A New Tonal World:

The Bohlen-Pierce Scale

by

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Abstract

This paper compares the Bohlen-Pierce scale to other octave and non-octave-based tuning systems, drawing parallels between it and the widely used 12-equal temperament. These parallels lead to the hypothesis that there can be a set of harmonic rules applicable to the Bohlen-Pierce scale that are analogous to the current musical practice. Those theorized rules are then applied to some examples of the growing body of musical compositions written in the Bohlen-Pierce scale. Also included are supportive arguments for a preference of the use of odd-partial timbres in performance of this scale, which make the invention of the Bohlen-Pierce clarinet a major turning point in the scale's development. Some of the musical works studied were written specifically for this author's performance on the Bohlen-Pierce clarinet.

Abrégé

Ce document compare la gamme Bohlen-Pierce à d'autres systèmes d'accord octave et non-octave-basés, établissant des parallèles entre soi et le système la plus utilisé, tempérament de 12-égal. Ces parallèles mènent à la possibilité qu'il peut être des règles harmoniques applicables à la gamme Bohlen-Pierce qui sont analogues à la pratique musicale courante. Ces règles théorisées sont alors appliquées à quelques exemples du corps croissant de compositions musicales écrites à la gamme Bohlen-Pierce. Il-y-a aussi inclus des arguments qui support la préférence d'utilisation des timbres impair-partiels dans la performance de cette gamme, qui ferait l'invention de la clarinette Bohlen-Pierce un point tournant majeur dans la développement de cette gamme. Certains compositions etudié a été ecris pour la pérformance sur clarinet Bohlen-Pierce par cette auteure.

Acknowledgements

This paper represents more than a product of research, it also represents the culmination of a performance project. At the instigation of James Bergin of the Boston Microtonal Society and with the financial assistance of the McGill University Schulich Scholarship, I purchased a Bohlen-Pierce clarinet and began finding composers interested in working with this strange new scale. The result was six new works for the instrument, all of which were premiered in March 2010 at the first ever Bohlen-Pierce Symposium in Boston. My thanks go out to those composers: Julia Werntz, Roger Feria, Stratis Minakakis, Katarina Miljkovic, Anthony De Ritis and James Bergin.

I'd like to especially thank Stephen Fox for his innovation and dedication to the Bohlen-Pierce Clarinet Project. His work is always on demand in the clarinet world and yet he found the time to construct these instruments, which are a joy to play. Also, I must thank Georg Hajdu who first approached Stephen to build the horns, and who coordinated the entire Bohlen-Pierce Symposium. He tirelessly organized this three-day event with composers, researchers and performers spanning three countries: Canada, Germany and the United States. Because of his brainchild, I was able to meet and work with many other artists specializing in Bohlen-Pierce performance: Nora-Louise Müller, Ákos Hoffmann, Tilly Kooyman and Todd Harrop, just to name a few. These artists and our collaborations had a profound effect on me and my work with the Bohlen-Pierce scale.

The support of the faculty at McGill University has been invaluable. Many thanks to Jon Wild who's extended knowledge on the subject of alternative tuning systems was extraordinarily helpful. Also thanks to Julie Cumming for her assistance in organizing the paper. Both Jon and Julie had the daunting task of reading this paper a number of times throughout the writing and editing process. Bravo and thanks to both of you! Also, I am indebted to Simon Aldrich for his assistance preparing the pieces for performance. His musical knowledge shaped my experience throughout this project and the rest of my education at McGill University.

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Introduction

The Bohlen-Pierce scale is an alternative tuning system that offers a fresh new set of intervals and pitches completely unlike those in the familiar twelve-tone scale, yet certain parallels exist between both scales. For example, it can be used both chromatically and diatonically and has its own set of modes analogous to our traditional church modes. It is also a very consonant scale and is supported by the harmonic series in its own way. These parallels to 12-equal temperament suggest that the Bohlen-Pierce scale may prove to have its own set of harmonic rules parallel to the traditional music theory. It offers much to those wishing to explore the possibilities of new sounds both consonant and dissonant, and has spurred an increasingly growing repertoire ranging from avant-garde to pop music.

This paper will begin with an introduction to the harmonic series and how it relates to the construction of scales. Chapter 1 will compare various octave and non-octave scales with regards to their relationship to the harmonic series. Chapter 2 will cover the discovery and construction of the Bohlen-Pierce scale and how it relates to our current musical system, including the exploration of a possible Bohlen-Pierce harmonic theory. Chapter 3 will explore what timbres work best with the Bohlen-Pierce scale, followed by descriptions of the available Bohlen-Pierce instruments. Also included here are instruments that have been constructed for other alternative tuning systems. Chapter 4 will apply the so called Bohlen-Pierce harmonic theory to various musical examples, some of which were composed for and premiered by the author.

Chapter 1 - New Scales, New Possibilities

1.1 - Alternative tuning systems

Alternative tuning systems offer new expressive possibilities. Our ears have been conditioned to hear twelve notes per octave, and many listeners will be surprised by the use of new scales. Alternative tunings go against our harmonic expectations and can be used to create tensions that would not be possible in any of the most atonal pieces. Additionally, some tunings can be used to create pure consonances that would not be possible in our current twelve-tone system. The interest in alternative tunings can be purely due to the intervals themselves being interesting. As microtonalist Julia Werntz put it:

> "A concerted effort to avoid emphasis of the traditional intervals is based, of course, not on some presumed criteria regarding the quality of certain relationships, but rather upon the need simply to experience the new relationships, to become familiar with them and give them musical relevance. This … is not unlike Schoenberg's advice in 1923 to avoid major, minor and diminished triads, advice that may seem, especially to today's twelve-note composers, dogmatic and arbitrary."[1]

Microtonal music frees the composer from the tonal implications of diatonicism; rules are tossed aside and sound is explored for its own sake. Microtonality, "with its additional, historically unfamiliar intervallic relationships – its ready-made 'innocence' – can provide today's composer with a renewed, heightened sense of that freedom with which to compose."[2]

Twelve-equal temperament became the scale of choice today, but only

1 Werntz, 32.
2 Werntz, 43.

after centuries of development. It represents a compromise between intonation and convenience of performance. This system is useful because it uses a relatively small number of notes per octave, but it compromises the pure tuning of just intervals derived from the harmonic series.

1.2 - Just Intervals and the Harmonic Series

Why all the fuss about just intervals? Their mathematical relationship is simpler, which may deem them more consonant to the ear. Consider the octave, which is achieved by a doubling of frequency, giving it a ratio of 2:1. Play this interval on a piano exactly in tune and it sounds consonant, so consonant that our brain perceives both notes as equivalent, with one pitch higher than the other. If the octave is a bit too narrow or wide, the interval becomes dissonant to the ears. The false-octave creates discordance with the second harmonic of the bottom pitch (see Figure 1), which is a pure octave. For example: an octave below A440 is A220. The harmonics that ring above the fundamental A440 are at 880Hz, 1320 Hz, 1760 Hz, etc. and the harmonics above A220 are at 440 HZ, 660 Hz, 880 Hz, 1100 Hz, 1320 Hz, etc. When the octave between A220 and A440 is in tune, their harmonics line up creating a pure interval to the ear. However, if one of the pitches were to be off by a few hertz, there would be discordance among the harmonics, creating a rougher sounding interval. Intervals described in small-numbered ratios, like the pure octave (2:1), are deemed the most consonant by the ear.

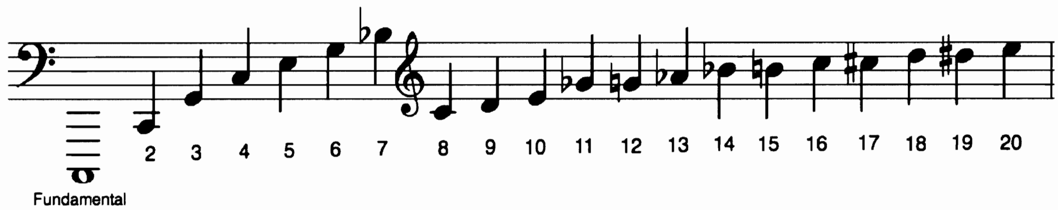


Figure 1.1: Harmonic Series

Looking at the harmonic series (Figure 1.1), one can see how the ratios describing intervals reflect their relationship in the harmonic series. For example, the octave is a 2:1 relationship and is also the interval between the second harmonic and the fundamental. Likewise, a pure perfect fifth (3/2) is the relationship between the third harmonic and the second harmonic. This interval is a third-partial interval, because of its placement in the harmonic series. Other third-partial intervals are ratios made up of only the numbers 2, 3, and their multiples. The pure perfect fourth (4/3) is another example of a third-partial interval. 4/3 can be achieved by multiplying 2/3 (a perfect fifth downwards from the fundamental) by two; multiplying an interval by two displaces a note by an octave: F-C is a perfect fifth and C-F is a perfect fourth. Therefore, fourth-partial intervals are related to intervals of previous partials by an octave. It is the prime partials that are of concern because higher prime partials introduce new frequencies not divisible by previous partials. The fifth-partial is the next prime partial; an example of a fifth-partial interval is the just major third, which is created by the relationship between the fifth-partial and the fourth-partial (5/4). The just minor sixth (8/5) is another fifth-partial interval that can be created by octave displacement, in the same way as the perfect fourth. The minor third (6/5) and the major sixth (5/3) are other third-partial intervals.

These intervallic relationships in the harmonic series have found their way into our traditional harmonic system. The current twelve-equal system makes use of a pure octave; the octave is the closest harmonic to the fundamental and is therefore treated with great importance. The third-partial intervals, the perfect fifth and perfect fourth, are not perfectly in tune; the tempered perfect fifth is 2 cents too narrow and the tempered perfect fourth is 2 cents too wide. The fifth-partial is more out of tune; the tempered major third is 16 cents too wide and the tempered minor sixth is 16 cents too narrow. The seventh-partial is so far off in the 12-equal scale that it is debatable whether or not it is part of our current musical system. The minor seventh created by the relationship between the seventh and fourth partial is approximated in 12-equal by an interval that is 31 cents too wide. The eleventh-partial and thirteenth-partials are even more out of tune and are not usually considered to be approximated by 12-equal.

1.3 - The Seventh Partial

Advocates of alternative tuning systems have argued that a large weakness of twelve-tone equal temperament is that it does not approximate the prime partials higher than the fifth-partial well.. In his PhD thesis, *Multiple division of the octave and the tonal resources of the 19-tone equal temperament*, microtonal composer Joel Mandelbaum said, "It is to those who regard a close approximation to the 5th partial or any kind of approximation of the 7th partial as desirable that 12-tone temperament must appear inadequate. For such musicians … multiple division of the octave is justified on acoustical grounds."[3]

The seventh partial is particularly important to many microtonalists because it is already implied by some harmonies in 12-equal, the 4:5:6:7 chord having some similarity to the dominant seventh chord. However, as Benson has discussed, a pure 4:5:6:7 chord would be extremely stable, whereas the dominant 7th chord wishes to move back to the tonic major triad (4:5:6).[4] Microtonal composer, Ivor Darreg agrees:

> "The errors for [seventh partial] intervals in the 12-tone system are so large it isn't even funny. In classical harmony one pretends that the seventh harmonic does not exist at all, carefully ignoring the greater dissonance of the interval substituted for it, and treating the dominant-seventh chord as a dissonance requiring resolution."[5]

For Mandelbaum, the seventh partial is "the prime reason for looking beyond the horizon of the 12-tone system.[6] Both Mandelbaum and Darreg have praised 31-equal temperament for its exceptional seventh partial.

1.4 - 19- and 31-Equal Temperaments

Mandelbaum's thesis focused greatly on 19-equal, perhaps because there are few enough notes for it to be a convenient choice, but his microtonal compositions were mainly in 31-equal. 19-equal approximates the fifth-partial

3 Mandelbaum, 77.
4 Benson, 162.
5 Darreg (1978).
6 Mandelbaum, 67-8.

better than 12-equal temperament (major thirds are only 7 cents flat), while sacrificing the purity of the third-partial (perfect fifth), which is 7 cents flat. The seventh partial fares only slightly better. 31-equal is a superior choice for its approximations of the third, fifth, and seventh partials. The perfect fifth is slightly over five cents flat and the major third is less than a cent sharp. The seventh partial is also good, with a 7:4 that is only a cent flat.

31-equal is probably even better known for its close approximation of 1/4-comma meantone tuning; its flat fifths are almost identical to meantone tempered fifths. Meantone tuning tempers the fifth to allow for purer thirds whereas a tuning such as Pythagorean tuning gives pure fifths while compromising the intonation of the thirds. There is a difference of 21.5 cents between the third achieved by stacking pure fifths and the pure major third; this difference is known as a syntonic comma. If the difference of the comma is divided among tempered fifths, purer thirds will be achieved. During the 16th-century 1/4-comma temperament was a very popular meantone tuning; it divides up the comma among the fifths, tempering each of them by a quarter-comma. This creates pure major thirds (stacking four pure fifths, say C-G-D-A-E, each tempered by a quarter comma, achieves a pure major third). This system is still not closed, however, 31-equal very nearly mimics 1/4-comma meantone tuning, creating a very useful closed system. 16th-century Italian musician, Nicola Vicentino and 17th-century Dutch mathematician, Christiaan Huygens, were among the first to make use of 31-equal as a very good approximation of 1/4-comma meantone.

1.5 - Just Intonation

For some composers, approximation of just ratios is not good enough. Harry Partch and his student Ben Johnston are contemporary pioneers of just intonation. To Partch, "the faculty – the prime faculty – of the ear is the perception of small-number intervals"[7] Partch blatantly disregards the traditional scale in his *Genesis of a Music* and with that the entire body of musical knowledge, choosing to begin anew on his own. He developed a 43-tone monophonic scale, which means it is derived directly from the overtones of a single pitch; it is an unequal symmetrical scale that stems from the just ratios of the harmonic series.

Partch uses ratios involving all prime numbers up to 11 in his 43-tone scale, so it can be said to be an 11-limit scale. The table below (see Table 1.1) shows that the ratio's integers contain prime numbers up to 11 and their multiples. Hindemith and Schoenberg believed that the 11th harmonic is implied in the tritone from the tonic however this pitch is so out of tune that one could not possibly hear the pitch as an eleventh-partial interval. Just intonation could go on to add more prime number limits, but Partch stops at the 11-limit. Of this he says, "When a hungry man has a large table of aromatic and unusual viands spread before him he is unlikely to go tramping along the seashore and in the woods for

7 Partch, 87.

still other exotic fare."[8] 11-limit provides enough for him in terms of new territory and he sees no reason to go on until that area has been explored fully.

Step	Ratio	Cents	Step	Ratio	Cents
1	1/1	0	23	10/7	617.5
2	81/80	21.5	24	16/11	648.7
3	33/32	53.2	25	40/27	680.5
4	21/20	84.5	26	3/2	702.0
5	16/15	111.7	27	32/21	729.2
6	12/11	150.6	2	14/9	764.9
7	11/10	165.0	29	11/7	782.5
8	10/9	182.4	30	8/5	813.7
9	9/8	203.9	31	18/11	852.6
10	8/7	231.2	32	5/3	884.4
11	7/6	266.9	33	27/16	905.9
12	32/27	294.1	34	12/7	933.1
13	6/5	315.6	35	7/4	968.8
14	11/9	347.4	36	16/9	996.1
15	5/4	386.3	37	9/5	1017.5
16	14/11	417.5	38	20/11	1035.0
17	9/7	435.1	39	11/6	1049.0
18	21/16	470.8	40	15/8	1088.3
19	4/3	498.0	41	40/21	1115.5
20	27/20	519.5	42	64/33	1146.8
21	11/8	551.3	43	160/81	1178.5
22	7/5	582.5			

Table 1.1: Partch's 43-tone Scale

Ben Johnston was also an advocate of just intonation. His biographer, Heidi von Gunden notes, “Since he was eleven years old Johnston had been concerned about pitch integrity, and his own good sense of hearing told him that equal temperament's consonances were not accurate.”[9] Johnston's use of just

8 Partch, 123.
9 Von Gunden, 89.

intonation expanded from Partch's to include higher partials, up to the 31-limit[10]. Johnston developed an intricate notational system (see Figure 1.2) which allowed him to write for traditional instruments. This involved special accidentals for deviations as small as the syntonic comma (notated with a plus or minus), as well as deviations allowing pitches to be heard as 7th or 13th partials and beyond (notated as a right-side-up and upside-down "7" or "13", etc. for overtone and undertone relationships, respectively). Using these symbols in combination, Johnston is able to obtain hundreds of pitches to the octave, allowing him to create the purest sonorities, the small-number ratios that Johnston and Partch, among other advocates of just intonation, believe that the ear prefers.

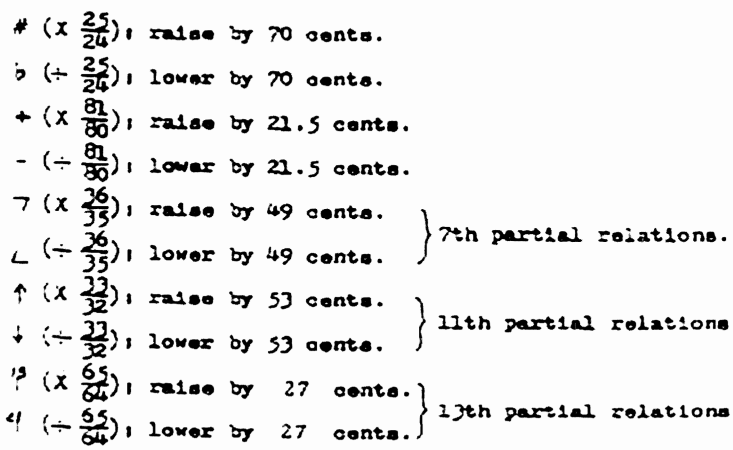


Figure 1.2: Ben Johnston's Just Intonation Notation[11]

1.6 - 24-Equal Temperament

Arguably the most convenient choice for microtonalists is 24-equal

10 Werntz.
11 Johnston, Trio

temperament, because of its close relationship to our 12-equal scale. It divides each traditional semitone of 100 cents in half, into quarter tones of 50 cents each; thus, it is more often known as the quarter-tone scale. This scale is playable on most traditional instruments (with the exception of keyboard instruments which would have to be specially constructed), by adjusting from the ear on stringed instruments, or using alternative fingerings and/or changing the embouchure on wind instruments.

24-equal offers a slightly better 7th partial; 7:4 (968.8 cents) is approximated by 950 (about 19 cents off) rather than 1000 (about 31 cents off). However, it is the improved 11th partial which was most noted by quarter-tone advocates. Twentieth-century microtonal composer, Ivan Wyschnegradsky and a number of others "have cited the 11th partial as the basis for quarter-tone music (24-equal temperament) owing to the close relationship between the intervals 12:11 (150.6 cents), 11:9 (347.4 cents) and 11:8 (551.3 cents) and the corresponding intervals in 24-tone temperament (150, 350 and 550 cents respectively.)"[12]

1.7 - 72-Equal Temperament

72-equal temperament is also related to 12-equal; because 72 is a multiple of 12, all of the pitches of the 12-equal scale are in the 72-equal scale. Each traditional semitone is divided into six steps; therefore, the smallest step in 72-

12 Mandelbaum, 69.

equal is a twelfth-tone. A number of composers are currently working in this scale; a large number of them reside in Boston, having studied or worked with the late 72-equal composer, Joe Maneri, at the New England Conservatory. One such composer, Ezra Sim intonation.

1.8 - 18- and 36-Equal Temperaments

36-equal is related to 12-equal in that it divides each semitone into thirds, creating a step-size of a sixth-tone. Italian composer, Ferruccio Busoni, first proposed the division of the octave into third-tones, which would make 18-equal temperament, in his paper *Sketch of a New Aesthetic of Music*, in 1907. The problem with 18-equal is that it does not contain anything resembling our fourths or fifths. According to microtonalist, Ivor Darreg, "Atonality had not yet taken over the place in serious music that it occupies today, so the resistance to such an affair as a fifthless scale was very strong."[13] Busoni divided the scale further into 36-equal and had a harmonium, whose manuals were in third-tones, separated by traditional semitones. 36-equal contains all of the aspects of 12-equal, with the additional close approximation of the seventh partial. Carrillo also worked closely with 18-equal and even had a third-tone piano built. Darreg describes his reaction to Carrillo's performance on the third-tone piano:

> "The effect is reminiscent of many efforts to carry on where Debussy left off. This because the 6-tone, or whole-tone scale has no fourths or fifths either--that gives a vague sound overall. For atonality, the absence of fourths and fifths would be an advantage – no longer necessary to avoid implying common

13 Darreg (1987).

> chords, the very reason why much "12-tone school" composition sounds so strained and awkward and made up of traditional composers' rejected material!"[14]

1.9 - Wendy Carlos' Scales

Wendy Carlos sought after just ratios, but disregarded the octave with her three non-octavian scales: *Alpha*, *Beta* and *Gamma*. Rather than use a whole number of notes to an octave, Carlos avoided the octave, but wanted to find equal-tempered scales that had nearly perfect 3/2, 5/4, 6/5, 7/4, and 11/8. She used a computer program to do this; the three scales listed above were the most consonant sounding to her ears. Of them her favorite was Alpha, which has 78 cents per step and 15.385 steps per octave. Alpha offers excellent seventh partials, and pure harmonies. Carlos describes Alpha as "amazingly exotic and fresh, like you've never heard before."[15]

14 Darreg (1987).
15 Carlos.

Chapter 2 - The Bohlen-Pierce Scale

2.1 - A Thirteen Step Scale

The Bohlen-Pierce scale is a thirteen step scale based upon, rather than an octave, the framework of the perfect twelfth (3:1). Its just form contains only odd-numbered ratios based upon the intervallic relationships among odd-numbered partials of the harmonic series (for ex: 5/3, 7/3, 9/7, etc.). The equal-tempered form also contains thirteen steps within the span of 3:1. Equal-tempered scales are the most convenient choice for instrument construction. Much like the prevalence of 12-equal temperament, the equal-tempered scale has become somewhat standard for Bohlen-Pierce performance.

The Bohlen-Pierce scale does not contain an octave; rather, the entire 13-step scale is based within the framework of a perfect twelfth (3:1), which was named a *tritave* by co-inventor John Pierce. The name's prefix *tri-* recalls the 3:1 relationship, and the suffix *-ave* is taken from the term *octave* suggesting the tritave's role as the interval of equivalence in the Bohlen-Pierce scale.

The tritave is the partial directly above the fundamental in an odd-partial harmonic series; therefore, it is treated with the same importance in Bohlen-Pierce as the octave in 12-equal. When playing a tritave with timbres of only odd partials, the higher pitch adds no new frequencies in its harmonics. For example, the harmonics that ring above A220 are at 660 Hz, 1100Hz, 1540Hz, 1980Hz, 3300Hz, etc. An E660, one tritave above A220, contains the harmonics 1980Hz, 3300Hz, etc. This would suggest that the tritave may be considered the interval of equivalence when played with timbres of only odd harmonics.

2.2 - The Discovery of the Bohlen-Pierce Scale

The Bohlen-Pierce scale was discovered independently by three different people: Heinz Bohlen, Kees van Prooijen and John Robinson Pierce. German microwave engineer, Heinz Bohlen (b. 1935) discovered the scale while experimenting with alternative musical systems in 1972, but he did not publish his paper *13 Tonstufen in der Duodezime* until 1978. This paper was only circulated among the German speaking population. A few months later, Dutch software engineer and music theorist Kees van Prooijen (b. 1952), not knowing of Bohlen's discovery, mentioned a scale with thirteen steps to the third harmonic in his *A Theory of Equal-tempered Scales,* published in a Dutch music journal. The scale had yet to travel to the United States until 1984 with the publication of *Four New Scales Based on Nonsuccessive-Integer-Ratio Chords.* Co-written by American microwave engineer John Robinson Pierce (1910-2002), the paper described a scale called P3579, which was later renamed the Bohlen-Pierce scale when Bohlen and Pierce realized they had discovered the same scale.

2.2.1 - Heinz Bohlen's Search for a New Scale

Heinz Bohlen never received musical training, due to a disruption of his education by WWII, and enjoyed a long career in electrical microwave and communications engineering. It was a friend and graduate student at the Hochschule für Musik und Theater (HfMT) in Hamburg who introduced him into the musical world. He made recordings for this friend and his classmates. During this time, Bohlen wondered why the pitches used were limited to the same twelve. His new musician friends could not answer this question, so he began to research it on his own. Paul Hindemith's studies about combination tones and their influence on interval relationships in his book, *Unterweisung im Tonsatz* (*The Craft of Musical Composition*) had a great influence on Bohlen's studies. He drafted a number of graphs (see Figures 2.1 and 2.2) showing how an interval's consonance depended upon their combination tones, and deduced which intervals might show the greatest consonance due to this phenomenon. He explains:

> "If combination tones exerted indeed so much influence on interval properties, then detecting intervals that were in consonance with the combination tones they generated should deliver the building blocks for a harmonic scale. Sure enough, it turned out that all just intervals known as consonant shared this feature of consonance with their related combination tones. Sorting them in accordance to their span resulted in a framework that only needed some fill-in following given examples in order to produce the just chromatic scale."[16]

He came to the conclusion that combination tones of the the pitches of the major chord (4:5:6) gave the chord "an almost unique 'gestalt' appearance."[17] This and the framework of the octave allowed the 12-tone just scale to be the inevitable choice. The equal-tempered version came from what Bohlen called the "principle

16 Bohlen, "An early document on a novel scale."
17 Bohlen, "The Bohlen-Pierce Site."

of equidistance", where a scale is aesthetically pleasing if its pitches "follow the equation, $f_n/f_0 = K^{n/N}$, with f_n meaning the pitch of step n of the scale, f_0 the fundamental tone, K the frame interval and N the total number of steps."[18] Twelve-tone equal temperaments follows this equation, demonstrated as $f_n/f_0 = 2^{n/12}$. Using this "principle of equidistance" and the aforementioned "principle of consonance" as the two requirements of an appealing musical scale, Bohlen set out to see if another scale could be built along those same requirements.

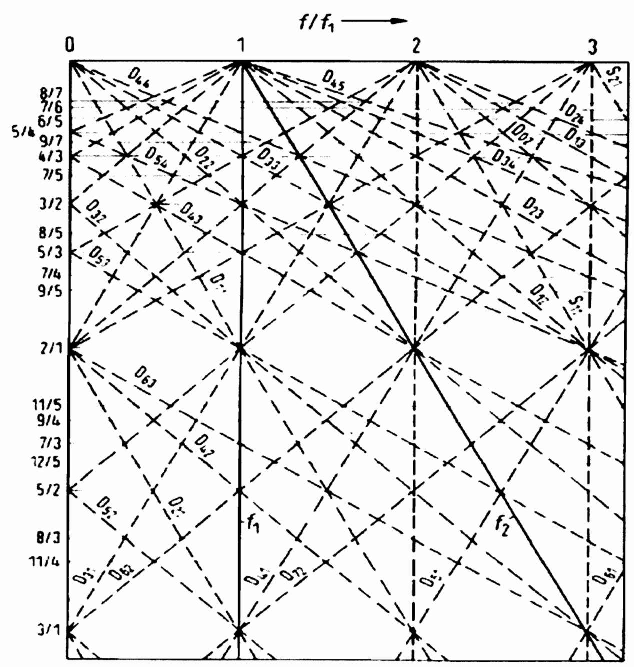


Fig. 1. **Frequenzverlauf der Kombinationstöne D_{mn}, S_{mn} (---) bis zur 8. Ordnung als Funktion zweier primärer Intervalltöne f_1 und f_2 (——) zwischen Einklang und Duodezime.**
(D_{mn} = Differenzton der Ordnung $mf_1 - nf_2$, S_{mn} = Summenton der Ordnung $mf_1 + nf_2$.)

Figure 2.1: Heinz Bohlen's Combination Tone Frequency Chart[19]

18 Bohlen, "The Bohlen-Pierce Site."
19 Bohlen, "An early document on a novel scale."

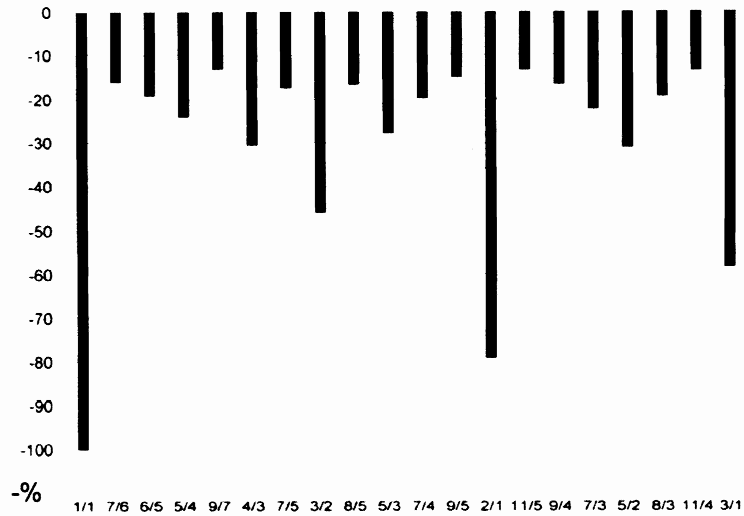


Figure 2.2: Bohlen's Consonance Chart[20]

Rather than use 4:5:6 to build the scale, Bohlen tested for consonant chords and found 3:5:7 appealing. Using the methodology he suspected had led to the intervals of our musical tradition, this chord yielded previously uncommon, but consonant intervals such as 9/7, 7/5, 15/7, and 7/3. This did not fit within the framework of the octave, but when placed within the framework of the perfect twelfth (3:1) yielded what would become known as the Bohlen-Pierce scale as the inevitable choice. The above intervals did not create a viable scale on their own, as there were large gaps that would need to be filled.

Table 2.1 shows the would-be scale steps in order, showing that there are three different gaps: two values for the smallest (1.08 or 27/25 and 1.0889 or 49/45), the next biggest (1.19 or 25/21) and the largest (1.28 or 9/7). These three

20 Bohlen, "An early document on a novel scale."

intervals are in the approximate proportion 1:2:3. Taking the smallest gap as the new scale's semitone, and filling in all of the larger gaps with it yields a 13-step chromatic scale, with odd ratios up to the 7-limit (see Table 2.2). There are two versions of the smallest gap, and thus two semitones to choose from in the creation of this chromatic scale. Bohlen (as did his co-creators after him) used the semitone that created the simplest fraction in each case. He filled the gap between 1/1 and 9/7 with a 25/21 whole tone and 27/25 semitone, filling in the whole tone with the same semitone. This creates a new semitone value between the first and second step of 625/567. This is because the relationship between the semitone and whole tone is not exactly 2:1. This happens again between the 7/5 and 5/3; Bohlen uses the 49/45 semitone value to avoid large fractions, and a fourth semitone value is added: 375/343.

Interval (from tonic)	Relation to previous tone
1/1	
9/7	9/7 = 1.2857
7/5	49/45 = 1.0889
5/3	25/21 = 1.1905
9/5	27/25 = 1.0800
15/7	25/21 = 1.1905
7/3	49/45 = 1.0889
3/1	9/7 = 1.2857

Table 2.1: Gaps in a Scale Created with the 3:5:7:9 Chord[21]

21 Bohlen, "The Bohlen-Pierce Site."

Step no.	Tone (relation to base)	Relation to previous step
0	1/1	-
1	27/25	27/25
2	25/21	625/567
3	9/7	27/25
4	7/5	49/45
5	75/49	375/343
6	5/3	49/45
7	9/5	27/25
8	49/25	49/45
9	15/7	375/343
10	7/3	49/45
11	63/25	27/25
12	25/9	625/567
13	3/1	27/25

Table 2.2: Gaps Filled in to Create Chromatic BP Scale[22]

2.2.2 - John Pierce's Discovery of the P3579 scale

John Pierce discovered the Bohlen-Pierce scale while testing intonation sensitivity of various triads with electrical engineer Max Mathews (b. 1926) at AT&T Bell Laboratories in Murray Hill, New Jersey. Mathews was also well known for his creation of computer music programs, which have inspired the creation of such modern-day programs as Csound and Max/MSP. Mathews is hailed as "The Father of Computer Music" on cSounds.com. His passion for computer music synthesis led him to invent the Radio Baton, with which the user can conduct performances from MIDI files, allowing for complete control over

22 Bohlen, "The Bohlen-Pierce Site."

tempo, dynamics and balance.[23] This instrument has since been used in the Bohlen-Pierce compositions by Berklee professor and composer Richard Boulanger (see Appendix, Nos. 8 and 9).

John Pierce is arguably most famous for his scientific work that led to the invention of the communications satellite but also had a musical background leading to his studies in music, speech and acoustics at Bell Labs. His work with Mathews led to an intense involvement with computer music, psychoacoustics and musical invention, including a new bridge for stringed instruments that has become the model for some digital instruments.[24]

Mathews and Pierce performed listening experiments testing intonation sensitivity of major triads, where intonation sensitivity is defined by a preference for a specific tuning or mistuning of the center note of a triad. The major triad is useful in this experiment because of the very sharp major third in 12-equal-temperament; the center note of the triad is off by 15 cents. In their results, two groups emerged: those who preferred the 12-equal major triad and those who preferred the just tuning of the triad (4:5:6). Looking to continue the study, they used the unfamiliar chords 3:5:7 and 5:7:9 and found that those who preferred 4:5:6 "mistuned" also preferred the 3:5:7 with its center note out of tune by 15 cents. Likewise, those that chose the justly tuned major triad also liked these

23 Marilungo and Boulanger.
24 Cannon.

unfamiliar triads tuned justly.[25]

Pierce wondered if a desirable scale could be made out of these chords. Since the listening subjects reacted similarly to the 3:5:7 and 5:7:9 triads as they did the familiar 4:5:6, perhaps a scale could be built that would be as musically viable as the current 12-equal system. Their paper *Four new scales based on nonsuccessive-integer-ratio chords* outlines their attempted scales: three of the scales were invented by Mathews and were based upon the octave. However, these created scales with three unique intervals rather than the desired two, which would be analogous to the semitone and whole-tone of the current musical system. Pierce discovered this issue was alleviated if a equal-tempered scale with thirteen steps was built upon the framework 3:1; he named his discovery P3579.

When Pierce discovered that Bohlen had already made the same discovery, he renamed the scale the Bohlen-Pierce scale. However, he often referred to it as the "Pierce scale":

> "Well, the most substantial thing I've done … was the Pierce scale. And either this will survive or it will not. You can write attractive music in it, but a good composer can write attractive music with any sounds. A bad composer can't write attractive music, not no how."[26]

2.2.3 - The Scale Studies of Kees van Prooijen

Kees van Prooijen came across the scale in his studies of equal temperaments about the octave and higher harmonics. His paper, *A Theory of*

25 Matthew and Pierce (1988).
26 Pierce.

Equal-tempered Scales is an extremely mathematical approach to tuning; the scale is demonstrated twice in his paper, but it is well-hidden from the public eye within all of his equations.

What is most interesting about van Prooijen's Bohlen-Pierce studies is that his diatonic scales closely mimic those of 12-equal. Much like the current diatonic system, van Prooijen's Bohlen-Pierce system has only two diatonic modes: major and minor, each with 7 steps.

He began by constructing a scale lattice for the diatonic scale in 12-equal temperament with the pitches related by a perfect fifth (3/2) horizontally and a major third (5/4) vertically. Figure 2.3 shows this lattice, expressed in ratios above and note names with chromatic step numbers below.[27]

5/3	5/4	15/8	
4/3	1	3/2	9/8

a:9	e:4	b:11	
f:5	c:0	g:7	d:2

Figure 2.3: 12-equal diatonic scale lattice[28]

van Prooijen used this same methodology to construct a Bohlen-Pierce diatonic scale, constructing a lattice based instead upon the intervals 5/3 and 7/3 (see Figure 2.4). The pitches are related by 5/3 horizontally and 7/3 vertically.

27 van Prooijen (2006).
28 van Prooijen (2006).

The resulting major scale (see Figure 2.5) is an asymmetrical scale with three different step sizes; the order of steps is 312 1 321. Figure 2.6 shows van Prooijen's minor Bohlen-Pierce scale, which has the steps 213 1 231. In his notational system a + or - represents a raising or lowering of pitch by one "semitone." A sharp or flat sign represents a raising or lowering of pitch by two "semitones." van Prooijen's work with the Bohlen-Pierce scale creates modes that are completely analogous to the 12-equal modes, but the equal-tempered chromatic and diatonic scales of Heinz Bohlen and John Pierce represent the standard in Bohlen-Pierce music.

7/5	7/3	35/27	
9/5	1	5/3	25/9

e:4	a:10	d:3	
g:7	c:0	f:6	b:12

Figure 2.4: van Prooijen's BP Diatonic Scale Lattice[29]

C			D	E		F	G			A		B	C
1			35/27	7/5		5/3	9/5			7/3		25/9	3

Figure 2.5: van Prooijen's Asymmetrical Major BP Scale[30]

E		F	G			A	B		C			D	E
1		25/21	9/7			5/3	9/5		15/7			25/9	3

Figure 2.6: van Prooijen's Asymmetrical Minor BP Scale[31]

29 van Prooijen (2006).
30 van Prooijen (2006).
31 van Prooijen (2006).

2.3 - The Just Bohlen-Pierce Scale

The Bohlen-Pierce scale is rather unique in that its just version contains intervals of small-numbered ratios which are not found in the just chromatic 12-tone scale. Those intervals deemed important for tonality such as the octave (2:1), perfect fifth (3:2) and perfect fourth (4:3), which have been the primary intervals used in 12-equal and all of the alternative scales described above, do not exist in the just Bohlen-Pierce scale. Somewhat reminiscent of Busoni's fifthless scale, the Bohlen-Pierce scale was born in a world far more likely to accept such an "atonal" idea.

Step	Interval from Tonic
1	27/25
2	25/21
3	9/7
4	7/5
5	75/49
6	5/3
7	9/5
8	49/25
9	15/7
10	7/3
11	63/25
12	25/9
13	3/1

Table 2.3: The Just Bohlen-Pierce Scale

The just Bohlen-Pierce scale is made up purely of odd-numbered ratios (see Table 2.3), with many small-numbered odd ratios that, although foreign to our 12-equal tuned ears, may be deemed consonant, such as 7:3, 7:5 and 9:7. This scale can be considered as analogous to our traditional twelve-tone scale; where the construction of the 12-equal scale reflects relationships in the harmonic series, the Bohlen-Pierce scale reflects relationships in a harmonic series that only contains odd partials. Thus odd-numbered intervals like 3/1, 5/3, 7/5, etc. may be said to have the same level of importance that intervals like the octave, perfect fifth, and perfect fourth, etc. have in 12-equal.

2.4 - The Equal-tempered Bohlen-Pierce Scale

The Bohlen-Pierce scale also has a 13-step equal-tempered version, which approximates its just ratios better than 12-equal approximates its set of just ratios (see Table 2.4). The fifth partial intervals in 12-equal are poorly approximated; the minor third and major sixth are 16 cents off and the major third and minor sixth are 14 cents off. In comparison, the majority of Bohlen-Pierce's intervals are no more than 7 cents off from just. This means that the approximated small-numbered intervals of the equal-tempered BP scale have fewer beats than do those of 12-equal, creating a harmonically viable and truly consonant scale.

The Bohlen-Pierce equal temperament would have been an even better approximation had Bohlen chosen different just ratios. For example, 25/23 (144 cents) is much closer to the equal tempered BP semitone of 146.3 cents. The second equal-tempered step is closer to the ratio 13/11 (289 cents), 33/13 (1613 cents) for step 11 and 69/25 (1758 cents) for step 12. Bohlen considered these, but chose to avoid the inclusion of 23-limit ratios in what would otherwise be a 7-limit scale. Also, including those replacement intervals would increase the number of just semitones to six.[32] In doing this, Bohlen consciously chose to keep the Bohlen-Pierce scale a 7-limit scale. This again places the scale at a parallel with the 12-equal scale, which implies, but poorly approximates, a 7-limit. The addition of 11-limit and 23-limit ratios was considered too much for ears that have been accustomed to, basically, a 7-limit scale.

Step no.	Equal temperament [cent]	Just tuning [cent]	Deviation (defect) [cent]
0	0	0	0
1	146.30	133.24	13.06
2	292.61	301.85	- 9.24
3	438.91	435.08	3.83
4	585.22	582.51	2.71
5	731.52	736.93	- 5.41
6	877.83	884.36	- 6.53
7	1024.13	1017.60	6.53
8	1170.44	1165.02	5.42
9	1316.74	1319.44	- 2.70
10	1463.05	1466.87	- 3.82
11	1609.35	1600.11	9.24
12	1756.66	1768.72	-13.06
13	1901.96	1901.96	0

Table 2.4: Comparison of BP-equal and Just BP Scale

2.5 - Closer Just Ratios to the Bohlen-Pierce-Equal Scale

However, if one ignores the requirement for ratios using only odd-numbered integers, one can find even closer just ratios to the equally tempered scale. German composer, Georg Hajdu includes even ratios (see Table 2.5) for the equally tempered Bohlen-Pierce scale in his research prior to composing his opera

32 Bohlen, “The Bohlen-Pierce Site.”

Der Sprung, which uses the scale in its first act. His ratios minimize the greatest error to only 6 cents, although one of them creates a larger deviation than its original counterpart: Hajdu's BP step 9 (32/15) has a deviation of 5 cents while Bohlen's BP step 9 (15/7) only deviates by 3 cents. Also, had Hajdu chosen 69/25 for BP step 12 rather than 11/4, the deviation would have only been 2 cents. However, Hajdu's ratios keep the axis of symmetry that exists between 5/3 and 9/5. Figure 10 shows that the distance between Bohlen's ratios is symmetrical over that axis. Hajdu's ratios (Table 2.5) are related over the axis of symmetry, sharing one common integer and relating to the other by multiples of 3: 5/3 and 9/5, 32/21 and 63/32, 32/27 and 81/32, 12/11 and 11/4, etc. The exception to this rule is his BP step 9, but had he kept the original 15/7 it would have paired well with 7/5. Figure 8 shows that Bohlen's ratios have this same property.

The contested intervals in each case thus far have been those outside the direct intervallic relationships formed by the 3:5:7:9 chord. These are what Hajdu refers to as the "strong" intervals of the scale.[33] The intervals made by this chord (for ex: 5/3, 9/7, 7/3, 9/5, etc.) are the small-numbered ratios of the scale (and therefore the most consonant) and are the best approximated ratios, using Bohlen's original just ratios, by the equal-tempered scale. The other intervals, as we can see, have closer just ratios, that aren't necessarily made up of odd numbers.

33 Georg Hajdu (2005), 15.

Step	Cents	Just Interval	Cents	Deviation
0	0	**1:1**	0	0
1	146	12:11	151	-5
2	293	32:27	294	-1
3	439	**9:7**	435	+4
4	585	**7:5**	583	+2
5	732	32:21	729	+3
6	878	**5:3**	884	-6
7	1024	**9:5**	1018	+6
8	1170	63:32	1173	-3
9	1317	32:15	1312	+5
10	1463	**7:3**	1467	-4
11	1609	81:32	1608	+1
12	1756	11:4	1751	+5
13	1902	**3:1**	1902	0

Bolded are small integer ratios based on odd numbers

Table 2.5: Hajdu's BP Ratios[34]

2.6 - Continued Fractions

To find even closer just ratios to the "weaker" Bohlen-Pierce equal-tempered intervals, one can use continued fractions. Continued fractions are used to approximate irrational numbers, and are therefore extremely useful in the creation of alternative scales for two reasons: they can find the closest approximating ratio of an irrational number, or they can be used to find where an interval will be approximated in an equal temperament. They are expressed as:

$$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \dots}}}$$

34 Hajdu, Georg. "Compositions: Der Sprung, Act I, Scene I."

where a_1 and a2, etc. are integers larger than 1. This can be written in shorthand[35]:

$$a_0 + \frac{1}{a_1+}\frac{1}{a_2+}\frac{1}{a_3+}\dots$$

To find approximated ratios for the equal tempered BP step, one can use continued fractions for the value $3^{(1/13)}$ which is ≈ 1.088182... The value for a_0 is taken to be the largest integer less than or equal to $3^{(1/13)}$, or 1. To find a_1 take the remainder (1.088182... - 1), which is a value less than 1, and invert it to obtain a value larger than one: a_1 = 1/(1.088182... - 1) = 11.340183... This continues, taking the largest integer less than or equal to a_1, which is 11: $a_2 = 1/(a_1-11)$ = 2.9395943... etc. In this case, because $3^{(1/13)}$ is an irrational number, the continued fraction expansion will go on indefinitely:

$$1+\frac{1}{11+}\frac{1}{2+}\frac{1}{1+}\frac{1}{15+}\frac{1}{1+}\frac{1}{1+}\frac{1}{1+}\frac{1}{2+}\frac{1}{2+}\frac{1}{1+}\frac{1}{27+}\dots$$

At every denominator, the expansion can be truncated to find the approximated ratios. Stopping at the first denominator gives the approximation 12/11 (1+1/11 = 12/11), which just so happens to be the interval Hajdu used for the BP-equal step. The resulting approximations are known as *convergents* and can be expressed thus for $3^{(1/13)}$: 12/11, 25/23, 37/34, 580/533... The most efficient approximations come just before large numbers in the expansion. In this case, the convergent with the denominator, 15, will be a closer approximation: 37/34. This also occurs right before a large gap in the convergents. To check our work: 37/34 = 146.39 cents,

35 Benson, 204.

which deviates from the equal tempered Bohlen-Pierce step by only 0.39 cents – an excellent approximation indeed! This is far better than both Hajdu's 12/11 and the 25/23, which was an interval considered by Heinz Bohlen.

Step	Cents	Just Interval	Cents	Deviation
0	0	1:1	0	0
1	146	37/34	146.39	-0.39
2	293	45/38	292.71	+0.29
3	439	9:7	435	+4
4	585	7:5	583	+2
5	732	29/19	732.06	-0.06
6	878	5:3	884	-6
7	1024	9:5	1018	+6
8	1170	59/30	1170.90	-0.9
9	1317	77/36	1316.23	+0.77
10	1463	7:3	1467	-4
11	1609	38/15	1609.24	-0.24
12	1756	91/33	1756.08	-0.08
13	1902	3:1	1902	0

Table 2.6: Closer Just Ratios to the Bohlen-Pierce Equal-Tempered Scale

By doing this for all of the weaker Bohlen-Pierce equal-tempered intervals, it was seen that they approximated certain just ratios almost exactly (see Table 2.6). The just ratios in this case are not made up of only odd integers, nor were they nearly as small, but they better represent the perceived intervallic relationships. All of the ratios are different than those studied by Hajdu and Bohlen; they never deviate from the equal-tempered Bohlen-Pierce scale by more than 1 cent. Now, in comparison, the "strong" intervals seem to be the weakest!

Ratio	**Convergents**							
27/25	1/14	3/43	4/57	7/100	11/157	40/571		
25/21	1/6	3/19	10/63	93/586				
9/7	1/4	2/9	**3/13**	8/35	35/153	778/3401		
7/5	1/3	3/10	**4/13**	15/49	34/111	49/160	83/271	
75/49	1/2	1/3	2/5	**5/13**	7/18	12/31	31/80	105/271
5/3	1/2	**6/13**	7/15	13/28	20/43	73/157	531/1142	
9/5	1/2	**7/13**	8/15	15/28	23/43	84/157	611/1142	
49/25	1/2	2/3	3/5	**8/13**	11/18	19/31	40/80	166/271
15/7	2/3	7/10	**9/13**	34/39	77/111	111/160	188/271	
7/3	3/4	7/9	**10/13**	27/35	118/153	2623/3401		
63/25	5/6	16/19	53/63	493/586				
25/9	13/14	40/43	53/57	93/100	146/157	531/571		

Table 2.7: BP Ratios and their Convergents

This same method, however, can prove that the strong ratios have been placed in the best possible equal-tempered scale over the tritave. An interval can be expressed logarithmically over a base harmonic. For example, a perfect fifth can be expressed: $\log_2(3/2)$. To find the convergents of a Bohlen-Pierce interval, for example: 9/7, we need to expand the value $\log_3(9/7)$. Table 2.7 shows the results. These convergents show where the interval would be approximated in an equal tempered scale using the third harmonic as its framework. The denominator tells us how many pitches per tritave there are in such a scale. The numerator tells us what step approximates the interval. So, for 9/7, we see that there is a convergent at 3/13, meaning that in an equally tempered scale with 13 pitches to the tritave, the 9/7 is approximated on the third step. Looking at this chart, we can see that choosing a scale with 13 equal steps to the tritave is the best choice, given the intervals provided. Bolded, are all the convergents with the denominator, 13. All but four of the intervals have convergents at 13. Going further into the expansion does not provide a better solution for the majority of the intervals, even when going into much higher divisions of the tritave.

2.7 - The Diatonic Bohlen-Pierce Scale

Bohlen discovered that a diatonic scale could also be created using his previously discussed method (see Table 2.8). By taking the medium-sized gap as the BP whole-tone, the large gaps could instead filled with one semitone and one whole tone, creating a scale made up of whole tones and half tones, analogous to the traditional 12-equal diatonic scales.

Tone no.	Relation to base tone	Name (*)	Distance to previous tone
1	1/1	C	-
2	25/21	D	25/21
3	9/7	E	27/25
4	7/5	F	49/45
5	5/3	G	25/21
6	9/5	H	27/25
7	15/7	J	25/21
8	7/3	A	49/45
9	25/9	B	25/21
10	3/1	C'	27/25

(*) Named in accordance with a suggestion by Manuel op de Coul, aiming at making memorizing easier.
The names previously used by Heinz Bohlen were: i, l, m, n, o, r, s, t, u, i'.

Table 2.8: Bohlen-Pierce Diatonic Just Ratios[36]

36 Bohlen, "The Bohlen-Pierce Site."

Table 2.8 shows the diatonic mode known as Lambda, which ends with a leading tone, creating motion and resolution towards the tonic pitch, making it a melodically viable scale. So far, it is the preferred diatonic reference for the Bohlen-Pierce system, much like Ionian is the diatonic reference for 12-equal. That is, playing only white keys starting on the reference pitch would sound the Lambda mode. Microtonal pop-music composer Elaine Walker prefers Lambda because it included all of the intervals in the chord 3:5:7:9 and contained a great number of pleasing chords. Walker did a listening test for her Master's thesis paper *The Bohlen-Pierce Scale: Continuing Research* in which Lambda mode scored as one of the top two most consonant modes. In an email correspondence with Walker, Heinz Bohlen agreed upon a keyboard "layout for the Lambda mode, which [he] strongly support(s) as a suitable reference."[37]

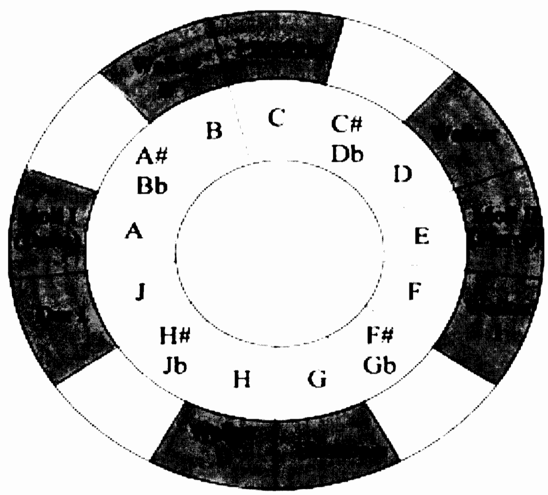


Figure 2.7: Lambda Family Wheel[38]

37 Walker (2010).
38 Bohlen, "The Bohlen-Pierce Site."

2.8 - Bohlen-Pierce Modes

Much like our traditional church modes, other modes exist and are related to each other in a very similar way. Creating something similar to our diatonic circle of fifths, Bohlen created a Lambda family wheel (see Figure 2.7) with related modes. The wheel shows the modes in position for a key signature with no accidentals. Lambda mode begins on C, Moll II begins on E, and so on. By rotating the outer wheel three steps in any direction, one can alter the key signature by one accidental (moving clockwise adds sharps, moving counterclockwise adds flats). So, for ex: an E Lambda scale has one sharp. All of the key signatures for Lambda are shown in Figure 2.8.

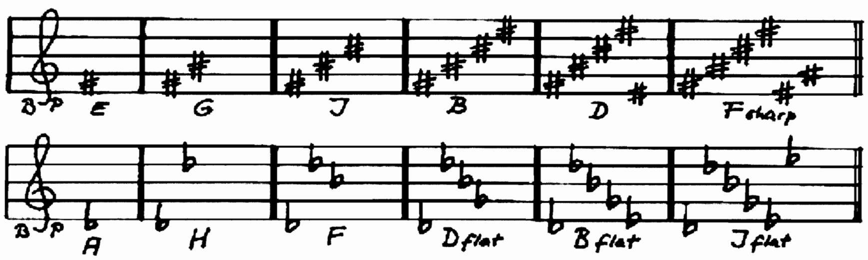


Figure 2.8: Lambda Key Signatures[39]

The modes on this wheel represent a collaboration among Bohlen-Pierce researchers. John Robinson Pierce, co-inventor of the Bohlen-Pierce scale, named his discovery P3579, not knowing at the time that the scale was one of the diatonic versions of a scale already discovered by Heinz Bohlen. This mode bears his name in honor of that shared discovery. All of the modes, but the Walker

39 Bohlen, "The Bohlen-Pierce Site."

modes, were discovered and named by Bohlen. Names like *Moll I*, *Moll II* and *Dur I* reflect a desire to parallel the 12-equal tonal language; in German, *Dur* and *Moll* translates to major and minor, respectively. More importantly, their construction almost directly mimics the 12-equal diatonic modes. The traditional diatonic scales are all made up of two tetrachords separated by a whole tone; in every mode both tetrachords are equal, spanning 5 semitones, or a perfect fourth. Bohlen's modes are similarly constructed; each mode is made up of two pentachords, spanning 6 semitones, or an interval of 5/3 (Major sixth) separated by a semitone. Elaine Walker proposed four new modes, named Walker I, II, A and B, all of which do not have equal pentachords. Bohlen added them to the chart on his website noting, "Walker's bold act of musical disobedience completed one family of possible basic BP modes"[40]

2.9 - Is there a Bohlen-Pierce Harmonic Theory?

The clear parallels between Bohlen-Pierce and 12-equal suggest that the Bohlen-Pierce scale can be used with harmonic rules analogous to the 12-equal scale. The BP diatonic modes are related to one another by a unique interval, the diatonic BP third, which may prove to be as useful for modulation as the dominant in 12-equal. Harmonies cannot be written only with scales, however, and Bohlen-Pierce offers a wide array of consonant intervals to choose from. The 3:5:7 and 5:7:9 chords, which formed the basis of the construction of the scale,

40 Bohlen, "The Bohlen-Pierce Site."

have also been chosen as the scale's most harmonically useful. The 3:5:7 can function in very much the same way as our traditional major chord, and 5:7:9 could relate to our minor chord. Figure 2.9 shows all of the possible diatonic chords of the Lambda scale, in this case the BP "major chord" is referred to as the *Wide Triad* and the "minor chord" is the *Narrow Triad* (these were the names attributed by Heinz Bohlen, in an attempt to avoid confusion with the 12-equal chords of the same name). Within a diatonic 12-equal scale, each diatonic pitch has its own triad, be it major, minor, or diminished. In the Bohlen-Pierce scale, there are many more chords to choose from within the diatonic limits; some pitches can have a "major" and a "minor" chord built upon them, without the use of accidentals.

Wide triads		**Narrow triads**	
I	C - G - A	i	C - F - H
-	-	ii	D - G - J
III	E - J - C	iii	E - H - A
-	-	-	-
V	G - B - E	v	G - A - C
VI	H - C - F	-	-
VII	J - D - G	vii	J - C - E
VIII	A - E - H	-	-
-	-	ix	B - E - G

Figure 2.9: Diatonic Chords in Lambda Mode[41]

The Bohlen-Pierce scale is fully equipped with all of the requirements of a tonal scale, but whether or not it develops its own tonality is in the hands of

41 Bohlen, "The Bohlen-Pierce Site."

today's composers and in the open-mindedness of the audience. Given the material, it is very possible that Bohlen-Pierce chords can be written so that they create the sense of a home key. Cadences and resolutions must be composed to allow the listener to overlook the lack of the octave, and actually hear the tritave as a consonant and final sounding interval. Whether this is possible is still being researched. Much music has been written, some of which will be analyzed later on in this paper.

2.10 - Bohlen-Pierce Notation

Bohlen-Pierce music is difficult to notate, both because of its extra diatonic and chromatic pitches and its lack of octaves. There are a few options, and as of yet there is no standardized notation. One option, proposed by Manuel Op de Coul, is a diatonic notation (see Figure 2.10) that gives each diatonic step its own line or space, using accidentals for the semitones when needed. This is no different from our standard 12-equal notation, except for the lack of octaves.

Figure 2.10: BP Diatonic Notation[42]

42 Bohlen, "The Bohlen-Pierce Site."

Op de Coul renamed all of the pitches of the scale; Bohlen's original note names (*i*, *l*, *m*, *n*, *o*, *r*, *s*, *t*, *u*, *i'*) were intended to reduce confusion with the traditional 12-equal note names, but they also made notation daunting. Bohlen himself had to write the note names below his pieces, as we'll see later. Op de Coul's new nomenclature made it much easier to set up a notation parallel to the one in current popular usage. The biggest hurdle one faces immediately with learning this notation is to adjust to the tritave relationship between C and C' and to be able to recognize the wider distance between those pitches on the staff.

This notation is useful for its representation of diatonic relationships, however, this also requires that the player learn those diatonic relationships. This involves knowing the distance between each diatonic pitch; some pitches are separated by a whole step, some by a half step. This is easily seen when performing on a keyboard instrument, but otherwise this information must be familiarized. All of this information has to be processed along with the memorization of new key signatures for new modes and, on top of all of this, the diatonic Bohlen-Pierce scale has two more pitches than does 12-equal. All of these difficulties are unfortunate because the diatonic scales are of high musical value; one would assume their notational representation to be preferable. Obviously, however, this could take some time to catch on.

An alternative notation (see Figure 2.11) was suggested by members of a Toronto-area based contemporary music group, *Transpectra*, which specializes in the performance of Bohlen-Pierce music. It is a chromatic notation, where each line and space of the staff is devoted to a different semitone. This notation is not unlike the chromatic notation proposed for 12-equal by Johann Ailler in 1904, (see Figure 2.12) and a very similar version by Albert Brennink in 1976, which has four line staves, the bottom line was an E, much like our traditional treble clef notation. Chromatic notations like these were created to better represent intervallic relationships without the necessity of accidentals, key signatures, or even clef signs.

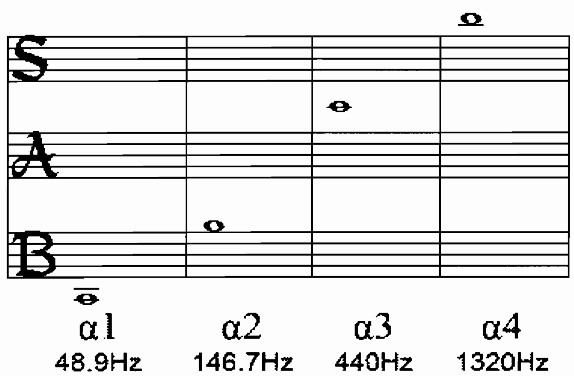


Figure 2.11: Transpectra's BP Notation[43]

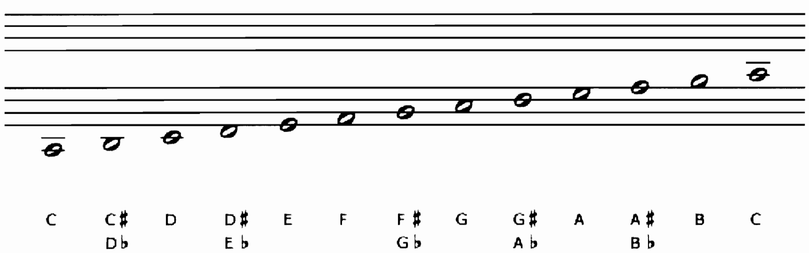


Figure 2.12: Ailler's Chromatic 12-Equal Notation[44]

43 Kooyman.
44 Reed, 7.

This notation also disregards the nomenclature by Op de Coul, employing new names that cannot be confused with those used in 12-equal: *alpha*, *beta*, *gamma*, *delta*, *epsilon*, *zeta*, *eta*, *theta*, *iota*, *kappa*, *lambda*, *mu*, *nu*. However, it is this author's opinion that these note names create confusion with mode names like gamma, lambda, delta, etc. which would bear no relationship to their respective note names. Perhaps the members of *Transpectra* have little interest in modal Bohlen-Pierce music. The diatonic relationship is lost in this notational system, but it might be the easiest to learn for those not playing on keyboard instruments.

Probably the most useful notation is the fingering-based notation, used for composing clarinet pieces. While this notation makes it difficult to judge both diatonic and chromatic relationships, it allows the performer to read as they would a traditional piece of music and match the fingering combination of the notated note. The sounded pitches are not what one would initially expect, but after practice the clarinetist can adjust their ears to the new scale. This type of notation allows for a quick transition from traditional clarinet to Bohlen-Pierce clarinet, and can be applicable to other instruments, once they become available.

Chapter 3 - Instruments and Timbres

3.1 - Heinz Bohlen's Organ

The Bohlen-Pierce scale has attracted much attention since its discovery, and a wide variety of instruments have been invented to play in the scale. The first such instrument was an electric organ constructed by Heinz Bohlen himself, which he used to experiment with the sound of the scale, and write a few short pieces. Bohlen built the organ from parts that he ordered from a do-it-yourself organ kit made by Dr. Böhm & Co. in Minden, Germany. Bohlen kept his invoice, which contains an order made in 1972 for 32 white keys and 14 black keys, a strange order which must have confounded the very traditional company.[45] Bohlen's organ was constructed at a time when synthesizers were extremely expensive, however the synthesizer evolved into a compact and affordable electronic instrument by the 1980's. Bohlen expressed some disappointment at this unfortunate timing; if the synthesizer had been developed a few years earlier he would have had a much easier time making Bohlen-Pierce music![46]

Bohlen described the first sounds he made on the organ as "frightening."[47] However, he had the organist, Andreas Rondthaler, come over to play the instrument. Bohlen described Rondthaler as the "first ever person to play *real* music in the novel scale"[48] and, after some time playing the instrument, he had

45 Bohlen, "An early document on a novel scale."
46 Bohlen, "An early document on a novel scale."
47 Bohlen, "An early document on a novel scale."
48 Bohlen, "The Bohlen-Pierce Site."

very little trouble finding consonant sounding harmonies.

3.2 - Timbre and the Bohlen-Pierce Scale

The Bohlen-Pierce scale is said to sound better when played with timbres containing strong odd harmonics. This is especially due to the discordance created by timbres that have an octave at the second partial. Bohlen himself chose square waves for his electronic organ; square waves consist only of odd harmonics. John Pierce also believed the odd harmonic timbre to be the ideal for Bohlen-Pierce. It wasn't until 2003 when German composer, Georg Hajdu began the Bohlen-Pierce Clarinet project with Canadian clarinet maker, Stephen Fox that this ideal was made possible on live instruments.

Some composers choose to explore Bohlen-Pierce with timbres containing both odd and even harmonics, relishing in the beating that occurs. Charles Carpenter and Elaine Walker are two popular music composers who have explored Bohlen-Pierce in this fashion. Carpenter's *Frog a la Peche* (see Appendix, #10) and *Splat* (see Appendix, #11) make use of the Bohlen-Pierce scale in a progressive rock band setting, using synthesizers and a TrapKat drum controller. Elaine Walker, who has four Bohlen-Pierce modes named after her, pushes microtonal boundaries, writing pop music in a variety of scales, but avoiding 12-equal at all costs. Her goal is to get microtonal music out to the masses and she is pleased when nobody seems to realize that her music is in an alternative tuning; it's just catchy music that has a different flavor.[49] She formed her band, ZIA, in 1991 and has since written songs in 10-equal, 19-equal and the Bohlen Pierce scale. Her music uses timbres consisting of both odd and even harmonics; she likes the sounds created by the full spectrum of harmonics, even if they create beats within the tuning.

3.3 - Instruments with Odd and Even Timbres

For those who are not interested in limiting the timbre to odd harmonics, a variety of instruments have been invented. Ron Sword is a major name in microtonal guitars; he currently sells guitars fretted to a wide array of tuning systems including meantone, microtonal and non-octavian scales. Some examples include 31-tone, 16-tone and 24-tone guitars as well as 13-tone Bohlen-Pierce and 39-tone Bohlen-Pierce guitars.[50] Bohlen also made a BP guitar in 1997 by refretting an acoustical guitar; he actually thought that the instrument worked well in the tuning, regardless of the even harmonics.[51] German mathematician Robert Schwartz experimented with a violin tuned to Bohlen-Pierce. He tuned the strings apart by 7/5, the BP third – yet another parallel between the 7/5 and the traditional dominant – and used training tape to make frets on the instrument[52]. The human voice also sounds fine in Bohlen-Pierce; Elaine Walker sings in her Bohlen-Pierce songs and it sounds completely natural, especially because she is

49 Walker (2010).
50 Sword.
51 Bohlen, "The Bohlen-Pierce Site."
52 Schwarz.

backed up by her extremely tonal harmonies.

3.4 - Triadalizer

Walker also invented a few electronic instruments, for which the timbre can be changed to suit the player's tastes. One such instrument is a computer tool called the Triadalizer (see Figure 3.1), which was programmed in Max/MSP. It plays individual pitches, intervals or triads in the Bohlen-Pierce scale with the click of a mouse. Its simple design is a graphic arrangement of pitches, allowing the user to pick notes off the graph and quickly arrange harmonies. The Triadalizer shows which harmonies are consonant or dissonant with its "heat matrix," where darker squares are more consonant than lighter squares.

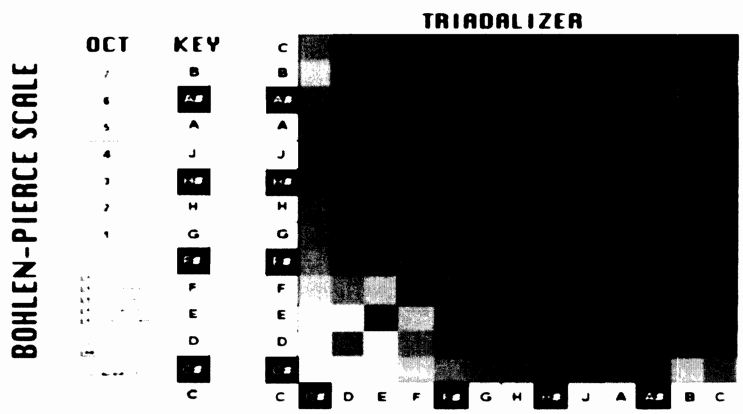


Figure 3.1: The Triadalizer[53]

3.5 - Bohlen-Pierce Keyboards

Walker built two Bohlen-Pierce electronic pianos, whose keyboard layouts were described her paper, *The Bohlen-Pierce Scale: Continuing Research*. She

53 Walker (2010).

chose Lambda mode as the diatonic reference mode (meaning that the white keys on the keyboard will play the Lambda mode) for its melodic properties and leading tone, and she built her Korg Poly-800 in this mode (See Figure 3.2). Bohlen found Walker's paper on the internet and emailed her to let her know that he agreed with her choice of Lambda as the diatonic reference.[54] His organ was tuned to Gamma, which he later realized was an error, as the Gamma family of modes only included two of the modes he had explored theoretically: Gamma and Dur II. The other seven modes in the Gamma family still remain unnamed (see Figure 3.3).

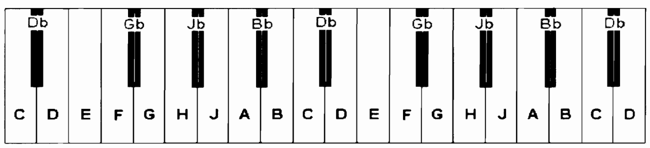


Figure 3.2: Elaine Walker's Lambda Keyboard Layout[55]

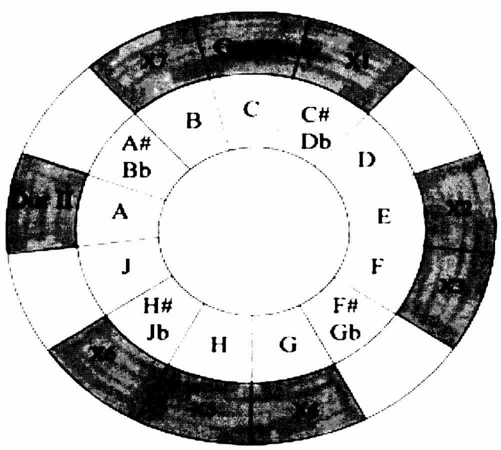


Figure 3.3: Gamma Family Wheel[56]

54 Walker (2010).
55 Walker (2001), 8.
56 Bohlen, "The Bohlen-Pierce Site."

However, even though Walker and Bohlen agreed that the Lambda mode was preferable for its melodic properties, Walker did not find the keyboard arrangement convenient for performance. She built a second keyboard, called a BP-tar, in Dur II (see Figure 3.4). The only difference between Dur II and Lambda mode is that the last interval of Lambda is a leading tone, while it is a whole step in Dur II. This may not be desirable from a melodic standpoint, but Walker prefers the way the black keys are arranged on the keyboard in Dur II, and plays an accidental to achieve the leading tone. Ironically, Walker's Dur II keyboard layout looks exactly the same as Bohlen's Gamma layout (see Figure 3.5, but take note that the yellow stripes are just covers for unused slots). This is because the two modes are in the Gamma family.

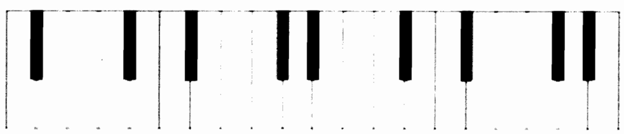


Figure 3.4: Elaine Walker's Dur II Keyboard Layout[57]

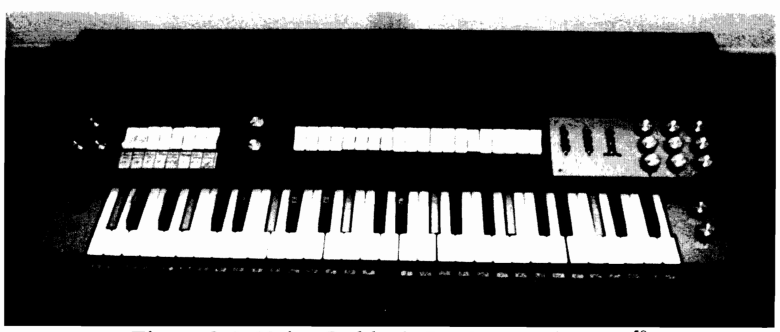


Figure 3.5: Heinz Bohlen's BP Organ, close-up[58]

57 Walker, "The Bohlen-Pierce Scale."
58 Bohlen, "The Bohlen-Pierce Site."

3.6 - Tritave Equivalence: John Pierce and Max Matthews

The argument for odd harmonic timbres is not only about discordance with the 2nd partial and the pseudo-octave; there is also the issue of hearing the tritave as the interval of equivalence. Traditionally we are accustomed to hearing octaves as equal, but some argue that this is a learned experience. If so, our ears could be trained over time to hear the Bohlen-Pierce tritave as an interval of equivalence. In the intonation sensitivity study by John Pierce and Max, a listening test of 3:5:7 and 5:7:9 chords were performed on a group of non-musicians and musicians.[59] In a section of the test, listeners were subjected to a series of chords and their inversions over the octave (for example the triad C-E-G and its inversion E-G-C') and were asked if they heard similarities. The results were compared to a similar listening test in Bohlen-Pierce, with chords inverted over the tritave. The timbre chosen for the listening test was only made up of odd harmonics. Matthews and Pierce explain that this is helpful in hearing the tritave as equivalent, extremely important in relating Bohlen-Pierce chord inversions. They add:

> "… for tones with both even and odd harmonics, the octave is musically empty in the sense that if we add a tone an octave above a sounding tone, we add no new frequency components; the harmonics of the added tone are all present in the tone already sounding. This is not so if the tones have odd harmonics only, for by adding a tone an octave up we add even harmonics of the tone already sounding. But if we add a tone whose fundamental frequency is 3 times (a tritave above) the sounding tone, the tone added will add no new frequency components. Hence, for tones with odd harmonics only, the tritave plays the role that the octave does for tones with both even and odd harmonics."[60]

59 Mathews and Pierce (1988).
60 Mathews and Pierce (1989), 168.

different from our current choice of scale. Our traditional 12-equal scale places a hierarchy on the very intervals that are the strongest on the majority of traditional instruments: octave, perfect fifth, perfect fourth, etc.

Sethares also proved that the perceived consonance of an interval was dependent upon spectra. He used stretched spectra, that is electronically manipulating the spectra to widen the intervals between partials, and found that when playing a just octave with a timbre whose spectrum has been stretched, the interval sounded dissonant. However, if the top pitch of the interval matched the stretched position of the second harmonic, this pseudo-octave was far preferable. This is extremely useful with regards to the Bohlen-Pierce scale; it suggests that the perceived consonance of Bohlen-Pierce music is highly dependent on its timbre. It also suggests that the octave can sound dissonant, so long as one chooses to make it so, with regard to timbre. If the second harmonic of a timbre was a tritave, that interval would be chosen as more consonant as the octave. Sethares adds, "consonance and dissonance are not inherent qualities of intervals, but they are dependent on the spectrum, timbre, or tonal quality of the sound"[62]

Creating the most consonance within the Bohlen-Pierce scale required more than just a stretching of spectrum; it worked best with spectra with predominantly odd harmonics. For the most agreeable sound, given the intervals of the scale, an instrument such as the clarinet or pan flute should be used, or any

62 Sethares, VI.

timbre in which the odd harmonics predominate. The spectral analysis of such instruments shows that the prominent frequencies are those that are 3, 5, 7, and 9 times as much as the fundamental. In Figure 3.6, the spectral analysis of a pan flute playing at 440Hz shows that frequencies with the highest magnitude are at 440Hz (fundamental, or *f*), 1322 (3*f*), 2200 (5*f*), etc. The Bohlen-Pierce scale utilizes intervals whose ratios are described by these same relationships: 7/3, 9/5, 9/7, etc. and would, therefore, sound the most consonant with timbres of predominantly odd harmonics.

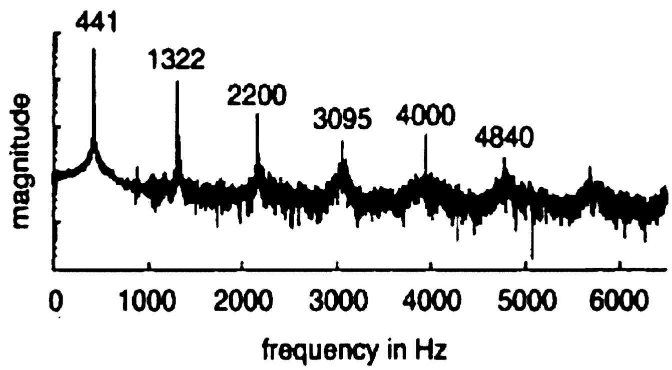


Figure 3.6: Spectral Analysis of Pan Flute[63]

Furthermore, a dissonance curve for varying spectra can be graphed whose minima show points of maximum sensory consonance.[64] This is a highly complex process, which will not be explained in full here; see Sethares' *Tuning, Timbre, Spectrum, Scale* for a far more detailed discussion. In short, dissonance curves can be used to find the intervals that will be perceived as the most consonant,

63 Sethares, 111.
64 Sethares, 97.

given a specific spectrum. As Sethares said, "A spectrum and a scale are said to be *related* if the dissonance curve for that spectrum has minima at scale positions."[65]

By taking a complex tone of a given spectrum at frequency, x, played against another whose frequency, for example, varies from $1x$ to $3x$ (the span of a tritave), one can calculate the total dissonance by adding the dissonance of the partials of each resulting pair of pitches. For example, Figure 3.7 shows the dissonance curve for spectra with predominant odd partials. The dips in the curve, or *minima*, provide the intervals that will be perceived as the most consonant with that timbre. Below the curve are the 13 equal steps of the Bohlen-Pierce scale; the minima occur at BP steps 3, 4, 6, 7, 10 and 13. Bohlen-Pierce is, again, an obvious choice scale for this timbre. At the top are the spatial representations of 12-equal divisions of 2, or our traditional 12-equal scale, and 12-equal divisions of 3, the result of spectral stretching so that the second harmonic lines up with the third. Notice that neither of these scales coincides with the minima, other than at the tritave. Also, notice that merely stretching the spectrum does not provide a consonant enough timbre. In the words of Sethares, "the Bohlen-Pierce scale is fundamentally different, and it requires a fundamentally new music theory."[66]

65 Sethares, 97.
66 Sethares, 112.

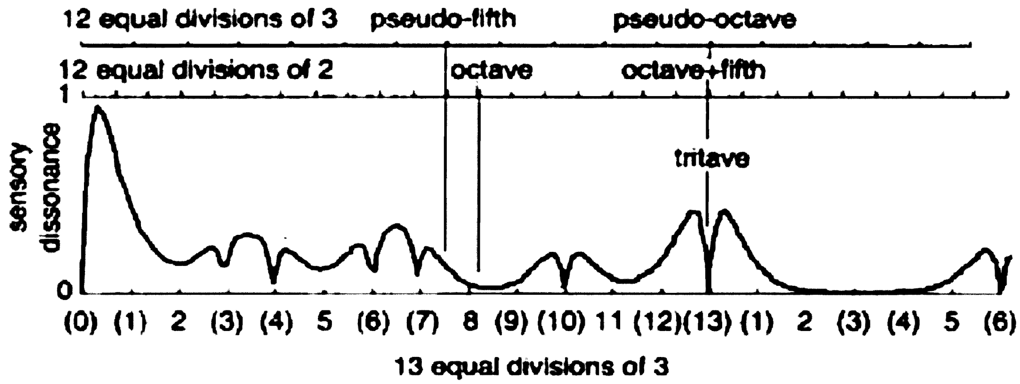


Figure 3.7: Sensory Dissonance Curve for Spectra with Odd Partials[67]

Though instruments with timbres consisting of both even and odd harmonics sound good to some Bohlen-Pierce trained ears, the study by Max Matthews and John Pierce as well as the spectral studies by William Sethares suggest that timbres of odd harmonics are preferable to reduce the sensory dissonance. These timbres might also make it easier to hear the tritave as the interval of equivalence. For this reason, the invention of the Bohlen-Pierce clarinet would be a major turning point in the development of the Bohlen-Pierce scale. The clarinet, which naturally has a timbre with strong odd harmonics and overblows at the tritave, may support tritave equivalence and reduce sensory dissonance while also providing a familiar live instrument for performance in the Bohlen-Pierce scale.

3.8 - Instruments for Other Alternative Scales

The Bohlen-Pierce scale is not the only scale that has spurred unique

67 Sethares, 112.

instruments. Many advocates of alternative tuning systems built instruments to experiment with their scales. This practice stems from the construction of meantone keyboards with extra keys from the usual twelve per octave. Keyboards can have improved intonation with an increase in the number of notes per octave. The first split key keyboard instrument is believed to be the organ of St. Martin's in Lucca, Italy, dated sometime before 1484. It had separate keys for D#/E-flat and G#/A-flat. This practice spread and became quite common in Germany during the late 17th/early 18th century.[68] These extra notes extend the meantone system beyond twelve notes. Huygens and Vicentino, however, explored the possibility of a 31-tone closed system that was a near match for quarter-comma meantone. Huygens built a 31-tone organ, which is currently held in a museum in Haarlem. Vicentino built the *Archicembalo* in 1555 which possessed 36 keys to the octave, and could accommodate 31-equal temperament.[69]

The nineteenth century saw many advocates of greater divisions of the octave. In 1872, Robert Bosanquet made a keyboard with 53 notes to the octave.[70] In 1875, Colin Brown built his *voice harmonium* which played just scales, containing over 40 notes per octave. Mexican composer, Julian Carrillo, divided the octave into as many as 96 equal parts, discovering in the late 1890's how to divide a string such that there are sixteen distinct tones within the span of a whole tone. He wrote a quarter-tone string quartet in 1895, and in 1940

68 Barbour, 107-8.
69 Mandelbaum, 105.
70 Benson, 228.

invented fifteen pianos that could play different multiple divisions from whole tones to sixteenth-tones. Around the same time Carrillo was first experimenting with smaller divisions of the whole tone, George Ives (Charles Ives' father) was stretching his children's ears with his own brand of music lessons. He believed in the beauty of pitches outside of our usual twelve. Charles, in his *Memos*, quotes him as saying:

> If the whole tones can be divided equally, why not half tones? That is, if one has twelve notes in an octave, why not more or less. If you can learn to like and use a consonance (so called), why not a dissonance (so called)? If the piano can be tuned out of tune to make it more practicable (that is, imperfect intervals), why can't the ear learn a hundred other intervals if it wants to try?[71]

George, after hearing a "new" chord played by the nearby church, invented a machine that played quarter-tones and used it to teach his children quarter-tone melodies. Ives recalls that the melodies were always easier to learn when they were supported by quarter-tone harmonies:

> It seems to me that a pure quarter-tone melody needs a pure quarter-tone harmony not only to back it up but to help generate it. This idea may be due to a kind of family prejudice, for my father had a weakness for quarter-tones--in fact he didn't stop even with them. ... He would pick out quarter-tone tunes and try to get the family to sing them, but I remember he gave that up except as a means of punishment – though we got to like some of the tunes which kept to the usual scale and had quarter-tone notes thrown in. But after working for some time he became sure that some quarter-tone chords must be learned before quarter-tone melodies would make much sense and become natural to the ear, and so for the voice.[72]

The 20th century saw a great deal of activity in the microtonal world, with many instruments invented. In 1929, composers Alois Hába and Ivan

71 Kirkpatrick, 140.
72 Ives, 110.

Wyschnegradsky collaborated on the construction of a quarter-tone piano with three manuals: the upper and lower manuals tuned normally and the middle one tuned a quarter-tone apart. Richard H. Stein experimented with a quarter-tone clarinet in 1911, which added tone holes and extra keys to the standard design. In 1937, Fritz Schüller invented a quarter-tone clarinet which attached two parallel bores, tuned a quarter-tone apart, to a single mouthpiece; the keywork covered tone holes on both bores simultaneously with a valve to switch from one to the other.[73] From 1928 until his death in 1974, Harry Partch invented many instruments that played in his unequal 43-tone scale; these instruments included a variety of string, keyboard and percussion instruments. Dutch musician, Adriaan Fokker invented a 31-tone organ in 1950, after studying Huygen's work with 31-equal temperament.

Many of these instruments were created for experimentation purposes only; some were simply unsuitable for standard performance. For example, the quarter-tone clarinets discussed above were far too complicated for practical use. There were far too many keys, and in the case of Schüller's clarinet there were two separate bores to manipulate! In direct contrast, Stephen Fox's Bohlen-Pierce clarinet's design is much simpler. In fact, the clarinet lends well to the Bohlen-Pierce tuning because it naturally overblows at the twelfth. Because of this and the larger step sizes, the Bohlen-Pierce clarinet has fewer keys than a traditional 12-equal clarinet.

73 E. Michael Richards (1992).

Currently, other conveniently constructed microtonal instruments are available for purchase. Ron Sword sells various microtonal guitars on his website (http://www.swordguitars.com). Elaine Walker's Bohlen-Pierce keyboards (although she can also construct keyboards for 10-equal and 19-equal temperament) can be purchased by contacting her directly through her website (http://www.ziaspace.com). Stephen Fox's Bohlen-Pierce clarinets, as well as many other custom designed instruments are available on his website (http://www.sfoxclarinets.com).

3.9 - The Bohlen-Pierce Clarinet

Stephen Fox's Bohlen-Pierce clarinet (see Figure 3.8) has only 10 keys compared with the Boehm clarinet's 17 keys and 6 rings and the Oehler clarinet's 22 keys, five rings and finger plate. There are no rings; there are six open finger holes and a thumb hole. The tritave is completed with keys to be activated by the little fingers (which mimic the standard Boehm design exactly) and a key to be activated by the first finger on the left hand (much like the standard clarinet's “A” key). A trill key was added for extra trilling capability over the break.

The traditional clarinet has played a major role in microtonal music; pieces for clarinet have been written in a variety of tunings including (but not limited to) 24-equal, 72-equal, 31-equal, just intonation, and Partch's 43-tone scale. Performance of alternative tunings on the traditional clarinet is possible because of the opening or closing of additional tone holes to raise or lower the pitch as necessary. There are many more possible fingering combinations on the clarinet than one would assume. According to Phillip Rehfeldt in his invaluable modern musician's resource, *New Directions for Clarinet*, "The computer tells that there are 373,248 possible finger combinations on the clarinet, not including half holes, most of which are capable of producing distinctive sounds."[74]

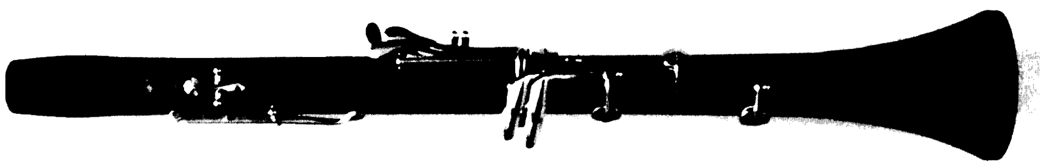


Figure 3.8: The Bohlen Pierce Clarinet[75]

Because the clarinet is so versatile, the question may come up, "Why does there have to be a different instrument to play in the Bohlen-Pierce scale at all?" For the microtonal clarinetist, there are many hurdles to face for a smooth performance on a traditional instrument. For example, there is the difficulty of memorizing various new and strange fingerings: in the performance of a 72-equal piece, for example, this task can be quite daunting. Some fingering combinations or half-hole fingerings create a muffled tone, causing a break in a melodic line. There are some fingerings that can't be played in succession with a legato sound; the fingers just can't jump around so much, especially if the passage is fast. An instrument dedicated to a specific tuning would benefit the performer, so long as

74 Rehfeldt, 21.
75 Fox.

the fingering system was similar to its traditional counterpart. The composer, too, is benefits from the creation of a new instrument, if only by knowing that all of the pitches can be produced with ease: in tune and with the same finger dexterity and virtuosity attributed to the standard clarinet when playing in twelve-equal. The Bohlen-Pierce clarinet is a major breakthrough in alternative instruments, because the fingering system is identical to the traditional clarinet, except for a lack of extra keys. Its simpler design is much preferable to the use of alternative fingering combinations on a standard clarinet.

3.10 - The Bohlen-Pierce Tenor Clarinet

Stephen Fox is currently expanding the Bohlen-Pierce clarinet family to include the tenor and contra clarinets. His most recent addition was the tenor clarinet, which was unveiled for the first time at the Bohlen-Pierce Symposium in March 2010. This three-day event in Boston was a collaboration among composers, scholars and performers with twenty lectures and three concerts. Fox's lecture focused on the construction of his Bohlen-Pierce instruments and introduced the tenor clarinet.

The Bohlen-Pierce tenor clarinet (see Figure 3.9) has a curved neck and bell much like the traditional alto and bass clarinets, but its size is about halfway between the two. It is pitched six BP “semitones” below the Bohlen-Pierce soprano clarinet. It features keywork similar to the conventional alto and bass clarinet; levers and raised keys which activate the tone holes allow for a comfortable spacing between the keys. The Bohlen-Pierce contra clarinet is currently unfinished, but will be about the same size as the conventional E-flat contrabass clarinet.

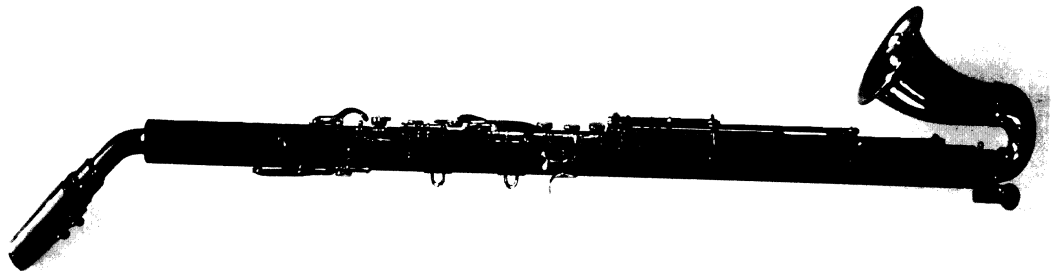


Figure 3.9: The Bohlen-Pierce Tenor Clarinet[76]

76 Fox (2010).

Chapter 4 - Bohlen-Pierce Music

4.1 - Composing in a New Language

The invention of the Bohlen-Pierce clarinet has created a flurry of new compositions for the instrument, many of which were unveiled at the first annual Bohlen-Pierce Symposium held in Boston in March of 2010. Before that, most compositions were for electronic instruments, like the ones described above. Compositions have varied in harmonic form; every composer has followed his/her own interpretation of the scale's material. Some have composed purely by ear, and others took on an incredibly academic approach, by focusing on harmonies that approximate small-numbered ratios or using specific modes and chords. The possibility of a harmonic theory for Bohlen-Pierce is there, but so many composers today are used to a very modern language; serious art music, for the most part, has avoided simple harmonies for some time now. A return to that simpler language in a new scale might be the best possible way to explore its harmonic capabilities. Heinz Bohlen's early pieces are an insight into what might be possible if we take the Bohlen-Pierce scale on a simpler harmonic route by using diatonic chords and learning what kinds of progressions can be created. With practice, it will become clearer what tensions and resolutions are possible. The traditional harmonic language in 12-equal temperament was developed in this way, with centuries of gradual acceptance of ideas of consonance.

4.2 - A Bohlen-Pierce Christmas Carol

Example 4.1 is an example of what might be the earliest Bohlen-Pierce composition, a 1973 transcription of a Christmas carol by Heinz Bohlen (see Appendix, #6). This piece employs a diatonic notation similar to that in Figure 2.10 but instead of the treble clef Bohlen uses a K clef, which marks the reference pitch of the keyboard. Bohlen's instrument was his electronic organ, whose reference scale was k-Gamma, hence the K clef. Bohlen originally used different note names (k, l, m, n, o, r, s, t, u, k' – although k was later changed to i). These note names have since been replaced with Op de Coul's nomenclature (see Figure 2.8).

At the top of the score, Bohlen marks the key of the piece (s-Δ or s-Delta) and the note names of the scale (according to Bohlen's nomenclature) are written below each pitch. Delta is a strange modal choice, as it is the only BP mode in the Lambda family (excluding two of Walker's additional modes) that omits the 7/5, one of the "strong" intervals of the scale.

This piece was written for Bohlen's organ which, as explained above, was centered around Gamma mode. As seen the Gamma Family Wheel (Figure 3.3), Delta mode is not directly related to Gamma as it is not in the Gamma family. This means that a Gamma keyboard would not play the Delta mode on any series of white keys. Therefore the accidentals used in the key signature are not "standard" for Delta mode, but were necessary to create the Delta mode on a

Gamma-centered keyboard layout.

Es kommt ein schiff geladen is a German folk carol dating back to the 14th century. The song has since been arranged in a variety of ways; the original melody is surely recognizable by most Germans. Bohlen keeps the contour of the melodic line, but changes the pitches to suit the most consonant aspects of the Bohlen-Pierce scale. Example 4.1 shows the piece with chord progression markings added, using the 3:5:7 as the “major chord” (created by an interval of 6 BP steps on the bottom and 4 BP steps on top; for example: C-G-A) and the 5:7:9 as the “minor chord” (4 BP steps on bottom and 3 BP steps on top; for example: C-F-H). Take note that in this figure, inversions utilize the same markings used in 12-equal theory, but do not hold any actual relation towards the intervals above the bass; they are merely used for familiarity's sake.

The piece demonstrates a clear tonic key, modulation and a return home with a cadence resolving on the tritave in the original key. The first five measures make up what may be called the first phrase of the piece. In those five bars, the “major chord” on the tonic is given plenty of weight harmonically. In the melodic line, the tonic (*s*), middle (*l*) and top note (*os*) of the triad appear a lot as well, giving the listener a clear sense of the home key. The first phrase ends on a III chord. This is significant if we return to look at the Lambda family wheel (Figure 2.7) and the Lambda key signatures (Figure 2.8). The modes are shown to be related by a third; movement up a third adds a sharp to the key signature of any

Lambda family mode. This is analogous to the circle of fifths in 12-equal; moving a fifth up adds a sharp to the key signature. Therefore, this cadence on the III chord may be a thought of as a half cadence.

Example 4.1: Bohlen, Es kommt ein Schiff geladen[77]

In the first three bars of the second phrase, the I or I6 chord alternates with the ix 6 chord. Then the harmony settle on the iii or III chords (look back at

77 Bohlen, BP Site.

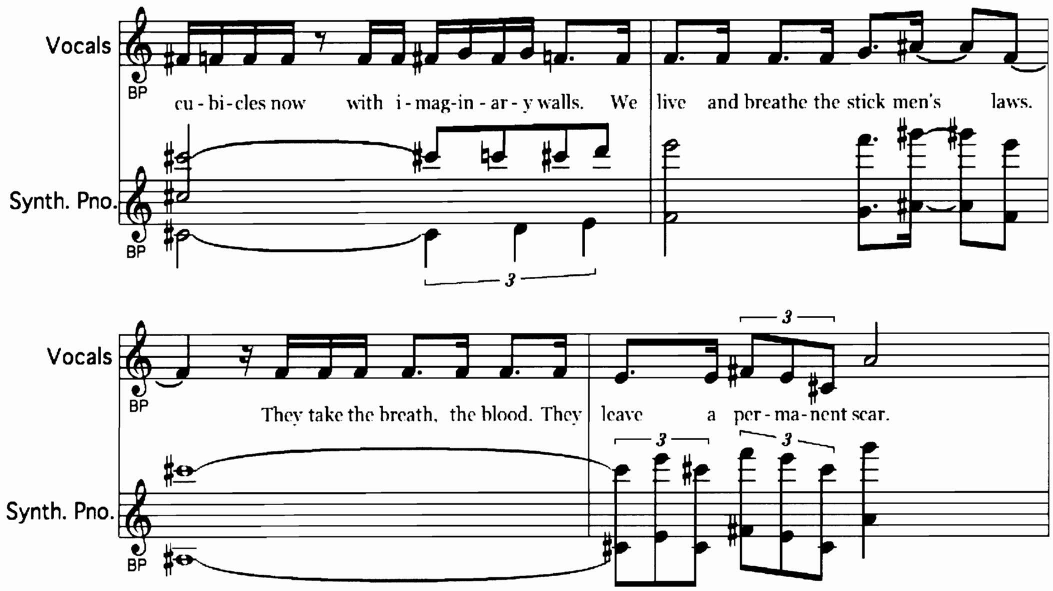
Figure 2.9 to see that there can be “major” and “minor chords” on the same diatonic pitches in Bohlen-Pierce modes). The final bars of this piece emphasize the “dominant”, as if we've modulated. In fact, if this were a modulation to the third, the ix6 or ix 6/4 chords would actually be vii6 or vii 6/4, and would mimic the opening. Ultimately, a cadence with a double leading tone brings us back to the tonic, ending the piece on a tritave.

Es kommt ein schiff geladen demonstrates how well the Bohlen-Pierce scale can be applied to harmonic theory. Bohlen takes the listener from the home key to a modulation to a “dominant” and then back home with a very strong final cadence. Probably this was all part of Bohlen's specific design. Bohlen's aims at finding a new scale were to find one that had the harmonic qualities analogous to the current musical system. In this piece, he is applying a similar set of harmonic rules to the new scale. In direct contrast, Elaine Walker's Bohlen-Pierce pop songs were all written exclusively by ear and do not follow any specific set of harmonic rules.

4.3 - Bohlen-Pierce Pop Songs

Stick men was Walker's first Bohlen-Pierce song, written in 1991 (See Apendix, #58). It was inspired by Berklee professor Dr. Richard Boulanger, who has also written a number of pieces in the tuning, and introduced Walker to alternative tuning systems. Walker describes her first experience listening to Bohlen-Pierce music: “My hair stood on end as the alien notes washed over me.

The notes were from outer space, yet it was still music. Wonderful music![78] *Stick men* is a direct response to this attachment of aliens and outer space to the scale; it describes a world which has been invaded, where "we live and breathe the stick men's laws."[79]

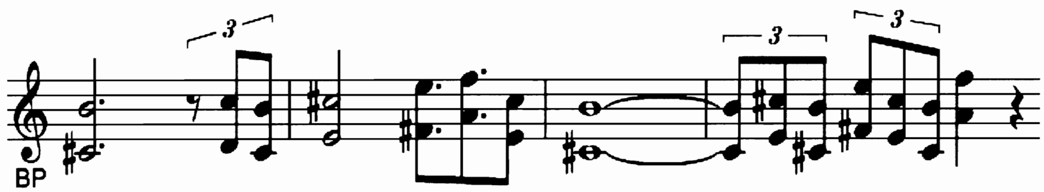


Example 4.2: *Stick men*: Dyadic Statement of Motive, in 9/5

Stick men utilizes a repeating motive, developed from fragments of vocal lines in the first minute of the song. Example 4.2 (notated in diatonic notation, as seen in Figure 2.10) shows the first statement of the motive, by the synthesized piano. The notes used do not appear to be in any Bohlen-Pierce diatonic mode; Walker wrote by ear and without regard to specific harmonic rules, so it is clear here that she is not yet hearing diatonically. The motive is stated dyadically in 7

78 Walker, "The Bohlen-Pierce Scale."
79 Walker, Lyrics to *Stick men.*

chromatic BP step intervals, or 9/5. The motive itself explores the melodic properties of the 2 chromatic BP step interval, 25/21, which measures 292.61 cents, making it very close to our traditional equal-tempered minor third (300 cents). These intervals are used in an arpeggiation at the end of the motive; C#-E, E-F#, F#-H are all 25/21.

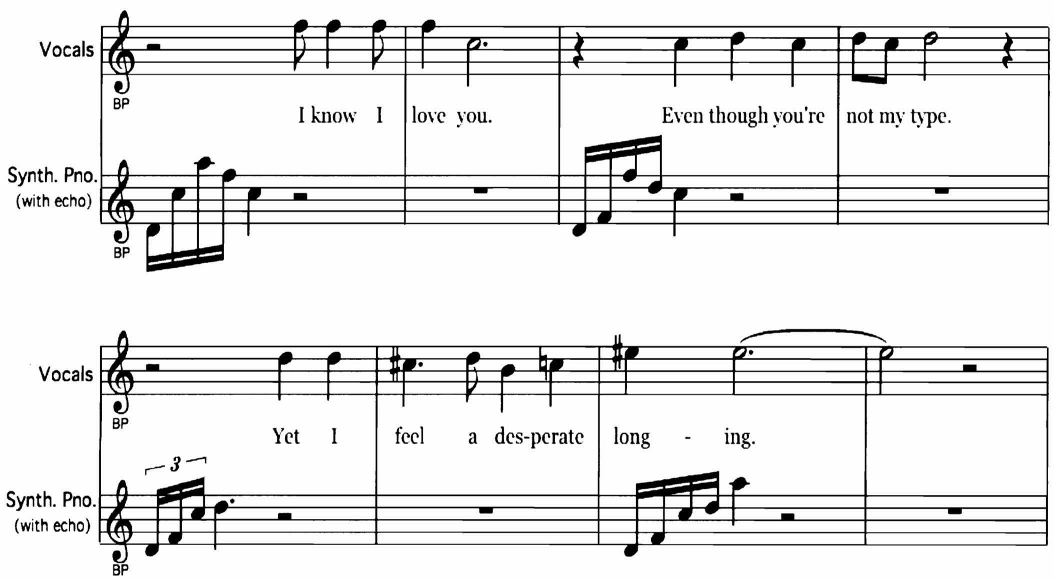


Example 4.3: *Stick men*: Dyadic Statement of Motive in False-octaves

The next motivic statement (see Example 4.3) includes a slightly altered opening; both versions are repeated in the song. Now, the motive occurs in dyads of eight chromatic BP steps (49/25), or false-octaves. At the close of this section is the same C#-E-F#-H arpeggiated figure as earlier. In this statement, the pitches C#, E, F#, and H are also incorporated into the opening of the motive. These false-minor thirds make up the most prominent interval in the song; the vocal line often spans no more than a false-minor third for long stretches (for ex: F-G in Example 4.2 is a false-minor third). This interval is flatter than the traditional minor third, which is already quite flat; the 12-equal minor third is 15.6 cents flatter than just. However, when these three "minor thirds" are stacked, we have an interval only 6.5 cents flat of a just major sixth, rather than its 12-equal counterpart, which is 15.6 cents flat.. The "false-minor thirds" are among the weaker intervals of the Bohlen-Pierce scale, those that were in question earlier in this paper. Walker uses these somewhat less desirable intervals, perhaps because they are somewhat "beaty," and she stacks them melodically to achieve the much purer major sixth.

The main intervals used in this song are the 25/21 (false minor thirds), 5/3 (major sixths), and the 49/25 (false octave). These intervals are very close to familiar intervals ins 12-equal, and may be deemed dissonant to the musically trained ear. According to the Bohlen-Pierce listening study by Matthews and Pierce, musically trained subjects judged chords that closely approximated 12-equal intervals as the most dissonant, while non-musically trained subjects did not. Their very familiarity caused them to be deemed dissonant, rather than seen as unfamiliar intervals that were consonant. Their extensive use in *Stick men* also suggests that while "composing by ear" Walker was still very much influenced by 12-equal theory.

Walker's *Love Song* (2007) is another Bohlen-Pierce song written for her microtonal pop group, ZIA (See Appendix, #63). It demonstrates how much her grasp of the scale has changed with the added time and research; by this time Walker had already written *Continuing Research on Perception and Performance of the Bohlen-Pierce Scale* (2001), in which she discovered four new Bohlen-Pierce modes. *Love Song* is far less chromatic than *Stick Men*; Example 4.4 shows that there are almost no accidentals used in its first vocal phrase. *Stick men* makes heavy use of the “semitone” in the vocals, while *Love Song's* vocal part is full of “whole tones” and larger skips. Walker's use of the Bohlen-Pierce scale has become more diatonic. Given enough exposure, when one becomes comfortable with this unique tonal language, it is possible that diatonic writing may become more popular with other composers.



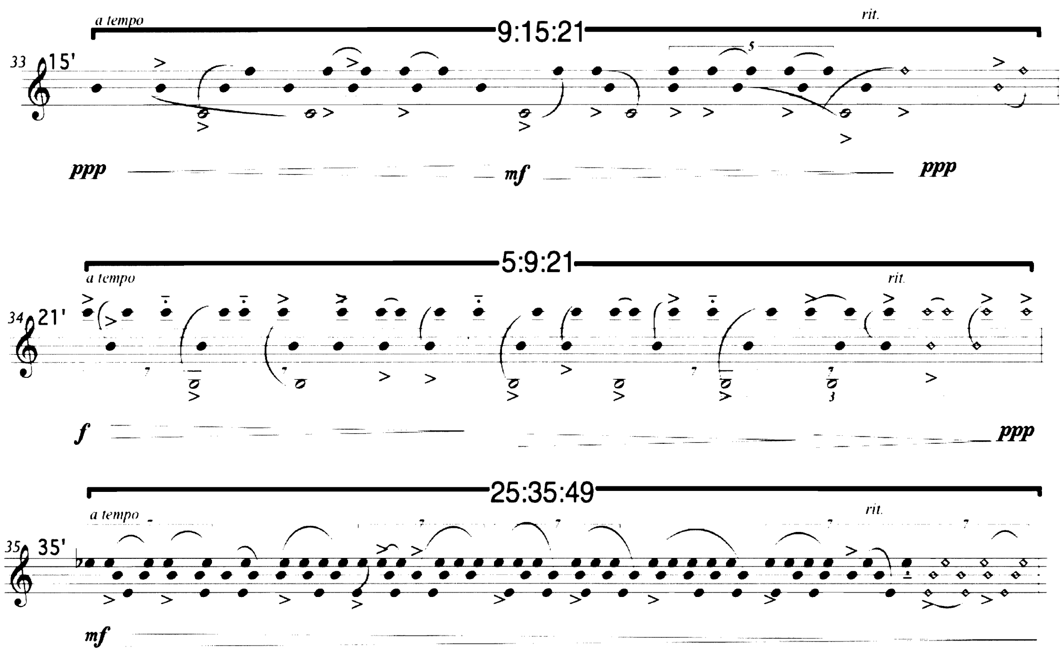
Example 4.4: *Love Song*: First Vocal Phrase

The use of accidentals in the opening phrase of *Love Song* is saved for the end of the phrase, pushing into the cadence, with the vocals on a c♯. This cadence leads us to the next phrase, which sounds very much like the first, but for a more final resolution. This c♯ leads to the d of the next phrase, much like any leading tone in 12-equal music. With this and the d's in the lowest note of each arpeggiated chord, one can assume that this piece is in d. No modes could be found that fit with the used pitches, starting on d or any other note in the piece.

However one can assume that the c♯ and the a♯ one measure earlier are chromatic pitches in a mode with no flats or sharps. Looking back at the Lambda family wheel (Figure 2.7), one can find the modes in position for keys with no accidentals. Walker A mode begins on d and therefore fits. Walker A is a problematic mode because it does not contain a leading tone. Walker gets around this by adding the aforementioned accidentals at the cadence; these benefit the melodic line and sense of home key.

4.4 - Katarina Miljkovic's *For Amy*

Katarina Miljkovic's *For Amy, for clarinet and electronics* (see Appendix, #31) makes use of odd numbers in every aspect of the piece, including pitch, time and rhythm. It was written for the 2010 Bohlen-Pierce Symposium and represents Miljkovic's first attempt composing in the scale. The electronics were composed with a computer program, *Mathematica*, which allowed Miljkovic to control the durations and pitch frequencies of events, basing them on odd ratios. Each event is notated by a single bar, whose duration is always an odd number of seconds. The clarinetist arpeggiates a different chord throughout each bar, and the duration of the bar is relative to the ratio of the approximated chord. Miljkovic begins with simpler relationships, chords approximating 3:5:7 and 5:7:9, Bars with 3:5:7 are given a duration of 3 seconds and bars with 5:7:9 are given 5 seconds. These chords are built on A440, the reference pitch upon which the Bohlen-Pierce clarinet is tuned. Later in the piece, chords are built around an axis of symmetry at A440. Example 4.5 shows symmetrical chords around A440; also note that the duration of the bars is related to the approximated ratio. (This piece utilizes clarinet fingering-based notation; A440 is located on the third line of the staff.) As the piece goes on, the chords are expanded outward from the axis, with ratios and durations increasing until the final measure, in which the chord approximates (in accordance with Bohlen's ratio chart) 105:125:189 and has a duration of 105 seconds.

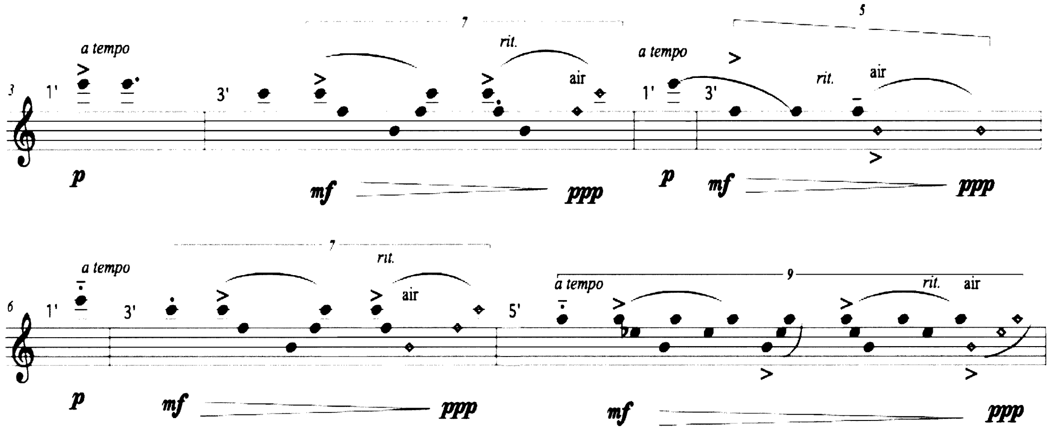


Example 4.5: Miljkovic, *For Amy*: Chords built symmetrically around A440

While experimenting mathematically with different chords based, in some form, upon A440, Miljkovic also happens to use diatonic pitches from the Lambda mode. Because A440 is stressed in this piece, it is assumed that this is the tonic pitch. For simplification of analysis, we'll equate this pitch with the *c* of the Lambda keyboard layout.

The piece begins on just one note, *c''* or one tritave above A440. Every time this event appears again, the duration of the bar is one second to match the 1:1 unison. The next event is a 3:5:7 approximation (see bar 4 of Example 4.6), given 3 seconds. This can be represented on the keyboard as C'-G'-A', or the "major" triad I. This chord is restated in number of ways throughout the next few measures: C'-G'-C' and C'-G'-A'-C" until the first statement of C-F-H, or I (see bar 8 of Example 4.6). Miljkovic uses I or i for the first 15 measures, allowing the listener to have a sense of the key. Almost all of the chords in *For Amy* utilize diatonic chords in "*c*" Lambda. There are various I chords and their inversions as well as III and several repetitions of a "seventh" chord on VII, spelled H-C-E-G (a major chord with an interval of 3 semitones on top might be called a "seventh" chord). Larger expansions towards the end of the piece become more difficult to analyze in this way, because the chords were derived from equidistant intervals from the axis of symmetry at A440. Chords like J-C-F, made up of two intervals of four semitones, do not relate to the prescribed "major" and "minor" chords studied here. The piece ends, surprisingly, on an *f* rather than return to the tonic *c*. This resolution to a far removed key – three keys away from *c* on the Lambda family – is prepared by chords with the accidental *J-flat*, which just so happens to be part of the three flat key signature for F-Lambda (see Figure 2.8). This piece, born from math, succeeds in representing the

Bohlen-Pierce scale in a diatonic fashion.



Example 4.6: *For Amy:* excerpt from opening

4.5 - Georg Hajdu's Opera, *Der Sprung*

Georg Hajdu's opera *Der Sprung* (1999) (See Appendix, #18) meticulously highlights intervals of the 3:5:7:9 chord in its opening scene. Hajdu uses the Bohlen-Pierce scale in an academic way to match academic context; this scene takes place at a university where two professors have just been shot by a student. Hajdu writes, "As this scale, not having any octaves, is an intellectual achievement in itself, it symbolizes the abstract, academic world of a university department."[80] The two professors sing a duet, accompanied by two microtonal keyboards called *secco*. During this scene, a solo wind instrument was also required, playing fast sweeping lines utilizing the "strong" intervals of the scale, those intervals based on 3:5:7:9. Example 4.7 shows that all pitches used are related to the tonic, *c*, by 9/7, 7/5, 5/3, etc. The premiere of the opera was in

80 Hajdu, Georg. "Compositions: Der Sprung, Act I, Scene I."

1999, and there were no instruments tuned to Bohlen-Pierce to choose from, so Hajdu wrote for a midi wind controller using a midi patch tuned to the Bohlen-Pierce scale. After the premiere Hajdu wanted to rewrite the part for a live instrument, and in 2004 contacted Stephen Fox, which began the Bohlen-Pierce Clarinet Project. [81]

Example 4.7: Hajdu, *Der Sprung*: excerpt

4.6 - Hajdu's *Beyond the Horizon*

Hajdu's next major Bohlen-Pierce composition, *Beyond the Horizon* (2008) for two Bohlen-Pierce clarinets accompanied by synthesizer and electronics (see Appendix, #19), is a far more chromatic work. The invention of the Bohlen-Pierce clarinet allowed Hajdu the freedom to make use of all of the scale's musical materials, in a virtuosic fashion.

Like his opera, this piece tells a story; Hajdu's musical realization is greatly affected by the context of the piece. The electronics part features narration, the text of which is about the Big Bang and the expansion of the universe.

81 Hajdu, Georg. "Starting Over – Chances Afforded by a New Scale."

> "We may be living in the only epoch in the history of the universe when scientists can achieve an accurate understanding of the true nature of the universe. A dramatic discovery almost a decade ago motivated our study. Two different groups of astronomers traced the expansion of the universe over the past five billion years and found that it appears to be speeding up. The source of this cosmic antigravity is thought to be some new form of "dark energy" associated with empty space. … We are led inexorably to a very strange conclusion. The window during which intelligent observers can deduce the true nature of our expanding universe might be very short indeed."[82]

If the universe is expanding at an ever increasing rate, the text suggests that we don't have much time to explore the universe in search of other, parallel worlds. Hajdu takes this concept of a parallel world and applies it musically by using a scale that may be parallel to our own. He explains:

> "What motivated this piece was the purely hypothetical and philosophical question of what the world would look like, if it consisted only of odd numbers as it is the case with the clarinet spectrum. But these are exactly the questions that inspire composers to create parallel worlds contrasting the omnipresence of 12-tone temperament. … The text … is an incentive to start thinking about the existence of a parallel (also tonal) world, that may eventually disappear from our view, if we don't catch the moment."[83]

Example 4.8: Hajdu, *Beyond the Horizon*: opening motive

Hajdu's use of the scale is highly chromatic. The opening has slow lyrical lines, expanding chromatically outward from a single pitch (see Example 4.8 – Note that the top line uses a clarinet fingering-based notation. The bottom line

82 Hajdu, Georg. Beyond the Horizon.
83 Hajdu, Georg, "Program Notes for *Beyond the Horizon*."

uses a chromatic notation, in accordance with the traditionally read pitches, where each traditional chromatic semitone represents a Bohlen-Pierce "semitone." For ex: a notated "middle-C" represents a middle C that is 22 cents sharp; the tritave above that is notated third space C#, or 13 traditional semitones higher). As the piece moves on, Hajdu speeds things up, reflecting the context of the ever-expanding universe. Fast chromatic scales are shared between the clarinets with the electronics playing clusters of chromatic pitches (see Example 4.9).

Example 4.9: *Beyond the Horizon*: excerpt

This chromatic and very "atonal" Bohlen-Pierce usage reflects the scientific and forward-thinking context of the piece. This is the present-day musical language at work in a very different tonal world. The listener is thrust into that new world, with sweeping lines displaying all that it has to offer. This piece is very effective at conveying mental images of the universe because of the use of the Bohlen-Pierce scale.

4.7 - Julia Werntz's *Imperfections*

Julia Werntz explores the melodic capabilities of the chromatic BP step in her piece for solo Bohlen-Pierce clarinet, *Imperfections* (see Appendix, # 65). Normally accustomed to composing in 72-equal temperament, using melodic lines that can span an interval as small as a whole tone, the large 146 cent "semitone" challenges Wertnz to compose melodic lines of a larger intervallic span. Her approach uses the scale in a very chromatic fashion; she mainly uses the "semitone" in each melodic line. Her chromatic approach is fueled by her interpretation of the scale as having only one step size, considering any interval larger than a "semitone" a "skip". She explains:

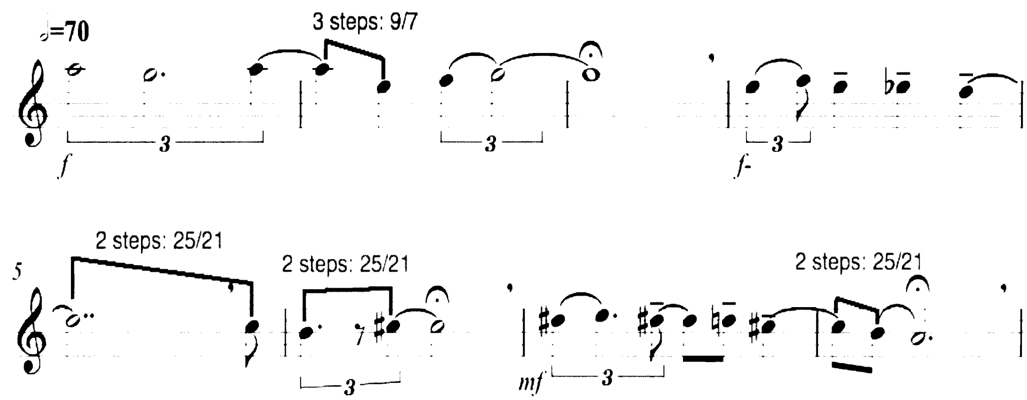
> "I took a straightforward linear approach, exploiting the special expressive melodic qualities of this scale which has only one "step" (the lovely 146 cents interval) and "skips" of rather ordinary sounding minor thirds (293 cents) and strikingly large and evocative major thirds (439 cents). I am used to composing microtonal music using 72 equal divisions of the octave (72 edo), which allows me seven kinds of minor third, seven kinds of major third, and so on. It was a welcome challenge to limit my melodic choices this way."[84]

This is a sensible interpretation from the stand-point of a composer who is used to an extraordinary number of "steps" to choose from in 72-equal temperament. The BP "whole tone" is 293 cents, far too large for Werntz to interpret as a "whole step." Rather, she hears this interval as a minor third.

Werntz's perception of the Bohlen-Pierce scale is heavily influenced by her work with fine-tuning various familiar intervals in 72-equal. Her choice of intervals reflects this; beyond the chromatic BP step she uses "skips" such as the

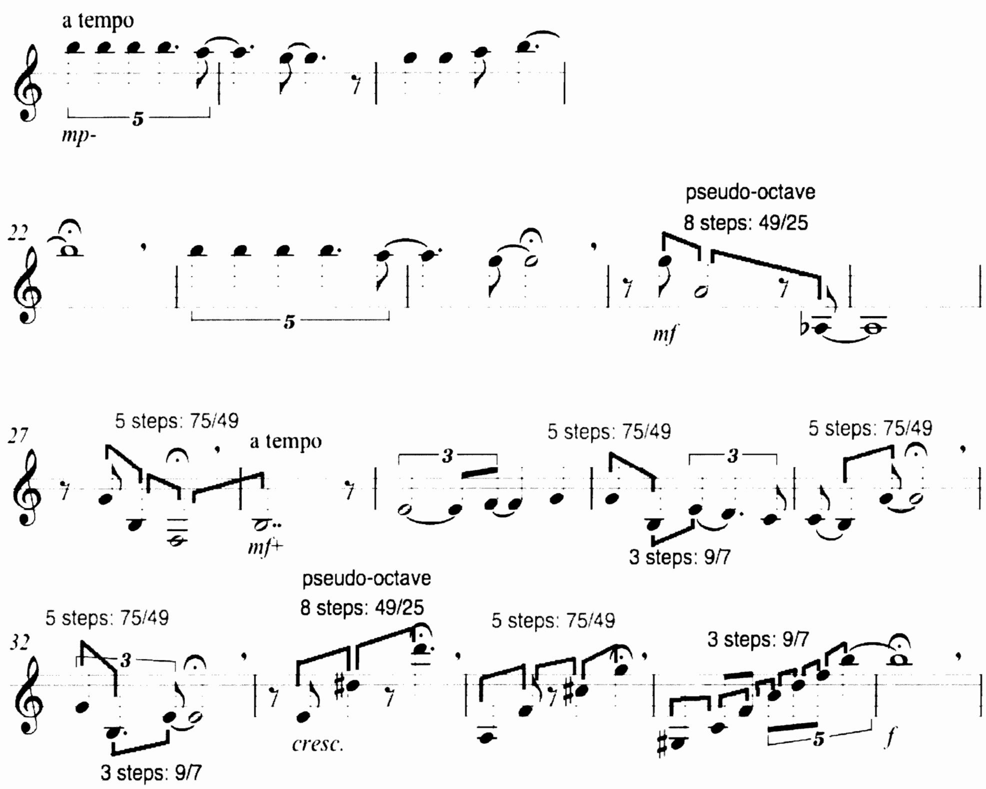
84 Werntz, "Program Notes for *Imperfections*."

293-cent "minor third" (2 BP chromatic steps, or 25/21), 439-cent "major third" (3 BP chromatic steps, or 9/7), the 732-cent "perfect fifth" (5 BP chromatic steps, or 75/49), and the 1170 cent false-octave (8 BP chromatic steps, or 49/25). Composing in Bohlen-Pierce scale removed her from her comfort zone where a different, and perhaps limiting, set of intervals were available. The only way Werntz could explore this new tonal world was by translating its pitches into a familiar language, using the closest available pitches in 72-equal and instead composing with those. When the piece was completed, she converted the music back into Bohlen-Pierce.



Example 4.10: Julia Werntz, *Imperfections*: Opening Phrase

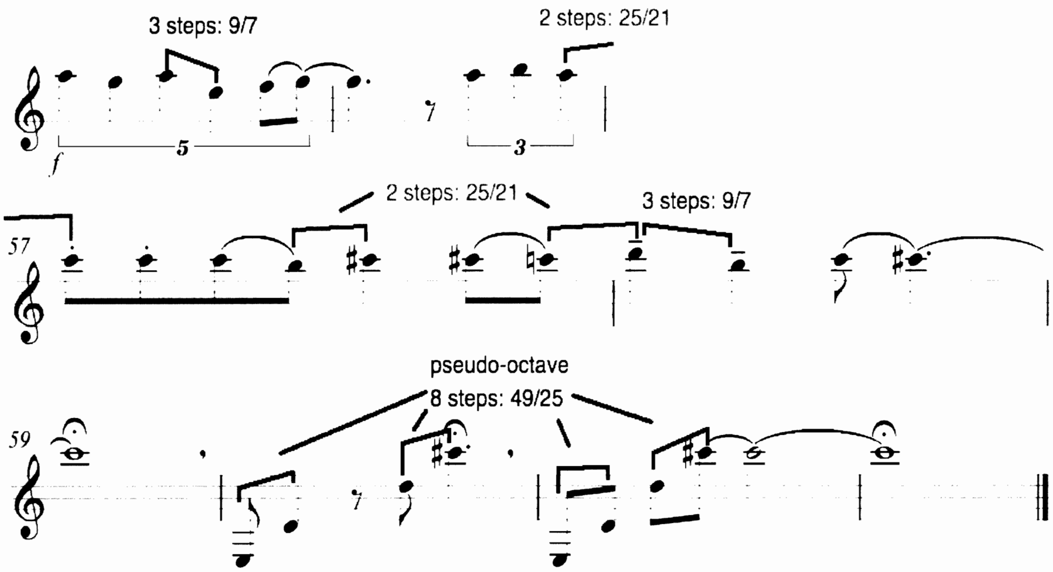
The opening phrase of *Imperfections* (see Example 4.10 – written in clarinet fingering-based notation. All intervals larger than a BP "semitone" are marked) allows the listener to appreciate and get accustomed to the large "semitone." Only two "skips" are used in this phrase: three intervals of 25/21 and one 9/7. This phrase has three parts, all of which close on pitches related to one another by commonly found intervals in the piece. The opening three measures focus on neighboring notes around the notated "g," using traditional notation because that is what would be read by the performer in this case. The next three measure section leads towards notated "c♯," which is related to "g" by an interval of 5 BP steps, a pseudo "perfect fifth". The last section closes the phrase on the notated "a," which is one false-octave (8 BP chromatic steps) below the "g". These are almost all of the intervals that will be used throughout the piece and are contained, either structurally or melodically, within the first phrase.



Example 4.11: *Imperfections:* Use of Larger Skips

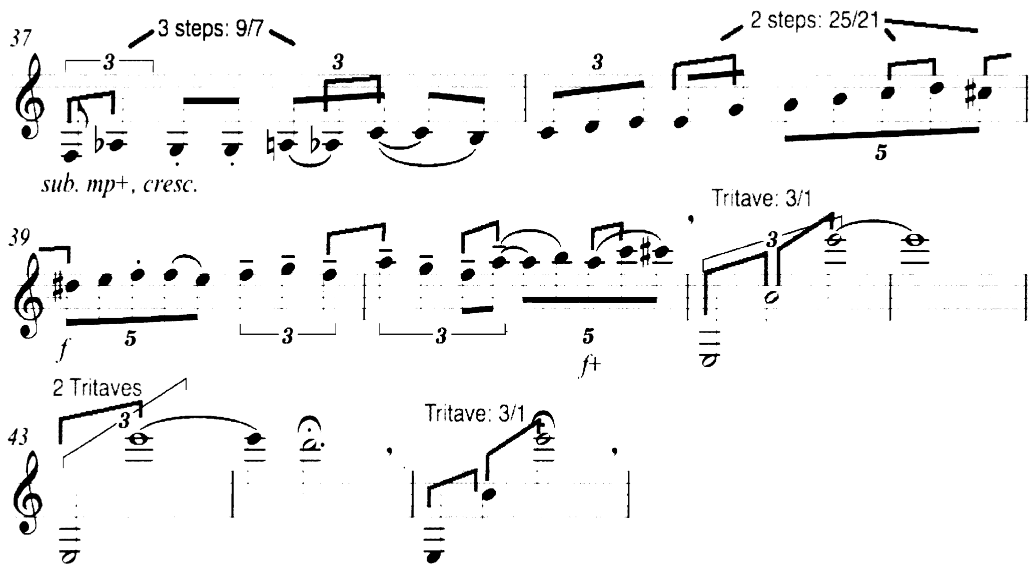
Later on in the piece, after the listener has adjusted to the new tuning, Wertnz begins to use larger intervals in a melodic fashion. Example 4.11 shows a phrase that begins purely chromatic-step-wise, but then includes skips of a false-octave as well as the pseudo "perfect fifth". At the end of the phrase, Werntz stacks all of these intervals, as well as the 9/7, opening up the melodic scope and creating tension.

Example 4.12 shows the aftermath of the tension created by those stacked intervals: a long rising, mostly chromatic line, which occasionally drops in pitch but increases in intensity all the while. The peak of all of this is the largest interval of the piece: the tritave. This moment is the climax of the piece so it is no coincidence that Werntz saves the only occurrences of the tritave for this moment.



Example 4.12: *Imperfections:* Climax

The final phrase (see Example 4.13) recalls the opening theme in its first two measures. Werntz has returned to using mainly step-wise motion throughout this phrase until the very end. The piece ends with pseudo-octaves. Although the framework of the Bohlen-Pierce scale is a tritave, Werntz chose to end on the 30 cent flat octave. Perhaps this piece can be seen as a struggle between the octave and tritave. The tritave was stressed with its placement at the climax of the piece. However, in the end, the octave wins out. Perhaps Werntz is commenting on the difficulty of hearing the tritave as the interval of equivalence, or maybe she is simply relishing the unique sound of this scale's pseudo-octave.



Example 4.13: *Imperfections*: Closing Phrase

Conclusion

This paper has discussed some reasons that alternative systems are constructed in order to better approximate: 1) familiar intervals, 2) higher partials or 3) unfamiliar intervals. Many alternative systems exist that create better approximations of the familiar third and fifth-partial intervals than those of the conventional 12-equal temperament. The seventh-partial, which is not well-approximated by 12-equal is more in tune with some alternative systems such as 19-equal and 31-equal. Some musical systems use just ratios with even higher partials, compromising the use of equal step sizes as is the case with Harry Partch's 43-tone scale or using hundreds of divisions of the octave, as with Ben Johnston. Other systems compromise the familiar framework of the octave in order to achieve extremely close approximations of familiar intervals, much like the scales of Wendy Carlos.

The Bohlen-Pierce scale is unique from the above discussed alternative systems because it does not use ratios that have been deemed important for tonality such as the octave, perfect fifth or major third. Rather, it acquires its intervals from the relationship of odd-partials of the harmonic system. Its construction is analogous to the 12-equal scale. The importance given to the octave (second-partial) and perfect fifth (third-partial) in 12-equal is relative to the importance given to the tritave (third-partial) and the fifth-partial of the Bohlen-Pierce scale. Of course, because the just Bohlen-Pierce scale contains only odd-numbered ratios, many of the intervals are unique to the familiar system.

Another parallel to the 12-equal scale, is the existence of a chromatic and diatonic Bohlen-Pierce scale. Many diatonic modes exist, but Lambda mode has been suggested by Elaine Walker and Heinz Bohlen as the ideal reference mode. A family of modes related to Lambda as well as key signatures for each have been created, showing that keys are related to one another by a diatonic BP third. The analogies between the diatonic BP third and the 12-equal dominant suggest that it may be used similarly in harmonic passages.

The relationship of the Bohlen-Pierce scale to an odd-partial harmonic series suggests that the scale would sound best with timbres of only odd-partials; this is supported by the studies of William Sethares, Max Mathews and John Pierce, to name a few. Thus, a Bohlen-Pierce clarinet, which overblows at the tritave and has a strong odd-partial timbre, has become an ideal vehicle for the scale. Other instruments include synthesizers, for which Elaine Walker has constructed keyboards in both Lambda and Dur II modes, Ron Sword's guitars and a number of computer programs that can be used to write music in the scale.

Various musical examples were explored, some of them supporting the possibility of a Bohlen-Pierce harmonic theory. The current musical language is atonal or avant-garde so not many works made use of the diatonic Bohlen-Pierce scale. A more atonal language can be very effective in the scale, as was the case with Georg Hajdu's *Beyond the Horizon*. The scale's unfamiliar intervals are effective at transporting the listener into the ever-expanding universe that Hajdu's piece contextualizes. Elaine Walker uses the scale similarly, writing Bohlen-Pierce songs about space and alien invasions, but also writes a simple love song that uses the scale diatonically. Julia Werntz focused on the scale's approximations of familiar intervals, although these approximations are not always close, such as the octave, perfect fifth, and minor third.

The wide variety of music written for the Bohlen-Pierce scale suggests that its musical language is as useful as that of the current musical system. New instruments are still being invented. More composers are learning about and using the scale in unique ways. The Bohlen-Pierce scale is far from exhausted; this exciting new musical world has much to be explored!

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Appendix – Bohlen-Pierce Compositions

1. Jon Appleton: *Eros Ex Machina* (1991)
   On the CD accompanying the book *Current Directions in Computer Music Research*, ed. Max Mathews and John R. Pierce (MATCCD 0-262-63121-0)

2. Clarence Barlow: *Pinball Play* (2010) for 4 Bohlen-Pierce clarinets
   Score available at <http://bohlenpierceconference.org/compositions/barlow-pinball-play/pinball-play/>
   Live recording with Amy Advocat, Tilly Kooyman, Ákos Hoffmann and Nora-Louise Müller, Bohlen-Pierce clarinets at: <http://recordings.bcdixon.com/10-03-07_BP/>

3. Constantin Basica: *If some kind of intelligent life form would find the Voyager Golden Record that we sent in space and their audio playing device would "translate" everything by default in Bohlen-Pierce scale, then...* (2010)

4. James Bergin, *Liebesleid* (2010), for solo BP clarinet
   Score available at <http://bohlen-pierce-conference.org/compositions/bergin-liebesleid/bergin-liebesleid-2>
   Live recording with Amy Advocat, Bohlen-Pierce clarinet at: <http://recordings.bcdixon.com/10-03-07_BP/>

5. Owen Bloomfield, *When the Ravens Descend* (2008/10) for soprano and Bohlen-Pierce soprano and tenor clarinets
   Live recording with Marion Samuel-Stevens, soprano, Tilly Kooyman, soprano Bohlen-Pierce clarinet and Stephen Fox, soprano and tenor Bohlen-Pierce clarinets at: <http://recordings.bcdixon.com/10-03-07_BP/>

6. Heinz Bohlen, trans, *Es kommt ein Schiff geladen* (1973)
   Score available at: <http://www.huygens-fokker.org/bpsite/tonality.html>

7. Sofia Borges, *Ilustre Desconhecido* (2010)
   Audio file available at: <http://bohlen-pierce-conference.org/compositions/borges-ilustre-desconhecido/borges-i-desconhecido/>

8. Richard Boulanger, *I Know of No Geometry* (1990, rev. 2010), for Radio Baton and Csound5:
   1. Introduction to Geometry
   2. Fractal Geometry
   3. Euclidean Geometry

4. Geometric Solution
   Audio file available at: <http://www.csounds.com/boulanger/music/baton/14geometry3.mp3> and <http://www.ziaspace.com/elaine/BP/The%20Bohlen-Pierce%20Scale.html>

9. Richard Boulanger, *Solemn Song for Evening* (1990), for soprano and Radio Baton. Lyrics after Herrmann Hesse, translated by Marjorie Mathews.
   On the CD "The Virtuoso in the Computer Age – V," CDCM Computer Music Series vol. 15. Centaur Records, CRC 2190, with Maureen Chowning, soprano and Richard Boulanger, Radio Baton
   Audio file available at: <http://www.csounds.com/boulanger/> and <http://www.ziaspace.com/elaine/BP/The%20Bohlen-Pierce%20Scale.html>

10. Charles Carpenter, *Frog à la Pêche* (1994) CD, released on Caterwaul Records.
    Available at amazon or at: <http://www.charlescarpenter.net/discography.html>

11. Charles Carpenter, *Splat* (1996), CD, released on Caterwaul Records.
    Available at amazon or at: <http://www.charlescarpenter.net/discography.html>

12. Lou Cohen, *Homage to Cage 2* (2010) (quadraphonic electronic composition employing Bohlen-Pierce temperament)

13. Anthony De Ritis, *Five Moods* (2010) for Bohlen-Pierce clarinet and tape
    Live recording with Amy Advocat, Bohlen-Pierce clarinet available at: <http://recordings.bcdixon.com/10-03-07_BP/>

14. Emily Doolittle, *Body of Wood* (2009) for soprano, Bohlen-Pierce clarinet, cello and percussion
    Live recording with Marion Samuel-Stevens, soprano, Stephen Fox, Bohlen-Pierce clarinet, Shane Neill, cello and Rick Stacks, percussion at: <http://recording.bcdixon.com/10-03-07_BP/>

15. Diane Douglas, *Spectrum of the Last Eclipse* (2010)

16. Roger Feria, *Re: Stinky Tofu* (2010), for BP clarinet and bass clarinet
    1. fermentation...
    2. deep fry!

    Live recording with Amy Advocat, Bohlen-Pierce clarinet and Stephen Davidson, bass clarinet at: <http://recordings.bcdixon.com/10-03-07_BP/>

17. John ffitch, *Universal Algebra* (2010)
    Sound files available at: <http://www.cs.bath.ac.uk/jpff/universalAlgebra/>

18. Georg Hajdu, *Ich kannte sie, Herr Kollege*, Act I Scene I from the opera *Der Sprung* (1999)
    CD of the entire opera distributed by NRW Vertrieb, with Wolfgang Tiemann, and Hans Hermann Jansen, tenors in the opening scene.

19. Georg Hajdu, *Beyond the Horizon* (2008) for two Bohlen-Pierce clarinets and synthesizer in Bohlen-Pierce tuning
    Score available at: <http://bohlen-pierce-conference.org/compositions/hajdu-beyond-the-horizon/beyond-the-horizon2>
    Live recording with Nora-Louise Müller and Ákos Hoffmann, Bohlen-Pierce clarinets at: <http://recordings.bcdixon.com/10-03-07_BP/>

20. Peter Michael Hamel, *Die Umkehrung der Mitte* (2008) for 2 Bohlen-Pierce clarinets, viola, marimba and vibraphone
    Score available at <http://bohlen-pierce-conference.org/compositions/hamel-die-umkehrung-der-mitte/udm2>
    Live recording with Nora-Louise Müller and Ákos Hoffmann, Bohlen-Pierce clarinets, Annette Klein, viola and Rick Sacks, percussion at: <http://recordings.bcdixon.com/10-03-07_BP/>

21. Peter Hannan, *No brighter sun : no darker night* (2009)
    Live recording with Marion Samuel-Stevens, soprano, Amy Advocat and Tilly Kooyman, Bohlen-Pierce clarinets, Shane Neill, cello, and Rick Sacks, malletKAT at: <http://recordings.bcdixon.com/10-03-07_BP/>

22. Todd Harrop, *Calypso* (2008)
    Live recording with Amy Advocat and Tilly Kooyman, Bohlen-Pierce clarinets, and Todd Harrop and Rick Sacks, percussion at: <http://recordings.bcdixon.com/10-03-07_BP/>

23. Jacob Joaquin, *Fragments* (2010)
    Mp3 and Csound score can be found at <http://csound.noisepages.com/2010/03/fragments/>

24. Jinku Kim, *Color Me Grey* (2010), Network Performance with LaunchPad/Monome and visuals

25. Pèter Köszegh, *UTOPIE XVII (chochma)* (2009/10) for Bohlen-Pierce clarinet and CD
    Live recording with Nora-Louise Müller and Ákos Hoffmann, Bohlen-Pierce clarinets at: <http://recordings.bcdixon.com/10-03-07_BP/>

26. Johannes Kretz, *Hoquetus II* (2009) for two Bohlen-Pierce clarinets and live

electronics
Score available at: <http://bohlen-pierce-conference.org/?attachment_id =996>
Live performance with Nora-Louise Müller and Ákos Hoffmann, Bohlen-Pierce clarinets at: <http://recordings.bcdixon.com/10-03-07_BP/>

27. Sascha Lino Lemke, *Pas de deux* (2008) for Bohlen-Pierce clarinet, B-flat clarinet and electronics
Live recording with Amy Advocat, B-flat clarinet and Nora-Louise Müller, Bohlen-Pierce clarinet at: <http://recordings.bcdixon.com/10-03-07_BP/>

28. Tim Lukens, *Permutations* (2010)

29. John Mallia, *The Larches* (2010), for laptop

30. Seiya Matsumiya, *The Melting Sun* (2010)

31. Katarina Miljkovic, *For Amy* (2010), for Bohlen-Pierce clarinet and electronics
Live recording with Amy Advocat, Bohlen-Pierce clarinet at: <http://recordings.bcdixon.com/10-03-07_BP/>

32. Herman Miller, *The Bent Vortex*, a study for synthesizer.

33. Herman Miller, *Warped Canon in BP* (just and tempered) for synthesizer
Recording available (with composer performing) at:
Just version: <http://www.io.com/~hmiller/midi/canonBP.mid>
Equal-tempered: <http://www.io.com/~hmiller/midi/canonBP-tempered.mid>

34. Stratis Minakakis, *Anarcharsis I* (2010), for BP clarinet, violin and dumdek
Live recording with Amy Advocat, clarinet, Samantha Bennett, violin and Gary Weaver, dumdek: <http://recordings.bcdixon.com/10-03-07_BP/>

35. Mike Moser-Booth, *A Place Between Dark and Light* (2010), for guitar and laptop

36. Jascha Narveson, *Wire* (2009) for two Bohlen-Pierce clarinets and percussion
Score can be found at <http://bohlen-pierce-conference.org/ compositions/ narveson-wire/wire-apr8-score/>
Live recording with Stephen Fox and Tilly Kooyman, clarinets at: <http:// recording.bcdixon.com/10-03-07_BP/>

37. Northeastern University's Ensemble: *Half The Plate*, using Quintet.net. Co-written by Andrew Cush, Ian Battenfield Headley, Dean Russell, Edward Young, Zachary Zukowski.

38. Mats Öljare, *Blue Rondo à la Thai* (2001), sythesizer
    Available in MIDI (with the composer performing) at: <http://groups .yahoo.com/group/tuning/files/oljare/brondo.mid

39. Joseph Pehrson, *Beepy* (2001), for synthesizer

40. John R. Pierce & Max Matthews, *Sea Songs Phasered* (1963, rev. 2010), for synthesizer:
    1. Original sound file
    2. Phaser filtered sound file

41. Kees van Prooijen, *Odd Piano* (2000), for synthesizer
    Audio file can be found at: <http://www.kees.cc/music/oddpiano/ oddpiano.html>

42. Kees van Prooijen, *Variations on a theme by Anton Webern*, for synthesizer.
    Score can be found at: <http://www.kees.cc/music/webvar.html>
    Audio file can be found at: <http://www.kees.cc/music/Kees%20van %20Prooijen%20-%20Variations%20on%20a%20Theme%20by %20Anton%20Webern.mp3>

43. Dario Quiñones, *BigPot for BP scale* (2010)
    Audio File can be found at <http://bohlen-pierce-conference.org/ compositions/quinones-bigpot/bigpot/>

44. Ami Radunskaya, *A Wild and Reckless Place* (1990), for electronic cello and Radio Baton.
    On the CD "The Virtuoso in the Computer Age – V," CDCM Computer Music Series vol. 15. Centaur Records, CRC 2190, with Ami Radunskaya, electric cello and Max Mathews, Radio Baton

45. Alyson Reeves, *Minuet* (1991)
    On the CD accompanying the book *Current Directions in Computer Music Research*, ed. Max Mathews and John R. Pierce (MATCCD 0-262-63121-0)

46. Alyson Reeves, *Canon 4* (1991)
    On the CD accompanying the book *Current Directions in Computer Music Research*, ed. Max Mathews and John R. Pierce (MATCCD 0-262-63121-0)

47. Juan Reyes and Cynthia Lawson, *Vientos de Los Santos Apóstoles* (2001), sound installation
    Presented at the mixed media and multimedia exposition TELE-vision at the Museum of Modern Art in Bogota, Colombia.

48. Juan Reyes, *ppP* (1999-2000), for piano and physical model of the piano (model developed by Scott Van Duyne)

49. Juan Reyes, *Chryseis* for BP-pitched Scan Synthesis.
    A CD containing ppP and Chryseis is available from the composer (http://www-ccrma.stanford.edu/~juanig)

50. Curtis Roads, *Purity* (1994)
    Can be found on Disc 2 of CCMIX Paris (New electroacoustic music from Paris), Mode Records 98/99

51. Fredrik Schwenk, *Night Hawks, Dark Scene* for for Bohlen-Pierce clarinet and Bb clarinet (2008)
    Live recording with Nora-Louise Müller and Ákos Hoffmann, Bohlen-Pierce clarinets at: <http://recordings.bcdixon.com/10-03-07_BP/>

52. Adam Shechter, *BP Piscium Star Fragments* (2010), for laptop

53. Manfred Stahnke, *Die Vogelmenschen von St. Kilda* (2008)
    Score can be found at <http://bohlen-pierce-conference.org/compositions/stahnke-die-vogelmenschen-von-st-kilda/vogelmenschen-gesamt>
    Live recording with Nora-Louise Müller and Ákos Hoffmann, Bohlen-Pierce clarinets at: <http://recordings.bcdixon.com/10-03-07_BP/>

54. Ron Sword, *"New Music" for the Bohlen Pierce Guitar (Eight Non-Octave Pieces for Heinz Bohlen)*
    Purchase sheet music directly from the composer at: <http://www.ronsword.com/scores.html>

55. Ron Sword, *Gamma Fantasy* for Solo Nylon String Guitar
    Purchase sheet music directly from the composer at: <http://www.ronsword.com/scores.html>

56. Ron Sword, *Triple Bohlen-Pierce Piece,* for Solo Nylon String Guitar
    Purchase sheet music directly from the composer at: <http://www.ronsword.com/scores.html>

57. Ron Sword, *Thirteen Bohlen Pierce Preludes for Guitar I-XIII* (2010)
    Purchase sheet music directly from the composer at: <http://www.ronsword.com/scores.html>

58. Elaine Walker, *Stick Men* (1991)
    Available on the CD "ZIAv1.5" (1992)* or at <http://www.ziaspace.com/ZIA/mp3s/StickMen.html

59. Elaine Walker, 1 - rx^2 (1992) for chaos controller (a hardware MIDI instrument of composer's invention).
    Available on accompanying CD to Walker's thesis paper, Chaos Melody Theory (NYU, 2001) or on her website: <http://www.ziaspace.com/elaine/chaos/>

60. Elaine Walker, *Space Time* (1994) (19tet and BP)
    Available on the CD "ZIAv1.5" (1992)*

61. Elaine Walker, *The Building* (1994) (19tet and BP)
    Available on the CD "ZIAv1.5" (1992)* or at <http://ziaspace.com/ZIA/mp3s/TheBuilding.mp3>

62. Elaine Walker, *Big Bang* (2000) for synthesizer
    Available on the CD "Big Bang" (2000)* or at <http://zww.ziaspace.com/ZIA/mp3s/BigBang.html

*Purchase all of Elaine Walker's CD's at <http://www.ziaspace.com/ZIA/sections/music.html>

63. Elaine Walker, *Love Song* (2007)
    Unreleased, but audio file is available on her site: <http://www.ziaspace.com/ZIA/sections/music.html>

64. Arash Waters, *Little Duet* (2010) for Bohlen-Pierce clarinet and cello
    Score can be found at <http://bohlen-pierce-conference.org/compositions/waters-little-duet/bp-arash>
    Live Recording with Tilly Kooyman, Bohlen-Pierce clarinet and Shane Neill, cello: <http://recordings.bcdixon.com/10-03-07_BP/>

65. Julia Werntz, *Imperfections* (2010), for solo Bohlen-Pierce clarinet
    Live Recording with Amy Advocat, Bohlen-Pierce clarinet: <http://recording.bcdixon.com/10-03-07_BP/>

66. David Wessel, *BP Compatible Spectra* (2010)

67. Randy Winchester, *Comets over Flatland* (1998), for synthesizer
    Audio file available at: <http://web.mit.edu/randy/www/Music/comets.html>

68. Steven Yi, *Reminiscences* (2010)
    Recording available at: <http://www.kunstmusik.com/2010/03/08/reminiscences/

69. Gayle Young, *Cross Current* for two Bohlen-Pierce clarinets, Bohlen-Pierce recorder, amaranth and percussion
    Live recording available at: <http://recording.bcdixon .com/10-03-07_BP/>