Stochastic Mining Supply Chain Optimization: A Study of Integrated Capacity Decisions and Pushback Design Under Uncertainty

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April, 2016

A thesis submitted to McGill University as a partial fulfilment of the requirements of the degree of Master of Engineering

Acknowledgements

I would like to thank my soon-to-be wife Sarah, my parents, two brothers, Cags, and Kobi for their patience, love and encouragement. Their support allowed me to complete this work.

I would also like to thank my thesis advisor, Professor Roussos G. Dimitrakopoulos for his unwavering encouragement, insights and advice. I am very glad he convinced me to come back to academia, even though my stay was brief. There are few people capable of introducing new things to the world, he is one of them.

Thanks is also owed to my fellow students and COSMO's research staff. Thanks to Dr. Luis Montiel and Dr. Ilnur Minniakmetov for answering my many questions on resource modelling and geostatistics. A special thanks to Ryan Goodfellow whose work forms the foundation of this thesis. I hope some day I can pay my debt to him for all his help. I would also like to thank Deb for all she does for the Lab; I'm sure it's not easy keeping track of us all.

Last but not least, I would like to thank the organizations that funded this research: the Natural Sciences and Engineering Research Council of Canada (NSERC) Discovery Grant 239019; and the COSMO Mining Industry Consortium: AngloGold Ashanti, Barrick Gold, BHP Billiton, De Beers Canada, Kinross Gold, Newmont Mining Corp and Vale, whose generous contributions make all this work possible.

Contribution of Authors

I, the author of this thesis and first author of both papers comprising this thesis, am the main contributor to the development of the proposed methods and the execution of the case studies. This work has been done under the supervision and advice of my supervisor, Prof. Roussos Dimitrakopoulos, co-author of both papers. The paper presented in Chapter 2 has an additional co-author, Dr. Ryan Goodfellow. He acted as an adviser on the implementation of capacity optimization.

The Simultaneous Optimization of Long-Term Production Schedules and Integrated Capacity Decisions Under Uncertainty by Iain Farmer, Ryan Goodfellow, and Roussos Dimitrakopoulos in Chapter 2 is yet to be submitted to a journal.

An Algorithmic Approach to Schedule-based Pushback Design by Iain Farmer and Roussos Dimitrakopoulos in Chapter 3 has been submitted to the International Journal of Mining Reclamation and Environment.

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Abstract

The mining value chain comprises many inter-related components. When the individual components are optimized separately the value that can be generated from the enterprise suffers. Global optimization of the mining value chain requires a shift away from conventional methods of optimization and towards the simultaneous optimization of all related aspects, including: the mine's extraction sequence, material destination decisions, material transport decisions, and equipment capacities. Further, if these decisions are to be robust, they must be made while considering sources of uncertainty.

The contributions included in this thesis are meant to help industry practitioners plan and evaluate mining projects under uncertainty. Specifically, the simultaneous integration of capacity decisions in long-term scheduling is meant to provide a tool that generates a NPV-optimal mine sequence and destination policy that is also synchronized with equipment capacities selected while being robust to two sources of uncertainty. Further, a schedule-based method of pushback design is developed in an effort to construct pushbacks from an optimal mine sequence. The method is applied in order to preserve as much of the sequence's optimality as possible while generating an operational mine plan.

Resumé

La chaîne de valeur minière comprenne de nombreuses composantes interdépendantes. Lorsque les composants individuels sont optimisés séparément, la valeur qui peut être générée par l'entreprise souffre. L'optimisation globale de la chaîne de valeur minière nécessite de passer des méthodes classiques d'optimisation et d'optimiser simultanément tous les aspects connexes, notamment la séquence d'extraction de la mine, les décisions de destination du matérielle, les décisions de distribution des matériaux, et les capacités des équipements. De plus, si ces décisions doivent être robuste, elles doivent être prises en tenant compte des sources d'incertitude.

Les contributions incluses dans cette thèse sont destinées à aider les praticiens de l'industrie à planifier et à évaluer les projets miniers en condition d'incertitude. Plus précisément, l'intégration simultanée des décisions de capacité dans l'ordonnancement à long terme est censée fournir un outil qui génère une séquence de mines et une tactique de destination optimales pour la VPN, qui est également synchronisée avec les capacités sélectionnées tout en étant robuste à deux sources d'incertitude. En outre, une méthode de crée des pushbacks basée sur la séquence brute est développée. La méthode est appliquée afin de préserver autant que possible l'optimalité de la séquence tout en générant un plan de mine opérationnel.

Chapter 1

Introduction and Literature Review

1.1 Introduction

The mining supply chain is the route by which raw materials are extracted from within the earth and transformed into marketable products. A mining complex is a set of mines, processors, transport mechanisms and stockpiles that can be considered as a stand-alone unit within the broader supply network. The global mining supply chain is comprised of many such mining complexes that vary in their degree of complexity and inter-connectivity.

Typically, a mining complex is controlled by a managing entity with the goal of maximizing the net present value (NPV) of cash flows generated by the complex as a whole. Cash flows are estimated based on a model of the mine plan which is optimized under certain assumptions and subject to certain constraints. The existing "best-practice" approach to optimization within the industry has focused on optimizing each component of the mining complex on its own rather than considering the enterprise as a whole. This approach ignores the important inter-dependence of components within the mining complex. The traditional mathematical programming formulations also ignore capital investment and its relationship to operational capacities, instead they consider fixed capacities as static constraints for the optimization. Further, the traditional approach does not incorporate various sources of uncertainty prohibiting any notion of risk-resiliency in design considerations. These past efforts fall short at any attempt at global optimization and the outputs are not robust to the inevitable uncertainties that underlie every mining endeavour. When it comes time to decide whether or not to invest in a mining project any action based on a mine plan derived from traditional optimization will be misguided.

A better approach is to simultaneously optimize the mining complex as a whole while considering the most important sources of uncertainty. This method leads to a globally-optimal design for the entire value chain that is robust under uncertainty. Goodfellow and Dimitrakopoulos (2016) refers to the result of this global risk-based optimization as a Smart Mining Complex. The objective of the Smart Mining Complex is to make the best use of non-renewable mineral resources by generating an operational plan that is economically robust over the distribution of uncertain inputs.

This chapter reviews traditional mine planning and past developments that have lead to the current state of the art in stochastic mine planning. Section 1.2 goes over conventional mine planning methodologies and tools, Section 1.3 outlines developments in stochastic mine optimization, Section 1.4 highlights the current state of the art in mine planning, Section 1.5 explains uncertainty and how it is modeled within the context of mine planning, and Section 1.6 covers the contribution of this thesis. The focus of this thesis is on open pit planning; where applicable the ideas and conclusions herein can also be extended to underground mine planning.

1.2 Conventional Mine Planning

This section reviews conventional mine planning methods (Wooller 2007; Whittle 2004; Wharton 2004; Dagdelen 2001) which include the methods that are employed by most commercially available software. The section also highlights some important shortcomings of the methods and how these impact corporate decision making and project valuation. Note that this section (and the ones that follow) reference "mine planning", this concept can easily be extended to "mine valuation" since the objective is generally the optimization of NPV. For this reason, the two concepts are inherently linked.

A notable drawback of conventional mine planning is that it is carried out sequentially rather than simultaneously (Goodfellow 2014). Each disjoint step in the mine planning process involves its own individual optimization, greatly reducing computational demand compared to global optimization, but also compromising on global optimality.

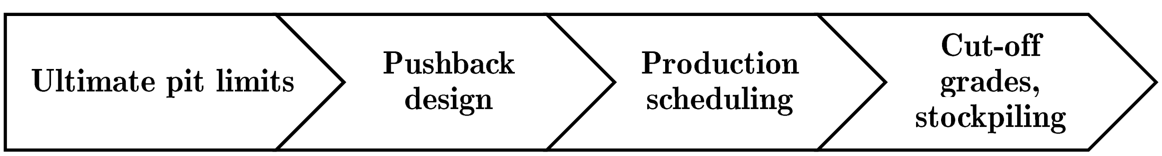

Figure 1: The steps followed in conventional mine planning

As shown in Figure 1, this sequential process generally starts out by defining the limits of the ultimate pit – the final volume that is estimated to generate the highest economic value. This step can be carried out using the maximum flow/minimum cut (Caccetta and Hill 2003; Meagher et al.

2009) or the Lerchs Grossman (Seymour 1995; Khalokakaie et al. 2000) algorithm. The next step is to generate pushbacks (or phases) which help to target higher-grade regions over the life of mine (LOM). Pushbacks are typically created by parametrizing the Maximum Flow or LG algorithms in order to create nested pits (or “pit shells”). These nested pits are grouped together to form a given number of pushbacks; mine scheduling is then carried out within this pre-determined phase design. Mathematical programming is used for the optimization of this mine scheduling step. Schedule value is maximized through sequencing metal, ore and waste extraction subject to operational targets and constraints (Dagdelen 2001; Underwood and Tolwinski 1998). An example of this type of conventional optimization of the mine schedule would see benches within pushbacks sequenced based on the averaged values of their constituent blocks; benches that contain blocks with higher values are preferentially extracted earlier in the mine life in an attempt to increase NPV. This step is accompanied by information loss as the values/attributes of blocks are averaged to generate a whole-bench value, which allows for sequencing proportions of benches. Allowing for linear decision variables (in this case proportions of benches instead of discrete blocks) represents another trade off between computational complexity and solution quality.

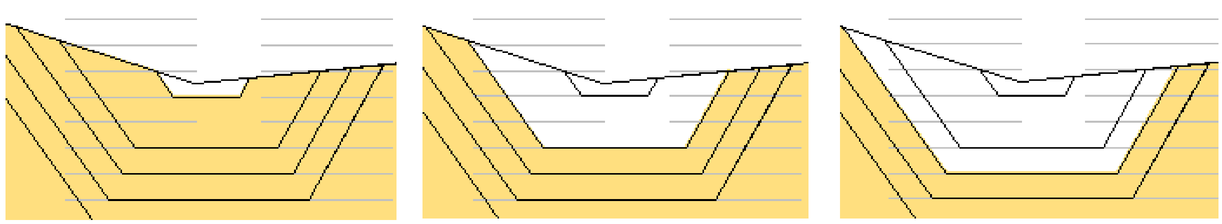

Figure 2: "Best Case" pitshell-by-pitshell conventional schedule

Newer works avoid some of these trade-offs by seeking to generate a block-wise mine schedule through mixed integer programming (MIP) (Gershon 1987; Bley et al. 2012).

As a final step in the conventional mine planning process, project economics can be improved by altering the cutoff grade policy as proposed by Lane (Lane 1964). Instead of considering these crucial destination decisions during schedule creation, these are left as an afterthought of the mine planning process. Goodfellow (2014) highlights the dependency between a mine’s cutoff grade policy and the destination policy for the various material types; stockpile use, blending, process capacities are all inter-related with the mine’s cutoff grade policy.

Attempts have been made to improve certain aspects of the conventional mine planning process. The widely-used Whittle™ software has included tools meant to incorporate blending and stockpiling, while optimizing cutoff grade and destination policies. However, even the most recent version of the software is still limited by the common pitfalls cited above. Blending, cutoff grades, and stockpiling variables are all optimized outside of the mine sequencing step which still relies on pitshell aggregation. Wooller (2007) provides an overview of the COMET scheduling tool. COMET simultaneously optimizes a mine's extraction schedule and cutoff grade/destination policy using a dynamic programming approach. Improvements of COMET over more sequential methods stem from its concurrent optimization strategy, however, the software is only able to consider pre-prescribed pushbacks, and it assumes attribute values at the block level. As such, COMET also suffers from many of the common conventional mine planning pitfalls.

In its current state, the latest-available commercial mine planning software is unable to jointly optimize all interrelated components within a mining complex, nor can it incorporate the various sources of uncertainty within the optimization. The following section focuses on the advances made in stochastic mine planning. First stochastic optimization is outlined, then, the inclusion of uncertainty into a global optimization framework is described. Finally, the modeling of geological uncertainty is explained.

1.3 Stochastic Mine Planning

1.3.1 Motivation for Stochastic Mine Planning

This section focuses on the incorporation of uncertainty within mine plan optimization. The most important source of uncertainty in mine planning and valuation is geological uncertainty (Vallee 2000; Douglas et al. 1994). Other sources of uncertainty are either Markov-like (e.g. commodity price uncertainty) or can be managed through various operational measures (e.g. capital or operating cost uncertainty).

Since the hard data used to build a geostatistical model is sparse in relation to the volume that needs to be estimated, the modelling of geological attributes is very difficult, thus ignoring uncertainty in any effort at quantification would be foolish. For instance, raw data usually comes in the form of diamond drillhole assays for which a "dense" exploration grid may only contain a single drill hole over an area of 2,500 square meters (a 50x50m pattern).

Conventional mine planning ignores uncertainty as it is based on a single estimated orebody model which contains the expected grade (or value) of each block in the model. The deterministic geological model is generated to be a locally-accurate predictor (Goovaerts 1997). It is created using weighted-average methods where samples (or "conditioning") data in the spatial vicinity of the node to be estimated are attributed a weight. The various interpolation methods differ in how these weights are calculated; some such methods include: nearest neighbour, inverse distance weighting, and kriging (Journel and Huijbregts 1978). Regardless of which technique is selected, all interpolation methods act as low-pass filters, smoothing the local spatial variation of the estimated attribute and failing to replicate connectivity of high grades which is an important drawback for mine planning. Statistical smoothing generated by traditional orebody modeling techniques results in a model that does not respect the univariate, multivariate and multivariate-spatial statistics of the data used to create it (Goovaerts 1997). This phenomenon is illustrated in the grade distributions and variograms presented in Figure 3 where the smoothing effect is readily visible. Further, as the term "interpolation" implies, node locations will never take on values outside the range of the conditioning data. This leads to what's called the "bullseye" effect where all extreme values are located at nodes containing hard data.

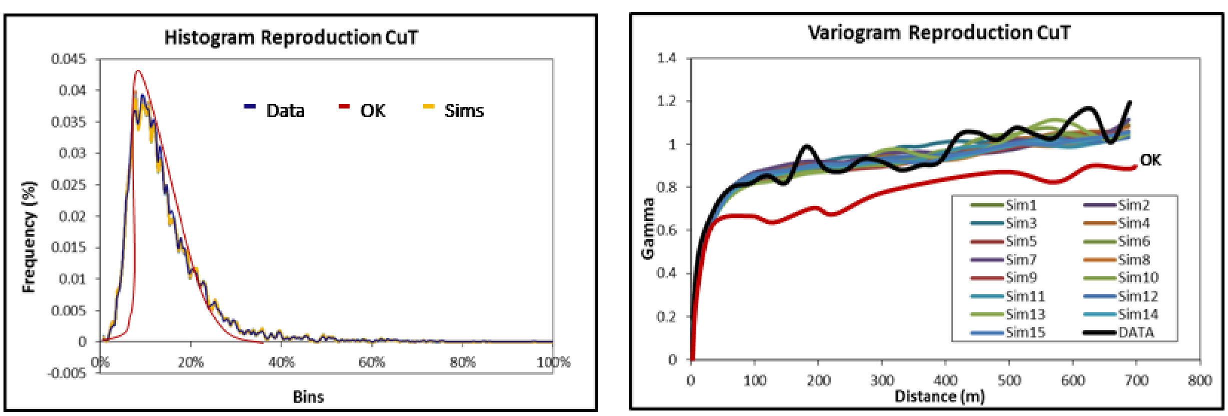

Figure 3: Grade histograms and variograms illustrating the smoothing effect

If a mine plan optimization is to contemplate uncertainty, then uncertainty must first be modeled. Unlike estimation techniques, each stochastic orebody simulation has the ability to reproduce the histogram and variograms of the raw data (Isaaks 1990). This is an important property, since in order to capture the model's uncertainty, a number of such equally-probable simulations (or "scenarios") must be considered jointly as a group. The ability of stochastic simulations to reproduce the hard data's statistics results in a model of the orebody that adequately represents the distribution of high and low-grade material – thus avoiding the smoothing effect seen in the conventional estimated model. This result has important implications for mine planning.

1.3.2 Simulations for Risk Analysis in Mine Plan Evaluation

Geostatistical simulations were used by Dimitrakopoulos et al. (2002) to quantify the risk of mine plans generated using traditional techniques. Simulated orebody models are input into a conventional mine plan generated using the estimated orebody model. The results show that the NPV predicted by the conventional schedule only has a 5% chance of being achieved, and that the expected NPV is 25% lower than anticipated. This shortcoming of the conventional approach is due to the non-linearity of the transfer function and the estimated model's inability to reproduce local variability as discussed above. Moving forward Ramazan and Dimitrakopoulos (2013) concludes that the mine planning process must consider multiple orebody simulations during sequence optimization in order to adequately account for uncertainty in connected high grade zones.

1.3.3 Steps Towards Risk-Based Mine Planning

In a first attempt at creating a LOM plan based on simulated orebody models Dimitrakopoulos et al. (2007), generated multiple conventional mine plans each based on a different simulated orebody model. Each of these schedules was then tested using the type of risk analysis described in Section 1.3.2 (above). The quality of the schedules was measured using different performance indicators and the schedule that maximized upside potential while minimizing downside risk was stated to be optimal. This method is straight forward and easily applied but it fails to directly incorporate uncertainty in the optimization process. Each schedule is only optimized for a single realization and is thus sub-optimal over the full range of possibility.

Another step in risk-based mine planning was taken when mixed integer programming (MIP) models were applied to probabilistic representations of the orebody. This is an attempt to incorporate geological uncertainty directly in the mine planning process. In the approach proposed by Dimitrakopoulos and Ramazan (2004) a number of schedules are created by optimizing the mine plans of multiple orebody simulations, then each block is assigned a probability distribution of being mined in a given period. Once the probability distributions are generated another optimization is run where the objective function seeks to maximize the project's probability-weighted NPV, where the weights are the probability of the block being extracted in the period selected. A drawback of this approach is the need to solve more than one MIP in order to obtain a final solution. Further, since the schedules generated in the first step are only optimal for the respective simulations used as inputs, the schedule obtained in the second step will not be optimal for the underlying uncertainty across all scenarios.

The same authors also employ another approach, this time instead focusing on reliably delivering the correct type of material (e.g. ore) to a process (Ramazan and Dimitrakopoulos 2004). Multiple simulations are again used to assess the probability that a given block has certain characteristics (e.g. is above a specified cutoff grade). In this way the MIP formulation used selects high value blocks that also have a high probability of being valuable. The concept of *risk discounting* is applied in order to defer riskier material – blocks with lower probability of having the desired property – to later in the mine life. In much the same way as a financial discount rate is applied to discount cash flows, a geological risk discount rate is applied within the optimization to push riskier material into the future at which time there could be more information available. Another contribution of the authors in this paper is the inclusion of a set of constraints meant to ensure "mineability" of the resulting schedule. Spatial connectivity of blocks mined in the same (or consecutive) periods is enforced through constraints that penalizes the objective function when the full set of blocks within a spatial template are not mined in the same period. This results in a schedule that is more efficient from an operational standpoint. The mine plan generated by this probability-based approach is shown to achieve targets more reliably while increasing value. This method can accommodate targets based on more than one attribute of interest and it acts to directly schedule blocks which has the advantage of eliminating the need to solve multiple MIPs. However, the method still fails to generate a solution that is optimal over all scenarios since it is not able to incorporate uncertainty of combinations of blocks, rather it only considers the blocks' individual probabilities.

Godoy and Dimitrakopoulos (2004) and Albor Consuegra and Dimitrakopoulos (2009) make an effort to incorporate joint uncertainty by combining individually-optimized schedules through the simulated annealing algorithm (Geman and Geman 1984; Kirkpatrick 1984; Cicirello 2007). Simulated annealing is a metaheuristic that strategically accepts or rejects perturbations of an initial solution with the goal of improving the objective function until convergence criteria is met. Godoy and Dimitrakopoulos (2004) and Albor Consuegra and Dimitrakopoulos (2009) use simulated annealing to swap blocks between periods in order to minimize deviations in planned ore and waste production. Both authors report improved NPV and a reduction in deviations, however their approach fails to incorporate the importance of material blending or direct maximization of NPV.

1.3.4 Stochastic Mine Planning with Stochastic Integer Programming

A stochastic integer programming (SIP) model Birge and Louveaux (2011) allows for multiple inputs (or "scenarios") to be used as a representation of uncertainty. These are two-stage optimization models where, in the first stage, scenario-independent variables are set to optimize a given objective and, in the second stage, scenario-dependant (or "recourse") variables are used to manage deviations from targets. This structure allows the optimizer to create a solution that simultaneously maximizes an objective while managing risk. The resulting solution has both high value and robustness to uncertainty.

In stochastic mine planning multiple orebody simulations are used as the set of scenarios input into the SIP. The group of scenarios represents geological uncertainty of the deposit. (Ramazan and Dimitrakopoulos 2013) use a SIP formulation for mine planning with a two-part objective that is (1) to maximize the project's NPV and (2) to minimize deviations from production targets. The concept of *risk discounting* is applied through the use of deviation penalties that decline through time by the application of a risk discount rate. There are a number of benefits to using this SIP approach in mine planning:

- The dual objective of maximizing NPV and minimizing deviation in production targets creates a high-value, risk-resilient solution.
- Joint local uncertainty is incorporated through the simultaneous use of multiple orebody scenarios.
- The multi-variate spatial statistics of the underlying data and connectivity of high grades are respected through the use of conditional simulation.
- Uncertainty can be pushed to later periods through the use of risk discounting.

Although the application of SIP formulations of the type proposed by Ramazan and Dimitrakopoulos (2013); Dimitrakopoulos and Ramazan (2013); Leite and Dimitrakopoulos (2007); Benndorf and Dimitrakopoulos (2013) address many of the shortcomings past optimization techniques have experienced, there are still certain limitations to the approach:

- Including multiple scenarios makes the optimization problem very large and difficult to solve using commercial optimizers.
- A certificate of optimality can only be provided for linear convex formulations. The inclusion of stockpiles and grade-recovery functions make the problem non-linear. Objective function convexity may also become an issue if more complex supply chain relationships are included.
- Value is attributed at the block scale meaning down-stream blending cannot be considered properly.

The first two limitations listed above can be dealt with through the use of metaheuristic solution methods, and the third limitation is discussed below in Section 1.4 which deals with Global Optimization. The metaheuristic approach to optimization is to strategically search a solution space that is not amenable to deterministic optimization techniques, or too large to search exhaustively (Blum and Roli 2003). Metaheuristic techniques employ methods of progressively improving a solution through perturbations to the solution vector (the set of decision variables). Depending on the type of metaheuristic used, local or global perturbations are made.

Metaheuristic optimization has been used to solve SIP mine planning formulations with a great deal of success. Although the method does not provide a guarantee of solution optimality, it provides high quality solutions in a reasonable amount of time. Lamghari et al. (2014); Lamghari and Dimitrakopoulos (2012) propose a Tabu search metaheuristic and a variable neighbourhood descent (VND) to solve a mine planning problem which uses a SIP formulation that incorporates geological uncertainty. Using Tabu search over a range of problem sizes, the authors show that the solution generated by the metaheuristic is consistently only 2-3% from the true optimal value with a solution time of less than 1% of the conventional deterministic optimizer. The authors show similar results when applying the VND metaheuristic. The simulated annealing metaheuristic has been used most prevalently in the field of stochastic mine planning and it has also been shown to produce very good results (Goodfellow and Dimitrakopoulos 2013; Montiel and Dimitrakopoulos 2013; Albor Consuegra and Dimitrakopoulos 2009; Godoy and Dimitrakopoulos 2004).

1.4 Developments in Stochastic Mine Planning – Global optimization under uncertainty

Global optimization in the mine planning context encompasses the simultaneous optimization of all components of the mineral value chain (Zhang and Dimitrakopoulos 2014; Goodfellow 2014; Montiel and Dimitrakopoulos 2015). A mineral value chain, or "mining complex", is comprised of the various components required to transform in-situ resources into cash flows. These include: the set of mines discretized into selective mining units (SMUs, or blocks), these blocks contain attributes of interest that flow through the value chain (e.g. metal quantity), the set of destinations that can be differentiated by their ability to transform and/or store materials, and the set of customers that will decide on the value to attribute to the final product. The optimization of a mining complex requires decisions involving: the block extraction sequence, material destination policy, transport options, quantity of material movement, and the allocation of products to customers. In order to properly account for the important non-linear interactions that occur along the mining supply chain, calculation of value must be based on the products sold rather than at the individual block level – this is one of the major contributions of global optimization. As soon as blocks are mined the material is blended and transformed and any concept of block-based value is lost.

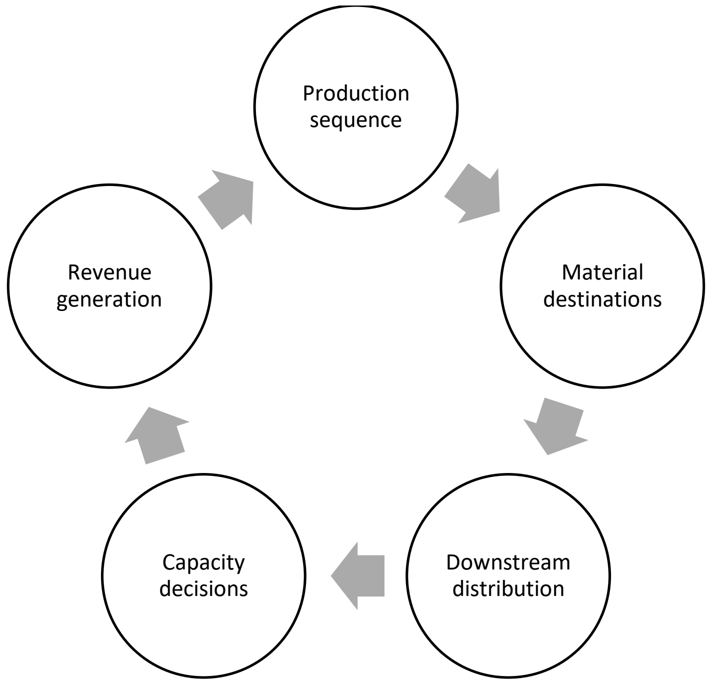

Figure 4: Global optimization approach

Acknowledging the above points makes it clear that, in order to generate a truly optimal plan for an entire mining complex the optimization must consider: (1) the material extraction sequence (when to extract blocks), (2) material destination policies (where to send the material), and (3) quantity of material movement between destinations (how much material to send). These fundamental decisions must account for time-value of money, material blending, and the value of final products. Goodfellow and Dimitrakopoulos (2016) and Montiel and Dimitrakopoulos (2015) both introduce global mine plan optimization models that can accommodate these integrated decisions.

Goodfellow and Dimitrakopoulos (2016) develop a mathematical programming model for the stochastic global optimization of mining complexes. The model employs an SIP formulation and is solved using a variety of metaheuristic methods. In order to be able to attribute value at the product-level rather than the block-level the authors develop an additive framework whereby attribute values can be propagated along the supply chain. In this way material is no longer seen as belonging to discrete blocks, but instead attributes are blended as they are processed; a better reflection of reality. In order to create robust, implementable destination decisions, a multi-attribute material destination policy is created which can be optimized under uncertainty. To do this the authors use the k-means++ clustering algorithm (Arthur and Vassilvitskii 2007) to group materials with similar characteristics. This procedure effectively allows for dynamic cutoff grade and material-type policy decisions to be made by the optimizer. The model proposed by the authors is able to account for non-linear supply chain interactions (e.g. grade-recovery relationships) through its ability to calculate attribute-based values at various destinations along the path to value creation. This capability unlocks the power of material blending.

Another application of global optimization developed by Montiel and Dimitrakopoulos (2015) can also accommodate important non-linear interactions while taking advantage of material blending. In this case the authors implement the ability for the optimizer to choose between two different processing modes. A higher throughput mode which can process more tonnage at a reduced recovery, and a lower throughput mode which processes less material with higher recovery. This model is able to create value through a better alignment between the production schedule and process capacity decisions. Notably, the throughput-recovery relationship is non-linear and dependant on the material characteristics being input each period. For this reason, it is necessary for the authors to move away from block-based attribute values and towards modelling blended attribute values at each point in the supply chain. Montiel and Dimitrakopoulos (2015) use the simulated annealing metaheuristic in order to solve the model with perturbations to three distinct neighbourhoods.

1.4.1 Integrating Capacity Decisions and Capital Costs in Optimization

Capacity decisions represent a trade-off between the amount of capital outlay required and the operational constraints of the project. The most common approach to mine design is to set mining and processing capacities based on experience and comparisons with existing projects with similar characteristics. This approach is not necessarily ideal since it does not tailor the investment decisions to the specific traits of the project at hand.

Only recently have there been attempts to incorporate capacity optimization in the stochastic mine planning framework. Elkington and Durham (2011) use the concept of "buying" incremental processing capacity within a MIP optimization based on pushback selection. A linear relationship is assumed between incremental capacity and cost. The formulation attempts to maximize NPV through the selection of intermediate pushbacks based on the scheduling of mining panels with deterministic geology. As noted above, Montiel and Dimitrakopoulos (2015) allowed the optimizer to alter the processing capacity between different operating modes whereby process throughput and recovery can change between two pre-set levels. This approach enabled better alignment between the mine schedule and the processor's capabilities under geological uncertainty, but it does not incorporate capital investment decisions. Goodfellow and Dimitrakopoulos (2016) implement dynamic mining capacities by allowing the optimizer the ability to purchase loading and hauling equipment. To do this the authors need to establish an additive attribute that can be used to model equipment cycle times and capacity requirements. For this, the concept of mill, truck, and shovel hours is developed. The optimizer is able to purchase trucks and shovels based on the number of total hours required which depend on cycle times which fluctuate during the operation and are determined by material type under geological uncertainty. NPV is optimized while penalizing deviations from mill targets and large fluctuations in fleet size. This model makes very good progress in capacity optimization but it falls short of integrating processing capacity.

The drawback to prior attempts to include capacity optimization in stochastic mine planning is that they approach the optimization problem from a pre-constrained starting point – either the processing or the mining capacity has already been fixed to some extent. This thesis aims to remedy this problem.

1.5 Stochastic Pushback Optimization

Once a mining schedule is developed mine planners group the sequence to form pushbacks (or "phases") which allows for the implementation of "mining widths" and equipment movement. Pushbacks optimization is typically performed semi-manually based on pitshell aggregation (Whittle 2004; Wharton 1997). Consuegra and Dimitrakopoulos (2010) perform a risk analysis based on the number and size of pushabacks under geological uncertainty. Nested pits are grouped and different combinations are evaluated. Meagher et al. (2009) optimizes a mine's pushback design under geological and market uncertainty using a parametrized version of the maximum flow algorithm. Asad et al. (2014) add to this by limiting variability in the relative sizes of the set of pushbacks using knapsack-type constraints are used to regulate the quantity of ore contained in each pushback. Goodfellow and Dimitrakopoulos (2013) also optimizes a pushback design under geological uncertainty while also including the ability to account for multiple material types, multiple elements, and multiple destinations (joint uncertainty). The authors employ an algorithmic approach that minimizes the deviation from a target tonnage in each pushback over the range of geological simulations. This work manages to create a pushback design that is robust under joint geological uncertainty, that can incorporate destination policy decisions, however the formulation does not directly create a mine schedule that maximizes NPV.

Stone et al. (2004) propose a method that generates a pushback design based on an optimized LOM schedule. The benefit of this method is that the schedule can be optimized by any formulation. Regardless of how the schedule is optimized, it is used as the input to create a series of pushbacks. This reverse in approach attempts to conserve as much of the qualities of the raw optimal schedule as possible within the pushback design. The authors propose a grouping based on temporal and spatial similarity of the scheduled blocks. Nearby blocks extracted in consecutive periods are clustered using the fuzzy c-means algorithm (Bezdek et al. 1984). After clustering, slope constraints are algorithmically imposed to generate pushbacks. The method is performed iteratively, and the resulting designs are evaluated until a final "best" design is chosen. The drawback of this method is that, although it creates a practical schedule, the effect of clustering blocks and algorithmically designing pushbacks means that the there is no guarantee of optimality based on the raw input schedule.

1.6 Modeling Uncertainty

In order to incorporate uncertainty in mine planning it must first be modeled. In this section two major sources of uncertainty are reviewed – geological and market price uncertainty.

1.6.1 Modeling Geological Uncertainty

Geological uncertainty is modeled through the use of multiple Monte-Carlo orebody simulations. The orebody is modeled as a spatial random field and each simulation is based on the random sampling of a conditional probability distribution. The most common approach to geostatistical simulation is the multi-Gaussian approach. This simulation method makes use of the assumption that the distribution of metal grades in the deposit follows a multivariate Gaussian distribution. The Gaussian assumption allows for the decomposition of a multivariate PDF into a series of conditional distributions. The conditional mean and variance of the distribution at each node is determined using kriging techniques (Goovaerts 1997; Journel and Huijbregts 1978; Krige 1976).

Two-point spatial correlations are respected by the use of the variogram in weighting the conditioning data.

$$\gamma(\boldsymbol{h}) = \frac{1}{2N(\boldsymbol{h})} \sum_{\alpha=1}^{N(\boldsymbol{h})} [z(\boldsymbol{u}_\alpha + \boldsymbol{h}) - z(\boldsymbol{u}_\alpha)]^2$$

Isaaks (1990) introduces the concept of Sequential Gaussian Simulation (SGS) which employs a different random path through the conditioning data for each simulation. As nodes are simulated, they are added to the set of conditioning data in order to conserve the underlying spatial correlation. This method makes use of the multivariate Gaussian distribution's ability to be decomposed into a product of conditional distributions.

$$Y(\boldsymbol{u}') = \sum_{a=1}^{n} \lambda_a Y(\boldsymbol{u}_a)$$

Dimitrakopoulos and Luo (2004) demonstrate the equivalence between SGS and the LU-decomposition simulation method which is slow when the covariance matrix is large because of the need for a memory-intensive matrix inversion. The authors propose the Generalized Sequential Gaussian Simulation method (GSGS) which sequentially simulates groups of nodes simultaneously. This method takes advantage of the benefits of both LU and SGS techniques. The Direct Block Simulation method is proposed by Dimitrakopoulos et al. (2002) which simulates at the block support and reduces memory requirements. It also does a better job at modeling spatial continuity, and at reproducing extreme values compared to GSGS. Desbarats and Dimitrakopoulos (2000) use min-max auto-correlation factors (Switzer and Green 1984) in order to simulate correlated pore size distributions. This simulation approach is also extended to mining by Boucher and Dimitrakopoulos (2009) in order to directly simulate correlated metal grades at the block scale.

1.6.2 Advances in Geostatistical Simulation

The geostatistical simulation methods described above all model spatial correlation using the (cross) variogram, a two-point measure of spatial variability. Strebelle (2002), Zhang et al. (2006) and Deutsch and Journel (1998) show that two-point statistics are not sufficient if more complex spatial structures are to be modelled. For this reason, a number of multiple-point simulation methods (MPS) have been proposed. These methods can be classified into two groups: (1) pattern-based, and (2) data driven. Pattern-based methods rely on the use of a training image (TI) in order to build a "library" of multiple point statistics that can be drawn upon during simulation. Algorithms of this type include: ENESIM (Guardiano and Srivastava 1993), SNESIM (Strebelle 2002), Direct Sampling (Mariethoz et al. 2010), and IMPALA (Mariethoz et al. 2010). These pattern-based methods all depend on the use of a training image to generate the multi-point statistic database. The training image is often developed based on geological interpretation and can be an arbitrary, subjective representation. This is a drawback of the pattern-based approach. The second technique is based on a data-driven approach. Unlike the pattern-based method, this approach does not completely rely on a training image. It instead attempts to model higher-order spatial cummulants based on the conditioning data. The TI is only used to supplement when there is data missing from the template. These methods employ different methods of describing the high-order spatial statistics of the data including: Legendre polynomials, spectral decomposition, computer vision and machine learning techniques among others (Dimitrakopoulos et al. 2010; Minniakhmetov and Pergament 2013; Minniakhmetov and Pergament 2012; Mu et al. 2015).

1.6.3 Metal Price Uncertainty

The price at which products can be sold from a mining operation has a very important impact on the project's value and the project's optimal design. For this reason, commodity price uncertainty is a desirable property to incorporate into the mine planning/evaluation process. Unlike geological uncertainty, commodity prices fluctuate in the temporal domain; in order to adequately quantify geological uncertainty Albor Consuegra and Dimitrakopoulos (2009) shows that only 15-20 simulations suffice (largely due to the volume-variance relationship), whereas in order to quantify metal price uncertainty significantly more simulations are needed (can be in the order of 100-1000) (Briggs et al. 2012). This requirement means that incorporating commodity price uncertainty in a stochastic optimization using the same approach as described for geological uncertainty would make the problem too big to solve.

Commodity price dynamics are governed by different models depending on the commodity in question. The prices of fungible commodities that are consumed in industrial (or other) processes are generally governed the laws of supply and demand. The prices of non-fungible industrial commodities are also governed by supply and demand but price elasticity is dependent on product quality. The price of commodities which are purchased for investment purposes are largely dependant on global interest rates and inflation. The differences in these commodity price drivers necessitates commodity-specific stochastic models.

Schwartz (1997) proposes three similar stochastic models of varying complexity: a one-factor Geometric Brownian Motion (GBM) model, a two-factor Mean-Reverting (MR) model that accounts for convenience yield, and a third model that incorporates stochastic interest rates. The authors determine that there is strong mean reversion in commercial commodity prices.

Deng (2000) proposes a MR model that incorporates market shocks modeled using the Poisson distribution. Random sampling of the Poisson distribution produces spikes in commodity price which are afterwards diffused by the MR process. This type of model is also referred to "MR with Poisson Jump Diffusion". The price model was used by Castillo and Dimitrakopoulos (2014) to incorporate price uncertainty into ultimate pit decisions for a copper project.

Bernard et al. (2008) test three models including Generalized Auto-Regressive Conditional Heteroscedasticity (GARCH) Process. The authors note that, depending on the time scale, different econometric components of the models dominate. The GARCH model accounts for autocorrelation of residuals which does a better job at modeling the change in variability (time-nonstationary) as the underlying price fluctuates.

The commodity price models described above fall into the category of "reduced-form" stochastic models; parsimonious representations of uncertainty. Pirrong (2011) develops a *structural* model of commodity price dynamics based on underlying economic theory. In this case the authors explicitly model supply, demand, and shocks to both. Although his approach builds off a more strict theoretical base, parameterizing the model is difficult. For this reason the reduced-form models continue to dominate due to their simplicity.

A review of the relevant literature shows that the MR GARCH Poisson Diffusion model works well for most industrial metals and commodities while the GBR-with-trend model works better for precious metals due to their positive correlation with inflation (Sick and Cassano 2012; Shafiee and Topal 2010; Schwartz 1997; Labys et al. 1998; Heal and Barrow 1980; Dooley and Lenihan 2005).

1.7 Thesis Outline

The remaining chapters of this thesis are organized as follows:

- Chapter 2 contains the first paper belonging to the thesis: *The simultaneous optimization of long-term production schedules and integrated capacity decisions under uncertainty*. An application at a large copper-gold mine is provided.
- Chapter 3 contains the second paper belonging to this thesis: *An algorithmic approach to schedule-based pushback design.* Two applications, one at a gold mine, and another at a REE project, are provided.
- Chapter 4 contains overview of contributions for each paper followed by conclusions and a review of future work.

The thesis terminates with appendices outlining various algorithms, tools, and notes relevant to the contained work but too detailed to be included in the main body of text.

1.8 Thesis Objectives and Contributions

The principal objective of this thesis is to advance stochastic mine planning methods for practical use within the mineral industry. The contributions included herein are meant to help industry practitioners plan and evaluate mining projects under uncertainty. Specifically, the simultaneous integration of capacity decisions in long-term scheduling is meant to provide a tool that generates a NPV-optimal mine sequence and destination policy, that is also synchronized with selected capacities, while being robust to two sources of uncertainty. Further, a schedule-based method of pushback design is developed in an effort to preserve as much of the sequence's optimality as possible while generating an operational mine plan.

These two contributions are applied to specific case studies. Both methods can easily be extended to other open pit mining operations.

Chapter 2

The Simultaneous Optimization of Long-Term Production Schedules and Integrated Capacity Decisions Under Uncertainty

2.1 Introduction

The mining supply chain is the route by which raw materials are extracted from within the earth and transformed into marketable products. A mining complex is a set of mines, processors, transport mechanisms and stockpiles that can be considered as a stand-alone unit within the broader supply network. The global mining supply chain is comprised of many such mining complexes that vary in their degree of complexity and inter-connectivity.

Typically, a mining complex is controlled by a managing entity with the goal of maximizing the net present value (NPV) of cash flows generated by the complex as a whole. Cash flows are estimated based on a model of the mine plan which is optimized under certain assumptions and subject to certain constraints. The conventional approach to optimization within the industry has focused on optimizing each component of the mining complex on its own (sequentially) rather than considering the enterprise as a whole (simultaneously). This approach ignores the important inter-dependence of components within the mining complex. The traditional approach also does not incorporate various sources of uncertainty prohibiting any notion of risk-resiliency in design considerations. Further, conventional mine planning ignores capital investments and their relationships to operational capacities, instead they consider fixed capacities as static constraints for the optimization.

This chapter builds off recent work in Stochastic Mine Planning which focuses on the simultaneous global optimization of mining complexes under uncertainty (Goodfellow and Dimitrakopoulos 2016; Montiel and Dimitrakopoulos 2015).

Within the mining complex framework, the task of creating an optimal mine plan involves determining the following: a block extraction sequence for each mine, a destination policy dependent on material type and attribute values, and the quantity of material flow between destinations. Notably, these decisions simultaneously contemplate material blending, stockpiling, and time value of money. In the stochastic context these decisions must also contemplate sources of uncertainty. If the optimization is extended to consider capital expenditure decisions, these also must be incorporated within the model along with the corresponding adjustments to the relevant constrains.

Goodfellow (2014) proposes an integrated model of the mining complex that is able to consider a unified optimization of the entire supply chain while including capacity decisions. Unlike conventional models, value is calculated based on products sold and not at the block level. This unlocks the power of material blending and allows for complex revenue calculations to be considered.

The model described by (Goodfellow) is extended in this paper to make three main contributions:

1. The integrated optimization of mining and processing capacities in the pre-production stage under geological uncertainty;
2. The incorporation of metal price uncertainty in optimization of second-stage material movement decisions;
3. The inclusion of various financial contracts in product-based revenue calculations.

2.2 Model and Formulation

The model is formulated as a two-stage mixed integer stochastic optimization (Birge and Louveaux 2011); where the first-stage variables are scenario-independent and the second-stage (recourse) variables are scenario dependant. The idea is to create a globally-optimal plan for the entire mineral value chain that will remain robust under uncertainty. The first-stage variables determine the decisions made in an uncertain environment, these decisions must remain robust over the distribution of possible outcomes. The values of the second-stage variables are determined after uncertainty has be unveiled, and the optimal value is chosen based on the revealed information.

One of the important contributions of (Goodfellow) and Montiel and Dimitrakopoulos (2015) was the method by which material flow is modeled within the mining complex. The formulation moves away from calculating attributes at the block level and instead tracks attributes as they move through the value chain. As materials move from one destination to another, blending and other interactions can easily be accounted for at a level of detail that far exceeds what was accomplished in prior works. This ability allows for economic values, and other characteristics of interest, to be calculated where they are realized instead of assigning each block values before materials are extracted, stockpiled, blended, transported, processed and sold.

2.2.1 Summary of Model Components

As described by Goodfellow (2014), a mining complex is comprised of a set of mines $m \in \mathbb{M}$, and processing destinations $\mathcal{P}$. The mines are discretized into blocks $b \in \mathbb{B}_m$ which are characterized by their material type and attribute values $\beta_{p,b,s}$, where $p \in \mathbb{P}$ refers to primary attributes and $s \in \mathbb{S}$ is one of many scenarios used to represent uncertainty.

Material flow is modeled using a graph structure with nodes $\mathcal{N}$ (sources/destinations in the mining complex) connected by arcs representing allowable incoming-outgoing ($I(i) \subseteq \mathcal{N}$, $O(i) \subseteq \mathcal{S} \cup \mathcal{P}$) pairs.

Each material type is subdivided into a number of different clusters $\mathcal{C}$, based on the values of the block's multiple attributes. For example, sulphide material, containing copper and gold, can be subdivided into the following four clusters: high-copper & high-gold, high-copper & low-gold, low-copper & high gold, and low-copper & low gold. Clustering is not limited to two attributes as in the example, but can extend to a number of attributes of interest. Material types are clustered using the k-means++ algorithm as a preprocessing step (Arthur and Vassilvitskii 2007). The Euclidian distance is used as the similarity metric and the number of clusters to use is a modelling decision. Clustering sets the value of $\Theta_{b,c,s} \in \{0,1\}$ which determines which cluster $c \in \mathcal{C}$ a block belongs to. These clusters are used within the optimization to determine material destinations policies.

The optimization model has four types of decision variables. The first-stage variable $x_{b,t} \in \{0,1\}$ is the mine sequencing variable that determines the period $t \in \mathbb{T}$ in which block $b \in \mathbb{B}_m$ is extracted. After a block is extracted a choice must be made regarding which destination it is sent to. Much like grade binning discussed in (Lane 1964; Wooller 2007), the pre-processing step of material clustering allows for robust destination policies to be established. Destination policy decisions are determined by the first-stage variable $z_{c,j,t} \in \{0,1\}$ which establishes where to send blocks belonging to each cluster in a given period. The second-stage variable $y_{i,j,t,s} \in [0,1]$ determines proportion of the tonnage held at location $i \in \mathcal{S} \cup \mathcal{P}$ is that is sent to location $j \in \mathcal{S} \cup \mathcal{P}$ period $t \in \mathbb{T}$ and under scenario $s \in \mathbb{S}$.

The variable $v_{h,t,s} \in \mathbb{R}$ is used to account for the quantity/value of hereditary attributes, derived as a function of primary attributes $v_{p,i,t,s} \in \mathbb{R}$ at locations within the mining complex. Notably, primary attributes must be additive (e.g., tonnage) to sum attributes over destinations in order to calculate the non-additive hereditary attributes (e.g., metal grade) as material moves through the value chain.

2.2.2 Integrating Capital Expenditure and Capacity Decisions in Long-Term Planning Under Uncertainty

Capacity decisions represent a trade-off between the amount of capital outlay required and the operational constraints of the project. The most common approach to mine design is to set mining and processing capacities based on experience and comparisons with existing projects with similar characteristics. This approach is not necessarily ideal since it does not tailor the investment decisions to the specific traits of the project at hand.

Only recently have there been attempts to incorporate capacity optimization in the stochastic mine planning framework. Montiel and Dimitrakopoulos (2015) allow the optimizer to alter the processing capacity between different operating modes whereby process throughput and recovery can change between two pre-set levels. This approach enables better alignment between the mine schedule and the processor's capabilities under geological uncertainty, but it does not incorporate capital investment decisions. Goodfellow and Dimitrakopoulos (2016) implement dynamic mining capacities by allowing the optimizer the ability to purchase loading and hauling equipment but the authors fall short of integrating processing capacity.

The drawback to these prior attempts at including capacity optimization in stochastic mine planning is that they approach the optimization problem from a pre-constrained starting point – either the processing or the mining capacity has already been fixed to some extent. Mining and processing capacities are inherently interrelated so it only makes sense to optimize them simultaneously; this is especially true for an operation that is in the planning stage while there is still the flexibility to make these important design choices. Setting mining capacity determines the maximum quantity of material (ore and waste) that can be extracted from the mines considered in the complex. The capital outlay required in order to produce at the desired mining capacity is generally related to the purchase of equipment such as trucks, shovels, loaders, etc. The amount of ore that can be mined under the mining capacity constraint will have an effect on the optimal processing capacity (and vice versa). Determining the processing capacity involves establishing the optimal trade-off between capital outlay and the size for each the (possibly many) processing streams. One of the contributions of this work is to allow the optimizer to decide the optimal mining and processing capacities simultaneously as the LOM schedule is created.

In order to do this both the cost, and the incremental increase contributed by the capacity decision must be considered in the objective function and constraints of the model. To accomplish this an additional variable is added to integrate capacity decisions within the optimization. The first-stage variable $w_{k,t} \in \{L_{k,t,}U_{k,t}\}$ establishes the amount of extra capacity gained from capital expenditure option $k \in \mathbb{K}$ which must be purchased at a price, $p_{k,t}$. The objective function can then be written as follows to account for the purchase of extra capacity. Notably this formulation assumes a linear relationship between incremental capacity and incremental cost, however $p_{k,t}$ can be made to be a function of the number of capacity increases in order to account for economies of scale.

Objective Function:

$$max\left\{\underbrace{\frac{1}{|\mathbb{S}|}\sum_{s\in\mathbb{S}}\sum_{t\in\mathbb{T}}\sum_{h\in\mathbb{H}} p_{h,t}\cdot v_{h,t,s}}_{\text{Expected NPV}} - \underbrace{\sum_{t\in\mathbb{T}}\sum_{k\in\mathbb{K}} p_{k,t}\cdot w_{k,t}}_{\text{Capital Expenditure}} - \underbrace{\frac{1}{|\mathbb{S}|}\sum_{s\in\mathbb{S}}\sum_{t\in\mathbb{T}}\sum_{h\subseteq\mathbb{H}} c^{+}_{h,t}\cdot d^{+}_{h,t,s} + c^{-}_{h,t}\cdot d^{-}_{h,t,s}}_{\text{Deviation Penalties}}\right\} \tag{2.1}$$

The traditional goal of mine complex optimization is to maximize NPV (identified by the first term in the above objective function. However, blindly maximizing a project's NPV without regard for uncertainty in the estimates is not ideal – there is little point in modelling risk without also managing risk. The third term is used to manage risk through the use of penalties that are assigned when deviations from targets occur. In this way the objective function searches for a solution that has both high expected NPV and low risk of failing to meet targets. This gives the stochastic solution its robustness under uncertainty.

Adjustments to the model's constraints are required to allow the available capacity to grow after a capacity decision is made. *Hereditary attribute constraints* provide the mechanism by which deviations from upper $U_{h,t}$ and lower $L_{h,t}$ target ranges are calculated. When capacity optimization is included in the model the target ranges can be adjusted by the increment of each capacity option $k \in \mathbb{K}$.

$$v_{h,t,s} - d^{+}_{h,t,s} \leq U_{h,t} + \sum_{t'=t-\lambda_k+\tau_k}^{t} \kappa_{k,h}\cdot w_{k,t'} \quad \forall h \in \mathbb{H}, t \in \mathbb{T}, s \in \mathbb{S} \tag{2.2}$$

$$v_{h,t,s} + d^{-}_{h,t,s} \geq L_{h,t} + \sum_{t'=t-\lambda_k+\tau_k}^{t} \kappa_{k,h}\cdot w_{k,t'} \quad \forall h \in \mathbb{H}, t \in \mathbb{T}, s \in \mathbb{S} \tag{2.3}$$

Capital expenditure constraints ensure that a capital expenditure option is exercised only once for one-time options, and between the lower $L_{k,t}$ and upper $U_{k,t}$ limits for multiple-purchase options.

$$\sum_{t\in\mathbb{T}} w_{k,t} \leq 1 \quad \forall k \in \mathbb{K}^1 \subseteq \mathbb{K} \tag{2.4}$$

$$L_{k,t} \leq w_{k,t} \leq U_{k,t} \quad \forall k \in \mathbb{K}, t \in \mathbb{T} \tag{2.5}$$

By including the above constraints, the model is able to handle multiple capital expenditure options. These decisions are considered simultaneously with the other mine planning variables in the optimization. The desired outcome of the methods proposed in this paper is an integrated mine plan that includes: a block extraction sequence, a material destination policy, a material flow plan, and an optimal capital allocation, all optimized in sync with each other while being robust under uncertainty as represented by the set of scenarios $s \in \mathbb{S}$. Ideally such an optimization would be carried out at the feasibility, or detailed engineering, stage of a project when there is flexibility to establish the mine's design before construction begins.

2.2.3 Solution Methods

The formulation of the mathematical optimization model used herein is structured as a SIP (Goodfellow 2014). Given the mine complex optimization model allows for non-linear attribute interactions, metaheuristic methods are the necessary tools used in order to obtain a solution. Metaheuristics do not guarantee convergence to mathematical optimality, but instead they are powerful tools that can generate good-quality solutions in an acceptable amount of time. For the purpose of mine complex optimization this is acceptable since many factors and input parameters are destined to change over the course of the long-term plan making the search for mathematical optimality impractical.

This paper employs the simulated annealing metaheuristic (Kirkpatrick 1984) with a number of various perturbation mechanisms that helps the algorithm explore the solution space as thoroughly as possible. Capacity optimization makes the solution more difficult since capacity decisions have a significant effect on the remainder of other decision variables. For this reason, including capacity decisions makes it necessary to consider measures in order to ensure the solution does not get trapped in local optima (Cicirello 2007).

Any further detailed discussion on metaheuristics and mathematical optimization methods inherent to the solution methods applied to the optimization performed are outside the scope of this work.

For a comprehensive discussion on metaheuristics and solution methods applied to mine complex optimization the reader is referred to: Lamghari and Dimitrakopoulos (2016); Blum and Roli (2003); Goodfellow (2014); Dimitrakopoulos and Montiel (2013); Caccetta and Hill (2003).

2.3 Project Financing's Effect on Optimization

Another contribution made in this work is the inclusion of flexibility in revenue calculation. In many instances a mining company is forced to raise external funds to build their project, this is especially true for "junior" or "mid-tier" companies that do not have the cash reserves for a large capital outlay. These funds are invariably accompanied by additional encumbrances impacting the project's revenue stream. In terms of the mathematical model used in this paper, the value of attributes $p_{h,t}$, may not be constant.

The mining industry's inability to manage uncertainty has been a major contributor in preventing it from generating attractive risk-adjusted rates of return (McClain et al. 1996; Ball and Brown 1980) (Tufano 1998). This has forced mining companies to increasingly seek less traditional sources of capital as the availability of debt and equity has become more and more scarce (Dionne and Garand 2003). Such sources include royalty, streaming and offtake agreements as outlined in Table 1.

Table 1: Non-traditional sources of capital that impact project revenue

Type	**Description**
Royalties	
Gross Revenue (GR)	The miner pays a percentage of gross (top-line) revenue to its royalty partner in exchange for an initial capital investment
Net Smelter Return (NSR)	The miner pays a percentage of net revenue to its royalty partner in exchange for an initial capital investment.
Net Profit Interest (NPI)	The miner pays a proportion of the project's net profits to its royalty partner (usually only after it has recovered its capital costs) in exchange for an initial capital investment.
Streaming Agreements	
Metal Streams	Streams provide the right to purchase a proportion of production of one or more of the mine's metals at a discounted price in exchange for an initial capital investment. Streams are well suited to mines with significant co-product production. Precious metals are the most common metals subject to streaming agreements; for example, a Cu-Au mine may wish to stream future gold production in order to fund production of the main metal, copper.
Offtake Agreements	
Metal Offtakes	Offtake agreements give the offtake buyer the right to purchase future metal or concentrate production from the mine in exchange for an upfront payment. The payment is usually intended either as pre-payment for a portion of the future metal delivery or to secure a joint-venture interest in the project.

Given that the financing alternatives outlined in Table 1 impact the revenue structure of the mineral complex, they must be accounted for in project optimization; the change in revenue calculation will impact the final LOM plan, resulting in different values for the decision variables. Due to the restrictive use of block values in traditional formulations, it was not possible accommodate a change in revenue calculations. The formulation set forth by Goodfellow (2014) can accommodate complex revenue structures due to the generality of its mechanism of hereditary attribute calculation. This paper extends the mechanism to include attribute-and-variable-dependant revenue calculations. The contribution allows for a mining complex optimization that considers project encumbrances brought on by the financing of the asset.

2.4 Including Metal Price Uncertainty in Mine Complex Optimization

By including uncertainty in the input prices the optimization is able to manage market volatility by making decisions that can take advantage of opportunities during high-price periods. Conversely, in weaker periods, stockpiles can act as a buffer to shield the operation from selling in low-price environments. Without the inclusion of market price uncertainty within the optimization, these operational flexibilities are wasted.

Past attempts at including metal price uncertainty have focused on determining pushback, or ultimate pit designs (Meagher et al. 2009; Castillo and Dimitrakopoulos 2014) and have fallen short of generating an integrated LOM plan robust to price uncertainty. The main reason that price uncertainty is not considered in simultaneous mine optimization is the large number of simulations required to represent uncertainty which can be in the order of 100-1000 (Briggs et al. 2012). Including this many scenarios increases the solution time prohibitively.

This paper proposes a two-part optimization whereby the entire mine complex is optimized first under geological uncertainty, then this schedule is fixed and used as an input to a second optimization where only down-stream decisions are optimized under metal price uncertainty. This procedure reflects a mine's operational reality in which the long-term LOM plan is optimized at a given starting point, and the down-stream decisions are taken in subsequent years when price uncertainty is revealed.

Notably, this approach to incorporating metal price uncertainty assumes knowledge of the entire path of each metal price scenario, i.e. material movement decisions made in a given year are based on the (assumed-to-be known) paths of metal prices in future years. Although there is a theoretical argument against this approach (Chib et al. 2002; Duan and Simonato 2001; Slade 2001; Dionne and Garand 2003; Smit and Trigeorgis 2012), and Markov Chain Monte Carlo (MCMC) methods are advocated, in practical terms the approach suffices as long as autocorrelation is not the driving force driving prices. Thus, the full set of metal price simulations is used and the model is solved as-is.

To model market uncertainty metal price simulations are generated using a stochastic reduced-form model. The simulation methods proposed are based on popular pricing models for each type of commodity (Schwartz 1997). Due to the important influence of supply and demand, base metals can be simulated using a mean reverting process with Poisson jump diffusion as shown in Equation 2.6.

$$S_t = S_{t-1} + \alpha \cdot (\bar{S} - S_{t-1}) + W \cdot S_{t-1} + \beta \cdot P \cdot S_{t-1} \tag{2.6}$$

Where S_t is the price of the metal at time t, α is the commodity's mean-reverting speed, W is a Weiner process, and β is the jump size of Poisson process P.

The typical model for precious metal prices is a trending Geometric Brownian Motion model with Poisson jump diffusion as shown in Equation 2.7. The trend component is used in precious metal price modeling to account for a positive correlation with inflation.

$$S_t = S_{t-1} \cdot exp\left(\eta \cdot t - \sigma^2 \cdot \frac{t}{2} + W + \beta \cdot P\right) \tag{2.7}$$

Where η is the average annual price drift and σ is the average annual volatility.

2.5 Case Study: Application at a Cu-Au mine with a precious metal streaming agreement

The case study presented below illustrates an application of simultaneous capacity optimization within the mining complex framework. Geological uncertainty is incorporated through the use of ten orebody simulations and a second optimization is run in order optimize downstream decisions under metal price uncertainty using 100 copper price simulations and 100 gold price simulations. Revenue is calculated based on a gold streaming agreement whereby a portion of the mine's gold production is sold to a customer at a fixed price. The case study applies to a copper-gold mining complex for which the orebody and processing stream data was supplied by an industrial sponsor.

2.5.1 Overview

The mining complex used in this case study produces two products, copper and gold. The complex comprises a single open pit mine that holds four main material types: oxide, sulphide, transition, and waste. Of these, the two materials containing the bulk of the mine's profit – sulphide and transition – are further split into two categories based on the grade of the main metal of interest, copper. The mining complex has six processing destinations and one stockpile. The processing destinations include: a sulphide and oxide dump leach; an oxide, sulphide and transition heapleach; and a sulphide processing plant. Each of these processing destinations is fed directly from the mine, with the sulphide plant accepting additional feed from a stockpile as shown in the material flow diagram in Figure 5. Copper electrolyte solution is produced at the sulphide dump leach and the sulphide heap leach, this solution is then converted into cathode copper at the solvent extraction electrowinning (SX-EW) plant. Gold metal is recovered from leachate which is produced at the oxide dump leach, oxide heap and transition heap leach. The sulphide processing plant produces a copper-gold concentrate containing 30% copper and 5-30g/t gold. This product accounts for the greatest proportion of the mine's revenue.

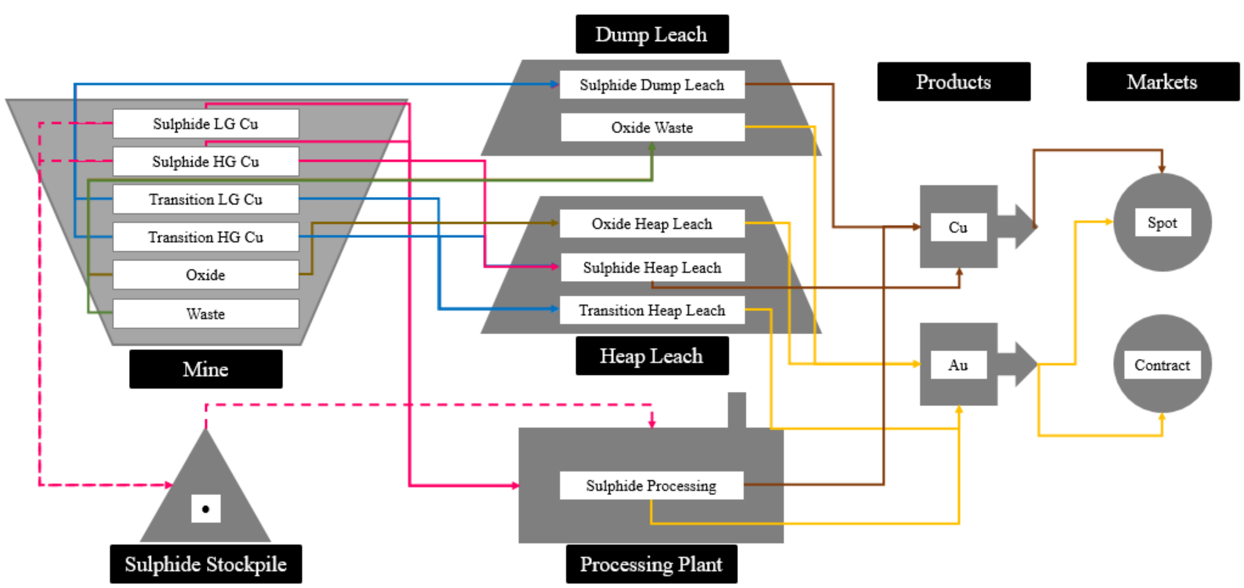

Figure 5: Material flow diagram for the mining complex used in the case study

Non-linear grade-recovery functions are used at the process destinations. These were also provided by the industry sponsor. The curves allow a realistic modelling of the relationship between the head-grade into a particular process and the amount of metal that can be effectively output from the process. This is a benefit of the formulation that is impossible to consider in the strict traditional SIP models that rely on predetermined block values. The copper and gold recovery curves used in this study are provided in Figure 6 and Figure 7. Looking at the copper grade-recovery curve, it is important to note that recoveries for grades in the range of 0-0.4% vary considerably between the different process destinations. The average copper grade of the project used in this case study is 0.3% which make the grade-recovery relationship an important factor in the optimization of the LOM schedule and capacity decisions.

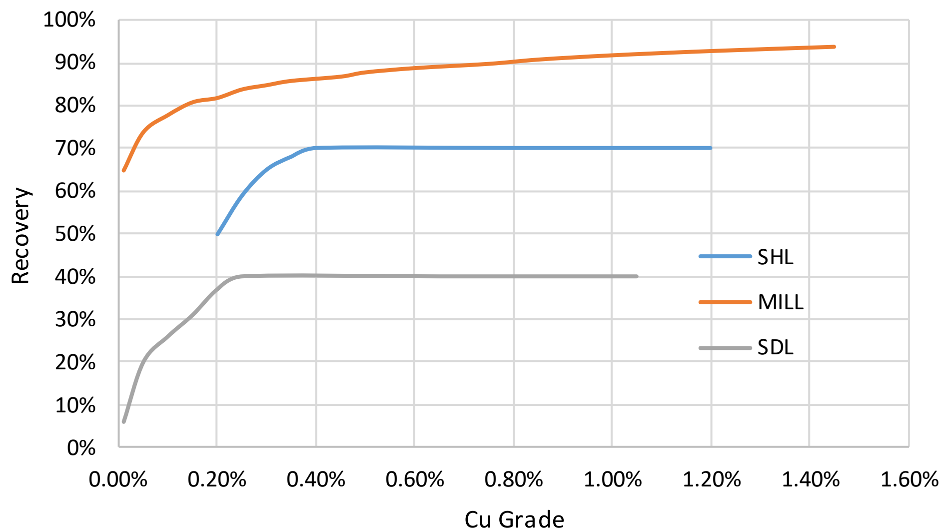

Figure 6: Grade-recovery curves for copper

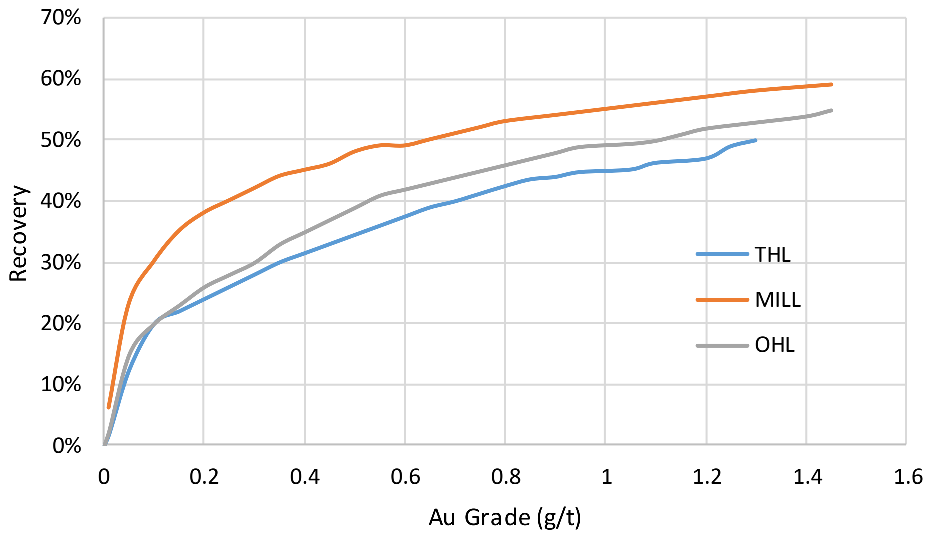

Figure 7: Grade-recovery curves for gold

The parameters used in this case study are presented in Table 2. Note that the mining and milling capacities outlined in the table are used a base case for illustrative purposes, these will be optimized within the model.

Table 2: Assumptions and inputs used in the mining complex optimization

Type	Value
Geological Model	
Number of blocks	493,290
Block dimensions	25x25x10m
Metals of interest	Cu, Au
Block tonnage	13,000 – 15,000 tonnes/block
Financial	
Copper price	\$2.88/lb
Gold Price	\$1,480/oz
Discount rate	8%
Mining cost	\$2.60/tonne
Milling cost	\$29.37/tonne
Heap leach cost	\$5.35-7.74/tonne
Dump leach cost	\$4.87/tonne
Optimization	
Geological discount rate	10%
Deviation penalties	5-100 per unit deviation (depending on constraint)
Objective function	Max NPV
Operational	
Mining capacity	20,000,000 tonnes/year
Milling capacity	6,000,000 tonnes/year
Mill stockpile capacity	10,000,000 tonnes
Sulphide leach capacity	8,100,000 tonnes/year
Metal recovery	Variable, based on recovery curves

2.5.2 *Stochastic Capacity Optimization Results*

For comparison purposes, a base case mine plan using a deterministic optimization model with fixed capacities (as shown in Table 2) was generated. Figure 8 shows the resulting sulphide mill input tonnage for the deterministic optimization. Notably the fixed capacity of six million tonnes per year is not expected to be exceeded over the life of the operation. However, the risk analysis, created by running a number of simulations through the base case schedule, shows that these expectations are likely not to be met. Contrary to the deterministic schedule, the stochastic schedule, incorporating geological uncertainty, is able to reliably meet production targets within the tight specified range.

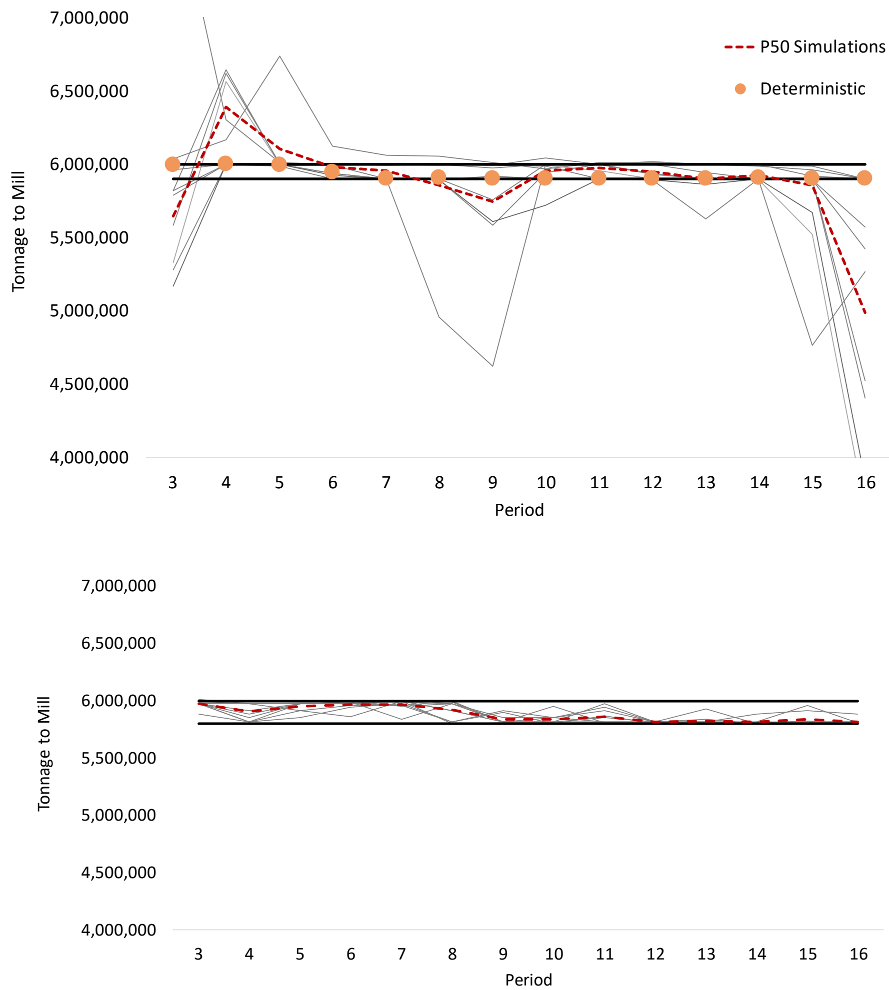

Figure 8: Top – risk analysis of deterministic optimization. Bottom – risk analysis of stochastic optimization incorporating geological uncertainty

Although the stochastic LOM plan shown in Figure 8 does a good job meeting production targets at the sulphide mill, the optimization is based on a fixed throughput level and mining capacity. By incorporating capacity decisions, the optimizer can see to maximize the trade-off of sending increasing or decreasing these capacities and sending the material elsewhere. The results presented below cover the following operational decisions: scheduling (and pit limits), destination policy, material flow, and capital expenditure/operational capacity selection. In this case, the optimization was no longer forced to abide by the mining and processing constraints laid out in

Table 2, instead these capacities were simultaneously optimized along with the rest of the mining complex. The milling capacity was modeled as a one-time decision made in year 1 with a two-year lag time. This allowed for a two-year pre-production period during which mill construction, stripping, stockpiling and leaching could occur, with milling only allowed to commence in year-3. During this 3-year ramp-up period mining capacity was allowed to increase, reaching its maximum level in year 3 at which point it is fixed for the remainder of the LOM. Table 3 provides the capital costs and capacity parameters used in the optimization model.

Table 3: Capital costs assumed for incremental mining and milling capacities

Mining	**Milling**
Incremental Capacity	
1,500,000 tonnes/year	200,000 tonnes/year
Cost per Increment	
\$4,000,000	\$10,000,000
Lead Time	
0 years	2 years
Life	
LOM	LOM

Using a 26-core 2.60GHz Intel Xeon CPU and 128GB of RAM the optimization took 37 hours and 7 minutes. Figure 9 shows the results from the stochastic optimization with capacity integration. The optimized design called for a 4,800,000 tonne per year milling capacity at the sulphide mill. The resulting mining capacity started at 5,000,000 tonnes in year 1 as construction was allowed to commence, increasing to 18,000,000 tonnes in year-2, and reaching a maximum capacity of 25,000,000 tonnes in year-3 until the remainder of the LOM in year-16. Notably the optimizer selected a higher mining rate and a lower milling rate than what otherwise would have been assumed for the capex-constrained model. This is due to the fact that the resulting optimal schedule is able to selectively feed the sulphide mill with high-grade material while making better use of the dump leach and heapleach facilities which have a lower operating cost. Notably, these decisions are being made in large part based on the grade-recovery relationship outlined in Figure 6 and Figure 7, something that is impossible if traditional block-based values are used.

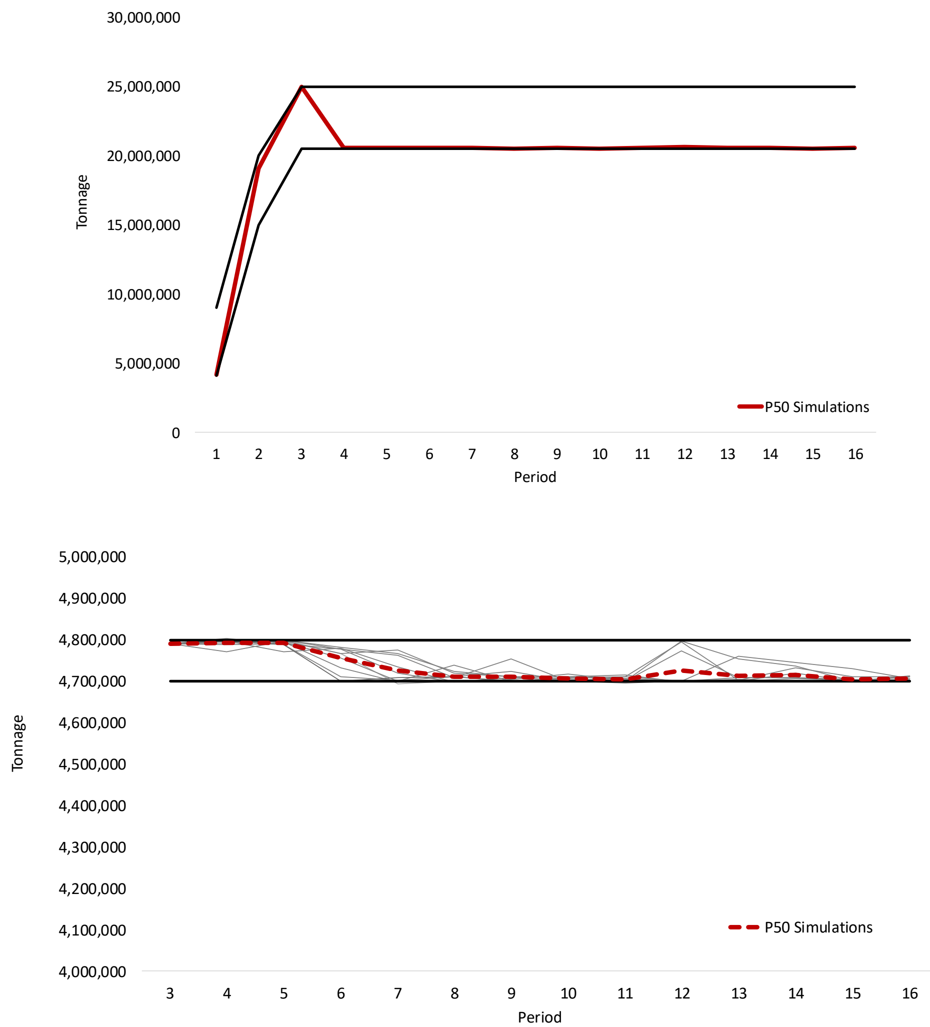

Figure 9: Mining capacity (top) and sulphide mill capacity (bottom) selection for stochastic optimization with integrated capacity decisions

The results of the optimization show that the stochastic schedule is robust to geological uncertainty and does a good job at feeding the mill within a narrow range. This avoids both financial losses due to missing targets and extra costs associated with overusing the sulphide stockpile. These benefits can be seen when comparing the financial results of the schedules as shown in Figure 10.

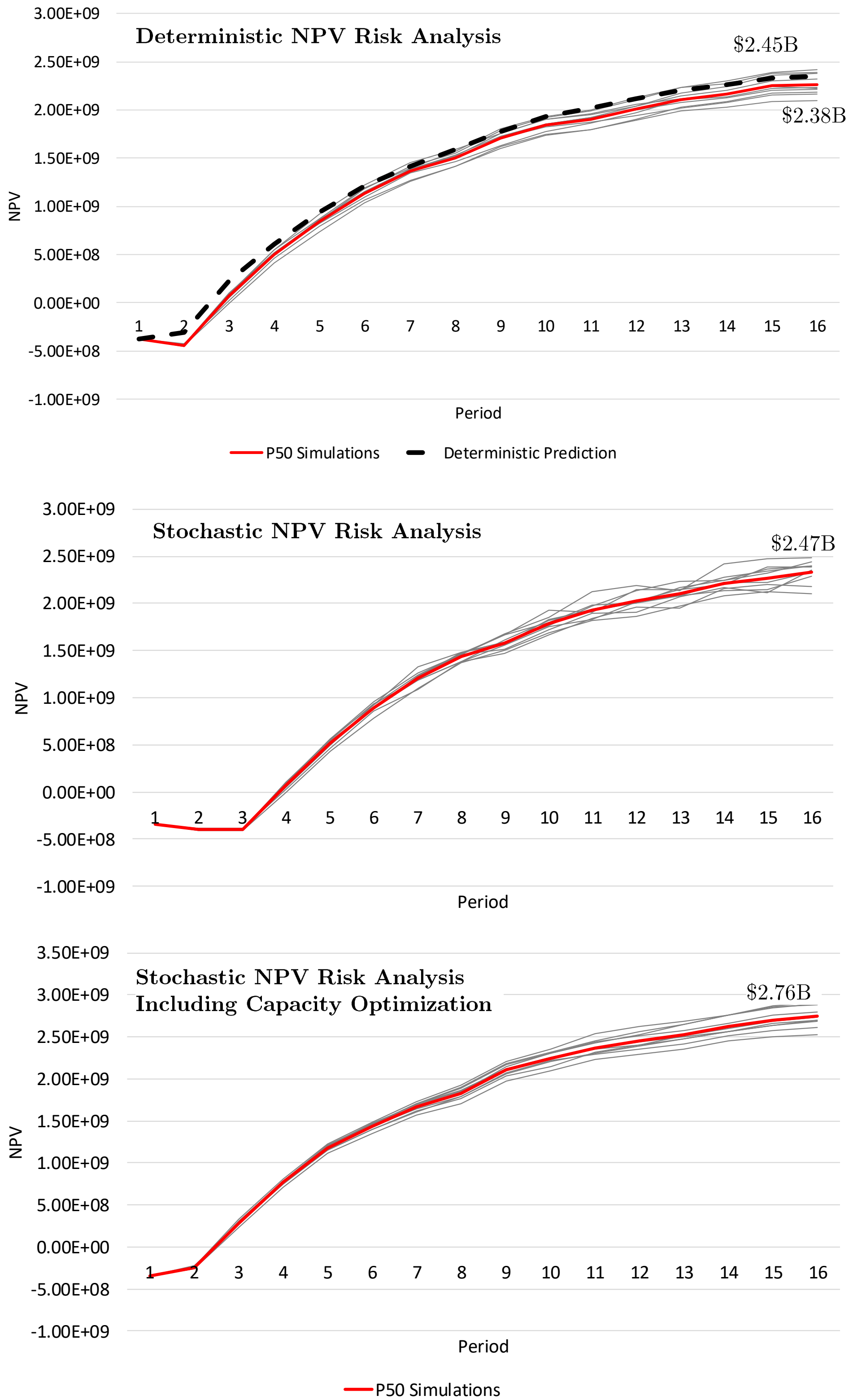

Figure 10: NPV results of capacity optimization applied to optimization of a mining complex under geological uncertainty

Based on the risk analysis, the NPV that the deterministic optimization predicts has an 80% chance of falling short of its estimate. The risk analysis forecasts an NPV that is 1% ($20,000,000) lower than the one predicted using the estimated model. This is due to its inability to manage the grade-related risk inherent to the underlying geology. This causes the deterministic schedule to largely misclassify material which leads to a misguided schedule, destination policy and material flow decisions. The schedule that is optimized simultaneously with capacity decisions shows a 12% ($290,000,000) increase over the stochastic schedule that does not integrate capacity optimization.

The pit design and extraction sequence generated by the optimization is presented in Figure 11. These designs can be compared to the stochastic schedule that did not integrate capacity optimization. Each period is represented by a different colour moving from cold to hot colours as years progress. The empty blocks at the bottom of the pit represent uneconomic material that was left behind. Also of note is that the final pit is larger than the one generated when the model was constrained to a fixed mining capacity of 20,000,000 tonnes per year. This again is due to the new schedule's ability to selectively send material to the sulphide mill and make better use of the upgraded 25,000,000 tonne per year mining capacity by sending material to stockpiles and leach pads which do not have a strict capacity requirements like the mill.

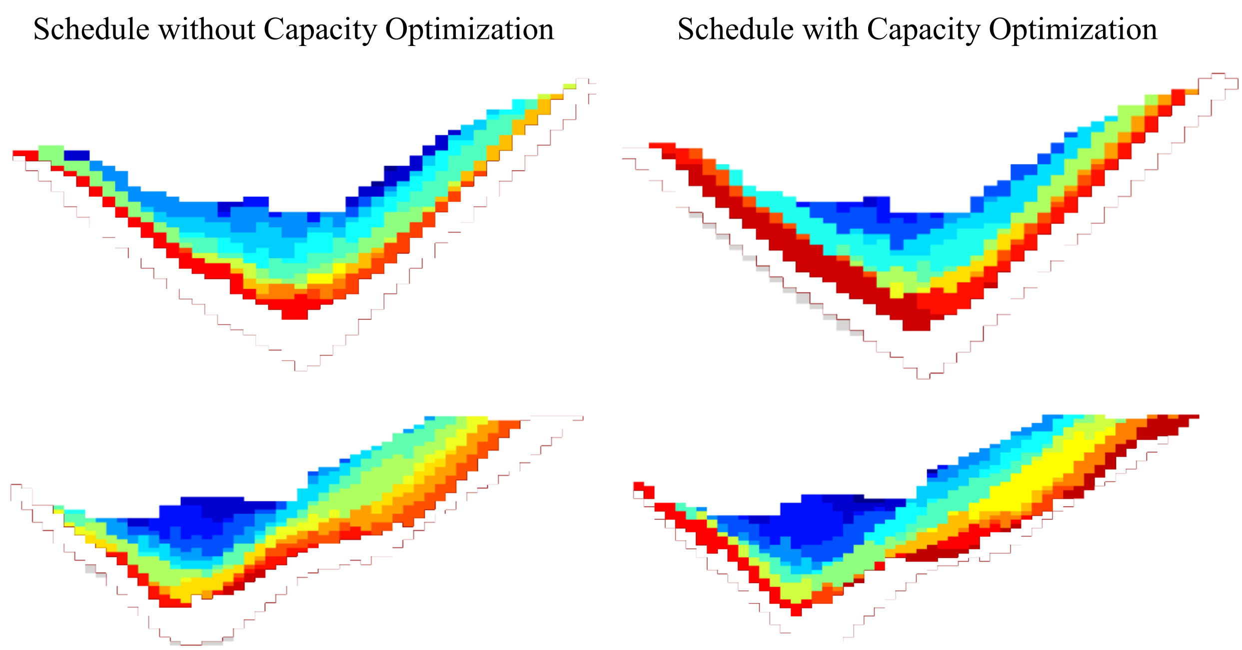

Figure 11: Cross-sections of the mining sequences and pit designs for long term mine schedules with and without capacity optimization under geological uncertainty

Two other important aspects of the resulting LOM plan generated by the optimization are the material destination policy ($z_{c,j,t}$) and the inter-destination decisions ($y_{i,j,t,s}$). As noted previously, the mine plan based on simultaneous capacity optimization decides to make more use of heap leach and stockpiling in order to favour higher grades at the mill. The impact of the optimization on these variables is shown in Appendix II.

2.5.3 Stochastic Optimization with Variable Capacities, a Precious Metal Streaming Agreement, and Two Types of Uncertainty

As noted in Section 2.4, selling contracts can be important encumbrances for many mining projects. If the commodities produced by a certain project are subject to such an agreement, it becomes necessary to account for the change in revenue calculation within the optimization. In this section the same mining complex is considered but a precious metal streaming agreement is applied to a proportion of the gold produced. In this section both geological and commodity price uncertainty is included.

The parameters used to model copper and gold price uncertainty are given in Table 4. Copper price is modeled using a mean-reverting process and gold price is predicted with a trend model.

Table 4: Parameters used to model metal price uncertainty

Parameter	Value and Comments
Copper	
Initial price, S_0	US$2.88/lb, the same as the reverting level
Reversion level, $\bar{S}$	$2.88/lb, 5-yr real reverting level in 2015 dollars
Annual volatility, σ	9%, average annual volatility over 25 years
Mean reverting speed, α	0.5,
Average jump frequency, μ_P	2 per year, 25-yr average number of Cu price shocks
Average jump size, β	3%
Gold	
Initial price, S_0	US$1480/oz, price assumption used by operation
Annual volatility, σ	5%, average annual volatility over 25 years
Annual drift, η	0.5%, 5-year moving average drift over 25 years
Average jump frequency, μ_P	2 per year
Average jump size, β	5%

Using the parameters from Table 4 produces the simulation profiles for copper and gold shown in Figure 12 and Figure 13 respectively.

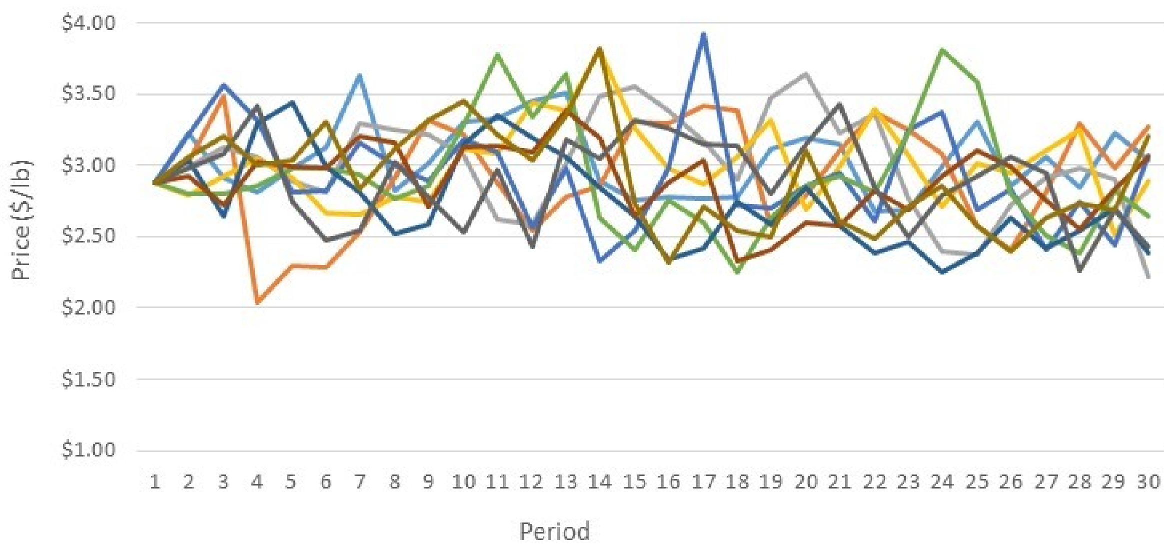

Figure 12: Copper price simulations

The copper price simualtions shown above show a clear pattern of mean reversion. This model is used for copper because the metal's price is heaviliy influenced by supply-demand dynamics which are driven by the metal's relatively stable marginal cost of production. Gold's use as an inflation hedge means that the real growth rate of its price can be expected to match the pace of inflation. The simulations below use only a small trend value in order to ensure that the ratio of metal values (Cu/Au) remains relatively stable over the life of mine.

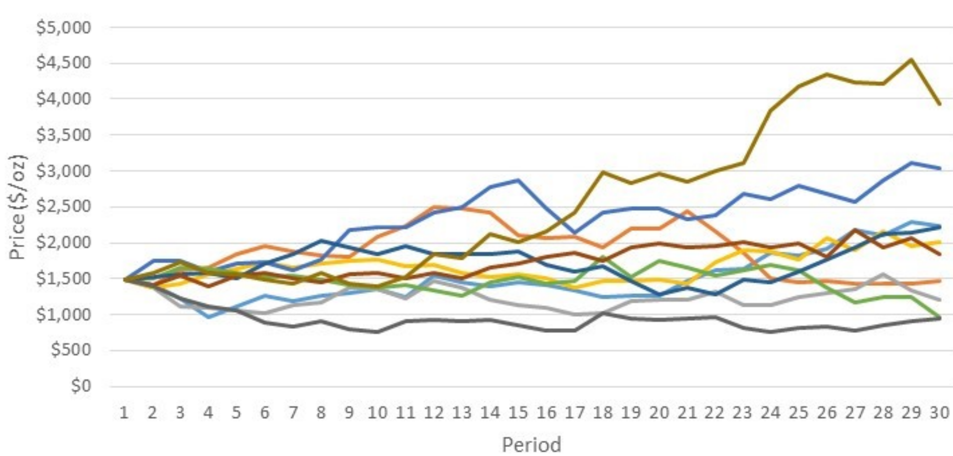

Figure 13: Gold price simulations

The streaming agreement considered herein provides the miner with an upfront payment of $800,000,000 in exchange for 80% of the mine's LOM gold production at a price of $620/oz. The remaining 20% of the mine's gold belongs to the operation and is assumed to be sold at the market price of $1,480/oz. The effect that this agreement has on the value resulting from the optimization can bee seen in Figure 14.

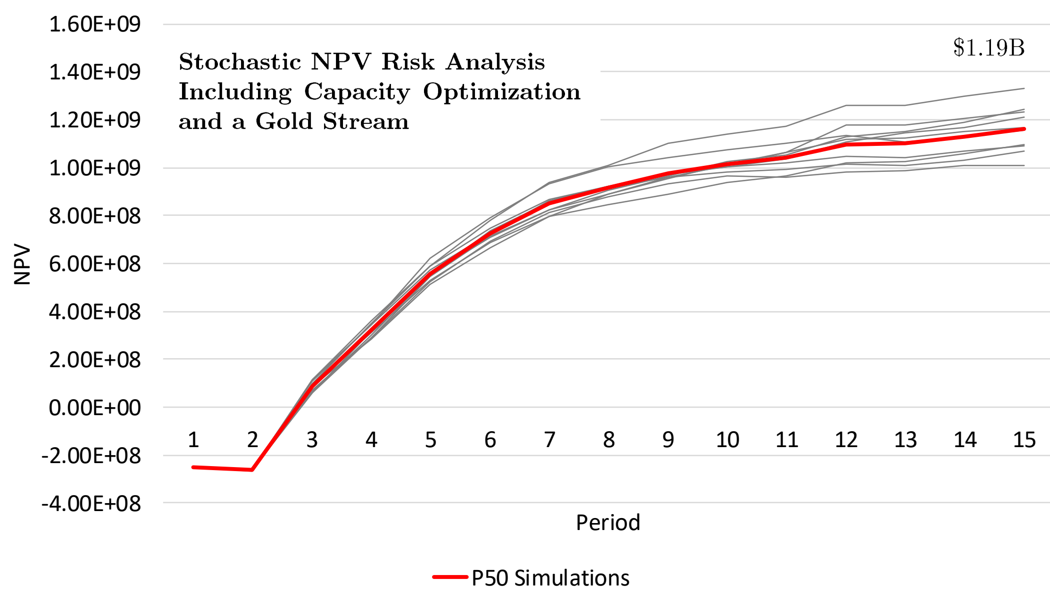

Figure 14: NPV risk analysis of optimization with gold stream under metal price and geological uncertainty

Notably, the expected NPV drops $1.57B due to the stream. This is reflected in the histogram shown in Figure 15 which illustrates the distribution of possible values of the gold stream based on the 200 (100 Cu and 100 Au) price simulations that used to optimize down-stream decision variables. Although the project has lost significant value through the gold stream contract, the inclusion of metal price uncertainty was able to shield a portion of the total potential loss. Without including metal price uncertainty, the stream would have accounted for an expected loss of $1.68B compared to the same optimization excluding a stream. This benefit is due the optimizer's ability to seek to recovery more metal during high-price periods and favor low-cost, lower-grade ore during low-price periods.

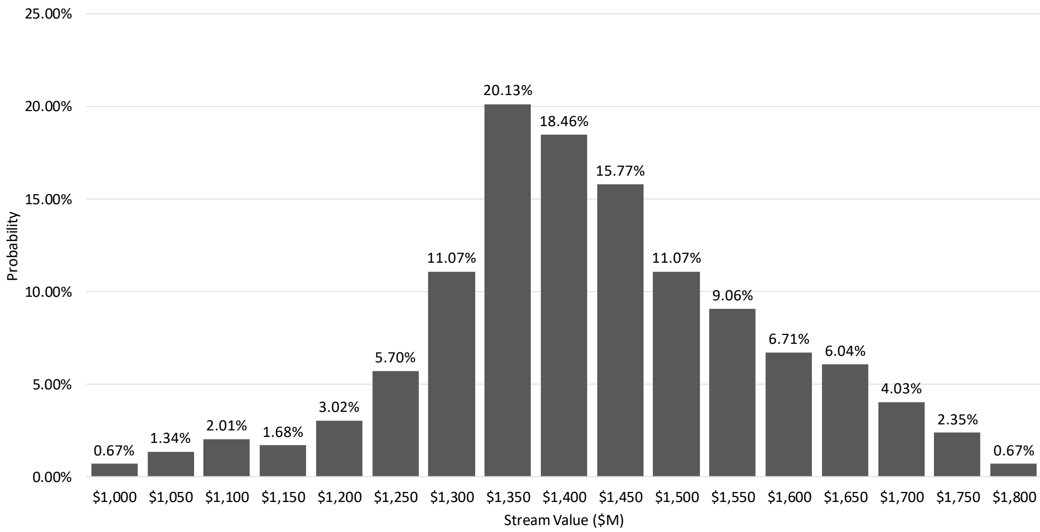

Figure 15: Distribution of values for the gold stream (to the stream owner)

2.5.4 Summary of results

Table 5 provides a summarized comparison between the results from different optimizations that were run. Notably, the mining and milling capacity selected under the streaming scenario are lower than without the stream. This can be expected due to the loss of revenue to the stream owner.

Table 5: Summary of Results

Result		**Det.**	**Det. Cap.**	**Stoch.**	**Stoch. Cap.**	**Stream Cap.**
Capacities						
Mining	(Mtpy)	20	21	20	25	20
Milling	(Mtpy)	6	5	6	4.8	3.2
Financial						
Capex mine	(US$M)	$67	$56	$67	$102	$57
Capex mill	(US$M)	$150	$300	$150	$240	$160
Total capex	(US$M)	$217	$356	$217	$342	$217
LOM	(yrs)	16	16	16	16	15
NPV	E{US$B}	$2.45	$2.49	2.47	$2.76	$1.19
Production						
Ore	E{Mt}	228	234	262	275	237
Copper	E{kt}	641	745	793	846	747
Gold	E{Moz}	0.9	1.3	1.1	1.3	0.8

2.6 Conclusions

This paper addresses the simultaneous integration of inter-related capacity decisions within the mining complex framework. Specifically, the mining complex is considered from the perspective of the planning stage and the optimizer is allowed to select both mining and milling capacities. The approach shows that eliminating arbitrary capacity restrictions at the outset of a project can unlock significant value that may otherwise have been loss. This study was conducted under geological uncertainty through the use of multiple orebody simulations in order to create a risk-robust schedule with higher probability of meeting the optimizer-set capacity targets.

Throughout this study it was discovered that progressively reducing the flexibility of the capacity decision within a narrowing realistic range greatly improved the stability of the final solution. However, this approach means that the optimization has to be attempted a number of times in order to ensure an adequate approximation of optimality. This is a trade-off that the practitioner must consider.

This paper also considered the impact that project financing can have on revenue calculation. A streaming contract was considered in the case study and the change in revenue calculation was included in the optimization. The resulting schedule was able to shield some of the potential losses to the stream owner by favouring high metal production in high-price periods and seeking low-cost, lower-grade tonnes during low-price periods.

In addition to mining and processing capacities, the capacities of auxiliary components of the mining complex can be incorporated in the optimization. These are typically components that are not critically related to production, but still play a part in the overall profitability of the operation. In most hard rock operations, the processing bottleneck is milling capacity, but other constraints of interest may include be: leach pad height, water use, down-stream transport, acid consumption, emission limits, fleet size fluctuation, equipment limits in various pits, or any combination of these factors. Regardless of the bottleneck, the model can incorporate it, and its corresponding capital cost, in the optimization.

Chapter 3

An algorithmic approach to schedule-based pushback design

3.1 Introduction

Mine planning is the practice of developing an optimal ultimate pit limit and extraction schedule. Optimality is usually defined in terms of net present value. The input for conventional mine planning is traditionally a single estimated- usually krigged- orebody model (Dimitrakopoulos, Farrellly and Godoy, 2001). This model, which does a good job of reproducing global-scale variability but a bad job of reproducing local-scale variability (Journel and Huijbregts 1978), forms the basis for the misguided desire to obtain the "best" mine plan – defined as the mine plan that optimizes this single input. The resulting plan generated from this deterministic approach is inherently flawed.

A brief look at the traditional, and current best-practice, mine planning procedure will provide the impetus for a new, more optimal approach to mine planning. More specifically the focus of this work will be phase design, where a "phase" or "pushback" refers to a large discretization of the open pit that can be developed independently with its own working face.

The purpose of phase design is to allow the mining engineer to plan equipment movement while respecting spatial and operational constraints. Figure 16 shows an orebody cross-section and the resulting traditional phase design. Traditionally phase design is performed based on assumed block values and engineering experience rather than through an objective theoretical process. Any phase design must be specified at the outset of mining operations thus limiting a project's flexibility. Only minor adjustments to the LOM extraction sequence can be made once a phase design has been pursued, this makes it a very important initial step in mine planning (Stone et al, 2007).

Orebody cross section and ultimate pit limit   Traditional Phase Design with 4 Phases

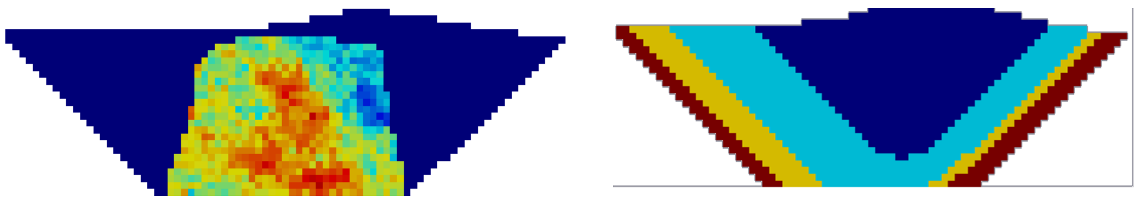

Figure 16: Orebody model with traditional ultimate pit and phase design

Figure 17 illustrates the traditional approach to mine planning which begins by defining ultimate pit limits either using Lerchs-Grossman or maximum flow methods. Through parameterizing either method, nested pit shells are created and then aggregated to form mining phases (Dagdelen 2001). Once a phase design is created, the next step in the conventional approach would be to create a bench-wise extraction schedule constrained within the phases. This long-term extraction schedule is meant to guide mining within each phase such that value is maximized. As a final step in the traditional framework, other components such as cutoff grades and blending strategies can be optimized.

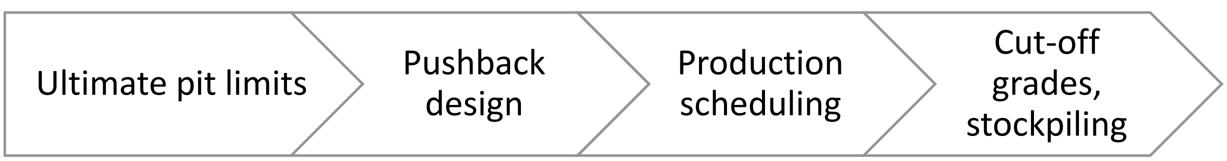

Figure 17: The traditional approach to long term mine planning.

Not only does the traditional approach described above arbitrarily constrain the mine's extraction schedule to a specific pit/phase configuration, but also it: makes a-priori assumptions on block values, largely ignores production and processing constraints, and it often ignores-or does not properly account for-time value of money. A more optimal approach to mine planning would be to base the phase design on an optimized extraction schedule – flipping the traditional approach on its head.

The optimal mine planning framework proposed in Figure 18 would first allow an optimizer to determine the best life of mine (LOM) production schedule (extraction sequence and destination policy etc.) using only an objective function, the orebody model (stochastic simulations), and constraints as inputs. This production schedule can simultaneously integrate: time value of money, capacity constraints, grade blending targets, and destination decisions while optimizing value based on the given objective. With this approach it becomes unnecessary to make a-priori assumptions on block value and risk reduction is integrated organically (Goodfellow 2014). Further, once the theoretically-optimal long term production schedule is created, its boundaries will naturally define the ultimate pit limit.

One of the main challenges with schedules generated by stochastic optimization techniques - such as SIPs and metaheuristics - is that they are not necessarily implementable (or “smooth”). Stochastic schedules often seek to mine patchy portions of the mine within the same period. While this is technically possible, it is almost always unrealistic since it often involves prohibitive cycle times and equipment movement costs. For this reason attempts have been made to account for spatial smoothness or connectivity during the schedule optimization stage. Ramazan and Dimitrakopoulos (2004) and Benndorf and Dimitrakopoulos (2013) both employ SIP formulations that use smoothness “windows” to penalize the objective function when adjacent blocks are mined in different periods. While this approach works well with small orebody models (~5,000 blocks), the formulation requires the addition of many additional constraints which makes the approach prohibitive for models of a more realistic size. Further, these methods do not contemplate the additional step of creating the mine’s optimal phase design after a schedule has been created.

From the schedule and within the pit limits a practical (or implementable) phase design must be generated in order to plan and control equipment movement. Once a phase design is established a mineable production schedule can be adapted. The point here is that the optimal production schedule should drive the pit/phase design, and not the other way around as with the traditional mine planning approach. Regardless of the impractical nature of the raw schedule, it remains a good basis for further design considerations (like phases) because it results in the highest theoretical value of our objective function. Thus, we would like to be able to create a phase design that inherits the major features from the initial schedule. Notably, the initial raw schedule can be the output from any one of the number of optimizers such as: the stochastic optimization of a single (Ramazan and Dimitrakopoulos 2004), a global mining complex optimization (Goodfellow and Dimitrakopoulos 2016), or the optimization of mineral supply chains (Zhang and Dimitrakopoulos 2014).

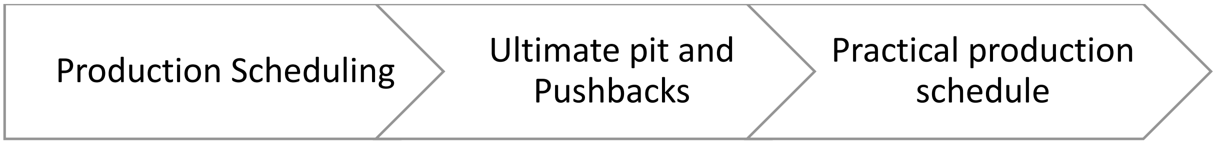

Figure 18: The optimal mine planning framework

According to best knowledge to date there exists only one known method that has been applied to create a phase design based on a pre-conceived schedule. Stone et al. (2004) propose a mine planning software called Blasor, the concept of which is to use an optimal extraction sequence to design ultimate pit limits and mining phases. Blasor performs a number of aggregation (and disaggregation) steps, both during the scheduling stage and during the phase design stage, which detract from the program's optimality. Below we propose our own method of schedule-based phase design.

3.2 A Proposed Method of Schedule-Based Phase Design

Taking the assumption that the raw schedule generated by the stochastic optimizer is the best basis for a phase design, the questions becomes: how to design phases that best mimic the LOM production schedule? The first step in the proposed process is to establish our building blocks – the constituent elements that will combine to form our phases. Since the goal of the phase design is to mimic the raw schedule as best as possible, what better source for the building blocks then the raw schedule itself?

At the left of Figure 19 is a cross-section from a raw LOM extraction sequence with the different colours representing different periods of extraction. The collection of blocks that form each 4D-contiguous (spatially-connected with the same period/colour) part of the raw schedule will constitute a component of the phase design. In Section 3.1 we discuss the method by which these components (herein called "shapes") are aggregated to build phases. The logic is that by building mineable phases out of the raw schedule's features, its major characteristics can be largely preserved, thus preserving as much of its optimality as possible.

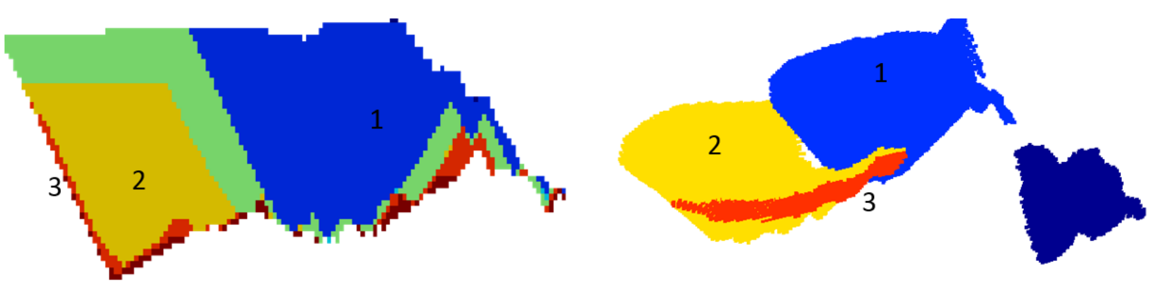

Figure 19: Cross-section (left) of a mine's schedule and some resulting 4D shapes (right)

The shapes shown at the right of Figure 19 above are created by amalgamating the orebody model's scheduled blocks using a Breadth First Search algorithm (BFS) whereby each block represents a vertex, and arcs (connectivity) are established between adjoining blocks belonging to the same period.

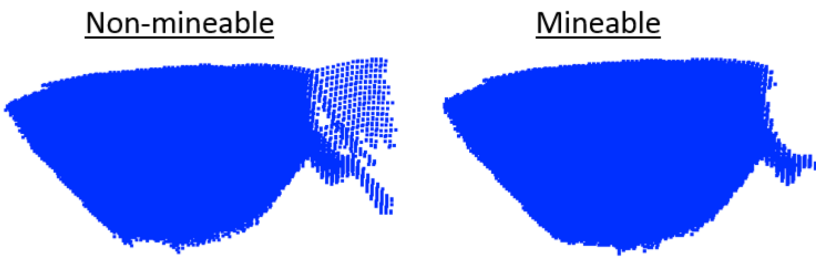

Figure 20: Separating non-mineable and mineable shapes

Once all shapes are built, they are disaggregated to form mineable and non-mineable groups (as shown in Figure 20 above) where mineability is determined by specifying a minimum number of connected blocks in each spatial dimension. This "minimum mining width" specification depends on the size of machinery and geotechnical considerations for the particular operation in question.

Algorithm 1: Building mineable and non-mineable shapes from the raw schedule

---

Initialization

$\chi(u) := \{u\}$, the set of blocks that will be formed into shapes consisting of all scheduled blocks in the orebody model

$\eta(u) \subset \chi(u)$, the set of blocks adjoining u with same period (t_u)

S_i, shape with ID i containing the set of blocks $\sigma_i(u) \subset \chi(u)$

S_i^m, mineable shape with ID i containing blocks $\sigma_i^m(u) \subset \chi(u)$

S_i^n, non-mineable shape with ID i containing blocks $\sigma_i^n(u) \subset \chi(u)$

$(x, y, z, t)_u$, the coordinates and period of the block u as determined by the raw schedule

(nx, ny, nz), the minimum number of blocks in the X, Y and Z dimensions that determine mineability

Perform Breadth-First-Search (BFS) to form initial shapes

$blocksForShapes = \chi(u)$

$i = 0$

while $blocksForShapes \neq \varnothing$ **do**

```
        Q = ∅
        Enqueue(Q, j ∈ blocksForShapes)
        S_i = S_i ∪ j
        while Q ≠ ∅ do
                b = Dequeue(Q)
                blocksForShapes.Erase(b)
                for each k ∈ η(b)
                        if blocksForShapes.Find(k) then
                                'adjoining block can be added to shape.
                                Enqueue(Q, k)
                                S_i = S_i ∪ k
                        end if
                end for
        end while
        i = i + 1
end while
```

<u>Disaggregate shapes into mineable and non-mineable groups</u>

```
while i < numShapes do
        blocksInShape = σ_i(u)
        while blocksInShape ≠ ∅ do
                j = Dequeue(blocksInShape)
                blocksInShape.Erase(j)
                if j satisfies minimum mining widths, S_i^m = S_i^m' ∪ j
                else S_i^n = S_i^n ∪ j
                for each l ∈ η(j)
                        if l satisfies minimum mining widths, S_i^m = S_i^m ∪ l
                        else S_i^n = S_i^n ∪ l
                end for
        end while
        i = i + 1
end while
```

Once created each shape can be defined by the following attributes:

- A unique shape ID which allows easy reference of a given shape and efficient memory/runtime
- The set of constituent blocks mapped to the shape ID
- Its period of extraction
- A mineability Boolean

- The set of neighbour shapes that determine valid connections during aggregation into phases
- Its centroid – a rough estimate of the shape's location within the proposed mine

After the 4D schedule-based shapes are generated the goal is to group them into an implementable phase design; the proposed procedure for this step is outlined in Section 3.1 below. The benefits of using the proposed schedule-based approach for phase design are that: slope constraints and temporal precedence relationships are automatically respected since the shapes are taken from the raw schedule, the problem size is reduced from the block scale to the shape scale, and any combination of shapes will remain theoretically feasible.

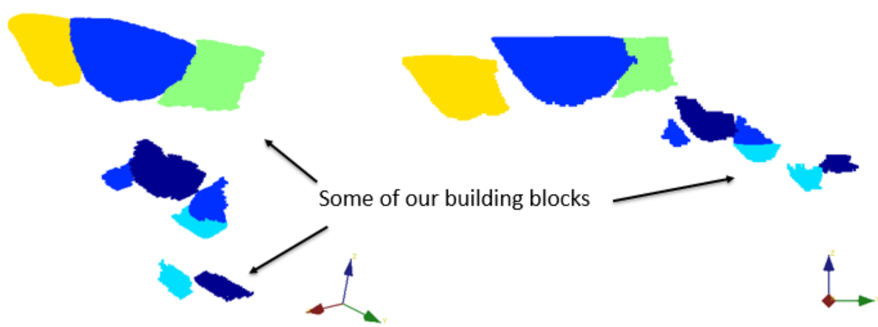

Figure 21: Some mineable shapes that we seek to combine into phases

Since any combination of 4D schedule-based shapes is feasible the next step is to decide the best way to combine the shapes to form a mineable phase design. The objective of this step is to preserve as much of the raw schedule's structure as possible while accounting for mineability. The proposed method comprises three main steps:

1. Aggregating mineable shapes through clustering
2. Using mineable clusters as seeds to grow phases
3. Post processing for minimum mining width

3.3 Aggregating 4D-Contiguous Shapes

The first step of the proposed method for schedule-based phase design is clustering of the mineable shapes. Clustering is a method whereby elements are assigned cluster memberships based on their similarity to one another. Similarity is defined using a distance function and attribute weightings. In our case, shapes are clustered in order to group similar elements of the raw schedule into starterphases (or “phase seeds”). Only mineable shapes are clustered to form these phase seeds because they, in theory, are the shapes that can be kept intact during the process of creating an implementable mine plan. Non-mineable shapes are saved and incorporated into phases later (as described in Section 3.2). This grouping process is what contributes to a less fragmented, or smoother, phase design.

Clustering is performed using the k-means++ algorithm which seeks to minimize the average distance between data within the same cluster (Arthur and Vassilvitskii 2007). The method is a more efficient extension of the k-means clustering problem which takes a given integer number of clusters k and a set of n data points $\chi \in R^d$. Each data point $x \in \chi$ has d attributes, each with their own weighting. In order to create clusters that best mimic the initial schedule, a heavy weighting is placed on the attribute corresponding to the period of extraction, thus urging shapes with similar periods of extraction to be grouped into the same seeds. The spatial attributes of each shape (its centroid coordinates) can also carry a weighting, but any spatial weighting should be minimal and will be deposit-dependant.

Algorithm 2: Aggregating mineable shapes into phase seeds

Initialization

S_i^m, mineable shape with ID i containing blocks $\sigma_i^m(u) \subset \chi(u)$

A_i, the attribute values (x, y, z, t) for shape i

K, number of phases to use

P_k, phase seed $k \in [0, K]$ with cluster center C_k

Perform k-means++ clustering on mineable shapes

'Randomly select initial cluster center

$C_0 = randomShape()$

do until $k = K$

    for each i

        let $minDist = \min\{dist(A_i, C_l)\}$

        randomly select $C_k = A_i$ with probability $\dfrac{minDist^2}{\sum_i dist(A_i, C_l)^2}$

    end for

end do

'initial cluster centroids have been chosen intelligently

while $membershipChange =$ true

    for each k

        for each shape i

            $dist[i][k] = dist(A_i, C_k)$

        end for

    end for

    for each shape i

        if $dist[i][k]$ is the minimum distance $\forall k \in K$

        then $P_k = P_k \cup S_i^m$

        'adding shapes to cluster with nearest centroid

    end for

    if there are no membership changes

        $membershipChange =$ false

    end if

    for each k

    'update cluster centers

        $$C_k = \frac{1}{|P_k|} \sum_{|P_k|} A_i \quad \forall S_i^m \in P_k, k \in K$$

    end for

end while

The result of the above k-means++ clustering is k clusters that will be used to seed our phase design. Together the clusters contain all mineable shapes as seen in the example presented in Figure 22 where each cluster is represented by a different colour.

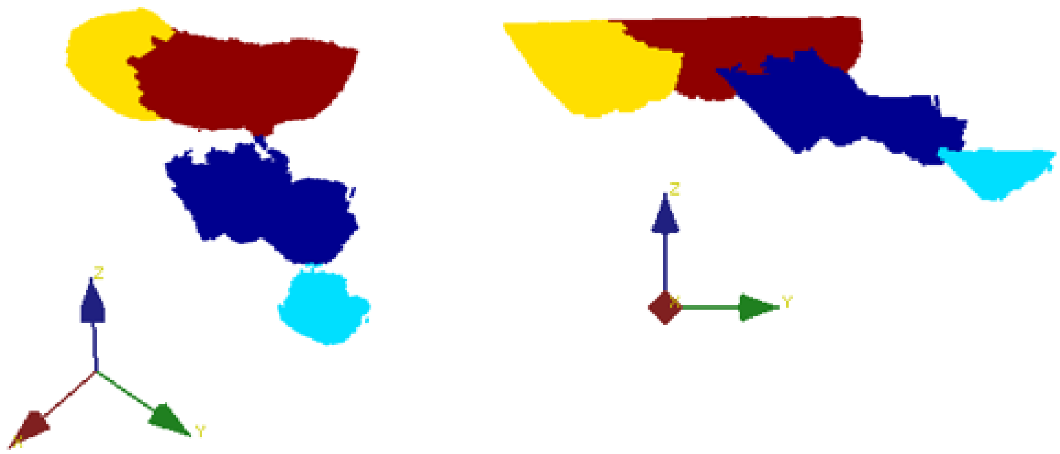

Figure 22: Mineable clusters used to seed our phase design

3.4 Growing Phases from Robust Seeds

Once the phase design has been seeded with the clusters of mineable shapes the remaining, non-mineable shapes, are added. The incorporation of the non-mineable shapes into the phase design is performed by first ordering the seeds based on their constituent shapes' average extraction period and then through applying slope/precedence constraints; adding predecessor shapes to blocks within each ordered seed. When adding predecessors, (moving up through the deposit) it is important to apply the slope constraints beginning with the youngest seed for two reasons: first, it tends to be the nearest seed to the surface and second, it tends to have the highest discounted per-block value. When adding successor shapes, the reverse is true and we begin with the oldest seed.

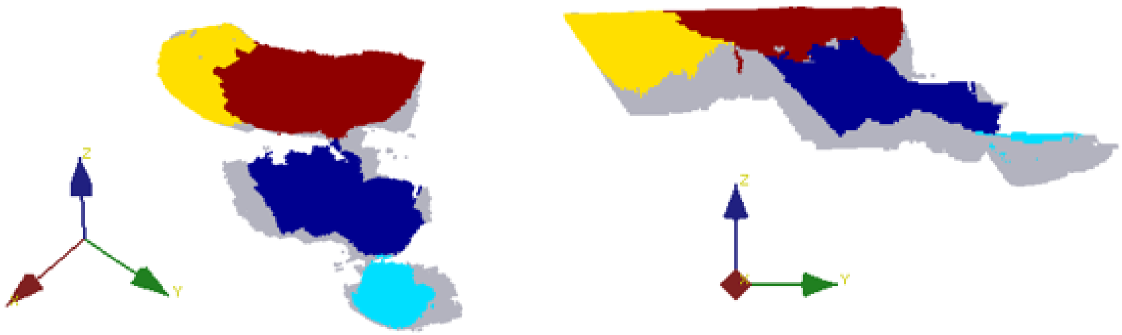

Figure 23: Phase seeds with non-mineable shapes (grey) that are to be added

Algorithm 3 which is used to add the predecessors and successors of each phase seed is presented below. For each seed there exists a list of predecessors, $preds_k$ and a list of successors, $succs_k$. These lists depend on the predecessors and successors for all the individual blocks located within each phase seed which, in turn, depend on the different geotechnical zones each having their accompanying slope constraints. The task of efficiently incorporating varying geotechnical zones is discussed in the appendix.

If a non-mineable shape can be added in its entirety to a phase without violating the slope constraints (i.e. all of its constituent blocks lie in the predecessor or successor list of a single seed), the shape is added to the phase. If it cannot be added as a whole, the non-mineable shape is disaggregated into blocks and the individual blocks are added to phases based on their predecessor/successor relationships.

Algorithm 3: Growing seeds into phases by adding remaining shapes using slope constraints

Initialization

P_k, phase seed containing the set of shapes $\zeta_k(S)$

$preds_k$, predecessors of all shapes in seed k

$succs_k$, successors of all shapes in seed k

$sortedSeeds$, sorted list of phase seeds such that $sortedSeeds[0]$ is the youngest seed

$remainingShapes$, initialized as all non-mineable shapes, S_i^n

Go through sorted phases adding predecessors and successors

```
for j = sortedSeeds[0] to j = sortedSeeds[K]
        for each shape S_i ∈ ζ_j
                if preds_k ∈ remainingShapes
                        then P_k = P_k ∪ preds_k
                        remainingShapes.erase( preds_k )
                end if
        end for
end for

for j = sortedSeeds[K] to j = sortedSeeds[0]
        for each shape S_i ∈ ζ_j
                if succs_k ∈ remainingShapes
                        then P_k = P_k ∪ succs_k
                        remainingShapes.erase( succs_k )
                end if
        end for
end for
```

Figure 24 compares the original raw schedule to the phase design generated by the proposed method. Notably, the phase design mimics the schedule well with similar portions of the schedule belonging to the same phase.

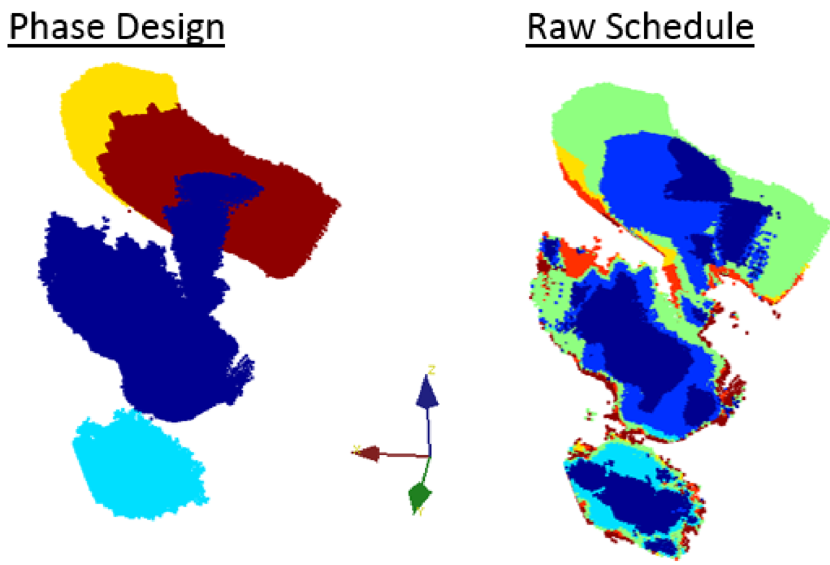

Figure 24: Schedule-based phase design compared with raw stochastic schedule

Due to the pre-defined geometries of what is already a partially-mined operation, the resulting phase design shown in Figure 24 may be fairly obvious and perhaps is not the best example to illustrate the benefits of the proposed method. The benefits of our approach are more recognizable in a second example shown in the case study below.

3.5 Post-Processing for Minimum Mining Width

After the seed phases have been grown into complete phases that incorporate all the blocks within the ultimate pit, it is still possible in certain places that the minimum mining dimensions are not respected. For this reason, a brief post-processing step is required which smooths out regions where a phase is too narrow by transferring offending blocks into the earliest adjoining phase (Wharton 1997).

There are two possibilities: either two separate phase boundaries sandwich a third phase boundary, or the space between the pit wall and a phase does not respect the minimum mining width. In the first instance offending blocks within the sandwiched phase are shifted into the earlier of the two surrounding phases, in the second instance the un-mineable portions near the pit wall are shifted into their nearest adjoining phase. In this way certain phases get slightly larger after smoothing whereas others get slightly smaller.

3.6 Evaluating and Selecting the Best Design

A first step towards evaluating the quality of the phase design that results from the method is by simple visual inspection. Cross-sections of the schedule and phase design can easily be compared and their similarity can be evaluated. However, this approach merely provides a qualitative measure. To quantify the quality of our schedule-based phase design we would need to generate a mineable extraction schedule confined within the phases and evaluate it using some metric (such as NPV or the objective function from the optimization that generated the initial raw schedule). The phase-constrained mineable schedule would most likely be some form of bench-wise schedule whereby the intersection of phases and benches are sequenced. Although this portion of the study is yet to be completed, the fact that the schedule-based phase design inherently mimics the raw schedule means the trade-off between practicality and value should lead to only a small reduction in value. It is indeed this trade-off that the proposed approach seeks to minimize.

With the assertion that the phase design's quality can be quantified, it only makes sense to see if the method can be iteratively improved upon during the initial optimization procedure. While it is likely that the optimal raw schedule will produce a good phase design and practical schedule, there is no guarantee that it will produce the *best* phase design and practical schedule. For this reason, perturbing the raw schedule is recommended, as well as the parameters used in the above method of phase design, in an attempt to find the combination that can iteratively lead to the optimal practical schedule.

A topic not contemplated is what sort of schedule perturbations would best suit the re-optimization process once an initial schedule is found. This decision is both highly optimizer and deposit-specific.

3.7 Case Study – Application of the Method to Two Different Mineral Projects

We apply the proposed method of schedule-based phase design to two orebodies with different characteristics. The first orebody has been operated for a number of years so it poses some pre-existing spatial restrictions on our phase design, the second has yet to be mined and thus represents a more typical example of the input for a long-term mine plan. Stochastic schedules (assumed to be optimal) for both orebodies have been generated and will form the main input for phase design. Cross-sections of each schedule are shown in Figure 25.

Orebody 1 Orebody 2

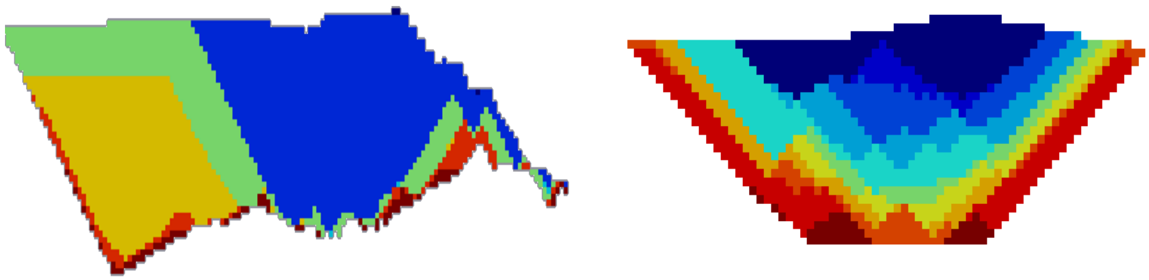

Figure 25: Cross-sections of raw schedules for two different orebodies

Step 1: The first step of the proposed method is to create mineable and non-mineable shapes from our schedules. This is done using a BFS and Algorithm 1. Figure 26 presents some the results for the two orebodies. The figure shows a non-exhaustive set of mineable shapes for each orebody; these shapes are then to be used in the creation of seed phases.

Orebody 1 Orebody 2

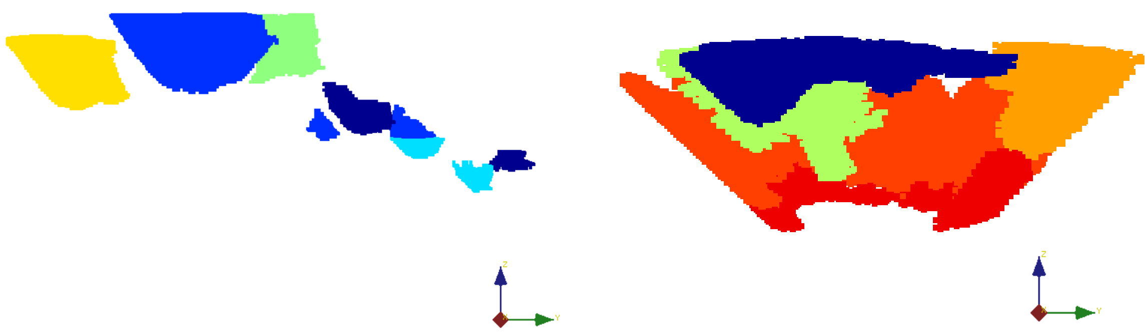

Figure 26: Some mineable shapes for each orebody

Step 2: The next step is to cluster these shapes using k-means++ and Algorithm 2 to form seed phases. The seed phases for the two schedules are shown in Figure 27. The collection of seeds contain all mineable shapes clustered based on the assigned clustering weights.

Orebody 1 Orebody 2

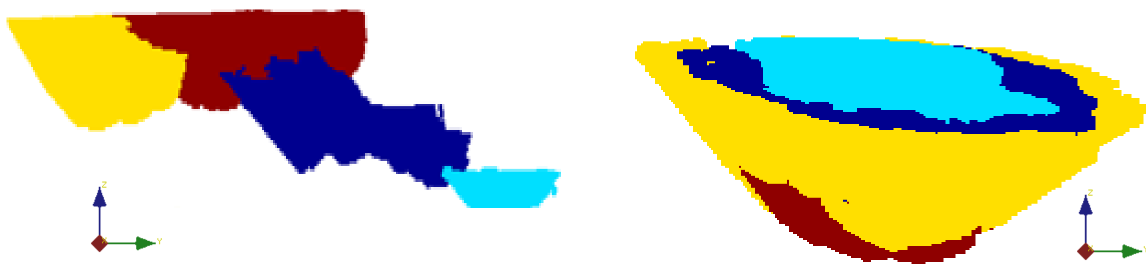

Figure 27: Initial phase seeds for each deposit

Step 3: Once the phase seeds are generated, they are grown into a final phase design by applying slope constraints, adding all remaining non-mineable shapes to the design. Comparisons between the final designs and the raw schedules for each deposit are shown in Figure 28.

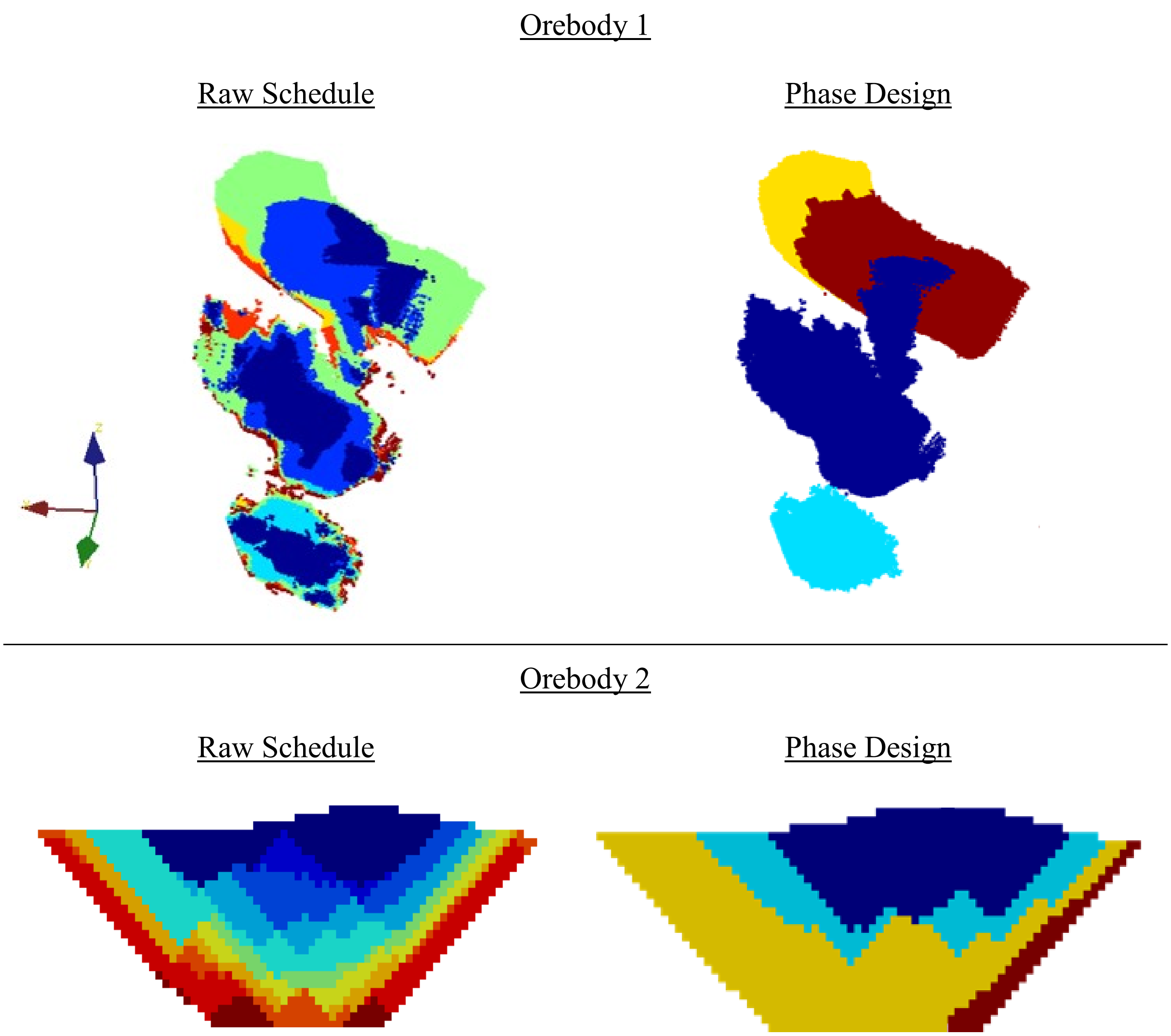

Figure 28: Final schedule-based phase design for each orebody

The above designs mimic the raw schedule well and can be shown to be mineable based on the specified minimum mining widths.

3.8 Conclusions Concerning the Proposed Method of Algorithmic Pushback Design

Traditional mine planning methods arbitrarily confine the LOM extraction schedule to pre-determined pit limits and phases, limiting the mine's optimality at the outset. A better approach is to generate an optimal schedule that serves as the basis for the physical pit design.

The proposed method for generating a schedule-based phase design seeks to minimize the trade-off between a LOM schedule's optimality and its ability to be implemented. By disaggregating the raw schedule into 4D-contiguous shapes we are able to easily retain the schedule's principal characteristics which can then be combined based on their similarity to generate mineable phases. The method has been shown to achieve a phase design that is different from the one generated using the conventional mine planning framework as seen in Figure 29, and since it is able to retain the raw schedule's features, the schedule-based design does a better job mimicking the optimal raw schedule.

Conventional Phase Design　　　　Schedule-Based Phase Design

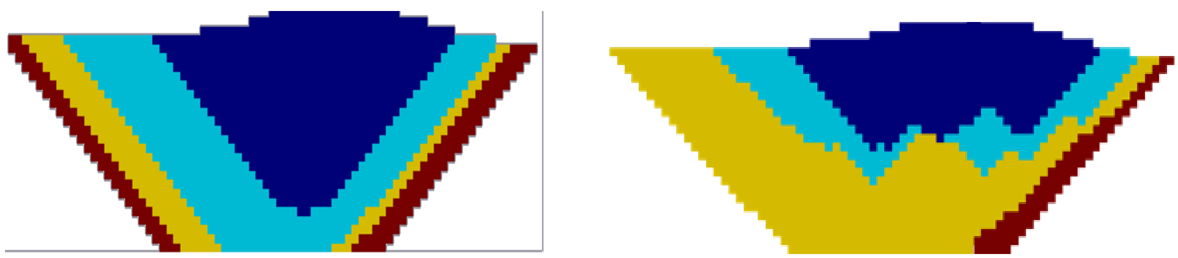

Figure 29: The schedule-based phase design differs from the traditional design

Chapter 4

Conclusions and future work

This thesis had three principal objectives. These are outlined below in order to demonstrate how the objectives have been met, what conclusions can be drawn from the results, and what further work may be undertaken to improve what was accomplished.

1. *Introduction to the field of Stochastic Mine Planning through a comprehensive literature review paying particular attention to past work of relevance to objectives 2 and 3.*

Chapter 1 provides an extensive literature review that highlights some of the drawbacks of traditional mine planning. The traditional approach is faulted for its sequential solution method and its inability to exploit connected high grade regions of the deposit. Geological simulations are introduced as a way to capture geological uncertainty. These simulations are shown to reproduce the multivariate spatial statistics of the underlying data; something that the conventional interpolation approach does not do. A review was given of early efforts of incorporating geological uncertainty in mine planning. Simulations were first used as a means of risk analysis to identify schedules that could generate value with higher certainty. The simulated annealing metaheuristic was introduced in a first effort towards using simulations directly in the scheduling process. Finally, SIPs were used to create a risk-robust schedule with high value. This approach was shown to be able to optimize the LOM extraction sequence while outputting an optimized cutoff grade policy and ultimate pit limits as byproducts. A subsequent advance in mine planning came from the application of metaheuristic optimization techniques which allowed for the modelling of non-linear interactions along the supply chain. This provoked the modelling of attributes, instead of block values, within the mathematical framework. The result was an ability to calculate the value of products, under the conditions under which they are sold, as opposed to attributing value at the block level. This concept unlocks the important value of blending.

Additionally, the concept of commodity price simulation is introduced as a method of accounting for price uncertainty. The main types of forecasting methods are highlighted along with the types of commodities that they apply to.

2. *Integrating simultaneous capacity optimization within the framework of mine complex optimization and also allowing for the ability to change how revenue is calculated depending on the terms under which products are sold.*

Chapter 2 proposes the simultaneous optimization of mining an milling capacity from the perspective of an operation in the planning stage where important capacity decisions can be made before construction begins. One of the contributions made in this chapter is allowing both capacity decisions to be made within the optimization. Past approaches at integrating capacity have always begun from a pre-constrained starting point. When comparing the LOM plan that included capacity optimization with a stochastic schedule that had fixed capacities, a significant increase in value was realized. This increase is attributable to the capacity-optimized schedule's ability to make more efficient use of the processing destinations at its disposal. Further, the tradeoff between available capacity and capital cost is explicitly accounted for in the objective function.

Metal price uncertainty and sales contract structures are also included using price simulations and by adapting the revenue calculation in the optimization model. A case study is presented whereby a gold stream applies. This example is optimized integrating this revenue structure, capital expenditure decisions, and metal price uncertainty, and the distribution of potential values of the gold stream are analysed. The results show that the model is able to account for the reduced revenue caused by the stream by decreasing equipment capacity, and further, the losses attributable to the stream are shielded through the inclusion of price uncertainty.

Further work might assess the capacity optimization of auxiliary components within the mining complex in addition to mining and processing capacities. These are typically components that are not critically related to production, but still play a part in the overall profitability of the operation. In most hard rock operations, the processing bottleneck is milling capacity, but other constraints of interest may include be: leach pad height, water use, down-stream transport, acid consumption, emission limits, fleet size fluctuation, equipment limits in various pits, or any combination of these factors. Regardless of the bottleneck, the proposed formulation can incorporate it, and its corresponding capital cost, in the optimization. Since these are not of critical importance they can be controlled through soft constraints by adding a penalty cost to the objective function equal to the incremental cost of deviating from within the target range.

3. *Generating an algorithmic approach to pushback design that best-mimics the raw schedule as output by the optimizer.*

Chapter 3 provides a method by which a practical pushback design can be built from a raw block-wise schedule. Unless burdensome smoothness constraints are included in the optimization model, the raw schedules that are typically output by the stochastic brand of optimizers are “patchy”. This lack of smoothness means that the schedules are not implementable in practice due to factors involving equipment movement and access. The method contributed in Chapter 3 attempts to preserve as much of the original schedule as possible while adhering to acceptable mineability parameters. This is performed through disaggregating the schedule into constituent “shapes”. Each shape is formed from contiguous blocks that share the same period. The method attempts to re-aggregate the shapes into pushbacks that can eventually be mined using a conventional bench-wise sequence. The method is applied to two case studies and shows promising results.

Further work is required to quantify the performance of the algorithm since, to date only a visual comparison has been performed.

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Appendix I: Formulation as per Goodfellow (2014)

Table 6: Notation used for sets and indices in the optimization model

Sets and Indices	
Set	Description
$\mathbb{P}$	Primary attributes
$\mathbb{H}$	Hereditary attributes
$\mathbb{T}$	Time Periods
$\mathbb{S}$	Joint scenarios considering all sources of uncertainty
$\mathbb{M}$	Mines
$\mathbb{B}_m$	Bocks belonging to mine $m \in \mathbb{M}$
$\mathbb{O}_b$	Predecessor blocks overlying block $b \in \mathbb{B}_m$, blocks belonging to this set must be extracted before block b
$\mathbb{K}$	Capital expenditure options
$\mathcal{C}$	Clusters of blocks with similar attributes (used in destination policy decisions)
$\mathcal{P}$	Processors in the mining complex. These cannot retain material over multiple periods; they must transfer all products in the period they are received
$\mathcal{N}$	Nodes within the mining complex, these are separated into two groups
$I(i) \subseteq \mathcal{N}$	Incoming: Nodes that destination $i \in \mathcal{S} \cup \mathcal{P}$ receives material from
$O(i) \subseteq \mathcal{S} \cup \mathcal{P}$	Outgoing: Nodes that destination $i \in \mathcal{S} \cup \mathcal{P}$ sends material to

Table 7: Optimization model variables

Decision variables	
$x_{b,t} \in \{0,1\}$	Variable that determines whether block $b \in \mathbb{B}_m$ is extracted in period $t \in \mathbb{T}$
$z_{c,j,t} \in \{0,1\}$	Variable that determines whether cluster $c \in \mathcal{C}$ is sent to destination $j \in \mathcal{O}(c)$ in period $t \in \mathbb{T}$
$y_{i,j,t,s} \in [0,1]$	Variable that determines the proportion of output material sent from location $i \in \mathcal{S} \cup \mathcal{P}$ to location in period $t \in \mathbb{T}$ and scenario $s \in \mathbb{S}$
$w_{k,t} \in \{L_{k,t,} U_{k,t}\}$	Variable that determines how many capital expenditure option increments of type $k \in \mathbb{K}$ are exercised in period $t \in \mathbb{T}$
Sate variables	
$v_{p,i,t,s} \in \mathbb{R}$	Value of the attribute $p \in \mathbb{P}$ at location (or cluster) $i \in \mathcal{N} \cup \mathbb{M}$ in period $t \in \mathbb{T}$ and scenario $s \in \mathbb{S}$
$v_{h,t,s} \in \mathbb{R}$	Value of attribute $h \in \mathbb{H}$ in period $t \in \mathbb{T}$ and scenario $s \in \mathbb{S}$
$r_{p,i,t,s} \in [0,1]$	Proportion of attribute $p \in \mathbb{P}$ recovered after processing at node $i \in \mathcal{P}$ in period $t \in \mathbb{T}$ and scenario $s \in \mathbb{S}$
$d^+_{h,t,s}, d^-_{h,t,s} \in \mathbb{R}$	Surplus or shortage values, respectively, for a deviation from the target value or range of attribute $h \in \mathbb{H}$ in period $t \in \mathbb{T}$ and scenario $s \in \mathbb{S}$

Table 8: Optimization parameters

Material flow parameters and attribute transformation functions	
$\beta_{p,b,s}$	Simulated value of attribute $p \in \mathbb{P}$ for block $b \in \mathbb{B}_m$ in scenario $s \in \mathbb{S}$
$\Theta_{b,c,s} \in \{0,1\}$	Determines if block $b \in \mathbb{B}_m$ belongs to cluster $c \in \mathcal{C}$ in scenario $s \in \mathbb{S}$
$f_h(p,i,k)$	A (Briggs et al.)-linear function used to calculate the value of hereditary attribute $h \in \mathbb{H}$ using primary attributes $p \in \mathbb{B}$ from incoming locations $i \in \mathcal{S} \cup \mathcal{P} \cup \mathbb{M}$ and capital expenditure options $k \in \mathbb{K}$
Optimization model parameters	
$U_{h,t}, L_{h,t}$	Respectively upper and lower bounds for attribute $h \in \mathbb{H}$ in period $t \in \mathbb{T}$
$p_{h,t}$	Price or cost of a unit of attribute $h \in \mathbb{H}$ in period $t \in \mathbb{T}$
$c^+_{h,t}, c^-_{h,t}$	Respectively unit surplus and shortage costs associated with deviations $d^+_{h,t,s}, d^-_{h,t,s}$ in attribute $h \in \mathbb{H}$ under scenario $s \in \mathbb{S}$ and in period $t \in \mathbb{T}$
Capital expenditure parameters	
$p_{k,t}$	Discounted purchase price for capital expenditure option $k \in \mathbb{K}$ in period $t \in \mathbb{T}$
$\kappa_{k,h}$	The per-unit increment in attribute $h \in \mathbb{H}$ added to a constraint when capex option $k \in \mathbb{K}$ is exercised
λ_k	The useful life of capital expenditure option $k \in \mathbb{K}$, once this time is up the additional capacity once provided is lost
τ_k	Lead time between when the capital expenditure decision is made for option $k \in \mathbb{K}$ and when the capacity is available (e.g. time to deliver and build)
$U_{k,t}, L_{k,t}$	Maximum and minimum purchase requirements (in terms of incremental capacity) for capital expenditure option $k \in \mathbb{K}$ in period $t \in \mathbb{T}$

Objective Function:

$$max\left\{\underbrace{\frac{1}{|\mathbb{S}|}\sum_{s\in\mathbb{S}}\sum_{t\in\mathbb{T}}\sum_{h\in\mathbb{H}} p_{h,t}\cdot v_{h,t,s}}_{\text{Expected NPV}} - \underbrace{\sum_{t\in\mathbb{T}}\sum_{k\in\mathbb{K}} p_{k,t}\cdot w_{k,t}}_{\text{Capital Expenditure}} - \underbrace{\frac{1}{|\mathbb{S}|}\sum_{s\in\mathbb{S}}\sum_{t\in\mathbb{T}}\sum_{h\subseteq\mathbb{H}} c_{h,t}^{+}\cdot d_{h,t,s}^{+} + c_{h,t}^{-}\cdot d_{h,t,s}^{-}}_{\text{Deviation Penalties}}\right\} \tag{A.1}$$

Subject to:

I. Reserve constraints ensure that a block can be mined at most once.

$$\sum_{t\in\mathbb{T}} x_{b,t} \leq 1 \quad \forall b \in \mathbb{B}_m \tag{A.2}$$

II. Slope *constraints* forbid mining a block if its overlying blocks (predecessors) are not already mined. A block's predecessor list is determined in a pre-processing step described in Apendix III.

$$x_{b,t} \leq \sum_{t'=1}^{t} x_{q,t'} \quad \forall\, b \in \mathbb{B}_m, q \in \mathbb{O}_b, t \in \mathbb{T} \tag{A.3}$$

III. *Destination policy constraints* ensure that each cluster is only sent to a single destination.

$$\sum_{j\in\mathcal{O}(c)} z_{c,j,t} = 1 \quad \forall c \in \mathcal{C}, t \in \mathbb{T} \tag{A.4}$$

IV. *Processing stream constraints* are used to track the quantities of primary attributes and to ensure all material is properly accounted for across destinations and over periods.

$$v_{p,j,(t+1),s} = v_{p,j,t,s}\cdot\left(1 - \sum_{k\in\mathcal{O}(j)} y_{i,k,t,s}\right) + \sum_{i\in I(j)\backslash\mathcal{C}} r_{p,i,t,s}\cdot v_{p,i,t,s}\cdot y_{i,j,t,s}$$
$$+ \sum_{c\in I(j)\cap\mathcal{C}}\left(\sum_{b\in\mathbb{B}_m} \Theta_{b,c,s}\cdot\beta_{p,b,s}\cdot x_{b,(t+1)}\right)\cdot z_{c,i,(t+1)}$$
$$\forall p \in \mathbb{P}, j \in \mathcal{S}\cup\mathcal{P}, t \in \mathbb{T}, s \in \mathbb{S}$$

(A.5)

V. & VI. *Material flow constraints* ensure that processor destinations do not store material over periods and that stockpiles cannot send more material than they contain.

$$\sum_{j\in\mathcal{O}(i)} y_{i,j,t,s} = 1 \quad \forall i \in \mathcal{P}, t \in \mathbb{T}, s \in \mathbb{S} \tag{A.6}$$

$\sum_{j \in \mathcal{O}(i)} y_{i,j,t,s} \leq 1 \quad \forall i \in \mathcal{S}, t \in \mathbb{T}, s \in \mathbb{S}$ (A.7)

VII. *Attribute calculation constraints* enable hereditary attributes to be calculated as a (user-defined) function of primary attributes.

$v_{h,t,s} = f_h(p, i, k) \quad \forall h \in \mathbb{H}, t \in \mathbb{T}, s \in \mathbb{S}$ (A.8)

VIII. *Primary attribute constraints* make use of the additivity of primary attributes to calculate attribute values at each destination from block values.

$v_{p,m,t,s} = \sum_{b \in \mathbb{B}_m} \beta_{p,b,s} \cdot x_{b,t} \quad \forall m \in \mathbb{M}, p \in \mathbb{P}, t \in \mathbb{T}, s \in \mathbb{S}$ (A.9)

IX. *Hereditary attribute constraints* provide the mechanism by which deviations from upper $U_{h,t}$ and lower $L_{h,t}$ target ranges are calculated. When capacity optimization is included in the model the target ranges can be influenced by the level of capital expenditure $k \in \mathbb{K}$ is exercised.

$v_{h,t,s} - d^{+}_{h,t,s} \leq U_{h,t} + \sum_{t'=t-\lambda_k+\tau_k}^{t} \kappa_{k,h} \cdot w_{k,t'} \quad \forall h \in \mathbb{H}, t \in \mathbb{T}, s \in \mathbb{S}$ (A.10)

$v_{h,t,s} + d^{-}_{h,t,s} \geq L_{h,t} + \sum_{t'=t-\lambda_k+\tau_k}^{t} \kappa_{k,h} \cdot w_{k,t'} \quad \forall h \in \mathbb{H}, t \in \mathbb{T}, s \in \mathbb{S}$ (A.11)

X. *Recovery constraints* ensure no material loss at stockpiles and allow for recovery curves to be used (a function of hereditary attributes) at processing destinations.

$r_{p,i,t,s} = 1 \quad \forall p \in \mathbb{P}, i \in \mathcal{P}, t \in \mathbb{T}, s \in \mathbb{S}$ (A.12)

$r_{p,i,t,s} = v_{h,t,s} \quad \forall p \in \mathbb{P}, i \in \mathcal{S}, t \in \mathbb{T}, s \in \mathbb{S}$ (A.13)

XI. *Stockpile balance constraints* enable ensure the balance of material into and out of the stockpiles between periods.

$v_{h,t,s} = v_{p,i,t,s} \cdot \left(1 - \sum_{j \in \mathcal{O}(i)} y_{i,j,t,s}\right) \quad \forall i \in \mathcal{S}, t \in \mathbb{T}, s \in \mathbb{S}$ (A.14)

XII. *Capital expenditure constraints* ensure that a capital expenditure option is exercised only once for one-time options, and between the lower $L_{k,t}$ and upper $U_{k,t}$ limits for multiple-purchase options.

$\sum_{t \in \mathbb{T}} w_{k,t} \leq 1 \quad \forall k \in \mathbb{K}^1 \subseteq \mathbb{K}$ (A.15)

$L_{k,t} \leq w_{k,t} \leq U_{k,t} \quad \forall k \in \mathbb{K}, t \in \mathbb{T}$ (A.16)

XIII. *Variable definitions*

$$x_{b,t} \in \{0,1\} \quad \forall b \in \mathbb{B}_m, t \in \mathbb{T} \tag{A.17}$$

$$z_{c,j,t} \in \{0,1\} \quad \forall c \in \mathcal{C}, j \in \mathcal{O}(c), t \in \mathbb{T} \tag{A.18}$$

$$\gamma_{p,c,t,s} \geq 0 \quad \forall p \in \mathbb{P}, c \in \mathcal{C}, t \in \mathbb{T}, s \in \mathbb{S} \tag{A.19}$$

$$y_{i,j,t,s} \in [0,1] \quad \forall i \in \mathcal{S} \cup \mathcal{P}, j \in \mathcal{O}(i), t \in \mathbb{T}, s \in \mathbb{S} \tag{A.20}$$

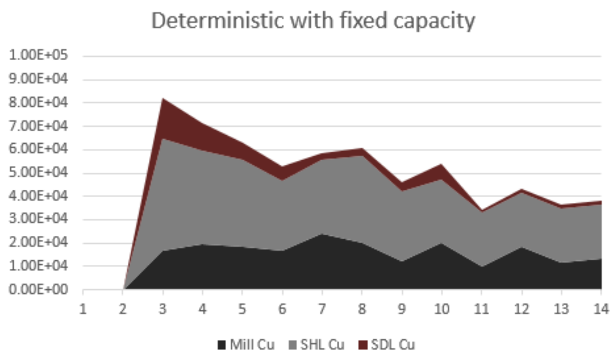
$$r_{p,i,t,s} \in [0,1] \quad \forall p \in \mathbb{P}, i \in \mathcal{S} \cup \mathcal{P}, t \in \mathbb{T}, s \in \mathbb{S} \tag{A.21}$$

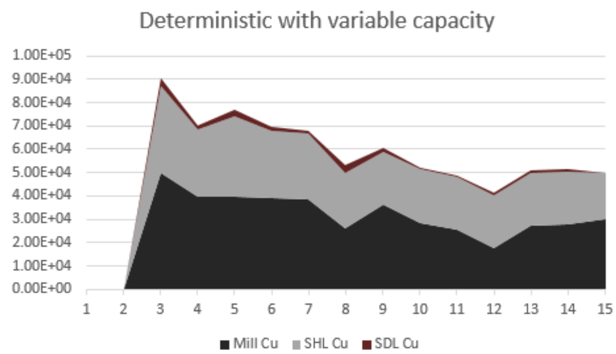
$$v_{p,i,t,s} \geq 0 \quad \forall p \in \mathbb{P}, i \in \mathcal{S} \cup \mathcal{P} \cup \mathbb{M}, t \in \mathbb{T}, s \in \mathbb{S} \tag{A.22}$$

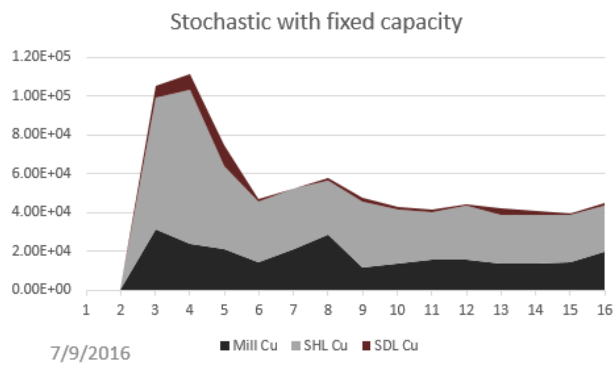
$$v_{h,t,s} \in \mathbb{R} \quad \forall h \in \mathbb{H}, t \in \mathbb{T}, s \in \mathbb{S} \tag{A.23}$$

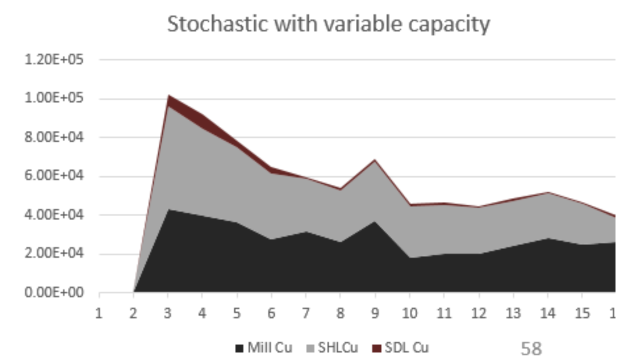
$$d^{+}_{h,t,s}, d^{-}_{h,t,s} \geq 0 \quad \forall h \in \mathbb{H}, t \in \mathbb{T}, s \in \mathbb{S} \tag{A.24}$$

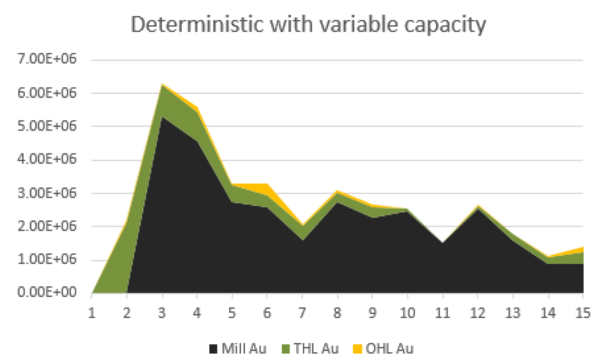
Appendix II: Material Flow of Optimized Schedules

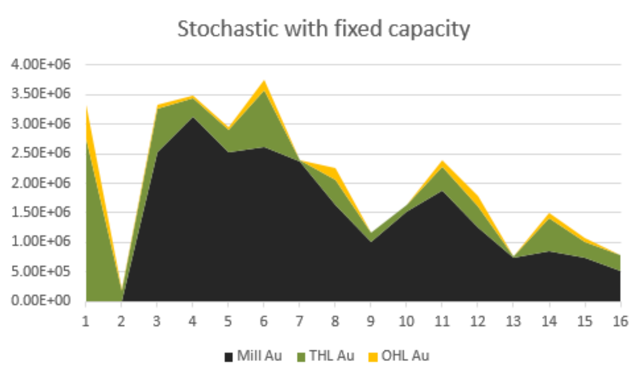


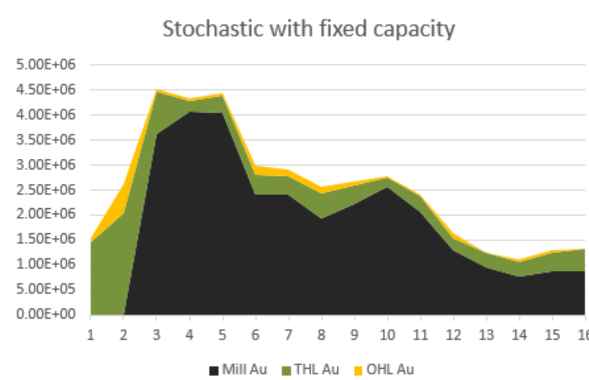


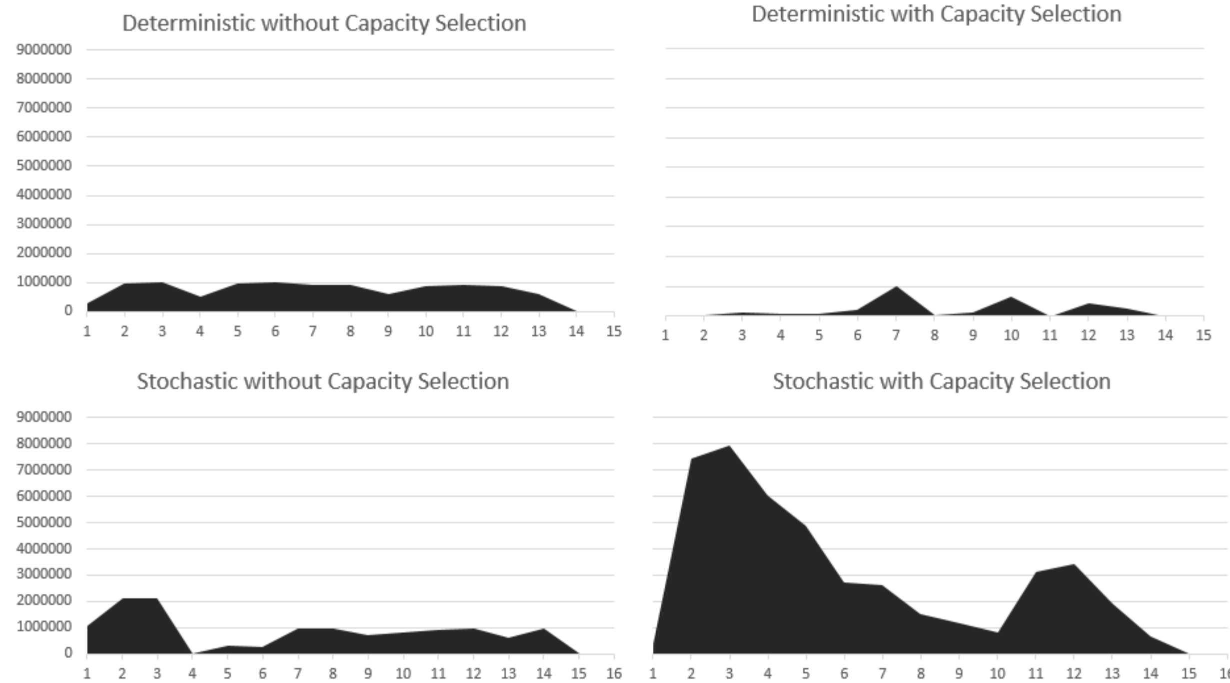











Deterministic without Capacity Selection
Deterministic with Capacity Selection
Stochastic without Capacity Selection
Stochastic with Capacity Selection
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1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

Appendix III: Efficient Algorithm for Generating Predecessor and Successor Sets

In any orebody there may be a number of geotechnical zones with various rock competencies. In more competent rock pit-wall slope angles can be steeper allowing for a lower strip ratio, whereas in poorer-quality rock shallower slope angles are required and strip ratios increase. These varying geotechnical zones determine slope constraints which dictate which overlying blocks (predecessors) must be mined before access can be granted to a target block (the successor).

As noted above, the procedure by which non-mineable shapes are incorporated into the seed phases requires a list of predecessors and successors for each block in the orebody model. When using metaheuristic optimization methods these same lists are also required when generating the raw schedule since a block's predecessors and successors determine how perturbations are allowed to proceed. Given that many perturbations are likely needed to reach a good solution, the most efficient representation of these lists is desired.

The exhaustive list of predecessors for a given block resembles a cone with an elliptical cross-section. The shape of the ellipse is determined by the minimum slope angles in four given directions. Instead of using the full predecessor cone, it is more efficient (in terms of computational time and memory) instead to store a more minimal set of predecessors for each block and algorithmically move up through the deposit while checking, when needed, this smaller set. The minimal predecessor set resembles an enveloping shell as shown in Figure 31. In some places the shell is thicker than others due to the interplay between the predecessor relationship and geotechnical variability. For example, in the simple case where we only consider a block and its overlying block; the predecessor envelope will become thinner as the minimum slope angles of the two blocks become more alike.

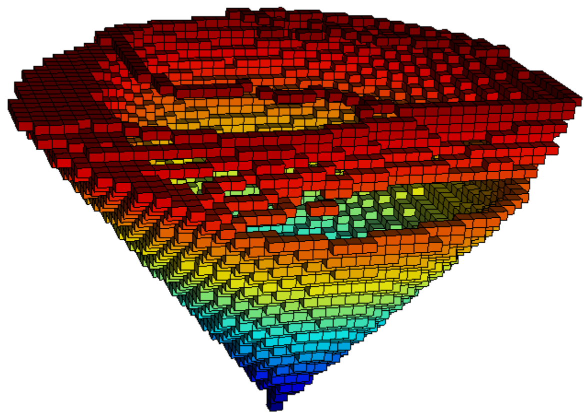

Figure 30: The predecessor envelope for a single block

Once each block's predecessor envelope is generated, it can be stored in a text file and accessed whenever needed. In this way, the predecessor list only has to be generated once and the algorithm for doing so can remain separate from the rest of the optimization/mine planning procedure. The following algorithm illustrates the process by which predecessor envelopes are generated.

Algorithm 4: Generating predecessor envelopes for deposits with varying geotechnical zones

Initialization

A, number of different geotechnical zones with angles α_A

$fullCone[a]$, the full cone of predecessor blocks for a block in zone a

Find full set for every zone

$fullCone[a] = getFullCone(a)$, as in Khalokakaie et al. (2000)

Perform set-subtraction with overlying block to generate minimal set

for each block u (descending z-coordinate)

$F(u) = fullCone[zone(u)]$

for each block $v \in F(u)$

if any $\alpha_a^v < \alpha_a^u$

$fullSet(u) \cup fullCone(v)$

'adding predecessors of predecessors if the block has any wider slope angle

end if

end for

$setSubtraction(u) = fullSet(u) \cap fullSet(u_{k+1})$

'set-subtraction with overlying block

end for

Add back "rings" to complete enveloping cone

for each block u

for each layer z in $fullCone(u)$

$rings(u) = rings(u) \cup extremeBlocks(z, fullCone(u))$

end for

$predEnvelope(u) = setSubtraction(u) \cup rings(u)$

end for

Now, in order to take best advantage of the envelope predecessor representation, we must generate an algorithm that will allow us to rigorously check for all possible violations as efficiently as possible. The method for doing so is illustrated in Algorithm 5.

Algorithm 5: Efficiently checking slope violations (R. Goodfellow, 2015)

<u>Identify and repair slope violations</u>

Stack $currentBlock = \{b\}$
'block who's predecessors are being examined

$blocksToChange = \{b\}$
'set of violating blocks

while $currentBlock \mathrel{!}= \varnothing$
    $u = currentBlock.$pop()
    $needsFixing =$ false

    for each $\{v\} \in preds[u]$

        if $period[v] \le period[u]$: continue
        'block v is ok
        end if

        $needsFixing =$ true
        $blocksToChange = blocksToChange \cup \{v\}$
    end for
    if $needsFixing ==$ true, $currentBlock.$push($u(x, y, z+1)$)
    'explore block above
end while