Attention and Math Proficiency in Children with Neurodevelopmental Conditions: Understanding and Remediating Learning

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Abstract

Math learning and achievement are integral to academic success and predict overall achievement at later grades. Therefore, methods for improving math ability are crucial. Domain-general attentional skills have demonstrated importance for math proficiency for typically developing (TD) children and children with neurodevelopmental conditions (NDCs). Given the role of attention in math proficiency, it follows that attention-based interventions have the potential to demonstrate *far-transfer* effects; that is, improved attention resulting in improved math proficiency. However, most current behavioural methods for remediation are limited to a task-specific approach of targeting and rehearsing specific math skills. The proposed research aims to understand the influence of cognitive factors on math learning for children with NDCs and explore how this information can be used for remediation. Study 1 aims to characterise cognitive correlates of math learning in students diagnosed with an NDC. Specifically, how it is influenced by attentional abilities and non-verbal reasoning skills. The objective of Study 2 was to explore the feasibility and appropriateness of domain-general and domain-specific training paradigms and rigorous study design for individuals diagnosed with NCDs. The intent of exploring the feasibility of such study design was to inform future research that could assess the efficacy of the unique and serial effects of the training paradigms. The potential findings of the proposed research have multiple implications on our knowledge of math learning and remediation for students with NDCs. Specifically, exploring the impact of multiple cognitive factors on math learning within youth with NDCs can inform practices for early math education and better practices working with neurodiverse students.

Résumé

L'apprentissage des mathématiques font partie intégrante de la réussite scolaire et prédisent la réussite générale tout au cours du parcours scolaire. Par conséquent, des méthodes pour améliorer les aptitudes en mathématiques sont cruciales. Des compétences attentionnelles générales ont prouvé leur importance pour la maîtrise des mathématiques chez les enfants au développement typique (DT) et les enfants atteints de troubles du neurodéveloppement (TND). Étant donné le rôle important de l'attention dans la maîtrise des mathématiques, il s'ensuit que des interventions axées sur l'attention ont le potentiel d'établir des effets de transfert lointain ; c'est-à-dire qu'une meilleure attention résultera en une meilleure maîtrise des mathématiques. Cependant, la plupart des méthodes comportementales actuelles de remédiation se limitent à une approche axée sur une tâche spécifique consistant à cibler et à répéter des aptitudes mathématiques spécifiques. La recherche proposée ici vise à comprendre l'influence des facteurs cognitifs sur l'apprentissage des mathématiques chez les enfants atteints de troubles du neurodéveloppement et à explorer comment ces informations peuvent être utilisées pour la remédiation. Plus précisément, la première étude vise à caractériser les corrélats cognitifs de l'apprentissage des mathématiques chez des élèves diagnostiqués avec un TND. Plus précisément, comment ce dernier est influencé par les capacités attentionnelles et les capacités de raisonnement non verbal. L'objectif de la deuxième étude est d'explorer la faisabilité et la pertinence d'utiliser un protocole d'étude rigoureux pour évaluer les effets d'interventions visant des habiletés de domaine général (attention) et les habiletés de domaine spécifique (mathématiques) avec des élèves diagnostiqués avec un TND.

Les résultats de cette étude permettront d'éclairer les futures études pour évaluer l'efficacité des effets de ces interventions.

Les résultats potentiels de la recherche proposée ont de multiples implications sur notre connaissance de l'apprentissage des mathématiques et de la remédiation de difficultés dans ce domaine chez les élèves présentant un TND. Plus précisément, une meilleure compréhension des multiples facteurs cognitifs ayant un impact sur l'apprentissage des mathématiques peut éclairer les interventions précoces en mathématiques.

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Contribution to Original Knowledge

The contents of this dissertation contributes to the current literature on cognitive factors that contribute to math learning, and current practices for remediation. Of interest specifically is the exploration of targeting cognitive skills to remediate math learning. This research was also conducted with students with NDCs, furthering our understanding of how to identify and support these exceptional learners. The findings of Study 1 expanded on extensive literature demonstrating the relationship between attentional abilities and math achievement. Specifically, while this relationship has been extensively evidenced in TD youth, Study 1 provided evidence that the cognitive factors of attention and non-verbal intelligence mediate math learning skills in youth with NDCs, regardless of diagnostic profile. This is significant as the relationship between these cognitive factors has not previously been explored amongst youth with NCDs who present with complex diagnostic profiles and lower cognitive functioning. Additionally, Study 1 used standardized assessment tools to measure attention, cognitive ability, and math achievement for the purpose of more meaningful comparison between math and these cognitive variables. This addresses a significant methodological void in existing literature as a large portion of research on math achievement explores test scores and overall grades, resulting in limited conclusions. By using the standardized measures, compared to previous studies that did not, we were now able to generalize the results and make meaningful comparisons between youth with NDCs and complex profiles and typically developing youth regarding the cognitive correlates of math learning.

Study 1 provided the rationale for Study 2, which explored the feasibility of using a domain-general (attention) and domain-specific (math) training program with students with

NDCs that have complex diagnostic profiles and lower cognitive functioning. Another aim of Study 2 was to determine if these youth could participate in a rigorous cross-over design study to explore the unique and serial effects of training. While our intent in Study 2 was not to observe transfer effects of training, it was nonetheless explored. The rationale proposed was that if youth with NDCs and complex profiles could engage with the tasks and rigorous study design, we could justify conducting a larger-scale study to assess the unique and serial effects of training. The findings of Study 2 revealed that implementing such a study is feasible with youth with NDCs. Specifically, youth with complex diagnostic profiles and lower cognitive ability were able to engage with both the domain-general (attention) and domain-specific (math) training tasks. Furthermore, they completed a cross-over study with multiple rounds of post-testing. Though we did not aim to demonstrate far transfer in the study, the results of this pilot study did not provide initial evidence that attentional training resulted in transfer to improvement in mathematical abilities. We hypothesized this was likely the results of the math measure not being sensitive enough to detect meaningful change with these youth, in addition to the small sample size used. The results of study 2 also led to recommendations for other future feasibility studies to explore different methodology before conducting a larger scale study assessing the transfer to math learning.

Overall, this research contributes new findings to the literature on understanding and remediating math learning for students with NDCs with complex diagnostic profiles and lower cognitive functioning. There are not studies that have directly assessed the interplay of attention and non-verbal cognitive skills for math achievement with youth with NDCs with complex profiles/lower cognitive ability. Additionally, we provide evidence in support of

assessing the efficacy of cognitive training and domain-specific (math) training for youth with NDCs, who are often excluded from the cognitive training literature. We conclude by making arguments in support of moving towards consistent use of standardized metrics in future research and clinical use.

Contribution of Authors

I, Emma Clark, am the primary author of this dissertation and have conceptualized, collected, and written this dissertation in its entirety. My doctoral research supervisor, Dr. Armando Bertone, played an integral role in the supervision of this project, and provided oversight for all projects, including conceptualization through to manuscript submission. Dr. Domenico Tullo also played an integral role in collaborating on these various research projects. Dr. Tullo was investigating attentional skills and training within students with NDCs, and as such introduced me to the 3D-Multiple Object Tracking intervention program, developed by Dr. Jocelyn Faubert. Given our shared interest in exploring attentional profiles in this youth with NDCs, we collaborated on participant recruitment, testing, and data entry. Dr. Tullo also recruited several undergraduate research students who supported the data collection for all studies that comprise this dissertation.

I am the principal author for all manuscripts. As co-authors, Drs. Bertone, Tullo, and Faubert provided support in formulating the research questions and finalizing the experimental methodology and procedure for the feasibility study. Dr. Tullo also provided guidance on the moderated mediation model used to describe the results in Study 1. Ms. Peremiter supported manuscript one by finding and compiling articles, editing content, and revising formatting. While I wrote the full drafts for each manuscript independently, my co-authors provided assistance with the editing of both manuscripts before submission.

Dr. Bertone and Dr. Adam Dubé served on my doctoral committee and helped me to ensure the rational for my proposed research was sound. They provided constructive feedback to my comprehensive literature review that provides the foundation for the research questions posed.

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List of Abbreviations

ADHD – Attention-Deficit/Hyperactivity Disorder
ASD – Autism Spectrum Disorder
CPT-3 – Conners Continuous Performance Task, Third Edition
d' – detectability score on the CPT-3
EasyCBM – Easy Curriculum-Based Measure
EF – Executive functioning
ERIC – Educational Resources Information Center
FRQS – Fonds de recherche du Québec – Santé
FSIQ – Full-scale intelligence quotient
IDR – Integrated dynamic representation
ID – Intellectual Disability
IQ – Intelligence quotient
KTEA-3 - Kaufman Test of Educational Achievement, Third Edition
LD – Language disorder
M - Mean
MD – Mathematics difficulties
MEES - Ministère de l'Éducation de l'Enseignement supérieur
MID – Mild Intellectual Disability
MLD – Math learning disorder
MLDs – Math learning disorders
MOT – Multiple object tracking
NDC(s) – Neurodevelopmental condition(s)
PAL-II - Process Assessment of the Learner, Second Edition
PNLab – Perceptual Neuroscience Lab for Autism and Development
PR – Perceptual reasoning
PRI – Perceptual reasoning index
PS – Processing speed
RD – Reading difficulties
RCT – Randomized controlled trial
RTI – Response to intervention
SCERT – Summit Centre for Research, Education, and Training
SD – Standard deviation
SLD – Specific learning disorder
TAU – Treatment as usual/passive control
TD – Typically developing
VCI – Verbal comprehension index
VSTM – Visual short-term memory
WASI-II – Weschler Abbreviated Scale of Intelligence, Second Edition
WIAT-III – Weschler Individual Achievement Test – Third Edition
WJ III ACH – Woodcock Johnson Test of Achievement – Third Edition
WM – Working memory
WRAT5 - Wide Range Achievement Test, Fifth Edition

Chapter I: Introduction

Development of Math Proficiency

Typical Development

Children's exposure to and development of mathematical thinking begins before they ever enter formal education (Geary, 1994). During play, they explore patterns, shapes, spatial relations, and count objects, all which provide foundational experiences that support the development of concepts important for mathematics (Kronholz, 2000). A large portion of children enter first grade with basic counting skills, understanding of relative quantities, and the ability to solve problems involving numbers (Bassok & Latham, 2017; Engel, 2016). By the end of first grade, most children understand relative size, sequence and ordinality, and proficiently add and subtract basic whole numbers (Denton & West, 2002). These skills fall under the domain of number sense and quantitative thinking, one of the most important areas of math learning for early childhood, and a large portion of the curriculum (Sarama & Clements, 2009). In their 2009 book, Samara and Clements extensively reviewed and presented details on the mathematics learning trajectories for young children, a relevant synopsis is presented in the following paragraphs. They highlighted that there are many smaller skills encompassed within this area of mathematics, and instruction becomes increasingly complex throughout education, building on earlier foundations. Thus, failing to understand or master the more fundamental skills could be detrimental to a child becoming mathematically proficient. They presented that first, a child must learn the logical foundations of quantity and number order, before they can progress to verbal and object counting. Students are then prepared to learn the concepts of comparing, ordering, and estimating discrete quantities. Samara and Clements (2009) proposed that children are finally equipped to begin formal instruction in arithmetic beginning with single-digit addition and subtraction. These basic arithmetic functions can be mastered with simple strategies, such as counting on or counting up (Geary et al., 2004), whereas later skills such as composition of number, place value, and multidigit addition and subtraction rely on more formal instruction and skill development (Sarama & Clements, 2009).

Though the research on the development of geometry skills is less consistent, it appears that most children can identify basic shapes at the beginning of kindergarten (Engel et al., 2016). Young children begin to employ strategies for determining shape congruence, such as edge matching (Beilin & Klein, 1982). Though Samara and Clements (2009) proposed that the early years of education are dominated with instruction in numerical and quantitative thinking, there is still exposure to and learning about spatial thinking, shapes, and composition and decomposition of standard geometrical shapes. They indicate that in later years, students move on to learning the more complex concepts of geometric measurement such as calculating length, area, volume and angle. All these later concepts evidentially rely on assumption that numerical and calculation skills have been adequately developed, as the formulas rely on an understanding of relative magnitude, as well as the ability to perform operations.

Throughout their learning of essential math concepts, children are exposed to pattern-related activities such as subitized word patterns, patterns in word counting, and spatial patterns (Sarama & Clements, 2009; Wu, 2007). In their book, Samara and Clements (2009) propose that the ability to recognize and analyze patterns is an integral skill, as it provides the foundation for algebraic thinking, a concept that is introduced later in the child’s education. They further suggest that algebraic thinking also relies on making generalizations, a skill that again is implicitly taught in earlier instruction. For example, children must generalize that adding 0 does not change a number (Carpenter & Levi, 2000). Thus, children learn these strategies implicitly, but it is integral that teachers begin to make these processes explicit as children approach the stages of algebraic instruction (Carpenter & Levi, 2000; Sarama & Clements, 2009).

Samara and Clements (2009) highlight another component of mathematical learning that is integral to proficiency and is interwoven throughout education, and that is mathematical concepts. While children learn facts and formulas, they also learn essential mathematical concepts such as problem solving, reasoning, connections, representation and patterning (Clements, 2004).

Evidentially, math learning is a complex process that has many intertwined concepts and processes that must be mastered in a progressive manner in order to facilitate future learning (Cargnelutti et al., 2017). Falling behind at any stage can hinder a child’s ability to become mathematically proficient. Samara and Clements (2009) highlight that this is especially true of earlier education, in which key concepts of number and quantitative knowledge are emphasized in learning so that the child can move on to arithmetic, geometry, and eventually algebra. A typical learning trajectory then, as measured by a standardized test based on grade-based material, would presume that children meet scores within the average range, annually. Above or below average scores would be indicated of advanced mathematical thinking or potential difficulty, respectively. But what happens for children who fail to grasp those essential early concepts? For children with neurodevelopmental conditions who experience several symptoms that impact their classroom functioning, this poses a significant risk for their development of mathematical proficiency.

Atypical Development

Neurodevelopmental conditions, of any origin, are often related to academic difficulties in many subjects (Barnes & Raghubar, 2014). Given that there are many neurocognitive processes involved in math learning and achievement, it can be assumed that childhood disorders that affect the brain have a direct impact on math proficiency (Barnes & Raghubar, 2014). Further, because mathematics is a complex process that relies of many cognitive factors, difficulties in just one area of cognitive functioning could impair learning (Cargnelutti et al., 2017). However, children with neurodevelopmental conditions often have difficulties in more than one area of cognitive functioning, making the impact on learning even more profound. To complicate matters further, children within the same diagnostic category can have varying cognitive profiles and symptomology (Charman, 2015; Finucane et al., 2016).

Research exploring the relationship between cognition and math proficiency in children with NCD has predominantly focused on relating specific cognitive skills to mathematical abilities. Though research is yet to explore the aforementioned learning trajectories, work with atypically developing youth has informed the literature relating cognitive abilities to math learning. Some of the most commonly identified cognitive correlates of math learning and performance are visual-spatial abilities, language processes, working memory, executive functioning, and attention (see Barnes & Raghubar, 2014; Cargnelutti et al., 2017; Tsal et al., 2005). For example, when estimating and completing basic operations of addition and subtraction, many children make use of a mental number line which relies on working memory and visual-spatial processes (Dehaene et al., 2003). Curriculum content and standards for teaching also promote the use of visual-spatial cognitive skills in many math problem-solving situations (Blair et al., 2005). However, for children who have limited visual working memory capacity or impaired visual-spatial cognition, they are at a disadvantage from the start (Barnes & Raghubar, 2014). This is particularly true considering the role of visual-spatial abilities in the development of these foundational skills. Difficulties in verbal working memory and phonological processing have also been related to math learning difficulties (Geary et al., 2007; Hecht et al., 2001; Simmons & Singleton, 2008).

Of most importance to the current research question is the importance of attentional abilities, as attention is impacted to different extents in many different neurodevelopmental conditions (Dekker & Koot, 2003; DuPaul et al., 2013; Garretson et al., 1990). Understanding the relationship between attention and math has the potential to greatly inform our understanding of the development of math proficiency in atypically developing youths, and better inform remediation.

The objectives of the proposed research are threefold. First, to analyze and summarize the key findings of the research that empirically demonstrate the relationship between attention, related executive skills, and mathematics. Secondly, to characterize the cognitive factors that contribute to math learning across children with various NDCs. Lastly, to assess the feasibility of domain-general (attention) and domain-specific (math) interventions for youth with NDCs in a rigorous cross-over design study.

Chapter II: Manuscript 1 - Review of the Literature

Understanding the relationship between attention, executive functions, and mathematics: Using a function-specific approach for understanding and remediating mathematics learning

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Abstract

Mathematics learning and achievement are integral to academic success and have been shown to predict overall achievement at later grades. Methods for improving mathematics ability are therefore crucial. Domain-general attentional skills are important for the development of mathematical proficiency for typically developing (TD) children and children with neurodevelopmental conditions (NDCs). Despite this, most current methods for remediation are limited to task-specific approaches of targeting and rehearsing specific mathematics skills. Given the evidence supporting the relationship between attentional abilities and mathematics learning and achievement, it is proposed that mathematics difficulties are function-specific and can be remediated as such. In this chapter, the relationship between attentional skills and mathematics learning and achievement in both TD children and those with NDCs will be presented. This literature will serve to contextualize the limited research that has evaluated cognitive training for mathematics remediation; the effectiveness of such cognitive-based interventions will also be evaluated. The chapter will conclude with a discussion regarding practical implications of understanding the role of attention in mathematics learning. This conclusion will aim to inform clinicians and educators about effective identification and remediation of mathematics learning using cognitive-based assessments and interventions. Suggestions for future research and potential cognitive interventions will be discussed.

Keywords: Mathematics Learning, Attention, Cognitive Training, Atypical Development, Intervention

3.1 Introduction

The development and mastery of school-entry mathematical skills are strongly related to later overall academic achievement (Duncan et al., 2007), suggesting early identification and remediation of mathematics difficulties are essential. However, to do so, an accurate understanding of the various factors that contribute to the development of mathematics proficiency in elementary and middle school is required. Commonly referred to as dyscalculia[1], mathematics learning disorders (MLDs) are characterized by a difficulty in acquiring arithmetic skills necessary to solve calculations or problems (Raja & Kumar, 2012). Specific impairments can include individual or combined deficits in number sense, memorization of arithmetic facts, fluent calculation, or mathematics reasoning (American Psychiatric Association, 2013). Specific learning disorders become apparent as children enter elementary school, as their ability to learn foundational skills is impacted (American Psychological Association, 2015). It then follows that proper identification and intervention are integral during the elementary and middle school years.

It is estimated that in the typically developing population (TD), 6-7% of school-aged children present with an MLD (Zentall, 2007). In children with neurodevelopmental conditions (NDCs), the rates of MLDs are almost three times that of TD individuals (Mayes & Calhoun, 2006). For instance, individuals with attention-deficit/hyperactivity disorder (ADHD) experience significant challenges in mathematics learning; in the ADHD population, the rate of MLDs is estimated at 31% (Zentall, 2007). Though there has been recent research into the etiology of

[1]The DSM-5 lists Dyscalculia as an alternative term to describe mathematics learning disabilities, however, it must be specified whether impairments with word-problem or problem-solving abilities are present (American Psychological Association, 2015). For this chapter, we will use each respective study's terminology in presenting findings.

MLDs (see Butterworth et al. 2011; Bartelet et al., 2014; Price & Ansari, 2013), there is much less known about numerical cognition and numeracy mastery, compared to our understanding of language acquisition and reading (Kwok & Ansari, 2019; Geary et al., 2011; Gersten et al., 2007; Zentall, 2007).

Mathematics difficulties are associated with cognitive deficits that are not explicitly related to numerical processing (Peng et al., 2018). There is substantial empirical literature demonstrating a relationship between various attentional abilities and mathematics proficiency across the lifespan (e.g., Blair & Razza, 2007; Peng et al., 2018). Various types of attentional abilities and related executive functioning skills have been strongly related to mathematics learning across periods of development and clinical populations (Cragg & Gilmore, 2014). For example, inattention, attentional switching, and inhibition have been linked to the development of mathematics skills in early elementary school years (Kindergarten to Grade 3) (Blair & Razza, 2007; Fuchs et al., 2005, Gold et al., 2013). Additionally, studies have demonstrated a functional relationship between working memory and numeracy in 5-year-olds (Kroesbergen et al., 2014) that has been further evidenced in middle childhood-aged samples by neuroimaging research (Dehaene et al., 1999; Kwok & Ansari, 2019; LeFevre et al., 2010; Menon, 2016). The studies mentioned above provide evidence for a relationship between mathematics learning, attention, and executive functions in childhood and thereby indicate that these cognitive skills should be considered in the assessment and remediation of mathematics learning. However, most remediation research centres around mathematics skill-based approaches such as repeated practice, exposure, and feedback (Mong & Mong, 2010; Templeton et al., 2008). While these interventions may be useful, it can be proposed that they are a *task*-specific approach to an arguably *function*-specific problem defined by deficits in attention and related executive functioning skills. Given the importance of mathematics to overall academic achievement (Duncan et al., 2007), and the increased rate of MLD amongst clinical populations, there is a need to understand the cognitive factors associated with mathematics learning to better inform function-specific remediation for both typically and atypically developing individuals (Zentall, 2007).

In this chapter, we will analyze and summarize the key findings of the research that empirically demonstrate the relationship between attention, related executive skills, and mathematics. We will then compare these results to intervention studies and assess whether the mathematics-related components of attention are adequately addressed in mathematics intervention research. Finally, we will assess the results of the interventions qualitatively and argue why they were effective, or not, based on the attentional skill targeted or the outcome measures used. To conclude, we will make recommendations for the future of attention-based interventions for mathematics remediation.

3.2 Attentional Abilities and Mathematics Proficiency

3.2.1 Attention

Broadly defined, attention is the ability to orient and sustain one's focus on stimuli while ignoring distracting information (Tsal et al., 2005). Various aspects of attention, including sustained attention, selective attention, and inattention have been related to mathematical abilities in students from third to eighth grade (e.g., Lindsay et al., 2001; Raghubar et al., 2009; Ruckert & Levy, 1995). For instance, numerosity, a fundamental skill in mathematics, is strongly related to both mathematics achievement and attentional abilities in second-grade children, suggesting a unique contribution of attention to the development of numerosity (Child et al., 2019). Furthermore, the development of attention from four to six years of age has been shown to contribute to mathematical proficiency above and beyond other vital skills such as visuomotor integration or fine motor coordination (Kim et al., 2018). These two studies provide initial examples of the integral role of attentional abilities in the development of key mathematical skills. While many more will be explored within the scope of this chapter, it is important to identify what is considered attention and related executive functioning skills.

Understanding the relationship between attentional and mathematical ability is complicated because of the often-conflicting operational definitions of “attention” and what are considered as related or distinct executive functioning skills. Sustained and selective attention, working memory, executive functioning, and processing speed are cognitive skills commonly associated with the general term “attention”. Many researchers consider attention, working memory (WM), and executive functioning (EF) to be distinct skills that are related to mathematics ability (Peng et al., 2018; Peterson et al., 2017).

WM can be considered as a mental storage unit that relies on the ability to focus one's attention and integrate information from several sources, all while inhibiting irrelevant information (Dahlin, 2013). However, these latter processes involved in WM have been described by Miyake et al (2000) as three interrelated executive functioning skills: shifting, updating, and inhibiting. The successful use of these executive function skills relies on attention being activated.

In addition to shifting, updating, and inhibiting, EF includes processes for self-regulated, goal-directed behaviours including goal identification, planning and initiation, self-regulation, cognitive flexibility, allocation of attention, and using feedback (Anderson, 2002). Attention can be considered a fundamental skill to EF, such that disruption to attention leads to distractibility, impulsivity, forgetfulness, and poor focus (May et al., 2013).

Finally, many studies consider processing speed (PS) as another factor that influences mathematics abilities (Peng et al., 2018). PS has been studied as a stand-alone variable, as well as a consideration within cognitive load theory (Conlin et al., 2005; Geary et al., 2007; Peng et al., 2018;). Cognitive load theory provides one method of conceptualizing the complicated relationship between attention, WM, EF, and PS. This theory proposes that an excess of information leads to inefficient processing of information, as attentional resources are being allocated to multiple pieces of information or processes (Paas et al., 2003). More efficient processing of information frees up cognitive space to perform higher-order functions encompassed by WM and EF (Paas et al., 2003).

Just as there is variability amongst researchers' beliefs about whether the aforementioned skills are related or distinct, there are two different schools of thought regarding whether executive functions themselves are a set of distinct domain-general skills, or a unilateral skill set. Conceptualized by Miyake (2000), one school of thought proposes that WM, EF, PS, and attention are distinct but related skills whose success depends on a top-down approach. That is, WM is a mental workspace that is integral to mathematical processing (Raghubar et al., 2010). The successful use of that workspace relies on intact executive functions, namely shifting, updating, and inhibiting (Miyake et al., 2000). Lastly, these skills require efficient information processing as well as attentional activation and control. Thus, the role of various related cognitive factors involved in mathematics relies initially on attention.

This conceptualization proposed by Miyake is widely cited and accepted in the literature. However, it's main critique is being an adult-centric framework by those proposing a more unilateral framework of EF in childhood (ex. Doebel, 2020; Garon et al., 2008; Karr et al., 2018). Given our interest in exploring the effects of general attention and the implications it has to math learning, our overview aligns most closely with that of Miyake (2000). However, it is important to highlight the alternative school of thought proposed by Doebel (2020), that is contextualized to a greater extent within a development framework.

Doebel (2020) counters Miyake (2000) by proposing that the development of executive function should be considered as related skills that emerge and coordinate in pursuit of specific goals. There is some research to suggest that the EF system changes as a function of development, with younger children showing less differentiation between skills, suggesting a unitary construct (Bull & Lee, 2014). Karr and colleagues (2018) suggest that a unidimensional approach is most appropriate for understanding EF in childhood particularly regarding isolating the skill of shifting (Karr et al., 2018). Along these lines, Garon, Bryson and Smith (2008) are also in agreement with a unilateral framework for conceptualizing EF in early childhood and proposes that the development of all EF can be attributed in part to development of the attention system.

The research exploring the relationship between these cognitive skill sets, and mathematics does not always adhere to one framework of EF conceptualization. Thus, for the purposes of isolating and exploring various EFs and the relationships to mathematics skills, we will explore them in separate sections, adhering most closely to Miyake's conceptualization. Following this top-down conceptualization, we will first explore the relationship between mathematics and attention (section 3.2.1), followed by EF, WM, and PS (sections 3.2.2, 3.2.3, and 3.2.4, respectively).

3.2.1.1 Attentional Abilities with Mathematics Learning Profiles.

Examining the attentional abilities of children with advanced mathematics skills, as well as those with MLDs, provides further insight into the math-attention relationship. Mathematically gifted adolescents (11- to 15-years-old) show significantly faster responses with fewer errors on a selective attention task when compared to typically developing (TD) peers (Rueckert & Levy, 1995). Conversely, compared to TD peers, fifth- to eight-grade students with dyscalculia performed significantly worse on a clinical task of sustained attention (Lindsay et al., 2001). Specifically, students with dyscalculia made more omission errors and demonstrated more inconsistency in reaction times (Lindsay et al., 2001). Dyscalculia has also been associated with elementary school children making more errors on sustained attention tasks even when compared to individuals with ADHD (Kuhn et al., 2016). Inattention has also been related to the severity of mathematics learning difficulties in Grades 3 to 4, with students with MLDs being rated as more inattentive than their low-achieving peers (Raghubar et al., 2009).

3.2.1.2 Mathematical Abilities within Attentional Profiles.

Further evidence for the relationship between attention and mathematics learning comes from assessing the mathematical abilities of individuals with impaired attention. Children in Grades 1 to 5 diagnosed with ADHD, without a comorbid learning disability, still demonstrate significant mathematics difficulties despite having intact numerical knowledge (Colomer et al., 2013). Elementary school children with ADHD have been observed to perform significantly lower than TD peers on standardized tests of mathematics (McConaughy et al., 2011). There is potential for a profound impact on future achievement if these children do not receive early intervention. This is evident within ADHD samples, where the proportion of children presenting with moderate to severe difficulties with mathematics increases with age (Colomer et al., 2013). However, for these children with ADHD, the nature of the difficulties shifts with age from counting errors and number dictation (Grades 1 to 2), which rely on automaticity, to more procedural skills such as mental calculation and counting (Grades 3 to 5) (Colomer et al., 2013).

Even within samples of children with impaired attention, the severity of attentional difficulties has an inverse relationship with mathematics performance. Third and fourth-grade students rated as more inattentive have been observed to answer fewer multi-digit computation questions correctly, perform more mathematics fact errors, and commit more procedural errors (Raghubar et al., 2009). Comparably, middle and high school students with lower ADHD-inattentive symptoms demonstrate increased mathematics achievement (Mattison & Blader, 2013). Ross and Randolph (2016) argue that the mathematics deficits exhibited by 7- to 11-year old children with ADHD can be accounted for by their task vigilance, specifically how easily they are distracted and how well they can disengage from distractions and return to tasks. Yet another example of the inverse relationship between attention and mathematics comes from a sample of children with Mild Intellectual Disability (MID). Djuric-Zdravkovic and colleagues (2011) assessed mathematics and attention in a sample of 12- to 14-year-old students with MID and reported that impaired mathematics learning was accounted for by difficulties with sustained attention, above and beyond what would be explained by general cognitive impairments.

Thus, whether you examine the attentional abilities of individuals with distinct mathematics learning profiles, or the mathematics abilities of those with attention difficulties, the mathematics and attention relationship exists. This is further supported by relationships among various attentional abilities, such as sustained and selective attention, inhibition, and the development of mathematics skills in TD children. The argument in support of the function-specific mathematics difficulties becomes stronger when considering the relationship of cognitive skills that rely on attention activation, such as executive functions.

3.2.2 Executive Functioning

The perceived influence of multiple EFs on mathematical ability is robust, with planning, updating, and inhibition explaining 45% of the variance in childhood numeracy in 5- and 6-year-olds, after controlling for intelligence quotient (IQ) (Kroesbergen et al., 2009). Poorghorban and colleagues (2018) even observed that high and low mathematics achieving fourth-grade students did not differ in their performance on sustained attention, but rather the difference in mathematics performance was accounted for by EF, specifically shifting. Compared to the *skill*-specific relationship of WM to mathematics skills (e.g., visual WM and magnitude estimation), Peng and colleagues (2018) conducted a meta-analytic review that identified more general EF deficits that have a global impact on mathematics learning for individuals with mathematics difficulties across the lifespan. Difficulties with inhibition and switching were identified amongst third-grade students with mathematics difficulties by Bull and Scerif (2001). Specifically, individuals with lower mathematical ability demonstrated significant difficulty inhibiting a learned strategy in favour of a new one (Bull & Scerif, 2001). Furthermore, these children were more impaired in their ability to inhibit prepotent responses (Bull & Scerif, 2001). This difficulty was thought to be accounted for by more irrelevant information being held in WM, and a lower WM span (Bull & Scerif, 2001). In addition, domain-general cognitive skills (i.e., inhibition, attention, and WM) were predictive of mathematics skills longitudinally and predicted growth in mathematics in a sample of children aged 3- to 4-years-old (Coolen et al., 2021). This suggests that executive skills serve as a predictor for foundational mathematics skills, before the onset of formal education. Aside from inhibiting and updating, attentional switching is a complex EF skill often impaired in individuals with neurodevelopmental conditions.

May and colleagues (2013) consider *attentional switching* to be integral to mathematics abilities. In a sample of 7- to 12-year-old children with an Autism Spectrum Disorder (ASD), they found that performance on an attentional switching task explained a significant amount of variance in mathematics achievement (May et al., 2013). A similar effect of attentional switching is observed in non-clinical samples, such that high mathematics achieving fourth-grade students perform significantly better on tasks of attentional shifting than their low mathematics achieving peers (Poorghorban et al., 2018). A meta-analytic review of the literature further supports the role that attentional shifting plays in mathematics ability, with an effect size of .33 (Yeniad et al., 2013). Alternatively, Bull and colleagues (2008) propose that skills such as attentional shifting, inhibition, goal planning, and monitoring influence learning more generally, as opposed to being specific to math. However, they do report that in early elementary grades, inhibition, planning, and monitoring provide some predictive power in addition to visual short-term memory (Bull et al., 2008). In contrast, the ability to shift attention may be critical in solving more complex mathematics problems in later elementary school (Bull et al., 2008).

A final EF related to mathematics achievement in middle childhood is *planning*: the mental representation of problem-solving to reach a goal (Friedman et al., 2014). Furthermore, better planning abilities are predicted by stronger short-term memory, sustained attention, and inhibition (Friedman et al., 2014). This exemplifies the complex relationship between many different cognitive domains that can be considered under the term of attention.

While general function-specific deficits in EF are related to general mathematics deficits, there are also distinct connections between distinct EFs and task-specific deficits, particularly in problem-solving. This point brings us back to the debate as to whether or not EF can be considered as a domain-specific skill set or a unilateral skillset within childhood. The unilateral framework aligns well with the first argument that general deficits in EF are related to general deficits in mathematics. However, considering EF as a set of distinct skills leads us to draw more nuanced conclusions between specific EF deficits and impacted mathematics skills. These specific relationships can help support intervention development, discussed later in this chapter. For example, attentional shifting or switching is related not only to general achievement in both typically and atypically developing populations, but also specifically to complex problem-solving skills. Additionally, function-specific deficits in inhibition result in task-specific difficulty with response inhibition and strategy selection during problem-solving. Finally, planning is also explicitly related to problem-solving. The development of one's planning abilities is directly connected to adequate ability in other attentional skills, demonstrating the interrelation of these cognitive processes. Thus, whether in global learning or specific skill development, EF skills are integral to the development of mathematical proficiency. Most importantly, attentional activation and successful use of executive functions support WM, the mental workspace essential for mathematical processing.

3.2.3 Working Memory

General mathematics competence requires the knowledge and flexible application of skills and procedures in different mathematical contexts (Raghubar et al., 2010). Furthermore, math-based problem-solving is dependent on the ability to hold and process information (Raghubar et al., 2010). Thus, WM skills are required to support the fundamentals of many mathematics tasks throughout life (Raghubar et al., 2010). In fact, visual WM has been shown to serve as a significant predictor of mathematics ability in 8- to 16-year-old students, even after controlling for reading and attentional difficulties (Peterson et al., 2017).

An in-depth exploration of the literature suggests the relationship between WM and mathematics ability is complex, with WM's impact on mathematics ability targeting specific mathematics skills only, as opposed to having a global effect on mathematics learning. For instance, Bull and colleagues (2008) noted that in TD school-aged children, visual short-term memory (VSTM) significantly predicted mathematics achievement, and by the end of third grade, visual WM was a unique predictor of mathematics skills. They suggest that the ability to represent visual-spatial information in WM is integral for non-numerical skills such as estimation and visualization of magnitude (Bull et al., 2008). This is as opposed to other mathematics skills, such as numeracy.

Further evidence for this complicated relationship comes from studies with participants presenting with atypical mathematics or attention abilities, or both. Elementary-school students with dyscalculia perform poorly on tasks of verbal, visual, and numerical WM (Kuhn et al., 2016; Peng et al., 2018). Peng and colleagues (2018) also noted that WM deficits seemed to be more strongly associated with calculation and comprehensive mathematics difficulties, as opposed to word problem-solving difficulties. Specifically, subtraction skills seem to be particularly impaired in elementary school students with dyscalculia and attention difficulties, suggesting the shared role of inattention and WM in solving subtraction problems (Kuhn et al., 2016). Geary and colleagues (2007) discovered central executive deficits as a core component that characterized early-elementary school students with mathematics learning disabilities across tasks but noted the phonological loop and visuospatial sketch pad as contributing to more specific deficits; estimation, addition, and counting (Geary et al., 2007). Additionally, deficits in visual WM result in task-specific difficulties with mathematics skills and non-numerical skills such as magnitude estimation for kindergarten and elementary school children (Geary et al, 2007; Raghubar et al., 2010).

Acknowledging the influence of many EFs in mathematics skills, including the important role of visual, but not verbal WM in mathematics processing, Szűcs et al. (2014) proposed an "executive memory function centric" (p. 518) model of mathematical processing in 9-year-old children. This model encompasses selective attention, shifting, updating, monitoring, and inhibition to be central executive memory processes involved during arithmetic tasks (Szűcs et al., 2014). Szűcs's model fits well with the framework of conceptualizing EF in childhood as unilateral. However, this poses challenges when considering remediation, as interventions would have to address each of these central executive memory processes, as opposed to potentially isolating just one.

As we have seen above, each distinct skill relies on attending and successful skill development and use for developing mathematical proficiency. Before exploring intervention, it is important to consider PS, a final cognitive skill related to attention that can impact attention, WM, and EF.

3.2.4 Processing Speed

Processing speed refers to the efficiency with which an individual perceives and processes information and produces a response (Forchelli et al., 2022). One possible explanation for the role of processing speed in mathematical learning is provided by the bottleneck theory. The bottleneck theory states that high-level cognitive skills such as WM and EF are supported by low-level cognitive skills such as processing speed and that deficits in the latter can constrict the information flow necessary for high-level processing during mathematics tasks (Peng et al., 2018). For example, for 7- to 9-year old children, performance on memory-based tasks is most severely impaired in situations where processing activity demands attentional resources (Conlin et al., 2005). This theory is one proposed explanation for the role of *processing speed* impacting all domains of mathematics proficiency. In a sample of 8- to 16-year old students with mathematics difficulties (MD), reading difficulties (RD), and ADHD, processing speed contributed to the relationship between mathematics and attention (Peterson et al., 2017), and has been noted to be a salient cognitive deficit in students with mathematics difficulties (Peng et al., 2018). Furthermore, processing speed deficits were observed across ages and different types of mathematics difficulties, indicating a possible fundamental cognitive correlate to all mathematics difficulties (Peng et al., 2018). The impact of more efficient processing of mathematics problems in the allocation of attentional resources has also been explored in cognitive neuroscience. It has been observed that in seventh-grade students, simple exposure to a "+" sign is related to increased activity in the hippocampus, a brain associated with spatial attention (Mathieu et al., 2018). It is suggested that this elicits a priming effect, resulting in more efficient processing of subsequent information (Mathieu et al., 2018). More efficient processing implies that more cognitive resources can then be allocated to incoming information, mental manipulation, execution of operations, and problem-solving.

Evidentially, function-specific deficits in PS have the potential to impact general mathematics learning and achievement and have task-specific relationships to mathematics processing and fluency. Clearly, many cognitive skills related to attention are involved in mathematics learning and performance. The bottleneck theory is one explanation for the relationship between processing speed to previously mentioned skills (attention, WM, EF). In the next section, Cognitive Load theory will be discussed as another framework that integrates attention-related cognitive skills in a way that explains the dynamic interplay between attention-related skills and how they can be considered harmoniously in mathematics learning.

3.2.5 Cognitive Load Theory

Cognitive load theory proposes that individuals have a limited WM capacity (Paas et al., 2003). The less automatic the processing of a given task is, the more attentional resources one has to allocate to attending to and processing of information during task completion, thus using valuable cognitive space that would otherwise be used for holding, manipulating, or integrating information (Paas et al., 2003). This theory integrates the many functions accounted for by the aforementioned attention-related cognitive skills (e.g., attention, WM, EF, and PS), and can help explain their interrelated contribution to mathematics abilities. For example, 13- to 17-

year old students with ADHD demonstrate a deficit in automatized retrieval of mathematics facts when their mathematics fluency is assessed (Zentall, 1990). In turn, their efforts to retrieve these mathematical facts consume already limited attentional resources, further impacting their mathematical performance (Zentall, 1990).

John Sweller is a pioneer of cognitive load theory and has informed the direction of research and application since coining the term. In this section, we will explore some of his early work that influenced instructional design changes that we continue to see today. Then, we will briefly review recent research that supports the role of cognitive load theory in mathematics learning. Tarmizi and Sweller (1988) propose that automaticity, and thus, cognitive load is significantly reduced by schema acquisition. A schema is a cognitive construct that enables someone to recognize a problem as belonging to a specific category and thus requiring specific steps to a solution (Tarmizi & Sweller, 1988). However, the development of these schemas relies on attentional regulation. Furthermore, it is not just an individual's inherent attentional abilities that influence mathematics performance. The way that educators present material to elementary-school students can considerably change the attentional demands of the task, suggesting that instructional design can greatly influence one's achievement (Bobis et al., 1993; Tarmizi & Sweller, 1988). They argue that traditional problem-solving with well-defined goals (e.g., solve for angle A) imposes a high cognitive load because it requires attentional splitting between multiple sources of information, integration of different operations and strategies, and maintenance of a goal state (Tarmizi & Sweller, 1988). The addition of a clear end goal forces the student to maintain this goal state while backward planning necessary steps and calculations (Tarmizi & Sweller, 1988). Thus, compared to open-ended problem solving (e.g., solve for as many angles as possible), what appear to be straightforward problems with clear end goals actually impose a heavy cognitive load (Tarmizi & Sweller, 1988). This then interferes with effective schema acquisition that would facilitate more fluent problem-solving skills (Tarmizi & Sweller, 1988). Additionally, providing students with redundant information to process, or presenting information in a way that physically separates information thus requiring split attention and mental integration of information, also increases extraneous cognitive load (Bobis et al., 1993). While the justification for providing this information is often to improve comprehension, it is argued that the demands on cognitive load eliminate any potential comprehension benefits (Bobis et al., 1993).

Tarmizi and Sweller (1988) discovered that removing goal states, reducing the amount of extraneous information given, and providing a worked-example with numerically ordered steps resulted in significantly more problems being answered when compared to control groups. Therefore, while the intrinsic complexity of a task cannot be reduced, how educators present material can significantly impact cognitive load (Tarmizi & Sweller, 1988; Bobis et al. 1993). In fact, changing instructional design flaws results in faster processing and fewer errors (Bobis et al., 1993).

Cognitive load theory has recently been used to explain more efficient problem-solving skills in second- to fourth-grade students with strong calculation abilities (Watchorn et al., 2014). Specifically, a relationship was observed between attentional flexibility, calculation skill, and the use of superior problem-solving strategies, but only for students with strong calculation skills (Watchorn et al., 2014). The proposed explanation is that better computation skills result in more attentional resources being available for evaluating and selecting more efficient strategies (Watchorn et al., 2014). There are also examples of instructional design being altered to reduce the demands on a child's cognitive load, resulting in improved performance in fourth grade and middle school students (Gillmor, Poggio & Embretson, 2015; Yung & Paas, 2015).

Given the evidence discussed above, it seems clear that the cognitive domain of attention plays a substantial role in mathematics learning and performance. Whether broken down to relate specific cognitive skills to components of mathematics or integrated with theory to account for its general contribution to math, the relationship is clear. However, this strong relationship is not being addressed in the literature on mathematics remediation. The majority of current remediation methods take a task-specific approach of targeting specific mathematics skills, as opposed to a function-specific approach that addresses underlying cognitive skills, such as those related to attention.

3.3 Intervention for Mathematics Remediation

A systematic approach was used to retrieve articles considered in this chapter. The studies reviewed were identified by searching electronic databases from the field of education and psychology (PsycINFO and Education Resources Information Center (ERIC)) and by conducting a backward search of relevant articles. Searches were limited to peer-reviewed materials. Two separate searches were conducted and are detailed below.

To gather literature on interventions, the following keywords were used to search PsycINFO: Mathematics OR Mathematics Education AND Remedial Education OR Intervention OR School-Based Intervention AND Attention. This search provided 7 results. In ERIC, the keywords Mathematics AND Attention were entered within the field of descriptors (SU) and the keyword Intervention with no specified search field was added. With the limit of <elementary

education>, <elementary and secondary education>, <middle schools>, and <primary education>, the search produced 38 papers.

For inclusion in this review, studies had to report on the relationship between sustained attention, selective attention, ADHD symptomology, executive functioning, working memory, processing speed, cognitive load, and mathematics learning or achievement. Correlational and experimental studies, as well as meta-analyses, were retained. Studies assessing populations with atypical mathematics or attentional skills were also included. Intervention literature needed to have mathematics remediation as a noted goal to be considered in this review. The following exclusion criteria were also used: (a) self-monitoring or meta-cognitive literature; (b) studies of teacher knowledge and characteristics; (c) progress-monitoring (response to intervention - RTI) program evaluation literature. After reviewing the titles and abstracts of all papers, 73 papers were retained based on the above criteria. After reviewing these 73 papers in more depth, a total of 34 papers were retained for the review of the relationship between attention and math, and 10 on mathematics remediation.

3.3.1 Task-Specific Intervention and Remediation

Despite overwhelming evidence for the role of attention in mathematical ability, many remediation methods are focused on mathematics instruction, as opposed to addressing the underlying cognitive mechanisms. Mathematics fluency has often been the target of intervention to improve mathematics proficiency. For instance, a program called Great Leaps was used as a supplemental intervention in addition to the daily mathematics curriculum for a small sample of 6 second and third-grade students with various intellectual, learning, and attentional difficulties (Jolivette et al., 2006). The sessions were brief, individualized instruction of single-digit basic operations, and as the students' fluency improved the tasks became more challenging. While three students made gains in their fluency for oral addition, only one student progressed to written fluency and began the oral subtraction component of the program. The other two students showed improvements in addition fluency however, they did not progress further in the program (Jolivette et al., 2006). It was noted that one of the students who did not progress to the next stage had significant attentional impairments that interfered with the one-on-one instruction. The authors noted that "attentional difficulties during any of the sessions would negatively affect student fluency performance" (Jolivette et al., 2006, p. 389).

Other evidence-based interventions, such as integrated dynamic representation (IDR), pose promising results for mathematics remediation for 6- to 9-year old students (González-Castro et al., 2016). While this intervention model appears to be math-knowledge-based, its instructional process addresses many components of attention. The IDR intervention involves a multilevel process carried out on a computer program that presents visual representations of concepts, links, questions, and processes that the participant must work through to reach the final solution of a situational mathematics problem. IDR attempts to remediate mathematics competencies and problem-solving skills using fragmented comprehension, representation, and integration of sets of representations (González-Castro et al., 2016).

However, a closer examination of the levels of representation of IDR reveals how cognitive factors may explain its success. For instance, key concepts (relevant information to solve the problem) are presented in circles, eliminating the demands of selective attention to orient to relevant information while ignoring distractors. There are pictograms that children can select to signify an operation to complete or a relationship between two key concepts. For example, a whole circle around a key concept to suggest adding, whereas a circle with a dotted line reflects subtraction. This reduces the amount of information to be held in WM. Ultimately, the fragmentation and reintegration of information in IDR reduce the demands of many attention skills, thus overall cognitive load. As such, more attentional resources can be allocated to solving the operation or word problem. Though IDR was effective in short-term remediation of mathematics competencies for children 6 to 9 years of age with MLD, ADHD, and MLD+ADHD, the most significant improvements were noted for children with MLD, followed by the comorbid group, and finally the ADHD group (González-Castro et al., 2016). We argue that the results can be accounted for by the degree to which cognitive strategies were made explicit. Specifically, mathematics knowledge was taught through the use of strategies that alleviated the demands of attentional skills; however, they did not directly intervene or train these skills. While IDR is a step in the right direction in addressing cognitive factors contributing to mathematics learning, the gains are likely to be short-lived without cognitive training.

3.3.2.1 Function-Specific Intervention and Remediation

As discussed, the research on cognitive interventions for mathematics learning difficulties is limited and often inconsistent. It is important to note that there is extensive literature on cognitive training in general, and while some studies report academic benefits, the goal of their interventions was not mathematics remediation (Peng & Miller, 2016). While they provide evidence for the effectiveness of cognitive training and thus justification for its use in remediating math, this body of literature is not included in this review, as intervention goals were not math-based. Rather, we will explore the small body of research on cognitive interventions with the specific goal of improving mathematics proficiency. We will also examine some studies where it can be argued that cognitive skills were indirectly targeted in attempts to improve mathematics performance. As all intervention studies are explored below, it is important to note that there is a significant gap in cognitive studies that explicitly target sustained attention. Intuitively, one would presume that successful use of processing skills such as EF and WM rely on one's ability to orient and sustain their attention to a task. We then must presume that the results are based on samples of students with relatively intact sustained attention skills. Alternatively, we could assume that results may have been more robust if interventions addressed sustained attention as well. This is certainly an area for future direction and will be discussed again later in this chapter.

Another important consideration before reviewing this research is revisiting the previously discussed debate in the field as to whether EF in childhood should be considered as a set of distinct skills, or a unilateral skillset that works together in pursuit of a goal. As we will see below, the existing literature assessing cognitive interventions for mathematics remediation do not systematically target each component of EF in the way that was previously outlined. In fact, the review did not return any studies that explicitly targeted distinct EF skills. As such, the sections below include studies that claimed to target attention-related skills and WM. This is perhaps a limitation on the existing research, and an area for future exploration that would provide significant knowledge to clinicians and educators working with these students.

3.3.2.2 Indirect Intervention of Attention-Related Skills.

Though there are instances of successful cognitive-based interventions specifically for math, it can be argued that the cognitive skills were often addressed within the context of a mathematics task, and not directly. For instance, fourth-grade girls at-risk for ADHD showed decreased off-task behaviours and improved problem-solving when given a mathematics task that had keywords highlighted for them (Kercood et al., 2012). The argument for this intervention was based on optimal stimulation theory and previous evidence that 8-year-old male students with ADHD benefited from having operation signs highlighted (Kercood & Grskovic, 2009). While the intervention used tools to address cognitive factors such as selective attention, the approach did not facilitate any training or development of attention; instead, it aimed to remove a barrier. In neglecting to train the cognitive skill of selective attention, there are unlikely to be long-term gains to the intervention that will transfer to future academic experiences or translate to real-world applications.

Kang and Zentall (2011) proposed a similar means of intervention, where they increased the stimulation of a task by adding novelty to relevant features through graphic representations, aiming to increase mathematics performance for second- to fourth-grade students with ADHD. Though one could argue against such intervention considering redundancy and split-attention effects (Bobis et al., 1993), the level of intensity was manipulated solely through visual input (e.g., contrast, shadows added to geographical shapes to provide light-source information) therefore not increasing cognitive load. It was noted that second- to fourth-grade students with diagnoses of ADHD and with at-risk inattentive behaviours performed significantly better than controls when questions were presented with high visual intensity (Kang & Zentall, 2011). However, similarly to Kercood and colleagues (2012), this study addressed cognitive factors related to mathematics performance in elementary school years without explicitly training any specific skill. Therefore, while this supports the effectiveness of cognitive-based interventions for remediating mathematics, these studies did not go further to evaluate more specific cognitive training. Given that children cannot manipulate or change the cognitive demands of real-world math-based tasks, it is logical that we again consider the potential of cognitive interventions to support cognitive load management.

3.3.2.2 Direct Intervention on Attention

As previously discussed, the intervention literature targeting sustained attention is limited. One exception to this is Barnes and colleagues (2016), who attempted to compare the effectiveness of task-specific mathematics interventions with and without attentional training (vigilance, switching) in very low mathematics performing pre-kindergarten students. They discovered that attention training provided near-transfer effects to improved attention but did not show far-transfer effects to math, as the intervention groups that received mathematics and attention training did not differ from the math-only group (Barnes et al., 2016). However, it is important to note that the improvements in attention that they noted were small (Barnes et al., 2016). The attention intervention was very low in intensity, with students only training once a week for eight minutes, which likely accounts for the minimal to null effects of this intervention and is a limitation recognized by the researchers (Barnes et al., 2016).

Paananen and colleagues (2018) addressed the question of sustained attention in their Maltii intervention for grade one to six students that targets EFs such as inhibition and attentional control. However, their methods for training sustained attention was through a token system that provided external reinforcement and motivation for on-task behaviour. It is known that once external motivators are removed, then intrinsic motivation decreases (Bénabou & Tirole, 2003), calling into question the long-term efficacy of this cognitive intervention. Furthermore, while the results indicated significant improvement for math, the effects were only seen in basic skills such as fluency, and not for more complex operations. One explanation for this is that post-testing was completed directly following the intervention; however, as previously discussed, the transfer effects of cognitive training to academic performance should increase over time. Therefore, it is possible the transfer to more complex skills would be observed at a later post-test. This is particularly true of classroom and testing environments where children are expected to regulate their attention over a prolonged period.

Aligned with Miyake and colleagues' (2000) conceptualization of EF, one needs to be able to effectively control their attention to adequately use WM and EF skills. The lack of research exploring the academic impacts of training attention is concerning, and has great potential to inform school-based interventions for elementary and middle school students. While we will continue to explore the interventions that target working memory, let us not forget that they may not be fruitful if the child has limited sustained attention capabilities.

3.2.3.2 Direct Intervention on Working Memory

Compared to the indirect studies explored earlier, transferrable gains are more likely when explicit strategies are taught, or when training is specific to cognitive skills. For example, embedding explicit instruction on WM strategies within a third-grade mathematics class (mean age = 8.5 years old) led to a significant increase in WM abilities, on-task behaviour, and therefore increased exposure to learning (Davis et al., 2014). While successful in training WM within the context of mathematics instruction, there was no explicit measure of pre- and post-test mathematics, making conclusions about the academic effectiveness speculative.

Dahlin (2013) answered some outstanding questions in a more individualized and targeted approach to WM training for 9- to 12-year-old children with ADHD and saw significant improvements in mathematics ability immediately and at short-term follow-up for both males and females. For males, the positive effects were still present at the seven-month follow-up. The authors speculate that WM training resulted in increased activity in the prefrontal cortex, and this increased their efficiency of processing and ability to focus (Dahlin, 2013). Though not explicitly stated, the results and interpretation of this intervention can be best understood by cognitive load theory, which integrates the many aspects of attention described earlier. Specifically, it can be speculated that more efficient processing and ability to focus means that more attentional resources can be allocated to learning new information within the classroom context. As there is increased exposure to learning, this will have a cumulative effect over time, such that more advanced concepts can be learned, thus test scores improving at longer-term follow-up.

Interestingly, whether or not WM training is domain-specific (numerically-based) or domain-general does not appear to impact the transfer to mathematics ability (Kroesbergen et al., 2014). For example, two groups of kindergarten children (mean age = 5.78) played games intended to increase WM (one number-based, one domain-general), and both demonstrated significant improvements in WM and numeracy skills compared to controls (Kroesbergen et al., 2014). The lack of difference between the two experimental groups suggests the unique influence of WM, as opposed to numerical WM or exposure to numbers during cognitive training. Despite promising results, this intervention did not take into account individuals with attentional or mathematics difficulties, thus cannot be generalized to these populations. Furthermore, they did not include follow-up assessments to determine the long-term effects of cognitive training. These findings are interesting when compared to Ramani and colleagues (2017), who targeted the improvement of 5- to 7-year-olds' foundational mathematics skills by utilizing two approaches: a domain-specific training of numerical knowledge and a domain-general training of WM. The results demonstrated that providing children with training on an iPad in both domain-specific knowledge and domain-general skills led to enhanced numerical magnitude knowledge, thus providing evidence for the conceptual framework of mathematics development proposed by Geary and Hoard (2005). These gains in numerical magnitude knowledge are proposed to support their mathematical proficiency through middle childhood. According to this framework, mathematical achievement depends on conceptual understanding and procedural knowledge, which are both supported by an array of cognitive systems, such as attention, inhibition, language, and visuospatial systems.

Finally, more intensive intervention has demonstrated long-term transfer effects to mathematics (Holmes et al., 2009). Children in middle childhood (8- to 11-years-old) who completed 35 minutes of WM training, for at least 20 days, over five to seven weeks, showed marked improvements in mathematics performance at a six-month follow-up (Holmes et al., 2009). Interestingly, these students did not show improvement in mathematics at immediate post-testing. However, it is argued that the effects of cognitive training for supporting learning take time to show significant improvement on standardized tests (Holmes et al., 2009). Thus, to have a more robust understanding of the effects of training-based intervention studies, there is a need for longer-term follow-up testing, as false conclusions could be drawn from only looking at immediate post-test. Such testing should be included in all intervention studies, not just cognitively based ones, to allow for meaningful comparisons of effects.

While the interventions explored throughout section 3.3.2 demonstrate potential for the utility of cognitive interventions to address mathematics learning difficulties, there is notably a substantial gap in addressing one cognitive skill: sustained attention. Intuitively, and in line with Miyake and colleagues' (2000) conceptualization of EF, higher-order attentional abilities such as WM and other EFs rely on one's ability to orient and sustain their attention to information over a prolonged period, especially in the classroom.

Considering the interventions discussed above, it is evident that certain elements of a cognitive intervention increase the likelihood of success and should be clearly identified to develop successful interventions for clinicians and educators. Specifically, interventions should directly target and train a cognitive skill and the training should be relatively rigorous and intensive. Furthermore, the training should ultimately result in more efficient processing of information and thus overall reduced cognitive load during mathematics tasks. The latter assumption rests on the success of cognitive load theory in not only conceptualizing the relationship of many attentional skills to mathematics learning and performance but also in explaining the success of the few cognitive interventions.

3.4 Discussion

The relationship between attentional abilities and mathematics learning is supported by empirical studies. Despite the nuanced definition of terminology, whether analyzed as domain-general attentional abilities or broken down into parts, there is overwhelming support for the contribution of attentional cognitive skills to mathematics proficiency in children. A large body of empirical work has demonstrated the unique and shared contribution of attentional control, selective attention, sustained attention, WM, EFs, and PS to performance on school-based mathematics tasks. Despite the evidence, the majority of literature and clinical practice still utilize a task-specific approach to remediation. However, with the awareness of mathematics difficulties in ADHD samples, as well as the attentional difficulties seen in mathematics LD profiles and conversely, the superior attentional abilities of individuals gifted in math, it seems the remediation literature is neglecting to address underlying cognitive factors. In other words, there is a trend of using task-specific solutions to a function-specific problem. It can be argued that the risk in this approach is that a task-specific solution is a surface-level, short-term approach that addresses the issue in the here and now. Targeting the function-specific problem through cognitive training is essential because it addresses the underlying mechanisms. This approach has the potential to create long-term, meaningful changes that can impact mathematics abilities, and thus, overall achievement. The question of how to target this function-specific problem becomes complicated by the debate within the literature as to whether EF is a unitary construct, or discrete skills, in childhood. Given the many links from specific EFs to mathematics skills, we argue that initial interventions should specifically target one skill. If these fail to be effective, then considering interventions that address EF as a unitary process should be explored. Most importantly, sustained attention should be the first skill addressed, as the adequate use of WM and EF relies on effortful control and sustained attention.

Despite the significant amount of literature demonstrating a relationship between mathematics and attention, the research exploring cognitive interventions to address this relationship is quite small. There are very few studies that assess the effectiveness of cognitive interventions for remediating math, and within those studies the results are mixed. That said, the interventions with the most immediate and long-term effects seem to align with cognitive load theory, which integrates many different attentional functions (see above, e.g., González-Casto et al., 2016; Kercood & Grskovic, 2009; Kang & Zentall, 2011; Dahlin, 2013). Unfortunately, there is still minimal investigation on the training of sustained attention on mathematics ability, a skill fundamental to academic success. Interventions targeting WM and EF are necessary, and they should follow primary intervention into attentional control skills, as they serve little utility unless a child can sustain their attention to the necessary and relevant information.

3.4.1 Future Research

Future studies should continue to explore and replicate interventions that target cognitive skills to remediate mathematics learning. Based on the review of the current literature, it is integral that the specific attentional skill is well operationalized. Furthermore, researchers should take careful consideration into selecting the means for their cognitive intervention, ensuring that it is adequately addressing the operationalized skill. The interventions should be intensive and should include immediate and follow-up testing to evaluate the impact of cognitive training over time.

Future research should also compare the effectiveness of the same intervention for typically developing students, students with mathematics learning disorders, students with attentional impairments, and students with comorbid mathematics learning and attentional difficulties. Doing so will provide further insight into the profiles of these populations, as well as help in understanding if intervention should be differentiated for different presentations. Finally, this could justify whether mathematics instruction for all students could be improved by addressing attentional components of instruction design or supplementing some cognitive training into the regular curriculum for the general education population.

3.4.2 Implications

While the relationship between cognition and mathematics achievement is not novel to many, it is widely ignored in the intervention literature. For the most part, cognitive training research has been broad, and intended to improve symptomatology for clinical populations (e.g., ADHD). This chapter highlights the lack of intervention training to follow up the empirically evidenced relationship between mathematics and attention. Understanding the unique relationship between attention and mathematics seen in multiple clinical and typically developing populations suggests an incredible potential for a new approach to mathematics remediation that is not being addressed. Furthermore, the implementation of such interventions in schools' clinical practice will require considerable evidence from research. For instance, searching for evidence-based practices for mathematics remediations on What Works Clearinghouse (a public website that reviews and identifies studies to support education) yields 150 results. However once combined with the keyword cognition, there are a total of 4 interventions, none of which are exclusively mathematics remediation programs.

For educators and clinicians, the research is strongly in support of early identification and intervention for students struggling with math, in support of their long-term academic growth and success (Duncan et al. 2007). The literature would suggest that educators should be keenly aware of their students who may be struggling with foundational mathematical skills, and/or those showing difficulties with attention, working memory, and executive functions. Flagging these difficulties in childhood has the potential for efficient identification and thus proper early remediation. For clinicians, students presenting for assessment who are experiencing attentional challenges should strongly be considered for academic testing that explores the acquisition of fundamental mathematics skills. Though the presenting concern may be attentional challenges or difficulties with executive functions, as we have read throughout this chapter, there is a strong likelihood that this child is susceptible to falling behind in mathematics learning. In the same regard, students presenting for assessment to explore mathematics learning challenges should have sufficient testing to explore their attentional profile and related skills (WM/EF) to identify any skills deficits. In investigating these related skill sets through standardized testing; it follows that at-risk students will receive thorough and individualized educational planning to support their immediate and long-term learning.

When considering interventions to support students with mathematics learning challenges, educators are encouraged to reflect on whether their programming is task-specific, and whether this is in the long-term interest of the student’s learning. Teachers and school professionals should review the current practices in place for remediation and explore potential for introducing function-specific programming. That said, it is essential that more research is conducted in the field to provide evidence in support of such interventions and programming. Far too often, schools are expected to implement intervention programs with limited research into its effectiveness, and a lack of explanation for the rationale. We strongly encourage that schools have a transparent understanding of their intervention programs, and selected cognitive-intervention programs are based in sound evidence.

3.5 Conclusion

The evidence supporting a relationship between domain-general attentional functions and mathematics learning and performance is clear. There are limited studies that look at using cognitive training as a means for mathematics remediation, despite the notable mathematics difficulties in children with ADHD, as well as attentional deficits in children with MLDs. Research on such interventions has the potential to inform and change current methods for clinical practice and remediation of mathematics difficulties. The value of doing so is pertinent, as effective mathematics intervention has the potential to influence overall academic achievement in later years.

Educators and clinicians are encouraged to use this framework in supporting their students, advocating for thorough assessment, and implementing evidenced-based interventions that address the underlying cognitive issues, as opposed to the observable deficits in learning.

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Chapter III: Bridging Between Manuscripts 1, 2 and 3

There is strong and extensive evidence demonstrating a relationship between attentional abilities and math learning for TD youth and some youth with NDCs (e.g., ADHD, MLD). However, the extent to which this relationship holds true for youth with NDCs and more complex diagnostic profiles and lower cognitive functioning remains unclear. Youth with NDCs are more at-risk for academic challenges than their peers (Wei et al., 2011). Understanding whether the relationship between attention and math abilities holds true for these more complex learners has the potential to better inform practices for remediation.

Regarding remediation, the exploration into function-specific (cognitive) training as a means for math remediation, even for TD youth, is limited. While there is certainly a need to continue to explore cognitive training, the question of how to target this function-specific problem becomes complicated by the debate within the literature as to whether EF is a unitary construct, or discrete skills, in childhood. Given Miyake's (2000) conceptualization of attentional control of a top-down process, we propose that sustained attention is the integral skill to address. Efficient information processing as well as attentional activation and control is required to use EF skills, namely shifting, updating, and inhibiting (Miyake et al., 2000). Having these executive functions intact promotes successful use of WM, a mental workspace that is integral to mathematical processing (Miyake et al., 2000; Raghubar et al., 2010). Thus, the role of various related cognitive factors involved in mathematics relies initially on attention, making it the integral skill for intervention.

The relationship between attentional abilities and math learning is strongly supported by several studies (see Child et al., 2019; Kim et al., 2018; Lindsay et al., 2001; Raghubar et al., 2009). Despite the nuanced definition of terminology, whether analyzed as domain-general attentional abilities or broken down into its parts, there is overwhelming support for the contribution of attention-based cognitive skills to math proficiency in children. Despite the evidence, most of the literature and clinical practice still utilizes a task-specific (math content based) approach to remediation. That is, programs such as repeated practice, number-lines, and intensive instruction that are intended to support learning and memorization. However, it seems the remediation literature is neglecting to address the underlying cognitive factors involved in math learning. In other words, remediation techniques are using a math-fact-based (task-specific) approaches to address a problem that is related to attentional abilities (function-specific), potentially missing the opportunity to remediate the underlying factor contributing to the math difficulties. It can be argued that the risk in this approach is that a task-specific solution is a surface-level, short-term approach that addresses the issue in the "here and now". Targeting the function-specific problem through cognitive training is essential because it addresses the underlying mechanisms that are involved in math learning, including those related to attentional ability. The function-specific remediation approach thus has the potential to create long-term, meaningful cognitive changes that can impact math abilities, and consequently, overall academic achievement.

The interventions with the most immediate and long-term effects seem to align with cognitive load theory functions (see chapter II, e.g., Dahlin, 2013; González-Casto et al., 2016; Kang & Zentall, 2011; Kercood & Grskovic, 2009), which integrates many different attentional functions. There is presently a dearth of knowledge regarding the effects of training sustained attention, a skill fundamental to academic success. Though intervention of skills such as WM and EF are necessary, they should follow primary intervention into attentional control skills, as they serve little utility unless a child can sustain their attention to the necessary and relevant information. One example of a task that targets sustained attentional abilities is multiple-object tracking (MOT). MOT tasks require individuals to track and identify a series of targets as they move among distractors. It is commonly used paradigm in measuring and assessing attention and is considered one of the most realistic measures of real-world visual attention (Scholl, 2009). Additionally, it can be designed such that there are limited social or language requirements and have low task complexity so that it is simple for individuals with any cognitive ability to engage on. Furthermore, MOT can be designed to be adaptive, fluctuating the difficulty based on a student's performance over the course of training. Given its real-world applicability and simple task designs, it is an intervention that favours the needs of youth with NDCs that have complex diagnostic profiles or lower cognitive abilities, and has demonstrate efficacy in producing near-transfer effects to improved attention for these youth (Tullo et al., 2018).

There are two significant gaps in the literature exploring the relationship between attention and math for youth with NDCs. First, there is no research that directly explores the cognitive correlates of math learning for youth with NDCs that have complex diagnostic profiles and lower cognitive functioning. Secondly, there is a general deficit in the exploration of cognitive intervention for math remediation for these exceptional learners. The proposed dissertation aims to address these gaps with two main aims.

First, I will explore the cognitive factors related to math learning for students with an NDC (Study 1). While students with NDCs often experience attentional difficulties, they also have a host of other symptoms that may impact academic functioning, such as cognitive ability and social-emotional challenges such as behavioural issues, or lack of intrinsic motivation towards schooling (Antshel et al., 2016; Wei et al., 2011). The objective of Study 1 is to build on the extensive existing literature relating attentional abilities to math learning within typically developing population (see Cragg & Gilmore, 2014; Fuchs et al., 2005) and to expand the research exploring the cognitive correlates of math learning in NDC populations (e.g., Czamara et al., 2013) to include individuals with lower cognitive functioning and complex diagnostic profiles. A better understanding of the cognitive correlates of math learning within NDC youth with lower cognitive functioning and complex diagnostic profiles will inform intervention that is best suited for this specific population. Furthermore, much of the existing literature on the cognitive correlates of math learning within youth with NDCs has not made use of standardized math assessment tools. Commonly, overall grades, test grades, or independently created measures have been used to evaluate math knowledge (e.g., Bartelet et al., 2014; Davis et al., 2014; Geary et al., 2007; Ross & Randolph, 2016). This has resulted in significant limitations in ability to compare to typically developing youth, or adequately objectify and measure learning and growth over time. To address this limitation, in Study 1 I will use standardized measures to measure cognitive, attentional, and mathematical abilities for youth with NDCs. I expect that for all youth, non-verbal reasoning abilities and attentional abilities will be strongly related to mathematical achievement. I will also explore whether diagnostic category (e.g., ASD vs. non-ASD NDC) impacts the relationship between attention, non-verbal intelligence, and math, given that individuals with ASD often demonstrate superior non-verbal intelligence (Dawson et al., 2007). I hypothesize that diagnostic profile will moderate the relationship between non-verbal reasoning, attention, and math, given the tendency of ASD youth to demonstrate strong non-verbal skills (Dawson et al., 2007).

The objective of Study 2 is to assess the appropriateness of a domain-general (attention) and domain-specific (math) tasks for youth with NDCs. Furthermore, to explore the feasibility of having these youth participate in an intervention using rigorous cross-over design study. The intent is to demonstrate that both the tasks and study design are feasible and appropriate for these youth and inform the design of future research to provide evidence for efficacy of the unique *and* serial effects of training. Since students with NDCs often exhibit impaired cognitive and attentional abilities, intervention with these students affords unique insight into how difficulties in, and remediation of attentional abilities has the potential to impact math proficiency. Furthermore, because students with NDCs are often academically disadvantaged compared to peers, it is integral to investigate methods for math remediation, particularly considering the predictive power of math proficiency to later overall academic achievement (Duncan et al., 2007).

Chapter IV: Manuscript 2 – Study 1

Perceptual reasoning skills mediate the relationship between attention and math proficiency in individuals with a neurodevelopmental condition

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Abstract

Background. An important component of academic success in typically developing students is the development of math skills, which is associated with attention and perceptual reasoning (PR) skills. For children with a neurodevelopmental condition (NDC), the relationship is confounded by diagnostic-specific cognitive characteristics. Specifically, enhanced PR is specific to individuals with Autism Spectrum Disorder (ASD).

Aims. The purpose of this study was to test: (i) a mediation model where PR skills would mediate the relationship between attention and math proficiency for students with an NCD, and (ii) whether this mediation model is moderated by a diagnostic profile.

Methods and Procedures. One hundred and thirty-seven students with an NDC participated in a school-based study examining the effectiveness of using a standardized measure of attention in predicting math capabilities.

Outcomes and Results. PR mediated the relationship between attention and math proficiency for students diagnosed with an NDC. However, the model was not moderated by diagnostic profile.

Conclusions and Implications. The results of this study provide a better understanding of the roles of higher-level cognitive ability specific to students with NDCs. Additionally, the superior PR skills demonstrated by the ASD sample further supports the research suggesting this population possesses cognitive strengths in this domain.

What this Paper Adds. Using standardized measures of attention, cognition, and mathematical proficiency, we have conducted a study that: (i) characterizes the relationship between attention, cognition, and math within children diagnosed with a neurodevelopmental condition and (ii) assesses whether the aforementioned relationship varies as a function of NDCs that present high non-verbal IQ (e.g., Autism Spectrum Disorder) compared to other neurodevelopmental conditions. Several key findings emerged from this study: first, we contributed valuable insight into understanding of the role of attention and higher-level cognitive ability in math proficiency specific to students with NDCs. Additionally, we observed superior non-verbal reasoning skills demonstrated within the ASD sample that further supports research suggesting this population possesses cognitive strengths in this domain. Overall, this study provided valuable information to school personnel and clinicians to better understand cognitive and educational profiles of their students and inform psychoeducational assessment and intervention.

Clinicians and researchers alike could benefit from the results of this study. As mentioned above, clinicians and school personnel could appreciate implications of the results, both in terms of the characterization of math capability in these populations and considerations for informing instruction and intervention. Additionally, we believe this research would appeal to individuals who investigate development within neurodevelopmental conditions. Finally, researchers who examine mathematical learning and cognition, as well as school-based interventions, could have interest in the results. Therefore, this study's integration of math, cognition, intervention, and unique populations has the potential to impact future research and clinical practice.
Keywords: Mathematics, Attention, Non-Verbal Intelligence, Perceptual Reasoning, Neurodevelopmental Disorders, Standardized Testing

Introduction

Relative to other skills for academic success, math has been demonstrated to have higher predictive validity for current and future overall levels of academic achievement (Claesens & Engel, 2013; Claessens et al., 2009; Duncan et al., 2007; Gottfried et al., 2013). Certain math skills, including quantitative knowledge (understanding and analyzing numerical information), number identification, sequence counting, pattern recognition, and value comparison, are consistent predictors of achievement (Geary, 2015; Claesens & Engel, 2013). For instance, Jordan et al. (2009) noted the importance of number competence (i.e., understanding of numbers and number relationships), as this skill predicted math growth above and beyond socioeconomic status or age. Furthermore, Geary et al. (2007) characterized differences in math-specific deficits across a range of math domains including number sets, number estimation, and counting knowledge between (i) high achieving children, (ii) low achieving children, and (iii) children with math-specific learning disabilities. While both studies compared deficits in specific skills between those with and without learning deficits, they developed their own math-based tasks to measure skill. Using a standardized measure would allow for more meaningful comparisons that could further contextualize these differences between groups as well as identify an association with other cognitive domains.

In addition to being a primary predictor of later academic achievement in typically developing children (Duncan et al., 2007; Mayes & Calhoun, 2007), attentional abilities also play a critical role in the development and utility of early math skills (Cragg & Gilmore, 2014). Fuchs et al., (2005) propose that attention is a “critical cognitive determinant” of math learning, as it is required for the development and automaticity of early math skills (p. 510). For instance, these authors demonstrated that attention persistently remained the single best predictor of ability for fact fluency, computation, and conceptual underpinnings of numbers (Fuchs et al., 2005). When processing a math problem, visual sustained attention and the ability to identify and inhibit attention to irrelevant information are also important processes (Anobile et al., 2013; LeFevre et al., 2013). High rates of comorbidities between attentional difficulties and math learning disabilities provide further evidence in support of the relationship between the two skills (Czamara et al., 2013; Gross-Tsuer, Maor & Shalev, 1996). However, the means by which attention is operationalized and measured is inconsistent across research and does not consistently specify domains of attention or executive functions. This inconsistency is in part due to its multiple subcomponents, which make defining this construct difficult (Heitz et al., 2005). For instance, working memory (WM) is a skill that is often considered when discussing attention, and it has been related to poor math skills in premature babies (Simms, Gilmore, Cragg, Clayton, Marlow & Johnson, 2015). However, some researchers would consider WM a stand-alone skill, while others have used it as an operational measure of attention. It is therefore imperative to define attention using standardized and validated measures when assessing and comparing differences in attention between individuals. When considering that attention is a skill that matures throughout development, and often faster in females, the use of standardized measures also allows for meaningful comparison of relationships between variables over time (Klenberg et al., 2001). Further, while there is a commonly reported relationship between attention and math performance in the classroom, standardized and validated measures of math must be used in order to understand the generalizability and extent of this relationship.

Given that early math skills are an integral component for academic success and can be used to predict overall achievement at later grades, it is important to identify the factors that contribute to the development of early math skills in students with Neurodevelopmental Conditions (NDCs) (Claesens & Engel, 2013) since children and adolescents with an NDC often present academic difficulties (Ashburner et al., 2010; DuPaul & Jimerson, 2014). While there has been some research conducted with clinical populations, the results are mixed, and conclusions do not consistently rely on comparisons drawn between standardized and widely used clinical measures. Fortunately, there is significant research on both the domain-general and domain-specific factors contributing to math-learning and achievement (see Geary, 2015 for review). However, much of what is understood about the relationship between attention and mathematics is based on studies sampling the neurotypical population. When assessing the relationship between these cognitive factors and math for children and adolescents with NDCs, it is imperative that unique cognitive attributes of children with NDCs are considered. Though some work has been conducted on math learning and growth in NDC populations, studies often did not use standardized measures or did not address domain-general cognitive mechanisms, a gap that the current study aims to fill. Titeca and colleagues have compared math learning in high-functioning ASD populations to typically developing peers and conclude that children with ASD demonstrate a profile with strengths, average abilities, and difficulties, and as such should be considered on an individual basis (Titeca, Roeyers, Josephy, Ceulemans & Desoete, 2014; Titeca, Roeyers, Loeys, Ceulemans & Desoete, 2015). However, neither study used a standardized measured to explore each area of math functioning that would facilitate ease of comparison to other standard measures. For instance, in addition to math and attention abilities, children with NDCs often present cognitive profiles that are characterized by unique attributes in higher-level cognition. For example, individuals with Autism Spectrum Disorder (ASD) often demonstrate average, and often above-average performance on perceptual reasoning tasks, including Weschler tests and the Raven's progressive matrices (Nader et al., 2016); this ability is specific to ASD and has not been reported in other NDCs.

Non-verbal intelligence, as measured by the Weschler tests and the Raven's progressive matrices, has been demonstrated to be related to mathematical performance in neurotypical students (Kyttälä & Lehto, 2008). Visual perception, motor, and visual-motor integration skills have been demonstrated to explain some variance in number fact retrieval and procedural calculation in both typically developing children and those with math learning disabilities (Pieters, Desoete, Roeyers, Vanderswalmen & Van Waelvelde, 2012). Furthermore, small samples of children with math learning disabilities have demonstrated significant difficulties completing these visual-based tasks (Pieters et al., 2012). The relationship between poor visual-spatial skills and weak math skills has also been seen in premature babies (Simms et al., 2015). Individuals with ASD have been reported to often exhibit relative cognitive strengths in the domain of perceptual reasoning (compared to other skills within their profile); therefore, an examination of how their non-verbal reasoning is involved in their math learning and achievement could provide unique insight into the separate roles of each cognitive ability (Dawson et al., 2007). While the relationship between these constructs have found to be related in neurotypicals, the use of standardized assessment tools to define and compare the relationship between these constructs in atypically developing individuals ensures sensitivity to different profile characteristics (e.g., ASD vs non-ASD NDCs). For this reason, the use of clinically validated measures (e.g., gender- and age-based norms) is crucial for assessing and understanding how math proficiency, attentional abilities, and perceptual reasoning relate to one another in students diagnosed with an NDC.

The objectives of the present study are three-fold; (i) to determine whether attention ability predicts math proficiency in relation to students diagnosed with an NDC, (ii) to measure the extent to which this predictive relationship is mediated by perceptual reasoning ability, and (iii) to explore whether this model is moderated by ASD and non-ASD diagnostic profiles. Therefore, this study aims to extend previous research on attention predicting math ability in typically developing students to assess a predictive model that examines this relationship in individuals with NDCs using standardized measures. Compared to previous research, this study will use standardized and clinically validated measures of attention Conners Continuous Performance Task - 3rd Edition (CPT-3), perceptual reasoning (Wechsler Abbreviated Scale of Intelligence – Second Edition [WASI-II]), and math proficiency (Pearson KeyMath). The use of these measures allows for meaningful comparisons and contextualization of relationships, as well as potential for generalization to other populations.

It is expected that within a sample of students with an NDC, superior attentional abilities, as measured by the CPT-3 will be predictive of higher math proficiency, as indicated by performance on the Pearson KeyMath task. Second, it is hypothesized that the aforementioned relationship between attention and math ability will be mediated by perceptual reasoning ability, as indicated by performance on the Perceptual Reasoning Index (PRI) score of the Wechsler Abbreviated Scale of Intelligence (WASI-II). Finally, diagnosis will be a significant moderator in the relationship, such that the mediational role of PRI will be greater for the ASD group compared to the non-ASD NDCs group. Theoretical and statistical representations of the hypothesized mediation (Figure 1) and moderated mediation (Figure 2) models are presented.

Material and Methods

Participants

One-hundred and forty (N = 140) children and adolescents (Age [6, 17], *M* = 13.84, *SD* = 2.47) with a diagnosis of a neurodevelopmental condition (100 male, 40 female; age: *M* = 13.84, *SD* = 2.47), enrolled in an alternative educational facility for specialized instruction participated in the study. The students were enrolled in a specific stream where enrollment is based on specific learning and academic-based criteria. Therefore, while the group presents with a range of primary diagnoses (see below), they were all tested and admitted based on like-criteria concerning their potential to function and learn in an educational setting. To explore the possible differences in the relationship between attention and math proficiency for students with ASD and other NDCs, 73 participants with a primary diagnosis of ASD were recruited and included in the ASD group, and participants with an NDC other then ASD were included in another group (non-ASD NDC). The non-ASD NDC group included students with a primary diagnosis of either ADHD (n = 9), intellectual disability (n = 41), and/or other rare-genetic based NDCs (Williams Syndrome, Down Syndrome, etc.; n = 17) (see sample characteristics presented in Table 1). Students were identified as ASD / non-ASD, according to the criteria and code established by the Ministère de l'Éducation de l'Enseignement supérieur (MEES or The Ministry of Education and Higher Education). The MEES sets out specific criteria to identify students with educational needs and neurodevelopmental conditions, and the set of criteria specifies a code for individual neurodevelopmental condition (Fombonne et al., 2006). Among the codes, codes

50 is of notable relevance to this study, since it “Pervasive Developmental Disorder” which includes Autism Spectrum Disorder. Students who received code 50 would have been evaluated in systematic observation and standardized assessment by a “psychologist who has expertise in this field and who works with a multidisciplinary or interdisciplinary team” and that they are limited in at least one of the following domains “communication, socialization, and learning at school” (Gouvernement du Québec Ministère de l’Éducation, du Loisir et du Sport, 2007). While the diagnosis of each participant was not verified in this study, participants were identified and selected based on their MEES codes.

Figure 1

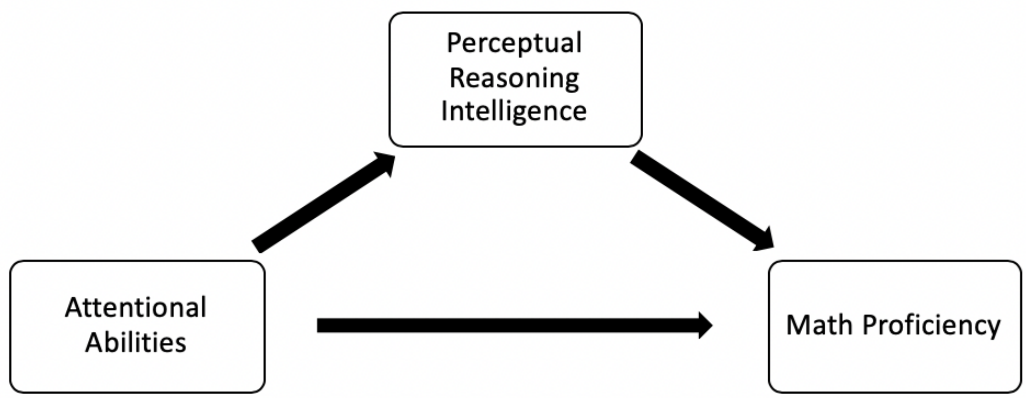

Figure 1. The mediation model examines whether attentional abilities predict scores on KeyMath, and whether perceptual reasoning mediates this relationship.

Figure 2

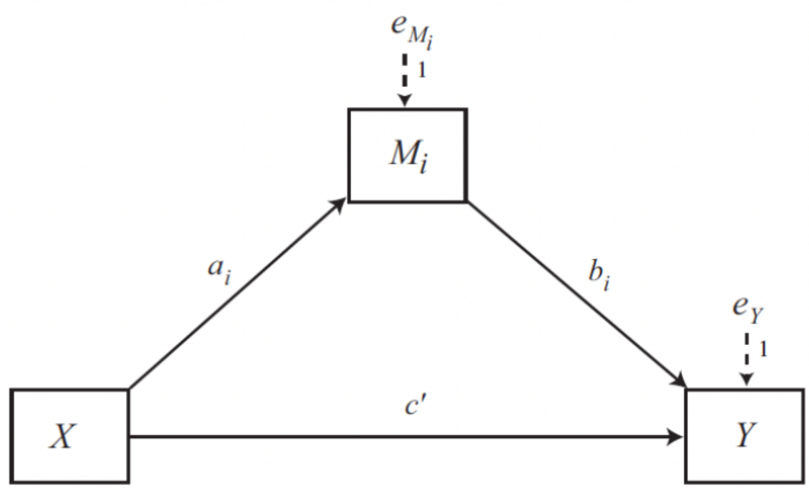

Figure 2. Statistical representation of the proposed mediational model: attention (X) predicting KeyMath (Y), mediated by PRI (M).

Figure 3

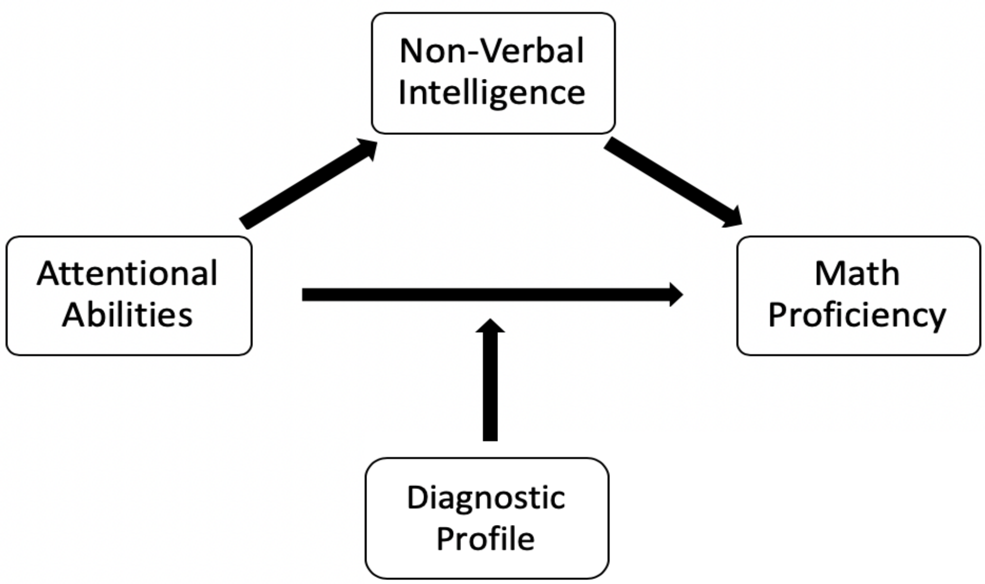

Figure 3. Moderated mediational models conceptual diagram divided amongst predictors of math ability. The model examines whether attentional abilities predicts scores on KeyMath, and whether perceptual reasoning mediates this relationship. Furthermore, whether diagnostic

profile (individuals with and ASD diagnosis vs. individuals with a diagnosis of a neurodevelopmental condition other than ASD) moderated the mediational model of attention, non-verbal intelligence and math proficiency.

Figure 4

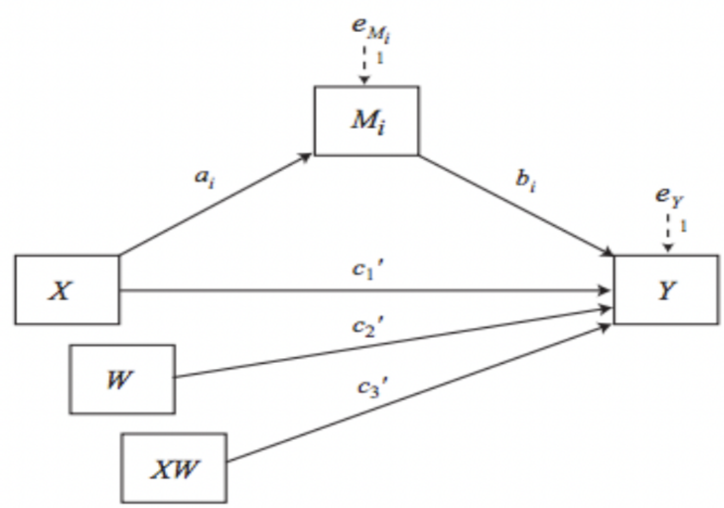

Figure 4. Statistical representation of the proposed moderation mediational model: attention (X) predicting KeyMath (Y), mediated by PRI (M), moderated by diagnostic profile (W).

Table 1

Means and standard deviations for sample across diagnostic groups

Group	N	M / F	Age	FSIQ*	PRI*	CPT-3 d' t-score	KeyMath*
ASD	73 (52.14%)	59/14	6.92 - 17.94 M =13.80 SD = 2.62	40 - 111 M = 75.99 SD = 17.45	45 - 124 M = 81.81 SD = 19.25	25 - 75 M = 58.67 SD = 10.77	0 - 10 M = 3.07 SD = 2.18
Non-ASD NDC	67 (47.86%)	41/26	7.25-17.12 M = 13.87 SD = 2.31	40 - 100 M = 63.70 SD = 12.21	45 - 116 M = 65.09 SD = 13.86	42 - 74 M = 61.51 SD = 10.77	0 – 6.5 M = 1.73 SD = 1.57
Total	140	100/40	6.92 – 17.94 M = 13.51 SD = 2.47	40 - 111 M = 70.12 SD = 16.32	45 - 124 M = 73.81 SD = 18.80	25 - 75 M = 60.03 SD = 9.72	0 - 10 M = 2.43 SD = 2.02

Note. Ranges, Means and Standard Deviations are reported. * $p < .05$ denotes a between-groups difference

Measures

Non-Verbal Intelligence: Wechsler Abbreviated Scale of Intelligence, Second Edition WASI-II - Perceptual Reasoning Index Subscale.

Non-Verbal Intelligence was measured using the Perceptual Reasoning Index from (PRI) the Wechsler Abbreviated Scale of Intelligence, Second Edition (WASI-II)(Wechsler, 2011). PRI is an overall measure of ability to "interpret and organize visually perceived material and to generate and test hypothesis related to problem solutions" (Sattler & Ryan, 2009, p. 48). Block design and matrix reasoning combine to create a score for the PRI. The subtests of the WASI-II began at the age-recommended norm, regardless of diagnosis, and were discontinued when ceilings were met.

Attention: Conners CPT-3.

Attention was measured using the Conners Continuous Performance Task, Third Edition (CPT-3; Conners, 2014). The CPT-3 is a clinical measure of sustained attention used for diagnostic purposes and has been standardized on a normative and clinical sample. The task requires children to attend to a select set of stimuli for a prolonged period of time, rapidly responding to stimuli while simultaneously inhibiting this response to inappropriate stimuli. Performance on the CPT-3 is defined by detectability: *d*' an overall measure of attentiveness and detectability. The score indicates how well the participant was able to discriminate non-targets from targets. The task, and the *d'* score in particular, have strong psychometric properties, with split-half reliability estimates of r = [-.96, .94] and test-rests reliability estimates of r = .74. Validity analyses reveal that the task has strong discriminative validity (p < .001), incremental validity, and generalizability. Higher *t*-scores on the CPT-3 are indicative poor attentional abilities, while lower *t*-scores represent superior attentional skills.

Math: Pearson KeyMath.

Math ability was assessed using the Pearson KeyMath test, an assessment of understanding and application of basic math skills (Connolly, 2007). The test has strong psychometric properties, with estimates for internal-consistency reliability ranging from r = [.93, .99], and reliability estimates ranging from r = [.65, .98]. The subtests are aligned to the curriculum content standards outlined by the National Council for Teachers of Mathematics. It is administered annually to all students enrolled at an educational institution for students diagnosed with neurodevelopmental conditions or other learning disorders by psychology interns who have training in administration of standardized tests. The purpose of the test administration within the institution is to monitor growth and choose class placement for the following year. As such, grade equivalent scores are calculated, as opposed to standard scores. Because the students vary in their age and ability level, the grade equivalent provides the most meaningful representation of their math proficiency for the institution's educational purposes. The results of the most recent assessment for each student was made available to us to determine current level of mathematics ability. The overall achievement grade equivalent was used to measure math proficiency in this study.

Procedure

Each student was provided a randomized ID number and then administered the WASI-II as well as the CPT-3 by trained research assistants. Students left their classrooms for 15-45 minutes to complete tasks with the principal investigator or research assistant. Tests were delivered on separate days to avoid over-testing and exhaustion. The administration order of the WASI-II and CPT-3 was randomized to avoid any potential testing bias. KeyMath scores were administered as part of school routine by school clinicians or trained clinical students.

Table 2

Descriptive Statistics and Variable Intercorrelations

Variable	M	SD	1	2	3
1. Attentional Ability	60.03	9.71	-		
2. Math Proficiency	2.43	2.02	-.492**	-	
3. Perceptual Reasoning IQ	73.81	18.80	-.414**	.588 **	-

Note. (1) Attentional ability, defined by the d-prime score of Conners Continuous Performance Task - 3rd Edition: (2) Math proficiency defined by performance on the Pearson KeyMath, (3) Perceptual Reasoning defined by the PRI scale of the Weschler Abbreviated Scale of Intelligence 2nd Edition.

Results

Descriptive Statistics and Intercorrelations between measures

Table 2 presents the means (M), standard deviations (SD), and intercorrelations for the CPT-3 d' t-score (i.e., attentional ability), KeyMath grade equivalent score (i.e., math proficiency), and the Perceptual Reasoning Index Subscale Score of the WASI-II, for all participants as well as divided between the two diagnostic groups (ASD and non-ASD NDC).

All variables were significantly related with KeyMath scores. Specifically, the CPT-3 and KeyMath had a moderate, negative correlation, indicating better attentional abilities (as indicated by a lower score on the CPT-3) are associated with higher grade equivalent math scores, $r(138) = -0.49$, $p < .001$. Additionally, Perceptual Reasoning had a large, positive correlation with KeyMath, indicating superior perceptual reasoning abilities are associated with higher math scores, $r(138) = 0.60$, $p < .001$. In addition to KeyMath, performance on the CPT-3 also had a moderate, negative correlation with Perceptual Reasoning, indicating better attention abilities are associated with higher math scores, $r(138) = -.42$, $p < .001$.

Preliminary analyses (one-way between subjects ANOVAs) revealed no statistically detectable differences between the ASD group and the non-ASD NDC group for age: $F(1, 138) = 0.29$, $p = .865$, $\eta_p^2 = 0$ and attention $F(1, 138) = 3.02$, $p = .085$, $\eta_p^2 = 0.02$. However, there was a statistically significant difference between diagnostic groups on non-verbal intelligence: $F(1, 138) = 34.24$, p < .001, $\eta_p^2 = 0.20$ and math proficiency: $F(1, 138) = 17.28$, $p < .001$, $\eta_p^2 = 0.11$.

Mediation Model

A simple mediation analysis was conducted to explore whether non-verbal intelligence (perceptual reasoning) mediates the relationship between attention and math proficiency. The conceptual model is presented in figure 1a along with the statistical model, presented in figure 1b (Hayes, 2018).

Results indicated that attention predicted non-verbal intelligence (path a_i: $b = -0.80$, $SE = 0.15$, $p < .001$), accounting for 17% of the variance: $F(1, 138) = 28.59$, $p < .001$. Furthermore, non-verbal intelligence (path b_i: $b = 0.05$, $SE = 0.01$, $p < .001$) predicted math proficiency. The total effect of attention on math proficiency was statistically detectable (path c_i: $b = -0.10$, $SE = 0.02$, $p < .001$); however the reduction in the direct effect (path c_i': $b = -0.06$, $SE = 0.01$, $p < .001$) suggests that perceptual reasoning acts as a partial mediator between attention and math proficiency. Approximately 42% of the variance in math proficiency was explained by non-verbal intelligence and attention: $F(2, 137) = 49.67$, $p < .001$. Moreover, the indirect effect was tested using a bootstrap estimation approach with 10,000 samples (Hayes, 2013). These results revealed a statistically detectable indirect effect of attention on math proficiency via non-verbal intelligence: ($b = -0.04$, $SE = 0.01$, 95%CI [-0.06, -0.02]) The results are presented in Table 3.

Moderated Mediational Model

Given the partial mediation between attention and math proficiency via perceptual reasoning, we assessed whether there would be an interaction with individuals diagnosed with ASD and those diagnosed with an NDC other than ASD given the significantly higher non-verbal intelligence (i.e., assessed by PRI score in the ASD group). Therefore, a moderated mediation model was conducted to explore the strength of perceptual reasoning as a mediator in the relationship between attention and math proficiency as moderated by diagnostic profile. The conceptual model is present in Figure 1a along with the statistical model, presented in Figure 2b (Hayes, 2018).

The results of the moderated mediation are presented in Table 4. Similarly, attention predicted non-verbal intelligence (path a_i: *b* = -0.80, *SE* = 0.15, $p < .001$), accounting for 17% of the variance: $F(1, 138) = 28.59$, $p < .001$. Again, non-verbal intelligence (path b_i: *b* = 0.04, *SE* = 0.01, $p < .001$) and attention (path c'_1: *b* = -0.09, *SE* = 0.04, $p = .037$) predicted math proficiency; however, diagnostic profile (path c'_2: *b* = -1.52, *SE* = 1.76, $p = .392$) and the interaction between attention and diagnostic profile (path c'_3: *b* = -0.02, *SE* = 0.03, $p = .529$) were not statistically detectable predictors of math proficiency in a model that accounted for 43% of the variance: $F(4, 135) = 28.59$, $p < .001$. These results suggest that there was no evidence of a moderated mediation model.

Table 3

Model coefficients for the mediation model

Predictor	*b*	SE	*t*	*p*
	Perceptual Reasoning IQ (PRI)			
Constant	121.92	9.11	13.38	.000
d’ *t*-score (a)	-0.80	0.15	-5.35	.000
	R^2 = 0.17			
	$F(1, 138) = 28.59$, $p < .001$			
	Math Proficiency (KeyMath Grade Equivalence)			
Constant	2.48	1.25	2.00	.048
PRI (b)	0.05	0.01	6.49	.000
d’ *t*-score (c’)	-0.06	0.01	-4.19	.000
	R^2 = .42			
	$F(2, 137) = 49.67$, $p < .001$			
	Total effect of attention on math proficiency			
d’ *t*-score (c)	-0.10	0.02	-6.64	.000
	Indirect effect of d’ *t*-score on Math Proficiency			
	Effect	Boot SE	Lower CI	Upper CI
PRI	-0.04	0.01	-0.06	-0.02

Note. N = 140. Unstandardized coefficients are reported. Boot sample size = 10,000.

Discussion

The objectives of this study were threefold: (i) to determine whether attention ability predicted math proficiency in relation to students with a NDC, (ii) to measure the extent to which the predictive relationship was mediated by perceptual reasoning ability, and (iii) to explore whether this model was moderated by ASD and non-ASD diagnostic profiles. Furthermore, this study used standardized and clinically validated measures that allowed for meaningful comparisons and conceptualization of the relationship between cognitive abilities and math proficiency between different diagnostic groups. Consistent with our hypothesis, results indicated that perceptual reasoning was as a significant, partial mediator in the relationship between attentional abilities predicting math proficiency. However, diagnosis (ASD vs non-ASD NDC) did not serve as a moderator in the model, indicating that diagnostic classification did not influence the relationship between attention, perceptual reasoning, and math proficiency.

Compared to previous research that has explored the contribution of attention or intelligence (non-verbal) to math proficiency in typically developing children, this study extends to young students with an NDCs, by integrating all three variables in a model. Consistent with previous research relating attention to math development and proficiency in Typically Developing (TD) children (Cragg & Gilmore, 2014; Fuchs et al., 2005), attention served as a significant predictor of math proficiency in children and adolescents with NDCs. Furthermore, the results are consistent with findings demonstrating the importance of sustained attention in predicting achievement after controlling for non-verbal intelligence (Steinmayr & Spinath, 2009). Additionally, the relationship between PRI and math abilities previously reported in typically developing children (Kyttälä & Lehto, 2008) was observed in this clinical sample. While PRI did mediate the relationship between attention and math proficiency, the fact that it served only as a partial mediator points to the unique and integral role of attention for predicting math proficiency in children and adolescents with NDCs. Furthermore, because diagnosis is not a significant moderator; this indicates that the interplay between the three variables holds true for all lower-functioning students with NDCs and is not exclusive to ASD populations despite their tendency to display enhanced PRI ability (Nader et al., 2016). The non-significant moderation may be a result of the superior non-verbal intelligence skills demonstrated by the higher-functioning ASD sample recruited for this study. Their superiority in this area further supports the existing research that higher-functioning individuals with ASD demonstrate higher non-verbal reasoning intelligence (Dawson et al., 2007; Nader et al., 2016). While these results are informative and helpful in characterizing the cognitive strengths of ASD populations, the use of a restricted range potentially impacted the application of the statistical model. Future research should recruit a larger range of ASD functioning where there is likely to be larger variability in cognitive skills. However, a larger range of functioning would likely include students enrolled in alternative educational contexts that may not translate to students with neurodevelopmental disabilities enrolled in other educational contexts. In addition to using a restricted range, it is important to note other limitations to this study. First, due to sample size, students presenting with non-ASD NDC diagnoses were not further classified by diagnosis. Additionally, because the KeyMath is administered annually for grade placement, grade equivalents were provided instead of standard scores. Use of standard scores would allow for more meaningful comparisons within and between individuals and groups. The use of standardized measures is integral in future studies that include a wide range of functioning, in order to best compare students of different diagnostic profiles.

There are differences in perspectives pertaining to the predictors of math learning in the typically developing population. It is possible that the contribution of attention and non-verbal intelligence to math learning within NDC populations would be better understood in the context of multiple variables. For instance, the opportunity-propensity model proposes that antecedent factors (demographics), opportunity factors (teaching style, education), and propensity factors (knowledge, motivation) play a large role in determining growth of math skills at kindergarten, first, third and eighth grades (Byrnes & Wasik, 2009; Byrnes & Miller, 2016). Other theories consider mathematics to be componential in nature and argue that it cannot be considered as a single characteristic (Dowker, 2015). As such, math ability is made up of several correlated and interdependent skills that dynamically evolve over time (Dowker, 2015). Henik, Rubinstein & Ashkenazi (2015) suggest that developmental dyscalculia is not a disorder that can be characterized by a single deficit, but rather present with deficits in various mental processes not exclusively related to numerical processing. The purpose of the current research was not to discredit either the opportunity-propensity model or the componential theory, but rather to explore two significant factors that could be considered in each: attention and visual-spatial processing. Future research could aim to assess the interplay of cognitive variables within these other proposed factors.

In addition to translating research on math proficiency in TD populations to an NDC sample, this study also made use of standardized measures that allows for meaningful relationship between variables and comparisons between groups. Previous research in both TD and NDC populations does not consistently make use of standardized measures for all variables, thus making replications and generalizations challenging. For example, research has often focused on custom math-based measures that assess specific skill (Geary et al., 2007; Jordan et al., 2009). However, the use of the KeyMath in this clinical population demonstrated the importance of using a standardized measure, as it was characterized by its relationship to other aspects of cognition.

Table 4

Regression coefficients for conditional direct effect of diagnosis and direct effect of PRI

Predictor	*b*	SE	*t*	*p*
	Perceptual Reasoning IQ (PRI)			
Constant	121.92	9.11	13.38	.000
d' t-score (a_i)	-0.80	0.15	-5.35	.000
	R^2 = 0.17			
	$F(1, 138) = 28.59$, $p < .001$			
	Math Proficiency (KeyMath Grade Equivalence)			
Constant	5.04	2.78	1.81	.072
PRI (b_i)	0.04	0.01	5.23	.000
d' *t*-score (c_1')	-0.09	0.04	-2.11	.037
DxProfile (c_2')	-1.52	1.76	-0.86	.392
d' *t*-score X DxProfile (c_3')	0.02	0.03	0.63	.529
	$R^2 = .43$			
	F(4, 135) = 28.59, p < .001			
Group	*b*	SE	*t*	*p*
	Conditional direct effect at levels of Dx Profile			
ASD	-0.07	0.02	-3.86	.002
non-ASD	-0.05	0.02	-2.20	.030
Predictor	Boot indirect effect	Boot SE	Boot LLCI	Boot ULCI
	Indirect effect of X on Y			
PRI	-0.04	0.01	-0.06	-0.02

Note. N = 140. Unstandardized coefficients are reported. Boot sample size = 10,000.

Conclusions

This study provided unique insight to the various cognitive domains involved in math ability for young students with an NDC. Future replications or modifications to this study have the potential to further develop this understanding and inform the course of mathematics instruction for these individuals. Furthermore, the superior non-verbal reasoning skills demonstrate by this average- to high-functioning ASD sample further supports the research suggesting that individuals with ASD possess cognitive strengths in the area of perceptual/non-verbal reasoning. The results of this study provide valuable information to school personnel and clinicians to better inform psychoeducational assessment and intervention. Specifically, the results suggest that if attention deficits are present within a student or patient, their math abilities should be evaluated using a standardized measure. This additional assessment component has the potential to identify students who are behind or at-risk of falling behind academically in math due to underlying attentional difficulties. Early detection and intervention can best support the student's success, as opposed to re-evaluating for math learning difficulties after a period of time with low grades and poor academic performance.

List of Abbreviations: Neurodevelopmental Condition (NDC), Autism Spectrum Disorder (ASD), Perceptual Reasoning (PR), Intelligence Quotient (IQ), Conners Continuous Performance Task – 3rd Edition (CPT-3), Weschler Abbreviated Scale of Intelligence – Second Edition (WASI-II), Perceptual Reasoning Index (PRI), Ministère de l'Éducation de l'Enseignement supérieur (MEES), Mean (M), Standard Deviation (SD), Typically Developing (TD).

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Availability of data and materials. The datasets used and/or analysed during the current study are available upon request.

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Chapter V: Bridging Manuscripts 2 and 3

Study 1 (manuscript 2) was centred on extensive literature relating attentional abilities to math learning within typically developing youth (see Cragg & Gilmore, 2014; Fuchs et al., 2005) and expanded the research exploring the cognitive correlates of math learning in youth with NDCs (e.g. Czamara et al., 2013) to include individuals with lower cognitive functioning and complex diagnostic profiles. Specifically, we were able to demonstrate that attentional abilities, defined by the detectability (*d'*) score on the CPT-3, are related to mathematical proficiency in youth with NDCs with lower cognitive ability, and connect this relationship to non-verbal intelligence, another cognitive variable heavily associated with math learning (Kyttälä & Lehto, 2008; Steinmayr & Spinath, 2009). Based on literature that demonstrates that individuals with ASD often demonstrate average to above-average performance on perceptual (non-verbal) reasoning tasks (Nader et al., 2016), we explored whether diagnosis moderated this relationship to further tease apart the effects of these cognitive factors. We found that diagnosis did not serve as a significant moderator, and though we explore potential explanations, one potential explanation is that attentional ability accounted for most of the predictive power in the relationship. In addition, based on the statistical role of non-verbal intelligence as a partial mediator, we were able to deduce the unique and integral role of attention. Most importantly, Study 1 used standardized assessment measures (e.g., math and attention) to make meaningful comparisons between factors, and to allow for more meaningful generalizations and the potential for replication and longitudinal studies of growth. This latter point is integral for the purposes of evaluating the effectiveness of future intervention work. To reliably draw conclusions regarding efficacy and recommend certain intervention programs, standardized measures need to be used to determine meaningful growth, as test scores, or end-of-term grades do not serve as a reliable metric (Willingham et al., 2016). In addition, there is significant variability between teaching styles, approaches to grading, and general content covered that could significantly impact the conclusions about the effectiveness of an intervention. Finally, the results of Study 1 also provided the justification to proceed with a larger-scale study to assess the efficacy of various training programs with students with NDCs with the purpose of remediating math with far-transfer effects of learning.

The purpose of Study 2 was to explore key messages pertaining to the feasibility of conducting rigorous research with lower-functioning youth with NDCs to explore various remediation techniques. Specifically, to assess the feasibility of conducting a rigorous school-based, cross-over design study with youth with NDCs, regardless of diagnosis (or transdiagnostic) given the non-significant moderating role of diagnosis found in Study 1. Study 2 also aimed to assess the feasibility of comparing the unique and serial effects of domain-general (attention) and domain-specific (math) training for the purposes of math remediation. We also aimed to explore the suitability of selected measures for their sensitivity in detecting change in mathematical proficiency with these youth. Though we previously highlighted the utility of standardized measures for meaningful comparisons and generalizations, these tests are lengthy and could hinder engagement in a clinical study. As such, we made use of a different math measure in Study 2, the Easy Curriculum Based Measure (EasyCBM) to balance accuracy and feasibility. We discuss the limitations of this measure. Although was not the goal to assess, nor did we did not expect to observe transfer effects in Study 2, it was nonetheless explored.

Chapter VI: Manuscript 3 – Study 2

Comparing domain-general (attention) vs. domain-specific (math) interventions for remediating math proficiency in youth with Neurodevelopmental Conditions: A feasibility study.

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Abstract

The purpose of this study was to assess the feasibility of conducting a school-based, cross-over design study with youth with neurodevelopmental conditions (NDCs). We also explored the feasibility of comparing the unique and serial effects of domain-general (attention) and domain-specific (math) training for math remediation. We recruited 36 students with an NDC enrolled at a school for neurodiverse learners. Pre-test measures of mathematical proficiency were obtained using the Easy Curriculum-Based Measure (EasyCBM). Students were randomly divided into 3 groups (math and attention treatment groups, and a passive control group). The attention training group used a multiple-object-tracking (MOT) task that was non-verbal, dynamic, and adaptive to individual performance. The math exposure group engaged in a visual-based patterning game. Following training, all groups completed post-testing (EasyCBM). A cross-over design was implemented following the first round of post-testing. Youth with NDCs were able to participate in a rigorous randomized controlled trial (RCT) cross-over design study. Regarding task feasibility, all participants adhered to the training tasks both before and after the cross-over, demonstrating both were appropriate for use with these youth. Though we did not expect to observe transfer effects, this was briefly explored. Transfer effects were not demonstrated within this study. We hypothesized this was due to the sensitivity of the selected math measure, and sample size that was not computed to explore these effects. Overall, the results of the study demonstrate the feasibility and appropriateness of the training paradigms and rigorous study design for individuals diagnosed with NDCs with below-average cognitive capabilities.

Keywords: mathematics, intervention, neurodevelopmental condition, learning, attention

Statements and Declarations

Competing Interests

JF is the director of Faubert Lab at the University of Montreal. He developed NeuroTracker, which is the commercial version of the Multiple Object Tracking task used in this study.

Ethics approval and consent to participate

The study protocol was approved by The Research Ethics Board of McGill University – REB# 303-1215. Signed parental consent was obtained prior to testing.

Consent for publication

Not applicable

Availability of data and materials

The dataset analysed in the current study is available upon request.

Authors' contributions

EC designed the study, collected data, analyzed, and interpreted the results, and was a major contributor in writing the manuscript. DT contributed to designing the study, organized data collection and support data analysis and interpretation, as well as significantly contributed to the manuscript. JF created the MOT task used and significant contributed to the manuscript. AB advised on the design of the study, data collection, and significantly contributed to the manuscript. All authors have seen and approved of the final manuscript and agree to the order of authors as listed on the title page.

Background

Cognitive training is a non-invasive and behavioral approach that involves repeated practice on one or more cognitive-based tasks, which have been designed to transfer benefits to non-trained domains. Cognitive training has gained significant attention as an alternative and/or supplemental approach to treat, improve, and enhance human cognition, behaviour, and academics (Green et al., 2019; Simons et al., 2016). Given the intervention's accessibility to clinical populations, researchers have targeted this approach for individuals that are theorized to benefit the most from this intervention, that is, individuals with alternative learning preferences and or lower levels of cognitive capabilities (Tullo & Jaeggi, 2022). Moreover, a commercialized and publicly available product, Endeavor RX by Akili Labs, has received clearance from the United States Food and Drug Administration (Commissioner, 2020). Nevertheless, there remains great contention throughout the cognitive sciences in regards to the validity of the approach (Simons et al., 2016).

One point driving the contention is the inconsistency in findings or lack of evidence suggesting transfer from training paradigms to non-trained domains (see Melby-Lervåg et al., 2016; Melby-Lervåg & Hulme, 2013; Sala & Gobet, 2017 for examples). While, meta-analyses have demonstrated small to moderate effects in transfer to proximal domains (i.e., near-transfer; Green et al., 2019; Rapport et al., 2013), that is, targeted outcomes that share several common characteristics with the training paradigm; meta-analyses have also demonstrated null to small effects to distal domains (Melby-Lervåg et al., 2016; Simons et al., 2016; Slattery et al., 2022), that is, targeted outcomes that share little common characteristics with the training paradigm. Similarly, the aggregation of cognitive training studies for clinical populations suggest small to moderate near-transfer effects and null to small far-transfer effects (Tullo & Jaeggi, 2022).

Attention is a commonly identified cognitive domain that is heavily related to academic achievement (Romano et al., 2010). This relationship has been repeatedly observed in both typically developing populations, and students with neurodevelopmental conditions (NDCs; Cragg & Gilmore, 2014; May et al., 2013). Moreover, learning difficulties observed in students with NDCs that implicate attention (e.g., ADHD), compared to that of neurotypical peers, further support this relationship (Zentall, 2007). Attention is a cognitive domain that has also commonly been the target of cognitive intervention (see Cortese et al., 2015 for review). Various forms of attention-based cognitive training have resulted in transfer to attentional abilities in clinical populations (Beck et al., 2010; Bigorra et al., 2016; Jones et al., 2020; Mezzacappa & Buckner, 2010; Roberts et al., 2016; Shalev et al., 2007). Thus, while there is evidence in support of cognitive training, much of this research has been conducted with students with average cognitive ability (Tullo & Jaeggi, 2022). The exclusion of students with lower cognitive abilities or complex diagnoses presents an area for valuable research to assess whether of cognitive-based training can be used by these unique individuals.

As such, there is limited research exploring cognitive training with clinical populations and thus more work is warranted to test the viability of cognitive training in clinical populations (Tullo & Jaeggi, 2022). Based on the lack of evidence in support of cognitive training in producing far-transfer in typically developing populations (see Melby-Lervåg et al., 2016; Melby-Lervåg & Hulme, 2013; Sala & Gobet, 2017 for examples), it is expected that the evidence, if any, would be minimal within NDC samples. The lack of supportive research for this population is concerning given that clinical populations, such as youth with NDCs, are already academically at-risk (Wei et al., 2011) and could have the greatest benefit from successful cognitive training (Tullo & Jaeggi, 2022). Moreover, clinical populations are typically underserved and underrepresented in cognitive training research, succumbing to stringent exclusionary criteria (e.g., Bikic et al., 2018; Chacko et al., 2018; Kofler et al., 2018; Orylska et al., 2019).

Recently, cognitive training has been successfully implemented in a classroom-based environment for students with cognitive functioning in the extremely low range (Archambault et al., 2021). Archambault and colleagues (2021) conducted a feasibility study with a sample of young students with moderate to severe intellectual disabilities to assess the success of implementing an attention-based cognitive training program based on a 3D Multiple Object-Tracking task (MOT). MOT tasks the individual with tracking a set of targets as they move among distractors that are physically indistinguishable (Meyerhoff et al., 2017). This paradigm is commonly used to measure and assess attention and is considered one of the most realistic measures of real-world visual attention (Scholl, 2009). The task is suitable for youth with NDCs, as it is non-verbal, conceptually simple, and adapts to the participant's capability to provide an ideal balance between current skill level and challenge (Archambault et al., 2021; Tullo, Guy, et al., 2018). Nevertheless, there is little exploration into the far-transfer effects on academic performance. It remains unknown whether cognitive training is feasible and appropriate for transfer to academic domains in clinical populations.

Given the relationship between attentional abilities and math learning, cognitive training has been proposed as a solution for math remediation that targets the functional deficits contributing to the challenges (Jones et al., 2020). Despite this evidence, current methods for remediation take a domain-specific approach; the focus is placed on intensive instruction in a specific skill area through repeated practice or explicit strategy instruction (e.g., problem solving, see Chodura et al., 2015; Gathercole et al., 2019). In fact, cognitive training efficacy is highly related to the degree of commonality between training paradigms and targeted outcomes (Rapport et al., 2013). Therefore, assessing the feasibility to measure the benefits of domain-general (i.e., attention) and domain-specific (i.e., math) for clinical populations both in part and in serial (consecutive) is highly valued and needed within the area of cognitive training research. There is currently a gap in the literature that explores the feasibility of implementing protocols that use domain-general (e.g., cognitive training) interventions to remediate learning for individuals with alternative learning preferences. It follows that early identification and remediation of math difficulties are essential, and perhaps this could be more adequately addressed using domain-general cognitive training, as opposed to domain-specific task-based training.

Aim

Prior to conducting a large-scale study to assess and compare the efficacy of either domain-general or domain-specific training, it is necessary to explore the feasibility of the methodology given the needs and preferences of youth with NDCs. Thus, the main aim of this study was to assess the feasibility of successfully implementing a school-based, cross-over intervention to explore the isolated effect of as well as the order of domain-specific versus domain-general training to improve mathematics capability. Three specific feasibility objectives were identified.

First, we assessed the feasibility of implementing a rigorous school-based, cross-over design study with this population by collecting data on the rate of retention. We hypothesized that attrition would be low, given the accessibility (convenience) and familiarity of the training site to participants. Second, we assessed the feasibility the cognitive intervention (3D-MOT) and math exposure tasks (2048) for use with this population by using data on task adherence and progress. Given the characteristics of the task (i.e., non-verbal and adaptable), the environment in which the intervention was delivered (i.e., controlled, familiar), and the success of similar designs (Archambault et al., 2021; Tullo, Guy, et al., 2018) we hypothesized that adherence would be high. By comparing the two different training conditions using a cross-over design, we also explored whether there were cumulative effects of the order of training. That is, did domain-general training followed by domain-specific exposure training (or vice versa) have cumulative effects that transferred to gains in math proficiency? Furthermore, did a serial training regimen (i.e., domain-specific and domain-general training) transfer to math proficiency? While we did not expect to observe far-transfer within this study, we briefly explored this to inform the design of future studies. Exploring the feasibility of this work with this population could provide grounds for continued research into cognitive training with youth with NDCs to increase our understanding and support of these individuals. Finally, we assessed the sensitivity of the selected math measure for detecting far-transfer based on pre-and post-intervention data. Given the purpose of assessing the feasibility of implementing this intervention for the eventual intention of remediating math learning, obtaining statistical significance for far-transfer effects was not expected. Significant improvements in math proficiency at post-test could provide evidence in support of one form of training, or a certain order (e.g., domain-general followed by domain-specific).

Methods

Participants

Recruitment

Participants were students enrolled at Summit School, an educational facility for students with exceptionalities, located in Montreal, Canada. Specifically, they were recruited through the Summit Centre for Research, Education and Training (SCERT). SCERT is a research component of the school that serves to support research projects for its students. This project was granted ethics approval by the McGill Ethics Review board (see Appendices A-E). The project also went through an ethical clearance at Summit School. A total of 44 (30 Male; Age: *M* = 10.63, *SD* = 1.86) participants assented to participate in this study, following parental consent. Using a transdiagnostic approach, all participants had a diagnosis of one or more NDC; however, it is important to note that no participants presented with a Specific Learning Disorder, with Impairment in Mathematics (i.e., dyscalculia). Once consent was obtained from parents the school collaborated with us with their most recent level of math achievement. Caution was taken when timing research activities so that participants were not missing essential class instruction, evaluation, or any other therapeutic services offered at their school. Participants were told they could leave the study at any point.

Inclusionary/exclusionary criteria

After obtaining informed consent from parents or legal guardians, participants were screened to assess their eligibility to engage in the intervention. Exclusionary criteria included participants: (i) with a personal or family history of seizures, and (ii) with known conditions that would impact their stereoscopic vision. Additionally, participants were excluded from the study if they demonstrated significant behavioural challenges (i.e., severe emotional distress, refusal to participate, or attentional impairments that interfered with completing baseline measures).

Assignment to Conditions

Participants were randomly assigned to one of three conditions: the attention training treatment condition, the math training treatment condition, or the treatment as usual condition. Classroom assignment or current math level was not related to condition assignment.

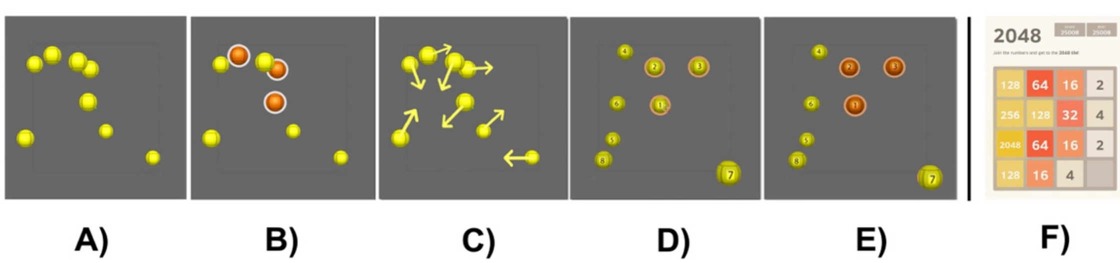
Training Tasks

Visuo-Attentive Cognitive Training – 3D-MOT

The task was controlled by a laptop and was presented on a 3D Smart Plasma HDTV (Panasonic TC-P65VT60 65-Inch, 600Hz). Participants wore 3D glasses to be able to perceive the stimuli in 3D. Each MOT training trial was comprised of five key components (see Fig. 1A–E & Tullo, Guy, et al., 2018 for more details). During a trial, the participant was first presented with eight identical yellow spheres. Three of these spheres then briefly illuminated orange, identifying them as the target objects. The targets then turned back to yellow, and all eight spheres moved around the screen. While the spheres moved, the participants had to track the targets amongst the distractors for eight seconds. Once the spheres came to a stop, the participant had to respond and identify which of the eight yellow spheres were the three targets. Following their response, they received feedback regarding their accuracy. The task was adaptive, meaning that a participant’s performance determined how quickly the spheres would move on the subsequent trial. That is, correct identification of targets on a trial would result in speed increasing on the next trial, whereas incorrect identification would result in the speed of the spheres decreasing. Progress was monitored using the average daily threshold score, which was the average speed at which a participant accurately could track all three targets.

To ensure participants understood the objective of the 3D-MOT task, all participants were required to pass qualification trials prior to commencing the 3D-MOT training regimen. First, participants were read the task instructions and shown how to enter their responses and interact with the three-dimensional environment. Next, participants were administered three trials where they were asked to track 1 target item out of 8 total items (i.e., 7 distractors). To qualify for the training the participant needed to successfully track the target item in 2 of 3 trials. If they did not successfully complete the two of three trials, the instructions were readministered to the participant and they were given a new set of three trials, where it was expected they successfully complete two of three trials to qualify for training. A failure to achieve this threshold in the second set of trials would have resulted in exclusion from the training sessions.

Fig. 1 (a-e) *Steps of a MOT Trial (f) screen capture of 2048 game*



2048 Math Game Training

In the math-based task, participants were faced with numbers in a grid and had to combine identical numbers to create multiples, ultimately working to get the sum 2048 (see figure 1f). The task exposed students to multiples, thus having a direct relation to numerical operations. Each time the student made a move, a new tile appeared with the number 2 or 4. To prolong the length of the game and get a high score, it was essential to pair numbers and make multiples. The game ended once all the squares were full, and no more pairs could be made. The game was completed on a laptop under the supervision of a research assistant. The participants completed approximately 7 minutes of training each day (i.e., to match the time spent training with the 3D-MOT trial), and the highest score was retained to record their daily score.

Similarly, to ensure participants understood the objectives of the 2048 task to qualify for the study, participants were read the game instructions and given scenarios to make specific moves (i.e., use the directional pad to make logical moves). They were given two of three attempts to demonstrate their understanding of the task and were readministered the instructions as well as a new set of attempts to qualify. If the participant failed to complete the logical move on the second set of three trials, they were excluded from the training sessions.

General Procedure

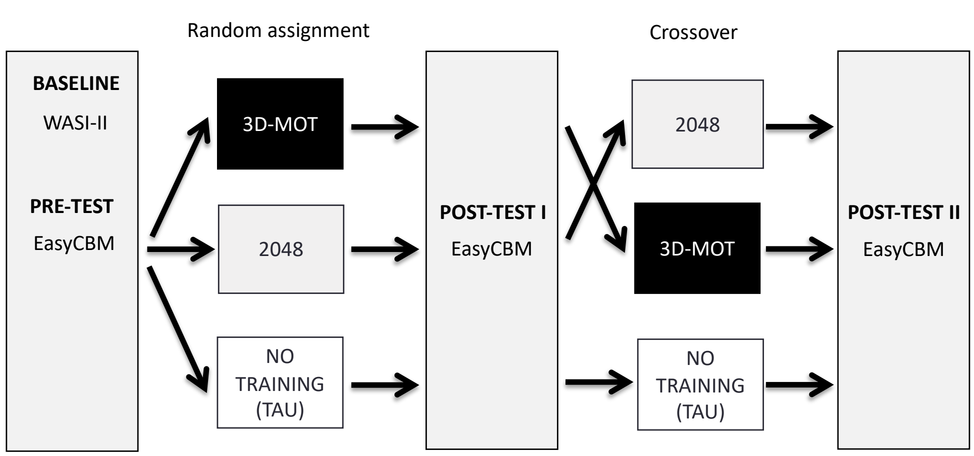
Pre-Post Data Collection

With the teacher's permission and consistent with caregiver consent, participants were individually removed from class to begin the pre-test phase. The WASI-II was administered first, followed by the EasyCBM. The set of math tests that the student was administered was based on their current math grade level, which was verified by educational staff at Summit School with their annual math testing. Tests were delivered on separate days to avoid over-testing and exhaustion. All testing was done in a room free from noise and distractions. All measures were completed by clinical graduate students or research assistants with appropriate training. The same procedure was followed for the second round of training, followed by post-testing on the EasyCBM for all participants.

Training

During the first training phase, participants individually trained on their respective programs three times a week for five weeks, for a total of 15 sessions. Immediately after the first round of training, post-test data was collected on the EasyCBM. Once post-testing was complete for all participants, including those in the passive control group, the training groups crossed over (See Figure 2). That is, those that had completed the attention training began math game training, and the previous math training group began attention training. The passive control group remained constant.

Fig. 2 Visual representation of cross-over design



Measures

Non-Verbal Intelligence – WASI-II

Non-Verbal Intelligence was measured using the perceptual reasoning index (PRI) of the Wechsler Abbreviated Scale of Intelligence, Second Edition (WASI-II; Weschler & Psychological Corporation, 2011). The WASI-II is one of the most used standardized tools in psychoeducational assessment. The PRI is comprised of two subtests, block design, and matrix reasoning, that combine to create an overall index score. PRI is an overall measure of the ability to “interpret and organize visually perceived material and to generate and test hypothesis related to problem solutions” (Sattler & Ryan, 2009, pp. 48). The subtests of the WASI-II began at the age-recommended norm, regardless of diagnosis, and were discontinued when ceilings were met.

The Easy Curriculum Based Measure [EasyCBM]

The EasyCBM (Alonzo et al., 2006) math battery is a standardized test that measures students’ performance on grade-based tasks that provides a percentile rank relative to their grade level. The test is designed to be administered three times in the year, fall, winter, and spring semester, and considers learning that would be expected as a result of progression through the curriculum. Each grade-level set consists of three tests, each that has 16 multiple-choice questions. All participants complete the grade-appropriate number-operations subtest; therefore, this test was used to monitor growth from pre to post-tests. The number-operations subtest was selected as it was the one content area that remained consistent across all grades.

Program adherence measures

Program feasibility (i.e., via adherence) was evaluated by collecting data indicative of task engagement for both domain-general and domain-specific training paradigms. For 3D-MOT training, engagement was defined as completing sufficient trials to generate an average speed score for that daily training session (see Tullo, Faubert, et al., 2018; Tullo, Guy, et al., 2018). To monitor progress, the daily average threshold score was calculated and recorded. For the 2048 task, participants had to complete sufficient moves to obtain a score greater than the minimum score, to demonstrate an engagement. Similarly, the daily score was used to track participants' progress.

Study timeline

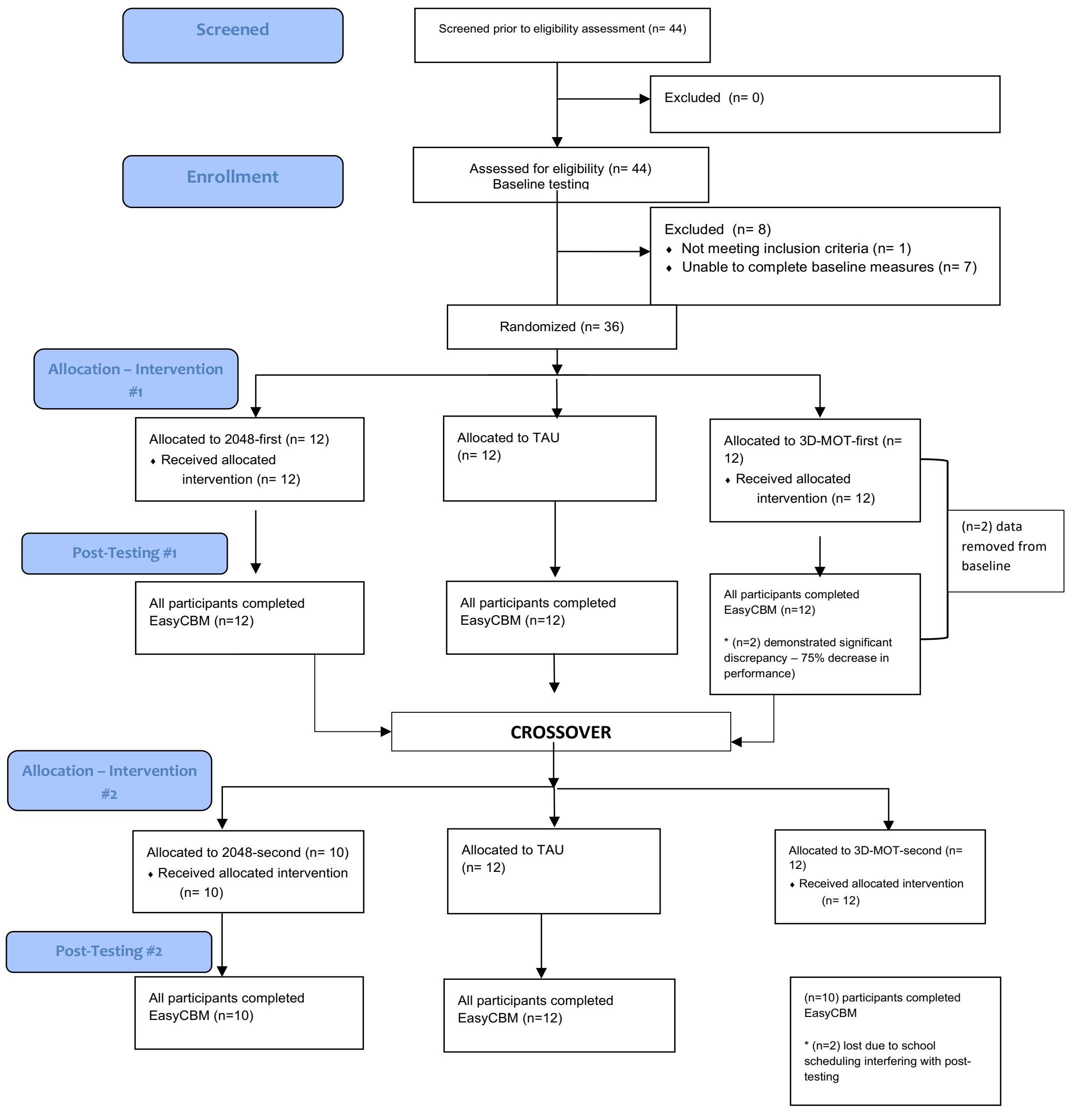
Recruitment, screening, and baseline assessments were conducted over a 10-week period. Training began concurrently for all participants across all training groups. This was done to ensure that all participants completed their respective training within the same five-week period. When that was not possible, due to student absences, make-up sessions were conducted within a week following the main five-week training period. Post-testing began immediately following training and was completed within a two-week period. This was then followed by another five-week training period and two weeks of post-testing proceeding the completion of the second round of training.

Data Analysis and Results

Recruitment and retention rate

We received informed consent for 44 (n_{male} = 30, n_{female} = 14) students. All 44 students participated in baseline assessments and initial trials of the two training tasks (3D-MOT or 2048). Of the 44 participants who consented to the study, 36 met the inclusion criteria outlined above and were retained for the study. One student was excluded from the study for demonstrating severe behavioural difficulties during the baseline assessment. Seven were excluded because they did not complete outcome measures at baselines due to incapability or lack of motivation. Of the 36 participants who met inclusion criteria, four more participants had some or all their data excluded from post-test analyses. Two were excluded due to significant reduction in performance that rendered them to be outliers and indicated low motivation (i.e., performance on a task dropped by more than 75%). These two participants had their data removed from all baseline and post-test analyses. Two other participants did not complete the second round of post-testing due to scheduling conflicts as the school year ended. Their data for the first-round of post-testing was included. 32 of the 36 participants included in the study completed all baseline testing, training, and post-testing, resulting in a retention rate of 88.89%. See figure 3 for graphic representation of participant retention using a Consort-style presentation.

Fig. 3 Visual representation of participant recruitment and retention



Adherence/feasibility of the intervention.

All participants who trained on MOT either before or after the cross-over design completed all trials (i.e., indicative of engagement) during each of the 15 training sessions within the five-week period, indicating 100% adherence to the training program. Perfect adherence was also observed for training with 2048 task for all participants. All participants were 100% adherent to their respective training programs following the cross-over.

Baseline Results

Table 1 outlines the mean Wechsler PRI (non-verbal intelligence) and baseline EasyCBM math measure scores for participants. We conducted two one-way independent samples ANOVA tests across measures of non-verbal intelligence and math to explore differences between groups at baseline. No statistically detectable difference between conditions was found for PRI: $F(2, 31) = 1.53$, $p = .232$, partial $\eta^2 = 0.09$. There was a significant difference in pre-intervention math achievement: $F(2, 31) = 6.39$, $p = .005$, partial $\eta^2 = 0.29$. Post-hoc independent samples *t*-tests with a Bonferroni correction (i.e. .05/3 = .016) revealed that the 2048 group was significantly different than the passive control group ($t(22) = -3.31$, $p = 0.003$). The MOT group and passive control group were not significantly different from each other ($t(20) = -1.78$, $p = .090$). The 2048 and MOT groups were also not significantly different from each other ($t(20) = -1.82$, $p = 0.083$.

Table 1

Means, standard deviations (parentheses) and ranges for sample across experimental groups at **Baseline** (pre-test)

Group	N	PRI	EasyCBM*
2048 1st	12 (35.29%)	75.67 (20.32) 49 - 114	8.75 (4.35) 2 - 15
3D - MOT 1st	10 (29.41%)	87.90 (16.26) 63 - 112	11.60 (2.55) 7 - 13
TAU	12 (35.29%)	86.83 (18.59) 60 - 116	13.33 (2.02) 9 - 15
Total	34	M = 70.12 (16.32) 49 - 116	11.21 3.65 2 - 15

Note. * $p < .05$ denotes a between-groups difference

Training Results

Table 2 outlines the baseline, post-test 1, and post-test 2 mean *EasyCBM* scores for participants.

Post-Test Time 1 (pre-test compared to results after first round of training)

Participants who completed the MOT task demonstrated improvements in the task over the course of training, $t(11) = 3.36$, $p = .006$. The same is not true for participants who trained on 2048, as they did not demonstrate significant improvement in their high score on 2048 over the course of training, $t(11) = -.90$, $p = .387$.

The group that trained on MOT did not demonstrate far-transfer effects to math, with no significant difference in EasyCBM scores from pre-test to post-test 1, t(9) = 0.48, p = .642. (See table 2). Conversely, participants who trained on 2048 did demonstrate near-transfer to improvement in math performance as measured by the EasyCBM, t(11) = 2.37, p = .037. (See table 2). Participants in the passive control (TAU) group did not demonstrate significant improvements in math proficiency t(11) = -0.50, p = .627 from pre-test to the first post-test. (See table 2). A two-way mixed-model ANOVA was conducted to assess group (3) by time (2) interaction for the effects of training on math proficiency. There was no significant main effect of time F(1, 31) = 3.02, p = .092, partial η^2 = .09 , but there was a significant effect of group F(2, 31) = 4.04 p=.028, partial η^2 = .207. Furthermore, there was a significant interaction of group and time F(2, 31) = 3.83, p = 0.033, partial η^2 = .198. Post-hoc independent samples t-tests with a Bonferroni correction (i.e. .05/3 = .016) revealed that the group that completed 2048 did not demonstrate significant improvements in performance on the EasyCBM at post-test compared to either the passive control group (t(22) = 1.35, p = 0.19) or the MOT group (t(20) = -0.37, p = .717). The MOT group and passive control group were not significantly different from each other (t(20) = -1.09, p = .288).

Post-Test Time 2 (post-test time 1 compared to results after the second round of training)

Participants who completed the MOT task after the cross-over did not demonstrate significant improvements in the task over the course of training *t*(9) = 1.64, *p* = .137. The same is true for participants who trained on 2048, as they did not demonstrate significant improvement in their high score on 2048 over the course of training, *t*(11) = -0.42, *p* = .686.

The group that trained on MOT did not demonstrate far-transfer effects to math, with no significant difference in EasyCBM scores from post-test 1 to post-test 2, $t(11) = -1.20$, $p = .256$. (See table 2). Participants who trained on 2048 following the cross-over also did not demonstrate near-transfer to improvement in math performance as measured by the EasyCBM, $t(9) = 1.69$, $p = .126$. (See table 2). Participants in the passive control (TAU) group did not demonstrate significant improvements in their math proficiency $t(11) = 0.43$, $p = .674$ from post-test 1 to post-test 2. (See table 2). A mixed-model ANOVA was conducted to assess group (3) by time (2) interaction for the effects of training on math proficiency following the cross-over. There was no significant main effect of time $F(1,31) = 0.13$, $p = .721$, $\eta^2 = .004$ or group $F(2,31) = 2.50$, $p = .099$, $\eta^2 = .139$. Furthermore, there was no significant interaction effect $F(2,31) = 2.29$, $p = .118$, $\eta^2 = .129$. Given the non-significant results, no post-hoc analyses were conducted.

Table 2

Average scores for math measures across groups at all test points.

Performance	***3D -MOT 1st*** **(n = 10)**	***2048 1st*** **(n = 12)**	**TAU (n = 12)**
	M (SD)	*M (SD)*	*M (SD)*
T1: Pre-test *EasyCBM*	11.60 (2.55)	8.75 (4.35)	13.33 (2.02)
T2 : Post-test I *EasyCBM*	11.80 (2.44)	11.33 (3.34)	13.00 (2.66)
T3 : Post-test II *EasyCBM*	13.00 (3.17)	10.33 (3.17)	13.25 (2.34)

Means and standard deviations (parentheses) for all experimental groups at the different test points: baseline, post-test time 1 and post-test time 2.

Effects of Serial Training (pre-test compared to results of post-test 2)

Participants who were assigned to MOT training, followed by training on 2048, did not demonstrate significant improvements in math proficiency $t(9) = 1.95$, $p = .083$ from pre-test to the final post-testing. It is worth noting the results are trending toward significance. Participants who trained first with 2048, followed by training on MOT, did not demonstrate significant improvements in math proficiency $t(11) = 1.52$, $p = .156$ from pre-test to the final post-testing. Lastly, participants in the treatment as usual condition did not demonstrate significant improvements in math proficiency $t(11) = -0.18$, $p = .862$ from pre-test to the final post-testing. A mixed-model ANOVA was conducted to assess group (3) by time (2) interaction for the serial effects of training on math proficiency. There was a significant main effect of time $F(1, 31) = 4.48$, $p=.042$, partial $\eta^2 = .126$, and a significant effect of group $F(2, 31) = 6.26$, $p=.005$, partial $\eta^2 = .288$. There was not a significant interaction effect $F(2, 31) = 1.40$, $p = 2.63$, $\eta^2 = .083$. Post-hoc independent samples *t*-tests with a Bonferroni correction (i.e. .05/3 = .016) revealed that the group that completed 2048 first did not demonstrate significant improvements in performance on the EasyCBM at post-test compared to either the passive control group ($t(20) = 1.970$, $p = 0.63$) or the MOT first group ($t(18) = -1.53$, $p = .144$). The MOT first group and passive control group were not significantly different from each other ($t(20) = -0.23$, $p = .823$).

Discussion

The aim of the current study was to evaluate the feasibility of the study design as well as obtain preliminary data comparing domain-general and domain-specific cognitive training approaches for individuals with NDCs. More specifically, we focused on characterizing the feasibility, appropriateness, and accessibility of domain-general and domain-specific training for improving mathematics capability for individuals with below average levels of cognitive functioning. For one, we aimed to characterize the feasibility of implementing a school-based, cross-over design study for assessing training for the population. Here, we demonstrated high adherence and relatively no attrition throughout the school-based training regimen. Second, we sought to obtain preliminary data on degree of transfer to mathematics capability as a function of (i) domain-general, attention training; (ii) domain-specific, math game training; and (iii) serial training both attention and math-based game training. While the objective of the study was to obtain preliminary data and demonstrate feasibility to progress through rigorous RCT design, the preliminary findings were inconclusive and thus, there is no preliminary evidence to suggest differences between domain training and/or a combination of both approaches. As a result, we obtained an evaluation of the outcome measure of math as an appropriate measure to capture training benefits. While there were advantages to using a paradigm with multiple versions and skill-level appropriate assessments (i.e., grade level), the multiple-choice nature of the outcome may have reduced the test's sensitivity to observe change between time points. Overall, the results of the study demonstrate feasibility and appropriateness of the training paradigms and rigorous study design for individuals diagnosed with NDCs, and further characterized by below average cognitive capabilities.

The study's main finding suggests that youth with diverse NDCs and learning capabilities can participate in a cross-over design trial and engage with domain general and domain specific game-like training paradigms. Specifically, the targeted population demonstrated a capability to transition between training paradigms with no drop-out. Regarding the feasibility of the intervention tasks, the results of the present study were consistent with what was observed by Archambault and colleagues (2021). Specifically, there was a low attrition rate and strong engagement on the MOT cognitive training program with youth with NDCs. Given that all participants were able to engage and complete all trials of the 3D-MOT and 2048 tasks, this provides further evidence that 3D-MOT is an appropriate program to use with this population as well as provides further evidence for 2048 as an appropriate active control (Tullo, Guy, et al., 2018) and novel evidence for a game-like domain-specific exposure paradigm. Taken together, these findings suggest that implementing this study methodology for youth with NDCs is feasible and is recommended to explore both the unique and serial effects of domain-general (attention) and domain-specific (math) game training. Moreover, this study is one of the first to demonstrate high adherence to training which allowed for the exploration of the feasibility of exploring the serial effects of intervention with youth with NDCs in a school-based setting.

With regards to the tasks, there are a few important considerations for the 2048 game task that should be considered for future research. Specifically, it is worth noting that although the 2048 exposed students to multiples, it did not explicitly train multiplication skills. The game, through exposure to multiples, also has components of visual planning skills. Specifically, success in the game relies in part on visual pattern recognition. Thus, the visual component of the task may have some overlap with the visual nature of the MOT task. Future research may look to differentiate between math exposure, or math-game training, and explicit mathematical training interventions with varied number of visual components.

Given the high retention rate and completion of post-testing by 88.89% of participants, the results suggest that youth with NDCs can participate in a rigorous cross-over design study to examine serial effects of training. We hypothesize that the high retention rate is in part due to the experiment taking place within a familiar environment to the participants. In conjunction with the results of Archambault et al. (2021) and Tullo, Guy, et al. (2018), it seems that conducting such intervention studies within schools promotes engagement. This is a phenomenon that has been observed in other realms of intervention, such as those school-based mental health interventions (Neil et al., 2009). All three studies have demonstrated consistently high task engagement and retention when conducting large-scale studies within the school environment. We hypothesize that the familiarity of the environment, in conjunction with the structure of the study being embedded within the school day were positive factors influencing engagement. Additionally, the convenience of having participants in an educational setting may have reduced the demands on parents to seek out research opportunities and arrange for transportation, as relevant lab-based studies. Future research will be well served to explore differences in adherence between training environment (i.e, lab-based, online, and school-based) for cognitive training programs targeting youth diagnosed with NDCs. Altogether, the success in obtaining high study adherence was essential, as it provided the opportunity to explore far transfer effects without concern for the accessibility of data.

A secondary aim of the study was to explore the effect of transfer to mathematics capability as a function of domain-general, domain-specific, and combined training approaches. Though we did not expect to observe transfer effects within the scope of the study, we briefly explored whether it occurred while assessing the suitability of the math measure. The results did not demonstrate that training on the 3D-MOT task results in transfer effects to improved mathematical ability. One possible explanation for this is that far-transfer effects can take longer to be observed than near-transfer effects, and the study design may have limited the time to observe such effects. Another possible explanation that can account for the lack of observed far-transfer was that the measure selected to monitor change was not sensitive enough. We selected the EasyCBM measure as it had nine test versions and multiple grade levels, affording the opportunity for multiple post-tests. The EasyCBM was also developed using Rasch modelling to control for item difficulty and item fit across test versions, and acceptable to strong psychometric properties were reported (Alonzo et al., 2006); the test was normed on a diverse 500 student sample across the United States (Alonzo et al., 2006). However, the test itself was not standardized, meaning that participants were only able to score a maximum 15 out of 15 points for each test. Within such a small range, it is possible that students hit ceiling effects, or meaningful growth was not detectable within a 15-point scale. We recommend that future research consider using a standardized measure of mathematical ability, one example being the Pearson KeyMath. Use of such measures could allow for more meaningful comparisons between other standardized measures of cognition and attentional abilities (Clark et al., 2021).

As such, a limitation of the present study was test selection. Current standardized measures for math achievement were considered but deemed inappropriate for this pilot. The reason that we did not use a test such as the KeyMath in the present study was to promote feasibility. That is, standardized measures that are often used in clinical assessments afford high accuracy but are lengthy to complete and often only have 1 alternate test, limiting re-test ability. We hypothesized that the use of a more intensive and rigorous test would impact retention and adherence. Future research should seek to balance the need between accuracy and feasibility. Potential alternatives for future research could explore using the Pearson KeyMath-3, Kaufman Test of Educational Achievement, Third Edition (KTEA-3), the Process Assessment of the Learner, Second Edition (PAL-II Math), or the Wide Range Achievement Test, Fifth Edition (WRAT5).

Implications for Future Studies

Domain-specific interventions targeting math skills have been explored to remediate math learning in children with NDCs (Cihak & Foust, 2008). However, research has demonstrated that attention is a cognitive correlate of math learning for youth with NDCs (e.g., Clark et al., 2021). While we hypothesize that cognitive training could afford a different means for math remediation, future research should obtain evidence for efficacy of a domain-general (attention) intervention for math proficiency in this population. Further, more research is needed to compare the viability of domain-general (i.e., attention) training to domain-specific (math) training. Comparing the efficacy of such interventions could further inform clinical practice to support long-term academic success and adult outcomes for these students (Stevens et al., 2018). Previous research has also suggested that similarity between the training and targeted tasks results in greater likelihood of transfer, pointing in favour of domain-specific training (Gathercole et al., 2019). Regarding domain-general training, there is reportedly no far-transfer to academics (Rapport et al., 2013). However, the unique exploration of the serial effects of training could answer whether domain-general training after domain-specific training could serve as a catalyst to enhance the effects of domain-specific training.

The information obtained through this pilot study extended the successful feasibility of conducting interventions using 3D-MOT with children and youth with NDCs. Like what was observed by Archambault and colleagues (2021), we demonstrated that this population can adequately engage with the MOT task and comply with training expectations. Furthermore, students diagnosed with different NDC can participate in a study with a cross-over design with multiple post-testing periods.

Future feasibility research may look to assess the interventions with a larger age-range of students, potentially including high-school students with diverse cognitive profiles. Mathematics performance decreases as students approach high-school (Stevens et al., 2015), making it a timepoint where many more students might benefit from intervention. One might also look to conduct a longitudinal study to characterize the math growth scale for youth with NDCs, taking non-verbal intelligence and baseline attentional abilities into account. Such research may provide insight as to whether there is a certain profile or timepoint in which interventions may be most effective and in turn, direct guidelines for assessment.

Conclusions

Despite strong evidence demonstrating the role of attentional abilities in math learning and achievement (e.g., Mong & Mong, 2010; Templeton et al., 2008), current methods for remediation use domain-specific (math) skill-based approaches. While these interventions may be useful, it can be argued that they are only a task-specific solution to the function-specific problem of the underlying cognitive deficits in attention. The present study assessed the feasibility of exploring the effects of domain-general training, domain-specific exposure training, and the serial effects of cumulative training in youth with NDCs. Overall, the results of the study demonstrate feasibility and appropriateness of the training paradigms and rigorous study design for individuals diagnosed with NDCs and characterized by below average cognitive capabilities. Although we did not intent to demonstrate transfer, we did observe high adherence and ability for youth with NDCs to complete a rigorous cross-over study design. We did not find transfer, but subsequent studies based on this one could explore transfer with a larger sample size and a more sensitive math measure. The difference between domain-specific and domain-general training for remediating math for youth with NDCs remains a question that needs to be explored. Further research will help reconcile inconsistencies in cognitive training literature to help in understanding the ability for some of the most vulnerable learners to benefit from cognitive training.

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Chapter VII: General Discussion

Early mathematical learning and achievement set the stage for future learning across subjects (Duncan et al., 2007). Within math learning, youth are exposed to skills that lend themselves outside of traditional academic math including patterning and shapes (Kronholz, 2000), sequencing and ordinality (Denton & West, 2002), quantitative thinking (Sarama & Clements, 2009), generalization (Sarama & Clements, 2009), and other processes such as problem solving, reasoning, connections, and representation (Clements, 2004). Given the demonstrated role that math plays in later academic achievement, it garnered our interest as an area of learning that warranted significant exploration into the mechanisms that support successful skill development, especially for students with NDCs.

I discussed the many cognitive factors that have been related to the development and proficiency of mathematical skills including working memory (see Bull et al, 2008; Peterson et al., 2017; Raghubar et al., 2010), executive functioning (see Kroesbergen et al., 2009; Peng et al., 2018; Poorghorban et al., 2018), and processing speed (see Peng et al, 2018; Peterson et al., 2017). Importantly, attentional abilities remained a constant cognitive correlate of math learning (see Kim et al., 2018; Lindsay et al., 2001; Raghubar et al., 2009; Rueckert & Levy, 1995) across developmental periods and specific math skills. For youth with NDCs who often have impacted attentional abilities, we recognized that this places them at even greater risk of learning delays compared to their TD peers. Thus, this work aimed to better understand the role of attention in mathematics learning for youth with NDCs with the goal of informing future research, in the form of larger-scale intervention studies using standardized diagnostic measures and assessing the serial effects of sequential training. At a clinical level, this work not only informs math learning and remediation, but it does so with a specific lens on our most vulnerable learners, with the goal of promoting long-term academic success.

This thesis explored the current literature investigating the influence of attention for math learning, identifying difficulties in generalization that originated from inconsistent definitions and conceptualizations of attention and related executive functioning skills. We characterized the cognitive correlates of math learning within students with NDCs and explored whether these relationships vary as a function of diagnostic profile. Finally, we established the feasibility of conducting a rigorous cross-over design study to assess the unique and serial efficacy of domain-general (attention) vs domain-specific (math) training for youth with NDCs and complex diagnostic profiles and lower cognitive abilities. Final conclusions, implications, and future directions that have resulted from this thesis are explored below.

Cognitive Correlates of Math Learning

The role of attention and related skills in supporting the development and mastery of math skills is undeniable (see Child et al., 2019; Kim et al., 2018; Lindsay et al., 2001; Raghubar et al., 2009). However, what clouds the results is the inconsistent definition of attention and related terms, including executive functioning, working memory and processing speed. There have been different theories and conceptualizations put forth to make sense of these interrelated systems. Cognitive Load theory (Sweller, 2011), for example, suggests that individuals have a limited working memory capacity, and the more attentional resources a task requires, the less cognitive space is available for executive processes (Paas et al., 2003). An alternative conceptualization is that attention, executive functioning, and working memory are all encompassed by a unilateral framework within childhood (Doebel, 2020; Garon et al., 2008;

Karr et al., 2018). We agree with the conceptualization put forth my Miyake and colleagues (2000) that attention and related skills can be conceptualized in a top-down approach. Specifically, that successful use of working memory relies on executive functioning, all of which require efficient attentional control (Miyake et al., 2000); it follows that successful use of executive functioning and working memory skills depend on one's ability to orient and sustain their attention to a target. It is within this context, that training global attentional abilities is our area of focus.

The initial literature review in this thesis argued for the importance of exploring attention-based cognitive training as a means for math remediation, thus setting the aim of Study 2. However, prior to exploring the feasibility of intervention research, it was first essential to explore the cognitive correlates of math learning within youth with NDCs (Study 1). Expanding on previous research relating attention to math with TD youth to include youth with NDCs provided necessary theoretical justification for exploring cognitive training for math remediation. Youth with NDCs have unique profiles that are often confounded with cognitive delays and attentional impairments that impact learning (Antshel et al., 2016; Wei et al., 2011). When included, the inclusion of neurodiverse participants in research has often used a deficit-view approach in which abilities are compared to TD youth as a way to measure and describe level of deficit. The research does not solely focus on characterizing their abilities and furthering understanding of cognitive determinants of learning (Dinishak, 2016). It was important for us to demonstrate a framework for math learning that represented the cognitive correlates of learning with youth with NDCs, as they need more sensitive diagnostic measures for math learning, and evidence-based interventions that are effective. It is also important for us to consider these youth from a strengths-based perspective. We recommend that future research exploring learning differences consider characterizing discrepancies as different styles and biases in learning and perceiving, as opposed to deficits.

Characterizing Learning

In Study 1, we aimed to characterize the cognitive correlates of math learning amongst youth with varying NDCs. Notably, these youth, presented with complex profiles that included lower cognitive ability and multiple diagnoses (e.g., transdiagnostic approach). Our results were consistent with what has been observed in TD youth (Child et al., 2019; Kim et al., 2018), and amongst youth with attentional (Colomer et al., 2013; McConaughy et al., 2011) or math learning challenges (Lindsay at el., 2001; Raghubar et al., 2009). That is, attentional abilities remained a strong predictor of mathematical learning, over and above non-verbal intelligence and regardless of diagnostic profile. We opted to include non-verbal intelligence in the model, as it too has been related to many mathematical concepts (Kyttälä & Lehto, 2008; Steinmayr & Spinath, 2009). Tests of non-verbal intelligence, such as the Raven's Progressive Matrices (Raven, 2003), have also provided comparable estimates of overall IQ in TD individuals (Mackintosh, 2011). This is not the case for individuals with NDCs (Nader et al., 2016), suggesting that traditional IQ measures (i.e. Wechsler scales) underestimate the general cognitive abilities of those with limited verbal skills. For these reasons, PRI served as a more reasonable measure of overall intelligence with our participants in Study 1. Furthermore, it is a cognitive area where autistic youth often demonstrate strengths (Nader et al., 2016). Therefore, we queried whether a diagnosis of autism influenced the predictive value of attention or non-verbal intelligence for math achievement as a function of superior PRI ability.

However, regardless of diagnostic profile, attention remained a predictor of mathematical achievement for these learners, with non-verbal intelligence serving as a significant moderator. This suggested that attention, above and beyond all other variables explored, served a significant role in math achievement. This is integral as it suggests that a universal approach to remediation may be suitable, given the lack of influence diagnosis carried.

In addition to confirming the cognitive mechanisms involved in math learning for youth with NDCs, Study 1 also emphasized the importance of using standardized measurement tools when working with youth with NDCs to facilitate meaningful comparisons between standardized tests, and comparisons to TD peers. However, it remains the case that for many youths with NDCs and lower cognitive abilities, standardized tests of math learning are challenging to engage with and often result in discontinuing subtests very early (floor effects). These floor effects translate to poor overall performance that can be explained by a few different reasons, including task demands, normative sampling, and sensitivity to the challenges these unique learners face. Thus, there is a need for more standardized mathematical achievement assessment tools that lend themselves well to research (e.g., multiple forms) and are appropriate for use with lower-functioning individuals (e.g., visually-based and intuitive) to provide the best estimate of their math achievement level.

Standardized Assessment

Standardized assessment tools are essential to clinical practice in assessing and diagnosing the academic, cognitive, and social-emotional abilities of youth. Presently, there are several standardized tools that are appropriate for the assessment of general cognitive abilities for youth with NDCs, such as the Wechsler Intelligence Scale for Children – Fifth Edition

(Wechsler, 2015), the Stanford-Binet – Fifth Edition (Roid, 2003), Woodcock Johnson – Fourth Edition (Schrank & Wendling, 2018), and Raven's Progressive Matrices (Raven, 2003). Furthermore, there are many standardized assessment measures for attention and executive functioning that are both feasible for use and sensitive enough to characterize individuals with significant difficulties in these domains (i.e. attention, EF), such as the CPT-3 (Conners 2014; see Tullo et al. 2018 for example). However, academic measures are an area in which standardized tools are not adequately designed for accurate assessment of youth with significant cognitive or attentional difficulties. The Weschler Individual Achievement Test – Third Edition (WIAT-III) and Woodcock Johnson Test of Achievement – Third Edition (WJ-III ACH) are two commonly used standardized measures of achievement, given their strong psychometric properties (Walters, 2020). Though they have utility in clinical practice, standardized academic tests could underrepresent the abilities of youth with NDCs for reasons explored above (task demands, language requirements, normative sampling, sensitivity to the challenges) (e.g. DiStefano et al., 2020; Rhodes et al., 2015; Tzuriel, 2001; Walsh et al., 2007). Furthermore, they do not provide in the in-depth exploration of specific math skills via specific subtests and indices which is more sufficiently address with a tool such as the Pearson KeyMath (Connolly, 2007). While standardized academic assessments are essential in clinical practice, they too are integral to sound research. By utilizing standardized measures as outcome measures in future research exploring cognitive correlates of learning and cognitive interventions, we can make meaningful comparisons between constructs (e.g., math, attention, IQ) and between groups (e.g., TD vs. NDC), and more readily discuss generalizability to clinical practice.

In Study 1, we demonstrated that standardized assessment tools permitted meaningful comparison between intelligence (WASI-II), attentional abilities (CPT-3), and mathematical proficiency (Pearson KeyMath). Furthermore, we were able to generalize these results and make comparisons to previous studies that explored the cognitive mechanisms involved in math learning amongst TD youth. In Study 2, we were met with limitations regarding the feasibility of math achievement measure. Specifically, we prioritized selecting a measure (e.g EasyCBM) that had multiple forms to accommodate baseline and multiple post-tests while not encountering practice effects. Additionally, it was essential that the measure selected balanced accuracy in measuring math achievement with feasibility (e.g., time to complete test). Specifically, given the difficulties posed by lengthy standardized assessments for youth with cognitive difficulties (Walsh et al., 2007), we wanted to avoid using an intensive math measure that may have led to participant attrition. Though the Easy CBM assessment has multiple forms, presenting sufficient options for repeated administration without practice effects, it lacked the ability to translate performance to a meaningful standardized score. The lack of standardized scores presented challenges for our statistical analysis, considering participants had varying grade levels. That is, a nine-year old youth who scored 10/15 on the grade five level test has very different abilities from a nine-year old youth who scored 10/15 on the grade one level test. Furthermore, for youth who scored highly on the first round of pre-testing, there was a very small margin for them to improve on subsequent test points (ceiling effects). These ceiling effects had the potential to impact accurately measuring growth in math proficiency for some participants. In turn, this may have influenced the overall results, and conclusions of the study. The limitations encountered in Study 2 by not using a standardized measure further reinforces the need to develop a new standardized measure of math achievement. Specifically, one that is sensitive to the needs of youth with NDCs and is appropriate for use in both research and clinical practice; a measure that balances accuracy and feasibility. It then follows that use of standardized assessments would be essential for adequately assessing the efficacy of future standardized or manualized interventions. Consistently using a standardized math achievement measure in efficacy studies would support work towards an intervention strongly based in evidence.

Feasibility of Comparing Domain-General and Domain-Specific Interventions

In Study 2, we explored the feasibility of conducting a rigorous RCT school-based, cross-over design study with youth with NDCs. Within Study 2 we also sought to explore the feasibility of comparing the unique and serial effects of domain-general (attention) and domain-specific (math) training for the purposes of math remediation. We did not intend to assess the efficacy of either training regiment, rather the capability of youth with NDCs to engage in the study design to inform future research assessing efficacy. An additional aim of Study 2 was to explore the suitability of the selected math achievement measure for its suitability for use with youth with NDCs. Specifically, whether the measure was accurate and sensitive in providing reasonable estimates of ability and detecting change in mathematical achievement. Overall, the results of Study 2 demonstrated that both the domain-general (attention) and domain-specific (math) intervention tasks were feasible for use with youth with NDCs; participants were able to engage in the entirety of each course of training without drop-out. Demonstrating the feasibility of these tasks was essential, as it can now inform the development of future studies that could assess the efficacy each task for domain-specific, domain-general, or serial training for youth with NDCs. Future research can work toward reconciling inconsistencies in the literature on cognitive training to inform the utility of cognitive training for some of our most vulnerable learners. While the intent of Study 2 was not to demonstrate efficacy of training, it was briefly explored.

Cognitive Training and Transfer Learning

Cognitive training is a highly contested topic with mixed results regarding efficacy amongst the recent literature (Simons et al., 2016). The results of Study 2 provide mixed results regarding the efficacy of attention-based cognitive training, specifically for intervention with youth with NDCs. Prior to the present research project, the 3D-MOT attention training paradigm had been validated for use with youth with NDCs and complex diagnostic profiles and lower cognitive ability (Archambault, 2021; Tullo et al., 2018). However, Study 2 did not provide evidence in support of the 3D-MOT attention training program for producing far-transfer effects to improvements in mathematics. The null results of Study 2 do not provide sufficient evidence that attention-based cognitive training is not effective. As previously mentioned, youth with similar profiles (lower cognitive ability, complex diagnostic profiles) have previously demonstrated successful engagement, task-based improvement, and near-transfer to attention when completing an identical study design (Tullo et al., 2018). Previous success is in part due to the accessibility of the task for youth with NDCs. Specifically, that the task has low language requirements, and is adaptive to the participant's capabilities throughout the course of training (Tullo et al., 2018). Given that the task in Study 2 and the Tullo et al. study (2018) had identical adaptive properties, the null results in Study 2 are somewhat surprising. When comparing the study designs, Tullo's 2018 study explored the 3D-MOT paradigm with a much larger sample size (n = 129) that encompassed a wider age range (6-18 years old). Given that the participants in Study 2 were 7-12 years old, it warrants isolated exploration as to age influences a participant's gains as a result of training on the 3D-MOT task. It is also possible that the results of Study 2 were influenced by the relatively smaller sample size and larger range in non-verbal intelligence (PRI). These factors may have affected variability on task performance and thus, overall significant effects of training. Participants in Study 2 also trained by tracking three out of eight spheres which may have been too challenging for some. Future studies using our paradigm may further the adaptability of the task to have participants track either one, or two of the eight target spheres to make it more accessible.

In addition to nuanced validation of the Tullo et al. (2018) study, there is a need for further exploration of the potential benefits of attention-based cognitive training with youth with NDCs and lower cognitive ability/complex diagnostic profiles. Particularly using the 3D-MOT task trialed in Study 2 that is designed to be accessible for youth with NDCs due to low language requirements and adaptive task difficulty. As aforementioned, the typical development of math learning is cumulative in nature. That is, strong foundations of early math skills are integral for the understanding of more complex concepts as one progresses through their education (Sarama & Clements, 2009). In addition, youth are being exposed to processes such as problem solving and reasoning that too become more challenging throughout education, requiring strong foundational skills (Clements et al., 2004; Sarama & Clements, 2009). Thus, this componential nature of learning explicit math skills and the related processes highlights the importance of addressing skill development early in education. One such way to explore whether early engagement in attention-based cognitive training has long-term benefits would be to implement training in early elementary years and assess youth's mathematical achievement developmentally, throughout elementary and secondary schooling. Early intervention combined with the proposed delay-onset of the benefits of attention training (Van der Molen et al., 2010) and increased exposure to learning may reveal slow and long-developing benefits of successful attention-based cognitive training.

Implications

While the relationship between attention and mathematics achievement has been demonstrated (see Kim et al., 2018; Lindsay et al., 2001; Raghubar et al., 2009), it remains widely under-investigated in the intervention literature. For the most part, attention-based cognitive training research has been broad, and intended to improve symptomatology for specific clinical groups (e.g., ADHD – Slattery et al., 2022). Exploration to far-transfer effects is often secondary, and not the focal point of studies (Slattery et al., 2022). In the studies that do target far-transfer learning, the results trend toward demonstrating minimal far-transfer of attention-based cognitive training to academic achievement (Slattery et al., 2022; Tullo & Jaeggi, 2022). While the results of Study 2 are consistent in demonstrating minimal far-transfer effects, the lack of evidence in support of this training does not align with the strong relationship between attention and mathematics. The introduction of this thesis highlighted the lack of sound intervention training to follow up the empirically evidenced relationship between mathematics and attention. We demonstrated the relationship between attention and mathematics strongly evidenced in TD youth is also evident in lower-functioning youth with NDCs, above and beyond non-verbal intelligence and regardless of diagnosis. This on its own is significant as it contributes to our understanding of these exceptional learners, and demonstrates that despite cognitive difficulties and other symptomology, the correlates of math learning remain the same as our TD youth. Thus, the larger body of literature exploring math learning in TD youth may be generalizable to youth with NDCs.

For educators and clinicians, the initial research findings are strongly in support of early identification and intervention for students struggling with math, in support of their long-term academic growth and success (Duncan et al. 2007). The literature would suggest that educators should be keenly aware of their students who may be struggling with foundational mathematical skills, and/or those showing difficulties with attention, working memory, and executive functions. Accurately identifying these difficulties in childhood has the potential for efficient identification and thus proper early remediation. For clinicians, youth presenting for assessment who experience attentional challenges should strongly be considered for academic testing that explores the acquisition of fundamental mathematics skills. Though the presenting concern may be attentional challenges or difficulties with executive functions, as discussed throughout this thesis, there is a strong likelihood that this child is susceptible to falling behind in mathematics learning. In the same regard, youth presenting for assessment to explore mathematics learning challenges should have sufficient testing to explore their attentional profile and related skills (WM/EF) to identify any skills difficulties. In investigating these related skill sets through standardized testing; it follows that at-risk youth will receive thorough and individualized educational planning to support their immediate and long-term learning.

When considering interventions to support youth with mathematics learning challenges, we encourage educators to reflect on whether their programming is task-specific, and whether this is in the long-term interest of the student’s learning. It is essential that more research is conducted in the field to provide evidence in support of intervention and programming for our most vulnerable learners. Far too often, school professionals are expected to implement intervention programs with limited research into its effectiveness, and a lack of explanation for the rationale. I strongly encourage that schools have a transparent understanding of their intervention programs and selected intervention programs are based in sound empirical evidence.

Future Research

Future studies should use the methodology demonstrated as feasible in Study 2 with a larger sample size to assess evidence in support of the unique and serial effects of domain-general (attention) vs. domain-specific (math) training programs. Continued exploration into cognitive training is needed to assess the ability of our most vulnerable learners to benefit. Based on the review of the current literature, it is integral that specific attentional skills being targeted are well operationalized. Furthermore, researchers should take careful consideration into selecting the means for their cognitive interventions, ensuring that it is adequately addressing the operationalized skill. The interventions should be intensive and should include immediate and follow-up testing to evaluate the impact of attention-based cognitive training over time. Like carefully operationalizing and selecting appropriate attention measures, future research should further explore appropriate math measures. That is, selecting a measure that maximizes accuracy and ability to draw meaningful comparisons and generalizations, but that also is feasible for use in a cross-over design with brief and multiple post-test time-points. There is a need for an appropriate math measure that balances both accuracy and feasibility for work with youth with NDCs presenting with complex diagnostic profiles and lower cognitive abilities.

Future research should also compare the effectiveness of the cognitive intervention for typically developing youth, youth with mathematics learning disorders, youth with attentional impairments, and youth with comorbid mathematics learning and attentional difficulties. Doing so will provide further insight into these specific profiles, as well as help in understanding if intervention should be differentiated for different presentations. Finally, this could justify whether mathematics instruction for all students could be improved by addressing attentional components of instruction design or supplementing some attention-based cognitive training into the regular curriculum for general education.

Chapter VIII: Conclusion

The evidence supporting a relationship between domain-general attentional functions and mathematics learning and performance is clear (e.g., Blair & Razza, 2007; Cragg & Gilmore, 2014; Fuchs et al., 2005; Gold et al., 2013; Peng et al., 2018). This thesis extended the strong body of literature to replicate this relationship amongst lower-functioning youth with NDCs and complex diagnostic profiles. We demonstrated that the relationship between attention and mathematics is transdiagnostic and is partially related to non-verbal intelligence. These results provided the theoretical justification to assess the feasibility of using an attention-based cognitive training program with youth with NDCs. The results of Study 1 provided the justification to explore the feasibility of using a rigorous cross-over design study to compare domain-general (attention) and domain-specific (math) interventions. Prior literature exploring cognitive training had mixed results, and research specifically exploring this training with youth with NDCs was very limited. Findings from our research demonstrated that the selected tasks and study design were appropriate for youth with NDCs, and though we did not intend to observe transfer effects, a brief exploration of the results demonstrated no transfer learning. The lack of significant findings in support of the efficacy of training within the feasibility study contrast with the evidenced relationship of cognitive mechanisms and learning. This suggests that further exploration into the methodology and feasibility of attention-based cognitive training is warranted. Some potential explanations for the current non-significant results include using a non-standardized measure, lack of long-term follow-up testing, and a small sample size.

Given the evidence cognitive correlates of math learning and success of the feasibility study, continued research exploring methodology, feasibility, and clinical utility of attention-based cognitive training for youth with NDCs remains integral. These learners have unique challenges, and early math intervention has the potential to influence their later academic trajectory. Furthermore, we encourage educators and clinicians to consider the factors related to math learning addressed in this thesis. Youth presenting with attentional challenges should be closely monitored regarding math learning. We suggest all school professionals critically evaluate the methods which with they remediate learning in their respective schools, and screen for any math learning challenges as soon as possible.

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Thus, I retain the right to include the manuscript in its submitted form in this dissertation.

Appendix A

McGill

Research Ethics Board Office
James Administration Bldg.
845 Sherbrooke Street West. Rm 325
Montreal, QC H3A 0G4

Tel: (514) 398-6831
Fax: (514) 398-4644
Website: www.mcgill.ca/research/researchers/compliance/human/

Research Ethics Board III
Certificate of Ethical Acceptability of Research Involving Humans

REB File #: 303-1215

Project Title: Assessing the role of attention and motivation for math proficiency for adolescents with Autism Spectrum Disorder

Principal Investigator: Emma Clark **Department:** Educational&Counselling Psychology

Status: Master's student **Supervisor:** Prof. Armando Bertone

Approval Period: **June 21, 2016 – June 20, 2017**

The REB-III reviewed and approved this project by delegated review in accordance with the requirements of the McGill University Policy on the Ethical Conduct of Research Involving Human Participants and the Tri-Council Policy Statement: Ethical Conduct For Research Involving Humans.

Lynda McNeil
Associate Director, Research Ethics

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* All research involving human participants requires review on at least an annual basis. A Request for Renewal form should be submitted 2-3 weeks before the above expiry date. Research cannot be conducted without a current ethics approval.
* When a project has been completed or terminated, a Study Closure form must be submitted.
* Unanticipated issues that may increase the risk level to participants or that may have other ethical implications must be promptly reported to the REB. Serious adverse events experienced by a participant in conjunction with the research must be reported to the REB without delay.
* Modifications must be reviewed and approved by the REB before they can be implemented.
* The REB must be promptly notified of any new information that may affect the welfare or consent of participants.
* The REB must be notified of any suspension or cancellation imposed by a funding agency or regulatory body that is related to this project.
* The REB must be notified of any findings that may have ethical implications or may affect the decision of the REB.

Appendix B

ÉCOLE LE SOMMET
SUMMIT SCHOOL
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APPROVED JUN 2 1 2016

Director General:
HERMAN ERDOGMUS

Vice Principal:
BENA FINKELBERG

Dear Parent,

The ability to pay attention and concentrate is crucial for learning both in the classroom and while completing homework.

Like a muscle, one can "train the brain" to pay attention and focus for longer periods of time, ultimately resulting in an increased ability to learn, succeed in school, and prevent behaviour problems.

We invite you to take part in a study looking at whether or not training on 3D task for 5-10 minutes for multiple weeks can improve attention in young students.

How is this possible ?
For the last several years, our research group has developed a brain training technique that has shown to improve attention and concentration in adolescent students. It is safe, easy & fun. It is called the Multiple Object Tracking. The Multiple Object Tracking task is a mental gym where your child will train their ability to pay attention and concentrate.

How does it work ?
The training involves a method referred to as, Multiple Object Tracking. For 5-10 minutes every day your child will be asked to focus on moving spheres (round balls) presented on a computer screen. If your child's attention and concentration improves they will be asked to track more of these spheres as they move on the screen.

What are the benefits for my child ?
It is our hope that your child will improve their ability to pay attention and concentrate for longer periods of time. This may help your child maximize learning in school, improve their overall confidence, and help with completing homework. In order to fully understand if attention improves, some students who sign up will be placed in a control group that will play a computerized math game, "2048", instead of doing the attention training. If your child is placed in our study's control group, they will be given access to the attention training at your request following the completion of the study.

Yours sincerely,

Dr Armando Bertone
Assistant Professor
Department of Educational and Counselling Psychology
School/Applied Psychology program
McGill University

Appendix C

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Consent Form

Institution: Faculty of Education, McGill University

Title of Project: Assessing the role of attention and motivation for math proficiency for adolescents with Autism Spectrum Disorders

Project Leader: Emma Clark, MA candidate
School/Applied Child Psychology Program
Education and Counselling Psychology
Faculty of Education, McGill University

Project Supervisor: Armando Bertone, PhD
School/Applied Child Psychology
Department of Education and Counselling Psychology
Faculty of Education, McGill University

Introduction. Early math skills are an integral component to academic success and can be used to predict overall achievement at later grades. Attention and intrinsic motivation are critical in mathematic proficiency and achievement. We are interested in the degree to which math achievement can be improved through attention training. In the present study, we will assess the potential benefits of a visuo-attentive cognitive training program by measuring your child's ability to track multiple balls bouncing on a screen, or Multiple Object Tracking (MOT), presented to him/her on a screen via 3D glasses. Specifically, we are interested in your child's progression on this task over an extended period of time, and measure whether an improved **MOT** performance is associated with his/her improved mathematic proficiency, as well as how initial intrinsic motivation influences this learning transfer.

Procedures. The study will be carried out in you child's school during a dedicated time of the day. Your child will be brought to a classroom set-up for research purposes to complete their training and then will return to class. Children will be randomly assigned to the training or control group. In the Training group, children will spend 10-15 minutes sessions on the MOT task, between 3 and 4 times a week for a period of about 2 months. During each training session, your child will be asked to sit comfortably in front of a screen and pay attention to a number of moving balls on the screen, then respond to which objects were tracked. The difficulty of the task will be adapted to your child's ability, and his/her progression on the task will be monitored throughout the school year. If assigned to the control group, your child will spend an equivalent amount of time playing a strategic math-game, 2048. In order to see whether the training on the MOT task improved your child's attention and math achievement, a series of cognitive and curriculum-based math measures will be taken before and after the training sessions. Children in the control group will completed these measures as well to ensure any improvements in attention were a result of attention training. These measures will take approximately 30 minutes and will be done at the school. All students in the control group will be provided access to the attention training at the end of the study at the request of the caregivers.

Advantages of the proposed studies: This study will enable a better understanding of how attentional abilities develop and will help guide the creation of age-specific learning approaches, teaching methods and educational materials.

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Disadvantages of the proposed studies: There are no known side effects associated with the previously described visual and/or cognitive tasks. The training is very safe and fun – it is very similar to playing a game.

Confidentiality: All the information will be kept confidential, except as required or permitted by law. Your child will be assigned a study number and the information will be filed using this unique identifier code. Only this code will link the participant to the sample. Apart from our research staff, only members of regulatory agencies or members of the Research Ethics Board may have access to the data. If data from this study is published or presented at scientific meetings, personal identity will never be revealed. Data obtained from this study will be stored for the duration of 10 years, after which it will be rendered completely anonymous through the deletion of any identifiers that would allow for the participant to be retraced. Although the research findings may be disseminated at scholarly conferences and in the writing of scientific articles, results from individual participants will remain strictly confidential.

Participation: Your consent and child's participation is voluntary. You may refuse your child's participating or withdraw your child from the study at any time without any prejudice to your child's future involvement with school-based studies or activities. In the case that you do withdraw your child from the study, all previous data collected will be destroyed.

Incidental Findings: Although your child's cognitive and behavioural findings are clinically non-interpretable (i.e., used for diagnosis); any questions regarding their performance will be explained to you, upon your request.

Compensation: Your child will be compensated for his/her participation with a gift certificate valued at 10$ at the completion of the study.

Contact Numbers: If you have any questions about the research, please contact Dr. Armando Bertone at the Faculty of Education at (514) 398-3448 or armando.bertone@mcgill.ca or Ed Cukier at Summit School (514) 744-2867, ecukier@summit-school.com

If you have any ethical concerns or complaints about your participation in this study, and want to speak with someone not on the research team, please contact the McGill Ethics Manager at 514-398-6831 orlynda.mcneil@mcgill.ca"

Declaration of the participant:
Please sign below if you have read the above information and consent to your child's participation in this study. Agreeing to participate in this study does not waive any of your rights or release the researchers from their responsibilities. A copy of this consent form will be given to you and the researcher will keep a copy.

____
Name of Child

____
Name of Parent (Legal tutor) Signature of Parent (Legal tutor) Date

Appendix D

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Assent Form (6-13 years)

Institution: Faculty of Education, McGill University

Title of Project: Assessing the role of attention and motivation for math proficiency for adolescents with Autism Spectrum Disorders

Project Leader: Emma Clark, MA candidate
School/Applied Child Psychology Program
Education and Counselling Psychology
Faculty of Education, McGill University

Project Supervisor: Armando Bertone, PhD
School/Applied Child Psychology
Department of Education and Counselling Psychology
Faculty of Education, McGill University

Why are we doing these tests?
We are doing these tests and activities on the computer to better understand how we can help children and teenagers pay attention and concentrate in class. We want to understand if doing this task on the computer every day will help you pay attention and learn math.

What will happen during testing?
Today, I will ask you to do some activities on a laptop, answer some questions, and complete some puzzles and math questions. For the next two months, I will ask you to look at moving objects on a TV screen and try to keep track of those that I will point out to you. It is like a video game with moving balls on the screen. There is no right or wrong answer to this game, so just try your best every day. Later on in the year, we will do some of the tests that we will do today.

Can I decide if I want to do these tests?
Your parents have given me permission for you to participate in this project. You do not have to participate in these activities if you do not want to. If you do want to participate, you do not have to answer any of the questions and we can stop at any time.

Who will know what I did during these activities?
All of the responses given throughout these activities are confidential. This means that only myself and other researchers working with me on this project will see your answers. The results from this project may be published or presented, but your name will not be used and no one will know that you participated in this study.

Do you have any questions? Do you want to participate?

Assent - Completed by the Researcher

I read this to .... and acknowledge that he/she gave me verbal assent to participate.

Signature.. Date..

Appendix E

APPROVED JUN 2 1 2016

Assent Form (13 years +)

Institution: Faculty of Education, McGill University

Title of Project: Assessing the role of attention and motivation for math proficiency for adolescents with Autism Spectrum Disorders

Project Leader: Emma Clark, MA candidate
School/Applied Child Psychology Program
Education and Counselling Psychology
Faculty of Education, McGill University

Project Supervisor: Armando Bertone, PhD
School/Applied Child Psychology
Department of Education and Counselling Psychology
Faculty of Education, McGill University

Why are we doing these tests on the computer?
We want to know if practicing or training your attention abilities by following objects on a screen for two months will help you pay attention and concentrate in school. We want to find out if improved ability to do this task will also help you learn and perform math skills. This study will therefore look at how you perform before training and after training, to see if there is an improvement.

What will happen during testing?
Today, I will ask you to do some activities on a laptop, answer some questions, and complete some puzzles and math questions. For the next two months, I will ask you to look at moving objects on a TV screen and try to keep track of those that I will point out to you. It is like a video game with moving balls on the screen. There is no right or wrong answer to this game, so just try your best every day. Later on in the year, we will do some of the tests that we will do today.

Can I decide if I want to do these tests?
Your parents have allowed for you to participate in this research project. You do not have to participate in these activities if you do not want to. If you do want to participate, you do not have to answer any of the questions and we can stop at any time.

Who will know what I did during these activities?
All of the responses given throughout these activities are kept confidential. This means that only myself and other researchers working with me on this project will see your answers. The answers from all of other kids who participated in this project may be presented at meetings or written in articles, but your name will not be used and no one will know that you participated in this study.

Do you have any questions? Do you want to take part in the study?