A Novel Method for the Determination of Direct Tensile Strength of Brittle Materials Using Expansive Cement

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November 2023

A thesis submitted to McGill University in partial fulfillment of the requirements of a degree of Master of Engineering in the Department of Mining and Materials Engineering.

Abstract

Currently, the two main testing methods available for tensile strength determination of brittle materials such as rocks and concrete are the Glued Ends Direct Tensile Strength (GEDTS) test and the Brazilian Tensile Strength (BTS) test. Although the BTS test is popularly used for indirect tensile strength measurement due to its simplicity, it suffers several validity issues that cause it to overestimate the tensile strength of the tested material. The GEDTS test, on the other hand, is a direct testing method that is internationally recognized and acknowledged as the correct method for determining the tensile strength of brittle materials. However, challenges involving eccentric loading due to the misalignment of the specimen-load axes and poor adhesive performance during hard rock tests render the GEDTS test unappealing to the research community. As a result, developing an alternative direct testing method remains an interesting research endeavor. Therefore, this thesis focuses on developing a novel testing method known as the Expansive Cement Direct Tensile Strength (ECDTS) test to directly determine the tensile strength of brittle materials. Test principle and methodology of the ECDTS test are discussed in detail and its merits over the conventional testing methods are highlighted.

This thesis is a two-phase research study. As part of the first phase which is the method validation phase, a simple yet effective testing system known as the Load Centering Device (LCD) is developed to eliminate eccentricity from the loading process of the conventional GEDTS test. The description and modus operandi of the LCD are outlined in detail for broader comprehension. The effectiveness of the LCD is examined and verified by conducting a series of direct tensile strength tests on Basaltic komatiite using the GEDTS test. Also, the veracity of the proposed ECDTS test is examined by conducting a series of numerical modeling analyses using Abaqus XFEM software as well as experimental studies on typical rock specimens from low-tensile marble. Comparison of ECDTS test experimental test results to the results of the conventional methods including the BTS and the GEDTS demonstrates the validity of the ECDTS test.

The second phase which is the method application phase examines the applicability of the ECDTS test on hard rocks. Sudbury breccia and Gneiss which are known to have high tensile capacities are tested using the ECDTS, GEDTS, and BTS

tests. Since the bonding capacity of the glue is lower than the tensile capacity of the tested rocks, the GEDTS test failed at the glued interface and, thus, did not produce any results. As such, the applicability of the proposed ECDTS test is verified by comparing its results to the results of the BTS test. It is found that the average value of the BTS results for the two rock types is 14-15% higher than the average value obtained from the proposed ECDTS test. This is expected since the BTS test is known to overestimate the tensile strength of brittle materials such as rocks. This result demonstrates the effectiveness and consistency of the newly proposed ECDTS test.

Résumé

Actuellement, les deux principales méthodes de test disponibles pour déterminer la résistance à la traction des matériaux cassants tels que les roches et le béton sont le test de résistance à la traction directe avec extrémités collées (GEDTS) et le test de résistance à la traction brésilien (BTS). Bien que le test BTS soit couramment utilisé pour la mesure indirecte de la résistance à la traction en raison de sa simplicité, il présente plusieurs problèmes de validité qui le conduisent à surestimer la résistance à la traction du matériau testé. Le test GEDTS, en revanche, est une méthode de test directe reconnue internationalement et considérée comme la méthode correcte pour déterminer la résistance à la traction des matériaux cassants. Cependant, des défis liés au chargement excentrique en raison du désalignement des axes spécimen-charge et à la mauvaise performance adhésive lors des tests sur roche dure rendent le test GEDTS peu attrayant pour la communauté de recherche. Par conséquent, le développement d'une méthode de test directe alternative reste un projet de recherche intéressant. Ainsi, cette thèse se concentre sur le développement d'une nouvelle méthode de test connue sous le nom de test de résistance à la traction directe avec ciment expansif (ECDTS) pour déterminer directement la résistance à la traction des matériaux cassants. Le principe et la méthodologie du test ECDTS sont discutés en détail et ses avantages par rapport aux méthodes de test conventionnelles sont mis en évidence. Cette thèse est une recherche en deux phases. Dans le cadre de la première phase, qui est la phase de validation de la méthode, un système de test simple mais efficace connu sous le nom de dispositif de centrage de charge (LCD) est développé pour éliminer l'excentricité du processus de chargement du test GEDTS conventionnel. La description et le mode opératoire du LCD sont détaillés pour une compréhension plus large. L'efficacité du LCD est examinée et vérifiée en effectuant une série de tests de résistance à la traction directe sur du komatiite basaltique en utilisant le test GEDTS. De plus, la véracité du test ECDTS proposé est examinée en réalisant une série d'analyses de modélisation numérique à l'aide du logiciel Abaqus XFEM ainsi que des études expérimentales sur des spécimens rocheux typiques en marbre à faible résistance à la traction. La comparaison des résultats des tests expérimentaux du test ECDTS avec les résultats des méthodes conventionnelles, y compris le BTS et le GEDTS, démontre la validité du test ECDTS. La deuxième phase, qui est la phase d'application de la méthode, examine l'applicabilité du test ECDTS sur des roches dures. La brèche de Sudbury et le gneiss, connus pour avoir des capacités de traction élevées, sont testés à l'aide des tests ECDTS, GEDTS et BTS. Étant donné que la capacité de liaison de la colle est inférieure à la capacité de traction des roches testées, le test GEDTS a échoué au niveau de l'interface collée et n'a donc pas produit de résultats. En conséquence, l'applicabilité du test ECDTS proposé est vérifiée en comparant ses résultats avec ceux du test BTS. Il est constaté que la valeur moyenne des résultats BTS pour les deux types de roches est de 14-15 % plus élevée que la valeur moyenne obtenue à partir du test ECDTS proposé. Ceci est attendu étant donné que le test BTS est connu pour surestimer la résistance à la traction des matériaux cassants tels que les roches. Ce résultat démontre l'efficacité et la cohérence du test ECDTS nouvellement proposé.

Acknowledgements

First, I would like to express my profound gratitude to my research supervisor, Professor Hani Mitri for the opportunity to be a part of his astute research group. I am utterly thankful for his professional guidance and invaluable mentorship which have contributed immensely to my academic and professional growth. His exceptional patience has been a cornerstone during challenging moments. I'm deeply appreciative of his understanding and support. A special thank you to his wife, Bella, for her warm welcome during our annual MDL BBQ.

To my research group, I am deeply grateful for your collaborative spirit which has been instrumental in achieving our collective goals and fostering a sense of unity. It has been an amazing experience working with you. A special thanks to you, Kelly and Tuo, for the time you spent with me on Abaqus and Flac3D which were the two important numerical modeling tools in my research.

I extend my deepest thanks to Dr. William Cook, Systems Manager, and Mr. John Bartczak, Laboratory Coordinator in the Department of Civil Engineering for their unwavering support during rock sample preparations and tests for my thesis.

Finally, I would like to express my heartfelt gratitude to my darling wife, Mabel, for being a strong woman, and staying solidly behind me with the support and love I so much needed to pursue graduate studies in Canada outside of our home country, Ghana. I'm exceptionally grateful for your tireless efforts in nurturing our two daughters, Halona and Gretchen, while I was away; your devotion and care have laid the foundation for their growth and happiness. I and the girls say, “Thank you, mummy”.

Contributions of the Author

All chapters of this thesis are the contributions of the author in their entirety. The author performed all the experiments by himself. He was responsible for the conceptualization, methodology, analysis, visualization, and production of the first thesis draft. Dr. William Cook, Systems Manager, and John Bartczak, Laboratory Coordinator, in the Department of Civil Engineering helped with sample preparation and Testing. Professor Hani Mitri's role included supervision and technical review. The author performed each experiment and reported the results as is.

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Chapter 1 Introduction

1.1 Tensile Strength of Brittle Materials

The tensile strength of brittle materials such as rock and concrete is an essential parameter in the design and stability analysis of many geotechnical engineering projects. It has been defined by Tufekci et al. (2016) as the failure stress of a rock element in pure uniaxial tensile loading. Understanding a material's tensile strength is significant in determining the roof span limit of underground openings, the internal pressure limit of storage structures, how tunnel boring is carried out, and the stability of boreholes (Fuenkajorn et al., 2010).

Generally, the tensile strength of rock materials is about a tenth of their compressive strength (Briševac et al., 2015). This is the primary reason why most rock failures occur in tensile zones (Kerbati et al., 2020). As such, the tensile capacity of a rock mass plays a significant role in its resistance to failure (Perras and Diederichs, 2014). While tensile strength is critically important to controlling many failure processes, its measurement is often neglected in engineering practice due to difficulty in obtaining reliable results. Much attention has been given to the determination of the unconfined compressive strength of intact rock (Perras and Diederichs, 2014). However, research has shown that crack initiation in brittle materials such as rocks under compression is initiated by a tensile cracking phenomenon, as demonstrated by many researchers (Griffith, 1924; Horii and Nemat-Nasser, 1985; Brace, 1960; Hoek and Bieniawski, 1984; McClintock, 1962; Brace and Bombolakis, 1963). Therefore, a good understanding of the tensile behavior of rocks is beneficial in intact rock and rock mass analysis (Liao et al., 1997). Accordingly, several papers have been published in the area of rock mechanics to estimate and determine the tensile strength of brittle materials such as rocks (Aliha et al., 2021).

The tensile strength of rocks is evaluated by subjecting a test specimen to either of the two main testing methods – Direct Tensile Strength (DTS) and Indirect Tensile Strength (ITS) testing methods. The DTS methods are generally acknowledged as the correct way of measuring the tensile capacity of rocks since test specimens are usually subjected to pure uniaxial tensile loads (ISRM, 2021; ASTM D 2936). A more recognized DTS testing method by the International Society for Rock Mechanics (ISRM) and the American Society for Testing and Materials (ASTM) as the standard procedure to directly determine the tensile strength of brittle materials is the Glued Ends Direct Tensile Strength (GEDTS) test where an adhesive is used between the specimen end surfaces and the pulling heads. However, challenges involving poor performance of adhesive material during hard rock tests as well as eccentric loading conditions due to specimen-load axis misalignment render this method unappealing to researchers. Another DTS method is the dog bone test where a clamping mechanism is used to grip the ends of the specimen. This method also suffers challenges such as expensive and time-consuming specimen preparations as well as the introduction of stress concentrations into the test procedure due to the clamping mechanism at the specimen ends (Zhang et al, 2021; Swaddiwudhipong et al, 2003).

The ITS methods are widely used for their easy sample preparation, repeatability, and simple test procedure. These methods generally aim to generate tensile stress in the sample through far-field compression, a much simpler, cheaper, and more common method to determine rock tensile strength due to the high availability of Unconfined Compressive Strength (UCS) devices in present laboratories (Demirdag et al., 2019). Researchers have proposed several ITS methods to determine the tensile strength of brittle materials. The Brazilian Tensile Test (BTS) has been the most popular ITS method since its introduction by Carneiro in 1943 (Carneiro, 1943), even though test results are mostly influenced by internal and external factors. It has been recognized by ISRM and the ASTM as the standard procedure to measure the tensile strength of brittle materials indirectly (ISRM, 1978; ASTM, 2008b). Some researchers have also introduced other ITS methods to determine the tensile strength of brittle materials in the past. Hobbs (1965) introduced the ring tensile testing method; the double core testing (i.e., Luong test) was suggested by Luong (1990); Hoek and Brown (1980) suggested the confined tensile testing method; etc. Yet most of these methods are not accepted worldwide since they are either impractical or do not provide accurate and consistent results compared to the DTS methods (Unlu and Yilmaz, 2014). For example, Wijk (1978) found that during a BTS test, tensile strength values are considerably higher than the true tensile strength values due to the large compressive stress(es) acting perpendicular to the tensile stress (biaxial stress state) at the disc center. Aliha (2014) also reported that the flexural bend test produced higher tensile values than the BTS test. In 2021, Aliha et al. again reported that the lack of appropriate shape factor considerations in tensile strength formulations for indirect ring tests significantly influences the tensile strength values of rock specimens.

For this reason, several researchers have questioned the validity of the ITS methods for measuring the tensile strength of brittle materials such as rocks based on their analytical, numerical, and experimental findings. Factors including rock anisotropy and inhomogeneity, test sample geometry, loading types, and lack of accurate tensile strength formulations have been found to influence the tensile strength values of rock specimens subjected to ITS tests (Briševac et al., 2015; Chen et al. 1998; Aliha et al., 2021). It is therefore evident that developing new and suitable alternative methods for estimating the tensile strength of brittle materials remains an important research challenge.

1.2 Scope and Objectives

This thesis is focused on highlighting and providing effective solutions to the two main limitations associated with the Glued Ends Direct Tensile Strength (GEDTS) test suggested by ISRM and ASTM for direct tensile strength determination of brittle materials such as rocks and concrete. Firstly, a simple yet effective testing system is developed to eliminate eccentricity from the loading process of GEDTS tests. Secondly, a novel testing method using expansive cement is proposed for testing hard rocks whose tensile capacities are higher than the bonding capacity of the glue.

In summary, this thesis concerns itself with the following objectives:

- Mechanical determination of the direct tensile strength of rocks following the internationally recognized (ISRM & ASTM) guidelines using a new and improved testing system.
- Determination of the tensile strength of the same rock types with a novel testing method using expansive cement and a well-designed instrumentation setup.

- Ascertain the veracity of the results from the novel testing method using Extended Finite Element Method (XFEM) analysis and experimental observations on typical rock materials.
- Comparison and establishment of a relation between the results of the mechanical test and the newly proposed method.

1.3 Thesis Structure

The thesis has five chapters. Chapter 1 provides an overview of the research together with the scope and objectives. Chapter 2 presents a state-of-the-art literature review of the existing tensile strength testing methods. This chapter primarily focuses on highlighting and discussing the limitations of the testing methods recognized by ISRM and ASTM namely the BTS test and the GEDTS test. Other testing methods proposed by some researchers are also discussed in this chapter for completeness. Chapter 3 presents a detailed description and the modus operandi of the novel testing system for GEDTS tests known as the "Load-centering device". Chapter 4 presents the novel testing method known as the Expansive Cement Direct Tensile Strength (ECDTS) test. This chapter covers a detailed description of the ECDTS test in terms of test principle, sample configuration, instrumentation setup, and test methodology. The chapter also reports the results of method validation through numerical modeling and experimental programs on low-tensile strength marble. Chapter 5 presents the application of the ECDTS testing method to hard rocks, namely Gneiss and Sudbury Breccia, whose tensile strengths are too high to be determined by the conventional GEDTS test. Chapter 6, the final chapter, concludes the study and provides recommendations for future studies on determining the tensile strength of rocks and concrete using the GEDTS test.

Chapter 2 Literature Review

2.1 Introduction

This literature aims to discuss the internationally recognized Glued Ends Direct Tensile Strength (GEDTS) test and the Brazilian Tensile Strength (BTS) test. Other variations, including the indirect compression ring test, the dog bone test, and the direct “seesaw” test, are discussed. Additionally, the limitations of these testing methods are highlighted and discussed.

Many researchers resort to indirect testing methods because of the difficulties associated with direct testing procedures. As mentioned earlier, the indirect methods aim to generate tensile stress in the sample through far-field compression. This choice is primarily due to the high availability of UCS devices in present laboratories (Perras and Diederichs, 2014). Several indirect testing methods have been proposed to determine the tensile strength of brittle materials such as rocks, including the BTS test, ring test, and the three- or four-point flexural test (Demirdag et al., 2019).

2.2. The Brazilian Tensile Strength (BTS) Test

The BTS test is an indirect testing method introduced by Caneiro (Caneiro, 1943). It has been the most popular test employed by many researchers as a simple and efficient way to measure the tensile strength of brittle materials (Aliha et al., 2021). The International Society for Rock Mechanics (ISRM) has recognized this method as the standard procedure to indirectly measure the tensile strength of brittle materials such as rocks (ISRM, 1978). This method involves diametrically compressing a thin circular disc to failure (Li and Wong, 2013). Fig. 2.1 shows Four typical loading configurations associated with indirect tensile strength determination using the BTS test.

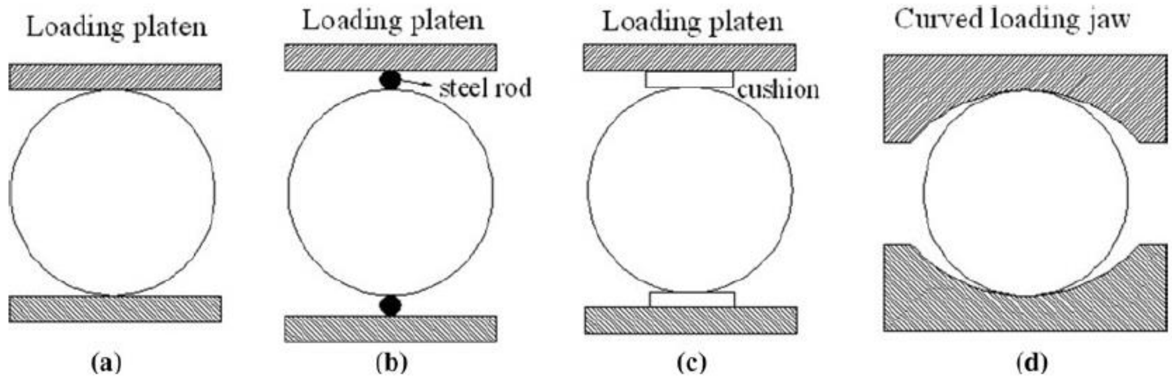


Figure 2.1 Brazilian disc loading configurations. (a) Flat loading platens (b) Flat loading platens with two small-diameter steel rods (c) Flat loading platens with cushion (d) Curved loading jaws. (Li and Wong, 2013).

It has been shown by Tsutsumi et al. (2016) that the Brazilian test is performed by putting a disk-shaped sample between two rigid platens with a thickness double its diameter. The sample is placed diametrically between two steel platens which transfer the compressive load through the sample. The applied load induces a biaxial state of stress at the center of the circular plane of the sample in which the stress in the loading direction is compressive and tensile in the lateral (horizontal) direction. Since the tensile strength of the sample is far less than its compressive strength, the sample fails in a tensile manner along the loading direction, usually in the form of a vertical diametrical fracture, as demonstrated in Fig. 2. Further, the results of a stress distribution numerical modeling study is shown in Figure 2.3. The ISRM (2007) suggested formula for calculating splitting tensile strength, which assumes that until the onset of failure, the brittle rock is isotropic, homogeneous, and linearly elastic medium is given by:

$$\text{Tensile strength, } \sigma_t^{BTS} = \frac{2P}{\pi t d} \qquad (2.1)$$

Where *P* is the ultimate load, *t* and *d* are the thickness and diameter of the disc specimen, respectively.

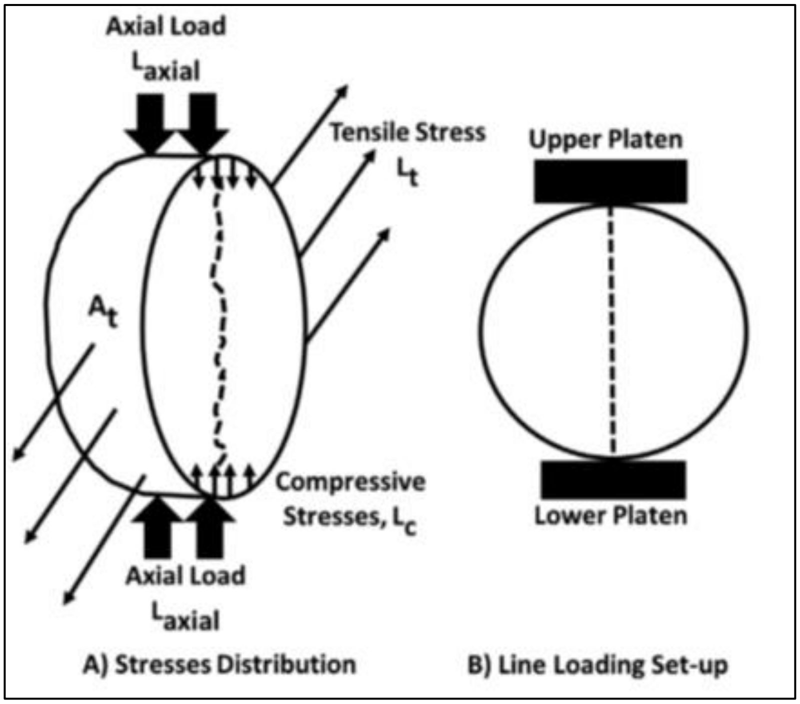


Figure 2.2 Schematic diagram of the Brazilian Tensile test (AlAwad, 2020)

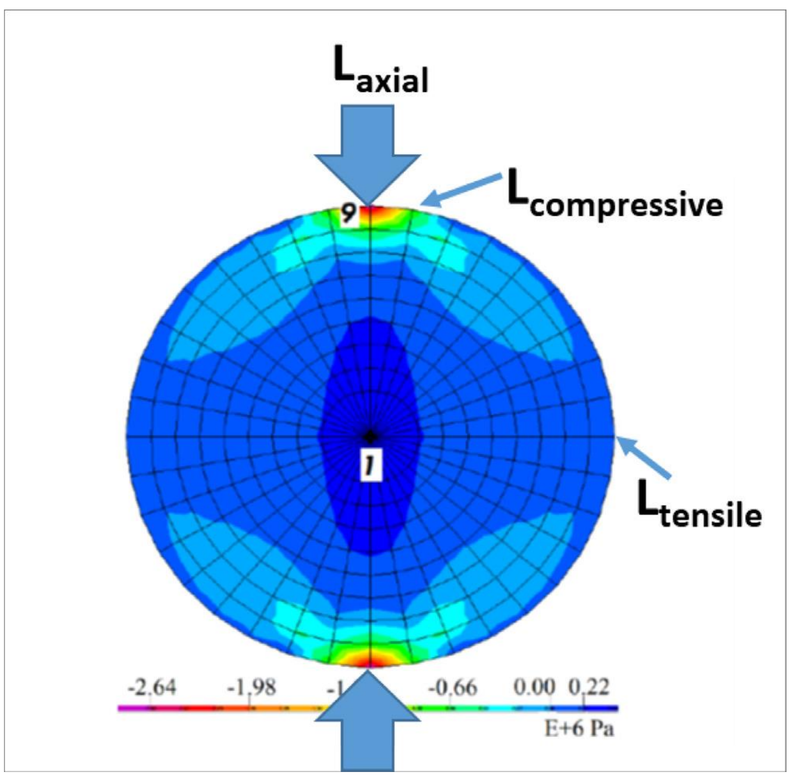


Figure 2.3 Stress distribution in the Brazilian disc during a BTS test (Rocha and Wahrhaftig, 2016).

Despite the popularity of the BTS test due to its simplicity and absence of the shortcomings encountered in the direct testing methods, the background of this method suggests the use of several assumptions that have triggered debates by several researchers over the years after its inception (Li and Wong, 2013).

According to Li and Wong (2013), It is assumed that the thin disc specimen is loaded with uniform pressure by radially applying pressure over a short strip of the circumference at both ends of the diameter. Frictional stresses between the loading platens and the specimen are assumed to have no effect. However, after conducting a test to determine the influence of friction on the stress field in a Brazilian disc, Markides et al. (2011) reported that although the effect of friction is negligible in the vicinity around the center of the disc, the stress field immediately near the loading platens is altered significantly by frictional stresses. The stress components due to friction become more pronounced as one approaches the loaded rim, even higher than stresses due to radial pressure. This could impact the tensile stress value.

Another critical assumption employed by the Brazilian test to determine the tensile strength of rocks and rock-like materials is that the material is homogeneous, isotropic, and linearly elastic before the brittle material fails (Mellor and Hawkes, 1971). It is also assumed that the diametrical compression of the disc generates tensile stresses normal to the diameter, which are relatively constant over a section about the center (Fairhurst, 1964). On the contrary, Yue et al. (2003) reported that numerical results from their finite element modeling of geomaterials using digital image processing demonstrated that the inhomogeneity of rock materials significantly affects tensile stress distribution along the vertical loading diameter during the Brazilian test. As such, the stress distribution is not constant at the center of the disc.

Also, it is assumed that the Brazilian disc specimen should split along the compressive diametral line; otherwise, it is considered invalid. In other words, rock failure should occur from the central part of the Brazilian disc, where the maximum tensile stress is generated (Colback, 1966). This was argued by Yu et al. (2005) who reported that initiation of tensile crack might not occur from the center due to the inhomogeneous nature of rocks. Using the finite element method, Yue et al. (2003) also reported that the

Brazilian test could not be used to measure the tensile strength of rock-like materials because the greatest equivalent stress was not located at the center of the disc, as the method assumes, but at the loading point.

Furthermore, in calculating the tensile strength, the assumption is that failure occurs at the point of maximum tensile stress (i.e., at the center of the disc) as such radial compressive stress has no effect (Fairhurst, 1964). However, when materials with low compression-tension ratios are loaded at small angles of the loading contact area, Fairhurst (1964) observed that failure might occur away from the center of the Brazilian disc and reported that the tensile strength calculated from the test results was usually lower than the actual value.

One of the main drawbacks of the BTS testing method is that the stress state existing at the center of the disc is not purely tensile (Hondros, 1959). Due to rock materials' anisotropic and inhomogeneous nature, the compressive stress at the center of the disc under diametrical loading is three times greater than the tensile stress. As such, the tensile values from BTS tests may be considerably overestimated. Also, due to the possibility of shear failure along the laminations in anisotropic materials, the Brazilian test cannot give a true reflection of the tensile strength of rocks when the disc inclination angle is in the range of 60^0 to 90^0 (Chen et al. 1998). Briševac et al. (2015) also argued that the inhomogeneity of the material significantly impacts the distribution of tensile stress along the loading axis. This phenomenon may substantially influence the tensile values reported. Furthermore, the approximation of the tensile strength of rock by an equation based on isotropic elasticity theory renders the method inaccurate since stress distribution in both isotropic and anisotropic discs can be expected to differ (Chen et al. 1998).

2.3 The Indirect Compression Ring Test

The indirect compression ring test, which uses a circular disc with a small hole in the center, is another common indirect method proposed by Hobbs (1965) to determine the tensile strength of brittle materials such as rocks. This method was primarily developed to solve the issue of the biaxial stress state existing at the center of the disc specimen during the BTS test (Hudson, 1969).

The principle of tensile strength determination based on the compression ring test has been shown by Zhang et al. (2018). In their work, they demonstrated that a ring specimen configuration of inner diameter d_1 and outer diameter d_2 is loaded in the y-axis direction by a compressive force, P, as shown in Fig. 2.4. Upon applying the compressive force to the ring specimen, tensile stress appears in both the upper and lower zones of the interior ring.

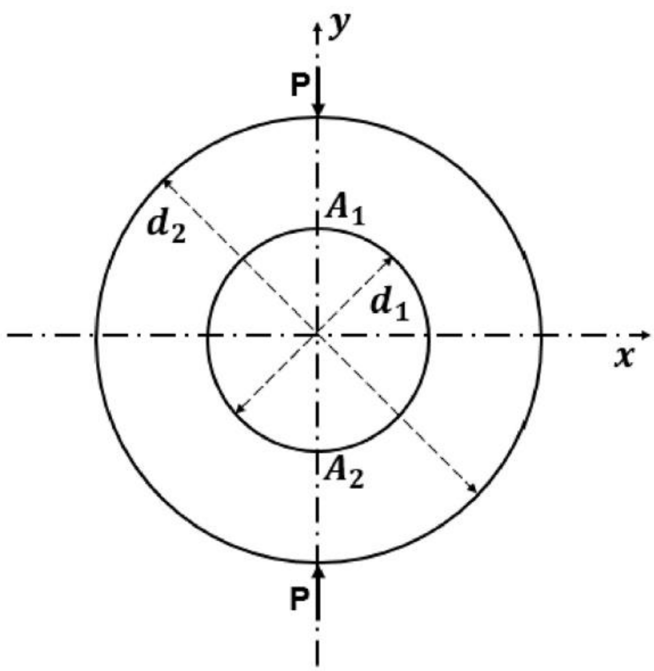


Figure 2.4 Schematic of ring specimen under diametrical compression (Zhang et al., 2018).

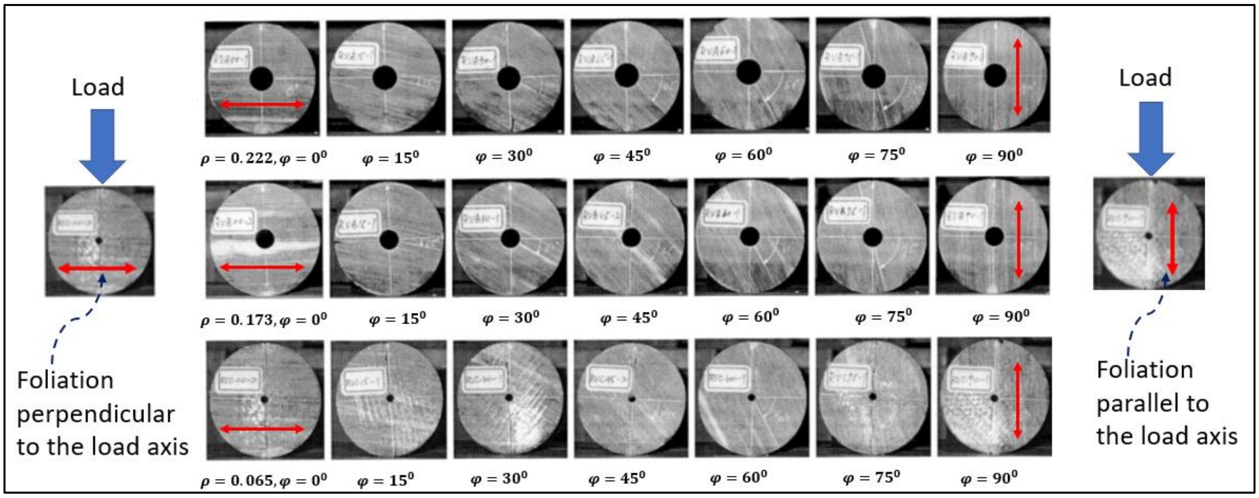
The specimen cracks at the upper and lower apexes of the inner ring as the load increases. Using an analytical or numerical model, the tensile stress at the top and bottom of the inner ring under compression can be calculated. The calculated tensile stress under maximum load during the test is regarded as the tensile strength of the brittle material.

The formula proposed by Hobbs (1965) to determine the tensile strength of rocks using the IR test is given by Equation 2.2.

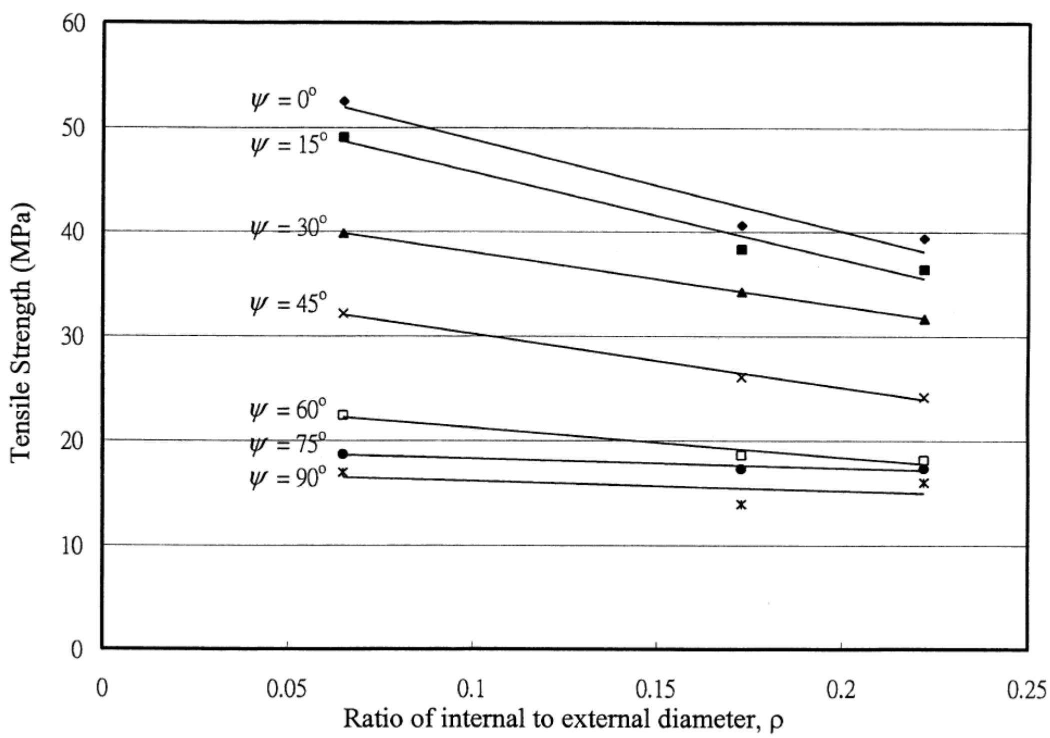
$$\sigma_t = 12\frac{P}{\pi d_o t} \qquad (2.2)$$

Where P is the maximum applied load, d_o and t are the outer diameter and thickness of the disc respectively.

Despite the advantages of the compression ring test including simplicity of test procedure, simple sample geometry and preparation, elimination of biaxial stress state, and application of relatively minimum compressive load to failure, it has been under considerable scrutiny since its introduction by Hobbs (1965) to determine the tensile strength of brittle materials (Li et al., 2016; Abdullah and Tsutsumi, 2018; Chen and Hsu, 2001). For example, in the experimental studies by Li et al. (2016) to investigate both static and dynamic failure mechanisms in the splitting test of ring specimens, it was discovered that the ring diameter ratio significantly influenced the tensile strength of rock ring specimens. Chen and Hsu (2001) also used a combination of laboratory testing and boundary element analysis to measure the indirect tensile strength of anisotropic rocks by the ring test. Results indicated that the tensile strength determined from the ring specimen is not constant but varies according to, among other things, the angle between the plane of foliation and the horizontal as well as the size of the hole in the disc specimen. Variations of test results for rings of different values of diameter ratio, ρ, and foliation angle, ψ are presented in Fig. 2.5(b).



(a) Ring specimens with different values of ρ and ψ.



(b) Variation of tensile strength of Hualien marble for different values of ρ and ψ

Figure 2.5 Indirect compression ring test results (Chen and Hsu, 2001)

As illustrated in Fig. 2.5b, the tensile strength of the rock specimen is influenced significantly by both the ring disc diameter ratio and the angle of foliation. Low tensile strength values are produced when the angle of foliation is parallel to the loading axis.

However, when the angle of foliation is perpendicular to the loading axis, the test produces high tensile strength values for the same rock type.

Aliha et al. (2021) have also argued that inaccurate formulations or the lack of consideration of relevant shape factors in tensile strength formulations could significantly alter the tensile strength values determined from the IR test. According to Aliha et al. (2021), the shape factor considers the influence of the ring diameter ratio and the loading type on the test result. Examples of tensile values produced from different formulations are shown in Table 2.1.

Table 2.1. Estimated tensile strength with different formulations for the same rock type using indirect compression ring test (Aliha et al., 2021).

Rock type	Tensile Strength (σ_t) Formula	σ_t (MPa)	Reference
Tuffite	$\sigma_t = 12\frac{P}{\pi d_o t}$	2.1	Hobbs (1965)
Tuffite	$\sigma_t = 12\frac{P}{\pi d_o t}\left(6 + 38\frac{d_i^2}{d_o^2}\right)$	6.6	Li et al. (2017)
Tuffite	$\sigma_t = \frac{2P_f}{\pi t(d_o - d_i)}\left[35.05\left(\frac{d_o}{d_i}\right) - 9.22\right]$	9.5	Aliha et al. (2021)

As shown in Table 2.1, the tensile strength of the same rock type is significantly influenced when determined with different tensile strength formulations.

As a result of the sample shape and loading configuration being similar to the Brazilian test, the compression ring test suffers identical problems to the Brazilian test except that the biaxial stress state which would have been induced at the center of the disk has been eliminated by the presence of the hole (Komurlu et al., 2017). Also, other critical factors such as inconsistencies in tensile strength formulations and varying ring diameter ratios hugely influence the tensile strength values produced from the compression ring tests (Aliha et al., 2021).

In addition to the Brazilian and Ring tests, other indirect testing methods such as the three or four-point flexural strength tests, which had been used for many years before the introduction of the Brazilian test by Carneiro (1943), Luong's test suggested by Luong (1986), and Hoek and Brown's confined tensile strength test method (Hoek and Brown, 1980) have been proposed. However, these indirect testing methods have issues with the validity of test results and challenges resulting from the impracticality of sample preparation and loading mechanisms. For example, the compression ring test introduced by Hobbs (1965) still holds several limitations present in the Brazilian test. Also, the double core testing (Luong’s test) suggested by Luong (1986) faces many limitations in failure validity. Hoek and Brown’s test is also not a valid way to measure the uniaxial tensile strength of rocks due to the triaxial stress distribution produced by the hydraulic confinement pressure Komurlu et al. (2017). The three or four-point bending test has also been found to produce higher tensile strength values than the BTS test (Aliha, 2014). Clearly, the inconsistent results obtained from the indirect testing methods due to the influence of both internal and external factors mentioned above question their validity.

2.4 Direct Tensile Strength (DTS) Test

The DTS test is regarded as the most valid test to determine the true tensile strength of rocks because there are minimal internal and external influences during the test when it is conducted properly (Hoek, 1964; Zhang et al, 2021; Swaddiwudhipong et al, 2003). This method employs the application of a uniaxial tensile force along the axis of the material to be tested (Lu, 2015). The two most popular DTS tests are the Dog bone test and the Glued Ends Direct Tensile Strength (GEDTS) test as shown in Fig. 2.6. Some researchers have developed other direct testing methods, including the “Seesaw” test proposed by Aliha et al. (2021) where compression load converter is used to induce tensile failure in a ring disc, as shown in Fig. 2.7.

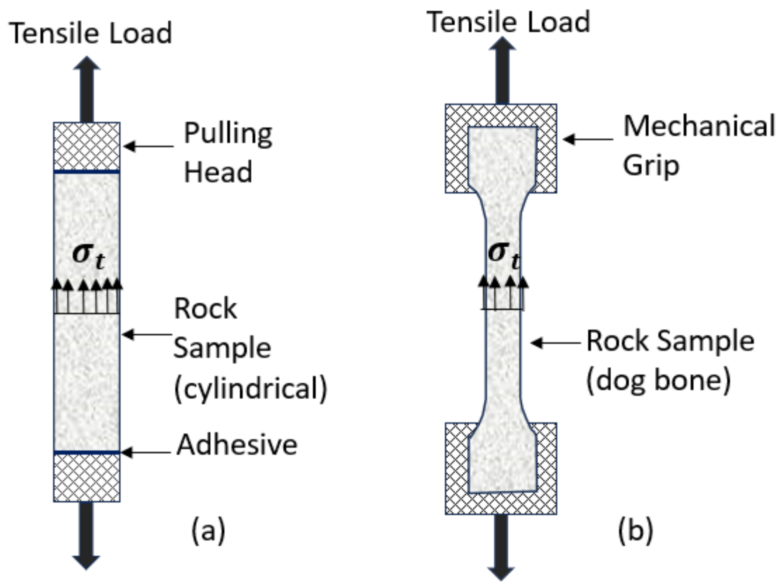


Figure 2.6 DTS test variation. (a) Dog bone test (b) Glued ends direct tensile strength test.

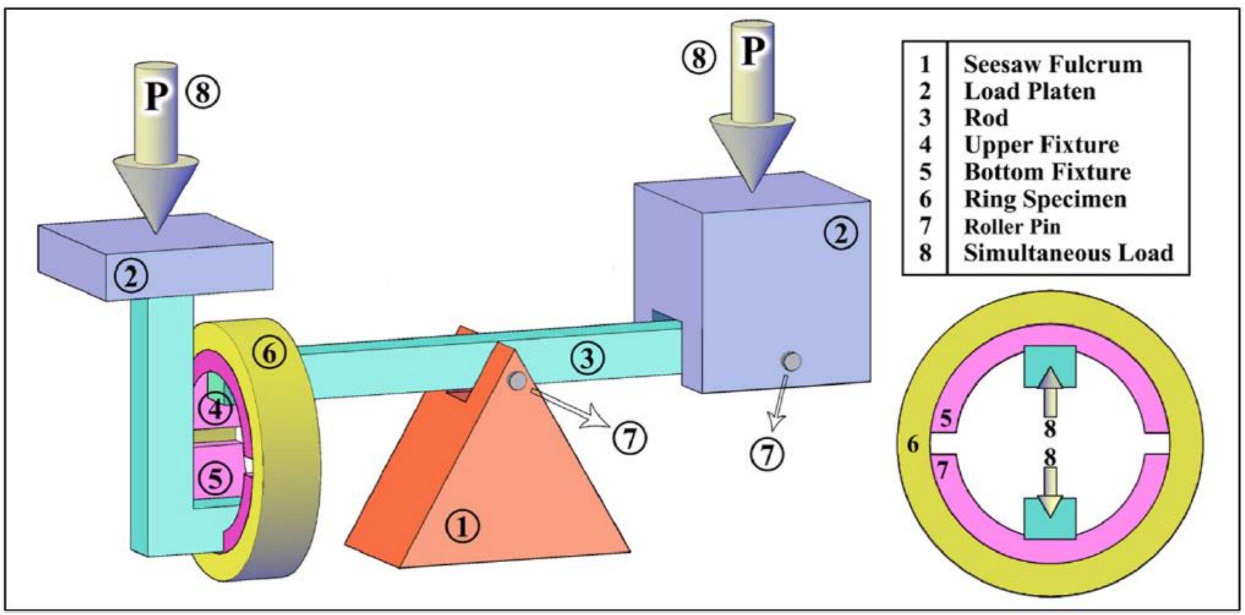


Figure 2.7 Schematic presentation of the seesaw device and its mechanism (Aliha et al., 2021).

2.5 The Seesaw Test

The "seesaw" test device was proposed by Aliha et al. (2021) to directly determine the tensile strength of rock using a ring-shaped specimen. The authors employed FEM analysis to formulate a shape factor to consider the effects of geometry and loading type on the stress state of the tested specimen. The method uses a seesaw mechanism to apply tensile forces on the inner surface of the ring specimen. The seesaw mechanism converts the compressive load to tensile based on the function of a seesaw, as illustrated in Fig. 2.7. During the experiment, Aliha et al. (2021) demonstrated that a servo-hydraulic tension-compression test machine was used for loading at a constant rate of 0.5 mm/min. By recording the critical splitting load and corresponding shape factor value for any given (d_1/d_o) ratio, the tensile strength of the specimen was determined from Equation 2.3.

$$\sigma_t^{ST} = \frac{2P_f}{\pi t(d_o - d_i)}\left[19.35\left(\frac{d_i}{d_o}\right) - 2.115\right] \qquad (2.3)$$

Where P_f is the critical applied load at the onset of fracture, d_i and d_o are the internal and external diameters of the ring specimen, respectively. Aliha et al. (2021) investigated the validity of the "Seesaw" test by comparing the test results to Brazilian test results and test results from other researchers. They concluded that the proposed "Seesaw" test is valid since the test result agrees with that of the Brazilian test, as presented in Fig. 2.8. However, the problem with this validation is that the BTS test is well-known to overestimate the tensile strength due to the internal and external factors mentioned earlier. As such, the results of a BTS test (8.2 MPa) may not provide a good basis for validating the results of a DTS test (8.3 MPa) when they both produce such tensile strength values. In fact, the average tensile strength value of a series of DTS tests should be less than that of a series of BTS tests because as a rule of thumb, DTS $\approx$ 0.67 BTS (Perras and Diederichs, 2014). For this reason, the seesaw test is considered more like an indirect rather than a direct testing method.

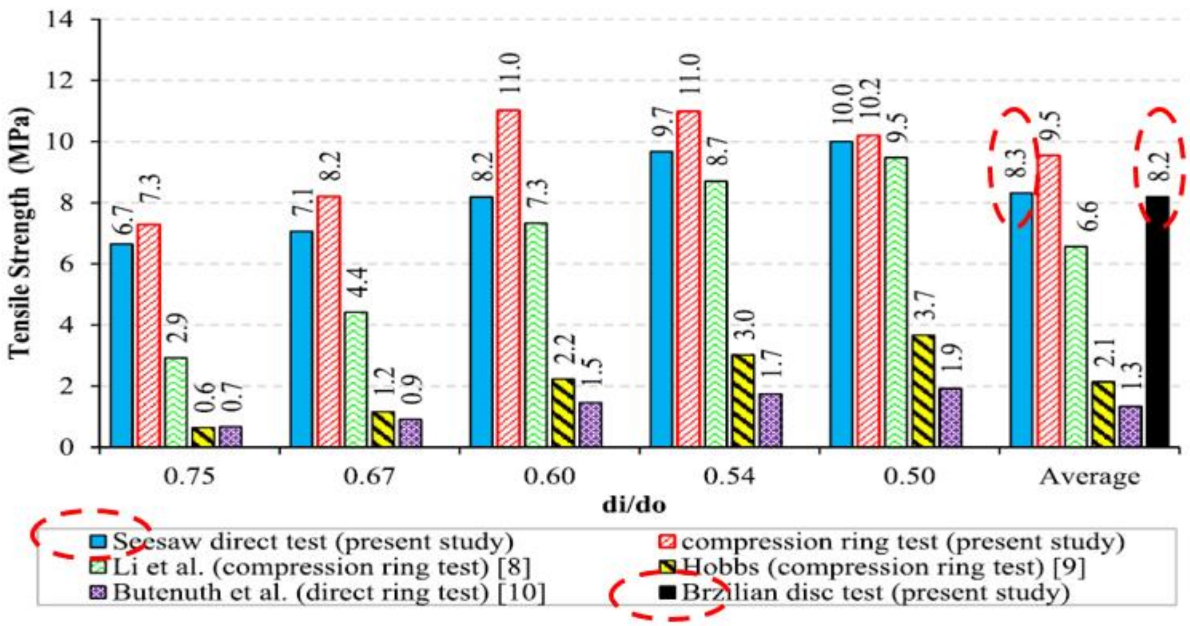


Figure 2.8 Tensile strength from Brazilian Disc, Indirect compression ring, and Seesaw tests (Aliha et al., 2021).

2.6 The Dog-bone Test

For direct tensile strength test specimens, Brace (1964) prescribed the dog bone, also known as 'dumbbell', as the best shape where the height-to-diameter ratio should be 2.0–3.0 of the central test region as shown in Fig 2.6b. The radius of fillet curvature should be approximately 1 – 2 times the diameter. According to Brace (1964), the test procedure involves mechanically gripping the lip and applying uniaxial tensile load at both ends of the specimen to cause failure at the midsection area. The author argued that the curved radius of the specimen reduces stress concentrations caused by the mechanical grip at the ends of the specimen. Perras and Diederichs (2014) have reported that the dog bone can still create stress concentrations at the ends of the specimen during loading, causing invalid failures away from the central area of the specimen.

Even though the dumbbell-shaped specimen is subjected to pure uniaxial tensile loading, Chen and Hsu (2001) have argued that difficulties with sample preparation and test procedure make its application unattractive. According to Zhao et al. (2021), there are three main drawbacks associated with the use of dumbbell specimens for testing the tensile strength of rocks. Firstly, specimen preparation is highly time-consuming due to its small dimensions and unique shape, which is required to reduce stress concentrations during loading. Also, specimen preparation is not always feasible for soft rocks. Secondly, it is necessary to have a precise machine. A conventional hydraulic servo-drive machine will be in a nonlinear section when a small specimen fails with a load of less than 200 N. Generally, the failure displacement of small specimens is around 100 μm, which requires a device that can measure displacement with greater accuracy. Lastly, the complex shape of the specimen presents complex strain distributions, which require more sophisticated or more professional strain analysis. For these reasons, Unlu (2014) argued that the dumbbell test is less practical for frequent laboratory use.

2.7 The Glued Ends Direct Tensile Strength (GEDTS) Test

Aside from the introduction of stress concentrations in the test procedure, the gripping of cylindrical rock specimens usually leads to local fractures at the gripped ends of the rock specimen that ultimately fail the direct tensile strength tests. To avoid this problem, the ends of cylindrical specimens are glued to loading plates or pulling heads. The tensile force is then applied to these pulling heads, as illustrated in Fig. 2.6a (Aliha et al., 2021). The International Society for Rock Mechanics (ISRM) has recognized this method as a standard procedure to directly measure the tensile strength of brittle materials such as rocks.

According to ISRM (2021), the ends of the right circular cylindrical specimen shall be generally smooth and flat during sample preparation. The sides of the specimen shall be smooth and free of abrupt irregularities and straight to within 0.1 mm over the entire length of the specimen. The height-to-diameter ratio shall be 2.5 - 3.0: 1.0. An NQ core size of 47 mm or NX core size of 54 mm could be used.

Figure 2.9 The GEDTS Test Specimen. (Zhao et al., 2021).

To conduct the GEDTS test, cylindrical pulling heads are glued to the specimen ends. It is important to ensure that the diameter of the pulling head is not less than that of the test specimen, nor should it exceed the test specimen diameter by more than 2 mm. The thickness of the glue layer should be at most 1.5 mm at each end (ISRM 2021). The pulling heads are bonded to the specimen in such a manner as to ensure alignment of the pulling head axis with the longitudinal axis of the specimen to avoid bonding eccentricity. After curing of the glue is achieved, usually after 24 – 48 hours, the bonded specimen is placed in a suitable tensile load machine. This is achieved by using a suitable linkage system known as load transmitter, to transfer the tensile load to the specimen without inducing any bending or torsional stresses into the test procedure. According to (ISRM, 2021), the length of the linkages at each end should be at least twice the diameter of the pulling head. The tensile load is then applied continuously at a constant rate such that failure will occur within 5 minutes of loading. The tensile strength of the specimen can be determined by Equation 2.4.

$$Tensile\ Strength, \sigma_t = \frac{4P}{\pi D^2} \qquad (2.4)$$

Where P and D are the maximum tensile force applied before the specimen fails and the diameter of the specimen, respectively. The use of adhesive material (glue) as a connection medium between the specimen and the applied load creates a more uniform stress distribution within the specimen than the use of a mechanical grip (Liao et al., 1997). However, the GEDTS has rarely been employed in rock mechanics laboratories because while the glue does not perform well when testing hard rocks, loading and bonding eccentricity could introduce bending and torsional stresses in the test sample when the test is not conducted correctly (Komurlu et al., 2017; Aliha et al., 2021; Liao et al., 1997). Wijk (1978) defined eccentricity as “a phenomenon that occurs in the loading process resulting in considerable bending stresses compared to the constant axial stress that the test is supposed to create.” According to Zhang et al. (2021), since bonding of the pulling head to the specimen end surface is done manually, bonding eccentricity is likely to occur when the pulling head and axial load are not coaxially aligned with the axis of the specimen as illustrated in Fig. 2.10(b). On the other hand, loading eccentricity is also likely to occur and induce bending or torsion (depending on the type of load transmitter) in the specimen when the loading direction is different from the axis of the specimen, as illustrated in Fig. 2.10(c). According to Liao et al. (1997), two types of load transmitters are used to transfer the tensile load from the testing machine to the specimen. These are the flexible type, such as the chain link (ASTM D 2936), and the stiff type, such as the steel rod with ball joints (Zhang et al., 2021). These are illustrated in Fig. 2.11. The use of the flexible type reduces bending in the specimen. Still, a torsional moment may be induced in the specimen, and the unpredictable movement of the flexible chain may endanger users or damage the equipment if the specimen fails. Even though adopting the stiff type may induce bending in the specimen, Liao et al. (1997) suggest the use of this type of load transmitter since bending in the specimen can be overcome by employing a well-designed steel rod with ball joint parts to eliminate loading eccentricity.

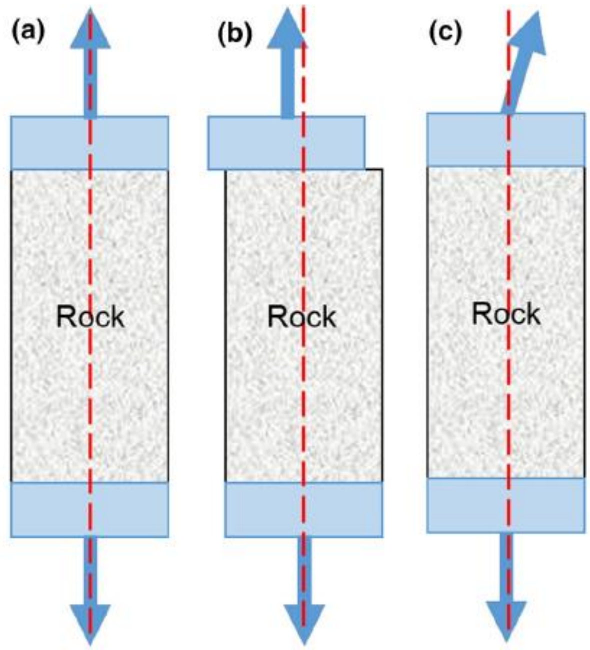


Figure 2.10 Eccentricity (a) Loading cap and tensile load precisely coaxial with the specimen (b) Eccentric bonding (c) Eccentric loading (Zhang et al., 2021).

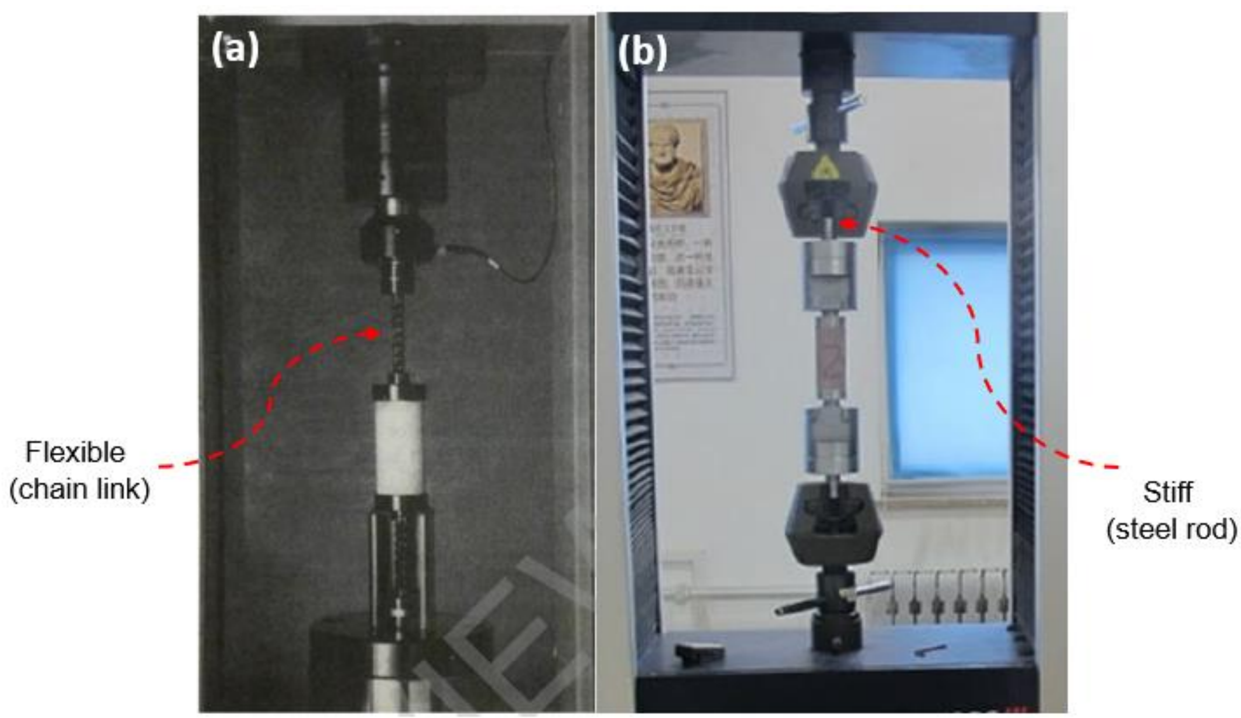


Figure 2.11 Types of load transmitters (a) Flexible (ASTM D 2936) (b) Stiff (Zhang et al., 2021).

Even though the glued ends method is widely accepted and recognized by ISRM and ASTM to directly determine the tensile strength of brittle materials such as rocks, it is rarely used by researchers due to three main reasons:

1. Application of the glued ends testing method to high-strength rocks may be a challenge due to the bonding limitations of the adhesive between the pulling heads and specimen end surfaces (Fuenkajorn et al., 2010). For this reason, many researchers resort to the classical Brazilian test, which has nothing to do with adhesive bonding.
2. In many laboratories, tensile (pull) loading frames are unavailable (Perras and Diederichs, 2014). This has compelled most researchers to resort to compression load converters. Pulling mechanisms are developed around compression loading machines to produce a tensile loading effect for direct tensile strength tests. A typical example is the seesaw test developed by Aliha et al. (2021).
3. Bonding and loading eccentricity could impact test results significantly if the test setup is not done correctly (Zhang et al., 2021).

2.8 Summary

It is clear from the reviewed literature that the inconsistencies in tensile strength results produced by the indirect testing methods due to factors such as sample geometry, loading type, rock foliation, rock inhomogeneity, biaxial stress state, and varying tensile strength formulations, the indirect testing methods cannot provide an accurate measure of the tensile strength of brittle materials such as rocks. In fact, per the definition of tensile strength, test specimens must be subjected to pure uniaxial tensile loads to achieve desirable results. As such, the determination of the tensile strength of brittle materials using direct testing methods is of utmost importance.

However, there are specific challenges associated with the use of the two conventional direct testing methods as discussed previously. The cost of the Dog-bone test is high, specimen preparation is laborious and stress concentrations influence the test results due to the clamping mechanism. On the other hand, the GEDTS test which is internationally accepted and recognized by ISRM and ASTM suffers issues of eccentricity and weak adhesive bonding when testing hard rocks. This study therefore aims to develop a testing system that eliminates eccentricity in the loading process of GEDTS tests and also develop a novel direct testing method that does not require the use of an adhesive when testing hard rocks. By resolving these challenges, it is anticipated that the affinity towards the use of a suitable direct testing method to determine the true tensile strength of brittle materials will increase in the research community.

This study also aims at developing a novel method for direct tensile strength testing of all hard rocks, regardless of the tensile strength values, even if it exceeds that of the glue bond strength that is commonly used with the conventional GEDTS test.

Chapter 3 Eliminating Eccentricity from the GEDTS Test Using the Newly Developed Load Centering Device

3.1 Introduction

In this chapter, the description and modus operandi of a newly developed testing system known as the Load Centering Device (LCD) is presented. As discussed previously in the literature review, the GEDTS test is prone to eccentric loading when the pulling heads and the rock specimen are not coaxially aligned. This phenomenon introduces bending or torsional stresses in the rock specimen which could influence the tensile strength results.

3.2 LCD Description

As shown in Fig. 3.1, the Load Centering Device is mainly composed of a hoist ring, a Clevis rod, and a threaded stud. The 360^0 swivel movement of the hoist ring eliminates torsion while the 180^0 pivot movement of both the hoist ring and clevis rod eliminates bending from the loading process. The threaded stud, which serves as a stiff load transmitter, connects the Clevis rod to the tensile loading machine while the threaded end of the hoist ring connects the Load centering device assembly to the pulling heads of the bonded specimen.

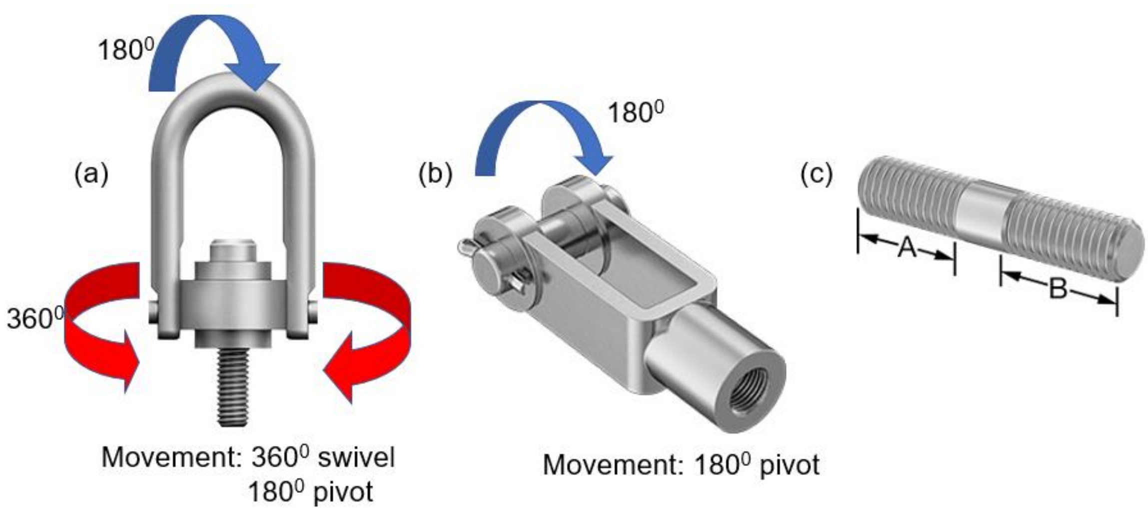


Figure 3.1 Components of the LCD. (a) Hoist ring. (b) Clevis rod. (c) Threaded stud.

Fig. 3.2 shows the LCD assembly, pulling heads, and the bonded specimen. It is worthy to note, that the pulling heads were designed and manufactured in-house. Its dimensions were numerically determined using ABAQUS FEM analysis to ensure that stress distribution and transfer to the rock specimen is uniform at the contact surface between the pulling head and the specimen.

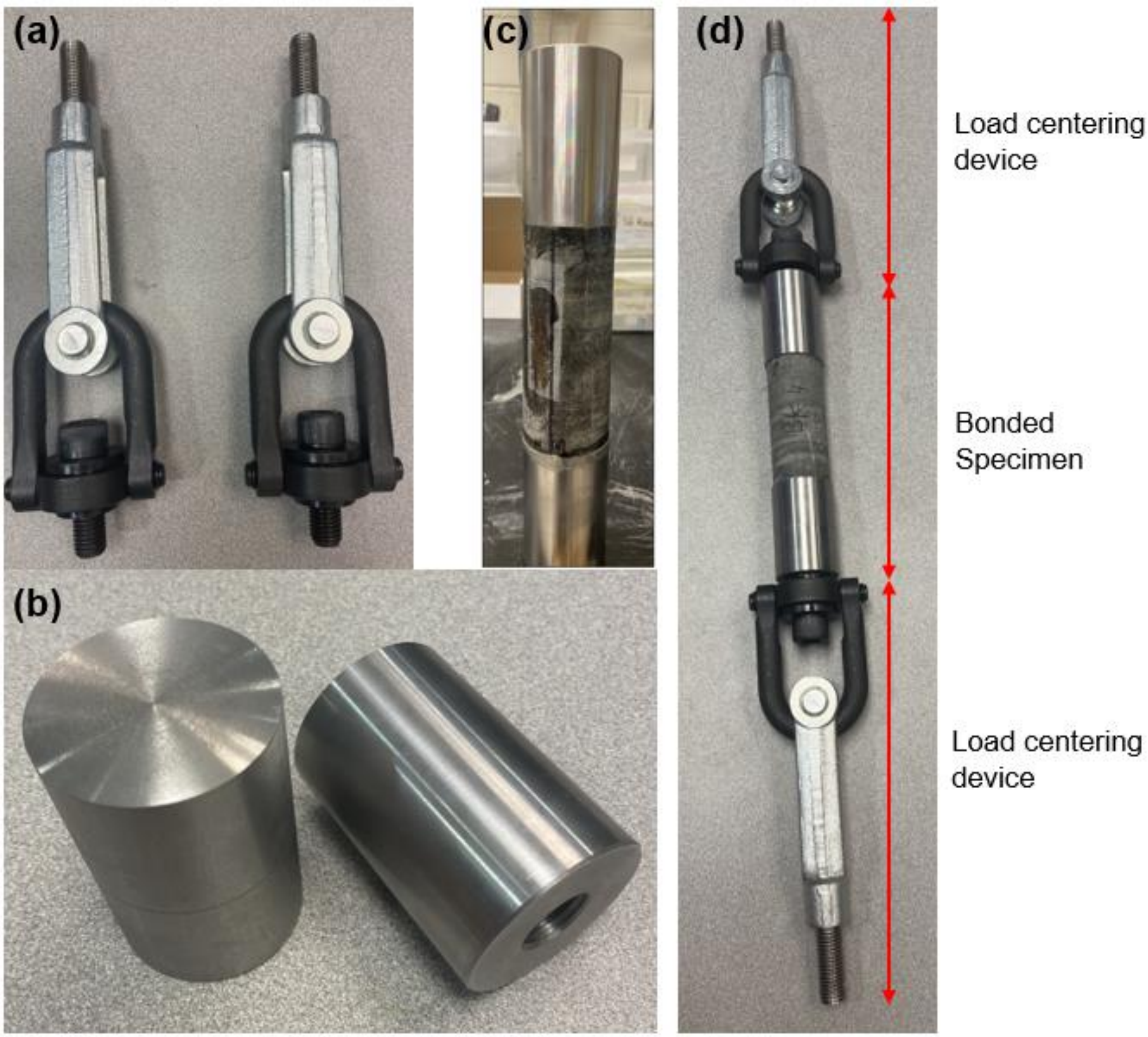


Figure. 3.2 Connection of the LCD for GEDTS test. (a) LCD assembly. (b) Pulling heads. (c) Bonded specimen. (d) LCD assembly connected to the bonded specimen.

3.3 GEDTS Test Using the LCD

To investigate the effectiveness of the LCD, three GEDTS tests were conducted on Basaltic Komatiite as shown in Fig. 3.3(a). The specimen preparation and test procedure were performed in accordance with ASTM (2008a) guidelines. Fig. 3.3(b) shows the failure mode of the specimen after the GEDTS test.

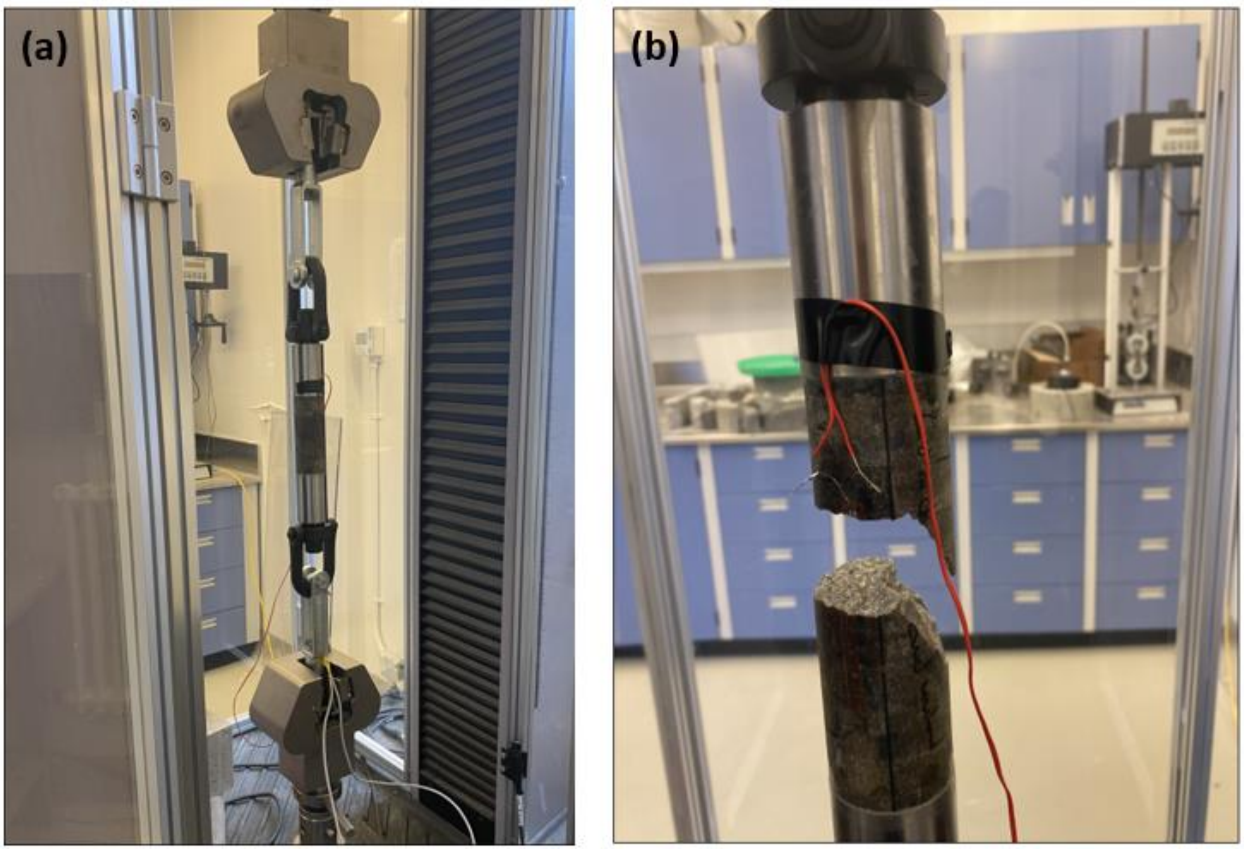


Figure 3.3 The GEDTS test of Basaltic Komatiite. (a) The GEDTS test setup. (b) Failure mode of the specimen.

3.4 Results and Discussions

The failure characteristics of the specimens are shown in Fig. 3.4(a). It can be observed that the tensile fracture planes are oriented diagonally to the axis of the applied tensile load. The morphology of the fracture plane is fresh and uneven with small pits as seen in Fig. 3.4(c). Also, it is evident that there is no trace of friction or accumulation of rock fragments caused by shear failure or torsional failure. Although the fracture planes are predominantly diagonal, Test 2 in Fig. 3.4(a) shows that for different rock specimens, certain randomness exists in the location and orientation of the fracture plane. This is largely due to the inherent heterogeneity of rock materials. Also, the diagonal orientations of the fracture planes are clearly due to the natural orientation of the foliation or weak planes associated with the rock specimen as shown in Fig. 3.4(b). From the failure characteristics, it can therefore be inferred, that with the aid of the load-centering device, the rock was subjected to pure uniaxial tensile load and the results can correctly represent the true tensile strength of the Basaltic Komatiite rock.

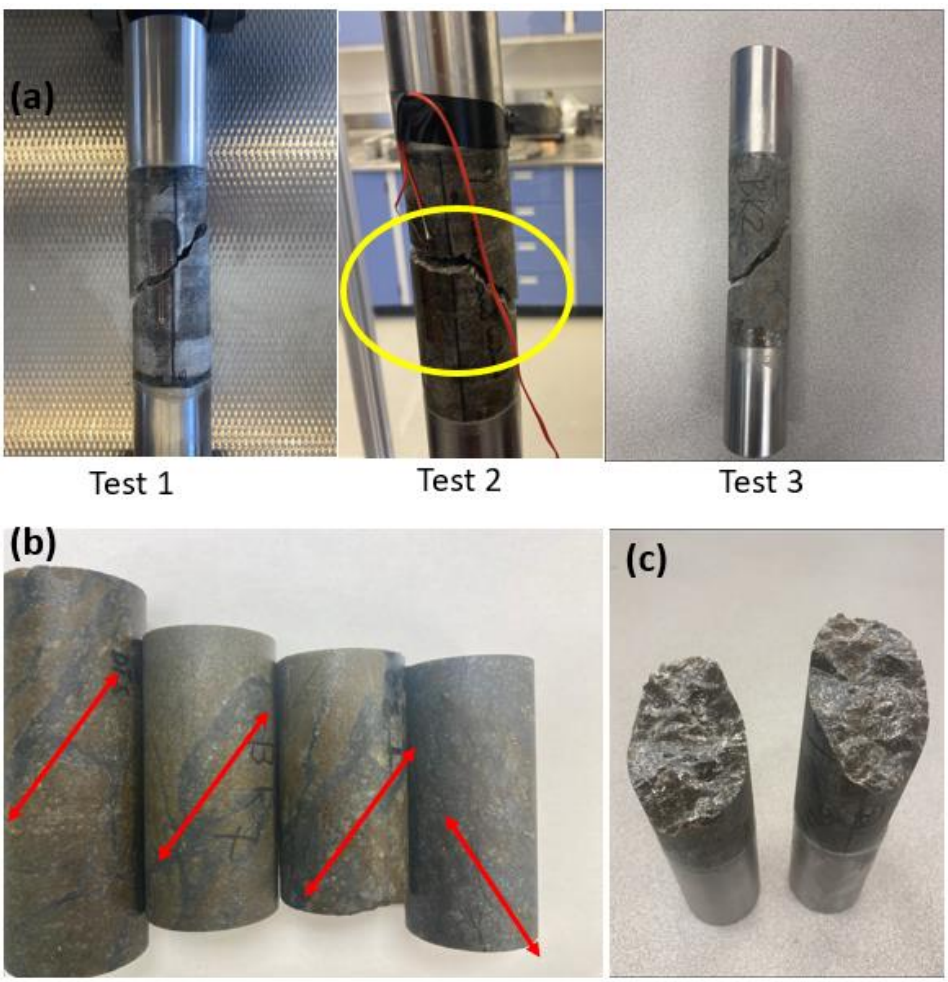


Figure. 3.4 Failure Characteristics of Basaltic Komatiite. (a) Specimen fracture patterns. (b) Orientation of weak planes. (c) Morphology of fracture surface.

The peak tensile load and their corresponding tensile strength values obtained from the GEDTS test for the Basaltic komatiite are presented in Table 3.1. It can be seen that there is some variation in the tensile strength values obtained for the three specimens with a coefficient of variation of 22.87% indicating the inherent heterogeneity of Basaltic komatiite as reported in literature (Malki, 2022).

Table 3.1 Results of GEDTS tests on Basaltic Komatiite.

Test #	Peak Load (kN)	Tensile Strength (MPa)
1	19.23	11
2	14.0	8.1
3	10.88	6.3
	Average	**8.46**
	Standard Deviation (SD)	**1.9362**
	Coefficient of Variation (COV)	**22.87%**

A series of Brazilian tests was also conducted to indirectly determine the tensile strength of the basaltic komatiite rock samples which were prepared in compliance with the ASTM D3967 Standard. The results give a mean value of 10.34 MPa as shown in Table 3.2. This confirms the overestimation of the tensile strength of rock materials using the BTS testing method. Additionally, similar to the GEDTS test, the heterogeneous nature of the Basaltic komatiite is evident in the 16.24% coefficient of variation obtained for the BTS tests.

Table 3.2 Results of BTS tests on Basaltic komatiite.

Test #	BTS (MPa)
1	9.77
2	13.16
3	8.87
4	8.93
5	10.99
6	9.11

Test #	BTS (MPa)
7	10.8
8	13.01
9	8.42
Average	10.34
Standard Deviation (SD)	1.6796
Coefficient of Variation (COV)	16.24%

Bridging text between manuscripts

Following the development of the Load Centering Device (LCD) to eliminate eccentricity from the loading process of the Glued Ends Direct Tensile Strength (GEDTS) test in the previous chapter, the next goal of this study is to develop a novel direct testing method known as the Expansive Cement Direct Tensile Strength (ECDTS) Test to determine the tensile strength of hard rocks whose tensile capacities are higher than the bonding capacity of adhesives. This novel method is developed to eliminate the issue of weak adhesive bonding that characterizes the GEDTS method during hard rock tests.

The ECDTS method is designed on the principle of inducing tensile stress in a thick-walled cylindrical rock specimen subjected to internal borehole pressure generated by expansive cement. A detailed description of the method including test principle, specimen configuration, instrumentation setup, and test methodology are discussed in the next chapter for thorough understanding. Validation of this novel method is carried out through numerical modeling and rock testing experiments on low-tensile strength marble. The choice of rock for the experimental works was deliberate to ensure that all three testing methods including BTS, GEDTS, and ECDTS are performed on the same rock type without any issue of weak adhesive bonding. A comparison of results from these three tests demonstrates the effectiveness and validity of the ECDTS test.

The following chapter is a paper manuscript that is ready to be submitted to a journal for publication.

Chapter 4

A Novel Method for The Determination of Direct Tensile Strength of Brittle Materials Using Expansive Cement

Abstract

A novel method for determining the direct tensile strength of brittle materials such as rock and concrete is proposed in this study. By using an instrumented thick-walled cylindrical specimen filled with expansive cement (EC), the tangential (tensile) strain induced in the specimen as a result of the expansive pressure exerted on the inner wall of the cylinder is captured. Lame's thick-walled cylinder formulation for pressure and tensile stress is adopted for determining the tensile strength of the material using the recorded peak tensile strain. Finite element modeling is carried out to determine the location of uniaxial tensile strain along the outer surface of the cylindrical specimen to satisfy the primary condition of direct tensile strength measurements. Also, the Extended Finite Element Method (XFEM) is used to simulate and predict the fracturing pattern of the thick-walled cylindrical specimen. To ascertain the practical ability of the proposed direct tensile strength testing method, conventional tensile strength experiments are conducted on marble for comparison. It is found that the results obtained from the newly proposed test method are in good agreement with the conventional testing methods including the Glued Ends Direct Tensile Test (GEDTS) and the Brazilian Tensile Strength (BTS) test. In addition, a suitable specimen configuration is proposed for testing the thick-walled cylindrical specimen with the new method.

4.1 Introduction

The initiation and control of brittle failure in rock materials are significantly influenced by their tensile strength. This is one of the primary reasons that most rock failures occur in tensile zones [1]. As such, the tensile strength of the material needs to be estimated and utilized accurately in any geotechnical study, especially for projects where the tensile strength parameter is expected to have a significant impact.

Generally, the estimation of the tensile capacity of rock materials is achieved by following one of the two main testing methods – Direct Tensile Strength (DTS) and Indirect Tensile Strength (ITS) tests. While the ITS methods aim to generate tensile stress in the intact sample through far-field compression, notably the Brazilian Tensile Strength (BTS) test [2] as shown in Fig. 4.1a, the DTS methods employ the application of pure uniaxial tensile load along the axis of the tested material [3]. A typical DTS method is the Glued Ends Direct Tensile Strength (GEDTS) Test as shown in Fig. 4.1b.

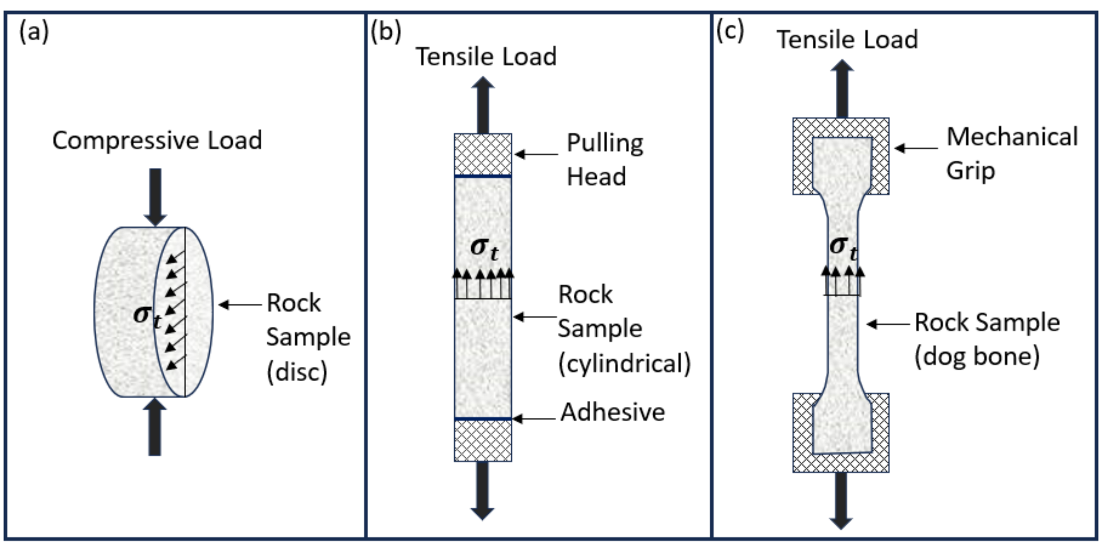


Figure 4.1: Tensile strength test methods (a) Brazilian Tensile Strength Test (b) Glued Ends Direct Tensile Strength Test (c) Direct Tensile Strength Test using the dog bone specimen.

Across literature, the DTS method is acknowledged as the most valid for estimating the true tensile strength of rocks since the test sample is subjected to pure uniaxial tensile load [4,5,6]. Unfortunately, due to limited testing budget and difficulties with sample preparation, the use of DTS tests is not popular [7]. As a result, most tensile strength estimates for rocks are obtained by utilizing indirect testing methods, favorably the BTS test owing to its simplicity and ease of sample preparation. Nonetheless, BTS test is notorious for introducing both internal and external factors into the testing procedure. These factors include the influence of rock anisotropy and inhomogeneity on stress distribution in the rock sample, the effect of foliation angle with respect to the loading axis, and the generation of biaxial stress state in the sample among others. All such factors can influence the tensile strength test results [8,9,10,11]. The inherent validity issues associated with the BTS tests reflect the importance of applying DTS testing methods to estimate the tensile strength of brittle materials such as rocks.

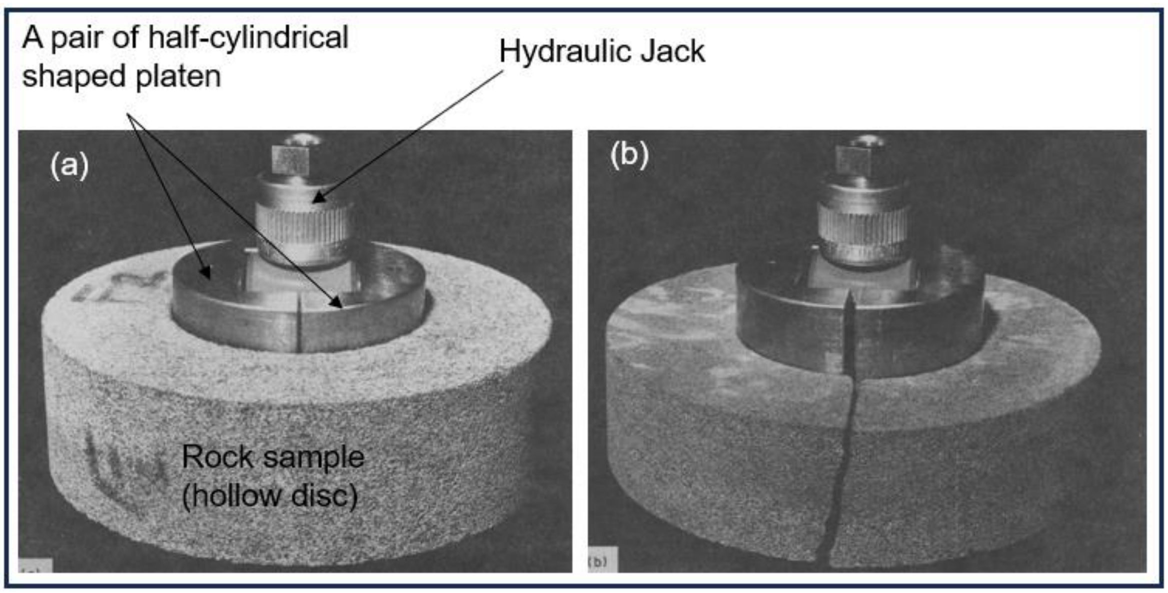


Figure 4.2 (a) The Hoop Test (b) Fracture pattern of the hollow disc (Butenuth et. Al 1993).

For this reason, several DTS methods have been proposed by some researchers in the past. In 1964, Brace [12] proposed the dog bone test and described the test procedure as mechanically gripping the lip and applying tensile force at both ends of the specimen to cause tensile failure at the midsection as shown in Fig. 4.1c. Butenuth et al. [13] introduced the Hoop Test on the principle of diametral tension where the direct tensile strength of sandstone was estimated by applying radial pressure to the inner surface of a ring specimen using a hydraulic jack as shown in Fig. 4.2. This approach was also adopted by Aliha et al. [8] for their direct tensile strength estimation of tuffite using a seesaw device.

Although the DTS testing methods proposed by these researchers employ the application of pure tensile force to induce failure in the test material, they suffer from certain limitations. According to Brace [12], the curved radius of the dog bone fillets reduces stress concentrations caused by the mechanical grip at the specimen ends. However, Perreira and Diederichs [14] argued that the mechanical grip could still cause stress concentrations at the specimen ends during loading, resulting in local fractures at the gripped regions. Zhao et al. [15] have also argued that the cumbersome preparation and the mechanical gripping of the dog bone ends are not always feasible for soft rocks. As such, the dog bone test is less practical for frequent laboratory use [16]. Also from Fig. 4.2, it is obvious that instead of allowing the sample to fail at its weakest plane during the hoop test proposed by Butenuth et al. [13], the tensile fracture is forced to occur at a predetermined location defined by the meeting point of the two half-cylindrical shaped platens positioned against the inner surface of the ring specimen.

For these reasons, to date, the Glued Ends Direct Tensile Strength (GEDTS) test suggested by ISRM [17] remains the most valid direct testing method for estimating the tensile strength of rocks when conducted properly. However, this direct testing approach also suffers a major limitation when applied to hard rocks. During hard rock tests, the bonding capacity of the glue between the specimen end surfaces and the pulling heads is not high enough to withstand the pulling force required to induce tensile failure in the rock specimen. This leads to failure occurring at the glued interface rendering the test invalid. Additionally, this testing approach requires the design of special fixtures to eliminate eccentricity from the loading procedure. Consequently, for the sake of simplicity and good test repeatability, many researchers resort to the classical BTS testing method despite its inherent validity issues.

In light of the above review, it is obvious that the development of new and suitable alternative methods for estimating the direct tensile strength of rocks remains an interesting challenge in the research community. This paper, therefore, presents a novel direct tensile strength testing approach for brittle materials such as rock and concrete using Soundless Chemical Demolition Agents otherwise known as Expansive Cement (EC). The EC, which is a self-stressing cement that contains Ordinary Portland Cement (OPC) and an expansive agent, generates high expansion pressures when confined [18]. These high confined pressures induce tensile stress in the surrounding rock material causing tensile fractures when the material's tensile capacity is exceeded. The EC is commonly used for the demolition of concrete foundations in rehabilitation projects as well as for rock fragmentation in stone quarries [19, 20]. Recently, the application of EC for the fragmentation of rock mass under biaxial confinement such as the face of development drifts in underground mining operations has been investigated [21, 22]. In this study, the use of EC for rock breakage in direct tension mode is evaluated both numerically and experimentally for the estimation of its tensile strength.

4.2 Expansive Cement Direct Tensile Strength (ECDTS) Test

4.2.1 Test Principle

The fundamental principle of the ECDTS test is the generation of tangential (tensile) stress in a thick-walled cylindrical rock specimen subjected to internal borehole pressure generated by expansive cement. A number of strain gauges placed on the outer surface of the cylindrical rock specimen measure the peak tensile strain before the tensile fracture occurs. This peak strain value is the main output to estimate the corresponding tensile stress at failure using Lame's thick-walled cylinder formulations as shown in Equations 4.1 and 4.2 respectively. The calculated tangential (tensile) stress value at failure corresponds to the rock's tensile strength.

$$p_i = \frac{E\varepsilon_o(r_o^2 - r_i^2)}{2r_i^2} \qquad (4.1)$$

$$\sigma_t = \frac{p_i r_i^2}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right) \qquad (4.2)$$

Where p_i = internal pressure generated by the expansive cement, E = modulus of elasticity of the rock material, ε_o = tensile strain on the outer surface of the cylindrical rock specimen, r_o and r_i = outer and inner radii of the cylindrical specimen respectively, and r = the distance from the center of the cylindrical specimen to the strain gauge. As the goal of the experiment is to calculate the tensile stress σ_t at $r = r_0$, and by eliminating p_i from (2), it can be shown that:

$$\sigma_t = E\varepsilon_0 \qquad (4.3)$$

Equation (4.3), also known as Hooke's law, is used for the estimation of the tensile strength ($\sigma_t = \sigma_t^{EC}$) of the specimen based on the average peak strain gauge reading $\varepsilon_0 = \varepsilon_p$ prior to failure.

4.2.2 Finite Element Modeling

A triaxial stress/strain state including radial, axial, and tangential (tensile) are induced in a thick-walled cylindrical material subjected to internal borehole pressure. However, there exists a specific location around the outer surface of the cylindrical specimen where the stress/strain condition is uniaxial and maximum. The goal of the numerical analysis is to ascertain this specific location for uniaxial strain measurement to satisfy the primary condition for direct tensile strength determination.

First, a 3-D finite element (FE) model of the cylindrical rock specimen of an NQ core size (48 mm or 1.9 inches) is constructed in Abaqus/CAE (2019). To ensure that the influence of borehole size on the location of maximum uniaxial strain around the outer surface of the cylinder is adequately examined, three borehole sizes namely 15.9 mm (5/8“), 19 mm (3/4“), and 25.4 mm (1“) are investigated. An aspect ratio (ratio of borehole length to its diameter) greater than 4 is employed to avoid the influence of the closed end of the cylindrical rock specimen on the tangential strain readings [23]. As shown in Fig. 4.3a, the model consists of two parts – a cylindrical rock part and an expansive cement

(EC) part. The rock-to-expansive cement interface is treated as hard contact in the normal direction and frictional in the tangential direction with a friction penalty of 0.3. Cylindrical coordinates system is used to fit the cylindrical specimen model. Fig. 4.3c shows a 3D finite element mesh pattern generated in Abaqus for both EC and rock parts using over 700,000 4-node tetrahedron elements. Elastic material properties of typical rock material with a modulus of elasticity, E_R = 8.92 Gpa, and Poisson ratio, υ_R = 0.2 are considered for analyzing the rock specimen. The peak elastic material properties of commercially available EC (Betonamit), i.e., after complete hydration, are obtained from Habib et al. [23]. They are modulus of elasticity, E_{EC} = 8.2 Gpa , and Poisson ratio, υ_{EC} = 0.2.

For model boundary conditions, as shown in Fig. 4.3b, the bottom of the Cylinder is fixed in all directions to prevent rotational and translational displacements as the EC expands. The EC pressure is modeled as initial stress and subsequently released in the second static step. An EC pressure of 10 MPa is assumed. From the model output, a tangential strain query path taken from the top edge to the bottom along the outer surface of the specimen as seen in Fig. 4.4a shows that the maximum tangential strain is located at the top edge of the specimen. As shown in Fig. 4.4b, a vertical section taken through the specimen shows an s-shaped tangential strain contour profile with the maximum value at the location marked 'X'. It is noteworthy that this location remains unchanged irrespective of the borehole size.

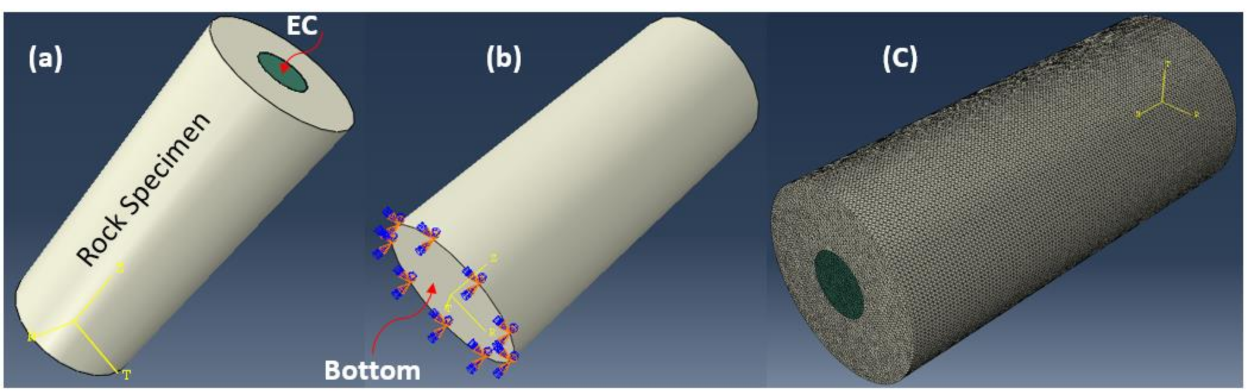


Figure. 4.3 3D FE model configuration. (a) Materials, (b) Boundary conditions, and (c) FE Mesh.

Also, as mentioned earlier, central to the ECDTS testing method is ensuring that the stress state at the location of strain measurement on the rock specimen is uniaxial to satisfy the requirement for direct tensile strength determination. As shown in Fig. 4.5, it is clear that the stress state at the top outer edge of the cylindrical rock specimen is uniaxial as the values of radial and axial stresses are 0 MPa. Thus, the tangential (tensile) stress which has a value of 3 MPa (Fig. 4.5) is a uniaxial stress. This numerically determined location confirms the authors' suggestion of positioning the strain gauges at the top edge of the cylindrical specimen to capture the highest possible uniaxial strain before fracture occurs. It also indicates that fracturing is more likely to be initiated at that same location before propagating downwards.

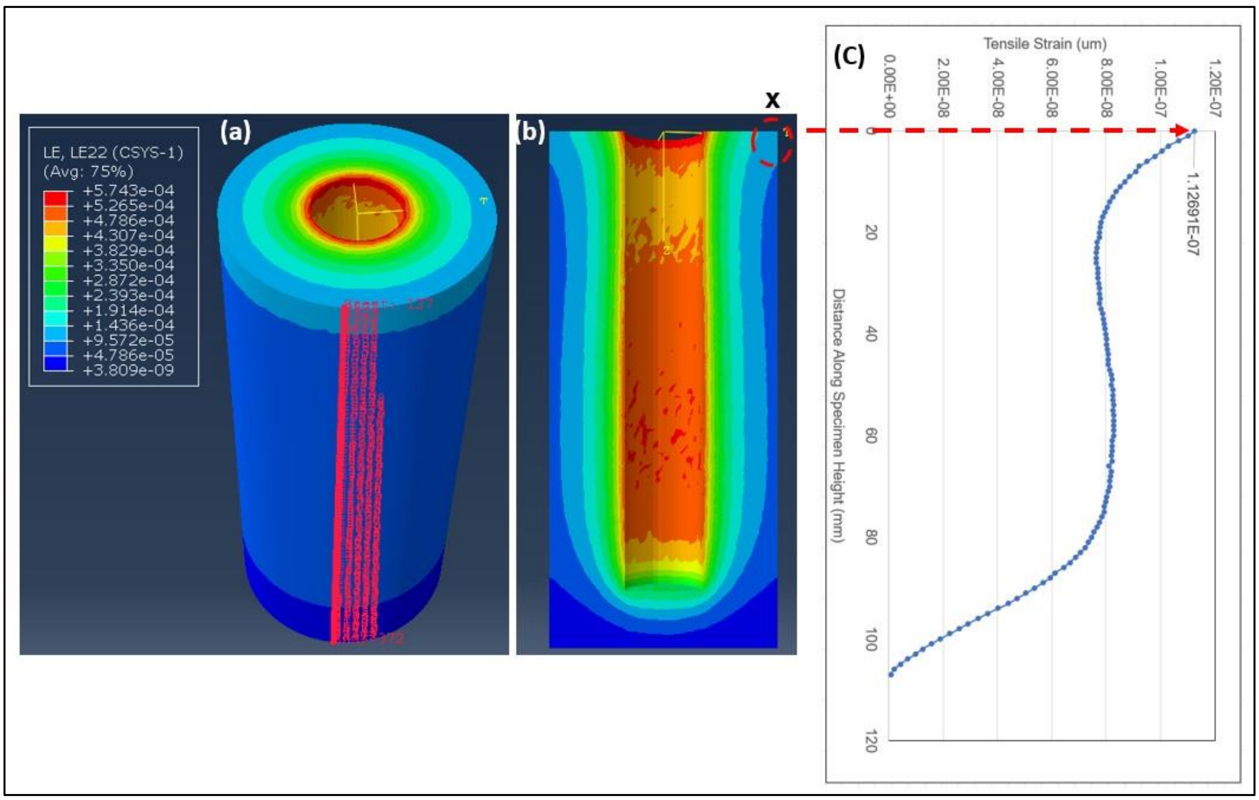


Figure. 4.4 Location of maximum tangential strain. (a) FE model with tangential strain query path along the outer surface of the rock specimen, (b) Vertical section showing the s-shaped contour profile of the tangential strain and the location of its maximum value, (c) Tangential strain distribution on the outer surface of the cylinder along its height.

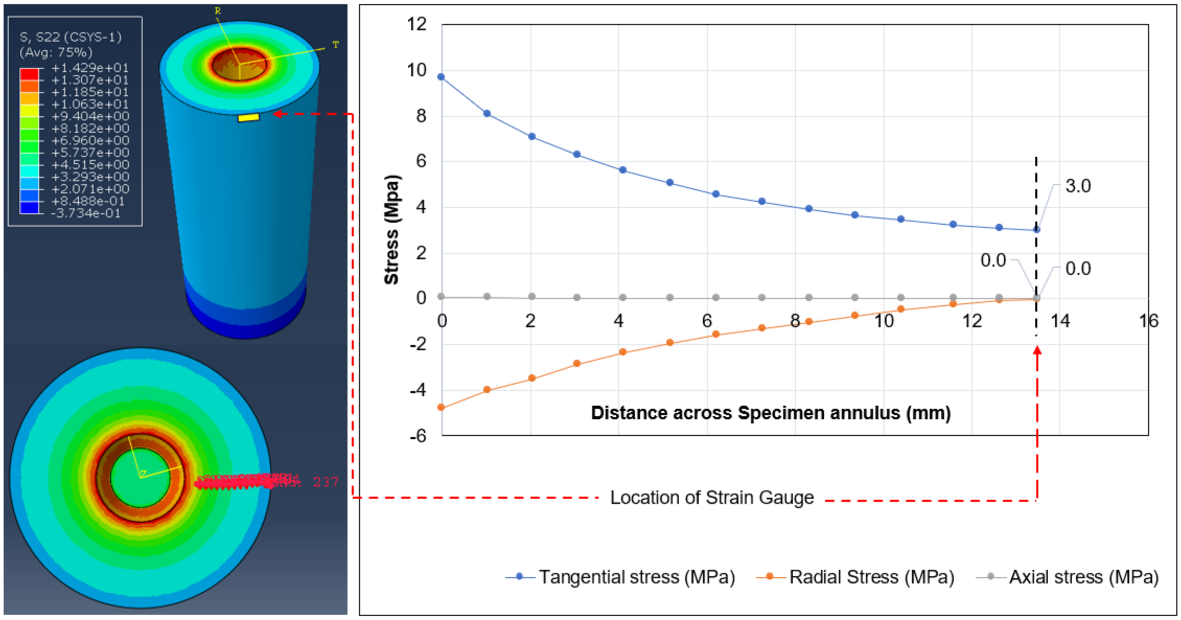


Figure. 4.5 Uniaxial stress state at the location of strain measurement.

Furthermore, the generation of tensile crack in the rock specimen due to the expansion pressure from the EC was adequately examined by simulating and predicting the fracture trajectory using the Extended Finite Element Method (XFEM). The XFEM method which was proposed by Dobow [24] is an effective simulation tool to characterize the behavior of crack propagation in solid media. The simulation of crack propagation using the XFEM method can be done without creating an initial crack and crack path definitions. To simulate the initiation and growth of the crack path, the maximum principal stress criterion was used. If the material's tensile strength at any integration point is exceeded by the principal stress after each equilibrium step, a new or additional crack is generated. A 2D XFEM model was used to simulate the initiation and propagation of the tensile crack. The 4-node quadrilateral element was adopted to generate a mesh of 6981 elements. The elastic properties of the rock and EC are the same as in the 3D model, and the tensile strength of the rock is 2.2 MPa. An EC pressure of 9 MPa is initialized. From Fig. 4.6, the predicted path of crack growth simulated using the XFEM method shows that the crack initiates from the inner surface towards the outer surface of the rock specimen which is expected.

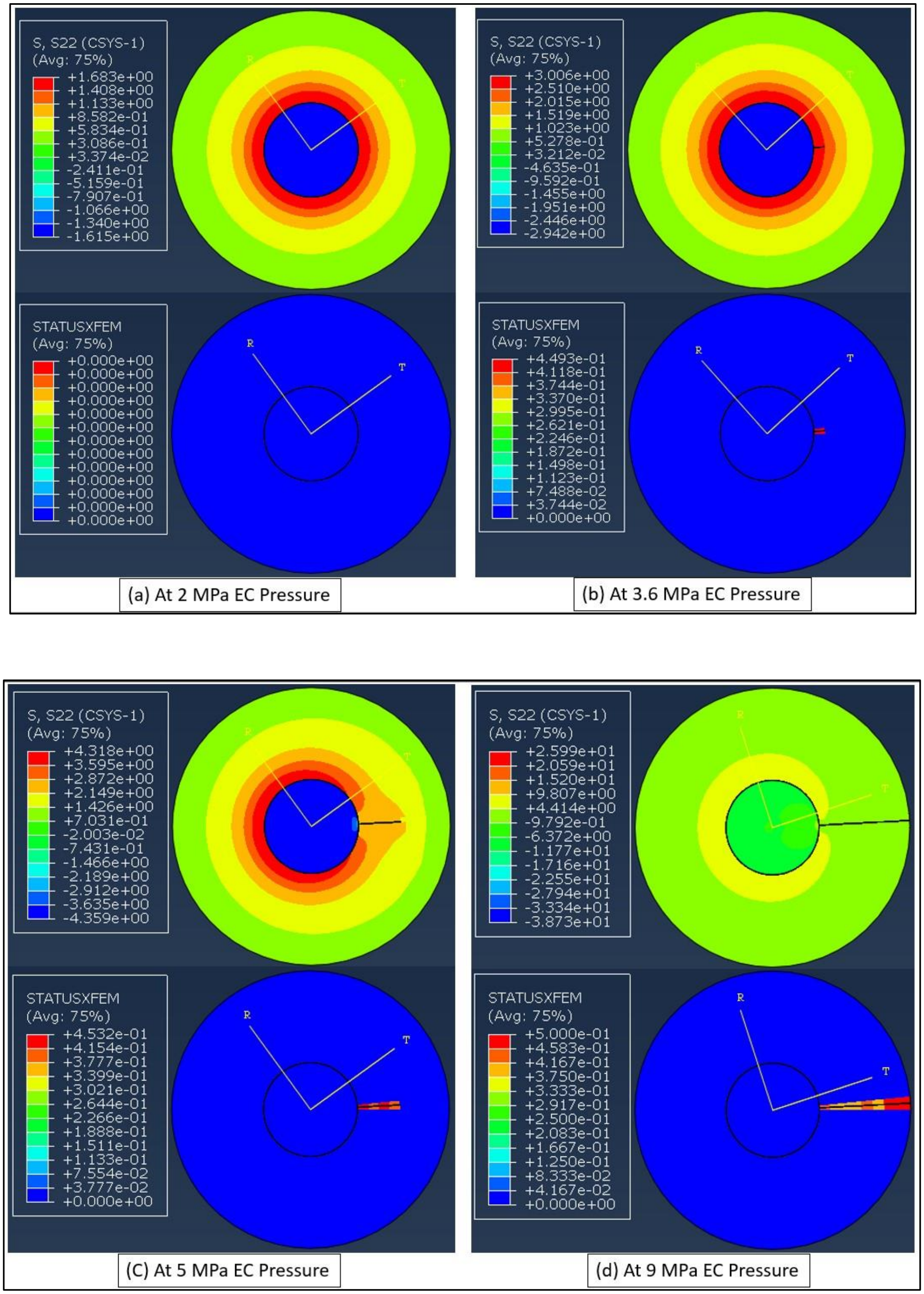


Figure. 4.6 Stages of crack growth using the XFEM method showing tensile stress (S22) contours and the corresponding XFEM status at a given pressure.

4.2.3 Specimen Configuration and Instrumentation Setup

Fig. 4.7 shows the configuration of the rock specimen used for the ECDTS test. As can be seen, a base thickness of 10-12 mm and an aspect ratio (EC borehole length to diameter ratio) of 5 is considered [23]. A well-designed instrumentation system is also required for the strain measurement. As shown in Fig. 4.8, The strain gauges are positioned in the tangential direction on the outer surface around the top edge of the rock specimen and are wired to a data acquisition (DAQ) system to record the tangential (tensile) strain values. A minimum of four strain gauges (SG) are recommended since the line of fracture is not predetermined. A time-lapse camera is mounted on a tripod and positioned over the rock specimen for crack visualization to identify the lead strain gauge (strain gauge which records the highest strain value). An acoustic emission (AE) sensor is positioned on the outer surface close to the vicinity of crack initiation and propagation at the top part of the rock specimen to record the Time of First Crack (TFC). Data from the AE and DAQ as well as images from the Time-Lapse camera are examined to read the peak tensile strain value.

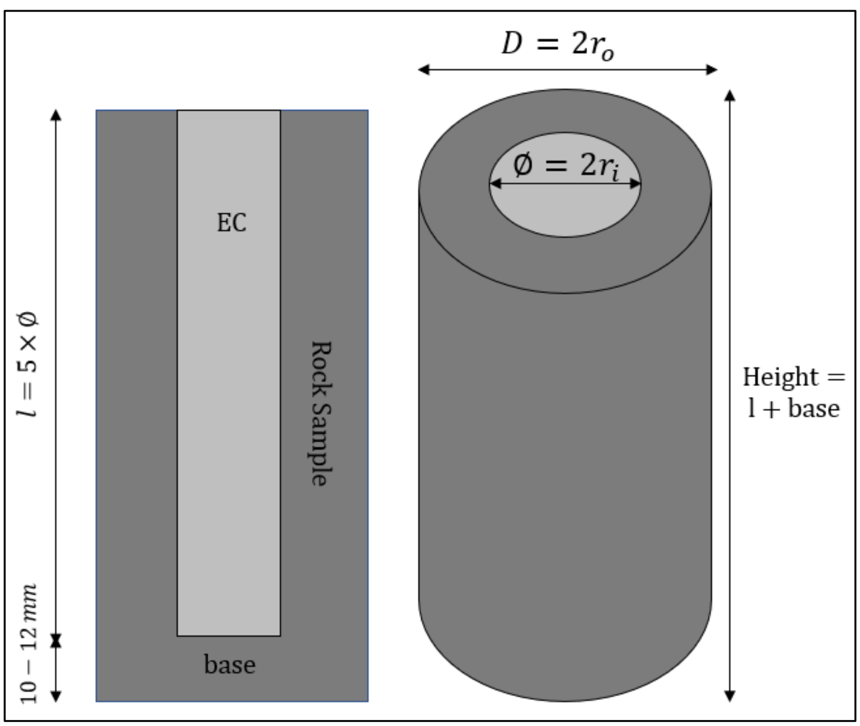


Figure. 4.7 The ECDTS Specimen Configuration.

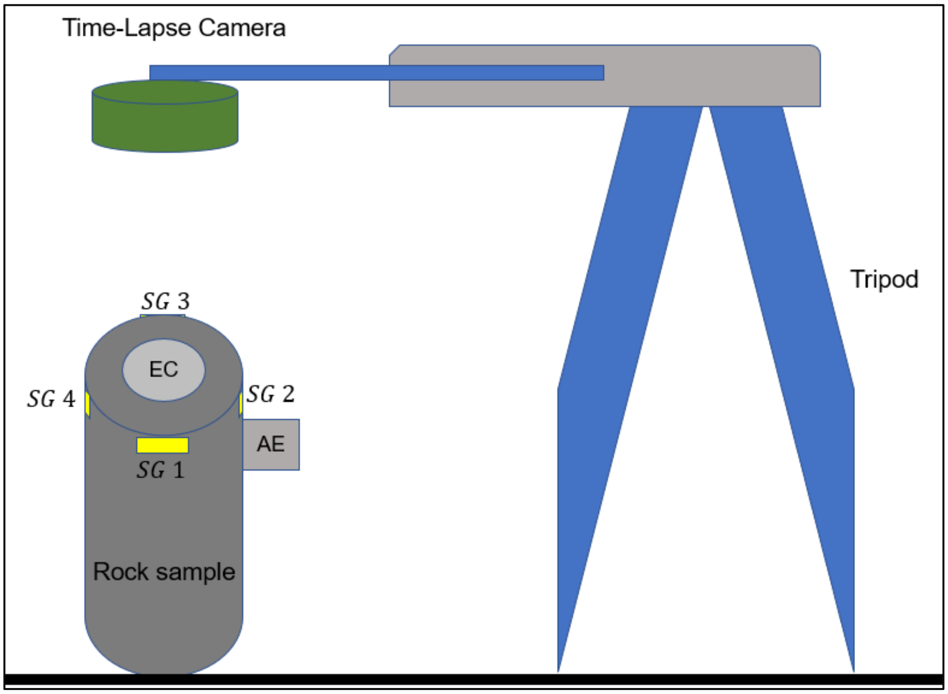


Figure. 4.8 The ECDTS Instrumentation Setup.

4.2.4 Rock Tensile Strength Experiments

For the sake of method validation by comparing the proposed ECDTS method to the internationally recognized GEDTS test and the classical BTS test, a low tensile strength marble is considered in this study. This was done to ensure that all three testing approaches were easily applied to the same rock type without any issues of weak adhesive bonding that limits the application of the GEDTS method when rocks with high tensile strengths are tested.

4.2.4.1 The ECDTS Test Methodology

To examine the practical ability of the ECDTS test proposed for direct tensile strength determination of rock materials, some experimental tests were conducted. Cylindrical rock specimens were cored from marble cubes and trimmed to the required height based on the recommended aspect ratio and sample base thickness. The outer diameter was kept constant and equal to $d_o = 48.3\ mm\ or\ 1.9''$ while varying the inner diameter, d_i as 15.9 mm (5/8''), 19.1 mm (3/4''), and 25.4 mm (1''). As such, the d_i/d_o ratios for the 15.9 mm, 19.1 mm, and 25.4 mm specimens are 0.33, 0.44, and 0.53 respectively. Fig. 4.9 shows the three specimen configurations used in the ECDTS test. The boreholes were blind drilled to the required length, l, in the rock samples leaving a solid base of 12 mm. Drilling was done with a lathe machine to ensure concentricity for uniform EC pressure distribution.

The experimental program is set up as outlined in section 2.3. Betonamit Type R, which is a commercially available expansive cement, is used in this test with a water-to-cement ratio of 0.2 as per the manufacturer's instructions. The expansive cement slurry is then poured by gravity into the borehole and the test is allowed to run as shown in Fig. 4.10. For the sake of repeatability and reliability, three ECDTS tests were conducted for each borehole size. On average, the tests lasted a period of 4.7, 4.4, and 3.3 hours for the 5/8", 3/4", and 1'' borehole sizes respectively. The rock's tensile strength was determined using eq. 3 after obtaining the peak strain value recorded by the lead strain gauge. Fig. 4.11 shows the strain curves obtained from the data acquisition system for the three 5/8'' specimens namely ECT-5/8''-T1, ECT-5/8''-T2, and ECT-5/8''-T3, with their corresponding fracture patterns.

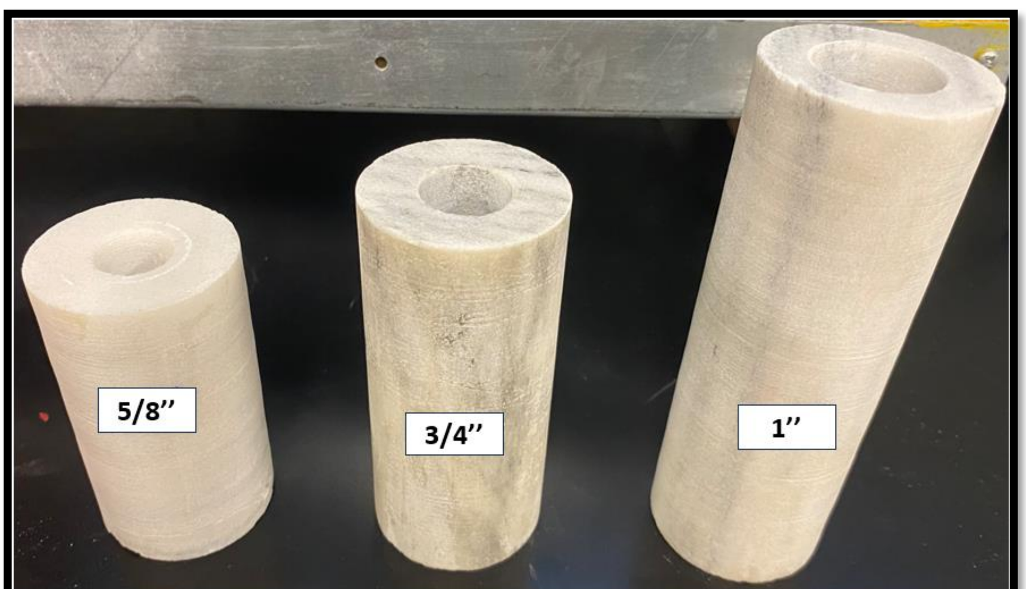


Figure. 4.9 ECDTS test specimen configurations with different borehole diameters.

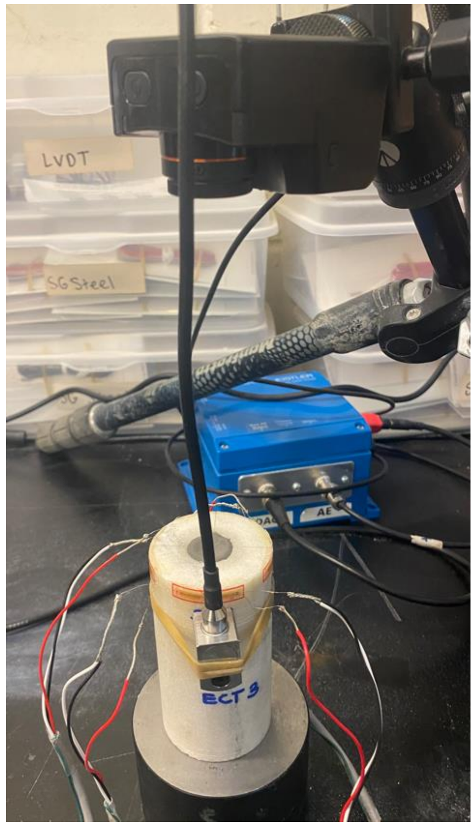


Figure. 4.10 The ECDTS test setup.

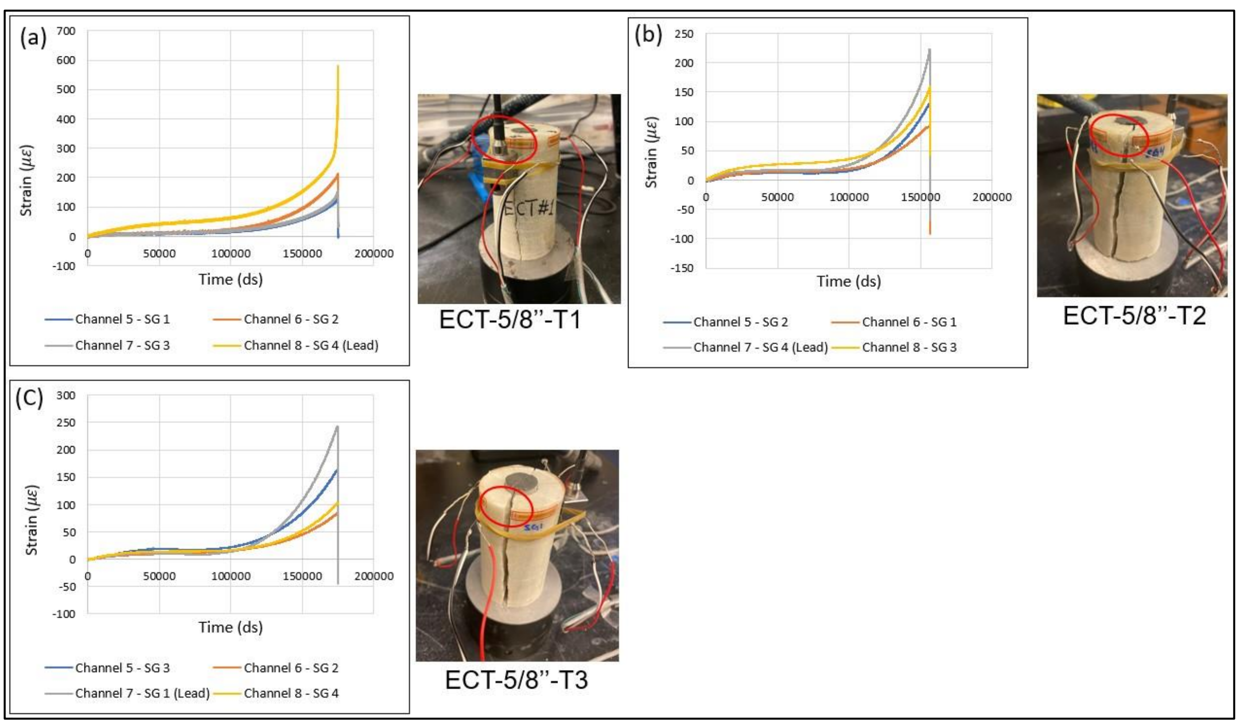


Figure. 4.11 Strain curves from data acquisition system and failure pattern.

It can be observed that at the onset of fracture, there is a sudden drop in the strain curve. However, this sudden drop is not seen for the lead strain gauge of test ECT-5/8''-T1 as shown in Fig. 4.11a, even though fracture had already occurred. This phenomenon occurs when the lead strain gauge bridges the line of fracture leading to a perpetual increase in strain values. For such scenarios, the average value of all four strain gauge readings is considered. It is also seen that the strain curves are nonlinear. This is due to varying loading rate as the expansive cement cures with time. Linearity is observed only at the onset of fracture which is an indication of steady pressure increase rate. The sudden drop in the strain curve demonstrates the brittleness of the rock failure in tension.

4.2.4.2 Results and Discussions

The peak strain values and their corresponding peak loads (pressures) and tensile strength values obtained from the ECDTS tests for different d_i/d_o ratios are presented in Table 1. It is seen that the tensile strength value slightly increases with increasing borehole size. As such, there is a bias as a function of specimen borehole diameter. It is however noteworthy that the tensile strength values obtained from the 5/8'' borehole specimens closely match those obtained from the conventional GEDTS tests presented in Table 4.2. It is also evident that the rock material (marble) is fairly homogeneous with tensile strength coefficient of variation below 8% for each specimen size.

The nonlinear strain curve behaviour depicted in Fig 4.11 is attributed to the time-dependent behaviour of EC as it cures and hardens. Regardless, the local material behaviour at the strain gauge location remains linear. This can be seen from Equation 4.3 where $\sigma_t = E\varepsilon_0$. Considering test ECT-5/8"-T1 in Table 1, the recorded peak strain value before tensile failure is 257 *um.* The estimated modulus of elasticity, E of the tested marble is 8.922 GPa. Using Hooke's equation 3, $\sigma_t = E\varepsilon_p$ produces a tensile strength value of 2.3.

Table 4.1 Tensile strength values obtained for the ECDTS specimens.

Test #	d_i	d_o	d_i/d_o	Peak Strain ($\mu\varepsilon$)	Pressure (MPa)	σ_t^{EC} (MPa)
ECT-5/8"-T1	15.9	48	0.33	257	9.6	2.3
ECT-5/8"-T2	15.9	48	0.33	223	8.3	2.0
ECT-5/8"-T3	15.9	48	0.33	243	9.0	2.2
					Mean =	**2.2**
					Standard Deviation=	**0.12**
					Coefficient of Variation=	**5.76 %**
ECT-3/4"-T1	19.0	48	0.40	317	7.8	2.8
ECT-3/4"-T2	19.0	48	0.40	311	7.7	2.8
ECT-3/4"-T3	19.0	48	0.40	290	7.2	2.6
					Mean =	**2.7**
					Standard Deviation=	**0.09**
					Coefficient of Variation=	**3.45 %**
ECT-1"-T1	25.4	48	0.53	322	3.8	2.9
ECT-1"-T2	25.4	48	0.53	386	4.6	3.4
ECT-1"-T3	25.4	48	0.53	385	4.6	3.4
					Mean =	**3.2**
					Standard Deviation=	**0.24**
					Coefficient of Variation=	**7.29 %**

Also, the tensile strength results from the proposed ECDTS tests were numerically verified using the XFEM method. To do this, an example is demonstrated using the results of test ECT-5/8"-T3 in Table 1. The calculated pressure of 9.0 MPa was set as the initial stress in the ABAQUS XFEM constitutive model, and the maximum principal stress criterion was used to initiate and propagate the crack as explained in the previous section. From the model output, as seen in Fig. 4.12, the average tensile stress value computed from two nodes across the crack at the rock's outer surface is 2.19 MPa which is in better agreement with the calculated tensile strength value of 2.2 MPa from the analytical model. This means that the XFEM method and the numerically determined location for uniaxial strain measurements for direct tensile strength determination validate the ECDTS experimental approach.

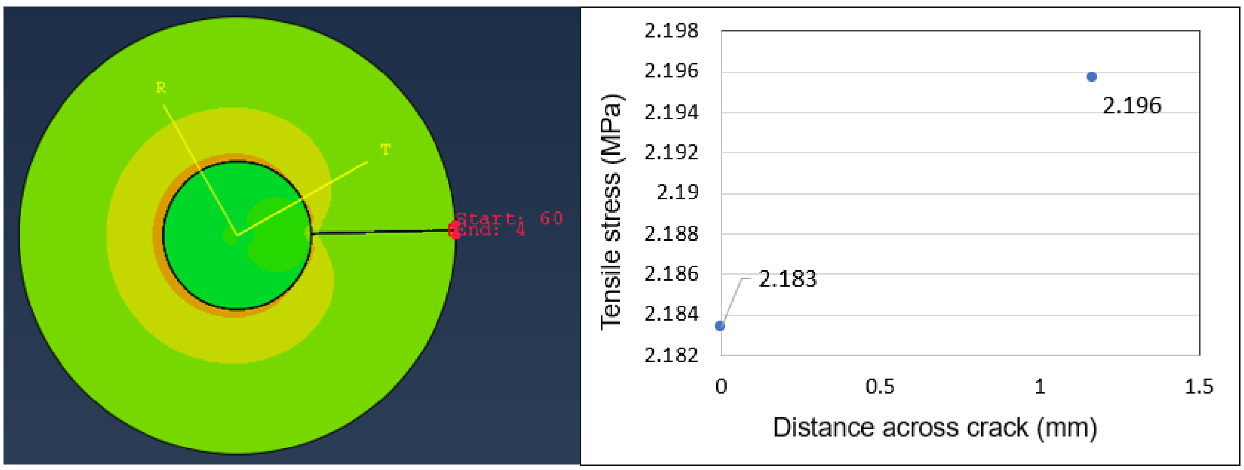


Figure 4.12 Computed tensile stress values across the crack at the rock specimen's outer surface.

4.3 Direct and Brazilian Tension Tests

In this section, the validity of the proposed ECDTS testing method is investigated by comparing the experimental results to those obtained from the internationally recognized glued end direct tension test (GEDTS) and Brazilian tension test (BTS). As such, four GEDTS and ten BTS samples from the same marble cubes were prepared and tested by the ISRM and ASTM suggested methods. Fig. 4.13 and Fig. 4.14 show the test setups, load-displacement and load-time curves, and fracture patterns for the GEDTS and BTS tests respectively.

In Table 2, the maximum tensile loads, and the corresponding tensile strengths for the four GEDTS samples obtained from Equation 4.4 are presented. Table 3 also presents the maximum splitting loads and the corresponding tensile strengths for the ten BTS samples estimated from Equation 4.5.

$$Tensile\ Strength, \sigma_t^{DTS} = \frac{4P}{\pi D^2} \tag{4.4}$$

$$Tensile\ Strength, \sigma_t^{BTS} = \frac{2P}{\pi D t} \tag{4.5}$$

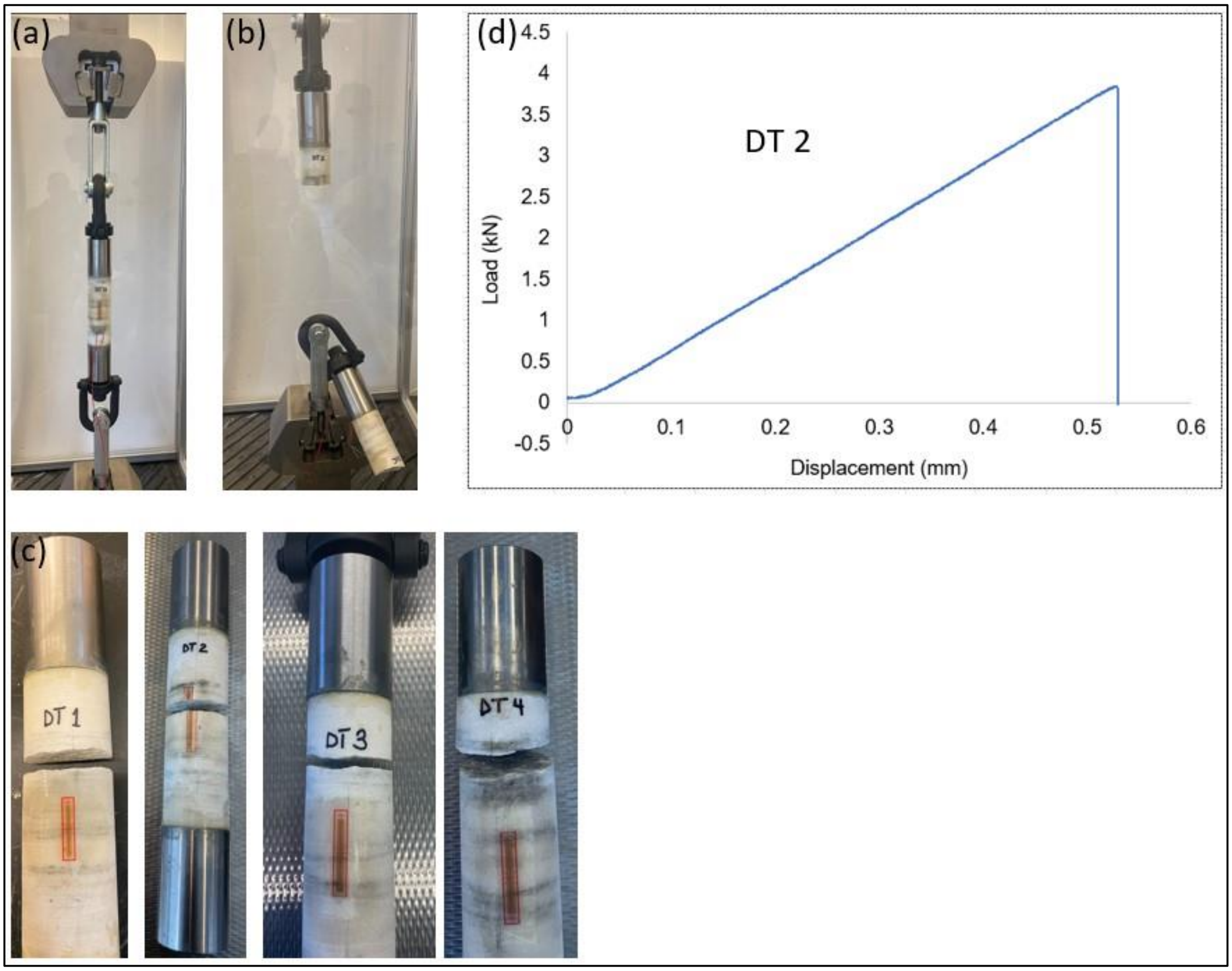


Figure. 4.13 GEDTS - Direct tension test apparatus (a) test set up, (b) Sample failure mode, (c) Fracture pattern of marble samples, (d) Load-displacement plot for GEDTS test 2.

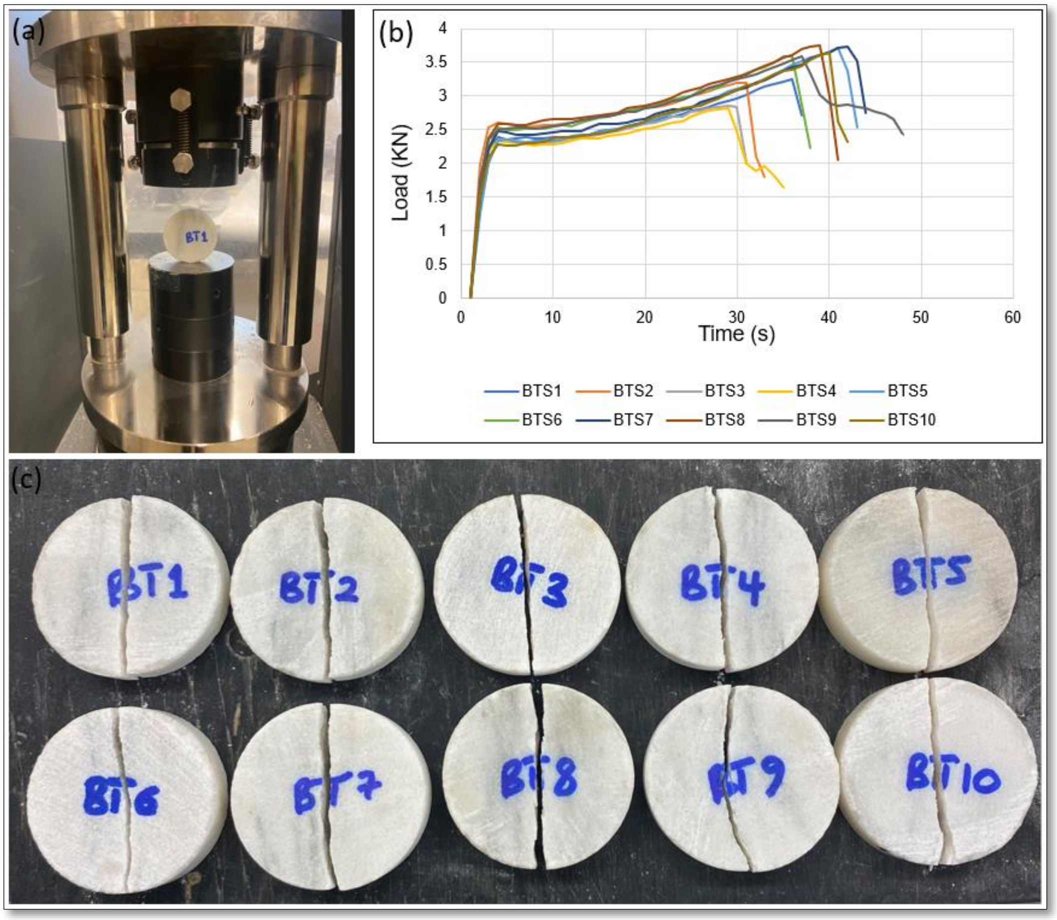


Figure. 4.14 Brazilian tensile strength (BTS) test. (a) Test setup, (b) Load-time curves for all ten samples, (c) Fracture patterns for BTS marble samples.

Table 4.2 Direct Tensile strength values obtained for the GEDTS Samples.

Test #	Tensile Load (kN)	σ_t^{DTS} (MPa)
GEDTS-1	4.15	2.2
GEDTS-2	4.76	2.6
GEDTS-3	3.98	2.2
GEDTS-4	3.85	2.1
	Average =	**2.3**
	Standard Deviation (SD) =	**0.192**
	Coefficient of Variation (COV) =	**8.44 %**

Table 4.3 Tensile strength values obtained for the BTS samples.

Test #	Splitting Load (KN)	σ_t^{BTS} (MPa)
BTS-1	3.3	4.0
BTS-2	3.2	3.7
BTS-3	2.9	3.6
BTS-4	2.8	3.3
BTS-5	3.7	4.0
BTS-6	3.6	3.9
BTS-7	3.7	3.9
BTS-8	3.8	4.2
BTS-9	3.6	4.4
BTS-10	3.6	4.4
	Average =	**3.9**
	Standard Deviation (SD) =	**0.329**
	Coefficient of Variation =	**8.36 %**

4.4 Comparison of Test Methods

From Fig. 4.15, comparing the average tensile strength values from the ECDTS test with different d_i/d_o ratios to the ISRM and ASTM suggested GEDTS and BTS tests indicate the validity and accuracy of the ECDTS testing method. Clearly, the BTS test overestimates the tensile strength of rock materials as has been argued by many researchers [e.g., 14]. As a rule of thumb, DTS $\approx$ 0.67 BTS [14]. However, it is noteworthy that the ECDTS test is proposed as an alternative testing method for the direct tensile strength determination of rock materials. Although the average tensile strength values obtained from the ECDTS test slightly increases with increasing d_i/d_o ratio, the ECDTS specimen configuration with 0.33 d_i/d_o ratio produces tensile values that are in better agreement with the GEDTS test results demonstrating its suitability as an alternative for the direct tensile strength determination of brittle materials such as rocks.

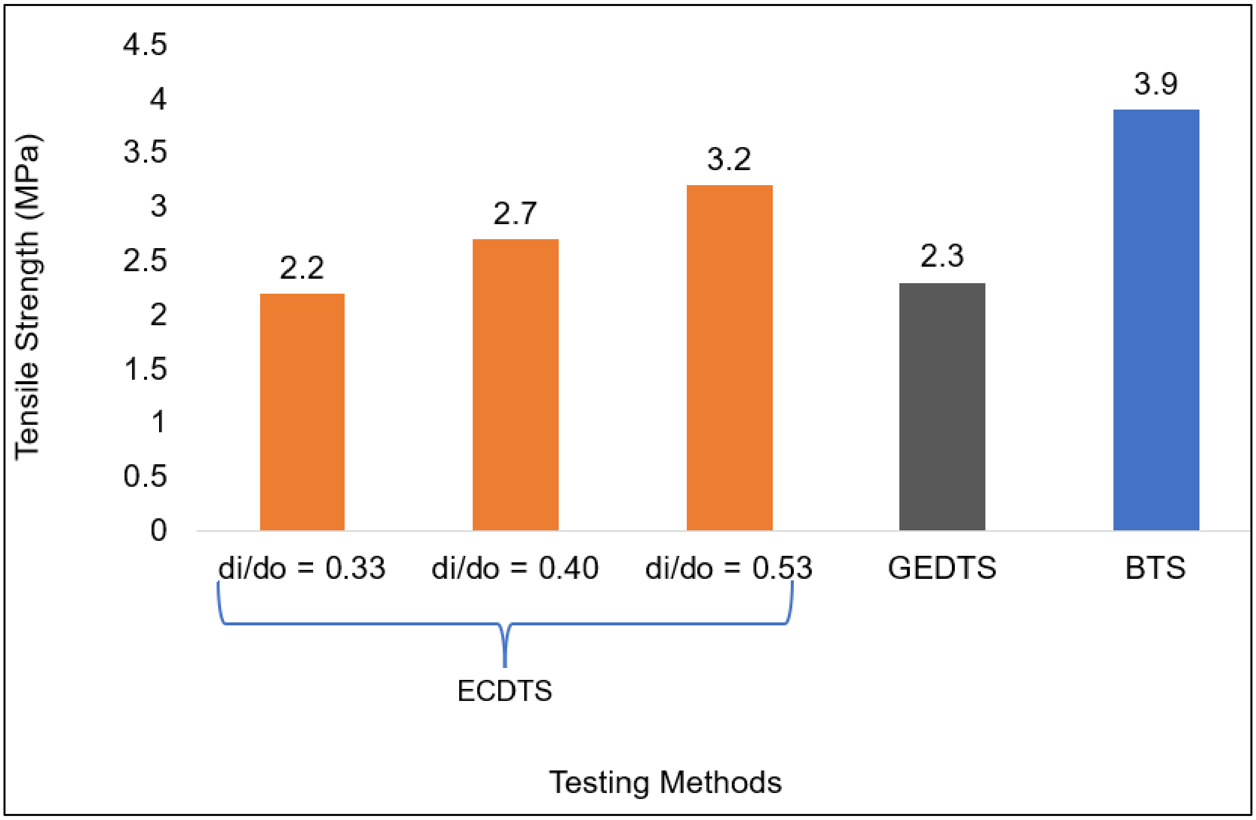


Figure. 4.15 Tensile strength values of marble obtained from different testing methods.

The ECDTS testing approach of applying pressure generated by expansive cement to the inner walls of a borehole to induce tensile fractures is somewhat similar to the hydraulic fracturing process used in oil and gas wells. In the field of hydraulic fracturing, the tensile strength parameter is an important input parameter that helps to determine the fluid pressure required to fracture the rock mass surrounding the borehole. Accurately determining the true splitting resistance or tensile strength of the rock mass is therefore critical to the overall success of the hydraulic fracturing operation. Because hydraulic fracturing and the ECDTS test have similar tensile fracturing mechanisms, the proposed ECDTS test appears to be a better candidate for providing the true tensile strength value of the rock surrounding the borehole for accurate engineering estimates of the minimum fluid pressure required to induce tensile fractures.

4.5 Conclusions and Recommendations

A novel direct tensile strength testing approach that subjects a thick-walled cylindrical specimen to internal borehole pressure generated by expansive cement is proposed in this research to estimate the tensile strength of rocks and geomaterials. The practical ability of the proposed method has been demonstrated and validated using cylindrical specimens made from marble with different inner diameters while keeping the outer diameter constant. The results are compared to that obtained from the ISRM and ASTM internationally recognized GEDTS and BTS suggested methods. The following conclusions can be drawn from this study.

- The overall average of the tensile strength values obtained for the marble specimens with different d_i/d_o ratios using the ECDTS testing method is in good agreement with the result obtained from the ISRM and ASTM suggested GEDTS method, with a coefficient of variation of only 0.08 demonstrating the validity and practical ability of the proposed ECDTS method.
- The tensile strength values are directly proportional to the borehole sizes of the tested cylindrical rock specimen. Increasing the d_i/d_o ratio slightly increases the tensile strength value albeit with a coefficient of variation of only 0.1336. Among the tested rock specimens, the tensile strength value obtained for specimen configuration with d_i/d_o ratio of 0.33 appears to be in very good agreement with the result obtained for the GEDTS testing method. For this reason, the d_i/d_o ratio of 0.33 is recommended as the best specimen configuration for the ECDTS test taking into consideration the sample solid base thickness of 12 mm and outer diameter of an NQ core size (48 mm).
- The linearity of the load-displacement at the strain gauge location prior to tensile failure is validated, hence Hooke's law is used to calculate the tensile strength.
- Compared to the GEDTS test, the new ECDTS method does not require highly sophisticated tensile loading machine. This makes the ECDTS test a much more affordable option for directly determining the tensile strength of brittle materials such as rock and concrete.

4.6 References

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Bridging text between manuscripts

In Chapter 4, the ECDTS testing method was described and validated both numerically and experimentally by conducting a series of tests on low-tensile strength marble. In the following chapter, the applicability of the ECDTS test to hard rocks with tensile strengths that are too high to be tested with the conventional GEDTS test is demonstrated. To do this, experimental tests are conducted on Sudbury breccia and Intermediate Gneiss which are known to be hard rocks with high tensile strength values. Since the ECDTS method has been verified and validated in the previous chapter, the test principle and methodology remain the same in this chapter. Specimens from each rock type were prepared for the BTS, GEDTS, and the ECDTS tests. This time however, comparison of results was done only between the ECDTS and BTS tests. This is because the conventional GEDTS test failed when applied to these hard rocks. Failure occurred at the glued interface due to the low bonding capacity of the adhesive compared to the high tensile capacity of the tested rock specimens. From the comparison, it is shown that the values for Sudbury breccia and Intermediate Gneiss determined by the direct (ECDTS) method are 14.4% and 14.2% less than those obtained by the indirect (BTS) method for the same rock types, respectively. This is to be expected since it is well-known that the BTS test overestimates the tensile strength of brittle materials as mentioned in the previous chapters. This demonstrates the consistency and effectiveness of the ECDTS test for direct tensile strength measurement.

The work reported in Chapter 5 is a paper manuscript that is in preparation to be submitted to a journal for publication.

Chapter 5

Determination of the Tensile Strength of Hard Rocks Using the Expansive Cement Direct Tensile Strength Testing Method

Abstract

The glued ends direct tensile strength (GEDTS) test is the testing method that is recognized by the international testing standard bodies as the correct way of determining the tensile strength of brittle materials such as rocks and concrete. However, during this test, it is not only difficult to ensure proper alignment of the specimen and load axis to avoid eccentricity, but also difficult to achieve results when the method is applied to hard rocks with tensile capacities higher than the bonding capacity of the adhesive. The goal of this research, therefore, is to develop an alternative testing method that eliminates the use of adhesives for direct tensile strength determination of rocks and concrete. This paper is the second of two-phase research which aims to directly determine the tensile strength of rock and concrete using expansive cement. In the first phase, a novel testing method known as the Expansive Cement Direct Tensile Strength (ECDTS) test was developed. The ECDTS method was designed on the principle of inducing tensile stress in a thick-walled cylindrical rock specimen subjected to internal borehole pressure generated by the expansive cement. The method was numerically validated and verified by conducting experimental test on low-tensile strength marble. Comparison of results to the conventional Brazilian Tensile Strength (BTS) and GEDTS tests demonstrated the validity of the ECDTS test. In this paper, the applicability of the ECDTS test to hard rocks is examined by conducting some experiments on two high-tensile strength rocks namely Sudbury breccia and Intermediate Gneiss. A series of BTS tests was also performed and comparison of results showed that the ECDTS test results were 14-15% lower than the BTS results which was expected.

5.1 Introduction

The tensile strength of rock, defined by Tufekci et al. [1] as “the failure stress of a rock element in pure uniaxial tensile loading”, is a critical parameter that plays a significant role in the design and stability analysis of many geotechnical projects such as stability of underground openings, hydraulic fracturing, and tunnel boring [2,3]. However, due to the difficulties in obtaining reliable results, the determination of the tensile strength parameter is often overlooked despite its significance in controlling many failure processes. It is common for engineering studies to focus on the unconfined compressive strength (UCS) of intact rock specimens even though many researchers have demonstrated that the initiation of cracks in brittle materials such as rocks is a tensile phenomenon [4,5,6,7,8,9]. Currently, the two main testing methods available for tensile strength determination of rocks are the indirect and direct testing methods [10]. The indirect testing method, particularly the Brazilian Tensile Strength (BTS) test, aims to generate tensile stress in the specimen by compressing a thin circular disk to failure [11]. Although the BTS test is widely used due to its simplicity and efficiency, its validity has been questioned by many researchers owing to several factors that could influence the test results [12,13,14,15]. Among these factors is the influence of friction on the stress field in the Brazilian disc immediately near the loading platens due to the diametrical compression [16]. Additionally, the compressive stress induced in the disc has been found to increase a rock material’s tensile strength compared to the result obtained from direct testing [17]. Chen et al. [14] also argued that when the disc foliation angle to the loading axis is between 60^o and 90^o degrees, there is a possibility of shear failure along the laminations in anisotropic materials rather than tensile failure. As such the Brazilian test is incapable of providing a true indication of the rock's tensile strength.

There are a number of direct methods for determining a rock’s tensile strength, including clamping and bonding, and their results are generally more accurate than indirect methods [11,18]. However, the clamping methods do not only introduce stress concentrations into the test procedure but also most often lead to fracture at the gripped region, especially for soft rocks, rendering the test invalid. This led to the introduction of the dog bone test proposed by Brace [19] but specimen preparation for this test is expensive and cumbersome [20]. As such, some researchers have reported that the dog bone test is less practical for frequent laboratory use [21]. The bonding method on the other hand could take 24-48 hours for the glue to cure before testing is conducted. Aside from that, failure at the glue interface usually occurs especially when testing hard rocks because the bonding capacity of the glue is limited and usually is lower than the tensile capacity of the tested rock specimen as shown in Fig. 5.1. Moreover, bad parallelism of specimen end surfaces and small misalignment of the tensile load axis could introduce bending and torsional stresses in the test procedure [22,23].

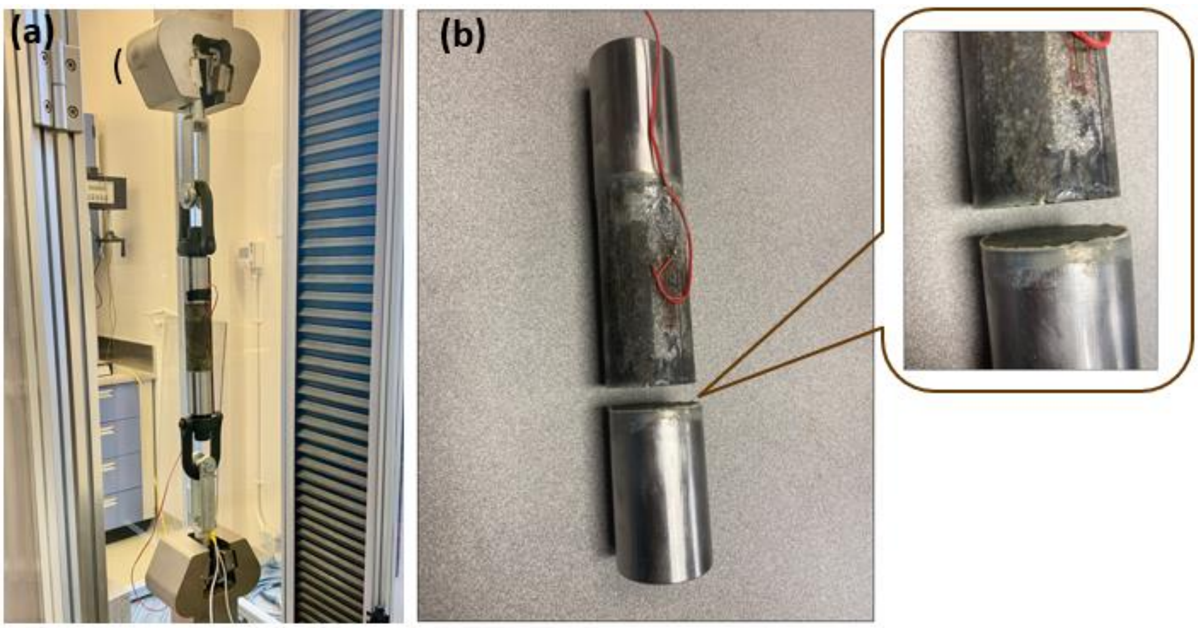

Figure 5.1. The Glued Ends Direct Tensile Strength (GEDTS) test of hard rock. (a) GEDTS Test Setup (b) Failure at glued interface due to weak adhesive bond.

As a result of the above limitations associated with the conventional testing methods, a novel direct tensile strength test using Expansive Cement (EC) has been proposed in our previous study [24]. The EC, also known as Soundless Chemical Demolition Agent (SCDA), is a self-stressing cement that contains ordinary Portland cement and an expansive additive that generates high expansion pressures when confined [25]. These expansive pressures induce tensile stress in the surrounding material leading to breakage. In terms of application, the EC is used in construction works to demolish concrete in rehabilitation projects and to split rock blocks in dimension stone quarries [26,27]. More recently, EC has been used to break a biaxially confined rock panel to examine the application of EC for drift development in underground mining environments [28,29].

In our previous work, the proposed Direct Tensile Strength (DTS) test named the Expansive Cement Direct Tensile Strength (ECDTS) test was validated both numerically and experimentally by conducting some tests on a low-tensile strength marble. This was to ensure that the newly proposed ECDTS test, the conventional Glued Ends Direct Tensile Strength (GEDTS) test, and the Brazilian Tensile Strength (BTS) test were easily conducted on the same rock type since the application of the GEDTS test to hard rocks is limited. This paper focuses on the determination of the tensile strength of hard rocks namely Gneiss and Sudbury Breccia using the ECDTS test. This testing approach does not require any of the expensive conventional testing machines, and the results from the different rock types demonstrate the effectiveness and reliability of the ECDTS test over existing methods.

5.2 Experimental Setup

The specimen configuration for the ECDTS test is shown in Fig. 5.2. It is a thick-walled cylinder-shaped specimen with an outer diameter, $d_o = 47.6\, mm\, (1.87'')$ - the size of an NQ core – and a borehole diameter, $d_i = 15.9\, mm\, (5/8'')$. This is to achieve a d_i/d_o ratio of 0.33 which has been demonstrated to produce optimum results in our previous study [24]. The sample has a minimum aspect ratio (borehole length to diameter ratio) of 4 and a base height of 10-12 mm to offset the influence of the closed end of the cylindrical specimen on the strain readings [30].

The experimental setup is shown in Fig. 5.3. As can be seen, a well-designed instrumentation system is set up for strain measurement which is the main output of the ECDTS experimental procedure. Strain gauges are placed in the tangential direction around the outer surface at the top edge of the cylindrical specimen. This location for strain measurement has been proven numerically using Abaqus FE analysis by Adams and Mitri [24] as the only location around the outer surface of the cylindrical specimen where maximum uniaxial strain for direct tensile strength determination exists. The gauges are wired to a data acquisition (DAQ) system to record the strain induced in the specimen as the EC expands. Since the line of fracture is not predetermined, four 20 mm length strain gauges are recommended. Also, the setup has a time-lapse camera mounted on top of the specimen to identify the lead strain gauge (strain gauge through or near which fracture occurs). The lead strain gauge records the highest strain value. To capture the time taken for the first crack in the specimen to occur, an acoustic emission (AE) sensor is positioned on the outer surface at the top part of the specimen where fracture is more likely to be initiated [24].

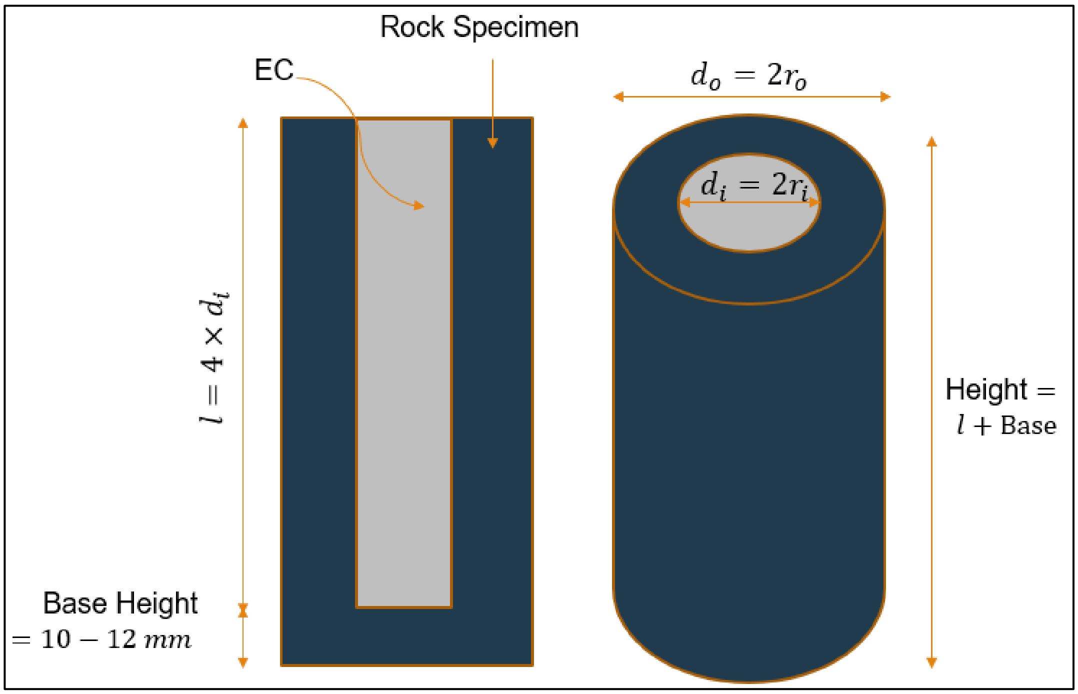

Figure 5.2. The ECDTS Specimen Configuration.

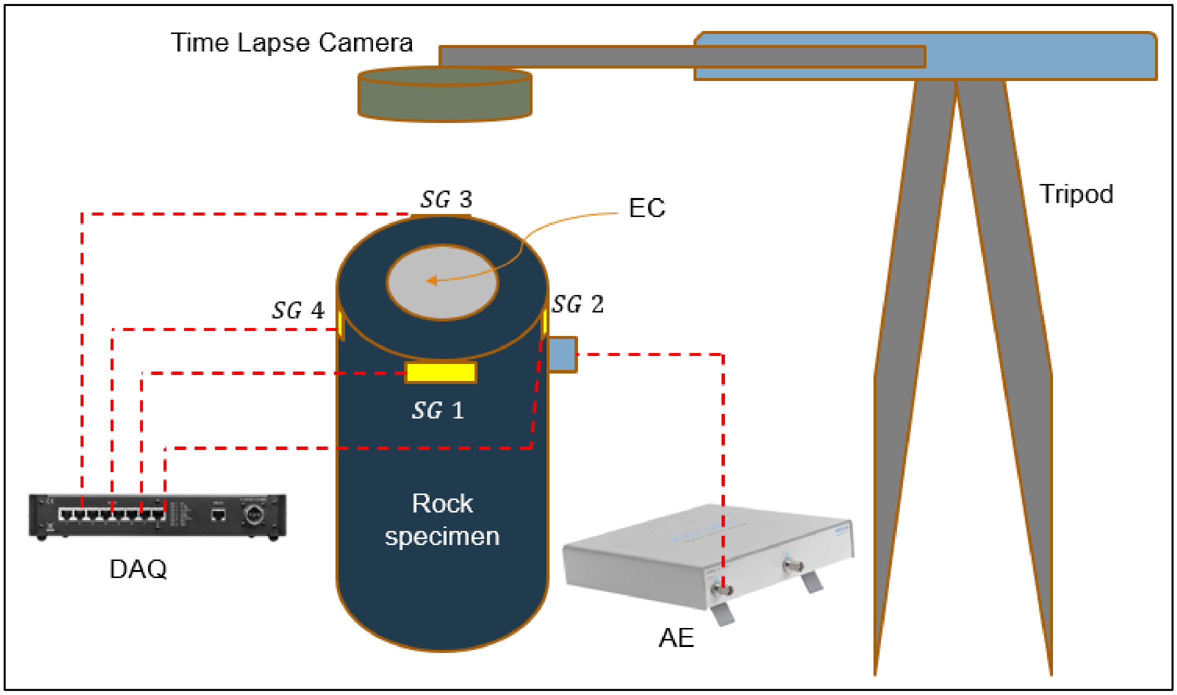

Figure 5.3. Experimental Setup of the ECDTS Test

5.3 Test Methodology

To begin, the experiment is set up as outlined in section 2.1. The expansive cement is mixed with a water-to-cement ratio 0.2 per the manufacturer's recommendations. The slurry is then poured into the borehole by gravity and the test is allowed to run until fracture occurs. The tensile strength of two hard rock types, including Sudbury Breccia and Gneiss are tested in this study. Three ECDTS specimens are prepared and tested for each rock type as shown in Fig. 5.4.

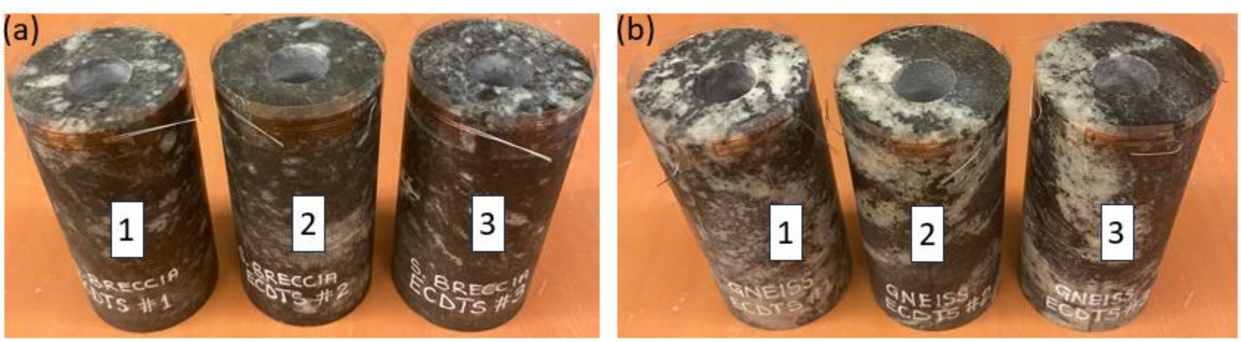

Figure 5.4 Tested Rock Types. (a) Sudbury Breccia. (b) Intermediate Gneiss.

Fig. 5.5 shows the failure pattern of the specimens after the ECDTS test. As can be seen, the failure mode of the specimens for both rock types is purely tensile with fracturing occurring at the section with minimum tensile bearing capacity.

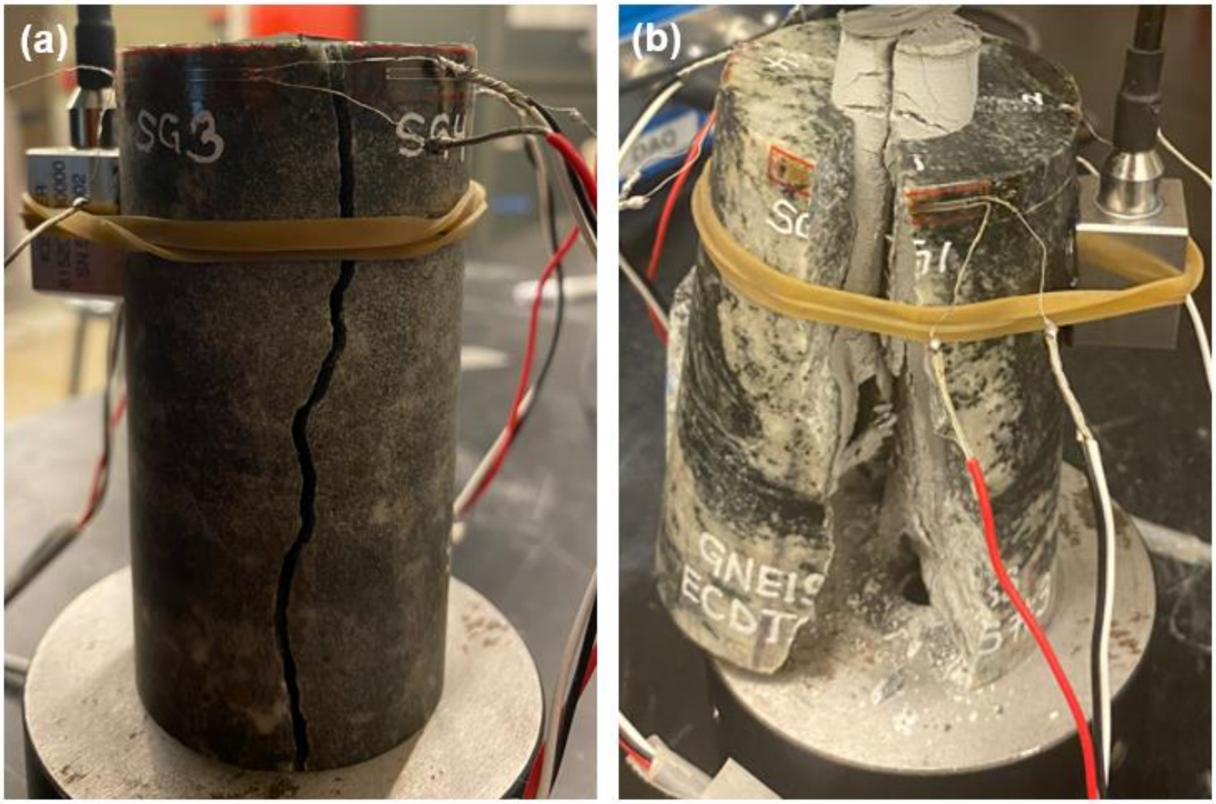

Figure 5.5 Specimen Failure Pattern. (a) Sudbury Breccia Specimen. (b) Intermediate Gneiss Specimen.

Fig. 5.6 shows the strain curves for all tested specimens namely ECDTS-SB-1, ECDTS-SB-2, and ECDTS-SB-3 for Sudbury Breccia and ECDTS-IG-1, ECDTS-IG-2, and ECDTS-IG-3 for Intermediate Gneiss. The linear behavior and sudden drop in the curve at the onset of fracture show the brittleness of the rock material.

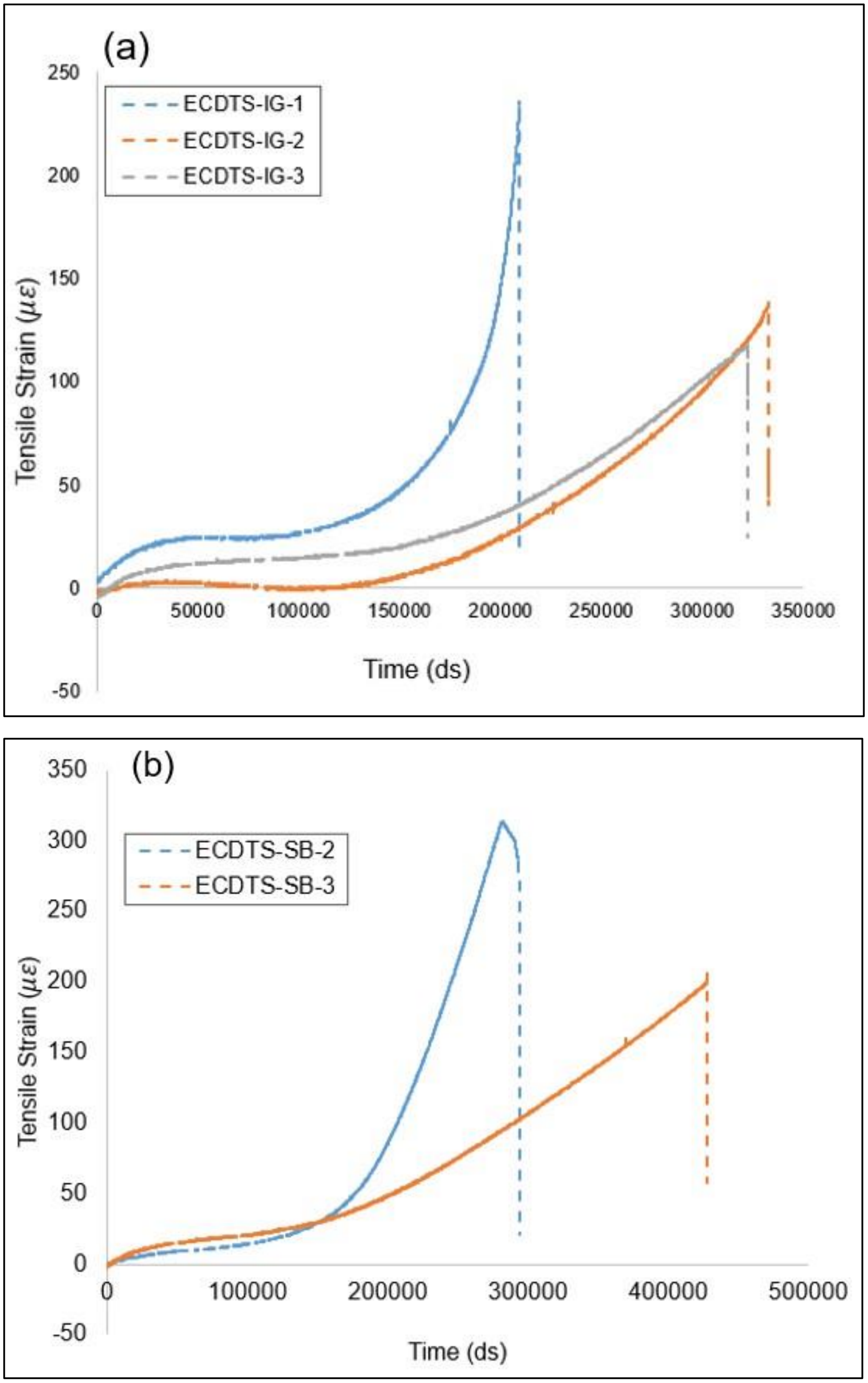

Figure 5.6 Strain Curves for Different Rock Types. (a) Strain Curves for Intermediate Gneiss Specimens. (b) Strain Curves for Sudbury Breccia Specimens.

5.4 Results and Discussions

From the specimen configuration, the tensile strength, σ_t can be determined by applying Lame's thick-walled cylinder formulations. However, in the authors' previous work, analytical and experimental results showed that the local material behaviour at the strain gauge location is linear [24]. As such, the tensile strength can be estimated using Hooke's law as shown in equation 1.

$$\sigma_t = E\varepsilon_0 \qquad (5.1)$$

Where E is the rock material's modulus of elasticity and ε_0 is the recorded peak strain, $\varepsilon_0 = \varepsilon_p$ prior to tensile fracture. Table 5.1 presents the peak strain values of the tested rock specimens and the corresponding tensile strength values obtained from the ECDTS test. To validate these results, a series of tests were conducted using the conventional BTS testing method based on ASTM guidelines. It is noteworthy that tests were also conducted using the conventional GEDTS testing method. However, since the rocks' tensile capacity was much higher than the bonding capacity of the adhesive, failure occurred at the glued interface as shown in Fig. 5.1, rendering the tests invalid.

The Tensile strength values of the BTS specimens are calculated using Equation 5.2 and presented in Table 5.2. Fig. 5.7 shows the test setup, load-time curves, and failure patterns of the BTS specimens for the different rock types.

$$\sigma_t^{BTS} = \frac{2P}{\pi D t} \qquad (5.2)$$

Table 5.1 Tensile strength Values of Tested ECDTS Rock Specimens.

Rock Type	Test #	d_i	d_o	d_i/d_o	Peak Strain ($\mu\varepsilon$)	Pressure (MPa)	Tensile Strength (MPa)
Sudbury Breccia (SB)	ECDTS-SB-T1	15.9	48	0.33	313	104.5	25.0
	ECDTS-SB-T2	15.9	48	0.33	206	68.7	16.5
	ECDTS-SB-T3	15.9	48	0.33	254	84.7	20.3
						Mean =	**20.6**
						Standard Deviation (SD) =	**4.25**
						Coefficient of Variation =	**20.48 %**
Intermediate Gneiss (IG)	ECDTS-IG-T1	15.9	48	0.33	237	80.1	19.2
	ECDTS-IG-T2	15.9	48	0.33	138	46.6	11.2
	ECDTS-IG-T3	15.9	48	0.33	117	39.5	9.5
						Mean =	**13.3**
						Standard Deviation (SD) =	**4.23**
						Coefficient of Variation =	**31.80 %**

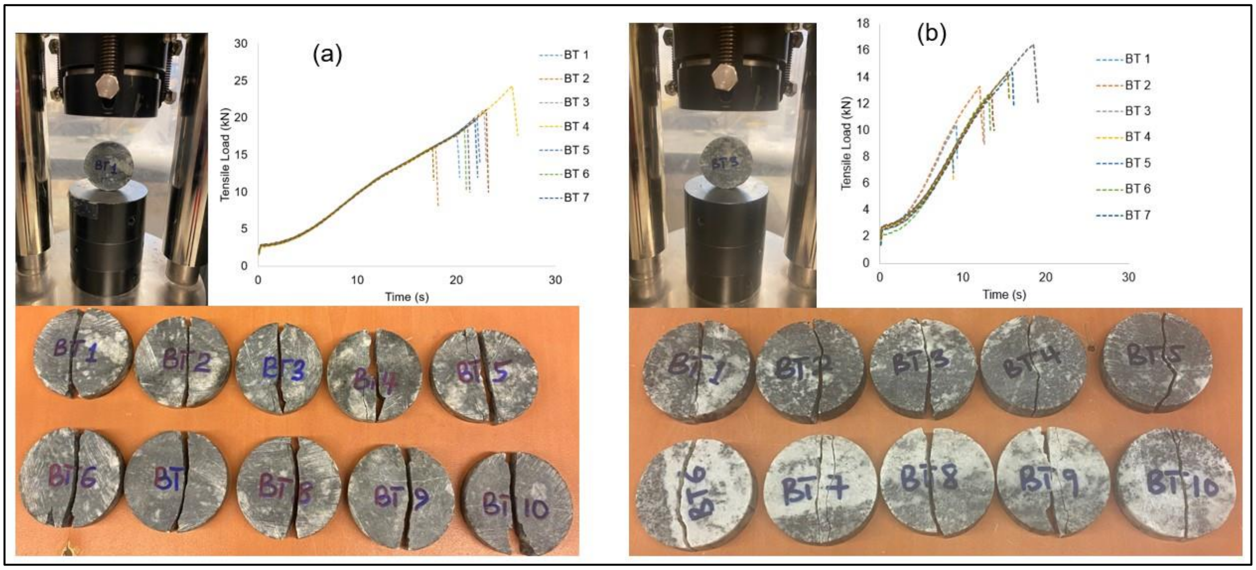

Figure 5.7 Brazilian Test setup, load-time curves, and failure patterns of the BTS specimens. (a) Sudbury Breccia BTS Test. (b) Intermediate Gneiss BTS Test.

Table 5.2 Tensile Strength Values of Tested BTS Rock Specimens.

Rock Type	Test #	Splitting Load (KN)	Tensile Strength (MPa)
Sudbury Breccia (SB)	BTS-1	18.0	23.4
	BTS-2	16.3	21.2
	BTS-3	21.1	27.2
	BTS-4	24.4	29.8
	BTS-5	20.0	24.8
	BTS-6	18.7	23.7
	BTS-7	20.0	24.4
	BTS-8	21.2	27.1
	BTS-9	18.9	21.9
	BTS-10	16.0	19.9
		Average =	**24.3 MPa**
		Standard Deviation (SD) =	**2.875**
		Coefficient of Variation (COV) =	**8.81 %**
Intermediate Gneiss (IG)	BTS-1	10.4	13.1
	BTS-2	13.3	16.1
	BTS-3	11.9	15.0
	BTS-4	8.1	10.5
	BTS-5	8.3	11.0
	BTS-6	12.8	14.9
	BTS-7	14.4	18.5
	BTS-8	12.8	17.1
	BTS-9	16.5	21.6
	BTS-10	14.3	17.4
		Average =	**15.5**
		Standard Deviation (SD) =	**3.230**
		Coefficient of Variation (COV) =	**20.81 %**

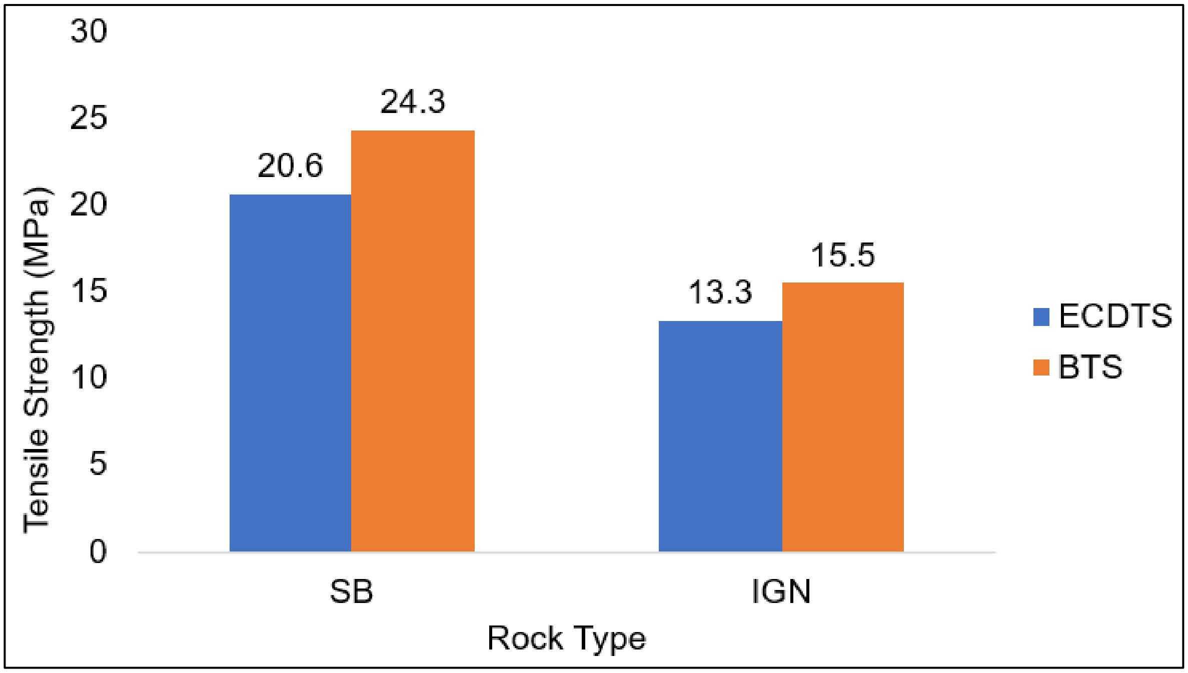

Figure 5.8 Tensile strength values of Sudbury breccia (SB) and Intermediate Gneiss (IG) obtained from different testing methods.

Comparing Tables 5.1 and 5.2, it is seen that the tensile strength values obtained from both Testing methods show significant heterogeneity in the two rock materials being tested, with the intermediate Gneiss exhibiting pronounce heterogeneous behavior than the Sudbury breccia. Also, Fig. 5.8 compares the average tensile strength values from both testing methods for the two rock types. It is seen that the values for Sudbury breccia and Intermediate Gneiss determined by the direct (ECDTS) method are 15.2% and 14.2% less than those obtained by the indirect (BTS) method respectively. This is expected since the tensile strength of rock materials is well-known to be overestimated by the BTS testing method as has been reported by many researchers [31,17].

5.5 Conclusions

This paper presents the tensile strength results of Sudbury Breccia and Intermediate Gneiss obtained for hard rocks using a novel test method known as the Expansive Cement Direct Tensile Strength (ECDTS) test. The results show the tensile strength of the Sudbury Breccia is 20.6 MPa and that of the Intermediate Gneiss is 13.3

MPa. Such high tensile strength values are not possible to obtain from the conventional glued end direct tensile strength (GEDTS) test due to the bond strength limitation of the glue. A series of tests were performed on both rock types using the BTS test to serve as the basis for comparison. The results showed that the ECDTS results are 14-15% lower than BTS as expected. The newly developed method has significant merits over the existing test methods. In contrast to the conventional indirect testing methods, the ECDTS method produces results that realistically reflect the true tensile strength of brittle materials since failure in the specimen occurs in a pure uniaxial tensile model and the plane of failure is not pre-determined as in some methods like BTS. Unlike the glue in direct tension methods, the expansive pressure generated by the EC is high enough to test any high-tensile strength hard rock. Additionally, this method is far more affordable than current testing methods requiring specialized setups and loading equipment.

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Chapter 6 Conclusions

6.1 Summary and Conclusions

This study is a two-phase research that aims to provide an alternative means to determine the tensile strength of brittle materials such as rocks and concrete. The primary goal of this research was to develop a testing method that eliminates the use of adhesives for direct tensile strength determination of brittle materials with high tensile capacities. To do this, a novel testing method known as the Expansive Cement Direct Tensile Strength (ECDTS) test was developed.

The first phase of the research, termed as method validation phase, was to ascertain the veracity of the proposed method by conducting both numerical modeling analyses and a series of experimental programs. Abaqus FEM software was used to design and analyze the ECDTS test model which has a thick-walled cylinder configuration filled with expansive cement. Multiple series of experiments on typical rock specimens from low-tensile marble cubes were carried out to corroborate the numerical findings. The experimental program involved the determination of the tensile strength of marble using the BTS, GEDTS, and the proposed ECDTS tests. To validate the new method, a comparison of the test results was made. It is found that the results obtained from the newly proposed ECDTS test are in good agreement with the conventional testing methods including the GEDTS and BTS tests. In addition, a suitable specimen configuration was proposed in the first phase for determining the direct tensile strength of brittle materials with the new method.

The second phase, termed as method application phase, was to examine the applicability of the ECDTS test on hard rocks by conducting some experiments on high-tensile strength rocks. The rock types selected for this phase were Sudbury breccia and Intermediate Gneiss which are known to have high tensile capacities. Again, a series of experimental programs were carried out to determine the tensile strength of these rock types using the BTS, GEDTS, and the new ECDTS tests. The GEDTS tests produced no results as specimens failed at the glued interface. This was because the bonding capacity of the glue was lower than the tensile capacity of the rock specimen. To verify the applicability of the ECDTS test to hard rocks, its results were compared to the results from the BTS test. It was found that the ECDTS results for the two rock types are 14-15% lower than those from the BTS test demonstrating the effectiveness and consistency of the ECDTS test. This was to be expected as the BTS test is known to overestimate the tensile strength of brittle materials such as rocks by 10 - 40% according to literature.

Based on the outcomes of this research, the following conclusions can be drawn:

- The tensile strength values are directly proportional to the borehole sizes of the tested cylindrical rock specimen. Increasing the d_i/d_o ratio slightly increases the tensile strength value albeit with a coefficient of variation of only 0.1336. Among the tested rock specimens, the tensile strength values obtained for specimen configuration with d_i/d_o ratio of 0.33 appear to be in very good agreement with the results obtained for the ISRM and ASTM suggested GEDTS testing method demonstrating the validity and practical ability of the proposed method. For this reason, the d_i/d_o ratio of 0.33 is recommended as the best specimen configuration for the ECDTS test taking into consideration the sample base thickness of 12 mm and outer diameter of an NQ core size (48 mm).
- The linearity of the strain curve at the strain gauge location prior to tensile failure is validated, hence Hooke's law is used to calculate the tensile strength.
- Compared to the GEDTS test, the new ECDTS method does not require highly sophisticated tensile loading machine. This makes the ECDTS test a much more affordable option for directly determining the tensile strength of brittle materials such as rock and concrete.

6.2 Recommendations for Future Research

Based on the findings of the research, some recommendations for future research are as follows:

- Expand the experimental program to include a wider range of rock types to further validate the consistency of the ECDTS testing method.
- Errors associated with strain gauge readings could influence the tensile strength estimation. It will therefore be interesting to adopt a more advanced strain measurement technique such as Digital Image Correlation to measure deformations occurring on the surface of the ECDTS specimen as the expansive cement expands.
- It was found during the experimental program that increasing the borehole size increases the critical strain, which in turn increases the calculated tensile strength value because the analytical model is strain dependent. It will be interesting to investigate numerically, analytically, and/or experimentally why the critical strain increases when the specimen annulus reduces in thickness.

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