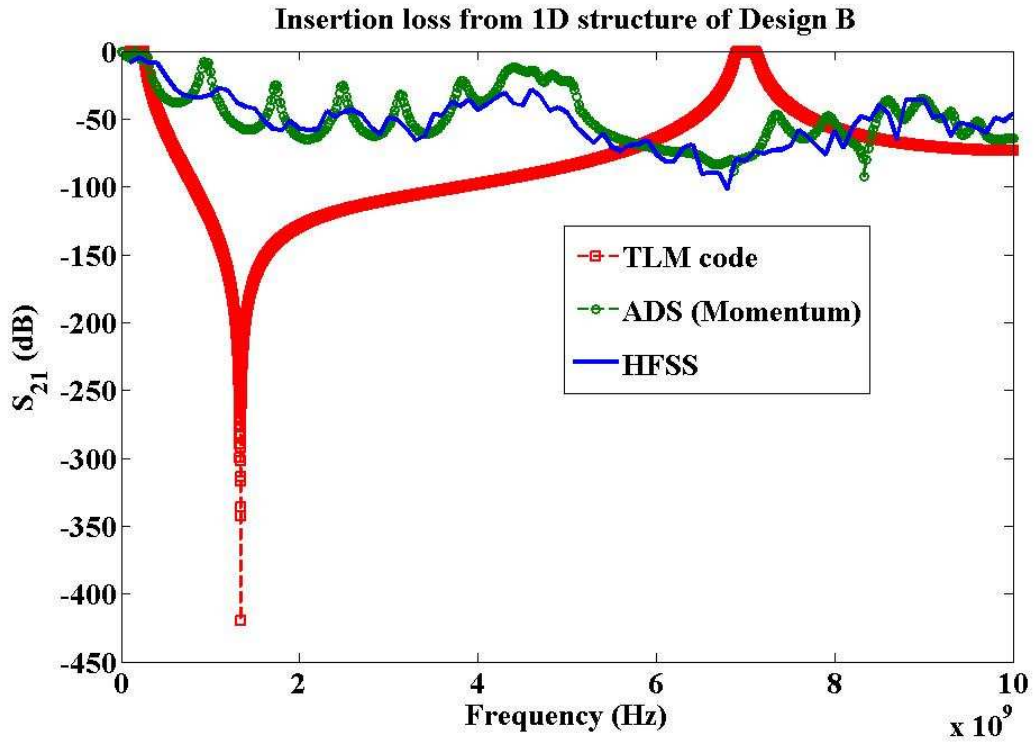


Figure 6.13: Insertion loss plot of the structure shown in Figure 6.10 (a)

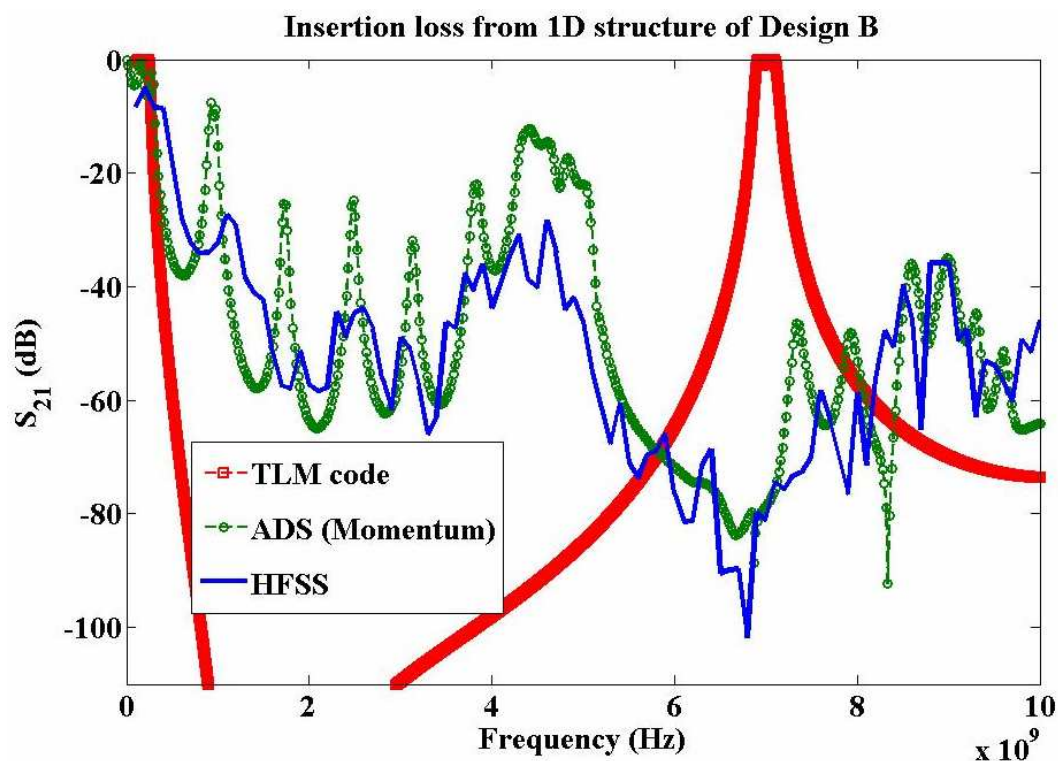


Figure 6.14: Zoomed in version of the graph shown in Figure 6.13.

Table 6.3: Comparison between the TLM code and full-wave solver results for 1D EBG designs

EBG geometry	Bandgap from TLM based code (BW_{code}) (GHz)	Bandgap from full wave solver HFSS ($BW_{full\ wave}$) (GHz)	Time to simulate the results using TLM based code	Time to simulate the results using HFSS	Percentage match in bandgap $\left(\frac{BW_{full\ wave}}{BW_{code}}\right) \times 100$
Design A	0.631~2.19	0.904~1.965	45 seconds	1 hour and 56 minutes	68.1
Design B	0.282~1.33	0.364~0.947	52 seconds	2 hours and 44 minutes	55.63

6.4 2D EBG Design A

Subsequently, the unit cell designs are used to develop 2D EBG structures. The model shown in Figure 6.11(b) is extended to y direction to conduct the 2D transmission line simulation. Figure 6.15 shows the volumetric view of the Design A unit cell. Since it is an open structure, perfectly matched layer (PML) boundary is used to define the top boundary condition in HFSS simulations. It should be mentioned that PML is defined for the top boundaries of the 1D and 2D periodic structures discussed in this Chapter and Chapter 4 in HFSS eigenvalue analysis.

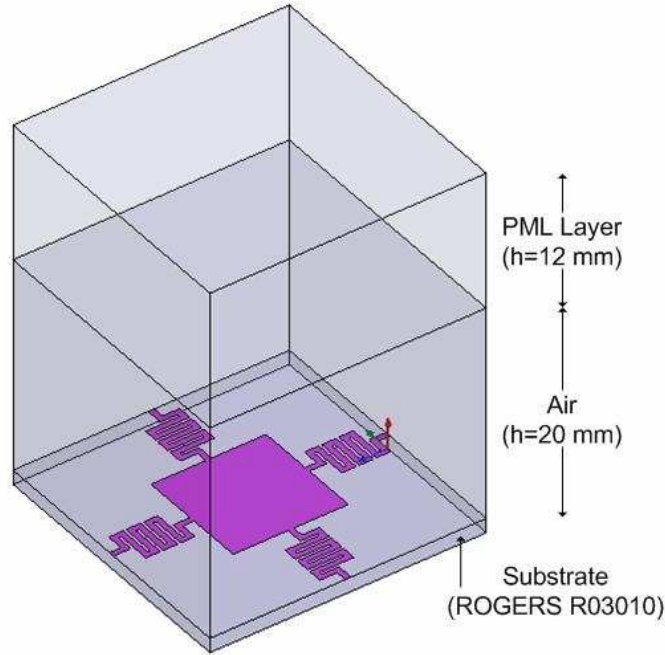


Figure 6.15: Simulation setup for the unit cell of 2D EBG structure of Design A.

The results obtained from the full wave solver are compared with the dispersion diagram obtained from the TLM based code. The dispersion equation for the 2D EBG structure is the same as (4.34) with just one modification. The meander model now replaces the dc link in (4.34) and hence the impedance of the narrow section is replaced by

$$Z_h = \frac{\omega L}{1 - \omega^2 LC} \quad (6.3)$$

Where L and C are the values of inductance and capacitance of the meander line obtained in Table 6.1 and Table 6.2.

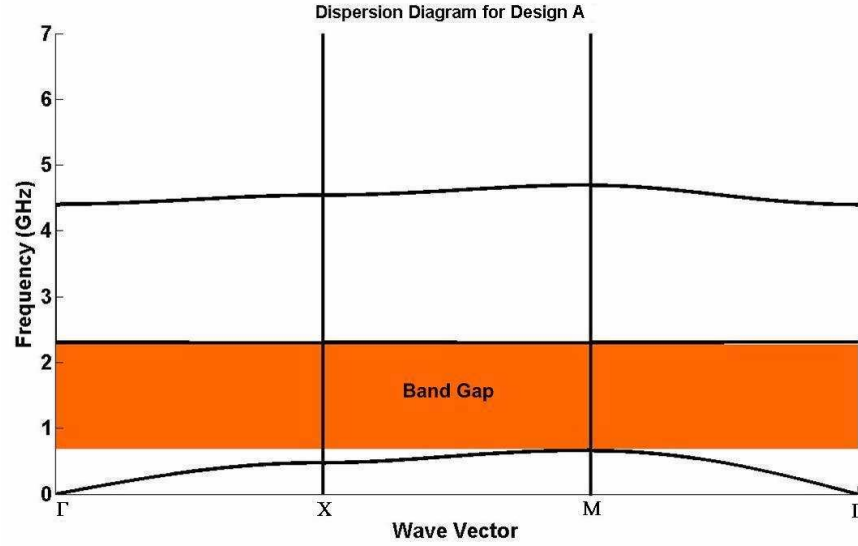


Figure 6.16: Dispersion diagram of the 2D EBG Design A obtained from the TLM code.

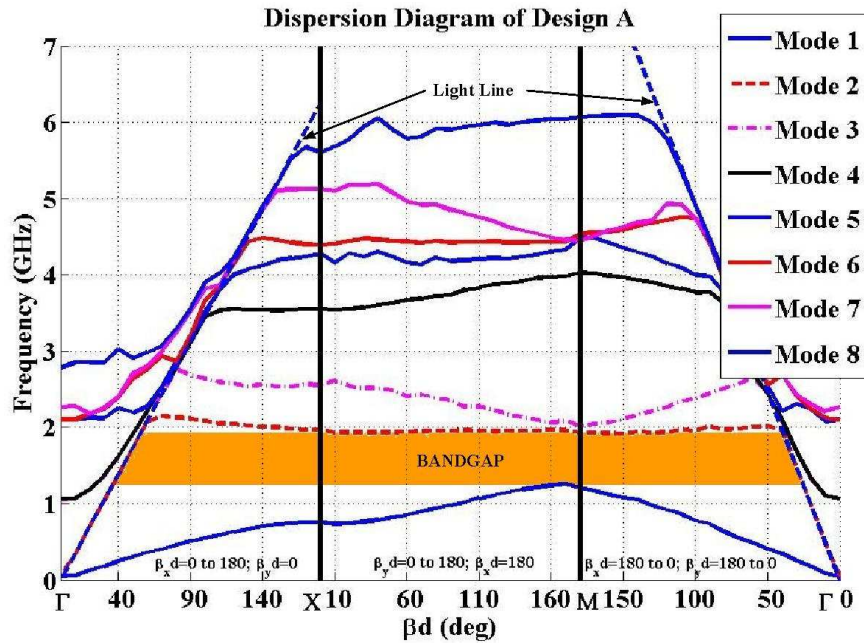


Figure 6.17: Dispersion diagram of the 2D EBG Design A obtained from full-wave analysis.

Figure 6.16 and Figure 6.17 present the dispersion diagrams obtained from the TLM code and HFSS eigenvalue solver. Considering the wide frequency range of simulation, a large number of modes are predicted from HFSS simulations. The first mode is a TM_0 mode that is captured by the transmission line modeling approach as well.

The next step is extraction of insertion loss characteristics. For this a finite size 2D EBG structure is considered and simulated using various tools; HFSS, ADS and approximation using the TLM method's code.

The finite 2D EBG structure used for the simulation consists of 5 rows and 5 columns of EBG unit cells cascaded along x- and y-directions. Hence, the entire setup consists of 25 unit cells of the EBG with a total area of 120 mm^2 along with a finite continuous ground plane. Two excitation ports are used at $(12 \text{ mm}, 12 \text{ mm})$ and $(108 \text{ mm}, 108 \text{ mm})$, respectively. Figure 6.19 shows the simulation setup for obtaining the insertion loss between port 1 and port 2.

Another way to approximate the S_{21} quickly is by setting up the equivalent circuit of the finite size EBG in ADS schematic window and including the model of Figure 6.4 for the meander section. The unit cell of this circuit is shown in Figure 6.18.

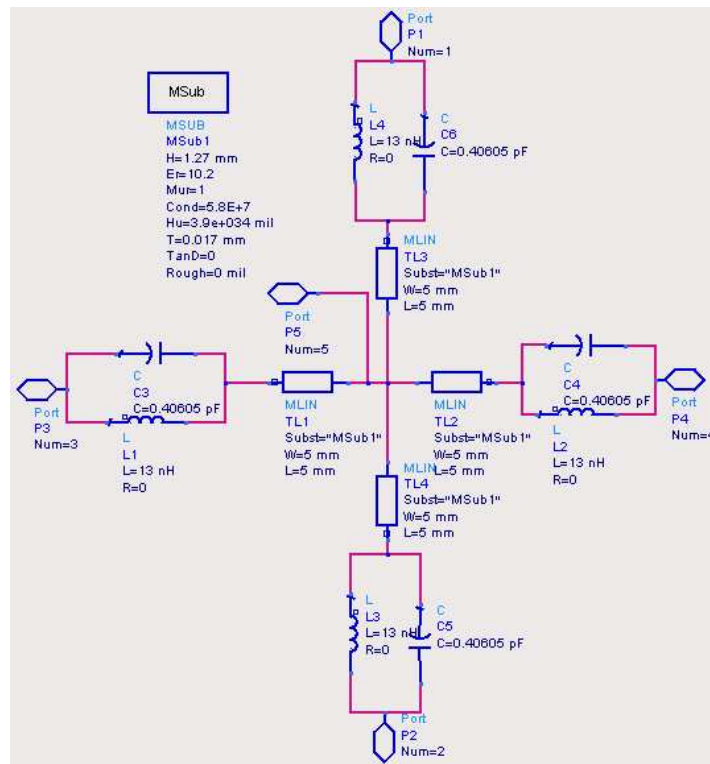


Figure 6.18: Unit cell of the 2D meander loaded EBG structure in ADS schematic window.

The magnitude of S_{21} obtained from all the aforementioned simulation approaches is shown in Figure 6.20 (a) (Figure 6.20 (b) is the zoomed in version).

For comparison a parallel-plate structure with an area of 120 mm^2 and with Rogers RO3010 dielectric between them, is used as reference. Two ports at exactly the same location, i.e. at $(12 \text{ mm}, 12 \text{ mm})$ and $(108 \text{ mm}, 108 \text{ mm})$, have been used for excitation. As expected with the parallel-plate structure, there is no stopband as opposed to the EBG based geometry and this has been clearly depicted in Figure 6.20 (a) and Figure 6.20 (b).

In Figure 6.20, it can be observed that the isolation obtained using Design A is approximately 60 dB higher compared to the parallel-plate structure. The S_{21} predicted from ADS schematic simulations closely follows the TLM code results but fails to capture the detailed variations of S_{21} as predicted by HFSS simulations.

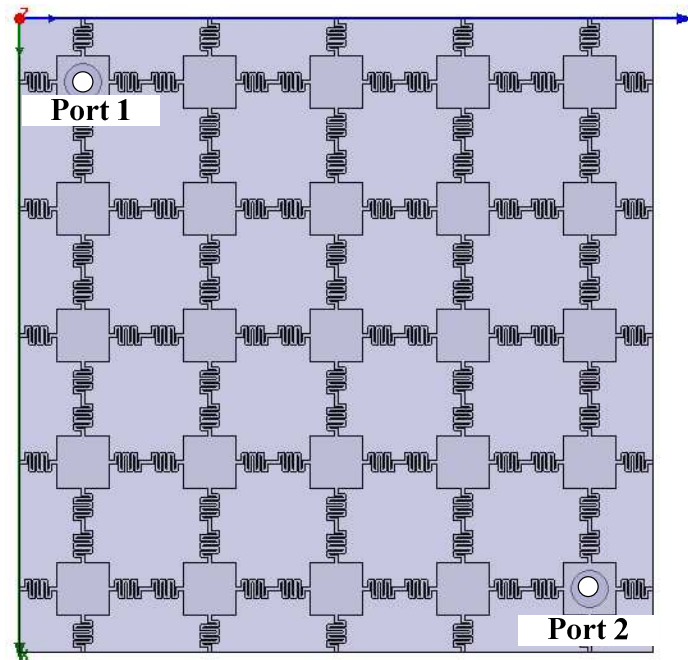


Figure 6.19: Illustration of the EBG structure constituted of several cascaded unit cells along with the port definitions for the calculation of the Insertion Loss

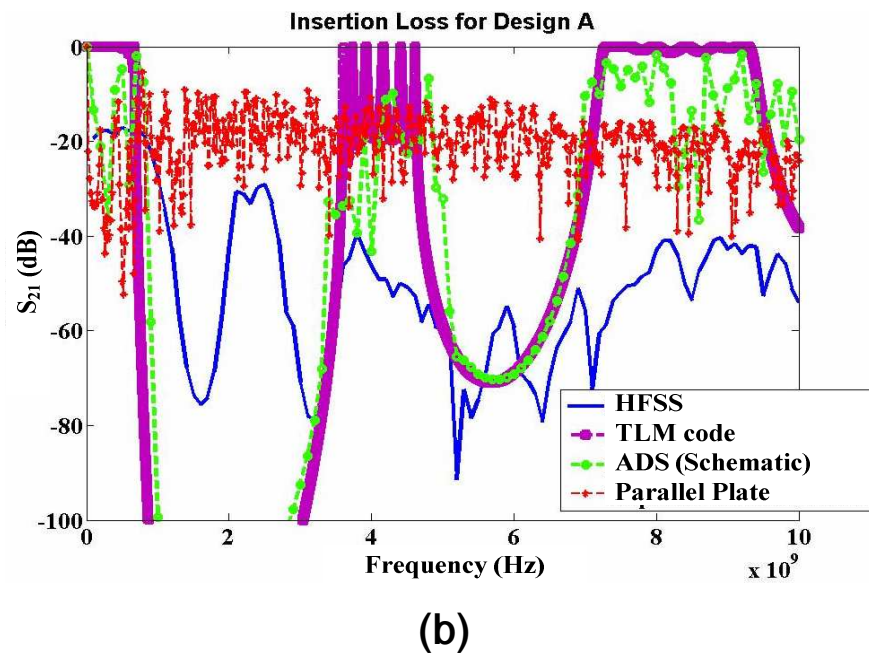
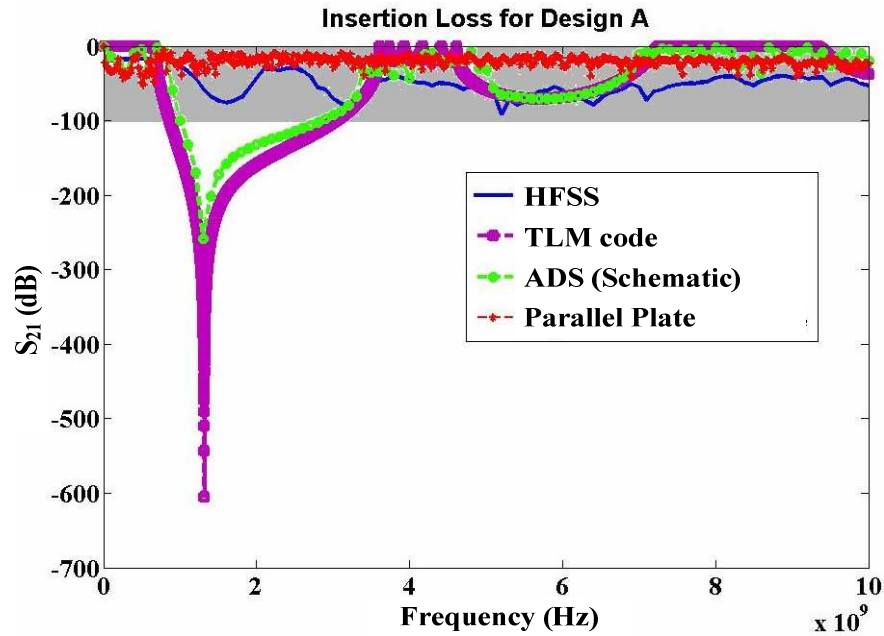


Figure 6.20 (a) Magnitude of S_{21} for a finite size 2D meander loaded EBG structure Design A. (b) Magnified scale of the shaded region shown in (a).

6.5 2D EBG Design B

The parameters used for this design are based on a structure presented in . Here, FR4 of thickness 0.02 cm is used as the dielectric substrate of choice. The dimensions of the patch are 1.5 cm x 1.5cm and the total dimension of the unit cell is 2.26cm x 2.26 cm. The geometrical parameters of the meander line are shown in Figure 6.2.

The dispersion diagram predicted from the transmission line modeling method is shown in Figure 6.22 and the one obtained from HFSS eigenvalue simulations is presented in Figure 6.23. It can be observed that there is a very good match between the bandgap obtained by both the processes. The bandgap predicted from the TLM code is from 0.442 GHz to 1.185 GHz and the slow-wave bandgap from HFSS extends from 0.618 GHz to 1.004 GHz. Table 6.4 lists these values along with the percentage difference.

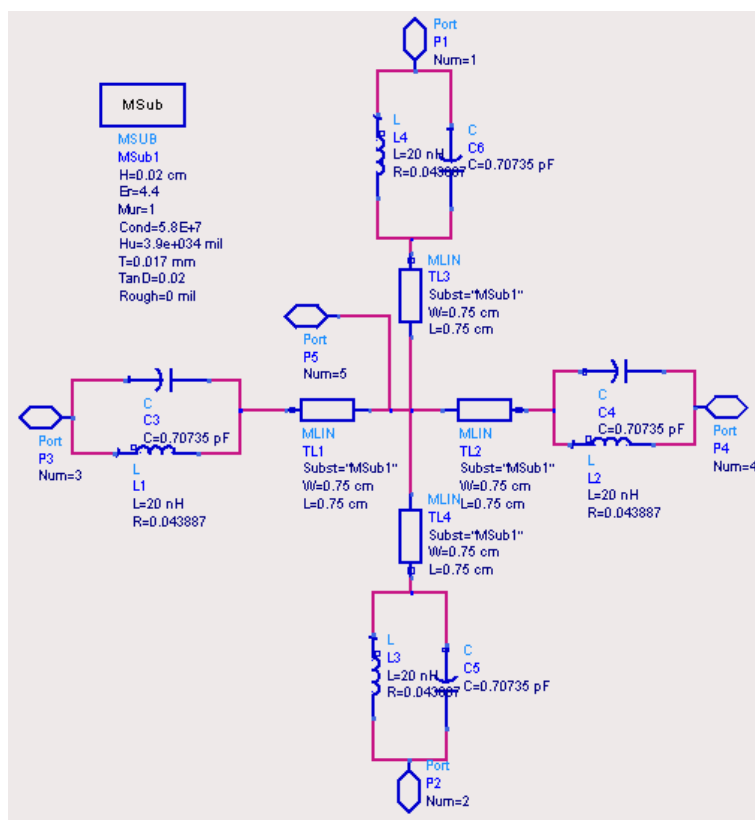


Figure 6.21: Unit cell of the 2D meander loaded EBG structure in ADS schematic window.

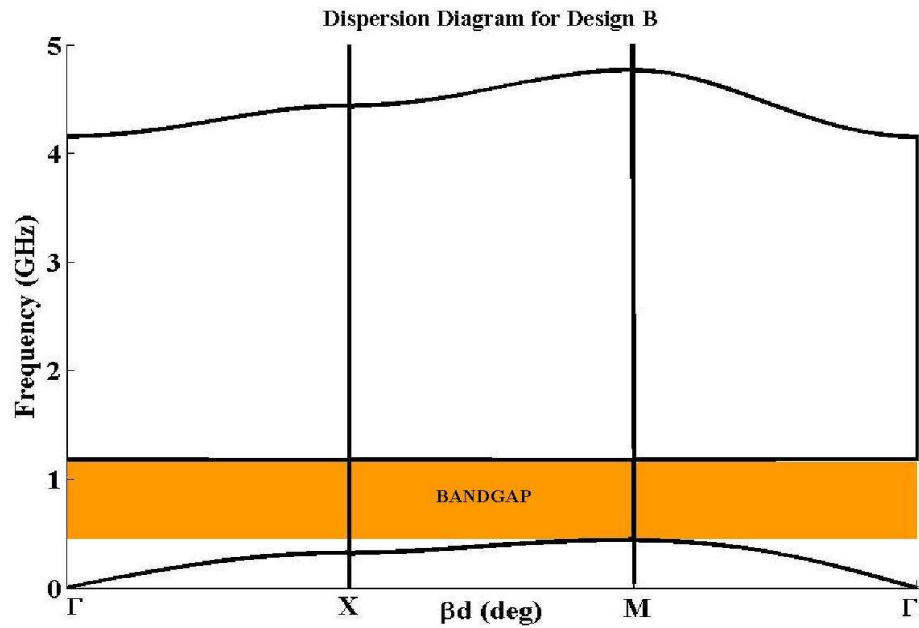


Figure 6.22: Dispersion diagram of the 2D EBG Design B obtained from the TLM code.

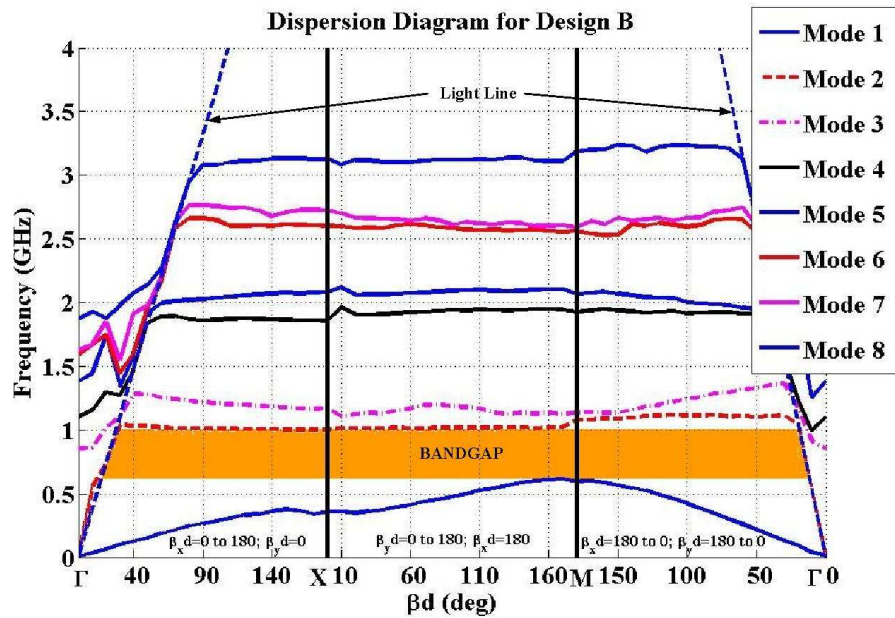


Figure 6.23: Dispersion diagram of the 2D EBG Design B obtained from full-wave analysis.

The transmission characteristic of this structure is evaluated through full-wave simulations and TLM code analysis. For this a finite size 2D EBG structure is considered and simulated.

The finite 2D EBG structure used for the simulation consists of 10 rows and 10 columns of EBG unit cells cascaded along x- and y-directions. Hence, the entire setup consists of 100 unit cells of the EBG with a total area of $22.6 \times 22.6 \text{ cm}^2$ along with a finite continuous ground plane. Two excitation ports are used at $(5.65 \text{ cm}, 10.17 \text{ cm})$ and $(16.95 \text{ cm}, 10.17 \text{ cm})$, respectively. Figure 6.24 shows the simulation setup for obtaining the insertion loss between port 1 and port 2. The location of the ports is not the same as described in [38].

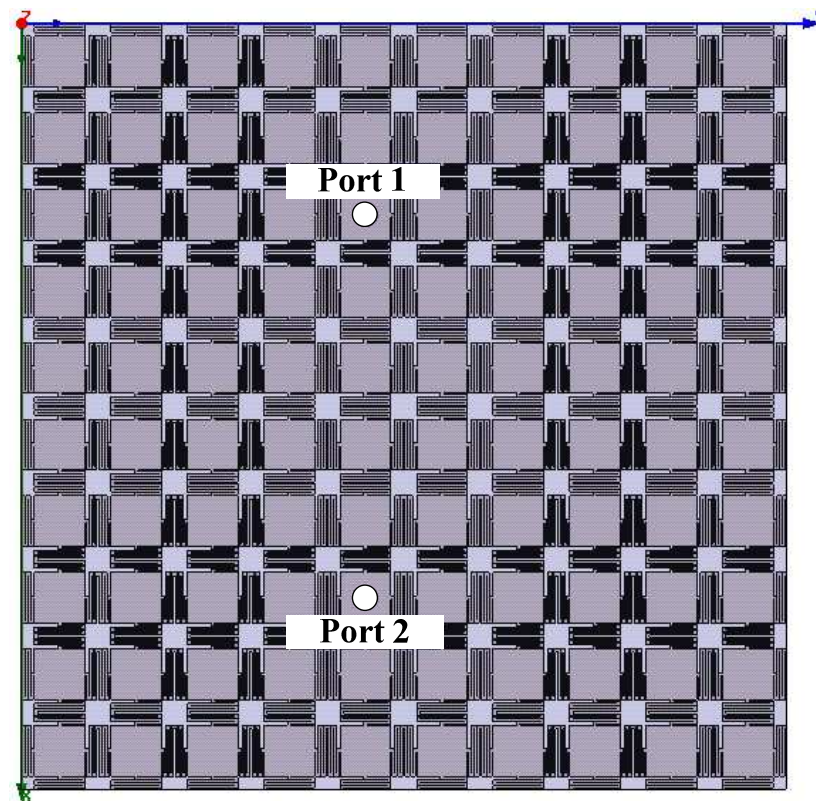
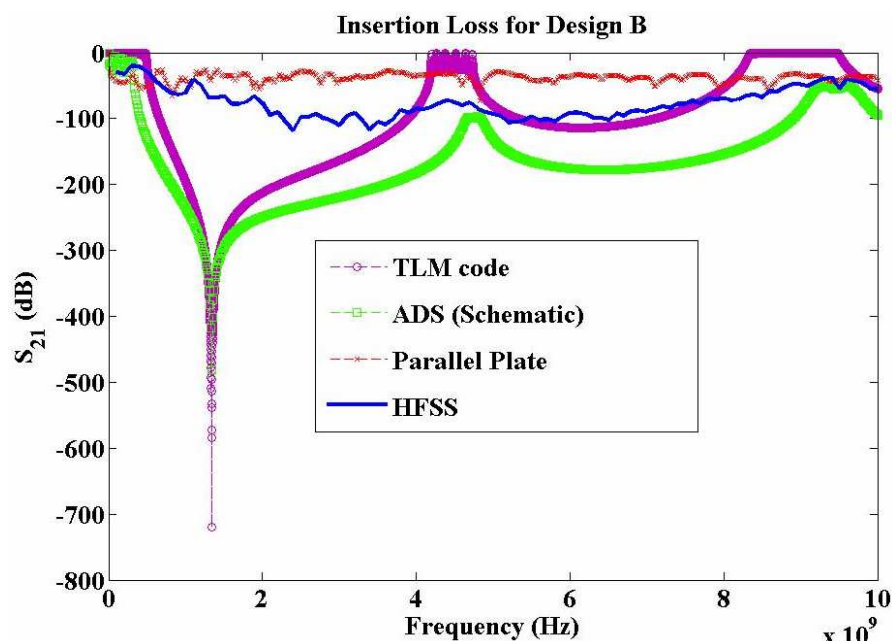


Figure 6.24: Illustration of the EBG structure constituted of several cascaded unit cells along with the port definitions for the calculation of the Insertion Loss

The simulation results are shown in Figure 6.25 (a) (Figure 6.25 (b) is the zoomed in version). Similar to the case of Design A, a circuit is set up in the ADS schematic window and scattering parameter simulations are conducted.

Again, like in the previous case, for comparison a parallel-plate structure with FR4 dielectric between them, is used as reference. Two ports at exactly the same location, i.e. at (5.65 mm,10.17 mm) and (16.95 mm,10.17 mm), have been used for excitation. As expected with the parallel-plate structure, there is no stopband as opposed to the EBG based geometry and this has been clearly depicted in Figure 6.25 (a) and Figure 6.25 (b).

From Figure 6.25, it can be observed that the insertion loss in the bandgap for this design is 80 dB higher than that for a continuous power plane geometry. Similar to Design A results, the S_{21} predicted from ADS schematic simulations closely follows the TLM code results. These S_{21} graphs do not provide the realistic magnitudes and variations predicted from full-wave simulations due to the fact that losses are ignored, limited modes are captured and approximations are used in modeling.



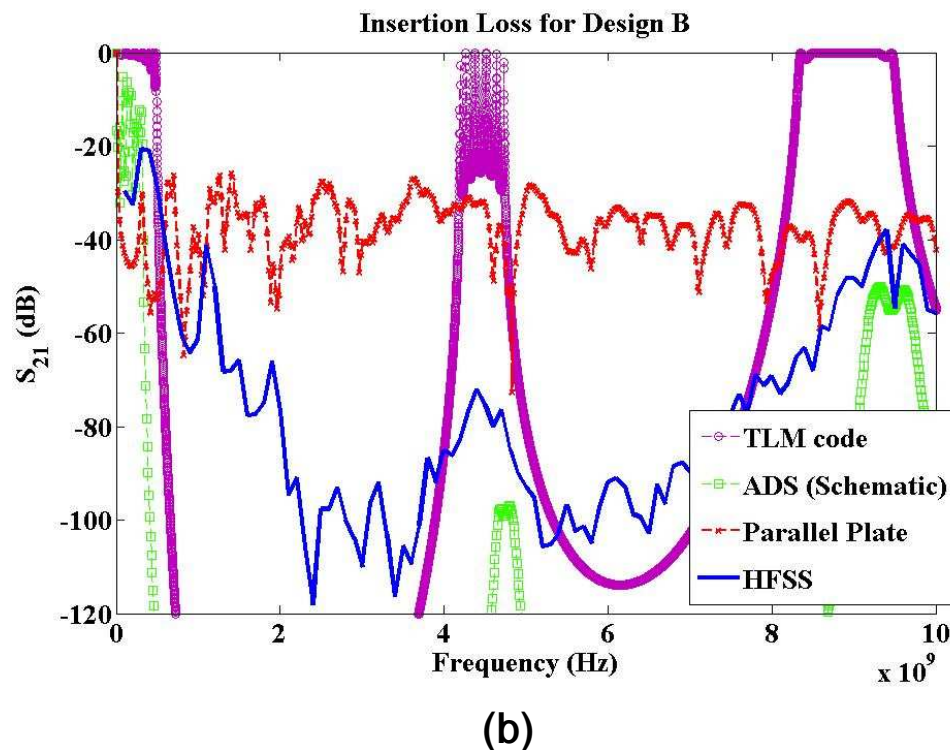


Figure 6.25: (a) Magnitude of S_{21} for a finite size 2D meander loaded EBG structure Design B. (b) Magnified scale of the shaded region shown in (a).

Table 6.4: Comparison between the TLM code and full-wave solver results for 2D EBG Design A and Design B from dispersion simulations

EBG geometry	Bandgap from TLM based code (BW_{code}) (GHz)	Bandgap from full wave solver HFSS ($BW_{full\ wave}$) (GHz)	Time to simulate the results using TLM based code	Time to simulate the results using HFSS	Percentage match in bandgap $\left(\frac{BW_{full\ wave}}{BW_{code}}\right) \times 100$
Design A	0.665~2.309	1.26~1.935	105 seconds	4 hours and 28 minutes	41.0584
Design B	0.442~1.185	0.618~1.004	113 seconds	6 hours and 32 minutes	51.9650

Finally, to show the impact of adding the meander sections to the EBG design and compare the designs of this chapter and the one presented in Chapter 4, Table 6.5 is presented.

Table 6.5 provides information about the reduction in the lower edge of the stopband for the two new EBG designs with respect to the stepped impedance based EBG design discussed in Section 4.4 of Chapter 4. Table 6.5 also sheds light on the reduction of the width of the stopbands for the meander based designs with respect to the stepped impedance EBG implementation. This is due to the added parasitic capacitances because of the meander sections in the EBG designs discussed in this chapter (refer to Equation 5.33 in Chapter 5).

Table 6.5: Comparison between the TLM code and full wave solver results for 1D EBG Designs

Heading	Lower end of the bandgap	Upper end of the bandgap	Bandwidth
Units	(GHz)	(GHz)	(GHz)
Stepped Impedance EBG of Chapter 4	2.368	3.836	1.468
Meander EBG Design A	1.26	1.935	0.675
Meander EBG Design B	0.618	1.004	0.386

6.6 Conclusion

In this chapter, two EBG structures consisting of metal patches linked by meander lines are investigated. The two designs are uniplanar structures chosen over textured geometries due to their more economical fabrication costs. The primary goal of using meander loading is to achieve a reduced lower edge of the bandgap. Compared to the design discussed in Chapter 4, the lower edge of the bandgap is brought down by almost half. This enables application of this type of EBG structure as a filter or noise suppression

component at low frequencies. Dispersion diagrams and transmission coefficients are obtained from the TLM based code and compared with full-wave simulation results. The TLM results are created in less than a couple of minutes while full-wave simulations can extend to few hours using the same computational platform. However, the accuracy of the TLM result at cases is compromised due to the simplifications made in the modeling.

CHAPTER 7

Conclusions

7.1 Concluding Remarks

Planar EBG structures are used extensively for wireless applications and microwave circuit design. The properties arising from the periodicity of these structures provide solutions to numerous engineering problems. The analysis of EBG structures is based on solving Maxwell's equations for the pertinent boundary conditions, depending on the nature of the application. Numerous CAD tools and analytical methods provide driven port and eigenvalue solutions needed for the analysis of EBG structures.

The thesis discusses the application of transmission line modeling approach in analysis of planar EBG structures. Multiple designs are analyzed and their frequency selective behaviour is predicted by this modeling method. The generated dispersion diagrams and insertion loss results are validated with full-wave simulations.

To develop the transmission line modeling method, a particular group of planar EBG structure, which is composed of periodic wide microstrip sections (low characteristic impedance) interconnected with narrow (high characteristic impedance) microstrips or meander microstrip lines, is studied. The method is applied to 1D periodic layouts, and then it is extended to 2D EBG structures. First, the EBG structures are modeled using transmission line circuits. For the case of the structures with interconnecting meander lines a lumped element equivalent circuit composed of L and C components is developed to represent the meander loading. The values of the lumped components in the equivalent circuit are found by using closed-form formulas. By applying Floquet's Theorem to the representative network of the unit cell of these periodic circuits, the characteristic equations or the complex dispersion equations are derived. A code is developed to plot the dispersion equation and extract the approximate insertion loss characteristic for the case of a finite size 1D or 2D EBG structure.

It is observed that the transmission line modeling allows for an extremely fast estimation of the bandgap of the studied EBG structures when compared to full-wave numerical simulations while providing a reasonable accuracy. Full-wave solvers generate complete modal information, accurate dispersion plots and full scattering parameters but they are expensive in terms of simulation time and computational resources.

In the cases studied in this thesis, the transmission line models and derived formulations are obtained with some simplifications and approximations, due to which the accuracy of the dispersion and insertion loss results is limited. For example, TE modes and higher order TM modes are not captured in the employed transmission line modeling technique. Moreover, the code does not capture the behaviour of the modes in the fast wave region. As well, it should be mentioned that transmission line and lumped circuit models that can be easily determined by using analytical formulas are not always available for all types of EBG structures, especially the ones with asymmetric unit cell or with complex parasitics. The cases studied in this thesis are among the most popular EBG structures and can serve as the starting design layout for more complicated geometries.

Owing to the fast nature of the presented method in predicting the behaviour of these EBG structures, it can be concluded that it is an efficient means for designing and optimizing transmission line based EBG structures for target applications.

The replacement of the high impedance microstrip patches by meander lines leads to an appreciable reduction in the lower edge of the bandgap. This is of major importance because it enables the designers to incorporate this method in low frequency operations too, an area which predominantly uses discrete decoupling capacitors to provide isolation.

7.2 Recommendations for Future Work

The present thesis can be extended in two major directions; first, generalization of the code, and second, investigation of other examples of EBG structures and their applications.

In the studied cases, a simple LC circuit was used to represent the meander loading. The TLM code can be further extended by including higher order lumped models to represent periodic loadings and achieve more accuracy in predicting the results. Moreover, the code can be enhanced by including multi-conductor transmission line formulations. This enables investigation of multilayer systems and covered EBG structures.

EBG structures have proven to provide an efficient means for the mitigation of power/ground noise, and every year new designs emerge to improve the noise suppression characteristics [102]. The possibility of modeling these structures using the presented technique should be investigated.

Furthermore, signal and power integrity issues associated with the structures presented in Chapters 4 and 6 should be studied. Many design modifications, like adjustment of the geometrical features of the layout and changing the dielectric constant can be employed to control the width and location of the bandgap.

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