A NEW BRIDGE FOR COMPARING CAPACITANCE, MUTUAL INDUCTANCE AND SELF INDUCTANCE

Thesis

by

ARTHUR MUNRO THURSTON, B.Eng.

Submitted in part fulfilment of the requirements
for the degree of Master of Science in the
Faculty of Graduate Studies and Research
at McGill University

The Macdonald Physics Laboratory
McGill University August, 1948

INDEX

A NEW BRIDGE FOR COMPARING CAPACITANCE, MUTUAL INDUCTANCE AND SELF INDUCTANCE

INTRODUCTION.

It was felt that a bridge was missing from the family of bridges in the literature dealing with the comparison of capacitance, mutual inductance and/or self inductance. This new bridge would bear a similar relationship to Maxwell's bridge that the Heydweiller did to the Owen. Reference to Appendix I will show these relationships.

To keep the study to reasonably comprehensive limits particular attention is to be paid to the comparison of the new bridge with Maxwell's bridge.

Frequently the effects of quadrature components of bridge units on the final balance conditions are glossed over in the study of simple bridges in elementary work. These components do not always affect, appreciably, the results for moderately accurate work.

However, in the case of this new bridge, it is realized that these components can be an important factor and special attention will be given to them.

As is usual in these bridges there are two conditions of balance due to the real and quadrature terms and the proper choice of balancing operations is important for simplest manipulation, best interpretation and optimum use of the apparatus at hand.

In the case of the new bridge the intention is to study the overall effects of:-

(a) variation of the resistance values of the resistors with frequency,

(b) variation of reactance values of the resistors with resistor dial settings,

(c) variation of the resistance of the variable self inductance with dial setting and frequency,

(d) variation of mutual inductance and resistance of the Standard Mutual Inductometer (Campbell Patent) with frequency,

(e) bridge connections and leads on the correction term,

(f) different power factors of various condensers used and the change of condenser conductance with frequency,

(g) the Q values of the inductors under test,

(h) the alteration of components of the correction term to give desired results, that is, r_2 and g_3.

In making comparison it will be necessary to study the Maxwell bridge from the point of view of:-

(a) the proper choice of individual resistance values for the product term R_2R_3 to give the desired results similar to the new bridge product term $\frac{M}{C_3}$.

(b) effect of inverted Q terms for similar type balance,

(c) comparative magnitude of correction term components,

(d) effect of frequency change on correction term components.

SIMPLE THEORETICAL CONSIDERATIONS.

The new bridge is set up as represented schematically in figure 1 with the Wagner earthing device shown connected for one balance condition.

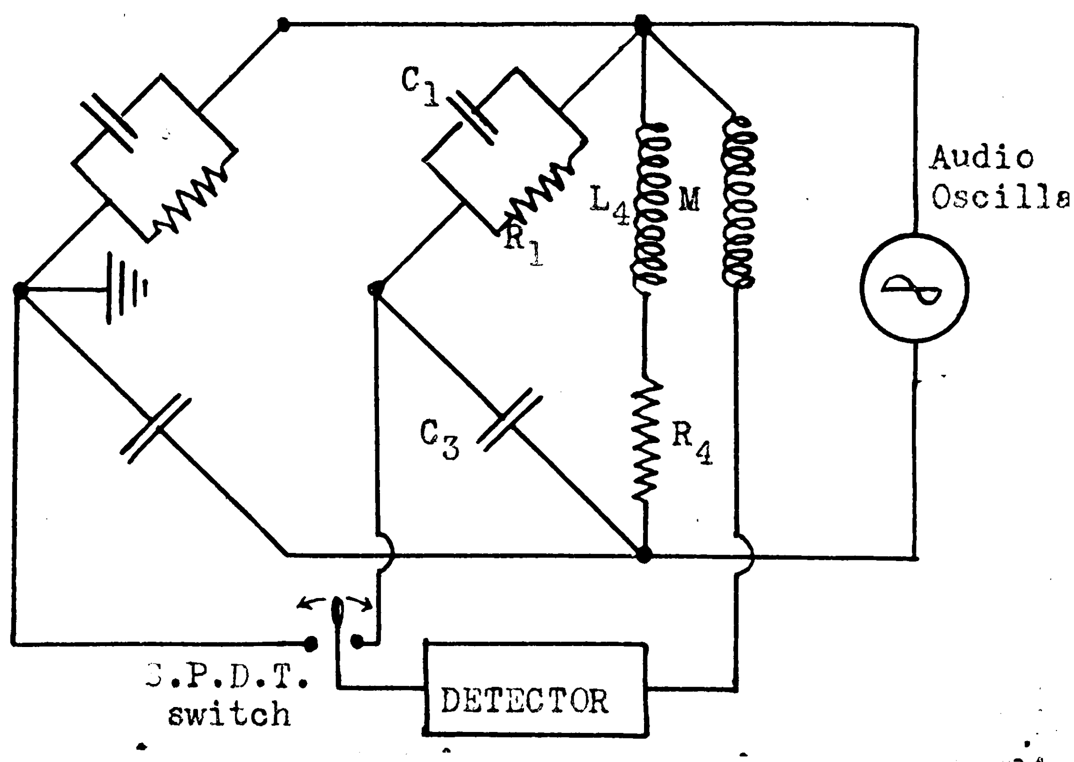
Employing the well known star-mesh transformation principle (1)[2], the bridge, when properly connected, is represented by four impedance arms which on balance give:-

$$\frac{Z_2}{Z_1} = \frac{Z_4}{Z_3}$$

where the Z's represent the total arm impedance and the subscripts indicate the arm referred to (see figure 2).

Figure 1

Showing schematic layout of the new bridge including the Wagner earth device connected for one of the balance conditions. In fact connections were made to a skeleton bridge and use was made of shielding transformers at the input and at the detector.



More conveniently, $\frac{Z_2}{Y_3} = \frac{Z_4}{Y_1}$ ....(1)

where the Y's refer to the arm admittances. So this gives

$$\frac{jwM + r_2}{jwC_3 + g_3} = \frac{jw(L-M) + R_4}{jwC_1 + G_1} \quad \text{...........(2)}$$

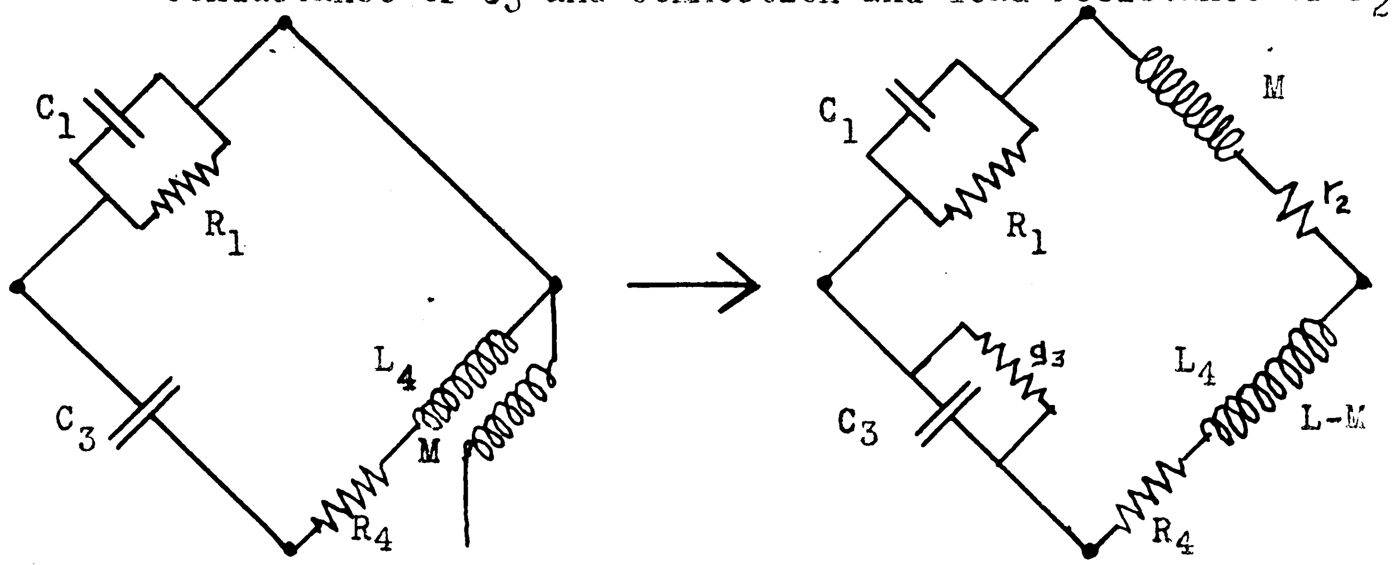
where $w = 2\pi f$ (f is frequency in cycles per second) and j is

---

(1)[2] Terms ( )[2] refer to bibliography the quadrature operator $\sqrt{-1}$, the other terms are as shown in figure 2.

Figure 2

Showing: (a) the new bridge circuit as connected
(b) the equivalent circuit which also includes conductance of C_3 and connection and lead resistance of r_2



(a) (b)

Equating the real terms and quadrature terms separately gives the following relationships:-

$$-w^2MC_1 + r_2G_1 = -w^2C_3(L-M) + R_4g_3 \qquad \dots\dots\dots\dots(3)$$

$$MG_1 + C_1r_2 = (L-M)g_3 + C_3R_4 \qquad \dots\dots\dots\dots(4)$$

Equation (3) can be rewritten as

$$MC_1 - (L-M)C_3 = \frac{1}{w^2}\left[r_2G_1 - R_4g_3\right] = \frac{1}{w^2}\left[\frac{r_2}{M}MG_1 - \frac{g_3}{C_3}C_3R_4\right] \qquad \dots\dots(5)$$

and equation (4) can be rewritten as

$$-MG_1 + C_3R_4 = \left[C_1r_2 - (L-M)g_3\right] = \left[\frac{r_2}{M}MC_1 - \frac{g_3}{C_3}C_3(L-M)\right] \qquad \dots\dots(6)$$

At this point certain logical approximations can be made. Ideally, the right hand members of equations (5) and (6) would be zero if r_2 and g_3 were zero, that is if there were no connection or lead resistances and the condenser C_3 had zero power factor. Then from equations (5) and (6) respectively there evolves:-

$$MC_1 \doteq (L-M)C_3 \qquad \qquad \ldots\ldots\ldots\ldots(7)$$

and

$$MG_1 \doteq C_3R_4 \qquad \qquad \ldots\ldots\ldots\ldots(8)$$

Introduce (7) and (8) as substitutions in (5) and (6) and get

$$MC_1 - (L-M)C_3 \doteq \frac{MG_1}{w^2}\left[\frac{r_2}{M} - \frac{g_3}{C_3}\right] \doteq \frac{C_3R_4}{w^2}\left[\frac{r_2}{M} - \frac{g_3}{C_3}\right] \qquad \ldots\ldots\ldots\ldots(9)$$

$$-MG_1 + C_3R_4 \doteq MC_1\left[\frac{r_2}{M} - \frac{g_3}{C_3}\right] \doteq C_3(L-M)\left[\frac{r_2}{M} - \frac{g_3}{C_3}\right] \qquad \ldots\ldots\ldots(10)$$

These equations may now be written in another form which will be used in this study. It is in this form that the equations for the various bridges are listed in Appendix I.

$$\frac{MC_1}{(L-M)C_3} - 1 \doteq \frac{R_4}{w^2(L-M)}\left[\frac{r_2}{M} - \frac{g_3}{C_3}\right] \qquad \ldots\ldots\ldots(11)$$

$$\frac{-MG_1}{C_3R_4} + 1 \doteq \frac{(L-M)}{R_4}\left[\frac{r_2}{M} - \frac{g_3}{C_3}\right] \qquad \ldots\ldots\ldots(12)$$

For bridge comparison purposes there is listed in Appendix I similar final balance conditions of the "new", Maxwell, Heydweiller and Owen bridges. These results were determined in a like manner and special reference will be made to those for the Maxwell bridge.

In the case of the new bridge a study of the bracketed term gives the following results:-

(a) r_2. The r_2 term is dependent on lead and connection resistances plus any resistance inserted intentionally to change the value of the bracketed term as a whole. With ordinary wiring practice this value of r_2 may be in the order of 10^{-2} ohms.

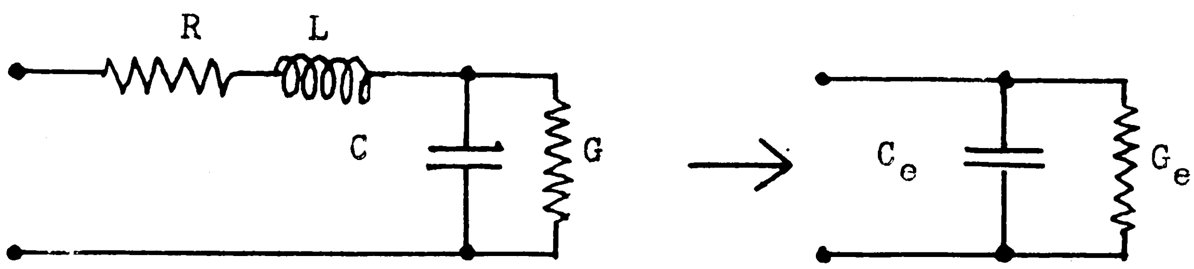
(b) M. The value of M may be chosen at will within the range of the instrument used. Depending on the condition of balance it can be adjusted to give either M or (L-M) a convenient round number. Let the value of M be considered at the present in the order of 10^{-2} henrys.

(c) $\frac{r_2}{M}$. The combined value of $\frac{r_2}{M}$ is thereforein the order of $\frac{10^{-2}}{10^{-2}}$ = 1 ohm/henry.

(d) g_3 The term g_3 depends partially on the quality of the condenser. A study of manufacturer's data (2)[2] and (3)[2] gives power factors in the order of 0.0001. g_3 can be altered readily to a larger value by the inclusion of a resistor in parallel with the condenser C_3. Equivalent circuits of a condenser are given in figure 3, (3)[2].

Figure 3

Two equivalent circuits of a condenser
(parallel resistance type)



(a) (b)

R = metallic resistance
L = lead inductance
G = dielectric conductance
C = static capacitance
G_e = circuit equivalent conductance
C_e = circuit equivalent capacitance
S_e = circuit equivalent elastance

$$G_e = \frac{w^2C^2R + G}{[1 - w^2LC]^2}$$

$$C_e = \frac{C}{(1 - w^2LC)} = \frac{1}{S_e}$$

The power factor of a condenser remains nearly constant over a wide range of voltages and frequencies (5)[2]. The power consumed in a condenser may be shown by the approximation

$$P \doteq KfE^2 \doteq E^2G_e \qquad \underline{\text{or}} \qquad G_e \doteq Kf \qquad \ldots\ldots\ldots(13)$$

so that g_3 is seen to be proportional to frequency where P is the power consumed in watts, K is a proportionality constant, and E is the voltage applied to the condenser terminals in volts. In (13), capacitor power factor becomes $K/2\pi C$ - a constant.

(e) $\left[\frac{r_2}{M} - \frac{g_3}{C_3}\right]$. From the foregoing it can be seen, readily, that the value of the bracketed term may be greater or less than unity and the algebraic sign may be positive or negative.

To make the right hand sides of equations (11) and (12) practically zero, the product of the bracketed and unbracketed terms must be nearly zero. First thought may suggest that $\frac{R_4}{w^2(L-M)}$ and $\frac{(L-M)}{R_4}$ both be made very much less than unity. As far as R_4 and $(L-M)$ alone are concerned these requirements are at variance because the two conditions to be satisfied at balance have these two terms in reciprocal positions. At a high frequency, it is conceivable that the requirements can tend to be satisfied. Another thought is to reduce the bracketed term to zero (or as near zero as accuracy requires).

Theoretically this may be accomplished readily by the inclusion of a resistance in arm #2 (see figure 2b) or by the addition of conductance to the g_3 term by insertion of a resistor in parallel with C_3, which can produce an increasingly positive trend or an increasingly negative trend, respectively, to the bracketed term.

For convenience in visualizing the results of these actions it is advisable at this point to consider equations (11) and (12) in the familiar straight line form, y = mx + c.

$$C_1 \doteq \frac{C_3}{w^2 M}\left[\frac{r_2}{M} - \frac{g_3}{C_3}\right] R_4 + \frac{(L-M)C_3}{M} \qquad \text{..........(14)}$$

where:- C_1 is plotted as the ordinate

R_4 is plotted as the abscissa

$\frac{(L-M)C_3}{M}$ is the resultant intercept on the y axis and $\frac{C_3}{w^2 M}\left[\frac{r_2}{M} - \frac{g_3}{C_3}\right]$ is the resultant slope.

Providing the value of C_3 is so great compared to the extra capacitance added to it by any resistor put in parallel with it, the factor $\frac{C_3}{w^2 M}$ may be considered constant when one or both of r_2 or g_3 are varied. So by these variations the slope may be made theoretically positive, negative or zero. When the slope is zero equation (14) reduces to

$$C_1 = \frac{(L-M)C_3}{M} \qquad \ldots\ldots\ldots\ldots(15)$$

which is just equation (7) in another form.

A similar treatment of equation (12) gives

$$G_1 \doteq -\frac{C_3}{M}\left[\frac{r_2}{M} - \frac{g_3}{C_3}\right](L-M) + \frac{C_3 R_4}{M} \qquad \ldots\ldots\ldots\ldots(16)$$

where:- G_1 is plotted as the ordinate

$(L-M)$ is plotted as the abscissa

$-\frac{C_3}{M}\left[\frac{r_2}{M} - \frac{g_3}{C_3}\right]$ is the resultant slope

and $\frac{C_3 R_4}{M}$ is the resultant intercept on the y axis.

When the slope is zero equation (16) becomes

$$G_1 = \frac{C_3 R_4}{M} \qquad \ldots\ldots\ldots\ldots(17)$$

which is just equation (8) in another form.

The choice of bridge operation to satisfy equation (12) introduces an unnecessary complication which can be overcome, though, by further corrective measures, as will be shown later. In this case the variation of the (L-M) term to be plotted as the abscissa may also introduce a change in R_4 and the two results will be superposed when plotting against G_1. Were a variable inductance available which did not change its effective resistance with dial setting and frequency this difficulty would not exist

The value of (L-M) itself, or actually ($L-M + \ell - cR^2$) as will be shown later, does not remain constant as R_4 is varied when using the operation to satisfy equation (11).

Next, the bracketed term of the Maxwell bridge is considered.

(a) ℓ_2. The value of the inductance ℓ of the so called "non-reactive" resistances may be in the order of 10^{-7} henrys (2)[2].

(b) R_2. In order to satisfy the requirements of the bridge and have an inductively reactive term in arm #2, R_2 must be small, as will be shown when discussing experimental apparatus, in the order of 10^2 ohms, otherwise the quadrature term shows up as capacitively reactive.

(c) $\frac{\ell_2}{R_2}$. So the combined expression $\frac{\ell_2}{R_2}$ approximates the value $\frac{10^{-7}}{10^2} \doteq 10^{-9}$ henrys/ohm.

(d) R_3. For the same reason as stated in (b) above, R_3 is chosen large, 10^3 ohms or greater, to have a capacitively reactive term.

(e) c_3. This c_3 is a very small quantity and in the order of 10^{-11} farad (2)[2] since a time constant approaching 10^{-9} is given for the hundreds dial.

(f) $\frac{c_3}{G_3}$. At low frequencies $\frac{c_3}{G_3} = c_3R_3$ and this approximates $10^{-11} \times 10^3 = 10^{-8}$ farads/mho so that the bracketed term is extremely small.

(g) $\left[\frac{\ell_2}{R_2} - \frac{c_3}{G_3}\right]$. It is noted here that the components of this bracketed term are neither readily nor conveniently adjustable for slope changing purposes, though some alteration can be made to the slope by varying R_2 and R_3 separately, keeping the product R_2R_3 unchanged but thereby changing the values of ℓ_2 and c_3. (As will be shown later c_3 can be altered leaving R_3 fixed).

Recapitulating, it is seen that the value of $\left[\frac{r_2}{M} - \frac{g_3}{C_3}\right]$ is in the order of 10^0, and $\left[\frac{\ell_2}{R_2} - \frac{c_3}{G_3}\right]$ is in the order of 10^{-8}, and negative.

However, the values of the multiplying factors must be considered too. Referring to Appendix I , equations $(N_1)'$, $(N_2)'$, $(M_1)'$, and $(M_2)'$, the following variables may be considered roughly to have the values:-

R_4	$\doteqdot 10^2$	ohms
$(L-M)$	$\doteqdot 10^{-2}$	henrys
w^2	$\doteqdot 4 \times 10^7$	radians/second
L	$\doteqdot 10^{-2}$	henrys

so that the right hand sides of these equations approximate the values

$(N_1)'$	2.5×10^{-4}
$(N_2)'$	10^{-4}
$(M_1)'$	4×10^{-5}
$(M_2)'$	10^{-4}

and are the error terms; the deviation from unity.

If it were not for the fact that the leads to the mutual inductometer are a fair length, in normal assemblage, to keep the field effect down, then r_2 could be made smaller and thus the two bridges could have nearly the same inherent accuracy. But as it is, a factor of nearly 10 shows up above for $(N_1)'$ and $(M_1)'$. At a definite frequency, by adjustment, $(N_1)'$ can be made to agree with $(M_1)'$ in a manner already explained, but $(M_1)'$ has the advantage of not varying to the same extent with frequency as $(N_1)'$ does (see equation 13), since this bracketed term does not effectively vary with frequency.

Further reference to the same equations will show that

$(N_1)'$. Terms ( )' refer to Appendix I.

the product $\frac{M}{C_3}$ in $(N_1)'$ and $(N_2)'$ plays the part of the product R_2R_3 in the equations $(M_1)'$ and $(M_2)'$. It can also be seen that w^2 plays an inverse role in the two bridges.

It is expected in the case of $(N_1)'$ that as the w value is made greater the error term will be less in spite of the fact that g_3 increases, for whether its increase either aids or opposes the tendency of the bracketed term to become zero its action varies as the frequency whereas the w^2 term varies as the square of the frequency. Unfortunately, the standard mutual inductometer (1)[3] available is not useable over a wide range of frequencies since it is not dependable above 2000 cycles/second; but a practical difference due to w variation may be shown.

The values used for R_4, L_4, (L-M), and f give a Q value of about 1. Considering the bracketed term for the new bridge to be adjusted to give an overall error term similar to that of the Maxwell bridge, and using equations $(N_1)'$ and $(M_2)'$ since L variable is to be avoided as will be seen later, the following statements can be made in tabular form from the simple theoretical considerations so far:-

Variable Considered	Value of Variable	Best Bridge to Use
Q	high	Maxwell
Q	low	new
f	high	new
f	low	Maxwell

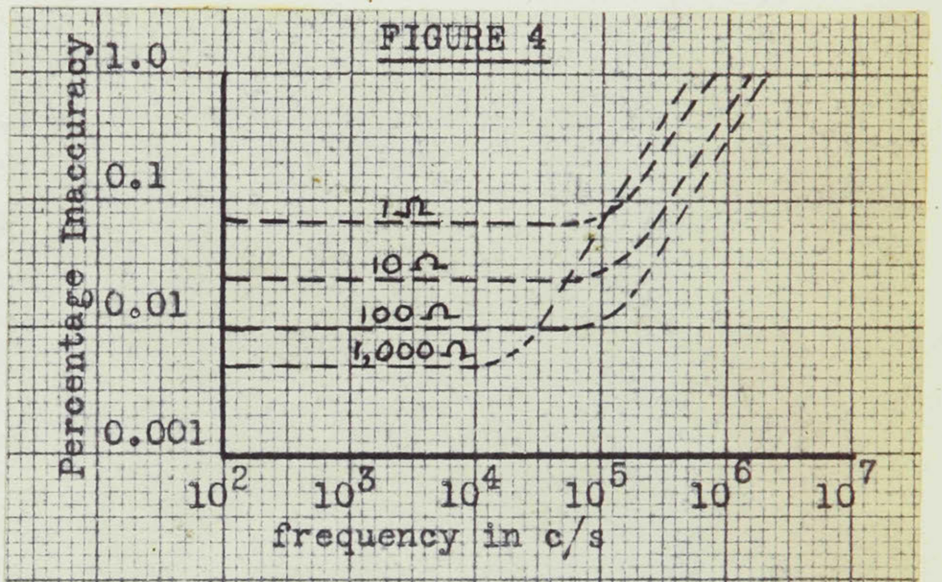
It should be born in mind, however, that these comparisons are not conclusive because of the bracketed term assumption and because accuracy requirements have not been shown yet.

EXPERIMENTAL APPARATUS

Trouble due to frequency variation should not be expected with the resistance boxes used since the first derivative of percentage inaccuracy on a frequency base for finest decade resistance standards is zero up to 10^4 c/s for 1,000 ohm decades and nearly up to 10^5 c/s for 100, 10, and 1 ohm decades (2)[2]. See figure (4). While the resistances used were A.C. standards, second grade, no great deviation is expected from the curves shown.

Further, the percentage inaccuracy of adjustment to nominal value on an ohmic base gives a lessening inaccuracy (2)[2] of 0.7% to 0.07% from 10^{-1} to 10^2 ohms and a fairly constant value in the order of 0.07% between 10^2 and 10^4 ohms. Refer to figure (5).

FIGURE 4



The effect of frequency upon the direct reading accuracy of the finest decade A.C. type resistance standards.

(Reproduced from (2)[2]

The "phase quality" of a resistance standard is conveniently expressed as a residual self inductance whether the residual reactance is positive or negative. The "box" values of a decade standard are those of the electrical values at the terminals of the standard when all decades are at zero.

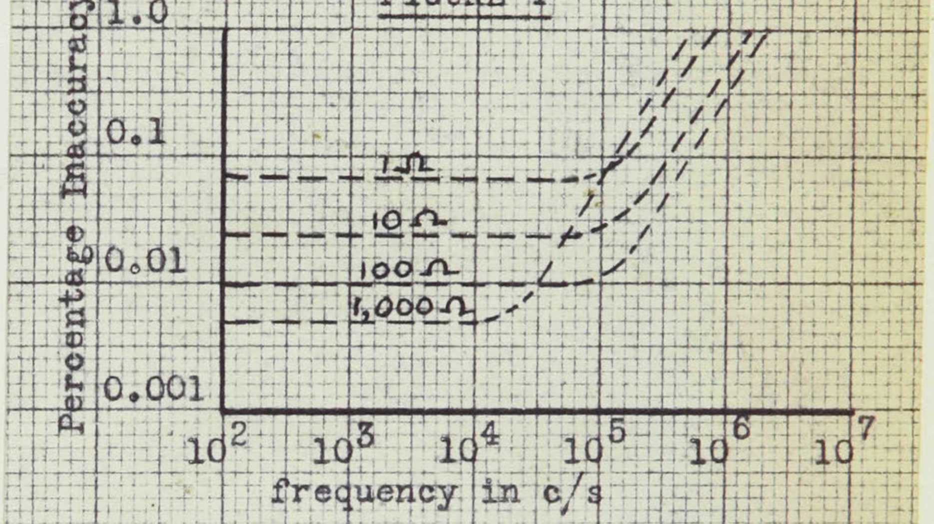
The equivalent circuit of a resistor (4)[2] is shown in figure (6). L is the inductance between the terminals and is effectively in series with the resistance, while the capacitance C appears across the terminals of the resistor. The expressions

EXPERIMENTAL APPARATUS

Trouble due to frequency variation should not be expected with the resistance boxes used since the first derivative of percentage inaccuracy on a frequency base for finest decade resistance standards is zero up to 10^4 c/s for 1,000 ohm decades and nearly up to 10^5 c/s for 100, 10, and 1 ohm decades (2)[2]. See figure (4). While the resistances used were A.C. standards, second grade, no great deviation is expected from the curves shown.

Further, the percentage inaccuracy of adjustment to nominal value on an ohmic base gives a lessening inaccuracy (2)[2] of 0.7% to 0.07% from 10^{-1} to 10^2 ohms and a fairly constant value in the order of 0.07% between 10^2 and 10^4 ohms. Refer to figure (5).

FIGURE 4



The effect of frequency upon the direct reading accuracy of the finest decade A.C. type resistance standards.

(Reproduced from (2)[2]

The "phase quality" of a resistance standard is conveniently expressed as a residual self inductance whether the residual reactance is positive or negative. The "box" values of a decade standard are those of the electrical values at the terminals of the standard when all decades are at zero.

The equivalent circuit of a resistor (4)[2] is shown in figure (6). L is the inductance between the terminals and is effectively in series with the resistance, while the capacitance C appears across the terminals of the resistor. The expressions for the effective terminal resistance R_e and effective reactance X_e of this circuit are (4)[2]:

$$R_e = \frac{R}{(1 - w^2LC)^2 + (wCR)^2} \quad \text{..........(18)}$$

$$X_e = \frac{w[L(1 - w^2LC) - CR^2]}{(1 - w^2LC)^2 + (wCR)^2} \quad \text{..........(19)}$$

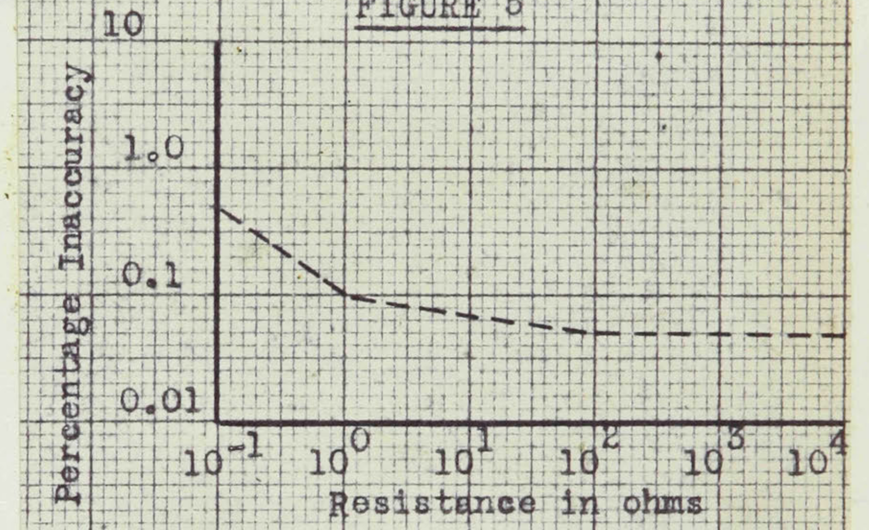
At the lower frequencies where the terms in w^2 are negligible compared to unity the following approximations hold:-

$$R_e \doteqdot R[1 + w^2C(2L - CR^2)] \quad \text{..........(20)}$$

$$L_e \doteqdot (L - CR^2)[1 + w^2C(2L-CR^2)] \doteqdot (L - CR^2) \quad \text{........(21)}$$

So it is seen that the effect of a rise in frequency is to increase both R_e and L_e by a factor of w^2LC. However, it has been shown that these changes are negligible in the frequency range to be used.

FIGURE 5



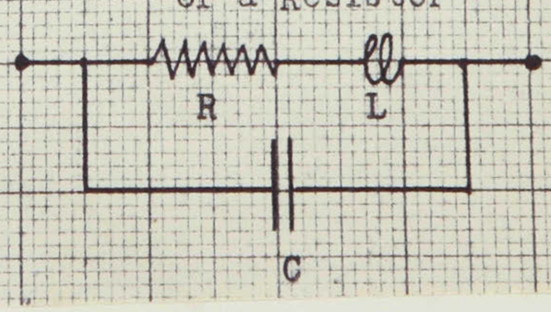
The inaccuracy of direct reading of decade resistance standards.

Reproduced from (2)[2]

The impedance errors of the resistor boxes used are most noticeable with resistance units of high and low values; intermediate values of, say, 10 to 300 ohms are invariably the best for both errors of effective resistance and reactance (2)[2]. Residual inductance (incremental) values may vary from +1 μH to more than -300 μH (2)[2].

FIGURE 6

Equivalent Circuit of a Resistor

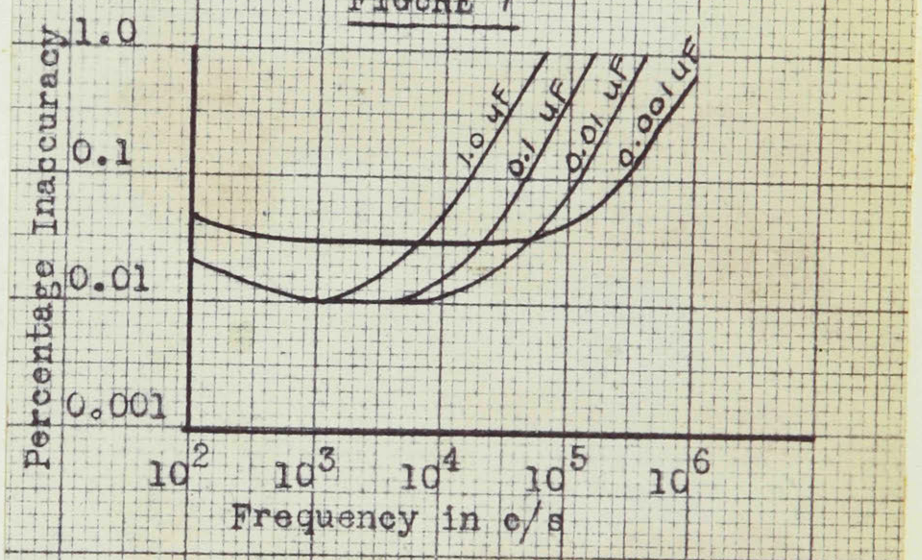


The temperature coefficient of all dials is given as of order 0.0025% per degree Centigrade.

Noting equation (21) and writing it as $L_e \doteq (L - R^2C)$ at low frequency (6)[2] it is readily understood that the effective inductance can be positive when R is small, negative when R is large, or zero when $\frac{L}{R} = CR$, that is, when the time constants are equal. Thus the manufacturer has a measure of control over the frequency design of the resistances.

The standard mica condensers used, see Appendix II (5)[3] (6)[3] (7)[3] , have a power factor of 0.0001 as stated by the manufacturer (2)[2] and a temperature coefficient of 1 part in 10^6 at all frequencies. The inaccuracy of effective capacitance of these fixed mica condensers (standards) is less affected by frequency than those shown in figure (7), (2)[2].

FIGURE 7



The inaccuracy of effective capacitance of the finest mica condenser standards of the multi-decade type.

(Reproduced from (2)[2]

The General Radio Type 505 capacitors used in preliminary experiments (not listed separately in Appendix II) have a value of less than 0.0005 for the dissipation factor (3)[2].

The true values which were used for each piece of apparatus were the ones given in the guarantee certificates on file in the Macdonald Physics Laboratory.

---

(5)[3] Terms ( )[3] refer to Appendix II.

EXPERIMENTAL SET-UP

The apparatus was set up as shown in figure 1 using a skeleton bridge to facilitate the interchange from the new to the Maxwell bridge. Shielded wires were used throughout and the inductors were kept well separated and away from metal objects. The Wagner earth was set up as shown and of course components were changed to suit each bridge.

EXPERIMENTAL PRELIMINARY RUNS

A preliminary test was made using formula (14) giving the results shown in table #1 and graph #1.

TABLE #1

C_1 µµF	±	R_1 ohms	R_4 ohms	Remarks
128421	2	6972.0	11	M = 7.77 mH (set)
128755	3	774.81	100	L-M = 10.00 mH (1)[3]
128915	3	387.47	200	f = 800 c/s
129081	9	258.43	300	C_3 = 0.1 µF (8)[3]
129264	9	193.83	400	
129423	6	155.12	500	
129592	12	129.29	600	

General Remarks -- Abbreviations

±	The ± columns give the uncertainty of balance and the figures refer to the last significant figures given in the pertinent column.
H	Henry
mH	millihenry
µH	microhenry
F	Farad
µF	microfarad
µµF	micromicrofarad
f	frequency
c/s	cycles per second

Temperature. During all of the experiments the temperature range was 22.2 °C to 23.6 °C. Most of the work was done between 23.0 °C and 23.2 °C. During any one experiment the temperature remained constant. D.C. resistance values were corrected from the given and measured values using a temperature coefficient of resistivity of +0.00382 ohms/ohm/ ^{o}C.

From graph #1 the slope is found to be 1.68×10^{-12} F/ohm. The term $\left[\frac{r_2}{M} - \frac{g_3}{C_3}\right]$ is therefore $1.68 \div \frac{C_3}{Mw^2} \times 10^{-12}$ which gives 3.34

To reduce the slope to nearly zero add extra conductance g_e to C_3 such that $\frac{g_e}{C_3} \doteq 3.34$

$$g_e \doteq 3.34 \times 10^{-7} \text{ mhos}$$

$$\text{so } r_e = \frac{1}{g_e} \doteq 3 \times 10^{6} \quad (3 \text{ megohms})$$

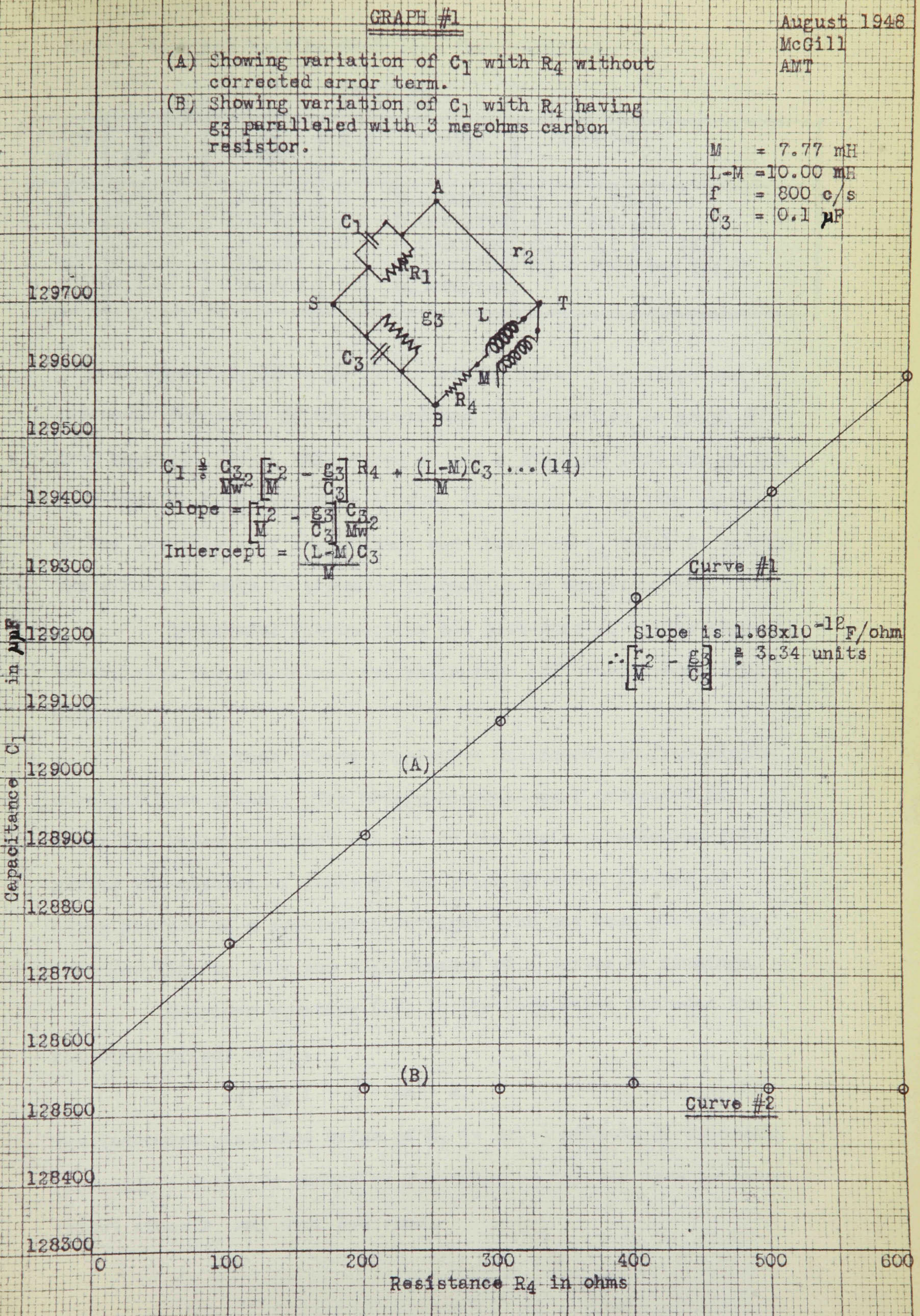
A carbon resistor of this value was added in parallel with C_3 and the results of table #1a were found and plotted on graph #1. This carbon resistor does not add appeciable capacitance to the arm #3 as was pointed out as a requirement in the theory. It is seen that the results are indicative of expectations. *

Next a test was made using equation (14) again but this time treating ($L + \ell - M$) as the independent variable. The ℓ used was an astatically connected variable self inductance (3)[a]: M was maintained constant. The results are given in table #2 and graph #2.

* TABLE #1a

C_1 $\mu\mu$F	±	R_1 ohms	R_4 ohms	Remarks
128545	2	774.07	100	Settings similar to table #1.
128539	3	387.49	200	C_3 paralleled by a
128534	4	258.43	300	3 megohm carbon
128541	6	193.85	400	resistor
128535	19	155.14	500	
128535	19	129.31	600	

The slope of curve #1, graph #2, from formula (14) is $\frac{C_3}{M}$ and the intercept is $\frac{C_3}{Mw^2}\left[\frac{r_2}{M} - \frac{g_3}{C_3}\right]R_4$. It is readily realized that in this form the results are of no help since C_3 and M (giving the slope) are known to much greater than plotting accuracy and the intercept


GRAPH #1
(A) Showing variation of C1 with R4 without corrected error term.
(B) Showing variation of C1 with R4 having g3 paralleled with 3 megohms carbon resistor.
August 1948
McGill
AMT
M = 7.77 mH
L-M = 10.00 mH
f = 800 c/s
C3 = 0.1 µF
A
C1
R1
r2
S
T
g3
L
C3
M
R4
B
C1 ≑ C3/Mw² [r2/M - g3/C3] R4 + (L-M)C3/M ...(14)
Slope = [r2/M - g3/C3] C3/Mw²
Intercept = (L-M)C3/M
Curve #1
Slope is 1.68x10^-12 F/ohm
∴ [r2/M - g3/C3] ≑ 3.34 units
(A)
(B)
Curve #2
Capacitance C1 in µµF
129700
129600
129500
129400
129300
129200
129100
129000
128900
128800
128700
128600
128500
128400
128300
0
100
200
300
400
500
600
Resistance R4 in ohms

reading accuracy will be much less than required.

TABLE #2

$L+\ell-M$ mH	C_1 μF	R_1 ohms	±	G_1 mhos	Remarks
20	0.2570	110.761	0	0.00902845	$M = 7.77 mH$
30	0.3860	110.777	0	0.00902714	$f = 800$ c/s
40	0.5156	110.798	0	0.00902543	$C_3 = 0.1\ \mu F$ (8)[s]
50	0.6454	110.822	0	0.00902348	$R_4 = 700$ ohms (14)[s]
60	0.7737	110.835	0	0.00902242	

As expected, curve #1 is a straight line. The slope and intercept for curve #2 are determined for insertion on graph #2 from equation (16). This curve is not a straight line due to the fact that as ℓ is varied in $(L+\ell-M)$ the value of R_ℓ (the resistance of the variable self inductance) and therefore R_4 total is varied: the effect of this is shown on inspection of equation (16).

By careful reconnection and placement of bridge elements, sensitivity was maintained and extraneous influences reduced to a minimum.

Tests made showed repetitive discrepancies from simple straight line graphs. The bridge elements were suspected of being the cause so a test was run on the basis of equation (14) giving the results of table #3 and graph #3 which instead of being plotted as a continuous straight line has been plotted as a broken line to show the significance of a series of findings.

Consider equation (15) in the form which includes the reactance effect of R_4.

$$C_1 \doteqdot \frac{C_3}{M}(L + \ell - CR^2 - M) \qquad \text{..........(22)}$$

The inductive reactance value of R_1 (+ or -) affects C_1 inversely (assuming a negative inductive reactance as was the case in these tests) since R_1 was over 300 ohms.

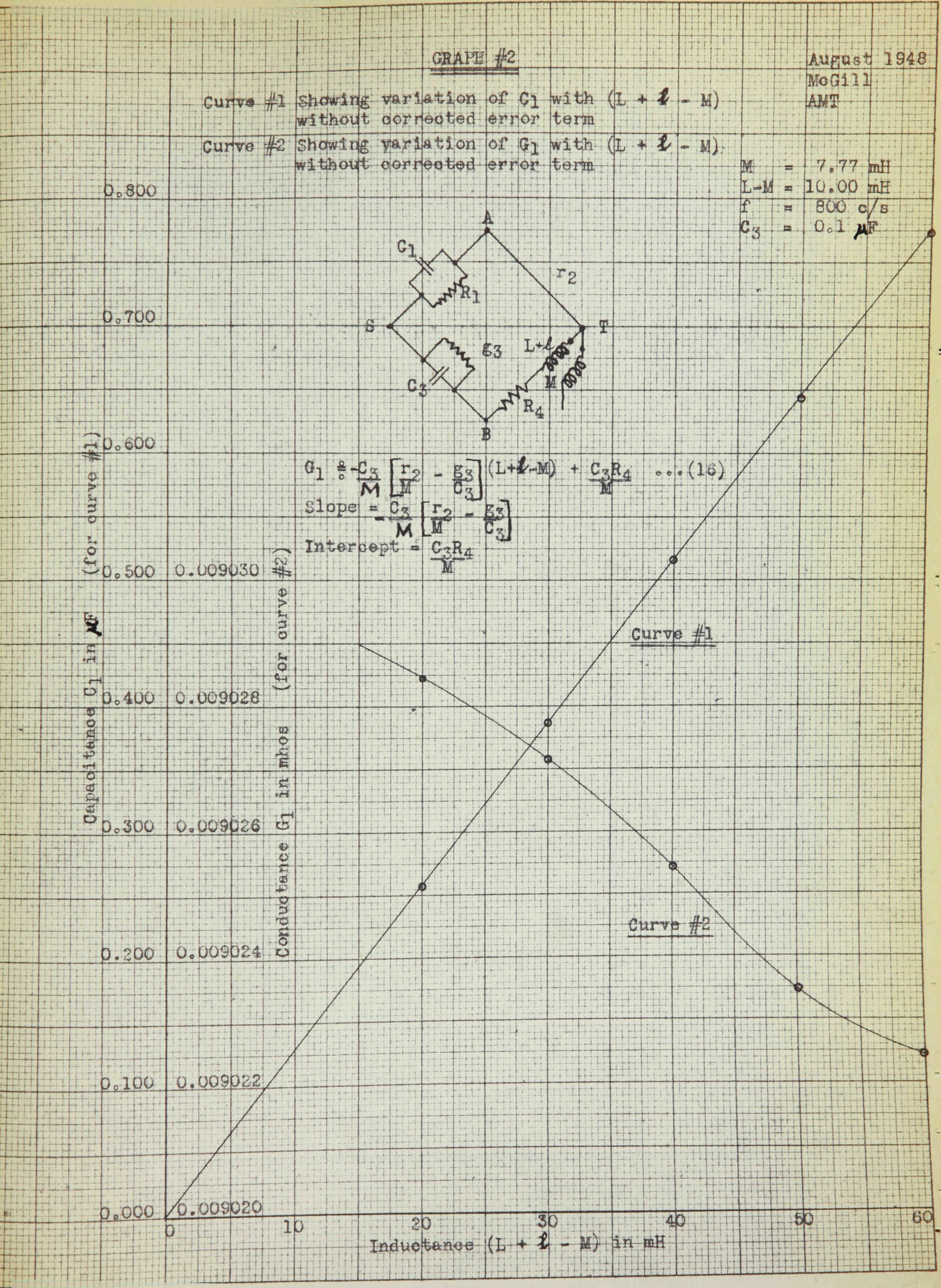
As R_4 is increased from a small value the $\ell - CR^2$ term diminishes and so the $(L + \ell - CR^2 - M)$ term diminishes and thus C_1 is less.

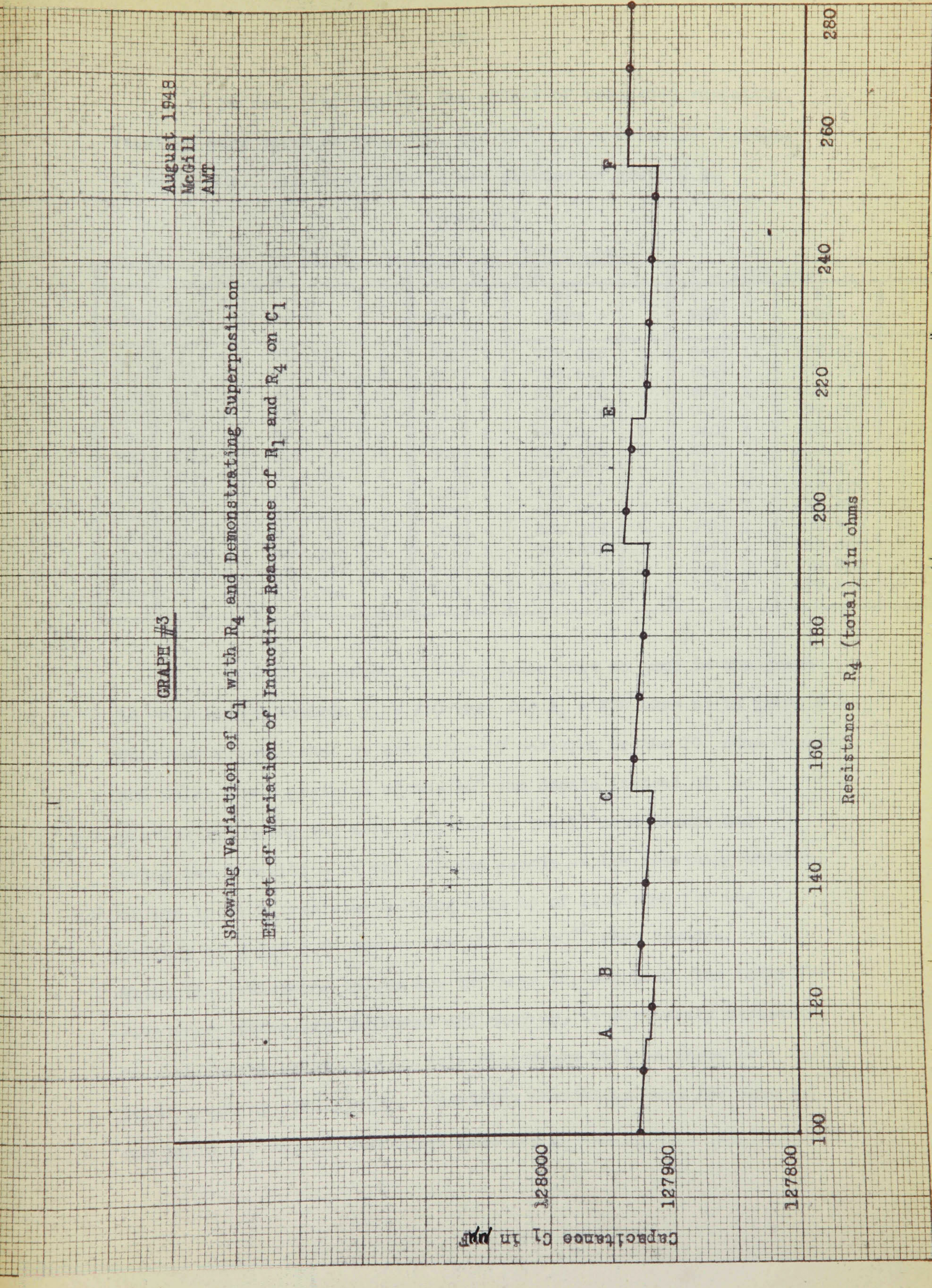
Recapitulating:- In each case for a balance condition the greater R_1 the smaller C_1 will be; the greater R_4 the smaller C_1 will be. This assumes that balancing is done by means of R_1, R_4 and C_1 only.

TABLE #3

R_4 ohms	C_1 $\mu\mu F$	$\pm$	R_1 ohms	$\pm$	Remarks
100	127929	1	778.547	3	M = 7.77 mH
110	127925	2	707.934	3	f = 800 c/s
120	127918	2	648.653	3	C_3 = 0.1 μF (8)[3]
130	127926	1	598.835	3	R_1....(15)[3]
140	127922	2	556.147	3	R_4....(14)[3]
150	127919	3	519.126	3	L-M = 10.00 mH
160	127932	4	486.660	3	
170	127928	3	458.083	2	
180	127925	2	432.654	2	
190	127924	2	409.914	2	
200	127940	3	389.322	2	
210	127936	2	370.790	2	
220	127924	2	353.887	1	
230	127922	2	338.539	0	
240	127920	2	324.436	0	
250	127916	3	311.461	0	
260	127939	3	299.516	0	
270	127938	3	288.431	0	
280	127936	3	278.141	0	

Referring to graph #3, at each of the points A, B, C, D, and F the graph tends to jump up since R_1 is is lowered by a "hundreds" dial setting. This requires more C_1 to balance. However, at points A and E where the "hundreds" dial of R_4 was changed, the graph tends to jump down. This requires less C_1 to balance. At A the two effects tend to balance out. A liberty was taken here in drawing the straight line through one point between A and B. The R_4 box settings were the values given in table #3, column R_4, minus 11.33 ohms for M and leads.


GRAPH #2
August 1948
McGill
AMT
Curve #1 Showing variation of C1 with (L + ℓ - M) without corrected error term
Curve #2 Showing variation of G1 with (L + ℓ - M) without corrected error term
M = 7.77 mH
L-M = 10.00 mH
f = 800 c/s
C3 = 0.1 µF
A
B
S
T
C1
R1
r2
g3
C3
L+ℓ
M
R4
G1 ≑ -C3/M [r2/M - g3/C3] (L+ℓ-M) + C3R4/M ...(16)
Slope = -C3/M [r2/M - g3/C3]
Intercept = C3R4/M
Curve #1
Curve #2
Capacitance C1 in µF (for curve #1)
Conductance G1 in mhos (for curve #2)
0.800
0.700
0.600
0.500
0.400
0.300
0.200
0.100
0.000
0.009030
0.009028
0.009026
0.009024
0.009022
0.009020
0
10
20
30
40
50
60
Inductance (L + ℓ - M) in mH


GRAPH #3
Showing Variation of C_1 with R_4 and Demonstrating Superposition
Effect of Variation of Inductive Reactance of R_1 and R_4 on C_1
August 1948
McGill
AMT
A
B
C
D
E
F
128000
127900
127800
100
120
140
160
180
200
220
240
260
280
Resistance R_4 (total) in ohms
Capacitance C_1 in $\mu\mu F$

To see, graphically, the effect of frequency variation, readings were taken at 400, 800, and 1200 c/s and listed together in table # 4. All three results are plotted on graph # 4.

TABLE # 4

In this table, to all C_1 readings add 127000 $\mu\mu F$.

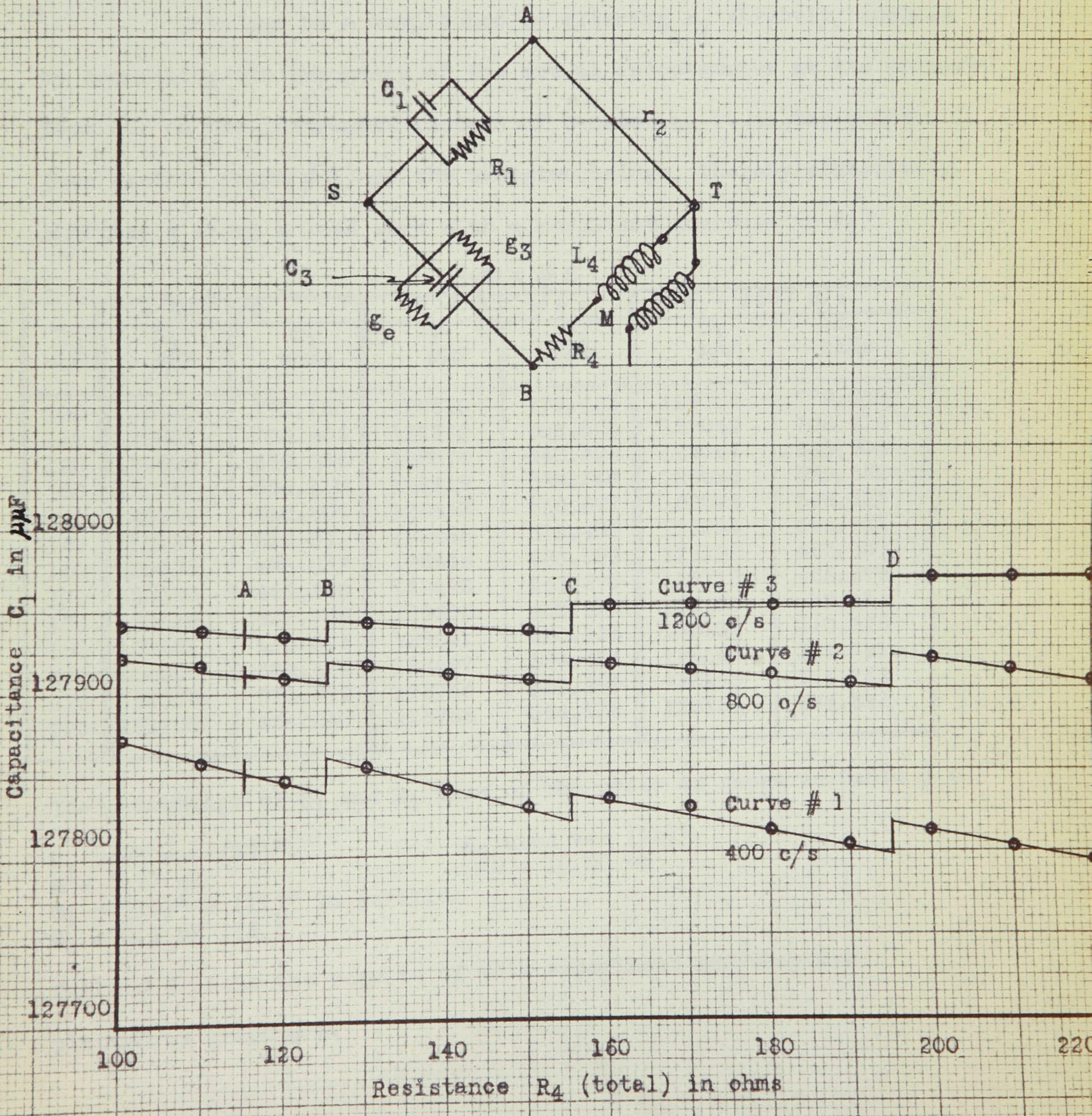
	f = 400 c/s				f = 800 c/s				f = 1200 c/s			
R_4 ohms	C_1 $\mu\mu F$	±	R_1 ohms	±	C_1 $\mu\mu F$	±	R_1 ohms	±	C_1 $\mu\mu F$	±	R_1 ohms	±
100	871	4	778.325	5	922	2	776.049	3	942	1	778.614	2
110	859	4	707.735	5	917	2	705.654	3	939	1	707.999	2
120	847	4	648.478	5	910	2	647.119	3	935	1	648.718	2
130	855	6	598.672	5	916	3	597.604	3	943	1	598.897	2
140	840	6	555.987	4	911	2	555.067	3	940	1	556.196	2
150	828	7	518.972	5	908	2	518.133	3	939	1	519.172	2
160	834	3	486.514	4	916	3	485.817	3	953	1	486.703	2
170	827	5	457.944	4	912	3	457.312	3	952	2	458.125	2
180	814	6	432.523	4	909	2	431.931	2	950	2	432.698	2
190	805	4	409.789	4	904	4	409.236	3	952	2	409.954	2
200	813	4	389.205	3	918	3	388.831	2	968	1	389.365	2
210	804	4	370.677	2	912	3	370.320	2	968	2	370.828	2
220	795	6	353.854	2	904	3	353.699	2	969	1	353.999	2

It will be noted in these cases that no jump in the graph is experienced between R_4 = 210 ohms and R_4 = 220 ohms as in graph # 3. This was accomplished by using the 10 position on the 10^1 ohms decade and using the 1 position of the 10^2 ohm decade instead of, as previously, using the zero position on the 10^1 ohm decade and using the 2 position on the 10^2 ohm decade -- the remaining amount of the total R_4 resistance being made up of 11.2 ohms for the primary of M and 8.8 ohms on the 10^0 and 10^{-1} ohms decades. While the resistance values are the same in each case, the inductive reactance values are different and show up, as previously explained, in a different value of C . This difference will be shown in a (p.33) later graph by overlapping. Operating conditions were the same as for table # 3 except that boxes used for R_1 and R_4 were interchanged.

GRAPH # 4

August 1948
McGill
AMT

Showing Variation of C_1 with R_4 for Frequencies of 400, 800, and 1200 c/s with Superposed Effects of Variation of Inductive Reactance of R_1 and R_4 on C_1 (with extra conductance g_3 added to C_3 in form of a 3 megohm carbon resistor).



So far only g_3 in the bracketed term of equation (14) has been altered, and this to correct the slope which was positive on first trial (see graph # 1). As will be seen later on, using a standard condenser with a poor power factor for C_3 gives a negative slope. At this point a trial was made at 400, 800 and 1600 c/s with the present set-up to make the slope more positive (while leaving the 3 megohm resistor across C_3).

This can be accomplished by altering r_2 slightly, such as by connecting the secondary of the mutual inductometer at the primary terminal on the instrument instead of connecting it to the point T (see sketch, graph # 1) of the skeleton bridge, thereby effectively shifting T along a lead wire with a resultant increase in r_2

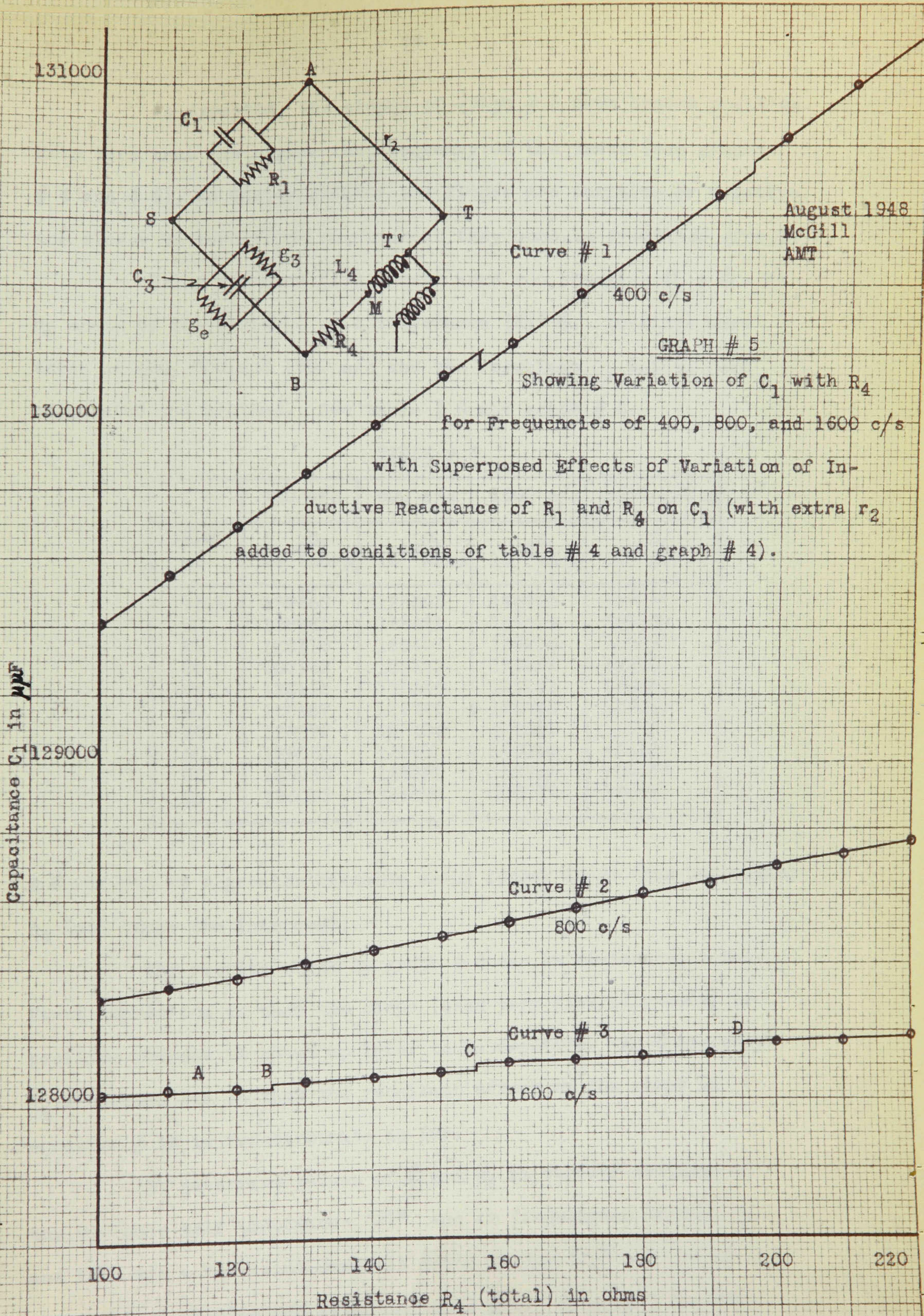
TABLE # 5

	f = 400 c/s		f = 800 c/s		f = 1600 c/s	
R_4 ohms	C_1 $\mu\mu F$	R_1 ohms	C_1 $\mu\mu F$	R_1 ohms	C_1 $\mu\mu F$	R_1 ohms
100	129410	779.136	128310	779.423	128035	779.688
110	129552	708.406	128342	708.660	128042	708.916
120	129695	649.035	128374	649.265	128047	649.504
130	129851	599.150	128419	599.348	128066	599.582
140	129995	556.406	128453	556.577	128076	556.811
150	130137	519.341	128488	519.498	128084	519.721
160	130236	486.837	128536	486.992	128108	487.207
170	130380	458.233	128573	458.373	128119	458.579
180	130519	432.788	128608	432.914	128126	433.114
190	130661	410.015	128641	410.136	128136	410.328
200	130828	389.424	128697	389.534	128165	389.721
210	130978	370.879	128734	370.985	128173	371.162
220	131109	354.040	128768	354.138	128184	354.311

The ± settings of C_1 and R_1 were of the order as shown in table # 4.

R_1 used was $(12)^3$. R_4 used was $(15)^3$
$M = 7.77$ mH $L-M = 10.00$ mH
$C_1 = (10)^3 + (11)^3$ in parallel
$C_3 = 0.1\ \mu F$ $(8)^3$

Table # 5 lists the results of this experiment and they are plotted as graph # 5.



GRAPH # 5

Showing Variation of C_1 with R_4 for Frequencies of 400, 800, and 1600 c/s with Superposed Effects of Variation of Inductive Reactance of R_1 and R_4 on C_1 (with extra r_2 added to conditions of table # 4 and graph # 4).

As expected, the slopes of graph # 5 have become positive. Note here that the higher the frequency of the tests (see graphs # 4 and # 5), no matter whether the slopes are positive or negative, the closer to zero slope requirement they come, as predicted (for.14).

Using equation (16), tests were run changing $(L + \ell - M)$ by means of the variable self inductance (3)[3] ℓ. Readings were taken at 400, 800, and 1600 c/s and the resultant G_1 values were calculated and plotted. See table # 6 and graph # 6.

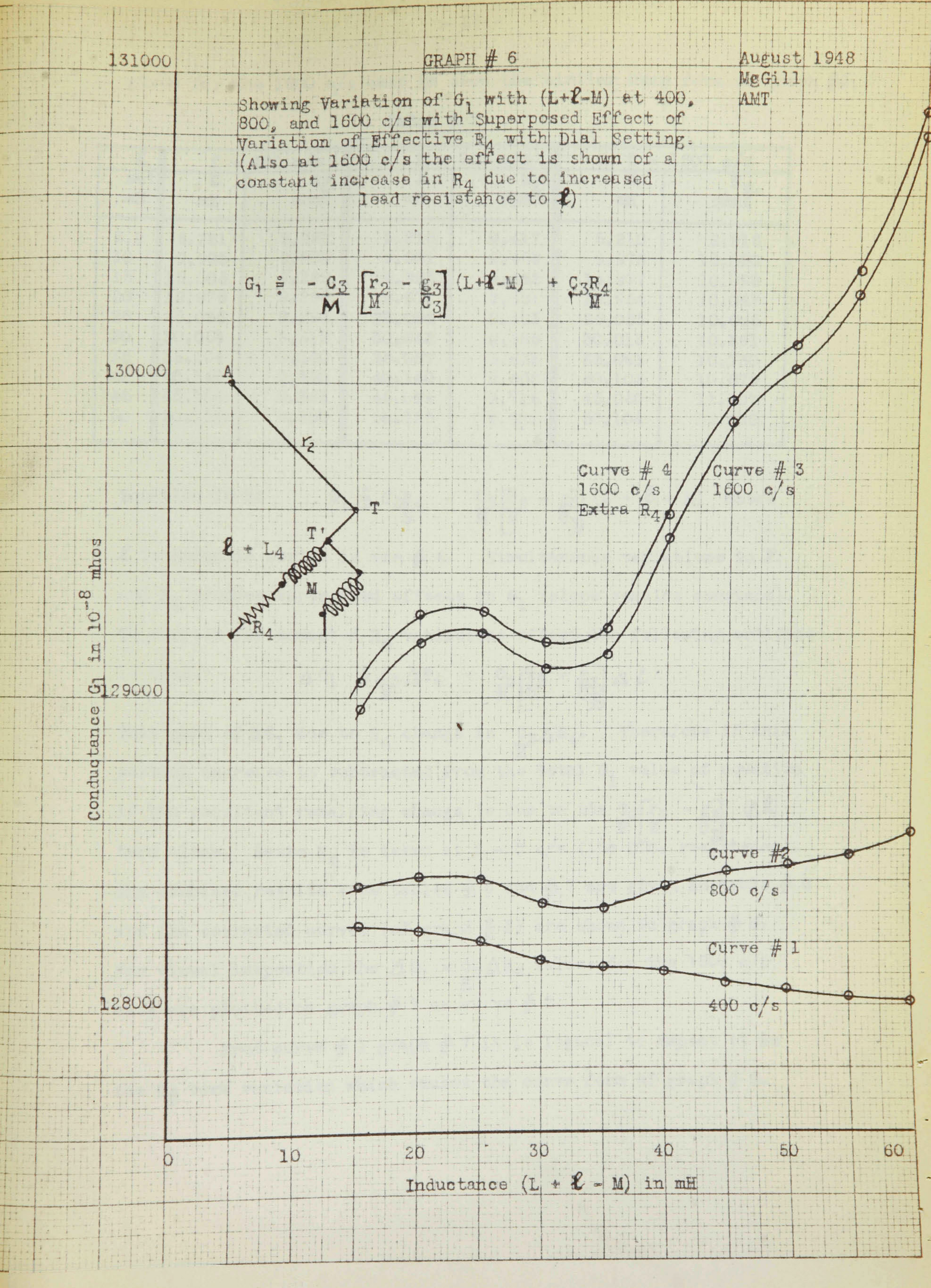
TABLE # 6

f =	400 c/s	800 c/s	1600 c/s	1600 c/s *
$L+\ell-M$ mH	G_1 10^{-8} mhos	G_1 10^{-8} mhos	G_1 10^{-8} mhos	G_1 10^{-8} mhos
15.2	128269	128393	128963	129051
20	128245	128421	129179	129266
25	128215	128407	129206	129278
30	128152	128333	129098	129180
35	128123	128316	129141	129222
40	128107	128385	129513	129589
45	128075	128434	129881	129956
50	128053	128456	130054	130129
55	128028	128481	130285	130363
60	128008	128555	130787	130861

* ℓ was put at a greater distance using longer leads thus increasing R_4 (and hence the intercept).

The results shown in graph # 6 can be misleading. The first impression is that the slope (used very liberally here) is positive whereas it shows up later to be negative. As ℓ increases R_4 increases too so that the resultant effect on the slope in equation # 16 is indeterminate so far.

To determine values of ℓ and R_1 at various settings of the variable self inductance (3)[3] and at frequencies of 400, 800, and 1600 c/s a Heaviside-Campbell Ratio Bridge (1)[2] was set up and the combined results are given in table # 7. Graph # 7 is also


GRAPH # 6
August 1948
McGill
AMT
Showing Variation of G1 with (L+ℓ-M) at 400, 800, and 1600 c/s with Superposed Effect of Variation of Effective R4 with Dial Setting. (Also at 1600 c/s the effect is shown of a constant increase in R4 due to increased lead resistance to ℓ)
$G_1 \doteqdot -\frac{C_3}{M}\left[\frac{r_2}{M} - \frac{g_3}{C_3}\right](L+\ell-M) + \frac{C_3 R_4}{M}$
131000
130000
129000
128000
A
r2
T
T'
ℓ + L4
M
R4
Curve # 4
1600 c/s
Extra R4
Curve # 3
1600 c/s
Curve #2
800 c/s
Curve # 1
400 c/s
Conductance G1 in 10^-8 mhos
0
10
20
30
40
50
60
Inductance (L + ℓ - M) in mH

given for the 1600 c/s case to show the similar hump form, of graph #6.

TABLE # 7

ℓ	f = 400 c/s		f = 800 c/s		f f = 1600 c/s	
set mH	ℓ mH	R_ℓ ohms	ℓ mH	R_ℓ ohms	ℓ mH	R_ℓ ohms
5.2	5.221	9.292	5.218	9.417	5.210	9.916
10	9.995	9.299	9.991	9.458	9.976	10.090
15	15.005	9.299	14.993	9.462	14.977	10.098
20	19.978	9.296	19.971	9.451	19.954	10.047
25	25.096	9.299	25.084	9.463	25.055	10.095
30	30.054	9.319	30.043	9.536	30.013	10.391
35	35.104	9.340	35.087	9.623	35.061	10.730
40	40.140	9.350	40.140	9.662	40.105	10.879
45	45.104	9.361	45.094	9.716	45.060	11.107
50	50.139	9.390	50.123	9.822	50.104	11.538

Equation 16 is $G_1 \doteq \frac{C_3 R_4}{M} - \frac{C_3}{M}\left[\frac{r_2}{M} - \frac{g_3}{C_3}\right](L + \ell - M)$ where

ℓ is added in series in arm # 4. Simultaneous variations in ℓ and R_4 produce superposed effects on G_1 (slope remains constant). To correct for unwanted R_4 variations effect the following artifice is used.

$$\Delta G_1 \doteq \frac{C_3}{M}\Delta R_4 - \frac{C_3}{M}\left[\frac{r_2}{M} - \frac{g_3}{C_3}\right]\Delta \ell$$

That part of ΔG_1 due to R_4 change is $\frac{C_3}{M}\Delta R_4$. Therefore if this partial increase is subtracted from the total G_1 value of equation 16 the resultant remaining change is due to the $\frac{C_3}{M}\left[\frac{r_2}{M} - \frac{g_3}{C_3}\right](\Delta\ell)$ term alone. Basic R_4 is taken at f = 0 c/s (the D.C. resistance). The combined results from tables # 6 and # 7 are given in table # 8 and the corrected curves (of graph # 6) are shown in graph # 8. For visual inspection the $\Delta G_1 = \frac{C_3}{M}\Delta R_4$ values for the 1600 c/s case are plotted on graph # 7 as curve # 2.

From curve # 1 graph # 7 it is logical to expect it is the R_ℓ term variation which causes the curve form of graph # 6.

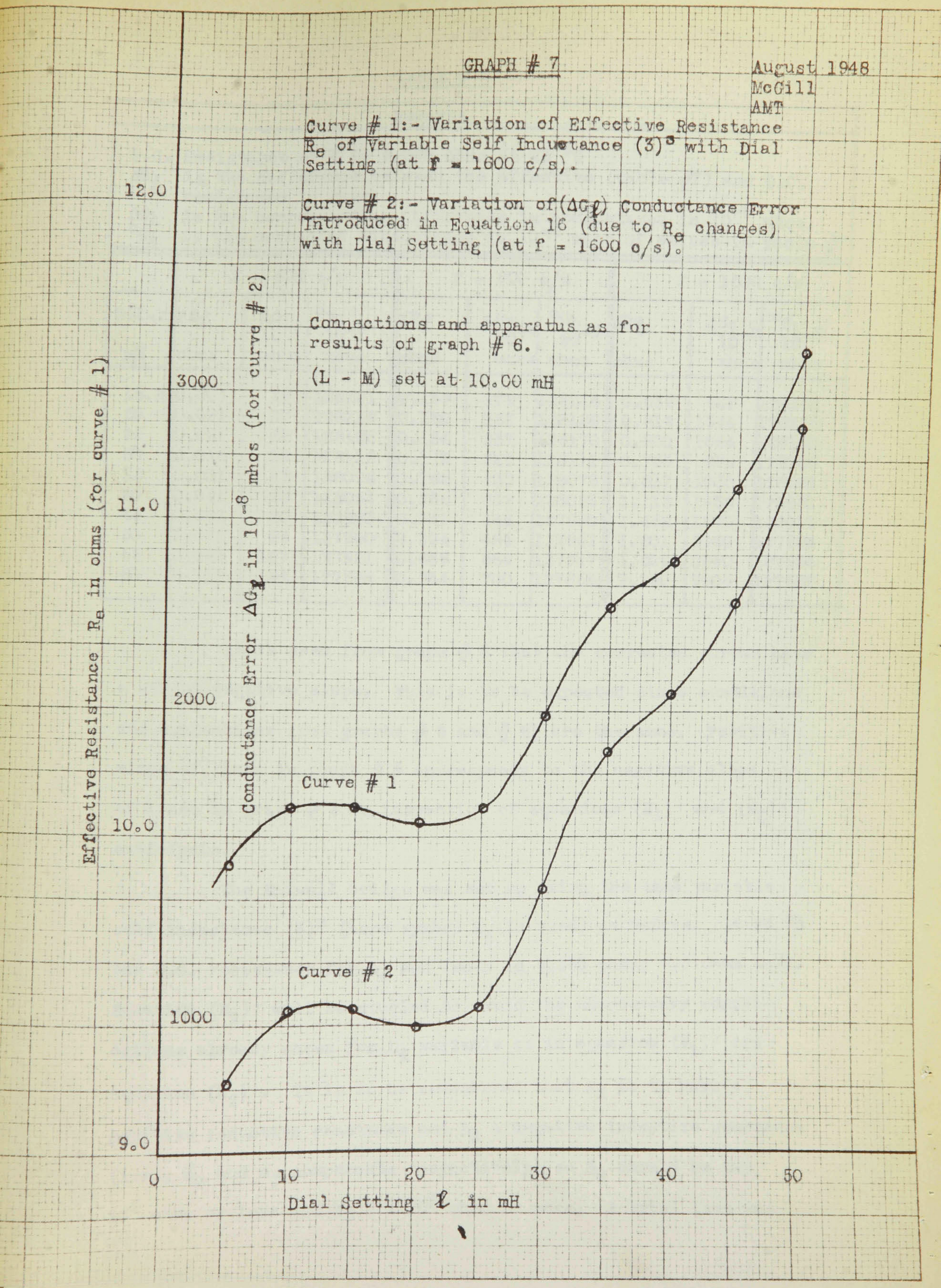

GRAPH # 7
August 1948
McGill
AMT
Curve # 1:- Variation of Effective Resistance R_e of Variable Self Inductance (3)[a] with Dial Setting (at f = 1600 c/s).
Curve # 2:- Variation of (ΔG_ℓ) Conductance Error Introduced in Equation 16 (due to R_e changes) with Dial Setting (at f = 1600 c/s).
Connections and apparatus as for results of graph # 6.
(L - M) set at 10.00 mH
Curve # 1
Curve # 2
Effective Resistance R_e in ohms (for curve # 1)
Conductance Error ΔG_ℓ in 10^{-8} mhos (for curve # 2)
12.0
11.0
10.0
9.0
3000
2000
1000
0
10
20
30
40
50
Dial Setting ℓ in mH

TABLE # 8

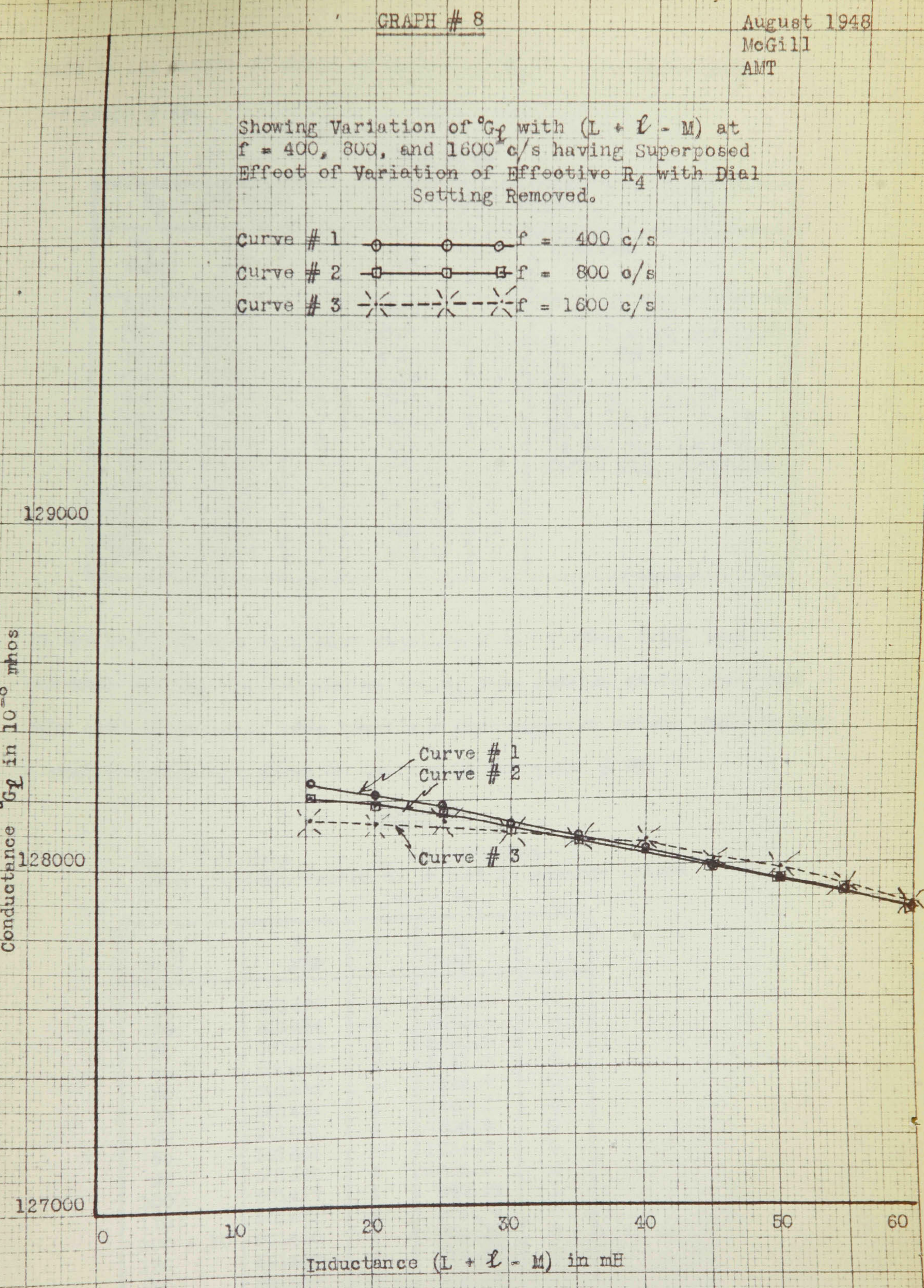
Remarks

D.C. Resistance of ℓ is 9.278 ohms at 23 °C
ΔR_ℓ is the difference between the effective (table #7) and D.C. resistance in ohms.
ΔG_1 is the corresponding difference in conductance in 10^{-8} mhos.
$^{o}G_1$ is the corrected conductance in 10^{-8} mhos of values in Table #7

	f = 400 c/s			f = 800 c/s			f = 1600 c/s		
$L+\ell-M$ mH	ΔR_ℓ ohms	ΔG_1 10^{-8} mhos	$^{o}G_1$ 10^{-8} mhos	ΔR_ℓ ohms	ΔG_1 10^{-8} mhos	$^{o}G_1$ 10^{-8} mhos	ΔR_ℓ ohms	ΔG_1 10^{-8} mhos	$^{o}G_1$ 10^{-8} mhos
15.2	0.014	18	128251	0.139	179	128214	0.638	821	128142
20	0.021	27	128218	0.180	232	128189	0.812	1044	128135
25	0.021	27	128188	0.184	237	128170	0.820	1054	128152
30	0.018	23	128129	0.173	222	128111	0.769	989	128109
35	0.021	27	128096	0.185	238	128078	0.817	1050	128091
40	0.041	53	128054	0.258	332	128053	1.113	1432	128081
45	0.062	80	127995	0.345	444	127990	1.452	1870	128011
50	0.072	93	127960	0.384	494	127962	1.601	2060	127994
55	0.083	107	127921	0.438	564	127917	1.829	2350	127935
60	0.112	144	127864	0.544	700	127855	2.260	2905	127882

It is seen from graph # 8 that the corrected curves have a slight negative slope. This is to be expected since conditions and connections (see graphs # 5 and # 6) are the same. Positive slope of lines in graph # 5 corresponds to the negative slope of lines in graph # 8 as inspection of equations $(N_1)'$ and $(N_2)'$ anticipate.

The Maxwell bridge was set up using the same variable self inductance (3)[3] (here symbol L_4 is used) as before. At 23 °C the D.C. resistance for L_4 and leads is 9.288 ohms: 9.3 ohms used. Equation $(M_2)'$ will be studied to avoid the unnecessary complications already shown for L_4 variable as in equation $(M_1)'$ and equation $(N_2)'$. It is to be remembered that R_2 is to have a positive inductive reactance and R_3 a negative inductive reactance (i.e., ℓ_2 and c_3 components respectively) so R_2 should be say 10^2 ohms or less and R_3 5×10^2 ohms or more. A trial test was



GRAPH # 8
August 1948
McGill
AMT
Showing Variation of $^{o}G_{\ell}$ with $(L + \ell - M)$ at
f = 400, 800, and 1600 c/s having Superposed
Effect of Variation of Effective R_4 with Dial
Setting Removed.
Curve # 1 f = 400 c/s
Curve # 2 f = 800 c/s
Curve # 3 f = 1600 c/s
Curve # 1
Curve # 2
Curve # 3
129000
128000
127000
Conductance $^{o}G_{\ell}$ in 10^{-8} mhos
0
10
20
30
40
50
60
Inductance $(L + \ell - M)$ in mH

made to see nature of slope of equation $(M_2)'$. See table # 9A and graph # 9A.

TABLE # 9A

R_2 = 100 ohms		R_3 = 10,000 ohms			
R_4 ohms	C_1 µµF	±	R_1 ohms	±	Remarks
100	50089	1	9903.6	1	f = 800 c/s
200	50075	2	4975.2	1	L = 50.0 mH set (3)[3]
300	50061	2	3321.3	0	R_1...(12)[3]
400	50047	2	2493.1	0	R_L...9.3 ohms
500	50029	2	1996.0	0	$R_4 = R_L$ + (14)[3]
600	50012	2	1663.5	0	C_1...(10)[3] + (11)[3]
700	49996	2	1425.8	0	R_2 and R_3..General
800	49979	2	1247.5	0	Radio plug resistors
900	49953	2	1109.9	1	
1000	49932	3	998.73	2	
1100	49945	6	908.70	2	
1200	49926	3	832.97	2	

In the light of the resultant negative slope another trial test was made with R_3 smaller making the c_3 term smaller so the bracketed term (and slope) will tend towards a less negative value. Results for a smaller range are shown in table # 9B and graph # 9B.

TABLE 9B

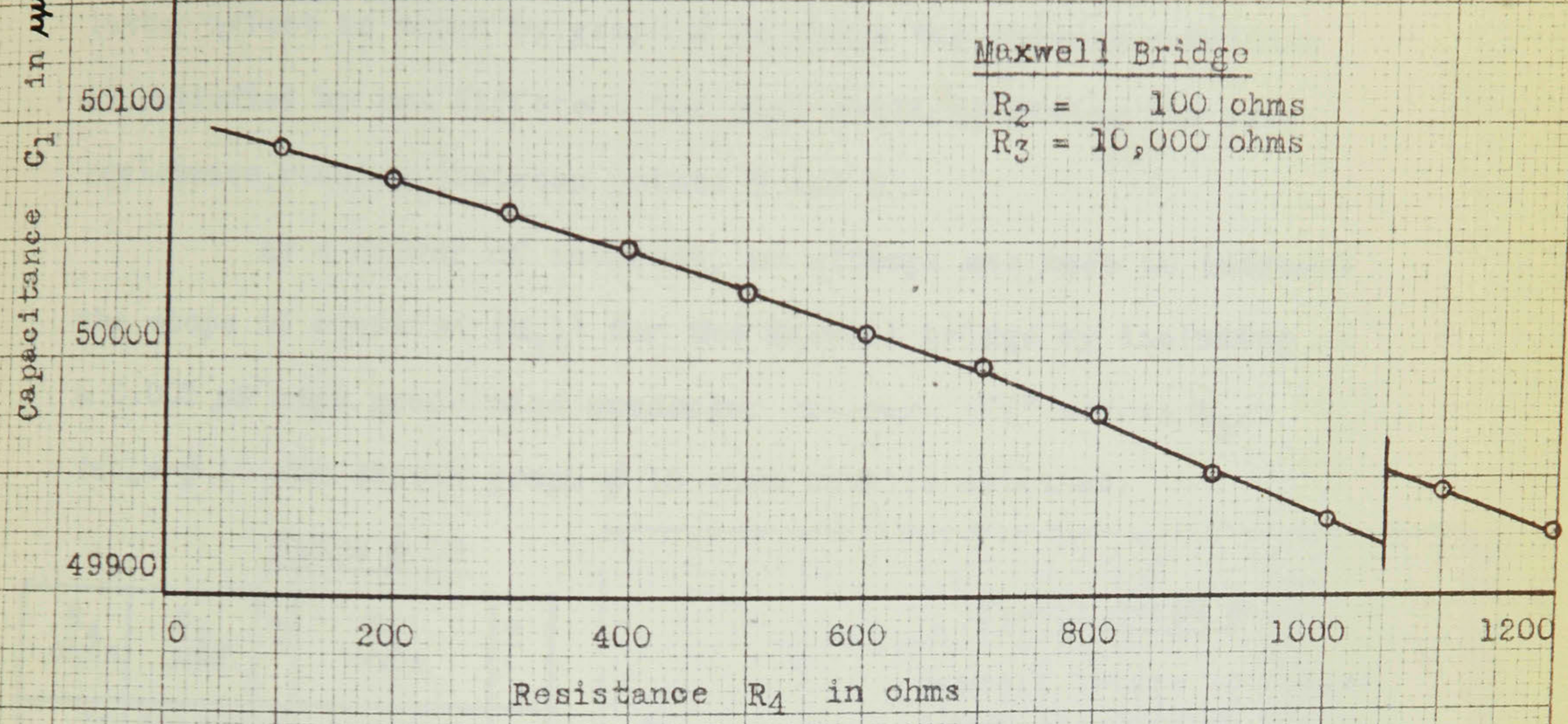
R_2 = 100 ohms R_3 = 1,000 ohms
Other factors remain as listed in Remarks column of table 9A

R_4 ohms	C_1 µµF	±	R_1 ohms	±
10	501478	4	9124.0	10
20	501484	4	4772.4	4
30	501484	4	3230.0	2
40	501484	4	2441.3	2
50	501482	4	1962.7	0
60	501481	4	1640.3	0
70	501481	4	1408.9	0
80	501481	4	1234.8	0
90	501482	4	1098.9	0
100	501472	4	990.89	1
110	501471	6	901.64	2
100 + 10	501460	4	901.32	2
120	501460	5	826.86	2
130	501457	4	763.65	1
140	501456	5	709.54	1

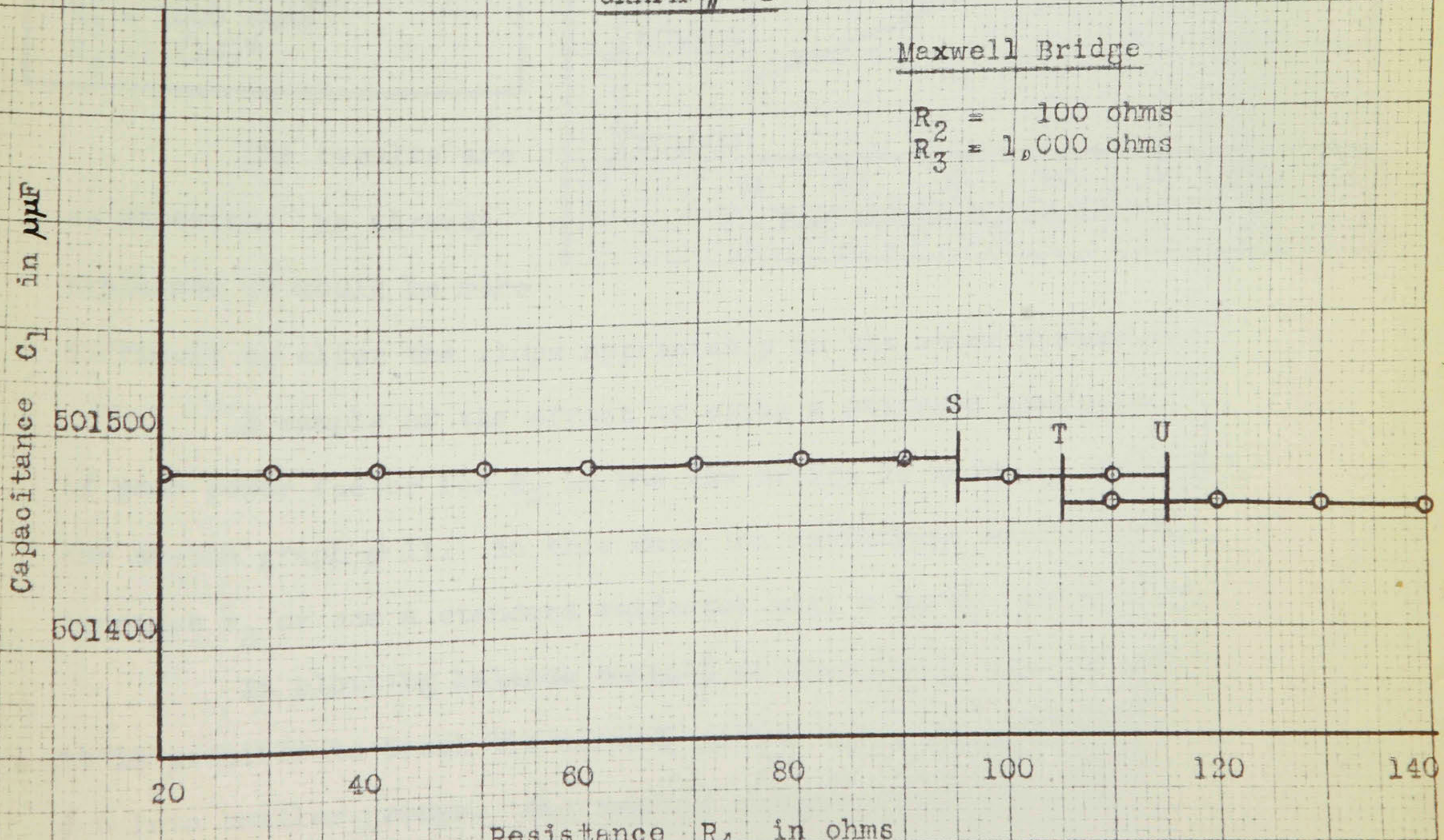
GRAPH # 9A

August 1948
McGill
AMT

Showing Variation of C_1 with R_4 for the Maxwell Bridge (conditions as stated in table # 9A) with Superposed Reactance Effect of Decade Resistances R_1 and R_4.



GRAPH # 9B



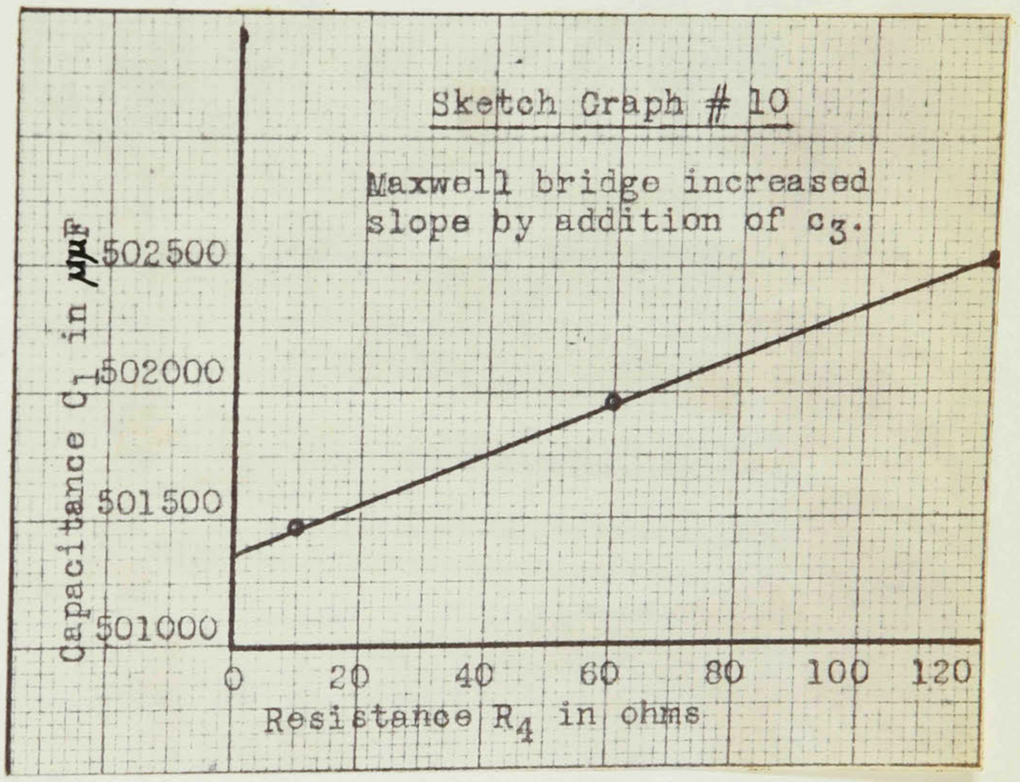
The slope of graph # 9A is again a superposition of two effects, the true slope and the inductive reactance changes of the dial resistance boxes. The direction of slope due to this latter effect is shown by graph # 9B where the balance condition is satisfied by two different box dial settings giving the same resistance reading (between points T and U).

As a matter of interest, an attempt was made to increase the slope of equation $(M_2)'$ for the Maxwell bridge by including a 0.001 μF high grade mica condenser standard (7)[3] across R_3. Table # 10 and sketch graph # 10 show results obtained.

TABLE # 10

R_4 ohms	C_1 $\mu\mu F$	±	R_1 ohms	±
10	501466	3	10322.8	4
60	501958	2	1675.2	0
120	502546	3	835.55	2

R_1....(12)[3]
R_2 = 100 ohms
R_3 = 1000 ohms
R_4....(14)[3]



The results are as expected. As already explained it would be more difficult to alter the slope appreciably in the other direction.

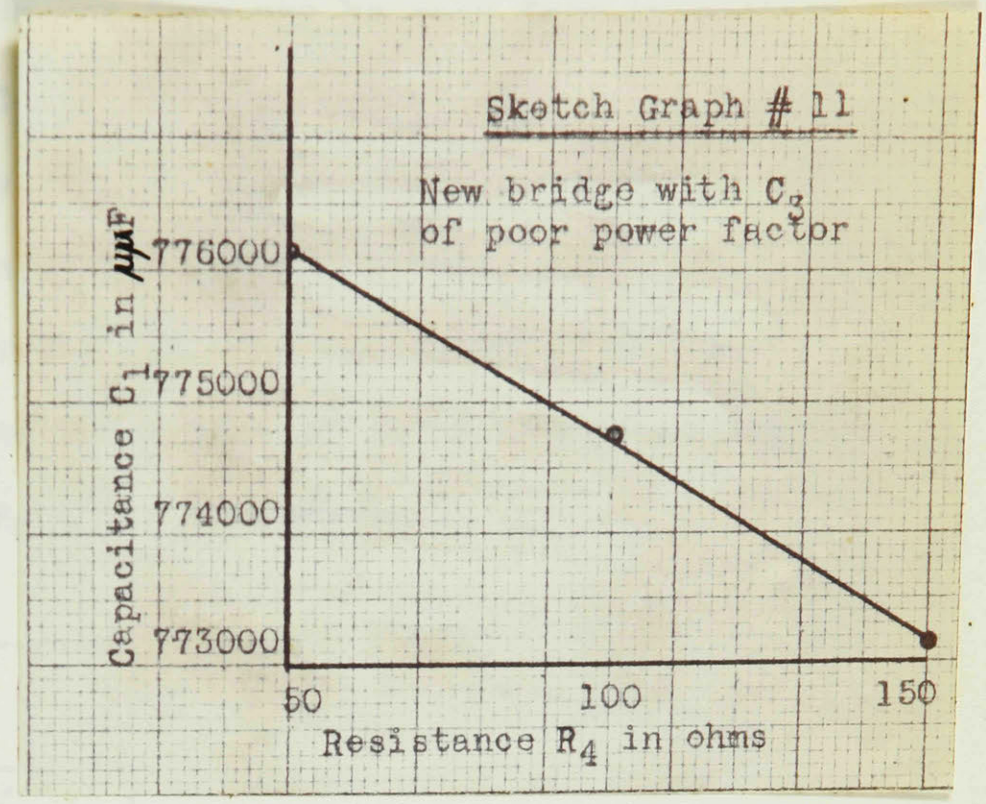
A sample of the effect of using a standard condenser of poor power factor for C_3 in the new bridge is given in table # 11 and sketch graph # 11. In this case the rectifying action is to increase r_2 or use a standard condenser with a better power factor.

In plotting balance conditions results of the new bridge it is possible to break the already broken lines of graphs # 4 and # 5 into smaller groups. One example showed definitely series of straight lines broken at every 10^1 dial change. It might be possible to check the same effect on the 10^0 dial. This last was not tried.

TABLE # 11

R_4 ohms	C_1 $\mu\mu F$	$\pm$	R_1 ohms
50	776122	15	203.26
100	774772	30	99.151
150	773183	90	66.055

f = 800 c/s
C_3 = 1.0 μF (9)[3]
M = 10.00 mH
L-M = 7.77 mH
R_4....(13)[3]
R_1....(12)[3]



EXPERIMENTAL RESULTS

A. Determination of primary inductance L_4 of the Campbell Standard Mutual Inductometer (1)[3].

$$\begin{aligned} f &= 800 \text{ c/s} \\ R_2 &= 100 \text{ ohms}) \text{ (giving practically)} \\ R_3 &= 1000 \text{ ohms}) \text{ (zero slope)} \\ C_1 &= 0.177308\ \mu F \text{ (set reading)} \end{aligned}$$

$L_4 \doteq R_2R_3C_1 = 17.73_1$ mH which agrees favourably with the value given for the instrument of 17.77 mH since R_2 and R_3 are 0.1% grade.

B. Determination of inductance L of 1 H nominal self inductance (2)[3]. See table # 12. (Maxwell's Bridge).

TABLE # 12

R_4 ohms	C_1 μF	$\pm$	R_1 ohms	$\pm$	Remarks
116.4	1.000141	15	8515	2	f = 800 c/s
180	1.000159	12	5526	2	R_2= 100 ohms
240	1.000165	21	4152	2	R_3= 10,000 ohms

Since R_3 is raised of necessity the slope leaves its zero value. The resultant effect of the error term can be seen from inspection of table # 12. $L_4 \doteq R_2R_3C_1 = 1.000$ H

Given value $= 1.0002$ H at 800 c/s and 24 °C.

C. A test was run on the Maxwell bridge, which, while not conclusive, is indicative of limiting measurements of low value. The L_4 leads were shorted and the bridge balanced to get table # 13 neglecting requirements of R_3 (over 300 ohms).

TABLE # 13

R_2 ohms	R_3 ohms	C_1 $\mu\mu F$	Remarks
10	100	2090	The slope of the error term will not be zero since R_3 not set at 1000 ohms
10	10	24400	
10	5	49000	

$$L_x = 10^2(244 \times 10^{-10} + x_s)$$
$$L_x = 50\ (490 \times 10^{-10} + x_s)$$

So $x_s = 200\ \mu\mu F$, where x_s is the effective stray capacitance in arm # 1.

Now considering R_3 to be 1000 ohms, then the minimum L_4 measurable is $L_4 = 10^4 \times 200 \times 10^{-12}$ or $2\ \mu H$.

D. Check on constancy of resistance of the inductometer used (1)[3] showed that over a range of frequencies 200 c/s to 2000 c/s the resistance did not vary more than 0.01 ohms in 11.2 ohms which matches the accuracy of the resistance boxes used.

Some comparative results on some inductances measured are given in table # 14. Pertinent information is given here.

Inductance	Remarks
1 H	(2)[3] D.C. resistance of 116.32 ohms at 23 °C A.C. resistance of 120 ohms at 24 °C and 800 c/s Inductance 1.0002 H at 24 °C and 800 c/s
1.75 μH	(4)[3] D.C. resistance 0.17 ohms Dial setting error of 1% possible
R.F. coil	D.C. resistance of 0.02 ohms

TABLE # 14

Nominal Inductance	Bridge Used	Units	Frequency in c/s 400	800	1600	2000
1 H	new	H	1.00_1	1.00_4	1.01_6	1.02_4
1 H	Maxwell	H	0.999_3	1.00_1	1.00_8	1.01_4
1.75 μH	new	μH	1.93	1.91	1.92	1.91
1.75 μH	Maxwell	μH	----	1.89	1.92	1.82
R.F.coil	new	μH	0.76	0.80	0.79	0.82
R.F.coil	Maxwell	μH	----	0.87	0.81	0.82

CONCLUSIONS.

Range:- Theory predicts and experiment tends to indicate that using the apparatus listed and aiming for an accuracy of 0.1% with normal operation, the range of the new bridge is 10 μH to 10 H while the range of the Maxwell bridge is 10 μH to 1 H, keeping in mind that the adjustment sensitivity of C_1 is nearly 1 part in 10^5 as shown in the tables. It is easier to measure large inductors with the new bridge since it is easier to render the "error" term negligible by corrective measures, and the product term M/C_3 is more readily variable over a greater range with less variation in this error term than the R_2R_3 term in the Maxwell bridge.

For intermediate ranges the Maxwell bridge seems superior in that the error term is more likely to be smaller especially at low frequencies and for high L/R ratios.

Error term adjustments:- Both bridges require some error term adjustments for changes in range settings. The error term of the new bridge has greater flexibility in adjustments than the Maxwell though it requires more adjustment. Both the r_2 and g_3 terms of the new bridge can be altered readily by the inclusion of an added resistance in arm 2 in the first case and by the addition of a conductance in arm 3 in the second case. Unfortunately, g_3

is frequency dependent being smallest at low frequencies. Hence readjustment should be made if the frequency is changed.

Of the two terms ℓ_2 and c_3 of the Maxwell, the second is readily altered by the inclusion of extra capacitance. However, the alteration of the first term introduces unwanted extraneous complications as shown in the text and is a practice to be avoided. This complication is usually met at the upper range of the bridge.

The adjustment requirements for the new bridge error term may vary from condenser to condenser even if of like capacitance. Two different C_3's, each of 1.0 µF, gave two different correction terms as shown by the slopes of graphs # 1 and # 11. This is not so likely to happen in the case of replacement resistor units used in the Maxwell bridge.

For both bridges the value of the error term x 10^2 is the percentage error to be expected in the results due to imperfect adjustments of slope. Experiment shows that without adjustments the new bridge is inferior to the Maxwell except at higher frequencies and lower L/R ratios. But, if adjustment of the error term is made, it is easily accomplished, positively and negatively, with the new bridge as mentioned above.

Sensitivity:- The sensitivity of the two bridges, as used, is the same in that a balance is detectable to ± 1 part in 10^5 as shown in the tables at higher frequencies, and further exemplified in the graphs. It has been shown, distinctly, that the residual changes of so-called non-reactive resistors are readily discernable even though these are far smaller than the guaranteed accuracy (0.1%) of the bridge components, and so the bridge sensitivity is considered to be ample.

The sensitivity of the new bridge is somewhat frequency dependent (see equations of Appendix I) in that Z_2 and Z_3 are frequency dependent whereas R_2 and R_3 in the Maxwell are not. Generally speaking, for maximum sensitivity the four arm impedances should be equal. This condition is reached with a rapidity proportional to w^2 in the new bridge and to w in Maxwell's bridge as a study of $Z_4/Z_3 = Z_2/Z_1$ shows in these two cases.

Measurement of L's and C's:- Essentially, both bridges measure L's in terms of C's more easily than C's in terms of L's. This fact results from the unreliability of the effective resistance of the variable self inductances with various dial settings. If capacitances are to be measured, the best method would seem to be by substitution in Y_1 in which case the unknown C is really measured in terms of another capacitance.

Equipment:- The equipment for the new bridge is more expensive than for the Maxwell bridge for similar use.

A refinement suggests a mutual inductor with one of its L's equal to M so that the term L-M in arm #4 equals zero. Thus the Z_1 arm is called upon to balance out the added L_4 only.

In the new bridge a knowledge of the exact value of the L of the primary of the mutual is not necessary when measuring an inductance since a zero reading can be made and then a difference value used to determine the unknown L.

D.C. in inductors. On completing this thesis it was realized that this new bridge seems to have marked advantages over other bridges (Owen's and Hay's) in the measurement of inductors with superimposed direct current. One is that it seems easier to get accuracy with large inductors; the other that heating troubles are less because the direct current flows through circuits of much lower resistance (the mutual inductor and the unknown).

ACKNOWLEDGMENT

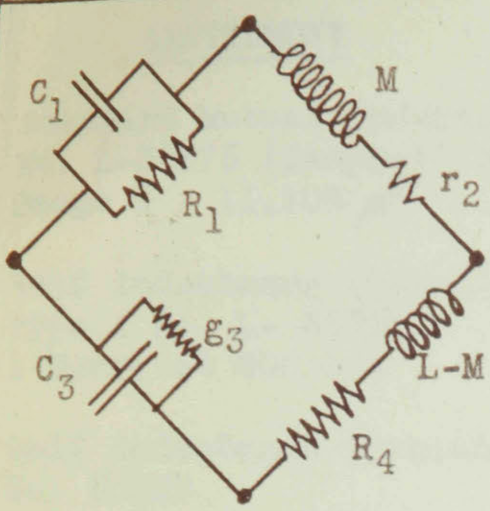
The author wishes to extend his sincere thanks to Dr. H.G.I. Watson for his enlivening interest and direction in this work.

ACKNOWLEDGMENT

The author wishes to extend his sincere thanks to Dr. H.G.I. Watson for his enlivening interest and direction in this work.

APPENDIX I

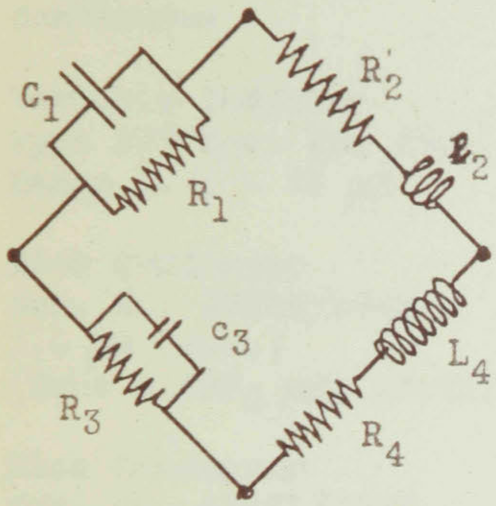
Real and quadrature balance condition relationships for the bridges listed following. Text references shown as ( )'



NEW BRIDGE

$$C_1\frac{M}{C_3(L-M)} - 1 \doteq \frac{R_4}{w^2(L-M)}\left[\frac{r_2}{M} - \frac{g_3}{C_3}\right] \ldots\ldots(N_1)'$$

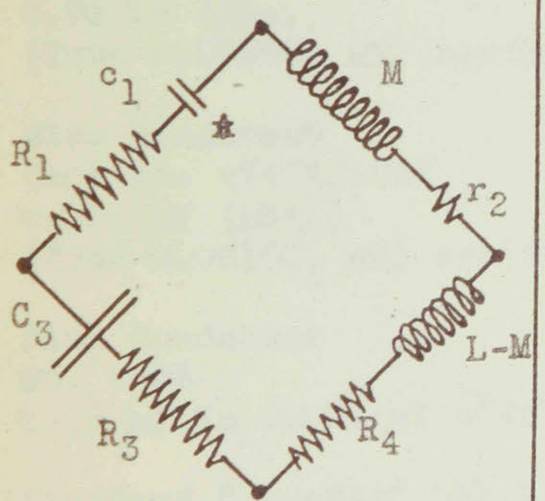
$$G_1\frac{M}{C_3R_4} - 1 \doteq \frac{-(L-M)}{R_4}\left[\frac{r_2}{M} - \frac{g_3}{C_3}\right] \ldots\ldots(N_2)'$$



MAXWELL

$$G_1\frac{R_2}{R_4G_3} - 1 \doteq \frac{w^2L_4}{R_4}\left[\frac{\ell_2}{R_2} - \frac{c_3}{G_3}\right] \ldots\ldots(M_1)'$$

$$C_1\frac{R_2}{L_4G_3} - 1 \doteq \frac{-R_4}{L_4}\left[\frac{\ell_2}{R_2} - \frac{c_3}{G_3}\right] \ldots\ldots(M_2)'$$

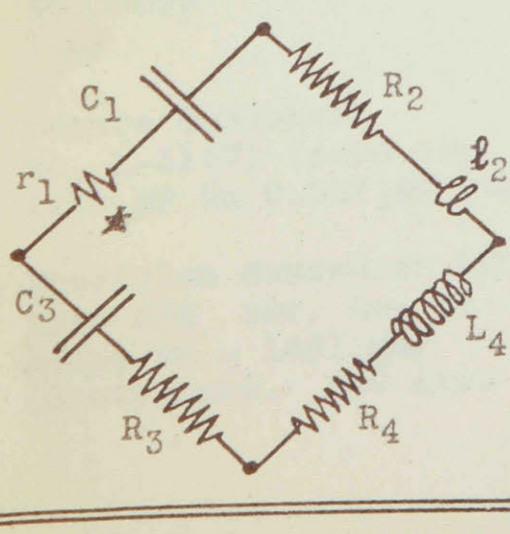


HEYDWEILLER

$$R_3\frac{M}{R_1(L-M)} - 1 \doteq \frac{R_4}{w^2(L-M)}\left[\frac{r_2}{M} - \frac{G_1}{c_1}\right] \ldots\ldots(H_1)'$$

$$\frac{1}{C_3}\cdot\frac{M}{R_1R_4} - 1 \doteq \frac{-(L-M)}{R_4}\left[\frac{r_2}{M} - \frac{G_1}{c_1}\right] \ldots\ldots(H_2)'$$

★ ($1/c_1$ small or c_1 large)



OWEN

$$\frac{1}{C_3}\cdot\frac{C_1R_2}{R_4} - 1 \doteq \frac{w^2L_4}{R_4}\left[\frac{\ell_2}{R_2} - \frac{C_1}{g_1}\right] \ldots\ldots(O_1)'$$

$$\frac{R_3C_1R_2}{L_4} - 1 \doteq \frac{-R_4}{L_4}\left[\frac{\ell_2}{R_2} - \frac{C_1}{g_1}\right] \ldots\ldots(O_2)'$$

★ ($1/g_1$ small or g_1 large)

APPENDIX II

Following is a list of the apparatus used for experimental work.

INSTRUMENT	MAKER
1. Standard Mutual Inductometer No. L-35175 (Campbell Patent) Range 0 - 11,105 μH	Cambridge Instrument Co., Ltd. England
2. Self Inductance (Fixed) Type A No. L- 32721 1 Henry at 800 c/s	Cambridge Instrument Co., Ltd. England
3. Self Inductance (Variable) No. 56529 Range 5.2 - 52 mH Continuous	Leeds & Northrup Co., Philadelphia, U.S.A.
4. Variable Inductor Type 107 Ser. No. 680 Range 1.75 - 53 μH	General Radio Co., Cambridge, Mass., U.S.A.
5. Mica Condenser Ser. No. 45229/1948 1.0 μF (Abs.) (True 0.999_6 μF) certificate	H.W. Sullivan Ltd., London
6. Mica Condenser Ser. No. 47471/1947 0.01 μF (Abs.) (True 0.00999_7 μF) certificate	H.W. Sullivan Ltd., London
7. Mica Condenser Ser. No. 47472/1947 0.001 μF (Abs.) (True 0.00100_3 μF) certificate	H.W. Sullivan Ltd., London
8. Plug Condenser No. 3663 1.11 μF in 0.001 μF steps	Nalder Bros. & Co., London
9. Standard Condenser (Mica) No. 8893 1 μF	H. Tinsley & Co., London, S.E.
10. Decade Condenser No. L-31571 (zero cap. of 24 $\mu\mu$F) 1.11 μF in 0.001 μF steps	Cambridge Instrument Co., Ltd., England
11. Precision Condenser 1500 $\mu\mu$F Type 222 Ser. No. 44 Range 48 - 1491 $\mu\mu$F (Continuous. 100 div. $\doteq$ 60 $\mu\mu$F)	General Radio Co., Cambridge, Mass., U.S.A.

APPENDIX II (concluded)

INSTRUMENT	MAKER
12. Dual Dial Non-Reactive Resistance Ser. No. 1146/1947 0.1% grade 111,110 ohms in 1.0 ohm steps or 11,111.0 ohms in 0.1 ohm steps or 1,111.10 ohms in 0.01 ohm steps or 111.110 ohms in 0.001 ohm steps	H.W. Sullivan Ltd., London
13. Non-Reactive Resistance Ser. No. 1138/1947 0.1% grade 11,111.10 ohms in 0.01 ohm steps	H.W. Sullivan Ltd., London
14. Decade Resistance Type A-25-N No. 140904 11,111.0 ohms in 0.1 ohm steps	Muirhead & Co. Ltd.
15. Decade Resistance (6 dial) Ser. No. 374332 11,111.10 ohms in 0.01 ohm steps	Leeds & Northrup Co., Philadelphia, U.S.A.
16. Audio Oscillator Model 200-I Range 5.9 - 6,300 c/s (Continuous)	Hewlett Packard California, U.S.A.
17. Audio Oscillator Mod. 79 D Ser. 2217A Range 25 - 15,000 c/s (Continuous)	The Clough-Brengle Co., Chicago, U.S.A.
18. 1000 c/s Vacuum-Tube Fork Type No. 723-A Ser. No. 171	General Radio Co., Cambridge, Mass., U.S.A.
19. Amplifier & Null Detector Type No. 1231-B Ser. No. 375	General Radio Co., Cambridge, Mass., U.S.A.
20. Wheatstone Bridge No. 294690 Dial and Ratio Arm range of 10^8 to 10^{-5} ohms	Leeds & Northrup Co., Philadelphia, U.S.A.

Auxiliary apparatus such as:- Sensitive galvanometer, battery, decade resistance boxes, decade capacitance boxes, shielding transformers, thermometer, plug-in unit capacitors, plug-in unit resistors.

Text references are shown as ( )

BIBLIOGRAPHY

1.	Hague, B.	Alternating Current Bridge Methods 4th edition 1938 ----- Pitman
2.	Sullivan Ltd.	Electrical Standards for Research and Industry. -- Catalogue 1946 H.W. Sullivan, Ltd., London, S.E.15
3.	General Radio Company	Catalogue "L" -- 1948 General Radio Company, Cambridge 39, Mass., U.S.A.
4.	Owen, David	Alternating Current Measurements 1937 edition -- Methuen & Co. Ltd., London.
5.	Steinmetz, C.P.	Alternating Current Phenomena 5th edition 1916 -- McGraw Hill
6.	Campbell, A. and Childs, E.C.	The Measurement of Inductance, Capacitance, and Frequency. 1935 edition --- Macmillan and Co., Ltd.

Text references are shown as ( )^{a}