OPTIMIZATION OF CONCRETE BEAMS WITH RELIABILITY CONSTRAINTS

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By

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ABSTRACT

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Submitted to the Department of Civil Engineering, McGill University, on August 5th, 1968, in partial fulfillment of the requirements for the degree of Master of Civil Engineering.

This thesis attempts to investigate the relationship between safety and optimum cost of reinforced concrete beams as related to simple-span highway bridges based on probability.

There have been several proposals for a realistic evaluation of safety based on probability studies as the traditional approach to safety does not ensure uniform safety levels. In general, probabilistic approaches to safety are based on statistical distributions of design variables and the operational characteristics of a structural mechanism.

The optimization problem is solved by an iterative-search method. Failure probabilities and the statistical parameters of certain design variables constitute the input variables whose values determine the optimum configuration of the beam characteristics.

As a result of this study, it may be concluded that the cost-safety relationship is linear, or nearly linear, but practical application of probabilistic formulations are limited because of a lack of knowledge of several components of design.

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NOTATIONS

The following notations are used in this study:

R	=	Resistance
S	=	Load
$\overline{R}$	=	Mean of Resistance
$\overline{S}$	=	Mean of Load
σ_R	=	Standard Deviation of Resistance
σ_S	=	Standard Deviation of Load
V_R	=	Coefficient of Variation of Resistance
V_S	=	Coefficient of Variation of Load
n	=	Safety Parameter (R/S)
r	=	Safety Parameter (R-S)
P_F	=	Probability of Failure
M_L	=	Live Load Moment
M_D	=	Dead Load Moment
M_A	=	Actual Moment Capacity of Beam
M_S	=	Total Applied Moment ($M_L + M_D$)
m	=	M_D/M_L = dead load/live load ratio.
B	=	Ratio of Effective Depth to Width of Beam

All other notations used in this study are either slight modifications of the above or are the standard notations used in A.C.I. Code (A.C.I. 318-63). In cases where the notations are none of the above, they are defined as it becomes necessary.

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INTRODUCTION

A fundamental concern of all structural designers and their clients is this: that structures be both safe and economical in the sense that they fulfill their functional purposes without being a hazard to society, and the cost must be the minimum that can be possibly achieved consistent with safety.

With the development of new methods in mathematics, the compilation of new data and the increasing application of modern techniques to engineering design and construction, it is obvious that the classical maxim: maximum safety at minimum cost, is contradictory.

Basically, the major variables under the control of the structural designer are form, detailed geometry, materials and proportions. A designer may use structural steel, reinforced concrete or pre-stressed concrete. Each structural member, say, a reinforced concrete beam, consists of a number of design variables such as beam shape and size, percentage of steel, strengths of concrete and steel, etc. The arrangement of the structural members, the materials used and the values and proportions of the design variables determine the overall operation of the structural mechanism.

Also, the level of structural operation depends on the applied forces. Thus the design variables and the applied forces are not independent. On the one hand, the combination of design variables depends on the applied forces; and on the other, the arrangement of the structural members is limited by the purposes or goals of the structure, and also influences the intensity and distribution of the applied forces. The general design process, therefore, must be considered as a totality taking all the important parameters into consideration.

1.1 Safety Factor in Design.

The concept of "factor of safety" or "safety factor" forms a fundamental premise on which the design of structures is based, but the basis of this concept has only been investigated within recent years.

Pugsley,[47] in 1951, placed both the design process and the concept of "safety factor" in historical perspective. That is, he showed how the design process evolved and how the "safety factor" became a fundamental part of the design process.

> "It was early apparent to reinforced concrete engineers that, - - - , as soon as the external loads became such that yielding of the steel reinforcing-rods occurred, then large cracks and some breakdown of adhesion between concrete and steel arose. It was thus essential to secure that such conditions did not arise under working loads; and to do this, it became customary to adopt limiting working stress in the steel that was only half the estimated yield stress."[47]

Pugsley further showed "how one ad hoc case after another has been dealt with until an engineering tradition has been set up."

Thus, the "factor of safety" is an arbitrarily chosen figure based on stress-strain relation of the materials and fitted into the design process in an attempt to prevent the collapse or failure of the structure. The adoption of this method of using only "half the estimated yield stress" reveals on the part of the engineer the subjective striving for "an adequate measure of safety as well as a consciousness of the limitations of his knowledge and the arbitrariness of his assumptions."[24]

Since the concept of "safety factor" in the design process has been discussed in great depth by Pugsley[47], Baker[7,8], Freudenthal[23,24], Turkstra[61] and others,[6,34,55,68] it is not necessary to dwell on it in this study. However, an exposition of the unrealistic basis of this concept in the face of new data and accumulated knowledge of structural behaviour will be given.

There are three main stages at which the "safety factor" enters the design process.

(1) The choise of the strength of materials.

In choosing the strength of materials to be used, the "permissible stress" allowed by the Code[4] is only a fraction of the strength of the materials, be it steel or concrete. And this "permissible stress" is arbitrarily chosen.[24]

(2) The choice of the design loads.

The lack of an objective basis for the determination of design loads is clearly shown in the variety of loadings now used in design.[7] Actual loading conditions are approximated by "standard loads" which at critical points in the operation of a structure underestimate sometimes the actual loading conditions and at other times overestimate the actual loading conditions to such an extent that the loads used result in a very conservative design.

(3) The determination of design bending moment and shear.

Finally, the "safety factor" used in the operational characteristics (bending, shear, torsion, etc.) vary from one characteristic to another. For example, the "safety factor" used in the computation of flexural resistance is different to that used for shear[4]. In fact, the "safety factor" can vary from 1.0 to 10.0 for structural members[52] and even one particular design possesses more than one "safety factor."[7,8].

The fact that conventionally designed structures almost always appear safe does not necessarily show the validity of the present concept of "safety factor" and the reliability of the design process, but more often indicates the caution and conservatism exercised by conventional designers and expressed in the code.[52]

In actual practice, the variables in a design can be shifted around to obtain a safe design on the basis of experience without having any comprehensive knowledge of the factors influencing the operation of the structure, so much so that the structure may collapse due to conditions not considered or known.[61] Thus the "safety factor" as presently used compensates for the designer's ignorance and uncertainty. It is only by a re-examination of the concept of safety can structural design be founded on a more realistic basis.

1.2 The Concept of Safety.

The first real insight into the concept of safety was given by Freudenthal[23,24] in his analysis of the present design process and the contemporary basis of the "safety factor". Since then, many engineers have attempted to formulate this concept, both philosophically[6,12,31,60] and mathematically.[14,44,61]

The safety of a structure involves two basic parameters:

(1) Load

(2) Resistance

Thus, any concept of safety must be based on the interrelationship of these two parameters; any mathematical formulation of safety must be a formulation involving these two basic parameters; any problems encountered in the determination of a value of safety must be problems resulting from the characteristics of load and resistance and the relationship existing between them in the operation of a structure.

Due to the goals of structural design, many writers have dealt with safety as a two concept formulation. Freudenthal[24], for example, says that the safety of a structure involves two aspects -- serviceability and failure.

In the progress report of the A.S.C.E. Committee on safety factors, Julian[34] puts forward the following concept of safety:

(1) "Minimum Required Factor of Safety to assume that a given probability of failure P_F of the structure is not exceeded, is defined as the ratio (greater than unity) of R_o, the mean (arithmetic average) estimated resistance to collapse during the anticipated life of a large number of structures meant to be identical with the subject structure, and W_o the mean load effect for which the subject structure is designed."

(2) "Minimum Required Factor of Serviceability to assume that a given probability of the structure becoming unserviceable, for the purpose and during the anticipated life for which it is designed, is not exceeded, is defined as a similar ratio but with respect to serviceability rather than collapse."[34]

The use of these two criteria for determining the safety of structures can be quite misleading and confusing when taken without question. It may apply to some structures and not to others, depending on the operation of the structure and the purposes for which it is used. In some cases, both concepts will be one and the same thing; that is, there will be no distinction between them. In others, they will have to be considered separately as two criteria. By examining the nature of the load and resistance parameters, a better understanding of the concept of safety and the safety criteria can be obtained.

Both load and resistance are in themselves very complex parameters, for in attempting to find their "true" character, other variables have to be considered. In a mathematical analysis of load and resistance, all the variables which influence these two parameters cannot be taken into account. Firstly, if all the variables are considered, the analysis would become quite complex, and, secondly, there is always an "unknown" in structural design. If the exact value of each variable was known before the construction of, say, a beam, then the precise value of safety could be easily computed. However, this is not the case. There is always some variation in the values of variables which Freudenthal[24] attributes to:

(1) the imperfections of human observations and actions (uncertainty),

(2) the imperfections of intellectual concepts devised to reproduce physical phenomenon (ignorance).

The causes of the factor of ignorance can be said to be attributable causes, while the causes of the factor of uncertainty can be called chance causes.

These two factors influence the design variables in both the load and resistance parameters. It is as a consequence of this that the load applied to and the resistance of a structural mechanism can only be realistically expressed in a probabilistic framework. The method of evaluating the safety of a structure, therefore, must involve "prediction within limits" by using the accumulated wealth of data available and predicting the way in which a certain structural phenomenon may be expected to vary. As Freudenthal [23] says: "Prediction within limits means that one can state the probability that an individual value will fall within given limits." Thus, safety cannot be predicted with certainty but only with a high degree of probability.

1.3 Safety and Failure Probability.

Laws of the operation of structural systems can be considered as a combination of functional and statistical relationships; functional in the sense of the theory of structural behaviour and statistical in the sense that real physical properties appear only as variables in the functional relationships. Further, most functional relationships in structural design are by their very nature and derivation statistic. It is logical, therefore, that frequency distributions form an integral part of the information required and relationships to be determined in the evaluation of safety in a probabilistic manner.

A structural mechanism is considered safe when its resistance is greater than the load applied, But implicit in this definition of safety is its opposite, that is, the failure of the structure. The one implies the other. As Asplund[6] pointed out: "the fundamental phenomenon connected with what is called safety is not safety at all but lack of safety and failure." Whatever terms might be coined to express structural safety - "lack of safety", "failure", "risk of failure", etc., the fundamental point is that the safety of structures can only be truly formulated as a statistical relationship between load and resistance, for both are random variables. This statistical relationship can only be realistically expressed by the probability of failure or probability of survival of the structural mechanism.

1.4 Economy

Within the present design procedure, any structure, from the just strong enough to the infinitely strong, is considered adequate or safe, and economy is based purely on cost.

Contemporary designers, no doubt, consider economy a major factor in design. But on what basis? How should a designer compare the costs of alternative designs in order to make a choice? Can a less costly structure serve the required purposes as well as the more costly? These questions, present design methods cannot answer.

If one structure functioned better than another but cost more, then there is an obvious conflict between performance and cost. In fact, designers juggle with section properties to achieve economy relative to fixed code requirements. But there is no guarantee that the code requirements give designs that are equally safe.

In order to make a realistic choice from a number of alternative designs, economy must be considered in the light of the relation between cost and safety and, on this basis, an optimum balance can be achieved.

1.5 Object and Scope.

The original purpose of this thesis was to obtain the optimum cost of simple-span bridges and to relate this cost to the safety of the bridge structure. However, neither cost nor safety is absolute; for the designer's control is severely limited due to uncertainties inherent in the problem of design. And, further, such an analysis would be very complex and exhaustive.

The object of this present study, therefore, is to determine the optimum combination of design variables, based on cost criteria, of a simple-span reinforced concrete bridge to satisfy certain safety requirements expressed in a probabilistic manner.

Even with such an objective, there are certain natural limitations to the range of the analysis. There are a number of geometric arrangements, shapes of structural elements and almost an infinite combination of design variables that can be used. Consequently, only a beam and slab bridge will be considered; the beam being of rectangular cross-section. Also, the analysis will be limited to a consideration of the beam cross-section at mid-span, and a specified set of alternative combinations of certain design variables will be considered. The above-described bridge layout is chosen as it is perhaps the simplest and most popular pattern encountered in actual design problems.

The cost of a bridge structure consists of both initial and long-term costs. This study will be limited to an investigation based on initial cost, the reasons for which will become obvious in Chapter Three. However, the initial cost of a bridge structure depends to a great extent on the cost of materials, formwork, falsework and erection. The other determinants in the initial cost function, such as cost of design, can be considered constant regardless of beam depth, percentage reinforcement, etc. Herein, only materials cost will be considered. The inclusion of formwork, falsework and erection costs will not only make the problem much more complex, but these costs vary significantly with methods used. And the revolution in formwork, falsework and erection methods is proceeding at such a rapid pace that what was considered standard yesterday is today obsolete. A clearer insight into the complex relationship between cost and safety might therefore be obtained by considering only materials cost at this stage.

A bridge beam can fail in a number of modes - flexure, shear, torsion and fatigue. In this study, only flexural failure will be considered.

Although shear failure is important in bridge structures, the mechanism of shear failure remains a riddle to engineers. Many studies[5,21,50,65,66] have been carried out to investigate the phenomena of shear failure, but no satisfactory theoretical model has been formulated. This is clearly shown by the number of studies that have recently been done and the wide variation in the results obtained[11,35,37,38,63]. The formula most widely used at present is extremely conservative and shows no definite correlation with test results (Fig.1-1)[3]. It gives only a lower limit.

Failure due to bond, torsion and fatigue is not considered. Torsion is unimportant in bridge structures as a result of the monolithic construction of beams and slab. Unfortunately, no practically applicable and satisfactory theoretical model has been formulated to represent fatigue failure as shown by the A.A.S.H.O. tests[1], and with the development of deformed bars, methods of detailing, etc., bond failure has been shown to occur only after the tension steel has yielded.

It is not the purpose of this study to yield results directly applicable to actual design problems but by analysing a reinforced concrete bridge cross-section in as detailed a manner as possible, a method is developed which relates cost to safety on a realistic and practical basis and which gives some insight into the properties and application of the cost-safety relationship.

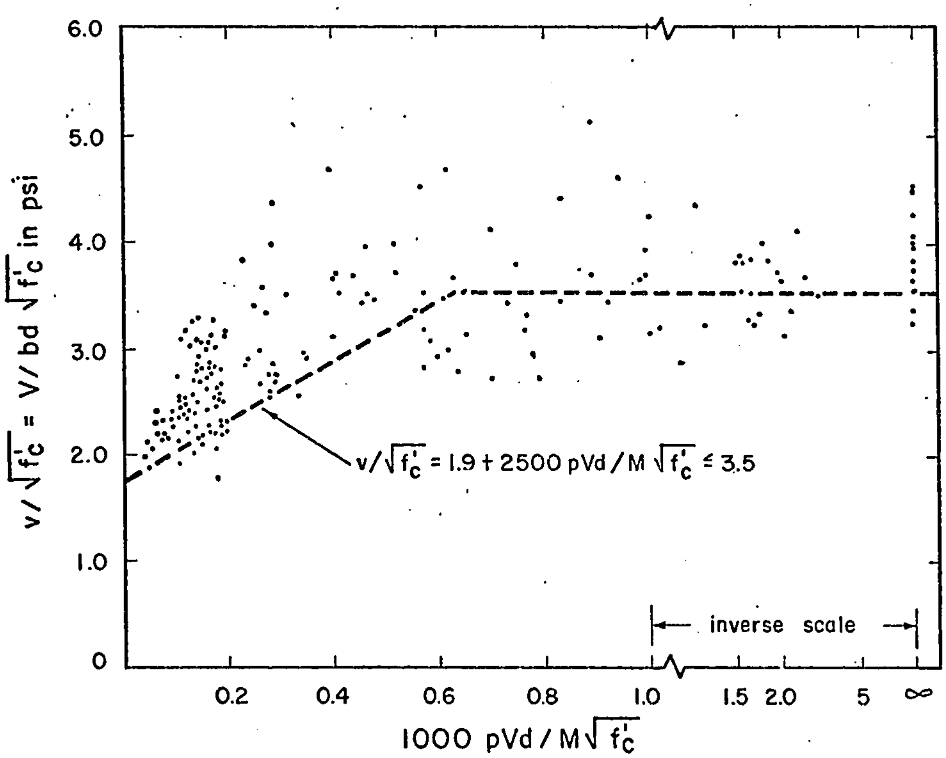


FIG. 1.1 ASCE – ACI COMMITTEE SHEAR FORMULA

II

LOAD AND RESISTANCE PARAMETERS

Recent writers have dealt with load and resistance in a generalised manner when discussing and evaluating the safety parameter or the probability of failure, but such generalisations can only indicate a broad application of the concept of safety to structural design. However, in actual practice, the problems encountered in the computation of the safety parameter varies for different classes of structures. Therefore each particular class of structures must be investigated from basic principles, examining the nature and complexity of the problem and developing a procedure that is directly applicable to that specific class of structures. In this chapter, therefore, the factors that influence the load and resistance parameters and, as a result, the frequency distributions of these parameters, will be investigated.

2.1 Loading Conditions.

It is not possible to take into consideration all the factors influencing the frequency distribution or statistical variation of the load parameter. Also, some of the factors which influence the load parameter are only vaguely known and understood. In general, the conditions of loading and the factors influencing these conditions are perhaps the least known variables in the design process.

The basic classes of loads to which a structure is subjected during its operational life are:

(1) Dead Loads.

(2) Live Loads.

2.1.1 Dead Loads.

The dead load can be categorised as the weights of the materials which make up the permanent features of the structure; such as concrete, steel, timber, railings, fittings, etc. These loads can be considered as fixed in intensity and location; that is, they are not movable except when alterations are made to the structure. The factors which influence the variation of dead loads are mainly the dimensions of members and the specific weights of materials.

Although in the design process both the specific weights of the materials and the dimensions of members are fixed, yet in the construction process one can never obtain the precise design values.

The specific weights of the materials are often different from the designed values as a result of errors in workmanship, quality of materials, proportioning of materials, etc. There are variations in the specific weights even within one particular member of the structure. On the other hand, the variations in the dimension of members are mainly due to errors in workmanship.

With the development of modern methods, the influence of these factors can be greatly reduced. As a result, the dead load of a structure is presently considered fixed; that is, as a non-statistic.

2.1.2. Live Loads

Live loads, unlike dead loads, consist of movable loads, or loads which are not a permanent feature of the structure. Such loads, for example, are chairs, desks, and people in an office building, or cars, buses and trucks on a highway bridge. These loads, in general, vary in a statistical manner, except in the case of maximum load intensity of relatively high frequency of occurrence, where the load can be specified in a non-statistical manner (e.g. warehouses, storage tanks, train loads, etc.) The main factors which influence live load effects, particularly those on highway bridges, are:

(1) Intensity and variation of intensity of loads.

(2) Duration of loads.

(3) Frequency and sequence of load applications.

(4) Mechanical properties of the structure.

The influence of these factors on load effects would vary for different classes of structures. The evaluation of all these variables would require a very complex analysis, but for a particular class of structures some variables have negligible effect and can be eliminated.

2.1.3. Bridge Loads.

Perhaps the most complicated analysis of live loads and their effects is that required for highway bridges. Stephenson and Jakkula[57] have carried out a comprehensive analysis of vehicle loads and their effects on bridge structures. As a result they have formulated a method which converts heavy vehicle loads in terms of "standardised equivalent loads" and they show how the frequency distributions of various intensities of these equivalent loads provide a simple, precise and yet rational means for measuring the level or levels of heavy motor vehicle operation corresponding to various traffic conditions and their effects on the operational characteristics of highway bridges. However, Stephenson's results are old and as such are only used as a guide.

According to Stephenson, the critical operational characteristics produced in bridges by heavy vehicle loads are influenced by no less than six variables:

(1) Span length of bridge.

(2) Gross weight of vehicle.

(3) Wheel base length of vehicle.

(4) Number of axles.

(5) Spacing of axles.

(6) Distribution of gross weight of vehicles.

It can thus be seen how complex is the analysis of vehicle loads on highway bridges. By converting all heavy vehicle loads into "standard loads" and by using various types of standard loads, Stephenson was able to simplify the problem, to a certain extent, so that the frequency distributions of these loads could be determined and the operational characteristics computed. This aspect of the analysis will be dealt with in greater detail in section 2.3.5 of this chapter.

The problem of the evaluation of loading conditions and their effects on structural operation is not an easy one. Although many studies have been made on the factors influencing the loads applied to structures, knowledge is still lacking as to the nature of load applications and more data is required for accurate analysis of load distribution.

2.2 Resistance Parameter.

There are many factors which influence the resistance of a structure such as strengths of materials, dimensions and spacing of members, percentage of steel in reinforced concrete members, etc. However, the influence of these design variables is expressed in the more general operational characteristics of flexure, shear, bond and torsion. In this analysis only flexure will be considered. Finally, the effects of these operational characteristics are all combined in what may be termed the overall behaviour of the structural mechanism.

2.2.1 Overall Structural Behaviour.

Although there have been many experimental studies on small concrete beams to examine their structural behaviour, very few prototype experiments have been carried out on bridge response to vehicle loads. Perhaps the most comprehensive experimental analysis on the structural behaviour of prototype bridges has been that undertaken by the American Association of State Highway Officials (A.A.S.H.O.)[1].

The object of that study was to determine the behaviour of certain simple-span highway bridges to the repeated application of vehicle loads and to test the reliability of the ultimate strength theory for predicting the capacity of bridge structures. The important results of the A.A.S.H.O. tests relevant to this present study can be summarised as follows:

(1) Concrete bridge structures subjected to vehicle loads do not collapse or fall apart suddenly, but gradually approach total collapse after extensive cracking of the beams accompanied by increasing permanent deformation and crushing of the compression concrete.

(2) In the case of equally reinforced bridge beams, the whole bridge behaves as one beam for all practical purposes and the loads were equally distributed to the beams. (That is, for one lane bridges.)

(3) The ultimate strength formula shows good correlation with experi-

mental results.

(4) The dynamic response of highway bridges to moving loads is a complex phenomenon and depends on the approach profile and surface conditions of the bridge deck, the variation of pressure in the tyres of vehicles, the suspension system of the vehicle, the frequency of vibration of the vehicle and of the bridge, the weight on the axles of the vehicle, etc. In fact, no definite correlation was obtained between the experimental results and theoretical models.

The A.A.S.H.O. experimental studies, therefore, indicated the behaviour of a bridge structure as a whole under increasing vehicle loads. However, the beams failed in flexure, more particularly in one mode of flexural failure - tension.

2.2.2 Flexural Failure.

Flexural failure of beams consists, basically, in the crushing of concrete in the compression zone. However, this may be a primary or secondary compression failure.

Firstly, the reinforcing steel in the tensile zone may yield and cause extensive tensile cracking. This tensile yielding of the steel will continue and a redistribution of stresses will take place until the ultimate capacity of the concrete in the compression zone is reached. This mode of failure was clearly shown in the A.A.S.H.O. tests[1].

On the other hand, the conditions might be the reverse. That is, failure could be caused by the crushing of the concrete compression zone while the stress of the tensile steel is still below the yield point; the yield point being finally reached after increasing permanent deformation and extensive cracking of the beams. The former is secondary compression failure and the latter primary compression failure.

There are, therefore, two modes of flexural failure:

(1) Yielding of tensile steel accompanied by extensive cracking of concrete followed by secondary failure of the concrete compression zone.

(2) Crushing of the concrete compression zone followed by extensive tensile cracking and yielding of the tensile steel.

Most present day designers attempt to avoid the occurrence of the second mode as there is no previous warning of failure; the concrete crushing suddenly.

2.2.3 Ultimate Capacity

The formula which best predicts flexural capacity of reinforced concrete beams is the ultimate strength formula developed by C.S. Whitney[64]. Since Whitney's pioneering work, this theory has been refined and developed by many writers[29,30,41]. It is not necessary to discuss the basic assumptions and formulation of the ultimate strength theory as it is well known and used in everyday design problems.

The basic formulas used in ultimate strength design are:

$$M_u = Asfyd\,(1-0.59q) \quad - - - - - - - - - \quad (2.1)$$

where $q = Pfy/f^1c$

for failure in tension; and for failure in compression, when compression steel is used,

$$M_u = (A_s - A^1_s)\,fy\,(d-a/2) + A^1_s\,f^1_y\,(d-d^1) \qquad (2.2)$$

where $a \doteq (As - A^1_s)\,fy/0.85\,f^1_c\,b.$

When there is no steel in the compression zone of the concrete, Whitney[64] showed that the ultimate capacity in compression depends only on the strength of concrete and the dimension of the beam. Thus, as long as there is sufficient tensile steel to develop the full capacity of the compression concrete, excess steel in the tensile zone does not contribute to the flexural capacity of the beam. By carrying out a lest squares analysis of the results for compression failure, Whitney obtained the formula:

$$M_u = f'_c\, bd^2 / 3 \qquad --- \qquad (2.3)$$

for $f'_c \gtrapprox 2000$ psi. Results obtained by Cox[16] and Evans[19] confirmed Whitney's compression theory. Fig.2.1 shows the correlation between formula (2.3) and test results obtained by Evans. The results of 364 beam tests[41] for tension failure are shown in Fig.2.2A. Fig. 2-2B shows the relationship between $\alpha = \overline{M}_u / M_u$ and q, where $\overline{M}_u$ is an empirical formula representing the mean of the test results for tension failure. It is clearly seen from the figure that Equation (2.1) is biased with q. Thus for the purpose of probabilistic studies a statistical analysis of test results is undertaken in the next section.

2.3 Statistical Analysis.

Repeated measurements of design variables seldom give identical values and, in fact, samples taken, say, from a particular mix of concrete or grade of steel, would give values that vary within a certain range. This situation makes exact prediction impossible, but by repeated measurements, an estimate can be made of the relative frequencies of the possible values of the design variables.

On the other hand, safety, or lack of safety, can only be expressed as a relationship between load and resistance which are, in themselves, variables. Thus, not only

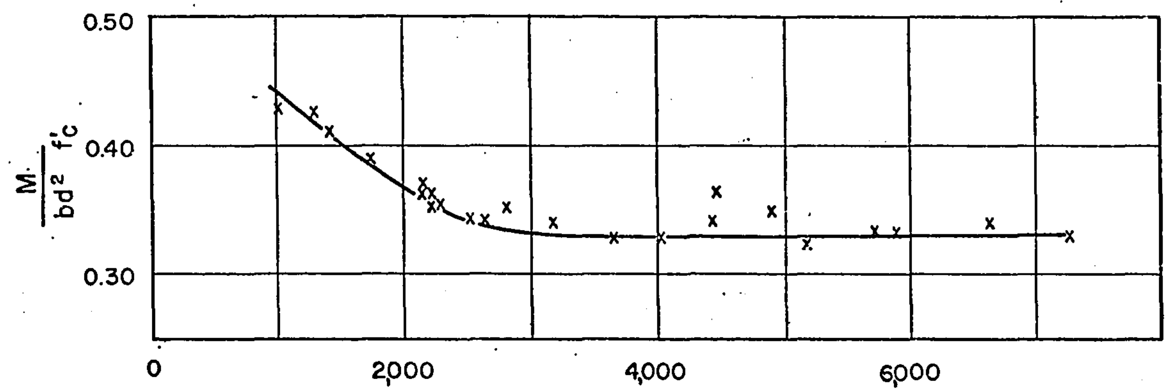


FIG. 2-1 FLEXURAL CAPACITY OF BEAMS CONTROLLED BY COMPRESSION

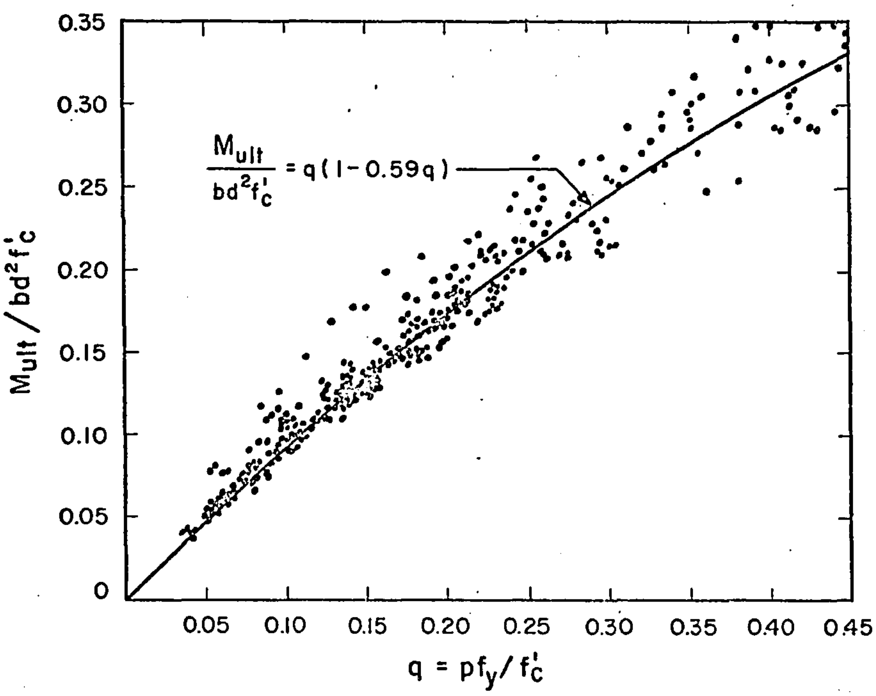


FIG. 2-2A-TEST OF 364 BEAMS CONTROLLED BY TENSION

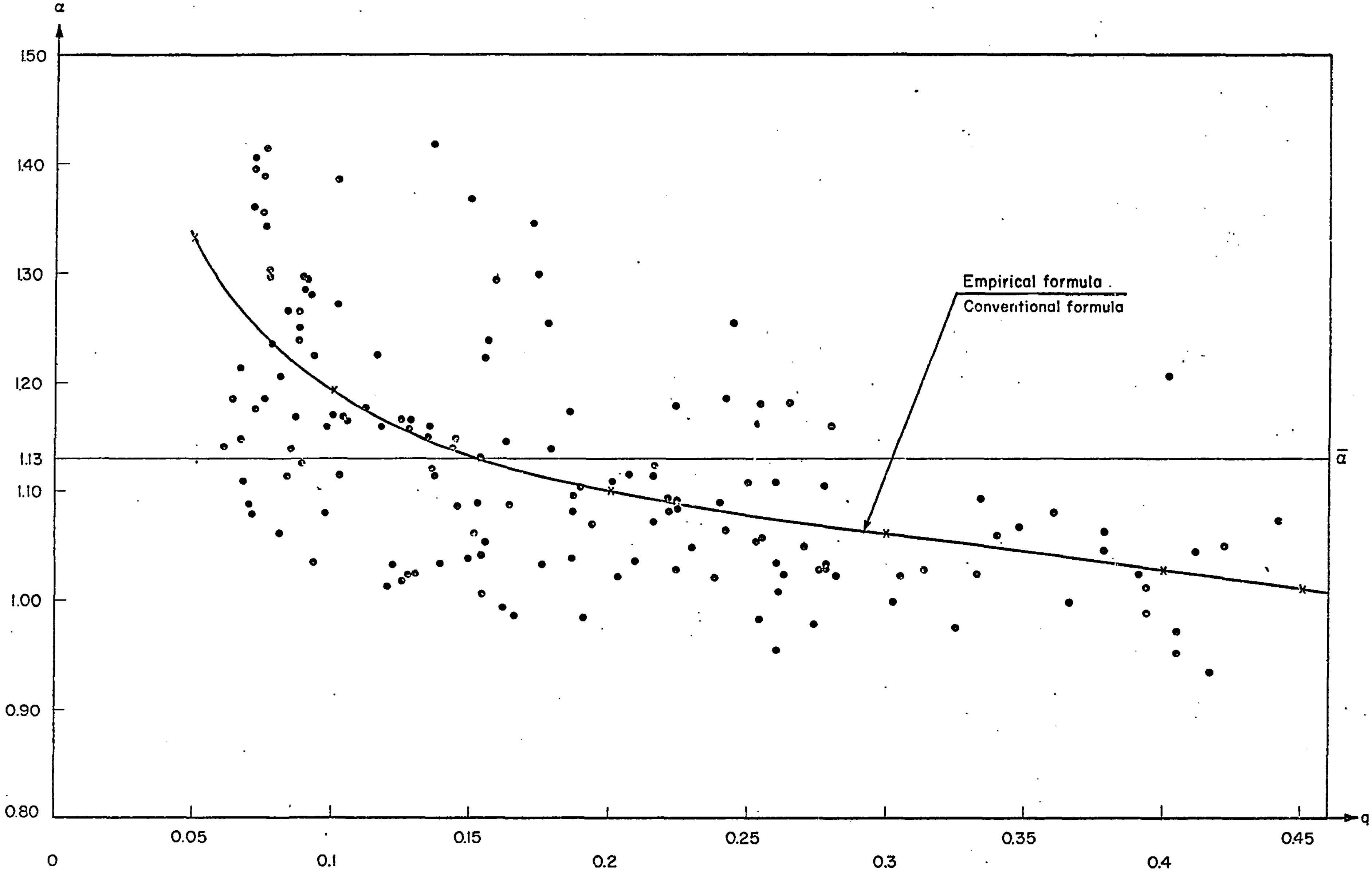


FIG. 2-2B - EXPERIMENTAL α's,— CONVENTIONAL FORMULA the material strengths, but the other design variables, the operational characteristics and the loads must be analysed in a statistical framework and the resulting frequency distributions combined in the safety parameter to obtain the probability of failure or survival.

In this section, then, the statistical analysis of load and resistance, and the design variables influencing these parameters, is undertaken.

2.3.1 Distribution Functions.

In the value measurements of certain material phenomena, the plotted points show a great scatter about what may be called an "average". Theoretical frequency distributions, as all other intellectual concepts, are only ideal formulations devised to represent the shape of and trend in the observed data. Basically, a distribution function is only an expression, represented by a curve which describes in a compact and simplified way the experimental scatter of a material phenomenon. As such, there are probably several alternative distribution functions that can fit an experimental scatter equally well.

In any one experiment, the number of measurements that can be made is limited and, therefore, a good estimate of the frequency distribution of the experimental data is difficult to obtain. If a particular distribution fits the data, then it is accepted and used. Moreover, the tail of the distribution curve is usually not known as data in that range is difficult to obtain by experiment. Thus it cannot be known how well a particular distribution fits a material phenomenon outside the range of experimental measure. This has led to considerable uncertainty as to the form of distribution that should be used for certain experimental data. In some cases, writers[24,25,34,58] have used two and sometimes four distribution functions to represent the same data.

One of the methods of reducing the extent of this problem is the use of confidence intervals or confidence limits. It can be assumed that a particular frequency distribution, say a normal distribution, fits the data. A range of values of the normal distribution can be obtained and tested with the experimental data for "fitness". On the basis of the "fitness test", the theoretical distribution can be accepted or rejected. However, this "fitness test" does not solve the problem encountered when considering the tail of the curve and no specific frequency interpretation can be given to confidence intervals. They only indicate the degree of confidence one can have in predicting the frequency of values related to the material phenomenon within a specific range by using the theoretical function.

One of the problems which causes a great deal of confusion among research engineers is that of an upper and a lower bound on distributions. Take, for example, the measurement of concrete cylinder strengths. Everyone agrees that the compressive strength of concrete cannot be infinite. Further, engineers would be startled if a sample of concrete mix designed for 3000 psi. specified strength and under controlled conditions were to show a strength of 10,000 psi. Yet, engineers would just not agree on what the upper or lower bound should be. Instead, the distribution curve is used over the whole range from 0 to $+\infty$.

The condition that the curve extends over an infinite range limits the number of alternative distributions that can be used. There are, however, many distributions that can represent the experimental data equally well. In fact, it is unlikely that there will be any uniquely determined distribution function for a particular set of experimental data.

2.3.2 Distribution of Concrete Strength.

The variation in strength of a particular concrete mix depends on the degree of control exercised in the mixing and placing of the concrete. The degree of control depends on the level of supervision exercised during the mixing and placing stages and on the choice of the quality of materials used and their quantitative ratios. Depending on the degree of control exercised, the frequency distribution of concrete strengths can follow a symmetrical or skewed curve.

Freudenthal[24] has fitted a log-normal distribution to data obtained from 673 tests for concrete under good control (Ref.No.1 Table 2.2). For concrete under poor control he fitted an extremal distribution to the results of 296 tests (Ref. No.4 Table 2.2). From the results, he concluded that inadequate or poor control increases the range of variation of the values and the number of low test results while it sharply reduces the number of high values.

On the other hand, Julian[34] showed that a normal distribution fits the test results of concrete under good control equally well (Fig.2.3) by analysing the results of 861 tests at 28 days (Ref.No.2, Table 2.2). In the case of poor control, he showed that the results followed a skewed distribution (Fig.2.4). From Freudenthal's and Julian's analysis, it can be concluded that both a normal and log-normal distribution can be employed to represent the variation in the compressive strength of concrete, manufactur under good control.

Fig.2.5 shows a histogram of 164 field tests taken by a commercial testing laboratory for concrete supplied by a ready-mixed concrete company from January 1958 through January 1959[2]. The mix was standard commercial concrete proportioned for 3,000 psi. specified strength at 28 days, using 1-inch maximum size aggregates. The

TABLE 2-1

COMPARISON OF OBSERVED WITH THEORETICAL FREQUENCIES

Cell Boundaries	Coded Mid-Cell Z	Observed Frequency f_o	$Z\ f_o$	$Z^2\ f_o$	Deviation of Class Limit x from Mean	Deviation x/σ	% Area between Class Limit & Mean	% Area in Class Interval	Theoretical Frequency f_t	$(f_o - f_t)$	$(f_o-f_t)^2$	χ^2 $\frac{(f_o-f_t)^2}{f_t}$
2700-2899	14	1	14	196	-1138	-2.55	49.46	1.25				
2900-3099	15	6	90	1350	- 938	-2.10	48.21	3.16	18.0	-1.0	1.0	.056
3100-3299	16	10	160	2560	- 738	-1.65	45.05	6.56				
3300-3499	17	19	323	5491	- 538	-1.20	38.49	11.15	18.3	0.7	0.49	.027
3500-3699	18	30	540	9720	- 338	-0.75	27.34	13.66	22.4	7.6	58.0	2.59
					- 138	-0.35	13.68					
3700-3899	19	27	513	9747	+ 61	+0.14	5.57	19.25	31.6	4.6	21.1	.668
3900-4099	20	26	520	10400	261	0.58	21.90	16.33	26.8	-0.7	0.49	.018
4100-4299	21	19	399	8379	461	1.03	34.85	12.95	21.2	-2.2	4.85	.228
4300-4499	22	17	374	8228	661	1.48	43.06	8.21	13.5	3.5	12.3	.910
4500-4699	23	5	115	2645	861	1.93	47.32	4.26				
4700-4899	24	2	48	1152	1061	2.38	49.13	1.81				
4900-5099	25	1	25	625	1261	2.82	49.76	0.63	11.4	-2.4	5.75	.505
5100-5299	26	1	26	676	1461	3.27	49.94	0.18				
5300-5499	27	0	0	0	1661	3.72	49.99	0.05				
		164	3147	61169					163.2			5.002

$\bar{T}^{1}{}_{c} = 3.84$ $\sigma_c = 0.45$ $V_c = 0.116$ $\chi^2_{obs} = 5.002$ $\chi^2_{.05} = 1.145$

TABLE 2-2

STATISTICAL DATA FOR CONCRETE

Ref. No	No of Tests	$\bar{f}'_c$	σ_c	V_c	f'_c	H.V/M	M/L.V.
1	673	5.32	-	-	3.825	1.31	1.35
2	861	5.18	0.54	.104	3.830	1.35	1.36
3	164	3.84	0.45	.116	3,000	1.35	1.33
4	296	5.85	1.54	.263	4.300	1.55	3.90

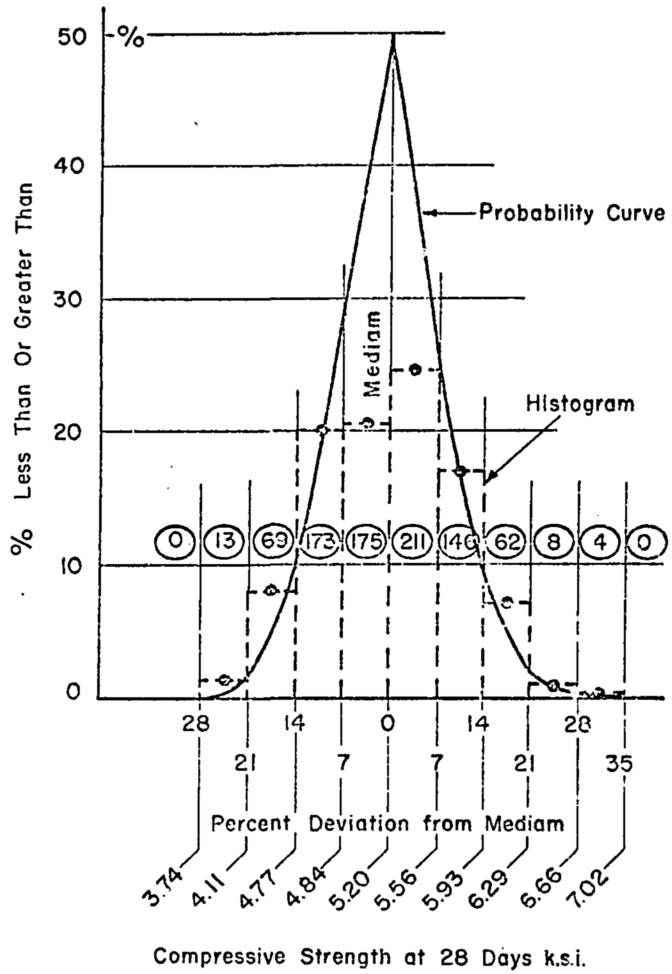


FIG 2-3 PROBABILITY CURVE FOR GOOD CONTROL

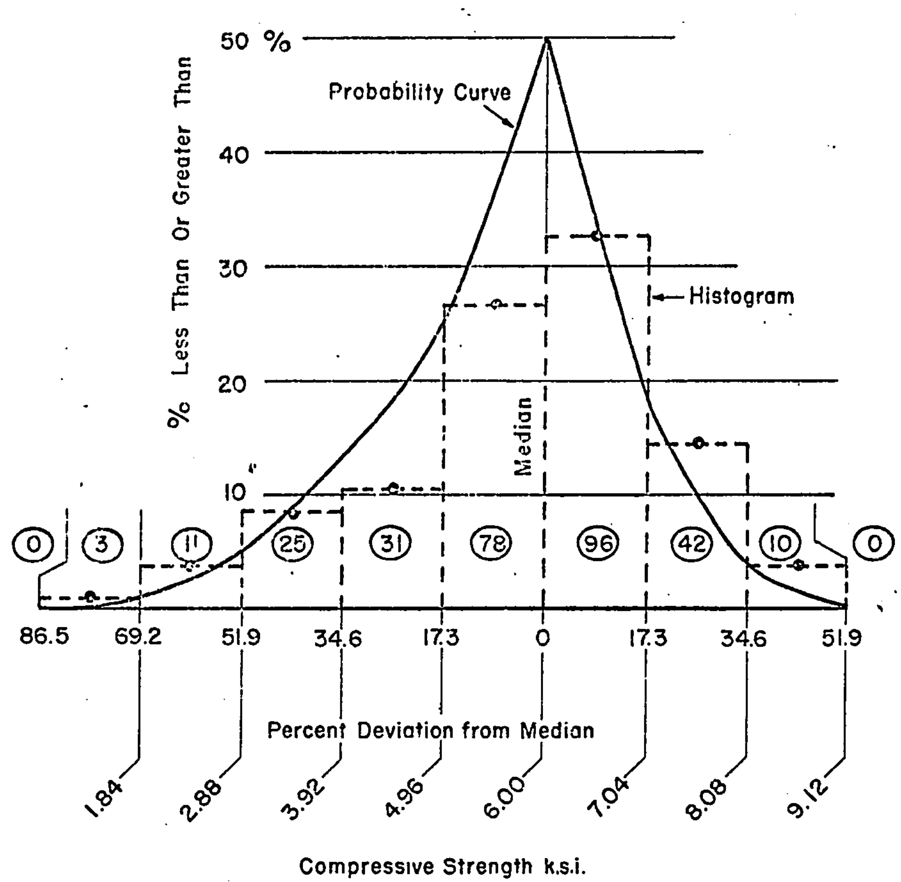


FIG. 2-4 PROBABILITY CURVE FOR POOR CONTROL

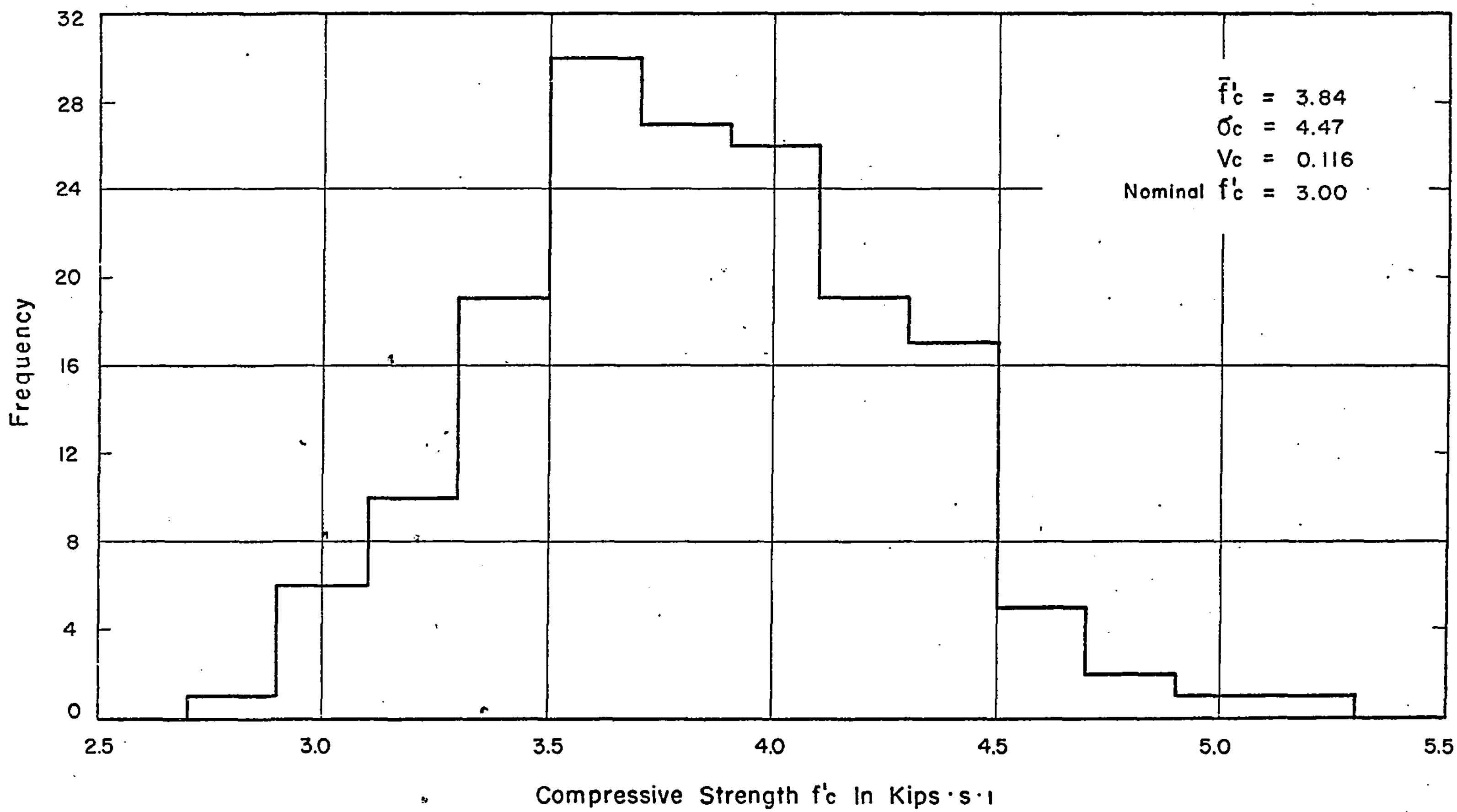


FIG. 2-5 — HISTOGRAM OF 164 CONCRETE TESTS

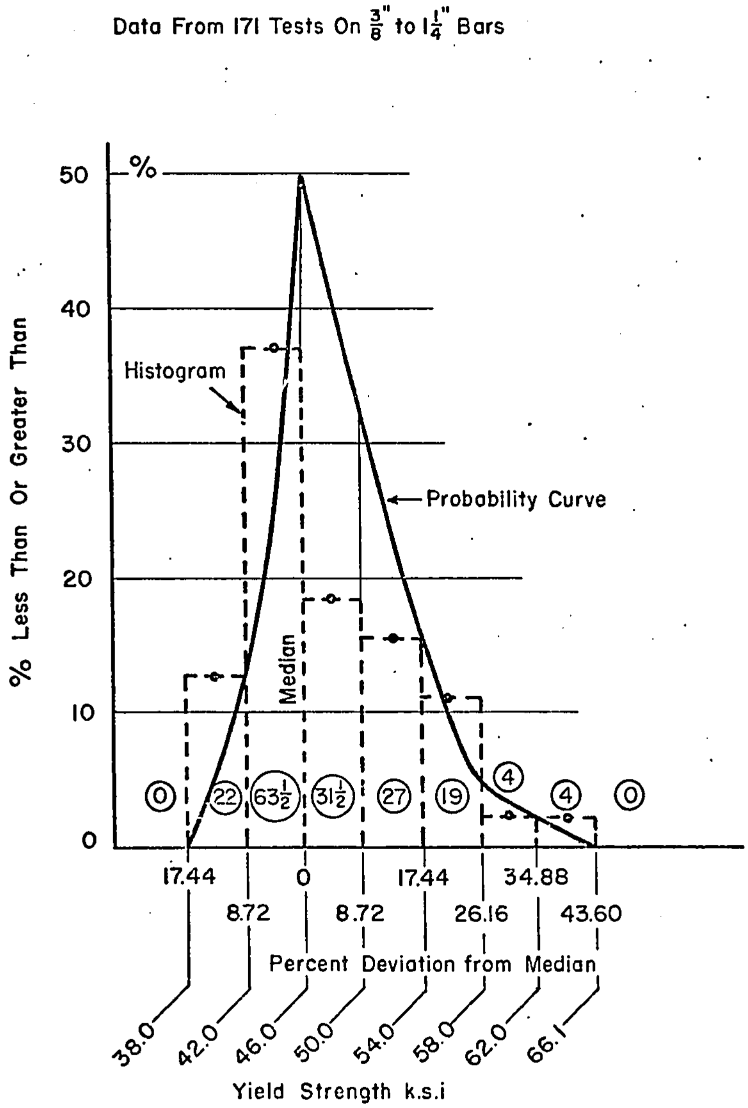


FIG. 2-6 HISTOGRAM & PROBABILITY CURVE FOR INTERMEDIATE GRADE NEW BILLET STEEL REINFORCING BARS

strengths are based on an average of three specimens. The chi-squared test is used to compare the observed test data to a normal frequency distribution in Table 2.1 and indicates a very good fit.

A summary of the statistical data for concrete strengths is given in Table 2.2. Ref. Nos. 1, 2 and 3 are for good control and Ref. No. 4 is for poor control. From the table, it can be seen that the highest value/mean and the mean/lowest value, given in the last two columns, are almost constant and equal for concrete under good control; whereas for concrete under poor control, the frequency curve can be seen from the table to be skewed towards the low values. The variance ratio or the coefficient of variation for concrete under good control is about 0.12.

2.3.3. Distribution of Steel Strengths.

Steel, unlike concrete, is subjected to rigid control as it is manufactured under factory conditions. As such, the frequency distribution of the yield points of reinforcing bars are all mostly similar in shape, varying only in magnitude of the statistical parameters.

Although steel is manufactured under factory conditions, not very many studies have been done on the distribution of steel strengths. As a result, the volume of test data required for a good estimate of the frequency distribution is not readily available. However, from analysis carried out so far, it can be concluded that the distribution follows a skewed pattern.

Freudenthal[24] has fitted a log-normal curve to the results of 121 tests of eye bars. The results of 171 tests of new billet steel reinforcing bars varying in diameter from 3/8-inch to 1-1/4-inches have been plotted by Julian[34] in the form of a frequency distribution curve as shown in Fig. 2.6. There is no doubt that the distribution is skewed towards the higher values. Using data obtained by Rice[49] as a guide, the statistical parameters employed in this study are $\bar{f}_y = 50.0$ and $V_y = .09, 0.12$.

2.3.4. Analysis of Flexural Resistance

In Section 2.2.3 it was shown that the contemporary ultimate strength equation is biased with q. Therefore, a series of test results obtained by various research engineers was analysed by the following method:

(1) The results of 152 tests from eight research experiments representing a cross-section of the tests carried out on the flexural capacity of R.C. beams failing in tension were collected and the dimensionless parameters, $M_u/bd^2f^1{}_c$ and $Pfy/f^1{}_c$, in the ultimate strength theory were computed.

(2) A least squares analysis was then carried out to determine the average curve passing through the results (Fig.2.7). The curve was assumed to be a second degree parabola of the form

$$y = A_o + A_1x + A_2x^2$$

From the analysis, the following values of the unknown parameters were obtained:

$$A_o = 0.011, \quad A_1 = 1.114, \quad A_2 = -0.90$$

Thus, giving the least squares equation:

$$\frac{\bar{M}_u}{bd^2f^1{}_c} = 0.011 + 1.114q - 0.90q^2 \qquad \text{- - - - (2.4)}$$

The analysis was only done for beams failing in tension. It was not necessary to do the analysis for beams failing in compression as will be shown in the next chapter.

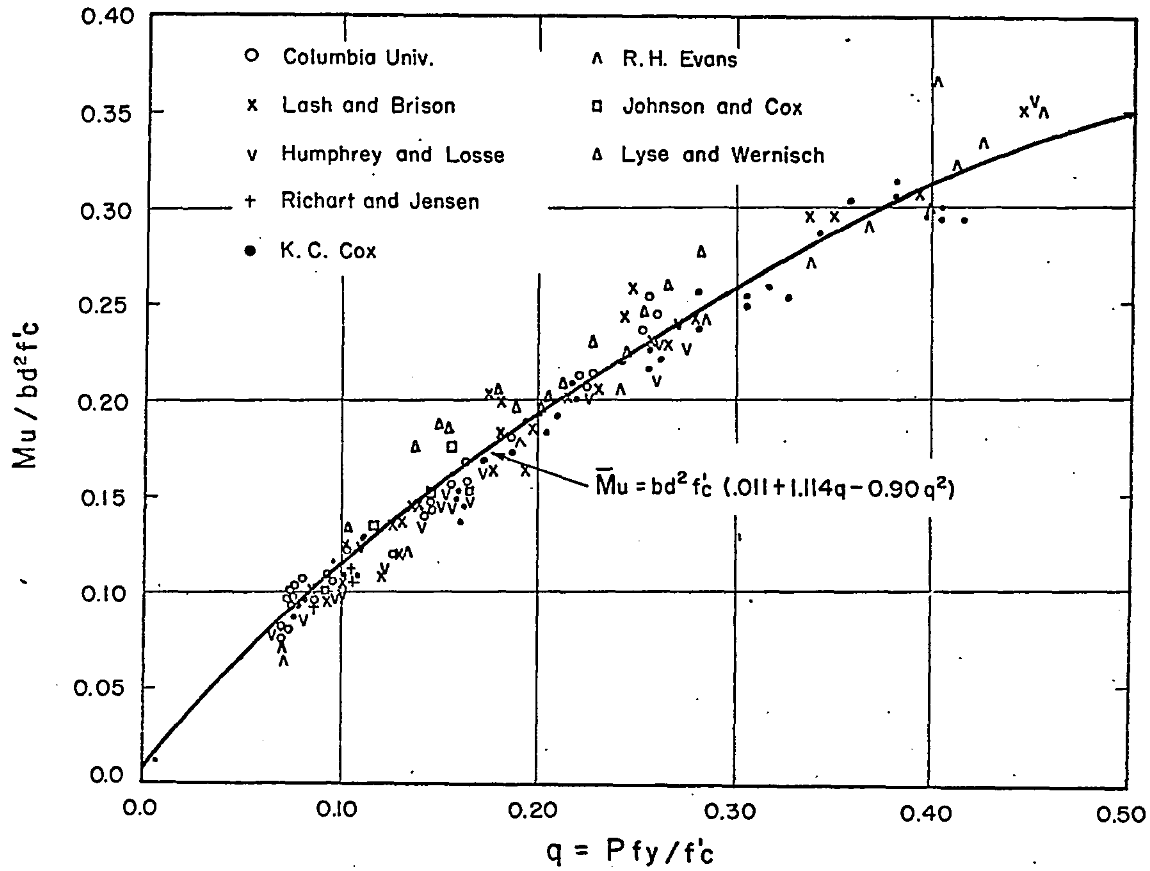


FIG. 2-7 - RESULTS OF 152 BEAM TESTS

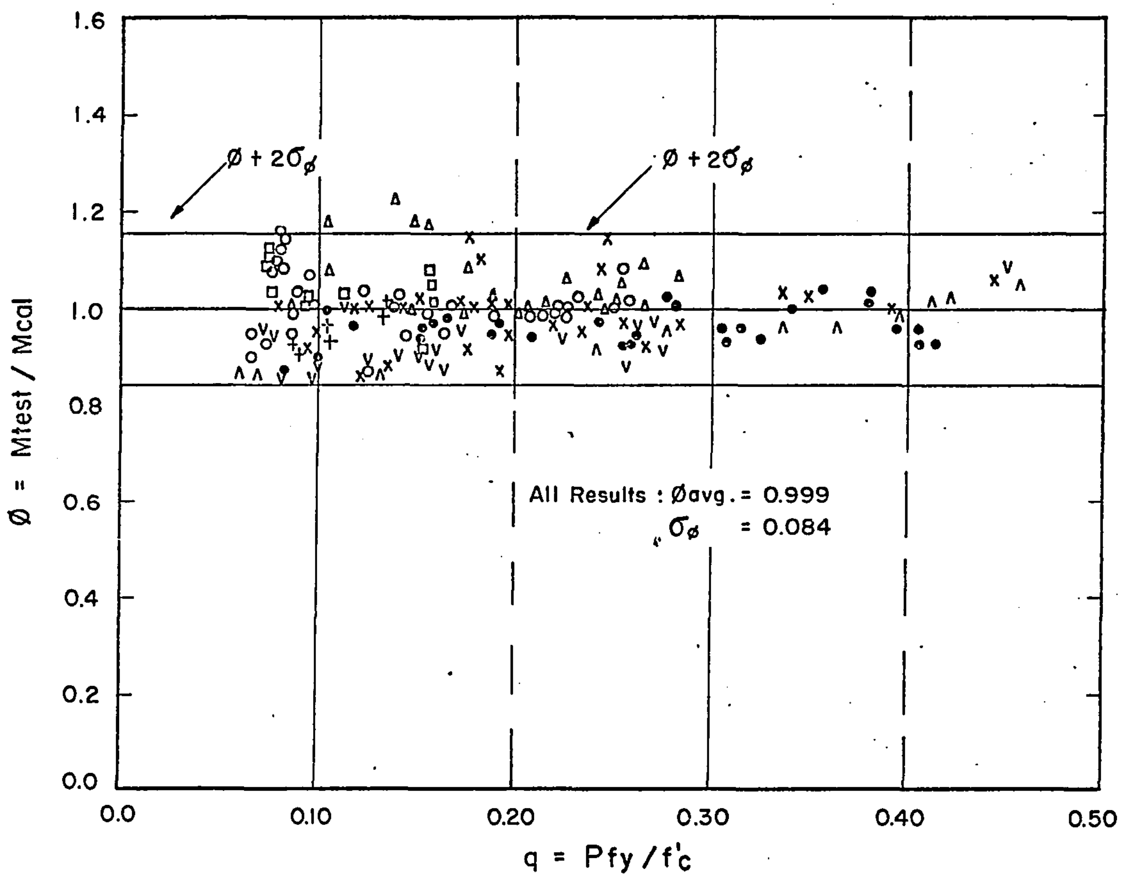


FIG. 2-8 – TEST RESULTS AND THEORY (152 TEST)

(3) Finally, it was assumed that the test Moment capacity, M_{test}, is of the form

$$M_{test} = \phi \bar{M}_u \qquad \text{- - - -} \qquad (2.5)$$

where ϕ is a random variable. If Equations (2.4) and (2.5) are acceptable assumptions, the distribution of ϕ does not depend on q. Test values of ϕ are plotted in Fig.2-8 together with calculated confidence intervals $\bar{\phi} \pm 2\sigma_{\phi}$. Theoretically, for 95% confidence, a maximum of 8 points out of 152 tests are allowed to fall outside the limits. From Fig. 2.8 only five points fall outside the limits.

A brief discussion of the various test series from which the results were obtained is given hereunder:

COLUMBIA UNIVERSITY (1935 and '41)[32] -- The tests were carried out for low percentages of steel. Basically, two grades of steel of nominal strength, 36 and 56 kips s.i. were used. For one set of beams, the concrete mix was designed to be of specified strength 3,000 psi. The actual concrete strengths used in the calculations was the average of three specimens taken from each batch of concrete used in any one beam. For the rest of the beams, the concrete strengths were 3550 and 3510 psi. It appears that half the number of beams tested in this set were cast from one batch of concrete and the other half from another batch. Thus samples were taken from two batches instead of the specified batch used in each beam.

LASH AND BRISON (1949-50)[39] -- These tests were carried out for a variety of concrete and steel strengths and percentages of tensile reinforcement. The concrete varied from 2000 to 5000 psi. nominal strength. Two grades of steel of nominal strenghts, 40 and 65 kips s.i., were used. The concrete strength was taken as the average of three specimens.

The reinforcing steel consisted of plain bars rusted to provide adequate bond resistance. The percentage of steel varied between 0.5 and 4.7 percent.

A few of the results showed either very high $M_u/bd^2f^1_c$ values or very low ones in comparison to the average value obtained from the least squares equation. On close examination, it was found that the high-valued results were produced in beams with a high f_y/f^1_c ratio, with steel of low yield point strength and with a high concrete strength.

HUMPHREY AND LOSSE (1912) - Very little can be said on these tests as the results have been taken from Jensen[32]. However, these tests give relatively low flexural capacity compared to the least squares curve. One point that should be mentioned is that the tests were done with lean concrete mixes, that is, with concrete of relatively low compressive strength - as low as 1500 psi. As for the tests by Lash and Brison, the low results were observed only for concrete of very high strengths and beams of a low f_y/f^1_c ratio. In this case, the nominal steel strength was 40 kips s.i.

RICHART AND JENSEN (1931)[51] - In these tests, the beam size, steel ratio and steel strength were all constant while the actual concrete strengths varied from 3000 psi. to 4800 psi. No explanation was given for the fixed steel strength used in the calculation.

COX (1941)[15] - Most of the beams of intermediate strength concrete were tested in triplicate, whereas others were tested singly. Why? Cox does not say. However, some uncertainty exists with respect to concrete and steel strengths which were not reported for the individual beams, but for each class of concrete and grade of steel. There were four classes of concrete, varying from 1700 psi. to 5800 psi. compressive strength. Similarly, the strengths given for the steel were: 48.1 and 53.4 kips s.i.

The results as shown in Fig. 2.8 indicate a consistently low flexural capacity relative to equation 2.4. Perhaps the explanation of this phenomenon is to be found in the steel and concrete strengths used by Cox in the computation of the dimensionless parameters q and M_u/bd^2f^1c.

EVANS (1943-44)[19] -- Evans tested samples of concrete used in each beam for both cube and cylinder strength. For each beam, four cubes and one cylinder were tested. He established a relationship between the cylinder and cube strength and showed that for high concrete strength, the cube and cylinder strengths are almost equal. However, in the computation of flexural capacity he used the cylinder strength.

Here, again, the low results observed around q = 0.06 and q = 0.13, on examination, reveal that in the particular beams a high concrete strength and relatively low steel strength giving a low f_y/f^1c ratio was evident.

JOHNSON AND COX (1939)[33] -- In these tests, four grades of steel and one mix of concrete were used. The reinforcing steel consisted of nickel, hard grade, square twisted, and cold twisted and stretched bars. The yield point was measured for each type or grade of steel; the number of samples tested for any one grade varying from 2 to 18.

For the concrete, samples were taken from two batches. Ten control specimens were made for each pair of beams cast from the first batch, and five specimens for each beam from the second batch. The average of all the specimens from the first batch was 3190 psi. and from the second batch 3220 psi. The value of the concrete strength used for all beams was 3200 psi., the average of 3190 psi. and 3220 psi. This method of obtaining the concrete strength is highly questionable. With such a large number of samples tested, the actual strength of concrete in the beam can be far different to the average value used.

LYSE AND WERNISCH (1937)[32] - The concrete strengths used in these tests were generally low; about 2500 psi. For such low strengths, the $f_y/f^1{}_c$ ratio was very high resulting in high values of $M_u/bd^2f^1{}_c$ for small q. This is clearly shown in Fig. 2-8.

Three grades of steel varying from 48.0 to 93.0 kipss.i. nominal strength were used. The concrete strength taken was the average of three specimens for each beam.

From the foregoing discussion, it can be concluded that the ultimate strength equation developed within a statistical framework fits the test data quite well. However, like all other theories, there are limitations to its use. One of the important points observed in the analysis is that scatter of the data points seems to depend upon the ratio of $f_y/f^1{}_c$. For high $f_y/f^1{}_c$ ratios, the least squares equation underestimates the flexural capacity, whereas for low $f_y/f^1{}_c$ ratios, it overestimates the capacity of the beam. It seems that a moderate $f_y/f^1{}_c$ ratio gives least scatter and allows prediction to be made with greater confidence.

2.3.5. Distribution of Vehicle Loads.

As with live loads in general, the nature of vehicle loadings and their effects on highway bridges have only recently been subjected to a comprehensive investigation. Stephenson[57,58] was perhaps the first to carry out a systematic and methodical analysis of the nature of vehicle loadings and to relate this to the operational characteristics of highway bridges. It is not possible to discuss his approach in detail, but a brief summary of his method and results are necessary.

By collating and analysing the results of the 1942 loadometer survey, Stephenson was able to show that only the heavy vehicles have any significant influence on the operational characteristics and, as a result, the design of highway bridge structures. The heavy vehicles are defined as "those with one or more axles weighing 18,000 lbs. or more; " or,"based on gross weight,all simple-unit trucks weighing 26 kips. or more, and all other combinations weighting 34 kips. or more."[58]

The method of finding the nature of vehicle operation is as follows:

(1) Firstly, the heavy vehicle loadings were converted into equivalent H, H-S and concentrated loads. These equivalent loads can be defined as the loads which will produce in a bridge of given span the same stress as that produced by the heavy vehicle from which the equivalent load was obtained.

(2) Once all the heavy vehicles reported by the loadometer survey were converted into equivalent loads for a given span, the relative frequencies of various intensities of these loading equivalents were then obtained by arranging them into groups or cells of increasing magnitude and computing the percentages of vehicles thus found in each cell respectively. The observed frequency of equivalent concentrated load (E.C.L.) on various span-lengths, as computed by Stephenson[58], are given in Table 2.3A. These frequencies relate to the bending moment produced by the E.C.L. on various spans. The constant k is Poisson's coefficient.

The results of Stephenson's analysis were based on the 1942 traffic survey which was taken in the summer of 1942. As such, the survey cannot be considered asa yearly survey for 1942. Stephenson pointed out, however, that heavy vehicular traffic occurs mostly during the summer. In this study, Stephenson's results are only used as a guide and the statistical characteristics chosen for the load parameter are considered to be based on yearly traffic data.

(3) Although any one of several frequency distribution functions might give comparable results for the observed data, Stephenson found that the Poisson distribution formula represented the observed data reasonably well (Table 2.3B) and, at the same time, provided "the most satisfactory procedure for solving such traffic problems, mainly because it is perhaps the simplest to apply in practice when the sample size is large."

By developing this method of analysis, Stephenson laid the basis of a new and realistic approach to the evaluation of vehicle operation and loadings on highway bridges. Thus, with a sufficient backlog of observed heavy vehicle frequency data in a given geographical area, the engineer is provided with a rational procedure for estimating the level of heavy vehicle operation that would likely obtain at a new location within the area.

Although the analysis was also carried out for both equivalent H and H-S loadings, the equivalent concentrated loading has the greatest potential for practical application. First of all, the maximum moment produced by a single concentrated load on a simple span bridge can be expressed by a very simple equation; namely,

$$M = PL/4$$

in which M is the maximum moment, P the concentrated load and L the span length. Secondly, it will be noted that this equation allows the moment M for any given load P to be expressed as a continuous function which varies directly with the span length. This is not possible with the H and H-S loadings. Thus the equivalent concentrated loadings provide both an absolute basis for comparing the operational characteristics produced by

TABLE 2-3A - OBSERVED FREQUENCIES

Equiv. Conc. Loads in Kips.	Span-Feet							
	10	20	30	40	50	60	80	100
4	.5							
5	.8	.5	.1					
6	2.5	1.7	.4	.1				
7	9.6	7.8	3.1	.4	.1			
8	19.5	17.3	11.1	1.9	.3	.1		
9	24.5	22.3	18.4	7.3	1.2	.3		
10	20.5	20.2	21.1	14.5	5.1	1.6	.3	
11	11.6	12.9	16.6	19.5	12.7	6.3	1.4	.8
12	5.7	8.0	11.2	18.2	18.1	12.6	4.4	2.1
13	2.7	4.6	7.4	13.7	19.0	17.6	11.5	6.4
14	1.2	2.6	4.7	9.3	14.1	17.0	15.7	12.1
15	.5	1.3	3.0	6.3	10.1	13.4	17.1	15.7
16	.2	.5	1.5	4.1	7.0	9.6	12.8	15.4
17	.1	.2	.8	2.3	5.0	7.1	10.0	12.1
18	.1	.1	.3	1.2	3.2	5.2	6.9	8.8
19			.2	.7	1.8	3.6	5.4	6.1
20			.1	.3	1.1	2.3	4.0	4.5
21				.2	.6	1.5	3.4	3.9
22					.3	.8	2.4	3.3
23					.1	.5	1.8	2.6
24					.1	.2	1.2	2.0
25					.1	.1	.7	1.5
26						.1	.4	1.1
27						.1	.2	.7
28							.1	.4
29							.1	.2
30							.1	.1
31							.1	.1
32								.1
Total	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Avg. E.C.L.	9.3	9.7	10.6	12.2	13.5	14.6	16.2	17.1
K	5.3	4.7	5.6	6.2	6.5	6.6	6.2	6.1

TABLE 2-3B - CALCULATED FREQUENCIES

Equiv. Conc. Loads in Kips.	Span-Feet							
	10	20	30	40	50	60	80	100
4	.5							
5	2.6	.9	.4					
6	7.0	4.3	2.1	.2				
7	12.4	10.0	5.8	1.3	.2			
8	16.4	15.7	10.8	3.9	1.0	.1		
9	17.4	18.5	15.2	8.1	3.2	.9		
10	15.4	17.4	17.0	12.5	6.9	3.0	.2	
11	11.6	13.6	15.8	15.5	11.2	6.5	1.3	.2
12	7.7	9.1	12.7	15.9	14.5	10.8	3.9	1.4
13	4.5	5.4	8.9	14.2	15.6	14.2	8.1	4.2
14	2.4	2.8	5.5	11.0	14.6	15.6	12.5	8.5
15	1.2	1.3	3.1	7.6	11.9	14.7	15.5	12.9
16	.5	.6	1.6	4.7	8.6	12.1	15.9	15.8
17	.2	.2	.7	2.6	5.6	8.9	14.2	16.0
18	.1	.1	.3	1.4	3.3	5.9	11.0	14.0
19	.1	.1	.1	.7	1.8	3.5	7.6	10.7
20				.3	.9	1.9	4.7	7.2
21				.1	.4	1.0	2.6	4.4
22					.2	.5	1.4	2.4
23					.1	.2	.7	1.2
24						.1	.3	.6
25						.1	.1	.3
26								.1
27								.1
28								
29								
30								
31								
32								
Total	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Avg.E.C.L.	9.3	9.7	10.6	12.2	13.5	14.6	16.2	17.1
K	5.3	4.7	5.6	6.2	6.5	6.6	6.2	6.1
Std. Dev.	2.302	2.168	2.366	2.490	2.550	2.569	2.490	2.470

A COMPARISON OF OBSERVED WITH THEORETICAL FREQUENCIES OF EQUIVALENT CONCENTRATED LOADS FOR ALL TYPE HEAVY VEHICLES ON SIMPLE SPANS OF VARIOUS LENGTHS

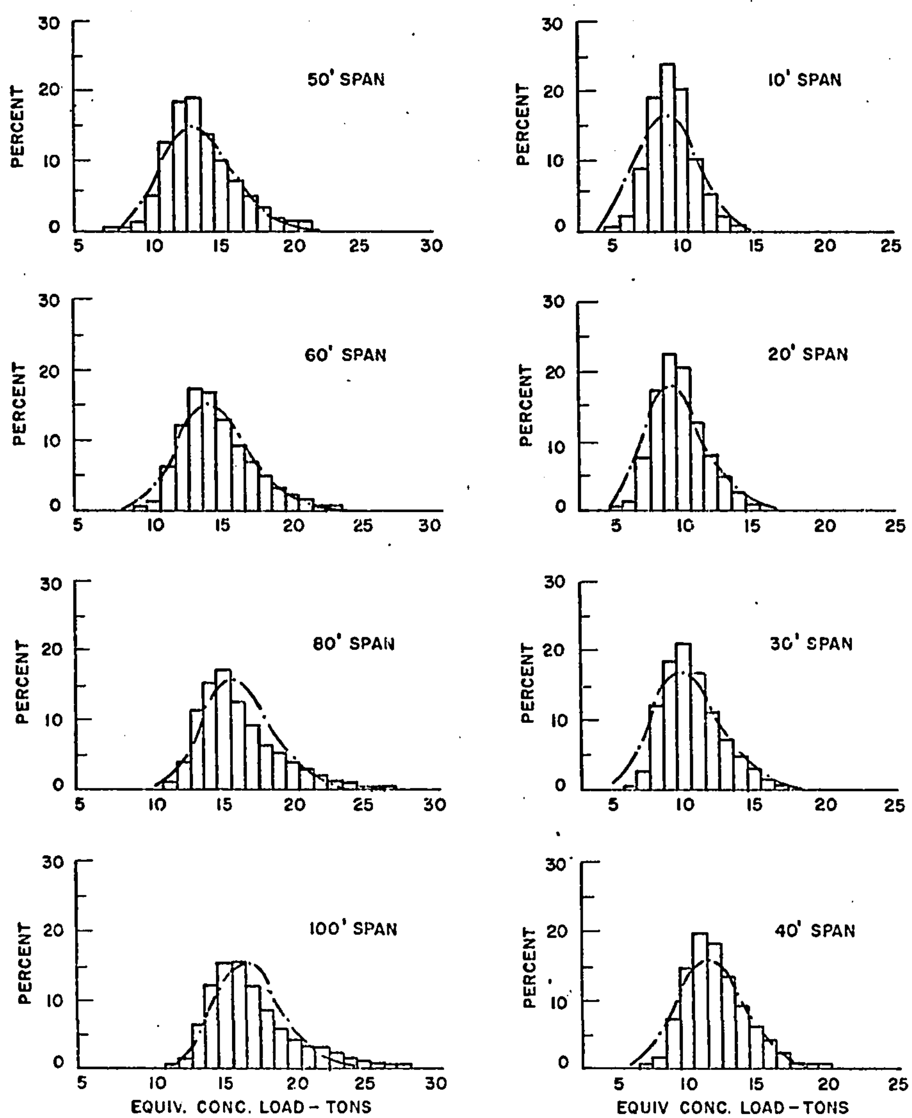


FIG. 2-9 - STEPHENSON'S FREQUENCY CURVES

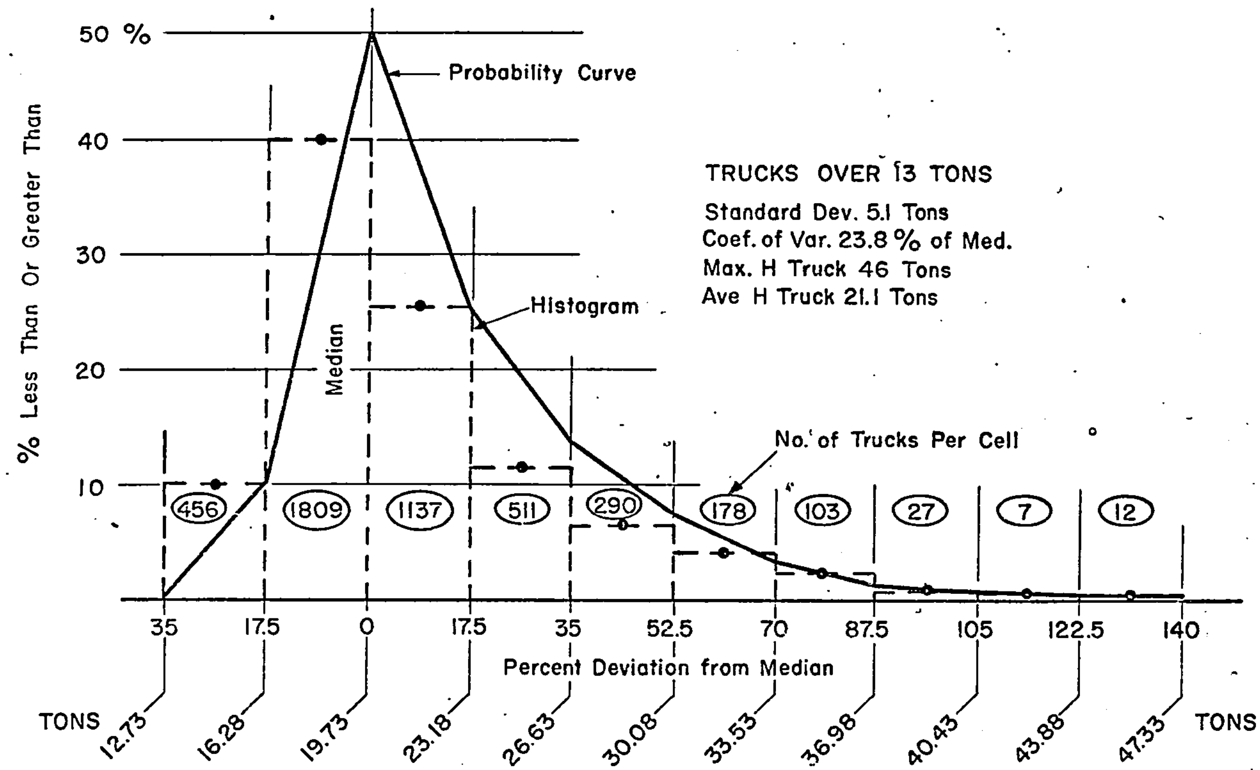


FIG. 2-10 EQUIV. H TRUCK LOADINGS - TONS 4530 HEAVY TRUCKS - ALL TYPES INDEPENDENT OF SPAN

effects from one span to another.

However, from Fig. 2.9, it can be seen that the Poisson distribution fits the observed data for the lower spans (10' to 50') reasonably well. But for the higher span lengths, the fit is disappointing.

Freudenthal[24], on the other hand, has fitted a log-normal distribution to all equivalent H truck loading for spans of 50' and 100' and to the gross weight of vehicles. He showed, by plotting the cumulative distribution function, that the poisson formula gives disappointing agreement with the observed data for high-load values, whereas the log-normal distribution gave a "good" fit for all load-values. The explanation given was that the definition of heavy trucks was not narrow enough to justify analysis by the rare-event approach; that is, by the poisson law.

Julian[34] plotted the distribution of all equivalent H truck loadings and showed that the frequency curve is skewed towards the high-load values (Fig. 2.10).

These studies have only shown that all loads on highway bridges have a statistical distribution. Precisely what distribution should be used is not yet certain. Moreover the distributions relative to fatigue loadings and dynamic effects have not yet been fully investigated. It may be a long time before sufficient data can be accumulated to make a comprehensive analysis of vehicle load distributions and their application to bridge design. At the present time, the data obtained and studies carried out so far can only act as a guide. In this study, values of $\bar{M}_L$ between 50 and 90 in increments of 10 and values of $V_L = 0.15, 0.20$ will be used.

2.3.6. Analysis of other Variables.

Besides the parameters discussed above, there are other variables in the design process which vary in a statistical manner. Those most relevant to this study are the dimensions of structural members and the dead load of the structure.

In the design of a rectangular R.C. beam, the designer obtains certain specific values for the effective depth and the width of the beams. However, these values are not necessarily realised in the construction process, because of errors of workmanship.

Although the dimensions (or size) of the beams do vary, yet, with the development of modern techniques, this variation is very small and, as such, the dimensions can be considered constant. The effective depth, on the other hand, can vary tremendously due to errors in the placing of the steel. It is therefore necessary to take account of this variation. Data on the variation of effective depth, however, is not available and in the absence of such data, a designer can only choose statistical parameters on a subjective basis.

As a result of variations, however small, in the dimension of beams, and, also, variation in the specific weights of materials, the dead load of a structure varies in a statistical manner. Here, again, no data are available and the designer will have to use his subjective judgment. From preliminary investigations, values of V_d and V_D of .05 and .10 were found to be realistic. These values imply a 95% confidence within 20% of the mean.

III

FAILURE PROBABILITY AND COST FUNCTION

The safety of a structure can only be realistically expressed in a probabilistic sense. That is, by formulating the problem of safety in terms of the probability of failure or survival. In order to evaluate the probability of failure the distributions of the design variables involved in the load and resistance parameters must be convoluted.

Economy in structural design cannot be considered on a cost basis, for cost is not absolute, but relative to the level of safety adopted. As such, economy can only be expressed as a relationship between cost and safety and on this basis an economic choice is made by finding the optimum balance between cost and safety for the particular design problem.

In this chapter, therefore, the mathematical formulation of the probability of failure in terms of the distribution of the design variables, and the relationship between cost and failure probability will be presented.

3.1 Formulation of the Probability Problem.

In the actual process of bridge operation, circumstances surrounding human needs and necessities choose one value of the load and nature, partially controlled by human skill, chooses one value of the resistance from the respective distributions of possible values.

If R is the resistance of the bridge structure, and S is the load, then the relationship between load and resistance can be expressed by the safety parameter

$$r = R - S$$

or $$\text{-------} \qquad (3.1)$$

$$n = R / S$$

These two forms are not independent, but, on the contrary, they are interrelated. For, from (3.1)

$$R = nS$$

and

$$r = nS - S = S(n-1)$$

or

$$S = R / n$$

and

$$r = R - R/n = R(n-1)/n$$

Since failure occurs when resistance is less than load, then, in terms of the safety parameters, the bridge structure has failed when r is less than zero or when n is less than one. The probability of failure, therefore, is the probability of r being less than zero, or the probability of n being less than one. That is,

$$P_F = \text{Prob.}\ (r \leq o) = \text{Prob.}\ (n \leq 1)$$

Either one of the two forms of expressing the safety parameter can be used in determining the probability of failure. The problem, then, is to obtain the distribution of the safety parameter by formulating it mathematically in terms of the load and resistance distribution.

3.2 The General Expression.

Freudenthal[24] in 1956 developed a geometric method of finding the probability of failure or survival. He obtained a three dimensional geometric figure on which he superimposed lines of constant P_F and from which he developed surfaces of survival. However, his analysis, besides being only on the macro level, involved a rather complex formulation.

Lawrence[40] and Corso[17] in a discussion of the Freudenthal method showed that the probability of failure can be determined by a very simple computation for relatively simple distribution functions.

In the general case, the probability of failure can be formulated as follows:

If $P_1(R)$ is the frequency function of R and $P_2(S)$ is the frequency function of S, such that the areas under the distribution curves are each unity, then the probability of obtaining a value S of the load within the interval ds is $P_2(S)\,dS$, and the probability that $R \leq S$ is

$$P\,(R \leq S) = \int_{-\infty}^{S} P_1(R)\,dR$$

The probability of both events occuring simultaneously is the $P\,(S = S)$ and $P\,(R \leq S)$.

That is,

$$\begin{aligned} P_{F_S} &= P\,(S = S) \text{ and } P\,(R \leq S) \\ &= P_2(S) \left(\int_{-\infty}^{S} P_1(R)\,dR \right) dS \end{aligned}$$

Thus, for P_{F_S} over all values of S, we obtain the double integral

$$P_F = \int_{-\infty}^{+\infty} \int_{-\infty}^{S} P_2(S)\ P_1(R)\,dR\,dS \qquad \text{------} \qquad (3.2)$$

Freudenthal[25] in 1961 formulated P_F in relation to the safety parameter n. If $n = R/S$ and the distribution function of n is $P(n)$, then $R = nS$ and by the probability law of quotients,

$$P(n) = \int_{o}^{\infty} P_1(nS)\, P_2(S)\, dS$$

thus,

$$P_F = \int_{-\infty}^{1} \int_{o}^{\infty} P_1(nS)\, P_2(S)\, S\, dS\, dn \quad --- \quad (3.3)$$

Both (3.2) and (3.3) are the same, for by a suitable transformation (3.2) can be reduced to 3.3).

Freudenthal[25,26] and others[14,61] have formulated the failure probability on the macro level for the different and varied aspects of structural design such as repeated loading, static loading, multiple member structures, etc. In this present formulation, however, only the initial probability of failure will be discussed.

3.3 Initial Probability of Flexural Failure.

The determination of the probability of failure can only be based on the criteria of failure adopted in a particular situation.

The bridge structure is considered to have failed, when either

(1) The tensile steel yields; that is, the load is greater than the flexural capacity in tension.

or (2) The concrete in the compression zone is crushed; that is, the load greater than the resistance, the flexural capacity, in compression.

Thus, the failure of the bridge is related only to its flexural capacity. Also, only the bridge beams are considered. This is a rational criterion atthis stage of the analysis as the capacity of R.C. bridges is mostly dependent on the flexural resistance of the beams[1]. Shear is not considered for the purpose of simplicity in the analysis and, also, because of the lack of understanding of shear failure.

In evaluating the probability of failure related to flexural failure, only tension failure is considered. Compression failure is not taken into account. There are two reasons for this. Firstly, compression failure occurs suddenly and without previous warning. As such, designers attempt to avoid compression failures by designing beams for low values of q. Consequently, the probability of compression failure is very low compared to that of tension failure. Secondly, the cost of a beam increases rapidly as q increases above the optimum as is shown in Chapter IV.

The probability of failure is given by Equation (3.2) where $P_1(R)$ is the frequency function of the beam moment capacity and $P_2(S)$ is the frequency function of applied moment.

Theoretically, the frequency function of the true beam capacity M_A can be determined by combining Equations (2.4) and (2.5) to obtain

$$M_A = \phi\, bd^2 f^1{}_c ((a_o + a_1\, Pfy/f^1{}_c + a_2 P^2 (fy/f^1{}_c)^2)) \quad - - - - \quad (3.4)$$

While the only random variable in experimental work is ϕ, in design the variables d, $f^1{}_c$ and fy are also random variables.

Thus, if M_A is the actual flexural resistance of the beam and M_S is the applied moment, then

$$P_F = P\,(M_A \leq M_S)$$

$$= \int_{-\infty}^{+\infty} \int_{-\infty}^{M_S} P_2\,(M_S)\;P_1\,(M_A)\; d\,M_A\, M_S \qquad - - - - \quad (3.5)$$

or $$P_F = \int_{0}^{\infty} P_2(M_S)\;P_1\,(M_S)\;\; d\,(M_S)$$

The incomplete knowledge of the frequency functions of all these variables and the complexity of Equation (3.4) prohibit computation of the frequency function of M_A, and, therefore, of the failure probability given by Equation (3.5). However, by making certain statistical assumptions and approximations, P_F can be evaluated in a relatively simple manner.

3.4 Approximate Method – Cornell's Approximation.

The preceding developments suggest that meaningful frequency distributions of the random variables involved in safety analysis of moment capacity are not reasonably obtained. At best, one can hope to establish the means and variances of the random variables involved and only a qualitative estimate of the shape of frequency functions.

A number of expressions for failure probability involving only the means and variances of design variables have been developed based on Tchebycheff's inequalities.[15] These simple expressions yield very conservative estimates of failure probability.

An alternative approach proposed by Cornell[14] seems to provide a reasonable measure of likelihood of failure although the results are necessarily approximate. For any structural mechanism,

$$PF = P\,(R \leq S) = P\,(R/S \leq 1)$$

$$= P\,(\!(\log\,(R/S) < 0\,)\!)$$

Assuming that R/S is log-normally distributed, then the standardized variate can be expressed as

$$X = \frac{\log\,(R/S) - \overline{\log(R/S)}}{\sigma_{\log R/S}}$$

in which $\overline{\log\,(R/S)}$ is the mean and $\sigma_{\log\, R/S}$ is the standard deviation of the natural logarithm of the ratio R/S. Then,

$$\log\,(R/S) = X\,\sigma_{\log R/S} + \overline{\log\,(R/S)} \qquad --- \quad (3.6)$$

and

$$P_F = P\,(\!(\,X < \frac{-\,\overline{\log\,(R/S)}}{\sigma_{\log R/S}}\,)\!) \qquad --- \quad (3.7)$$

That is,

$$P_F = F_X \left(- \frac{\overline{\log (R/S)}}{\sigma_{\log R/S}} \right) \quad --- \quad (3.7)$$

where F_X is the cumulative distribution function. In general, by a law of probability, if $z = xy$, then

$$\bar{z} = \overline{xy} = \bar{x}\,\bar{y} + \text{cov}(x, y)$$

When x and y are independent, cov (x, y) is zero. However, when x and y are not independent, but cov (x, y) is small and $\bar{x}\,\bar{y}$ is large, the approximation

$$\bar{z} = \bar{x} \cdot \bar{y}$$

can be made. The smaller the covariance, the better the approximation. Extending this approximation principle to statistical functions, it can be said that the mean of a function is approximately the function of the means, and the variance of a function can be approximated[15,36] by the formula

$$\sigma_z^2 = \bar{x}^2 \sigma_y^2 + \bar{y}^2 \sigma_x^2$$

Thus,

$$\overline{\log R/S} \approx \log (\bar{R}/\bar{S})$$

and $\sigma^2_{\log R/S} = \sigma^2_{(\log_R - \log_S)} = V^2_R + V^2_S$

Therefore:

$$\frac{\overline{\log R/S}}{\sigma_{\log R/S}} \approx \frac{-\log (\bar{R} / \bar{S})}{\sqrt{V_R^2 + V_S^2}}$$

assuming that the lower tail of the cumulative distribution function of X can be approximated by an exponentional of the form [14]

$$F_X(X) \approx k e^{b'X} \quad ---- \quad (3.8)$$

thus,

$$P_F = F_X \left(- \frac{\overline{\log R/S}}{\sigma_{\log R/S}} \right) \quad ---- \quad (3.9)$$

$$= k \exp \left(- \frac{b' \log (\bar{R}/\bar{S})}{\sqrt{V_R^2 + V_S^2}} \right) \quad --- \quad (3.9)$$

$$\cong k (\bar{R}/\bar{S})^{-b'/\sqrt{V_R^2 + V_S^2}} \quad \text{(Cornell[14])}$$

and

$$\log P_F = \log k - \frac{b' \log (\bar{R}/\bar{S})}{\sqrt{V_R^2 + V_S^2}} \quad --- \quad (3.10)$$

from which,

$$\log (\bar{R}/\bar{S}) = \frac{\sqrt{V_R^2 + V_S^2}}{b'} \left(\log k - \log P_F \right)$$

and

$$\bar{R}/\bar{S} = \text{antilog} \left(\frac{\sqrt{V_R^2 + V_S^2}}{b'} (\log k - \log P_F) \right) --- \quad (3.11)$$

For equations (3.10) and(3.11) to be suitable for application to practical problems, k and b must be determined. By plotting $F_X(-X)$ versus $-X$, values of $\log k = 7.5$ and $b' = 4.5$ were obtained[14], for R/S log normally distributed.

In the development of this approximate method, Cornell recognised that the assumptions and approximations made could and should be improved and extended by further investigations. The validity of the approximate analysis depends mainly on the reliability of the assumption that R/S is log-normally distributed. Firstly, it must be noted that the "true" distribution is not critical. It is the distribution of the values in the range of the tail of the frequency distribution that is of importance in evaluating the probability of structural failure. Although a large number of "common" distributions can be approximated by the exponential form for a wide range of values in the region of the tail of the frequency distribution, this fact does not make the use of the approximation valid for the "true" distribution is not known. Since only direct measurement along the tail of the distribution can determine the "true" distribution function, it is quite impossible at this stage to prove the validity of using one distribution function above another. It is only by investigating the sensitivity of the total cost to changes in P_F and $\bar{n}$ can the reliability of the assumption be tested. This aspect of the problem is discussed in Section 3.6.2 of this chapter.

3.5 Application of Approximate Method.

If M_A is the actual capacity of a bridge beam, M_T, the capacity given by the theoretical formula, and M_S the total load acting on the beam, then formula (3.11) can be rewritten by substituting for R and S.

Let $\bar{n} = \bar{M}_A / \bar{M}_S$

Where $\bar{n}$ is the central safety factor, then from (3.11)

$$\bar{n} = \text{antilog} \left(\left(\frac{\sqrt{V_{M_A}^2 + V_{M_S}^2}}{4.5} (7.5 - \log P_F) \right) \right) \quad - - - \quad 3.12)$$

To obtain a relationship between $\bar{n}$ and $\log P_F$, the problem is resolved to the determination of V_{M_A} and V_{M_S}. With the evaluation of V_{M_A} and V_{M_S}, $\bar{n}$ can be obtained for various values of P_F.

3.5.1 Evaluation of V_{M_A}:

Writing Equation (3.4) in the form

$$M_A = \phi M_T$$

Then, by the same approximation methods of Section 3.4,

$$\bar{M}_A \approx \bar{\phi}\ \bar{M}_T$$

and
$$\sigma^2_{M_A} \approx \bar{\phi}^2\ \sigma^2_{M_T} + \bar{M}_T^2\ \sigma^2_{\phi}$$

therefore,
$$V^2_{M_A} \approx \frac{\phi^2\ \sigma^2_{M_T} + M_T^2\ \sigma^2_{\phi}}{\bar{\phi}^2\ \bar{M}_T^2}$$

$$\approx V^2_{M_T} + V^2_{\phi} \qquad --- \qquad (3.13)$$

Using the formula obtained by the least squares analysis for the theoretical moment capacity,

$$M_T = bd^2f^1{}_c\ \left(a_o + a_1 Pfy/f^1{}_c + a_2 P^2\ (fy/f^1{}_c)^2 \right)$$

$$= bd^2f^1{}_c a_o + d\ A_s fy\ a_1 + \frac{A_s^2\ fy^2}{b\ f^1{}_c}\ a_2$$

Assuming that b and A_s have no distribution function; that is, they are fixed or can be found from other design variables, then

$$M_T = b\ d^2\ f^1{}_c\ a_o + d\ fy\ A_s\ a_1 + \frac{fy^2\ A_s^2}{f^1{}_c\ b}\ a_2$$

and

$$\frac{\partial M_T}{\partial d} = 2\ b\ d\ f^1{}_c\ a_o + A_s\ fy\ a_1$$

$$\frac{\partial M_T}{\partial fy} = d\ A_s\ a_1 + 2\ \frac{A^2{}_s\ fy}{b\ f^1{}_c}\ a_2$$

$$\frac{\partial M_T}{\partial f^1{}_c} = b\ d^2\ a_o - \frac{A_s^2\ fy^2}{b\ f^1{}_c{}^2}\ a_2$$

$$\sigma_{M_T}{}^2 \approx \left(\frac{\partial M_T}{\partial d}\right)^2\ \sigma_d{}^2 + \left(\frac{\partial M_T}{\partial fy}\right)^2\ \sigma_{fy}{}^2 + \left(\frac{\partial M_T}{\partial f^1{}_c}\right)^2\ \sigma_{f^1{}_c}{}^2$$

Evaluating the partial derivatives at $\bar{d}$, $\bar{f}y$ and $\bar{f}^1{}_c$, we obtain

$$\sigma_{M_T}^{\;2} = (2\,b\,\bar{d}\,\bar{f}^1{}_c\,a_o + A_s\,\bar{f}y\,a_1)^2\,\sigma_d^{\;2} + \left(\bar{d}\,A_s\,a_1 + 2\,\frac{A_s^2\,\bar{f}y}{b\,\bar{f}^1{}_c}\,a_2\right)^2 \sigma_{fy}^{\;2} + \left(b\,\bar{d}^2\,a_o - \frac{A_s^2\,\bar{f}y^2}{b\,\bar{f}^1{}_c{}^2}\,a_2\right)^2 \sigma_{f^1{}_c}^{\;2}$$

By putting $\bar{q} = \dfrac{A_s}{b\,\bar{d}}\,\dfrac{\bar{f}y}{\bar{f}^1{}_c}$,

$$V_{M_T}^{\;2} = \frac{(2\,a_o + a_1\,\bar{q})^2\,V_d^{\;2} + (a_1\,\bar{q} + 2\,a_2\,\bar{q}^2)^2\,V_y^2 + (a_o - a_2\,\bar{q}^2)^2\,V_c^{\;2}}{(a_o + a_1\,\bar{q} + a_2\,\bar{q}^{\,2})^2} \quad \text{------} \quad (3.14)$$

where $V_d = \dfrac{\sigma d}{\bar{d}}$, $V_y = \dfrac{\sigma fy}{\bar{f}y}$, $V_c = \dfrac{\sigma f^1{}_c}{\bar{f}^1{}_c}$

Thus:

$$V_{M_A}^{\;2} \approx \frac{(2\,a_o + a_1\,\bar{q})^2\,V_d^{\;2} + (a_1\,\bar{q} + 2\,a_2\,\bar{q}^2)^2\,V_y^2 + (a_o - a_2\bar{q}^2)^2\,V_c^{\;2}}{(a_o + a_1\bar{q} + a_2\,\bar{q}^{\,2})^2} + V^2_{\emptyset} \quad \text{-----} \quad (3.15)$$

Once the statistical parameters of d, fy, $f^1{}_c$ and $\emptyset$ are determined, values of $V_{M_A}^{\;2}$ can be obtained for various values of $\bar{q}$.

3.5.2. Evaluation of V_{M_S}.

Let M_L and M_D be the live and dead load respectively.

Then,

$$M_S = M_L + M_D \quad \text{----} \quad (3.16)$$

$$\overline{M}_S = \overline{M}_L + \overline{M}_D$$

and

$$\sigma_{M_S}^{\ 2} \approx \sigma_{M_L}^{\ 2} + \sigma_{M_D}^{\ 2}$$

thus,

$$V_{M_S}^{\ 2} = \frac{\sigma_{M_S}^{\ 2}}{\overline{M}_S^{\ 2}} \approx \frac{\sigma_{M_L}^{\ 2} + \sigma_{M_D}^{\ 2}}{(\overline{M}_L + \overline{M}_D)^2} \qquad \text{--- (3.16)}$$

Putting $\overline{m} = \overline{M}_D / \overline{M}_L$, $V_L = \dfrac{\sigma_{M_L}}{\overline{M}_L}$ and $V_D = \sigma_{M_D} / \overline{M}_D$

we have,

$$V_{M_S}^{\ 2} \approx \frac{V_L^{\ 2} + \overline{m}^2 V_D^{\ 2}}{(1 + \overline{m})^2} \qquad \text{--- (3.17)}$$

V_L is obtained from the distribution of vehicle loads. With an estimate of V_D and an initial value for $\overline{m}$, V_{M_S} can be evaluated.

3.6 Cost.

Cost is undoubtedly one of the main factors in the economic considerations of bridge design and construction.

With the development of the computer and the evolution of new methods and techniques, there are three main reasons for placing economy on a truly realistic basis:

(1) It is as easy to optimise cost based on sound engineering principles as it is to approximate,

(2) From a national and international point of view, the resources of the construction industry must be deployed to the maximum advantage in the face of increasing human needs, and

(3) Since design for certain purposes is being standardised, - industrial blocks and offices, apartment buildings, highway bridges, etc.,- considerable attention should be paid in the design stage to economy, with which structural engineers are already preoccupied.

3.6.1 General Cost Function.

The cost of a structure depends on many factors that can vary from country to country and even within one country[52]. The overall cost of a structure can be categorised as follows:

Cost of
- (a) design
- (b) materials
- (c) construction
- (d) maintenance
- (e) unserviceability or failure
- (f) demolition

For a particular class of structure, however, some of these cost factors are constant, such as cost of maintenance and demolition. The cost of design, construction and materials can be considered as the initial cost of the structure and cost (e) is the cost associated with failure. Thus, the two basic cost factors which are of particular importance to structural designers are initial cost and failure cost.

Many research engineers[12,24,25,61] have attempted to incorporate these two cost factors into an expression for the total cost of a structure. The generally accepted expression for the total cost function is:

$$T = I + P_F C_F \quad --- \quad (3.18)$$

in which T is the total cost; I, the initial cost and C_F, the cost associated with failure. Engineers have attempted to include in C_F, not only interest rates, cost of material loss, cost of repair or reconstruction, but also the cost due to loss of life. The inclusion of the latter cost, however, is questionable as life cannot be measured in dollars and cents or in gold.

3.6.2 Cost and Failure Probability.

One of the main problems encountered in obtaining a good estimate of the total cost of a structure is the evaluation of the cost of failure, $P_F C_F$. Since P_F has to be approximated as discussed in Section 3.4, it is necessary to investigate the sensitivity of the total cost and the central safety factor (central safety factor $\bar{n}$ depends on P_F) to changes in order of magnitude in P_F.

The basic form of the total cost function or total utility losses[62] is

$$T = I + P_F C_F \qquad --- \quad (3.18)$$

The initial cost, I, is a function of the strength; that is, it is dependent upon the central safety factor and can be expressed as

$$I = f(\bar{n})$$

The precise relationship between I and $\bar{n}$ is not known, but in view of the small range of strengths and form in a basic structure, a nearly linear $I - \bar{n}$ relationship is expected.[62] Paez and Toroga (Turkstra[62,62]) have studied the relationship for simple bridge structures and found the linear relationship

$$I = C_1 \bar{n} + C_0 \qquad --- \quad (3.19)$$

where C_1 and C_0 are constants. One of the aspects of this present study is to investigate the $I - \bar{n}$ relationship for a bridge beam.

Since the constant C_0 has no effect on the minimization of the total cost expression, formula 3.18 can be rewritten in the form

$$T = C_1\bar{n} + P_F C_F \qquad --- \quad (3.20)$$

Equation 3.9 for P_F can be rewritten as

$$P_F \approx k\,\bar{n}^{-b'/V} \qquad --- \quad (3.21)$$

where $\bar{n} = \bar{R}/\bar{S}$ and $V = \sqrt{V_R^2 + V_S^2}$.

Substituting in 3.20 for P_F, the expression for T becomes

$$T = C_1\bar{n} + C_F k\,\bar{n}^{-b'/V}$$

$$= C_1\bar{n}\left(\left(1 + \frac{C_F k}{C_1}\,\bar{n}^{-(b'+V)/V}\right)\right) \qquad --- \quad (3.22)$$

Differentiating and solving for $\bar{n}_{opt.}$,

$$\frac{\partial T}{\partial \bar{n}} = C_1 - C_F k \frac{b'}{V}\,\bar{n}^{-(b'+V)/V}$$

$$\bar{n}_{opt.} = \left(\frac{C_F}{C_1}\right)^{\frac{V}{V+b'}} \left(\frac{k\,b'}{V}\right)^{\frac{V}{V+b'}} \qquad --- \quad (3.23)$$

Substituting $\bar{n}_{opt.}$ for $\bar{n}$ in 3.22,

$$T_{opt.} = C_1\bar{n}_{opt.}\left(1 + \frac{V}{b'}\right) \qquad --- \quad (3.24)$$

Considering values of $\bar{n}$ near the optimum and putting $\bar{n}_{opt.} = \bar{n}^*$, the expression

$$\bar{n} = \bar{n}^*(1 + D) \qquad --- \quad (3.25)$$

represents a divergence of $\bar{n}$ from $\bar{n}_{opt.}$ in the vicinity of the optimum; where D is small compared to one. Then,

$$T = C_1\bar{n}^* \ (1 + D) \left(\left(1 + k \ \frac{C_F}{C_1} \ \bar{n}^{*\,-(b'+V)/V} \ (1 + D)^{-(b'+V)/V} \right)\right)$$

$$= C_1\bar{n}^* \ (1 + D) \left(\left(1 + \frac{V}{b'} (1 + D)^{-\frac{(b'+V)}{V}} \right)\right) \qquad ---- \quad (3.26)$$

$$\approx C_1\bar{n}^* (1 + D) \left(\left(1 + \frac{V}{b'} \ (1 - D \frac{b'}{V} \right)\right)$$

Since $V << b'$, by simplification

$$T \approx C_1\bar{n}^* (1 + \frac{V}{b'}) + \frac{V}{b'} D \ C_1 \ \bar{n}^* \qquad ---- \quad (3.27)$$

$$\doteq T_{opt.} + \frac{V}{b'} D \ C_1 \ \bar{n}^*$$

Since $V/b' << 1$ it can be seen that the total cost is relatively insensitive to changes in $\bar{n}$ near the optimum.

By substituting the value for $\bar{n}^*$ in the equation for P_F,

$$P_{F_{opt.}} = \frac{V}{b'} \ \frac{C_1}{C_F} \left(\frac{V \ C_1}{b' k \ C_F} \right)^{V/b'} \qquad ---- \quad (3.28)$$

Also, substituting the value for $\bar{n}$ near the optimum given by equation 3.21 in the formula for P_F,

$$P_F = k \ \bar{n}^{*\,-b'/V} \ (1 + D)$$

$$= P_{F_{opt.}} \ (1 - \frac{b'}{V} D)^{-b'/V} \qquad ---- \quad (3.29)$$

Since $b'/V >> 1$, it can be concluded that $\bar{n}$ is relatively insensitive to changes in P_F around the optimum point.

Paaz and Toroja (Turkstra[61]) has shown that there is a wide range of strength within which the computed value of the total cost shows little variation. Computations done by Turkstra have given similar results. The general conclusions that can be drawn from this theoretical analysis are:

(1) The total cost is relatively insensitive to changes in $\bar{n}$ near the optimum.

(2) There is a range of P_F's that can be used with only small variations in $\bar{n}$ near the optimum. On the other hand, P_F is extremely sensitive to small changes in $\bar{n}$.

(3) As a result of (1) and (2), it can be concluded that approximations such as that proposed by Cornell are sufficiently reliable at this stage of engineering knowledge for computing the probability of failure for a wide range of distribution functions of load and resistance.

It must be emphasised, however, that these conclusions only hold when $V << b$. As V gets larger; that is, as the level of control exercised on the design variables decreases and the factor of ignorance increases, the conclusions become less valid.

3.6.3 Initial Cost.

For the purposes of this study, only the initial cost is required. Since all other costs, except material cost, are reasonably constant for a particular class of structure, such as simple-span reinforced concrete bridges, only the material cost will be considered.

The material cost of a bridge beam is a function of the quantities of concrete and steel and the unit costs of these materials. Thus,

$$C_t = F(C_c \cdot Q_c) + G(C_s \cdot Q_s)$$

where C_t is the total initial cost per unit length; C_c and C_s, the unit costs of concrete and steel respectively; and Q_c and Q_s the quantities of concrete and steel.

For a rectangular concrete beam,

$$Q_c = b\,\bar{d} \times 1$$

$$Q_s = A_s \times 1$$

Substituting, and converting all quantities to the same unit of measure,

$$C_t = 0.00694\, b\,\bar{h}\, C_c + 3.40\, A_s\, C_s \qquad --- \qquad (3.30)$$

The values of C_c and C_s employed in this study are .50 and .07 respectively. These values are based on prices in Montreal, Canada.

A formulation of the cost function and the probability of failure in terms of the distributions of load and resistance has been presented. Due to the complexity involved in the evaluation of the failure probability in the general formulation, an approximate method, developed by Cornell, has been applied to the specific problem. The final proposed formulation requires only the means and variances of the distributions of the design variables to solve the problem.

3.7 Summary.

On the basis of the preceding discussion, the following are adopted:

(1) The relationship between $\bar{n}$ and P_F is given by Equation (3.11) with $k = 7.5$ and $b' = 4.5$.

(2) The mean moment capacity $\bar{M}_A$ is given by Equation (3.4) with the constants a_o, a_1 and a_2 equal to 0.011, 1.114 and -0.90 respectively, and $\bar{\phi} = 0.999$.

(3) The coefficient of variation of moment capacity V_{M_A} is given by Equation (3.15) and of applied moment V_{M_S} by Equation (3.17).

(4) Construction costs of concrete related to the cost of materials are given by Equation (3.30), with unit costs C_c and C_s equal to 0.50 and 0.07 respectively.

IV

OPTIMAL SOLUTION

The optimization techniques of operations research have been applied to structural design within recent years to obtain a "minimum-weight design" or a "minimum-cost design". In general, writers have referred to the combination of design variables resulting from such optimization procedures as the optimum combination. But the term optimum can be very misleading unless it is defined in relation to the specific problem; that is, the framework within which the optimal problem is to be solved. Herein, a method of obtaining the optimal solution of a reinforced concrete bridge problem based on cost factors and subject to certain safety requirements is presented.

4.1 Formulation of the Optimal Problem.

Several writers[13,43,48] within recent years have formulated the optimal problem as a non-linear programming problem and by employing certain approximations and iterative procedures, they obtain an "optimal solution". This optimization technique has been applied to plate girders[48], prestressed concrete structures[53], framed structures[18,45], and, in some cases, a general formulation applicable to different classes of structures has been attempted[13]. However, these formulations are made within the present design framework; that is, by adhering to the limitations and conditions set by the present design code. On this basis, an absolute optimum is sought.

This approach is unrealistic for two main reasons. Firstly, it does not relate cost to safety on a realistic basis. Safety or lack of safety can only be expressed in a probabilistic sense as discussed in previous chapters. By basing the optimal solution on fixed "allowable stresses" and "allowable load", the formulation ignores the statistical variation of these design variables.

Secondly, economy in structural design cannot be absolute. In other words, a designer cannot obtain an absolute optimum in a design problem. For example, rectangular R.C. beams have no finite optimum that is absolute. The cost of materials of such beams decrease as the depth of the beam increases. Thus the absolute optimum is at d equal to infinity. Further, the solution of the non-linear programming problem involves a complex procedure[18,43,44,45] and the results are questionable.

Economy in structural design, therefore, can only be considered in the context of a relationship between cost and safety. The problem here is to compute the optimum initial cost of a R.C. bridge beam cross-section for different values of the probability of failure and to examine the relationship between them.

4.2 Optimization Procedure.

An optimization procedure for solving the optimal problem is formulated as a computer programming problem. The known variables and constants are the input and the unknown variables are determined in logical sequence.

The optimal solution is obtained for a rectangular beam cross-section at mid-span of a simple span R.C. bridge. The bridge geometry is considered as a simple beam and slab arrangement. The bridge span is 50 feet and the superstructure is supported for one lane of traffic on three rectangular beams spaced on a width of

15 feet at 5 foot centres. The procedural steps are best understood by discussing each step as the formulation is developed.

4.2.1. Computational Steps.

The optimization procedure consists of the following computational steps:

Step 1: Constant Input - This input consists of the constants and all variables kept fixed throughout the computation.

Step 2: Variable Input - This consists of variables, the values of which are not constant, but are varied in the analysis.

Step 3: Central Safety Factor - The statistical parameters of f^{1}_{c}, fy, $\emptyset$ and M_L are obtained from the frequency distributions of these variables. On the other hand, the statistical parameters of V_D and V_d are estimated. This estimation is based on design experience. On the basis of these determined parameters and by using an initial value for $\bar{m} \doteq \bar{m}_i$, $V^2{}_{M_A}$ and $V^2{}_{M_S}$ are computed from the formulas developed in Section 3.5 of Chapter III for various values of $\bar{q}$. Thus, with $V^2{}_{M_A}$ and $V^2{}_{M_S}$ known, $\bar{n}$ is determined for different values of P_F from the formula:

$$\bar{n} = \exp.\left(Bk\sqrt{V^2{}_{M_A} + V^2{}_{M_S}} \right) \quad ----- \quad (4.1)$$

where $Bk = (7.5 - \log P_F) / 4.5$. A check is made in Step 6 for the value of $\bar{m}$.

Step 4: Design Variables - With $\bar{n}$ computed and an initial value of $\bar{m}$ estimated, the design moment is obtained from:

$$\bar{M}_A = \bar{n} \left(\left(\bar{M}_L \, (1 + \bar{m}) \right)\right)$$

and $\bar{d}$ can then be computed from

$$\bar{d} = \left(\left(\frac{12.0 \quad B \quad \bar{M}_A}{\bar{\emptyset} \; \bar{f}'_c \, (a_o + a_1 \bar{q} + a_2 \bar{q}^2)} \right)\right)^{1/3}$$

where $B = \bar{d} / b$. This equation for $\bar{d}$ is developed in Appendix A. See Appendix C for assumptions on $\bar{m}$.

It is necessary at this stage to choose values of one of the variables, $\bar{d}$, b or A_s, or a combination of these in order to compute the others, for there can be no absolute optimum. A preliminary computation was carried out to investigate which variable or variables' combination would be best suited to the analysis. The combination, $B = \bar{d}/b$, was chosen because in practical problems B would rarely be greater than three and, as a result, only three or four values need to be used. Also, the volume of computations to be carried out using B is comparatively small and therefore the computational time is reduced and the results less exhausting. The other design variables, b and A_s, are easily obtained from $B = \bar{d}/b$ and $\bar{q} = A_s \bar{f}_y / b \, \bar{d} \, \bar{f}'_c$.

Step 5: Total Depth - It is necessary to know the total depth of the beam before the cost of materials can be determined. It is, therefore, required to choose bar sizes and compute the number of rows of steel for a particular total area of steel. By investigating the influence of the bar sizes for various steel areas on the total depth of a rectangular beam, an automatic method was developed for determining the total depth, $\bar{h}$. This method is explained in Appendix B.

Step 6: Checking $\bar{m}$ - With the width and total depth of the beam determined and the bridge geometry given as explained earlier, the actual dead load moment, M_D, can be computed. $\bar{m}_{i+1}$ is then computed and compared with $\bar{m}_i$; a tolerance limit of 0.05 being allowed. If the deviation is greater than 0.05, the procedure is repeated again from Step 3 until convergence is obtained.

Step 7: Cost - Having obtained b, $\bar{h}$ and A_s, the materials' cost can be determined. To do this, the unit cost of both steel and concrete was chosen, based on delivered prices in Montreal, Canada. The cost per foot length of the beam, as developed in Section 3.6 of Chapter III, is expressed as

$$C_t = 0.00694\, b\bar{h}C_c + 3.40\, A_s\, C_s \quad - - - - \quad (7.2)$$

where C_c is the unit cost of concrete and C_s the unit cost of steel.

Step 8: Repeat - The procedure is repeated from Step 3 for values of $\bar{q}$ between .05 and .40.

Step 9: Repeat - The entire procedure is repeated from Step 2 for various values of the variables.

The numerical computations were performed on the IBM-7044 Computer of McGill University. A list of the values of the input data is given in Appendix C. The output data consisted mainly of the computed values of V^2M_A, V^2M_S, $\bar{n}$, $\bar{m}$,

$\bar{M}_A$, $\bar{d}$, b, A_s, $\bar{h}$ and C_t.

4.3 Optimum Cost and Failure Probability.

The output data was analysed in order to investigate the relationship between optimum initial cost and strength, and the influence, if any, this relationship has on the design variables.

It was observed early in the analysis of the output data that the optimum cost generally, occurred at an almost constant value of $\bar{q}$ for all values of the input variables. The output data gave the optimum in some cases at $\bar{q} = 0.20$ and in others at $\bar{q} = 0.25$. This was as a result of the increment of 0.05 used for $\bar{q}$.

Figs. (4-1A) to (4-1B) show the variation in cost with $\bar{q}$ for various values of P_F, $\bar{M}_L$, VM_A and VM_S. It can be observed from the curves that, in fact, the optimum lies somewhere between $\bar{q} = 0.20$ and $\bar{q} = 0.25$; perhaps closer to $\bar{q} = 0.25$. This constancy in the values of $\bar{q}$ for optimum cost can be explained by the fact that $\bar{q} = A_s\bar{f}_y / b\bar{d}\,\bar{f}^1{}_c$; thus if $\bar{f}_y$ and $\bar{f}^1{}_c$ are constant or $(\bar{f}_y/\bar{f}^1{}_c)$ is constant, then $\bar{q}_{opt.}$ can be expected to be constant. Further investigation is required to establish the relationship between $\bar{q}_{opt.}$ and $(\bar{f}_y/\bar{f}^1{}_c)$.

The relationship between optimum cost and strength is summarised in a series of tables and curves. Tables (4-1A) to (4-1D) summarises the optimum cost for four combinations of V_{M_A} and V_{M_S}. It can be observed from the tables that a large number of combinations of P_F, $\bar{M}_L$ and B will give the same cost for a particular V_{M_A} and V_{M_S} and even for various combinations of VM_A and VM_S. In fact, on each table, lines of constant cost can be drawn in a contour-like fashion.

In order to show the general relationship obtained, a number of curves are drawn of $\log P_F$ against C_t for $\overline{M}_L = 50$. Fig. 4/2 shows one such set of curves for varying V_{M_A}, V_{M_S} and B. For any particular P_F, the optimum cost increases for decreasing B and for increasing V_{M_A} and V_{M_S}, as is expected. Also, as the probability of failure decreases, the cost increases, but the rate of increase decreases as V_{M_A} and V_{M_S} increases; that is, for increasing V_{M_A} and V_{M_S}, the slope of the curve $\log P_F/C_t$ decreases.

This influence of V_{M_A} and V_{M_S} on the $\log P_F$ vs. C_t relationship is further shown by Figs. 4.3A, 4.3B and 4.3C.

This general tendency of the $\log P_F$ vs C_t curves for $\overline{M}_L = 50$ is typical for all $\overline{M}_L$'s. Figs. 4.4A and 4.4B give $\log P_F$ with C_t for various $\overline{M}_L$'s, V_{M_A} and V_{M_S}. The only influence of the increasing live load moment, $\overline{M}_L$, is to increase the magnitude of the optimum cost. It can be observed that the slope of the curves for all $\overline{M}_L$ and for a particular combination of V_{M_A} and V_{M_S} is almost constant. Also, the increase in the magnitude of C_t is almost constant for equal increases in the live load moment; P_F being kept constant. Fig.4-5 shows the relationship between $\log P_F$ and $\overline{n}$. The trend in the curves can be easily deduced from equation (3-11)

The most interesting and important relationship obtained was that between initial cost and strength or the central safety factor, $\overline{n}$. The curves shown in Figs. 4-6A to 4-6C indicate that there is a linear relationship of the form

$$C_t = A_o\overline{n} + A_1$$

between initial cost and $\overline{n}$ for a bridge beam; the constants A_o and A_1 depending on the live load moment and the depth to width ratio of the beam. It can be seen from the $C_t - \overline{n}$ curves that the relationship is independent of P_F, V_{M_A} and V_{M_S}. However, the value of $\overline{n}$ is directly dependent on P_F, V_{M_A} and V_{M_S}.

TABLE - 4-1A

OPTIMUM INITIAL COST

$V_{M_A} = .10$ $\qquad$ $V_{M_S} = .05$

P_F	$\bar{n}$	$\bar{M}_L = 50$			$\bar{M}_L = 60$			$\bar{M}_L = 70$			$\bar{M}_L = 80$			$\bar{M}_L = 90$		
		B			B			B			B			B		
		2.0	2.5	3.0	2.0	2.5	3.0	2.0	2.5	3.0	2.0	2.5	3.0	2.0	2.5	3.0
10^{-2}	1.35	1.948	1.809	1.677	2.008	1.841	1.729	2.068	1.899	1.770	2.115	1.943	1.821	2.179	1.999	1.875
10^{-3}	1.42	2.037	1.877	1.751	2.100	1.924	1.806	2.164	1.980	1.848	2.209	2.021	1.903	2.302	2.084	1.960
10^{-4}	1.50	2.131	1.964	1.830	2.197	2.022	1.888	2.300	2.052	1.931	2.344	2.103	1.990	2.429	2.165	2.048
10^{-5}	1.59	2.260	2.039	1.912	2.334	2.078	1.973	2.408	2.161	2.033	2.455	2.199	2.079	2.545	2.291	2.141
10^{-6}	1.69	2.378	2.117	1.999	2.451	2.181	2.063	2.523	2.282	2.126	2.548	2.349	2.163	2.652	2.395	2.230

TABLE - 4-1B

OPTIMUM INITIAL COST

$V_{M_A} = .10 \quad V_{M_S} = .10$

P_F	$\bar{n}$	$\bar{M}_L = 50$			$\bar{M}_L = 60$			$\bar{M}_L = 70$			$\bar{M}_L = 80$			$\bar{M}_L = 90$		
		B			B			B			B			B		
		2.0	2.5	3.0	2.0	2.5	3.0	2.0	2.5	3.0	2.0	2.5	3.0	2.0	2.5	3.0
10^{-2}	1.42	2.039	1.880	1.752	2.096	1.920	1.801	2.153	1.981	1.836	2.193	2.033	1.885	2.321	2.061	1.937
10^{-3}	1.52	2.162	1.986	1.846	2.211	2.036	1.897	2.311	2.054	1.948	2.369	2.114	1.985	2.427	2.155	2.038
10^{-4}	1.62	2.319	2.061	1.947	2.380	2.105	2.000	2.441	2.171	2.052	2.502	2.262	2.090	2.541	2.301	2.145
10^{-5}	1.73	2.469	2.177	2.054	2.518	2.272	2.109	2.556	2.331	2.164	2.586	2.389	2.194	2.653	2.446	2.290
10^{-6}	1.86	2.575	2.342	2.168	2.614	2.403	2.215	2.679	2.478	2.271	2.739	2.539	2.325	2.805	2.600	2.362

TABLE - 4-1C

OPTIMUM INITIAL COST

$V_{M_A} = .15 \qquad V_{M_S} = .05$

P_F	$\bar{n}$	$\bar{M}_L = 50$			$\bar{M}_L = 60$			$\bar{M}_L = 70$			$\bar{M}_L = 80$			$\bar{M}_L = 90$		
		B			B			B			B			B		
		2.0	2.5	3.0	2.0	2.5	3.0	2.0	2.5	3.0	2.0	2.5	3.0	2.0	2.5	3.0
10^{-2}	1.52	2.159	1.978	1.853	2.248	2.035	1.909	2.312	2.073	1.965	2.375	2.122	2.007	2.438	2.182	2.064
10^{-3}	1.65	2.330	2.090	1.974	2.399	2.142	2.034	2.468	2.232	2.093	2.535	2.294	2.187	2.577	2.339	2.175
10^{-4}	1.79	2.502	2.246	2.105	2.563	2.314	2.168	2.598	2.381	2.185	2.675	2.488	2.253	2.753	2.535	2.303
10^{-5}	1.94	2.638	2.399	2.212	2.716	2.513	2.279	2.789	2.585	2.342	2.867	2.657	2.409	2.969	2.727	2.496
10^{-6}	2.10	2.851	2.606	2.372	2.900	2.685	2.430	3.013	2.762	2.502	3.095	2.817	2.614	3.176	2.853	2.684

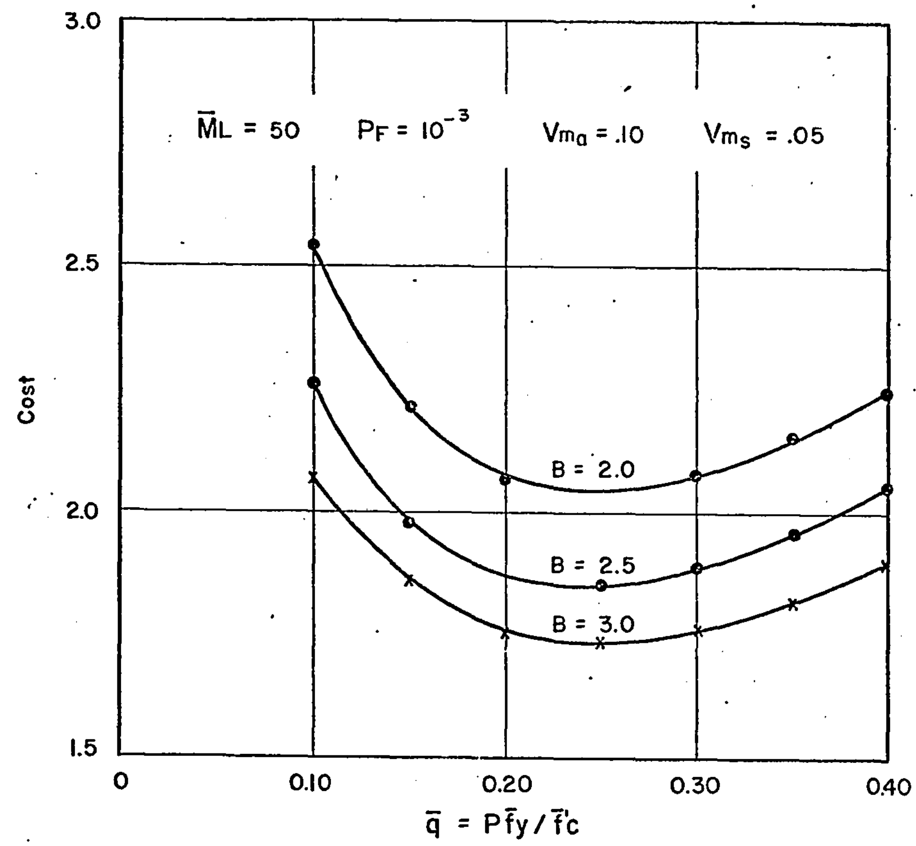
TABLE - 4-1D

OPTIMUM INITIAL COST

$V_{M_A} = .15 \qquad V_{M_S} = .10$

P_F	$\bar{n}$	$\bar{M}_L = 50$			$\bar{M}_L = 60$			$\bar{M}_L = 70$			$\bar{M}_L = 80$			$\bar{M}_L = 90$		
		B			B			B			B			B		
		2.0	2.5	3.0	2.0	2.5	3.0	2.0	2.5	3.0	2.0	2.5	3.0	2.0	2.5	3.0
10^{-2}	1.63	2.326	2.075	1.959	2.393	2.123	2.017	2.458	2.194	2.074	2.524	2.284	2.116	2.550	2.326	2.163
10^{-3}	1.79	2.520	2.270	2.112	2.570	2.335	2.170	2.601	2.400	2.200	2.676	2.489	2.273	2.753	2.534	2.315
10^{-4}	1.97	2.698	2.486	2.262	2.755	2.547	2.327	2.825	2.617	2.410	2.900	2.686	2.475	3.025	2.755	2.520
10^{-5}	2.16	2.918	2.677	2.460	3.045	2.753	2.531	3.105	2.827	2.581	3.196	2.886	2.671	3.276	2.938	2.741
10^{-6}	2.35	3.213	2.905	2.661	3.320	2.969	2.736	3.320	3.021	2.786	3.467	3.120	2.886	3.512	3.198	2.960

FIG. 4 - 1A



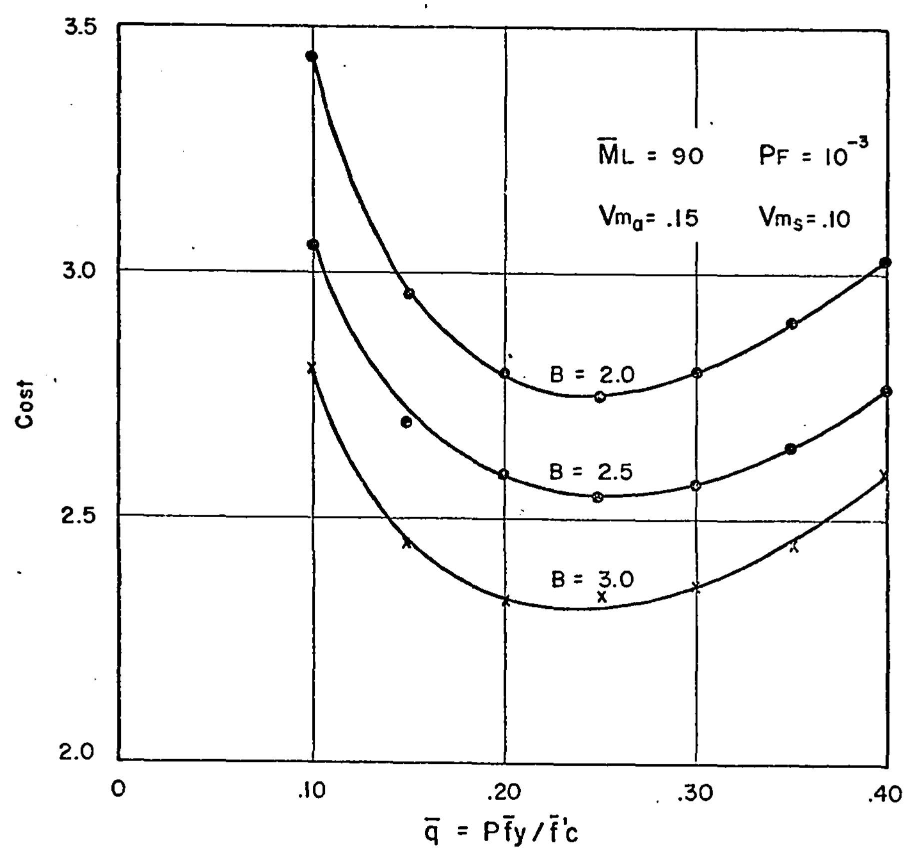


FIG. 4 - 1B

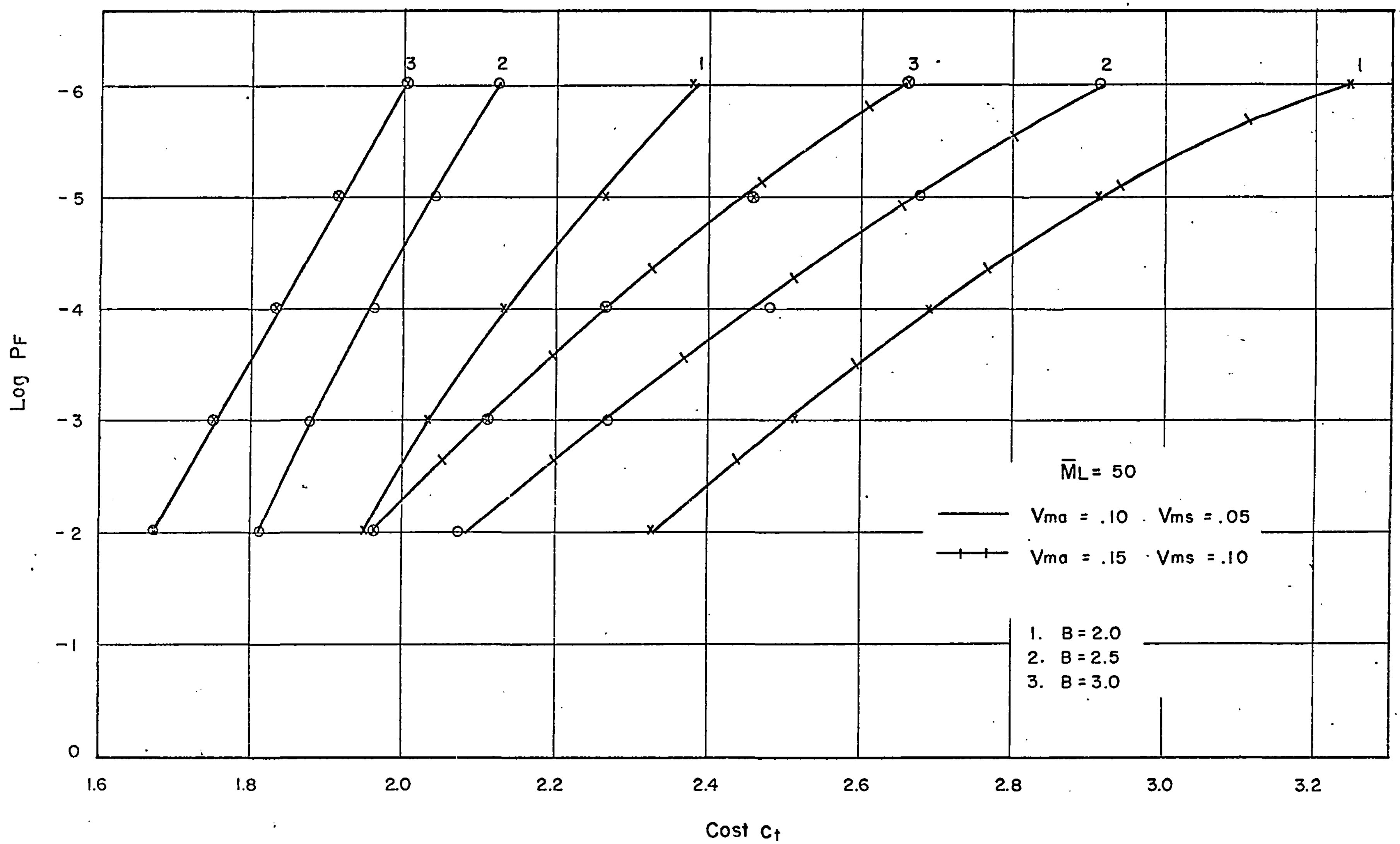


FIG. 4-2 PROBABILITY OF FAILURE WITH COST

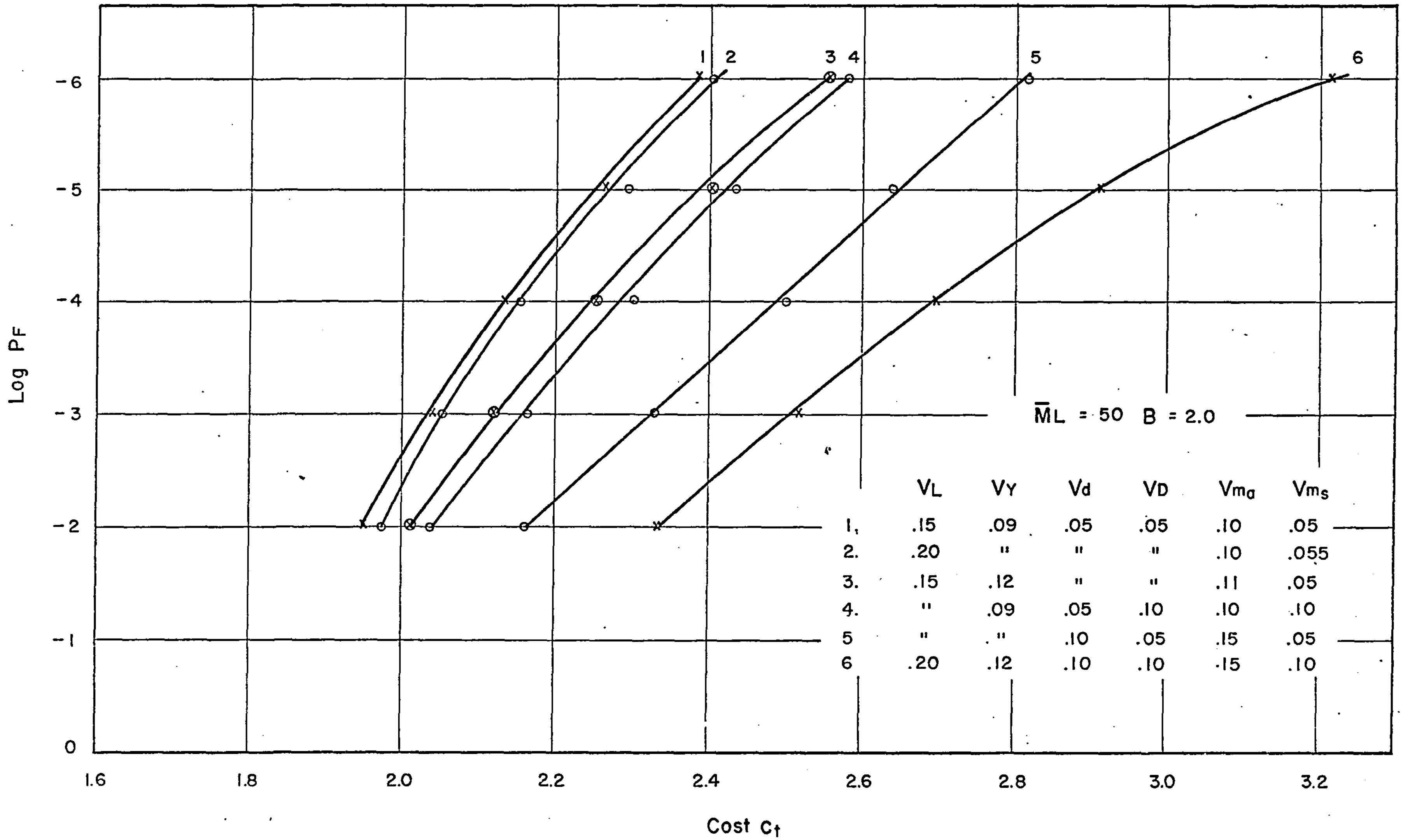


FIG. 4-3A PROBABILITY OF FAILURE WITH COST

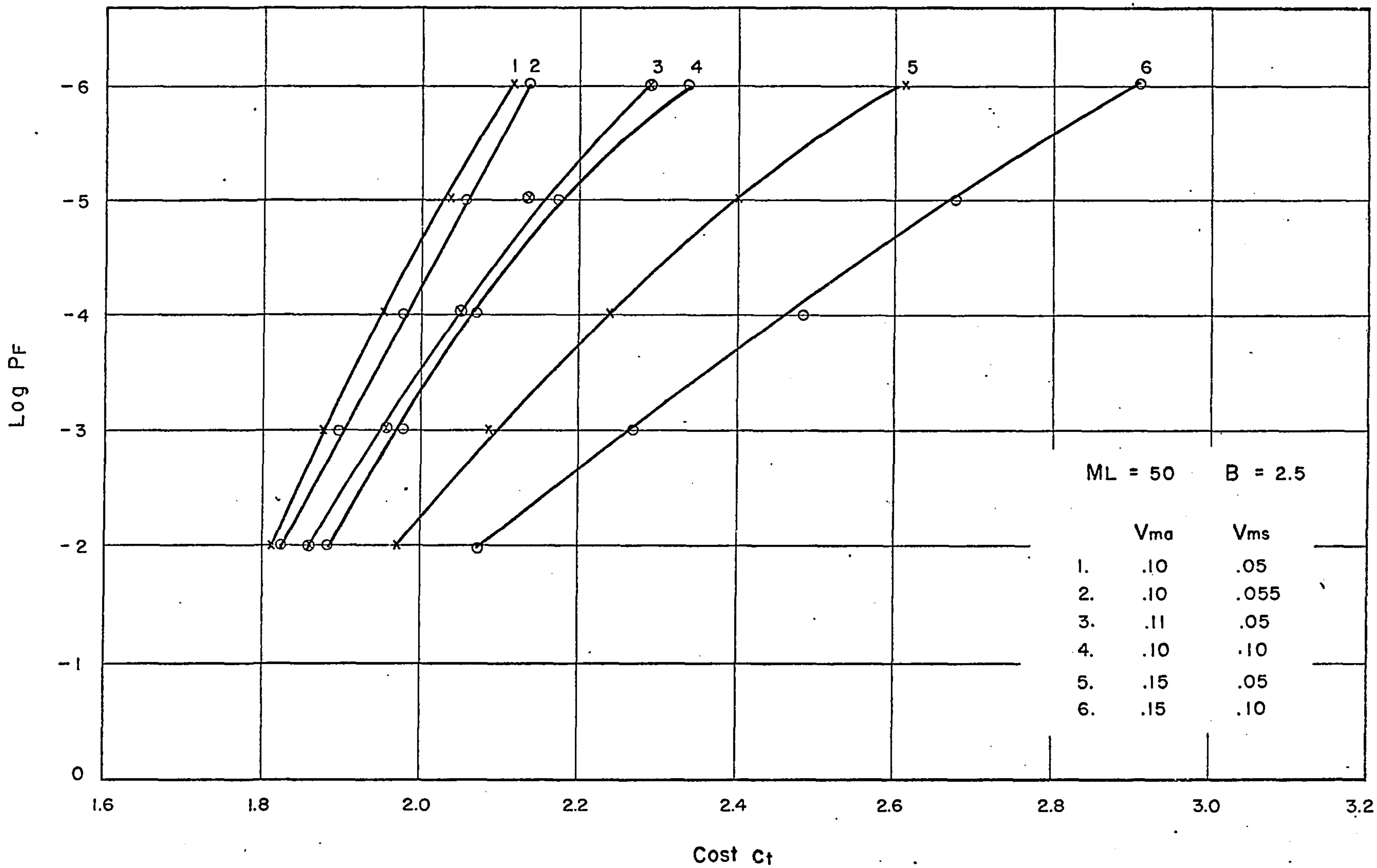


FIG. 4-3B PROBABILITY OF FAILURE WITH COST

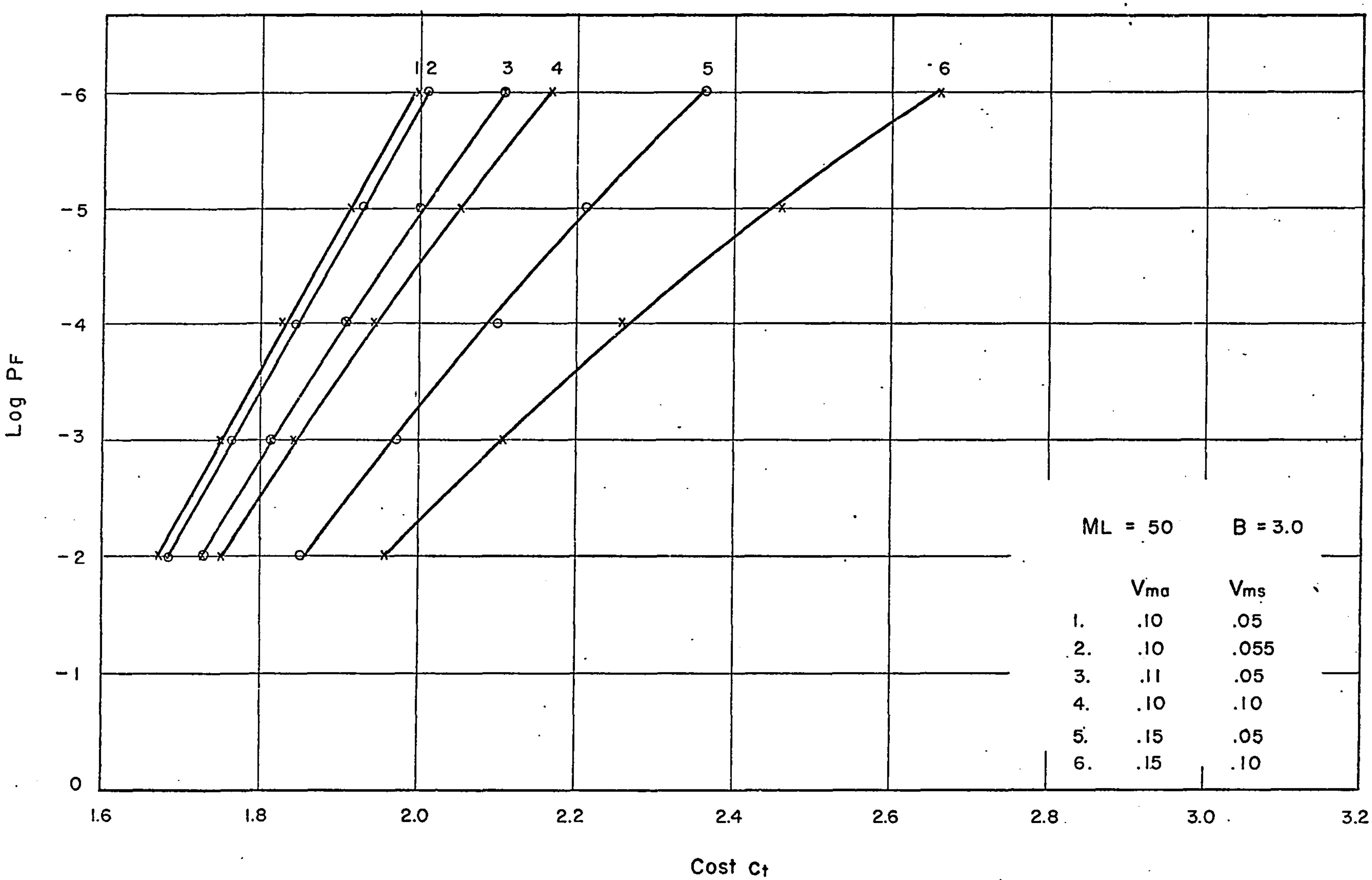


FIG. 4-3C PROBABILITY OF FAILURE WITH COST

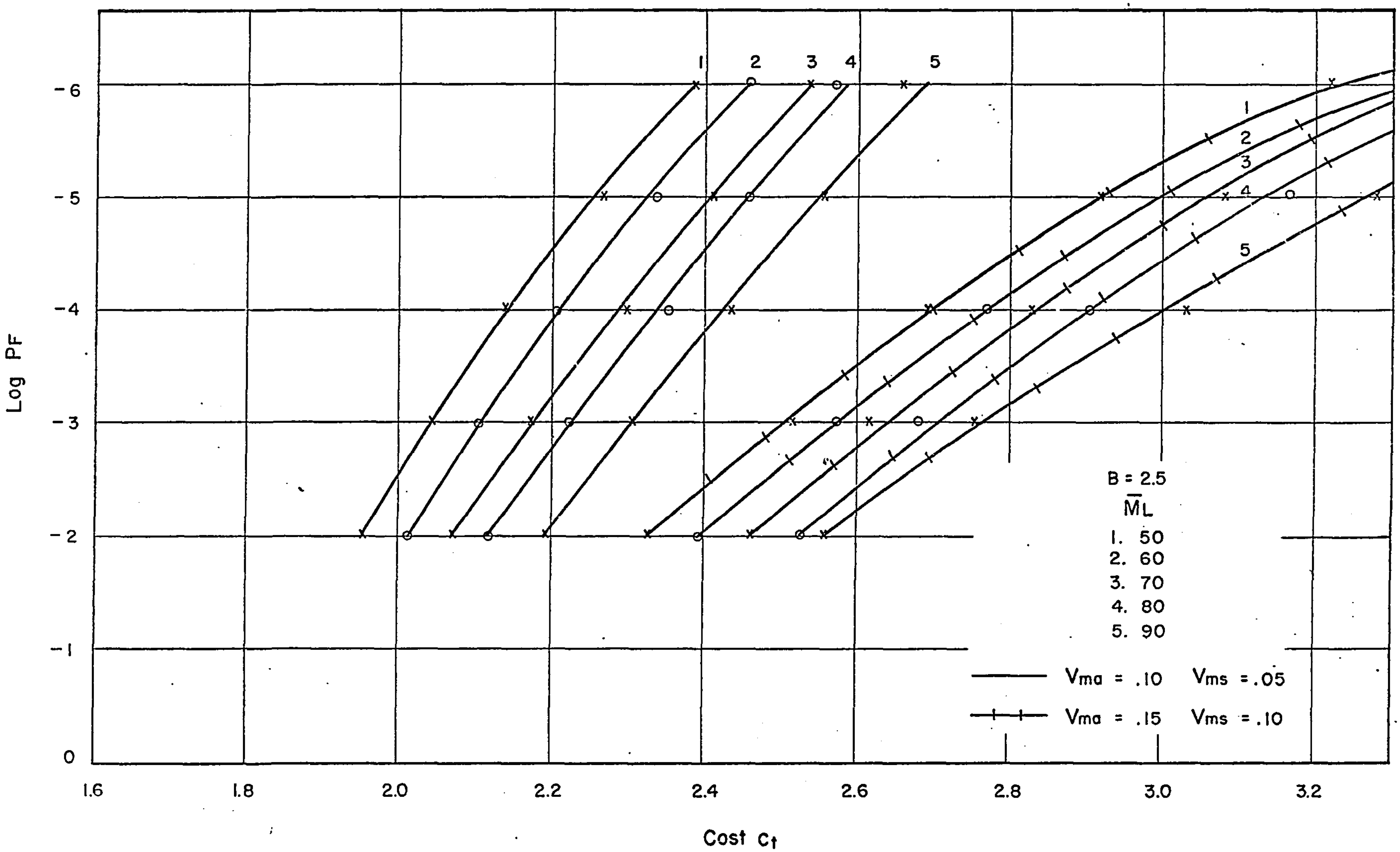


FIG. 4-4A PROBABILITY OF FAILURE WITH COST

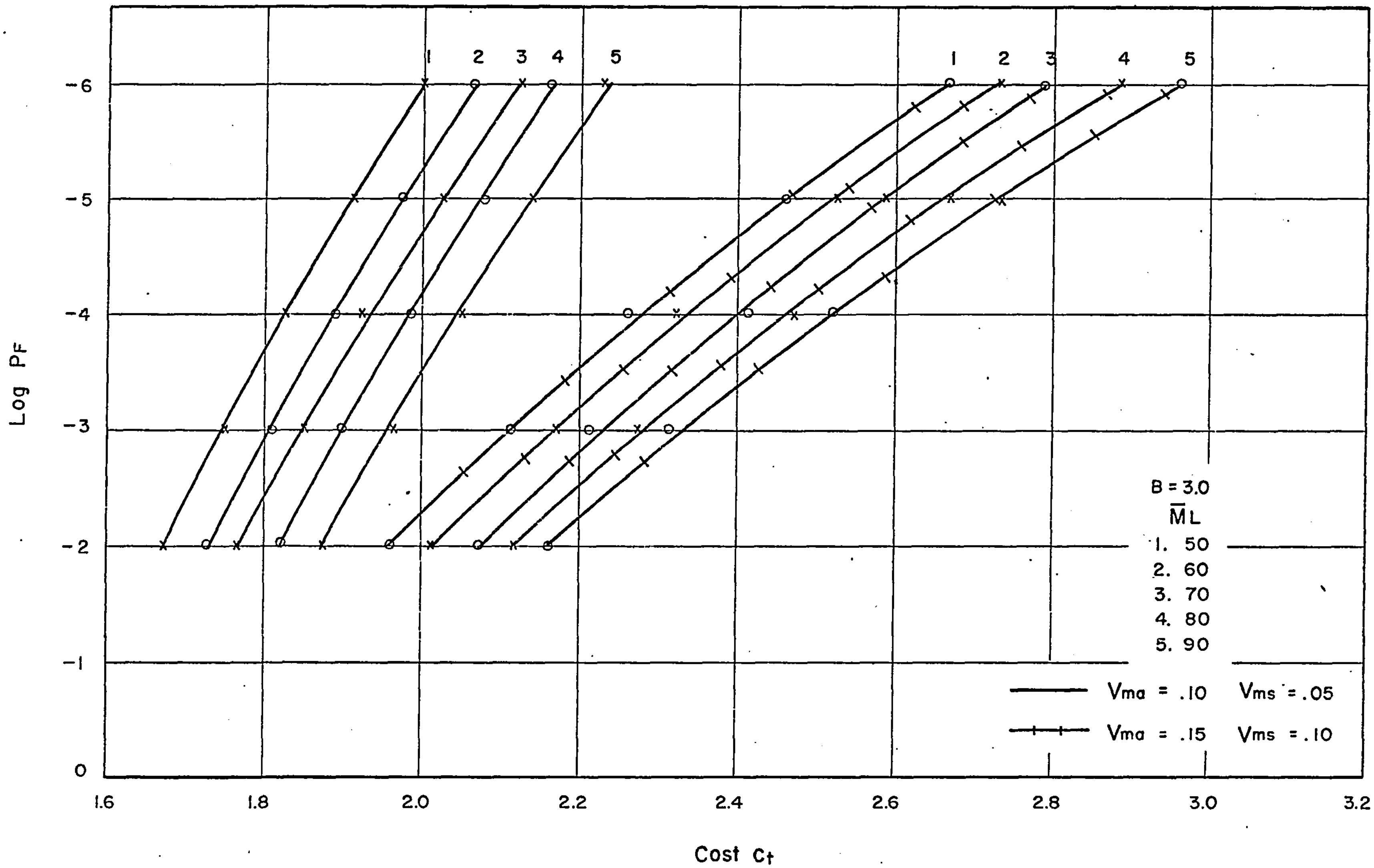


FIG. 4-4B PROBABILITY OF FAILURE WITH COST

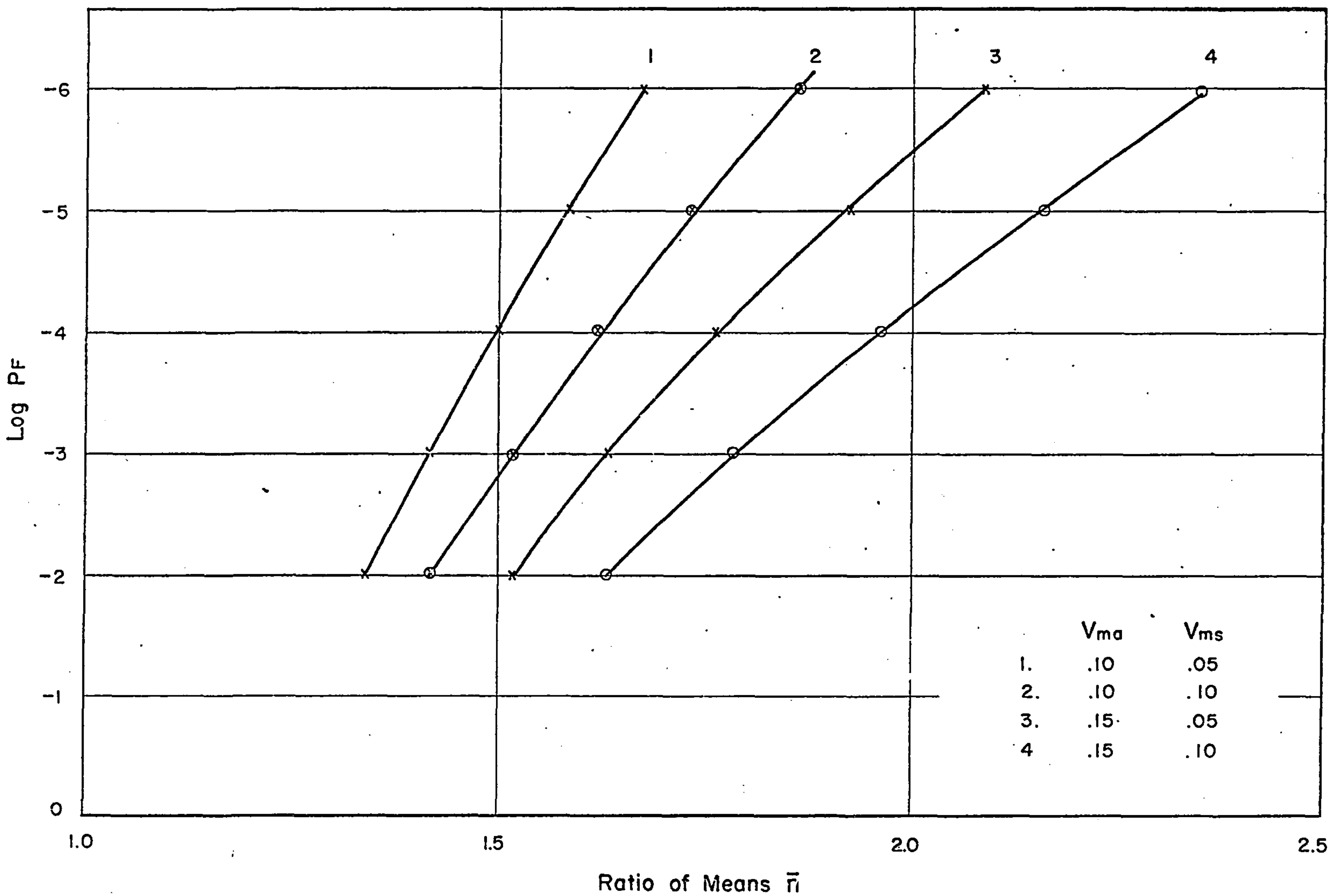


FIG. 4-5 PROBABILITY OF FAILURE WITH $\bar{n}$

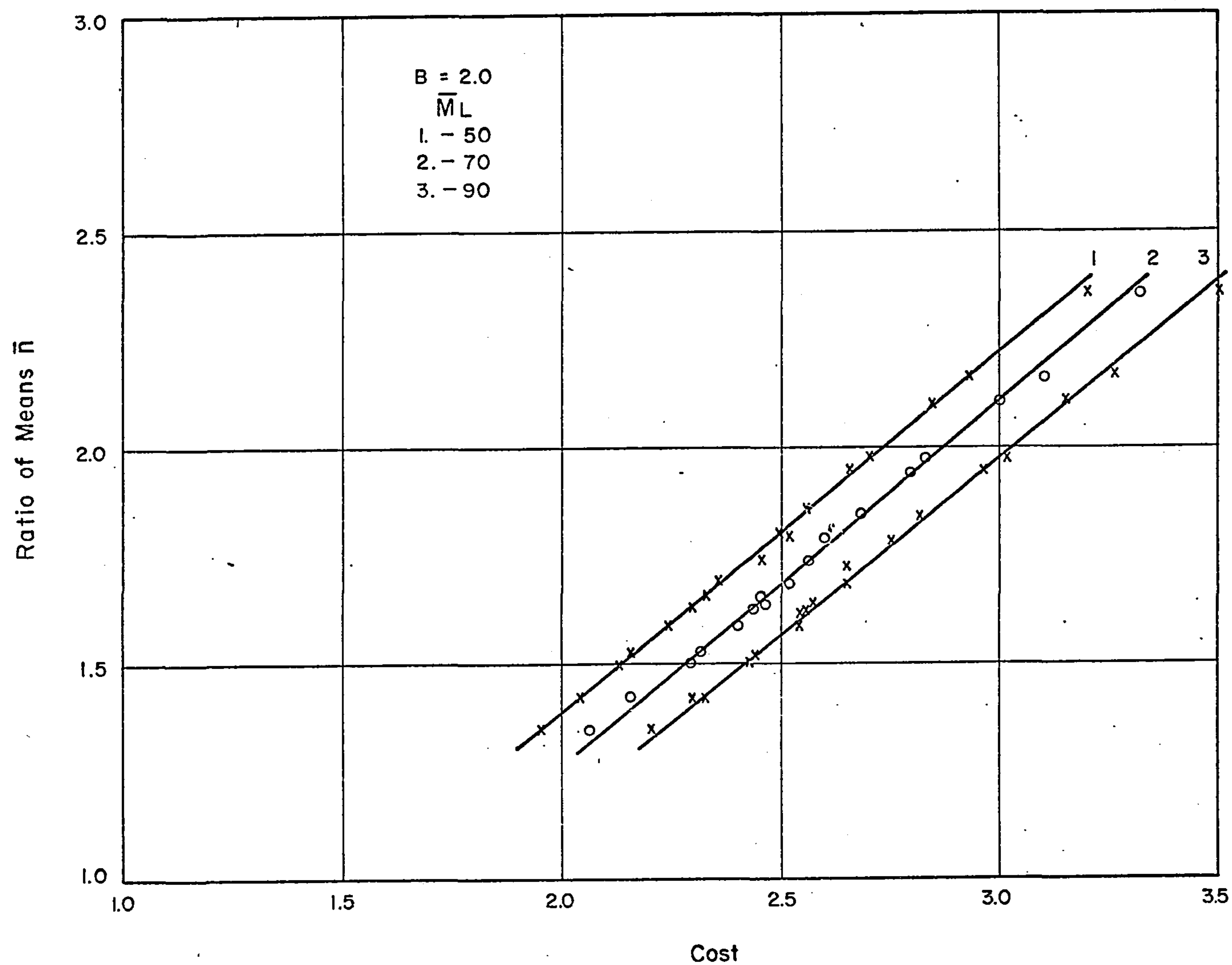


FIG. 4-6A RELATIONSHIP BETWEEN INITIAL COST AND $\bar{n}$

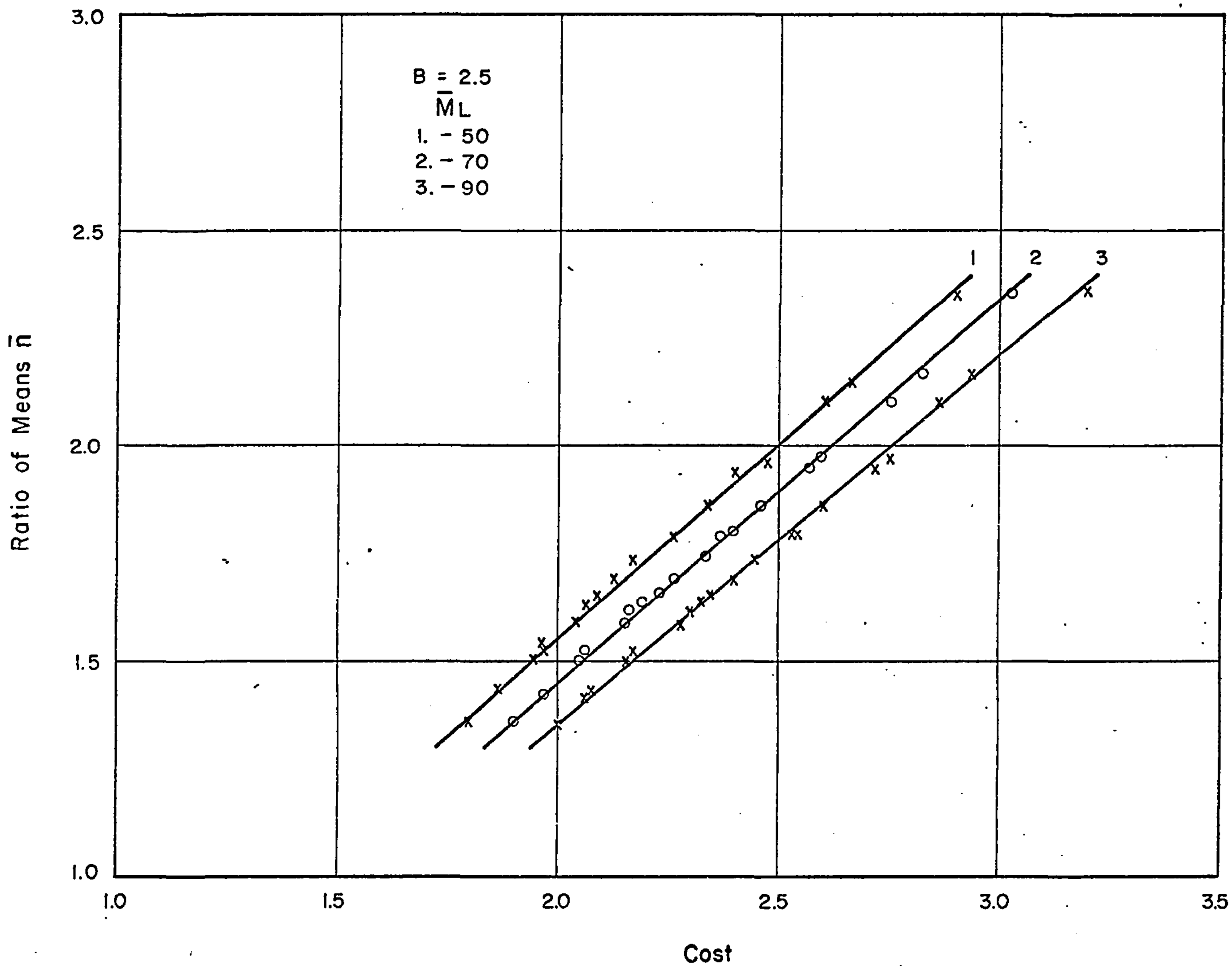


FIG. 4-6B RELATIONSHIP BETWEEN INITIAL COST AND $\bar{n}$

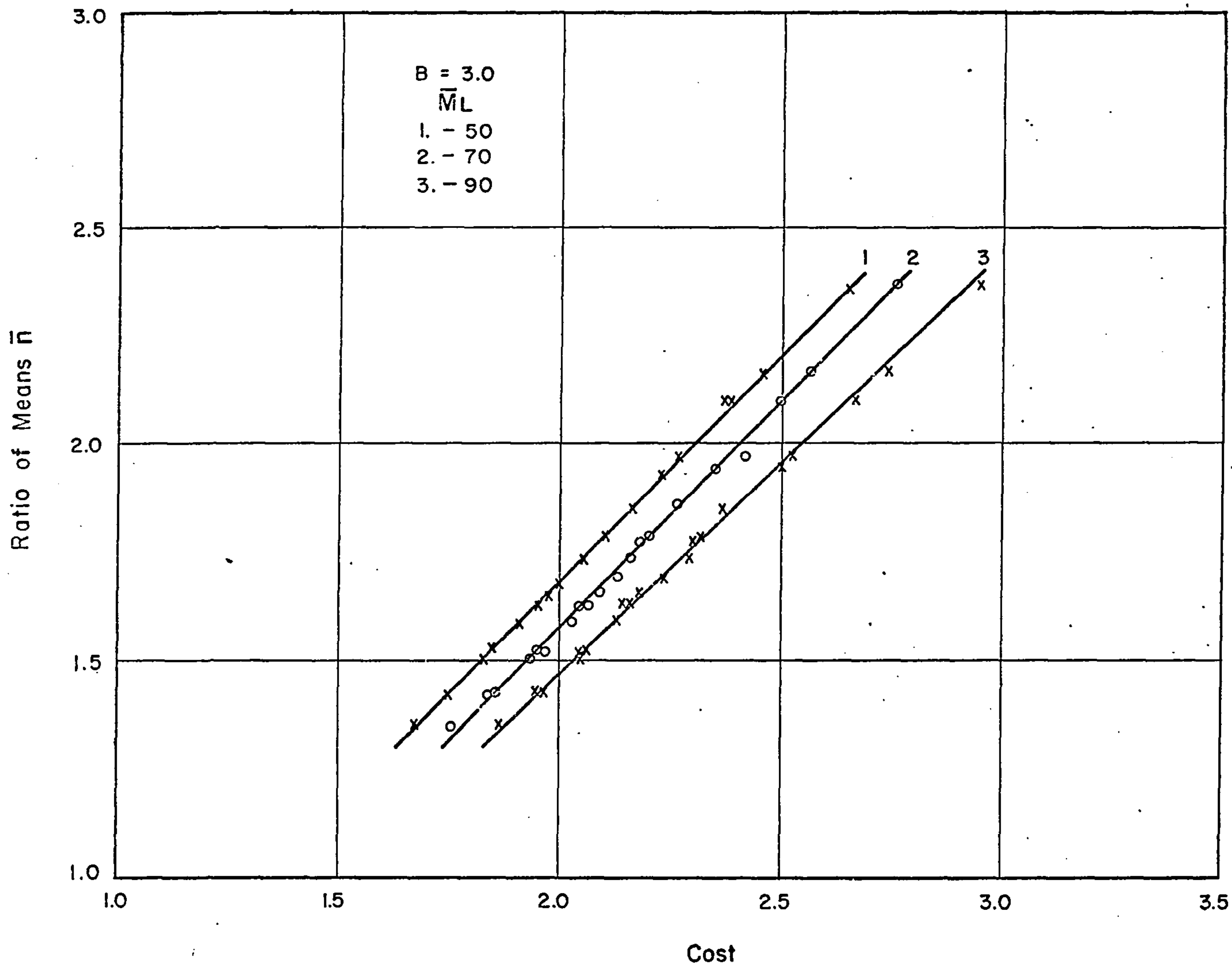


FIG. 4-6C RELATIONSHIP BETWEEN INITIAL COST AND $\bar{n}$

It was observed from the results of the computation that the dead to live load ratio $\bar{m}$ varied widely for various values of $\bar{M}_L$, V_{M_A} and V_{M_S}. The most pronounced variation in $\bar{m}$ was observed for varying $\bar{M}_L$. As $\bar{M}_L$ increased from 50 to 90, $\bar{m}$ decreased from 6.0 to 3.0 approximately. However, the effect on $\bar{m}$ of variation in V_{M_A} and V_{M_S} is not as pronounced. No systematic variation in $\bar{m}$ for any one variable or combination of variables was observed.

4.4 New Design Approach.

Since both cost and safety are the fundamental concern of designer, client and society, it is necessary and urgent that a new approach to the design problem be presented on a realistic, yet simple basis. Hereunder, such a formulation is presented for a bridge beam of rectangular cross-section, based on the results of this present study, but which formulation can act as a basis for future investigations.

(1) Say $\bar{f}_y$, V_y, $\bar{f}'_c$, V_c, $\bar{M}_L$, V_L, $\bar{\phi}$, V_ϕ, V_D and V_d are obtained from statistical data and chosen on a subjective basis from experience (the professional code can be a guide in such a choice).

(2) Probability of Failure – It is not meant here to begin a discussion on the choice of failure probabilities. However, the choice of the failure probability is undoubtedly related to the specific class of structure under consideration and the relative importance of the particular member to the structural operation of the entire structure. For certain members and certain classes of structures, the designer may be allowed a degree of flexibility in choosing the failure probability. Suffice it to say that a designer can be and must be guided in his choice by a new code. Say that a choice is made.

(3) Variance Ratios of Resistance and Load – From the statistical parameters determined in (1) and by calculating the ratio $(\overline{f}_y/\overline{f}^1{}_c)$, $\overline{q}_{opt.}$ for the particular class of structure, in this case an R.C. simple-span bridge, can be obtained from graphs and as a result V_{M_A} and V_{M_S} can be computed from the formulas given in Chapter III, or any other such formulas for the class of structure under consideration.

The only unknown is $\overline{m}$. At this stage, $\overline{m}$ can be guesstimated. The results of the computation showed that $\overline{m}$ varied from 3.0 to 6.0 approximately as $\overline{M}_L$ decreased from 90 to 50. Thus, knowing $\overline{M}_L$, say 90, $\overline{m}$ can be guesstimated to be 3.0. A check for $\overline{m}$ is made in Step (6).

It might be mentioned that with further investigation a new code might guide a designer by giving values of V_{M_A} and V_{M_S} for certain classes of structures and levels of control.

(4) Central Safety Factor – Having V_{M_A}, V_{M_S} and P_F, $\overline{n}$ can be obtained from a series of curves such as shown in Fig. 4-5. Turkstra[61] has drawn a series of these curves for various types of frequency distributions.

(5) Design Variables – With $\overline{n}$ known, $\overline{M}_A/\overline{m}$ can be taken from a curve such as that shown in Fig. 4-7, since $\overline{M}_L$ is known from (1). With the initial estimate of $\overline{m}$ from (3), $\overline{M}_A$ can be obtained or, instead, $\overline{M}_A/\overline{m}\,\overline{M}_L$ can be evaluated.

Figs. 4-8A, 4-8B and 4-9 show curves for finding the effective depth, $\overline{d}$. Since $\overline{q}_{opt.}$ has been determined in Step (3), values of $\overline{d}$ can be obtained from one of these curves for various values of $B = \overline{d}/b$, thus giving $b = \overline{d}/B$ and $A_s = b\overline{d}\,\overline{q}\,(\overline{f}^1{}_c/\overline{f}_y)$. In this way, values of b, $\overline{d}$ and A_s can be obtained for various values of B.

It might be necessary to limit, say, $\overline{d}$ because of clearance required. This constraint can be taken into consideration at this stage quite simply.

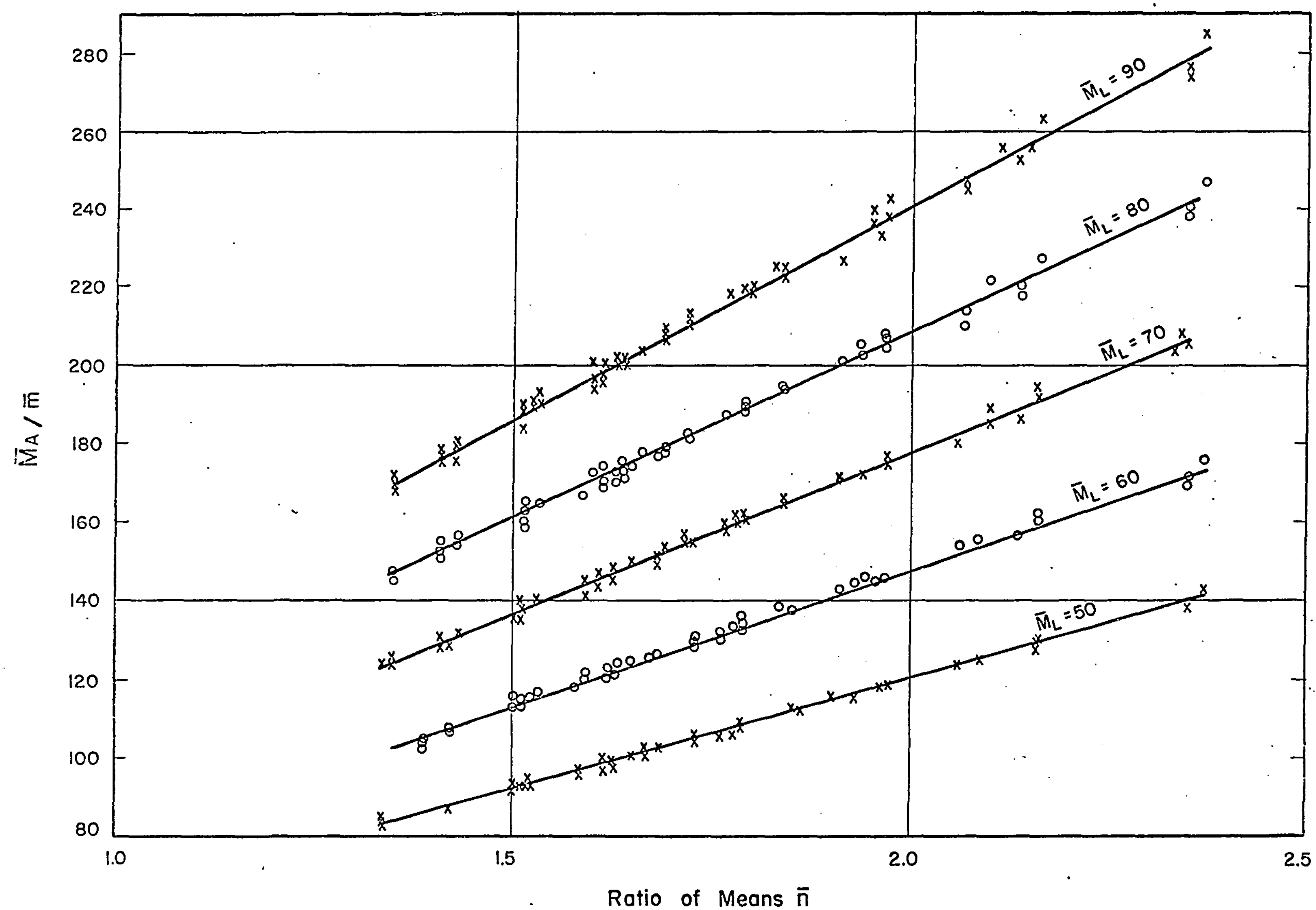


FIG. 4-7 CURVES FOR DETERMINING $\bar{M}_A$

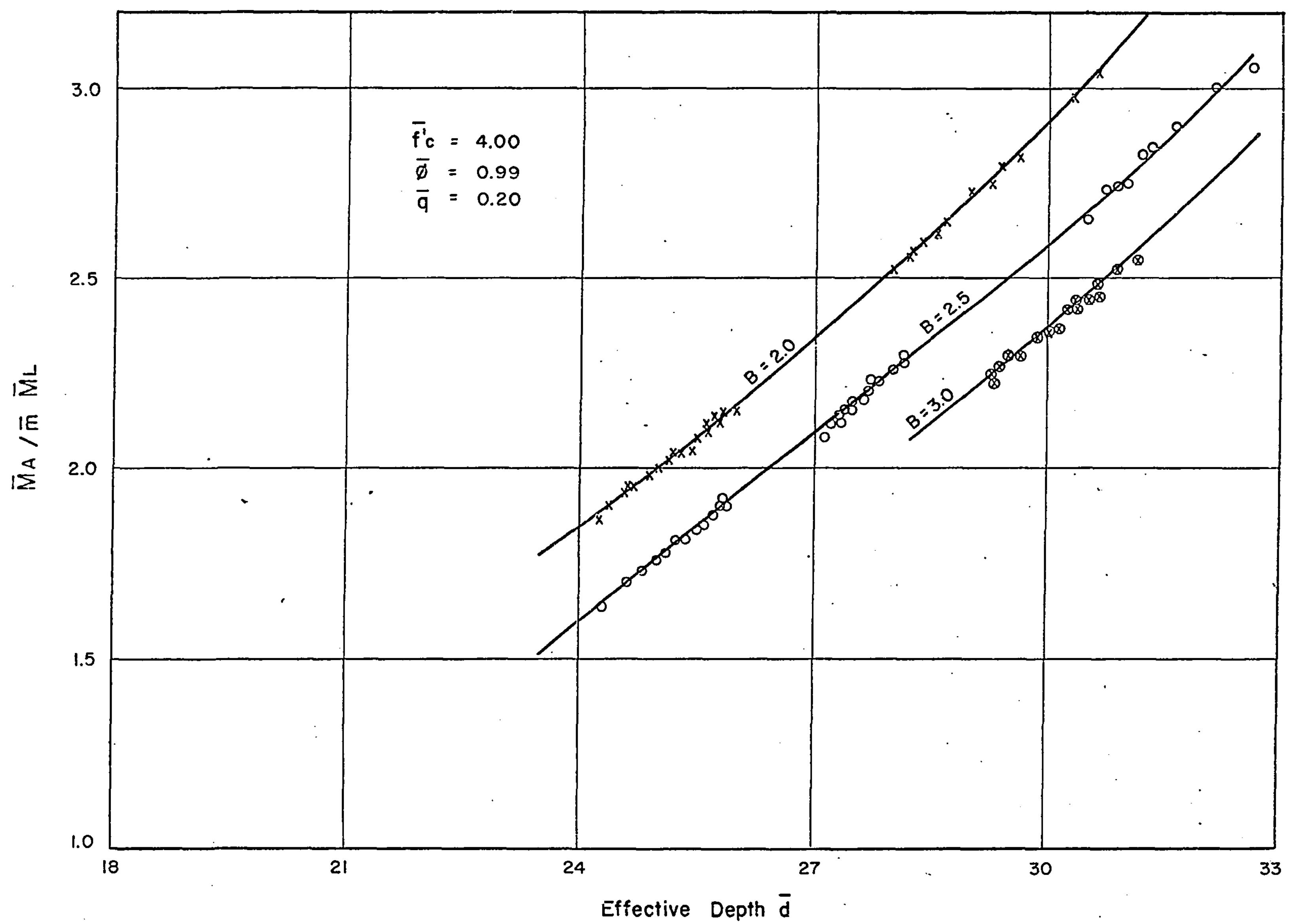


FIG 4-8A CURVES FOR DETERMINING $\bar{d}$

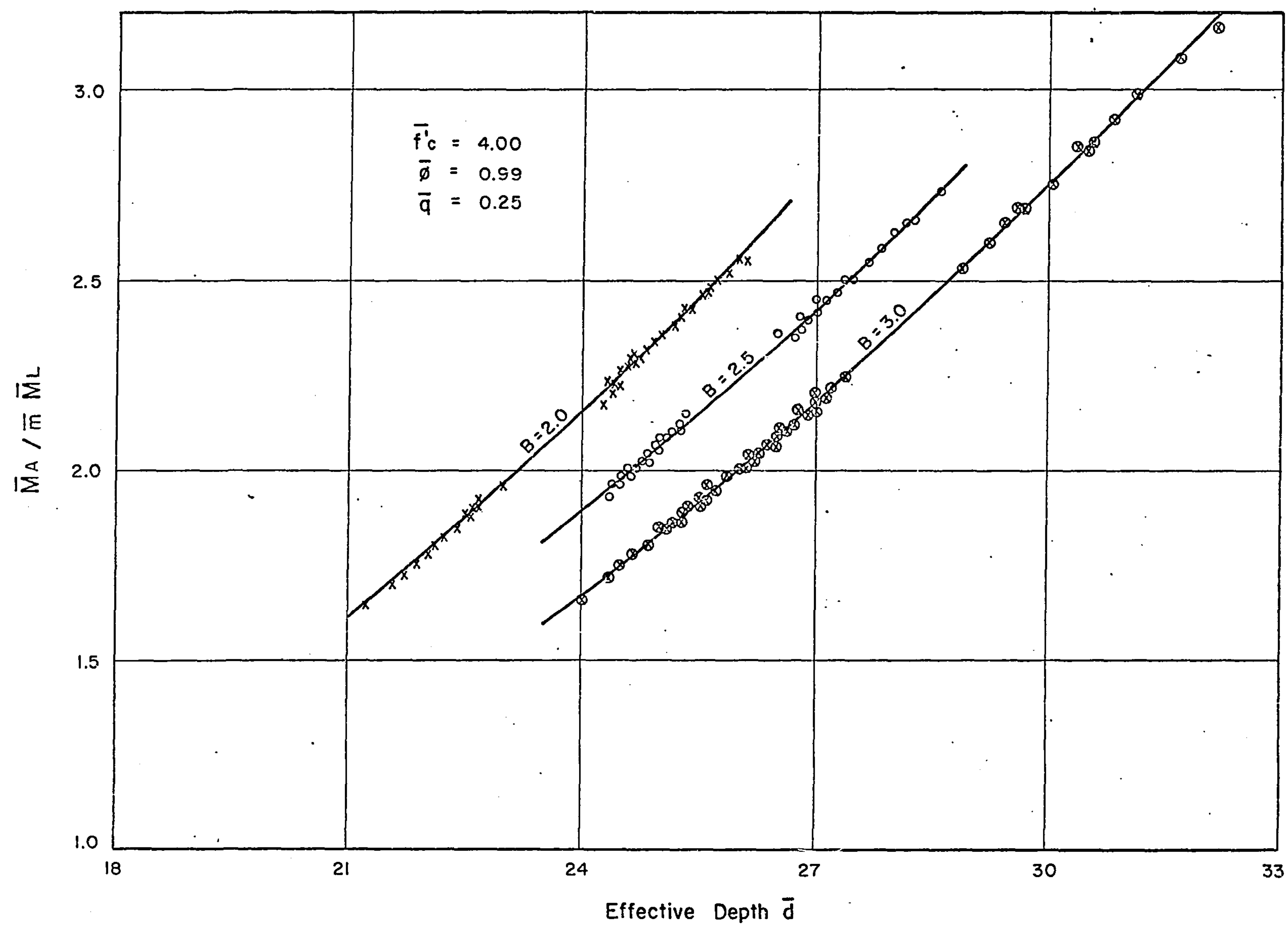


FIG 4-8B CURVES FOR DETERMINING $\bar{d}$

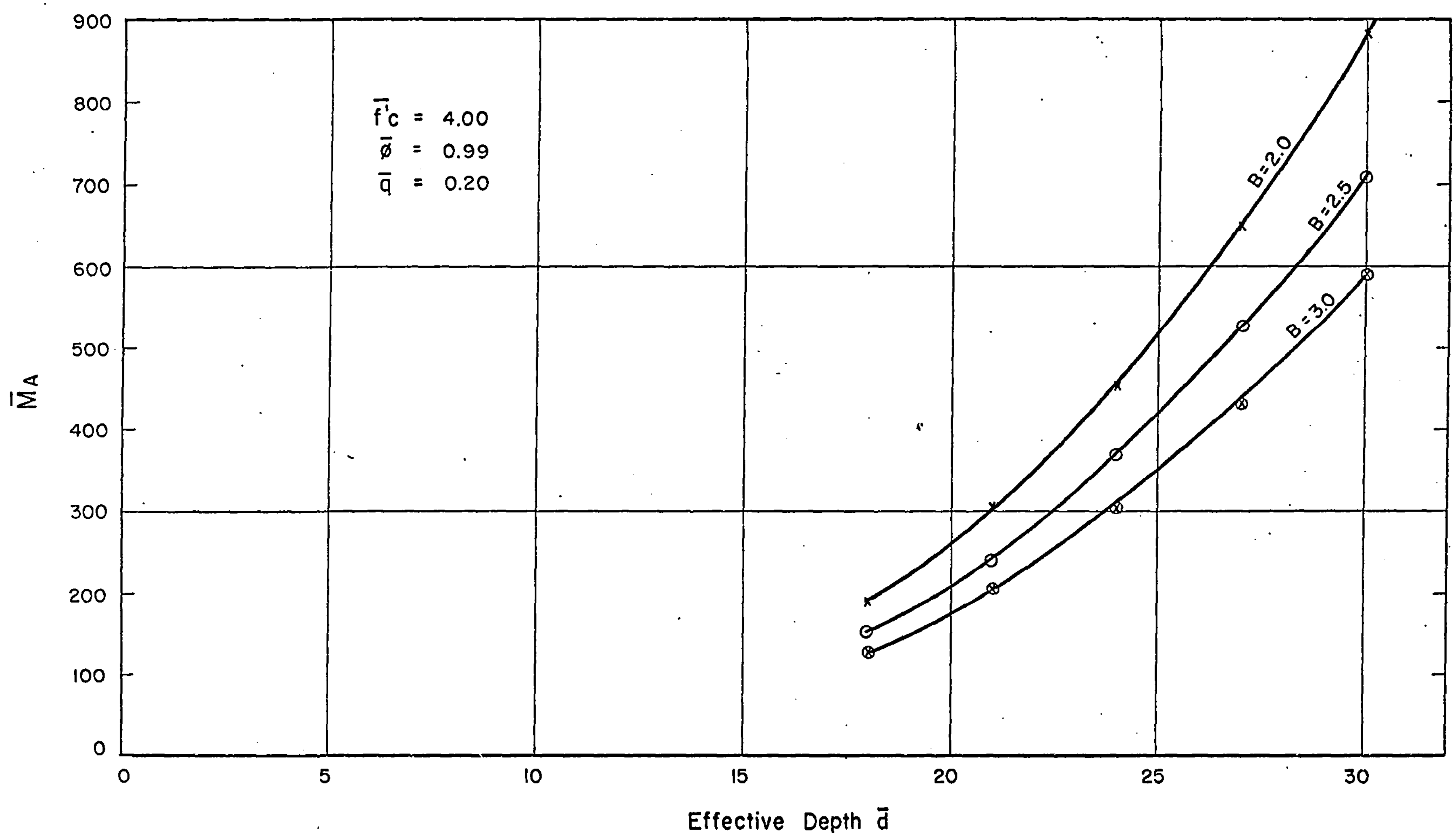


FIG 4-9 DESIGN MOMENT VERSUS EFFECTIVE DEPTH

(6) Check for $\bar{m}$ - Now, the value of $\bar{M}_D$ can be determined and thus $\bar{m} = \bar{M}_D/\bar{M}_L$ can be evaluated and checked with the initial estimate of $\bar{m}$. If necessary, the procedure can be repeated from Step (3) for the new value of $\bar{m}$.

(7) Cost - The cost depends, as observed earlier, on many variables. The procedure at this stage is to minimise cost. With V_{M_A} and V_{M_S}, P_F and $\bar{M}_L$ known, the cost can be investigated for various B's. Further, if flexibility is allowed in the choice of P_F, the cost can be examined for changes in P_F.

The method is equally reversable if an existing structure is to be rated. This new design approach cuts, to a considerable extent, the time spent on computation. Consequently, the designer can devote more time to the art of the design and the consideration of economy and safety.

There is no doubt that different classes of structures will have to be treated differently; but the basic procedure outlined can be improved and developed for a wide range of the popular design problems. The new procedure varies little in actual procedural steps from the contemporary design process. In fact, although the whole basis of the design method is changed, a designer can feel familiar with the procedure presented within a very short time.

V

RESULTS AND CONCLUSION

The main purpose of this study was to investigate the relationship between optimum initial cost and safety on a probabilistic basis as related to simple-span reinforced concrete bridge beams, and to examine the properties of the cost-safety relationship. Basically, the problem consisted of two parts: (a) formulation of the cost function and of the probability of failure in terms of the central safety factor n, and (b) the development of a method for solving the optimum problem.

5.1 Summary of Procedure.

A formulation of the probability problem is presented. This formulation is an approximation as a result of the lack of knowledge of the parameters of the exact formulation. In order to obtain the statistical parameters required for the evaluation of the approximate formula, a statistical analysis of data on load and flexural resistance was carried out. The loading data was obtained from the 1942 loadometer survey as analysed by Stephenson. The ultimate strength theory for moment capacity of beams was employed but the conventional ultimate strength formula was shown to be biased with q. As such, an empirical formula based on a statistical analysis of test data was developed. Data required for a statistical analysis of some of the variables were not available and a reasonable estimate of their values was made based on previous investigation.

The cost function was formulated for the cross-section of the beams at mid-span and, therefore, was expressed in terms of the sectional characteristics of the beams and the specific weights and unit costs of materials.

The optimization technique adopted was based on an iterative search method. A set of means, $\overline{M}_L$, $\overline{f}'_c$, $\overline{f}_y$, and coefficients of variations V_d, V_D, V_c, V_y, V_L, were specified and V_{M_A} was computed for a specified value of $\overline{q}$. Then for a particular value of P_F, $\overline{n}$ was computed and the sectional properties were obtained using an iterative procedure involving the dead load to live load ratio. The computation was carried out for various values of $\overline{q}$ and the optimum section obtained by a search method. The procedure was repeated for a variety of sets of basic parameters.

5.2 Properties of the Cost-Safety Relationship.

The results of this study indicate that the relationship between cost and safety for the class of problem and parameters studied exhibit certain properties that may give some insight into the probabilistic basis of design.

The optimum value of the ultimate strength parameter $\bar{q}$ was observed to vary between 0.20 and 0.25 for all values of variables used. The percentage difference in cost between $\overline{q}$ equal to 0.20 and 0.25 was of the order of 1.0 percent. This indicates that initial cost is relatively insensitive to changes in $\overline{q}$ and therefore in $\overline{p}$ around the optimum for a specific value of the $\overline{f}_y/\overline{f}'_c$ ratio.

As predicted by theoretical analysis, the central safety factor $\overline{n}$ was observed to be relatively insensitive to large changes in P_F around the optimum. For the smallest values of V_{M_A} and V_{M_S} used (V_{M_A} = .10, V_{M_S} = .05) a 100 percent increase in log P_F only gives a change in $\overline{n}$ of 9 percent; whereas for the largest V_{M_A} and V_{M_S} (V_{M_A} = .15, V_{M_S} = .10) a 100 percent increase in log P_F gives 18 percent change in $\overline{n}$. It is, therefore, obvious that as V_{M_A} and V_{M_S} increases $\overline{n}$ becomes more sensitive to changes in log P_F. This increase in sensitivity of $\overline{n}$ with increasing V_{M_A} and V_{M_S}

may be shown by considering the parameter $V = \sqrt{V_{M_A}^2 + V_{M_S}^2}$. For a 100 percent increase in log P_F, the change in $\bar{n}$ increases by 100 percent for 50 percent increase in V. Thus the sensitivity of $\bar{n}$ to changes in P_F around the optimum depends to a great extent on the values of the coefficients of variation of load and resistance.

A similar relationship was observed between failure probability and optimum cost. For 100 percent increase in log P_F, the optimum cost only increased by 9 percent. But for an increase in V of 50 percent, the change in optimum cost for a 100 percent increase in P_F increases to 17 percent. It was also observed that the relationship between optimum initial cost and $\bar{n}$ is a linear one of the form, $C_t = A_o\bar{n} + A_1$ where A_o and A_1 are constants. The ration A_1/A_o varied between 0.25 and 0.35 as the depth to width ratio increased from 2.0 to 3.0 with A_o remaining almost constant.

One of the most interesting properties of the cost-safety relationship is the effect on optimum cost of increasing $\bar{M}_L$. For any set of values of P_F, V_{M_A} and V_{M_S}, the absolute increase in the optimum cost was almost constant for an absolute constant increase in $\bar{M}_L$. As $\bar{M}_L$ increases from 50 to 90, that is an increase of 80 percent, the optimum cost increased by only 12 percent. A similar effect was observed in the $C_t - \bar{n}$ relationship. Thus the effect of increasing $\bar{M}_L$ on the cost-safety relationship is to increase the optimum cost proportionately for any P_F or $\bar{n}$.

The influence of the depth to width ratio of the beams was as expected. As the depth to width ratio increased the optimum cost decreased for any given set of values of the basic parameters. For example, as depth to width ratio increased from 2.0 to 3.0 or 50 percent, the optimum initial cost decreased by 15 percent. However, the dead load to live load ratio did not show any systematic variation. In general, it seems that $\bar{m}$ decreased with increasing $\bar{M}_L$ and $\bar{n}$ but it also varied to a lesser extent with changes in

V_{M_A}, V_{M_S} and the depth to width ratio.

5.3 Remarks.

The basic formulation of the probability problem has been developed by many writers and has been expressed as a statistical relationship between load and resistance based on the frequency distributions of these parameters. However, knowledge is lacking as to the precise nature of these distributions. Data in the range of the tail of the distribution curve is not available. Only the general trend of the distribution within the range of experimental measure is known. It is doubtful if sufficient data would ever be accumulated to allow engineers to specify the précise nature of the load and resistance distributions. Further, data required for a realistic estimate of some of the variables are not available and engineers will have to make a reasonable estimate based on experience and the level of control exercised. Under such circumstances, it seems that only the means and variances of the parameters can be reasonably estimated. Also, application of the probabilistic approach to design problems give a complex set of equations which cannot be solved at the present state of knowledge. It is as a consequence of this that an approximate formulation of the probability problem is necessary and may remain so for quite some time.

The optimization procedure developed in this study avoids many of the pitfalls of previous optimization methods. Whereas in previous programming problems of this nature research engineers have found it necessary to use mcve limits, adaptive move limits, accumulation of constraint equations, etc., which could give sub-optima points instead of optima, the method developed herein is a straightforward iterative search method.

The preceding discussion on the properties of the cost-safety relationship shows that many interesting phenomena were observed but further investigation is required to confirm their generality to the class of problem considered. The constancy in the value of $\bar{q}_{opt.}$ is one such phenomenum but it may be that $\bar{q}_{opt.}$ is related to the $(\bar{f}_y/\bar{f}^1{}_c)$ ratio. Also, further investigation is required into the effect of the statistical parameters of f_y and $f^1{}_c$ on the cost-safety relationship.

This study shows that a great deal of work is required to be done on the probabilistic approach to safety and economy in structural design before the concept is institutionalised and applied to everyday design problems. However, it also shows that the process of evaluating accumulated data, combining them with experience, examining the results and making a realistic choice can be formulated in a simple and logical manner.

APPENDIX A

EFFECTIVE DEPTH OF BEAM

From Section 3.5 of Chapter III:

$$\bar{n} = \bar{M}_A / \bar{M}_S$$

and from Section 3.5.2:

$$\bar{M}_S = \bar{M}_L + \bar{M}_D$$

Therefore,

$$\bar{M}_A = \bar{n}\,\bar{M}_S = \bar{n}\,(\bar{M}_L + \bar{M}_D)$$

Since $\bar{m} = \bar{M}_D / \bar{M}_L$

$$\bar{M}_A = \bar{n}\,(\!(\bar{M}_L\,(1 + \bar{m})\,)\!)$$

Also,

$$\bar{M}_A = \emptyset\,\bar{M}_T = \emptyset\,(\!(\,b\,\bar{d}^2\,\bar{f}'_c\,(a_o + a_1\bar{q} + a_2\bar{q}^2)\,)\!)$$

Thus,

$$\bar{d}^2 = \frac{\bar{M}_A}{\bar{\emptyset}\,\bar{f}'_c\,(a_o + a_1\bar{q} + a_2\bar{q}^2)\,b}$$

$$\bar{d}^3 = \frac{B \times \bar{M}_A}{\bar{\emptyset}\,\bar{f}'_c\,(a_o + a_1\bar{q} + a_2\bar{q}^2)}$$

APPENDIX B

TOTAL DEPTH OF BEAM

By investigating the relationship between the reinforcing steel bar sizes, number of rows and total depth of beam for various areas of steel and widths of beam, it was found that the following method of determining the total depth of the beam is both realistic and reliable. A check was made on the results and the method was shown to give accurate values of h, the total depth.

(a) Bar Sizes -- Assume the bar size to be No. 11.

(b) Total No. of Bars - If RN is the total number of bars required for a total area of steel A_s, then

$$RN = 0.64 A_s$$

(c) No. of Bars in One Row - Let ROW be the number of bars in one row; b^1 the required width of beam for ROW; R the total number of rows, then by putting

$$ROW = ROW_1$$

and assuming a 2" clear cover with 1" clear spacing between bars, the equation for b^1 is

$$b^1 = 2.41\, ROW_1 + 3.0$$

Using a tolerance limit of 0.5", if

$$(b + 0.5 - b^1)$$

is less than zero, $(ROW_1 - 1)$ is used; if it is zero, ROW_1 is used, and if it is greater than zero, the cycle is repeated with $ROW = ROW_1 + 1.0$. The result of this step is a value for ROW.

(d) Number of Rows - Now,

$$R = RN/ROW$$

Using a tolerance limit of 0.1; if NR is the nearest lower digit of R, and if

$$\langle\langle R - (NR + o.1) \rangle\rangle$$

is less than or equal to zero, then NR is used as the number of rows; if it is greater than zero, then NR + 1 is used.

(e) Total Depth - With the number of rows, NOR, determined, by a simple computation, $\overline{h}$ is obtained, thus

$$\overline{h} = \overline{d} + NOR + 1.5$$

There might be other methods that can be developed, but for the popular bar sizes used in bridge and structural design in general, whenever heavy loads are the applied loads, this method is simple and gives accurate results.

APPENDIX C

VALUES OF INPUT CONSTANTS AND VARIABLES

The values of the constants and variables which form the input data are given below. Only one value each for $\bar{f}^1{}_c$ and $\bar{f}y$ is used. If two or three values were used, the output data would be too voluminous and, consequently, too time-consuming to analyse. One value of V_c is employed as preliminary investigation showed that variation in V_c has little effect on the value of V_{M_A}. In computing $\bar{m}$ in Section 4.2.1 of Chapter IV, the following assumptions are made:

1. the thickness of the slab is 6";
2. the beams are spaced at 5 feet centres;
3. the bridge span is 50'.

(a) Constants:

$a_o = 0.011$; $a_1 = 1.114$; $a_2 = -0.90$

$\bar{\phi} = 0.999$; $V_\phi = 0.085$

$\bar{f}^1{}_c = 4.0$; $V_c = 0.120$

$\bar{f}y = 50.0$

$C_c = 0.50$

$C_s = 0.07$

(b) Variables:

$P_F = 10^{-2};\ 10^{-3};\ 10^{-4};\ 10^{-5};\ 10^{-6}$

$\bar{M}_L = 50\ ;\ 60\ ;\ 70\ ;\ 80\ ;\ 90$

$V_L = 0.15\ ;\ 0.20$

V_d = 0.05 ; 0.10

V_D = 0.05 ; 0.10

V_y = 0.09 ; 0.12

B = 2.0 ; 2.5 ; 3.0

REFERENCES

1. A.A.S.H.O. Road Test, 1961: "Bridge Research"; Report 4, Highway Research Board, Special Report 61D.

2. A.C.I. Committee 214, 1964: "Recommended Practice for Evaluation of Compression Test Results of Field Concrete", Journal, A.C.I., Vol.60, 1964.

3. A.C.I. - A.S.C.E. Committee 326, 1962: "Shear and Diagonal Tension", J.A.C.I., Vol.59, 1962.

4. A.C.I.: "Building Code Requirements for Reinforced Concrete", A.C.I. 318-63, 1963.

5. AL-ALUSI, A.F.: "Diagonal Tension Strength of R.C. T-beams with Varying Shear Span", J.A.C.I., Vol.28, 1957.

6. ASPLUND, S.O.: "The Risk of Failure", Structural Engineer, Vol.36, No.8, 1958.

7. BAKER, J.F.: "The Steel Skeleton", Vol.1, Cambridge Univ. Press, 1954.

8. BAKER, J.F., HORNE, M.R., HEYMAN, J.; "The Steel Skeleton", Vol.II, Cambridge Univ. Press, 1956.

9. BIGGS, J.M., SUER, H.S., LOUW, J.M.: "Bridge Vibration", Progress Report No.2, Special Report No.19, Dept. of Civil Eng., M.I.T., July 1956.

10. BLOEM, D.L.: "Studies of Uniformity of Compressive Strength Tests of Ready-Mixed Concrete", A.S.T.M., Bull.206, May 1955.

11. BROCK, G.: "Effect of Shear on Ultimate Strength", J.A.C.I., Vol.31, Jan. 1960.

12. BROWN, C.B.: "Concepts of Structural Safety", A.S.C.E. Proc. Struc. Div., Vol.86, 1960.

13. BROWN, C.B., and H-S. ANG, A.: "Structural Optimization by Non Linear Programming", private correspondence.

14. CORNELL, C.A.: "A Proposal for a Reliability-based Code Suitable for Immediate Implementation", private correspondence, Aug. 1967.

15. CORNELL, C.A.: "Bounds on the Reliability of Structural Systems", A.S.C.E. Proc.Struc.Div., Vol.93, Pt.1, Feb.1967.

16. COX, K.C.: "Tests of R.C. Beams with Recommendations for Attaining Balanced Design", J.A.C.I., Vol.38, Sept. 1941.

17. CORSO, J.M. : "Disc. on Freudenthal's 'Safety and Probability of Structural Failure'", A.S.C.E., Trans., Vol. 121, 1956.

18. ESTRADA-VILLEGAS, J.E.: "Optimum Design of Planar Trusses using Linear Programming", M.S.Thesis, School of Eng., M.I.T., Sept. 1965.

19. EVANS, R.H.: "Strength of R.C. Beams", I.C.E., Jour., Vol.21-22, 1943-44.

20. FELLER, W.: "An Introduction to Probability Theory and its Applications", John Wiley and Sons, Inc., N.Y.

21. FERGUSON, P.M.: "Diagonal Tension in T-beams without Stirrups", A.C.I., Jour. Vol.24, March 1953.

22. FERGUSON, P.M.: "Some Implications of Recent Diagonal Tension Tests", A.C.I. Jour. Vol.53, 1956.

23. FREUDENTHAL, A.: "The Safety of Structures", A.S.C.E., Trans., Vol.112, 1947.

24. FREUDENTHAL, A.: "Safety and Probability of Structural Failure", A.S.C.E., Trans., Vol.121, 1956.

25. FREUDENTHAL, A.: "Safety, Reliability and Structural Design", A.S.C.E., Proc. Struc. Div., Vol.87, 1961.

26. FREUDENTHAL, A., GARRELTS, J.M., SHINOZUKA, M.: "The Analysis of Structural Safety", A.S.C.E. Proc. Struc. Div., Vol.92, Pt.1, 1966.

27. GREENSHIELDS, B.D. and WEIDA, F.M.: "Statistics with Application to Highway Traffic Analyses", The Eno Foundation, Columbia Univ.Press, N.Y., 1952.

28. HIMSWORTH, F.R.: "The Variability of Concrete and its Effect on Mix Design", Inst. Civ.Eng., Proc., Pt.1, Vol.3, No.2, March 1954.

29. HOGNESTAD, E., HANSON, N.W., McHENRY, D.: "Concrete Stress Distribution in Ultimate Strength Design", A.C.I. Jour., Vol.52, 1955-56.

30. HOGNESTAD, E.: "Confirmation of Inelastic Stress Distribution in Concrete", A.S.C.E., Proc.Struc.Div., Vol.83, 1957.

31. INSTITUTION OF STRUCTURAL ENGINEERS: "Report on Structural Safety", The Structural Engineer, Vol.33, No.5, May 1955.

32. JENSEN, V.P.: "Ultimate Strength of R.C. Beams as Related to the Plasticity Ratio of Concrete:" Univ. of Ill., Eng.Exp.Sta., Bull, No.345, June 1943.

33. JOHNSTON, B. and COX, K.C.: "High Yield-Point Steel as Tension Reinforcement in Beams", A.C.I. Jour., Vol.11, Sept.1939-40

34. JULIAN, O.G.: "Synopsis of the First Progress Report of the Committee on Factors of Safety", A.S.C.E., Proc. Struc. Div. Vol.83, 1957.

35. KANI, G.N.J.: "Riddle of Shear Failure and its Solution", A.C.I., Jour. Vol.61 April 1964,

36. KENDALL, M.G. and STUART, A.: "The Advanced Theory of Statistics", Vol.I, London, Charles Griffin, 1958.

37. KREFELD, W.J. and THURSTON, C.W.: "Contribution of Longitudinal Steel to Shear Resistance of R.C. Beams", A.C.I. Jour. Vol. 63 , March 1966.

38. KREFELD, W.J. and THURSTON, C.W.: "Studies of the Shear and Diagonal Tension Strength of Simply-Supported R.C. Beams", A.C.I. Jour., Vol.63 , 1966.

39. LASH, S.D. and BRISON, J.W.: "The Ultimate Strength of R.C. Beams", A.C.I. Jour., Vol.38, Feb. 1950.

40. LAWRENCE, L.S. : "Disc. on Freudenthal's 'Safety and Probability of Structural Failure'", A.S.C.E. Trans., Vol.121, 1956.

41. MATTOCK, A.H., KRIZ, L.B. and HOGNESTAD, E.: "Rectangular Concrete Stress Distribution in Ultimate Strength Design", A.C.I. Jour., Vol.57, Feb.1951.

42. MITCHELL, S. and BORRMANN, G.: "Vehicle Loads and Highway Bridge Design", A.S.C.E. Proc. Struc. Div., Vol.83, 1957.

43. MOSES, F.: "Optimum Structural Design Using Linear Programming", A.S.C.E. Proc. Struc. Div., Vol.90, Pt.2, 1964.

44. MOSES, F. and KINSER, D.E.: "Optimum Structural Design with Failure Probability Constraints", Private Correspondence, 1967.

45. NAKAMURA, Y.: "Optimum design of Framed Structures Using Linear Programming", M.S. Thesis, M.I.T., Jan. 1966.

46. PRICE, W.H.: "Factors Influencing Concrete Strength", A.C.I. Jour., Vol.22, 1951.

47. PUGSLEY, A.G.: "Concepts of Safety in Structural Engineering", Inst. of Civ. Engs., Jour., Vol.35, 1951.

48. RAZANI, R. and GOBLE, G.G.: "Optimum Design of Constant-depth Plate Girders", A.S.C.E., Proc. Struc. Div., Vol.92, No.ST2, April 1966.

49. RICE, P.F.: "Ultimate Strength Design Factors for Reinforcement", Private Correspondence, Dec.1959.

50. RICHART, F.E.: "An Investigation of Web Stresses in R.C. Beams", Univ.Ill. Eng. Exp. Sta., Bull. No.166, 1927.

51. RICHART, F.E. and JENSEN, V.P.: "Tests of Plain and Reinforced Concrete Made with Haydite Aggregates", Eng. Exp. Sta., Univ.Ill., Bull.237, Oct.1931.

52. ROWE, R.E., CRANSTON, W.B. and BEST, B.C.: "New Concepts in the Design of Structural Concrete", Inst. of Struc.Engs., Jour., Vol.43, Dec.1965.

53. ROZVANY, G.I.N.: "Optimum Synthesis of Prestressed Structures", A.S.C.E. Proc. Struc. Div., Vol.90, Pt.2, 1964.

54. SAWYER, H.A.: "Economy and Concrete Beams", A.C.I. Jour., Vol.23, 1951-52.

55. SNODGRASS, J.G.: "Statistical Approach to Working Stress for Timber", A.S.C.E., Proc. Struc. Div. Vol.85, 1959.

56. SOKOLNIKOFF, I.S. and REDHEFFER, R.M.: "Mathematics of Physics and Modern Engineering", McGraw-Hill, Inc., Toronto, 1958.

57. STEPHENSON, H.K. and JAKKULA, A.A.: "Highway Loads and Their Effects on Highway Structures based on Traffic Data of 1942", Texas Eng.Exp.Sta., Bull.116, Jan.1950.

58. STEPHENSON, H.K. and CLONINGER, K.: "Methods of Converting Heavy Motor Vehicle Loads into Equivalent Design Loads on the Basis of Max. Bending Moments", Texas Eng.Exp.Sta., Bull.No.127, Oct.1952.

59. STEPHENSON, H.K. and CLONINGER, K.: "Methods of Converting Heavy Motor Vehicle Loads into Equivalent Design Loads on the Basis of Maximum Shears", Texas Eng.Exp.Sta., Bull.131, July 1953.

60. SU, H.L.: "Philosophical Aspects of Structural Design", A.S.C.E. Proc. Struc. Div., Vol.87, 1961.

61. TURKSTRA, C.J.: "A Formulation of Structural Design Decisions", Ph.D. Thesis, Waterloo, Canada, 1962.

62. TURKSTRA, C.J.: "The Choice of Failure Probabilities", A.S.C.E., Proc. Struc. Div., Vol.93, Pt.2, Dec. 1967.

63. VAN DEN BERG, F.J.: "Shear Strength of R.C. Beams without Web Reinforcement", A.C.I. Jour., Vol.59, Pt.2, 1962.

64. WHITNEY, C.S.: "Plastic Theory of R.C. Design, A.S.C.E. Proc., Vol.66, Dec.1940.

65. WHITNEY, C.S.: "Disc. on Cox's'Tests of R.C. Beams'", A.C.I. Jour. Vol.38, 1941.

66. WHITNEY, C.S.: "Ultimate Strength of R.C. Slabs, Footings, Beams and Framed Members without Shear Reinforcement", A.C.I Jour. Vol.29, Oct.1957.

67. WINTER, G.: "Properties of Steel and Concrete and the Behaviour of Structures", A.S.C.E., Proc.Struc.Div., Vol.86, Feb.1960.

68. WOOD, L.W.: "Factor of Safety in the Design of Timber Structures", A.S.C.E. Trans. Vol.125, 1960.