Dynamic Stability of Plane Structures

## S.Z.H. Burney

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Ph.D. March 1971

#### ABSTRACT

The finite element method has been used to determine the natural frequencies and modal shapes of plane structures such as beams, grids, trusses and frames. A computer program has been written for this purpose. The members of the structure may be of non-uniform cross-section and varying properties. Several examples are given to show comparison with results obtained experimentally and using other methods.

A numerical method of determining the regions of dynamic instability of a column subjected to periodic axial force and periodic support motion has been developed. The column is idealized as consisting of massless springs and lumped masses. Comparison with known results and with experimental work, which was carried out as a part of the investigation is presented.

An experimental investigation of the dynamic instability of a six storey portal frame is reported. The analytical solution of the problem was attempted by the continuum method and the results obtained are presented. Stabilité Lynamique des Structures Planes

S.Z.H. Burney

Département de Génie Civil et de Méchanique Appliquée Ph.D. Mars 1971

# Résumé

La méthode des éléments finis a été utilisée afin de déterminer les fréquences et les mouvements propres de structures planes telles que poutres, treilles et cadres rigides.

Un programme pour calcutrice électronique a été développé à cette fin. Les membrures de la structure peuvent être de sections et propriétés variables.

Plusieurs examples sont traités; leurs résultats sont comparés avec des valeurs expérimentales ou obtenues par d'autres méthodes.

Une méthode numérique pour la détermination des domaines d'instabilité dynamique d'une colonne soumise à une charge et à un déplacement d'appui axiaux et périodiques a été développée.

La colonne est idéalisée par une série de masses concentrées, et de ressorts de masse nulle.

Les résultats sont comparés avec des valeurs connues ou expérimentales, obtenues dans le cadre du présent travail.

L'étude expérimentale de l'instabilité dynamique d'un cadre rigide de six étages est incluse.

Une solution analytique de ce probleme fut tentée en remplacant le portique par un milieu continu équivalent, et les resultats obtenus sont présentés.

## DYNAMIC STABILITY OF PLANE STRUCTURES

A Thesis,

by

S.Z.H. Burney, B.Sc. (Hons.), M.Sc.

Submitted to the Faculty of Graduate Studies and Research in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

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1971

McGill University

March 1971

S.Z.H. Burney

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# To my wife and to my parents

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## SYMBOLS

Unless of	cherwise defined, the following symbols are used.
a	length of link
a	coefficient of Fourier series
A	cross-sectional area
b	breadth
b	coefficient of Fourier series
Ĕ	strain matrix
с	amplitude of support motion
c <sub>x</sub> , c <sub>y</sub>	direction cosines
ğ	displacement vector
D	elasticity matrix
e	strain
e <sub>o</sub>	initial strain
E	Young's modulus of elasticity
F	force
G	shear modulus
H	height
I	moment of inertia
k	element of stiffness matrix
K	spring stiffness, overall stiffness matrix
L	length
m	element of mass matrix

<sup>m</sup> o	mass per unit length
M	mass
M	mass matrix
n, N	number of beams
Ň	shape matrix
P	distributed load
P	load, force
Pe	approximate Euler buckling load
r <sub>n</sub>	K <sub>n</sub> /a <sub>n</sub>
Q ~	force matrix
đ	amplitude of displacement
<b>ď</b> *	inertia force
R ~	external modal force vector
s <sub>n</sub>	K <sub>n</sub> ∕a <sub>n-1</sub>
S,	stiffness matrix
t	time
T	time period
т. e	Kinetic Energy
u, v	displacements
us	support displacement.
υ, ν	Force
U e	Potential Energy
w	frequency

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- x, y coordinates or displacements
- X, Y Body Force Components
  - α stiffness coefficient for a continuum
- $\beta$   $\alpha$  H,a measure of the relative stiffness of beams w.r.t. columns
- β. natural frequency of the bar loaded by a longitudinal compressive force.
- $\gamma$  shear strain
- δ parameter of Mathieu's equation
- e parameter of Mathieu's equation
- \$ acceleration of the support motion
- 2 coefficients associated with V, m and K
- $\theta$  angular displacement,  $\frac{dy}{dx}$
- $\lambda$  coefficients associated with V<sub>0</sub>, m, and K
  - $v_{t}/2(P_{k}-v_{c})$

μ

- v Poisson's ratio
- ρ mass per unit volume
- $\rho$  mass per unit height
- σ stress
- $\phi$  modal shape function
- $\Omega$  frequency of external load, or exciting frequency of base motion

#### CHAPTER I

#### INTRODUCTION

## 1.1. General Review

Earthquakes occur in various parts of the world and occasionally they cause considerable damage to life and In order to build economic and attractive structures property. which are resistant to earthquake effects engineers and scientists have been studying their causes and behaviour. From records of ground motion it has been observed that during an earthquake the principal component is in the . On this basis in the design of horizontal direction structures subjected to seismic loading the vertical acceleration is usually ignored or else its effect is combined with that of the horizontal acceleration in an arbitrary manner. Very little is known about the response of structures when subjected to fluctuating vertical base Therefore, before one can attempt to study the motion. response of the structure to the combined effects of horizontal and vertical motions it is necessary to first resolve the problem of vertical motion. The first step in attacking the problem of seismic vibrations, which are of a random nature, is to seek solutions for cases of periodic support motion. Even with this simplifying assumption it

will be observed that the formulation and solution of the problem is difficult. It is hoped that this work will help in the study of the response of structures to random fluctuating base motion.

The differential equations of motion of structures subjected to periodic horizontal support motion have constant (2) coefficients . Consider the structure shown in Figure 1.1 and let the support motion be  $y_s(t)$ . The equations of motion write as:-

 $m_1 \ddot{y}_1 + k_{11} (y_1 - y_s) + k_{12} (y_2 - y_s) + \dots + k_{1n} (y_n - y_s) = 0$ 

$m_2 \ddot{y}_2 + k_{21} (y_1 - y_s) + k_{22} (y_2 - y_s) + \dots + k_{2n} (y_n - y_s) = 0$
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• • • • • • • • • • • • • • • • • • • •
$m_n \ddot{y}_n + k_{n1} (y_1 - y_s) + k_{n2} (y_2 - y_s) + \dots + k_{nn} (y_n - y_s) = 0$
${}^{m_{1}}\ddot{y}_{1} + {}^{k_{11}}y_{1} + {}^{k_{12}}y_{2} + \cdots + {}^{k_{1n}}y_{n} = f_{1}(k)y_{s}$
${}^{m}_{2}\ddot{y}_{2} + {}^{k}_{21}y_{1} + {}^{k}_{22}y_{2} + \cdots + {}^{k}_{n2}y_{2} = f_{2}(k)y_{s}$
•••••••
• • • • • • • • • • • • • • • • • • • •
$m_n \ddot{y}_n + k_{n1} y_1 + k_{n2} y_n + \dots + k_{nn} y_n = f_n (k) y_s$

i.e.









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where k<sub>ij</sub> are the stiffness coefficients and represent the force at station i due to a unit displacement at station j, the dots indicate differentiation w.r.t. time.

In matrix form the above equations may be written as:-

$$\begin{bmatrix} \mathbf{M} \\ \mathbf{Y} \end{bmatrix} \left\{ \mathbf{Y} \right\} + \begin{bmatrix} \mathbf{K} \end{bmatrix} \left\{ \mathbf{Y} \right\} = \mathbf{Y}_{\mathbf{S}} \left\{ \mathbf{f} \left( \mathbf{k} \right) \right\}$$
(1.1)

Equations 1.1 are second order linear differential equations with constant coefficients.

By contrast, if the cantilever column of Figure 1.2 is subjected to fluctuating vertical support motion then vertical accelerations are present and this causes the axial force in the cantilever to vary with time; the effective column stiffnesses are no longer constant but become functions of time. The resulting differential equations have varying coefficients. If the vertical support motion is periodic, the differential equation will also have periodic coefficients. Such differential (3) equations are termed Mathieu-Hill equations and are encountered in various areas of physics, engineering and celestial mechanics. Therefore, the crucial difference between those structures subjected to horizontal support motion and those subjected to vertical support motion is that in the former case we get differential equations with constant coefficients whilst the latter gives rise to



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Figure 1.4: <u>Stable and unstable regions for the Mathieu equation</u>. (Hatched areas represent stable regions)

differential equations with time-varying coefficients. As a result the characteristics of the response of structures to the two types of support motion are different and will be discussed later. In particular, when the governing equations are of the Mathieu-Hill type it is possible to have solutions which grow without limit as time progresses. Such behaviour is dynamic instability and is termed "parametric" to distinguish it from ordinary resonance phenomena.

The problem of the stability of a uniform bar subjected to time-varying axial forces was first studied (8) and has been reported by Timoshenko and by Beliaev . Beliaev studied the case of and Bolotin Gere a column with hinged ends (Figure 1.3) subjected to the action of an axial compressive force,  $v_c + v_t \cos \Omega t$ . Experience has shown that for certain cases slender bars can, without buckling, sustain instantaneously greater loads,  $(V_c+V_t)$  than the Euler buckling load for the column. Also, at certain values of the frequency  $\Omega$  the lateral motion y(t) of the column becomes unstable. The column then tends to oscillate with an amplitude which increases with time, being finally limited by system changes such as the nonlinearity of the restoring force at large amplitude. Such dynamically unstable behaviour occurs when the exciting

frequency  $\Omega$  is double the response frequency w of the system; this contrasts with the more familiar resonance behaviour of a forced oscillatory system, which occurs when the exciting frequency and response frequency are equal. Another distinguishing feature of parametric resonance lies in the possibility of producing resonance with exciting frequencies less than the natural frequency of the structure. Also parametric resonance differs from ordinary resonance by the fact that there exist continuous regions of dynamic instability. To illustrate the general nature of parametric resonance, consider the problem of the stability of a uniform bar with hinged ends and subjected to an axial pulsating load  $v_c + v_t \cos \Omega$  t as shown in Figure 1.3. Assuming that the bar is initially straight and perfectly elastic, and ignoring the rotational inertia of the bar, the differential equation of motion is

$$EI \frac{\partial^4 y}{\partial x^4} + (V_c + V_t \cos Q t) \frac{\partial^2 y}{\partial x^2} + m_o \frac{\partial^2 y}{\partial t^2} = 0$$
(1.2)

where  $m_0$  is the mass per unit length of the bar. If we seek the solution of Equation 1.2 in the form

$$y(x,t) = f_k(t) \sin \frac{k \pi x}{L}, (k = 1,2,3...)$$
 (1.3)

Equation 1.2 reduces to

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$$\frac{d^{2}f_{k}}{dt^{2}} + w_{k}^{2} \left(1 - \frac{V_{c} + V_{t} \cos \Omega t}{P_{k}}\right) f_{k} = 0, \quad (k = 1, 2, 3..) \quad (1.4)$$

where

$$r_{k} = \frac{k^{2} \pi^{2}}{L^{2}} = \frac{EI}{m_{o}}$$
 (1.5)

is the kth frequency of the free vibrations of an unloaded bar and

$$P_{k} = \frac{k^{2}\pi^{2} EI}{2}$$
L (1.6)

is the kth Euler buckling load.

Equation 1.4 may be further written as:-

$$\frac{d^{2}f_{k}}{dt} + \beta o_{k}^{2}(1 - 2\mu_{k} \cos \Omega t) f_{k} = 0, \quad (k = 1, 2, 3...) \quad (1.7)$$

where  $\beta_{ok}$  is the frequency of the free vibrations of the bar loaded by a constant longitudinal force V<sub>c</sub>, i.e.

$$\beta_{\rm ok}^2 = w_{\rm k}^2 (1 - V_{\rm c/P_{\rm k}})$$
 (1.8)

and 
$$2\mu_{k} = \frac{V_{t}}{(P_{k} - V_{c})}$$
 (1.9)

Since Equation 1.7 is identical for all forms of vibrations, the suffix k may be omitted without loss of generality and Equation 1.7 then writes as:-

$$\vec{f} + \beta_0^2 (1 - 2 \mu \cos \Omega t) f = 0 \qquad (1.10)$$
or
$$\vec{f} + (\delta + \varepsilon \cos \Omega t) f = 0 \qquad (1.10a)$$
where
$$\delta = \beta_0^2 \text{ and } \varepsilon = -2 \mu \beta_0^2$$

Equation 1.10 is the well-known Mathieu equation which has an extensive literature, for example the book by (3) McLachlan

To investigate the stability of the motion, the solutions of the Mathieu equation have to be studied. A given motion is stable if all the solutions of the Mathieu equation are bounded for all positive values of t, and unstable if the equation has an unbounded solution. The boundedness of the solutions depend upon the relationship between  $\delta$  and  $\in$ . For certain values of  $\delta$  and  $\in$  bounded solutions are obtained, whilst for other values unbounded solutions result. The regions of bounded and unbounded solutions in the  $\delta - \epsilon$  plane are shown in Figure 1.4.

Lubkin and Lubkin and Stoker studied the stability of columns and strings under periodically varying forces, sinusoidal in nature. Apparently, they were not (8) aware of the work of Beliaev They showed that the problem of columns and strings subjected to periodically varying axial forces reduces to a Mathieu equation. They studied the stability of the Mathieu equation and determined the co-ordinates of the boundary points of the regions of instability. Utida and Sezawa also independently studied the dynamic stability of columns under periodic longitudinal forces. Their study was both theoretical and experimental in







character. They also showed that the problem of a column subjected to periodic longitudinal forces reduces to a Mathieu equation. Experimentally they investigated the parametric stability of brass strips 0.5 mm thick and 20.7 mm wide with one end being clamped and the other having a concentrated mass, which was supported by piano wires to permit axial movement as shown in Figure 1.5. They observed that the deflection and slope at the end carrying the concentrated mass were almost zero. The length of the strip measured from its clamped end to the centre of the concentrated mass was 353 mm. Concentrated mass weights of 1000 gms and 250 gms were used. The longitudinal force  $v = v_t \cos \Omega t$  ( $v_c = 0$ ) was generated electromagnetically. The concentrated mass consisted of a heavy coil around which a constant magnetic field was created by passing D.C. current. The passage of an alternating current gave rise to the axial load. Utida and Sezawa were the first to verify experimentally the existence of secondary regions of instability.

Bolotin carried out an experimental investigation of the dynamic stability of columns. Unlike Utida and Sezawa the periodic axial force consisted of a constant part together with a harmonic portion, i.e.  $V_c \neq 0$ . A schematic diagram of his experimental set-up is shown in

(5)

Figure 1.6. The lower end of the column is pinned and fixed in position; the upper end is also pinned but allowed to move vertically. A set of weights placed at the end of a lever supplied the constant term  $V_c$ ; the fluctuating component  $V_t \cos \Omega$  t was generated by a rotating eccentric mass situated at the upper end of the column. From the tests carried out, Bolotin determined the principal region of dynamic instability for pinned columns and this was found to be in good agreement with theory. However, he failed to verify experimentally the existence of the higher (secondary) regions.

(10)

Weingarten also conducted experiments to investigate the parametric instability of columns subjected to periodic axial loads of the form  $V = V_t \cos \Omega t$ . His results confirmed the existence of secondary regions of instability. He also investigated the effect of boundary conditions on the stability of the rod and studied the following two cases.

(a) Column simply-supported at both ends.

(b) Column clamped at both ends (for which he presented a theoretical formulation).

He found out that the instability regions for the two cases are similar, i.e. they are independent of the boundary conditions, and obtained an experimental verification. Somerset and Evan-Iwanowski

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out an extensive experimental investigation of the principal region of dynamic instability of columns pinned at both ends subjected to periodic axial forces of the type  $V = V_c + V_t$  cos  $\Omega$  t. The experiments were so designed as to vary independently the parameters  $V_c$ ,  $V_t$  and  $\Omega$ . They also studied the effect of damping and it was achieved by immersing the column in a fluid. Light oil and water were used as damping fluids.

(11)

also carried

#### (2, 12)

Jaeger and Barr investigated the problem of the stability of a cantilever column subjected to periodic vertical support motion. They showed that the governing equations of motion, initially a partial differential equation, may be reduced, as an approximation, to an ordinary differential equation of the Mathieu type by (13,14) employing the Galerkin Method

1.2. Purpose and Scope of Investigation

The purpose of the investigation is to study the dynamic stability of structures subjected to periodic axial forces and periodic support motion. A numerical method of determining the regions of dynamic stability of a uniform column subjected to a periodic longitudinal force and different boundary conditions has been developed. All the previous investigations had been confined to the testing of columns with periodic longitudinal forces only. The

author carried out an experimental investigation of the parametric stability of columns subjected to periodic support motion. A numerical method formulation is also presented.

An experimental investigation of the dynamic stability of portal frames subjected to periodic support motion was undertaken. The study of the dynamic stability of frames requires a knowledge of their natural frequencies. A study was undertaken to find a suitable method for the determination of the natural frequencies of structures. It was found that the finite element method was an efficient and powerful method and was suitable for computer programming. Computer programs were written and the results obtained for different types of structures were compared with known experimental results.

#### CHAPTER II

## NATURAL FREQUENCIES OF PLANE STRUCTURES

#### BY A FINITE ELEMENT METHOD

2.1. Introduction

A number of methods are available in the literature for determining the natural frequencies of plane structures and some of them employ computer techniques. For simple structures like beams and columns various classical methods are available, a good account of which may be found in the texts of Timoshenko (15), Biggs (16), and Minhinnick (17). A brief review of the major methods available for portal frames will now be presented.

Bishop <sup>(18,19,20)</sup> used the method of receptances to determine the natural frequencies. The forced vibrations of the system are expressed in terms of the receptances of its component parts, which have been tabulated for reference. At the natural frequencies the overall structure receptance vanishes and by a trial and error method the desired frequencies are obtained. Bishop's example involved only single-bay, single storey frames. Gladwell <sup>(21)</sup> extended Bishop's method of receptances to the analysis of multi-bay, multi-storey portal frames. Hurty <sup>(22)</sup> determined the natural modes and frequencies of structural systems by an energy method approach, which is a variation of the Rayleigh-Ritz method, using mode functions applicable to the complete system or subsystems.

Newmark <sup>(23)</sup> proposed a numerical integration technique for the computation of the dynamic response of structures. Chaudhury et al <sup>(24)</sup> also used a numerical integration method for the dynamic analysis of structural frameworks. However, numerical integration methods are not suitable for the determination of the natural frequency and the normal modes. The finite difference method has also been used for the determination of the natural frequencies, e.g. Livesley <sup>(25)</sup> employed it to obtain the natural frequencies of beams, Allman and Brotton <sup>(26)</sup> for plane structures, Cox and Denke <sup>(27)</sup> and Filington and McCallion <sup>(28)</sup> for grillages. A good review of the other available methods for grillages is given by Hendry <sup>(29)</sup> and Rogers <sup>(30)</sup>.

Ariaratnam <sup>(31)</sup> presented a method for analysing the dynamic behaviour of a plane structure which takes into account its distributed mass. The method is based on the stiffness approach of Livesley <sup>(32)</sup> for the elastic stability analysis of frameworks. Force-displacement equations of an individual member are obtained by combining the solutions of the differential equations governing the longitudinal and the transverse vibrations of the member. The natural frequencies are the roots of the resulting transcendental equation.

The matrix formulation of the general dynamic problem without damping leads to the equation

$$\begin{bmatrix} M \\ \sim \end{bmatrix} \begin{bmatrix} u \\ \sim \end{bmatrix} + \begin{bmatrix} K \\ \sim \end{bmatrix} \begin{bmatrix} u \\ \sim \end{bmatrix} = \begin{bmatrix} Q \\ \sim \end{bmatrix}$$
(2.1)

where M is the mass matrix

K is the stiffness matrix

u is the displacement matrix

Q is the force matrix

In the early attempts to deal with dynamic problems the lumped mass procedure was used, and the mass matrix M, therefore, was constructed by lumping the masses at nodal or station points. The mass matrix M obtained was a diagonal matrix. The natural frequencies may then be obtained by substituting Q = 0 in Equation (2.1) and this technique is well covered in the texts by Biggs <sup>(16)</sup> and Hurty and Rubinstein <sup>(33)</sup>.

It is known that when the distributed mass of a body is replaced by an equal mass concentrated at the centre of gravity of the section or at node points, two primary inaccuracies are introduced into the analysis through the implied suppositions that

(i) The resultant of inertia actions of the elementary masses always passes through the centre of gravity of these masses.

(ii) A concentrated load produces the same deflection as a distributed load of which it is the resultant. define the state of strain and stress within the element. By applying the principle of virtual work the work done by the internal forces may be equated to that done by the external forces, and thereby the solution for the unknown displacements may be obtained.

Different types of elements and various displacement fields may be chosen depending upon the type of the problem. The mathematical formulation is as follows:-

# (a) <u>Displacement field</u>

Let the displacements at any point within the element be defined by  $\underline{f}$ , a column vector.

$$\underbrace{f}_{\sim}(\mathbf{x},\mathbf{y}) = \begin{bmatrix} \mathbf{N} \\ \mathbf{N} \end{bmatrix} \left\{ \underbrace{d}_{\sim} \right\}$$
 (2.2)

where  $\underline{d}$  is the column vector of nodal displacements

 $\stackrel{N}{\sim}$  is the 'shape' matrix and is a function of (x,y) and depends upon the assumed displacement field.

e.g. for a flat triangular element in plane stress (see fig. 2.1).

$$\begin{split} f &\equiv \begin{cases} u(\mathbf{x}, \mathbf{y}) \\ \mathbf{v}(\mathbf{x}, \mathbf{y}) \end{cases} &\equiv \begin{cases} \alpha_1 + \alpha_2 \mathbf{x} + \alpha_3 \mathbf{y} \\ \alpha_4 + \alpha_5 \mathbf{x} + \alpha_6 \mathbf{y} \end{cases} \\ \begin{cases} d_1 \\ d_j \\ d_m \end{cases} &\equiv \begin{cases} u_1 \\ \mathbf{v}_1 \\ \mathbf{v}_j \\ \mathbf{v}_j \\ \mathbf{v}_m \\ \mathbf{v}_m \end{cases} \\ & & & \\ \end{bmatrix} \\ & & \\ \begin{bmatrix} \mathbf{N} \\ \mathbf{z} \end{bmatrix} = \begin{bmatrix} \mathbf{N}_1 & \mathbf{N}_j & \mathbf{N}_m \end{bmatrix} . \end{split}$$

Archer <sup>(34)</sup> and Leckie and Lindberg <sup>(35)</sup> showed that an equivalent mass matrix, also called a consistent mass matrix, gives better results than the method of lumped masses. A good account of equivalent mass matrices is given by Przemieniecki <sup>(36)</sup>. The vibrations of beams using finite elements has been shown to yield good results <sup>(34,35,37,38,39)</sup>.

The author has used the finite element method for determining the natural frequencies of non-uniform plane structures and the results obtained show good agreement with known experimental results (40).

## 2.2. The Finite Element Method

The theory of the finite element method is well covered in the texts by Zienkiewicz and Cheung  $^{(41)}$  and Przemieniecki  $^{(36)}$ . A brief account of its characteristics will be given.

In the finite element method the actual structure is divided into elements interconnected only at a finite number of points, called nodal points, at which some fictitious forces, representative of the distributed stresses actually acting on the element boundaries are supposed to act. The displacements of the nodal points, d, are the basic unknown parameters of the problem. A displacement function is assumed to define the displacement field within the element in terms of the nodal displacements. The displacement function serves to





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The strain at any point within the element may be expressed as

$$\begin{cases} e \\ e \end{cases} = \begin{bmatrix} B \\ e \end{bmatrix} \begin{cases} d \\ e \end{cases}$$
(2.3)

where e is the strain vector

B is the strain matrix and may be easily obtained

from matrix 
$$\begin{bmatrix} N \\ \sim \end{bmatrix}$$
.

For the plane stress case

$$\left\{ \underbrace{\mathbf{e}}_{\mathbf{x}} \right\} = \left\{ \begin{array}{c} \mathbf{e}_{\mathbf{x}} \\ \mathbf{e}_{\mathbf{y}} \\ \mathbf{e}_{\mathbf{y}} \\ \mathbf{e}_{\mathbf{x}\mathbf{y}} \end{array} \right\} = \left\{ \begin{array}{c} \underbrace{\partial \mathbf{u}} \\ \partial \mathbf{x} \\ \frac{\partial \mathbf{v}}{\partial \mathbf{y}} \\ \frac{\partial \mathbf{u}}{\partial \mathbf{y}} + \frac{\partial \mathbf{v}}{\partial \mathbf{x}} \end{array} \right\}$$

(2.3a)

## (c) <u>Stresses</u>

If the initial stresses within the element are denoted by  $\left\{ \stackrel{e}{=} \right\}$ , assuming general elastic behaviour, the relationship between stresses and strains may be expressed as

$$\left\{ \stackrel{\circ}{\sim} \right\} = \left[ \stackrel{\mathsf{D}}{\underset{\sim}{\sim}} \right] \left( \left\{ \stackrel{\mathsf{e}}{\underset{\sim}{\sim}} \right\} - \left\{ \stackrel{\mathsf{e}}{\underset{\sim}{\sim}} \right\} \right)$$
 (2.4)

Again, for the case of plane stress, we have

$$\left\{ \stackrel{\circ}{\sim} \right\} = \left\{ \begin{array}{c} \stackrel{\circ}{\sim} \mathbf{x} \\ \stackrel{\circ}{\sigma} \mathbf{y} \\ \stackrel{\circ}{\sigma} \mathbf{xy} \end{array} \right\}$$
(2.4a)
For an isotropic material

$$\underbrace{\mathbf{D}}_{\mathcal{D}} = \frac{\mathbf{E}}{1 - v^2} \begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & (\underline{1 - v}) \\ & & 2 \end{bmatrix}$$
 (2.4b)

(d) Equivalent Nodal Forces

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define the nodal forces which are statically equivalent to the free boundary stresses and distributed loads on the element. The distributed loads  $\left\{ p \right\}$  are defined as those acting on a unit volume of the material within the element.

For the case of plane stress

$$\begin{cases} F_{i} \\ \sim \end{cases} = \begin{cases} U_{i} \\ V_{i} \end{cases}$$
(2.5a)

where  $U_{i}$  and  $V_{i}$  are the components of forces in the x and y direction respectively and the distributed load is

$$\begin{bmatrix} \mathbf{p} \\ \mathbf{x} \end{bmatrix} = \begin{bmatrix} \mathbf{X} \\ \mathbf{y} \end{bmatrix}$$
(2.5b)

in which X, Y are the 'body force' components in the x and y directions respectively.

The principle of virtual work is used to determine the fictitious nodal forces statically equivalent to the actual boundary stresses and distributed loads. By equating the work done by the external and internal forces it may be shown that

$$\mathbf{\tilde{F}}^{\mathbf{e}} = \left( \int \left[ \mathbf{B} \right]^{\mathbf{T}} \left[ \mathbf{D} \right] \left[ \mathbf{B} \right] \mathbf{d} \quad (\text{Vol}) \right) \left\{ \mathbf{d} \right\} - \int \left[ \mathbf{B} \right] \left[ \mathbf{D} \right] \left\{ \mathbf{e}_{\mathbf{o}} \right\} \mathbf{d} \quad (\text{Vol}) - \int \left[ \mathbf{N} \right]^{\mathbf{T}} \left\{ \mathbf{p} \right\} \mathbf{d} \quad (\text{Vol}) \quad (2.6)$$

where the superscript e refers to the element and

$$\begin{bmatrix} \end{bmatrix}^{T} \text{ defines the transpose of a matrix.} \\ \text{Let } \begin{bmatrix} k \\ \sim \end{bmatrix}^{e} = \int \begin{bmatrix} B \\ \sim \end{bmatrix}^{T} \begin{bmatrix} D \\ \sim \end{bmatrix} \begin{bmatrix} B \\ \sim \end{bmatrix}^{d} \text{ (Vol)}$$
(2.7a)

and is called the stiffness matrix.

**.** 

$$\left\{ \underline{F} \right\}_{p}^{e} = -\int \left[ \underline{N} \right]^{T} \left\{ \underline{p} \right\} d \quad (Vol)$$
(2.7b)

and is called the distributed nodal force vector.

$$\left\{ \mathbf{F} \right\}_{\mathbf{O}}^{\mathbf{e}} = -\int \left[ \mathbf{B} \right]^{\mathbf{T}} \left[ \mathbf{D} \right] \left\{ \mathbf{e}_{\mathbf{O}} \right\} d \quad (\text{Vol})$$
 (2.7c)

and is called the initial nodal force vector.

Having determined the element stiffness matrix, the structure stiffness matrix, S, may be obtained by the appropriate addition of the element stiffness matrices. Since the element stiffness matrix is generated in the local (member) axis system it must be transformed into the global axis system before the addition for the formation of the overall structure stiffness matrix, K.

For the equilibrium of the assembled structure, external nodal forces must be equal and opposite to the equivalent nodal forces

i.e. 
$$\left\{ \begin{array}{c} R \\ \end{array} \right\} = \left\{ \begin{array}{c} F \\ \end{array} \right\}$$

where  $\underset{\sim}{\mathbb{R}}$  is the external nodal force vector. But  $\left\{ \underset{\sim}{\mathbb{F}} \right\} = \begin{bmatrix} K \\ \sim \end{bmatrix} \left\{ \underset{\sim}{\mathbb{d}} \right\} + \left\{ \underset{\sim}{\mathbb{F}} \right\}_{\mathbb{P}} + \left\{ \underset{\sim}{\mathbb{F}} \right\}_{\mathbb{O}}$ 

Hence 
$$\begin{bmatrix} \mathbf{K} \\ \mathbf{K} \end{bmatrix} \left\{ \mathbf{d} \\ \mathbf{d$$

Solving Equation (2.8) the unknown nodal displacements d may be determined. Once the nodal displacements are known the strains and stresses may be obtained using relations (2.3) and (2.4) respectively.

If masses  $M_i$  are attached to the nodes of the structure, with no external forces acting there, we have

$$\left\{ \begin{array}{c} \mathbf{R}_{i} \\ \mathbf{\tilde{z}}_{i} \end{array} \right\} = -\mathbf{M}_{i} \frac{\mathbf{d}^{2}}{2} \left\{ \begin{array}{c} \mathbf{\tilde{d}}_{i} \\ \mathbf{\tilde{d}}_{i} \end{array} \right\} = -\mathbf{M}_{i} \left\{ \begin{array}{c} \mathbf{\tilde{d}}_{i} \\ \mathbf{\tilde{d}}_{i} \end{array} \right\}$$
(2.9)

Let the inertia loading be represented by  $\left\{ \substack{p \\ c} \right\}$ . If the mass per unit volume is  $\ell$ , then again with no external forces acting we have

$$\left\{ \begin{array}{c} \mathbf{p} \\ \mathbf{p} \\ \mathbf{d} \end{array} \right\} = - \left\{ \begin{array}{c} \frac{\mathbf{d}^2 \mathbf{f}}{\mathbf{d} t^2} &= - \left\{ \begin{array}{c} \mathbf{f} \\ \mathbf{f} \\ \mathbf{d} \end{array} \right\} \right\}$$
(2.10)

Since 
$$\{f\} = [N] \{d\}$$
, we obtain  
 $\{p\} = -\ell[N] \{d\}$  (2.11)

and since the equivalent nodal forces due to the inertia loading  $\begin{cases} p \\ p \\ p \end{cases} \text{ are} \\ \begin{cases} F_{p}^{e} = -\int [N]^{T} \{p\} d (vol) \\ \text{we have } \{F_{p}^{e} = \int [N]^{T} \ell [N] d (vol) \{\ddot{d}\} \\ = [m]_{e} \{\breve{d}\} \end{cases}$ (2.12)

where  $\begin{bmatrix} m \end{bmatrix}_e$  is the element mass matrix.

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A typical element  $m_{ij}$  of the mass matrix represents the mass inertia load at point i developed by a unit acceleration at point j with all other points stationary.

The equivalent nodal forces due to the inertia loading,  

$$\begin{cases}
F_{p}^{e}, \text{ may now be added to those already present at the nodes, thus} \\
F_{p}^{e} = -\left(\left[\underline{M}^{O}\right] + \left[\underline{M}_{s}\right]\right)\left\{\overset{\bullet}{d}\right\} + \left\{\underline{R}\right\} \\
Where \left[\underline{M}^{O}\right] = \begin{bmatrix}M_{1} & 0 & 0 \cdot \cdot 0 \\ 0 & M_{2} & 0 \cdot \cdot 0 \\ 0 & 0 & 0 \cdot \cdot M_{n}\end{bmatrix}$$
(2.13)

is the matrix of external masses actually attached to the nodes.  $M_{s}$  is the overall mass matrix obtained from the assembly of the element mass matrices,  $[m]_{e}$ .

The general formulation now writes as

$$\begin{bmatrix} \mathbf{K} \\ \mathbf{K} \end{bmatrix} \left\{ \mathbf{d} \right\} = -\left( \begin{bmatrix} \mathbf{M}^{\mathbf{O}} \\ \mathbf{K} \end{bmatrix} + \begin{bmatrix} \mathbf{M}_{\mathbf{S}} \end{bmatrix} \right) \left\{ \mathbf{d} \\ \mathbf{d} \end{bmatrix} + \left\{ \mathbf{R} \\ \mathbf{K} \end{bmatrix}$$
(2.14)

 $\left\{ \begin{array}{c} R \\ \sim \end{array} \right\}$  has been retained in case actual external forces are still active during the motion.

# For the particular case of free vibrations, the above equation becomes

$$\begin{bmatrix} \mathbf{K} \\ \mathbf{M} \end{bmatrix} \left\{ \mathbf{d} \\ \mathbf{M} \end{bmatrix}^{*} = -\begin{bmatrix} \mathbf{M} \\ \mathbf{M} \end{bmatrix} \left\{ \mathbf{d} \\ \mathbf{M} \end{bmatrix}^{*} = -\begin{bmatrix} \mathbf{M} \\ \mathbf{M} \end{bmatrix} \left\{ \mathbf{d} \\ \mathbf{M} \end{bmatrix}^{*}$$
(2.15)  
where  $\begin{bmatrix} \mathbf{M} \\ \mathbf{M} \end{bmatrix}^{*} = \left( \begin{bmatrix} \mathbf{M}^{\mathbf{O}} \\ \mathbf{M} \end{bmatrix}^{*} + \begin{bmatrix} \mathbf{M} \\ \mathbf{M} \\ \mathbf{M} \end{bmatrix} \right)$ (2.15a)

Assuming the free vibrations to be harmonic, the displacements d may be written as

$$\left\{ d_{i} \right\} = q_{i} e^{i w t}$$
(2.16)

where q is a column matrix of the amplitudes of the displace-

ments d

w is the natural frequency of vibration

t is the time

Substituting Equation (2.16) in (2.15) it reduces to

$$\left(\underbrace{K} - w^{2} \underbrace{M}\right) \underbrace{q} = 0$$
 (2.17)

This is now in the standard form of eigenvalue determination.

#### 2.3 Development of Element Characteristics

a) <u>Plane Frames</u>

For the case of plane frameworks a beam element with three degrees of freedom (two translations and one rotation\*) at each node, as shown in Figure 2.2, has been used. The mass matrix for a beam element, referred to member coordinate axes and ignoring its rotary inertia is

$$\mathbf{m}' = \frac{\ell \,\mathrm{AL}}{420} \qquad \begin{vmatrix} 140 & 0 & 0 & 70 & 0 & 0 \\ 0 & 156 & 22\mathrm{L} & 0 & 54 & -13\mathrm{L} \\ 0 & 22\mathrm{L} & 4\mathrm{L}^2 & 0 & 13\mathrm{L} & -3\mathrm{L}^2 \\ 70 & 0 & 0 & 140 & 0 & 0 \\ 0 & 54 & 13\mathrm{L}_2 & 0 & 156 & -22\mathrm{L} \\ 0 & -13\mathrm{L} & -3\mathrm{L}^2 & 0 & -22\mathrm{L} & 4\mathrm{L}^2 \end{vmatrix}$$

$$(2.18)$$

where  $\ell$  is the density of the material of the element

A is the cross-sectional area of the element

L is the length of the element.

If the element is oriented as shown in Figure 2.3 the element mass matrix referred to the global coordinate axes is given by Equation (2.19) where  $C_x = \cos \theta$  is the direction cosine of \* Rotation about an axis is represented by a double headed arrow.



Figure 2.2: Beam element in member coordinate axes.



Figure 2.3: Beam element in global coordinate axes.

the member with respect to the X axis,  $C_y = \sin \theta$  is the direction cosine of the member with respect to the Y axis.

In a similar way the element stiffness matrix referred to global coordinate axes is given by Equation (2.20) where  $I_z$  = Moment of inertia about the Z axis.

b) <u>Grids</u>

In the case of grillages, a beam element with three degrees of freedom (two rotations and one translation) at each node, as shown in Figure 2.4a has been used. The mass matrix for a beam element with respect to member coordinate axes and ignoring its rotary inertia is

$$\mathbf{m}' = \frac{\varrho \mathbf{AL}}{420} \qquad \begin{bmatrix} 140 \ \mathbf{J}_{\mathbf{X}}/\mathbf{A} & 0 & 0 & 70 \ \mathbf{J}_{\mathbf{X}}/\mathbf{A} & 0 & 0 \\ 0 & 4\mathbf{L}^2 - 22\mathbf{L} & 0 & -3\mathbf{L}^2 & -13\mathbf{L} \\ 0 & -22\mathbf{L} & 156 & 0 & 13\mathbf{L} & 63 \\ 70 \ \mathbf{J}_{\mathbf{X}}/\mathbf{A} & 0 & 0 & 140 \ \mathbf{J}_{\mathbf{X}}/\mathbf{A} & 0 & 0 \\ 0 & -3\mathbf{L}^2 & 13\mathbf{L} & 0 & 4\mathbf{L}^2 & 22\mathbf{L} \\ 0 & -13\mathbf{L} & 63 & 0 & 22\mathbf{L} & 156 \end{bmatrix}$$
(2.21)

where  $J_x$  is the Torsion constant.

If the element is oriented as shown in Figure 2.4b the element mass matrix in global coordinate axes is given by Equation (2.22); and the element stiffness matrix by Equation (2.23).

Based on the above formulation a computer program was written to obtain the natural frequencies and nodal shapes of plane structures such as beams, grids, frames and trusses. The members of the structure may be of non-uniform cross-section and varying properties. A listing of the program is given in

 $140c_{x}^{2}+156c_{y}^{2}$  $-16C_{x}C_{y}$   $-22LC_{y}$   $70C_{x}^{2}+54C_{y}^{2}$ 160, Cy 13LCy  $-16\dot{c}_{x}c_{y}$  140 $c_{y}^{2}$ +156 $c_{x}^{2}$  22L $c_{x}$  16 $\dot{c}_{x}c_{y}$  70 $c_{y}^{2}$ +54 $c_{x}^{2}$  -13L $c_{x}$ 221.C<sub>x</sub> 41.<sup>2</sup> -131.C<sub>y</sub> 131.C<sub>x</sub> -31<sup>2</sup>  $m = \frac{\rho_{AL}}{420}$ -22LCy  $70C_{x}^{2}+54C_{y}^{2}$  $16C_{x}C_{y}$  -13LC<sub>y</sub>  $140C_{x}^{2}$ +156C<sub>y</sub><sup>2</sup> -16C<sub>x</sub>C<sub>y</sub> 22LC<sub>v</sub>  $70c_y^2 + 54c_x^2$  13LC<sub>x</sub>  $-16c_xc_y$  140c<sub>y</sub><sup>2</sup> + 156c<sub>x</sub><sup>2</sup> - 22LC<sub>x</sub> 16C<sub>x</sub>C<sub>y</sub> 131.C<sub>y</sub>  $-13LC_x$   $-3L^2$ 221.Cy  $4L^2$  $-22LC_{x}$ 

 $\left( \frac{\mathrm{AL}^2 \mathrm{C}_{\mathrm{X}}^2 + 12\mathrm{C}_{\mathrm{y}}^2}{\mathrm{I}_{\mathrm{z}}} \right) \left( \frac{\mathrm{AL}^2 - 12}{\mathrm{I}_{\mathrm{z}}} \right)^{\mathrm{C}_{\mathrm{X}}\mathrm{C}_{\mathrm{y}}} - \frac{\mathrm{CL}^2 \mathrm{C}_{\mathrm{y}}}{\mathrm{C}_{\mathrm{z}}^2 + 12\mathrm{C}_{\mathrm{y}}^2} - \left( \frac{\mathrm{AL}^2 \mathrm{C}_{\mathrm{z}}^2 + 12\mathrm{C}_{\mathrm{y}}^2}{\mathrm{I}_{\mathrm{z}}} \right) - \left( \frac{\mathrm{AL}^2 - 12}{\mathrm{I}_{\mathrm{z}}} \right)^{\mathrm{C}_{\mathrm{x}}\mathrm{C}_{\mathrm{y}}} - \frac{\mathrm{CL}^2 \mathrm{C}_{\mathrm{y}}}{\mathrm{C}_{\mathrm{z}}^2 + 12\mathrm{C}_{\mathrm{y}}^2} - \frac{\mathrm{CL}^2 \mathrm{C}_{\mathrm{z}}}{\mathrm{C}_{\mathrm{z}}^2 + 12\mathrm{C}_{\mathrm{y}}^2} - \frac{\mathrm{CL}^2 \mathrm{C}_{\mathrm{z}}}{\mathrm{C}_{\mathrm{z}}} - \frac{\mathrm{CL}^2 \mathrm{C}_{\mathrm{z}}}{\mathrm{C}} - \frac{\mathrm{CL}^2 \mathrm{C}_{\mathrm{z}}}{\mathrm{C}_{\mathrm{z}}} - \frac{\mathrm{CL}^2 \mathrm{C}_{\mathrm{z}}}{\mathrm{C}} - \frac{\mathrm{CL}^2 \mathrm{C}}{\mathrm{C}} - \frac{\mathrm{CL}^2 \mathrm{C}}{\mathrm{C}} - \frac{\mathrm{CL}^2 \mathrm{C}}}{\mathrm{C}} - \frac{\mathrm{CL}^2 \mathrm{C}} - \frac{\mathrm{CL}^2 \mathrm{C}}{\mathrm{C}} - \frac{\mathrm{CL}^2 \mathrm{C}}{\mathrm{C}}}{- \frac{\mathrm{CL}^2 \mathrm{C}}}{\mathrm{C}} - \frac{\mathrm{CL}^2 \mathrm{C}$  $\begin{pmatrix} \underline{AL}^2 - 12 \\ \overline{I_z} \end{pmatrix}^{C_x C_y} \begin{pmatrix} \underline{AL}^2 C_y^2 + 12C_x^2 \\ \overline{I_z} \end{pmatrix}^{C_x C_y} - \begin{pmatrix} \underline{AL}^2 - 12 \\ \overline{I_z} \end{pmatrix}^{C_x C_y} - \begin{pmatrix} \underline{AL}^2 C_y^2 + 12C_x^2 \\ \overline{I_z} \end{pmatrix}^{C_x C_y} = \begin{pmatrix} \underline{AL}^2 C_y^2 + 12C_x^2 \\ \overline{I_z} \end{pmatrix}^{C_x C_y}$  $\frac{k = EI_{z}}{L^{3}} \begin{vmatrix} -6LC_{y} & 6LC_{x} & 4L^{2} & 6LC_{y} & -6LC_{x} & 2L^{2} \\ -\left(\frac{AL^{2}C_{x}^{2}+12C_{y}^{2}}{I_{z}}\right) - \left(\frac{AL^{2}-12}{I_{z}}\right)^{C} C_{x}C_{y} & 6LC_{y} & \left(\frac{AL^{2}C_{x}^{2}+12C_{y}^{2}}{I_{z}}\right) & \left(\frac{AL^{2}-12}{I_{z}}\right)^{C} C_{x}C_{y} & 6LC_{y} \end{vmatrix}$ 21<sup>2</sup> (2.20)  $-\left(\frac{\mathrm{AL}^2-12}{\mathrm{I}_z}\right)^{\mathrm{C}_x\mathrm{C}_y} - \left(\frac{\mathrm{AL}^2\mathrm{C}_y^2+12\mathrm{C}_x^2}{\mathrm{I}_z}\right)^{-6\mathrm{LC}_x} - \left(\frac{\mathrm{AL}^2-12}{\mathrm{I}_z}\right)^{\mathrm{C}_x\mathrm{C}_y} - \left(\frac{\mathrm{AL}^2\mathrm{C}_y^2+12\mathrm{C}_x^2}{\mathrm{I}_z}\right)^{-6\mathrm{LC}_x}$ 6LC<sub>x</sub> 2L<sup>2</sup> -elcy 6LC -6LC<sub>x</sub>  $\cdot$  4L<sup>2</sup>

(2.19)

 ${}^{140J_{x}C_{x}^{2}+4L^{2}C_{y}^{2}}_{A}$  ${}^{140J_{x}C_{x}C_{y}-4L^{2}C_{x}C_{y}}_{A}$  $70J_{x}C_{x}^{2}-3L^{2}C_{y}^{2}$ . 221°  $70 \operatorname{J}_{A^{x}} C_{x} C_{y} + 3L^{2} C_{x} C_{y} \quad 13LC_{y}$  ${}^{140J_xC_xC_y-4L^2C_xC_y}_A$  ${}^{140J_{x}C_{y}^{2}+4L^{2}C_{x}^{2}}_{A}$  ${}^{70J_xC_xC_y+3L^2C_xC_y}_{A}$  70  ${}^{J_xC_y^2-3L^2C_x^2}_{X}$ -221.C<sub>x</sub> -13LC<sub>x</sub> -221C<sub>x</sub> 221.C. 156 -13LCy 131C<sub>x</sub>  $\frac{m}{420}$ 63  $70J_x O_x^2 3L^2 O_y^2$ (2.22)<sup>70J</sup><sub>x</sub>C<sub>x</sub>C<sub>y</sub>+3L<sup>2</sup>C<sub>x</sub>C<sub>y</sub> -131Cy  $140J_{x}c_{x}^{2}+4L^{2}c_{y}^{2}$ 140J<sub>x</sub>C<sub>x</sub>C<sub>y</sub>-4L<sup>2</sup>C<sub>x</sub>C<sub>y</sub> -22LC<sub>y</sub>  $70J_x c_y^2 - 3L^2 c_x^2$  $70J_{x}C_{x}C_{y}+3L^{2}C_{x}C_{y}$ 131C<sub>x</sub> 140J<sub>x</sub>C<sub>x</sub>C<sub>y</sub>-4L<sup>2</sup>C<sub>x</sub>C<sub>y</sub>  $140J_{x}C_{y}^{2}+4L^{2}C_{x}^{2}$ 221.C<sub>x</sub> -131C<sub>x</sub> 13LCy -221Cy 63 22LC<sub>x</sub> 156  $\frac{\mathrm{GI}_{x}\mathrm{C}_{x}^{2}+4\mathrm{EI}_{y}\mathrm{C}_{y}^{2}}{\mathrm{L}}$  $\begin{array}{c} -\underline{\text{GI}}_x c_x^2 + \underline{2\text{EI}}_y c_y^2 \\ \underline{\text{L}} \end{array}$  $\left( \frac{\mathbf{GI}_{\mathbf{X}}}{\mathbf{L}}^{-\underline{\mathbf{4}} \mathbf{EI}} \mathbf{y} \right)^{\mathbf{C}_{\mathbf{X}} \mathbf{C}_{\mathbf{y}}}$  $\frac{6 E I}{L^2} y^C y$  $-\left(\frac{GI}{L} + \frac{2EI}{L}y\right) C_{x}C_{y} - \frac{6EI}{L^{2}}yC_{y}$  $\frac{\mathrm{GI}}{\mathrm{L}} \mathrm{C}_{\mathrm{y}}^{2} + \frac{\mathrm{4EI}}{\mathrm{L}} \mathrm{y} \mathrm{C}_{\mathrm{x}}^{2}$ -6EI Cx  $-\underline{\operatorname{GI}}_{\mathbf{L}} \mathbf{C}_{\mathbf{y}}^{2} + \underline{\operatorname{2EI}}_{\mathbf{y}} \mathbf{C}_{\mathbf{x}}^{2}$  $\left( \frac{\text{GI}_{\mathbf{X}}}{L} - \frac{4\text{EI}_{\mathbf{y}}}{L} \right) C_{\mathbf{X}} C_{\mathbf{y}}$  $-\left(\frac{\mathrm{GI}_{\mathbf{X}}}{\mathrm{L}}+\frac{\mathrm{2EI}_{\mathbf{Y}}}{\mathrm{L}}\right)^{\mathrm{C}_{\mathbf{X}}\mathrm{C}_{\mathbf{Y}}}$  $\frac{6EI}{L^2}y^{C_X}$ 12EI L<sup>3</sup>y (2.23) $\frac{6 E I_y C_y}{L^2}$ -<u>6EIy</u>C<sub>x</sub>  $\frac{\mathbf{6EI}_{\mathbf{y}}\mathbf{C}_{\mathbf{y}}}{\mathbf{L}^{\mathbf{2}}\mathbf{y}}\mathbf{C}_{\mathbf{y}}$  $-\frac{6EI_y}{L^2}C_x$ -<u>12EI</u>y k =  $-\underline{\operatorname{GI}}_{\mathrm{L}} \mathrm{C}_{\mathrm{X}}^{2} + \underline{\operatorname{SEI}}_{\mathrm{L}} \mathrm{C}_{\mathrm{Y}}^{2}$  $\frac{\operatorname{GI}_{x}\operatorname{C}_{x}^{2}}{\operatorname{L}}^{+}\frac{\operatorname{4EI}_{y}\operatorname{C}_{y}^{2}}{\operatorname{L}} \quad \left( \frac{\operatorname{GI}_{x}}{\operatorname{L}}^{-}\frac{\operatorname{4EI}_{y}}{\operatorname{L}} \right) \operatorname{C}_{x}\operatorname{C}_{y} \quad - \frac{\operatorname{6EI}_{y}\operatorname{C}_{y}}{\operatorname{L}^{2}}\operatorname{C}_{y}$  $-\left(\frac{\mathbf{GI}_{\mathbf{X}}}{\mathbf{L}} + \frac{\mathbf{2EI}_{\mathbf{y}}}{\mathbf{L}}\right) \mathbf{C}_{\mathbf{X}} \mathbf{C}_{\mathbf{y}}$ <u>6EIy</u>Cy L<sup>2</sup>  $\frac{\mathrm{GI}_{\mathbf{X}}}{\mathrm{L}} \mathbf{C}_{\mathbf{y}}^{2} + \frac{\mathrm{4EI}_{\mathbf{y}}}{\mathrm{T}} \mathbf{C}_{\mathbf{X}}^{2}$  $-\underline{\operatorname{GI}}_{\mathrm{I},\mathrm{X}}\mathrm{C}_{\mathrm{y}}^{2} + \underline{\operatorname{2EI}}_{\mathrm{I},\mathrm{y}}\mathrm{C}_{\mathrm{X}}^{2}$  $-\left(\frac{GI_{x}+2EI_{y}}{L}\right)C_{x}C_{y}$  $\left(\frac{\mathrm{GI}_{\mathbf{x}}}{\mathrm{L}} - \frac{4\mathrm{EI}_{\mathbf{y}}}{\mathrm{L}}\right)^{\mathrm{C}_{\mathbf{x}}\mathrm{C}_{\mathbf{y}}}$ -<u>eeiy</u>cx  $\frac{6EI}{L^2}y^C_x$ -<u>6ELy</u>Cy  $\frac{6 EI_y C_x}{L^2}$ -<u>12EI</u>y -<u>6EIy</u>Cy <u>6EIy</u>C<sub>X</sub>  $\frac{12EI}{L}y$ 



Figure 2.4a: Grid element in member coordinate axes.

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axes

# Figure 2.4b: Grid element in global coordinate axes.

Appendix C.

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The program was used to determine the natural frequencies of the structures shown in Figures 2.5 to 2.8 and the results obtained have been reported in (39). The results of the program were compared with those obtained experimentally and analytically by Bishop and Johnson (20), Rieger and McCallion (42), Cheng (43), and Zeidan (44).

2.4. DISCUSSION OF RESULTS

Bishop tested the frames shown in Fig. 2.5. The theoretical natural frequencies were obtained by the method of receptances. Rieger and McCallion employed a finite differences approach in obtaining the theoretical natural frequencies. The frames tested by them are shown in Fig. 2.6. Cheng determined the theoretical natural frequencies of the frame of Fig. 2.7 by a stiffness approach but employed the lumped mass technique in generating the mass matrix. Zeidan tested a 3-girder and a 4-girder grid as shown in Fig. 2.8.

For the analysis by the finite element method the frames of Fig. 5 were divided into elements as shown in Fig. 2.9. In Fig. 2.9(i) and 2.9(iv) each member of the frame is considered as one element, whilst in Fig. 2.9(ii) and 2.9(v) each member is subdivided into two elements, and in Fig. 2.9(iii) into 3 elements. Fig. 2.9(vi) shows the frame divided in such a way that each element is 2" long. The comparison of results is

illustrated in Table 2.1.

<u>.</u>

Figure 2.10 shows two different subdivisions for the structure of Fig. 2.6. In the first one each member is considered as one element, whilst in the second it is divided into two elements. Since Rieger and McCallion did not quote material density for the frames tested by them, the author has used a value of  $\ell A = .000725$  lbs.  $\sec^2/in^4$  based on a specific weight of 480 lbs./ft<sup>3</sup> for steel. The results for Fig. 2.6(a) to 6(d) and 6(h) are shown in Table 2.2.

Three cases of subdivision as shown in Fig. 2.11 are analysed for the frame of Fig. 2.7 and the results are shown in Table 2.3.

For the 3-girder grid of Fig. 2.8(a) two subdivisions are considered, with each member being taken as one element or as two elements as shown in Fig. 2.12(i) and 2.12(ii) respectively. For the 4-girder grid of Fig. 2.8(b) only one case, that of each member as one element, is considered as is shown in Fig. 2.12(iii). The results are shown in Table 2.4.

From the results illustrated in Tables 2.1 to 2.4 it can be observed that very good results may be obtained by the finite element method. If only the first few frequencies are required a coarse subdivision can yield sufficiently accurate results. Finer subdivision results in a rapid convergence towards the "exact" answer. From Table 2.4 it will be observed

that there are no experimental values for the second harmonic both for the 3-girder and the 4-girder grids. This is due to the fact that Zeidan's experimental work was concerned with those nodes that are symmetric with respect to the central longitudinal axis of the grid. Thus, the second frequency reported by him is actually the third frequency of the complete set.

The program also gives the eigenvectors together with the eigenvalues, from which the modal shapes may be determined. The modal shapes corresponding to Fig. 2.9 (iii) are illustrated in Fig. 2.13.

Due to the coarse idealization of the structure the points of contraflexure cannot be determined unless a finer subdivision is used. Such a fine idealization was not undertaken since the frequencies compared favourably with the experimental values.

#### 2.5. CONCLUSIONS

From the examples presented in this Chapter it may be concluded that a simple beam element may be used to obtain the natural frequencies and modal shapes of structures of non-uniform cross-section and varying properties. The presence of different boundary conditions presents no basic difficulty and may be easily incorporated. Though the effects of rotary inertia and shear

deformations have not been included they can be easily incorporated. For most structures in which the ratio of the length to the depth is large, ignoring the rotary inertia does not appreciably affect the results (40). This was confirmed for a number of cases and the program for portal frames is presented in Appendix C and it includes the effect of rotary inertia.

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Figure 2.7: Cheng's plane truss.

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Figure 2.8: Ziedan's 3-girder and 4 girder grids.



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Figure 2.12: Finite element idealizations of Ziedan's grids.

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# Figure 2.12: Finite element idealization's of Ziedan's grids.







Figure 2.13: Modal shapes of portal frame.



Figure 2.13: Modal shapes of portal frame.

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## SYMMETRICAL FRAME

Bishop and Johnson		Finite Element Solution		
Calculated	Experimental	Case (i)	Case (ii)	Case (iii)
cps.	cps.	cps.	cps.	cps.
36	36	36	36 -	36
145	145	171	143	143
233	233	368	235	233
256	254	4021	256	253
513	517	4318	579	5ï7
628	626	7349	757	634
	•			

### UNSYMMETRICAL FRAME

Calculated	Experimental	Case (iv)	Case (v)	Case (vi)
cps.	cps.	cps.	cps.	cps.
· 38	39	39	39	39
134	135	167	135	134
194	197	378	196	194
346	346 346		351	345
439	436	5151	512	432
606	598	7426	717	585

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TABLE	2	.2	
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No.	of	Bays	Rieger and	McCallion	Finite	Element	Solution
	·		Calc.	Exper.	Case	(a) Ca	se (b)
			cps.	cps.	cps	·.	cps.
	1		152	152.7	152	}	152
			602.8	602.8	714	÷	601
				980	1544	•	986
				1069	6756	5 1	071
	2		142.3	142.3	140	.5	140.1
F			583	583	682		581
			734	736	963		733
•			990	986	1600	1	994
			1072	1067	5181	1	067
	3		138.3	138.6	137		137
			575	574	669	ł	573
			663	663	818	6	660
			812	808.5	1148		810
			994	986	1626		997
•			1072	1064	4527	1	064
	4	-	136	137.3	135		135
			571	571.5	663		569
			626	614.5	754	I	615
			736	739.5	965		734
		•	850	849	1258		850
			995	989	1640	1	999
			1072	1064	3651	10	062
•	8		132.5	132	132		132
	•		565	565	653	1	562
			584	580	682	•	581
			620	619	742	(	517
			673	672	835		569
·			736	742	964		733
			801	792	1118		799
			862	827	1290	1	860
			904	871	1342		889
			998	998	1663	10	002
			1072	1069	2037	10	057

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TABLE 2.3

Cheng	Finit	e Element S	Solution
Calculated	Case (i)	Case (ii)	Case (iii)
rad/sec.	rad/sec.	rad/sec.	rad/sec.
737.5	760.9	743.6	739.7
1116.5	1164.5	1123.5	1120.0
1519.4	1712.1	1573.3	1534.0
	2129.2	1780.0	1758.7
	3616.6	2713.3	2636.6
	7940.6	3017.8	2840.1

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# TABLE 2.4

# 3-Girder Grid

Experimental	Finite Element Solution		
	Case (i)	Case (ii)	
cps.	cps.	cps.	
· 30	29	29	
-	35.5	35	
49	51	50	

## 4-Girder Grid

Experimental	Finite Element Solution
	Case (iii)
cps.	cps.
29.8	26.4
-	31
40.0	40.5

#### CHAPTER III

#### PARAMETRIC INSTABILITIES OF COLUMNS

#### 3.1 Introduction

When a column is subjected to a periodic longitudinal force, or when its base is given a periodic vertical motion the differential equation of motion reduces to a Mathieu equation. In Section 1.1 it was shown that for the case of a simply supported column carrying a periodic axial load ( $V_c + V_t$  cos  $\Omega t$ ), the equation of motion, Equation (1.2), may be reduced to the following form of the Mathieu equation

$$f + \beta^2 (1-2\mu \cos \Omega t) f = 0$$
 (1.10)

To investigate the stability of the motion the solutions of the Mathieu equation have to be studied. Since Equation (1.10) is a second order linear differential equation, it will have two independent non-zero solutions. (A brief account of some of the properties of the Mathieu equation is given in Appendix A.) If all the solutions of Equation (1.10) are bounded for all positive values of t, the corresponding motion is regarded as stable; however, for an unbounded solution, the resulting motion will continue to grow with time and is termed as unstable.

One may seek the periodic solutions of the Mathieu equations in the form of a Fourier series. For a periodic solution with period 2T, where T is defined as  $\frac{2\Lambda}{\Omega}$ , the solution may be obtained in the form

$$f(t) = \sum_{k = 1,3,5,...} (\bar{a}_k \sin \frac{k\Omega t}{2} + \bar{b}_k \cos \frac{k\Omega t}{2}) \quad (3.1)$$

Substituting the series (3.1) into Equation (1.10) and equating the coefficients of  $\sin \frac{k\Omega}{2} t$  and  $\cos \frac{k\Omega}{2} t$ , the following system of linear homogeneous algebraic equations is obtained

$$\begin{pmatrix} 1 + \mu - \frac{\Omega^2}{4\beta_0} 2 \end{pmatrix} \bar{a}_1 - \mu \bar{a}_3 = o$$

$$\begin{pmatrix} 1 - \frac{k^2 \Omega^2}{4\beta_0} 2 \end{pmatrix} \bar{a}_k - \mu (\bar{a}_{k-2} + \bar{a}_{k+2}) = o \quad (k = 3, 5, 7, ...)$$

$$\begin{pmatrix} 1 - \mu - \frac{\Omega^2}{4\beta_0} 2 \end{pmatrix} \bar{b}_1 - \mu \bar{b}_3 = o$$

$$\begin{pmatrix} 1 - k^2 \frac{\Omega^2}{4\beta_0} 2 \end{pmatrix} \bar{b}_k - \mu (\bar{b}_{k-2} + \bar{b}_{k+2}) = o \quad (k = 3, 5, 7, ...)$$

The non-zero values of  $\bar{a}_k$  and  $\bar{b}_k$  require that

For periodic solutions of period T, one may seek the solution in the form

$$f(t) = \overline{b}_{0} + \sum_{k=2,4,6,\ldots} (\overline{a}_{k} \sin \frac{k\Omega t}{2}) + \overline{b}_{k} \cos \frac{k\Omega t}{2}$$
(3.3)

which again, after equating coefficients, results in the following system of equations

$$\begin{pmatrix} 1 - \Omega^2 \\ \overline{\beta_0}^2 \end{pmatrix} \bar{a}_2 - \mu a_4 = o$$

$$\begin{pmatrix} 1 - \frac{k^2 \Omega^2}{4\beta_0^2} \end{pmatrix} \bar{a}_k - \mu (\bar{a}_{k-2} + \bar{a}_{k+2}) = o$$

$$\bar{b}_0 - \mu \bar{b}_2 = o$$

$$\begin{pmatrix} 1 - \Omega^2 \\ \overline{\beta_0}^2 \end{pmatrix} \bar{b}_2 - \mu (2 \bar{b}_0 + \bar{b}_4) = o$$

$$\begin{pmatrix} 1 - \frac{k^2 \Omega^2}{\beta_0^2} \end{pmatrix} \bar{b}_k - \mu (\bar{b}_{k-2} + \bar{b}_{k+2}) = o$$

For non-trivial solutions

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Equations (3.2) and (3.4) are called the equations of boundary frequencies. The boundary frequencies are defined as the frequencies of the fluctuation of the external loading corresponding to the boundaries of the region of instability.

As a first approximation, we may determine the boundaries of the principal region of instability, by retaining the upper diagonal element only, and equating it to zero.

$$1 \pm \mu - \frac{\Omega^2}{4\beta_0^2} = 0$$

which gives

$$\Omega = 2\beta_0 \sqrt{1 \pm \mu}$$
(3.5)

The accuracy of Equation (3.5) may be improved by considering the terms contained in the first two columns and rows of Equation (3.2)

$$\begin{vmatrix} 1 \pm \mu & -\frac{Q^2}{4\beta_0^2} & -\mu \\ -\mu & & 1 -9 \frac{Q^2}{4\beta_0^2} \end{vmatrix} = 0$$

and

which gives

$$\Omega = 2\beta_0 \sqrt{1 \pm \mu + \frac{\mu^2}{8 \pm 9\mu}}$$
(3.6)

The last term under the square root takes into account the correction for the second approximation. This correction increases with  $\mu$  but even for a value of  $\mu = 0.3$ , the error is less than 1%. Hence, for all practical purposes, Equation (3.5) is sufficiently accurate.

Similarly, for the second region of instability, we get the following approximate formulas for the boundary frequencies  $^{(6)}$ .

$$\Omega = \beta_0 \sqrt{1 + \frac{1}{3}\mu^2} \qquad \Omega = \beta_0 \sqrt{1 - 2\mu^2} \qquad (3.7)$$

and for the third regions of instability we have

$$\Omega = \frac{2}{3} \beta_0 \sqrt{1 - \frac{9\mu^2}{8 + 9\mu}}$$
(3.8)

The typical diagram for the regions of instability is shown in Figure 3.1.

#### 3.2 The Proposed Method

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The author has developed a numerical method approach to obtain the regions of dynamic stability <sup>(45)</sup>. A uniform column is idealized by a system of lumped masses and massless springs to represent the stiffness of the column.

It is assumed that the mass and stiffness of each . segment is concentrated at its end. For instance, if the length




of the segment is a, and its mass per unit length is A $\beta$ , then the mass of the segment is replaced by two equal concentrated masses A $\beta$  a/2 at the ends. It is shown that the equations of motion are of the Mathieu type and from their solutions one may establish the regions of dynamic stability. From the equation of motion it is also possible to obtain the natural frequencies and buckling loads; these when compared with known values serve as a measure for the validity of the idealization. The following three cases have been studied.

Case A : Column hinged at both ends and subjected to a periodically fluctuating axial compressive force. (Figure 1.3) Case B : Column fixed at the base and free at the top and subjected to a periodically fluctuating compressive force. (Figure 3.2)

Case C : Column fixed at the base and free at the top and subjected to a periodic support motion (Figure 3.3)

3.2A. : Case A: Simply Supported Column Subjected to

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## Periodically Varying Axial Force

The mathematical development is based on the following assumptions:



compressive force.





support motion.

1. The column consists of links of equal length, a, except the first and last links which are of length a/2. It may be noted that the total mass and stiffness of the first and last links is concentrated at node 1 and n-1 respectively.

2. Vertical accelerations are so small as to be neglected. Therefore, at any instant we have the same vertical force at each end of . link.

The idealization is shown in Figure 3.4. The spring stiffness is designated by K. The forces acting on the ith link are shown in Figure 3.5. Clearly

$$\begin{aligned} x_{i} &= a \left( \frac{1}{2} \cos \theta_{1} + \cos \theta_{2} + \dots + \cos \theta_{i} \right) \\ y_{i} &= a \left( \frac{1}{2} \sin \theta_{1} + \sin \theta_{2} + \dots + \sin \theta_{i} \right) \quad (3.9) \\ H_{i-1} - H_{i} &= M_{i-1} \ddot{y}_{i-1} \end{aligned}$$

where H represents the horizontal shear forces.

If  $\theta$ 's are small, the equation of motion for the i-th member writes as

$$-V_{0}a \frac{\theta}{i} + H_{i-1}a - M_{i-1}a \frac{y}{i-1} = K_{i-1}\left(\theta_{i-1} - \theta_{i}\right) - K_{i}\left(\theta_{i} - \theta_{i+1}\right) \quad (3.10)$$
  
Similarly, for the (i + 1) th member, we get

$$-V_{o}a\theta_{i+1} + H_{i}a-M_{i}a\ddot{y}_{i} = K_{i}\left(\theta_{i}-\theta_{i+1}\right) - K_{i+1}\left(\theta_{i+1}-\theta_{i+2}\right)$$
(3.11)

Subtracting Equation (3.11) from (3.10) and simplifying gives

$$-\mathbf{V}_{o}^{a}\left(\theta_{i},\theta_{i+1}\right) + \mathbf{M}_{i}^{a^{2}}\left(\theta_{1/2}^{i} + \theta_{2}^{i} + \theta_{3}^{i} + \ldots + \theta_{i}^{i}\right) = \mathbf{K}_{i-1}\left(\theta_{i-1},\theta_{i}^{i}\right)$$

$$-2K_{i}\left(\theta_{i} - \theta_{i+1}\right) + K_{i+1}\left(\theta_{i+1} - \theta_{i+2}\right)$$
(3.12)  
If all masses and springs are equal, Equation (3.12) reduces to

$$-v_{oa}\left(\theta_{i} - \theta_{i+1}\right) + Ma^{2}\left(\frac{\ddot{\theta}_{1}}{2} + \ddot{\theta}_{2} + \dots + \ddot{\theta}_{i}\right)$$

$$= K \left( \theta_{i-1} - 3 \theta_i + 3 \theta_{i+1} - \theta_{i+2} \right) \quad (i=2,3, \ldots n-2), \quad (3.13)$$
  
where n is the number of members.

Similarly, the first and second members yield

$$-V_{0}a(\theta_{1}-\theta_{2}) + M_{1}\frac{a^{2}}{2}\ddot{\theta}_{1} = K(-3\theta_{1} + 4\theta_{2} - \theta_{3})$$
(3.13a)  
and the (n - 1) and n-th members yield

 $-v_0a(\theta_{n-1} - \theta_n) + M_{n-1}a^2(\frac{\ddot{\theta}_1}{2} + \ddot{\theta}_2 + \ldots + \ddot{\theta}_{n-1}) = K(\theta_{n-2} - 4\theta_{n-1} + 3\theta_n)(3.13b)$ Since there are n coordinates and since there is an equation of constraint, arising from the geometry,

$$\frac{a}{2} \sin \theta + a \sin \theta + \dots + \frac{a}{2} \sin \theta = 0$$
(3.14)

there are only (n - 1) degrees of freedom and thus (n - 1) equations of type (3.12). It may be noted that the above equations may be also obtained by the Lagrangian formulation.









# In matrix form the equations become

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θ<sub>2</sub> θ<sub>3</sub> 0.5  $\bar{\theta}_1\\ \bar{\theta}_2$ 0 θ2-1 .0 -voª  $+M_a^2$ • 0 0 1.0 1.0 1.0 0 1.0 1.0 0.5 1.0 0.5 1.0 0.5 1.0  $\theta_{n-1}$ . 1.0 .

	<b>~</b>									-				
	-3	4	-1	0	ο	•	• •	• •	• .•	• 0		$\left( \theta_{1} \right)$	)	
	1	-3	3	-1	0					0	ĺ	$\theta_2$		
	0	1	-3	3	-1			۰,		0		₿ <sub>3</sub>		
	•									•		•	}	
	•					•				•		•		12
= K	•									•	٦	•	7	(3
	•									•		•		
	. <b>o</b>	Ο.	0	0	0	•	•	1	-33	-i		$\theta_{n-3}$		
	1	2	2	2	2	•	•	2	3-1	5		$\theta_{n-2}$		
	3	-6	-6	-6	-6	•	•	-6	-6 -5	-10		θ n-1;		

(3.15)

In the matrix which appears above on the right hand side of Equation (3.15) the last two rows have a non-typical form which arises from substitution for  $\theta_n$  in terms of  $\theta_1$ ,  $\theta_2$ , ...  $\theta_{n-1}$  utilizing Equation (3.14).

3.

By applying Galerkin's Method as applicable to discrete systems (see Appendix B) equations (3.15) may be reduced to a single equation for each mode of the type

 $\lambda_{10}^{Va\theta} + \lambda_{2}^{Ma^{2}\theta} + \lambda_{3}^{K\theta} = 0 \qquad (3.16)$ 

where the  $\lambda$  's are the coefficients associated with  $\mathtt{V}_{o},$  m and k.

From Appendix B and in particular from Equation (B.5) it may be seen that the application of the Galerkin Method requires the knowledge of the modal shape vector,  $\hat{\theta}$ . To determine  $\hat{\theta}$  one sets  $V_0 = 0$ , then Equations (3.15) represent the free vibration case. If  $\hat{\theta} = q e^{i wt}$ , where q is a column matrix of the amplitudes of the displacement  $\hat{\theta}$ , w is the natural frequency, there results

$$(\underline{S} - w^2 \underline{M}) = 0 \tag{3.17}$$

Equation (3.17) is in the standard form for eigenvalue problems, and its solution yields the required modal vector .

The natural frequency may now be determined from Equation (3.16) as

$$w^2 = \frac{\lambda_3}{\lambda_2} = \frac{K}{Ma^2}$$

(3.18)



-, n= ∞.

The Euler buckling load may also be obtained from Equation (3.16) by putting  $\ddot{\theta} = 0$ ,

$${}^{P}e = -\frac{\lambda_{3}}{\lambda_{1}} \quad \frac{\kappa}{a} \tag{3.19}$$

Substituting Equations (3.18) and (3.19) in Equation (3.16) we get

$$\ddot{\theta} + w^{2} \left(1 - v_{0}\right) \theta = 0 \qquad (3.20)$$
  
Since  $v_{0} = v_{c} + v_{t} \cos \Omega t$ , Equation (3.20) gives  
 $\ddot{\theta} + \beta_{0}^{2} (1 - 2\mu \cos \Omega t) \theta = 0 \qquad (3.21)$ 

Equation (3.21) is the differential equation of motion for the particular mode and is of the Mathieu type.

A uniform column of length L was analyzed by this procedure and the results obtained are shown in Table 3.1. From this it may be seen that as the column is divided into more parts, the values for the natural frequency and the Euler buckling load converge to their true values. As regards the mapping of the principal zone of instability each subdivision of the column into elements yields its associated Mathieu equation. Figure 3.6 illustrates the rapid convergence to the true configuration for the principal region of instability. Even for the case of n = 6, though the results obtained for the natural frequency and the Euler buckling load are not very

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accurate, the error for the region of dynamic instability is not very great.

3.2B: <u>Case B</u>: <u>Cantilever Column Subjected to Periodic Axial Force</u>.

The mathematical formulation for Case B is based on the following assumptions:

- On account of the base fixity the first link does not rotate and remains vertical.
- Vertical accelerations may be neglected as for Case A.
   Consider the column idealization of figure 3.7.

Clearly

$$\begin{aligned} y_{i} &= a_{1} \sin \theta_{1} + a_{2} \sin \theta_{2} + \dots + a_{i} \sin \theta_{i} \quad (i = 1, 2, \dots, n) \\ T_{e} &= \frac{1}{2} M_{1} \dot{y}_{1}^{2} + \frac{1}{2} M_{2} \dot{y}_{2}^{2} + \dots + \frac{1}{2} M_{n} \dot{y}_{n}^{2} \\ &= \frac{1}{2} M_{1} (a_{1} \dot{\theta}_{1})^{2} + \frac{1}{2} M_{2} (a_{1} \dot{\theta}_{1} + a_{2} \dot{\theta}_{2})^{2} + \dots \\ &+ \frac{1}{2} M_{n} (a_{1} \dot{\theta}_{1} + a_{2} \dot{\theta}_{2} + \dots + a_{n} \dot{\theta}_{n})^{2} \end{aligned} (3.23) \\ U_{e} &= \frac{1}{2} K_{1} \theta_{1}^{2} + \frac{1}{2} K_{2} (\theta_{1} - \theta_{2})^{2} + \dots + \frac{1}{2} K_{n} (\theta_{n-1} - \theta_{n})^{2} + \int V_{0} dh \\ &\qquad (3.24) \end{aligned}$$

where Te is the kinetic energy of the system



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Figure 3.7: Column idealization for cases B and C ( $\theta_i$  assumed to be zero).

$$\begin{array}{l} \frac{2^{\mathrm{T}}\mathbf{e}}{\partial \theta_{1}} = \mathbf{o} \quad (\mathbf{i} = 1, 2, \dots, \mathbf{n}) \\ \end{array} (3.23b) \\ \frac{2^{\mathrm{T}}\mathbf{e}}{\partial \theta_{1}} = -\mathbf{K}_{1}\theta_{1-1} + (\mathbf{K}_{1} + \mathbf{K}_{1+1}) \quad \theta_{1} \cdot \mathbf{K}_{1} \quad \theta_{1+1} - \mathbf{v}_{0}\mathbf{a}_{1} \quad (\mathbf{i} = 2, 3, \dots, \mathbf{n}-1) \\ \qquad (3.24a) \\ \frac{2^{\mathrm{T}}\mathbf{e}}{2\theta_{1}} = (\mathbf{K}_{1} + \mathbf{K}_{2}) \quad \theta_{1} - \mathbf{K}_{2} \quad \theta_{2} - \mathbf{v}_{0}\mathbf{a}_{1} \quad (3.24b) \\ \frac{2^{\mathrm{T}}\mathbf{e}}{2\theta_{1}} = (\mathbf{K}_{1} + \mathbf{K}_{2}) \quad \theta_{1} - \mathbf{K}_{2} \quad \theta_{2} - \mathbf{v}_{0}\mathbf{a}_{1} \quad (3.24b) \\ \frac{2^{\mathrm{T}}\mathbf{e}}{2\theta_{1}} = (\mathbf{K}_{1} - \mathbf{K}_{2} \quad \theta_{1} - \mathbf{K}_{2} \quad \theta_{2} - \mathbf{v}_{0}\mathbf{a}_{1} \quad (3.24c) \\ \text{Substituting in Lagrange's equation} \\ \frac{d}{dt} \left( \frac{\partial \mathbf{T}_{\mathbf{e}}}{\partial \theta_{1}} \right) = \frac{2^{\mathrm{T}}\mathbf{e}}{\partial \theta_{1}} + \frac{2^{\mathrm{T}}\mathbf{e}}{\partial \theta_{1}} = \mathbf{o}, \quad (\mathbf{i} = 1, 2, \dots, \mathbf{n}), \text{ we obtain} \\ \frac{a_{1}^{2}\mathbf{M}_{1}^{*} \quad a_{1}\mathbf{a}_{2}\mathbf{M}_{2}^{*} \quad a_{2}\mathbf{a}_{3}\mathbf{M}_{3}^{*} \quad \dots \quad a_{2}\mathbf{a}_{n}\mathbf{M}_{n}^{*} \\ a_{2}\mathbf{a}_{1}\mathbf{M}_{2}^{*} \quad a_{2}^{2} \quad \mathbf{M}_{2}^{*} \quad a_{2}\mathbf{a}_{3}\mathbf{M}_{3}^{*} \quad \dots \quad a_{2}\mathbf{a}_{n}\mathbf{M}_{n}^{*} \\ \frac{a_{1}\mathbf{a}_{1}\mathbf{a}_{1}\mathbf{a}_{2}\mathbf{M}_{2}^{*} \quad a_{2}\mathbf{a}_{3}\mathbf{M}_{3}^{*} \quad \dots \quad a_{2}\mathbf{a}_{n}\mathbf{M}_{n}^{*} \\ \frac{a_{1}\mathbf{a}_{1}\mathbf{a}_{1}\mathbf{a}_{2}\mathbf{M}_{n}^{*} \quad a_{n}\mathbf{a}_{3}\mathbf{M}_{n}^{*} \quad \dots \quad a_{n}^{2}\mathbf{n}_{n}^{*} \\ \frac{a_{n}\mathbf{a}_{n}\mathbf{M}_{n}^{*} \quad a_{n}\mathbf{a}_{2}\mathbf{M}_{n}^{*} \quad a_{n}\mathbf{a}_{3}\mathbf{M}_{n}^{*} \quad \dots \quad a_{n}^{2}\mathbf{M}_{n}^{*} \\ \frac{a_{n}\mathbf{a}_{n}\mathbf{M}_{n}^{*} \quad a_{n}\mathbf{a}_{2}\mathbf{M}_{n}^{*} \quad a_{n}\mathbf{a}_{3}\mathbf{M}_{n}^{*} \quad \dots \quad a_{n}^{2}\mathbf{M}_{n}^{*} \\ \frac{a_{n}\mathbf{a}_{n}\mathbf{n}_{n}\mathbf{n}_{n}\mathbf{n}_{n}\mathbf{n}_{n}\mathbf{n}_{n}\mathbf{n}_{n}\mathbf{n}_{n}\mathbf{n}_{n}\mathbf{n}_{n}\mathbf{n}_{n}^{*} \\ \frac{a_{n}\mathbf{a}_{n}\mathbf{n}_{n}\mathbf{n}_{n}\mathbf{n}_{n}\mathbf{n}_{n}\mathbf{n}_{n}\mathbf{n}_{n}\mathbf{n}_{n}\mathbf{n}_{n}\mathbf{n}_{n}\mathbf{n}_{n}\mathbf{n}_{n}\mathbf{n}_{n}\mathbf{n}_{n}\mathbf{n}_{n}\mathbf{n}_{n}^{*} \\ \frac{a_{1}\mathbf{a}_{1}\mathbf{a}_{1}\mathbf{a}_{1}\mathbf{a}_{1}\mathbf{a}_{1}\mathbf{a}_{1}\mathbf{a}_{2}\mathbf{a}_{2}\mathbf{a}_{1}\mathbf{a}_{1}\mathbf{a}_{n}\mathbf{n}_$$

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where  $M_{i}^{*} = M_{i} + M_{i+1} + \dots + M_{n}$  (i = 1,2, ..., n) In abbreviated form we may write

$$\underbrace{M}_{0} \underbrace{\hat{\theta}}_{0} - V_{0} \underbrace{a}_{0} \underbrace{\theta}_{1} + \underbrace{S}_{0} \underbrace{\theta}_{1} = 0 \qquad (3.25a)$$

The results for the idealization of Figure 3.7 are shown in Table 2. It may be noted that if one were to follow the procedure for Case A for the determination of the Euler buckling load the results obtained would not be very good. This is because the modal shapes for buckling under a tip load and for free vibration of a cantilever column are significantly different. The error in the buckling loads determined on the basis of the free vibration modal shapes is quite appreciable even after applying the Galerkin Method.

A superior method is due to the fact that the buckling loads are the latent roots of the matrix <u>S</u> and this procedure has been used in obtaining Table 3.2.This is a more general method; however, there is no advantage in using it for Cases A and C because the free vibration modal shape vector itself yields good results.

Figure 3.8 shows the region of principal instability for a uniform column of length L free at the top and fixed at the base. It may again be observed that as the number of segments increases the curves converge rapidly to the true one.

—, n=6 ; Î 0.35 µ , n=5;---Figure 3.8: Instability diagram for cantilever column (case B).... 0.30 0.25 0.20 -, n=∞. 0.15 n=11; --0.10 0.05 1**.**2 1.0 6.0 0.8 1.1 0.7 C1 20

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## 3.2C. : Case C: Cantilever Column Subjected to Periodically

## Varying Vertical Support Motion.

In addition to the assumption of base fixity (as in Case B) it is now assumed that the vertical acceleration throughout the length of the column remains constant. A step-wise variation of the axial force in the column, resulting from this latter assumption, is shown in Figure 3.9. The mathematical 'development is based on the idealization shown in Figure 3.7.

Consider the top link of the column shown in Figure 3.7. The forces acting on it are shown in Figure 3.10. Taking moments about mass  $M_{n-1}$  gives  $M_{n} \overset{\ddot{x}}{n} a_{n} \theta_{n} - M_{n} \overset{\ddot{y}}{n} a_{n} = K_{n} \left( \theta_{n} - \theta_{n-1} \right)$ (3.26)The vertical equilibrium of free-body of mass  $M_n$  gives  $M_n \ddot{x}_n = V_n$ (3.27) From (3.26) and (3.27) we get  $\mathbf{v}_{n} \mathbf{a}_{n} \theta_{n} - \mathbf{M}_{n} \mathbf{\ddot{y}}_{n} \mathbf{a}_{n} = \mathbf{K}_{n} \left( \theta_{n} - \theta_{n-1} \right)$ (3.28) Similarly, for the next link, taking moments about mass  $M_{n-2}$ (Figure 3.11) gives  $\frac{M_{n}\ddot{x}}{n} \left( a_{n} \theta_{n} + a_{n-1} \theta_{n-1} \right) + \frac{M_{n-1}}{n-1} \ddot{x}_{n-1} \left( a_{n-1} \theta_{n-1} \right) - \frac{M_{n}\ddot{y}}{n} \left( a_{n} + a_{n-1} \right)$  $-M_{n-1}\ddot{y}_{n-1} = K_{n-1} + K_{n-1} \left(\theta_{n-2} - \theta_{n-1}\right) = 0$ (3.29)From the vertical equilibrium of the free-body of mass  $M_{n-1}$  we get

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Figure 3.10: Free body diagram of n-th link.

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Figure 3.11: Free body diagram of n-th and (n-1)th links.

$$V_{n-1} - V_n = M$$
  $\ddot{x}_{n-1}$  (3.30)  
Substituting Equations (3.27) and (3.30) in Equation (3.29)

and subtracting Equation (3.26) from it results in

$$v_n(y_n-y_{n-1}) - M_n y_n a_n = r_n y_n - (r_n + s_n) y_{n-1} + s_n y_{n-2}$$
 (3.26a)

and

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$$v_{n-1}(y_{n-1} - y_{n-2}) - (M_n \ddot{y}_n + M_{n-1} \ddot{y}_{n-1}) a_{n-1} = s_{n-1} y_{n-3} - (r_{n-1} + s_{n-1} + s_n) y_{n-2}$$
  
+  $(r_{n-1} + r_n + s_n) y_{n-1} - r_n y_{n-1}$  (3.31a)

where

$$r_{n} = \frac{K_{n}}{\frac{a_{n}}{a_{n}}}$$

$$S_{n} = \frac{K_{n}}{\frac{a_{n-1}}{a_{n-1}}}$$

$$r_{n-1} = \frac{K_{n-1}}{\frac{a_{n-1}}{a_{n-1}}}$$

$$S_{n-1} = \frac{K_{n-1}}{a_{n-2}}, \text{ etc.}$$

In this way, the equations of motion in matrix form are written as



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Since a step-wise variation has been assumed for the axial forces, they may be expressed in terms of the base axial force, V. As before, applying the Galerkin Method, Equations (3.32) may be reduced to a single equation

$${}^{\eta}1^{\nabla}y + {}^{\eta}2 \stackrel{Ma}{}^{}^{}_{y} + {}^{\eta}3 \stackrel{K}{=} y = o$$
 (3.33)

where l's are the coefficients associated with V , m, and K.

By putting V = o, the natural frequency is obtained as

$$v^2 = \frac{\pi}{n_2} \frac{K}{Ma^2}$$
 (3.34)

and if y = 0, we get

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$$V^{\star} = -\frac{\eta_3}{\eta_7} \frac{K}{a}$$
(3.35)

where V\* denotes the inertia force which if applied at the base would cause the column to buckle (cf. buckling of the column under its own weight). If the base acceleration varies, the axial force in the column will also vary. If the base acceleration varies in a periodic manner the resulting equations of motion (Equation (3.33) will be of the Mathieu type).

Substituting Equations (3.34) and (3.35) in Equation (3.33) yields

$$\ddot{y} + w^2 \left(1 - \frac{v}{v^*}\right) y = 0$$
 (3.36)

Now  $V = \ell LA \xi$ 

where

A = cross-sectional area

 $\xi$  = acceleration

Let the support motion be  $u_s = C \cos \Omega t$  where C is the amplitude of the base motion, then  $\ddot{u}_s = -\Omega^2 C \cos \Omega t$ , whence substituting in Equation (3.36) we get

$$\ddot{y} + w^{2} \left( 1 + \frac{\rho_{LA}}{V^{*}} \Omega^{2} c \cos \Omega t \right) y = o$$

or

Let

$$\ddot{y} + w^{2} \left( 1 + b\Omega^{2}C \cos \Omega t \right) y = 0 \qquad (3.37)$$

$$b = \frac{\rho_{LA}}{v_{\star}}$$
(3.38)

$$\epsilon = b\Omega^2 c \tag{3.39}$$

Then we get  $\ddot{y} + w^2$  (1 +  $\varepsilon \cos \Omega$  t) y = 0 (3.40) Equation (3.40) is the familiar Mathieu equation.

The principal region of instability corresponding to the first mode for a cantilever was determined by the proposed method and is shown by full lines in Figure 3.12. The column was divided into 11 parts as shown in Figure 3.13.

An experimental investigation was also carried out to determine the principal region of instability of such a cantilever column. An aluminum specimen 23.5" long, 0.75" wide and 0.040" thick was used. The circled points in Figure 3.12 are experimental observations and it may be seen that these results are in good agreement with the theory. A comprehensive account of the experimental set-up and results is given in Chapter IV.





The principal region of instability corresponding to the second mode for the cantilever of Figure 3.13 is shown in Figure 3.14. As before the full lines refer to the proposed method whilst the experimental points are represented by circles. It may be observed that the agreement between theoretical and experimental results is good.

### 3.3 CONCLUSIONS

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The analysis presented shows that the idealization of columns by massless springs and lumped masses may be used for the construction of the regions of dynamic stability. The equation of motion for each mode may be transformed into its associated Mathieu equation whose solution yields the stability zones. It is shown that as the column is divided into more parts, the region of instability converges to the true one. The method is general and it may be applied to columns with different end conditions. A non-uniform column may be treated easily by an appropriate subdivision.

The above analysis was carried out by computer programs. The programs for Cases A and C are presented in Appendix D. The program for Case B is similar with only slight modifications. For a particular subdivision the values of the natural frequencies and the buckling load are given for Cases A and B. For Case C, the program gives the value of the natural frequency and the

inertia force,  $V_{*}^{*}$  which if applied at the base would cause the column to buckle. From these values the stability regions may be obtained as discussed above.

1)

.No. of Parts	3		4		6		10		15		EXACT	
Mode	Nat <sup>1</sup> Freq	Buck <sup>2</sup> Load	Nat Freq	Buck Load	Nat Freq	Buck Load	Nat Freq	Buck Load	Nat Freq	Buck Load	Nat Freq	Buck Load
1	64.0	8.0	81.0	9.0	91.19	9.55	95.48	9.77	97.28	9,89	97.41	9.87
2	256.0	16:0	729.0	27.0	1193.65	34.55	1436.47	37.90	1507.23	38.83	1558.56	39.48
3	• .		1296.0	36.0	4283.81	65.45	. 6561.18	81.00	7313.67	85.52	7890.21	88.83
4					8181.39	90.45	17920.85	133.87	21781.91	147.58	24936.96	157.91
5					10000.0	100.0	36149 <b>.</b> 39	199.13	49242.28	221.90	60881.25	246.74

1 Nat Freq

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- = Natural Frequency (Coefficient  $x \sqrt{\frac{ET}{m_{L}}}$ )
- = Buckling Load (Coefficient x  $\frac{EI}{L^2}$ ) Buck Load

TABLE 3.2

No. of Parts	3		4		· 6		11		16		EXACT	
Mode	Nat Freq	Buck Load	Nat Freq	Buck Load	Nat Freq	Buck Load	Nat Freq	Buck Load	Nat Freq	Buck Load	Nat Freq	Buck Load
. 1	3.455	2.343	3.479	2.411	3.499	2.447	3.511	2.462	3.514	2.465	3.516	2.467
2	18.39	13.657	19.47	18.00	20.86	20.61	21.68	21.80	21.87	. 22.02	22.03	22.20
3	•		49.70	33.59 <sup>·</sup>	52.57	50.00	59.03	58.58	60.44	60.29	61.70	- 61.68
4					84.91	79.39	110.70	109.20	116.09	115.58	120.64	120.90
5					138.42	97.55	172.23	168.71	186.77	185.50	199.81	199.83

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#### CHAPTER IV

#### EXPERIMENTAL INVESTIGATION OF PARAMETRIC

#### INSTABILITY OF COLUMNS

### 4.1 Introduction

For the most part previous experimental investigations have been directed towards studying the parametric instability of columns subjected to periodically varying axial forces. The author performed tests on a cantilever column to determine experimentally the regions of dynamic instability when its base is subjected to periodically varying vertical motion. The test specimen used was an aluminum alloy, 24S-T3, 3/4" wide and 0.040" thick. The specimen was clamped by means of screws and nuts as shown in Figure 4.1. The length of the column from its free tip to the centre of the 'clamps' was 23.5". Since an aluminum alloy was used the "deformations" at instability were fairly large and could be observed visually as evident from Figures 4.3 to 4.5. As the investigation was of a qualitative nature, i.e. it was intended only to establish whether the motion was stable or unstable, no elaborate instrumentation was The motion was termed as unstable when the column necessary. departed from its initial configuration. This was quite easy



Figure 4.1: Cantilever column mounted on shaker

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Figure 4.1: Cantilever column mounted on shaker



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to detect.

To measure the amplitude of oscillation of the base a Sanborn displacement transducer (D.C. excited) was used. A schematic diagram of the experimental set-up is shown in Figure 4.2.

### 4.2 Equipment

The equipment used in the experiments is described below:-

## 1. Function Generator

The function generator used was the Hewlett-Packard Model 3300A which is capable of generating three types of waveforms, namely sine, square and triangular at frequencies ranging from 0.01 Hz to 100 kHz. For frequencies below 10 kHz the sine wave distortion is less than 1%.

2. Power Amplifiers

A Ling Model 300-2 power amplifier was used to drive the shaker manufactured by the same firm. The power amplifier was matched to the shaker used.

#### 3. Shaker

A Goodman's V50 shaker manufactured by Pye-Ling was employed. The output force is generated when a current flows through a moving coil placed in a magnetic field produced by a permanent magnet. The coil is driven by the amplifiers and the motion of the coil is determined by the frequency and mode of the generator. The output force is imparted to the structure by means of a flexible diaphragm which moves with a maximum displacement of 0.7 in. between stops. The shaker can generate a maximum force of 48 lbs. A major drawback of this type of shaker is that the amplitude is inversely proportional to the applied frequency and hence at high frequencies the amplitude is very small.

### 4. <u>Amplitude Measurement Device</u>

A Sanborn displacement transducer, denomination 7DCDT-1000, was used to measure the amplitude of the movement of the base of the cantilever column. The moving core was connected to the base. A 6 volts D.C. power supply was used to excite the transducer. The device is capable of measuring displacements of up to  $\pm 1$  inch with an output of 4.8 volts for full scale displacement. Linearity of the output is guaranteed to 0.5% of full scale under the conditions used in the test.

## 5. <u>Recording Unit</u>

The output from the displacement transducer was recorded on photographic paper by an ultra-violet recorder, type 1050 manufactured by New Electronics Products Ltd., London. This recorder

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is of a galvanometric type; therefore, the input impedance is low (typically around 300 ohms). To retain the linearity of the transducer output with displacement an amplifier, Budd-Brown type 1631, was used so as to provide the required high input impedance, i.e. to match the galvo-impedance to the transducer.

#### 6. Strobotac

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A Type 1538A strobotac manufactured by the General Radio Company was used. The Strobotac consists of a power supply, an oscillator for controlling the rate at which the lamp is flashed and a Strobotac or flashing lamp. The frequency of the oscillator and hence the flashing speed of the lamp can be adjusted to any desired value between 100 c.p.m. to 400,000 c.p.m. The frequency controller is graduated directly in c.p.m. and was used to measure the frequency of the various instability modes.

#### 4.3 Test Procedure

A sinusoidal waveform at the required frequency was generated by the function generator. This signal was amplified by the power amplifier and fed into the shaker thereby imparting a vertical sinusoidal motion to the base of the cantilever column. The base was directly mounted on to the shaker spindle.

The determination of the regions of dynamic instability

entailed the measurement of the frequency and the amplitude of the base motion. For each frequency increment the amplitude was increased slowly until the column became unstable. If no instability occurred at that frequency, the frequency was increased and the whole procedure repeated. When instability was encountered at a certain frequency the determination of the 'critical' amplitude which caused instability was repeated at least five times to get an average reading. It was observed that the variation from the average was small. The record of the output from the displacement transducer was used to obtain the frequency and amplitude of the motion. The frequency of the instability mode was measured by the strobotac.

The modulus of elasticity was obtained experimentally from four tension test specimens made from the same strip as the column. The average value obtained was  $E = 9.82 \times 10^6$  psi.

The sample was weighed and its density was found to be .00025273 lbs-sec $^2/in^4$ .

4.4 Test Results

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The average values of the amplitude of the support motion corresponding to the first mode are given in Table 4.1, and those corresponding to the second mode are given in Table 4.2.

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Figures 4.3, 4.4, 4.5 respectively show the first mode, second mode and the third mode instabilities of the column corresponding to the first region of instability.

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The amplitude of the base motion corresponding to the third mode was very small and could not be measured. Hence no readings are reported.



Figure 4.3: First Mode Instability

Figure 4.4: Second Mode Instability





Figure 4.5: Third mode instability

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Figure 4.5: Third mode instability

## TABLE 4.1

AVERAGE READINGS OF THE AMPLITUDE OF THE SUPPORT MOTION CORRESPONDING TO THE FIRST MODE

•	
FREQUENCY (c.p.s.)	AMPLITUDE (inches)
4.52	0.2786
4.54	0.2053
4.56	0.1542
4.58	0.1120
4.60	0.0794
4.62	0.0538
4.64	0.0740
4.66	0.1538
4.68	0.1955
4.70	0.2411
4.71	0.2728

## TABLE 4.2

AVERAGE VALUES OF THE AMPLITUDE OF THE SUPPORT MOTION CORRESPONDING TO THE SECOND MODE

FREQUENCY (c.p.s.)	AMPLITUDE (inches)
29.4	0.08860
29.6	0.07735
29.8	0.06327
30.0	0.03836
30.2	0.01240
30.4	0.01072
30.6	0.02001
30.8	0.03600
31.0	0.05170
31.2	0.07541
31.4	0.08326

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#### CHAPTER V

# EXPERIMENTAL INVESTIGATION OF PARAMETRIC INSTABILITY OF PORTAL FRAMES

#### 5.1 Introduction

An experimental investigation of the parametric instability of portal frames was also undertaken. The same criterion of instability as for the columns was adopted in this case. This necessitated that the structure be such that its instability modes could be observed visually. To achieve this, different types of frames of various materials were constructed and tested. As remarked earlier the characteristics of the shaker are such that at high frequencies the available output amplitude is very small. Therefore, the frame had to be designed to permit the possibility of occurrence of as many of the modes as possible. The finite element method was used in selecting an appropriate shape and size. A six storey, single bay portal frame was considered to be a suitable shape.

The first frame was made from spring steel and was of the type shown in Figure 5.1. It was observed that this frame was very stiff and only the first mode could be observed visually.





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The second frame, similar to the previous one was made from aluminum, denomination 24S-TO. This frame was found to be too soft and it suffered inelastic changes of configuration when disturbed violently. Consequently, the third frame was made from aluminum of a slightly stiffer variety, denomination 24S-T3. This frame was used for the determination of the regions of dynamic instability. From the tension test on six specimens the modulus of elasticity was found to be 9.5 x  $10^6$  p.s.i.

All the above frames were constructed from strips 3/4" wide and 0.040" thick. The same section was used for the columns and beams. The beams were spaced at 6" centre to centre and were joined by screws and nuts to the columns by 1/2" x 1/2" x 1/16" aluminum brackets as shown in Figure 5.1. International locking washers were used to ensure that the nuts did not become loose during vibration and were found to perform satisfactorily. The frame was clamped to an aluminum plate 5-1/4" x 3" x 3/8" which was directly mounted on the shaker.

The testing procedure adopted was the same as for the cantilever column. For a particular frequency the amplitude was increased till the frame became unstable; the corresponding frequency and the amplitude were measured by means of the Sanborn displacement transducer. The experimental set-up is shown in Figure 5.2.



Figure 5.2(a): Experimental set-up



Figure 5.2(a): Experimental set-up





Figure 5.2(b): Experimental set-up

The first four modes were observed visually and are shown in Figures 5.3. The above photographs are for the natural modes corresponding to the principal region of instability. The instabilities corresponding to the first secondary region (i.e. the second region of instability) were also detected visually for the above four natural modes, but the amplitude of the instabilities was smaller, as was to be expected. The frequency of the instability mode was measured by the strobotac. A movie of the development of the instability modes was also taken; for which the author is greatly indebted to Professor B. Gersovitz and Mr. W. Hillgartner for their assistance and advice.

The amplitudes could only be measured for the first three modes. For the fourth mode the amplitudes were so small that they could not be detected by the deflection transducer. The experimental readings are given in Tables 5.1 to 5.3 and are plotted in Figures 5.4, 5.5 and 5.6.

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The natural frequencies for the first four modes in c.p.s. as estimated by the finite element method and as obtained experimentally were as follows:

Mode	1	2	·3	4
Finite Element	3.83	12.75	22.77	34.33
Experimental	3.80	13.7	23.5	36.0











Figure 5.3: Instability Modes of portal frame

















d) Fourth Mode







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## TABLE 5.1

AVERAGE VALUES OF THE AMPLITUDE OF THE SUPPORT MOTION

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FREQUENCY	AMPLITUDE
(c.p.s.)	(inches)
7.30	0.1499
7.35	0.1021
7.40	0.0629
7.45	0.0473
7,50	0.0348
7.55	0.0289
7.60	0.0261
7.65	0.0355
7.70	0.0622
7.75	0.0835
7.80	0.1028
7.85	0.1142
7.90	0.1254
7.95	0.1391

## TABLE 5.2

# AVERAGE VALUES OF THE AMPLITUDE OF THE SUPPORT MOTION

CORRESPONDING TO THE SECOND MODE

FREQUENCY (c.p.s.)	AMPLITUDE (inches)	
27.0	0.0701	
27.2	0.0365	
27.4	0.0102	
27.6	0.0113	
27.8	0.0286	
28.0	0.0610	

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### TABLE 5.3

# AVERAGE VALUES OF THE AMPLITUDE OF THE SUPPORT MOTION

## CORRESPONDING TO THE THIRD MODE

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FREQUENCY	AMPLITUDE		
(c.p.s.)	(inches)		
46.0	0.0239		
46.2	0.0185		
46.4	0.0143		
46.6	0.011		
46.8	0.0082		
47.0	0.0058		
47.2	0.0071		
47.4	0.0126		
47.6	0.0170		
47.8	0.0221		
48.0	0.0274		

#### CHAPTER VI

#### PARAMETRIC INSTABILITY OF PORTAL FRAMES

### 6.1 Introduction

The experimental investigation of the parametric instability of portal frames showed that the relationship between the frequency of the base motion and the natural frequencies of the frame is similar to that for the case of columns. When  $\ell \rightarrow 0$ for instability to occur in the principal region the frequency of the base motion must be twice that of the natural frequency. Similarly, for the instability corresponding to the second region the frequency must be equal to the natural frequency of the structure. In other words, the following equation of boundary frequencies still holds good

$$\theta = \frac{2\Omega}{k}$$

To determine the equation of the principal region of instability a continuum approach was attempted and is discussed in detail in the next section.

### 6.2 The Continuum Approach

Consider the frame shown in Figure 6.1. Let the axial changes of length of the column be ignored and let the

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Figure 6.1: Multi-storey portal frame.





actual horizontal beams be replaced by a medium of bending stiffness  $\frac{\text{NEI}_{T}}{H}$  per unit length. Consider a slice of thickness dx of the medium at height x above the base. Let end moment per unit height be called m. From Figure 6.2 and using the moment area method we have:

$$\frac{1}{2} (mdx) \frac{b}{2} \frac{2}{3} \frac{b}{dx} = \frac{NEI_{T}}{H} \frac{dx}{dx} \frac{b}{dx} \frac{dy}{dx}$$
so m = 6  $\frac{NEI_{T}}{Hb} \frac{dy}{dx}$ 
(6.1)

Let  $q^*(x)$  be a static external force acting per unit height on the left hand column. Consider the free-body of a differential element of the column as shown in Figure 6.3.

Horizontal equilibrium gives 
$$\frac{dV}{dx} = -\frac{q^*}{2}$$
 (6.2)

Moment equilibrium gives V dx = dM + mdx

or 
$$V = \frac{dM}{dx} + m$$
 (6.3)

Differentiating Equation (6.3) w.r.t. x and eliminating V gives

$$\frac{d^2 M}{dx^2} + \frac{dm}{dx} = -\frac{q*}{2}$$

Now 
$$M = -EI \frac{d^2 y}{dx^2}$$
,  $m = 6 \frac{NEI_T}{Hb} \frac{dy}{dx}$   
so  $-EI \frac{d^4 y}{dx^4} + 6 \frac{NEI_T}{Hb} \frac{d^2 y}{dx^2} = -\frac{q^*}{2}(x)$  (6.4)

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(6.4a)

(6.4b)

or 
$$\frac{d^4y}{dx^4} - \frac{6NEI_T}{HbEI} \frac{d^2y}{dx^2} = \frac{q^*(x)}{2EI}$$
  
or  $\frac{d^2M}{dx^2} - \frac{6NEI_T}{HbEI} M = -\frac{q^*(x)}{2}$ 

Put  $\alpha^2 = 6 \frac{\text{NEI}_{T}}{\text{HbEI}}$ 

$$let q^*(x) = \underline{q} x$$

Equation (6.4b) then becomes

$$\frac{d^2 M}{dx^2} - \alpha^2 M = - \frac{q_o}{2H} x$$
(6.4c)

The solution of Equation (6.4c) may be readily obtained as

$$M = A* \sinh \alpha x + B* \cosh \alpha x + \frac{q_{\bullet}}{2H\alpha^2} x$$
(6.5)

and since  $M = - EI \frac{d^2y}{dx^2}$ , one obtains

$$\frac{d^2 y}{dx^2} = C^* \sinh \alpha x + D^* \cosh \alpha x - \frac{q}{2H\alpha^2 EI} x$$
(6.6)

whence

$$\frac{dy}{dx} = \frac{C^*}{\alpha} \cosh \alpha x + \frac{D^*}{\alpha} \sinh \alpha x - \frac{q}{2H\alpha^2 EI} \frac{x^2}{2} + G^*$$

$$y = \frac{C^*}{\alpha^2} \sinh \alpha x + \frac{D^*}{\alpha^2} \cosh \alpha x - \frac{g}{2H\alpha^2 EI} \frac{x^3}{6} + G^*x + S^*$$

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Conditions at the bottom end require  $y = \frac{dy}{dx} = 0$  at x = 0, from which ·  $S^* = -\frac{D^*}{\sqrt{2}}$  $G^* = - \underline{C^*}$ and so  $y = \frac{C^*}{\alpha^2}$   $(\sinh \alpha x - \alpha x) + \frac{D^*}{\alpha^2} (\cosh \alpha x - 1) - \frac{q_* x^3}{12H\alpha^2 EI}$  (6.7) Conditions at the top end require M = o i.e.  $\frac{d^2y}{dx^2} = o$ (6.8a) and also V = 0, i.e. - EI  $\frac{d^3y}{dx^3}$  +  $6\frac{NEI_T}{H_D}$   $\frac{dy}{dx}$  = 0  $\frac{d^3y}{dx^3} - \alpha^2 \frac{dy}{dx} = 0$ or (6.8b) For brevity one puts  $\alpha H = \beta$  and then Equation (6.8a) yields  $\frac{q_{\bullet}}{2\alpha^{2}EI} = C* \sinh\beta + D* \cosh\beta$ (6.9)whilst Equation (6.8b) gives  $\alpha C^* - \underline{q}_{\bullet} + \underline{q}_{\bullet} \frac{\beta^2}{4H\alpha^2 ET} = 0$  $C^* = \frac{q_e}{2H\alpha^3 EI} \left(1 - \frac{\beta^2}{2}\right)$ Hence (6.10)and  $D^* = \underline{q} \cdot \left\{ \frac{\beta}{\beta} - (1 - \beta^2/2) \sinh \beta \right\}$ 2H $\alpha^3$ ET cosh  $\beta$ (6.11)Substituting Equations (6.10) and (6.11) in Equation (6.7) gives  $y = \frac{q}{2H\alpha^{5}EI} \left\{ \frac{(1-\beta^{2})(\sinh\frac{\beta_{x}}{H} - \frac{\beta_{x}}{H}) + \left[\beta - (1-\beta^{2})\sinh\beta\right](\cosh\beta_{x}-1) - \frac{\beta_{x}}{H}}{H} - \frac{\beta_{x}}{6} \right\}$ cosh A (6.12)

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Figure 6.4: Comparison of deflection by continuum and finite element methods.

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Equation (6.12) gives the deflected shape of the frame and is a function of  $\beta$ , a measure of the relative stiffness of the beams w.r.t. the columns. The case  $\beta = 0$  represents a free cantilever column.

The modal shape for the frame shown in Figure (5.1) was determined using Equation (6.12) and is drawn in Figure 6.4 by the solid line. The modal shape was also obtained using the finite element method and is indicated by triangular markers in Figure 6.4, from which it may be concluded that the continuum approach gives a good prediction.

We are now in a position to study the dynamic instability of a portal frame. Let the vertical support motion be  $u_{g}(t)$ . Consider an element of the column shown in Figure 6.5.

Vertical mass-acceleration gives  $dP = -c_u u_s dx$ 

$$P = -\int_{\mathbf{x}}^{\mathbf{H}} \int_{\mathbf{x}}^{\mathbf{u}} \mathbf{u}_{\mathbf{s}} d\mathbf{x}$$
$$= \int_{\mathbf{x}}^{\mathbf{H}} (\mathbf{H} - \mathbf{x}) \mathbf{u}_{\mathbf{s}}$$
(6.13)

where l = mass per unit height for half of the structure. $i.e. <math>l = \frac{1}{2} l^*$  where  $l^*$  is the mass per unit height for the whole structure.

Horizontal resolution gives  $\frac{dV}{dx} = -\frac{q^*}{2}$  (6.14)

Moment equilibrium gives P dy + V dx = dM + m dxor

$$V = \underline{dM} - P(x) \underline{dy} + m \qquad (6.15)$$
  
$$dx \qquad dx$$

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Differentiating Equation (6.15) w.r.t. x we get

$$\frac{dV}{dx} = \frac{d^2M}{dx^2} - P \frac{d^2y}{dx^2} - \frac{dP}{dx} \frac{dy}{dx} + \frac{dm}{dx}$$
(6.16)

Substituting for V and m and simplifying gives

$$EI \frac{d^4y}{dx^4} + P \frac{d^2y}{dx^2} + \frac{dP}{dx} \frac{dy}{dx} - \frac{6NEI_T}{Hb} \frac{d^2y}{dx^2} = \frac{q^*(x)}{2}$$

Since  $\alpha^2 = 6 \frac{\text{NEI}}{\text{HbEI}}$ , we have

$$\frac{d^{4}y}{dx^{4}} + \frac{\rho}{EI} (H-x) \ddot{u}_{s} \frac{d^{2}y}{dx^{2}} - \frac{\rho}{EI} \ddot{u}_{s} \frac{dy}{dx} - \alpha^{2} \frac{d^{2}y}{dx^{2}} = \frac{q^{*}(x)}{2EI}$$
Now  $q^{*}(x) = -\rho^{*} \frac{\partial^{2}y}{\partial t^{2}}$ 
(6.16a)

Substituting in the above equation gives

$$\frac{\partial^4 y}{\partial x^4} + \frac{\rho^*}{2EI} \qquad (H-x) \quad \ddot{u}_s \quad \frac{\partial^2 y}{\partial x^2} - \frac{\rho^*}{2EI} \quad \ddot{u}_s \quad \frac{\partial y}{\partial x} - \alpha^2 \frac{\partial^2 y}{\partial x^2} + \frac{\rho^*}{2EI} \quad \frac{\partial^2 y}{\partial t^2} = 0 \qquad (6.17)$$

From Equation (6.12) we get

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$$\frac{\partial y}{\partial x} = \frac{q}{2ET} \frac{H^4}{\beta^5} \left\{ \frac{(1-\frac{\beta^2}{2})\left(\frac{\beta}{H}\cosh\frac{\beta x}{H} - \frac{\beta}{H}\right) + \frac{\beta}{H} + \frac{\beta}{H} + \frac{\beta}{(1-\frac{\beta^2}{2})\sinh\beta} \left(\frac{\beta}{H}\sinh\frac{\beta x}{H}\right) - \frac{\beta}{H} - \frac{\beta}{H} + \frac{\beta}{H} +$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{q_e H^4}{2EI\beta^5} \left\{ \frac{(1-\beta^2)}{2} \left(\frac{\beta}{H}\right)^2 \sinh \frac{\beta x}{H} + \frac{\left[\beta - (1-\beta_2^2) \sinh \beta\right]}{\cosh \beta} \left(\frac{\beta^2 \cosh \frac{\beta x}{H}}{H^2} - \left(\frac{\beta}{H}\right)^3 x\right\} \right\}$$

$$\frac{\partial^{4} y}{\partial x^{4}} = \frac{q_{e} H^{4}}{2E I \beta^{5}} \left\{ \frac{(1-\beta^{2})(\frac{\beta}{H})^{4} \sinh \frac{\beta x}{H} + \frac{\left[\beta - (1-\beta^{2}_{2}) \sinh \beta\right](\frac{\beta}{H})^{4} \cosh \frac{\beta x}{H}}{\cosh \beta}}{(\frac{\beta}{H})^{4} \cosh \frac{\beta x}{H}} \right\} (6.12c)$$

$$\frac{\partial^{4} y}{\partial x^{4}} - \alpha^{2} \frac{\partial^{2} y}{\partial x^{2}} = \frac{q_{e} H^{4}}{2E I \beta^{5}} \left(\frac{\beta}{H}\right)^{5} x$$

$$= R^{*} \alpha^{5} x \qquad (6.18)$$

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where 
$$R^* = \frac{q_{, H}^4}{2EI\beta^5}$$

Substituting in Equation (6.17) yields

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$$\frac{\ell^{*}}{2\mathrm{EI}} \quad (\mathrm{H-x}) \quad \ddot{\mathrm{u}}_{\mathrm{S}} \quad \frac{\partial^{2}}{\partial \mathrm{x}^{2}} \quad - \quad \frac{\ell^{*}}{2\mathrm{EI}} \quad \ddot{\mathrm{u}}_{\mathrm{S}} \quad \frac{\partial \mathrm{y}}{\partial \mathrm{x}} + \\ \mathrm{R}^{*} \propto^{5} \quad \mathrm{x} + \quad \frac{\ell^{*}}{2\mathrm{EI}} \quad \frac{\partial^{2} \mathrm{y}}{\partial \mathrm{t}^{2}} = 0 \quad (6.20)$$

$$\mathrm{Let} \quad \mathrm{y}(\mathrm{x},\mathrm{t}) = \sum_{i=1}^{\infty} \\ \mathrm{R}^{*} \quad \phi_{i} \quad (\mathrm{x}) \quad \mathrm{f}_{i}(\mathrm{t})$$

where  $\phi_{i}(x)$  are a complete set of functions satisfying the boundary conditions, for instance the various modes of free vibration of the problem, and  $f_{i}(t)$  are generalized coordinates. An approximate solution for y may be obtained by curtailing the series at some integer i = n. This approximation will not now satisfy Equation (6.20), but the error introduced by the substitution of the curtailed series may be minimized by applying the Galerkin Method.

$$\int_{R}^{R} \left[ \frac{\rho^{*}}{2EI} (H-x) \ddot{u}_{s} \sum_{i=1}^{n} f_{i} \phi_{i}^{*} - \frac{\rho^{*}}{2EI} \ddot{u}_{s} \sum_{i=1}^{n} f_{i} \phi_{i}^{i} + \alpha^{5} x f_{i} + \frac{\rho^{*}}{2EI} \right]_{i=1}^{R} \ddot{f}_{i} \phi_{i}^{j} dx = 0 \text{ for } j = 1, 2, \dots n \quad (6.22)$$

Consider a one term series approximation for y.

i.e. 
$$y = R^* \phi_1(x) f_1(t)$$

Equation (6.22) then writes as

$$\int \left[ \left\{ \frac{\rho^*}{2\mathrm{EI}} (\mathrm{H-x}) \ddot{\mathrm{u}}_{\mathrm{S}} \frac{\mathrm{d}^2 \phi}{\mathrm{dx}^2} - \frac{\rho^*}{2\mathrm{EI}} \ddot{\mathrm{u}}_{\mathrm{S}} \frac{\mathrm{d} \phi_1}{\mathrm{dx}} + \overset{5}{\mathrm{dx}} \right\} f_1(\mathrm{t}) + \frac{\rho^*}{2\mathrm{EI}} \frac{\mathrm{d}^2 f_1}{\mathrm{dt}^2} \phi_1 \right] \phi_1 \, \mathrm{dx} = \mathrm{o}$$

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(6.19)

$$\operatorname{or} \int \left[ \left\{ \frac{\rho^{*}}{2\mathrm{EI}} \left( \mathrm{H-x} \right) \ddot{\mathrm{u}}_{\mathrm{S}} \phi_{1} \frac{\mathrm{d}^{2} \phi_{1}}{\mathrm{dx}^{2}} - \frac{\rho^{*}}{2\mathrm{EI}} \ddot{\mathrm{u}}_{\mathrm{S}} \phi_{1} \frac{\mathrm{d} \phi_{1}}{\mathrm{dx}} + \overset{<}{\overset{5}} \mathrm{x} \phi_{1} \right\} f_{1}(\mathsf{t}) \\ + \frac{\rho^{*}}{2\mathrm{EI}} \frac{\mathrm{d}^{2} f_{1}}{\mathrm{dt}^{2}} \phi_{1}^{2} \right] \mathrm{dx} = 0 \qquad (6.23)$$

The integrals 
$$\int \phi_1 \frac{d^2 \phi_1}{dx^2} dx$$
,  $\int x \phi_1 \frac{d^2 \phi_1}{dx^2} dx$ ,  $\int \phi_1 \frac{d \phi_1}{dx} dx$ ,  
 $\int x \phi_1 dx$ ,  $\int \phi_1^2 dx$ 

may be evaluated numerically using Equations (6.12).

Equation (6.23) is a Mathieu equation and its solution would yield the region of dynamic instability.

The frame shown in Figure 5.1 was analysed by this procedure. Using Simpson's rule the above integrals were determined and the following values were obtained.

$$\int \phi_1'' \quad \phi_1 \, dx = - \ 0.06772 \ x \ 10^5$$
$$\int \phi_1^2 \, dx = 43.7656 \ x \ 10^6$$
$$\int x \ \phi_1 \, dx = 1.4131 \ x \ 10^6$$
$$\int \phi_1' \quad \phi_1 \, dx = 0.8369 \ x \ 10^6$$
$$\int x \ \phi_1'' \quad \phi_1 \, dx = - 1.9173 \ x \ 10^6$$

The frame was weighed and its mass was found to be 0.465 lbs. Substituting the value of the integrals in Equation (6.23) yields

$$21.8826 \frac{d^2 f}{dt^2} + (17600 - 0.9805 \ddot{u}_s) f(t) = 0$$

Letting  $u_s = C \cos Q t$ , results in

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21.8826 
$$\frac{d^2f}{dt^2}$$
 + (17600 + 0.9805  $\Omega^2$ C cos  $\Omega$  t) f = 0  
The solution of this Mathieu equation gives

$$\Omega^2 = \frac{3230}{1 \pm 0.09C}$$
(6.24)

Equation (6.24) gives the value of the disturbing frequency required to cause the frame to go into instability in the first mode. C = o, refers to the hypothetical case when instability will occur even if the amplitude of the base motion is zero. From Equation (6.24)  $\Omega = 57.0$  rad./sec. when C = o. However, from the experimental tests, as reported in Chapter V, the value of the disturbing frequency, $\Omega$ , corresponding to the least amplitude of the base motion was 47.8 rad./sec. Eq. (6.24) does not give a good prediction of the width of the instability region. This is possibly due to the fact that the above analysis considered only a one term series approximation for y. If more terms are included a better estimate of the width of the instability region would be obtained.

### 6.3 CONCLUSIONS

The continuum approach has been used to determine the natural frequencies of structures e.g. grids (29,46). It was felt that this method could be used to obtain the region of dynamic instability of portal frames. As shown above, the method gives a good estimate of the deflected shape and the

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natural frequency of the first mode of the structure. However, the prediction of the width of the instability zone is not satisfactory. This may be attributed to several factors. In the above analysis a one-term series approximation for the deflected shape was considered. The inclusion of more terms would yield better results. The accuracy of the results depends upon the values of the integrals in Equation 6.23. Since the integrals were evaluated numerically, and even though the deflected shape was shown to be in good agreement with that given by the finite element solution, the relationship for the slope and curvature may not be good. This may account for the poor estimate of the region of instability.

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## Chapter VII

# CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

This study deals with the parametric instability of plane structures. The dynamic stability of structures requires a knowledge of their natural frequencies. The finite element method was used for this purpose and has been shown to yield good results. One of the major advantages of the finite element method is that the governing differential equations need not be known and different boundary conditions may be easily handled. The method is applicable to structures of non-uniform crosssection and varying properties. Computer programs to obtain the natural frequencies and modal shapes of structures are also given.

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A simple numerical method for determining the regions of dynamic instability of columns subjected to periodic axial forces or periodic support motion is presented. The validity of the results has been checked against experimental work and available classical solutions. A non-uniform column or one with varying properties can be easily handled.

Finally, the dynamic instability of portal frames due to periodic support motion was investigated. An extensive experimental program was carried out and the results have been

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reported in Chapter V. The continuum method gives a good estimate of the deflected shape and the natural frequency of the frame for the first mode, but the prediction of the width of the instability region was found to be unsatisfactory. By taking more terms in the series for the equation of the deflected shape a better relationship for the instability region could be obtained.

As the ground motion in earthquakes consists of horizontal and vertical movements the study of the response of structures subjected to continued horizontal and vertical support motions needs to be investigated and is proposed for future research. The present investigation was confined to the response of structures to periodic support motion. Since seismic vibrations are random in nature the study of the response of structures to random fluctuating base motion needs to be looked into.

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#### APPENDIX A

#### SOME PROPERTIES OF THE MATHIEU EQUATION

Consider the Mathieu equation

$$\frac{d^2y}{dz^2} + (\delta + \epsilon \cos 2z) y = o \qquad (A.1)$$

which is periodic in z.

The general solution of the Mathieu equation containing two arbitrary constants has not yet been found. However, periodic coefficients lead to periodic solutions and the above linear equation may be solved by Floquet's theory for the solution of linear differential equations with periodic coefficients. Since Equation (A.1) is a linear second-order differential equation, two independent non-zero solutions may be found as  $y_1(z)$  and  $y_2(z)$ . These solutions may be combined linearly to give any other solution, say y(z) so that

$$y(z) = C_1 y_1(z) + C_2(y_2) z$$
 (A.2)

where  $C_1$  and  $C_2$  are constants.

For  $y_1(z)$  and  $y_2(z)$  to be independent, they must satisfy the determinant

$$W (Y_1, Y_2) = \begin{vmatrix} Y_1 & Y_2 \\ \\ Y_1 & Y_2 \end{vmatrix} \neq 0$$

The replacement of y(z) by y(-z) does not alter Equation (A.1). Therefore, if y(z) is a solution of Equation (A.1) then y(-z) is also a solution. If y(z) has no symmetry about the point z = 0, it can be split into components having even and odd symmetry by the usual process.

Even: 
$$\frac{1}{2} \begin{bmatrix} y(z) + y(-z) \end{bmatrix} = C_1 y_1(z)$$
  
odd :  $\frac{1}{2} \begin{bmatrix} y(z) - y(-z) \end{bmatrix} = C_2 y_2(z)$ 

Since y(z) is periodic, y(z + T) = y(z), and since  $y_1(z)$  and  $y_2(z)$  are solutions of Equation (A.1), then  $y_1(z + T)$ and  $y_2(z + T)$  also are solutions. It does not mean that  $y_1$  (z + T) is the same as  $y_1(z)$ . Rather it means that  $y_1(z + T)$ and  $y_2(z + T)$  may be represented in the form of a linear combination of the primary functions.

$$y_{1}(z + T) = a_{11}y_{1}(z) + a_{12}y_{2}(z)$$
  

$$y_{2}(z + T) = a_{21}y_{1}(z) + a_{22}y_{2}(z)$$
(A.3)

where  $a_{ik}$  are the constants called coefficients of transformations. Now  $y(z + T) = C_1 y_1 (z + T) + C_2 y_2 (z + T)$ substituting from Equations (A.3) we get

 $y(z + T) = (C_1a_{11} + C_2a_{21})y_1(z) + (C_1a_{12} + C_2a_{22})y_2(z)$ If y(z) is suitably chosen, we can have

$$c_{1}a_{11} + c_{2}a_{21} = \rho c_{1}$$

$$c_{1}a_{12} + c_{2}a_{22} = \rho c_{2}$$
(A.4)

and hence we get,  $y(z + T) = \rho y(z)$  (A.5)

A solution that possesses the above property is called a normal solution e.g. if y denotes displacement, it means that the displacement at the end of the period is (<sup>9</sup> times that at the beginning of the period.

If constants  $C_1$  and  $C_2$  are not to be zero in Equation (A.4) the determinant of their coefficients must be zero, i.e.

$$\begin{vmatrix} a_{11} - \rho & a_{21} \\ a_{12} & a_{22} - \rho \end{vmatrix} = o$$
 (A6)

This is called the characteristic equation.

We can select the two linearly independent solutions  $y_1(z)$  and  $y_2(z)$  such that they satisfy the initial conditions at z = 0.

$$y_{1}(o) = 1 \qquad y'_{1}(o) = 0$$

$$y_{2}(o) = 0 \qquad y'_{2}(o) = 1$$
(A.7)

From these conditions we can determine the coefficients  $a_{ik}$  and the characteristic equation becomes

$$e^{2} - 2 A e + B = 0$$
 (A.8)  
 $A = \frac{1}{2} \left[ y_{1}(T) + y_{2}'(T) \right]$ 

where

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It can be readily shown that B is always equal to 1.

Hence, Equation (A.8) becomes

 $\rho^2 - 2A\rho + 1 = 0$ 

or 
$$\ell = A \pm \sqrt{A^2 - 1}$$
 (A.9)

and the roots are related by  $\rho_1 \cdot \rho_2 = 1$  (A.10) From Equation (A.5) we have

$$y_k (z + T) = {\rho_k y_k(z)} (k = 1, 2)$$

These solutions may also be expressed as

$$y_{k}(z) = \chi_{k}(z) e^{z/T \ln \frac{2}{k}}$$
 (k = 1,2) (A.11)

where  $\frac{\chi}{k}(z)$  are periodic functions of period T.

From Equation (A.11) it may be observed that the behaviour of the solution as  $t \rightarrow \infty$  depends upon the values of  $\ell_1$ ,  $\ell_2$ .

Since  $\log \rho = \log |\rho| + i \arg \rho$ 

Equation (A.11) may be written as

$$y_{k}(z) = \phi_{k}(z) e^{\frac{z}{T} \ln \left| \frac{\rho_{k}}{k} \right|}$$

$$(k = 1, 2)$$

$$(A.12)$$

$$\phi_{k} = \chi_{k}(t) e^{\frac{iz}{T} \arg \ell}$$

where

Equation (A.12) shows that if  $\ell_k > 1$ , the corresponding solution will have an unbounded exponential multiplier. If  $\ell_k < 1$  the solution is damped as z increases; and if  $\ell = 1$  then the solution is periodic (or almost periodic), i.e. it will be bounded in time. It may be readily shown that if A > 1, the roots of the characteristic equation will be real and one of them will be greater than unity and hence unbounded solutions will result. However, if A < 1, the characteristic equation has conjugate complex roots, and since their product must be equal to unity, their modulus will be equal to unity. The case of complex roots corresponds to the region of bounded solutions. On the boundaries separating the regions of bounded solutions from those of unbounded solutions we must have A = 1.

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i.e. 
$$y_1(T) + y'_2(T) = 2$$
 (A.13)

The regions of bounded solutions correspond to the regions of dynamic stability in the physical sense, and unbounded solutions to the regions of dynamic instability. Therefore, Equation (A.13) may be used to determine the regions of dynamic instability.

Since  $\ell_1 \cdot \ell_2 = 1$ , at the boundaries separating the regions of stability from that of instability we must have either  $\ell_1 = \ell_2 = 1$  or  $\ell_1 = \ell_2 = -1$ . The former case corresponds to periodic solution of period T, while for the latter case the period is 2T.

Therefore, the regions of unbounded solutions are separated from those of bounded solutions by the periodic solutions with periods T and 2T. In fact, the regions of 138

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stability are bounded by solutions of different periods, while those of instability are bounded by solutions of identical period. Therefore, the determination of the boundaries of the regions of instability is reduced to finding the condition under which the given differential equation has periodic solutions with periods T and 2T.

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#### APPENDIX B

#### 1. GALERKIN'S METHOD

Let us represent the motion of a body by considering the motion of some discrete points. The equations of motion write as

where  $\{ \emptyset \}$  is the displacement vector  $\{ F \}$  is the external force vector ex Let us suppose that

$$\begin{array}{c} \theta_{1} \\ \theta_{2} \\ \vdots \\ \vdots \\ \theta_{n} \end{array} = \begin{cases} y_{1} \\ y_{2} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ y_{n} \end{cases} = \\ y_{n} \end{cases}$$

(B.2)

where  $\{y\}$  is the modal shape or eigenvector. If the eigenvector,

 $\{ \underbrace{j} \}, \text{ is only approximately known, then in general the forces}$  will not be in equilibrium. Equation (B.1) then becomes  $\sum_{F_{1}} = \begin{bmatrix} m_{11} & m_{12} & \cdots & m_{1n} \\ m_{21} & m_{22} & \cdots & m_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{F_{n}} = \begin{bmatrix} m_{n1} & m_{n2} & \cdots & m_{nn} \end{bmatrix} \begin{pmatrix} y_{1} \\ y_{2} \\ \vdots \\ y_{n} \end{pmatrix} \begin{pmatrix} k_{11} & k_{12} & \cdots & k_{1n} \\ k_{21} & k_{22} & \cdots & k_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ k_{n1} & k_{n2} & k_{nn} \end{bmatrix} \begin{pmatrix} y_{1} \\ y_{2} \\ \vdots \\ y_{n} \end{pmatrix} \begin{pmatrix} F_{1} \\ F_{2} \\ \vdots \\ F_{n} \end{pmatrix} \begin{pmatrix} e_{1} \\ e_{2} \\ \vdots \\ e_{n} \end{pmatrix}$  (B.3)

The i-th row of this equation represents the sum of the generalized forces  $\sum F_i$  associated with the coordinate  $\theta_i$ . The quantity  $e_i$  on the right-hand side of Equation (B.3) represents the error in equilibrium of the forces associated with  $\theta_i$ .

Galerkin's method requires that the weighted sum of the errors  $\{e\}$  be zero; each of the terms in  $\{e\}$  is weighted by the corresponding term in the modal shape vector  $\{y\}$ . Thus it requires that

$$\begin{bmatrix} y_1 & y_2 & \cdots & y_n \end{bmatrix} \begin{cases} e_1 \\ e_2 \\ \vdots \\ \vdots \\ n \end{cases} = 0$$

(B.4)

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If we take  $\{\underbrace{y}\}$  to be a displacement vector and  $\{e\}$  as a force vector, we can say that Galerkin's Method requires the error in the forces to be orthogonal to the assumed shape of the displacements. Substitution of Equation (B.3) into Equation (B.4) gives

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 $\begin{bmatrix} \underline{y} \end{bmatrix} \begin{bmatrix} \underline{M} \end{bmatrix} \{ \underline{y} \} \ddot{\theta} + \begin{bmatrix} \underline{y} \end{bmatrix} \begin{bmatrix} \underline{s} \end{bmatrix} \{ \underline{y} \} \theta + \begin{bmatrix} \underline{y} \end{bmatrix} \{ \underline{F} \}_{ex} = 0$ (B.5)

## APPENDIX C

# PROGRAM TO FIND THE NATURAL FREQUENCIES OF

## PLANE FRAMES AND GRIDS

The program is coded in Fortran IV and can be run either on the RAX or O/S systems of IBM 360/65 or IBM 360/75. Nodal points and elements should be numbered in a consecutive order. The size of the problem to be solved can be adjusted by suitably changing the dimension statement.

The input to the program consists of the nodal coordinates, member properties and the boundary conditions. The program generates the stiffness and the mass matrices from which the natural frequencies and the modal amplitudes are computed. The computer print-out includes

- a) Reprint of input data
- b) Natural frequencies
- c) Normalized modal amplitudes

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	• .	
1. Input	data for pl	ane frames.
Cols.	Notation	Description
A Structu	re Paramete	rs and Material Properties (415, Fl0.0, Fl0.8 one card
1-5	М	Number of elements or members
6-10	NJ	Number of nodes or joints
11-15	NR	Number of restraints
16-20	NRJ	Number of restrained joints
21-30	E	Modulus of Elasticity
31-40	R	Member density
B Joint Co	ordinates (	I5, 2F10.2) NJ cards
1-5	J	Counter
6–15	x	X coordinate
16-25	Y	Y coordinate
C Member De	esignations	and Properties (315, F10.5, F10.8) M cards
1-5	J	Counter
6-10	ររ	Designation of J end of member
1-15	JK	Designation of K end of member
L6-25	AX	Area of member
<b>!6–35</b>	IZ	Moment of inertia about 7-avia

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# 2. Input data for grids

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Cols.	Notation	Description
A Structure	Parameters and	Material Properties (415, 2F10.0, F10.8) one card
1-5	М	Number of members
6-10	ŊJ	Number of joints
11-15	NR .	Number of restraints
16-20	NRJ	Number of restrained joints
21-30	E	Modulus of Elasticity
31-40	G	Shear Modulus
41-50	RA	Member density
B Joint Coo	rdinates (15, 2	F10.2) NJ cards
1-5	J	Counter
6-15	X .	X coordinate
16-25	Y	Y coordinate
C Member De	signations and	Properties (315, 3F10.6) M cards
1-5	J	Counter
6-10	JJ	Designation of J end of member
11-15	JK	Designation of K end of member
16-25	IX	Torsion constant
26-35	IY	Moment of inertia about Y axis
36-45	AX	Area of member

		•
\$.0001	DIMENSION L(20),X(20),Y(20),JJ(20),JK(20),AX(20),IZ(20),CX(20),CY( 120),RL(50),CRL(50),SMD(6,6),S(50,50),MM(50,50),MD(6,6),	
5.0002	LXL(15),XX(15,15),MSN(15,15),SN(15,15)	·
S.0003		•
S.0004	WRITE(6,1)	· ·
		•
\$.0005	1 FORMAT('1','NATURAL FREQUENCIES OF FRAME INCLUDING ROT. INERTIA') C************************************	• •
	C 1. INPUT AND STRUCTURE DATA	· ·
χ.	C 1.A STRUCTURE PARAMETERS, ELASTIC MODULUS AND DENSITY	• •
5.0006	WRITE (6,2)	• •
S.0007	2 FORMAT(//'0', STRUCTURE DATA')	
\$.0009	NKIIT (0,5) 3 FORMAT (404.4 M. N. N.I. NR. NR. E	•
5.0010	READ (5,4) H,NJ,NR,NRJ,E,R	•
5.0011	4 FORMAT(415,F10.0,F10.8)	· · · ·
\$.0013	WRITE (6.5) M.N.NJ.NR.NRJ.E.R	•
5.0014	5 FORMAT(515,F15.0,F12.8) C	•
	C 18. JOINT COORDINATES	· · · · · ·
S-0015	WRITE (6,6)	
5.0017	WRITE (6.7)	
\$.0018	7 FORMAT ('O', 'JOINT , X COORD Y COORD')	
5.0019	DD 101 J=1+NJ DEAD /E-A) / V/11-V/11	
5.0021	8 FORMAT (15,2710.2)	· •
5.0023	WRITE (6,9) J.X(J).Y(J)	
5.0023	9 FORMAT (15,2F10,2) ·	•
	C IC. MEMBER DESIGNATIONS AND PROPERTIES	
S.0025	WRITE (6,10)	
\$.0027	WRITE (6.11)	•
S.0028	11 FORMAT (1H0,6HMEMBER,2X,2HJJ,3X,2HJK,5X,2HAX,9X,2H12,12X,1HL,9X,2H	
\$ .00 29		
\$.0030	00 τνε τ≠τρπ READ (5+12) J+JJ(Σ}+JK(Σ}+ΔΧ(Σ)+ΓΖ(Σ)	
5.0031	12 FORMAT(315,F10.5,F10.8)	
S-0032		
S.0034	K1=JK(1) XC1=X{K1}-X{1}T1	
S.0035	YCL=Y(KI)-Y(JI)	
5.0036	L(I)=SQRT(XCL++2&YCL++2)	
3.0031 5.0038	LK[]=XUL/L[]) CV/T}-VCI/I/T	<b>Н</b>
5.0039	WRITE (6,13) I.JJ(I).JK(I).AX(I).17(T).(/// /////////////////////////////////	5
340037		

S.0040 S.0041	• .1 10	3 FORMAT(315,F10.5,F14.8,3F10.2) 2 Continue	Nosse V	•		Yruge Y	
	Ċ	1D. JOINT RESTRAINT LIST, C	UMULATIVE RESTRAINT LIST				
	C C	INITIALISE RL(I) TO ZERO					
5.0042	C	NJ3=3+NJ			· •	·	
5.0043	•	DD 104 J3=1,NJ3	· •				
S-0044	. 10	RL(J3)=0					
\$.0046	10	WRITE (6,14)		•			
5.0047	1	4 FORMAT( '0', 'JOINT RESTRAINTS')					
5.0048	1	WRITE (6,15) 5 FORMAT (101,1)OINT - X RSTRT - Y RST	RT 7 RSTRT+1	•	•	•	
S.0050	•	DD 103 J=1,NRJ				•	
S.0051		READ (5,16) K,RL(3+K-2),RL(3+K-1),RL(	3+K)		•		
5.0052	1	5 FURMAT (4110) WRITE(6.17) K.R![3*K-2].R![3*K-1].R![	3.46)	•			
\$.0054	1	7 FORMAT (15,3110)	· · · · ·	•	•	•	
S.0055	10	3 CONTINUE					
5.0056		CRL(1)=RL(1) DD 106 K=2-N13	· · ·				
5.0058		CRL(K)=CRL(K-1)&RL(K)			,		
5.0059	10	5 CONTINUE				•	
	00	2.A GENERATION OF OVERALL STI	FNESS AND MASS MATRICES.				•
5.0060	L.	NJ2=3*NJ					
5.0061		DD 106 J3=1,NJ2					
S.0062		DO 106 K3=1,NJ2					
S.0064		MM{J3.K3}=0.0					
5.0065	10	6 CONTINUE					
S.0066		DD 51 I=l+M					
5.0068		J2=3+JJ([]-2 J2=3+JJ([])-1					
5.0069		J3=3*JJ([)	•				
5.0070		K1=3+JK(I)-2					
5.0072		K2=3+JK(1)-1 K3=3+.K{(1)				•	
5.0073		SCM1=(E+AX(1))/L(1)	•		•		
S.0074	÷	SCM2={4,04E+IZ(I)}/L(I)		•			
5.0075		50M3=(1+7+50M2)/1(1) 50M4=(2+0+50M3)/1(1)	. •				
S.0077	•	Z=R+AX(I)+L(I)/420.0			•	· /	
5.0078	_	WRITE (6,73)					
5.0079	7	3 FORMAT (*0*+*SGM*) UDITE 14.72\ SCM1.SCM2.SCM3.SCM4					
S.0081	7	2 FORMAT (1H0.4F10.2)	•				
\$.0082	-	1F (RL(J1)) 18,19,18				•	
S.0083	1	9 J1=J1-CRL(J1)			•		
5.0084 S.0085	1	50 10 20 8 J1=N&CRL(J1)					
5.0086	2	0 IF (RL(J2)) 21,22,21		•			
5.0087	2	2 J2=J2-CRL(J2)					
2.0088	•	GU TU Z3 1 12-NECRI ( 12)	•				
5.0090	2	3 IF (RL(J3)) 24,25,24				-1 1	4
5.0091	2	5 J3=J3-GRL(J3)					Ĵ.

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S.0092	. GO TO 26		
5.0093	24 J3=N&CRL(J3) ,		•
5.0094	26 IF (RL(K1)) 27,28,27		
5.0095	28 K1=K1-CRL(K1)		
5.0098	GO TO 29		• •
5.0097	27 KI*N&CRL(K1)		
5.0090	29 IF (RL(R2)) 30,31,30		•
5.0100	21 N2=N2-LKL(K2) CO TO 22		•.
S.0101	30 K2=N(CD) (K3)	· .	
5.0102	32 IF (RI(K3)) 33.34.33		-
\$.0103	34 K3#K3-CRI/K31		
S.0104	GO TO 35		
S.0105	33 K3=NGCRL(K3)		
S.0106	35 WRITE(6,200) J1.J2.J3.K1.K2.K3.7		•
S.0107	200 FORMAT(1H0,615,E15.8)		•
S.0108	CY2=CY(1)+CY(1)		
5.0109	CX2=CX([)+CX([)		·
5.0110	CXY=CX(I)+CY(I)		
5.0111	SHD(1,1)=SCH1+CX2&SCH4+CY2		•
5.0112	SMD(4+4)=SMD(1+1)		•
5.0114	240(4,1)=-CH0(1,1) 240(4,1)=-CH0(1,1)	•	
5.0115	SHD(1,2)=(CCH1_CCH4)+CVV	·	
5.0116			
5.0117	SMD(4.5)#SMD(1.2)		•
S.0118	SMD(5+4)=SMD(1+2)		
S.0119	SMD(1,5) =- SMD(1.2)		
5.0120	SMD(5,1) = -SMD(1,2)		
5.0121	SHD(2,4)=-SHD(1,2)		
5.0122	SMD(4,2)=-SMD(1,2)		
5.0123	SMD(1,3) = -SCH3 + CY(1)		
5.0125	SMD(3,1)=SMD(1,3)		
5.0126	SMD(1+0)=SMD(1+3)		
S-0127	SMD(3,6)		
5.0128	SMD14,21=-SMU11,21		
5.0129	SMD(4.6)=-SMD(1.3)		
S.0130	SMD(6,4) = -SMD(1,3)		
S-0131	SMD(2,2)=SCM1+CY263CH4+CX2	,	
S.0132	SHD(5,5)=SHC(2,2)		•
S-0133	SMD(2,5) =- SMD(2,2)		
5.0134	SHD(5,2)=-SMD(2,2)		
5.0135	SMD(2,3)=SCH3+CX(1)		
5-0137	SMD(3+2)=SMD(2+3)		
5.0138	SHD(2+0)+SHD(2+3) SHD(4-3)-SHD(2+3)	•	·
5.0139	SMD(0)//#SMD(2)5)		
5.0140	SMD(5.3)=-SMD(2.3)		
5.0141	SMD(5.6)=-SMD(2.3)		
5.0142	SND(6.5)=-SMD(2.3)		
S.0143	SMD(3,3) = SCM2		•
5.0144	SMD(6+6)=S4D(3+3)		
S-0145	SHD(3,6)=SCH2/2.0		
5.0146	SMD(6,3)=SMD(3,6)		
5-0147		· ·	•
5.0149	し ス /	·	
5.0140	5772=572= 7473=744=3		
S.0150	6812-68772 FX12-68715417552		4
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.0161 .		1350 V
01 63	CYLZ=CY([)+L(])+Z	
•VL72	4L2=L(I)=L([)=Z	
C		
.0173	WRITE(6,800)	
0154 80	FORMAT( *0 *, *CXY*)	•
01 55	WRITE(6,801) CX2+CY2+CX2+CY22+CXY2+CXL2+CYL2+ZL2	• •
0156 80	FORMAT (1H0,9E12.6)	
•0157 ·	Z1=R/420.0	•
0158	ZL=1L+Z1 ,	
•0159	CYIZ=CY(I)+IZ(I)+Z1	·
•0160	CX12=CX([)+[2(])+21	
•0161	ZIL=Z1+17(I)+L(I)	
•0162	CX22L=CX2+ZL	·
•0163	CY22L=CY2+ZL	
•0164	CXYZL=CXY+ZL	
•0165	MD(1,1)=140.0+CX2Z&156.0+CY2Z&504.0+CY2ZL	
•0160	MD(4,4)=MD(1,1)	•
•0167	MD{1,2}=-16.0+CXYZ-504.0+CXYZL	
•0168	MD(2,1)=MD(1,2)	
.0169	ND{1.3}=-22.0+CYLZ-42.0+CYIZ	•
•0170	MD(3,1)=M7(1,3)	•
.0171	HD(4,6)=-KD(1,3)	
.0172	HO(6,4)=-HD(1,3)	
.0173	HD(1+4)=70.0+CX22654.0+CY2Z-504.0+CY2Z1	
0174	MD(4,1)=MD(1,4)	· · ·
0175	ND(1,5)=-ND(1,2)	
0176	MD(5,1)=-ND(1,2)	· · ·
.0177	HD(2,4)=-HD(1,2)	•
.01 78	HD(4,2)=-HD(1,2)	
0179	MD(1+6)=13.0*CYLZ-42.0*CYLZ	•
0180	MD(6,1)=MD(1,6)	
0181	MU (2,2)=140,0+CY2Z &156,0+CX2Z &504,0+CX271	,
0182	ND(5,5)=MD(2,2)	
01 83	MD(2,3)=22.0+CXLZ642.0+CXLZ	
0184	HD(3-2)=HD(2-3)	•
0185	MD(2+5)=70+0+6 Y2Z&54+0+6X27-504+0+6X271	
01 86	ND(2+6)=-13-0+CX17642-0+CX17	
0187	MD (6+2) = MD (2+6)	
01 88	ND (3.3) =4, 0+71 2656, 0+71	
01 89	ND(6.6)=ND(3.3)	•
0190	HD(3,4) = -HD(1,6)	·
0191 '	MD(4,3) = -MD(1,6)	
0192	MD(3,5)=-MD(2,6)	
0193	MD(5,3) = -MD(2,6)	
0194	MD(3,6) = 3.04712 = 16.04711	
0195		
0196	MD(5,6) = -MD(2,3)	
0197		•
0198		
0199		
0200		•
0201 2001		
02 02		•
203 2002		
	AUTIFICATITY (UNITA+1A1+1A=T+0)	
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~ ~	JJ7-JTJ1(11	
120E		· · · · · · · · · · · · · · · · · · ·
	IT INLIJJ-211 41.40.41	4
	S(J1+J1)=S(J1+J1)&SMD(1+1)	ΰ
1007	S(JZ+JI)=S(J2+J1)&SMD(2+1)	
** -* *******		
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	V Phy			₩66 <b>₩</b>		V 141- V
S	.0208 '	S(J3.J	1)=S(J3,J1)&SMD(3,1)		·····	Managana ang kanana ang
Ś	.0209	S(K1.J	1)=SMD(4+1)	•	•	
s	6.0210	S(K2+J	1) = SMD(5, 1)			•
S	.0211	SIK3.J	1) = SMD(6,1)			
s	.0212	MM(J1.	J1)=MM(J1,J1)&MD(1.1)			•
İš	.0213	MH(J2)	$J1) = MM(J2, J1) \in MD(2, 1)$	•	·	
Ī	.0214	MM(J3.	J1)=MM(J3.J1)6MD(3.1)		•	
	-0215 ·	HMIK1.	$J1) = MD(4 \cdot 1)$	•		
İš	6.0216 <b>`</b>	MM(K2.	J1)=MD(5,1)			· · ·
l s	5.0217	MM (K3.	$(11) = MO(6 \cdot 1)$	4	•	
Š	.0218	41 1F (RL	(113-1)) 43.42.43			
l s	5-0219	42 S(J1+J	2)=S(J1+J2)&SMD(1+2)			
s	5.0220	StJ2+J	$2) = S(J_2 \cdot J_2) \in SMD(2 \cdot 2)$			• •
	5.0221	S(J3.J	2) = S(J3, J2) & SMD(3, 2)			
	5.0222	S(K1.J	2)=SMD(4,2)		•	
Š	5.0223	Stk2.J	(2) = SMD(5, 2)	· •		
l s	5.0224	S(K3.J	2)=SMD(6,2)		•	
l s	5.0225	MM(J1.	$J_{2} = MM(J_{1}, J_{2}) \in MD(1, 2)$		•	
5	5.0226	MMIJ2.	$J_{2}$ = MM ( $J_{2}$ , $J_{2}$ ) & MD ( $2, 2$ )		•	
	5.0227	MM(J3.	$J_{2} = MM(J_{3}, J_{2}) \in MD(3, 2)$		•	
	5.0228	MM(K1.	J2)=MD(4.2)	· .		
i	5.0229	NM(K2.	J2)=MD(5+2)	· ·	. •	•
	5.0230	MM(K3.	J2)=MD(6,2)			•
<b>1</b>   5	5.0231	43 IF (RL	(JJ3)) 45.44.45			
<u>.</u>	\$.0232	44 S(J1.J	3)=S(J1,J3)(SMD(1,3)	•		
	\$.0233	S(J2,J	3) = S(J2, J3) CSMD(2, 3)			
	5.0234	S(J3,J	3)=S(J3,J3)&SMD(3,3)		· ·	.*.
	S.0235	SIK1.J	3)=SMD (4,3)		•	
	5.0236	S(K2.J	3)=SMD(5+3)			
	5.0237	S(K3, J	3)=SMD (6,3)			· .
<b>X</b>   s	S•0238	MM(J1+	J3)=MH(J1+J3)&MD(1+3)		•	•
1 :	\$.0239	MM(J2.	J3)=MM(J2+J3)&MD(2+3)	•		•
SII :	S.0240	MM(J3.	J3)=MM(J3,J3)&MD(3,3) .		•	
	S.0241	MM(K1.	J3)=MD(4+3)			
3   5	5.0242	MM(K2+	J3)=MD(5,3)			•
21 :	S•0243	MM(K3.	J3)=MD(6,3)	•		
		C				•
	5.0244	45 JK3=3+	JK(I)			
	5.0245	L 15 /BI	(183-21) 47.46.47			
	5.0246	46 51.11.8	1)=SMD(1-4)			
	5.0247	S( 12.K	1)=SHD(2.4)			
3  3	5.0248	SL 13.K	(1)=SMD(3.4)			· ·
	5.0249	S(K).K	11=S(K1-K1)ESMD(4-4)		•	
	5.0250	SIK2.K	(1)=S(K2.K1)(SMD(5.4)			·
	5.0251	C/K3-K	11=S(K3-K1) & SMD(6.4)			•
	5.0252	MMC.11.	K1)=MD(1.4)			
	5 02 52	MM / 12.	K11=ND(2.4)			
	5.0255	MML13.	K1)=MD[3.4]		•	
	5.0255	NM(K)	K1)=MM(K1.K1)EMD(4.4)			
	5.0256	MMEK 2	K11=MM(K2.K1)END(5.4)			
	5 02 57	MMIK2.	K11=MM(K3.K1)END(6.4)			<u>, ~ · · · · · · · · · · · · · · · · · · </u>
	5.0269	47 IS (D)	( IK 3-11) 49.48.49			
	3 + VE 70 5 . A7 86	47 17 1KL	2)= SMD(1,5)		•	· · · · · · · ·
	5.0260	40 31014N	2)=540(2,5)		•	
	00200	51 JZ #N		• •		
	3 •U 291	217342	21 - 2101 2727 21 - 2101 2727			
		21214	21-31N11N21434014793/ .			•
	5.0205	2182+8	21-318486763638013431			
	3.0204	21K3+K	NJ)-NU() 61	• .		
	3.UZ03	MMIJL	NC1=NU(1+21			

.

<b>T</b> : • <b>F</b>	

	<b>A</b>		B	. •	•	,		
Γ	S.0266 S.0267 S.0268	MM(J2,K2)=MD(2,5) MM(J3,K2)=MD(3,5) MM(K1,K2)=MM(K1,K2)&MD(4,5)		· ·		•	·.	
	S.0269 S.0270 S.0271	MN(K2,K2)=MM(K2,K2)&MD(5,5) MM(K3,K2)=MM(K3,K2)&MD(6,5) 49 [F (RL(JK3)) 51,50,51	: .			· .		•
	S.0272 S.0273 S.0274	50 S(J1,K3)=SMD(1,6) S(J2,K3)=SMD(2,6) S(J3,K3)=SMD(3,6)	• •					
	S.0275 S.0276 S.0277	S(K1,K3)=S(K1,K3)&SMD(4+6) S(K2,K3)=S(K2,K3)&SMD(5+6) S(K3,K3)=S(K3,K3)&SMD(6+6)				• •		
	5.0278 5.0279 5.0280	MM(J1,K3)=MD(1,6) MM(J2,K3)=MD(2,6) MM(J3,K3)=ND(3,6)	· .	·		·		
	5.0281 5.0282 5.0283	MH(K1;K3)=MH(K2;K3)&MD(5;6) MH(K3;K3)=MH(K3;K3)&MD(6;6) S) CONTINUE		•			•	
U.S.	- 3.02.04	C 2.B RETAINING OF REDUCED STU	FNESS AND MAS	S MATRICES	<b>i</b> •		· ·	
Jeff	S•0285 S•0286 S•0287	C DO 150 [=1,N DO 150 J=1,N SM(I,J)=S(I,J)					•	. :
puting	S.0288 S.0289 S.0290	MSM(I,J)=MM(I,J) 150 CONTINUE WRITE(6,100)	•			•	•	
Join	5.0291 5.0292 5.0293	100 FORMAT (INITIONSTIFFICESS HARRAN DD 110 I=1+N WRITE (6,111) (S(I+J)+J=1+N)		·		· · ·	•	
resity.	5.0295 5.0295 5.0296 5.0297	WRITE(6,250) 250 FORMAT (1H0,11HMASS MATRIX) DD 310 [=1.N	•	· ·		•		
Unive	S.0298 S.0299 S.0300	WRITE(6,111)(MSM(I,J),J=1,N) 310 CONTINUE 111 FORMAT (1H0,8E15.6)	• .				•	•
લા		C 3. CALCULATION OF EIGENVALUE	S AND EIGENVE	CTORS.	•			
W	5.0301	CALL, NROOT (N. SM. MSM. XL. XX) C C PRINTING OF NATURAL FREQUENCIES.						•
	\$.0302 \$.0303	C WRITE(6.500) 500 FORMAT(//'1'. NATURAL FREQUENCIES IN	RADIANS PER	SECOND •)		•		
	S.0304 S.0305 S.0306 S.0307	DD 600 [=1.N Evalue=Sort(XL(I)) 600 Write(6.400) Evalue 600 Format(1H0.F20.8)						
•	5.0308	END		:		• •	•• •	
•	•		· ·					•
•		• ••			·			151
-								· .

	101	PROGRAM TO FIND THE NATURAL FREQUENCIES OF GRIDS.
0001		DIMENSICN L(20),X(20),Y(20),JJ(20),JK(20),IX(20),IY(20),CX(20),CY( 12C),RL(50),CRL(50),SMD(6,6),S(50,50),MM(50,50),MD(6,6),AX(20),
		1XL(39),XX(39,39),MSM(39,39),SM(39,39)
002		REAL IX; IY; L; MD; MM; MSM INTEGED CDI DI
505		RALENCE CALINE
	i i i	C IX TIRSION CONSTANT
•		C IY M.O.I. ABOUT Y AXIS
004		WRITE(6.1)
	(	
005	(	C#####################################
•	·	C C C 1. INPUT AND STRUCTURE DATA
		C LA STRUCTURE PARAMETERS ELASTIC MODULUS AND DENSITY
004	č	
007		2 FORMAT(// 102+ STRUCTURE CATAD)
009		WRITE (6,3)
009		3 FORMAT ('OA,' M N NJ NR NRJ E G
010		READ (5+4) M+NJ+NRJ+E+G+RA
011		4 FCRMAT(415,2F10.0,F10.8)
012		
014	•	5 FORMAT(515,F15.0,F10.0,F12.8)
•	(	
		C IB. JOINT COORCINATES
015	,	hRITE (0,6)
016		6 FURMAT ('00, 'JOINT COORDINATESA)
017		NRITE (0)7) 7 Eordat (103.1)01NT - X COORD - Y COORDA)
019		DD 101 J=1+NJ
020		READ (5,8) J,X(J),Y(J)
021		8 FORMAT (15,2F10.2)
023		9 FORNAT (15.2F10.2)
024		101 CENTINUE
	9	C IC. MENNED DESIGNATIONS AND DRODEDTIES
	č	v - IGE HEHDER VEGIGHHIIDHG HHV FROFERIIEG
025	•	WRITE (6,10)
026		IO FORMAT ('00, MEMBER CESIGNATIONS AND PROPERTIESO)
28		11 FORMAT (1HU, 6HMEMBER, 2X, 2HJJ, 3X, 2HJK, 5X, 2HIX, 9X, 2HIY, 6X, 2HAX, 10X, 1
		1HL,9X,2HCX,8X,2HCY)
029		$DO 102 I = 1 \bullet M$ $PEAD (5-12) I = 1 (1) \bullet (1) \bullet (1) \bullet (1) \bullet (2) (1) \bullet (2) (1)$
031		12 FORMAT(315,3F10.6)
32		JI=JJ(I)
		50 N
		MCGILL UNIVERSITY COMPUTING CENTRE
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approximation for the second of the second second second second second second second second second second second

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0033		(T.m.)K(T)
0034		
0035		
036		
37		
120		
30	•	
40	12	
40	102	
741	C 102 (	
		IN THIS RESTRAINT LIST. CHNILLATIVE DESTRAINT LIST
	č	
	č	INITIALISE REALS TO ZERO
	č	
042	•	v.J3=3+N.I
943		10 104 J3=1+NJ3
044		31(13)=0
45	104	ONTINUE
46		(RITE (5.14)
47	14	ORMAT ( 400. JDINT RESTRAINTSO)
48		APITE (6.15)
69	15	ORMAT (100-1JCINT X RSTRT Y RSTRT Z RSTRT2)
50		0 103 J=1-NRJ
51	N	3EAD (5.16) K.RL(3*K-2).RL(3*K-1).RL(3*K)
52	16	FORMAT (4110)
53		4RITE(6.17) K.RL(3*K-2).RL(3*K-1).RL(3*K)
54	17	FCRMAT (15.3110)
55	103	
56		RL(1)=RL(1)
57	1	DD 105 K=2,NJ3
58		CRL(K)=CRL(K-1)+RL(K)
59	105	CONTINUE
	C	
	C C	2.A GENERATION OF OVERALL STIFNESS AND MASS MATRICES.
060	v	
061		CD 106 J3#1-NJ2
)62		00 106 K3=1-NJ2
63		5(13.K3)=0.0
64		YM(J3.K3)=0.C
65	106	CONTINUE
66 .		00 51 [=1,M
67		J1=3+JJ(I)-2
68		J2=3+JJ(I)-1
69		J3=3+JJ(I)
70		<1=3+JK(1)-2
71		<2=3+JK(I)→1
72		(3=3+ JK(1)
73	•	SCM1=G*IX(I)/L(I)
74		SCM2=4.0*E*IY(I)/L(I)
75		SCN3=1.5+SCM2/L(1)
76		SCM4=2.0+SCM3/L(I)
77		Z=RA+L(I)/420.0
78		IF (RL(J1)) 18,15,18
	•	
		· · · · · · · · · · · · · · · · · · ·
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•	•• • • • •					•	1
0079	19 J1=J1-CRÍ(J1)		•			· .	
0080					•	•	
0081				•		• •	
0082	20 IE /01/ (2)\ 21 22 21						
0083	20 IF (KL(J2)) 21922921 22 12-12-CD1(12)	•	. • •				
0085	22 J2=J2=LRL(J2)				•	,	
0084	GU TU 23					•	
0085	21 J2=N+CRL(J2)	•••					
0086	23 IF (RL(J3)) 24,25,24						
008,7	25 J3=J3-CRL(J3)			•			
0088	GO TO 26						
0089	24 J3=N+CRL(J3)	•					
0090	26 IF (R) (K1)) 27-28-27					•	•
0091	28 KlaKlaCVI/Kl)					.*	
0092				•		•	•
0093		•			<u>.</u>		
0004	27 NI#NYUKLINIJ .				•	•	
0094	29 IF (RL(K2)) 30,31,30						
0095	31 K2=K2~CRL(K2)						
0096	GO TU 32		•			•	
0097	30 K2=N+CRL(K2)						
0098	32 [F (RL(K3)) 33.34.33					•	
0099	34 K3=K3-CKL(K3)	• •					
0100	60 TO 35					•	
	JJ NJ4NTURLINJ					•	
0102	35 WRITE(6,200) J1,J2,J3,K1,K2,K3,Z	•			•		
0103	200 FORMAT(1H0,6I5,E15.8)		•				
0104	CY2=CY(1)+CY(1)				,		
0105	C×2=C×(1)+C×(1)				•		
0106	CXY=CX(I)+CY(I)		• • •				• •
0107	SMD(1.1)#SCM1#CX2+SCM2#CV2	•					
0108							•
0100	SHD(1 9)-/CCN1 CCN2+CNN		•				
0110	5HD12 11+2/*15CH1-5CH2/+CX1			•			•
	SMU(2+1)#SMU(1+2)		•	•			
. 0111	SMD(1,3)=SCM3+CY(I)						
0112	SMD(3+1)=SMD(1+3)					•	
0113	SMD(1,4)=-SCM1+CX2+SCM2+CY2/2.0	•.					
0114	SMD(4,1)=SMD(1,4)						
0115	SPD(1+5) =- (SCP1+SCM2/2-01+CXV		•				
0116	SMD(5,1)#SND(1,5)		•		· .	•	•.
0117						•	
	Smull + 01 == Smull + 31			•		•	
0118	SMD(6,1)=-SMD(1,3)		-		,*		•
0119	SMD(2,2) #SCM1#CY2+SCM2#CX2			•	· · · · · · · · · · · · · · · · · · ·		
0120	SMD(5,5)=SMD(2,2)						
0121	SMD(2+3)=-SCM3+CX(I)	.,	•				
0122	SMD(3.2)=SKD(2.3)						
0123	SMD(2.4)=SMD(1.5)			27.0 N			
0124	SND(4,2)=SND(1,5)	•			•		•
0126	SHD(3 5)			1.54		• •	•
0125			•	•			
0120	SMD(5,2)=SMD(2,5)			· · i		•	•
0127	SMD(2,6) =- SMD(2,3)						
0128	SMD(6,2)=-SMD(2,3)						•
0129	SMC(3,3) = SCM4						
0130	SMD(6.6)=SMD(3.3)			•			
0131	SMD(3.4)#SMD(1.2)		: •				
0132	SNO(4,2)=SNO(1,2)						
1	JUN 793/-JUN 193/						
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		•••
0133	SMD(3,5)=SMD(2,3)	•
0134 .	SMO(5,3)=SMD(2,3)	
0135	SMD(3,6)=-SCM4	
0136	SMD(6.3)=SMD(3.6)	
0137	SMD(4.5)=SMD(1.2)	
0137	CHD/5 ()-CHD/1 2)	
0128	· SPU(S)4)=SPU(1)2/	
0139	SMD(4,6) = -SMD(1,3)	
0140	SMC(6,4)=-SMD(1,3)	
0141 ·	SMD(5,6)=-SMD(2,3)	
0142	SMD(6,5)=-SMD(2,3)	
	C	
0143	AT=1X(1)/AX(1)	
0144	C¥2A1=A1=C¥2	
0144	CV2A1-A1+CV2	
0143		
0140	GATAL=AL+CAT	
0147	CX2L=CX2+L([)+L([)	
0148	CY2L=CY2+L(I)+L(I)	
0149	CXYL=CXY+L(I)+L(I)	
	C	
0150	MD(1,1)=(140,0*CX241+4,0*CY2L)*Z	
0150		
0151	ML(49414FU(191) MA(1) D) -///A A+CVVA1-A A+CVVI) 1+7	
0152	MU(1+2)=(140+0+0ATA1=4+0+0ATE)+2	
-0153	MD(2,1)=MD(1,2)	
0154	MD(1,3)=22.0+L(I)+CY(I)+Z	
0155	MD(3,1)=MD(1,3)	•
0156	MD(1,4)=(70.0*CX2AI-3.0*CY2L)*Z	
0157	MD(4.1) = MD(1.4)	
0159	MD(1,5)=(70,0+CXYAI+3,0+CXYL)+Z	
0150	NO(5.1) + NO(1.5)	
0137	ND() () () () () () () () () () () () () (	
0100	MU(1+0)=13.0+L(1)+C(1)+2	
0161	PD(6,1)=PD(1,0)	
0162	MD(2,2)=(140.0*CY2AI+4.0*CX2LJ*Z	
0163	ND(5,5)=ND(2,2)	•
0164	MD(2,3)=-22.0*L(I)*CX(I)*Z	
0165	MD(3,2)=MD(2,3)	
0166	MD(2.4)=ND(1.5)	
0167	HO(4,2) = HO(1,5)	
0101	NO(2,5)=170,0±CV241-3,0±CV21)±7	
0100	MD(E - 2) = (10 + 0 + 0) = E + 0	
0169		
0170	MD(2,6)=-13.0*L(1)+CX(1)+2	
0171	MD(6,2)=MD(2,6)	
0172	MD(3,3)=156.0+Z	
0173	MD(6,6)=>D(3,3)	
0174	MD(3.4) = -MD(1.6)	
0175	MD(4,3) = -MD(1,6)	·
0174	MD(3.5) == MD(2.6)	
0170		
0177		
0178	MD(3+6)=63+0+2	
0179	MD(6,3)=ND(3,6)	
0180	MD(4,6)=-MD(1,3)	
0181	MD(6,4)=-MD(1,3)	
0182	PD(4.5) = PD(1.2)	
0193	MC(5.4)=WD(1.2)	•
0103	HD15.41==HD12.31	
0104	muloto/mul2to/	
	•	
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	•	,				4 ) 2	.•
0185		MD(6.5)=-MD(2.3)		·			
0107	c				•	• • .	
0186	-	JJ3=3+JJ(I)					
	С		•			•	
0187		IF (RL(JJ3-2)) 41,40,41				•	•
0188	. 40	$0  S(JI_{+}JI) = S(JI_{+}JI) + SMU(I_{+}I)$	•	•			
0189		5(J2+J1)=5(J2+J1)+5HD(2+1)	,				*
0190		2(J3+J1)=2(J3+J1)=200/3+1/		· · · ·			
0192		S(K2.11)=SMD(5.1)				·	
0193		$S(K3 \cdot J1) = SMD(6 \cdot 1)$					
0194		MM(J1,J1)=MM(J1,J1)+MD(1,1)				· · · ·	
0195		MM(J2, J1) = MM(J2, J1) + MD(2, 1)		•		-	
0196		MM(J3,J1)=MM(J3,J1)+MD(3,1)	•				
0197		MM(K1, J1) = MD(4, 1)				•	
0198		MM(K2, J1) = MD(5, 1)	•	•			
0199		MM(K3+J1)=MD(0+1)		۰.		• •	
0200	4	1 1F (KL(JJ3-1)) 43142443			•		
0201	. 4	(12, 12) = (12, 12) + (12, 2)	· .	· ·			
0202		$S(J_3, J_2) = S(J_3, J_2) + SMD(3, 2)$			• •		•
0204		S(K1, J2) = SMD(4, 2)			•.		
0205		S(K2, J2) = SMD(5, 2)				· · · ·	
0206		S(K3, J2) = SMD(6, 2)					
0207		MM(J1,J2) = MM(J1,J2) + MD(1,2)		•			
0208		MM(J2, J2) = MM(J2, J2) + MD(2, 2)					
0209		MM(J3, J2) = MM(J3, J2) + MD(3, 2)			•		
0210		MM(K1,J2)=M0(4,2)	•				
0211		MM(K2+J2)=MU(2+2)			•	•	
0212	4	A TE (RI (.1.13)) 45.44.45	•				
0214	4	4 S(J1.J3)=S(J1.J3)+SMC(1.3)				•	
0215	-•	S(J2,J3) = S(J2,J3) + SMD(2,3)		•			
0216	•	S(J3, J3) = S(J3, J3) + SMC(3, 3)					
0217	•	S(K1, J3) = SMD(4, 3)					
0218		S(K2, J3) = SMD(5, 3)		•			
0219		S(K3,J3)=SMD(6,3)					•
0220		MM(J1, J3) = MM(J1, J3) + MU(1, 3)			•	•	
0221		MM(12,12)=MM(12,13)+ND(3,3)		-		•	
0222		MM(K), 13)=MD(4,3)				•	
0225		$MM(K2 \cdot .13) = MD(5 \cdot 3)$		•			
0225		MM(K3,J3)=MD(6,3)					:
	C	•			•	·	•
0226	4	5 JK3=3+JK(I)			•	•	
	С						
0227		IF (RL(JK3-2)) 47,46,47					
0228	4	6 5(J1,K1)=SMD(1,4)			•		
0229		5(J2+K1)=5MU(2+4)			. •		
0230		2(J3+K1)#5MU(3+4) 2(J3+K1)#5MU(3+4)				•	
0231		C1K2*K1)=C1K2*K1)+CMU(2*7)			-		•
0232		S(K3.K1)#S(K3.K1)+SMD(544)		•			
0234		MM(J1.K1)=MD(1.4)		· ·			
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0235	MM(J2,K1)=MD(2,4)		•
0236	MH(J3+K1)=MD(3+4)		•
0237	MM(K1,K1) = MM(K1,K1) + MD(4,4)	·	
0238	MM(K2,K1) = MM(K2,K1) + MD(5,4)		•
0239	MM(K3,K1) = MM(K3,K1) + MD(6,4)		
0241	4/ IF (RL(JK3-1)) 49,48,49		•
0242	40 S(J1+K2)=SMD(1+5)		#5
0243	S(13,K2)*SMD(2,5)		
0244	S(K) (K2) = S(K) (K2) (KD(/ E)	•	-
0245	S(K2+K2)=S(K2+K2)+SMC(4+5)		
0246	S(K3+K2)=S(K3+K2)+SMC(6-5)		
0247	MP(J1,K2)=MD(1,5)		
0248	MM(J2,K2)=MD(2,5)		
0249	MM(J3,K2)=MD(3,5)		
0250	MM(K1,K2)=MM(K1,K2)+MD(4,5)		
0252	MM(K2,K2)=MM(K2,K2)+MD(5,5)		
0253	49 15 /PL/ K211 51 50 51	· · · ·	•
0254	50 S(J1.K3)=SMD(1.K)		
0255	S(J2+K3)=SMD(2+6)		
0256	S(J3,K3) = SHD(3,6)	·	
0257	S(K1,K3)=S(K1,K3)+SMU(4.6)		
0258	S(K2,K3)=S(K2,K3)+SMD(5,6)	· · · · ·	
0259	S(K3,K3) = S(K3,K3) + SMD(6,6)		
0260	MM(J1,K3)=MD(1,6)	•	
0261	MM(J2+K3)=MD(2+6)	•	
0263	TTIJJ4KJJ4MU(J40]		
0264	MM1K2_K3)=MM1K2_K3)+MU1K_40]		
0265	MM(K3+K3)=MM(K3+K3)+MD(6+6)		
0266	51 CONTINUE		
0267	C 2.B RETAINING OF REDUCE	STIFFNESS AND MASS MATRICES.	
0268	DO 150 1=1.N		· .
0269			:
0270	MSM(1.J)=MM(1.1)		•
0271	150 CONTINUE		
0272	WRITE(6,250)	•	
0273	250 FORMAT (IHO, 11HMASS MATRIX)		
UZ 14 0276	DD 310 I=1,N	•	
0276	$\frac{\text{KRIE}\{0, \text{III}\}(\text{MSM}(\text{I}, \text{J}), \text{J}=1, \text{N})}{310}$		
0277	111 FORMAT (100,9515 4)		
	C	· · ·	
	C 3. CALCULATION OF EIGEN	ALUES AND EIGENVECTORS.	
0278	CALL NROOT (N,SM,FSF,XL,XX) C		
	C PRINTING OF NATURAL FREQUENC C	÷S•	
UZ [Y	WRITE(6,500)		
· . ·	*		Ц
		· · · · ·	7
<u> </u>		· ·	
		MeGHI I HNIVEDELTY	

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0280				500	FORMAT(//'1a, 'NATURAL FREQUENCIES IN RADIANS PER SECOND a)
0281					DD 600 I=1,N
0282					EVALUE=SQKT(XL(I))/6.283184
0283				600	WRITE(6,400) EVALUE
0284				400	FORMAT(1H0,F20.8)
		•	Ç		PRINTING OF NORMALIZED NODAL AMPLITUDES.
			C		•
0285					WRITE(6,550)
0286	•			550	FURMAT(//'10,'NORMALIZEC NODAL AMPLITUDES@)
0287					DO 650 I=1,N
0288				650	WRITE(6,651) (XX(I,J),J=1,N)
0289				651	FORMAT(1H0,10F10.4)
0290					END

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## APPENDIX D

## PROGRAM FOR THE PARAMETRIC INSTABILITY OF COLUMNS

The programs in this section are also coded in Fortran IV and are used to obtain the natural frequencies and the buckling loads for a particular idealization of the column.

Dl: Program to find the natural frequencies and buckling load of a pinned-ended column.

Input

Cols. Notation Description A Number of Subdivisions (I5) one card

1-5 NM Number of parts in which the column

is divided

B Stiffness Matrix (8F10.3) (NM-1) cards

1-80 S Elements of stiffness matrix

C Mass Matrix (8F10.3) (NM-1) cards

1-80 M Elements of mass matrix

## Output

The output consists of the input data, the modal shape, the natural frequencies and the buckling loads for all the modes.

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	r	•	•	·
	č		0.00	
	L L	DIANDAM TA CING THE FIGHT WALKER AND FIGHT WETTER	KHS 1	•
		PROGRAM TO FIND THE EIGEN VALUES AND EIGEN VECTORS OF A PINNED	KIIS Z	, · · ·
	Ļ	ENDED COLUMN	RHS 3	
	C	**********	RHS 4	
2001	•	DOUGLE PRECISION A,B,C,U,X,Y,Y1,EIGV,DELT,DIV,EPS,R	RHS 5	
002		DIMENSION SS(8,8)		•
003		DIALNSIUN A(8,8),U(8),DF(8),DV(8)	RHS 6	
U04		DIMENSION B(8,8).C(8,8).X(8).Y1(8).Y(8).EIGV(8).	RHS 7	
		1M(2,2),S(2,2),L1(2),M1(2)		•
005		EQUIVALENCE (X(1).Y1(1))	8HS 9	
006		CALL PGMCHK	PHS 10	
007				
001	•		KHS 11	
000		READIST 201 NM	RHS 12	
704	20	FURMATELS	RHS 13	
110		WRIFE(6,21) NM	RHS	
4 <b>11</b> -	. 21	FURMAT(1H1,5X,3HNN=,I3)	RHS 15	
12		$DU \ge I = 1, NM$	RHS 16	
13	2	REAU(5,1) (S(1,J),J=1,NM)	RHS 17	
14		FURMAT(10F10-3)	845 18	
15	-	WRITE(6.3)	DUC 10	
16	2		000 20	
17		FORMATION F STIFFNESS MATKIN F	KH3 20	
		DU 4 ITI, NM	RHS 21	
18	•	WRITE(C+1) (S([+J]+J=1+NM)	RHS 22	2
9		DO 40 I=1,NM	RHS 23	• • •
20	• 40	READ(5,1) (M(1,J),J=1,NM)	RHS 24	
1	• .	WKITE(6,71)	RHS 25	5
2	· 71	FURMAT( "0", " MASS MATRIX")	RHS 26	
3		DO 8 1=1-NM	RHS 27	
4	· 8	WRITE(A,1) (M(1,J), J=1, NM)	RHS 28	
14	•			<b>)</b>
24				•
20				
21	10			
28		CALL MINV(S,NM,D,LI,MI)	RHS 29	
129		DO 31 I=1,NM	RHS 30	)
30	• •	DO 31 K=1, N/4	RHS 31	
31 .		SUM=0.	RHS 32	2
32		00 30 J=1.NM	RHS 33	
33	30	SLM=SUM+S([.J])+M(J.K)	RHS 34	
34	. 31	6(1.K)=SUM	RHS 35	
35			DUC 24	
24		NEIGOFJJ Erdantienie – dornict matoty ei	KN3 20	
20		FORMATIVU'; FRODUCI MATRIA ')	RHS 37	
31	• •	DU 32 I=I,NM	RHS 38	3
38	32	WRITE(6,1) (B(1,J),J=1,NM)	RHS 39	Э
	C	HATKIX C=MATRIX B	RHS 40	) • •
i39 -	121	DO 98 1=1,NM	RHS 41	
40		DU 98 J=1,NM	RHS 42	
41		C(1,J) = B(1,J)	RHS 47	- , A
	C I			
	č	S SMALL MUMBER TO THAT WHETHER ANY DIACOMAL TO 2000 OD NOT		
		S SHALL NUMBER TO LEST WHETHER ANT DIAGUNAL IS ZERU UK NUT	KH5 45	
	L		KHS 46	
72		EPS=.00000001	RHS 47	7
193		WRIIE(6,64) EPS	RHS 48	8
	**	FURMAT(1H0.5X.4HEPS=.F10.9)	DHC 40	a
			NII.J 777	

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•	•		· · · · · · · · · · · · · · · · · · ·	•		`*	
		C		RHS	50	•	
		č	TEST TO FIND IF LAST EIGENVALUE REACHED	RHS	<sup>5</sup> .51		• •
0045			00 110 II=1.NM	RHS	52		
		C		RHS	53		•
0046			IF(II-NM) 116,117,116	RHS	54		
0047	•	- 116	• N=NM-11+1	RHS	55 °		
0048			N1=N-1	RHS	56		
		С		RHS	57		
	•	c	· SET YI(I) AS A UNIT CCLUMN MATRIX	RHS	58	•	
		C		RHS	59	•	
0049		100		RHS,	60	•	
0050		100		RH3 DUC	61		
0091		· · · ·		PHC	62	•	
		č	UP TO STATEMENT 115. SUCCESSIVE APPROXIMATION IN FINDING THE	RHS	64		
		ă	EIGENVALUES XAND THE FIRST FIGENVECTOR DNLY < IS CARRIED OUT	RHS	65	•	
0052		•	DU 101 I=1.N	RHS	66	•	
0053			Y(1)=0.	RHS	67		
0054			DD 101 J=1,N	RHS	68	· · ·	
0055		101	Y(I)=Y(I)+C(I,I)+YI(J)	RHS	69		
0056			J=1	RHS	70		
0057		92	IF (Y(J)) 89,91,89	RHS	71		
0058		69	· · · · · · · · · · · · · · · · · · ·	RHS	72		· ·
0059			GU TO 93	RHS	73		
0060		91		RHS	74	• • • • • • • • • • • • • • • • • • •	
0061		•	GU 10 92	RHS	75		
			STALMAN HE THE ETDET EVENENT OF THE CLASHWOOTON	RHS	16		• •
0.06.2		63	DO GO LENALUE IS THE FIRST ELEMENT OF THE EIGENVECTUR	- KIIS	74	•	. <u>.</u>
0063		73		DHC	70		
0064			$1 = \{1 = 0, 1 $	RHS	80	· .	
0065		102		RHS	81		
0066		90	CONTINUE	RHS	82		
0067			IF(KC) 103,103,104	RHS	83		
		C		RHS	84		
		C	TRANSFER X(M+1)TH EIGENVECTOR TU X(M)TH EIGENVECTOR	RHS	85	· ·	
0068		104	• DO 115 I=1,N	RHS	86		
0069		115	• Y1(I)=Y(I)	RHS	87		
0070			GŬ TŬ 99	RHS	88		
		C		RHS	87		
		6	THRE EIGENVALUE IS THE FIRST ELEMENT IN THE EIGENVECTOR	RHS	90		
0071		103	I GUVIII)#R	RHS	91		
0072		L.	DO TO STATEMENT 112, THE STZE OF MATRIX C IS REDUCED BY UNE	KHS	92		
0072		100		RIS DUC	93		
0074		101			94		
0075				RHS	96		
0076		112	A(1-1, J-1) = C(1, J) + C(1, J) + X(1-1)	RHS	97		
		C	REDUCTION OF SIZE OF MATRIX C BY ONE COMPLETED	RHS	98		
0077			DO 113 1=1,N1	RHS	99		
0078			DD 113 J=1,N1	RHS	100		•
0079		113	C(I,J)=A(I,J)	RHS	101		
0800			GU TU 118	RHS	102		
0081		117	EIGV(11)=C(1,1)	RHS	103		
		• • •		•	•		16
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118 WRITE(6,54) II.EIGV(11) 54 FURMAT(12,16HTH. EIGENVALUE=,E12.5)	RHS 105 RHS 106	•	•
C UP 10 STATEMENT 9013, THE EIGENVECTORS ARE SOLVED	RHS 107		•
	RHS 108	۰.	
NLA 31=NFF1 tc/tt_N/ 1/1/1/1/2/2	RHS 109	•	
17117171717171717171717171717171717171	RHS 110		
	RHS 111		
	RHS 112		
16(1-1) 107.139.107	RHS 113		•
139 A(1,J) = A(1,J) = EIGV(11)	RHS 114	•	
107 CUNTINUE	RHS 115		•
105 J(I)=-B(I,NM)	RHS 116		·
C	RHS 117		
C TEST TO SEE WHETHER DIAGONAL ELEMENT IS ZERO OR NOT	KU2 110	•	
C	RHS 120		
DU 9015 [#1,NN	RHS 121	•	• .
[F[[-NN] 4021,4007,4021	RHS 122		
9021 1F141411-EP31 9003490049007	RHS 123		
YUUJ 171-711/1/-273/ YUJ0/70/0/700/	RHS 124		
	RHS 125		
	RHS 126	•	• • • •
	RHS 127		
	RHS 128	•	
	RHS 129		-
C · · · · · · · · · · · · · · · · · · ·	RHS 130		
C DIVIDE ALL ELEMENTS OF I-TH EQUATION BY A(I,I)	RHS 131		
C	RHS 132		•
DU 9009 J=1+NN	KHS 133		
9009 A([,J)=A([,J)/DIV	KH5 134	•	
C THE REPORT OF THE OTHER FOUNTIONS TO JERO	DHC 136		• •
C REDUCE THE I-TH ELEMENT OF THE DIMER EQUATIONS TO ZERO	DHG 137		
	RHS 138		
	RHS 139		
UELI=A(MM,I) 0014 IELWH_I) 0010-0015-0010	RHS 140		
9010 1F(MM-17 9010)901399010	RHS 141		•••
	RHS 142		
(1) = (1)	RHS 143		
9015 CUNTINUE	RHS 144		
Y(NM)=1.0	RHS 145		
00 108 f=1,NN	RHS 146		
108 Y(1)=U(I)	RHS 147		
C	RH5 148		
C CALCULATION OF " LAST ANGLE "	KH2, 160 -		
C	NH3 19V 940 161		
141  Y(NLAST) = Y(I)	RHS 152		• •
	RHS 153	•	
160 Y{NLASIJ=T{NLASIJ+2.047111	RHS 154	4	
	· RHS 155		-
	RHS 156		
	RHS 157		
			•

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	L.	AND I LOUGH		•						
	0120		1)V/1)#()_5*Y(1)	RHS	158			•		
	0121		D(1631=2.NM	RHS	159					
	0122	163	$D_{V}(1) = D_{V}(1-1) + Y(1)$	RHS	160		. •	•		
	0122	105		RHS	161			•		•
	0123		FLOWAT(1) AV. ICHTH. FLOEN VECTOR.5X.19HDISPLACEMENT VECTOR)	RHS	162		•		°4	
	0124	-		RHS	163					
	0125		DU IU KAIIMA	RHS	164					
	0126	10		RHS	165					
	0127	• • •	NRIJELUJIOIJ TUNENJIJ	RHS	166			:		
	0128+	101		RHS	167					
	0129	79	- FURMA1 (F22+0+F12+0)	RHS	168					
	0130		DD 201 1=1,NM	2HG	169	•				
	0131		SUM=-EIGV(II) #Y(I)		170			•		
	0132		DC 200 J=L,NM	кнэ рыс	171					•
	0133		STW=2NW+B(1*)*4(1)	040	172			•		
	0134	200	CLNTINUE	KI S	172	· ·	•	•		
	0135	264	FORMAT(1H ,2HI=,12,4X,4HSUM=,F15.8)	KHS	173					
	0136	201	WRITE(6,204) 1,SUM	RHS	174					
	0137		IF(11-1) 151,151,152	RHS	175					
	0138	15	. DU 143 [=1,NM	RHS	176					
	0139	14	5 DF(1)=DV(1)	RHS	177					
	0140		GU TÙ 310					•		
	••••	C		RHS	179					
		č	LEST FOR ORTHOGONALITY OF EIGEN VECTORS	RHS	180					•
		č	•	RHS	181				•	
	0141	15		RHS	182					
•	0142		DO 154 LELANM	RHS	183					
	0142	15	DO 175 1-1900 • Chumschuwahf (11+DV(1)	RHS	184				•	
	0145	\$ Z.		RHS	185	. •				
	0144		NTIE(0)101 1100000	RHS	186					
	0142	12							•	
		L L	A ANNA MARKAN DE CALEGRINIS METUDO							
		C C	APPLICATION OF GALERATIN'S HEIHOU			•				
		C	***************************************							
		C	MULTIPLICATION OF MASS MATRIA							
	0146	• 31	D D0 300 1=1,NM							
	0147	•	DV(I)=0.0							
	0148		DD 300 J=L,NM							
	0149		{L}Y+{L+L}+A(L+L)+A(L+L)+A(L+L)+A(L+L)+A(L+L)+A(L+L)+A(L)+A(	7						•
	0150	30	O CONTINUE			•			•	
	0151		GMASS=0.0							
	0152		DG 301 1=1,NM							
	0153	30	1 CMASS=CMASS+DV(1) +Y(1)							
	0154		WRITE(6,302) GMASS							
	0155	30	2 FURMAT('0','CCEFFICIENT OF MASS MATRIX',5X,6HCMASS=,E20.8)							•
		C	MULTIPLICATION OF STIFFNESS MATRIX							
	0156		DG 303 I=1,NM							
	0157		Dv([]=0.0							
	0158		00 303 J=1.NM							
	0159		UV(1) = DV(1) + SS(1, J) + Y(J)							
	0160	30	3 CONTINUE							
	0161		CSTIFE=0.0				•			
	0142		D() 104 (1=1-NM							
	0102		CSTIFF=CSTIFF+DV(1)+Y(1)			•			•	
	0103	20		• *						
	0104	30	- UNITENSE - STIEF							
	0102	•	ANTIE (0)3037 COLLER							
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305 FORMAT( '0', 'COEFFICIENT OF STIFFNESS MATRIX', 5X, 7HCSTIFF=, E20.8) 0166 MULTIPLICATION OF FORCE VECTOR С CFORCE=0.0 0167 00 306 I=1,NM 0168 306 CFORCE=CFORCE+(Y(1)-Y(1+1))\*Y(1) 0169 WRITE(6,307) CFORLE 307 FURMAT( 101, LOUFFICIENT OF FORCE MATRIX , 5X, 7HCFORCE=, E20.8) RHS 0170 0171 GFREQ=CSTIFF/CMASS .0172 GLUAD=CSTIFF/CFORCE 0173 308 FORMAT( '0', 'FREQUENCY BY GALERKINS METHOD', 5X, 6HGFREQ=, E20.8) WRITE(6,308) GFREQ RHS 0174 0175 WRITE(6,309) GLOAD 309 FURMAT( '0', 'BUCKLING LOAD BY GALERKINS METHOD', 5X, 6HGLUAD=, E20.8) 0176 RHS 187 0177 110 CONTINUE 188 RHS U178 END 0179

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TOTAL MEMORY REQUIREMENTS COLCRE BYTES

D2: Program to find the natural frequencies and the base inertia force for a column subjected to periodic support motion.

## Input

Cols. Notation Description A Number of Subdivisions (I5) one card

1-5 NMM Number of parts in which the column

is divided

B Member Properties (I5, 3F10.3) NMM cards

1-5	I	Counter
6-15	MS	Mass of link
16-25	KM	Stiffness of spring
26-35	L	Length of spring

C Axial Force Array (10F10.6) one card

1-80 FM Variation of the axial force along

the length of the column

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			~									
			L L				•			•		• •
-		0001		DIMENSION MS(40) +L(40) +KM(40) +PM(40) +SM(40) +RL(40) +CRL(40)	LIN	2		. •	•			
		0002		DIMENSION A(40,40),U(40),DF(40),DV(40),S(40,40)	1 TN							
		0003	•	DIMENSION B(40,40), C(40,40), X(40), Y1(40), Y(40), FIGV(40), FIGG(40),								•
				155(10.10).M(10.10).L1(10).M1(10).FM(10)		-		•	•		•	
•		0004		DOUBLE PRECISION L.KM.SM.RM.MS		5						
		0005		DOUBLE PRECISION A.B.C.U.X.Y.Y. FIGV.DELT. DIV. ED. D		0						
		0006		EQUIVALENCE (X(1), Y1(1))	LIN	7						•
•		0007	•	CALL PGMCHK	LIN	8						
1	ſ	0008	•	REAL M	LIN	9						•
		0009			LIN	·10		•				
•		0010			LIN	11			•	•		
1		0011	20		LIN	12						
)	1	0012			LIN	13						
-		0013		WRITE(0,21) NMM	LIN	14	·					
ł		0014	21	FUKMA1(1H1+5X,6HNMM±,I3)	LIN	15				•		
	1	0014		DU 252 I=1,NYM	LIN	16						
-	11.11	.0015	252	RL(I)=0	LIN	17			•			
_	1	0016		CRL(1)=0	LIN	18				•		
		0017		DQ 253 I=2,NMM	1 TN	10				•		
_	ic;	0018	253	CRL([)=CRL([-1)+RL([]	I TN	20						
-	0.01	0019	•	NM=NMM-CRL(NMM)	T T M	20				•		
		0020		WRITE(G.14) NMM.NM		21						
	1.1	0021	14	FORMAT(1H0+5X+4HNMM=+I3+5X+3HNM=+I3)		2.2		•			•	•
-	4 <b>G</b>	0022	•	WRITE (6,51)		23						
		0023	51	FORMAT("0"," JOINT RL CRI+)	LIN	24					•	•
		0024		DO 9 1=1.NMM	LIN	25						
**	6 <u>-</u>	0025	9	WRITE (6.6) I.BL(1).CRI(1)	LIN	26						•
		0026	6	FORMAT(316)	LIN	27	•					
	<b>6</b> 5)	0027	•		LIN	28						
••	K. 11	0028	20.6	READ(S.207) I. AS(I) MAILER IN	LIN	29				•		
		0029	207		LIN	30			•			
	500	0030		MD176/4 000	LIN	31			•			
•••	1999 I.	0031	224		LIN	32						
		0072		COMMAILING ORNOMBER 5X + 4HMASS + 6X + 9HSTIFFNESS + 6X + 6HLENGTH)	LIN	33			•			
		0032	20.0		LIN	34						
	الإتناع	0034	208	WRITE(6,209) I,MS(I),KM(I),L(I)	LIN	35			•			
		0034	209	FCRMAT(1H0,15,F15,11,2F10,5)	LIN	36					•	
		0035		R[AD (5,81) (FM(1),I=1,NM)	LIN	37						
		0030	81	FORMAT(10F8.5)	LIN	38						•
		0037		WRITE(6,82)	LIN	39						
		0038	82	FORMAT(*0*,*FORCE MATRIX*)	1 TN	Å0						
		0039		WRITE(6,83) (FM(I),I=1,NM)	ITN	40						
		0040	83	FORMAT(1H0,10F10.6)	I TN	A 2						
		0041	•	DO 210 I=1.0NMM								
		0042	•	DO 210 J=1.NMM		43						
••		0043		S(I,J)=0.0		44.						
		0044	210	CONTINUE		45						
		0045	•	DO 1 I=1,NM		46					•	
		0046		DO 1 J=1 NM	LIN	47	•					
		0047		$M(1 \cdot J) = 0 \cdot 0$	LIN	48						
		0048	1	SS(1,J)=0.0	LIN	49		•			-	
• ••	í	0049	•		LIN	50					•	
		0050	250		LIN	51				•	•	
		0051			LIN	52						
<b></b>		0052	944	·· ···································	LIN	53						
	· • •		677	ME=MET 8	LIN	54						
	· ·	•									Ĕ	
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	•					•••••••	•••••••		······································	•			·	
						•					••••			
0054			GO TO 211 TCDI = 1CDI ( 1 )		· · ·			55		• •				
0055	<b>-</b>		LL=ML-CRL(T)	•			E I N	57		• •				- 1
0056			DO 212 J=1.NMM	•	. ·		LIN	58					•	
.0057			IF (1.L-J) 244,244,243				LIN	59			•		. •	1
0058	24	13	GN TO 212	· .	•		LIN	60			•			
0059	. 24	4	JRL=J+CPL(I)				LIN	61						
0060		_	IF (NMM-JRL) 212.242.242			• •	LIN	62		· .				
0061	• 24	. 2	JGRL=J-CRL(JRL)+CRL(I)	•		·	LIN	63		· · ·				
0063	97		IF (RL(JRL)) 230+231+230 M(1CD) - ICD) - 45(1D) M(1T)					64						1
0014	<b>6</b> •		$\frac{1}{10} = \frac{1}{2}$			•		60			•			
0065	2:	30	M(ICRL.JCRL)=M(ICRL.JCRL)+MS(JRL)+L(I)	•			1.1 N	67					•	
0066	. 21	2	CONTINUE	.•				68						
0067			14L = 4L + 1				LIN	69			•	•		
0068	21	1	CONTINUE	•	•		LIN	70				•		
0069			DO 215 I=1.NMM				LIN	71			-			
0070	. 21	5	RM(I)=KM(I)/L(I)			•	LIN	72						
0071	••		DO 216 [=2,NMM				LIN	73						1
0072		6	SM(I)=KM(I)/L(I-1)				LIN	74						
0073		• •	DO 18 I=1,NMM				LIN	75	•			•		
0074	•	19	WRITE(0,11) 1,4M(1),6M(1)				LIN	76	•	•.				1
0076	<b>.</b> .	L	CC(1.1)+UM(1)+DW(2)+CM(2)		•		LIN	77			•			
0077			SS(1,2)==DM(2)					78						
0078	•		SS(2,1) = (RM(2) + SM(2) + SM(3))		• •			90						
0079			55(2,2)=RM(2)+RM(3)+SM(3)					81					• .	
0080			SS(2,3)=-SM(3)		·		LIN	82						
0091			S(1,1)=SS(1,1)				LIN	83 '			• '			1
0082			S(1,2)=SS(1,2)				LIN	84		• •				
0083			S(2,1)=SS(2,1)			•	LIN	85						1
0084			5(2,2)=55(2,2)				LIN	86						
0085			5(2,3)=55(2,3)				LIN	87			•			
0086			5(N'14,NMM) =RM(NMM)	٠			LIN	88				•		
0087	•		5(NMM+NMM-1)=-(RM(NMM)+5M(NMM)) 5(NUV+NVAN-2)=CM(NVAV)				LIN	89						
0089			3(1819) (1810) - 2 / 2 3 4(1814) MN=NMM-1				LIN	90					•	
0090						•	1.10	.41					•	
0091			DO 217 I=3.MN				1 1 1	92			•			1
0092			$S(I_{LL-2}) = SM(I)$				LIN	94		•				- 1
0093			S(I,LL-1) = -(RM(I) + SM(I) + SM(I+1))				LIN	93						
0094			S([,LL)=PM(])+RM([+1)+SM([+1)	•		•	LIN	36						
0095			$S(I_{+}LL+1) = -RM(I+1)$				LIN	97						- 1
0096		•	LL=LL+1				LIN	98						
0097	. 51	7	CONTINUE		•		LIN	99	•					
0008	•		DD 2110 I=3,NMM			•	LIN	100						
0099		~~	IF(RL(I)) 2400+2410+2400				LIN	101						- 1
0100	24	10					LIN	102						
0102	6.7	10						103						
0103			JRL=J+CRL(1)	•				104		-				
0104			IF (NMM-JRL) 2120, 2420.2420			•		105					•	1
0105	24	20	JCRL=J-CRL(JRL)+CRL(I)				LIM	107						1
0106			IF (RL(JRL)) 2300,2310,2300	· ·	•		LIN	108		•				1
			· · ·					- • •						
				•			•						6	1
				•									<b>7</b> ·	1
• •														
· ·										•		•		

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0107	2310 SS(ICRL,JCRL)=S(I,JRL)	LIN	109			
0108	GU TO 2120	LIN	110			
0109	2300 SS(ICRL+JCRL)=SS(ICRL+JCRL)+S(I+JRL)	LIN	111			
0110	2120 CONTINUE	LIN	112			•
0111	2110 CONTINUE	LIN	113		•	
0112	WRITE(6,3001)	LIN	114			
0113	3001 FURMAT(+0+,+STIFFNESS MATRIX+)	LIN	115	·		
0114	DD 3002 I=1.NM	LIN	116		:	
0115 •	3002 WPITE(6.3003) (SS(I.J).J=1.NM)	LIN	117	. :		
0116	WRITE(6,3004)	LIN	118	•		
0117	3004 FORMAT(+0+,+MASS MATRIX+)	LIN	119			
0118	DD 3005 I=1.NM	LIN	120			•
0119	3005 WRITE(6.3003) (M(I.J).J=1.NM)	LIN	121			
0120	3003 FURMAT(1H .9F13.5)	LIN	122			
0121	DD 3006 I=1.NMM	LIN	123			
0122	3006 WPITE(6.3003) (S(1.1).Jat. MMN)	LIN	124			
0123		LIN	125			•
0123		LIN	126			
0164		LTN	127			
0125		LIN	128			
0126		1 7 65	120			
0127	DU 30 J=1, NM		147			
0128	30 SUM=SUM+SS(L+J)=M(J+K)		130			
0129	31 B(1•K)=SUM	LIN	131			
	C MATRIX C=MATRIX B	LIN	132			
0130	121 DO 98 I=1,NM	LIN	133			
0131	00 98 J=1,NM	LIN	134			
0132	C(1,1)8≖C(1,1)	LIN	135			
	C	LIN	136	•		
	C EPS SMALL NUMBER TO TEST WHETHER ANY DIAGONAL IS ZERO OR NOT	LIN	137 ·			
	Č	LIN	138	•		
0133	FPS=.00000001	LIN	139	·.		
0134	WDITE(6.64) EPS	LIN	140			
0134	64 FORMAT (1H0-5X-4HFPS==E12-5)	LIN	141			
0133		LIN	142			
	C TEST TO FIND THIAST FIGENVALUE REACHED	LIN	143			
•		I TN	144	•		
		1.1.1.	145			
0136			145			
0137	IF(I[-NM) 116.117.116					
0138	116 N=NM-II+1		147			•
0139	N1=N-1	LIN	148			
	c	LIN	149			
	C SFT YI(I) AS A UNIT COLUMN MATRIX	LIN	150			
•	c	LIN	151			
0140	DD 100 I=1.N	LIN	152		÷	
0141	100 ¥1(1)=1.0	LIN	153			
0142	99 KC=0	LIN	154			
••••	c	LIN	155		•	
	C UP TO STATEMENT 115, SUCCESSIVE APPROXIMATION IN FINDING THE	LIN	156			
	C FIGENVALUES (AND THE FIRST EIGENVECTOR ONLY ) IS CARRIED OUT	LIN	157			
ALA 3		LIN	158	•	•	
0145		LIN	159			
0144 0148		LIN	160			•
0140		LIN	161			
0140		1 7 1	162			
0147	n=0	<b>L L F4</b>				
•						
•						
	•					

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0144       62       1F (Y(1)) DX-91.00       Lin       163         0145       67       10       11       11       164         0146       67       10       11       164       164         0147       67       11       164       164         0148       67       11       164       164         0149       67       168       11       164         0148       67       168       167       111         0148       67       168       11       164         0148       67       168       111       164       171         0148       102       160       103       111       111       111         0148       102       160       111		3.1°		V as V	· · · ·					
0100         100,01,80         Lin 103           0101         010,00         Lin 103           0102         00,00         Lin 104           0103         00,00         Lin 105           0103         00,00         Lin 105           0103         00,00         Lin 105           0103         00,00         Lin 105           0103         C         FIGENVALUE 15 THE FIRST BLEMENT OF THE FIGENVECTOR         Lin 105           0103         C         FIGENVALUE 15 THE FIRST BLEMENT OF THE FIGENVECTOR         Lin 175           0103         C         FIGENVALUE 15 THE FIRST BLEMENT IN THE FIGENVECTOR         Lin 175           0103         FIGENVALUE 15 THE FIRST BLEMENT IN THE FIRST BLEMENT IN THE CLOENVECTOR         Lin 175           0104         C         THW FIGENVALUE 15 THE FIRST BLEMENT IN THE CLOENVECTOR         Lin 175           0104         C         THW FIGENVALUE 15 THE FIRST BLEMENT IN THE CLOENVECTOR         Lin 175           0105         C         THW FIGENVALUE 15 THE FIRST BLEMENT IN THE CLOENVECTOR         Lin 176           0105         C         THW FIGENVALUE 15 THE FIRST BLEMENT IN THE CLOENVECTOR         Lin 176           0105         C         THW FIGENVALUE 15 THE FIRST BLEMENT IN THE CLOENVECTOR         Lin 176	0148					• 7			1.78×1.1	•
0100000000000000000000000000000000000	0140	97	1F (Y(J)) 89.91.89			1 TN 167			•	
0151 <td>0144</td> <td>. 84</td> <td>R≡Y(J)</td> <td></td> <td></td> <td>1 TN 164</td> <td></td> <td></td> <td></td> <td></td>	0144	. 84	R≡Y(J)			1 TN 164				
0132       01 J-J-1 C       11 J-J-1 C       11 J-J-1 C       11 J-J-1 C         0133       02 DO 90 Te1, M T       12 STAC FIRST ELEMENT OF THE ÉIGENVECTOR T       11 H 165 LIN 105 LIN 105 LIN 107 LIN 107	0150		GO TO 93		· ·	LIN 104		. •		
00000         00000         000000         00000000         0000000000         000000000000000000000000000000000000	0151	91	J=J-1			LIN 165				
C EIGENVALUE IS THE FIRST ELEMENT OF THE EIGENVECTOR C EIGENVALUE IS THE FIRST ELEMENT IN THE EIGENVECTOR ARE SOLVED C EIGENVALUES AND THE EIGENVECTORS ARE SOLVED C EIGENVALUES AND THE EIGENVECTORS ARE SOLVED C EIGENVALUES INTO INTO INTO EIGENVECTORS ARE SOLVED C EIGENVALUES EIGENVECTORS ARE SOLVED C EIGENVALUES EIGENVECTORS ARE SOLVED C EIGENVELTION EIGENVELTON C EIGENVALUES EIGENVECTORS ARE SOLVED C EIGENVELTION EIGENVELTON C EIGENVELTION EIGENVELTON EIGENVELTON EIGENVECTORS ARE SOLVED C EIGENVALUES EIGENVELTER EIGENVELTON EIGENVELTON C EIGENVELTION EIGENVELTON EIGENVELTON EIGENVELTON EIGENVELTON EIGENVELTON EIGENVELTON	0152		GO TO 92		1	LIN 166				
0133         C         FIGENVALUE IS THE FIRST ELEMENT OF THE FIGENVECTOR         LIN         168           0134         TF (OABSIN)         LIN         169         LIN         169           0135         TF (OABSIN)         LIN         171         LIN         172           0135         TF (OABSIN)         LIN         172         LIN         172           0136         DO CONTINUE         LIN         173         LIN         174           0137         DO CONTINUE         LIN         174         LIN         174           0136         C         TRANSPER X(H4) 17H EIGENVECTOR         LIN         176           0136         TO TO TO TO         LIN         176         LIN         176           0140         TO TO TO TO         LIN         176         LIN         176           0140         TO TO TO TO TO         LIN         176         LIN         168           0141         TO TO TO TO TO         TO TO TO TO TO         LIN         168         LIN         168           0142         TO TO TO TO TO TO TO TO TO TO TO TO TO T		С				LIN 167				
1153       92       DD 00 1011/04       LIN 1104       LIN 1104         1154       1104 (1) / (1	1	· C	FIGENVALUE IS THE EXDEX	ELENENT OF THE OPPORT	· . /	LIN 168				
0155       Trilowiny (1)-Y1(1))-EPS3 90.90.102       LIN 170         0157       102 XC11       LIN 171         0157       102 XC11       LIN 173         0158       TP(RC) 103.103.104       LIN 174         0159       C       TRANSPER X(N+1)TH EIGENVECTOR TO X(N)TH EIGENVECTOR       LIN 175         0158       C       TRANSPER X(N+1)TH EIGENVECTOR TO X(N)TH EIGENVECTOR       LIN 176         0159       C       TRANSPER X(N+1)TH EIGENVECTOR TO X(N)TH EIGENVECTOR       LIN 176         0150       C       TTHE EIGENVALUE IS THE FIRST ELEMENT IN THE EIGENVECTOR       LIN 176         0162       C       TTHE EIGENVALUE IS THE FIRST ELEMENT IN THE EIGENVECTOR       LIN 176         0163       DO NO STATEMENT 112. THE SIZE OF MATRIX C IS REDUCED BY ONE       LIN 186         0164       DO X(1)Y(1)       LIN 176       LIN 186         0165       DO X(1)Y(1)       LIN 176       LIN 186         0166       DO 113 TE1+N1       LIN 186       LIN 186         0167       112 A(1)-J)-C(1)-J)       LIN 186       LIN 186         0166       DO 113 TE1+N1       LIN 186       LIN 186         0167       DI 13 TE1+N1       LIN 186       LIN 186         0168       C       DI 13 TE1+N1       LIN 1	0153	93	DO 90 TEL.N	CLEMENT OF THE EIGENVECTO	n i	LIN 169				
0155       12 (0.000)       1.01 (1)-(1(1))-EPS) 90.90.102       1.01 (1)-(1)-(1)-(1)-EPS) 90.90.102         0157       90 CONTINUE       1.01 (1)-(1)-(1)-(1)-(1)-(1)-(1)-(1)-(1)-(1)-	0154					LIN 170				
0155       102 (c+1)       111 FT(L))-EPS 3 00-00.102       11N 172         0157       90 CONTINUE       115 TT(L) 03.104       11N 173         0158       115 TT(L) 03.104       11N 173         0159       C       TRAMEER X(N+1)TH EIGENVECTOR TO X(M)TH EIGENVECTOR       11N 173         0150       G0 TO 90       11N 175         0151       G0 TO 90       11N 175         0152       C       THF EIGENVALUE IS THE FIRST ELEMENT IN THE EIGENVECTOR       11N 175         0152       C       THF EIGENVALUE IS THE FIRST ELEMENT IN THE EIGENVECTOR       11N 175         0152       C       THF EIGENVALUE IS THE FIRST ELEMENT IN THE EIGENVECTOR       11N 175         0152       C       THF EIGENVALUE IS THE FIRST ELEMENT IN THE EIGENVECTOR       11N 175         0153       C       W TO STATEMENT 112. THE SIZE OF MATRIX C IS REDUCED BY DNE       11N 156         0154       DO 113 TE1-41       11N 156       11N 156         0155       DO 113 TE1-41       11N 156       11N 156         0156       DO 113 TE1-41       11N 156       11N 156         0157       112 AT1.3 + 11.4 + 11.4 + 12.6 + 11.1 + 13.6 + 11.4 + 13.6 + 11.1 + 13.6 + 11.1 + 13.6 + 11.1 + 13.6 + 11.1 + 13.6 + 11.1 + 13.6 + 11.1 + 13.6 + 11.1 + 13.6 + 11.1 + 13.6 + 11.1 + 13.6 + 11.1 + 13.6 + 11.1 + 13.6 + 11.1 + 13.6 + 11.1 + 13.6 +	0155		IF (DADC/V(I)-WI/II)		· · ·	LIN 171	1. A 1. A 1. A 1. A 1. A 1. A 1. A 1. A			•
0187       190 CONTINUE       LIN 173         0189       IFRC) 103.103.104       LIN 176         0180       110 DO 115 101.N       LIN 176         0181       103 EIGUXIDHR       C       THF EIGENVALUE IS THE FIRST ELEMENT IN THE EIGENVECTOR       LIN 176         0182       103 EIGUXIDHR       THF EIGENVALUE IS THE FIRST ELEMENT IN THE EIGENVECTOR       LIN 186         0183       103 EIGUXIDHR       THF EIGENVALUE IS THE FIRST ELEMENT IN THE EIGENVECTOR       LIN 186         0184       100 TISTERMENT 112. THE SIZE OF MATRIX C IS REDUCED BY DNE       LIN 186         0185       100 112 TH:NI       LIN 186         0186       DO 112 TH:NI       LIN 186         0186       DO 113 JH:NI       LIN 186         0186       DO 113 JH:NI       LIN 186         0186       DO 113 JH:NI       LIN 186         0187       LIN 190       LIN 190         0186       DO 113 JH:NI       LIN 190         0186       DO 113 JH:NI       LIN 190         0187       HIM EIGG(1111.1074       LIN 190         0187 </td <td>0156</td> <td>. 10.9</td> <td>17 (DANS(T(1)=T1(1))=EPS) 9</td> <td>/0+90+102</td> <td>1</td> <td>LIN 172</td> <td></td> <td>÷</td> <td></td> <td></td>	0156	. 10.9	17 (DANS(T(1)=T1(1))=EPS) 9	/0+90+102	1	LIN 172		÷		
0155         00 CHNTINE:         114           0155         C         TRANSPER X(M+1)TH EIGENVECTOR TO X(M)TH EIGENVECTOR         LIN 175           0160         115 Y1(1)+Y(1)         LIN 175         LIN 175           0161         115 Y1(1)+Y(1)         LIN 175         LIN 175           0162         TTHE FIGENVALUE IS THE FIRST ELEMENT IN THE EIGENVECTOR         LIN 175           0162         TTHE FIGENVALUE IS THE FIRST ELEMENT IN THE EIGENVECTOR         LIN 176           0163         C         0100 TSTEMENT 112, THE SIZE OF MATRIX C IS REDUCED BY DNE         LIN 186           0163         C         0100 TSTEMENT 112, THE SIZE OF MATRIX C IS REDUCED BY DNE         LIN 186           0163         C         0100 TSTEMENT 112, THE SIZE OF MATRIX C IS REDUCED BY DNE         LIN 186           0163         D112 JT:NI         LIN 186         LIN 186           0164         D0 112 JT:NI         LIN 186         LIN 186           0165         D0 113 JT:NI         LIN 186         LIN 186           0166         D113 JT:NI         LIN 186         LIN 186           0170         LIN 197         LIN 196         LIN 196           0171         LIN 197         LIN 196         LIN 196           0172         TF OVITISCIN.JJANTALISAL FEGUINY TH RATI	0187	102				TN 172				
C TANSFER X(MAI)TH EIGENVECTOR TO X(M)TH EIGENVECTOR C TANSFER X(MAI)TH EIGENVECTOR TO X(M)TH EIGENVECTOR C THE EIGENVALUE IS THE FIRST ELEMENT IN THE EIGENVECTOR C THE EIGENVALUE IS THE FIRST ELEMENT IN THE EIGENVECTOR C THE EIGENVALUE IS THE FIRST ELEMENT IN THE EIGENVECTOR C THE EIGENVALUE IS THE FIRST ELEMENT IN THE EIGENVECTOR C THE EIGENVALUE IS THE SIZE OF MATRIX C IS REDUCED BY ONE C THE EIGENVALUE IS THE SIZE OF MATRIX C IS REDUCED BY ONE C THE EIGENVALUE IS THE SIZE OF MATRIX C IS REDUCED BY ONE C THE EIGENVALUE IS THE FIRST ELEMENT IN THE EIGENVECTOR C D 107 112 AIR JOINT C HEDUCTION OF SIZE OF MATRIX C BY ONE COMPLETED C HEDUCTION OF SIZE OF MATRIX C BY ONE COMPLETED C HEDUCTION OF SIZE OF MATRIX C BY ONE COMPLETED C HEDUCTION OF SIZE OF MATRIX C BY ONE COMPLETED C HEDUCTION OF SIZE OF MATRIX C BY ONE COMPLETED C HEDUCTION OF SIZE OF MATRIX C BY ONE COMPLETED C HEDUCTION OF SIZE OF MATRIX C BY ONE COMPLETED C HEDUCTION OF SIZE OF MATRIX C BY ONE COMPLETED C HEDUCTION OF SIZE OF MATRIX C BY ONE COMPLETED C HEDUCTION OF SIZE OF MATRIX C BY ONE COMPLETED C HEDUCTION OF SIZE OF MATRIX C BY ONE COMPLETED C HEDUCTION OF SIZE OF MATRIX C BY ONE COMPLETED C HIN 105 C THE FIGURATION OF SIZE OF MATRIX C BY ONE COMPLETED C HIN 105 C HEDUCTION OF SIZE OF MATRIX C BY ONE COMPLETED C HIN 105 C THE FIGURATION OF SIZE OF MATRIX C BY ONE COMPLETED C HIN 105 C HIN 105 C TO SIZE OF MATRIX PEGGUENCY IN RADIANS PEG SECONDUM-EDD C HIN 205 C HIN 205 C HIN 105 CHINHALL INVERSE SECONDUM-EIGENVELUE-SECONDUM-EDD C HIN 205 C HIN 205 C HIN 105 CHINHAL INVERSE OILDON CHIN AGE SOLVED C HIN 205 C HIN 105 CHINHAL INVERSE OILDON CHIN 205 C HIN 105 CHINHAL INVERSE OILDONAL ELEMENT IS ZERO OR NOT C HIN 205 C HIN 105 SICHTHER DIAGONAL ELEMENT IS ZERO OR NOT C HIN 216 C HIN 105 SICHTHER DIAGONAL ELEMENT IS ZERO OR NOT C HIN 216 C HIN 105 SICHTHER DIAGONAL ELEMENT IS ZERO OR NOT C HIN 216 C HIN 105 SICHTHER DIAGONAL ELEMEN	0150	90	CUNTINUE					•	•	
C TRANSFER X(N+1)TH EIGENVECTOR TO X(H)TH EIGENVECTOR 100 0 113 0 113 114 117 0160 113 0 113 114 114 114 C THE EIGENVALUE IS THE FIRST ELEMENT IN THE EIGENVECTOR 103 EIGENVALUE IS THE FIRST ELEMENT IN THE EIGENVECTOR 103 EIGENVECTOR 103 EIGENVECTOR 103 EIGENVECTOR 103 EIGENVECTOR 104 100 100 111 112, THE SIZE OF MATRIX C IS REDUCED BY ONE 114 103 0160 112 1141 105 0112 1141 105 0112 1141 106 113 JELINI 0167 113 JELINI 0167 113 JELINI 0168 C DO 113 JELINI 0169 DO 113 JELINI 0169 DO 113 JELINI 0170 113 CILINI 0171 113 CILINI 0171 113 CILINI 0172 CILINI 0173 114 EIGG(111)1, EIGG(11) 114 EIGG(1101-10/R) 0174 JIA EIGG(1101-10/R) 0175 JIA EIGG(1101-10/R) 0175 JIA EIGG(1101-10/R) 0176 C I UT STATEMENT SOLS. FRIDANS PER SECONDE, F2D, 3] LIN 200 0177 CILINI 0179 CILINI 0179 CILINI 0170 CIL		-	IF(KC) 103,103,104					•		
C TRANSFER X(M+1)TH EIGENVECTOR TO X(M)TH EIGENVECTOR LIN 176 10400 115 TF 11-N 0160 115 TF 11-N C TTHF EIGENVALUE IS THE FIRST ELEMENT IN THE EIGENVECTOR LIN 136 C TTHF EIGENVALUE IS THE FIRST ELEMENT IN THE EIGENVECTOR LIN 136 C TUE TO STATEMENT 112. THE SIZE OF MATRIX C IS REDUCED BY ONE LIN 136 100 TO 10 TO STATEMENT 112. THE SIZE OF MATRIX C IS REDUCED BY ONE LIN 136 100 TO 10 TO STATEMENT 112. THE SIZE OF MATRIX C IS REDUCED BY ONE LIN 136 100 TO 112 T=1.NI 0166 DD 112 J=1.NI 0167 112 A11.J)C(I.J)SX(I) C REDUCTING OF SIZE OF MATRIX C BY ONE COMPLETED LIN 136 0166 DD 112 J=1.NI 0167 112 A11.J)C(I.J)SX(I) C REDUCTING OF SIZE OF MATRIX C BY ONE COMPLETED LIN 136 0160 DD 112 J=1.NI 0160 DD 113 J=1.NI 0160 DD 113 J=1.NI 0170 CI 13 CI .J)SX(I).EIGG(II) C IN CI .J)SX(I).EIGG(		C .	•		· L	-IN 175				
0183       104 DO 115 1=1,N       LIN 104 DO 115 1=1,N       LIN 105         0184       115 Y(1)=Y(1)       LIN 105       LIN 105         0185       GD TO 90       LIN 105         0186       D3 FIGV(1)=PR       LIN 105         0187       C       UP TO STATEMENT 112. THE SIZE OF MATRIX C IS REDUCED BY ONE       LIN 105         0183       100 Y(1)=-Y(1)       LIN 105       LIN 105         0184       100 Y(1)=-Y(1)       LIN 105       LIN 105         0186       D0 113 L=1,NI       LIN 105       LIN 105         0187       112 K(1)=/K(1)+/K(1),EIG(11)       LIN 105       LIN 105         0187       GO TO 113       LIN 105       LIN 105         0187       GO TO 113 S-11/N       LIN 105       LIN 105         0187       GO TO 113 S-11/N       LIN 105       LIN 105         0187       GO TO 113 S-11/N       LIN 105       LIN 105         0187       GO TO 113       LIN 1		с	TRANSFER X (M+1)TH EIGENV	ECTOR TO XINITH ELCENVECT		IN 176	•	•		
0160       115 Y1(1)=Y(1)       LIN       178         0161       G TO 90       LIN       170         0162       C       THE ELGENVALUE IS THE FIRST ELEMENT IN THE LIGENVECTOR       LIN       180         0163       103 ELGENTIAL       TEMENT 112, THE SIZE OF MATRIX C IS REDUCED BY ONE       LIN       183         0164       109 X(1)=-Y(1)       LIN       184       LIN       184         0164       109 X(1)=-Y(1)       LIN       184       LIN       185         0164       109 X(1)=-Y(1)       LIN       184       LIN       185         0164       109 X(1)=-Y(1)       LIN       184       LIN       185         0165       D0 112 J=1.N1       LIN       185       LIN       185         0166       D0 112 J=1.N1       LIN       186       LIN       197         0167       112 X(1)=C(1)       LIN       188       LIN       198         0166       D0 113 J=1.N1       LIN       191       LIN       191         0170       LIS C(1)+LIN       LIN       183       LIN       193         0171       G TO 105       LIN       191       LIN       192         0170       FIGC(11)	0159	104	DO 115 I=1+N	ACIDA TO ATMITTA EIGENVECT	אר אנ	_IN 177		•		
0161         C         TWF ELGENVALUE IS THE FIRST ELEMENT IN THE ELGENVECTOR         LIN         113           0162         C         TWF ELGENVALUE IS THE FIRST ELEMENT IN THE ELGENVECTOR         LIN         163           0163         C         UTTSTATEMENT 112. THE SIZE OF MATRIX C IS REDUCED BY ONE         LIN         163           0164         100 KTIS-VIIN         LIN         164         104           0165         D0 112 t=1.NI         LIN         165           0166         D0 112 t=1.NI         LIN         166           0167         112 Aff.JJ=C(I,J)+X(I)         LIN         167           0168         C         REDUCTION OF SIZE OF MATRIX C BY ONE COMPLETED         LIN         168           0169         C         REDUCTION OF SIZE OF MATRIX C BY ONE COMPLETED         LIN         169           0166         C         REDUCTION OF SIZE OF MATRIX C BY ONE COMPLETED         LIN         169           0178         GO TO 113         J=1.NI         LIN         164         164           0166         C         REDUCTION OF SIZE OF MATRIX C BY ONE COMPLETED         LIN         164           0177         GO TO 113         LIN         164         164         164           0177         FRECONDATIONES SIZE	0160	115	Y1(1) = Y(1)		L	.IN 178				
C THE FIGENVALUE IS THE FIRST ELEMENT IN THE EIGENVECTOR C THE FIGENVALUE IS THE FIRST ELEMENT IN THE EIGENVECTOR LIN 180 103 EIGVIII)=R UP TO STATEMENT 112, THE SIZE OF MATRIX C IS REDUCED BY ONE LIN 183 0163 DO 110 TEI.NI 0164 DO 112 JEI.NI 0165 DO 113 JEI.NI 0166 DO 113 JEI.NI 0167 LIN 183 0168 DO 113 JEI.NI 0167 LIN 183 0169 DO 113 JEI.NI 0167 LIN 183 0170 DO 113 JEI.NI 0171 LIN 183 0171 LIN 183 0172 LIN 183 0173 LIN 193 0174 WRITCIG.LSJ FIREO 0175 SFORMATING STATEMENT 9015. THE EIGENVECTORS ARE SOLVED 110 DI 015 TEI.NI 0177 C NOTATEMENT 9015. THE EIGENVECTORS ARE SOLVED 111 ZEI.NI 0170 LIN 2015. THE EIGENVECTORS ARE SOLVED 112 CONTACTION 2015. THE EIGENVECTORS ARE SOLVED 113 CFILINAL 0174 LIN 2015 0175 LIN 2015. THE EIGENVECTORS ARE SOLVED 110 DI 015 TEI.NI 0175 LIN 2015 0176 LIN 2015 0177 LIN 2015. THE EIGENVECTORS ARE SOLVED 111 DI 105 LIN.NI 0179 LIN 2015 0179 LIN 2015. THE EIGENVECTORS ARE SOLVED 111 DI 105 LIN.NI 0179 LIN 2015 0179 LIN 2015. THE EIGENVECTORS ARE SOLVED 112 DI 015 TEI.NN 0179 LIN 2015 0179 LIN 2015 0179 LIN 2015 0170 LIN 2015 0170 LIN 2015 0170 LIN 2015 0171 LIN 2015 0172 LIN 2015 0174 LIN 2015 0175 LIN 2015 0175 LIN 2015 0175 LIN 2015 0176 LIN 2015 0177 LIN 2015 0177 LIN 2015 0177 LIN 2015 0177 LIN 2015 0177 LIN 2015 0178 LIN 2015 0179 LIN 2015 0179 LIN 2015 0179 LIN 2015 0179 LIN 2015 0179 LIN 2015 0170 LIN 2015 017	0161		GO TO GO		L	-IN 179				
C THF EIGENVALUE IS THE FIRST ELEMENT IN THE EIGENVECTOR LIN IA 103 ETGV(11)=0 103 ETGV(11)=0 104 LIN IA 105 X(1)=-V(1) 105 X(1)=-V(1) 105 LIL =1:N1 106 LIL =1:N1 106 LIL =1:N1 107 LIL =1:N1 108 LIN IA 108 LIN IA 109 X(1)=-V(1) 112 A(1,J) = J(1,N) 112 A(1,J) = J(1,N) 112 A(1,J) = J(1,N) 112 A(1,J) = J(1,N) 113 (T,J) = J(1,N) 114 A(1,J) = J(1,N) 115 (T,J) = J(1,N) 116 (T,J) = J(1,N) 117 EIGV(11)=C(1,1) 117 EIGV(11)=C(1,1) 117 EIGV(11)=C(1,1) 117 EIGV(11)=C(1,1) 117 EIGV(11)=C(1,1) 118 (G(11)=1:A) 117 EIGV(11)=C(1,1) 117 EIGV(11)=C(1,1) 117 EIGV(11)=C(1,1) 118 (G(11)=1:A) 117 (G(11)=1:A) 119 (LIN 103 119 (LIN 103 119 (LIN 103 117 EIGV(11)=C(1,1) 118 (G(11)=1:A) 118 (G(11)=C(1,1)) 119 (LIN 103 119 (LIN 103 110 (CIN WHITE(0,3)) 110 (LIN 103 111 (LIN 103 113 (LIN 103 113 (LIN 103 113 (LIN 103 113 (LIN 103 113 (LIN 103 113 (LIN 103 113 (LIN 103 113 (LIN 103 113 (LIN 103 113 (LIN 103 113 (LIN 103 114 (LIN 103 115 (LIN 103 115 (LIN 103 116 (LIN 103 117 (LIN 103 118 (LIN 103 118 (LIN 103 118 (LIN 103 119 (LIN 103 119 (LIN 103 119 (LIN 103 119 (LIN 103 119 (LIN 103 110 (LIN 103	1	c			L	IN 180				
0162       103 ETOULTI LIGENAUDE IS THE FIRST ELEMENT IN THE LIGENVECTOR       LIN       163         0163       00 100 1=1.NI       112, THE SIZE OF MATRIX C IS REDUCED BY ONE       LIN       164         0164       100 X(1)=-V(1)       LIN       164       165         0165       00 112 1=1.NI       LIN       166       LIN       167         0166       00 112 1=1.NI       LIN       167       LIN       167         0167       112 A(1,J)=C(1,J)+C(N,J)*X(T)       LIN       167       LIN       168         0168       00 112 J=1.NI       LIN       167       LIN       169         0169       D0 113 J=1.NI       LIN       167       LIN       169         0160       D0 113 J=1.NI       LIN       167       LIN       169         0160       D0 113 J=1.NI       LIN       169       LIN       169         0170       113 GO TO 113       LIN       160       LIN       160         0171       GO TO 113       LIN       163       LIN       163         0171       HIT FIGVET119C(11)       LIN       164       LIN       165         0171       HIT FIGVET119C(11)       LIN       164       167       16		č			ĩ	IN 181	•			•
0163       C       UP TO STATEMENT 112, THE SIZE OF MATRIX C IS REDUCED BY ONE       LIN       163         0164       109 XIII=xii       LIN       166       LIN       166         0165       DO 112 J=1,wii       LIN       166       LIN       166         0166       DO 112 J=1,wii       LIN       167       LIN       167         0167       DI 13 J=1,wii       LIN       168       LIN       169         0166       DO 113 J=1,wii       LIN       169       LIN       169         0167       DI 13 J=1,wii       LIN       169       LIN       169         0166       DO 113 J=1,wii       LIN       169       LIN       169         0113 J=1,wii       LIN       169       LIN       169         0170       113 GTGN11,0/R       LIN       160       LIN       169         0171       G TO 113       LIN       160       LIN       160         0173       HF FROMISTICK(HIL), EIGG(IT)       LIN       160       LIN       160         0174       HF FROMISTICK FROM	0162		THE EIGENVALUE IS THE F	IRST ELEMENT IN THE EIGENV	VECTOR I	TAL 102				
0163       C       UP TO STATEMENT 112. THE SIZE OF MATRIX C IS REDUCED BY ONE       Lin       183         0164       109 XI 112. THE SIZE OF MATRIX C IS REDUCED BY ONE       Lin       184         0164       109 XI 112. THE SIZE OF MATRIX C IS REDUCED BY ONE       Lin       185         0165       D0 112 J=1.NI       Lin       186         0166       D0 113 J=1.NI       Lin       189         0167       112 All.J=C(I.J)+C(IN.J)*X(I)       Lin       189         0168       D0 113 J=1.NI       Lin       189         0169       D0 113 J=1.NI       Lin       190         0169       D0 113 J=1.NI       Lin       191         0170       I13 C(I.J)+C(IN.J)*X(I)       Lin       192         0174       TH ETG(A.D)       Lin       193         0175       FRCORTALASS (FIGG(III)       Lin       194         0176       FSCORSORT(ABS (FIGG(III))       Lin       196         0177       B3 FORMAT(IN-HORSE VALUE=:E12:ESCA:GATIONE:E20.J3)       Lin       199         0177       B3 FORMAT(IN-HORSE VALUE::E12:ESCA:GATIONE:E20.SIL       Lin       203         0178       INNENE-I       Lin       204       Lin       205         0179       S3 FORMAT(	VIUZ	103	EIGV(II)=R			.10 102				
013.1       D0 109 1=1:N1       Link 100 K (1)=-V(1)         0144       100 K (1)=-V(1)       Link 145         0145       D0 112 1=1:N1       Link 145         0146       D0 112 1=1:N1       Link 145         0147       D1 112 A(1,1)=C(1,1)+X(1)       Link 145         0148       D0 113 J=1:N1       Link 145         0149       D0 113 J=1:N1       Link 145         0140       D0 113 J=1:N1       Link 145         0140       D0 113 J=1:N1       Link 145         0141       G0 T0 113       Link 145         0143       G0 T0 113       Link 145         0144       D0 113 J=1:N1       Link 145         0145       D0 113 J=1:N1       Link 145         0146       D0 113 J=1:N1       Link 145         0177       G0 T0 115       Link 145         0178       FRC9KSRT(ABS(FIGG(11)))       Link 145         0179       S3 FORMATING-RX-KUESE VALUES-KEL2S-SK-164TH-EIGEN VALUES-KEL2S-LIN POI       Link 145         0179       S3 FORMATING-RX-KUESE VALUES-KEL2S-SK-164TH-EIGEN VALUES-KEL2S-LIN POI       Link 205         0179       C       UT TO STATEMENT 9015. THE EIGENVECTORS ARE SOLVED       Link 205         0179       C       UT TO STATEMENT 9015. THE EIGENVECTORS		C	UP TO STATEMENT 112, THE	SIZE OF MATRIX C IS DEDUC		IN 183				
0144       109 X(1)=-V(1)       LIN       145         0165       00 112 J=1:N1       LIN       166         0166       00 112 J=1:N1       LIN       167         0167       112 A(1,J)=C(1,J)+C(N,J)*X(1)       LIN       169         0166       01 113 J=1:N1       LIN       169         0167       01 113 J=1:N1       LIN       169         0168       00 113 J=1:N1       LIN       169         0170       113 G(1) J=C(1,1)       LIN       169         0171       113 G(1) J=C(1,1)       LIN       169         0177       117 FIGV(11)=C(1,1)       LIN       160         0177       117 FIGV(11)=C(1,1)       LIN       160         0177       117 FIGV(11)=C(1,1)       LIN       165         0177       117 FIGV(11)=C(1,1)       LIN       165         0177       53 FORMAT(14,163; K400HATURAL FPEQUENCY IN RADIAMS PER SECOND=, F20.3)       LIN       169         0178       54 FORMAT(14,163; K400HATURAL FPEQUENCY IN RADIAMS PER SECOND=, F20.3)       LIN       205         0179       01 TO STATEMENT 9015. THE EIGENVECTORS ARE SOLVED       LIN       206         0179       01 TO STATEMENT 9015. THE EIGENVECTORS ARE SOLVED       LIN       206 <td>0163</td> <td></td> <td>DO 109 I=1+N1</td> <td></td> <td>LED BY UNE L</td> <td>.IN 184</td> <td></td> <td></td> <td></td> <td></td>	0163		DO 109 I=1+N1		LED BY UNE L	.IN 184				
0165       DD 112 t=1.N1       LIN 166         0166       DD 112 t=1.N1       LIN 167         0167       112 A(1.J)=C(1.J)*X(1)       LIN 169         0167       113 t=1.N1       LIN 190         0168       DO 113 t=1.N1       LIN 190         0169       DO 113 t=1.N1       LIN 191         0170       113 C(1.J)=A(1.J)       LIN 192         0171       GO TO 113       LIN 193         0172       117 FICV(1)=C(1.1)       LIN 193         0173       117 FICV(1)=C(1.1)       LIN 193         0174       WRITC(16.64) 11.CIGV(11),EIGG(11)       LIN 193         0175       117 FICV(1)=C(1.1)       LIN 193         0176       FRITEGO(11).ST       LIN 193         0177       FRITEGO(11).ST       LIN 193         0176       FRITEGO(11).ST       LIN 193         0177       S3 FORMAT(14.STA0MHAURAL FPEQUENCY IN RADIANS PER SECOND=,F20.3) LIN 200       LIN 203         0179       INNEWEN       S4 FORMAT(14.1)       LIN 203         0179       UP TO STATEMENT 9015, THE EIGENVECTORS ARE SOLVED       LIN 203         0180       I.G(1-1) 101.141.142       LIN 203         0181       I.G(1-1) 101.141.141.142       LIN 205         0181	0164	109	X(I) = -Y(I)		L	.IN 185				
0165       DD 112 J=1,N1       LIN 167         0167       112 A(1,J) = C(1,J) + X(1)       LIN 168         0166       DD 113 J=1,N1       LIN 169         0167       D1 13 J=1,N1       LIN 191         0169       DO 113 J=1,N1       LIN 192         0170       I13 G(1,J) = L(1,J)       LIN 193         0171       GO TO 113       LIN 193         0173       I1A EIGG(11)=C(1,1)       LIN 195         0174       WRTCIG(4,6) I1.cIGV(11),EIGG(11)       LIN 195         0175       FRCO-SGNRT(ABG(FIGG(11)))       LIN 196         0176       S3 FORMAT(10.6,8X:AOHNATURAL FPEQUENCY IN RADIANS PER SECOND=,F20,3) LIN 190       DIN 200         0177       S3 FORMAT(10.6,8X:AOHNATURAL FPEQUENCY IN RADIANS PER SECOND=,F20,3) LIN 190       DIN 200         0177       C       UP to StateMent 9015. THE EIGENVECTORS ARE SOLVED       LIN 202         0170       C       UP to StateMent 9015. THE EIGENVECTORS ARE SOLVED       LIN 202         0180       Inf J=1,NN       LIN 205       LIN 205         0181       If (1,1) 141,141,142       LIN 205       LIN 205         0181       If (1,1) 107,16,107       LIN 205       LIN 205         0183       DI 05,1-FI,NN       LIN 210       LIN 210	0165		DD 112 1=1.N1		L	.IN 186				
0167       112 ATL:J=C(1,J)+C(N,J)*X(1)       LIN       188         0167       C       REDUCTION OF SIZE OF MATRIX C BY ONE COMPLETED       LIN       189         0168       DO 113 T=1,N1       LIN       190       LIN       190         0170       113 C11,J)=ATL:J       LIN       191       LIN       192         0171       GO TO 113 J=1,N1       LIN       192       LIN       193         0177       117 FIGUTIJ=C(1,1)       LIN       193       LIN       194         0177       117 FIGUTIJ=C(1,1)       LIN       193       LIN       194         0177       117 FIGUTIJ=C(1,1)       LIN       193       LIN       194         0177       117 FIGUTIJ=C(1,1)       LIN       194       LIN       194         0178       WRITC(6,64) FILCEG(11))       LIN       195       LIN       196         0179       FRCa=SGRIT(AS)(FIEGG(11))       LIN       110       200       200       201	0166		00 112 J=1-N1	,	L	.IN 187				
C III PEDUCTION DEVENTS C III PEDUCTION DEVENTS D IIJ JELANI D IIJ JE	0167	112			ι	.IN 188				
0166       D       D113 [1:1:N] DF SIZE OF MATRIX C BY ONE COMPLETED       LIN 190         0169       D0 113 [1:1:N]       LIN 190         0170       113 (1:1:1:1:1:1:1:1:1:1:1:1:1:1:1:1:1:1:1:		c			· ·	IN 189				
0166       00 113 TEL.NI       113 C(1.J)FA(1.J)         0170       113 C(1.J)FA(1.J)       114 193         0171       117 FICV(11)=C(1.1)       114 193         0173       117 FICV(11)=C(1.1)       114 193         0174       WRITE(6.6A) 11.CIGV(11),EIGG(11)       114 193         0175       FRCG=SOR(ASS(FIGG(11)))       114 193         0176       wRITE(6.6A) 11.CIGV(11),EIGG(11)       114 193         0176       wRITE(6.6A) 11.CIGV(11),EIGG(11)       114 193         0176       wRITE(6.6A) FRCG       113 114         0176       wRITE(6.6A) FRCG       114 193         0176       wRITE(6.6A) FRCG       114 193         0176       wRITE(6.6A) FRCG       118 193         0176       bar DRMAT(14.064.142       114 193         0177       c       UP TO STATEMENT 9015. THE EIGENVECTORS ARE SOLVED       118 203         0180       NLASTENHA1       LIN 205       118 203         0181       IF(11-1) 141.141.142       118 203       118 203         0181       IF(1-1) 141.141.142       118 207       118 205         0183       D0 105 I=1.NN       LIN 205       118 205         0184       A1(.J)=A1(.J)=FIGV(11)       LIN 210       LIN 213 <t< td=""><td>0168</td><td>•</td><td>REDUCTION OF SIZE OF MATE</td><td>RIX C BY ONE COMPLETED</td><td>ī</td><td>TN 100</td><td></td><td></td><td></td><td>•</td></t<>	0168	•	REDUCTION OF SIZE OF MATE	RIX C BY ONE COMPLETED	ī	TN 100				•
0170       D0 113 J=1,NI       LIN 191         0170       113 C(1,J)=A(1,J)       LIN 192         0171       G0 TO 113       LIN 193         0172       117 FICV(11)=C(1,1)       LIN 194         0173       118 EIGG(11)=L(16/R)       LIN 194         0174       WRITE(6.6.3) I.CIGV(11),EIGG(11)       LIN 196         0175       FRC0=SORT(ADS(FIGG(11)))       LIN 196         0176       WRITE(6.6.3) FRE0       LIN 196         0177       B3 FORMAT(104,185,4-60HATURAL FPEQUENCY IN RADIANS PER SECOND=+F20.3] LIN 200       LIN 196         0176       FORMAT(104,185,4-60HATURAL FPEQUENCY IN RADIANS PER SECOND=+F20.3] LIN 200       LIN 196         0177       B3 FORMAT(104,185,4-60HATURAL FPEQUENCY IN RADIANS PER SECOND=+F20.3] LIN 200       LIN 200         0177       B3 FORMAT(104,187,4-60HATURAL FPEQUENCY IN RADIANS PER SECOND=+F20.3] LIN 200       LIN 200         0177       B3 FORMAT(104,187,4-60HATURAL FPEQUENCY IN RADIANS PER SECOND=+F20.4] LIN 200       LIN 200         0177       B3 FORMAT(104,187,4-60HATURAL FPEQUENCY IN RADIANS PER SECOND=+F20.4] LIN 200       LIN 200         0170       C UP TO STATEMENT 9015. THE EIGENVECTORS ARE SOLVED       LIN 200         0181       IF(11-1) 141,161,142       LIN 200         0182       IF(11-1) 141,161,142       LIN 200 <tr< td=""><td>0160</td><td></td><td>DU 113 T=1+N1</td><td></td><td>-</td><td>114 190</td><td></td><td></td><td></td><td></td></tr<>	0160		DU 113 T=1+N1		-	114 190				
0170       113 C(1,,)=A(1,,)       LIN 192         0171       GG TG 119       LIN 193         0172       117 FICV(1)=C(1,1)       LIN 195         0173       118 EIGG(11)=1,0/R       LIN 195         0174       WRITC(6,6,6,4) 11,CIGV(11),EIGG(11)       LIN 196         0175       FREGGSORI(ADS(FIGG(11))))       LIN 197         0176       WRITC(6,53) FREG       LIN 199         0177       63 FORMAT(14,6,40HATURAL FPEQUENCY IN RADIANS PER SECOND=,F20.3) LIN 200       LIN 199         0176       64 FORMAT(14,6,40HATURAL FPEQUENCY IN RADIANS PER SECOND=,F20.3) LIN 200       LIN 202         0177       63 FORMAT(14,6,40HATURAL FPEQUENCY IN RADIANS PER SECOND=,F20.3) LIN 200       LIN 202         0177       64 FORMAT(14,6,40HATURAL FPEQUENCY IN RADIANS PER SECOND LIN 200       LIN 202         0179       C       UP TO STATEMENT 9015, THE EIGENVECTORS ARE SOLVED       LIN 203         0179       C       UP TO STATEMENT 9015, THE EIGENVECTORS ARE SOLVED       LIN 203         0180       NL=SAN+1       LIN 205       LIN 205         0181       IF(1-1) 141:141;142       LIN 205       LIN 206         0182       IO 105 I=1;NN       LIN 205       LIN 206         0184       IF(1-j) 107:139;107       LIN 210       LIN 210 <tr< td=""><td>0104</td><td></td><td>DO 113 J=1.N1</td><td>·</td><td>L</td><td>IN 191</td><td></td><td></td><td>•</td><td></td></tr<>	0104		DO 113 J=1.N1	·	L	IN 191			•	
0171       G0 TO 113       LIN 193         0172       117 FIGVIIJEC(1,1)       LIN 194         0173       118 EIGG(II)=LO/R       LIN 195         0174       WRITC(6,64) II.CIGV(II),EIGG(II)       LIN 196         0175       FREQ@SORT(ADS(FIGG(II)))       LIN 196         0176       WRITC(6,53) FREQ       LIN 199         0177       53 FORMAT(140,15X,40HNATURAL FPE QUENCY IN RADIANS PER SECOND#,F20,3) LIN 200       LIN 199         0176       54 FORMAT(140,16X,40HNATURAL FPE QUENCY IN RADIANS PER SECOND#,F20,3) LIN 200       LIN 199         0177       53 FORMAT(140,16X,40HNATURAL FPE QUENCY IN RADIANS PER SECOND#,F20,3) LIN 200       LIN 200         0177       53 FORMAT(140,16X,40HNATURAL FPE QUENCY IN RADIANS PER SECOND#,F20,3) LIN 200       LIN 199         0176       UP TO STATEMENT 9015. THE EIGENVECTORS ARE SOLVED       LIN 200         0177       01 STATEMENT 9015. THE EIGENVECTORS ARE SOLVED       LIN 200         0180       NLASTENH1       LIN 200         0181       IF(11-1) 101,114,14,142       LIN 200         0182       IF(11-1) 101,139,107       LIN 206         0183       IG TO 107, J=1,NN       LIN 206         0184       IG TO 41,1,01-F1GV(II)       LIN 210         0185       IG TO 57 CONTINUE       LIN 210	0170	113	C{I+J}=A(I+J)		. L	IN 192			•	
0172       117 FICV(11)=C(1,1)       LIN       194         0173       118 E1GG(11)=1.0/2R       LIN       195         0174       WRITE(6,6.3) I1.CIGV(II).EIGG(II)       LIN       196         0175       FRC0=SORT(ABS(FIGG(II))       LIN       197         01.6       WRITE(6,5.3) FRE0       LIN       199         0176       54 FORMAT(140,5.X.40HNATURAL FPEQUENCY IN RADIANS PER SECOND=,F20.3)       LIN       200         0176       54 FORMAT(141,101+1. INVERBE VALUE=,E12.5.5X.5X.60HTH-EIGEN VALUE=,E12.5.5LN       201       11         0177       53 FORMAT(141,11,11,101,11,11,11,11,11,11,11,11,11,1	0171		GO TO 113		· L	IN 193				
0173       118 EIGG(11)#:0/8       LIN       195         0174       WRITT(6.5.6.4)       11.6.1GV(11).EIGG(11)       LIN       196         0175       FREGESORT(ADS(FIGG(11)))       LIN       197         0176       WRITT(6.5.3) FREG       LIN       196         0177       D3 FORMAT(140.53.4 AONHATURAL FPEQUENCY IN RADIANS PER SECOND=,F20.3)       LIN       200         0176       D3 FORMAT(140.53.4 AONHATURAL FPEQUENCY IN RADIANS PER SECOND=,F20.3)       LIN       200         0177       D3 FORMAT(140.53.4 AONHATURAL FPEQUENCY IN RADIANS PER SECOND=,F20.3)       LIN       200         0177       D3 FORMAT(140.53.4 AONHATURAL FPEQUENCY IN RADIANS PER SECOND=,F20.3)       LIN       200         0178       D4 FORMAT(140.53.4 AONHATURAL FPEQUENCY IN RADIANS PER SECOND=,F20.3)       LIN       200         0179       D4 FORMAT(140.51.4 AONHATURAL FPEQUENCY IN RADIANS PER SECOND=,F20.3)       LIN       200         0179       D7       D7       D7       LIN       200       201         0179       D7       D7       D1       LIN       201       203       203         0180       NESUMAT       D1       D1       LIN       204       205       111       206       207       111       201       111	0172	117	FIGV(11)=C(1.1)		L	IN 194				
0174       WRITE(6,5.4) IT.E[6y(II),EIGG(II)       LIN       196         0175       FRCGASCRT(AASIFIGG(II))       LIN       197         01.6       WRITE(6,5.3) FRE0       LIN       198         0176       DS FORMAT(IN,5.X,40FINATURAL FPEQUENCY IN RADIANS PER SECOND=,F20,3) LIN       200         0177       DS FORMAT(IN,5.X,40FINATURAL FPEQUENCY IN RADIANS PER SECOND=,F20,3) LIN       200         0178       DS FORMAT(IN,5.X,40FINATURAL FPEQUENCY IN RADIANS PER SECOND=,F20,3) LIN       200         0179       DS FORMAT(IN,5.X,40FINATURAL FPEQUENCY IN RADIANS PER SECOND=,F20,3) LIN       200         0179       DS FORMAT(IN,5.X,40FINATURAL FPEQUENCY IN RADIANS PER SECOND=,F20,3) LIN       200         0179       DS FORMAT(IN,5.X,40FINATURAL FPEQUENCY IN RADIANS PER SECOND=,F20,3) LIN       200         0179       DP TO STATEMENT 9015. THE EIGENVECTORS ARE SOLVED       LIN       200         0170       DIP TO STATEMENT 9015. THE EIGENVECTORS ARE SOLVED       LIN       201         0180       IF(II-1) 141.141.142       LIN       203         0181       IF(II-1) 141.141.142       LIN       206         0182       IF(II-1) 141.141.142       LIN       206         0183       D1 07 J=1.NN       LIN       208         0184       IF(II-1) 141.141.141.142 <t< td=""><td>0173</td><td>118</td><td>E1GG(11)=1.0/R</td><td></td><td>L</td><td>IN 195</td><td></td><td>•</td><td>•</td><td></td></t<>	0173	118	E1GG(11)=1.0/R		L	IN 195		•	•	
0175       FREGSORTIANS(FIGG(11)))       LIN       197         01.66       WRITE(6,53) FREG       LIN       198         01.77       53 FORMAT(140,553) FREG       LIN       198         01.76       54 FORMAT(140,53) FREG       VALUE=.F12.5,5X.16HTH. EIGEN VALUE=.F12.5LIN       200         0176       017       017       LIN       198         0177       017       017       LIN       200         0178       017       017       LIN       200         0179       01       TO STATEMENT 9015. THE EIGENVECTORS ARE SOLVED       LIN       203         0170       NH=WH-1       LIN       203       LIN       203         0170       NH=WH-1       LIN       204       LIN       205         0181       IF(II-1)       141.141.142       LIN       205       LIN       206         0182       142 DD 105 I=1.NN       LIN       206       LIN       206         0183       ID 107.139.107       LIN       206       LIN       210         0184       A(I.J)=R(I.J)       LIN       210       LIN       211         0187       107 CONTINUE       LIN       LIN       211       LIN       214	0174		WRITE(6.54) II.EIGV(II).EICC		L	IN 196				
01.6       LIN       198         01.77       53 FORMAT(110,53,40HNATURAL FPEQUENCY IN RADIANS PER SECOND=,F20.3)       LIN       190         0178       54 FORMAT(14),184785       VALUE=,E12.5,5X,16HTH.       EIGEN VALUE=,E12.5LIN       201         17       UP TO STATEMENT 9015, THE EIGENVECTORS ARE SOLVED       LIN       202         0179       UP TO STATEMENT 9015, THE EIGENVECTORS ARE SOLVED       LIN       202         0179       UP TO STATEMENT 9015, THE EIGENVECTORS ARE SOLVED       LIN       203         0180       NLAST=NH+1       LIN       204         0181       IF(11-1)       141,141,142       LIN       205         0181       IF(11-1)       141,141,142       LIN       205         0181       IF(11-1)       141,141,142       LIN       206         0182       142 DD 105 I=1,NN       LIN       205         0184       IO1 007 J=1,NN       LIN       207         0184       A(1,J)=A(1,J)-EIGV(II)       LIN       209         0185       IF(1-J) 107.159,107       LIN       LIN       210         0186       139 A(1,J)=A(1,J)-EIGV(II)       LIN       LIN       216         0187       105 U(I)=-D(1,NM)       LIN       LIN       214 </td <td>0175</td> <td>1</td> <td>FREQUESORT LARS LET CCLITAN</td> <td>2111</td> <td>L</td> <td>IN 197</td> <td></td> <td></td> <td></td> <td></td>	0175	1	FREQUESORT LARS LET CCLITAN	2111	L	IN 197				
0177       53 FORMAT(1H0,5X,40MATURAL FPEQUENCY IN RADIANS PER SECOND=,F20.3) LIN 199       LIN 199         0178       54 FORMAT(14,18HTH. INVERSE VALUE=,E12.5,5X.16HTH. EIGEN VALUE=,E12.5LIN 200       LIN 200         11       C       UP TO STATEMENT 9015. THE EIGENVECTORS ARE SOLVED       LIN 202         0179       NLENN-1       LIN 203       LIN 203         0180       NLAST=NH+1       LIN 204       LIN 204         0181       IF(1I-1) 141.141.142       LIN 205       LIN 205         0181       IF(1I-1) 141.141.142       LIN 206       LIN 206         0181       IF(1I-1) 141.141.142       LIN 206       LIN 206         0182       142 DD 105 I=1.NN       LIN 206       LIN 206         0183       DD 107 J=1.NN       LIN 206       LIN 206         0184       If(I-J) 107.139.107       LIN 210       LIN 210         0185       IF(I-J) 107.139.107       LIN 210       LIN 210         0186       130 A(I.J)=A(I.J)-FIGV(II)       LIN 210       LIN 211         0187       105 U(I)=-U(I.NN)       LIN 212       LIN 212         0188       105 U(I)=-U(I.NN)       LIN 213       C         0189       D0 9015 I=1.NN       LIN 216       LIN 216	01/6		WRITE(6.53) EDEO		L	IN 198				•
0178       D1 TO STATEMENT NOVERSE VALUE=, E12.5.5 X.10HTH. EIGEN VALUE=, E12.5.5 X.10HTH. EIGEN VALUE=, E12.5.5 X.10HTH. EIGEN VALUE=, E12.5.5 X.10HTH. EIGEN VALUE=, E12.5.5 X.10HTH. EIGEN VALUE=, E12.5.5 X.10HTH. EIGEN VALUE=, E12.5.5 X.10HTH. EIGEN VALUE=, E12.5.5 X.10HTH. EIGEN VALUE=, E12.5.5 X.10HTH. EIGEN VALUE=, E12.5.5 X.10HTH. EIGEN VALUE=, E12.5.5 X.10HTH. EIGEN VALUE=, E12.5.5 X.10HTH. EIGEN VALUE=, E12.5.5 X.10HTH. EIGEN VALUE=, E12.5.5 X.10HTH. EIGEN VALUE=, E12.5.5 X.10HTH. EIGEN VALUE=, E12.5.5 X.10HTH. EIGEN VALUE=, E12.5.5 X.10HTH. EIGEN VALUE=, E12.5.5 X.10HTH. EIGEN VALUE=, E12.5.5 X.10HTH. EIGEN VALUE=, E12.5 X.10HTH. EIGEN VALUE EIGEN VALUE EIGEN VALUE EIGEN VALUE EIGEN VALUE EIGEN VALUE EIGEN VALUE EIGEN VALUE EIGEN VALUE EIGEN VALUE EIGEN VALUE EIGEN VALUE EIGEN VALUE EIGEN VALUE EI	0177	53 (	FORMATILMA EV ADUMATURA		L	IN 190		,	·	
1)       C       UP TO STATEMENT 9015. THE EIGENVECTORS ARE SOLVED       LIN 202         0179       0170       NIASTENNE1       LIN 203         0180       NLASTENNE1       LIN 203         0181       IF(II-1) 141.141.142       LIN 205         0182       142 DD 105 I=1.NN       LIN 206         0183       DD 107 J=1.NN       LIN 206         0184       IF(II-1) 101.139.107       LIN 206         0185       IF(I-1) 107.139.107       LIN 206         0186       139 A(I.J)=A(II.J)-EIGV(II)       LIN 210         0187       107 CONTINUE       LIN 210         0188       105 U(I)=-U(I.NM)       LIN 211         0189       DD 9015 I=1.NN       LIN 213         C       TEST TO SEE WHETHER DIAGONAL ELEMENT IS ZERO OR NOT       LIN 214         C       TEST TO SEE WHETHER DIAGONAL ELEMENT IS ZERO OR NOT       LIN 216         0189       DO 9015 I=1.NN       LIN 216       G	0178	5.	CONATCINUISA 40HNATURAL FPE	QUENCY IN RADIANS PER SEC	OND=+F20.31 1	IN 200				
C       UP TO STATEMENT 9015, THE EIGENVECTORS ARE SOLVED       LIN 202         0170       NN=NM-1       LIN 203         0180       NLAST=NM+1       LIN 205         0181       IF(II-1) 141,141,142       LIN 205         0182       142 DD 105 I=1,NN       LIN 206         0183       DO 107 J=1,NN       LIN 206         0184       AIT,JPR(I,J)       LIN 206         0185       IF(I-J) 107,139,107       LIN 206         0186       139 A(I,J)=R(I,J)       LIN 206         0186       139 A(I,J)=A(I,J)       LIN 206         0186       139 A(I,J)=A(I,J)       LIN 206         0187       107 CONTINUE       LIN 210         0188       105 U(I]=-D(I,NM)       LIN 212         0189       D0 9015 I=1,NN       LIN 213         C       Test to SEE WHETHER DIAGONAL ELEMENT IS ZERO OR NOT       LIN 214         C       O189       DO 9015 I=1,NN       LIN 216		54.1	WRATTIN TONTH - INVERSE VAL	_UE=+E12+5+5X+16HTH+ EIGEN	VALUES-F12-5	TN 201				•
0179         C         UP TO STATEMENT 9015, THE EIGENVECTORS ARE SOLVED         LIN 202           0100         NLAST=NH+1         LIN 203           0110         NLAST=NH+1         LIN 204           0181         IF(II-1) 141,141,142         LIN 205           0182         142 D0 105 I=1,NN         LIN 205           0183         ND 107 J=1,NN         LIN 207           0184         A(I,J)=H(I,J)         LIN 207           0185         IF(I-J) 107,139,107         LIN 209           0186         139 A(I,J)=A(I,J)=FIGV(II)         LIN 210           0187         107 CONTINUE         LIN 210           0188         105 U(I)=-U(I,NM)         LIN 212           C         TEST TO SEE WHETHER DIAGONAL ELEMENT IS ZERO OR NOT         LIN 213           C         TEST TO SEE WHETHER DIAGONAL ELEMENT IS ZERO OR NOT         LIN 214           C         DO 9015 I=1,NN         LIN 215           O189         DO 9015 I=1,NN         LIN 216		1	/			11 201				
0179       NN=NM-1       LIN 203         0180       NLAST=NM+1       LIN 205         0181       IF(II-1) 141.141.142       LIN 205         0182       142 DD 105 I=1.NN       LIN 206         0183       DD 07 J=1.NN       LIN 206         0184       A(I.J)=R(I,J)       LIN 206         0185       IF(I-J) 107.139.107       LIN 206         0186       139 A(I.J)=R(I,J)=FIGV(II)       LIN 210         0187       107 CONTINUE       LIN 210         0188       105 U(I)==0(I.NM)       LIN 213         C       TEST TO SFE WHETHER DIAGONAL ELEMENT IS ZERO OR NOT       LIN 213         C       0189       DD 9015 I=1.NN       LIN 216         790       790       LIN 216       LIN 216		e	UP TO STATEMENT 9015. THE	EIGENVECTORS ARE SOLVED	L	IN 202				
0180       NLASTENH+1       LIN 204         0181       IF(II-1) 141+141,142       LIN 205         0182       142 DD 105 I=1,NN       LIN 206         0183       DD 107 J=1,NN       LIN 207         0184       A(I,J)=I(I,J)       LIN 207         0185       IF(I-J) 107,139,107       LIN 209         0186       139 A(I,J)=FIGV(II)       LIN 210         0187       107 CONTINUE       LIN 211         0188       105 U(I)==0(I,NM)       LIN 212         C       TEST TO SEE WHETHER DIAGONAL ELEMENT IS ZERO OR NOT       LIN 213         C       0189       D0 9015 I=1,NN         0189       D0 9015 I=1,NN       LIN 216	V1 / Y	(	4M=/W-1		L	IN 203				
0181       IF(II-1) 141,141,142       LIN 205         0182       142 DD 105 I=1,NN       LIN 206         0193       DD 107 J=1,NN       LIN 207         0184       A(I,J)=P(I,J)       LIN 208         0185       IF(I-J) 107,139,107       LIN 209         0186       139 A(I,J)=FIGV(II)       LIN 210         0187       107 CONTINUE       LIN 211         0188       105 U(I)=-U(I,NM)       LIN 212         C       TEST TO SEE WHETHER DIAGONAL ELEMENT IS ZERO OR NOT       LIN 213         C       TEST TO SEE WHETHER DIAGONAL ELEMENT IS ZERO OR NOT       LIN 216         0189       D0 9015 I=1,NN       LIN 216         790       2790       107	0100		NLAST=NM+1		L.	IN 204				
01A2       142 DD 105 I=1,NN       LIN 206         01A3       DD 107 J=1,NN       LIN 207         01A4       A(I,J)=H(I,J)       LIN 208         01A5       IF(I-J) 107,139,107       LIN 209         01A6       139 A(I,J)=A(I,J)-EIGV(II)       LIN 210         01A7       107 CONTINUE       LIN 211         01A8       105 U(I)=-D(I,NM)       LIN 212         01A9       C       TEST TO SEE WHETHER DIAGONAL ELEMENT IS ZERO OR NOT       LIN 213         C       TEST TO SEE WHETHER DIAGONAL ELEMENT IS ZERO OR NOT       LIN 214         C       0189       DO 9015 I=1,NN       LIN 216	0181	1	(F(II-1) 141,141.142		E.	IN 205	· ·			
0193       DD 107 J=1,NN       LIN 207         0164       A(I,J)=R(I,J)       LIN 208         0135       IF(I-J) 107,139,107       LIN 209         0166       139 A(I,J)=EIGV(II)       LIN 210         0187       107 CONTINUE       LIN 211         0188       105 U(I)==D(I,NM)       LIN 212         0189       C       TEST TO SEE WHETHER DIAGONAL ELEMENT IS ZERO OR NOT       LIN 215         0189       D0 9015 I=1,NN       LIN 216       LIN 216         790       290       290       200       200	0182	142 [	00 105 L=1.NN		L'	IN 206		•		
0184       A(1,J)=H(1,J)       LIN 208         0185       IF(I-J) 107,139,107       LIN 209         0186       139 A(1,J)=A(1,J)=EIGV(II)       LIN 210         0187       107 CONTINUE       LIN 211         0188       105 U(I)=U(I,NM)       LIN 212         C       TEST TO SEE WHETHER DIAGONAL ELEMENT IS ZERO OR NOT       LIN 213         C       0189       DO 9015 I=1,NN         0189       DO 9015 I=1,NN       LIN 216         790       790       I	0193	n	00 107 J=1.NN	•	· L'	IN 207				
0135       IF(I-J) 107,139,107       LIN 209         0186       139 A(I,J)=A(I,J)=EIGV(II)       LIN 210         0187       107 CONTINUE       LIN 211         0188       105 U(I)==0(I,NM)       LIN 213         C       TEST TO SEE WHETHER DIAGONAL ELEMENT IS ZERO OR NOT       LIN 214         C       0189       D0 9015 I=1,NN         O189       D0 9015 I=1,NN       LIN 216         790       790       Intervention	0184			•	L ·	IN 208	•			
0186     139 A(1,J)=A(1,J)=EIGV(II)     LIN 210       0187     107 CONTINUE     LIN 211       0188     105 U(1)=-B(1,NM)     LIN 213       C     TEST TO SEE WHETHER DIAGONAL ELEMENT IS ZERO OR NOT     LIN 214       C     0189     DD 9015 I=1,NN       790     790	0135					IN 200				
0187 INFACT, J)-EIGV(II) 0187 INFACT, J)-EIGV(II) 0187 LIN 210 LIN 211 LIN 212 LIN 213 C TEST TO SEE WHETHER DIAGONAL ELEMENT IS ZERO OR NOT C 0189 DO 9015 I=1,NN LIN 216 LIN 216 LIN 216 LIN 216 LIN 217 LIN 216 LIN 216 LIN 216 LIN 216 LIN 217 LIN 216 LIN 216	0186	170		•		IN SIA				
0188 107 CONTINUE 0188 105 U(I)==U(I.NM) C TEST TO SEE WHETHER DIAGONAL ELEMENT IS ZERO OR NOT C 0189 DO 9015 I=1.NN LIN 215 LIN 216 4 790	0107	139 4	((1+J)=A(1+J)=EIGV(II)							•
C TEST TO SEE WHETHER DIAGONAL ELEMENT IS ZERO OR NOT C 0189 DO 9015 I=1,NN LIN 216 T90 790	0100	107 0	UNTINUE		L	11 211	· ·			
C TEST TO SEE WHETHER DIAGONAL ELEMENT IS ZERO OR NOT C 0189 DO 9015 I=1.NN LIN 215 LIN 216 790	V103	105 0	(I)=-B(I.NM)	٠,	L ?	IN 212				
C DO 9015 (=1, NN LIN 2260 DR NOT LIN 214 LIN 215 LIN 216 790		C TES	T TO SEE WHETHER DIAGONAL F	LEMENT IS ZERO OD NOT	L7	IN 213				
0189' DO 9015 I=1,NN LIN 216 790		С		THE IS LERO UN NUT	LI	IN 214				
LIN 216 790	0189	۵	0 9015 I=1.NN	· .	· L7	IN 215				
				• •	LI	IN 216				
790 j	•		• •	· · ·						
<del>7</del> 90										· •
7 <u>90</u>				•						5
790				• •	•		•		1. S.	Ū į
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130	700							•	·	•
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019)	IF(I-NN) 9021.9007.9021				4 9 61	217					
0191	9021 IF(A(I+I)-EPS) 9005-0006-000	7			1 P · ·	E17					
0192	9005 IF(-A(1+1)-EPS) 0006-006-00	7		•		619		. •	•		
0193	9006 U(I)=U(I)+11(I+1)	· ·	• • •		LIN	219					
0194	DD 9023 1=1 ANN	-	•		LIN	220					
0196	007050 041100 0023 4/1-11-4/1 114/1-11				LIŅ	221					•
01.04	TO TO COST		•	•	LIN	222					
01.00	GU 10 9021				LIN	223		-			
V197	VU( DIV#A(I.I)	•		••	LIN	224					
01.44	U(1)=U(1)/DTV			-	LIN	225				•	
	, C	· ·	• •		LIN	226					
	C DIVIDE ALL ELEMENTS OF I-TH EQ	JATION BY A(I,I)			LIN	227		•			
	c				LIN	228					
0199	00 9009 J=1,NN					220					
0200	9009 A(1,J)=A(1,J)/DIV			•	1 7 1 1	227				•	
	C			•		230			•		
	C REDUCE THE ISTH ELEMENT OF THE				LIN	231					
		UTALK EQUALITINS	IU ZERO		LIN	232					
0201					LIN	233			•		
0202			•		LIN	234				•	
0202	UCLIMAINMII) 0016 Tringet off off off				LIN	235					
0203	TVID IPTMM-IJ 9010,9015,9010				LIN	236			•		
VEU4	AATA A(WW)=A(WW)-A(I)+DELL				LIN	237					
0205	DO 9011 J=1.NN				LIN	238					
0206	9011 A(MM.J)=A(NM.J)-A(I.J)+DELT				LIN	239		•			
02 07	9015 CONTINUE		•		LIN	240					
0208	Y ( NM ) =1 . 0				5 TA1	241					
0209	DO 108 J=1.NN					r 71 040					
0210	108 Y(I)=U(I)					242	• .				
	C	•			LIN	243					
	č ·	•			LIN	244		-			
					LIN	245				•	
	C CALCULATION OF DISPLACEMEN	IT VECTOR			LIN	246					
0211				•	LIN	247			•		
VELL	141 ()V(1)=0.5#Y(1)				LIN	248					
0212	DCI 163 I=2,NM				LIN	249					
6120	163 DV([)¤DV([−1)+Y([)				LIN	250		•			
0214	DIV=DV(NM)				LIN	251					
0215	DD 190 I=1.NM				LIN	252			•		
0216 .	190 DV([)=DV([)/DIV			•.	1 7 14	267					
0217	WRITE(6,5) II					203					
0218	5 FORMAT(12.8X.16HTH. EIGEN VEC	TOR . 5% . 19HD TEDL 40	CHENT VES			254					•
0219	DO 10 Kut.NM	TOT TOT TOTOLOPLAC	THEME ARCI	UR) ,	LIN	255			•		
0220	10 WITE (6.70) V/VI.DV/VI			•	LIN	256					
0221	TO BOUNAT/200 4 212 41			•	LIN	257					
N 5 4 1	77 FURMAILF22.05+13+0} .	•			LIN	258					
V266	DU 201 T=1.NM	•	. •		LIN	259	•				
V283	SUM=-EIGV(II)*Y(I)				LIN	260					•
0274	DD 200 J=1.NM				LIN	261					
0225	SUN=SUM+B(I,J)+Y(J)				I IN	262					
0226	200 CONTINUE					267					
0227	204 FORMAT(1H +2HI=+12+4X+4HSUM=+	F15.8)			1 7 1	044					
0228	201 WRITE(6,204) 1.5UM					204					
0229	IF(11-1) 151,151,152				LIN	205		•			
0230	151 DD 143 T#1-NM				LIN	266					
0231	143 DE(1)=DV(1)	,			LIN	267					
0232					LIN	268	·				
VEUF		· ·	•		LIN	269					
		•	•	•	LIN	27.0					
		-	•	•		-					
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	c		-	•		· •		· .
	č	TEST FU- U-T+OGONAL+TY OF E+GEN VECTO-S	LIN	271				•
0233	152		LIN	272		. •		.•
0234			LIN	273	•			
0235	153		LIN	274				
0236		WRIFF(5,154) 11.00M	LIN	275		•		
0237	154		LIN	276		•		
	c	**************************************	LIN	277				
•	Ċ	APPLICATION OF CALENT AND APPLICATION	LIN	278				
	с		LIN	279				
	c		LIN	280				
0238	31.0	DO 300 I = 1.NM	LIN	281		•		
0237			LIN	282			•	
0240			LIN	283				
0241			LIN	284				
0242 .	30.0		LIN	285	•			
0243			LIN	286				
0244			LIN	287			•	
0245	30 1		LIN	288				
0246			LIN	239				
0247	302		LIN	201				
	с "	MULTID ICAL PICTURE STICKERS MATRIX • 5X+6HCMASS=+E20+8)	LIN	291				
0248	-	DO 303 LENN	LIN	292				
0249			LIN	293		•		
0250			LIN	294				
0251			LIN	295				
0252	30.3		LIN	296				
0253	003		LIN	297				
0254			LIN	208				
0255			LIN	200				
0256	304	CONTERPECTIEF+DV(I)+V(I)	LIN	300				
0257	304		LIN	301				
0258	304	WRITE (0,305) CSTIFF	I TN	302				
1200	200	FURMATION COEFFICIENT OF STIFFNESS MATRIX +5X+7HCSTIFF=+E20+8)	I TN	302				
1250	C	MULTIPLICATION OF FORCE MATRIX		303	•			
0260				305				
1261		CrURCL=Y([]#Y(])#FM(])	I TN	306				
1262	707	10 306 I=2,NM		300				•
263	306	CFURCE=CFURCE+(Y(I)-Y(I-1))*Y(I)*FM(I)		307				
264	707	WRITE (6, 307) GFORCE	ITN	308				
265	307	FURMAT( '0', COEFFICIENT OF FORCE MATRIX', 5X, 7HCFORCE=, E20, 8)	A TM	309				
245		GFRF0=CSTIFF/CMASS	1 T.N	310	•	• •		
267		GLUAD CST IFF/CFORCE	1.7.5	311				
1268	200	WRITE(6,308) GFREQ	1 7 51	312				
269	304	FURNALLEDT, FREQUENCY BY GALERKINS METHOD +5X+6HGFREQ=+F20+81	L IN	313				
270	30.0	WRITE(6,309) GLUAD	1 11	316				
271	309	TURMAT( 0 ", BUCKLING LOAD BY GALERKINS METHOD ,5X .6HGLOAD = 520 - 8		315				
272	110		1 1 1	310				
			L TM	317				
•				219				
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