A Systematic Approach to Setting Underfrequency Relays in Electric Power Systems

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A thesis submitted to the Department of Electrical Engineering
in partial fulfillment of the requirements for the degree of
Master of Engineering

McGill University
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June 2008

Your file Votre référence
ISBN: 978-0-494-66958-7
Our file Notre référence
ISBN: 978-0-494-66958-7

ABSTRACT

Generation loss contingencies in electric power systems result in a deviation of system frequency from nominal, a condition which must be corrected promptly in order to prevent further degradation of the power system. Automatic load-shedding using underfrequency relays is one of the techniques used to correct abnormal frequency deviations and prevent the risk of uncontrolled outages. If sufficient load is shed following a contingency to preserve interconnections and keep generators on-line, the system can be restored with relative speed and ease. On the other hand, if a declining frequency condition is not dealt with adequately, a cascading disconnection of generating units may develop, leading to a possible total system blackout.

This thesis develops and tests a new systematic method for setting underfrequency relays offering a number of advantages over conventional methods. A discretized swing equation model is used to evaluate the system frequency following a contingency, and the operational logic of an underfrequency relay is modeled using mixed integer linear programming (MILP) techniques. The proposed approach computes relay settings with respect to a subset of all plausible contingencies for a given system. A method for selecting the subset of contingencies for inclusion in the MILP is presented. The goal of this thesis is to demonstrate that given certain types of degrees of freedom in the relay setting problem, it is possible to obtain a set of relay settings that limits damage or disconnection of generating units for each and every possible generation loss outage in a given system, while attempting to shed the least amount of load for each contingency.

ABRÉGE

Une méthode systématique pour établir des relais de fréquences dans un réseau électrique

Les éventualités de pertes de génération dans les réseaux électriques aboutissent à des changements de fréquence du système. Ce problème devrait être corrigé rapidement pour prévenir la dégradation du réseau électrique. En utilisant des relais de fréquences, les systèmes automatisés pour réguler la charge peuvent corriger les déviations de fréquence et prévenir les pannes de courant.

Cette thèse propose et évalue une nouvelle méthode systématique pour établir des relais de fréquences. Un modèle des équations d'oscillation discretisé dans le temps est utilisé pour estimer la fréquence de système suivant une perte de géneration; Des techniques de programmation linéaire mixtes sont utilisées pour étudier la logique d'opération des relais. Cette méthode calcule les conditions des relais quant à un groupe d'éventualités et diminue la quantité de charge delesté avec chaque éventualité.

For my family, who offered me unconditional love and support throughout the course of this thesis.

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LIST OF SYMBOLS

Indices

i Generating units, $i = 1,....,ng$

k Loads, $k = 1,....,nd$

j Contingencies, $j = 1,....,nc$

l Generator underfrequency thresholds, $l = 1,....,nl$

s Load-shedding stages, $s = 1,....,ns$

p Frequency setpoints for multiple setpoint relays, $p = 1,....,np$

n Integration time steps, $n = 1,....,nt$

Parameters

g_0^i Generation setpoint of unit i in the pre-contingency state (MW)

H_i Inertia constant of unit i (MJ/MW)

R_i Primary frequency regulation constant of unit i (MW/Hz)

d_k^0 Demand of load k in the pre-contingency state (MW)

f_i^l Underfrequency threshold l of generating unit i (Hz)

Δt_i^l Maximum permissible times below underfrequency threshold level l of unit i

Continuous Variables

Δg^j Generation loss resulting from contingency j

Δf_n^j System frequency deviation from nominal of trajectory j at time step n

Δr_n^j Primary frequency regulation available for trajectory j at time step n

f_s Frequency setpoint of underfrequency relay s

Continuous variables (continued)

Δd_s Amount of load shed by underfrequency relay s

Δt_s Underfrequency relay time delay for load-shedding stage s before load Δd_s is shed

Δt_{sn}^{j} Time spent by system frequency below frequency setpoint f_s of underfrequency relay s during contingency j at time step n

Δt_n^{jl} Time spent by system frequency below generator frequency threshold l during contingency j at time step n

0/1 Binary Variables

u_{sn}^{j} Equal to 1 if and only if relay s operates under contingency j at time step n

v_{sn}^{j} Equal to 1 if and only if the frequency deviates below underfrequency relay setpoint s under contingency j at time step n

w_n^{jl} Equal to 1 if and only if the frequency deviates below generator underfrequency threshold l under contingency j at time step n

I. INTRODUCTION

1.1 Motivation and Thesis Objectives

One of the primary concerns of the electric utility industry is the maintenance of maximum service reliability [1]. To achieve this goal, power systems are designed and operated in order to provide adequate generation and transmission capacity for any predicted system condition. Even under emergency conditions requiring corrective control actions [2], power systems are devised to provide electricity with an appropriate degree of reliability in terms of the amount of load served and the duration of the outage. System operators frequently conduct reliability assessments [3] based on lists of specific emergency conditions and make provisions for the dispatch of preventive control actions [4] that eliminate violations of operational constraints. These assessments are typically based on deterministic criteria such as the "$N-1$" criterion, introduced after the 1965 Northeast USA blackout [5]. In its simplest form, the $N-1$ criterion dictates that power systems should be able to withstand the loss of any single element without jeopardizing system operation. This criterion is prevalent in power system operation today.

However, regardless of how meticulous the design procedure is, the unpredictable nature of power systems entails a low yet finite probability of the occurrence of contingencies not included in the postulated security criterion, such as the loss of multiple network elements, the alleged "$N-k$" contingencies. In addition, it is possible that preventive control methods will be inadequate when dealing with severe $N-1$ contingencies such as the loss of a critical tie-line in an interconnected power system, or the loss of a large generating unit in an isolated power system. These contingencies usually result in an imbalance between generation and load, a condition which is characterized by a decaying system frequency [1]. Declining system frequency affects the performance of the remaining generating units and power plant auxiliaries [6], and must be corrected promptly in order to prevent further degradation of the power system. Corrective control actions [4] such as the deployment of generation dispatch or automatic load-shedding [7],[8] must be used to recover the active power imbalance and arrest the decline of system frequency. The first alternative, increasing generation, can never be deployed fast enough to prevent a substantial decrease in system frequency; in the extreme case, there may not be sufficient spinning reserves [9] to meet the additional demand. The second alternative, automatic load-shedding based on low frequency, is a quick and effective technique for attaining generation-load balance and restoring the system frequency to normal. Load-shedding is achieved using underfrequency relays that shed increments of load at specific frequency thresholds. The goal of an underfrequency load-shedding scheme is to disconnect the smallest amount of load possible that corrects the frequency to a safe range rapidly enough so that generators are not subjected to excessively low operation frequencies for extended periods of time.

This thesis developed from discussions pertaining to the modeling of underfrequency relays in the analysis of contingencies conducted by Zaag et al in [10]. In that study, the frequency deviations following a severe contingency were approximated using a quasi steady-state assumption, and underfrequency relays were modeled as switches that disconnected loads (in a preset priority order) when the system frequency in an island dropped below a safe operating limit (59.5 Hz). The load-shedding model used in [10] ignored frequency transients following a contingency, and was therefore acknowledged to be overly optimistic; it was noted that such frequency transients might trigger more load-shedding than predicted by the quasi-steady state model.

A literature review on appropriate modeling of underfrequency relays in power systems revealed that a standard for setting these relays does not exist [7], [11]. The approach followed by most utilities to obtain appropriate relay settings is to perform an iterative series of transient stability study trials based on historical experience and heuristics [11].

The objective of this dissertation is to develop and test an innovative systematic approach for setting under-frequency relays that does not require repeated trial-and-error studies, relying instead on the solution of a mixed integer linear program (MILP). Another original contribution of the proposed approach is to set the relays so as to ensure that for any one contingency from a set of plausible severe contingencies (in contrast to the traditional approach where a single severe contingency is used to set the relays), the frequency stabilizes through primary frequency regulation [9] and load-shedding to a safe level. The relay setting approach also ensures that, during the frequency transient, the generators are not stressed by excessively low frequency for extended periods. Finally, the proposed approach sets the relays so as to minimize the accompanying load shedding action.

The new formulation requires as an input the initial generation and load levels, the set of plausible contingencies, the available primary frequency regulation spinning reserves, generation characteristics such as inertia and damping, and generator under-frequency limitations. The MILP approach then determines the setting of each underfrequency relay. In this thesis, such settings are very flexible and are defined by a number of frequency threshold levels and corresponding amounts of load shed and time delays. Particular attention is paid to the time delay parameters, the purpose of which is to ride out short frequency transients and avoid unnecessary load-shedding.

Since power systems are highly non-linear by nature, the use of a mixed integer linear programming formulation is inherently complex. The thesis overcomes this challenge by the discretization of the governing dynamic equations such as the generator swing equation as well as by utilizing special binary arithmetic techniques. It is then possible to establish the validity of a linear equivalence for certain nonlinearities. The accuracy of this model is debated in a later section.

1.2 History of Underfrequency Events

Following the 1965 Northeast blackout [5], the North American Electric Reliability Council (NERC) recommended the implementation of underfrequency load-shedding in each region of the United States power system. All regions within NERC now utilize underfrequency load-shedding as a mechanism for preventing system collapse. As an example, the Northeast Power Coordinating Council (NPCC) defines the following [11]:

"The intent of the Automatic Underfrequency Load Shedding program is to stabilize the system frequency in an area during an event leading to declining frequency while recognizing the generation characteristics in each area. The goal of the program is to arrest the system frequency decline and to return the frequency to at least 58.5 Hertz in ten seconds or less and to at least 59.5 Hertz in thirty seconds or less, for a generation deficiency of up to 25% of the load."

A number of catastrophic disturbances have occurred over the past few years leading to severe underfrequency conditions where automatic load-shedding programs have been critical in maintaining system stability. The following recounts some of these events.

The Western Electric Coordinating Council (WECC) experienced three underfrequency disturbances on December 14th 1994, July 2nd 1996, and August 10th 1996, all resulting from generation outages. The disturbance on August 10th 1996 had the most severe impact as almost 7.5 million customers experienced power outages ranging from a few minutes to about 7 hours. Four electrical islands formed as a result of cascading outages; the lowest system frequency recorded was 58.3 Hz. In each island, the decline in system frequency was arrested by automatically shedding load using underfrequency relays; for a total generation loss of 27.3 GW, the amount of load shed was unusually high at 30.5 GW. Figure 1 shows the variation in frequency in the North California island during the August 10th disturbance.

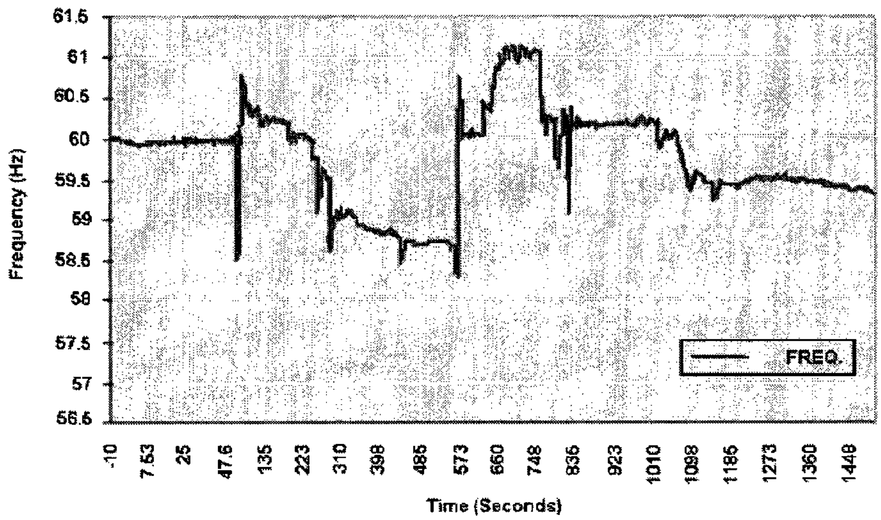

Figure 1: System frequency in the Northern California island during the August 10th 1996 disturbance

The underfrequency load-shedding scheme was effective in preventing a blackout of the island; however excess load was shed, causing the frequency to rise to 61.2 Hz (748 seconds after the onset of the event). The over-frequency condition led to tripping of more generators before the frequency settled at around 59.5 Hz. This particular case highlights the importance of setting underfrequency relays properly, as excess load-shedding can lead to an overfrequency condition, which can lead to further generator tripping.

On September 28^{th} 2003, a series of cascading outages occurred in the Italian power system: a tree flashover caused the tripping of a major tie-line between Italy and Switzerland, resulting in overloads on parallel transmission lines, which in turn tripped all major interconnections between Italy and her neighbors. The outages left the Italian system with a deficit of 6400 MW of power and a rapidly decaying system frequency; the automatic load-shedding program did not arrest the decay of system frequency quickly enough, and all generators in the power system tripped due to the severe underfrequency condition.

On May 15th 2003, lightning struck and damaged a high voltage 345 kV transmission circuit near a large power plant in North Texas. This resulted in the loss of about 4500 MW of generation, causing the frequency to deviate to 59.25 Hz. The automatic load-shedding scheme of ERCOT operated successfully by reducing load by 2020 MW, affecting over 400,000 customers, yet preventing a full-scale blackout.

1.3 The Underfrequency Relay

Underfrequency load-shedding relays are designed to prevent total system collapse by detecting the underfrequency condition and disconnecting some amount of load. Underfrequency load-shedding is usually carried out at every substation, where the bus frequency is continuously monitored. When the bus frequency dips below a certain setpoint, a timer is started as shown in Figure 2. If the timer reaches its preset value, then a trip signal is sent to the circuit breaker, which disconnects a load feeder in six cycles or less (~ 0.1 seconds) [11].

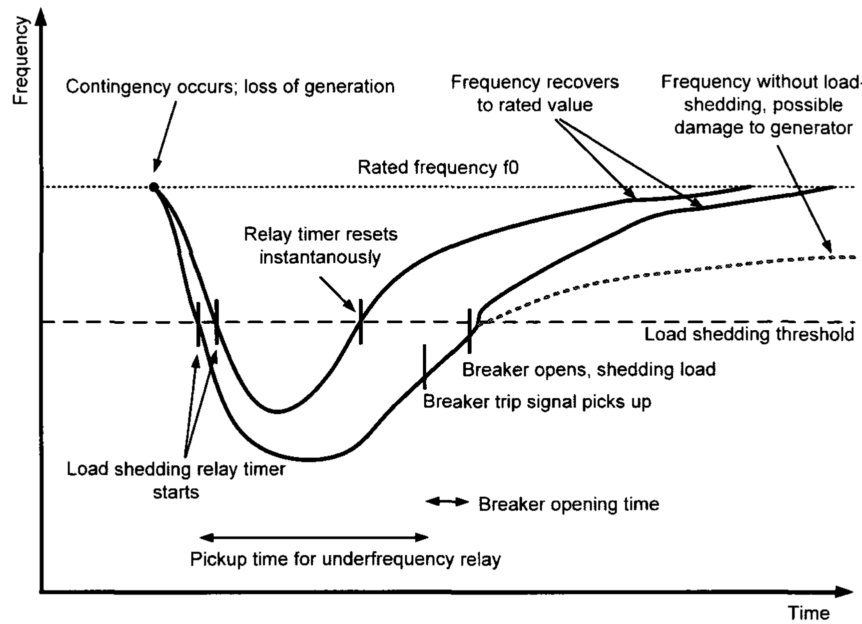

Figure 2: Simple example showing operation of load-shedding underfrequency relays

In this thesis, a generic single setpoint underfrequency relay s is defined by the following three parameters: (i) the frequency setpoint f_s; (ii) the amount of load shed Δd_s (iii) the time delay Δt_s spent by the frequency below f_s at which the shedding action occurs. The logic behind this single setpoint load-shedding relay is as follows [12],

$$\begin{gathered} if\ f(t_0) = f_s\ and\ f(t) < f_s\ for\ t_0 \le t \le t_0 + \Delta t_s \\ then\ shed\ \Delta d_s \end{gathered}$$

A more general relay, also considered in this thesis, allows a relay to use multiple frequency setpoints f_s^p, and associated time delays Δt_s^p for $p = 1, \ldots, n_p$ in order to trigger a given load Δd_s. The logic for this multiple setpoint relay is,

$$\begin{gathered} if\ for\ any\ p = 1, \ldots, np, f(t_0) = f_s^p\ and\ f(t) < f_s^p\ for\ t_0 \le t \le t_0 + \Delta t_s^p \\ then\ shed\ \Delta d_s \end{gathered}$$

Note that the multiple frequency setpoints must satisfy $f_s^{p+1} < f_s^p$ to indicate that the frequency must fall below level p before the next lower level $p+1$ becomes active.

The decision to trip at a certain frequency level can be based on other parameters such as the rate of change of frequency [8]; however, in this dissertation, the relays are assumed to react only to crossing of frequency setpoints, f_s, and to the amount of time spent below such setpoints, Δt_s. It is important to note however that the multi-level relay has a characteristic similar to that of an adaptive relay in which the decision to shed is based on the rate of change of frequency [8].

In additon to load shedding, underfrequency relays also trip generating units if the local frequency drops below certain critical thresholds for specified time intervals. These stringent so-called *generator underfrequency/time limitations* are imposed by generator manufacturers so as to protect the equipment from being damaged by extended off-nominal frequency operation. Table 1 shows some typical examples of these limitations.

Table 1: Typical generator off-nominal frequency/time limitations

Underfrequency limit (Hz)	Overfrequency limit (Hz)	Maximum permissible time
60.5-59.5	60.0-60.5	N/A (continuous operating range)
59.4-58.5	60.6-61.5	30 seconds - 3 minutes
58.4-57.9	61.6-61.7	7.5 seconds
57.8-57.4		45 cycles
56.8-56.5		7.2 cycles
Less than 56.4	Greater than 61.7	Instantaneous trip

1.4 Power System Dynamics

An electric power system behaves like a rotating mechanical system (Figure 3). Mechanical power produced from a hydro or steam turbine generates a mechanical torque T_m, which is used to rotate the shaft of a synchronous generator. A synchronous generator transforms mechanical power into electrical power and the load connected to the generator causes an electrical torque T_e on the generator shaft. A sudden change in power demand or production causes a deviation in speed of the turbine-generator, resulting in a fluctuation in frequency of the power system. In order to accurately predict the frequency response of a power system to active power unbalances, it is necessary to identify a set of such equations of motion, called swing equations.

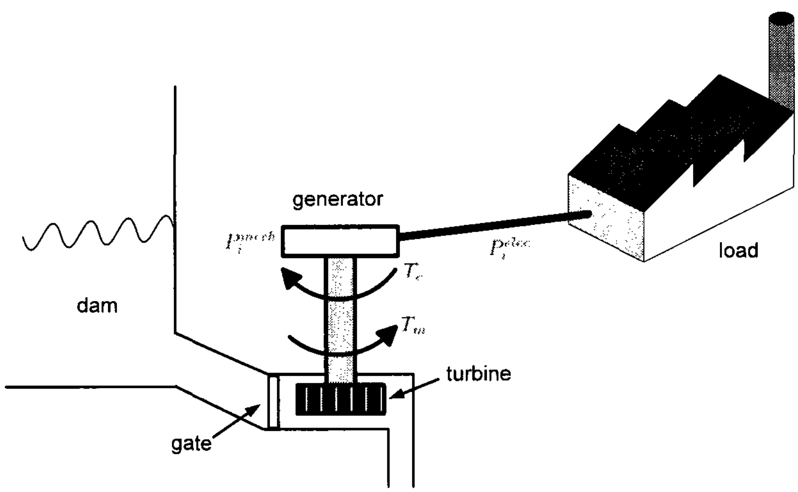

Figure 3: Simplified power system model

The swing equation of a generating unit relates the frequency of its generated voltage to the balance of mechanical power input and electrical power output [13]. For a system with ng generators, $i = \{1, \ldots, ng\}$, the simplest form of the i^{th} generator swing equation is:

$$\frac{2H_i}{f_0}\frac{d\Delta f_i}{dt} = p_i^{mech} - p_i^{elec} \quad (1.1)$$

In Equation (1.1), p_i^{mech} is the mechanical turbine power input in per unit, p_i^{elec} is the electrical power output in per unit, H_i is the generator inertia constant in seconds, Δf_i is the frequency deviation from nominal in Hz, and f_0 is the nominal or rated frequency (60 Hz in North American systems).

In steady state operation, $\Delta f_i = 0$, and all synchronous generators rotate at nominal frequency. In the event of any change in the active power balance, the frequency will deviate from f_0. For example, if a generator trips, then $p_i^{mech} = 0$, and the frequency will decline rapidly with slope $\frac{df}{dt} = -\frac{p_i^{elec}}{2H_i}$. The electrical output as well as the inertia constant H_i then determine the initial rate of change of frequency following such a disturbance. The value of H_i varies between 3 seconds for hydraulic turbines and 10 seconds for steam turbines; a larger inertia constant implies a slower response of the system frequency to power imbalances.

The frequency response of a generating unit following a disturbance is however affected by parameters others than those in Equation (1.1); two factors that have a significant impact on the frequency trajectory of a generating unit are (i) the frequency dependence of loads, and (ii) primary frequency regulation through governor action. Both of these factors are discussed in the following sections.

1.5 Frequency-dependence of Loads

Power system loads are composed of a variety of electrical devices. For resistive loads such as incandescent lighting and heating, the active power consumed is independent of frequency. Motor loads, however, are dependent on frequency because motor speeds depend on the frequency of the input power supply. A lower system frequency usually results in a reduction of the active power consumed by motor loads, an effect which is expressed mathematically as:

$$\Delta p_i^{freq} = D_i \Delta f \tag{1.2}$$

where Δp_i^{freq} is the change in active power consumed by frequency sensitive loads, D_i is the load-damping constant, and Δf is the frequency deviation from normal. The damping constant is usually expressed as a percent change in load for a one percent change in frequency; a typical value of D_i is 2%, implying that a 1% change in frequency would cause a 2% change in load. The frequency dependence of loads is modeled in the swing equation as follows:

$$\frac{2H_i}{f_0}\frac{d\Delta f_i}{dt} = p_i^{mech} - p_i^{elec} - D_i \Delta f_i \tag{1.3}$$

Figure 4 illustrates the effect of load damping on the frequency response for a 50% generation loss on a system with an inertia constant H_i of 8.6 seconds. If the effect of frequency dependence of loads is ignored, then the frequency response is too conservative and does not accurately reflect the frequency response of a system to an outage. Studies such as [1, 8, 14] have stressed the importance of modeling the load-frequency response when conducting load shedding studies.

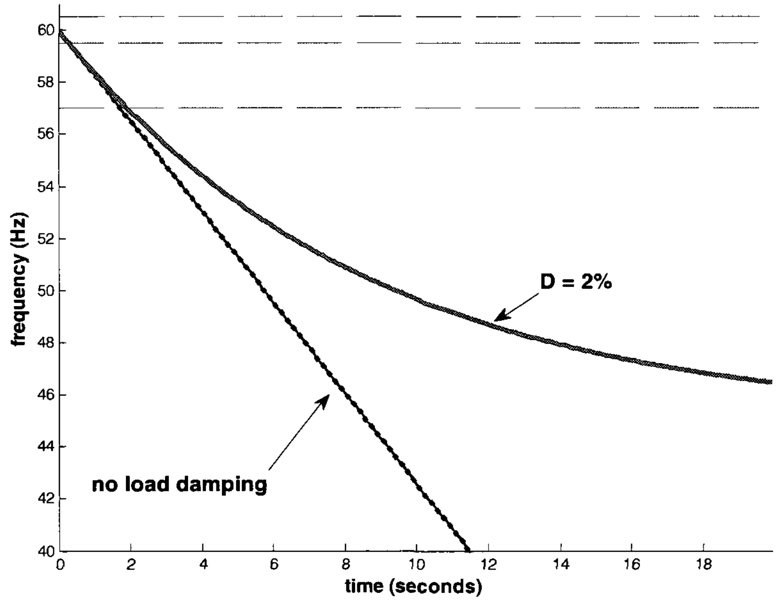

Figure 4: Effect of frequency-dependence of loads due to a 50% generation loss

1.6 Primary Frequency Regulation through Governor Action

Primary frequency regulation [9] is the automatic change in active power generation of generating units following a change in system frequency. After the loss of a large generating unit (or a critical intertie in interconnected systems), the system frequency decreases from its nominal level as the kinetic energy of the rotating masses decreases. Each generating unit i located in a synchronous zone with frequency deviation Δf is fitted with speed governors that automatically respond to such a deviation by incrementing its active power generation by $-\frac{\Delta f}{R_i}$, subject to capacity and ramp limits (see Figure 5). The parameter R_i is the frequency regulation constant or governor droop of unit i in Hz/MW; typical values of governor droop lie between 4 and 6 Hz for the loss of the rated power.

As an example, following a contingency that leads to a steady-state frequency deviation of 5% or 3 Hz, governors will act to increase their active power generation by 100% of rated power. Primary frequency control therefore helps to stabilize the system frequency following a generation deficiency, and has a significant effect on system frequency following a disturbance.

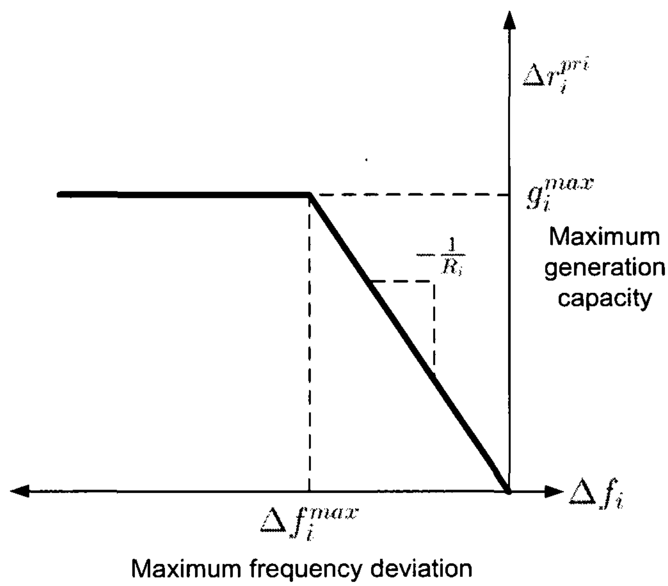

Figure 5: Primary frequency regulation characteristics of unit i [15]

During the transient following a disturbance, the primary reserve of unit i is not available immediately; there are time constants of about 5-10 seconds associated with the opening of valves in the governor. This delay can be modeled as shown in Figure 6, where Δr_i is the primary frequency regulation of unit i due to a frequency deviation of Δf, R_i is the frequency regulation constant, and T_i is the time constant representing governor action.

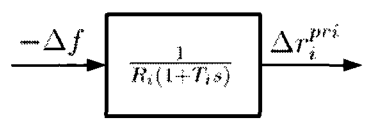

Figure 6: Time constants associated with primary frequency regulation

In the time domain, the primary frequency regulation with governor action takes the form:

$$T_i \frac{d\Delta r_i}{dt} = -\frac{\Delta f_i}{R_i} - \Delta r_i \tag{1.3}$$

which is then incorporated into the swing equation as follows:

$$\frac{2H_i}{f_0}\frac{d\Delta f_i}{dt} = p_i^{mech} + \Delta r_i^{pri} - p_i^{elec} - D_i \Delta f_i \tag{1.4}$$

The complete transfer function relating frequency and active power is shown in Figure 7 below:

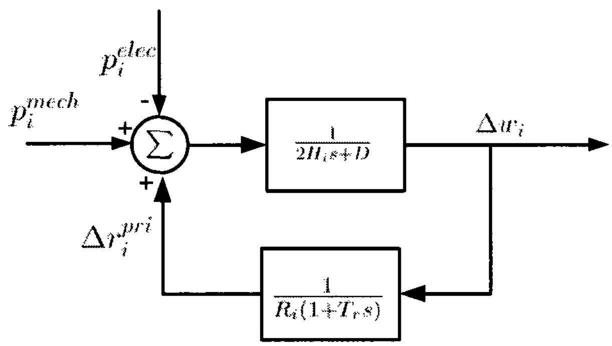

Figure 7: Block diagram representation of power system dynamics including governor action

Studies such [1] and [14] choose to ignore the effect of the governor when conducting load-shedding studies, and only model the load-frequency sensitivity when computing the frequency decay. This assumption is usually based on the reasoning that all load-shedding must take place within the first three seconds of a disturbance, during which time frame governor action is assumed to be negligible.

Nonetheless, Figure 8 shows why modeling governor action accurately is important. In the figure, frequency trajectories using three distinct frequency decay models are plotted for the same generation loss and inertia constant. Trajectory 3 is a representation of frequency decay with only load-frequency sensitivity. Trajectory 2, which includes the primary frequency reserve model of Figure 7, has the same gradient as Trajectory 3 for the first three seconds, but the regulation provided by the governor aids the recovery of Trajectory 2. Therefore, instead of the frequency decaying to the critical level indicated by Trajectory 3, primary frequency regulation plays a significant role in arresting the frequency decline and subsequent recovery of Trajectory 2.

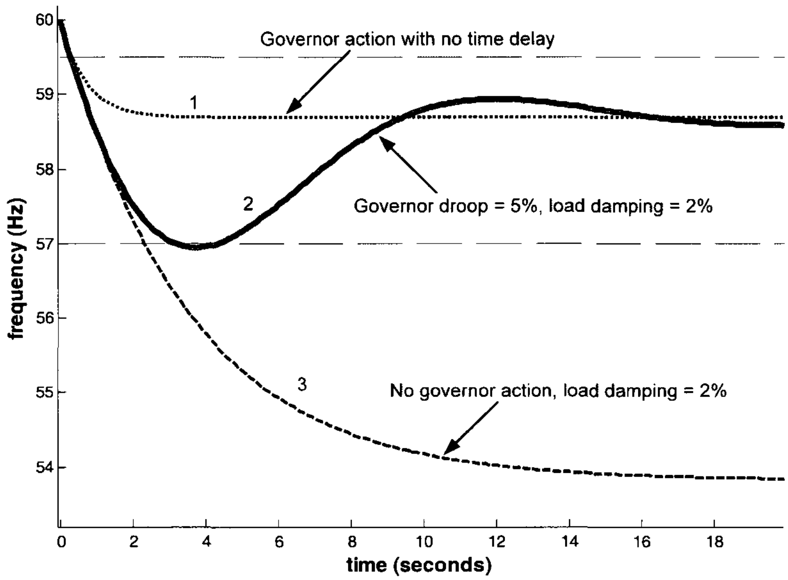

Figure 8: Effect of primary frequency regulation on system frequency response

It should be evident from Figure 8 that by accounting for primary frequency regulation, less load-shedding need take place in order to correct the same contingency. One of the goals of this thesis is to consider all available primary frequency regulation following a contingency in order to shed less load without compromising the underfrequency time limitations of generators.

Trajectory 1 in Figure 8 is a representation of the 'quasi steady state' model used in [10]. Even though Trajectory 1 and Trajectory 2 settle at the same steady state frequency, Trajectory 1 ignores the 'swing' due to the action of delayed primary frequency reserves, and hence its model cannot be used to determine the settings of underfrequency relays as accurately as those modelling trajectory 2.

1.7 Discrete-Time Frequency Response Model

In the previous section, the swing equation and frequency response model was developed for a single generating unit. In a multimachine network, each generator has a different inertia value, and a unique frequency response to every generation loss contingency. An approximation that is often made to simplify the calculation of the frequency decay following a power imbalance is to lump all generators together to create a single machine equivalent by assuming that they all swing synchronously at a common frequency, f.

For a system with ng generators, the per unit swing equation for the equivalent system is then given by the following expression:

$$\frac{2H^{eq}}{f_0}\frac{d\Delta f}{dt} = \sum_i g_i^0 - \Delta g + \Delta r - \sum_k d_k^0 - D\Delta f \tag{1.5}$$

where

$$T\frac{d\Delta r}{dt} = -\frac{\Delta f}{R^{eq}} - \Delta r \tag{1.6}$$

The total intial generation is given by $\sum_{i=1}^{ng} g_i^0$ where the g_i^0 are the precontingency generation setpoints. Similarly, the total demand in the precontingency state is given by $\sum_{k=1}^{nd} d_k^0$. The equivalent inertia constant is computed using:

$$H^{eq} = \frac{\sum_i H_i S_i^{base}}{\sum_i S_i^{base}} \tag{1.7}$$

and the equivalent primary frequency regulation is calculated using:

$$\frac{1}{R^{eq}} = \sum_i \frac{1}{R_i} \tag{1.8}$$

In addition, in Equation (1.5) the Δg term refers to the per unit generation loss defining a contingency.

The equations above can be solved using the Euler method, which says that a differential equation of the form $\frac{dy}{dt} = f(t, y(t));\ y(t_0) = y_0$ can be solved using the iterative form:

$$y_n = y_{n-1} + hf(t_{n-1}, y_{n-1})$$

where h is the integration step size. Therefore, Equations (1.5) and (1.6) can be approximately solved using the following expressions:

$$\Delta f_n = \Delta f_{n-1} + \Delta t\left(\sum_i g_i^0 - \Delta g + \Delta r - \sum_k d_k^0 - D\Delta f\right)\frac{f_0}{2H^{eq}} \tag{1.9}$$

and

$$\Delta r_n = \Delta r_{n-1} + \Delta t\left(-\frac{\Delta f}{R^{eq}} - \Delta r\right)\frac{1}{T} \tag{1.10}$$

1.8 Testing the Discrete-Time Frequency Response Model

In order to test the accuracy of the piecewise linear frequency response model discussed in the previous section, the effect of a 25% generation loss in the simple 3 bus network of Figure 9 was simulated using the power system simulation tool PSS/E [16].

Each generator was characterized using the round rotor generator model 'GENROU' with governor model 'TGOV1'. Loads were modeled as constant real power loads, and tranmission lines had nominal resistance and admittance values. In addition, to ensure that voltage collapse was not encountered, shunt elements were placed at each bus to provide sufficient voltage support.

The single machine equivalent of Figure 9 is a generator with inertia computed using Equation (1.7) and with primary frequency regulation computed using Equation (1.8).

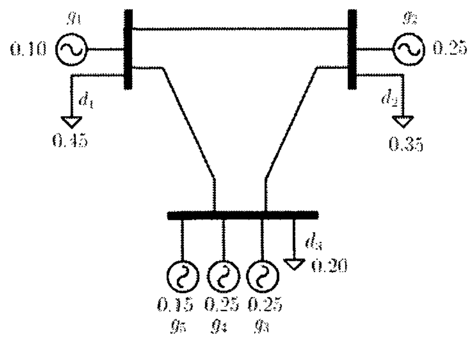

Figure 9: A simple 3-bus test network

The frequencies at each of the three buses following the contingency, along with the frequency response computation using Equations (1.9) and (1.10) are plotted in Figure 10.

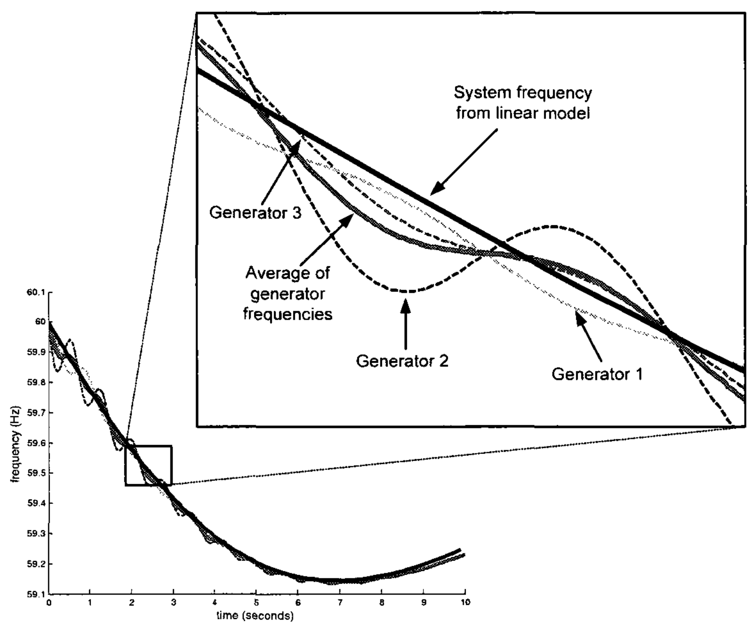

Figure 10: Comparison of results from PSS/E and discrete-time frequency response model

The frequency trajectories in Figure 10 indicate that the resulting frequency response computations, denoted by the thick curve in the figure, provide a reasonable first order approximation of the average frequency response of a system. Such an approximation will be used in this thesis to develop the proposed scheme to set the underfrequency relays. More general models could be conceived in which every single generator is described by its own discretized swing equation and in which the network equations are modeled by a DC load flow. The drawback of such a general formulation is the corresponding sharp increase in the number of variables and computation time.

1.9 Note on the number and size of time steps

The accuracy of the discrete-time frequency model is dependent on the number of time steps nt, and the size of the time step Δt. Quite clearly, smaller time steps will yield a better approximation of the frequency response, but will result in a larger number of variables. Even though computation of relay settings is not intended to be carried out online, it is important to choose a reasonable step size. Since the smallest time constants in the relay setting problem are of the order of 6 cycles, a step size of 0.1 seconds was chosen for the frequency model. Figure 11 highlights the effect of step size on the discrete frequency model: when a step size of 1 second is used, the frequency response of a 25% generation loss is exaggerated.

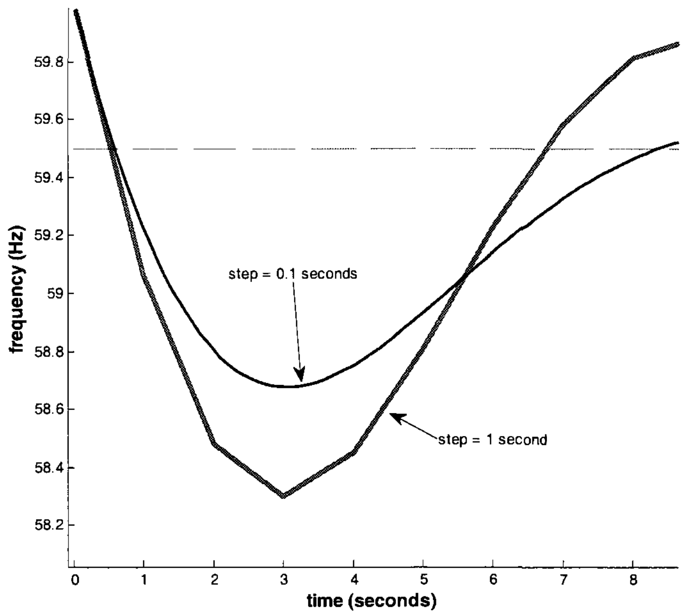

Figure 11: Effect of time steps on frequency response

The number of time steps for which frequency computations are carried out is also significant. The frequency response to each contingency, Equations (1.9) and (1.10), should be evaluated until all contingencies reach steady state. If a small number of time steps are used, then it is possible that the trajectories will be 'squeezed' along the time horizon, and the relay settings will be more sensitive than required. In the worst case, the problem may become infeasible.

It was noted that most frequency trajectories reach steady state within 20 seconds. Therefore, the number of time steps used for computing relay settings was decided to be: (total time/step size) = 20/0.1 = 200.

II. HISTORICAL REVIEW OF THE UNDERFREQUENCY RELAY SETTING PROBLEM

2.1 Conventional Methods of Setting Relays

Underfrequency relays have been widely employed in bulk power systems since the 1965 Northeast blackout. Numerous studies have attempted to determine ideal relay settings for given systems, and various others have researched the effectiveness of established load-shedding schemes. The conventional method of setting relays was introduced by Lokay and Burtnyk in [14], and this study has been the basis for the relay setting methods utilized in [1], [7], and [17], among others. This section describes the conventional method of setting underfrequency relays, and highlights some of the advantages and deficiencies of the method.

As the basis for applying underfrequency relays is the expected decay of the system frequency, the first step in determining relay settings involves accurately predicting the frequency response to a generation loss. Reference [14], for example, suggests the use of a simple first order model of generator dynamics. The effect of inter-machine oscillations is ignored, and all machines in the network are lumped into a single machine equivalent. Governor action is also ignored based on the argument that load shedding relays must operate within the first two seconds of a disturbance, and that during such time periods governor action will be insignificant [1]. The conventional method used to set underfrequency relays is usually based on the load-shedding scheme suggested in [14]. It involves the following steps:

(1) **Determination of the maximum generation loss event**: The generator outage that results in the highest initial rate of frequency decay is selected as the worst case contingency that the underfrequency relaying scheme must protect against. Alternatively, such an event could be defined by the most pessimistic loss of generation expected [14].

(2) **Calculation of the total amount of load that must be shed**: This amount is computed to ensure that the frequency does not deviate below a specified minimum permissible value. The amount of load that must be shed can be estimated from Equation 1.9 in steady-state under the assumption of the maximum generation loss event. Some studies recommend that an even more conservative estimate be used by rounding up this number.

(3) **Determination of the number and size of load shedding steps**: The amount calculated in step (2) is not shed all at once but in a number of steps. The number and size of such load shedding steps is generally a function of the total amount of load to be shed and the frequency range between the highest frequency which may be used for load shedding (e.g. 59.5 Hz) and the minimum permissible frequency (e.g. 58.4 Hz). Most utilities use three to five load-shedding steps, but the number chosen is usually arbitrary. Once the total amount of required load shed is calculated from step (2), a trial load shedding schedule is suggested. In [14], a three step shedding schedule of 10%, 10%, and 15% is used to shed a total of 35% for a worst case generation loss of 33%. Other combinations of load shedding totaling 35% could also be used; if the current scheme is not successful in arresting the frequency decline above the minimum permissible frequency, then a new trial load shedding schedule is tried.

Some studies such as [1] also retrieve an estimate of the time available for shedding actions. In the example above, a conservative estimate of time available for load shedding is the time taken for the frequency to drop from 60 Hz to the minimum permissible level of 58.4 Hz assuming the frequency always decays at the same initial rate without damping. This time then defines the time frame within which the relays must operate following the disturbance.

(4) **Calculation of relay settings**: A number of general rules are provided in [14] for selecting the shedding frequencies for the various load

shedding steps. The frequency of the first load shedding step should be below any frequency from which the system could recover without dropping load and without damaging equipment. Most load shedding programs use a frequency between 59.0 to 59.4 Hz for the first step because steam turbines can operate continuously in the frequency range of 59.5 to 60.5 Hz.

The setting of the next load shedding step is chosen at a low enough frequency to prevent its operation during contingencies that could have been relieved by the operation of the first step. As an example, suppose that the first step of 10% is shed at 59.1 Hz and that the frequency is decaying at -1.0 Hz/s. Because of the inherent time delay in executing the load shedding step (0.1 seconds for the underfrequency relay and another 0.1 seconds for the circuit breaker), the circuit breaker will only open after the frequency has declined to 58.9 Hz. Therefore, the second shedding step must be initiated below 58.9 Hz.

In addition, in order to account for random frequency oscillations and frequency differences between buses, it is desirable to have a coordinating margin of 0.1 Hz between the frequency at which the circuit breaker opens for one step and the setting of the next load shedding relay. Relay coordination refers to the proper separation of load shedding steps; there must be a sufficiently large frequency margin between the operations of two consecutive load-shedding steps to ensure that the steps do not overlap. Therefore, the pickup frequency of the second stage of the second 10% step in the example above is selected to be 58.8 Hz.

The last load shedding step should also have some coordinating margin to ensure that the circuit breaker opens before the frequency goes below the minimum permissible frequency. In the example, the frequency for the last shedding step of 15% is chosen to be 58.6 Hz.

These relay settings obtained from the conventional approach are summarized in Table 1.

Table 2: Trial relay settings determined using conventional approach

Step	Frequency (Hz)	Time Delay (secs)	Load Shed (%)
1	59.1	0.2	10
2	58.8	0.2	10
3	58.6	0.2	15

The above settings are considered trial settings until the disturbance is simulated using either a full transient stability program (such as PSS/E) or Equations (1.9) and (1.10) to observe how they perform in a more realistic environment. The goal is to ensure that the frequency decline is arrested, and that relay coordination [14] is maintained.

Figure 12 provides an example of how these types of relay operate in time. The point R_1 denotes the time and frequency at which the relay for the first load shedding step initiates the shedding process. Point T_1 is the time and frequency at which the relay actually sheds the first load after the breaker time and coordinating delays are added. Note from Figure 12 that although the point at which the second step is initiated, denoted by R_2, is before T_1, after its delays are added, the second step is implement at T_2, which does not interfere with T_1.

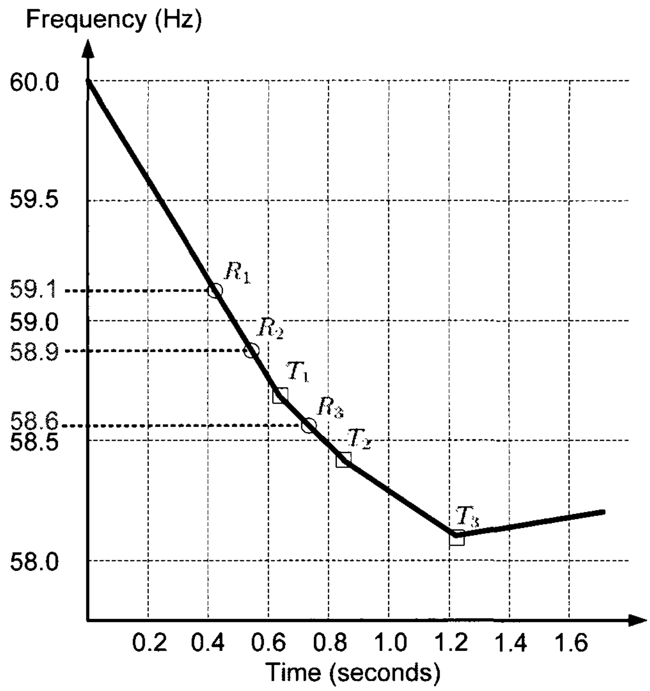

Figure 12: Frequency decline following contingency

It is evident from Figure 12 that since the frequency decline is not arrested before the lowest admissible frequency of 58.4 Hz, the trial relay settings of Table 2 are not acceptable. In order to correct these settings there are two options: (i) repeat the above procedure with a different shedding schedule, perhaps by shedding a larger block of load at an earlier step, or by using four instead of three load shedding steps, and (ii) change the relay settings by varying the frequency set-points. Therefore, steps (3) and (4) have to be repeated until relay settings that meet all conditions are found. In general, this is a tedious and non-systematic trial-and-error approach.

2.2 *Limitations of the Conventional Method of Setting Underfrequency Relays*

From the procedure for setting underfrequency relays outlined in the previous section, some of the limitations of the conventional method can be identified. The approach is a cumbersome trial-and-error process. First, it is necessary to guess at the "worst" contingency, a decision that, as will be shown later, is not evident. In addition, if a trial 'guess' of the relay frequency settings and the amount of load to shed at each step, results in an unsatisfactory frequency response when the actual relay operation is simulated, there is no systematic way of modifying this guess. In addition, the following deficiencies of the conventional method can be noted:

The amount of load curtailment computed in Step 2 is sufficient to arrest the frequency decline and maintain it above a minimum specified level. However, this method does not elaborate on how long it takes for the frequency to return to the acceptable continuous operating range. The assumption made by the conventional method is that if all load-shedding actions succeed in stopping the decline of frequency, secondary frequency regulation via Automatic Generation Control will assist in the frequency recovery process [17]. However, as described in Chapter I, generator manufacturers provide stringent underfrequency/time limitations that must be adhered to; as an example, most steam turbine generators can only operate below 58.8 Hz for a maximum time of ten seconds. Therefore, simply arresting the decline of frequency may not be enough.

Under the conventional method, relays are set with respect to the maximum possible generation loss event. Most studies however acknowledge that doing so results in unnecessary load shedding for milder contingencies, and dismiss such occurrences as the cost of preventing blackouts during severe contingencies. However, since there are enough degrees of freedom in selecting the relay settings, namely the frequency set-points and the time delays, this thesis will show that, by appropriately utilizing these degrees of freedom, it is possible to eliminate unnecessary load-shedding for both severe and mild contingencies.

2.3 *Adaptive Underfrequency Relays*

Over the past decade, the topic of adaptive underfrequency relays has received significant attention in the research community. Adaptive schemes evolved from studies such as [7], where relays measure $\frac{df}{dt}$ when a certain frequency threshold is reached. The amount of load to be shed is determined based on the value of the slope. Usually, the rate of change of frequency is measured only at the first threshold.

The adaptive method, formally introduced by Anderson and Mirheydar in [8], relies on frequency sensing relays that measure the gradient of the frequency trajectory. The initial gradient is given by the expression $\left.\frac{df}{dt}\right|_{t=0^+} = \frac{\Delta P}{2H}$; a higher value of initial gradient indicates a more severe contingency, and therefore the load-shedding scheme can 'adapt' appropriately by shedding more load. Numerous studies such as [4, 12, 18-21] have been conducted in the past few years on adaptive underfrequency relays, and the general conclusion is that such relays shed less load than their traditional counterparts.

In [22], Thompson proposes an adaptive relaying scheme, where the relay incorporates a microcontroller and receives information from the SCADA system such as system demand, spinning reserve, system kinetic energy, and amount of lower-priority load available for shedding. The scheme uses the measured post-outage rate of change of frequency to estimate the magnitude of the generation loss, and to determine whether a particular relay should operate. Simulations on a representative system using this adaptive scheme indicate improved load-shedding compared to the conventional approach. The scheme however requires relays to be fitted with communications and microcontroller technology.

2.4 *Optimization Techniques Used In Setting Underfrequency Relays*

The study in [23] described the development of a load-shedding scheme for a small isolated power system. Development and validation of the scheme was done in three stages: the first stage, a screening stage, used a lumped network model (single equivalent generator and a single equivalent load) to test a wide range of underfrequency load-shedding plans. The model was subjected to a set of generation loss events for different levels of load and spinning reserve. For each candidate load-shedding schedule, the maximum frequency excursion over the scenario set was estimated, along with root-mean square value of the maximum deviation, and the standard deviation of the post-shedding steady state frequency. These statistics were used to measure the performance of each test schedule in minimizing frequency deviations over load-shedding actions; the test load-shedding schedules were selected with the objective of minimizing the above statistical measures. Once the screening process generated a candidate scheme, a more detailed model of the power system consisting of dynamic models of all generating units and load-flow representations of the network was used to evaluate the effectiveness of that scheme. Finally, a fully detailed transient stability program was used to evaluate the stability of the system during the course of the disturbance. The paper also demonstrated an application of the method to develop a robust load-shedding scheme for the Electricity Authority of Cyprus (EAC).

The conclusions outlined in [23] are significant to this thesis: the author confirmed the benefits of utilizing a lumped network model in developing load-shedding schedules, and iterated that full transient stability programs should be used to validate the preferred load-shedding scheme. The author also illustrated the importance of including spinning reserves in the formulation, and showed that, in general, the effectiveness of load-shedding increases with load levels, and therefore that it is expedient to compute relay settings for high load levels.

III. THE NATURE OF THE UNDERFREQUENCY RELAY SETTING PROBLEM

3.1 Impact of Loss-of-Generation Contingencies

Consider the simple three-bus network shown in Figure 1. The pre-contingency per unit real power generation and demand levels are indicated in the figure.

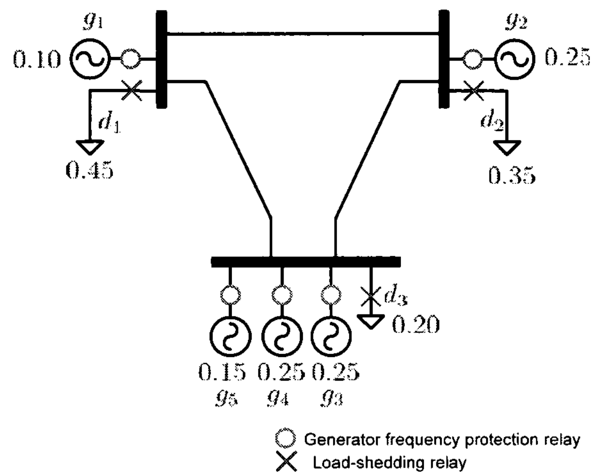

Figure 13: A simple 3-bus network

Frequency excursions (also referred to in this dissertation as frequency trajectories) for various levels of generation deficiencies in the 3-bus test network of Figure 13 are plotted in Figure 14. The loss of the small generator g_1 (generating 10% of the total load) does not cause a significant deviation in frequency. The loss of the larger generator g_2 results in a small deviation below the safe operating limit of 59.5 Hz, but the frequency quickly recovers through primary frequency regulation to 59.62 Hz. In addition, the time spent by the resulting trajectory below the safe operating limit of 59.5 Hz is about 5.3 seconds, which is within the maximum permissible level of 30 seconds discussed in Section 2.4.

In contrast, the simultaneous loss of either three (65% loss) or four (90% loss) generators results in a substantial deviation of the system frequency below 57 Hz. Without load shedding, these larger contingencies would trigger the generator underfrequency protection relays, thus disconnecting the remaining units and resulting in a complete blackout of the system. Note that a simplifying assumption made in this thesis, as in other similar studies, is that the loss of a generator and any accompanying load-shedding does not significantly affect the bus voltages. As a result, the remaining loads remain constant, unaffected by voltage variations.

It is clear therefore that some amount of load shedding is required in order to successfully counter extreme generation loss contingencies. On the other hand, mild generation loss contingencies may not require any load shedding. The nature of the underfrequency relay setting problem is therefore a compromise between ensuring that the system is protected against the worst contingencies while ensuring that this is done without unnecessary load shedding when mild contingencies occur.

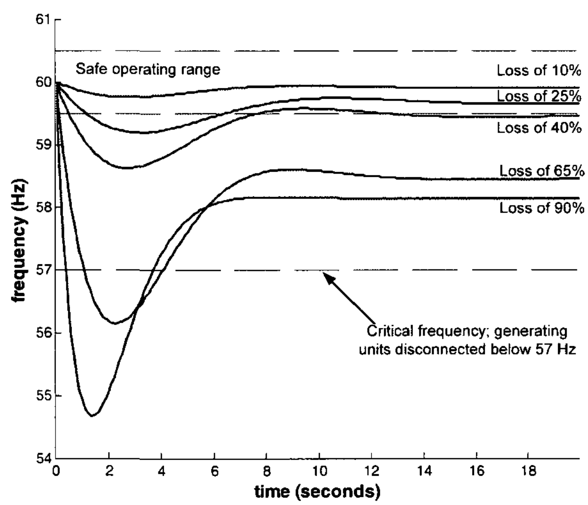

Figure 14: Typical frequency trajectories following various levels of generation deficiency

3.2 General Criteria for Underfrequency Relay Setting

Motivated by the above example, this thesis proposes that the load shedding process governed by the underfrequency relay settings be based on the following three criteria:

(A) Sufficient load must be shed to ensure that the steady state frequency lies within the safe operating range;

(B) Sufficient load must be shed at the right time following a contingency to ensure that the generator underfrequency/time limitations are not violated;

(C) Unnecessary load shedding must be minimized.

Criterion A states that sufficient load must be shed, but just enough. Too little will lead to a steady-state frequency below the safe operating range. On the other hand, excess load shedding, which is a distinct possibility, must be avoided as it will lead to an over-frequency condition which is as damaging as underfrequency excursions [8].

Typically, the relays are set so that load shedding takes place so as to satisfy criteria A for the "most severe" contingency. By so doing, it is expected that criteria A will also be met when any other less severe contingencies occur, although how to prove this irrefutably is an open issue. What can definitely be stated is that the relay settings based on the most severe contingency will be too conservative in terms of criterion C; in other words, too much load will be shed when milder contingencies occur.

One purpose of this thesis is to propose and examine more general underfrequency relay models that will allow us to meet criterion C and shed less load without sacrificing the absolute requirements of criterion A as well as B. Note that criterion B is not considered explicitly in the conventional relay setting approaches. It is simply assumed that by arresting the frequency decline, the frequency will return to the safe operating range in an appropriate time via secondary frequency regulation.

The implications of the three criteria of the underfrequency relay setting problem are now illustrated using the example of Section 3.1. From the conventional method described in Chapter II, a set of relay settings is computed under the following assumptions: (i) all generators are equipped with speed governors with 5% droop. Thus, if the load-frequency sensitivity factor is 2.0, the total damping factor is $D = 2 + \frac{1}{0.05} = 22$, (ii) the underfrequency load-shedding plan must attempt to return the frequency to the safe operating range of 59.5-60.5 Hz, not simply arrest the decline of frequency. Thus, in this example, criteria B and C are not explicitly enforced. Following the conventional method, the relay settings are computed as follows:

(i) Identification of the maximum generation deficiency. Here, the loss of generators {2, 3, 4, 5} represents the worst possible contingency, corresponding to a loss of 90% generation.

(ii) The remaining amount of primary frequency regulation is $\Delta P = D\Delta f$ = 18%. Therefore, a total of 72% of the original load must be shed. Since the usual practice is to be conservative in the amount of load that is shed, the total load shedding required is revised to 75%.

(iii) A proposed starting load-shedding schedule is {15%, 20%, 20%, and 20%}.

(iv) The following relay frequency setpoints are obtained after repeated iterations of Steps (3) and (4) in Section 2.1:

Table 3: Final relay settings to correct 90% generation loss

Step	Frequency (Hz)	Time Delay (secs)	Load Shed (%)
1	59.4	0.2	15
2	58.9	0.2	20
3	58.6	0.2	20
4	58.3	0.2	20

Figure 15 shows the results of a simulation of the 90% generation loss scenario with the underfrequency relays configured as in Table 3. It should be evident that the load-shedding schedule meets criteria A and B: the final steady state frequency lies within the safe operating range, and the generator underfrequency/time limitations are respected.

Consider now the loss of the 25% generator, a simulation which is depicted in Figure 16 with the relay settings of Table 2 and load shedding action (solid line) and without relays and load shedding (dotted line). Given that there is sufficient primary frequency reserve, no load shedding should be required for this particular contingency since, as seen by the dotted curve in Figure 16, the frequency recovers to the safe region in 4 seconds without violating the underfrequency/time limitations of about 30 seconds. However, the relay settings of Table 2 result in the relay controlling the first 15% block to shed this load, as the solid line indicates. This is an unnecessary load shedding action, clearly violating Criterion C.

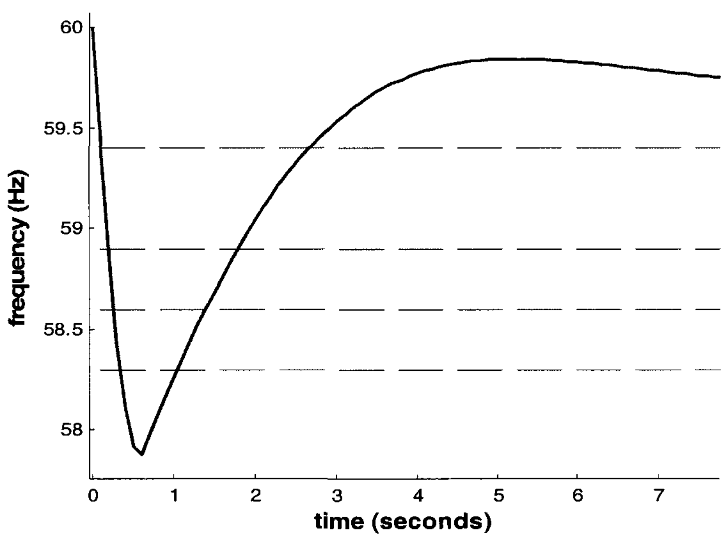

Figure 15: Simulation of 90% generation loss with relay settings from Table 2

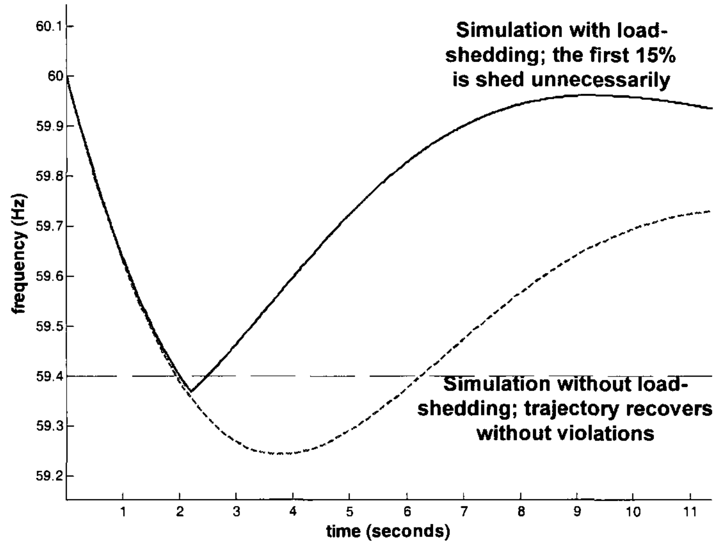

Figure 16: Simulation of 25% generation loss; excess load-shedding

The above example demonstrates one of the limitations of the conventional method of setting underfrequency relays. Since the relay settings are computed with respect to the most severe contingency, the relays may be unnecessarily 'sensitive'. As a result, for mild contingencies such as the 25% generation loss in Figure 16, excess load is shed. The relay settings in Table 3 satisfy Criterion A and B, but shed too much load for the less severe contingencies, thereby violating Criterion C. In addition, although this example does not illustrate it, trajectories corresponding to contingencies other than the most severe, may violate the underfrequency/time limitations.

Since the number of possible generation loss contingencies on a system is very large, it is desirable to have a systematic way of setting relays that respond appropriately to all contingencies.

3.3 *The Effect of Load Shedding Priority with Discrete-Valued Loads*

In underfrequency load-shedding schemes, loads are usually curtailed in a specific priority order, a requirement that can have a major impact on relay settings. To illustrate the implications of load-shedding priority in defining the shedding strategy and the setting of the underfrequency relays, consider again the 3-bus example of Figure 13. Assume that loads have to be shed in the order {20,35,45}, and that fractions of load cannot be shed.

In order to compute a sufficient amount of load-shedding for a given contingency, the available primary frequency regulation must be estimated. For example, following a 65% generation loss, the amount of deployable primary reserve with 4% governor droop is 20%. Therefore, the net generation deficiency is (65 - 20) = 45%. In order to arrest the frequency deviation, shedding the smallest load (20%) is not enough. There are two solutions which result in a steady state frequency within the safe operating range (Criterion A): shedding both the 20% and the 35% loads for a total of 55%, or shedding only the 45% load. If load shedding is obliged to follow the given priority list, here {20%, 35%, 45%}, then in the above example there would be no choice but to shed both the 20% and the 35% loads for a total of 55%, instead of the lower option of 45%. Load shedding with priority generally leads to higher load shedding levels than if the loads can be shed without following a priority list. Similarly, for a contingency with 90% generation loss, the 35% and 45% loads have to be shed to correct the system frequency for a total of 80% load shedding. If the priority list has to be respected, 100% of system load must be shed. These results are summarized in Table 4.

Table 4: Analysis of contingencies in 3-bus network with and without load shedding priority

Contingency	Generation loss (%)	Primary frequency reserve used (%)		Load shed (%)	
		Without priority	With priority	Without priority	With priority
1	10.0	0.0	0.0	0.00	0.00
2	25.0	0.0	0.0	0.00	0.00
3	40.0	20.0	20.0	20.0	20.0
4	65.0	20.0	10.0	45.0	55.0
5	90.0	10.0	0.0	80.0	100.0

Note that the above discussion is valid if the loads to be shed in a given priority order correspond to physical load blocks located at specific buses as shown in Figure 14. As seen, this results in significantly more load being shed than without priority ordering. However, if the loads do not represent physical loads at a particular substation, but a fraction of the total demand, then the effect of priority does not affect the load shedding scheme, which remains the same as without priority. The fraction of the variable load shed during each stage is then distributed by the relay engineers throughout the network in accordance with the priority list.

3.4 Load acting as a Reserve (LaaR)

Electricity utilities such as the Electric Reliability Council of Texas or ERCOT have introduced the use of loads acting as responsive reserves to enhance security and reliability of the grid [24]. Customers with interruptible loads that can meet various performance requirements are eligible to provide operating reserves under the Load Acting as a Resource (LaaR) program. The general idea is that during generation loss contingencies, loads under the LaaR program can be disconnected when the frequency drops to a preset value, thereby acting as fast spinning reserves in an attempt to restore the active power balance. Most LaaRs in the ERCOT grid are controlled using an underfrequency relay with a high frequency setpoint, and therefore the use of LaaRs in a grid will have an impact on the underfrequency relay settings. LaaRs participants usually receive a capacity payment and an energy payment based on the actual deployment and the market clearing prices for capacity and energy [24].

In the example of Figure 14, assume that the loads are not distributed as {20%, 35%, 45%}, but as 10 blocks of 10% each, with the loads numbered from 1 to 10. Also assume that the load with index 5 submits the cheapest offer to participate in the LaaRs program. As a result, the underfrequency relay controlling load 5 will have the highest frequency setpoint, and will be the first load to be tripped in the priority order. Therefore, the effect of the LaaRs program is to assign priorities to the load-shedding action.

3.5 Systematic Underfrequency Relay Setting Approach

The discussions in this chapter indicate that there is a need for a more systematic method for computing underfrequency relay settings that overcomes the deficiencies of the conventional method. Specifically, the method should:

(i) Take into account the effect of a set of plausible contingencies, not just the most severe one;

(ii) Allow load-shedding priority if needed;

(iii) Explicitly adhere to the generator underfrequency-time constraints;

(iv) Minimize load shedding actions

The subsequent chapters in this dissertation develop and test such a systematic method for setting relays based on mixed integer linear programming (MILP) techniques.

IV. FORMULATION OF THE UNDERFREQUENCY RELAY SETTING PROBLEM AS A MIXED INTEGER LINEAR PROGRAM (MILP)

4.1 Generation Loss Contingencies

Consider a lossless power system model with ng generators, each denoted by the index i. The total number of generation loss contingencies is therefore $(2^{ng}-2)$ where the instances of 0% generation loss and 100% generation loss have trivial solutions and can be ignored. In general, a generation loss contingency j is defined by the loss of all units belonging to a set C^j. If each i^{th} generator has a pre-contingency generation level of g_i^0, then the generation loss resulting from each contingency j can be computed using the following expression:

$$\Delta g^j = \sum_{i \in C^j} g_i^0 \tag{4.1}$$

The equivalent system inertia constant following contingency j is:

$$H^j = \sum_{i \notin C^j} H_i \tag{4.2}$$

when all values H_i are expressed in MJ/MW. Similarly, the equivalent governor droop value is computed using:

$$\frac{1}{R^j} = \sum_{i \notin C^j} \frac{1}{R_i} \tag{4.3}$$

In this thesis, the relay setting problem is based not on a single contingency j (possibly the most severe one) but on a set of nc contingencies S (see Section 5.2 on how to choose S). Thus, $S = \left\{C^j; j = 1,\ldots,nc\right\}$.

4.2 Implicit Formulation of the Underfrequency/Time Limitations

For each generator i, the underfrequency/time limitations require that the frequency during a contingency should not deviate below any of a set of progressively more stringent thresholds f_i^l for longer than Δt_i^l seconds, where $l = 1,\ldots,nl$ (see Table 1, reproduced below for convenience). These thresholds and time constraints are specified by the manufacturer so as to protect the generator against excessively long periods of low frequency operation. If any of these threshold levels is violated, the generator is tripped according to the following logic,

$$\textit{if for any } l = 1,\ldots,nl,\ f(t_0) = f_i^l \textit{ and } f(t) \le f_i^l \textit{ for } t_0 \le t \le t_0 + \Delta t_i^l$$
$$\textit{then trip generating unit } i$$

Table 5: Typical generator off-nominal frequency/time limitations

Underfrequency limit (Hz)	Overfrequency limit (Hz)	Maximum permissible time
60.5-59.5	60.0-60.5	N/A (continuous operating range)
59.4-58.5	60.6-61.5	30 seconds - 3 minutes
58.4-57.9	61.6-61.7	7.5 seconds
57.8-57.4		45 cycles
56.8-56.5		7.2 cycles
Less than 56.4	Greater than 61.7	Instantaneous trip

4.3 Load Shedding Models

In this thesis, we make use of the conventional continuous load shedding model as well as what we term discrete load shedding.

The continuous model of underfrequency load-shedding assumes ns shedding stages, where, at each stage s, a variable amount Δd_s is shed, which is a continuous fraction of the known pre-contingency total system load level. This amount does not necessarily correspond to a specific load. It is rather an amount that is approximately reached by adding up a number of specific discrete loads.

Under the single setpoint relay model (see Section 1.3), load is shed if the frequency drops below setpoint f_s for a time greater than Δt_s seconds. All the variables $\{ f_s, \Delta t_s, \Delta d_s, \; s = 1,\ldots,ns \}$ are unknown decision variables that must be computed by the relay setting problem.

In the discrete load model, the subscript s now stands for a specific sub-station load. Each such load d_s^0, for $s = 1,\ldots,ns$ is assumed to be known and interruptible by an underfrequency relay s. Therefore, the amount of load shed at every stage in the discrete load model is no longer a variable, but is equal to the pre-contingency block size d_s^0. In addition, the symbol ns here denotes the number of discrete loads. Under the single setpoint relay model (Section 1.3), the known load d_s^0 is shed if the frequency drops below setpoint f_s for a time greater than Δt_s seconds. The unknown variables that must be computed by this relay setting problem are $\{ f_s, \Delta t_s \;, \; s = 1,\ldots,ns \}$.

Note that the frequency setpoints f_s and corresponding time delays Δt_s defining the settings of the load shedding relay s should not be confused with the frequency thresholds f_i^l and time delays Δt_i^l defining the underfrequency/time limitations of generating unit i.

4.4 Lower Bound on the Amount of Load-Shedding Required for the Frequency to Recover after a Contingency

Criterion A of the relay setting problem says that, for each contingency j, a sufficient amount of load must be shed to ensure that the steady-state frequency lies within the safe operating range. A lower bound on this "sufficient" amount can be found for each contingency j by minimizing the amount of load-shedding Δd^j that ensures that the resulting steady-state frequency $f_0 + \Delta f^j$ will lie within the safe operating range. This range is defined by $f_0 + \Delta f^j \geq f^1$, where f^1 is the highest generator threshold or, equivalently, the lower limit of the safe operating range, usually 59.5 Hz.

Mathematically, this lower bound can be found from:

$$\min \Delta d^j \tag{4.4}$$

subject to the post-contingency power balance,

$$\sum_i g_i^0 - \Delta g^j - \sum_k d_k^0 + \Delta d^j - \Delta f^j (D + \frac{1}{R^j}) = 0 \tag{4.5}$$

and to,

$$\Delta f^j + f_0 \geq f^1 \tag{4.6}$$

The term $(D + \frac{1}{R^j})$ in Equation (4.5) represents the total damping factor from load-frequency sensitivity and from primary frequency regulation.

If the amount of load-shed after each contingency is assumed to be a continuous quantity and the frequency regulation terms do not reach their maximum capacity or ramp limit then the optimization problem above has an analytical solution of the form:

$$\Delta d^j = \Delta f^j (D + \frac{1}{R^j}) + \Delta g^j \tag{4.7}$$

where we assumed that $\sum_i g_i^0 = \sum_k d_k^0$ and that Δf^j is set to -0.5 Hz corresponding to a minimum permissible continuous steady-state frequency of 59.5 Hz.

As an example, for a generation loss of 35%, a system with 5% primary frequency regulation and 2% load-frequency sensitivity will require 16% of the original load to be shed in order to ensure a steady state frequency of 59.5 Hz.

Note that if the ramp limits are active, it is still possible to solve the lower bound problem but it would then require the use of mixed integer linear programming, such as the approach used in [15].

This measure of the minimum required amount of load-shedding for every contingency is a lower bound since, in general, when we consider frequency trajectory dynamics as well as underfrequency/time limitations, additional load shedding may be required. Nonetheless, this bound is used in this thesis as a simple-to-compute estimate of the amount of load shedding required for any given contingency.

4.5 Incorporating Load-Shedding in the Discrete-Time Frequency Response Model of the Power System through Binary Variables

From Section 1.7, the discrete-time frequency response model following contingency j for the time steps $n = 0,1,\ldots,nt$ is given by the following expression:

$$\Delta f_n^j = \Delta f_{n-1}^j + S_{n-1}^j \Delta t \tag{4.8}$$

where Δt is the pre-defined time step length and where the gradient of the trajectory is given by,

$$S_n^j = \frac{f_0}{2H^j}\left(\sum_i g_i^0 + \Delta r_n^j - \Delta g^j - \sum_k d_k^0 - D\Delta f_n^j\right) \tag{4.9}$$

with primary frequency regulation,

$$\Delta r_n^j = \Delta r_{n-1}^j + \frac{\Delta t}{T}\left(-\frac{\Delta f_n^j}{R^j} - \Delta r_{n-1}^j\right) \tag{4.10}$$

Note that the initial conditions at $n=0$ prior to the contingency are $\Delta f_0^j = 0$ and $\Delta r_0^j = 0$.

If at time step n of trajectory j, load s, Δd_s, is shed, then the effect is to change the gradient S_n^j. Such a load shedding action by an underfrequency relay is modeled here through a binary variable u_{sn}^j , which is defined as follows:

$$u_{sn}^j = \begin{cases} 1\,; \textit{if during contingency } j,\ \Delta d_s \textit{ is shed at time step } n \\ 0\,; \textit{if during contingency } j,\ \Delta d_s \textit{ is not shed at time step } n \end{cases}$$

Using this binary variable, the total amount of load shed during trajectory j at time step n is $\sum_s u_{sn}^j \Delta d_s$, a quantity that can then be incorporated into the discrete time frequency model by modifying Equation (4.9) as follows:

$$S_n^j = \frac{f_0}{2H^j}\left(\sum_i g_i^0 + \Delta r_n^j - \Delta g^j - \left(\sum_k d_k^0 - \sum_s u_{sn}^j \Delta d_s\right) - D\Delta f_n^j\right) \tag{4.11}$$

The introduction of binary variables into the dynamic frequency response model may seem at first glance unnecessarily intricate. However, as shown below, binary variables greatly facilitate the formulation of the generator underfrequency/time limitations, of the relay operation logic, and of a number of relay operational constraints. Without binary variables a systematic formulation of the relay setting problem would not be possible in an explicit form suitable for analysis using MILP.

As a first demonstration of the power of binary variables, consider that, for reasons of coordination, the relay engineer may require that no two relays should operate at the same time, and that there should be a minimum time delay τ between two successive load shedding operations. Using the binary variable model, this type of constraint can easily be expressed explicitly as follows:

$$\sum_s u_{sn}^j - \sum_s u_{s,n-\tau}^j \le 1 \quad \forall j,n \tag{4.12}$$

Another operational constraint dictates that once a relay operates, it cannot change its on/off state within the time span being considered for the relay setting problem, that is, within the range $n = 0,1,\ldots,nt$, typically 20 seconds or 200 time steps at 0.1 seconds each. Using binary variables, this condition requires that,

$$u_{sn}^j \ge u_{s,n-1}^j \quad \forall j,s,n \tag{4.13}$$

in other words, if the binary variable is 1 at time step $n-1$ then it must also be 1 for any subsequent time step.

If a load shedding priority is imposed according to the order $s = 1,\ldots,ns$ the higher the index, the higher the priority, then it must follow that:

$$u_{s-1,n}^j \le u_{sn}^j \quad \forall j,s,n \tag{4.14}$$

which implies that if load s has been shed at or before time n then load $s-1$ must also have been shed at time n or before.

4.6 Relay Timer Model Using Binary Variables

As described in Section 1.3, an underfrequency relay s operates to disconnect a load s under contingency j at time step n when the frequency trajectory Δf_n^j as computed by Equation 4.8 remains below a frequency setpoint f_s for an interval of time greater than Δt_s (the more general case of relays with multiple setpoints is treated in Appendix A).

To describe this relay logic, the definition of timers measuring the time spent below a frequency setpoint is required for every load s. This can be efficiently accomplished through the following *implicit* binary variable definition:

$$v_{sn}^{j} = \begin{cases} 1 & if\ f_0 + \Delta f_n^j \le f_s; \\ 0 & if\ f_0 + \Delta f_n^j > f_s \end{cases} \tag{4.15}$$

which says that if the frequency trajectory j reaches or falls below the frequency setpoint f_s at time step n, then the binary variable v_{sn}^{j} is equal to 1 and 0 otherwise.

Equation (4.15) can be expressed in an equivalent *explicit* linear form through:

$$\frac{f_s - \left(f_0 + \Delta f_n^j\right)}{L} \le v_{sn}^{j} \le 1 + \frac{f_s - \left(f_0 + \Delta f_n^j\right)}{L} \tag{4.16}$$

where L is a sufficiently large positive number (e.g. 60 Hz). The equivalence between the implicit relation (4.15) and its explicit form (4.16) can be verified as follows: if $f_0 + \Delta f_n^j < f_s$, then from (4.16) it can be seen that $\epsilon \le v_{sn}^{j} \le 1 + \epsilon$ where ϵ is a positive number much smaller than 1. Since v_{sn}^{j} is a binary variable, the only choice is $v_{sn}^{j} = 1$. Similarly, when $f_0 + \Delta f_n^j > f_s$, then $-\epsilon \le v_{sn}^{j} \le 1 - \epsilon$, implying that $v_{sn}^{j} = 0$.

Thus, the timer corresponding to relay s starts counting when the binary variable v_{sn}^{j} is equal to 1. Therefore, the total time in seconds spent by trajectory j below the frequency setpoint f_s at time step n is given by:

$$\Delta t_{sn}^{j} = \sum_{m=0}^{n} v_{sm}^{j} \Delta t \tag{4.17}$$

The binary variables v_{sn}^{j} must also satisfy the condition $v_{s0}^{j} = 0; \forall j, s$ in order to represent explicitly the fact that all frequency trajectories begin at the nominal frequency where no relay frequency setpoints have yet been crossed.

In addition, for each relay s, the frequency setpoint f_s must be greater than or equal to the lowest permissible generator frequency threshold, f^{nl}, typically 57 Hz. Thus,

$$f_s \geq f^{nl} \tag{4.18}$$

Finally, if standard practice requires that consecutive frequency setpoints be separated by a coordinating margin of 0.1 Hz, then this can be imposed through the following inequalities for all s,

$$f_s - f_{s+1} \geq 0.1\ Hz \tag{4.19}$$

4.7 Relay Operation Logic Using Binary Variables

The logic that determines the operation of each relay s says that Δd_s is shed when the amount of time spent by trajectory j below the setpoint f_s at time step n, Δt^j_{sn}, reaches a value Δt_s. Note that whereas in conventional relays the relay time delays Δt_s are usually fixed at 0.2 seconds, in this thesis, these delays are decision variables that add new degrees of freedom to the relay setting problem.

The load shedding binary variable u^j_{sn} defined in Section 4.5 can now be related to the variables Δt_s and Δt^j_{sn} through the implicit relation:

$$u^j_{sn} = \begin{cases} 0 & \text{if } \Delta t^j_{sn} < \Delta t_s; \\ 1 & \text{if } \Delta t^j_{sn} \geq \Delta t_s \end{cases} \tag{4.20}$$

which can be put in an explicit form suitable for a MILP formulation through,

$$\frac{\Delta t^j_{sn} - \Delta t_s}{L} \leq u^j_{sn} \leq 1 + \frac{\Delta t^j_{sn} - \Delta t_s}{L} \tag{4.21}$$

where L is a sufficiently large positive number (e.g. 20 seconds).

In addition to inequality (4.21), the relay time delay decision variables Δt_s must be greater than the minimum time required for the circuit breaker to open , Δt_{min}, typically 0.2 seconds, that is,

$$\Delta t_s \geq \Delta t^{min} \quad \forall s \tag{4.22}$$

4.8 Constraints on the Load Shedding Variables

If the discrete load shedding model is used, then the amount of load to be shed at each stage is known, that is,

$$\Delta d_s = d_s^0; \ \forall s \tag{4.23}$$

Under the continuous load shedding model, the total amount of load-shedding over the ns stages cannot be greater than the pre-contingency system load,

$$\sum_s \Delta d_s \leq d^0 \tag{4.24}$$

In addition the decision variables Δd_s must all be non-negative.

$$\Delta d_s \geq 0 \tag{4.25}$$

4.9 Explicit Formulation of the Generator Underfrequency/Time Limitations Using Binary Variables

The relay settings must also be coordinated with the underfrequency/time limitations defined by the specified generator frequency thresholds f^l and maximum permissible times below these thresholds Δt^l. This must be satisfied for all thresholds $l = 1,\ldots,nl$ and all contingencies $j = 1,\ldots,nc$.

To ensure this coordination, we first define a new set of binary variables equal to 1 if trajectory j reaches or falls below frequency threshold l at time step n and zero otherwise:

$$w_n^{jl} = \begin{cases} 1 & if\ f_0 + \Delta f_n^j \le f^l; \\ 0 & if\ f_0 + \Delta f_n^j > f^l \end{cases} \quad (4.26)$$

As before, (4.27) can be expressed explicitly via:

$$\frac{f^l - (f_0 + \Delta f_n^j)}{L} \le w_n^{jl} \le 1 + \frac{f^l - (f_0 + \Delta f_n^j)}{L} \quad (4.27)$$

With these new binary variables, the time spent by trajectory j at time n at or below frequency threshold l can be expressed explicitly as:

$$\Delta t_n^{jl} = \sum_{r=0}^{n} w_r^{jl} \Delta t \quad (4.28)$$

(note that $w_0^{jl} = 0$ since at $n = 0$ no frequency threshold has been crossed by any trajectory).

The generator underfrequency/time limitations can now be expressed through the following explicit constraints:

$$\Delta t_{nt}^{jl} \le \Delta t^l \ \ \forall j, l \quad (4.29)$$

where Δt_{nt}^{jl} is the total time spent by the frequency during trajectory j (of duration nt steps) at or below generator threshold l.

4.10 Converting Non-linearities into Equivalent Linear Forms

Since both variables u_{sn}^j and Δd_s in the non-linear expression $u_{sn}^j \Delta d_s$ of Equation (4.11) are variables, we now convert this expression into an equivalent linear form, thus rendering the problem formulation compatible with MILP. Note that when the discrete load shedding model is used, Δd_s is a known constant and there is no non-linearity in the term $u_{sn}^j \Delta d_s$.

For the continuous load shedding model, consider that from binary mathematics, the product of a binary and continous variable can be expressed as an equivalent pair of linear relations [25]. Denoting the product $u_{sn}^j \Delta d_s$ by a new continuous variable x_{sn}^j, these explicit linear relations are:

$$0 \le x_{sn}^j \le u_{sn}^j d^0$$
$$0 \le \Delta d_s - x_{sn}^j \le (1 - u_{sn}^j) d^0 \qquad (4.30)$$

where we used the condition that $0 \le \Delta d_s \le d^0$. To show that (4.30) is equivalent to $u_{sn}^j \Delta d_s$, consider the following: if $u_{sn}^j = 1$, then the linear relations reduce to:

$$0 \le x_{sn}^j \le d^0$$
$$0 \le \Delta d_s - x_{sn}^j \le 0$$

which imply correctly that $x_{sn}^j = \Delta d_s$. Alternatively, if $u_{sn}^j = 0$, then reduces to,

$$x_{sn}^j = 0$$
$$0 \le \Delta d_s \le d^0$$

as required.

4.11 Degrees of Freedom in Setting the Underfrequency Relays and MILP Formulation

There are three types of degrees of freedom (also called decision variables) in the relay setting problem defined in this chapter: the frequency setpoints, the time delays, and the amount of load to be shed at every stage. If a discrete load model is used, then the number of types of degrees of freedom reduces to two as the amount of load shed in every stage is known.

The previous sections of this chapter show that there are numerous constraints among these degrees of freedom, both in the form of equalities and inequalities. Because of the large number of decision variables and constraints, just finding a feasible solution is a non-trivial problem that is very hard to solve in general by heuristic means. However, the fact that the constraints on the relay setting decision variables are linear, permit us to use systematic and efficient linear programming approaches to find feasible solutions. Such MILP approaches not only allow us to find feasible solutions but also feasible and optimal ones if we can define an acceptable objective function to be minimized that reflects how underfrequency relays should behave ideally. Since objective functions are open to interpretation, a suitable choice is discussed in Section 4.12.

Given an objective function to be minimized, the relay setting problem can now be formulated as a MILP subject to:

(i) The discrete time frequency response through Equations (4.8), (4.10) and (4.11), with the non-linearity in Equation (4.11) expressed in the linear form of (4.30) ;

(ii) The load-shedding model through relations (4.12) to (4.14);

(iii) The relay timer model through relations (4.16) to (4.19);

(iv) The underfrequency relay operational logic of relations (4.21) through (4.25);

(v) The generator underfrequency/time limitations of relations (4.27) to (4.29).

An important characteristic of this MILP relay setting solution is that the underfrequency/time constraints will be satisfied for all contingencies belonging to S (the set of contingencies explicitly considered in the MILP relay setting problem). Although there is no guarantee that contingencies outside S will also meet the underfrequency/time constraints, experimental results suggest that by choosing S appropriately, most contingencies will satisfy the underfrequency/time constraints (see Chapter V).

4.12 A Suitable Objective Function for MILP Formulation

The relay time delays, Δt_s, are an important decision variable in the relay setting problem. Maximizing the sum of all time delays, for example, may seem sensible in relation to criterion C of Section 3.2. The reasoning here is that by delaying load shedding action as much as possible, unnecessary load shedding may be avoided for milder contingencies. This reasoning however turned out to be flawed since it leads to relay settings that do not respond quickly enough to contingencies not belonging to S, even to some contingencies that are relatively mild.

Instead, we concluded that a more effective objective function of the MILP is to minimize a weighted combination of the relay times before load-shedding and the amount of load shed for all contingencies in S.

By minimizing, instead of maximizing, relay time delays, the resulting relay settings are more conservative, shedding loads more quickly, in some cases, at the minimum time delay of 0.2 seconds. This more conservative approach seems to protect the system more effectively against contingencies not included in S.

On the other hand, if only time delays are minimized, the resulting MILP settings tend to become unnecessarily conservative, shedding some loads even for mild contingencies not belonging to S that could recover without load shedding. This observation justified modifying the objective function by adding a weighted load shedding term whose purpose is to prevent unnecessary load shedding. As a result, a suitable objective function of the MILP relay setting problem is of the form,

$$\min\left\{\sum_{s}\kappa_1\Delta t_s+\sum_{j}\sum_{s}\kappa_2 u_{s,nt}^{j}\Delta d_s\right\} \tag{4.31}$$

where κ_1 and κ_2 are normalization and weighting constants. These parameters were chosen so that κ_1 = estimated average percent load shed per stage, and κ_2 = estimated average time delay to shedding in seconds per contingency.

4.13 Testing the MILP Formulation against the Conventional Approach

In order to test the mixed integer formulation, a set of relay settings are computed using the MILP method for the worst case contingency in the 3-bus network of Figure 13, and compared with the results obtained using the conventional method (see Section 3.1).

In the 3-bus network, the worst case contingency is the loss of 90% of the pre-contingency generation level, and is the only contingency in the set S. As in Section 3.1, a four-stage load-shedding schedule is used, assuming continuous load-shedding. Time delays are fixed at 0.2 seconds. The MILP is solved using GAMS [26] to calculate the frequency setpoints and amount shed at each load-shedding stage. Table 6 shows the relay settings, which turn out to be in close agreement with those computed using the conventional method of Section 3.2.

Table 6: Relay settings obtained from MILP for 90% contingency compared with conventional settings computed in Section 3.2

Stage	Frequency (Hz)		Delay	Load Shed (%)	
	MILP	Conventional	(seconds)	MILP	Conventional
1	59.50	59.4	0.20	12.7	15.0
2	58.99	58.9	0.20	18.6	20.0
3	58.08	58.6	0.20	18.6	20.0
4	57.81	58.3	0.20	20.0	20.0

Figure 17 plots the frequency trajectory with the load shedding strategy found using MILP, which is also in close agreement with the response of Figure 15 when the relays are set by the conventional method.

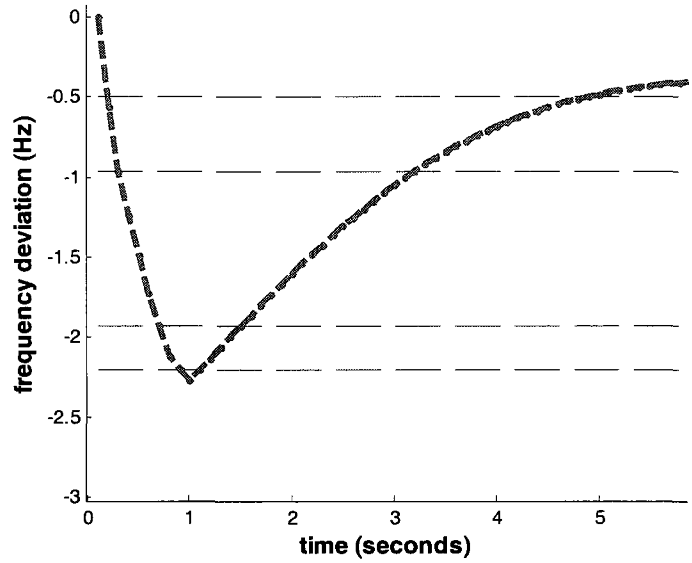

Figure 17: Frequency trajectory from MILP for 90% contingency

It is interesting to observe the performance of these MILP relay settings for other contingencies outside those considered in the set S. As an example, consider the loss of 25% and 60% generation, possible contingencies which are not in the set S. These two contingencies were simulated using the discrete-time frequency model with the MILP relay settings of Table 6.

First, we examine whether the corresponding frequency responses to the contingencies meet the underfrequency/time limitations. From Table 7, we see that the larger contingency violates the 59.5 Hz threshold in the sense that since it never reaches a safe operating level, the amount of time spent below this threshold is infinite. The same underfrequency/time violation occurs if the relays are set according to the conventional method.

The advantage of the MILP method is that the loss of 60% generation can be added to the contingency set S in order to re-compute the relay settings and avoid violating the underfrequency/time constraints when this contingency occurs. This contingency set updating process is discussed in extensive detail in the next chapter.

Table 7: Performance of trajectories outside *S* relative to underfrequency/time limitations

Frequency thresholds	Maximum permissible time below threshold (s)	Time spent below threshold in s for	
		Loss of 25%	Loss of 60%
59.5	30.0	0	∞
58.5	10.0	0	2.5
57.5	1.0	0	0

The contingency corresponding to the loss of 25% of the pre-contingency generation does not violate any of the underfrequency/time limitations. However, with the current relay settings, the amount of load shed under this contingency should be compared with the lower bound computed in Section 4.4 to judge whether unnecessary load shedding has taken place. This comparison is shown in Table 8 which suggests that the 25% generation loss contingency may be shedding too much load since the lower bound is zero while the actual load shedding is 12%.

Table 8: Comparison of simulated load shed with lower bound

Contingency	Lower Load Shedding Bound (%)	Simulated LoadShed (%)
Loss of 25%	0	12
Loss of 60%	Violates underfrequency/time constraints	

Chapter V presents a case study illustrating the MILP approach to relay setting. It also builds on the previous example by discussing an algorithm for updating the contingency set S with the intention of meeting all three load-shedding criteria for all contingencies not in S.

V. A SYSTEMATIC APPROACH TO SELECTING CONTINGENCIES IN THE MILP RELAY SETTING FORMULATION: A CASE STUDY

5.1 Selecting the Contingency Set S

In this thesis, a contingency j is defined as the loss of a pre-specified combination of generating units belonging to the set C^j. In the mixed integer formulation of the relay setting problem discussed in the previous chapter, relays are set with respect to a set of contingencies S defined by $S = \{C^j; j = 1,...,nc\}$. The motivation dictating the process of selecting the contingencies that compose the set S is discussed in this section.

As discussed in Chapter 3, if the underfrequency relays are set on the basis on severe contingencies only as in the conventional method, then when a less severe contingency occurs, the relays may be too sensitive and trip too much load. Conversely, if the relays are set on the basis of mild contingencies only, then when a more severe contingency occurs, the relays may not be sensitive enough, leading to insufficient load shedding, or worse still, to violations of the underfrequency/time limitations of generators. As a result, the contingency set S should be composed of both mild and severe contingencies.

Even for a set of judiciously chosen contingencies covering both mild and severe generation loss scenarios, one remaining question is whether the resulting relay settings will work properly for contingencies not belonging to S. Improper relay operation means that following such a contingency, either of the following occurs: (i) none of the generator underfrequency limitations are violated, satisfying criterion B, but an unnecessary amount of load is shed, which is a violation of criterion C or (ii) irrespective of whether or not the amount of load shed is excessive, a violation of criterion B occurs (load is not shed quickly enough, leading to violations of the generator underfrequency/time limitations).

The first concern is important, but not as critical as the second; the time spent below the fixed generator thresholds must be less than the maximum permissible limits for any contingency, lest it lead to damaged generation equipment.

Thus, the relay settings computed using the MILP approach devised in this thesis *ideally* should conform to *all* three load-shedding criteria A, B, and C. However, in extreme cases, it may be expedient to relax the conditions of criterion C in order to accommodate the more critical criterion B.

It should be mentioned that it is theoretically possible to include all generation loss contingencies in the set S. The practical obstacle to doing this is, obviously, limited available computational time and resources. Although optimization software such as CPLEX [27] in GAMS is capable of handling large numbers of continuous and binary variables, consider this: for a network with 20 generators, the number of possible generation loss events is $(2^{20} - 2) = 1{,}048{,}574$. If a four stage load-shedding schedule with single level relay timers is to be devised for this network, and frequency computations carried out every 0.1 seconds for 20 seconds, the number of variables increases to 838,859,200. Although CPLEX can solve this type of problem in reasonable time, it is unreasonable to include all possible generation loss events in the MILP formulation if the number of generators is very large. There must be a systematic method of selecting a relatively small set of contingencies that best represent outages on the network. Recall that this requirement is not unique to the MILP method since the conventional method also typically relies on one single contingency to set the relays, namely the "worst" or most severe one.

5.2 *An Algorithm for Selecting Contingencies in the Set S*

A systematic choice of the contingency set S to compute relay operation consistent with criteria A, B and C (see Section 3.2) is as follows:

(i) Define an initial set of contingencies S_N, where N is the number of contingencies in the set, including the most severe contingency in which all but one generator trip, as well as the least severe contingency in which only one generator trips. The most severe can be defined by the smallest non-zero value of $\frac{\Delta g^j}{H^j}$, giving the steepest intial frequency drop. The least severe contingency can similarly be defined by the largest non-zero value of $\frac{\Delta g^j}{H^j}$, giving the shallowest initial slope.

(ii) Solve the MILP and set the underfrequency relays.

(iii) For these relay settings, find the loss of generation contingency not belonging to S_N that violates the underfrequency/time conditions in the worst way, that is, under which the frequency transient spends the maximum amount of time below one of the generator underfrequency thresholds. If this time is greater than zero, then add this contingency to S_N and restart the process, else stop.

(iv) If there are no violations of underfrequency/time limitations, then find the contingency not belonging to S_N that sheds the largest amount of excess load compared to its lower bound. If this value is positive and above a certain threshold, then add the contingency to S_N, and restart the process, else stop.

Steps (iii) and (iv) can be formulated and solved as MILP, however, this more systematic approach is beyond the scope of this thesis. Here, for simplicity, these steps are carried out using full enumeration for a small system.

The following section describes a case study on the MILP approach to underfrequency relay setting with updating of the set S.

5.3 Case Study: 3-bus Test Network

Figure 18 shows a 3-bus network, with generator data as in Table 9.

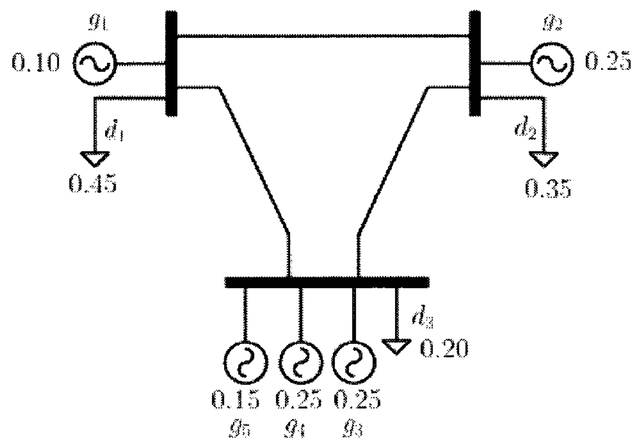

Figure 18: Three-bus test network

Table 9: Case study input data

Unit	g_i^0 (p.u.)	R_i (p.u.)	H_i (MJ/MW)
g_1	0.10	0.05	2.8
g_2	0.25	0.05	5.0
g_3	0.25	0.05	5.0
g_4	0.25	0.05	3.0
g_5	0.15	0.05	2.8

Given that there are 5 generators in this network, the number of plausible generation loss events is $(2^5 - 2) = 30$. However, the nature of the data here is such that there are 11 distinct generation loss scenarios, as indicated in Table 10:

Table 10: All possible contingencies in the example

Contingency #	Units lost under C^j	Generation loss Δg^j (%)	Equivalent inertia H^j (MJ/MW)	Initial slope $\frac{\Delta g^j}{H^j}$ (puMW/s)
1	$\{g_1\}$	10	15.8	0.006
2	$\{g_5\}$	15	15.8	0.009
3	$\{g_2\}$	25	12.8	0.019
4	$\{g_1, g_2\}$	35	10.8	0.032
5	$\{g_2, g_5\}$	40	10.8	0.037
6	$\{g_2, g_3\}$	50	8.6	0.058
7	$\{g_1, g_2, g_3\}$	60	5.8	0.103
8	$\{g_2, g_3, g_5\}$	65	5.8	0.112
9	$\{g_2, g_3, g_4\}$	75	3.6	0.208
10	$\{g_1, g_2, g_3, g_4\}$	85	2.8	0.304
11	$\{g_2, g_3, g_4, g_5\}$	90	2.8	0.321

In this example, all generators are assumed to have the same underfrequency/time limitations as specified in Table 11 (values are based on typical generator underfrequency/time limitations presented in [6]).

Table 11: Generator underfrequency/time limitations for example

Frequency threshold f^l (Hz)	Maximum permissible time Δt^l (seconds)
59.5	30.0
59.0	20.0
58.5	10.0
58.0	5.0
57.5	1.0

Following the algorithm of Section 5.2, the initial set of contingencies S_2 (the subscript denotes the number of contingencies in the set) contains the generation loss events that result in the highest and lowest initial slopes of the frequency trajectories. From Table 10, the highest and lowest initial slopes contingencies 11 and 1 respectively, corresponding to generation losses of 90% and 10%. Therefore, the initial set of contingencies is selected as $S_2 = \{10\%, 90\%\}$.

Using the continuous load-shedding model, where load is to be shed in four stages (the number four is picked to be the same as in the conventional method), the MILP was solved with S_2 as the contingency set. The computational time for this small system using GAMS on an Intel® Core™2Duo 1.66GHz processor was of the order of 20 seconds. The following relay settings were obtained:

Table 12: Results from MILP with contingency set S2

Stage	Frequency (Hz)	Delay (seconds)	Load Shed (%)
1	59.04	0.20	3.01
2	58.54	0.20	16.9
3	58.22	0.20	23.7
4	57.62	0.20	26.4

Next, we found the contingencies not belonging to S_2 that violated the generator underfrequency/time limitations in the worst way. The nine contingencies not belonging to S_2 were simulated using the discrete-time frequency model. The times spent by the system frequency below each generator underfrequency threshold under each contingency are listed in Table 13.

Table 13: Performance of trajectories outside S_2 relative to underfrequency/time limitations

Frequency Thresholds	Maximum permissible time (s)	Time spent below threshold in seconds for contingency with generation loss of:								
		15% (2)	25% (3)	35% (4)	40% (5)	50% (6)	60% (7)	65% (8)	75% (9)	85% (10)
59.5	30.0	0.0	5.37	6.08	∞	∞	3.96	31.5	∞	2.61
59.0	20.0	0.0	0.0	2.21	3.3	5.86	2.31	3.40	4.59	1.66
58.5	10.0	0.0	0.0	0.0	0.0	2.99	1.11	1.68	2.85	1.11
58.0	5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.38	0.70
57.5	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.35

From Table 13, it is clear that the relay settings based on S_2 result in violations of underfrequency/time limitations for contingencies 5, 6, 8 and 9 (corresponding to generation losses of 40%, 50%, 65% and 75% respectively), as these trajectories shed an insufficient amount of load and fail to recover above the lower bound of the safe operating range of 59.5 Hz within the maximum permissible time of 30 seconds. The trajectory of contingency 6 (the highlighted column in Table 13) is identified as the one violates the underfrequency/time limitations in the worst way, as it spends the largest amount time below all frequency threshold levels, even though the maximum time constraints for some threshold were not violated.

Following the systematic approach outlined in Section 5.2, we added contingency 6 to the set S to yield $S_3 = \{10\%, 90\%, 50\%\}$. After re-solving the MILP, the relay settings were obtained:

Table 14: Relay settings from MILP using S_3 as the contingency set

Stage	Frequency (Hz)	Delay (seconds)	Load Shed (%)
1	59.04	0.20	15.7
2	58.48	0.20	15.7
3	57.93	0.20	15.7
4	57.57	0.20	24.1

As in step (iii) of the algorithm, to identify the contingency that violates the underfrequency/time constraints in the worst way, the eight contingencies not belonging to S_3 were simulated using the discrete-time frequency response model with the underfrequency relays set as in Table 14. The times spent by each new trajectory below the threshold levels are summarized in Table 15.

Table 15: Performance of trajectories outside S_3 relative to underfrequency/time limitations

Frequency Thresholds (Hz)	Maximum permissible time (s)	Time spent below threshold in seconds for contingency with generation loss:							
		15% (2)	25% (3)	35% (4)	40% (5)	60% (7)	65% (8)	75% (9)	85% (10)
59.5	**30.0**	0.0	5.37	3.59	4.46	∞	∞	∞	2.43
59.0	**20.0**	0.0	0.0	0.3	1.07	4.83	6.48	4.17	1.58
58.5	**10.0**	0.0	0.0	0.0	0.0	2.68	3.88	2.56	1.06
58.0	**5**	0.0	0.0	0.0	0.0	0.0	0.0	0.88	0.67
57.5	**1**	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.32

From Table 15, we observed that contingency 8 (corresponding to a 65% generation loss) violates the underfrequency/time limitations in the worst way. Therefore the set S was updated to $S_4 = \{10\%, 90\%, 50\%, 65\%\}$. The MILP was solved again, and a new set of relay settings was obtained, which are listed in Table 16.

Table 16: Relay settings from MILP using S_4

Stage	Frequency (Hz)	Delay (seconds)	Load Shed (%)
1	59.36	0.20	22.2
2	58.42	0.20	22.8
3	58.17	0.20	25.0
4	58.06	0.20	25.0

The contingencies outside S_4 were simulated with the relays set according to Table 16. The times spent by each trajectory below the threshold levels are listed in Table 17.

Table 17: Performance of trajectories outside S_4 relative to underfrequency/time limitations

Frequency Thresholds (Hz)	Maximum permissible time (s)	Time spent below threshold in seconds for each contingency with generation loss:						
		15% (2)	25% (3)	35% (4)	40% (5)	60% (7)	75% (9)	85% (10)
59.5	**30.0**	0.0	1.32	2.22	3.31	4.11	2.21	1.01
59.0	**20.0**	0.0	0.0	0.0	0.0	2.26	1.41	0.75
58.5	**10.0**	0.0	0.0	0.0	0.0	0.75	0.75	0.52
58.0	**5**	0.0	0.0	0.0	0.0	0.0	0.0	0.32
57.5	**1**	0.0	0.0	0.0	0.0	0.0	0.0	0.08

The results in Table 17 indicate that no violations of the underfrequency/time constraints occur for simulations of the seven contingencies not belonging to S_4. This is a significant result, and indicates that the relay settings listed in Table 16 were successful in meeting the generation underfrequency/time violations not only for the four contingencies in S_4, but also for the seven contingencies not belonging to S_4.

In accordance with step (iv) of the systematic approach discussed in Section 5.2, for each of the seven contingencies not belonging to S_4, the difference between the amounts of load shed and the lower bound of load required in the steady state were computed. The results are summarized in Table 18.

Table 18: Comparison of simulated load shed with lower bound for contingencies not belonging to S_4

Contingency #	Generation loss (%)	Lower Load Shedding Bound (%)	Simulated Load Shed (%)
2	**15.0**	0.0	0.0
3	**25.0**	0.0	0.0
4	**35.0**	0.0	22.2
5	**40.0**	3.3	22.2
7	**60.0**	40.0	45.0
9	**75.0**	55.0	70.0
10	**85.0**	65.0	70.0

The amounts of load shed for each of the contingencies S_N from the MILP solution are presented in Table 19 for convenience.

***Table 19:** Comparison of MILP load shed with lower bound for contingencies in S_4*

Contingency#	Generation loss (%)	Lower Load Shedding Bound (%)	Load Shed computed using MILP (%)
1	**10.0**	0.0	0.0
6	**50.0**	30.0	45.0
8	**65.0**	45.0	45.0
11	**90.0**	70.0	70.0

From Table 18, we note that the differences between the amounts of simulated load shed and the lower bound for every contingency are nonzero.

The question now is whether by adding more contingencies to the contingency set S, a lower amount of load can be shed, or whether we are satisfied with the amount of load shed with the current relays settings, in which case the process could be stopped here. An alternative approach to reduce the amount of load shed may be to increase the number of load shedding steps without altering S.

To examine the first alternative, we now update the set S_N with the contingency that results in the greatest difference between the simulated load shed and lower bound. In Table 18, contingency 4 (highlighted row) results in the largest difference between the simulated amount of load shedding and the lower bound of load-shedding required. Therefore, the MILP contingency set was updated to include contingency 4 to $S_5 = \{10\%, 90\%, 50\%, 65\%, 35\%\}$, and a new set of MILP relay settings were computed as listed in Table 20.

Table 20: Relay settings from MILP using S_5

Stage	Frequency (Hz)	Delay (seconds)	Load Shed (%)
1	58.75	0.20	23.3
2	58.36	0.20	23.3
3	58.18	0.20	23.4
4	57.62	0.20	23.4

Using the above relay settings, the contingencies not belonging to S_5 were simulated, and the difference between the simulated load shed and the lower bound were recorded, as listed in Table 21. Note that there were no violations of the underfrequency/time limitations for this set of simulations. In addition, the amounts of load shedding computed using the MILP for all contingencies in S_5 are summarized in Table 22.

Table 21: Comparison of simulated load shed with lower bound for contingencies not belonging to S_5

Contingency #	Generation loss (%)	Lower Load Shedding Bound (%)	Simulated Load Shed (%)
2	15.0	0.0	0.0
3	25.0	0.0	0.0
5	40.0	3.3	23.3
7	60.0	40.0	46.6
9	75.0	55.0	70.0
10	85.0	65.0	70.0

Table 22: Comparison of MILP load shed with lower bound for contingencies in S_5

Contingency #	Generation loss (%)	Lower Load Shedding Bound (%)	Load Shed computed using MILP (%)
1	10.0	0.0	0.0
4	35.0	0.0	0.0
6	50.0	30.0	46.6
8	65.0	45.0	46.6
11	90.0	70.0	70.0

From Table 21, it is evident that contingency 5 (highlighted row) now results in the largest difference between the simulated load-shed and the lower bound. Since this difference is now 23.3%-3.3% = 20% whereas the worst difference with S_4 was 22.2%-0% = 22.2%, then according to step (iv) of the algorithm, we can stop the process, as the new MILP settings do not yield a significant improvement in the amount of load shed.

The MILP relay setting process was nonetheless repeated in this case study for the sets S_7, up to S_{11} to determine whether the amount of load shed would ever equal to the lower bound, something that, not unexpectedly, did not happen.

5.4 Note on the Number of Load-Shedding Stages Used in the MILP

In the case study presented above, the MILP was solved for a four stage load-shedding scheme. This number was chosen in accordance with the conventional method outlined in Section 2.1. However, increasing the number of stages may result in relay settings that shed a lower amount of load for the contingencies not considered in the MILP set S_N, than when using a lower number of stages.

To test this hypothesis, the MILP was resolved for contingency set S_4 with five load-shedding stages instead of four. It was noted that the average amount of load shed decreased from 7.85 to 6.21 %. The gain in the amount of load shed by increasing the number of load shedding stages was offset by an increase in computation time from something of the order of 20 seconds to about 20 minutes on an Intel Core 2 Duo 1.66 GHz processor.

Given sufficient computing power, once the contingency set selection process is finalised, it may be expedient to resolve the MILP using a higher number of load-shedding stages in an attempt to further reduce the amount of load shed.

5.6 Comments on the Variation of Relay Settings with the Set S_N

An analysis of the relay settings computed using the MILP over the number of contingencies N for the 3-bus case study revealed some interesting trends regarding the three decision variables of the relay setting problem. These findings are summarised in this section.

It can be noted from the relay settings in Tables 11, 12, 15, 19 and 25 that the time delays are always set to the minimum value of 0.2 seconds. This implies that there are enough degrees of freedom in choosing the frequency setpoints such that there is no benefit derived from delaying load-shedding actions. In other words, in order to avoid load-shedding during a mild contingency, the MILP will choose to lower the frequency setpoint instead of increasing time delays. This is an obvious consequence given the chosen form of objective function (Equation 4.30). This result is also in agreement with studies such as [12, 14, 23], in which it is claimed that there is no significant advantage to delaying load-shedding actions.

The variation in the frequency setpoints for each of the four stages across the sets S_N is plotted in Figure 19.

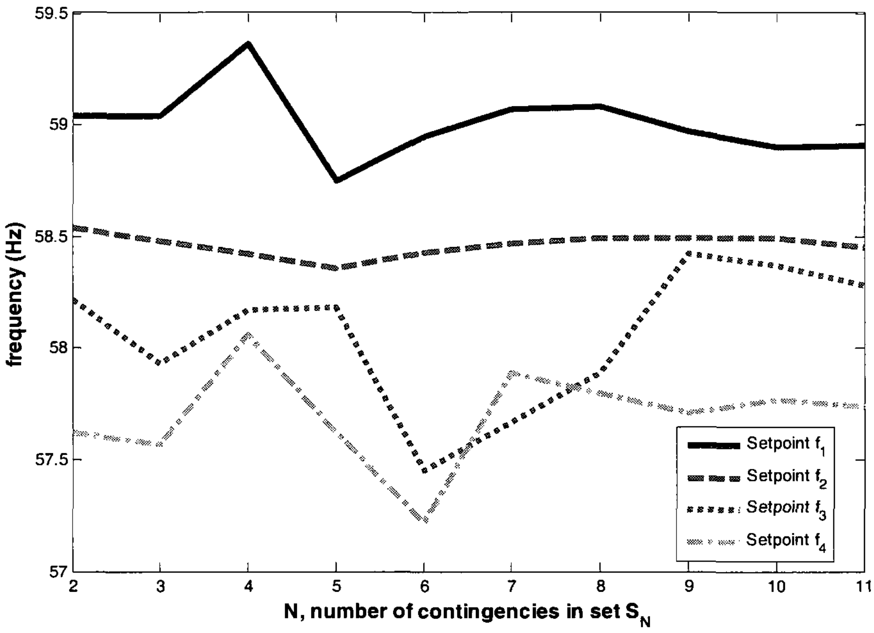

Figure 19: Variation of frequency setpoints with N

Figure 19 indicates that there is generally greater variation in the lower frequency setpoints f_3 and f_4 than in the higher setpoints f_1 and f_2. This implies that the settings of the lower frequency setpoints generally determine the performance of the relay in ensuring that the frequency recovers within the generator underfrequency/time constraints.

From Figure 19, we can also observe the effect of the severity of the added contingency on the computed relay settings. As an example, when going from *N=3* to $N = 4$ by adding a severe contingency of 65% generation loss, there is a sharp upward kink for three of the four setpoints. This implies that the relays must start shedding load at an earlier time in order to accommodate the severe contingency within the underfrequency/time limitations. In contrast, when going from *N=4* to $N = 5$ by adding a mild contingency of 35%, slightly lower values resulted for all frequency setpoints compared to the values computed with $N = 4$. We infer from this that the frequency setpoints were lowered in order to delay load-shedding for the mild contingency. Mild contingencies therefore tend to lower the frequency setpoints while severe contingencies have the opposite effect.

Another observation from Figure 19 is that for $N = 11$, when all contingencies are considered in the MILP, the relay setpoints seem to "converge" to the values computed for the initial set S_2. Recall that the set S_2 only included the mildest and the most severe contingency, and simulations of the contingencies outside S_2 resulted in violations of the underfrequency/time limitations; the relay settings computed using S_2 were therefore unacceptable. However, we now observe that the frequency setpoints for the sets S_2 are within 1% of the values computed using S_{11}. This however may be a coincidence.

Another interesting observation is the amount of load shed at every stage in terms of N as shown in Figure 20.

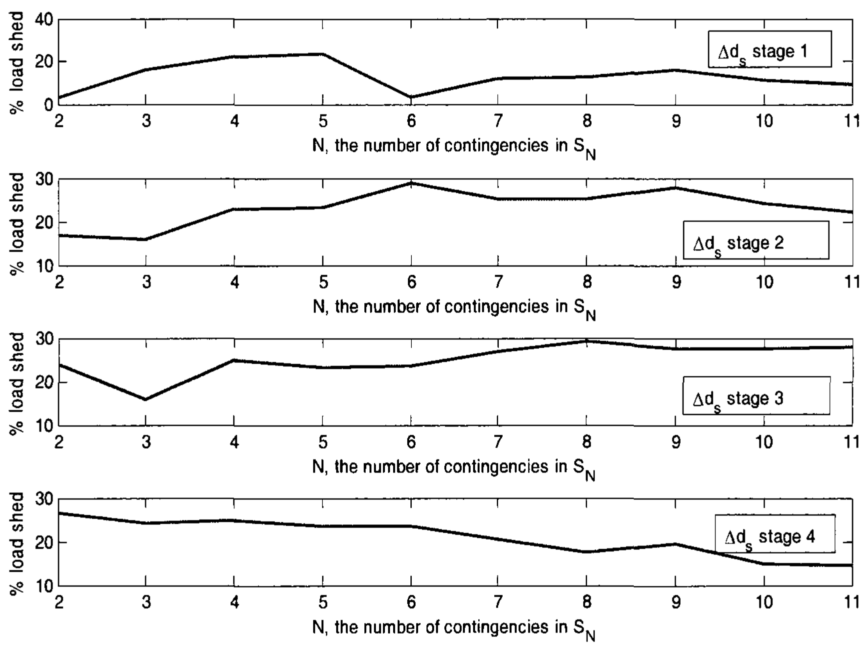

Figure 20: Comparison of % load shed for each stage over N

From Figure 20, while it is difficult to explain the variation of load-shed at every stage over N, we observe the following: (i) a small amount of load is shed at the first stage (which corresponds to the highest frequency threshold), with an average of 12.1% over the eleven contingencies; (ii) an average of about 25% load is shed during each of the second and third stages; and (iii) the last stage sheds a smaller amount of load than either the second or third stages.

Another interesting observation is that the total load shed over all four stages for each N usually adds up to the amount of load-shedding required to correct the worst contingency.

We conclude that there is no apparent benefit to increasing time delays beyond the minimum value of 0.2 seconds. At least in this case study, it appears that there are enough degrees of freedom in the relay definition to set the frequency setpoints to specified values (e.g the generator underfrequency thresholds) and allow the amounts of load shed to vary, or, alternatively, set the amounts of load shed and allow the frequency setpoints to vary.

CONCLUSIONS

In this thesis, we have designed and tested a systematic method for determining the settings of underfrequency relays in power systems. This relay setting problem has been formulated as a mixed integer linear program (MILP), which computes the system frequency response at each time step following a contingency using a discrete-time approximation. The use of binary variables facilitates the formulation of relay operational logic in an explicit form suitable for solution using standard mixed-integer optimization software. The proposed approach involves a systematic iterative process to identify the smallest subset of all possible generation loss contingencies that yield the "best" set of relay settings. In this thesis, the evaluation of the performance of a set of relay settings is based on whether or not the settings can be used to successfully protect the system against *all* possible generation loss contingencies while shedding the least amount of load.

A discussion on conventional methods of setting relays is also provided in this thesis. The conventional approach is to set the relays to protect the system against a single contingency, usually the most pessimistic generation loss event. This approach is usually too conservative, and results in shedding too much load for milder generation losses. We believe that the innovative MILP approach presented here is a significant improvement on most conventional methods of setting underfrequency relays, as the relay settings are computed with respect to a small subset of all possible generation loss contingencies, where the subset can include both mild and severe generation losses. Using a simple case study, we show that relays can be set to ensure that the generator underfrequency/time limitations can be respected for all plausible generation loss events while shedding the least amount of load.

One of the assumptions made in this thesis is that generation loss contingencies in any given network are equiprobable, and therefore the number of plausible generation loss contingencies is an astronomically large number for networks with a large number of generators. Vulnerability analysis, such as the work conducted in [28-30] could be carried out to determine the set of credible contingencies that can occur with high probability, and then compute underfrequency relay settings with respect to only these contingencies.

Another assumption made in this thesis is that the frequency response of each generator is identical. However, in a more general formulation, the frequency response of each generator could be represented using its own discretized swing equation. Moreover, the operation of the network could also be modeled using a DC load flow.

In this MILP formulation of the relay setting problem, no caps were placed on the generation capacity and ramp limits of each unit. Such limits can be introduced using mixed integer programming techniques such as the work done in [15]. It is expected that introducing these limits will result in less primary frequency reserve available following each generation loss contingency, and therefore more load will have to be shed for each contingency. Modeling these limits will also permit the computation of relay settings for different levels of demand in the system.

The proposed MILP formulation involves three decision variables, namely frequency setpoints, time delays, and amount of load to shed at each stage. One additional decision variable that could potentially add significant degrees of freedom to the relay setting problem is the rate of change of frequency, a parameter that has recently received a lot of interest in underfrequency relay literature. In this thesis, we propose multiple setpoint relays that in essence models the rate of change of frequency (Appendix A). However, the MILP formulation is flexible enough to permit the addition of rate of change of frequency as the fourth decision variable, and future work could look into this possibility.

APPENDIX A

A.1 Re-formulation of the MILP for Multiple-Setpoint Relays

In Section 1.3, the concept of multiple-setpoint relays was discussed. The motivation behind multiple setpoints is similar to the arguments put forward by advocates of adaptive underfrequency relays [8, 19, 21]: by sensing the frequency at various times during a frequency trajectory before making the decision to shed a load, the severity of a contingency can be estimated. We argue that if there were an infinite number of setpoints for each relay, then the relay operation logic would be based on the rate of change of frequency and not just the frequency.

Multiple frequency setpoints will add a new type of degree of freedom to the three already present in the MILP formulation discussed in Chapter 4, which are the frequency setpoints, time delays, and amount of load to be shed for each relay.

To describe the logic for multiple setpoints relays, the definition of timers measuring the time spent below a frequency setpoint (Equation 4.15) must be modified to Equation (A.1) as follows,

$$v_{sn}^{jp} = \begin{cases} 1 & \text{if } f_0 + \Delta f_n^j \le f_s^p; \\ 0 & \text{if } f_0 + \Delta f_n^j > f_s^p \end{cases} \tag{A.1}$$

which says that if the frequency trajectory j reaches or falls below the frequency setpoint f_s^p at time step n for $p = 1,\ldots,np$, then the binary variable v_{sn}^{jp} is equal to 1 and 0 otherwise.

Equation (A.1) can be expressed in an equivalent *explicit* linear form through:

$$\frac{f_s^p - \left(f_0 + \Delta f_n^j\right)}{L} \le v_{sn}^{jp} \le 1 + \frac{f_s^p - \left(f_0 + \Delta f_n^j\right)}{L} \tag{A.2}$$

where L is a sufficiently large positive number (e.g. 60 Hz). The equivalence between the implicit relation (A.1) and its explicit form (A.2) has been discussed in Section 4.6.

Thus, the timer corresponding to setpoint p of relay s starts counting when the binary variable v_{sn}^{jp} is equal to 1. Therefore, the total time in seconds spent by trajectory j below the frequency setpoint f_s^p at time step n is given by:

$$\Delta t_{sn}^{jp} = \sum_{m=0}^{n} v_{sm}^{jp} \Delta t \qquad (A.3)$$

The binary variables v_{sn}^{jp} must also satisfy the condition $v_{s0}^{jp} = 0; \forall j, s, p$ in order to represent explicitly the fact that all frequency trajectories begin at the nominal frequency where no relay frequency setpoints have yet been crossed.

In addition, for each relay s, the frequency setpoints f_s^p must be greater than or equal to the lowest permissible generator frequency threshold, f^{nl},

$$f_s^p \geq f^{nl} \qquad (A.4)$$

The logic that determines the operation of each relay s says that Δd_s is shed when, for any $p = 1,...,np$, the amount of time spent by trajectory j below the setpoint f_s^p at time step n, Δt_{sn}^{jp}, reaches a value Δt_s^p.

The load shedding binary variable u_{sn}^j defined in Section 4.5 can now be expressed through the relay logic,

$$u_{sn}^j = \begin{cases} 0 & \text{if } \Delta t_{sn}^{jp} < \Delta t_s^p; \\ 1 & \text{if } \Delta t_{sn}^{jp} \geq \Delta t_s^p \end{cases} \qquad (A.5)$$

The other relations describing the relay setting problem, which were summarized in Section 4.11, are the same for the multiple-setpoint formulation.

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