EFFECTS OF UNBALANCED VOLTAGE ON SYNCHRONOUS MOTOR OPERATION

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I wish to express my appreciation for the advice and encouragement given me by the staff of the Department of Electrical Engineering at McGill University during the writing of this paper.

R. L. S.

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INTRODUCTION

i) Introducing the subject of this paper we must first explain what is meant by the terms "unbalanced voltage" and "effects", and then show why this problem arises.

The synchronous motor is operated on three phase power supply and the "voltages" referred to in the title are the voltages of each phase at the motor terminals. The term "balanced" is applied to these voltages when they are of equal magnitude and when the voltage of fundamental frequency of one phase passes through its peak value 120 degrees ahead of the peak value of one of the other phases and 120 degrees behind the peak value of the remaining phase. Thus at any instant the voltages are balanced if their vectors are equal in magnitude and 120 degrees apart. If this relation does not hold in either or both respects the voltages are said to be "unbalanced".

These voltage conditions arise from a number of causes. Thus the voltages may not be generated equally in the three phases and at 120 degrees apart due to faulty generator design. The generator may be balanced but unbalance may be introduced in the transmission line or the transformers. The voltage may be unbalanced in the distribution system by connecting single phase loads such as induction furnaces, single phase motors, or refining tanks (with rectifiers) across lines of the three phase system.

Synchronous and induction motors are designed to operate with balanced applied voltages. The problem of this paper is to determine what effects unbalanced voltages will have on the starting, pull-in, and pull-out torques, and the heating and efficiency of a synchronous motor.

The unbalance may be corrected by phase balancing equipment or by changes such as to eliminate the cause of the unbalance. This may be done only at some cost and this cost should be less than the additional cost of unbalanced over balanced operation. When we know the effects of the unbalance on the synchronous motor we can estimate the annual cost of the addional losses, and decreased life of the windings, and find the total cost of the unbalance.

Methods of calculating the characteristics of a synchronous motor under unbalanced voltage conditions will be developed in this paper. These methods will be applied to a 15 kilowatt laboratory motor and the results supplemented by direct measurement.

HISTORICAL REVIEW

ii) The question of three phase machines operating with unbalanced applied voltages was first introduced about 1909. At that time the question was of greater importance than it is today as line regulation was not as good as it is today and unbalanced loads introduced different voltage drops in the lines, actually unbalancing the system voltages. Today through the interconnection of systems and the consequent pooling of capacity unbalanced loads produce little change in system voltages. In 1909 Messrs. Charters and Hillebrand published in the Transactions of the A.I.E.E. experimental data on the" Reduction in Capacity of Polyphase Motors Due to Unbalancing in Voltages". These early investigators lacked mathematical methods of dealing with unbalanced voltages and currents.

E.F.Alexanderson and L.G.Stokvis in 1913 developed Ferraris' two reaction theory for the single phase induction motor (1893) and applied it to three phase motors. Their development used the conception of positively and counter rotating fluxes. They successfully applied their principles to phase balancers and generator voltage regulation.

In 1918 Dr.C.L.Fortescue approached the general problem of unbalanced systems from a different point of view and developed the "Theory of Symmetrical Components". This clarified the whole question of calculating unbalance and was applied by research engineers to a number of problems. In 1918 Dr.

Slepian took up the question of induction motors on unbalanced voltage and applied the concept of symmetrical components. His conclusions and treatment with a few refinements are those accepted today.

Before a mathematical treatment of unbalance of synchronous machines could be made considerable information concerning the many reactances necessary for such calculations had to be acquired. C.F.Wagner and A.Dovjikov discussed the impedance of synchronous machines to negative sequence voltage.in 1927. They suggested methods by which this impedance could be measured but did not investigate the fundamental nature of the impedance in its relation to the other machine constants. The most outstanding work done in recent years is that of the research engineers of the General Electric Company. These men R.H.Park, B.L.Robertson, R.E.Doherty ,and C.A.Nickle have derived expressions relating flux conditions and constants for transient and stable states in synchronous machinery. In a beautiful piece of analysis Park and Robertson show the consequences of applied negative sequence currents on the fluxes present and definitely relate the negative sequence impedance to the subtransient reactances of the motor. Today these investigators have developed the theory of synchronous machine operation to such detail that any problem,such as unbalance,has its soution in their fundamental flux equations.

CHAPTER 1.

THE SYMMETRICAL COMPONENTS

1. In balanced three phase systems, since each phase has the same impedances, currents, and voltages it is only necessary to make a solution for one phase and then multiply the value obtained by some factor to get the resultant for the three phases. This cannot be done under unbalanced conditions. The method used in this case is that of symmetrical components. This form of analysis divides any unbalanced system of currents and voltages into three balanced systems. Any system of balanced impedances will offer to each of these systems a constant impedance. Solutions may then be made as before using the values per phase for each system.

These systems are known as the sequence components and are of three kinds:

1) Positive Sequence Components- Fig 1(a) in which the resultant waves for the three phases pass through their peaks in the order a,b,c,. This sequence is indicated by the subscript (1).

2) Negative Sequence Components- Fig.1(b)in which the resultant waves for the three phases pass through their peaks in the order a,c,b. and is indicated by the subscript (2).

3) Zero Sequence Components- Fig. 1(c). This system consists of three equal vectors in phase with each other and indicated by the subscript (0).

THE SYMMETRICAL COMPONENTS

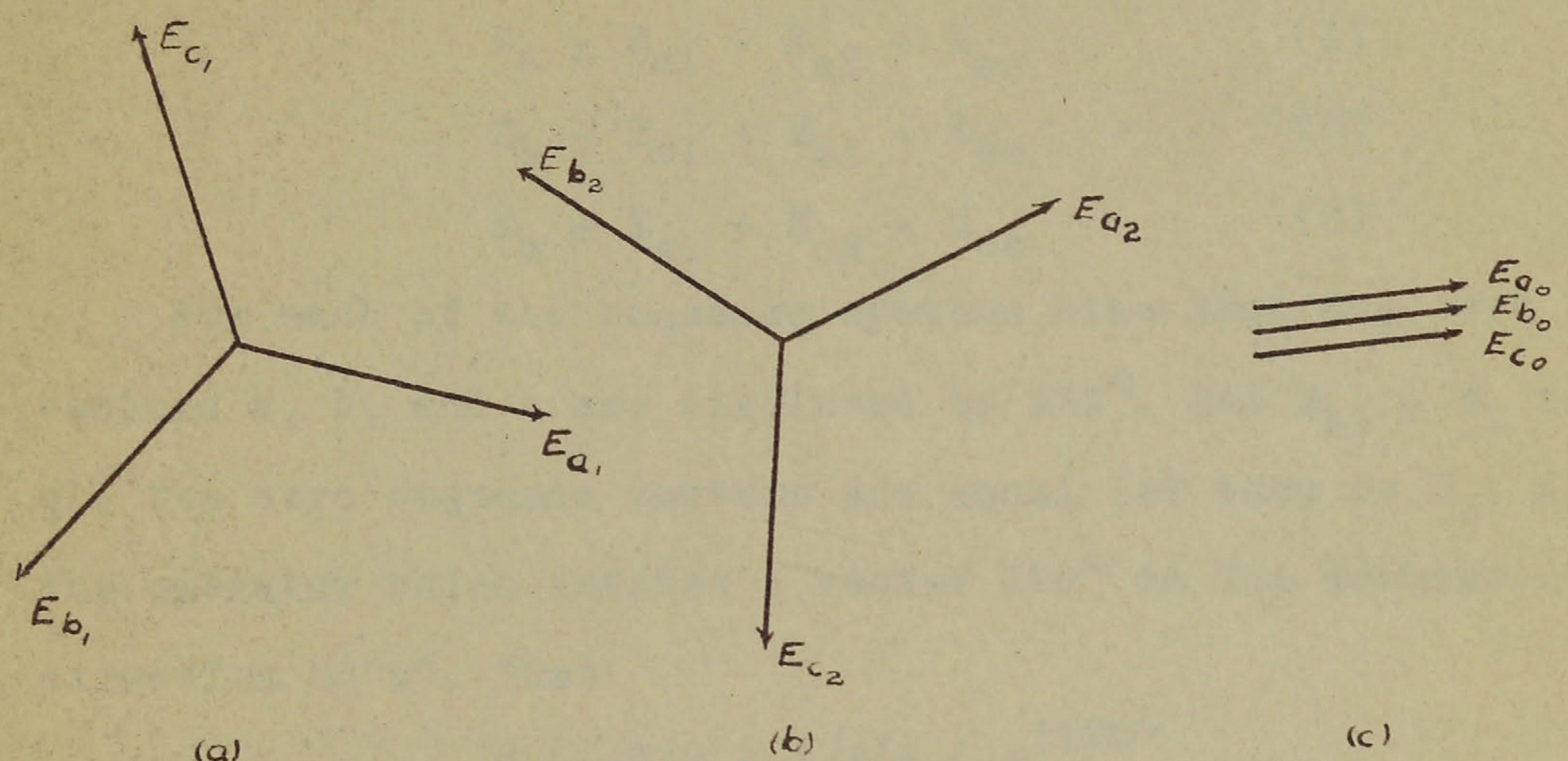


Fig 1

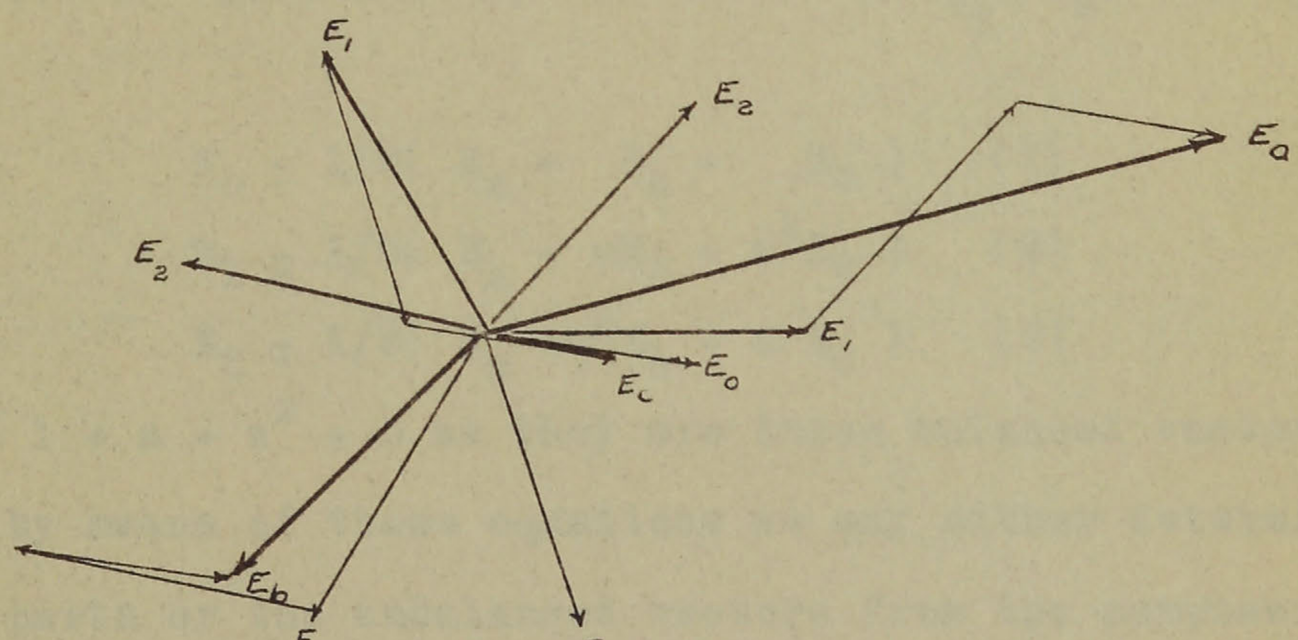


Fig 2

Since these systems are equivalent to a single unbalanced system, if E_a, E_b, and E_c are the unbalanced vectors then;

$$E_a = E_{a1} + E_{a2} + E_{ao} \qquad (1)$$

$$E_b = E_{b1} + E_{b2} + E_{bo} \qquad (2)$$

$$E_c = E_{c1} + E_{c2} + E_{co} \qquad (3)$$

For each of the sequence systems,other than the zero sequence vectors a, b, and c are displaced by 120^o. Let $E_{a1} = E_1$ and since all the zero sequence vectors are equal let them be E_o. Also let the operator which rotates a vector 120^o in the counter-clockwise direction be"a". Then:

$$a = -1/2 + j\sqrt{3}/2 = e^{j120^o}$$

$$E_a = E_1 + E_2 + E_o \qquad (4)$$

$$E_b = a^2E_1 + aE_2 + E_o \qquad (5)$$

$$E_c = a\,E_1 + a^2E_2 + E_o \qquad (6)$$

This set of equations may be solved for E_1, E_2, and E_o to give;

$$E_o = 1/3(\ E_a + E_b + E_c\ ) \qquad (7)$$

$$E_1 = 1/3(\ E_a + aE_b + a^2E_c\ ) \qquad (8)$$

$$E_2 = 1/3(\ E_a + a^2E_b + a\,E_c\ ) \qquad (9)$$

\- Since $1 + a + a^2 = 0$ as they are three balanced vectors.

Thus by means of these equations we may either determine the component parts or the unbalanced vectors from the components.

Notice that if the vector E_o be absent the unbalanced vectors will form a closed system(there will be no resultant when they are added) since E_1 and E_2 form closed systems.

The use of these sequence components depends on the fact that in any symmetrical system these sequences are independent of each other . In other words, positive sequence currents will not introduce voltage drops in the negative sequence network, and negative sequence currents will not introduce voltage drops in the positive sequence. This was originally proven by Dr. C.L.Fortescue.

2. Power in Terms of Sequence Quantities

Alternators supply only positive sequence currents and voltages when properly designed. When there is any dissymmetry in the three phases it acts as a generator for negative and zero sequence power quantities. Thus a line to ground fault gives rise to negative and zero sequence components of current which flow through their respective impedances and give rise to the voltage of their own sequence. These negative and zero sequence components of current result in a total power loss in the system supplied by the generators.

The expression for the total power:

$$P_T + jQ_T = E_a I_a + E_b I_b + E_c I_c$$

if the sequence components for the voltages and currents are substitutèd in this equation it becomes;

$$P_T + jQ_T = 3(E_1 I_1 + E_2 I_2 + E_o I_o)$$

It might also be added that the two wattmetær connection will give the correct power reading on unbalance if the zero sequence currents and voltages are absent.

CHAPTER 11.

MEASUREMENT OF SEQUENCE COMPONENTS

Two methods of obtaining sequence components are in common use. The first of these is that of obtaining the components from the phase values by the use of some sort of graphical construction. The other method is to use instruments designed to measure the components directly.

3. Graphical Methods

Certain phase quantities are required to define the sequence components. Thus if the three sequence components are present we require six phase quantities, either three amplitudes and three phase angles ,or six amplitudes. If only the positive and negative sequence quantities are present three amplitudes are required.

A few of the graphical methods in use are shown in Figs. 3 4, and 5. The analytical proofs upon which they depend are shown in the next few pages.

The Graphical Determination of the Phase Values of Voltage from the Line to Line Values

Zero Sequence Present

See Fig. 3. In this figure E_A E_B and E_C, the line to line voltage vectors form a closed triangle. Arcs of radii E_a, E_b, and E_c, the phase values are swung from the corners of this triangle. The intersection of these arcs gives the neutral point of the system. This completely determines the phase vectors.

GRAPHICAL DETERMINATION OF SEQUENCE QUANTITIES

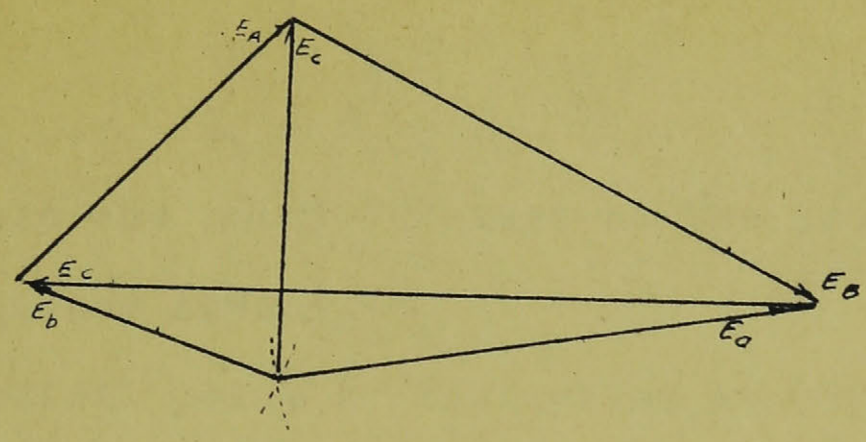


Fig 3

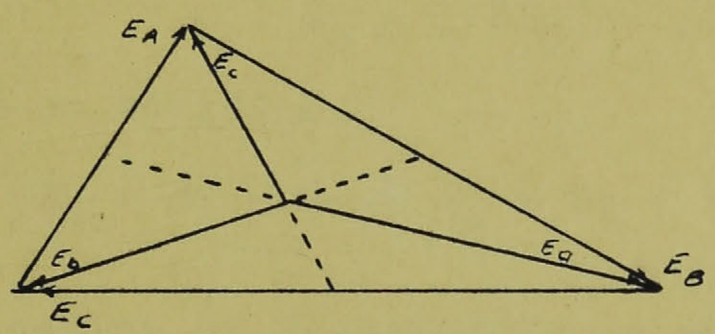


Fig 4

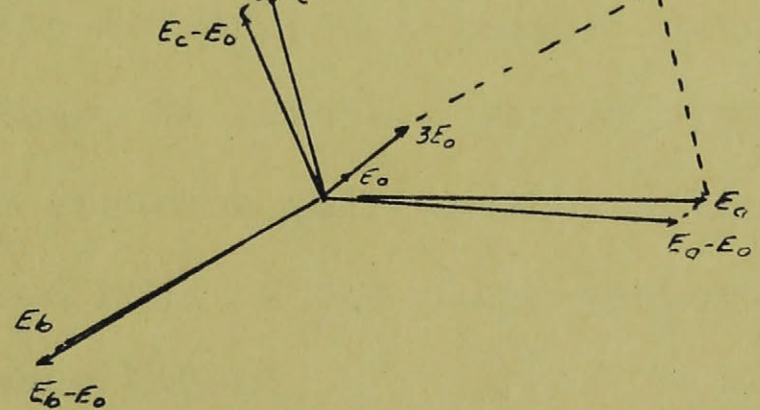


Fig 5

Proof $E_a - E_c = E_B$

$E_c - E_b = E_A$

$E_b - E_a = E_C$

Therefore the point "O" must be the neutral point.

Zero Sequence Absent

The neutral point of the system is defined by the intersection of the medians of the line to line triangle.

Since:
$$E_a = 1/3E_A + 2/3E_B$$
$$E_b = 1/3E_B + 2/3E_C$$
$$E_c = 1/3E_C + 2/3E_A$$

which gives on addition:

$$E_a + E_b + E_c = E_A + E_B + E_C = 0.$$

Determination of Sequence Components from Phase Values

Three Sequence Components Present

The first essential in this case is to eliminate the zero sequence components. This is done by adding the three phase vectors If zero sequence is present there will be a resultant. This is the sum of the three zero sequence vectors which are in phase with each other. We obtain a closed system consisting of positive and negative sequence only by subtracting one third of this resultant from each of the phase vectors. See Fig. 6.

Zero Sequence Absent

Two methods of separating positive and negative sequence are shown below.

1) Direct Method

This method is based on equations 7,8,and 9. Each of the phase vectors has its operator applied to it and one third the sum

THE SEPARATION OF POSITIVE AND NEGATIVE SEQUENCES

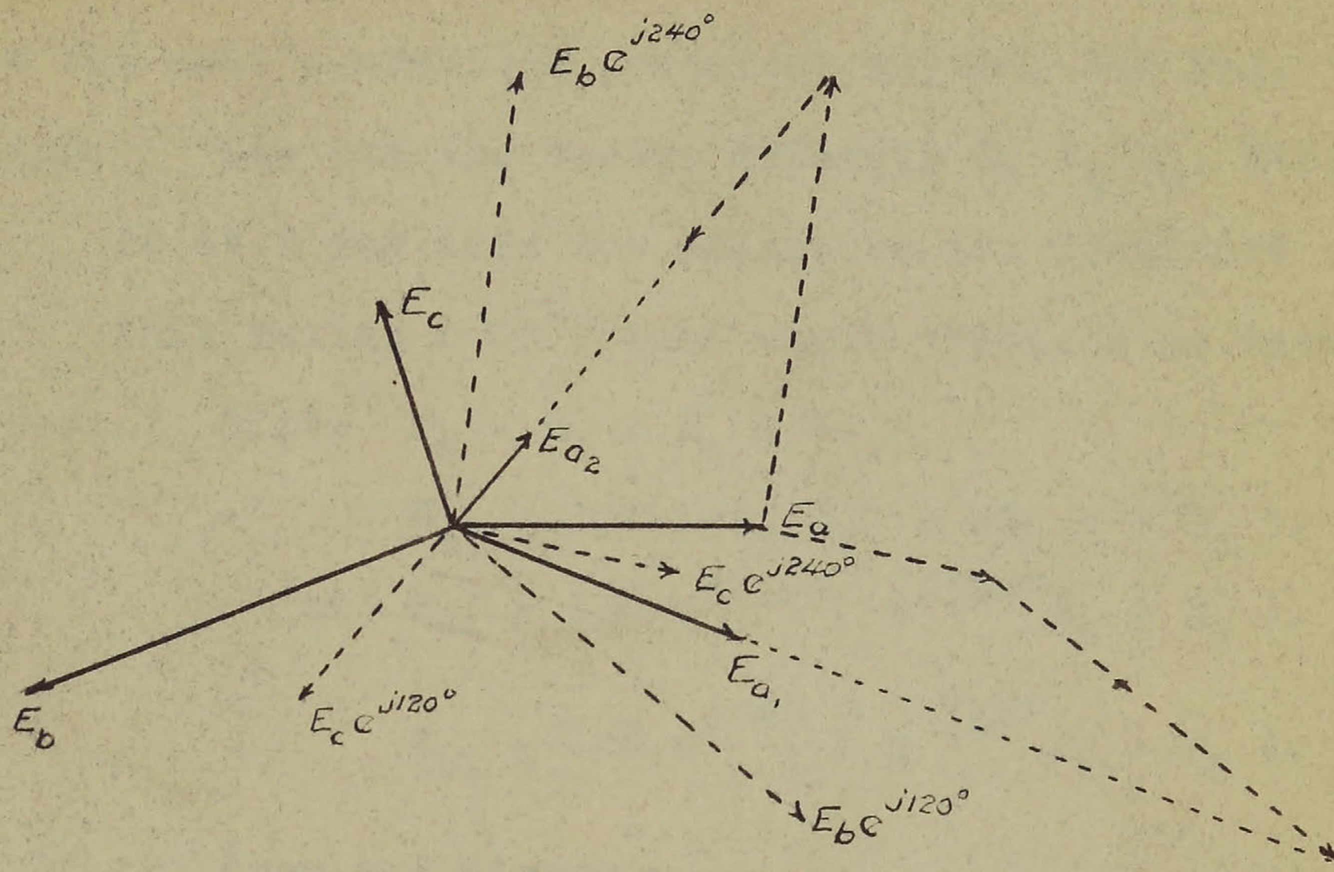


Fig 6.

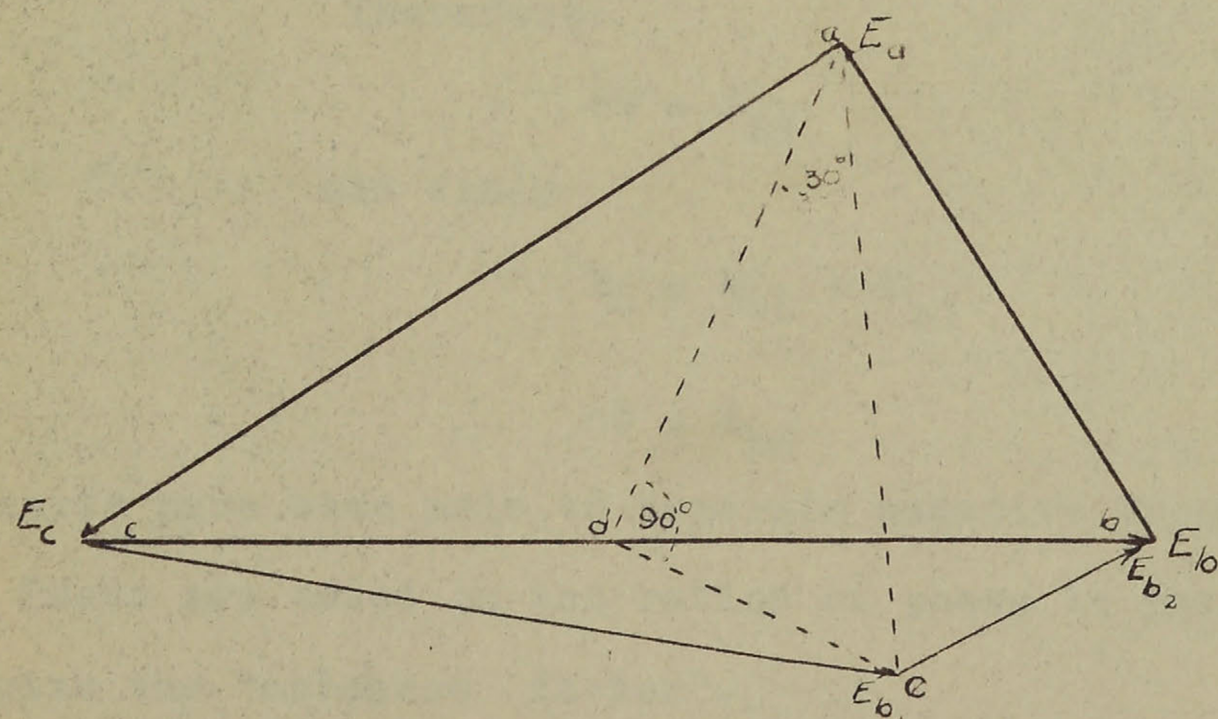


Fig 7.

then equals the sequence component. See Fig. 6.

2) Dubusc's Method

This is the most convenient graphical method. See Fig. 7.

Construction: Lay out the vector triangle E_a E_b E_c. Bisect cb at d and draw the median to it. Construct on this median a 30° right angle triangle as shown.

Proof: Since $E_a + E_b + E_c = 0.$

$$E_{a1} = 1/3((-E_b - E_c) + aE_b + a^2E_c).$$

$$\text{or} \quad E_{a1} = 1/\sqrt{3}.e^{j210^\circ}(e^{-j60^\circ}E_b + E_c).$$

$$E_{b1} = 1/\sqrt{3}.e^{j90^\circ}(e^{-j60^\circ}E_b + E_c).$$

From the diagram:

$$ad = E_c + \tfrac{1}{2}.E_b$$

$$ae = 2/\sqrt{3}.e^{j30^\circ}(E_c + \tfrac{1}{2}E_b)$$

$$ce = 2/\sqrt{3}.e^{j30^\circ}(E_c + \tfrac{1}{2}E_b) - E_c$$

$$= 1/\sqrt{3}.e^{j90^\circ}(e^{-j60^\circ}E_b + E_c)$$

Therefore:

$$ce = E_{b1}$$

and since

$$E_b = E_{b1} + E_{b2}$$

$$\therefore \quad eb = E_{b2}$$

Charts have been made to separate negative from positive sequence. These are based on the ratios of phase to phase voltages and also give the "unbalance factor".

The"Unbalance Factor" is the ratio of the value of the negative sequence component to the value of the positive sequence value.

4. The Direct Measurements of Sequence Components

In the experiments to follow as there were to be a considerable number of sequence components required it was considered advantageous to construct instruments to read these quantities directly. The methods outlined by workers in this field were of two distinct types; There were those instruments requiring special meter windings and those requiring only static networks external to the meters. The second of these methods was chosen as it was not desired to interfere with the windings of the ammeters and voltmeters used. Since the measurements were to be taken on a synchromous motor with an ungrounded neutral it was not necessary to eliminate and measure the zero sequence components.

5 Sequence Voltmeters

In Figs. 8,9,10, and 11 are shown static networks for measureing positive and negative sequence components when the zero sequence component is absent.

Fundamentally the segregating action of these networks depends on the ratio pf the impedances of the two branches of the network. A preliminary check on the wave characteristics of the alternator used showed the voltage wave to consist of several prominent tooth harmonics. It was found that using networks containing a condenser the presence of these harmonics introduced considerable error in the value of the meter current. Thus for the network of Fig.9. for measuring positive sequence voltage:

SEQUENCE NETWORKS

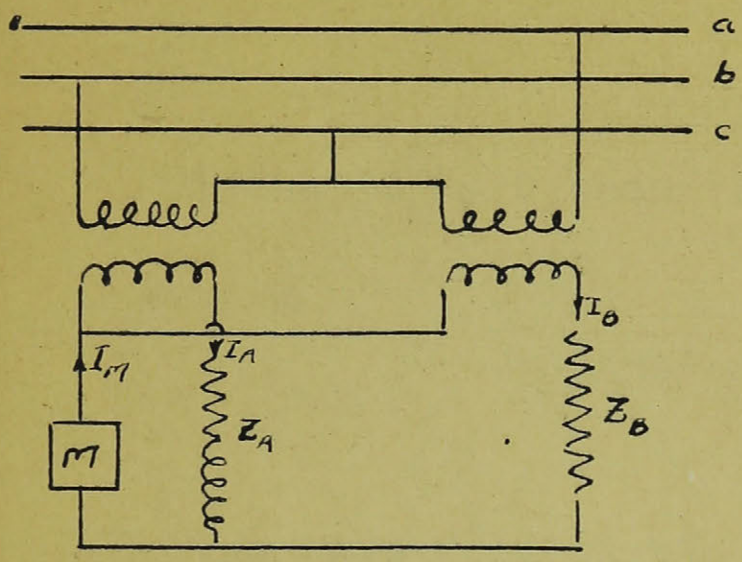


Fig 8 (a)

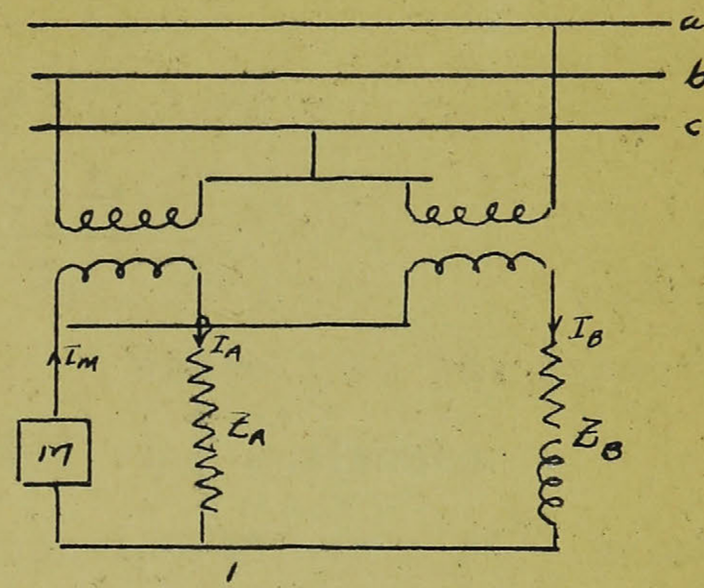


Fig 8 (b)

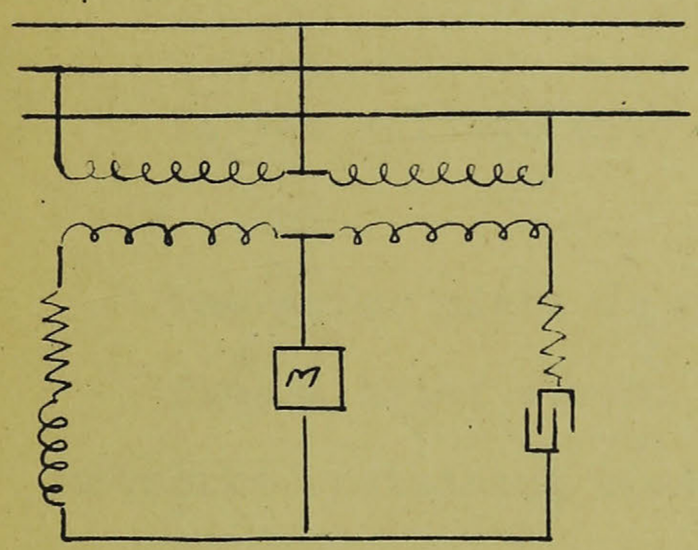


Fig 9 (a)

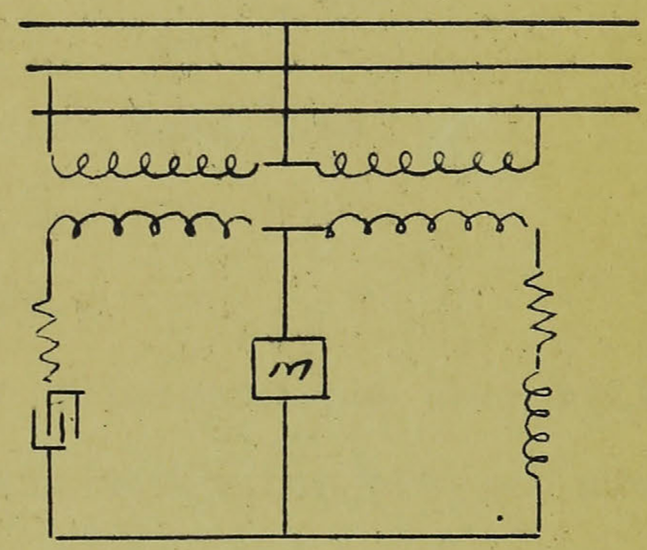


Fig. 9 (b)

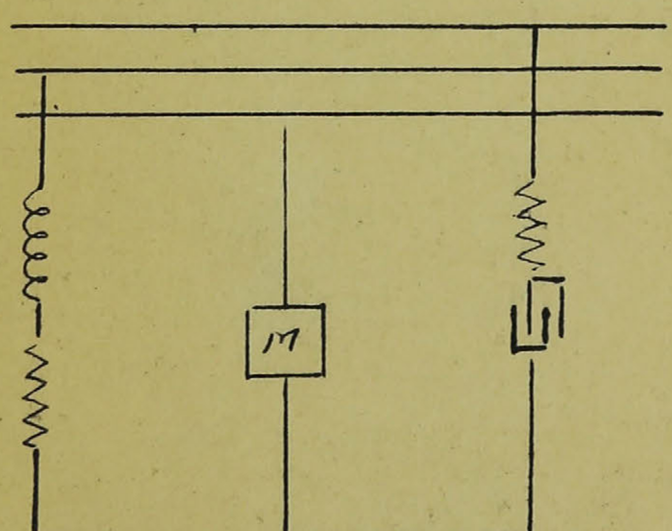


Fig 10

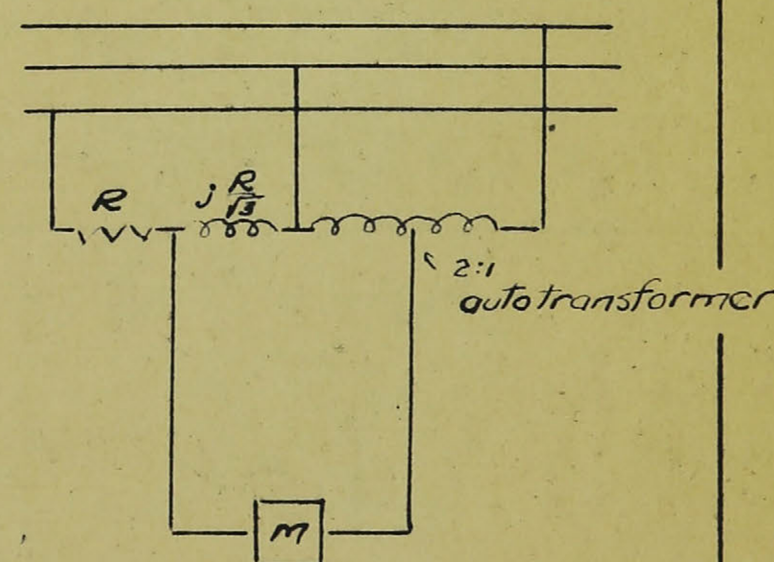


Fig. 11.

The meter current $I_m = - \dfrac{j2\sqrt{3}.E_1}{\sqrt{3}\, Z_m e^{-j30^\circ} + Z_c}$

Assume $Z_m = 0$ then $I_m = - \dfrac{j2\sqrt{3}.E_1}{Z_c}$

and $Z_c = \dfrac{1}{j2\pi fC}$ where C is the capacity of the condenser.

The wave form of the alternator considered has a 25th harmonic of 1%.

$$I_m = 2.\sqrt{3}\ .99(\ 2\pi 60C\ ) + 2.\sqrt{3}\ .1(\ 2\pi 1500C\ )$$

$$= 4.\sqrt{3}\ .60C\ (\ 114\ )$$

If the voltage wave were a pure sine wave then

$$I_m = 100(\ 4.\sqrt{3}\ .\pi.60C\ )$$

The error might be as large as 14%..

Thus a brief survey would seem to eliminate the possibility of networks containing condensers when the harmonics present are large.

The network shown in Fig. 8.was used. The basic equations derived from Kirchoff's Laws follow:

$$E_a - E_c = Z_B I_B + Z_M(\ I_A + I_B\ )$$

$$E_c - E_b = Z_A I_A + Z_M(\ I_A + I_B\ )$$

$$I_M = \frac{Z_A(\ E_a - E_c\ ) + Z_B(\ E_c - E_b\ )}{Z_A.Z_B + Z_M.Z_A + Z_M.Z_B}$$

$$E_a = E_1 + E_2$$

$$E_b = a^2E_1 + aE_2$$

$$E_c = aE_1 + a^2E_2$$

$$I_m = \frac{(1-a)(Z_A + aZ_B).E_1}{Z_A.Z_B + Z_M(Z_A + Z_B)} + \frac{(1-a^2)(Z_A + a^2Z_B).E_2}{Z_A.Z_B + Z_M(Z_A + Z_B)}$$

and if I_m is to measure positive sequence only then

$$\frac{(1-a^2).(Z_A + a^2Z_B)}{Z_A.Z_B + Z_M(Z_A + Z_B)} = 0.$$

$$\text{and } Z_A = -a^2 Z_B = e^{j60^\circ} Z_B$$

$$I_{m1} = \frac{3.E_1}{Z_B + Z_M(1-a)}$$

Similarily for the negative sequence

$$Z_B = e^{j60^\circ} Z_A$$

$$I_{m2} = \frac{3.E_2}{Z_B + Z_M(1-a^2)}$$

Actual Design

Voltage across Z_A and Z_B is $E_B - E_A = 382$ volts. In order to limit the current in the network to the capacity of the available resistance a 2:1 power transformer was used. A high inductance, low resistance coil was used , minimizing stray field effects, and change in resistance due to temperature rise, for the impedance branch. See Fig. 12.

AIR CORE INDUCTANCE COIL
SEQUENCE NETWORK

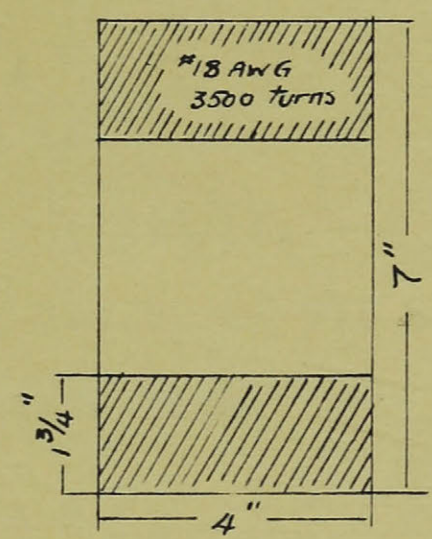


Fig. 12

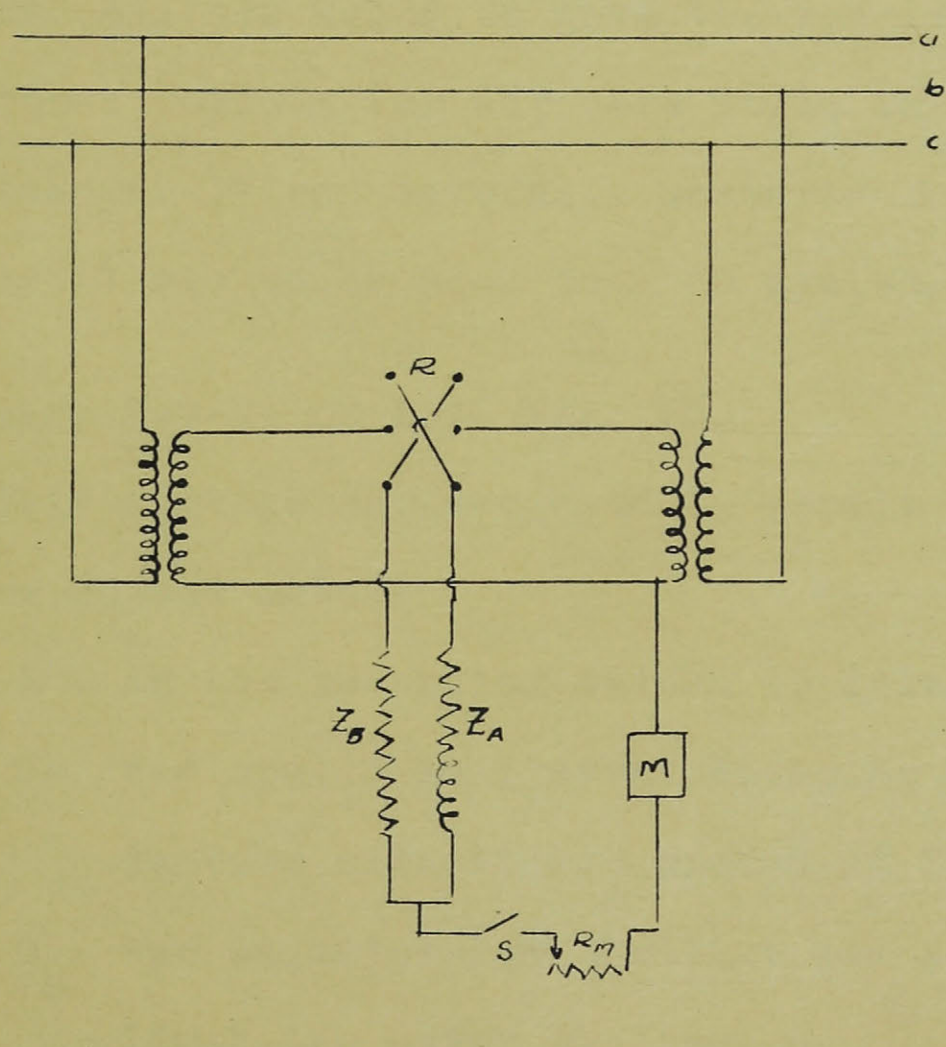


Fig. 13

$n = 3500$ turns of #18 wire

$Z = 21.9 + j194$ ohms

Added $R = 90.1$

$Z_A = 112 + j194$ ohms

Approx. current in Z_A

$= 191/(Z_A + Z_B) = .49$ amps.

and the working c.c.c. of #18 is 3amps. so we can expect a negligible temperature rise.

Since $Z_A = Z_B(.5 + j0.866)$

$Z_B = 224$ ohms

a non inductive resistance was used.

The coil was wound to give the required inductance by using Prof. Brooks formula for air core coils in the Standard Handbook for Engineers. It was carefully measured on an A.C. bridge at the frequency it was to be used on - 60 cycles.

Preliminary Set-Up of Net- Fig. 13.

This arrangement was used to permit the measurement ofboth sequences on one meter.

R - is the reversing swithh to interchange Z_A and Z_B.

S- was opened to prevent damaging the meter when the inductive circuit was opened at R.

R_M- was adjusted for positive and negative sequence to limit the meter reading.

The final adjustment on Z_A and Z_B was made when it was assembled. The drop across the two should be equal and the negative sequence connection should give zero deflection on balanced voltage.

The multipying factors for the meter readings are determined from the network equations

$$E_1 = 1/3.I_m(Z_B + Z_m(1-a))$$

where Zm is the sum of the meter impedance and R_m the external resistance.

$$E_2 = 1/3.I_m(Z_B + Z_m(1-a^2))$$

Care was taken to keep the inductance coil well away from stray fields and iron objects.

It was intended at first that standard Weston milliammeters available in the laboratory should be used in the meter circuit. It was found ,however, that, although these meters were satisfactory to measure the positive sequence voltage they were not suitable to measure the much smaller value of negative sequence voltage. The difficulty arises from the fact that the ratio of the internal impedance of the meter to its lowest readable current is too high. Thus if a 75 range milliammeter ,which has an impedance of 98.8 + j109 ohms is used $E_2 = .316I_m$. The minimum current for which the meter is calibrated is 20 milliamps. The smallest voltage which could be read would be 6.5 volts and since this reading would be at the end of the scale its accuracy would be no better than 4%. Similarily a 30 milliamp. meter has an impedance of 624 + j415 and the smallest voltage which could be detected on it was about 6 volts. Thus to obtain low values of negative sequence voltage a thermocouple with a microammeter

CURRENT SEQUENCE NETWORK

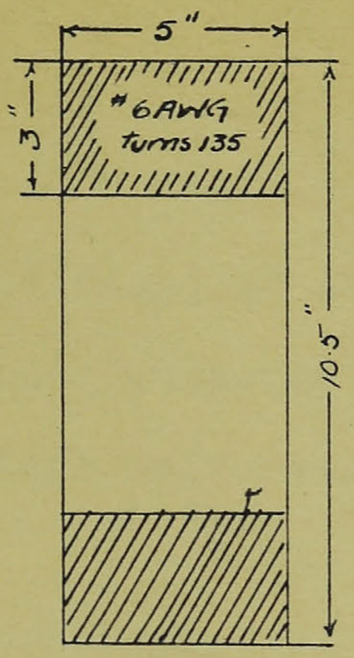


Fig. 14

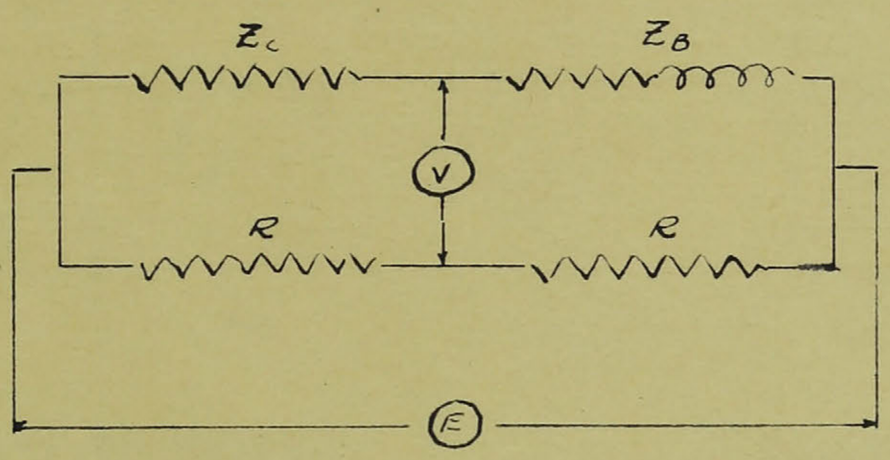


Fig. 15

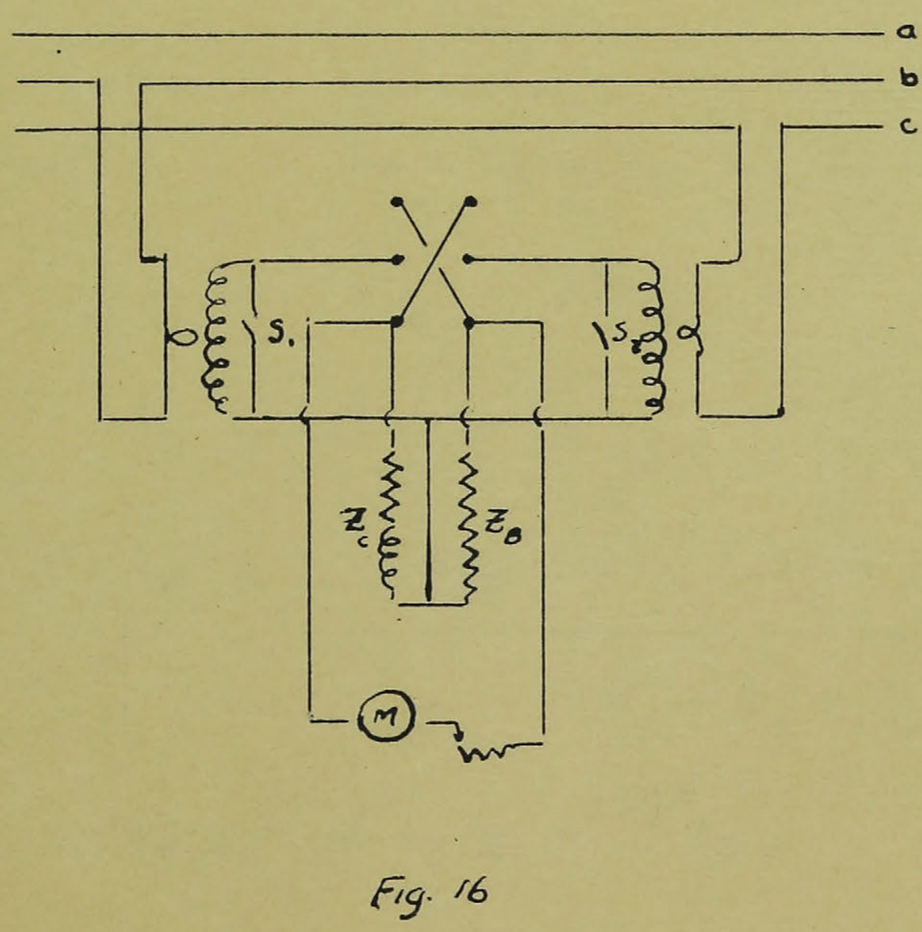
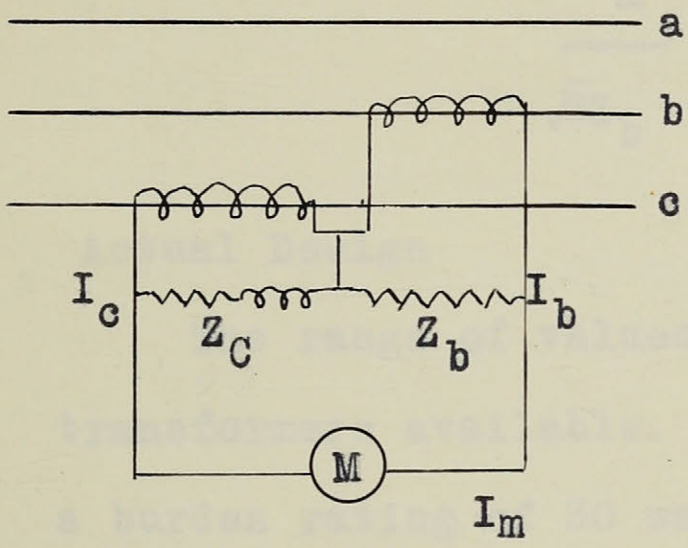


Fig. 16

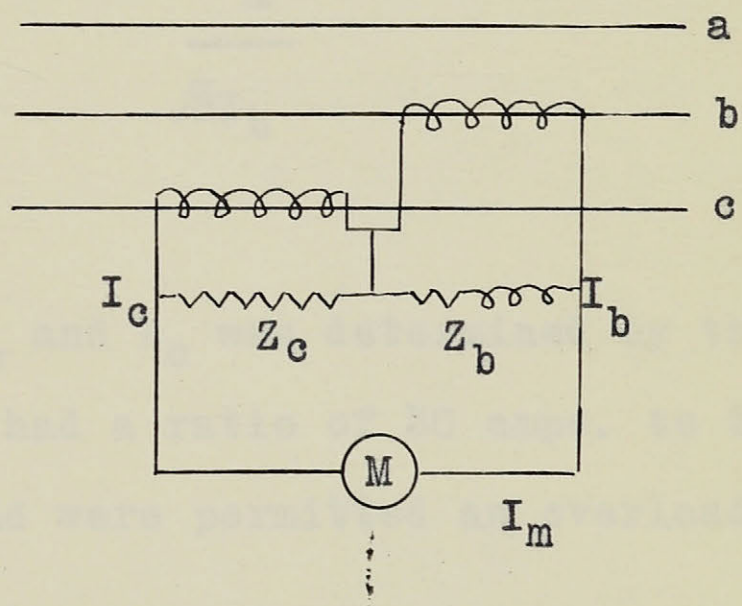
was used. The thermocouple had a resistance of 5 ohms and could be used over a range of values by inserting in series with it a Deacade resistance box

6. The Measurement of Sequence Currents

When zero sequence currents are absent a similar network to that shown for voltage measurements may be used Fig. 17.



Positive Sequence
Fig.17(a)



Positive Sequence
Fig.17(b)

$$I_m = \frac{Z_c}{Z_b + Z_c + Z_m} . I_c - \frac{Z_b}{Z_b + Z_c + Z_m} . I_b$$

and since $I_c = aI_1 + a^2 I_2$

$$I_b = a^2 I_1 + a I_2$$

$$I_m = \frac{-(aZ_c + a^2 Z_b).I_1 + (a^2 Z_c + aZ_b).I_2}{Z_b + Z_c + Z_m}$$

$$Z_c = -a^2 Z_b = e^{j60^o} Z_c$$

For the negative sequence

$$Z_b = -a^2 Z_c = e^{j60^\circ} Z_c$$

$$I_{m1} = \frac{1}{1 + \frac{Z_m}{\sqrt{3}Z_b} \cdot e^{-j30^\circ}} = \frac{1}{1 + \frac{Z_m}{\sqrt{3}Z_b} \cdot e^{j(\theta_m - \theta_{Z_b} - 30^\circ)}}$$

$$I_{m2} = \frac{1}{1 + \frac{Z_m}{\sqrt{3}Z_b} \cdot e^{j30^\circ}} = \frac{1}{1 + \frac{Z_m}{\sqrt{3}Z_b} \cdot e^{j(\theta_m - \theta_{Z_b} + 30^\circ)}}$$

Actual Design

The range of values for Z_b and Z_c was determined by the transformers available. These had a ratio of 30 amps. to 5 a burden rating of 30 watts, and were permitted an overload of 50% for one hour.

Since the rated load current for the machine to be used was 40 amps, if the meter circuit is neglected:

$$I_b = 40/6 = 6.67 \text{ amps.}$$

and $$Z_b = 30/6.67^2 = .675 \text{ ohms.}$$

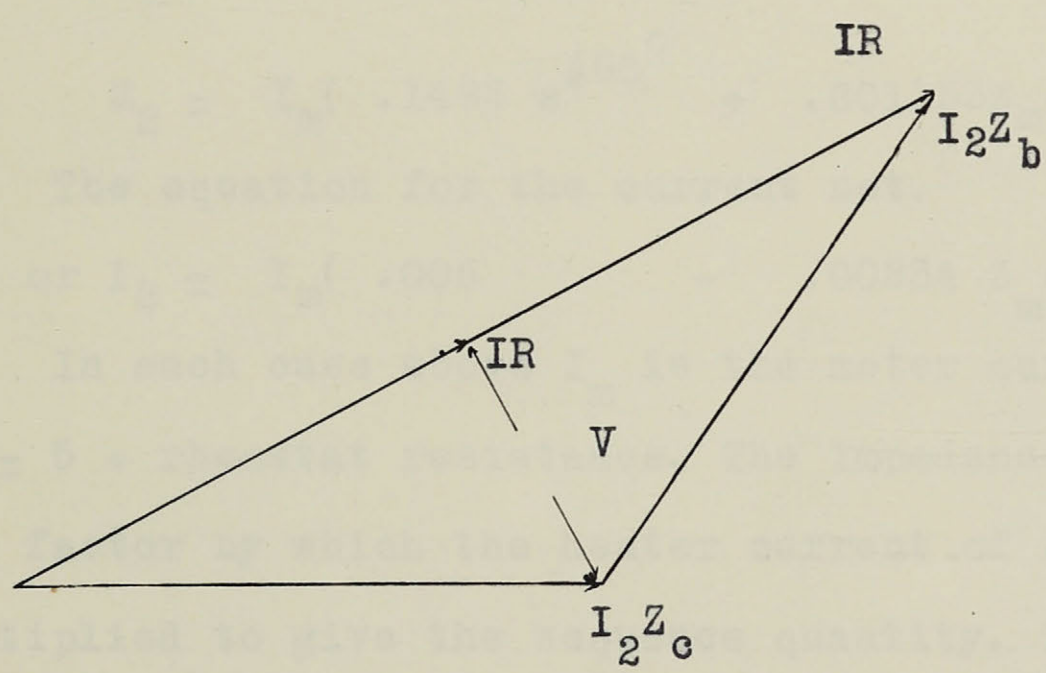
The impedance of the coil was made larger than the value shown above to give a coil which could be accurately measured and which would be influenced less by external fields. The resistance corresponding to a 50% increase in burden is 1.00 ohms. This necessitated an inductance coil of approximately 2.29 millihenrys. A coil as shown in Fig. 14. was constructed by using the formula in the handbook.

No. 6 rubber covered wire was used.

The number of turns was 135.

The reactance of the constructed coil was measured on an A.C. bridge as being 1.28 ohms.

Due to the wide divergence of resistance and reactance , and the small value of resistance, it was necessary to make the final adjustments in the network shown in Fig.15. The resistance branch Z_c was made and measured to be 1.28 x $2/\sqrt{3}$ = 1.47 (a D.C. bridge was used). In Fig.15. the R'_s were two equal standard resistances. The voltmeter measuring V was of high resistance. When the correct amount of resistance was in the Z_c and Z_b E = $\sqrt{3}$ V. ,since the following vector relations hold



The resistance of Z_b was adjusted until the two voltages had this relation to each other. Approximately 10 amps were passed through branch Z_b - Z_c giving a drop of about 30 volts.

The network as used is shown in Fig. 16.The switches S_1 and S_2 are to short circuit the current transformers when the reversing switch is used and thus prevent them from being damaged.

It was found necessary to use a thermocouple meter for the measurement of negative sequence current. The lowest readable current was about 4 amps. with a standard meter of 150 milliamp. range. There was but one thermocouple available and this was used for the voltage and current networks. It was connected across poles of a double throw switch and the current network was connected across the one set of jaws with the voltage network across the other. This arrangement did not permit the taking of simultaneous readings and required a large range of multiplying factors for the meter corresponding to the resistance used in the meter circuit.

Multiplying Factors Used

The equation for E_2 with the values of the network impedances used inserted in it is:

$$E_2 = I_m(.1493\ e^{j60^\circ} + .001153 Z_m\ e^{j30^\circ}) \quad \text{volts}$$

The equation for the current net.

$$I_1 \text{ or } I_2 = I_m(.006 - .00234\ Z_m e^{-j30^\circ}) \quad \text{amps.}$$

In each case above I_m is the meter current in milliamps, and $Z_m = 5 +$ rheostat resistance. The impedance term in each case is t the factor by which the heater current of the thermocouple must be multiplied to give the sequence quantity. These factors were obtained by making a graphical addition for the two parts of the multiplying factors. These factors are plotted for various values of series resistance. See Fig. 18 and 19.

The thermocouple and microammeter used were carefully calibrated together

7. Performance of the Meters

The results obtainable with the sequence meters are shown in the following tables. The check on the meters was made graph-

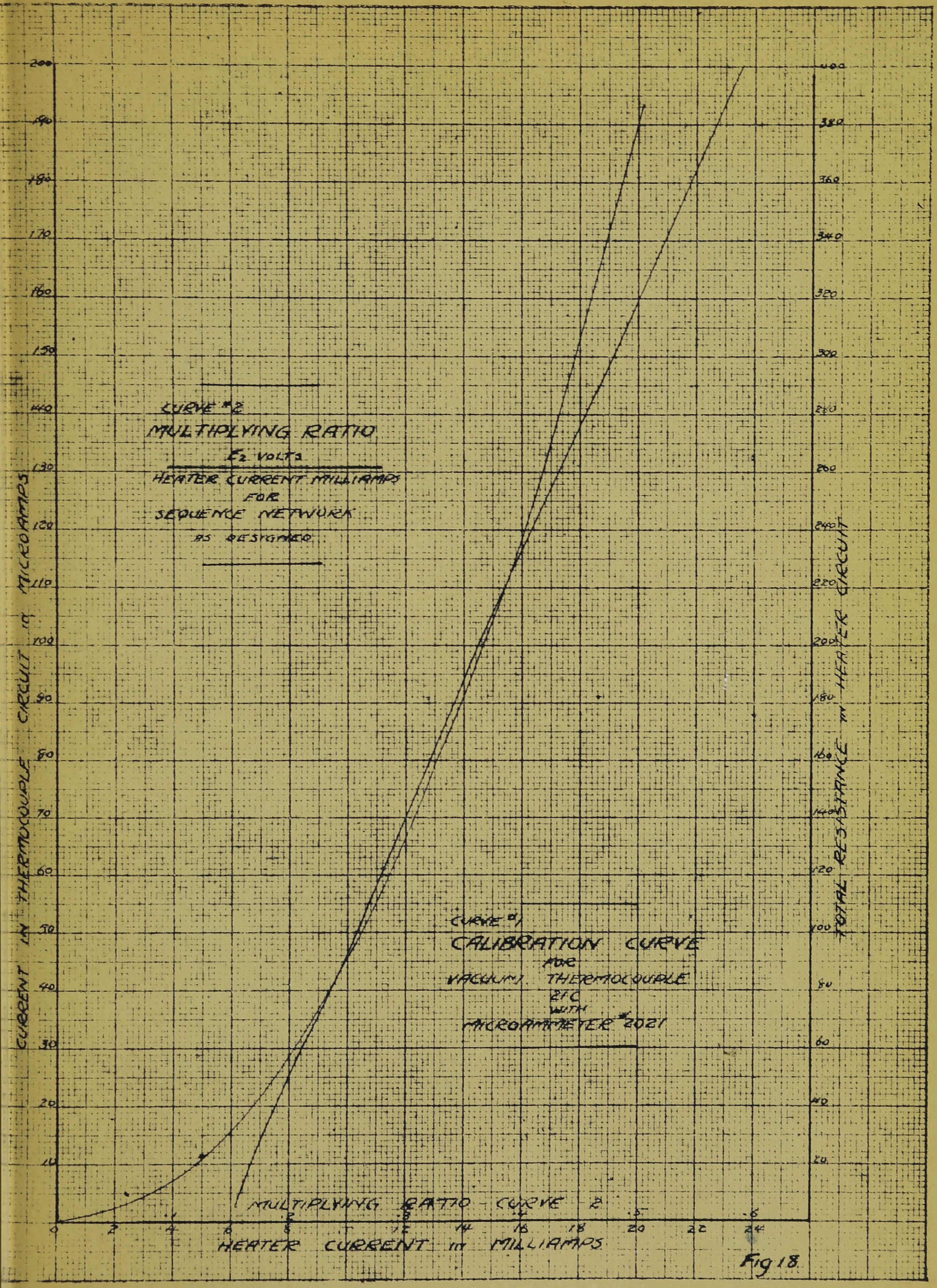
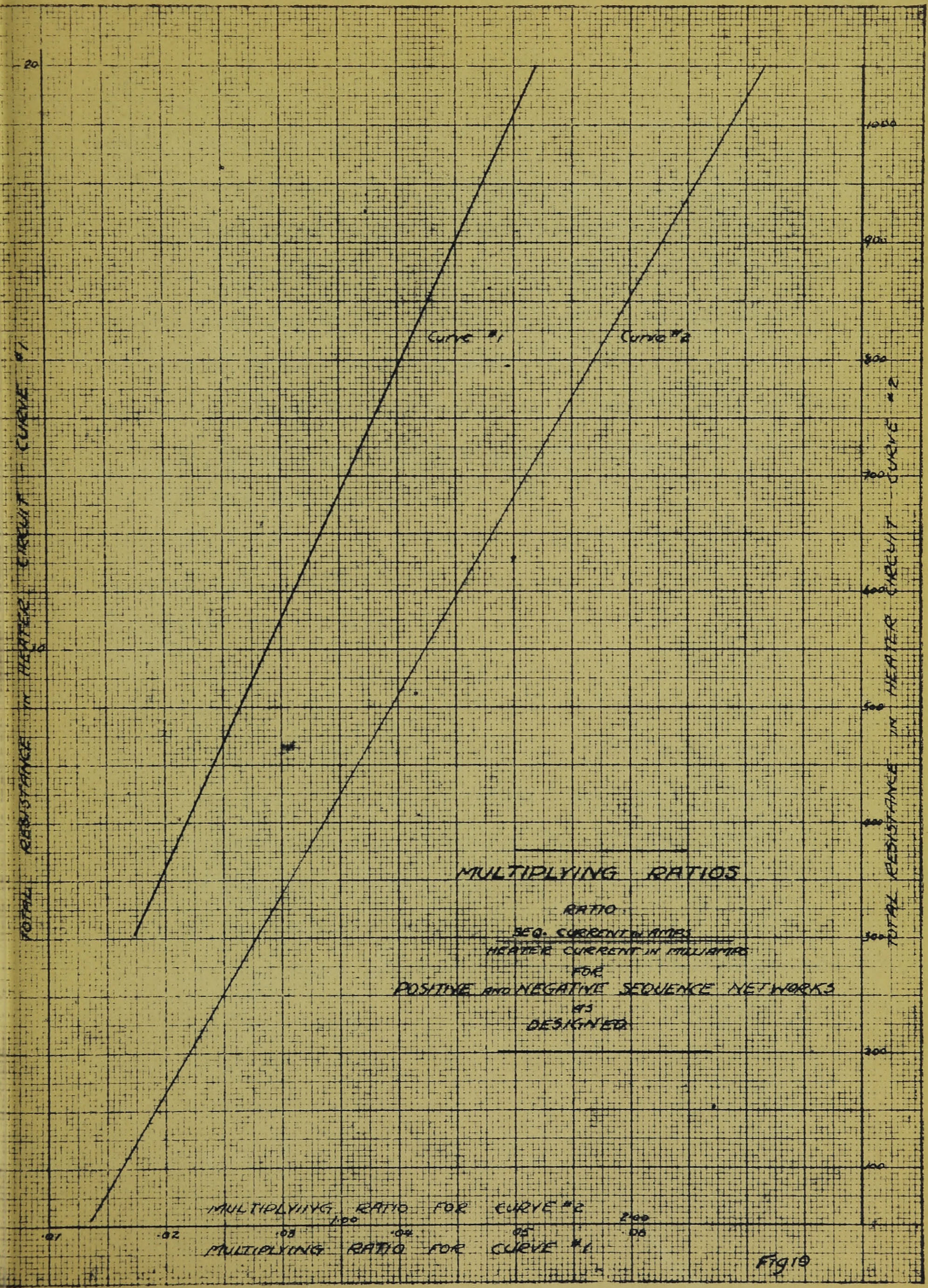

CURVE #2
MULTIPLYING RATIO
E2 VOLTS
HEATER CURRENT MILLIAMPS
FOR
SEQUENCE NETWORK
AS DESIGNED
CURVE #1
CALIBRATION CURVE
FOR
VACUUM THERMOCOUPLE
2/C
WITH
MICROAMMETER #2021
MULTIPLYING RATIO CURVE 2
CURRENT IN THERMOCOUPLE CIRCUIT IN MICROAMPS
TOTAL RESISTANCE IN HEATER CIRCUIT
HEATER CURRENT IN MILLIAMPS
0
2
4
6
8
12
14
16
18
20
22
24
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
40
60
80
100
120
140
160
180
200
220
240
260
280
300
320
340
360
380
400

Fig 18.


TOTAL RESISTANCE IN HEATER CIRCUIT CURVE #1
20
10
Curve #1
Curve #2
TOTAL RESISTANCE IN HEATER CIRCUIT CURVE #2
1000
900
800
700
600
500
400
300
200
100
MULTIPLYING RATIOS
RATIO = SEQ. CURRENT IN AMPS / HEATER CURRENT IN MILLIAMPS
FOR
POSITIVE AND NEGATIVE SEQUENCE NETWORKS
AS
DESIGNED
MULTIPLYING RATIO FOR CURVE #2
1.00
2.00
.01
.02
.03
.04
.05
.06
MULTIPLYING RATIO FOR CURVE #1
Fig 19

ically as explained.

Probable Sources of Error-

1) One source of error with the sequence voltmeter is due to the type of circuit in which it was measuring the components. It was connected across the terminals of a synchronous motor operating on unbalance. It will be shown later in this paper in Chapt. 111. that there is produced a third harmonic voltage as well as those of fundamental frequency when unbalanced currents flow in the motor. The networks are calibrated to give the correct values for fundamental frequency only and the presence of this third harmonic will result in an error. This effect will be noticed in the measurement of the negative sequence voltage.

2) The variation in current transformer ratio with load is the most probable source of error with the current net. It is not possible to apply any correction for the increase in turns ratio on light load because, on unbalance, the load on the two transformers is not equal. Thus the burden on the current transformer in the line carrying the larger unbalanced current will be greater than that on the line carrying the smaller.

It was found necessary for accuracy to replace the original current transformers, which were of rather low quality, with standard C.T.s These had different ratio blocks which could be used for various primary currents.

SEQUENCE METER ACCURACY

AMMETER

I_a	I_b	I_c	I_1	I_2	Meter			Meter			Tap#
					R	I_h	I_1	R	I_h	I_2	
31.2	29.6	28.0	29.4	1.8	720	173	30.2	120	144	4.75	1
26.5	25.2	24.1	25.1	1.4	620	170	26.1	120	135	4.58	1
21.4	20.4	19.5	20.4	1.0	520	114	21.3	120	104	4.04	1
13.2	13.0	13.0	13.0	0	320	181	13.0	120	116	4.30	1
43.0	35.7	32.6	36.9	6.6	880	173	37.1	180	156	7.53	4
35.7	29.8	26.8	30.5	6.5	740	170	30.4	220	111	75.4	4
27.7	27.0	17.2	23.6	6.5	610	131	22.5	180	161	7.6	4

VOLTMETER

E_A	E_B	E_C	E_1	E_2	Meter			Meter			Tap#
					R	I_h	E_1	R	I_h	E_2	
220	220	220	127	0	6570	99	127.0	5	92	2.35	1
218	226	218	129	3	6570	99	127.0	30	137	3.4	4
216	230	216	127	5	6570	99	127.0	100	180	5.68	6
214	235.5	214	127	7.5	6570	99	127.0	360	84	8.15	8
216	230	216	127	5	6570	99	127.0	160	122	5.82	6 NL
214	230	2216	126.5	6.5	6570	99	127.0	160	106	5.41	6 T40

CHAPTER 111.

SYNCHRONOUS MACHINE CONSTANTS

Any detailed study of synchronous machine operation requires the use of a number of constants. The various "reactances" are of especial interest. A large number of these is required to obtain accurate results because a separate reactance must be used for every flux condition and each of these must have separate values for the direct (polar) and quadrature (interpolar) axes. Separate values must be used as under different flux values different circuits are affected and the reactances are not a linear function of the fluxes present. It is proposed in the next few pages to outline these reactances, definitely define the flux conditions under which they are effective, and briefly describe how they may be obtained.

9. Per Unit Values

In analytical work on synchronous machine constants it is found that mÅch labor can be saved by expressing the various quantities encountered on a "per unit " basis. Thus they are expressed as decimal fractions of some "normal" or base value.

$$\text{Thus "per unit" reactance} = \frac{\text{reactance in ohms}}{\text{normal ohms}}.$$

$$\text{Where "normal ohms"} = \frac{\text{normal voltage to neutral}}{\text{normal full load current}}$$

This use has been extended to apply to other values both fundamental and derived. Thus per unit current is rate current and is "1" when used with other per unit quantities. A calcul-

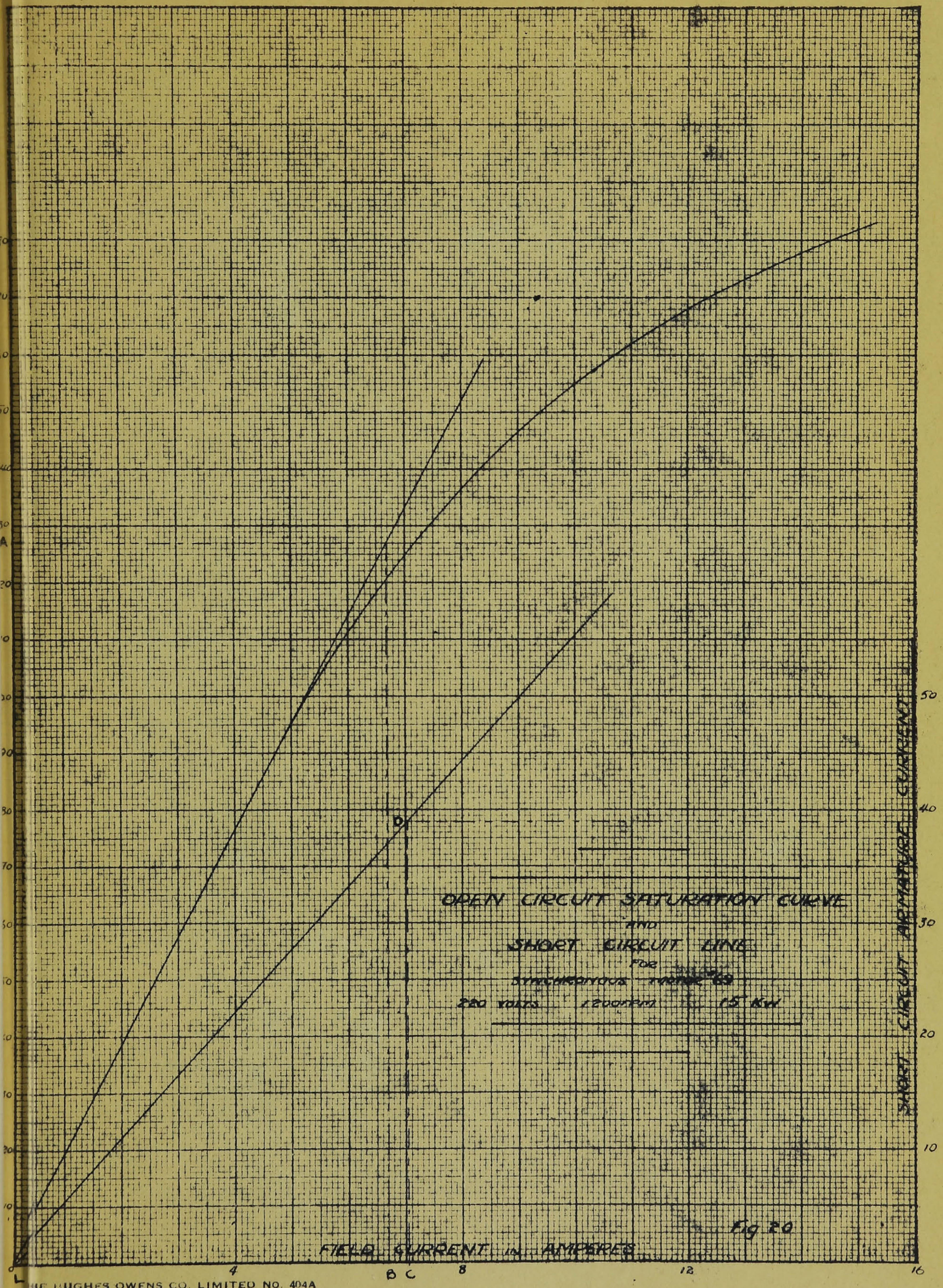

OPEN CIRCUIT SATURATION CURVE
AND
SHORT CIRCUIT LINE
FOR
SYNCHRONOUS
220 VOLTS 1200 RPM 15 KW
SHORT CIRCUIT ARMATURE CURRENT
10
20
30
40
50
FIELD CURRENT IN AMPERES
4
8
12
16
A
B
C
D
L

Fig 20

ation involving the revolving M.M.F. set up by balanced three phase currents, which in ordinary units $= 3/2nI_m$ will be represented in per unit values as being 3/2n at full load.

Examples of derived per unit values are:

ψ_o = normal armature flux linkages(rated voltage)

$= \frac{e_o}{2\pi f} 10^8$

ϕ_o = normal armature flux .

E_o = per unit field voltage.

10. X_d - Direct Axis Synchronous Reactance

This constant is defined in the A.I.E.E. Standards as;

"Synchronous impedance is the ratio of the field current required to circulate rated current on sustained three phase short circuit to the field current which would produce rated voltage at no load if there were no saturation". This value is the per unit value.

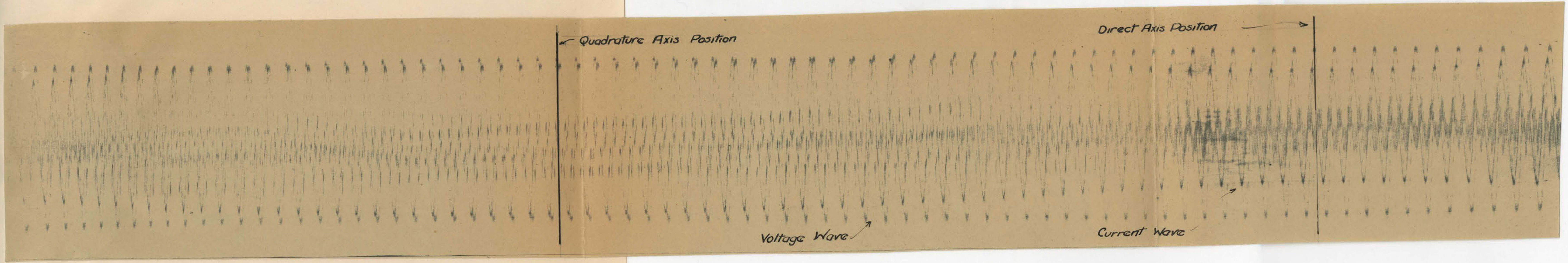
This definition may be expressed;

"Synchronous impedance at any field current is the ratio of the voltage off the air gap line for the no-load saturation curve to the value of the three phase short circuit current corresponding to the same excitation".

Thus in Fig. 20. $\frac{LC}{LB}$ is the direct axis synchronous reactance.

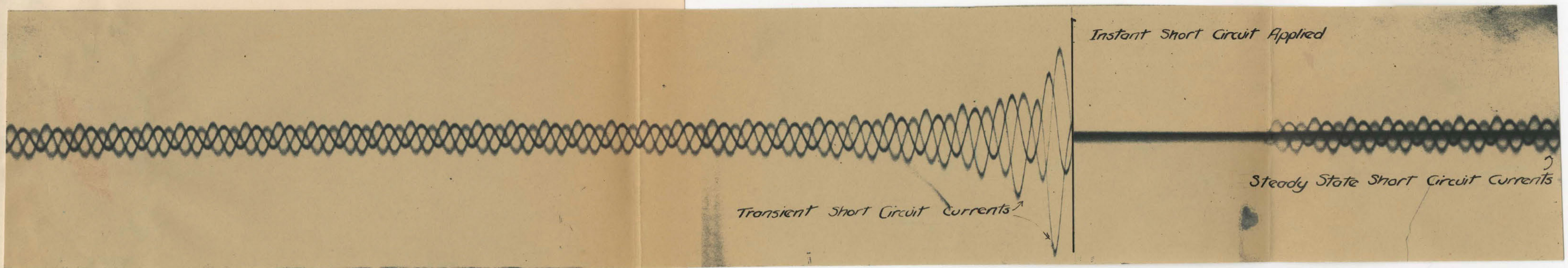
Test

The open circuit saturation curve is run. The short circuit current is then plotted against the field current for a



OSCILLOGRAM FOR SLIP TEST METHOD

Fig. 21.



OSCILLOGRAM OF A THREE PHASE SHORT CIRCUIT

Fig. 22.

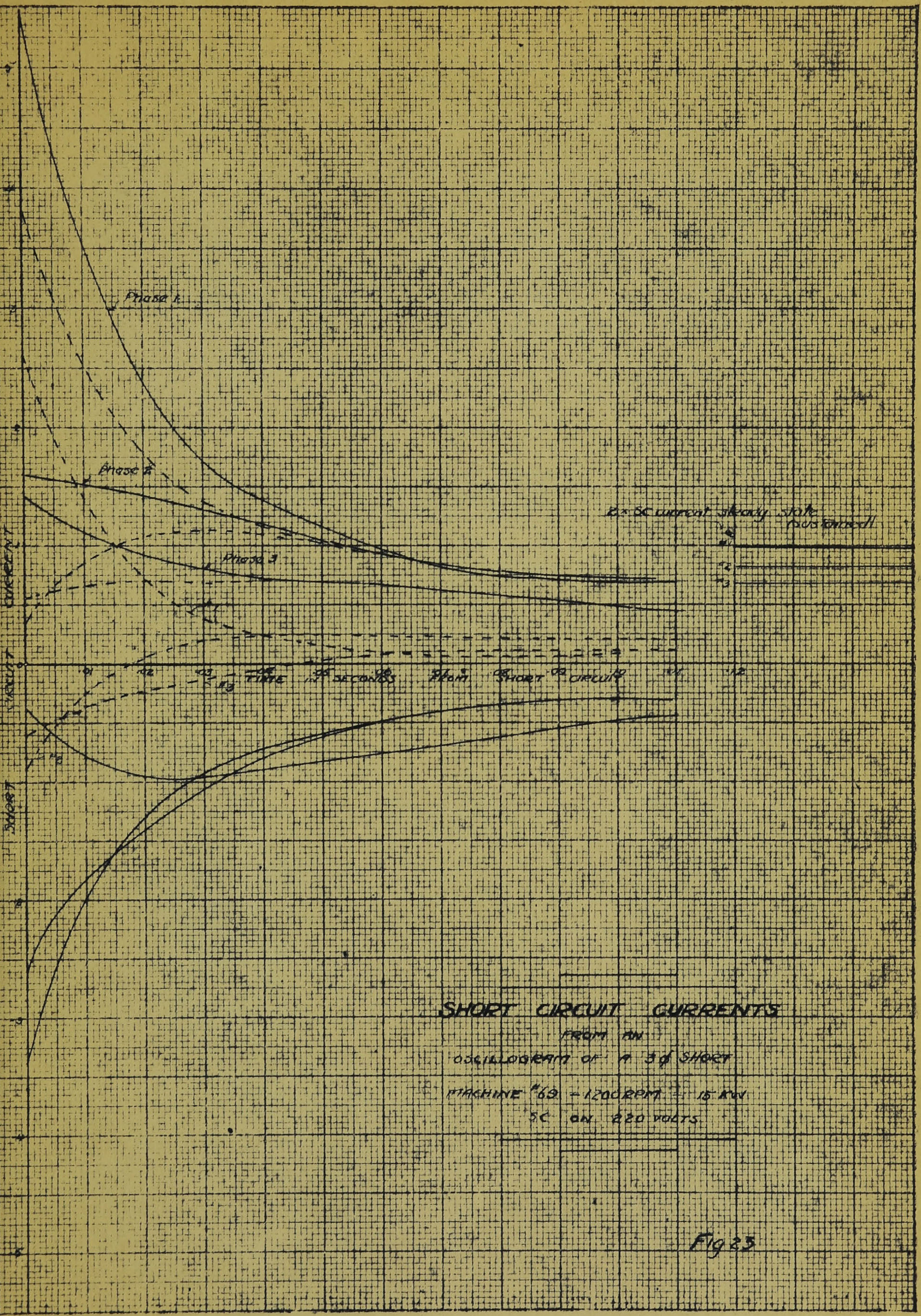

Phase 1
Phase 2
Phase 3
Ex SC current steady state (sustained)
Time seconds from short circuit
Short circuit current
SHORT CIRCUIT CURRENTS
FROM AN
OSCILLOGRAM OF A 3 ϕ SHORT
MACHINE #69 - 1200 RPM - 15 KV
SC ON 220 VOLTS

Fig 23

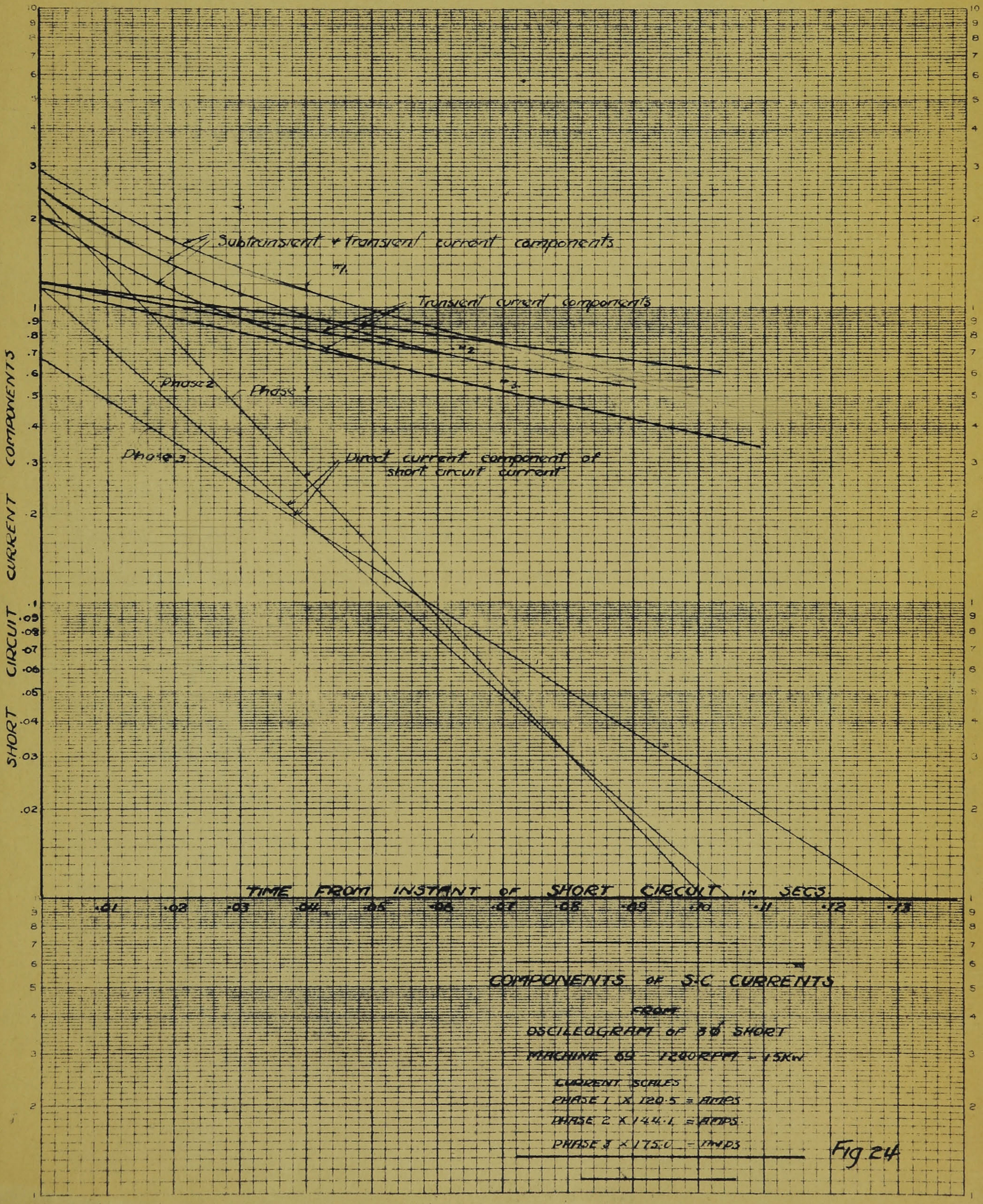

Subtransient + transient current components
Transient current components
Phase 2
Phase 1
Phase 3
Direct current component of short circuit current
SHORT CIRCUIT CURRENT COMPONENTS
TIME FROM INSTANT OF SHORT CIRCUIT IN SECS
COMPONENTS OF S.C CURRENTS
FROM
OSCILLOGRAM OF 3Ø SHORT
MACHINE 69 - 1280 RPM - 15KW
CURRENT SCALES
PHASE 1 X 120·5 = AMPS
PHASE 2 X 144·1 = AMPS
PHASE 3 X 175·0 = AMPS

Fig 24

sustained three-phase short circuit. The quantities required in the definition above are picked from these curves.

11. X_q- Quadrature Axis Synchronous Reactance

Test

The following slip test may be used to determine X_d and X_q. The machine tested is driven at a speed slightly less than that corresponding to rated frequency. The field of the machine is left open and about one-quarter voltage is applied. This voltage is of a frequency slightly greater than rated frequency. In other words the field poles are slowly slipping past the revolving armature M.M.F. Due to the varying reluctance of the flux path, a minimum at the pole face and a maximum between poles, the magnetizing current will vary for different rotor positions. By taking an oscillogram of armature current and voltage we can determine the ratio of the two at any instant. See Fig.21. Note that when the magnetizing current is a maximum the flux will be between the poles and X_q is determined from this point.

Note: These values of X_q and X_d in both cases are taken at conditions of low saturation. Under saturated conditions there is liable to be a considerable variation in X_d with slightly less in X_q due to the pole face iron in the direct axis. X_d is used when the current reaches a maximum as the field pole axis and axis of magnetization for each individual phase are in line. X_q is effective one-quarter of a cycle later when the current reaches a maximum since the current is a maximum when passing over the interpolar space.

The next group of reactances is taken under transient conditions by the analysis of the oscillogram of a three phase short circuit.

Test for X_d' and X_d''

12. Analysis of a Three-Phase Short Circuit

Fig.22. shows an oscillogram of a symmetrical three-phase short circuit taken at 220 volts on a laboratory motor. An element of the oscillograph measures the current in each phase. The photograph was taken at the instant of short circuit and exposed for one-half the length of the film. After a few seconds had lapsed the second half of the film was exposed and from this we get the steady state short -circuit current. The elements of the oscillograph were not calibrated but the scale for the current was determined from the steady state ordinate. This ordinate is determined from the relation:

$$I_{sc} = \frac{E}{X_d} = \frac{1.00}{X_d}$$

The time scale is known since the distance along this axis for one wave length can be measured under the steady state short circuit condition, This distance corresponds to a time lapse of 1/60 of a second with the 60 cycle supply frequency.

The envelope of the current ordinate was plotted for the region in which they were measurable (all except the first few cycles). These envelopes are shown in Fig.23. Each ordinate consists of an A.C. component and an assymetrical D.C. component. It is necessary to separate these. The centre line about which the current for each phase was oscillating was marked in by bisecting the ordinates between the two envelopes . The asymmetrical D.C. component will die out exponentially. Thus if we plot the ordinates on a logarithmic scale we will get a straight line. This was done in Fig.24. in which the D.C. component of current in each case is the ordinate measured from the steady state value of the centre line of the current envelope to the value of this centre line at the instant considered. The projection of these D.C. components back to the instant of short circuit gives a value which may be plotted back on the short circuit curve. The A.C. components of current were then plotted on a logarithmic scale. They are the difference between the ordinate and the D.C. component of current.

Under the action of the transient reactance the current dies out exponentially and for this reason the logarithmic plot gives a straight line for all A.C. values except the first few cycles. This straight line was projected back to the instant of short circuit. This ordinate = I' and

$$X'_d = \frac{E}{I_o}$$

The subtransient component of current diminishes at a greater rate than that given by a logarithmic function. The ordinates of I'' I' are plotted and the curve so formed continued back to the instant of short circuit.

$$X''_d = \frac{E}{I'_o}$$

-where E in each case above was the voltage before short circuit.

Thus from the oscillogram we obtain values of X'_d and X''_d for each phase. The average of, these values is shown in Fig.30.

The constants $X_d^{'}$ $X_q^{'}$ $X_D^{''}$ $X_q^{''}$ are all reactances effective during transient conditions. Since these reactances are definitely related to the fluxes it is well to consider flux conditions in an alternator, running at open circuit, when it is suddenly short circuited across its three phases.

13. $X_d^{''}$- Direct Axis Subtransient Reactance

Following the application of a short circuit, for the first few cycles, the flux set up by the demagnetizing armature current will not greatly cut down the field linkages present. This is due to the fact that it requires an appreciable time to build up or destroy the magnetic lines in an iron circuit. The effective circuit will be the damper windings in which short circuit currents will be induced. The flux set up by these currents will be equal and opposite , except for a small leakage flux in the damper bars, to that which induced the currents. The resultant flux will be small and the limiting reactance small. The effective reactance is in the direct axis and is written $X_d^{''}$.

14. $X_d^{'}$- Direct Axis Transient Reactance

After the first few cycles the demagnetizing armature flux will begin to penetrate the pole body and link the field windings. This linkage in the field windings will result in a decrease in the net flux in the pole body and air gap. Thus the effective reactance offered to the armature current will be larger than before. This condition will be that under which X_d the transient reactance is effective. Ultimately the demagnetizing action of armature reaction becomes effective and the air gap flux will be that determined by the vector sum of the field M.M.F. and the armature current M.M.F.

15. X''_q -Quadrature Axis Subtransient Reactance

Test

The field winding of the motor or generator is short circuited, the rotor is blocked, and a single phase A.C. voltage is applied across two of the line terminals. Measurements are taken of E the applied voltage and I the current flowing at different rotor positions. When the axis of the pulsating single phase flux coincides with the direct axis of the field poles the ratio of E/2 to I will give X''_d. When it coincides with the interpolar space the ratio gives X''_q. This variation in X'' is shown in Fig.25.

Note that the flux conditions in this test are those which are present in the 1/10th. second after the short circuit has been applied in the three phase short circuit test except for saturation effects due to current. In this test the flux is rapidly pulsating at supply frequency and the closed damper windings permit no penetration beyond them. In the case of the three-phase short circuit in the first few cycles there is a very rapid change in the flux set up by the armature current with the damper bars providing the only effective circuit. The X''_d of turbo-alternators variesm greatly with the armature current but with salient pole machines with damper windings it remains constant for all loads.

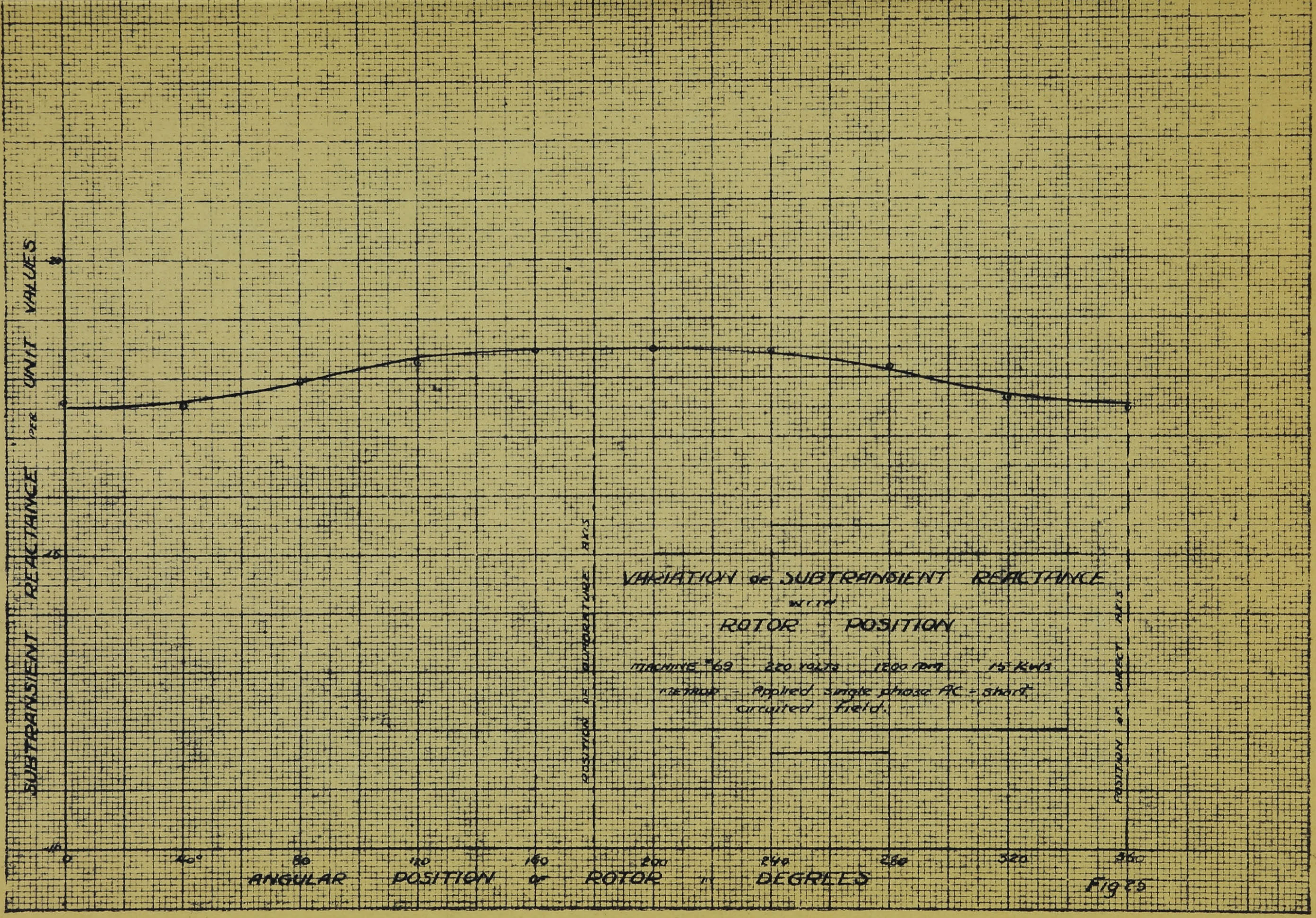

VARIATION of SUBTRANSIENT REACTANCE
WITH
ROTOR POSITION
MACHINE #69 220 VOLTS 1200 RPM 15 KWS
METHOD - Applied single phase AC - short circuited field.
SUBTRANSIENT REACTANCE PER UNIT VALUES
ANGULAR POSITION OF ROTOR IN DEGREES
POSITION OF QUADRATURE AXIS
POSITION OF DIRECT AXIS
0
40°
80
120
160
200
240
280
320
360
.15

Fig 25

Negative Sequence Resistance and Reactance

R_2 and X_2

Since these quantities are the fundamental ones behind the behaviour of synchronous motors under unbalanced conditions they have been investigated in some detail.

16. X_2 - Negative Sequence Reactance

Negative sequence reactance is the reactance offered by a synchronous motor to the flow of negative sequence current. The internal flux conditions under unbalance in the synchronous motor have been thoroughly investigated by Messrs. Park and Robertson. Their analysis is reproduced here as it is of such a fundamental nature as to require our attention.

Consider the application of negative sequence currents to the stator of a synchronous motor . At rated current values

$$i_a = \cos t$$
$$i_b = \cos (t + 120^\circ)$$
$$i_c = \cos (t - 120^\circ)$$

using per unit values.

These currents will give a resultant M.M.F. rotating at synchronous speed in the opposite direction to the positive sequence M.M.F. Thus the fluxes due to the negative sequence will pass over the rotor at a rate equal to twice applied frequency. These fluxes will vary rapidly at different positions of the rotor due to the variations in the path reluctance.

This path will consist of the stator air gap ,amortisseur winding, field collars, spider, metal wedges, rotor iron and field winding. These are studied by considering the leakage fluxes for the individual circuits and superimposing them to obtain the resultant fluxes. Equations have been derived for the general variation of rotor inductance with position and these equations are used with different reactances (X X' X'') depending on the flux conditions,

When three phase currents are applied to a synchronous motor a M.M.F. of constant value is set up.

$$A = 3/2nI_m$$

where A is the M.M.F.

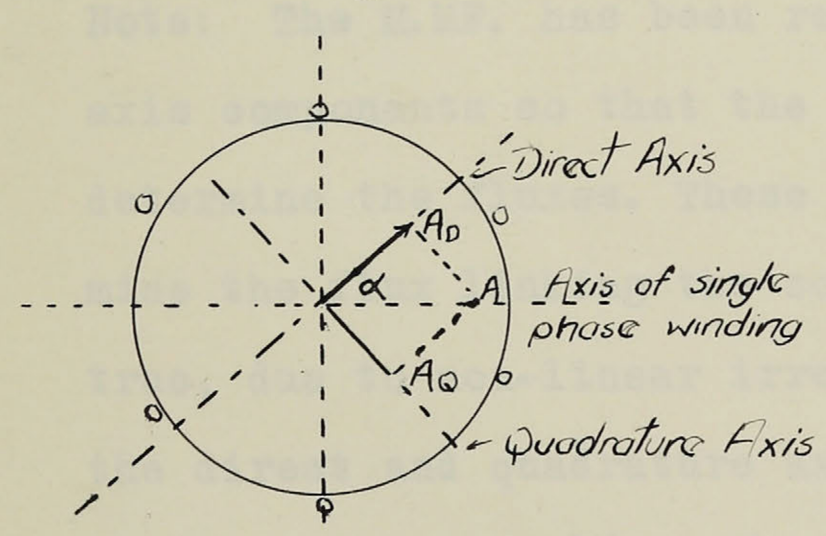
The M.M.F. of each phase considered individually is

$$A_1 = nI_m$$

If a single phase voltage is applied from line to neutral it will set up a pulsating flux. If the current circulated is equal in value to the three phase current then:

$$A_1 = 2/3A = 2/3I$$

This single phase M.M.F. may be broken into a direct axis component A_D and a quadrature axis component A_Q .



Referring to the figure to the left which shows a three phase armature winding with single phase applied voltage:

$$A_1 = A_D + A_Q$$

Let t be the per unit space angle (rated frequency is unity) measured from any arbitrary point.

α is the angle the direct axis of the poles made with the axis of the coil at $t = 0$

The actual value of A varies as a sine function with different rotor positions.

$$A_1 = \bar{A}_1 . \cos t.$$

Consider $t = 0$

Then $$A_D = 2/3I.\cos\alpha.\cos t.$$

$$A_Q = 2/3I.\sin\alpha.\cos t.$$

These M.M.F.'s will set up fluxes of values depending on the reluctance of the path in which they are effective. Since we are considering a component of three phase balanced M.M.F. the reactances used are the values per phase. For the present discussion X_d'' and X_q'' will be used and the explanation as to why they are used will be given later.

X_d'' -the voltage or flux at rated frequency per unit of armature current.

$$\psi_D = 2/3I.X_d''.\cos\gamma.\cos t.$$

$$\psi_Q = 2/3I.X_q''.\sin\gamma.\cos t.$$

γ is the electric space angle of the "a" phase component at $t = 0$ corresponding to α for any phase.

Note: The M.MF. has been resolved into quadrature and direct axis components so that the true reactances can be used to d determine the fluxes. These fluxes can now be combined to determine the flux linking the coil. This method is not rigorously true, due to non-linear irregularities in the flux path between the direct and quadrature axes.

Then $$\psi_T = \psi_D\cos\gamma + \psi_Q\sin\gamma.$$

$$= 2/3I\left(\frac{X_d'' + X_q''}{2} + \frac{X_d'' - X_q''}{2}.\cos 2\gamma\right).\cos t.$$

for one phase.

In the case of the negative sequence rotation the flux passes over the pole face at a rate corresponding to twice rated frequency. The pole face circuits will offer a high reactance to the flow of induced currents. The flux present will consequently be largely a leakage flux ni damper and armature windings. The similarity between this flux condition and that effective in the initial stages of a short circuit indicates that the subtransient reactances are those offered to negative sequence currents. This is also confirmed by direct measurements of X_2. We can now write down the flux equation of each phase.

$$\psi_a = \frac{X_d'' + X_q''}{2}\cdot\cos t + \frac{X_d'' - X_q''}{2}\cdot\cos(2\theta + t)$$

$$\psi_b = \frac{X_d'' + X_q''}{2}\cdot\cos(t+120^\circ) + \frac{X_d'' - X_q''}{2}\cdot\cos(2\theta + t - 120^\circ)$$

$$\psi_c = \frac{X_d'' + X_q''}{2}\cdot\cos(t-120^\circ) + \frac{X_d'' - X_q''}{2}\cdot\cos(2\theta + t + 120^\circ)$$

Where θ is the space angle between the axis of the "a" phase and the polar axis. The back voltages are given by the rate of change of flux.

Then for phase "a"

$$e = \frac{d\psi}{dt} = -\frac{X_d'' + X_q''}{2}\cdot\sin t - \frac{X_d'' - X_q''}{2}\cdot\sin(2\theta + t)\left(2\frac{d\theta}{dt} + t\right).$$

Thus there are two voltages induced, one of fundamental frequency, and the other of three times fundamental $\left(2\frac{d\theta}{dt} + t\right)$. To supply these a negative sequence current and a third harmonic current must flow. If $I_{1(3)}$ is the magnitude of the third harmonic current and $I_{2(1)}$ that of the negative sequence current then:

$$\psi_a = I_{2(1)}\cdot\frac{X_d'' + X_q''}{2}\cdot\cos t + I_{1(3)}\cdot\frac{X_d'' + X_q''}{2}\cdot\cos(3t + \alpha)$$

$$+ I_{2(1)} \cdot \frac{X_d'' - X_q''}{2} \cdot \cos(3t + 2\theta_0) + I_{1(3)} \cdot \frac{X_d'' - X_q''}{2} \cdot \cos(2\theta_0 - t - \alpha).$$

where $\theta = \theta_0 + t$.

If there is to be a fundamental of flux only then $\alpha = 2\theta_0$ and third harmonic currents will flow in such phase relations as to make this true. Also:

$$I_{2(1)} \cdot \frac{X_d'' - X_q''}{2} = -I_{1(3)} \cdot \frac{X_d'' + X_q''}{2} .$$

and

$$\psi_a = I_{2(1)} \left(\frac{X_d'' + X_q''}{2} - \frac{(X_d'' - X_q'')^2}{2(X_d'' + X_q'')} \right) \cos t.$$

$$= I_{2(1)} \cdot \frac{2X_d'' X_q''}{X_d'' + X_q''} \cdot \cos t.$$

If the fluxes are of the form

$$\psi_a = \cos t.$$

$$\psi_b = \cos(t + 120^\circ)$$

$$\psi_c = \cos(t - 120^\circ)$$

Then the amplitude of the flux is given by

$$I_{2(1)} \cdot \frac{2X_d'' X_q''}{X_d'' + X_q''} = 1$$

$$\text{or } I_2 = 1/2\left(\frac{1}{X_d''} + \frac{1}{X_q''}\right)$$

$$\text{and } I_3 = -1/2\left(\frac{1}{X_d''} + \frac{1}{X_q''}\right)\left(\frac{X_d'' - X_q''}{X_d'' + X_q''}\right)$$

The phase currents will be

$$I_a = 1/2\left(\frac{1}{X_d''} + \frac{1}{X_q''}\right)\left(\cos t - \frac{X_d'' - X_q''}{X_d'' + X_q''} \cdot \cos(3t + 2\theta_0)\right)$$

$$I_b = 1/2\left(\frac{1}{X_d''} + \frac{1}{X_q''}\right)\left(\cos(t - 120^\circ) - \frac{X_d'' - X_q''}{X_d'' + X_q''} \cdot \cos(3t + 2\theta_0 - 120^\circ)\right)$$

$$I_c = 1/2\left(\frac{1}{X_d''} - \frac{1}{X_q''}\right) \cdot \left(\cos(t + 120^\circ) - \frac{X_d'' - X_q''}{X_d'' + X_q''} \cdot \cos(3t + 2\theta_o + 120^\circ)\right)$$

These are the currents which must flow in each phase to give a balanced negative sequence terminal voltage.

The per unit value of reactance given above is the negative sequence reactance and is given by:

$$X_2 = \frac{1}{\frac{1}{2}\left(\frac{1}{X_d''} - \frac{1}{X_q''}\right)}$$

If there is external reactance which is great compared to X_d'' and X_q'' the value of X_2 becomes

$$X_2 = 1/2(X_d'' + X_q'')$$

This external reactance offers considerable reactance to the flow of third harmonic currents. By so preventing the flow of the third harmonic current necessary to give a balanced negative sequence voltage and flux, the voltage will contain a third harmonic and the current will become sinusoidal. In most cases sufficient external reactance is provided to satisfy this condition. Because the current is of pure sine wave form the value of X_2 is usually associated with the current I_2 rather than with E_2 the voltage.

17. R_2 - Negative Sequence Resistance

Negative sequence resistance is the resistance associated with the loss due to the flow of negative sequence current. This loss will consist of a copper loss in the armature winding, core loss in the rotor, copper loss in the rotor, and of a counter-rotational torque.

The relative values of these losses may be brought out by a study of the equivalent diagram for an induction motor. We can apply the conclusions of this study to the synchronous motor if it is realized that the rotor circuits in each case are of a different nature. The rotor circuit of an induction motor consists of the rotor bars and end rings. The effective rotor circuit consists of field collars, dampers etc.

The equivalent circuit of the induction motor is shown below:

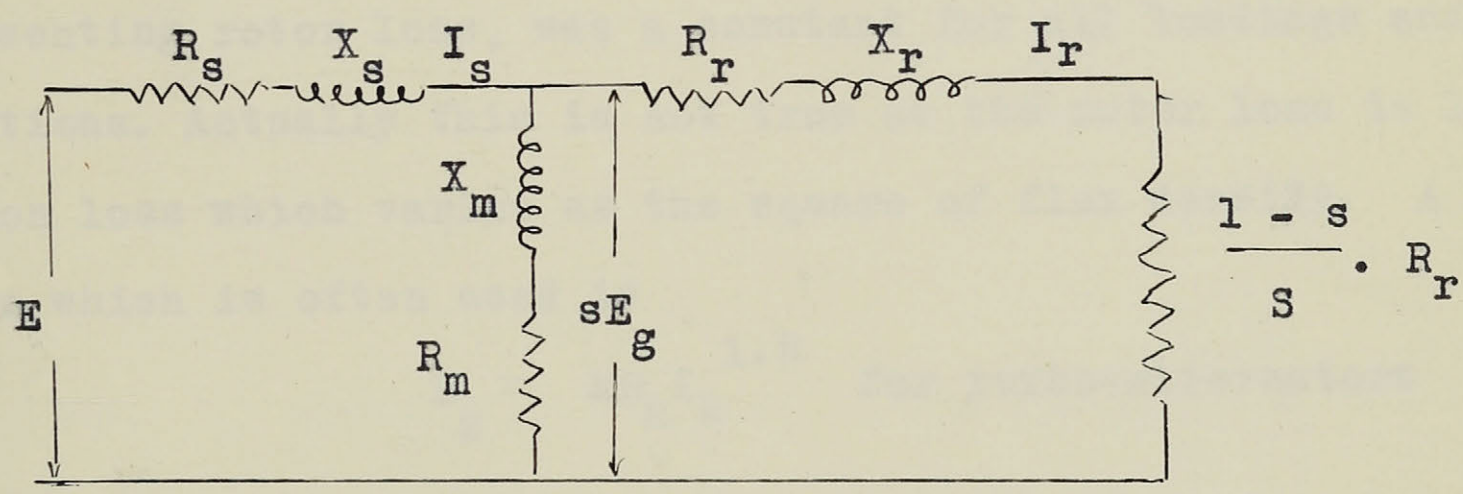


Fig. 26.

In this circuit:

s is the slip.

R_r the rotor resistance.

X_r the rotor reactance.

R_s the stator resistance.

X_s the stator reactance.

sE_g the effective voltage in the rotor winding.

$\left(\frac{1-s}{s}\right).R_r$ represents the shaft load.

$R_m + jX_m$ is the exciting impedance.

If this circuit is converted to the circuit for negative sequence currents E will then be E_2 and since the slip for negative sequence is equal and opposite to the synchronous speed it will equal "2".

Then the shaft load $= -1/2 R_r I_{2r}^2$

The total rotor loss which must be supplied is $R_r I_2^2$. The input from the shaft is $1/2 R_r I_{2r}^2$. We conclude therefore that half the rotor copper loss due to negative sequence current is supplied from the stator and half from the shaft.

One of the approximations used in the establishing of the equivalent diagram as used above was that R_r, the resistance representing rotor loss, was a constant for all loadings and flux conditions. Actually this is not true as the rotor loss is largely an iron loss which varies as the square of flux density. A figure which is often used is

$$R_2 = kR_2 I_2^{1.8} \quad \text{for turbo-alternators}$$

where "k" is the constant of proportionality.

Note that in the equivalent circuit above if we neglect the exciting current

$$R_2 = R_s + \frac{R_r}{2}$$

The Measurement of R_2 and X_2

The value of R_2 which is usually used is that effective when I_2 is equal to rated current. With negative sequence currents of this order there will be considerable heating in the high resistance damper bars. The consequent rise in temperature will result in an increase in R_r and therefore R_2. For this reason

R_2 should be measured shortly after I_2 is applied.

Three methods of determining R_2 and X_2 will be outlined.

Method 1

Two Generator Method

This is perhaps more obvious and less accurate than are the other two methods. It offers two main difficulties in that it requires two identical synchronous generators mechanically coupled with other driving means on the same shaft and does not lend itself to the obtaining of a value of I_2 equal to rated current.

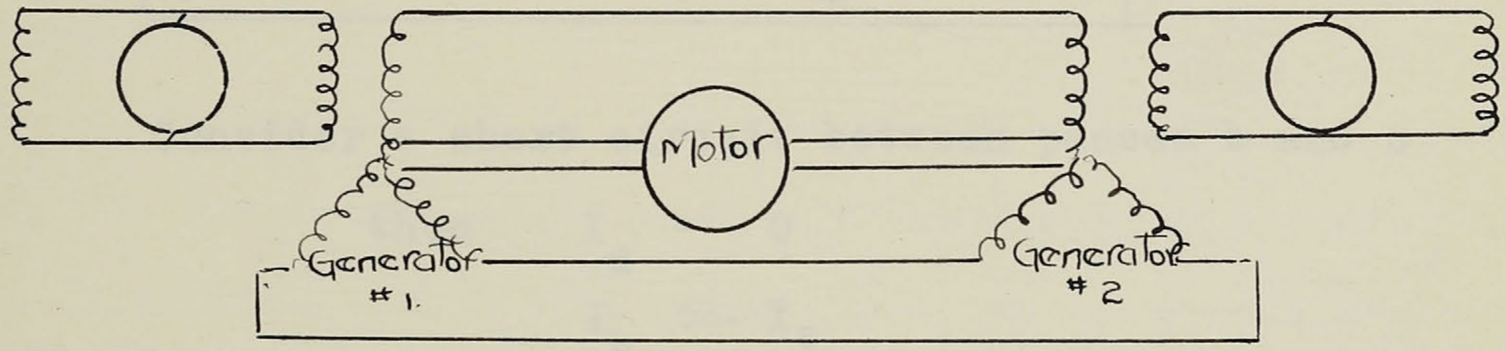


Fig. 27.

Fig. 27. shows the arrangement. The connections are made:

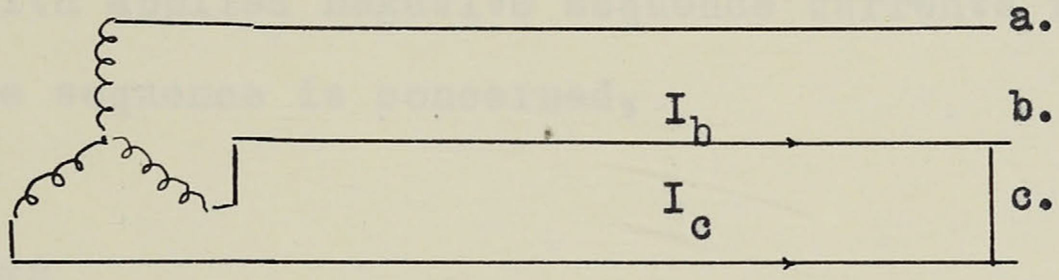
Gen. #1 1. 2. 3.
Gen. #2 1. 2. 3.

Thus which ever generator is taken as setting up a positive phase rotation of flux the other is applying to it a negative phase rotation. If the excitation of the one is varied negative and positive sequence currents will flow between the two machines. Measurements can be made to determine I_2 E_2 I_1 E_1 and the necessary angles between them. Then:

$$\frac{E_2}{I_2} = Z_2$$

Methods 2 and 3

The next two methods are those commonly used. They depend on the fact that when a line to line short circuit is put on an alternator the negative sequence current which will flow is equal and opposite to the positive sequence current. Thus the machine supplies its own negative sequence voltage.



Consider a short circuit between phases b and c

then $I_a = 0$

$I_b = - I_c$

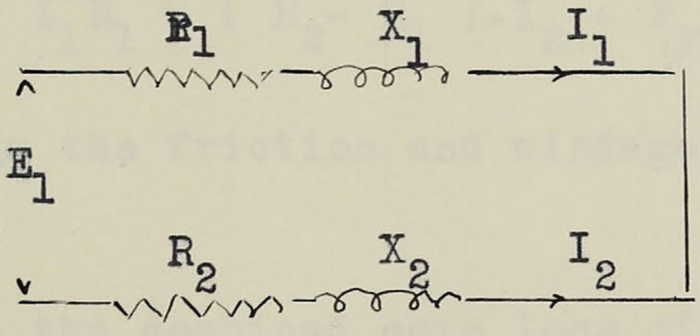
$E_b = E_c$

$$I_1 = 1/3(I_a + aI_b + a^2 I_c) = 1/3 I_c(-a + a^2).$$

$$I_2 = 1/3(I_a + a^2 I_b + a I_c) = 1/3 I_c(a - a^2).$$

or $I_1 = -I_2$

We can then show the equivalent circuit



- As seen from the vector diagram of this circuit.

It has been shown that $R_2 = R_s + \frac{R_r}{2}$ for an induction motor with applied negative sequence current.[2] When the generator is shorted from line to line it is supplying itself with negative sequence currents.

The negative shaft power $= -\frac{R_r}{2} I_2^2 = -(R_2 - R_s) . I_2^2$

Notice that $R_s = R_1$ is the stator resistance and is the only resistance offered to positive sequence current when the machine is in synchronism.

When short circuited the generator is the equivalent of a motor with applied negative sequence currents in so far as the negative sequence is concerned,

Method #2

R_2 is determined in this method by measuring the input to the motor with the generator short circuited and by by deducting various losses to obtain the actual shaft input to the generator. The shaft must supply:

Negative sequence shaft power $= (R_2 - R_1) . I_2^2$

Generator positive sequence copper loss $= R_1 I_1^2.$

Negative sequence power $= R_2 I_2^2.$

The total input to the motor:

$$P_m = I_2^2 R_2 + I_1^2 R_1 + (R_2 - R_1) . I_2^2 + P_{FW} + CL + I^2 R_m.$$

where P_{FW} is the friction and windage loss pf motor and generator.

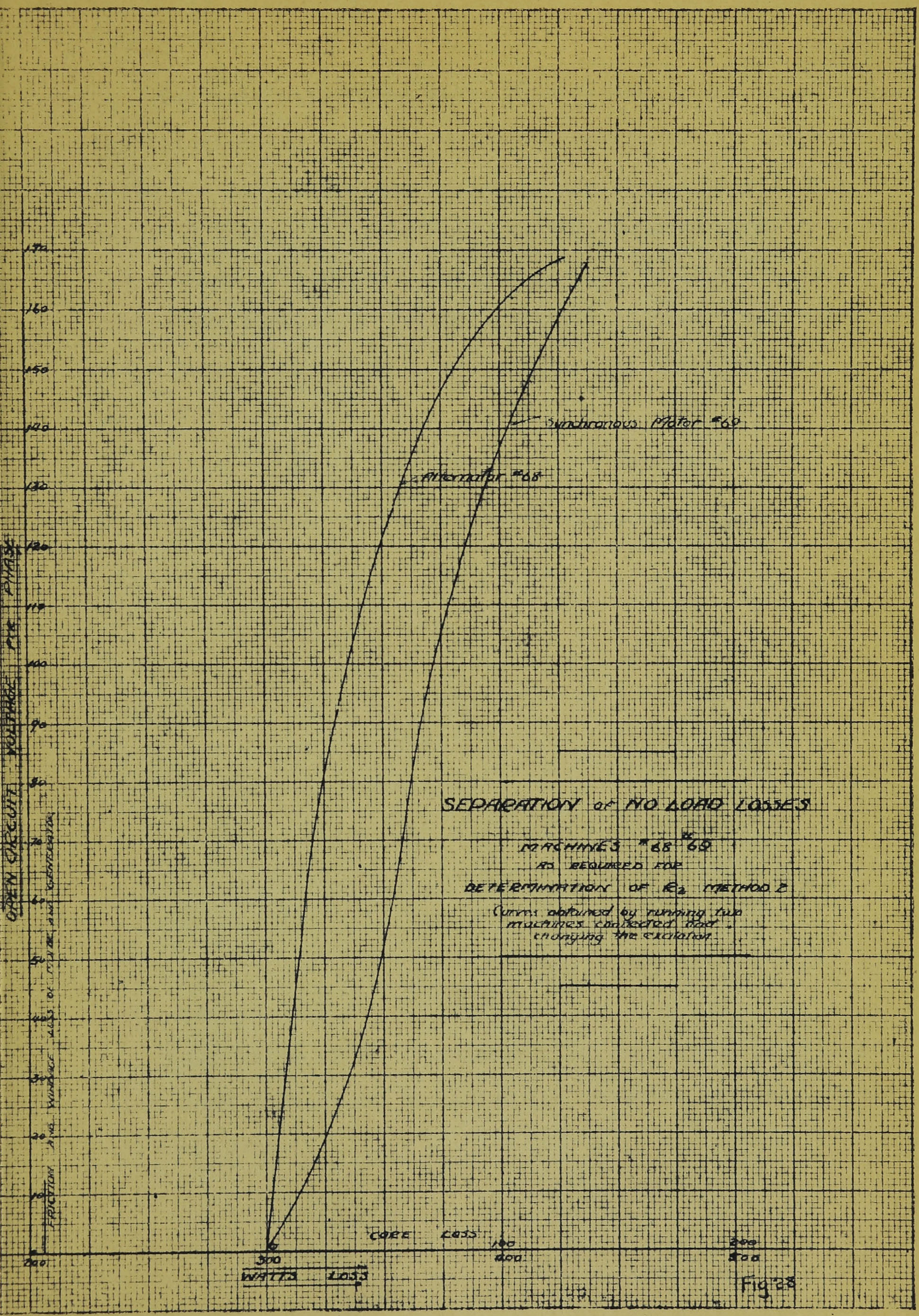
CL is the combined core loss of motor and generator.

$I^2 R_m$ is the motor copper loss.

Then since $I_1 = -I_2$

$$P_m = 2 I_2^2 R_2 + P_{FW} + CL + I^2 R_m.$$

The accuracy of the value obtained for R_2 is determined by the accuracy with which the losses are determined.


SEPARATION of NO LOAD LOSSES
MACHINES #68 & 69
AS REQUIRED FOR
DETERMINATION OF R_2 METHOD 2
Curves obtained by running two machines connected and changing the excitation
Synchronous Motor #69
Alternator #68
OPEN CIRCUIT VOLTAGE PER PHASE
FRICTION AND WINDAGE LOSS OF MOTOR AND GENERATOR
CORE LOSS
WATTS LOSS
Fig 28

Readings were taken at no load to obtain the core losses and the friction and windage. The motor was driven at constant excitation and the excitation on the generator was raised from zero to rated value. The generator was then used to drive the motor for various excitations. Curves were plotted for the core loss of the two machines (against internal voltage) and the friction and windage determined by using the input measurements.

Then

$$2I_2^2R_2 = P_m - P_{FW} - CL - I^2R_m.$$

Shaft power or power transferred to the shaft:

$$P_s = P_m - CL - I^2R_m$$

$$2I_2^2R_2 = P_s - P_{FW}$$

$$R_2 = \frac{P_s - P_{FW}}{2I_2^2}$$

It has been shown that:

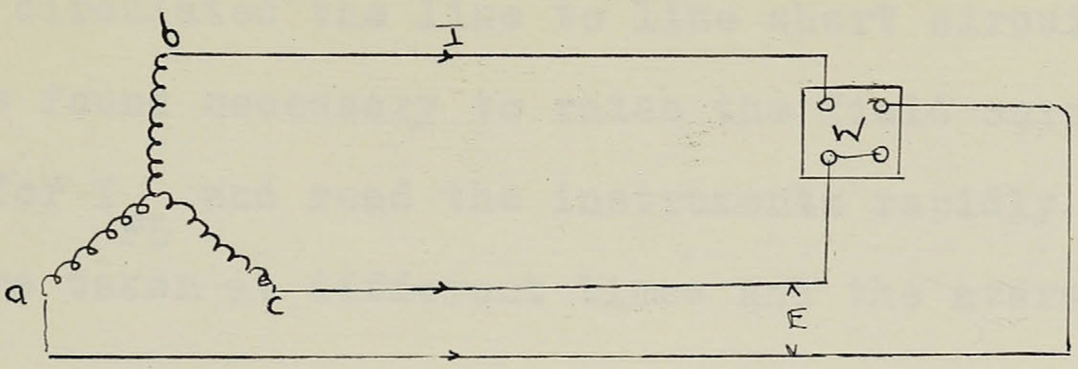
$$I_2 = 1/3(a-a^2).I_c = j.577I_c$$

$$R_2 = \frac{P_s - P_{FW}}{2(.577)^2.I_c^2} = \frac{1.5(Ps - P_{FW})}{I_c^2}$$

Method #3

The value of R_2 in this case is determined on a line to line short circuit by measuring the negative sequence output of the generator.

The wattmeter connection is shown below:



On short circuit-

$$I_b = -I_c = I$$

$$E_B = -E_C = E$$

$$I_{a2} = \frac{a^2 - a.I}{3}$$

$$E_{A2} = \frac{a^2 - a.E}{3}$$

$$E_{a2} = \frac{jE_{A2}}{\sqrt{3}} = \frac{ja^2 - a}{3\sqrt{3}} .E$$

$$Z_2 = \frac{E_{a2}}{I_{a2}} = \frac{jE}{\sqrt{3}I}$$

If W is the wattmeter reading then:

$$W = \overline{E}\overline{I}\cos\phi$$

since $E_{a2} = I_{a2}(R_2 + jX_2) = jE = \sqrt{3}I(R_2 + jX_2)$

and $\cos\phi$ is the angle between the vectors E and I.

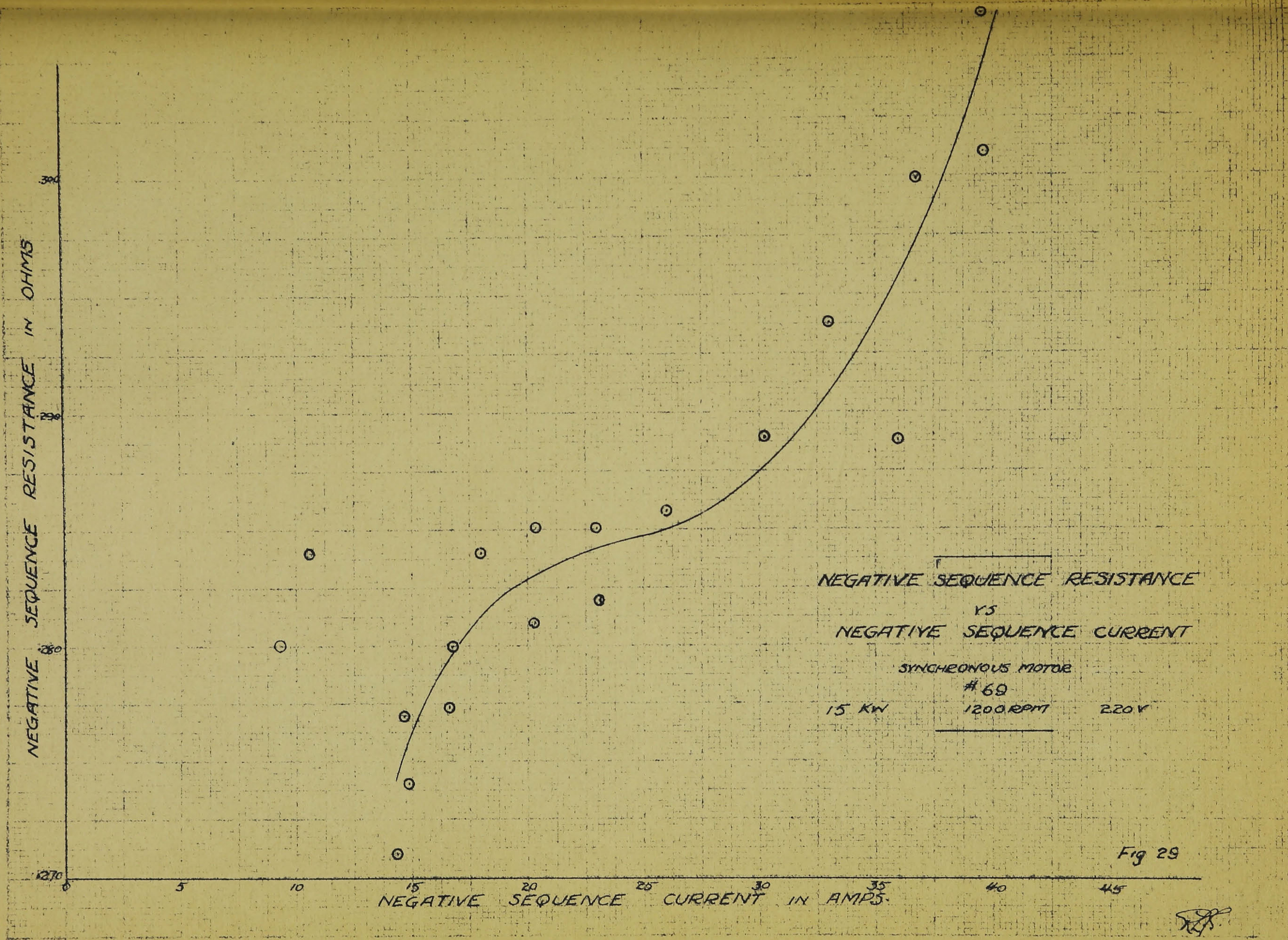
$$Z_2 = \frac{\overline{E}}{\sqrt{3}\overline{I}} (\sin\phi - j\cos\phi) = R_2 + jX_2$$

$$R_2 = \frac{E}{\sqrt{3}I} \left(\sqrt{1 - \left(\frac{W}{EI} \right)^2} \right) \text{ ohms}$$

$$X_2 = \frac{W}{\sqrt{3}I^2} \text{ ohms}$$

The test was run to give rated negative sequence current so $\sqrt{3}.I_{FL}$ was circulated through the line to line short circuit.

It was found necessary to raise the field current to the value required for I_{FL} and read the instruments rapidly. A number of values were taken at different times and the average taken. See table 30.


NEGATIVE SEQUENCE RESISTANCE
VS
NEGATIVE SEQUENCE CURRENT
SYNCHRONOUS MOTOR
#69
15 KW
1200 RPM
220 V
NEGATIVE SEQUENCE RESISTANCE IN OHMS
NEGATIVE SEQUENCE CURRENT IN AMPS.
0
5
10
15
20
25
30
35
40
45
270
280
290
300
Fig 29

18. TYPICAL SYNCHRONOUS MACHINE CONSTANTS

Type of Machine		Small high speed salient pole motor Damper winding	Larger salient pole medium speed motor Damper winding	Generator Dampers	Generator no Dampers	Turbo Generator
Rating		15 KVA 220 volts 1200 rpm	240 KVA 2200 volts 277 rpm	100 KVA 2300 volts 1200 rpm	187 KVA 240 volts [illegible] rpm	1875 KVA 6000 volts 3600
Constant	Method of Determination					
R_{DC}		.0445				
R_{AC}		.0578				
X_d	AIEE Definition	1.06	1.05	1.25	.94	1.00
X_q	Slip Test	.70				
X_d'	1. Three Phase SC	.426	.42	.24	.41	.15
	2. Relation $X_d' = .02 + 1.4 X_d''$	.254				
X_q'		.70				
X_d''	1. Three Phase S.C.	.179	.32	.165	.348	.12
	2. Applied L-L voltage	.175				
X_q''	Applied L-L voltage	.185				
R_2	1. Input at L-L SC	.0987	.089	.017		.035
	2. Measure W_2 L-L SC	.0969				
X_2	1. Measure W_2 L-L SC	.156	.33	.170	.55	.12
	2. $\frac{X_d'' + X_q''}{2}$	.180				
X_0	Apply single phase coils in series	.044	.06	.033	.07	.03

Fig 30.

CHAPTER IV.

EFFECTS of UNBALANCE on SYNCHRONOUS MOTOR PERFORMANCE

19. The Effect on Starting Torque

The self-starting synchronous motor starts as an induction motor. Copper, bronze, or nickle alloy bars are imbedded in slots in the pole face. The bars are welded to end rings running around the periphery of the field poles to form a complete squirrel cage winding. The torques developed on starting under balanced conditions are largely dependent on the design of this squirrel cage winding and on the speed of the motor when operating synchronously. Thus motors up to 400 r.p.m. started on full voltage will develop about 40% of full load torque. Motors with normal speeds ranging from 600 to 1800 r.p.m. will develop starting torques of from 100% to 150% of full load torque with rated impressed voltage. This torque depends on the resistance of the bars, their shape and arrangement. Nickel alloy bars are used for high starting torque.

The Equivalent Diagram of the Induction Motor Applied to Negative Sequence Currents

Since the motor starts as an induction motor it may be treated mathematically as such. It was shown on page that the output term in the positive sequence equivalent diagram was equal to $\left(\frac{1-s}{s}\right) \cdot R_r$ or

$$P_1 = 3\left(\frac{1-s}{s}\right) \cdot R_r I_{r1}^2$$

This term may be used for negative sequence currents if the corresponding negative sequence slip is used. If S is the synchronous speed then at positive sequence slip s_1, the speed of the rotor is $S(1-s_1)$. Since the negative sequence sets up a flux rotating in the opposite direction at the same speed its velocity is $-S(1-s_2)$. If these speeds are equal:

$$S(1-s_1) = -S(1-s_2)$$

$$s_2 = 2-s_1$$

Thus a negative sequence slip s_2 is equivalent to a slip of $2-s_1$ referred to the positive sequence diagram.

$$P_2 = -3.\frac{1-s}{2-s}.R_r I_{r2}^2$$

If both positive and negative sequence currents are present then the shaft power will be:

$$P = 3\left(\left(\frac{1-s}{s}\right).R_r I_{r1}^2 - \frac{(1-s)}{(2-s)}.R_r I_{r2}^2\right).$$

The resultant torque will be

$$T = \frac{3\ .\ 33000}{2\pi\ .746}.\frac{1}{S(1-s)}.\left(\frac{1-s}{s}.R_r I_{r1}^2 - \frac{1-s}{2-s}.R_r I_{r2}^2\right)$$

$$= \frac{21.1}{s}.\left(R_r I_{r1}^2 - \frac{R_r}{2-s}.I_{r2}^2\right).$$

Torque from Starting to Synchronous Speed

Curves wre drawn showing the torques under balanced and un balanced conditions for all speeds during the starting period. Values of R_r, X_s, X_r, X_m, and R_m are needed to establish the equivalent diagram shown in Fig. 26. As it was desired to obtain the complete curves it was considered that much labor could be saved by using the circle diagram rather than the equivalent diagram.

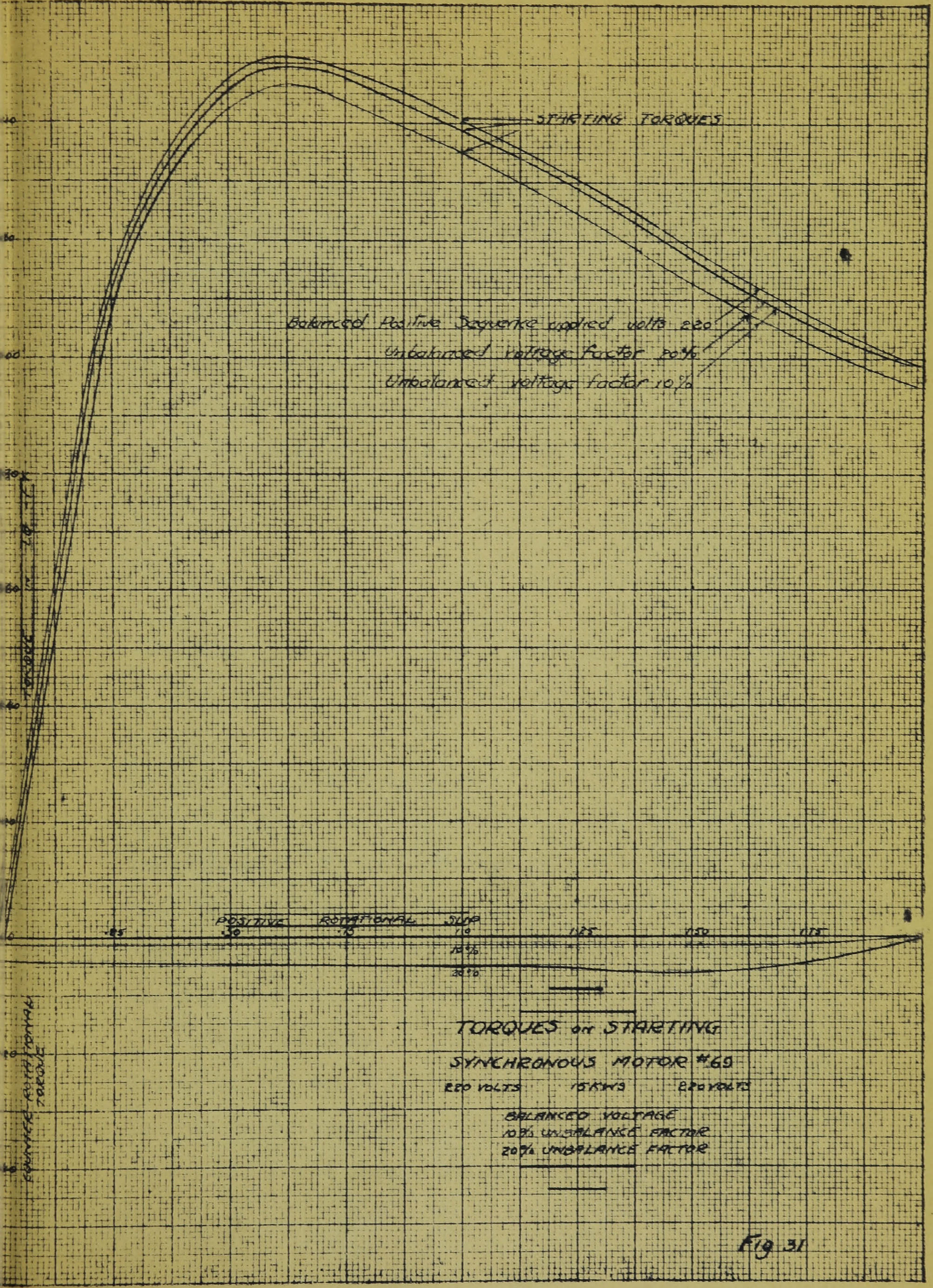

STARTING TORQUES
Balanced Positive Sequence applied volts 220
Unbalanced voltage factor 20%
Unbalanced voltage factor 10%
POSITIVE ROTATIONAL SLIP
TORQUE
COUNTER ROTATIONAL TORQUE
TORQUES on STARTING
SYNCHRONOUS MOTOR #69
220 VOLTS
15 KWS
220 VOLTS
BALANCED VOLTAGE
10% UNBALANCE FACTOR
20% UNBALANCE FACTOR

Fig 31

Notice however that the equivalent diagram has served its purpose in that it has shown the relation between counter-rotational and direct rotational torques. It has indicated for any given value of positive rotational slip s, at equal voltages ,the counter-rotational torque will be equal to the positive rotational torque at a slip of 2-s. We may construct the circle diagram for the one sequence and from it obtain the values for the other by using this relation between the slips. The circle diagram was constructed from a no load run and a locked rotor test. Construction data:

Test	Volts	Amps.	Cos∅	Input Watts	Stator Loss	Rotor Loss
No Load	220	24	.287	480		
Locked Rotor	220	155.2	.653	12930	5150	7780

Output constant = T/W = .00587

The circle diagram was constructed on a scale of 1" =10 amps. Details of the construction may be found in any text book dealing with the induction motor. It is found necessary however to use this diagram beyond its usual range. Values for slips greater than "1" are needed.

When the slip is greater than unity the rotor is being driven in the opposite direction to the applied M.M.F. The stator in this case supplies one half the rotor loss and the shaft the other half (see Chapt. 111). The torque curves obtained are shown in Fig. 31. The resultant torques with 10% and 20% voltage unbalance factor are shown. Since the torque varies as the square of the voltage the small negative sequence voltage does not greatly decrease the torque.

A 10% unbalance factor reduces the starting torque 1.08%

A 20% unbalance factor reduces the starting torque 3.57%

We would conclude that the decrease in starting torque for unbalance of the order of less than 10% will be less than 1%. The curves were drawn for full voltage starting. For reduced voltage starting the relative magnitudes of positive and negative sequence voltages will be constant and the percentage reduction in torque the same.

20. The Effect on Pull-in Torque

In designing a synchronous motor three factors enter into the determination of the pull-in torque namely:

1) resistance of the damper windings
2) field resistance - when the motor is started with the field closed through a resistance.
3) field strength.

It is customary when starting to apply the field when the motor reaches about 94% of synchronous speed. The application of the field will supply the torque necessary to accelerate the rotor to synchronous speed. The first two factors enter into the obtaining of a high torque in the neighborhood of 94% of speed. According to design curves, with an increase of damper resistance, the pull-in torque decreases. An increase in field resistance results in an increase in pull-in torque.

The laboratory motor, as seen from the torque curves at 94% of synchronous speed, gives only 5 lbs. ft. torque. This is due to the fact that the motor was started with its field open. The effect of the unbalance is to decrease the torque available to 4 lbs. ft. at 10% voltage unbalance factor and 1lbs.ft. at 20% voltage unbalance factor. Most of the pull-in torque is provided by the field current.

Pull-in Torque = sufficient torque to accelerate the rotor and load on the generator to synchronous speed plus torque made available through the design of the motor.

This torque is not readily calculated from the constants of the machine but an approximation to the effect of unbalance may be made by assuming the pull-in torque to be about 40% of the full load torque (open field starting). Then at 10% unbalance factor the reduction in pull-in torque $= \frac{100}{87} = 1.1\%$ and at 20% unbalance factor a reduction of 4.6%.

21. Effect of Unbalance on Motor Efficiency

Since the effect of unbalance is greatest on the efficiency of a motor a complete theoretical and experimental study will be made.

Efficiency by the Loss Method

The efficiency of a motor is the ratio of the output to the input. The output is equal to the input minus the losses of the motor.

The losses are:

a) Armature copper loss.

b) Stray load or core loss due to armature current.

c) Core loss due to the field flux

d) Friction and Windage loss.

e) Field copper loss

a) In predetermining the efficiency the armature current I is assumed at a certain value and the efficiency determined for that value. Thus loss (a) will be given by I^2R_a where R_a is the D.C. armature resistance corresponding to a temperature of 75°C. as recommended in the A.I.E.E. specifications.

b) The core loss due to the leakage flux set up by the current in the armature conductore varies with the armature current. It is common practice for this reason to include this loss with the loss (a) by increasing the resistance R_a by a value which is found to give the combined loss. For machines of the type of the laboratory motor a 30% increase in resistance is found to give accurate results. Them the effective resistance $R_e = 1.30 R_a$.

c) To get the core loss C.L. due to the field flux a curve must be taken for the motor giving C.L. vs. field current. This was done in Method 2 of getting R_2. The I_f to be used for a given value of I is determined by using the Blondel Two Reaction Theory with a saturation correction. This gives a more accurate result than the A.I.E.E. and synchronous impedance methods. The construction and procedure are given below and further explanation may be found in " Alternating Current Machinery" -Bryant.

DETERMINATION OF INTERNAL VOLTAGES OF A SYNCHRONOUS MOTOR

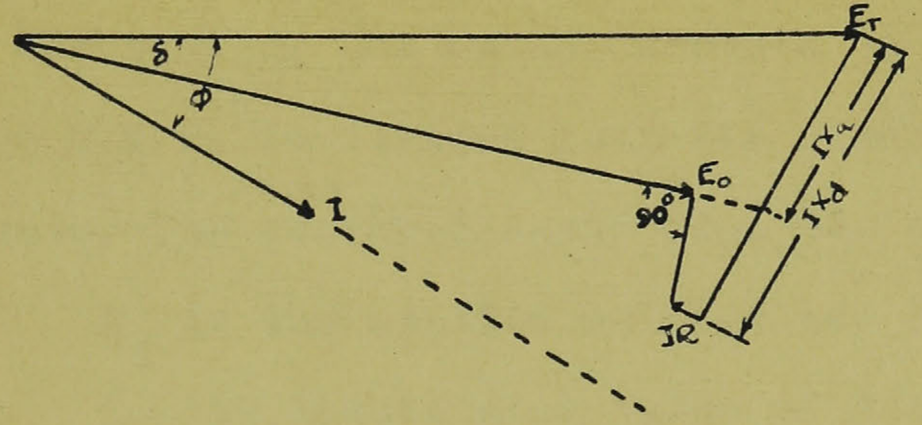


Fig 32

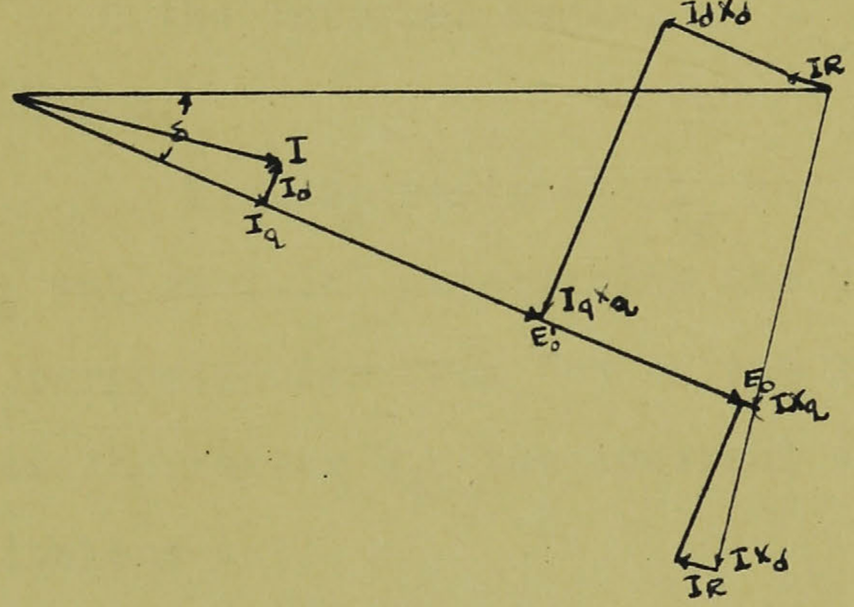


Fig 33

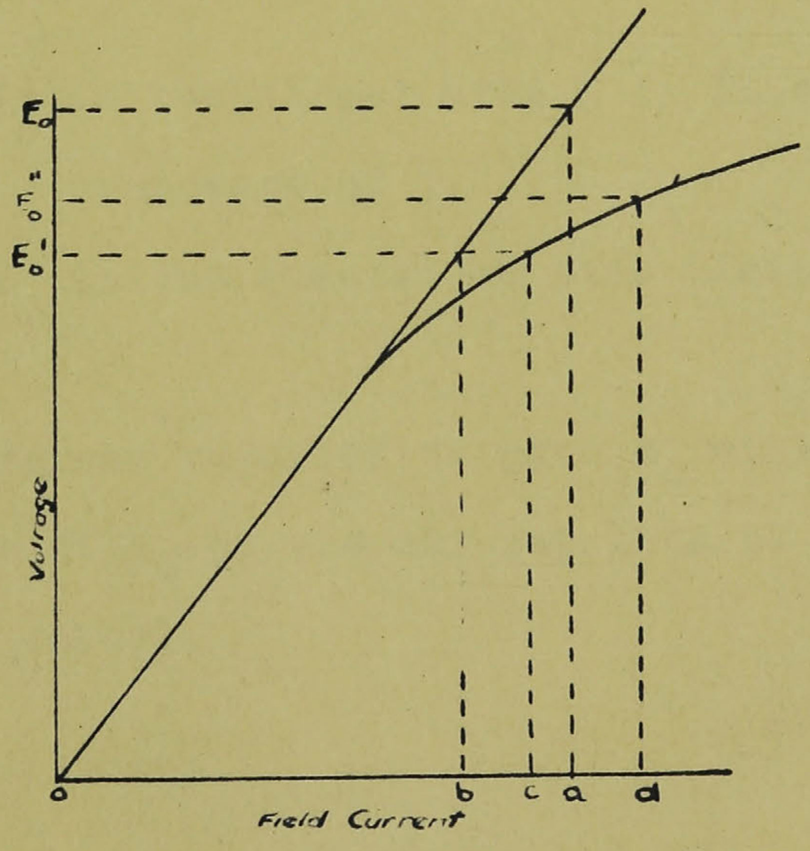


Fig 34

The Determination of the Unsaturated Vlue of Internal Voltage

The accuracy of the method depends on the fact that the armature M.M.F. is divided into its quadrature and direct axis components. The vector diagram Fig. 32. shows the method.

E_t is the applied voltage per phase.

E_o the unsaturated value of the internal voltage

δ opposed by an equal flux set up by the fi eld.

δ the interaal angle.

Construction:

From E_t subtract IX_d and IR_a determining point "b" From E_t subtract IX_q determining the point "a". Then the foot of the perpendicular from "b" on the line "oa" marks the terminus of the vector E_o, the internal voltage. This value of E_o is equivalent to:

$$E_o = E_t - I_q(X_q - R_e) - I_d(X_d - R_e).$$

The proof of this may be simply shown from the geometry of the diagram.

I_d is the direct axis (in line with the poles) component of I.

I_q is the quadrature axis (interpolar) component of I.

The field current required to give E_o will be that corresponding to the value of E_o on the air gap line of the motor (it is an unsaturated value).

DETERMINATION OF INTERNAL VOLTAGES

Synchronous Motor

#69

$X_q = 2.25\ \Omega$ $X_d = 3.41\ \Omega$ $X'_d = 1.37\ \Omega$ $R_e = 0.1865\ \Omega$

	Armature Current in Amperes					
	5	10	20	30	40	50
E_t	127	127	127	127	127	127
IX_d	17.05	34.1	68.2	102.3	136.4	170.5
IX_q	11.25	22.5	45.0	67.5	90.0	112.5
IR_e	0.93	1.9	3.7	5.6	7.5	9.3
I_q	4.9	9.9	18.9	26.5	32.8	37.3
I_d	0.5	1.8	6.5	14.0	23.0	33.0
$I_d X'_d$	0.7	2.5	8.9	19.2	31.5	45.2
E_o	127.5	129.5	140.0	156.0	177.1	202.1
E'_o		126.0	126.5	127.5	131.0	135.0
oa		6.8	7.3	8.2	9.3	10.6
ob		6.6	6.5	6.7	6.9	7.1
oc		7.1	7.0	7.2	7.5	7.9
od	7.1	7.3	7.8	8.7	10.0	11.4
E''_o	127.5	128.3	135.0	143.8	154.5	164.5

Note; These values were obtained from the Blondel Two Reaction Theory Method using a graphical solution and the no load saturation curve.

b) The Correction for Saturation

This value of E_o is the unsaturated value of the induced E.M.F. opposing that set up by I_f, the field current. Due to the effect of saturation a smaller E.M.F. will be effective. It is necessary in applying this correction to separate the direct axis M.M.F. into two components, the one representing the reaction effect which sets up a flux in the air gap where there is no saturation ,and the other supplying leakage fluxes in the irom paths surrounding the conductors where saturation is effective.

The construction in Fig.33. is used. Thus on Fig.33. X_d' is the transient reactance in the direct axis.It will be seen from Chapt. 111. that X_d' is the reactance effective when the armature current is supplying the leakage fluxes only and does not include the effect of armature reaction. X_q in the quadrature axis represents a leakage flux only. Then:

$$E_o' = E_T - IR_e - I_d X_d' - I_q X_q$$

$$= E_T - I_d(R_e + X_d') - I_q(R_e + X_q)$$

-where E_o' is the internal voltage which would be effective if the armature reaction M.M.F. did not exist. The difference in field currents oc and ob is the extra current required because of saturation. Then od = oa - bc is the field current required to give E_o'' the saturated value of the internal voltage corresponding to an unsaturated value E_o.

This procedure was followed for various values of the armature current and the core loss used was that corresponding to the field current od above.

EFFICIENCY CALCULATED FROM LOSSES

Motor #69

Unity PF 220 volts

I_a	E''_o	3EI	$3I^2R$	FW	CL	If	E_fI_f	Input	Output	Eff.
5	127.5	1905	14	105	91	7.15	393	2298	1695	73.8
10	128.3	3810	56	105	92	7.29	402	4212	3442	81.8
20	135.0	7620	224	105	98	7.85	432	8052	7193	89.4
30	143.8	11430	503	105	108	8.70	478	11908	10714	90.2
40	154.5	15240	895	105	117	9.96	548	15788	14123	89.4
50	164.5	19050	1400	105	132	11.40	627	19677	17413	88.5

Note; The stray load losses have been taken into account By using an effective value of armature resistance 30% greater than the DC armature resistance.

d) The friction and windage loss FW was determined in the determination of R_2.

e) The field copper loss was determined by the product of field current and field voltage $E_f.I_f$. The field current in each case has been determined in the method outlined above.

The efficiency under unbalanced conditions was plotted for, constant voltage unbalance factor. Since the negative sequence impedance for the motor has been determined by test the negative sequence current flowing for any given negative sequence voltage may be calculated. Thus for 10% unbalance factor:

$$E_2 = 12.7 \text{ volts}$$

$$Z_2 = R_2 + jX_2$$

$$= .0987 + j0.156 \text{ in per unit values}$$

$$= .318 + j0.502 \text{ ohms}$$

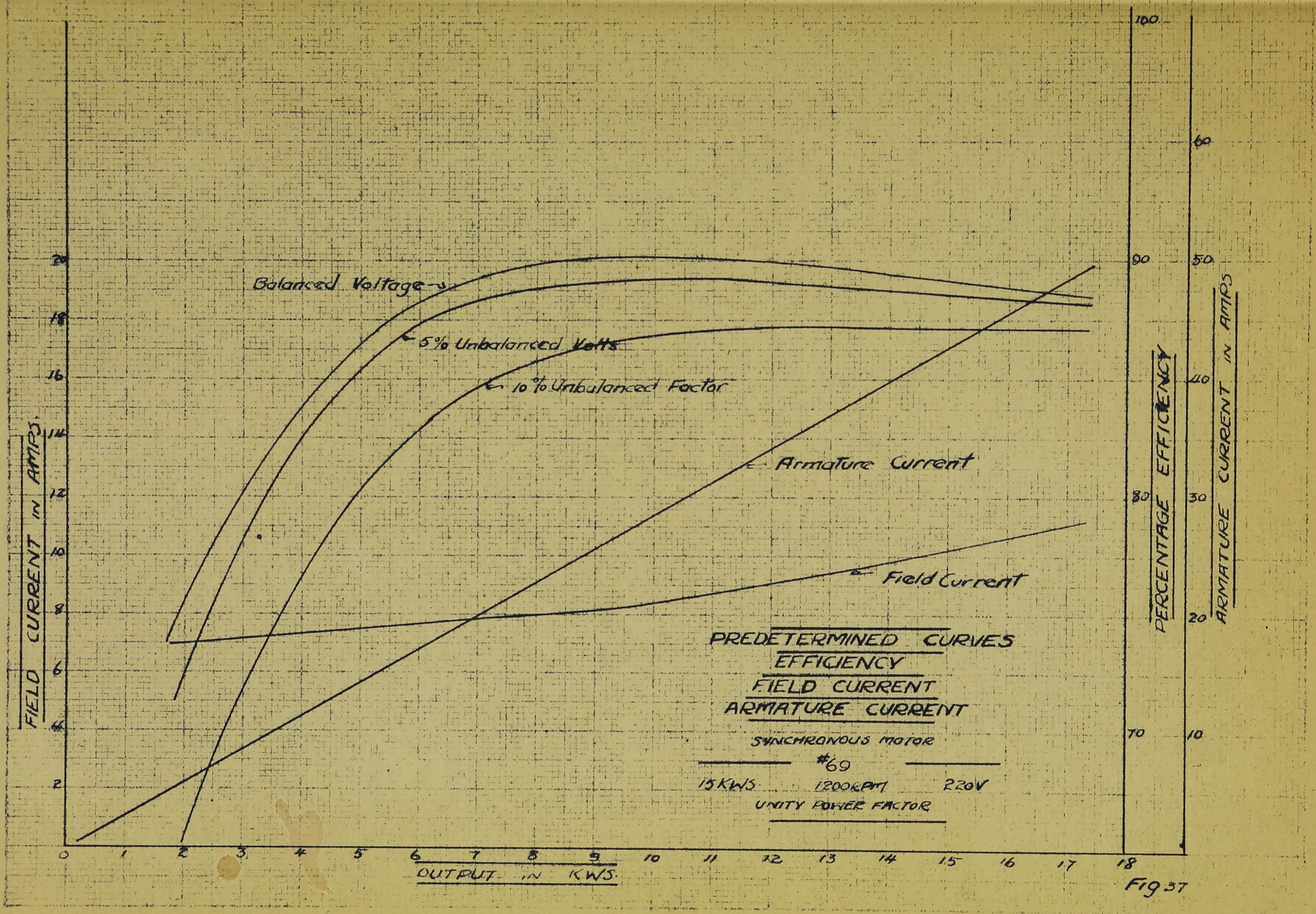
$$\text{and } I_2 = \frac{j12.7}{Z_2} = 21.4 \text{ amps.}$$

The total nagative sequence loss will then be

$$= 3I_2^2.R_2 = 435 \text{ watts.}$$

The efficiency of the motor will than be

$$= \frac{\text{Output}}{\text{Input(positive sequence)} + 435}$$



FIELD CURRENT IN AMPS.
OUTPUT IN KWS.
PERCENTAGE EFFICIENCY
ARMATURE CURRENT IN AMPS
Balanced Voltage
5% Unbalanced Volts
10% Unbalanced Factor
Armature Current
Field Current
PREDETERMINED CURVES
EFFICIENCY
FIELD CURRENT
ARMATURE CURRENT
SYNCHRONOUS MOTOR
#69
15 KWS. 1200 RPM 220V
UNITY POWER FACTOR

Fig 37

Notice from the efficiency curves

Load	Balanced Eff.	5% Unbal. Factor Eff.	10% Unbal. Factor Eff.
1	89.2	88.6	87.2
3/4	90.0	89.2	87.1
1/2	89.7	88.6	85.2
1/4	83.0	81.6	75.4

22. Laboratory Measurements of Efficiency

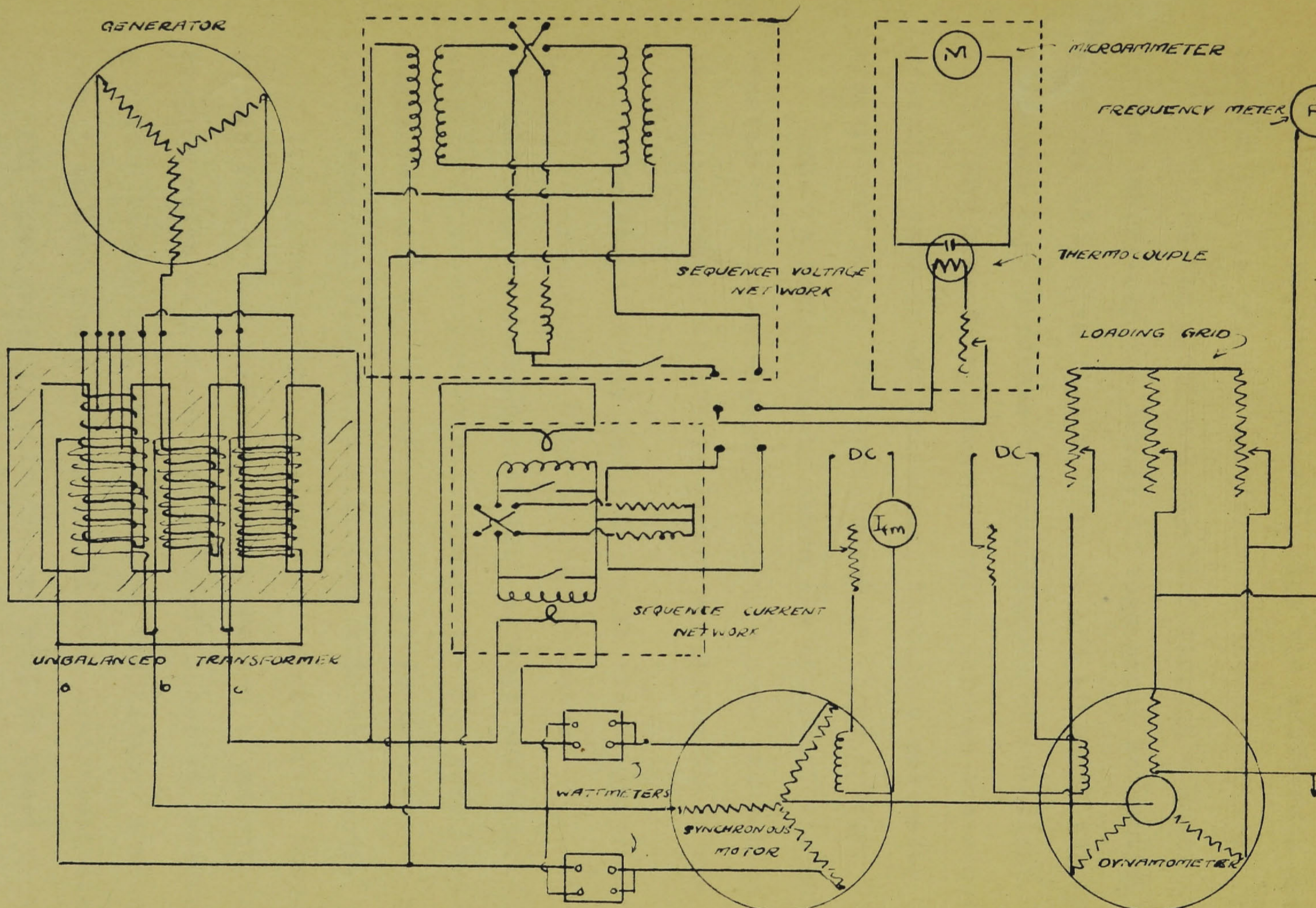
The objects of the tests on the laboratory motor were to create and accurately measure unbalanced voltage conditions, apply these voltages to the synchronous motor, and measure the efficiency by use of a dynamometer. We should get a check from these results on ;

a) The fundamental assumption that the sequences are independent .

b) Predetermined efficiency curves.

c) The method of calculating loss due to negative sequence.

Unbalance was created in the laboratory by the use of an unequal turn ratio on the power transformers serving the motor. The transformer used was a three phase core type. The core consisted of five legs , all of which were accessible. Twenty turns of flexible wire were wound on one of the phase coils and weree connected in series with the primary winding of the transformer. From every second turn a tap was taken. Since



SCHEMATIC DIAGRAM of SET-UP USED to OBTAIN EFFICIENCY UNDER UNBALANCE

Fig 35

the transformer was rated at 110:220 volts and 220 volts line to line were required for the motor the units were connected in star on the primary and delta on the secondary.

Since the power was supplied through a motor generator set in the laboratory the supply voltage could be controlled to the required 100 volts line to line on the primary. The use of the different taps produced varying degrees of unbalance.

The transformer was connected through the sequence measuring ammeter and voltmeter, through a wattmeter to the synchronous motor. The arrangement is shown in Fig. 35.

The following theory shows the nature of the unbalance created by this arrangement. The unbalance was not calculated as it was necessary to obtain direct measurements in any case.

23. Relation of the Sequences with the Transformer Arrangement Used in the Laboratory

It will be noticed that this arrangement to produce unbalance involves unsymmetrical impedances. For this reason the sequences will not be independent. It is purposed to show how these sequence components will vary with each other.

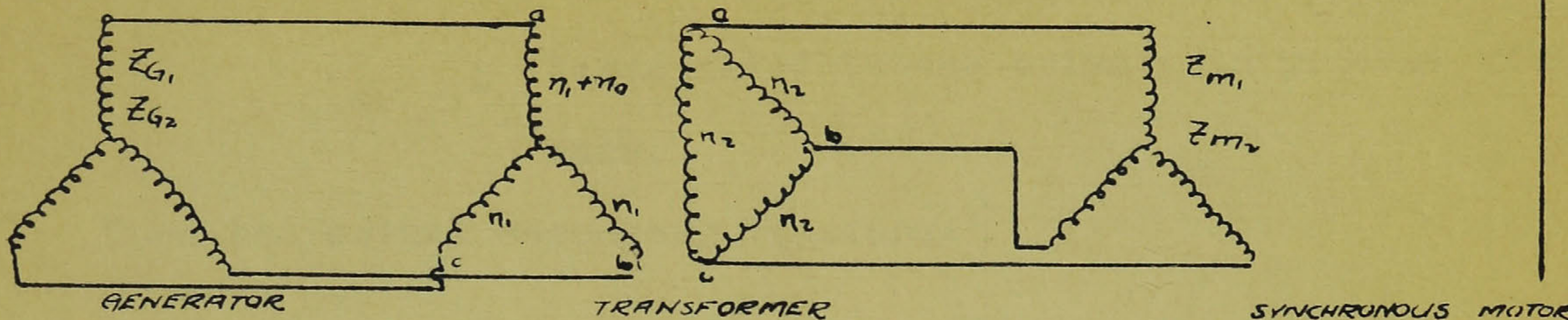
It has been shown in elementary transformer theory that a transformer and load can be represented in an equivalent diagram which will be in the form of a series impedance with a return to neutral. The unbalanced transformer impedances will result in each phase having a different equivalent diagram.

All impedances in this diagram will be based on the applied primary voltage base.

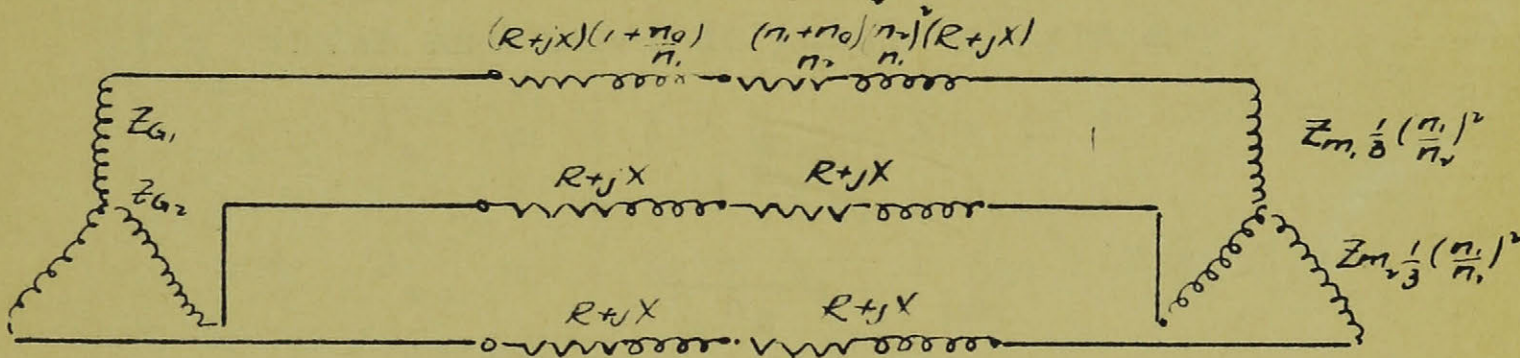
Then let:

$$R_1 + jX_1 = \text{primary impedance}$$

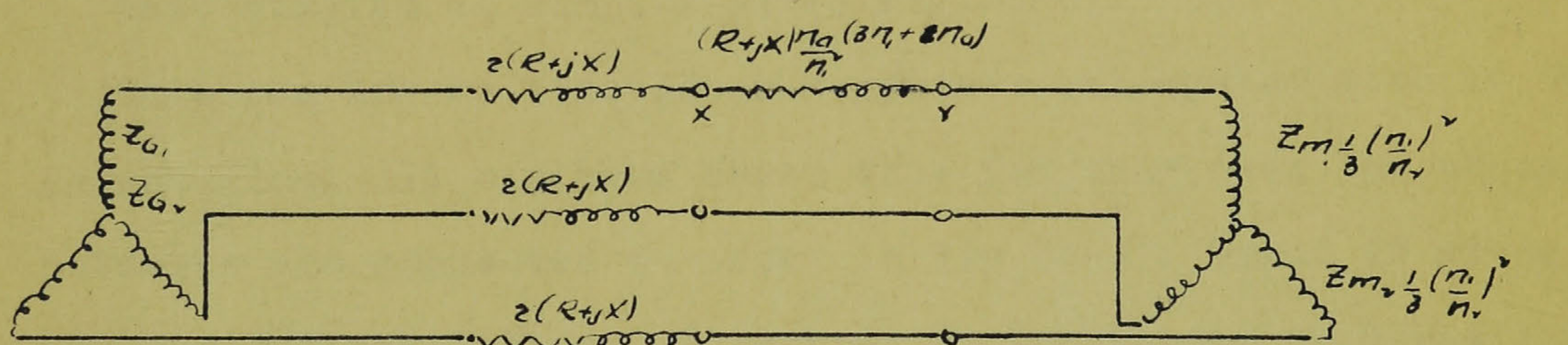
UNBALANCED TRANSFORMER CONNECTION



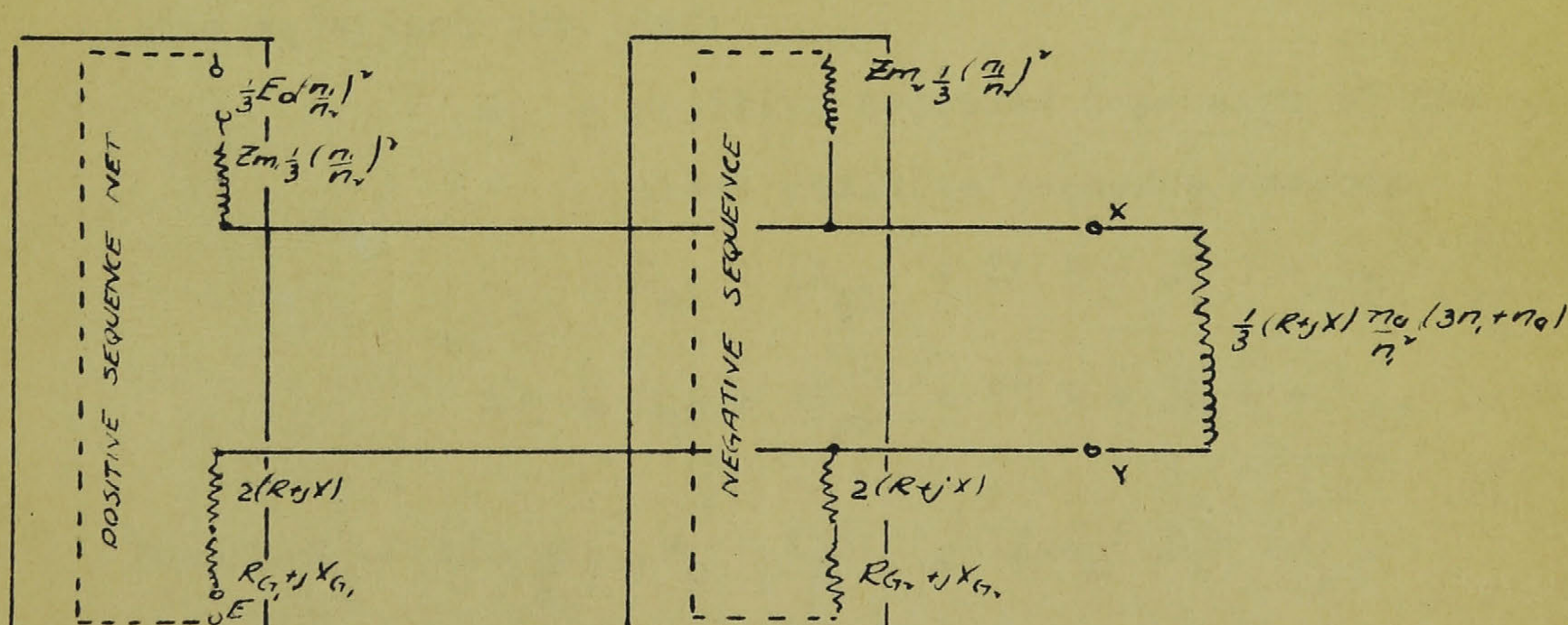
CIRCUIT



EQUIVALENT CIRCUIT



SIMPLIFIED CIRCUIT



CONNECTION OF THE SEQUENCE NETWORKS Fig 36

n_1 = primary transformer turns.

n_2 = secondary transformer turns,

n_a = turns added to the primary to produce unbalance.

Then the actual secondary impedance

$$= \left(\frac{n_2}{n_1}\right)^2 \cdot (R + jX).$$

If n_a turns are added to phase "a" primary

$$Z'_a = \left(1 + \frac{n_a}{n_1}\right)(R + jX)$$

and

$$Z''_{a\,equiv.} = \left(\frac{n_1 + n_a}{n_2}\right)^2 \cdot \left(\frac{n_2}{n_1}\right)^2 (R + jX).$$

We cam now draw the equivalent circuit. See Fig.36 (a). This diagram is simplified to Fig.36(b).

This can now be treated as a symmetrical system with an impedance in one line. It has been shown that for this case the sequence networks are connected as shown in Fig.36(d). Proof is given in the paper "Peak Voltage on Saturating Reactances in Three Phase Circuits"- Electrical Journal Mar. '32.

The equations are then:

Z_1 = positive sequence impedance of the equivalent positive sequence network.

$$Z_1 = R_{g1} + jX_{g1} + 2(R + jX) + \frac{Z_m}{3} \cdot \left(\frac{n1}{n_2}\right)^2$$

$$Z2 = R_{g2} + jX_{g2} + 2(R + jX) + \frac{Z_{m2}}{3} \cdot \left(\frac{n1}{n_2}\right)^2$$

Then $E_g - 1/3\left(\frac{n1}{n_2}\right)^2 \cdot E_o - I_1 Z_1 = 1/3 Z_a (I_1 + I_2)$

where $Z_a = (R + jX).\frac{n_{a2}}{n_1}.(3n_1 + n_a)$

and

$$I_2Z_2 = 1/3Z_a(I_1 + I_2)$$

$$I_2 = \frac{Z_a}{3Z_2 - Z_a + Z_2Z_a}.(E_g - 1/3\left(\frac{n_1}{n_2}\right)^2.E_o).$$

The negative sequence current will depend on the generated voltage and the excitation of the motor.

Procedure:

A preliminary run was made to determine the field currents required to give unity power factor under various loads. In all runs it was considered essential that the machine be run under full load for at least a half hour to stabilize friction and windage losses and minimize changes in armature resistance due to heating. The procedure in each case was to first set the balance scale for a given torque. The field current in the dynamometer was then increased until the scale beam just balanced. A check was then made on the applied voltage and the frequency. These were carefully adjusted and the field current in the motor was adjusted to give unity power factor. Since the two wattmeter method of measuring powern aws used an accurate setting for unity power factor on balanced applied voltages could be made by adjusting the field current to give equal readings on the two meters. In each case the procedure outlined above ,start ing with the balancing of the beams,was repeated until all the desired relationships existed together. Readings were taken under these conditions of applied voltage, armature current, field current,torque, and total input for scale settings of 10, 20, 30, 40, and 50 lbs. The arm on the dynamometer was 18 inches in length so these correspond to torques from 15 to 75 lbs. ft.

Readings were taken under unbalanced conditions. Great difficulty was encountered in getting accurate readings in this case. The difference in input, under equivalent conditions of excitation, torque, and positive sequence voltage between balanced and 10% unbalnce factor is approximately 200 watts. Thus on rated input of approximately 16000 watts we are measuring accurately a quantity which is about 1.2% of the total. Some of the difficulties encountered in the test run are listed below.

a). The greatest hinderance to obtaining accuracy was due to the variations in supply voltage. The motor generator set used to provide the A.C. was supplied from the McGill University D.C. power plant. A large proportion of the load connected to this system cosists of elevators. The heavy starting currents taken by these caused irregularities in the line voltage. On the motor generator set this resulted in variations in the D.C. motor speed and consequent irregularities in the frequency of the voltage supplied to the test motor. These irregularities occurred so frequently that greater stability was tried for by use of a balancer set but the attempt was unsatisfactory. The results shown here were obtained on a Saturday afternoon when the voltage was comparatively steady.

b) Difficulty was had with the accuracy of the sequence meters. These difficulties have been outlined in Chapt. 11. Finally the sequence current meter was abandoned and simultaneous readings were made of the line currents. The values of I_1 and I_2 were obtained from these by using Dubusc's Method.

c) One of the most difficult factors to control was the change in armature copper loss due to temperature rise. The importance of this factor was not realized at first and three sets of unsatisfactory results were obtained in which it was ignored. The problem was to get measurements under balanced and unbalanced conditions at the same machine temperature. This was accomplished by taking balanced readings before and after each set of unbalanced readings for the same load and excitation. These readings were taken at equal time intervals. The average of the balanced watts input before and after an unbalanced reading, was taken to be the input for balanced conditions, corresponding to the temperature at which the unbalanced input was measured. It was assumed that the machine was cooling or heating on a uniform temperature gradient.

d) Instrument accuracy was found to be of the utmost importance. When the magnitude of the change of input is considered it will be realized how important instrument errors will be. The sequence voltmeter depends on the frequency for its accuracy Thus it was found that in the measurement of the negative sequence that a 1% variation from normal frequency produced a 1% variation in E_2. The frequency meter was recalibrated with a revolution counter. The wattmeters were accurate to within one tenth of 1%.

e) Difficulty was experienced in obtaining "simultaneous readings" with a single observer.

MEASURED EFFICIENCIES UNDER BALANCED AND

UNBALANCED CONDITIONS

Tap	E_1	E_2	I_1	I_2	I_f	$E_f I_f$	W_i	Cor.W_b	Φ	F&W	OUTPUT	EFF.
Bal	127		41.0		9.80	539	16275		55	105	14205	87.3
Bal	127		35.6		9.55	526	14646		50	105	12905	88.1
Bal	127		29.5		8.80	484	11940		40	105	10345	86.6
Bal	127		28.9		8.80	484	11876		40	105	10345	87.2
#4	127	5.00	29.7	5.9	8.80	484	12036	11940	40	105	10345	85.9
Bal	127		30.2		8.80	484	12004		40	105	10345	86.2
#6	127	5.63	30.1	9.7	8.80	484	12100	11984	40	105	10345	85.5
Bal	127		30.0		8.80	484	11964		40	105	10345	86.6
Bal	127		22.0		8.10	446	9062		30	105	7785	85.8
Bal	127		22.2		8.20	446	9174		30	105	7785	84.8
#4	127	3.72	22.9	5.9	8.20	446	9246	9174	30	105	7785	84.2
Bal	127		22.0		8.20	446	9174		30	105	7785	84.8
#6	127	5.78	22.3	9.7	8.20	446	9286	9174	30	105	7785	83.8
Bal	127		22.0		8.20	446	9174		30	105	7785	84.8
#8	127	7.00	22.3	12.1	8.20	446	9334	9158	30	105	7785	83.4
Bal	127		22.0		8.20	446	9142		30	105	7785	85.1

Table continued:-

Tap	E_1	E_2	I_1	I_2	I_f	E_fI_f	W_i	Cor. W_b	T	F&W	OUTPUT	EFF.
Bal	127		15.3		7.60	418	6462		20	105	5225	80.8
Bal	127		15.5		7.60	418	6578		20	105	5225	79.4
#8	127	7.10	15.5	11.8	7.60	418	6722	6566	20	105	5225	77.8
Bal	127		15.6		7.60	418	6554		20	105	5225	79.7
#6	127	5.87	15.6	9.5	7.60	418	6642	6542	20	105	5225	78.8
Bal	127		15.5		7.60	418	6530		20	105	5225	80.0
#4	127	4.00	15.5	5.5	7.60	418	6514	6474	20	105	5225	80.2
Bal	127		15.2		7.60	418	6418		20	105	5225	81.3
Bal	127		8.8		7.10	390	3812		10	105	2665	69.8

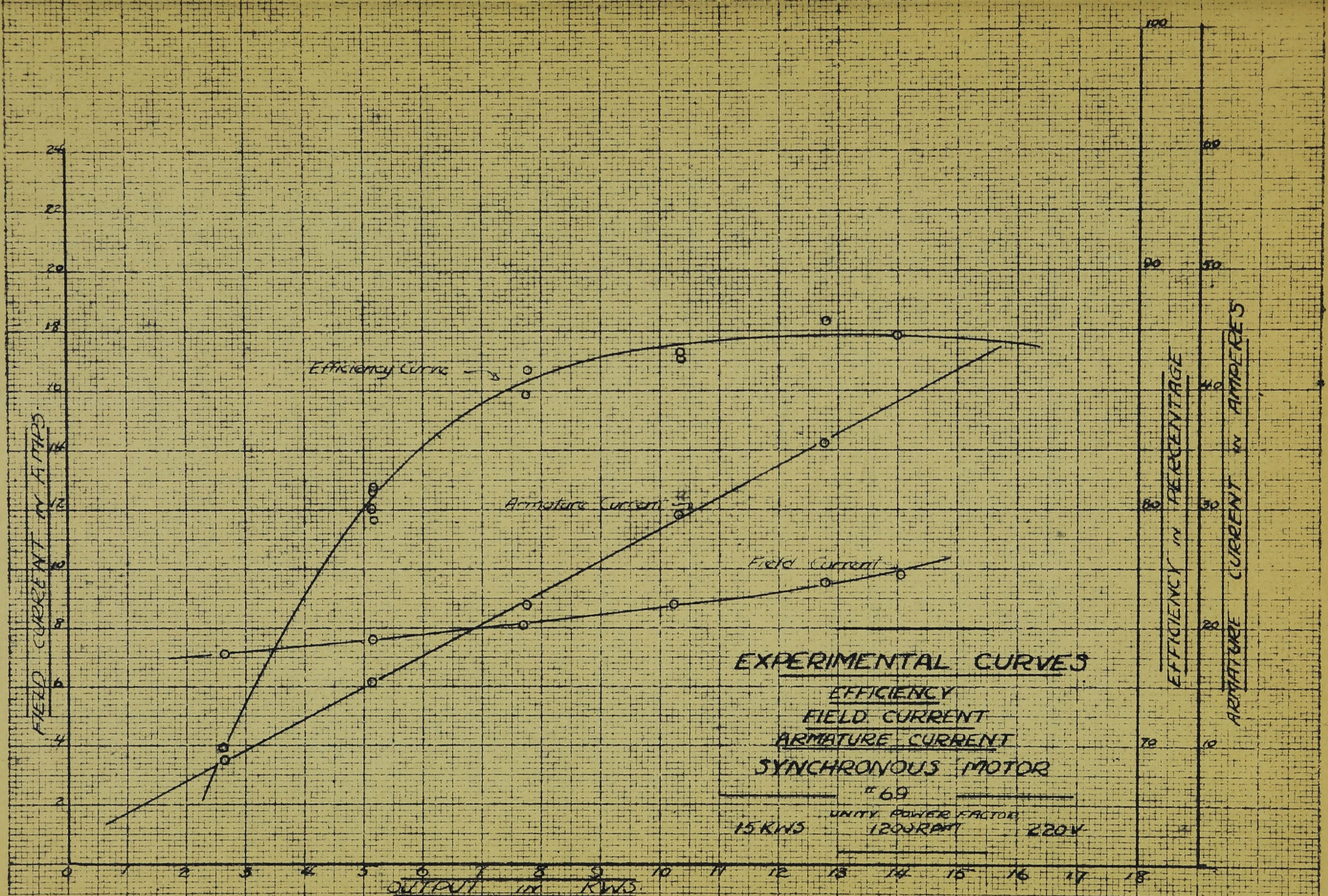

EXPERIMENTAL CURVES
EFFICIENCY
FIELD CURRENT
ARMATURE CURRENT
SYNCHRONOUS MOTOR
#69
UNITY POWER FACTOR
15 KWS
1200 RPM
220 V
Efficiency Curve
Armature Current
Field Current
FIELD CURRENT IN AMPS
EFFICIENCY IN PERCENTAGE
ARMATURE CURRENT IN AMPERES
OUTPUT IN KWS
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
2 4 6 8 10 12 14 16 18 20 22 24
70 80 90 100
10 20 30 40 50 60

Fig 38

Discussion of Results

1) Efficiency with Balanced Voltages

It will be noticed that the efficiency as predetermined was consistently higher than that obtained from the dynamometer test. It will also be observed that the calculated efficiency at 10% unbalance factor exactly coincides with the curve obtained from test for balanced conditions. This signifies that the discrepancy is caused by some loss which is constant for all loadings of the machine. There is a possibility that the motor has an unusually large stray load loss but this would give an added loss which would vary as the square of the load current and would not be constant at all loadings. There are three possibilities of this type of error,namely :

a) An error in the value used for friction and windage in the predetermined curves.

b) An error in the zero setting of the wattmeter used in the test run.

c) A constant error in the dynamometer scale.

The error which must be accounted for is 435 watts and there is no possibility of an error in a) of this order. The tendency towards error in friction and windage is to obtain too large a value. The zero scale settings of the wattmeters were closely checked and there was no chance for errors of this order. This leaves us with the conclusion that there was a constant error in the scales of the dynamometer.

2) Field Current Curve

The check between calculated and actual field currents is very close, the maximum difference being 0.2 amperes. This gives an indication of the high order of accuracy which can be obtained by the use of the Blondel Two Reaction Theory with the saturation correction.

3) Loss Due to Unbalance

The value of the loss due to negative sequence currents from the direct measurement of the difference in input from balanced to unbalanced conditions and from the calculation of $I_2^2 \cdot R_2$ are shown in the table and graph Fig.39. The calculated values are shown for two values of R_2, the one measured at $I_2 = 40$ amps and the other measured at 12.5 amps. Notice that the three curves are approximately parallel. We would conclude from this that for this type of synchromous motor the negative sequence loss varies as the square of the negative sequence current (with turbo-generators it varies as the 1.8th power). The calculated loss is about 20 watts less than that measured for all values of I_2. This would be expected when the difference in conditions is considered under which R_2 was measured and under which the direct measurements were made. When R_2 was measured the synchronous motor was short circuited from one line terminal to the other. The excitation required under these conditions was quite low (supplying only a small armature reaction) and the flux threading through the rotor bars in the pole face was quite small. When operating normally on load this flux set up by the field will be large. The negative sequence current flowing in the

LOSS DUE TO NEGATIVE SEQUENCE CURRENT

Synchronous Motor

#69

15Kws. 1200 rpm 220 volts

Measured				Calculated $Z_2 = .270+j.502$			Calculated $Z_2 = .317+j.502$		
Tap	E_2	I_2	Loss	E_2	I_2	Loss	E_2	I_2	Loss
#4	5.00	5.9	96	5.00	8.8	62	5.00	8.4	67
#6	5.63	9.7	116	5.63	9.9	79	5.63	9.5	85
#4	3.72	5.9	72	3.72	6.5	35	3.72	6.3	38
#6	5.78	9.7	112	5.78	10.1	82	5.78	9.7	90
#8	7.00	12.1	176	7.00	12.2	120	7.00	11.8	132
#8	7.10	11.8	156	7.10	12.5	125	7.10	12.0	146
#6	6.87	9.5	100	6.87	10.3	86	6.87	9.9	93
#4	4.00	5.5	40	4.00	7.0	40	4.00	6.7	43

Note; The value $Z_2 = .270 + j.502$ is that effective at $I_2 = 40$ amperes.

The value $Z_2 = .317 + j.502$ is that effective at $I_2 = 12.5$ amperes.

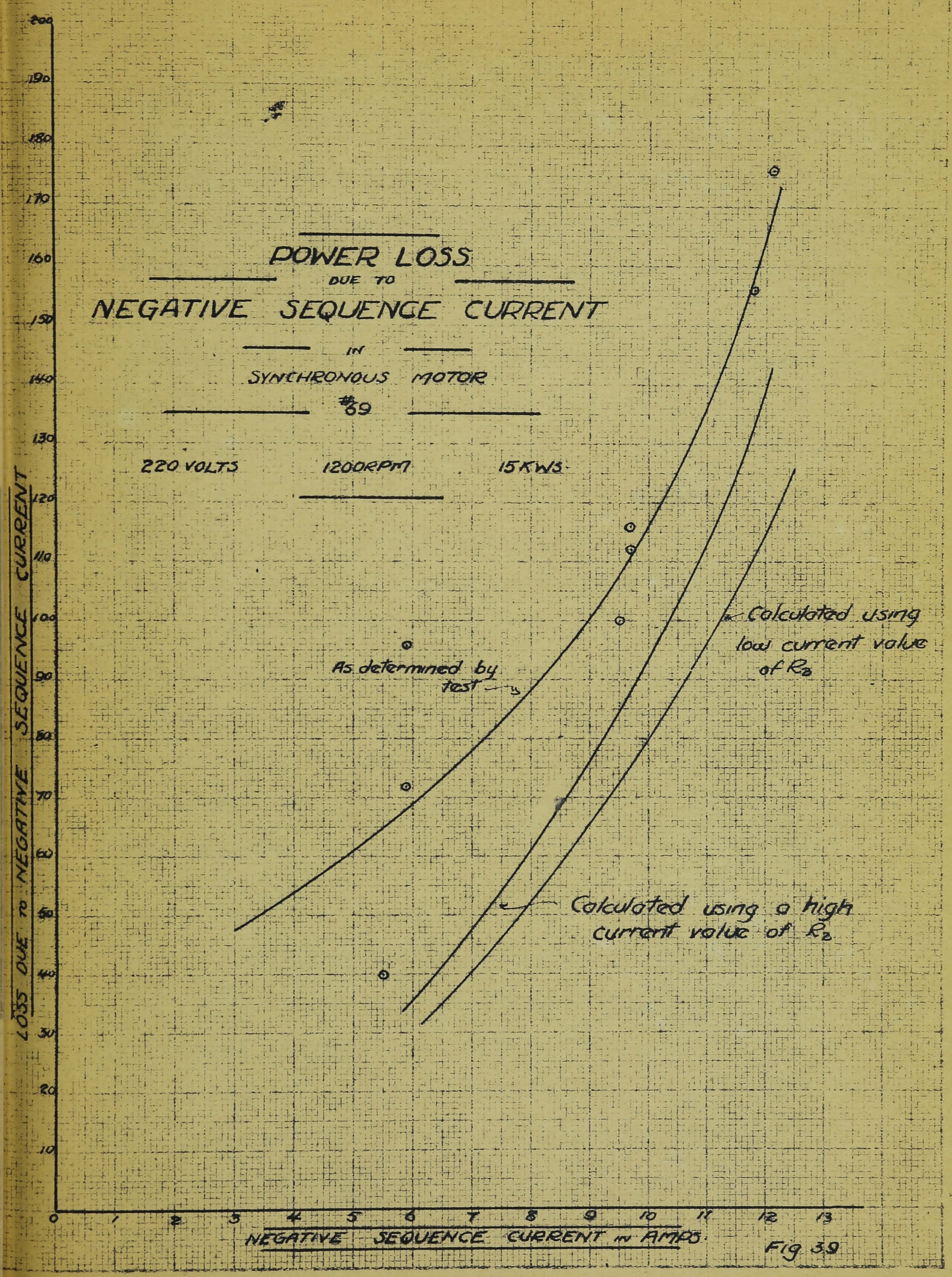

POWER LOSS
DUE TO
NEGATIVE SEQUENCE CURRENT
IN
SYNCHRONOUS MOTOR
#59
220 VOLTS
1200RPM
15KWS
As determined by test
Calculated using low current value of R_2
Calculated using a high current value of R_2
LOSS DUE TO NEGATIVE SEQUENCE CURRENT
NEGATIVE SEQUENCE CURRENT IN AMPS.
0
1
2
3
4
5
6
7
8
9
10
11
12
13
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200

Fig 39

rotor bars will thus be flowing in a magnetic field of small value in the first case and of high value in the second.

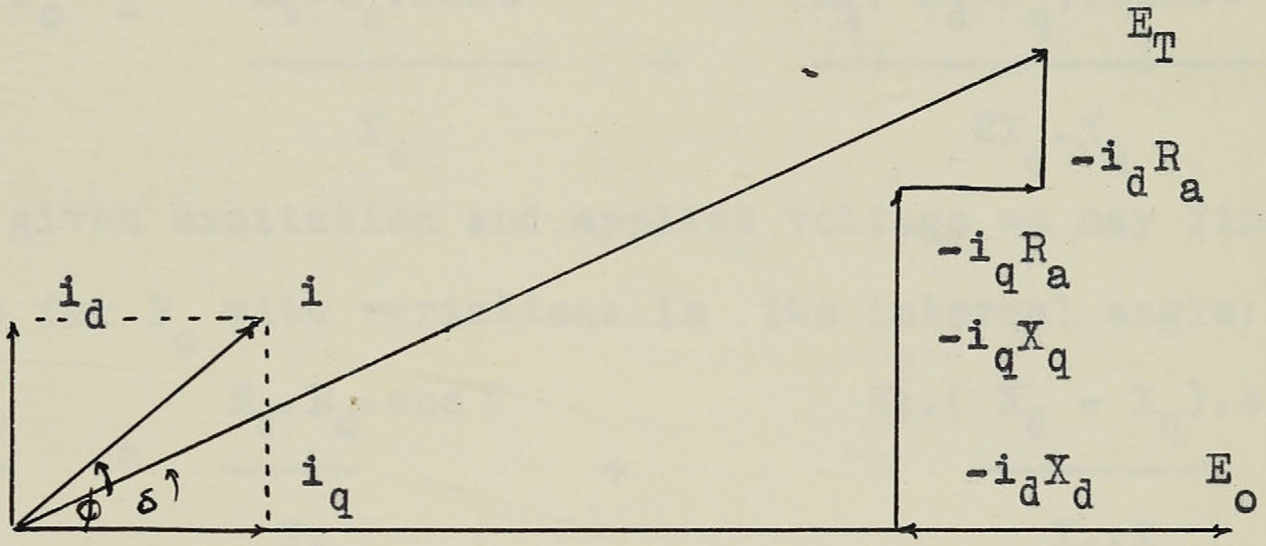
Thus if we considered a centimeter cube in the iron circuit on each side of the conductor and call them (a) and (b) if there were no constant external flux the flux density $B_a = B_b$ (due to the rotor current) and the core loss in the two cubes would be approximately equal to $k(B_a^2 + B_b^2)$. If a constant internal flux was imposed of just sufficient strength to cancel the flux due to the rotor current on the one side of the bar then the total loss $= k(0 + (B_a + B_b)^2)$. Notice that $(B_a + B_b)^2 > B_a^2 + B_b^2$. Thus the presence of the large field flux will increase the effective loss due to the negative sequence current by increasing the core loss due to it.

Due to this added loss more accurate results will be obtained in calculating the negative sequence loss for small values of I_2 if the value of R_2 obtained at $I_2 =$ rated current is used. This recommended by S.H.Wright in "Synchronous Machine Constants" - A.I.E.E. Journal 1931 and is corroborated by these test results.

24. Effect of Unbalance on Pull-out Torque

Pull-out torque under balanced and unbalanced conditions for slowly changing loads was calculated. An attempt was made to measure these torques in the laboratory. It was found that, due to, irregularities in power supply, coarseness of field control and inability to measure the changing torque on the scale balance ,satisfactory results could not be obtained.

The pull-out torque for any given excitation of the motor and load will be the torque corresponding to the maximum power output. ,This maximum power output may be obtained by differentiating the general expression for power output with respect to the internal angle.



$$E_o = E_t \cdot \cos\delta - i_q R_a + i_d X_d$$

$$0 = E_t \cdot \sin\delta - i_d R_a - i_q X_q$$

Solving these we get:

$$i_d = \frac{R_a \cdot E_t \sin\delta - X_q \cdot (E_o - E_t \cdot \cos\delta)}{R_a^2 + X_d \cdot X_q}$$

$$i_q = \frac{X_d \cdot E_t \sin\delta - R_a \cdot (E_o - E_t \cdot \cos\delta)}{R_a^2 + X_d \cdot X_q}$$

Power input (P_i) is equal to the sum of the products of the in phase components of i_d and i_q and the voltage E_t.

$$P_i = E_t \cdot i_d \sin\delta + E_t \cdot i_q \cos\delta$$

or $$P_i = \frac{E_t \cdot (X_q \cdot E_o \sin\delta - R_a \cdot E_o \cdot \cos\delta)}{R_a^2 + X_d \cdot X_q} + \frac{E_t^2 (2R_a + (X_d - X_q) \sin\delta)}{2(R_a^2 + X_d \cdot X_q)}$$

Then ,neglecting the constant losses the power output (P_o)

$$P_o = P_i - I^2R_a = P_i - R_a(i_d^2 + i_q^2)$$

This simplifies to the following form if the R_a terms which are small compared to the X terms are neglected.

$$P_o = \frac{E_t.E_o.\sin\delta}{X_d} + \frac{E_t^2(X_d-X_q).\sin 2\delta}{2X_d.X_q}$$

Then at a given excitation and applied voltage we may find the maximum value for P_o with variations in the internal angle:

$$\frac{dP_o}{d\delta} = \frac{E_t.E_o.\cos\delta}{X_d} + \frac{E_t^2.(X_d - X_q).\cos 2\delta}{X_d.X_q}$$

or $$\frac{2E_t^2.(X_d - X_q).\cos^2\delta}{X_d.X_q} + \frac{E_t.E_o.\cos\delta}{X_d} - \frac{E_t^2.(X_d - X_q)}{X_d.X_q} = 0$$

This equation becomes for the laboratory motor used

$$4870\cos^2\delta + 37.2E_o.\cos\delta - 2435 = 0.$$

This value of δ may then be substituted in the equation for P_q above and the maximum power output thus determined.

$$T_{max.} = P_q. \frac{33000}{2\pi.746.S} \text{ lbs. ft.}$$

The torque due to negative sequence current is

$$T_2 = \frac{3I_2^2.R_r}{2} . \frac{33000}{2\pi.S.746} \text{ lbs. ft.}$$

acting in the opposite direction to $T_{max.}$ and

$$R_r = 2(R_2 - R_s)$$

PULL- OUT TORQUE CALCULATIONS - UNBALANCE

Synchronous Motor

#69

220 volts 1200 rpm 15 kws.

Balanced										
E_0	I_f	$\cos\delta$	$\sin\delta$	$\sin 2\delta$	P_0	CL	FW	P_0 Net	T_{max}	I
137	7.2	.3565	.9343	.6661	15723	100	105	15518	91.2	59.6
168	8.8	.3125	.9499	.5938	20022	137	105	19780	116.0	64.7
186	9.8	.2915	.9565	.5577	21960	164	105	21691	127.2	68.3
200	10.5	.2775	.9608	.5343	23460	180	105	22175	130.0	71.3

I_1	I_2 10%	T_2	T_2/T_1%	I_2 20%	T_2	T_2/T_1%
59.6	5.96	14	.09%	11.92	56	.4%
64.7	6.47	16	"	12.94	66	"
68.3	6.83	18	"	13.66	73	"
71.3	7.13	20	"	14.26	80	"

Note; No correction has been made for saturation above.

The internal angle corresponding to the maximum output for any given value of E_0 is given by;

$$4870 \cos^2\delta + 37.2E_0\cos\delta - 2435 = 0$$

and the maximum output;

$$P_0 = 37.2E_0\sin\delta + 1215\sin 2\delta$$

The rotor resistance R_r is $2(R_2 - R_a)$

These calculations are for slowly varying loads.

These relationships have been shown in the following table. We would conclude from these results that unbalance has a negligible effect on the pull-out torque of the motor.

25. Rotor Heating on Unbalance

The currents induced in the rotor bars by the negative sequence M.M.F. will cause heating. This heating will be proportional to the square of I_2. The actual currents flowing in the rotor bars will be considerably larger than the stator negative sequence current but will be proportional to it. A marked temperature rise was observed on the laboratory motor rotor circuit when operating under unbalance. If the applied currents are balanced there will be no current flowing in the damper windings when the motor is operating synchronously unless the motor is supplying a load with a cyclic variation. The large currents in the bars on unbalance will result in heating which might be severe enough to burn the winding out. This heating is liable to cause damage to the rotor winding due to expansion and to the stator winding due to radiation from it.

26. Stator Heating on Unbalance

The overheating of individual phases is the greatest objection to unbalance. The heating on a balanced circuit is proportional to the square of the load current. When unbalanced the currents in each phase will be different. The actual current in any one phase will be determined by the values of the positive and negative sequence currents and the angle between their vectors (determined by the method the unbalance is created and the

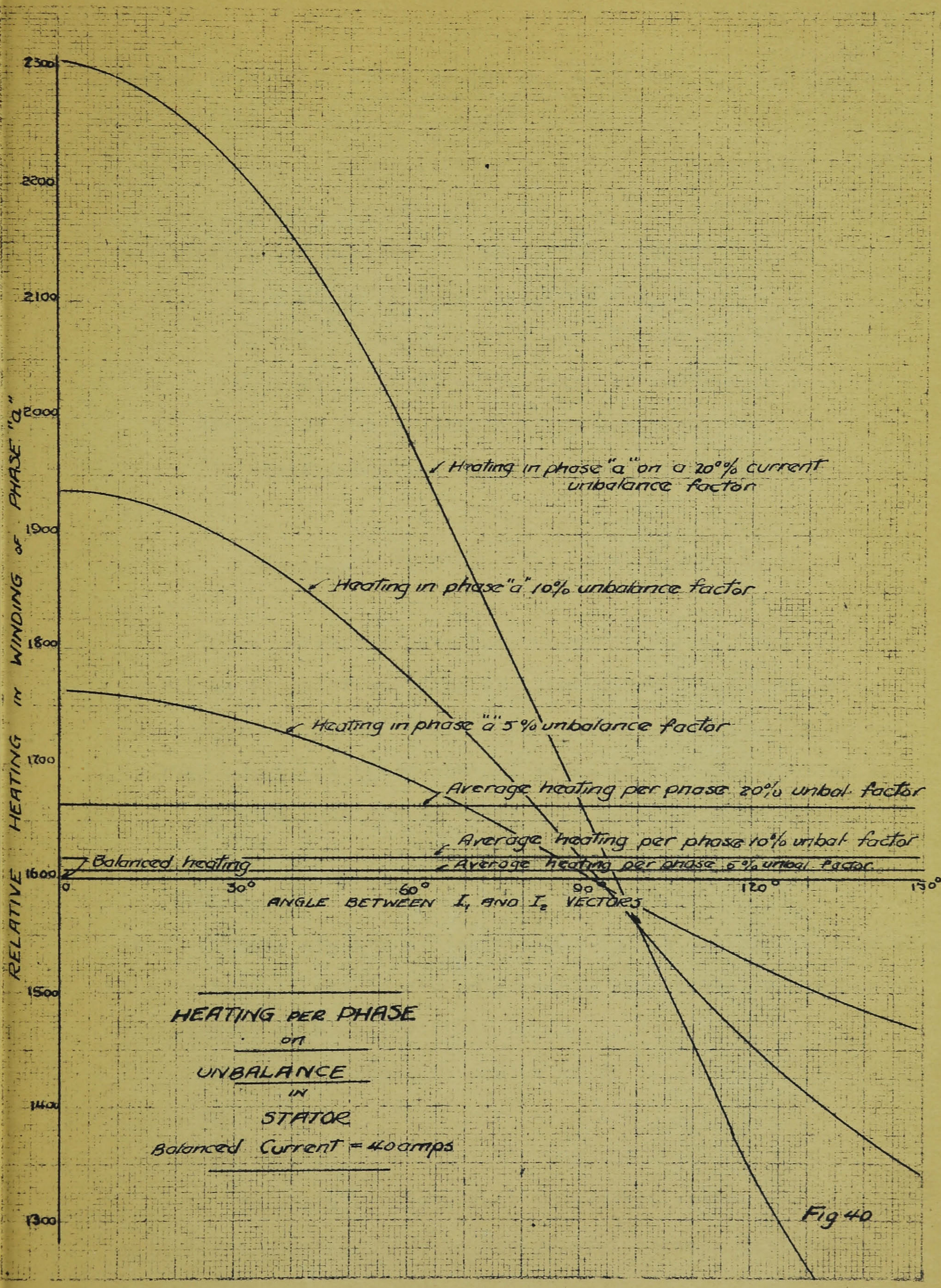

2300
2200
2100
2000
1900
1800
1700
1600
1500
1400
1300
RELATIVE HEATING IN WINDING OF PHASE "a"
Heating in phase "a" on a 20% current unbalance factor
Heating in phase "a" 10% unbalance factor
Heating in phase "a" 5% unbalance factor
Average heating per phase 20% unbal. factor
Average heating per phase 10% unbal. factor
Average heating per phase 5% unbal. factor
Balanced heating
0
30°
60°
90°
120°
150°
ANGLE BETWEEN I_1 AND I_2 VECTORS
HEATING PER PHASE
on
UNBALANCE
IN
STATOR
Balanced Current = 40 amps

Fig 40

phases involved). Let θ be the angle between I_{a1} and I_{a2}.

$$I_a = \sqrt{(I_1 + I_2\cos\theta)^2 + (I_2\sin\theta)^2}$$
$$I_b = \sqrt{(I_2\cos(120^\circ - \theta) + I_1)^2 + (I_2\sin(120^\circ - \theta))^2}$$
$$I_c = \sqrt{(I_2\cos(\ 60^\circ - \theta) - I_2)^2 + (I_1\sin(\ 60^\circ - \theta))^2}$$

Considering the heating per phase these become

$$H_a = kI_a^2 = k(\ I_1^2 + 2I_1I_2\cos\theta + I_2^2\ ).$$
$$H_b = kI_b^2 = k(\ I_1^2 + 2I_1I_2\sin(\ \theta - 60^\circ) + I_2^2\ )$$
$$H_c = kI_c^2 = k(\ I_1^2 - 2I_1I_2\sin(\ \theta - 60^\circ) + I_2^2\ )$$

where k is the proportionality constant.

The average heating for the motor considered as a unit will be:

$$H_{average} = \frac{H_a + H_b + H_c}{3} = k(\ I_1^2 + I_2^2\ ).$$

These relations have been shown in Fig.40. for the motor operating at rated positive sequence current and with various degrees of unbalance. Notice from these curves that if normal heating exists at rated current balanced voltages applied at

5% unbalance factor there will be 10.3% overheating.
10% " " " " " 20.9% "
20% " " " " " 43.8% "

These figures are for a single phase in the extreme case in which the vectors I_1 and I_2 are coincident. The excessive heating of the machine as a unit will be

.37% for an unbalance factor of 5%
1.25% " " " " " 10%
4.06% " " " " " 20%

We would conclude that individual phases are liable to marked overheating although the general heating will not be excessive on small unbalances. This heating will be increased by heat received from the rotor by radiation. The actual heating will depend largely on the ability of the stator to distribute its heating over the winding through conductivity and dissipate heat by convection.

CHAPTER V.

PREDETERMINATION OF THE UNBALANCE CAUSED BY UNSYMMETRICAL LOADINGS

27. It is often necessary to know before hand just what unbalance will be set up in an otherwise symmetrical system by the addition of single phase loads. These loads will be those connected from line to line and those from line to neutral. It might also be pointed out that unbalance might also be caused by unsymmetrical series impedances such as was dealt with in the unbalanced transformer connection used in the tests outlined in Chapt. IV. This type of problem is not usually encountered and the following treatment will not consider it.

It must be realized that the sequences are independent only as long as the system remains symmetrical in all respects. The addition of these single phase loads thus destroys the independence of the sequences. There are two methods of treating the unsymmetrical systems mathematically.

The Terminal Condition Method

This is the method used in most short circuit calculations and is that used in the transformer problem mentioned above. The method is to isolate the unsymmetrical portion from the rest of the system. When this has been done the system may be represented by its three sequence networks. These networks may be simplified to their equivalent impedances and then connected to the unsymmetrical portion. The current which will flow in the unsymmetrical portion will cause drops in each of the networks. The method of connection of these networks with the unbalanced impedance is determined by the analysis of the phase currents and voltages and is always the same for the same type of unbalance. This method will be used in the following discussions.

The Method Using Special Impedance Terms

This method is applicable in all cases of unbalance and may be applied when the other method becomes too difficult to apply. It is used in problems in which unsymmetrical series impedances are present. The equations for the voltage drops are written for each individual coductor. The sequence components for the voltage and current are then substituted for the phase values. It will then be found that the voltage of each sequence is equal to the sum of the products of the sequence currents and composite impedance terms of the following form (usually the "B" terms are written as betas):

$$E_{ao} = B_o I_{ao} + B_2 I_{a1} + B_1 I_{a2}$$

$$E_{a1} = B_1 I_{ao} + B_o I_{a1} + B_2 I_{a2}$$

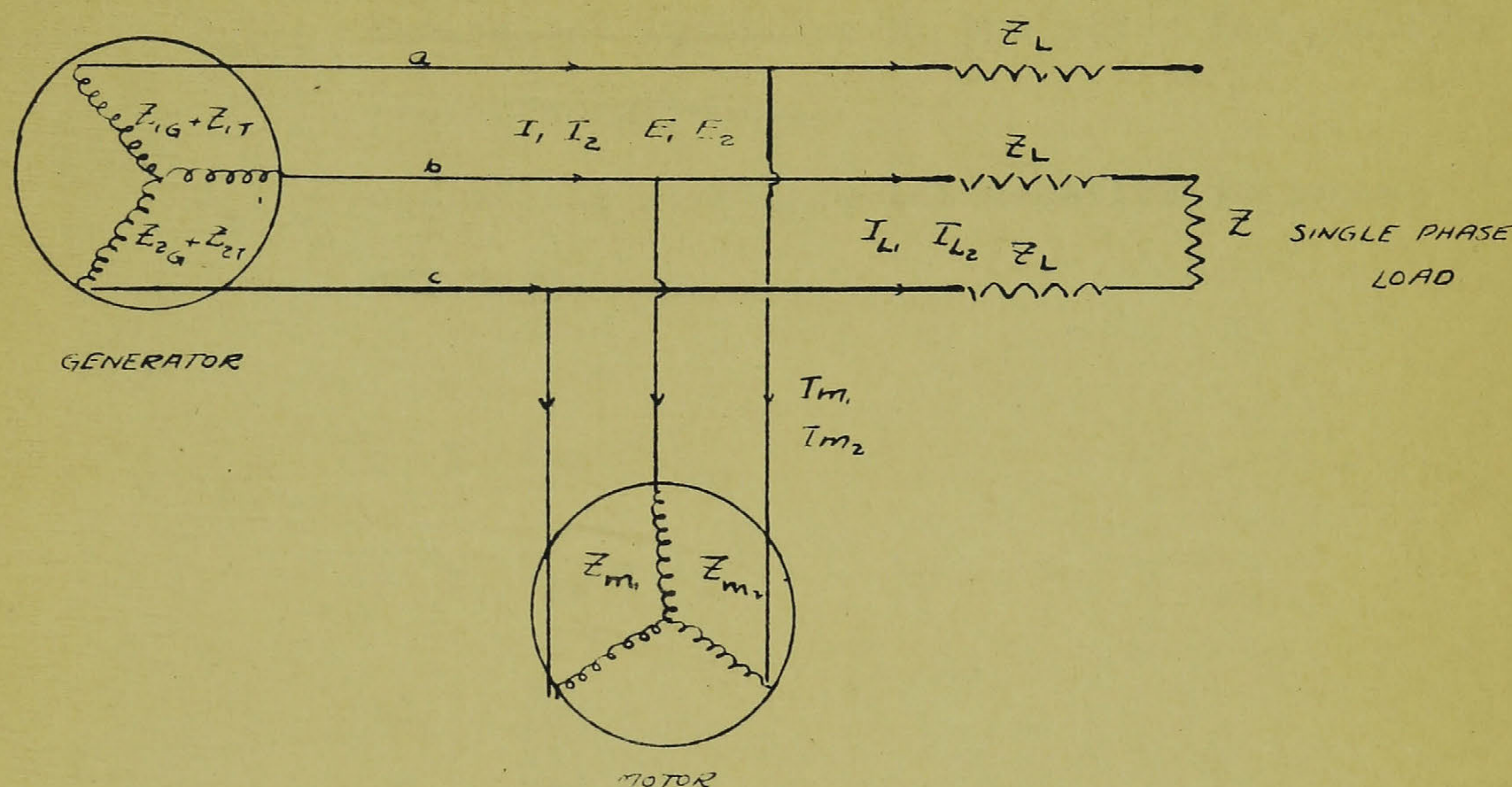
$$E_{a2} = B_2 I_{ao} + B_1 I_{a1} + B_o I_{a2}$$

This method is fully developed in Wagner and Evan's Symmetrical Components"..

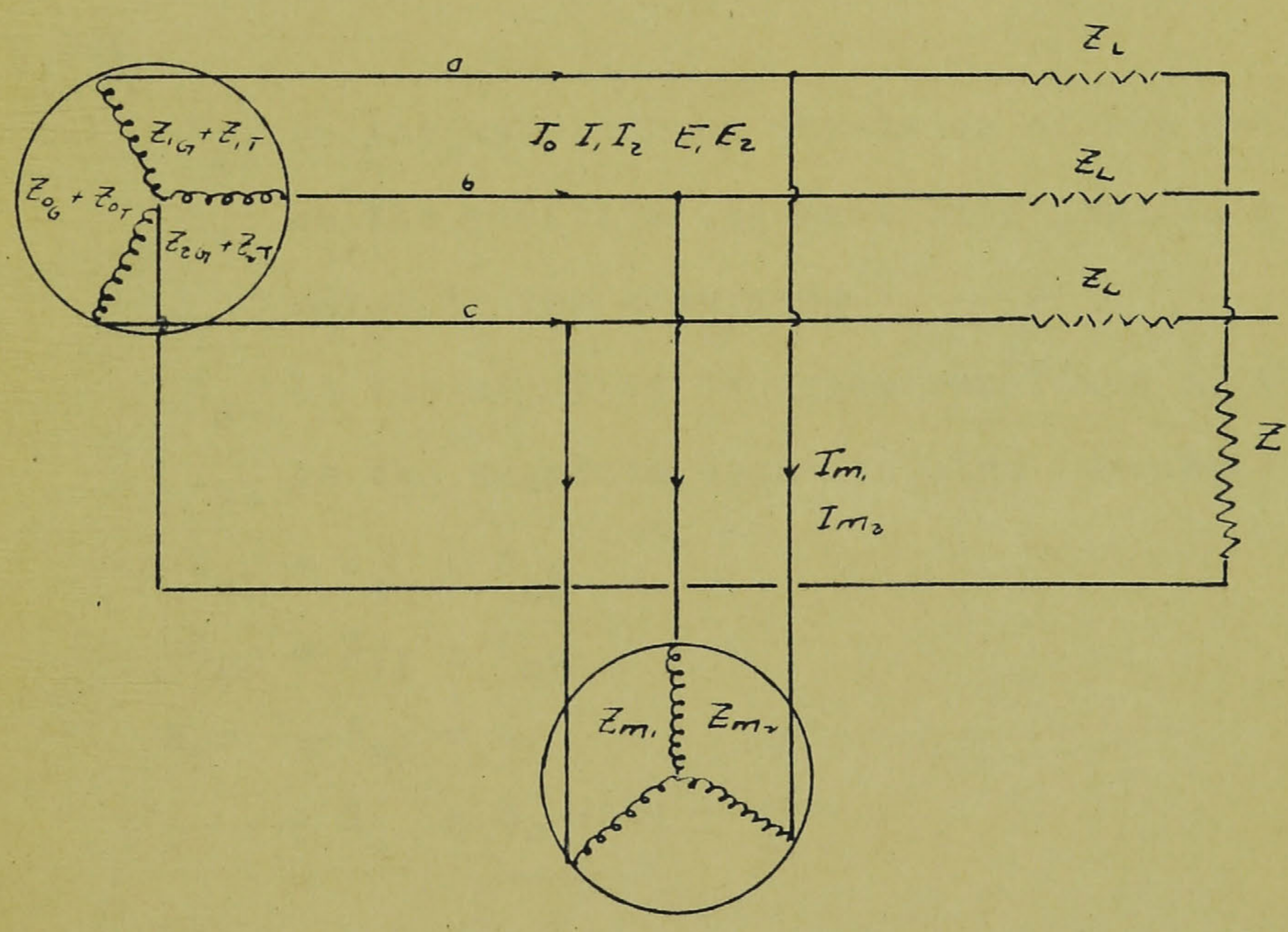
28. 1) Unbalance Due to a Load Connected from Line to Line

This case is shown in Fig. 41. in which a single motor is shown connected through a line impedance to a single phase load.

SINGLE PHASE LOADS ON THREE PHASE LINES



LINE TO LINE CONNECTION
Fig 41



LINE TO NEUTRAL CONNECTION
Fig 42

Consider first of all that the motor is an induction motor and that

Z_{m1} = equivalent impedance of the motor for positive sequence currents.

Z_{m2} = equivalent impedance for negative sequence currents.

Then

$$I_1 = I_{m1} + I_{L1}$$

$$I_2 = I_{m2} + I_{L2}$$

$$I_{m1} = \frac{E_1}{Z_{m1}}$$

$$I_{m2} = \frac{E_2}{Z_{m2}}$$

Let

Z_{g1} be the synchronous impedance of the generator.

Z_{t_1} be the positive sequence line impedance from the motor to the generator.

Z_{g2} be the negative sequence reactance of the generator

Z_{t2} be the negative sequence line impedance to the motor

$$Z_{g1} + Z_{t1} = Z_{a1}$$

$$Z_{g2} + Z_{t2} = Z_{a2}$$

$$E_o - I_I \cdot Z_{a1} = E_1$$

$$I_1 = \frac{E_o - E_1}{Z_{a1}}$$

$$E_2 = -I_2 Z_{a2}$$

Then

$$\frac{E_o - E_1}{Za_1} = I_{m1} + I_{L1}$$

$$= \frac{E_1}{Z_{m1}} + I_{L1}$$

or

$$E_1 = \frac{Z_{m1}}{Z_{a1} + Z_{m1}} \cdot E_o - \frac{Z_{a1} \cdot Z_{m1}}{Z_{a1} + Z_{m1}} \cdot I_{L1}$$

and

$$E_2 = - \frac{Z_{a2} \cdot Z_{m2}}{Z_{a2} + Z_{m2}} \cdot I_{L2}$$

If E_{L1} and E_{L2} are the voltages at the single phase load terminals then:

$$E_{L1} = E_1 - I_{L1} \cdot Z_L$$

$$E_{L2} = E_2 - I_{L2} \cdot Z_L$$

The connecting link between the two sequences is provided at the terminal load. Consider the terminal voltages and currents (dropping the subscript L)

$$I_a = 0 \qquad I_b = -I_c$$

$$E_b = E_c - Z \cdot I_c$$

$$I_c = \frac{3}{-a + a^2} \cdot I_{a1}$$

$$E_b = E_c - \frac{3}{-a + a^2} \cdot I_{a1} \cdot Z$$

$$E_{a1} - E_{a2} = 1/3(a - a^2) \cdot E_b + 1/3(a^2 - a) \cdot E_c = Z \cdot I_{a_1}$$

Ressuming our notation

$$E_{L1} = E_{L2} - Z \cdot I_{L1}$$

and $I_{L1} = - I_{L2}$

Then $E_1 - E_2 = I_{L1} \cdot (Z + 2Z_L)$

We may now substitute in the values obtained for E_1 and E_2 in equations (1) and (2).

$$Z_{L_1} \cdot \left(Z + 2Z_L + \frac{Z_{a1} \cdot Z_{m1}}{Z_{a1} + Z_{m1}} + \frac{Z_{a2} \cdot Z_{m2}}{Z_{a2} + Z_{m2}} \right) = \frac{Z_{m1}}{Z_{a1} + Z_{m1}} \cdot E_o$$

This equation gives the most general case for a line to line load. We consider that the presence of the balanced and unbalanced load does not affect the system voltage when they are connected to a distribution system of considerable capacity.

Then

$$E_1 = I_{L1} \left(Z + 2Z_L + \frac{Z_{a2} \cdot Z_{m2}}{Z_{a2} + Z_{m2}} \right)$$

substituting back to get I_2

$$I_2 = - \frac{E_1}{(Z + 2Z_L + Z_{a2}) + \frac{Z_{a2}}{Z_{m2}} \cdot (Z + 2Z_L)}$$

Several conclusions may be taken from this equation:

1) The induction motor supplies negative sequence current for the single phase load and by so doing prevents system unbalance. Thus in the equation above if there were no induction motor

$Z_{m2} = \infty$ and

$$I_2 = -\frac{E_1}{Z + 2Z_L + Z_{a2}}$$

2) The proportion of the negative sequence current which will be supplied by the system depends on the type of generators used. Average values of negative sequence reactances are:

Turbo-generators .12

Water wheel generators .50

We may conclude that the induction motor will be more effective in preventing line unbalance on systems supplied by waterwheel generators.

3) Finally we conclude that the amount of the negative sequence current supplied by the induction motor depends on the distance it is separated from the single phase load. Thus with the induction motor directly at the single phase tap the system will supply

$$I_2 = -\frac{E_1}{Z + Z_{a2} + \frac{Z_{a2} \cdot Z}{Z_{m2}}}$$

which represents the maximum it will have to supply. If the induction motor is moved away from the line to line tap it will supply an increasing amount of the required negative sequence current.

Now consider the balanced load to be a synchronous motor We can use the relation

$$E_{om} = E_1 - I_{m1} \cdot Z_{m1}$$

where E_{om} is the E.M.F. generated by the field poles (corresponding to the no load flux for any one excitation).

Z_{m1} is the synchronous impedance

$\doteq R_1 + jX_d$

This gives the equation

$$I_{L1} \cdot \left(Z + 2Z_L + \frac{Z_{a1} \cdot Z_{m1}}{Z_{a1} + Z_{m1}} + \frac{Z_{a2} Z_{m2}}{Z_{a2} + Z_{m2}} \right) = \frac{E_o \cdot Z_{m1} + E_{om} \cdot Z_{a1}}{Z_{m1} + Z_{a1}}$$

The expression for the negative sequence current which must be supplied by the system is

$$I_2 = \frac{E_o \cdot Z_m + E_{om} \cdot Z_{a1}}{Z_{m1} + Z_{a1}} \cdot \left(\frac{1}{Z + 2Z_L + \frac{Z_{a1} \cdot Z_{m1}}{Z_{m1} + Z_{a1}} + \frac{Z_{a2} \cdot Z_{m2}}{Z_{m2} + Z_{a2}}} \right) \cdot \left(1 - \frac{Z_{a2}}{Z_{m2} + Z_{a2}} \right)$$

If this is simplified by considerihg, as was done for the induction motor, that the terminal voltage is constant we get the same expression as for the induction motor. We can, therefore, apply the conclusions reached for the the induction motor. It should also be noted from the expression for I_2 above that if the excitation of the synchronous motor is increased E_{om} will increase and the proportion of negative sequence current which must be supplied by the system will increase. Thus the device of raising the plant power factor by overexciting the synchronous motors will increase the ratio of I_2 to I_1 which must be supplied from the system.

29. Unbalance Due to a Load Connected from Line to Neutral

The first conditions apply as before

$$E_1 = \frac{Z_{m1}}{Z_{a1} + Z_{m1}} \cdot E_o - \frac{Z_{a1} \cdot Z_{m1} \cdot I_{L1}}{Z_{a1} + Z_{m1}}$$

$$E_2 = - \frac{Z_{a2} \cdot Z_{m2}}{Z_{m2} + Z_{a2}} \cdot I_{L2}$$

The terminal conditions may now be applied. At the point where the single phase load is applied

$$I_{Lb} = 0.$$

$$I_{Lc} = 0.$$

$$E_a = Z I_L$$

and $I_{La1} = I_{La2} = I_{Lao} = 1/3\ I_a$

or $E_a = 3ZI_{a1}$

$$I_{La1} = \frac{E1}{Z_o + Z_2 + Z_L + 3Z} = \frac{E1}{Z_o + 2Z_L + 3Z}$$

where Z_o is the zero sequence impedance of the system from this point. Since induction motors and synchronous motors are not operated with a grounded neutral point the zero sequence current will not flow in the motor branch.

Substituting back we get

$$\frac{Z_{m1} \cdot E_o}{Z_{a1} + Z_{m1}} = +I_{L1} \cdot \left(Z_o + 2Z_L + 3Z + \frac{Z_{a1} \cdot Z_{m1}}{Z_{a1} + Z_{m1}} \right)$$

The negative sequence current taken from the line is given by: -

$$I_2 = I_{L2} + I_{m2}$$

The equation now becomes

$$\frac{Z_{m1} \cdot E_o}{Z_{a1} + Z_{m1}} \cdot \left(\frac{Z_{m2}}{Z_{m2} + Z_{a2}} \right) = I_2 \left(Z_o + 2Z_L + 3Z + \frac{Z_{a1} \cdot Z_{m1}}{Z_{a1} + Z_{m1}} \right)$$

and

$$\frac{Z_{m1} \cdot E_o}{Z_{a1} + Z_{m1}} \cdot \left(\frac{Z_{a2}}{Z_{m2} + Z_{a2}} \right) = I_{m2} \left(Z_o + 2Z_L + 3Z + \frac{Z_{a1} \cdot Z_{m1}}{Z_{a1} + Z_{m1}} \right)$$

Notice that the negative sequence current flowing in the motor circuit will depend on the zero sequence impedance. If Z_o is large there will little negative sequence current flowing in the motor branch.

It should be noted that for single phase loads power pulsating at double frequency must be supplied.

The preceding chapter suggests another important phsae of the problem of unbalance, that of the correction of unbalance. This problem might well be dealt with, the material of this chapter providing a theoretical basis.

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