

# **Topics in mechanics of helices**

By

**Hossein Shahsavari**

Department of Civil Engineering and Applied Mechanics

McGill University

Montréal, Québec

August, 2004

A thesis submitted to  
the Faculty of Graduate Studies and Research  
in partial fulfillment of the requirements for the degree of  
Master of Engineering

© Hossein Shahsavari, 2004



Library and  
Archives Canada

Published Heritage  
Branch

395 Wellington Street  
Ottawa ON K1A 0N4  
Canada

Bibliothèque et  
Archives Canada

Direction du  
Patrimoine de l'édition

395, rue Wellington  
Ottawa ON K1A 0N4  
Canada

*Your file* *Votre référence*  
ISBN: 0-494-06584-2

*Our file* *Notre référence*  
ISBN: 0-494-06584-2

#### NOTICE:

The author has granted a non-exclusive license allowing Library and Archives Canada to reproduce, publish, archive, preserve, conserve, communicate to the public by telecommunication or on the Internet, loan, distribute and sell theses worldwide, for commercial or non-commercial purposes, in microform, paper, electronic and/or any other formats.

The author retains copyright ownership and moral rights in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author's permission.

In compliance with the Canadian Privacy Act some supporting forms may have been removed from this thesis.

While these forms may be included in the document page count, their removal does not represent any loss of content from the thesis.

#### AVIS:

L'auteur a accordé une licence non exclusive permettant à la Bibliothèque et Archives Canada de reproduire, publier, archiver, sauvegarder, conserver, transmettre au public par télécommunication ou par l'Internet, prêter, distribuer et vendre des thèses partout dans le monde, à des fins commerciales ou autres, sur support microforme, papier, électronique et/ou autres formats.

L'auteur conserve la propriété du droit d'auteur et des droits moraux qui protège cette thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

Conformément à la loi canadienne sur la protection de la vie privée, quelques formulaires secondaires ont été enlevés de cette thèse.

Bien que ces formulaires aient inclus dans la pagination, il n'y aura aucun contenu manquant.

\*\*  
**Canada**

## **Abstract**

This thesis treats three topics in mechanics of helices, each being covered in a separate part. The first part focuses on the determination of a spectral (i.e., frequency dependent) finite element of a helix. The helix is treated as a straight, linear elastic element, exhibiting coupling of axial with torsional responses. Explicit forms of all the coefficients of the stiffness matrix are derived and their dependencies on the frequency and the parameter describing the said coupling are plotted. In general, the growth of that parameter leads to a progressively denser occurrence of the resonances of both axial and torsional motions.

In the second part, mechanics of helically shaped elastic strands is studied. Explicit forms of four constitutive coefficients of the helix are derived according to the formulation of Costello (1990). By performing parametric studies of the derived coefficients, it is shown that the coupling coefficients relating axial with torsional responses are not in perfect agreement. Thus, the reciprocity relations dictated by the Maxwell-Betti theorem are not recovered exactly, whereby an improvement on the theory of Costello remains an open challenge.

In the third part, the viscoelastic response of the helix is studied and its constitutive differential equations are derived by assuming three specific types of viscoelastic models (Kelvin, Maxwell, or Zener) for a material at the micro level. Solutions to these constitutive differential equations are expressed for creep and relaxation behaviors of the helix. It is found that the effective viscoelastic response of the helix is generally different, and more complex in type than the material response of viscoelastic helical strands themselves.

## Résumé

Le présent mémoire traite de trois facettes de la mécanique des hélices qui sont présentées séparément. La première partie concerne l'élaboration d'un élément fini spectral (i.e., dépendent de la fréquence) d'un élément de type hélice. L'hélice est ici considérée comme étant un élément droit, élastique linéaire et ayant un comportement à la fois axial et de torsion. Une formulation détaillée de tous les coefficients de la matrice de rigidité est démontrée et les corrélations de ces coefficients par rapport aux fréquences et le paramètre décrivant le couplage sont montrés graphiquement. En règle générale, la croissance de ce paramètre tend à démontrer une incidence progressivement plus dense des résonances de mouvements axiaux et de torsion.

En deuxième lieu, la mécanique de torons élastiques hélicoïdaux est étudiée. Les formulations détaillées des quatre coefficients d'une telle hélice sont démontrées selon la formulation de Costello (1990). En effectuant des études paramétriques sur les coefficients étudiés, il a été démontré que les coefficients de couplage ne correspondent pas parfaitement. Ainsi, les relations de réciprocité telles que démontrées par le théorème de Maxwell-Betti ne sont pas respectées et l'amélioration de la théorie de Costello demeure un défi.

Dans la troisième et dernière section de ce mémoire, le comportement viscoélastique de l'hélice est étudié et ses équations différentielles sont démontrées en supposant trois types de modèles viscoélastiques (Kelvin, Maxwell, ou Zener) du matériau à un niveau microscopique. Les solutions à ces équations différentielles sont exprimées pour le fluage et l'état de relaxation d'une hélice. Il est démontré que le comportement viscoélastique effectif de l'hélice est généralement différent et plus complexe que le comportement viscoélastique du matériau constituant les torons hélicoïdaux.

## **Acknowledgements**

I would like to greatly thank my thesis supervisor, Professor Martin Ostoja-Starzewski for introducing me to the world of research and study, and teaching me many new topics. His past and present contributions in mechanics have set the foundation for many innovative methods and approaches. The resulting effect on this thesis and the few publications resulting from it are due to his will to continuously improve what others have accepted as being sufficient.

I also wish to thank my co-supervisor, Professor Ghislaine McClure for her kind help and guidance in my research career and life. Both my supervisors taught me who I really want to be, and I believe this will be extremely valuable in my future.

My gratitude is also extended to my best friend Jonathan who has always volunteered to help me correct the thesis texts.

Lastly, a special thanks goes out to my father and all my family and friends. Without their support and encouragement, this thesis would not have been possible.

## CONTENTS

Abstract	i
Resume	ii
Acknowledgments	iii
Contents	iv
List of Figures	vi
<b>1 INTRODUCTION .....</b>	<b>1</b>
1.1 Background .....	1
1.2 Literature review .....	2
1.2.1 Problem definition and terminology .....	3
1.2.2 Strands under axial loads .....	4
1.2.2.1 Purely tensile or fiber models .....	4
1.2.2.2 Helical rod models .....	5
1.2.2.3 Comparative evaluation of strands models .....	7
1.2.3 Strand bending mechanical models .....	9
1.2.3.1 Purely tensile or fiber models .....	9
1.2.3.2 Curved rod models .....	9
1.3 Motivations .....	11
<b>2 SPECTRAL FINITE ELEMENT OF AN ELASTIC HELIX.....</b>	<b>12</b>
2.1 Introduction .....	12
2.2 Spectral Stiffness Matrix .....	14
2.3 Results .....	23

<b>3</b>	<b>CONSTITUTIVE COEFFICIENTS OF AN ELASTIC HELIX</b>	.27
3.1	Explicit forms of the constitutive helix coefficients .....	27
3.2	Parametric studies of the constitutive helix coefficients .....	38
3.3	Results .....	44
<b>4</b>	<b>RESPONSE OF VISCOELASTIC HELICES</b>	.46
4.1	Viscoelastic Models .....	46
4.2	Correspondence Principle in Viscoelasticity .....	47
4.3	Differential equations of viscoelastic helix using Kelvin model .....	48
4.4	Differential equations of viscoelastic helix using Maxwell model .....	50
4.5	Differential equations of viscoelastic helix using Zener model .....	51
4.6	Creep and relaxation phases .....	52
4.6.1	Relaxation phase .....	52
4.6.2	Creep phase .....	54
4.7	Results .....	56
<b>5</b>	<b>CONCLUSIONS</b>	.57
5.1	Conclusion .....	57
5.2	Future research.....	59
<b>6</b>	<b>REFERENCES</b>	.60
	<b>APPENDICES</b>	.64
A	Discrepancy between $C_2$ and $C_3$ .....	65
B	Coefficients of the differential equations of a viscoelastic helix .....	67
C	Maple inverse Laplace transforms of the differential equations.....	150

## LIST OF FIGURES

1.1	Strand basic geometry and wire internal forces and moments .....	3
2.1	A wire rope structure shows the coupling of axial and torsional responses .....	13
2.2	Dependence of spectral stiffnesses on the axial-torsional coupling .....	24
2.3	Dependence of the spectral stiffness $k_{11}$ on frequency and axial-torsional coupling .....	25
2.4	Dependence of the spectral stiffness $k_{12}$ on frequency and axial-torsional coupling .....	25
2.5	Dependence of the spectral stiffness $k_{13}$ on frequency and axial-torsional coupling .....	26
2.6	Dependence of the spectral stiffness $k_{14}$ on frequency and axial-torsional coupling .....	26
3.1	Loaded simple strand (helix) .....	28
3.2	Dependence of $\frac{C_3}{C_2}$ on Poisson's ratio $\nu$ .....	38
3.3	Dependence of $\frac{C_3}{C_2}$ on helix angle $\alpha$ (in radians) .....	39
3.4	Dependence of $\frac{C_3}{C_2}$ on $R_1$ .....	40
3.5	Dependence of $\frac{C_3}{C_2}$ on $R_2$ .....	41
3.6	Dependence of constitutive helix coefficients on the helix angle $\alpha$ .....	42
3.7	Dependence of constitutive helix coefficients on $R_1$ .....	43
3.8	Dependence of constitutive helix coefficients on Poisson's ratio $\nu$ .....	44
4.1	Three viscoelastic models .....	46

## **2.11 INTRODUCTION**

### **1.1 Background**

A property common to structural elements such as rope, yarn, cord, cable, and strand – all of which are complex helical assemblies - is the ability to resist relatively large axial loads in comparison to bending and torsional loads. Rope, because of this property, is one of the oldest tools that men have used in their efforts to improve the living conditions. For example, a copper cable found in the ruins of Nemeveh near Babylon indicates that wire rope was used as a structural element about 700 B.C.

This thesis, as the title indicates, is concerned with several mechanical responses of helices. In recent years, considerable progress has been made in the development of models to predict the response of a helix or any helically wound bundle like a wire rope. Since there are several parameters that may vary in the construction of a helix, such models can be used to determine the effects of possible variations of the parameters on the performance of a rope. In these models, the underlying geometry is generally that of a core strand surrounded by one or several wire layers.

Wires or helically wound fibers have many important engineering applications. Steel cables which have helical shapes are used in suspension bridges. These cables might be 1 m in diameter. This, of course, is made by more than just one cable. There are thousands of smaller helical strands that are combined together to make such a large cable. All transmission lines and power lines are also helical strands. They are basically aluminum strands twisted around a steel center wire. Wire strands are used as braces for teeth as well. Wire rope has applications in superconductivity. It can also be used to strengthen the rubber tires. In biology, DNA has a helical shape and the microstructure of a bone is made of helices. A list of a helical strand's applications may be endless.

## 1.2 Literature Review

Helically wound fibers or wires constitute a wide class of important engineering components. It is well known that a major advantage of such elements is their capacity to support large axial loads with comparatively small bending or torsional stiffness. Some of the applications often require a quantitative evaluation of the relevant mechanical parameters. Two important fields of application are cables and overhead electrical conductors.

During the last decades, each field has developed a specific body of knowledge, based on the previous work and extensive testing experience, leading to empirical rules for particular practical applications. However, unifying these rules under some general physical and mechanical theory should allow a better understanding and prediction of the mechanical responses of these elements, thus reducing the need for expensive tests under varying parameters and operating conditions.

A review of helical strand models was published 26 years ago by Costello (1978). As indicated by its title, this review was oriented toward wire rope applications. However, most of the 30 references in the review were of a general nature. A few years later, another review on wire ropes was published by Utting and Jones (1984). Among its 77 references, a large proportion were also of general mechanical interest (both analytical and experimental). This review has then been updated by Utting (1995), with 110 new references. In the overhead electrical conductor field, the only general reference on mechanical behavior is EPRI's *Transmission Line Reference Book* (1979), whose contents are mostly oriented towards bending vibration phenomena. In these studies, the conductor is simply treated as a taut string having some bending stiffness and damping coefficients, which have to be found experimentally.

The objective of the present review is to concentrate on available strand models, irrespective of their field of application. Thus, the system is called a helical strand, independently of any applied connotation. A tentative classification of these models

based on the types of hypotheses employed is proposed. Emphasis is placed on the basic problem of the behavior of strands under small deformations.

### 1.2.1 Problem definition and terminology

The basic strand geometry is shown in Fig.1.1. In general, it consists of a core, radius  $R_1$ , surrounded by  $n$  layers of round section wires. In layer (i),  $m_i$  wires (radius  $R_i$ ) are wound helically. The helix angle is  $\alpha_i$  and the lay angle  $\beta_i = 90^\circ - \alpha_i$ . Wire centerlines are helices on a cylinder of radius  $r_i$ . The material properties are represented by the elastic constants  $(E_i, \nu_i)$ . In some cases, a friction coefficient  $\mu$  is also introduced. In fact, many publications deal with a one layer strand, consisting of six wires in the layer and a single wire for the core. This will be called a 1/6 strand. Often, all seven wires are identical, as regards both diameter and material.

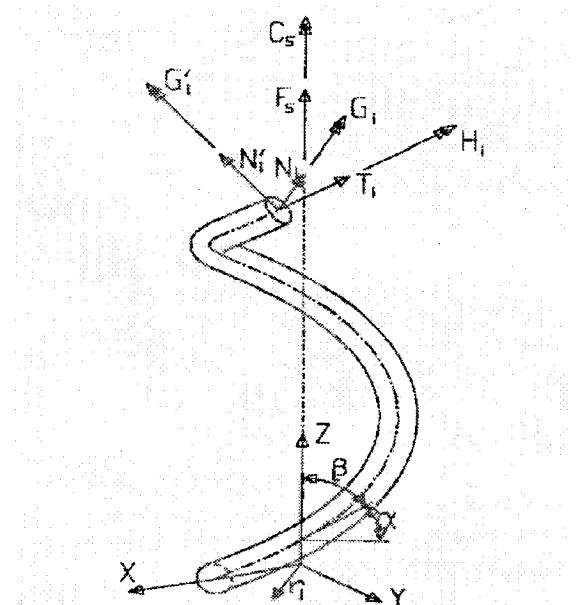


Fig. 1.1 Strand basic geometry and wire internal forces and moments

A strand may be loaded axially with a tensile force  $F_s$  and a twisting moment  $C_s$ . While under tension, it may also be subjected to a bending moment  $M_s$ . In some cases, a

uniform lateral pressure exerted on the outer layer may also be considered. The corresponding strand strains are: axial strain  $\varepsilon_s$ , torsion per unit length  $\theta_s$ , and curvature  $k_s = \frac{1}{\rho_s}$ . It is often easier to work with imposed generalized strains (which may include curvature and twist) and to calculate the corresponding loads.

### 1.2.2 Strands under axial loads

Under pure axial loading, an axial force  $F_s$  and twisting moment  $C_s$  are imposed on the strand. In such cases, all the wires in a given layer are assumed to carry exactly the same loads. Global strand strains are designated by  $\varepsilon_s$  and  $\theta_s$ .

#### 1.2.2.1 Purely tensile or fiber models

The simplest model is due to Hruska (1951, 1952, 1953). It is based on six simple hypotheses:

- a) No end condition effects (i.e., unbounded strand length); however, zero end rotation is assumed.
- b) The contact mode is purely radial.
- c) Radial contraction is neglected.
- d) Wires are assumed to be subjected to pure tensile forces (no moments).
- e) Friction can be neglected,
- f) The global strand strain is small.

With these hypotheses, Hruska obtained: wire stress, interlayer pressure and unwinding torque. He also extended his model to multi-strand wire ropes.

Hruska's equations have also been extended by Knapp (1979), to include the case of a compressible core, and possible large strand strain, and by Lanteigne (1985) (as part of a paper on strand bending).

### 1.2.2.2 Helical rod models

Wire bending and twisting can be added to the model. This was done by Machida and Durelli (1973) and Knapp (1979). However, a more rigorous derivation was presented by Philips and Costello (1974), based on Love's equation for the equilibrium of a curved rod (Love, 1944). In the first stage, lateral contact force has to be calculated. This requires helical contact line localization, carried out by assuming that the wire cross-section in strand normal section are ellipses (Costello and Phillips, 1974), which implies small lay angles ( $\beta < 30^\circ$ ). Large variations of the helix angle are acceptable and smooth contact is again assumed. Wire strain is neglected and strand strain arises only from helix angle variations. This last hypothesis is removed in Costello and Phillips (1976). The previous paper shows normalized  $F_s$  vs  $\varepsilon_s$  curves involving up to 10% strain for 1/3 (one core wire and one 3 wire layer) and 1/6 strands, with no end-rotation or end-moment. The curves are slightly nonlinear for practical strain levels.

Using the same approach, Costello (1983) subsequently studied a 1/6 strand with radial wire contact. The equations were linearized by Velinsky (1984), still considering radial contraction (Poisson's ratio only). These equations can also be found in Costello's monograph (1990). Kumar and Cochran (1987) also proposed a linearized version of Costello's equations, including radial contraction. Recently, Jiang (1995) has proposed a general formulation of this theory, including all these aspects, and extended it to more complex situations, such as multi-strand ropes. This paper has been discussed by Jolicœur (1996) and by Sathikh, Jayakumar (1996).

Contact pressure conditions were evaluated by Philips (1980) in a 1/6/12 strand, with the outer layer in radial contact with the inner layer, and the wires in the latter layer in lateral contact, i.e, no contact with the core. Line contact is assumed between both layers. The contact force per unit length is then transformed into point loads on the outer layer. As

expected, it is found that these forces are slightly smaller than those calculated using Hruska's model (1952).

Using the same rod equilibrium and constitutive equations, Huang (1978) studied the contact mode conditions (mixed, radial or lateral) for a 1/6 strand. Local contact deformation is neglected whilst the Poisson's ratio effect is included. Initially, mixed contact (in the unloaded state) is assumed. It is assumed that, under axial load, slippage exists between adjacent wires (with friction), with no friction between the wire and the core. It is found that lateral contact is lost in all the end conditions (free or blocked rotation). Thus, under these conditions, radial contact seems to be the prevailing case, even when no initial gap exists between wires in the layer. The analysis is valid for finite strand extension, but assumes small lay angle. Utting and Jones (1987) have extended Costello analysis to include wire flattening (contact deformation) and friction effects.

All the previous papers, based on the curved rod theory, use the following constitutive beam equations:

$$G' = EI\Delta\kappa' \quad (1.1)$$

$$H = GJ\Delta\tau \quad (1.2)$$

with Costello's notation (Fig1.1):

$G'$ : local bending moment on wire section about the binormal direction

$H$ : local twisting moment about the wire centerline tangential direction

$EI$ : wire bending stiffness

$GJ$ : wire twisting stiffness

$\Delta\kappa'$ : variation in the centerline curvature between the deformed and undeformed helices

$\Delta\tau$ : variation of centerline torsion

Ramsey (1988) notes that a more rigorous analysis is required, particularly for wire twist. For example, one cannot deduce the straight bar torsion formula from eqs. (1.1) and (1.2).

Using a more general and accurate theory, he derived new constitutive equations. In the axial loading case, and after some linearization, they reduce to:

$$G' = EI(\Delta\kappa' + \kappa'_0\varepsilon) \quad (1.3)$$

$$H = GJ(\Delta\tau + \tau_0\varepsilon) \quad (1.4)$$

where  $\kappa'_0$ ,  $\tau_0$  are the undeformed helix curvature and torsion, respectively,  $\varepsilon$  is the axial strain on the wire, and  $\Delta\tau$  must be considered as the variation of a twist of a wire, not just of the centerline tortuosity (Love, 1944).  $G'$  and  $H$  are interpreted as the moment components in directions close to the current centerline binormal and tangential directions. In a subsequent paper, Ramsey (1990) extended his analysis further, leading to a non-zero distributed moment component in the radial direction, thus revising his own earlier assumption (Ramsey, 1988). Clamped end stress and strain conditions have also been studied by Ramsey (1991). Numerical applications to the Bersimis electrical conductor, a conductor also used by Lanteigne (1985), have been carried out. The results show a small stress increase, compared with the free-field values.

### 1.2.2.3 Comparative evaluation of strand models

The complexity of the strand model increases with the inclusion of various secondary aspects. The performance of the various published models has generally been assessed either in relation: a) to one another, or b) to experimental results. The principle behind the evaluation is, in general, to compare global strand response to axial loading under various end conditions. A typical helical strand property is the coupling which appears between the extension and torsion responses. In the linear case, the global strand stiffness and coupling effect can be conveniently expressed by the matrix equation:

$$\begin{Bmatrix} F_s \\ C_s \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} \varepsilon_s \\ \theta_s \end{Bmatrix} \quad (1.5)$$

The axial stiffness constant  $K_{11}$  is sometimes represented by ( $AE$ ), the stiffness of an equivalent solid bar of section A and Young's modulus E. Similarly, the torsion stiffness constant  $K_{22}$  may be expressed as ( $GJ$ ).

Durelli and Machida (1973) studied the response of 1/6 epoxy oversized models under axial loading (axial force and torque) and compared the experimental results with their theoretical predictions. The correlation is very good for the global strand response  $(\varepsilon_s, \theta_s)$  under axial force  $F_s$ , but less satisfactory when torque  $C_s$  is applied. Strain gage measurements on individual wires show a great deal of scatter, which indicates how difficult it is to equalize the loads to each of wires.

McConnell and Zemke (1982) carried out an experimental program on a number of standard bi-metallic (steel and aluminum) overhead electrical conductors. They compared their results with a slightly modified version of Hruska's fiber model: their torsional stiffness  $K_{22}$  includes the tensile ( $AE$ ) and torsion stiffnesses of the wire ( $GJ$ ). Except for the initial nonlinear behavior, they found that this simple model is quite accurate.

Utting and Jones (1985,1987) tested 1/6 steel strands. They compared their results with the Machida and Durelli (1973) model and, also, with their improved version of Costello model. Axial stiffness  $K_{11}$  correlates very well. They also obtained good free end or partial restraint equivalent stiffness results.

Typical fiber and rod models have been compared by Jolicoeur and Cardou (1991). For each model, stiffness matrix coefficients were calculated for three cables and two conductors. Comparisons were made with the experimental results of Knapp (1979) and Utting and Jones (1987). The general effect of wire bending and twisting is to bring a very slight decrease (of the order of 1%) in axial stiffness  $K_{11}$  but a marked increase in torsional stiffness  $K_{22}$ . As mentioned by Samras (1974), linearized models should satisfy the Maxwell-Betti reciprocity theorem. Thus, the stiffness matrix eq. (1.5) should be

symmetric. Jolicoeur and Cardou (1991) showed that fiber models do satisfy this condition, while rod models deviate by a few percent.

### 1.2.3 Strand bending mechanical models

Strand bending is generally studied by assuming a strand centerline radius of curvature  $\rho_s$  and calculating the corresponding bending moment  $M_s$ . Then, the strand bending stiffness  $B_s$  (in fact, a secant modulus, also expressed by  $(EI)_s$ ) is defined as  $M_s/\rho_s$ .

#### 1.2.3.1 Purely tensile or fiber models

Such a model has been proposed by Lutchansky (1969). It is assumed that wire diameter is very small compared to the helix radius. No distinction is made between wire centerline and contact helix radius. Contact is purely radial and there is no interaction between wires in the same layer. The core on which a wire is wrapped is merely a geometrically defined cylinder. This cylinder is then transformed into a torus, with centerline radius  $\rho$ . It is assumed that a given wire sticks to a curve drawn on the torus surface which is a mathematical extension of the helix. It is determined by assuming a tangential elastic constant  $k$  between wire and core. The constant  $K$  has to be obtained experimentally, for example, by performing constant radius bending on the strand and measuring the bending stiffness  $B_s$ . Raoof and Hobbs (1984) have proposed an analytical determination of  $K$  based on Deresiewicz's (1957) contact mechanics equations. The resulting values of  $K$  are much larger than those used by Lutchansky. The same approach has been developed in detail by Raoof (1990). With lateral contact neglected, the method was used to evaluate core-wire microslip in the region of the clamp.

#### 1.2.3.2 Curved rod models

In practical cable or conductor bending stress calculations, the independent wire behavior hypothesis is often made, to which one adds the small lay angle hypothesis, that is to say,

the wires are considered as independent, parallel, straight rods (Drucker and Tachau 1945; EPRI, 1979). This is Reuleaus's formula for a cable bent over a sheave. A more accurate evaluation should take into account helix angle and wire interactions. This was done by Carsarphen (1931), considering the wires as independent helical springs (as in Timoshenko, 1956).

Costello and Butson (1982) proposed a model in which wires were considered as independent curved rods, with no contact restraint. Lanteigne (1985) proposed a model in which the bending moment contribution comes mostly from the fiber effect ( $T_i$ ) calculated using the Bernoulli-Euler hypothesis, and also from independent wire bending. Similar wire bending equations were used by Papailiou (1996), except that the slip criterion is based on the  $T_i$  variation along a given wire and partial slip can be considered. Partial slip due to bending was also studied by Vinogradov and Atatekin (1986) under the assumption that a strand of finite length is clamped at one end and is loaded by a transverse force at the free end. The resulting twisting moment on each wire section is thus periodic and induces wire torsion which is used to evaluate energy dissipation. This model is discussed by Sathikh and Parthasarathy (1988). Partial slip under constant radius bending has been investigated along the same lines by Huang and Vinogradov (1994).

In the preceding models, it is assumed that contact conditions between the wire and the layer underneath are either stick or slip (full or partial). Another contact mode was analyzed by Goudreau (1990): a wire may roll on the core, with or without slipping, when constant curvature bending is imposed on the core.

### 1.3 Motivations

As mentioned in the first chapter (literature review), many studies of helical strands have been carried out. However, there are still some more complex situations for which no previous work has been reported in any journal or book. A spectral finite element (SFE) of a helix is one of these complex situations. Doyle (1997) has derived SFE of a rod in pure axial loading, and this carries over by analogy to pure torsional loading. However, the SFE of a helix and its dynamic stiffness matrix are still open questions that will be discussed in the second chapter of this thesis.

Based on previous studies of wire rope (Milburn and Rendler 1972), the constitutive equations relating the rope tension and torque to the rope deformations are postulated as

$$\frac{F}{AE} = C_1 \varepsilon + C_2 \beta$$

$$\frac{M}{ER^3} = C_3 \varepsilon + C_4 \beta$$

where  $C_1, C_2, C_3, C_4$  are constitutive constants dependent on both the rope material and construction. Finding explicit forms of these constants and their parametric studies, which show how they depend on the material and geometry, is the subject of the third chapter.

Most of the research that has been done for a helical strand is purely elastic and only a limited amount of experimental inelastic research has been reported. In the last chapter the viscoelastic response of a strand is derived theoretically and the respective explicit differential equations are expressed completely. To achieve this goal, the explicit forms of the derived constants from chapter three, as well as the correspondence principle of viscoelasticity, are used.

## 2 SPECTRAL FINITE ELEMENT OF AN ELASTIC HELIX

Determination of a spectral (i.e. frequency dependent) finite element of a helix is the focus of this chapter. The helix is treated as straight, linear elastic element, exhibiting coupling of axial with torsional responses. We derive explicit forms of all the coefficients of the stiffness matrix and plot their dependencies on the frequency and the parameter describing the said coupling. In general, the growth of that parameter leads to a progressively denser occurrence of the resonances of both axial and torsional motions.

### 2.1 Introduction

To a mechanician, a helix is a structural element coupling axial and torsional responses. This comes about through some kind of a helically wound microstructure, such as in the very well known wire ropes, Fig. 2.1. The book by Costello [4] discusses the latter subject extensively, albeit in the static setting. The said coupling is expressed by two constitutive equations

$$F = A_1 \varepsilon + A_2 \beta \quad (2.1a)$$

$$M = A_3 \varepsilon + A_4 \beta \quad (2.1b)$$

where  $F$  is tensile force and  $M$  is torque, while  $\varepsilon = \frac{\partial u}{\partial z}$  is axial strain and  $\beta = \frac{\partial \theta}{\partial z}$  is angle of twist per unit length. The parameter  $z$  is the measure of length parallel to the axis of a helix. Furthermore,  $A_1, A_2, A_3, A_4$  are constitutive constants dependent on both the rope material and construction. These constants are necessarily positive, and also these relations hold

$$A_2 = A_3 \quad (2.2)$$

$$A_1 A_4 \pm A_2 A_3 > 0 \quad (2.3)$$

Notably, there are various other examples of helices in engineering and in nature, but they are all characterized by the same equation system as eqs. (2.1).

The equations of motion of a helix, referred to the unstressed rope length, and in the absence of body forces, become

$$\frac{\partial F}{\partial z} = m \frac{\partial^2 u}{\partial t^2} \quad (2.4a)$$

$$\frac{\partial M}{\partial z} = J \frac{\partial^2 \theta}{\partial t^2} \quad (2.4b)$$

Here  $m$  and  $J$  are, respectively, mass and mass moment of inertia per unit unstressed length. Substituting eqs. (2.1) into (2.4), the equations governing coupled extensional-torsional oscillations of wire rope become

$$A_1 \frac{\partial^2 u}{\partial z^2} + A_2 \frac{\partial^2 \theta}{\partial z^2} = m \frac{\partial^2 u}{\partial t^2} \quad (2.5a)$$

$$A_3 \frac{\partial^2 u}{\partial z^2} + A_4 \frac{\partial^2 \theta}{\partial z^2} = J \frac{\partial^2 \theta}{\partial t^2} \quad (2.5b)$$

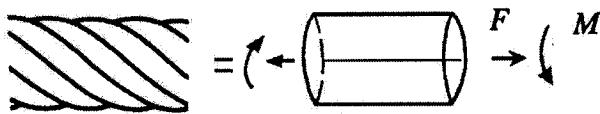


Fig. 2.1

A wire rope structure (left) shows the coupling of axial and torsional responses, typical of a helix.

## 2.2 Spectral Stiffness Matrix

Let us now consider harmonic motions according to:

$$u(z,t) = \hat{u}(z, \omega) e^{i\omega t} \quad (2.6a)$$

$$\theta(z,t) = \hat{\theta}(z, \omega) e^{i\omega t} \quad (2.6b)$$

where  $\omega$  is the frequency, and the hat stands for a quantity in the frequency space. Now, of interest is the derivation of a spectral stiffness matrix expressing a connection between the kinematic and dynamic quantities at both ends of a helical element:

$$\begin{Bmatrix} \hat{F}_1 \\ \hat{F}_2 \\ \hat{M}_1 \\ \hat{M}_2 \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \begin{Bmatrix} \hat{u}_1 \\ \hat{u}_2 \\ \hat{\theta}_1 \\ \hat{\theta}_2 \end{Bmatrix} \quad (2.7)$$

Here subscripts 1 and 2 of  $F$  and  $M$  denote the left and right ends of the rope, respectively.

The derivation of the spectral stiffness matrix follows these steps. First, substituting eqs. (2.6) into eqs. (2.5) one obtains:

$$A_1 \frac{\partial^2 \hat{u}}{\partial z^2} + A_2 \frac{\partial^2 \hat{\theta}}{\partial z^2} = -m \omega^2 \hat{u} \quad (2.8a)$$

$$A_3 \frac{\partial^2 \hat{u}}{\partial z^2} + A_4 \frac{\partial^2 \hat{\theta}}{\partial z^2} = -j\omega^2 \hat{\theta} \quad (2.8b)$$

To solve eqs. (2.8) we consider:

$$\hat{u}(z, \omega) = ue^{ikz} \quad (2.9a)$$

$$\hat{\theta}(z, \omega) = \theta e^{ikz} \quad (2.9b)$$

Here,  $u$  and  $\theta$  are constant wave amplitudes, and  $k$  is the wave number. Introducing eqs. (2.9) into eqs. (2.8) yields the two homogeneous equations

$$(A_1 k^2 - m\omega^2)u + (A_2 k^2)\theta = 0 \quad (2.10a)$$

$$(A_3 k^2)u + (A_4 K^2 - J\omega^2)\theta = 0 \quad (2.10b)$$

For a nontrivial solution of eqs. (2.10) to exist, the determinant of the coefficients must vanish. This requirement gives the dispersive relation for eqs (2.10) as

$$(A_1 A_4 - A_2 A_3)k^4 - (A_1 J + A_4 m)\omega^2 k^2 + m J \omega^4 = 0 \quad (2.11)$$

and solution for  $k^2$  becomes

$$k^2 = \omega^2 \left( \frac{(A_1 J + A_4 m) \pm \sqrt{[(A_1 J - A_4 m)^2 + 4m J A_2 A_3]}}{2(A_1 A_4 - A_2 A_3)} \right) \quad (2.12)$$

Since, from equation (2.3),  $A_1 A_4 - A_2 A_3 > 0$ , the square root of the discriminant in equation (2.12) is less than  $A_1 J + A_4 m$ . The choice of either sign in the braces then gives a positive value for  $k^2$  which correspond to propagating wave solutions with

$$k^2 = \frac{\omega^2}{c_1^2} \quad (2.13a)$$

or

$$k^2 = \frac{\omega^2}{c_2^2} \quad (2.13b)$$

The wave speeds  $c_1$  and  $c_2$  are given by

$$c_{1,2}^2 = \frac{2(A_1 A_4 - A_2 A_3)}{(A_1 J + A_4 m) \pm \sqrt{(A_1 J - A_4 m)^2 + 4mJA_2 A_3}} \quad (2.13c)$$

while the ratios  $R_1$  and  $R_2$  of torsional to extensional oscillations are obtained from equation (2.10a) as

$$R_{1,2} = \frac{\theta}{u} = \frac{(mc_{1,2}^2 - A_1)}{A_2} \quad (2.13d)$$

By substituting eqs (2.13a) through (2.14b) into eqs. (2.9), the solution for extensional-torsional oscillations of wire rope becomes

$$\hat{u}(z, \omega) = U_1 e^{i\frac{\omega}{c_1} z} + U_2 e^{-i\frac{\omega}{c_1} z} + U_3 e^{i\frac{\omega}{c_1} z} + U_4 e^{-i\frac{\omega}{c_1} z} \quad (2.14a)$$

$$\hat{\theta}(z, \omega) = R_1 \left[ U_1 e^{i\frac{\omega}{c_1} z} + U_2 e^{-i\frac{\omega}{c_1} z} \right] + R_2 \left[ U_3 e^{i\frac{\omega}{c_1} z} + U_4 e^{-i\frac{\omega}{c_1} z} \right] \quad (2.14b)$$

where  $U_1$  through  $U_4$  are arbitrary constants. By taking

$$k_1 = \frac{\omega}{c_1} \quad (2.15a)$$

$$k_2 = \frac{\omega}{c_2} \quad (2.15b)$$

and using the fact that the solution of a differential equation which is in the form of a linear combination of exponential functions can be written as a linear combination of trigonometric functions, we can write eqs. (2.14) as

$$\hat{u}(z, \omega) = U_1 \sin(k_1 z) + U_2 \sin(k_1(L-z)) + U_3 \sin(k_2 z) + U_4 \sin(k_2(L-z)) \quad (2.16a)$$

$$\hat{\theta}(z, \omega) = R_1 [U_1 \sin(k_1 z) + U_2 \sin(k_1(L-z))] + R_2 [U_3 \sin(k_2 z) + U_4 \sin(k_2(L-z))] \quad (2.16b)$$

Thus, in effect, there are two waves: a predominantly extensional wave traveling at a speed  $c_1$ , and a predominantly torsional wave traveling at a speed  $c_2$ . Next, by taking the boundary conditions as

$$u(0) = \hat{u}_1 \quad (2.17a)$$

$$u(L) = \hat{u}_2 \quad (2.17b)$$

$$\theta(0) = \hat{\theta}_1 \quad (2.17c)$$

$$\theta(L) = \hat{\theta}_2 \quad (2.17d)$$

and substituting eqs.(2.17) into eqs. (2.16) we find

$$\begin{bmatrix} 0 & \sin(k_1 L) & 0 & \sin(k_2 L) \\ \sin(k_1 L) & 0 & \sin(k_2 L) & 0 \\ 0 & R_1 \sin(k_1 L) & 0 & R_2 \sin(k_2 L) \\ R_1 \sin(k_1 L) & 0 & R_2 \sin(k_2 L) & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} \hat{u}_1 \\ \hat{u}_2 \\ \hat{\theta}_1 \\ \hat{\theta}_2 \end{bmatrix} \quad (2.18)$$

with the constants in terms of nodal deformations being

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \frac{1}{(R_2 - R_1)} \begin{bmatrix} (\hat{u}_2 R_2 - \hat{\theta}_2) \csc(k_1 l) \\ (\hat{u}_1 R_2 - \hat{\theta}_1) \csc(k_1 l) \\ -( \hat{u}_2 R_1 - \hat{\theta}_2) \csc(k_2 l) \\ -( \hat{u}_1 R_1 - \hat{\theta}_1) \csc(k_2 l) \end{bmatrix} \quad (2.19)$$

Now we differentiate eqs. (2.16) with respect to  $z$  to get

$$\hat{\epsilon} = \frac{\partial \hat{u}}{\partial z} = U_1 k_1 \cos(k_1 z) - U_2 k_1 \cos(k_1(L-z)) + U_3 k_2 \cos(k_2 z) - U_4 k_2 \cos(k_2(L-z)) \quad (2.20a)$$

$$\hat{\beta} = \frac{\partial \hat{\theta}}{\partial z} = R_1 [U_1 k_1 \cos(k_1 z) - U_2 k_1 \cos(k_1(L-z))] + R_2 [U_3 k_2 \cos(k_2 z) - U_4 k_2 \cos(k_2(L-z))] \quad (2.20b)$$

using eqs. (2.1),  $F$  and  $M$  can be written as dynamic quantities as

$$\hat{F} = A_1 \hat{\epsilon} + A_2 \hat{\beta} \quad (2.21a)$$

$$\hat{M} = A_3 \hat{\epsilon} + A_4 \hat{\beta} \quad (2.21b)$$

The periodic boundary conditions for the dynamic quantities are set as

$$\hat{F}_1 = -\hat{F}(0) \quad (2.22a)$$

$$\hat{F}_2 = \hat{F}(L) \quad (2.22b)$$

$$\hat{M}_1 = -\hat{M}(0) \quad (2.22c)$$

$$\hat{M}_2 = \hat{M}(L) \quad (2.22d)$$

Using eqs. (2.20) and (2.21a)  $\hat{F}$  will be

$$\begin{aligned} \hat{F} = & A_1 \left[ U_1 k_1 \cos(k_1 z) - U_2 k_1 \cos(k_1(L-z)) + U_3 k_2 \cos(k_2 z) - U_4 k_2 \cos(k_2(L-z)) \right] \\ & + A_2 \left[ R_1 (U_1 k_1 \cos(k_1 z) - U_2 k_1 \cos(k_1(L-z))) + R_2 (U_3 k_2 \cos(k_2 z) - U_4 k_2 \cos(k_2(L-z))) \right] \end{aligned} \quad (2.23)$$

By putting  $z=0$  in eq. (2.23) the first B.C. is satisfied and  $\hat{F}_1$  will be (note that  $U_1$  through  $U_4$  are replaced by eqs. (2.19))

$$\begin{aligned} \hat{F}_1 = -\hat{F}(0) = & -\frac{k_1(A_1 + A_2 R_1) \csc(k_1 L)}{(R_2 - R_1)} \left[ (\hat{u}_2 R_2 - \hat{\theta}_2) - (\hat{u}_1 R_2 - \hat{\theta}_1) \cos(k_1 L) \right] \\ & - \frac{k_2(A_1 + A_2 R_2) \csc(k_2 L)}{(R_2 - R_1)} \left[ (\hat{u}_2 R_1 - \hat{\theta}_2) - (\hat{u}_1 R_1 - \hat{\theta}_1) \cos(k_2 L) \right] \end{aligned} \quad (2.24)$$

To simplify the notations consider

$$E_1 = A_1 + A_2 R_1 \quad (2.25)$$

$$E_2 = A_1 + A_2 R_2 \quad (2.26)$$

Rearranging eq. (2.25)

$$\begin{aligned} \hat{F}_1 = & \frac{-1}{(R_2 - R_1)} \left[ \begin{aligned} & \{-k_1 R_2 E_1 \cot(k_1 L) + k_2 R_1 E_2 \cot(k_2 L)\} \hat{\mu}_1 + \{k_1 R_2 E_1 \csc(k_1 L) - k_2 R_1 E_2 \csc(k_2 L)\} \hat{\mu}_2 \\ & + \{k_1 E_1 \cot(k_1 L) - k_2 E_2 \cot(k_2 L)\} \hat{\theta}_1 + \{-k_1 E_1 \csc(k_1 L) + k_2 E_2 \csc(k_2 L)\} \hat{\theta}_2 \end{aligned} \right] \end{aligned} \quad (2.27)$$

The other boundary condition for  $\hat{F}$  is at  $z=L$  as

$$\begin{aligned} \hat{F}_2 = \hat{F}(L) = & A_1 [U_1 k_1 \cos(k_1 L) - U_2 k_1 + U_3 K_2 \cos(k_2 L) - U_4 k_2] \\ & + A_2 [R_1 (U_1 k_1 \cos(k_1 L) - U_2 k_1) + R_2 (U_3 K_2 \cos(k_2 L) - U_4 k_2)] \end{aligned} \quad (2.28)$$

By using eqs. (2.19), eq. (2.28) can be written

$$\begin{aligned} \hat{F}_2 = & \frac{k_1 (A_1 + A_2 R_1) \cos(k_1 L)}{(R_2 - R_1)} [(\hat{\mu}_2 R_2 - \hat{\theta}_2) \cos(k_1 L) - (\hat{\mu}_1 R_2 - \hat{\theta}_1)] \\ & + \frac{k_2 (A_1 + A_2 R_2) \csc(k_2 L)}{(R_2 - R_1)} [(-\hat{\mu}_2 R_1 + \hat{\theta}_2) \cos(k_2 L) - (-\hat{\mu}_1 R_1 + \hat{\theta}_1)] \end{aligned} \quad (2.29)$$

or

$$\hat{F}_2 = \frac{-1}{(R_2 - R_1)} \left[ \begin{aligned} & \{k_1 R_2 E_1 \csc(k_1 L) - k_2 R_1 E_2 \csc(k_2 L)\} \hat{u}_1 + \{-k_1 R_2 E_1 \cot(k_1 L) + k_2 R_1 E_2 \cot(k_2 L)\} \hat{u}_2 \\ & + \{-k_1 E_1 \csc(k_1 L) + k_2 E_2 \csc(k_2 L)\} \hat{\theta}_1 + \{k_1 E_1 \cot(k_1 L) - k_2 E_2 \cot(k_2 L)\} \hat{\theta}_2 \end{aligned} \right] \quad (2.30)$$

So far, the boundary conditions are satisfied for  $\hat{F}$ . In order to satisfy them for  $\hat{M}$ , let us look at the constitutive equations again but this time in frequency space

$$\hat{F} = A_1 \hat{\epsilon} + A_2 \hat{\beta} \quad (2.31a)$$

$$\hat{M} = A_3 \hat{\epsilon} + A_4 \hat{\beta} \quad (2.31b)$$

Since  $\hat{\epsilon}$  and  $\hat{\beta}$  are equal in both eqs. (2.31a) and (2.31b), it seems that if we replace  $A_1$  and  $A_2$  and  $\hat{F}$  in eqs. (2.27) and (2.30) by  $A_3$  and  $A_4$  and  $\hat{M}$ , respectively, all the boundary conditions for  $\hat{M}$  are automatically satisfied. Therefore, using the following transformations

$$\begin{cases} A_1 \Rightarrow A_3 \\ A_2 \Rightarrow A_4 \end{cases} \rightarrow \begin{cases} A_1 + A_2 R_1 = E_1 \Rightarrow A_3 + A_4 R_1 = E_3 \\ A_1 + A_2 R_2 = E_2 \Rightarrow A_3 + A_4 R_2 = E_4 \end{cases} \quad (2.32)$$

One can easily find  $\hat{M}_1$  and  $\hat{M}_2$  as follows

$$\hat{M}_1 = \frac{-1}{(R_2 - R_1)} \left[ \begin{aligned} & \{ -k_1 R_2 E_3 \cot(k_1 L) + k_2 R_1 E_4 \cot(k_2 L) \} \hat{u}_1 + \{ k_1 R_2 E_3 \csc(k_1 L) - k_2 R_1 E_4 \csc(k_2 L) \} \hat{u}_2 \\ & + \{ k_1 R_2 E_3 \csc(k_1 L) - k_2 R_1 E_4 \csc(k_2 L) \} \hat{\theta}_1 + \{ -k_1 E_3 \csc(k_1 L) + k_2 E_4 \csc(k_2 L) \} \hat{\theta}_2 \end{aligned} \right] \quad (2.33)$$

$$\hat{M}_2 = \frac{-1}{(R_2 - R_1)} \left[ \begin{aligned} & \{ k_1 R_2 E_3 \csc(k_1 L) - k_2 R_1 E_4 \csc(k_2 L) \} \hat{u}_1 + \{ -k_1 R_2 E_3 \cot(k_1 L) + k_2 R_1 E_4 \cot(k_2 L) \} \hat{u}_2 \\ & + \{ -k_1 E_3 \csc(k_1 L) + k_2 E_4 \csc(k_2 L) \} \hat{\theta}_1 + \{ k_1 E_3 \cot(k_1 L) - k_2 E_4 \cot(k_2 L) \} \hat{\theta}_2 \end{aligned} \right] \quad (2.34)$$

By putting eqs. (2.27), (2.30), (2.33), (2.34) in a matrix form, we determine the four-by-four spectral stiffness matrix of (2.7) as follows:

$$K = \frac{1}{R_2 - R_1} \begin{bmatrix} k_1 R_2 E_1 \cot(k_1 L) - k_2 R_1 E_2 \cot(k_2 L) & -k_1 R_2 E_3 \csc(k_1 L) + k_2 R_1 E_4 \csc(k_2 L) \\ -k_1 R_2 E_1 \csc(k_1 L) + k_2 R_1 E_2 \csc(k_2 L) & k_1 R_2 E_1 \cot(k_1 L) - k_2 R_1 E_2 \cot(k_2 L) \\ k_1 R_2 E_3 \cot(k_1 L) - k_2 R_1 E_4 \cot(k_2 L) & -k_1 R_2 E_3 \csc(k_1 L) + k_2 R_1 E_4 \csc(k_2 L) \\ -k_1 R_2 E_3 \csc(k_1 L) + k_2 R_1 E_4 \csc(k_2 L) & k_1 R_2 E_3 \cot(k_1 L) - k_2 R_1 E_4 \cot(k_2 L) \\ \\ -k_1 E_1 \cot(k_1 L) + k_2 E_2 \cot(k_2 L) & k_1 E_1 \csc(k_1 L) - k_2 E_2 \csc(k_2 L) \\ k_1 E_1 \csc(k_1 L) - k_2 E_2 \csc(k_2 L) & -k_1 E_1 \cot(k_1 L) + k_2 E_2 \cot(k_2 L) \\ -k_1 E_3 \cot(k_1 L) + k_2 E_4 \cot(k_2 L) & -k_1 R_2 E_3 \csc(k_1 L) + k_2 R_1 E_4 \csc(k_2 L) \\ k_1 E_3 \csc(k_1 L) - k_2 E_4 \csc(k_2 L) & -k_1 E_3 \cot(k_1 L) + k_2 E_4 \cot(k_2 L) \end{bmatrix} \quad (2.35)$$

Symmetry of the stiffness matrix requires that, for example  $k_{41} = k_{14}$ , and so, by comparing these components, we note that

$$R_2 E_3 = -E_1 \quad \Rightarrow \quad R_2(A_3 + A_4 R_1) = -(A_1 + A_2 R_1) \quad (2.36a)$$

$$R_1 E_4 = -E_2 \quad \Rightarrow \quad R_1(A_2 + A_4 R_2) = -(A_1 + A_2 R_2) \quad (2.36b)$$

One can easily verify that eqs. (2.19) holds for arbitrary values of  $A_1$  through  $A_4$ .

## Results

- With the above formulas, in Fig. 2.2 we now plot all four coefficients  $k_{11}$ ,  $k_{12}$ ,  $k_{13}$ ,  $k_{14}$  in function of the helical coupling parameter  $A_2 = A_3$  at one frequency:  $\omega = 40$  KHz. Note that these coefficients cover all the distinct dependencies of the spectral stiffness matrix on  $\omega$ . The progressively denser location of ‘hills’ and ‘valleys’ with the coupling increasing – indicative of the occurrence of resonance – is evident here. To display the entire dependence of  $k_{11}$  through  $k_{14}$  on  $A_2 = A_3$  and  $\omega$ , we next plot Figs. 2.3, 2.4, 2.5, and 2.6. We observe here the wavy-surface, albeit somewhat irregular, character of all these plots, and the tendency to always (i.e. for any  $\omega$ ) ‘densify’ the resonances of both axial and torsional motions as the coupling increases.
- Taking  $A_2 = A_3 = 0$  (absence of the coupling effect) and partitioning the four-by-four spectral stiffness matrix into four two-by-two sub-matrices as

$$K = \begin{bmatrix} \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} & \begin{bmatrix} k_{13} & k_{14} \\ k_{23} & k_{24} \end{bmatrix} \\ \begin{bmatrix} k_{31} & k_{32} \\ k_{41} & k_{42} \end{bmatrix} & \begin{bmatrix} k_{33} & k_{34} \\ k_{43} & k_{44} \end{bmatrix} \end{bmatrix},$$

we see that the first and second sub-matrices on the diagonal represent the spectral stiffness matrices of a straight bar under a pure axial and torsional load, respectively, with the two off-diagonal sub-matrices being zero identically. By increasing  $A_2$  from zero up, the two sub-matrices on the diagonal get modified and the two off-diagonal sub-matrices (equal to each other due to the already mentioned symmetry) come into existence.

Results of this chapter are published in (Shahsavari and Ostoja-Starzewski, 2004)

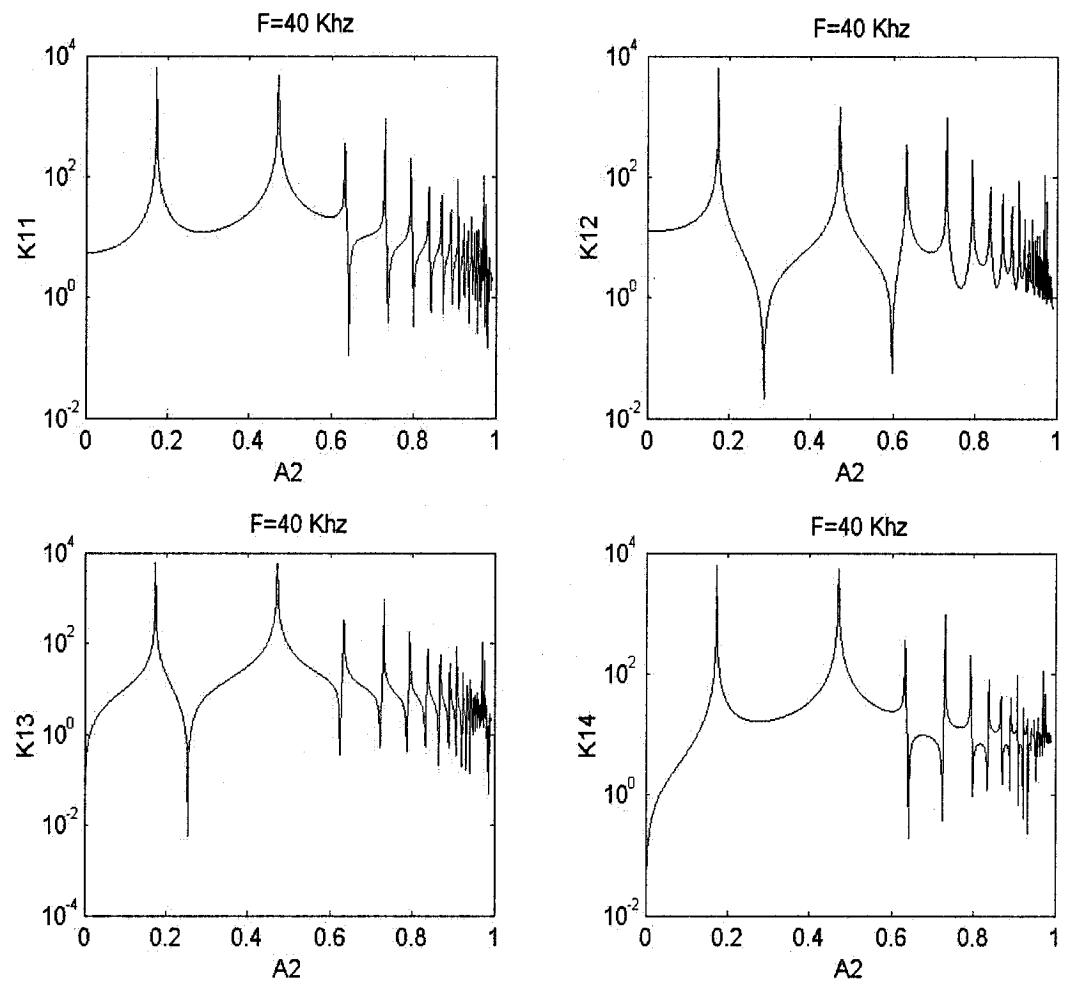


Fig. 2.2  
Dependence of spectral stiffnesses  $k_{11}$ ,  $k_{12}$ ,  $k_{13}$ , and  $k_{14}$  on the axial-torsional coupling.

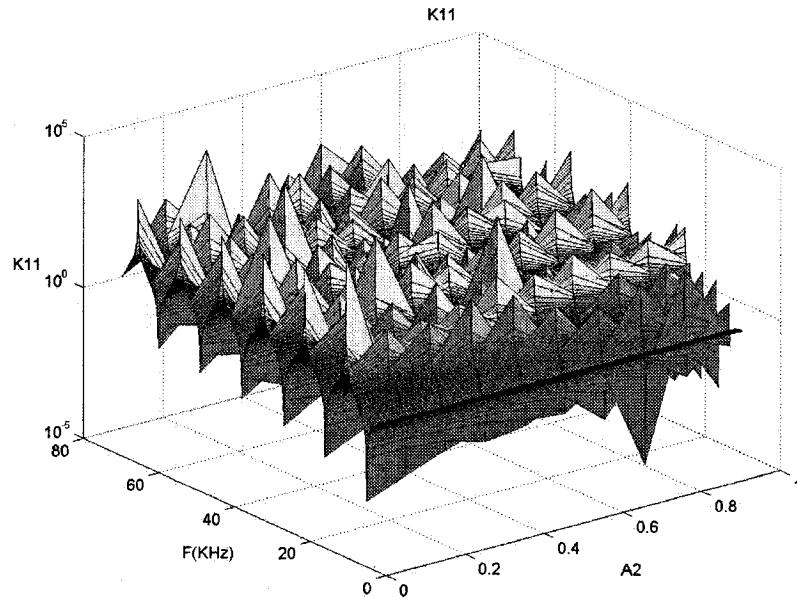


Fig. 2.3  
Dependence of the spectral stiffness  $k_{11}$  on frequency and axial-torsional coupling.

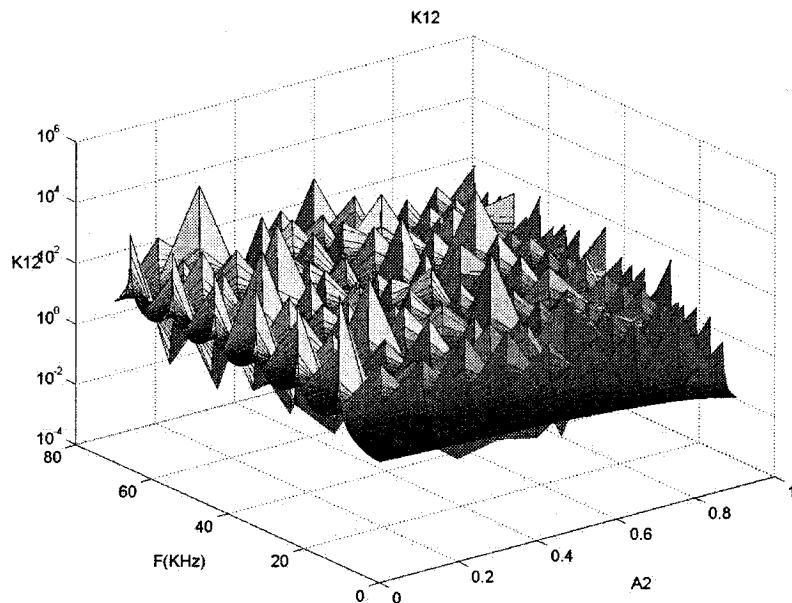
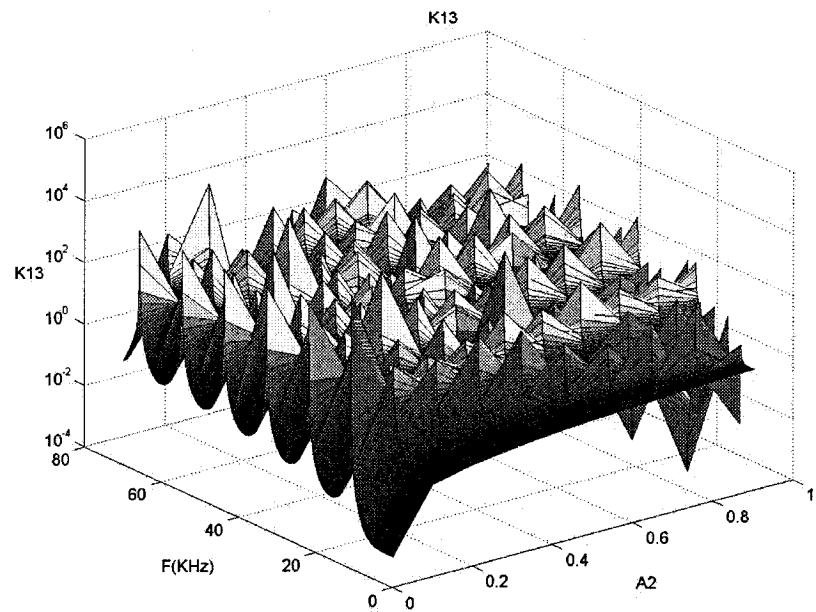
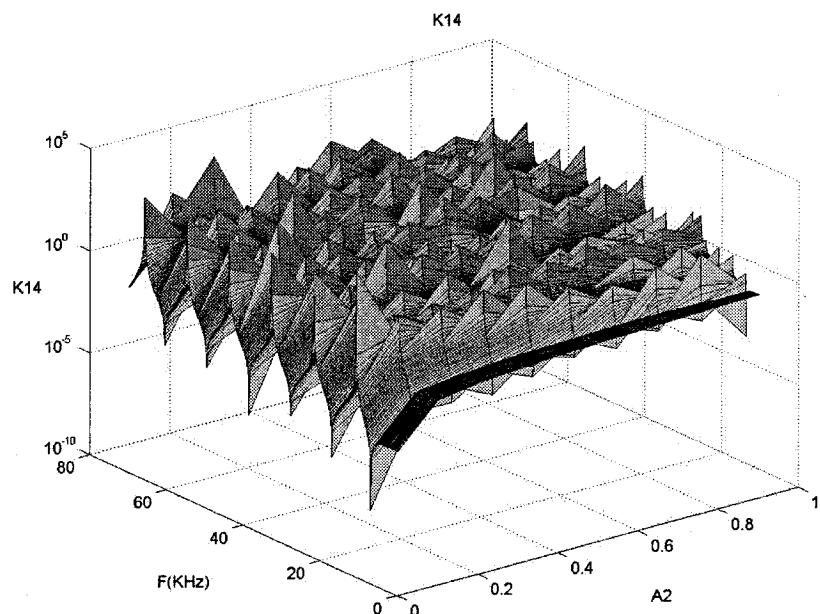


Fig. 2.4  
Dependence of the spectral stiffness  $k_{12}$  on frequency and axial-torsional coupling.



**Fig. 2.5**  
Dependence of the spectral stiffness  $k_{13}$  on frequency and axial-torsional coupling.



**Fig. 2.6**  
Dependence of the spectral stiffness  $k_{14}$  on frequency and axial-torsional coupling.

### 3 CONSTITUTIVE COEFFICIENTS OF AN ELASTIC HELIX

An exact analytical determination of the mechanical behavior of a helical elastic bundle is very difficult if not impossible. Various approximations and assumptions have been made to make an analytical solution more tractable. Note that in this chapter we refer to the helix as a bundle consisting of a center strand and  $m$  outer helically wound strands.

#### 3.1 Explicit forms of the constitutive helix coefficients

We work under the following assumptions (Fig. 3.1):

- 1) The bundle is composed of  $m$  strands, uniformly spaced along the perimeter of a circle of radius  $r_2 = R_1 + R_2$  thus forming a ring in a plane perpendicular to the fiber axis without touching each other.
- 2) Each strand's equilibrium configuration is a helix of constant radius  $r_2 = R_1 + R_2$  and constant helix angle  $\alpha_2$  (the subscript 2 refers to the deformed configuration).
- 3) Strands are linear elastic (with a given axial modulus,  $E$ , and Poisson's ratio) and undergo very small strains.
- 4) Friction and contact deformations are neglected.

The axial strain of a straight fiber is defined as

$$\varepsilon = \frac{\bar{h} - h}{h} \quad (3.1)$$

where  $h$  and  $\bar{h}$  are the original and final lengths of the strand, respectively, Fig. 3.1. The rotational strain of the strand is defined as

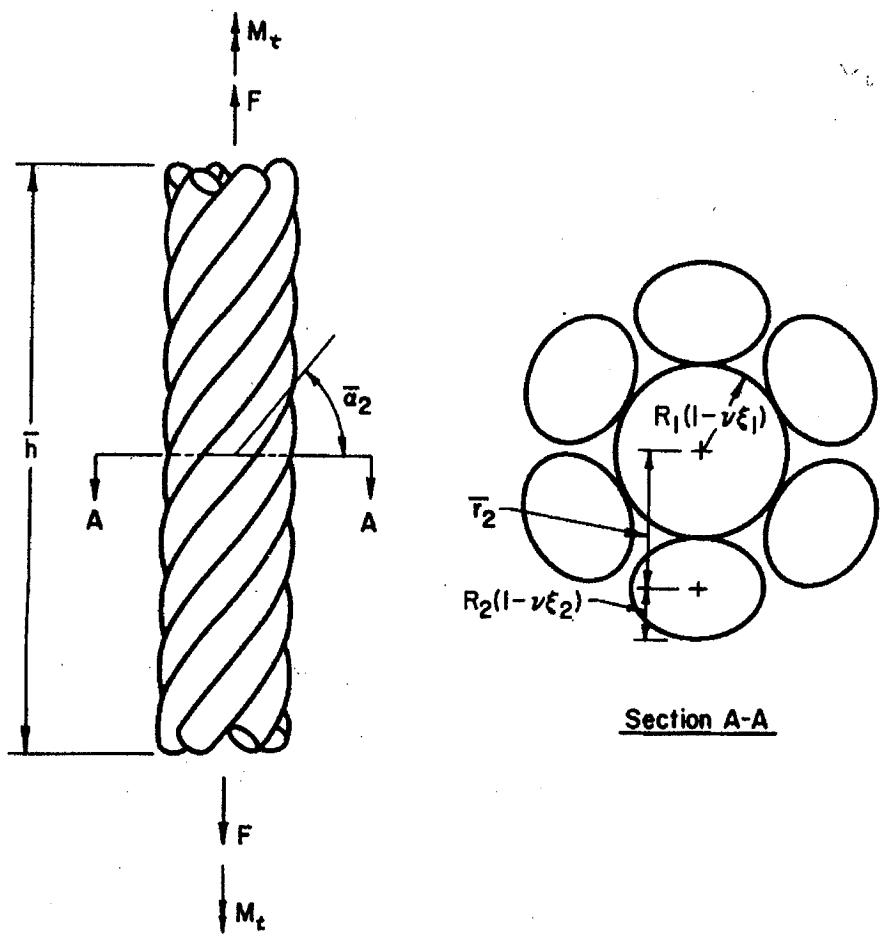


Fig. 3.1. Loaded simple strand (after Costello, 1997).

$$\beta_2 = r_2 \frac{\bar{\theta} - \theta}{h} \quad (3.2)$$

where  $\theta$  and  $\bar{\theta}_2$  are the initial and final angles that a strand sweeps out in a plane perpendicular to the strand's axis, respectively. Finally, the angle of twist per unit length of the strand is defined as (note  $\beta_2 = r_2\tau_2$ )

$$\tau_2 = \frac{\bar{\theta} - \theta}{h} \quad (3.3)$$

The constitutive equations of a helix can be written as

$$\frac{F}{AE} = C_1 \varepsilon + C_2 \tau$$

$$\frac{M}{ER^3} = C_3 \varepsilon + C_4 \tau \quad (3.4a,b)$$

where

$$F = F_1 + F_2 \quad (3.5a)$$

$$M = M_1 + M_2 \quad (3.5b)$$

$$R = R_1 + 2R_2 \quad (3.6)$$

and  $F_1$  and  $M_1$  are the contributions of the center strand and  $F_2$  and  $M_2$  are the contributions of the outer strands. Note that eqs. (3.4a,b) are the same constitutive equations as in chapter two (eqs.(2.1a,b)) except that here  $F$  and  $M$  are divided by some constants for convenience, and that is why we changed all  $A$ -coefficients to  $C$ -coefficients.

The following formulas can be derived from the equilibrium equations (Costello 1990)

$$\frac{F_1}{ER_1^2} = \pi \varepsilon_1 \quad (3.7a)$$

$$\frac{M_1}{ER_1^3} = \frac{\pi}{4(1+v)} R_1 \tau_s \quad (3.7b)$$

$$\frac{F_2}{ER_2^2} = m \left[ \frac{T_2}{ER_2^2} \sin \alpha_2 + \frac{N'_2}{ER_2^2} \cos \alpha_2 \right] \quad (3.8a)$$

$$\frac{M_2}{ER_2^3} = m \left[ \frac{H_2}{ER_2^3} \sin \alpha_2 + \frac{G'_2}{ER_2^3} \cos \alpha_2 + \frac{T_2}{ER_2^2} \frac{r_2}{R_2} \cos \alpha_2 - \frac{N'_2}{ER_2^2} \frac{r_2}{R_2} \sin \alpha_2 \right] \quad (3.8b)$$

where  $N'_2$  is the components of the shearing force on a strand cross section;  $T_2$  is the axial tension in the strand;  $G'_2$  is the component of the bending moment on a strand cross section;  $H$  is the twisting moment in the strand and their subscript 2 refers to the outer strands and the prime symbol means perpendicular to the axis of the strand.

In order to find  $C$  coefficients, first assume  $\beta = R\tau_s = 0$  and  $\varepsilon = \varepsilon_1$  and substituting eq. (3.7) and (3.8) into (3.4) one may obtain:

$$\frac{F}{AE} = \frac{1}{AE} (\pi\varepsilon_1 ER_1^2 + m[T_2 \sin \alpha_2 + N'_2 \cos \alpha_2]) = C_1 \varepsilon_1 \quad (3.9a,b)$$

$$\frac{M}{ER^3} = 0 + \frac{m}{ER^3} [H_2 \sin \alpha_2 + G'_2 \cos \alpha_2 + T_2 r_2 \cos \alpha_2 - N'_2 r_2 \sin \alpha_2] = C_3 \varepsilon_1$$

therefore

$$C_1 = \frac{1}{AE} \left( \pi ER_1^2 + \frac{m}{\varepsilon_1} [T_2 \sin \alpha_2 + N'_2 \cos \alpha_2] \right) \quad (3.10)$$

$$C_3 = \frac{m}{ER^3 \varepsilon_1} [H_2 \sin \alpha_2 + G'_2 \cos \alpha_2 + T_2 r_2 \cos \alpha_2 - N'_2 r_2 \sin \alpha_2] \quad (3.11)$$

With analysis of the configuration of an outer strand, one can find the following formulas for an outer strand (Costello 1990)

$$\varepsilon_1 = \varepsilon_2 + \frac{\Delta \alpha_2}{\tan \alpha_2} \quad (3.12a,b)$$

$$\beta_2 = r_2 \tau_s = \frac{\varepsilon_2}{\tan \alpha_2} - \Delta \alpha_2 + \nu \frac{(R_1 \varepsilon_1 + R_2 \varepsilon_2)}{r_2 \tan \alpha_2}$$

where  $\varepsilon_1$  is the axial strain in the center strand ( $\varepsilon = \varepsilon_1$ ) and  $\varepsilon_2$  is the axial strain in an outer strand. Also, the following can be written down for an outer strand (Costello 1990):

$$G'_2 = \frac{\pi}{4} R_2 \Delta \kappa'_2 E R_2^3 \quad (3.13)$$

$$H_2 = \frac{\pi}{4(1+v)} R_2 \Delta \tau_2 E R_2^3 \quad (3.14)$$

$$N'_2 = \frac{H_2}{R_2} \frac{\cos^2 \alpha_2}{r_2/R_2} - \frac{G'_2}{R_2} \frac{\sin \alpha_2 \cos \alpha_2}{r_2/R_2} \quad (3.15)$$

$$T_2 = \pi \varepsilon_2 E R_2^2 \quad (3.16)$$

$$R_2 \Delta k'_2 = -\frac{2 \sin \alpha_2 \cos \alpha_2}{r_2/R_2} \Delta \alpha_2 + v \left( \frac{R_1 \varepsilon_1 + R_2 \varepsilon_2}{r_2} \right) \frac{\cos^2 \alpha_2}{r_2/R_2} \quad (3.17)$$

$$R_2 \Delta \tau_2 = \frac{(1 - 2 \sin^2 \alpha_2)}{r_2/R_2} \Delta \alpha_2 + v \left( \frac{R_1 \varepsilon_1 + R_2 \varepsilon_2}{r_2} \right) \frac{\sin \alpha_2 \cos \alpha_2}{r_2/R_2} \quad (3.18)$$

where  $k'_2$  is the curvature of the outer strand and  $\tau_2$  is the twist per unit length of the outer strand. Thus, using eq. (3.15),  $C_3$  can be rearranged as

$$C_3 = \frac{m}{\varepsilon_1 E R^3} [H_2 \sin^3 \alpha_2 + G'_2 \cos \alpha_2 (1 + \sin^2 \alpha_2) + T_2 r_2 \cos \alpha_2] \quad (3.19)$$

Considering the assumption  $\beta = R \tau_s = 0$  ( $\Rightarrow \tau_s = 0$ ) and  $\varepsilon = \varepsilon_1$ , eq. (3.12) becomes

$$\varepsilon_1 = \varepsilon_2 + \frac{\Delta \alpha_2}{\tan \alpha_2} \quad (3.20a,b)$$

$$0 = \frac{\varepsilon_2}{\tan \alpha_2} - \Delta \alpha_2 + v \frac{(R_1 \varepsilon_1 + R_2 \varepsilon_2)}{r_2 \tan \alpha_2}$$

Eq. (3.20a) and (3.20b) are two equations for the two unknowns  $\Delta \alpha_2$  and  $\varepsilon_2$ . Using  $\Delta \alpha_2 = (\varepsilon_1 - \varepsilon_2) \tan \alpha_2$  from eq. (3.20a), eq. (3.20b) becomes

$$0 = \varepsilon_2 \left( \frac{1}{\tan \alpha_2} + \tan \alpha_2 + \frac{\nu R_2}{r_2 \tan \alpha_2} \right) - \varepsilon_1 \tan \alpha_2 + \frac{\nu R_1 \varepsilon_1}{r_2 \tan \alpha_2} \quad (3.21)$$

or

$$\varepsilon_2 = \frac{\left( \tan \alpha_2 - \frac{\nu R_1}{r_2 \tan \alpha_2} \right) \varepsilon_1}{\left( \frac{1}{\tan \alpha_2} + \tan \alpha_2 + \frac{\nu R_2}{r_2 \tan \alpha_2} \right)} = \frac{(r_2 \tan^2 \alpha_2 - \nu R_1)}{r_2 + r_2 \tan^2 \alpha_2 + \nu R_2} \quad (3.22)$$

$$\Delta \alpha_2 = \left( 1 - \frac{(r_2 \tan^2 \alpha_2 - \nu R_1)}{r_2 + r_2 \tan^2 \alpha_2 + \nu R_2} \right) \varepsilon_1 \tan \alpha_2 = \left( \frac{r_2 + \nu R_2 + \nu R_1}{r_2 + r_2 \tan^2 \alpha_2 + \nu R_2} \right) \varepsilon_1 \tan \alpha_2$$

$$= \left( \frac{r_2(1+\nu)}{r_2 + r_2 \tan^2 \alpha_2 + \nu R_2} \right) \varepsilon_1 \tan \alpha_2 \quad (3.23)$$

Now using eq. (3.22) and eq. (3.23), eqs. (3.17) and (3.18) become

$$R_2 \Delta k'_2 = - \frac{2 \sin \alpha_2 \cos \alpha_2}{r_2/R_2} \left( \frac{r_2(1+\nu) \varepsilon_1 \tan \alpha_2}{r_2 + r_2 \tan^2 \alpha_2 + \nu R_2} \right) + \nu \frac{\left( R_1 \varepsilon_1 + R_2 \varepsilon_1 \frac{(r_2 \tan^2 \alpha_2 - \nu R_1)}{r_2 + r_2 \tan^2 \alpha_2 + \nu R_2} \right)}{r_2} \frac{\cos^2 \alpha_2}{r_2/R_2}$$

$$= \frac{-2 \sin^2 \alpha_2 (1+\nu) \varepsilon_1 R_2}{r_2 + r_2 \tan^2 \alpha_2 + \nu R_2} + \nu \varepsilon_1 R_2 \frac{(R_1(r_2 + r_2 \tan^2 \alpha_2 + \nu R_2) + R_2(r_2 \tan^2 \alpha_2 - \nu R_1))}{r_2^2(r_2 + r_2 \tan^2 \alpha_2 + \nu R_2)} \cos^2 \alpha_2$$

$$(3.24)$$

$$R_2 \Delta \tau_2 = \frac{(1-2 \sin^2 \alpha_2)}{r_2/R_2} \left( \frac{r_2(1+\nu) \varepsilon_1 \tan \alpha_2}{r_2 + r_2 \tan^2 \alpha_2 + \nu R_2} \right) + \nu \left( \frac{R_1 \varepsilon_1 + R_2 \frac{(r_2 \tan^2 \alpha_2 - \nu R_1)}{r_2 + r_2 \tan^2 \alpha_2 + \nu R_2} \varepsilon_1}{r_2} \right) \frac{\sin \alpha_2 \cos \alpha_2}{r_2/R_2}$$

$$= \tan \alpha_2 \frac{(1-2\sin^2 \alpha_2)}{r_2 + r_2 \tan^2 \alpha_2 + vR_2} R_2 (1+v) \epsilon_1 + v \epsilon_1 R_2 \left( \frac{R_1(r_2 + r_2 \tan^2 \alpha_2 + vR_2) + R_2(r_2 \tan^2 \alpha_2 - vR_1)}{r_2^2(r_2 + r_2 \tan^2 \alpha_2 + vR_2)} \right) \sin \alpha_2 \cos \alpha_2$$

(3.25)

By using eqs. (3.13) through (3.16) and eqs. (3.24) and (3.25),  $C_1$  and  $C_3$  can be easily found as follows:

$$C_1 = \frac{\pi R_1^2}{A} + \frac{m}{AE\epsilon_1} \left[ \begin{aligned} & \pi E R_2^2 \frac{(r_2 \tan^2 \alpha_2 - vR_1)}{r_2 + r_2 \tan^2 \alpha_2 + vR_2} \epsilon_1 \sin \alpha_2 - \frac{\pi E R_2^3 \sin \alpha_2 \cos^2 \alpha_2}{4r_2} \frac{(-2 \sin^2 \alpha_2 (1+v) \epsilon_1 R_2)}{(r_2 + r_2 \tan^2 \alpha_2 + vR_2)} \\ & + \frac{\pi}{4(1+v)} (E R_2^3) \frac{(1-2\sin^2 \alpha_2)}{r_2 + r_2 \tan^2 \alpha_2 + vR_2} R_2 (1+v) \epsilon_1 \tan \alpha_2 \\ & + v \epsilon_1 R_2 \sin \alpha_2 \cos \alpha_2 \left( \frac{R_1(r_2 + r_2 \tan^2 \alpha_2 + vR_2) + R_2(r_2 \tan^2 \alpha_2 - vR_1)}{r_2^2(r_2 + r_2 \tan^2 \alpha_2 + vR_2)} \right) \frac{\cos^3 \alpha_2}{r_2} \\ & + v \epsilon_1 R_2 \left( \cos^2 \alpha_2 \left( \frac{R_1(r_2 + r_2 \tan^2 \alpha_2 + vR_2) + R_2(r_2 \tan^2 \alpha_2 - vR_1)}{r_2^2(r_2 + r_2 \tan^2 \alpha_2 + vR_2)} \right) \right) \end{aligned} \right] \quad (3.26)$$

$$C_3 = \frac{m}{ER^3 \epsilon_1} \left[ \begin{aligned} & \frac{\pi E R_2^3}{4(1+v)} \left( \frac{(1-2\sin^2 \alpha_2) R_2 (1+v) \epsilon_1 \tan \alpha_2}{r_2 + r_2 \tan^2 \alpha_2 + vR_2} + \frac{\pi E R_2^3}{4} \frac{(-2 \sin^2 \alpha_2 (1+v) \epsilon_1 R_2)}{r_2 + r_2 \tan^2 \alpha_2 + vR_2} \right. \\ & \left. + v \epsilon_1 R_2 \left( \frac{R_1(r_2 + r_2 \tan^2 \alpha_2 + vR_2) + R_2(r_2 \tan^2 \alpha_2 - vR_1)}{r_2^2(r_2 + r_2 \tan^2 \alpha_2 + vR_2)} \right) \sin \alpha_2 \cos \alpha_2 \right) \sin^3 \alpha_2 \\ & + v \epsilon_1 R_2 \left( \frac{R_1(r_2 + r_2 \tan^2 \alpha_2 + vR_2) + R_2(r_2 \tan^2 \alpha_2 - vR_1)}{r_2^2(r_2 + r_2 \tan^2 \alpha_2 + vR_2)} \cos^2 \alpha_2 \right) \cos \alpha_2 (1 + \sin^2 \alpha_2) \\ & + \pi E R_2^2 \left( \frac{r_2 \tan^2 \alpha_2 - vR_1}{r_2 + r_2 \tan^2 \alpha_2 + vR_2} \right) \epsilon_1 r_2 \cos \alpha_2 \end{aligned} \right] \quad (3.27)$$

where  $A = \pi R_1^2 + m\pi R_2^2$  in eq. (3.26).

In order to find  $C_2$  and  $C_4$ , assume  $\varepsilon = 0$  and  $\beta = \beta_2 = r_2 \tau_s$ . With this eqs (12a,b) become

$$0 = \varepsilon_2 + \frac{\Delta\alpha_2}{\tan\alpha_2} \quad \Rightarrow \quad \Delta\alpha_2 = -\varepsilon_2 \tan\alpha_2 \quad (3.28)$$

$$\beta_2 = r_2 \tau_s = \frac{\varepsilon_2}{\tan\alpha_2} - \Delta\alpha_2 + v \frac{R_2 \varepsilon_2}{r_2 \tan\alpha_2} \quad (3.29)$$

$$\beta_2 = \left( \frac{1}{\tan\alpha_2} + \tan\alpha_2 + \frac{vR_2}{r_2 \tan\alpha_2} \right) \varepsilon_2 \quad (3.30)$$

Using similar procedures as before, one can easily find the following

$$\varepsilon_2 = \frac{\beta_2 r_2 \tan\alpha_2}{r_2 + r_2 \tan^2\alpha_2 + vR_2} \quad (3.31)$$

$$\Delta\alpha_2 = \frac{-\beta_2 r_2 \tan^2\alpha_2}{r_2 + r_2 \tan^2\alpha_2 + vR_2} \quad (3.32)$$

$$R_2 \Delta k'_2 = \frac{2 \sin\alpha_2 \cos\alpha_2}{r_2 + r_2 \tan^2\alpha_2 + vR_2} (\beta_2 R_2 \tan^2\alpha_2) + v \frac{R_2^2 \beta_2 \tan\alpha_2 \cos^2\alpha_2}{r_2 (r_2 + r_2 \tan^2\alpha_2 + vR_2)} \quad (3.33)$$

$$R_2 \Delta \tau_2 = \frac{-(1 - 2 \sin^2\alpha_2) \beta_2 R_2 \tan^2\alpha_2}{r_2 + r_2 \tan^2\alpha_2 + vR_2} + \frac{v R_2^2 \beta_2 \sin^2\alpha_2}{r_2 (r_2 + r_2 \tan^2\alpha_2 + vR_2)} \quad (3.34)$$

$$\frac{F}{AE} = 0 + \frac{m}{AE} [T_2 \sin\alpha_2 + N'_2 \cos\alpha_2] = C_2 \tau \quad (3.35)$$

where  $\beta = R \tau_s = (R_1 + 2R_2) \tau_s$

$$\frac{M}{ER^3} = \frac{\pi}{4(1+v)} ER_1^3 R_1 \tau_s + m [H_2 \sin\alpha_2 + G'_2 \cos\alpha_2 + T_2 r_2 \cos\alpha_2 - N'_2 r_2 \sin\alpha_2] = C_4 \tau \quad (3.36)$$

$$C_2 = \frac{m}{AE\beta} [T_2 \sin\alpha_2 + N'_2 \cos\alpha_2] \quad (3.37)$$

$$\begin{aligned}
C_4 &= \frac{1}{ER^3} \left( \frac{\pi}{4(1+v)} ER_1^3 \frac{R_1}{R} + \frac{m}{R\tau_s} [H_2 \sin \alpha_2 + G'_2 \cos \alpha_2 + T_2 r_2 \cos \alpha_2 - N'_2 r_2 \sin \alpha_2] \right) \\
&= \frac{1}{ER^3} \left( \frac{\pi}{4(1+v)} ER_1^3 \frac{R_1}{R} + \frac{m}{R\tau_s} [H_2 \sin^3 \alpha_2 + G'_2 \cos \alpha_2 (1 + \sin^2 \alpha_2) + T_2 r_2 \cos \alpha_2] \right)
\end{aligned} \tag{3.38}$$

$$C_2 = \frac{m}{AER \tau_s} \left[ \begin{aligned} &\left[ \pi ER_2^2 \frac{(\beta_2 r_2 \tan \alpha_2) \sin \alpha_2}{r_2 + r_2 \tan^2 \alpha_2 + vR_2} + \frac{\pi ER_2^3}{4(1+v)} \left( \frac{(2 \sin^2 \alpha_2 - 1) \beta_2 R_2 \tan^2 \alpha_2}{r_2 + r_2 \tan^2 \alpha_2 + vR_2} \right. \right. \\ &\left. \left. + \frac{vR_2^2 \beta_2 \sin^2 \alpha_2}{r_2(r_2 + r_2 \tan^2 \alpha_2 + vR_2)} \right) \frac{\cos^3 \alpha_2}{r_2} - \frac{\pi}{4} ER_2^3 \left( \frac{2 \sin \alpha_2 \cos \alpha_2 (\beta_2 R_2 \tan^2 \alpha_2)}{r_2 + r_2 \tan^2 \alpha_2 + vR_2} \right. \right. \\ &\left. \left. \left. + \frac{vR_2^2 \beta_2 \tan \alpha_2 \cos^2 \alpha_2}{r_2(r_2 + r_2 \tan^2 \alpha_2 + vR_2)} \right) \frac{\sin \alpha_2 \cos^2 \alpha_2}{r_2} \right] \end{aligned} \right] \tag{3.39}$$

$$C_4 = \frac{\pi}{4(1+v)} \frac{R_1^4}{R^4} + \frac{m}{R^4 \tau_s} \left[ \begin{aligned} &\left[ \frac{\pi}{4(1+v)} R_2^3 \left( \frac{(2 \sin^2 \alpha_2 - 1) \beta_2 R_2 \tan^2 \alpha_2}{r_2 + r_2 \tan^2 \alpha_2 + vR_2} + \frac{vR_2^2 \beta_2 \sin^2 \alpha_2}{r_2(r_2 + r_2 \tan^2 \alpha_2 + vR_2)} \right) \sin^3 \alpha_2 \right. \\ &\left. + \frac{\pi}{4} R_2^3 \left( \frac{2 \sin \alpha_2 \cos \alpha_2 (\beta_2 R_2 \tan^2 \alpha_2)}{r_2 + r_2 \tan^2 \alpha_2 + vR_2} + \frac{vR_2^2 \beta_2 \tan \alpha_2 \cos^2 \alpha_2}{r_2(r_2 + r_2 \tan^2 \alpha_2 + vR_2)} \right) \cos \alpha_2 (1 + \sin^2 \alpha_2) \right. \\ &\left. \left. + \frac{\pi R_2^2}{r_2 + r_2 \tan^2 \alpha_2 + vR_2} \frac{\beta_2 r_2^2 \sin \alpha_2}{r_2 + r_2 \tan^2 \alpha_2 + vR_2} \right] \end{aligned} \right] \tag{3.40}$$

Upon substituting  $\beta_2 = r_2 \tau_s$  in the above formulas and factoring out common terms, one can find  $C_1, C_2, C_3, C_4$  as:

$$C_1 = \frac{R_1^2}{(R_1^2 + mR_2^2)} + \frac{mR_2^2}{(R_1^2 + mR_2^2)(r_2 + r_2 \tan^2 \alpha_2 + vR_2)} \left[ \begin{aligned} & (r_2 \tan^2 \alpha_2 - vR_1) \sin \alpha_2 + \frac{R_2^2 (1 - 2 \sin^2 \alpha_2) \sin \alpha_2 \cos^2 \alpha_2}{4r_2} \\ & + \frac{R_2^2 \sin^3 \alpha_2 \cos^2 \alpha_2 (1+v)}{2r_2} - \frac{v^2 R_2^2 \sin \alpha_2 \cos^4 \alpha_2 (R_1 + r_2 \tan^2 \alpha_2)}{4(1+v)r_2^2} \end{aligned} \right] \quad (3.41)$$

$$C_2 = \frac{mr_2 R_2^2}{(R_1 + 2R_2)(R_1^2 + mR_2^2)(r_2 + r_2 \tan^2 \alpha_2 + vR_2)} \left[ \begin{aligned} & r_2 \tan \alpha_2 \sin \alpha_2 + \frac{R_2^2}{4(1+v)r_2} ((2 \sin^2 \alpha_2 - 1) \sin^2 \alpha_2 \cos \alpha_2) \\ & - \frac{R_2^2}{2r_2} \sin^4 \alpha_2 \cos \alpha_2 - \frac{v^2 R_2^3 \sin^2 \alpha_2 \cos^3 \alpha_2}{4r_2^2 (1+v)} \end{aligned} \right] \quad (3.42)$$

$$C_3 = \frac{m\pi R_2^2}{(R_1 + 2R_2)^3(r_2 + r_2 \tan^2 \alpha_2 + vR_2)} \left[ \begin{aligned} & \frac{(1 - 2 \sin^2 \alpha_2) R_2^2 \tan \alpha_2 \sin^3 \alpha_2}{4} + \frac{vR_2^2}{4(1+v)r_2} (R_1 + r_2 \tan^2 \alpha_2) \sin^4 \alpha_2 \cos \alpha_2 \\ & - \frac{(1+v)}{2} R_2^2 \sin^2 \alpha_2 \cos \alpha_2 (1 + \sin^2 \alpha_2) + \frac{vR_2^2}{4r_2} (R_1 + r_2 \tan^2 \alpha_2) (1 + \sin^2 \alpha_2) \cos^3 \alpha_2 \\ & + (r_2 \tan^2 \alpha_2 - vR_1) r_2 \cos \alpha_2 \end{aligned} \right] \quad (3.43)$$

$$C_4 = \frac{\pi R_1^4}{4(R_1 + 2R_2)^4(1+v)} + \frac{r_2 \pi m R_2^2}{(R_1 + 2R_2)^4(r_2 + r_2 \tan^2 \alpha_2 + vR_2)} \left[ \begin{aligned} & \frac{R_2^2 \sin^3 \alpha_2 \tan^2 \alpha_2 (2 \sin^2 \alpha_2 - 1)}{4(1+v)} + \frac{R_2^2 (1 + \sin^2 \alpha_2) \sin^3 \alpha_2}{2} \\ & \frac{vR_2^3 \sin^5 \alpha_2}{4(1+v)r_2} + \frac{vR_2^3 \sin \alpha_2 (1 + \sin^2 \alpha_2) \cos^2 \alpha_2}{4r_2} + r_2^2 \sin \alpha_2 \end{aligned} \right] \quad (3.44)$$

Note that eqs. (3.41) through (3.44) are in very general forms. By putting  $m=2$  and neglecting the first terms in  $C_1$  and  $C_4$  (which are the contributions of the core strand), one may obtain the constitutive coefficients of a double helix, like the DNA which consists of two helically wound strands.

### 3.2 Parametric studies of the constitutive helix coefficients

In this section we quantitatively assess the dependence of the helix' constitutive coefficients eqs (3.41-3.44) on its geometric and material properties. In what follows, one parameter will be chosen to vary at a time with all the other parameters being kept constant. Values of parameters are the same as in the example 3.1 in Costello (1997), that is: (for figures 3.3 to 3.5 Poisson's ratio  $\nu = 0.25$ )

$$R_1 = 0.103 \text{ in (or } 0.26 \text{ cm)}$$

$$R_2 = 0.101 \text{ in (or } 0.25 \text{ cm)}$$

$$m = 6$$

$$\alpha = 82.5106^\circ$$

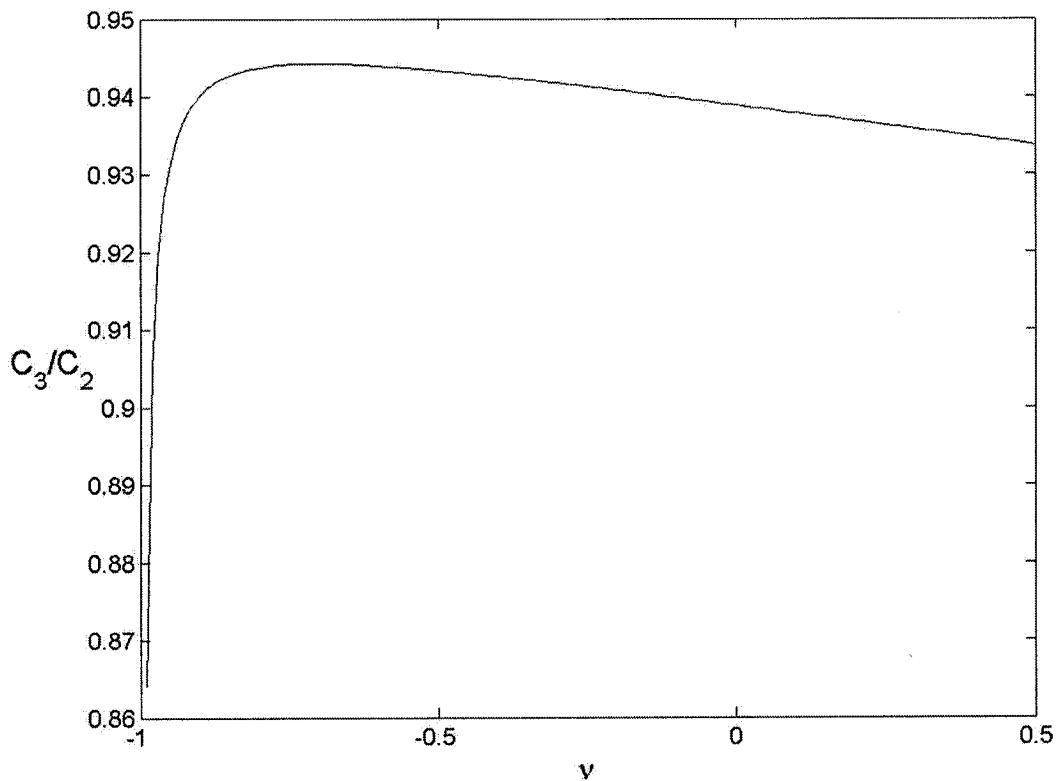


Fig. 3.2. Dependence of  $\frac{C_3}{C_2}$  on Poisson's ratio  $\nu$ .

First, the dependence of  $\frac{C_3}{C_2}$  on different parameters is derived. Note that to compare  $C_2$  and  $C_3$ , they have been multiplied by  $\pi(R_1^2 + mR_2^2)$  and  $(R_1 + 2R_2)^2$ , respectively. (See Appendix A).

Due to the reciprocity theorem of Maxwell-Betti,  $C_2$  and  $C_3$  are supposed to be ideally equal. But, as it turns out from Fig 3.2, because of the aforementioned assumptions (page 27), they are not exactly equal. However, for a certain range of the Poisson ratio, between -0.75 to 0.5, the ratio of  $\frac{C_3}{C_2}$  is 0.93 or higher, indicating a maximum error of 7%.

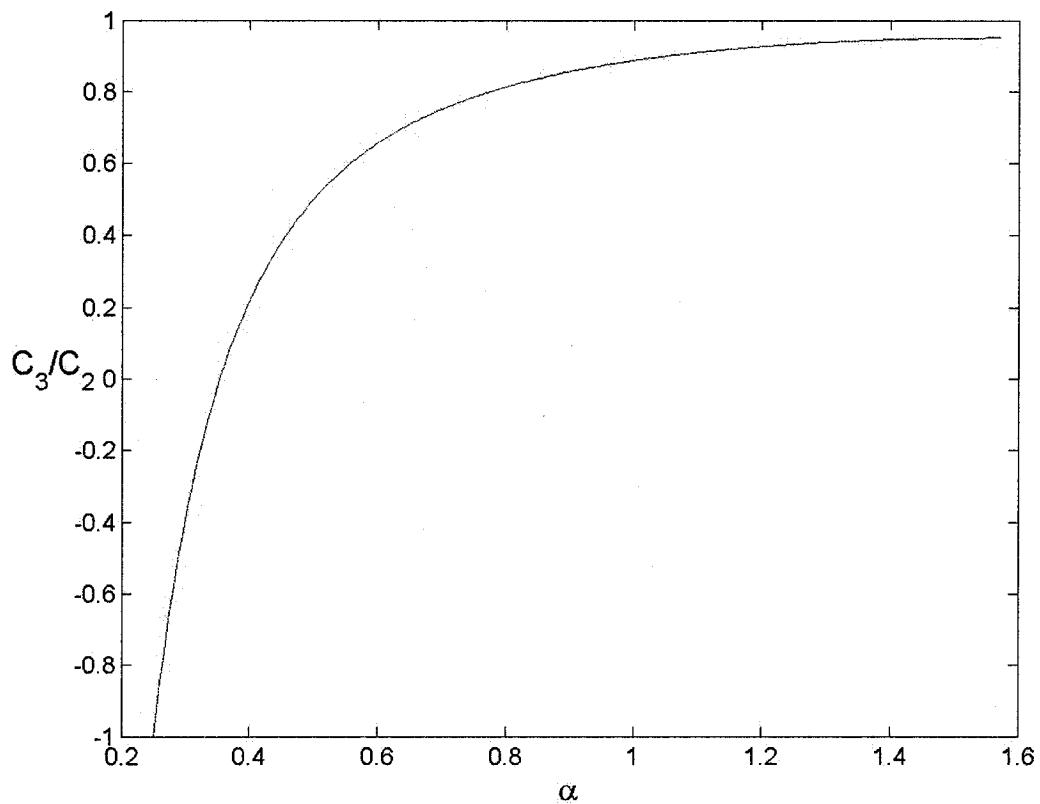


Fig. 3.3. Dependence of  $\frac{C_3}{C_2}$  on helix angle  $\alpha$  (in radians).

Fig. 3.3 shows that when  $\alpha$  is less than a certain value (about 0.4 radian)  $C_3$  is less than  $C_2$ , and it is larger elsewhere. Also when  $\alpha$  approaches  $\frac{\pi}{2}$  (no helical effect),  $C_2$  and  $C_3$  are getting closer (in fact, they are both approaching zero).

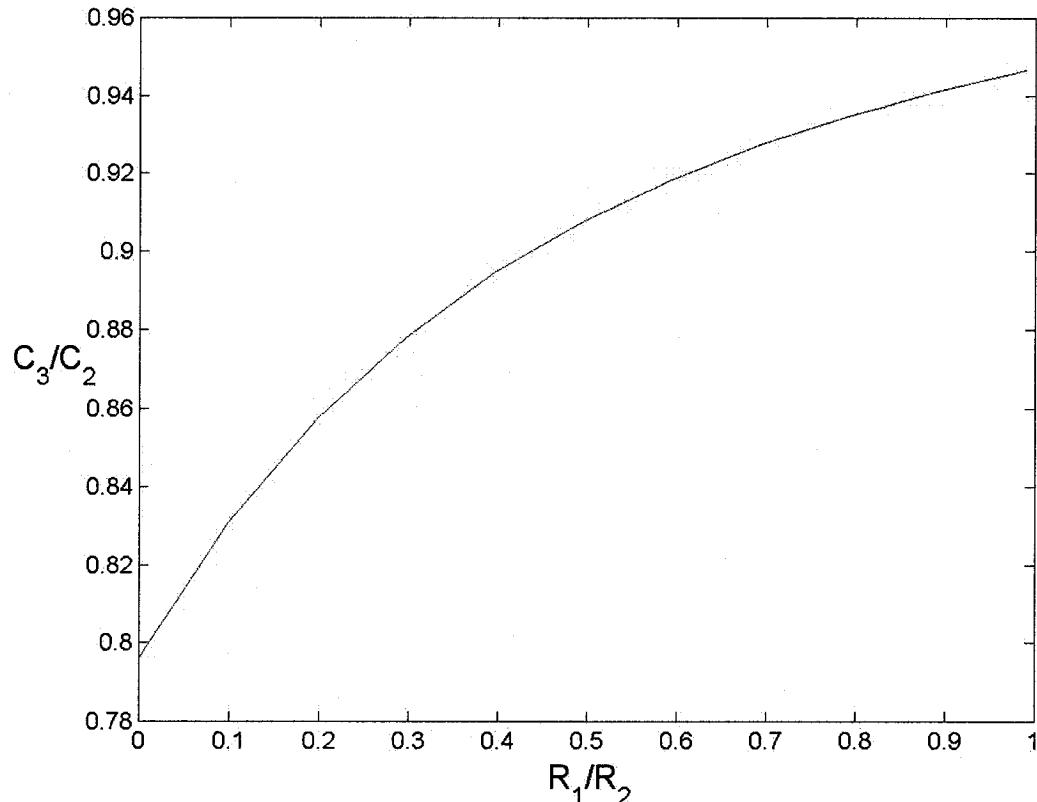


Fig. 3.4 Dependence of  $\frac{C_3}{C_2}$  on  $R_1$ .

Looking at Fig. 3.4, by increasing  $R_1$  (note that  $R_1$  is divided by a constant value  $R_2$  to make it dimensionless), the center strand becomes more dominant than the outer strands, and therefore the bundle is more like a straight bar than a helix. This is why the ratio of  $C_3/C_2$  approaches to unity. In the case of Fig 3.5, by increasing  $R_2$  (the radius of the

outer strands),  $\frac{C_3}{C_2}$  diverges from unity since this time the effect of the outer strands, or in other words the effect of the helix, is more dominant than that of the center strand.

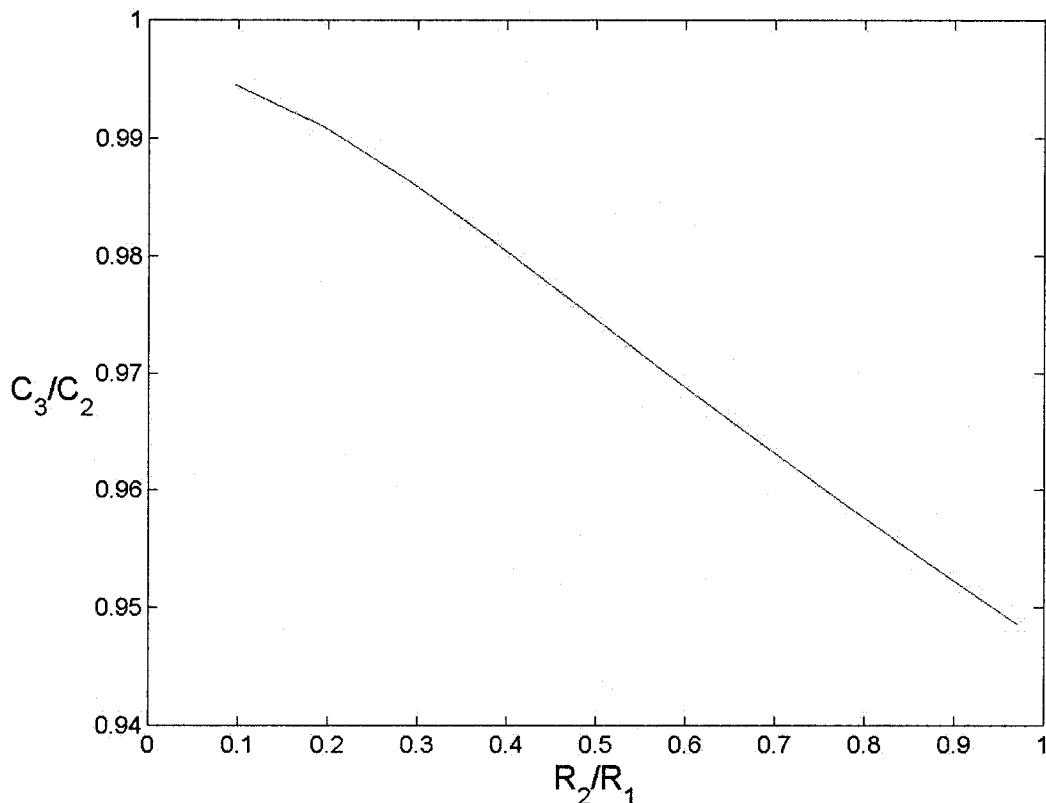


Fig. 3.5. Dependence of  $\frac{C_3}{C_2}$  on  $R_2$ .

Figure 3.6. shows the dependence of all helix coefficients on the helix angle  $\alpha$ , which is in terms of radian. Note that when  $\alpha$  approaches  $\pi/2$ , the following can be observed:

- 1) The bundle is a bundle of straight rods and the coupling coefficients  $C_2$  and  $C_3$  are zero.

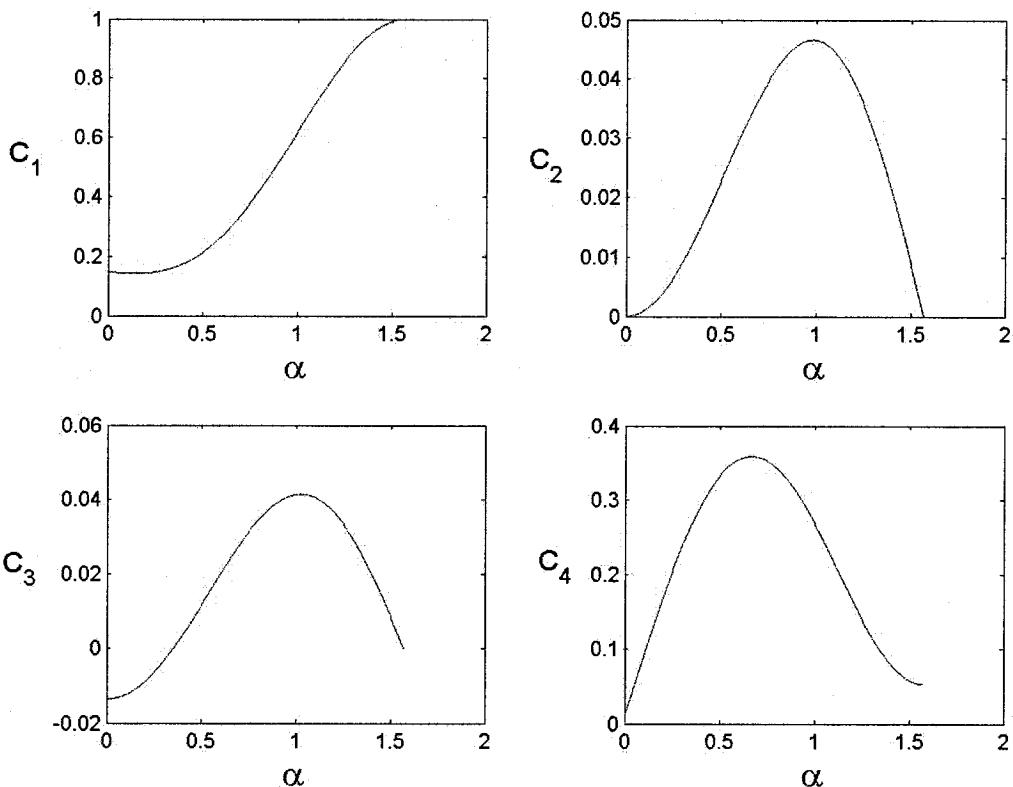


Fig. 3.6. Dependence of constitutive helix coefficients on the helix angle  $\alpha$ .

- 2)  $C_1$  approaches its maximum value since all the strands become straight (no reduction in the axial load carrying capacity).
- 3)  $C_4$  approaches the value proportional to shear modulus,  $G$ . However, its maximum occurs at an angle  $\alpha$  about 0.6 or 0.7 radian. This is because it is much easier to rotate a system of parallel strands than that of helically wound strands or, in other words, the torsional stiffnesses of curved beams are greater than those of straight beams.
- 4) The maximum effect of the coupling coefficients  $C_2$  and  $C_3$  occurs when  $\alpha$  is around 1 radian (or  $\alpha = 57^\circ$ ). This means that the greatest reduction in the axial capacity of the helical bundle occurs at this angle. Note that the helix angles, which maximize the helix coefficients, are identical for different constant parameters

The following graphs show further interesting results.

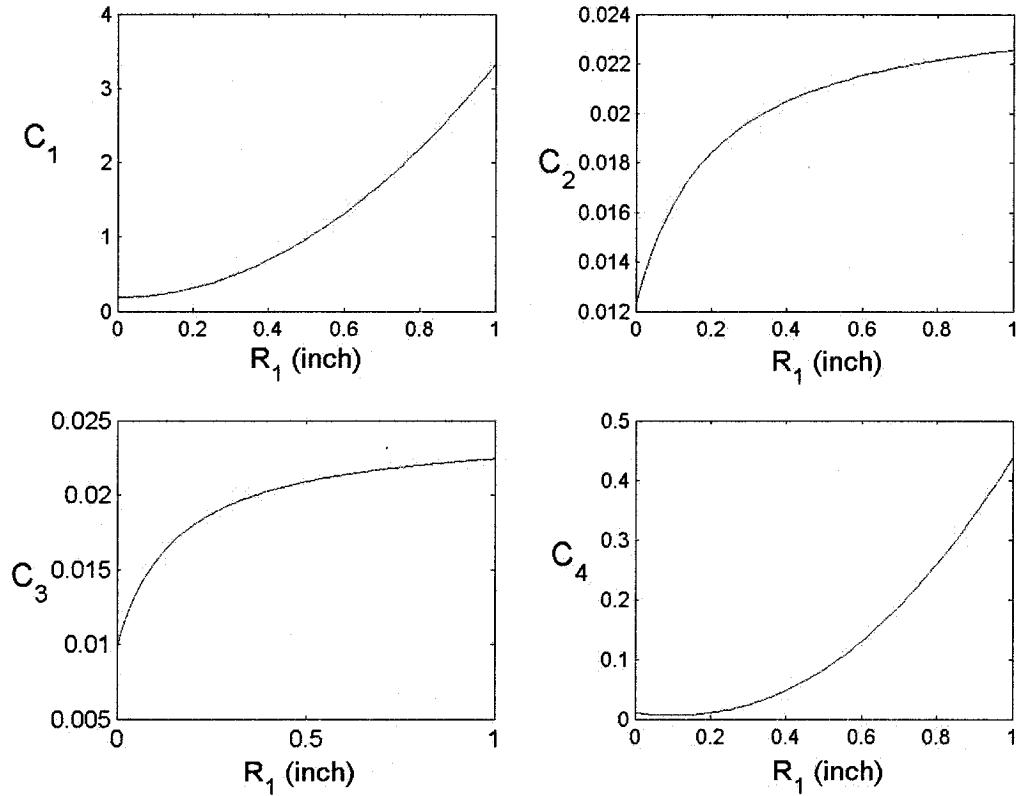


Fig. 3.7. Dependence of constitutive helix coefficients on  $R_1$ .

In Fig. 3.7 all the helix coefficients increase as the radius of the center strand increases. This makes sense since each of the coefficients can be considered as indicating stiffness, and by enlarging the radius of the center strand the area has been increased and therefore the bundle becomes stiffer. Note that the increasing behaviors of  $C_1$  and  $C_4$  are similar, and so are the increasing behaviors of  $C_2$  and  $C_3$ .

Figure 3.8 shows how the coefficients depend on Poisson's ratio. As it turns out, all the helix coefficients decrease with increasing Poisson's ratio. In order to explain this, it is enough to note that, by increasing Poisson's ratio, the strands become more deformable and flexible, and therefore less stiff.

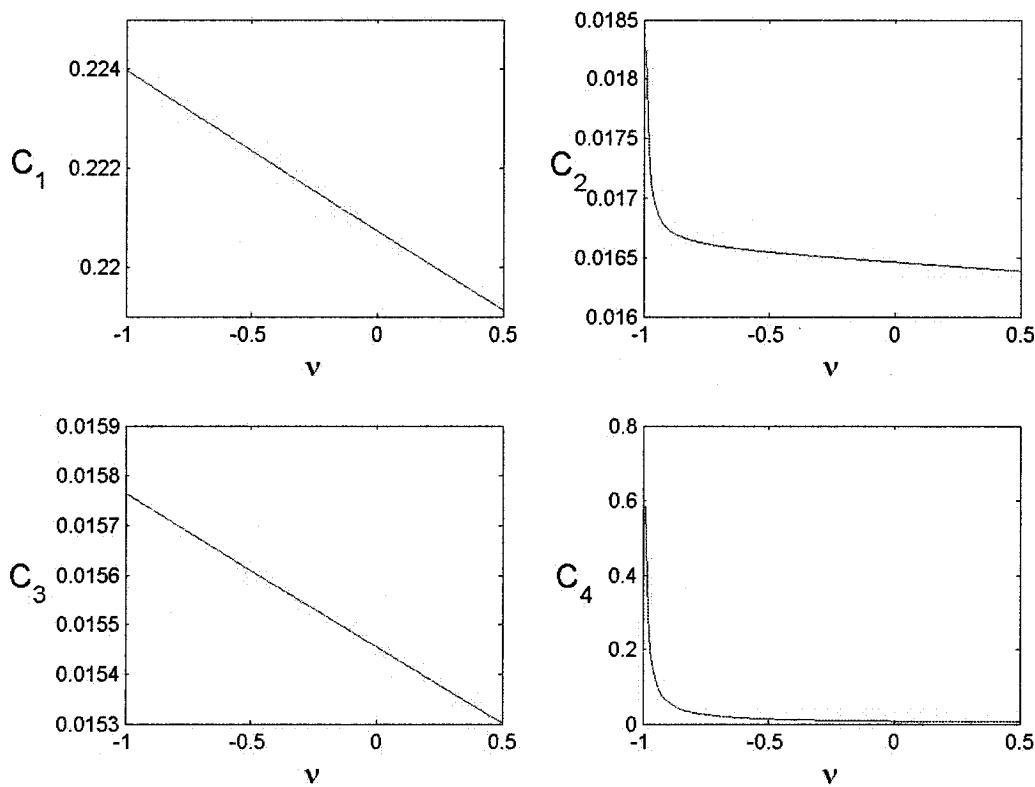


Fig. 3.8. Dependence of constitutive helix coefficients on Poisson 's ratio  $v$ .

## Results

- By making certain assumptions, all the constitutive coefficients of the helix have been derived explicitly.
- The ratio  $C_3/C_2$  is never equal to unity, though in some cases it is very close.
- $C_2$  and  $C_3$  are independent of the center strand and the only contribution of the center strand is the first terms in  $C_1$  and  $C_4$ .

- When  $\alpha$  approaches  $\frac{\pi}{2}$ ,  $C_1$  and  $C_4$  approach values proportional to  $E$  and  $G$  respectively and  $C_2$  and  $C_3$  approach zero. Also, the ratio  $\frac{C_3}{C_2}$  approaches unity as  $\alpha$  approaches  $\frac{\pi}{2}$ .
- The maximum value of  $C_1$  occurs at  $\alpha = \frac{\pi}{2}$  while the maximum values of  $C_2$  and  $C_3$  occur approximately when  $\alpha = 1$  radian. Also, the maximum value of  $C_4$  occurs when  $\alpha$  is about 0.6 or 0.7 radians.
- All the helix coefficients increase by increasing  $R_1$  and decrease by increasing  $v$ .

## 4 RESPONSE OF VISCOELASTIC HELICES

### 4.1 Viscoelastic Models

In general, the differential equation of any linear viscoelastic model can be written as

$$\sigma + P_1\dot{\sigma} + P_2\ddot{\sigma} + \dots = Q_0\varepsilon + Q_1\dot{\varepsilon} + Q_2\ddot{\varepsilon} + \dots \quad (4.1)$$

where  $\dot{\sigma}$  and  $\dot{\varepsilon}$  are the first time derivatives of the stress and strain, and more dots indicate corresponding higher derivatives. Parameters  $P_1, P_2, \dots, Q_0, Q_1, \dots$  are the constants of the model depending on the particular elastic moduli and viscosity coefficients.

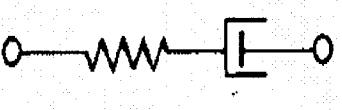
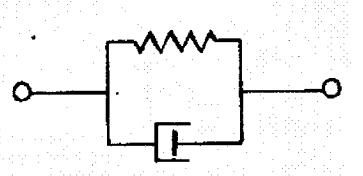
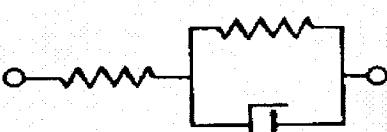
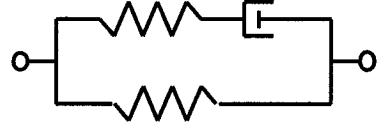
	Maxwell model	$\sigma + P_1\dot{\sigma} = Q_1\dot{\varepsilon}$
	Kelvin model	$\sigma = Q_0\varepsilon + Q_1\dot{\varepsilon}$
 Or 	Zener model (3-Parameter model)	$\sigma + P_1\dot{\sigma} = Q_0\varepsilon + Q_1\dot{\varepsilon}$

Fig. 4.1. Three viscoelastic models.

In Fig. 4.1 three types of common viscoelastic models and their differential equations are shown. It is worth mentioning that the Zener model (i.e, 3-Parameter model) is the simplest acceptable model of real viscoelastic solids (Ziegler, 1983).

There are two possible ways to study the viscoelastic response of a solid body and to find its differential equations:

1. Direct Method:

It is based on the superposition of stresses and strains in the springs and dashpots, and is a very useful technique for simple problems.

2. Correspondence Principle in Viscoelasticity:

A method for complex situations where the elastic solution is known.

Here, due to the complexity of the helix geometry, the second method has been chosen.

## 4.2 Correspondence Principle in Viscoelasticity

In general, any stress tensor can be split into two components: 1) the dilatation (spherical) stress tensor, 2) the shear (deviatoric) stress tensor. These two stress tensors have been used in the derivation of the correspondence principle. Thus in general, an isotropic material can be postulated as one viscoelastic model in dilatation and another (different) viscoelastic model in shear.

By taking the Laplace transform of the general differential equation of a viscoelastic model (eq. (4.1)) and assuming all initial boundary conditions are zero, one may obtain

$$\underbrace{(1 + P_1 s + P_2 s^2 + \dots)}_{P(s)} \bar{\sigma} = \underbrace{(Q_0 + Q_1 s + Q_2 s^2 + \dots)}_{Q(s)} \bar{\epsilon} \quad (4.2)$$

where the bar sign in  $\bar{\sigma}$  and  $\bar{\varepsilon}$  refers to the Laplace field.  $P(s)$  and  $Q(s)$ , which will be used in the correspondence principle, are the coefficients of  $\bar{\sigma}$  and  $\bar{\varepsilon}$ , respectively, and are polynomial functions of  $s$ .

The most general form of the correspondence principle for an isotropic material is as follows: If the solution of an elastic problem is known, the Laplace transform of the solution to the corresponding viscoelastic problem may be found by replacing the constants  $E$  and  $\nu$  according to the equations

$$E = \frac{3qQ}{2pQ+qP} \quad \nu = \frac{pQ-qP}{2pQ+qP} \quad (4.3a,b)$$

by quotients of operator polynomials, and the actual loads by their Laplace transforms.  $P$  and  $Q$  are the coefficients of  $\bar{\sigma}$  and  $\bar{\varepsilon}$  in the chosen viscoelastic model in dilatation, and  $p$  and  $q$  are those in shear. By using the correspondence principle and assuming certain viscoelastic models for the material at the micro level, one can find differential equations of the viscoelastic helix.

### 4.3 Differential equations of viscoelastic helix using Kelvin model

Using two different Kelvin models for dilatation (eq (4.4)) and shear (eq. (4.5)) deformations of the material at the micro level as

$$\sigma = Q_0\varepsilon + Q_1\dot{\varepsilon} \quad (4.4)$$

$$\sigma = q_0\varepsilon + q_1\dot{\varepsilon} \quad (4.5)$$

and taking the Laplace transforms of eqs. (4.4) and (4.5)

$$P(s) = 1 \quad (4.6)$$

$$Q(s) = Q_0 + Q_1 s \quad (4.7)$$

$$p(s) = 1 \quad (4.8)$$

$$q(s) = q_0 + q_1 s \quad (4.9)$$

Substituting eqs. (4.6) through (4.9) into the eqs. (4.2) and (4.3) and then substituting  $E$  and  $\nu$  into the elastic constitutive equations of the helix (eqs. (3.4a,b) in which  $C$  coefficients have explicit forms) and rearranging the terms, one can find the Laplace transforms of the desired differential equations as

$$(D_0 + D_1 s + D_2 s^2) \bar{\sigma} = (E_0 + E_1 s + E_2 s^2 + E_3 s^3) \bar{\epsilon} + (B_0 + B_1 s + B_2 s^2 + B_3 s^3) \bar{\tau} \quad (4.10)$$

$$(D'_0 + D'_1 s + D'_2 s^2) \bar{\mu} = (E'_0 + E'_1 s + E'_2 s^2 + E'_3 s^3) \bar{\epsilon} + (B'_0 + B'_1 s + B'_2 s^2 + B'_3 s^3) \bar{\tau} \quad (4.11)$$

where  $\sigma = \frac{F}{A}$ ,  $\mu = \frac{M}{A}$  and the bar sign refers to the Laplace field; all the constant coefficients,  $D's, Es, Bs$  and  $D's, E's, B's$ , are functions of the geometry and the Kelvin model parameters (See Appendix B for explicit forms of all the coefficients).

To find the governing differential equations it is sufficient to compare eqs. (4.10) and (4.11) to eqs (4.1) and (4.2) in order to easily obtain

$$D_0 \sigma + D_1 \sigma^{(1)} + D_2 \sigma^{(2)} = E_0 \epsilon + E_1 \epsilon^{(1)} + E_2 \epsilon^{(2)} + E_3 \epsilon^{(3)} + B_0 \tau + B_1 \tau^{(1)} + B_2 \tau^{(2)} + B_3 \tau^{(3)} \quad (4.12)$$

$$D'_0 \mu + D'_1 \mu^{(1)} + D'_2 \mu^{(2)} = E'_0 \epsilon + E'_1 \epsilon^{(1)} + E'_2 \epsilon^{(2)} + E'_3 \epsilon^{(3)} + B'_0 \tau + B'_1 \tau^{(1)} + B'_2 \tau^{(2)} + B'_3 \tau^{(3)} \quad (4.13)$$

where  $\sigma^{(1)}$  and  $\sigma^{(2)}$  are the first and second time derivatives of stress, ...etc. Eqs. (4.12) and (4.13) are the differential equations of a viscoelastic helix with  $\nu \neq 0$  for the material at the micro level. If  $\nu = 0$  (which is equivalent to saying that the two dilatation and distortion models are equal, since  $P(s)=p(s)$ ,  $Q(s)=q(s)$  and eqs. (4.3a,b) become

$E = \frac{q(s)}{p(s)}$  and  $\nu = 0$  ), then we will get much simpler forms of differential equations of this viscoelastic helix as

$$D_0\sigma = E_0\varepsilon + E_1\varepsilon^{(1)} + B_0\tau + B_1\tau^{(1)} \quad (4.14)$$

$$D'_0\mu = E'_0\varepsilon + E'_1\varepsilon^{(1)} + B'_0\tau + B'_1\tau^{(1)} \quad (4.15)$$

In this special case of  $\nu = 0$ , eqs (4.14) and (4.15) are similar in-type (analogous) to the Kelvin differential equations except that due to the coupling nature of the helix, cross terms are also added.

#### 4.4 Differential equations of viscoelastic helix using Maxwell model

By using two different Maxwell models for the material at the micro level as:

$$\sigma + P_1\dot{\sigma} = Q_1\dot{\varepsilon} \quad (4.16)$$

$$\sigma + p_1\dot{\sigma} = q_1\dot{\varepsilon} \quad (4.17)$$

and doing similar procedures like those for the Kelvin model mentioned in section 4.1.1, one can find the differential equations of a viscoelastic helix as (See Appendix B for the details):

$$D_2\sigma^{(2)} + D_3\sigma^{(3)} + D_4\sigma^{(4)} + D_5\sigma^{(5)} = E_3\varepsilon^{(3)} + E_4\varepsilon^{(4)} + E_5\varepsilon^{(5)} + B_3\tau^{(3)} + B_4\tau^{(4)} + B_5\tau^{(5)} \quad (4.18)$$

$$D'_2\mu^{(2)} + D'_3\mu^{(3)} + D'_4\mu^{(4)} + D'_5\mu^{(5)} = E'_3\varepsilon^{(3)} + E'_4\varepsilon^{(4)} + E'_5\varepsilon^{(5)} + B'_3\tau^{(3)} + B'_4\tau^{(4)} + B'_5\tau^{(5)} \quad (4.19)$$

Note that if  $\nu = 0$ , then we get much simpler forms of differential equations of this viscoelastic helix as

$$D_0\sigma + D_1\sigma^{(1)} = E_1\varepsilon^{(1)} + B_1\tau^{(1)} \quad (4.20)$$

$$D'_0\mu + D'_1\mu^{(1)} = E'_1\varepsilon^{(1)} + B'_1\tau^{(1)} \quad (4.21)$$

## 4.5 Differential equations of viscoelastic helix using Zener model

By using two different Zener models for the material at the micro level as:

$$\sigma + P_1\dot{\sigma} = Q_0\varepsilon + Q_1\dot{\varepsilon} \quad (4.22)$$

$$\sigma + p_1\dot{\sigma} = q_0\varepsilon + q_1\dot{\varepsilon} \quad (4.23)$$

and following similar procedures as those in section 4.1.1, one can find differential equations of the viscoelastic helix as:

$$\begin{aligned} D_0\sigma + D_1\sigma^{(1)} + D_2\sigma^{(2)} + D_3\sigma^{(3)} + D_4\sigma^{(4)} + D_5\sigma^{(5)} &= E_0\varepsilon + E_1\varepsilon^{(1)} + E_2\varepsilon^{(2)} + E_3\varepsilon^{(3)} + E_4\varepsilon^{(4)} + E_5\varepsilon^{(5)} \\ &\quad + B_0\tau + B_1\tau^{(1)} + B_2\tau^{(2)} + B_3\tau^{(3)} + B_4\tau^{(4)} + B_5\tau^{(5)} \end{aligned} \quad (4.24)$$

$$\begin{aligned} D'_0\mu + D'_1\mu^{(1)} + D'_2\mu^{(2)} + D'_3\mu^{(3)} + D'_4\mu^{(4)} + D'_5\mu^{(5)} &= E'_0\varepsilon + E'_1\varepsilon^{(1)} + E'_2\varepsilon^{(2)} + E'_3\varepsilon^{(3)} + E'_4\varepsilon^{(4)} + E'_5\varepsilon^{(5)} \\ &\quad + B'_0\tau + B'_1\tau^{(1)} + B'_2\tau^{(2)} + B'_3\tau^{(3)} + B'_4\tau^{(4)} + B'_5\tau^{(5)} \end{aligned} \quad (4.25)$$

If  $\nu = 0$ , then we get much simpler forms of the differential equations of this viscoelastic helix as

$$D_0\sigma + D_1\sigma^{(1)} = E_0\varepsilon + E_1\varepsilon^{(1)} + B_0\tau + B_1\tau^{(1)} \quad (4.26)$$

$$D'_0\mu + D'_1\mu^{(1)} = E'_0\varepsilon + E'_1\varepsilon^{(1)} + B'_0\tau + B'_1\tau^{(1)} \quad (4.27)$$

## 4.6 Creep and Relaxation Phases

Thus far, we have described the viscoelastic helix by its differential equations. To study the behavior of the viscoelastic helix, a standard test consisting of a creep and a relaxation phase has been used. In the relaxation phase constant strains are applied and the time dependent stresses are derived, and in the creep phase constant stresses are applied and time dependent strains are derived.

### 4.6.1 Relaxation phase

In the relaxation phase both  $\varepsilon$  and  $\tau$  are assumed to be constant. Therefore,

$$\varepsilon^{(1)} = \varepsilon^{(2)} = \dots = \tau^{(1)} = \tau^{(2)} = \dots = 0 \quad (4.28)$$

The differential equations of a viscoelastic helix using two Kelvin models (eqs. (4.12) and (4.13)) will simplify to

$$D_0\sigma + D_1\sigma^{(1)} + D_2\sigma^{(2)} = E_0\varepsilon + B_0\tau \quad (4.29)$$

$$D'_0\mu + D'_1\mu^{(1)} + D'_2\mu^{(2)} = E'_0\varepsilon + B'_0\tau \quad (4.30)$$

Eqs. (4.29) and (4.30) are two independent linear second order differential equations with constant coefficients and it is obvious that the solution is either of the form of exponential functions or hyperbolic functions. The solution for the Kelvin model, using Maple 9.0 Software, is found as (See Appendix C)

$$\sigma_{\text{Kelvin}} = \frac{(\varepsilon E_0 + \beta B_0)}{D_0} \left[ 1 + e^{\left(\frac{-D_1 t}{2D_2}\right)} \left( -\cosh\left(\frac{t\sqrt{D_1^2 - 4D_0D_2}}{2D_2}\right) - \frac{D_1 \sinh\left(\frac{t\sqrt{D_1^2 - 4D_0D_2}}{2D_2}\right)}{\sqrt{D_1^2 - 4D_0D_2}} \right) \right] \quad (4.31)$$

$$\mu_{\text{Kelvin}} = \frac{(\varepsilon E'_0 + \beta B'_0)}{D'_0} \left[ 1 + e^{\left( \frac{-D'_1 t}{2D'_2} \right)} \left( -\cosh \left( \frac{t\sqrt{D'^2 - 4D'_0 D'_2}}{2D'_2} \right) - \frac{D'_1 \sinh \left( \frac{t\sqrt{D'^2 - 4D'_0 D'_2}}{2D'_2} \right)}{\sqrt{D'^2 - 4D'_0 D'_2}} \right) \right] \quad (4.32)$$

The solution for the Zener models, with the help of a similar procedure is found as

$$\sigma_{\text{Zener}} = \frac{-(\varepsilon E'_0 + \beta B'_0)}{D'_0} \left[ \left( \sum_{i=1}^5 \frac{(D_5 \lambda_i^4 + D_4 \lambda_i^3 + D_3 \lambda_i^2 + D_2 \lambda_i + D_1)}{5D_5^4 \lambda_i^4 + 4D_4^3 \lambda_i^3 + 3D_3^2 \lambda_i^2 + 2D_2 \lambda_i + D_1} e^{\lambda_i t} \right) - 1 \right] \quad (4.33)$$

$$\mu_{\text{Zener}} = \frac{-(\varepsilon E'_0 + \beta B'_0)}{D'_0} \left[ \left( \sum_{i=1}^5 \frac{(D'_5 \lambda_i^4 + D'_4 \lambda_i^3 + D'_3 \lambda_i^2 + D'_2 \lambda_i + D'_1)}{5D'_5^4 \lambda_i^4 + 4D'_4^3 \lambda_i^3 + 3D'_3^2 \lambda_i^2 + 2D'_2 \lambda_i + D'_1} e^{\lambda_i t} \right) - 1 \right] \quad (4.34)$$

where in eq. (4.33)

$$\lambda_i = \text{Root of } (D_5 z^5 + D_4 z^4 + D_3 z^3 + D_2 z^2 + D_1 z + D_0) \quad (4.35)$$

and in eq. (4.34)

$$\lambda_i = \text{Root of } (D'_5 z^5 + D'_4 z^4 + D'_3 z^3 + D'_2 z^2 + D'_1 z + D'_0) \quad (4.36)$$

The relaxation response of the Maxwell model may be found as a special case of that of the Zener model above.

In general, in the relaxation phase of the viscoelastic helix, since both strains  $\varepsilon$  and  $\tau$  are constant and stresses are not coupled, each of the differential equations can be

independently solved to find the time dependent stresses. In the creep phase however, as we will see in the next section, coupled differential equations should be solved to find the time dependent strains.

#### 4.6.2 Creep Phase

In the creep phase, as mentioned earlier, both stresses  $\sigma$  and  $\mu$  are assumed to be constant. Therefore,

$$\sigma^{(1)} = \sigma^{(2)} = \dots = \mu^{(1)} = \mu^{(2)} = \dots = 0 \quad (4.37)$$

and in order to find the solution, despite the relaxation case, we have to solve a system of two coupled differential equations. For example, for a Kelvin model eqs. (4.12) and (4.13) reduce to

$$D_0\sigma = E_0\varepsilon + E_1\varepsilon^{(1)} + E_2\varepsilon^{(2)} + E_3\varepsilon^{(3)} + B_0\tau + B_1\tau^{(1)} + B_2\tau^{(2)} + B_3\tau^{(3)} \quad (4.38)$$

$$D'_0\mu = E'_0\varepsilon + E'_1\varepsilon^{(1)} + E'_2\varepsilon^{(2)} + E'_3\varepsilon^{(3)} + B'_0\tau + B'_1\tau^{(1)} + B'_2\tau^{(2)} + B'_3\tau^{(3)} \quad (4.39)$$

Taking Laplace transforms of both eqs. (4.38) and (4.39) gives:

$$D_0 \frac{\sigma}{s} = (E_0 + E_1s + E_2s^2 + E_3s^3)\bar{\varepsilon} + (B_0 + B_1s + B_2s^2 + B_3s^3)\bar{\tau} \quad (4.40)$$

$$D'_0 \frac{\mu}{s} = (E'_0 + E'_1s + E'_2s^2 + E'_3s^3)\bar{\varepsilon} + (B'_0 + B'_1s + B'_2s^2 + B'_3s^3)\bar{\tau} \quad (4.41)$$

where the bar sign in the  $\bar{\varepsilon}, \bar{\tau}$  refers to the Laplace field. Note, (4.40) and (4.41) are two equations for two unknowns  $\bar{\varepsilon}, \bar{\tau}$ . Thus

$$\bar{\varepsilon} = \frac{D_0(B'_3s^4 + B'_2s^3 + B'_0s)\sigma - D'_0(B_3s^4 + B_2s^3 + B_1s^2 + B_0s)\tau}{(E_3s^4 + E_2s^3 + E_1s^2 + E_0s)(B'_3s^4 + B'_2s^3 + B'_1s^2 + B'_0s) - (E'_3s^4 + E'_2s^3 + E'_1s^2 + E'_0s)(B_3s^4 + B_2s^3 + B_1s^2 + B_0s)} \quad (4.42)$$

$$\bar{\tau} = \frac{D'_0(E_3s^4 + E_2s^3 + E_0s)\tau - D_0(E'_3s^4 + E'_2s^3 + E'_1s^2 + E'_0s)\sigma}{(E_3s^4 + E_2s^3 + E_1s^2 + E_0s)(B'_3s^4 + B'_2s^3 + B'_1s^2 + B'_0s) - (E'_3s^4 + E'_2s^3 + E'_1s^2 + E'_0s)(B_3s^4 + B_2s^3 + B_1s^2 + B_0s)} \quad (4.43)$$

By performing the inverse Laplace transform on eqs. (4.42) and (4.43), the solution forms of  $\varepsilon$  and  $\tau$  will be

$$\varepsilon = \sum_{i=1}^8 \frac{F(\lambda_i)}{G'(\lambda_i)} e^{\lambda_i t} \quad (4.44)$$

$$\tau = \sum_{i=1}^8 \frac{H(\lambda_i)}{G'(\lambda_i)} e^{\lambda_i t} \quad (4.45)$$

where  $G$  is the denominator of eqs. (4.42) or (4.43) (which is a polynomial function of  $s$ ) and  $\lambda_i$ s are the roots of this polynomial;  $G'(\lambda_i)$  is the derivative of  $G$  with respect to  $s$  calculated at  $\lambda_i$ ;  $F$  and  $H$  are numerators of eqs. (4.42) and (4.43), respectively, which are polynomial functions of  $s$  calculated at  $\lambda_i$ .

By using the same procedure, we can find  $\varepsilon$  and  $\tau$  for any other system, like the Maxwell model or the Zener model.

## 4.7 Results

- The differential equations of a linear viscoelastic helix have been derived, along with explicit forms of all the coefficients, for three basic types of linear differential models (Kelvin, Maxwell, Zener) for the material at the micro level.

- The linear viscoelastic responses of a helix are generally different in type (and more complex) from those of the material at the micro level.
- When  $\nu = 0$  the type of the differential equations of the helix are analogous to the differential equation of the material at the micro level. Thus, in the relaxation phase in which both the differential equations of the helix can be solved independently, the viscoelastic response of the helix at the macro level is qualitatively the same as the viscoelastic response of the assumed model at the micro level.
- Although we did not work with an arbitrary-order differential equation (eq. 4.1), our study clearly indicates that direct viscoelastic generalizations of effective constitutive equations of helices, not based on systematic analyses such as those presented here, are likely going to be invalid. This is due to the fact that, for a given complexity of the material model, higher order derivatives are showing up in the differential equation governing the helix.
- The foregoing observation relating to uniaxial helices also provides guidance for admissible *vis-à-vis* inadmissible models of three-dimensional chiral (i.e. helically structured) materials. It is well known that the constitutive relation of a linear elastic chiral material involves two coupled equations (Nowacki, 1986)

$$\sigma_{ij} = C_{ijkl}^{(1)} \varepsilon_{kl} + C_{ijkl}^{(2)} \kappa_{kl} \quad \mu_{ij} = C_{ijkl}^{(3)} \varepsilon_{kl} + C_{ijkl}^{(4)} \kappa_{kl} \quad (4.46a,b)$$

where  $\mu_{ij}$  is the couple-stress tensor, while  $\kappa_{kl}$  is the torsion-curvature tensor. However, in view of our results, one cannot arbitrarily postulate viscoelastic generalizations of eqs. (4.46a,b) in terms of differential tensorial equations. Another case study where a uniaxial helix has provided guidance on thermomechanics of three-dimensional chiral materials has recently been presented in (Ostoja-Starzewski, 2003).

## 5 CONCLUSIONS

### 5.1 Conclusions

In the first chapter, after a complete literature review study about helices, it was shown that there are a lot of studies that need to be done. In the second chapter, Spectral Finite Element of an Elastic Helix, we derived explicit forms of all the coefficients of the stiffness matrix and plotted their dependencies on the frequency and the parameter describing the said coupling. In general, the growth of that parameter leads to a progressively denser occurrence of the resonances of both axial and torsional motions.

Taking  $A_2 = A_3 = 0$  in a helix (absence of the coupling effect) and partitioning its four-by-four spectral stiffness matrix into four two-by-two sub-matrices as

$$K = \begin{bmatrix} \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} & \begin{bmatrix} k_{13} & k_{14} \\ k_{23} & k_{24} \end{bmatrix} \\ \begin{bmatrix} k_{31} & k_{32} \\ k_{41} & k_{42} \end{bmatrix} & \begin{bmatrix} k_{33} & k_{34} \\ k_{43} & k_{44} \end{bmatrix} \end{bmatrix},$$

we see that the first and second sub-matrices on the diagonal represent the spectral stiffness matrices of a straight bar under a pure axial and torsional load, respectively, with the two off-diagonal sub-matrices being zero identically.

In the third chapter, by making certain assumptions, all the constitutive coefficients of a helix have been derived explicitly and from the parametric studies over different parameters, it is shown that

- The ratio  $C_3/C_2$  is never equal to unity, although in some cases it is very close.

- When  $\alpha$  approaches  $\frac{\pi}{2}$ ,  $C_1$  and  $C_4$  approach to values that are proportional to  $E$  and  $G$  respectively and  $C_2$  and  $C_3$  approach zero. Also the ratio  $C_3/C_2$  approaches unity as  $\alpha$  approaches  $\frac{\pi}{2}$ .
- $C_2$  and  $C_3$  do not depend on the center fibril and the only contribution of the center fibril is in the first terms of  $C_1$  and  $C_4$ .
- The Maximum value of  $C_1$  occurs at  $\alpha = \frac{\pi}{2}$  while the maximum values of  $C_2$  and  $C_3$  occur approximately when  $\alpha = 1$  radian. Also, the maximum value of  $C_4$  occurs when  $\alpha$  is about 0.6 or 0.7 radians.
- All the helix coefficients increase by increasing  $R_1$  and decrease by increasing  $\nu$

Finally, in the last chapter, the viscoelastic response of a helix was studied. Having the elastic solution from chapter 3 and, by using the correspondence principle of viscoelasticity, the differential equations of a linear viscoelastic helix have been derived with all explicit coefficients. In the derivation of these equations, three types of linear viscoelastic models (Kelvin, Maxwell, Zener models) have been used for the material at the micro level.

It was shown that the linear viscoelastic responses of a helix are different than that of the material at the micro level. However, when  $\nu = 0$  the type of the differential equations of the helix are analogous to the differential equation of the material at the micro level. Thus, in the relaxation phase in which both the differential equations of the helix can be solved independently, the viscoelastic response of the helix at the macro level is qualitatively the same as the viscoelastic response of the assumed model at the micro level.

## 5.2 Future research

Regarding the literature review and the work that has been done in this thesis, it seems that the overall elastic behavior of a helix (or any helically stranded wires) submitted to axial loading (forces and moments) is well represented by variety of approaches. Depending on the application, models of varying degree of freedom are available. However there are some important issues that need further work. For instance, considering  $C_2$  and  $C_3$ , which are not exactly equal from current micromechanics analysis, more work on the analysis of geometric configuration and the mechanics assumptions is needed. This should allow a better assessment of the validity or influence of the various hypotheses. Various related studies can be carried out in the future:

- Adding another DOF on helix as a vertical deflection, and finding the coupled behavior of the new DOF in interaction with the axial and torsional deflections and deriving the three governing coupled differential equations.
- Extending available theories of the helical strands from small deformations to finite deformations.
- Study of nonlinear viscoelastic, or even plastic, responses of helices.
- Improving basic strand bending models to be applied to much more complex situations such as multi-strand cables, fatigue, contact, friction and stick-slip conditions.
- Predicting different behaviors of the individual strands.
- Deriving fractional order differential equations of a linear viscoelastic stress relaxation in helical polymers.

## REFERENCES

- Cardou, A. and Jolicoeur, C. (January 1997), Mechanical models of helical strands, *Appl. Mech. Rev.* **50**, No 1.
- Carstarphen, F.C. (1931), Effects of bending wire rope, *Trans. ASCE* **57**, 1439-1466.
- Christenson, R.M. (1982), *Theory of Viscoelasticity*, Academic Press, London.
- Costello, G. A. (1990), *Theory of WireRrope*. Springer-Verlag, New York.
- Costello, G.A. (1974), Stresses in multilayered cables, *J. Energy Resour. Tech.* **105**, 337-340.
- Costello, G.A. (1978), Analytical investigation of wire rope, *Appl. Mech. Rev.* **31**(7), 897-900.
- Costello, G.A. and Butson, G.J. (1982), Simplified bending theory for wire rope, *ASCE J. Eng. Mech. Div.* **108**(EM2), 219-227.
- Costello, G.A. and Phillip, J.W. (1974), A more exact theory for twisted wire cables, *ASCE J. Eng. Mech. Div.* **100**(EM5), 1096-1099.
- Costello, G.A. and Phillip, J.W. (1976), Effective modulus of the twisted wire cables, *ASCE J. Eng. Mech. Div.* **102**(EM1), 171-181.
- Deresiewicz, H. (1957), Oblique contact of non-spherical elastic bodies, *J. Appl. Mech.* **24**, 623-624.
- Doyle, J.F. (1991), *Wave Propagation in Structures*, Springer-Verlag, New York.
- Drucker, D.C. and Tachau, H. (1945), A new design criterion for wire rope, *J. Appl. Mech.* **24**, 623-624.
- Durelli, A.J. and Machida, S. (1973), Response of epoxy oversized model of strands to axial and torsional loads, *Exp. Mech.* **13**, 313-321.
- Flügge, W. (1975), *Viscoelasticity*, Springer-Verlag, New York.
- Goudreau, S. (1990), *Study of Helical Strands in Bending*, Ph.D. thesis, Univ. Laval, Ste-Foy, QC, Canada (in French).
- Hruska, F.H. (1951), Calculation of stresses in wire ropes, *Wire Wire Prod.* **26**(9), 766-767, 799-801.
- Hruska, F.H. (1952), Radial forces in wire ropes, *Wire Wire Prod.* **27**(5), 459-463.

- Hruska, F.H. (1953), Tangential forces in wire ropes, *Wire Wire Prod* **28**(5), 455-460.
- Huang, N.C. (1978), Finite extension of an elastic strand with a central core, *J. Appl. Mech.* **45**(4), 852-857.
- Huang, X. and Vinogradov, O. (1994), Analysis of dry friction hysteresis in a cable under uniform bending, *Struct. Eng. Mech.* **2**(1), 63-80.
- Jiang, W. (1995), General formulation of the theories of wire ropes, *J. Appl. Mech.* **62**, 747-755.
- Jolicoeur, C. (1996), Discussion of: A general formulation of the theories of the wire ropes, *J. Appl. Mech.* **63**, 1053.
- Jolicoeur, C. and Cardou, A. (1991), Numerical comparison of current mathematical models of twisted wire cables under axisymmetric loads, *J. Energy Resources Tech.* **113**(4), 241-249.
- Knapp, R.H. (1979), Derivation of a new stiffness matrix for helically armored cables considering tension and torsion, *Int. J. Numer. Methods Eng.* **14**, 515-529.
- Kumar, K. and Cochran, J.E. (1987), Closed-form analysis of elastic deformations of multilayered strands, *J. Appl. Mech.* **54**, 898-903.
- Lanteigne, J. (1985), Theoretical estimation of the response of helically armored cables to tension, torsion and bending, *ASME J. Appl. Mech.* **52**, 423-432.
- Love A.E.H. (1944), *Mathematical Theory of Elasticity*, Dover Pub, New York.
- Lutchansky, M. (1969), Axial stresses in armor wires of bent submarine cables, *J. Eng. Ind.* **91**, 687-693.
- Machida, S. and Durelli, A.J. (1973), Response of a strand to axial and torsional displacements, *J. Mech. Eng. Sci.* **15**(4), 241-251.
- McConell, K.G. and Zemke, W.P. (1982), Model to predict the coupled axial torsion properties of ACSR electrical conductors, *Exp. Mech.* **22**(7), 237-244.
- Milburn, D.A., and Rendler, N. J. (July 1972), Methods of measuring the mechanical behavior of wire rope, *Memorandum Report 2459*, Naval Research Laboratory, Washington, D.C.
- Nowacki, W., (1986), *Theory of Asymmetric Elasticity*, Pergamon Press, Oxford/PWN - Polish Sci. Publ., Warsaw.

Ostoja-Starzewski, M. (2003), Thermoelastic waves in a helix with parabolic or hyperbolic heat conduction, *J. Thermal Stresses* (in honour of R.B. Hetnarski), **26**, 1205-1219.

Ostoja-Starzewski, M. and Woods, A.N. (2003), Spectral finite elements for vibrating rods and beams with random field properties, *J. Sound. Vib.* **268**, 779-797, 2003.

Papailliou, K.O. (1996), Bending stiffness of transmission line conductors, *IEEE/PES Summer meeting*, July 28-Aug. 1<sup>st</sup>, Denver Co. Paper No. 96-SM-444-0-PWRD, 8 pp.

Philips, J.W., Miller, R.E., and Costello, G.A. (1980), Contact stresses in a straight cross-lay wire rope, *Proc. first Annual Wire Rope Conf.* Denver Co, 177-199.

Ramsey, H. (1988), theory of thin rods with application to helical constituent wires in cables, *Int. J. Mech. Sci.* **30**(8), 559-570.

Ramsey, H. (1990), Analysis of interwire friction in multilayered cables under uniform extension and twisting, *Int. J. Mech. Sci.* **30**(8), 709-716.

Ramsey, H. (1991), Localized effect of clamp or socket and connections on helical wires in multilayered cables, *Int. J. Solids Struct.* **28**(6), 779-790.

Raoof, M. (1990), Free bending of spiral strands, *ASCE J. Eng. Mech.* **116**(3), 512-530.

Raoof, M. and Hobbs, R.E. (1984), Bending of spiral strand and armored cables close to terminations, *J. Energy Resource Tech.* **106**, 349-355.

Samras, R.K., Skop, R.A. and Milburn, D.A. (1974), An analysis of coupled extensional-torsional oscillations in wire rope, *ASME J. Eng. Industry* **96**, 1130-1135.

Sathikh, S. and Parthasarathy, N.S. (1988), Discussion of: Internal friction due to the wire twist in bent cable, *ASCE J. Eng. Mech.* **114**(4), 727-730.

Sathikh, S., Jayakumar, and Jebaraj, C. (1996), Discussion of: A general formulation of the theories of the wire ropes, *ASME J. Appl. Mech.* **63**, 1053-1054.

Shahsavari, H. and Ostoja-Starzewski, M. (2004), Spectral finite element of a helix, *Mechanics Research Communications*, in press.

Timoshenko, S. (1956), *Strength of Materials*: part II, Van Nostrand, New York.

Utting, W.S. (1995), Survey of the literature on the behavior of steel wire ropes-part III, *Wire Ind.*, 269-270.

Utting, W.S. and Jones, N. (1984), Survey of literature on the behavior of wire ropes, *Wire Ind.*, 623-629.

Utting, W.S. and Jones, N. (1985), Tensile testing of a wire rope strand, *J. Strain. Anal. Eng. Des.* **20**(3), 151-164.

Utting, W.S. and Jones, N. (1987), Response of wire rope strands to axial loads tensile loads, Part I: Experimental results and theoretical predictions, *Int. J. Mech. Sci.* **29**(9), 605-619.

Velsicky, S.A., Anderson, G.L. and Costello, G.A. (1984), Wire rope with complex cross sections, *ASCE J. Eng. Mech.* **110**(3), 380-391.

Vinogradov, O.G. and Atatekin, I.S. (1986), Internal friction due to the wire twist in bent cable, *ASCE J. Eng. Mech.* **112**(9), 859-873.

Ziegler, H. (1983), *An Introduction to Thermomechanics*, North-Holland Pub. Co., Amesterdam.

## **APPENDICES**

## APPENDIX A

### Discrepancies between $C_2$ and $C_3$

There are some discrepancies between the coefficients  $C_2$  and  $C_3$  as discussed below.

The work done on a straight bundle by the loads  $F$  and  $M$  can be written as

$$W = \int F dx + M d\phi$$

where  $x = h\varepsilon$  and  $\phi = h\tau = \frac{h\beta}{R}$ . Hence

$$dx = hd\varepsilon$$

and

$$d\phi = \frac{h}{R} d\beta$$

where  $R = R_1 + 2R_2$  is the radius of the bundle. Now  $W$  can be written as

$$W = \int \left( (AEC_1\varepsilon + AEC_2\beta)hd\varepsilon + \left(ER^3C_3\varepsilon + ER^3C_4\beta\right)\frac{h}{R}d\beta \right)$$

This integral should be independent of the path. Hence,

$$\frac{\partial}{\partial\beta} (AEC_1\varepsilon + AEC_2\beta)h = \frac{\partial}{\partial\varepsilon} \left(ER^3C_3\varepsilon + ER^3C_4\beta\right) \frac{h}{R}$$

$$\frac{\partial}{\partial \varepsilon} (AEC_1\varepsilon + AEC_2\beta)h = \frac{\partial}{\partial \beta} (ER^3C_3\varepsilon + ER^3C_4\beta) \frac{h}{R}$$

which results in

$$AC_2 = R^2 C_3$$

$$AC_1 = R^2 C_4$$

or

$$\pi(R_1^2 + mR_2^2)C_2 = (R_1 + 2R_2)^2 C_3$$

$$\pi(R_1^2 + mR_2^2)C_1 = (R_1 + 2R_2)^2 C_4$$

Note that due to the assumptions made to derive the constitutive helix coefficients, the above equations which are based on the energy concept, are not exactly satisfied.

## APPENDIX B

### Coefficients of the differential equations of a viscoelastic helix

There are six programs written by Maple 9.0 software for the two stresses (axial and shear) of a helix while considering three viscoelastic models for each:

- 1) Program No. 1: Finding the coefficients of the differential equation of the axial stress of a helix using two Kelvin models
- 2) Program No. 2: Finding the coefficients of the differential equation of the shear stress of a helix using two Kelvin models
- 3) Program No. 3: Finding the coefficients of the differential equation of the axial stress of a helix using two Maxwell models
- 4) Program No. 4: Finding the coefficients of the differential equation of the shear stress of a helix using two Kelvin models
- 5) Program No. 5: Finding the coefficients of the differential equation of the axial stress of a helix using two Zener (3-Parameter) models
- 6) Program No. 6: Finding the coefficients of the differential equation of the shear stress of a helix using two Zener (3-Parameter) models

In order to shorten the long coefficients, the following notations are used

$$s = \sin \alpha_2$$

$$c = \cos \alpha_2$$

$$ta = \tan \alpha_2$$

In all the programs , the Laplace parameter is shown by s1;  $\tau$  and  $\mu$  in the constitutive equations are replaced by  $\beta$  and  $\tau$  respectively. Due to length limitations on a master's thesis only the final results are given for the programs No. 5 and No. 6.

&gt;

## Program No. 1

Coefficients of the diff. eq. of the axial  
stress of a helix using two Kelvin models

```
restart;
```

```
>
>
>
>
```

```
with(linalg):
```

Warning, the protected names norm and trace have been redefined and unprotected

```
> c1:=E*R1^2/(R1^2+m*R2^2)+m*E*R2^2/((R1^2+m*R2^2)*(r2+r2*ta^2+nu*R2))*((r
2*ta^2-nu*R1)*s+R2^2*(1-2*s^2)*s*c^2/4/r2+R2^2*s^3*c^2*(1+nu)/2/r2-nu^2*
R2^2*s*c^4*(R1+r2*ta^2)/4/r2^2/(1+nu));
c1 := 
$$\frac{ER1^2}{R1^2 + m R2^2} + \frac{m E R2^2 \left( (r2 ta^2 - \nu RI) s + \frac{R2^2 (1 - 2 s^2) s c^2}{4 r2} + \frac{R2^2 s^3 c^2 (1 + \nu)}{2 r2} - \frac{\nu^2 R2^2 s c^4 (RI + r2 ta^2)}{4 r2^2 (1 + \nu)} \right)}{(R1^2 + m R2^2) (r2 + r2 ta^2 + \nu R2)}$$

```

```
> c2:=r2*E*m*R2^2/(R1+2*R2)/(R1^2+m*R2^2)/(r2+r2*ta^2+nu*R2)*(r2*ta*s+R2^2
/4/(1+nu)/r2*((2*s^2-1)*s^2*c)-R2^2/2/r2*s^4*c-nu^2*R2^3*s^2*c^3/4/r2^2/
(1+nu));
c2 := 
$$\frac{r2 E m R2^2 \left( r2 ta s + \frac{R2^2 (2 s^2 - 1) s^2 c}{4 (1 + \nu) r2} - \frac{R2^2 s^4 c}{2 r2} - \frac{\nu^2 R2^3 s^2 c^3}{4 r2^2 (1 + \nu)} \right)}{(RI + 2 R2) (R1^2 + m R2^2) (r2 + r2 ta^2 + \nu R2)}$$

```

$$\begin{aligned}
> \text{sigma} := & c1 * \text{epsilon} + c2 * \beta; \\
\sigma := & \left( \frac{\frac{E R I^2}{R I^2 + m R 2^2}}{m E R 2^2 \left( (r2 t a^2 - v R I) s + \frac{R 2^2 (1 - 2 s^2) s c^2}{4 r2} + \frac{R 2^2 s^3 c^2 (1 + v)}{2 r2} - \frac{v^2 R 2^2 s c^4 (R I + r2 t a^2)}{4 r2^2 (1 + v)} \right)} \right. \\
& + \frac{r2 E m R 2^2 \left( r2 t a s + \frac{R 2^2 (2 s^2 - 1) s^2 c}{4 (1 + v) r2} - \frac{R 2^2 s^4 c}{2 r2} - \frac{v^2 R 2^3 s^2 c^3}{4 r2^2 (1 + v)} \right) \beta}{(R I + 2 R 2) (R I^2 + m R 2^2) (r2 + r2 t a^2 + v R 2)} \\
& \left. \varepsilon + \frac{r2 E m R 2^2 \left( r2 t a s + \frac{R 2^2 (2 s^2 - 1) s^2 c}{4 (1 + v) r2} - \frac{R 2^2 s^4 c}{2 r2} - \frac{v^2 R 2^3 s^2 c^3}{4 r2^2 (1 + v)} \right) \beta}{(R I + 2 R 2) (R I^2 + m R 2^2) (r2 + r2 t a^2 + v R 2)} \right)
\end{aligned}$$

$$\begin{aligned}
> \text{sigmal} := & \text{subs}(E=3*Q*q/(2*p*Q+q*P), nu=(p*Q-q*P)/(2*p*Q+q*P), \text{sigma}); \\
\sigma l := & \left( \frac{\frac{3 Q q R I^2}{(2 p Q + q P) (R I^2 + m R 2^2)} + \frac{1}{(2 p Q + q P) (R I^2 + m R 2^2) \left( r2 + r2 t a^2 + \frac{(p Q - q P) R 2}{2 p Q + q P} \right)} \right. \\
& q R 2^2 \left( \left( r2 t a^2 - \frac{(p Q - q P) R I}{2 p Q + q P} \right) s + \frac{R 2^2 (1 - 2 s^2) s c^2}{4 r2} + \frac{R 2^2 s^3 c^2 \left( 1 + \frac{p Q - q P}{2 p Q + q P} \right)}{2 r2} \right. \\
& \left. \left. - \frac{(p Q - q P)^2 R 2^2 s c^4 (R I + r2 t a^2)}{4 (2 p Q + q P)^2 r2^2 \left( 1 + \frac{p Q - q P}{2 p Q + q P} \right)} \right) \varepsilon + \frac{1}{(2 p Q + q P) (R I + 2 R 2) (R I^2 + m R 2^2) \left( r2 + r2 t a^2 + \frac{(p Q - q P) R 2}{2 p Q + q P} \right)} \right. \\
& \left. \left( 3 r2 Q q m R 2^2 \left( r2 t a s \right. \right. \right. \\
& \left. \left. + \frac{R 2^2 (2 s^2 - 1) s^2 c}{4 \left( 1 + \frac{p Q - q P}{2 p Q + q P} \right) r2} - \frac{R 2^2 s^4 c}{2 r2} - \frac{(p Q - q P)^2 R 2^3 s^2 c^3}{4 (2 p Q + q P)^2 r2^2 \left( 1 + \frac{p Q - q P}{2 p Q + q P} \right)} \right) \beta \right)
\end{aligned}$$

> sigma2 := simplify(sigmal);

$$\begin{aligned}
\sigma^2 := & ((24 \varepsilon Q^2 R1^3 r2^3 p^2 + 12 \varepsilon Q R1^3 r2^3 p q P + 24 \varepsilon Q^2 R1^3 r2^3 p^2 t a^2 + 12 \varepsilon Q R1^3 r2^3 p t a^2 q P \\
& + 12 \varepsilon Q^2 R1^3 r2^2 p^2 R2 + 24 \varepsilon Q^2 R1^2 r2^2 p^2 R2^2 - 12 \varepsilon Q R1^3 r2^2 p R2 q P \\
& - 24 \varepsilon Q R1^2 r2^2 p R2^2 q P + 48 \varepsilon m R2^3 s r2^3 p^2 Q^2 t a^2 + 24 \varepsilon m R2^3 s r2^3 p Q t a^2 q P \\
& - 12 \varepsilon m R2^2 s r2^2 p^2 Q^2 R1^2 - 24 \varepsilon m R2^3 s r2^2 p^2 Q^2 R1 + 12 \varepsilon m R2^2 s r2^2 p Q R1^2 q P \\
& + 24 \varepsilon m R2^3 s r2^2 p Q R1 q P + 12 \varepsilon m R2^5 s^3 c^2 p^2 Q^2 r2 - 12 \varepsilon m R2^5 s^3 c^2 r2 p Q q P \\
& + 12 \varepsilon m R2^5 s c^2 r2 p^2 Q^2 + 6 \varepsilon m R2^5 s c^2 r2 p Q q P - \varepsilon m R2^4 s c^4 p^2 Q^2 R1^2 \\
& - 2 \varepsilon m R2^5 s c^4 p^2 Q^2 R1 - 2 \varepsilon m R2^5 s c^4 p^2 Q^2 r2 t a^2 + 2 \varepsilon m R2^4 s c^4 p Q q P R1^2 \\
& + 4 \varepsilon m R2^5 s c^4 p Q q P R1 + 4 \varepsilon m R2^5 s c^4 p Q q P r2 t a^2 - \varepsilon m R2^4 s c^4 q^2 P^2 R1^2 \\
& - 2 \varepsilon m R2^5 s c^4 q^2 P^2 R1 - 2 \varepsilon m R2^5 s c^4 q^2 P^2 r2 t a^2 + 48 \varepsilon Q^2 R1^2 r2^3 p^2 R2 \\
& + 24 \varepsilon Q R1^2 r2^3 p q P R2 + 48 \varepsilon Q^2 R1^2 r2^3 p^2 t a^2 R2 + 24 \varepsilon Q R1^2 r2^3 p t a^2 q P R2 \\
& + 24 \varepsilon m R2^2 s r2^3 p^2 Q^2 t a^2 R1 + 12 \varepsilon m R2^2 s r2^3 p Q t a^2 q P R1 + 6 \varepsilon m R2^4 s^3 c^2 p^2 Q^2 r2 R1 \\
& - 6 \varepsilon m R2^4 s^3 c^2 r2 p Q q P R1 + 6 \varepsilon m R2^4 s c^2 r2 p^2 Q^2 R1 + 3 \varepsilon m R2^4 s c^2 r2 p Q q P R1 \\
& - \varepsilon m R2^4 s c^4 p^2 Q^2 r2 t a^2 R1 + 2 \varepsilon m R2^4 s c^4 p Q q P r2 t a^2 R1 - \varepsilon m R2^4 s c^4 q^2 P^2 r2 t a^2 R1 \\
& + 24 r2^4 m R2^2 s \beta t a p^2 Q^2 + 12 r2^4 m R2^2 s \beta t a p Q q P - 4 r2^2 m R2^4 s^4 \beta c p^2 Q^2 \\
& - 4 r2^2 m R2^4 s^2 \beta c p^2 Q^2 + 2 r2^2 m R2^4 s^4 \beta c p Q q P - 4 r2^2 m R2^4 s^2 \beta c p Q q P \\
& + 2 r2^2 m R2^4 s^4 \beta c q^2 P^2 - r2^2 m R2^4 s^2 \beta c q^2 P^2 - r2 m R2^5 s^2 \beta c^3 p^2 Q^2 \\
& + 2 r2 m R2^5 s^2 \beta c^3 p Q q P - r2 m R2^5 s^2 \beta c^3 q^2 P^2) q) / (4 p r2^2 (2 r2 p Q + r2 q P + 2 r2 t a^2 p Q \\
& + r2 t a^2 q P + R2 p Q - R2 q P) (R1^2 + m R2^2) (R1 + 2 R2) (2 p Q + q P))
\end{aligned}$$

> LHS1:=denom(sigma2);

$$LHS1 := 4 p r2^2 (2 r2 p Q + r2 q P + 2 r2 t a^2 p Q + r2 t a^2 q P + R2 p Q - R2 q P) (R1^2 + m R2^2) (R1 + 2 R2) (2 p Q + q P)$$

> LHS2:=subs(p=1,P=1,q=q0+q1\*s1,Q=Q0+Q1\*s1,LHS1);

$$\begin{aligned}
LHS2 := & 4 r2^2 (2 r2 (Q0 + Q1 s1) + r2 (q0 + q1 s1) + 2 r2 t a^2 (Q0 + Q1 s1) + r2 t a^2 (q0 + q1 s1) \\
& + R2 (Q0 + Q1 s1) - R2 (q0 + q1 s1)) (R1^2 + m R2^2) (R1 + 2 R2) (2 Q0 + 2 Q1 s1 + q0 + q1 s1)
\end{aligned}$$

> LHS3:=collect(LHS2,s1);

$$\begin{aligned}
LHS3 := & 4 r2^2 (2 r2 Q1 + r2 q1 + 2 r2 t a^2 Q1 + r2 t a^2 q1 + R2 Q1 - R2 q1) (R1^2 + m R2^2) (R1 + 2 R2) (2 \\
& Q1 + q1) s1^2 + (4 r2^2 (2 r2 Q0 + r2 q0 + 2 r2 t a^2 Q0 + r2 t a^2 q0 + R2 Q0 - R2 q0) (R1^2 + m R2^2) (R1 \\
& + 2 R2) (2 Q1 + q1) + 4 r2^2 (2 r2 Q1 + r2 q1 + 2 r2 t a^2 Q1 + r2 t a^2 q1 + R2 Q1 - R2 q1) (R1^2 \\
& + m R2^2) (R1 + 2 R2) (2 Q0 + q0)) s1 + 4 r2^2 (2 r2 Q0 + r2 q0 + 2 r2 t a^2 Q0 + r2 t a^2 q0 + R2 Q0 \\
& - R2 q0) (R1^2 + m R2^2) (R1 + 2 R2) (2 Q0 + q0)
\end{aligned}$$

> RHS1:=numer(sigma2);

$$\begin{aligned}
RHS1 := & q (24 \varepsilon Q^2 R1^3 r2^3 p^2 + 12 \varepsilon Q R1^3 r2^3 p q P + 24 \varepsilon Q^2 R1^3 r2^3 p^2 t a^2 \\
& + 12 \varepsilon Q R1^3 r2^3 p t a^2 q P + 12 \varepsilon Q^2 R1^3 r2^2 p^2 R2 + 24 \varepsilon Q^2 R1^2 r2^2 p^2 R2^2 \\
& - 12 \varepsilon Q R1^3 r2^2 p R2 q P - 24 \varepsilon Q R1^2 r2^2 p R2^2 q P + 48 \varepsilon m R2^3 s r2^3 p^2 Q^2 t a^2 \\
& + 24 \varepsilon m R2^3 s r2^3 p Q t a^2 q P - 12 \varepsilon m R2^2 s r2^2 p^2 Q^2 R1^2 - 24 \varepsilon m R2^3 s r2^2 p^2 Q^2 R1 \\
& + 12 \varepsilon m R2^2 s r2^2 p Q R1^2 q P + 24 \varepsilon m R2^3 s r2^2 p Q R1 q P + 12 \varepsilon m R2^5 s^3 c^2 p^2 Q^2 r2 \\
& - 12 \varepsilon m R2^5 s^3 c^2 r2 p Q q P + 12 \varepsilon m R2^5 s c^2 r2 p^2 Q^2 + 6 \varepsilon m R2^5 s c^2 r2 p Q q P
\end{aligned}$$

$$\begin{aligned}
& -\varepsilon m R2^4 s c^4 p^2 Q^2 R1^2 - 2\varepsilon m R2^5 s c^4 p^2 Q^2 R1 - 2\varepsilon m R2^5 s c^4 p^2 Q^2 r2 ta^2 \\
& + 2\varepsilon m R2^4 s c^4 p Q q P R1^2 + 4\varepsilon m R2^5 s c^4 p Q q P R1 + 4\varepsilon m R2^5 s c^4 p Q q P r2 ta^2 \\
& - \varepsilon m R2^4 s c^4 q^2 P^2 R1^2 - 2\varepsilon m R2^5 s c^4 q^2 P^2 R1 - 2\varepsilon m R2^5 s c^4 q^2 P^2 r2 ta^2 \\
& + 48\varepsilon Q^2 R1^2 r2^3 p^2 R2 + 24\varepsilon Q R1^2 r2^3 p q P R2 + 48\varepsilon Q^2 R1^2 r2^3 p^2 ta^2 R2 \\
& + 24\varepsilon Q R1^2 r2^3 p ta^2 q P R2 + 24\varepsilon m R2^2 s r2^3 p^2 Q^2 ta^2 R1 + 12\varepsilon m R2^2 s r2^3 p Q ta^2 q P R1 \\
& + 6\varepsilon m R2^4 s^3 c^2 p^2 Q^2 r2 R1 - 6\varepsilon m R2^4 s^3 c^2 r2 p Q q P R1 + 6\varepsilon m R2^4 s c^2 r2 p^2 Q^2 R1 \\
& + 3\varepsilon m R2^4 s c^2 r2 p Q q P R1 - \varepsilon m R2^4 s c^4 p^2 Q^2 r2 ta^2 R1 + 2\varepsilon m R2^4 s c^4 p Q q P r2 ta^2 R1 \\
& - \varepsilon m R2^4 s c^4 q^2 P^2 r2 ta^2 R1 + 24r2^4 m R2^2 s \beta tap^2 Q^2 + 12r2^4 m R2^2 s \beta tap Q q P \\
& - 4r2^2 m R2^4 s^4 \beta c p^2 Q^2 - 4r2^2 m R2^4 s^2 \beta c p^2 Q^2 + 2r2^2 m R2^4 s^4 \beta c p Q q P \\
& - 4r2^2 m R2^4 s^2 \beta c p Q q P + 2r2^2 m R2^4 s^4 \beta c q^2 P^2 - r2^2 m R2^4 s^2 \beta c q^2 P^2 \\
& - r2 m R2^5 s^2 \beta c^3 p^2 Q^2 + 2r2 m R2^5 s^2 \beta c^3 p Q q P - r2 m R2^5 s^2 \beta c^3 q^2 P^2
\end{aligned}$$

```
> RHS2:=subs(p=1,P=1,q=q0+q1*s1,Q=Q0+Q1*s1,RHS1);
```

$$\begin{aligned}
RHS2 := & (q0 + q1 s1) (24 \varepsilon (Q0 + Q1 s1)^2 R1^3 r2^3 - \varepsilon m R2^4 s c^4 (q0 + q1 s1)^2 R1^2 \\
& - 2\varepsilon m R2^5 s c^4 (q0 + q1 s1)^2 R1 - \varepsilon m R2^4 s c^4 (q0 + q1 s1)^2 r2 ta^2 R1 \\
& - 2\varepsilon m R2^5 s c^4 (q0 + q1 s1)^2 r2 ta^2 - r2 m R2^5 s^2 \beta c^3 (q0 + q1 s1)^2 \\
& + 2r2^2 m R2^4 s^4 \beta c (q0 + q1 s1)^2 - r2^2 m R2^4 s^2 \beta c (q0 + q1 s1)^2 \\
& + 12\varepsilon (Q0 + Q1 s1) R1^3 r2^3 (q0 + q1 s1) + 12\varepsilon (Q0 + Q1 s1) R1^3 r2^3 ta^2 (q0 + q1 s1) \\
& + 24\varepsilon (Q0 + Q1 s1)^2 R1^3 r2^3 ta^2 + 12\varepsilon (Q0 + Q1 s1)^2 R1^3 r2^2 R2 \\
& + 24\varepsilon (Q0 + Q1 s1)^2 R1^2 r2^2 R2^2 + 48\varepsilon m R2^3 s r2^3 (Q0 + Q1 s1)^2 ta^2 \\
& - 12\varepsilon m R2^2 s r2^2 (Q0 + Q1 s1)^2 R1^2 - 24\varepsilon m R2^3 s r2^2 (Q0 + Q1 s1)^2 R1 \\
& + 12\varepsilon m R2^5 s^3 c^2 (Q0 + Q1 s1)^2 r2 + 12\varepsilon m R2^5 s c^2 r2 (Q0 + Q1 s1)^2 \\
& - \varepsilon m R2^4 s c^4 (Q0 + Q1 s1)^2 R1^2 - 2\varepsilon m R2^5 s c^4 (Q0 + Q1 s1)^2 R1 \\
& - 2\varepsilon m R2^5 s c^4 (Q0 + Q1 s1)^2 r2 ta^2 + 48\varepsilon (Q0 + Q1 s1)^2 R1^2 r2^3 R2 \\
& + 48\varepsilon (Q0 + Q1 s1)^2 R1^2 r2^3 ta^2 R2 + 24\varepsilon m R2^2 s r2^3 (Q0 + Q1 s1)^2 ta^2 R1 \\
& + 6\varepsilon m R2^4 s^3 c^2 (Q0 + Q1 s1)^2 r2 R1 + 6\varepsilon m R2^4 s c^2 r2 (Q0 + Q1 s1)^2 R1 \\
& - \varepsilon m R2^4 s c^4 (Q0 + Q1 s1)^2 r2 ta^2 R1 + 24r2^4 m R2^2 s \beta ta (Q0 + Q1 s1)^2 \\
& - 4r2^2 m R2^4 s^4 \beta c (Q0 + Q1 s1)^2 - 4r2^2 m R2^4 s^2 \beta c (Q0 + Q1 s1)^2 \\
& - r2 m R2^5 s^2 \beta c^3 (Q0 + Q1 s1)^2 - 12\varepsilon (Q0 + Q1 s1) R1^3 r2^2 R2 (q0 + q1 s1) \\
& - 24\varepsilon (Q0 + Q1 s1) R1^2 r2^2 R2^2 (q0 + q1 s1) + 24\varepsilon m R2^3 s r2^3 (Q0 + Q1 s1) ta^2 (q0 + q1 s1) \\
& + 12\varepsilon m R2^2 s r2^2 (Q0 + Q1 s1) R1^2 (q0 + q1 s1) + 24\varepsilon m R2^3 s r2^2 (Q0 + Q1 s1) R1 (q0 + q1 s1) \\
& - 12\varepsilon m R2^5 s^3 c^2 r2 (Q0 + Q1 s1) (q0 + q1 s1) + 6\varepsilon m R2^5 s c^2 r2 (Q0 + Q1 s1) (q0 + q1 s1) \\
& + 2\varepsilon m R2^4 s c^4 (Q0 + Q1 s1) (q0 + q1 s1) R1^2 + 4\varepsilon m R2^5 s c^4 (Q0 + Q1 s1) (q0 + q1 s1) R1 \\
& + 4\varepsilon m R2^5 s c^4 (Q0 + Q1 s1) (q0 + q1 s1) r2 ta^2 + 24\varepsilon (Q0 + Q1 s1) R1^2 r2^3 (q0 + q1 s1) R2 \\
& + 24\varepsilon (Q0 + Q1 s1) R1^2 r2^3 ta^2 (q0 + q1 s1) R2 + 12\varepsilon m R2^2 s r2^3 (Q0 + Q1 s1) ta^2 (q0 + q1 s1) R1 \\
& - 6\varepsilon m R2^4 s^3 c^2 r2 (Q0 + Q1 s1) (q0 + q1 s1) R1 + 3\varepsilon m R2^4 s c^2 r2 (Q0 + Q1 s1) (q0 + q1 s1) R1 \\
& + 2\varepsilon m R2^4 s c^4 (Q0 + Q1 s1) (q0 + q1 s1) r2 ta^2 R1 + 12r2^4 m R2^2 s \beta ta (Q0 + Q1 s1) (q0 + q1 s1) \\
& + 2r2^2 m R2^4 s^4 \beta c (Q0 + Q1 s1) (q0 + q1 s1) - 4r2^2 m R2^4 s^2 \beta c (Q0 + Q1 s1) (q0 + q1 s1) \\
& + 2r2 m R2^5 s^2 \beta c^3 (Q0 + Q1 s1) (q0 + q1 s1))
\end{aligned}$$

```
> RHS3epsilon:=collect(RHS2, epsilon);
```

$$RHS3epsilon := (q0 + q1 s1) (24 (Q0 + Q1 s1)^2 R1^3 r2^3 + 12 (Q0 + Q1 s1) R1^3 r2^3 (q0 + q1 s1)$$

$$\begin{aligned}
& + 24 (Q0 + Q1 sI)^2 R1^3 r2^3 ta^2 + 12 (Q0 + Q1 sI)^2 R1^3 r2^2 R2 + 24 (Q0 + Q1 sI)^2 R1^2 r2^2 R2^2 \\
& + 48 (Q0 + Q1 sI)^2 R1^2 r2^3 R2 - m R2^4 s c^4 (q0 + q1 sI)^2 R1^2 - 2 m R2^5 s c^4 (q0 + q1 sI)^2 R1 \\
& - m R2^4 s c^4 (q0 + q1 sI)^2 r2 ta^2 R1 - 2 m R2^5 s c^4 (q0 + q1 sI)^2 r2 ta^2 \\
& + 12 (Q0 + Q1 sI) R1^3 r2^3 ta^2 (q0 + q1 sI) + 48 m R2^3 s r2^3 (Q0 + Q1 sI)^2 ta^2 \\
& + 12 m R2^5 s c^2 r2 (Q0 + Q1 sI)^2 - m R2^4 s c^4 (Q0 + Q1 sI)^2 R1^2 \\
& - 12 m R2^2 s r2^2 (Q0 + Q1 sI)^2 R1^2 - 24 m R2^3 s r2^2 (Q0 + Q1 sI)^2 R1 \\
& + 12 m R2^5 s^3 c^2 (Q0 + Q1 sI)^2 r2 + 6 m R2^4 s^3 c^2 (Q0 + Q1 sI)^2 r2 R1 \\
& + 6 m R2^4 s c^2 r2 (Q0 + Q1 sI)^2 R1 - 12 (Q0 + Q1 sI) R1^3 r2^2 R2 (q0 + q1 sI) \\
& - 2 m R2^5 s c^4 (Q0 + Q1 sI)^2 R1 - 2 m R2^5 s c^4 (Q0 + Q1 sI)^2 r2 ta^2 \\
& + 48 (Q0 + Q1 sI)^2 R1^2 r2^3 ta^2 R2 + 24 m R2^2 s r2^3 (Q0 + Q1 sI)^2 ta^2 R1 \\
& + 12 m R2^2 s r2^2 (Q0 + Q1 sI) R1^2 (q0 + q1 sI) + 24 m R2^3 s r2^2 (Q0 + Q1 sI) R1 (q0 + q1 sI) \\
& - 12 m R2^5 s^3 c^2 r2 (Q0 + Q1 sI) (q0 + q1 sI) + 6 m R2^5 s c^2 r2 (Q0 + Q1 sI) (q0 + q1 sI) \\
& - m R2^4 s c^4 (Q0 + Q1 sI)^2 r2 ta^2 R1 - 24 (Q0 + Q1 sI) R1^2 r2^2 R2^2 (q0 + q1 sI) \\
& + 24 m R2^3 s r2^3 (Q0 + Q1 sI) ta^2 (q0 + q1 sI) + 2 m R2^4 s c^4 (Q0 + Q1 sI) (q0 + q1 sI) R1^2 \\
& + 4 m R2^5 s c^4 (Q0 + Q1 sI) (q0 + q1 sI) R1 + 4 m R2^5 s c^4 (Q0 + Q1 sI) (q0 + q1 sI) r2 ta^2 \\
& + 24 (Q0 + Q1 sI) R1^2 r2^3 (q0 + q1 sI) R2 + 24 (Q0 + Q1 sI) R1^2 r2^3 ta^2 (q0 + q1 sI) R2 \\
& + 12 m R2^2 s r2^3 (Q0 + Q1 sI) ta^2 (q0 + q1 sI) R1 - 6 m R2^4 s^3 c^2 r2 (Q0 + Q1 sI) (q0 + q1 sI) R1 \\
& + 3 m R2^4 s c^2 r2 (Q0 + Q1 sI) (q0 + q1 sI) R1 + 2 m R2^4 s c^4 (Q0 + Q1 sI) (q0 + q1 sI) r2 ta^2 R1 \\
& \varepsilon + (q0 + q1 sI) (-r2 m R2^5 s^2 \beta c^3 (Q0 + Q1 sI)^2 + 24 r2^4 m R2^2 s \beta ta (Q0 + Q1 sI)^2 \\
& - 4 r2^2 m R2^4 s^4 \beta c (Q0 + Q1 sI)^2 - 4 r2^2 m R2^4 s^2 \beta c (Q0 + Q1 sI)^2 \\
& + 2 r2^2 m R2^4 s^4 \beta c (Q0 + Q1 sI) (q0 + q1 sI) - r2 m R2^5 s^2 \beta c^3 (q0 + q1 sI)^2 \\
& + 2 r2^2 m R2^4 s^4 \beta c (q0 + q1 sI)^2 - r2^2 m R2^4 s^2 \beta c (q0 + q1 sI)^2 \\
& - 4 r2^2 m R2^4 s^2 \beta c (Q0 + Q1 sI) (q0 + q1 sI) + 2 r2 m R2^5 s^2 \beta c^3 (Q0 + Q1 sI) (q0 + q1 sI) \\
& + 12 r2^4 m R2^2 s \beta ta (Q0 + Q1 sI) (q0 + q1 sI))
\end{aligned}$$

> RHS3beta:=collect(RHS2,beta);

$$\begin{aligned}
RHS3beta := & (q0 + q1 sI) (-r2^2 m R2^4 s^2 c (q0 + q1 sI)^2 - 4 r2^2 m R2^4 s^4 c (Q0 + Q1 sI)^2 \\
& - 4 r2^2 m R2^4 s^2 c (Q0 + Q1 sI)^2 - r2 m R2^5 s^2 c^3 (Q0 + Q1 sI)^2 + 24 r2^4 m R2^2 s ta (Q0 + Q1 sI)^2 \\
& - r2 m R2^5 s^2 c^3 (q0 + q1 sI)^2 + 2 r2^2 m R2^4 s^4 c (q0 + q1 sI)^2 \\
& + 12 r2^4 m R2^2 s ta (Q0 + Q1 sI) (q0 + q1 sI) + 2 r2^2 m R2^4 s^4 c (Q0 + Q1 sI) (q0 + q1 sI) \\
& - 4 r2^2 m R2^4 s^2 c (Q0 + Q1 sI) (q0 + q1 sI) + 2 r2 m R2^5 s^2 c^3 (Q0 + Q1 sI) (q0 + q1 sI)) \beta + (q0 \\
& + q1 sI) (24 \varepsilon (Q0 + Q1 sI)^2 R1^3 r2^3 - \varepsilon m R2^4 s c^4 (q0 + q1 sI)^2 R1^2 \\
& - 2 \varepsilon m R2^5 s c^4 (q0 + q1 sI)^2 R1 - \varepsilon m R2^4 s c^4 (q0 + q1 sI)^2 r2 ta^2 R1 \\
& - 2 \varepsilon m R2^5 s c^4 (q0 + q1 sI)^2 r2 ta^2 + 12 \varepsilon (Q0 + Q1 sI) R1^3 r2^3 (q0 + q1 sI) \\
& + 12 \varepsilon (Q0 + Q1 sI) R1^3 r2^3 ta^2 (q0 + q1 sI) + 24 \varepsilon (Q0 + Q1 sI)^2 R1^3 r2^3 ta^2 \\
& + 12 \varepsilon (Q0 + Q1 sI)^2 R1^3 r2^2 R2 + 24 \varepsilon (Q0 + Q1 sI)^2 R1^2 r2^2 R2^2 \\
& + 48 \varepsilon m R2^3 s r2^3 (Q0 + Q1 sI)^2 ta^2 - 12 \varepsilon m R2^2 s r2^2 (Q0 + Q1 sI)^2 R1^2 \\
& - 24 \varepsilon m R2^3 s r2^2 (Q0 + Q1 sI)^2 R1 + 12 \varepsilon m R2^5 s^3 c^2 (Q0 + Q1 sI)^2 r2 \\
& + 12 \varepsilon m R2^5 s c^2 r2 (Q0 + Q1 sI)^2 - \varepsilon m R2^4 s c^4 (Q0 + Q1 sI)^2 R1^2 \\
& - 2 \varepsilon m R2^5 s c^4 (Q0 + Q1 sI)^2 R1 - 2 \varepsilon m R2^5 s c^4 (Q0 + Q1 sI)^2 r2 ta^2 \\
& + 48 \varepsilon (Q0 + Q1 sI)^2 R1^2 r2^3 R2 + 48 \varepsilon (Q0 + Q1 sI)^2 R1^2 r2^3 ta^2 R2
\end{aligned}$$

$$\begin{aligned}
& + 24 \varepsilon m R2^2 s r2^3 (Q0 + Q1 s1)^2 ta^2 R1 + 6 \varepsilon m R2^4 s^3 c^2 (Q0 + Q1 s1)^2 r2 R1 \\
& + 6 \varepsilon m R2^4 s c^2 r2 (Q0 + Q1 s1)^2 R1 - \varepsilon m R2^4 s c^4 (Q0 + Q1 s1)^2 r2 ta^2 R1 \\
& - 12 \varepsilon (Q0 + Q1 s1) R1^3 r2^2 R2 (q0 + q1 s1) - 24 \varepsilon (Q0 + Q1 s1) R1^2 r2^2 R2^2 (q0 + q1 s1) \\
& + 24 \varepsilon m R2^3 s r2^3 (Q0 + Q1 s1) ta^2 (q0 + q1 s1) + 12 \varepsilon m R2^2 s r2^2 (Q0 + Q1 s1) R1^2 (q0 + q1 s1) \\
& + 24 \varepsilon m R2^3 s r2^2 (Q0 + Q1 s1) R1 (q0 + q1 s1) - 12 \varepsilon m R2^5 s^3 c^2 r2 (Q0 + Q1 s1) (q0 + q1 s1) \\
& + 6 \varepsilon m R2^5 s c^2 r2 (Q0 + Q1 s1) (q0 + q1 s1) + 2 \varepsilon m R2^4 s c^4 (Q0 + Q1 s1) (q0 + q1 s1) R1^2 \\
& + 4 \varepsilon m R2^5 s c^4 (Q0 + Q1 s1) (q0 + q1 s1) R1 + 4 \varepsilon m R2^5 s c^4 (Q0 + Q1 s1) (q0 + q1 s1) r2 ta^2 \\
& + 24 \varepsilon (Q0 + Q1 s1) R1^2 r2^3 (q0 + q1 s1) R2 + 24 \varepsilon (Q0 + Q1 s1) R1^2 r2^3 ta^2 (q0 + q1 s1) R2 \\
& + 12 \varepsilon m R2^2 s r2^3 (Q0 + Q1 s1) ta^2 (q0 + q1 s1) R1 \\
& - 6 \varepsilon m R2^4 s^3 c^2 r2 (Q0 + Q1 s1) (q0 + q1 s1) R1 + 3 \varepsilon m R2^4 s c^2 r2 (Q0 + Q1 s1) (q0 + q1 s1) R1 \\
& + 2 \varepsilon m R2^4 s c^4 (Q0 + Q1 s1) (q0 + q1 s1) r2 ta^2 R1
\end{aligned}$$

> beta1:=coeff(RHS3beta,beta,1);

$$\begin{aligned}
\beta1 := & (q0 + q1 s1) (-r2^2 m R2^4 s^2 c (q0 + q1 s1)^2 - 4 r2^2 m R2^4 s^4 c (Q0 + Q1 s1)^2 \\
& - 4 r2^2 m R2^4 s^2 c (Q0 + Q1 s1)^2 - r2 m R2^5 s^2 c^3 (Q0 + Q1 s1)^2 + 24 r2^4 m R2^2 s ta (Q0 + Q1 s1)^2 \\
& - r2 m R2^5 s^2 c^3 (q0 + q1 s1)^2 + 2 r2^2 m R2^4 s^4 c (q0 + q1 s1)^2 \\
& + 12 r2^4 m R2^2 s ta (Q0 + Q1 s1) (q0 + q1 s1) + 2 r2^2 m R2^4 s^4 c (Q0 + Q1 s1) (q0 + q1 s1) \\
& - 4 r2^2 m R2^4 s^2 c (Q0 + Q1 s1) (q0 + q1 s1) + 2 r2 m R2^5 s^2 c^3 (Q0 + Q1 s1) (q0 + q1 s1))
\end{aligned}$$

> beta2:=collect(beta1,s1);

$$\begin{aligned}
\beta2 := & q1 (2 r2^2 m R2^4 s^4 c q1^2 + 12 r2^4 m R2^2 s ta Q1 q1 - r2 m R2^5 s^2 c^3 Q1^2 - 4 r2^2 m R2^4 s^2 c Q1^2 \\
& + 2 r2^2 m R2^4 s^4 c Q1 q1 + 24 r2^4 m R2^2 s ta Q1^2 - r2^2 m R2^4 s^2 c q1^2 - 4 r2^2 m R2^4 s^2 c Q1 q1 \\
& - r2 m R2^5 s^2 c^3 q1^2 - 4 r2^2 m R2^4 s^4 c Q1^2 + 2 r2 m R2^5 s^2 c^3 Q1 q1) s1^3 + (q0 (2 r2^2 m R2^4 s^4 \\
& c q1^2 + 12 r2^4 m R2^2 s ta Q1 q1 - r2 m R2^5 s^2 c^3 Q1^2 - 4 r2^2 m R2^4 s^2 c Q1^2 \\
& + 2 r2^2 m R2^4 s^4 c Q1 q1 + 24 r2^4 m R2^2 s ta Q1^2 - r2^2 m R2^4 s^2 c q1^2 - 4 r2^2 m R2^4 s^2 c Q1 q1 \\
& - r2 m R2^5 s^2 c^3 q1^2 - 4 r2^2 m R2^4 s^4 c Q1^2 + 2 r2 m R2^5 s^2 c^3 Q1 q1) + q1 ( \\
& - 2 r2 m R2^5 s^2 c^3 Q0 Q1 - 8 r2^2 m R2^4 s^2 c Q0 Q1 + 48 r2^4 m R2^2 s ta Q0 Q1 - 2 r2^2 m R2^4 s^2 c q0 q1 \\
& - 4 r2^2 m R2^4 s^2 c Q0 q1 - 4 r2^2 m R2^4 s^2 c Q1 q0 - 2 r2 m R2^5 s^2 c^3 q0 q1 \\
& - 8 r2^2 m R2^4 s^4 c Q0 Q1 + 2 r2 m R2^5 s^2 c^3 Q0 q1 + 2 r2 m R2^5 s^2 c^3 Q1 q0 \\
& + 4 r2^2 m R2^4 s^4 c q0 q1 + 2 r2^2 m R2^4 s^4 c Q0 q1 + 2 r2^2 m R2^4 s^4 c Q1 q0 \\
& + 12 r2^4 m R2^2 s ta Q0 q1 + 12 r2^4 m R2^2 s ta Q1 q0) s1^2 + (q0 (-2 r2 m R2^5 s^2 c^3 Q0 Q1 \\
& - 8 r2^2 m R2^4 s^2 c Q0 Q1 + 48 r2^4 m R2^2 s ta Q0 Q1 - 2 r2^2 m R2^4 s^2 c q0 q1 \\
& - 4 r2^2 m R2^4 s^2 c Q0 q1 - 4 r2^2 m R2^4 s^2 c Q1 q0 - 2 r2 m R2^5 s^2 c^3 q0 q1 \\
& - 8 r2^2 m R2^4 s^4 c Q0 Q1 + 2 r2 m R2^5 s^2 c^3 Q0 q1 + 2 r2 m R2^5 s^2 c^3 Q1 q0 \\
& + 4 r2^2 m R2^4 s^4 c q0 q1 + 2 r2^2 m R2^4 s^4 c Q0 q1 + 2 r2^2 m R2^4 s^4 c Q1 q0 \\
& + 12 r2^4 m R2^2 s ta Q0 q1 + 12 r2^4 m R2^2 s ta Q1 q0) + q1 (-r2^2 m R2^4 s^2 c q0^2 \\
& - 4 r2^2 m R2^4 s^2 c Q0 q0 - r2 m R2^5 s^2 c^3 q0^2 - 4 r2^2 m R2^4 s^4 c Q0^2 + 2 r2 m R2^5 s^2 c^3 Q0 q0 \\
& + 2 r2^2 m R2^4 s^4 c q0^2 - 4 r2^2 m R2^4 s^2 c Q0^2 + 2 r2^2 m R2^4 s^4 c Q0 q0 \\
& + 12 r2^4 m R2^2 s ta Q0 q0 - r2 m R2^5 s^2 c^3 Q0^2 + 24 r2^4 m R2^2 s ta Q0^2)) s1 + q0 ( \\
& - r2^2 m R2^4 s^2 c q0^2 - 4 r2^2 m R2^4 s^2 c Q0 q0 - r2 m R2^5 s^2 c^3 q0^2 - 4 r2^2 m R2^4 s^4 c Q0^2 \\
& + 2 r2 m R2^5 s^2 c^3 Q0 q0 + 2 r2^2 m R2^4 s^4 c q0^2 - 4 r2^2 m R2^4 s^2 c Q0^2 \\
& + 2 r2^2 m R2^4 s^4 c Q0 q0 + 12 r2^4 m R2^2 s ta Q0 q0 - r2 m R2^5 s^2 c^3 Q0^2 \\
& + 24 r2^4 m R2^2 s ta Q0^2)
\end{aligned}$$

```

> epsilon1:=coeff(RHS3epsilon,epsilon,1);

$$\varepsilon_1 := (q0 + q1 s1) (24 (Q0 + Q1 s1)^2 R1^3 r2^3 + 12 (Q0 + Q1 s1) R1^3 r2^3 (q0 + q1 s1) + 24 (Q0 + Q1 s1)^2 R1^3 r2^3 ta^2 + 12 (Q0 + Q1 s1)^2 R1^3 r2^2 R2 + 24 (Q0 + Q1 s1)^2 R1^2 r2^2 R2^2 + 48 (Q0 + Q1 s1)^2 R1^2 r2^3 R2 - m R2^4 s c^4 (q0 + q1 s1)^2 R1^2 - 2 m R2^5 s c^4 (q0 + q1 s1)^2 R1 - m R2^4 s c^4 (q0 + q1 s1)^2 r2 ta^2 R1 - 2 m R2^5 s c^4 (q0 + q1 s1)^2 r2 ta^2 + 12 (Q0 + Q1 s1) R1^3 r2^3 ta^2 (q0 + q1 s1) + 48 m R2^3 s r2^3 (Q0 + Q1 s1)^2 ta^2 + 12 m R2^5 s c^2 r2 (Q0 + Q1 s1)^2 - m R2^4 s c^4 (Q0 + Q1 s1)^2 R1^2 - 12 m R2^2 s r2^2 (Q0 + Q1 s1)^2 R1^2 - 24 m R2^3 s r2^2 (Q0 + Q1 s1)^2 R1 + 12 m R2^5 s^3 c^2 (Q0 + Q1 s1)^2 r2 + 6 m R2^4 s^3 c^2 (Q0 + Q1 s1)^2 r2 R1 + 6 m R2^4 s c^2 r2 (Q0 + Q1 s1)^2 R1 - 12 (Q0 + Q1 s1) R1^3 r2^2 R2 (q0 + q1 s1) - 2 m R2^5 s c^4 (Q0 + Q1 s1)^2 R1 - 2 m R2^5 s c^4 (Q0 + Q1 s1)^2 r2 ta^2 + 48 (Q0 + Q1 s1)^2 R1^2 r2^3 ta^2 R2 + 24 m R2^2 s r2^3 (Q0 + Q1 s1)^2 ta^2 R1 + 12 m R2^2 s r2^2 (Q0 + Q1 s1) R1^2 (q0 + q1 s1) + 24 m R2^3 s r2^2 (Q0 + Q1 s1) R1 (q0 + q1 s1) - 12 m R2^5 s^3 c^2 r2 (Q0 + Q1 s1) (q0 + q1 s1) + 6 m R2^5 s c^2 r2 (Q0 + Q1 s1) (q0 + q1 s1) - m R2^4 s c^4 (Q0 + Q1 s1)^2 r2 ta^2 R1 - 24 (Q0 + Q1 s1) R1^2 r2^2 R2^2 (q0 + q1 s1) + 24 m R2^3 s r2^3 (Q0 + Q1 s1) ta^2 (q0 + q1 s1) + 2 m R2^4 s c^4 (Q0 + Q1 s1) (q0 + q1 s1) R1^2 + 4 m R2^5 s c^4 (Q0 + Q1 s1) (q0 + q1 s1) R1 + 4 m R2^5 s c^4 (Q0 + Q1 s1) (q0 + q1 s1) r2 ta^2 + 24 (Q0 + Q1 s1) R1^2 r2^3 (q0 + q1 s1) R2 + 24 (Q0 + Q1 s1) R1^2 r2^3 ta^2 (q0 + q1 s1) R2 + 12 m R2^2 s r2^3 (Q0 + Q1 s1) ta^2 (q0 + q1 s1) R1 - 6 m R2^4 s^3 c^2 r2 (Q0 + Q1 s1) (q0 + q1 s1) R1 + 3 m R2^4 s c^2 r2 (Q0 + Q1 s1) (q0 + q1 s1) R1 + 2 m R2^4 s c^4 (Q0 + Q1 s1) (q0 + q1 s1) r2 ta^2 R1$$

> epsilon2:=collect(epsilon1,s1);

$$\varepsilon_2 := q1 (24 Q1^2 R1^3 r2^3 + 12 Q1 R1^3 r2^3 q1 + 24 Q1^2 R1^3 r2^3 ta^2 + 12 Q1^2 R1^3 r2^2 R2 + 24 Q1^2 R1^2 r2^2 R2^2 + 48 Q1^2 R1^2 r2^3 R2 - m R2^4 s c^4 q1^2 R1^2 - 2 m R2^5 s c^4 q1^2 R1 + 12 m R2^5 s c^2 r2 Q1^2 - m R2^4 s c^4 q1^2 r2 ta^2 R1 - 2 m R2^5 s c^4 q1^2 r2 ta^2 + 12 Q1 R1^3 r2^3 ta^2 q1 + 48 m R2^3 s r2^3 Q1^2 ta^2 - m R2^4 s c^4 Q1^2 R1^2 - 12 m R2^2 s r2^2 Q1^2 R1^2 - 24 m R2^3 s r2^2 Q1^2 R1 + 12 m R2^5 s^3 c^2 Q1^2 r2 + 6 m R2^4 s^3 c^2 Q1^2 r2 R1 + 6 m R2^4 s c^2 r2 Q1^2 R1 - 12 Q1 R1^3 r2^2 R2 q1 - 2 m R2^5 s c^4 Q1^2 R1 - 2 m R2^5 s c^4 Q1^2 r2 ta^2 + 48 Q1^2 R1^2 r2^3 ta^2 R2 + 4 m R2^5 s c^4 Q1 q1 R1 + 24 m R2^2 s r2^3 Q1^2 ta^2 R1 + 12 m R2^2 s r2^2 Q1 R1^2 q1 + 24 m R2^3 s r2^2 Q1 R1 q1 - 12 m R2^5 s^3 c^2 r2 Q1 q1 + 6 m R2^5 s c^2 r2 Q1 q1 - m R2^4 s c^4 Q1^2 r2 ta^2 R1 - 24 Q1 R1^2 r2^2 R2^2 q1 + 24 m R2^3 s r2^3 Q1 ta^2 q1 + 2 m R2^4 s c^4 Q1 q1 R1^2 + 4 m R2^5 s c^4 Q1 q1 r2 ta^2 + 24 Q1 R1^2 r2^3 q1 R2 + 24 Q1 R1^2 r2^3 ta^2 q1 R2 + 12 m R2^2 s r2^3 Q1 ta^2 q1 R1 - 6 m R2^4 s^3 c^2 r2 Q1 q1 R1 + 3 m R2^4 s c^2 r2 Q1 q1 R1 + 2 m R2^4 s c^4 Q1 q1 r2 ta^2 R1) s1^3 + (q0 (24 Q1^2 R1^3 r2^3 + 12 Q1 R1^3 r2^3 q1 + 24 Q1^2 R1^3 r2^3 ta^2 + 12 Q1^2 R1^3 r2^2 R2 + 24 Q1^2 R1^2 r2^2 R2^2 + 48 Q1^2 R1^2 r2^3 R2 - m R2^4 s c^4 q1^2 R1^2 - 2 m R2^5 s c^4 q1^2 R1 + 12 m R2^5 s c^2 r2 Q1^2 - m R2^4 s c^4 q1^2 r2 ta^2 R1 - 2 m R2^5 s c^4 q1^2 r2 ta^2 + 12 Q1 R1^3 r2^3 ta^2 q1 + 48 m R2^3 s r2^3 Q1^2 ta^2 - m R2^4 s c^4 Q1^2 R1^2 - 12 m R2^2 s r2^2 Q1^2 R1^2 - 24 m R2^3 s r2^2 Q1^2 R1 + 12 m R2^5 s^3 c^2 Q1^2 r2 + 6 m R2^4 s^3 c^2 Q1^2 r2 R1 + 6 m R2^4 s c^2 r2 Q1^2 R1 - 12 Q1 R1^3 r2^2 R2 q1 - 2 m R2^5 s c^4 Q1^2 R1 - 2 m R2^5 s c^4 Q1^2 r2 ta^2 + 48 Q1^2 R1^2 r2^3 ta^2 R2 + 4 m R2^5 s c^4 Q1 q1 R1)$$


```

$$\begin{aligned}
& + 24 m R2^2 s r2^3 Q1^2 ta^2 R1 + 12 m R2^2 s r2^2 Q1 R1^2 q1 + 24 m R2^3 s r2^2 Q1 R1 q1 \\
& - 12 m R2^5 s^3 c^2 r2 Q1 q1 + 6 m R2^5 s c^2 r2 Q1 q1 - m R2^4 s c^4 Q1^2 r2 ta^2 R1 \\
& - 24 Q1 R1^2 r2^2 R2^2 q1 + 24 m R2^3 s r2^3 Q1 ta^2 q1 + 2 m R2^4 s c^4 Q1 q1 R1^2 \\
& + 4 m R2^5 s c^4 Q1 q1 r2 ta^2 + 24 Q1 R1^2 r2^3 q1 R2 + 24 Q1 R1^2 r2^3 ta^2 q1 R2 \\
& + 12 m R2^2 s r2^3 Q1 ta^2 q1 R1 - 6 m R2^4 s^3 c^2 r2 Q1 q1 R1 + 3 m R2^4 s c^2 r2 Q1 q1 R1 \\
& + 2 m R2^4 s c^4 Q1 q1 r2 ta^2 R1) + q1 (4 (m R2^5 s c^4 Q0 q1 + m R2^5 s c^4 Q1 q0) r2 ta^2 \\
& + 48 Q0 Q1 R1^3 r2^3 + 12 Q0 R1^3 r2^3 q1 + 12 Q1 R1^3 r2^3 q0 + 48 Q0 Q1 R1^3 r2^3 ta^2 \\
& + 24 Q0 Q1 R1^3 r2^2 R2 + 48 Q0 Q1 R1^2 r2^2 R2^2 + 96 Q0 Q1 R1^2 r2^3 R2 - 2 m R2^4 s c^4 q0 q1 R1^2 \\
& - 4 m R2^5 s c^4 q0 q1 R1 - 2 m R2^4 s c^4 q0 q1 r2 ta^2 R1 - 4 m R2^5 s c^4 q0 q1 r2 ta^2 \\
& + 12 Q0 R1^3 r2^3 ta^2 q1 + 12 Q1 R1^3 r2^3 ta^2 q0 + 96 m R2^3 s r2^3 Q0 Q1 ta^2 \\
& + 24 m R2^5 s c^2 r2 Q0 Q1 - 2 m R2^4 s c^4 Q0 Q1 R1^2 - 24 m R2^2 s r2^2 Q0 Q1 R1^2 \\
& - 48 m R2^3 s r2^2 Q0 Q1 R1 + 24 m R2^5 s^3 c^2 Q0 Q1 r2 + 12 m R2^4 s^3 c^2 Q0 Q1 r2 R1 \\
& + 12 m R2^4 s c^2 r2 Q0 Q1 R1 - 12 Q0 R1^3 r2^2 R2 q1 - 12 Q1 R1^3 r2^2 R2 q0 \\
& - 4 m R2^5 s c^4 Q0 Q1 R1 - 4 m R2^5 s c^4 Q0 Q1 r2 ta^2 \\
& + 2 (m R2^4 s c^4 Q0 q1 + m R2^4 s c^4 Q1 q0) r2 ta^2 R1 + 96 Q0 Q1 R1^2 r2^3 ta^2 R2 \\
& + 48 m R2^2 s r2^3 Q0 Q1 ta^2 R1 + 12 m R2^2 s r2^2 Q0 R1^2 q1 + 12 m R2^2 s r2^2 Q1 R1^2 q0 \\
& + 24 m R2^3 s r2^2 Q0 R1 q1 + 24 m R2^3 s r2^2 Q1 R1 q0 - 12 m R2^5 s^3 c^2 r2 Q0 q1 \\
& - 12 m R2^5 s^3 c^2 r2 Q1 q0 + 6 m R2^5 s c^2 r2 Q0 q1 + 6 m R2^5 s c^2 r2 Q1 q0 \\
& - 2 m R2^4 s c^4 Q0 Q1 r2 ta^2 R1 + 2 (m R2^4 s c^4 Q0 q1 + m R2^4 s c^4 Q1 q0) R1^2 \\
& + 4 (m R2^5 s c^4 Q0 q1 + m R2^5 s c^4 Q1 q0) R1 + 24 (Q0 R1^2 r2^3 q1 + Q1 R1^2 r2^3 q0) R2 \\
& + 24 (Q0 R1^2 r2^3 ta^2 q1 + Q1 R1^2 r2^3 ta^2 q0) R2 \\
& + 12 (m R2^2 s r2^3 Q0 ta^2 q1 + m R2^2 s r2^3 Q1 ta^2 q0) R1 - 24 Q0 R1^2 r2^2 R2^2 q1 \\
& - 24 Q1 R1^2 r2^2 R2^2 q0 + 24 m R2^3 s r2^3 Q0 ta^2 q1 + 24 m R2^3 s r2^3 Q1 ta^2 q0 \\
& - 6 (m R2^4 s^3 c^2 r2 Q0 q1 + m R2^4 s^3 c^2 r2 Q1 q0) R1 \\
& + 3 (m R2^4 s c^2 r2 Q0 q1 + m R2^4 s c^2 r2 Q1 q0) R1)) s1^2 + (q0 (4 (m R2^5 s c^4 Q0 q1 \\
& + m R2^5 s c^4 Q1 q0) r2 ta^2 + 48 Q0 Q1 R1^3 r2^3 + 12 Q0 R1^3 r2^3 q1 + 12 Q1 R1^3 r2^3 q0 \\
& + 48 Q0 Q1 R1^3 r2^3 ta^2 + 24 Q0 Q1 R1^3 r2^2 R2 + 48 Q0 Q1 R1^2 r2^2 R2^2 + 96 Q0 Q1 R1^2 r2^3 R2 \\
& - 2 m R2^4 s c^4 q0 q1 R1^2 - 4 m R2^5 s c^4 q0 q1 R1 - 2 m R2^4 s c^4 q0 q1 r2 ta^2 R1 \\
& - 4 m R2^5 s c^4 q0 q1 r2 ta^2 + 12 Q0 R1^3 r2^3 ta^2 q1 + 12 Q1 R1^3 r2^3 ta^2 q0 \\
& + 96 m R2^3 s r2^3 Q0 Q1 ta^2 + 24 m R2^5 s c^2 r2 Q0 Q1 - 2 m R2^4 s c^4 Q0 Q1 R1^2 \\
& - 24 m R2^2 s r2^2 Q0 Q1 R1^2 - 48 m R2^3 s r2^2 Q0 Q1 R1 + 24 m R2^5 s^3 c^2 Q0 Q1 r2 \\
& + 12 m R2^4 s^3 c^2 Q0 Q1 r2 R1 + 12 m R2^4 s c^2 r2 Q0 Q1 R1 - 12 Q0 R1^3 r2^2 R2 q1 \\
& - 12 Q1 R1^3 r2^2 R2 q0 - 4 m R2^5 s c^4 Q0 Q1 R1 - 4 m R2^5 s c^4 Q0 Q1 r2 ta^2 \\
& + 2 (m R2^4 s c^4 Q0 q1 + m R2^4 s c^4 Q1 q0) r2 ta^2 R1 + 96 Q0 Q1 R1^2 r2^3 ta^2 R2 \\
& + 48 m R2^2 s r2^3 Q0 Q1 ta^2 R1 + 12 m R2^2 s r2^2 Q0 R1^2 q1 + 12 m R2^2 s r2^2 Q1 R1^2 q0 \\
& + 24 m R2^3 s r2^2 Q0 R1 q1 + 24 m R2^3 s r2^2 Q1 R1 q0 - 12 m R2^5 s^3 c^2 r2 Q0 q1 \\
& - 12 m R2^5 s^3 c^2 r2 Q1 q0 + 6 m R2^5 s c^2 r2 Q0 q1 + 6 m R2^5 s c^2 r2 Q1 q0 \\
& - 2 m R2^4 s c^4 Q0 Q1 r2 ta^2 R1 + 2 (m R2^4 s c^4 Q0 q1 + m R2^4 s c^4 Q1 q0) R1^2 \\
& + 4 (m R2^5 s c^4 Q0 q1 + m R2^5 s c^4 Q1 q0) R1 + 24 (Q0 R1^2 r2^3 q1 + Q1 R1^2 r2^3 q0) R2 \\
& + 24 (Q0 R1^2 r2^3 ta^2 q1 + Q1 R1^2 r2^3 ta^2 q0) R2 \\
& + 12 (m R2^2 s r2^3 Q0 ta^2 q1 + m R2^2 s r2^3 Q1 ta^2 q0) R1 - 24 Q0 R1^2 r2^2 R2^2 q1 \\
& - 24 Q1 R1^2 r2^2 R2^2 q0 + 24 m R2^3 s r2^3 Q0 ta^2 q1 + 24 m R2^3 s r2^3 Q1 ta^2 q0
\end{aligned}$$

$$\begin{aligned}
& -6(mR2^4s^3c^2r2Q0q1 + mR2^4s^3c^2r2Q1q0)R1 \\
& + 3(mR2^4sc^2r2Q0q1 + mR2^4sc^2r2Q1q0)R1 + q1(24Q0^2R1^3r2^3 - 12Q0R1^3r2^2R2q0 \\
& + 12Q0R1^3r2^3q0 + 24Q0^2R1^3r2^3ta^2 + 12Q0^2R1^3r2^2R2 + 24Q0^2R1^2r2^2R2^2 \\
& + 48Q0^2R1^2r2^3R2 - mR2^4sc^4q0^2R1^2 - 2mR2^5sc^4q0^2R1 + 12mR2^5sc^2r2Q0^2 \\
& - mR2^4sc^4q0^2r2ta^2R1 - 2mR2^5sc^4q0^2r2ta^2 + 12Q0R1^3r2^3ta^2q0 \\
& + 48mR2^3sr2^3Q0^2ta^2 - mR2^4sc^4Q0^2R1^2 - 12mR2^2sr2^2Q0^2R1^2 \\
& - 24mR2^3sr2^2Q0^2R1 + 12mR2^5s^3c^2Q0^2r2 + 6mR2^4s^3c^2Q0^2r2R1 \\
& + 6mR2^4sc^2r2Q0^2R1 - 2mR2^5sc^4Q0^2R1 - 2mR2^5sc^4Q0^2r2ta^2 \\
& + 48Q0^2R1^2r2^3ta^2R2 + 4mR2^5sc^4Q0q0R1 + 24mR2^2sr2^3Q0^2ta^2R1 \\
& + 12mR2^2sr2^2Q0R1^2q0 + 24mR2^3sr2^2Q0R1q0 - 12mR2^5s^3c^2r2Q0q0 \\
& + 6mR2^5sc^2r2Q0q0 - mR2^4sc^4Q0^2r2ta^2R1 - 24Q0R1^2r2^2R2^2q0 \\
& + 24mR2^3sr2^3Q0ta^2q0 + 2mR2^4sc^4Q0q0R1^2 + 4mR2^5sc^4Q0q0r2ta^2 \\
& + 24Q0R1^2r2^3q0R2 + 24Q0R1^2r2^3ta^2q0R2 + 12mR2^2sr2^3Q0ta^2q0R1 \\
& - 6mR2^4s^3c^2r2Q0q0R1 + 3mR2^4sc^2r2Q0q0R1 + 2mR2^4sc^4Q0q0r2ta^2R1) s1 + q0 \\
& (24Q0^2R1^3r2^3 - 12Q0R1^3r2^2R2q0 + 12Q0R1^3r2^3q0 + 24Q0^2R1^3r2^3ta^2 \\
& + 12Q0^2R1^3r2^2R2 + 24Q0^2R1^2r2^2R2^2 + 48Q0^2R1^2r2^3R2 - mR2^4sc^4q0^2R1^2 \\
& - 2mR2^5sc^4q0^2R1 + 12mR2^5sc^2r2Q0^2 - mR2^4sc^4q0^2r2ta^2R1 - 2mR2^5sc^4q0^2r2ta^2 \\
& + 12Q0R1^3r2^3ta^2q0 + 48mR2^3sr2^3Q0^2ta^2 - mR2^4sc^4Q0^2R1^2 \\
& - 12mR2^2sr2^2Q0^2R1^2 - 24mR2^3sr2^2Q0^2R1 + 12mR2^5s^3c^2Q0^2r2 \\
& + 6mR2^4s^3c^2Q0^2r2R1 + 6mR2^4sc^2r2Q0^2R1 - 2mR2^5sc^4Q0^2R1 \\
& - 2mR2^5sc^4Q0^2r2ta^2 + 48Q0^2R1^2r2^3ta^2R2 + 4mR2^5sc^4Q0q0R1 \\
& + 24mR2^2sr2^3Q0^2ta^2R1 + 12mR2^2sr2^2Q0R1^2q0 + 24mR2^3sr2^2Q0R1q0 \\
& - 12mR2^5s^3c^2r2Q0q0 + 6mR2^5sc^2r2Q0q0 - mR2^4sc^4Q0^2r2ta^2R1 \\
& - 24Q0R1^2r2^2R2^2q0 + 24mR2^3sr2^3Q0ta^2q0 + 2mR2^4sc^4Q0q0R1^2 \\
& + 4mR2^5sc^4Q0q0r2ta^2 + 24Q0R1^2r2^3q0R2 + 24Q0R1^2r2^3ta^2q0R2 \\
& + 12mR2^2sr2^3Q0ta^2q0R1 - 6mR2^4s^3c^2r2Q0q0R1 + 3mR2^4sc^2r2Q0q0R1 \\
& + 2mR2^4sc^4Q0q0r2ta^2R1)
\end{aligned}$$

```

> D5:=coeff(LHS3,s1^5);
D5 := 0

> D4:=coeff(LHS3,s1^4);
D4 := 0

> D3:=coeff(LHS3,s1^3);
D3 := 0

> D2:=coeff(LHS3,s1^2);
D2 := 4r2^2(2r2Q1+r2q1+2r2ta^2Q1+r2ta^2q1+R2Q1-R2q1)(R1^2+mR2^2)(R1+2R2)(2Q1
+q1)

> D1:=coeff(LHS3,s1);
D1 := 4r2^2(2r2Q0+r2q0+2r2ta^2Q0+r2ta^2q0+R2Q0-R2q0)(R1^2+mR2^2)(R1+2R2)(2Q1
+q1)+4r2^2(2r2Q1+r2q1+2r2ta^2Q1+r2ta^2q1+R2Q1-R2q1)(R1^2+mR2^2)(R1+2R2)

```

(2  $Q0 + q0$ )

```

> D0:=coeff(LHS3,s1,0);
D0 := 4  $r2^2$  (2  $r2 Q0 + r2 q0 + 2 r2 ta^2 Q0 + r2 ta^2 q0 + R2 Q0 - R2 q0$ ) ( $R1^2 + m R2^2$ ) ( $R1 + 2 R2$ ) (2  $Q0 + q0$ )

```

```

> E5:=coeff(epsilon2,s1^5);
E5 := 0

```

```

> E4:=coeff(epsilon2,s1^4);
E4 := 0

```

```

> E3:=coeff(epsilon2,s1^3);
E3 := q1 (24  $Q1^2 R1^3 r2^3 + 12 Q1 R1^3 r2^3 q1 + 24 Q1^2 R1^3 r2^3 ta^2 + 12 Q1^2 R1^3 r2^2 R2 + 24 Q1^2 R1^2 r2^2 R2^2 + 48 Q1^2 R1^2 r2^3 R2 - m R2^4 s c^4 q1^2 R1^2 - 2 m R2^5 s c^4 q1^2 R1 + 12 m R2^5 s c^2 r2 Q1^2 - m R2^4 s c^4 q1^2 r2 ta^2 R1 - 2 m R2^5 s c^4 q1^2 r2 ta^2 + 12 Q1 R1^3 r2^3 ta^2 q1 + 48 m R2^3 s r2^3 Q1^2 ta^2 - m R2^4 s c^4 Q1^2 R1^2 - 12 m R2^2 s r2^2 Q1^2 R1^2 - 24 m R2^3 s r2^2 Q1^2 R1 + 12 m R2^5 s^3 c^2 Q1^2 r2 + 6 m R2^4 s^3 c^2 Q1^2 r2 R1 + 6 m R2^4 s c^2 r2 Q1^2 R1 - 12 Q1 R1^3 r2^2 R2 q1 - 2 m R2^5 s c^4 Q1^2 R1 - 2 m R2^5 s c^4 Q1^2 r2 ta^2 + 48 Q1^2 R1^2 r2^3 ta^2 R2 + 4 m R2^5 s c^4 Q1 q1 R1 + 24 m R2^2 s r2^3 Q1^2 ta^2 R1 + 12 m R2^2 s r2^2 Q1 R1^2 q1 + 24 m R2^3 s r2^2 Q1 R1 q1 - 12 m R2^5 s^3 c^2 r2 Q1 q1 + 6 m R2^5 s c^2 r2 Q1 q1 - m R2^4 s c^4 Q1^2 r2 ta^2 R1 - 24 Q1 R1^2 r2^2 R2^2 q1 + 24 m R2^3 s r2^3 Q1 ta^2 q1 + 2 m R2^4 s c^4 Q1 q1 R1^2 + 4 m R2^5 s c^4 Q1 q1 r2 ta^2 + 24 Q1 R1^2 r2^3 q1 R2 + 24 Q1 R1^2 r2^3 ta^2 q1 R2 + 12 m R2^2 s r2^3 Q1 ta^2 q1 R1 - 6 m R2^4 s^3 c^2 r2 Q1 q1 R1 + 3 m R2^4 s c^2 r2 Q1 q1 R1 + 2 m R2^4 s c^4 Q1 q1 r2 ta^2 R1)$ 
```

```

> E2:=coeff(epsilon2,s1^2);
E2 := q0 (24  $Q1^2 R1^3 r2^3 + 12 Q1 R1^3 r2^3 q1 + 24 Q1^2 R1^3 r2^3 ta^2 + 12 Q1^2 R1^3 r2^2 R2 + 24 Q1^2 R1^2 r2^2 R2^2 + 48 Q1^2 R1^2 r2^3 R2 - m R2^4 s c^4 q1^2 R1^2 - 2 m R2^5 s c^4 q1^2 R1 + 12 m R2^5 s c^2 r2 Q1^2 - m R2^4 s c^4 q1^2 r2 ta^2 R1 - 2 m R2^5 s c^4 q1^2 r2 ta^2 + 12 Q1 R1^3 r2^3 ta^2 q1 + 48 m R2^3 s r2^3 Q1^2 ta^2 - m R2^4 s c^4 Q1^2 R1^2 - 12 m R2^2 s r2^2 Q1^2 R1^2 - 24 m R2^3 s r2^2 Q1^2 R1 + 12 m R2^5 s^3 c^2 Q1^2 r2 + 6 m R2^4 s^3 c^2 Q1^2 r2 R1 + 6 m R2^4 s c^2 r2 Q1^2 R1 - 12 Q1 R1^3 r2^2 R2 q1 - 2 m R2^5 s c^4 Q1^2 R1 - 2 m R2^5 s c^4 Q1^2 r2 ta^2 + 48 Q1^2 R1^2 r2^3 ta^2 R2 + 4 m R2^5 s c^4 Q1 q1 R1 + 24 m R2^2 s r2^3 Q1^2 ta^2 R1 + 12 m R2^2 s r2^2 Q1 R1^2 q1 + 24 m R2^3 s r2^2 Q1 R1 q1 - 12 m R2^5 s^3 c^2 r2 Q1 q1 + 6 m R2^5 s c^2 r2 Q1 q1 - m R2^4 s c^4 Q1^2 r2 ta^2 R1 - 24 Q1 R1^2 r2^2 R2^2 q1 + 24 m R2^3 s r2^3 Q1 ta^2 q1 + 2 m R2^4 s c^4 Q1 q1 R1^2 + 4 m R2^5 s c^4 Q1 q1 r2 ta^2 + 24 Q1 R1^2 r2^3 q1 R2 + 24 Q1 R1^2 r2^3 ta^2 q1 R2 + 12 m R2^2 s r2^3 Q1 ta^2 q1 R1 - 6 m R2^4 s^3 c^2 r2 Q1 q1 R1 + 3 m R2^4 s c^2 r2 Q1 q1 R1 + 2 m R2^4 s c^4 Q1 q1 r2 ta^2 R1) + q1 (4 (m R2^5 s c^4 Q0 q1 + m R2^5 s c^4 Q1 q0) r2 ta^2 + 48 Q0 Q1 R1^3 r2^3 + 12 Q0 R1^3 r2^3 q1 + 12 Q1 R1^3 r2^3 q0 + 48 Q0 Q1 R1^3 r2^3 ta^2 + 24 Q0 Q1 R1^3 r2^2 R2 + 48 Q0 Q1 R1^2 r2^2 R2^2 + 96 Q0 Q1 R1^2 r2^3 R2 - 2 m R2^4 s c^4 q0 q1 R1^2 - 4 m R2^5 s c^4 q0 q1 R1 - 2 m R2^4 s c^4 q0 q1 r2 ta^2 R1 - 4 m R2^5 s c^4 q0 q1 r2 ta^2 + 12 Q0 R1^3 r2^3 ta^2 q1 + 12 Q1 R1^3 r2^3 ta^2 q0 + 96 m R2^3 s r2^3 Q0 Q1 ta^2)$ 
```

$$\begin{aligned}
& + 24 m R2^5 s c^2 r2 Q0 Q1 - 2 m R2^4 s c^4 Q0 Q1 R1^2 - 24 m R2^2 s r2^2 Q0 Q1 R1^2 \\
& - 48 m R2^3 s r2^2 Q0 Q1 R1 + 24 m R2^5 s^3 c^2 Q0 Q1 r2 + 12 m R2^4 s^3 c^2 Q0 Q1 r2 R1 \\
& + 12 m R2^4 s c^2 r2 Q0 Q1 R1 - 12 Q0 R1^3 r2^2 R2 q1 - 12 Q1 R1^3 r2^2 R2 q0 \\
& - 4 m R2^5 s c^4 Q0 Q1 R1 - 4 m R2^5 s c^4 Q0 Q1 r2 ta^2 \\
& + 2 (m R2^4 s c^4 Q0 q1 + m R2^4 s c^4 Q1 q0) r2 ta^2 R1 + 96 Q0 Q1 R1^2 r2^3 ta^2 R2 \\
& + 48 m R2^2 s r2^3 Q0 Q1 ta^2 R1 + 12 m R2^2 s r2^2 Q0 R1^2 q1 + 12 m R2^2 s r2^2 Q1 R1^2 q0 \\
& + 24 m R2^3 s r2^2 Q0 R1 q1 + 24 m R2^3 s r2^2 Q1 R1 q0 - 12 m R2^5 s^3 c^2 r2 Q0 q1 \\
& - 12 m R2^5 s^3 c^2 r2 Q1 q0 + 6 m R2^5 s c^2 r2 Q0 q1 + 6 m R2^5 s c^2 r2 Q1 q0 \\
& - 2 m R2^4 s c^4 Q0 Q1 r2 ta^2 R1 + 2 (m R2^4 s c^4 Q0 q1 + m R2^4 s c^4 Q1 q0) R1^2 \\
& + 4 (m R2^5 s c^4 Q0 q1 + m R2^5 s c^4 Q1 q0) R1 + 24 (Q0 R1^2 r2^3 q1 + Q1 R1^2 r2^3 q0) R2 \\
& + 24 (Q0 R1^2 r2^3 ta^2 q1 + Q1 R1^2 r2^3 ta^2 q0) R2 \\
& + 12 (m R2^2 s r2^3 Q0 ta^2 q1 + m R2^2 s r2^3 Q1 ta^2 q0) R1 - 24 Q0 R1^2 r2^2 R2^2 q1 \\
& - 24 Q1 R1^2 r2^2 R2^2 q0 + 24 m R2^3 s r2^3 Q0 ta^2 q1 + 24 m R2^3 s r2^3 Q1 ta^2 q0 \\
& - 6 (m R2^4 s^3 c^2 r2 Q0 q1 + m R2^4 s^3 c^2 r2 Q1 q0) R1 \\
& + 3 (m R2^4 s c^2 r2 Q0 q1 + m R2^4 s c^2 r2 Q1 q0) R1
\end{aligned}$$

> E1:=coeff(epsilon2,s1);

$$\begin{aligned}
E1 := & q0 (4 (m R2^5 s c^4 Q0 q1 + m R2^5 s c^4 Q1 q0) r2 ta^2 + 48 Q0 Q1 R1^3 r2^3 + 12 Q0 R1^3 r2^3 q1 \\
& + 12 Q1 R1^3 r2^3 q0 + 48 Q0 Q1 R1^3 r2^3 ta^2 + 24 Q0 Q1 R1^3 r2^2 R2 + 48 Q0 Q1 R1^2 r2^2 R2^2 \\
& + 96 Q0 Q1 R1^2 r2^3 R2 - 2 m R2^4 s c^4 q0 q1 R1^2 - 4 m R2^5 s c^4 q0 q1 R1 \\
& - 2 m R2^4 s c^4 q0 q1 r2 ta^2 R1 - 4 m R2^5 s c^4 q0 q1 r2 ta^2 + 12 Q0 R1^3 r2^3 ta^2 q1 \\
& + 12 Q1 R1^3 r2^3 ta^2 q0 + 96 m R2^3 s r2^3 Q0 Q1 ta^2 + 24 m R2^5 s c^2 r2 Q0 Q1 \\
& - 2 m R2^4 s c^4 Q0 Q1 R1^2 - 24 m R2^2 s r2^2 Q0 Q1 R1^2 - 48 m R2^3 s r2^2 Q0 Q1 R1 \\
& + 24 m R2^5 s^3 c^2 Q0 Q1 r2 + 12 m R2^4 s^3 c^2 Q0 Q1 r2 R1 + 12 m R2^4 s c^2 r2 Q0 Q1 R1 \\
& - 12 Q0 R1^3 r2^2 R2 q1 - 12 Q1 R1^3 r2^2 R2 q0 - 4 m R2^5 s c^4 Q0 Q1 R1 - 4 m R2^5 s c^4 Q0 Q1 r2 ta^2 \\
& + 2 (m R2^4 s c^4 Q0 q1 + m R2^4 s c^4 Q1 q0) r2 ta^2 R1 + 96 Q0 Q1 R1^2 r2^3 ta^2 R2 \\
& + 48 m R2^2 s r2^3 Q0 Q1 ta^2 R1 + 12 m R2^2 s r2^2 Q0 R1^2 q1 + 12 m R2^2 s r2^2 Q1 R1^2 q0 \\
& + 24 m R2^3 s r2^2 Q0 R1 q1 + 24 m R2^3 s r2^2 Q1 R1 q0 - 12 m R2^5 s^3 c^2 r2 Q0 q1 \\
& - 12 m R2^5 s^3 c^2 r2 Q1 q0 + 6 m R2^5 s c^2 r2 Q0 q1 + 6 m R2^5 s c^2 r2 Q1 q0 \\
& - 2 m R2^4 s c^4 Q0 Q1 r2 ta^2 R1 + 2 (m R2^4 s c^4 Q0 q1 + m R2^4 s c^4 Q1 q0) R1^2 \\
& + 4 (m R2^5 s c^4 Q0 q1 + m R2^5 s c^4 Q1 q0) R1 + 24 (Q0 R1^2 r2^3 q1 + Q1 R1^2 r2^3 q0) R2 \\
& + 24 (Q0 R1^2 r2^3 ta^2 q1 + Q1 R1^2 r2^3 ta^2 q0) R2 \\
& + 12 (m R2^2 s r2^3 Q0 ta^2 q1 + m R2^2 s r2^3 Q1 ta^2 q0) R1 - 24 Q0 R1^2 r2^2 R2^2 q1 \\
& - 24 Q1 R1^2 r2^2 R2^2 q0 + 24 m R2^3 s r2^3 Q0 ta^2 q1 + 24 m R2^3 s r2^3 Q1 ta^2 q0 \\
& - 6 (m R2^4 s^3 c^2 r2 Q0 q1 + m R2^4 s^3 c^2 r2 Q1 q0) R1 \\
& + 3 (m R2^4 s c^2 r2 Q0 q1 + m R2^4 s c^2 r2 Q1 q0) R1) + q1 (24 Q0^2 R1^3 r2^3 - 12 Q0 R1^3 r2^2 R2 q0 \\
& + 12 Q0 R1^3 r2^3 q0 + 24 Q0^2 R1^3 r2^3 ta^2 + 12 Q0^2 R1^3 r2^2 R2 + 24 Q0^2 R1^2 r2^2 R2^2 \\
& + 48 Q0^2 R1^2 r2^3 R2 - m R2^4 s c^4 q0^2 R1^2 - 2 m R2^5 s c^4 q0^2 R1 + 12 m R2^5 s c^2 r2 Q0^2 \\
& - m R2^4 s c^4 q0^2 r2 ta^2 R1 - 2 m R2^5 s c^4 q0^2 r2 ta^2 + 12 Q0 R1^3 r2^3 ta^2 q0 \\
& + 48 m R2^3 s r2^3 Q0^2 ta^2 - m R2^4 s c^4 Q0^2 R1^2 - 12 m R2^2 s r2^2 Q0^2 R1^2 \\
& - 24 m R2^3 s r2^2 Q0^2 R1 + 12 m R2^5 s^3 c^2 Q0^2 r2 + 6 m R2^4 s^3 c^2 Q0^2 r2 R1
\end{aligned}$$

$$\begin{aligned}
& + 6 m R2^4 s c^2 r2 Q0^2 RI - 2 m R2^5 s c^4 Q0^2 R1 - 2 m R2^5 s c^4 Q0^2 r2 ta^2 \\
& + 48 Q0^2 RI^2 r2^3 ta^2 R2 + 4 m R2^5 s c^4 Q0 q0 RI + 24 m R2^2 s r2^3 Q0^2 ta^2 RI \\
& + 12 m R2^2 s r2^2 Q0 RI^2 q0 + 24 m R2^3 s r2^2 Q0 RI q0 - 12 m R2^5 s^3 c^2 r2 Q0 q0 \\
& + 6 m R2^5 s c^2 r2 Q0 q0 - m R2^4 s c^4 Q0^2 r2 ta^2 RI - 24 Q0 RI^2 r2^2 R2^2 q0 \\
& + 24 m R2^3 s r2^3 Q0 ta^2 q0 + 2 m R2^4 s c^4 Q0 q0 RI^2 + 4 m R2^5 s c^4 Q0 q0 r2 ta^2 \\
& + 24 Q0 RI^2 r2^3 q0 R2 + 24 Q0 RI^2 r2^3 ta^2 q0 R2 + 12 m R2^2 s r2^3 Q0 ta^2 q0 RI \\
& - 6 m R2^4 s^3 c^2 r2 Q0 q0 RI + 3 m R2^4 s c^2 r2 Q0 q0 RI + 2 m R2^4 s c^4 Q0 q0 r2 ta^2 RI
\end{aligned}$$

> E0:=coeff(epsilon2,s1,0);

$$\begin{aligned}
E0 := & q0 (24 Q0^2 RI^3 r2^3 - 12 Q0 RI^3 r2^2 R2 q0 + 12 Q0 RI^3 r2^3 q0 + 24 Q0^2 RI^3 r2^3 ta^2 \\
& + 12 Q0^2 RI^3 r2^2 R2 + 24 Q0^2 RI^2 r2^2 R2^2 + 48 Q0^2 RI^2 r2^3 R2 - m R2^4 s c^4 q0^2 RI^2 \\
& - 2 m R2^5 s c^4 q0^2 RI + 12 m R2^5 s c^2 r2 Q0^2 - m R2^4 s c^4 q0^2 r2 ta^2 RI - 2 m R2^5 s c^4 q0^2 r2 ta^2 \\
& + 12 Q0 RI^3 r2^3 ta^2 q0 + 48 m R2^3 s r2^3 Q0^2 ta^2 - m R2^4 s c^4 Q0^2 RI^2 \\
& - 12 m R2^2 s r2^2 Q0^2 RI^2 - 24 m R2^3 s r2^2 Q0^2 RI + 12 m R2^5 s^3 c^2 Q0^2 r2 \\
& + 6 m R2^4 s^3 c^2 Q0^2 r2 RI + 6 m R2^4 s c^2 r2 Q0^2 RI - 2 m R2^5 s c^4 Q0^2 RI \\
& - 2 m R2^5 s c^4 Q0^2 r2 ta^2 + 48 Q0^2 RI^2 r2^3 ta^2 R2 + 4 m R2^5 s c^4 Q0 q0 RI \\
& + 24 m R2^2 s r2^3 Q0^2 ta^2 RI + 12 m R2^2 s r2^2 Q0 RI^2 q0 + 24 m R2^3 s r2^2 Q0 RI q0 \\
& - 12 m R2^5 s^3 c^2 r2 Q0 q0 + 6 m R2^5 s c^2 r2 Q0 q0 - m R2^4 s c^4 Q0^2 r2 ta^2 RI \\
& - 24 Q0 RI^2 r2^2 R2^2 q0 + 24 m R2^3 s r2^3 Q0 ta^2 q0 + 2 m R2^4 s c^4 Q0 q0 RI^2 \\
& + 4 m R2^5 s c^4 Q0 q0 r2 ta^2 + 24 Q0 RI^2 r2^3 q0 R2 + 24 Q0 RI^2 r2^3 ta^2 q0 R2 \\
& + 12 m R2^2 s r2^3 Q0 ta^2 q0 RI - 6 m R2^4 s^3 c^2 r2 Q0 q0 RI + 3 m R2^4 s c^2 r2 Q0 q0 RI \\
& + 2 m R2^4 s c^4 Q0 q0 r2 ta^2 RI)
\end{aligned}$$

> B5:=coeff(beta2,s1^5);

$$B5 := 0$$

> B4:=coeff(beta2,s1^4);

$$B4 := 0$$

> B3:=coeff(beta2,s1^3);

$$\begin{aligned}
B3 := & q1 (2 r2^2 m R2^4 s^4 c q1^2 + 12 r2^4 m R2^2 s ta Q1 q1 - r2 m R2^5 s^2 c^3 Q1^2 - 4 r2^2 m R2^4 s^2 c Q1^2 \\
& + 2 r2^2 m R2^4 s^4 c Q1 q1 + 24 r2^4 m R2^2 s ta Q1^2 - r2^2 m R2^4 s^2 c q1^2 - 4 r2^2 m R2^4 s^2 c Q1 q1 \\
& - r2 m R2^5 s^2 c^3 q1^2 - 4 r2^2 m R2^4 s^4 c Q1^2 + 2 r2 m R2^5 s^2 c^3 Q1 q1)
\end{aligned}$$

> B2:=coeff(beta2,s1^2);

$$\begin{aligned}
B2 := & q0 (2 r2^2 m R2^4 s^4 c q1^2 + 12 r2^4 m R2^2 s ta Q1 q1 - r2 m R2^5 s^2 c^3 Q1^2 - 4 r2^2 m R2^4 s^2 c Q1^2 \\
& + 2 r2^2 m R2^4 s^4 c Q1 q1 + 24 r2^4 m R2^2 s ta Q1^2 - r2^2 m R2^4 s^2 c q1^2 - 4 r2^2 m R2^4 s^2 c Q1 q1 \\
& - r2 m R2^5 s^2 c^3 q1^2 - 4 r2^2 m R2^4 s^4 c Q1^2 + 2 r2 m R2^5 s^2 c^3 Q1 q1) + q1 ( \\
& - 2 r2 m R2^5 s^2 c^3 Q0 Q1 - 8 r2^2 m R2^4 s^2 c Q0 Q1 + 48 r2^4 m R2^2 s ta Q0 Q1 - 2 r2^2 m R2^4 s^2 c q0 q1 \\
& - 4 r2^2 m R2^4 s^2 c Q0 q1 - 4 r2^2 m R2^4 s^2 c Q1 q0 - 2 r2 m R2^5 s^2 c^3 q0 q1 \\
& - 8 r2^2 m R2^4 s^4 c Q0 Q1 + 2 r2 m R2^5 s^2 c^3 Q0 q1 + 2 r2 m R2^5 s^2 c^3 Q1 q0 \\
& + 4 r2^2 m R2^4 s^4 c q0 q1 + 2 r2^2 m R2^4 s^4 c Q0 q1 + 2 r2^2 m R2^4 s^4 c Q1 q0 \\
& + 12 r2^4 m R2^2 s ta Q0 q1 + 12 r2^4 m R2^2 s ta Q1 q0)
\end{aligned}$$

> B1:=coeff(beta2,s1);

```

B1 := q0 (-2 r2 m R25 s2 c3 Q0 Q1 - 8 r22 m R24 s2 c Q0 Q1 + 48 r24 m R22 s ta Q0 Q1
      - 2 r22 m R24 s2 c q0 q1 - 4 r22 m R24 s2 c Q0 q1 - 4 r22 m R24 s2 c Q1 q0
      - 2 r2 m R25 s2 c3 q0 q1 - 8 r22 m R24 s4 c Q0 Q1 + 2 r2 m R25 s2 c3 Q0 q1
      + 2 r2 m R25 s2 c3 Q1 q0 + 4 r22 m R24 s4 c q0 q1 + 2 r22 m R24 s4 c Q0 q1
      + 2 r22 m R24 s4 c Q1 q0 + 12 r24 m R22 s ta Q0 q1 + 12 r24 m R22 s ta Q1 q0) + q1 (
-r22 m R24 s2 c q02 - 4 r22 m R24 s2 c Q0 q0 - r2 m R25 s2 c3 q02 - 4 r22 m R24 s4 c Q02
      + 2 r2 m R25 s2 c3 Q0 q0 + 2 r22 m R24 s4 c q02 - 4 r22 m R24 s2 c Q02
      + 2 r22 m R24 s4 c Q0 q0 + 12 r24 m R22 s ta Q0 q0 - r2 m R25 s2 c3 Q02
      + 24 r24 m R22 s ta Q02)

```

> B0:=coeff(beta2,s1,0);

```

B0 := q0 (-r22 m R24 s2 c q02 - 4 r22 m R24 s2 c Q0 q0 - r2 m R25 s2 c3 q02 - 4 r22 m R24 s4 c Q02
      + 2 r2 m R25 s2 c3 Q0 q0 + 2 r22 m R24 s4 c q02 - 4 r22 m R24 s2 c Q02
      + 2 r22 m R24 s4 c Q0 q0 + 12 r24 m R22 s ta Q0 q0 - r2 m R25 s2 c3 Q02
      + 24 r24 m R22 s ta Q02)

```

[>

[>

[>

&gt;

## Program No. 2

Coefficients of the diff. eq. of the shear  
stress of a helix using two Kelvin models

[&gt;

```
> restart;
> with(linalg):
```

Warning, the protected names norm and trace have been redefined and unprotected

```
> c3:=2*E*m*R2^2/(R1+2*R2)^3/(r2+r2*ta^2+nu*R2)*(ta*(1-2*s^2)/4*R2^2*s^3+n
u*R2^2/4/(1+nu)/r2*(R1+r2*ta^2)*s^4*c-(1+nu)/2*R2^2*s^2*c*(1+s^2)+nu*R2^
2/4/r2*(R1+r2*ta^2)*c^3*(1+s^2)+(r2*ta^2-nu*R1)*r2*c);
c3:=
$$\frac{1}{(R1 + 2 R2)^3 (r2 + r2 ta^2 + \nu R2)} \left( 2 E m R2^2 \left( \frac{1}{4} ta (1 - 2 s^2) R2^2 s^3 + \frac{\nu R2^2 (R1 + r2 ta^2) s^4 c}{4 (1 + \nu) r2} \right. \right.$$


$$\left. \left. - \frac{1}{2} (1 + \nu) R2^2 s^2 c (1 + s^2) + \frac{\nu R2^2 (R1 + r2 ta^2) c^3 (1 + s^2)}{4 r2} + (r2 ta^2 - \nu R1) r2 c \right) \right)$$

> c4:=2*E*R1^4/4/(R1+2*R2)^4/(1+nu)+2*E*r2*m*R2^2/(R1+2*R2)^4/(r2+r2*ta^2+
nu*R2)*((R2^2*(2*s^2-1)*ta^2*s^3)/4/(1+nu)+nu*R2^3*s^5/4/(1+nu)/r2+R2^2/
2*s^3*(1+s^2)+nu*R2^3/4/r2*s*(1+s^2)*c^2+r2^2*s);
c4:=
$$\frac{E R1^4}{2 (R1 + 2 R2)^4 (1 + \nu)} + \frac{1}{(R1 + 2 R2)^4 (r2 + r2 ta^2 + \nu R2)} \left( 2 E r2 m R2^2 \left( \frac{R2^2 (2 s^2 - 1) ta^2 s^3}{4 (1 + \nu)} \right. \right.$$


$$\left. \left. + \frac{\nu R2^3 s^5}{4 (1 + \nu) r2} + \frac{1}{2} R2^2 s^3 (1 + s^2) + \frac{\nu R2^3 s (1 + s^2) c^2}{4 r2} + r2^2 s \right) \right)$$

> tau:=c3*epsilon+c4*beta;
```

$$\tau := \frac{1}{(R1 + 2 R2)^3 (r2 + r2 ta^2 + v R2)} \left( 2 E m R2^2 \left( \frac{1}{4} ta (1 - 2 s^2) R2^2 s^3 + \frac{v R2^2 (R1 + r2 ta^2) s^4 c}{4 (1 + v) r2} \right. \right. \\ \left. \left. - \frac{1}{2} (1 + v) R2^2 s^2 c (1 + s^2) + \frac{v R2^2 (R1 + r2 ta^2) c^3 (1 + s^2)}{4 r2} + (r2 ta^2 - v R1) r2 c \right) \varepsilon \right) + \left( \right. \\ \left. \frac{E RI^4}{2 (R1 + 2 R2)^4 (1 + v)} + \frac{1}{(R1 + 2 R2)^4 (r2 + r2 ta^2 + v R2)} \left( 2 E r2 m R2^2 \left( \frac{R2^2 (2 s^2 - 1) ta^2 s^3}{4 (1 + v)} \right. \right. \right. \\ \left. \left. \left. + \frac{v R2^3 s^5}{4 (1 + v) r2} + \frac{1}{2} R2^2 s^3 (1 + s^2) + \frac{v R2^3 s (1 + s^2) c^2}{4 r2} + r2^2 s \right) \right) \beta \right)$$

> tau1:=subs(E=3\*Q\*q/(2\*p\*Q+q\*P),nu=(p\*Q-q\*P)/(2\*p\*Q+q\*P),tau);

$$\tau1 := \frac{1}{(2 p Q + q P) (R1 + 2 R2)^3 \left( r2 + r2 ta^2 + \frac{(p Q - q P) R2}{2 p Q + q P} \right)} \left( 6 Q q m R2^2 \left( \frac{1}{4} ta (1 - 2 s^2) R2^2 s^3 \right. \right. \\ \left. \left. + \frac{(p Q - q P) R2^2 (R1 + r2 ta^2) s^4 c}{4 (2 p Q + q P) \left( 1 + \frac{p Q - q P}{2 p Q + q P} \right) r2} - \frac{1}{2} \left( 1 + \frac{p Q - q P}{2 p Q + q P} \right) R2^2 s^2 c (1 + s^2) \right. \right. \\ \left. \left. + \frac{(p Q - q P) R2^2 (R1 + r2 ta^2) c^3 (1 + s^2)}{4 (2 p Q + q P) r2} + \left( r2 ta^2 - \frac{(p Q - q P) R1}{2 p Q + q P} \right) r2 c \right) \varepsilon \right) + \left( \right. \\ \left. \frac{3 Q q RI^4}{2 (2 p Q + q P) (R1 + 2 R2)^4 \left( 1 + \frac{p Q - q P}{2 p Q + q P} \right)} + \right. \\ \left. \frac{1}{(2 p Q + q P) (R1 + 2 R2)^4 \left( r2 + r2 ta^2 + \frac{(p Q - q P) R2}{2 p Q + q P} \right)} \left( 6 Q q r2 m R2^2 \left( \frac{R2^2 (2 s^2 - 1) ta^2 s^3}{4 \left( 1 + \frac{p Q - q P}{2 p Q + q P} \right)} \right. \right. \right. \\ \left. \left. \left. + \frac{(p Q - q P) R2^3 s^5}{4 (2 p Q + q P) \left( 1 + \frac{p Q - q P}{2 p Q + q P} \right) r2} + \frac{1}{2} R2^2 s^3 (1 + s^2) + \frac{(p Q - q P) R2^3 s (1 + s^2) c^2}{4 (2 p Q + q P) r2} + r2^2 s \right) \right) \beta \right)$$

```

> tau2:=simplify(tau1);
tau2 := 
$$\frac{1}{2 r^2 p (2 r^2 p Q + r^2 q P + 2 r^2 t a^2 p Q + r^2 t a^2 q P + R^2 p Q - R^2 q P) (R^1 + 2 R^2)^4 (2 p Q + q P)} \quad ((4)$$


$$r^2 \beta R^1 p^2 Q^2 - 24 m R^2 s^5 p^2 Q^2 r^2 - 12 m R^2 s^5 p Q r^2 q P$$


$$+ 12 m R^2 s^3 p^2 Q^2 r^2 + 6 m R^2 s^3 p Q r^2 q P + 2 m R^2 s^4 c p^2 Q^2 R^1$$


$$+ 4 m R^2 s^4 c p^2 Q^2 R^1 + 4 m R^2 s^4 c p^2 Q^2 r^2 t a^2 - m R^2 s^4 c p Q q P R^1$$


$$- 2 m R^2 s^4 c p Q q P R^1 - 2 m R^2 s^4 c p Q P r^2 t a^2 - m R^2 s^4 c q^2 P^2 R^1$$


$$- 2 m R^2 s^4 c q^2 P^2 R^1 - 2 m R^2 s^4 c q^2 P^2 r^2 t a^2 - 36 m R^2 s^4 c p^2 Q^2 s^2 c r^2$$


$$- 36 m R^2 s^4 c p^2 Q^2 s^4 c r^2 + 3 m R^2 s^4 c^3 p^2 Q^2 R^1 - 6 m R^2 s^4 c^3 p^2 Q^2 R^1$$


$$+ 3 m R^2 s^4 c^3 p^2 Q^2 R^1 - 6 m R^2 s^4 c^3 p^2 Q^2 R^1 s^2 + 6 m R^2 s^4 c^3 p^2 Q^2 r^2 t a^2$$


$$+ 6 m R^2 s^4 c^3 p^2 Q^2 r^2 t a^2 s^2 - 3 m R^2 s^4 c^3 p Q R^1 - 6 m R^2 s^4 c^3 p Q R^1 q P$$


$$- 3 m R^2 s^4 c^3 p Q R^1 q P s^2 - 6 m R^2 s^4 c^3 p Q R^1 q P s^2 - 6 m R^2 s^4 c^3 p Q r^2 t a^2 q P$$


$$- 6 m R^2 s^4 c^3 p Q r^2 t a^2 q P s^2 - 12 m R^2 s^4 c p^2 Q^2 R^1 - 12 m R^2 s^4 c p^2 Q^2 R^1$$


$$+ 12 m R^2 s^4 c p^2 Q^2 r^2 R^1 - 6 m R^2 s^4 c p Q r^2 q P R^1 + 6 m R^2 s^4 c p^2 Q^2 r^2 R^1$$


$$+ 3 m R^2 s^4 c p^2 Q^2 r^2 q P R^1 + 2 m R^2 s^4 c p^2 Q^2 r^2 t a^2 R^1 - m R^2 s^4 c p Q q P r^2 t a^2 R^1$$


$$- m R^2 s^4 c q^2 P^2 r^2 t a^2 R^1 - 18 m R^2 s^4 c p^2 Q^2 s^2 c r^2 R^1 - 18 m R^2 s^4 c p^2 Q^2 s^4 c r^2 R^1$$


$$+ 3 m R^2 s^4 c^3 p^2 Q^2 r^2 t a^2 R^1 + 3 m R^2 s^4 c^3 p^2 Q^2 r^2 t a^2 s^2 R^1 - 3 m R^2 s^4 c^3 p Q r^2 t a^2 q P R^1$$


$$- 3 m R^2 s^4 c^3 p Q r^2 t a^2 q P s^2 R^1 + 24 m R^2 s^4 c p^2 Q^2 t a^2 R^1 + 48 m R^2 s^4 c r^2 s^2 c p^2 Q^2 t a^2$$


$$+ 12 m R^2 s^4 c r^2 s^2 c p Q t a^2 q P R^1 + 24 m R^2 s^4 c r^2 s^2 c p Q t a^2 q P - 24 m R^2 s^4 c r^2 s^2 c p^2 Q^2 R^1$$


$$+ 24 m R^2 s^4 c r^2 s^2 c p Q R^1 q P + 2 r^2 \beta m R^2 s^5 p^2 Q^2 + 2 r^2 \beta R^1 p^2 Q^2 R^2 + r^2 \beta R^1 q^2 P^2$$


$$- r^2 \beta R^1 q^2 P^2 R^2 + 4 r^2 \beta R^1 p^2 Q^2 t a^2 + 4 r^2 \beta R^1 p^2 Q^2 t a^2 + 8 r^2 \beta m R^2 s^5 t a^2 p^2 Q^2$$


$$+ 8 r^2 \beta m R^2 s^5 t a^2 p Q q P + 2 r^2 \beta m R^2 s^5 t a^2 q^2 P^2 - 4 r^2 \beta m R^2 s^3 t a^2 p^2 Q^2$$


$$- 4 r^2 \beta m R^2 s^3 t a^2 p Q q P - r^2 \beta m R^2 s^3 t a^2 q^2 P^2 - r^2 \beta m R^2 s^5 p Q q P$$


$$- r^2 \beta m R^2 s^5 q^2 P^2 + 12 r^2 \beta m R^2 s^3 p^2 Q^2 + 6 r^2 \beta m R^2 s^3 p Q q P$$


$$+ 12 r^2 \beta m R^2 s^5 p^2 Q^2 + 6 r^2 \beta m R^2 s^5 p Q q P + 3 r^2 \beta m R^2 s^3 c^2 p^2 Q^2$$


$$+ 3 r^2 \beta m R^2 s^3 c^2 p^2 Q^2 - 3 r^2 \beta m R^2 s^3 c^2 p Q q P - 3 r^2 \beta m R^2 s^3 c^2 p Q q P$$


$$+ 24 r^2 \beta m R^2 s^3 p^2 Q^2 + 12 r^2 \beta m R^2 s^3 p Q q P) q)$$


```

[

[

```

> LHS1:=denom(tau2);

LHS1 := 
$$2 r^2 p (2 r^2 p Q + r^2 q P + 2 r^2 t a^2 p Q + r^2 t a^2 q P + R^2 p Q - R^2 q P) (R^1 + 2 R^2)^4 (2 p Q + q P)$$


```

[> LHS2:=subs(p=1,P=1,q=q0+q1\*s1,Q=Q0+Q1\*s1,LHS1);

```

LHS2 := 
$$2 r^2 (2 r^2 (Q_0 + Q_1 s^1) + r^2 (q_0 + q_1 s^1) + 2 r^2 t a^2 (Q_0 + Q_1 s^1) + r^2 t a^2 (q_0 + q_1 s^1)$$


$$+ R^2 (Q_0 + Q_1 s^1) - R^2 (q_0 + q_1 s^1)) (R^1 + 2 R^2)^4 (2 Q_0 + 2 Q_1 s^1 + q_0 + q_1 s^1)$$


```

[> LHS3:=collect(LHS2,s1);

```

LHS3 := 
$$2 r^2 (2 r^2 Q_1 + r^2 q_1 + 2 r^2 t a^2 Q_1 + r^2 t a^2 q_1 + R^2 Q_1 - R^2 q_1) (R^1 + 2 R^2)^4 (2 Q_1 + q_1) s^1 - (2$$


$$r^2 (2 r^2 Q_0 + r^2 q_0 + 2 r^2 t a^2 Q_0 + r^2 t a^2 q_0 + R^2 Q_0 - R^2 q_0) (R^1 + 2 R^2)^4 (2 Q_1 + q_1)$$


$$+ 2 r^2 (2 r^2 Q_1 + r^2 q_1 + 2 r^2 t a^2 Q_1 + r^2 t a^2 q_1 + R^2 Q_1 - R^2 q_1) (R^1 + 2 R^2)^4 (2 Q_0 + q_0)) s^1$$


```

```

+ 2 r2 (2 r2 Q0 + r2 q0 + 2 r2 ta2 Q0 + r2 ta2 q0 + R2 Q0 - R2 q0) (R1 + 2 R2)4 (2 Q0 + q0)

> RHS1:=numer(tau2);

RHS1 := q (4 r22 β RI4 p2 Q2 - 24 m R25 ε tas5 p2 Q2 r2 - 12 m R25 ε tas5 p Q r2 q P
+ 12 m R25 ε tas3 p2 Q2 r2 + 6 m R25 ε tas3 p Q r2 q P + 2 m R24 ε s4 c p2 Q2 R12
+ 4 m R25 ε s4 c p2 Q2 RI + 4 m R25 ε s4 c p2 Q2 r2 ta2 - m R24 ε s4 c p Q q P R12
- 2 m R25 ε s4 c p Q q P R1 - 2 m R25 ε s4 c p Q q P r2 ta2 - m R24 ε s4 c q2 P2 R12
- 2 m R25 ε s4 c q2 P2 R1 - 2 m R25 ε s4 c q2 P2 r2 ta2 - 36 m R25 ε p2 Q2 s2 c r2
- 36 m R25 ε p2 Q2 s4 c r2 + 3 m R24 ε c3 p2 Q2 R12 + 6 m R25 ε c3 p2 Q2 RI
+ 3 m R24 ε c3 p2 Q2 RI2 s2 + 6 m R25 ε c3 p2 Q2 RI s2 + 6 m R25 ε c3 p2 Q2 r2 ta2
+ 6 m R25 ε c3 p2 Q2 r2 ta2 s2 - 3 m R24 ε c3 p Q R12 q P - 6 m R25 ε c3 p Q R1 q P
- 3 m R24 ε c3 p Q R12 q P s2 - 6 m R25 ε c3 p Q R1 q P s2 - 6 m R25 ε c3 p Q r2 ta2 q P
- 6 m R25 ε c3 p Q r2 ta2 q P s2 - 12 m R22 ε r22 c p Q R12 q P
- 12 m R24 ε tas5 p2 Q2 r2 RI - 6 m R25 ε tas5 p Q r2 q P R1 + 6 m R24 ε tas3 p2 Q2 r2 RI
+ 3 m R24 ε tas3 p Q r2 q P R1 + 2 m R24 ε s4 c p2 Q2 r2 ta2 RI - m R24 ε s4 c p Q q P r2 ta2 RI
- m R24 ε s4 c q2 P2 r2 ta2 RI - 18 m R24 ε p2 Q2 s2 c r2 RI - 18 m R24 ε p2 Q2 s4 c r2 RI
+ 3 m R24 ε c3 p2 Q2 r2 ta2 RI + 3 m R24 ε c3 p2 Q2 r2 ta2 s2 RI - 3 m R24 ε c3 p Q r2 ta2 q P R1
- 3 m R24 ε c3 p Q r2 ta2 q P s2 RI + 24 m R22 ε r23 c p2 Q2 ta2 RI + 48 m R23 ε r23 c p2 Q2 ta2
+ 12 m R22 ε r23 c p Q ta2 q P R1 + 24 m R23 ε r23 c p Q ta2 q P - 24 m R23 ε r22 c p2 Q2 RI
+ 24 m R23 ε r22 c p Q R1 q P + 2 r2 β m R25 s5 p2 Q2 + 2 r2 β R14 p2 Q2 R2 + r22 β R14 q2 P2
- r2 β R14 q2 P2 R2 + 4 r22 β R14 p Q q P + 4 r22 β R14 p2 Q2 ta2 + 4 r22 β R14 p Q q P ta2
- r2 β R14 p Q R2 q P + r22 β R14 q2 P2 ta2 + 8 r22 β m R24 s5 ta2 p2 Q2
+ 8 r22 β m R24 s5 ta2 p Q q P + 2 r22 β m R24 s5 ta2 q2 P2 - 4 r22 β m R24 s3 ta2 p2 Q2
- 4 r22 β m R24 s3 ta2 p Q q P - r22 β m R24 s3 ta2 q2 P2 - r2 β m R25 s5 p Q q P
- r2 β m R25 s5 q2 P2 + 12 r22 β m R24 s3 p2 Q2 + 6 r22 β m R24 s3 p Q q P
+ 12 r22 β m R24 s5 p2 Q2 + 6 r22 β m R24 s5 p Q q P + 3 r2 β m R25 s c2 p2 Q2
+ 3 r2 β m R25 s3 c2 p2 Q2 - 3 r2 β m R25 s c2 p Q q P - 3 r2 β m R25 s3 c2 p Q q P
+ 24 r24 β m R22 s p2 Q2 + 12 r24 β m R22 s p Q q P)

> RHS2:=subs(p=1,P=1,q=q0+q1*s1,Q=Q0+Q1*s1,RHS1);

RHS2 := (q0 + q1 s1) (r22 β RI4 (q0 + q1 s1)2 + 4 r22 β RI4 (Q0 + Q1 s1)2
- 2 m R25 ε s4 c (q0 + q1 s1)2 RI - m R24 ε s4 c (q0 + q1 s1)2 RI2
- 2 m R25 ε s4 c (q0 + q1 s1)2 r2 ta2 - m R24 ε s4 c (q0 + q1 s1)2 r2 ta2 RI
- r2 β R14 (q0 + q1 s1)2 R2 + r22 β R14 (q0 + q1 s1)2 ta2 + 2 r22 β m R24 s5 ta2 (q0 + q1 s1)2
- r22 β m R24 s3 ta2 (q0 + q1 s1)2 - r2 β m R25 s5 (q0 + q1 s1)2 - 24 m R25 ε tas5 (Q0 + Q1 s1)2 r2
+ 12 m R25 ε tas3 (Q0 + Q1 s1)2 r2 + 2 m R24 ε s4 c (Q0 + Q1 s1)2 RI2
+ 4 m R25 ε s4 c (Q0 + Q1 s1)2 RI + 4 m R25 ε s4 c (Q0 + Q1 s1)2 r2 ta2
- 36 m R25 ε (Q0 + Q1 s1)2 s2 c r2 - 36 m R25 ε (Q0 + Q1 s1)2 s4 c r2
+ 3 m R24 ε c3 (Q0 + Q1 s1)2 RI2 + 6 m R25 ε c3 (Q0 + Q1 s1)2 RI
+ 3 m R24 ε c3 (Q0 + Q1 s1)2 RI2 s2 + 6 m R25 ε c3 (Q0 + Q1 s1)2 RI s2
+ 6 m R25 ε c3 (Q0 + Q1 s1)2 r2 ta2 + 6 m R25 ε c3 (Q0 + Q1 s1)2 r2 ta2 s2
- 12 m R22 ε r22 c (Q0 + Q1 s1)2 RI2 - 12 m R24 ε tas5 (Q0 + Q1 s1)2 r2 RI
+ 6 m R24 ε tas3 (Q0 + Q1 s1)2 r2 RI + 2 m R24 ε s4 c (Q0 + Q1 s1)2 r2 ta2 RI

```

$$\begin{aligned}
& -18mR2^4\varepsilon(Q0+Q1s1)^2s^2cr2R1 - 18mR2^4\varepsilon(Q0+Q1s1)^2s^4cr2R1 \\
& + 3mR2^4\varepsilon c^3(Q0+Q1s1)^2r2ta^2R1 + 3mR2^4\varepsilon c^3(Q0+Q1s1)^2r2ta^2s^2R1 \\
& + 24mR2^2\varepsilon r2^3c(Q0+Q1s1)^2ta^2R1 + 48mR2^3\varepsilon r2^3c(Q0+Q1s1)^2ta^2 \\
& - 24mR2^3\varepsilon r2^2c(Q0+Q1s1)^2R1 + 2r2\beta mR2^5s^5(Q0+Q1s1)^2 + 2r2\beta R1^4(Q0+Q1s1)^2R2 \\
& + 4r2^2\beta R1^4(Q0+Q1s1)^2ta^2 + 8r2^2\beta mR2^4s^5ta^2(Q0+Q1s1)^2 \\
& - 4r2^2\beta mR2^4s^3ta^2(Q0+Q1s1)^2 + 12r2^2\beta mR2^4s^3(Q0+Q1s1)^2 \\
& + 12r2^2\beta mR2^4s^5(Q0+Q1s1)^2 + 3r2\beta mR2^5s^2c^2(Q0+Q1s1)^2 \\
& + 3r2\beta mR2^5s^3c^2(Q0+Q1s1)^2 + 24r2^4\beta mR2^2s(Q0+Q1s1)^2 \\
& + 4r2^2\beta R1^4(Q0+Q1s1)(q0+q1s1) - 12mR2^5\varepsilon tas^5(Q0+Q1s1)r2(q0+q1s1) \\
& + 6mR2^5\varepsilon tas^3(Q0+Q1s1)r2(q0+q1s1) - mR2^4\varepsilon s^4c(Q0+Q1s1)(q0+q1s1)R1^2 \\
& - 2mR2^5\varepsilon s^4c(Q0+Q1s1)(q0+q1s1)R1 - 2mR2^5\varepsilon s^4c(Q0+Q1s1)(q0+q1s1)r2ta^2 \\
& - 3mR2^4\varepsilon c^3(Q0+Q1s1)R1^2(q0+q1s1) - 6mR2^5\varepsilon c^3(Q0+Q1s1)R1(q0+q1s1) \\
& - 3mR2^4\varepsilon c^3(Q0+Q1s1)R1^2(q0+q1s1)s^2 - 6mR2^5\varepsilon c^3(Q0+Q1s1)R1(q0+q1s1)s^2 \\
& - 6mR2^5\varepsilon c^3(Q0+Q1s1)r2ta^2(q0+q1s1) - 6mR2^5\varepsilon c^3(Q0+Q1s1)r2ta^2(q0+q1s1)s^2 \\
& + 12mR2^2\varepsilon r2^2c(Q0+Q1s1)R1^2(q0+q1s1) - 6mR2^4\varepsilon tas^5(Q0+Q1s1)r2(q0+q1s1)R1 \\
& + 3mR2^4\varepsilon tas^3(Q0+Q1s1)r2(q0+q1s1)R1 - mR2^4\varepsilon s^4c(Q0+Q1s1)(q0+q1s1)r2ta^2R1 \\
& - 3mR2^4\varepsilon c^3(Q0+Q1s1)r2ta^2(q0+q1s1)R1 \\
& - 3mR2^4\varepsilon c^3(Q0+Q1s1)r2ta^2(q0+q1s1)s^2R1 \\
& + 12mR2^2\varepsilon r2^3c(Q0+Q1s1)ta^2(q0+q1s1)R1 \\
& + 24mR2^3\varepsilon r2^3c(Q0+Q1s1)ta^2(q0+q1s1) + 24mR2^3\varepsilon r2^2c(Q0+Q1s1)R1(q0+q1s1) \\
& + 4r2^2\beta R1^4(Q0+Q1s1)(q0+q1s1)ta^2 - r2\beta R1^4(Q0+Q1s1)R2(q0+q1s1) \\
& + 8r2^2\beta mR2^4s^5ta^2(Q0+Q1s1)(q0+q1s1) - 4r2^2\beta mR2^4s^3ta^2(Q0+Q1s1)(q0+q1s1) \\
& - r2\beta mR2^5s^5(Q0+Q1s1)(q0+q1s1) + 6r2^2\beta mR2^4s^3(Q0+Q1s1)(q0+q1s1) \\
& + 6r2^2\beta mR2^4s^5(Q0+Q1s1)(q0+q1s1) - 3r2\beta mR2^5s^2c^2(Q0+Q1s1)(q0+q1s1) \\
& - 3r2\beta mR2^5s^3c^2(Q0+Q1s1)(q0+q1s1) + 12r2^4\beta mR2^2s(Q0+Q1s1)(q0+q1s1))
\end{aligned}$$

```

> RHS3epsilon:=collect(RHS2,epsilon);
RHS3epsilon:=(q0+q1s1)(-2mR2^5s^4c(q0+q1s1)^2R1 - mR2^4s^4c(q0+q1s1)^2R1^2
- 2mR2^5s^4c(q0+q1s1)^2r2ta^2 - 24mR2^5tas^5(Q0+Q1s1)^2r2
- mR2^4s^4c(q0+q1s1)^2r2ta^2R1 + 12mR2^5tas^3(Q0+Q1s1)^2r2
- 36mR2^5(Q0+Q1s1)^2s^4cr2 + 3mR2^4c^3(Q0+Q1s1)^2R1^2
+ 2mR2^4s^4c(Q0+Q1s1)^2R1^2 + 4mR2^5s^4c(Q0+Q1s1)^2R1
+ 4mR2^5s^4c(Q0+Q1s1)^2r2ta^2 - 36mR2^5(Q0+Q1s1)^2s^2cr2
- 12mR2^2r2^2c(Q0+Q1s1)^2R1^2 - 12mR2^4tas^5(Q0+Q1s1)^2r2R1
+ 6mR2^4tas^3(Q0+Q1s1)^2r2R1 + 6mR2^5c^3(Q0+Q1s1)^2R1
+ 3mR2^4c^3(Q0+Q1s1)^2R1^2s^2 + 6mR2^5c^3(Q0+Q1s1)^2R1s^2
+ 6mR2^5c^3(Q0+Q1s1)^2r2ta^2 + 6mR2^5c^3(Q0+Q1s1)^2r2ta^2s^2
- 24mR2^3r2^2c(Q0+Q1s1)^2R1 + 2mR2^4s^4c(Q0+Q1s1)^2r2ta^2R1
- 18mR2^4(Q0+Q1s1)^2s^2cr2R1 - 18mR2^4(Q0+Q1s1)^2s^4cr2R1
+ 3mR2^4c^3(Q0+Q1s1)^2r2ta^2R1 + 3mR2^4c^3(Q0+Q1s1)^2r2ta^2s^2R1
+ 24mR2^2r2^3c(Q0+Q1s1)^2ta^2R1 + 48mR2^3r2^3c(Q0+Q1s1)^2ta^2

```

$$\begin{aligned}
& -m R2^4 s^4 c(Q0 + Q1 s1) (q0 + q1 s1) R1^2 - 2 m R2^5 s^4 c(Q0 + Q1 s1) (q0 + q1 s1) R1 \\
& - 2 m R2^5 s^4 c(Q0 + Q1 s1) (q0 + q1 s1) r2 ta^2 - 3 m R2^4 c^3 (Q0 + Q1 s1) R1^2 (q0 + q1 s1) \\
& - 6 m R2^5 c^3 (Q0 + Q1 s1) R1 (q0 + q1 s1) - 12 m R2^5 ta s^5 (Q0 + Q1 s1) r2 (q0 + q1 s1) \\
& + 6 m R2^5 ta s^3 (Q0 + Q1 s1) r2 (q0 + q1 s1) - 3 m R2^4 c^3 (Q0 + Q1 s1) R1^2 (q0 + q1 s1) s^2 \\
& - 6 m R2^5 c^3 (Q0 + Q1 s1) R1 (q0 + q1 s1) s^2 - 6 m R2^5 c^3 (Q0 + Q1 s1) r2 ta^2 (q0 + q1 s1) \\
& - 6 m R2^5 c^3 (Q0 + Q1 s1) r2 ta^2 (q0 + q1 s1) s^2 + 12 m R2^2 r2^2 c(Q0 + Q1 s1) R1^2 (q0 + q1 s1) \\
& - 6 m R2^4 ta s^5 (Q0 + Q1 s1) r2 (q0 + q1 s1) R1 + 3 m R2^4 ta s^3 (Q0 + Q1 s1) r2 (q0 + q1 s1) R1 \\
& - m R2^4 s^4 c(Q0 + Q1 s1) (q0 + q1 s1) r2 ta^2 R1 - 3 m R2^4 c^3 (Q0 + Q1 s1) r2 ta^2 (q0 + q1 s1) R1 \\
& - 3 m R2^4 c^3 (Q0 + Q1 s1) r2 ta^2 (q0 + q1 s1) s^2 R1 \\
& + 12 m R2^2 r2^3 c(Q0 + Q1 s1) ta^2 (q0 + q1 s1) R1 + 24 m R2^3 r2^3 c(Q0 + Q1 s1) ta^2 (q0 + q1 s1) \\
& + 24 m R2^3 r2^2 c(Q0 + Q1 s1) R1 (q0 + q1 s1)) \varepsilon + (q0 + q1 s1) (r2^2 \beta R1^4 (q0 + q1 s1)^2 \\
& + 4 r2^2 \beta R1^4 (Q0 + Q1 s1)^2 - r2 \beta R1^4 (q0 + q1 s1)^2 R2 + r2^2 \beta R1^4 (q0 + q1 s1)^2 ta^2 \\
& + 2 r2^2 \beta m R2^4 s^5 ta^2 (q0 + q1 s1)^2 - r2^2 \beta m R2^4 s^3 ta^2 (q0 + q1 s1)^2 \\
& - r2 \beta m R2^5 s^5 (q0 + q1 s1)^2 + 2 r2 \beta m R2^5 s^5 (Q0 + Q1 s1)^2 + 2 r2 \beta R1^4 (Q0 + Q1 s1)^2 R2 \\
& + 4 r2^2 \beta R1^4 (Q0 + Q1 s1)^2 ta^2 + 8 r2^2 \beta m R2^4 s^5 ta^2 (Q0 + Q1 s1)^2 \\
& - 4 r2^2 \beta m R2^4 s^3 ta^2 (Q0 + Q1 s1)^2 + 12 r2^2 \beta m R2^4 s^3 (Q0 + Q1 s1)^2 \\
& + 12 r2^2 \beta m R2^4 s^5 (Q0 + Q1 s1)^2 + 3 r2 \beta m R2^5 s c^2 (Q0 + Q1 s1)^2 \\
& + 3 r2 \beta m R2^5 s^3 c^2 (Q0 + Q1 s1)^2 + 24 r2^4 \beta m R2^2 s (Q0 + Q1 s1)^2 \\
& + 4 r2^2 \beta R1^4 (Q0 + Q1 s1) (q0 + q1 s1) + 4 r2^2 \beta R1^4 (Q0 + Q1 s1) (q0 + q1 s1) ta^2 \\
& - r2 \beta R1^4 (Q0 + Q1 s1) R2 (q0 + q1 s1) + 8 r2^2 \beta m R2^4 s^5 ta^2 (Q0 + Q1 s1) (q0 + q1 s1) \\
& - 4 r2^2 \beta m R2^4 s^3 ta^2 (Q0 + Q1 s1) (q0 + q1 s1) - r2 \beta m R2^5 s^5 (Q0 + Q1 s1) (q0 + q1 s1) \\
& + 6 r2^2 \beta m R2^4 s^3 (Q0 + Q1 s1) (q0 + q1 s1) + 6 r2^2 \beta m R2^4 s^5 (Q0 + Q1 s1) (q0 + q1 s1) \\
& - 3 r2 \beta m R2^5 s c^2 (Q0 + Q1 s1) (q0 + q1 s1) - 3 r2 \beta m R2^5 s^3 c^2 (Q0 + Q1 s1) (q0 + q1 s1) \\
& + 12 r2^4 \beta m R2^2 s (Q0 + Q1 s1) (q0 + q1 s1))
\end{aligned}$$

> RHS3beta:=collect(RHS2,beta);

$$\begin{aligned}
RHS3beta := & (q0 + q1 s1) (r2^2 R1^4 (q0 + q1 s1)^2 + 4 r2^2 R1^4 (Q0 + Q1 s1)^2 - r2 R1^4 (q0 + q1 s1)^2 R2 \\
& + r2^2 R1^4 (q0 + q1 s1)^2 ta^2 + 2 r2 R1^4 (Q0 + Q1 s1)^2 R2 + 4 r2^2 R1^4 (Q0 + Q1 s1)^2 ta^2 \\
& + 4 r2^2 R1^4 (Q0 + Q1 s1) (q0 + q1 s1) + 2 r2^2 m R2^4 s^5 ta^2 (q0 + q1 s1)^2 \\
& + 2 r2 m R2^5 s^5 (Q0 + Q1 s1)^2 - r2^2 m R2^4 s^3 ta^2 (q0 + q1 s1)^2 - r2 m R2^5 s^5 (q0 + q1 s1)^2 \\
& + 8 r2^2 m R2^4 s^5 ta^2 (Q0 + Q1 s1)^2 - 4 r2^2 m R2^4 s^3 ta^2 (Q0 + Q1 s1)^2 \\
& + 12 r2^2 m R2^4 s^3 (Q0 + Q1 s1)^2 + 12 r2^2 m R2^4 s^5 (Q0 + Q1 s1)^2 + 3 r2 m R2^5 s c^2 (Q0 + Q1 s1)^2 \\
& + 3 r2 m R2^5 s^3 c^2 (Q0 + Q1 s1)^2 + 24 r2^4 m R2^2 s (Q0 + Q1 s1)^2 \\
& + 4 r2^2 R1^4 (Q0 + Q1 s1) (q0 + q1 s1) ta^2 - r2 R1^4 (Q0 + Q1 s1) R2 (q0 + q1 s1) \\
& + 8 r2^2 m R2^4 s^5 ta^2 (Q0 + Q1 s1) (q0 + q1 s1) - 4 r2^2 m R2^4 s^3 ta^2 (Q0 + Q1 s1) (q0 + q1 s1) \\
& - r2 m R2^5 s^5 (Q0 + Q1 s1) (q0 + q1 s1) + 6 r2^2 m R2^4 s^3 (Q0 + Q1 s1) (q0 + q1 s1) \\
& + 6 r2^2 m R2^4 s^5 (Q0 + Q1 s1) (q0 + q1 s1) - 3 r2 m R2^5 s c^2 (Q0 + Q1 s1) (q0 + q1 s1) \\
& - 3 r2 m R2^5 s^3 c^2 (Q0 + Q1 s1) (q0 + q1 s1) + 12 r2^4 m R2^2 s (Q0 + Q1 s1) (q0 + q1 s1)) \beta + (q0 \\
& + q1 s1) (-2 m R2^5 \varepsilon s^4 c (q0 + q1 s1)^2 R1 - m R2^4 \varepsilon s^4 c (q0 + q1 s1)^2 R1^2 \\
& - 2 m R2^5 \varepsilon s^4 c (q0 + q1 s1)^2 r2 ta^2 - m R2^4 \varepsilon s^4 c (q0 + q1 s1)^2 r2 ta^2 R1 \\
& - 24 m R2^5 \varepsilon ta s^5 (Q0 + Q1 s1)^2 r2 + 12 m R2^5 \varepsilon ta s^3 (Q0 + Q1 s1)^2 r2
\end{aligned}$$

$$\begin{aligned}
& + 2 m R2^4 \varepsilon s^4 c (Q0 + Q1 s1)^2 R1^2 + 4 m R2^5 \varepsilon s^4 c (Q0 + Q1 s1)^2 R1 \\
& + 4 m R2^5 \varepsilon s^4 c (Q0 + Q1 s1)^2 r2 ta^2 - 36 m R2^5 \varepsilon (Q0 + Q1 s1)^2 s^2 c r2 \\
& - 36 m R2^5 \varepsilon (Q0 + Q1 s1)^2 s^4 c r2 + 3 m R2^4 \varepsilon c^3 (Q0 + Q1 s1)^2 R1^2 \\
& + 6 m R2^5 \varepsilon c^3 (Q0 + Q1 s1)^2 RI + 3 m R2^4 \varepsilon c^3 (Q0 + Q1 s1)^2 RI^2 s^2 \\
& + 6 m R2^5 \varepsilon c^3 (Q0 + Q1 s1)^2 RI s^2 + 6 m R2^5 \varepsilon c^3 (Q0 + Q1 s1)^2 r2 ta^2 \\
& + 6 m R2^5 \varepsilon c^3 (Q0 + Q1 s1)^2 r2 ta^2 s^2 - 12 m R2^2 \varepsilon r2^2 c (Q0 + Q1 s1)^2 RI^2 \\
& - 12 m R2^4 \varepsilon ta s^5 (Q0 + Q1 s1)^2 r2 RI + 6 m R2^4 \varepsilon ta s^3 (Q0 + Q1 s1)^2 r2 RI \\
& + 2 m R2^4 \varepsilon s^4 c (Q0 + Q1 s1)^2 r2 ta^2 RI - 18 m R2^4 \varepsilon (Q0 + Q1 s1)^2 s^2 c r2 RI \\
& - 18 m R2^4 \varepsilon (Q0 + Q1 s1)^2 s^4 c r2 RI + 3 m R2^4 \varepsilon c^3 (Q0 + Q1 s1)^2 r2 ta^2 RI \\
& + 3 m R2^4 \varepsilon c^3 (Q0 + Q1 s1)^2 r2 ta^2 s^2 RI + 24 m R2^2 \varepsilon r2^3 c (Q0 + Q1 s1)^2 ta^2 RI \\
& + 48 m R2^3 \varepsilon r2^3 c (Q0 + Q1 s1)^2 ta^2 - 24 m R2^3 \varepsilon r2^2 c (Q0 + Q1 s1)^2 RI \\
& - 12 m R2^5 \varepsilon ta s^5 (Q0 + Q1 s1) r2 (q0 + q1 s1) + 6 m R2^5 \varepsilon ta s^3 (Q0 + Q1 s1) r2 (q0 + q1 s1) \\
& - m R2^4 \varepsilon s^4 c (Q0 + Q1 s1) (q0 + q1 s1) RI^2 - 2 m R2^5 \varepsilon s^4 c (Q0 + Q1 s1) (q0 + q1 s1) RI \\
& - 2 m R2^5 \varepsilon s^4 c (Q0 + Q1 s1) (q0 + q1 s1) r2 ta^2 - 3 m R2^4 \varepsilon c^3 (Q0 + Q1 s1) RI^2 (q0 + q1 s1) \\
& - 6 m R2^5 \varepsilon c^3 (Q0 + Q1 s1) RI (q0 + q1 s1) - 3 m R2^4 \varepsilon c^3 (Q0 + Q1 s1) RI^2 (q0 + q1 s1) s^2 \\
& - 6 m R2^5 \varepsilon c^3 (Q0 + Q1 s1) RI (q0 + q1 s1) s^2 - 6 m R2^5 \varepsilon c^3 (Q0 + Q1 s1) r2 ta^2 (q0 + q1 s1) \\
& - 6 m R2^5 \varepsilon c^3 (Q0 + Q1 s1) r2 ta^2 (q0 + q1 s1) s^2 + 12 m R2^2 \varepsilon r2^2 c (Q0 + Q1 s1) RI^2 (q0 + q1 s1) \\
& - 6 m R2^4 \varepsilon ta s^5 (Q0 + Q1 s1) r2 (q0 + q1 s1) RI + 3 m R2^4 \varepsilon ta s^3 (Q0 + Q1 s1) r2 (q0 + q1 s1) RI \\
& - m R2^4 \varepsilon s^4 c (Q0 + Q1 s1) (q0 + q1 s1) r2 ta^2 RI \\
& - 3 m R2^4 \varepsilon c^3 (Q0 + Q1 s1) r2 ta^2 (q0 + q1 s1) RI \\
& - 3 m R2^4 \varepsilon c^3 (Q0 + Q1 s1) r2 ta^2 (q0 + q1 s1) s^2 RI \\
& + 12 m R2^2 \varepsilon r2^3 c (Q0 + Q1 s1) ta^2 (q0 + q1 s1) RI \\
& + 24 m R2^3 \varepsilon r2^3 c (Q0 + Q1 s1) ta^2 (q0 + q1 s1) + 24 m R2^3 \varepsilon r2^2 c (Q0 + Q1 s1) RI (q0 + q1 s1))
\end{aligned}$$

> **beta1:=coeff(RHS3beta,beta,1);**

$$\begin{aligned}
\beta1 := & (q0 + q1 s1) (r2^2 R1^4 (q0 + q1 s1)^2 + 4 r2^2 R1^4 (Q0 + Q1 s1)^2 - r2 R1^4 (q0 + q1 s1)^2 R2 \\
& + r2^2 R1^4 (q0 + q1 s1)^2 ta^2 + 2 r2 R1^4 (Q0 + Q1 s1)^2 R2 + 4 r2^2 R1^4 (Q0 + Q1 s1)^2 ta^2 \\
& + 4 r2^2 R1^4 (Q0 + Q1 s1) (q0 + q1 s1) + 2 r2^2 m R2^4 s^5 ta^2 (q0 + q1 s1)^2 \\
& + 2 r2 m R2^5 s^5 (Q0 + Q1 s1)^2 - r2^2 m R2^4 s^3 ta^2 (q0 + q1 s1)^2 - r2 m R2^5 s^5 (q0 + q1 s1)^2 \\
& + 8 r2^2 m R2^4 s^5 ta^2 (Q0 + Q1 s1)^2 - 4 r2^2 m R2^4 s^3 ta^2 (Q0 + Q1 s1)^2 \\
& + 12 r2^2 m R2^4 s^3 (Q0 + Q1 s1)^2 + 12 r2^2 m R2^4 s^5 (Q0 + Q1 s1)^2 + 3 r2 m R2^5 s c^2 (Q0 + Q1 s1)^2 \\
& + 3 r2 m R2^5 s^3 c^2 (Q0 + Q1 s1)^2 + 24 r2^4 m R2^2 s (Q0 + Q1 s1)^2 \\
& + 4 r2^2 R1^4 (Q0 + Q1 s1) (q0 + q1 s1) ta^2 - r2 R1^4 (Q0 + Q1 s1) R2 (q0 + q1 s1) \\
& + 8 r2^2 m R2^4 s^5 ta^2 (Q0 + Q1 s1) (q0 + q1 s1) - 4 r2^2 m R2^4 s^3 ta^2 (Q0 + Q1 s1) (q0 + q1 s1) \\
& - r2 m R2^5 s^5 (Q0 + Q1 s1) (q0 + q1 s1) + 6 r2^2 m R2^4 s^3 (Q0 + Q1 s1) (q0 + q1 s1) \\
& + 6 r2^2 m R2^4 s^5 (Q0 + Q1 s1) (q0 + q1 s1) - 3 r2 m R2^5 s c^2 (Q0 + Q1 s1) (q0 + q1 s1) \\
& - 3 r2 m R2^5 s^3 c^2 (Q0 + Q1 s1) (q0 + q1 s1) + 12 r2^4 m R2^2 s (Q0 + Q1 s1) (q0 + q1 s1))
\end{aligned}$$

> **beta2:=collect(beta1,s1);**

$$\begin{aligned}
\beta2 := & q1 (r2^2 R1^4 q1^2 + 4 r2^2 R1^4 Q1^2 - r2 R1^4 q1^2 R2 + r2^2 R1^4 q1^2 ta^2 + 2 r2 R1^4 Q1^2 R2 \\
& + 4 r2^2 R1^4 Q1^2 ta^2 + 4 r2^2 R1^4 Q1 q1 + 2 r2 m R2^5 s^5 Q1^2 + 2 r2^2 m R2^4 s^5 ta^2 q1^2 \\
& - 4 r2^2 m R2^4 s^3 ta^2 Q1^2 - r2^2 m R2^4 s^3 ta^2 q1^2 - r2 m R2^5 s^5 q1^2 + 8 r2^2 m R2^4 s^5 ta^2 Q1^2 \\
& + 12 r2^2 m R2^4 s^3 Q1^2 + 12 r2^2 m R2^4 s^5 Q1^2 + 3 r2 m R2^5 s c^2 Q1^2 + 6 r2^2 m R2^4 s^3 Q1 q1
\end{aligned}$$

$$\begin{aligned}
& + 3 r_2 m R2^5 s^3 c^2 Q1^2 + 24 r_2^4 m R2^2 s Q1^2 + 4 r_2^2 R1^4 Q1 q1 ta^2 - r_2 R1^4 Q1 R2 qI \\
& + 8 r_2^2 m R2^4 s^5 ta^2 Q1 q1 - 4 r_2^2 m R2^4 s^3 ta^2 Q1 q1 - r_2 m R2^5 s^5 Q1 q1 \\
& + 6 r_2^2 m R2^4 s^5 Q1 q1 - 3 r_2 m R2^5 s c^2 Q1 q1 - 3 r_2 m R2^5 s^3 c^2 Q1 q1 + 12 r_2^4 m R2^2 s Q1 q1) \\
& s1^3 + (q0 (r_2^2 R1^4 q1^2 + 4 r_2^2 R1^4 Q1^2 - r_2 R1^4 q1^2 R2 + r_2^2 R1^4 q1^2 ta^2 + 2 r_2 R1^4 Q1^2 R2 \\
& + 4 r_2^2 R1^4 Q1^2 ta^2 + 4 r_2^2 R1^4 Q1 q1 + 2 r_2 m R2^5 s^5 Q1^2 + 2 r_2^2 m R2^4 s^5 ta^2 q1^2 \\
& - 4 r_2^2 m R2^4 s^3 ta^2 Q1^2 - r_2^2 m R2^4 s^3 ta^2 q1^2 - r_2 m R2^5 s^5 q1^2 + 8 r_2^2 m R2^4 s^5 ta^2 Q1^2 \\
& + 12 r_2^2 m R2^4 s^3 Q1^2 + 12 r_2^2 m R2^4 s^5 Q1^2 + 3 r_2 m R2^5 s c^2 Q1^2 + 6 r_2^2 m R2^4 s^3 Q1 q1 \\
& + 3 r_2 m R2^5 s^3 c^2 Q1^2 + 24 r_2^4 m R2^2 s Q1^2 + 4 r_2^2 R1^4 Q1 q1 ta^2 - r_2 R1^4 Q1 R2 qI \\
& + 8 r_2^2 m R2^4 s^5 ta^2 Q1 q1 - 4 r_2^2 m R2^4 s^3 ta^2 Q1 q1 - r_2 m R2^5 s^5 Q1 q1 \\
& + 6 r_2^2 m R2^4 s^5 Q1 q1 - 3 r_2 m R2^5 s c^2 Q1 q1 - 3 r_2 m R2^5 s^3 c^2 Q1 q1 + 12 r_2^4 m R2^2 s Q1 q1) \\
& q1 (2 r_2^2 R1^4 q0 q1 + 8 r_2^2 R1^4 Q0 Q1 + 4 r_2^2 R1^4 Q0 q1 + 4 r_2^2 R1^4 Q1 q0 - 2 r_2 R1^4 q0 q1 R2 \\
& + 2 r_2^2 R1^4 q0 q1 ta^2 + 4 r_2 R1^4 Q0 Q1 R2 + 8 r_2^2 R1^4 Q0 Q1 ta^2 + 4 r_2^2 m R2^4 s^5 ta^2 q0 q1 \\
& + 4 r_2 m R2^5 s^5 Q0 Q1 - 2 r_2^2 m R2^4 s^3 ta^2 q0 q1 - 2 r_2 m R2^5 s^5 q0 q1 \\
& + 16 r_2^2 m R2^4 s^5 ta^2 Q0 Q1 - 8 r_2^2 m R2^4 s^3 ta^2 Q0 Q1 + 24 r_2^2 m R2^4 s^3 Q0 Q1 \\
& + 24 r_2^2 m R2^4 s^5 Q0 Q1 + 6 r_2 m R2^5 s c^2 Q0 Q1 + 6 r_2^2 m R2^4 s^3 Q0 q1 \\
& + 6 r_2^2 m R2^4 s^3 Q1 q0 + 6 r_2 m R2^5 s^3 c^2 Q0 Q1 + 4 (r_2^2 R1^4 Q0 q1 + r_2^2 R1^4 Q1 q0) ta^2 \\
& + 48 r_2^4 m R2^2 s Q0 Q1 - r_2 R1^4 Q0 R2 q1 - r_2 R1^4 Q1 R2 q0 + 8 r_2^2 m R2^4 s^5 ta^2 Q0 q1 \\
& + 8 r_2^2 m R2^4 s^5 ta^2 Q1 q0 - 4 r_2^2 m R2^4 s^3 ta^2 Q0 q1 - 4 r_2^2 m R2^4 s^3 ta^2 Q1 q0 \\
& - r_2 m R2^5 s^5 Q0 q1 - r_2 m R2^5 s^5 Q1 q0 + 6 r_2^2 m R2^4 s^5 Q0 q1 + 6 r_2^2 m R2^4 s^5 Q1 q0 \\
& - 3 r_2 m R2^5 s c^2 Q0 q1 - 3 r_2 m R2^5 s c^2 Q1 q0 - 3 r_2 m R2^5 s^3 c^2 Q0 q1 - 3 r_2 m R2^5 s^3 c^2 Q1 q0 \\
& + 12 r_2^4 m R2^2 s Q0 q1 + 12 r_2^4 m R2^2 s Q1 q0)) s1^2 + (q0 (2 r_2^2 R1^4 q0 q1 + 8 r_2^2 R1^4 Q0 Q1 \\
& + 4 r_2^2 R1^4 Q0 q1 + 4 r_2^2 R1^4 Q1 q0 - 2 r_2 R1^4 q0 q1 R2 + 2 r_2^2 R1^4 q0 q1 ta^2 \\
& + 4 r_2 R1^4 Q0 Q1 R2 + 8 r_2^2 R1^4 Q0 Q1 ta^2 + 4 r_2^2 m R2^4 s^5 ta^2 q0 q1 + 4 r_2 m R2^5 s^5 Q0 Q1 \\
& - 2 r_2^2 m R2^4 s^3 ta^2 q0 q1 - 2 r_2 m R2^5 s^5 q0 q1 + 16 r_2^2 m R2^4 s^5 ta^2 Q0 Q1 \\
& - 8 r_2^2 m R2^4 s^3 ta^2 Q0 Q1 + 24 r_2^2 m R2^4 s^3 Q0 Q1 + 24 r_2^2 m R2^4 s^5 Q0 Q1 \\
& + 6 r_2 m R2^5 s c^2 Q0 Q1 + 6 r_2^2 m R2^4 s^3 Q0 q1 + 6 r_2^2 m R2^4 s^3 Q1 q0 \\
& + 6 r_2 m R2^5 s^3 c^2 Q0 Q1 + 4 (r_2^2 R1^4 Q0 q1 + r_2^2 R1^4 Q1 q0) ta^2 + 48 r_2^4 m R2^2 s Q0 Q1 \\
& - r_2 R1^4 Q0 R2 q1 - r_2 R1^4 Q1 R2 q0 + 8 r_2^2 m R2^4 s^5 ta^2 Q0 q1 + 8 r_2^2 m R2^4 s^5 ta^2 Q1 q0 \\
& - 4 r_2^2 m R2^4 s^3 ta^2 Q0 q1 - 4 r_2^2 m R2^4 s^3 ta^2 Q1 q0 - r_2 m R2^5 s^5 Q0 q1 - r_2 m R2^5 s^5 Q1 q0 \\
& + 6 r_2^2 m R2^4 s^5 Q0 q1 + 6 r_2^2 m R2^4 s^5 Q1 q0 - 3 r_2 m R2^5 s c^2 Q0 q1 - 3 r_2 m R2^5 s c^2 Q1 q0 \\
& - 3 r_2 m R2^5 s^3 c^2 Q0 q1 - 3 r_2 m R2^5 s^3 c^2 Q1 q0 + 12 r_2^4 m R2^2 s Q0 q1 + 12 r_2^4 m R2^2 s Q1 q0 \\
& ) + q1 (4 r_2^2 R1^4 Q0^2 + r_2^2 R1^4 q0^2 + 2 r_2 m R2^5 s^5 Q0^2 - r_2 R1^4 q0^2 R2 + r_2^2 R1^4 q0^2 ta^2 \\
& + 2 r_2 R1^4 Q0^2 R2 + 4 r_2^2 R1^4 Q0^2 ta^2 + 4 r_2^2 R1^4 Q0 q0 + 2 r_2^2 m R2^4 s^5 ta^2 q0^2 \\
& - 4 r_2^2 m R2^4 s^3 ta^2 Q0^2 - r_2^2 m R2^4 s^3 ta^2 q0^2 - r_2 m R2^5 s^5 q0^2 + 8 r_2^2 m R2^4 s^5 ta^2 Q0^2 \\
& + 12 r_2^2 m R2^4 s^3 Q0^2 + 12 r_2^2 m R2^4 s^5 Q0^2 + 3 r_2 m R2^5 s c^2 Q0^2 + 6 r_2^2 m R2^4 s^3 Q0 q0 \\
& + 3 r_2 m R2^5 s^3 c^2 Q0^2 + 24 r_2^4 m R2^2 s Q0^2 + 4 r_2^2 R1^4 Q0 q0 ta^2 - r_2 R1^4 Q0 R2 q0 \\
& + 8 r_2^2 m R2^4 s^5 ta^2 Q0 q0 - 4 r_2^2 m R2^4 s^3 ta^2 Q0 q0 - r_2 m R2^5 s^5 Q0 q0 \\
& + 6 r_2^2 m R2^4 s^5 Q0 q0 - 3 r_2 m R2^5 s c^2 Q0 q0 - 3 r_2 m R2^5 s^3 c^2 Q0 q0 + 12 r_2^4 m R2^2 s Q0 q0)) \\
& s1 + q0 (4 r_2^2 R1^4 Q0^2 + r_2^2 R1^4 q0^2 + 2 r_2 m R2^5 s^5 Q0^2 - r_2 R1^4 q0^2 R2 + r_2^2 R1^4 q0^2 ta^2 \\
& + 2 r_2 R1^4 Q0^2 R2 + 4 r_2^2 R1^4 Q0^2 ta^2 + 4 r_2^2 R1^4 Q0 q0 + 2 r_2^2 m R2^4 s^5 ta^2 q0^2 \\
& - 4 r_2^2 m R2^4 s^3 ta^2 Q0^2 - r_2^2 m R2^4 s^3 ta^2 q0^2 - r_2 m R2^5 s^5 q0^2 + 8 r_2^2 m R2^4 s^5 ta^2 Q0^2 \\
& + 12 r_2^2 m R2^4 s^3 Q0^2 + 12 r_2^2 m R2^4 s^5 Q0^2 + 3 r_2 m R2^5 s c^2 Q0^2 + 6 r_2^2 m R2^4 s^3 Q0 q0
\end{aligned}$$

$$\begin{aligned}
& + 3 r2 m R2^5 s^3 c^2 Q0^2 + 24 r2^4 m R2^2 s Q0^2 + 4 r2^2 R1^4 Q0 q0 ta^2 - r2 R1^4 Q0 R2 q0 \\
& + 8 r2^2 m R2^4 s^5 ta^2 Q0 q0 - 4 r2^2 m R2^4 s^3 ta^2 Q0 q0 - r2 m R2^5 s^5 Q0 q0 \\
& + 6 r2^2 m R2^4 s^5 Q0 q0 - 3 r2 m R2^5 s c^2 Q0 q0 - 3 r2 m R2^5 s^3 c^2 Q0 q0 + 12 r2^4 m R2^2 s Q0 q0
\end{aligned}$$

```

> epsilon1:=coeff(RHS3epsilon,epsilon,1);
e1 := (q0 + q1 s1) (-2 m R2^5 s^4 c (q0 + q1 s1)^2 R1 - m R2^4 s^4 c (q0 + q1 s1)^2 R1^2
- 2 m R2^5 s^4 c (q0 + q1 s1)^2 r2 ta^2 - 24 m R2^5 ta s^5 (Q0 + Q1 s1)^2 r2
- m R2^4 s^4 c (q0 + q1 s1)^2 r2 ta^2 R1 + 12 m R2^5 ta s^3 (Q0 + Q1 s1)^2 r2
- 36 m R2^5 (Q0 + Q1 s1)^2 s^4 c r2 + 3 m R2^4 c^3 (Q0 + Q1 s1)^2 R1^2
+ 2 m R2^4 s^4 c (Q0 + Q1 s1)^2 R1^2 + 4 m R2^5 s^4 c (Q0 + Q1 s1)^2 R1
+ 4 m R2^5 s^4 c (Q0 + Q1 s1)^2 r2 ta^2 - 36 m R2^5 (Q0 + Q1 s1)^2 s^2 c r2
- 12 m R2^2 r2^2 c (Q0 + Q1 s1)^2 R1^2 - 12 m R2^4 ta s^5 (Q0 + Q1 s1)^2 r2 R1
+ 6 m R2^4 ta s^3 (Q0 + Q1 s1)^2 r2 R1 + 6 m R2^5 c^3 (Q0 + Q1 s1)^2 R1
+ 3 m R2^4 c^3 (Q0 + Q1 s1)^2 R1^2 s^2 + 6 m R2^5 c^3 (Q0 + Q1 s1)^2 R1 s^2
+ 6 m R2^5 c^3 (Q0 + Q1 s1)^2 r2 ta^2 + 6 m R2^5 c^3 (Q0 + Q1 s1)^2 r2 ta^2 s^2
- 24 m R2^3 r2^2 c (Q0 + Q1 s1)^2 R1 + 2 m R2^4 s^4 c (Q0 + Q1 s1)^2 r2 ta^2 R1
- 18 m R2^4 (Q0 + Q1 s1)^2 s^2 c r2 R1 - 18 m R2^4 (Q0 + Q1 s1)^2 s^4 c r2 R1
+ 3 m R2^4 c^3 (Q0 + Q1 s1)^2 r2 ta^2 R1 + 3 m R2^4 c^3 (Q0 + Q1 s1)^2 r2 ta^2 s^2 R1
+ 24 m R2^2 r2^3 c (Q0 + Q1 s1)^2 ta^2 R1 + 48 m R2^3 r2^3 c (Q0 + Q1 s1)^2 ta^2
- m R2^4 s^4 c (Q0 + Q1 s1) (q0 + q1 s1) R1^2 - 2 m R2^5 s^4 c (Q0 + Q1 s1) (q0 + q1 s1) R1
- 2 m R2^5 s^4 c (Q0 + Q1 s1) (q0 + q1 s1) r2 ta^2 - 3 m R2^4 c^3 (Q0 + Q1 s1) R1^2 (q0 + q1 s1)
- 6 m R2^5 c^3 (Q0 + Q1 s1) R1 (q0 + q1 s1) - 12 m R2^5 ta s^5 (Q0 + Q1 s1) r2 (q0 + q1 s1)
+ 6 m R2^5 ta s^3 (Q0 + Q1 s1) r2 (q0 + q1 s1) - 3 m R2^4 c^3 (Q0 + Q1 s1) R1^2 (q0 + q1 s1) s^2
- 6 m R2^5 c^3 (Q0 + Q1 s1) R1 (q0 + q1 s1) s^2 - 6 m R2^5 c^3 (Q0 + Q1 s1) r2 ta^2 (q0 + q1 s1)
- 6 m R2^5 c^3 (Q0 + Q1 s1) r2 ta^2 (q0 + q1 s1) s^2 + 12 m R2^2 r2^2 c (Q0 + Q1 s1) R1^2 (q0 + q1 s1)
- 6 m R2^4 ta s^5 (Q0 + Q1 s1) r2 (q0 + q1 s1) R1 + 3 m R2^4 ta s^3 (Q0 + Q1 s1) r2 (q0 + q1 s1) R1
- m R2^4 s^4 c (Q0 + Q1 s1) (q0 + q1 s1) r2 ta^2 R1 - 3 m R2^4 c^3 (Q0 + Q1 s1) r2 ta^2 (q0 + q1 s1) R1
- 3 m R2^4 c^3 (Q0 + Q1 s1) r2 ta^2 (q0 + q1 s1) s^2 R1
+ 12 m R2^2 r2^3 c (Q0 + Q1 s1) ta^2 (q0 + q1 s1) R1 + 24 m R2^3 r2^3 c (Q0 + Q1 s1) ta^2 (q0 + q1 s1)
+ 24 m R2^3 r2^2 c (Q0 + Q1 s1) R1 (q0 + q1 s1))

```

```

> epsilon2:=collect(epsilon1,s1);
e2 := q1 (-2 m R2^5 s^4 c q1^2 R1 - m R2^4 s^4 c q1^2 R1^2 - 2 m R2^5 s^4 c q1^2 r2 ta^2 - 24 m R2^5 ta s^5 Q1^2 r2
- m R2^4 s^4 c q1^2 r2 ta^2 R1 + 12 m R2^5 ta s^3 Q1^2 r2 - 36 m R2^5 Q1^2 s^4 c r2 + 3 m R2^4 c^3 Q1^2 R1^2
+ 2 m R2^4 s^4 c Q1^2 R1^2 + 4 m R2^5 Q1^2 s^4 c R1 + 4 m R2^5 Q1^2 s^4 c r2 ta^2 - 36 m R2^5 Q1^2 s^2 c r2
- 12 m R2^2 r2^2 c Q1^2 R1^2 - 12 m R2^4 ta s^5 Q1^2 r2 R1 + 6 m R2^4 ta s^3 Q1^2 r2 R1
+ 6 m R2^5 c^3 Q1^2 R1 + 3 m R2^4 c^3 Q1^2 R1^2 s^2 + 6 m R2^5 c^3 Q1^2 R1 s^2 + 6 m R2^5 c^3 Q1^2 r2 ta^2
+ 6 m R2^5 c^3 Q1^2 r2 ta^2 s^2 - 24 m R2^3 r2^2 c Q1^2 R1 + 2 m R2^4 s^4 c Q1^2 r2 ta^2 R1
- 18 m R2^4 Q1^2 s^2 c r2 R1 - 18 m R2^4 s^4 c Q1^2 r2 R1 + 3 m R2^4 c^3 Q1^2 r2 ta^2 R1
+ 3 m R2^4 c^3 Q1^2 r2 ta^2 s^2 R1 + 24 m R2^2 r2^3 c Q1^2 ta^2 R1 + 48 m R2^3 r2^3 c Q1^2 ta^2
- m R2^4 s^4 c Q1 q1 R1^2 - 2 m R2^5 s^4 c Q1 q1 R1 - 2 m R2^5 s^4 c Q1 q1 r2 ta^2
- 3 m R2^4 c^3 Q1 R1^2 q1 - 6 m R2^5 c^3 Q1 R1 q1 - 12 m R2^5 ta s^5 Q1 r2 q1 + 6 m R2^5 ta s^3 Q1 r2 q1

```

$$\begin{aligned}
& -3mR2^4c^3Q1R1^2q1s^2 - 6mR2^5c^3Q1R1q1s^2 - 6mR2^5c^3Q1r2ta^2q1 \\
& - 6mR2^5c^3Q1r2ta^2q1s^2 + 12mR2^2r2^2cQ1R1^2q1 - 6mR2^4tas^5Q1r2q1R1 \\
& + 3mR2^4tas^3Q1r2q1R1 - mR2^4s^4cQ1q1r2ta^2R1 - 3mR2^4c^3Q1r2ta^2q1R1 \\
& - 3mR2^4c^3Q1r2ta^2q1s^2R1 + 12mR2^2r2^3cQ1ta^2q1R1 + 24mR2^3r2^3cQ1ta^2q1 \\
& + 24mR2^3r2^2cQ1R1q1)sl^3 + (q0(-2mR2^5s^4cq1^2R1 - mR2^4s^4cq1^2R1^2 \\
& - 2mR2^5s^4cq1^2r2ta^2 - 24mR2^5tas^5Q1^2r2 - mR2^4s^4cq1^2r2ta^2R1 \\
& + 12mR2^5tas^3Q1^2r2 - 36mR2^5Q1^2s^4cr2 + 3mR2^4c^3Q1^2R1^2 + 2mR2^4s^4cQ1^2R1^2 \\
& + 4mR2^5Q1^2s^4cR1 + 4mR2^5Q1^2s^4cr2ta^2 - 36mR2^5Q1^2s^2cr2 \\
& - 12mR2^2r2^2cQ1^2R1^2 - 12mR2^4tas^5Q1^2r2R1 + 6mR2^4tas^3Q1^2r2R1 \\
& + 6mR2^5c^3Q1^2R1 + 3mR2^4c^3Q1^2R1^2s^2 + 6mR2^5c^3Q1^2R1s^2 + 6mR2^5c^3Q1^2r2ta^2 \\
& + 6mR2^5c^3Q1^2r2ta^2s^2 - 24mR2^3r2^2cQ1^2R1 + 2mR2^4s^4cQ1^2r2ta^2R1 \\
& - 18mR2^4Q1^2s^2cr2R1 - 18mR2^4s^4cQ1^2r2R1 + 3mR2^4c^3Q1^2r2ta^2R1 \\
& + 3mR2^4c^3Q1^2r2ta^2s^2R1 + 24mR2^2r2^3cQ1^2ta^2R1 + 48mR2^3r2^3cQ1^2ta^2 \\
& - mR2^4s^4cQ1q1R1^2 - 2mR2^5s^4cQ1q1R1 - 2mR2^5s^4cQ1q1r2ta^2 \\
& - 3mR2^4c^3Q1R1^2q1 - 6mR2^5c^3Q1R1q1 - 12mR2^5tas^5Q1r2q1 + 6mR2^5tas^3Q1r2q1 \\
& - 3mR2^4c^3Q1R1^2q1s^2 - 6mR2^5c^3Q1R1q1s^2 - 6mR2^5c^3Q1r2ta^2q1 \\
& - 6mR2^5c^3Q1r2ta^2q1s^2 + 12mR2^2r2^2cQ1R1^2q1 - 6mR2^4tas^5Q1r2q1R1 \\
& + 3mR2^4tas^3Q1r2q1R1 - mR2^4s^4cQ1q1r2ta^2R1 - 3mR2^4c^3Q1r2ta^2q1R1 \\
& - 3mR2^4c^3Q1r2ta^2q1s^2R1 + 12mR2^2r2^3cQ1ta^2q1R1 + 24mR2^3r2^3cQ1ta^2q1 \\
& + 24mR2^3r2^2cQ1R1q1) + q1(-3(mR2^4c^3Q0r2ta^2q1 + mR2^4c^3Q1r2ta^2q0)s^2R1 \\
& - 2(mR2^5s^4cQ0q1 + mR2^5s^4cQ1q0)r2ta^2 - 4mR2^5s^4cq0q1R1 - 2mR2^4s^4cq0q1R1^2 \\
& - 4mR2^5s^4cq0q1r2ta^2 - 48mR2^5tas^5Q0Q1r2 - 2mR2^4s^4cq0q1r2ta^2R1 \\
& + 24mR2^5tas^3Q0Q1r2 - 72mR2^5Q0Q1s^4cr2 + 6mR2^4c^3Q0Q1R1^2 \\
& + 4mR2^4s^4cQ0Q1R1^2 + 8mR2^5Q0Q1s^4cR1 + 8mR2^5Q0Q1s^4cr2ta^2 \\
& - 72mR2^5Q0Q1s^2cr2 - 24mR2^2r2^2cQ0Q1R1^2 - 24mR2^4tas^5Q0Q1r2R1 \\
& + 12mR2^4tas^3Q0Q1r2R1 + 12mR2^5c^3Q0Q1R1 + 6mR2^4c^3Q0Q1R1^2s^2 \\
& + 12mR2^5c^3Q0Q1R1s^2 + 12mR2^5c^3Q0Q1r2ta^2 + 12mR2^5c^3Q0Q1r2ta^2s^2 \\
& - 48mR2^3r2^2cQ0Q1R1 + 4mR2^4s^4cQ0Q1r2ta^2R1 - 36mR2^4Q0Q1s^2cr2R1 \\
& - (mR2^4s^4cQ0q1 + mR2^4s^4cQ1q0)r2ta^2R1 - 36mR2^4s^4cQ0Q1r2R1 \\
& + 6mR2^4c^3Q0Q1r2ta^2R1 + 6mR2^4c^3Q0Q1r2ta^2s^2R1 + 48mR2^2r2^3cQ0Q1ta^2R1 \\
& - (mR2^4s^4cQ0q1 + mR2^4s^4cQ1q0)R1^2 - 2(mR2^5s^4cQ0q1 + mR2^5s^4cQ1q0)R1 \\
& + 96mR2^3r2^3cQ0Q1ta^2 - 3mR2^4c^3Q0R1^2q1 - 3mR2^4c^3Q1R1^2q0 \\
& - 6mR2^5c^3Q0R1q1 - 6mR2^5c^3Q1R1q0 - 12mR2^5tas^5Q0r2q1 - 12mR2^5tas^5Q1r2q0 \\
& + 6mR2^5tas^3Q0r2q1 + 6mR2^5tas^3Q1r2q0 \\
& - 3(mR2^4c^3Q0R1^2q1 + mR2^4c^3Q1R1^2q0)s^2 \\
& - 6(mR2^5c^3Q0R1q1 + mR2^5c^3Q1R1q0)s^2 \\
& - 6(mR2^5c^3Q0r2ta^2q1 + mR2^5c^3Q1r2ta^2q0)s^2 \\
& - 6(mR2^4tas^5Q0r2q1 + mR2^4tas^5Q1r2q0)R1 \\
& + 3(mR2^4tas^3Q0r2q1 + mR2^4tas^3Q1r2q0)R1 - 6mR2^5c^3Q0r2ta^2q1 \\
& - 6mR2^5c^3Q1r2ta^2q0 + 12mR2^2r2^2cQ0R1^2q1 + 12mR2^2r2^2cQ1R1^2q0 \\
& - 3(mR2^4c^3Q0r2ta^2q1 + mR2^4c^3Q1r2ta^2q0)R1 \\
& + 12(mR2^2r2^3cQ0ta^2q1 + mR2^2r2^3cQ1ta^2q0)R1 + 24mR2^3r2^3cQ0ta^2q1
\end{aligned}$$

$$\begin{aligned}
& + 24 m R2^3 r2^3 c Q1 ta^2 q0 + 24 m R2^3 r2^2 c Q0 RI q1 + 24 m R2^3 r2^2 c Q1 RI q0)) s1^2 + (q0 (-3 (m R2^4 c^3 Q0 r2 ta^2 q1 + m R2^4 c^3 Q1 r2 ta^2 q0) s^2 RI \\
& - 2 (m R2^5 s^4 c Q0 q1 + m R2^5 s^4 c Q1 q0) r2 ta^2 - 4 m R2^5 s^4 c q0 q1 RI - 2 m R2^4 s^4 c q0 q1 RI^2 \\
& - 4 m R2^5 s^4 c q0 q1 r2 ta^2 - 48 m R2^5 tas^5 Q0 Q1 r2 - 2 m R2^4 s^4 c q0 q1 r2 ta^2 RI \\
& + 24 m R2^5 tas^3 Q0 Q1 r2 - 72 m R2^5 Q0 Q1 s^4 c r2 + 6 m R2^4 c^3 Q0 Q1 RI^2 \\
& + 4 m R2^4 s^4 c Q0 Q1 RI^2 + 8 m R2^5 Q0 Q1 s^4 c RI + 8 m R2^5 Q0 Q1 s^4 c r2 ta^2 \\
& - 72 m R2^5 Q0 Q1 s^2 c r2 - 24 m R2^2 r2^2 c Q0 Q1 RI^2 - 24 m R2^4 tas^5 Q0 Q1 r2 RI \\
& + 12 m R2^4 tas^3 Q0 Q1 r2 RI + 12 m R2^5 c^3 Q0 Q1 RI + 6 m R2^4 c^3 Q0 Q1 RI^2 s^2 \\
& + 12 m R2^5 c^3 Q0 Q1 RI s^2 + 12 m R2^5 c^3 Q0 Q1 r2 ta^2 + 12 m R2^5 c^3 Q0 Q1 r2 ta^2 s^2 \\
& - 48 m R2^3 r2^2 c Q0 Q1 RI + 4 m R2^4 s^4 c Q0 Q1 r2 ta^2 RI - 36 m R2^4 Q0 Q1 s^2 c r2 RI \\
& - (m R2^4 s^4 c Q0 q1 + m R2^4 s^4 c Q1 q0) r2 ta^2 RI - 36 m R2^4 s^4 c Q0 Q1 r2 RI \\
& + 6 m R2^4 c^3 Q0 Q1 r2 ta^2 RI + 6 m R2^4 c^3 Q0 Q1 r2 ta^2 s^2 RI + 48 m R2^2 r2^3 c Q0 Q1 ta^2 RI \\
& - (m R2^4 s^4 c Q0 q1 + m R2^4 s^4 c Q1 q0) RI^2 - 2 (m R2^5 s^4 c Q0 q1 + m R2^5 s^4 c Q1 q0) RI \\
& + 96 m R2^3 r2^3 c Q0 Q1 ta^2 - 3 m R2^4 c^3 Q0 RI^2 q1 - 3 m R2^4 c^3 Q1 RI^2 q0 \\
& - 6 m R2^5 c^3 Q0 RI q1 - 6 m R2^5 c^3 Q1 RI q0 - 12 m R2^5 tas^5 Q0 r2 q1 - 12 m R2^5 tas^5 Q1 r2 q0 \\
& + 6 m R2^5 tas^3 Q0 r2 q1 + 6 m R2^5 tas^3 Q1 r2 q0 \\
& - 3 (m R2^4 c^3 Q0 RI^2 q1 + m R2^4 c^3 Q1 RI^2 q0) s^2 \\
& - 6 (m R2^5 c^3 Q0 RI q1 + m R2^5 c^3 Q1 RI q0) s^2 \\
& - 6 (m R2^5 c^3 Q0 r2 ta^2 q1 + m R2^5 c^3 Q1 r2 ta^2 q0) s^2 \\
& - 6 (m R2^4 tas^5 Q0 r2 q1 + m R2^4 tas^5 Q1 r2 q0) RI \\
& + 3 (m R2^4 tas^3 Q0 r2 q1 + m R2^4 tas^3 Q1 r2 q0) RI - 6 m R2^5 c^3 Q0 r2 ta^2 q1 \\
& - 6 m R2^5 c^3 Q1 r2 ta^2 q0 + 12 m R2^2 r2^2 c Q0 RI^2 q1 + 12 m R2^2 r2^2 c Q1 RI^2 q0 \\
& - 3 (m R2^4 c^3 Q0 r2 ta^2 q1 + m R2^4 c^3 Q1 r2 ta^2 q0) RI \\
& + 12 (m R2^2 r2^3 c Q0 ta^2 q1 + m R2^2 r2^3 c Q1 ta^2 q0) RI + 24 m R2^3 r2^3 c Q0 ta^2 q1 \\
& + 24 m R2^3 r2^3 c Q1 ta^2 q0 + 24 m R2^3 r2^2 c Q0 RI q1 + 24 m R2^3 r2^2 c Q1 RI q0) + q1 (-2 m R2^5 s^4 c q0^2 RI - m R2^4 s^4 c q0^2 RI^2 - 2 m R2^5 s^4 c q0^2 r2 ta^2 - 24 m R2^5 tas^5 Q0^2 r2 \\
& - m R2^4 s^4 c q0^2 r2 ta^2 RI + 12 m R2^5 tas^3 Q0^2 r2 - 36 m R2^5 Q0^2 s^4 c r2 + 3 m R2^4 c^3 Q0^2 RI^2 \\
& + 2 m R2^4 s^4 c Q0^2 RI^2 + 4 m R2^5 Q0^2 s^4 c RI + 4 m R2^5 Q0^2 s^4 c r2 ta^2 - 36 m R2^5 Q0^2 s^2 c r2 \\
& - 12 m R2^2 r2^2 c Q0^2 RI^2 - 12 m R2^4 tas^5 Q0^2 r2 RI + 6 m R2^4 tas^3 Q0^2 r2 RI \\
& + 6 m R2^5 c^3 Q0^2 RI + 3 m R2^4 c^3 Q0^2 RI^2 s^2 + 6 m R2^5 c^3 Q0^2 RI s^2 + 6 m R2^5 c^3 Q0^2 r2 ta^2 \\
& + 6 m R2^5 c^3 Q0^2 r2 ta^2 s^2 - 24 m R2^3 r2^2 c Q0^2 RI + 2 m R2^4 s^4 c Q0^2 r2 ta^2 RI \\
& - 18 m R2^4 Q0^2 s^2 c r2 RI - 18 m R2^4 s^4 c Q0^2 r2 RI + 3 m R2^4 c^3 Q0^2 r2 ta^2 RI \\
& + 3 m R2^4 c^3 Q0^2 r2 ta^2 s^2 RI + 24 m R2^2 r2^3 c Q0^2 ta^2 RI + 48 m R2^3 r2^3 c Q0^2 ta^2 \\
& - m R2^4 s^4 c Q0 q0 RI^2 - 2 m R2^5 s^4 c Q0 q0 RI - 2 m R2^5 s^4 c Q0 q0 r2 ta^2 \\
& - 3 m R2^4 c^3 Q0 RI^2 q0 - 6 m R2^5 c^3 Q0 RI q0 - 12 m R2^5 tas^5 Q0 r2 q0 + 6 m R2^5 tas^3 Q0 r2 q0 \\
& - 3 m R2^4 c^3 Q0 RI^2 q0 s^2 - 6 m R2^5 c^3 Q0 RI q0 s^2 - 6 m R2^5 c^3 Q0 r2 ta^2 q0 \\
& - 6 m R2^5 c^3 Q0 r2 ta^2 q0 s^2 + 12 m R2^2 r2^2 c Q0 RI^2 q0 - 6 m R2^4 tas^5 Q0 r2 q0 RI \\
& + 3 m R2^4 tas^3 Q0 r2 q0 RI - m R2^4 s^4 c Q0 q0 r2 ta^2 RI - 3 m R2^4 c^3 Q0 r2 ta^2 q0 RI \\
& - 3 m R2^4 c^3 Q0 r2 ta^2 q0 s^2 RI + 12 m R2^2 r2^3 c Q0 ta^2 q0 RI + 24 m R2^3 r2^3 c Q0 ta^2 q0 \\
& + 24 m R2^3 r2^2 c Q0 RI q0) s1 + q0 (-2 m R2^5 s^4 c q0^2 RI - m R2^4 s^4 c q0^2 RI^2 \\
& - 2 m R2^5 s^4 c q0^2 r2 ta^2 - 24 m R2^5 tas^5 Q0^2 r2 - m R2^4 s^4 c q0^2 r2 ta^2 RI \\
& + 12 m R2^5 tas^3 Q0^2 r2 - 36 m R2^5 Q0^2 s^4 c r2 + 3 m R2^4 c^3 Q0^2 RI^2 + 2 m R2^4 s^4 c Q0^2 RI^2
\end{aligned}$$

$$\begin{aligned}
& + 4 m R2^5 Q0^2 s^4 c R1 + 4 m R2^5 Q0^2 s^4 c r2 ta^2 - 36 m R2^5 Q0^2 s^2 c r2 \\
& - 12 m R2^2 r2^2 c Q0^2 R1^2 - 12 m R2^4 tas^5 Q0^2 r2 R1 + 6 m R2^4 tas^3 Q0^2 r2 R1 \\
& + 6 m R2^5 c^3 Q0^2 R1 + 3 m R2^4 c^3 Q0^2 R1^2 s^2 + 6 m R2^5 c^3 Q0^2 R1 s^2 + 6 m R2^5 c^3 Q0^2 r2 ta^2 \\
& + 6 m R2^5 c^3 Q0^2 r2 ta^2 s^2 - 24 m R2^3 r2^2 c Q0^2 R1 + 2 m R2^4 s^4 c Q0^2 r2 ta^2 R1 \\
& - 18 m R2^4 Q0^2 s^2 c r2 R1 - 18 m R2^4 s^4 c Q0^2 r2 R1 + 3 m R2^4 c^3 Q0^2 r2 ta^2 R1 \\
& + 3 m R2^4 c^3 Q0^2 r2 ta^2 s^2 R1 + 24 m R2^2 r2^3 c Q0^2 ta^2 R1 + 48 m R2^3 r2^3 c Q0^2 ta^2 \\
& - m R2^4 s^4 c Q0 q0 R1^2 - 2 m R2^5 s^4 c Q0 q0 R1 - 2 m R2^5 s^4 c Q0 q0 r2 ta^2 \\
& - 3 m R2^4 c^3 Q0 R1^2 q0 - 6 m R2^5 c^3 Q0 R1 q0 - 12 m R2^5 tas^5 Q0 r2 q0 + 6 m R2^5 tas^3 Q0 r2 q0 \\
& - 3 m R2^4 c^3 Q0 R1^2 q0 s^2 - 6 m R2^5 c^3 Q0 R1 q0 s^2 - 6 m R2^5 c^3 Q0 r2 ta^2 q0 \\
& - 6 m R2^5 c^3 Q0 r2 ta^2 q0 s^2 + 12 m R2^2 r2^2 c Q0 R1^2 q0 - 6 m R2^4 tas^5 Q0 r2 q0 R1 \\
& + 3 m R2^4 tas^3 Q0 r2 q0 R1 - m R2^4 s^4 c Q0 q0 r2 ta^2 R1 - 3 m R2^4 c^3 Q0 r2 ta^2 q0 R1 \\
& - 3 m R2^4 c^3 Q0 r2 ta^2 q0 s^2 R1 + 12 m R2^2 r2^3 c Q0 ta^2 q0 R1 + 24 m R2^3 r2^3 c Q0 ta^2 q0 \\
& + 24 m R2^3 r2^2 c Q0 R1 q0
\end{aligned}$$

> D5:=coeff(LHS3,s1^5);

$$D5 := 0$$

> D4:=coeff(LHS3,s1^4);

$$D4 := 0$$

> D3:=coeff(LHS3,s1^3);

$$D3 := 0$$

> D2:=coeff(LHS3,s1^2);

$$D2 := 2 r2 (2 r2 QI + r2 qI + 2 r2 ta^2 QI + r2 ta^2 qI + R2 QI - R2 qI) (R1 + 2 R2)^4 (2 QI + qI)$$

> D1:=coeff(LHS3,s1);

$$\begin{aligned}
D1 := & 2 r2 (2 r2 Q0 + r2 q0 + 2 r2 ta^2 Q0 + r2 ta^2 q0 + R2 Q0 - R2 q0) (R1 + 2 R2)^4 (2 QI + qI) \\
& + 2 r2 (2 r2 QI + r2 qI + 2 r2 ta^2 QI + r2 ta^2 qI + R2 QI - R2 qI) (R1 + 2 R2)^4 (2 Q0 + q0)
\end{aligned}$$

> D0:=coeff(LHS3,s1,0);

$$D0 := 2 r2 (2 r2 Q0 + r2 q0 + 2 r2 ta^2 Q0 + r2 ta^2 q0 + R2 Q0 - R2 q0) (R1 + 2 R2)^4 (2 Q0 + q0)$$

> E5:=coeff(epsilon2,s1^5);

$$E5 := 0$$

> E4:=coeff(epsilon2,s1^4);

$$E4 := 0$$

> E3:=coeff(epsilon2,s1^3);

$$\begin{aligned}
E3 := & qI (-2 m R2^5 s^4 c qI^2 R1 - m R2^4 s^4 c qI^2 R1^2 - 2 m R2^5 s^4 c qI^2 r2 ta^2 - 24 m R2^5 tas^5 QI^2 r2 \\
& - m R2^4 s^4 c qI^2 r2 ta^2 R1 + 12 m R2^5 tas^3 QI^2 r2 - 36 m R2^5 QI^2 s^4 c r2 + 3 m R2^4 c^3 QI^2 R1^2 \\
& + 2 m R2^4 s^4 c QI^2 R1^2 + 4 m R2^5 QI^2 s^4 c R1 + 4 m R2^5 QI^2 s^4 c r2 ta^2 - 36 m R2^5 QI^2 s^2 c r2 \\
& - 12 m R2^2 r2^2 c QI^2 R1^2 - 12 m R2^4 tas^5 QI^2 r2 R1 + 6 m R2^4 tas^3 QI^2 r2 RI \\
& + 6 m R2^5 c^3 QI^2 R1 + 3 m R2^4 c^3 QI^2 R1^2 s^2 + 6 m R2^5 c^3 QI^2 R1 s^2 + 6 m R2^5 c^3 QI^2 r2 ta^2 \\
& + 6 m R2^5 c^3 QI^2 r2 ta^2 s^2 - 24 m R2^3 r2^2 c QI^2 R1 + 2 m R2^4 s^4 c QI^2 r2 ta^2 R1
\end{aligned}$$

$$\begin{aligned}
& -18mR2^4QI^2s^2c r2 RI - 18mR2^4s^4c QI^2r2 RI + 3mR2^4c^3QI^2r2 ta^2 RI \\
& + 3mR2^4c^3QI^2r2 ta^2 s^2 RI + 24mR2^2r2^3c QI^2ta^2 RI + 48mR2^3r2^3c QI^2ta^2 \\
& - mR2^4s^4c QI q1 RI^2 - 2mR2^5s^4c QI q1 RI - 2mR2^5s^4c QI q1 r2 ta^2 \\
& - 3mR2^4c^3QI RI^2q1 - 6mR2^5c^3QI RI q1 - 12mR2^5ta^5QI r2 q1 + 6mR2^5ta^3QI r2 q1 \\
& - 3mR2^4c^3QI RI^2q1 s^2 - 6mR2^5c^3QI RI q1 s^2 - 6mR2^5c^3QI r2 ta^2 q1 \\
& - 6mR2^5c^3QI r2 ta^2 q1 s^2 + 12mR2^2r2^2c QI RI^2q1 - 6mR2^4ta^5QI r2 q1 RI \\
& + 3mR2^4ta^3QI r2 q1 RI - mR2^4s^4c QI q1 r2 ta^2 RI - 3mR2^4c^3QI r2 ta^2 q1 RI \\
& - 3mR2^4c^3QI r2 ta^2 q1 s^2 RI + 12mR2^2r2^3c QI ta^2 q1 RI + 24mR2^3r2^3c QI ta^2 q1 \\
& + 24mR2^3r2^2c QI RI q1)
\end{aligned}$$

> E2:=coeff(epsilon2,s1^2);

$$\begin{aligned}
E2 := & q0 (-2mR2^5s^4c q1^2 RI - mR2^4s^4c q1^2 RI^2 - 2mR2^5s^4c q1^2 r2 ta^2 - 24mR2^5ta^5QI^2r2 \\
& - mR2^4s^4c q1^2 r2 ta^2 RI + 12mR2^5ta^3QI^2r2 - 36mR2^5QI^2s^4c r2 + 3mR2^4c^3QI^2RI^2 \\
& + 2mR2^4s^4c QI^2RI^2 + 4mR2^5QI^2s^4c RI + 4mR2^5QI^2s^4c r2 ta^2 - 36mR2^5QI^2s^2c r2 \\
& - 12mR2^2r2^2c QI^2RI^2 - 12mR2^4ta^5QI^2r2 RI + 6mR2^4ta^3QI^2r2 RI \\
& + 6mR2^5c^3QI^2RI + 3mR2^4c^3QI^2RI^2s^2 + 6mR2^5c^3QI^2RI s^2 + 6mR2^5c^3QI^2r2 ta^2 \\
& + 6mR2^5c^3QI^2r2 ta^2 s^2 - 24mR2^3r2^2c QI^2RI + 2mR2^4s^4c QI^2r2 ta^2 RI \\
& - 18mR2^4QI^2s^2c r2 RI - 18mR2^4s^4c QI^2r2 RI + 3mR2^4c^3QI^2r2 ta^2 RI \\
& + 3mR2^4c^3QI^2r2 ta^2 s^2 RI + 24mR2^2r2^3c QI^2ta^2 RI + 48mR2^3r2^3c QI^2ta^2 \\
& - mR2^4s^4c QI q1 RI^2 - 2mR2^5s^4c QI q1 RI - 2mR2^5s^4c QI q1 r2 ta^2 \\
& - 3mR2^4c^3QI RI^2q1 - 6mR2^5c^3QI RI q1 - 12mR2^5ta^5QI r2 q1 + 6mR2^5ta^3QI r2 q1 \\
& - 3mR2^4c^3QI RI^2q1 s^2 - 6mR2^5c^3QI RI q1 s^2 - 6mR2^5c^3QI r2 ta^2 q1 \\
& - 6mR2^5c^3QI r2 ta^2 q1 s^2 + 12mR2^2r2^2c QI RI^2q1 - 6mR2^4ta^5QI r2 q1 RI \\
& + 3mR2^4ta^3QI r2 q1 RI - mR2^4s^4c QI q1 r2 ta^2 RI - 3mR2^4c^3QI r2 ta^2 q1 RI \\
& - 3mR2^4c^3QI r2 ta^2 q1 s^2 RI + 12mR2^2r2^3c QI ta^2 q1 RI + 24mR2^3r2^3c QI ta^2 q1 \\
& + 24mR2^3r2^2c QI RI q1) + q1 (-3(mR2^4c^3Q0 r2 ta^2 q1 + mR2^4c^3Q1 r2 ta^2 q0)s^2 RI \\
& - 2(mR2^5s^4c Q0 q1 + mR2^5s^4c Q1 q0)r2 ta^2 - 4mR2^5s^4c q0 q1 RI - 2mR2^4s^4c q0 q1 RI^2 \\
& - 4mR2^5s^4c q0 q1 r2 ta^2 - 48mR2^5ta^5Q0 Q1 r2 - 2mR2^4s^4c q0 q1 r2 ta^2 RI \\
& + 24mR2^5ta^3Q0 Q1 r2 - 72mR2^5Q0 Q1 s^4c r2 + 6mR2^4c^3Q0 Q1 RI^2 \\
& + 4mR2^4s^4c Q0 Q1 RI^2 + 8mR2^5Q0 Q1 s^4c RI + 8mR2^5Q0 Q1 s^4c r2 ta^2 \\
& - 72mR2^5Q0 Q1 s^2c r2 - 24mR2^2r2^2c Q0 Q1 RI^2 - 24mR2^4ta^5Q0 Q1 r2 RI \\
& + 12mR2^4ta^3Q0 Q1 r2 RI + 12mR2^5c^3Q0 Q1 RI + 6mR2^4c^3Q0 Q1 RI^2s^2 \\
& + 12mR2^5c^3Q0 Q1 RI s^2 + 12mR2^5c^3Q0 Q1 r2 ta^2 + 12mR2^5c^3Q0 Q1 r2 ta^2 s^2 \\
& - 48mR2^3r2^2c Q0 Q1 RI + 4mR2^4s^4c Q0 Q1 r2 ta^2 RI - 36mR2^4Q0 Q1 s^2c r2 RI \\
& - (mR2^4s^4c Q0 q1 + mR2^4s^4c Q1 q0)r2 ta^2 RI - 36mR2^4s^4c Q0 Q1 r2 RI \\
& + 6mR2^4c^3Q0 Q1 r2 ta^2 RI + 6mR2^4c^3Q0 Q1 r2 ta^2 s^2 RI + 48mR2^2r2^3c Q0 Q1 ta^2 RI \\
& - (mR2^4s^4c Q0 q1 + mR2^4s^4c Q1 q0)RI^2 - 2(mR2^5s^4c Q0 q1 + mR2^5s^4c Q1 q0)RI \\
& + 96mR2^3r2^3c Q0 Q1 ta^2 - 3mR2^4c^3Q0 RI^2q1 - 3mR2^4c^3Q1 RI^2q0 \\
& - 6mR2^5c^3Q0 RI q1 - 6mR2^5c^3Q1 RI q0 - 12mR2^5ta^5Q0 r2 q1 - 12mR2^5ta^5Q1 r2 q0 \\
& + 6mR2^5ta^3Q0 r2 q1 + 6mR2^5ta^3Q1 r2 q0 \\
& - 3(mR2^4c^3Q0 RI^2q1 + mR2^4c^3Q1 RI^2q0)s^2 \\
& - 6(mR2^5c^3Q0 RI q1 + mR2^5c^3Q1 RI q0)s^2 \\
& - 6(mR2^5c^3Q0 r2 ta^2 q1 + mR2^5c^3Q1 r2 ta^2 q0)s^2
\end{aligned}$$

$$\begin{aligned}
& -6(mR2^4 tas^5 Q0 r2 q1 + mR2^4 tas^5 Q1 r2 q0) R1 \\
& + 3(mR2^4 tas^3 Q0 r2 q1 + mR2^4 tas^3 Q1 r2 q0) R1 - 6mR2^5 c^3 Q0 r2 ta^2 q1 \\
& - 6mR2^5 c^3 Q1 r2 ta^2 q0 + 12mR2^2 r2^2 c Q0 R1^2 q1 + 12mR2^2 r2^2 c Q1 R1^2 q0 \\
& - 3(mR2^4 c^3 Q0 r2 ta^2 q1 + mR2^4 c^3 Q1 r2 ta^2 q0) R1 \\
& + 12(mR2^2 r2^3 c Q0 ta^2 q1 + mR2^2 r2^3 c Q1 ta^2 q0) R1 + 24mR2^3 r2^3 c Q0 ta^2 q1 \\
& + 24mR2^3 r2^3 c Q1 ta^2 q0 + 24mR2^3 r2^2 c Q0 R1 q1 + 24mR2^3 r2^2 c Q1 R1 q0
\end{aligned}$$

> E1:=coeff(epsilon2,s1);

$$\begin{aligned}
E1 := & q0 (-3(mR2^4 c^3 Q0 r2 ta^2 q1 + mR2^4 c^3 Q1 r2 ta^2 q0) s^2 R1 \\
& - 2(mR2^5 s^4 c Q0 q1 + mR2^5 s^4 c Q1 q0) r2 ta^2 - 4mR2^5 s^4 c q0 q1 R1 - 2mR2^4 s^4 c q0 q1 R1^2 \\
& - 4mR2^5 s^4 c q0 q1 r2 ta^2 - 48mR2^5 tas^5 Q0 Q1 r2 - 2mR2^4 s^4 c q0 q1 r2 ta^2 R1 \\
& + 24mR2^5 tas^3 Q0 Q1 r2 - 72mR2^5 Q0 Q1 s^4 c r2 + 6mR2^4 c^3 Q0 Q1 R1^2 \\
& + 4mR2^4 s^4 c Q0 Q1 R1^2 + 8mR2^5 Q0 Q1 s^4 c R1 + 8mR2^5 Q0 Q1 s^4 c r2 ta^2 \\
& - 72mR2^5 Q0 Q1 s^2 c r2 - 24mR2^2 r2^2 c Q0 Q1 R1^2 - 24mR2^4 tas^5 Q0 Q1 r2 R1 \\
& + 12mR2^4 tas^3 Q0 Q1 r2 R1 + 12mR2^5 c^3 Q0 Q1 R1 + 6mR2^4 c^3 Q0 Q1 R1^2 s^2 \\
& + 12mR2^5 c^3 Q0 Q1 R1 s^2 + 12mR2^5 c^3 Q0 Q1 r2 ta^2 + 12mR2^5 c^3 Q0 Q1 r2 ta^2 s^2 \\
& - 48mR2^3 r2^2 c Q0 Q1 R1 + 4mR2^4 s^4 c Q0 Q1 r2 ta^2 R1 - 36mR2^4 Q0 Q1 s^2 c r2 R1 \\
& - (mR2^4 s^4 c Q0 q1 + mR2^4 s^4 c Q1 q0) r2 ta^2 R1 - 36mR2^4 s^4 c Q0 Q1 r2 R1 \\
& + 6mR2^4 c^3 Q0 Q1 r2 ta^2 R1 + 6mR2^4 c^3 Q0 Q1 r2 ta^2 s^2 R1 + 48mR2^2 r2^3 c Q0 Q1 ta^2 R1 \\
& - (mR2^4 s^4 c Q0 q1 + mR2^4 s^4 c Q1 q0) R1^2 - 2(mR2^5 s^4 c Q0 q1 + mR2^5 s^4 c Q1 q0) R1 \\
& + 96mR2^3 r2^3 c Q0 Q1 ta^2 - 3mR2^4 c^3 Q0 R1^2 q1 - 3mR2^4 c^3 Q1 R1^2 q0 \\
& - 6mR2^5 c^3 Q0 R1 q1 - 6mR2^5 c^3 Q1 R1 q0 - 12mR2^5 tas^5 Q0 r2 q1 - 12mR2^5 tas^5 Q1 r2 q0 \\
& + 6mR2^5 tas^3 Q0 r2 q1 + 6mR2^5 tas^3 Q1 r2 q0 \\
& - 3(mR2^4 c^3 Q0 R1^2 q1 + mR2^4 c^3 Q1 R1^2 q0) s^2 \\
& - 6(mR2^5 c^3 Q0 R1 q1 + mR2^5 c^3 Q1 R1 q0) s^2 \\
& - 6(mR2^5 c^3 Q0 r2 ta^2 q1 + mR2^5 c^3 Q1 r2 ta^2 q0) s^2 \\
& - 6(mR2^4 tas^5 Q0 r2 q1 + mR2^4 tas^5 Q1 r2 q0) R1 \\
& + 3(mR2^4 tas^3 Q0 r2 q1 + mR2^4 tas^3 Q1 r2 q0) R1 - 6mR2^5 c^3 Q0 r2 ta^2 q1 \\
& - 6mR2^5 c^3 Q1 r2 ta^2 q0 + 12mR2^2 r2^2 c Q0 R1^2 q1 + 12mR2^2 r2^2 c Q1 R1^2 q0 \\
& - 3(mR2^4 c^3 Q0 r2 ta^2 q1 + mR2^4 c^3 Q1 r2 ta^2 q0) R1 \\
& + 12(mR2^2 r2^3 c Q0 ta^2 q1 + mR2^2 r2^3 c Q1 ta^2 q0) R1 + 24mR2^3 r2^3 c Q0 ta^2 q1 \\
& + 24mR2^3 r2^3 c Q1 ta^2 q0 + 24mR2^3 r2^2 c Q0 R1 q1 + 24mR2^3 r2^2 c Q1 R1 q0) + q1 \\
& - 2mR2^5 s^4 c q0^2 R1 - mR2^4 s^4 c q0^2 R1^2 - 2mR2^5 s^4 c q0^2 r2 ta^2 - 24mR2^5 tas^5 Q0^2 r2 \\
& - mR2^4 s^4 c q0^2 r2 ta^2 R1 + 12mR2^5 tas^3 Q0^2 r2 - 36mR2^5 Q0^2 s^4 c r2 + 3mR2^4 c^3 Q0^2 R1^2 \\
& + 2mR2^4 s^4 c Q0^2 R1^2 + 4mR2^5 Q0^2 s^4 c R1 + 4mR2^5 Q0^2 s^4 c r2 ta^2 - 36mR2^5 Q0^2 s^2 c r2 \\
& - 12mR2^2 r2^2 c Q0^2 R1^2 - 12mR2^4 tas^5 Q0^2 r2 R1 + 6mR2^4 tas^3 Q0^2 r2 RI \\
& + 6mR2^5 c^3 Q0^2 R1 + 3mR2^4 c^3 Q0^2 R1^2 s^2 + 6mR2^5 c^3 Q0^2 R1 s^2 + 6mR2^5 c^3 Q0^2 r2 ta^2 \\
& + 6mR2^5 c^3 Q0^2 r2 ta^2 s^2 - 24mR2^3 r2^2 c Q0^2 R1 + 2mR2^4 s^4 c Q0^2 r2 ta^2 R1 \\
& - 18mR2^4 Q0^2 s^2 c r2 R1 - 18mR2^4 s^4 c Q0^2 r2 R1 + 3mR2^4 c^3 Q0^2 r2 ta^2 R1 \\
& + 3mR2^4 c^3 Q0^2 r2 ta^2 s^2 R1 + 24mR2^2 r2^3 c Q0^2 ta^2 R1 + 48mR2^3 r2^3 c Q0^2 ta^2 \\
& - mR2^4 s^4 c Q0 q0 R1^2 - 2mR2^5 s^4 c Q0 q0 R1 - 2mR2^5 s^4 c Q0 q0 r2 ta^2
\end{aligned}$$

$$\begin{aligned}
& -3mR2^4c^3Q0RI^2q0 - 6mR2^5c^3Q0RIq0 - 12mR2^5tas^5Q0r2q0 + 6mR2^5tas^3Q0r2q0 \\
& - 3mR2^4c^3Q0RI^2q0s^2 - 6mR2^5c^3Q0RIq0s^2 - 6mR2^5c^3Q0r2ta^2q0 \\
& - 6mR2^5c^3Q0r2ta^2q0s^2 + 12mR2^2r2^2cQ0RI^2q0 - 6mR2^4tas^5Q0r2q0RI \\
& + 3mR2^4tas^3Q0r2q0RI - mR2^4s^4cQ0q0r2ta^2RI - 3mR2^4c^3Q0r2ta^2q0RI \\
& - 3mR2^4c^3Q0r2ta^2q0s^2RI + 12mR2^2r2^3cQ0ta^2q0RI + 24mR2^3r2^3cQ0ta^2q0 \\
& + 24mR2^3r2^2cQ0RIq0
\end{aligned}$$

> E0:=coeff(epsilon2,s1,0);

$$\begin{aligned}
E0 := & q0(-2mR2^5s^4c q0^2 RI - mR2^4s^4c q0^2 RI^2 - 2mR2^5s^4c q0^2 r2ta^2 - 24mR2^5tas^5Q0^2r2 \\
& - mR2^4s^4c q0^2 r2ta^2 RI + 12mR2^5tas^3Q0^2r2 - 36mR2^5Q0^2s^4cr2 + 3mR2^4c^3Q0^2RI^2 \\
& + 2mR2^4s^4c Q0^2RI^2 + 4mR2^5Q0^2s^4c RI + 4mR2^5Q0^2s^4cr2ta^2 - 36mR2^5Q0^2s^2cr2 \\
& - 12mR2^2r2^2cQ0^2RI^2 - 12mR2^4tas^5Q0^2r2RI + 6mR2^4tas^3Q0^2r2RI \\
& + 6mR2^5c^3Q0^2RI + 3mR2^4c^3Q0^2RI^2s^2 + 6mR2^5c^3Q0^2RI s^2 + 6mR2^5c^3Q0^2r2ta^2 \\
& + 6mR2^5c^3Q0^2r2ta^2s^2 - 24mR2^3r2^2cQ0^2RI + 2mR2^4s^4cQ0^2r2ta^2RI \\
& - 18mR2^4Q0^2s^2cr2RI - 18mR2^4s^4cQ0^2r2RI + 3mR2^4c^3Q0^2r2ta^2RI \\
& + 3mR2^4c^3Q0^2r2ta^2s^2RI + 24mR2^2r2^3cQ0^2ta^2RI + 48mR2^3r2^3cQ0^2ta^2 \\
& - mR2^4s^4cQ0q0RI^2 - 2mR2^5s^4cQ0q0RI - 2mR2^5s^4cQ0q0r2ta^2 \\
& - 3mR2^4c^3Q0RI^2q0 - 6mR2^5c^3Q0RIq0 - 12mR2^5tas^5Q0r2q0 + 6mR2^5tas^3Q0r2q0 \\
& - 3mR2^4c^3Q0RI^2q0s^2 - 6mR2^5c^3Q0RIq0s^2 - 6mR2^5c^3Q0r2ta^2q0 \\
& - 6mR2^5c^3Q0r2ta^2q0s^2 + 12mR2^2r2^2cQ0RI^2q0 - 6mR2^4tas^5Q0r2q0RI \\
& + 3mR2^4tas^3Q0r2q0RI - mR2^4s^4cQ0q0r2ta^2RI - 3mR2^4c^3Q0r2ta^2q0RI \\
& - 3mR2^4c^3Q0r2ta^2q0s^2RI + 12mR2^2r2^3cQ0ta^2q0RI + 24mR2^3r2^3cQ0ta^2q0 \\
& + 24mR2^3r2^2cQ0RIq0)
\end{aligned}$$

> B5:=coeff(beta2,s1^5);

$$B5 := 0$$

> B4:=coeff(beta2,s1^4);

$$B4 := 0$$

> B3:=coeff(beta2,s1^3);

$$\begin{aligned}
B3 := & q1(r2^2RI^4q1^2 + 4r2^2RI^4Q1^2 - r2RI^4q1^2R2 + r2^2RI^4q1^2ta^2 + 2r2RI^4Q1^2R2 \\
& + 4r2^2RI^4Q1^2ta^2 + 4r2^2RI^4Q1q1 + 2r2mR2^5s^5Q1^2 + 2r2^2mR2^4s^5ta^2q1^2 \\
& - 4r2^2mR2^4s^3ta^2Q1^2 - r2^2mR2^4s^3ta^2q1^2 - r2mR2^5s^5q1^2 + 8r2^2mR2^4s^5ta^2Q1^2 \\
& + 12r2^2mR2^4s^3Q1^2 + 12r2^2mR2^4s^5Q1^2 + 3r2mR2^5sc^2Q1^2 + 6r2^2mR2^4s^3Q1q1 \\
& + 3r2mR2^5s^3c^2Q1^2 + 24r2^4mR2^2sQ1^2 + 4r2^2RI^4Q1q1ta^2 - r2RI^4Q1R2q1 \\
& + 8r2^2mR2^4s^5ta^2Q1q1 - 4r2^2mR2^4s^3ta^2Q1q1 - r2mR2^5s^5Q1q1 \\
& + 6r2^2mR2^4s^5Q1q1 - 3r2mR2^5sc^2Q1q1 - 3r2mR2^5s^3c^2Q1q1 + 12r2^4mR2^2sQ1q1)
\end{aligned}$$

> B2:=coeff(beta2,s1^2);

$$\begin{aligned}
B2 := & q0(r2^2RI^4q1^2 + 4r2^2RI^4Q1^2 - r2RI^4q1^2R2 + r2^2RI^4q1^2ta^2 + 2r2RI^4Q1^2R2 \\
& + 4r2^2RI^4Q1^2ta^2 + 4r2^2RI^4Q1q1 + 2r2mR2^5s^5Q1^2 + 2r2^2mR2^4s^5ta^2q1^2 \\
& - 4r2^2mR2^4s^3ta^2Q1^2 - r2^2mR2^4s^3ta^2q1^2 - r2mR2^5s^5q1^2 + 8r2^2mR2^4s^5ta^2Q1^2 \\
& + 12r2^2mR2^4s^3Q1^2 + 12r2^2mR2^4s^5Q1^2 + 3r2mR2^5sc^2Q1^2 + 6r2^2mR2^4s^3Q1q1 \\
& + 3r2mR2^5s^3c^2Q1^2 + 24r2^4mR2^2sQ1^2 + 4r2^2RI^4Q1q1ta^2 - r2RI^4Q1R2q1
\end{aligned}$$

$$\begin{aligned}
& + 8 r2^2 m R2^4 s^5 ta^2 Q1 q1 - 4 r2^2 m R2^4 s^3 ta^2 Q1 q1 - r2 m R2^5 s^5 Q1 q1 \\
& + 6 r2^2 m R2^4 s^5 Q1 q1 - 3 r2 m R2^5 s c^2 Q1 q1 - 3 r2 m R2^5 s^3 c^2 Q1 q1 + 12 r2^4 m R2^2 s Q1 q1) \\
& q1 (2 r2^2 RI^4 q0 q1 + 8 r2^2 RI^4 Q0 Q1 + 4 r2^2 RI^4 Q0 q1 + 4 r2^2 RI^4 Q1 q0 - 2 r2 RI^4 q0 q1 R2 \\
& + 2 r2^2 RI^4 q0 q1 ta^2 + 4 r2 RI^4 Q0 Q1 R2 + 8 r2^2 RI^4 Q0 Q1 ta^2 + 4 r2^2 m R2^4 s^5 ta^2 q0 q1 \\
& + 4 r2 m R2^5 s^5 Q0 Q1 - 2 r2^2 m R2^4 s^3 ta^2 q0 q1 - 2 r2 m R2^5 s^5 q0 q1 \\
& + 16 r2^2 m R2^4 s^5 ta^2 Q0 Q1 - 8 r2^2 m R2^4 s^3 ta^2 Q0 Q1 + 24 r2^2 m R2^4 s^3 Q0 Q1 \\
& + 24 r2^2 m R2^4 s^5 Q0 Q1 + 6 r2 m R2^5 s c^2 Q0 Q1 + 6 r2^2 m R2^4 s^3 Q0 q1 \\
& + 6 r2^2 m R2^4 s^3 Q1 q0 + 6 r2 m R2^5 s^3 c^2 Q0 Q1 + 4 (r2^2 RI^4 Q0 q1 + r2^2 RI^4 Q1 q0) ta^2 \\
& + 48 r2^4 m R2^2 s Q0 Q1 - r2 RI^4 Q0 R2 q1 - r2 RI^4 Q1 R2 q0 + 8 r2^2 m R2^4 s^5 ta^2 Q0 q1 \\
& + 8 r2^2 m R2^4 s^5 ta^2 Q1 q0 - 4 r2^2 m R2^4 s^3 ta^2 Q0 q1 - 4 r2^2 m R2^4 s^3 ta^2 Q1 q0 \\
& - r2 m R2^5 s^5 Q0 q1 - r2 m R2^5 s^5 Q1 q0 + 6 r2^2 m R2^4 s^5 Q0 q1 + 6 r2^2 m R2^4 s^5 Q1 q0 \\
& - 3 r2 m R2^5 s c^2 Q0 q1 - 3 r2 m R2^5 s c^2 Q1 q0 - 3 r2 m R2^5 s^3 c^2 Q0 q1 - 3 r2 m R2^5 s^3 c^2 Q1 q0 \\
& + 12 r2^4 m R2^2 s Q0 q1 + 12 r2^4 m R2^2 s Q1 q0)
\end{aligned}$$

> B1:=coeff(beta2,s1);

$$\begin{aligned}
B1 := & q0 (2 r2^2 RI^4 q0 q1 + 8 r2^2 RI^4 Q0 Q1 + 4 r2^2 RI^4 Q0 q1 + 4 r2^2 RI^4 Q1 q0 - 2 r2 RI^4 q0 q1 R2 \\
& + 2 r2^2 RI^4 q0 q1 ta^2 + 4 r2 RI^4 Q0 Q1 R2 + 8 r2^2 RI^4 Q0 Q1 ta^2 + 4 r2^2 m R2^4 s^5 ta^2 q0 q1 \\
& + 4 r2 m R2^5 s^5 Q0 Q1 - 2 r2^2 m R2^4 s^3 ta^2 q0 q1 - 2 r2 m R2^5 s^5 q0 q1 \\
& + 16 r2^2 m R2^4 s^5 ta^2 Q0 Q1 - 8 r2^2 m R2^4 s^3 ta^2 Q0 Q1 + 24 r2^2 m R2^4 s^3 Q0 Q1 \\
& + 24 r2^2 m R2^4 s^5 Q0 Q1 + 6 r2 m R2^5 s c^2 Q0 Q1 + 6 r2^2 m R2^4 s^3 Q0 q1 \\
& + 6 r2^2 m R2^4 s^3 Q1 q0 + 6 r2 m R2^5 s^3 c^2 Q0 Q1 + 4 (r2^2 RI^4 Q0 q1 + r2^2 RI^4 Q1 q0) ta^2 \\
& + 48 r2^4 m R2^2 s Q0 Q1 - r2 RI^4 Q0 R2 q1 - r2 RI^4 Q1 R2 q0 + 8 r2^2 m R2^4 s^5 ta^2 Q0 q1 \\
& + 8 r2^2 m R2^4 s^5 ta^2 Q1 q0 - 4 r2^2 m R2^4 s^3 ta^2 Q0 q1 - 4 r2^2 m R2^4 s^3 ta^2 Q1 q0 \\
& - r2 m R2^5 s^5 Q0 q1 - r2 m R2^5 s^5 Q1 q0 + 6 r2^2 m R2^4 s^5 Q0 q1 + 6 r2^2 m R2^4 s^5 Q1 q0 \\
& - 3 r2 m R2^5 s c^2 Q0 q1 - 3 r2 m R2^5 s c^2 Q1 q0 - 3 r2 m R2^5 s^3 c^2 Q0 q1 - 3 r2 m R2^5 s^3 c^2 Q1 q0 \\
& + 12 r2^4 m R2^2 s Q0 q1 + 12 r2^4 m R2^2 s Q1 q0) + q1 (4 r2^2 RI^4 Q0^2 + r2^2 RI^4 q0^2 \\
& + 2 r2 m R2^5 s^5 Q0^2 - r2 RI^4 q0^2 R2 + r2^2 RI^4 q0^2 ta^2 + 2 r2 RI^4 Q0^2 R2 + 4 r2^2 RI^4 Q0^2 ta^2 \\
& + 4 r2^2 RI^4 Q0 q0 + 2 r2^2 m R2^4 s^5 ta^2 q0^2 - 4 r2^2 m R2^4 s^3 ta^2 Q0^2 - r2^2 m R2^4 s^3 ta^2 q0^2 \\
& - r2 m R2^5 s^5 q0^2 + 8 r2^2 m R2^4 s^5 ta^2 Q0^2 + 12 r2^2 m R2^4 s^3 Q0^2 + 12 r2^2 m R2^4 s^5 Q0^2 \\
& + 3 r2 m R2^5 s c^2 Q0^2 + 6 r2^2 m R2^4 s^3 Q0 q0 + 3 r2 m R2^5 s^3 c^2 Q0^2 + 24 r2^4 m R2^2 s Q0^2 \\
& + 4 r2^2 RI^4 Q0 q0 ta^2 - r2 RI^4 Q0 R2 q0 + 8 r2^2 m R2^4 s^5 ta^2 Q0 q0 - 4 r2^2 m R2^4 s^3 ta^2 Q0 q0 \\
& - r2 m R2^5 s^5 Q0 q0 + 6 r2^2 m R2^4 s^5 Q0 q0 - 3 r2 m R2^5 s c^2 Q0 q0 - 3 r2 m R2^5 s^3 c^2 Q0 q0 \\
& + 12 r2^4 m R2^2 s Q0 q0)
\end{aligned}$$

> B0:=coeff(beta2,s1,0);

$$\begin{aligned}
B0 := & q0 (4 r2^2 RI^4 Q0^2 + r2^2 RI^4 q0^2 + 2 r2 m R2^5 s^5 Q0^2 - r2 RI^4 q0^2 R2 + r2^2 RI^4 q0^2 ta^2 \\
& + 2 r2 RI^4 Q0^2 R2 + 4 r2^2 RI^4 Q0^2 ta^2 + 4 r2^2 RI^4 Q0 q0 + 2 r2^2 m R2^4 s^5 ta^2 q0^2 \\
& - 4 r2^2 m R2^4 s^3 ta^2 Q0^2 - r2^2 m R2^4 s^3 ta^2 q0^2 - r2 m R2^5 s^5 q0^2 + 8 r2^2 m R2^4 s^5 ta^2 Q0^2 \\
& + 12 r2^2 m R2^4 s^3 Q0^2 + 12 r2^2 m R2^4 s^5 Q0^2 + 3 r2 m R2^5 s c^2 Q0^2 + 6 r2^2 m R2^4 s^3 Q0 q0 \\
& + 3 r2 m R2^5 s^3 c^2 Q0^2 + 24 r2^4 m R2^2 s Q0^2 + 4 r2^2 RI^4 Q0 q0 ta^2 - r2 RI^4 Q0 R2 q0 \\
& + 8 r2^2 m R2^4 s^5 ta^2 Q0 q0 - 4 r2^2 m R2^4 s^3 ta^2 Q0 q0 - r2 m R2^5 s^5 Q0 q0 \\
& + 6 r2^2 m R2^4 s^5 Q0 q0 - 3 r2 m R2^5 s c^2 Q0 q0 - 3 r2 m R2^5 s^3 c^2 Q0 q0 + 12 r2^4 m R2^2 s Q0 q0)
\end{aligned}$$

[>

&gt;

### Program No. 3

Coefficients of the diff. eq. of the axial  
stress of a helix using two Maxwell models

```
restart;
```

```
> with(linalg):
```

Warning, the protected names norm and trace have been redefined and unprotected

```
> c1:=E*R1^2/(R1^2+m*R2^2)+m*E*R2^2/((R1^2+m*R2^2)*(r2+r2*ta^2+nu*R2))*((r2*ta^2-nu*R1)*s+R2^2*(1-2*s^2)*s*c^2/4/r2+R2^2*s^3*c^2*(1+nu)/2/r2-nu^2*R2^2*s*c^4*(R1+r2*ta^2)/4/r2^2/(1+nu));
```

$$c1 := \frac{ER1^2}{R1^2 + m R2^2} + \frac{m ER2^2 \left( (r2 ta^2 - \nu RI) s + \frac{R2^2 (1 - 2 s^2) s c^2}{4 r2} + \frac{R2^2 s^3 c^2 (1 + \nu)}{2 r2} - \frac{\nu^2 R2^2 s c^4 (RI + r2 ta^2)}{4 r2^2 (1 + \nu)} \right)}{(R1^2 + m R2^2) (r2 + r2 ta^2 + \nu R2)}$$

```
> c2:=r2*E*m*R2^2/(R1+2*R2)/(R1^2+m*R2^2)/(r2+r2*ta^2+nu*R2)*(r2*ta*s+R2^2/4/(1+nu)/r2*((2*s^2-1)*s^2*c)-R2^2/2/r2*s^4*c-nu^2*R2^3*s^2*c^3/4/r2^2/(1+nu));
```

$$c2 := \frac{r2 E m R2^2 \left( r2 t a s + \frac{R2^2 (2 s^2 - 1) s^2 c}{4 (1 + v) r2} - \frac{R2^2 s^4 c}{2 r2} - \frac{v^2 R2^3 s^2 c^3}{4 r2^2 (1 + v)} \right)}{(R1 + 2 R2) (R1^2 + m R2^2) (r2 + r2 t a^2 + v R2)}$$

```
> sigma:=c1*epsilon+c2*beta;
```

$$\sigma := \left( \frac{\frac{E R1^2}{R1^2 + m R2^2}}{} + \frac{m E R2^2 \left( (r2 t a^2 - v R1) s + \frac{R2^2 (1 - 2 s^2) s c^2}{4 r2} + \frac{R2^2 s^3 c^2 (1 + v)}{2 r2} - \frac{v^2 R2^2 s c^4 (R1 + r2 t a^2)}{4 r2^2 (1 + v)} \right)}{(R1^2 + m R2^2) (r2 + r2 t a^2 + v R2)} \right) \varepsilon + \frac{r2 E m R2^2 \left( r2 t a s + \frac{R2^2 (2 s^2 - 1) s^2 c}{4 (1 + v) r2} - \frac{R2^2 s^4 c}{2 r2} - \frac{v^2 R2^3 s^2 c^3}{4 r2^2 (1 + v)} \right) \beta}{(R1 + 2 R2) (R1^2 + m R2^2) (r2 + r2 t a^2 + v R2)}$$

```
> sigmal:=subs(E=3*Q*q/(2*p*Q+q*P),nu=(p*Q-q*P)/(2*p*Q+q*P),sigma);
```

$$\sigma I := \left( \frac{\frac{3 Q q R1^2}{(2 p Q + q P) (R1^2 + m R2^2)} + \frac{1}{(2 p Q + q P) (R1^2 + m R2^2) \left( r2 + r2 t a^2 + \frac{(p Q - q P) R2}{2 p Q + q P} \right)}}{3 m Q} \right. \\ \left. q R2^2 \left( \left( r2 t a^2 - \frac{(p Q - q P) R1}{2 p Q + q P} \right) s + \frac{R2^2 (1 - 2 s^2) s c^2}{4 r2} + \frac{R2^2 s^3 c^2 \left( 1 + \frac{p Q - q P}{2 p Q + q P} \right)}{2 r2} \right. \right. \\ \left. \left. - \frac{(p Q - q P)^2 R2^2 s c^4 (R1 + r2 t a^2)}{4 (2 p Q + q P)^2 r2^2 \left( 1 + \frac{p Q - q P}{2 p Q + q P} \right)} \right) \right) \varepsilon + \\ \frac{1}{(2 p Q + q P) (R1 + 2 R2) (R1^2 + m R2^2) \left( r2 + r2 t a^2 + \frac{(p Q - q P) R2}{2 p Q + q P} \right)} \left( 3 r2 Q q m R2^2 \left( r2 t a s \right. \right.$$

$$+ \frac{R2^2 (2 s^2 - 1) s^2 c}{4 \left( 1 + \frac{p Q - q P}{2 p Q + q P} \right) r2} - \frac{R2^2 s^4 c}{2 r2} - \frac{(p Q - q P)^2 R2^3 s^2 c^3}{4 (2 p Q + q P)^2 r2^2 \left( 1 + \frac{p Q - q P}{2 p Q + q P} \right)} \beta \Bigg)$$

```
> sigma2:=simplify(sigma1);
```

$$\begin{aligned} \sigma^2 := & ((24 \varepsilon Q^2 R I^3 r2^3 p^2 + 12 \varepsilon Q R I^3 r2^3 p q P + 24 \varepsilon Q^2 R I^3 r2^3 p^2 t a^2 + 12 \varepsilon Q R I^3 r2^3 p t a^2 q P \\ & + 12 \varepsilon Q^2 R I^3 r2^2 p^2 R2 + 24 \varepsilon Q^2 R I^2 r2^2 p^2 R2^2 - 12 \varepsilon Q R I^3 r2^2 p R2 q P \\ & - 24 \varepsilon Q R I^2 r2^2 p R2^2 q P + 48 \varepsilon m R2^3 s r2^3 p^2 Q^2 t a^2 + 24 \varepsilon m R2^3 s r2^3 p Q t a^2 q P \\ & - 12 \varepsilon m R2^2 s r2^2 p^2 Q^2 R I^2 - 24 \varepsilon m R2^3 s r2^2 p^2 Q^2 R I + 12 \varepsilon m R2^2 s r2^2 p Q R I^2 q P \\ & + 24 \varepsilon m R2^3 s r2^2 p Q R I q P + 12 \varepsilon m R2^5 s^3 c^2 p^2 Q^2 r2 - 12 \varepsilon m R2^5 s^3 c^2 r2 p Q q P \\ & + 12 \varepsilon m R2^5 s c^2 r2 p^2 Q^2 + 6 \varepsilon m R2^5 s c^2 r2 p Q q P - \varepsilon m R2^4 s c^4 p^2 Q^2 R I^2 \\ & - 2 \varepsilon m R2^5 s c^4 p^2 Q^2 R I - 2 \varepsilon m R2^5 s c^4 p^2 Q^2 r2 t a^2 + 2 \varepsilon m R2^4 s c^4 p Q q P R I^2 \\ & + 4 \varepsilon m R2^5 s c^4 p Q q P R I + 4 \varepsilon m R2^5 s c^4 p Q q P r2 t a^2 - \varepsilon m R2^4 s c^4 q^2 P^2 R I^2 \\ & - 2 \varepsilon m R2^5 s c^4 q^2 P^2 R I - 2 \varepsilon m R2^5 s c^4 q^2 P^2 r2 t a^2 + 48 \varepsilon Q^2 R I^2 r2^3 p^2 R2 \\ & + 24 \varepsilon Q R I^2 r2^3 p q P R2 + 48 \varepsilon Q^2 R I^2 r2^3 p^2 t a^2 R2 + 24 \varepsilon Q R I^2 r2^3 p t a^2 q P R2 \\ & + 24 \varepsilon m R2^2 s r2^3 p^2 Q^2 t a^2 R I + 12 \varepsilon m R2^2 s r2^3 p Q t a^2 q P R I + 6 \varepsilon m R2^4 s^3 c^2 p^2 Q^2 r2 R I \\ & - 6 \varepsilon m R2^4 s^3 c^2 r2 p Q q P R I + 6 \varepsilon m R2^4 s c^2 r2 p^2 Q^2 R I + 3 \varepsilon m R2^4 s c^2 r2 p Q q P R I \\ & - \varepsilon m R2^4 s c^4 p^2 Q^2 r2 t a^2 R I + 2 \varepsilon m R2^4 s c^4 p Q q P r2 t a^2 R I - \varepsilon m R2^4 s c^4 q^2 P^2 r2 t a^2 R I \\ & + 24 r2^4 m R2^2 s \beta t a p^2 Q^2 + 12 r2^4 m R2^2 s \beta t a p Q q P - 4 r2^2 m R2^4 s^4 \beta c p^2 Q^2 \\ & - 4 r2^2 m R2^4 s^2 \beta c p^2 Q^2 + 2 r2^2 m R2^4 s^4 \beta c p Q q P - 4 r2^2 m R2^4 s^2 \beta c p Q q P \\ & + 2 r2^2 m R2^4 s^4 \beta c q^2 P^2 - r2^2 m R2^4 s^2 \beta c q^2 P^2 - r2 m R2^5 s^2 \beta c^3 p^2 Q^2 \\ & + 2 r2 m R2^5 s^2 \beta c^3 p Q q P - r2 m R2^5 s^2 \beta c^3 q^2 P^2) q) / (4 p r2^2 (2 r2 p Q + r2 q P + 2 r2 t a^2 p Q \\ & + r2 t a^2 q P + R2 p Q - R2 q P) (R I^2 + m R2^2) (R I + 2 R2) (2 p Q + q P)) \end{aligned}$$

```
> LHS1:=denom(sigma2);
```

$$LHS1 := 4 p r2^2 (2 r2 p Q + r2 q P + 2 r2 t a^2 p Q + r2 t a^2 q P + R2 p Q - R2 q P) (R1^2 + m R2^2) (R1 + 2 R2) \\ (2 p Q + q P)$$

```
> LHS2 :=subs(p=1+p1*s1,P=1+P1*s1,q=q1*s1,Q=Q1*s1,LHS1);
```

$$LHS2 := 4(1 + p1 s1) r2^2 (2 r2 (1 + p1 s1) Q1 s1 + r2 q1 s1 (1 + P1 s1) + 2 r2 t a^2 (1 + p1 s1) Q1 s1 + r2 t a^2 q1 s1 (1 + P1 s1) + R2 (1 + p1 s1) Q1 s1 - R2 q1 s1 (1 + P1 s1)) (R1^2 + m R2^2) (R1 + 2 R2) (2 (1 + p1 s1) Q1 s1 + q1 s1 (1 + P1 s1))$$

```
> LHS3:=collect(LHS2,s1);
```

$$\begin{aligned}
LHS3 := & 4 p1 r2^2 (2 r2 p1 Q1 + r2 q1 P1 + 2 r2 ta^2 p1 Q1 + r2 ta^2 q1 P1 + R2 p1 Q1 - R2 q1 P1) (R1^2 \\
& + m R2^2) (R1 + 2 R2) (2 p1 Q1 + q1 P1) s1^5 + (4 (r2^2 (2 r2 p1 Q1 + r2 q1 P1 + 2 r2 ta^2 p1 Q1 \\
& + r2 ta^2 q1 P1 + R2 p1 Q1 - R2 q1 P1) \\
& + p1 r2^2 (2 r2 Q1 + r2 q1 + 2 r2 ta^2 Q1 + r2 ta^2 q1 + R2 Q1 - R2 q1)) (R1^2 + m R2^2) (R1 + 2 R2) (2 \\
& p1 Q1 + q1 P1) + 4 p1 r2^2 (2 r2 p1 Q1 + r2 q1 P1 + 2 r2 ta^2 p1 Q1 + r2 ta^2 q1 P1 + R2 p1 Q1 \\
& - R2 q1 P1) (R1^2 + m R2^2) (R1 + 2 R2) (2 Q1 + q1)) s1^4 + (4 r2^2 (2 r2 Q1 + r2 q1 + 2 r2 ta^2 Q1
\end{aligned}$$

```

+  $r2 ta^2 qI + R2 Q1 - R2 qI) (R1^2 + m R2^2) (R1 + 2 R2) (2 p1 Q1 + q1 PI) + 4 (r2^2 (2 r2 p1 Q1$ 
+  $r2 q1 PI + 2 r2 ta^2 p1 Q1 + r2 ta^2 q1 PI + R2 p1 Q1 - R2 q1 PI)$ 
+  $p1 r2^2 (2 r2 Q1 + r2 q1 + 2 r2 ta^2 Q1 + r2 ta^2 q1 + R2 Q1 - R2 q1)) (R1^2 + m R2^2) (R1 + 2 R2) (2$ 
 $Q1 + q1)) s1^3 + 4 r2^2 (2 r2 Q1 + r2 q1 + 2 r2 ta^2 Q1 + r2 ta^2 q1 + R2 Q1 - R2 q1) (R1^2 + m R2^2) (R1$ 
+  $2 R2) (2 Q1 + q1) s1^2$ 

> RHS1:=numer(sigma2);

RHS1 := q (24 ε Q^2 R1^3 r2^3 p^2 + 12 ε Q R1^3 r2^3 p q P + 24 ε Q^2 R1^3 r2^3 p^2 ta^2
+ 12 ε Q R1^3 r2^3 p ta^2 q P + 12 ε Q^2 R1^3 r2^2 p^2 R2 + 24 ε Q^2 R1^2 r2^2 p^2 R2^2
- 12 ε Q R1^3 r2^2 p R2 q P - 24 ε Q R1^2 r2^2 p R2^2 q P + 48 ε m R2^3 s r2^3 p^2 Q^2 ta^2
+ 24 ε m R2^3 s r2^3 p Q ta^2 q P - 12 ε m R2^2 s r2^2 p^2 Q^2 R1^2 - 24 ε m R2^3 s r2^2 p^2 Q^2 R1
+ 12 ε m R2^2 s r2^2 p Q R1^2 q P + 24 ε m R2^3 s r2^2 p Q R1 q P + 12 ε m R2^5 s^3 c^2 p^2 Q^2 r2
- 12 ε m R2^5 s^3 c^2 r2 p Q q P + 12 ε m R2^5 s c^2 r2 p^2 Q^2 + 6 ε m R2^5 s c^2 r2 p Q q P
- ε m R2^4 s c^4 p^2 Q^2 R1^2 - 2 ε m R2^5 s c^4 p^2 Q^2 R1 - 2 ε m R2^5 s c^4 p^2 Q^2 r2 ta^2
+ 2 ε m R2^4 s c^4 p Q q P R1^2 + 4 ε m R2^5 s c^4 p Q q P R1 + 4 ε m R2^5 s c^4 p Q q P r2 ta^2
- ε m R2^4 s c^4 q^2 P^2 R1^2 - 2 ε m R2^5 s c^4 q^2 P^2 R1 - 2 ε m R2^5 s c^4 q^2 P^2 r2 ta^2
+ 48 ε Q^2 R1^2 r2^3 p^2 R2 + 24 ε Q R1^2 r2^3 p q P R2 + 48 ε Q^2 R1^2 r2^3 p^2 ta^2 R2
+ 24 ε Q R1^2 r2^3 p ta^2 q P R2 + 24 ε m R2^2 s r2^3 p^2 Q^2 ta^2 R1 + 12 ε m R2^2 s r2^3 p Q ta^2 q P R1
+ 6 ε m R2^4 s^3 c^2 p^2 Q^2 r2 R1 - 6 ε m R2^4 s^3 c^2 r2 p Q q P R1 + 6 ε m R2^4 s c^2 r2 p^2 Q^2 R1
+ 3 ε m R2^4 s c^2 r2 p Q q P R1 - ε m R2^4 s c^4 p^2 Q^2 r2 ta^2 R1 + 2 ε m R2^4 s c^4 p Q q P r2 ta^2 R1
- ε m R2^4 s c^4 q^2 P^2 r2 ta^2 R1 + 24 r2^4 m R2^2 s β tap^2 Q^2 + 12 r2^4 m R2^2 s β tap Q q P
- 4 r2^2 m R2^4 s^4 β cp^2 Q^2 - 4 r2^2 m R2^4 s^2 β cp^2 Q^2 + 2 r2^2 m R2^4 s^4 β cp Q q P
- 4 r2^2 m R2^4 s^2 β cp Q q P + 2 r2^2 m R2^4 s^4 β c q^2 P^2 - r2^2 m R2^4 s^2 β c q^2 P^2
- r2 m R2^5 s^2 β c^3 p^2 Q^2 + 2 r2 m R2^5 s^2 β c^3 p Q q P - r2 m R2^5 s^2 β c^3 q^2 P^2)

> RHS2:=subs(p=1+p1*s1,P=1+P1*s1,q=q1*s1,Q=Q1*s1,RHS1);

RHS2 := q1 s1 (-ε m R2^4 s c^4 q1^2 s1^2 (1 + P1 s1)^2 R1^2 - 2 ε m R2^5 s c^4 q1^2 s1^2 (1 + P1 s1)^2 R1
- 2 ε m R2^5 s c^4 q1^2 s1^2 (1 + P1 s1)^2 r2 ta^2 - ε m R2^4 s c^4 q1^2 s1^2 (1 + P1 s1)^2 r2 ta^2 R1
+ 2 r2^2 m R2^4 s^4 β c q1^2 s1^2 (1 + P1 s1)^2 - r2^2 m R2^4 s^2 β c q1^2 s1^2 (1 + P1 s1)^2
- r2 m R2^5 s^2 β c^3 q1^2 s1^2 (1 + P1 s1)^2 + 24 ε Q1^2 s1^2 R1^3 r2^3 (1 + p1 s1)^2
+ 24 ε Q1^2 s1^2 R1^3 r2^3 (1 + p1 s1)^2 ta^2 + 12 ε Q1^2 s1^2 R1^3 r2^2 (1 + p1 s1)^2 R2
+ 24 ε Q1^2 s1^2 R1^2 r2^2 (1 + p1 s1)^2 R2^2 + 48 ε m R2^3 s r2^3 (1 + p1 s1)^2 Q1^2 s1^2 ta^2
- 12 ε m R2^2 s r2^2 (1 + p1 s1)^2 Q1^2 s1^2 R1^2 - 24 ε m R2^3 s r2^2 (1 + p1 s1)^2 Q1^2 s1^2 R1
+ 12 ε m R2^5 s^3 c^2 (1 + p1 s1)^2 Q1^2 s1^2 r2 + 12 ε m R2^5 s c^2 r2 (1 + p1 s1)^2 Q1^2 s1^2
- ε m R2^4 s c^4 (1 + p1 s1)^2 Q1^2 s1^2 R1^2 - 2 ε m R2^5 s c^4 (1 + p1 s1)^2 Q1^2 s1^2 R1
- 2 ε m R2^5 s c^4 (1 + p1 s1)^2 Q1^2 s1^2 r2 ta^2 + 48 ε Q1^2 s1^2 R1^2 r2^3 (1 + p1 s1)^2 R2
+ 48 ε Q1^2 s1^2 R1^2 r2^3 (1 + p1 s1)^2 ta^2 R2 + 24 ε m R2^2 s r2^3 (1 + p1 s1)^2 Q1^2 s1^2 ta^2 R1
+ 6 ε m R2^4 s^3 c^2 (1 + p1 s1)^2 Q1^2 s1^2 r2 R1 + 6 ε m R2^4 s c^2 r2 (1 + p1 s1)^2 Q1^2 s1^2 R1
- ε m R2^4 s c^4 (1 + p1 s1)^2 Q1^2 s1^2 r2 ta^2 R1 + 24 r2^4 m R2^2 s β ta (1 + p1 s1)^2 Q1^2 s1^2
- 4 r2^2 m R2^4 s^4 β c (1 + p1 s1)^2 Q1^2 s1^2 - 4 r2^2 m R2^4 s^2 β c (1 + p1 s1)^2 Q1^2 s1^2
- r2 m R2^5 s^2 β c^3 (1 + p1 s1)^2 Q1^2 s1^2 + 12 ε Q1 s1^2 R1^3 r2^3 (1 + p1 s1) q1 (1 + P1 s1)
+ 12 ε Q1 s1^2 R1^3 r2^3 (1 + p1 s1) ta^2 q1 (1 + P1 s1)
- 12 ε Q1 s1^2 R1^3 r2^2 (1 + p1 s1) R2 q1 (1 + P1 s1)

```

$$\begin{aligned}
& -24 \varepsilon Q1 s1^2 R1^2 r2^2 (1+p1 s1) R2^2 q1 (1+P1 s1) \\
& +24 \varepsilon m R2^3 s r2^3 (1+p1 s1) Q1 s1^2 ta^2 q1 (1+P1 s1) \\
& +12 \varepsilon m R2^2 s r2^2 (1+p1 s1) Q1 s1^2 R1^2 q1 (1+P1 s1) \\
& +24 \varepsilon m R2^3 s r2^2 (1+p1 s1) Q1 s1^2 R1 q1 (1+P1 s1) \\
& -12 \varepsilon m R2^5 s^3 c^2 r2 (1+p1 s1) Q1 s1^2 q1 (1+P1 s1) \\
& +6 \varepsilon m R2^5 s c^2 r2 (1+p1 s1) Q1 s1^2 q1 (1+P1 s1) \\
& +2 \varepsilon m R2^4 s c^4 (1+p1 s1) Q1 s1^2 q1 (1+P1 s1) R1^2 \\
& +4 \varepsilon m R2^5 s c^4 (1+p1 s1) Q1 s1^2 q1 (1+P1 s1) R1 \\
& +4 \varepsilon m R2^5 s c^4 (1+p1 s1) Q1 s1^2 q1 (1+P1 s1) r2 ta^2 \\
& +24 \varepsilon Q1 s1^2 R1^2 r2^3 (1+p1 s1) q1 (1+P1 s1) R2 \\
& +24 \varepsilon Q1 s1^2 R1^2 r2^3 (1+p1 s1) ta^2 q1 (1+P1 s1) R2 \\
& +12 \varepsilon m R2^2 s r2^3 (1+p1 s1) Q1 s1^2 ta^2 q1 (1+P1 s1) R1 \\
& -6 \varepsilon m R2^4 s^3 c^2 r2 (1+p1 s1) Q1 s1^2 q1 (1+P1 s1) R1 \\
& +3 \varepsilon m R2^4 s c^2 r2 (1+p1 s1) Q1 s1^2 q1 (1+P1 s1) R1 \\
& +2 \varepsilon m R2^4 s c^4 (1+p1 s1) Q1 s1^2 q1 (1+P1 s1) r2 ta^2 R1 \\
& +12 r2^4 m R2^2 s \beta ta (1+p1 s1) Q1 s1^2 q1 (1+P1 s1) \\
& +2 r2^2 m R2^4 s^4 \beta c (1+p1 s1) Q1 s1^2 q1 (1+P1 s1) \\
& -4 r2^2 m R2^4 s^2 \beta c (1+p1 s1) Q1 s1^2 q1 (1+P1 s1) \\
& +2 r2 m R2^5 s^2 \beta c^3 (1+p1 s1) Q1 s1^2 q1 (1+P1 s1)
\end{aligned}$$

```

> RHS3epsilon:=collect(RHS2,epsilon);
RHS3epsilon:=q1 s1 (-m R2^4 s c^4 q1^2 s1^2 (1+P1 s1)^2 R1^2 - 2 m R2^5 s c^4 q1^2 s1^2 (1+P1 s1)^2 R1
- 2 m R2^5 s c^4 q1^2 s1^2 (1+P1 s1)^2 r2 ta^2 - m R2^4 s c^4 q1^2 s1^2 (1+P1 s1)^2 r2 ta^2 R1
+ 24 Q1^2 s1^2 R1^3 r2^3 (1+p1 s1)^2 ta^2 + 24 Q1^2 s1^2 R1^3 r2^3 (1+p1 s1)^2
- 12 m R2^2 s r2^2 (1+p1 s1)^2 Q1^2 s1^2 R1^2 + 12 Q1^2 s1^2 R1^3 r2^2 (1+p1 s1)^2 R2
+ 24 Q1^2 s1^2 R1^2 r2^2 (1+p1 s1)^2 R2^2 + 48 m R2^3 s r2^3 (1+p1 s1)^2 Q1^2 s1^2 ta^2
- 2 m R2^5 s c^4 (1+p1 s1)^2 Q1^2 s1^2 R1 + 48 Q1^2 s1^2 R1^2 r2^3 (1+p1 s1)^2 R2
- 24 m R2^3 s r2^2 (1+p1 s1)^2 Q1^2 s1^2 R1 + 12 m R2^5 s^3 c^2 (1+p1 s1)^2 Q1^2 s1^2 r2
+ 12 m R2^5 s c^2 r2 (1+p1 s1)^2 Q1^2 s1^2 - m R2^4 s c^4 (1+p1 s1)^2 Q1^2 s1^2 R1^2
- m R2^4 s c^4 (1+p1 s1)^2 Q1^2 s1^2 r2 ta^2 R1 + 12 Q1 s1^2 R1^3 r2^3 (1+p1 s1) q1 (1+P1 s1)
- 2 m R2^5 s c^4 (1+p1 s1)^2 Q1^2 s1^2 r2 ta^2 + 48 Q1^2 s1^2 R1^2 r2^3 (1+p1 s1)^2 ta^2 R2
+ 24 m R2^2 s r2^3 (1+p1 s1)^2 Q1^2 s1^2 ta^2 R1 + 6 m R2^4 s^3 c^2 (1+p1 s1)^2 Q1^2 s1^2 r2 R1
+ 6 m R2^4 s c^2 r2 (1+p1 s1)^2 Q1^2 s1^2 R1 + 12 m R2^2 s r2^2 (1+p1 s1) Q1 s1^2 R1^2 q1 (1+P1 s1)
+ 24 m R2^3 s r2^2 (1+p1 s1) Q1 s1^2 R1 q1 (1+P1 s1)
- 12 m R2^5 s^3 c^2 r2 (1+p1 s1) Q1 s1^2 q1 (1+P1 s1)
+ 12 Q1 s1^2 R1^3 r2^3 (1+p1 s1) ta^2 q1 (1+P1 s1)
- 12 Q1 s1^2 R1^3 r2^2 (1+p1 s1) R2 q1 (1+P1 s1)
- 24 Q1 s1^2 R1^2 r2^2 (1+p1 s1) R2^2 q1 (1+P1 s1)
+ 24 m R2^3 s r2^3 (1+p1 s1) Q1 s1^2 ta^2 q1 (1+P1 s1)
+ 6 m R2^5 s c^2 r2 (1+p1 s1) Q1 s1^2 q1 (1+P1 s1)
+ 2 m R2^4 s c^4 (1+p1 s1) Q1 s1^2 q1 (1+P1 s1) R1^2

```

$$\begin{aligned}
& + 4 m R2^5 s c^4 (1 + p1 sl) Q1 sl^2 q1 (1 + P1 sl) RI \\
& + 4 m R2^5 s c^4 (1 + p1 sl) Q1 sl^2 q1 (1 + P1 sl) r2 ta^2 \\
& + 24 Q1 sl^2 RI^2 r2^3 (1 + p1 sl) q1 (1 + P1 sl) R2 \\
& + 24 Q1 sl^2 RI^2 r2^3 (1 + p1 sl) ta^2 q1 (1 + P1 sl) R2 \\
& + 12 m R2^2 s r2^3 (1 + p1 sl) Q1 sl^2 ta^2 q1 (1 + P1 sl) RI \\
& - 6 m R2^4 s^3 c^2 r2 (1 + p1 sl) Q1 sl^2 q1 (1 + P1 sl) RI \\
& + 3 m R2^4 s c^2 r2 (1 + p1 sl) Q1 sl^2 q1 (1 + P1 sl) RI \\
& + 2 m R2^4 s c^4 (1 + p1 sl) Q1 sl^2 q1 (1 + P1 sl) r2 ta^2 RI \varepsilon + q1 sl ( \\
& - 4 r2^2 m R2^4 s^4 \beta c (1 + p1 sl)^2 Q1^2 sl^2 - 4 r2^2 m R2^4 s^2 \beta c (1 + p1 sl)^2 Q1^2 sl^2 \\
& - r2 m R2^5 s^2 \beta c^3 (1 + p1 sl)^2 Q1^2 sl^2 + 12 r2^4 m R2^2 s \beta ta (1 + p1 sl) Q1 sl^2 q1 (1 + P1 sl) \\
& + 2 r2^2 m R2^4 s^4 \beta c q1^2 sl^2 (1 + P1 sl)^2 - r2^2 m R2^4 s^2 \beta c q1^2 sl^2 (1 + P1 sl)^2 \\
& - r2 m R2^5 s^2 \beta c^3 q1^2 sl^2 (1 + P1 sl)^2 + 2 r2^2 m R2^4 s^4 \beta c (1 + p1 sl) Q1 sl^2 q1 (1 + P1 sl) \\
& - 4 r2^2 m R2^4 s^2 \beta c (1 + p1 sl) Q1 sl^2 q1 (1 + P1 sl) \\
& + 2 r2 m R2^5 s^2 \beta c^3 (1 + p1 sl) Q1 sl^2 q1 (1 + P1 sl) + 24 r2^4 m R2^2 s \beta ta (1 + p1 sl)^2 Q1^2 sl^2 )
\end{aligned}$$

```

> RHS3beta:=collect(RHS2,beta);
RHS3beta:=q1 sl (-4 r2^2 m R2^4 s^2 c (1 + p1 sl)^2 Q1^2 sl^2 - r2 m R2^5 s^2 c^3 (1 + p1 sl)^2 Q1^2 sl^2
+ 24 r2^4 m R2^2 s ta (1 + p1 sl)^2 Q1^2 sl^2 - 4 r2^2 m R2^4 s^4 c (1 + p1 sl)^2 Q1^2 sl^2
+ 2 r2^2 m R2^4 s^4 c q1^2 sl^2 (1 + P1 sl)^2 - r2^2 m R2^4 s^2 c q1^2 sl^2 (1 + P1 sl)^2
- r2 m R2^5 s^2 c^3 q1^2 sl^2 (1 + P1 sl)^2 + 12 r2^4 m R2^2 s ta (1 + p1 sl) Q1 sl^2 q1 (1 + P1 sl)
+ 2 r2^2 m R2^4 s^4 c (1 + p1 sl) Q1 sl^2 q1 (1 + P1 sl)
- 4 r2^2 m R2^4 s^2 c (1 + p1 sl) Q1 sl^2 q1 (1 + P1 sl)
+ 2 r2 m R2^5 s^2 c^3 (1 + p1 sl) Q1 sl^2 q1 (1 + P1 sl)) \beta + q1 sl (
- \epsilon m R2^4 s c^4 q1^2 sl^2 (1 + P1 sl)^2 RI^2 - 2 \epsilon m R2^5 s c^4 q1^2 sl^2 (1 + P1 sl)^2 RI
- 2 \epsilon m R2^5 s c^4 q1^2 sl^2 (1 + P1 sl)^2 r2 ta^2 - \epsilon m R2^4 s c^4 q1^2 sl^2 (1 + P1 sl)^2 r2 ta^2 RI
+ 24 \epsilon Q1^2 sl^2 RI^3 r2^3 (1 + p1 sl)^2 + 24 \epsilon Q1^2 sl^2 RI^3 r2^3 (1 + p1 sl)^2 ta^2
+ 12 \epsilon Q1^2 sl^2 RI^3 r2^2 (1 + p1 sl)^2 R2 + 24 \epsilon Q1^2 sl^2 RI^2 r2^2 (1 + p1 sl)^2 R2^2
+ 48 \epsilon m R2^3 s r2^3 (1 + p1 sl)^2 Q1^2 sl^2 ta^2 - 12 \epsilon m R2^2 s r2^2 (1 + p1 sl)^2 Q1^2 sl^2 RI^2
- 24 \epsilon m R2^3 s r2^2 (1 + p1 sl)^2 Q1^2 sl^2 RI + 12 \epsilon m R2^5 s^3 c^2 (1 + p1 sl)^2 Q1^2 sl^2 r2
+ 12 \epsilon m R2^5 s c^2 r2 (1 + p1 sl)^2 Q1^2 sl^2 - \epsilon m R2^4 s c^4 (1 + p1 sl)^2 Q1^2 sl^2 RI^2
- 2 \epsilon m R2^5 s c^4 (1 + p1 sl)^2 Q1^2 sl^2 RI - 2 \epsilon m R2^5 s c^4 (1 + p1 sl)^2 Q1^2 sl^2 r2 ta^2
+ 48 \epsilon Q1^2 sl^2 RI^2 r2^3 (1 + p1 sl)^2 R2 + 48 \epsilon Q1^2 sl^2 RI^2 r2^3 (1 + p1 sl)^2 ta^2 R2
+ 24 \epsilon m R2^2 s r2^3 (1 + p1 sl)^2 Q1^2 sl^2 ta^2 RI + 6 \epsilon m R2^4 s^3 c^2 (1 + p1 sl)^2 Q1^2 sl^2 r2 RI
+ 6 \epsilon m R2^4 s c^2 r2 (1 + p1 sl)^2 Q1^2 sl^2 RI - \epsilon m R2^4 s c^4 (1 + p1 sl)^2 Q1^2 sl^2 r2 ta^2 RI
+ 12 \epsilon Q1 sl^2 RI^3 r2^3 (1 + p1 sl) q1 (1 + P1 sl)
+ 12 \epsilon Q1 sl^2 RI^3 r2^3 (1 + p1 sl) ta^2 q1 (1 + P1 sl)
- 12 \epsilon Q1 sl^2 RI^3 r2^2 (1 + p1 sl) R2 q1 (1 + P1 sl)
- 24 \epsilon Q1 sl^2 RI^2 r2^2 (1 + p1 sl) R2^2 q1 (1 + P1 sl)
+ 24 \epsilon m R2^3 s r2^3 (1 + p1 sl) Q1 sl^2 ta^2 q1 (1 + P1 sl)
+ 12 \epsilon m R2^2 s r2^2 (1 + p1 sl) Q1 sl^2 RI^2 q1 (1 + P1 sl)
+ 24 \epsilon m R2^3 s r2^2 (1 + p1 sl) Q1 sl^2 RI q1 (1 + P1 sl)

```

$$\begin{aligned}
& -12 \varepsilon m R2^5 s^3 c^2 r2 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) \\
& + 6 \varepsilon m R2^5 s c^2 r2 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) \\
& + 2 \varepsilon m R2^4 s c^4 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) R1^2 \\
& + 4 \varepsilon m R2^5 s c^4 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) R1 \\
& + 4 \varepsilon m R2^5 s c^4 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) r2 ta^2 \\
& + 24 \varepsilon Q1 s1^2 R1^2 r2^3 (1 + p1 s1) q1 (1 + P1 s1) R2 \\
& + 24 \varepsilon Q1 s1^2 R1^2 r2^3 (1 + p1 s1) ta^2 q1 (1 + P1 s1) R2 \\
& + 12 \varepsilon m R2^2 s r2^3 (1 + p1 s1) Q1 s1^2 ta^2 q1 (1 + P1 s1) R1 \\
& - 6 \varepsilon m R2^4 s^3 c^2 r2 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) R1 \\
& + 3 \varepsilon m R2^4 s c^2 r2 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) R1 \\
& + 2 \varepsilon m R2^4 s c^4 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) r2 ta^2 R1
\end{aligned}$$

> **beta1:=coeff(RHS3beta,beta,1);**

$$\begin{aligned}
\beta1 := & q1 s1 (-4 r2^2 m R2^4 s^2 c (1 + p1 s1)^2 Q1^2 s1^2 - r2 m R2^5 s^2 c^3 (1 + p1 s1)^2 Q1^2 s1^2 \\
& + 24 r2^4 m R2^2 s ta (1 + p1 s1)^2 Q1^2 s1^2 - 4 r2^2 m R2^4 s^4 c (1 + p1 s1)^2 Q1^2 s1^2 \\
& + 2 r2^2 m R2^4 s^4 c q1^2 s1^2 (1 + P1 s1)^2 - r2^2 m R2^4 s^2 c q1^2 s1^2 (1 + P1 s1)^2 \\
& - r2 m R2^5 s^2 c^3 q1^2 s1^2 (1 + P1 s1)^2 + 12 r2^4 m R2^2 s ta (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) \\
& + 2 r2^2 m R2^4 s^4 c (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) \\
& - 4 r2^2 m R2^4 s^2 c (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) \\
& + 2 r2 m R2^5 s^2 c^3 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1))
\end{aligned}$$

> **beta2:=collect(beta1,s1);**

$$\begin{aligned}
\beta2 := & q1 (-r2 m R2^5 s^2 c^3 q1^2 P1^2 + 12 r2^4 m R2^2 s ta p1 Q1 q1 P1 - 4 r2^2 m R2^4 s^4 c p1^2 Q1^2 \\
& + 24 r2^4 m R2^2 s ta p1^2 Q1^2 + 2 r2^2 m R2^4 s^4 c p1 Q1 q1 P1 + 2 r2^2 m R2^4 s^4 c q1^2 P1^2 \\
& - 4 r2^2 m R2^4 s^2 c p1^2 Q1^2 - 4 r2^2 m R2^4 s^2 c p1 Q1 q1 P1 - r2^2 m R2^4 s^2 c q1^2 P1^2 \\
& - r2 m R2^5 s^2 c^3 p1^2 Q1^2 + 2 r2 m R2^5 s^2 c^3 p1 Q1 q1 P1) s1^5 + q1 (-8 r2^2 m R2^4 s^4 c p1 Q1^2 \\
& + 48 r2^4 m R2^2 s ta p1 Q1^2 + 4 r2^2 m R2^4 s^4 c q1^2 P1 - 8 r2^2 m R2^4 s^2 c p1 Q1^2 \\
& - 4 r2^2 m R2^4 s^2 c Q1 q1 P1 - 4 r2^2 m R2^4 s^2 c p1 Q1 q1 - 2 r2^2 m R2^4 s^2 c q1^2 P1 \\
& - 2 r2 m R2^5 s^2 c^3 p1 Q1^2 + 2 r2 m R2^5 s^2 c^3 Q1 q1 P1 + 2 r2 m R2^5 s^2 c^3 p1 Q1 q1 \\
& - 2 r2 m R2^5 s^2 c^3 q1^2 P1 + 2 r2^2 m R2^4 s^4 c Q1 q1 P1 + 2 r2^2 m R2^4 s^4 c p1 Q1 q1 \\
& + 12 r2^4 m R2^2 s ta Q1 q1 P1 + 12 r2^4 m R2^2 s ta p1 Q1 q1) s1^4 + q1 (-4 r2^2 m R2^4 s^2 c Q1^2 \\
& - 4 r2^2 m R2^4 s^2 c Q1 q1 - r2^2 m R2^4 s^2 c q1^2 - r2 m R2^5 s^2 c^3 Q1^2 + 2 r2 m R2^5 s^2 c^3 Q1 q1 \\
& - r2 m R2^5 s^2 c^3 q1^2 + 24 r2^4 m R2^2 s ta Q1^2 + 2 r2^2 m R2^4 s^4 c Q1 q1 + 12 r2^4 m R2^2 s ta Q1 q1 \\
& - 4 r2^2 m R2^4 s^4 c Q1^2 + 2 r2^2 m R2^4 s^4 c q1^2) s1^3
\end{aligned}$$

> **epsilon1:=coeff(RHS3epsilon,epsilon,1);**

$$\begin{aligned}
\epsilon1 := & q1 s1 (-m R2^4 s c^4 q1^2 s1^2 (1 + P1 s1)^2 R1^2 - 2 m R2^5 s c^4 q1^2 s1^2 (1 + P1 s1)^2 R1 \\
& - 2 m R2^5 s c^4 q1^2 s1^2 (1 + P1 s1)^2 r2 ta^2 - m R2^4 s c^4 q1^2 s1^2 (1 + P1 s1)^2 r2 ta^2 R1 \\
& + 24 Q1^2 s1^2 R1^3 r2^3 (1 + p1 s1)^2 ta^2 + 24 Q1^2 s1^2 R1^3 r2^3 (1 + p1 s1)^2 \\
& - 12 m R2^2 s r2^2 (1 + p1 s1)^2 Q1^2 s1^2 R1^2 + 12 Q1^2 s1^2 R1^3 r2^2 (1 + p1 s1)^2 R2 \\
& + 24 Q1^2 s1^2 R1^2 r2^2 (1 + p1 s1)^2 R2^2 + 48 m R2^3 s r2^3 (1 + p1 s1)^2 Q1^2 s1^2 ta^2 \\
& - 2 m R2^5 s c^4 (1 + p1 s1)^2 Q1^2 s1^2 R1 + 48 Q1^2 s1^2 R1^2 r2^3 (1 + p1 s1)^2 R2
\end{aligned}$$

$$\begin{aligned}
& -24 m R2^3 s r2^2 (1 + p1 s1)^2 Q1^2 s1^2 R1 + 12 m R2^5 s^3 c^2 (1 + p1 s1)^2 Q1^2 s1^2 r2 \\
& + 12 m R2^5 s c^2 r2 (1 + p1 s1)^2 Q1^2 s1^2 - m R2^4 s c^4 (1 + p1 s1)^2 Q1^2 s1^2 R1^2 \\
& - m R2^4 s c^4 (1 + p1 s1)^2 Q1^2 s1^2 r2 ta^2 R1 + 12 Q1 s1^2 R1^3 r2^3 (1 + p1 s1) q1 (1 + P1 s1) \\
& - 2 m R2^5 s c^4 (1 + p1 s1)^2 Q1^2 s1^2 r2 ta^2 + 48 Q1^2 s1^2 R1^2 r2^3 (1 + p1 s1)^2 ta^2 R2 \\
& + 24 m R2^2 s r2^3 (1 + p1 s1)^2 Q1^2 s1^2 ta^2 R1 + 6 m R2^4 s^3 c^2 (1 + p1 s1)^2 Q1^2 s1^2 r2 R1 \\
& + 6 m R2^4 s c^2 r2 (1 + p1 s1)^2 Q1^2 s1^2 R1 + 12 m R2^2 s r2^2 (1 + p1 s1) Q1 s1^2 R1^2 q1 (1 + P1 s1) \\
& + 24 m R2^3 s r2^2 (1 + p1 s1) Q1 s1^2 R1 q1 (1 + P1 s1) \\
& - 12 m R2^5 s^3 c^2 r2 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) \\
& + 12 Q1 s1^2 R1^3 r2^3 (1 + p1 s1) ta^2 q1 (1 + P1 s1) \\
& - 12 Q1 s1^2 R1^3 r2^2 (1 + p1 s1) R2 q1 (1 + P1 s1) \\
& - 24 Q1 s1^2 R1^2 r2^2 (1 + p1 s1) R2^2 q1 (1 + P1 s1) \\
& + 24 m R2^3 s r2^3 (1 + p1 s1) Q1 s1^2 ta^2 q1 (1 + P1 s1) \\
& + 6 m R2^5 s c^2 r2 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) \\
& + 2 m R2^4 s c^4 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) R1^2 \\
& + 4 m R2^5 s c^4 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) R1 \\
& + 4 m R2^5 s c^4 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) r2 ta^2 \\
& + 24 Q1 s1^2 R1^2 r2^3 (1 + p1 s1) q1 (1 + P1 s1) R2 \\
& + 24 Q1 s1^2 R1^2 r2^3 (1 + p1 s1) ta^2 q1 (1 + P1 s1) R2 \\
& + 12 m R2^2 s r2^3 (1 + p1 s1) Q1 s1^2 ta^2 q1 (1 + P1 s1) R1 \\
& - 6 m R2^4 s^3 c^2 r2 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) R1 \\
& + 3 m R2^4 s c^2 r2 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) R1 \\
& + 2 m R2^4 s c^4 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) r2 ta^2 R1)
\end{aligned}$$

> epsilon2:=collect(epsilon1,s1);

$$\begin{aligned}
\epsilon2 := & q1 (24 Q1^2 R1^3 r2^3 p1^2 - m R2^4 s c^4 q1^2 P1^2 R1^2 - 2 m R2^5 s c^4 q1^2 P1^2 R1 \\
& - 2 m R2^5 s c^4 q1^2 P1^2 r2 ta^2 - m R2^4 s c^4 q1^2 P1^2 r2 ta^2 R1 + 24 Q1^2 R1^3 r2^3 p1^2 ta^2 \\
& - 12 m R2^2 s r2^2 p1^2 Q1^2 R1^2 + 12 Q1^2 R1^3 r2^2 p1^2 R2 + 24 Q1^2 R1^2 r2^2 p1^2 R2^2 \\
& + 48 m R2^3 s r2^3 p1^2 Q1^2 ta^2 - 2 m R2^5 s c^4 p1^2 Q1^2 R1 + 48 Q1^2 R1^2 r2^3 p1^2 R2 \\
& - 24 m R2^3 s r2^2 p1^2 Q1^2 R1 + 12 m R2^5 s^3 c^2 p1^2 Q1^2 r2 + 12 m R2^5 s c^2 r2 p1^2 Q1^2 \\
& - m R2^4 s c^4 p1^2 Q1^2 R1^2 - m R2^4 s c^4 p1^2 Q1^2 r2 ta^2 R1 + 12 Q1 R1^3 r2^3 p1 q1 P1 \\
& - 2 m R2^5 s c^4 p1^2 Q1^2 r2 ta^2 + 48 Q1^2 R1^2 r2^3 p1^2 ta^2 R2 + 24 m R2^2 s r2^3 p1^2 Q1^2 ta^2 R1 \\
& + 6 m R2^4 s^3 c^2 p1^2 Q1^2 r2 R1 + 6 m R2^4 s c^2 r2 p1^2 Q1^2 R1 + 12 m R2^2 s r2^2 p1 Q1 R1^2 q1 P1 \\
& + 24 m R2^3 s r2^2 p1 Q1 R1 q1 P1 - 12 m R2^5 s^3 c^2 r2 p1 Q1 q1 P1 + 12 Q1 R1^3 r2^3 p1 ta^2 q1 P1 \\
& - 12 Q1 R1^3 r2^2 p1 R2 q1 P1 - 24 Q1 R1^2 r2^2 p1 R2^2 q1 P1 + 24 m R2^3 s r2^3 p1 Q1 ta^2 q1 P1 \\
& + 6 m R2^5 s c^2 r2 p1 Q1 q1 P1 + 2 m R2^4 s c^4 p1 Q1 q1 P1 R1^2 + 4 m R2^5 s c^4 p1 Q1 q1 P1 R1 \\
& + 4 m R2^5 s c^4 p1 Q1 q1 P1 r2 ta^2 + 24 Q1 R1^2 r2^3 p1 q1 P1 R2 + 24 Q1 R1^2 r2^3 p1 ta^2 q1 P1 R2 \\
& + 12 m R2^2 s r2^3 p1 Q1 ta^2 q1 P1 R1 - 6 m R2^4 s^3 c^2 r2 p1 Q1 q1 P1 R1 \\
& + 3 m R2^4 s c^2 r2 p1 Q1 q1 P1 R1 + 2 m R2^4 s c^4 p1 Q1 q1 P1 r2 ta^2 R1) s1^5 + q1 (4 (m R2^5 s c^4 \\
& Q1 q1 P1 + m R2^5 s c^4 p1 Q1 q1) r2 ta^2 + 48 Q1^2 R1^3 r2^3 p1 \\
& + 2 (m R2^4 s c^4 Q1 q1 P1 + m R2^4 s c^4 p1 Q1 q1) r2 ta^2 R1 - 2 m R2^4 s c^4 q1^2 P1 R1^2 \\
& - 4 m R2^5 s c^4 q1^2 P1 R1 - 4 m R2^5 s c^4 q1^2 P1 r2 ta^2 - 2 m R2^4 s c^4 q1^2 P1 r2 ta^2 R1 \\
& + 48 Q1^2 R1^3 r2^3 p1 ta^2 - 24 m R2^2 s r2^2 p1 Q1^2 R1^2 + 24 Q1^2 R1^3 r2^2 p1 R2 \\
& + 48 Q1^2 R1^2 r2^2 p1 R2^2 + 96 m R2^3 s r2^3 p1 Q1^2 ta^2 - 4 m R2^5 s c^4 p1 Q1^2 R1
\end{aligned}$$

$$\begin{aligned}
& + 96 Q1^2 R1^2 r2^3 p1 R2 - 48 m R2^3 s r2^2 p1 Q1^2 R1 + 24 m R2^5 s^3 c^2 p1 Q1^2 r2 \\
& + 24 m R2^5 s c^2 r2 p1 Q1^2 - 2 m R2^4 s c^4 p1 Q1^2 R1^2 - 2 m R2^4 s c^4 p1 Q1^2 r2 ta^2 R1 \\
& + 12 Q1 R1^3 r2^3 p1 q1 + 12 Q1 R1^3 r2^3 q1 P1 - 4 m R2^5 s c^4 p1 Q1^2 r2 ta^2 \\
& + 96 Q1^2 R1^2 r2^3 p1 ta^2 R2 + 48 m R2^2 s r2^3 p1 Q1^2 ta^2 R1 + 12 m R2^4 s^3 c^2 p1 Q1^2 r2 R1 \\
& + 12 m R2^4 s c^2 r2 p1 Q1^2 R1 + 12 m R2^2 s r2^2 p1 Q1 R1^2 q1 + 12 m R2^2 s r2^2 Q1 R1^2 q1 P1 \\
& + 24 m R2^3 s r2^2 p1 Q1 R1 q1 + 24 m R2^3 s r2^2 Q1 R1 q1 P1 \\
& + 2 (m R2^4 s c^4 Q1 q1 P1 + m R2^4 s c^4 p1 Q1 q1) R1^2 \\
& + 4 (m R2^5 s c^4 Q1 q1 P1 + m R2^5 s c^4 p1 Q1 q1) R1 - 12 m R2^5 s^3 c^2 r2 p1 Q1 q1 \\
& - 12 m R2^5 s^3 c^2 r2 Q1 q1 P1 + 12 Q1 R1^3 r2^3 p1 ta^2 q1 + 12 Q1 R1^3 r2^3 ta^2 q1 P1 \\
& - 12 Q1 R1^3 r2^2 p1 R2 q1 - 12 Q1 R1^3 r2^2 R2 q1 P1 - 24 Q1 R1^2 r2^2 p1 R2^2 q1 \\
& - 24 Q1 R1^2 r2^2 R2^2 q1 P1 + 24 m R2^3 s r2^3 p1 Q1 ta^2 q1 + 24 m R2^3 s r2^3 Q1 ta^2 q1 P1 \\
& + 6 m R2^5 s c^2 r2 p1 Q1 q1 + 6 m R2^5 s c^2 r2 Q1 q1 P1 \\
& + 24 (Q1 R1^2 r2^3 q1 P1 + Q1 R1^2 r2^3 p1 q1) R2 \\
& + 24 (Q1 R1^2 r2^3 ta^2 q1 P1 + Q1 R1^2 r2^3 p1 ta^2 q1) R2 \\
& + 12 (m R2^2 s r2^3 Q1 ta^2 q1 P1 + m R2^2 s r2^3 p1 Q1 ta^2 q1) R1 \\
& - 6 (m R2^4 s^3 c^2 r2 Q1 q1 P1 + m R2^4 s^3 c^2 r2 p1 Q1 q1) R1 \\
& + 3 (m R2^4 s c^2 r2 Q1 q1 P1 + m R2^4 s c^2 r2 p1 Q1 q1) R1) s1^4 + q1 (24 Q1^2 R1^3 r2^3 \\
& + 24 Q1^2 R1^3 r2^3 ta^2 + 12 Q1^2 R1^3 r2^2 R2 + 24 Q1^2 R1^2 r2^2 R2^2 + 48 Q1^2 R1^2 r2^3 R2 \\
& + 12 Q1 R1^3 r2^3 q1 - m R2^4 s c^4 q1^2 R1^2 - 2 m R2^5 s c^4 q1^2 R1 - 2 m R2^5 s c^4 q1^2 r2 ta^2 \\
& - m R2^4 s c^4 q1^2 r2 ta^2 R1 - 12 m R2^2 s r2^2 Q1^2 R1^2 + 48 m R2^3 s r2^3 Q1^2 ta^2 \\
& - 2 m R2^5 s c^4 Q1^2 R1 - 24 m R2^3 s r2^2 Q1^2 R1 + 12 m R2^5 s^3 c^2 Q1^2 r2 + 12 m R2^5 s c^2 r2 Q1^2 \\
& - m R2^4 s c^4 Q1^2 R1^2 - m R2^4 s c^4 Q1^2 r2 ta^2 R1 - 2 m R2^5 s c^4 Q1^2 r2 ta^2 \\
& + 48 Q1^2 R1^2 r2^3 ta^2 R2 + 24 m R2^2 s r2^3 Q1^2 ta^2 R1 + 6 m R2^4 s^3 c^2 Q1^2 r2 R1 \\
& + 6 m R2^4 s c^2 r2 Q1^2 R1 + 12 m R2^2 s r2^2 Q1 R1^2 q1 + 24 m R2^3 s r2^2 Q1 R1 q1 \\
& - 12 m R2^5 s^3 c^2 r2 Q1 q1 + 12 Q1 R1^3 r2^3 ta^2 q1 - 12 Q1 R1^3 r2^2 R2 q1 - 24 Q1 R1^2 r2^2 R2^2 q1 \\
& + 24 m R2^3 s r2^3 Q1 ta^2 q1 + 6 m R2^5 s c^2 r2 Q1 q1 + 2 m R2^4 s c^4 Q1 q1 R1^2 \\
& + 4 m R2^5 s c^4 Q1 q1 R1 + 4 m R2^5 s c^4 Q1 q1 r2 ta^2 + 24 Q1 R1^2 r2^3 q1 R2 \\
& + 24 Q1 R1^2 r2^3 ta^2 q1 R2 + 12 m R2^2 s r2^3 Q1 ta^2 q1 R1 - 6 m R2^4 s^3 c^2 r2 Q1 q1 R1 \\
& + 3 m R2^4 s c^2 r2 Q1 q1 R1 + 2 m R2^4 s c^4 Q1 q1 r2 ta^2 R1) s1^3
\end{aligned}$$

> D5:=coeff(LHS3,s1^5);

$$\begin{aligned}
D5 := & 4 p1 r2^2 (2 r2 p1 Q1 + r2 q1 P1 + 2 r2 ta^2 p1 Q1 + r2 ta^2 q1 P1 + R2 p1 Q1 - R2 q1 P1) (R1^2 \\
& + m R2^2) (R1 + 2 R2) (2 p1 Q1 + q1 P1)
\end{aligned}$$

> D4:=coeff(LHS3,s1^4);

$$\begin{aligned}
D4 := & 4 (r2^2 (2 r2 p1 Q1 + r2 q1 P1 + 2 r2 ta^2 p1 Q1 + r2 ta^2 q1 P1 + R2 p1 Q1 - R2 q1 P1) \\
& + p1 r2^2 (2 r2 Q1 + r2 q1 + 2 r2 ta^2 Q1 + r2 ta^2 q1 + R2 Q1 - R2 q1)) (R1^2 + m R2^2) (R1 + 2 R2) (2 \\
& p1 Q1 + q1 P1) + 4 p1 r2^2 (2 r2 p1 Q1 + r2 q1 P1 + 2 r2 ta^2 p1 Q1 + r2 ta^2 q1 P1 + R2 p1 Q1 \\
& - R2 q1 P1) (R1^2 + m R2^2) (R1 + 2 R2) (2 Q1 + q1)
\end{aligned}$$

> D3:=coeff(LHS3,s1^3);

$$D3 := 4 r2^2 (2 r2 Q1 + r2 q1 + 2 r2 ta^2 Q1 + r2 ta^2 q1 + R2 Q1 - R2 q1) (R1^2 + m R2^2) (R1 + 2 R2) (2 p1$$

$$Q1 + q1 P1) + 4(r2^2(2r2 p1 Q1 + r2 q1 P1 + 2r2 ta^2 p1 Q1 + r2 ta^2 q1 P1 + R2 p1 Q1 - R2 q1 P1) + p1 r2^2(2r2 Q1 + r2 q1 + 2r2 ta^2 Q1 + r2 ta^2 q1 + R2 Q1 - R2 q1)) (R1^2 + m R2^2) (R1 + 2 R2) (2 Q1 + q1)$$

```

> D2:=coeff(LHS3,s1^2);
D2 := 4 r2^2 (2 r2 Q1 + r2 q1 + 2 r2 ta^2 Q1 + r2 ta^2 q1 + R2 Q1 - R2 q1) (R1^2 + m R2^2) (R1 + 2 R2) (2 Q1 + q1)

> D1:=coeff(LHS3,s1);
D1 := 0

> D0:=coeff(LHS3,s1,0);
D0 := 0

> E5:=coeff(epsilon2,s1^5);
E5 := q1 (24 Q1^2 R1^3 r2^3 p1^2 - m R2^4 s c^4 q1^2 P1^2 R1^2 - 2 m R2^5 s c^4 q1^2 P1^2 R1
- 2 m R2^5 s c^4 q1^2 P1^2 r2 ta^2 - m R2^4 s c^4 q1^2 P1^2 r2 ta^2 R1 + 24 Q1^2 R1^3 r2^3 p1^2 ta^2
- 12 m R2^2 s r2^2 p1^2 Q1^2 R1^2 + 12 Q1^2 R1^3 r2^2 p1^2 R2 + 24 Q1^2 R1^2 r2^2 p1^2 R2^2
+ 48 m R2^3 s r2^3 p1^2 Q1^2 ta^2 - 2 m R2^5 s c^4 p1^2 Q1^2 R1 + 48 Q1^2 R1^2 r2^3 p1^2 R2
- 24 m R2^3 s r2^2 p1^2 Q1^2 R1 + 12 m R2^5 s^3 c^2 p1^2 Q1^2 r2 + 12 m R2^5 s c^2 r2 p1^2 Q1^2
- m R2^4 s c^4 p1^2 Q1^2 R1^2 - m R2^4 s c^4 p1^2 Q1^2 r2 ta^2 R1 + 12 Q1 R1^3 r2^3 p1 q1 P1
- 2 m R2^5 s c^4 p1^2 Q1^2 r2 ta^2 + 48 Q1^2 R1^2 r2^3 p1^2 ta^2 R2 + 24 m R2^2 s r2^3 p1^2 Q1^2 ta^2 R1
+ 6 m R2^4 s^3 c^2 p1^2 Q1^2 r2 R1 + 6 m R2^4 s c^2 r2 p1^2 Q1^2 R1 + 12 m R2^2 s r2^2 p1 Q1 R1^2 q1 P1
+ 24 m R2^3 s r2^2 p1 Q1 R1 q1 P1 - 12 m R2^5 s^3 c^2 r2 p1 Q1 q1 P1 + 12 Q1 R1^3 r2^3 p1 ta^2 q1 P1
- 12 Q1 R1^3 r2^2 p1 R2 q1 P1 - 24 Q1 R1^2 r2^2 p1 R2^2 q1 P1 + 24 m R2^3 s r2^3 p1 Q1 ta^2 q1 P1
+ 6 m R2^5 s c^2 r2 p1 Q1 q1 P1 + 2 m R2^4 s c^4 p1 Q1 q1 P1 R1^2 + 4 m R2^5 s c^4 p1 Q1 q1 P1 R1
+ 4 m R2^5 s c^4 p1 Q1 q1 P1 r2 ta^2 + 24 Q1 R1^2 r2^3 p1 q1 P1 R2 + 24 Q1 R1^2 r2^3 p1 ta^2 q1 P1 R2
+ 12 m R2^2 s r2^3 p1 Q1 ta^2 q1 P1 R1 - 6 m R2^4 s^3 c^2 r2 p1 Q1 q1 P1 R1
+ 3 m R2^4 s c^2 r2 p1 Q1 q1 P1 R1 + 2 m R2^4 s c^4 p1 Q1 q1 P1 r2 ta^2 R1)

```

```

> E4:=coeff(epsilon2,s1^4);
E4 := q1 (4(m R2^5 s c^4 Q1 q1 P1 + m R2^5 s c^4 p1 Q1 q1) r2 ta^2 + 48 Q1^2 R1^3 r2^3 p1
+ 2(m R2^4 s c^4 Q1 q1 P1 + m R2^4 s c^4 p1 Q1 q1) r2 ta^2 R1 - 2 m R2^4 s c^4 q1^2 P1 R1^2
- 4 m R2^5 s c^4 q1^2 P1 R1 - 4 m R2^5 s c^4 q1^2 P1 r2 ta^2 - 2 m R2^4 s c^4 q1^2 P1 r2 ta^2 R1
+ 48 Q1^2 R1^3 r2^3 p1 ta^2 - 24 m R2^2 s r2^2 p1 Q1^2 R1^2 + 24 Q1^2 R1^3 r2^2 p1 R2
+ 48 Q1^2 R1^2 r2^2 p1 R2^2 + 96 m R2^3 s r2^3 p1 Q1^2 ta^2 - 4 m R2^5 s c^4 p1 Q1^2 R1
+ 96 Q1^2 R1^2 r2^3 p1 R2 - 48 m R2^3 s r2^2 p1 Q1^2 R1 + 24 m R2^5 s^3 c^2 p1 Q1^2 r2
+ 24 m R2^5 s c^2 r2 p1 Q1^2 - 2 m R2^4 s c^4 p1 Q1^2 R1^2 - 2 m R2^4 s c^4 p1 Q1^2 r2 ta^2 R1
+ 12 Q1 R1^3 r2^3 p1 q1 + 12 Q1 R1^3 r2^3 q1 P1 - 4 m R2^5 s c^4 p1 Q1^2 r2 ta^2
+ 96 Q1^2 R1^2 r2^3 p1 ta^2 R2 + 48 m R2^2 s r2^3 p1 Q1^2 ta^2 R1 + 12 m R2^4 s^3 c^2 p1 Q1^2 r2 R1
+ 12 m R2^4 s c^2 r2 p1 Q1^2 R1 + 12 m R2^2 s r2^2 p1 Q1 R1^2 q1 + 12 m R2^2 s r2^2 Q1 R1^2 q1 P1
+ 24 m R2^3 s r2^2 p1 Q1 R1 q1 + 24 m R2^3 s r2^2 Q1 R1 q1 P1
+ 2(m R2^4 s c^4 Q1 q1 P1 + m R2^4 s c^4 p1 Q1 q1) R1^2
+ 4(m R2^5 s c^4 Q1 q1 P1 + m R2^5 s c^4 p1 Q1 q1) R1 - 12 m R2^5 s^3 c^2 r2 p1 Q1 q1
- 12 m R2^5 s^3 c^2 r2 Q1 q1 P1 + 12 Q1 R1^3 r2^3 p1 ta^2 q1 + 12 Q1 R1^3 r2^3 ta^2 q1 P1

```

$$\begin{aligned}
& -12 Q1 RI^3 r2^2 p1 R2 q1 - 12 Q1 RI^3 r2^2 R2 q1 P1 - 24 Q1 RI^2 r2^2 p1 R2^2 q1 \\
& - 24 Q1 RI^2 r2^2 R2^2 q1 P1 + 24 m R2^3 s r2^3 p1 Q1 ta^2 q1 + 24 m R2^3 s r2^3 Q1 ta^2 q1 P1 \\
& + 6 m R2^5 s c^2 r2 p1 Q1 q1 + 6 m R2^5 s c^2 r2 Q1 q1 P1 \\
& + 24 (Q1 RI^2 r2^3 q1 P1 + Q1 RI^2 r2^3 p1 q1) R2 \\
& + 24 (Q1 RI^2 r2^3 ta^2 q1 P1 + Q1 RI^2 r2^3 p1 ta^2 q1) R2 \\
& + 12 (m R2^2 s r2^3 Q1 ta^2 q1 P1 + m R2^2 s r2^3 p1 Q1 ta^2 q1) R1 \\
& - 6 (m R2^4 s^3 c^2 r2 Q1 q1 P1 + m R2^4 s^3 c^2 r2 p1 Q1 q1) R1 \\
& + 3 (m R2^4 s c^2 r2 Q1 q1 P1 + m R2^4 s c^2 r2 p1 Q1 q1) R1
\end{aligned}$$

> E3:=coeff(epsilon2,s1^3);

$$\begin{aligned}
E3 := & q1 (24 Q1^2 RI^3 r2^3 + 24 Q1^2 RI^3 r2^3 ta^2 + 12 Q1^2 RI^3 r2^2 R2 + 24 Q1^2 RI^2 r2^2 R2^2 \\
& + 48 Q1^2 RI^2 r2^3 R2 + 12 Q1 RI^3 r2^3 q1 - m R2^4 s c^4 q1^2 RI^2 - 2 m R2^5 s c^4 q1^2 RI \\
& - 2 m R2^5 s c^4 q1^2 r2 ta^2 - m R2^4 s c^4 q1^2 r2 ta^2 RI - 12 m R2^2 s r2^2 Q1^2 RI^2 \\
& + 48 m R2^3 s r2^3 Q1^2 ta^2 - 2 m R2^3 s c^4 Q1^2 RI - 24 m R2^3 s r2^2 Q1^2 RI \\
& + 12 m R2^5 s^3 c^2 Q1^2 r2 + 12 m R2^5 s c^2 r2 Q1^2 - m R2^4 s c^4 Q1^2 RI^2 \\
& - m R2^4 s c^4 Q1^2 r2 ta^2 RI - 2 m R2^5 s c^4 Q1^2 r2 ta^2 + 48 Q1^2 RI^2 r2^3 ta^2 R2 \\
& + 24 m R2^2 s r2^3 Q1^2 ta^2 RI + 6 m R2^4 s^3 c^2 Q1^2 r2 RI + 6 m R2^4 s c^2 r2 Q1^2 RI \\
& + 12 m R2^2 s r2^2 Q1 RI^2 q1 + 24 m R2^3 s r2^2 Q1 RI q1 - 12 m R2^5 s^3 c^2 r2 Q1 q1 \\
& + 12 Q1 RI^3 r2^3 ta^2 q1 - 12 Q1 RI^3 r2^2 R2 q1 - 24 Q1 RI^2 r2^2 R2^2 q1 + 24 m R2^3 s r2^3 Q1 ta^2 q1 \\
& + 6 m R2^5 s c^2 r2 Q1 q1 + 2 m R2^4 s c^4 Q1 q1 RI^2 + 4 m R2^5 s c^4 Q1 q1 RI \\
& + 4 m R2^5 s c^4 Q1 q1 r2 ta^2 + 24 Q1 RI^2 r2^3 q1 R2 + 24 Q1 RI^2 r2^3 ta^2 q1 R2 \\
& + 12 m R2^2 s r2^3 Q1 ta^2 q1 RI - 6 m R2^4 s^3 c^2 r2 Q1 q1 RI + 3 m R2^4 s c^2 r2 Q1 q1 RI \\
& + 2 m R2^4 s c^4 Q1 q1 r2 ta^2 RI)
\end{aligned}$$

> E2:=coeff(epsilon2,s1^2);

$$E2 := 0$$

> E1:=coeff(epsilon2,s1);

$$E1 := 0$$

> E0:=coeff(epsilon2,s1,0);

$$E0 := 0$$

> B5:=coeff(beta2,s1^5);

$$\begin{aligned}
B5 := & q1 (-r2 m R2^5 s^2 c^3 q1^2 P1^2 + 12 r2^4 m R2^2 s tap1 Q1 q1 P1 - 4 r2^2 m R2^4 s^4 c pl^2 Q1^2 \\
& + 24 r2^4 m R2^2 s tap1^2 Q1^2 + 2 r2^2 m R2^4 s^4 c pl Q1 q1 P1 + 2 r2^2 m R2^4 s^4 c q1^2 P1^2 \\
& - 4 r2^2 m R2^4 s^2 c pl^2 Q1^2 - 4 r2^2 m R2^4 s^2 c pl Q1 q1 P1 - r2^2 m R2^4 s^2 c q1^2 P1^2 \\
& - r2 m R2^5 s^2 c^3 pl^2 Q1^2 + 2 r2 m R2^5 s^2 c^3 pl Q1 q1 P1)
\end{aligned}$$

> B4:=coeff(beta2,s1^4);

$$\begin{aligned}
B4 := & q1 (-8 r2^2 m R2^4 s^4 c pl Q1^2 + 48 r2^4 m R2^2 s tap1 Q1^2 + 4 r2^2 m R2^4 s^4 c q1^2 P1 \\
& - 8 r2^2 m R2^4 s^2 c pl Q1^2 - 4 r2^2 m R2^4 s^2 c Q1 q1 P1 - 4 r2^2 m R2^4 s^2 c pl Q1 q1 \\
& - 2 r2^2 m R2^4 s^2 c q1^2 P1 - 2 r2 m R2^5 s^2 c^3 pl Q1^2 + 2 r2 m R2^5 s^2 c^3 Q1 q1 P1 \\
& + 2 r2 m R2^5 s^2 c^3 pl Q1 q1 - 2 r2 m R2^5 s^2 c^3 q1^2 P1 + 2 r2^2 m R2^4 s^4 c Q1 q1 P1)
\end{aligned}$$

```

+ 2 r22 m R24 s4 c pI QI qI + 12 r24 m R22 s ta QI qI P1 + 12 r24 m R22 s tapI QI qI)

> B3:=coeff(beta2,s1^3);
B3 := qI (-4 r22 m R24 s2 c QI2 - 4 r22 m R24 s2 c QI qI - r22 m R24 s2 c qI2 - r2 m R25 s2 c3 QI2
+ 2 r2 m R25 s2 c3 QI qI - r2 m R25 s2 c3 qI2 + 24 r24 m R22 s ta QI2 + 2 r22 m R24 s4 c QI qI
+ 12 r24 m R22 s ta QI qI - 4 r22 m R24 s4 c QI2 + 2 r22 m R24 s4 c qI2)

> B2:=coeff(beta2,s1^2);
B2 := 0

> B1:=coeff(beta2,s1);
B1 := 0

> B0:=coeff(beta2,s1,0);
B0 := 0

>
>
>

```

&gt;

## Program No. 4

Coefficients of the diff. eq. of the shear  
stress of a helix using two Maxwell models

```
restart;
```

```
> with(linalg):
```

Warning, the protected names norm and trace have been redefined and unprotected

```
> c3 := 2*E*m*R2^2/(R1+2*R2)^3/(r2+r2*ta^2+nu*R2)*(ta*(1-2*s^2)/4*R2^2*s^3+n
u*R2^2/4/(1+nu)/r2*(R1+r2*ta^2)*s^4*c-(1+nu)/2*R2^2*s^2*c*(1+s^2)+nu*R2^
2/4/r2*(R1+r2*ta^2)*c^3*(1+s^2)+(r2*ta^2-nu*R1)*r2*c);
```

$$c3 := \frac{1}{(R1 + 2 R2)^3 (r2 + r2 ta^2 + v R2)} \left( 2 E m R2^2 \left( \frac{1}{4} ta (1 - 2 s^2) R2^2 s^3 + \frac{v R2^2 (R1 + r2 ta^2) s^4 c}{4 (1 + v) r2} \right. \right.$$

$$\left. \left. - \frac{1}{2} (1 + v) R2^2 s^2 c (1 + s^2) + \frac{v R2^2 (R1 + r2 ta^2) c^3 (1 + s^2)}{4 r2} + (r2 ta^2 - v R1) r2 c \right) \right)$$

```
> c4 := 2*E*R1^4/4/(R1+2*R2)^4/(1+nu)+2*E*r2*m*R2^2/(R1+2*R2)^4/(r2+r2*ta^2+
nu*R2)*((R2^2*(2*s^2-1)*ta^2*s^3)/4/(1+nu)+nu*R2^3*s^5/4/(1+nu)/r2+R2^2/
2*s^3*(1+s^2)+nu*R2^3/4/r2*s*(1+s^2)*c^2+r2^2*s);
```

$$c4 := \frac{E R1^4}{2 (R1 + 2 R2)^4 (1 + v)} + \frac{1}{(R1 + 2 R2)^4 (r2 + r2 ta^2 + v R2)} \left( 2 E r2 m R2^2 \left( \frac{R2^2 (2 s^2 - 1) ta^2 s^3}{4 (1 + v)} \right. \right.$$

$$+ \frac{v R2^3 s^5}{4(1+v)r2} + \frac{1}{2} R2^2 s^3 (1+s^2) + \frac{v R2^3 s (1+s^2) c^2}{4r2} + r2^2 s \Bigg) \Bigg)$$

> tau:=c3\*epsilon+c4\*beta;

$$\tau := \frac{1}{(RI+2R2)^3(r2+r2ta^2+vR2)} \left( 2EmR2^2 \left( \frac{1}{4}ta(1-2s^2)R2^2s^3 + \frac{vR2^2(RI+r2ta^2)s^4c}{4(1+v)r2} \right. \right. \\ \left. \left. - \frac{1}{2}(1+v)R2^2s^2c(1+s^2) + \frac{vR2^2(RI+r2ta^2)c^3(1+s^2)}{4r2} + (r2ta^2-vRI)r2c \right) \varepsilon \right) + \left( \right. \\ \left. \frac{ERI^4}{2(RI+2R2)^4(1+v)} + \frac{1}{(RI+2R2)^4(r2+r2ta^2+vR2)} \left( 2Er2mR2^2 \left( \frac{R2^2(2s^2-1)ta^2s^3}{4(1+v)} \right. \right. \right. \\ \left. \left. \left. + \frac{vR2^3s^5}{4(1+v)r2} + \frac{1}{2}R2^2s^3(1+s^2) + \frac{vR2^3s(1+s^2)c^2}{4r2} + r2^2s \right) \right) \beta \right)$$

> tau1:=subs(E=3\*Q\*q/(2\*p\*Q+q\*P),nu=(p\*Q-q\*P)/(2\*p\*Q+q\*P),tau);

$$\tau_1 := \frac{1}{(2pQ+qP)(RI+2R2)^3 \left( r2+r2ta^2 + \frac{(pQ-qP)R2}{2pQ+qP} \right)} \left( 6QqmR2^2 \left( \frac{1}{4}ta(1-2s^2)R2^2s^3 \right. \right. \\ \left. \left. + \frac{(pQ-qP)R2^2(RI+r2ta^2)s^4c}{4(2pQ+qP)} - \frac{1}{2} \left( 1 + \frac{pQ-qP}{2pQ+qP} \right) R2^2s^2c(1+s^2) \right. \right. \\ \left. \left. + \frac{(pQ-qP)R2^2(RI+r2ta^2)c^3(1+s^2)}{4(2pQ+qP)r2} + \left( r2ta^2 - \frac{(pQ-qP)RI}{2pQ+qP} \right) r2c \right) \varepsilon \right) + \left( \right. \\ \left. \frac{3QqRI^4}{2(2pQ+qP)(RI+2R2)^4 \left( 1 + \frac{pQ-qP}{2pQ+qP} \right)} + \right. \\ \left. \frac{1}{(2pQ+qP)(RI+2R2)^4 \left( r2+r2ta^2 + \frac{(pQ-qP)R2}{2pQ+qP} \right)} \left( 6Qqr2mR2^2 \left( \frac{R2^2(2s^2-1)ta^2s^3}{4 \left( 1 + \frac{pQ-qP}{2pQ+qP} \right)} \right. \right. \right. \\ \left. \left. \left. + \frac{vR2^3s^5}{4(1+v)r2} + \frac{1}{2}R2^2s^3(1+s^2) + \frac{vR2^3s(1+s^2)c^2}{4r2} + r2^2s \right) \right) \beta \right)$$

$$\left. \left( \left. \left. + \frac{(pQ - qP)R2^3 s^5}{4(2pQ + qP)} + \frac{1}{2} R2^2 s^3 (1 + s^2) + \frac{(pQ - qP)R2^3 s (1 + s^2) c^2}{4(2pQ + qP)r2} + r2^2 s \right) \frac{1}{r2} \right) \beta \right)$$

> tau2:=simplify(tau1);

$$\begin{aligned}
\tau_2 := & \frac{1}{2r2 p (-2r2 p Q - r2 q P - 2r2 t a^2 p Q - r2 t a^2 q P - R2 p Q + R2 q P) (R1 + 2R2)^4 (2pQ + qP)} (( \\
& -4r2^2 \beta R1^4 p Q q P - 4r2^2 \beta R1^4 p^2 Q^2 t a^2 - 4r2^2 \beta R1^4 p Q q P t a^2 + r2 \beta R1^4 p Q R2 q P \\
& - r2^2 \beta R1^4 q^2 P^2 t a^2 - 8r2^2 \beta m R2^4 s^5 t a^2 p^2 Q^2 - 8r2^2 \beta m R2^4 s^5 t a^2 p Q q P \\
& - 2r2^2 \beta m R2^4 s^5 t a^2 q^2 P^2 + 4r2^2 \beta m R2^4 s^3 t a^2 p^2 Q^2 + 4r2^2 \beta m R2^4 s^3 t a^2 p Q q P \\
& + r2^2 \beta m R2^4 s^3 t a^2 q^2 P^2 - 2r2 \beta m R2^5 s^5 p^2 Q^2 + r2 \beta m R2^5 s^5 p Q q P \\
& + r2 \beta m R2^5 s^5 q^2 P^2 - 12r2^2 \beta m R2^4 s^3 p^2 Q^2 - 6r2^2 \beta m R2^4 s^3 p Q q P \\
& - 12r2^2 \beta m R2^4 s^5 p^2 Q^2 - 6r2^2 \beta m R2^4 s^5 p Q q P - 3r2 \beta m R2^5 s^2 c^2 p^2 Q^2 \\
& - 3r2 \beta m R2^5 s^3 c^2 p^2 Q^2 + 3r2 \beta m R2^5 s^3 c^2 p Q q P + 3r2 \beta m R2^5 s^3 c^2 p Q q P \\
& - 24r2^4 \beta m R2^2 s p^2 Q^2 - 12r2^4 \beta m R2^2 s p Q q P - 4r2^2 \beta R1^4 p^2 Q^2 - 2r2 \beta R1^4 p^2 Q^2 R2 \\
& - r2^2 \beta R1^4 q^2 P^2 + r2 \beta R1^4 q^2 P^2 R2 + 24m R2^5 \epsilon t a s^5 p^2 Q^2 r2 + 12m R2^5 \epsilon t a s^5 p Q r2 q P \\
& - 12m R2^5 \epsilon t a s^3 p^2 Q^2 r2 - 6m R2^5 \epsilon t a s^3 p Q r2 q P - 2m R2^4 \epsilon s^4 c p^2 Q^2 R1^2 \\
& - 4m R2^5 \epsilon s^4 c p^2 Q^2 R1 - 4m R2^5 \epsilon s^4 c p^2 Q^2 r2 t a^2 + m R2^4 \epsilon s^4 c p Q q P R1^2 \\
& + 2m R2^5 \epsilon s^4 c p Q q P R1 + 2m R2^5 \epsilon s^4 c p Q q P r2 t a^2 + m R2^4 \epsilon s^4 c q^2 p^2 R1^2 \\
& + 2m R2^5 \epsilon s^4 c q^2 P^2 R1 + 2m R2^5 \epsilon s^4 c q^2 P^2 r2 t a^2 + 36m R2^5 \epsilon p^2 Q^2 s^2 c r2 \\
& + 36m R2^5 \epsilon p^2 Q^2 s^4 c r2 - 3m R2^4 \epsilon c^3 p^2 Q^2 R1^2 - 6m R2^5 \epsilon c^3 p^2 Q^2 R1 \\
& - 3m R2^4 \epsilon c^3 p^2 Q^2 R1^2 s^2 - 6m R2^5 \epsilon c^3 p^2 Q^2 R1 s^2 - 6m R2^5 \epsilon c^3 p^2 Q^2 r2 t a^2 \\
& - 6m R2^5 \epsilon c^3 p^2 Q^2 r2 t a^2 s^2 + 3m R2^4 \epsilon c^3 p Q R1^2 q P + 6m R2^5 \epsilon c^3 p Q R1 q P \\
& + 3m R2^4 \epsilon c^3 p Q R1^2 q P s^2 + 6m R2^5 \epsilon c^3 p Q R1 q P s^2 + 6m R2^5 \epsilon c^3 p Q r2 t a^2 q P \\
& + 6m R2^5 \epsilon c^3 p Q r2 t a^2 q P s^2 + 12m R2^2 \epsilon r2^2 c p^2 Q^2 R1^2 - 12m R2^2 \epsilon r2^2 c p Q R1^2 q P \\
& + 12m R2^4 \epsilon t a s^5 p^2 Q^2 r2 R1 + 6m R2^4 \epsilon t a s^5 p Q r2 q P R1 - 6m R2^4 \epsilon t a s^3 p^2 Q^2 r2 R1 \\
& - 3m R2^4 \epsilon t a s^3 p Q r2 q P R1 - 2m R2^4 \epsilon s^4 c p^2 Q^2 r2 t a^2 R1 + m R2^4 \epsilon s^4 c p Q q P r2 t a^2 R1 \\
& + m R2^4 \epsilon s^4 c q^2 P^2 r2 t a^2 R1 + 18m R2^4 \epsilon p^2 Q^2 s^2 c r2 R1 + 18m R2^4 \epsilon p^2 Q^2 s^4 c r2 R1 \\
& - 3m R2^4 \epsilon c^3 p^2 Q^2 r2 t a^2 R1 - 3m R2^4 \epsilon c^3 p^2 Q^2 r2 t a^2 s^2 R1 + 3m R2^4 \epsilon c^3 p Q r2 t a^2 q P R1 \\
& + 3m R2^4 \epsilon c^3 p Q r2 t a^2 q P s^2 R1 - 24m R2^2 \epsilon r2^3 c p^2 Q^2 t a^2 R1 - 48m R2^3 \epsilon r2^3 c p^2 Q^2 t a^2 \\
& - 12m R2^2 \epsilon r2^3 c p Q t a^2 q P R1 - 24m R2^3 \epsilon r2^3 c p Q t a^2 q P + 24m R2^3 \epsilon r2^2 c p^2 Q^2 R1 \\
& - 24m R2^3 \epsilon r2^2 c p Q R1 q P) q
\end{aligned}$$

> LHS1:=denom(tau2);

```

LHS1 := 2 r2 p (-2 r2 p Q - r2 q P - 2 r2 ta2 p Q - r2 ta2 q P - R2 p Q + R2 q P) (R1 + 2 R2)4 (2 p Q + q P)

> LHS2 := subs(p=1+p1*s1, P=1+P1*s1, q=q1*s1, Q=Q1*s1, LHS1);

LHS2 := 2 r2 (1 + p1 s1) (-2 r2 (1 + p1 s1) Q1 s1 - r2 q1 s1 (1 + P1 s1) - 2 r2 ta2 (1 + p1 s1) Q1 s1
- r2 ta2 q1 s1 (1 + P1 s1) - R2 (1 + p1 s1) Q1 s1 + R2 q1 s1 (1 + P1 s1)) (R1 + 2 R2)4 (2 (1 + p1 s1) Q1
s1 + q1 s1 (1 + P1 s1))

> LHS3 := collect(LHS2, s1);

LHS3 := 2 r2 p1 (-2 r2 p1 Q1 - r2 q1 P1 - 2 r2 ta2 p1 Q1 - r2 ta2 q1 P1 - R2 p1 Q1 + R2 q1 P1) (R1 + 2 R2)4
(2 p1 Q1 + q1 P1) s15 + (2 (r2 (-2 r2 p1 Q1 - r2 q1 P1 - 2 r2 ta2 p1 Q1 - r2 ta2 q1 P1 - R2 p1 Q1
+ R2 q1 P1) + r2 p1 (-2 r2 Q1 - r2 q1 - 2 r2 ta2 Q1 - r2 ta2 q1 - R2 Q1 + R2 q1)) (R1 + 2 R2)4 (2 p1
Q1 + q1 P1) + 2 r2 p1 (-2 r2 p1 Q1 - r2 q1 P1 - 2 r2 ta2 p1 Q1 - r2 ta2 q1 P1 - R2 p1 Q1 + R2 q1 P1)
(R1 + 2 R2)4 (2 Q1 + q1)) s14 + (2 r2 (-2 r2 Q1 - r2 q1 - 2 r2 ta2 Q1 - r2 ta2 q1 - R2 Q1 + R2 q1)
(R1 + 2 R2)4 (2 p1 Q1 + q1 P1) + 2 (r2 (-2 r2 p1 Q1 - r2 q1 P1 - 2 r2 ta2 p1 Q1 - r2 ta2 q1 P1
- R2 p1 Q1 + R2 q1 P1) + r2 p1 (-2 r2 Q1 - r2 q1 - 2 r2 ta2 Q1 - r2 ta2 q1 - R2 Q1 + R2 q1))
(R1 + 2 R2)4 (2 Q1 + q1)) s13
+ 2 r2 (-2 r2 Q1 - r2 q1 - 2 r2 ta2 Q1 - r2 ta2 q1 - R2 Q1 + R2 q1) (R1 + 2 R2)4 (2 Q1 + q1) s12

> RHS1 := numer(tau2);

RHS1 := q (-4 r22 β R14 p Q q P - 4 r22 β R14 p2 Q2 ta2 - 4 r22 β R14 p Q q P ta2 + r2 β R14 p Q R2 q P
- r22 β R14 q2 P2 ta2 - 8 r22 β m R24 s5 ta2 p2 Q2 - 8 r22 β m R24 s5 ta2 p Q q P
- 2 r22 β m R24 s5 ta2 q2 P2 + 4 r22 β m R24 s3 ta2 p2 Q2 + 4 r22 β m R24 s3 ta2 p Q q P
+ r22 β m R24 s3 ta2 q2 P2 - 2 r2 β m R25 s5 p2 Q2 + r2 β m R25 s5 p Q q P
+ r2 β m R25 s5 q2 P2 - 12 r22 β m R24 s3 p2 Q2 - 6 r22 β m R24 s3 p Q q P
- 12 r22 β m R24 s5 p2 Q2 - 6 r22 β m R24 s5 p Q q P - 3 r2 β m R25 s c2 p2 Q2
- 3 r2 β m R25 s3 c2 p2 Q2 + 3 r2 β m R25 s c2 p Q q P + 3 r2 β m R25 s3 c2 p Q q P
- 24 r24 β m R22 s p2 Q2 - 12 r24 β m R22 s p Q q P - 4 r22 β R14 p2 Q2 - 2 r2 β R14 p2 Q2 R2
- r22 β R14 q2 P2 + r2 β R14 q2 P2 R2 + 24 m R25 ε tas5 p2 Q2 r2 + 12 m R25 ε tas5 p Q r2 q P
- 12 m R25 ε tas3 p2 Q2 r2 - 6 m R25 ε tas3 p Q r2 q P - 2 m R24 ε s4 c p2 Q2 R12
- 4 m R25 ε s4 c p2 Q2 R1 - 4 m R25 ε s4 c p2 Q2 r2 ta2 + m R24 ε s4 c p Q q P R12
+ 2 m R25 ε s4 c p Q q P R1 + 2 m R25 ε s4 c p Q q P r2 ta2 + m R24 ε s4 c q2 P2 R12
+ 2 m R25 ε s4 c q2 P2 R1 + 2 m R25 ε s4 c q2 P2 r2 ta2 + 36 m R25 ε p2 Q2 s2 c r2
+ 36 m R25 ε p2 Q2 s4 c r2 - 3 m R24 ε c3 p2 Q2 R12 - 6 m R25 ε c3 p2 Q2 R1
- 3 m R24 ε c3 p2 Q2 R12 s2 - 6 m R25 ε c3 p2 Q2 R1 s2 - 6 m R25 ε c3 p2 Q2 r2 ta2
- 6 m R25 ε c3 p2 Q2 r2 ta2 s2 + 3 m R24 ε c3 p Q R12 q P + 6 m R25 ε c3 p Q R1 q P
+ 3 m R24 ε c3 p Q R12 q P s2 + 6 m R25 ε c3 p Q R1 q P s2 + 6 m R25 ε c3 p Q r2 ta2 q P
+ 6 m R25 ε c3 p Q r2 ta2 q P s2 + 12 m R22 ε r22 c p2 Q2 R12 - 12 m R22 ε r22 c p Q R12 q P
+ 12 m R24 ε tas5 p2 Q2 r2 R1 + 6 m R24 ε tas5 p Q r2 q P R1 - 6 m R24 ε tas3 p2 Q2 r2 R1
- 3 m R24 ε tas3 p Q r2 q P R1 - 2 m R24 ε s4 c p2 Q2 r2 ta2 R1 + m R24 ε s4 c p Q q P r2 ta2 R1
+ m R24 ε s4 c q2 P2 r2 ta2 R1 + 18 m R24 ε p2 Q2 s2 c r2 R1 + 18 m R24 ε p2 Q2 s4 c r2 R1
- 3 m R24 ε c3 p2 Q2 r2 ta2 R1 - 3 m R24 ε c3 p2 Q2 r2 ta2 s2 R1 + 3 m R24 ε c3 p Q r2 ta2 q P R1
+ 3 m R24 ε c3 p Q r2 ta2 q P s2 R1 - 24 m R22 ε r23 c p2 Q2 ta2 R1 - 48 m R23 ε r23 c p2 Q2 ta2
- 12 m R22 ε r23 c p Q ta2 q P R1 - 24 m R23 ε r23 c p Q ta2 q P + 24 m R23 ε r22 c p2 Q2 R1
- 24 m R23 ε r22 c p Q R1 q P)

```

```

> RHS2:=subs(p=1+p1*s1,P=1+P1*s1,q=q1*s1,Q=Q1*s1,RHS1);
RHS2 := q1 s1 (6 m R25 ε c3 (1 + p1 s1) Q1 s12 RI q1 (1 + P1 s1)
+ 3 m R24 ε c3 (1 + p1 s1) Q1 s12 RI2 q1 (1 + P1 s1) s2 - r22 β RI4 q12 s12 (1 + P1 s1)2 ta2
- r22 β RI4 q12 s12 (1 + P1 s1)2 + 6 m R25 ε c3 (1 + p1 s1) Q1 s12 RI q1 (1 + P1 s1) s2
+ 6 m R25 ε c3 (1 + p1 s1) Q1 s12 r2 ta2 q1 (1 + P1 s1)
+ 6 m R25 ε c3 (1 + p1 s1) Q1 s12 r2 ta2 q1 (1 + P1 s1) s2
- 12 m R22 ε r22 c (1 + p1 s1) Q1 s12 RI2 q1 (1 + P1 s1)
+ 6 m R24 ε ta s5 (1 + p1 s1) Q1 s12 r2 q1 (1 + P1 s1) RI
- 3 m R24 ε ta s3 (1 + p1 s1) Q1 s12 r2 q1 (1 + P1 s1) RI
+ m R24 ε s4 c (1 + p1 s1) Q1 s12 q1 (1 + P1 s1) r2 ta2 RI
+ 3 m R24 ε c3 (1 + p1 s1) Q1 s12 r2 ta2 q1 (1 + P1 s1) RI
+ 3 m R24 ε c3 (1 + p1 s1) Q1 s12 r2 ta2 q1 (1 + P1 s1) s2 RI
- 12 m R22 ε r23 c (1 + p1 s1) Q1 s12 ta2 q1 (1 + P1 s1) RI
- 24 m R23 ε r23 c (1 + p1 s1) Q1 s12 ta2 q1 (1 + P1 s1)
- 24 m R23 ε r22 c (1 + p1 s1) Q1 s12 RI q1 (1 + P1 s1) - 4 r22 β RI4 (1 + p1 s1)2 Q12 s12 ta2
- 8 r22 β m R24 s5 ta2 (1 + p1 s1)2 Q12 s12 + 4 r22 β m R24 s3 ta2 (1 + p1 s1)2 Q12 s12
- 2 r2 β m R25 s5 (1 + p1 s1)2 Q12 s12 - 12 r22 β m R24 s3 (1 + p1 s1)2 Q12 s12
- 12 r22 β m R24 s5 (1 + p1 s1)2 Q12 s12 - 3 r2 β m R25 s c2 (1 + p1 s1)2 Q12 s12
- 3 r2 β m R25 s3 c2 (1 + p1 s1)2 Q12 s12 - 24 r24 β m R22 s (1 + p1 s1)2 Q12 s12
- 4 r22 β RI4 (1 + p1 s1)2 Q12 s12 - 2 r2 β RI4 (1 + p1 s1)2 Q12 s12 R2
+ 24 m R25 ε ta s5 (1 + p1 s1)2 Q12 s12 r2 - 12 m R25 ε ta s3 (1 + p1 s1)2 Q12 s12 r2
- 2 m R24 ε s4 c (1 + p1 s1)2 Q12 s12 RI2 - 4 m R25 ε s4 c (1 + p1 s1)2 Q12 s12 RI
- 4 m R25 ε s4 c (1 + p1 s1)2 Q12 s12 r2 ta2 + 36 m R25 ε (1 + p1 s1)2 Q12 s12 s2 cr2
+ 36 m R25 ε (1 + p1 s1)2 Q12 s12 s4 cr2 - 3 m R24 ε c3 (1 + p1 s1)2 Q12 s12 RI2
- 6 m R25 ε c3 (1 + p1 s1)2 Q12 s12 RI - 3 m R24 ε c3 (1 + p1 s1)2 Q12 s12 RI2 s2
- 6 m R25 ε c3 (1 + p1 s1)2 Q12 s12 RI s2 - 6 m R25 ε c3 (1 + p1 s1)2 Q12 s12 r2 ta2
- 6 m R25 ε c3 (1 + p1 s1)2 Q12 s12 r2 ta2 s2 + 12 m R22 ε r22 c (1 + p1 s1)2 Q12 s12 RI2
+ 12 m R24 ε ta s5 (1 + p1 s1)2 Q12 s12 r2 RI - 6 m R24 ε ta s3 (1 + p1 s1)2 Q12 s12 r2 RI
- 2 m R24 ε s4 c (1 + p1 s1)2 Q12 s12 r2 ta2 RI + 18 m R24 ε (1 + p1 s1)2 Q12 s12 s2 cr2 RI
+ 18 m R24 ε (1 + p1 s1)2 Q12 s12 s4 cr2 RI - 3 m R24 ε c3 (1 + p1 s1)2 Q12 s12 r2 ta2 RI
- 3 m R24 ε c3 (1 + p1 s1)2 Q12 s12 r2 ta2 s2 RI - 24 m R22 ε r23 c (1 + p1 s1)2 Q12 s12 ta2 RI
- 48 m R23 ε r23 c (1 + p1 s1)2 Q12 s12 ta2 + 24 m R23 ε r22 c (1 + p1 s1)2 Q12 s12 RI
- 4 r22 β RI4 (1 + p1 s1) Q1 s12 q1 (1 + P1 s1) - 4 r22 β RI4 (1 + p1 s1) Q1 s12 q1 (1 + P1 s1) ta2
+ r2 β RI4 (1 + p1 s1) Q1 s12 R2 q1 (1 + P1 s1)
- 8 r22 β m R24 s5 ta2 (1 + p1 s1) Q1 s12 q1 (1 + P1 s1)
+ 4 r22 β m R24 s3 ta2 (1 + p1 s1) Q1 s12 q1 (1 + P1 s1)
+ r2 β m R25 s5 (1 + p1 s1) Q1 s12 q1 (1 + P1 s1)
- 6 r22 β m R24 s3 (1 + p1 s1) Q1 s12 q1 (1 + P1 s1)
- 6 r22 β m R24 s5 (1 + p1 s1) Q1 s12 q1 (1 + P1 s1)
+ 3 r2 β m R25 s c2 (1 + p1 s1) Q1 s12 q1 (1 + P1 s1)
+ 3 r2 β m R25 s3 c2 (1 + p1 s1) Q1 s12 q1 (1 + P1 s1)
- 12 r24 β m R22 s (1 + p1 s1) Q1 s12 q1 (1 + P1 s1)

```

$$\begin{aligned}
& + 12 m R2^5 \varepsilon t a s^5 (1 + p1 s1) Q1 s1^2 r2 q1 (1 + P1 s1) \\
& - 6 m R2^5 \varepsilon t a s^3 (1 + p1 s1) Q1 s1^2 r2 q1 (1 + P1 s1) \\
& + m R2^4 \varepsilon s^4 c (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) R1^2 \\
& + 2 m R2^5 \varepsilon s^4 c (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) R1 \\
& + 2 m R2^5 \varepsilon s^4 c (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) r2 t a^2 \\
& + 3 m R2^4 \varepsilon c^3 (1 + p1 s1) Q1 s1^2 R1^2 q1 (1 + P1 s1) - 2 r2^2 \beta m R2^4 s^5 t a^2 q1^2 s1^2 (1 + P1 s1)^2 \\
& + r2^2 \beta m R2^4 s^3 t a^2 q1^2 s1^2 (1 + P1 s1)^2 + r2 \beta m R2^5 s^5 q1^2 s1^2 (1 + P1 s1)^2 \\
& + r2 \beta R1^4 q1^2 s1^2 (1 + P1 s1)^2 R2 + m R2^4 \varepsilon s^4 c q1^2 s1^2 (1 + P1 s1)^2 R1^2 \\
& + 2 m R2^5 \varepsilon s^4 c q1^2 s1^2 (1 + P1 s1)^2 R1 + 2 m R2^5 \varepsilon s^4 c q1^2 s1^2 (1 + P1 s1)^2 r2 t a^2 \\
& + m R2^4 \varepsilon s^4 c q1^2 s1^2 (1 + P1 s1)^2 r2 t a^2 R1
\end{aligned}$$

> **RHS3epsilon:=collect(RHS2, epsilon);**

$$\begin{aligned}
RHS3epsilon := & q1 s1 (6 m R2^5 c^3 (1 + p1 s1) Q1 s1^2 R1 q1 (1 + P1 s1) \\
& + 3 m R2^4 c^3 (1 + p1 s1) Q1 s1^2 R1^2 q1 (1 + P1 s1) s^2 \\
& + 6 m R2^5 c^3 (1 + p1 s1) Q1 s1^2 R1 q1 (1 + P1 s1) s^2 \\
& + 6 m R2^5 c^3 (1 + p1 s1) Q1 s1^2 r2 t a^2 q1 (1 + P1 s1) \\
& + 6 m R2^5 c^3 (1 + p1 s1) Q1 s1^2 r2 t a^2 q1 (1 + P1 s1) s^2 \\
& - 12 m R2^2 r2^2 c (1 + p1 s1) Q1 s1^2 R1^2 q1 (1 + P1 s1) \\
& - 24 m R2^3 r2^3 c (1 + p1 s1) Q1 s1^2 t a^2 q1 (1 + P1 s1) \\
& + 6 m R2^4 t a s^5 (1 + p1 s1) Q1 s1^2 r2 q1 (1 + P1 s1) R1 \\
& - 3 m R2^4 t a s^3 (1 + p1 s1) Q1 s1^2 r2 q1 (1 + P1 s1) R1 \\
& + m R2^4 s^4 c (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) r2 t a^2 R1 \\
& + 3 m R2^4 c^3 (1 + p1 s1) Q1 s1^2 r2 t a^2 q1 (1 + P1 s1) R1 + 24 m R2^5 t a s^5 (1 + p1 s1)^2 Q1^2 s1^2 r2 \\
& + 3 m R2^4 c^3 (1 + p1 s1) Q1 s1^2 r2 t a^2 q1 (1 + P1 s1) s^2 R1 \\
& - 12 m R2^2 r2^3 c (1 + p1 s1) Q1 s1^2 t a^2 q1 (1 + P1 s1) R1 \\
& - 24 m R2^3 r2^2 c (1 + p1 s1) Q1 s1^2 R1 q1 (1 + P1 s1) - 12 m R2^5 t a s^3 (1 + p1 s1)^2 Q1^2 s1^2 r2 \\
& - 2 m R2^4 s^4 c (1 + p1 s1)^2 Q1^2 s1^2 R1^2 - 6 m R2^5 c^3 (1 + p1 s1)^2 Q1^2 s1^2 R1 \\
& - 3 m R2^4 c^3 (1 + p1 s1)^2 Q1^2 s1^2 R1^2 s^2 - 6 m R2^5 c^3 (1 + p1 s1)^2 Q1^2 s1^2 R1 s^2 \\
& - 4 m R2^5 s^4 c (1 + p1 s1)^2 Q1^2 s1^2 R1 - 4 m R2^5 s^4 c (1 + p1 s1)^2 Q1^2 s1^2 r2 t a^2 \\
& + 36 m R2^5 (1 + p1 s1)^2 Q1^2 s1^2 s^2 c r2 + 36 m R2^5 (1 + p1 s1)^2 Q1^2 s1^2 s^4 c r2 \\
& - 24 m R2^2 r2^3 c (1 + p1 s1)^2 Q1^2 s1^2 t a^2 R1 - 48 m R2^3 r2^3 c (1 + p1 s1)^2 Q1^2 s1^2 t a^2 \\
& + 24 m R2^3 r2^2 c (1 + p1 s1)^2 Q1^2 s1^2 R1 - 3 m R2^4 c^3 (1 + p1 s1)^2 Q1^2 s1^2 R1^2 \\
& - 6 m R2^5 c^3 (1 + p1 s1)^2 Q1^2 s1^2 r2 t a^2 - 6 m R2^5 c^3 (1 + p1 s1)^2 Q1^2 s1^2 r2 t a^2 s^2 \\
& + 12 m R2^2 r2^2 c (1 + p1 s1)^2 Q1^2 s1^2 R1^2 + 12 m R2^4 t a s^5 (1 + p1 s1)^2 Q1^2 s1^2 r2 R1 \\
& - 6 m R2^4 t a s^3 (1 + p1 s1)^2 Q1^2 s1^2 r2 R1 - 2 m R2^4 s^4 c (1 + p1 s1)^2 Q1^2 s1^2 r2 t a^2 R1 \\
& + 18 m R2^4 (1 + p1 s1)^2 Q1^2 s1^2 s^2 c r2 R1 + 18 m R2^4 (1 + p1 s1)^2 Q1^2 s1^2 s^4 c r2 R1 \\
& - 3 m R2^4 c^3 (1 + p1 s1)^2 Q1^2 s1^2 r2 t a^2 R1 - 3 m R2^4 c^3 (1 + p1 s1)^2 Q1^2 s1^2 r2 t a^2 s^2 R1 \\
& + m R2^4 s^4 c q1^2 s1^2 (1 + P1 s1)^2 R1^2 + 2 m R2^5 s^4 c q1^2 s1^2 (1 + P1 s1)^2 R1 \\
& + 2 m R2^5 s^4 c q1^2 s1^2 (1 + P1 s1)^2 r2 t a^2 + m R2^4 s^4 c q1^2 s1^2 (1 + P1 s1)^2 r2 t a^2 R1 \\
& + 12 m R2^5 t a s^5 (1 + p1 s1) Q1 s1^2 r2 q1 (1 + P1 s1) \\
& - 6 m R2^5 t a s^3 (1 + p1 s1) Q1 s1^2 r2 q1 (1 + P1 s1)
\end{aligned}$$

$$\begin{aligned}
& + m R2^4 s^4 c (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) R1^2 \\
& + 2 m R2^5 s^4 c (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) R1 \\
& + 2 m R2^5 s^4 c (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) r2 ta^2 \\
& + 3 m R2^4 c^3 (1 + p1 s1) Q1 s1^2 R1^2 q1 (1 + P1 s1) \varepsilon + q1 s1 (-r2^2 \beta R1^4 q1^2 s1^2 (1 + P1 s1)^2 ta^2 \\
& - r2^2 \beta R1^4 q1^2 s1^2 (1 + P1 s1)^2 - 4 r2^2 \beta R1^4 (1 + p1 s1)^2 Q1^2 s1^2 ta^2 \\
& - 8 r2^2 \beta m R2^4 s^5 ta^2 (1 + p1 s1)^2 Q1^2 s1^2 + 4 r2^2 \beta m R2^4 s^3 ta^2 (1 + p1 s1)^2 Q1^2 s1^2 \\
& - 2 r2 \beta m R2^5 s^5 (1 + p1 s1)^2 Q1^2 s1^2 - 12 r2^2 \beta m R2^4 s^3 (1 + p1 s1)^2 Q1^2 s1^2 \\
& - 12 r2^2 \beta m R2^4 s^5 (1 + p1 s1)^2 Q1^2 s1^2 - 3 r2 \beta m R2^5 s c^2 (1 + p1 s1)^2 Q1^2 s1^2 \\
& - 3 r2 \beta m R2^5 s^3 c^2 (1 + p1 s1)^2 Q1^2 s1^2 - 24 r2^4 \beta m R2^2 s (1 + p1 s1)^2 Q1^2 s1^2 \\
& - 4 r2^2 \beta R1^4 (1 + p1 s1)^2 Q1^2 s1^2 - 2 r2 \beta R1^4 (1 + p1 s1)^2 Q1^2 s1^2 R2 \\
& - 4 r2^2 \beta R1^4 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) - 4 r2^2 \beta R1^4 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) ta^2 \\
& + r2 \beta R1^4 (1 + p1 s1) Q1 s1^2 R2 q1 (1 + P1 s1) \\
& - 8 r2^2 \beta m R2^4 s^5 ta^2 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) \\
& + 4 r2^2 \beta m R2^4 s^3 ta^2 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) \\
& + r2 \beta m R2^5 s^5 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) \\
& - 6 r2^2 \beta m R2^4 s^3 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) \\
& - 6 r2^2 \beta m R2^4 s^5 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) \\
& + 3 r2 \beta m R2^5 s c^2 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) \\
& + 3 r2 \beta m R2^5 s^3 c^2 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) \\
& - 12 r2^4 \beta m R2^2 s (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) - 2 r2^2 \beta m R2^4 s^5 ta^2 q1^2 s1^2 (1 + P1 s1)^2 \\
& + r2^2 \beta m R2^4 s^3 ta^2 q1^2 s1^2 (1 + P1 s1)^2 + r2 \beta m R2^5 s^5 q1^2 s1^2 (1 + P1 s1)^2 \\
& + r2 \beta R1^4 q1^2 s1^2 (1 + P1 s1)^2 R2
\end{aligned}$$

```

> RHS3beta:=collect(RHS2,beta);
RHS3beta:=q1 s1 (-r2^2 R1^4 q1^2 s1^2 (1 + P1 s1)^2 ta^2 - r2^2 R1^4 q1^2 s1^2 (1 + P1 s1)^2
- 4 r2^2 R1^4 (1 + p1 s1)^2 Q1^2 s1^2 ta^2 - 8 r2^2 m R2^4 s^5 ta^2 (1 + p1 s1)^2 Q1^2 s1^2
+ 4 r2^2 m R2^4 s^3 ta^2 (1 + p1 s1)^2 Q1^2 s1^2 - 4 r2^2 R1^4 (1 + p1 s1)^2 Q1^2 s1^2
- 24 r2^4 m R2^2 s (1 + p1 s1)^2 Q1^2 s1^2 - 2 r2 R1^4 (1 + p1 s1)^2 Q1^2 s1^2 R2
- 2 r2 m R2^5 s^5 (1 + p1 s1)^2 Q1^2 s1^2 - 12 r2^2 m R2^4 s^3 (1 + p1 s1)^2 Q1^2 s1^2
- 12 r2^2 m R2^4 s^5 (1 + p1 s1)^2 Q1^2 s1^2 - 3 r2 m R2^5 s c^2 (1 + p1 s1)^2 Q1^2 s1^2
- 3 r2 m R2^5 s^3 c^2 (1 + p1 s1)^2 Q1^2 s1^2 - 4 r2^2 R1^4 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1)
- 4 r2^2 R1^4 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) ta^2 + r2 R1^4 (1 + p1 s1) Q1 s1^2 R2 q1 (1 + P1 s1)
- 8 r2^2 m R2^4 s^5 ta^2 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1)
+ 4 r2^2 m R2^4 s^3 ta^2 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1)
+ r2 m R2^5 s^5 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) - 6 r2^2 m R2^4 s^3 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1)
- 2 r2^2 m R2^4 s^5 ta^2 q1^2 s1^2 (1 + P1 s1)^2 + r2^2 m R2^4 s^3 ta^2 q1^2 s1^2 (1 + P1 s1)^2
+ r2 m R2^5 s^5 q1^2 s1^2 (1 + P1 s1)^2 + r2 R1^4 q1^2 s1^2 (1 + P1 s1)^2 R2
- 6 r2^2 m R2^4 s^5 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1)
+ 3 r2 m R2^5 s c^2 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1)
+ 3 r2 m R2^5 s^3 c^2 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1)
- 12 r2^4 m R2^2 s (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1)) \beta + q1 s1 (6 m R2^5 \varepsilon c^3 (1 + p1 s1) Q1 s1^2 R1 q1

```

$$\begin{aligned}
& (1 + P1 sI) + 3 m R2^4 \varepsilon c^3 (1 + p1 sI) Q1 sI^2 R1^2 qI (1 + P1 sI) s^2 \\
& + 6 m R2^5 \varepsilon c^3 (1 + p1 sI) Q1 sI^2 R1 qI (1 + P1 sI) s^2 \\
& + 6 m R2^5 \varepsilon c^3 (1 + p1 sI) Q1 sI^2 r2 ta^2 qI (1 + P1 sI) \\
& + 6 m R2^5 \varepsilon c^3 (1 + p1 sI) Q1 sI^2 r2 ta^2 qI (1 + P1 sI) s^2 \\
& - 12 m R2^2 \varepsilon r2^2 c (1 + p1 sI) Q1 sI^2 R1^2 qI (1 + P1 sI) \\
& + 6 m R2^4 \varepsilon ta s^5 (1 + p1 sI) Q1 sI^2 r2 qI (1 + P1 sI) R1 \\
& - 3 m R2^4 \varepsilon ta s^3 (1 + p1 sI) Q1 sI^2 r2 qI (1 + P1 sI) R1 \\
& + m R2^4 \varepsilon s^4 c (1 + p1 sI) Q1 sI^2 qI (1 + P1 sI) r2 ta^2 R1 \\
& + 3 m R2^4 \varepsilon c^3 (1 + p1 sI) Q1 sI^2 r2 ta^2 qI (1 + P1 sI) R1 \\
& + 3 m R2^4 \varepsilon c^3 (1 + p1 sI) Q1 sI^2 r2 ta^2 qI (1 + P1 sI) s^2 R1 \\
& - 12 m R2^2 \varepsilon r2^3 c (1 + p1 sI) Q1 sI^2 ta^2 qI (1 + P1 sI) R1 \\
& - 24 m R2^3 \varepsilon r2^3 c (1 + p1 sI) Q1 sI^2 ta^2 qI (1 + P1 sI) \\
& - 24 m R2^3 \varepsilon r2^2 c (1 + p1 sI) Q1 sI^2 R1 qI (1 + P1 sI) + 24 m R2^5 \varepsilon ta s^5 (1 + p1 sI)^2 Q1^2 sI^2 r2 \\
& - 12 m R2^5 \varepsilon ta s^3 (1 + p1 sI)^2 Q1^2 sI^2 r2 - 2 m R2^4 \varepsilon s^4 c (1 + p1 sI)^2 Q1^2 sI^2 R1^2 \\
& - 4 m R2^5 \varepsilon s^4 c (1 + p1 sI)^2 Q1^2 sI^2 R1 - 4 m R2^5 \varepsilon s^4 c (1 + p1 sI)^2 Q1^2 sI^2 r2 ta^2 \\
& + 36 m R2^5 \varepsilon (1 + p1 sI)^2 Q1^2 sI^2 s^2 c r2 + 36 m R2^5 \varepsilon (1 + p1 sI)^2 Q1^2 sI^2 s^4 c r2 \\
& - 3 m R2^4 \varepsilon c^3 (1 + p1 sI)^2 Q1^2 sI^2 R1^2 - 6 m R2^5 \varepsilon c^3 (1 + p1 sI)^2 Q1^2 sI^2 R1 \\
& - 3 m R2^4 \varepsilon c^3 (1 + p1 sI)^2 Q1^2 sI^2 R1^2 s^2 - 6 m R2^5 \varepsilon c^3 (1 + p1 sI)^2 Q1^2 sI^2 R1 s^2 \\
& - 6 m R2^5 \varepsilon c^3 (1 + p1 sI)^2 Q1^2 sI^2 r2 ta^2 - 6 m R2^5 \varepsilon c^3 (1 + p1 sI)^2 Q1^2 sI^2 r2 ta^2 s^2 \\
& + 12 m R2^2 \varepsilon r2^2 c (1 + p1 sI)^2 Q1^2 sI^2 R1^2 + 12 m R2^4 \varepsilon ta s^5 (1 + p1 sI)^2 Q1^2 sI^2 r2 R1 \\
& - 6 m R2^4 \varepsilon ta s^3 (1 + p1 sI)^2 Q1^2 sI^2 r2 R1 - 2 m R2^4 \varepsilon s^4 c (1 + p1 sI)^2 Q1^2 sI^2 r2 ta^2 R1 \\
& + 18 m R2^4 \varepsilon (1 + p1 sI)^2 Q1^2 sI^2 s^2 c r2 R1 + 18 m R2^4 \varepsilon (1 + p1 sI)^2 Q1^2 sI^2 s^4 c r2 R1 \\
& - 3 m R2^4 \varepsilon c^3 (1 + p1 sI)^2 Q1^2 sI^2 r2 ta^2 R1 - 3 m R2^4 \varepsilon c^3 (1 + p1 sI)^2 Q1^2 sI^2 r2 ta^2 s^2 R1 \\
& - 24 m R2^2 \varepsilon r2^3 c (1 + p1 sI)^2 Q1^2 sI^2 ta^2 R1 - 48 m R2^3 \varepsilon r2^3 c (1 + p1 sI)^2 Q1^2 sI^2 ta^2 \\
& + 24 m R2^3 \varepsilon r2^2 c (1 + p1 sI)^2 Q1^2 sI^2 R1 + 12 m R2^5 \varepsilon ta s^5 (1 + p1 sI) Q1 sI^2 r2 qI (1 + P1 sI) \\
& - 6 m R2^5 \varepsilon ta s^3 (1 + p1 sI) Q1 sI^2 r2 qI (1 + P1 sI) \\
& + m R2^4 \varepsilon s^4 c (1 + p1 sI) Q1 sI^2 qI (1 + P1 sI) R1^2 \\
& + 2 m R2^5 \varepsilon s^4 c (1 + p1 sI) Q1 sI^2 qI (1 + P1 sI) R1 \\
& + 2 m R2^5 \varepsilon s^4 c (1 + p1 sI) Q1 sI^2 qI (1 + P1 sI) r2 ta^2 \\
& + 3 m R2^4 \varepsilon c^3 (1 + p1 sI) Q1 sI^2 R1^2 qI (1 + P1 sI) + m R2^4 \varepsilon s^4 c qI^2 sI^2 (1 + P1 sI)^2 R1^2 \\
& + 2 m R2^5 \varepsilon s^4 c qI^2 sI^2 (1 + P1 sI)^2 R1 + 2 m R2^5 \varepsilon s^4 c qI^2 sI^2 (1 + P1 sI)^2 r2 ta^2 \\
& + m R2^4 \varepsilon s^4 c qI^2 sI^2 (1 + P1 sI)^2 r2 ta^2 R1)
\end{aligned}$$

```

> beta1:=coeff(RHS3beta,beta,1);
beta1 := q1 sI (-r2^2 R1^4 qI^2 sI^2 (1 + P1 sI)^2 ta^2 - r2^2 R1^4 qI^2 sI^2 (1 + P1 sI)^2
- 4 r2^2 R1^4 (1 + p1 sI)^2 Q1^2 sI^2 ta^2 - 8 r2^2 m R2^4 s^5 ta^2 (1 + p1 sI)^2 Q1^2 sI^2
+ 4 r2^2 m R2^4 s^3 ta^2 (1 + p1 sI)^2 Q1^2 sI^2 - 4 r2^2 R1^4 (1 + p1 sI)^2 Q1^2 sI^2
- 24 r2^4 m R2^2 s (1 + p1 sI)^2 Q1^2 sI^2 - 2 r2 R1^4 (1 + p1 sI)^2 Q1^2 sI^2 R2
- 2 r2 m R2^5 s^5 (1 + p1 sI)^2 Q1^2 sI^2 - 12 r2^2 m R2^4 s^3 (1 + p1 sI)^2 Q1^2 sI^2
- 12 r2^2 m R2^4 s^5 (1 + p1 sI)^2 Q1^2 sI^2 - 3 r2 m R2^5 s c^2 (1 + p1 sI)^2 Q1^2 sI^2
- 3 r2 m R2^5 s^3 c^2 (1 + p1 sI)^2 Q1^2 sI^2 - 4 r2^2 R1^4 (1 + p1 sI) Q1 sI^2 qI (1 + P1 sI)
- 4 r2^2 R1^4 (1 + p1 sI) Q1 sI^2 qI (1 + P1 sI) ta^2 + r2 R1^4 (1 + p1 sI) Q1 sI^2 R2 qI (1 + P1 sI)
- 8 r2^2 m R2^4 s^5 ta^2 (1 + p1 sI) Q1 sI^2 qI (1 + P1 sI)

```

$$\begin{aligned}
& + 4 r2^2 m R2^4 s^3 ta^2 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) \\
& + r2 m R2^5 s^5 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) - 6 r2^2 m R2^4 s^3 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) \\
& - 2 r2^2 m R2^4 s^5 ta^2 q1^2 s1^2 (1 + P1 s1)^2 + r2^2 m R2^4 s^3 ta^2 q1^2 s1^2 (1 + P1 s1)^2 \\
& + r2 m R2^5 s^5 q1^2 s1^2 (1 + P1 s1)^2 + r2 R1^4 q1^2 s1^2 (1 + P1 s1)^2 R2 \\
& - 6 r2^2 m R2^4 s^5 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) \\
& + 3 r2 m R2^5 s c^2 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) \\
& + 3 r2 m R2^5 s^3 c^2 (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1) \\
& - 12 r2^4 m R2^2 s (1 + p1 s1) Q1 s1^2 q1 (1 + P1 s1)
\end{aligned}$$

```

> beta2:=collect(beta1,s1);
β2 := q1 (-3 r2 m R2^5 s^3 c^2 p1^2 Q1^2 - 4 r2^2 R1^4 p1 Q1 q1 P1 ta^2 + r2 R1^4 p1 Q1 R2 q1 P1
+ r2 R1^4 q1^2 P1^2 R2 - 8 r2^2 m R2^4 s^5 ta^2 p1 Q1 q1 P1 + 4 r2^2 m R2^4 s^3 ta^2 p1 Q1 q1 P1
+ r2 m R2^5 s^5 p1 Q1 q1 P1 - 6 r2^2 m R2^4 s^3 p1 Q1 q1 P1 - 2 r2^2 m R2^4 s^5 ta^2 q1^2 P1^2
+ r2^2 m R2^4 s^3 ta^2 q1^2 P1^2 + r2 m R2^5 s^5 q1^2 P1^2 - 6 r2^2 m R2^4 s^5 p1 Q1 q1 P1
+ 3 r2 m R2^5 s c^2 p1 Q1 q1 P1 + 3 r2 m R2^5 s^3 c^2 p1 Q1 q1 P1 - 12 r2^4 m R2^2 s p1 Q1 q1 P1
- 4 r2^2 R1^4 p1^2 Q1^2 - r2^2 R1^4 q1^2 P1^2 ta^2 - 4 r2^2 R1^4 p1^2 Q1^2 ta^2
- 8 r2^2 m R2^4 s^5 ta^2 p1^2 Q1^2 + 4 r2^2 m R2^4 s^3 ta^2 p1^2 Q1^2 - 24 r2^4 m R2^2 s p1^2 Q1^2
- 2 r2 R1^4 p1^2 Q1^2 R2 - 2 r2 m R2^5 s^5 p1^2 Q1^2 - 12 r2^2 m R2^4 s^3 p1^2 Q1^2
- 12 r2^2 m R2^4 s^5 p1^2 Q1^2 - 3 r2 m R2^5 s c^2 p1^2 Q1^2 - r2^2 R1^4 q1^2 P1^2 - 4 r2^2 R1^4 p1 Q1 q1 P1
s1^5 + q1 (-6 r2 m R2^5 s^3 c^2 p1 Q1^2 - 4 r2^2 R1^4 p1 Q1 q1
- 4 (r2^2 R1^4 Q1 q1 P1 + r2^2 R1^4 p1 Q1 q1) ta^2 - 4 r2^2 R1^4 Q1 q1 P1 + r2 R1^4 p1 Q1 R2 q1
+ r2 R1^4 Q1 R2 q1 P1 + 2 r2 R1^4 q1^2 P1 R2 - 8 r2^2 m R2^4 s^5 ta^2 p1 Q1 q1
- 8 r2^2 m R2^4 s^5 ta^2 Q1 q1 P1 + 4 r2^2 m R2^4 s^3 ta^2 p1 Q1 q1 + 4 r2^2 m R2^4 s^3 ta^2 Q1 q1 P1
+ r2 m R2^5 s^5 p1 Q1 q1 + r2 m R2^5 s^5 Q1 q1 P1 - 6 r2^2 m R2^4 s^3 p1 Q1 q1
- 6 r2^2 m R2^4 s^3 Q1 q1 P1 - 4 r2^2 m R2^4 s^5 ta^2 q1^2 P1 + 2 r2^2 m R2^4 s^3 ta^2 q1^2 P1
+ 2 r2 m R2^5 s^5 q1^2 P1 - 6 r2^2 m R2^4 s^5 p1 Q1 q1 - 6 r2^2 m R2^4 s^5 Q1 q1 P1
+ 3 r2 m R2^5 s c^2 p1 Q1 q1 + 3 r2 m R2^5 s c^2 Q1 q1 P1 + 3 r2 m R2^5 s^3 c^2 p1 Q1 q1
+ 3 r2 m R2^5 s^3 c^2 Q1 q1 P1 - 12 r2^4 m R2^2 s p1 Q1 q1 - 12 r2^4 m R2^2 s Q1 q1 P1
- 2 r2^2 R1^4 q1^2 P1 - 8 r2^2 R1^4 p1 Q1^2 - 2 r2^2 R1^4 q1^2 P1 ta^2 - 8 r2^2 R1^4 p1 Q1^2 ta^2
- 16 r2^2 m R2^4 s^5 ta^2 p1 Q1^2 + 8 r2^2 m R2^4 s^3 ta^2 p1 Q1^2 - 48 r2^4 m R2^2 s p1 Q1^2
- 4 r2 R1^4 p1 Q1^2 R2 - 4 r2 m R2^5 s^5 p1 Q1^2 - 24 r2^2 m R2^4 s^3 p1 Q1^2 - 6 r2 m R2^5 s c^2 p1 Q1^2
- 24 r2^2 m R2^4 s^5 p1 Q1^2) s1^4 + q1 (-4 r2^2 R1^4 Q1^2 - 3 r2 m R2^5 s^3 c^2 Q1^2
- 4 r2^2 R1^4 Q1 q1 ta^2 + r2 R1^4 Q1 R2 q1 + 4 r2^2 m R2^4 s^3 ta^2 Q1 q1 + r2 m R2^5 s^5 Q1 q1
- 6 r2^2 m R2^4 s^3 Q1 q1 - 2 r2^2 m R2^4 s^5 ta^2 q1^2 + r2^2 m R2^4 s^3 ta^2 q1^2 + r2 m R2^5 s^5 q1^2
- 6 r2^2 m R2^4 s^5 Q1 q1 + 3 r2 m R2^5 s c^2 Q1 q1 + 3 r2 m R2^5 s^3 c^2 Q1 q1 - 12 r2^4 m R2^2 s Q1 q1
- r2^2 R1^4 q1^2 ta^2 - 4 r2^2 R1^4 Q1^2 ta^2 - 2 r2 R1^4 Q1^2 R2 - 4 r2^2 R1^4 Q1 q1 + r2 R1^4 q1^2 R2
- 8 r2^2 m R2^4 s^5 ta^2 Q1^2 + 4 r2^2 m R2^4 s^3 ta^2 Q1^2 - 24 r2^4 m R2^2 s Q1^2 - 2 r2 m R2^5 s^5 Q1^2
- 12 r2^2 m R2^4 s^5 Q1^2 - 3 r2 m R2^5 s c^2 Q1^2 - 8 r2^2 m R2^4 s^5 ta^2 Q1 q1 - r2^2 R1^4 q1^2
- 12 r2^2 m R2^4 s^3 Q1^2) s1^3

```

```

> epsilon1:=coeff(RHS3epsilon,epsilon,1);
ε1 := q1 s1 (6 m R2^5 c^3 (1 + p1 s1) Q1 s1^2 R1 q1 (1 + P1 s1)

```

$$\begin{aligned}
& + 3mR2^4c^3(1+p1sl)Q1sl^2RI^2q1(1+P1sl)s^2 \\
& + 6mR2^5c^3(1+p1sl)Q1sl^2RIq1(1+P1sl)s^2 \\
& + 6mR2^5c^3(1+p1sl)Q1sl^2r2ta^2q1(1+P1sl) \\
& + 6mR2^5c^3(1+p1sl)Q1sl^2r2ta^2q1(1+P1sl)s^2 \\
& - 12mR2^2r2^2c(1+p1sl)Q1sl^2RI^2q1(1+P1sl) \\
& - 24mR2^3r2^3c(1+p1sl)Q1sl^2ta^2q1(1+P1sl) \\
& + 6mR2^4tas^5(1+p1sl)Q1sl^2r2q1(1+P1sl)RI \\
& - 3mR2^4tas^3(1+p1sl)Q1sl^2r2q1(1+P1sl)RI \\
& + mR2^4s^4c(1+p1sl)Q1sl^2q1(1+P1sl)r2ta^2RI \\
& + 3mR2^4c^3(1+p1sl)Q1sl^2r2ta^2q1(1+P1sl)RI + 24mR2^5tas^5(1+p1sl)^2Q1^2sl^2r2 \\
& + 3mR2^4c^3(1+p1sl)Q1sl^2r2ta^2q1(1+P1sl)s^2RI \\
& - 12mR2^2r2^3c(1+p1sl)Q1sl^2ta^2q1(1+P1sl)RI \\
& - 24mR2^3r2^2c(1+p1sl)Q1sl^2RIq1(1+P1sl) - 12mR2^5tas^3(1+p1sl)^2Q1^2sl^2r2 \\
& - 2mR2^4s^4c(1+p1sl)^2Q1^2sl^2RI^2 - 6mR2^5c^3(1+p1sl)^2Q1^2sl^2RI \\
& - 3mR2^4c^3(1+p1sl)^2Q1^2sl^2RI^2s^2 - 6mR2^5c^3(1+p1sl)^2Q1^2sl^2RI s^2 \\
& - 4mR2^5s^4c(1+p1sl)^2Q1^2sl^2RI - 4mR2^5s^4c(1+p1sl)^2Q1^2sl^2r2ta^2 \\
& + 36mR2^5(1+p1sl)^2Q1^2sl^2s^2cr2 + 36mR2^5(1+p1sl)^2Q1^2sl^2s^4cr2 \\
& - 24mR2^2r2^3c(1+p1sl)^2Q1^2sl^2ta^2RI - 48mR2^3r2^3c(1+p1sl)^2Q1^2sl^2ta^2 \\
& + 24mR2^3r2^2c(1+p1sl)^2Q1^2sl^2RI - 3mR2^4c^3(1+p1sl)^2Q1^2sl^2RI^2 \\
& - 6mR2^5c^3(1+p1sl)^2Q1^2sl^2r2ta^2 - 6mR2^5c^3(1+p1sl)^2Q1^2sl^2r2ta^2s^2 \\
& + 12mR2^2r2^2c(1+p1sl)^2Q1^2sl^2RI^2 + 12mR2^4tas^5(1+p1sl)^2Q1^2sl^2r2RI \\
& - 6mR2^4tas^3(1+p1sl)^2Q1^2sl^2r2RI - 2mR2^4s^4c(1+p1sl)^2Q1^2sl^2r2ta^2RI \\
& + 18mR2^4(1+p1sl)^2Q1^2sl^2s^2cr2RI + 18mR2^4(1+p1sl)^2Q1^2sl^2s^4cr2RI \\
& - 3mR2^4c^3(1+p1sl)^2Q1^2sl^2r2ta^2RI - 3mR2^4c^3(1+p1sl)^2Q1^2sl^2r2ta^2s^2RI \\
& + mR2^4s^4cq1^2sl^2(1+P1sl)^2RI^2 + 2mR2^5s^4cq1^2sl^2(1+P1sl)^2RI \\
& + 2mR2^5s^4cq1^2sl^2(1+P1sl)^2r2ta^2 + mR2^4s^4cq1^2sl^2(1+P1sl)^2r2ta^2RI \\
& + 12mR2^5tas^5(1+p1sl)Q1sl^2r2q1(1+P1sl) \\
& - 6mR2^5tas^3(1+p1sl)Q1sl^2r2q1(1+P1sl) \\
& + mR2^4s^4c(1+p1sl)Q1sl^2q1(1+P1sl)RI^2 \\
& + 2mR2^5s^4c(1+p1sl)Q1sl^2q1(1+P1sl)RI \\
& + 2mR2^5s^4c(1+p1sl)Q1sl^2q1(1+P1sl)r2ta^2 \\
& + 3mR2^4c^3(1+p1sl)Q1sl^2RI^2q1(1+P1sl))
\end{aligned}$$

> epsilon2:=collect(epsilon1, sl);

$$\begin{aligned}
\epsilon2 := & q1(-4mR2^5s^4cp1^2Q1^2RI + 36mR2^5s^4cp1^2Q1^2r2 - 4mR2^5s^4cp1^2Q1^2r2ta^2 \\
& + 36mR2^5p1^2Q1^2s^2cr2 - 24mR2^2r2^3cp1^2Q1^2ta^2RI - 48mR2^3r2^3cp1^2Q1^2ta^2 \\
& + 24mR2^3r2^2cp1^2Q1^2RI - 6mR2^5c^3p1^2Q1^2r2ta^2 - 6mR2^5c^3p1^2Q1^2r2ta^2s^2 \\
& + 12mR2^2r2^2cp1^2Q1^2RI^2 + 12mR2^4tas^5p1^2Q1^2r2RI + 2mR2^5s^4cp1Q1q1P1RI \\
& - 6mR2^4tas^3p1^2Q1^2r2RI - 2mR2^4s^4cp1^2Q1^2r2ta^2RI + 18mR2^4p1^2Q1^2s^2cr2RI \\
& + 6mR2^5c^3p1Q1RIq1P1 + 3mR2^4c^3p1Q1RI^2q1P1 + 3mR2^4c^3p1Q1RI^2q1P1s^2 \\
& + 6mR2^5c^3p1Q1RIq1P1s^2 + 6mR2^5c^3p1Q1r2ta^2q1P1 \\
& + 6mR2^5c^3p1Q1r2ta^2q1P1s^2 - 12mR2^2r2^2cp1Q1RI^2q1P1 \\
& - 24mR2^3r2^3cp1Q1ta^2q1P1 + 6mR2^4tas^5p1Q1r2q1P1RI
\end{aligned}$$

$$\begin{aligned}
& -3mR2^4 tas^3 p1 Q1 r2 q1 P1 R1 + mR2^4 s^4 cp1 Q1 q1 P1 r2 ta^2 R1 \\
& + 3mR2^4 c^3 p1 Q1 r2 ta^2 q1 P1 R1 + 24mR2^5 tas^5 p1^2 Q1^2 r2 \\
& + 3mR2^4 c^3 p1 Q1 r2 ta^2 q1 P1 s^2 R1 - 12mR2^2 r2^3 cp1 Q1 ta^2 q1 P1 R1 \\
& - 24mR2^3 r2^2 cp1 Q1 R1 q1 P1 - 12mR2^5 tas^3 p1^2 Q1^2 r2 - 2mR2^4 s^4 cp1^2 Q1^2 R1^2 \\
& - 6mR2^5 c^3 p1^2 Q1^2 R1 - 3mR2^4 c^3 p1^2 Q1^2 R1^2 - 3mR2^4 c^3 p1^2 Q1^2 R1^2 s^2 \\
& - 6mR2^5 c^3 p1^2 Q1^2 R1 s^2 + 18mR2^4 s^4 cp1^2 Q1^2 r2 R1 - 3mR2^4 c^3 p1^2 Q1^2 r2 ta^2 R1 \\
& - 3mR2^4 c^3 p1^2 Q1^2 r2 ta^2 s^2 R1 + mR2^4 s^4 c q1^2 P1^2 R1^2 + 2mR2^5 s^4 c q1^2 P1^2 R1 \\
& + 2mR2^5 s^4 c q1^2 P1^2 r2 ta^2 + mR2^4 s^4 c q1^2 P1^2 r2 ta^2 R1 + 12mR2^5 tas^5 p1 Q1 r2 q1 P1 \\
& - 6mR2^5 tas^3 p1 Q1 r2 q1 P1 + mR2^4 s^4 cp1 Q1 q1 P1 R1^2 + 2mR2^5 s^4 cp1 Q1 q1 P1 r2 ta^2 \\
& s1^5 + q1 (3(mR2^4 c^3 Q1 r2 ta^2 q1 P1 + mR2^4 c^3 p1 Q1 r2 ta^2 q1) s^2 R1 - 8mR2^5 s^4 cp1 Q1^2 R1 \\
& + 72mR2^5 s^4 cp1 Q1^2 r2 - 8mR2^5 s^4 cp1 Q1^2 r2 ta^2 + 72mR2^5 p1 Q1^2 s^2 cr2 \\
& - 48mR2^2 r2^3 cp1 Q1^2 ta^2 R1 - 96mR2^3 r2^3 cp1 Q1^2 ta^2 + 48mR2^3 r2^2 cp1 Q1^2 R1 \\
& - 12mR2^5 c^3 p1 Q1^2 r2 ta^2 - 12mR2^5 c^3 p1 Q1^2 r2 ta^2 s^2 + 24mR2^2 r2^2 cp1 Q1^2 R1^2 \\
& + 24mR2^4 tas^5 p1 Q1^2 r2 R1 - 12mR2^4 tas^3 p1 Q1^2 r2 R1 - 4mR2^4 s^4 cp1 Q1^2 r2 ta^2 R1 \\
& + 36mR2^4 p1 Q1^2 s^2 cr2 R1 + 3(mR2^4 c^3 Q1 R1^2 q1 P1 + mR2^4 c^3 p1 Q1 R1^2 q1) s^2 \\
& + 6(mR2^5 c^3 Q1 R1 q1 P1 + mR2^5 c^3 p1 Q1 R1 q1) s^2 \\
& + 6(mR2^5 c^3 Q1 r2 ta^2 q1 P1 + mR2^5 c^3 p1 Q1 r2 ta^2 q1) s^2 + 6mR2^5 c^3 p1 Q1 R1 q1 \\
& + 6mR2^5 c^3 Q1 R1 q1 P1 + 3mR2^4 c^3 p1 Q1 R1^2 q1 + 3mR2^4 c^3 Q1 R1^2 q1 P1 \\
& + 6mR2^5 c^3 p1 Q1 r2 ta^2 q1 + 6mR2^5 c^3 Q1 r2 ta^2 q1 P1 - 12mR2^2 r2^2 cp1 Q1 R1^2 q1 \\
& - 12mR2^2 r2^2 c Q1 R1^2 q1 P1 + 6(mR2^4 tas^5 Q1 r2 q1 P1 + mR2^4 tas^5 p1 Q1 r2 q1) R1 \\
& - 3(mR2^4 tas^3 Q1 r2 q1 P1 + mR2^4 tas^3 p1 Q1 r2 q1) R1 - 24mR2^3 r2^3 cp1 Q1 ta^2 q1 \\
& - 24mR2^3 r2^3 c Q1 ta^2 q1 P1 + 3(mR2^4 c^3 Q1 r2 ta^2 q1 P1 + mR2^4 c^3 p1 Q1 r2 ta^2 q1) R1 \\
& - 12(mR2^2 r2^3 c Q1 ta^2 q1 P1 + mR2^2 r2^3 cp1 Q1 ta^2 q1) R1 + 48mR2^5 tas^5 p1 Q1^2 r2 \\
& - 24mR2^3 r2^2 cp1 Q1 R1 q1 - 24mR2^3 r2^2 c Q1 R1 q1 P1 - 24mR2^5 tas^3 p1 Q1^2 r2 \\
& - 4mR2^4 s^4 cp1 Q1^2 R1^2 - 12mR2^5 c^3 p1 Q1^2 R1 - 6mR2^4 c^3 p1 Q1^2 R1^2 \\
& - 6mR2^4 c^3 p1 Q1^2 R1^2 s^2 - 12mR2^5 c^3 p1 Q1^2 R1 s^2 + 36mR2^4 s^4 cp1 Q1^2 r2 R1 \\
& - 6mR2^4 c^3 p1 Q1^2 r2 ta^2 R1 - 6mR2^4 c^3 p1 Q1^2 r2 ta^2 s^2 R1 + 2mR2^4 s^4 c q1^2 P1 R1^2 \\
& + 4mR2^5 s^4 c q1^2 P1 R1 + 4mR2^5 s^4 c q1^2 P1 r2 ta^2 + 2mR2^4 s^4 c q1^2 P1 r2 ta^2 R1 \\
& + (mR2^4 s^4 c Q1 q1 P1 + mR2^4 s^4 cp1 Q1 q1) R1^2 \\
& + 2(mR2^5 s^4 c Q1 q1 P1 + mR2^5 s^4 cp1 Q1 q1) R1 + 12mR2^5 tas^5 p1 Q1 r2 q1 \\
& + 12mR2^5 tas^5 Q1 r2 q1 P1 - 6mR2^5 tas^3 p1 Q1 r2 q1 - 6mR2^5 tas^3 Q1 r2 q1 P1 \\
& + (mR2^4 s^4 c Q1 q1 P1 + mR2^4 s^4 cp1 Q1 q1) r2 ta^2 R1 \\
& + 2(mR2^5 s^4 c Q1 q1 P1 + mR2^5 s^4 cp1 Q1 q1) r2 ta^2 s1^4 + q1 (-4mR2^5 s^4 c Q1^2 R1 \\
& + 36mR2^5 s^4 c Q1^2 r2 - 4mR2^5 s^4 c Q1^2 r2 ta^2 + 36mR2^5 Q1^2 s^2 cr2 \\
& - 24mR2^2 r2^3 c Q1^2 ta^2 R1 - 48mR2^3 r2^3 c Q1^2 ta^2 + 24mR2^3 r2^2 c Q1^2 R1 \\
& - 6mR2^5 c^3 Q1^2 r2 ta^2 - 6mR2^5 c^3 Q1^2 r2 ta^2 s^2 + 12mR2^2 r2^2 c Q1^2 R1^2 \\
& + 12mR2^4 tas^5 Q1^2 r2 R1 + 2mR2^5 s^4 c Q1 q1 R1 - 6mR2^4 tas^3 Q1^2 r2 R1 \\
& - 2mR2^4 s^4 c Q1^2 r2 ta^2 R1 + 18mR2^4 Q1^2 s^2 cr2 R1 + 6mR2^5 c^3 Q1 R1 q1 \\
& + 3mR2^4 c^3 Q1 R1^2 q1 + 3mR2^4 c^3 Q1 R1^2 q1 s^2 + 6mR2^5 c^3 Q1 R1 q1 s^2 \\
& + 6mR2^5 c^3 Q1 r2 ta^2 q1 + 6mR2^5 c^3 Q1 r2 ta^2 q1 s^2 - 12mR2^2 r2^2 c Q1 R1^2 q1 \\
& - 24mR2^3 r2^3 c Q1 ta^2 q1 + 6mR2^4 tas^5 Q1 r2 q1 R1 - 3mR2^4 tas^3 Q1 r2 q1 R1 \\
& + mR2^4 s^4 c Q1 q1 r2 ta^2 R1 + 3mR2^4 c^3 Q1 r2 ta^2 q1 R1 + 24mR2^5 tas^5 Q1^2 r2
\end{aligned}$$

$$\begin{aligned}
& + 3mR2^4c^3Q1r2ta^2q1s^2R1 - 12mR2^2r2^3cQ1ta^2q1R1 - 24mR2^3r2^2cQ1R1q1 \\
& - 12mR2^5tas^3Q1^2r2 - 2mR2^4s^4cQ1^2R1^2 - 3mR2^4c^3Q1^2R1^2 - 3mR2^4c^3Q1^2R1^2s^2 \\
& - 6mR2^5c^3Q1^2R1s^2 + 18mR2^4s^4cQ1^2r2R1 - 3mR2^4c^3Q1^2r2ta^2R1 \\
& - 3mR2^4c^3Q1^2r2ta^2s^2R1 + mR2^4s^4cq1^2R1^2 + 2mR2^5s^4cq1^2R1 \\
& + 2mR2^5s^4cq1^2r2ta^2 + mR2^4s^4cq1^2r2ta^2R1 + 12mR2^5tas^5Q1r2q1 \\
& - 6mR2^5tas^3Q1r2q1 + mR2^4s^4cQ1q1R1^2 + 2mR2^5s^4cQ1q1r2ta^2 - 6mR2^5c^3Q1^2R1 \\
& )sl^3
\end{aligned}$$

```

> D5:=coeff(LHS3,s1^5);
D5 := 2 r2 p1 (-2 r2 p1 Q1 - r2 q1 P1 - 2 r2 ta^2 p1 Q1 - r2 ta^2 q1 P1 - R2 p1 Q1 + R2 q1 P1) (R1 + 2 R2)^4 (2
p1 Q1 + q1 P1)

> D4:=coeff(LHS3,s1^4);
D4 := 2 (r2 (-2 r2 p1 Q1 - r2 q1 P1 - 2 r2 ta^2 p1 Q1 - r2 ta^2 q1 P1 - R2 p1 Q1 + R2 q1 P1)
+ r2 p1 (-2 r2 Q1 - r2 q1 - 2 r2 ta^2 Q1 - r2 ta^2 q1 - R2 Q1 + R2 q1)) (R1 + 2 R2)^4 (2 p1 Q1 + q1 P1) +
2 r2 p1 (-2 r2 p1 Q1 - r2 q1 P1 - 2 r2 ta^2 p1 Q1 - r2 ta^2 q1 P1 - R2 p1 Q1 + R2 q1 P1) (R1 + 2 R2)^4 (2
Q1 + q1)

> D3:=coeff(LHS3,s1^3);
D3 := 2 r2 (-2 r2 Q1 - r2 q1 - 2 r2 ta^2 Q1 - r2 ta^2 q1 - R2 Q1 + R2 q1) (R1 + 2 R2)^4 (2 p1 Q1 + q1 P1) + 2 (r2
(-2 r2 p1 Q1 - r2 q1 P1 - 2 r2 ta^2 p1 Q1 - r2 ta^2 q1 P1 - R2 p1 Q1 + R2 q1 P1)
+ r2 p1 (-2 r2 Q1 - r2 q1 - 2 r2 ta^2 Q1 - r2 ta^2 q1 - R2 Q1 + R2 q1)) (R1 + 2 R2)^4 (2 Q1 + q1)

> D2:=coeff(LHS3,s1^2);
D2 := 2 r2 (-2 r2 Q1 - r2 q1 - 2 r2 ta^2 Q1 - r2 ta^2 q1 - R2 Q1 + R2 q1) (R1 + 2 R2)^4 (2 Q1 + q1)

> D1:=coeff(LHS3,s1);
D1 := 0

> D0:=coeff(LHS3,s1,0);
D0 := 0

> E5:=coeff(epsilon2,s1^5);
E5 := q1 (-4 m R2^5 s^4 c p1^2 Q1^2 R1 + 36 m R2^5 s^4 c p1^2 Q1^2 r2 - 4 m R2^5 s^4 c p1^2 Q1^2 r2 ta^2
+ 36 m R2^5 p1^2 Q1^2 s^2 c r2 - 24 m R2^2 r2^3 c p1^2 Q1^2 ta^2 R1 - 48 m R2^3 r2^3 c p1^2 Q1^2 ta^2
+ 24 m R2^3 r2^2 c p1^2 Q1^2 R1 - 6 m R2^5 c^3 p1^2 Q1^2 r2 ta^2 - 6 m R2^5 c^3 p1^2 Q1^2 r2 ta^2 s^2
+ 12 m R2^2 r2^2 c p1^2 Q1^2 R1^2 + 12 m R2^4 tas^5 p1^2 Q1^2 r2 R1 + 2 m R2^5 s^4 c p1 Q1 q1 P1 R1
- 6 m R2^4 tas^3 p1^2 Q1^2 r2 R1 - 2 m R2^4 s^4 c p1^2 Q1^2 r2 ta^2 R1 + 18 m R2^4 p1^2 Q1^2 s^2 c r2 R1
+ 6 m R2^5 c^3 p1 Q1 R1 q1 P1 + 3 m R2^4 c^3 p1 Q1 R1^2 q1 P1 + 3 m R2^4 c^3 p1 Q1 R1^2 q1 P1 s^2
+ 6 m R2^5 c^3 p1 Q1 R1 q1 P1 s^2 + 6 m R2^5 c^3 p1 Q1 r2 ta^2 q1 P1
+ 6 m R2^5 c^3 p1 Q1 r2 ta^2 q1 P1 s^2 - 12 m R2^2 r2^2 c p1 Q1 R1^2 q1 P1
- 24 m R2^3 r2^3 c p1 Q1 ta^2 q1 P1 + 6 m R2^4 tas^5 p1 Q1 r2 q1 P1 R1
- 3 m R2^4 tas^3 p1 Q1 r2 q1 P1 R1 + m R2^4 s^4 c p1 Q1 q1 P1 r2 ta^2 R1
+ 3 m R2^4 c^3 p1 Q1 r2 ta^2 q1 P1 R1 + 24 m R2^5 tas^5 p1^2 Q1^2 r2
+ 3 m R2^4 c^3 p1 Q1 r2 ta^2 q1 P1 s^2 R1 - 12 m R2^2 r2^3 c p1 Q1 ta^2 q1 P1 R1

```

$$\begin{aligned}
& -24mR2^3r2^2cp1Q1RIq1P1 - 12mR2^5tas^3p1^2Q1^2r2 - 2mR2^4s^4cp1^2Q1^2RI^2 \\
& - 6mR2^5c^3p1^2Q1^2RI - 3mR2^4c^3p1^2Q1^2RI^2 - 3mR2^4c^3p1^2Q1^2RI^2s^2 \\
& - 6mR2^5c^3p1^2Q1^2RI s^2 + 18mR2^4s^4cp1^2Q1^2r2RI - 3mR2^4c^3p1^2Q1^2r2ta^2RI \\
& - 3mR2^4c^3p1^2Q1^2r2ta^2s^2RI + mR2^4s^4cq1^2P1^2RI^2 + 2mR2^5s^4cq1^2P1^2RI \\
& + 2mR2^5s^4cq1^2P1^2r2ta^2 + mR2^4s^4cq1^2P1^2r2ta^2RI + 12mR2^5tas^5p1Q1r2q1P1 \\
& - 6mR2^5tas^3p1Q1r2q1P1 + mR2^4s^4cp1Q1q1P1RI^2 + 2mR2^5s^4cp1Q1q1P1r2ta^2)
\end{aligned}$$

> E4:=coeff(epsilon2,s1^4);

$$\begin{aligned}
E4 := & q1(3(mR2^4c^3Q1r2ta^2q1P1 + mR2^4c^3p1Q1r2ta^2q1)s^2RI - 8mR2^5s^4cp1Q1^2RI \\
& + 72mR2^5s^4cp1Q1^2r2 - 8mR2^5s^4cp1Q1^2r2ta^2 + 72mR2^5p1Q1^2s^2cr2 \\
& - 48mR2^2r2^3cp1Q1^2ta^2RI - 96mR2^3r2^3cp1Q1^2ta^2 + 48mR2^3r2^2cp1Q1^2RI \\
& - 12mR2^5c^3p1Q1^2r2ta^2 - 12mR2^5c^3p1Q1^2r2ta^2s^2 + 24mR2^2r2^2cp1Q1^2RI^2 \\
& + 24mR2^4tas^5p1Q1^2r2RI - 12mR2^4tas^3p1Q1^2r2RI - 4mR2^4s^4cp1Q1^2r2ta^2RI \\
& + 36mR2^4p1Q1^2s^2cr2RI + 3(mR2^4c^3Q1RI^2q1P1 + mR2^4c^3p1Q1RI^2q1)s^2 \\
& + 6(mR2^5c^3Q1RIq1P1 + mR2^5c^3p1Q1RIq1)s^2 \\
& + 6(mR2^5c^3Q1r2ta^2q1P1 + mR2^5c^3p1Q1r2ta^2q1)s^2 + 6mR2^5c^3p1Q1RIq1 \\
& + 6mR2^5c^3Q1RIq1P1 + 3mR2^4c^3p1Q1RI^2q1 + 3mR2^4c^3Q1RI^2q1P1 \\
& + 6mR2^5c^3p1Q1r2ta^2q1 + 6mR2^5c^3Q1r2ta^2q1P1 - 12mR2^2r2^2cp1Q1RI^2q1 \\
& - 12mR2^2r2^2cQ1RI^2q1P1 + 6(mR2^4tas^5Q1r2q1P1 + mR2^4tas^5p1Q1r2q1)RI \\
& - 3(mR2^4tas^3Q1r2q1P1 + mR2^4tas^3p1Q1r2q1)RI - 24mR2^3r2^3cp1Q1ta^2q1 \\
& - 24mR2^3r2^3cQ1ta^2q1P1 + 3(mR2^4c^3Q1r2ta^2q1P1 + mR2^4c^3p1Q1r2ta^2q1)RI \\
& - 12(mR2^2r2^3cQ1ta^2q1P1 + mR2^2r2^3cp1Q1ta^2q1)RI + 48mR2^5tas^5p1Q1^2r2 \\
& - 24mR2^3r2^2cp1Q1RIq1 - 24mR2^3r2^2cQ1RIq1P1 - 24mR2^5tas^3p1Q1^2r2 \\
& - 4mR2^4s^4cp1Q1^2RI^2 - 12mR2^5c^3p1Q1^2RI - 6mR2^4c^3p1Q1^2RI^2 \\
& - 6mR2^4c^3p1Q1^2RI^2s^2 - 12mR2^5c^3p1Q1^2RI s^2 + 36mR2^4s^4cp1Q1^2r2RI \\
& - 6mR2^4c^3p1Q1^2r2ta^2RI - 6mR2^4c^3p1Q1^2r2ta^2s^2RI + 2mR2^4s^4cq1^2P1RI^2 \\
& + 4mR2^5s^4cq1^2P1RI + 4mR2^5s^4cq1^2P1r2ta^2 + 2mR2^4s^4cq1^2P1r2ta^2RI \\
& + (mR2^4s^4cQ1q1P1 + mR2^4s^4cp1Q1q1)RI^2 \\
& + 2(mR2^5s^4cQ1q1P1 + mR2^5s^4cp1Q1q1)RI + 12mR2^5tas^5p1Q1r2q1 \\
& + 12mR2^5tas^5Q1r2q1P1 - 6mR2^5tas^3p1Q1r2q1 - 6mR2^5tas^3Q1r2q1P1 \\
& + (mR2^4s^4cQ1q1P1 + mR2^4s^4cp1Q1q1)r2ta^2RI \\
& + 2(mR2^5s^4cQ1q1P1 + mR2^5s^4cp1Q1q1)r2ta^2)
\end{aligned}$$

> E3:=coeff(epsilon2,s1^3);

$$\begin{aligned}
E3 := & q1(-4mR2^5s^4cQ1^2RI + 36mR2^5s^4cQ1^2r2 - 4mR2^5s^4cQ1^2r2ta^2 \\
& + 36mR2^5Q1^2s^2cr2 - 24mR2^2r2^3cQ1^2ta^2RI - 48mR2^3r2^3cQ1^2ta^2 \\
& + 24mR2^3r2^2cQ1^2RI - 6mR2^5c^3Q1^2r2ta^2 - 6mR2^5c^3Q1^2r2ta^2s^2 \\
& + 12mR2^2r2^2cQ1^2RI^2 + 12mR2^4tas^5Q1^2r2RI + 2mR2^5s^4cQ1q1RI \\
& - 6mR2^4tas^3Q1^2r2RI - 2mR2^4s^4cQ1^2r2ta^2RI + 18mR2^4Q1^2s^2cr2RI \\
& + 6mR2^5c^3Q1RIq1 + 3mR2^4c^3Q1RI^2q1 + 3mR2^4c^3Q1RI^2q1s^2 \\
& + 6mR2^5c^3Q1RIq1s^2 + 6mR2^5c^3Q1r2ta^2q1 + 6mR2^5c^3Q1r2ta^2q1s^2 \\
& - 12mR2^2r2^2cQ1RI^2q1 - 24mR2^3r2^3cQ1ta^2q1 + 6mR2^4tas^5Q1r2q1RI \\
& - 3mR2^4tas^3Q1r2q1RI + mR2^4s^4cQ1q1r2ta^2RI + 3mR2^4c^3Q1r2ta^2q1RI
\end{aligned}$$

$$\begin{aligned}
& + 24 m R2^5 t a s^5 Q1^2 r2 + 3 m R2^4 c^3 Q1 r2 t a^2 q1 s^2 R1 - 12 m R2^2 r2^3 c Q1 t a^2 q1 R1 \\
& - 24 m R2^3 r2^2 c Q1 R1 q1 - 12 m R2^5 t a s^3 Q1^2 r2 - 2 m R2^4 s^4 c Q1^2 R1^2 - 3 m R2^4 c^3 Q1^2 R1^2 \\
& - 3 m R2^4 c^3 Q1^2 R1^2 s^2 - 6 m R2^5 c^3 Q1^2 R1 s^2 + 18 m R2^4 s^4 c Q1^2 r2 R1 \\
& - 3 m R2^4 c^3 Q1^2 r2 t a^2 R1 - 3 m R2^4 c^3 Q1^2 r2 t a^2 s^2 R1 + m R2^4 s^4 c q1^2 R1^2 \\
& + 2 m R2^5 s^4 c q1^2 R1 + 2 m R2^5 s^4 c q1^2 r2 t a^2 + m R2^4 s^4 c q1^2 r2 t a^2 R1 \\
& + 12 m R2^5 t a s^5 Q1 r2 q1 - 6 m R2^5 t a s^3 Q1 r2 q1 + m R2^4 s^4 c Q1 q1 R1^2 \\
& + 2 m R2^5 s^4 c Q1 q1 r2 t a^2 - 6 m R2^5 c^3 Q1^2 R1
\end{aligned}$$

> E2:=coeff(epsilon2,s1^2);  
 $E2 := 0$

> E1:=coeff(epsilon2,s1);  
 $E1 := 0$

> E0:=coeff(epsilon2,s1,0);  
 $E0 := 0$

> B5:=coeff(beta2,s1^5);  
 $B5 := q1 (-3 r2 m R2^5 s^3 c^2 p1^2 Q1^2 - 4 r2^2 R1^4 p1 Q1 q1 P1 t a^2 + r2 R1^4 p1 Q1 R2 q1 P1$   
 $+ r2 R1^4 q1^2 P1^2 R2 - 8 r2^2 m R2^4 s^5 t a^2 p1 Q1 q1 P1 + 4 r2^2 m R2^4 s^3 t a^2 p1 Q1 q1 P1$   
 $+ r2 m R2^5 s^5 p1 Q1 q1 P1 - 6 r2^2 m R2^4 s^3 p1 Q1 q1 P1 - 2 r2^2 m R2^4 s^5 t a^2 q1^2 P1^2$   
 $+ r2^2 m R2^4 s^3 t a^2 q1^2 P1^2 + r2 m R2^5 s^5 q1^2 P1^2 - 6 r2^2 m R2^4 s^5 p1 Q1 q1 P1$   
 $+ 3 r2 m R2^5 s c^2 p1 Q1 q1 P1 + 3 r2 m R2^5 s^3 c^2 p1 Q1 q1 P1 - 12 r2^4 m R2^2 s p1 Q1 q1 P1$   
 $- 4 r2^2 R1^4 p1^2 Q1^2 - r2^2 R1^4 q1^2 P1^2 t a^2 - 4 r2^2 R1^4 p1^2 Q1^2 t a^2$   
 $- 8 r2^2 m R2^4 s^5 t a^2 p1^2 Q1^2 + 4 r2^2 m R2^4 s^3 t a^2 p1^2 Q1^2 - 24 r2^4 m R2^2 s p1^2 Q1^2$   
 $- 2 r2 R1^4 p1^2 Q1^2 R2 - 2 r2 m R2^5 s^5 p1^2 Q1^2 - 12 r2^2 m R2^4 s^3 p1^2 Q1^2$   
 $- 12 r2^2 m R2^4 s^5 p1^2 Q1^2 - 3 r2 m R2^5 s c^2 p1^2 Q1^2 - r2^2 R1^4 q1^2 P1^2 - 4 r2^2 R1^4 p1 Q1 q1 P1)$

> B4:=coeff(beta2,s1^4);  
 $B4 := q1 (-6 r2 m R2^5 s^3 c^2 p1 Q1^2 - 4 r2^2 R1^4 p1 Q1 q1 - 4 (r2^2 R1^4 Q1 q1 P1 + r2^2 R1^4 p1 Q1 q1) t a^2$   
 $- 4 r2^2 R1^4 Q1 q1 P1 + r2 R1^4 p1 Q1 R2 q1 + r2 R1^4 Q1 R2 q1 P1 + 2 r2 R1^4 q1^2 P1 R2$   
 $- 8 r2^2 m R2^4 s^5 t a^2 p1 Q1 q1 - 8 r2^2 m R2^4 s^5 t a^2 Q1 q1 P1 + 4 r2^2 m R2^4 s^3 t a^2 p1 Q1 q1$   
 $+ 4 r2^2 m R2^4 s^3 t a^2 Q1 q1 P1 + r2 m R2^5 s^5 p1 Q1 q1 + r2 m R2^5 s^5 Q1 q1 P1$   
 $- 6 r2^2 m R2^4 s^3 p1 Q1 q1 - 6 r2^2 m R2^4 s^3 Q1 q1 P1 - 4 r2^2 m R2^4 s^5 t a^2 q1^2 P1$   
 $+ 2 r2^2 m R2^4 s^3 t a^2 q1^2 P1 + 2 r2 m R2^5 s^5 q1^2 P1 - 6 r2^2 m R2^4 s^5 p1 Q1 q1$   
 $- 6 r2^2 m R2^4 s^5 Q1 q1 P1 + 3 r2 m R2^5 s c^2 p1 Q1 q1 + 3 r2 m R2^5 s c^2 Q1 q1 P1$   
 $+ 3 r2 m R2^5 s^3 c^2 p1 Q1 q1 + 3 r2 m R2^5 s^3 c^2 Q1 q1 P1 - 12 r2^4 m R2^2 s p1 Q1 q1$   
 $- 12 r2^4 m R2^2 s Q1 q1 P1 - 2 r2^2 R1^4 q1^2 P1 - 8 r2^2 R1^4 p1 Q1^2 - 2 r2^2 R1^4 q1^2 P1 t a^2$   
 $- 8 r2^2 R1^4 p1 Q1^2 t a^2 - 16 r2^2 m R2^4 s^5 t a^2 p1 Q1^2 + 8 r2^2 m R2^4 s^3 t a^2 p1 Q1^2$   
 $- 48 r2^4 m R2^2 s p1 Q1^2 - 4 r2 R1^4 p1 Q1^2 R2 - 4 r2 m R2^5 s^5 p1 Q1^2 - 24 r2^2 m R2^4 s^3 p1 Q1^2$   
 $- 6 r2 m R2^5 s c^2 p1 Q1^2 - 24 r2^2 m R2^4 s^5 p1 Q1^2)$

> B3:=coeff(beta2,s1^3);  
 $B3 := q1 (-4 r2^2 R1^4 Q1^2 - 3 r2 m R2^5 s^3 c^2 Q1^2 - 4 r2^2 R1^4 Q1 q1 t a^2 + r2 R1^4 Q1 R2 q1$

$$\begin{aligned}
& + 4 r2^2 m R2^4 s^3 ta^2 Q1 q1 + r2 m R2^5 s^5 Q1 q1 - 6 r2^2 m R2^4 s^3 Q1 q1 - 2 r2^2 m R2^4 s^5 ta^2 q1^2 \\
& + r2^2 m R2^4 s^3 ta^2 q1^2 + r2 m R2^5 s^5 q1^2 - 6 r2^2 m R2^4 s^5 Q1 q1 + 3 r2 m R2^5 s c^2 Q1 q1 \\
& + 3 r2 m R2^5 s^3 c^2 Q1 q1 - 12 r2^4 m R2^2 s Q1 q1 - r2^2 R1^4 q1^2 ta^2 - 4 r2^2 R1^4 Q1^2 ta^2 \\
& - 2 r2 R1^4 Q1^2 R2 - 4 r2^2 R1^4 Q1 q1 + r2 R1^4 q1^2 R2 - 8 r2^2 m R2^4 s^5 ta^2 Q1^2 \\
& + 4 r2^2 m R2^4 s^3 ta^2 Q1^2 - 24 r2^4 m R2^2 s Q1^2 - 2 r2 m R2^5 s^5 Q1^2 - 12 r2^2 m R2^4 s^5 Q1^2 \\
& - 3 r2 m R2^5 s c^2 Q1^2 - 8 r2^2 m R2^4 s^5 ta^2 Q1 q1 - r2^2 R1^4 q1^2 - 12 r2^2 m R2^4 s^3 Q1^2
\end{aligned}$$

```

> B2:=coeff(beta2,s1^2);
B2 := 0

> B1:=coeff(beta2,s1);
B1 := 0

> B0:=coeff(beta2,s1,0);
B0 := 0

>
>
>

```

&gt;

## Program No. 5

Coefficients of the diff. eq. of the axial  
stress of a helix using two 3-Para models

```
restart;
```

&gt;

&gt;

&gt;

```
> with(linalg):
```

Warning, the protected names norm and trace have been redefined and unprotected

```
> c1:=E*R1^2/(R1^2+m*R2^2)+m*E*R2^2/((R1^2+m*R2^2)*(r2+r2*ta^2+nu*R2))*((r2*ta^2-nu*R1)*s+R2^2*(1-2*s^2)*s*c^2/4/r2+R2^2*s^3*c^2*(1+nu)/2/r2-nu^2*R2^2*s*c^4*(R1+r2*ta^2)/4/r2^2/(1+nu));
```

$$c1 := \frac{ERI^2}{RI^2 + m R2^2} + \frac{m E R2^2 \left( (r2 ta^2 - \nu RI) s + \frac{R2^2 (1 - 2 s^2) s c^2}{4 r2} + \frac{R2^2 s^3 c^2 (1 + \nu)}{2 r2} - \frac{\nu^2 R2^2 s c^4 (RI + r2 ta^2)}{4 r2^2 (1 + \nu)} \right)}{(RI^2 + m R2^2) (r2 + r2 ta^2 + \nu R2)}$$

```
> c2:=r2*E*m*R2^2/(R1+2*R2)/(R1^2+m*R2^2)/(r2+r2*ta^2+nu*R2)*(r2*ta*s+R2^2/4/(1+nu)/r2*((2*s^2-1)*s^2*c)-R2^2/2/r2*s^4*c-nu^2*R2^3*s^2*c^3/4/r2^2/(1+nu));
```

$$c2 := \frac{r2 E m R2^2 \left( r2 ta s + \frac{R2^2 (2 s^2 - 1) s^2 c}{4 (1 + \nu) r2} - \frac{R2^2 s^4 c}{2 r2} - \frac{\nu^2 R2^3 s^2 c^3}{4 r2^2 (1 + \nu)} \right)}{(RI + 2 R2) (RI^2 + m R2^2) (r2 + r2 ta^2 + \nu R2)}$$

$$\begin{aligned}
& + 24 (Q0 RI^2 r2^3 q0 P1 + Q0 RI^2 r2^3 q1 + (Q0 RI^2 r2^3 p1 + Q1 RI^2 r2^3) q0) R2 \\
& + 24 (Q0 RI^2 r2^3 ta^2 q0 P1 + Q0 RI^2 r2^3 ta^2 q1 + (Q0 RI^2 r2^3 p1 + Q1 RI^2 r2^3) ta^2 q0) R2 + 12 \\
& (m R2^2 s r2^3 Q0 ta^2 q0 P1 + m R2^2 s r2^3 Q0 ta^2 q1 + (m R2^2 s r2^3 Q1 + m R2^2 s r2^3 p1 Q0) ta^2 q0 \\
& ) R1 - 6 (m R2^4 s^3 c^2 r2 Q0 q0 P1 + m R2^4 s^3 c^2 r2 Q0 q1 \\
& + (m R2^4 s^3 c^2 r2 Q1 + m R2^4 s^3 c^2 r2 p1 Q0) q0) R1 + 3 (m R2^4 s c^2 r2 Q0 q0 P1 \\
& + m R2^4 s c^2 r2 Q0 q1 + (m R2^4 s c^2 r2 Q1 + m R2^4 s c^2 r2 p1 Q0) q0) R1 + q1 (24 Q0^2 RI^3 r2^3 \\
& + 24 Q0^2 RI^3 r2^3 ta^2 + 12 Q0^2 RI^3 r2^2 R2 + 24 Q0^2 RI^2 r2^2 R2^2 + 48 Q0^2 RI^2 r2^3 R2 \\
& + 12 Q0 RI^3 r2^3 q0 - m R2^4 s c^4 q0^2 RI^2 - 2 m R2^5 s c^4 q0^2 R1 - 2 m R2^5 s c^4 q0^2 r2 ta^2 \\
& - m R2^4 s c^4 q0^2 r2 ta^2 RI - 12 m R2^2 s r2^2 Q0^2 RI^2 + 48 m R2^3 s r2^3 Q0^2 ta^2 \\
& - 2 m R2^5 s c^4 Q0^2 RI - 24 m R2^3 s r2^2 Q0^2 RI + 12 m R2^5 s^3 c^2 Q0^2 r2 + 12 m R2^5 s c^2 r2 Q0^2 \\
& - m R2^4 s c^4 Q0^2 RI^2 - m R2^4 s c^4 Q0^2 r2 ta^2 RI - 2 m R2^5 s c^4 Q0^2 r2 ta^2 \\
& + 48 Q0^2 RI^2 r2^3 ta^2 R2 + 24 m R2^2 s r2^3 Q0^2 ta^2 RI + 6 m R2^4 s^3 c^2 Q0^2 r2 RI \\
& + 6 m R2^4 s c^2 r2 Q0^2 RI + 12 m R2^2 s r2^2 Q0 RI^2 q0 + 24 m R2^3 s r2^2 Q0 RI q0 \\
& - 12 m R2^5 s^3 c^2 r2 Q0 q0 + 12 Q0 RI^3 r2^3 ta^2 q0 + 3 m R2^4 s c^2 r2 Q0 q0 RI \\
& - 12 Q0 RI^3 r2^2 R2 q0 - 24 Q0 RI^2 r2^2 R2^2 q0 + 24 m R2^3 s r2^3 Q0 ta^2 q0 + 6 m R2^5 s c^2 r2 Q0 q0 \\
& + 2 m R2^4 s c^4 Q0 q0 RI^2 + 4 m R2^5 s c^4 Q0 q0 RI + 4 m R2^5 s c^4 Q0 q0 r2 ta^2 \\
& + 24 Q0 RI^2 r2^3 q0 R2 + 24 Q0 RI^2 r2^3 ta^2 q0 R2 + 12 m R2^2 s r2^3 Q0 ta^2 q0 RI \\
& - 6 m R2^4 s^3 c^2 r2 Q0 q0 RI + 2 m R2^4 s c^4 Q0 q0 r2 ta^2 RI)) s1 + q0 (24 Q0^2 RI^3 r2^3 \\
& + 24 Q0^2 RI^3 r2^3 ta^2 + 12 Q0^2 RI^3 r2^2 R2 + 24 Q0^2 RI^2 r2^2 R2^2 + 48 Q0^2 RI^2 r2^3 R2 \\
& + 12 Q0 RI^3 r2^3 q0 - m R2^4 s c^4 q0^2 RI^2 - 2 m R2^5 s c^4 q0^2 R1 - 2 m R2^5 s c^4 q0^2 r2 ta^2 \\
& - m R2^4 s c^4 q0^2 r2 ta^2 RI - 12 m R2^2 s r2^2 Q0^2 RI^2 + 48 m R2^3 s r2^3 Q0^2 ta^2 \\
& - 2 m R2^5 s c^4 Q0^2 RI - 24 m R2^3 s r2^2 Q0^2 RI + 12 m R2^5 s^3 c^2 Q0^2 r2 + 12 m R2^5 s c^2 r2 Q0^2 \\
& - m R2^4 s c^4 Q0^2 RI^2 - m R2^4 s c^4 Q0^2 r2 ta^2 RI - 2 m R2^5 s c^4 Q0^2 r2 ta^2 \\
& + 48 Q0^2 RI^2 r2^3 ta^2 R2 + 24 m R2^2 s r2^3 Q0^2 ta^2 RI + 6 m R2^4 s^3 c^2 Q0^2 r2 RI \\
& + 6 m R2^4 s c^2 r2 Q0^2 RI + 12 m R2^2 s r2^2 Q0 RI^2 q0 + 24 m R2^3 s r2^2 Q0 RI q0 \\
& - 12 m R2^5 s^3 c^2 r2 Q0 q0 + 12 Q0 RI^3 r2^3 ta^2 q0 + 3 m R2^4 s c^2 r2 Q0 q0 RI \\
& - 12 Q0 RI^3 r2^2 R2 q0 - 24 Q0 RI^2 r2^2 R2^2 q0 + 24 m R2^3 s r2^3 Q0 ta^2 q0 + 6 m R2^5 s c^2 r2 Q0 q0 \\
& + 2 m R2^4 s c^4 Q0 q0 RI^2 + 4 m R2^5 s c^4 Q0 q0 RI + 4 m R2^5 s c^4 Q0 q0 r2 ta^2 \\
& + 24 Q0 RI^2 r2^3 q0 R2 + 24 Q0 RI^2 r2^3 ta^2 q0 R2 + 12 m R2^2 s r2^3 Q0 ta^2 q0 RI \\
& - 6 m R2^4 s^3 c^2 r2 Q0 q0 RI + 2 m R2^4 s c^4 Q0 q0 r2 ta^2 RI)
\end{aligned}$$

```

> D5:=coeff(LHS3,s1^5);
D5 := 4 p1 r2^2 (r2 ta^2 q1 P1 + r2 q1 P1 + R2 p1 Q1 + 2 r2 p1 Q1 + 2 r2 ta^2 p1 Q1 - R2 q1 P1) (R1^2
+ m R2^2) (R1 + 2 R2) (2 p1 Q1 + q1 P1)

```

```

> D4:=coeff(LHS3,s1^4);
D4 := 4 (r2^2 (r2 ta^2 q1 P1 + r2 q1 P1 + R2 p1 Q1 + 2 r2 p1 Q1 + 2 r2 ta^2 p1 Q1 - R2 q1 P1) + p1 r2^2 (2 r2
Q1 + 2 r2 p1 Q0 + 2 r2 ta^2 Q1 + 2 r2 ta^2 p1 Q0 + r2 q0 P1 + r2 q1 + R2 Q1 + R2 p1 Q0
+ r2 ta^2 q0 P1 + r2 ta^2 q1 - R2 q0 P1 - R2 q1)) (R1^2 + m R2^2) (R1 + 2 R2) (2 p1 Q1 + q1 P1) + 4 p1
r2^2 (r2 ta^2 q1 P1 + r2 q1 P1 + R2 p1 Q1 + 2 r2 p1 Q1 + 2 r2 ta^2 p1 Q1 - R2 q1 P1) (R1^2 + m R2^2) (
R1 + 2 R2) (2 Q1 + 2 p1 Q0 + q0 P1 + q1)

```

```
> D3:=coeff(LHS3,s1^3);
```

$$\begin{aligned}
D3 := & 4(r2^2(2r2Q1 + 2r2p1Q0 + 2r2ta^2Q1 + 2r2ta^2p1Q0 + r2q0P1 + r2q1 + R2Q1 + R2p1Q0 \\
& + r2ta^2q0P1 + r2ta^2q1 - R2q0P1 - R2q1) \\
& + p1r2^2(2r2Q0 + r2ta^2q0 - R2q0 + r2q0 + R2Q0 + 2r2ta^2Q0))(R1^2 + mR2^2)(R1 + 2R2)(2 \\
& p1Q1 + q1P1) + 4(r2^2(r2ta^2q1P1 + r2q1P1 + R2p1Q1 + 2r2p1Q1 + 2r2ta^2p1Q1 \\
& - R2q1P1) + p1r2^2(2r2Q1 + 2r2p1Q0 + 2r2ta^2Q1 + 2r2ta^2p1Q0 + r2q0P1 + r2q1 \\
& + R2Q1 + R2p1Q0 + r2ta^2q0P1 + r2ta^2q1 - R2q0P1 - R2q1))(R1^2 + mR2^2)(R1 + 2R2)(2Q1 \\
& + 2p1Q0 + q0P1 + q1) + 4p1r2^2(r2ta^2q1P1 + r2q1P1 + R2p1Q1 + 2r2p1Q1 \\
& + 2r2ta^2p1Q1 - R2q1P1)(R1^2 + mR2^2)(R1 + 2R2)(2Q0 + q0)
\end{aligned}$$

> D2:=coeff(LHS3,s1^2);

$$\begin{aligned}
D2 := & 4r2^2(2r2Q0 + r2ta^2q0 - R2q0 + r2q0 + R2Q0 + 2r2ta^2Q0)(R1^2 + mR2^2)(R1 + 2R2)(2p1 \\
& Q1 + q1P1) + 4(r2^2(2r2Q1 + 2r2p1Q0 + 2r2ta^2Q1 + 2r2ta^2p1Q0 + r2q0P1 + r2q1 \\
& + R2Q1 + R2p1Q0 + r2ta^2q0P1 + r2ta^2q1 - R2q0P1 - R2q1) \\
& + p1r2^2(2r2Q0 + r2ta^2q0 - R2q0 + r2q0 + R2Q0 + 2r2ta^2Q0))(R1^2 + mR2^2)(R1 + 2R2)(2 \\
& Q1 + 2p1Q0 + q0P1 + q1) + 4(r2^2(r2ta^2q1P1 + r2q1P1 + R2p1Q1 + 2r2p1Q1 \\
& + 2r2ta^2p1Q1 - R2q1P1) + p1r2^2(2r2Q1 + 2r2p1Q0 + 2r2ta^2Q1 + 2r2ta^2p1Q0 \\
& + r2q0P1 + r2q1 + R2Q1 + R2p1Q0 + r2ta^2q0P1 + r2ta^2q1 - R2q0P1 - R2q1))(R1^2 + mR2^2) \\
& (R1 + 2R2)(2Q0 + q0)
\end{aligned}$$

> D1:=coeff(LHS3,s1);

$$\begin{aligned}
D1 := & 4r2^2(2r2Q0 + r2ta^2q0 - R2q0 + r2q0 + R2Q0 + 2r2ta^2Q0)(R1^2 + mR2^2)(R1 + 2R2)(2Q1 \\
& + 2p1Q0 + q0P1 + q1) + 4(r2^2(2r2Q1 + 2r2p1Q0 + 2r2ta^2Q1 + 2r2ta^2p1Q0 + r2q0P1 \\
& + r2q1 + R2Q1 + R2p1Q0 + r2ta^2q0P1 + r2ta^2q1 - R2q0P1 - R2q1) \\
& + p1r2^2(2r2Q0 + r2ta^2q0 - R2q0 + r2q0 + R2Q0 + 2r2ta^2Q0))(R1^2 + mR2^2)(R1 + 2R2)(2 \\
& Q0 + q0)
\end{aligned}$$

> D0:=coeff(LHS3,s1,0);

$$\begin{aligned}
D0 := & 4r2^2(2r2Q0 + r2ta^2q0 - R2q0 + r2q0 + R2Q0 + 2r2ta^2Q0)(R1^2 + mR2^2)(R1 + 2R2)(2Q0 \\
& + q0)
\end{aligned}$$

> E5:=coeff(epsilon2,s1^5);

$$\begin{aligned}
E5 := & q1(24Q1^2R1^3r2^3p1^2 + 24Q1^2R1^3r2^3p1^2ta^2 - mR2^4sc^4q1^2P1^2R1^2 \\
& - 2mR2^5sc^4q1^2P1^2R1 - 2mR2^5sc^4q1^2P1^2r2ta^2 + 12Q1^2R1^3r2^2p1^2R2 \\
& - mR2^4sc^4q1^2P1^2r2ta^2R1 - 12mR2^2s^2r2^2p1^2Q1^2R1^2 + 24Q1^2R1^2r2^2p1^2R2^2 \\
& + 48mR2^3s^2r2^3p1^2Q1^2ta^2 - 2mR2^5sc^4p1^2Q1^2R1 + 48Q1^2R1^2r2^3p1^2R2 \\
& - 24mR2^3s^2r2^2p1^2Q1^2R1 + 12mR2^5s^3c^2p1^2Q1^2r2 + 12mR2^5sc^2r2p1^2Q1^2 \\
& - mR2^4sc^4p1^2Q1^2R1^2 - mR2^4sc^4p1^2Q1^2r2ta^2R1 + 12Q1R1^3r2^3p1q1P1 \\
& - 2mR2^5sc^4p1^2Q1^2r2ta^2 + 48Q1^2R1^2r2^3p1^2ta^2R2 + 24mR2^2s^2r2^3p1^2Q1^2ta^2R1 \\
& + 6mR2^4s^3c^2p1^2Q1^2r2R1 + 6mR2^4sc^2r2p1^2Q1^2R1 + 12mR2^2s^2r2^2p1Q1R1^2q1P1 \\
& + 24mR2^3s^2r2^2p1Q1R1q1P1 - 12mR2^5s^3c^2r2p1Q1q1P1 + 12Q1R1^3r2^3p1ta^2q1P1 \\
& - 12Q1R1^3r2^2p1R2q1P1 - 24Q1R1^2r2^2p1R2^2q1P1 + 24mR2^3s^2r2^3p1Q1ta^2q1P1 \\
& + 6mR2^5sc^2r2p1Q1q1P1 + 2mR2^4sc^4p1Q1q1P1R1^2 + 4mR2^5sc^4p1Q1q1P1R1 \\
& + 4mR2^5sc^4p1Q1q1P1r2ta^2 + 24Q1R1^2r2^3p1q1P1R2 + 24Q1R1^2r2^3p1ta^2q1P1R2 \\
& + 12mR2^2s^2r2^3p1Q1ta^2q1P1R1 - 6mR2^4s^3c^2r2p1Q1q1P1R1 \\
& + 3mR2^4sc^2r2p1Q1q1P1R1 + 2mR2^4sc^4p1Q1q1P1r2ta^2R1)
\end{aligned}$$

```

> E4:=coeff(epsilon2,s1^4);
E4 := q0 (24 Q1^2 R1^3 r2^3 p1^2 + 24 Q1^2 R1^3 r2^3 p1^2 ta^2 - m R2^4 s c^4 q1^2 P1^2 R1^2
- 2 m R2^5 s c^4 q1^2 P1^2 R1 - 2 m R2^5 s c^4 q1^2 P1^2 r2 ta^2 + 12 Q1^2 R1^3 r2^2 p1^2 R2
- m R2^4 s c^4 q1^2 P1^2 r2 ta^2 R1 - 12 m R2^2 s r2^2 p1^2 Q1^2 R1^2 + 24 Q1^2 R1^2 r2^2 p1^2 R2^2
+ 48 m R2^3 s r2^3 p1^2 Q1^2 ta^2 - 2 m R2^5 s c^4 p1^2 Q1^2 R1 + 48 Q1^2 R1^2 r2^3 p1^2 R2
- 24 m R2^3 s r2^2 p1^2 Q1^2 R1 + 12 m R2^5 s^3 c^2 p1^2 Q1^2 r2 + 12 m R2^5 s c^2 r2 p1^2 Q1^2
- m R2^4 s c^4 p1^2 Q1^2 R1^2 - m R2^4 s c^4 p1^2 Q1^2 r2 ta^2 R1 + 12 Q1 R1^3 r2^3 p1 q1 P1
- 2 m R2^5 s c^4 p1^2 Q1^2 r2 ta^2 + 48 Q1^2 R1^2 r2^3 p1^2 ta^2 R2 + 24 m R2^2 s r2^3 p1^2 Q1^2 ta^2 R1
+ 6 m R2^4 s^3 c^2 p1^2 Q1^2 r2 R1 + 6 m R2^4 s c^2 r2 p1^2 Q1^2 R1 + 12 m R2^2 s r2^2 p1 Q1 R1^2 q1 P1
+ 24 m R2^3 s r2^2 p1 Q1 R1 q1 P1 - 12 m R2^5 s^3 c^2 r2 p1 Q1 q1 P1 + 12 Q1 R1^3 r2^3 p1 ta^2 q1 P1
- 12 Q1 R1^3 r2^2 p1 R2 q1 P1 - 24 Q1 R1^2 r2^2 p1 R2^2 q1 P1 + 24 m R2^3 s r2^3 p1 Q1 ta^2 q1 P1
+ 6 m R2^5 s c^2 r2 p1 Q1 q1 P1 + 2 m R2^4 s c^4 p1 Q1 q1 P1 R1^2 + 4 m R2^5 s c^4 p1 Q1 q1 P1 R1
+ 4 m R2^5 s c^4 p1 Q1 q1 P1 r2 ta^2 + 24 Q1 R1^2 r2^3 p1 q1 P1 R2 + 24 Q1 R1^2 r2^3 p1 ta^2 q1 P1 R2
+ 12 m R2^2 s r2^3 p1 Q1 ta^2 q1 P1 R1 - 6 m R2^4 s^3 c^2 r2 p1 Q1 q1 P1 R1
+ 3 m R2^4 s c^2 r2 p1 Q1 q1 P1 R1 + 2 m R2^4 s c^4 p1 Q1 q1 P1 r2 ta^2 R1) + q1 (4 (((m R2^5 s c^4 Q1
+ m R2^5 s c^4 p1 Q0) q1 + m R2^5 s c^4 p1 Q1 q0) P1 + m R2^5 s c^4 p1 Q1 q1) r2 ta^2
+ 48 (2 Q0 Q1 R1^2 r2^3 p1^2 + 2 Q1^2 R1^2 r2^3 p1) ta^2 R2
- 2 (2 m R2^5 s c^4 p1 Q1^2 + 2 m R2^5 s c^4 p1^2 Q0 Q1) r2 ta^2
+ 24 (2 m R2^2 s r2^3 p1 Q1^2 + 2 m R2^2 s r2^3 p1^2 Q0 Q1) ta^2 R1
+ 6 (2 m R2^4 s^3 c^2 p1 Q1^2 + 2 m R2^4 s^3 c^2 p1^2 Q0 Q1) r2 R1 + 48 Q0 Q1 R1^3 r2^3 p1^2
+ 48 Q1^2 R1^3 r2^3 p1 - (2 m R2^4 s c^4 q0 q1 P1^2 + 2 m R2^4 s c^4 q1^2 P1) r2 ta^2 R1
- (2 m R2^4 s c^4 p1 Q1^2 + 2 m R2^4 s c^4 p1^2 Q0 Q1) r2 ta^2 R1
+ 24 (2 Q0 Q1 R1^3 r2^3 p1^2 + 2 Q1^2 R1^3 r2^3 p1) ta^2
- (2 m R2^4 s c^4 q0 q1 P1^2 + 2 m R2^4 s c^4 q1^2 P1) R1^2
- 2 (2 m R2^5 s c^4 q0 q1 P1^2 + 2 m R2^5 s c^4 q1^2 P1) R1
- 2 (2 m R2^5 s c^4 q0 q1 P1^2 + 2 m R2^5 s c^4 q1^2 P1) r2 ta^2
+ 12 (2 Q0 Q1 R1^3 r2^2 p1^2 + 2 Q1^2 R1^3 r2^2 p1) R2
- 12 (2 m R2^2 s r2^2 p1 Q1^2 + 2 m R2^2 s r2^2 p1^2 Q0 Q1) R1^2
+ 24 (2 Q0 Q1 R1^2 r2^2 p1^2 + 2 Q1^2 R1^2 r2^2 p1) R2^2
+ 48 (2 m R2^3 s r2^3 p1 Q1^2 + 2 m R2^3 s r2^3 p1^2 Q0 Q1) ta^2
- 2 (2 m R2^5 s c^4 p1 Q1^2 + 2 m R2^5 s c^4 p1^2 Q0 Q1) R1
+ 48 (2 Q0 Q1 R1^2 r2^3 p1^2 + 2 Q1^2 R1^2 r2^3 p1) R2
- 24 (2 m R2^3 s r2^2 p1 Q1^2 + 2 m R2^3 s r2^2 p1^2 Q0 Q1) R1
+ 12 (2 m R2^5 s^3 c^2 p1 Q1^2 + 2 m R2^5 s^3 c^2 p1^2 Q0 Q1) r2
- (2 m R2^4 s c^4 p1 Q1^2 + 2 m R2^4 s c^4 p1^2 Q0 Q1) R1^2 + 24 m R2^5 s c^2 r2 p1 Q1^2
+ 24 m R2^5 s c^2 r2 p1^2 Q0 Q1 + 12 ((Q0 R1^3 r2^3 p1 + Q1 R1^3 r2^3) q1 + Q1 R1^3 r2^3 p1 q0) P1
+ 12 Q1 R1^3 r2^3 p1 q1 + 6 (2 m R2^4 s c^2 r2 p1 Q1^2 + 2 m R2^4 s c^2 r2 p1^2 Q0 Q1) R1
+ 12 ((m R2^2 s r2^2 Q1 + m R2^2 s r2^2 p1 Q0) R1^2 q1 + m R2^2 s r2^2 p1 Q1 R1^2 q0) P1
+ 12 m R2^2 s r2^2 p1 Q1 R1^2 q1
+ 24 ((m R2^3 s r2^2 Q1 + m R2^3 s r2^2 p1 Q0) R1 q1 + m R2^3 s r2^2 p1 Q1 R1 q0) P1
- 12 ((m R2^5 s^3 c^2 r2 Q1 + m R2^5 s^3 c^2 r2 p1 Q0) q1 + m R2^5 s^3 c^2 r2 p1 Q1 q0) P1
+ 12 ((Q0 R1^3 r2^3 p1 + Q1 R1^3 r2^3) ta^2 q1 + Q1 R1^3 r2^3 p1 ta^2 q0) P1

```

$$\begin{aligned}
& + 24 m R2^3 s r2^2 p1 Q1 R1 q1 - 12 m R2^5 s^3 c^2 r2 p1 Q1 q1 + 12 Q1 R1^3 r2^3 p1 ta^2 q1 \\
& - 12 ((Q0 R1^3 r2^2 p1 + Q1 R1^3 r2^2) R2 q1 + Q1 R1^3 r2^2 p1 R2 q0) P1 \\
& - 24 ((Q0 R1^2 r2^2 p1 + Q1 R1^2 r2^2) R2^2 q1 + Q1 R1^2 r2^2 p1 R2^2 q0) P1 \\
& + 24 ((m R2^3 s r2^3 Q1 + m R2^3 s r2^3 p1 Q0) ta^2 q1 + m R2^3 s r2^3 p1 Q1 ta^2 q0) P1 \\
& + 6 ((m R2^5 s c^2 r2 Q1 + m R2^5 s c^2 r2 p1 Q0) q1 + m R2^5 s c^2 r2 p1 Q1 q0) P1 \\
& - 12 Q1 R1^3 r2^2 p1 R2 q1 + 2 (((m R2^4 s c^4 Q1 + m R2^4 s c^4 p1 Q0) q1 + m R2^4 s c^4 p1 Q1 q0) P1 \\
& + m R2^4 s c^4 p1 Q1 q1) r2 ta^2 R1 - 24 Q1 R1^2 r2^2 p1 R2^2 q1 + 24 m R2^3 s r2^3 p1 Q1 ta^2 q1 \\
& + 6 m R2^5 s c^2 r2 p1 Q1 q1 \\
& + 2 (((m R2^4 s c^4 Q1 + m R2^4 s c^4 p1 Q0) q1 + m R2^4 s c^4 p1 Q1 q0) P1 + m R2^4 s c^4 p1 Q1 q1) \\
R1^2 & \\
& + 4 (((m R2^5 s c^4 Q1 + m R2^5 s c^4 p1 Q0) q1 + m R2^5 s c^4 p1 Q1 q0) P1 + m R2^5 s c^4 p1 Q1 q1) R1 \\
& + 24 (((Q0 R1^2 r2^3 p1 + Q1 R1^2 r2^3) q1 + Q1 R1^2 r2^3 p1 q0) P1 + Q1 R1^2 r2^3 p1 q1) R2 \\
& + 24 (((Q0 R1^2 r2^3 p1 + Q1 R1^2 r2^3) ta^2 q1 + Q1 R1^2 r2^3 p1 ta^2 q0) P1 + Q1 R1^2 r2^3 p1 ta^2 q1) \\
R2 & + 12 (((m R2^2 s r2^3 Q1 + m R2^2 s r2^3 p1 Q0) ta^2 q1 + m R2^2 s r2^3 p1 Q1 ta^2 q0) P1 \\
& + m R2^2 s r2^3 p1 Q1 ta^2 q1) R1 - 6 (((m R2^4 s^3 c^2 r2 Q1 + m R2^4 s^3 c^2 r2 p1 Q0) q1 \\
& + m R2^4 s^3 c^2 r2 p1 Q1 q0) P1 + m R2^4 s^3 c^2 r2 p1 Q1 q1) R1 + 3 (((m R2^4 s c^2 r2 Q1 \\
& + m R2^4 s c^2 r2 p1 Q0) q1 + m R2^4 s c^2 r2 p1 Q1 q0) P1 + m R2^4 s c^2 r2 p1 Q1 q1) R1
\end{aligned}$$

> E3:=coeff(epsilon2,s1^3);

$$\begin{aligned}
E3 := & q0 (4 (((m R2^5 s c^4 Q1 + m R2^5 s c^4 p1 Q0) q1 + m R2^5 s c^4 p1 Q1 q0) P1 + m R2^5 s c^4 p1 Q1 q1) r2 \\
& ta^2 + 48 (2 Q0 Q1 R1^2 r2^3 p1^2 + 2 Q1^2 R1^2 r2^3 p1) ta^2 R2 \\
& - 2 (2 m R2^5 s c^4 p1 Q1^2 + 2 m R2^5 s c^4 p1^2 Q0 Q1) r2 ta^2 \\
& + 24 (2 m R2^2 s r2^3 p1 Q1^2 + 2 m R2^2 s r2^3 p1^2 Q0 Q1) ta^2 R1 \\
& + 6 (2 m R2^4 s^3 c^2 p1 Q1^2 + 2 m R2^4 s^3 c^2 p1^2 Q0 Q1) r2 R1 + 48 Q0 Q1 R1^3 r2^3 p1^2 \\
& + 48 Q1^2 R1^3 r2^3 p1 - (2 m R2^4 s c^4 q0 q1 P1^2 + 2 m R2^4 s c^4 q1^2 P1) r2 ta^2 R1 \\
& - (2 m R2^4 s c^4 p1 Q1^2 + 2 m R2^4 s c^4 p1^2 Q0 Q1) r2 ta^2 R1 \\
& + 24 (2 Q0 Q1 R1^3 r2^3 p1^2 + 2 Q1^2 R1^3 r2^3 p1) ta^2 \\
& - (2 m R2^4 s c^4 q0 q1 P1^2 + 2 m R2^4 s c^4 q1^2 P1) R1^2 \\
& - 2 (2 m R2^5 s c^4 q0 q1 P1^2 + 2 m R2^5 s c^4 q1^2 P1) R1 \\
& - 2 (2 m R2^5 s c^4 q0 q1 P1^2 + 2 m R2^5 s c^4 q1^2 P1) r2 ta^2 \\
& + 12 (2 Q0 Q1 R1^3 r2^2 p1^2 + 2 Q1^2 R1^3 r2^2 p1) R2 \\
& - 12 (2 m R2^2 s r2^2 p1 Q1^2 + 2 m R2^2 s r2^2 p1^2 Q0 Q1) R1^2 \\
& + 24 (2 Q0 Q1 R1^2 r2^2 p1^2 + 2 Q1^2 R1^2 r2^2 p1) R2^2 \\
& + 48 (2 m R2^3 s r2^3 p1 Q1^2 + 2 m R2^3 s r2^3 p1^2 Q0 Q1) ta^2 \\
& - 2 (2 m R2^5 s c^4 p1 Q1^2 + 2 m R2^5 s c^4 p1^2 Q0 Q1) R1 \\
& + 48 (2 Q0 Q1 R1^2 r2^3 p1^2 + 2 Q1^2 R1^2 r2^3 p1) R2 \\
& - 24 (2 m R2^3 s r2^2 p1 Q1^2 + 2 m R2^3 s r2^2 p1^2 Q0 Q1) R1 \\
& + 12 (2 m R2^5 s^3 c^2 p1 Q1^2 + 2 m R2^5 s^3 c^2 p1^2 Q0 Q1) r2 \\
& - (2 m R2^4 s c^4 p1 Q1^2 + 2 m R2^4 s c^4 p1^2 Q0 Q1) R1^2 + 24 m R2^5 s c^2 r2 p1 Q1^2 \\
& + 24 m R2^5 s c^2 r2 p1^2 Q0 Q1 + 12 ((Q0 R1^3 r2^3 p1 + Q1 R1^3 r2^3) q1 + Q1 R1^3 r2^3 p1 q0) P1 \\
& + 12 Q1 R1^3 r2^3 p1 q1 + 6 (2 m R2^4 s c^2 r2 p1 Q1^2 + 2 m R2^4 s c^2 r2 p1^2 Q0 Q1) R1 \\
& + 12 ((m R2^2 s r2^2 Q1 + m R2^2 s r2^2 p1 Q0) R1^2 q1 + m R2^2 s r2^2 p1 Q1 R1^2 q0) P1 \\
& + 12 m R2^2 s r2^2 p1 Q1 R1^2 q1
\end{aligned}$$

$$\begin{aligned}
& + 24 ((m R2^3 s r2^2 Q1 + m R2^3 s r2^2 p1 Q0) R1 q1 + m R2^3 s r2^2 p1 Q1 R1 q0) P1 \\
& - 12 ((m R2^5 s^3 c^2 r2 Q1 + m R2^5 s^3 c^2 r2 p1 Q0) q1 + m R2^5 s^3 c^2 r2 p1 Q1 q0) P1 \\
& + 12 ((Q0 R1^3 r2^3 p1 + Q1 R1^3 r2^3) ta^2 q1 + Q1 R1^3 r2^3 p1 ta^2 q0) P1 \\
& + 24 m R2^3 s r2^2 p1 Q1 R1 q1 - 12 m R2^5 s^3 c^2 r2 p1 Q1 q1 + 12 Q1 R1^3 r2^3 p1 ta^2 q1 \\
& - 12 ((Q0 R1^3 r2^2 p1 + Q1 R1^3 r2^2) R2 q1 + Q1 R1^3 r2^2 p1 R2 q0) P1 \\
& - 24 ((Q0 R1^2 r2^2 p1 + Q1 R1^2 r2^2) R2^2 q1 + Q1 R1^2 r2^2 p1 R2^2 q0) P1 \\
& + 24 ((m R2^3 s r2^3 Q1 + m R2^3 s r2^3 p1 Q0) ta^2 q1 + m R2^3 s r2^3 p1 Q1 ta^2 q0) P1 \\
& + 6 ((m R2^5 s c^2 r2 Q1 + m R2^5 s c^2 r2 p1 Q0) q1 + m R2^5 s c^2 r2 p1 Q1 q0) P1 \\
& - 12 Q1 R1^3 r2^2 p1 R2 q1 + 2 (((m R2^4 s c^4 Q1 + m R2^4 s c^4 p1 Q0) q1 + m R2^4 s c^4 p1 Q1 q0) P1 \\
& + m R2^4 s c^4 p1 Q1 q1) r2 ta^2 R1 - 24 Q1 R1^2 r2^2 p1 R2^2 q1 + 24 m R2^3 s r2^3 p1 Q1 ta^2 q1 \\
& + 6 m R2^5 s c^2 r2 p1 Q1 q1 \\
& + 2 (((m R2^4 s c^4 Q1 + m R2^4 s c^4 p1 Q0) q1 + m R2^4 s c^4 p1 Q1 q0) P1 + m R2^4 s c^4 p1 Q1 q1) \\
& RI^2 \\
& + 4 (((m R2^5 s c^4 Q1 + m R2^5 s c^4 p1 Q0) q1 + m R2^5 s c^4 p1 Q1 q0) P1 + m R2^5 s c^4 p1 Q1 q1) RI \\
& + 24 (((Q0 R1^2 r2^3 p1 + Q1 R1^2 r2^3) q1 + Q1 R1^2 r2^3 p1 q0) P1 + Q1 R1^2 r2^3 p1 q1) R2 \\
& + 24 (((Q0 R1^2 r2^3 p1 + Q1 R1^2 r2^3) ta^2 q1 + Q1 R1^2 r2^3 p1 ta^2 q0) P1 + Q1 R1^2 r2^3 p1 ta^2 q1) \\
& R2 + 12 (((m R2^2 s r2^3 Q1 + m R2^2 s r2^3 p1 Q0) ta^2 q1 + m R2^2 s r2^3 p1 Q1 ta^2 q0) P1 \\
& + m R2^2 s r2^3 p1 Q1 ta^2 q1) R1 - 6 (((m R2^4 s^3 c^2 r2 Q1 + m R2^4 s^3 c^2 r2 p1 Q0) q1 \\
& + m R2^4 s^3 c^2 r2 p1 Q1 q0) P1 + m R2^4 s^3 c^2 r2 p1 Q1 q1) R1 + 3 (((m R2^4 s c^2 r2 Q1 \\
& + m R2^4 s c^2 r2 p1 Q0) q1 + m R2^4 s c^2 r2 p1 Q1 q0) P1 + m R2^4 s c^2 r2 p1 Q1 q1) R1) + q1 (12 (m \\
& R2^2 s r2^2 Q1 + m R2^2 s r2^2 p1 Q0) RI^2 q1 + 24 (m R2^3 s r2^2 Q1 + m R2^3 s r2^2 p1 Q0) RI q1 \\
& + 24 (m R2^3 s r2^3 Q1 + m R2^3 s r2^3 p1 Q0) ta^2 q1 + 12 (Q0 R1^3 r2^3 p1 + Q1 R1^3 r2^3) ta^2 q1 \\
& + 24 Q1^2 R1^3 r2^3 + 4 ((m R2^5 s c^4 Q0 q1 + (m R2^5 s c^4 Q1 + m R2^5 s c^4 p1 Q0) q0) P1 \\
& + (m R2^5 s c^4 Q1 + m R2^5 s c^4 p1 Q0) q1 + m R2^5 s c^4 p1 Q1 q0) r2 ta^2 \\
& - 2 (m R2^5 s c^4 q0^2 PI^2 + 4 m R2^5 s c^4 q0 q1 P1 + m R2^5 s c^4 q1^2) r2 ta^2 \\
& - 12 (Q0 R1^3 r2^2 p1 + Q1 R1^3 r2^2) R2 q1 - 24 Q1 R1^2 r2^2 p1 R2^2 q0 \\
& + 24 (m R2^2 s r2^3 Q1^2 + 4 m R2^2 s r2^3 p1 Q0 Q1 + m R2^2 s r2^3 p1^2 Q0^2) ta^2 R1 \\
& - 24 (Q0 R1^2 r2^2 p1 + Q1 R1^2 r2^2) R2^2 q1 + 24 Q0^2 R1^3 r2^3 p1^2 + 96 Q0 Q1 R1^3 r2^3 p1 \\
& - (m R2^4 s c^4 q0^2 PI^2 + 4 m R2^4 s c^4 q0 q1 P1 + m R2^4 s c^4 q1^2) r2 ta^2 R1 \\
& - (m R2^4 s c^4 Q1^2 + 4 m R2^4 s c^4 p1 Q0 Q1 + m R2^4 s c^4 p1^2 Q0^2) r2 ta^2 R1 \\
& + 24 (Q0^2 R1^3 r2^3 p1^2 + 4 Q0 Q1 R1^3 r2^3 p1 + Q1^2 R1^3 r2^3) ta^2 \\
& - (m R2^4 s c^4 q0^2 PI^2 + 4 m R2^4 s c^4 q0 q1 P1 + m R2^4 s c^4 q1^2) RI^2 \\
& - 2 (m R2^5 s c^4 q0^2 PI^2 + 4 m R2^5 s c^4 q0 q1 P1 + m R2^5 s c^4 q1^2) RI \\
& + 12 (Q0^2 R1^3 r2^2 p1^2 + 4 Q0 Q1 R1^3 r2^2 p1 + Q1^2 R1^3 r2^2) R2 \\
& - 12 (m R2^2 s r2^2 Q1^2 + 4 m R2^2 s r2^2 p1 Q0 Q1 + m R2^2 s r2^2 p1^2 Q0^2) RI^2 \\
& + 24 (Q0^2 R1^2 r2^2 p1^2 + 4 Q0 Q1 R1^2 r2^2 p1 + Q1^2 R1^2 r2^2) R2^2 \\
& + 48 (m R2^3 s r2^3 Q1^2 + 4 m R2^3 s r2^3 p1 Q0 Q1 + m R2^3 s r2^3 p1^2 Q0^2) ta^2 \\
& - 2 (m R2^5 s c^4 Q1^2 + 4 m R2^5 s c^4 p1 Q0 Q1 + m R2^5 s c^4 p1^2 Q0^2) RI \\
& + 48 (Q0^2 R1^2 r2^3 p1^2 + 4 Q0 Q1 R1^2 r2^3 p1 + Q1^2 R1^2 r2^3) R2 \\
& - 24 (m R2^3 s r2^2 Q1^2 + 4 m R2^3 s r2^2 p1 Q0 Q1 + m R2^3 s r2^2 p1^2 Q0^2) RI \\
& + 12 (m R2^5 s^3 c^2 Q1^2 + 4 m R2^5 s^3 c^2 p1 Q0 Q1 + m R2^5 s^3 c^2 p1^2 Q0^2) R2 \\
& - (m R2^4 s c^4 Q1^2 + 4 m R2^4 s c^4 p1 Q0 Q1 + m R2^4 s c^4 p1^2 Q0^2) RI^2 + 12 m R2^5 s c^2 r2 Q1^2
\end{aligned}$$

$$\begin{aligned}
& + 12 m R2^5 s c^2 r2 p1^2 Q0^2 + 48 m R2^5 s c^2 r2 p1 Q0 Q1 + 12 (Q0 R1^3 r2^3 p1 + Q1 R1^3 r2^3) q1 \\
& + 12 (Q0 R1^3 r2^3 q1 + (Q0 R1^3 r2^3 p1 + Q1 R1^3 r2^3) q0) P1 + 12 Q1 R1^3 r2^3 p1 q0 \\
& - 2 (m R2^5 s c^4 Q1^2 + 4 m R2^5 s c^4 p1 Q0 Q1 + m R2^5 s c^4 p1^2 Q0^2) r2 ta^2 \\
& + 48 (Q0^2 R1^2 r2^3 p1^2 + 4 Q0 Q1 R1^2 r2^3 p1 + Q1^2 R1^2 r2^3) ta^2 R2 \\
& + 6 (m R2^4 s c^2 r2 Q1^2 + 4 m R2^4 s c^2 r2 p1 Q0 Q1 + m R2^4 s c^2 r2 p1^2 Q0^2) RI \\
& + 12 (m R2^2 s r2^2 Q0 R1^2 q1 + (m R2^2 s r2^2 Q1 + m R2^2 s r2^2 p1 Q0) R1^2 q0) P1 \\
& + 6 (m R2^4 s^3 c^2 Q1^2 + 4 m R2^4 s^3 c^2 p1 Q0 Q1 + m R2^4 s^3 c^2 p1^2 Q0^2) r2 RI \\
& + 12 m R2^2 s r2^2 p1 Q1 R1^2 q0 \\
& + 24 (m R2^3 s r2^2 Q0 R1 q1 + (m R2^3 s r2^2 Q1 + m R2^3 s r2^2 p1 Q0) R1 q0) P1 \\
& - 12 (m R2^5 s^3 c^2 r2 Q1 + m R2^5 s^3 c^2 r2 p1 Q0) q1 \\
& - 12 (m R2^5 s^3 c^2 r2 Q0 q1 + (m R2^5 s^3 c^2 r2 Q1 + m R2^5 s^3 c^2 r2 p1 Q0) q0) P1 \\
& + 12 (Q0 R1^3 r2^3 ta^2 q1 + (Q0 R1^3 r2^3 p1 + Q1 R1^3 r2^3) ta^2 q0) P1 + 24 m R2^3 s r2^2 p1 Q1 R1 q0 \\
& - 12 m R2^5 s^3 c^2 r2 p1 Q1 q0 + 12 Q1 R1^3 r2^3 p1 ta^2 q0 \\
& - 12 (Q0 R1^3 r2^2 R2 q1 + (Q0 R1^3 r2^2 p1 + Q1 R1^3 r2^2) R2 q0) P1 \\
& - 24 (Q0 R1^2 r2^2 R2^2 q1 + (Q0 R1^2 r2^2 p1 + Q1 R1^2 r2^2) R2^2 q0) P1 \\
& + 24 (m R2^3 s r2^3 Q0 ta^2 q1 + (m R2^3 s r2^3 Q1 + m R2^3 s r2^3 p1 Q0) ta^2 q0) P1 \\
& + 6 (m R2^5 s c^2 r2 Q1 + m R2^5 s c^2 r2 p1 Q0) q1 \\
& + 6 (m R2^5 s c^2 r2 Q0 q1 + (m R2^5 s c^2 r2 Q1 + m R2^5 s c^2 r2 p1 Q0) q0) P1 \\
& - 12 Q1 R1^3 r2^2 p1 R2 q0 + 2 ((m R2^4 s c^4 Q0 q1 + (m R2^4 s c^4 Q1 + m R2^4 s c^4 p1 Q0) q0) P1 \\
& + (m R2^4 s c^4 Q1 + m R2^4 s c^4 p1 Q0) q1 + m R2^4 s c^4 p1 Q1 q0) r2 ta^2 RI \\
& + 24 m R2^3 s r2^3 p1 Q1 ta^2 q0 + 6 m R2^5 s c^2 r2 p1 Q1 q0 + 2 ((m R2^4 s c^4 Q0 q1 \\
& + (m R2^4 s c^4 Q1 + m R2^4 s c^4 p1 Q0) q0) P1 + (m R2^4 s c^4 Q1 + m R2^4 s c^4 p1 Q0) q1 \\
& + m R2^4 s c^4 p1 Q1 q0) RI^2 + 4 ((m R2^5 s c^4 Q0 q1 + (m R2^5 s c^4 Q1 + m R2^5 s c^4 p1 Q0) q0) P1 \\
& + (m R2^5 s c^4 Q1 + m R2^5 s c^4 p1 Q0) q1 + m R2^5 s c^4 p1 Q1 q0) RI + 24 ((Q0 R1^2 r2^3 q1 \\
& + (Q0 R1^2 r2^3 p1 + Q1 R1^2 r2^3) q0) P1 + (Q0 R1^2 r2^3 p1 + Q1 R1^2 r2^3) q1 + Q1 R1^2 r2^3 p1 q0) \\
& R2 + 24 ((Q0 R1^2 r2^3 ta^2 q1 + (Q0 R1^2 r2^3 p1 + Q1 R1^2 r2^3) ta^2 q0) P1 \\
& + (Q0 R1^2 r2^3 p1 + Q1 R1^2 r2^3) ta^2 q1 + Q1 R1^2 r2^3 p1 ta^2 q0) R2 + 12 ((m R2^2 s r2^3 Q0 ta^2 q1 \\
& + (m R2^2 s r2^3 Q1 + m R2^2 s r2^3 p1 Q0) ta^2 q0) P1 + (m R2^2 s r2^3 Q1 + m R2^2 s r2^3 p1 Q0) ta^2 q1 \\
& + m R2^2 s r2^3 p1 Q1 ta^2 q0) R1 - 6 ((m R2^4 s^3 c^2 r2 Q0 q1 \\
& + (m R2^4 s^3 c^2 r2 Q1 + m R2^4 s^3 c^2 r2 p1 Q0) q0) P1 \\
& + (m R2^4 s^3 c^2 r2 Q1 + m R2^4 s^3 c^2 r2 p1 Q0) q1 + m R2^4 s^3 c^2 r2 p1 Q1 q0) RI + 3 ((m R2^4 s c^2 \\
& r2 Q0 q1 + (m R2^4 s c^2 r2 Q1 + m R2^4 s c^2 r2 p1 Q0) q0) P1 \\
& + (m R2^4 s c^2 r2 Q1 + m R2^4 s c^2 r2 p1 Q0) q1 + m R2^4 s c^2 r2 p1 Q1 q0) RI
\end{aligned}$$

> E2:=coeff(epsilon2,s1^2);

$$\begin{aligned}
E2 := & q0 (12 (m R2^2 s r2^2 Q1 + m R2^2 s r2^2 p1 Q0) R1^2 q1 \\
& + 24 (m R2^3 s r2^2 Q1 + m R2^3 s r2^2 p1 Q0) R1 q1 \\
& + 24 (m R2^3 s r2^3 Q1 + m R2^3 s r2^3 p1 Q0) ta^2 q1 + 12 (Q0 R1^3 r2^3 p1 + Q1 R1^3 r2^3) ta^2 q1 \\
& + 24 Q1^2 R1^3 r2^3 + 4 ((m R2^5 s c^4 Q0 q1 + (m R2^5 s c^4 Q1 + m R2^5 s c^4 p1 Q0) q0) P1 \\
& + (m R2^5 s c^4 Q1 + m R2^5 s c^4 p1 Q0) q1 + m R2^5 s c^4 p1 Q1 q0) r2 ta^2 \\
& - 2 (m R2^5 s c^4 q0^2 P1^2 + 4 m R2^5 s c^4 q0 q1 P1 + m R2^5 s c^4 q1^2) r2 ta^2 \\
& - 12 (Q0 R1^3 r2^2 p1 + Q1 R1^3 r2^2) R2 q1 - 24 Q1 R1^2 r2^2 p1 R2^2 q0 \\
& + 24 (m R2^2 s r2^3 Q1^2 + 4 m R2^2 s r2^3 p1 Q0 Q1 + m R2^2 s r2^3 p1^2 Q0^2) ta^2 RI
\end{aligned}$$

$$\begin{aligned}
& -24(Q0 R1^2 r2^2 p1 + Q1 R1^2 r2^2) R2^2 q1 + 24 Q0^2 R1^3 r2^3 p1^2 + 96 Q0 Q1 R1^3 r2^3 p1 \\
& - (m R2^4 s c^4 q0^2 P1^2 + 4 m R2^4 s c^4 q0 q1 P1 + m R2^4 s c^4 q1^2) r2 t a^2 R1 \\
& - (m R2^4 s c^4 Q1^2 + 4 m R2^4 s c^4 p1 Q0 Q1 + m R2^4 s c^4 p1^2 Q0^2) r2 t a^2 R1 \\
& + 24 (Q0^2 R1^3 r2^3 p1^2 + 4 Q0 Q1 R1^3 r2^3 p1 + Q1^2 R1^3 r2^3) t a^2 \\
& - (m R2^4 s c^4 q0^2 P1^2 + 4 m R2^4 s c^4 q0 q1 P1 + m R2^4 s c^4 q1^2) R1^2 \\
& - 2 (m R2^5 s c^4 q0^2 P1^2 + 4 m R2^5 s c^4 q0 q1 P1 + m R2^5 s c^4 q1^2) R1 \\
& + 12 (Q0^2 R1^3 r2^2 p1^2 + 4 Q0 Q1 R1^3 r2^2 p1 + Q1^2 R1^3 r2^2) R2 \\
& - 12 (m R2^2 s r2^2 Q1^2 + 4 m R2^2 s r2^2 p1 Q0 Q1 + m R2^2 s r2^2 p1^2 Q0^2) R1^2 \\
& + 24 (Q0^2 R1^2 r2^2 p1^2 + 4 Q0 Q1 R1^2 r2^2 p1 + Q1^2 R1^2 r2^2) R2^2 \\
& + 48 (m R2^3 s r2^3 Q1^2 + 4 m R2^3 s r2^3 p1 Q0 Q1 + m R2^3 s r2^3 p1^2 Q0^2) t a^2 \\
& - 2 (m R2^5 s c^4 Q1^2 + 4 m R2^5 s c^4 p1 Q0 Q1 + m R2^5 s c^4 p1^2 Q0^2) R1 \\
& + 48 (Q0^2 R1^2 r2^3 p1^2 + 4 Q0 Q1 R1^2 r2^3 p1 + Q1^2 R1^2 r2^3) R2 \\
& - 24 (m R2^3 s r2^2 Q1^2 + 4 m R2^3 s r2^2 p1 Q0 Q1 + m R2^3 s r2^2 p1^2 Q0^2) R1 \\
& + 12 (m R2^5 s^3 c^2 Q1^2 + 4 m R2^5 s^3 c^2 p1 Q0 Q1 + m R2^5 s^3 c^2 p1^2 Q0^2) r2 \\
& - (m R2^4 s c^4 Q1^2 + 4 m R2^4 s c^4 p1 Q0 Q1 + m R2^4 s c^4 p1^2 Q0^2) R1^2 + 12 m R2^5 s c^2 r2 Q1^2 \\
& + 12 m R2^5 s c^2 r2 p1^2 Q0^2 + 48 m R2^5 s c^2 r2 p1 Q0 Q1 + 12 (Q0 R1^3 r2^3 p1 + Q1 R1^3 r2^3) q1 \\
& + 12 (Q0 R1^3 r2^3 q1 + (Q0 R1^3 r2^3 p1 + Q1 R1^3 r2^3) q0) P1 + 12 Q1 R1^3 r2^3 p1 q0 \\
& - 2 (m R2^5 s c^4 Q1^2 + 4 m R2^5 s c^4 p1 Q0 Q1 + m R2^5 s c^4 p1^2 Q0^2) r2 t a^2 \\
& + 48 (Q0^2 R1^2 r2^3 p1^2 + 4 Q0 Q1 R1^2 r2^3 p1 + Q1^2 R1^2 r2^3) t a^2 R2 \\
& + 6 (m R2^4 s c^2 r2 Q1^2 + 4 m R2^4 s c^2 r2 p1 Q0 Q1 + m R2^4 s c^2 r2 p1^2 Q0^2) R1 \\
& + 12 (m R2^2 s r2^2 Q0 R1^2 q1 + (m R2^2 s r2^2 Q1 + m R2^2 s r2^2 p1 Q0) R1^2 q0) P1 \\
& + 6 (m R2^4 s^3 c^2 Q1^2 + 4 m R2^4 s^3 c^2 p1 Q0 Q1 + m R2^4 s^3 c^2 p1^2 Q0^2) r2 R1 \\
& + 12 m R2^2 s r2^2 p1 Q1 R1^2 q0 \\
& + 24 (m R2^3 s r2^2 Q0 R1 q1 + (m R2^3 s r2^2 Q1 + m R2^3 s r2^2 p1 Q0) R1 q0) P1 \\
& - 12 (m R2^5 s^3 c^2 r2 Q1 + m R2^5 s^3 c^2 r2 p1 Q0) q1 \\
& - 12 (m R2^5 s^3 c^2 r2 Q0 q1 + (m R2^5 s^3 c^2 r2 Q1 + m R2^5 s^3 c^2 r2 p1 Q0) q0) P1 \\
& + 12 (Q0 R1^3 r2^3 t a^2 q1 + (Q0 R1^3 r2^3 p1 + Q1 R1^3 r2^3) t a^2 q0) P1 + 24 m R2^3 s r2^2 p1 Q1 R1 q0 \\
& - 12 m R2^5 s^3 c^2 r2 p1 Q1 q0 + 12 Q1 R1^3 r2^3 p1 t a^2 q0 \\
& - 12 (Q0 R1^3 r2^2 R2 q1 + (Q0 R1^3 r2^2 p1 + Q1 R1^3 r2^2) R2 q0) P1 \\
& - 24 (Q0 R1^2 r2^2 R2^2 q1 + (Q0 R1^2 r2^2 p1 + Q1 R1^2 r2^2) R2^2 q0) P1 \\
& + 24 (m R2^3 s r2^3 Q0 t a^2 q1 + (m R2^3 s r2^3 Q1 + m R2^3 s r2^3 p1 Q0) t a^2 q0) P1 \\
& + 6 (m R2^5 s c^2 r2 Q1 + m R2^5 s c^2 r2 p1 Q0) q1 \\
& + 6 (m R2^5 s c^2 r2 Q0 q1 + (m R2^5 s c^2 r2 Q1 + m R2^5 s c^2 r2 p1 Q0) q0) P1 \\
& - 12 Q1 R1^3 r2^2 p1 R2 q0 + 2 ((m R2^4 s c^4 Q0 q1 + (m R2^4 s c^4 Q1 + m R2^4 s c^4 p1 Q0) q0) P1 \\
& + (m R2^4 s c^4 Q1 + m R2^4 s c^4 p1 Q0) q1 + m R2^4 s c^4 p1 Q1 q0) r2 t a^2 R1 \\
& + 24 m R2^3 s r2^3 p1 Q1 t a^2 q0 + 6 m R2^5 s c^2 r2 p1 Q1 q0 + 2 ((m R2^4 s c^4 Q0 q1 \\
& + (m R2^4 s c^4 Q1 + m R2^4 s c^4 p1 Q0) q0) P1 + (m R2^4 s c^4 Q1 + m R2^4 s c^4 p1 Q0) q1 \\
& + m R2^4 s c^4 p1 Q1 q0) R1^2 + 4 ((m R2^5 s c^4 Q0 q1 + (m R2^5 s c^4 Q1 + m R2^5 s c^4 p1 Q0) q0) P1 \\
& + (m R2^5 s c^4 Q1 + m R2^5 s c^4 p1 Q0) q1 + m R2^5 s c^4 p1 Q1 q0) R1 + 24 ((Q0 R1^2 r2^3 q1 \\
& + (Q0 R1^2 r2^3 p1 + Q1 R1^2 r2^3) q0) P1 + (Q0 R1^2 r2^3 p1 + Q1 R1^2 r2^3) q1 + Q1 R1^2 r2^3 p1 q0) \\
& R2 + 24 ((Q0 R1^2 r2^3 t a^2 q1 + (Q0 R1^2 r2^3 p1 + Q1 R1^2 r2^3) t a^2 q0) P1 \\
& + (Q0 R1^2 r2^3 p1 + Q1 R1^2 r2^3) t a^2 q1 + Q1 R1^2 r2^3 p1 t a^2 q0) R2 + 12 ((m R2^2 s r2^3 Q0 t a^2 q1
\end{aligned}$$

$$\begin{aligned}
& + (m R2^2 s r2^3 Q1 + m R2^2 s r2^3 p1 Q0) ta^2 q0 P1 + (m R2^2 s r2^3 Q1 + m R2^2 s r2^3 p1 Q0) ta^2 q1 \\
& + m R2^2 s r2^3 p1 Q1 ta^2 q0 RI - 6 ((m R2^4 s^3 c^2 r2 Q0 q1 \\
& + (m R2^4 s^3 c^2 r2 Q1 + m R2^4 s^3 c^2 r2 p1 Q0) q0) P1 \\
& + (m R2^4 s^3 c^2 r2 Q1 + m R2^4 s^3 c^2 r2 p1 Q0) q1 + m R2^4 s^3 c^2 r2 p1 Q1 q0) RI + 3 ((m R2^4 s c^2 \\
& r2 Q0 q1 + (m R2^4 s c^2 r2 Q1 + m R2^4 s c^2 r2 p1 Q0) q0) P1 \\
& + (m R2^4 s c^2 r2 Q1 + m R2^4 s c^2 r2 p1 Q0) q1 + m R2^4 s c^2 r2 p1 Q1 q0) RI) + q1 ( \\
& - 2 (2 m R2^5 s c^4 Q0 Q1 + 2 m R2^5 s c^4 p1 Q0^2) r2 ta^2 - 24 (Q0 RI^2 r2^2 p1 + Q1 RI^2 r2^2) R2^2 q0 \\
& - 12 (Q0 RI^3 r2^2 p1 + Q1 RI^3 r2^2) R2 q0 + 6 (2 m R2^4 s^3 c^2 Q0 Q1 + 2 m R2^4 s^3 c^2 p1 Q0^2) r2 RI \\
& - 2 (2 m R2^5 s c^4 q0^2 P1 + 2 m R2^5 s c^4 q0 q1) r2 ta^2 \\
& + 4 (m R2^5 s c^4 Q0 q0 P1 + m R2^5 s c^4 Q0 q1 + (m R2^5 s c^4 Q1 + m R2^5 s c^4 p1 Q0) q0) r2 ta^2 \\
& + 48 (2 Q0^2 RI^2 r2^3 p1 + 2 Q0 Q1 RI^2 r2^3) ta^2 R2 \\
& + 24 (2 m R2^2 s r2^3 Q0 Q1 + 2 m R2^2 s r2^3 p1 Q0^2) ta^2 RI \\
& + 12 (m R2^2 s r2^2 Q1 + m R2^2 s r2^2 p1 Q0) RI^2 q0 + 12 (Q0 RI^3 r2^3 p1 + Q1 RI^3 r2^3) ta^2 q0 \\
& + 24 (m R2^3 s r2^2 Q1 + m R2^3 s r2^2 p1 Q0) RI q0 \\
& + 24 (m R2^3 s r2^3 Q1 + m R2^3 s r2^3 p1 Q0) ta^2 q0 + 48 Q0 Q1 RI^3 r2^3 + 48 Q0^2 RI^3 r2^3 p1 \\
& - (2 m R2^4 s c^4 q0^2 P1 + 2 m R2^4 s c^4 q0 q1) r2 ta^2 RI \\
& - (2 m R2^4 s c^4 Q0 Q1 + 2 m R2^4 s c^4 p1 Q0^2) r2 ta^2 RI + 12 Q0 RI^3 r2^3 q1 \\
& + 24 (2 Q0^2 RI^3 r2^3 p1 + 2 Q0 Q1 RI^3 r2^3) ta^2 - (2 m R2^4 s c^4 q0^2 P1 + 2 m R2^4 s c^4 q0 q1) RI^2 \\
& - 2 (2 m R2^5 s c^4 q0^2 P1 + 2 m R2^5 s c^4 q0 q1) RI + 12 (2 Q0^2 RI^3 r2^2 p1 + 2 Q0 Q1 RI^3 r2^2) R2 \\
& - 12 (2 m R2^2 s r2^2 Q0 Q1 + 2 m R2^2 s r2^2 p1 Q0^2) RI^2 \\
& + 24 (2 Q0^2 RI^2 r2^2 p1 + 2 Q0 Q1 RI^2 r2^2) R2^2 \\
& + 48 (2 m R2^3 s r2^3 Q0 Q1 + 2 m R2^3 s r2^3 p1 Q0^2) ta^2 \\
& - 2 (2 m R2^5 s c^4 Q0 Q1 + 2 m R2^5 s c^4 p1 Q0^2) RI + 48 (2 Q0^2 RI^2 r2^3 p1 + 2 Q0 Q1 RI^2 r2^3) R2 \\
& - 24 (2 m R2^3 s r2^2 Q0 Q1 + 2 m R2^3 s r2^2 p1 Q0^2) RI \\
& + 12 (2 m R2^5 s^3 c^2 Q0 Q1 + 2 m R2^5 s^3 c^2 p1 Q0^2) r2 \\
& - (2 m R2^4 s c^4 Q0 Q1 + 2 m R2^4 s c^4 p1 Q0^2) RI^2 + 24 m R2^5 s c^2 r2 Q0 Q1 \\
& + 24 m R2^5 s c^2 r2 p1 Q0^2 + 12 (Q0 RI^3 r2^3 p1 + Q1 RI^3 r2^3) q0 + 12 Q0 RI^3 r2^3 q0 P1 \\
& + 6 (2 m R2^4 s c^2 r2 Q0 Q1 + 2 m R2^4 s c^2 r2 p1 Q0^2) RI + 12 m R2^2 s r2^2 Q0 RI^2 q1 \\
& - 12 (m R2^5 s^3 c^2 r2 Q1 + m R2^5 s^3 c^2 r2 p1 Q0) q0 + 12 m R2^2 s r2^2 Q0 RI^2 q0 P1 \\
& + 24 m R2^3 s r2^2 Q0 RI q1 + 24 m R2^3 s r2^2 Q0 RI q0 P1 - 12 m R2^5 s^3 c^2 r2 Q0 q1 \\
& - 12 m R2^5 s^3 c^2 r2 Q0 q0 P1 + 12 Q0 RI^3 r2^3 ta^2 q1 + 12 Q0 RI^3 r2^3 ta^2 q0 P1 \\
& + 6 (m R2^5 s c^2 r2 Q1 + m R2^5 s c^2 r2 p1 Q0) q0 - 12 Q0 RI^3 r2^2 R2 q1 - 12 Q0 RI^3 r2^2 R2 q0 P1 \\
& + 2 (m R2^4 s c^4 Q0 q0 P1 + m R2^4 s c^4 Q0 q1 + (m R2^4 s c^4 Q1 + m R2^4 s c^4 p1 Q0) q0) r2 ta^2 RI \\
& - 24 Q0 RI^2 r2^2 R2^2 q1 - 24 Q0 RI^2 r2^2 R2^2 q0 P1 + 24 m R2^3 s r2^3 Q0 ta^2 q1 \\
& + 24 m R2^3 s r2^3 Q0 ta^2 q0 P1 + 6 m R2^5 s c^2 r2 Q0 q1 + 6 m R2^5 s c^2 r2 Q0 q0 P1 \\
& + 4 (m R2^5 s c^4 Q0 q0 P1 + m R2^5 s c^4 Q0 q1 + (m R2^5 s c^4 Q1 + m R2^5 s c^4 p1 Q0) q0) RI \\
& + 2 (m R2^4 s c^4 Q0 q0 P1 + m R2^4 s c^4 Q0 q1 + (m R2^4 s c^4 Q1 + m R2^4 s c^4 p1 Q0) q0) RI^2 \\
& + 24 (Q0 RI^2 r2^3 q0 P1 + Q0 RI^2 r2^3 q1 + (Q0 RI^2 r2^3 p1 + Q1 RI^2 r2^3) q0) R2 \\
& + 24 (Q0 RI^2 r2^3 ta^2 q0 P1 + Q0 RI^2 r2^3 ta^2 q1 + (Q0 RI^2 r2^3 p1 + Q1 RI^2 r2^3) ta^2 q0) R2 + 12 \\
& (m R2^2 s r2^3 Q0 ta^2 q0 P1 + m R2^2 s r2^3 Q0 ta^2 q1 + (m R2^2 s r2^3 Q1 + m R2^2 s r2^3 p1 Q0) ta^2 q0 \\
& ) RI - 6 (m R2^4 s^3 c^2 r2 Q0 q0 P1 + m R2^4 s^3 c^2 r2 Q0 q1 \\
& + (m R2^4 s^3 c^2 r2 Q1 + m R2^4 s^3 c^2 r2 p1 Q0) q0) RI + 3 (m R2^4 s c^2 r2 Q0 q0 P1
\end{aligned}$$

$$+ m R2^4 s c^2 r2 Q0 q1 + (m R2^4 s c^2 r2 Q1 + m R2^4 s c^2 r2 p1 Q0) R1)$$

> E1:=coeff(epsilon2,s1);

$$\begin{aligned}
E1 := & q0 (-2 (2 m R2^5 s c^4 Q0 Q1 + 2 m R2^5 s c^4 p1 Q0^2) r2 ta^2 \\
& - 24 (Q0 R1^2 r2^2 p1 + Q1 R1^2 r2^2) R2^2 q0 - 12 (Q0 R1^3 r2^2 p1 + Q1 R1^3 r2^2) R2 q0 \\
& + 6 (2 m R2^4 s^3 c^2 Q0 Q1 + 2 m R2^4 s^3 c^2 p1 Q0^2) r2 R1 \\
& - 2 (2 m R2^5 s c^4 q0^2 P1 + 2 m R2^5 s c^4 q0 q1) r2 ta^2 \\
& + 4 (m R2^5 s c^4 Q0 q0 P1 + m R2^5 s c^4 Q0 q1 + (m R2^5 s c^4 Q1 + m R2^5 s c^4 p1 Q0) q0) r2 ta^2 \\
& + 48 (2 Q0^2 R1^2 r2^3 p1 + 2 Q0 Q1 R1^2 r2^3) ta^2 R2 \\
& + 24 (2 m R2^2 s r2^3 Q0 Q1 + 2 m R2^2 s r2^3 p1 Q0^2) ta^2 R1 \\
& + 12 (m R2^2 s r2^2 Q1 + m R2^2 s r2^2 p1 Q0) R1^2 q0 + 12 (Q0 R1^3 r2^3 p1 + Q1 R1^3 r2^3) ta^2 q0 \\
& + 24 (m R2^3 s r2^2 Q1 + m R2^3 s r2^2 p1 Q0) R1 q0 \\
& + 24 (m R2^3 s r2^3 Q1 + m R2^3 s r2^3 p1 Q0) ta^2 q0 + 48 Q0 Q1 R1^3 r2^3 + 48 Q0^2 R1^3 r2^3 p1 \\
& - (2 m R2^4 s c^4 q0^2 P1 + 2 m R2^4 s c^4 q0 q1) r2 ta^2 R1 \\
& - (2 m R2^4 s c^4 Q0 Q1 + 2 m R2^4 s c^4 p1 Q0^2) r2 ta^2 R1 + 12 Q0 R1^3 r2^3 q1 \\
& + 24 (2 Q0^2 R1^3 r2^3 p1 + 2 Q0 Q1 R1^3 r2^3) ta^2 - (2 m R2^4 s c^4 q0^2 P1 + 2 m R2^4 s c^4 q0 q1) R1^2 \\
& - 2 (2 m R2^5 s c^4 q0^2 P1 + 2 m R2^5 s c^4 q0 q1) R1 + 12 (2 Q0^2 R1^3 r2^2 p1 + 2 Q0 Q1 R1^3 r2^2) R2 \\
& - 12 (2 m R2^2 s r2^2 Q0 Q1 + 2 m R2^2 s r2^2 p1 Q0^2) R1^2 \\
& + 24 (2 Q0^2 R1^2 r2^2 p1 + 2 Q0 Q1 R1^2 r2^2) R2^2 \\
& + 48 (2 m R2^3 s r2^3 Q0 Q1 + 2 m R2^3 s r2^3 p1 Q0^2) ta^2 \\
& - 2 (2 m R2^5 s c^4 Q0 Q1 + 2 m R2^5 s c^4 p1 Q0^2) R1 + 48 (2 Q0^2 R1^2 r2^3 p1 + 2 Q0 Q1 R1^2 r2^3) R2 \\
& - 24 (2 m R2^3 s r2^2 Q0 Q1 + 2 m R2^3 s r2^2 p1 Q0^2) R1 \\
& + 12 (2 m R2^5 s^3 c^2 Q0 Q1 + 2 m R2^5 s^3 c^2 p1 Q0^2) r2 \\
& - (2 m R2^4 s c^4 Q0 Q1 + 2 m R2^4 s c^4 p1 Q0^2) R1^2 + 24 m R2^5 s c^2 r2 Q0 Q1 \\
& + 24 m R2^5 s c^2 r2 p1 Q0^2 + 12 (Q0 R1^3 r2^3 p1 + Q1 R1^3 r2^3) q0 + 12 Q0 R1^3 r2^3 q0 P1 \\
& + 6 (2 m R2^4 s c^2 r2 Q0 Q1 + 2 m R2^4 s c^2 r2 p1 Q0^2) R1 + 12 m R2^2 s r2^2 Q0 R1^2 q1 \\
& - 12 (m R2^5 s^3 c^2 r2 Q1 + m R2^5 s^3 c^2 r2 p1 Q0) q0 + 12 m R2^2 s r2^2 Q0 R1^2 q0 P1 \\
& + 24 m R2^3 s r2^2 Q0 R1 q1 + 24 m R2^3 s r2^2 Q0 R1 q0 P1 - 12 m R2^5 s^3 c^2 r2 Q0 q1 \\
& - 12 m R2^5 s^3 c^2 r2 Q0 q0 P1 + 12 Q0 R1^3 r2^3 ta^2 q1 + 12 Q0 R1^3 r2^3 ta^2 q0 P1 \\
& + 6 (m R2^5 s c^2 r2 Q1 + m R2^5 s c^2 r2 p1 Q0) q0 - 12 Q0 R1^3 r2^2 R2 q1 - 12 Q0 R1^3 r2^2 R2 q0 P1 \\
& + 2 (m R2^4 s c^4 Q0 q0 P1 + m R2^4 s c^4 Q0 q1 + (m R2^4 s c^4 Q1 + m R2^4 s c^4 p1 Q0) q0) r2 ta^2 R1 \\
& - 24 Q0 R1^2 r2^2 R2^2 q1 - 24 Q0 R1^2 r2^2 R2^2 q0 P1 + 24 m R2^3 s r2^3 Q0 ta^2 q1 \\
& + 24 m R2^3 s r2^3 Q0 ta^2 q0 P1 + 6 m R2^5 s c^2 r2 Q0 q1 + 6 m R2^5 s c^2 r2 Q0 q0 P1 \\
& + 4 (m R2^5 s c^4 Q0 q0 P1 + m R2^5 s c^4 Q0 q1 + (m R2^5 s c^4 Q1 + m R2^5 s c^4 p1 Q0) q0) R1 \\
& + 2 (m R2^4 s c^4 Q0 q0 P1 + m R2^4 s c^4 Q0 q1 + (m R2^4 s c^4 Q1 + m R2^4 s c^4 p1 Q0) q0) R1^2 \\
& + 24 (Q0 R1^2 r2^3 q0 P1 + Q0 R1^2 r2^3 q1 + (Q0 R1^2 r2^3 p1 + Q1 R1^2 r2^3) q0) R2 \\
& + 24 (Q0 R1^2 r2^3 ta^2 q0 P1 + Q0 R1^2 r2^3 ta^2 q1 + (Q0 R1^2 r2^3 p1 + Q1 R1^2 r2^3) ta^2 q0) R2 + 12 \\
& (m R2^2 s r2^3 Q0 ta^2 q0 P1 + m R2^2 s r2^3 Q0 ta^2 q1 + (m R2^2 s r2^3 Q1 + m R2^2 s r2^3 p1 Q0) ta^2 q0 \\
&) R1 - 6 (m R2^4 s^3 c^2 r2 Q0 q0 P1 + m R2^4 s^3 c^2 r2 Q0 q1 \\
& + (m R2^4 s^3 c^2 r2 Q1 + m R2^4 s^3 c^2 r2 p1 Q0) q0) R1 + 3 (m R2^4 s c^2 r2 Q0 q0 P1 \\
& + m R2^4 s c^2 r2 Q0 q1 + (m R2^4 s c^2 r2 Q1 + m R2^4 s c^2 r2 p1 Q0) q0) R1) + q1 (24 Q0^2 R1^3 r2^3
\end{aligned}$$

$$\begin{aligned}
& + 24 Q0^2 R1^3 r2^3 ta^2 + 12 Q0^2 R1^3 r2^2 R2 + 24 Q0^2 R1^2 r2^2 R2^2 + 48 Q0^2 R1^2 r2^3 R2 \\
& + 12 Q0 R1^3 r2^3 q0 - m R2^4 s c^4 q0^2 R1^2 - 2 m R2^5 s c^4 q0^2 R1 - 2 m R2^5 s c^4 q0^2 r2 ta^2 \\
& - m R2^4 s c^4 q0^2 r2 ta^2 R1 - 12 m R2^2 s r2^2 Q0^2 R1^2 + 48 m R2^3 s r2^3 Q0^2 ta^2 \\
& - 2 m R2^5 s c^4 Q0^2 R1 - 24 m R2^3 s r2^2 Q0^2 R1 + 12 m R2^5 s^3 c^2 Q0^2 r2 + 12 m R2^5 s c^2 r2 Q0^2 \\
& - m R2^4 s c^4 Q0^2 R1^2 - m R2^4 s c^4 Q0^2 r2 ta^2 R1 - 2 m R2^5 s c^4 Q0^2 r2 ta^2 \\
& + 48 Q0^2 R1^2 r2^3 ta^2 R2 + 24 m R2^2 s r2^3 Q0^2 ta^2 R1 + 6 m R2^4 s^3 c^2 Q0^2 r2 R1 \\
& + 6 m R2^4 s c^2 r2 Q0^2 R1 + 12 m R2^2 s r2^2 Q0 R1^2 q0 + 24 m R2^3 s r2^2 Q0 R1 q0 \\
& - 12 m R2^5 s^3 c^2 r2 Q0 q0 + 12 Q0 R1^3 r2^3 ta^2 q0 + 3 m R2^4 s c^2 r2 Q0 q0 R1 \\
& - 12 Q0 R1^3 r2^2 R2 q0 - 24 Q0 R1^2 r2^2 R2^2 q0 + 24 m R2^3 s r2^3 Q0 ta^2 q0 + 6 m R2^5 s c^2 r2 Q0 q0 \\
& + 2 m R2^4 s c^4 Q0 q0 R1^2 + 4 m R2^5 s c^4 Q0 q0 R1 + 4 m R2^5 s c^4 Q0 q0 r2 ta^2 \\
& + 24 Q0 R1^2 r2^3 q0 R2 + 24 Q0 R1^2 r2^3 ta^2 q0 R2 + 12 m R2^2 s r2^3 Q0 ta^2 q0 R1 \\
& - 6 m R2^4 s^3 c^2 r2 Q0 q0 R1 + 2 m R2^4 s c^4 Q0 q0 r2 ta^2 R1)
\end{aligned}$$

> E0:=coeff(epsilon2,s1,0);

$$\begin{aligned}
E0 := & q0 (24 Q0^2 R1^3 r2^3 + 24 Q0^2 R1^3 r2^3 ta^2 + 12 Q0^2 R1^3 r2^2 R2 + 24 Q0^2 R1^2 r2^2 R2^2 \\
& + 48 Q0^2 R1^2 r2^3 R2 + 12 Q0 R1^3 r2^3 q0 - m R2^4 s c^4 q0^2 R1^2 - 2 m R2^5 s c^4 q0^2 R1 \\
& - 2 m R2^5 s c^4 q0^2 r2 ta^2 - m R2^4 s c^4 q0^2 r2 ta^2 R1 - 12 m R2^2 s r2^2 Q0^2 R1^2 \\
& + 48 m R2^3 s r2^3 Q0^2 ta^2 - 2 m R2^5 s c^4 Q0^2 R1 - 24 m R2^3 s r2^2 Q0^2 R1 \\
& + 12 m R2^5 s^3 c^2 Q0^2 r2 + 12 m R2^5 s c^2 r2 Q0^2 - m R2^4 s c^4 Q0^2 R1^2 \\
& - m R2^4 s c^4 Q0^2 r2 ta^2 R1 - 2 m R2^5 s c^4 Q0^2 r2 ta^2 + 48 Q0^2 R1^2 r2^3 ta^2 R2 \\
& + 24 m R2^2 s r2^3 Q0^2 ta^2 R1 + 6 m R2^4 s^3 c^2 Q0^2 r2 R1 + 6 m R2^4 s c^2 r2 Q0^2 R1 \\
& + 12 m R2^2 s r2^2 Q0 R1^2 q0 + 24 m R2^3 s r2^2 Q0 R1 q0 - 12 m R2^5 s^3 c^2 r2 Q0 q0 \\
& + 12 Q0 R1^3 r2^3 ta^2 q0 + 3 m R2^4 s c^2 r2 Q0 q0 R1 - 12 Q0 R1^3 r2^2 R2 q0 - 24 Q0 R1^2 r2^2 R2^2 q0 \\
& + 24 m R2^3 s r2^3 Q0 ta^2 q0 + 6 m R2^5 s c^2 r2 Q0 q0 + 2 m R2^4 s c^4 Q0 q0 R1^2 \\
& + 4 m R2^5 s c^4 Q0 q0 R1 + 4 m R2^5 s c^4 Q0 q0 r2 ta^2 + 24 Q0 R1^2 r2^3 q0 R2 \\
& + 24 Q0 R1^2 r2^3 ta^2 q0 R2 + 12 m R2^2 s r2^3 Q0 ta^2 q0 R1 - 6 m R2^4 s^3 c^2 r2 Q0 q0 R1 \\
& + 2 m R2^4 s c^4 Q0 q0 r2 ta^2 R1)
\end{aligned}$$

> B5:=coeff(beta2,s1^5);

$$\begin{aligned}
B5 := & q1 (2 r2^2 m R2^4 s^4 c pl Q1 q1 P1 - 4 r2^2 m R2^4 s^4 c pl^2 Q1^2 - r2 m R2^5 s^2 c^3 pl^2 Q1^2 \\
& - r2 m R2^5 s^2 c^3 q1^2 P1^2 - r2^2 m R2^4 s^2 c q1^2 P1^2 - 4 r2^2 m R2^4 s^2 c pl Q1 q1 P1 \\
& - 4 r2^2 m R2^4 s^2 c pl^2 Q1^2 + 2 r2^2 m R2^4 s^4 c q1^2 P1^2 + 12 r2^4 m R2^2 s tapl Q1 q1 P1 \\
& + 24 r2^4 m R2^2 s tapl^2 Q1^2 + 2 r2 m R2^5 s^2 c^3 pl Q1 q1 P1) + q1 (-8 r2^2 m R2^4 s^4 c pl Q1^2 \\
& - 8 r2^2 m R2^4 s^4 c pl^2 Q0 Q1
\end{aligned}$$

> B4:=coeff(beta2,s1^4);

$$\begin{aligned}
B4 := & q0 (2 r2^2 m R2^4 s^4 c pl Q1 q1 P1 - 4 r2^2 m R2^4 s^4 c pl^2 Q1^2 - r2 m R2^5 s^2 c^3 pl^2 Q1^2 \\
& - r2 m R2^5 s^2 c^3 q1^2 P1^2 - r2^2 m R2^4 s^2 c q1^2 P1^2 - 4 r2^2 m R2^4 s^2 c pl Q1 q1 P1 \\
& - 4 r2^2 m R2^4 s^2 c pl^2 Q1^2 + 2 r2^2 m R2^4 s^4 c q1^2 P1^2 + 12 r2^4 m R2^2 s tapl Q1 q1 P1 \\
& + 24 r2^4 m R2^2 s tapl^2 Q1^2 + 2 r2 m R2^5 s^2 c^3 pl Q1 q1 P1) + q1 (-8 r2^2 m R2^4 s^4 c pl Q1^2 \\
& - 8 r2^2 m R2^4 s^4 c pl^2 Q0 Q1 \\
& + 2 ((r2^2 m R2^4 s^4 c Q1 + r2^2 m R2^4 s^4 c pl Q0) q1 + r2^2 m R2^4 s^4 c pl Q1 q0) P1 \\
& + 2 r2^2 m R2^4 s^4 c pl Q1 q1 \\
& - 4 ((r2^2 m R2^4 s^2 c Q1 + r2^2 m R2^4 s^2 c pl Q0) q1 + r2^2 m R2^4 s^2 c pl Q1 q0) P1 \\
& - 4 r2^2 m R2^4 s^2 c pl Q1 q1 - 8 r2^2 m R2^4 s^2 c pl Q1^2 - 8 r2^2 m R2^4 s^2 c pl^2 Q0 Q1
\end{aligned}$$

$$\begin{aligned}
& -2r2mR2^5s^2c^3q0q1P1^2 - 2r2mR2^5s^2c^3q1^2P1 - 2r2^2mR2^4s^2cq0q1P1^2 \\
& - 2r2^2mR2^4s^2cq1^2P1 + 48r2^4mR2^2sTap1Q1^2 + 48r2^4mR2^2sTap1^2Q0Q1 \\
& + 2((r2mR2^5s^2c^3Q1 + r2mR2^5s^2c^3p1Q0)q1 + r2mR2^5s^2c^3p1Q1q0)P1 \\
& + 2r2mR2^5s^2c^3p1Q1q1 - 2r2mR2^5s^2c^3p1Q1^2 - 2r2mR2^5s^2c^3p1^2Q0Q1 \\
& + 4r2^2mR2^4s^4cq0q1P1^2 + 4r2^2mR2^4s^4cq1^2P1 \\
& + 12((r2^4mR2^2sTaQ1 + r2^4mR2^2sTap1Q0)q1 + r2^4mR2^2sTap1Q1q0)P1 \\
& + 12r2^4mR2^2sTap1Q1q1
\end{aligned}$$

> B3:=coeff(beta2,s1^3);

$$\begin{aligned}
B3 := & q0(-8r2^2mR2^4s^4cp1Q1^2 - 8r2^2mR2^4s^4cp1^2Q0Q1 \\
& + 2((r2^2mR2^4s^4cQ1 + r2^2mR2^4s^4cp1Q0)q1 + r2^2mR2^4s^4cp1Q1q0)P1 \\
& + 2r2^2mR2^4s^4cp1Q1q1 \\
& - 4((r2^2mR2^4s^2cQ1 + r2^2mR2^4s^2cp1Q0)q1 + r2^2mR2^4s^2cp1Q1q0)P1 \\
& - 4r2^2mR2^4s^2cp1Q1q1 - 8r2^2mR2^4s^2cp1Q1^2 - 8r2^2mR2^4s^2cp1^2Q0Q1 \\
& - 2r2mR2^5s^2c^3q0q1P1^2 - 2r2mR2^5s^2c^3q1^2P1 - 2r2^2mR2^4s^2cq0q1P1^2 \\
& - 2r2^2mR2^4s^2cq1^2P1 + 48r2^4mR2^2sTap1Q1^2 + 48r2^4mR2^2sTap1^2Q0Q1 \\
& + 2((r2mR2^5s^2c^3Q1 + r2mR2^5s^2c^3p1Q0)q1 + r2mR2^5s^2c^3p1Q1q0)P1 \\
& + 2r2mR2^5s^2c^3p1Q1q1 - 2r2mR2^5s^2c^3p1Q1^2 - 2r2mR2^5s^2c^3p1^2Q0Q1 \\
& + 4r2^2mR2^4s^4cq0q1P1^2 + 4r2^2mR2^4s^4cq1^2P1 \\
& + 12((r2^4mR2^2sTaQ1 + r2^4mR2^2sTap1Q0)q1 + r2^4mR2^2sTap1Q1q0)P1 \\
& + 12r2^4mR2^2sTap1Q1q1 + q1(-r2mR2^5s^2c^3q1^2 - r2mR2^5s^2c^3q0^2P1^2 \\
& - 4r2mR2^5s^2c^3q0q1P1 - r2^2mR2^4s^2cq1^2 - r2^2mR2^4s^2cq0^2P1^2 \\
& - 4r2^2mR2^4s^2cq0q1P1 + 24r2^4mR2^2sTaQ1^2 + 24r2^4mR2^2sTap1^2Q0^2 \\
& + 96r2^4mR2^2sTap1Q0Q1 - 4r2^2mR2^4s^4cq1^2 - 4r2^2mR2^4s^4cp1^2Q0^2 \\
& - 16r2^2mR2^4s^4cp1Q0Q1 + 12(r2^4mR2^2sTaQ1 + r2^4mR2^2sTap1Q0)q1 \\
& + 12(r2^4mR2^2sTaQ0q1 + (r2^4mR2^2sTaQ1 + r2^4mR2^2sTap1Q0)q0)P1 \\
& + 2(r2^2mR2^4s^4cQ1 + r2^2mR2^4s^4cp1Q0)q1 \\
& + 2(r2^2mR2^4s^4cQ0q1 + (r2^2mR2^4s^4cQ1 + r2^2mR2^4s^4cp1Q0)q0)P1 \\
& - 4r2^2mR2^4s^2cQ1^2 - 4r2^2mR2^4s^2cp1^2Q0^2 - 16r2^2mR2^4s^2cp1Q0Q1 \\
& - r2mR2^5s^2c^3Q1^2 - r2mR2^5s^2c^3p1^2Q0^2 - 4r2mR2^5s^2c^3p1Q0Q1 \\
& + 2r2^2mR2^4s^4cq1^2 + 2r2^2mR2^4s^4cq0^2P1^2 + 8r2^2mR2^4s^4cq0q1P1 \\
& + 12r2^4mR2^2sTap1Q1q0 + 2r2^2mR2^4s^4cp1Q1q0 \\
& - 4(r2^2mR2^4s^2cQ1 + r2^2mR2^4s^2cp1Q0)q1 \\
& - 4(r2^2mR2^4s^2cQ0q1 + (r2^2mR2^4s^2cQ1 + r2^2mR2^4s^2cp1Q0)q0)P1 \\
& + 2(r2mR2^5s^2c^3Q1 + r2mR2^5s^2c^3p1Q0)q1 \\
& + 2(r2mR2^5s^2c^3Q0q1 + (r2mR2^5s^2c^3Q1 + r2mR2^5s^2c^3p1Q0)q0)P1 \\
& - 4r2^2mR2^4s^2cp1Q1q0 + 2r2mR2^5s^2c^3p1Q1q0
\end{aligned}$$

> B2:=coeff(beta2,s1^2);

$$\begin{aligned}
B2 := & q0(-r2mR2^5s^2c^3q1^2 - r2mR2^5s^2c^3q0^2P1^2 - 4r2mR2^5s^2c^3q0q1P1 \\
& - r2^2mR2^4s^2cq1^2 - r2^2mR2^4s^2cq0^2P1^2 - 4r2^2mR2^4s^2cq0q1P1 \\
& + 24r2^4mR2^2sTaQ1^2 + 24r2^4mR2^2sTap1^2Q0^2 + 96r2^4mR2^2sTap1Q0Q1 \\
& - 4r2^2mR2^4s^4cq1^2 - 4r2^2mR2^4s^4cp1^2Q0^2 - 16r2^2mR2^4s^4cp1Q0Q1
\end{aligned}$$

$$\begin{aligned}
& + 12(r2^4 m R2^2 s ta Q1 + r2^4 m R2^2 s tap1 Q0) q1 \\
& + 12(r2^4 m R2^2 s ta Q0 q1 + (r2^4 m R2^2 s ta Q1 + r2^4 m R2^2 s tap1 Q0) q0) P1 \\
& + 2(r2^2 m R2^4 s^4 c Q1 + r2^2 m R2^4 s^4 c p1 Q0) q1 \\
& + 2(r2^2 m R2^4 s^4 c Q0 q1 + (r2^2 m R2^4 s^4 c Q1 + r2^2 m R2^4 s^4 c p1 Q0) q0) P1 \\
& - 4 r2^2 m R2^4 s^2 c Q1^2 - 4 r2^2 m R2^4 s^2 c p1 Q0^2 - 16 r2^2 m R2^4 s^2 c p1 Q0 Q1 \\
& + 2 r2^2 m R2^4 s^4 c q1^2 + 2 r2^2 m R2^4 s^4 c q0^2 P1^2 + 8 r2^2 m R2^4 s^4 c q0 q1 P1 \\
& + 12 r2^4 m R2^2 s tap1 Q1 q0 + 2 r2^2 m R2^4 s^4 c p1 Q1 q0 \\
& - 4(r2^2 m R2^4 s^2 c Q1 + r2^2 m R2^4 s^2 c p1 Q0) q1 \\
& - 4(r2^2 m R2^4 s^2 c Q0 q1 + (r2^2 m R2^4 s^2 c Q1 + r2^2 m R2^4 s^2 c p1 Q0) q0) P1 \\
& + 2(r2 m R2^5 s^2 c^3 Q1 + r2 m R2^5 s^2 c^3 p1 Q0) q1 \\
& + 2(r2 m R2^5 s^2 c^3 Q0 q1 + (r2 m R2^5 s^2 c^3 Q1 + r2 m R2^5 s^2 c^3 p1 Q0) q0) P1 \\
& - 4 r2^2 m R2^4 s^2 c p1 Q1 q0 + 2 r2 m R2^5 s^2 c^3 p1 Q1 q0) + q1 (-2 r2 m R2^5 s^2 c^3 q0 q1 \\
& - 2 r2 m R2^5 s^2 c^3 q0^2 P1 - 2 r2^2 m R2^4 s^2 c q0 q1 - 2 r2^2 m R2^4 s^2 c q0^2 P1 \\
& + 48 r2^4 m R2^2 s ta Q0 Q1 + 48 r2^4 m R2^2 s tap1 Q0^2 - 8 r2^2 m R2^4 s^4 c Q0 Q1 \\
& - 8 r2^2 m R2^4 s^4 c p1 Q0^2 + 12(r2^4 m R2^2 s ta Q1 + r2^4 m R2^2 s tap1 Q0) q0 \\
& + 2(r2^2 m R2^4 s^4 c Q1 + r2^2 m R2^4 s^4 c p1 Q0) q0 - 8 r2^2 m R2^4 s^2 c Q0 Q1 \\
& - 8 r2^2 m R2^4 s^2 c p1 Q0^2 - 2 r2 m R2^5 s^2 c^3 Q0 Q1 - 2 r2 m R2^5 s^2 c^3 p1 Q0^2 \\
& + 4 r2^2 m R2^4 s^4 c q0 q1 + 4 r2^2 m R2^4 s^4 c q0^2 P1 + 12 r2^4 m R2^2 s ta Q0 q1 \\
& + 12 r2^4 m R2^2 s ta Q0 q0 P1 + 2 r2^2 m R2^4 s^4 c Q0 q1 \\
& - 4(r2^2 m R2^4 s^2 c Q1 + r2^2 m R2^4 s^2 c p1 Q0) q0 \\
& + 2(r2 m R2^5 s^2 c^3 Q1 + r2 m R2^5 s^2 c^3 p1 Q0) q0 + 2 r2^2 m R2^4 s^4 c Q0 q0 P1 \\
& - 4 r2^2 m R2^4 s^2 c Q0 q1 - 4 r2^2 m R2^4 s^2 c Q0 q0 P1 + 2 r2 m R2^5 s^2 c^3 Q0 q1 \\
& + 2 r2 m R2^5 s^2 c^3 Q0 q0 P1
\end{aligned}$$

```

> B1:=coeff(beta2,s1);
B1 := q0 (-2 r2 m R2^5 s^2 c^3 q0 q1 - 2 r2 m R2^5 s^2 c^3 q0^2 P1 - 2 r2^2 m R2^4 s^2 c q0 q1
- 2 r2^2 m R2^4 s^2 c q0^2 P1 + 48 r2^4 m R2^2 s ta Q0 Q1 + 48 r2^4 m R2^2 s tap1 Q0^2
- 8 r2^2 m R2^4 s^4 c Q0 Q1 - 8 r2^2 m R2^4 s^4 c p1 Q0^2
+ 12(r2^4 m R2^2 s ta Q1 + r2^4 m R2^2 s tap1 Q0) q0
+ 2(r2^2 m R2^4 s^4 c Q1 + r2^2 m R2^4 s^4 c p1 Q0) q0 - 8 r2^2 m R2^4 s^2 c Q0 Q1
- 8 r2^2 m R2^4 s^2 c p1 Q0^2 - 2 r2 m R2^5 s^2 c^3 Q0 Q1 - 2 r2 m R2^5 s^2 c^3 p1 Q0^2
+ 4 r2^2 m R2^4 s^4 c q0 q1 + 4 r2^2 m R2^4 s^4 c q0^2 P1 + 12 r2^4 m R2^2 s ta Q0 q1
+ 12 r2^4 m R2^2 s ta Q0 q0 P1 + 2 r2^2 m R2^4 s^4 c Q0 q1
- 4(r2^2 m R2^4 s^2 c Q1 + r2^2 m R2^4 s^2 c p1 Q0) q0
+ 2(r2 m R2^5 s^2 c^3 Q1 + r2 m R2^5 s^2 c^3 p1 Q0) q0 + 2 r2^2 m R2^4 s^4 c Q0 q0 P1
- 4 r2^2 m R2^4 s^2 c Q0 q1 - 4 r2^2 m R2^4 s^2 c Q0 q0 P1 + 2 r2 m R2^5 s^2 c^3 Q0 q1
+ 2 r2 m R2^5 s^2 c^3 Q0 q0 P1) + q1 (-r2 m R2^5 s^2 c^3 q0^2 + 2 r2^2 m R2^4 s^4 c Q0 q0
- 4 r2^2 m R2^4 s^4 c Q0^2 - 4 r2^2 m R2^4 s^2 c Q0^2 + 2 r2^2 m R2^4 s^4 c q0^2 - r2^2 m R2^4 s^2 c q0^2
- 4 r2^2 m R2^4 s^2 c Q0 q0 + 2 r2 m R2^5 s^2 c^3 Q0 q0 - r2 m R2^5 s^2 c^3 Q0^2
+ 12 r2^4 m R2^2 s ta Q0 q0 + 24 r2^4 m R2^2 s ta Q0^2)

```

```
> B0:=coeff(beta2,s1,0);
```

```

B0 := q0 (-r2 m R25 s2 c3 q02 + 2 r22 m R24 s4 c Q0 q0 - 4 r22 m R24 s4 c Q02 - 4 r22 m R24 s2 c Q02
+ 2 r22 m R24 s4 c q02 - r22 m R24 s2 c q02 - 4 r22 m R24 s2 c Q0 q0 + 2 r2 m R25 s2 c3 Q0 q0
- r2 m R25 s2 c3 Q02 + 12 r24 m R22 s ta Q0 q0 + 24 r24 m R22 s ta Q02)

```

[>  
[>  
[>

> #

## Program No. 6

Coefficients of the diff. eq. of the shear  
stress of a helix using two 3-Para models

restart;

> with(linalg):

Warning, the protected names norm and trace have been redefined and unprotected

> c3:=2\*E\*m\*R2^2/(R1+2\*R2)^3/(r2+r2\*ta^2+nu\*R2)\*(ta\*(1-2\*s^2)/4\*R2^2\*s^3+n u\*R2^2/4/(1+nu)/r2\*(R1+r2\*ta^2)\*s^4\*c-(1+nu)/2\*R2^2\*s^2\*c\*(1+s^2)+nu\*R2^2/4/r2\*(R1+r2\*ta^2)\*c^3\*(1+s^2)+(r2\*ta^2-nu\*R1)\*r2\*c);

$$c3 := \frac{1}{(R1 + 2 R2)^3 (r2 + r2 ta^2 + v R2)} \left( 2 E m R2^2 \left( \frac{1}{4} ta (1 - 2 s^2) R2^2 s^3 + \frac{v R2^2 (R1 + r2 ta^2) s^4 c}{4 (1 + v) r2} \right. \right. \\ \left. \left. - \frac{1}{2} (1 + v) R2^2 s^2 c (1 + s^2) + \frac{v R2^2 (R1 + r2 ta^2) c^3 (1 + s^2)}{4 r2} + (r2 ta^2 - v R1) r2 c \right) \right)$$

> c4:=2\*E\*R1^4/4/(R1+2\*R2)^4/(1+nu)+2\*E\*r2\*m\*R2^2/(R1+2\*R2)^4/(r2+r2\*ta^2+nu\*R2)\*((R2^2\*(2\*s^2-1)\*ta^2\*s^3)/4/(1+nu)+nu\*R2^3\*s^5/4/(1+nu)/r2+R2^2/2\*s^3\*(1+s^2)+nu\*R2^3/4/r2\*s\*(1+s^2)\*c^2+r2^2\*s);

$$c4 := \frac{E R1^4}{2 (R1 + 2 R2)^4 (1 + v)} + \frac{1}{(R1 + 2 R2)^4 (r2 + r2 ta^2 + v R2)} \left( 2 E r2 m R2^2 \left( \frac{R2^2 (2 s^2 - 1) ta^2 s^3}{4 (1 + v)} \right. \right. \\ \left. \left. \right) \right)$$

$$\begin{aligned}
& Q0 ta^2 q0 P1 + m R2^2 r2^3 c Q0 ta^2 q1 + (m R2^2 r2^3 c Q1 + m R2^2 r2^3 c p1 Q0) ta^2 q0) RI \\
& + 24 m R2^3 r2^3 c Q0 ta^2 q1 + 24 m R2^3 r2^3 c Q0 ta^2 q0 P1 + 24 m R2^3 r2^2 c Q0 RI q1 \\
& + 24 m R2^3 r2^2 c Q0 RI q0 P1) + q1 (-m R2^4 s^4 c q0^2 RI^2 - 2 m R2^5 s^4 c q0^2 RI \\
& + 6 m R2^5 c^3 Q0^2 RI - 2 m R2^5 s^4 c q0^2 r2 ta^2 - m R2^4 s^4 c q0^2 r2 ta^2 RI - 24 m R2^5 ta s^5 Q0^2 r2 \\
& - 36 m R2^5 s^4 c Q0^2 r2 + 4 m R2^5 s^4 c Q0^2 r2 ta^2 + 3 m R2^4 c^3 Q0^2 RI^2 + 12 m R2^5 ta s^3 Q0^2 r2 \\
& + 2 m R2^4 s^4 c Q0^2 RI^2 + 4 m R2^5 s^4 c Q0^2 RI + 6 m R2^5 c^3 Q0^2 r2 ta^2 \\
& + 6 m R2^5 c^3 Q0^2 r2 ta^2 s^2 - 36 m R2^5 Q0^2 s^2 c r2 + 3 m R2^4 c^3 Q0^2 RI^2 s^2 \\
& + 6 m R2^5 c^3 Q0^2 RI s^2 + 24 m R2^2 r2^3 c Q0^2 ta^2 RI + 48 m R2^3 r2^3 c Q0^2 ta^2 \\
& - 24 m R2^3 r2^2 c Q0^2 RI - 12 m R2^2 r2^2 c Q0^2 RI^2 - 12 m R2^4 ta s^5 Q0^2 r2 RI \\
& + 6 m R2^4 ta s^3 Q0^2 r2 RI + 2 m R2^4 s^4 c Q0^2 r2 ta^2 RI - 18 m R2^4 Q0^2 s^2 c r2 RI \\
& - 18 m R2^4 s^4 c Q0^2 r2 RI + 3 m R2^4 c^3 Q0^2 r2 ta^2 RI + 3 m R2^4 c^3 Q0^2 r2 ta^2 s^2 RI \\
& - 2 m R2^5 s^4 c Q0 q0 RI - 2 m R2^5 s^4 c Q0 q0 r2 ta^2 - 3 m R2^4 c^3 Q0 RI^2 q0 - 6 m R2^5 c^3 Q0 RI q0 \\
& - 12 m R2^5 ta s^5 Q0 r2 q0 + 6 m R2^5 ta s^3 Q0 r2 q0 - m R2^4 s^4 c Q0 q0 RI^2 \\
& - 3 m R2^4 c^3 Q0 RI^2 q0 s^2 - 6 m R2^5 c^3 Q0 RI q0 s^2 - 6 m R2^5 c^3 Q0 r2 ta^2 q0 \\
& - 6 m R2^5 c^3 Q0 r2 ta^2 q0 s^2 + 12 m R2^2 r2^2 c Q0 RI^2 q0 - 6 m R2^4 ta s^5 Q0 r2 q0 RI \\
& + 3 m R2^4 ta s^3 Q0 r2 q0 RI - m R2^4 s^4 c Q0 q0 r2 ta^2 RI - 3 m R2^4 c^3 Q0 r2 ta^2 q0 RI \\
& - 3 m R2^4 c^3 Q0 r2 ta^2 q0 s^2 RI + 12 m R2^2 r2^3 c Q0 ta^2 q0 RI + 24 m R2^3 r2^3 c Q0 ta^2 q0 \\
& + 24 m R2^3 r2^2 c Q0 RI q0) s1 + q0 (-m R2^4 s^4 c q0^2 RI^2 - 2 m R2^5 s^4 c q0^2 RI \\
& + 6 m R2^5 c^3 Q0^2 RI - 2 m R2^5 s^4 c q0^2 r2 ta^2 - m R2^4 s^4 c q0^2 r2 ta^2 RI - 24 m R2^5 ta s^5 Q0^2 r2 \\
& - 36 m R2^5 s^4 c Q0^2 r2 + 4 m R2^5 s^4 c Q0^2 r2 ta^2 + 3 m R2^4 c^3 Q0^2 RI^2 + 12 m R2^5 ta s^3 Q0^2 r2 \\
& + 2 m R2^4 s^4 c Q0^2 RI^2 + 4 m R2^5 s^4 c Q0^2 RI + 6 m R2^5 c^3 Q0^2 r2 ta^2 \\
& + 6 m R2^5 c^3 Q0^2 r2 ta^2 s^2 - 36 m R2^5 Q0^2 s^2 c r2 + 3 m R2^4 c^3 Q0^2 RI^2 s^2 \\
& + 6 m R2^5 c^3 Q0^2 RI s^2 + 24 m R2^2 r2^3 c Q0^2 ta^2 RI + 48 m R2^3 r2^3 c Q0^2 ta^2 \\
& - 24 m R2^3 r2^2 c Q0^2 RI - 12 m R2^2 r2^2 c Q0^2 RI^2 - 12 m R2^4 ta s^5 Q0^2 r2 RI \\
& + 6 m R2^4 ta s^3 Q0^2 r2 RI + 2 m R2^4 s^4 c Q0^2 r2 ta^2 RI - 18 m R2^4 Q0^2 s^2 c r2 RI \\
& - 18 m R2^4 s^4 c Q0^2 r2 RI + 3 m R2^4 c^3 Q0^2 r2 ta^2 RI + 3 m R2^4 c^3 Q0^2 r2 ta^2 s^2 RI \\
& - 2 m R2^5 s^4 c Q0 q0 RI - 2 m R2^5 s^4 c Q0 q0 r2 ta^2 - 3 m R2^4 c^3 Q0 RI^2 q0 - 6 m R2^5 c^3 Q0 RI q0 \\
& - 12 m R2^5 ta s^5 Q0 r2 q0 + 6 m R2^5 ta s^3 Q0 r2 q0 - m R2^4 s^4 c Q0 q0 RI^2 \\
& - 3 m R2^4 c^3 Q0 RI^2 q0 s^2 - 6 m R2^5 c^3 Q0 RI q0 s^2 - 6 m R2^5 c^3 Q0 r2 ta^2 q0 \\
& - 6 m R2^5 c^3 Q0 r2 ta^2 q0 s^2 + 12 m R2^2 r2^2 c Q0 RI^2 q0 - 6 m R2^4 ta s^5 Q0 r2 q0 RI \\
& + 3 m R2^4 ta s^3 Q0 r2 q0 RI - m R2^4 s^4 c Q0 q0 r2 ta^2 RI - 3 m R2^4 c^3 Q0 r2 ta^2 q0 RI \\
& - 3 m R2^4 c^3 Q0 r2 ta^2 q0 s^2 RI + 12 m R2^2 r2^3 c Q0 ta^2 q0 RI + 24 m R2^3 r2^3 c Q0 ta^2 q0 \\
& + 24 m R2^3 r2^2 c Q0 RI q0)
\end{aligned}$$

> D5:=coeff(LHS3,s1^5);

$$D5 := 2 r2 p1 (r2 ta^2 q1 P1 + r2 q1 P1 + R2 p1 Q1 + 2 r2 p1 Q1 + 2 r2 ta^2 p1 Q1 - R2 q1 P1) (RI + 2 R2)^4 (2 p1 Q1 + q1 P1)$$

> D4:=coeff(LHS3,s1^4);

$$\begin{aligned}
D4 := & 2 (r2 (r2 ta^2 q1 P1 + r2 q1 P1 + R2 p1 Q1 + 2 r2 p1 Q1 + 2 r2 ta^2 p1 Q1 - R2 q1 P1) + r2 p1 (2 r2 Q1 \\
& + 2 r2 p1 Q0 + 2 r2 ta^2 Q1 + 2 r2 ta^2 p1 Q0 + r2 q0 P1 + r2 q1 + R2 Q1 + R2 p1 Q0 + r2 ta^2 q0 P1 \\
& + r2 ta^2 q1 - R2 q0 P1 - R2 q1)) (RI + 2 R2)^4 (2 p1 Q1 + q1 P1) + 2 r2 p1 (r2 ta^2 q1 P1 + r2 q1 P1
\end{aligned}$$

```

+ R2 p1 Q1 + 2 r2 p1 Q1 + 2 r2 ta2 p1 Q1 - R2 q1 P1) (R1 + 2 R2)4 (2 Q1 + 2 p1 Q0 + q0 P1 + q1)

> D3:=coeff(LHS3,s1^3);

D3 := 2 (r2 (2 r2 Q1 + 2 r2 p1 Q0 + 2 r2 ta2 Q1 + 2 r2 ta2 p1 Q0 + r2 q0 P1 + r2 q1 + R2 Q1 + R2 p1 Q0
+ r2 ta2 q0 P1 + r2 ta2 q1 - R2 q0 P1 - R2 q1)
+ r2 p1 (2 r2 Q0 + r2 ta2 q0 - R2 q0 + r2 q0 + R2 Q0 + 2 r2 ta2 Q0)) (R1 + 2 R2)4 (2 p1 Q1 + q1 P1) +
2 (r2 (r2 ta2 q1 P1 + r2 q1 P1 + R2 p1 Q1 + 2 r2 p1 Q1 + 2 r2 ta2 p1 Q1 - R2 q1 P1) + r2 p1 (2 r2 Q1
+ 2 r2 p1 Q0 + 2 r2 ta2 Q1 + 2 r2 ta2 p1 Q0 + r2 q0 P1 + r2 q1 + R2 Q1 + R2 p1 Q0 + r2 ta2 q0 P1
+ r2 ta2 q1 - R2 q0 P1 - R2 q1)) (R1 + 2 R2)4 (2 Q1 + 2 p1 Q0 + q0 P1 + q1) + 2 r2 p1 (r2 ta2 q1 P1
+ r2 q1 P1 + R2 p1 Q1 + 2 r2 p1 Q1 + 2 r2 ta2 p1 Q1 - R2 q1 P1) (R1 + 2 R2)4 (2 Q0 + q0)

> D2:=coeff(LHS3,s1^2);

D2 := 2 r2 (2 r2 Q0 + r2 ta2 q0 - R2 q0 + r2 q0 + R2 Q0 + 2 r2 ta2 Q0) (R1 + 2 R2)4 (2 p1 Q1 + q1 P1) + 2
(r2 (2 r2 Q1 + 2 r2 p1 Q0 + 2 r2 ta2 Q1 + 2 r2 ta2 p1 Q0 + r2 q0 P1 + r2 q1 + R2 Q1 + R2 p1 Q0
+ r2 ta2 q0 P1 + r2 ta2 q1 - R2 q0 P1 - R2 q1)
+ r2 p1 (2 r2 Q0 + r2 ta2 q0 - R2 q0 + r2 q0 + R2 Q0 + 2 r2 ta2 Q0)) (R1 + 2 R2)4 (2 Q1 + 2 p1 Q0
+ q0 P1 + q1) + 2 (r2 (r2 ta2 q1 P1 + r2 q1 P1 + R2 p1 Q1 + 2 r2 p1 Q1 + 2 r2 ta2 p1 Q1 - R2 q1 P1) +
r2 p1 (2 r2 Q1 + 2 r2 p1 Q0 + 2 r2 ta2 Q1 + 2 r2 ta2 p1 Q0 + r2 q0 P1 + r2 q1 + R2 Q1 + R2 p1 Q0
+ r2 ta2 q0 P1 + r2 ta2 q1 - R2 q0 P1 - R2 q1)) (R1 + 2 R2)4 (2 Q0 + q0)

> D1:=coeff(LHS3,s1);

D1 := 2 r2 (2 r2 Q0 + r2 ta2 q0 - R2 q0 + r2 q0 + R2 Q0 + 2 r2 ta2 Q0) (R1 + 2 R2)4 (2 Q1 + 2 p1 Q0
+ q0 P1 + q1) + 2 (r2 (2 r2 Q1 + 2 r2 p1 Q0 + 2 r2 ta2 Q1 + 2 r2 ta2 p1 Q0 + r2 q0 P1 + r2 q1
+ R2 Q1 + R2 p1 Q0 + r2 ta2 q0 P1 + r2 ta2 q1 - R2 q0 P1 - R2 q1)
+ r2 p1 (2 r2 Q0 + r2 ta2 q0 - R2 q0 + r2 q0 + R2 Q0 + 2 r2 ta2 Q0)) (R1 + 2 R2)4 (2 Q0 + q0)

> D0:=coeff(LHS3,s1,0);

D0 := 2 r2 (2 r2 Q0 + r2 ta2 q0 - R2 q0 + r2 q0 + R2 Q0 + 2 r2 ta2 Q0) (R1 + 2 R2)4 (2 Q0 + q0)

> E5:=coeff(epsilon2,s1^5);

E5 := q1 (6 m R25 c3 p12 Q12 R1 - m R24 s4 c q12 P12 R12 - 2 m R25 s4 c q12 P12 R1
- 2 m R25 s4 c q12 P12 r2 ta2 - m R24 s4 c q12 P12 r2 ta2 R1 - 24 m R25 ta s5 p12 Q12 r2
- 36 m R25 s4 c p12 Q12 r2 + 4 m R25 s4 c p12 Q12 r2 ta2 + 3 m R24 c3 p12 Q12 R12
+ 12 m R25 ta s3 p12 Q12 r2 + 2 m R24 s4 c p12 Q12 R12 + 4 m R25 s4 c p12 Q12 R1
+ 6 m R25 c3 p12 Q12 r2 ta2 + 6 m R25 c3 p12 Q12 r2 ta2 s2 - 36 m R25 p12 Q12 s2 c r2
+ 3 m R24 c3 p12 Q12 R12 s2 + 6 m R25 c3 p12 Q12 R1 s2 + 24 m R22 r23 c p12 Q12 ta2 R1
+ 48 m R23 r23 c p12 Q12 ta2 - 24 m R23 r22 c p12 Q12 R1 - 12 m R22 r22 c p12 Q12 R12
- 12 m R24 ta s5 p12 Q12 r2 R1 + 6 m R24 ta s3 p12 Q12 r2 R1 + 2 m R24 s4 c p12 Q12 r2 ta2 R1
- 18 m R24 p12 Q12 s2 c r2 R1 - 18 m R24 s4 c p12 Q12 r2 R1 + 3 m R24 c3 p12 Q12 r2 ta2 R1
+ 3 m R24 c3 p12 Q12 r2 ta2 s2 R1 - 2 m R25 s4 c p1 Q1 q1 P1 R1
- 2 m R25 s4 c p1 Q1 q1 P1 r2 ta2 - 3 m R24 c3 p1 Q1 R12 q1 P1 - 6 m R25 c3 p1 Q1 R1 q1 P1
- 12 m R25 ta s5 p1 Q1 r2 q1 P1 + 6 m R25 ta s3 p1 Q1 r2 q1 P1 - m R24 s4 c p1 Q1 q1 P1 R12
- 3 m R24 c3 p1 Q1 R12 q1 P1 s2 - 6 m R25 c3 p1 Q1 R1 q1 P1 s2 - 6 m R25 c3 p1 Q1 r2 ta2 q1 P1
- 6 m R25 c3 p1 Q1 r2 ta2 q1 P1 s2 + 12 m R22 r22 c p1 Q1 R12 q1 P1
- 6 m R24 ta s5 p1 Q1 r2 q1 P1 R1 + 3 m R24 ta s3 p1 Q1 r2 q1 P1 R1

```

$$\begin{aligned}
& -m R2^4 s^4 c p1 Q1 q1 P1 r2 ta^2 R1 - 3 m R2^4 c^3 p1 Q1 r2 ta^2 q1 P1 R1 \\
& - 3 m R2^4 c^3 p1 Q1 r2 ta^2 q1 P1 s^2 R1 + 12 m R2^2 r2^3 c p1 Q1 ta^2 q1 P1 R1 \\
& + 24 m R2^3 r2^3 c p1 Q1 ta^2 q1 P1 + 24 m R2^3 r2^2 c p1 Q1 R1 q1 P1)
\end{aligned}$$

> E4:=coeff(epsilon2,s1^4);

$$\begin{aligned}
E4 := & q0 (6 m R2^5 c^3 p1^2 Q1^2 R1 - m R2^4 s^4 c q1^2 P1^2 R1^2 - 2 m R2^5 s^4 c q1^2 P1^2 R1 \\
& - 2 m R2^5 s^4 c q1^2 P1^2 r2 ta^2 - m R2^4 s^4 c q1^2 P1^2 r2 ta^2 R1 - 24 m R2^5 ta s^5 p1^2 Q1^2 r2 \\
& - 36 m R2^5 s^4 c p1^2 Q1^2 r2 + 4 m R2^5 s^4 c p1^2 Q1^2 r2 ta^2 + 3 m R2^4 c^3 p1^2 Q1^2 R1^2 \\
& + 12 m R2^5 ta s^3 p1^2 Q1^2 r2 + 2 m R2^4 s^4 c p1^2 Q1^2 R1^2 + 4 m R2^5 s^4 c p1^2 Q1^2 R1 \\
& + 6 m R2^5 c^3 p1^2 Q1^2 r2 ta^2 + 6 m R2^5 c^3 p1^2 Q1^2 r2 ta^2 s^2 - 36 m R2^5 p1^2 Q1^2 s^2 cr2 \\
& + 3 m R2^4 c^3 p1^2 Q1^2 R1^2 s^2 + 6 m R2^5 c^3 p1^2 Q1^2 R1 s^2 + 24 m R2^2 r2^3 c p1^2 Q1^2 ta^2 R1 \\
& + 48 m R2^3 r2^3 c p1^2 Q1^2 ta^2 - 24 m R2^3 r2^2 c p1^2 Q1^2 R1 - 12 m R2^2 r2^2 c p1^2 Q1^2 R1^2 \\
& - 12 m R2^4 ta s^5 p1^2 Q1^2 r2 R1 + 6 m R2^4 ta s^3 p1^2 Q1^2 r2 R1 + 2 m R2^4 s^4 c p1^2 Q1^2 r2 ta^2 R1 \\
& - 18 m R2^4 p1^2 Q1^2 s^2 cr2 R1 - 18 m R2^4 s^4 c p1^2 Q1^2 r2 R1 + 3 m R2^4 c^3 p1^2 Q1^2 r2 ta^2 R1 \\
& + 3 m R2^4 c^3 p1^2 Q1^2 r2 ta^2 s^2 R1 - 2 m R2^5 s^4 c p1 Q1 q1 P1 R1 \\
& - 2 m R2^5 s^4 c p1 Q1 q1 P1 r2 ta^2 - 3 m R2^4 c^3 p1 Q1 R1^2 q1 P1 - 6 m R2^5 c^3 p1 Q1 R1 q1 P1 \\
& - 12 m R2^5 ta s^5 p1 Q1 r2 q1 P1 + 6 m R2^5 ta s^3 p1 Q1 r2 q1 P1 - m R2^4 s^4 c p1 Q1 q1 P1 R1^2 \\
& - 3 m R2^4 c^3 p1 Q1 R1^2 q1 P1 s^2 - 6 m R2^5 c^3 p1 Q1 R1 q1 P1 s^2 - 6 m R2^5 c^3 p1 Q1 r2 ta^2 q1 P1 \\
& - 6 m R2^5 c^3 p1 Q1 r2 ta^2 q1 P1 s^2 + 12 m R2^2 r2^2 c p1 Q1 R1^2 q1 P1 \\
& - 6 m R2^4 ta s^5 p1 Q1 r2 q1 P1 R1 + 3 m R2^4 ta s^3 p1 Q1 r2 q1 P1 R1 \\
& - m R2^4 s^4 c p1 Q1 q1 P1 r2 ta^2 R1 - 3 m R2^4 c^3 p1 Q1 r2 ta^2 q1 P1 R1 \\
& - 3 m R2^4 c^3 p1 Q1 r2 ta^2 q1 P1 s^2 R1 + 12 m R2^2 r2^3 c p1 Q1 ta^2 q1 P1 R1 \\
& + 24 m R2^3 r2^3 c p1 Q1 ta^2 q1 P1 + 24 m R2^3 r2^2 c p1 Q1 R1 q1 P1) + q1 ( \\
& -3 (((m R2^4 c^3 Q1 + m R2^4 c^3 p1 Q0) r2 ta^2 q1 + m R2^4 c^3 p1 Q1 r2 ta^2 q0) P1 \\
& + m R2^4 c^3 p1 Q1 r2 ta^2 q1) s^2 R1 - 12 (2 m R2^4 ta s^5 p1 Q1^2 + 2 m R2^4 ta s^5 p1^2 Q0 Q1) r2 R1 \\
& - 2 (2 m R2^5 s^4 c q0 q1 P1^2 + 2 m R2^5 s^4 c q1^2 P1) r2 ta^2 \\
& + 4 (2 m R2^5 s^4 c p1 Q1^2 + 2 m R2^5 s^4 c p1^2 Q0 Q1) r2 ta^2 \\
& + 3 (2 m R2^4 c^3 p1 Q1^2 + 2 m R2^4 c^3 p1^2 Q0 Q1) R1^2 s^2 \\
& + 6 (2 m R2^5 c^3 p1 Q1^2 + 2 m R2^5 c^3 p1^2 Q0 Q1) R1 s^2 \\
& - 6 ((m R2^5 c^3 Q1 + m R2^5 c^3 p1 Q0) r2 ta^2 q1 + m R2^5 c^3 p1 Q1 r2 ta^2 q0) P1 - 2 (((m R2^5 s^4 c Q1 \\
& + m R2^5 s^4 c p1 Q0) q1 + m R2^5 s^4 c p1 Q1 q0) P1 + m R2^5 s^4 c p1 Q1 q1) r2 ta^2 \\
& + 24 (2 m R2^2 r2^3 c p1 Q1^2 + 2 m R2^2 r2^3 c p1^2 Q0 Q1) ta^2 R1 \\
& + 6 (2 m R2^4 ta s^3 p1 Q1^2 + 2 m R2^4 ta s^3 p1^2 Q0 Q1) r2 R1 \\
& - (2 m R2^4 s^4 c q0 q1 P1^2 + 2 m R2^4 s^4 c q1^2 P1) r2 ta^2 R1 \\
& - (2 m R2^4 s^4 c q0 q1 P1^2 + 2 m R2^4 s^4 c q1^2 P1) R1^2 \\
& - 2 (2 m R2^5 s^4 c q0 q1 P1^2 + 2 m R2^5 s^4 c q1^2 P1) R1 \\
& + 6 (2 m R2^5 c^3 p1 Q1^2 + 2 m R2^5 c^3 p1^2 Q0 Q1) R1 \\
& - 24 (2 m R2^5 ta s^5 p1 Q1^2 + 2 m R2^5 ta s^5 p1^2 Q0 Q1) r2 \\
& + 3 (2 m R2^4 c^3 p1 Q1^2 + 2 m R2^4 c^3 p1^2 Q0 Q1) R1^2 \\
& + 12 (2 m R2^5 ta s^3 p1 Q1^2 + 2 m R2^5 ta s^3 p1^2 Q0 Q1) r2 \\
& + 2 (2 m R2^4 s^4 c p1 Q1^2 + 2 m R2^4 s^4 c p1^2 Q0 Q1) R1^2 \\
& + 4 (2 m R2^5 s^4 c p1 Q1^2 + 2 m R2^5 s^4 c p1^2 Q0 Q1) R1 \\
& + 6 (2 m R2^5 c^3 p1 Q1^2 + 2 m R2^5 c^3 p1^2 Q0 Q1) r2 ta^2
\end{aligned}$$

$$\begin{aligned}
& + 6 (2 m R2^5 c^3 p1 Q1^2 + 2 m R2^5 c^3 p1^2 Q0 Q1) r2 ta^2 s^2 \\
& - 36 (2 m R2^5 p1 Q1^2 + 2 m R2^5 p1^2 Q0 Q1) s^2 c r2 \\
& - 36 (2 m R2^5 p1 Q1^2 + 2 m R2^5 p1^2 Q0 Q1) s^4 c r2 \\
& + 2 (2 m R2^4 s^4 c p1 Q1^2 + 2 m R2^4 s^4 c p1^2 Q0 Q1) r2 ta^2 RI \\
& + 48 (2 m R2^3 r2^3 c p1 Q1^2 + 2 m R2^3 r2^3 c p1^2 Q0 Q1) ta^2 \\
& - 24 (2 m R2^3 r2^2 c p1 Q1^2 + 2 m R2^3 r2^2 c p1^2 Q0 Q1) RI \\
& - 12 (2 m R2^2 r2^2 c p1 Q1^2 + 2 m R2^2 r2^2 c p1^2 Q0 Q1) RI^2 \\
& - 18 (2 m R2^4 p1 Q1^2 + 2 m R2^4 p1^2 Q0 Q1) s^2 c r2 RI \\
& - 18 (2 m R2^4 p1 Q1^2 + 2 m R2^4 p1^2 Q0 Q1) s^4 c r2 RI \\
& + 3 (2 m R2^4 c^3 p1 Q1^2 + 2 m R2^4 c^3 p1^2 Q0 Q1) r2 ta^2 RI - ((m R2^4 s^4 c Q1 + m R2^4 s^4 c p1 Q0) \\
& ) q1 + m R2^4 s^4 c p1 Q1 q0) P1 + m R2^4 s^4 c p1 Q1 q1) r2 ta^2 RI \\
& - 2 (((m R2^5 s^4 c Q1 + m R2^5 s^4 c p1 Q0) q1 + m R2^5 s^4 c p1 Q1 q0) P1 + m R2^5 s^4 c p1 Q1 q1) RI \\
& - 3 (((m R2^4 c^3 Q1 + m R2^4 c^3 p1 Q0) RI^2 q1 + m R2^4 c^3 p1 Q1 RI^2 q0) P1 \\
& + 3 (2 m R2^4 c^3 p1 Q1^2 + 2 m R2^4 c^3 p1^2 Q0 Q1) r2 ta^2 s^2 RI - 3 m R2^4 c^3 p1 Q1 RI^2 q1 \\
& - 6 (((m R2^5 c^3 Q1 + m R2^5 c^3 p1 Q0) RI q1 + m R2^5 c^3 p1 Q1 RI q0) P1 \\
& - 12 (((m R2^5 t a s^5 Q1 + m R2^5 t a s^5 p1 Q0) r2 q1 + m R2^5 t a s^5 p1 Q1 r2 q0) P1 \\
& + 6 (((m R2^5 t a s^3 Q1 + m R2^5 t a s^3 p1 Q0) r2 q1 + m R2^5 t a s^3 p1 Q1 r2 q0) P1 \\
& - 6 m R2^5 c^3 p1 Q1 RI q1 - 12 m R2^5 t a s^5 p1 Q1 r2 q1 + 6 m R2^5 t a s^3 p1 Q1 r2 q1 \\
& - (((m R2^4 s^4 c Q1 + m R2^4 s^4 c p1 Q0) q1 + m R2^4 s^4 c p1 Q1 q0) P1 + m R2^4 s^4 c p1 Q1 q1) RI^2 - \\
& 3 (((m R2^4 c^3 Q1 + m R2^4 c^3 p1 Q0) RI^2 q1 + m R2^4 c^3 p1 Q1 RI^2 q0) P1 \\
& + m R2^4 c^3 p1 Q1 RI^2 q1) s^2 \\
& - 6 (((m R2^5 c^3 Q1 + m R2^5 c^3 p1 Q0) RI q1 + m R2^5 c^3 p1 Q1 RI q0) P1 + m R2^5 c^3 p1 Q1 RI q1) \\
& s^2 - 6 m R2^5 c^3 p1 Q1 r2 t a^2 q1 - 6 (((m R2^5 c^3 Q1 + m R2^5 c^3 p1 Q0) r2 t a^2 q1 \\
& + m R2^5 c^3 p1 Q1 r2 t a^2 q0) P1 + m R2^5 c^3 p1 Q1 r2 t a^2 q1) s^2 \\
& + 12 (((m R2^2 r2^2 c Q1 + m R2^2 r2^2 c p1 Q0) RI^2 q1 + m R2^2 r2^2 c p1 Q1 RI^2 q0) P1 - 6 (((m R2^4 \\
& t a s^5 Q1 + m R2^4 t a s^5 p1 Q0) r2 q1 + m R2^4 t a s^5 p1 Q1 r2 q0) P1 + m R2^4 t a s^5 p1 Q1 r2 q1) RI \\
& + 12 m R2^2 r2^2 c p1 Q1 RI^2 q1 + 3 (((m R2^4 t a s^3 Q1 + m R2^4 t a s^3 p1 Q0) r2 q1 \\
& + m R2^4 t a s^3 p1 Q1 r2 q0) P1 + m R2^4 t a s^3 p1 Q1 r2 q1) RI - 3 (((m R2^4 c^3 Q1 + m R2^4 c^3 p1 Q0) \\
& r2 t a^2 q1 + m R2^4 c^3 p1 Q1 r2 t a^2 q0) P1 + m R2^4 c^3 p1 Q1 r2 t a^2 q1) RI + 12 (((m R2^2 r2^3 c Q1 \\
& + m R2^2 r2^3 c p1 Q0) t a^2 q1 + m R2^2 r2^3 c p1 Q1 t a^2 q0) P1 + m R2^2 r2^3 c p1 Q1 t a^2 q1) RI \\
& + 24 (((m R2^3 r2^3 c Q1 + m R2^3 r2^3 c p1 Q0) t a^2 q1 + m R2^3 r2^3 c p1 Q1 t a^2 q0) P1 \\
& + 24 m R2^3 r2^3 c p1 Q1 t a^2 q1 \\
& + 24 (((m R2^3 r2^2 c Q1 + m R2^3 r2^2 c p1 Q0) RI q1 + m R2^3 r2^2 c p1 Q1 RI q0) P1 \\
& + 24 m R2^3 r2^2 c p1 Q1 RI q1)
\end{aligned}$$

> E3:=coeff(epsilon2,s1^3);

E3 := q0 (

$$\begin{aligned}
& - 3 (((m R2^4 c^3 Q1 + m R2^4 c^3 p1 Q0) r2 t a^2 q1 + m R2^4 c^3 p1 Q1 r2 t a^2 q0) P1 \\
& + m R2^4 c^3 p1 Q1 r2 t a^2 q1) s^2 RI - 12 (2 m R2^4 t a s^5 p1 Q1^2 + 2 m R2^4 t a s^5 p1^2 Q0 Q1) r2 RI \\
& - 2 (2 m R2^5 s^4 c q0 q1 P1^2 + 2 m R2^5 s^4 c q1^2 P1) r2 t a^2 \\
& + 4 (2 m R2^5 s^4 c p1 Q1^2 + 2 m R2^5 s^4 c p1^2 Q0 Q1) r2 t a^2 \\
& + 3 (2 m R2^4 c^3 p1 Q1^2 + 2 m R2^4 c^3 p1^2 Q0 Q1) RI^2 s^2 \\
& + 6 (2 m R2^5 c^3 p1 Q1^2 + 2 m R2^5 c^3 p1^2 Q0 Q1) RI s^2
\end{aligned}$$

$$\begin{aligned}
& -6((m R2^5 c^3 Q1 + m R2^5 c^3 p1 Q0) r2 ta^2 q1 + m R2^5 c^3 p1 Q1 r2 ta^2 q0) P1 - 2(((m R2^5 s^4 c Q1 \\
& + m R2^5 s^4 c p1 Q0) q1 + m R2^5 s^4 c p1 Q1 q0) P1 + m R2^5 s^4 c p1 Q1 q1) r2 ta^2 \\
& + 24(2 m R2^2 r2^3 c p1 Q1^2 + 2 m R2^2 r2^3 c p1^2 Q0 Q1) ta^2 RI \\
& + 6(2 m R2^4 tas^3 p1 Q1^2 + 2 m R2^4 tas^3 p1^2 Q0 Q1) r2 RI \\
& - (2 m R2^4 s^4 c q0 q1 P1^2 + 2 m R2^4 s^4 c q1^2 P1) r2 ta^2 RI \\
& - (2 m R2^4 s^4 c q0 q1 P1^2 + 2 m R2^4 s^4 c q1^2 P1) RI^2 \\
& - 2(2 m R2^5 s^4 c q0 q1 P1^2 + 2 m R2^5 s^4 c q1^2 P1) RI \\
& + 6(2 m R2^5 c^3 p1 Q1^2 + 2 m R2^5 c^3 p1^2 Q0 Q1) RI \\
& - 24(2 m R2^5 tas^5 p1 Q1^2 + 2 m R2^5 tas^5 p1^2 Q0 Q1) r2 \\
& + 3(2 m R2^4 c^3 p1 Q1^2 + 2 m R2^4 c^3 p1^2 Q0 Q1) RI^2 \\
& + 12(2 m R2^5 tas^3 p1 Q1^2 + 2 m R2^5 tas^3 p1^2 Q0 Q1) r2 \\
& + 2(2 m R2^4 s^4 c p1 Q1^2 + 2 m R2^4 s^4 c p1^2 Q0 Q1) RI^2 \\
& + 4(2 m R2^5 s^4 c p1 Q1^2 + 2 m R2^5 s^4 c p1^2 Q0 Q1) RI \\
& + 6(2 m R2^5 c^3 p1 Q1^2 + 2 m R2^5 c^3 p1^2 Q0 Q1) r2 ta^2 \\
& + 6(2 m R2^5 c^3 p1 Q1^2 + 2 m R2^5 c^3 p1^2 Q0 Q1) r2 ta^2 s^2 \\
& - 36(2 m R2^5 p1 Q1^2 + 2 m R2^5 p1^2 Q0 Q1) s^2 c r2 \\
& - 36(2 m R2^5 p1 Q1^2 + 2 m R2^5 p1^2 Q0 Q1) s^4 c r2 \\
& + 2(2 m R2^4 s^4 c p1 Q1^2 + 2 m R2^4 s^4 c p1^2 Q0 Q1) r2 ta^2 RI \\
& + 48(2 m R2^3 r2^3 c p1 Q1^2 + 2 m R2^3 r2^3 c p1^2 Q0 Q1) ta^2 \\
& - 24(2 m R2^3 r2^2 c p1 Q1^2 + 2 m R2^3 r2^2 c p1^2 Q0 Q1) RI \\
& - 12(2 m R2^2 r2^2 c p1 Q1^2 + 2 m R2^2 r2^2 c p1^2 Q0 Q1) RI^2 \\
& - 18(2 m R2^4 p1 Q1^2 + 2 m R2^4 p1^2 Q0 Q1) s^2 c r2 RI \\
& - 18(2 m R2^4 p1 Q1^2 + 2 m R2^4 p1^2 Q0 Q1) s^4 c r2 RI \\
& + 3(2 m R2^4 c^3 p1 Q1^2 + 2 m R2^4 c^3 p1^2 Q0 Q1) r2 ta^2 RI - (((m R2^4 s^4 c Q1 + m R2^4 s^4 c p1 Q0 \\
& ) q1 + m R2^4 s^4 c p1 Q1 q0) P1 + m R2^4 s^4 c p1 Q1 q1) r2 ta^2 RI \\
& - 2(((m R2^5 s^4 c Q1 + m R2^5 s^4 c p1 Q0) q1 + m R2^5 s^4 c p1 Q1 q0) P1 + m R2^5 s^4 c p1 Q1 q1) RI \\
& - 3(((m R2^4 c^3 Q1 + m R2^4 c^3 p1 Q0) RI^2 q1 + m R2^4 c^3 p1 Q1 RI^2 q0) P1 \\
& + 3(2 m R2^4 c^3 p1 Q1^2 + 2 m R2^4 c^3 p1^2 Q0 Q1) r2 ta^2 s^2 RI - 3 m R2^4 c^3 p1 Q1 RI^2 q1 \\
& - 6((m R2^5 c^3 Q1 + m R2^5 c^3 p1 Q0) RI q1 + m R2^5 c^3 p1 Q1 RI q0) P1 \\
& - 12((m R2^5 tas^5 Q1 + m R2^5 tas^5 p1 Q0) r2 q1 + m R2^5 tas^5 p1 Q1 r2 q0) P1 \\
& + 6((m R2^5 tas^3 Q1 + m R2^5 tas^3 p1 Q0) r2 q1 + m R2^5 tas^3 p1 Q1 r2 q0) P1 \\
& - 6 m R2^5 c^3 p1 Q1 RI q1 - 12 m R2^5 tas^5 p1 Q1 r2 q1 + 6 m R2^5 tas^3 p1 Q1 r2 q1 \\
& - (((m R2^4 s^4 c Q1 + m R2^4 s^4 c p1 Q0) q1 + m R2^4 s^4 c p1 Q1 q0) P1 + m R2^4 s^4 c p1 Q1 q1) RI^2 - \\
& 3(((m R2^4 c^3 Q1 + m R2^4 c^3 p1 Q0) RI^2 q1 + m R2^4 c^3 p1 Q1 RI^2 q0) P1 \\
& + m R2^4 c^3 p1 Q1 RI^2 q1) s^2 \\
& - 6(((m R2^5 c^3 Q1 + m R2^5 c^3 p1 Q0) RI q1 + m R2^5 c^3 p1 Q1 RI q0) P1 + m R2^5 c^3 p1 Q1 RI q1) \\
& s^2 - 6 m R2^5 c^3 p1 Q1 r2 ta^2 q1 - 6(((m R2^5 c^3 Q1 + m R2^5 c^3 p1 Q0) r2 ta^2 q1 \\
& + m R2^5 c^3 p1 Q1 r2 ta^2 q0) P1 + m R2^5 c^3 p1 Q1 r2 ta^2 q1) s^2 \\
& + 12((m R2^2 r2^2 c Q1 + m R2^2 r2^2 c p1 Q0) RI^2 q1 + m R2^2 r2^2 c p1 Q1 RI^2 q0) P1 - 6(((m R2^4 \\
& ta^5 Q1 + m R2^4 tas^5 p1 Q0) r2 q1 + m R2^4 tas^5 p1 Q1 r2 q0) P1 + m R2^4 tas^5 p1 Q1 r2 q1) RI \\
& + 12 m R2^2 r2^2 c p1 Q1 RI^2 q1 + 3(((m R2^4 tas^3 Q1 + m R2^4 tas^3 p1 Q0) r2 q1 \\
& + m R2^4 tas^3 p1 Q1 r2 q0) P1 + m R2^4 tas^3 p1 Q1 r2 q1) RI - 3(((m R2^4 c^3 Q1 + m R2^4 c^3 p1 Q0)
\end{aligned}$$



$$\begin{aligned}
& -6(mR2^5c^3Q1 + mR2^5c^3p1Q0)r2ta^2q1 - ((mR2^4s^4cQ0q1 \\
& + (mR2^4s^4cQ1 + mR2^4s^4cp1Q0)q0)P1 + (mR2^4s^4cQ1 + mR2^4s^4cp1Q0)q1 \\
& + mR2^4s^4cp1Q1q0)r2ta^2R1 - 2((mR2^5s^4cQ0q1 + (mR2^5s^4cQ1 + mR2^5s^4cp1Q0)q0) \\
& P1 + (mR2^5s^4cQ1 + mR2^5s^4cp1Q0)q1 + mR2^5s^4cp1Q1q0)R1 \\
& - 3(mR2^4c^3Q0R1^2q1 + (mR2^4c^3Q1 + mR2^4c^3p1Q0)R1^2q0)P1 \\
& + 3(mR2^4c^3Q1^2 + 4mR2^4c^3p1Q0Q1 + mR2^4c^3p1^2Q0^2)r2ta^2s^2R1 \\
& - 3mR2^4c^3p1Q1R1^2q0 - 6(mR2^5c^3Q0R1q1 + (mR2^5c^3Q1 + mR2^5c^3p1Q0)R1q0)P1 \\
& - 12(mR2^5tas^5Q0r2q1 + (mR2^5tas^5Q1 + mR2^5tas^5p1Q0)r2q0)P1 \\
& + 6(mR2^5tas^3Q0r2q1 + (mR2^5tas^3Q1 + mR2^5tas^3p1Q0)r2q0)P1 \\
& - 6mR2^5c^3p1Q1R1q0 - 12mR2^5tas^5p1Q1r2q0 + 6mR2^5tas^3p1Q1r2q0 - ((mR2^4s^4c \\
& Q0q1 + (mR2^4s^4cQ1 + mR2^4s^4cp1Q0)q0)P1 + (mR2^4s^4cQ1 + mR2^4s^4cp1Q0)q1 \\
& + mR2^4s^4cp1Q1q0)R1^2 - 3((mR2^4c^3Q0R1^2q1 + (mR2^4c^3Q1 + mR2^4c^3p1Q0)R1^2q0) \\
& P1 + (mR2^4c^3Q1 + mR2^4c^3p1Q0)R1^2q1 + mR2^4c^3p1Q1R1^2q0)s^2 - 6((mR2^5c^3Q0R1 \\
& q1 + (mR2^5c^3Q1 + mR2^5c^3p1Q0)R1q0)P1 + (mR2^5c^3Q1 + mR2^5c^3p1Q0)R1q1 \\
& + mR2^5c^3p1Q1R1q0)s^2 - 6mR2^5c^3p1Q1r2ta^2q0 - 6((mR2^5c^3Q0r2ta^2q1 \\
& + (mR2^5c^3Q1 + mR2^5c^3p1Q0)r2ta^2q0)P1 + (mR2^5c^3Q1 + mR2^5c^3p1Q0)r2ta^2q1 \\
& + mR2^5c^3p1Q1r2ta^2q0)s^2 \\
& + 12(mR2^2r2^2cQ0R1^2q1 + (mR2^2r2^2cQ1 + mR2^2r2^2cp1Q0)R1^2q0)P1 - 6((mR2^4tas^5 \\
& Q0r2q1 + (mR2^4tas^5Q1 + mR2^4tas^5p1Q0)r2q0)P1 \\
& + (mR2^4tas^5Q1 + mR2^4tas^5p1Q0)r2q1 + mR2^4tas^5p1Q1r2q0)R1 \\
& + 12mR2^2r2^2cp1Q1R1^2q0 + 3((mR2^4tas^3Q0r2q1 \\
& + (mR2^4tas^3Q1 + mR2^4tas^3p1Q0)r2q0)P1 + (mR2^4tas^3Q1 + mR2^4tas^3p1Q0)r2q1 \\
& + mR2^4tas^3p1Q1r2q0)R1 - 3((mR2^4c^3Q0r2ta^2q1 \\
& + (mR2^4c^3Q1 + mR2^4c^3p1Q0)r2ta^2q0)P1 + (mR2^4c^3Q1 + mR2^4c^3p1Q0)r2ta^2q1 \\
& + mR2^4c^3p1Q1r2ta^2q0)R1 + 12((mR2^2r2^3cQ0ta^2q1 \\
& + (mR2^2r2^3cQ1 + mR2^2r2^3cp1Q0)ta^2q0)P1 + (mR2^2r2^3cQ1 + mR2^2r2^3cp1Q0)ta^2q1 \\
& + mR2^2r2^3cp1Q1ta^2q0)R1 \\
& + 24(mR2^3r2^3cQ0ta^2q1 + (mR2^3r2^3cQ1 + mR2^3r2^3cp1Q0)ta^2q0)P1 \\
& + 24mR2^3r2^3cp1Q1ta^2q0 \\
& + 24(mR2^3r2^2cQ0R1q1 + (mR2^3r2^2cQ1 + mR2^3r2^2cp1Q0)R1q0)P1 \\
& + 24mR2^3r2^2cp1Q1R1q0)
\end{aligned}$$

> E2:=coeff(epsilon2,s1^2);

$$\begin{aligned}
E2 := & q0 \left( \right. \\
& -2((mR2^5s^4cQ0q1 + (mR2^5s^4cQ1 + mR2^5s^4cp1Q0)q0)P1 \\
& + (mR2^5s^4cQ1 + mR2^5s^4cp1Q0)q1 + mR2^5s^4cp1Q1q0)r2ta^2 \\
& - 12(mR2^5tas^5Q1 + mR2^5tas^5p1Q0)r2q1 + 24(mR2^3r2^3cQ1 + mR2^3r2^3cp1Q0)ta^2q1 \\
& + 24(mR2^2r2^3cQ1^2 + 4mR2^2r2^3cp1Q0Q1 + mR2^2r2^3cp1^2Q0^2)ta^2R1 \\
& + 4(mR2^5s^4cQ1^2 + 4mR2^5s^4cp1Q0Q1 + mR2^5s^4cp1^2Q0^2)r2ta^2 \\
& + 6(mR2^5tas^3Q1 + mR2^5tas^3p1Q0)r2q1 \\
& + 6(mR2^5c^3Q1^2 + 4mR2^5c^3p1Q0Q1 + mR2^5c^3p1^2Q0^2)r2ta^2 \\
& \left. + 3(mR2^4c^3Q1^2 + 4mR2^4c^3p1Q0Q1 + mR2^4c^3p1^2Q0^2)R1^2s^2 - 3((mR2^4c^3Q0r2ta^2 \\
& q1 + (mR2^4c^3Q1 + mR2^4c^3p1Q0)r2ta^2q0)P1 + (mR2^4c^3Q1 + mR2^4c^3p1Q0)r2ta^2q1 \right)
\end{aligned}$$

$$\begin{aligned}
& + m R2^4 c^3 p1 Q1 r2 ta^2 q0) s^2 R1 - 6 (m R2^5 c^3 Q1 + m R2^5 c^3 p1 Q0) R1 q1 \\
& + 12 (m R2^2 r2^2 c Q1 + m R2^2 r2^2 c p1 Q0) R1^2 q1 - 3 (m R2^4 c^3 Q1 + m R2^4 c^3 p1 Q0) R1^2 q1 \\
& + 6 (m R2^5 c^3 Q1^2 + 4 m R2^5 c^3 p1 Q0 Q1 + m R2^5 c^3 p1^2 Q0^2) R1 s^2 \\
& + 24 (m R2^3 r2^2 c Q1 + m R2^3 r2^2 c p1 Q0) R1 q1 \\
& - 6 (m R2^5 c^3 Q0 r2 ta^2 q1 + (m R2^5 c^3 Q1 + m R2^5 c^3 p1 Q0) r2 ta^2 q0) P1 \\
& - 2 (m R2^5 s^4 c q0^2 P1^2 + 4 m R2^5 s^4 c q0 q1 P1 + m R2^5 s^4 c q1^2) r2 ta^2 \\
& - (m R2^4 s^4 c q0^2 P1^2 + 4 m R2^4 s^4 c q0 q1 P1 + m R2^4 s^4 c q1^2) r2 ta^2 R1 \\
& - (m R2^4 s^4 c q0^2 P1^2 + 4 m R2^4 s^4 c q0 q1 P1 + m R2^4 s^4 c q1^2) R1^2 \\
& - 2 (m R2^5 s^4 c q0^2 P1^2 + 4 m R2^5 s^4 c q0 q1 P1 + m R2^5 s^4 c q1^2) R1 \\
& + 6 (m R2^5 c^3 Q1^2 + 4 m R2^5 c^3 p1 Q0 Q1 + m R2^5 c^3 p1^2 Q0^2) R1 \\
& - 24 (m R2^5 tas^5 Q1^2 + 4 m R2^5 tas^5 p1 Q0 Q1 + m R2^5 tas^5 p1^2 Q0^2) r2 \\
& + 3 (m R2^4 c^3 Q1^2 + 4 m R2^4 c^3 p1 Q0 Q1 + m R2^4 c^3 p1^2 Q0^2) R1^2 \\
& + 12 (m R2^5 tas^3 Q1^2 + 4 m R2^5 tas^3 p1 Q0 Q1 + m R2^5 tas^3 p1^2 Q0^2) r2 \\
& + 2 (m R2^4 s^4 c Q1^2 + 4 m R2^4 s^4 c p1 Q0 Q1 + m R2^4 s^4 c p1^2 Q0^2) R1^2 \\
& + 4 (m R2^5 s^4 c Q1^2 + 4 m R2^5 s^4 c p1 Q0 Q1 + m R2^5 s^4 c p1^2 Q0^2) R1 \\
& + 6 (m R2^5 c^3 Q1^2 + 4 m R2^5 c^3 p1 Q0 Q1 + m R2^5 c^3 p1^2 Q0^2) r2 ta^2 s^2 \\
& - 36 (m R2^5 Q1^2 + 4 m R2^5 p1 Q0 Q1 + m R2^5 p1^2 Q0^2) s^2 c r2 \\
& - 36 (m R2^5 Q1^2 + 4 m R2^5 p1 Q0 Q1 + m R2^5 p1^2 Q0^2) s^4 c r2 \\
& + 2 (m R2^4 s^4 c Q1^2 + 4 m R2^4 s^4 c p1 Q0 Q1 + m R2^4 s^4 c p1^2 Q0^2) r2 ta^2 R1 \\
& + 48 (m R2^3 r2^3 c Q1^2 + 4 m R2^3 r2^3 c p1 Q0 Q1 + m R2^3 r2^3 c p1^2 Q0^2) ta^2 \\
& - 24 (m R2^3 r2^2 c Q1^2 + 4 m R2^3 r2^2 c p1 Q0 Q1 + m R2^3 r2^2 c p1^2 Q0^2) R1 \\
& - 12 (m R2^2 r2^2 c Q1^2 + 4 m R2^2 r2^2 c p1 Q0 Q1 + m R2^2 r2^2 c p1^2 Q0^2) R1^2 \\
& - 12 (m R2^4 tas^5 Q1^2 + 4 m R2^4 tas^5 p1 Q0 Q1 + m R2^4 tas^5 p1^2 Q0^2) r2 R1 \\
& + 6 (m R2^4 tas^3 Q1^2 + 4 m R2^4 tas^3 p1 Q0 Q1 + m R2^4 tas^3 p1^2 Q0^2) r2 R1 \\
& - 18 (m R2^4 Q1^2 + 4 m R2^4 p1 Q0 Q1 + m R2^4 p1^2 Q0^2) s^2 c r2 R1 \\
& - 18 (m R2^4 Q1^2 + 4 m R2^4 p1 Q0 Q1 + m R2^4 p1^2 Q0^2) s^4 c r2 R1 \\
& + 3 (m R2^4 c^3 Q1^2 + 4 m R2^4 c^3 p1 Q0 Q1 + m R2^4 c^3 p1^2 Q0^2) r2 ta^2 R1 \\
& - 6 (m R2^5 c^3 Q1 + m R2^5 c^3 p1 Q0) r2 ta^2 q1 - ((m R2^4 s^4 c Q0 q1 \\
& + (m R2^4 s^4 c Q1 + m R2^4 s^4 c p1 Q0) q0) P1 + (m R2^4 s^4 c Q1 + m R2^4 s^4 c p1 Q0) q1 \\
& + m R2^4 s^4 c p1 Q1 q0) r2 ta^2 R1 - 2 ((m R2^5 s^4 c Q0 q1 + (m R2^5 s^4 c Q1 + m R2^5 s^4 c p1 Q0) q0) \\
& P1 + (m R2^5 s^4 c Q1 + m R2^5 s^4 c p1 Q0) q1 + m R2^5 s^4 c p1 Q1 q0) R1 \\
& - 3 (m R2^4 c^3 Q0 R1^2 q1 + (m R2^4 c^3 Q1 + m R2^4 c^3 p1 Q0) R1^2 q0) P1 \\
& + 3 (m R2^4 c^3 Q1^2 + 4 m R2^4 c^3 p1 Q0 Q1 + m R2^4 c^3 p1^2 Q0^2) r2 ta^2 s^2 R1 \\
& - 3 m R2^4 c^3 p1 Q1 R1^2 q0 - 6 (m R2^5 c^3 Q0 R1 q1 + (m R2^5 c^3 Q1 + m R2^5 c^3 p1 Q0) R1 q0) P1 \\
& - 12 (m R2^5 tas^5 Q0 r2 q1 + (m R2^5 tas^5 Q1 + m R2^5 tas^5 p1 Q0) r2 q0) P1 \\
& + 6 (m R2^5 tas^3 Q0 r2 q1 + (m R2^5 tas^3 Q1 + m R2^5 tas^3 p1 Q0) r2 q0) P1 \\
& - 6 m R2^5 c^3 p1 Q1 R1 q0 - 12 m R2^5 tas^5 p1 Q1 r2 q0 + 6 m R2^5 tas^3 p1 Q1 r2 q0 - ((m R2^4 s^4 c \\
& Q0 q1 + (m R2^4 s^4 c Q1 + m R2^4 s^4 c p1 Q0) q0) P1 + (m R2^4 s^4 c Q1 + m R2^4 s^4 c p1 Q0) q1 \\
& + m R2^4 s^4 c p1 Q1 q0) R1^2 - 3 ((m R2^4 c^3 Q0 R1^2 q1 + (m R2^4 c^3 Q1 + m R2^4 c^3 p1 Q0) R1^2 q0) \\
& P1 + (m R2^4 c^3 Q1 + m R2^4 c^3 p1 Q0) R1^2 q1 + m R2^4 c^3 p1 Q1 R1^2 q0) s^2 - 6 ((m R2^5 c^3 Q0 R1 \\
& q1 + (m R2^5 c^3 Q1 + m R2^5 c^3 p1 Q0) R1 q0) P1 + (m R2^5 c^3 Q1 + m R2^5 c^3 p1 Q0) R1 q1 \\
& + m R2^5 c^3 p1 Q1 R1 q0) s^2 - 6 m R2^5 c^3 p1 Q1 r2 ta^2 q0 - 6 ((m R2^5 c^3 Q0 r2 ta^2 q1
\end{aligned}$$

$$\begin{aligned}
& + (m R2^5 c^3 Q1 + m R2^5 c^3 p1 Q0) r2 ta^2 q0) P1 + (m R2^5 c^3 Q1 + m R2^5 c^3 p1 Q0) r2 ta^2 q1 \\
& + m R2^5 c^3 p1 Q1 r2 ta^2 q0) s^2 \\
& + 12 (m R2^2 r2^2 c Q0 RI^2 q1 + (m R2^2 r2^2 c Q1 + m R2^2 r2^2 c p1 Q0) RI^2 q0) P1 - 6 ((m R2^4 ta s^5 \\
& Q0 r2 q1 + (m R2^4 ta s^5 Q1 + m R2^4 ta s^5 p1 Q0) r2 q0) P1 \\
& + (m R2^4 ta s^5 Q1 + m R2^4 ta s^5 p1 Q0) r2 q1 + m R2^4 ta s^5 p1 Q1 r2 q0) RI \\
& + 12 m R2^2 r2^2 c p1 Q1 RI^2 q0 + 3 ((m R2^4 ta s^3 Q0 r2 q1 \\
& + (m R2^4 ta s^3 Q1 + m R2^4 ta s^3 p1 Q0) r2 q0) P1 + (m R2^4 ta s^3 Q1 + m R2^4 ta s^3 p1 Q0) r2 q1 \\
& + m R2^4 ta s^3 p1 Q1 r2 q0) RI - 3 ((m R2^4 c^3 Q0 r2 ta^2 q1 \\
& + (m R2^4 c^3 Q1 + m R2^4 c^3 p1 Q0) r2 ta^2 q0) P1 + (m R2^4 c^3 Q1 + m R2^4 c^3 p1 Q0) r2 ta^2 q1 \\
& + m R2^4 c^3 p1 Q1 r2 ta^2 q0) RI + 12 ((m R2^2 r2^3 c Q0 ta^2 q1 \\
& + (m R2^2 r2^3 c Q1 + m R2^2 r2^3 c p1 Q0) ta^2 q0) P1 + (m R2^2 r2^3 c Q1 + m R2^2 r2^3 c p1 Q0) ta^2 q1 \\
& + m R2^2 r2^3 c p1 Q1 ta^2 q0) RI \\
& + 24 (m R2^3 r2^3 c Q0 ta^2 q1 + (m R2^3 r2^3 c Q1 + m R2^3 r2^3 c p1 Q0) ta^2 q0) P1 \\
& + 24 m R2^3 r2^3 c p1 Q1 ta^2 q0 \\
& + 24 (m R2^3 r2^2 c Q0 RI q1 + (m R2^3 r2^2 c Q1 + m R2^3 r2^2 c p1 Q0) RI q0) P1 \\
& + 24 m R2^3 r2^2 c p1 Q1 RI q0) + q1 \\
-3 & (m R2^4 c^3 Q0 r2 ta^2 q0 P1 + m R2^4 c^3 Q0 r2 ta^2 q1 + (m R2^4 c^3 Q1 + m R2^4 c^3 p1 Q0) r2 ta^2 q0) s^2 \\
& RI + 12 (m R2^2 r2^2 c Q1 + m R2^2 r2^2 c p1 Q0) RI^2 q0 \\
& - 12 (2 m R2^4 ta s^5 Q0 Q1 + 2 m R2^4 ta s^5 p1 Q0^2) r2 RI \\
& - 6 (m R2^5 c^3 Q1 + m R2^5 c^3 p1 Q0) RI q0 + 6 (m R2^5 ta s^3 Q1 + m R2^5 ta s^3 p1 Q0) r2 q0 \\
& + 24 (m R2^3 r2^2 c Q1 + m R2^3 r2^2 c p1 Q0) RI q0 \\
& + 24 (m R2^3 r2^3 c Q1 + m R2^3 r2^3 c p1 Q0) ta^2 q0 \\
& + 6 (2 m R2^5 c^3 Q0 Q1 + 2 m R2^5 c^3 p1 Q0^2) r2 ta^2 \\
& - 12 (m R2^5 ta s^5 Q1 + m R2^5 ta s^5 p1 Q0) r2 q0 - 3 (m R2^4 c^3 Q1 + m R2^4 c^3 p1 Q0) RI^2 q0 \\
& + 6 (2 m R2^4 ta s^3 Q0 Q1 + 2 m R2^4 ta s^3 p1 Q0^2) r2 RI \\
& - 2 (m R2^5 s^4 c Q0 q0 P1 + m R2^5 s^4 c Q0 q1 + (m R2^5 s^4 c Q1 + m R2^5 s^4 c p1 Q0) q0) r2 ta^2 \\
& - (2 m R2^4 s^4 c q0^2 P1 + 2 m R2^4 s^4 c q0 q1) r2 ta^2 RI \\
& - (2 m R2^4 s^4 c q0^2 P1 + 2 m R2^4 s^4 c q0 q1) RI^2 \\
& - 2 (2 m R2^5 s^4 c q0^2 P1 + 2 m R2^5 s^4 c q0 q1) RI + 6 (2 m R2^5 c^3 Q0 Q1 + 2 m R2^5 c^3 p1 Q0^2) RI \\
& - 24 (2 m R2^5 ta s^5 Q0 Q1 + 2 m R2^5 ta s^5 p1 Q0^2) r2 \\
& - 2 (2 m R2^5 s^4 c q0^2 P1 + 2 m R2^5 s^4 c q0 q1) r2 ta^2 \\
& + 4 (2 m R2^5 s^4 c Q0 Q1 + 2 m R2^5 s^4 c p1 Q0^2) r2 ta^2 \\
& + 3 (2 m R2^4 c^3 Q0 Q1 + 2 m R2^4 c^3 p1 Q0^2) RI^2 \\
& + 12 (2 m R2^5 ta s^3 Q0 Q1 + 2 m R2^5 ta s^3 p1 Q0^2) r2 \\
& + 2 (2 m R2^4 s^4 c Q0 Q1 + 2 m R2^4 s^4 c p1 Q0^2) RI^2 \\
& + 4 (2 m R2^5 s^4 c Q0 Q1 + 2 m R2^5 s^4 c p1 Q0^2) RI \\
& + 6 (2 m R2^5 c^3 Q0 Q1 + 2 m R2^5 c^3 p1 Q0^2) r2 ta^2 s^2 \\
& - 36 (2 m R2^5 Q0 Q1 + 2 m R2^5 p1 Q0^2) s^2 c r2 - 36 (2 m R2^5 Q0 Q1 + 2 m R2^5 p1 Q0^2) s^4 c r2 \\
& + 2 (2 m R2^4 s^4 c Q0 Q1 + 2 m R2^4 s^4 c p1 Q0^2) r2 ta^2 RI \\
& + 3 (2 m R2^4 c^3 Q0 Q1 + 2 m R2^4 c^3 p1 Q0^2) RI^2 s^2 \\
& + 6 (2 m R2^5 c^3 Q0 Q1 + 2 m R2^5 c^3 p1 Q0^2) RI s^2 \\
& + 24 (2 m R2^2 r2^3 c Q0 Q1 + 2 m R2^2 r2^3 c p1 Q0^2) ta^2 RI
\end{aligned}$$

$$\begin{aligned}
& + 48 (2 m R2^3 r2^3 c Q0 QI + 2 m R2^3 r2^3 c p1 Q0^2) ta^2 \\
& - 24 (2 m R2^3 r2^2 c Q0 QI + 2 m R2^3 r2^2 c p1 Q0^2) RI \\
& - 12 (2 m R2^2 r2^2 c Q0 QI + 2 m R2^2 r2^2 c p1 Q0^2) RI^2 \\
& - 18 (2 m R2^4 Q0 QI + 2 m R2^4 p1 Q0^2) s^2 c r2 RI \\
& - 18 (2 m R2^4 Q0 QI + 2 m R2^4 p1 Q0^2) s^4 c r2 RI \\
& + 3 (2 m R2^4 c^3 Q0 QI + 2 m R2^4 c^3 p1 Q0^2) r2 ta^2 RI \\
& - 6 (m R2^5 c^3 QI + m R2^5 c^3 p1 Q0) r2 ta^2 q0 \\
& - (m R2^4 s^4 c Q0 q0 PI + m R2^4 s^4 c Q0 q1 + (m R2^4 s^4 c QI + m R2^4 s^4 c p1 Q0) q0) r2 ta^2 RI \\
& - 2 (m R2^5 s^4 c Q0 q0 PI + m R2^5 s^4 c Q0 q1 + (m R2^5 s^4 c QI + m R2^5 s^4 c p1 Q0) q0) RI \\
& + 3 (2 m R2^4 c^3 Q0 QI + 2 m R2^4 c^3 p1 Q0^2) r2 ta^2 s^2 RI - 3 m R2^4 c^3 Q0 RI^2 q1 \\
& - 3 m R2^4 c^3 Q0 RI^2 q0 PI - 6 m R2^5 c^3 Q0 RI q1 - 6 m R2^5 c^3 Q0 RI q0 PI \\
& - 12 m R2^5 ta s^5 Q0 r2 q1 - 12 m R2^5 ta s^5 Q0 r2 q0 PI + 6 m R2^5 ta s^3 Q0 r2 q1 \\
& + 6 m R2^5 ta s^3 Q0 r2 q0 PI \\
& - (m R2^4 s^4 c Q0 q0 PI + m R2^4 s^4 c Q0 q1 + (m R2^4 s^4 c QI + m R2^4 s^4 c p1 Q0) q0) RI^2 \\
& - 3 (m R2^4 c^3 Q0 RI^2 q0 PI + m R2^4 c^3 Q0 RI^2 q1 + (m R2^4 c^3 QI + m R2^4 c^3 p1 Q0) RI^2 q0) s^2 \\
& - 6 (m R2^5 c^3 Q0 RI q0 PI + m R2^5 c^3 Q0 RI q1 + (m R2^5 c^3 QI + m R2^5 c^3 p1 Q0) RI q0) s^2 \\
& - 6 m R2^5 c^3 Q0 r2 ta^2 q1 - 6 (m R2^4 ta s^5 Q0 r2 q0 PI + m R2^4 ta s^5 Q0 r2 q1 \\
& + (m R2^4 ta s^5 QI + m R2^4 ta s^5 p1 Q0) r2 q0) RI - 6 m R2^5 c^3 Q0 r2 ta^2 q0 PI - 6 (m R2^5 c^3 Q0 r2 \\
& ta^2 q0 PI + m R2^5 c^3 Q0 r2 ta^2 q1 + (m R2^5 c^3 QI + m R2^5 c^3 p1 Q0) r2 ta^2 q0) s^2 \\
& + 12 m R2^2 r2^2 c Q0 RI^2 q1 + 12 m R2^2 r2^2 c Q0 RI^2 q0 PI + 3 (m R2^4 ta s^3 Q0 r2 q0 PI \\
& + m R2^4 ta s^3 Q0 r2 q1 + (m R2^4 ta s^3 QI + m R2^4 ta s^3 p1 Q0) r2 q0) RI - 3 (m R2^4 c^3 Q0 r2 ta^2 \\
& q0 PI + m R2^4 c^3 Q0 r2 ta^2 q1 + (m R2^4 c^3 QI + m R2^4 c^3 p1 Q0) r2 ta^2 q0) RI + 12 (m R2^2 r2^3 c \\
& Q0 ta^2 q0 PI + m R2^2 r2^3 c Q0 ta^2 q1 + (m R2^2 r2^3 c QI + m R2^2 r2^3 c p1 Q0) ta^2 q0) RI \\
& + 24 m R2^3 r2^3 c Q0 ta^2 q1 + 24 m R2^3 r2^3 c Q0 ta^2 q0 PI + 24 m R2^3 r2^2 c Q0 RI q1 \\
& + 24 m R2^3 r2^2 c Q0 RI q0 PI
\end{aligned}$$

```

> E1:=coeff(epsilon2,s1);
E1:=q0(
-3 (m R2^4 c^3 Q0 r2 ta^2 q0 PI + m R2^4 c^3 Q0 r2 ta^2 q1 + (m R2^4 c^3 QI + m R2^4 c^3 p1 Q0) r2 ta^2 q0) s^2
RI + 12 (m R2^2 r2^2 c QI + m R2^2 r2^2 c p1 Q0) RI^2 q0
- 12 (2 m R2^4 ta s^5 Q0 QI + 2 m R2^4 ta s^5 p1 Q0^2) r2 RI
- 6 (m R2^5 c^3 QI + m R2^5 c^3 p1 Q0) RI q0 + 6 (m R2^5 ta s^3 QI + m R2^5 ta s^3 p1 Q0) r2 q0
+ 24 (m R2^3 r2^2 c QI + m R2^3 r2^2 c p1 Q0) RI q0
+ 24 (m R2^3 r2^3 c QI + m R2^3 r2^3 c p1 Q0) ta^2 q0
+ 6 (2 m R2^5 c^3 Q0 QI + 2 m R2^5 c^3 p1 Q0^2) r2 ta^2
- 12 (m R2^5 ta s^5 QI + m R2^5 ta s^5 p1 Q0) r2 q0 - 3 (m R2^4 c^3 QI + m R2^4 c^3 p1 Q0) RI^2 q0
+ 6 (2 m R2^4 ta s^3 Q0 QI + 2 m R2^4 ta s^3 p1 Q0^2) r2 RI
- 2 (m R2^5 s^4 c Q0 q0 PI + m R2^5 s^4 c Q0 q1 + (m R2^5 s^4 c QI + m R2^5 s^4 c p1 Q0) q0) r2 ta^2
- (2 m R2^4 s^4 c q0^2 PI + 2 m R2^4 s^4 c q0 q1) r2 ta^2 RI
- (2 m R2^4 s^4 c q0^2 PI + 2 m R2^4 s^4 c q0 q1) RI^2
- 2 (2 m R2^5 s^4 c q0^2 PI + 2 m R2^5 s^4 c q0 q1) RI + 6 (2 m R2^5 c^3 Q0 QI + 2 m R2^5 c^3 p1 Q0^2) RI
- 24 (2 m R2^5 ta s^5 Q0 QI + 2 m R2^5 ta s^5 p1 Q0^2) r2

```

$$\begin{aligned}
& -2(2mR2^5s^4c q0^2 P1 + 2mR2^5s^4c q0 q1) r2 ta^2 \\
& + 4(2mR2^5s^4c Q0 Q1 + 2mR2^5s^4c p1 Q0^2) r2 ta^2 \\
& + 3(2mR2^4c^3 Q0 Q1 + 2mR2^4c^3 p1 Q0^2) RI^2 \\
& + 12(2mR2^5ta s^3 Q0 Q1 + 2mR2^5ta s^3 p1 Q0^2) r2 \\
& + 2(2mR2^4s^4c Q0 Q1 + 2mR2^4s^4c p1 Q0^2) RI^2 \\
& + 4(2mR2^5s^4c Q0 Q1 + 2mR2^5s^4c p1 Q0^2) RI \\
& + 6(2mR2^5c^3 Q0 Q1 + 2mR2^5c^3 p1 Q0^2) r2 ta^2 s^2 \\
& - 36(2mR2^5Q0 Q1 + 2mR2^5p1 Q0^2) s^2 c r2 - 36(2mR2^5Q0 Q1 + 2mR2^5p1 Q0^2) s^4 c r2 \\
& + 2(2mR2^4s^4c Q0 Q1 + 2mR2^4s^4c p1 Q0^2) r2 ta^2 RI \\
& + 3(2mR2^4c^3 Q0 Q1 + 2mR2^4c^3 p1 Q0^2) RI^2 s^2 \\
& + 6(2mR2^5c^3 Q0 Q1 + 2mR2^5c^3 p1 Q0^2) RI s^2 \\
& + 24(2mR2^2r2^3c Q0 Q1 + 2mR2^2r2^3c p1 Q0^2) ta^2 RI \\
& + 48(2mR2^3r2^3c Q0 Q1 + 2mR2^3r2^3c p1 Q0^2) ta^2 \\
& - 24(2mR2^3r2^2c Q0 Q1 + 2mR2^3r2^2c p1 Q0^2) RI \\
& - 12(2mR2^2r2^2c Q0 Q1 + 2mR2^2r2^2c p1 Q0^2) RI^2 \\
& - 18(2mR2^4Q0 Q1 + 2mR2^4p1 Q0^2) s^2 c r2 RI \\
& - 18(2mR2^4Q0 Q1 + 2mR2^4p1 Q0^2) s^4 c r2 RI \\
& + 3(2mR2^4c^3 Q0 Q1 + 2mR2^4c^3 p1 Q0^2) r2 ta^2 RI \\
& - 6(mR2^5c^3 Q1 + mR2^5c^3 p1 Q0) r2 ta^2 q0 \\
& - (mR2^4s^4c Q0 q0 P1 + mR2^4s^4c Q0 q1 + (mR2^4s^4c Q1 + mR2^4s^4c p1 Q0) q0) r2 ta^2 RI \\
& - 2(mR2^5s^4c Q0 q0 P1 + mR2^5s^4c Q0 q1 + (mR2^5s^4c Q1 + mR2^5s^4c p1 Q0) q0) RI \\
& + 3(2mR2^4c^3 Q0 Q1 + 2mR2^4c^3 p1 Q0^2) r2 ta^2 s^2 RI - 3mR2^4c^3 Q0 RI^2 q1 \\
& - 3mR2^4c^3 Q0 RI^2 q0 P1 - 6mR2^5c^3 Q0 RI q1 - 6mR2^5c^3 Q0 RI q0 P1 \\
& - 12mR2^5ta s^5 Q0 r2 q1 - 12mR2^5ta s^5 Q0 r2 q0 P1 + 6mR2^5ta s^3 Q0 r2 q1 \\
& + 6mR2^5ta s^3 Q0 r2 q0 P1 \\
& - (mR2^4s^4c Q0 q0 P1 + mR2^4s^4c Q0 q1 + (mR2^4s^4c Q1 + mR2^4s^4c p1 Q0) q0) RI^2 \\
& - 3(mR2^4c^3 Q0 RI^2 q0 P1 + mR2^4c^3 Q0 RI^2 q1 + (mR2^4c^3 Q1 + mR2^4c^3 p1 Q0) RI^2 q0) s^2 \\
& - 6(mR2^5c^3 Q0 RI q0 P1 + mR2^5c^3 Q0 RI q1 + (mR2^5c^3 Q1 + mR2^5c^3 p1 Q0) RI q0) s^2 \\
& - 6mR2^5c^3 Q0 r2 ta^2 q1 - 6(mR2^4ta s^5 Q0 r2 q0 P1 + mR2^4ta s^5 Q0 r2 q1 \\
& + (mR2^4ta s^5 Q1 + mR2^4ta s^5 p1 Q0) r2 q0) RI - 6mR2^5c^3 Q0 r2 ta^2 q0 P1 - 6(mR2^5c^3 Q0 r2 \\
& ta^2 q0 P1 + mR2^5c^3 Q0 r2 ta^2 q1 + (mR2^5c^3 Q1 + mR2^5c^3 p1 Q0) r2 ta^2 q0) s^2 \\
& + 12mR2^2r2^2c Q0 RI^2 q1 + 12mR2^2r2^2c Q0 RI^2 q0 P1 + 3(mR2^4ta s^3 Q0 r2 q0 P1 \\
& + mR2^4ta s^3 Q0 r2 q1 + (mR2^4ta s^3 Q1 + mR2^4ta s^3 p1 Q0) r2 q0) RI - 3(mR2^4c^3 Q0 r2 ta^2 \\
& q0 P1 + mR2^4c^3 Q0 r2 ta^2 q1 + (mR2^4c^3 Q1 + mR2^4c^3 p1 Q0) r2 ta^2 q0) RI + 12(mR2^2r2^3c \\
& Q0 ta^2 q0 P1 + mR2^2r2^3c Q0 ta^2 q1 + (mR2^2r2^3c Q1 + mR2^2r2^3c p1 Q0) ta^2 q0) RI \\
& + 24mR2^3r2^3c Q0 ta^2 q1 + 24mR2^3r2^3c Q0 ta^2 q0 P1 + 24mR2^3r2^2c Q0 RI q1 \\
& + 24mR2^3r2^2c Q0 RI q0 P1) + q1 (-mR2^4s^4c q0^2 RI^2 - 2mR2^5s^4c q0^2 RI \\
& + 6mR2^5c^3 Q0^2 RI - 2mR2^5s^4c q0^2 r2 ta^2 - mR2^4s^4c q0^2 r2 ta^2 RI - 24mR2^5ta s^5 Q0^2 r2 \\
& - 36mR2^5s^4c Q0^2 r2 + 4mR2^5s^4c Q0^2 r2 ta^2 + 3mR2^4c^3 Q0^2 RI^2 + 12mR2^5ta s^3 Q0^2 r2 \\
& + 2mR2^4s^4c Q0^2 RI^2 + 4mR2^5s^4c Q0^2 RI + 6mR2^5c^3 Q0^2 r2 ta^2 \\
& + 6mR2^5c^3 Q0^2 r2 ta^2 s^2 - 36mR2^5Q0^2 s^2 c r2 + 3mR2^4c^3 Q0^2 RI^2 s^2 \\
& + 6mR2^5c^3 Q0^2 RI s^2 + 24mR2^2r2^3c Q0^2 ta^2 RI + 48mR2^3r2^3c Q0^2 ta^2
\end{aligned}$$

$$\begin{aligned}
& -24 m R2^3 r2^2 c Q0^2 R1 - 12 m R2^2 r2^2 c Q0^2 R1^2 - 12 m R2^4 t a s^5 Q0^2 r2 R1 \\
& + 6 m R2^4 t a s^3 Q0^2 r2 R1 + 2 m R2^4 s^4 c Q0^2 r2 t a^2 R1 - 18 m R2^4 Q0^2 s^2 c r2 R1 \\
& - 18 m R2^4 s^4 c Q0^2 r2 R1 + 3 m R2^4 c^3 Q0^2 r2 t a^2 R1 + 3 m R2^4 c^3 Q0^2 r2 t a^2 s^2 R1 \\
& - 2 m R2^5 s^4 c Q0 q0 R1 - 2 m R2^5 s^4 c Q0 q0 r2 t a^2 - 3 m R2^4 c^3 Q0 R1^2 q0 - 6 m R2^5 c^3 Q0 R1 q0 \\
& - 12 m R2^5 t a s^5 Q0 r2 q0 + 6 m R2^5 t a s^3 Q0 r2 q0 - m R2^4 s^4 c Q0 q0 R1^2 \\
& - 3 m R2^4 c^3 Q0 R1^2 q0 s^2 - 6 m R2^5 c^3 Q0 R1 q0 s^2 - 6 m R2^5 c^3 Q0 r2 t a^2 q0 \\
& - 6 m R2^5 c^3 Q0 r2 t a^2 q0 s^2 + 12 m R2^2 r2^2 c Q0 R1^2 q0 - 6 m R2^4 t a s^5 Q0 r2 q0 R1 \\
& + 3 m R2^4 t a s^3 Q0 r2 q0 R1 - m R2^4 s^4 c Q0 q0 r2 t a^2 R1 - 3 m R2^4 c^3 Q0 r2 t a^2 q0 R1 \\
& - 3 m R2^4 c^3 Q0 r2 t a^2 q0 s^2 R1 + 12 m R2^2 r2^3 c Q0 t a^2 q0 R1 + 24 m R2^3 r2^3 c Q0 t a^2 q0 \\
& + 24 m R2^3 r2^2 c Q0 R1 q0
\end{aligned}$$

> E0:=coeff(epsilon2,s1,0);

$$\begin{aligned}
E0 := & q0 (-m R2^4 s^4 c q0^2 R1^2 - 2 m R2^5 s^4 c q0^2 R1 + 6 m R2^5 c^3 Q0^2 R1 - 2 m R2^5 s^4 c q0^2 r2 t a^2 \\
& - m R2^4 s^4 c q0^2 r2 t a^2 R1 - 24 m R2^5 t a s^5 Q0^2 r2 - 36 m R2^5 s^4 c Q0^2 r2 \\
& + 4 m R2^5 s^4 c Q0^2 r2 t a^2 + 3 m R2^4 c^3 Q0^2 R1^2 + 12 m R2^5 t a s^3 Q0^2 r2 \\
& + 2 m R2^4 s^4 c Q0^2 R1^2 + 4 m R2^5 s^4 c Q0^2 R1 + 6 m R2^5 c^3 Q0^2 r2 t a^2 \\
& + 6 m R2^5 c^3 Q0^2 r2 t a^2 s^2 - 36 m R2^5 Q0^2 s^2 c r2 + 3 m R2^4 c^3 Q0^2 R1^2 s^2 \\
& + 6 m R2^5 c^3 Q0^2 R1 s^2 + 24 m R2^2 r2^3 c Q0^2 t a^2 R1 + 48 m R2^3 r2^3 c Q0^2 t a^2 \\
& - 24 m R2^3 r2^2 c Q0^2 R1 - 12 m R2^2 r2^2 c Q0^2 R1^2 - 12 m R2^4 t a s^5 Q0^2 r2 R1 \\
& + 6 m R2^4 t a s^3 Q0^2 r2 R1 + 2 m R2^4 s^4 c Q0^2 r2 t a^2 R1 - 18 m R2^4 Q0^2 s^2 c r2 R1 \\
& - 18 m R2^4 s^4 c Q0^2 r2 R1 + 3 m R2^4 c^3 Q0^2 r2 t a^2 R1 + 3 m R2^4 c^3 Q0^2 r2 t a^2 s^2 R1 \\
& - 2 m R2^5 s^4 c Q0 q0 R1 - 2 m R2^5 s^4 c Q0 q0 r2 t a^2 - 3 m R2^4 c^3 Q0 R1^2 q0 - 6 m R2^5 c^3 Q0 R1 q0 \\
& - 12 m R2^5 t a s^5 Q0 r2 q0 + 6 m R2^5 t a s^3 Q0 r2 q0 - m R2^4 s^4 c Q0 q0 R1^2 \\
& - 3 m R2^4 c^3 Q0 R1^2 q0 s^2 - 6 m R2^5 c^3 Q0 R1 q0 s^2 - 6 m R2^5 c^3 Q0 r2 t a^2 q0 \\
& - 6 m R2^5 c^3 Q0 r2 t a^2 q0 s^2 + 12 m R2^2 r2^2 c Q0 R1^2 q0 - 6 m R2^4 t a s^5 Q0 r2 q0 R1 \\
& + 3 m R2^4 t a s^3 Q0 r2 q0 R1 - m R2^4 s^4 c Q0 q0 r2 t a^2 R1 - 3 m R2^4 c^3 Q0 r2 t a^2 q0 R1 \\
& - 3 m R2^4 c^3 Q0 r2 t a^2 q0 s^2 R1 + 12 m R2^2 r2^3 c Q0 t a^2 q0 R1 + 24 m R2^3 r2^3 c Q0 t a^2 q0 \\
& + 24 m R2^3 r2^2 c Q0 R1 q0)
\end{aligned}$$

> B5:=coeff(beta2,s1^5);

$$\begin{aligned}
B5 := & q1 (-r2 R1^4 q1^2 P1^2 R2 + 3 m R2^5 s^3 c^2 p1^2 Q1^2 r2 + 3 m R2^5 s c^2 r2 p1^2 Q1^2 \\
& - 3 m R2^5 s^3 c^2 r2 p1 Q1 q1 P1 - 3 m R2^5 s c^2 r2 p1 Q1 q1 P1 + r2^2 R1^4 q1^2 P1^2 \\
& + 4 r2^2 R1^4 p1^2 Q1^2 + 4 r2^2 R1^4 p1^2 Q1^2 t a^2 - r2 m R2^5 s^5 q1^2 P1^2 + r2^2 R1^4 q1^2 P1^2 t a^2 \\
& + 8 r2^2 m R2^4 s^5 t a^2 p1^2 Q1^2 + 2 r2^2 m R2^4 s^5 t a^2 q1^2 P1^2 - r2^2 m R2^4 s^3 t a^2 q1^2 P1^2 \\
& + 2 r2 R1^4 p1^2 Q1^2 R2 - 4 r2^2 m R2^4 s^3 t a^2 p1^2 Q1^2 + 2 r2 m R2^5 s^5 p1^2 Q1^2 \\
& + 12 r2^2 m R2^4 s^3 p1^2 Q1^2 + 12 r2^2 m R2^4 s^5 p1^2 Q1^2 + 24 r2^4 m R2^2 s p1^2 Q1^2 \\
& + 4 r2^2 R1^4 p1 Q1 q1 P1 + 4 r2^2 R1^4 p1 Q1 q1 P1 t a^2 - r2 R1^4 p1 Q1 R2 q1 P1 \\
& + 8 r2^2 m R2^4 s^5 t a^2 p1 Q1 q1 P1 - 4 r2^2 m R2^4 s^3 t a^2 p1 Q1 q1 P1 - r2 m R2^5 s^5 p1 Q1 q1 P1 \\
& + 6 r2^2 m R2^4 s^3 p1 Q1 q1 P1 + 6 r2^2 m R2^4 s^5 p1 Q1 q1 P1 + 12 r2^4 m R2^2 s p1 Q1 q1 P1)
\end{aligned}$$

> B4:=coeff(beta2,s1^4);

$$\begin{aligned}
B4 := & q0 (-r2 R1^4 q1^2 P1^2 R2 + 3 m R2^5 s^3 c^2 p1^2 Q1^2 r2 + 3 m R2^5 s c^2 r2 p1^2 Q1^2 \\
& - 3 m R2^5 s^3 c^2 r2 p1 Q1 q1 P1 - 3 m R2^5 s c^2 r2 p1 Q1 q1 P1 + r2^2 R1^4 q1^2 P1^2 \\
& + 4 r2^2 R1^4 p1^2 Q1^2 + 4 r2^2 R1^4 p1^2 Q1^2 t a^2 - r2 m R2^5 s^5 q1^2 P1^2 + r2^2 R1^4 q1^2 P1^2 t a^2)
\end{aligned}$$

$$\begin{aligned}
& + 8 r2^2 m R2^4 s^5 ta^2 p1^2 QI^2 + 2 r2^2 m R2^4 s^5 ta^2 q1^2 PI^2 - r2^2 m R2^4 s^3 ta^2 q1^2 PI^2 \\
& + 2 r2 R1^4 p1^2 QI^2 R2 - 4 r2^2 m R2^4 s^3 ta^2 p1^2 QI^2 + 2 r2 m R2^5 s^5 p1^2 QI^2 \\
& + 12 r2^2 m R2^4 s^3 p1^2 QI^2 + 12 r2^2 m R2^4 s^5 p1^2 QI^2 + 24 r2^4 m R2^2 s p1^2 QI^2 \\
& + 4 r2^2 R1^4 p1 QI q1 PI + 4 r2^2 R1^4 p1 QI q1 PI ta^2 - r2 R1^4 p1 QI R2 q1 PI \\
& + 8 r2^2 m R2^4 s^5 ta^2 p1 QI q1 PI - 4 r2^2 m R2^4 s^3 ta^2 p1 QI q1 PI - r2 m R2^5 s^5 p1 QI q1 PI \\
& + 6 r2^2 m R2^4 s^3 p1 QI q1 PI + 6 r2^2 m R2^4 s^5 p1 QI q1 PI + 12 r2^4 m R2^2 s p1 QI q1 PI) + q1 \\
& (48 r2^4 m R2^2 s p1 QI^2 + 24 r2^2 m R2^4 s^5 p1 QI^2 \\
& + 3 (2 m R2^5 s^3 c^2 p1 QI^2 + 2 m R2^5 s^3 c^2 p1^2 Q0 QI) r2 + 6 m R2^5 s c^2 r2 p1 QI^2 \\
& + 6 m R2^5 s c^2 r2 p1^2 Q0 QI \\
& - 3 ((m R2^5 s^3 c^2 r2 QI + m R2^5 s^3 c^2 r2 p1 Q0) q1 + m R2^5 s^3 c^2 r2 p1 QI q0) PI \\
& - 3 m R2^5 s^3 c^2 r2 p1 QI q1 \\
& - 3 ((m R2^5 s c^2 r2 QI + m R2^5 s c^2 r2 p1 Q0) q1 + m R2^5 s c^2 r2 p1 QI q0) PI \\
& - 3 m R2^5 s c^2 r2 p1 QI q1 + 2 r2^2 R1^4 q1^2 PI + 8 r2^2 R1^4 p1 QI^2 + 2 r2^2 R1^4 q0 q1 PI^2 \\
& + 4 (2 r2^2 R1^4 p1 QI^2 + 2 r2^2 R1^4 p1^2 Q0 QI) ta^2 - (2 r2 R1^4 q0 q1 PI^2 + 2 r2 R1^4 q1^2 PI) R2 \\
& + (2 r2^2 R1^4 q0 q1 PI^2 + 2 r2^2 R1^4 q1^2 PI) ta^2 + 8 r2^2 R1^4 p1^2 Q0 QI - 2 r2 m R2^5 s^5 q0 q1 PI^2 \\
& - 2 r2 m R2^5 s^5 q1^2 PI + 2 (2 r2 R1^4 p1 QI^2 + 2 r2 R1^4 p1^2 Q0 QI) R2 \\
& + 16 r2^2 m R2^4 s^5 ta^2 p1 QI^2 + 16 r2^2 m R2^4 s^5 ta^2 p1^2 Q0 QI + 4 r2^2 m R2^4 s^5 ta^2 q0 q1 PI^2 \\
& + 4 r2^2 m R2^4 s^5 ta^2 q1^2 PI - 2 r2^2 m R2^4 s^3 ta^2 q0 q1 PI^2 - 2 r2^2 m R2^4 s^3 ta^2 q1^2 PI \\
& + 4 ((r2^2 R1^4 QI + r2^2 R1^4 p1 Q0) q1 + r2^2 R1^4 p1 QI q0) PI + r2^2 R1^4 p1 QI q1) ta^2 \\
& - 8 r2^2 m R2^4 s^3 ta^2 p1 QI^2 - 8 r2^2 m R2^4 s^3 ta^2 p1^2 Q0 QI \\
& + 4 ((r2^2 R1^4 QI + r2^2 R1^4 p1 Q0) q1 + r2^2 R1^4 p1 QI q0) PI + 4 r2 m R2^5 s^5 p1 QI^2 \\
& + 4 r2 m R2^5 s^5 p1^2 Q0 QI + 24 r2^2 m R2^4 s^3 p1 QI^2 + 24 r2^2 m R2^4 s^3 p1^2 Q0 QI \\
& + 24 r2^2 m R2^4 s^5 p1^2 Q0 QI + 48 r2^4 m R2^2 s p1^2 Q0 QI + 4 r2^2 R1^4 p1 QI q1 \\
& + 12 r2^4 m R2^2 s p1 QI q1 - ((r2 R1^4 QI + r2 R1^4 p1 Q0) R2 q1 + r2 R1^4 p1 QI R2 q0) PI \\
& + 8 ((r2^2 m R2^4 s^5 ta^2 QI + r2^2 m R2^4 s^5 ta^2 p1 Q0) q1 + r2^2 m R2^4 s^5 ta^2 p1 QI q0) PI \\
& - 4 ((r2^2 m R2^4 s^3 ta^2 QI + r2^2 m R2^4 s^3 ta^2 p1 Q0) q1 + r2^2 m R2^4 s^3 ta^2 p1 QI q0) PI \\
& - ((r2 m R2^5 s^5 QI + r2 m R2^5 s^5 p1 Q0) q1 + r2 m R2^5 s^5 p1 QI q0) PI - r2 R1^4 p1 QI R2 q1 \\
& + 8 r2^2 m R2^4 s^5 ta^2 p1 QI q1 - 4 r2^2 m R2^4 s^3 ta^2 p1 QI q1 - r2 m R2^5 s^5 p1 QI q1 \\
& + 6 ((r2^2 m R2^4 s^3 QI + r2^2 m R2^4 s^3 p1 Q0) q1 + r2^2 m R2^4 s^3 p1 QI q0) PI \\
& + 6 ((r2^2 m R2^4 s^5 QI + r2^2 m R2^4 s^5 p1 Q0) q1 + r2^2 m R2^4 s^5 p1 QI q0) PI \\
& + 12 ((r2^4 m R2^2 s QI + r2^4 m R2^2 s p1 Q0) q1 + r2^4 m R2^2 s p1 QI q0) PI \\
& + 6 r2^2 m R2^4 s^3 p1 QI q1 + 6 r2^2 m R2^4 s^5 p1 QI q1)
\end{aligned}$$

> B3:=coeff(beta2,s1^3);

$$\begin{aligned}
B3 := & q0 (48 r2^4 m R2^2 s p1 QI^2 + 24 r2^2 m R2^4 s^5 p1 QI^2 \\
& + 3 (2 m R2^5 s^3 c^2 p1 QI^2 + 2 m R2^5 s^3 c^2 p1^2 Q0 QI) r2 + 6 m R2^5 s c^2 r2 p1 QI^2 \\
& + 6 m R2^5 s c^2 r2 p1^2 Q0 QI \\
& - 3 ((m R2^5 s^3 c^2 r2 QI + m R2^5 s^3 c^2 r2 p1 Q0) q1 + m R2^5 s^3 c^2 r2 p1 QI q0) PI \\
& - 3 m R2^5 s^3 c^2 r2 p1 QI q1 \\
& - 3 ((m R2^5 s c^2 r2 QI + m R2^5 s c^2 r2 p1 Q0) q1 + m R2^5 s c^2 r2 p1 QI q0) PI \\
& - 3 m R2^5 s c^2 r2 p1 QI q1 + 2 r2^2 R1^4 q1^2 PI + 8 r2^2 R1^4 p1 QI^2 + 2 r2^2 R1^4 q0 q1 PI^2 \\
& + 4 (2 r2^2 R1^4 p1 QI^2 + 2 r2^2 R1^4 p1^2 Q0 QI) ta^2 - (2 r2 R1^4 q0 q1 PI^2 + 2 r2 R1^4 q1^2 PI) R2 \\
& + (2 r2^2 R1^4 q0 q1 PI^2 + 2 r2^2 R1^4 q1^2 PI) ta^2 + 8 r2^2 R1^4 p1^2 Q0 QI - 2 r2 m R2^5 s^5 q0 q1 PI^2
\end{aligned}$$

$$\begin{aligned}
& -2 r2 m R2^5 s^5 q1^2 P1 + 2 (2 r2 RI^4 p1 Q1^2 + 2 r2 RI^4 p1^2 Q0 Q1) R2 \\
& + 16 r2^2 m R2^4 s^5 ta^2 p1 Q1^2 + 16 r2^2 m R2^4 s^5 ta^2 p1^2 Q0 Q1 + 4 r2^2 m R2^4 s^5 ta^2 q0 q1 P1^2 \\
& + 4 r2^2 m R2^4 s^5 ta^2 q1^2 P1 - 2 r2^2 m R2^4 s^3 ta^2 q0 q1 P1^2 - 2 r2^2 m R2^4 s^3 ta^2 q1^2 P1 \\
& + 4 ((r2^2 RI^4 Q1 + r2^2 RI^4 p1 Q0) q1 + r2^2 RI^4 p1 Q1 q0) P1 + r2^2 RI^4 p1 Q1 q1) ta^2 \\
& - 8 r2^2 m R2^4 s^3 ta^2 p1 Q1^2 - 8 r2^2 m R2^4 s^3 ta^2 p1^2 Q0 Q1 \\
& + 4 ((r2^2 RI^4 Q1 + r2^2 RI^4 p1 Q0) q1 + r2^2 RI^4 p1 Q1 q0) P1 + 4 r2 m R2^5 s^5 p1 Q1^2 \\
& + 4 r2 m R2^5 s^5 p1^2 Q0 Q1 + 24 r2^2 m R2^4 s^3 p1 Q1^2 + 24 r2^2 m R2^4 s^3 p1^2 Q0 Q1 \\
& + 24 r2^2 m R2^4 s^5 p1^2 Q0 Q1 + 48 r2^4 m R2^2 s p1^2 Q0 Q1 + 4 r2^2 RI^4 p1 Q1 q1 \\
& + 12 r2^4 m R2^2 s p1 Q1 q1 - ((r2 RI^4 Q1 + r2 RI^4 p1 Q0) R2 q1 + r2 RI^4 p1 Q1 R2 q0) P1 \\
& + 8 ((r2^2 m R2^4 s^5 ta^2 Q1 + r2^2 m R2^4 s^5 ta^2 p1 Q0) q1 + r2^2 m R2^4 s^5 ta^2 p1 Q1 q0) P1 \\
& - 4 ((r2^2 m R2^4 s^3 ta^2 Q1 + r2^2 m R2^4 s^3 ta^2 p1 Q0) q1 + r2^2 m R2^4 s^3 ta^2 p1 Q1 q0) P1 \\
& - ((r2 m R2^5 s^5 Q1 + r2 m R2^5 s^5 p1 Q0) q1 + r2 m R2^5 s^5 p1 Q1 q0) P1 - r2 RI^4 p1 Q1 R2 q1 \\
& + 8 r2^2 m R2^4 s^5 ta^2 p1 Q1 q1 - 4 r2^2 m R2^4 s^3 ta^2 p1 Q1 q1 - r2 m R2^5 s^5 p1 Q1 q1 \\
& + 6 ((r2^2 m R2^4 s^3 Q1 + r2^2 m R2^4 s^3 p1 Q0) q1 + r2^2 m R2^4 s^3 p1 Q1 q0) P1 \\
& + 6 ((r2^2 m R2^4 s^5 Q1 + r2^2 m R2^4 s^5 p1 Q0) q1 + r2^2 m R2^4 s^5 p1 Q1 q0) P1 \\
& + 12 ((r2^4 m R2^2 s Q1 + r2^4 m R2^2 s p1 Q0) q1 + r2^4 m R2^2 s p1 Q1 q0) P1 \\
& + 6 r2^2 m R2^4 s^3 p1 Q1 q1 + 6 r2^2 m R2^4 s^5 p1 Q1 q1) + q1 (- (r2 RI^4 Q1 + r2 RI^4 p1 Q0) R2 q1 \\
& + 4 r2^2 RI^4 Q1^2 - r2 m R2^5 s^5 q0^2 P1^2 + r2^2 RI^4 q1^2 - r2 m R2^5 s^5 q1^2 \\
& + 3 (m R2^5 s^3 c^2 Q1^2 + 4 m R2^5 s^3 c^2 p1 Q0 Q1 + m R2^5 s^3 c^2 p1^2 Q0^2) r2 \\
& + 3 m R2^5 s c^2 r2 Q1^2 + 3 m R2^5 s c^2 r2 p1^2 Q0^2 + 12 m R2^5 s c^2 r2 p1 Q0 Q1 \\
& - 3 (m R2^5 s^3 c^2 r2 Q1 + m R2^5 s^3 c^2 r2 p1 Q0) q1 \\
& - 3 (m R2^5 s^3 c^2 r2 Q0 q1 + (m R2^5 s^3 c^2 r2 Q1 + m R2^5 s^3 c^2 r2 p1 Q0) q0) P1 \\
& - 3 m R2^5 s^3 c^2 r2 p1 Q1 q0 - 3 (m R2^5 s c^2 r2 Q1 + m R2^5 s c^2 r2 p1 Q0) q1 \\
& - 3 (m R2^5 s c^2 r2 Q0 q1 + (m R2^5 s c^2 r2 Q1 + m R2^5 s c^2 r2 p1 Q0) q0) P1 \\
& - 3 m R2^5 s c^2 r2 p1 Q1 q0 + r2^2 RI^4 q0^2 P1^2 + 4 r2^2 RI^4 p1^2 Q0^2 + 4 r2^2 RI^4 q0 q1 P1 \\
& + 4 (r2^2 RI^4 Q1^2 + 4 r2^2 RI^4 p1 Q0 Q1 + r2^2 RI^4 p1^2 Q0^2) ta^2 \\
& - (r2 RI^4 q0^2 P1^2 + 4 r2 RI^4 q0 q1 P1 + r2 RI^4 q1^2) R2 \\
& + (r2^2 RI^4 q0^2 P1^2 + 4 r2^2 RI^4 q0 q1 P1 + r2^2 RI^4 q1^2) ta^2 + 16 r2^2 RI^4 p1 Q0 Q1 \\
& - 4 r2 m R2^5 s^5 q0 q1 P1 - 4 r2^2 m R2^4 s^3 ta^2 Q1^2 + 8 r2^2 m R2^4 s^5 ta^2 Q1^2 \\
& + 2 (r2 RI^4 Q1^2 + 4 r2 RI^4 p1 Q0 Q1 + r2 RI^4 p1^2 Q0^2) R2 + 8 r2^2 m R2^4 s^5 ta^2 p1^2 Q0^2 \\
& + 32 r2^2 m R2^4 s^5 ta^2 p1 Q0 Q1 + 2 r2^2 m R2^4 s^5 ta^2 q1^2 + 2 r2^2 m R2^4 s^5 ta^2 q0^2 P1^2 \\
& + 8 r2^2 m R2^4 s^5 ta^2 q0 q1 P1 - r2^2 m R2^4 s^3 ta^2 q1^2 - r2^2 m R2^4 s^3 ta^2 q0^2 P1^2 \\
& - 4 r2^2 m R2^4 s^3 ta^2 q0 q1 P1 + 4 ((r2^2 RI^4 Q0 q1 + (r2^2 RI^4 Q1 + r2^2 RI^4 p1 Q0) q0) P1 \\
& + (r2^2 RI^4 Q1 + r2^2 RI^4 p1 Q0) q1 + r2^2 RI^4 p1 Q1 q0) ta^2 - 4 r2^2 m R2^4 s^3 ta^2 p1^2 Q0^2 \\
& - 16 r2^2 m R2^4 s^3 ta^2 p1 Q0 Q1 + 2 r2 m R2^5 s^5 Q1^2 + 4 (r2^2 RI^4 Q1 + r2^2 RI^4 p1 Q0) q1 \\
& + 4 (r2^2 RI^4 Q0 q1 + (r2^2 RI^4 Q1 + r2^2 RI^4 p1 Q0) q0) P1 + 2 r2 m R2^5 s^5 p1^2 Q0^2 \\
& + 8 r2 m R2^5 s^5 p1 Q0 Q1 + 12 r2^2 m R2^4 s^3 Q1^2 + 12 r2^2 m R2^4 s^3 p1^2 Q0^2 \\
& + 48 r2^2 m R2^4 s^3 p1 Q0 Q1 + 12 r2^2 m R2^4 s^5 Q1^2 + 12 r2^2 m R2^4 s^5 p1^2 Q0^2 \\
& + 48 r2^2 m R2^4 s^5 p1 Q0 Q1 + 24 r2^4 m R2^2 s Q1^2 + 24 r2^4 m R2^2 s p1^2 Q0^2 \\
& + 96 r2^4 m R2^2 s p1 Q0 Q1 + 4 r2^2 RI^4 p1 Q1 q0 + 12 r2^4 m R2^2 s p1 Q1 q0 \\
& - (r2 RI^4 Q0 R2 q1 + (r2 RI^4 Q1 + r2 RI^4 p1 Q0) R2 q0) P1 \\
& + 8 (r2^2 m R2^4 s^5 ta^2 Q1 + r2^2 m R2^4 s^5 ta^2 p1 Q0) q1
\end{aligned}$$

$$\begin{aligned}
& + 8(r2^2 m R2^4 s^5 ta^2 Q0 q1 + (r2^2 m R2^4 s^5 ta^2 Q1 + r2^2 m R2^4 s^5 ta^2 p1 Q0) q0) P1 \\
& - 4(r2^2 m R2^4 s^3 ta^2 Q1 + r2^2 m R2^4 s^3 ta^2 p1 Q0) q1 \\
& - 4(r2^2 m R2^4 s^3 ta^2 Q0 q1 + (r2^2 m R2^4 s^3 ta^2 Q1 + r2^2 m R2^4 s^3 ta^2 p1 Q0) q0) P1 \\
& - (r2 m R2^5 s^5 Q1 + r2 m R2^5 s^5 p1 Q0) q1 \\
& - (r2 m R2^5 s^5 Q0 q1 + (r2 m R2^5 s^5 Q1 + r2 m R2^5 s^5 p1 Q0) q0) P1 - r2 R1^4 p1 Q1 R2 q0 \\
& + 8 r2^2 m R2^4 s^5 ta^2 p1 Q1 q0 - 4 r2^2 m R2^4 s^3 ta^2 p1 Q1 q0 - r2 m R2^5 s^5 p1 Q1 q0 \\
& + 6(r2^2 m R2^4 s^3 Q1 + r2^2 m R2^4 s^3 p1 Q0) q1 \\
& + 6(r2^2 m R2^4 s^3 Q0 q1 + (r2^2 m R2^4 s^3 Q1 + r2^2 m R2^4 s^3 p1 Q0) q0) P1 \\
& + 6(r2^2 m R2^4 s^5 Q1 + r2^2 m R2^4 s^5 p1 Q0) q1 \\
& + 6(r2^2 m R2^4 s^5 Q0 q1 + (r2^2 m R2^4 s^5 Q1 + r2^2 m R2^4 s^5 p1 Q0) q0) P1 \\
& + 12(r2^4 m R2^2 s Q1 + r2^4 m R2^2 s p1 Q0) q1 \\
& + 12(r2^4 m R2^2 s Q0 q1 + (r2^4 m R2^2 s Q1 + r2^4 m R2^2 s p1 Q0) q0) P1 \\
& + 6 r2^2 m R2^4 s^3 p1 Q1 q0 + 6 r2^2 m R2^4 s^5 p1 Q1 q0
\end{aligned}$$

> B2:=coeff(beta2,s1^2);

$$\begin{aligned}
B2 := & q0 (-(r2 R1^4 Q1 + r2 R1^4 p1 Q0) R2 q1 + 4 r2^2 R1^4 Q1^2 - r2 m R2^5 s^5 q0^2 P1^2 + r2^2 R1^4 q1^2 \\
& - r2 m R2^5 s^5 q1^2 + 3(m R2^5 s^3 c^2 Q1^2 + 4 m R2^5 s^3 c^2 p1 Q0 Q1 + m R2^5 s^3 c^2 p1^2 Q0^2) r2 \\
& + 3 m R2^5 s c^2 r2 Q1^2 + 3 m R2^5 s c^2 r2 p1^2 Q0^2 + 12 m R2^5 s c^2 r2 p1 Q0 Q1 \\
& - 3(m R2^5 s^3 c^2 r2 Q1 + m R2^5 s^3 c^2 r2 p1 Q0) q1 \\
& - 3(m R2^5 s^3 c^2 r2 Q0 q1 + (m R2^5 s^3 c^2 r2 Q1 + m R2^5 s^3 c^2 r2 p1 Q0) q0) P1 \\
& - 3 m R2^5 s^3 c^2 r2 p1 Q1 q0 - 3(m R2^5 s c^2 r2 Q1 + m R2^5 s c^2 r2 p1 Q0) q1 \\
& - 3(m R2^5 s c^2 r2 Q0 q1 + (m R2^5 s c^2 r2 Q1 + m R2^5 s c^2 r2 p1 Q0) q0) P1 \\
& - 3 m R2^5 s c^2 r2 p1 Q1 q0 + r2^2 R1^4 q0^2 P1^2 + 4 r2^2 R1^4 p1^2 Q0^2 + 4 r2^2 R1^4 q0 q1 P1 \\
& + 4(r2^2 R1^4 Q1^2 + 4 r2^2 R1^4 p1 Q0 Q1 + r2^2 R1^4 p1^2 Q0^2) ta^2 \\
& - (r2 R1^4 q0^2 P1^2 + 4 r2 R1^4 q0 q1 P1 + r2 R1^4 q1^2) R2 \\
& + (r2^2 R1^4 q0^2 P1^2 + 4 r2^2 R1^4 q0 q1 P1 + r2^2 R1^4 q1^2) ta^2 + 16 r2^2 R1^4 p1 Q0 Q1 \\
& - 4 r2 m R2^5 s^5 q0 q1 P1 - 4 r2^2 m R2^4 s^3 ta^2 Q1^2 + 8 r2^2 m R2^4 s^5 ta^2 Q1^2 \\
& + 2(r2 R1^4 Q1^2 + 4 r2 R1^4 p1 Q0 Q1 + r2 R1^4 p1^2 Q0^2) R2 + 8 r2^2 m R2^4 s^5 ta^2 p1^2 Q0^2 \\
& + 32 r2^2 m R2^4 s^5 ta^2 p1 Q0 Q1 + 2 r2^2 m R2^4 s^5 ta^2 q1^2 + 2 r2^2 m R2^4 s^5 ta^2 q0^2 P1^2 \\
& + 8 r2^2 m R2^4 s^5 ta^2 q0 q1 P1 - r2^2 m R2^4 s^3 ta^2 q1^2 - r2^2 m R2^4 s^3 ta^2 q0^2 P1^2 \\
& - 4 r2^2 m R2^4 s^3 ta^2 q0 q1 P1 + 4((r2^2 R1^4 Q0 q1 + (r2^2 R1^4 Q1 + r2^2 R1^4 p1 Q0) q0) P1 \\
& + (r2^2 R1^4 Q1 + r2^2 R1^4 p1 Q0) q1 + r2^2 R1^4 p1 Q1 q0) ta^2 - 4 r2^2 m R2^4 s^3 ta^2 p1^2 Q0^2 \\
& - 16 r2^2 m R2^4 s^3 ta^2 p1 Q0 Q1 + 2 r2 m R2^5 s^5 Q1^2 + 4(r2^2 R1^4 Q1 + r2^2 R1^4 p1 Q0) q1 \\
& + 4(r2^2 R1^4 Q0 q1 + (r2^2 R1^4 Q1 + r2^2 R1^4 p1 Q0) q0) P1 + 2 r2 m R2^5 s^5 p1^2 Q0^2 \\
& + 8 r2 m R2^5 s^5 p1 Q0 Q1 + 12 r2^2 m R2^4 s^3 Q1^2 + 12 r2^2 m R2^4 s^3 p1^2 Q0^2 \\
& + 48 r2^2 m R2^4 s^3 p1 Q0 Q1 + 12 r2^2 m R2^4 s^5 Q1^2 + 12 r2^2 m R2^4 s^5 p1^2 Q0^2 \\
& + 48 r2^2 m R2^4 s^5 p1 Q0 Q1 + 24 r2^4 m R2^2 s Q1^2 + 24 r2^4 m R2^2 s p1^2 Q0^2 \\
& + 96 r2^4 m R2^2 s p1 Q0 Q1 + 4 r2^2 R1^4 p1 Q1 q0 + 12 r2^4 m R2^2 s p1 Q1 q0 \\
& - (r2 R1^4 Q0 R2 q1 + (r2 R1^4 Q1 + r2 R1^4 p1 Q0) R2 q0) P1 \\
& + 8(r2^2 m R2^4 s^5 ta^2 Q1 + r2^2 m R2^4 s^5 ta^2 p1 Q0) q1 \\
& + 8(r2^2 m R2^4 s^5 ta^2 Q0 q1 + (r2^2 m R2^4 s^5 ta^2 Q1 + r2^2 m R2^4 s^5 ta^2 p1 Q0) q0) P1 \\
& - 4(r2^2 m R2^4 s^3 ta^2 Q1 + r2^2 m R2^4 s^3 ta^2 p1 Q0) q1 \\
& - 4(r2^2 m R2^4 s^3 ta^2 Q0 q1 + (r2^2 m R2^4 s^3 ta^2 Q1 + r2^2 m R2^4 s^3 ta^2 p1 Q0) q0) P1
\end{aligned}$$

$$\begin{aligned}
& - (r2 m R2^5 s^5 Q1 + r2 m R2^5 s^5 p1 Q0) q1 \\
& - (r2 m R2^5 s^5 Q0 q1 + (r2 m R2^5 s^5 Q1 + r2 m R2^5 s^5 p1 Q0) q0) P1 - r2 RI^4 p1 Q1 R2 q0 \\
& + 8 r2^2 m R2^4 s^5 ta^2 p1 Q1 q0 - 4 r2^2 m R2^4 s^3 ta^2 p1 Q1 q0 - r2 m R2^5 s^5 p1 Q1 q0 \\
& + 6 (r2^2 m R2^4 s^3 Q1 + r2^2 m R2^4 s^3 p1 Q0) q1 \\
& + 6 (r2^2 m R2^4 s^3 Q0 q1 + (r2^2 m R2^4 s^3 Q1 + r2^2 m R2^4 s^3 p1 Q0) q0) P1 \\
& + 6 (r2^2 m R2^4 s^5 Q1 + r2^2 m R2^4 s^5 p1 Q0) q1 \\
& + 6 (r2^2 m R2^4 s^5 Q0 q1 + (r2^2 m R2^4 s^5 Q1 + r2^2 m R2^4 s^5 p1 Q0) q0) P1 \\
& + 12 (r2^4 m R2^2 s Q1 + r2^4 m R2^2 s p1 Q0) q1 \\
& + 12 (r2^4 m R2^2 s Q0 q1 + (r2^4 m R2^2 s Q1 + r2^4 m R2^2 s p1 Q0) q0) P1 \\
& + 6 r2^2 m R2^4 s^3 p1 Q1 q0 + 6 r2^2 m R2^4 s^5 p1 Q1 q0 + q1 (-r2 RI^4 Q1 + r2 RI^4 p1 Q0) R2 q0 \\
& + 24 r2^2 m R2^4 s^3 p1 Q0^2 + 3 (2 m R2^5 s^3 c^2 Q0 Q1 + 2 m R2^5 s^3 c^2 p1 Q0^2) r2 \\
& + 6 m R2^5 s c^2 r2 Q0 Q1 + 6 m R2^5 s c^2 r2 p1 Q0^2 \\
& - 3 (m R2^5 s^3 c^2 r2 Q1 + m R2^5 s^3 c^2 r2 p1 Q0) q0 - 3 m R2^5 s^3 c^2 r2 Q0 q1 \\
& - 3 m R2^5 s^3 c^2 r2 Q0 q0 P1 - 3 (m R2^5 s c^2 r2 Q1 + m R2^5 s c^2 r2 p1 Q0) q0 \\
& - 3 m R2^5 s c^2 r2 Q0 q1 - 3 m R2^5 s c^2 r2 Q0 q0 P1 + 2 r2^2 RI^4 q0 q1 + 2 r2^2 RI^4 q0^2 P1 \\
& + 8 r2^2 RI^4 Q0 Q1 + 8 r2^2 RI^4 p1 Q0^2 + 4 (2 r2^2 RI^4 Q0 Q1 + 2 r2^2 RI^4 p1 Q0^2) ta^2 \\
& - (2 r2 RI^4 q0^2 P1 + 2 r2 RI^4 q0 q1) R2 + (2 r2^2 RI^4 q0^2 P1 + 2 r2^2 RI^4 q0 q1) ta^2 \\
& + 4 r2^2 RI^4 Q0 q1 - 2 r2 m R2^5 s^5 q0 q1 - 2 r2 m R2^5 s^5 q0^2 P1 \\
& + 2 (2 r2 RI^4 Q0 Q1 + 2 r2 RI^4 p1 Q0^2) R2 + 16 r2^2 m R2^4 s^5 ta^2 Q0 Q1 \\
& + 16 r2^2 m R2^4 s^5 ta^2 p1 Q0^2 + 4 r2^2 m R2^4 s^5 ta^2 q0 q1 + 4 r2^2 m R2^4 s^5 ta^2 q0^2 P1 \\
& - 2 r2^2 m R2^4 s^3 ta^2 q0 q1 - 2 r2^2 m R2^4 s^3 ta^2 q0^2 P1 - 8 r2^2 m R2^4 s^3 ta^2 Q0 Q1 \\
& - 8 r2^2 m R2^4 s^3 ta^2 p1 Q0^2 + 4 (r2^2 RI^4 Q1 + r2^2 RI^4 p1 Q0) q0 + 4 r2 m R2^5 s^5 Q0 Q1 \\
& + 4 r2 m R2^5 s^5 p1 Q0^2 + 24 r2^2 m R2^4 s^3 Q0 Q1 + 24 r2^2 m R2^4 s^5 Q0 Q1 \\
& + 24 r2^2 m R2^4 s^5 p1 Q0^2 + 48 r2^4 m R2^2 s Q0 Q1 + 48 r2^4 m R2^2 s p1 Q0^2 \\
& + 4 r2^2 RI^4 Q0 q0 P1 + 12 r2^4 m R2^2 s Q0 q1 + 12 r2^4 m R2^2 s Q0 q0 P1 \\
& + 4 (r2^2 RI^4 Q0 q0 P1 + r2^2 RI^4 Q0 q1 + (r2^2 RI^4 Q1 + r2^2 RI^4 p1 Q0) q0) ta^2 \\
& + 8 (r2^2 m R2^4 s^5 ta^2 Q1 + r2^2 m R2^4 s^5 ta^2 p1 Q0) q0 \\
& - 4 (r2^2 m R2^4 s^3 ta^2 Q1 + r2^2 m R2^4 s^3 ta^2 p1 Q0) q0 - (r2 m R2^5 s^5 Q1 + r2 m R2^5 s^5 p1 Q0) q0 \\
& - r2 RI^4 Q0 R2 q1 - r2 RI^4 Q0 R2 q0 P1 + 8 r2^2 m R2^4 s^5 ta^2 Q0 q1 + 8 r2^2 m R2^4 s^5 ta^2 Q0 q0 P1 \\
& - 4 r2^2 m R2^4 s^3 ta^2 Q0 q1 - 4 r2^2 m R2^4 s^3 ta^2 Q0 q0 P1 - r2 m R2^5 s^5 Q0 q1 \\
& - r2 m R2^5 s^5 Q0 q0 P1 + 6 (r2^2 m R2^4 s^3 Q1 + r2^2 m R2^4 s^3 p1 Q0) q0 \\
& + 6 (r2^2 m R2^4 s^5 Q1 + r2^2 m R2^4 s^5 p1 Q0) q0 + 12 (r2^4 m R2^2 s Q1 + r2^4 m R2^2 s p1 Q0) q0 \\
& + 6 r2^2 m R2^4 s^3 Q0 q1 + 6 r2^2 m R2^4 s^3 Q0 q0 P1 + 6 r2^2 m R2^4 s^5 Q0 q1 \\
& + 6 r2^2 m R2^4 s^5 Q0 q0 P1
\end{aligned}$$

> B1:=coeff(beta2,s1);

$$\begin{aligned}
B1 := & q0 (-r2 RI^4 Q1 + r2 RI^4 p1 Q0) R2 q0 + 24 r2^2 m R2^4 s^3 p1 Q0^2 \\
& + 3 (2 m R2^5 s^3 c^2 Q0 Q1 + 2 m R2^5 s^3 c^2 p1 Q0^2) r2 + 6 m R2^5 s c^2 r2 Q0 Q1 \\
& + 6 m R2^5 s c^2 r2 p1 Q0^2 - 3 (m R2^5 s^3 c^2 r2 Q1 + m R2^5 s^3 c^2 r2 p1 Q0) q0 \\
& - 3 m R2^5 s^3 c^2 r2 Q0 q1 - 3 m R2^5 s^3 c^2 r2 Q0 q0 P1 \\
& - 3 (m R2^5 s c^2 r2 Q1 + m R2^5 s c^2 r2 p1 Q0) q0 - 3 m R2^5 s c^2 r2 Q0 q1 - 3 m R2^5 s c^2 r2 Q0 q0 P1 \\
& + 2 r2^2 RI^4 q0 q1 + 2 r2^2 RI^4 q0^2 P1 + 8 r2^2 RI^4 Q0 Q1 + 8 r2^2 RI^4 p1 Q0^2 \\
& + 4 (2 r2^2 RI^4 Q0 Q1 + 2 r2^2 RI^4 p1 Q0^2) ta^2 - (2 r2 RI^4 q0^2 P1 + 2 r2 RI^4 q0 q1) R2
\end{aligned}$$

$$\begin{aligned}
& + (2 r2^2 R1^4 q0^2 P1 + 2 r2^2 R1^4 q0 q1) ta^2 + 4 r2^2 R1^4 Q0 q1 - 2 r2 m R2^5 s^5 q0 q1 \\
& - 2 r2 m R2^5 s^5 q0^2 P1 + 2 (2 r2 R1^4 Q0 Q1 + 2 r2 R1^4 p1 Q0^2) R2 + 16 r2^2 m R2^4 s^5 ta^2 Q0 Q1 \\
& + 16 r2^2 m R2^4 s^5 ta^2 p1 Q0^2 + 4 r2^2 m R2^4 s^5 ta^2 q0 q1 + 4 r2^2 m R2^4 s^5 ta^2 q0^2 P1 \\
& - 2 r2^2 m R2^4 s^3 ta^2 q0 q1 - 2 r2^2 m R2^4 s^3 ta^2 q0^2 P1 - 8 r2^2 m R2^4 s^3 ta^2 Q0 Q1 \\
& - 8 r2^2 m R2^4 s^3 ta^2 p1 Q0^2 + 4 (r2^2 R1^4 Q1 + r2^2 R1^4 p1 Q0) q0 + 4 r2 m R2^5 s^5 Q0 Q1 \\
& + 4 r2 m R2^5 s^5 p1 Q0^2 + 24 r2^2 m R2^4 s^3 Q0 Q1 + 24 r2^2 m R2^4 s^5 Q0 Q1 \\
& + 24 r2^2 m R2^4 s^5 p1 Q0^2 + 48 r2^4 m R2^2 s Q0 Q1 + 48 r2^4 m R2^2 s p1 Q0^2 \\
& + 4 r2^2 R1^4 Q0 q0 P1 + 12 r2^4 m R2^2 s Q0 q1 + 12 r2^4 m R2^2 s Q0 q0 P1 \\
& + 4 (r2^2 R1^4 Q0 q0 P1 + r2^2 R1^4 Q0 q1 + (r2^2 R1^4 Q1 + r2^2 R1^4 p1 Q0) q0) ta^2 \\
& + 8 (r2^2 m R2^4 s^5 ta^2 Q1 + r2^2 m R2^4 s^5 ta^2 p1 Q0) q0 \\
& - 4 (r2^2 m R2^4 s^3 ta^2 Q1 + r2^2 m R2^4 s^3 ta^2 p1 Q0) q0 - (r2 m R2^5 s^5 Q1 + r2 m R2^5 s^5 p1 Q0) q0 \\
& - r2 R1^4 Q0 R2 q1 - r2 R1^4 Q0 R2 q0 P1 + 8 r2^2 m R2^4 s^5 ta^2 Q0 q1 + 8 r2^2 m R2^4 s^5 ta^2 Q0 q0 P1 \\
& - 4 r2^2 m R2^4 s^3 ta^2 Q0 q1 - 4 r2^2 m R2^4 s^3 ta^2 Q0 q0 P1 - r2 m R2^5 s^5 Q0 q1 \\
& - r2 m R2^5 s^5 Q0 q0 P1 + 6 (r2^2 m R2^4 s^3 Q1 + r2^2 m R2^4 s^3 p1 Q0) q0 \\
& + 6 (r2^2 m R2^4 s^5 Q1 + r2^2 m R2^4 s^5 p1 Q0) q0 + 12 (r2^4 m R2^2 s Q1 + r2^4 m R2^2 s p1 Q0) q0 \\
& + 6 r2^2 m R2^4 s^3 Q0 q1 + 6 r2^2 m R2^4 s^3 Q0 q0 P1 + 6 r2^2 m R2^4 s^5 Q0 q1 \\
& + 6 r2^2 m R2^4 s^5 Q0 q0 P1) + q1 (4 r2^2 R1^4 Q0^2 + r2^2 R1^4 q0^2 - r2 m R2^5 s^5 q0^2 \\
& + 3 m R2^5 s^3 c^2 Q0^2 r2 + 3 m R2^5 s c^2 r2 Q0^2 - 3 m R2^5 s^3 c^2 r2 Q0 q0 - 3 m R2^5 s c^2 r2 Q0 q0 \\
& + 4 r2^2 R1^4 Q0^2 ta^2 - r2 R1^4 q0^2 R2 + r2^2 R1^4 q0^2 ta^2 + 2 r2 R1^4 Q0^2 R2 + 4 r2^2 R1^4 Q0 q0 \\
& - 4 r2^2 m R2^4 s^3 ta^2 Q0^2 + 8 r2^2 m R2^4 s^5 ta^2 Q0^2 + 2 r2^2 m R2^4 s^5 ta^2 q0^2 \\
& - r2^2 m R2^4 s^3 ta^2 q0^2 + 2 r2 m R2^5 s^5 Q0^2 + 12 r2^2 m R2^4 s^3 Q0^2 + 12 r2^2 m R2^4 s^5 Q0^2 \\
& + 24 r2^4 m R2^2 s Q0^2 + 12 r2^4 m R2^2 s Q0 q0 + 4 r2^2 R1^4 Q0 q0 ta^2 - r2 R1^4 Q0 R2 q0 \\
& + 8 r2^2 m R2^4 s^5 ta^2 Q0 q0 - 4 r2^2 m R2^4 s^3 ta^2 Q0 q0 - r2 m R2^5 s^5 Q0 q0 \\
& + 6 r2^2 m R2^4 s^3 Q0 q0 + 6 r2^2 m R2^4 s^5 Q0 q0
\end{aligned}$$

> B0:=coeff(beta2,s1,0);

$$\begin{aligned}
B0 := & q0 (4 r2^2 R1^4 Q0^2 + r2^2 R1^4 q0^2 - r2 m R2^5 s^5 q0^2 + 3 m R2^5 s^3 c^2 Q0^2 r2 \\
& + 3 m R2^5 s c^2 r2 Q0^2 - 3 m R2^5 s^3 c^2 r2 Q0 q0 - 3 m R2^5 s c^2 r2 Q0 q0 + 4 r2^2 R1^4 Q0^2 ta^2 \\
& - r2 R1^4 q0^2 R2 + r2^2 R1^4 q0^2 ta^2 + 2 r2 R1^4 Q0^2 R2 + 4 r2^2 R1^4 Q0 q0 \\
& - 4 r2^2 m R2^4 s^3 ta^2 Q0^2 + 8 r2^2 m R2^4 s^5 ta^2 Q0^2 + 2 r2^2 m R2^4 s^5 ta^2 q0^2 \\
& - r2^2 m R2^4 s^3 ta^2 q0^2 + 2 r2 m R2^5 s^5 Q0^2 + 12 r2^2 m R2^4 s^3 Q0^2 + 12 r2^2 m R2^4 s^5 Q0^2 \\
& + 24 r2^4 m R2^2 s Q0^2 + 12 r2^4 m R2^2 s Q0 q0 + 4 r2^2 R1^4 Q0 q0 ta^2 - r2 R1^4 Q0 R2 q0 \\
& + 8 r2^2 m R2^4 s^5 ta^2 Q0 q0 - 4 r2^2 m R2^4 s^3 ta^2 Q0 q0 - r2 m R2^5 s^5 Q0 q0 \\
& + 6 r2^2 m R2^4 s^3 Q0 q0 + 6 r2^2 m R2^4 s^5 Q0 q0)
\end{aligned}$$

[>  
[>  
[>

## **APPENDIX C**

**Maple inverse Laplace transforms of the differential equations**

```

[> restart:
> with(linalg):
Warning, the protected names norm and trace have been redefined and
unprotected

[> sigma1:=((E0)*epsilon/s+((B0)*beta/s))/(D2*s^2+D1*s+D0);

$$\sigma_1 := \frac{\frac{E_0 \varepsilon}{s} + \frac{B_0 \beta}{s}}{D_2 s^2 + D_1 s + D_0}$$


[> with(inttrans):
Warning, the name hilbert has been redefined

[> sigmakelvin:=invlaplace(sigma1,s,t);
sigmakelvin :=


$$(E_0 \varepsilon + B_0 \beta) \frac{\left( 1 + e^{-\frac{t D_1}{2 D_2}} \right) \left( -\cosh\left(\frac{t \sqrt{D_1^2 - 4 D_2 D_0}}{2 D_2}\right) - \frac{D_1 \sinh\left(\frac{t \sqrt{D_1^2 - 4 D_2 D_0}}{2 D_2}\right)}{\sqrt{D_1^2 - 4 D_2 D_0}} \right)}{D_0}$$


[> sigma3:=((E0)*epsilon/s+((B0)*beta/s))/(D5*s^5+D4*s^4+D3*s^3+D2*s^2+D1*s+D0);

$$\sigma_3 := \frac{\frac{E_0 \varepsilon}{s} + \frac{B_0 \beta}{s}}{D_5 s^5 + D_4 s^4 + D_3 s^3 + D_2 s^2 + D_1 s + D_0}$$


[> sigma3parameter:=invlaplace(sigma3,s,t);
sigma3parameter := -  $\frac{1}{D_0} \left( \left( \left( \left( \text{Product} \left( \left( \frac{(D_5 \alpha^4 + D_4 \alpha^3 + D_3 \alpha^2 + D_2 \alpha + D_1) e^{(-\alpha t)}}{5 D_5 \alpha^4 + 4 D_4 \alpha^3 + 3 D_3 \alpha^2 + 2 D_2 \alpha + D_1}} \right)^{-\alpha} \right) .. \right) - 1 \right) (E_0 \varepsilon + B_0 \beta) \right)$  = RootOf( $D_5 Z^5 + D_4 Z^4 + D_3 Z^3 + D_2 Z^2 + D_1 Z + D_0$ ,  $\alpha$ )
```