SYSTEM OF SYSTEMS APPROACH TO AIR TRANSPORTATION DESIGN

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Abstract

Aircraft sizing, route network design, demand estimation and allocation of aircraft to routes are different facets of the air transportation optimization problem that can be viewed as individual systems, since they can be conducted independently. There is a large body of literature that investigates each of these as a stand-alone problem. In this regard, the air transportation design optimization problem can be viewed as an optimal system-of-systems (SoS) design problem. The resulting mixed variable programming problem may not be solvable using an all-in-one (AiO) approach because its size and complexity grow rapidly with increasing number of network nodes. A decomposition-based nested formulation and the Mesh Adaptive Direct Search (MADS) optimization algorithm are presented to solve the optimal SoS design problem. The two-stage expansion of a regional Canadian airline's network to enable national operations is considered as a demonstrating example.

Résumé

La taille des avions, les réseaux d'itinéraire, l'estimation de la demande et l'affectation des avions aux itinéraires sont différentes facettes d'un problème d'optimisation de transport aérien, pouvant être considérés comme des systèmes individuels puisqu'ils peuvent être résolus indépendamment. Un nombre important de travaux étudie chacune de ces facettes comme un problème isolé. Dans cette optique, le problème présenté peut être vu comme l'optimisation d'un système de systèmes. Le problème couplé en résultant n'est pas solvable en utilisant une approche tout-en-un, car sa taille et sa complexité augmente rapidement avec le nombre de noeuds du réseau. Une formulation par décomposition, et l'optimisation par un algorithme de recherche directe par maillage adaptatif (Mesh Adaptive Direct Search, MADS) sont présentées pour le résoudre. L'expansion en deux étapes d'un réseau aérien régional canadien vers un réseau national est considérée à titre d'exemple.

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Contents

List of Figures

List of Tables

CHAPTER 1

Introduction

1.1 System of Systems

The world is composed of an innumerable number of systems - from computers to automobiles. Before we introduce 'System of Systems', it is important to understand what constitutes a 'system'. The International Council on Systems Engineering (INCOSE) Systems Engineering Handbook [1] defines systems as 'a combination of interacting elements organized to achieve one or more stated purposes'. Thus a system can be anything from a simple pulley mechanism to a large commercial aircraft.

With the increase in complexity of systems, the need has arisen for developing a holistic perspective to system development. This need for a composite design process forms the basic setting for a System of Systems (SoS) approach.

While there is no single universal definition of the term 'System of Systems' several characteristics of what constitutes an SoS have been proposed. Jamshidi [2] defines SoS as a 'class of complex systems whose components are themselves complex'. The INCOSE handbook defines an SoS as 'an inter-operating collection of component systems that produce results unachievable by the individual systems alone'. This definition is very similar to that of 'complex systems' and therefore implies that SoS is a complex system. Other definitions in literature emphasize the independent operational capability of individual systems that constitute an SoS. The term System-of-Systems (SoS) can thus be used to describe a large system composed of multiple individual systems capable of independent operation, that together provide capabilities beyond those of individual constituent systems. Maier [3] prescribes five properties a majority of which must be satisfied for a system of systems: operational and managerial independence, geographic distribution, emergent behaviour and evolutionary development.

1.2 Air Transportation

Air transportation is a complex field that involves multidisciplinary objectives and constraints. Since the deregulation of airlines in America in late 1970s, the airline industry has become increasingly competitive. Revenue margins have shrunk and airlines have to ensure lean, cost-effective operation to stay in business. Massoud Bazargan [4] describes how the current US airline industry illustrates the 'survival of the fittest' motto. Operations research as well as system design (aircraft, airports etc) have been key factors in ensuring that their financial goals are met.

Operations research for airlines has been a key focus area since 1950s. The advances in technology - design tools, manufacturing processes, new materials - and computing power have allowed more complex problems to be tackled in shorter times resulting in an enormous impact on airline managing and planning operations. Similarly, aircraft design has also improved immensely during the last three decades.

Traditionally, majority of research in this area has focused on discrete areas of airline operations - operations, aircraft design. Given the complex nature of air transportation, a broader, cumulative approach to tackling the problem might be more beneficial. This can be effected through a systems-of-systems methodology. The separate problems are simultaneously considered for modelling and characterizing the additional functionalities provided by the SoS. A few of these SoS models are discussed in the next section.

1.3 Air Transportation Design

The different elements of air transportation SoS are shown in Fig. 1.1. Each of these individual systems is capable of independent operation and can be optimized independently. The application of Maier's criteria to the air transportation problem is reported in Table 1.1.

The goal of this research is to model an air transportation system as an SoS. We want to use the SoS model to identify key steps required for airline expansion. This involves identifying possible areas of expansion (airports, routes) and designing/acquiring new resources (aircrafts, airports) for servicing the increased demand. There are several practical considerations that limit the choices available for each system (sub-problem) - these include explicit constraints related to system design such as aircraft performance characteristics, airport landing facilities and also more abstract constraints related to airline competitiveness, demand forecasting etc. It is our aim to build a mathematical

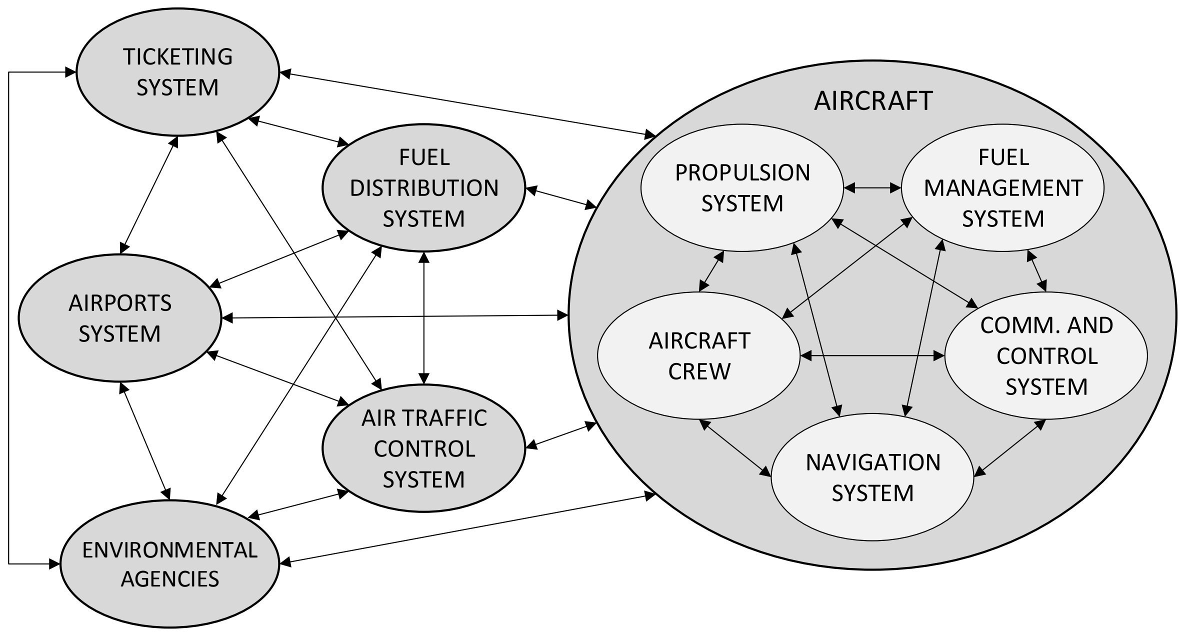


Figure 1.1: Components of an air transportation system

model than can capture the dynamic nature of these interactions and their effect on the emergent behaviour of our system.

The air transportation SoS is assumed to consist of the following systems:

1. *Demand Estimation* Like most service industries, airline operations and revenues are highly dependent on passenger demand. Further, airlines operations require extensive investment in terms of infrastructure development, capital expenditure in acquiring resources etc. Given the scale of these operations, airline companies plan for long term operations. Demand estimation plays an important role in this planning. It is desired to incorporate a realistic forecasting model into the SoS to obtain practical results.

2. *Network Design* The airline routes can be modeled as a network, the structure of which greatly impacts airline operations. Designing a network calls upon the areas of graph theory, complex network theory and multi-commodity flow networks. Further, constraints specific to the airline transportation need to be defined in order to obtain a solution that can be applied to practical scenarios.

3. *Aircraft Design* The aircraft design problem is a complex problem in itself - Sobieszczanski and Agte [5] used the MDO SoS approach for the aircraft design problem and it has since been studied extensively in current literature. The different aspects

Table 1.1: Maier's criteria applied to air transportation problem

Criterion	Equivalence in current problem & description
Operational Independence	The aircrafts, route network design algorithm, the scheduling authority and the allocation algorithm are all capable of independent functioning
Managerial Independence	Shared variables are exchanged for the system-wide optimization algorithm but each system is a discrete functional entity capable of independent operation that can reach its optimal state individually
Evolutionary Development	The SoS can evolve in response to change in needs over time and as new technologies become available
Emergent Behavior	The system-wide coordination between different subproblems leads to an improved performance compared to individual systems acting alone
Geographic Distribution	The different components of the SoS are geographically discrete - airports are fixed while the aircrafts, the scheduling authority occupy variable geographic locations. The network design system, allocation system are computer algorithms which do not occupy physical space

of aircraft design include aerodynamics, aircraft performance, structural mechanics and engine performance. Generally, the airline company does not have a significant role to play in aircraft design and it is undertaken solely by the manufacturer.

4. *Aircraft Allocation* Aircraft allocation involves assigning aircraft to different routes in the network to satisfy passenger demand. It is a complex exercise that forms a significant part of an airline's operations and can have a considerable impact on its revenues. Several models have been suggested in literature to solve the aircraft allocation problem.

Each of these systems and the corresponding formulations are discussed in more details in following chapters.

1.4 Theoretical Background

Transportation problems have been studied extensively in the literature. Air transportation systems form a growing subset in this area of study. Several formulations have been proposed that differ in the different systems/sub-systems considered each with their own assumptions. We study these approaches in an attempt to arrive at a general model for air transportation.

Mane, Crossley and Nusawardhana [6] formulated a ‘variable resource allocation’ problem that involved finding an appropriate mix of yet-to-be-designed and existing aircraft. They employed a multidisciplinary design optimization (MDO) based decomposition approach for solving the coupled aircraft design and allocation problem for a fixed network configuration (hub-spoke) of increasing sizes. Mane et al. reported that: i) decomposing the problem provides a computational advantage over conventional methods, and ii) the computational advantage becomes more significant as problem size grows. They further extended their approach to incorporate uncertainty in operations for a problem with only yet-to-be-designed aircraft for a fractional management company [7]. For both [6] and [7], the aircraft design variables are passenger capacity, aspect ratio, wing loading and thrust to weight ratio.

Similar to Mane et al., Taylor and De Weck [8] proposed a method that coupled vehicle (aircraft) design and network flow (allocation) through a multi-disciplinary design optimization (MDO) approach. Both vehicle design and network flow are treated as ‘subsystems’ that are optimized to minimize overall operating cost. Their method differs from Mane et al. in terms of i) formulation of the two problems and ii) how they are coupled. Aircraft design is defined empirically in terms of range, passenger capacity, cruise velocity, wing loading, thrust-to-weight ratio and the number of engines. A simplified model, similar to [9], is used for vehicle allocation model.

A combined vehicle design and allocation problem is also solved by Hidalgo and Kim [10]. They use analytical target cascading (ATC) to formulate the problem. A key difference in their approach from Mane et al. and Taylor et al. lies in the treatment of vehicle routing problem - allocation for larger networks is broken down into smaller subproblems in their formulation. They solve an eight-route problem by decomposing it into four subproblems with two routes each.

The works described above illustrate the benefit of coupling vehicle design and operations for air transportation systems. Airline operations are intrinsically linked with passenger demand and it is interesting to study this relationship. Davendralingam and

Crossley [11] analyze the impact of passenger demand in an air transportation system. They formulate a recursive model for airline operations and passenger demand that builds upon [6, 7] by accounting for changes in demand and ticket prices over time. The demand is obtained from an econometric model based on the Federal Aviation Administration models.

Majority of air transportation models proposed in literature are based on a known static network configuration. Liu et al. [12] study the network structure for a real-world transportation problem using a network theory approach. They employ a genetic algorithm for optimizing network properties in order to facilitate airline operations in Europe.

Similar to Liu et al., Kotegawa [13] analyzes air-transportation SoS using network theory. He formulates a model to study evolutionary behavior in air transportation networks in US based on network properties and machine learning. Network topology models are investigated and applied to a multilayer network with distinct levels for demand, mobility and capacity. The approach is used to predict addition and removal of routes in an air-transportation with greater accuracy compared to existing approaches, using historical data for calibration. Kotegawa's model underscores the impact of network configuration on different aspects of airline transportation.

Ayyalasomayajula [14] investigates further into the airline networks and operations for geographic locations with airports in close proximity to each other (metroplexes). Metroplexes are studied for influence of competing airports on passenger demand - this is particularly useful for dense passenger networks, e.g. Europe and Western Canada.

Each of the above mentioned studies investigate key aspect of airline transportation. The goal of this research is to build upon these methods and develop a general SoS model for air transportation SoS.

1.5 Thesis Organization

This thesis is organized as follows. The next chapter provides an overview of the conventional approach to air transportation design and the system of systems design problem formulation. Thereafter, different components of the SoS problem are introduced. Chapter 3 describes the demand estimation problem for air transportation. Section 3.2 discusses the principal demand models for air transportation. The SoS demand estimation model and calibration results are given in Section 3.3. A brief introduction to the aircraft sizing problem is given in Chapter 4. Different components of the sizing problem are described in Section 4.1. Sections 4.2 and 4.3 contain a description of the sizing tool for the SoS problem and the corresponding results. The nested network design problem and aircraft allocation are discussed in Chapter 5 and Chapter 6 respectively. Chapter 7 provides a brief introduction to derivative-free optimization using MADS - a direct search algorithm. MADS is used for the outer-loop SoS problem (discussed in Chapter 2.2) and the network design problem. The results for the SoS air transportation problem are presented in Chapter 8. Section 8.1 and Section 8.2 discuss the results for the different stages of airline expansion described in Chapter 2.2 alongwith comparisons to models suggested in literature and actual data. The last part of this thesis, Chapter 9 summarizes the SoS problem and discusses the scope of future work.

CHAPTER 2

Air Transportation System Design

In this chapter, we discuss the conventional approach to air transportation design and then propose a system of systems design problem formulation.

2.1 Conventional Approach

Traditionally, majority of the research in the area of air transportation design has focused on improving performance in individual disciplines separately by treating extra-disciplinary quantities as fixed parameters. Each of the elements discussed in Section 1.3 are studied and optimized separately. For example, allocate aircraft to routes assuming fixed aircraft design and route network configuration. To a large extent, this also holds true for actual airline practices. Airline companies purchase off-the-shelf stock aircraft from manufacturers and fly these aircraft along fixed routes. This approach is not optimal since the coupling between different elements of air transportation can impact airline operations. Bazargan [4] discusses how increased competition in the airline industry has led to shrinking profit margins. Return on invested capital (ROIC) is used as a benchmark for measuring an industry's profitability. Studies such as [15] indicate that the airline industry has one of the lowest reported ROIC - an argument that favors a more detailed analysis of different components of airline operations.

The system of systems approach provides a means to build a comprehensive airline transportation model. This can allow airline companies to collaborate with manufacturers to design aircraft that better meets their needs and plan their operations more efficiently. Information about actual airline practices is not available publicly and a good approximate is obtained from publications such as [15] and existing research. A brief discussion about existing research was presented in Section 1.4 which is continued below.

Conventional aircraft design studies focus only on designing the best aircraft for achieving a given objective - most commonly to minimize operating cost or fuel consumption e.g. [5]. The impact of existing resources has only been investigated by Mane et al. [6].

Other works in literature study the combined aircraft design and allocation problem. Taylor and de Weck [8] consider aircraft design and allocation in their problem but their problem does not include existing aircraft fleet. Jansen and Perez investigate [16] solve the coupled aircraft design and fleet allocation problem under uncertain demand to minimize the environmental impact of the airline. Their work involves coupled aircraft design and allocation but does not consider existing fleet or network design. They extend their approach in [17] for coupled design and optimization of an aircraft belonging to a given family, to minimize fuel burnt and cost of allocation. Lastly, they simultaneously optimize the airframe design and aircraft trajectory [18].

The structure of airline networks is another area that has not been considered in existing research. Airline networks evolve over a period of time in response to changing business needs and external factors. Hub-spoke models are the most commonly adopted network topology for dense networks with competing airlines. These networks offer limited control over travel itineraries, and consequently, the capacity utilization of airline resources. Yang and Kornfeld investigated the optimality of hub-spoke networks in context of an overnight package delivery system [20]. Although their work does not include aircraft design, their findings indicate that it has significant impact on the optimal network structure in addition to other factors such as city location and cargo demand. They report how the optimal network structure changes from hub-spoke configuration to a point-to-point network (fully connected) and back to hub-spoke configuration in response to these factors. This is similar to [13, 14] and underscores the changing nature of optimal network configuration. An example of this can be found in mature markets (North America, Europe) that are witnessing a greater proliferation of low-cost carriers (LCCs) [21, 22]. The increased competition has an impact on profitability that can cause airlines to modify their operations along different routes (in terms of flight frequency and total revenue-passenger-miles[1]). A system of systems approach that can help determine the best network configuration considering existing resources. This becomes especially useful as newer technologies (for aircraft design) become available.

Operations planning in airline industry has improved significantly with the advent of increased computing power. Allocation of aircraft to different routes along with scheduling is the end-point of airline operations that is directly related to airline revenue. A system of systems approach can be used to couple allocation with the other systems discussed above and optimize operations to yield savings.

[1]Revenue-passenger-miles or passenger-air-miles is a fundamental unit of measuring air traffic. It is obtained as the product of an aircraft's passenger capacity (revenue seats), no. of trips and route distance summed for the section of aircraft fleet being considered.

2.2 System of Systems Formulation

In order to illustrate the concept of system of systems applied to the airline system design and resource allocation problem, we consider a sample problem in which a Canadian regional carrier wishes to expand its operations to become a nationwide carrier. The airline originally operates in five cities in western Canada: Vancouver, Kelowna, Edmonton, Calgary and Victoria and expands to a network of fifteen cities.

We begin by first simplifying the air transportation model of Fig. 1.1. We omit the environmental agencies, air traffic control system and the fuel distribution system as shown in Fig. 2.1. The aim of this exercise is to arrive at a simplified model that captures all essential aspects of air transportation to which additional systems (e.g. environmental agencies) can then be added for a more comprehensive model.

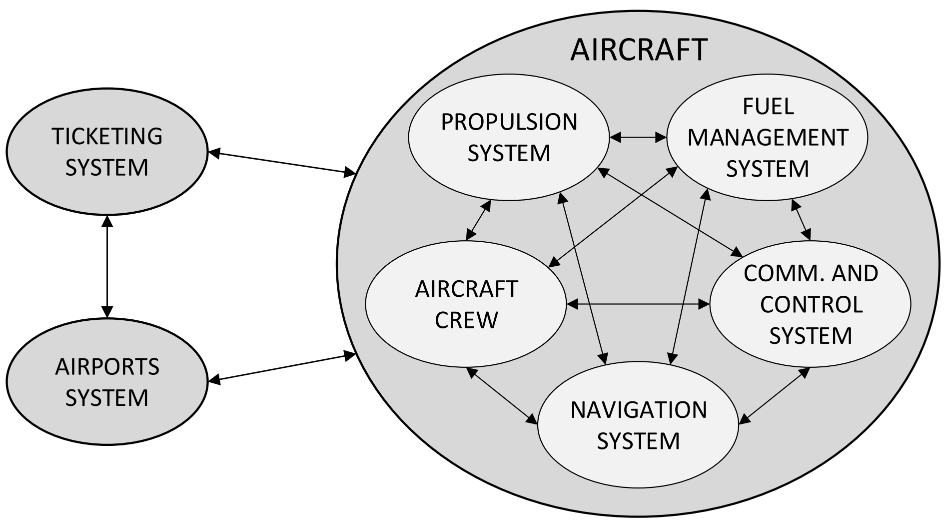


Figure 2.1: Simplified air transportation problem

In order to expand operations, the airline needs to accomplish the following:

1. *Estimate demand:* The airline needs to estimate the increase in demand due to addition of new cities to the network.

2. *Acquire new aircraft:* The airline needs to purchase new aircraft that can fly longer routes and to expand its total seat capacity to service demand in the extended network.

3. *Plan operational routes:* The airline needs to plan which routes will be operational in the extended network.

4. *Allocate aircraft:* Finally, the airline needs to allocate its fleet along the operational route to meet the total demand.

We consider a two-stage expansion for the airline. Figure 2.2 shows the original five cities in the network. In the first stage we add two cities to the network as shown in Fig. 2.3. The network is further expanded to include fifteen cities nationwide in the next stage - Fig. 2.4.

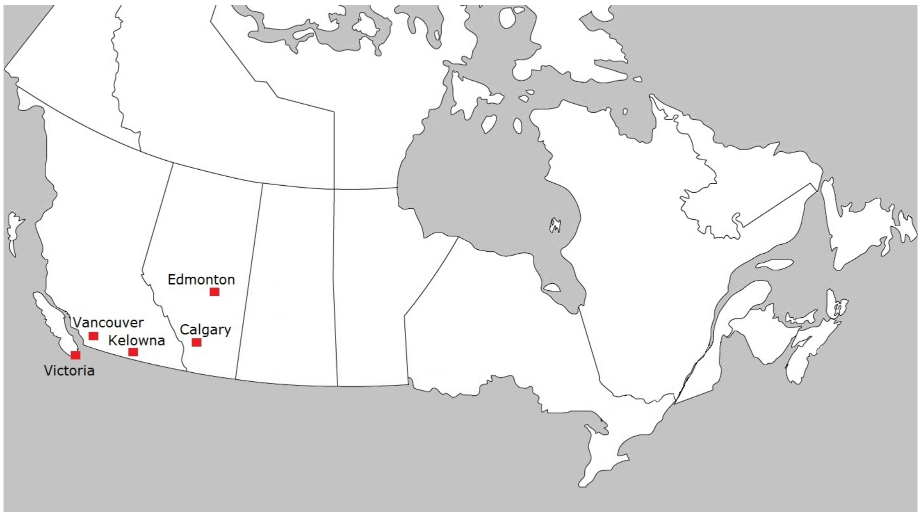


Figure 2.2: Regional network

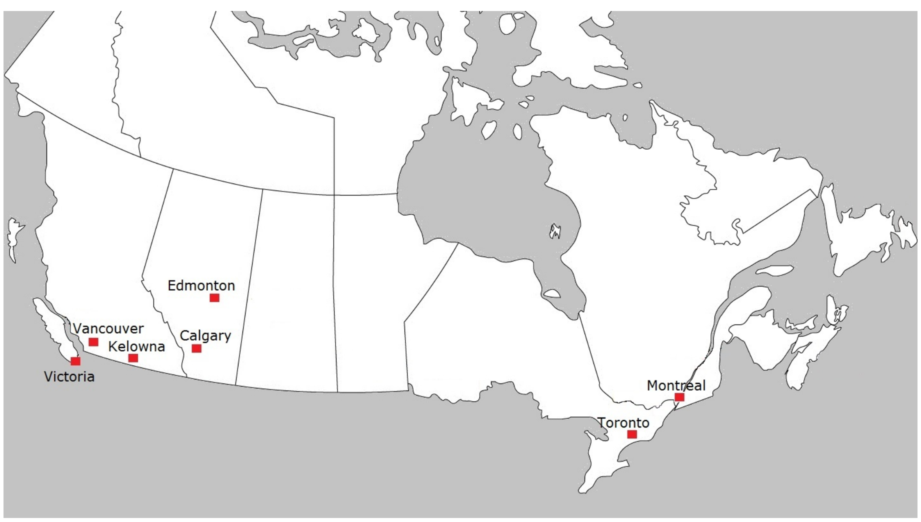


Figure 2.3: Stage 1: expansion to seven cities

The airline is assumed to have an existing fleet which is comprised of two aircraft types, A and B, whose range and passenger capacity are reported in Table 2.1.

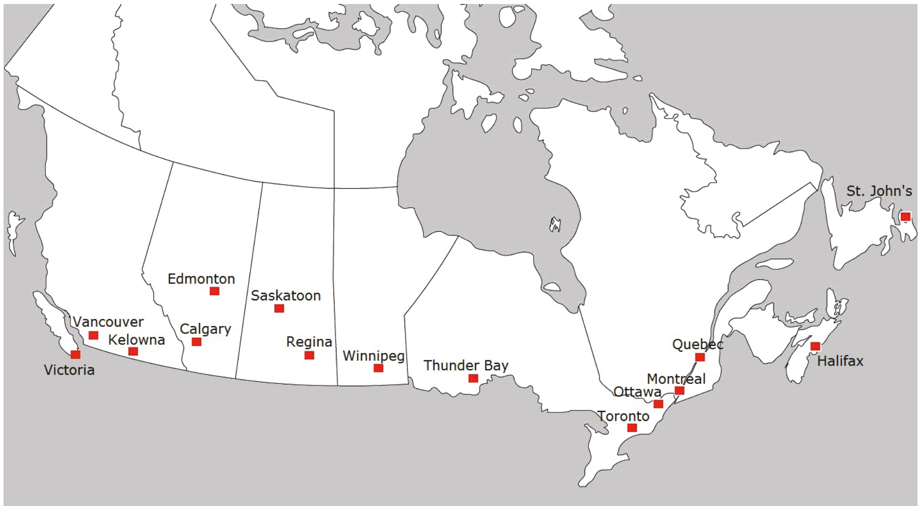


Figure 2.4: Stage 2: expansion to fifteen cities

Table 2.1: Range and passenger capacity of existing aircraft

Aircraft Type	Range (nmi)	Passenger Capacity
A	2000	100
B	2400	140

We are interested to model all aspects of the air transportation mentioned above and depicted in Fig. 2.1, namely, demand estimation, aircraft design, network design (airports system) and aircraft allocation (ticketing system). A systems of systems approach is implemented using the formulation shown in Fig. 2.5. It is different from [6, 7, 8] in terms

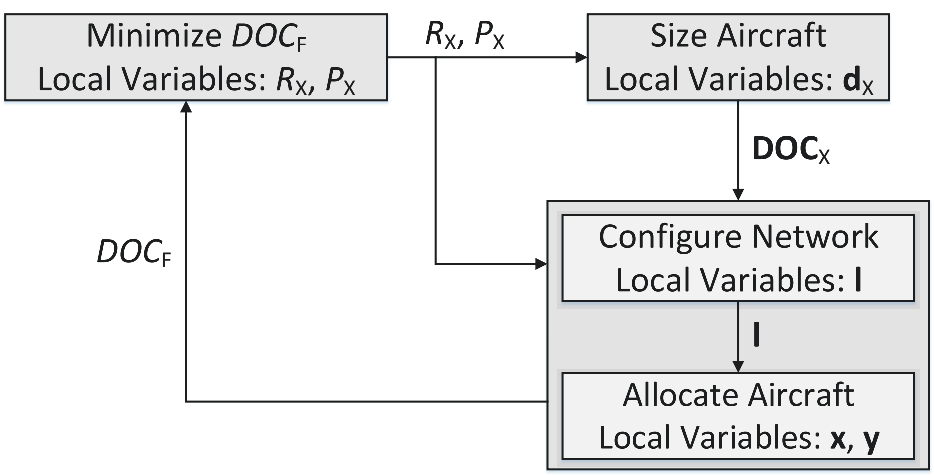


Figure 2.5: Systems of systems model for airline transportation

of: i) treating range of new aircraft as an additional SoS outer-loop design variable, and ii) the inclusion of network design as a coupled system.

The SoS optimization problem is solved using a nested formulation. The SoS outer-loop optimization problem minimizes direct operating cost of the fleet (DOC_{F}) and has range R_{X} and passenger capacity P_{X} of the new aircraft as its design variables. This objective is a surrogate for the more realistic objective of maximizing profit, as the latter requires revenue and pricing models based on data that are usually proprietary. For each iteration of the SoS outer-loop optimization problem, the algorithm performs aircraft sizing and solves the network design and aircraft allocation problems using a nested structure. Demand along different routes in the network is a fixed parameter in this model.

Passenger demand estimates along each route are obtained using the 'gravity model' formulation described in Section 3.3. The demand estimation problem has all continuous variables which are obtained using nonlinear least squares. The aircraft design problem has three design variables that are continuous and are bounded at both ends. An inequality constraint is included in the aircraft design problem for takeoff and landing field length considerations. The mathematical formulation for the aircraft design problem is presented in Section 4.3.

The network design problem has all binary variables with four inequality constraints. The number of variables increases with increase in network size and additional constraints are added for the larger second stage of airline expansion - described in Section 8.2. The allocation problem is a linear program with all integer variables with inequality and equality constraints. The allocation problem formulation is described in Chapter 6.

The SoS outer-loop problem and the network design problem are solved using the Mesh Adaptive Direct Search algorithm described in Chapter 7. The aircraft sizing problem is solved as an optimization problem using NASA developed Flight Optimization System as explained in Chapter 4. The aircraft allocation problem is solved using GNU Linear Programming Kit (glpk) and IBM ILOG Cplex for the seven-cities and fifteen-cities problems, respectively. Each of these systems is described in more detail in the following chapters.

CHAPTER 3

Demand Estimation

Airline transportation involves flow of goods and/or people from one place to another. Demand forecasts are used by airline companies to predict this movement. Accurate forecasts are, therefore, crucial to successful airline operations. As first step towards solving the air transportation problem, we present a model for demand estimation.

3.1 Background

Rodrigue [23] terms flow of demand between two locations as a spatial interaction, defined as 'realized movement of people, freight or information between an origin and a destination. It is a transport demand/supply relationship expressed over a geographical space. He goes on to define three independent conditions necessary for a spatial interaction to occur:

1. *Complementarity:* there must be a supply and a demand between the interacting locations. For example, residential zones and industrial zones are complimentary.

2. *Intervening opportunity:* there must not be another location that may offer a better alternative as a point of origin or a point of destination - this follows from simple laws of economics.

3. *Transferability:* Freight, persons or information being transferred must be supported by transport infrastructures.

The problem presented here deals with domestic civil aviation and therefore only passenger movement is considered. The following section offers a brief introduction to the demand estimation models available in literature and the corresponding selection criteria.

Based on the type of interactions considered for demand estimation, models can be broadly classified into two categories:

1. Models that take into account the effect of neighboring cities, such as the Potential model [23].

2. Models that ignore the effect of neighboring cities, such as the Gravity model [24].

Mathematical formulations for both model types take into account socio-economic attributes and location attributes of participating destinations. The general form is given by

$$
\begin{aligned}
T_{ij} &= f(V_i, W_i, S_{ij}), \qquad (3.1)\\
\text{where} \quad T_{ij} &= \text{interaction between location } i \text{ and location } j,\\
V_i &= \text{attributes of location } i\\
W_j &= \text{attributes of location } j\\
S_{ij} &= \text{attributes of separation between location } i \text{ and location } j.
\end{aligned}
$$

The attributes for either location can include socio-economic and geographical attributes such as population, gross domestic product and industrial output. These attributes are explained in more details in section 3.2.

Figure 3.1 illustrates the difference in the formulation of the types of models explained above. Another basic interaction model - retail model - is presented with the mathematical formulation. Although it does not give the interaction directly, it gives the boundary B_{ij} of two locations competing over the same market which is used for subsequent demand estimation analyses.

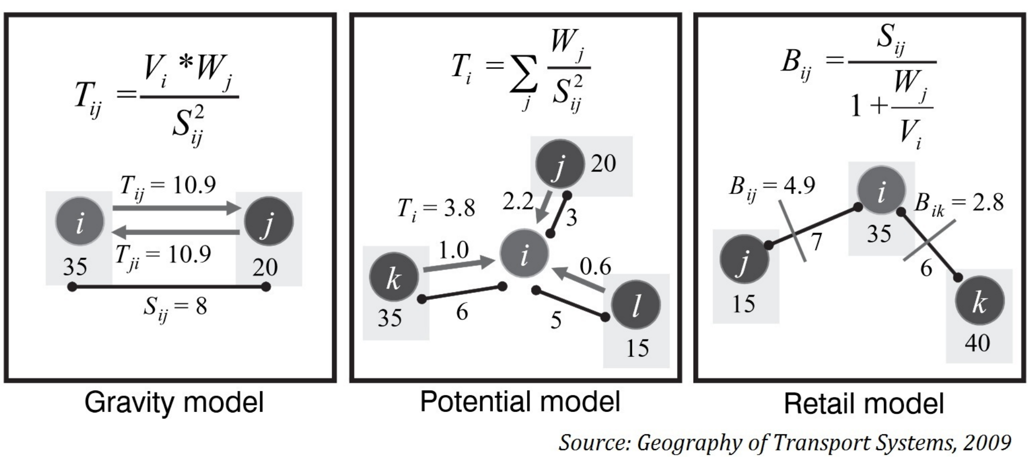


Figure 3.1: Basic spatial interaction models - (figure adapted from [23])

Since no single forecasting model can guarantee accuracy, airline companies employ several models and compare the results. Within this set, gravity models are a widely used subset. In this context a brief summary of gravity model development and application is presented below.

3.2 Gravity Model

Gravity models were one of the earliest causal models developed for travel forecasting and are one of the most commonly used methods for predicting flow of demand between two cities. They are so named due to close resemblance of the mathematical expression in its basic form with Newton's gravitational law. The generalized formulation (as shown in Fig. 3.1) is given by

$$T_{ij} = \frac{(V_i * W_j)^{\alpha}}{{S_{ij}}^{\gamma}} \tag{3.2}$$

As explained above, V_i and W_j are location attributes of the two destinations and S_{ij} represents the friction of separation. Similar to the gravitational law, the demand is directly proportional to the location attributes of the two cities and varies inversely with the separation between them. The parameters α and γ control the two factors respectively.

3.2.1 Extended Gravity Model

In the most basic form, location attributes in the formulation given by Eq. (3.2) are indicated by the population of both destination cities and the separation is represented by the intervening distance. This basic formulation can be extended to include other parameters as described below in order to develop a more accurate model - depending on the application area. For transportation models, the location attributes are expanded to include socio-economic and geographical attributes. These are the driving forces for the gravity model and can be divided into two groups: geo-economic factors and service-related factors [25].

Geo-economic factors, as the name suggests, involve economic activities and geographical characteristics of the cities served. Commonly used geo-economic factors include: income and population of the area served, population of the catchment area, income distribution, gross domestic product, telephone connectivity and tourism destinations. A principal geographic factor affecting demand between two cities is the geographical distance between two destinations. Since we are dealing with air-transportation, direct distance between destinations is considered. Geographical distance (hereafter referred to as ‘distance’) has two conflicting effects: longer distances lead to lower interactions between destination cities while increasing competitiveness of air-transportation compared to other means of transport.

Service-related factors are primarily related to the airline itself [26]. These factors focus on the quality and time of the airline service and include: travel time between cities, average on-time arrivals, ticket pricing, load factors, frequency of flights and aircraft equipment. Several of these factors are outside the control of the airline companies, such as operational routes and frequency of flights are affected by competitors. Airfare being dependent on distance and travel time can also be considered as an exogenous factor. The airline has limited control over air travel prices in a competitive market and it is highly dependent on air fuel prices that are highly volatile and hard to forecast accurately. Due to these reasons, airfare is often omitted from the model [25].

3.2.2 Existing Formulations

Calderón’s Model

Calderón [26] presented a formulation of the extended gravity model that incorporates the factors described in Sec. 3.2. Equation (3.3) gives the mathematical formulation of this model applied to individual routes in the network. The inclusion of service-related factors reported in Table 3.2 attains special significance as explained below.

$$\begin{aligned} TRAFFIC_i = {} & DISTANCE_i^{\beta_1} \times POPULATION_i^{\beta_2} \times INCOME_i^{\beta_3} ... \\ & \times FREQ_i^{\beta_4} \times ASIZE_i^{\beta_5} \times ECONOMY_i^{\beta_6} ... \\ & \times exp(\alpha + \gamma_1 MODISC_i \times \gamma_2 HIDISC_i \times \gamma_3 PROX1_i ... \\ & \times \gamma_4 PROX2_i \times \gamma_5 SEAX_i \times \gamma_6 TOURISM_i ... \\ & \times \gamma_7 HUB1_i \times \gamma_8 HUB2_i + \epsilon) \end{aligned} \tag{3.3}$$

This formulation was applied to European airspace with high coefficient of determination and the following results:

1. Incorporating service-related factors improves the accuracy of the model.
2. Demand is inelastic to ECONOMY fares - used by business travellers.
3. Aircraft size becomes more important as route distance increases.
4. Aircraft frequency and on-time arrivals are important for short-haul routes in the network.

Table 3.1: Geo-Economic variables in Jorge Calderón's mode

Attribute	Notation	Description
Distance	DISTANCE	Geographical distance between two cities
Population	POPULATION	Population of the origin and destination cities
Income	INCOME	Overall average (population weighted) of origin and destination cities
Economy fare	ECONOMY	Cheapest unrestricted economy class fare
Proximity to hubs	PROX1	One of the cities is close to a hub city (< 108 nmi)
	PROX2	Both cities are close to a hub city (< 108 nmi)

Table 3.2: Service-related variables in Jorge Calderón's model

Attribute	Notation	Description
Frequency	FREQ	Return flights on a route (per week)
Aircraft Size	ASIZE	Obtained as the number of seats per aircraft averaged over total flights per week
Airfare	ECONOMY	Cheapest unrestricted economy fare
	MODISC	Moderate discount over ECONOMY ($\leq$ 49%)
	HIDISC	High discount over ECONOMY ($\geq$ 49%)

Grosche's Model

Grosche, Rothlauf and Heinzl [24] proposed a model similar to Calderón's, also adapted for Europe. Grosche et al. presented two formulations of the extended gravity model for air transportation: a basic model (BM) that does not take into account competing airports and an extended model (EM) that includes multi-airport locations. For both models, medium-haul and long-haul flights were used for calibration while excluding tourist destinations. Formulation of the Basic Model is given by Eq. (3.4). Table 3.4 reports the functional form of these parameters.

$$V_{ij} = P_{ij}^{\pi} C_{ij}^{\chi} B_{ij}^{\beta} G_{ij}^{\gamma} D_{ij}^{\delta} T_{ij}^{\tau} \tag{3.4}$$

- *Population:* Population figures for the city where the airport is located. Catchment

Table 3.3: Special control variables in Jorge Calderón's model

Attribute	Notation	Description
Tourism	TOURISM	Active for tourism destinations
Hub cities	HUB1 HUB2	One city is an airline hub Both cities are airline hubs ($\geq$ 49%)
Marine transport	SEAX	Dummy variable - assumes a value of 1 whenever a route flies over water

Table 3.4: Independent variables used in the Grosche's gravity model

Notation	Functional Form	Factor
P_{ij}	$P_i P_j$	Population
C_{ij}	$C_i C_j$	Catchment
B_{ij}	$B_i + B_j$	Buying power index
G_{ij}	$G_i G_j$	Gross domestic product
D_{ij}		Geographical distance
T_{ij}		Average travel time

area population is included separately.

- *Catchment:* This refers to population of airport vicinity - defined as the area within a specified driving time (usually assumed to be 60 minutes).
- *Buying power index:* Buying power index of the catchment area.
- *Gross domestic product:* Gross domestic product (GDP) is used in place of income distribution as a representative indicator of economic activity.
- *Geographical distance:* Geographical distance is the great circle distance (used by aircraft) between the two airports.
- *Average travel time:* Average travel time between two cities - obtained from air traffic bookings or empirically from average aircraft speed.

Bhadra's Model

Bhadra [27] presented a detailed model for semi-log linear demand relationship for origin-destination (OD) travel in the United States. Bhadra's model takes into account local characteristics of the OD cities as opposed to a 'top-down' approach commonly used in the industry and Federal Aviation Administration and focuses on an econometric approach to demand forecasting for OD pairs.

3.3 Proposed Model for System of Systems Problem

Models proposed by Calderón and Grosche et al. are calibrated for European airspace. Bhadra's model follows an econometric approach to demand estimation. Each of these models requires changes in the formulation before they can be used for Canadian airspace.

Considering the constraints of the current problem, service-related factors may be omitted from the network. A key consideration for doing so derives from the fact that airline transportation industry is highly competitive and as such the market condition is always changing. This volatility is not particularly useful for long-term forecasting. Calderón's model is the only formulation that discusses several-related factors. The sample European network used is dense with several large and medium hubs. Most airlines follow similar practices with high passenger volumes on short-haul flights between important business centers. Additionally, the North-American market medium and long-haul flights that necessitate a different modeling approach. Factors such as proximity to hub-cities (PROX1, PROX2) cannot be used for the current problem since the hub-cities (HUB1, HUB2) are not previously known and the network topology is indeterminate. The flight data and fares are also unavailable and corresponding factors can not be used (ECONOMY, HIDISC, MODISC). Tourism affects seasonal demand between cities and is in turn affected by network topology i.e. if the OD pair has a direct flight. Since network topology in our problem is not known and we are not interested in seasonal demand [25], tourism (TOURISM) is omitted in the proposed model.

The model proposed by Grosche et al. considers geo-economic factors only and is taken as the basic model for our problem. Two additional factors are added to this model based on the results from calibration (discussed in 3.3.1). These factors are incorporated to improve the accuracy for cities close to the ocean (O_{ij}) and language factor for Québec (L_{ij}). The formulation is given by Eq. (3.5):

$$V_{ij} = P_{ij}^{\pi} C_{ij}^{\chi} B_{ij}^{\beta} G_{ij}^{\gamma} D_{ij}^{\delta} T_{ij}^{\tau} L_{ij}^{\lambda} O_{ij} \frac{1}{c} \tag{3.5}$$

Tables 3.5 reports the additional factors and the cities for which these factors are active.

Table 3.5: Additional Factors in proposed model

Variable	Description	Cities
L_{ij}	Language Factor	Montréal, Québec
O_{ij}	Ocean Influence	Victoria, St. Johns

3.3.1 Calibration and Results

A key challenge related to the use of any forecasting model is calibration. Calibration is the process of obtaining optimal values for all parameters in the formulation in order to minimize the difference between the estimated values and real data. Actual data for calibrating a demand model can be obtained through a detailed market research and survey. Since large scale surveys would be very expensive, most models are calibrated using airline traffic figures as substitutes for unconstrained demand. These figures are influenced by the fleet size and capacity of various airlines and do not equal actual unconstrained demand. Further, the models discussed above are calibrated for European and American networks and there are little recent data available for domestic aviation in Canada. A range-based calibration methodology proposed by Grosche et al. [24] is implemented using linear least squares. Three distance intervals are considered: short (less than 432 nmi), moderate (between 432 nmi and $1,080$ nmi) and long (greater than $1,080$ nmi). The language and ocean factors are held constant for each interval.

The most recent data for all domestic OD pairs in Canada was released in 1999. However, in its 2011 report, Transport Canada released daily seat numbers for the top 25 city-pairs based on domestic demand [28]. We used these data to calibrate the model of Eq. (3.5) and then estimated demand for the routes considered in the two stage expansion (the 2011 data do not include all routes considered in the problem presented here). We used both linear (using log transformations) and non-linear least squares to determine the coefficients of this model, including the normalizing constant c. We also explore the option of fitting different models for each distance interval as well as reducing the number of variables (e.g., by using the correlation of travel time to geographic distance). All models returned demand values that we overall very close to each other as quantified by the root mean square (RMS) error. We report here the coefficients of the model of Eq. (3.5) obtained using non-linear least squares since it had the smallest RMS error reported in Table 3.6.

Table 3.6: Root-mean-square error for different demand model formulations

	Model 1	Model 2	Model 3	Model 4
RMS Error	186	288	276	390

Where,

1. Model 1 - Nonlinear least squares with 3 range-intervals and 9 unknowns in each interval

2. Model 2 - Linear least squares with 2 range-intervals and 9 unknowns in each interval

3. Model 3 - Linear least squares with 3 range-intervals and 9 unknowns in each interval

4. Model 4 - Linear least squares with 3 range-intervals and 6 unknowns in each interval

Table 3.7 reports the numerical values of the coefficients of Eq. (3.5) obtained through non-linear least squares. Note that we discuss the demand component because of its importance to the problem. However, in this work demand is estimated a-priori and held fixed throughout the SoS optimization process; in this regard we could have used directly the 1999 demand values as they included all routes considered in our examples.

Table 3.7: Demand model coefficients for different distance intervals

Distance	π	χ	β	γ	δ	τ	λ	c
< 432 nmi	1.393	9.123E-6	3.528	-0.859	0.259	-0.240	-2.504	1.491E+6
432-1080 nmi	1.565E-5	0.478	0.240	-0.112	3.578	-6.385	-2.504	1.499E+8
> 1080 nmi	0.351	4.223E-5	0.0001	0.409	1.732	-1.547	-2.504	1903E+8

Using these values of the coefficients, the demand figures for the top 40 routes obtained from the model are reported in Table 3.8. The values obtained from the model follow the trend reported in [28]. It must also be noted that gravity models report reciprocal demand for an OD pair i.e. demand from city i to city j is the same as the demand from city j to city i which is practically very nearly the case. Cases where this is not applicable become especially relevant for the allocation problem (Chapter 6) since difference in demand for either direction would leave balance aircraft at one airport.

Route distances and demand for all possible OD pairs in the network are reported in Table 3.9 and Table 3.10, respectively.

Table 3.8: Top 40 domestic routes by daily air traffic demand using Eq. (3.5)

City pair	Daily demand	City pair	Daily demand
Toronto-Montreal	5602	Vancouver-Winnipeg	901
Toronto-Ottawa	3717	Ottawa-Winnipeg	896
Toronto-Vancouver	3433	Vancouver-Montreal	893
Vancouver-Calgary	2870	Vancouver-Victoria	875
Toronto-Calgary	2457	Calgary-Saskatoon	828
Toronto-Edmonton	2101	Calgary-Regina	816
Vancouver-Edmonton	1904	Calgary-Ottawa	796
Calgary-Edmonton	1812	Edmonton-Regina	773
Toronto-Halifax	1655	Edmonton-Saskatoon	766
Toronto-Winnipeg	1553	Toronto-Quebec	734
Ottawa-Halifax	1253	Toronto-Victoria	720
Calgary-Winnipeg	1153	Edmonton-Ottawa	678
Edmonton-Winnipeg	1138	Montreal-Halifax	659
Vancouver-Ottawa	1107	Ottawa-Thunder Bay	654
Toronto-Thunder Bay	1086	Montreal-Calgary	646
Calgary-Victoria	1059	Vancouver-Kelowna	630
Vancouver-Saskatoon	1041	Halifax-St Johns	595
Edmonton-Victoria	1022	Toronto-Saskatoon	586
Vancouver-Regina	914	Kelowna-Saskatoon	581
Montreal-Ottawa	901	Calgary-Kelowna	574

Table 3.9: Route distances (nmi) for all possible OD pairs in the network

	VAN	MTL	CAL	EDM	OTT	HAL	WIN	VIC	STJ	KEL	QBE	SKT	REG	TBY
TOR	1813	272	1463	1460	190	699	817	1830	1140	1670	393	1201	1102	497
VAN	–	1992	364	442	1913	2403	1008	50	2706	146	2049	647	718	1330
MTL	–	–	1633	1607	90	441	985	2014	869	1847	126	1357	1274	665
CAL	–	–	–	151	1555	2039	650	394	2343	219	1687	283	361	973
EDM	–	–	–	–	1534	1998	645	483	2277	313	1651	261	377	963
OTT	–	–	–	–	–	530	906	1934	956	1768	204	1281	1195	584
HAL	–	–	–	–	–	–	1402	2428	465	2257	356	1758	1687	1090
WIN	–	–	–	–	–	–	–	1029	1741	862	1046	385	289	323
VIC	–	–	–	–	–	–	–	–	2737	176	2074	677	741	1350
STJ	–	–	–	–	–	–	–	–	–	2562	756	2061	2012	1457
KEL	–	–	–	–	–	–	–	–	–	–	1904	502	573	1185
QBE	–	–	–	–	–	–	–	–	–	–	–	1407	1333	735
SKT	–	–	–	–	–	–	–	–	–	–	–	–	128	705
REG	–	–	–	–	–	–	–	–	–	–	–	–	–	612

Table 3.10: Passenger demand for all possible OD pairs in the network

	VAN	MTL	CAL	EDM	OTT	HAL	WIN	VIC	STJ	KEL	QBE	SKT	REG	TBY
TOR	3433	5602	2457	2101	3717	1655	1553	720	435	491	734	587	476	1086
VAN	–	893	2870	1904	1107	489	901	875	311	630	206	1041	914	145
MTL	–	–	646	549	902	659	408	187	272	128	380	156	128	303
CAL	–	–	–	1812	796	361	1153	1059	232	574	149	828	816	398
EDM	–	–	–	–	678	307	1138	1022	196	560	127	766	773	395
OTT	–	–	–	–	–	1253	895	232	485	159	359	191	157	655
HAL	–	–	–	–	–	–	161	103	595	71	187	89	75	34
WIN	–	–	–	–	–	–	–	479	108	532	167	553	514	359
VIC	–	–	–	–	–	–	–	–	73	242	43	550	486	31
STJ	–	–	–	–	–	–	–	–	–	45	138	58	49	24
KEL	–	–	–	–	–	–	–	–	–	–	30	581	524	20
QBE	–	–	–	–	–	–	–	–	–	–	–	36	30	124
SKT	–	–	–	–	–	–	–	–	–	–	–	–	260	343
REG	–	–	–	–	–	–	–	–	–	–	–	–	–	360

CHAPTER 4

Aircraft Sizing

Aircraft design (or aircraft sizing) refers to the process of obtaining a conceptual design of an aircraft in terms of selected design variables and subject to relevant constraints. Aircraft design has been studied as a standalone MDO problem by several researchers in recent literature [5]. Clearly the level of detail in aircraft design depends on the choice of design variables. This chapter presents a brief description of the sizing variables for aircraft design. Finally a mathematical formulation for the SoS problem using NASA's Flight Optimization System (FLOPS) is presented.

4.1 Aircraft Sizing Variables

4.1.1 Aerodynamic Variables

Aerodynamics is a principal part of aircraft design. Table 4.1 reports important aerodynamic variables used for aircraft sizing. It is possible to go beyond the small list reported in Table 4.1, however, we limit our focus to principal design variables. The scope of discussion is limited to subsonic jet design for civil (domestic) aviation. The variables in Table 4.1 are coupled together through empirical relations based on existing design practices. These relations are available in most standard textbooks on aircraft design such as [29], [30] and [31]. For example, aspect ratio (AR), wingspan (b) and planform area (S_W) are related as shown in Eq. (4.1).

$$AR = \frac{b^2}{S_W} \tag{4.1}$$

Fuselage and Cabin design variables represent structural design of the aircraft which affects the computation of weights and finally the aerodynamic variables. Design data for horizontal and vertical tails is used for calibration of the aircraft sizing algorithm (Sec 4.2).

Table 4.1: Aerodynamic design variables

Variable	Notation	Description
Wing Data	S_W	Planform area - projected area of the wing (including that buried in the fuselage)
	Λ	Wing sweep angle - angle of the quarter chord line extended from the line perpendicular to the centerline
	λ	Wing taper ratio - ratio of wing tip chord (c_{tip}) to wing root chord (c_{root})
	AR	Wing aspect ratio
	b	Wing span
H-Tail	C_{HT}	H-tail volume coefficient
	S_{HT}	Theoretical H-tail area
	AR_{HT}	H-tail aspect ratio
V-Tail	C_{VT}	V-tail volume coefficient
	S_{VT}	Theoretical V-tail area
	AR_{VT}	V-tail aspect ratio
Fuselage Data	X_L	Length of aircraft fuselage
	H	Fuselage height - max. distance of the fuselage from its underside to the top in the vertical plane
	W	Fuselage width - max. width of the fuselage - equal to diameter for a circular cross section
Cabin Data	H_{cab}	Internal cabin height from the floor
	W_{cab}	Internal cabin width
	L_P	Length of passenger compartment

4.1.2 Propulsion System Variables

Propulsion system provides the motive power for the aircraft. For the problem described here, we consider variables relevant to subsonic turbofan aircraft. A basic turbofan engine consists of the following: a fan, a compressor, a combustion chamber, a turbine and finally an expansion nozzle. Each of the rotary units may further consist of multiple stages (for example, a multi-stage turbine). A key difference between turbofans and turbojets is that all the exhaust from the fan does not enter the combustion chamber and turbine. This portion of air is termed as ‘bypass’ and the corresponding ratio, the ‘bypass ratio’. Figure 4.1 shows the basic turbofan engine. Key variables related to the propulsion

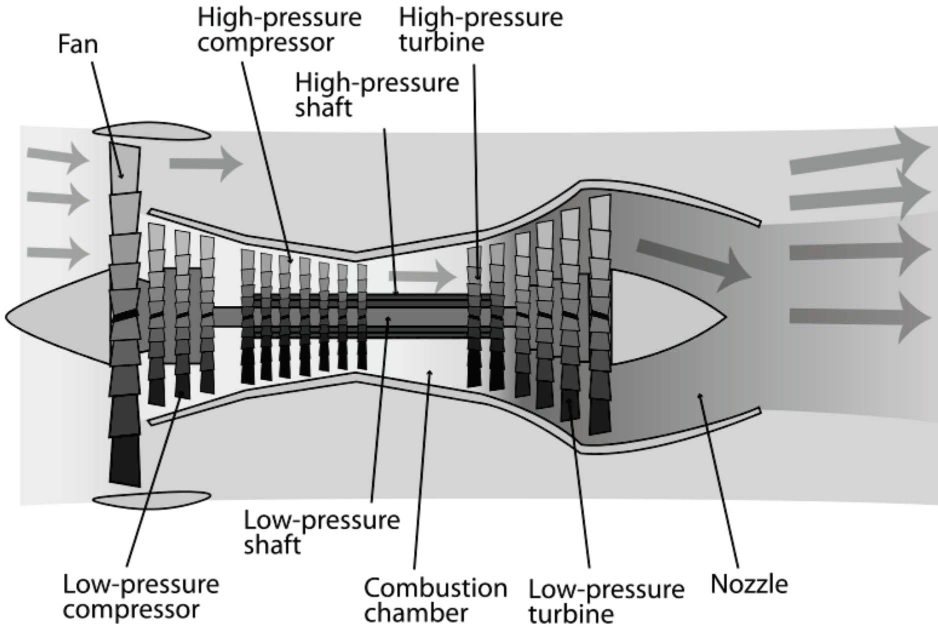


Figure 4.1: Schematic of a turbofan engine

system are listed in Table 4.2[1]. Of these variables engine weight and engine thrust are of particular importance. The sizing approach is described in sections 4.2 and 4.3.

4.1.3 Weights

Weights estimation is a key component of aircraft design. The aircraft weight is typically defined in terms of maximum take-off mass (MTOM) or maximum take-off weight (MTOW) which is also expressed maximum take-off weight i.e. with the effect of acceleration due to gravity (g). Three approaches have been discussed in literature [30, 29]:

1. *Statistical average method* - also known as the 'rapid method' - in this method mass of all components and component systems is represented as a percentage of MTOM.

2. *Graphical method* - this method consists of plotting weights of various manufacture aircraft to fit into a regression curve.

3. *Semi-empirical method* - this method combines empirical derived from theory and real data from various manufactured aircraft.

[1] These design variables are relevant for subsonic turbofan engine. Additional variables are required for turboprops and rotary wing aircraft.

Table 4.2: Propulsion system variables

Variable	Notation	Description
Thrust	T	Rated thrust of each baseline engine
Engine Weight	$\mathrm{Wt}_{\mathrm{Engine}}$	Weight of each baseline engine
Nacelle Length	L_{NAC}	Length of engine nacelle
Specific Fuel Consumption	SFC	Engine specific fuel consumption
Engine Pressure Ratio	EPR	Overall engine pressure ratio
Bypass Ratio	BPR	Ratio of volume of air that bypasses engine core to the volume of air used in combustion

Equation (4.2) gives the different components used for weight estimation. The different terms are detailed in Table 4.3.

$$\begin{aligned}\mathrm{MTOM} = {} & \mathrm{M_{STR}} + \mathrm{M_{PP}} + \mathrm{M_{SYS}} + \mathrm{M_{FUR}} + \mathrm{M_{CONT}} \\ & + \mathrm{M_{CREW}} + \mathrm{M_{CON}} + \mathrm{M_{PL}} + \mathrm{M_{FUEL}}\end{aligned} \tag{4.2}$$

4.1.4 Take-off and Landing Performance

The variables related to take-off and landing are used to ensure that the aircraft can operate from all airports. The corresponding weights MTOW and MLW (maximum landing

Table 4.3: Individual weights used for MTOM computation

Term	Description
M_{STR}	Structure group: Fuselage + Wing + H-tail + V-tail + Nacelle + Undercarriage + Misc. $M_{STR} = M_{FU} + M_{W} + M_{HT} + M_{VT} + M_{N} + M_{UC} + M_{MISC}$
M_{PP}	Power plant group: Engine (dry) + Thrust reverser + Control system + Fuel System + Oil system $M_{PP} = M_{E} + M_{TR} + M_{EC} + M_{FS} + M_{OIL}$
M_{SYS}	Systems group: Environmental control system + Flight control system + Hydraulic system + Electrical system + Electrical system + Instrument System + Avionics $M_{SYS} = M_{ECS} + M_{FC} + M_{HP} + M_{ELEC} + M_{INS} + M_{AV}$
M_{FUR}	Furnishing group: Seats + Oxygen system + Paint $M_{FUR} = M_{SEAT} + M_{OX} + M_{PN}$
M_{CONT}	Contingencies: Allowance for unspecific weight growth
M_{CREW}	Crew: Flight crew + Cabin crew $M_{CREW} = M_{FLC} + M_{CCR}$
M_{PL}	Payload: Passengers + Baggage
M_{FUEL}	Mass of fuel

weight) and the weight ratio W_R are used for payload estimation and cost analyses.

$$\begin{aligned}
\mathrm{MTOW} &= \mathrm{MTOM} \times g \qquad (4.3)\\
&= \mathrm{MRW} - (\mathrm{M}_{\text{Taxi Fuel}} \times g)\\
&= (\mathrm{M}_{\text{Empty}} + \mathrm{M}_{\text{CREW}} + \mathrm{M}_{\text{CONT}} + \mathrm{M}_{\text{PL}} + \mathrm{M}_{\text{FUEL}}) \times g \qquad (4.4)\\
\mathrm{MLW} &= \mathrm{MTOW} - (\mathrm{M}_{\text{Mission Fuel}} + \mathrm{M}_{\text{Reserve Fuel}}) \times g \qquad (4.5)\\
\mathrm{W_R} &= \frac{\mathrm{MLW}}{\mathrm{MTOW}}, \qquad (4.6)\\
\text{where} \quad \mathrm{M_{PL}} &= \mathrm{N}_{\text{Passengers}} * (\mathrm{M}_{\text{Passenger}} + \mathrm{M}_{\text{Baggage}}) + \mathrm{M}_{\text{Cargo}} \qquad (4.7)\\
\mathrm{M_{FUEL}} &= 2 * \mathrm{Fuel}_{\text{Wing}} + (\mathrm{N}_{\text{Fuselage}} * \mathrm{Fuel}_{\text{Fuselage}})\\
&= \mathrm{M}_{\text{Mission Fuel}} + \mathrm{M}_{\text{Reserve Fuel}} \qquad (4.8)
\end{aligned}$$

Aircraft performance variables are commonly expressed in fps units and the corresponding empirical relations are expressed in terms of aircraft weights. Table 4.4 provides a description of these weights.

Table 4.4: Take-off and landing variables

Variable	Notation	Description
Approach Velocity	$V_{Approach}$	Max. allowable approach velocity for landing
Weights	MTOW	Max. take-off weight
	MRW	Max. ramp weight
	MLW	Max. landing weight
	W_R	Ratio of max. landing wt. to max. take-off wt.
Runway Length	FL_{TO}	Max. allowable takeoff field length
	FL_{LDG}	Max. allowable landing field length

4.1.5 Mission Profile

The mission profile is a complete description of the aircraft operations that are performed to execute the mission - from takeoff to landing. Figure 4.2 depicts the mission profile used for the problem described here. The mission profile variables are calculated through

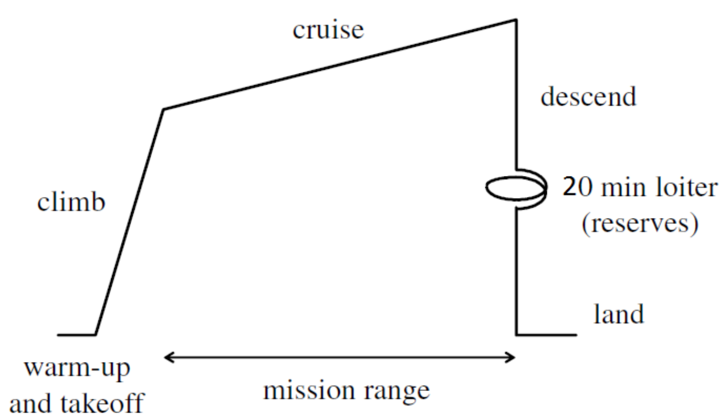


Figure 4.2: Aircraft mission profile

empirical relations described in [30] and are reported in table 4.5. Additionally time available for takeoff can also be prescribed. Parameters related to aircraft noise are omitted for the problem described here.

Table 4.5: Aircraft mission profile

Variable	Notation	Description
Time	T_{Takeoff}	Takeoff time, min
	$T_{\text{Taxi-out}}$	Taxi out time, min
	T_{Approach}	Approach time, min
	$T_{\text{Taxi-in}}$	Taxi in time, min
Climb Information	V_{Climb}	Rate of climb
	$(C_L)_{\text{Climb}}$	Lift coefficient during climb
	CL_{Min}	Min. mach number
	CL_{Max}	Max. mach number
	CH_{Min}	Min. altitude
	CH_{Max}	Max. altitude

4.1.6 Cost Estimation Parameters

Cost analysis relates to the process of obtaining aircraft cost coefficients along different routes in the network. Liebeck et al. [32] proposed a methodology for DOC estimation for subsonic aircraft that incorporates the cost elements listed below:

1. Flight Crew
2. Cabin Crew
3. Landing Fees
4. Navigation Fees
5. Maintenance - Airframe
6. Maintenance - Engine
7. Fuel Costs
8. Depreciation - Aircraft & Spares
9. Insurance
10. Interest

Elements 1 through 7 are commonly referred to as 'cash costs' and elements 8 through 10 are referred to as 'ownership costs'.

4.2 Flight Optimization System

4.2.1 Introduction

The Flight Optimization System (FLOPS) is a NASA developed code for aircraft design and cost analyses that is routinely used in government and academic studies. It provides a good multidisciplinary platform for conceptual aircraft design as well as for evaluation of advanced concepts. FLOPS consists of nine primary modules - reported in Table 4.6. Through the program control option (9), it is possible to run a variety of analyses on FLOPS. FLOPS performs the aircraft sizing operation based on empirical relations such

Table 4.6: FLOPS modules for aircraft design and cost analysis

Module	FLOPS Namelist (primary)
Weights	$WTIN
Aerodynamics	$AERIN
Engine cycle analysis	$ENGINE
Propulsion data	$ENGDIN
Mission performance	$MISSIN
Takeoff and landing	$MISSIN
Noise footprint	$NOISIN
Cost analysis	$COSTIN
Program control	$OPTION

as those presented in [29]. It is possible to perform sizing analyses with the least amount of input from the user with unspecified variables computed using internal defaults. This is particularly useful as much information about aircraft structure and performance is not readily available. It is also possible to run FLOPS in 'optimization mode' to optimize aircraft design for optimizing performance of one or more of the systems discussed above. FLOPS offers a choice of five optimization algorithms for minimizing the objective function - listed in Table 4.7. The objective function for the sizing optimization problem is given by Eq. (4.9) and Table 4.8 provides a description of the corresponding weights.

$$\begin{aligned} OBJ = {} & OFG * GW + OFF * \mathrm{M_{Fuel}} + OFM * VCMN * (\text{Lift/Drag}) \\ & + OFR * \text{Range} + OFC * \text{Cost} + OSFC * SFC \\ & + OFNOX * NOx + OFNF * (\text{Flyover Noise}) + OFNS * (\text{Sideline Noise}) \\ & + OFNFOM * (\text{Noise Figure of Merit}) + OFH * (\text{Hold Time}) \end{aligned} \tag{4.9}$$

Table 4.7: Optimization routines available in FLOPS

Davidon-Fletcher-Powell (DFP) Algorithm
Broyden-Fletcher-Goldfarb-Shano (BFGS) Algorithm
Conjugate Gradient (Polak-Ribiere) Algorithm
Steepest Descent Algorithm
Univariate Search Algorithm

Table 4.8: Weights for FLOPS objective function

Parameter	Description
OFG	Objective function weighting factor for gross weight
OFF	Objective function weighting factor for mission fuel
OFM	Objective function weighting factor for Mach*(L/D)
OFR	Objective function weighting factor for Range,
OFC	Objective function weighting factor for Cost
OSFC	Objective function weighting factor for Specific Fuel Consumption at the engine design point
OFNOX	Objective function weighting factor for NOx emissions
OFNF	Objective function weighting factor for flyover noise
OFNS	Objective function weighting factor for sideline noise
OFNFOM	Objective function weighting factor for noise figure of merit
OAREA	Objective function weighting factor for area of noise footprint
OFH	Objective function weighting factor for hold time

4.2.2 Aircraft Sizing Using FLOPS

For the SoS air transportation problem, FLOPS is used to perform both aircraft sizing and DOC estimation. A series of aircraft are sized to validate the results obtained from FLOPS. For the problem presented here, the network has only short and medium haul routes and the validation is carried out for short and medium range aircraft. Mane [33] reported that validation results are more accurate for aircraft empty weight. Sizing for empty weight was performed for four aircraft using the information obtained from Jane's All the World's Aircraft [34]. The known sizing parameters were input to FLOPS and the remaining were computed internally. The results obtained are shown in Fig. 4.3. Although FLOPS does not offer very high accuracy, it performs better compared the empirical approach reported in [33] and is considered to be adequate for the problem

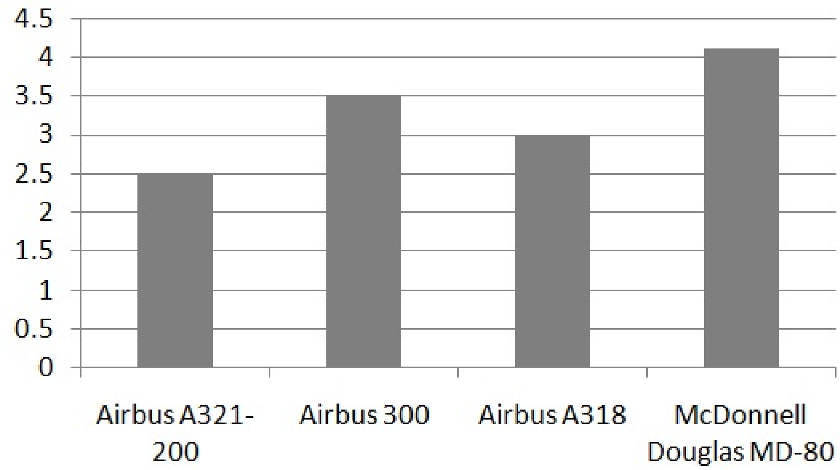


Figure 4.3: Percentage error in aircraft empty wt. computation using FLOPS

presented here. The Airbus A321-200 aircraft is chosen for modeling the FLOPS code for the SoS problem. Results of more detailed sizing operation for this aircraft are shown in Fig. 4.4.

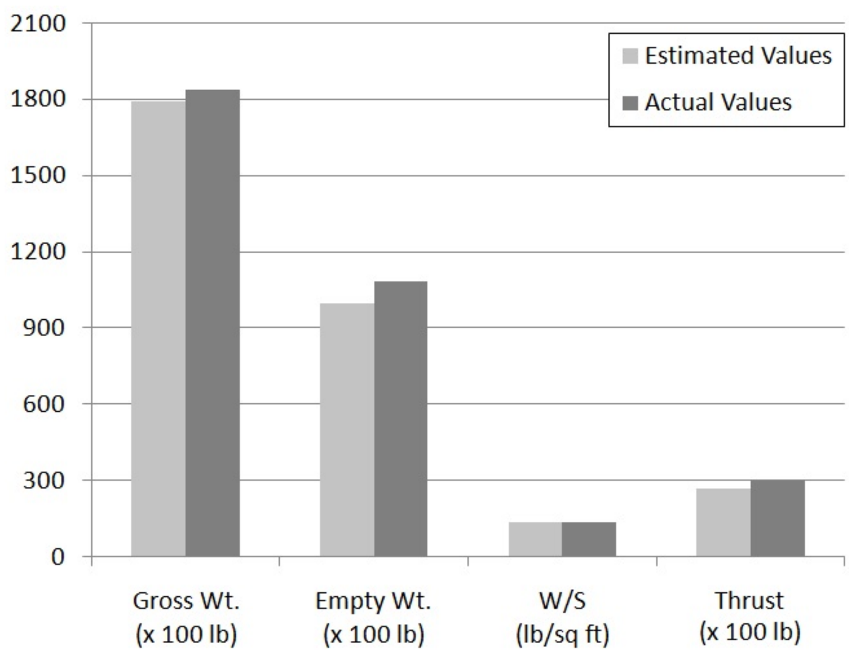


Figure 4.4: Error in sizing Airbus A321-200 aircraft using FLOPS

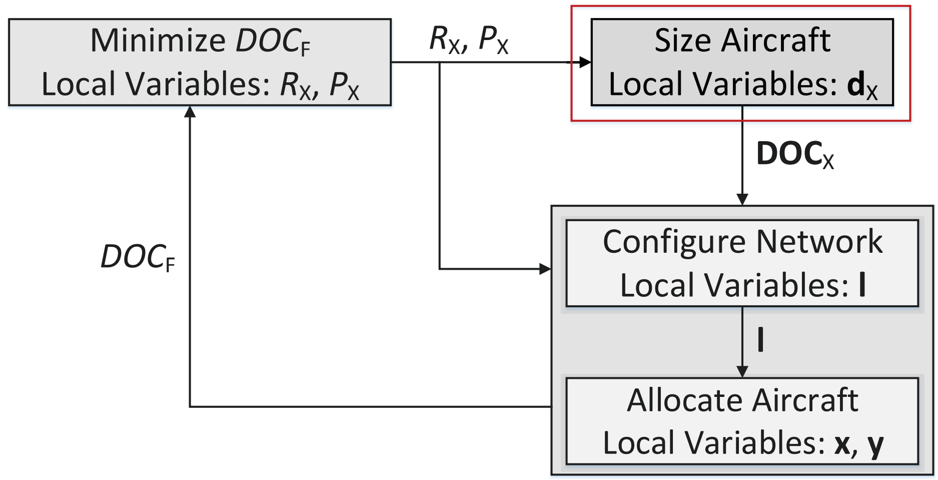


Figure 4.5: Aircraft sizing in the SoS problem

4.3 Aircraft Sizing Problem

Figure 4.5 highlights the aircraft sizing problem in the SoS problem described in Chapter 2.2. The aircraft sizing problem is formulated to minimize aircraft operating cost $\mathbf{DOC}_X$ as a function of a set of aircraft design variables $\mathbf{d}_X$, namely wing aspect ratio $(AR)_X$, thrust-to-weight ratio $(T/W)_X$ and wing loading $(W/S)_X$. These are reported to have the most impact on aircraft sizing and cost of operation [8, 6, 13, 33]. Figure 4.6 depicts the FLOPS aircraft sizing optimization problem. Note that aircraft range R_X

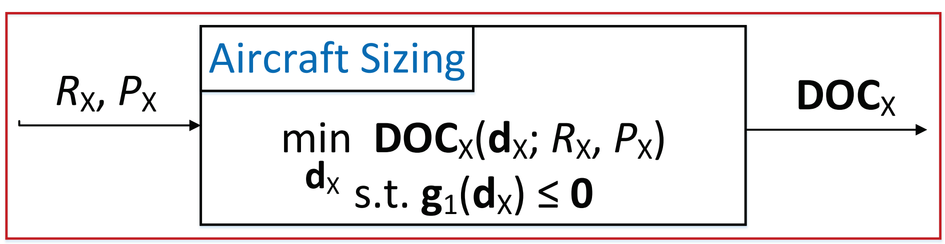


Figure 4.6: Schematic for aircraft sizing problem

and passenger capacity P_X are held constant for the sizing step - as they are determined in the outer-loop SoS problem. FLOPS is run in optimization mode using the Broyden-Fletcher-Goldfarb-Shano (BFGS) algorithm.

Mathematically the aircraft sizing problem is described by Equations (4.10) - (4.14):

$$\min_{\mathbf{d}_\mathrm{X}} \mathbf{DOC}_\mathrm{X}(\mathbf{d}_\mathrm{X}; R_\mathrm{X}, P_\mathrm{X}) \tag{4.10}$$

subject to

$$6.5 \leq (AR)_\mathrm{X} \leq 10.5 \tag{4.11}$$

$$98 \leq (W/S)_\mathrm{X} \leq 150 \text{ (lb / sq. ft)} \tag{4.12}$$

$$0.3 \leq (T/W)_\mathrm{X} \leq 0.5 \tag{4.13}$$

$$S_{TO}\left(R_\mathrm{X}, P_\mathrm{X}, (AR)_\mathrm{X}, (W/S)_\mathrm{X}, (T/W)_\mathrm{X}\right) \leq 8,990 \text{ ft} \tag{4.14}$$

The constraint set $\mathbf{g_1}$ depicted in Fig. 4.6 is described by Eqs. (4.11) - (4.14). These bounds are based on the observed values for medium-range aircraft reported in [30, 31]. Since wing-loading can not be bound directly in FLOPS optimization mode, other variables are so chosen that wing-loading always lies within the bounds given by Eq. (4.12). A bound on the runway length (S_{TO}) is imposed to ensure that the aircraft can take-off and land from all cities in the network and is given by Eq. (4.14).

FLOPS minimizes the aircraft gross weight as a surrogate for minimizing aircraft operating cost. The sized aircraft is then flown along all feasible routes in the network (regardless of whether or not they are operational) to obtain the corresponding cost coefficients. The basic mission profile used includes a quick ascent to cruise altitude of $30,000$ ft, cruise-climb at constant velocity of Mach 0.82, and a quick descent to the airport for landing. An allowance is made for a loiter time of 20 minutes for the scenario where the aircraft does not get immediate clearance for landing.

Cost coefficients for the existing airline aircraft types (A and B) in the fleet are obtained using the relations described in [32, 35]. Additional parameters used for these calculations are reported in Table 4.9. The results obtained for aircrafts A and B along all feasible routes are given by Tables 4.10 and 4.11 respectively[2]. As described previously, DOC_A and DOC_B are constant parameters in our problem. All these cost coefficients are passed on to the network design and aircraft allocation problems.

[2]The existing fleet can not operate along routes greater than aircraft range. Additionally, aircraft B can not operate from Thunder Bay (TBY) as explained in Sec 8.2.

Table 4.9: Additional cost estimation variables for existing fleet

Variable	Notation	Description
Development Year	DEVST	Year in which aircraft design was initiated
Depreciation Period	DEPPER	Period over which aircraft value depreciates
Residual Value	RESID	Residual value at the end of lifetime (percentage)
Load Factor	LF	Ratio of no. of passengers flown to total available seats
Operation Type	$\mathrm{I_{OP}}$ $\mathrm{I_{RNG}}$ $\mathrm{I_{BDY}}$	Domestic or international operation Range indicator - short, medium or long range aircraft Body type indicator - narrow-body or wide-body aircraft
Aircraft Development	$\mathrm{N_{FLT}}$ $\mathrm{N_{PROT}}$	No. of flight test aircraft No. of prototype aircraft

4.4 Summary and MATLAB Implementation

The outer loop of the SoS optimization problem considers candidates for capacity and range. For each R_{X} and P_{X}, FLOPS attempts to design an aircraft while satisfying constraints. At each iteration of the outer-loop SoS problem, MATLAB creates an input script for execution in FLOPS (optimization mode). The output from FLOPS is recorded on an external file which is read into MATLAB to get the design characteristics of aircraft along with $\mathbf{DOC}_{\mathrm{X}}$ for all feasible routes. Since it is not possible to constrain wing-loading explicitly in FLOPS, any design obtained from the FLOPS output file that does not satisfy Eq. (4.12) is rejected at this stage. The entire process takes nearly 4 seconds to compute on a 64-bit Intel i7 processor with 4 cores and 8192 MB of RAM.

It is observed that $\mathbf{DOC}_{\mathrm{X}}$ follows trends expected from economies of scale - the cost coefficient of aircraft X for a fixed R_{X} increases with increase in P_{X}, however, the cost per passenger decreases. Due to this, the optimal P_{X} is expected to be close to the upper bound of 240. This is described in more detail in Chapter 8.

Table 4.10: DOC_{A} for all routes in the extended network

	VAN	MTL	CAL	EDM	OTT	HAL	WIN	VIC	STJ	KEL	QBE	SKT	REG	TBY
TOR	42,793	11,604	35,709	35,654	9,936	20,254	22,627	43,127	29,164	39,897	14,061	30,402	28,404	16,164
VAN	–	46,419	13,467	15,045	44,810	–	26,489	7,121	–	9,046	–	19,194	20,639	33,008
MTL	–	–	39,136	38,629	7,913	15,017	26,032	–	23,680	43,478	8,645	33,552	31,879	19,558
CAL	–	–	–	9,149	37,558	–	19,249	14,067	–	10,532	40,230	11,829	13,411	25,791
EDM	–	–	–	–	37,146	46,537	19,161	15,864	–	12,429	39,504	11,388	13,733	25,597
OTT	–	–	–	–	–	16,829	24,424	45,230	25,449	41,875	10,235	32,019	30,277	17,909
HAL	–	–	–	–	–	–	34,463	–	15,518	–	13,297	41,675	40,240	28,167
WIN	–	–	–	–	–	–	–	26,928	41,333	23,552	27,274	13,885	11,953	12,642
VIC	–	–	–	–	–	–	–	–	–	9,652	–	19,790	21,098	33,420
STJ	–	–	–	–	–	–	–	–	–	–	21,389	–	–	35,587
KEL	–	–	–	–	–	–	–	–	–	–	44,622	16,262	17,699	30,078
QBE	–	–	–	–	–	–	–	–	–	–	–	34,563	33,067	20,974
SKT	–	–	–	–	–	–	–	–	–	–	–	–	8,680	20,366
REG	–	–	–	–	–	–	–	–	–	–	–	–	–	18,488

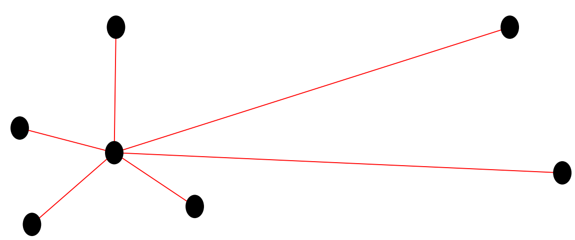
Table 4.11: DOC_{B} for all routes in the extended network

	VAN	MTL	CAL	EDM	OTT	HAL	WIN	VIC	STJ	KEL	QBE	SKT	REG	TBY
TOR	38,443	10,930	32,194	32,145	9,458	18,560	20,654	38,737	26,420	35,888	13,097	27,513	25,749	–
VAN	–	41,642	12,574	13,965	40,223	–	24,061	6,975	–	8,673	42,657	17,625	18,900	–
MTL	–	–	35,217	34,770	7,674	13,940	23,658	42,029	21,582	39,048	8,320	30,292	28,816	–
CAL	–	–	–	8,764	33,825	42,477	17,674	13,103	47,907	9,984	36,182	11,129	12,523	–
EDM	–	–	–	–	33,462	41,746	17,597	14,687	46,726	11,658	35,542	10,739	12,808	–
OTT	–	–	–	–	–	15,539	22,239	40,593	23,143	37,634	9,722	28,939	27,403	–
HAL	–	–	–	–	–	–	31,094	–	14,383	46,363	12,423	37,457	36,191	–
WIN	–	–	–	–	–	–	–	24,448	37,156	21,470	24,753	12,942	11,238	–
VIC	–	–	–	–	–	–	–	–	–	9,207	43,092	18,151	19,305	–
STJ	–	–	–	–	–	–	–	–	–	–	19,562	42,863	41,984	–
KEL	–	–	–	–	–	–	–	–	–	–	40,057	15,039	16,307	–
QBE	–	–	–	–	–	–	–	–	–	–	–	31,183	29,864	–
SKT	–	–	–	–	–	–	–	–	–	–	–	–	8,351	–
REG	–	–	–	–	–	–	–	–	–	–	–	–	–	–

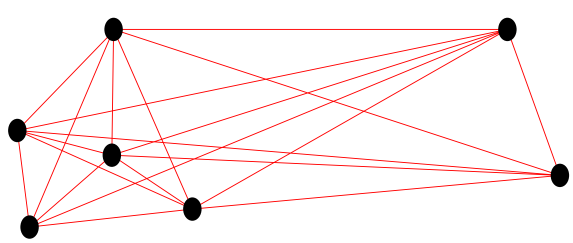
CHAPTER 5

Network Design

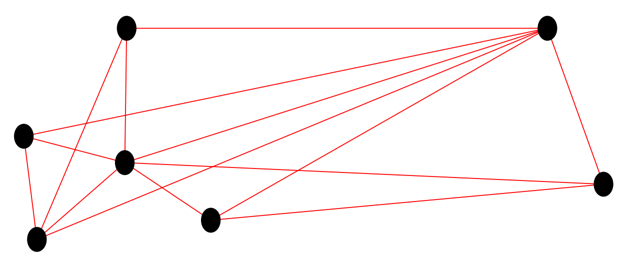
An airline route network[1] is the complete set of direct flights operated by the airline to meet passenger demand. All cities in the airline network may not be connected to each other through direct flights. Depending on which routes are active in the network, there are several feasible topologies for the same set of destination cities. Different net-



(a) Hub-spoke network



(b) Fully connected network



(c) Multi-hub network

Figure 5.1: Different network topologies

work topologies have evolved in response to airline needs and operations as depicted in Figure 5.1. A brief description of three common topologies is presented below:

1. *Hub-spoke networks:* This network type has a central hub-city to which all other non-hub cities are connected. There are no connections between non-hub cities and all passenger demand must be routed through the hub-city. These networks evolved

[1] Hereafter referred to as 'network'

as the airline industry was deregulated in the 1970s. Direct flights between small and medium-sized cities were increasingly routed through larger, central hubs allowing airlines to benefit from cost and volume advantages. This topology is suitable for networks with high passenger demand - Fig. 5.1a

2. *Fully connected networks:* Also known as point-to-point networks, this topology is typically used by low-cost carriers that have low marginal cost per passenger and are typically regional in operation. In a fully connected network all cities are connected to each other - Fig. 5.1b - however, some routes may not be operational in an actual airline network.

3. *Multi-hub networks:* These networks have a structure between hub-spoke networks and fully-connected networks. It is possible to have more than one hub-city as shown in Fig. 5.1c although they may not be connected to *all* other cities in the network.

5.1 Network Topology Design

The cost of operation for a network is a function of the network topology/configuration which is represented by the direct flights or active routes. Direct flights represent two conflicting factors: generally, direct flights are less expensive than flights routed through other cities, other factors remaining constant, however, practically this is not feasible due to the associated costs involved. These costs include the fees the airline must pay to the air traffic control authority, navigation authority, airport fees related to aircraft ground operations and cost of developing infrastructure for the new routes. Finally, the airline also needs to plan for future operations which includes revenue estimation. Collectively these costs are termed as Indirect Operating Costs (IOC).

There is no method reported in current literature for IOC estimation. Further, each airline has its own revenue generation model details of which are not easily available. An alternative approach to modeling airline network topology is therefore needed.

Liu et al. [12] proposed a model based on complex network theory. The network design problem is treated as an optimization problem to maximize the weighted sum of network properties and solved using a genetic algorithm. However, the model does not take into account the cost of operation of the network. The approach described in [8] incorporates some elements of operating costs in network design but omits network properties. It is desired to have a more encompassing design for the SoS problem.

We consider a model that minimizes the cost of operation while also ensuring that it is ‘well-connected’. The latter is accomplished by including network properties as constraints. A complex network theory approach [36] is followed wherein the nodes represent different cities and the arcs between these nodes denote the existence of a direct flight. The following simplifying assumptions are made:

1. The network is an undirected, simple and connected graph i.e. all nodes are connected without any self-loops and that there can be no more than one arc connecting two nodes.

2. Not all arcs are active - feasibility of a direct flight depends on range and distance constraints.

3. Reciprocal demand - number of passengers flying between a pair of two cities is the same in either direction.

4. Each city (airport) has sufficient resources to handle all the air-traffic directed to/from that node.

Before we provide the mathematical formulation for network design, a brief description of some terms used and network properties is presented:

1. *Adjacency matrix* - this is a binary matrix of the order n ($n =$ no. of cities in the network) that represents network configuration. Equation (5.1) gives the mathematical description of adjacency matrix $\mathbf{l}$.

$$l_{ij} = \begin{cases} 1, & \text{if there exists an edge connecting } i \text{ and } j \\ 0, & \text{otherwise} \end{cases} \tag{5.1}$$

   Based on assumption 1 described above, the adjacency matrix is symmetric i.e. $\mathbf{l}^\top = \mathbf{l}$ with all diagonal entries as zero.

2. *Edge betweenness* - Girvan and Newman [37] extended the concept of betweenness from vertices to edges. It has since been used to measure relative importance of an edge in the network and identify clusters (communities) in social and biological networks. In context of the problem described here, it helps to identify critical routes for the network. For any edge in the network, it is measured as the total number of shortest paths between all possible node pairings in the network that pass through that edge. For example, for the configuration shown in Fig. 5.2, the highlighted edge has the highest betweenness since most number of shortest are expected to

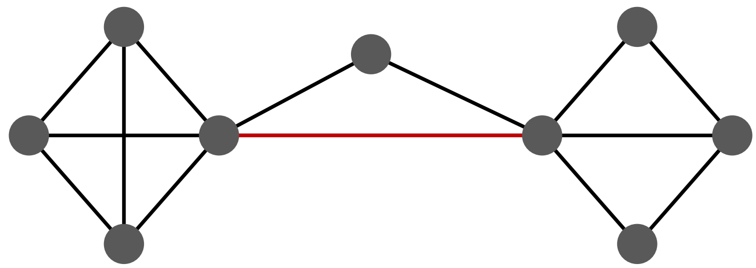


Figure 5.2: Edge betweenness

pass through it. Equation (5.2) gives the mathematical relation for calculating edge betweenness.

$$B_{ij}^{e} = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\sigma_{ij}(e)}{\sigma_{ij}}, \tag{5.2}$$

$$\text{where} \quad B_{ij}^{e} = \text{Betweenness of edge } e \text{ connecting nodes } i \text{ and } j \tag{5.3}$$

$$\sigma_{ij}(e) = \text{No. of shortest paths between nodes } i \text{ and } j$$

$$\text{that pass through edge } e \tag{5.4}$$

$$\sigma_{ij} = \text{Total no. of shortest paths between } i \text{ and } j \tag{5.5}$$

3. *Eigen-centrality* - Boncnich [38, 39] postulated that nodal degree and degree distribution (for nodes) in a network fail to capture all aspects of importance of a node in the network and proposed eigen-centrality as an alternate measure. Eigen-centrality is obtained from the eigenvector corresponding to the largest positive eigenvalue of the adjacency matrix as given by Eq. (5.6).

$$\mathbf{l}x = \lambda x \tag{5.6}$$

$$\lambda x_i = \sum_{j=1}^{n} l_{ij} x_j, \qquad i = 1, \dots, n \tag{5.7}$$

$$\text{where} \quad \mathbf{l} = \text{Adjacency matrix (binary)} \tag{5.8}$$

$$\lambda = \text{Largest eigenvalue of } \mathbf{l} \tag{5.9}$$

5.2 Network Design Problem Formulation

For a network of n cities, it is possible to have up to $n(n-1)/2$ active routes in the network. The network design problem aims to find the best network configuration that minimizes the cost of operating the airline fleet DOC_{F} as a surrogate for maximizing profits. By omitting pricing and IOC estimates, we are able to develop a model that can be universally adopted. Figure 5.3 highlights the network design problem in the SoS problem.

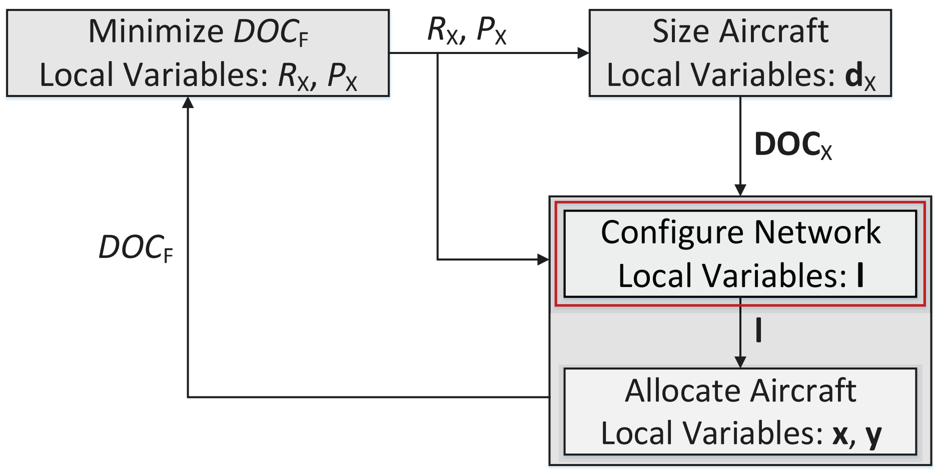


Figure 5.3: Network design problem in the SoS problem

The objective function value for the network design problem DOC_{F} is obtained from the aircraft allocation problem which takes as input the network configuration and allocates aircraft to minimize DOC_{F}. The network design and aircraft allocation problems are thus nested. Figure 5.4 depicts the network design problem schematic.

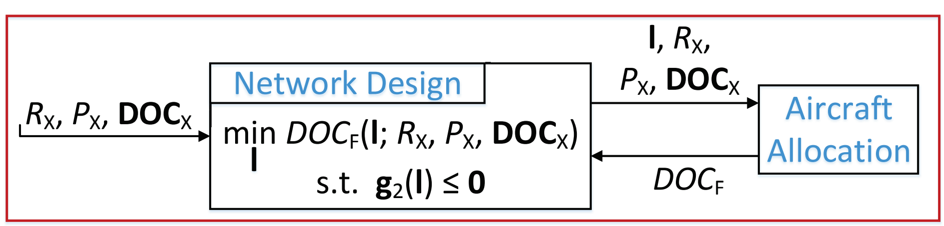


Figure 5.4: Nested network design and aircraft allocation problems

Mathematically the network design problem is given by equations (5.10) - (5.16):

$$\min_{\mathbf{l}} DOC_{\mathrm{F}}(\mathbf{l}; R_{\mathrm{X}}, P_{\mathrm{X}}, \mathbf{DOC}_{\mathrm{X}}) \tag{5.10}$$

subject to

$$m_l \leq \sum_{i=1}^{n} \sum_{j=i+1}^{n} l_{ij} \leq m_u \tag{5.11}$$

$$\sum_{i=1}^{n} l_{ij} \geq 1 \tag{5.12}$$

$$l_{ij} = 1 \quad \forall i, j \in \Pi \tag{5.13}$$

$$\mathrm{LFC}(\mathbf{l}) \leq \lambda \tag{5.14}$$

$$b_l \leq \sum_{i=1}^{n} \sum_{j=i+1}^{n} b_{ij}(\mathbf{l}) \leq b_u \tag{5.15}$$

$$c_l \leq \sum_{i=1}^{n} \sum_{j=i+1}^{n} c_{ij}(\mathbf{l}) \leq c_u \tag{5.16}$$

Equation (5.10) describes the objective function for the network design problem, the design variable of which is the binary adjacency matrix $\mathbf{l}$. The cost of operation DOC_{F} is a function of network configuration and design of new aircraft along with its cost coefficients. The latter two are obtained from the aircraft sizing step described in Chapter 4 and are held constant for the network design problem.

The first constraint (Eq. (5.11)) sets the bounds for the number of active routes in the network. For the problem described here, a lower bound is selected that is greater than the number of edges in the minimum spanning network [36]. The upper bound is based on actual aircraft itineraries and observed aircraft movement reported in [28]. It is possible to define a more rigorous procedure for selecting these bounds but not doing so does not affect the usefulness or the validity of the methodology presented here.

The second constraint (Eq. (5.12)) ensures that each city has at least one direct flight. If a city is to serve as a hub, it must be an element of the set Π. The third constraint (Eq. (5.13)) ensures that hub-cities are connected to all other cities (since Π can be an empty set).

The fourth constraint limits the max. allowable least no. of flight changes (LFC) for any pair of cities in the network (5.14). This ensures that the network configuration allows for realistic modeling of passenger movement. The last two constraints Eqs. (5.15)-5.16

are added to ensure that critical links in the network are preserved.

We use MADS to solve outer-loop of the nested optimal network configuration problem. The nested aircraft allocation problem is discussed in greater detail along with the mathematical formulation in Chapter 6.

CHAPTER 6

Aircraft Allocation

The airline fleet consists of aircraft of different types, each with a unique cost associated with it for flying different routes in the network. The goal of the allocation problem is to assign aircraft to all active routes in a given network for minimizing overall cost of operation DOC_F while satisfying passenger demand. The network configuration, design of new aircraft and the corresponding cost coefficients are fixed parameters for the allocation problem.

Since the number of aircraft allocated and unfulfilled demand are integers, the resulting problem is an integer programming problem. We solve this problem using the model described in [9] with the following modifications:

1. Not all routes in the network are active and it is possible that passenger demand between two cities may need to be routed through other active routes. Aircraft allocation routes this demand along the shortest path determined using Dijkstra's algorithm [40].

2. Unlike the method employed by Mane et al. [6, 33], an allowance is made for unfulfilled demand. Further, unfulfilled demand along each route and total unfulfilled demand are bounded from above.

3. Aircraft allocation along each route is bounded from above; this is necessary because in practice each aircraft requires a different maintenance crew and other operations before take-off and after landing

Figure 6.1 highlights the aircraft allocation problem in the SoS problem. The representative schematic of the aircraft allocation problem is depicted in Figure 6.2. The objective function is to minimize the fleet operating cost, and the optimization variables are:

1. Number of aircraft x_{pij} of a given type $p \in T = \{A, B, X\}$ assigned along each active route, and

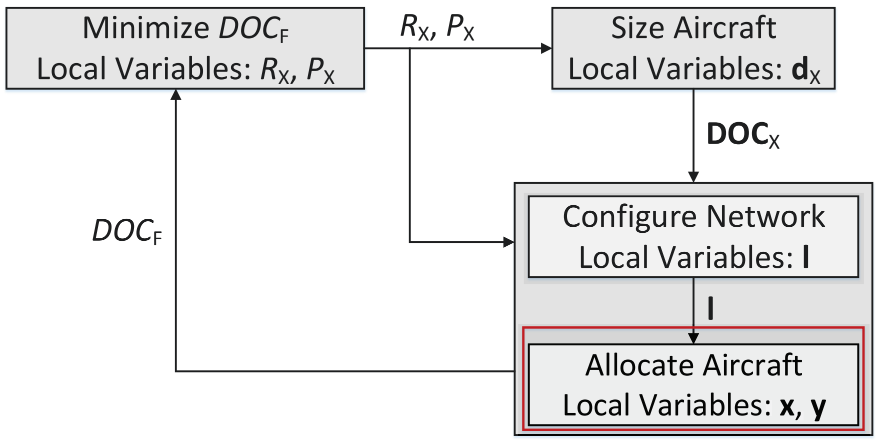


Figure 6.1: Aircraft Allocation in the SoS problem

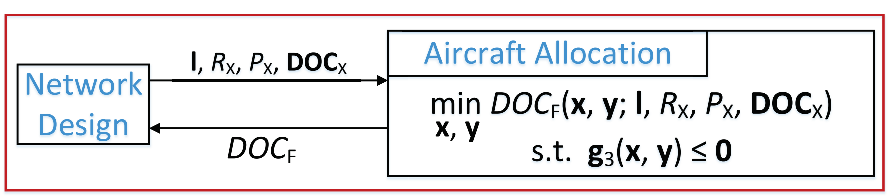


Figure 6.2: Allocation problem schematic

2. Unfulfilled demand y_{ij} along each active route.

The total number of aircraft allocated is bounded by the available fleet size. Not all aircraft can fly each route in the network and an additional constraint is included to ensure no aircraft is allocated to a route that is longer than the aircraft range. The unfulfilled route demand is penalized by a high penalty weight w_{ij}. The airline must meet demand along all active routes. The allocation problem formulation is given by Eqs. (6.1) - (6.8):

$$\min_{\mathbf{x},\mathbf{y}} \sum_{p \in T} \sum_{i=1}^{n} \sum_{j=i+1}^{n} [l_{ij} DOC_{pij} x_{pij} + w_{ij} y_{ij}], \quad T = \{A, B, X\} \tag{6.1}$$

subject to

$$\sum_{p \in T} \sum_{i=1}^{n} \sum_{j=i+1}^{n} [P_{pij} + y_{ij}] \geq V_{ij} \tag{6.2}$$

$$\sum_{i=1}^{n} \sum_{j=i+1}^{n} x_{pij} \leq s_p \quad \forall \;\; p \in T \tag{6.3}$$

$$\sum_{i=1}^{n} \sum_{j=i+1}^{n} y_{ij} \leq Y \tag{6.4}$$

$$x_{pij} \leq h_{pij} \quad \forall \;\; p, i, j \text{ with } i > j \tag{6.5}$$

$$y_{ij} \leq \delta_{ij} \quad \forall \;\; i, j \text{ with } i > j \tag{6.6}$$

$$x_{pij} = 0 \;\; \forall \; \langle p, i, j \rangle \in \xi(R_{\mathrm{A}}, R_{\mathrm{B}}, R_{\mathrm{X}}) \tag{6.7}$$

where

$$w_{ij} = \frac{2.5}{\sum\limits_{p \in T} f_{pij}} \left[\sum_{p \in T} f_{pij} \left(\frac{DOC_{pij}}{P_p} \right) \right] \tag{6.8}$$

and $\xi(R_{\mathrm{A}}, R_{\mathrm{B}}, R_{\mathrm{X}}) = \{\langle A, 3, 6 \rangle, \ldots\}$ is the set of 3-tuples describing infeasible aircraft range of an aircraft type for a given route.

For the problem presented here w_{ij} is set to 2.5 times the DOC-per-passenger averaged over all aircraft as shown in Eq. (6.8). The variable f_{pij} in Eq. (6.8) is a binary variable whose default value is 1 but becomes 0 for any $\langle p, i, j \rangle$ in the set $\xi(R_{\mathrm{A}}, R_{\mathrm{B}}, R_{\mathrm{X}})$.

The right hand side of Eq (6.2) represents the routed demand V_{ij} along that route. For routes that are not active in $\mathbf{l}$ this demand becomes zero. For an active link l_{ij}, routed demand V_{ij} is the sum of demand between cities i and j and the demand for any other pair of cities r and s that are not connected directly and the shortest path between them includes the link l_{ij}.

The constraint given by Eq. (6.3) ensures that the number of aircraft allocated does not exceed fleet size. Equation (6.4) ensures total unfulfilled demand does not exceed allowable limit. Equations (6.5) and (6.6) prescribe bounds on x_{pij} and y_{ij}, respectively. Equation (6.7) ensures that aircraft range feasibility constraints are always met. Allocation x_{pij} where $\langle p, i, j \rangle$ belongs to the set of forbidden 3-tuples is always zero. For example, $\xi(R_{\mathrm{A}}, R_{\mathrm{B}}, R_{\mathrm{X}})$ includes $\langle A, 3, 6 \rangle$ - this is because the corresponding route distance (Montréal-Toronto) is greater than range R_{A} of aircraft A.

Table 6.1 lists the bound values for the allocation problem (seven-cities). A constant value for δ_{ij} is prescribed for all routes but it is possible to bound y_{ij} along individual routes. Similarly, allocation of aircraft of a given type along all routes has a constant upper bound which may be replaced with individual route bounds.

Table 6.1: Bound values for aircraft allocation problem

Bound	s_p	Y	$\delta_{ij}\ \forall i, j$	$h_{pij}\ \forall i, j$
Value	$\{70, 100, 100\}$	100	20	$\{25, 10, 20\}$

The allocation problem is solved using the revised simplex method described in [9] and implemented using GNU Linear Programming Kit (glpk) adapted for Matlab [41].

CHAPTER 7

Mesh Adaptive Direct Search

Mesh Adaptive Direct Search (MADS) is a direct search optimization algorithm. It is used for blackbox optimization of discontinuous functions where gradient information is not available and the problem is subject to nonlinear constraints. The MADS algorithm extends upon the generalized pattern search (GPS) class of algorithms [42] by allowing for local exploration termed as *polling*. This allows to overcome the limitation imposed by finite exploration directions offered by GPS algorithms. Without going into much detail, the convergence of MADS is based on Clarke's calculus for non-smooth functions [43]. A brief description of the MADS algorithm is presented below.

7.1 MADS Algorithm

MADS is an iterative feasible-point algorithm. Given an initial starting point $x_0 \in \Omega$, the algorithm attempts to located a local minimizer of the objective function f over the feasible domain Ω by evaluating some trial points f_Ω. The algorithm does not require an additional derivative information. This is useful for problems where derivative information is difficult to obtain and may be affected by noise in the objective function. At each iteration a finite number of feasible trial points are generated and the infeasible trial points are discarded. The algorithm compares the objective function value at the feasible trial points and the current iterate $f_\Omega(x_k)$ - the best feasible value found so far. Each of these trial points lies on a *mesh* constructed from a finite set of n_D directions $D \subset \mathbb{R}^n$ and scaled by a mesh size parameter $\Delta_k^m \in R_+$. The set D must be a positive spanning set in $\mathbb{R}^n$.

The mesh is defined as the union of sets over the set of points where the objective function has already been evaluated S_k. The mesh is conceptual in the sense that it is not actually constructed and only underlies the algorithm. The evaluation of f at a trial point x is preceded by the evaluation of constraints which may be ordered from easy to expensive based on the computation. If the constraints are not satisfied $x \notin \Omega$, then $f_\Omega(x)$

is set to $+\infty$ without evaluating $f(x)$ and (possibly) other constraints. This essentially discards infeasible trial points without evaluating the computationally expensive objective function.

Each MADS iteration is divided into two steps: the *search* step and the *poll* step: The search step allows evaluation of f_Ω at any number of trial points constrained to lie on the mesh. The search is said to be empty when no trial points are considered. When an improved mesh point is found, the iteration may be stopped or can be continued to search for a better mesh point. In either case, the the next iteration is initiated with the new incumber $f_\Omega(x_{k+1}) < f_\Omega(x_k)$ and a mesh size parameter Δ^m_{k+1} set equal to or larger than Δ^m_k.

When the search step fails to find an improved mesh point, the second step called *poll* is invoked. The poll step involves local exploration around the current incumbent x_k and determines the magnitude of distance of the trial points from the current incumbent. This distance is called the poll size parameter and is denoted as Δ^m_k. A key difference between the GPS and MADS algorithm is that in GPS algorithms, there is only one quantity equal to both mesh size parameter and poll size parameter $\Delta_k = \Delta^m_k = \Delta^m_p$ whereas the MADS strategy prescribes $\Delta^m_k < \Delta^m_p$ for all k. At MADS iteration k, the set of trial points P_k (also called *frame*) is given by Eq. (7.1)

$$P_k = \{x_k + \Delta^m_k d \ : \ d \ \in \ D_k\} \subset M_k \tag{7.1}$$

where D_k is a positive spanning set. When the poll step fails to find an improved mesh point, the mesh is refined as $\Delta_{k+1} < \Delta_k$. Figure 7.1 depicts the general MADS algorithm.

7.2 Implementing MADS

The MADS algorithm is implemented for the SoS problem using NOMAD (Nonlinear Optimization with the MADS algorithm) software [44, 45, 46]. NOMAD has been under development since the year 2000 and is capable of running GPS algorithms in addition to MADS. It allows for use of surrogates (non-adaptive) for evaluation of function trial points before the actual objective function evaluation. This is particularly useful for problems where the objective function is expensive to compute. The surrogate functions may be defined by the user.

For the SoS air transportation problem, a Matlab interface to the C++ implementation of MADS offered by OPTI toolbox [47] was used. MADS is used for the SoS outer-loop

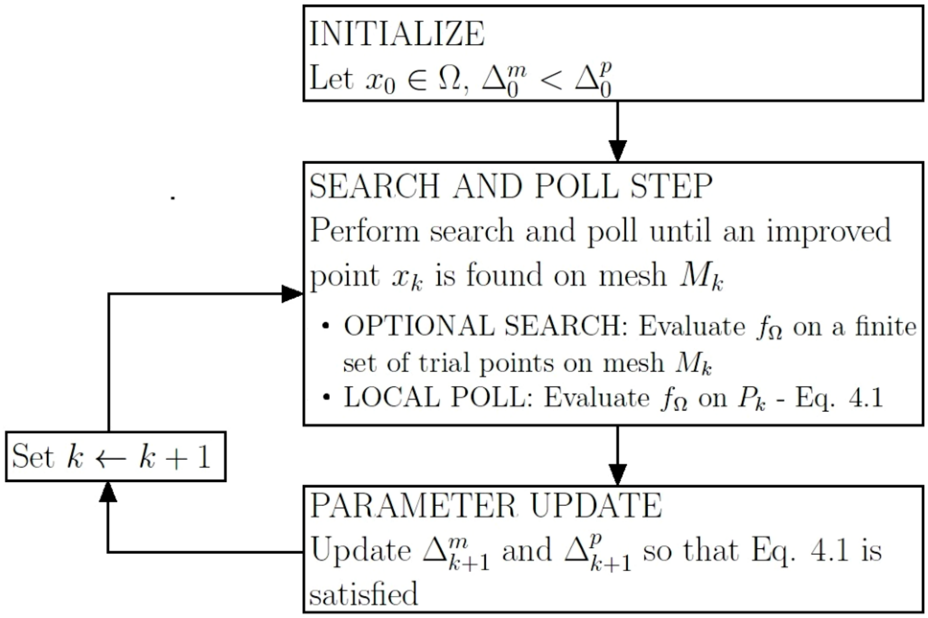


Figure 7.1: MADS algorithm

optimization problem and the network design problems depicted in Fig. 2.5.

CHAPTER 8

Results

We consider a two stage expansion of airline's network such that the final network is comprised of fifteen cities. The formulation given in Chapters 4-5 is slightly modified for the second stage as explained in Section 8.2.

8.1 Stage 1: expansion to 7 cities

In the first stage two cities in the eastern subnetwork (Toronto and Montreal) are added to the original regional network. The behaviour of DOC_{F} is studied through a modest full-factorial design-of-experiments with respect to range and passenger capacity using points visited by NOMAD in preliminary design space explorations. The obtained response surface is depicted in Fig. 8.1. Note that the network configuration and aircraft allocation

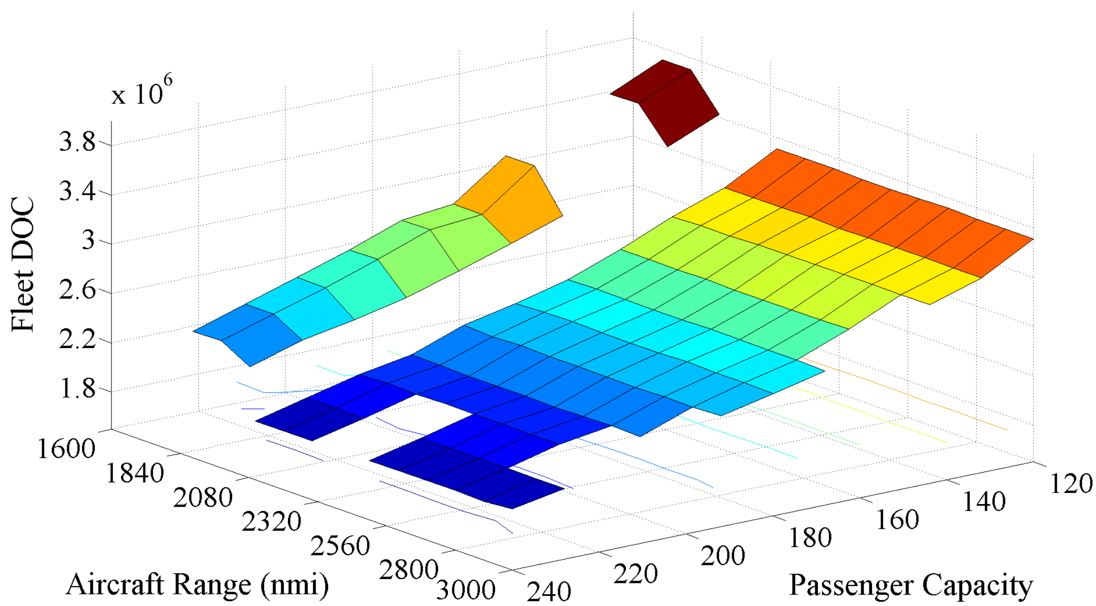


Figure 8.1: DOC_{F} response surface for seven cities problem

problems are solved for each feasible range-capacity point on that surface. The network problem is not necessarily convex; local optimal solutions are this possible. However, the allocation problem is linear and thus yields globally optimal solutions.

The empty regions of the plot represent regions where the SoS problem becomes infeasible. This can happen for the following reasons:

1. The aircraft sizing problem fails to

    (a) satisfy its constraints, or

    (b) perform a cost analysis for some route(s).

2. The allocation problem becomes integer infeasible.

It can be seen that the DOC_{F} response surface is monotonically decreasing in passenger capacity of new aircraft and is extremely flat in the range dimension for a large part of the graph. Investigating further, in the 1-d projection of the response surface on the design space, DOC_{F} becomes insensitive with respect to range a little after 2000 nmi as shown in Fig. 8.2. As R_{X} increases, the new aircraft can fly a greater number of routes. Since the new aircraft has a lower operating cost $\mathbf{DOC}_{\mathrm{X}}$ compared to existing fleet, the overall fleet cost DOC_{F} also decreases. The longest route in the network is 2015 nmi, so R_{X} values greater than 2015 do not affect DOC_{F} significantly. The two local minima around $2,150$ and $2,900$ nmi shown in Fig. 8.2 are most likely attributed to FLOPS numerical noise.

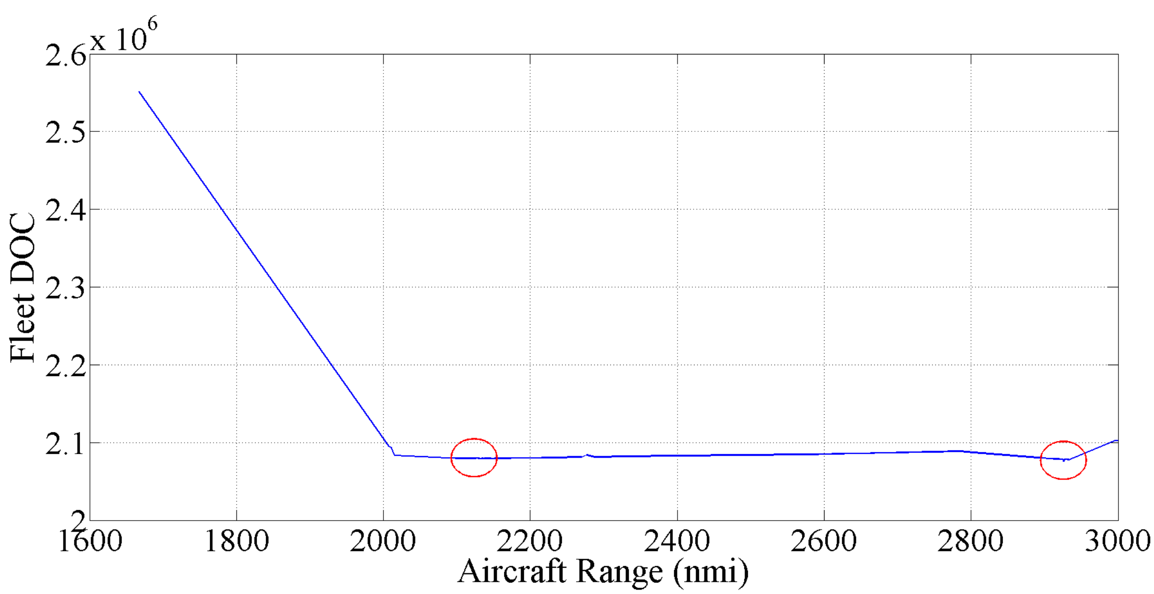


Figure 8.2: DOC_{F} vs aircraft range for fixed passenger capacity

The outer-loop SoS optimization problem depicted in Fig. 2.5 was solved using 4 different initial guesses for range and capacity. The initial network configuration for each of these runs was as shown in Fig. 8.3. This initial configuration was obtained from a stand-alone optimization problem using only the existing fleet (aircrafts A and B).

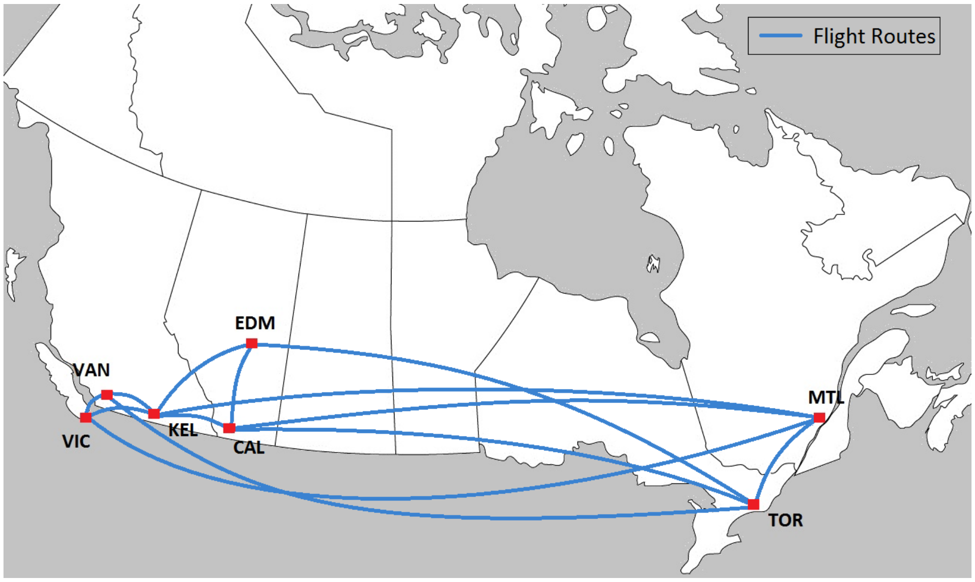


Figure 8.3: Initial network configuration for seven cities problem

Table 8.1 reports the results obtained for each optimization run. The 4 runs produced two local optima, confirming the information obtained by investigating the DOC_{F} response surface (Fig. 8.1). This validates the choice of MADS as the optimization algorithm which is able to find minima when the objective function is extremely flat in one of the variables.

Table 8.1: Results for the seven cities problem

Variable	Run 1	Run 2	Run 3	Run 4
Initial R_{X} (nmi)	2,015	2,933	2,500	1,800
Initial P_{X}	240	240	170	170
Initial DOC_{F} (\$)	3,090,187	2,114,174	2,816,993	3,028,026
Optimal R_{X} (nmi)	2147	2926	2147	2147
Optimal P_{X}	240	240	240	240
Optimal DOC_{F} (\$)	2,079,355	2,075,566	2,079,355	2,079,355
No. of active routes	14	14	14	14

Both optima reported in Table 8.1 have the same optimal network configuration with

14 active routes (out of a total of 21 possible routes) - depicted in Fig. 8.4. There are 3 new routes in the optimal network configuration and two routes that were active in the initial configuration were removed: KEL-VAN and KEL-CAL.

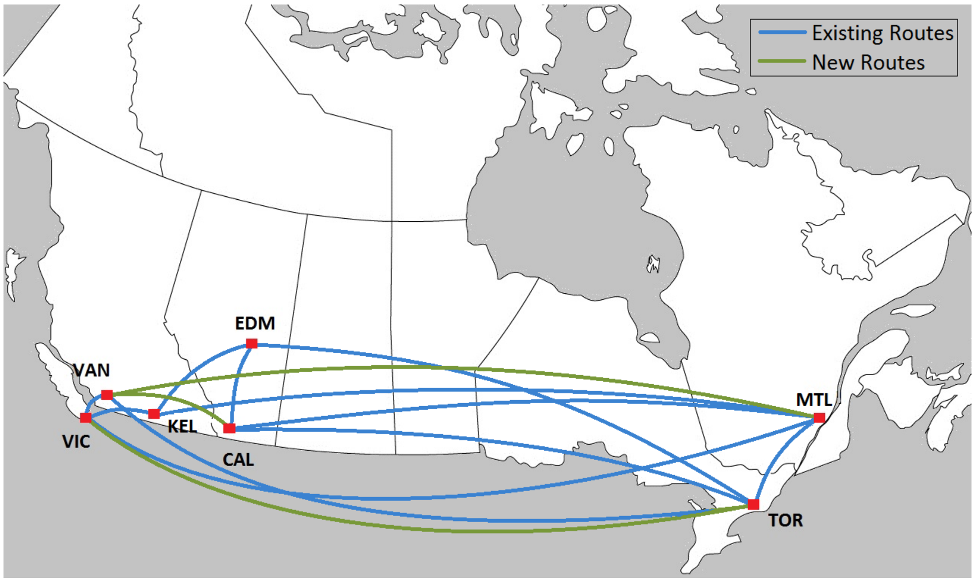


Figure 8.4: Optimal network configuration for seven cities problem

The second minima reported in Table 8.1 at $R_{\mathrm{X}} = 2926$ nmi is slightly better compared to the minima at $R_{\mathrm{X}} = 2147$ nmi - the difference being nearly $\$3,800$. However, we choose the latter[1] because a lower aircraft range is desirable for the network size considered here. The optimal aircraft design $\mathbf{d}_{\mathrm{X}}$ for this minimum is reported in Table 8.2. The corresponding $\mathbf{DOC}_{\mathrm{X}}$ is listed in Table 8.3 for all routes, regardless of whether they are active in the optimal network. The optimal aircraft allocation is given in Table 8.4. The allocation algorithm attempts to make maximum allocations for aircraft X since it has a lower operating cost compared to existing fleet.

[1]highlighted in Table 8.1.

Table 8.2: Optimal design of aircraft X for the 7-city problem

Variable	Value
Range (nmi)	2,147
Passenger capacity	240
Gross weight (lb)	181,600
Wing aspect ratio (-)	9.69
Thrust per engine (lb)	27,000
Wing area (sq. ft)	1,311
Cruise velocity (Mach)	0.82
Wing loading (lb / sq. ft)	138.50
Thrust to weight ratio (-)	0.31
Take-off distance (ft)	8,990

Table 8.3: Optimal $\mathbf{DOC}_{\mathrm{X}}$ (\$) for the 7-city problem

	VAN	MTL	CAL	EDM	VIC	KEL
TOR	25,461	7,979	21,461	21,430	25,650	23,822
VAN	–	27,518	9,020	9,900	5,471	6,548
MTL	–	–	23,393	23,107	27,768	25,850
CAL	–	–	–	6,606	9,355	7,380
EDM	–	–	–	–	10,357	8,440
VIC	–	–	–	–	–	6,887

Table 8.4: Optimal aircraft allocation and unfulfilled demand for the 7-city problem (Inactive routes left out for the sake of brevity)

Route Number	OD Pair	Aircraft A	Aircraft B	Aircraft X	Unfulfilled Demand
1	TOR–VAN	—	—	15	—
2	TOR–MTL	9	1	20	—
3	TOR–CAL	—	—	11	—
4	TOR–EDM	—	—	11	11
5	TOR–VIC	—	—	3	—
7	VAN–MTL	—	—	4	—
8	VAN–CAL	—	—	20	—
10	VAN–VIC	14	—	—	—
12	MTL–CAL	—	—	3	—
14	MTL–VIC	—	—	1	—
15	MTL–KEL	—	—	1	—
16	CAL–EDM	25	—	1	—
20	EDM–KEL	—	—	7	—
21	VIC–KEL	4	—	3	—

8.1.1 Solving the SoS Problem All in One

It is possible to combine the aircraft sizing, network design and allocation subproblems to solve the SoS design optimization problem using an all-in-one (AiO) approach. The SoS outer-loop optimization problem in AiO formulation shown in Fig. 8.5 finds the optimal R_{X}, P_{X} and $\mathbf{l}$ to minimize DOC_{F}.

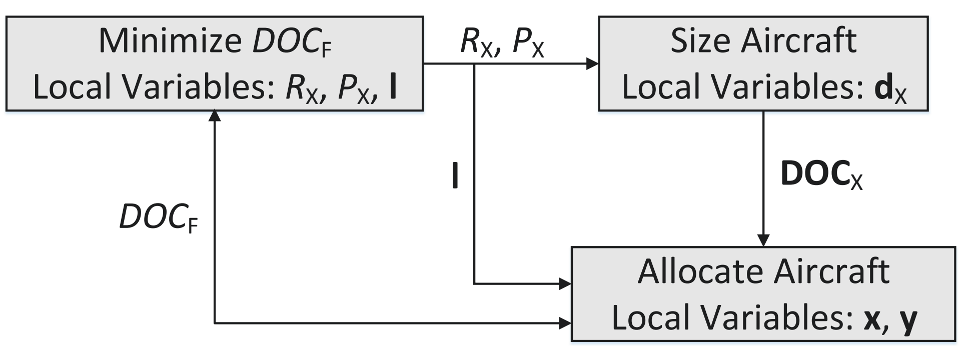


Figure 8.5: Schematic for all-in-once (AiO) problem

This is mathematically given by Eqs. (8.1) - (8.9). While the SoS outer-loop formulation changes, the formulations for aircraft sizing and aircraft allocation problem remain unchanged. The AiO problem formulation is a mixed integer nonlinear program (MINLP), which can be more difficult to solve and require longer computation time.

$$\min_{R_{\mathrm{X}}, P_{\mathrm{X}}, \mathbf{l}} DOC_{\mathrm{F}}(R_{\mathrm{X}}, P_{\mathrm{X}}, \mathbf{l}) \tag{8.1}$$

subject to

$$1600 \leq R_{\mathrm{X}} \leq 3000 \tag{8.2}$$

$$120 \leq P_{\mathrm{X}} \leq 240 \tag{8.3}$$

$$m_l \leq \sum_{i=1}^{n} \sum_{j=i+1}^{n} l_{ij} \leq m_u \tag{8.4}$$

$$\sum_{i=1}^{n} l_{ij} \geq 1 \tag{8.5}$$

$$l_{ij} = 1 \quad \forall \;\; i, j \in \Pi \tag{8.6}$$

$$LFC(\mathbf{l}) \leq \lambda \tag{8.7}$$

$$b_l \leq \sum_{i=1}^{n} \sum_{j=i+1}^{n} b_{ij}(\mathbf{l}) \leq b_u \tag{8.8}$$

$$c_l \leq \sum_{i=1}^{n} c_i(\mathbf{l}) \leq c_n, \tag{8.9}$$

The seven cities AiO problem was solved using NOMAD with the same initial guesses used to solve the aircraft sizing, network design and allocation subproblems in the nested formulation. The obtained results differed only for the optimal range, i.e., optimal capacity, aircraft sizing, active network routes and allocation of aircraft to routes were the same as for the decomposed problem formulation. As summarized in Table 8.5, the local optimal range values for the AiO problem are very near to the ones for the decomposed problem; the best "AiO" objective value is only 0.1% worse then the best "SoS" objective value. However, the AiO problem required more than 4 times longer computation time compared to the nested SoS formulation (80 and 17 minutes, respectively, on a 64-bit Intel i7 processor with 4 cores and 8192 MB of RAM). This discrepancy grows rapidly when considering larger-size problems.

The AiO formulation is not suited for problems of this magnitude. Although it takes

Table 8.5: Results for the AiO problem formulation (7-city problem)

Variable	Run 1	Run 2
Initial R_{X} (nmi)	2015	1800
Initial P_{X}	240	240
Initial DOC_{F}(\$)	3,090,187	3,090,187
Optimal R_{X} (nmi)	2124	2920
Optimal P_{X}	240	240
Optimal DOC_{F}(\$)	2,124,600	2,078,618

less time to set up, the computation time increases exponentially with increase in problem size and this method often fails to yield a solution. The failure of AiO approaches for larger problems has been investigated in literature [6, 7] and is one of the motivating factors for the SoS approach described here. In fact, we were not able to obtain a solution to the AiO problem for the fifteen cities network - described in Section 8.2.1.

8.1.2 Comparison to Hub-Spoke Networks

Mane et al. considered fixed hub-spoke network configurations in a thirty-one route problem [6]. They employed a sequential decomposition approach that has only P_{X} as the outer-loop SoS variable - shown in Fig. 8.6. The aircraft range R_{X} is assumed to be the longest route distance in the network. Further, their allocation algorithm does not allow for unfulfilled demand. Mathematically this implies that an aircraft is allocated to a route for even small residual demand. Allowing for unfulfilled demand allows for more realistic modeling of airline transportation since airlines tend to ensure highest possible capacity utilization for each aircraft.

Following the approach of Mane et al., the seven cities problem was solved for fixed hub-spoke networks and aircraft sizing for a fixed range of 2015 nmi. Two different hub-spoke configurations were considered centered at Calgary and Toronto as shown in Figs 8.7a and 8.7b respectively. Since non-hub cities are not connected to each other, the demand between them is routed through the hub-cities.

The allocation and DOC_{F} results are given in Tables 8.6 and 8.7, respectively. While the hub network with Calgary as the hub city is 7% less expensive to operate, it is dramatically (almost three times) more expensive than the optimal "free" network configuration.

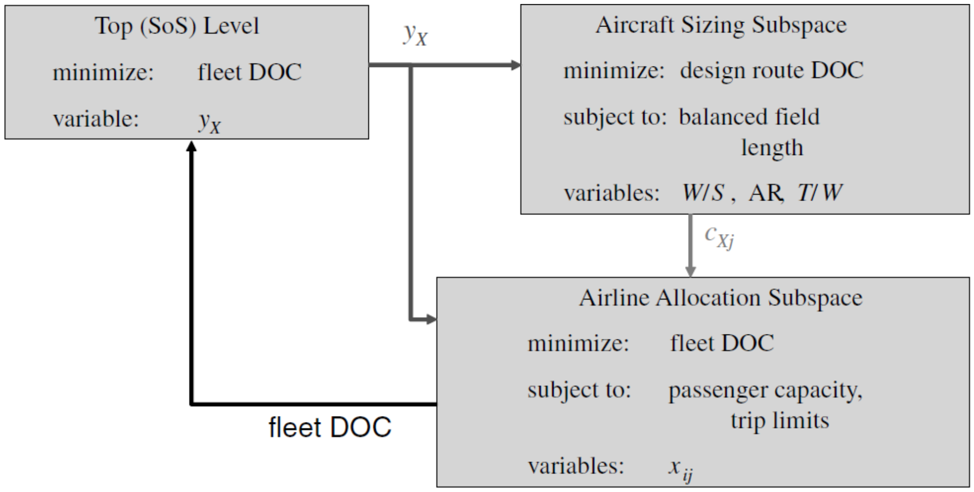
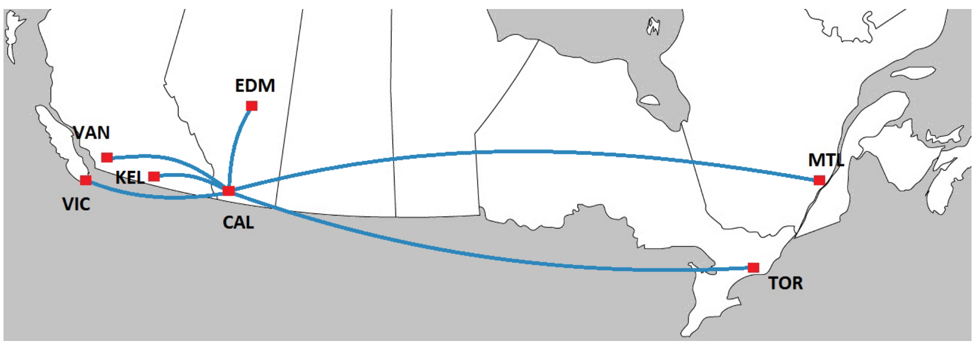
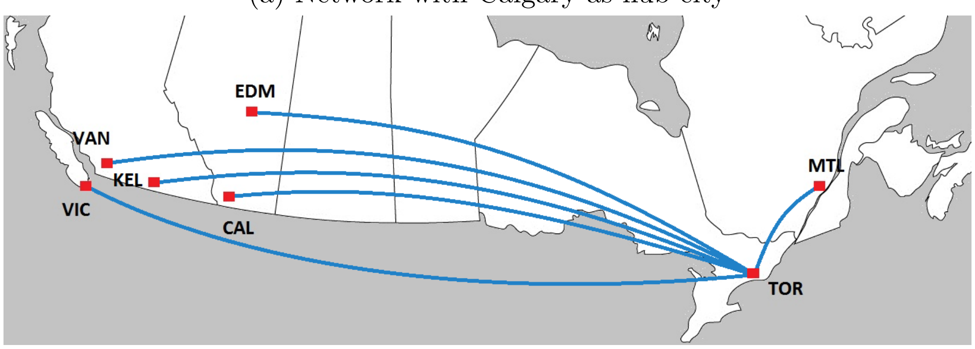


Figure 8.6: Mane's model schematic



(a) Network with Calgary as hub-city



(b) Network with Toronto as hub-city

Figure 8.7: Single-hub networks

Table 8.6: Optimal aircraft allocation and unfulfilled demand for Calgary and Toronto single-hub networks for the 7-city problem (route numbers kept same but only possible routes shown)

Route No.	OD Pair	Calgary				Toronto			
		A_j	B_j	X_j	y_j	A_j	B_j	X_j	y_j
1	TOR–VAN	—	—	—	—	40	2	20	4
2	TOR–MTL	—	—	—	—	36	1	12	—
3	TOR–CAL	50	30	20	4	33	—	20	—
4	TOR–EDM	—	—	—	—	22	1	20	—
5	TOR–VIC	—	—	—	—	—	—	17	25
6	TOR–KEL	—	—	—	—	—	—	11	—
8	VAN–CAL	40	2	20	4	—	—	—	—
12	MTL–CAL	23	—	20	—	—	—	—	—
16	CAL–EDM	36	—	12	29	—	—	—	—
17	CAL–VIC	—	—	17	25	—	—	—	—
18	CAL–KEL	—	—	11	—	—	—	—	—

Table 8.7: DOC_{F} for single-hub networks for the 7-city problem

Hub City	DOC_{F}
Calgary	$5,957,316
Toronto	$6,399,722

8.2 Stage 2: expansion to 15 cities

To this point, the analysis presented here is a simplistic representation of actual airline operations. Medium-sized airline companies fly several hundred routes per day using different aircraft. For example, Westjet Airlines operates nearly 425 flights daily using 114 aircraft of 4 different types. It is therefore interesting to apply the nested formulation to a larger problem.

We consider an extended network consisting of fifteen cities and 105 possible routes as shown in Figure 8.8. While the number of variables for the outer-loop SoS problem and aircraft sizing remains unchanged, the number of variables for the network design and aircraft allocation problems increases 5-fold from 21 and 84 to 105 and 420, respectively.

The estimated demand (computed using the model outlined in Chapter 3) for the new possible routes is listed in Table 8.8.

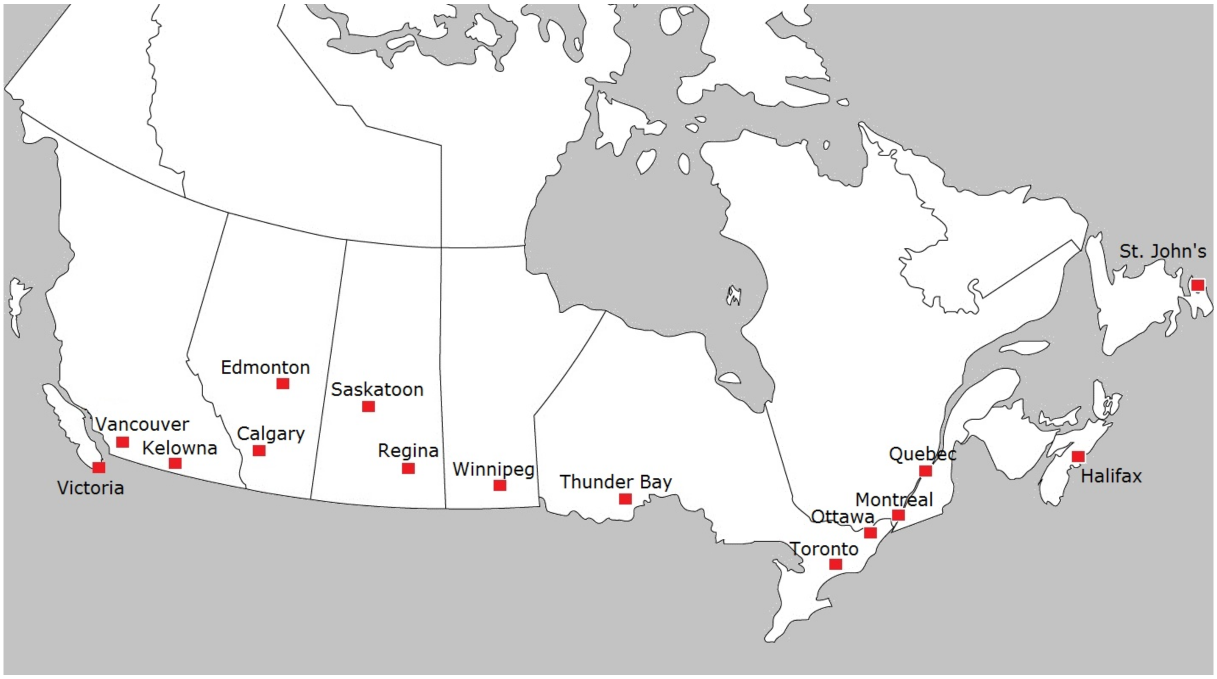


Figure 8.8: Fifteen-city network

The cities are divided into eastern and western sub-networks given by sets π_E and π_W and Eqs. (8.10) - (8.11) respectively in order to draw a comparison between the results and actual passenger movement.

$$\pi_E \equiv \{\text{Toronto, Ottawa, Montreal, Quebec, Halifax, St. Johns}\} \tag{8.10}$$

$$\pi_W \equiv \{\text{Victoria, Vancouver, Kelowna, Calgary, Edmonton, Saskatoon, Regina, Winnipeg}\} \tag{8.11}$$

Additional constraints are included in the nested network design and aircraft allocation problems to account for the following.

1. Aircraft B cannot operate from Thunder Bay (Ontario), and aircraft X can not use all airports because of runway length limitations[2]. Table 8.9 reports the runway lengths available at new cities added to the network.

2. There exist several routes that are longer than the range of the existing fleet (aircraft A and B) but lie within the bounds of R_X.

[2]We ignore Halifax otherwise the number of feasible routes for aircraft X falls to 45.

Table 8.8: Demand for the additional possible routes of the 15-city problem

	OTT	HAL	WIN	STJ	QBE	SKT	REG	TBY
TOR	3717	1655	1553	435	734	587	476	1086
VAN	1107	489	901	311	206	1041	914	145
MTL	902	659	408	272	380	156	128	303
CAL	796	361	1153	232	149	828	816	398
EDM	678	307	1138	196	127	766	773	395
OTT	–	1253	895	485	359	191	157	655
HAL	–	–	161	595	187	89	75	34
WIN	–	–	–	108	167	553	514	359
VIC	–	–	–	73	43	550	486	31
STJ	–	–	–	–	138	58	49	24
KEL	–	–	–	–	30	581	524	20
QBE	–	–	–	–	–	36	30	124
SKT	–	–	–	–	–	–	260	343
REG	–	–	–	–	–	–	–	360

Table 8.9: Airport runway length data

Airport	Halifax	Winnipeg	St. Johns	Kelowna	Quebec	Saskatoon	Regina	Thunder Bay
S_{TO} (ft)	8800	11000	8500	9000	9000	8300	7900	7300

These constraints are implement by expanding the set of impermissible 3-tuples -

$$\langle p, i, j\rangle \ \in \ \xi(R_p, s_{TO_p}) \ \text{ if } \ s_{TOp} > \{r_i, r_j\} \ \text{ or } \ R_p < D_{ij} \ \ \forall \ p \ \in \{A, B, X\}, \qquad (8.12)$$

where the parameters r_i and r_j denote runway lengths for cities i and j, respectively. Based on the aircraft range and runway length constraints, the maximum number of feasible routes for each aircraft is given in Table 8.10 (upper bound values were used for range and runway length of aircraft X). The set ξ contains at least 83 3-tuples of infeasible aircraft-route combinations. Additional tuples are introduced to this set when R_{X} is smaller than the distance of an otherwise feasible route.

Table 8.10: Maximum possible routes for each aircraft type for the 15-city problem

Aircraft Type	A	B	X
No. of Routes	91	86	55

The sharp drop in possible routes for aircraft X is due to:

1. The runway length constraint.
2. The fact that it can serve a maximum of 11 cities (depending on R_{X}).

The network design problem is also modified to include feasibility constraints that ensure that routes common to all three aircraft in the set ξ are always inactive. Mathematically, this is formulated by

$$l_{ij} = 0 \quad \forall \;\; i, j \quad \text{such that} \;\; \langle p, i, j \rangle \in \xi \;\; \forall \; p \in \{A, B, X\}. \tag{8.13}$$

Finally, for the larger-sized 15-city problem we used the Mixed Integer Linear Programming (MILP) solver in IBM ILOG CPLEX for increased computational efficiency [48]. The bound values for the allocation problem were modified as listed in Table 8.11 to ensure fulfilling the increased demand.

Table 8.11: Revised bound values for the allocation problem (15-city problem)

Bound	s_p	$\delta_{ij}\ \forall i, j$	$h_{pij}\ \forall i, j$
Value	$\{350, 500, 500\}$	40	$\{50, 20, 40\}$

8.2.1 Results

Figure 8.9 depicts the DOC_{F} response surface for the 15-city problem. The empty regions indicate points where the aircraft sizing problem fails to perform a cost analysis for one or more routes. Figure 8.10 depicts a contour map of the same response surface. We can see that the response surface is substantially more multi-modal than the one for the seven cities problem, and can pose a large challenge to an optimization algorithm.

We solved the 15-city problem with an initial guess for capacity and range equal to the optimal values of the seven cities problem. The minimum spanning network was used as the initial network configuration for the nested optimal network design problem. Each iteration of the MADS algorithm for the SoS design optimization problem (including aircraft sizing, network design and aircraft allocation) takes nearly 90 minutes to compute on a 64-bit Intel 7 processor with 4 cores and 8192 Mb of RAM. This is in contrast to 17 minutes for finding the optimal solution to the seven cities problem (using the nested formulation). As mentioned earlier, we could not solve the problem using the AiO approach

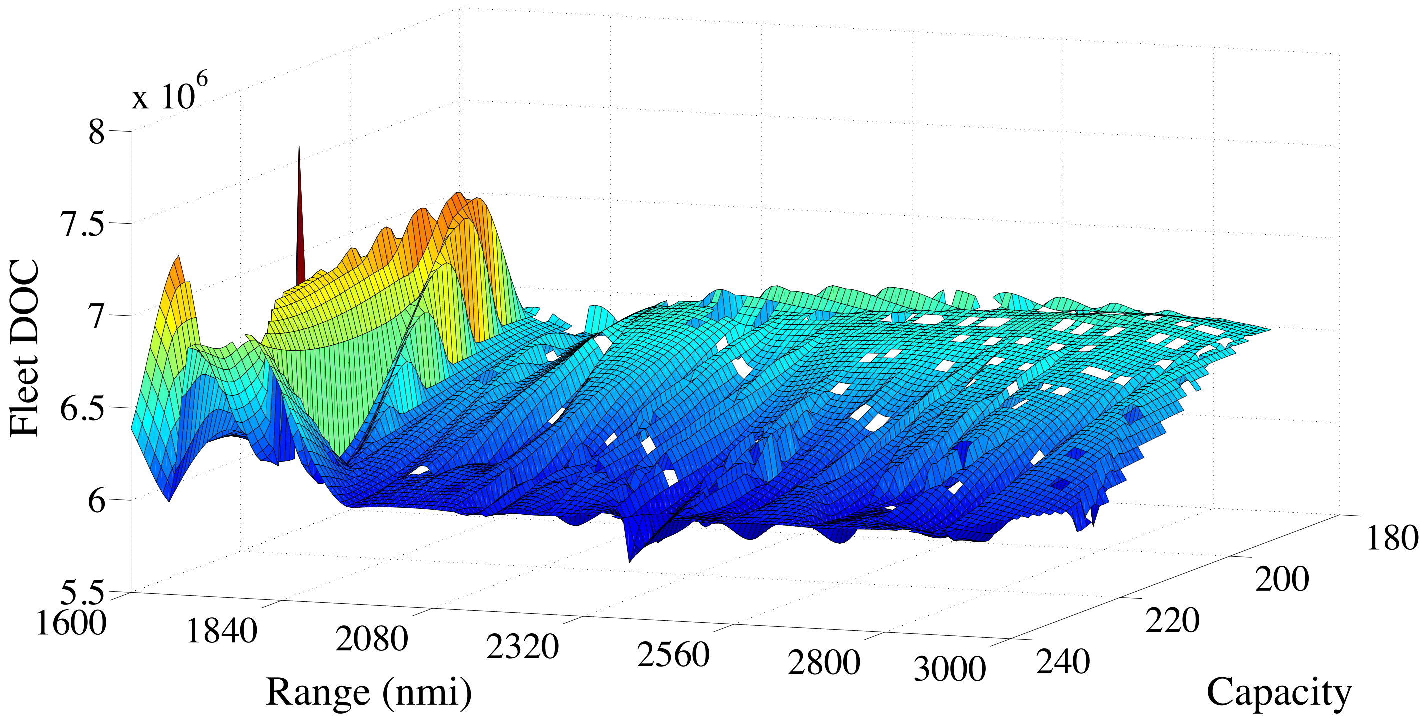


Figure 8.9: DOC_{F} response surface for the 15-city problem

(which took 80 minutes for the seven cities problem) despite several attempts as computations were exceeding 24 hours forcing Matlab to crash due to memory allocation problems. The MADS algorithm generated 134 feasible solutions for the decomposed formulation of the 15-city problem (as opposed to 62 for the 5-city problem). The best feasible solution was obtained for $R_{\mathrm{X}} = 2388$ and $P_{\mathrm{X}} = 240$. The different points investigated by the MADS algorithm are shown in Fig. 8.11

Table 8.12: Best feasible solution for the 15-city problem

R_{X}	P_{X}	DOC_{F}	Active routes	Aircraft allocated	Total unfulfilled demand
2388	240	\$5,642,754	55	418	215

This solution is confirmed by inspecting the response surface, and is similar to the seven cities problem where the optimal solution was found at the upper bound for passenger capacity. The optimal aircraft design $\mathbf{d}_{\mathrm{X}}$ is reported in Table 8.13 and contrasted to the design obtained for the seven cities problem. The corresponding $\mathbf{DOC}_{\mathrm{X}}$ is listed in Table 8.14 for all routes regardless of whether they are active in the optimal network.

Figure 8.12 depicts the optimal network configuration. Table 8.15 lists the optimal aircraft allocation and the unfulfilled demand for all routes in the network.

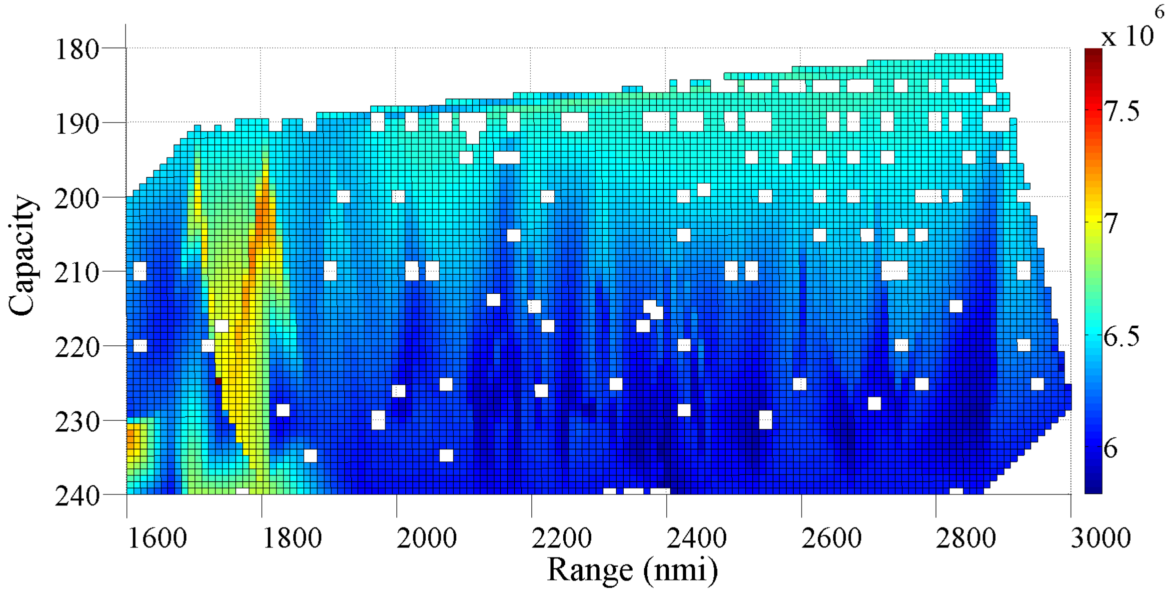


Figure 8.10: Contour map of the DOC_{F} response surface for the 15-city problem

8.2.2 Comparison to Real Data

The results obtained for the fifteen cities problem are compared to real data for WestJet Airlines, which expanded from being a regional operator to a national carrier. Since the SoS problem formulation does not take into account aircraft scheduling and intensive revenue models that affect airline operations, we compare the principal aviation hubs and routes of the network.

Caldéron [26] defines hub-cities as "top cities two cities in the country where the airline carries out operations and where the airport is among the top twenty destinations in the sample in terms of throughput". Calgary and Edmonton in the western sub-network (Π_W)and Toronto in the eastern sub-network (Π_E)emerge as principal aviation hubs for the network. The corresponding nodes have the highest nodal degree, nodal betweenness and number of passengers enplaned - reported in Table 8.16. This is in agreement with WestJet's operations statistics that list Calgary, Toronto and Vancouver as the principal operating bases accounting for nearly 46% of its domestic market share in terms of daily flights [49].

Edmonton and Winnipeg are other important hubs that collectively account for 14% of WestJet Airlines' market share. The best configuration obtained from the nested formulation returns similar results, however, Winnipeg has greater number of flight connections due to network configuration. This is due to network configuration - Winnipeg is the only city in Π_W connected to St. Johns - and therefore, all demand originating in the western

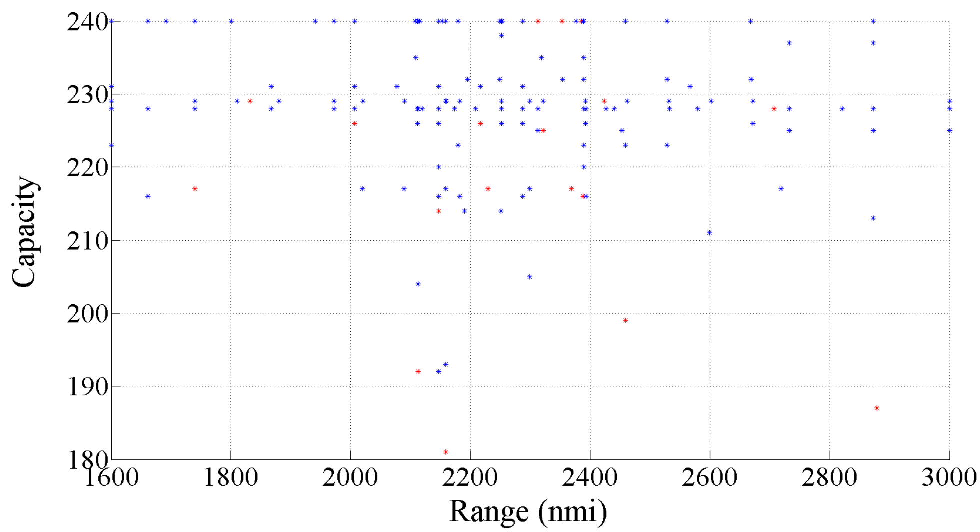


Figure 8.11: Different points investigated by MADS algorithm for 15-city problem

Table 8.13: Optimal design of aircraft X

Variable	15-city problem	7-city problem
Range (nmi)	2,388	2,147
Gross weight (lb)	193,133	181,600
Wing aspect ratio (-)	9.20	9.69
Thrust per engine (lb)	33,000	27,000
Wing area (sq. ft)	1,289	1,311
Cruise velocity (Mach)	0.82	0.82
Wing loading (lb / sq. ft)	149.83	138.50
Thrust to weight ratio (-)	0.34	0.31
Take-off distance (ft)	8,990	8,990

sub-network Π_W for St. Johns is routed through it.

In the absence of route-specific data for WestJet Airlines, we make a comparison with the overall observed passenger movement (for all airline carriers in Canada). Table 8.17 reports the four busiest routes (domestic non-stop flights) in Canada [28] and those obtained from solving the SoS design optimization problem. The agreement for three out of these four busiest routes is remarkable: the ranking dissimilarities occur because our method assigned a significantly higher passenger number to the CAL-TOR route, which can be attributed to demand that is routed through for other routes of "our" optimal network configuration.

Table 8.14: Optimal $\mathbf{DOC}_{\mathrm{X}}$ for all possible routes of the 15-city problem

	VAN	MTL	CAL	EDM	OTT	HAL	WIN	VIC	STJ	KEL	QBE	SKT	REG	TBY
TOR	25,522	7,996	21,513	21,482	7,061	12,835	14,163	25,712	17,828	23,880	9,371	18,524	17,401	10,548
VAN	–	27,586	9,039	9,922	26,670	–	16,327	5,482	–	6,562	4,908	12,242	13,050	19,990
MTL	–	–	23,449	23,163	5,926	9,906	16,071	27,835	14,752	25,912	6,337	20,296	19,355	12,446
CAL	–	–	–	6,620	22,556	28,125	12,272	9,375	31,650	7,395	24,068	8,122	9,007	15,936
EDM	–	–	–	–	22,324	27,653	12,223	10,380	30,881	8,458	23,657	7,875	9,188	15,827
OTT	–	–	–	–	–	10,919	15,169	26,908	15,743	25,001	7,229	19,433	18,454	11,524
HAL	–	–	–	–	–	–	20,809	–	10,186	30,645	8,943	24,888	24,074	17,269
WIN	–	–	–	–	–	–	–	16,574	24,694	14,681	16,767	9,273	8,191	8,577
VIC	–	–	–	–	–	–	–	–	–	6,902	28,523	12,575	13,307	20,222
STJ	–	–	–	–	–	–	–	–	–	–	13,470	28,375	27,806	21,443
KEL	–	–	–	–	–	–	–	–	–	–	26,563	10,602	11,406	18,342
QBE	–	–	–	–	–	–	–	–	–	–	–	20,866	20,023	13,238
SKT	–	–	–	–	–	–	–	–	–	–	–	–	6,356	12,897
REG	–	–	–	–	–	–	–	–	–	–	–	–	–	11,847

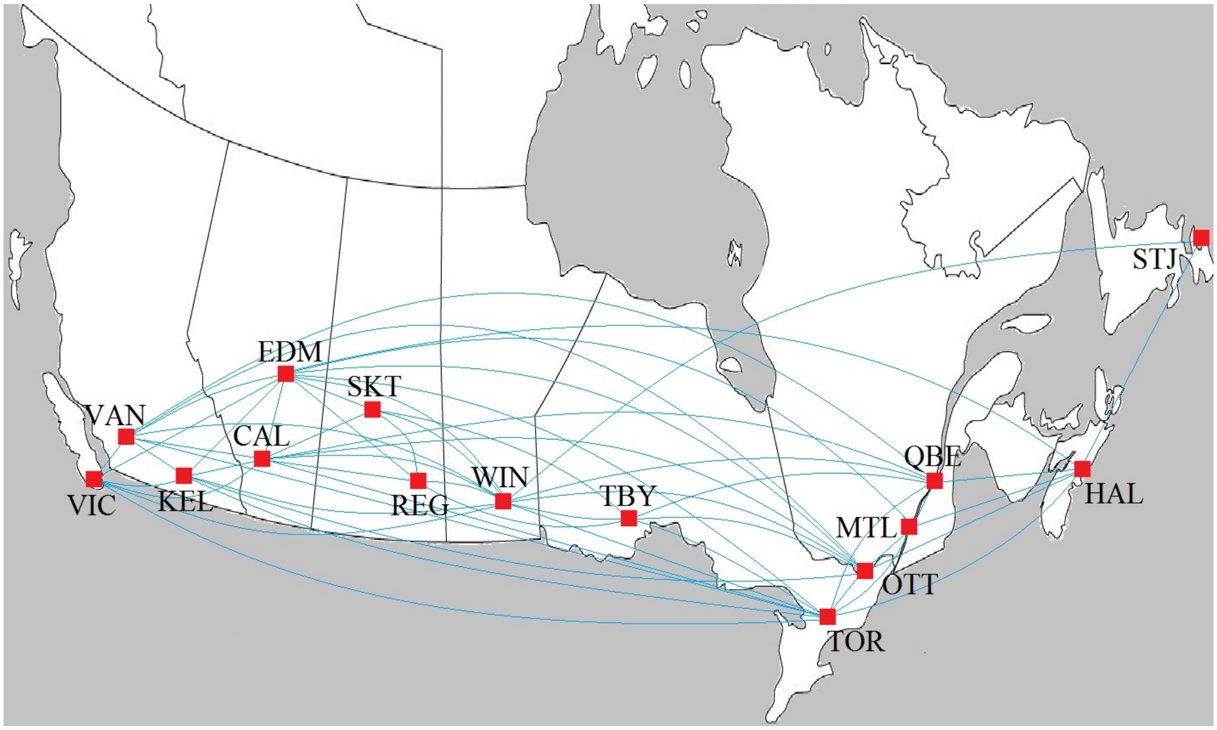


Figure 8.12: Optimal network configuration for the 15-city problem

8.2.3 Comparison to Hub-Spoke Networks

Similar to the comparison presented for the seven-cities problem, we considered two different fixed hub-spoke network configurations for the 15-city problem. For each case the new aircraft was sized for a fixed range equal to the longest route distance in the corresponding network. The DOC_{F} for hub-spoke network is given in Table 8.18. The Calgary and Toronto hub-spoke networks are now approximately 65% and 135% more expensive, respectively, for the 15-city problem (as opposed to both being approximately 200% more expensive for the seven cities problem), i.e., the Toronto hub-spoke network is now twice more expensive to operate than the Calgary hub-spoke network; this may be due to a large number of cities being located in the western part of the network.

Table 8.15: Optimal aircraft allocation and unfulfilled demand for the 15-city problem (Inactive routes left out for the sake of brevity)

Route	OD Pair	A	B	X	U	Route	OD Pair	A	B	X	U
1	TOR–VAN	–	–	15	–	48	CAL–SKT	20	2	–	–
2	TOR–MTL	–	–	24	–	49	CAL–REG	19	1	–	–
3	TOR–CAL	–	–	11	–	51	EDM–OTT	–	–	3	–
4	TOR–EDM	–	–	9	–	52	EDM–HAL	–	–	2	24
5	TOR–OTT	–	–	16	–	53	EDM–WIN	–	–	7	–
6	TOR–HAL	–	–	9	–	54	EDM–VIC	–	–	5	–
7	TOR–WIN	–	–	11	–	56	EDM–KEL	–	–	3	–
8	TOR–VIC	–	–	3	–	58	EDM–SKT	5	1	–	–
10	TOR–KEL	–	–	2	11	59	EDM–REG	5	1	–	–
11	TOR–QBE	–	–	3	15	61	OTT–HAL	–	–	8	–
14	TOR–TBY	8	–	–	–	62	OTT–WIN	–	–	9	8
16	VAN–CAL	–	–	20	26	65	OTT–KEL	–	–	2	–
17	VAN–EDM	–	–	8	–	69	OTT–TBY	7	–	–	36
18	VAN–OTT	–	–	5	–	72	HAL–STJ	18	–	–	7
20	VAN–WIN	–	–	5	–	74	HAL–QBE	–	–	9	–
21	VAN–VIC	–	–	6	–	78	WIN–VIC	–	–	3	22
23	VAN–KEL	–	–	3	–	79	WIN–STJ	3	1	–	–
24	VAN–QBE	–	–	10	–	80	WIN–KEL	–	–	3	–
29	MTL–EDM	–	–	3	–	81	WIN–QBE	–	–	3	–
30	MTL–OTT	–	–	13	–	82	WIN–SKT	12	–	–	–
31	MTL–HAL	–	–	4	–	83	WIN–REG	10	–	–	29
36	MTL–QBE	–	–	8	–	84	WIN–TBY	10	–	–	–
40	CAL–EDM	–	–	8	–	86	VIC–KEL	–	–	1	3
41	CAL–OTT	–	–	6	3	102	QBE–TBY	1	–	–	–
43	CAL–WIN	–	–	8	–	103	SKT–REG	2	–	–	–
44	CAL–VIC	–	–	9	–	104	SKT–TBY	3	–	–	–
46	CAL–KEL	–	–	7	–	105	REG–TBY	3	–	–	–
47	CAL–QBE	–	–	2	31						

Table 8.16: Nodal degree and passengers enplaned for Calgary and Toronto (15-city problem)

City	Passengers Enplaned	Degree
Calgary	22,343	10
Toronto	25,045	11
Vancouver	16,588	8

Table 8.17: Principal routes in the 15-city network

	Real Data		Air Transportation SoS Design	
Route	Daily Passengers	Rank	Daily Passengers	Rank
MTL–TOR	5,547	1	5,602	1
OTT–TOR	3,743	2	3,717	3
VAN–TOR	3,496	3	3,433	4
CAL–VAN	3,101	4	4,826	2

Table 8.18: DOC_{F} for single-hub networks (15-city problem)

Hub City	DOC_{F}
Calgary	$\$8,925,466$
Toronto	$\$13,102,547$

CHAPTER 9

Summary

This research presents a new, enhanced model for air transportation as an SoS design optimization problem formulation. Compared to the existing literature, the SoS model introduces range as a SoS outer-loop design variable, conducts optimal network configuration and treats unfulfilled demand as additional optimization variable in the aircraft to routes allocation problem. The proposed methodology was demonstrated using the example of a two-stage expansion of a regional airline network consisting of 5 cities to a small national network consisting of 7 cities that was then more than doubled in size to a network of 15 cities.

Given the rapid grow of problem size and complexity, the SoS design optimization problem was solved using a decomposition approach with a nested formulation. The Mesh Adaptive Direct Search (MADS) algorithm was used to solve the outer-loop SoS optimization problem, NASA's FLOPS code was used for aircraft sizing and a combination of MADS with either GNU's linear programming kit (for the seven cities problem) or IBM ILOG CPLEX (for the 15-city problem) was used for the nested network configuration / aircraft allocation optimization problem.

For the seven cities problem, both the nested formulation and the all-in-one (AiO) approach two local optima that differ in the range value. The respective local optima of the two approaches are almost identical. However, the nested formulation reduced computation time by a factor of more than 4 compared to the AiO approach. A comparison to fixed hub-spoke networks demonstrated that the free network configuration can decrease daily fleet direct operating cost by a factor of almost 3. At the same time, the choice of single-hub city does not seem to alter the fleet direct operating cost for small networks dramatically.

For the 15-city problem, computational challenges prevented the solution of the AiO problem. The results of the nested formulation agree closely with actual observed passenger movement and show that the formulation presented here can be used to obtain practical results. Moreover, comparisons to single-hub networks showed that while the latter are still significantly more expensive to operate, the cost ratio of single-hub to free networks decreases with increasing network size. At the same time, the location of the cities in the network, in conjunction with demand, can have a large impact on which city should be considered as the single hub.

For the allocation problem it is assumed that each airline can arrive or depart from any airport throughout the day. In practice, there are strict guidelines for aircraft traffic close to an airport for safety reasons. Consequently, each aircraft gets a limited window of time for take-off and landing and associated operations such as taxi-in, taxi-out, deplaning etc. Therefore, for a more accurate model, aircraft scheduling needs to be considered as a part of the allocation problem. This can be be done for example by incorporating block-hours availability as constraints in the allocation step. As a preliminary attempt in this direction, we limit the number of aircraft (of a given type) that can fly along a given route; this is especially important since different aircraft have different crew handling ground operations after landing and before take-off. A possible means of incorporating scheduling into the problem could be modeling the scheduling problem as a network where arcs indicate the block-hours available for aircraft movement.

The symmetric demand assumption in the model described here is quite realistic. However, it has been observed that deviation from the symmetric demand assumption can significantly impact operating costs. This becomes especially important when considering aircraft scheduling since unsymmetrical demand would leave balance aircraft at an airport, leading to reduction fluctuations in daily demand that can be serviced with a given fleet size.

Finally, the cost analysis estimates presented here consider only the direct operating cost. In practice, an airline would also need to factor other indirect costs such cost of acquisition, maintenance costs, crew requirements and costs for flying additional (new) routes. Of these costs, the first and the last are typically of higher order of magnitude, and need to be accounted for in operating costs. As the airline expands operations to include new routes, these costs are expected to rise along with additional fees which may be levied by relevant regulatory authorities. The airline may adjust its revenue and ticket pricing models based on its planned return on investment (ROI) [15]. However, to our best knowledge, there does not exist a comparable method for assigning cost of including a route in the network in the literature.

Despite these limitations, the presented model illustrates the usefulness of an SoS approach to air transportation and provides a starting point for the development of higher fidelity models. Further, this type of analysis would hold well for air-cargo services where the assumptions described above are more likely to be satisfied - unlike passengers, air-cargo traffic is not sensitive to flight-timings, shipment quantities are usually known beforehand and pricing model follows simpler constraints.

Other topics of possible future work include considering more than one type of new aircraft to be designed and allocated as well as incorporating demand uncertainty in cost analysis for airline operations (variable ticket pricing is a good example of how airlines account for uncertain demand in civil aviation).

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