SWELLING AND EXCLUSION BEHAVIOR OF HYDROGELS

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In the Name of God
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ABSTRACT

The swelling and exclusion behavior of crosslinked hydrogels were determined in aqueous solutions of electrolytes and in aqueous solutions of proteins and enzymes. The gels were ionic copolymers of acrylamide, ionic copolymers of N-isopropylacrylamide (NIPA) and homopolymers of NIPA. Both anionic (weak acid and strong acid) and cationic (strong base) monomers were used. The following new gels were synthesized:

(a) - copolymer of acrylamide and 2-acrylamido-2-methyl-1-propanesulfonic acid ($R\text{-}SO_3^-\ H^+$).

(b) - copolymer of NIPA and $R\text{-}SO_3^-\ H^+$.

(c) - copolymer of NIPA and sodium vinylsulfonate.

The gel structure was varied by changing the following variables at gel preparation: total monomer concentration, fraction of ionizable monomer and proportion of crosslinker.

The swelling behavior of the gels at equilibrium was interpreted through a thermodynamic model developed from Flory's theory and an additivity rule for the osmotic pressure of polyelectrolyte–salt solutions. The kinetics of gel swelling were Fickian for NIPA–based gels, but non–Fickian for acrylamide–based gels. The rate of one–dimensional swelling or collapse was described by a mathematical model which used a material coordinate and a chemical potential driving force. Deformed copolymer gels of acrylamide and sodium acrylate exhibited a volume overshoot during swelling.

Extensive exclusion measurements were made for NIPA-based gels using uncharged solutes (polyethylene glycol and dextran), ionic solutes (sodium polyvinylsulfonate and dextran sulfate), proteins (ovalbumin, bovine albumin, lactoglobulin and cytochrome C) and enzymes (lipase, α–amylase and β-galactosidase). Exclusion is primarily by size and net charge although some proteins adsorbed onto the surfaces of hydrogels having the same charge.

RESUME

Le gonflement et le comportement d'exclusion des hydrogels réticulés ont été déterminés dans des solutions aqueuses d'électrolytes et dans des solutions aqueuses de protéines et d'enzymes. Les gels sont constitués de copolymères ioniques d'acrylamide, de copolymères ioniques de N–isopropylacrylamide (NIPA) et d'homopolymères de NIPA. Des monomères anioniques (acide faible, acide fort) et cationiques (base forte) ont été utilisés. La synthèse des nouveaux gels suivants a été réalisée:

(a) – copolymère d'acrylamide et d'acide 2–acrylamido–2–methyl–1–propane-sulfonique ($R–SO_3^-$ H^+).

(b) – copolymère de NIPA et de $R–SO_3^-$ H^+.

(c) – copolymère de NIPA et de vinylsulfonate de sodium.

La structure de gel varie avec les variables suivantes durant la préparation du gel: concentration totale en monomères, la fraction de monomère ionisable et la proportion d'agent réticulant.

Un modèle thermodynamique basé sur la théorie de Flory et la règle d'additivité de la pression osmotique des solutions de sels contenant plusieurs electrolytes a permis d'interpréter le phénomène de gonflement des gels à l'équilibre. Les cinétiques de gonflement des gels à base de NIPA suivent la loi de Fick, contrairement aux gels à base d'acrylamide. Un modèle mathématique qui utilise une coordonnée du matériau et le potentiel chimique comme force motrice décrit le taux de gonflement unidimensionel ou le taux de contraction Lors du gonflement des copolymères déformés de gels d'acrylamide et d'acrylate de sodium, une augmentation temporaire du volume a été mise en évidence.

De nombreuses mesures de l'exclusion ont été effectuées sur des gels de NIPA à l'aide de solutés non chargés (polyéthylène glycol et dextrane), de solutés ioniques (polyvinylsulfonate de sodium et sulfate de dextrane), de protéines (ovalbumine, albumine de bovin, lactoglobuline, et cytochrome C) et d'enzymes (lipase, α–amylase, et β-galactosidase). Le phénomène d'exclusion est dû à la taille et la charge nette quoique certaines protéines se soient adsorbées sur les surfaces des hydrogels ayant la même charge.

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Chapter 1

INTRODUCTION

Separation processes are a key aspect of the chemical industry. The common separation processes, however, are rarely suitable for the separation of biological products from dilute aqueous solutions. The separation and purification of protein products of genetically engineered microorganisms such as enzymes and vaccines require gentle separation processes which will not alter biological activity.

One promising technique involves the use of hydrophilic polymer gels as extraction solvents. The process consists of an equilibrium step followed by a regeneration step. A simplified diagram is shown in Fig. 1.1. Gel particles of low water content are added to the dilute aqueous solution of biological molecules. The gel swells by absorbing water and molecules of low molecular weight while large molecules and ionic solutes (if gel is polyelectrolyte) are excluded. The swollen gel is regenerated by exposing it to different pH, temperature and salt concentration which collapses the gel. The collapsed gel is separated from the released solution, the extract, and reused. The first step of this process, i.e. concentrating dilute aqueous solutions of macromolecules, was first reported by Flodin et al. (1960), but its application was limited due to the lack of a simple, low cost method for regeneration of the gel.

The second step of this process was developed by Cussler et al. (1984) after the discovery of phase transitions in polymeric gels by Dušek and Patterson (1968), Tanaka (1978, 1981) and Ilavský (1981). Stokar (1983) was the first to use crosslinked polymer gels whose swelling is a dramatic function of pH as extraction solvents.

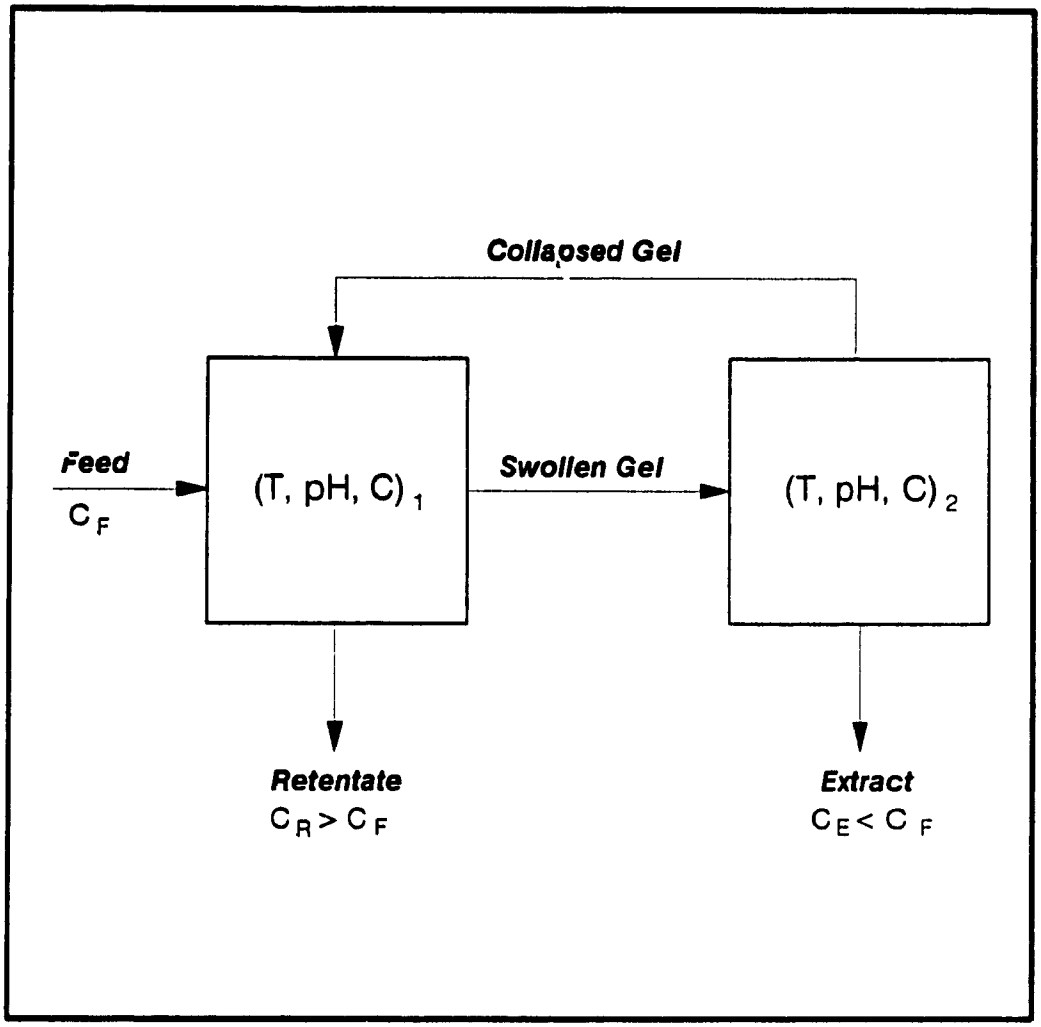


Figure 1.1: Proposed Gel Extraction Process.

The candidate gels for the process under consideration should have the following properties:

(a) – swelling by an order of magnitude in dilute aqueous solutions containing salts found in fermentation broths.

(b) – complete exclusion of large molecules.

(c) – no effect on the activity of biological molecules.

(d) – complete collapse upon exposure to mild conditions during regeneration.

(e)– stability over repeated cycles of swelling and collapse.

To meet these criteria, synthetic gels must be hydrophilic and they must be crosslinked sufficiently to maintain integrity but still allow considerable volume change. Crosslinked gels of acrylamide and its derivatives exhibit most of the desirable properties.

1.1 Potential Application of Hydrogels

Synthetic hydrogels are versatile materials which are of value in biomedical (Law et al., 1986; Lyndon, 1986; Tighe, 1986; Peppas, 1987; Heller et al., 1988; Sefton, 1989), biotechnological (Gharapetian et al., 1986; Afrassiabi et al., 1987) and industrial applications. Gel-based systems have been proposed for applications ranging from hazardous waste treatment (Huang et al., 1988) and food processing (Trank et al., 1989) to the removal of water from transformer oil (Staniewski et al., 1988) and agricultural use (Andreopoulos, 1989). Current application of highly swollen hydrogels (super absorbent polymers) include dewatering coal fines, agriculture in dry areas where the polymer can bind moisture to plant roots, and improving the efficiency of phase–change heat storage systems which use ionic salts as the working material (Anon, 1989).

The most promising hydrogels are those which respond to changes in the surrounding medium. Physiologically sensitive hydrogels have been used as controlled–release devices for drug delivery (Good and Mueller, 1981; Graham and McNeill, 1984; Hoffman, 1987; Lee and Good, 1987) and biosensors which are devices that translate a biological phenomenon into a measurable form such as an electrical signal which can then be monitored (Guilbeau et al., 1987; Regnault and Picciolo, 1987; Varanasi et al., 1987).

Recently, a novel electrochemical transducing system was demonstrated using

synthetic polyelectrolyte gels, in which the conversion of electrochemical energy into mechanical energy was stimulated by an electric field (Osada, 1987; Kishi and Osada, 1989). Polymer gel fingers driven by an electric field were constructed to catch or release an object inside an aqueous solution (Shiga et al., 1989).

pH-sensitive hydrogels (Gehrke, 1986) and temperature sensitive hydrogels (Freitas, 1986) were used to concentrate dilute aqueous solution of macromolecules including proteins. Trank et al. (1989) demonstrated the superiority of a gel extraction process, using temperature sensitive hydrogels, to conventional methods for separation of edible proteins from soybeans.

1.2 Objectives

The objectives of the present work were:

(i) – To develop hydrophilic gels which are suitable for concentrating dilute aqueous solutions of biological products.

(ii) – To investigate the swelling, collapse and exclusion behavior of these hydrogels.

(iii) – To develop a thermodynamic model which describes equilibrium swelling.

(iv) – To develop a mathematical model which describes the kinetics of gel swelling.

1.3 Organization of the Thesis

The thesis is organized in the following manner. Chapter 2 describes the experimental procedure which includes gel preparation, swelling experiments, selectivity experiments and reproducibility of experimental results. The principles of the thermodynamic formulations describing the equilibrium swelling of polyelectrolyte gels are presented in Chapter 3. Chapter 4 opens with a review of the literature pertinent to the kinetics of gel volume change. A general mathematical model describing the kinetics of gel swelling for different geometries is derived, using polymer material coordinates. The swelling behavior of hydrogels which include equilibrium swelling results, comparison of the thermodynamic model and experiment, collapse of hydrogels and pattern formation are presented in Chapter 5. This chapter also analyzes the dynamic swelling results in light of the background given in Chapter 4. The

mathematical model of kinetics of gel swelling is compared to experimental results. Chapter 6 describes the exclusion behavior of hydrogels. The interactions of ionic and nonionic hydrogels with different solutes are examined. The conclusions are presented in Chapter 7.

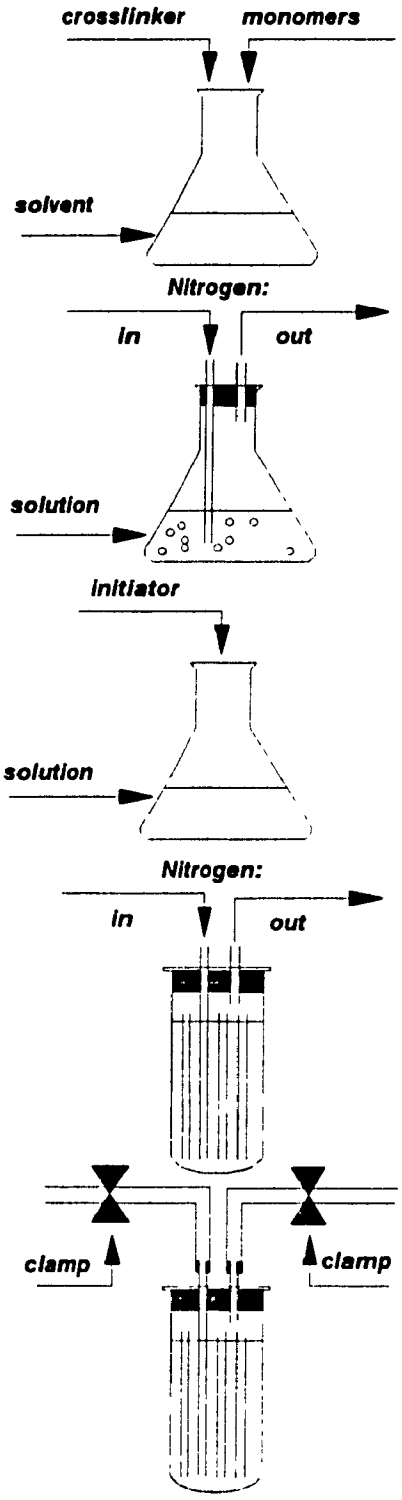
Chapter 2

GEL PREPARATION AND EXPERIMENTAL PROCEDURES

For convenience the experimental work is considered in three parts. The first part is the synthesis of the polymeric gels. The second part is the swelling behavior of these gels in response to changes in the surrounding medium. The third part involves the use of the gels for concentrating dilute aqueous solutions of macromolecules.

2.1 Gel Preparation

Copolymers of acrylamide and sodium acrylate, 3-(methacrylamido)propyltrimethylammonium chloride or 2-acrylamido-2-methyl-1-propanesulfonic acid ($R\text{–}SO_3^-\ H^+$) were prepared. Gels were also prepared as homopolymers of N-isopropylacrylamide (NIPA), copolymers of NIPA and sodium acrylate or $R\text{–}SO_3^-\ H^+$, sodium vinylsulfonate and copolymers of N,N-diethylacrylamide and sodium methacrylate. The hydrogels were prepared by free radical solution polymerization in distilled water (pH$\sim$ 5.7), under a nitrogen atmosphere. The procedure is shown in Fig. 2.1. Monomers and crosslinking agent were added to sufficient nitrogen-sparged distilled water in a 250 mL Erlenmeyer flask to make 50 mL of solution after all materials were added. This solution was flushed with nitrogen until the materials dissolved completely (step 2). The initiator and accelerator were added (step 3) and the solution was transferred to a large test tube (22 mm i.d.) which contained a number of glass tubes of 2.4 mm i.d. The solution was flushed with nitrogen for 5 to 10 minutes and then sealed (step 4). The polymerization time was 24 hr (step 5). The polymerization temperature was 23°C ($\pm$2°C) except for copolymers of N,N-diethylacrylamide and sodium methacrylate where it was 6°C ($\pm$1°C).



Step 1: Add monomer(s) and crosslinking agent to N_2-sparged distilled water in a 125 mL Erlenmeyer to make 50 mL of solution (after Step 3).

Step 2: Flush solution with nitrogen until monomer(s) and crosslinking agent dissolve.

Step 3: Add initiator and accelerator.

Step 4: Transfer solution to a large test tube containing a number of smaller glass tubes and flush with N_2 for five to ten minutes.

Step 5: Clamp plastic tubes and allow polymerization to proceed for 24 hr at a specific temperature.

Figure 2.1: Procedure for gel preparation.

After polymerization, the gels were forced from the tubes and cut into cylinders having lengths roughly equal to their diameters. In some extraction experiments gel particles of different length-to-diameter ratios were used.

Acrylamide (A & C Chemicals), N-isopropylacrylamide (Polyscience), N,N diethylacrylamide (Polyscience), sodium acrylate (Polyscience), sodium methacrylate (Polyscience), 3-(methacrylamido)propyltrimethylammonium chloride (Aldrich), 2-acrylamido-2-methyl-1-propanesulfonic acid (Aldrich), sodium vinylsulfonate (Aldrich), ammonium persulfate (Aldrich), sodium metabisulfite (Aldrich) and N,N,N',N'-tetramethylethylenediamine (Aldrich) were reagent grade and were used as received. The N,N'-methylenebisacrylamide (Aldrich) and the ethylene diacrylate (Aldrich) were electrophoresis grade. The structures and molecular weights of these materials are shown in Table 2.1. The gel composition is specified in terms of percent

Table 2.1: Gel Materials.

Material	Function	Formula	MW
Acrylamide	Monomer	$H_2C{=}CHCONH_2$	71.08
N-isopropylacrylamide (NIPA)	Monomer	$H_2C{=}CHCONHCH(CH_3)_2$	113.16
N,N-diethylacrylamide (NDEA)	Monomer	$H_2C{=}CHCON(C_2H_5)_2$	127.19
Sodium acrylate	Monomer (ionizable)	$H_2C{=}CHCOO^- \ Na^+$	94.08
Sodium methacrylate	Monomer (ionizable)	$H_2C{=}C(CH_3)COO^- \ Na^+$	108.08
2-acrylamido-2-methyl-1-propanesulfonic acid ($R\text{-}SO_3^- \ H^+$)	Monomer (ionizable)	$H_2C{=}CHCONHC(CH_3)_2CH_2SO_3H$	207.25
Sodium vinylsulfonate	Monomer (ionizable)	$H_2C{=}CHSO_3^- \ Na^+$	130.1
3-methacrylamidopropyl-trimethylammonium chloride (MAPTAC)	Monomer (ionizable)	$H_2C{=}C(CH_3)CONH(CH_2)_3N(CH_3)_3Cl$	220.74
N,N'-methylenebisacrylamide	Crosslinker	$(H_2C{=}CHCONH)_2CH_2$	154.17
Ethylene diacrylate	Crosslinker	$[H_2C{=}CHCO_2CH_2\ ]_2$	170.16
N,N,N',N'-tetramethyl-ethylenediamine	Accelerator	$(CH_3)_2NCH_2CH_2N(CH_3)_2$	116.21
Sodium metabisulfite	Accelerator	$Na_2S_2O_5$	190.10
Ammonium persulfate	Initiator	$(NH_4)_2S_2O_8$	228.20

(w/v) of total monomers at preparation (%T), percent (w/w) of crosslinker (%C) and mole percent ionizable monomer (%I). These quantities are defined by (Chrambach and Rodbard, 1971; Ilavský, 1982):

$$\%T = \frac{\text{mass of all monomers}(g)}{\text{volume of solution}(cm^3)} \times 100 \tag{2.1}$$

$$\%C = \frac{\text{mass of crosslinking agent}}{\text{mass of all monomers}} \times 100 \tag{2.2}$$

$$\%I = \frac{\text{moles of ionizable monomer}}{\text{moles of all monomers}} \times 100 \tag{2.3}$$

For example, steps 1 and 2 in the formation of a copolymer gel of acrylamide and sodium acrylate (5.3%T, 2.5%C, 10.0%I) were the dissolution of 2.2460 g of acrylamide, 0.3348 g of sodium acrylate and 0.0665 g of N,N'–methylenebisacrylamide in distilled water to make 49 mL solution. Step 3 involved the addition of 0.5 mL of 4 w/v% of ammonium persulfate and 0.5 mL of 4 w/v% of sodium metabisulfite to make 50 mL of solution. All gels were prepared using N,N'–methylenebisacrylamide as crosslinking agent except for an anionic gel which was prepared using ethylene diacrylate (0.0734 g in 50 mL solution). Sodium metabisulfite was used as accelerator except for the cationic gel which was prepared by addition of 140 μL N,N,N',N'–tetramethylethylenediamine to the solution as an accelerator. Table 2.2 shows the

Table 2.2: Classification of Hydrogels and Monomers.

Type of Gel	Monomers	
	nonionic	ionic
Nonionic	NIPA	—
Anionic	NIPA	sodium acrylate
Anionic	NIPA	$R–SO_3^-\ H^+$
Anionic	NIPA	sodium vinylsulfonate
Anionic	NDEA	sodium methacrylate
Anionic	Acrylamide	sodium acrylate
Anionic	Acrylamide	$R–SO_3^-\ H^+$
Cationic	Acrylamide	MAPTAC

types of gels used in this work and the monomers required to prepare them.

To prepare the cationic gel, whose composition is given in Table 2.3, the small glass tubes were first immersed in nitrogen-sparged paraffin oil and then transferred to the test tube, where polymerization took place. If this were not done, it was difficult to remove the gel from these tubes after polymerization.

Table 2.3: Composition of the cationic copolymer gel of acrylamide and 3-(methacrylamido)propyltrimethylammonium chloride (MAPTAC).

Gel Identification	Acrylamide (g)	MAPTAC (g)	Crosslinker (g)
6.2%T, 2.2%C, 10.0%I	2.2460	0.7855	0.0665

Different copolymer gels of acrylamide and sodium acrylate were prepared. The compositions of these anionic gels are given in Table 2.4.

Table 2.4: Composition of the anionic copolymer gel of acrylamide and sodium acrylate.

Gel Identification	Acrylamide (g)	Na-acrylate (g)	Crosslinker (g)
2.7%T, 2.5%C, 10.0%I	1.1230	0.1674	0.0333
3.7%T, 2.5%C, 10.0%I	1.5720	0.2344	0.0465
5.3%T, 2.5%C, 10.0%I	2.2460	0.3348	0.0665
5.3%T, 2.8%C, 10.0%I	2.2460	0.3348	0.0734*
10.6%T, 2.5%C, 10.0%I	4.4920	0.6696	0.1330
21.2%T, 2.5%C, 10.0%I	8.9840	1.3392	0.2660
5.2%T, 2.6%C, 5.0%I	2.3730	0.1674	0.0665

*Crosslinker: Ethylene diacrylate

The compositions of the anionic copolymer gels of acrylamide and 2-acrylamido-2-methyl-1-propanesulfonic acid ($R\text{-}SO_3^- H^+$) are given in Table 2.5.

Table 2.5: Composition of copolymer gels of acrylamide and R-SO_3^- H^+.

Gel Identification	Acrylamide (g)	R-SO_3^- H^+ (g)	Crosslinker (g)
6.1%T, 2.2%C, 10.0%I	2.2460	0.7377	0.0665
6.2%T, 3.2%C, 10.0%I	2.2460	0.7377	0.1000

An anionic gel was prepared which contained equal mole percent of sodium acrylate and highly acidic monomer, R-SO_3^- H^+. The composition is given in Table 2.6.

Table 2.6: Composition of anionic copolymer gel of acrylamide which contained both weakly and highly acidic monomers.

Gel Identification	Acrylamide (g)	R-SO_3^- H^+ (g)	Na-acrylate (g)
5.6%T, 2.4%C, 5.0%$I_{SO_3^-}$, 5.0%I_{COO^-}	2.2460	0.3689	0.1674

Table 2.7 shows the composition of an anionic copolymer gel of N,N'-diethylacrylamide (NDEA) and sodium methacrylate which was prepared at 6°C (±1°C).

Table 2.7: Composition of anionic copolymer gel of NDEA and sodium methacrylate.

Gel Identification	NDEA (g)	Na-methacrylate (g)	Crosslinker (g)
9.0%T, 5.6%C, 6.7%I	4.0000	0.2379	0.2500

Nonionic and ionic gels of N-isopropylacrylamide (NIPA) were prepared by the same general method except that smaller amounts of initiator and accelerator (0.0025 g of each) were used for preparation of the nonionic gel. This was done to allow the

reaction to proceed slowly, otherwise the increased rate of heat release affects the solubility of the temperature sensitive monomer resulting in a heterogereous network. The compositions of nonionic NIPA gels as well as their ionic counterparts are given in Tables 2.8 and 2.9.

Table 2.8: Composition of nonionic and ionic gels of NIPA.

Gel Identification	NIPA (g)	Na-acrylate (g)	Crosslinker (g)
8.1%T, 1.6%C	3.9800	—	0 0665
10.0%T, 1.6%C	4.9200	—	0.0822
12.0%T, 1.6%C	5.9000	—	0.0986
14.0%T, 1.6%C	6.8900	—	0.1151
20.0%T, 1.6%C	9.8400	—	0.1644
8.1%T, 1.7%C, 5.0%I	3.7920	0.1674	0.0665
8.0%T, 1.7%C, 10.0%I	3.5760	0.3348	0 0665
8.9%T, 1.7%C, 15.0%I	3.3760	0.5022	0.0665

Table 2.9: Composition of copolymer gels of NIPA and R-SO_3^- H^+.

Gel Identification	NIPA (g)	R-SO_3^- H^+ (g)	Crosslinker (g)
8.2%T, 1.6%C, 1.0%I	3.9460	0 0738	0 0665
8.3%T, 1.6%C, 2.5%I	3 8790	0.1845	0.0665
8.4%T, 1.6%C, 5.0%I	3.7780	0 3689	0 0665
8.5%T, 2.4%C, 5 0%I	3.7540	0.3689	0 1000
8.6%T, 1.3%C, 7.5%I	3.6780	0.5534	0.0665
8.8%T, 2.3%C, 10.0%I	3.5520	0.7378	0 1000
8.3%T, 3.2%C, 2 5%I	3.8310	0.1845	0.1330
8.3%T, 5 0%C, 2.5%I	3.7740	0.1845	0 2080
8.4%T, 7.0%C, 2.5%I	3.7130	0.1845	0.2930
10.0%T, 1.6%C, 2.5%I	4.6940	0 2232	0 0805
12.5%T, 1.6%C, 2 5%I	5.8680	0 2790	0.1006
16.0%T, 1.6%C, 2 5%I	7.5100	0.3571	0 1288
20.0%T, 1.6%C, 2.5%I	9.3880	0.4405	0.1610

A second strong acid gel was a copolymer of NIPA (3.7780 g) and sodium vinylsulfonate (0.2316 g). In contrast to 2-acrylamido-2-methyl-1-propanesulfonic acid (R-SO_3^- H^+), which may cause cancer, sodium vinylsulfonate is not toxic.

2.2 Swelling Experiments

The second part of the experimental work was aimed at obtaining information about the swelling behavior of the hydrogels. The experiments were divided into equilibrium swelling experiments and dynamic swelling experiments.

In the equilibrium swelling experiments, the original mass of each gel piece at preparation, M_o, was determined and then each piece was dialyzed for 48 hr against a large amount of distilled water to remove minute quantities of impurities and unreacted monomers or oligomers trapped in the network. The degree of swelling was measured by immersing a piece of dialyzed gel (original mass, M_o, of approximately 0.01 g) in 2 L of a solution of known composition, pH and temperature. The solution pH was varied by addition of either NaOH or HNO_3. In experiments which involved the effect of both pH and salt, the solution was replaced with fresh solution every 6 hours. After equilibration for at least 48 hr, the pH was measured and the swollen gel was removed from the solution and weighed after blotting with tissue paper. The swelling ratio was defined as the ratio of swollen mass, M, to the original mass, M_o.

A Metrohm pH–meter (Brinkmann, Model 632) was used to measure the pH. The salts studied include: sodium nitrate (Fisher), sodium chloride (Anachemia), sodium phosphate (Anachemia), sodium sulfate (Fisher), silver nitrate (Fisher), potassium permanganate (Fisher), calcium nitrate (Anachemia), calcium chloride (BDH chemicals), cobaltous chloride (Anachemia), cupric nitrate (Fisher) and lanthanum nitrate (Anachemia). All salts were reagent grade and were used as received. The concentrations of the solutions ranged from 0 to 0.1 M except for cupric nitrate and lanthanum nitrate (maximum 10^{-3} M), silver nitrate and cobaltous chloride (maximum 10^{-2} M) due to the precipitation at higher concentrations at pH 7.

Swollen hydrogels were collapsed by exposure to different pH, temperature and/or salt concentration. The collapse experiments were undertaken to explore the conditions required to regenerate the gels by expelling solvent so that they might be reused.

The dynamic swelling experiments involved determination of transient dimensional changes of hydrogels at 23°C (±2°C) in solutions of known pH and salt concentration. The swelling as a function of time was determined by removing the gel from the solution, weighing it after blotting with tissue paper and returning it to

the solution. The equilibrium time depended upon the size, prehistory and structure of the gel, solution pH and ionic strength, and the rate of stirring of the solution.

2.3 Extraction Experiments

To investigate gel selectivity, ionized copolymer gels of acrylamide and ionic and nonionic N-isopropylacrylamide (NIPA) gels were used. The effects of monomer concentration, crosslinking ratio, network and solute charge, pH and ionic strength of the solution on the exclusion efficiency of hydrogels were studied.

These experiments were similar to the equilibrium swelling experiments. Collapsed gel particles of known weight (0.2 - 2 g) were brought into contact with a solution of known volume, pH and concentration. The initial volume of the solution was approximately twice the volume of the swollen gel at equilibrium. The swollen gel was separated from the retentate or raffinate and weighed. The volume and concentration of retentate were also determined.

The solutes used for this study included: polyethylene glycols of different molecular weights (Aldrich and Polyscience), sodium polyvinylsulfonate (Aldrich), β-galactosidase (Aldrich), dextran, dextran sulfate, ovalbumin, bovine albumin, β-lacto- globulin, cytochrome C, α–amylase and lipase, which were bought from Sigma Chemical Company.

Polyethylene glycol (PEG), with a stoichiometric formula of $H(OCH_2CH_2-)_nOH$, is available commercially in a range of molecular weights. The concentration of polyethylene glycol solutions was determined by refractometry.

Sodium polyvinylsulfonate, the sodium salt of poly(vinylsulfonic acid), is an anionic linear polymer with an approximate molecular weight of 3400 (based on personal communication with the supplier). Its concentration was determined by refractometry.

Dextran is a polysaccharide produced by the bacterium *Leuconostoc mesenteroids*, strain no. B-512, growing on a sucrose substrate. An industrial grade dextran (MW = 19500) and a laboratory grade (MW = 9400) as well as dextran sulfate (sodium salt) with an average molecular weight of approximately 8000 were used The concentration of dextran and dextran sulfate was measured using a Polax-D polarimeter manufactured by Atago. The optical rotation of a solution (Ψ) is a function

of its concentration in weight/volume, C, its specific optical activity, $[\alpha]_D^{25}$, and the length of the polarimeter tube (L). The specific optical activity of dextran is 199° (Albertsson, 1986). The dextran concentration, C (%w/v), was calculated from

$$C = \Psi/1.99\mathrm{L} \tag{2.4}$$

Ovalbumin, the major protein of egg white, has a molecular weight of approximately 45000 and an isoelectric pH of 4.7. The ovalbumin used here was Grade V.

Bovine albumin, a widely used protein as nutrient, has a molecular weight of approximately 66000 with an isoelectric point of 4.8. Sigma Fraction V powder with 96–99% protein was used.

β–Lactoglobulin (from Bovine milk) was a 3 × crystallized and lyophilized protein with a molecular weight of approximately 37000. This protein contains subunits A and B with a molecular weight of 18400.

Cytochrome C (Sigma Type V–A) from bovine heart had a molecular weight of 12330. Its concentration was determined by spectrophotometry at 550 nm.

α–Amylase occurs in nearly all plants, animals and microorganisms. It hydrolyzes starch to oligosaccharides and in turn slowly to maltose and glucose. The enzyme used here was a 4 × crystallized and lyophilized powder from *Bacillus* species (Sigma Type II–A), with an approximate molecular weight of 50000–55000. Its isoelectric point is about 5.3 (Bernfeld, 1955). The enzyme activity of α–amylase was measured using the Somogyi (1960) method with a Sigma Diagnostic Kit (Procedure No. 700). The assay measures the time required to hydrolyze starch in a standard substrate. The procedure takes advantage of the interaction of starch with iodine to yield a blue color whereas the oligosaccharides produced by the hydrolysis yield a red color in the presence of iodine. The color change from blue to reddish–brown is sufficiently pronounced to permit visual detection. The endpoint was determined by removing portions of amylase–starch reaction mixture at timed intervals and adding them to iodine solution. As long as starch is present, purplish color develops. As the incubation proceeds, the color changes from blue to blue–purple, to red purple and finally to reddish–brown at the endpoint. The amylase activity was calculated

as follows:

$$\text{Amylase activity [Somogyi Units/dL]} = \frac{\text{Factor}}{\text{Endpoint time(min.)}} \qquad (2.5)$$

where Factor at 37°C = 1800; at 40°C = 1600.

The lipases used in this study were from Wheat Germ (Sigma Type I) and *Candida cylindracea* (Sigma Type VII). Lipase from wheat germ was a lyophilized powder containing 95% protein. It contained acid phosphates and its molecular weight is not available.

Candida cylindracea is available as an impure preparation. Tomizuka et al. (1966) determined the amino acid composition of the purified enzyme and a molecular weight of 100000–120000 was estimated from physical constants. However, two calculated values of molecular weight were not in agreement with each other and larger than that conjectured from gel filtration on a Sephadex G–100 column. The value obtained by Sephadex gel was not reported. The enzyme has a large number of hydrophilic residues and considerable leucine. The lipase used in this study contained some material insoluble in distilled water which was removed by filtration through Whatman filter paper (No. 4). The enzymatic activity of the lipase from *Candida cylindracea* was measured using a Sigma procedure (Sigma Diagnostic Kit, No. 800) which is based on the method of Tietz and Fiereck (1966). The method involves hydrolysis of triglycerides in olive oil into fatty acids, diglycerides and to some small extent into monoglycerides and glycerol. The amount of fatty acids formed under the specific conditions of the test is a measure of lipase activity in the sample. The fatty acids were determined by titration with sodium hydroxide. The required incubation time at 37°C was 3 hours using 10 mL substrate or 6 hours using 3 mL substrate. Sigma–Tietz Units/mL of lipase activity was calculated by subtracting the volume of 0.05 N NaOH used for the titration of blank substrate (no lipase) from the volume of 0.05 N NaOH used for the titration of the test substrate containing lipase.

β–Galactosidase consists of four subunits forming an active tetramer. Goldberg (1969) reported a molecular weight of 595000 for this enzyme. Erickson (1970) demonstrated discrepancies in former studies concerning the molecular weight and suggested a subunit weight of 91000. Kalnins et al. (1983), using DNA sequencing techniques, predicted that β-galactosidase consists of 1023 amino acid residues,

resulting in a protein with a molecular weight of 116353 per subunit.

β-galactosidase activity was assayed as described by Miller (1972). An appropriate volume (5–10 mL) of 0.25–0.5 mg/L enzyme solution was diluted 5 times in BB buffer: 0.2 M Tris-HCl, 0.2 M NaCl, 0.01 M MgAc, 0.01 M 2-mercaptoethanol, 5% glycerol. 5 mL of diluted sample was added to 1 mL of Z buffer: 0.06 M Na_2HPO_4, 0.04 M NaH_2PO_4, 0.01 M KCl, 0.001 M $MgSO_4$, 0.05 M 2–mercaptoethanol at pH 7, and equilibrated at 28°C. To one test tube 1 mL of Z buffer and 5 mL of BB buffer alone were added. This was a control for the spontaneous hydrolysis of O–nitrophenyl-β–D–galactopyranoside (ONPG). The reaction began by adding to each tube 0.2 mL of ONPG at 4 mg/mL, which had also been equilibrated to 28°C. The samples were incubated at 28°C for at least 5 minutes until a faint yellow color developed. The reaction was stopped by adding 0.5 mL of 1 M Na_2CO_3, and the length of time of incubation for each was recorded. The absorbance of the sample at 420 nm was read against the control containing buffer alone. The total amount of enzyme in the reaction mixture was calculated from the original sample concentration in mg/mL. The specific activity of the β–galactosidase was calculated in Units/mg as:

$$\text{Units/mg} = \frac{A_{420} \times 380}{\text{min. at } 28° \times \text{mg enzyme in reaction}} \tag{2.6}$$

The method used for the analysis of the proteins (enzymes) was spectrophotometry. This method is accurate for concentrations of proteins up to 1 g/L. The absorbance of a dilute solution at a specific wavelength is a linear function of its concentration. The accuracy of this method depends on the selection of appropriate wavelength, λ, such that the change of absorbance with wavelength is small–see Day and Underwood (1980).

The absorption maximum of proteins at 280 nm is due to the presence of the aromatic amino acids: tyrosine and tryptophan. This two amino acids are present in nearly all proteins, and their proportions relative to other amino acids usually vary over a narrow range. The absorption of ultraviolet light is suitable for estimating protein concentrations if the solution does not contain more than 20% by weight of other ultraviolet light–absorbing compounds, such as nucleic acids or phenols and if the solution is not turbid.

2.4 Reproducibility of the Experimental Results

Tables 2.10–2.12 show the extent to which the experimental results for swelling of an anionic copolymer gel of acrylamide and sodium acrylate (5.3%T, 2.5%C, 10.0%I) were reproducible. The swelling ratio was defined as the ratio of swollen mass at equilibrium, M, to the original mass of gel at preparation, M_o. The 95% confidence intervals for the effect of pH on swelling equilibrium are given in Table 2.10. The intervals range from 5.2 to 9.4% of the mean values. The 95% confidence interval for

Table 2.10: Reproducibility of the swelling ratio at different pHs.

pH	number of replicates	mean swelling ratio	sample standard deviation	95% confidence interval
4.0	4	2.5	0.081	2.5 ± 0.13
5.7	4	6.1	0.28	6.1 ± 0.45
7.0	4	26	1.1	26 ± 1.8
8.7	4	31	1.8	31 ± 2.9

the swelling ratio of a copolymer gel of acrylamide and sodium acrylate, computed from data for three different batches of gel at pH 7, is given in Table 2.11. The confidence interval is ±15.4% of sample mean. The confidence interval is wider here

Table 2.11: Reproducibility of the swelling ratio of three batches of copolymer gel of acrylamide and sodium acrylate at pH 7.

number of replicates	mean swelling ratio	sample standard deviation	95% confidence interval
3	26	1.6	26 ± 4.0

than in Table 2.10 (± 4.0 vs ± 1.8) because the value in Table 2.11 includes batch-to-batch variations. Nevertheless, the data exhibit good reproducibility. Table 2.12 shows the 95% confidence intervals for the mean swelling ratio of copolymer gels of

acrylamide and sodium acrylate at pH 7 in 10^{-3} M salt solutions of different cations. the intervals are within ±7.7–14% of the sample means.

Table 2.12: Reproducibility of the swelling ratio of copolymer gels of acrylamide and sodium acrylate at pH 7 in presence of different cations at 10^{-3} M.

cation	number of replicates	mean swelling ratio	sample standard deviation	95% confidence interval
Na^{+}	3	18	0.68	18 ± 1.7
Ca^{+2}	3	5.2	0.16	5.2 ± 0.40
La^{+3}	3	0.30	0.017	0.30 ± 0.042

The 95% confidence intervals, computed for the mean swelling ratio of a copolymer gel of N-isopropylacrylamide (NIPA) and sodium acrylate (10%I) at different temperatures, are given in Table 2.13. The intervals are within ±8.4–14.1% of the sample means.

Table 2.13: Reproducibility of the swelling ratio of copolymer gel of NIPA and sodium acrylate for different temperatures at pH 7.

temperature °C	number of replicates	mean swelling ratio	sample standard deviation	95% confidence interval
23	3	10	0.35	10 ± 0.87
35	3	9.2	0.52	9.2 ± 1.3
40	3	8.7	0.48	8.7 ± 1.2
50	3	7.7	0.26	7.7 ± 0.65
58	3	1.0	0.035	1.0 ± 0.087

Table 2.14 shows the 95% confidence interval computed for cyclic swelling/collapse of ionic copolymer gels of NIPA. The gels were swollen for 1 hr at 23°C (±2°C) in 10^{-3} M $Ca(NO_3)_2$ at pH 7 and then collapsed for 1 hr at 35°C in 1 M NaCl solution. The sample mean is the ratio of swollen mass, M_s to the collapsed mass, M_r, of the gel. The confidence intervals were within ±19.3-34.7% of the sample means. The larger

Table 2.14: Reproducibility of the swelling ratio, M_s/M_r, of copolymer gels of NIPA in swelling/collapse cycles.

ionizable monomer (5.0%I)	number of replicates	mean swelling ratio	sample standard deviation	95% confidence interval
Na-acrylate	3	14	1.1	14 ± 2.7
$R\text{-}SO_3^-\ H^+$	3	15	2.1	15 ± 5.2

confidence intervals here reflect the nonhomogeneous shrinking of the gel particles in the collapsing step.

The 95% confidence intervals for the swelling ratio in dynamic swelling of a copolymer gel of acrylamide and sodium acrylate (5.3%T, 2.5%C, 10.0%I) swollen at 23°C (±2°C) in distilled water at pH 7 are given Table 2.15. The swelling ratio is defined as the swollen mass at time t, M_t to the original mass of the gel at preparation, M_s, at t=0. The confidence intervals were within ±8.8–44.6% of the sample mean.

Table 2.15: Reproducibility of dynamic swelling of copolymer gel of acrylamide and sodium acrylate in distilled water at pH 7.

time (minute)	number of replicates	mean swelling ratio	sample standard deviation	95% confidence interval
7.5	3	5.6	1.0	5.6 ± 2.5
15	3	8.9	0.80	8.9 ± 2.0
30	3	13	0.80	13 ± 2.0
60	3	17	0.71	17 ± 1.8
120	3	22	0.90	22 ± 2.2
180	3	24	0.83	24 ± 2.1
240	3	25	0.95	25 ± 2.4

The least reproducible result was at t=7.5 minutes where the swelling ratio was the smallest. This may be due to the difficulty of blotting the small gel particles of irregular shape, in the early stages of swelling.

Table 2.16 shows the 95% confidence intervals for the distribution coefficient,

K_d, for exclusion behavior of anionic and cationic copolymer gels of acrylamide (10.0%I) in bovine albumin solution at pH 5.6. The distribution coefficient is defined as the ratio of the solute concentration in the gel phase, $\overline{C}$, to that in the retentate (raffinate), C_R. The values of K_d were evaluated from experimental measurements by

$$K_d = (V_F C_F - V_R C_R)/(V C_R) \tag{2.7}$$

where V_F and C_F are feed volume and concentration, respectively; V_R and V are retentate and swollen gel volume, respectively. This equation is derived in Chapter 6. The 95% confidence interval was within ±22.0% and ±38.8% of the sample mean for anionic and cationic copolymer gels of acrylamide, respectively. The large confidence interval for the cationic gel is a reflection of the adsorption of the protein on the gel surface. The confidence interval for the anionic gel, where no surface adsorption occurred, is more typical of the reproducibility of K_d.

Table 2.16: Reproducibility of the exclusion behavior of anionic and cationic copolymer gels of acrylamide at 6°C (±1°C) in 0.25 g/L bovine albumin solution at pH 5.6.

ionic gel	number of replicates	mean K_d	sample standard deviation	95% confidence interval
anionic	3	0.82	0.071	0.82 ± 0.18
cationic	4	8.5	2.1	8.5 ± 3.3

The 95% confidence intervals for the distribution coefficient, K_d, and the activity of lipase (*Candida cylindracea*) are given in Table 2.17 for an anionic copolymer gel of NIPA and sodium vinylsulfonate. The gel was used to concentrate 0.25 g/L of lipase solution of pH 5.8 at 6°C (±1°C). The 95% confidence interval was ±6.0% and ±8.3% of the sample mean for K_d and enzymatic activity (Sigma–Tietz Units/mL), respectively.

Table 2.17: Reproducibility of the distribution coefficient, K_d, of lipase and its activity in exclusion experiments with NIPA–sodium vinylsulfonate gel.

variable	number of replicates	sample mean	sample standard deviation	95% confidence interval
K_d	9	0.20	0.015	0.20 ± 0.012
activity	3	3.0	0.10	3.0 ± 0.25

Chapter 3

THERMODYNAMICS OF GEL SWELLING

3.1 Introduction

Crosslinked polymer gels may exhibit large changes in volume in response to changes in the surrounding medium. The swelling behavior of gels has been investigated for many years (Procter and Wilson, 1916; Flory, 1941; Katchalsky, 1954; Katchalsky and Zwick, 1955; Sussman and Katchalsky, 1970). Interest in this subject accelerated in the late seventies upon reports by Tanaka and his coworkers of swelling/collapse phenomena in polyacrylamide gels reminiscent of vapor/liquid phase transitions (Tanaka, 1978; Tanaka, 1979; Hochberg et al., 1979). This behavior was anticipated earlier by Dušek and Patterson (1968). The volume change in the phase transition is sometimes as large as 1000-fold (Ilavský, 1982; Ilavský and Hrouz, 1983) resulting in a dramatic change in physical properties (Ulbrich and Kopeček, 1979). Hydrogels containing ionizable monomers exhibit large volume changes in the transition (Tanaka, 1981; Nicoli et al., 1983; Hirotsu et al., 1987). The collapse of the gel occurred upon a lowering temperature in acetone/water solution (Tanaka, 1978). Further investigation showed another type of phase transition, in which shrinking of a gel was induced by an increase in temperature (Ilavský et al., 1982; Hirokawa and Tanaka, 1984; Hirose et al., 1987; Hirotsu et al., 1987; Huang et al., 1988). Furthermore, a "convexo type" transition which the volume first increased and then decreased with temperature, was reported by Katayama and Ohata (1985).

Solvent effects were examined with aqueous solutions containing several kinds of polar organic components, mainly alcohols, acetone and dimethyl-sulfoxide (Ilavský, 1982; Ilavský et al., 1982 and 1983; Ilavský and Hrouz, 1983; Hirokawa and Tanaka, 1984; Amiya and Tanaka, 1987; Hirotsu, 1987 and 1988). Although the addition of or-

ganic solvents to hydrogel systems usually led to shrinkage of the gel, N-isopropylacrylamide gels exhibited a peculiar swelling behavior, termed the "reentrant type" (Katayama et al., 1984), characterized by two transitions: a discontinuous collapse followed by a discontinuous swelling with a monotonic change in solvent composition. The concentration-dependent collapse of polymer gels in solutions of incompatible polymers was studied by Momii and Nose (1989). Lee et al. (1990) reported a pressure–dependent phase transition in NIPA hydrogel. A photoinduced phase transition was observed for copolymer gels of NIPA and bis(4–(dimethylamino)phenyl)(4–vinylphenyl)methyl leucocyanide under ultraviolet irradiation (Mamada et al., 1990).

Changes in pH and electrolyte concentration have a greater effect on the transition of hydrogels containing ionizable groups (Katchalsky and Zwick, 1955; Ohmine and Tanaka, 1982; Hirokawa et al., 1984; Rička and Tanaka, 1984; Rička and Tanaka, 1985) than on nonionic hydrogels (Huang et al., 1988). Hydrogels containing either anionic or cationic groups usually swell at middle pH values whereas hydrogels containing both anionic and cationic groups swell at high and low pH and collapse at middle pH-values (Katayama et al., 1988).

More recently, Saito (1989) has reviewed both an outline of fundamental concepts and the most recent developments in phase transition and equilibrium of small molecules, polymers and gel systems. An attempt has been made to explain the particular phenomena involved in these systems by reference to the concept of the connectivity of molecular chains.

The large changes in volume of hydrogels in response to changes in the surrounding environment have led to the use of such gels as extraction solvents in chemical separation systems (Cussler et al., 1984 ; Freitas and Cussler, 1987) and as physiologically sensitive compounds in biomedical and pharmaceutical applications (Peppas, 1987).

In this work the swelling behavior of anionic and cationic copolymer gels of acrylamide in aqueous electrolyte solutions of various pH and ionic strength was studied. The swelling behavior was interpreted using theory for ionic gels which accounts for the non-Gaussian behavior of rubber-like elasticity and the dissociation behavior of polyelectrolytes.

3.2 Theory

Early theories describing the swelling of polyelectrolyte gels have been reviewed by Helfferich (1962). Flory's (1953) treatment of the thermodynamics of gel swelling was the basis for much subsequent work. Many researchers (Hasa et al., 1975; Rička and Tanaka, 1984; Gehrke et al., 1986; Sun, 1986) attempted to modify Flory's theory for quantitative prediction of swelling effects. Erman and Flory (1986) reexamined this theory with the primary objective of elucidating the requirements for discontinuous shrinkage through a first order transition in a poor solvent. The parameter χ, which characterizes the solvent-polymer interactions, was presented as a function of composition. The elastic free energy was formulated to account for the nonaffine displacements of network junctions under strain. Vasilevskaya and Khokhlov (1986) presented a theory for the effect of salts on the collapse of charged polymeric networks which resulted in a system of non-linear equations containing empirical parameters. Among the most recent efforts, Brannon-Peppas and Peppas (1988) presented a derivation in which the osmotic coefficients were set equal to unity, the distribution of the polymer chains was Gaussian, and other effects were accounted for by a structural term. Unfortunately, their results contain miscalculations of mobile ion concentrations within the polyelectrolyte gel. Morro and Müller (1988) showed how the application of a load or clamping affected swelling and collapse of polyelectrolyte gels. A lattice model was proposed to describe the swelling of polymer gels in solution (Netemeyer and Glandt, 1989). Prange et al. (1989) proposed a model based on the quasi-chemical theory (Guggenheim, 1952; Panayiotou and Vera, 1980) suitable for the study of temperature sensitive hydrogels which exhibit lower critical solution temperatures. This theory was applied to describe swelling equilibria for cationic copolymer gels of acrylamide in aqueous NaCl solution (Hooper et al., 1990). Ideal Donnan theory was used to account for polyelectrolyte effects on swelling. Marchetti, Prager and Cussler (1990) presented a molecular theory of swollen gels based on a compressible lattice model of Lacombe and Sanchez (1976) for mixing of polymer solutions through the addition of the Flory–Rehner theory of rubber elasticity (1953). The four basic types of phase behavior predicted for swollen gels include near-critical lower consolute boundaries, low-temperature upper consolute boundaries, and closed-loop miscibility gaps potentially containing additional lower and upper consolute boundaries.

Koňák and Bansil (1989) modified the elasticity term by including an effect of the electrostatic persistence length. They included an explicit electrostatic term and used a simplified osmotic term. Otake et al. (1989) presented a model which considered a hydrophobic interaction for the thermally induced discontinuous shrinkage of hydrogels. In this thesis Flory's treatment as described by Hasa et al. (1975), is applied to the swelling of gels.

The theoretical description of the swelling of the polyelectrolyte gels at equilibrium is based on the minimization of the Gibbs free energy of the gel. The free energy change corresponding to the volume change during swelling of a gel, ΔG, is the sum of contributions due to mixing of pure solvent with an initially pure, amorphous, unstrained gel network, ΔG_1, due to configurational changes of the gel structure, ΔG_2, and due to mixing of ions with solvent, ΔG_3. An ionic gel is subjected to a swelling pressure, π, which is expressed as the sum of three components corresponding to each contribution to ΔG:

$$\pi = \pi_1 + \pi_2 + \pi_3 = -(\frac{\partial \Delta G}{\partial V})_{T,n_i} \tag{3.1}$$

The equilibrium condition is obtained when π is set equal to zero.

The osmotic pressure of a polymer solution, π_1, is given by the Flory–Huggins theory (Flory, 1953):

$$\pi_1 = -\frac{RT}{V_1}[ln(1-v) + v + \chi v^2] \tag{3.2}$$

where v is the polymer volume fraction and V_1 is the molar volume of solvent. The polymer–solvent interaction parameter, χ, can be expressed as (Hasa and Ilavský, 1975):

$$\chi = 0.44 + 0.6v \tag{3.3}$$

Expanding the logarithmic term in Eq. (3.2) in a power series, neglecting terms higher than cubic and combining the result with Eq. (3.3) gives

$$\pi_1 = -\frac{RT}{V_1}(v_o/X)^2(-0.06 + 0.267v_o/X) \tag{3.4}$$

where $X = v_o/v$ is the swelling ratio and v_o is the polymer volume fraction at gel formation.

The configurational contribution, π_2, is evaluated from the configurational free energy change, ΔG_2, including the finite extensibility of polymer chains during swelling. Assuming isotropic swelling, ΔG_2 can be expressed by

$$\Delta G_2 = N_p kT\{n[\gamma_4 A_4 + ln\frac{A_4}{sinhA_4} - \gamma_5 A_5 - ln\frac{A_5}{sinhA_5}] + \frac{1}{2}lnv\} \tag{3.5}$$

where

k = Boltzmann's constant
n = number of statistical segments in the polymer chain
N_p = number of the polymer chains
$\gamma_4 = < \alpha_o^2 >^{1/2} v^{-1/3} n^{-1/2}$
$\gamma_5 = < \alpha_o^2 >^{1/2} n^{-1/2}$
A_i = $L^{-1}(\gamma_i)$, where $L^{-1}(\gamma)$ is the inverse Langevin function of γ.

The dilation factor in the dry isotropic state, $< \alpha_o^2 >$, is defined as the ratio of the mean-square end-to-end distance in the dry isotropic state to the same quantity in the reference state, when no force acts on the chain ends (Dušek and Prins, 1969). The factor $(N_p kTlnv/2)$ is used by analogy with the procedure employed in the kinetic theory of rubber elasticity, even though its presence does not come from the three-chain model (Smith et al., 1964).

By differentiating ΔG_2 with respect to volume and expanding the inverse Langevin function in a power series (Treloar, 1958 and Hasa et al., 1975), π_2 is obtained as

$$\pi_2 = -\nu_d RT[< \alpha_o^2 > v^{1/3} - \frac{1}{2}v] - \nu_d RT[\frac{3}{5} < \alpha_o^2 >^2 v^{-1/3}\frac{1}{n}$$

$$+\frac{99}{175}(<\alpha_o^2>^3 v^{-1}\frac{1}{n^2})+\frac{513}{875}(<\alpha_o^2>^4 v)^{-5/3}\frac{1}{n^3}+....] \qquad (3.6)$$

where $\nu_d = v\nu$ is the molar concentration of chains related to the dry state. The molar concentration of chains in swollen state, ν, is given by

$$\nu = \frac{N_p}{N_A V} \qquad (3.7)$$

where N_A is Avogadro's number. The first term in Eq. (3.6) is the contribution of the Gaussian configurational free energy and the second term corresponds to the non-Gaussian free energy.

Substitution of the dilation factor of the dry state, $<\alpha_o^2>=v_o^{2/3}$ into Eq. (3.6) gives

$$\begin{aligned}\pi_2 &= -\nu_o RT[(v/v_o)^{1/3}-\frac{1}{2}(v/v_o)]-\nu_o RT[\frac{3}{5}(v_o/v)^{1/3}\frac{1}{n} \\ &\quad +\frac{99}{175}(v_o/v)\frac{1}{n^2}+\frac{513}{875}(v_o/v)^{5/3}\frac{1}{n^3}+....] \end{aligned} \qquad (3.8)$$

where $\nu_o = v_o\nu_d$ is the concentration of constituent chains at gel formation. The osmotic pressure, π_3, attributed to the difference between the osmotic pressure of the mobile ions in the gel and in the external solution, is given by (Flory, 1953):

$$\pi_3 = RT[\Phi\Sigma_i\overline{C_i}-\phi\Sigma_i C_i] \qquad (3.9)$$

where C_i and $\overline{C_i}$ are the concentrations of mobile ions in the external solution and in the gel, respectively. The osmotic coefficients, Φ and ϕ, are discussed in detail below

3.2.1 Osmotic Coefficients

Charged groups attached to the network play an essential role in swelling phenomena. To account for the non-ideal behavior of the polyelectrolyte gel an osmotic coefficient for the gel phase, Φ, is defined

$$\Phi = \pi_p/\pi_{ideal} \qquad (3.10)$$

The ideal osmotic pressure of salt-free polyelectrolyte solution is given by the Van't Hoff expression

$$\pi_{ideal} = RT(n_m\alpha + n_p) \tag{3.11}$$

where n_m is the molarity of the monomer, α is the degree of ionization and n_p is the molarity of the polymer.

Since polyelectrolytes are strongly non-ideal, a correction factor, ϕ_p, called the osmotic coefficient, is introduced (Alexandrowicz, 1960)

$$\pi_p = RT(n_m\alpha\phi_p + n_p) \tag{3.12}$$

Combination of Eqs. (3.10)-(3.12) gives

$$\Phi = (\phi_p + \frac{n_p}{n_m\alpha})/(1 + \frac{n_p}{n_m\alpha}) \tag{3.13}$$

When $n_m\alpha >> n_p$, Eq. (3.12) becomes

$$\pi_p = RTn_m\alpha\phi_p \tag{3.14}$$

According to Katchalsky (1971), the dissociation behavior of polyelectrolyte solutions can be summarized as follows. It is often found that at higher degree of dissociation $\alpha\phi_p$ is approximately constant, or that the osmotic coefficient decreases somewhat as the degree of ionization increases. It is also observed that the dilution of a polyelectrolyte solution does not lead to stronger dissociation of counterions and often the opposite effect is observed, i.e. ϕ_p decreases upon dilution which indicates a strong binding of the counterions to the polyion with lower osmotic activity.

The stronger electrostatic attraction of small polyvalent ions to the polyion should reduce the fraction of free counterions, hence ϕ_p for bivalent counterions should be about one-half of the value for the monovalent counterions. This expectation was confirmed experimentally. Measurements of ϕ_p for Mg-alginate gave a value of

0.15, compared with ϕ_p=0.4 for Na-alginate or ϕ_p=0.35 for K-alginate (Katchalsky et al., 1961). For mixed salts the results are more complex. Dolar and Peterlin (1969) extended the rod-like model for the evaluation of the osmotic coefficient of mixed polyelectrolyte salts in which the polyion was neutralized by both mono- and bivalent ions. The theoretical predictions were tested qualitatively by Dolar and Kozak (1970). Both theory and experiment indicated the existence of a maximum value of ϕ_p at a certain ratio of monovalent to bivalent ions.

Alexandrowicz (1960, 1962) found that the osmotic pressure of a polyelectrolyte and a low molecular weight mono–monovalent salt system could be represented by an additivity rule as the sum of the osmotic pressure of the salt-free polyelectrolyte solution, π_p, and the polyelectrolyte-free salt solution, π_s:

$$\Pi = \pi_p + \pi_s \tag{3.15}$$

Following Alexandrowicz for the polyelectrolyte/mono–monovalent salt system:

$$\pi_s = RT(2n_s\phi_s) \tag{3.16}$$

where n_s is the molarity of the salt and the factor 2 accounts for the dissociation of the salt into two ionic species. Combination of Eqs. (3.14)-(3.16) yields

$$\Pi = RT(n_m\alpha\phi_p + 2n_s\phi_s) \tag{3.17}$$

From Eq. (3.10) the osmotic pressure, π, can be written as

$$\Pi = RT(n_m\alpha + 2n_s)\Phi \tag{3.18}$$

Combination of Eqs. (3.17) and (3.18) yields

$$\Phi = \frac{x\phi_p + 2\phi_s}{x + 2} \tag{3.19}$$

where $x = n_m\alpha/n_s$. For $0.5 \leq x \leq 8$ the deviation of Φ from experimental results can be as large as 15% (Alexandrowicz, 1960).

In the presence of a bi-monovalent salt, the concentration of counterions in a solution containing a negatively charged polyion is

$$n_+ = n_s + \frac{1}{2}n_m\alpha \tag{3.20}$$

while the concentration of co-ions is

$$n_- = 2n_s \tag{3.21}$$

Using the additivity rule for this system gives

$$\Phi = \frac{x\phi_p + 6\phi_s}{x + 6} \tag{3.22}$$

Equations (3.19) and (3.22) also hold for a mixture of positively charged polyions with mono-monovalent and mono-bivalent salts, respectively.

When the external solution contains both mono- and bivalent counterions, ideal Donnan theory is used to determine the concentration of counterions in the gel phase. Donnan equilibrium develops when ions are confined to fixed locations in a solution, usually by an impermeable membrane or by being bound to an insoluble matrix (e.g. a gel), while other ions can freely distribute between the two phases. In such a system the concentration of co-ions (the ion with the same charge as the polyion) will be greater in the free-ion solution than in the solution containing the fixed ions. The unequal concentration of fixed ions forced upon the system is responsible for such a phenomenon. For electroneutrality, freely diffusible counterions (those with charge opposite to the polyion) must stay near the polyion. Because of the electroneutrality of both phases, unequal concentration of all ionic species will develop in the two phases.

Consider a polyelectrolyte gel/free solution system. At equilibrium, the electrochemical potential of each electrolyte within the gel and external solution must be

the same (Helfferich, 1962), thus:

$$\eta_i = \overline{\eta_i} \tag{3.23}$$

where η_i is the electrochemical potential of species 'i' and the overbar indicates the gel phase. The electrochemical potential is

$$\eta_i = \mu_i + Z_i F\psi \tag{3.24}$$

where

μ_i = chemical potential of species 'i'
Z_i = charge of species 'i'
F = Faraday constant
ψ = electric potential

The chemical potential can be written as

$$\mu_i(\mathrm{P,C}) = \mu_i^\circ(\mathrm{P}^\circ) + RT ln a_i \tag{3.25}$$

where

P = pressure
C = concentration
$\mu_i^\circ(\mathrm{P}^\circ)$ = chemical potential of species 'i' in the standard state
a_i = activity of species 'i'

Any swelling pressure effect (for P≠P°) is placed into the activity coefficient. For typical ion exchange processes these effects are minor (Helfferich, 1962). Combination of Eqs. (3.23) to (3.25) gives

$$\mu_i^\circ(\mathrm{P}^\circ) + RT ln a_i + Z_i F\psi = \mu_i^\circ(\mathrm{P}^\circ) + RT ln \overline{a_i} + Z_i F\overline{\psi} \tag{3.26}$$

Rearrangement of Eq.(3.26) gives the Donnan Potential E :

$$E = \overline{\psi} - \psi = (\frac{RT}{Z_i F}) ln(a_i/\overline{a_i}) \tag{3.27}$$

which can be written as

$$(a_i/\overline{a_i})^{1/Z_i} = \exp(FE/RT) \tag{3.28}$$

Since the Donnan potential is the same for all ions, the right hand side of Eq. (3.28) is a constant. The assumption of ideal solutions gives

$$(a_i/\overline{a_i})^{1/Z_i} = (C_i/\overline{C_i})^{1/Z_i} = \text{constant} \tag{3.29}$$

When the polyion is neutralized by both Na^+ and M^{2+} ions, Eq. (3.29) can be written:

$$(C_{Na^+}/\overline{C}_{Na^+})^2 = (C_{M^{2+}}/\overline{C}_{M^{2+}}) \tag{3.30}$$

The electroneutrality of the gel phase, when the salt concentration within the gel is negligible compared with the concentration of fixed charges, $n_m \alpha$, implies that

$$\overline{C}_{Na^+} + 2\overline{C}_{M^{2+}} = n_m \alpha \tag{3.31}$$

Defining f as the fraction of the ionic sites on the polyion neutralized by Na^+

$$f = \overline{C}_{Na^+}/n_m \alpha \tag{3.32}$$

and combining Eqs. (3.30) and (3.31) yield

$$2an_m \alpha f^2 + f - 1 = 0 \tag{3.33}$$

where a = $C_{M^{2+}}/(C_{Na^+})^2$. The ion swelling pressure, π_3, for $C_{M^{2+}} < 10^{-4}$ M in the presence of Na^+ is then obtained through Eq. (3.9) as

$$\pi_3 = RT\{\Phi[n_m\alpha(1+f)/2] + \phi\Sigma_i C_i\} \tag{3.34}$$

To obtain the osmotic coefficient of the gel phase the concentration of mobile co-ions in the gel phase must be determined. In the absence of data these concentrations are estimated from Donnan equilibrium (Kitchener, 1957). For a gel with a concentration of fixed charges $n_m\alpha$ in an ideal solution of cation 'C' with a charge of Z_C and anion 'A' with a charge of Z_A, Donnan theory gives (Gehrke, 1986)

$$(\overline{C_A}/C_A)^{1/|Z_A|} = (\frac{C_A}{n_m\alpha/|Z_A| + \overline{C_A}})^{1/Z_C} \tag{3.35}$$

For a cationic gel the following result is obtained

$$(\overline{C_C}/C_C)^{1/Z_C} = (\frac{C_C}{n_m\alpha/Z_C + \overline{C_C}})^{1/|Z_A|} \tag{3.36}$$

Katchalsky and Michaeli (1955) concluded that the concentrations of co ions in the gel calculated from ideal Donnan equilibrium were close to experimental values when $\alpha \leq 0.1$. For $\alpha > 0.1$ there was an increasing discrepancy between theory and experiment as the degree of ionization increased.

Chapter 4

KINETICS OF GEL SWELLING

The sorption of small molecules in polymeric materials is accompanied by swelling and other bulk, morphological changes which may result in anomalous phenomena such as dual sorption, constant rate or overshoot sorption kinetics, non-monotonically increasing dimensions, swelling stress and cracking. The uptake of penetrant molecules enhances polymer chain mobility resulting in polymer dissolution if the polymer is not crosslinked or swelling if the material is crosslinked. Studies of some polymers have shown that crosslinking does not alter the swelling mechanism; it affects only the rate of swelling (Hayes and Park, 1955; Chen and Ferry, 1968)

The swelling mechanism for a polymer–solvent pair is different above and below the glass transition temperature, T_g. Above T_g, a polymer is hard and stiff, resisting applied stresses. Liquid penetrant transport in glassy polymers is associated with a transition from the glassy to the rubbery state due to the increased mobility of the polymer chains.

4.1 Penetrant Sorption in Glassy Polymers

It is well known that sorption processes for polymer–solvent systems frequently do not conform to the behavior expected from the classical theory of diffusion (Park, 1968). Although penetrant sorption by rubbery polymers may be described by Fickian transport with a concentration dependent diffusion coefficient, this description usually is not successful for glassy polymers. The slow reorientation of polymer molecules can led to a wide variety of anomalous effects for both permeation and sorption experiments, particularly when such experiments are conducted near or below the glass transition temperature.

The three basic categories of the sorption phenomena in polymers may be described as follows. Fickian or Case I transport is characterized by the single parameter

D, the diffusion coefficient. Molecular relaxation is either faster than diffusion (well above T_g) or so slow that is not observed on the time scale of the experiment (well below T_g). Case II transport is characterized by the single parameter V, the velocity of the advancing penetrant front (Alfrey et al., 1966). Diffusion is very rapid compared to relaxation, with relaxation occurring at an observable rate. Non-Fickian or anomalous transport is observed when the diffusion and relaxation rates are comparable.

Modes of transport are generally distinguished by fitting sorption data over approximately the first half of the sorption curve to the following empirical equation (Petropoulos and Roussis, 1969; Enscore and Hopfenberg, 1980; Frisch, 1980, Millar, 1983; Ritger and Peppas, 1987; Gehrke et al., 1989).

$$\frac{M(t)}{M(\infty)} = Kt^{n} \tag{1.1}$$

where

$M(t)$ = mass of penetrant at time t
$M(\infty)$ = mass of penetrant sorbed at equilibrium
K, n = parameters

For Fickian diffusion, n=1/2; for anomalous transport, 1/2<n<1, and for Case II transport, n=1. Although this method is widely used it fails to account for the required dimensional dependence. A more rigorous test is provided by the following relationship (Fujita, 1968; Crank, 1975; Davidson and Peppas, 1986; Gehrke and Cussler, 1989).

$$\frac{M(t)}{M(\infty)} = Kt^{n}/l \tag{1.2}$$

where l is the characteristic sample dimension. For different values of l, the parameters K and n should be invariant for Case I and Case II transport. If they change, anomalous transport is indicated even if n=1 or 1/2. Near the end of a Case II transport process, a rapid increase in the penetrant sorption rate is sometimes observed. In

this situation Case II transport is said to have evolved into super-Case II transport. This change is attributed to the expansion forces exerted by the swollen gel on the glassy core (Peterlin, 1979). Structural defects can also lead to the superposition of a hole-filling adsorption mechanism upon ordinary Fickian diffusion resulting in a two stage sorption (Vieth et al., 1976; Pace and Datyner, 1980). In a penetrant/polymer system, if there is an initial increase in the amount sorbed followed by a decrease to a steady state value, the phenomenon is called overshoot (Frisch, 1980). The different modes of sorption are shown schematically in Fig. 4.1.

Fickian diffusion can be identified from a plot of the fractional approach to equilibrium, $\frac{M(t)}{M(\infty)}$, against $t^{1/2}/l$ by the following characteristics (Fujita, 1968; Crank, 1975).

(1) - The relationship is linear in its initial portion: generally up to 60% of equilibrium, depending upon the geometry (higher for the slab, lower for the sphere or cylinder).

(2) - Above the linear portion, the curve is concave to the time axis.

(3) - The curves for different thicknesses (l) superimpose.

Criterion (3) is the most important one, but it is not always tested. If only criteria (1) and (2) hold, the transport is considered "Pseudo- Fickian" (Rogers, 1965; Fujita, 1968; Crank, 1975).

Numerous mathematical models have been proposed for Case I and Case II sorption (Crank, 1953; Long and Richman, 1960; Kishimoto and Kitahara, 1967; Frisch et al., 1969; Wang et al., 1969; Astarita and Sarti, 1978; Berens and Hopfenberg, 1978; Petropoulos and Rossis, 1978; Joshi and Astarita, 1979; Hansen, 1980; Peterlin, 1980; Skaarup and Hansen, 1980, Gostoli and Sarti, 1982; Thomas and Windle, 1982; Cohen, 1983; Peppas and Sinclair, 1983; Tosun and Yilmaz, 1983; Yilmaz et al., 1983; Petropoulos, 1984; Lee, 1985; Korsmeyer et al., 1986; Sarti et al., 1986; Singh and Fan, 1986; Yang et al., 1986; Cohen and Goodhart, 1987; Cohen and White, 1989; Subramanian et al., 1989). It is beyond the present scope to review this extensive literature. Reviews are available by Rogers (1965), Frisch (1980) and Lusting et al.(1989).

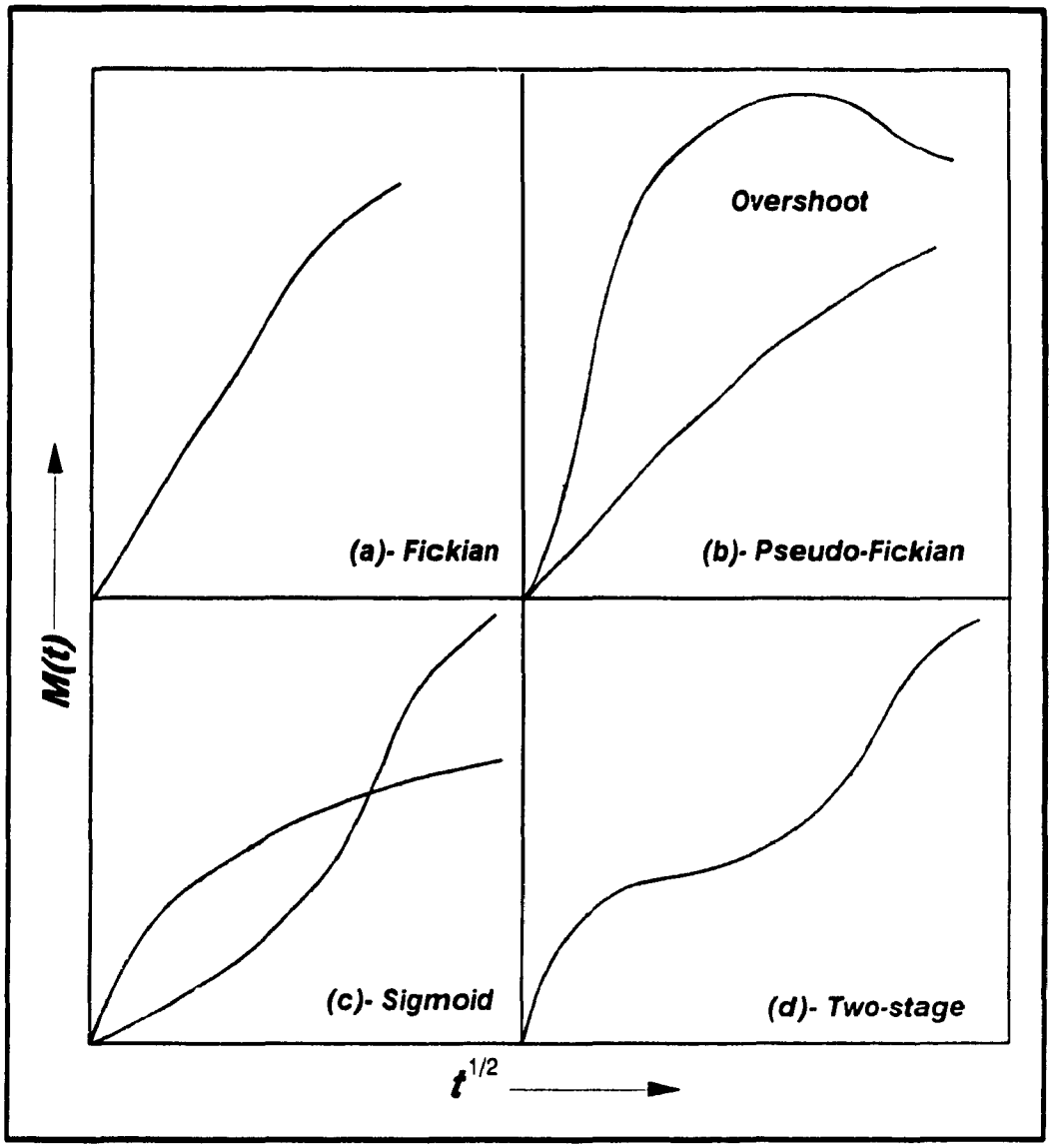


Figure 4.1: Different modes of sorption phenomena in solvent/ polymer systems (a-d, Rogers 1965).

4.2 Penetrant Sorption in Rubbery Polymers

The understanding of kinetics of the absorption and desorption of solvent by rubbery polymeric materials is of considerable practical importance (Tanaka et al., 1985; Peppas 1987; Thomas and Muniandy, 1987; Tong et al., 1989). The diffusion of low molecular weight species in rubbery polymers is normally described by Fick's law with a concentration dependent diffusion coefficient (Buckley and Berger, 1962; Rogers, 1965; Crank and Park, 1968; Duda and Vrentas, 1971; Crank, 1975; Vrentas et al., 1975; Frisch, 1980; Ju et al., 1981; Duda et al., 1982; Vrentas et al., 1984). In traditional Fickian diffusion, the concentration gradient of the penetrant is usually assumed to provide the driving force necessary for diffusion; however, it has been suggested that it is the chemical potential gradient of the penetrant which provides the driving force (Onsager and Fouss, 1932; Hartley, 1946; Hartley and Crank, 1949; Park, 1950). Departures from Fickian behavior have also been found (Petropoulos and Roussis, 1969; Gehrke and Cussler, 1989). Also it is recognized that polymer itself dominates the transport process (Fujita, 1968). Gehrke (1986) demonstrated that the solvent penetration is much faster than the volume change due to swelling. The rate of gel volume change was determined by a cooperative diffusion coefficient which was found to be an order of magnitude less than the tracer (D_2O) diffusion coefficient. The gel network cannot move instantaneously; it must move in concert with solvent. The difference between the cooperative diffusion coefficient, D_c, and the tracer diffusion coefficient (self-diffusion), D_t, may be due to the influence of different modes of polymer relaxation (Gehrke, 1986). For penetration by D_2O, only local chain movement is required which is usually fast relative to solute penetration in polymers above the glass transition temperature. In contrast, the long-range rearrangement of polymer chains is responsible for gel volume change. This may be indicated by the rapid development of opacity relative to volume change in temperature sensitive hydrogels. The opacity is caused by the development of heterogeneity in the gel, which requires conformational change of polymer chains at a local level (Dušek and Sedláček, 1969; Sedláček and koňák, 1982). Non–Fickian behavior may be observed if the surface concentration slowly relaxes to an equilibrium value (Frisch, 1980), however the surface concentration in rubbery polymers reaches equilibrium on a time scale much faster than diffusion.

4.2.1 Mathematical Modeling of Gel Swelling

Buckley and Berger (1962) used a simple Fickian analysis to calculate diffusion coefficients of cyclohexane into butyl gum vulcanizates from swelling data. Tanaka, Hocker, and Benedek (1973) derived an equation of motion of the gel network which was applied to the swelling of polyacrylamide gels in water (Tanaka and Fillmore, 1979) and to the temperature–induced swelling and collapse of poly N-isopropylacrylamide gels in water (Tanaka et al., 1985). Tanaka and coworkers claimed that the approach taken by Buckley and Berger is invalid because they consider the solvent to diffuse into the gel instead of the gel into the solvent. The polymer/solvent diffusion coefficient is a characteristic of the polymer solvent pair and should not be associated with either the polymer or solvent exclusively, although it is primarily controlled by the polymer (Gehrke, 1986).

In Tanaka and Fillmore's theory, the local motion of a polymer network obeys a diffusion equation in which the diffusion coefficient, D, is defined by the ratio of the elastic modulus, k, to the frictional coefficient, f, between the network and the fluid This diffusion coefficient for the process of gel swelling is called the collective diffusion coefficient of a gel. Gehrke (1986) demonstrated that these diffusion coefficients are not the same by comparison of the equations by which D is calculated in each approach. For a spherical gel Tanaka and Fillmore (1979) obtained

$$\frac{r(\infty) - r(t)}{r(\infty) - r(0)} = \frac{6}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} exp(-n^2\pi^2 \mathrm{D}t/\mathrm{r}^2) \tag{4.3}$$

The corresponding Fickian equation is (Crank, 1975)

$$\frac{r^3(\infty) - r^3(t)}{r^3(\infty) - r^3(0)} \simeq 1 - \frac{M(t)}{M(\infty)} = \frac{6}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} exp(-n^2\pi^2 \mathrm{D}t/\mathrm{r}^2) \tag{4.4}$$

where

$r(t)$ = gel radius at time t

$r(0)$ = initial gel radius

$r(\infty)$ = gel radius at equilibrium

Since the left–hand sides of Eqs. (4.3) and (4.4) are different, the diffusion coefficients are different.

Gehrke (1986) studied the kinetics of gel volume change for pH–sensitive hydrogels based on the well established ion–exchange theories developed by Helfferich (1965). The collapse of these gels was almost Fickian at pH 4 whereas the swelling phenomenon was anomalous for swelling of a collapsed gel at pH 9. The theory of Tanaka and Fillmore (TF theory), extended to include the effect of shear modulus, σ, was used for cylindrical gels (Peters and Candau, 1986 and 1988). Li and Tanaka (1990) pointed out that the TF theory is insufficient for gels with a nonspherical shape, and hence the calculations by Peter and Candau (1988) for cylindrical gels should be modified. This insufficiency is due to the existence of the shear modulus of the network. A new relation, in addition to the differential equation of TF theory, was formulated for the swelling of nonspherical gels. These solutions predict that the diffusion coefficient of long cylinder and large disk gels are 1.5 and 3 times smaller than that of a spherical gel, respectively. Yoshio et al. (1986) and Schosseler et al. (1987), using TF theory to analyze experimental results for swelling of ionic gels, concluded that the diffusion coefficient increased with increasing degree of ionization of the polymeric network.

Komori and Sakamoto (1989) presented a theoretical model different from TF theory. They coupled the diffusion equation for the excess concentration of the penetrant with an expression for the distribution of local strain to drive an expression for the incremental radius of a spherical gel. Komori, Takahashi, and Okamoto (1988) studied the swelling behavior of sodium acrylate gels. The results were discussed and compared with theory of Komori and Sakamoto. Considerable discrepancy was found between the theoretical and experimental values, especially in an early period of swelling where the rate of solvent uptake was overestimated. As noted earlier, the diffusion coefficient in the rubbery polymers is highly concentration dependent. This concentration dependence was ignored in these theories.

4.2.2 A Model of Gel Swelling Using Polymer Material Coordinates

This section describes a general framework for the mathematical modeling of gel swelling in planar (slab), cylindrical, and spherical geometries. A material coordinate transformation is used to recast the moving boundary problem from Eulerian to Lagrangian (material) form. The driving force for the swelling of the polymeric network is taken to be the chemical potential of the solvent, μ, which accounts for all factors contributing to the gel volume change as discussed in Chapter 3.

Unsteady diffusion problems are usually formulated in an Eulerian format. An alternative approach, first presented by Hartley and Crank (1949) for slabs, uses a Lagrangian format employing polymer material coordinates based on the distribution of polymer in a reference configuration. The studies by Duda and Vrentas (1968, 1971) show that when substantial polymer displacement accompanies sorption, the polymer material coordinate (PMC) approach is more convenient than the Eulerian approach. The latter requires application of a boundary condition at a surface whose position varies with time. In the PMC approach, the position of the surface is constant. These coordinates have been applied to specific problems in a limited number of investigations (Duda and Vrentas, 1971; Ware and Cohen, 1980, Vandijk et al., 1984; Durning and Tabor, 1986; Lusting and Peppas, 1987). In a series of papers, Rajagopal, Wineman, and coworkers (1981, 1983, 1986, 1987) used a combination of Eulerian and PMC systems to analyze steady transport through an elastic solid subject to large multidimensional displacements.

Previous use of polymer material coordinates was limited to one dimensional problems in slab geometry. In what follows, a general mathematical description of the kinetics of gel swelling in a coordinate system fixed with respect to the reference configuration of the polymer is presented[1].

The gel consists of two phases: the polymer matrix (p) and the solvent (s). Each phase is incompressible. During one-dimensional unsteady penetration of solvent into a polymeric network accompanied by swelling of the network, the continuity equations for each phase are the following.

[1] While this work was in progress, Billovits and Durning (1989) presented a general mathematical formulation for mutual diffusion in polymer-penetrant systems using polymer material coordinates. The approach presented in this thesis involves much simpler mathematics.

For the polymeric matrix (p)

$$(\frac{\partial \theta_p}{\partial t})_r = -\frac{1}{r^i}\frac{\partial}{\partial r}(r^i u_p)_t \tag{4.5}$$

For the solvent (s)

$$(\frac{\partial \theta_s}{\partial t})_r = -\frac{1}{r^i}\frac{\partial}{\partial r}(r^i u_s)_t \tag{4.6}$$

where i takes the following values

$i = 0$ for a slab

$i = 1$ for a cylinder

$i = 2$ for a sphere

and r is the Eulerian spatial coordinate, t is the time and θ_j and u_j are the volume fraction and superficial velocity of each phase, respectively. The volume fraction of each phase is constrained by

$$\theta_p + \theta_s = 1 \tag{4.7}$$

The superficial velocity of phase j is defined as the volumetric flow rate of phase j per unit area normal to the flow, i.e. the volumetric flux density. The velocity of phase j is given by u_j/θ_j.

The velocity of the solvent, u_s/θ_s, is considered to be the sum of two components. The first component is a velocity identical to that of the matrix, u_p/θ_p. The second component is a velocity relative to the matrix, u/θ_s:

$$\frac{u_s}{\theta_s} = \frac{u_p}{\theta_p} + \frac{u}{\theta_s} \tag{4.8}$$

or

$$u_s = u + \vartheta u_p \tag{4.9}$$

where $\vartheta = (\frac{\theta_s}{\theta_p})$ is called solvent ratio. These equations define the relative velocity, u. This quantity will be considered subsequently to be diffusive.

Combination of Eqs. (4.6) and (4.9), having used the definition of the solvent ratio, ϑ, gives

$$\frac{\partial}{\partial t}(\vartheta\theta_p)_r = -\frac{1}{r^i}\frac{\partial}{\partial r}[r^i(u + \vartheta u_p)]_t \tag{4.10}$$

Differentiation of Eq. (4.10) by parts gives

$$\vartheta\frac{\partial\theta_p}{\partial t} + \theta_p\frac{\partial\vartheta}{\partial t} = -\frac{1}{r^i}\frac{\partial}{\partial r}(r^i u) - \frac{\vartheta}{r^i}\frac{\partial}{\partial r}(r^i u_p) - u_p\frac{\partial\vartheta}{\partial r} \tag{4.11}$$

The elimination of two terms in Eq. (4.11) using Eq. (4.5) and division by θ_p yield

$$(\frac{\partial\vartheta}{\partial t})_r = -\frac{1}{r^i\theta_p}\frac{\partial}{\partial r}(r^i u)_t - \frac{u_p}{\theta_p}(\frac{\partial\vartheta}{\partial r})_t \tag{4.12}$$

In Eq. (4.12) the first term on the right-hand side represents the contribution of flow relative to the polymer. The left-hand side of this equation together with the second term on the right represent the differential of ϑ following the motion of the polymer phase. Employing a material coordinate, m, based on the distribution of the polymer phase, gives

$$(\frac{\partial\vartheta}{\partial t})_r = -\frac{u_p}{\theta_p}(\frac{\partial\vartheta}{\partial r})_t + (\frac{\partial\vartheta}{\partial t})_m \tag{4.13}$$

with $m(r,t)$, defined by the following equations:

$$(\frac{\partial m}{\partial r})_t = r^i\theta_p \tag{4.14}$$

and

$$(\frac{\partial m}{\partial t})_r = -r^i u_p \tag{4.15}$$

A derivation of Eq. (4.13) is given in Appendix A. The material coordinate, which is based on the distribution of the polymer phase, satisfies the continuity equation for that phase. This is shown by differentiating Eqs. (4.14) and (4.15) with respect

to t and r, respectively and comparing the results with continuity equation for the polymer phase. Combination of Eqs. (4.12)–(4.14) yields the continuity equation for the solvent in the material coordinate:

$$(\frac{\partial \vartheta}{\partial t})_m = -\frac{\partial}{\partial m}(r^i u)_t \tag{4.16}$$

To complete the description a constitutive equation is needed for the relative velocity, u. It is postulated that u is described by Fick's law with the chemical potential of the solvent, μ, as the driving force:

$$u = -\frac{D_T}{RT}\frac{\partial \mu}{\partial r} \tag{4.17}$$

The mobility coefficient, D_T, is a function of the local thermodynamic variables, ϑ, the absolute temperature, T, and the total impressed hydrostatic pressure difference, ΔP, but not the time (Frisch, 1980).

The chemical potential of a solvent within the gel can be written as

$$\mu - \mu^\circ(\mathrm{P}^\circ) = -\pi V_1 \tag{4.18}$$

where π is the swelling pressure and V_1 is the molar volume of the solvent – as discussed in Section 3.2. Combination of Eqs. (4.17) and (4.18) yields

$$u = \frac{D_T}{RT} V_1 \frac{\partial \pi}{\partial r} \tag{4.19}$$

In the material coordinate Eq. (4.19) can be written as

$$u = \frac{D_T}{RT} V_1 (\frac{1}{1+\vartheta}) r^i (\frac{\partial \pi}{\partial \vartheta})(\frac{\partial \vartheta}{\partial m}) \tag{4.20}$$

where $(\frac{1}{1+\vartheta}) = \theta_p$.

Combination of Eqs. (4.16) and (4.20) gives

$$(\frac{\partial \vartheta}{\partial t})_m = -\frac{\partial}{\partial m}[r^{2i}\frac{D_T}{RT}V_1(\frac{1}{1+\vartheta})r^i(\frac{\partial \pi}{\partial \vartheta})(\frac{\partial \vartheta}{\partial m})]_t \tag{4.21}$$

The material coordinate is calculated by integrating the differential of $m(r,t)$ for the relevant geometry:

$$dm = r^i\theta_p dr - r^i u_p dt \tag{4.22}$$

as

$$m(r,t) = \int_0^r \zeta^i \theta_p(\zeta,t)d\zeta - \int_0^t \zeta^i u_p(0,\tau)d\tau \tag{4.23}$$

where ζ and τ are the spatial and time integration variables. This integration is most conveniently performed by noting that the flux of polymer is zero at $r = 0$. Since $u_p(0,t) = 0$,

$$m(r,t) = \int_0^r (1+\vartheta)^{-1}\zeta^i d\zeta \tag{4.24}$$

For a slab, m is simply the cumulative volume of the solid polymeric component per unit area of cross section measured away from plane of symmetry. For cylindrical geometry, m is proportional to the cumulative volume of solid polymeric network per unit length, measured away from the axis of the cylinder, with a constant of proportionality equal to $\frac{1}{2\pi}$. For a sphere, m is proportional to the cumulative volume of solid polymeric component, measured away from the center of the sphere, with a constant of proportionality equal to $\frac{1}{4\pi}$. Therefore, the cumulative volume m, is related to the material space coordinate, x, as follow

$$m = (\frac{1}{i+1})x^{i+1} \tag{4.25}$$

or

$$dm = x^i dx \tag{4.26}$$

Combination of Eqs. (4.24) and (4.25) gives a relationship between the PMC, x, and the conventional laboratory (Eulerian) coordinate, r, as

$$x^{\iota+1} = (\iota + 1) \int_0^r (1 + \vartheta)^{-1} \zeta^{\iota} d\zeta \tag{4.27}$$

Inserting Eq. (4.26) into Eq. (4.21) yields

$$(\frac{\partial \vartheta}{\partial t})_x = \frac{1}{x^{\iota}} \frac{\partial}{\partial x} [-x^{\iota} (\frac{r}{x})^{2\iota} \frac{D_T V_1}{RT} (\frac{1}{1+\vartheta}) (\frac{\partial \pi}{\partial \vartheta}) (\frac{\partial \vartheta}{\partial x})]_t \tag{4.28}$$

Equation (4.28) describes the kinetics of gel swelling in PMC. The initial and boundary conditions are:

$$\vartheta(x, 0) = \vartheta_0 \tag{4.29}$$

$$\frac{\partial \vartheta}{\partial x}|_{x=0} = 0 \qquad (\text{for } t \geq 0) \tag{4.30}$$

$$\vartheta(x_0, t) = \vartheta_e \qquad (\text{for } t > 0) \tag{4.31}$$

where ϑ_0 and ϑ_e are the initial and equilibrium values of solvent ratio, respectively. It is assumed that at the boundary between the swelling gel and the surrounding fluid, the equilibrium degree of swelling, X, is reached instantaneously. The boundary of the swelling gel in PMC, x_0, is calculated from Eq. (4.27) as

$$x_0 = r_0 (1 + \vartheta_0)^{-\frac{1}{\iota+1}} \tag{4.32}$$

where r_0 is the initial half thickness of the slab and/or the initial radius of the cylindrical or spherical gel, in laboratory coordinates.

For the slab ($i = 0$) Eq. (4.28) involves only the material coordinate, but for the cylinder ($i = 1$) and sphere ($i = 2$) the Eulerian coordinate, r, appears. An approximation will be used for $i = 1$ and 2, so that this equation can be written as follows for all values of i:

$$(\frac{\partial \vartheta}{\partial t})_x = \frac{1}{x^i}\frac{\partial}{\partial x}(Dx^i\frac{\partial \vartheta}{\partial x})_t \tag{4.33}$$

For a slab D can be written as

$$D = -\frac{D_T V_1}{RT}(\frac{1}{1+\vartheta})(\frac{\partial \pi}{\partial \vartheta}) \tag{4.34}$$

The following approximation is made for cylindrical and spherical geometry

$$(r/x) \simeq (1+\vartheta)^{\frac{1}{i+1}} \tag{4.35}$$

resulting in following expressions for diffusion coefficient, D:
For a cylinder

$$D = -\frac{D_T V_1}{RT}(\frac{\partial \pi}{\partial \vartheta}) \tag{4.36}$$

and for a sphere

$$D = -\frac{D_T V_1}{RT}(1+\vartheta)^{1/3}(\frac{\partial \pi}{\partial \vartheta}) \tag{4.37}$$

To test the validity of this assumption, the relationships between the diffusion coefficients in PMC and Eulerian coordinates are examined. For one-dimensional diffusion in a Cartesian system, the diffusion coefficient in PMC, D, is related to that in Eulerian coordinate, D_c, as follows (Hartley and Crank, 1949).

$$D = (1+\vartheta)^{-2}D_c = \theta_p^2 D_c \tag{4.38}$$

Comparison of Eqs. (4.34) and (4.38) gives

$$-\frac{D_T V_1}{RT}\left(\frac{\partial \pi}{\partial \vartheta}\right) = \theta_p D_c \tag{4.39}$$

Substitution of Eq. (4.39) into Eqs. (4.36) and (4.37) yields:
For a cylinder

$$D = (1+\vartheta)^{-1} D_c = \theta_p D_c \tag{4.40}$$

and for a sphere

$$D = (1+\vartheta)^{-2/3} D_c = \theta_p^{2/3} D_c \tag{4.41}$$

Equation (4.41) gives the relation for isotropic swelling in spherical geometry which matches the result for the isotropic three-dimensional diffusion in a Cartesian system derived by Crank and Park (1968). In Eq. (4.40), D/D_c scales with the first power of the polymer volume fraction, θ_p, a result intermediate between that for one-dimensional swelling of a slab [Eq. (4.38)] and that for isotropic swelling of a sphere [Eq. (4.41)]. Equation (4.40) matches the result for two-dimensional swelling of a slab in a Cartesian system derived by Billovits and Durning (1989). The expressions derived above show that the diffusion coefficient is concentration dependent and that this dependence is related to the swelling pressure of the network during volume change and to the thermodynamic diffusion or mobility coefficient.

Chapter 5

SWELLING BEHAVIOR

5.1 Introduction

Interest in the swelling and collapsing of polyacrylamide gels was generated by Tanaka's 1978 report of a discrete change in volume with an infinitesimal change in solvent composition. Subsequent studies have considered the phenomenon as a phase transition (Tanaka, 1981; Ilavský, 1981 and 1982). A phase transition can be induced by changes in solvent composition (Tanaka, 1981), salt concentration (Ohmine and Tanaka, 1982), pH (Katchalsky, 1949; Firestone and Siegel, 1988), temperature (Hirokawa and Tanaka, 1984), electric field strength (Tanaka et al., 1982) and external deformation (Starodubtsev et al., 1985). Yoshio et al. (1986) have noted discontinuous swelling in copolymer gels of acrylamide and acrylic acid. Similar behavior was observed for positively charged gels upon changes in the composition of a water/acetone mixture (Katayama and Ohata, 1985; Ilavský et al., 1985). A discontinuity in volume is not limited to polyacrylamide hydrogels. Styrene derivatives (Hirokawa et al., 1985) and natural polymers (Amiya and Tanaka, 1987) exhibited similar behavior.

The elastic behavior (Oppermann et al., 1985; Geissler et al., 1988; Hirotsu, 1990), the diffusional behavior (Brown and Johnsen, 1981; Sellen, 1986), physical structure (Hsu et al., 1983, Ilavský et al., 1984), the extent and effect of hydrolysis (Ilavský et al., 1984; Mallo et al., 1985) have been also studied for polyacrylamide gels and their derivatives. Self- sustained oscillations of hydrogen ions were detected in water-swollen polyelectrolyte gels when an electric potential was applied for a certain period of time and then removed (Umezawa and Osada, 1987). It was also found that repetitive oscillation of electric current occurred in water-swollen crosslinked gels made of synthetic polyelectrolytes, as well as in proteins and sugars, provided

they had ionized groups in the macromolecular chains (Osada et al., 1988).

For the ionic copolymers of acrylamide and its derivatives, the degree of ionization and the nature of the medium surrounding these polymeric networks are extremely important. Some variables which affect the behavior of the polymer are the pH of the polymerization reaction (Rajan et al., 1987), the degree of ionization of the polymer (Bednar et al., 1985; Kou et al., 1988) and the concentration of the electrolytes in the swelling medium (Kowblansky and Zema, 1982). All of these variables have been shown to affect the network structure of the polymer (Hirokawa et al., 1984) and the rate of diffusion through these hydrogels (Gehrke and Cussler, 1989). When crosslinked polymer gels containing ionizable groups are placed in an aqueous solution, the sorption of water can either ionize or deionize these groups, depending on the solution pH and ionic composition. As a result gel properties such as hydration and equilibrium swelling become sensitive to the pH, composition and ionic strength of the external solution. The ionic strength of the solution depends on both the concentration of mobile ions and their valency. Small quantities of divalent or trivalent cations can decrease drastically the swelling of anionic gels containing carboxylic groups. This decrease is due to the complexing ability of carboxylate groups which induces intramolecular and intermolecular complex formation. Consequently, the crosslink density of the network increases resulting in reduced swelling of the gel. Polymers containing carboxylic acid groups formed ionic crosslinks when treated in aqueous media with salts such as calcium chloride, copper bromide, copper sulfate and aluminum acetate (Allcock and Kwon, 1989). Aluminum ions were more efficient crosslinking reagents than the divalent cations while Cu^{2+} was a more effective crosslinking agent than Ca^{2+}. The ionically crosslinked gels were stable in neutral or strongly acidic media, but the crosslinking process was reversed in basic aqueous solutions of excess monovalent cations. Jang et al. (1989) calculated the polymer subphase volume of alginic acid based on the phase-partition model of Marinsky et al. (1982) and Donnan equilibrium theory. The aqueous region surrounding the polymer chain, where strong electrostatic attractive forces for counterions exist, was defined as separate polymer subphase within the colloidal phase enclosed by a polymer coil. The reduction of polymer subphase volume in the presence of copper was ascribed to intramolecular and intermolecular crosslinking of the polymer chains by cupric ion.

In this chapter the equilibrium swelling data, the cyclic swelling/collapse data

and dynamic swelling data for polyacrylamide gels and their derivatives are presented. The modified Flory theory, presented in Chapter 3, is tested against the experimental results for the effect of salts on the equilibrium swelling of anionic and cationic copolymer gels of acrylamide. The dynamic swelling results are analyzed in light of the principles of penetrant sorption in polymers presented in the previous chapter. The theoretical model for the kinetics of gel swelling, developed in Chapter 4, is compared with dynamic swelling data for acrylamide and for N-isopropylacrylamide hydrogels.

5.2 Equilibrium Swelling

5.2.1 pH Effect

Figure 5.1 shows the swelling behavior of an anionic copolymer gel of acrylamide and sodium acrylate (5.3% T, 2.8% C, 10.0%I), crosslinked with ethylene diacrylate, as the pH was varied by addition of either NaOH or HNO_3. This gel swelled to a maximum of 170 times its original mass at preparation, M_o, (about 3200 times its dry mass, M_d) at pH 10. Further increases of pH resulted in disintegration of the network. When the pH of the external solution was lowered to about one, the polymer was precipitated out of the solution, but it did not regain its original structure when the pH was raised. In spite of its high swelling ratio, this gel is not suitable for repeated use as a sorbent because it is too fragile.

Figure 5.2 shows the swelling behavior of cationic and anionic copolymer gels of acrylamide as the pH was varied by addition of either NaOH or HNO_3. It should be noted that the mass of polymer in the gel at preparation was about 5% of M_o, therefore the mass of water absorbed per unit mass of dry polymer was approximately 20 times larger than the swelling ratio, M/M_o. The solution pH has a profound effect on the balance of forces that determines swelling equilibrium of gels which contain weakly acidic or basic groups (Grignon and Scallan, 1980; Ishihara et al., 1984). The swelling ratio of the anionic gel prepared with sodium acrylate increased with increasing pH of the external solution up to a maximum at pH about 9 where ionization of the charged network was complete. The increase in mobile counterion content within the gel that accompanied gel ionization sharply increased the gel swelling pressure relative to the solution. This induced the observed increase of gel volume.

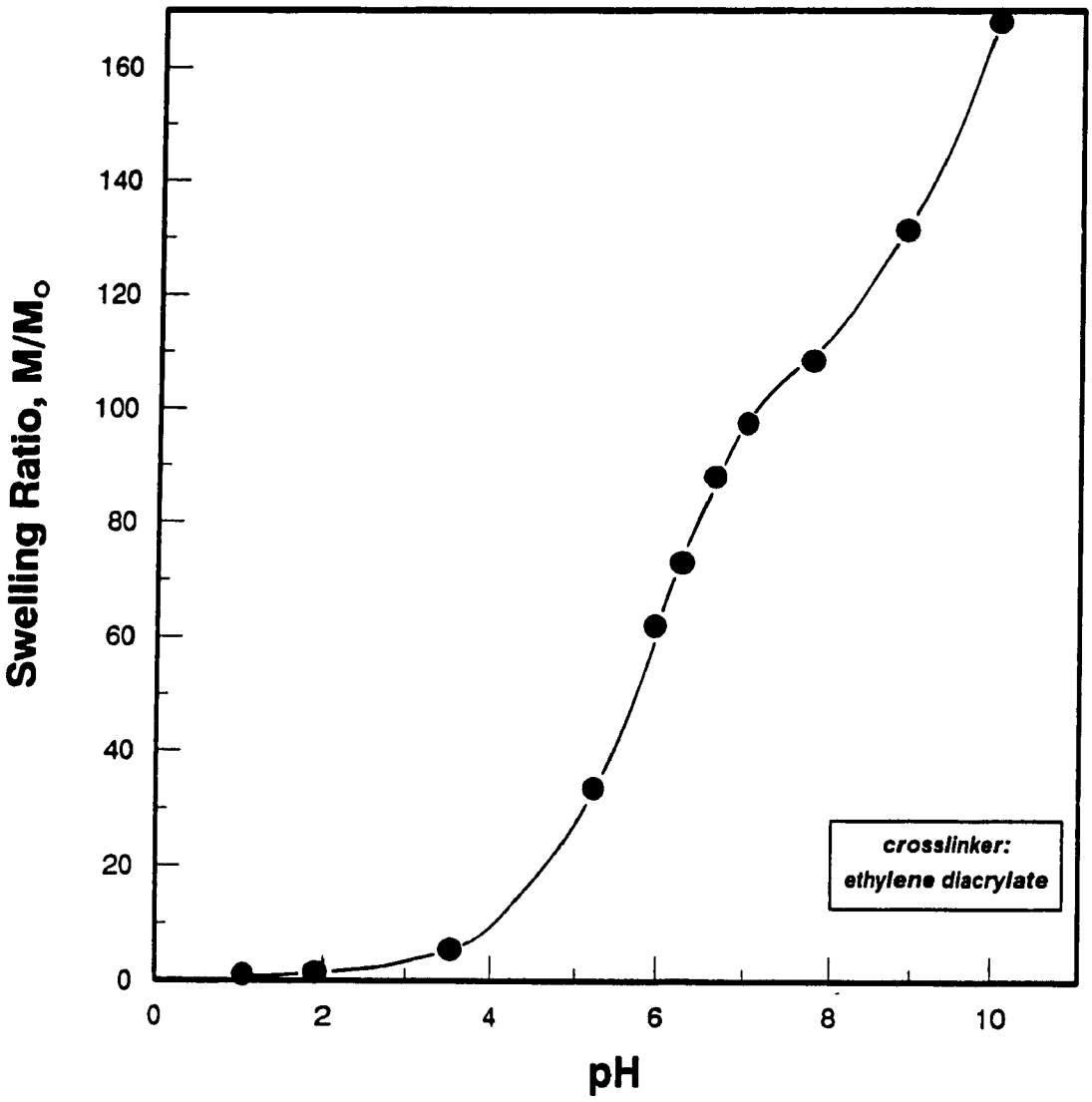


Figure 5.1: Swelling ratio of copolymer gel of acrylamide and sodium acrylate as a function of pH.

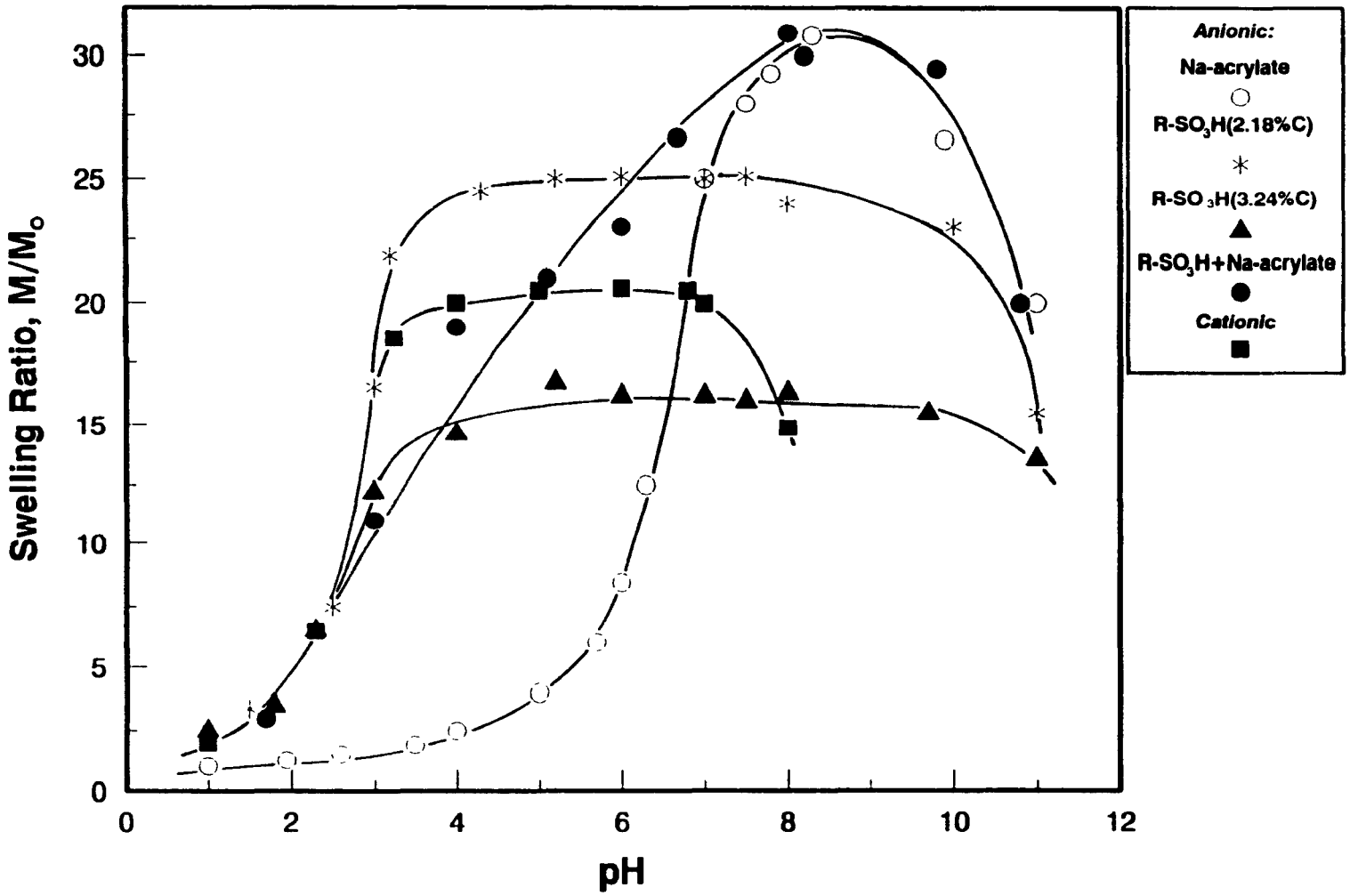


Figure 5.2: Swelling ratio of polyelectrolyte gels as a function of pH.

Further increase of pH increased the osmotic pressure of the external solution, hence decreasing the ion swelling pressure and reducing the degree of swelling [see Eq. (3.9)]. The anionic copolymer gels of acrylamide and 2-acrylamido-2-methyl-1-propanesulfonic acid ($R\text{–}SO_3^-\ H^+$), a strong acid monomer, exhibited a large and constant swelling ratio for pHs between 3.5 and 10, but gel with less crosslinking agent (6.1%T, 2.2%C, 10.0%I) was very fragile. The swelling ratio of these ionic gels, which are completely ionized, decreased at either low or high pH values due primarily to increased osmotic pressure of the external solution. The swelling ratio for the copolymer gel of acrylamide, sodium acrylate and $R\text{–}SO_3^-\ H^+$, with equal mole % of each ionizable monomer, was intermediate between the ratios for gels containing only one type of anionic monomer for pH values between 3 and 6. The cationic copolymer gel of acrylamide and 3-(methacrylamido)propyltrimethylammonium chloride (6.2%T, 2.2%C, 10.0%I) exhibited swelling behavior similar to that of highly acidic copolymer gels up to pH 8, but its swelling ratio decreased at pH $>$ 8 and it was not reversible. Irreversible swelling was also observed for cationic polyvinylamine gels for pH values between 3 and 10 (Kobayashi et al., 1989). The swelling behavior of cationic gels suggests that a gel containing a weakly basic ionizable group might be suitable for gel extraction because most fermentations are carried out at pH $\leq$ 7. However, extraction experiments presented in the following chapter, indicated such gels did not exhibit desirable exclusion behavior.

Figure 5.3 shows the variation of the swelling ratio with pH for copolymer gels of acrylamide and sodium acrylate, with different monomer concentrations at gel preparation. The monomer concentration had a dramatic effect on the swelling behavior of these hydrogels. The gel with the lowest monomer concentration (2.7%T), had the highest swelling ratio. The dependence of the physical properties of the crosslinked polymers on the concentration at which the network was formed was noted by a number of researchers (Flory, 1979; Ross-Murphy and McEvoy, 1986). Ilavský and Hrouz (1983) reported that the efficiency of the crosslinking reaction increased with increasing monomer concentration at network formation. Vasiliev et al. (1985) reported that more chemical and physical bonds were formed at higher concentrations.

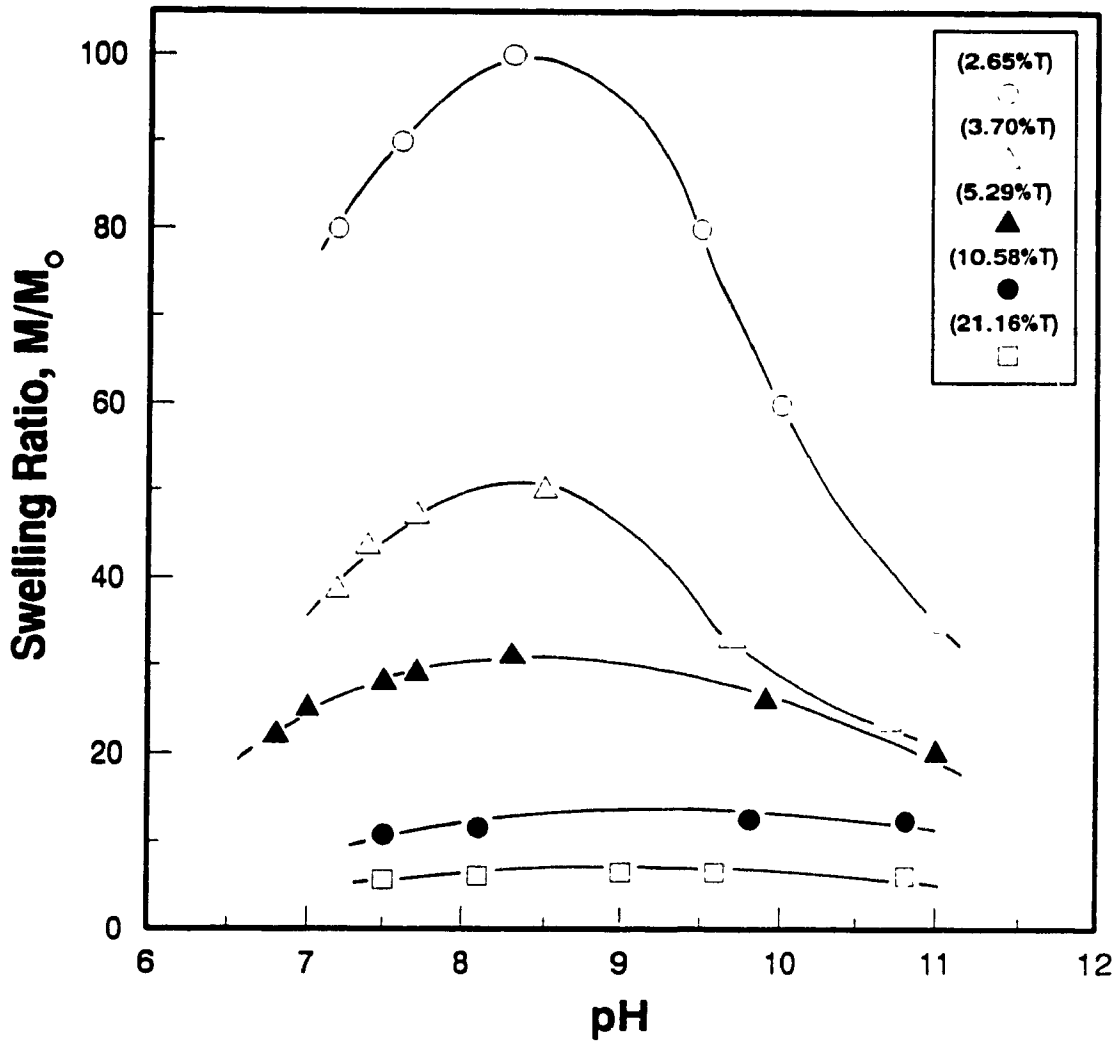


Figure 5.3: Swelling ratio of copolymer gels of acrylamide and sodium acrylate, with different monomer concentrations, as a function of pH.

5.2.2 Salt Effect

Anionic Gel

Figure 5.4 shows the swelling of the anionic gel at equilibrium in aqueous solutions containing a single salt. All salts depressed the degree of swelling if the concentrations were sufficiently high, but some were more effective than others. The monovalent sodium, potassium and silver ions had the same influence on the gel at the same molarity. The effect of bivalent calcium and cobalt was more pronounced than that of the monovalent cations. The trivalent lanthanum ion had the largest effect on the swelling of the anionic gel. Since some ions form complexes with charged macromolecules containing carboxylate groups, such ions are expected to have larger effects on swelling than noncomplexing ions of the same charge. Cupric ions decreased swelling more than other bivalent ions–see Fig. 5.4 and Rička and Tanaka (1985). Early work with this gel demonstrated that it formed a strong complex with Cu^{2+} but not with Ni^{2+}, Co^{2+} or Zn^{2+} (Wall and Gill, 1954). This anomaly is not a problem for biological application because copper is toxic to microorganisms and thus it is not a component of fermentation broths. Silver ions are capable of complex formation with some negatively charged macromolecules (Katchalsky et al., 1961), but no difference was observed between Ag^{+} and other monovalent cations.

Cationic Gel

Figure 5.5 shows the swelling behavior of the cationic gel in the presence of various salts. Nitrate and chloride ions were introduced with either monovalent sodium or bivalent calcium ions. All nitrates and chlorides gave the same swelling ratio at the same anion concentrations. At pH 5 7, for the concentration of sodium sulfate used here, the major anionic species in solution was SO_4^{2-}. This bivalent ion had a much stronger effect on the swelling of the gel than monovalent ions–as noted for the anionic gel. This was also demonstrated by the studies with mixtures of Na_2SO_4 and $NaNO_3$ for which the swelling was intermediate between the monovalent and bivalent data. The theoretical curves in Fig. 5.5 are discussed subsequently.

In a typical fermentation medium phosphate is the most concentrated anion. At pH 7 the major ions are HPO_4^{2-} and $H_2PO_4^{-}$. The balance between the monovalent and bivalent ions is changed by pH. Figure 5.6 shows the swelling behavior for pH

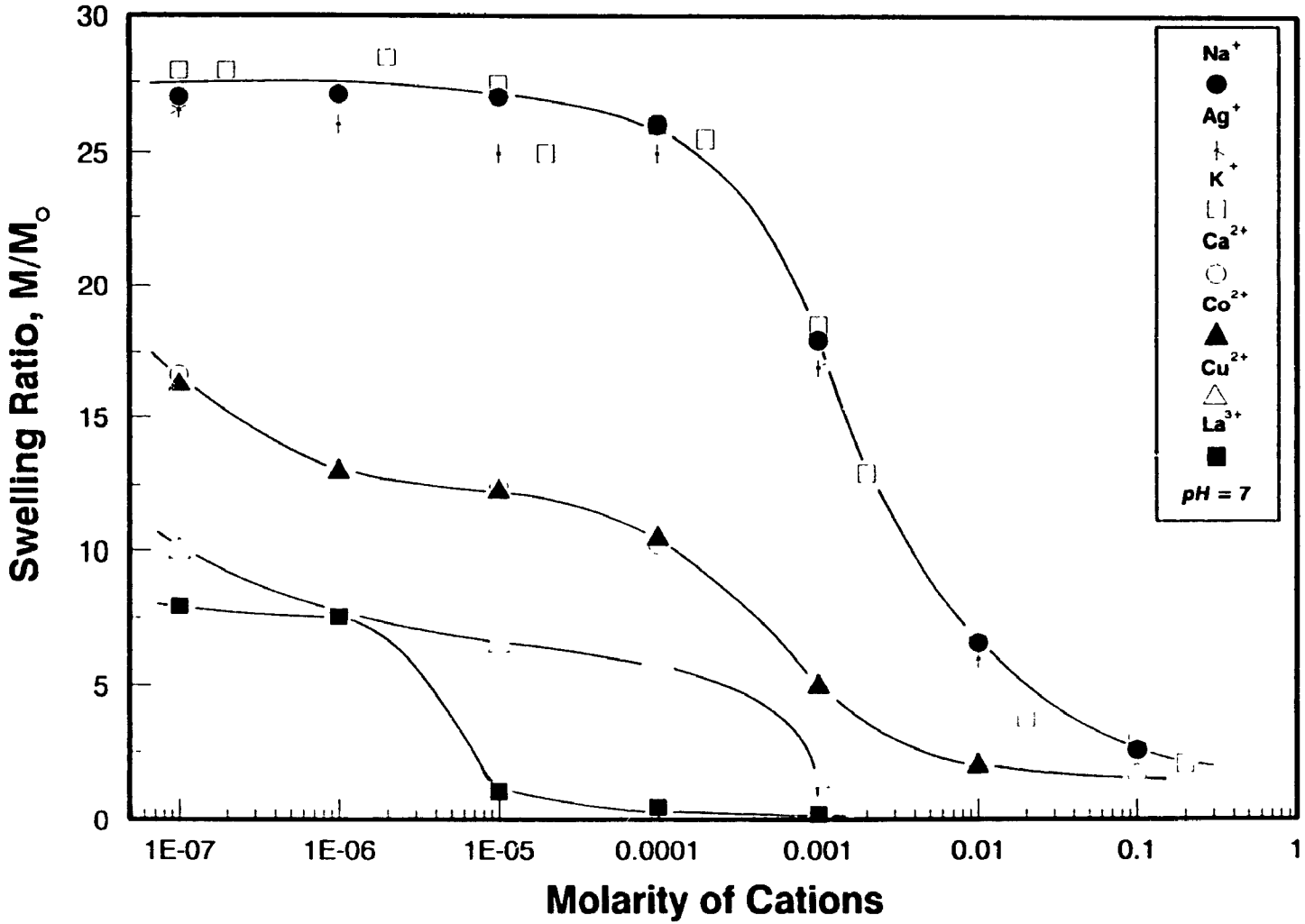


Figure 5.4: Swelling of copolymer gels of acrylamide and sodium acrylate in salt solutions.

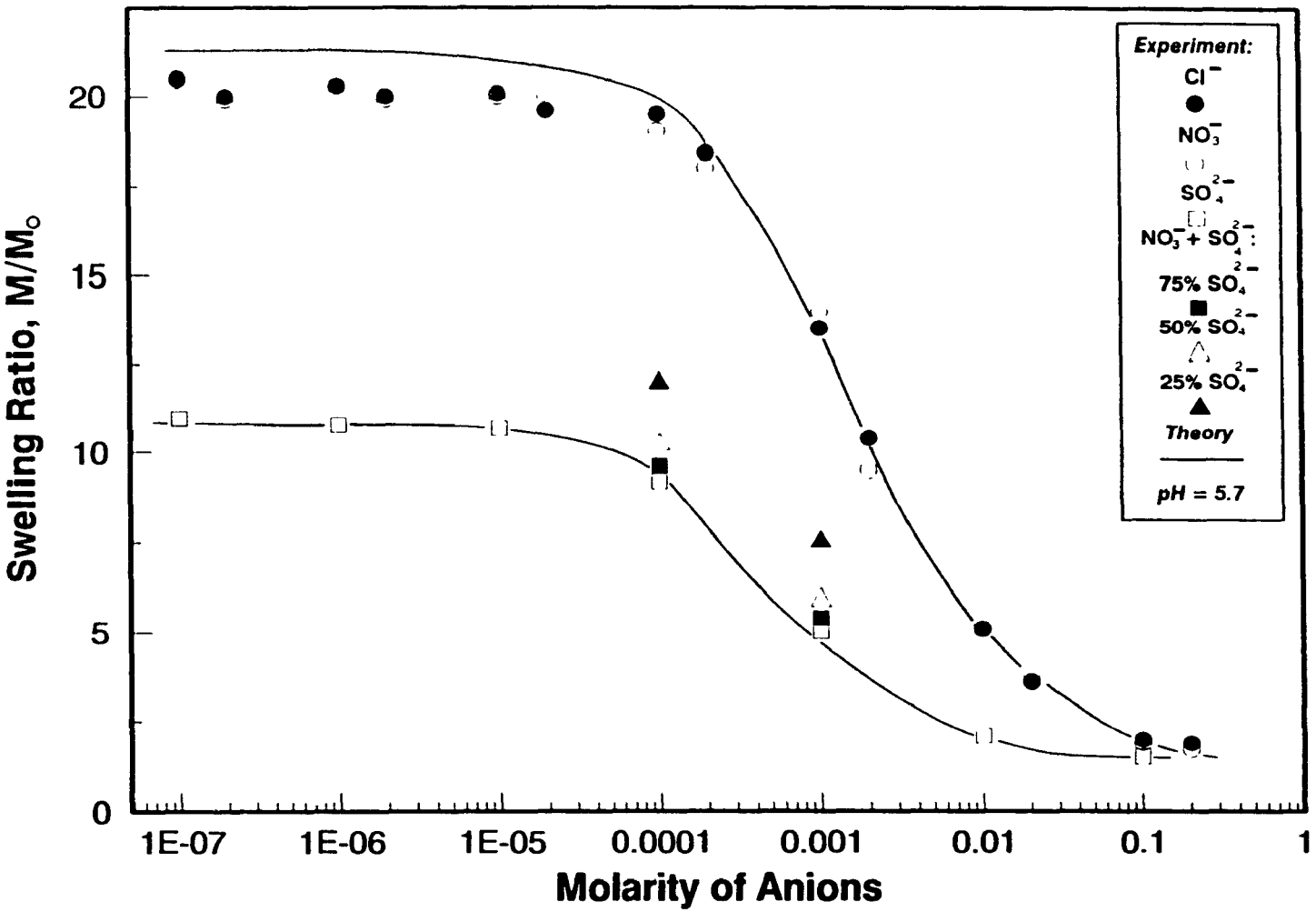


Figure 5.5: Comparison of theory with experimental results for swelling ofcationic gel in salt solutions.

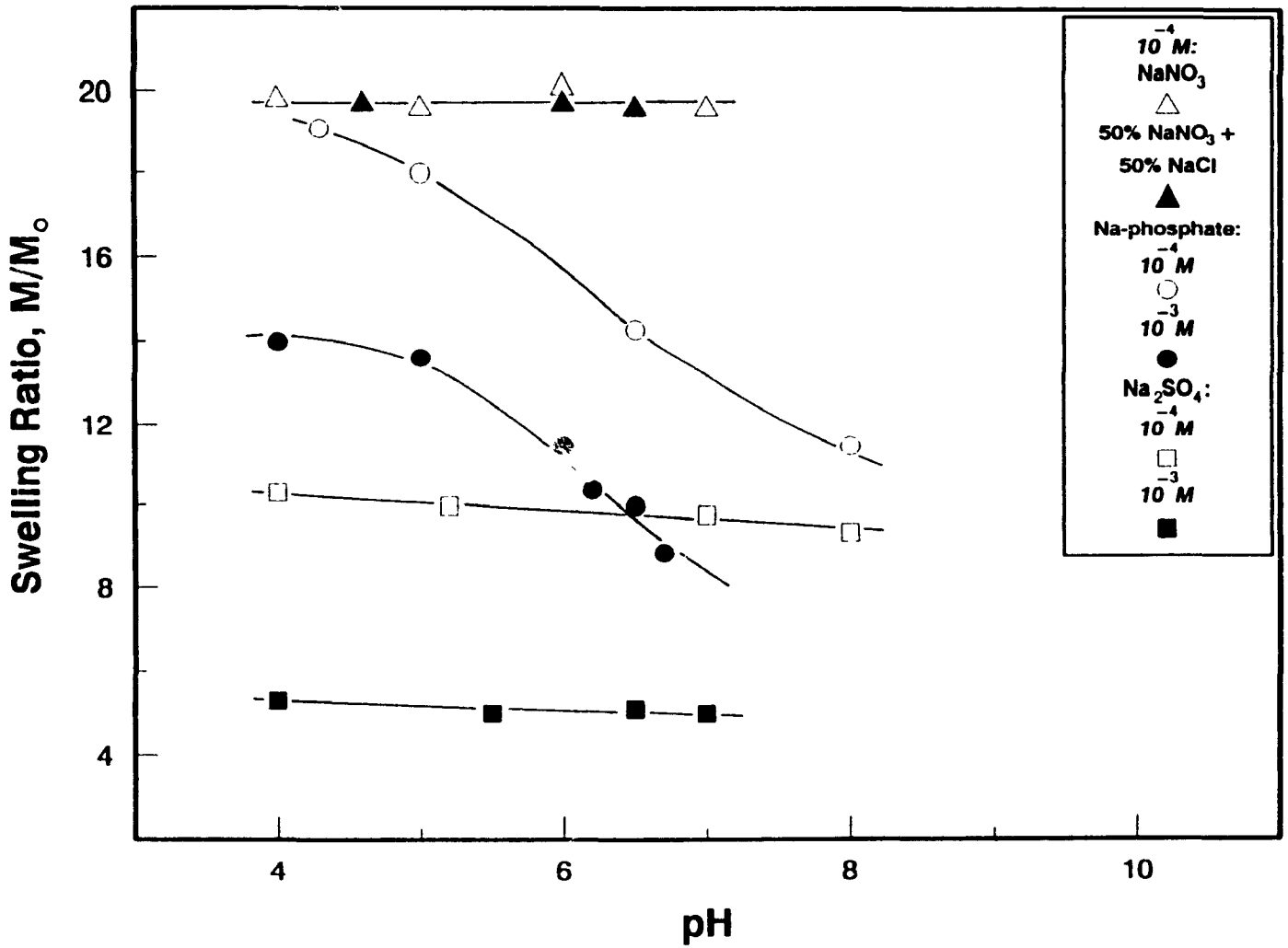


Figure 5.6: Swelling of cationic gel as a function of pH in salt solutions.

values between 4 and 8 for sodium phosphate concentrations of 10^{-3} and 10^{-4} M and for three other salts. In Fig. 5.2 the swelling ratio of the cationic gel without salt is constant at a value of 20 between pH 4 and pH 7.5. When NaCl, $NaNO_3$ or Na_2SO_4 is added, Fig. 5.6 shows that pH has little effect on swelling. For phosphate, the swelling ratio decreases as the pH increases because the phosphate equilibrium shifts from $H_2PO_4^-$ to HPO_4^{2-}.

5.2.3 Comparison of Theory and Experiment

Both anionic and cationic gels contained 10 mole % ionizable monomer and the carboxylic groups of the anionic gel are assumed to be completely ionized at pH 7, i.e., $\alpha = 0.1$. The molarity of monomeric units in the polymeric network, n_m, is related to the concentration of monomers at gel formation, n_m°, by

$$n_m = n_m^\circ / X \tag{5.1}$$

where n_m°=0.712 M.

Determination of Parameters

The molarity of salt in the gel phase, n_s, is related to the co-ion concentration in the gel which can be obtained from Eqs. (3.35) and (3.36). For a mono-monovalent salt n_s is equal to the co-ion concentration, whereas for bi-monovalent salts (anionic gel) or mono-bivalent salts (cationic gel) n_s is one-half of the co-ion concentration.

The osmotic coefficients, ϕ and ϕ_s, were obtained from experimental results for $NaNO_3$ (Hamer and Wu, 1972), $CaCl_2$ (Staples and Nuttal, 1977), and Na_2SO_4 (Goldberg, 1981). The osmotic coefficient, ϕ_p, of Na-polyacrylate solution changed from 0.55 to 0.65 at α= 0.1 as n_m was varied from 0.01 to 0.25 (Kern, 1939). For $0.5 \leq x \leq 8$ the osmotic coefficient of the gel phase, Φ, was obtained from the experimental results of Alexandrowicz (1960). In the absence of data, the osmotic coefficient of Ca-polyacrylate solution was assumed initially to be half of that for Na-polyacrylate solution. However, use of this value underestimated the swelling of the gel, particularly at C$\leq 10^{-3}$ M, as discussed below. The osmotic coefficient, ϕ_p, of a positively charged polyelectrolyte solution (cationic gel) was assumed to be equal

to that of a negatively charged polyelectrolyte solution (anionic gel). The values of osmotic coefficients are given in Tables 5.1–5.4.

Table 5.1: Osmotic coefficients for swelling of anionic gel in $NaNO_3$ solution.

C_i, (M)	X	$n_m\alpha$	n_s	ϕ	ϕ_p	ϕ_s	Φ
10^{-7}	27	2.6×10^{-3}	3.8×10^{-12}	1.00	0.550	1.00	0.550
10^{-6}	27	2.6×10^{-3}	3.8×10^{-10}	1.00	0.550	1.00	0.550
10^{-5}	27	2.6×10^{-3}	3.8×10^{-8}	1.00	0.550	1.00	0.550
10^{-4}	26	2.7×10^{-3}	3.7×10^{-6}	0.998	0.550	1.00	0.550
10^{-3}	18	4.0×10^{-3}	2.4×10^{-4}	0.988	0.600	0.998	0.650
10^{-2}	6.6	1.1×10^{-2}	6.3×10^{-3}	0.967	0.630	0.976	0.821
10^{-1}	2.6	2.7×10^{-2}	8.7×10^{-2}	0.921	0.650	0.928	0.891

Table 5.2: Osmotic coefficients for swelling of anionic gel in $CaCl_2$ solution.

C_i, (M)	X	$n_m\alpha$	n_s	ϕ	ϕ_p	ϕ_s	Φ
10^{-7}	13	5.5×10^{-3}	6.0×10^{-10}	1.00	0.440	1.00	0.140
10^{-6}	13	5.7×10^{-3}	1.9×10^{-8}	0.990	0.440	1.00	0.140
10^{-5}	13	5.7×10^{-3}	5.9×10^{-7}	0.980	0.440	1.00	0.440
10^{-4}	11	6.8×10^{-3}	1.7×10^{-5}	0.970	0.440	0.980	0.150
10^{-3}	5.0	1.4×10^{-2}	3.7×10^{-4}	0.960	0.440	0.970	0.515
10^{-2}	2.1	3.4×10^{-2}	6.5×10^{-3}	0.907	0.440	0.920	0.693
10^{-1}	1.6	4.5×10^{-2}	9.3×10^{-2}	0.850	0.440	0.852	0.764

Table 5.3: Osmotic coefficients for swelling of cationic gel in $NaNO_3$ solution.

C_s, (M)	X	$n_m\alpha$	n_s	ϕ	ϕ_p	ϕ_s	Φ
10^{-7}	20	3.6×10^{-3}	2.8×10^{-12}	1.00	0.550	1.00	0.550
10^{-6}	20	3.6×10^{-3}	2.8×10^{-10}	1.00	0.550	1.00	0.550
10^{-5}	20	3.6×10^{-3}	2.9×10^{-8}	1.00	0.550	1.00	0.550
10^{-4}	19	3.8×10^{-3}	2.7×10^{-6}	0.998	0.570	1.00	0.600
10^{-3}	14	5.1×10^{-3}	1.9×10^{-4}	0.988	0.610	0.998	0.637
10^{-2}	5.2	1.4×10^{-2}	5.3×10^{-3}	0.967	0.630	0.975	0.823
10^{-1}	1.9	3.8×10^{-2}	8.3×10^{-2}	0.921	0.650	0.928	0.877

Table 5.4: Osmotic coefficients for swelling of cationic gel in Na_2SO_4 solution.

C_s, (M)	X	$n_m\alpha$	n_s	ϕ	ϕ_p	ϕ_s	Φ
10^{-7}	11	6.5×10^{-3}	5.6×10^{-10}	1.00	0.440	1.00	0.440
10^{-6}	11	6.6×10^{-3}	1.7×10^{-8}	0.990	0.440	1.00	0.440
10^{-5}	11	6.6×10^{-3}	5.5×10^{-7}	0.980	0.440	1.00	0.440
10^{-4}	9.2	7.7×10^{-3}	1.6×10^{-5}	0.970	0.440	0.980	0.447
10^{-3}	5.0	1.4×10^{-2}	3.7×10^{-4}	0.960	0.440	0.970	0.515
10^{-2}	2.1	3.4×10^{-2}	6.5×10^{-3}	0.896	0.440	0.910	0.693
10^{-1}	1.5	4.8×10^{-2}	9.3×10^{-2}	0.786	0.440	0.792	0.762

The concentration of constituent chains, ν_o is related to the concentration of constituent chains per unit volume in the dry state, ν_d, by

$$\nu_o = \nu_d . v_o \tag{5.2}$$

where v_o is the polymer volume fraction at gel formation; here $v_o = 0.048$. A value of $\nu_d = 7 \times 10^{-5}$ mole/cm^3 was used based on previous work on ionic copolymer gels of acrylamide (Ilavský and Hrouz , 1982).

The number of statistical segments per chain, n, which is a function of fixed charge density, quality of the solvent, added salt, temperature, etc., was used as a

free parameter to fit the experimental data. The best values of n are approximated by

$$n = 40(X)^{-1/3} \qquad \text{(anionic gel)} \tag{5.3}$$

and

$$n = 24(X)^{-1/3} \qquad \text{(cationic gel)} \tag{5.4}$$

These expressions were obtained using only swelling data for monovalent counterions. They were then used for bivalent counterions.

Theoretical Predictions

The theoretical predictions for the swelling of the anionic gel in mono-monovalent ($NaNO_3$) and bi-monovalent ($CaCl_2$) salt solutions are compared to experimental data in Fig. 5.7. This figure also includes the theoretical predictions for the mono-monovalent salt in the ideal case ($\Phi = \phi = 1$) and for the case where the non-Gaussian distribution of chain extension is neglected, i.e. $n = \infty$. For high swelling ratio the contribution to swelling pressure due to mixing of polymer with solvent, π_1, is negligible. Using n given by Eq. (5.3), the theoretical predictions agree well with experiment for mono-monovalent salts. If the non-Gaussian term is neglected the predictions are very poor at low concentrations ($C_s < 10^{-3}$ M). If the osmotic coefficients are assumed to be unity, the swelling ratio is also badly overpredicted.

In Fig. 5.7 three sets of experimental data are presented for $CaCl_2$. For the open circles and filled triangles the pH was adjusted with NaOH, hence these systems contained small concentrations of monovalent as well as bivalent cations. Air was excluded from the vessels for the former system, but not for the latter one during pH adjustment. The filled circles represent experiments in which a single bivalent ion, Ca^{2+}, was present. To obtain these data dialyzed gel particles (which were formed from sodium acrylate) were first equilibrated with 10^{-3} M $CaCl_2$ solution to replace the residual sodium. The particles were then swollen in 10^{-4} to 10^{-7} M $CaCl_2$ solutions while the pH was adjusted with $Ca(OH)_2$.

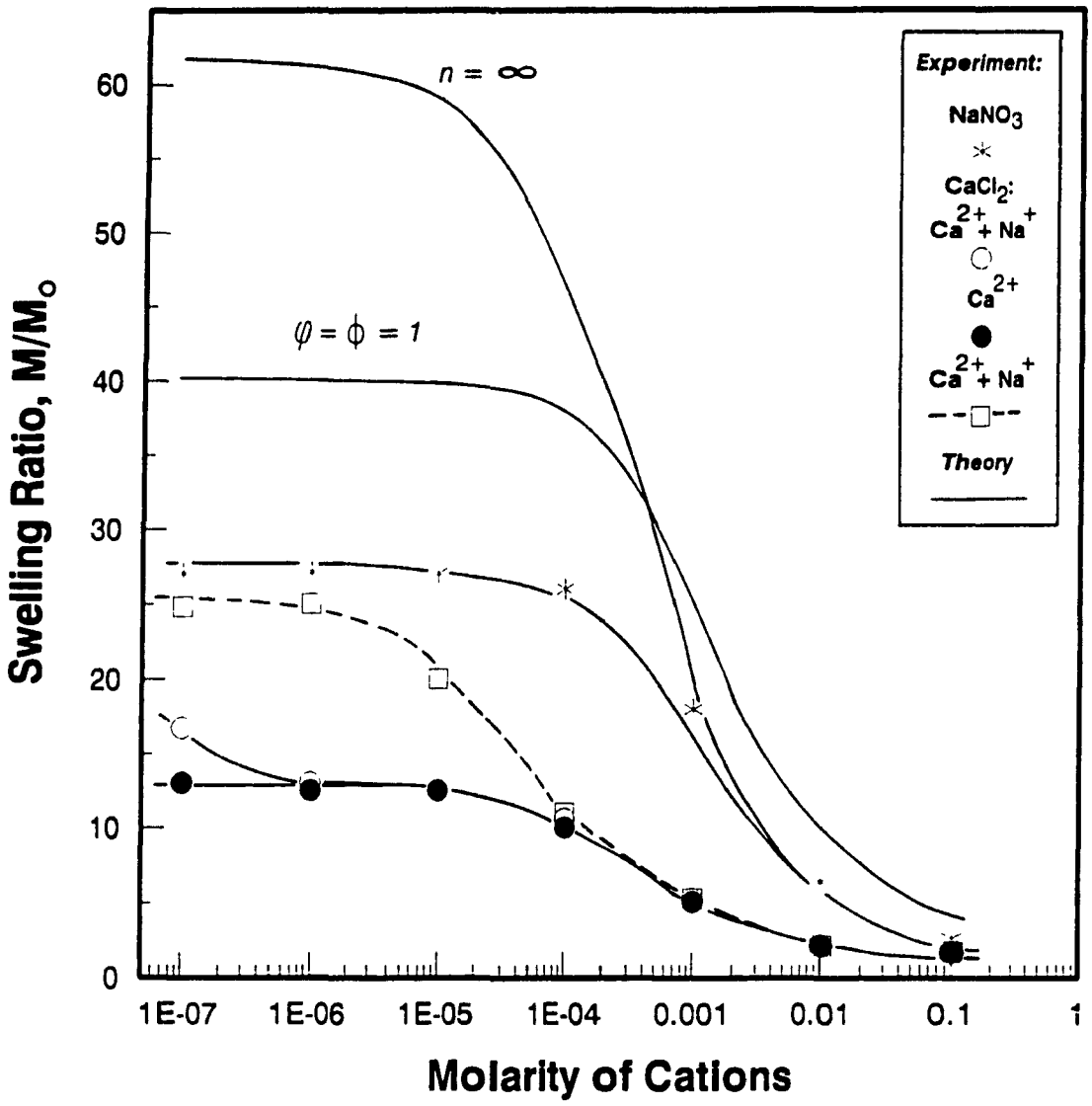


Figure 5.7: Comparison of the theory with experimental results for swelling of anionic gel in salt solutions.

When the pH was adjusted with NaOH and air was excluded from the vessels, about 3×10^{-6} M NaOH was required to bring the pH to 7. Figure 5.7 shows that at concentrations of $CaCl_2$ below 10^{-6} M the swelling ratio increased above the value for the system containing only Ca^{2+}. In the system containing Na^+ and Ca^{2+} the swelling ratio approached the value for monovalent counterions as the Ca^{2+} concentration decreased. This behavior was predicted by the theory as shown in Fig. 5.7. When the pH was adjusted with NaOH without air exclusion, higher amounts of monovalent cations were present in the system and swelling of the gel approached its behavior in the presence of monovalent counterions at concentrations of $CaCl_2$ below 10^{-5} M. Similar behavior in mixed mono- and bivalent systems was exhibited by polymethacrylic acid gels (Katchalsky and Zwick, 1955), by copolymer gels of acrylic acid (Rička and Tanaka, 1984), and by the cationic gel in Fig. 5.5. If air were not carefully excluded, additional NaOH was required to maintain the pH. At concentrations below 10^{-4} M $CaCl_2$ any swelling ratio between the pure Ca^{2+} and Na^+-values could then be obtained depending upon the experimental procedure. The data for the system containing only Ca^{2+} were well represented by the theory with n from Eq. (5.3) and $\phi_p = 0.44$–see Fig. 5.7. The latter value was somewhat larger than the expected value of 0.33, which is one-half of the value for Na-polyacrylate solutions.

In the mixed cation system the gel polyion was neutralized by a mixture of monovalent and bivalent cations. The swelling behavior was investigated by noting that the concentration of NaOH in the external solution was approximately 3×10^{-6} M. The equivalent fraction of Na^+ for the gel phase, obtained through Eq. (3.33) for various $CaCl_2$ solutions is shown in Table 5.5. For $CaCl_2$ concentrations of 10^{-5} M and above, there is too little sodium present inside the gel to affect swelling. At lower concentrations, where the equivalent fraction increases, the osmotic coefficient Φ was used as a free parameter to match the data using Eq. (3.34) and the theory described earlier. The results are shown in Table 5.5. The osmotic coefficient, Φ, for the mixture of monovalent and bivalent counterions is higher than that for bivalent ones. These results are consistent with theoretical expectations as discussed earlier.

Figure 5.5 also shows the comparison of the theoretical predictions with experimental data for swelling of the cationic gel in salt solutions. There is good agreement between theory and experiment for the effect of mono- and bivalent anions on the swelling of the cationic gel. The osmotic coefficient of bivalent counterions, ϕ_p, equal

Table 5.5: Equivalent fraction of Na^+ in gel phase, f, and osmotic coefficient, Φ, of the anionic gel

$CaCl_2$, (M)	X	f	Φ
10^{-7}	16.8	0.10	0.55
10^{-6}	13.0	0.03	0.44
10^{-5}	12.5	0.01	0.44

to 0.44 was used to fit the experimental results.

5.3 Collapse of Hydrogels

The success of the proposed separation process is heavily dependent on the regeneration step because there must be a large volume change from the collapsed (regenerated) state to the swollen state. After regeneration the gel may be much larger than it was at preparation. As a result, a single swelling experiment after preparation is insufficient to characterize a gel for use in a separation process. Cyclic experiments in which swelling follows regeneration are required.

Ilavský et al. (1985) reported that a copolymer gel of N,N-diethylacrylamide and sodium methacrylate underwent a sharp phase transition at 47.5°C, with a volume change of approximately 130 - fold. However in preliminary experiments with this gel no sharp phase transition was found even at 50°C - see Fig. 5.8. Freitas and Cussler (1987) also reported that discontinuous phase transition did not occur for this gel.

Hirokawa and Tanaka (1984) found that nonionic N-isopropylacrylamide (NIPA) gel underwent a discontinuous phase transition upon changing solvent composition or temperature with a phase transition temperature of approximately 33°C. Linear poly(N,N-diethylacrylamide) and linear poly(N-isopropylacrylamide), readily soluble in water at low temperature, precipitate at 29 - 30°C and 31°C, respectively (Heskins and Guillet, 1968; Ulbrich and Kopeček, 1979). Priest et al. (1986) showed

that copolymerization of NIPA with acrylamide or other N – substituted acrylamide altered its lower critical solution temperature (LCST), the temperature above which the polymer precipitates. The poly NIPA gel behaved in an analogous way with shrinking as the gel–equivalent of the precipitation of the uncrosslinked polymer. Plestil et al. (1987) and Fujishige et al. (1989) reported that in this type of phase transition phenomena, especially in the vicinity of the phase transition temperature, an abrupt conformational change occurs from a state of well–solvated random coils below the LCST to a state of tightly packed globular chains above the LCST. In subsequent works with NIPA gels discontinuous phase transition (Amiya et al., 1987; Freitas, 1986; Hirotsu, 1987; Matsuo and Tanaka, 1988) and sharp but continuous phase transition (Hirotsu, 1985; Hirotsu et al., 1987; Hirose et al., 1987; Gehrke et al., 1989) around 34°C were observed.

Hirotsu (1985) studied the effect of an electric field on the phase transition of copolymer gels of NIPA and acrylic acid. He observed a discontinuous phase transition of ionized NIPA gels which contained up to 7% ionizable monomer. The phase transition temperature decreased with increasing electric field strength. In most subsequent studies on ionic copolymers of NIPA gel discontinuous phase transitions were observed. But Hirose et al. (1987) reported that submicron gels underwent a fairly sharp, but continuous volume phase transition in water in response to temperature changes. They hypothesized that each gel bead had a volume discontinuity at a slightly different temperature which resulted from the variation of ionic group density, crosslinkage, size and shape from bead to bead. Amiya et al. (1987) reported that at 42°C ionized NIPA gels containing more than 4.7% sodium acrylate were still swollen, but gels with fewer ionizable groups were collapsed. Hirotsu et al. (1987) found that the transition temperature of ionic NIPA gels increased as the ionic concentration was increased, but all volumes were essentially the same in the shrunken state. The ionic gel having 10% ionizable monomer underwent a discontinuous volume change around 42°C but the gel having the highest ionic concentration (18.8%I) did not shrink up to 80°C due to the high ionic swelling pressure. On the other hand, a discontinuous volume phase transition was observed around 60°C for spherical ionized NIPA gels, prepared by inverse suspension polymerization, containing 18.8% sodium acrylate (Matsuo and Tanaka, 1988). This literature review indicates some discrepancies about the phase transition behavior of ionic NIPA gels. These

discrepancies are probably associated with the pH of the external solution which was not reported in most studies.

5.3.1 Equilibrium Collapse of Temperature Sensitive Hydrogels

Figure 5.8 shows the effect of temperature or the swelling behavior of a copolymer gel of N,N–diethylacrylamide and sodium methacrylate (6.7%I) in distilled water at pH 7. The swelling ratio, M/M_o, continuously decreased as temperature increased.

The swelling behavior as a function of temperature of ionized NIPA gels in distilled water and sodium chloride solutions (pH =7) is shown in Fig. 5.9. Ionic gels which contained more than 5.0% ionizable monomer did not collapse even at 50°C. These results are in agreement with Amiya et al. (1987). Nonionic gels collapsed by a factor of ten from 23 to 35°C which agreed with Freitas' result (1986).

Table 5.6 shows equilibrium swelling results for ionized NIPA gels in 10^{-3} M calcium nitrate solution at 23°C (±2°C) and pH 7. The swelling ratios were about the same for ionic gels which contained 10 0% ionizable monomer, but copolymer gels of sodium acrylate exhibited a larger volume change from their collapsed state than copolymer gels of 2–acrylamido–2-methyl-1–propanesulfonic acid (R–SO_3^- H^+).

Table 5.6: Equilibrium swelling of ionized NIPA gels in 10^{-3} M $Ca(NO_3)_2$ solution at pH 7*.

Swelling Ratio	NIPA + R–SO_3^- H^+ (10.0%I)	NIPA + Na–acrylate (10.0%I)	NIPA + Na–acrylate (15.0%I)
M/M_o	4.3	4.1	5.3
M/M_r	25	32	34
M/M_d	50	52	67

*M_r = regenerated mass; M_d = dry mass of polymer

The extent of volume change from the collapsed state was almost the same for copolymer gels of NIPA and sodium acrylate because of the higher shrunken volume of the gel with 15.0% ionizable monomer.

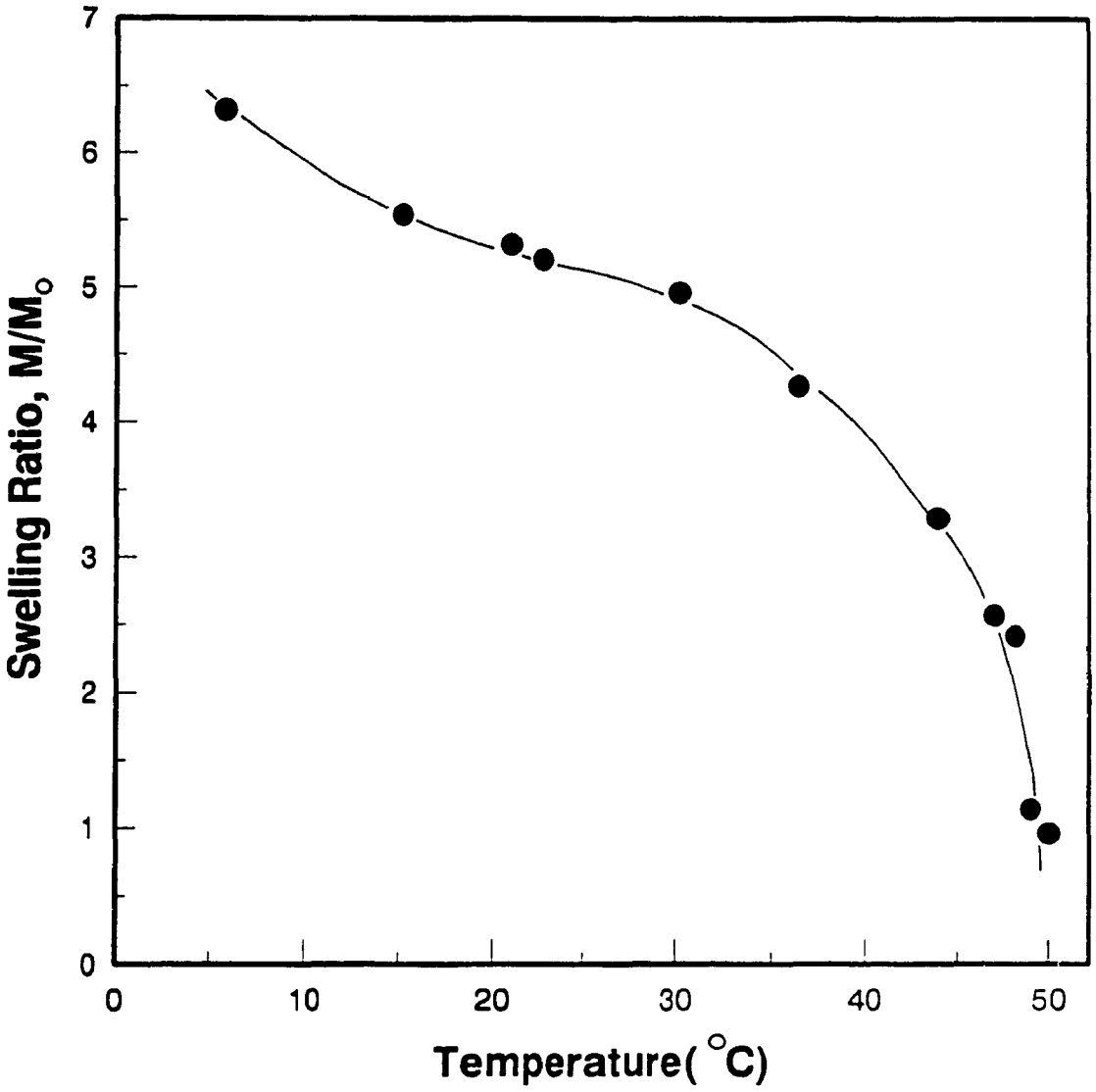


Figure 5.8: Temperature effect on the swelling behavior of copolymer gel of N,N-diethylacrylamide and sodium methacrylate at pH 7.

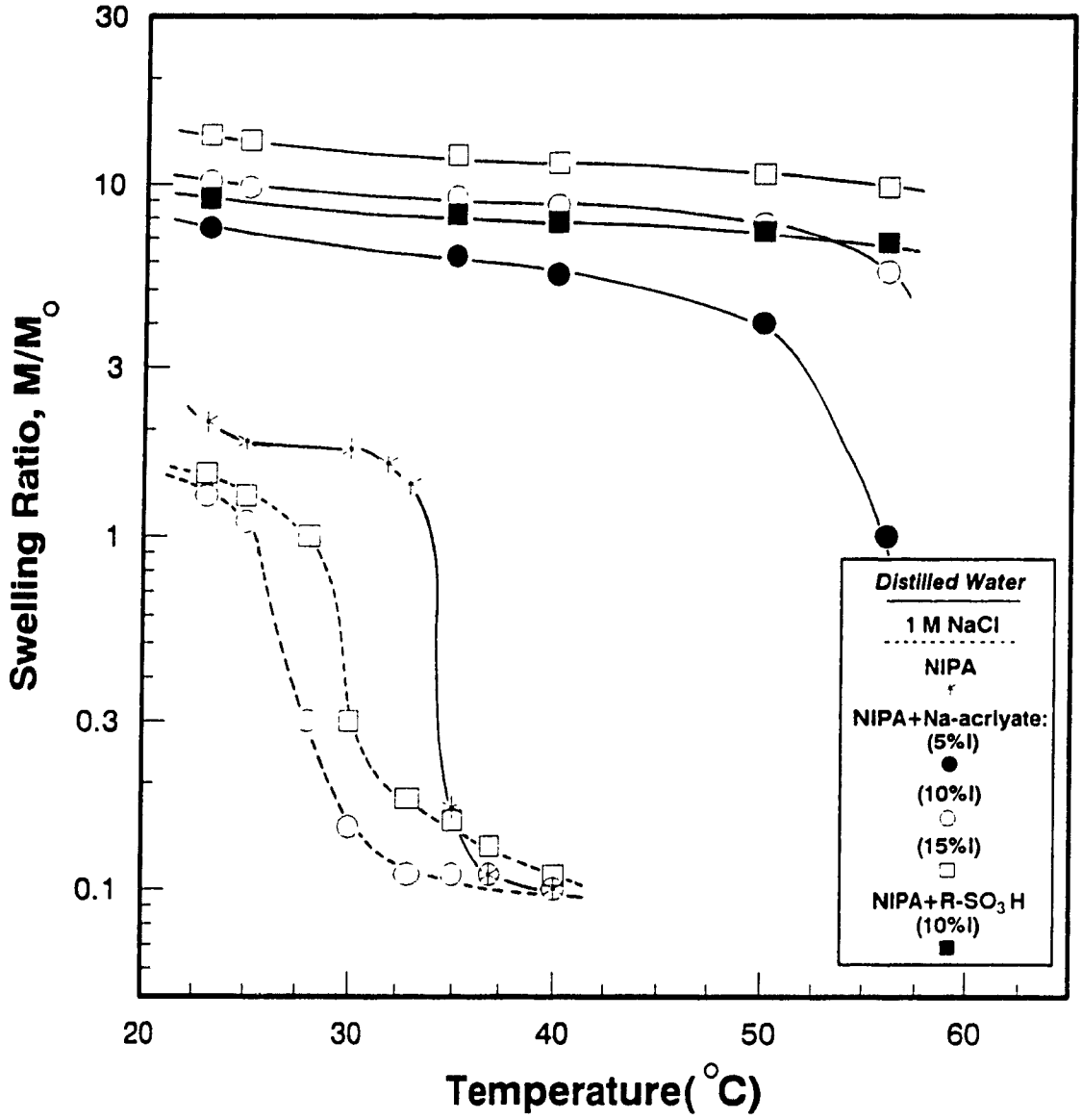


Figure 5.9: Temperature effect on the swelling behavior of NIPA gels at pH 7.

5.3.2 Cyclic Swelling/Collapse of Hydrogels

It is convenient to present the experimental results so that the amount of gel required for the sorption of a specific volume of water can be determined, thus the ratio of swollen mass, M_s, to the regenerated mass, M_r, is appropriate. On the other hand the sorption capacity of a polymeric network can be determined by the ratio of the mass of imbibed solvent, $M_s - M_r$, to the dry mass of polymer, M_d. Having this parameter, one can obtain the dry mass of monomers at gel formation which is required for the sorption of a specific amount of solvent. The relationship between these quantities and the swelling ratio after swelling, M_s/M_o, and after regeneration, M_s/M_r, is

$$\frac{M_s}{M_o} = \left(\frac{M_s - M_r}{M_d}\right)\left(\frac{M_d}{M_o}\right)\frac{M_s/M_r}{\frac{M_s}{M_r} - 1} \tag{5.5}$$

It should be noted that in cyclic experiments gel particles, swollen to equilibrium at pH 5.7 after gel preparation, were collapsed in different regeneration media for a certain period of time (6 hr for copolymer gels of acrylamide and 1 hr for NIPA gels) and then returned to the swelling media where swelling occurred for 1 hr. Swollen gel particles were collapsed in a regeneration step followed by a swelling step in next cycle. If swelling and regeneration were for long time, then M_s and M_r were at equilibrium, otherwise these quantities were dependent on the swelling and regeneration time. The time required to achieve any given M_s and M_r depends upon the gel particle size and the swelling medium. Times are shorter for smaller particles. Table 5.7 shows data for the cyclic swelling of copolymer gels of acrylamide and sodium acrylate (5.3%T, 2.5%C, 10.0%I) in distilled water (DW) at pH 5.7 and in 10^{-3} M $Ca(NO_3)_2$ solution at pH 7 after regeneration in different media. Regeneration was accomplished with a combination of pH and salt effects using NaCl solutions and swelling time was 1 hr. This gel did not exhibit a large volume change when it was brought in contact with solutions containing 10^{-3} M bivalent cations, a typical concentration of fermentation broth.

Table 5.8 shows data for the cyclic swelling of copolymer gels of NIPA and sodium acrylate after regeneration (collapse) in different media. The sorption capacity per unit dry weight of ionized NIPA gel decreased somewhat with either increasing temperature above 35°C or salt concentration above 1 M NaCl in regeneration. This

Table 5.7: Cyclic swelling of copolymer gel of acrylamide and sodium acrylate.*

Regeneration medium	$d_r^{\dagger}/d_o^{\ddagger}$	Swelling in DW at pH 5.7		Swelling in 10^{-3} M $Ca(NO_3)_2$ at pH 7	
		$\frac{M_s-M_r}{M_d}$	$\frac{M_s}{M_r}$	$\frac{M_s-M_r}{M_d}$	$\frac{M_s}{M_r}$
2 M NaCl, pH=5.7	1.25	366	10.8		
2 M NaCl, pH=6.4	1.26	373	11.1		
2 M NaCl, pH=8.7	1.28	448	12.5	70.4	2.80
2 M NaCl, pH=2.0	1.05	186	9.40	49.3	3.80
DW, pH=2	1.09	66.2	3.72		

*$\frac{M_o}{M_d} = 18.9$ $^{\dagger}d_r$= collapsed gel diameter $^{\ddagger}d_o$= gel diameter at preparation(0 24cm)

is related to the larger shrunken volume of the gel at these conditions. It is possible that a thick layer of dense, collapsed polymer network which is nearly impermeable to water formed at the surface of the particles. This barrier temporarily prevented the gel from further shrinking. High salt concentration or temperature may result in a thicker layer of dense polymer. The process of shrinking of temperature sensitive hydrogels is more complicated than that of swelling. Bulges formed on the surface of the gel during shrinking in a manner similar to that reported by Matsuo and Tanaka (1988) This will be discussed in Section 5.4.

Table 5.8: Cyclic swelling of copolymer gels of NIPA and sodium acrylate in 10^{-3} M $Ca(NO_3)_2$ at 23°C (±2°C) and pH 7, after regeneration in different media*.

Regeneration medium	(5.0%I)			(10.0%I)		
	$d_r^{\dagger}/d_o^{\ddagger}$	$\frac{M_s-M_r}{M_d}$	$\frac{M_s}{M_r}$	d_r/d_o	$\frac{M_s-M_r}{M_d}$	$\frac{M_s}{M_r}$
1 M NaCl, T=35°C	0.58	25	14	0.63	42	15
1 M NaCl, T=40°C	—-	—	—	0.79	36	7.8
2 M NaCl, T=35°C	0.71	23	6.4	—	—	—
2 M NaCl, T=40°C	0.82	21	4.7	0.84	33	6.3

*Swelling time = 1 hr; Regeneration time = 1 hr; $\frac{M_o}{M_d} = 13$

$^{\dagger}d_r$= collapsed gel diameter

$^{\ddagger}d_o$= gel diameter at preparation (0.24cm)

Table 5.9: Cyclic swelling of copolymer gels of NIPA and R-SO_3^- H^+ (5.0%I) in 10^{-3} M $Ca(NO_3)_2$ at 23°C (±2°C) and pH 7, after regeneration in different media*.

Regeneration medium	(1.6%C)			(2.4%C)		
	d_r/d_o	$\frac{M_a-M_r}{M_d}$	$\frac{M_a}{M_r}$	$d_r^{\dagger}/d_o^{\ddagger}$	$\frac{M_a-M_r}{M_d}$	$\frac{M_a}{M_r}$
1 M NaCl, T=35°C	0.67	28	15	0.67	24	12
2 M NaCl, T=35°C	0.77	24	9.0	—	—	-
2 M NaCl, T=40°C	0.79	24	7.8	—	—	-

*Swelling time = 1 hr; Regeneration time = 1 hr; $\frac{M_a}{M_d} = 12$

$^{\dagger}d_r$= collapsed gel diameter

$^{\ddagger}d_o$= gel diameter at preparation (0.24cm)

Table 5.9 shows cyclic swelling of the ionic copolymer gels of NIPA and R SO_3^- H^+, collapsed in different media. Swelling occurred at 23°C (±2°C) in 10^{-3} M calcium nitrate solution of pH 7. The gel containing the lower amount of crosslinking agent exhibited larger swelling. Sorption capacity decreased with increasing either temperature above 35°C or salt concentration above 1 M NaCl, as for copolymers of NIPA and sodium acrylate.

The sorption capacity of the copolymer gel of acrylamide and sodium acrylate and that of nonionic NIPA gel and its ionic counterparts are compared in Table 5.10. Ionic gels were regenerated at 35°C in 1 M NaCl solutions whereas the NIPA gel was collapsed at 35°C in distilled water. Copolymer gels of acrylamide and sodium acrylate did not show a considerable volume change and therefore are not suitable for the process under consideration. Among the ionic copolymers of NIPA, the one which contained the strong acid monomer (R-SO_3^- H^+) is preferred because of its constant swelling in response to a wide range of pH change in the external solution – see Fig. 5.2. Although the nonionic NIPA gel did not swell as much as its ionic copolymer gels, it has the advantage of easy regeneration in distilled water. Based upon these results, most of the extraction experiments were performed using a NIPA gel and NIPA copolymer gels containing a strong acid

Table 5.10: Comparison of sorption capacity of polymeric gels in 10^{-3} M $Ca(NO_3)_2$ at 23°C (±2°C) and pH 7*.

Type of Gel	d_r/d_o	$(M_s - M_r)/M_d$	M_s/Mr
Copolymer of acrylamide+ and sodium acrylate (10.0%I)	2.2	29	1.5
N-isopropylacrylamide (NIPA)×	0.67	25	14
Copolymer of NIPA† and sodium acrylate (10.0%I)	0.63	42	15
Copolymer of NIPA‡ and $R\text{-}SO_3^-\ H^+$ (1.6%C, 5.0%I)	0.67	28	15

*Swelling time = 1 hr; Regeneration time = 1 hr

+ $\frac{M_s}{M_d} = 18.9$ × $\frac{M_s}{M_d} = 12.4$ † $\frac{M_s}{M_d} = 12.6$ ‡ $\frac{M_s}{M_d} = 11.8$

5.4 Pattern Formation

When ionic copolymer gels were immersed in distilled water, a peculiar regular pattern appeared on the surface of the swelling gel and changed in form and size with time. Similar behavior was observed by Hirokawa et al. (1985), Tanaka (1986), Tanaka et al. (1987) and Komori et al. (1988). Patterns evolve in the form of surface cells or cusps, usually of regular hexagonal or pentagonal shape which grow with time. As times goes on, the thickness of swollen layer increases, as does the wavelength of the pattern. The swollen gel layer on the gel surface undergoes buckling because the surface tries to expand, whereas the gel core acts to prevent expansion. For a gel slab with one free surface and the other surface covalently crosslinked to a film, pattern evolution eventually stopped but the patterns never disappeared (Tanaka, 1986). For a thin film of copolymer gel of acrylamide and sodium acrylate (5.3%T, 2.5%C, 10.0%I), prepared on the bottom of a beaker, buckling of the confined swollen layer on the surface of the swelling gel in cytochrome C solution was so severe it resulted in tearing of the gel, as shown in Fig. 5.10. There has been no report of such behavior upon constrained swelling.

Figure 5.10: Tearing of the swollen layer of gel film during confined swelling of copolymer gel of acrylamide and sodium acrylate in cytochrome C solution.

Elastic theories have been developed to describe the mechanical instability of gel surface during pattern formation (Sekimoto and Kawasaki, 1987 and 1988, Tanaka et al., 1987; Onuki, 1988) More recently, Onuki (1989) developed a mathematical model by incorporating non-linear elasticity into the Ginzburge-Landau theory of phase transition, to clarify the elastic structure of the pattern and conditions of its observation. The free energy was shown to be lowered below its value in the homogeneous state when the surface folds itself periodically. This framework resulted in a number of predictions which cannot be described by the usual elastic theory of isotropic bodies (Sekimoto and Kawasaki, 1989).

The process of shrinking of temperature sensitive gels is more com, licated than that of swelling Bulges were formed on the surface of the gel during the shrinking process This phenomenon was also observed by Matsuo and Tanaka (1988) who studied the kinetics of discontinuous volume phase transition of ionic and nonionic N-isopropylacrylamide gels. After some initial shrinking when the gel kept its original shape, shrinking stopped for a certain period of time called the plateau period. At the end of this period, bulges appeared on the surface of the gel and the gel resumed shrinking. Initially the bulges swelled as the gel shrunk, but eventually they stopped swelling, shrunk and finally disappeared as the gel approached its final equilibrium size. The process of shrinking was described as follows. At first, the gel shrunk only near the surface resulting in a layer of dense, collapsed polymer network which was impermeable to the inner fluid. This barrier temporarily prevented the gel from further shrinking. At this moment, the density of the polymer network and the swelling pressure inside the gel were nonhomogeneous. This nonhomogeneity relaxed during the plateau period, reforming the sphere into a dense, thin surface layer and a homogeneous dilute inner sphere When the wave front of the osmotic pressure reached the core of the gel, an outward osmotic pressure was exerted on the impermeable gel layer which was trying to shrink. This inner pressure blew up some portions of the dense surface layer resulting in bulge formation. These bulges were permeable to the fluid and hence the gel started to shrink again. The occurrence of this phenomenon was more pronounced for ionic copolymer gels of NIPA

5.5 Dynamic Swelling

5.5.1 General Results

Figure 5.11 shows the time dependence of the swelling ratio of copolymer gels of acrylamide and sodium acrylate (5.3%T, 2.5%C, 10.0%I) at different pHs. The swelling ratio, M_t/M_o, is defined as the ratio of swollen mass at any time, M_t, to the original mass of gel at preparation, M_o. The gels were swollen in fresh unstirred solutions of pH 4, 5.7, and 9. At pH 9 the swelling ratio increased monotonically from unity to the equilibrium value of 34. During swelling at lower pH gel volume went through a maximum followed by a gradual approach to equilibrium. Similar transient dimensional changes have been observed for a nonionic glassy hydrogel during drug release (Lee, 1983) and for anionic gels (Schosseler et al., 1987; Sakohara et al., 1990) An increase in pH of the external solution was measured in the initial stage of swelling indicating transfer of H^+ to the gel and release of Na^+. Ionic copolymer gels of acrylamide and sodium acrylate are almost completely ionized at gel formation, where $M_t/M_o = 1$ and the gel is approximately 95% water. As the gel swells Na^+ diffuses out and H^+ enters. The H^+ deionizes some of the carboxylate groups ($-COO^-$) except at pH 9. In response to this change in the number of charged groups on the gel network, the gel swells to a maximum and then reduces its volume by desorption of water in accordance with thermodynamic requirements discussed in Chapter 3.

The transport of small molecules in polymers is known to be affected by the mechanical history of the material. This effect can be manifested by variations of transport kinetics, the equilibrium solubility, dimensional change and deformation caused by stress during swelling (de Candia et al., 1980; Jameel et al , 1981, Holden et al., 1985). Early studies indicated that the behavior varies dramatically with the nature of the material. To check the effect of an external force on the swelling behavior, preswollen gel particles were subjected to a centrifugal force. Anionic copolymer gels of acrylamide and sodium acrylate (5.3%T, 2.5%C, 10.0%I) were first treated with acidic solution at pH 2 to deionize the ionizable monomers and then equilibrated with unstirred distilled water at pH 5.7. These gel particles were placed in cone-shaped centrifuge tubes and centrifuged for two hours at 15000 rpm in an Eppendorf centrifuge (model 5414). The swelling ratios during the 2 hours of centrifugation were measured by stopping the centrifuge, removing the gel particles, weighing them and

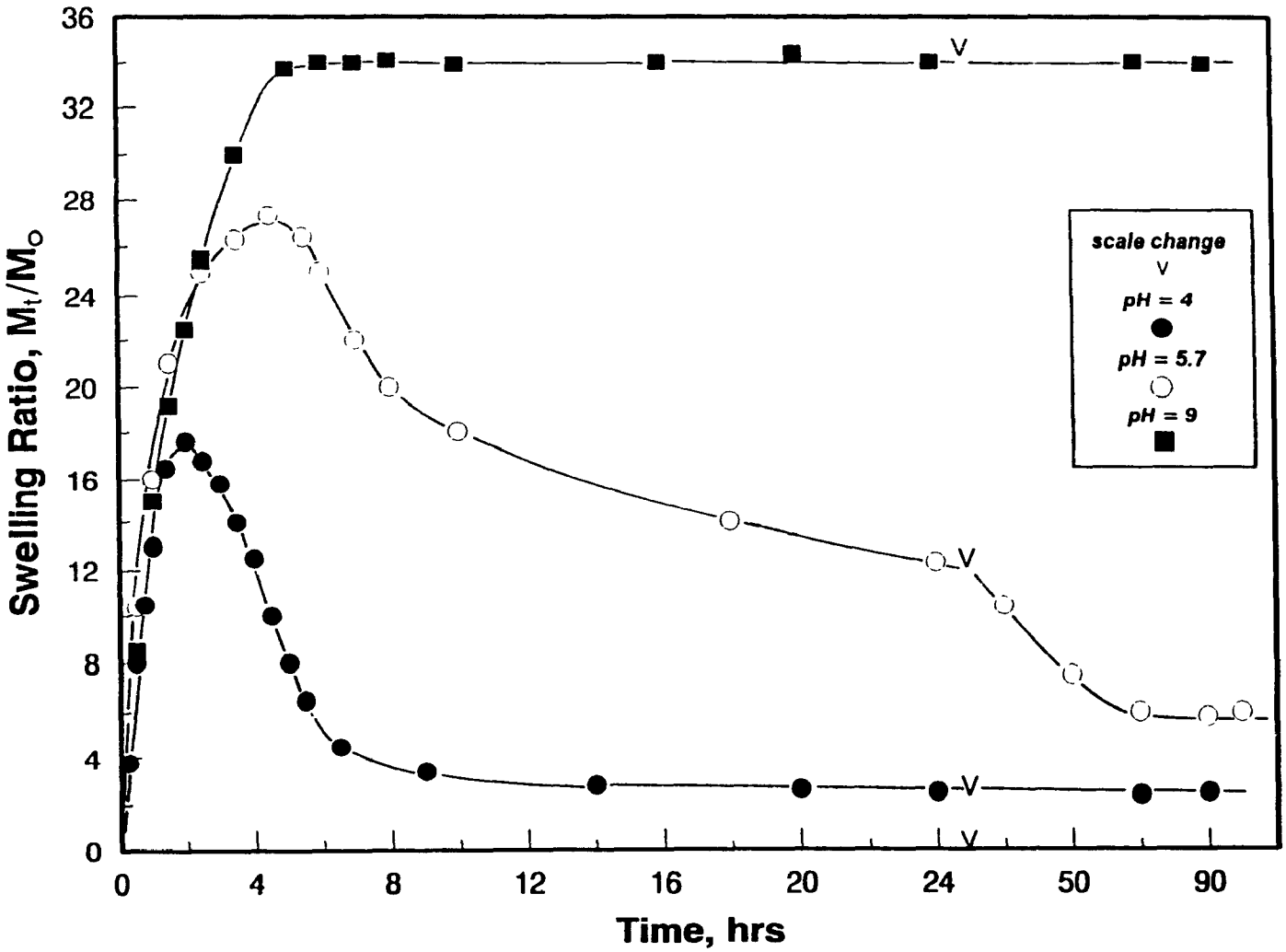


Figure 5.11: Volume change of copolymer gel of acrylamide and sodium acrylate with time for different pHs.

returning them to the centrifuge. After 2 hours the gels were swollen to equilibrium in unstirred solutions of pH 5.7. Figure 5.12 shows three successive cycles of collapsing/swelling for copolymer gels of acrylamide and sodium acrylate. In this figure, M_t is the equilibrium swollen mass of the hydrogel before application of centrifugal force in the first cycle. The time scale for the swelling step differs from that in collapsing step in Fig. 5.12. Gel collapsed under centrifugal force went through a maximum during swelling, which was about twice its former equilibrium value, followed by a gradual approach to a new equilibrium state lower than previous cycle. The reduction of equilibrium mass at each cycle was due to the loss of small fragments of the fragile gel particles especially in handling during centrifugation. This loss was observed clearly at the cut edges of cylindrical gel particles and was confirmed in experiments with gel particles which were collapsed for two hours without weighing. The loss of mass suggests that the data in Fig. 5.12 would be brought together by multiplying the swelling results of each cycle by the inverse of M_t/M_e at equilibrium swelling, i.e., multiplying each point of swelling step in the first cycle by 1/0.87, in the second cycle by 1/0.8 and in the third cycle by 1/0.7. These normalized results for each cycle collapse onto a single curve as shown in Fig. 5.13. Ionic and nonionic N-isopropylacrylamide gels did not exhibit such overshoot behavior, they reached the swelling equilibrium state monotonically.

For glassy polymers deformation caused reduced rate of sorption and diffusion when the direction of transport was normal to the chain orientation in some cases and enhanced response in others. This effect was demonstrated by methanol sorption in compressed poly(methyl methacrylate) (Li, 1984) and in uniaxially drawn poly(ether sulphone) (Chau et al., 1989) in which the transport of methanol was faster in the deformed material. Vrentas et al. (1984) observed an overshoot during the vapor sorption by an amorphous polymer, poly(ethyl methacrylate) at a temperature far above the glass transition temperature (rubbery state). They proposed that the overshoot in the sorption curves might be due to the structural rearrangements produced by relaxation of polymer chains. There has been no report of anomalous behavior during the swelling of predeformed hydrogels. A possible explanation for the results shown in Figs. 5.12 and 5.13 is polymer chain relaxation proposed by Vrentas et al. (1984).

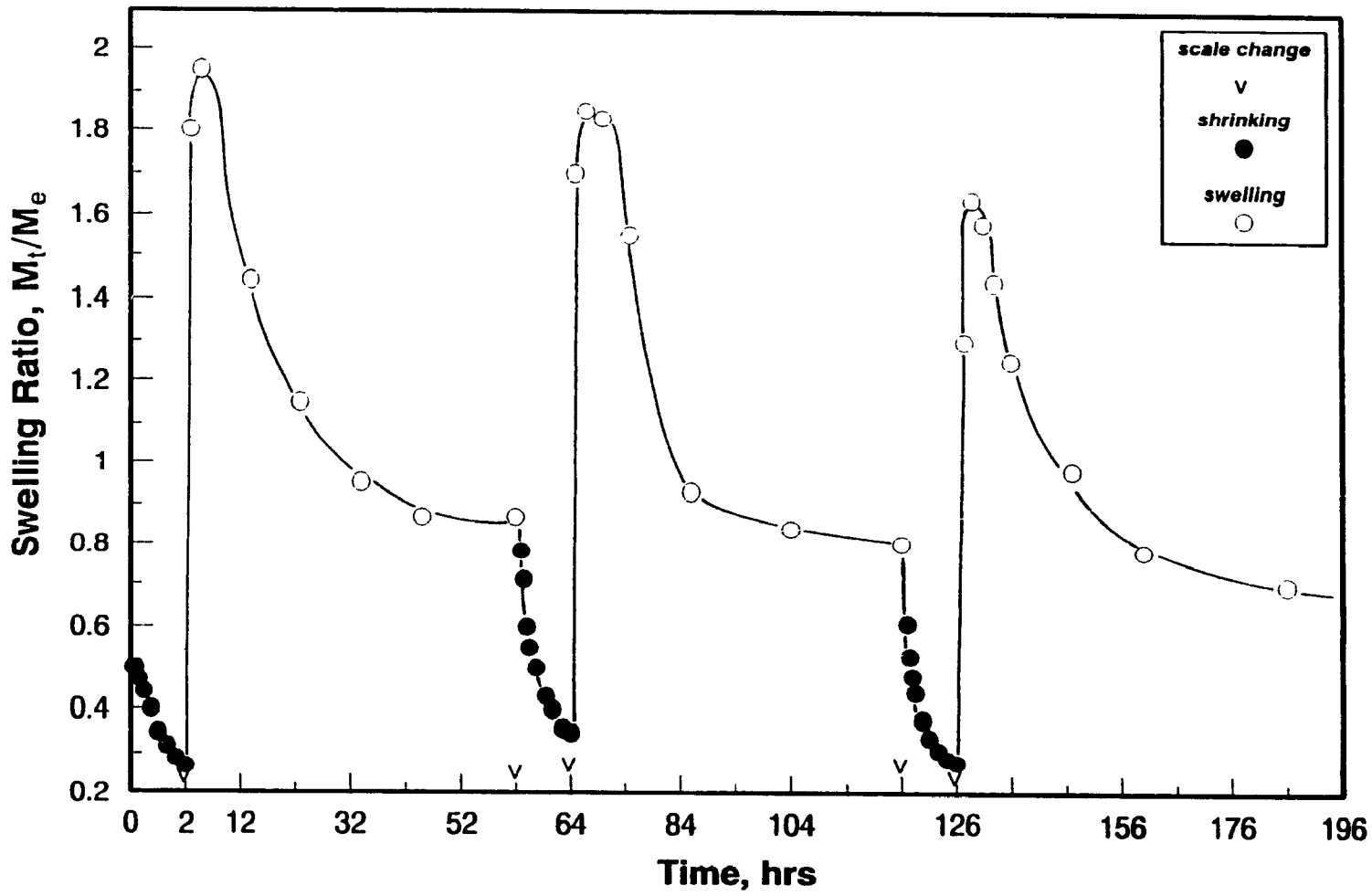


Figure 5.12: Volume change of copolymer gel of acrylamide and sodium acrylate with time during cyclic shrinking under centrifugal force and swelling in distilled water at pH 5.7.

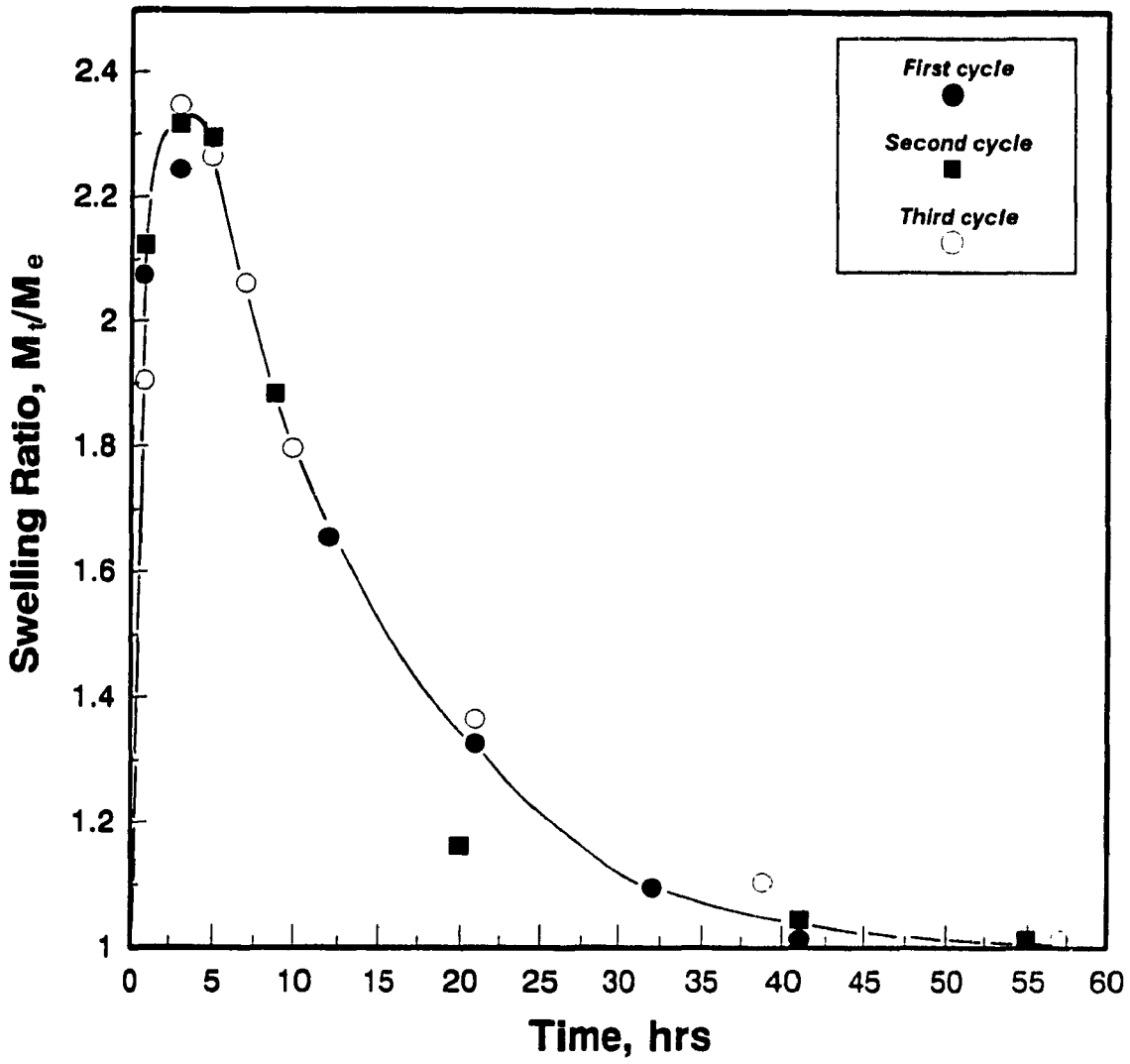


Figure 5.13: Volume change of copolymer gel of acrylamide and sodium acrylate in three successive cycles of swelling after partial collapsing under centrifugal force (normalized data).

Figure 5.14 shows the fractional approach to swelling equilibrium, F, versus the ratio of square root of time to gel diameter at t=0, $\sqrt{t}/d_0$, for deformed and undeformed N-isopropylacrylamide (NIPA) gels. Deformation was induced by centrifugal force on preswollen gels as described above. The fractional approach to swelling equilibrium, F, is defined as

$$F = \frac{M_t - M_{t=0}}{M - M_{t=0}} \tag{5.6}$$

where

M_t = swollen mass at time t

$M_{t=0}$ = mass of gel particle at time equal to zero

M = equilibrium swollen mass

As discussed in Chapter 4, a Fickian diffusion mechanism should produce a linear relationship up to $F \sim 0.5$. Both deformed and undeformed gels show Fickian behavior in Fig. 5.14, but the rate of water uptake is higher for the deformed gel. This may be due to increased physical crosslinking of the polymer chains or to cracking effects around the cut edges of the gel particles induced by applying centrifugal force during the collapsing step.

Figure 5.15 shows the fractional approach to maximum swelling VS $\sqrt{t}/d_0$ for deformed and undeformed copolymer gels of acrylamide and sodium acrylate. The ordinate was calculated from Eq. (5.6) using the maximum value of the swelling ratio in place of the equilibrium value. Although a single curve was generated, swelling was non-Fickian. The rate of swelling was the same for both deformed and undeformed gels. These results were analyzed according to Eq. (4.2). The diffusional exponent, n, was 0.48 and 0.63 for N-isopropylacrylamide gels and copolymer gels of acrylamide and sodium acrylate, respectively. Swelling was Fickian for NIPA gels and non-Fickian for copolymer gels of acrylamide and sodium acrylate. These results agree with those of Gehrke (Gehrke and Cussler, 1989; Gehrke et al., 1989).

Figures 5.16 and 5.17 show the effect of stirring on the swelling of anionic copolymer gels of acrylamide and NIPA in solutions of pH 5.7. In Fig. 5.16 the time scale changes after 22 hours while in Fig. 5.17 the scale changes after 4 hours. The

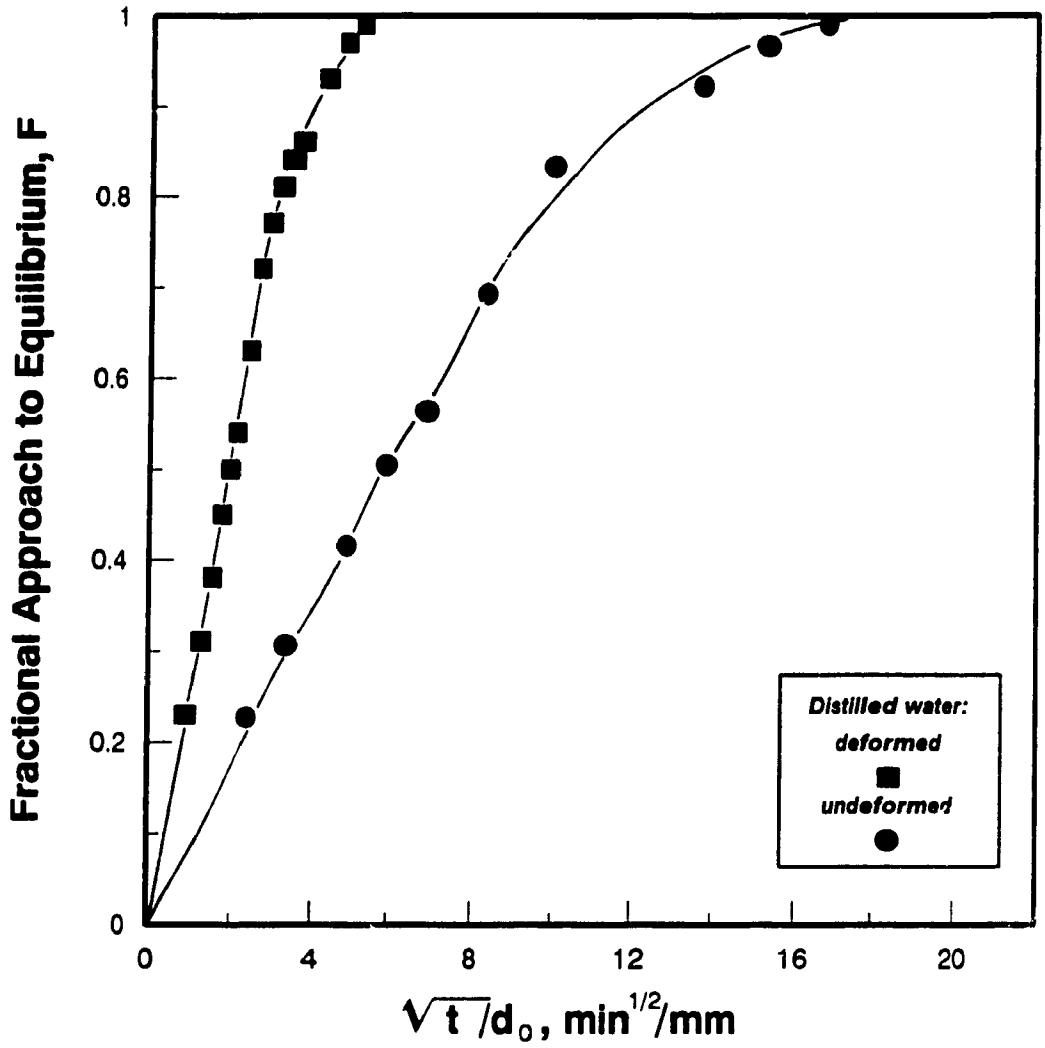


Figure 5.14: Fractional approach to equilibrium vs $\sqrt{t}/d_0$ for swelling of NIPA gels.

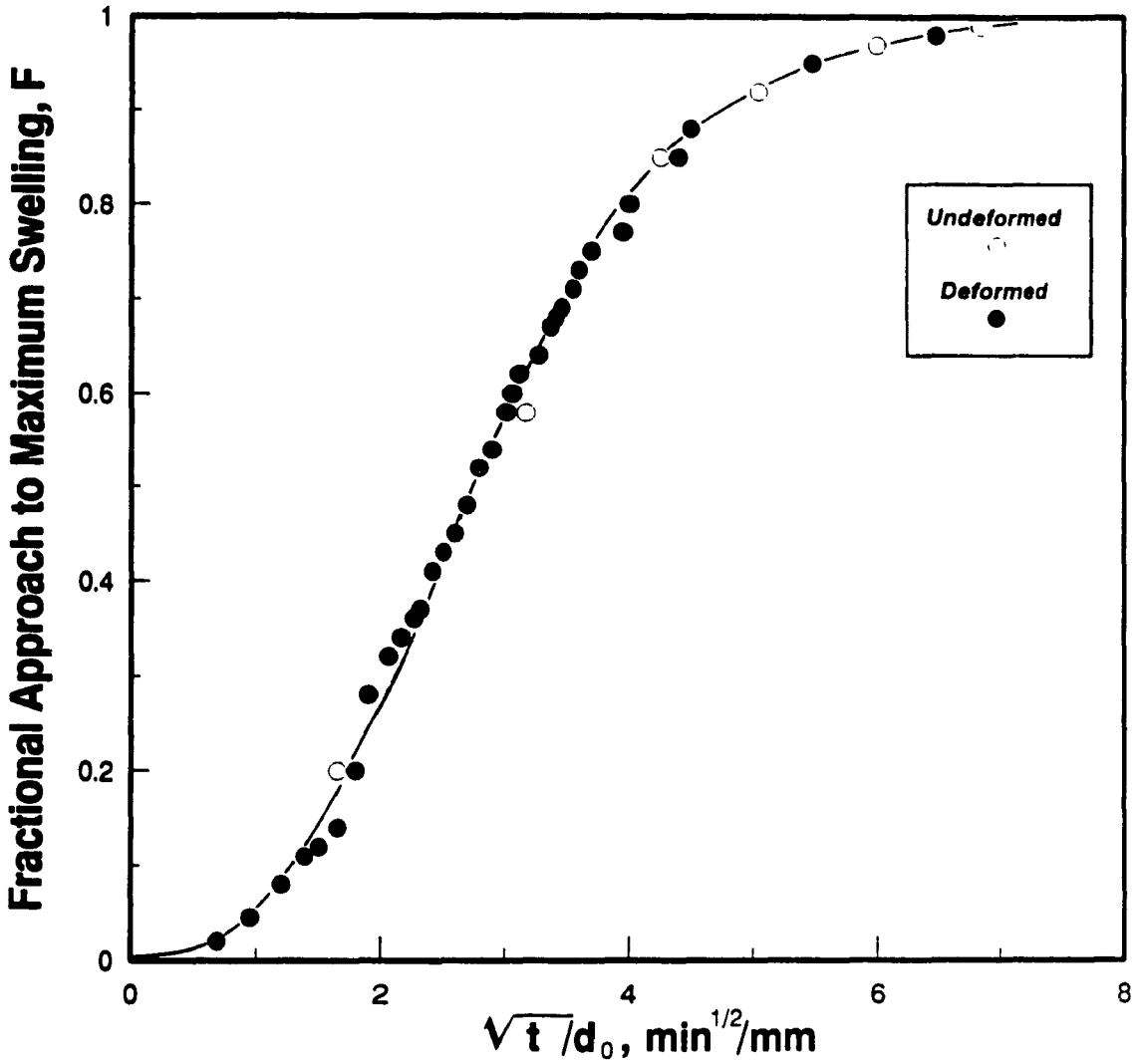


Figure 5.15: Fractional approach to maximum swelling vs $\sqrt{t}/d_0$ for swelling of copolymer gels of acrylamide and sodium acrylate in distilled water at pH 5.7.

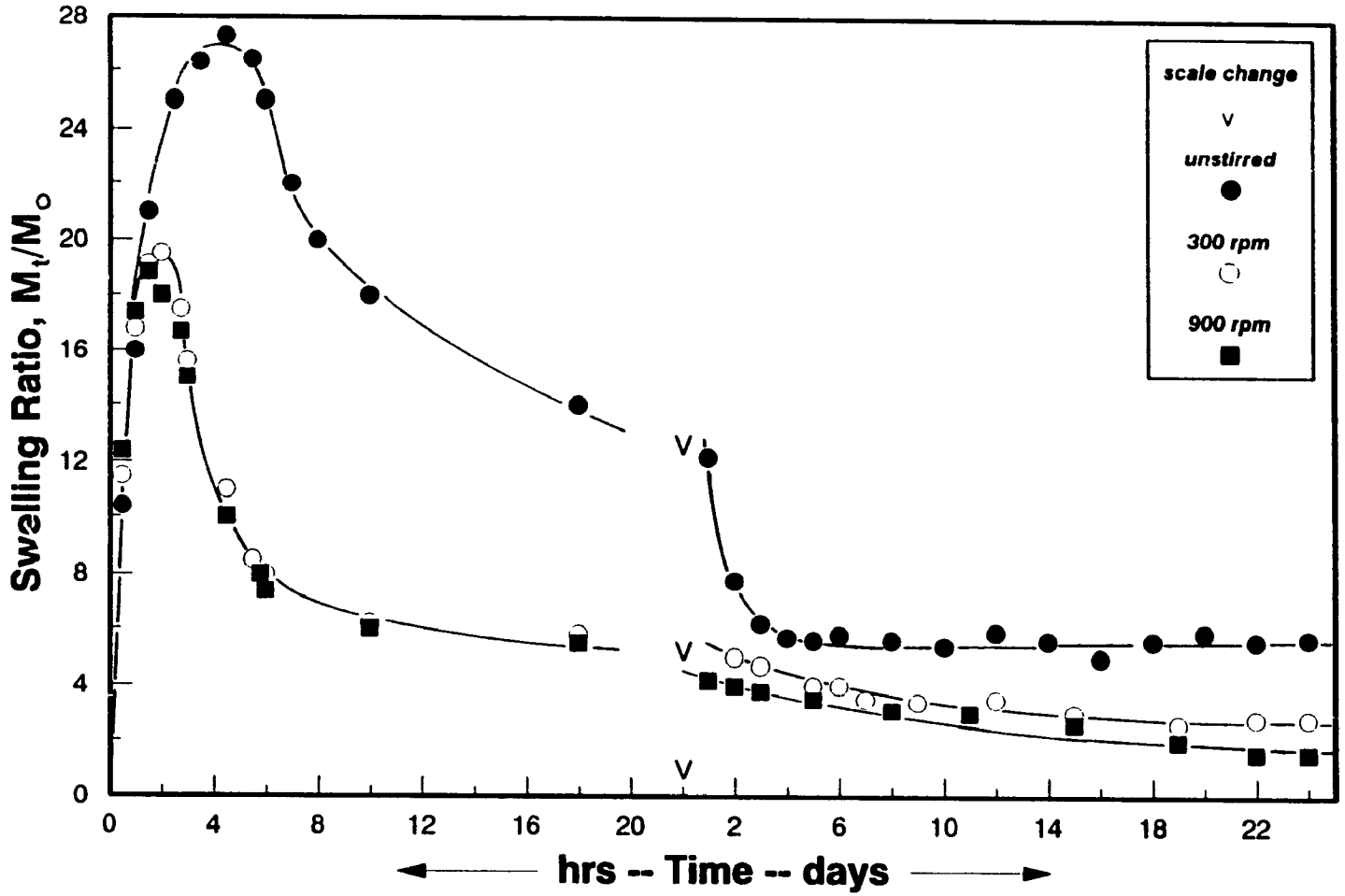


Figure 5.16: Volume change of copolymer gel of acrylamid and sodium acrylate with time for different rates of solution stirring at pH 5.7.

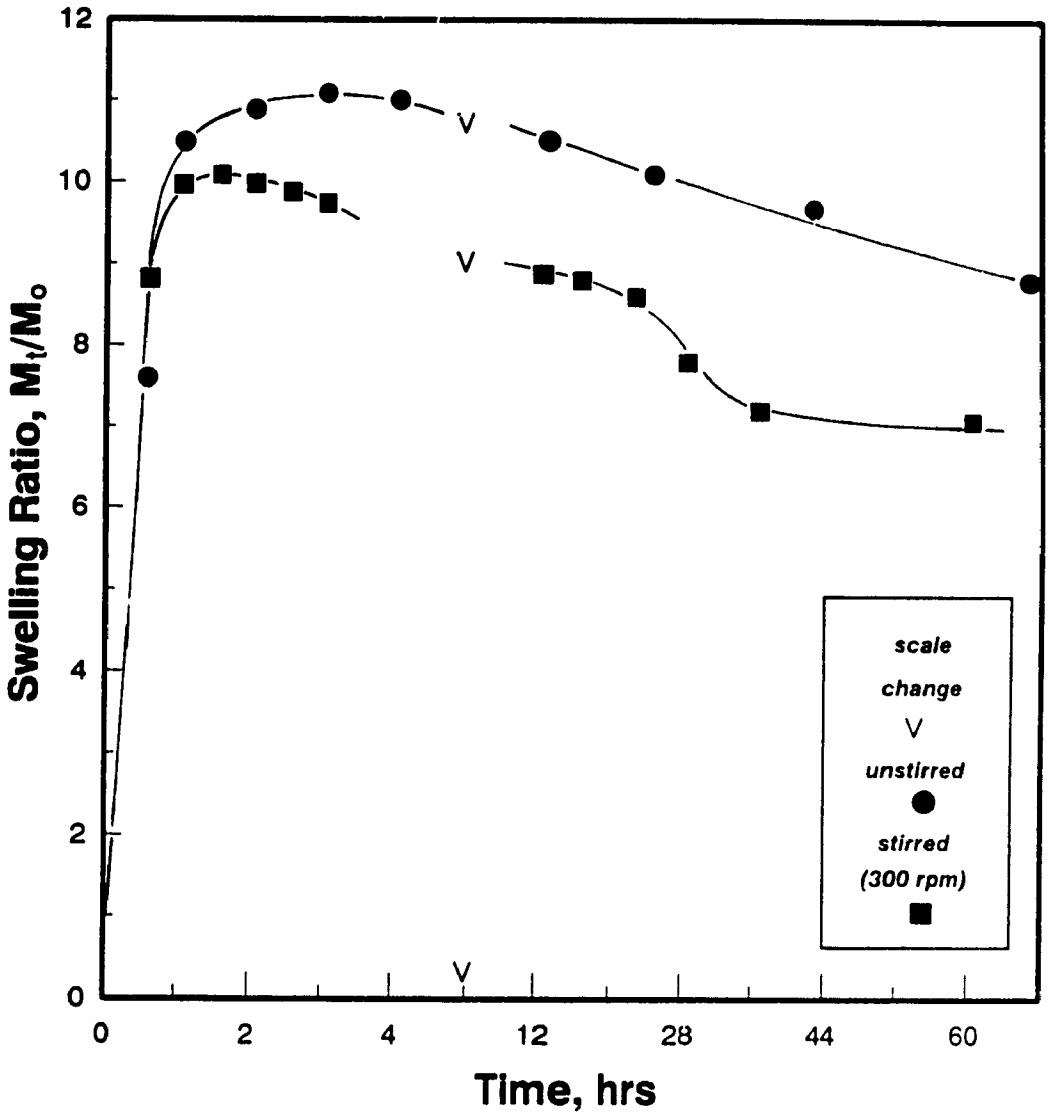


Figure 5.17: Volume change of copolymer gel of NIPA and R-SO_3H (10%I) with time in a solution of pH 5.7

gel particles were placed in containers made of nylon or stainless steel mesh which were suspended in the liquid to prevent collision of the particles with stirring bar or the vessel walls which might tear fragments from the gel. The maximum value of the swelling ratio in the unstirred solution was greater than that in the stirred solution, thus indicating that external film diffusion had some influence. Stirring reduces the external resistance to the transfer of mobile ions and impurities and unreacted monomers including ionic monomers from the gel into the solution resulting in a decrease of osmotic pressure difference between the gel and surrounding medium which in turn reduces the extent of maximum swelling. The differences between the 24 day swelling ratios in Fig. 5.16 may be due to the following:

(a) - small fragments of fragile gel particles were lost upon swelling in stirred solutions.

(b) - more unreacted ionic monomers were left in the network at the early stage of swelling in unstirred solution and they might react later resulting in higher amounts of ionizable groups in the gel.

(c) - bivalent ferrous ions originating from the stainless steel mesh might have some effect on the swelling ratio.

(d) - mechanical force resulting from stirring of the solution might induce some structural change in the gel.

The swelling ratio for preswollen gel particles equilibrated with stirred (900 rpm) solution was 4 in unstirred solution. Equilibrium was not achieved after 60 hr for swelling of copolymer gel of NIPA in unstirred solution, but it would approach equilibrium if swelling proceeded for more than 3 days.

Figure 5.18 shows time dependent swelling of ionic copolymer gels of NIPA in a finite volume of solution (250 mL with M_o = 0.1 g) with an initial pH of 5.7 at a stirring rate of 300 rpm. Anionic gels which contained weakly acidic monomer (sodium acrylate) exhibited a swelling maximum followed by a gradual approach to a minimum value and then a gradual increase to the equilibrium value. For these weakly acidic copolymer gels the pH increased from 5.7 to a final value of 6.4 after 60 hours. The cause of the overshoot or swelling maximum for these ionic gels was described earlier. There have been no reports of a swelling increase after an overshoot in the early stages of swelling as shown in Fig. 5.18. This behavior is associated with the ion-exchange nature of these pH-sensitive hydrogels. The final swelling increase

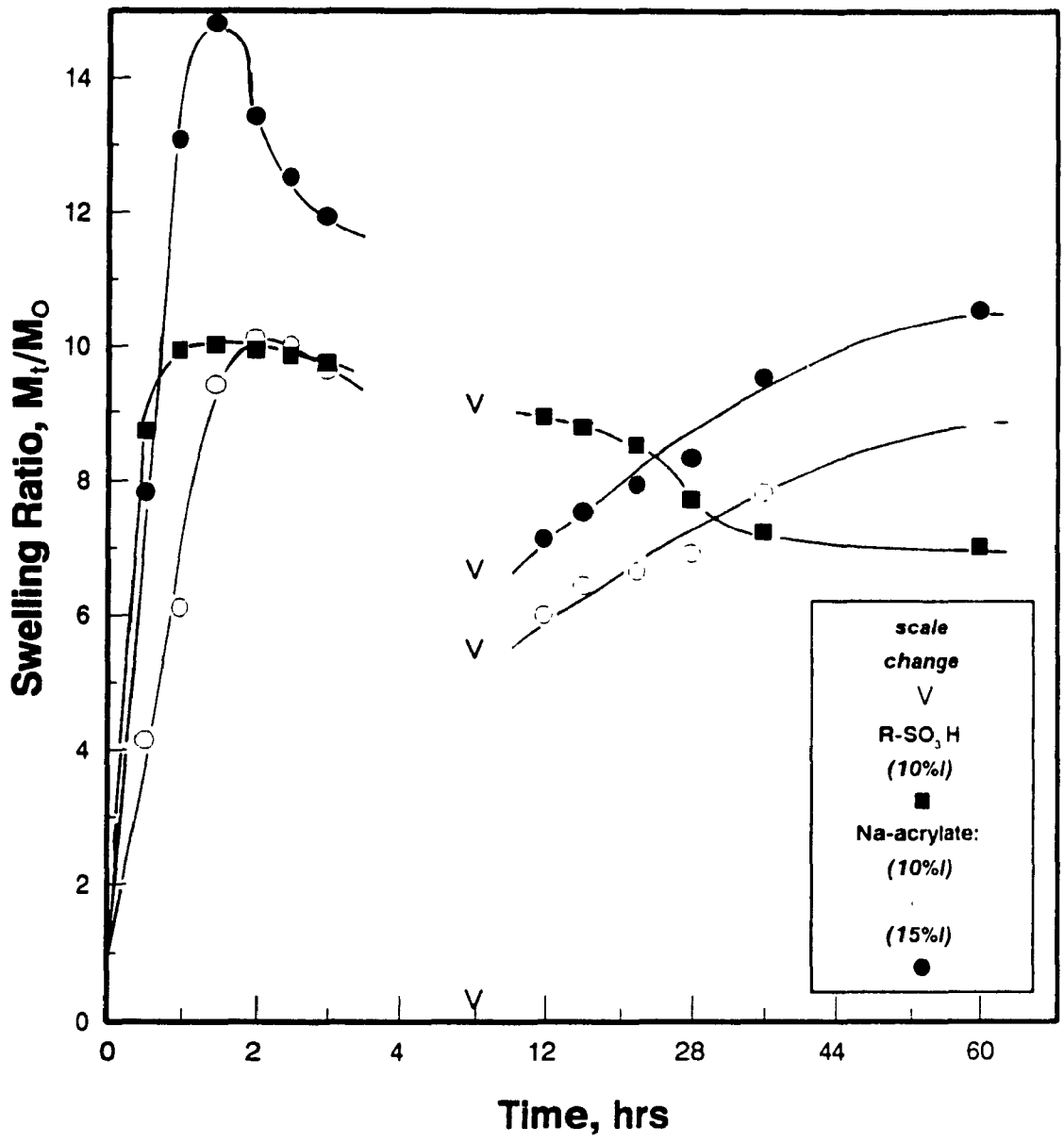


Figure 5.18: Volume change of ionic copolymer gels of NIPA in a solution of finite volume (250 mL) at 300 rpm stirring rate.

was mainly due to the increase of the pH of the external solution. For the highly acidic copolymer gel the maximum which was less pronounced than for the weak acid gel, could be caused by the diffusion out of the gel of mobile ions originated from initiator and accelerator, impurities and unreacted monomers. To test this hypothesis, highly acidic copolymer gel was swollen several times. The data are shown in Fig 5.19 in the form of swollen mass at time t, M_t, divided by the mass of dry polymer, M_d, versus time. The time scale changes after 12 hours The first cycle of swelling in 250 mL of distilled water at pH 5.7 shows an initial overshoot to $M_t/M_d = 120$ followed by a slow approach to an equilibrium value of $M/M_d = 80$. This gel was then dried in oven and reswollen in water at pH 5.7. In this second cycle the gel swelled to its previous equilibrium value without exhibiting the overshoot observed in the first cycle. This swollen gel was then collapsed in 2 M NaCl solution at 35°C When the collapsed gel was returned to the original solution it exhibited an overshoot in swelling similar to that in the first cycle. This overshoot resulted from the presence of NaCl which was left in the network during collapsing step, i.e., sodium and chlorine ions diffused out of the gel as swelling proceeded in the third cycle. Diffusion of these ions out of the gel, which resembles diffusion of mobile ions and ionic monomers in the first cycle, was accompanied by an overshoot in swelling in accordance with thermodynamic requirements discussed in Chapter 3.

Figure 5.20 shows time dependent swelling of ionic copolymer gels of acrylamide and ionic and nonionic NIPA gels in 10^{-3} M $Ca(NO_3)_2$ solution at pH 7 These gels were collapsed at different conditions Copolymer gels of acrylamide exhibited higher swelling equilibrium than NIPA gels and the equilibrium swelling increased as the amount of ionizable monomer increased as described by the thermodynamic treatment in Chapter 3 These experimental results are replotted in Fig 5.21 as M_t/M_r where M_r is the regenerated (collapsed) mass of the gel. NIPA gels exhibited larger volume changes from the collapsed state upon swelling than acrylamide gels, although the latter exhibited larger equilibrium swelling ratios, M/M_d. When the data from Fig. 5.21 are plotted in reduced Fickian form in Fig 5.22, essentially a single curve is generated for each group of gels. This plot also includes the swelling of copolymer gels of acrylamide and sodium acrylate at pH 9 The data for ionic and nonionic NIPA gels are well represented by a straight line over the first 70% of the volume change, thus indicating Fickian behavior The swelling of copolymer gels of

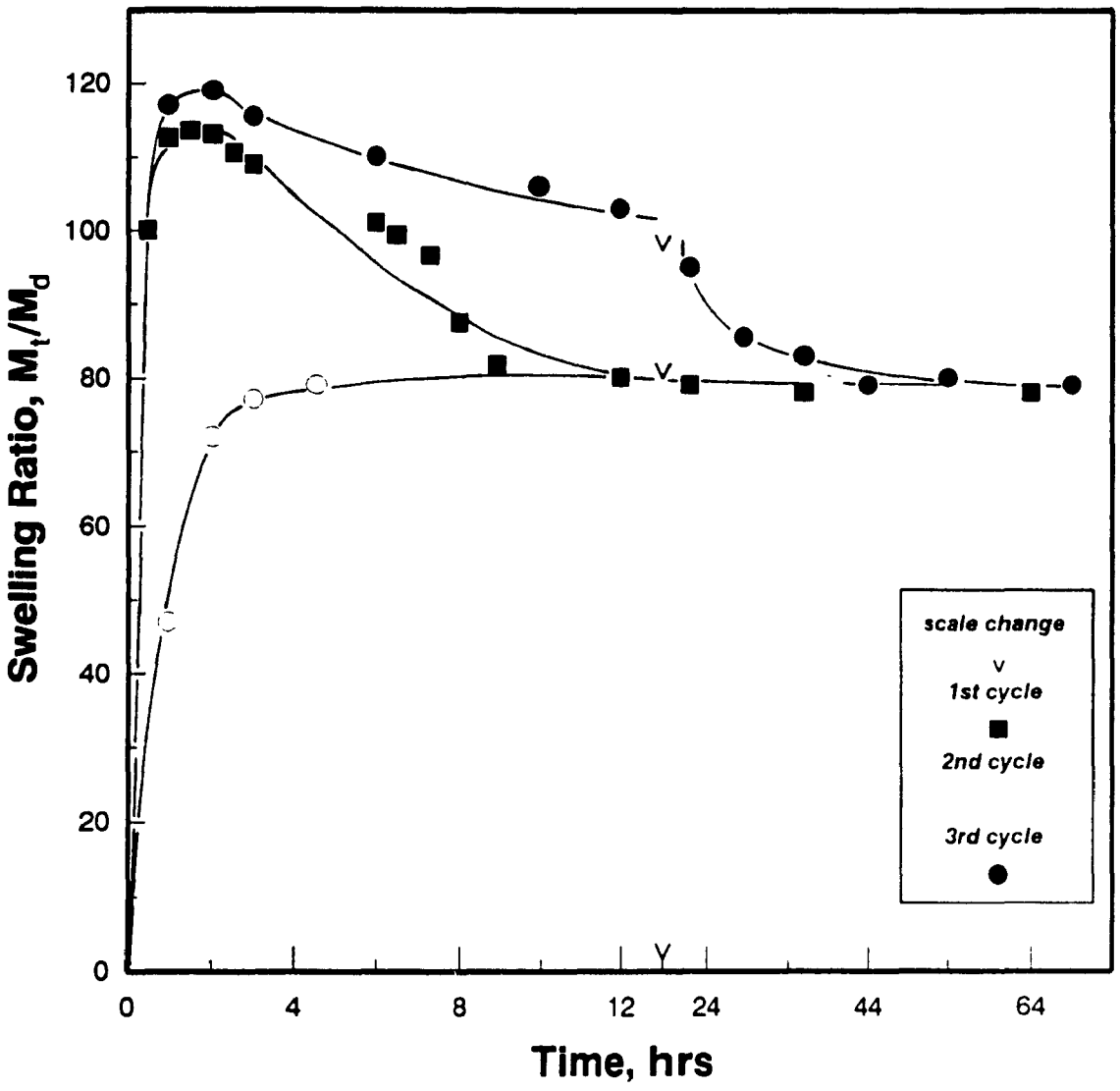


Figure 5.19: Volume change of ionic copolymer gel of NIPA and R-SO_3H (10%I) with time in a solution of pH 5.7 at 300 rpm stirring rate.

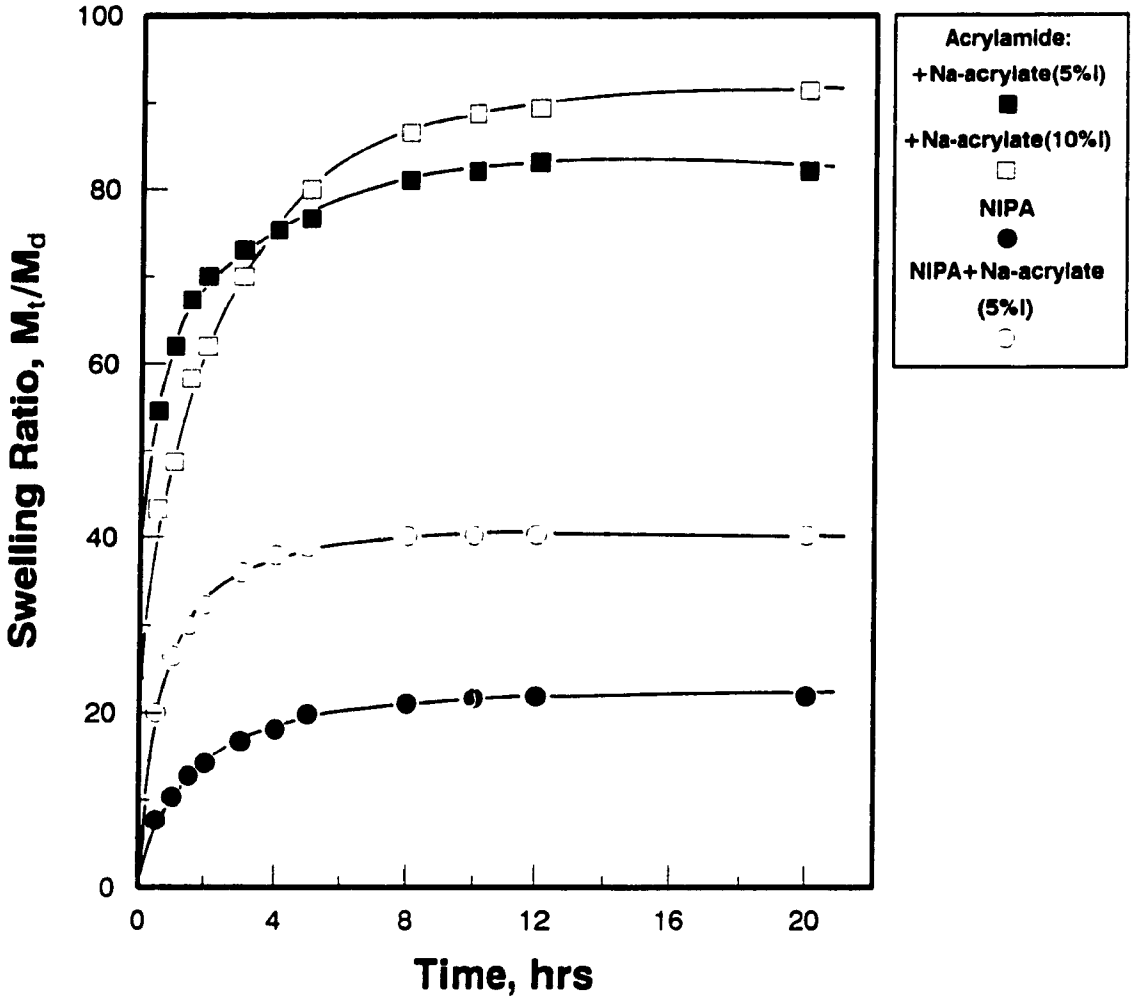


Figure 5.20: Volume change of hydrogels with time, after regeneration in different media, in 0.001 M $Ca(NO_3)_2$ solution at pH 7.

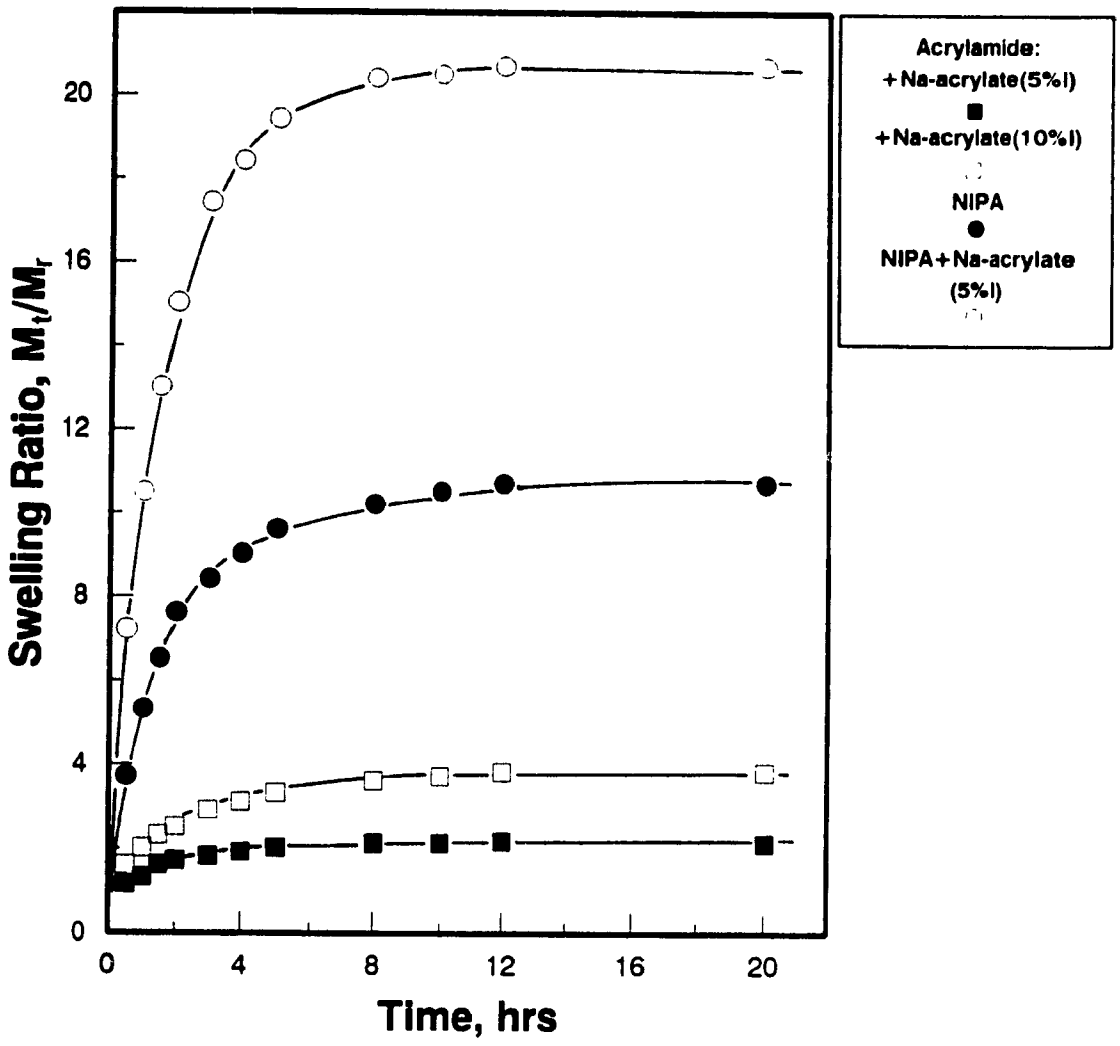


Figure 5.21: Volume change of hydrogels with time from collapsed state during swelling in 0.001 M $Ca(NO_3)_2$ solution at pH 7.

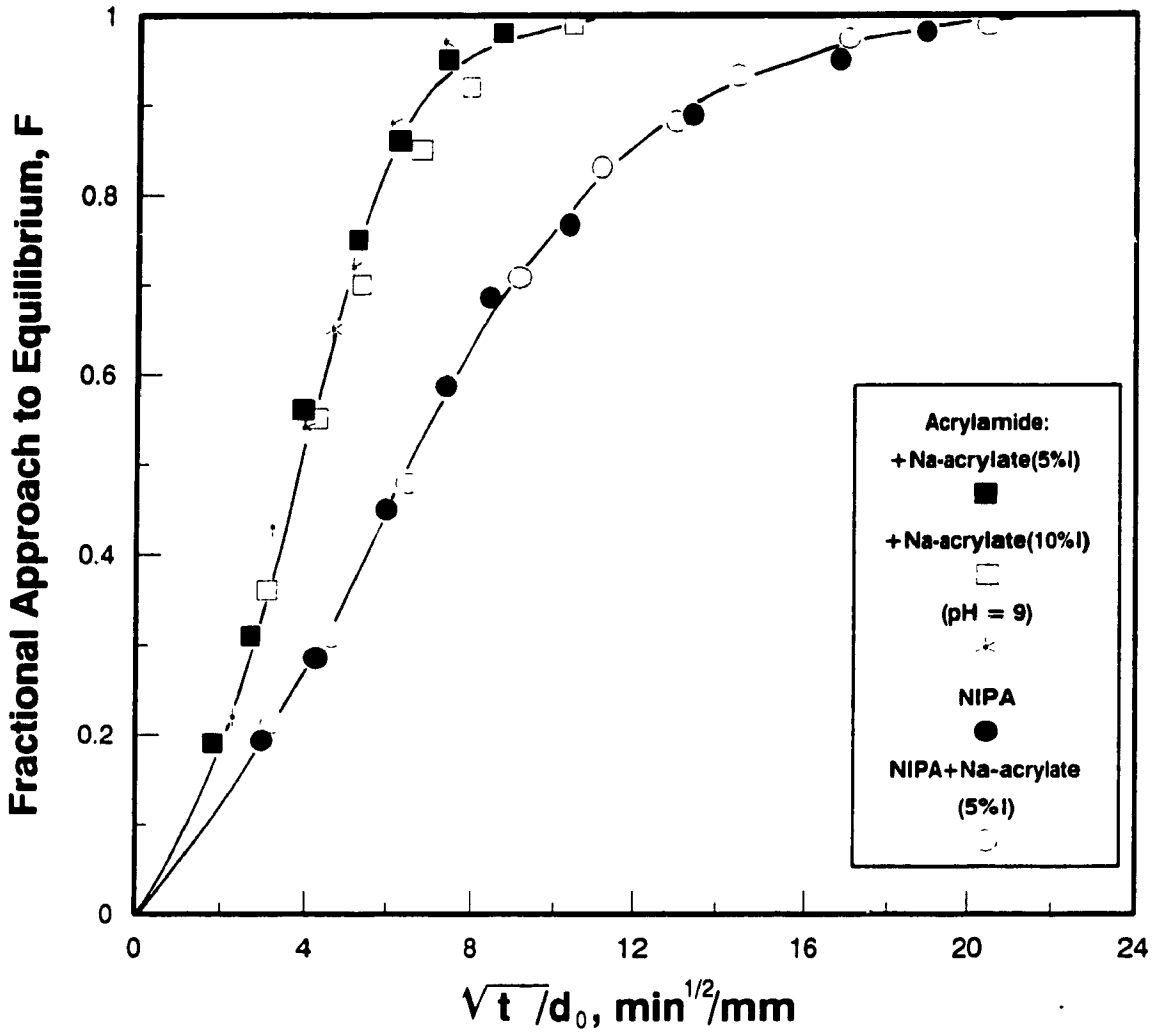


Figure 5.22: Fractional approach to equilibrium vs $\sqrt{t}/d_0$ for swelling of acrylamide and NIPA hydrogels in 0.001 M $Ca(NO_3)_2$ at pH 7.

acrylamide was non–Fickian but faster than that of NIPA gels. The diffusional exponents in Eq. (4.2) were 0.51 and 0.75 for NIPA and acrylamide gels, respectively. Although the rate of water uptake for copolymer gels of acrylamide was faster than that of NIPA gels, they were not easily regenerated. For the water removal process under consideration, the nonionic NIPA gel, which requires mild conditions for collapse (T = 35°C in distilled water), is more promising.

Figure 5.23 shows the fractional approach to equilibrium, F, VS $\sqrt{t}/d_0$ for the swelling/collapse cycle of NIPA gel. Swelling and collapse were carried out in distilled water at 23°C (±2°C) and 35°C, respectively. Swelling was Fickian whereas collapse was non–Fickian and the rate of swelling was slower than the rate of collapse. Similar behavior for this gel was observed by Gehrke et al. (1989).

Figure 5.24 shows three consecutive swelling cycles for a copolymer gel of N,N–diethylacrylamide and sodium methacrylate in distilled water at 23°C and pH 5.7. Gel particles after preparation and swollen gel particles at each cycle were dried in an oven at 70°C. The rate of gel volume change decreased with each cycle. Diffusion of impurities and minute amounts of unreacted monomers plus deionization of carboxylic ($-COO^-$) groups at pH < 7 were responsible for such behavior. The data of Fig. 5.24 are plotted in pseudo–Fickian form in Fig. 5.25. Here the maximum swelling ratio was used in place of the equilibrium value. The data for all cycles collapse onto a single curve. The sigmodial shape of the curve is evidence of non–Fickian behavior. This behavior is expected because the polymer is glassy when dried.

5.5.2 Comparison of Theory with Experiment

The mathematical model for the kinetics of gel swelling presented in Chapter 4 is compared with experimental results for an ionic copolymer gel of acrylamide and sodium acrylate (5.3%T, 2.5%C, 10.0%I) swollen at pH 7 in distilled water, for the collapsed NIPA gels swollen at 23°C (±2°C) in 10^{-3} M $Ca(NO_3)_2$ solution of pH 7 and for deformed and undeformed NIPA gels swollen at 23°C (±2°C) in distilled water at pH 5.7. Cylindrical particles of diameter 0.24 cm at gel formation having a length about equal to their diameter were used. The radius of the volume–equivalent sphere was used to compare the data to the theory.

The model is also compared with experimental results of Gehrke (1986) for swelling (pH = 9) and collapsing (pH = 4) of cylindrical copolymer gels of acrylamide

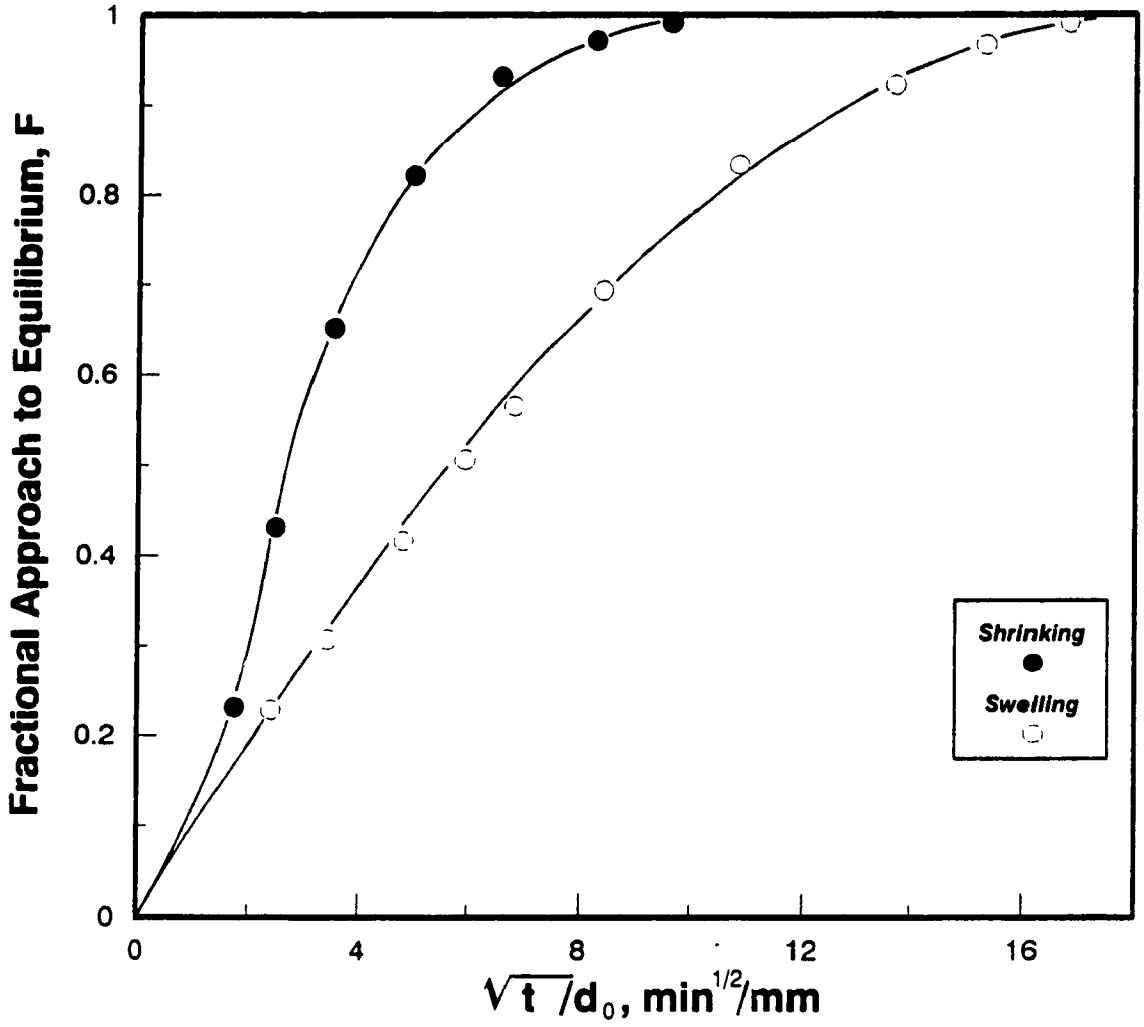


Figure 5.23: Fractional approach to equilibrium vs $\sqrt{t}/d_0$ for swelling/collapsing cycle of NIPA gel in distilled water.

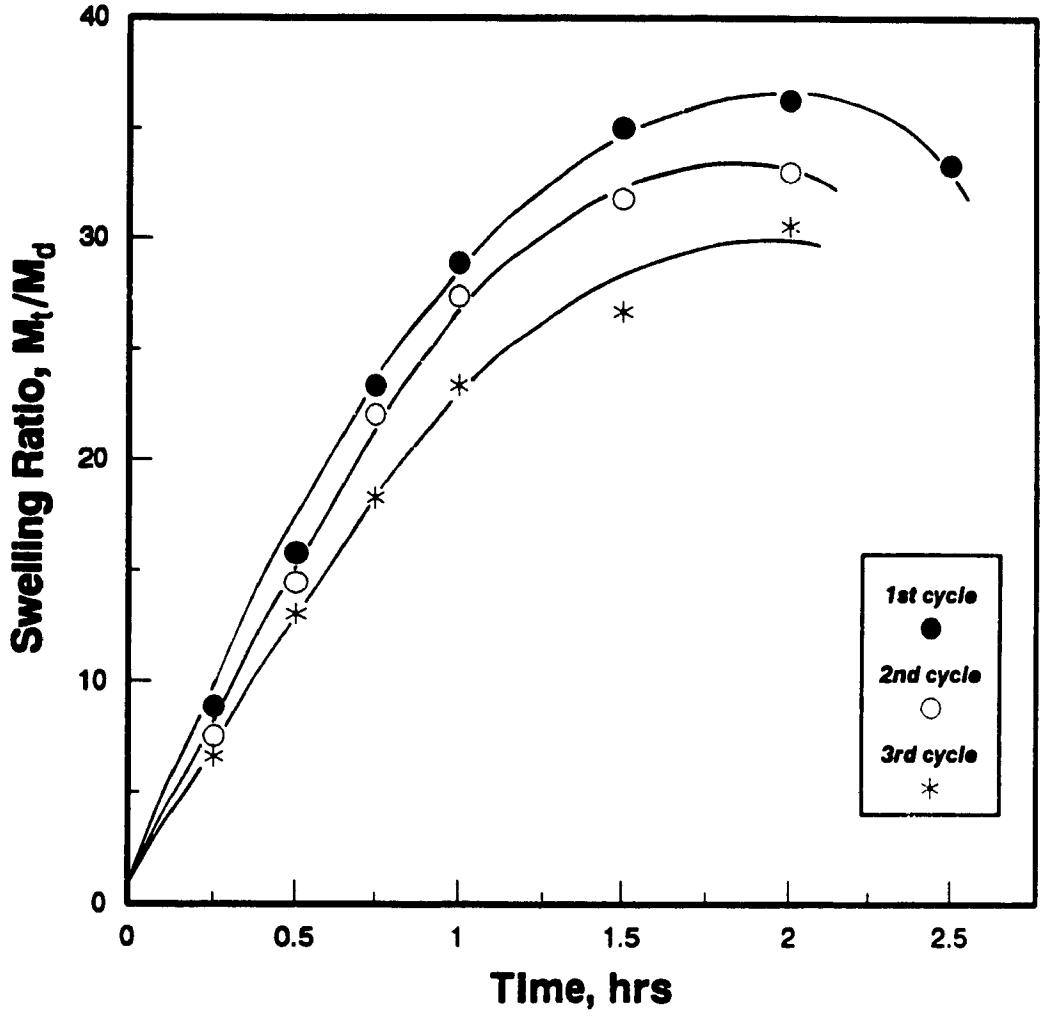


Figure 5.24: Swelling of copolymer gel of N,N-diethylacrylamide and sodium methacrylate at 23 °C in distilled water at pH 5.7.

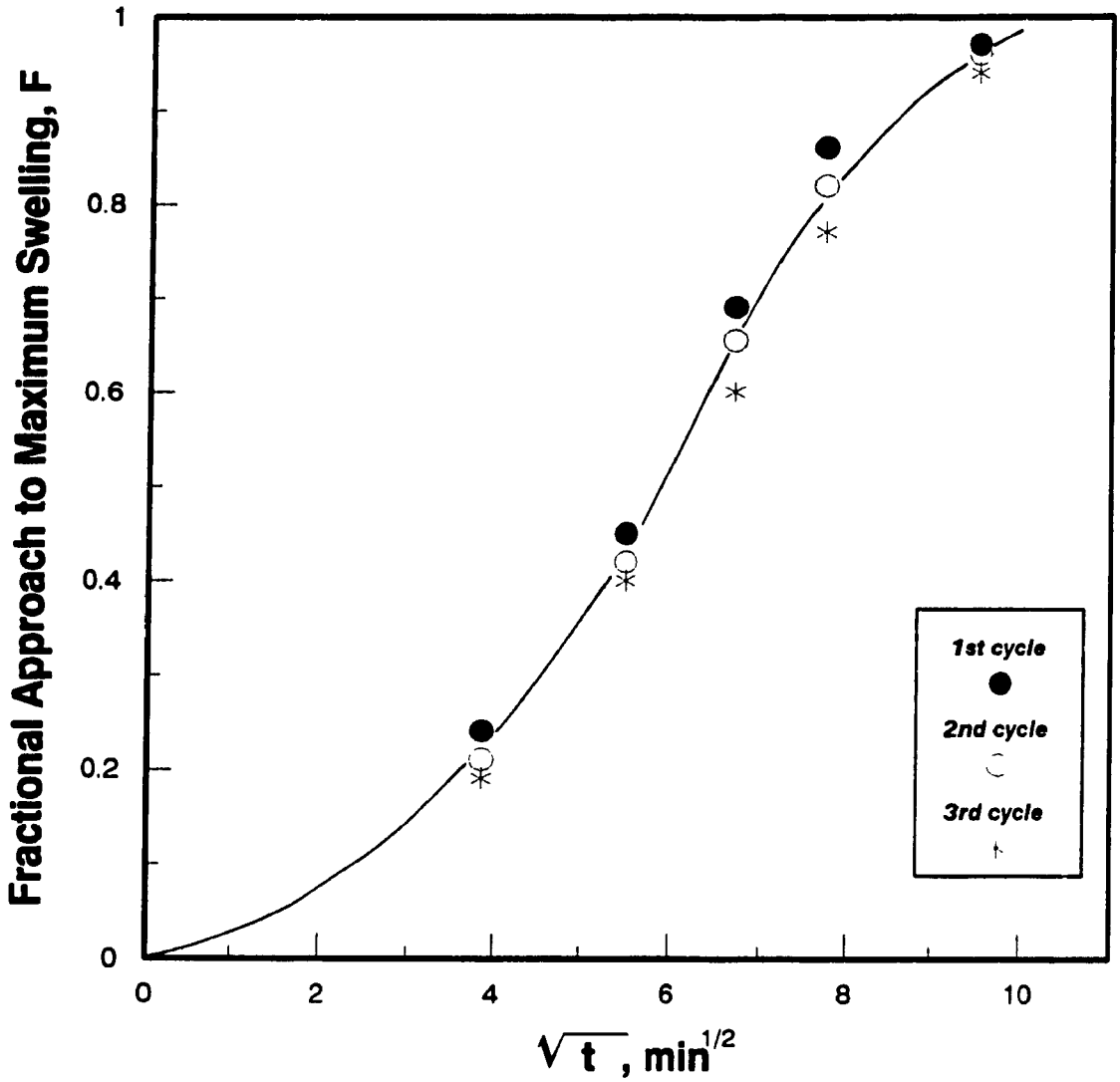


Figure 5.25: Fractional approach to maximum swelling of glassy copolymer of N,N-diethylacrylamide and sodium methacrylate in distilled water at pH 5.7.

and sodium methacrylate and with experimental results of Gehrke et al. (1989) for swelling of planar NIPA gel (16%T, 1%C) at 5°C in distilled water. The diameter of copolymer gel (16%T, 4%C, 2.4%I) used for the swelling experiment was 0.628 cm at preparation. The swollen diameter of copolymer gel (16%T, 4%C, 0.5%I) at the start of collapsing experiments was 0.468 cm. The thickness of the planar NIPA gel was 0.125 cm at preparation.

Formulation of Diffusion Coefficient

Consider the governing equation for solvent transport in planar (i=0), cylindrical (i=1) and spherical (i=2) geometries, given by Eq. (4.33), which is repeated here:

$$\frac{\partial \vartheta}{\partial t} = \frac{1}{x^i}\frac{\partial}{\partial x}(Dx^i\frac{\partial \vartheta}{\partial x}) \tag{5.7}$$

where D is the diffusion coefficient in material coordinates given by

$$D = -\frac{D_T V_1}{RT}(1+\vartheta)^{\frac{i+1}{i+1}}(\frac{\partial \pi}{\partial \vartheta}) \tag{5.8}$$

The swelling pressure, π, can be related to the solvent ratio, ϑ, through the swelling ratio, X, as follows

$$X = \frac{\vartheta + \rho_p/\rho_s}{\vartheta_0 + \rho_p/\rho_s} \tag{5.9}$$

where

ϑ_0 = solvent ratio at t=0
ρ_p = polymer density ($\sim$ 1.16 g/cm^3)
ρ_s = density of water (1 g/cm^3)

The expressions for swelling pressure given in the thermodynamic treatment of gel swelling in Chapter 3 must be used to differentiate swelling pressure, π, with respect to solvent ratio, ϑ, in Eq. (5.8). To simplify this differentiation , Eq. (5.9) can be well approximated for $1 < \rho_p/\rho_s < 1.2$ as

$$X \simeq \frac{\vartheta + 1}{\vartheta_0 + 1} \tag{5.10}$$

Since the parameters in these expressions were determined only for ionic copolymer gels of acrylamide in Section 5.3, the discussion of diffusion coefficient associated with swelling pressure is limited to copolymer gels of acrylamide and sodium acrylate (5.3%T, 2.5%C, 10.0%I). The osmotic pressure of distilled water (external solution) at pH 7 is negligible compared with that of gel phase and hence the ionic swelling pressure, π_3, from Eq. (3.9) can be written as

$$\pi_3 = RT(\Phi \Sigma_i \overline{C_i}) = RT(\phi_p n_m \alpha) \tag{5.11}$$

Substitution of parameters, obtained in Section 5.3, and Eq. (5.10) into Eqs. (3.4), (3.8) and (5.11) and taking the derivative of the resulting equations with respect to ϑ yield

$$\begin{aligned} D \;=\; & D_T[1.745 \times 10^{-13}(1+\vartheta)^2 + 1.137 \times 10^{-10}(1+\vartheta) \\ & -3.974 \times 10^{-5} + 1.977 \times 10^{-2}(1+\vartheta)^{-5/3} \\ & +0.12(1+\vartheta)^{-8/3} - 0.7(1+\vartheta)^{-11/3} + 5.209 \times 10^{-8}] \end{aligned} \tag{5.12}$$

Evaluation of Eq. (5.12) shows that it can be well approximated by

$$D = 2.918 \times 10^{-2} D_T (1+\vartheta)^{-1.747} \tag{5.13}$$

Since the mobility coefficient, D_T, is an increasing function of solvent content in the network, the increase of solvent ratio has two opposite effects on the diffusion coefficient, D. The chemical potential gradient decreases with increasing the solvent ratio whereas the mobility coefficient increases with increasing solvent ratio. It is also known that the rate of swelling or collapse of polymeric networks is controlled by the cooperative diffusion coefficient, D_c, which is in general different from the tracer or self-diffusion coefficient, D_t. These coefficients coincide only in very dilute solutions (de Gennes, 1979).

To test the scaling theory on the concentration dependent diffusion coefficient of swollen networks, Munch et al. (1977) performed several experiments for different systems as described below:

(i) polystyrene networks swollen by benzene for which they obtained

$$D_c = (6.0 \pm 0.3) \times 10^{-6} C_e^{0.68 \pm 0.01} \quad cm^2/s \tag{5.14}$$

(ii) polystyrene networks swollen by ethylacetate,

$$D_c = 2 \times 10^{-6} C_e^{0.66} \quad cm^2/s \tag{5.15}$$

(iii) polydimethylsiloxane swollen by toluene,

$$D_c = (7.0 \pm 0.8) \times 10^{-6} C_e^{0.77 \pm 0.03} \quad cm^2/s \tag{5.16}$$

where C_e (g/cm^3) is the polymer concentration at equilibrium swelling at a given temperature. This concentration can be controlled by changing the crosslinking density of the network. Takebe et al. (1989) investigated the cooperative diffusion coefficient of polyacrylamide gels in water by quasielastic light scattering both in the isotropically swollen state and in the uniaxially stretched and swollen state. For isotropically swollen gels the results yielded

$$D_c = (3.4 \pm 0.5) \times 10^{-6} C_e^{0.76 \pm 0.03} \quad cm^2/s \tag{5.17}$$

The diffusion coefficient of sodium polyacrylate network, with different amounts of crosslinking agent, reported by Komori et al. (1988) may be correlated by

$$D_c = 6.386 \times 10^{-5} C_e^{0.75} \quad cm^2/s \tag{5.18}$$

These results are all in reasonable agreement with the scaling theory of Gennes (1979)

$$D_c \propto C_e^{3/4} \tag{5.19}$$

Based on the observed diffusional behavior of polymeric networks, the following expressions for the diffusion coefficient in material coordinates, D, were used to fit the experimental results.

(i) - A constant diffusion coefficient, i.e., one parameter:

$$D = a \tag{5.20}$$

(ii) - A concentration dependent diffusion coefficient having two parameters (a and b):

$$D = a(1 + \vartheta)^b \tag{5.21}$$

(iii) - A concentration dependent diffusion coefficient having three parameters (a, b and c):

$$D = a(1 + \vartheta)^{-17/12} exp[b(1 + \vartheta)^c(\frac{\vartheta}{\vartheta_e} - 1)] \tag{5.22}$$

The concentration dependent diffusion coefficients in PMC were only used for copolymer gels of acrylamide and sodium acrylate. The exponential term in Eq. (5.22) vanishes at equilibrium swelling, i.e., $\vartheta = \vartheta_e$, yielding an expression for the cooperative diffusion coefficient (Laboratory coordinates), D_c, through multiplying D by $(1 + \vartheta_e)^{2/3}$ using Eq. (4.11) in agreement with scaling theory. The unknown parameters, a,b and c in Eqs. (5.20)-(5.22) were determined from the experimental results.

Solution Method

The optimum value of the parameters a, b and c were obtained through minimization of an objective function, G, which is the square of the relative deviation between experiment and theory:

$$G = \sum_{i}^{NP}[(F_{exp}(t) - F(t))/F_{exp}(t)]^2 \tag{5.23}$$

where $F_{exp}(t)$ is determined through Eq. (5.10) using experimental values of swelling ratio, $X = M_t/M_{t=0}$, at each time; $F(t)$ is theoretical value of the fractional approach to equilibrium given by Eq. (5.28) and NP is the number of experimental points. For every set of parameters, the governing equation was solved by analytical (constant D) or numerical method. Direct search techniques with optimization programs from the IMSL library, called DBCPOL and DUMPOL, were used to minimize the objective function. The DBCPOL program minimizes a function of N variables subject to bounds on the variables using a direct search complex algorithm. DUMPOL minimizes a function of N variables using an unconstrained direct search algorithm.

One measure of the goodness of fit of the theoretical model to the experimental points is the correlation coefficient, r_c, defined as

$$r_c = \sqrt{1 - \frac{\sum_i^{NP}[F_{exp}(t) - F(t)]^2}{\sum_i^{NP}[F_{exp}(t) - F_{av}]^2}} \tag{5.24}$$

where F_{av} is the average value of the experimental points.

The analytical solution to Eq. (5.7) with a constant diffusion coefficient, D, is given by Crank (1975) as:

For a slab,

$$F(t) = 1 - \sum_{n=0}^{\infty} \frac{8}{(2n+1)^2\pi^2} exp\{-D(2n+1)^2\pi^2 t/(4x_0^2)\} \tag{5.25}$$

for a long cylinder,

$$F(t) = 1 - 4\sum_{n=1}^{\infty} \frac{1}{\beta_n^2} exp(-\beta_n^2 Dt/r_0^2) \tag{5.26}$$

where β_ns are the positive roots of the zero order Bessel function of the first kind, and for a sphere

$$F(t) = 1 - \frac{6}{\pi^2}\sum_{n=1}^{\infty} \frac{1}{n^2} exp(-n^2\pi^2 Dt/x_0^2) \tag{5.27}$$

where

$$F(t) = \frac{\vartheta_{Theory}(t) - \vartheta_0}{\vartheta_e - \vartheta_0} \tag{5.28}$$

and ϑ_0 and ϑ_e are the initial and the equilibrium value of the solvent ratio, respectively, and x_0 is the radius of a cylinder or sphere or the half-thickness of a slab in material coordinates. For a concentration dependent diffusion coefficient, $D = f(\vartheta)$, the nonlinear partial differential equation, [Eq. (5.7)], was solved numerically using an implicit finite difference method as described in Appendix B. The numerical solution gave the spatial distribution of solvent ratio at a specific time, $\vartheta(x,t)$.

The ratio of total volume of the solvent to that of the polymer, $\vartheta_{Theory}(t)$, is given by

$$\vartheta_{Theory}(t) = \frac{\int_0^{r_0} r^\iota \vartheta(r,t)\theta_p dr}{\int_0^{r_0} r^\iota \theta_p dr} \tag{5.29}$$

where r_0 is the maximum value of r and θ_p is the volume fraction of the polymer. Inserting Eq. (4.22) into Eq. (5.29) and noting that $u_p(0,t) = 0$ give

$$\vartheta_{Theory}(t) = \frac{1}{m_0}\int_0^{m_0} \vartheta(m,t)dm \tag{5.30}$$

where m_0 is the maximum value of the cumulative volume, m. Combination of Eqs. (4.25), (4.26) and (5.30) gives the following relationship between $\vartheta_{Theory}(t)$ and the spatial distribution of solvent ratio in PMCs.

$$\vartheta_{Theory}(t) = \frac{\iota+1}{x_0^{\iota+1}}\int_0^{x_0} x^\iota \vartheta(x,t)dx \tag{5.31}$$

Theoretical Predictions

The theoretical predictions for the fractional approach to equilibrium swelling, F, of ionic and nonionic NIPA gels are compared to experimental data in Fig. 5.26.

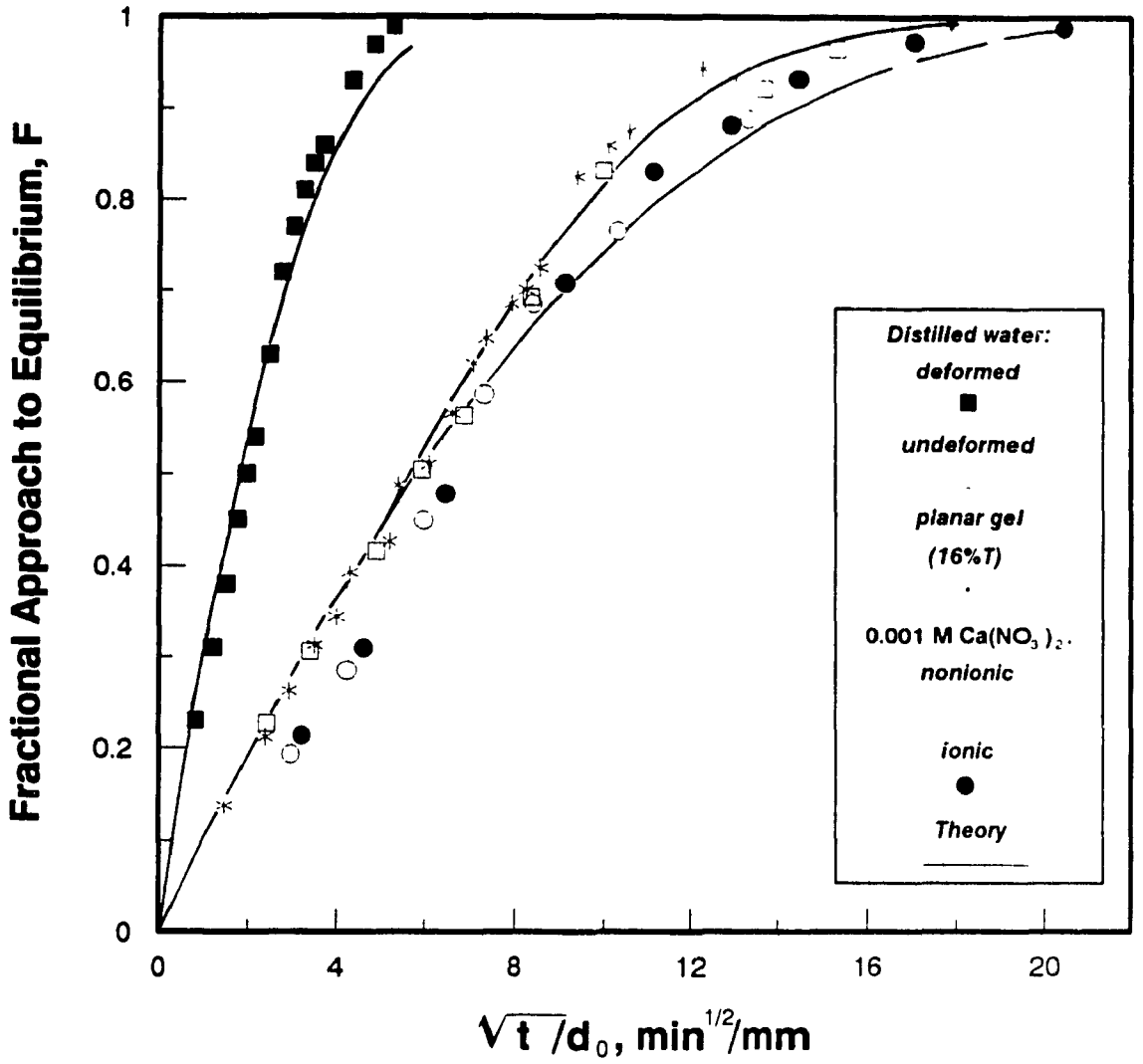


Figure 5.26: Comparison of theory with experimental results for swelling of NIPA gels.

Gel Shape	$D\ (cm^2/s) \times 10^8$	Correlation Coefficient
cylinder - deformed	6 87	0.988
cylinder	1.56	0.975
planar (slab)	0.65	0 997

The theoretical curve for swelling of ionic NIPA gel in 10^{-3} M $Ca(NO_3)_2$ which is slightly above that for nonionic gel (1% higher) is not shown in this figure. The optimum values of a constant diffusion coefficient in material coordinates, D, which fit the experimental results with a correlation coefficient equal to 0.975–0.997, are $1.45 - 6.87 \times 10^{-8} cm^2/s$ for spherical gels and $6.5 \times 10^{-9} cm^2/s$ for the planar gel. The cooperative diffusion coefficient, D_c, becomes:
For spherical gels

$$D_c = D(1+\vartheta)^{2/3} \quad cm^2/s \tag{5.32}$$

and for the planar gel

$$D_c = 6.5 \times 10^{-9}(1+\vartheta)^2 \quad cm^2/s \tag{5.33}$$

Using Eq. (5.32), the cooperative diffusion coefficients at equilibrium swelling are:

(a) - $D_c = 5.53 \times 10^{-7} cm^2/s$ for swelling of deformed NIPA (8.1%T, 1.6%C) gel in distilled water.

(b) - $D_c = 1.31 \times 10^{-7} cm^2/s$ for swelling of NIPA (8.1%T, 1.6%C) gel in distilled water.

(c) - $D_c = 1.28 \times 10^{-7} cm^2/s$ for swelling of NIPA (8.1%T, 1.6%C) gel in 10^{-3} M $Ca(NO_3)_2$.

(d) - $D_c = 2.07 \times 10^{-7} cm^2/s$ for swelling of NIPA (8.1%T, 1.7%C, 5.0%I) gel in 10^{-3} M $Ca(NO_3)_2$.

These diffusion coefficients agree with those of Tanaka et al. (1985) and Gehrke et al. (1989) for nonionic NIPA gels which were $1 - 2 \times 10^{-7} cm^2/s$. The cooperative diffusion coefficient is higher for the ionic gel in agreement with the results of Yoshio et al. (1986) and Schosseler (1987). The cooperative diffusion coefficient at equilibrium swelling for the planar nonionic NIPA gel (16%T, 1%C), obtained through Eq. (5.33), is $3.45 \times 10^{-6} cm^2/s$. This value is an order of magnitude higher than the values which were obtained for spherical NIPA gels. This discrepancy originates from the isotropic swelling of planar and long cylindrical gels observed here, by Gehrke (1986) and by Li and Tanaka (1990). The relation given by Eq. (5.33) is valid only for one dimensional, anisotropic swelling. According to Li and Tanaka (1990), the swelling and shrinking of gels in the form of disks of large diameter and in the form of

long cylinders occur isotropically with effective diffusion coefficient which are 3 and 1.5 times smaller, respectively, than that of a spherical gel. Therefore for isotropic swelling and shrinking of planar or large disk gels and long cylindrical gels the relation for spherical gels, i.e. Eq. (5.32), must be used to obtain the effective diffusion coefficient in laboratory coordinates. Using this relationship for the planar gel, an effective diffusion coefficient equal to $5.26 \times 10^{-8} cm^2/s$ was obtained. Multiplying this vale by 3 resulted in a cooperative diffusion coefficient equal to $1.58 \times 10^{-7} cm^2/s$ which agrees with those obtained for spherical gels.

Figure 5.27 shows the comparison of theoretical predictions with experimental data for non-Fickian swelling and collapse of copolymer gels of acrylamide and sodium methacrylate. The optimum values of diffusion coefficient, which fit experimental data with a correlation coefficient equal to 0.957 for swelling and collapse are $5.48 \times 10^{-8} cm^2/s$ and $5.22 \times 10^{-8} cm^2/s$, respectively. For anisotropic swelling and collapse of cylindrical gels the cooperative diffusion coefficient in laboratory coordinates, D_c, is related to the diffusion coefficient in PMC by

$$D_c = D(1 + \vartheta) \quad cm^2/s \tag{5.34}$$

Using Eq. (5.34), the cooperative diffusion coefficients for swelling (16%T, 4%C, 2.4%I) and collapsing (16%T, 4%C, 0.5%I) of copolymer gels of acrylamide and sodium acrylate at equilibrium are $9.98 \times 10^{-7} cm^2/s$ and $5.44 \times 10^{-7} cm^2/s$, respectively. As discussed above, the swelling and shrinking of cylindrical gels occur isotropically with an effective diffusion coefficient which is 1.5 times smaller than the cooperative diffusion coefficient for spherical gels (Li and Tanaka, 1990). Using Eq. (5.32) which is valid for isotropical swelling and shrinking, the effective diffusion coefficients for swelling and collapsing of these hydrogels are $3.75 \times 10^{-7} cm^2/s$ and $2.04 \times 10^{-7} cm^2/s$, respectively. Multiplying these values by 1.5 gives cooperative diffusion coefficients equal to $5.62 \times 10^{-7} cm^2/s$ and $3.06 \times 10^{-7} cm^2/s$ at equilibrium for ionic copolymer gels of acrylamide which contained 2.4% and 0.5% ionizable monomer, respectively. The cooperative diffusion coefficients measured by the quasielastic light scattering method (Takebe et al., 1989) were $0.83 - 2.7 \times 10^{-7} cm^2/s$ for nonionic acrylamide gels of different crosslinking.

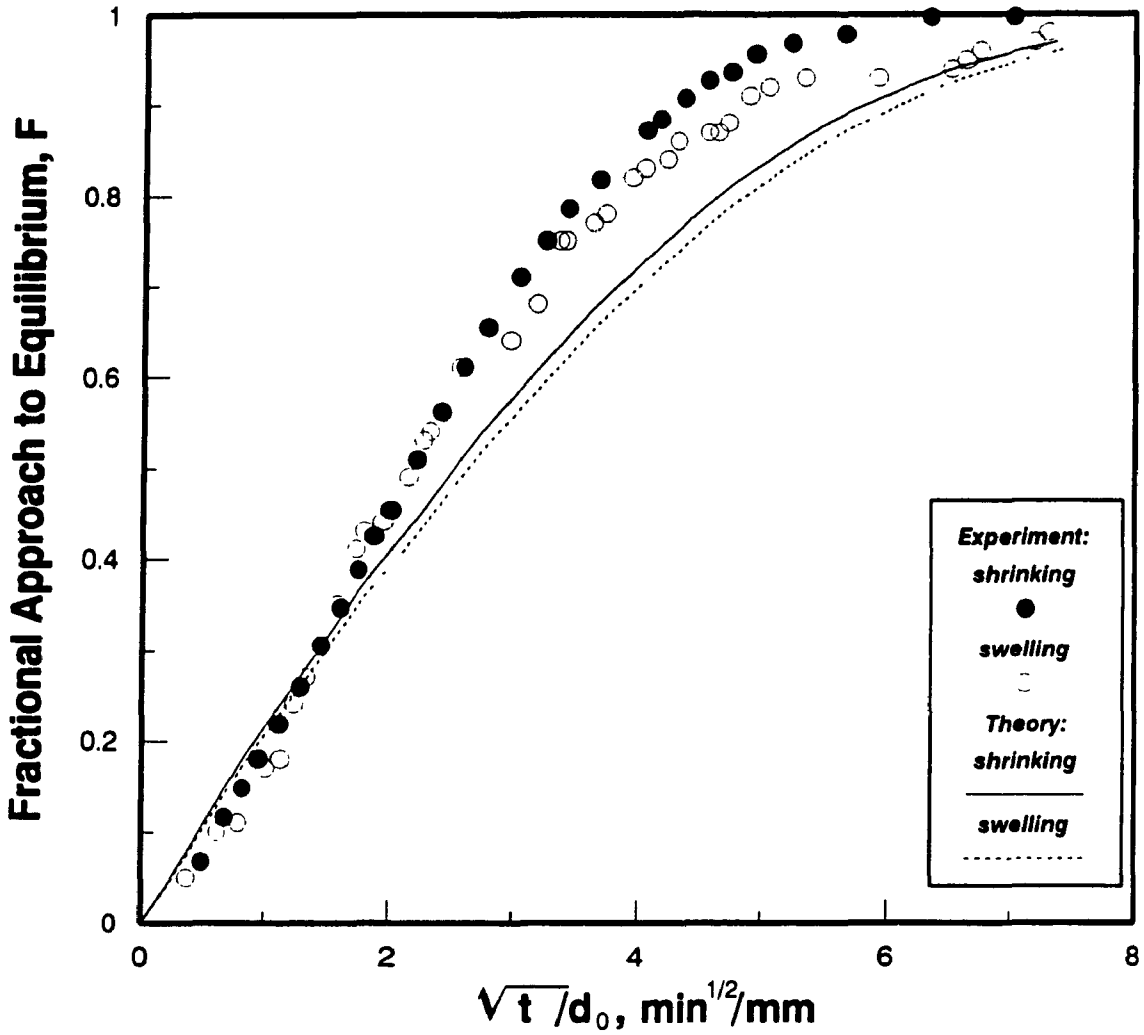


Figure 5.27: Comparison of theory with experimental results for swelling and collapse of cylindrical copolymer gels of acrylamide and sodium methacrylate.

Type of Experiment	$D\ (cm^2/s) \times 10^8$	Correlation Coefficient
Swelling	5.48	0.957
Shrinking	5.22	0.957

Introduction of ionic monomers into the network increased the diffusion coefficient of ionic copolymer gels of acrylamide. Based on the initial dimensions of the gel, i.e. at $t = 0$, the cooperative diffusion coefficients calculated by Gehrke (1986) from the rate of swelling and collapsing of these hydrogels were $6 \times 10^{-7} cm^2/s$ and $7 \pm 0.4 \times 10^{-7} cm^2/s$, respectively. These values gave a correlation coefficient of 0.987 but did not minimize the objective function, thus indicating that a higher correlation coefficient does not necessarily mean the best fit to the experimental results. The optimum values of the diffusion coefficients for swelling and collapsing of these hydrogels, based on the dimensions of gel at $t = 0$, which minimized the objective function, were $3.77 \times 10^{-7} cm^2/s$ and $3.43 \times 10^{-7} cm^2/s$, respectively. These values are close to values at equilibrium swelling calculated with PMC.

The theoretical predictions for dynamic swelling of copolymer gels of acrylamide and sodium acrylate are compared to experimental data in Fig. 5.28. The parameters were obtained using experimental data for the gel having a diameter equal to 2.4 mm at gel formation. The optimum value for a constant diffusion coefficient in material coordinates, $D = 2.8 \times 10^{-8} cm^2/s$, fits the experimental results with a correlation coefficient equal to 0.948. The cooperative diffusion coefficient, D_c, becomes

$$D_c = 2.8 \times 10^{-8}(1 + \vartheta)^{2/3} \quad cm^2/s \tag{5.35}$$

Using the two parameters form of Eq. (5.21) the optimized diffusion coefficient is

$$D = 4 \times 10^{-11}(1 + \vartheta)^{1.132} \quad cm^2/s \tag{5.36}$$

Equation (5.36) fits the experimental results with a correlation coefficient equal to 0.959. The cooperative diffusion coefficient, D_c, is then

$$D_c = 4 \times 10^{-11}(1 + \vartheta)^{1.799} \quad cm^2/s \tag{5.37}$$

Using the three parameters form of Eq. (5.22) the optimized diffusion coefficient is

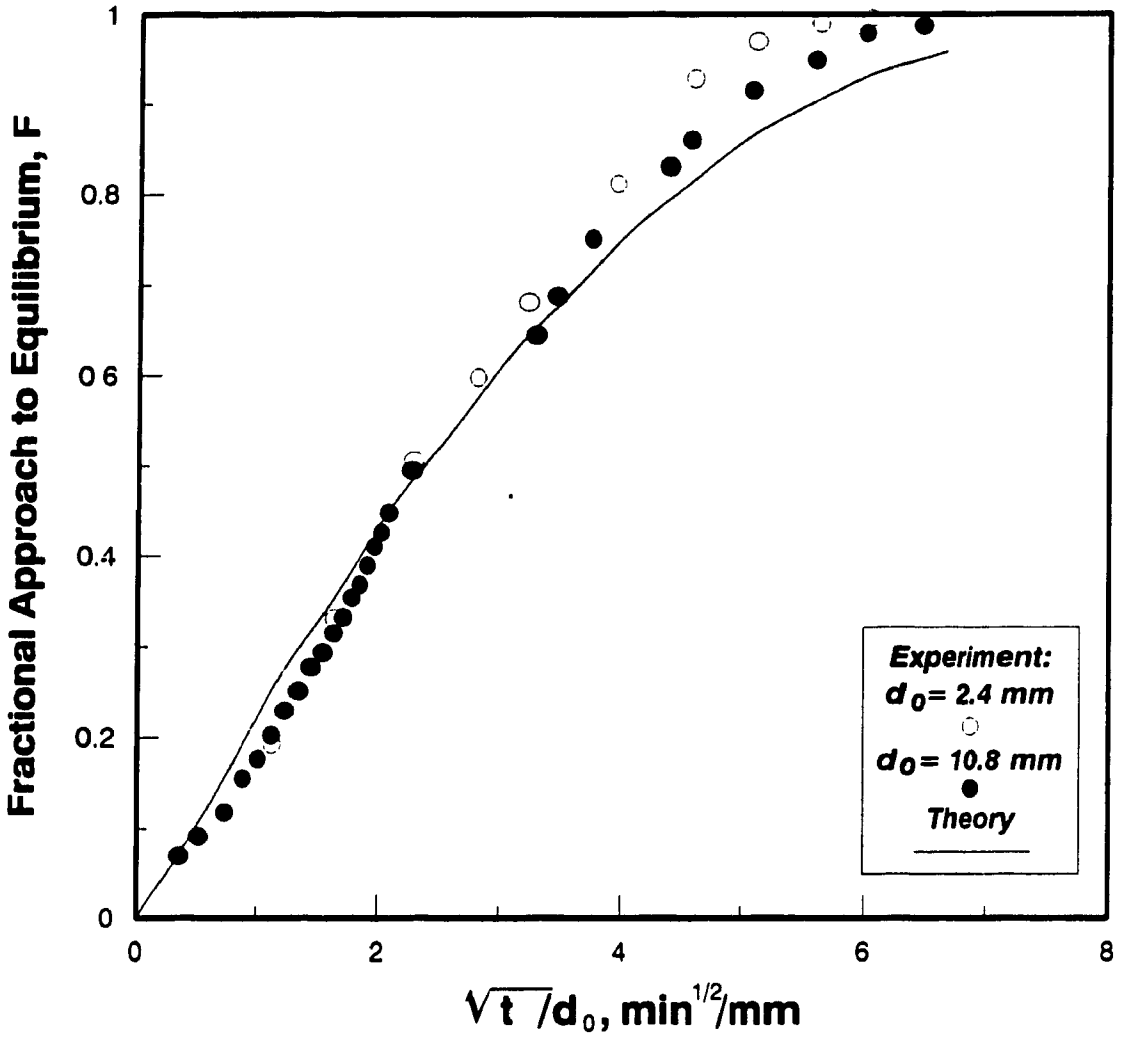


Figure 5.28: Comparison of theory with experimental results for swelling of copolymer gels of acrylamide and sodium acrylate at pH 7.

Gel Diameter at Formation	$D\ (cm^2/s)$	Correlation Coefficient
2.4 mm	Eq. (5.38)	0.961
10.8 mm	Eq. (5.38)	0.972

$$D = 3.115 \times 10^{-4}(1+\vartheta)^{-17/12} exp[61.57(1+\vartheta)^{-0.525}(\frac{\vartheta}{\vartheta_e} - 1)] \tag{5.38}$$

The correlation coefficient is 0.961. The cooperative diffusion coefficient is

$$D_c = 3.115 \times 10^{-4}(1+\vartheta)^{-3/4} exp[61.57(1+\vartheta)^{-0.525}(\frac{\vartheta}{\vartheta_e} - 1)] \tag{5.39}$$

At equilibrium swelling, i.e. $\vartheta = \vartheta_e$, Eq. (5.39) can be written as

$$D_c = 2.879 \times 10^{-4} C_e^{3/4} \tag{5.40}$$

where the polymer concentration at equilibrium swelling, C_e, is related to the polymer concentration, C_o, and solvent ratio, ϑ_o, at gel formation by

$$C_e = C_o(\frac{1+\vartheta_o}{1+\vartheta_e}) \tag{5.41}$$

The cooperative diffusion coefficient at equilibrium swelling, D_c, calculated from Eqs. (5.35), (5.37) and (5.39) ranges from $1.82 - 3.13 \times 10^{-6} cm^2/s$. Yoshio et al. (1986) reported a cooperative diffusion coefficient for copolymer gels of acrylamide and acrylic acid in the range of $2 - 4 \times 10^{-6} cm^2/s$. Cooperative diffusion of sodium acrylate gels ranged from $3.6 - 8.6 \times 10^{-6} cm^2/s$ (Komori et al., 1988). These values are similar in magnitude to those found here. The discrepancy between theory and experiment for ionic copolymer gels of acrylamide (Figs. 5.27 and 5.28) originates from non-Fickian behavior of acrylamide-based gels. The mathematical model was based on a form of Fick's law for the relative velocity, u,-see Eq. (4.17). In Fig. 5.28 the theory is essentially a straight line for $F < 0.4$ while the data show sigmoid shape characteristic of non-Fickian behavior. If sorption equilibrium cannot be achieved instantaneously at the surface of the swelling network, sorption curves exhibit an inflection point and are Sigma-shaped. Pattern formation on the surface of a gel (i.e. the roughening of the surface), which increases the rate of solvent uptake due to the increased surface area, may also cause non-Fickian behavior.

Chapter 6

EXCLUSION BEHAVIOR

The exclusion behavior of copolymer gels of acrylamide, and homopolymer and copolymer gels of N-isopropylacrylamide (NIPA) was studied. The effects of solute size, crosslinking ratio, monomer concentration, percent of ionizable monomer as well as solute charge on the exclusion behavior of hydrogels are described.

The selectivity results are reported in terms of the distribution coefficient, K_d, defined as the ratio of the solute concentration in the gel phase, $\overline{C}$, to that in the retentate or raffinate, C_R, i.e.

$$K_d = \overline{C}/C_R \tag{6.1}$$

A mass balance on the solute gives

$$V_F C_F = V_R C_R + V\overline{C} \tag{6.2}$$

where V_F and C_F are the feed volume and concentration respectively; and V_R and V are the retentate and the swollen gel volume, respectively. The distribution coefficient was calculated from experimental measurements by combining Eqs. (6.1) and (6.2) to give

$$K_d = (V_F C_F - V_R C_R)/(V C_R) \tag{6.3}$$

The fraction of solute which is excluded, η, is given by

$$\eta = \frac{V_R C_R}{V_F C_F} = \frac{1}{1 + \frac{K_d V}{V_R}} \tag{6.4}$$

Equation (6.4) indicates that complete exclusion occurs for $K_d = 0$ and complete removal from solution for $K_d \rightarrow \infty$. In addition to K_d the fraction of solute excluded depends upon the volume of gel as well as the volume of solution.

6.1 Effect of Solute Size

Size selective separation depends upon the relationship between the molecular size and shape of the solute and the pore size of the hydrogel. For polyethylene glycol (PEG), an uncharged solute, the solute size may be characterized by its molecular weight. Figure 6.1 shows the effect of PEG molecular weight on the swelling and exclusion behavior of a weak acid copolymer gel of NIPA and sodium acrylate (8.0%T, 1.7%C, 10.0%I) and a strong acid copolymer gel of NIPA and 2-acrylamido-2-methyl-1-propanesulfonic acid (R-$SO_3^-H^+$), (8.4%T, 1.6%C, 5.0%I). The swelling ratio is defined as the ratio of swollen gel mass, M, to dry mass of polymer, M_d. In spite of higher number of ionizable groups in weak acid gel, there was no significant difference in swelling ratios. This was mainly due to incomplete ionization of ionic groups of weak acid gel in a solution of pH 6. The distribution coefficient K_d, decreased as PEG molecular weight increased thus indicating that gels sorb low molecular weight solutes (MW = 800) but exclude high molecular weight solutes (MW = 18500). The distribution coefficient of low molecular weight PEG approaches neutrality (i.e. a value of $K_d = 1$), thus indicating equal concentrations of solute in both gel phase and external solution. Since the feed concentration of PEG, C_F, was 25 g/L in each case, the higher exclusion of solute (lower K_d) for higher molecular weight PEG resulted in an external solution (retentate) of high concentration at equilibrium which, in turn, lowered the swelling ratio of gels at higher molecular weight. This is in consistent with the thermodynamics of gel swelling presented in Chapter 3. These results indicate that the exclusion of an uncharged solute by ionic gels is affected by solute size and pore size of the gels in swollen state.

Solute size alone does not determine selectivity; the shape of the solute molecule is also important. An extended molecule like polyethylene glycol (MW = 18500) was almost completely excluded by a strong acid NIPA gel ($K_d = 0.01$ in Fig. 6.1) whereas a globular protein (enzyme) like lipase (MW $\simeq 10^5$) partitioned with a distribution coefficient of about 0.2 - see Table 6.4.

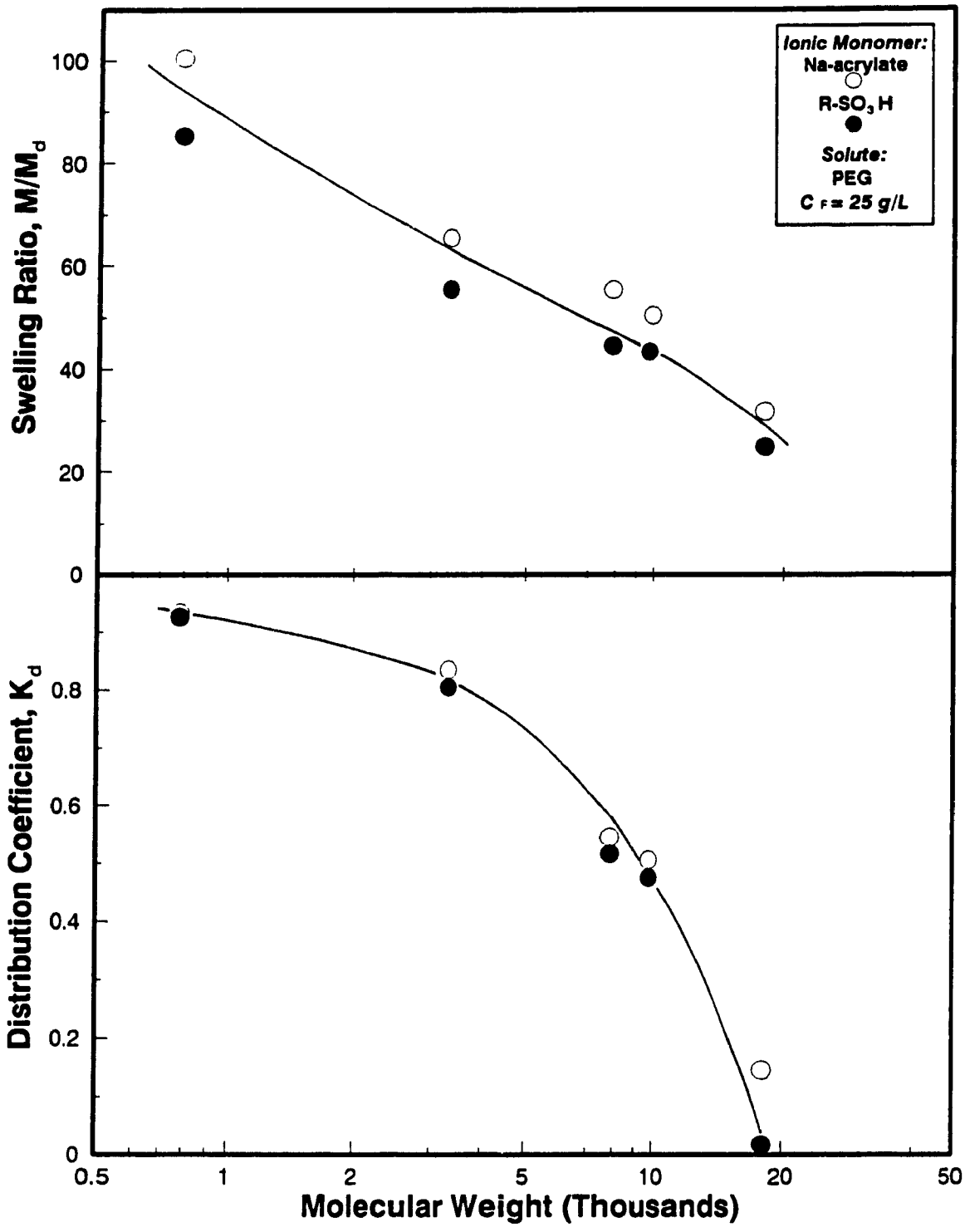


Figure 6.1: Effect of solute molecular weight on the swelling and exclusion behavior of ionized NIPA gels.

6.2 Effect of Monomer Concentration and Crosslinking Ratio

The structure of a synthetic gel is determined by the chemical (structure) properties of its components, the concentrations of the reactants and solubility conditions during gel formation. It is usually assumed that the number of crosslinks is proportional to the crosslinking ratio, but this is not universally true. The reactions involving crosslinking are affected by the functional reactivity of the monomers and their spatial orientation during gel formation. The voids produced during gel formation are closely related to the effective pore size of the gel.

From the aspect of pore size, Kun and Kunin (1964) classified gel structure into two principal types: microreticular (microporous) and macroreticular. Microreticular gels have more crosslinks, higher solid contents and lower specific solvent uptakes than macroreticular gels. The more uniform repetition of the crosslinks in microreticular gels produces smaller pores and renders the gel suitable for the separation of smaller molecules. The structure of the macroreticular gels is rather heterogeneous, the spatial distribution of the matrix being uneven. Gels may be considered to be swollen networks through which small molecules move freely, while large molecules are excluded. The system of crosslinks acts as a physical barrier for molecules of certain sizes and shapes.

The exclusion behavior of a gel changes if the pore size of the network is changed. Gels with smaller pore sizes can be prepared by increasing the concentration of the monomers or of the crosslinking agent at gel formation. To study the effect of monomer concentration and crosslinking ratio on the selectivity of hydrogels, N-isopropylacrylamide gel and its ionic counterparts with different concentrations of monomers and crosslinking ratios were prepared. Polyethylene glycol (MW = 3400) was used as the test solute. The experimental results for the effect of monomer concentration, (%T), on the swelling and exclusion behavior of homopolymer gels of NIPA (1.6%C) and copolymer gels of NIPA and R-$SO_3^-H^+$ (1.6%C, 2.5%I) are summarized in Fig. 6.2. Figure 6.3 shows the effect of crosslinking ratio, (%C), on the swelling and exclusion behavior of copolymer gel of NIPA and R-$SO_3^-H^+$ (2.5%I) having a total monomer content of approximately 8.3%T . The total monomer concentrations of these hydrogels ranged from 8.3 to 8.4%T –see Table 2.9.

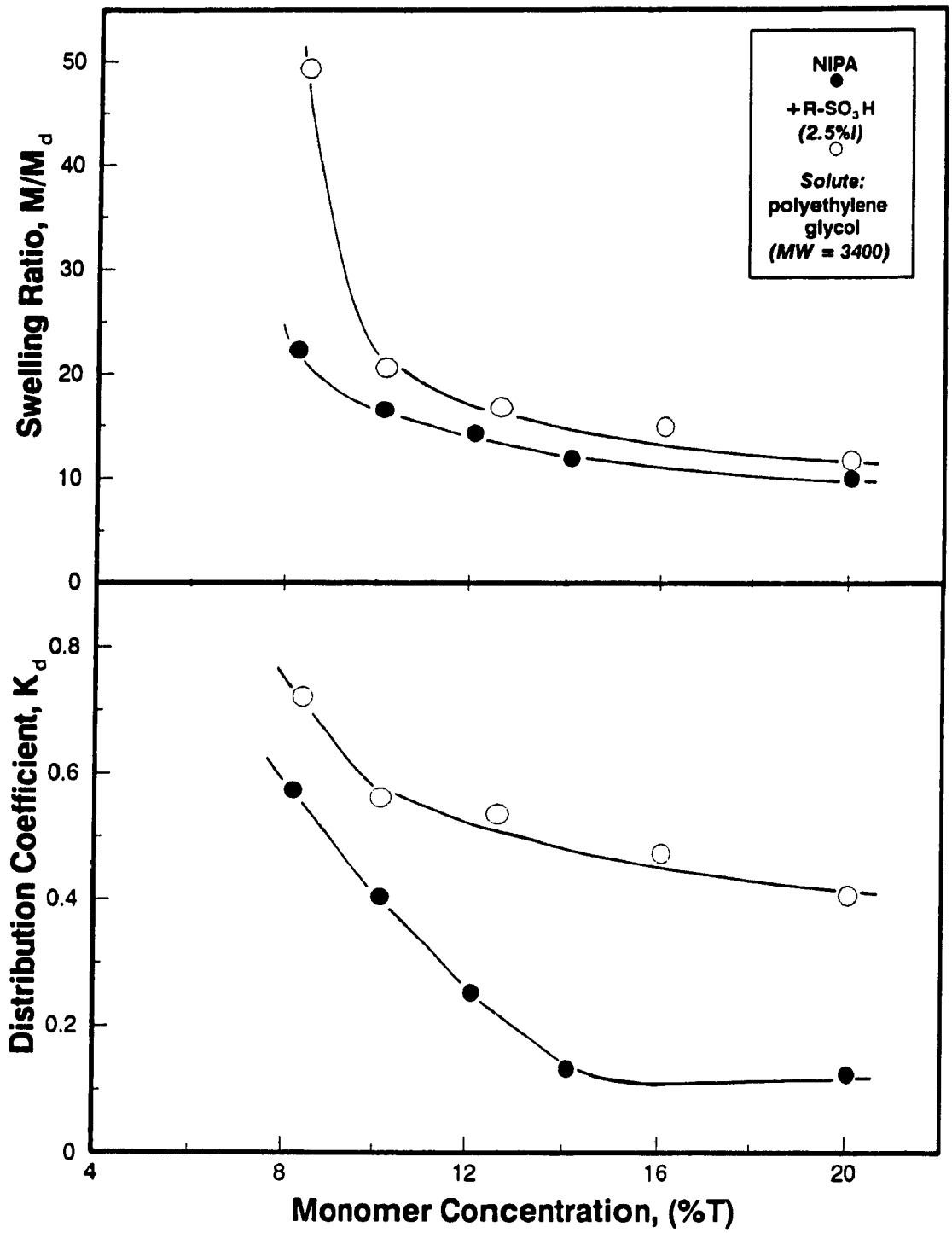


Figure 6.2: Effect of monomer concentration on the swelling and exclusion behavior of NIPA gels.

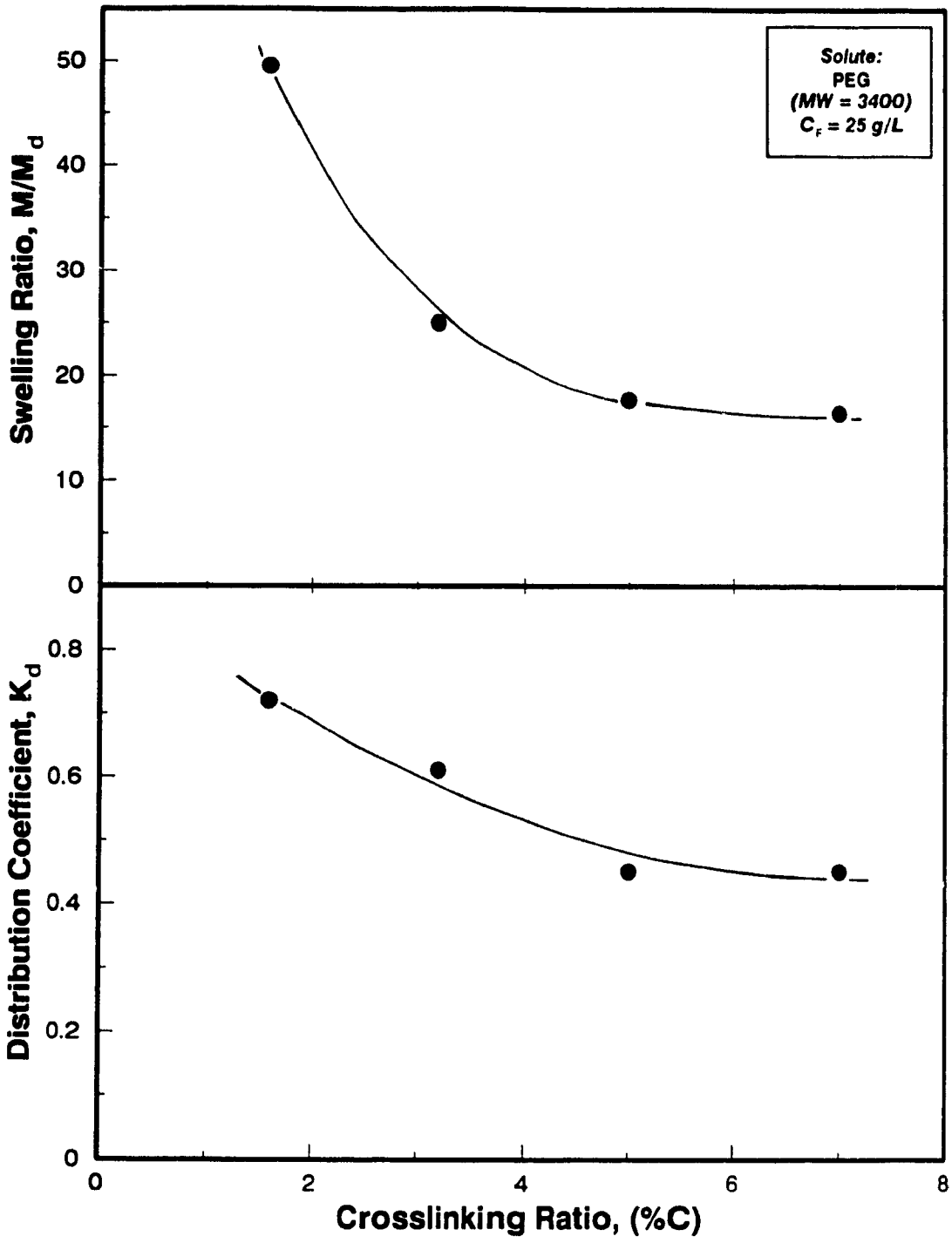


Figure 6.3: Effect of crosslinker on the swelling and exclusion behavior of copolymer gel of NIPA and R-SO_3H (2.5%I).

The extent of swelling and the distribution coefficient, K_d, decreased with increasing monomer concentration and with increasing percent crosslinker. These results can be explained in light of the structural properties of the network. At low concentration of network formation, ring formation is favored. At concentrations below 5 w/v %, ring formation is such a dominant process that no continuous network is formed (Oppermann et al., 1985). Ilavský and Hrouz (1983) reported that the crosslinking efficiency increases with increasing monomer content at gel formation mainly due to the lower cyclization at network formation. Vasiliev et al. (1985) found that both chemical crosslinking and physical crosslinking (chain entanglements) decreased with a decrease in monomer concentration at gel formation.

The value of K_d decreased with increasing monomer or crosslinker concentration, but it approached an asymptote at approximately 5 w/w % crosslinker or 16 w/v % monomer concentration. Increasing the percentage crosslinking increases the number of crosslinks in the gel network with a consequent decrease in pore size and swelling. According to Horkay et al., (1989) the reduced swelling at high crosslinking ratio is related to unfavorable interaction of polymer chains with water, i.e. chemical crosslinking alters the electronic configuration of the constituent polymer chains, so good solvents of the uncrosslinked polymer will tend to have a more unfavorable interaction parameter, χ (see Chapter 3). But the decrease of swelling ratio as a result of increased percentage of crosslinker may rather be a geometrical effect, i.e. the main polymer chains cannot separate as long as the crosslinks are well separated. Further increase of the amount of crosslinker does not create additional isolated crosslinks, but rather promotes the formation of inhomogeneities with regions where the matrix material is aggregated and regions where there is little gel matrix present.

It has been suggested that microsyneresis (Dušek and Prins, 1969), in which a dispersion is formed by the gel and separated liquid phases (e.g. droplets of liquid inside the gel), and other modes of phase separation (Geissler and Hecht, 1985) may produce heterogeneous networks. The aggregation of bisacrylamide (crosslinking agent) in water has also been invoked to explain the existence of highly crosslinked regions in polyacrylamide gels (Hsu and Cohen, 1984; Weiss et al., 1979 and 1981). Baselga et al. (1987) studied the effect of crosslinking agent on the properties of polyacrylamide gels and found that the swelling of hydrogels decreased with increasing percentage of crosslinker and reached a plateau at 7 w/w %. Baselga et al. (1989)

have shown that near this value, the probability of forming permanent intramolecular cycles becomes significant. They also concluded that network defects increased with further increase of the amount of crosslinking agent making the crosslinking efficiency progressively lower. The effect of crosslinker on the swelling and thermodynamic properties of polyacrylamide gels has been studied (Baselga et al., 1989). The polymer volume fraction, v, increased with increasing percentage of crosslinker up to 5–7 w/w % but at higher percentages it remained constant. Similar results were reported for this system by Hsu et al. (1983). Tanaka, Fukumori, and Nishi (1988) investigated the influence of monomer concentration and crosslinking ratio on the chemical gelation dynamics of acrylamide in water. They observed spatial heterogeneity of the network chains at high acrylamide and crosslinker concentrations. Heterogeneous networks are not transparent in the swollen state (Dušek, 1971; Tanaka et al., 1988). Turbidity was observed here for copolymer and homopolymer gels of N-isopropylacrylamide at monomer concentrations $\geq$ 12.0 w/v % or percentage crosslinker $\geq$ 5.0 w/w %. Turbidity was also found for NIPA gels by Freitas (1986).

Since size selective separation depends on the relationship between solute size and pore size, any variable which affects pore size will affect exclusion behavior. The ionizable monomer content of the gel affects the degree of swelling, and hence exclusion. Nonionic solutes like PEG (MW = 3400) will be less excluded as the swelling of the network increases. Figure 6.4 shows the swelling and exclusion behavior of copolymer gels of NIPA and R–SO_3 H^+ as a function of the percent ionizable monomer. The compositions of these ionic gels are given in Table 2.9. Considering the two gels with 5 mole % ionizable monomer (5%I), the one with lower amount of crosslinker (8.4%T, 1.6%C, 5.0%I) was used in all exclusion experiments. The composition of the nonionic NIPA gel (8.1%T, 1.6%C) used in all exclusion experiments is given in Table 2.8. The swelling ratio, M/M_d, and the distribution coefficient, K_d, increased with increasing content of ionizable monomer.

A compact presentation of the data from Figs. 6.2, 6.3, and 6.4 for the effect of crosslinker, monomer concentration and mole % of ionizable monomer on the exclusion behavior of NIPA gels is shown in Fig. 6.5. This figure also includes the data from Fig. 6.8 for NIPA gels in a solution of PEG (C_F = 25 g/L) and 0.038 M NaCl. The abscissa in Fig. 6.5 is the volume fraction of polymer in the swollen state, v. This volume fraction was calculated from the swelling ratio, M/M_d, by

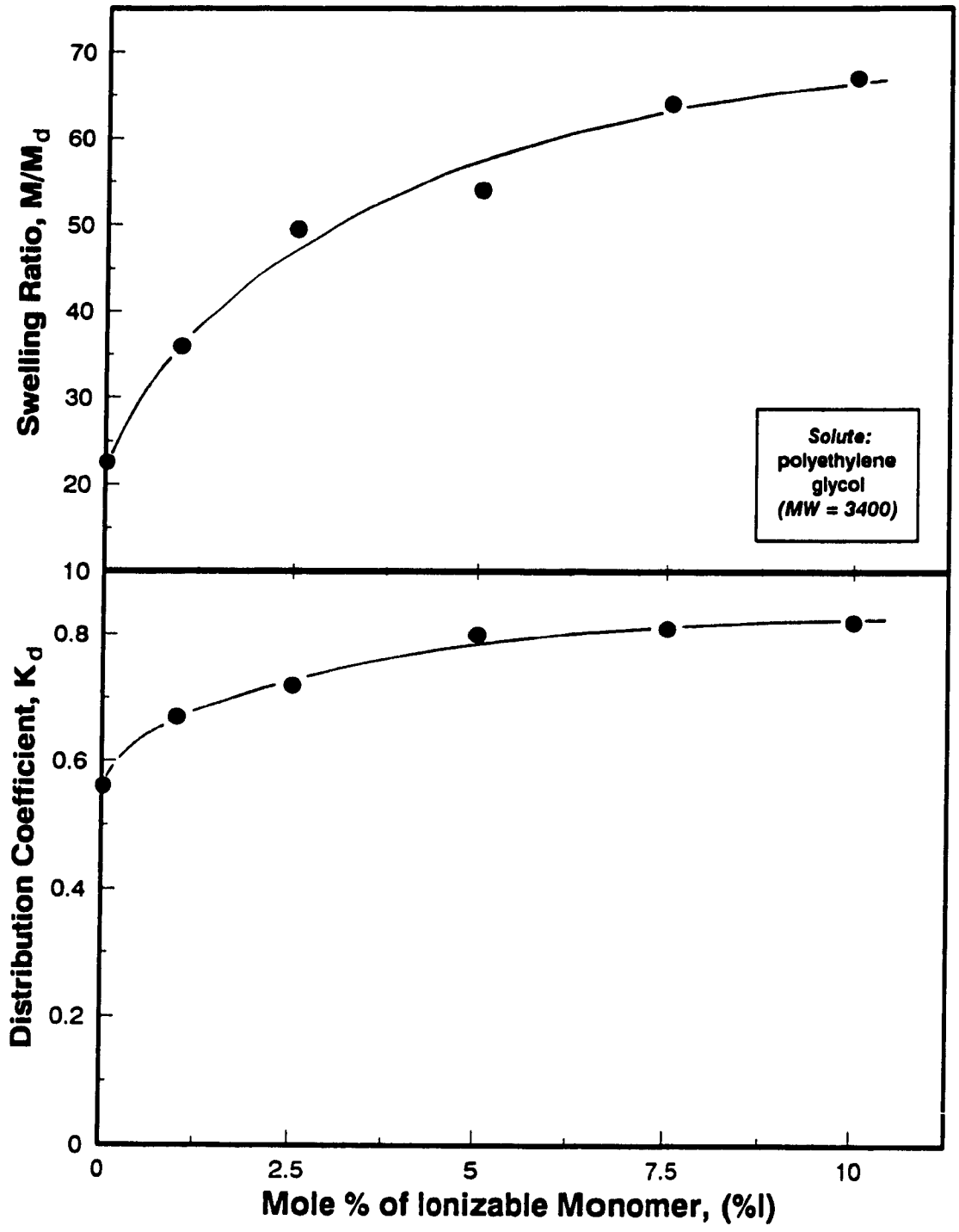


Figure 6.4: Effect of ionizable monomer on the swelling and exclusion behavior of copolymer gel of NIPA and R-SO_3H.

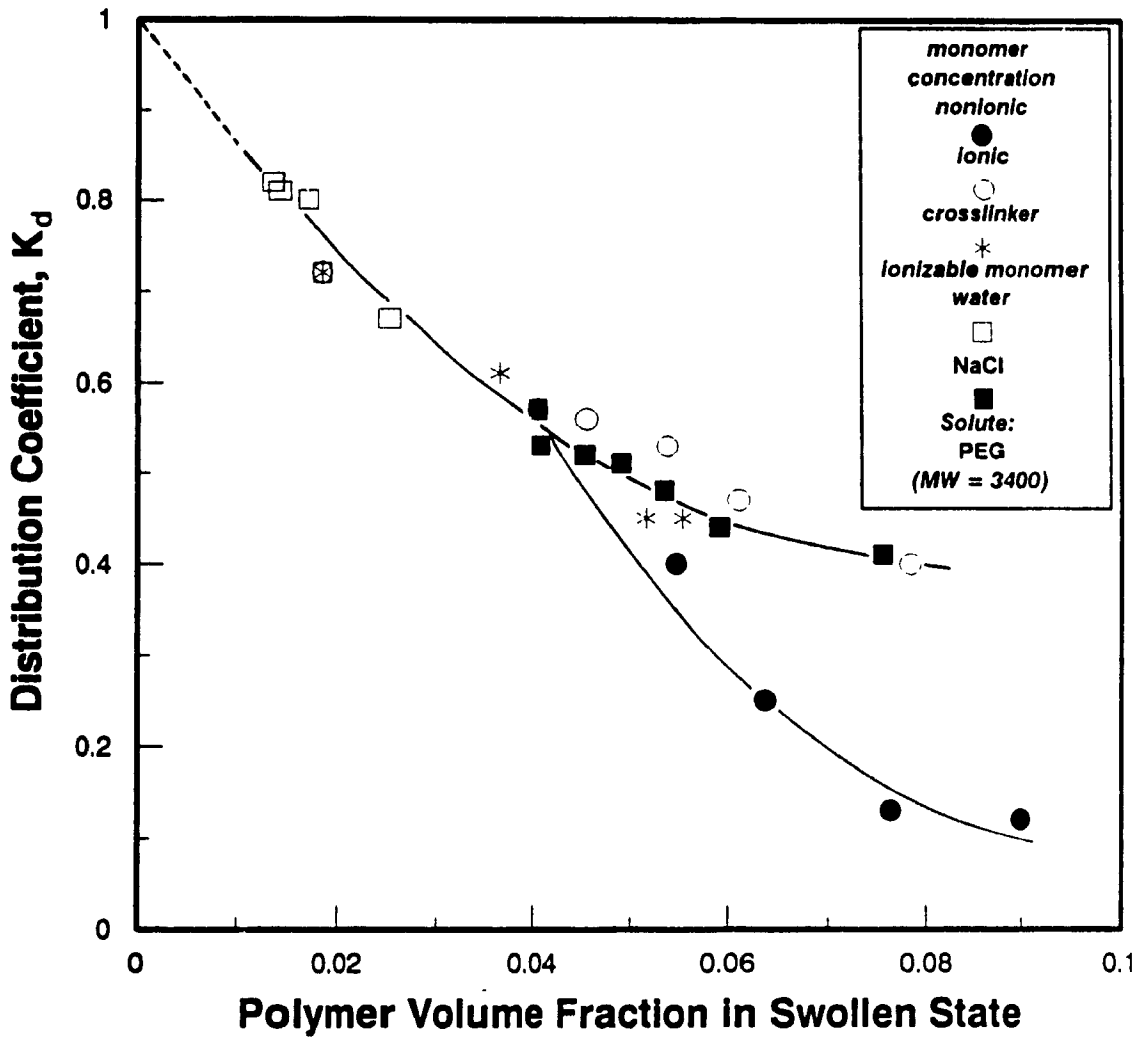


Figure 6.5: Distribution coefficient, K_d, for PEG (MW = 3400) as a function of polymer volume fraction of NIPA gels.

$$v = \frac{\rho_g/\rho_p}{M/M_d} \tag{6.5}$$

where ρ_g and ρ_p are the gel and polymer density, respectively. The values of ρ_g and ρ_p determined experimentally were 1.04 ± 0.04 and 1.16 ± 0.04, respectively.

The PEG distribution coefficient decreased as the polymer volume fraction increased. Highly swollen gels exhibited decreased exclusion. As $v \rightarrow 0$ the distribution coefficient should approach unity, i.e. the solute concentration is the same inside and outside the gel. Up to a polymer volume fraction of about 0.05 the data lie on a single curve. For higher volume fractions the data for the nonionic NIPA gels (from Fig. 6.2) lie below the data for the ionic gels. The effect of monomer concentration at gel formation on the exclusion behavior of hydrogels was more pronounced for nonionic NIPA gels.

To test the generality of the correlation presented in Fig. 6.5 the data of Freitas (1986) for PEG (MW = 3400) are plotted in the same format in Fig. 6.6. This figure also contains the curves from Fig. 6.5. Freitas varied the monomer concentration and the crosslinking ratio for nonionic NIPA gels. His data are in general agreement with those of Fig. 6.5 except that results for the variation of monomer concentration were more scattered. The results in Figs. 6.5 and 6.6 show that increasing the monomer concentration was the most effective way to lower K_d and thus to improve exclusion for nonionic gels.

6.3 Effect of Charge

The exclusion behavior of a polyelectrolyte gel is highly dependent on its interaction with charged solutes. An ionic solute like sodium pentachlorophenolate was almost completely excluded by a pH–sensitive ionic hydrogel (Gehrke et al., 1986) whereas its exclusion was negligible for the nonionic NIPA gel (Freitas and Cussler, 1987). Because of the fixed negative charges on the polymer backbone of the ionic gel, the negatively charged pentachlorophenolate ions were excluded, based on Donnan equilibrium. Use of ionic copolymer gels to concentrate dilute aqueous solutions of biological products seemed promising because of the large water uptake of these gels and their expected exclusion of charged macromolecules.

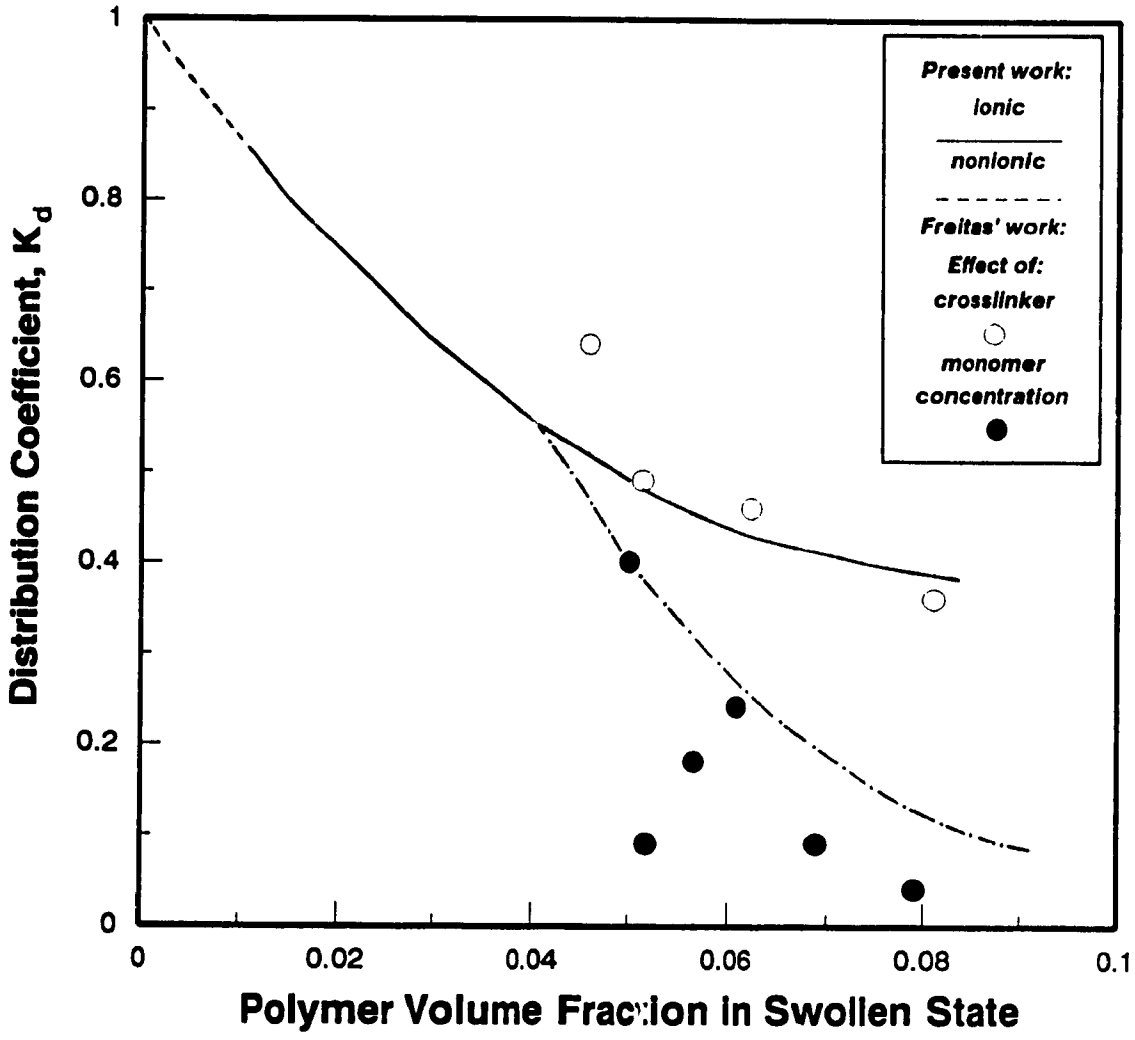


Figure 6.6: Distribution coefficient, K_d, for PEG (MW = 3400) as a function of polymer volume fraction of NIPA gels [data of Freitas (1986)].

The effect of solute charge on the swelling ratio and the distribution coefficient of copolymer gels of N-isopropylacrylamide with different amounts of ionizable strong acid monomer (R-SO_3^- H^+) is shown in Figs. 6.7 and 6.8. Figure 6.7 shows the swelling ratio and the distribution coefficient in dextran and dextran sulfate solutions. To isolate the effect of charge on exclusion behavior for the two solutes of comparable size, dextran (MW = 9400) and dextran sulfate (MW = 8000), NaCl was added to the feed dextran solution. In 0.5 M NaCl/dextran solution the swelling ratio was essentially equal to that in distilled water/dextran sulfate solution over the complete range of ionizable monomer. Figure 6.7 shows both the effect of size and the effect of charge. The data for dextran (MW =19500) and (MW = 9400) show that swelling increased with ionizable monomer content, but that the value of K_d changed only slightly as the % ionizable monomer increased for this nonionic solute. The dextran with the higher molecular weight was more completely excluded, i.e. it had a lower value of K_d. The effect of charge is illustrated by comparing the data for dextran (MW = 9400) in 0.5 M NaCl and dextran sulfate in distilled water. Identical swelling ratios were obtained, but the value of K_d was lower for dextran sulfate when the gel contained ionizable monomer. For a nonionic gel (0.0%I) the value of K_d was lower for the dextran because its molecular weight was higher. The value of K_d for dextran sulfate was essentially constant between 2.5 and 10.0% ionizable monomer. This unexpected behavior was caused by the increase in swelling as the %I increased, i.e. the increasing pore size resulting from the larger swelling counteracted the charge effect. To demonstrate the effect of charge more clearly, exclusion tests were performed using dextran sulfate in 0.5 M NaCl. In this solution the swelling ratio was much more nearly constant than it was in distilled water. In 0.5 M NaCl the distribution coefficient for dextran sulfate decreased from 0.37 for nonionic NIPA gel t almost zero for an ionic copolymer gel with 10.0% ionizable monomer. Donnan equilibrium is responsible for the enhanced exclusion of negatively charged solute by the anionic copolymer gel which carries the same charge as the backbone of the network.

Figure 6.8 contains experimental results for swelling and exclusion in solutions of polyethylene glycol (MW = 3400) and in solutions of sodium vinylsulfonate (MW $\sim$ 3400). The swelling ratios for nonionic NIPA gel and for ionic copolymer gels of NIPA and R-SO_3 H^+ decreased with increasing solute concentration or with the

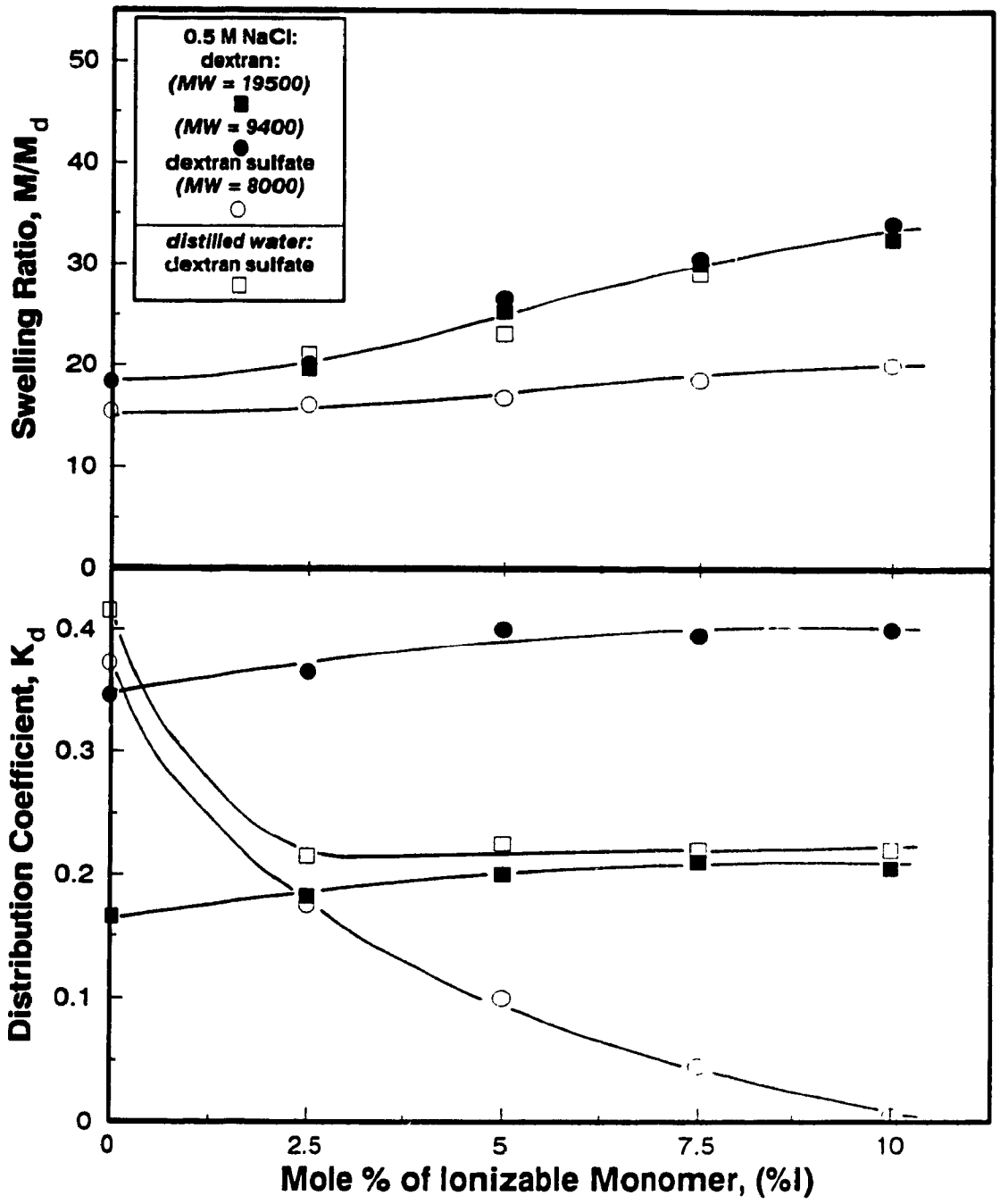


Figure 6.7: Charge effect on the swelling and exclusion behavior of NIPA gels.

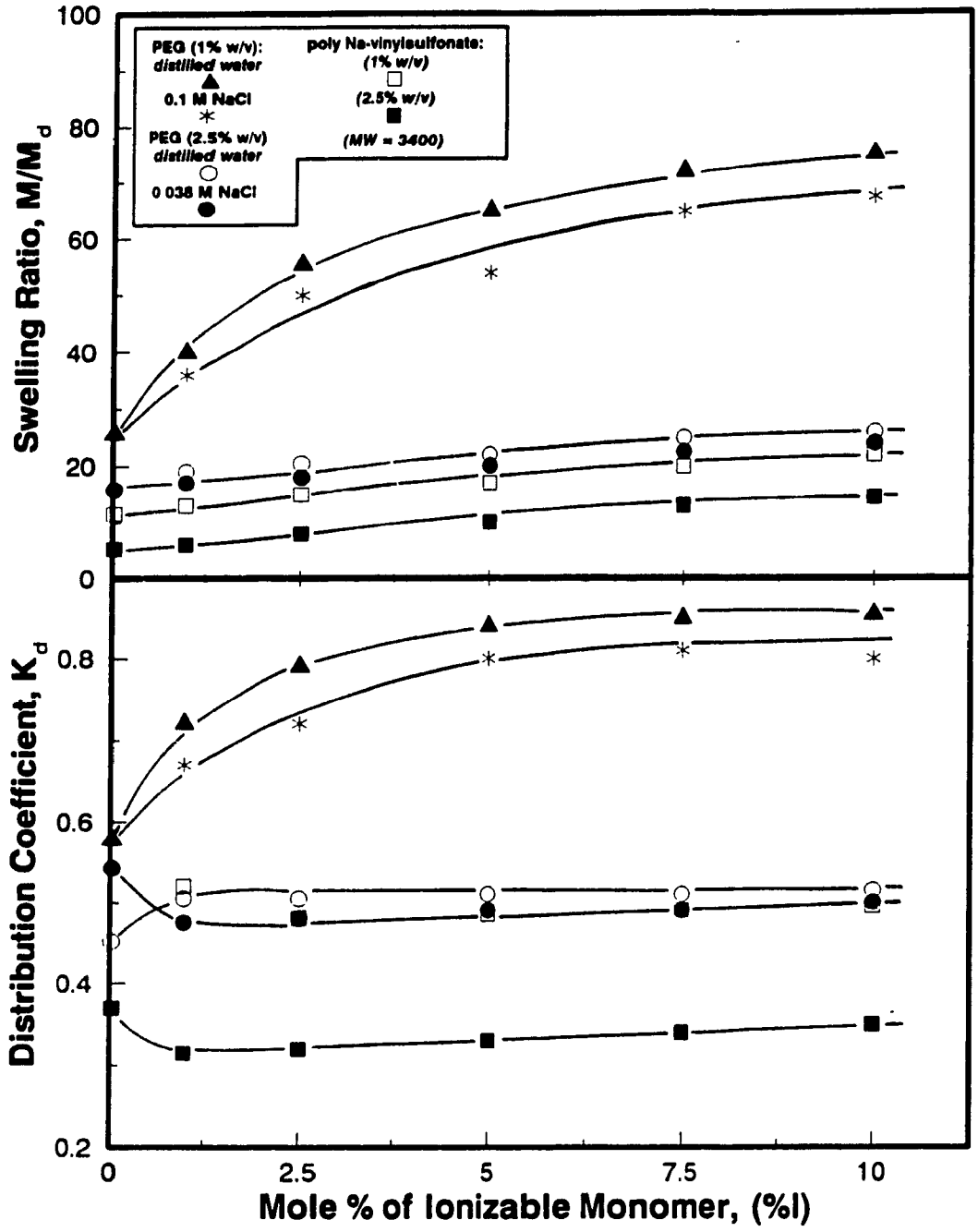


Figure 6.8: Charge effect on the swelling and exclusion behavior of NIPA gels.

addition of sodium chloride to the feed. The decreased swelling ratios resulted in higher exclusion of the solute. The swelling ratios of the ionic network increased with the percentage of ionizable monomer while the distribution coefficient of the ionic solute decreased somewhat. The effect of the %I on K_d for the ionic solute in Fig. 6.8 is less dramatic than that shown in Fig. 6.7, perhaps because the solutes in Fig. 6.7 are larger.

6.4 Exclusion of Proteins

Biological products, such as proteins, are more complex than the simple ionic macromolecules discussed above. Among the important characteristics of proteins are the pendant chemical groups such as amine ($-NH_2$) and carboxylic acid (-COOH) groups. In acidic solution, the amine group may protonate to yield the cationic group ($-NH_3^+$); in basic solution, the carboxylic acid group may ionize to yield the anionic group ($-COO^-$). By altering the pH, the net charge on the protein can be changed. At the isoelectric point, pI, a protein has an equal number of positive and negative charges and its net charge is zero.

Ion–exchange chromatography and hydrophobic interaction chromatography are protein purification techniques which are described in terms of the net charge on the protein and its hydrophobicity. Only recently has the role of heterogeneity on the protein surface been considered (Regnier. 1987; Ruckenstein and Lesins, 1988). If ionic or hydrophobic groups are distributed nonuniformly on the protein surface, interactions may occur between a sorbent and these ionic or hydrophobic regions on the protein surface. For example, retention in ion–exchange chromatography may be determined by a relatively small number of charged amino acids ($-COO^-$) on the protein surface. Regions of the surface containing the anionic group are electrostatically attracted to the sorbent even though the net charge of the protein might be positive. Consequently, if there is a nonuniform charge distribution on the protein surface, it is not necessary for the net charge of the protein to be opposite to that of the sorbent, but rather the occurrence of an oppositely charged patch on the protein surface is sufficient to cause adsorption. Lesins and Ruckenstein (1988) demonstrated that proteins can easily adsorb to surfaces of like charge. This "patch–controlled" interaction is dependent upon the characteristics of the surrounding medium (i.e. pH,

ionic strength) and the adsorbent.

The exclusion behavior of an anionic copolymer gel of acrylamide and sodium acrylate (5.3%T, 2.5%C, 10.0%I) and a cationic copolymer gel of acrylamide and 3-(methacrylamido)propyltrimethylammonium chloride (6.2%T, 2.2%C, 10.0%I) is summarized in Table 6.1 in terms of the distribution coefficient. This table also shows the swelling ratios for each gel. For a nonionic solute, polyethylene glycol, both anionic and cationic gels gave similar distribution coefficients. The higher molecular weight PEG was excluded more completely. For proteins the anionic and cationic gels gave markedly different values of K_d. At pH 5.8, which is above the isoelectric point (pI), both proteins carry a net negative charge. The interaction between the positively charged network and the negatively charged proteins is controlled by the global charge of the proteins. These results imply that the cationic gel is not suitable to concentrate protein solutions, although it showed desirable swelling characteristics-see Fig. 5.2.

Table 6.1: Exclusion behavior of anionic and cationic copolymer gels of acrylamide.

Solute	MW	Anionic gel		Cationic gel	
		M/M_d	K_d	M/M_d	K_d
PEG*	8000	380	0.85	246	0.82
PEG*	18500	172	0.30	120	0.28
Ovalbumin (pI= 4.6)†	45000	113	0.07	115	0.73
Bovine albumin (pI= 4.8)†	66000	157	0.82	113	0.73

*C_F=25 g/L, pH=6.1
†C_F=1 g/L, pH=5.8

Ovalbumin was more excluded, i.e. had a lower K_d, than bovine albumin, even though the latter has a higher molecular weight. One factor which may contribute to this behavior is the extended shape of ovalbumin which has dimensions of 33 x 33 x 96Å (Haurowitz, 1963). In spite of the high molecular weight of bovine albumin and its net negative charge at pH 5.8, its distribution coefficient was near unity for the anionic gel. This result suggests that nonuniform charge distribution on the surface of the protein controls the behavior (patch-controlled behavior). To test this hypothesis, cylindrical gel particles of different length to diameter ratios were used to concentrate protein solutions at different pH-values. The results are given in Table 6.2. Knowing

that the area to volume ratio for a cylinder is

$$\frac{area}{volume} = \frac{4}{d}[1 + \frac{1}{2(L/d)}] \tag{6.6}$$

the ratio of $(area)_{short}/(area)_{long}$ cylinders was about two. The distribution coefficient of β-lactoglobulin at pH 5.8 was the same for short or long gel particles, indicating that the net charge of the protein was responsible for the interaction between solute and ionic network. At pH 2.9, however, the net charge of the protein was positive and it adsorbed on the surface of the negatively charged hydrogel. This surface adsorption increased the distribution coefficient above that at pH 5.8.

Table 6.2: Exclusion behavior of NIPA gels for protein concentration*.

Solute	MW	pH	Nonionic gel			Ionic gel		
			$\frac{M}{M_d}$	K_d		$\frac{M}{M_d}$	K_d	
				$\frac{L_o}{d_o} \sim 5$	$\frac{L_o}{d_o} \sim 0.5$		$\frac{L_o}{d_o} \sim 5$	$\frac{L_o}{d_o} \sim 0.5$
β-Lactoglobulin	37000	5.8	–	–	–	150	0.19	0.19
(pI= 4.5–5.5)		2.9	–	–	–	52	0.66	3.22
Ovalbumin	45000	5.8	–	–	–	132	0.01	0.01
(pI= 4.6)		3.5	–	–	–	53	5.01	7.20
Bovine albumin	66000	11	28	0.0	0.0	105	0.05	0.07
(pI= 4.8)		5.8	28	0.0	0.0	127	0.82	1.85
		3.8	26	0.0	0.0	60	3.82	8.30

*C_F= 0.25 g/L; d_o=0.24 cm; Ionic gel: NIPA+R–SO_3^- H^+ (8.4%T, 1.6%C, 5.0%I)

In addition, the surface adsorption yielded increased K_d for the low length-to-diameter ratio particles. Similar behavior was exhibited by ovalbumin. Bovine albumin was completely excluded by the nonionic NIPA gel for both short and long gel particles, thus indicating no surface adsorption. With ionic gel at pH 5.8 the distribution coefficient of bovine albumin was larger for the shorter particles, indicating that the nonuniform charge distribution on the surface of the protein (patch-controlled mechanism) was important. At pH 3.8 the net charge of the bovine albumin was positive and there was strong adsorption by the negatively charged network. The distribution coefficient increased with decreasing length-to-diameter ratio because more surface area was available. However at pH 11, well above the isoelectric point

of bovine albumin, the solute was almost completely excluded, but there existed some surface adsorption which might be due to hydrophobic interactions. The effect of network charge on the exclusion behavior of cytochrome C (MW = 12330, pI = 9.3) with copolymer gels of NIPA and R-SO_3^- H^+ is shown in Fig. 6.9. At pH 5.5, which was well below the isoelectric point of solute, positively charged cytochrome C exhibited high affinity toward the ionic and nonionic gels which increased with increasing the amount of ionizable monomer. The large distribution coefficient of the solute for the nonionic gel (i.e. 0.0% ionizable monomer) indicates that in addition to the electrostatic interaction, hydrophobic interactions were involved between cytochrome C and the gel network.

6.5 Effect of Solute Concentration

It is of practical importance to show to what extent dilute aqueous solutions of biological products can be concentrated efficiently. The effect of solute concentration on the distribution coefficient of ovalbumin is shown in Fig. 6.10. At low concentrations (i.e. $C_R \leq 1g/L$) ovalbumin was completely excluded by copolymer gel of NIPA and R-SO_3^- H^+ (8.6%T, 1.3%C, 7.5%I). However, the distribution coefficient increased, i.e. the efficiency of solute exclusion decreased, with increasing ovalbumin concentration in the feed even though the swelling ratio decreased. This result implies that dilute solutions can be more concentrated in consecutive cycles, but that exclusion efficiency decreases as cycles continue. The increased distribution coefficient is attributed to the entrainment of small amounts of the retentate between gel particles due to the increased viscosity of more concentrated solutions. Increasing the gel bead size and washing the swollen gel should reduce the effect of solute entrainment.

6.6 Enzyme Concentration

The swelling and exclusion behavior of nonionic NIPA gel (8.1%T, 1.6%C) and copolymer gel of NIPA and R-SO_3^- H^+ (8.4%T, 1.6%C, 5.0%I) for concentration of 0.25 g/L lipase (wheat germ) solution at different pHs and salt concentrations are compared in Table 6.3. The swelling ratios, M/M_d, for both nonionic and ionic NIPA gels, swollen in enzyme solutions having salt concentrations typical of fermentation broths,

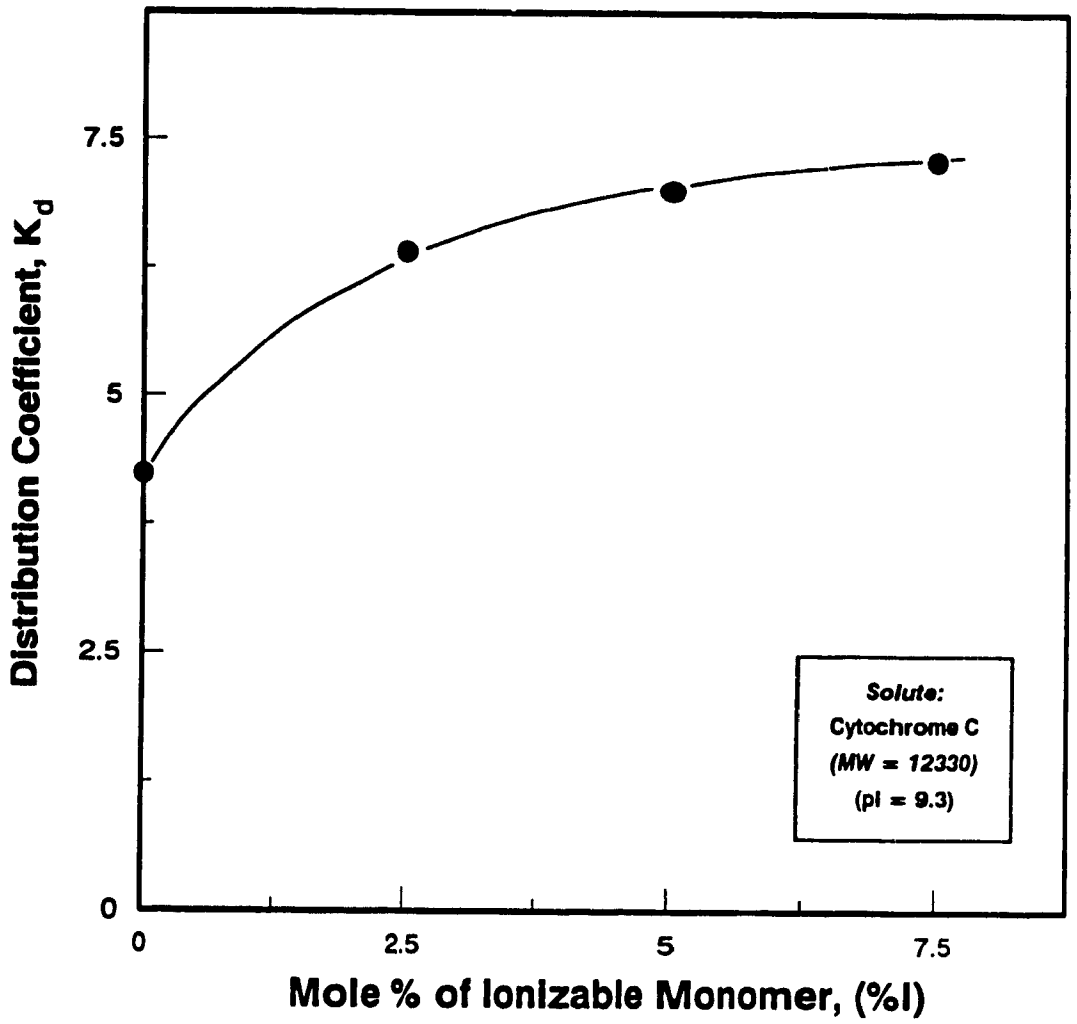


Figure 6.9: Charge effect on the exclusion behavior of NIPA gels.

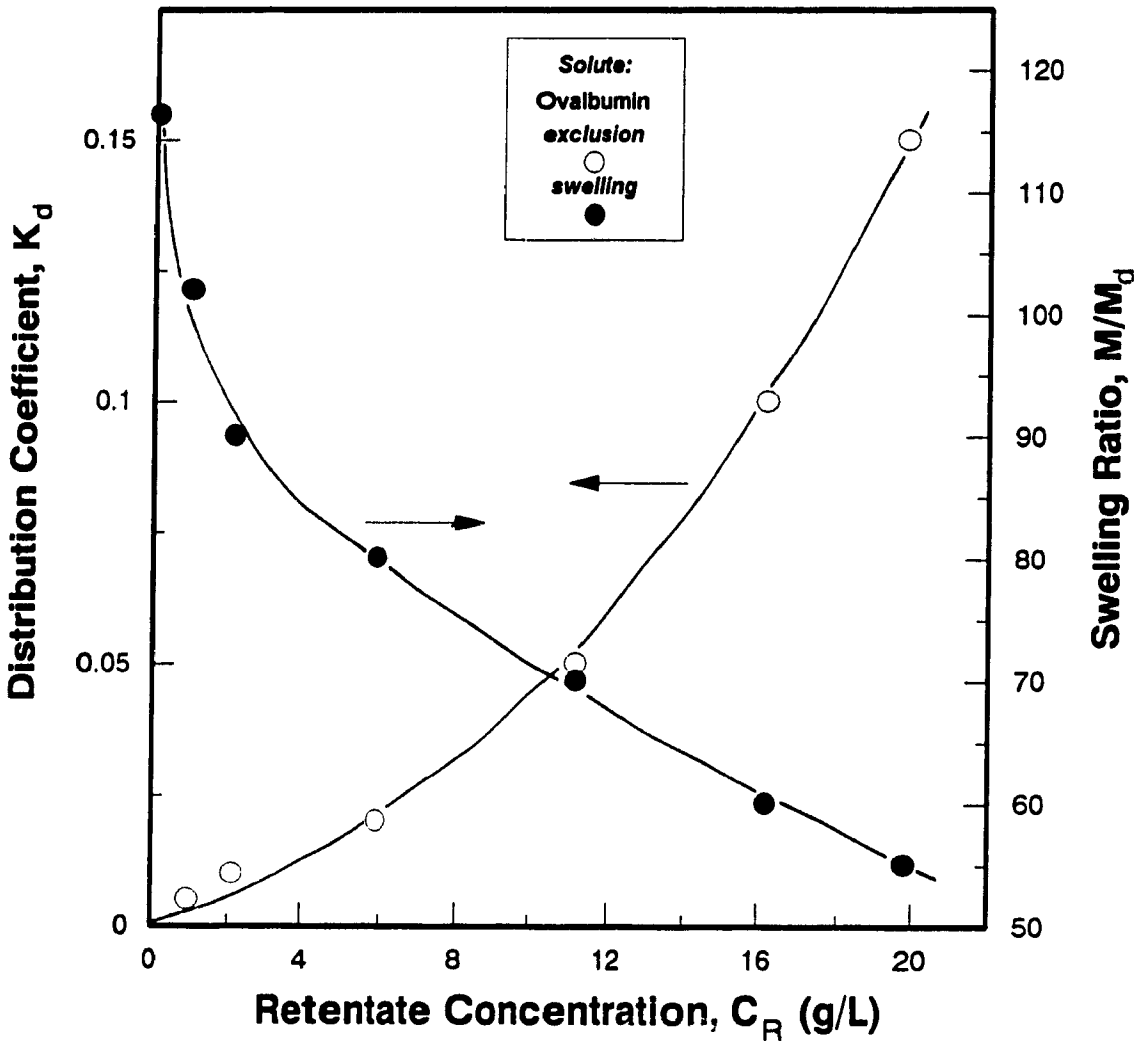


Figure 6.10: Effect of solute concentration on the swelling and exclusion behavior of copolymer gel of NIPA and R-SO_3H (7.5%l).

were essentially equal, but the lipase was more excluded by the nonionic gel.

Table 6.3: Swelling and Exclusion behavior of NIPA gels for concentration of lipase (wheat germ) at 6°C (±1°C).

pH	NIPA gel		NIPA+R-SO_3^- H^+ gel	
	M/M_d	K_d	M/M_d	K_d
8.0*	24	0.25	27	0.32
6.0*	23	0.20	24	0.38
4.0*	23	0.20	23	0.73
4.8†	27	0.27	28	0.77
4.4‡	22	0.20	24	0.75

*Tris-HCl buffer † 0.1 M NaCl + 0.005 M $CaCl_2$
‡0.1 M KH_2PO_4 + 0.004 M $MgSO_4$

Table 6.4 gives the swelling and exclusion behavior of nonionic NIPA gel (8.1%T, 1.6%C) and ionic copolymer gel of NIPA and sodium vinylsulfonate (8.2%T, 1.6%C, 5.0%I) in solutions of different enzymes in the presence of salts which are frequently encountered in buffered enzyme solutions.

Table 6.4: Exclusion behavior of NIPA gels for concentration of 0.25 g/L enzyme solutions at 6°C (±1°C).

Solute	MW	NIPA gel		NIPA+Na-vinylsulfonate gel	
		$\frac{M}{M_d}$	K_d	$\frac{M}{M_d}$	K_d
α-Amylase*	~ 52500	20	0.21	57	0.40
α-Amylase†	~ 52500	18	0.17	19	0.20
Lipase* (*Candida cylindracea*)	~ 100000	19	0.02	37	0.20
Lipase† (wheat germ)	unknown	19	0.27	23	0.28
β-Galactosidase*	466140	24	0.00	70	0.00
β-Galactosidase†	466140	17	0.00	20	0.00

*distilled water at pH 5.8
†9.3×10^{-2} M KH_2PO_4 + 3.9×10^{-2} M $MgSO_4$
‡5.6×10^{-2} M KH_2PO_4 + 2.3×10^{-2} M $MgSO_4$

The β–galactosidase was completely excluded by both ionic and nonionic gels. Lipase (wheat germ) was only partially excluded by both ionic and nonionic gels. Lipase (*Candida cylindracea*) was excluded almost completely by the nonionic gel, but it was partially ($K_d = 0.2$) excluded by the ionic gel. α–Amylase was more excluded by the nonionic gel, but complete exclusion was not achieved with either gel.

One criterion which must be met by a successful exclusion process is that the process not affect the activity of the enzyme upon concentration of dilute aqueous solution. To test the effect of the process on the activity of enzymes, the retentate (raffinate) concentration was determined and then this solution was diluted to yield a solution with a concentration equal to that of the feed. The activity of the diluted retentate and that of the feed were measured using standard assays as described in Chapter 2. The activity of α–amylase was about 720 $\frac{\text{Somogyi Units}}{\text{mg solid}}$ in both feed and retentate. The lipase activity in both feed and raffinate was about 12.4 $\frac{\text{Sigma–Tietz Units}}{\text{mg solid}}$ The enzymatic activity of the β–galactosidase was 105 $\frac{\text{Units}}{\text{mg solid}}$ in both feed and the retentate. These results indicate that the concentration process using NIPA gels has no adverse effect on the activity of these enzymes.

Chapter 7

CONCLUSIONS AND RECOMMENDATIONS

7.1 Conclusions

The present study was related to the potential application of hydrogels to concentrate dilute aqueous solutions of biological products in a gel-based filtration. The success of the proposed process is heavily dependent upon the swelling and exclusion behavior of crosslinked polymer gels which are used as extraction solvents. Copolymer gels of acrylamide and its derivatives were prepared and factors affecting their swelling and exclusion behavior were studied.

Crosslinked polymer gels, whose swelling is sensitive to the temperature or concentration of the surrounding medium, can be exploited to concentrate dilute aqueous solutions of macromolecular solutes including proteins (enzymes) in a gel extraction process. The efficiency of solute exclusion, based on the size of the solute and the pore size of the network, increased as the polymer volume fraction increased. The variable with the largest effect on the size exclusion behavior of hydrogels was the monomer concentration at gel formation.

The simple ionic solutes were more excluded by polyelectrolyte gels based on the Donnan equilibrium. The mechanism of interaction between biological products and polyelectrolyte gels was not solely determined by the net charge of the solute, but the nonuniform charge distribution on the surface of biological macromolecules plus hydrophobic interactions may play an important role in a gel-based separation processes.

Nonionic N-isopropylacrylamide (NIPA) gel, which exhibited the least interaction with biological materials, appeared promising because it is easily regenerated at 35°C without contamination of the gel by chemical additives. The low energy

requirements, low equipment cost and reusability of this gel might be exploited to develop a separation process which seems competitive with widely used ultrafiltration and precipitation methods. The potential superiority of the gel–based filtration can be easily imagined for the concentration of dilute aqueous solutions of biological products which are highly sensitive to process condition, e.g. enzyme concentration while the activity of the microorganisms are preserved. However, gel–based filtration have some limitations. Swelling of NIPA gels decreases in the presence of appreciable amount of organic solvents (Hirokawa and Tanaka, 1984; Otake et al., 1990) or small solutes (Freitas and Cussler, 1987). Multiple swelling which is required to get more concentrated solutions of large solutes is limited because of corresponding loss in yield at each cycle.

The solution pH had a profound effect on the swelling equilibrium of gels containing weakly acidic groups. The degree of swelling of an ionic gel in a salt solution was determined largely by the concentration of the mobile counterions. At a fixed composition counterions of higher charge caused a larger shrinkage of the gel. Complex formation increased this effect. Small concentrations of counterions, present from acid or base added to adjust pH determined the swelling behavior. NIPA gels exhibited larger volume changes from the collapsed state upon swelling in 10^{-3} M $Ca(NO_3)_2$ solution than acrylamide gels, although the latter exhibited larger equilibrium degree of swelling. The cationic copolymer gel of acrylamide and 3–(methacrylamido)propyltrimethylammonium chloride exhibited irreversible swelling at pH $>$ 8. The monomer concentration (%T), crosslinking ratio (%C) and proportion of ionizable monomer (%I) at gel formation affected the swelling behavior of hydrogels. The degree of swelling decreased with increasing monomer concentration or crosslinking ratio whereas it increased with increasing amounts of ionizable monomer at gel formation.

A thermodynamic model based on Flory's theory and on an additivity rule for the osmotic pressure of polyelectrolyte–salt solutions described the effect of salt on the swelling of ionic gels. Theoretical predictions agreed with the experimental results for swelling of ionic gels in mono–monovalent salt solutions. Assuming the osmotic coefficient equal to unity and neglecting the non–Gaussian distribution of chain extension gave predictions well above the data at low salt concentrations. The effect of bivalent counterions on the swelling of ionic gels was also well represented by

the theory if no complex formation occurred. The effect of a salt solution containing both mono– and bivalent counterions on swelling behavior was predicted at low salt concentration using ideal Donnan equilibrium.

The volume of anionic copolymer gels of acrylamide and NIPA swollen in distilled water at pH < 9 after preparation, went through a maximum followed by a gradual approach to equilibrium. The maximum value of the swelling ratio in an unstirred solution exceeded that in a stirred solution, thus indicating that external film diffusion had some influence. Anionic gels containing weakly acidic monomers exhibited a maximum volume upon swelling in a finite volume of solution of initial pH of 5.7, followed by a decrease in volume and then a gradual increase to the equilibrium value. Preswollen copolymer gels of acrylamide and sodium acrylate which were partially collapsed by subjecting them to a centrifugal force went through a maximum during the swelling step, which was about twice the former equilibrium value, followed by a gradual approach to equilibrium state.

Deformed and undeformed NIPA gels exhibited Fickian behavior upon swelling, but the rate of solvent uptake was higher for the deformed gel. Although the swelling of NIPA gels was Fickian their shrinking at 35°C was non-Fickian. The rate of swelling was slower than the rate of collapse. The swelling of copolymer gels of acrylamide was non-Fickian but faster than that of NIPA gels.

Pattern formation on the surface of the ionic gels was observed upon swelling. Buckling of a thin film of the gel during confined swelling was so intense it resulted in tearing of the gel.

A Fickian mathematical model, which describes kinetics of swelling and collapse, was developed through the use of a material coordinate and a chemical potential driving force. By an approximation, explicit relationships between diffusion coefficients in polymer material coordinates (Lagrangian) and in laboratory coordinates (Eulerian) for cylindrical and spherical geometries were obtained. The theoretical predictions agreed well with experimental data for swelling of gels which exhibited Fickian behavior.

7.2 Contributions to Knowledge

The contributions to knowledge resulting from this study are:

(1) - Concentration of dilute aqueous solutions of the following enzymes with no effect on the enzyme activities:

(a) - α-amylase

(b) - lipases from wheat germ and *Candida cylindracea*

(c) - β-galactosidase

(2) - Preparation of the following polymeric gels:

(a) - copolymer gels of acrylamide and 2-acrylamido-2-methyl-1-propanesulfonic acid ($R\text{-}SO_3^-\ H^+$)

(b) - copolymer gels of N-isopropylacrylamide (NIPA) and $R\text{-}SO_3^-\ H^+$

(c) - copolymer gel of NIPA and sodium vinylsulfonate

and studying their swelling and exclusion behavior.

(3) - Demonstration of anomalous behavior of deformed hydrogels upon swelling as follows:

(a) - copolymer gels of acrylamide and sodium acrylate, which were collapsed by applying centrifugal force on gels at equilibrium swelling, went through a maximum during the swelling step which was about twice its former equilibrium value.

(b) - the rate of solvent uptake was higher for deformed NIPA gel.

(4) - Observation of following phenomena upon swelling of anionic gels:

(a)-anionic copolymer gels of NIPA and sodium acrylate exhibited a volume overshoot, upon swelling in a finite volume of solution of initial pH of 5.7, followed by a gradual increase to the equilibrium value.

(b)-buckling of the surface of a thin film of copolymer gel of acrylamide and sodium acrylate upon confined swelling was so intense which resulted in tearing of the gel.

(5) - Extension of a thermodynamic model for swelling of polyelectrolyte gels using Flory's theory and an additivity rule for the osmotic pressure of polyelectrolyte-salt solutions with the inclusion of the effect of:

(a) - noncomplexing bivalent counterions

(b) - a salt solution containing both mono- and bivalent counterions

(6) - Development of a mathematical model for the kinetics of the swelling and shrinking of planar (slab), cylindrical and spherical gels, using polymer material coordinates (PMC), with the following features:

(a) – simple mathematical derivation

(b) – explicit expressions of diffusion coefficient in PMC for cylindrical and spherical gels

7.3 Recommendations

The results of this work suggest the following avenues for further research.

— Study of elastic and swelling behavior of nonionic NIPA gel and anionic and cationic copolymer gels of NIPA with different amounts of monomers, crosslinking agent and ionizable monomer at gel formation in the presence of mono– and bivalent counterions at different pHs. Comparison of resulting experimental data with thermodynamic and kinetic theories presented in this work.

— Determination of the phase behavior of ionic copolymer gels of N-isopropylacrylamide (NIPA), particularly cationic copolymers which can be prepared with weakly basic monomers.

— Study of the effect of pH on the irreversible swelling of cationic gel at pH > 8 which can be started by investigation of physical properties of the linear copolymer solution.

— Study of the effect of an electric field on the swelling behavior of cationic gels and on the swelling behavior of anionic gels prepared with highly acidic monomers.

Nomenclature

A_{420}	=	absorbance at 420 nm
A_i	=	$L^{-1}(\gamma_i)$, where $L^{-1}(\gamma)$ is the inverse Langevin function of γ
a	=	$C_{M^{2+}}/(C_{Na^+})^2$
a	=	unknown parameter
a_i	=	activity of species 'i'
$\overline{a_i}$	=	activity of species 'i' in the gel phase
b	=	unknown parameter
C	=	concentration
C'	=	dextran concentration (% w/v)
$\overline{C'}$	=	solute concentration in the gel phase
C'_A	=	molarity of anion in the external solution
$\overline{C'_A}$	=	molarity of anion in the gel phase
C'_C	=	molarity of cation in the external solution
$\overline{C'_C}$	=	molarity of cation in the gel phase
C'_E	=	extract concentration
C'_f	=	polymer concentration at equilibrium swelling (g/cm^3)
C'_F	=	feed concentration
C'_i	=	molarity of mobile ions in the external solution
$\overline{C'_i}$	=	molarity of mobile ions in the gel phase
C'_R	=	retentate concentration
C'_o	=	polymer concentration at gel formation (g/cm^3)
$\%C'$	=	percent (w/w) of crosslinker
c	=	unknown parameter
D	=	diffusion coefficient
D	=	diffusion coefficient in material coordinate
D_c	=	cooperative diffusion coefficient in Eulerian coordinate
D_T	=	mobility coefficient (Thermodynamic diffusion coefficient)
D_t	=	tracer or self-diffusion coefficient
d	=	diameter of gel particle
d_0	=	diameter of gel particle at t = 0
d_r	=	collapsed gel diameter
d_o	=	diameter of gel particle at formation
E	=	Donnan Potential
F	=	Faraday constant
F	=	fractional approach to equilibrium
F_a	=	average of the experimental values of the fractional approach to equilibrium
$F_{exp}(t)$	=	experimental value of the fractional approach to equilibrium
$F(t)$	=	theoretical value of the fractional approach to equilibrium
f	=	frictional coefficient
I	=	fraction of ionic sites neutralized by Na^+
O	=	objective function

ΔG = Gibbs free energy change
ΔG_1 = free energy change due to mixing of polymer with solvent
ΔG_2 = elastic free energy change
ΔG_3 = free energy change due to mixing of ions with solvent
%I = mole percent ionizable monomer
K = parameter
K_d = distribution coefficient
k = elastic modulus
k = Boltzmann's constant
L = length of the polarimeter tube
L = length of gel particle
L_o = length of gel particle at formation
l = characteristic sample dimension
M = swollen gel mass at equilibrium
M_d = dry mass of polymer
M_e = equilibrium swollen mass of the hydrogel before subjecting to centrifugal force in the first cycle
M_r = regenerated mass of gel
M_s = swollen gel mass
M_t = swollen gel mass at time t
$M_{t=0}$ = mass of gel particle at time equal to zero
M_o = original mass of gel at preparation
$M(t)$ = mass of penetrant at time t
$M(\infty)$ = mass of penetrant sorbed at equilibrium
m = material coordinate (cumulative volume)
m_0 = maximum value of m
N_A = Avogadro's number
N_p = number of the polymer chains
NP = number of experimental points
n = diffusional exponent
n = number of statistical segments per chain
n_i = molarity of component i
n_m = molarity of monomer
n_m° = molarity of monomer at gel formation
n_p = molarity of polymer
n_s = molarity of salt in polyelectrolyte salt mixture
n_+ = molarity of counterions for negatively charged polymer/bi-monovalent salt system
n_- = molarity of co-ion for negatively charged polymer /bi-monovalent salt system
P = pressure
P° = pressure at standard state
R = gas constant
r = gel radius
r = Eulerian spatial coordinate

r_0 = initial half thickness of the slab and/or the initial radius of the cylindrical or spherical gel, in laboratory coordinates
r_c = correlation coefficient
$r(0)$ = initial gel radius
$r(\infty)$ = gel radius at equilibrium
$r(t)$ = gel radius at time t
T = absolute temperature
T_g = glass transition temperature
%T = percent (w/v) of total monomers at preparation
t = time
u_s = velocity of solvent relative to the matrix
u_j = superficial velocity of each phase
u_p = superficial velocity of polymeric matrix
u_s = superficial velocity of solvent
V = gel volume
V_1 = molar volume of solvent
V_F = feed volume
V_R = retentate volume
v = polymer volume fraction
v_o = polymer volume fraction at gel formation
X = swelling ratio, M/M_o
χ = concentration of dissociated monomers/concentration of salt in polyelectrolyte–salt system
x = material space coordinate
x_0 = boundary of gel during swelling or shrinking in material coordinate
Z_A = charge of anion
Z_C = charge of cation
Z_i = charge of species 'i'

Greek Letters

α = degree of ionization
$[\alpha]_D^{25}$ = specific optical activity at 25°C
$< \alpha_o^2 >$ = dilation factor in the dry isotropic state
β_n = the positive roots of the zero order Bessel function of the first kind
γ_1 = $< \alpha_o^2 >^{1/2} v^{-1/3} n^{-1/2}$
γ_3 = $< \alpha_o^2 >^{1/2} n^{-1/2}$
ζ = spatial integration variable
η = fraction of solute excluded
η_i = electrochemical potential of each electrolyte in the external solution
$\overline{\eta_i}$ = electrochemical potential of each electrolyte in the gel
θ_j = volume fraction of each phase
θ_p = volume fraction of polymeric matrix
θ_s = volume fraction of solvent
ν = solvent ratio

ϑ_0 = initial value of solvent ratio
ϑ_e = equilibrium value of solvent ratio
ϑ_o = solvent ratio at gel formation
$\vartheta(r,t)$ = spatial distribution of solvent ratio at a specific time in Eulerian coordinate
$\vartheta(x,t)$ = spatial distribution of solvent ratio at a specific time in PMC
$\vartheta_{Theory}(t)$ = ratio of total volume of the solvent to that of polymer matrix
μ = chemical potential of solvent
$\mu^\circ(\mathrm{P}^\circ)$ = chemical potential of solvent in the standard state
μ_i = chemical potential of species 'i'
$\mu_i^\circ(\mathrm{P}^\circ)$ = chemical potential of species 'i' in the standard state
ν_d = concentration of constituent chains per unit volume in dry state
ν_o = concentration of constituent chains per unit volume at gel formation
Π = osmotic pressure of polyelectrolyte/salt solution
π = osmotic pressure
π_{ideal} = ideal osmotic pressure
π_p = osmotic pressure of salt–free polyelectrolyte solution
π_s = osmotic pressure of polyelectrolyte–free salt solution
π_1 = osmotic pressure of polymer solution
π_2 = elastic component of osmotic pressure
π_3 = osmotic pressure of mobile ions
ρ_g = gel density
ρ_p = polymer density
σ = shear modulus
τ = time integration variable
Φ = osmotic coefficient of gel phase
ϕ = osmotic coefficient of external solution
ϕ_p = osmotic coefficient of salt- free polyelectrolyte solution
ϕ_s = osmotic coefficient of polyelectrolyte–free salt solution
χ = polymer–solvent interaction parameter
Ψ = optical rotation of solution
ψ = electric potential of external solution
$\overline{\psi}$ = electric potential of gel

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Appendix A

DERIVATION OF EQ. (4.13)

Since $\vartheta = \vartheta(m,t)$,

$$d\vartheta = (\frac{\partial \vartheta}{\partial m})_t dm + (\frac{\partial \vartheta}{\partial t})_m dt \tag{A.1}$$

Because m = m(r,t),

$$dm = (\frac{\partial m}{\partial r})_t dr + (\frac{\partial m}{\partial t})_r dt \tag{A.2}$$

Substitution for dm from Eq. (A.2) into Eq. (A.1) yields

$$d\vartheta = (\frac{\partial \vartheta}{\partial m})_t (\frac{\partial m}{\partial r})_t dr + (\frac{\partial \vartheta}{\partial m})_t (\frac{\partial m}{\partial t})_r dt + (\frac{\partial \vartheta}{\partial t})_m dt \tag{A.3}$$

or

$$d\vartheta = (\frac{\partial \vartheta}{\partial r})_t dr + [(\frac{\partial \vartheta}{\partial m})_t (\frac{\partial m}{\partial t})_r + (\frac{\partial \vartheta}{\partial t})_m] dt \tag{A.4}$$

Since $\vartheta = \vartheta(r,t)$, Eq. (A.4) is an exact differential and it follows that

$$(\frac{\partial \vartheta}{\partial t})_r = (\frac{\partial \vartheta}{\partial m})_t (\frac{\partial m}{\partial t})_r + (\frac{\partial \vartheta}{\partial t})_m \tag{A.5}$$

or

$$(\frac{\partial \vartheta}{\partial t})_r = (\frac{\partial \vartheta}{\partial r})_t (\frac{\partial r}{\partial m})_t (\frac{\partial m}{\partial t})_r + (\frac{\partial \vartheta}{\partial t})_m \tag{A.6}$$

Substitution for $(\frac{\partial r}{\partial m})_t$ and $(\frac{\partial m}{\partial r})_r$ from Eqs. (4.14) and (4.15) into Eq. (A.6) yields

$$(\frac{\partial \vartheta}{\partial t})_r = -\frac{u_p}{\theta_p} (\frac{\partial \vartheta}{\partial r})_t + (\frac{\partial \vartheta}{\partial t})_m \tag{A.7}$$

Equation (A.7) is identical to Eq. (4.13).

Appendix B

NUMERICAL SOLUTION TECHNIQUES

The implicit finite difference method was used to solve the non-linear partial differential equation expressed by Eq. (5.7).

B.1 Implicit Method

Since stability problems may develop in Explicit finite difference method, small time steps must be employed resulting in long computational times. The use of an Implicit method is more practical. In this method all spatial derivatives were differenced at time level n+1 and the central difference scheme was used. The difference between the Explicit and the Implicit methods is that the spatial derivatives are differenced at the time level n in the former method.

Equation (5.7) for a spherical geometry is rearranged to

$$\frac{\partial \vartheta}{\partial t} = \frac{2}{x} D(\vartheta) \frac{\partial \vartheta}{\partial x} + \frac{\partial}{\partial x}[D(\vartheta) \frac{\partial \vartheta}{\partial x}] \tag{B.1}$$

Since $\frac{\partial \vartheta}{\partial x}|_{x=0} = 0$, the first term on the right-hand side of Eq. (B.1) must be determined by L'Hopital's rule:

$$\lim_{x \to 0}(\frac{\partial \vartheta}{\partial x}/x) = \frac{\partial^2 \vartheta}{\partial x^2} \tag{B.2}$$

Hence, for $x = 0$, Eq. (B.1) becomes

$$\frac{\partial \vartheta}{\partial t} = 2D(\vartheta) \frac{\partial^2 \vartheta}{\partial x^2} + \frac{\partial}{\partial x}[D(\vartheta) \frac{\partial \vartheta}{\partial x}] \tag{B.3}$$

For $x \neq 0$, Eq. (B.1) can be written in a finite difference form as follows.

$$\frac{\vartheta_i^{n+1} - \vartheta_i^n}{\delta t} = \frac{2}{x_i} D_i^{n+1} \frac{\vartheta_{i+1}^{n+1} - \vartheta_{i-1}^{n+1}}{2\delta x} + \frac{A}{(\delta x)^2} \tag{B.4}$$

where

$$A = D_{i+1/2}^{n+1} \vartheta_{i+1}^{n+1} - (D_{i+1/2}^{n+1} + D_{i-1/2}^{n+1}) \vartheta_i^{n+1} + D_{i-1/2}^{n+1} \vartheta_{i-1}^{n+1} \tag{B.5}$$

and

$$D_{i+1/2}^{n+1} = \frac{1}{2}(D_i^{n+1} + D_{i+1}^{n+1}) \tag{B.6}$$

and

$$D_{i-1/2}^{n+1} = \frac{1}{2}(D_i^{n+1} + D_{i-1}^{n+1}) \tag{B.7}$$

The subscript i and superscript n correspond to the x-direction and time index, respectively. For x=0, Eq. (B.3) can be written in a finite difference form as follows.

$$\frac{\vartheta_i^{n+1} - \vartheta_i^n}{\delta t} = 2D_i^{n+1}\frac{\vartheta_{i+1}^{n+1} - 2\vartheta_i^{n+1} + \vartheta_{i-1}^{n+1}}{(\delta x)^2} + \frac{A}{(\delta x)^2} \tag{B.8}$$

The boundary condition at x=0 implies that $\vartheta_{i+1}^{n+1} = \vartheta_{i-1}^{n+1}$, hence Eqs. (B.6)-(B.8) can be written as

$$D_{i+1/2}^{n+1} = D_{i-1/2}^{n+1} = \frac{1}{2}(D_i^{n+1} + D_{i+1}^{n+1}) \tag{B.9}$$

and

$$\frac{\vartheta_i^{n+1} - \vartheta^n}{\delta t} = \frac{4}{(\delta x)^2}D_i^{n+1}(\vartheta_{i+1}^{n+1} - \vartheta_i^{n+1}) + \frac{1}{(\delta x)^2}[(D_i^{n+1} + D_{i+1}^{n+1})(\vartheta_{i+1}^{n+1} - \vartheta_i^{n+1})] \tag{B.10}$$

In the above equations, δx and δt are the step length and the time interval, respectively. By rearrangement of the above equations, the following expressions are obtained.

$$AT_i\vartheta_{i-1}^{n+1} + BT_i\vartheta_i^{n+1} + CT_i\vartheta_{i+1}^{n+1}) = DT_i \tag{B.11}$$

where for $x \neq 0$

$$AT_i = \frac{D_i^{n+1}}{(x_i\delta x)} - \frac{(D_i^{n+1} + D_{i-1}^{n+1})}{2(\delta x)^2} \tag{B.12}$$

$$BT_i = \frac{1}{\delta t} + \frac{D_i^{n+1}}{(\delta x)^2} + \frac{(D_{i-1}^{n+1} + D_{i+1}^{n+1})}{2(\delta x)^2} \tag{B.13}$$

$$CT_i = -\frac{D_i^{n+1}}{(x_i\delta x)} - \frac{(D_i^{n+1} + D_{i+1}^{n+1})}{2(\delta x)^2} \tag{B.14}$$

$$DT_i = \vartheta_i^n/\delta t \tag{B.15}$$

and for x=0

$$AT_i = 0 \tag{B.16}$$

$$BT_i = \frac{1}{\delta t} + 5\frac{D_i^{n+1}}{(\delta x)^2} + \frac{D_{i+1}^{n+1}}{(\delta x)^2} \tag{B.17}$$

$$CT_i = -5\frac{D_i^{n+1}}{(\delta x)^2} - \frac{D_{i+1}^{n+1}}{(\delta x)^2} \tag{B.18}$$

$$DT_i = \vartheta_i^n/\delta t \tag{B.19}$$

Applying Eq. (B.11) to all nodes, a system of nonlinear algebraic equations in terms of the solvent ratios at time level $n+1$ is obtained. These equations can be solved by the Thomas-algorithm (Lapidus, 1962) with iteration to obtain the spatial distribution

of solvent ratio at a specific time, $\vartheta(x,t)$. The numerical values of $\vartheta_{Theory}(t)$ in Eq. (5.28) for a sphere are obtained by

$$\vartheta_{Theory}(t) = \frac{3}{x_0^3}\int_0^{x_0} \vartheta(x,t)x^2 dx \qquad \text{(B.20)}$$

where x is the radius of each grid point and x_0 is the maximum value of x, both in polymer material coordinates. This integration is performed using Simpsons' rule.

B.1.1 Thomas-algorithm with Iteration

The following steps were followed.

1- Guess the initial values for solvent ratios at time step $n+1$. Applying initial and boundary conditions is the most appropriate guess for the first time step.

2- For all nodes, determine the coefficients AT_i, BT_i, CT_i and DT_i in Eq. (B.12) based on the guessed values in step 1.

3- Solve the system of algebraic equations obtained by the application of Eq. (B.11) to all nodes using the Thomas-algorithm for a tridiagonal system of linear equations.

4- Use the calculated solvent ratios at time level $n+1$ from step 3 as a new guess starting from step 1 and then continue until convergence is achieved.

B.1.2 Convergence and Accuracy of Numerical Results

Convergence for calculation of solvent ratio of each grid point (node) by Thomas-algorithm was achieved with ≤ 4 iterations. To test the global convergence of the numerical procedure different step lengths, δx, and time intervals, δt, were used. The numerical results for the fractional approach to equilibrium [Eq. (5.28)], F, obtained with different δx-values are given in Table B.1. These results indicate good convergence achieved at $\delta x = 2.175 \times 10^{-3}$ cm. The largest relative deviation between F-values calculated with this δx and the smallest δx was 0.52% at t= 5 min. Table B.2 gives the numerical values of F for different time intervals. Convergence was satisfactory at $\delta t = 10s$ with the largest relative deviation equal to 0.24% at t = 5 min.

To check the accuracy of the numerical method, the computational results with constant diffusion coefficient were compared with exact analytical solution, [Eq. (5.27)] in Table B.3. The percentage relative deviations between the numerical results and the exact solution were less than 0.07. Based on these results the time interval and step length for the subsequent computations were set equal to 10 s and 1.45×10^{-3} cm, respectively.

Table B.1: Comparison of numerical values of the fractional approach to equilibrium for different step lengths*.

Time (min)	F			$\frac{F_2-F_1}{F_2}$ ×100%
	$\delta x_1 = 1.45 \times 10^{-3} cm$	$\delta x_2 = 2.175 \times 10^{-3}$ cm	$\delta x_3 = 4.35 \times 10^{-3} cm$	
5	0.2122	0.2133	0.2195	0.52
10	0.2922	0.2929	0.2962	0.24
15	0.3506	0.3511	0.3550	0.14
20	0.3977	0.3980	0.4000	0.08
25	0.4376	0.4379	0.4395	0.07
30	0.4724	0.4726	0.4740	0.04
35	0.5033	0.5035	0.5046	0.04
40	0.5312	0.5314	0.5323	0.04
45	0.5566	0.5567	0.5575	0.02

*$\delta t = 10s$; $D = 2.8 \times 10^{-8} cm^2/s$; $x_0 = 0.0435cm$

Table B.2: Comparison of numerical values of the fractional approach to equilibrium for different time intervals†.

Time (min)	F			$\frac{F_1-F_2}{F_1}$ ×100%
	$\delta t_1 = 5s$	$\delta t_2 = 10s$	$\delta t_3 = 20s$	
5	0.2127	0.2122	0.2113	0.24
10	0.2926	0.2922	0.2916	0.14
15	0.3508	0.3506	0.3500	0.06
20	0.3979	0.3977	0.3972	0.05
25	0.4378	0.4376	0.4372	0.05
30	0.4726	0.4724	0.4720	0.04
35	0.5035	0.5033	0.5030	0.04
40	0.5314	0.5312	0.5309	0.04
45	0.5568	0.5566	0.5563	0.04

†$\delta x = 1.45 \times 10^{-3} cm$; $D = 2.8 \times 10^{-8} cm^2/s$; $x_0 = 0.0435cm$

Table B.3: Comparison of the numerical results with the exact analytical solution†

Time (min)	F		$(F_A - F_N)/F_A$ ×100%
	Analytical	Numerical*	
5	0.2122	0.2122	0.00
10	0.2924	0.2922	0.07
15	0.3507	0.3506	0.03
20	0.3979	0.3977	0.05
25	0.4378	0.4376	0.05
30	0.4726	0.4724	0.04
35	0.5036	0.5033	0.06
40	0.5314	0.5312	0.04
45	0.5568	0.5566	0.04

† $D = 2.8 \times 10^{-8} cm^2/s$

* $\delta x = 1.45 \times 10^{-3} cm$; $\delta t = 10s$

The computer programs on a floppy diskette are available from Professor *M. E. Weber.*[1]

Literature Cited

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