THE DESIGN OF A RESONANT LINE IMPEDANCE MEASURING DEVICE

for the frequency range

from 100 to 200 megacycles.

Paul C. Boire, B. Eng.

M. Eng. Thesis, submitted to the

Faculty of Graduate Studies and Research,

McGill University.

April 1949

Acknowledgements

The author sincerely appreciates the efforts of those who have assisted in the preparation and completion of this work. Thanks are particularly due to Dr. F.S. Howes for his patient and understanding guidance.

To John Gnesko for his expert machining, and to Abe Gordon for his help with the photography, many thanks.

The assistance of Miss S.K. Moreland, formerly with the Sun Life Library, was an invaluable aid in the compilation of the bibliography. The final typing of the thesis was done by Mrs. R.A. Boire, who has given generously of her time.

Table of Contents

List of Illustrations

SYNOPSIS

STATEMENT OF PURPOSE

The problem of measuring impedance is an old one, but its currency is renewed every time there is an advance into the radio-frequency spectrum. The trend in the development of Radio science has been towards higher frequencies, and particularly during the recent war has there been a rapid advance in this direction.

The need for, and emphasis on, practical results from wartime development and research has led to the by-passing or at any rate incomplete solution of many problems in the frequency range from 100 to 200 mc. Because of the dearth of information and the lack of practical techniques to deal with measurements at these frequencies, a great part of the practical results were obtained by trial and error, cut and try methods. As a result of some of this work, it is now possible and undoubtedly profitable to review this portion of the spectrum, and to try to evolve, under less urgent conditions, some more practical and efficient methods of measurement.

This thesis deals with the problem of developing a device for the measurement of impedance, in the frequency range from 100 to 200 mc.

To gain a better understanding of the problem, and to set the stage for what is to come, the first part of the thesis will be devoted to a review of the methods that have been used to measure impedance in the past, and of how these methods have been modified or found unsuitable as the frequency became higher.

The second part will cover the design and development of a device for impedance measurement having adequate frequency, reactance and resistance ranges.

Part four will contain the experimental results of measurements made on a number of components designed for the frequency range from 100 to 200 mc., and a comparison of these results with those obtained by other means.

An extensive bibliography is included at the end of the thesis. References are grouped under separate headings indicating the general nature of the papers listed, and within each group the articles are in chronological order.

1 - Early Methods of Impedance Measurement

1-1 Current and Voltage Relations:

1-1.1 D.C. The passage of a D.C. current through a circuit is impeded or resisted by a property of that circuit which is called resistance. If the potential difference or voltage across the circuit is E volts and the current flowing through the circuit is I amperes, then the resistance of the circuit R ohms, is according to Ohm's Law, the quotient of the voltage divided by the current:

$$R = \frac{E}{I}$$

1-1.2 A.C. For an A.C. current, as the frequency is increased the potential difference across the circuit is affected more and more by the magnetic field of the current, and is no longer dependent solely upon the resistance. The current is opposed by a self-induced e.m.f. $e = -L\frac{di}{dt}$, where L henries is the self-inductance of the circuit. If the current has a maximum value I_m, and is of sine wave form, then $i = I_m \sin \omega t$, where $\omega = 2\pi f$ and f is the frequency. The potential difference across the circuit is now given by

$$E = Ri + L\frac{di}{dt}$$
$$= RI_m \sin \omega t + \omega L I_m \cos \omega t$$
$$= RI_m \sin \omega t + \omega L I_m \sin(\omega t + \tfrac{\pi}{2}).$$

Thus the potential difference consists of two parts, the first dependent upon the resistance R, and the second dependent upon the self-inductance L and the frequency.

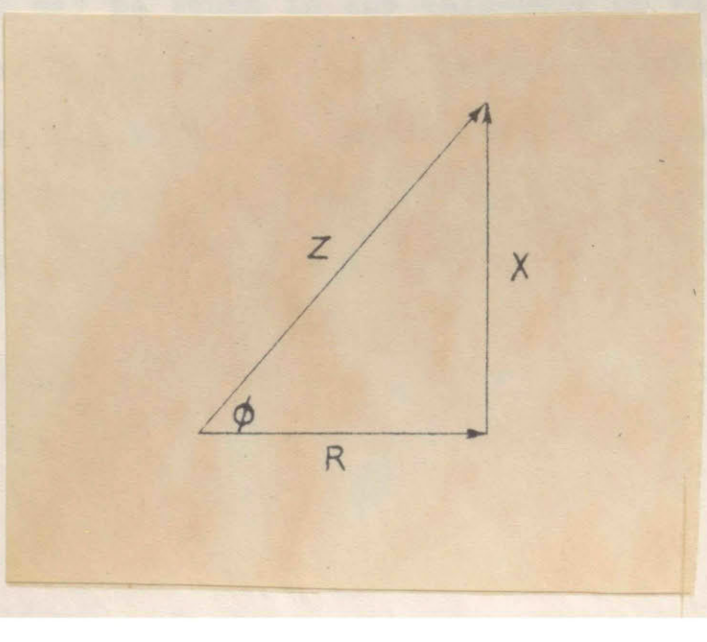
1-1.3 Resistance, Reactance and Impedance: If effective values of current are used, and $\omega L = X$, then

$$E = RI + jXI$$

Now

$$R + jX = \frac{E}{I}$$

The quantity X is the reactance of the circuit, and $(R + jX) = Z$ is the impedance of the circuit. The vectors RI and jXI add vectorially to form the potential vector IZ, or dividing by I, R and jX add vectorially to form Z as shown in the vector diagram Fig. 1.



<u>1-1.4 Phase Angle and Absolute Value:</u> The angle ϕ, the phase angle of the impedance, is given by

$$\phi = \tan^{-1} \frac{X}{R}$$

and the absolute value of the impedance is given by

$$Z = \sqrt{R^2 + X^2}$$

It follows from above that measurement of voltage and current in an A.C circuit may be used to determine the value of Z, in exactly the same way as D.C. voltages and currents are measured to determine the value of R, i.e., by application of Ohm's Law. This meter method however, gives only the absolute value of the circuit impedance, and has therefore a very limited usefulness.

If the absolute value of the impedance and its phase angle are measured by some means, then R and X may be determined; and if R and X are measured individually, then Z and ϕ may be determined from them.

Complete treatments of impedance and quantities related to it may be found in any text-book on Electricity, Electrical Engineering or Electrical Measurements. The above condensation is based on parts of Chapter 1 of "Radio-Frequency Measurements", by L. Hartshorn, John Wiley and Sons Inc., New York.

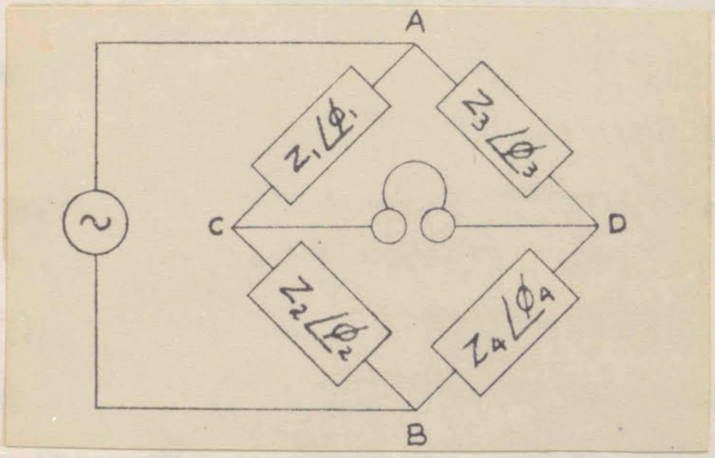
1-2 Bridge Methods:

1-2.1 Origin of Bridge Principle Accurate measurements of D.C. resistance are usually made by means of the Wheatstone bridge, and it was this principle that was first applied to the measurement of A.C. resistance and inductance. The bridge principle was introduced by S.H. Christie[1] in 1833, and in 1843 Sir Charles Wheatstone[2] applied the idea to the measurement of resistances. Wheatstone's resistance balance had a limited range due to the fact that the ratio arms were fixed and equal, but in 1848 Werner von Siemens[3] increased the range of the balance by giving the ratio arms values in any desired ratio. Maxwell,[4] in 1865, adapted the bridge for the measurement of inductance, using a ballistic galvanometer. Similar methods were also used for the measurement of capacitance.

The invention of the telephone by Alexander Graham Bell in 1875, provided a sensitive detector of alternating currents which greatly increased the utility of the induction balance. The modern A.C. bridge, however, is largely due to Max Wien[5], who, in 1891, developed the principles which are in use to-day. Since that time, of course, considerable improvements in technique have been made; but the fundamental ideas remain the same.

<u>1-2.2 Simple Theory of Bridges:</u> Most A.C. bridges are variations of the fundamental form shown in Fig.2. The four arms or current-paths have impedances $Z_1 \angle \phi_1$, $Z_2 \angle \phi_2$, $Z_3 \angle \phi_3$, $Z_4 \angle \phi_4$ respectively.

An A.C. source is connected to the two opposite junctions A and B, and a suitable detector of this A.C. is connected to the two other junctions C and D. In various bridges the impedances take different forms, being simple resistances, inductances, or capacitances, or combinations of them in one or more of the arms.



To balance the bridge, adjustments are made to the magnitudes and phase angles of the four impedances, until the two points C and D are at the same potential, as indicated by zero current through the detector. The conditions for balance of the bridge are

$$\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}$$

and

$$\phi_1 - \phi_2 = \phi_3 - \phi_4$$

1-2.3 Radio-Frequency Applications: While bridges of this general type give good accuracy at audio frequencies their application to radio-frequency measurements is definitely limited. For high-frequency work, the two ratio-arms Z_3 and Z_4 are usually two exactly similar impedances. The bridge is then symmetrical, and stray currents or e.m.fs. which are also symmetrical will not affect the balance.

Almost all radio-frequency bridges are based on the pioneer work of G.A. Campbell,[11,103] and generally they can be grouped under four main types:

(a) The capacitance-conductance bridge,
(b) The inductance-resistance bridge,
(c) The Schering bridge,
(d) The resonance bridge.

The details of these bridges are available in any book on radio frequency measurements, and it will not be necessary to describe them here. Suffice it to say that the capacitance-conductance bridge is generally preferred over the inductance-resistance type, and the Schering bridge over both.

1-2.4 Limitations Due to Nature of Components: The limitations which are imposed on bridge measurements at radio frequencies arise out of several factors. The first is the nature of the quantities to be measured themselves, as well as the nature of the components which make up the bridge. Whereas at audio frequencies we may say that a component is a resistance, or a capacitance, or an inductance; it is not possible to be so definite when dealing with high radio frequencies. A resistor may be predominantly resistive at a particular frequency, but it has always associated with it an inductance due to leads, etc., and a distributed capacity shunting it. The higher the frequency the more predominant these effects become, until at some frequency the character of the component may change completely, and instead of a resistance, we have effectively, a capacitance.

1-2.5 Limitations of Standards: Bridge measurements are essentially comparisons of the unknown to a standard component, and necessarily presuppose the the existance of such standards. Although radio-frequency standards can be and have been built, their construction is very difficult and costly, and at frequencies above 60 mc. they are virtually unheard of.

Air dielectric condensers are possibly the most reliable standards at radio frequencies, but even they are subject to troublesome errors which greatly restrict their usefulness. The construction of loss-free condensers and their proper calibration at high frequencies is rendered difficult by the inductance of binding-posts and leads, the effect of eddy currents in the condenser plates, redistribution of the dielectric field resulting in increased losses in the stacking and insulating supports, as well as possible changes in the properties of the dielectric material itself.

Standard inductances also suffer from many ills at high radio frequencies. The effects of self-capacitance, eddy currents in the wires, losses in the formers and coil supports, skin and proximity effects, all tend to make the calibration of the inductance somewhat uncertain. Inductances are consequently seldom used as standards, except at low frequencies.

In 1928, Zickner[13] described "A bridge for the measurement of inductance and capacity." The state of development of radio-frequency bridge measurements is indicated in the following quotation from his paper:

"Although sufficiently exact measurements of capacity can be performed without difficulty with the aid of the convenient capacity bridge which is generally used, there has not hitherto been available a corresponding arrangement for the measurement of inductance."

1-2.6 Stray Capacities and Shielding: Other factors which limit the application of A.C. bridges to radio-frequency measurements are the stray capacities from one component to another and to ground. These capacities result in stray displacement currents which create one of the main problems we have to contend with at high frequencies. To a large extent this difficulty can be overcome by electrostatic shidding.

The shidded bridge was first described in 1904, by G.A. Campbell.[11] Each component is encased in a separate metallic box, which is grounded or connected to some point on the system. Shielded transformers screen the source and detector from each other and from the rest of the bridge. The electrostatic shields change the distribution of the stray capacities, making them independent of the movements of the observer, and arranging them in such a way that they do not distrub the point of balance.

The extent to which this had been successfully applied by 1929, may be gauged by a statement appearing in a paper entitled "Shielding in high frequency measurements,"[14] where it is stated that a properly designed shielded bridge should be effective at 2,000 kilocycles.

1-2.7 Frequency Limit: In more recent times, bridges have been designed and built for limited applications up to frequencies of 60 mc. One such bridge is described by D.B. Sinclair.[18] It is a modification of the Schering bridge, and measures both resistance and reactance in terms of incremental values of capacitance. The bridge is suitable for the measurement of low impedances, and reads resistance directly in ohms from zero to one thousand, regardless of frequency. The reactance range is zero to five thousand ohms at a frequency of 1 mc., and varies inversely with the frequency.

In this article some of the reasons for the unsatisfactory performance of conventional bridge circuits at ultra-high frequencies are discussed, and the difficulty of obtaining transformers having adequate shielding is noted. Sinclair states that: "As in other types of high-frequency measurement equipment, the limitations imposed upon the bridge with respect to frequency range arise from the residual parameters in the circuit elements and in the wiring."

1-3 Bridged-T and Twin-T Methods:

1-3.1 Bridge and T-Network Equivalence A short discussion of bridged-T and parallel-T methods at this point may be out of place chronologically, but is included here, because of the similarity between bridge networks and T networks, both being capable of balance for a null indication. Bartlett,[12] in 1927, first developed the equivalence relationships between symmetrical lattice or bridge networks and bridged-T networks.

1-3.2 Application to Radio-Frequency Measurements The application of T- networks to radio-frequency measurements was developed by Tuttle,[15] in 1940. He pointed out several advantages of the bridged-T and parallel-T networks over conventional bridges. The existence of a common grounded terminal between the generator and the detector is particularly advantageous, as no Wagner ground or shielded transformer is necessary. For the same reason many of the stray capacities are balanced, and thus have little effect on the null point. Several other advantages of this type of circuit are mentioned by Pavlasek.[22] Due to the fact that the instrument is a null device, accurate radio-frequency voltmeters required in certain resonance methods, are not necessary. Similarly, the output voltage stability requirements of the generator are not very stringent. These features of course apply equally to the bridge methods mentioned previously.

1-3.3 Disadvantages Against these advantages, however must be noted the fact that when the circuit is balanced for the frequency of measurement, it is out of balance for harmonics. The source must therefore by very free from harmonics, or a selective detector must be used. Other difficulties lie in the residual series resistance and inductance of the condensers, and in the fact that there must be sufficient capacity range to tune any coil that is measured.

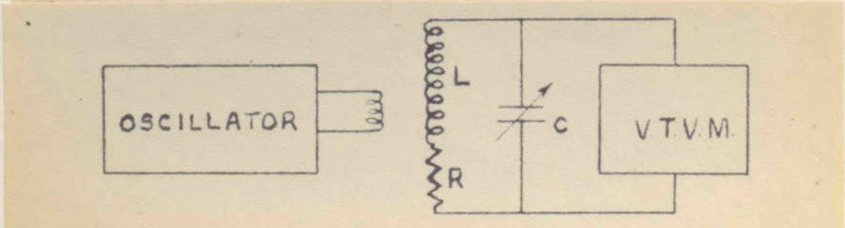
Essentially, the circuit elements themselves impose the high-frequency limit of this type of circuit. An instrument of this type, capable of measurements up to 30 mc. is described by Sinclair.[17]

1-4 Resonance Methods:

1-4.1 Simplicity At high radio frequencies, the exact distribution of current is difficult to determine due to stray e.m.fs., and stray displacement currents. Because of this, it is desireable to keep the measuring circuit as simple, and with as few meshes as possible. The earliest and simplest circuit for radio-frequency measurements, the resonant circuit, is still very much in use at the present time. Its main virtue is in its simplicity, and in this respect it is more satisfactory than bridge methods. There is only one circulating current, which, having its maximum value at resonance, causes stray displacement currents to become insignificant by comparison. Moullin,[97] in his book on radio-frequency measurements, has this to say:

"There is much to be said for breaking away boldly from the methods of low frequencies: let us seek to perfect resonance methods which lend themselves to high frequencies rather than attempt to adapt methods which were developed to suit very different conditions."

1-4.2 Detection of Resonance The general circuit arrangement used in making most resonance measurements is shown below.



Resonance may be detected by observing current variations by means of a suitable thermo-ammeter, or by observing voltage changes across the condenser by means of a vacuum-tube voltmeter having negligible input losses. The conditions for resonance are slightly different in each case, but when R is very small compared to ωL "the condition of resonance is practically the same whether it is obtained by variation of capacitance, inductance or frequency and whether it is observed as current or voltage."[103]

Most measurements involve the tuning of C, L, or ω until the current in the circuit is a maximum, or the voltage across the condenser is a maximum, and the circuit is then said to be resonant.

It is also possible to detect current resonance in the measuring circuit by observations made on the oscillator plate current, and this has the advantage that it does not require the connection of instruments to the resonant circuit.[27,30.]

<u>1-4.3 Methods in Use</u> The various resonance methods of making impedance measurements at radio frequencies may be divided into five main classifications:

1. Simple substitution of one component for another,
2. Resistance variation,

3. Reactance variation,
4. Frequency variation,
5. Voltage step-up or Q-meter method.

1-4.4 Substitution In measuring resistance by substitûtion method, the circuit is first tuned to resonance with the unknown resistance in the circuit, the current flowing being noted. The component being tested is then replaced by a known variable resistance, and the circuit retuned to resonance at the same frequency. The current previously noted is again obtained by varying the resistance until it has the necessary value, this being the value of the unknown resistance.

Capacitance may be measured in a manner similar to that described above. The capacitance to be measured is placed in parallel with a calibrated variable condenser which is used to tune the circuit to resonance. The unknown capacity is then disconnected, and the calibrated condenser varied until the circuit is again resonant. The change in capacity of the calibrated condenser gives the value of the unknown capacitance. This method may also be used to determine the value of inductances,[25] and in this case the change in capacitance of the variable condenser will be in the opposite sense to that above.

<u>1-4.5 Resistance Variation</u> The resistance-variation method, also known as the added-resistance method and he change of resistance method, was originally described by R. Lindemann[9] in 1909. The simplest application of this method consists of inserting a known variable resistance in the resonant circuit, and adjusting its value until the current is halved. Since the current at resonance is equal to the applied voltage divided by the circuit resistance, the resistance required to halve the current is equal to the inherent resistance of the circuit.

<u>1-4.6 Reactance Variation</u> The reactance-variation method, also known as the distuning method and the capacity-variation method, was first described by V. Bjerknes[6] in 1895. It was not until 1907, however, that von Traubenberg and Monasch[8] adapted it for use with continuous oscillations. In this method, after the circuit has been tuned for resonance, either by observing the current or the voltage across the condenser, the capacitance of the circuit is changed until the current or voltage has decreased to 0.707 of the resonant value. This results in two values of capacitance C_1 and C_2, one on either side of the value C_r required for resonance. The apparent series resistance of the circuit is then given by

$$R_s = \frac{C_2 - C_1}{2\omega C_1 C_2}$$

<u>1-4.7 Frequency Variation</u> In the frequency-variation method, the response at resonance is noted. The frequency of the oscillator is then decreased to a value f_1 at which the response has dropped to 0.707 of its resonant value. The frequency is then increased to a value f_2 above the resonant frequency, at which the response is also 0.707 of its maximum value. This method gives the actual series resistance of the curcuit

$$r_s = 2\pi L\ (f_2 - f_1)$$

<u>1-4.8 Q Meter</u> Accordingly to the commonly used definition, the symbol Q designates the ratio of reactance to resistance of a coil, condenser, or other two terminal circuit element. Thus

$$Q = \frac{\omega L}{R} = \frac{1}{\omega CR} \quad .$$

Q is a figure of merit for a reactance, and generally the higher the Q of the reactive elements the better the circuit performance obtained.

The Q meter is an instrument designed to measure the Q of reactive circuit elements directly, and indirectly, by calculation, the reactance and resistance of these elements. The circuit consists of a variable-frequency oscillator which delivers a measured controllable current to a known fixed small resistance. The small resistance is in series with either a standard inductance or an unknown inductance connected in place of the standard. The resistance and inductance in series have connected across them a calibrated condenser and a high impedance vacuum-tube voltmeter in parallel. It is assumed that the small fixed resistance can be neglected by comparison to the other resistances in the circuit, or if necessary corrected for.

The oscillator current flowing through the known resistance causes a known voltage to be introduced in series with the inductance and the variable capacitance, the resistance of the latter being negligible. When the circuit is tuned to resonance by varying either the frequency or the condenser setting, the capacitive reactance will balance the inductive reactance and the current will be

$$I = \frac{E}{R}$$

where E is the known voltage introduced, and R is the effective circuit resistance, i.e. the resistance of the inductance coil. The voltage E_c across the condenser is measured by means of the voltmeter, and is

$$E_c = \frac{I}{\omega C}$$

Thus the ratio of $E_c/R = \frac{1}{\omega CR}$, and at resonance

$$\frac{1}{\omega C} = \omega L$$

therefore $$\frac{E_c}{E} = \frac{\omega L}{R} = Q .$$

If E is made one volt, the voltmeter across C reads the Q directly. Since the capacitance C can be read off the calibrated condenser, the reactance $\frac{1}{\omega C} = \omega L$ can be calculated for the known frequency of measurement. From a knowledge of the reactance and the Q of the coil, its resistance R can be obtained from $R = \frac{\omega L}{Q}$.

This method of impedance measurement is subject to the same limitations as other resonance methods. These limitations are discussed in the next few pages. The best commercial instruments of this type have an accuracy of plus or minus 10°/o up to 100 mc./sec., which decreases with increasing frequency.

1-4.9 Duals of Resistance and Reactance Variation Methods The resistance variation and reactance-variation methods described are best suited for the measurement of small impedances, and determine the effective series resistance of series tuned circuits. Sinclair in 1938,[37] described two methods which he called the "duals" of those already mentioned. These new methods are called the susceptance-variation method, and the conductance-variation method respectively, and are essentially for determining the effective parallel conductance of a parallel-tuned circuit.

The susceptance-variation method requires two readings of voltage across the circuit (parallel resonance), and the other at some other value of capacitance. The second point in all these measurements is conveniently made that at which the response has fallen to 0.707 of its resonant value, as this simplifies the calculations.

The conductance-variation method requires, in addition to the calibrated variable condenser, a conductance standard. Two readings of voltage at parallel resonance are taken: one with the standard conductance in parallel with the circuit, and one with it disconnected. Since reliable high-resistance standards are difficult to obtain, this method is largely of theoretical value only.

<u>1-4.10 Resonant Impedance</u> One other method, described by Iinuma,[28,31] is to be noted at this point. A tuned circuit consisting of a capacitance and an inductance is connected in parallel with the negative resistance of a screen-grid tube operating as a dynatron. The grid bias of the tube is varied until the negative resistance of the dynatron is just equal to the resonant impedance of the tuned circuit

at which point oscillations begin. The negative resistance can be determined from the static characteristics of the screen-grid tube, and the resonant impedance accordingly determined. Iinuma used this arrangement at frequencies up to 30 mc. in 1931, and suggested that higher frequencies might be reached by using two or more tubes in parallel.

1-4.11 Necessary Conditions and Precautions In summing up this discussion, it must be noted that resonance methods require certain precautions to be taken, but have no serious drawbacks sufficient to outweigh the advantageousness and the simplicity of such methods, assuming that suitable circuit components are available.

The oscillator frequency and output voltage must be stable and as far as possible independent of the resistance and reactance of the oscillating circuit under measurement. This usually requires the generator to be well shielded and capable of considerable power output.

The source must generally be free from harmonics, although in certain cases their effect is negligible, as long as the circuit under measurement is not complex.

The coupling of the resonant circuit to the oscillator must be very weak in order that variations of impedance will not be reflected back into the source, and cause changes in the frequency and the amplitude of the output.

Similarly the detector or indicating device must be loosely coupled to the circuit, or have an effect which is negligible or can be easily calculated and accounted for. It must also be capable of being accurately calibrated at the required frequency.

1-4.12 Limitations A certain difficulty is experienced in determining the exact response at resonance, due to the small slope of the resonance curve near its peak; but this should not cause any great trouble when the Q of the circuit is reasonably high. There will, however, be certain deviations from the simple circuit due to stray capacities which cannot be balanced out as they are in certain null methods.

The resistance-variation method has obvious drawbacks, and becomes "inconveniently difficult at frequencies above 10 mc."[36] In this connection, however, and in the case of the conductance-variation method, the use of thermistors,[129,133,134,135] might be effective in increasing the frequency range.

In the reactance-variation method the results obtained must be corrected for the distributed capacity of the coil and the resistance of the thermocouple meter, and whenever possible, the stray circuit capacities. The tuning condenser must be accurately calibrated, and shunted by a vernier condenser.

The frequency-variation method is probably the simplest and most accurate at ultra-high frequencies, and it requires no correction for the distributed capacity of the coil being measured. However the effects of connecting leads and binding posts become increasingly great at high frequencies due to their large physical dimensions with respect to the wavelength.

In most of these methods the total circuit resistance is measured, the condenser losses being considered negligible. While this is true at low frequencies, it can lead to serious errors at ultra-high frequencies. However, in certain cases, condenser losses can be evaluated using condensers designed so that their losses are independent of the capacity.[25]

1-5 Summary

All that has been written so far has been included to form a historical background of impedance measurement, as well as to point out the direction that measurements at ultra-high frequencies must take. The main difficulties have been with components which, because they are "lumped", demand, for satisfactory operation, that their physical size be small in comparison to the wave-length at which they are used. This is necessary in order that the current be uniform throughout their length. As the wave-lengths at high frequencies are relatively small (3 meters at 100 mc.), it is increasingly difficult to satisfy this condition with lumped-constant circuits. The use of circuits with distributed constants is therefore indicated, and it is to a consideration of these circuits that the next part of this thesis will be largely devoted.

A fitting summation for this chapter on early methods of impedance measurement is supplied by Nergaard:[51]

> "Workers in the ..... field of ultra-short waves...... have found that the devices and techniques which served ......at longer wavelengths have failed to give reliable results at the shorter wavelengths."

2 - TRANSMISSION LINE METHODS OF IMPEDANCE MEASUREMENT

2 - 1 Review of Methods Used

2-1.1 Early Uses of Lines:

In the frequency range from 100 to several thousand megacycles, measurement on transmission lines has become the standard method of determining impedance. Although many variations of this method are known to-day, in most cases the apparatus consists of:

(a) a transmission line, either parallel or coaxial, near or at one end of which the unknown impedance is connected,

(b) a signal generator or source of ultra-high-frequency e.m.f. to which the line is loosely coupled, and

(c) a detector to determine current or voltage amplitudes on the line.

Most of the developments of this method from its beginnings over 50 years ago, have taken place in the last ten to fifteen years. Chipman[55] attributes the first use of transmission lines for radio-frequency measurements to Drude[7], who in 1897 used the properties of lines as the basis for measurements of dielectric constants. For some years after that, references to lines in radio-frequency circuits are not very numerous. Around 1920, Southworth[70] and Englund[43,107], pointed out the necessity of substituting circuits with distributed inductance and capacity for those with lumped constants, in order to obtain satisfactory operation at wavelengths below 10 meters. Then in 1924, Wuckel[10] measured the resistance of the conductors of a transmission line, using the relative amplitudes of standing waves of current along the line.

Generally, very little was known about ultra-high-frequency techniques, as indicated in a quotation from a paper by Smith-Rose and McPetrie[76] in 1929:

"At the commencement of this investigation some two years ago, there was little, if any, published information on the generation, transmission and reception of wireless waves of lengths below about 10 meters."

Shortly after this, however, quite a number of patents were granted covering the use of line sections in ultra-high-frequency circuits. One such, British patent 283651, was issued to Standard Telephones and Cables[71]

"In order to avoid undesireable capacity leakage, and to keep circuit induction within the necessary limits in generating and amplifying very short wavelengths, the input and output leads of the generating and amplifying valves are constructed in the form of telescopic wave circuits....... A wave circuit is defined to be one in which the conductors are of a length comparable with that of the waves being handled."

2-1.2 LABUS J. W. Labus,[44] in 1931, measured the absolute value of impedances using a copper transmission line at a wavelength of 21.8 meters. In this method, the unknown impedance was connected across one end of a transmission line which was energized at the other end. The currents at both ends of the line were measured by radio-frequency ammeters, and the absolute value of the impedance given by

$$Z_x = Z_o \quad (I_1/I_2),$$

I_1 and I_2 being the r.m.s. values of the currets and Z_o the characteristic impedance of the line.

According to the author:

"In general, this method lends itself to measurement of impedances of any kind; but it only gives the absolute value of the unknown impedance. However, by means of a known capacity or resistance, connected in series or in parallel with the impedance to be determined, the phase of the latter and, therefore, its real and imaginary components are found."

This method is not necessarily limited to the lower radio frequencies, for since the ratio of the currents is all that is required, accurate radio-frequency meters are not necessary. The range of impedance values which can be measured is, however, limited by the necessity of having a known capacitance or resistance for the purpose of determining the phase angle.

2-1. 3 SCHMIDT O. Schmidt,[47] in 1933, described an absolute method for the determination of any unknown impedance. A parallel-wire system to which the unknown was connected as a termination was used, and the impedance measured in terms of current or voltage distribution along the line.

2-1. 4 BARROW Barrow[49] pointed out that at low frequencies, from about 50 kc. to several thousand kc., lines may often be replaced advantageously by equivalent networks, using the methods of Labus and others. He also presented a method, for use with networks at low frequencies, and with lines at high frequencies, for the absolute determination of impedances.

In Barrow's method, the unknown impedance Z_x is connected to the output terminals of a line, and the voltages or currents at three fixed points on the line are measured. The resistive and reactive components of the impedance Z_X are then determined in terms of the characteristic impedance of the line, and the ratios of these voltages. In order to simplify the calculations, the measurements are usually made at the load, at a distance $\lambda/8$ from the load, and at a distance $\lambda/4$ from the load.[60]

This method has the advantages of simplicity and, as in other absolute methods, the fact that frequency and the constants of the line are the reference standards. Unfortunately the range of impedance values is limited to from about 10 ohms to about 50,000 ohms.

2-1.5 KING A comparative method of impedance measurement employing parallel lines, and a detailed theoretical study of the approximations and conditions implied in transmission line measurements was presented by R. King[50] in 1935. King's paper contains a complete solution for a pair of parallel wires bridged at each end by a general impedance. The usual transmission line equations are derived from the Maxwell field equations, and the solution of the equations is in the form of expressions giving the voltage across and the current through one of these impedances. Certain special cases are discussed, and expressions are derived for use in the measurement of reactance and resistance.

The method of King involves the measurement of voltage distribution curves on a line which is tuned to resonance, with the unknown impedance connected as a termination. The unknown impedance is bridged across the line, and a short-circuiting bar placed $\lambda/4$ behind it on the line. With this arrangement the input admittance of the secondary length of line in parallel with the unknown is, then, always a minimum. The possible reaction on the load of the tertiary length of parallel wires beyond the tandem bridge is assumed negligible.

For the measurement of any size of susceptances the method is convenient and accurate, and it was one of the first methods developed for the accurate measurement of ultra-high frequency resistance. The resistance measurement however, is a comparison with an assumed known resistance, and this fact limits its usefulness considerably. To determine an unknown resistance it is necessary to know the value of gamma; the experimentally measureable amplitude ratio p; and the quantity ρ_s of a standard impedance. Gamma is the quantity $\alpha' s + \rho d$, and is assumed constant; where α' is the effective attenuation constant of the line, s is the length of line necessary for resonance, and

ρ_d is $\dfrac{g_d}{1 + b^2_d}$, ρ_s is $\dfrac{g_s}{1 + b^2_s}$

g_d and b_d being the conductance and susceptance of the detector termination, and g_s and b_s relating to the standard impedance.

The low frequency concept of inductance and capacitance is examined by King, and the conclusion reached that a component measured at ultra-high frequencies using a parallel-wire method bears very little relation to its inductance or capacity measured at low frequencies. Reactance, which can be measured to a high degree of accuracy, is frequency dependent; and the values obtained at any particular frequency are meaningful at that frequency only, regardless of whether they be divided or multiplied by ω and called inductance or capacitance.

<u>2-1.6 NERGAARD</u> Nergaard[51] described in 1936, some means for measuring power and voltage at ultra-high frequencies. In the wavelength range from 20 cm. to

200 cm., the characteristics of a 'good' signal generator and of a 'good' voltmeter are specified, in his paper. In conclusion, he suggested that the measurement problems currently requiring solution were:

" 1. The measurement of current.

Thermocouples seem most promising for the measurement of small currents. ....
For large currents the measurement of voltage drop across a resistor seems feasible.

2. The measurement of impedance.

Measurements of impedance have been made by placing the unknown impedance across a transmission line loosely coupled to an oscillator and measuring the length of the line for resonance and the sharpness of the resonance. This method seems capable of considerable accuracy and may be a satisfactory solution to the problem."

Nergaard did not have long to wait for the solution to the latter problem, for several methods of impedance measurement were developed one after another in the next few years.

<u>2-1.7 HEMPEL</u> In 1937, Hempel evolved a scheme for the determination of impedance by voltage measurement at certain spots along the line. The line is terminated in a reflecting plate, the dimensions of which are sufficiently great to ensure that the wave is completely reflected at this point. The unknown impedance Z_x is connected across the line 5 $\lambda/4$ from the shorted end at an antinode of the stationary waves produced by the reflector. The impedance of the line beyond Z_x is extremely large, and Z_x is in effect the terminating impedance. The voltage maximum and minimum, the voltage at the load, and the distance x_1 of the first voltage maximum from the load are measured. In the general case the magnitude of the terminal impedance is given by:

$$\frac{Z_x}{Z_o}^2 = b^2 = \frac{V_x^2}{V_{max}^2 + V_{min}^2 - V_x^2}$$

and the reactance by

$$X_x = {}^1/_2 Z_o \,(1-b^2) \quad \tan \quad (4 \pi x_1/\lambda).$$

This method, as is the case for all distribution curve methods, suffers from the fact that when the resistive component of the terminating impedance is small compared to the characteristic impedance Z_o, the voltage minimum becomes very difficult to determine due to the limitations of voltage detectors.

2-1.8 BRUCKMANN Working at 100 mc., Bruckmann[53] in 1938 developed a method of impedance measurement using open-wire lines. His was the first absolute method in which formulae for the phase angle of the terminating impedance were developed. The method is somewhat similar to that of Hempel, the unknown being connected $\lambda/4$ or some odd multiple thereof from the end of the line, and the voltage maximum and minimum as well as the voltages at $\lambda/4$ and $\lambda/2$ from the load are measured. The absolute value of the terminating impedance is given by

$$\frac{Z_x}{Z_o} = \frac{V^1/_2}{V^1/_4}$$

and the phase angle is given by

$$\cos \theta_X = \frac{V_{max.}V_{min}}{V^1/_2 . V^1/_4}$$

It has been pointed out by Bruckmann, King, and others, that the impedance value assigned to a termination on a line depends fundamentally on the point at which the termination is regarded as being located. This concept resulted in the tandem bridges of King and the methods of positioning the termination used by Hempel and Bruckman. These various schemes all have the same result in view, namely, to make the unknown impedance the effective termination of the line and render the space beyond the unknown ineffective in determining its impedance. It is also to be noted that the effective impedance of a termination may be determined to a considerable extent by the details of the connection.

2-1.9 NERGAARD Nergaard[54] described lines suitable for use with standard condensers, where the line replaces the inductance coils used in lower frequency resonance methods. A voltmeter and a standard condenser are connected to the end of the line in parallel with the unknown impedance to be measured. The line is driven by an oscillator, the coupling between the two being loose enough to ensure that no frequency pulling of the oscillator takes place. The line is tuned to resonance by varying the capacity at the end. The reactance variation method described previously may be used to obtain the resistance and reactance of the circuit. Standard variable air condensers suitable for use up to 300mc., have been described by Nergaard.

2-1.10 CHIPMAN Nergaard's prediction that resonance curve bandwidth method might be a satisfactory solution to the problem of impedance measurement came true in 1939, with the publication of Chipman's[54] work. With reference to what is now known as the Chipman method, Smith[64] made the following remarks:

> "....it is not often that the research worker sees the result of his researches brought into extensive practice within three years or so of their publication. ......workers with this very simple method, covered as it is so completely by exact mathematics, must feel a touch of romance behind the scientific achievement."

Chipman used an open wire parallel line terminated at one end by the impedance to be measured, and at the other end by a thermocouple. Heavy brass tandem bridges are placed approximately $\lambda/4$ and $3\lambda/4$ behind the thermocouple, in a manner similar to that described by King[50]. A micrometer adjustment is attached to the second brass bridge, for varying the line length. An oscillator loosely coupled to the end of the line at which the unknown impedance is connected, supplies the high-frequency e.m.f.

To measure an impedance connected to the end of the line, two resonance curve widths are measured; one with the unknown connected, to determine the product of the reflection coefficients of the thermocouple and the impedance to be measured; and the other with the line short circuited, to determine the reflection coefficient of the thermocouple. The ratio of these quantities is used in the final impedance calculation.

Chipman develops an expression for the current at any point on a resonant line from a consideration of the successive reflections, of a wave travelling down the line, that take place at the two terminations. This expression is a function of the propogation constants of the line, the reflection coefficients of the terminations, and the length of the line. By varying l and observing the current, the shape of a resonance curve is obtained, and the length of the line for current resonance. An expression is obtained for the product of the reflection coefficients in terms of the width of the resonance curve, and the length of the line for resonance. This length is usually less than $\lambda/2$.

Equations giving the resistance and reactance of the unknown in terms the reflection coefficients and the characteristic impedance of the measuring line, are derived in section III of Chipman's paper in the Journal of Applied Physics. There is a typographical error in lines 8 and 12 of this section, where (17) should be replaced by (18); the numbers referring to equations given in the text. Possibly this has been pointed out before, and in any case it should be obvious to anyone who has followed the devebpment up to that point. When questioned on this point recently, Dr. Chipman stated that he had noticed the error in the published paper, but that no one else had pointed it out to him.

2-1.11 KAUFMANN Kaufmann's arrangement for measurements by voltage resonance was published also in 1939. An open-wire line consisting of two brass rods is shorted at its input end, and coupled loosely to a shielded oscillator. At the receiving end the line is terminated in a perfectly reflecting copper disc. A diode voltmeter is connected to a $\lambda/4$ auxiliary line which is at right angles to the measuring line. The detector line is terminated in a variable condenser, and the sensitivity of the voltmeter is varied by moving the diode along the detector line.

or by changing the setting of the variable condenser. The open end of the detector line is placed near the measuring line but does not touch it. The line is tuned to resonance by moving the reflecting disc, and the unknown impedance to be measured is connected at a voltage maximum. The detector may be located in the same position as the unknown, or $\lambda/2$ from it along the line. With the test object removed, the line is tuned to resonance and the distance from the reflecting disc to the first voltage maximum recorded as l_o. The unknown is then connected across the line at the voltage maximum, and the reflecting disc moved to re-resonate the line, the distance from the load to the disc now being l_r, and the change in length (l_o-l_r). The width of the resonance curve at the points where the response is 0.707 of the maximum is recorded.

The unknown conductance is then given by

$$G \doteq \frac{1}{Z_o}\left\{\tan\frac{2\pi}{\lambda}(l_o-l_r) - \tan\frac{2\pi}{\lambda}(l_o-l_r-\tfrac{1}{2}\Delta)\right\}$$

where Δ is the width of the resonance curve.

The susceptance is given by

$$\omega C = \frac{1}{Z_o}\tan\frac{2\pi}{\lambda}(l_o-l_r).$$

The above equations are applicable only when the power losses in the measuring circuit are negligible, and this condition can usually be met in practice, except in cases where the Q of the unknown exceeds several thousand. The range of impedances which can be measured using this method is limited only by the accuracy with which the bandwidth of the resonance curve can be measured. The main drawbacks would seem to arise out of radiation from the unshielded line, and the awkwardness of the detector system.

A useful equation for the determination of α is derived by Kaufmann, when the line is terminated in perfect reflectors at both ends

$$\Delta = \frac{\alpha\, n\lambda^2}{2\pi}$$

where α is the attenuation constant in nepers/meter and n is the number of half wavelengths in the line, λ and Δ being expressed in metres.

<u>2-1.12 NERGAARD AND SALZBERG</u> The assumption usually made in ultra-high frequency transmission line theory, that the characteristic impedance of the line is purely resistive, was shown to be invalid for short line calculations, by Nergaard and Salzberg in 1939. The usual

assumption is that the resistance per unit length of line R_0 is negligible with respect to the loop inductance per unit line length. In their joint paper[56] Nergaard and Salzberg reserved this assumption for second order quantities, i.e. $\left(\frac{R_0}{\omega L_0}\right)^2$ is considered negligible. The result of including the first order of $\frac{R_0}{\omega L_0}$ in the theoretical calculations of Z_0 and P the propagation constant, is to give Z_0 an imaginary component and P a real component. The real component of P is, of course, the attentuation constant. It was shown theoretically that taking into account the imaginary component of Z_0 resulted in more logical expressions for the input impedance of the line than resulted from the old theory.

To demonstrate the validity of their new theory experimentally, Nergaard and Salzberg used an open wire line terminated at one end in a moveable short-circuiting bar. The sending end was terminated in an essentially lossless condenser. The impedance of the line was determined by the line-length-variation method.[54] This consists of loosely coupling the line to a signal generator; measuring the length of line which corresponds to resonance; and then measuring the change in line length required to reduce the voltage across the line to 0.707 of its resonant value. The resonant impedance is given by (see Table No. 4)

$$\frac{r}{Z_o} = \frac{\sin^2 \theta_r}{\Delta\theta}$$

where

Z_o is the real part of the characteristic impedance

θ_r is the electrical length of the line at resonance

$\Delta\theta$ is the change in electrical length of the line required to reduce the voltage to 0.707 of its resonant value.

It was found that the experimental values for the resonant impedance coincided almost exactly with the values calculated on the basis of the new theory, and differed widely from those predicted by the old theory.

2-1.13 HAMBURGER AND MILLER Apparatus for inductance coil reactance measurement was described in 1940, by Hamburger and Miller,[58] using a quarter wave resonant parallel line. The line was first tuned to self-resonance, and then again with the coil to be measured connected across the end of the line. The unknown reactance is then given by the expression

$$X = Z_o \cot \left(90^\circ . \frac{l_o}{\lambda/4} \right)$$

where l_o is the necessary change in the length of the line to restore resonance. In their measurements Hamburger and Miller found it necessary to take into account the proximity effect of the parallel rods, as they used large diameter tubing and small axial spacing, to reduce radiation from the line. A more satisfactory solution to the radiation problem would probably be to use a shielded line. The conductor spacing can then be given a larger value, so as to accomodate components to be measured without the use of connecting leads.

2-1.14 KIBLER In 1944 Kibler[60] extended the method of Barrow previously mentioned, and developed an impedance chart from which the values of R_t and X_t can be read off in terms of the measured quantities $\frac{V_1}{V_3}$ and $\frac{V_2}{V_3}$.

If V_1 is the load voltage, V_2 the voltage at $\lambda/8$ from the load, and V_3 the voltage at $\lambda/4$ from the load, then

$$R_t = \left\{ A_2(A_1 - A_2 + 1) - 1/_4 (A_1 - 1)^2 \right\}^{\frac{1}{2}} . Z_o$$

and $X_t = \left(A_2 - \frac{1}{2} (A_1 + 1) \right) . Z_o$

where $A_1 = \left\{\frac{V_1}{V_3}\right\}^2$ and $A_2 = \left\{\frac{V_2}{V_3}\right\}^2$

The range of measurement using this method is limited to values of resistance from about 0.3 Z_o to 2 Z_o, and values of reactance not far removed from Z_o. These limitations can be seen, from the impedance chart, to be due to the fact that when the above voltage ratios are large, the A circles on the chart intersect at small angles. The point of intersection, which determines the R and the X components of the unknown impedance is therefore indefinite. This method is most useful in cases where it is desired to match an impedance to a transmission line, as in these cases the measured voltage ratios are small.

2-1.15 ESSEN The results of Chipman's work were very quickly put to use in the determination of the propogation constants of cables. Many workers developed measuring lines based on the Chipman method of impedance measurement. Essen[61] described in 1944 shielded balanced and unbalanced lines which had been in use since 1941. The main points of interest in this paper are the use of a shielded rather than an open wire line, and the development of the unbalanced or coaxial measuring line.

It was found that although reliable results could be obtained on the open-wire line, movements of the observer caused instability in the current readings, and made the operation of the line inconvenient. Although it is not mentioned in Essen's paper, radiation from the shielded line would be much reduced, and thus the overall accuracy of the apparatus improved, and the necessity of trying to allow for this radiation loss removed.

2-1.16 JONES AND SEAR The methods and apparatus of Jones and Sear[63,64] are essentially the same as those of Essen, and both are based on the Chipman or current-resonance-curve-width method of impedance measurement. Shielded lines, both balanced and unbalanced were used, and some interesting details of shorting piston construction, contact difficulties, and different types of detectors are given. Methods for checking the looseness of coupling to the oscillator, and factors determining asymmetry of the response curve are discussed. In a later paper,[64] changes in resonant line length and resonance curve width due to the presence of insulating supports on a balanced line are investigated. It is concluded that insulating supports should be reduced to the absolute minimum required for mechanical strength.

<u>2-1.17 SIMMONDS</u> A comparison method for the measurement of ultra-high frequency resistance was developed by Simmonds[67] in 1945. A shielded parallel-brass-rod line was used, the rods being supported by a brass disc at one end, and an ebonite insulator at the other. The line was excited by means of a coupling loop projecting through a hole in the face of a moveable shorting piston, A diode voltmeter was connected to the open end of the line, in parallel with a thermistor and one arm of a wheatstone bridge. The wheatstone bridge measures the D.C. value of the thermistor resistance, and is isolated from the U.H.F. voltage on the line by means of chokes. The line is tuned to resonance with the unknown impedance connected across the open end of the line, by moving the shorting piston until maximum voltage is read on the diode voltmeter. The length of the line and the voltmeter reading are recorded. The unknown is then disconnected

and the line retuned to resonance. The voltmeter reading is returned to its former value by changing the resistance of the thermistor, this being accomplished by sending more or less current through the thermistor from the wheatstone bridge. It is claimed in the paper that the thermistor resistance is constant within 3°/o from D.C. to 200 mc., thus enabling the thermistor to be used as a resistance standard. From the above procedure the resistance of the unknown is given by

$$R_x = \frac{R_1 R_2 \sin \beta l_1}{R_1 \sin \beta l_2 - R_2 \sin \beta l_1}$$

where R_1 and R_2 are the thermistor resistances with the load connected and disconnected respectively, and l_1 and l_2 the corresponding line lengths.

The reactance is obtained in the usual way from the difference in length of the line at resonance with the unknown connected and disconnected, and is given by

$$X_x = \frac{Z_o}{\tan \frac{\omega}{v} (l_2 - l_1) \left(\cot \frac{\omega}{v} l_1 \cot \frac{\omega}{v} l_2 + 1 \right)}$$

The accuracy of this method is claimed to be about 3°/o, but the range of values of resistance which can be measured is restricted by the fact that the thermistor resistance cannot be greater than 4000 ohms. This restriction is due to the fact that the resistance of the thermistor is not constant from D.C. to 200 mc. for values larger than about 4000 ohms.

2-1.18 CONLEY An apparatus for impedance measurement using a single conductor and its image in a semi-infinite conducting plane has been described by Conley.[145] The line is fed symmetrically from both ends, and the unknown impedance is located at the centre of the line. It is claimed for this method that many disadvantages, notably radiation and unbalance, are overcome by replacing one of the line conductors by its image. Feeding the line from both ends does away with the necessity for dielectric supports, and permits the line conductors to continue past the load, without their having any effect on the terminating impedance value.

Perhaps the main disadvantage of this type of line lies in its large physical dimensions. In order for the conducting plane to approach semi-infinity, it must stretch on either side of the single conductor, a distance of at least three or four wavelengths. At a frequency of 100 mc. this requirement would involve a conducting plane about 70 feet square, which is obviously impracticable. At frequencies around 750 mc., as used by Conley, however, the image line seems to be a useful development. The order of accuracy attainable is given as from 5% to 15%, using either the voltage distribution along the line, or the width of the minimum dip in the standing wave pattern to calculate the unknown impedance.

TABLE No.1

CHART OF IMPEDANCE MEASUREMENTS ON TRANSMISSION LINES

METHOD	QUANTITIES OBSERVED	AUTHOR	YEAR	MAIN APPLICATION	FREQ. MC.	ACCURACY %	TYPE OF LINE USED
STANDING WAVE DISTRIBUTION CURVE	Current Length	Drude	1897	Dielectric constants			Parallel open-wire
		Wuckel	1924	Conductor resistance			Parallel open-wire
		Schmidt	1933	Impedance			Parallel open wire
		Labus	1931	Absolute value of Z	14		Parallel open-wire
	Voltage Length	Barrow	1935	10-50,000 ohms	1.5		Networks simulating lines
		King	1935	Impedance	160		Parallel open-wire
		Hempel	1937	Impedance	600		" " "
		Bruckmann	1938	Impedance	100		" " "
		Kibler	1944	Values near Z_0			Coaxial line
		Conley	1948	Antenna impedance	750	5%-15%	Image line
RESONANCE CURVE PEAK	Current Length Bandwidth	Nergaard & Salzberg	1938	Impedance	150	5%	Parallel open-wire
		Chipman	1939	Impedance	377	1%	" " "
		Hamburger & Miller	1940	Reactance	100		" " "
		Essen	1944	Cable Const.	500	2%-5%	Shielded twin-wire
		Jones&Sear	1944	" "			Coaxial line
	Voltage Length Bandwidth	Kaufmann	1939	Impedance			Parallel open-wire
		Simmonds	1945	Cable Const.	10-200	2%-3%	Shielded twin-wire
MIN. DIP WIDTH	"	Conley	1946	Antenna Impedance	750	5%-15%	Image line

3 - DESIGN OF IMPEDANCE MEASURING LINE

3-1 Theory of Short Lines:

3-1.1 Errors in Old Theory

It has been shown by Nergaard and Salzberg[56] that certain theoretical formulae commonly used in treating short ultra-high-frequency transmission lines are in error, and that the theory presented in their paper is more accurate. As a logical extension of the more precise theory outlined by Nergaard and Salzberg, it is proposed to show in this thesis that the theoretical formula for the Q of a non-resonant line, as derived by Terman[83] is also in error, and that the results obtained on the basis of this formula are therefore not to be realized in practice. Expressions for the Q of non-resonant lines are derived:

a) in terms of the constants of the line, and its electrical length,

b) in terms of the resonant Q of the line, and its electrical length,

c) in terms of the length of the line, and the change in length necessary to reduce the response to 0.707 of the maximum response, when the line is tuned to resonance with a lossless condenser.

3 - 1.2 Imaginary component of Z_o In conventional transmission line theory[146] the input impedance of a uniform line of length l, which is short-circuited at its distant end is

$$Z_{in} = Z_o' \tanh Pl$$

where $Z_o' = \frac{(R_o + j\omega L_o)^{\frac{1}{2}}}{(G_o + j\omega C_o)^{\frac{1}{2}}}$ is defined as the characteristic impedance of the line, and $P = (R_o + j\omega L_o)^{\frac{1}{2}}(G+j\omega C_o)^{\frac{1}{2}}$ is defined as the propagation constant of the line.

R_o, L_o, G_o and C_o, are the series resistance, inductance, shunt conductance, and capacitance (all per unit length of line), respectively.

Nergaard and Salzberg have shown that each of the quantities Z_o' and P may be expressed as the sum of a real and an imaginary component, so that

$$Z_o' = Z_o\ (1-jk)$$

and

$$P = \beta\ (k + j)$$

where

$$Z_o = \left(\frac{L_o}{C_o}\right)^{\frac{1}{2}} \cdot \left\{ \frac{1}{2}\left(1 + \left(\frac{R_o}{\omega L_o}\right)^2\right)^{\frac{1}{2}} + \frac{1}{2}\right\}^{\frac{1}{2}}$$

$$\beta = \omega(L_o C_o)^{\frac{1}{2}} \cdot \left\{ \frac{1}{2}\left[1 + \left(\frac{R_o}{\omega L_o}\right)^2\right]^{\frac{1}{2}} + \frac{1}{2}\right\}^{\frac{1}{2}}$$

and $$k = \left(\frac{R_o}{\omega L_o}\right) \left\{ 1 + \left[1 + \left(\frac{R_o}{\omega L_o}\right)^2 \right]^{\frac{1}{2}} \right\}^{-1}$$

3 - 1.3 Input Impedance of Line The foregoing expressions are exact in form, involving no approximations other than that $G_o = 0$ which is true for an air dielectric line.

The usual assumption at this point is that $\frac{R_o}{\omega L_o}$ is negligible with respect to one, and it is this assumption which causes conventional transmission line theory to be inapplicable to very short lines.

If now, we assume that $\left(\frac{R_o}{\omega L_o}\right)^2$ is much less than unity, the foregoing expressions may be written as

$$Z_o = \left(\frac{L_o}{C_o}\right)^{\frac{1}{2}}$$

$$\beta = \omega (L_o C_o)^{\frac{1}{2}} = \frac{2\pi}{\lambda}$$

$$k = \frac{R_o}{2\omega L_o}$$

The input impedance may now be written as

$$Z_{in} = Z_o(1 - jk) \tanh (k + j)\beta l .$$

Expanding this trigonometrically so as to express the resistive and reactive components of the input impedance

$$Z_{in} = \frac{Z_o((\sinh 2k\beta l + k\sin 2\beta l) + j(\sin 2\beta l - k \sinh 2k\beta l))}{\cosh 2k\beta l + \cos 2\beta l}$$

<u>3 - 1.4 Q of Non-resonant Line</u> Terman has assumed that the ratio of reactance to resistance of the input impedance may be called the Q of the non-resonant line. Using this definition of Q we obtain from the foregoing expression for the input impedance, that

$$Q = \frac{\sin 2\beta l - k \sinh 2k\beta l}{\sinh 2k\beta l + k \sin 2\beta l}.$$

Since $k = \frac{R_o}{2\omega L_o}$ is a small quantity for radio-frequency transmission lines, $2k\beta l$ is a small angle, and the expression for Q may be written

$$Q = \frac{\sin 2\beta l - 2k^2\beta l}{2k\beta l + k \sin 2\beta l}$$

and since k^2 is much less than unity

$$Q = \frac{\sin 2\beta l}{k(2\beta l + \sin 2\beta l)}$$

Substituting values for β and k given in section 3-1.3 in the above, we have

$$Q = \frac{2\omega L_o}{R_o} \cdot \frac{\sin 4\pi \frac{l}{\lambda}}{4\pi \frac{l}{\lambda} + \sin 4\pi \frac{l}{\lambda}}$$

$$= \frac{\frac{\omega L_o}{R_o}}{\frac{1}{2}\left\{\frac{4\pi \frac{l}{\lambda}}{\sin 4\pi \frac{l}{\lambda}} + 1\right\}}$$

3 - 1.5 Comparison with Old Theory For a very short length of line, $\sin 4\pi \frac{l}{\lambda} = 4\pi\frac{l}{\lambda}$, since for small angles the sin of the angle is equal to the angle in radians. Therefore $Q = \frac{\omega L_o}{R_o} = \frac{\text{reactance per unit length}}{\text{resistance per unit length}}$

and this is the limiting value of Q one would expect as l approaches zero.

Terman's expression for Q given by equation (17) in his paper[83] is

$$Q = \frac{Z_o f}{R_o c} \cdot \frac{\sin 4\pi \frac{l}{\lambda}}{\frac{l}{\lambda}}$$

which may be written $Q = \frac{\omega L_o}{R_o} \frac{\sin 4\pi \frac{l}{\lambda}}{2\pi \frac{l}{\lambda}}$

For a very short length of line, i.e. as l tends to zero, this gives $Q = \frac{2\omega L_o}{R_o}$

which is not the Q or ratio of reactance to resistance for a very short length of line which one must expect from the definition of the quantities.

3 - 1.6 Q of resonant lines The Q of a resonant line section i.e. $(2n + 1)\frac{\lambda}{4}$ in length, has been derived by Terman, and is given by

$$Q_o = \frac{2\pi Z_o f}{R_o c}$$

and since $Z_o = \left(\frac{L_o}{C_o}\right)^{\frac{1}{2}}$ and $c = \frac{1}{(L_o C_o)^{\frac{1}{2}}}$

since $\beta = 2\pi f \quad (L_o C_o)^{\frac{1}{2}}$ and $\quad f\lambda = c$

the above expression for Q_o may be written

$$Q_o = \frac{\omega L_o}{R_o}$$

This can also be shown to be

$$Q_o = \frac{\beta}{2\alpha}$$

since $\alpha = \frac{R_o}{2Z_o}$ and $\beta = \frac{2\pi f}{c}$

The Q of a non-resonant line section may therefore be written in terms of the resonant Q_o

$$Q = \frac{Q_o}{\frac{1}{2}\left\{\frac{4\pi \frac{l}{\lambda}}{\sin 4\pi \frac{l}{\lambda}} + 1\right\}}$$

This expression becomes identical with Terman's expression (equation 17) when $\frac{4\pi \frac{l}{\lambda}}{\sin 4\pi \frac{l}{\lambda}}$ is much greater than unity, but this is only true for lengths of line near $\frac{\lambda}{4}$ or multiples thereof. This is clearly shown in Figure 4, where $\frac{1}{2}\left\{\frac{4\pi \frac{l}{\lambda}}{\sin 4\pi \frac{l}{\lambda}} + 1\right\}$ is plotted

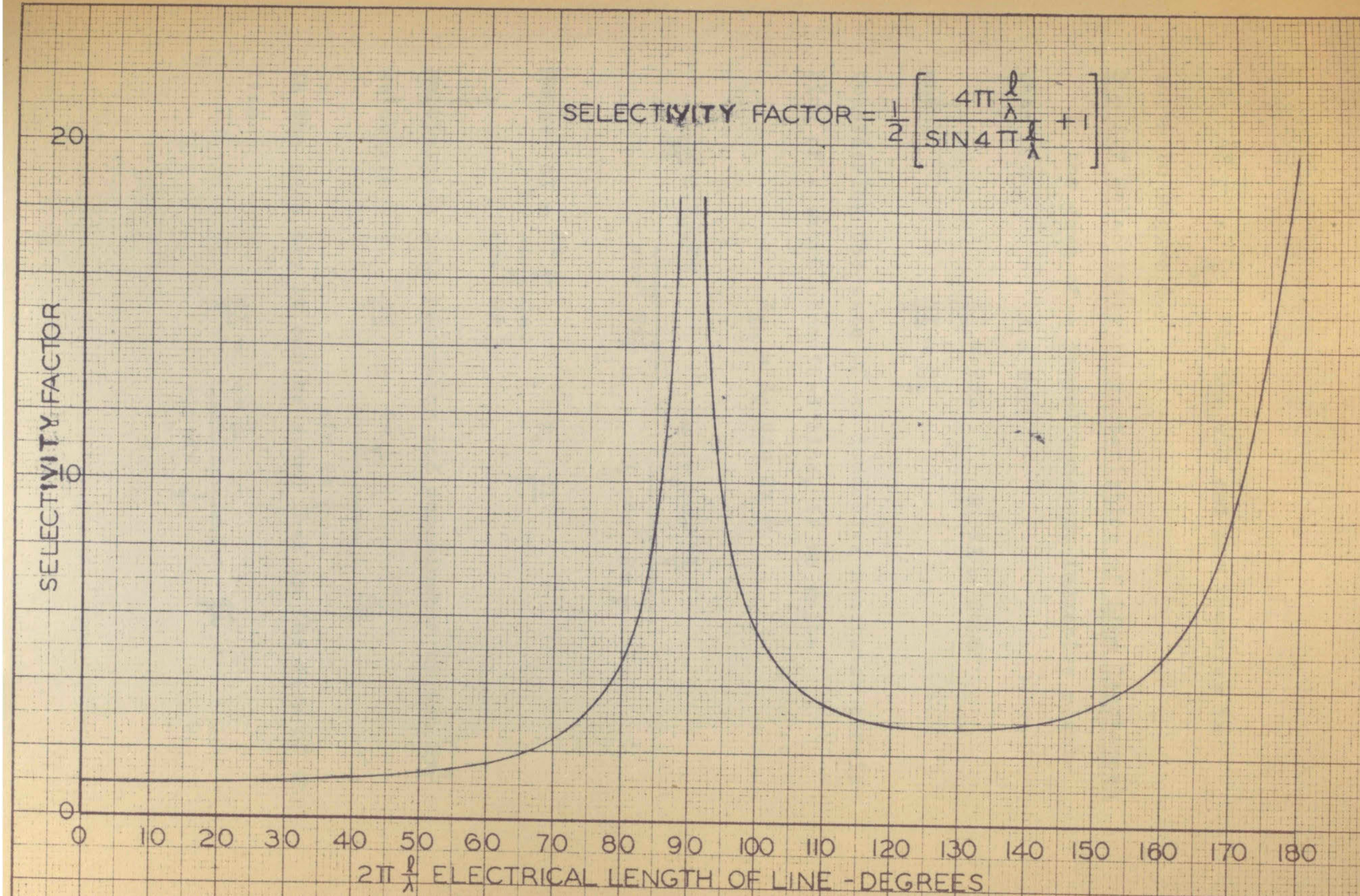

SELECTIVITY FACTOR = $\frac{1}{2}\left[\frac{4\pi\frac{\ell}{\lambda}}{\sin 4\pi\frac{\ell}{\lambda}}+1\right]$
SELECTIVITY FACTOR
20
10
0
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180
$2\pi\frac{\ell}{\lambda}$ ELECTRICAL LENGTH OF LINE - DEGREES

FIGURE 4

vs. $2\pi \frac{l}{\lambda} \cdot \frac{Q}{Q_o}$ for the old and new theories is shown in Figure 5.

3 - 1.7 Selectivity Characteristics of Line Reactances

In the case of a lumped constant inductive reactance

$$X = 2\pi f L$$

$$\frac{dX}{df} = 2\pi L \qquad \text{so } dX = 2\pi L df$$

and

$$\frac{dX}{X} = \frac{df}{f}$$

Considering the case of an inductive reactance consisting of a short circuited line, if we write

$$X_{in} = Z_o \tan 2\pi \frac{l}{\lambda}$$

$$\frac{dX}{dl} = Z_o \frac{\frac{2\pi}{\lambda}}{\cos^2 2\pi\frac{l}{\lambda}} = \frac{Z_o \; 2\pi \; \frac{l}{\lambda}}{l \; \cos^2 \; 2\pi \; \frac{l}{\lambda}}$$

Hence $dX = \dfrac{2\pi \frac{l}{\lambda}}{\cos^2 2\pi \frac{l}{\lambda}} \cdot Z_o \dfrac{dl}{l}$

Multiplying both numerator and denominator by $\tan 2\pi\frac{l}{\lambda}$ we obtain

$$\frac{dX}{X} = \frac{4\pi \frac{l}{\lambda}}{\sin 4\pi \frac{l}{\lambda}} \cdot \frac{dl}{l}$$

In the same way it can be shown that in the case of the line

$$\frac{dX}{X} = \frac{4\pi \frac{l}{\lambda}}{\sin 4\pi \frac{l}{\lambda}} \cdot \frac{df}{f}$$

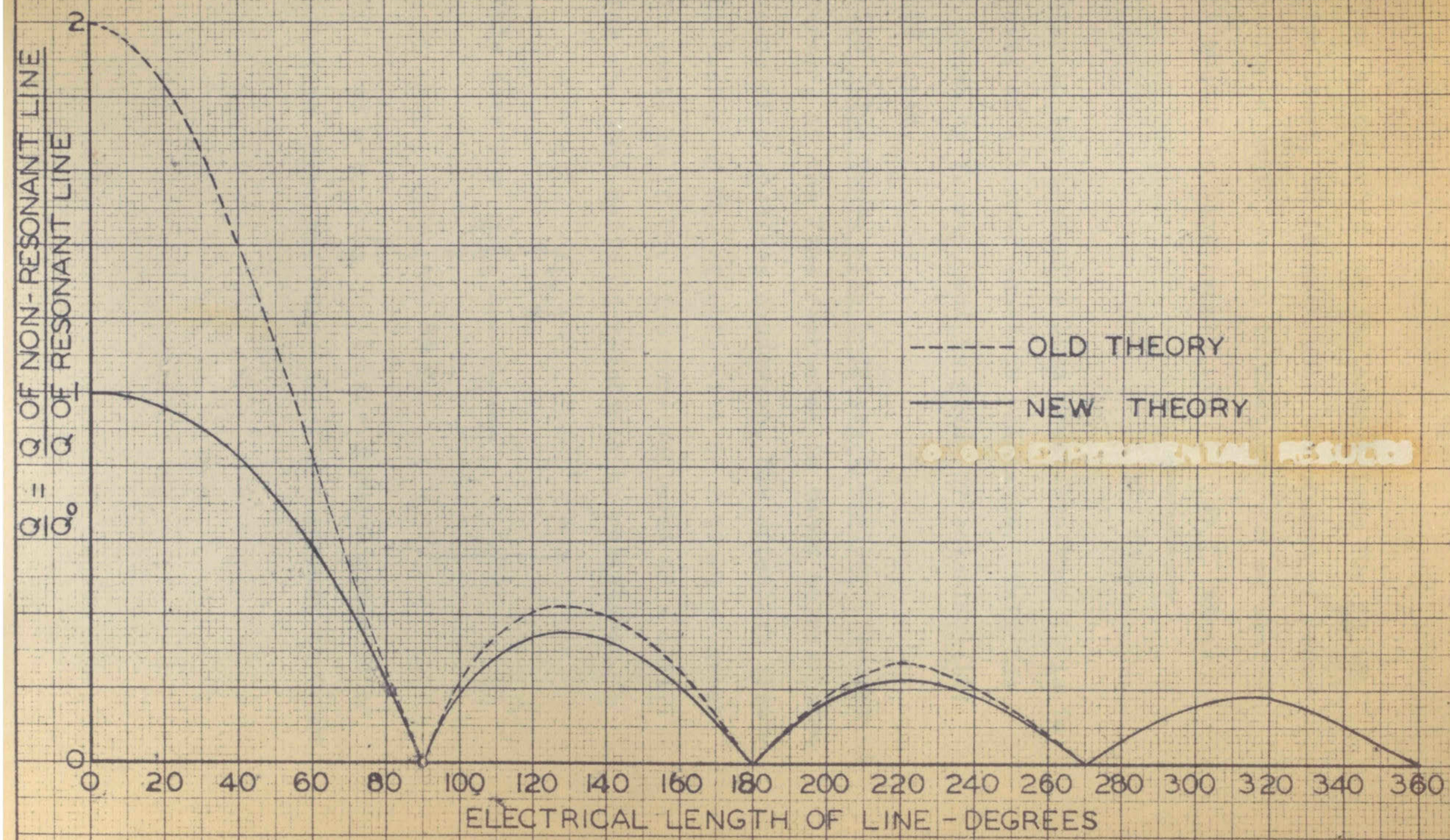

$\frac{Q}{Q_0} = \frac{Q \text{ OF NON-RESONANT LINE}}{Q \text{ OF RESONANT LINE}}$
2
0
0 20 40 60 80 100 120 140 160 180 200 220 240 260 280 300 320 340 360
ELECTRICAL LENGTH OF LINE - DEGREES
------ OLD THEORY
—— NEW THEORY

FIGURE 5

From a comparison of the expressions for $\frac{dX}{X}$ in the two cases, it is seen that the change of reactance for a small change in frequency or an equivalent change in line length, is $\dfrac{4\pi\frac{l}{\lambda}}{\sin 4\pi\frac{l}{\lambda}}$ times greater in the case of a line reactance than it is in the case of a lumped constant reactance. This has been shown by Terman, and is given by equation (18) in his paper.

<u>3 - 1.8 Line Tuned with Lossless Condenser</u> According to Terman, if the line reactance is tuned to resonance by means of a lossless condenser, the effective slectivity factor of the circuit will be S times as great as it would be if the line were replaced by a coil having the same ratio of reactance to resistance.

In this case $S = \frac{1}{2}\left\{\dfrac{4\pi\frac{l}{\lambda}}{\sin 4\pi\frac{l}{\lambda}} + 1\right\}$

If $\Delta X = S.\ X\frac{\Delta l}{l}$ is the change in reactance required to lower the response of the circuit to 0.707

maximum response, such that the real and imaginary components of the input impedance are equal, then

$$\frac{1}{2\Delta l} = S \cdot \frac{X}{2\Delta X}$$

But in the case of a lumped constant resonant circuit

$\frac{X}{2\Delta X} = \frac{f}{2\Delta f}$ = the Q of the coil. Since, in the case of the line reactance, $\frac{l}{2\Delta l} = \frac{f}{2\Delta f}$ we may write $\frac{l}{2\Delta l} = S \cdot Q_l$ where Q_l is the Q of the line reactance. In section 3 - 1.6 it is shown that $Q_l = \frac{Q_o}{S}$, so that $\frac{l}{2\Delta l} = Q_o$.

Similarly $\frac{f}{2\Delta f} = Q_o$

3 - 1.9 Comparison with Old Theory Terman has stated that the effective selectivity or $\frac{f}{2\Delta f}$ of a line tuned to resonance with a lossless condenser is S times greater than it would be if the line were replaced by a coil having the same Q or ratio of reactance to resistance. Since $\frac{l}{2\Delta l}$ expresses the same effective selectivity, we may, using Terman's expression for the

Q of a line reactance, write

$$\frac{1}{2\Delta l} = \frac{Z_o f}{R_o c} \cdot \frac{\sin 4\pi \frac{l}{\lambda}}{\frac{l}{\lambda}} \cdot S$$

Multiplying both numerator and denominator by 2π,

$$\frac{1}{2\Delta l} = \frac{2\pi Z_o f}{R_o c} \cdot \frac{\sin 4\pi \frac{l}{\lambda}}{2\pi \frac{l}{\lambda}} \cdot \frac{4\pi \frac{l}{\lambda} + \sin 4\pi \frac{l}{\lambda}}{2 \sin 4\pi \frac{l}{\lambda}}$$

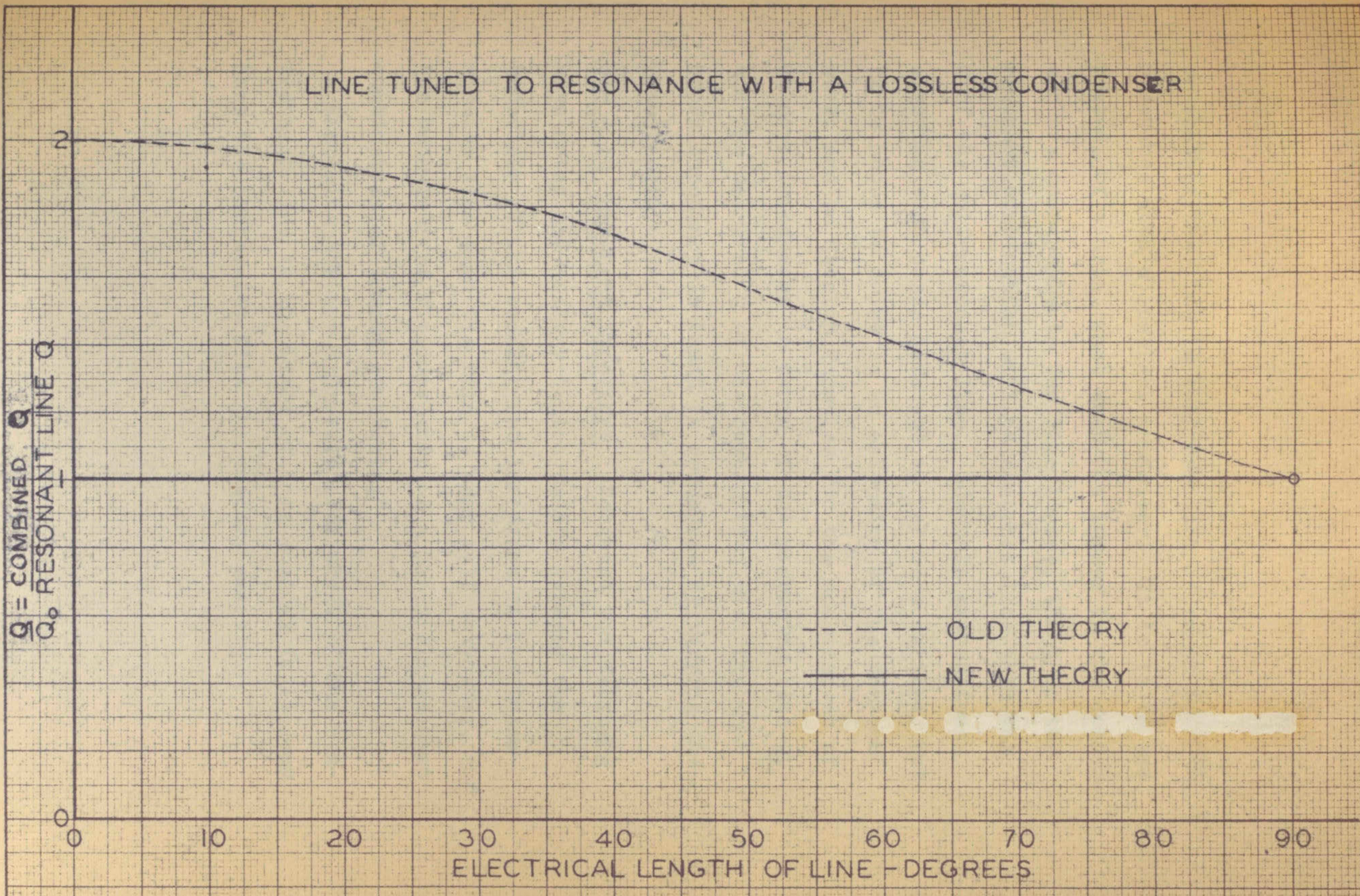
So that $$\frac{1}{2dl} = Q_o \left\{ 1 + \frac{\sin 4\pi \frac{l}{\lambda}}{4\pi \frac{l}{\lambda}} \right\}$$

For short lengths of line this approaches the limit

$$\frac{1}{2dl} = 2Q_o$$

The ratio of $\frac{1}{2dl}$ to Q_o is plotted in Figure 6, for lengths of line from 0 to $\frac{\lambda}{4}$ i.e. from 0 to 90° electrical length, and the curves show clearly the wide difference between the results obtained from the above expression, and those predicted by the expression derived in section 3 - 1.8, on the basis of the new expression for the Q of a non-resonant line.

It is seen that what Terman called the effective selectivity is actually the Q of the resonant circuit, and is the same regardless of the length of the line



LINE TUNED TO RESONANCE WITH A LOSSLESS CONDENSER
$\frac{Q}{Q_o} = \frac{\text{COMBINED } Q}{\text{RESONANT LINE } Q}$
2
1
0
0
10
20
30
40
50
60
70
80
90
ELECTRICAL LENGTH OF LINE - DEGREES
OLD THEORY
NEW THEORY

FIGURE 6

or whether the line is self-resonant or tuned to resonance with a lossless condenser. It is to be expected, of course, that when the losses in the condenser are comparable to the line losses, the Q of the resonant combination will be somewhat less than the Q of the self-resonant line. Adequate expressions covering such cases have not as yet been derived, but it is hoped to do so as an extension of the present work; and to develop formulae for the measurement of unknown impedances in terms of the measuring line Q, and the measured Q of the combination when the unknown impedance is tuned to resonance by varying the line length.

3 - 1.10 Resonant Impedance Nergaard and Salzberg have noted the analogy "which exists between the behaviour of the sending-end impedance of the transmission line when the line is near self resonance, and the behaviour of the impedance of a parallel-resonant combination of lumped constants."

Examining the new expression for the Q of the non-resonant line in the light of this analogy it is seen that the expression fits the analogy exactly.

In the case of a lumped constant parallel-resonant circuit the resonant impedance is given by

$$r = \frac{L}{CR}$$

where L is the series inductance of the coil

R is the series resistance of the coil

C is the capacitance of the condenser which is assumed to be lossless. This may be written in terms of the Q of the coil as

$$r = \omega L\, Q = X_1\, Q$$

In the case of the transmission line tuned to resonance with a lossless condenser then

$$r = Z_o \tan \beta l \cdot \frac{Q_o}{S}$$

is the analogous expression, since $Q_{line} = \frac{Q_o}{S}$ and

$$S = \frac{1}{2}\left(\frac{2\beta l}{\sin 2\beta l} + 1\right).$$

Since $Q_o = \frac{\omega L_o}{R_o} = \frac{1}{2k}$ from sections 3 - 1.3 and 3 - 1.6

$$\frac{r}{Z_o} = \frac{\tan \beta l}{2kS} = \frac{1}{k} \cdot \frac{\tan \beta l \sin 2\beta l}{2\beta l + \sin 2\beta l}$$

$$= \frac{1}{k} \cdot \frac{\tan \beta l\; 2\sin \beta l \cos \beta l}{2\beta l + \sin 2\beta l}$$

$$= \frac{1}{k} \cdot \frac{2(1 - \cos^2\beta l)}{2\beta l + \sin 2\beta l}$$

$$= \frac{1}{k} \cdot \frac{2\left[1 - \left(\frac{1 + \cos 2\beta l}{2}\right)\right]}{2\beta l + \sin 2\beta l}$$

$$= \frac{1}{k} \cdot \frac{1 - \cos 2\beta l}{2\beta l + \sin 2\beta l}$$

The above expression is exactly the same as the expression for the input impedance of a line tuned to resonance with a lossless condenser as derived by Nergaard and Salzberg. It is equation (11) in their paper.[56] The ratio $\frac{r}{r_o}$, where r_o is the input impedance of the self-resonant line, is plotted vs. βl in Figure 7.

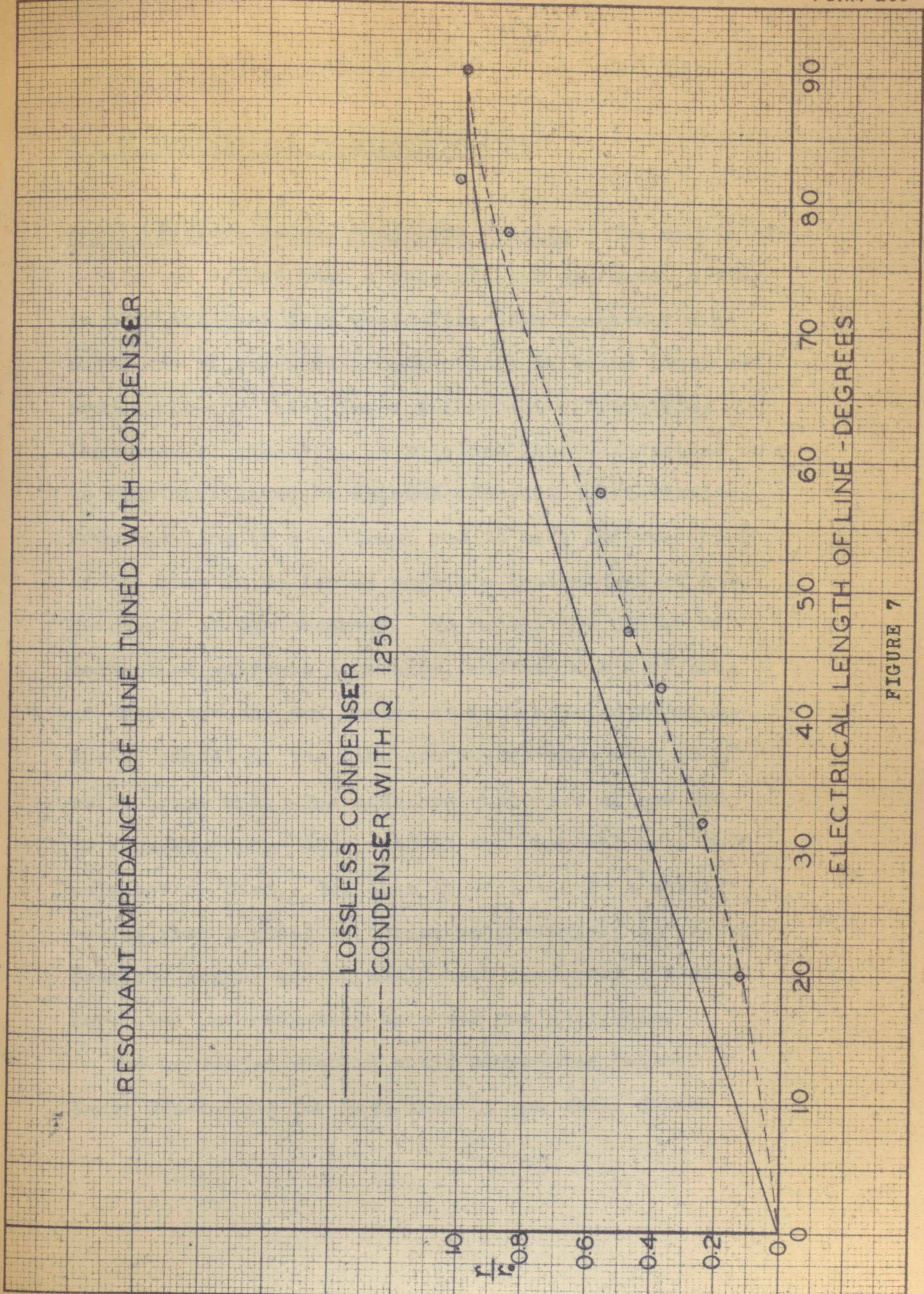

RESONANT IMPEDANCE OF LINE TUNED WITH CONDENSER
LOSSLESS CONDENSER
CONDENSER WITH Q 1250
ELECTRICAL LENGTH OF LINE - DEGREES
0
10
20
30
40
50
60
70
80
90
$\frac{r}{r_o}$
0
0.2
0.4
0.6
0.8
1.0

FIGURE 7

3 - 2 DESCRIPTION OF MEASURING LINE:

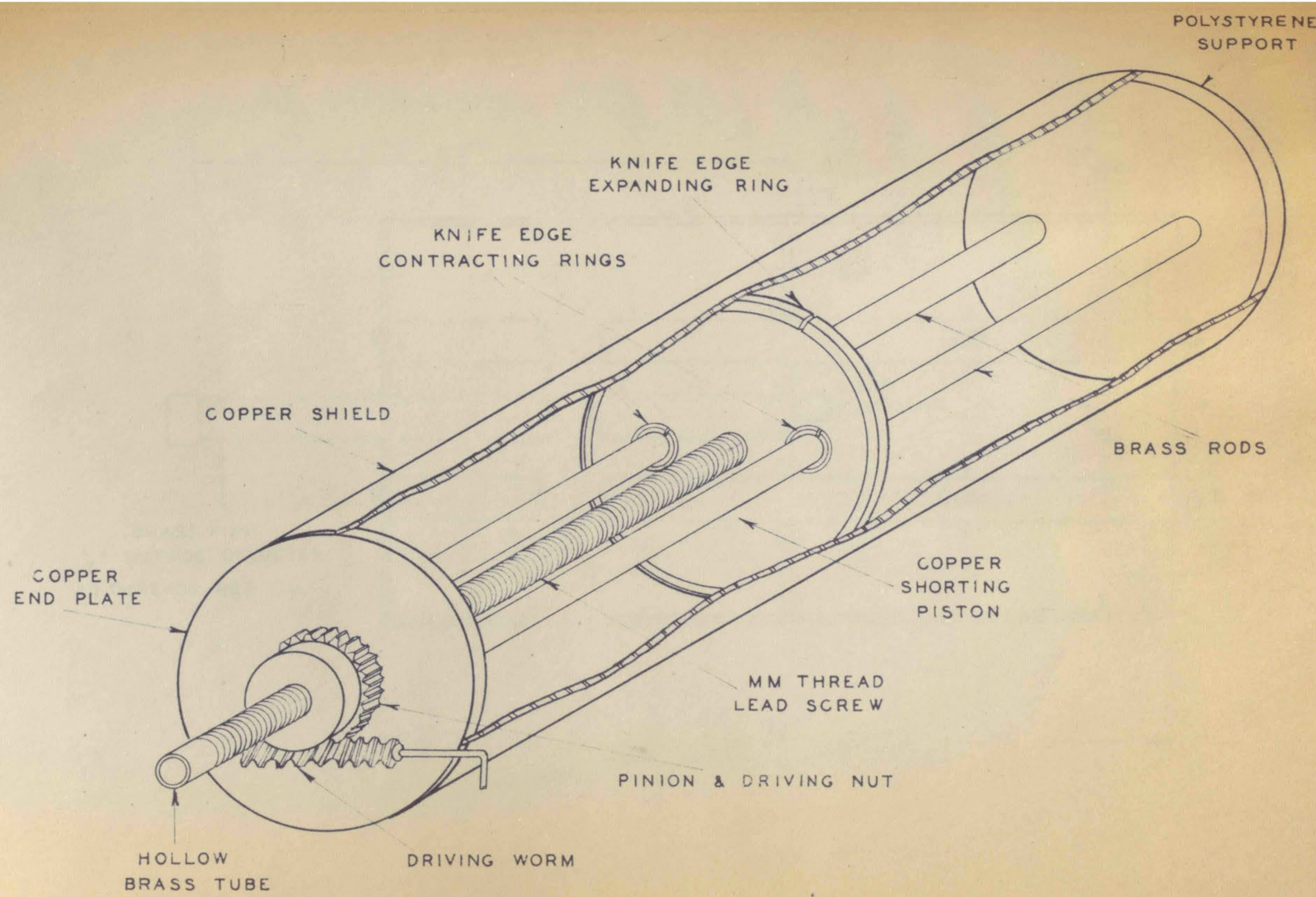
3-2.1 Factors determining line details The choice of a balanced rather than a coaxial line was made, having in mind the fact that most ultra-high frequency circuits are of balanced construction; balanced lines and components involving fewer mechanical difficulties than coaxial circuits. A shielded line was decided upon to keep unbalanced or antenna currents and radiation from the line to a minimum; and to make the operation of the line in so far as possible independent of the movements of the observer. To reduce the number of mechanical supports necessary, and avoid the effects of conductor sag and change of mechanical dimensions due to handling, the line conductors were made of $\frac{3}{8}$" solid brass rod. The rods are supported at one end by a heavy copper disc, and at the open end by a thin polystyrene disc. The line is enclosed in a shield rolled from a sheet of copper approximately $\frac{1}{16}$" thick. Seamless copper tubing would have been more satisfactory, but unfortunately this was not available.

The conductor spacing was a compromise between large dimensions for high line Q, and small dimensions for low radiation loss from the open end of the line. The dimensions of components likely to be measured on the line was also a factor in determining the conductor spacing. The details of the line construction and the dimensions of the various parts are shown in Figures 8 and 9. Figures 10 and 11 are photographs of the line, broadside, and looking at the open end, respectively.

3-2.2 Measuring Line Constants: Expressions for R_o, L_o, G_o and C_o, the resistance, inductance, shunt conductance and capacitance (all per unit length of line), have been derived by Curtis,[147] for a screened twin line with continuous dielectric. Approximate forms of these complex expressions are given in Jackson,[65] page 44, which are stated to be sufficiently accurate for most practical cases. Sacrificing some thoroughness for brevity, these expressions will not be derived here, but merely stated and used to calculate the constants of the measuring line which has been constructed.

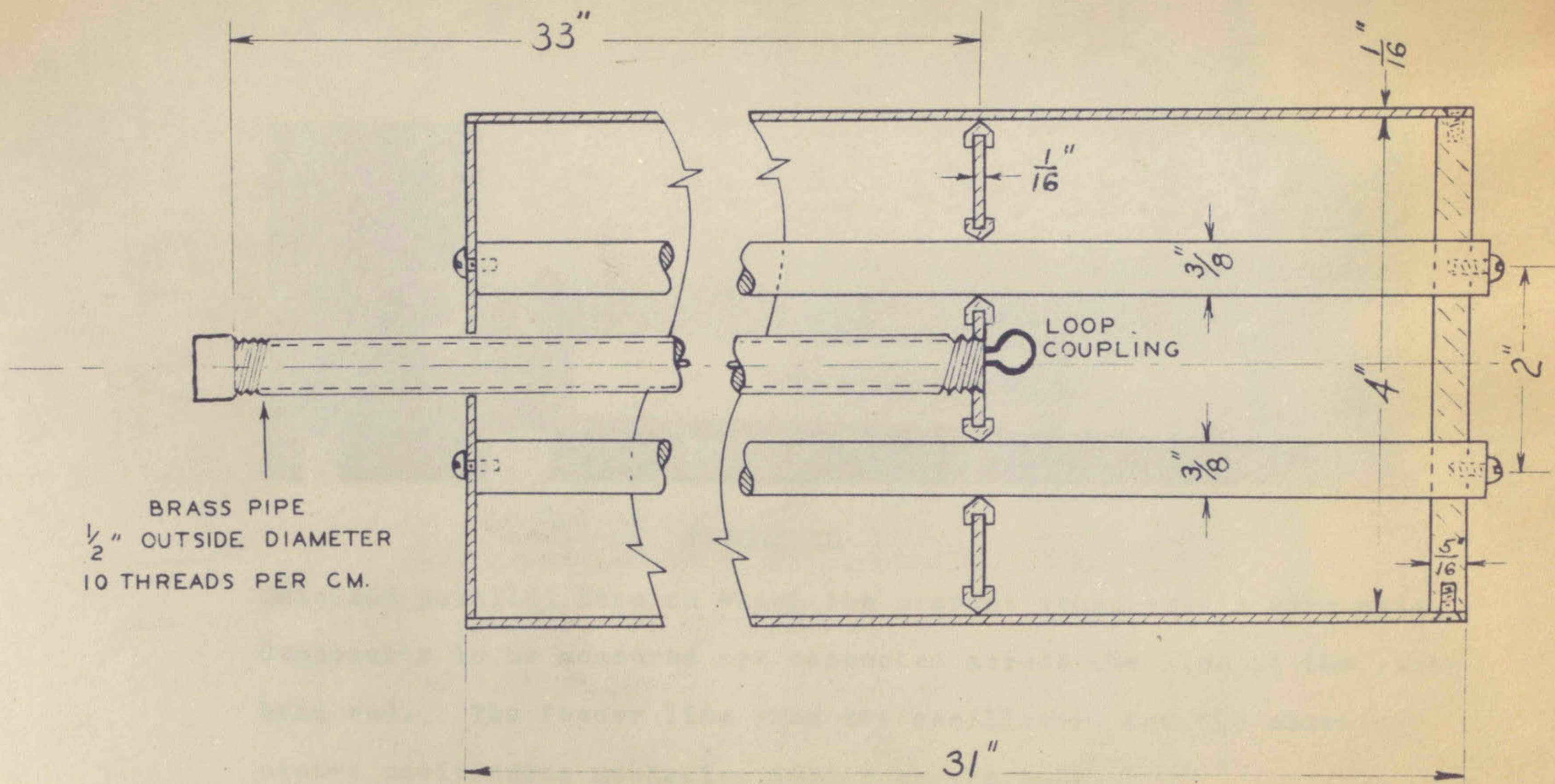
The dimensions of the measuring line used in the following formulas are:

a - radius of the line conductors	0.479×10^{-2} meters
d - centre to centre spacing of conductors	5.14×10^{-2} meters



MEASURING LINE ASSEMBLY

FIGURE 8



MEASURING LINE DETAIL

FIGURE 9

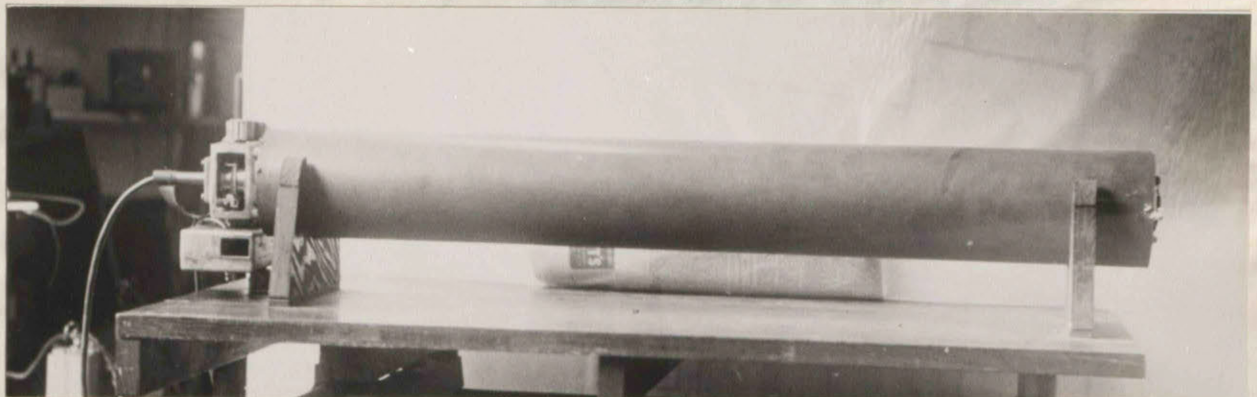


FIGURE 10

Shielded parallel line on which the present measurements were made. Components to be measured are connected across the line at the right hand end. The feeder line from the oscillator, and the shorting piston positioning mechanism appear at the left.

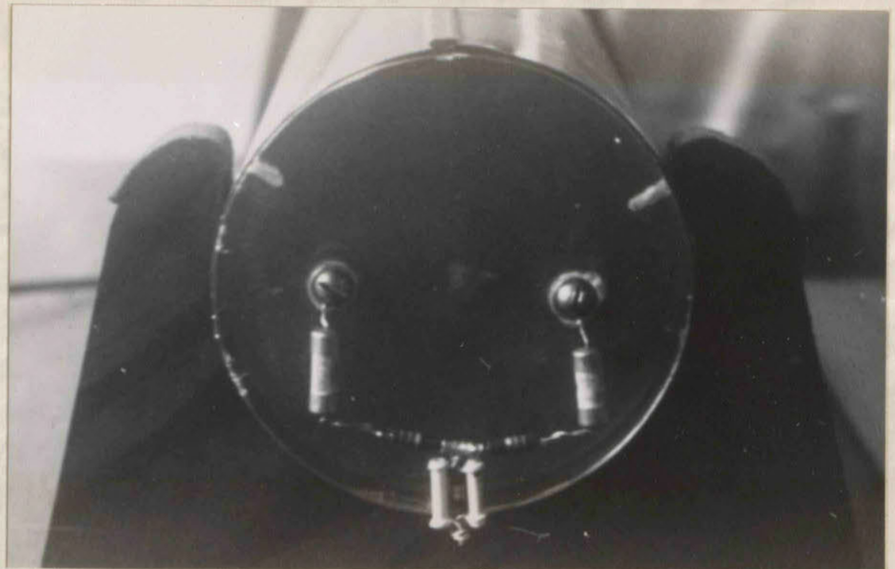


FIGURE 11

End view of measuring line showing copper shield, polystyrene end support, connecting screws on ends of parallel brass rods, and balanced (1N34 crystal) detector.

2s – inside diameter of shield	10.16×10^{-2} meters
s_a – conductivity of conductors (brass)	1.5×10^{7} mhos/meter cube
s_s – conductivity of shield (copper)	5.9×10^{7} mhos/meter cube
f – frequency, cycles/sec.	

The resistance per unit length is given by

$$R_o = \frac{2}{a}\,\frac{f}{10^7 s_a} + \frac{8}{s}\,\frac{f}{10^7 s_s}\,\frac{\left(\frac{d}{2s}\right)^2}{1 - \left(\frac{d}{2s}\right)^4} \text{ ohms/meter}$$

The first term relates to the centre conductors and the second term to the shield, and the lack of a proximity effect factor in the first term is due to the fact that the tendency of each centre conductor to disturb the uniformity of charge distribution on the surface of the other is counteracted by the presence of the screen. The complete solution for the resistance of conductors of circular cross-section involves Bessel functions, but for frequencies above 10 mc./sec., when the depth of current penetration is small compared with the radius of the conductor, the conductor surface may be treated as part of the surface of a flat conducting slab, and the usual skin effect formula applied.

The resistance per unit length for the measuring line has been computed from the above formula, and is

$$R_o = 0.0359\sqrt{f_{mc.}} \quad \text{ohms/meter}$$

The value of R_o thus ranges from 0.359 ohms/meter at 100 mc. to 0.509 ohms/meter at 200 mc.

<u>The inductance per unit length</u> of line is given by

$$L_o = 4 \times 10^{-7} \cdot \log_e \frac{d}{a} \cdot \frac{1 - (\frac{d}{2s})^2}{1 + (\frac{d}{2s})^2} \quad \text{henrys/meter}$$

Jackson has derived an expression for the internal inductance of the conductors, but since this part of the line inductance decreases as the square root of the frequency, it is negligible at ultra-high frequencies by comparison with that associated with the space between the conductors. It is not included in the above expression, and will be assumed negligible.

The value of inductance calculated for the measuring line is

$$7.4 \times 10^{-7} \quad \text{henrys/meter}$$

<u>The capacitance per unit length</u> of line is given

by

$$C_0 = \frac{1}{3.6 \times 10^{10}} \cdot \frac{\frac{e}{e_0}}{\log_e \frac{d}{a} \cdot \frac{1 - (\frac{d}{2s})^2}{1 + (\frac{d}{2s})^2}} \text{ farads/meter}$$

Since the line dielectric is air, the effect of the polystyrene end support being ignored in this calculation, the ratio of the permittivities $\frac{e}{e_0}$ is unity. The calculated value for the measuring line capacitance is

$$0.15 \times 10^{-10} \text{ farads/meter}$$

The shunt conductance per unit length of line with a continuous dielectric of loss angle d is given by

$$G_0 = \omega C \tan d \text{ mhos/meter}$$

Since the line dielectric is air, tan d and therefore the shunt conductance, will be assumed to be zero.

3-2.3 Effect of Polystyrene Support The dielectric constant of Polystyrene is 2.52 at 100 mc. From the formula on the preceding page the capacitance of the length of line containing the polystyrene end support is calculated to be 0.378×10^{-10} farads/meter. The width of the end support is 0.794 cm., and the loss factor tan d for polystyrene is 0.0003. The conductance in the region of the polystyrene support is calculated from the above expression to be

$$7.11 \times 10^{-6} \text{ mhos/meter.}$$

This conductance is equivalent to a resistance of

17.6 megohms

connected across the end of the line, at 100 mc., and a resistance of 8.8 megohms at 200 mc.

This shunt conductance across the end of the line can be corrected for by subtracting it from the unknown conductance when measurements are made on components connected to the line. A more satisfactory solution to the problem would be to reduce the width of the polystyrene support, and cut away as much of it as is consistent with sufficient mechanical support of the line conductors.

<u>3-2.4 Characteristic Impedance</u> The characteristic impedance for a transmission line with air dielectric is given in section 3 1.2 as

$$Z_o' = Z_o\ (1 - jk)$$

and from section 3-1.3

$$Z_o = \left(\frac{L}{C}\right)^{1/2} \qquad \text{and} \qquad k = \frac{R_o}{2\omega L_o}$$

Using the expressions for R_o L_o and C_o given in section 3 - 2.2, we obtain

$$Z_o = \left(\frac{e_o}{e}\right)^{1/2} .\ 276 \log_{10} \frac{d}{a} \cdot \frac{1 - \left(\frac{d}{2s}\right)^2}{1 + \left(\frac{d}{2s}\right)^2} \text{ ohms}$$

$$\text{and } k = \frac{54.6}{\sqrt{f}} \cdot \frac{\frac{1}{a\sqrt{s_a}} + \frac{4}{s\sqrt{s_s}} \frac{\left(\frac{d}{2s}\right)^2}{1 - \left(\frac{d}{2s}\right)^4}}{\log_{10} \frac{d}{a} \cdot \frac{1 - \left(\frac{d}{2s}\right)^2}{1 + \left(\frac{d}{2s}\right)^2}}$$

Substituting the dimensions of the measuring line in the above expressions we obtain

$$Z_o = 222 \text{ ohms}$$

and

$$kZ_o = \frac{1.71}{\sqrt{f_{mc}}} \text{ ohms}$$

The value of the imaginary component of the Characteristic impedance is thus seen to range from 0.171 ohms at 100 mc. to 0.121 ohms at 200 mc.

<u>3 - 2.5 Phase Constant</u> The expression for the propagation constant P is given in section 3-1.2 as

$$P = \beta(k + j) = \alpha + j\beta$$

The imaginary component of P, or phase constant is β, and is given in section 3 - 1.3 as

$$\beta = \omega(LC)^{\frac{1}{2}} \text{ radians/meter}$$

Using the expressions for L and C given in section 3 - 2.2 the above becomes

$$\beta = \frac{2\pi f}{3 \times 10^8} = \frac{2\pi f}{c}$$

The wavelength $\lambda = \frac{2\pi}{\beta} = \frac{v}{f}$, where v is the velocity of principal mode propagation along the line. The line velocity is thus seen to be equal to the free space velocity c, for an air dielectric line, when the internal inductance of the conductors relative to the inductance of the dielectric space is neglected.

<u>3 - 2.6 Attenuation Constant</u> The real part of the expression for the propagation constant is

$$\alpha = \beta k$$

Multiplying the expression for k given in 3 - 2.4 by $\beta = \frac{2\pi}{\lambda}$, we obtain for the attenuation constant

$$\alpha = 1.15 \times 10^{-6}\sqrt{f}\; \frac{\dfrac{1}{a\sqrt{s_a}} + \dfrac{4}{s\sqrt{s_s}}\; \dfrac{\left(\frac{d}{2s}\right)^2}{1 - \left(\frac{d}{2s}\right)^4}}{\log_{10} \dfrac{d}{a} \cdot \dfrac{1 - \left(\frac{d}{2s}\right)^2}{1 + \left(\frac{d}{2s}\right)^2}}$$

This is in nepers/meter, and to obtain the attenuation constant in db/meter, results obtained from the above expression must be multiplied by the factor 8.66 . The attenuation constant of the measuring line is calculated to be

$$\alpha = 7.04 \times 10^{-4} \sqrt{f_{mc}} \qquad \text{db/meter}$$

and the value of α is thus seen to range from 7×10^{-3} db/meter at 100 mc. to 9.95×10^{-3} db/meter at 200 mc.

<u>3 - 2.7 Q Value of a Resonant Line</u> In the case of a lumped constant resonant circuit consisting of an inductance coil with series resistance, and a condenser with negligible losses, the Q value of the circuit is commonly written

$$Q = \frac{\omega L}{R}$$

Multiplying both numerator and denominator by $\frac{1}{2} I^2$, where I is the current at resonance,

$$Q = 2\pi f \frac{\frac{1}{2}LI^2}{\frac{1}{2}RI^2}$$

$$= \frac{\text{Energy stored in magnetic field}}{\text{Energy loss per cycle}}$$

For a resonant transmission line, one end of which is shorted and the other end open, the standing wave of current has the form $I \cos \frac{2\pi}{\lambda} l$ where I is the current through the short-circuited end, and l is the distance from this end. The energy stored in the magnetic field is thus

$$= \frac{1}{2} L_o \int_0^{\frac{\lambda}{4}} \left(I \cos \frac{2\pi}{\lambda} l \right)^2 dl$$

$$= \frac{\lambda L_o I^2}{16}$$

The rate of energy loss in the conductors, the losses in the short circuit being ignored, is

$$= \frac{1}{2f} R_o \int_0^{\frac{\lambda}{4}} \left(I \cos \frac{2\pi}{\lambda} l \right)^2 dl$$

$$= \frac{R_o \lambda \ I^2}{16f}$$

Therefore $$Q_o = 2\pi \cdot \frac{\frac{1}{16} \quad L_o I^2}{\frac{R_o \quad I^2}{16f}} = \frac{\omega L_o}{R_o}$$

Terman [83] derived the expression in the form

$$Q_o = \frac{2\pi Z_o f}{R_o \ c}$$

but this is shown in section 3 - 1.6 to be identical with $\frac{\omega L_o}{R_o}$ and $\frac{\beta}{2d}$.

The resonant Q_o of the measuring line may therefore be determined from the expressions for L_o and R_o given in section 3 - 2.2 .

$$Q_o = \frac{2\pi f \times 4\times10^{-7} \log_e \frac{d}{a} \quad \dfrac{1 - (\frac{d}{2s})^2}{1 + (\frac{d}{2s})^2}}{\frac{2}{a}\sqrt{\frac{f}{10^7 s_a}} + \frac{8}{s}\sqrt{\frac{f}{10^7 s_s}} \quad \dfrac{(\frac{d}{2s})^2}{1 - (\frac{d}{2s})^4}}$$

The value of Q_o calculated from this formula is

$$Q_o = 129.5 \sqrt{f_{mc}}$$

and the value of Q_o is thus seen to range from 1295 at 100 mc. to 1815 at 200 mc.

Since this formula makes no allowance for the losses in the shorting plate, the contact resistances between the line conductors and the shorting plate, the contact resistances between the shield and the shorting plate, and the losses in the polystyrene end support, it is not to be expected that the above values will be realized in practice unless extreme care is taken to reduce these losses to negligible quantities. It is further to be noted that oxidation of the line conductors and of the shield will increase the value of R_o beyond that calculated, and thus lower the value of Q_o an amount which would be extremely difficult to calculate.

Another factor for which allowance has not been made is the radiation from the open end of the line. King[50] suggests the use of the Hertzian formula for the radiation resistance of a short straight conductor

$$R_r = 20\beta^2 \quad s^2$$

in which case s would be the centre to centre spacing of the line conductors. This problem reqires further investigation.

<u>3 - 2.8 Shorting Piston and Coupling Loop</u> The length of the measuring line is varied by means of a moveable solid copper disc or shorting piston about $\frac{1}{16}$ inch thick. The details of this piston are shown in Figures 8 and 9, and Figure 12 is a photograph of the open end of the line with the shorting piston pushed right out to the end of the line.

The copper disc makes contact with the line conductors and with the outer shield, by means of brass rings which are fitted to the disc. The inner rings

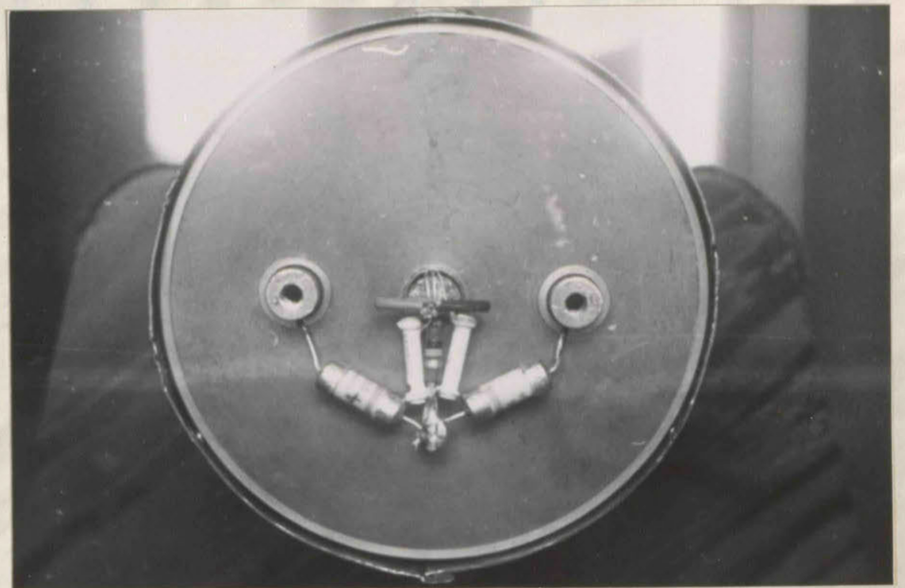


FIGURE 12

The shorting piston at the open end of the measuring line. The holes in the ends of the line conductors are for the connection of unknown impedances by means of 8-32 machine screws. The 1N34 crystals are attached to the shorting-piston rings, and the detector load, which appears directly below the coupling loop.

are grooved and split so that they snap into holes in the shorting piston. The inside diameter of these rings is slightly less than the diameter of the line conductors, so that when the piston is pressed on, the rings are expanded and thus make firm contact with the line conductors. The outer ring, which makes contact with the shield, is likewise grooved and split so as to fit over the outside edge of the shorting piston. This ring was machined to a diameter slightly larger than the inside diameter of the shield, and then a small piece was cut out of the circumference of the ring. When the piston is pressed into the shield this ring is contracted and thus presses out against the shield, ensuring firm contact. The inner edges of the inside rings, and the outer edges of the outside ring were machined to knife edges, in order to make the point of contact and thus the position of the shorting piston, quite definite. This definite point of contact is achieved at the expense of the line Q, as the knife edges undoubtedly have a higher contact resistance than would be obtained with broader contacts. This type of shorting piston construction has not, to the author's knowledge, been used before; and as the usual spring-finger type of piston was not tried on the present line, no comparison between the two types of shorting disc can as yet be made. It is hoped to make some comparisons of this nature as an extension of the present work. The smallest bandwidths measured on the line were of the order of 0.04 centimeters, and it is thought, consequently, that the width of the contacts between the shorting piston and the conductors may be of some importance.

The shorting piston is firmly attached to a hollow brass lead-screw which moves the piston along the line. Inside the lead-screw is the feeder cable which projects, in the form of a small loop, through a centrally located hole in the face of the shorting piston. The loop is located in the plane of the line conductors and symmetrically positioned with respect to them, so as to induce equal and opposite voltages in the two rods. In this way, unbalanced or antenna currents in the line conductors were kept to a minimum. The shield of the feeder cable is grounded to the shorting piston, but the centre point of the coupling loop is

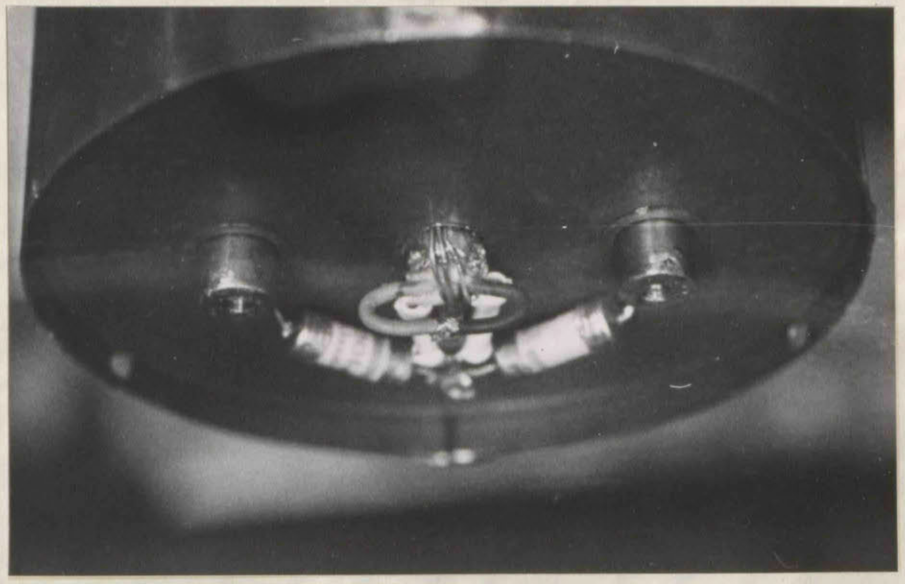


FIGURE 13

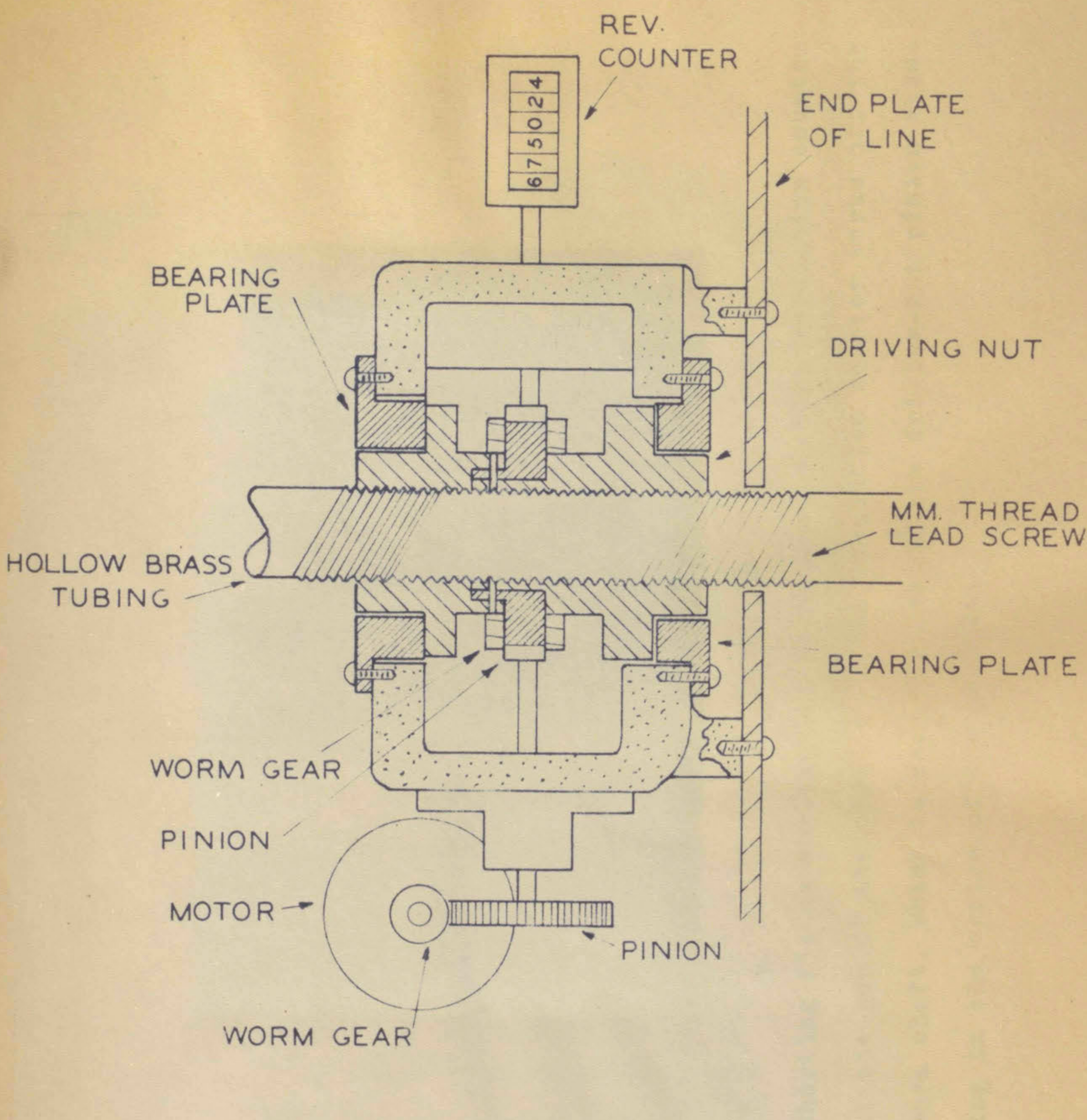
The shorting piston at the open end of the measuring line, showing the coupling loop (in the centre of the picture) and the detector directly below it.

not grounded. It was initially thought that grounding the centre point of the loop would help to balance the input, but it was found in practice that this procedure gave rise to considerable unbalance, as evidenced by double humps occurring in the response curve. The coupling loop projecting through the hole in the face of the shorting piston, can be seen as the half-black half-white oval in the centre of Figure 13.

The feeder cable is a shielded twin with rubber and rope insulation, and introduces a loss of approximately 30 db between the oscillator and the line. This large loss in the feeder cable is necessary to ensure that changes in the line impedance as the line is tuned through resonance will not be reflected back into the oscillator. As the line impedance is only a small part of the load on the oscillator, any variations due to line tuning will have a negligible effect on the frequency and output of the oscillator.

3 - 2.9 Device for Adjusting and Reading Line Length

The general arrangement for positioning the shorting piston is shown schematically in Figure 14. Figures 15 and 16 are photographs of the positioning mechanism attached to the shorted end of the measuring line.



ARRANGEMENT FOR POSITIONING SHORTING PISTON

FIGURE 14

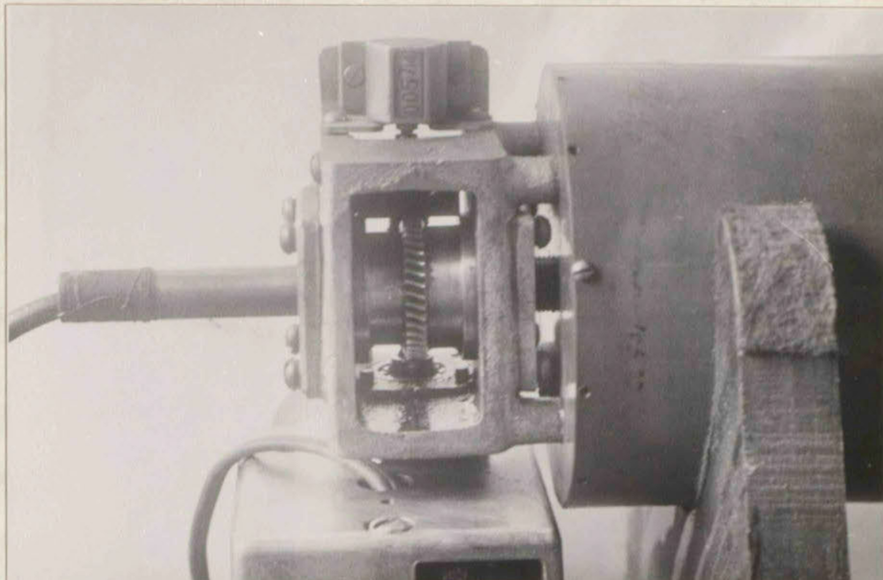


FIGURE 15

Shorting piston positioning mechanism, showing revolution counter at the top of the picture. The revolution counter works off the worm shaft, which is directly behind the driving-nut pinion showing in the centre of the picture.

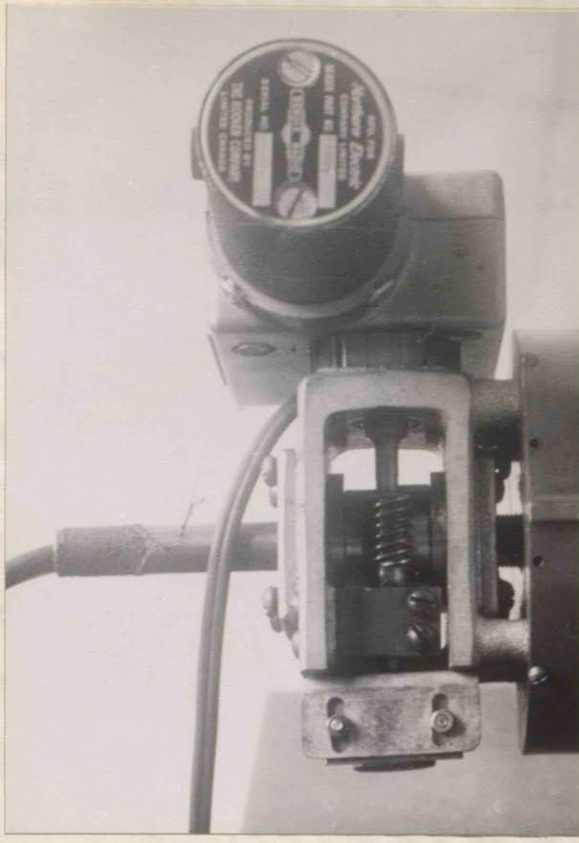


Figure 16

Shorting-piston positioning mechanism. The driving motor appears at the top. A worm and pinion inside the motor housing drives the worm showing in the picture. The driving nut is directly behind the worm gear, and the millimeter threads on the lead screw passing through this nut can be seen at the right.

The fundamental parts of this device are the driving nut, and the millimeter thread lead screw which moves in or out 1 mm. for each revolution of the driving nut. The driving nut is held between two end bearings in a cast aluminum housing. The thrust bearings are adjusted by means of shims, so that there is no end or lengthwise motion of the driving nut. Part of the driving nut is in the form of a pinion with 30 teeth which is driven by a helical worm gear so that ten revolutions of the worm turn the driving nut through one revolution. To one end of the worm shaft is attached another pinion with 30 teeth driven by a helical worm gear on the motor shaft. Thus 100 revolutions of the motor turn the driving nut through one revolution, and move the shorting piston 1 millimeter. To the other end of the main worm shaft is attached a revolution counter which reads directly the position of the shorting piston to 0.001 cm. Every effort has been made to reduce loose motion between the gears, and bandwidth readings are always taken with the motor turning in the same direction, so as to eliminate the effects of backlash. The motor is provided with a reversing switch so that the shorting piston may be moved in either direction, and the speed of the motor is controlled by means of a Variac.

The application of motor drive, and the use of a revolution counter to read the line length directly, are original developments which have not, to the author's knowledge, been used before, in connection with measuring lines. Most lines which have been built in the past have used either friction drive or a rack and pinion, with a vernier scale attached to the positioning rod. With friction drive, it is difficult to position the piston exactly, and even with the more positive drive obtained with a rack and pinion, a special light and magnifying glass are required to read the vernier scale. The present method of drive has the advantages of positive motion combined with extreme accuracy and ease of reading the line length. Furthermore the tedious process of moving the piston is turned over to a motor the speed of which may be varied at will. A series motor was used so as to obtain a wide range of speed control.

<u>3 - 2.10 Detector</u> Two types of detector were used on the measuring line. Initially the detector was

Figure 17

General arrangement of apparatus used in making measurements. At the left is the oscillator and wavemeter initially used. The variac at the bottom controls the speed of the positioning mechanism driving motor. The meter, top right, reads the output of the crystal detector, as the line is tuned through resonance.

connected across the open end of the line, as shown in Figure 11; but it was found that this resulted in serious loading of the measuring line, with a consequent lowering of the line Q. The final form of the detector was as shown in Figure 12, the location being changed from the end of the line to the plane of the shorting piston.

The detector consists of two 1N34 crystals soldered to the small rings in the shorting piston, and operating into a common load resistor of 100k ohms which is grounded at one end to the shorting piston. The load resistor is by-passed by two 1000$\mu\mu$f condensers. A lead is attached to the junction of the crystals and the load resistor, and runs out through the hollow lead screw to a vacuum-tube voltmeter. The detector reads the extremely small voltage existing across the resistance of the shorting piston, and to avoid stray pick-up, the lead to the voltmeter is shielded. This type of detector was found to give a good indication from a slightly amplitude-modulated radio-frequency signal, on the lower ranges of an ordinary Hewlett-Packard 400A vacuum-tube voltmeter. Since the detector is located in the plane of the shorting piston the loading effect on the measuring line is negligible. No direct pick-up between the coupling loop and the detector was noticeable.

The main disadvantage of this type of detector is due to the instability of the crystal calibration, as this changes with frequency and with signal amplitude.

<u>3 - 2.11 U.H.F. Oscillator</u> Several types of oscillator were used to feed the measuring line, two of these being shown in Figures 18 and 19. The oscillator finally decided upon is shown in Figure 19, and is a line-controlled I.F.F. Mk. III transmitter modified to operate continuous wave. The output of the oscillator is controlled by varying the plate voltage on the parallel 826 triodes, by means of a Variac on the front panel of the oscillator chassis. Monitoring of the output was accomplished by means of a crystal detector similar to that used on the measuring line. This detector was located at the output terminals of the oscillator, and the voltage read on a vacuum-tube voltmeter.

The oscillator frequency is controlled by parallel-line sections in the cathode and grid circuits of the triodes; and the frequency changed by moving the shorting bars on the line sections. The output coupling

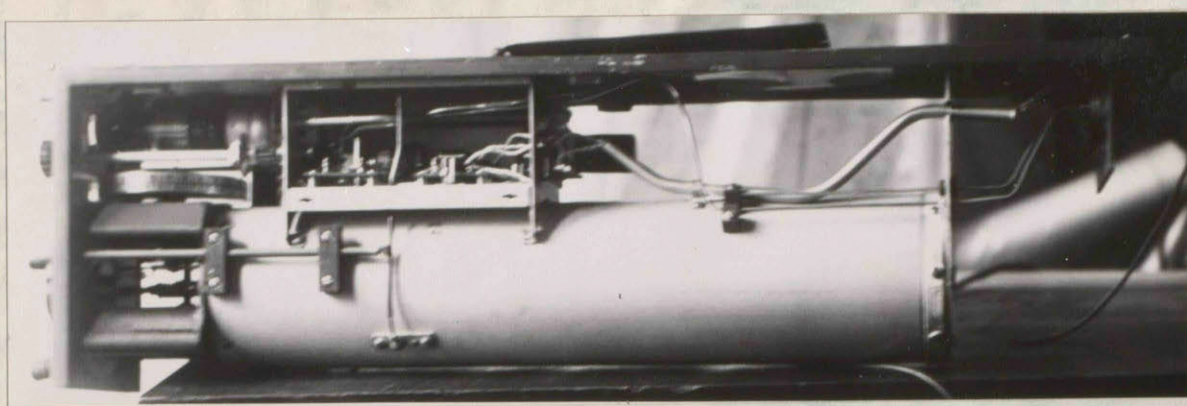


FIGURE 18

One of the oscillators used, removed from its case. The cylinder at the bottom of the picture is a variable line wavemeter. The oscillator is in the metal box sitting on top of the wavemeter. The horizontal rod at the left is a telescoping antenna, which is capacitively coupled to the vertical probe coupling from the oscillator to the wavemeter. A magic-eye tuning device is at the upper left. The output terminals are on the right.

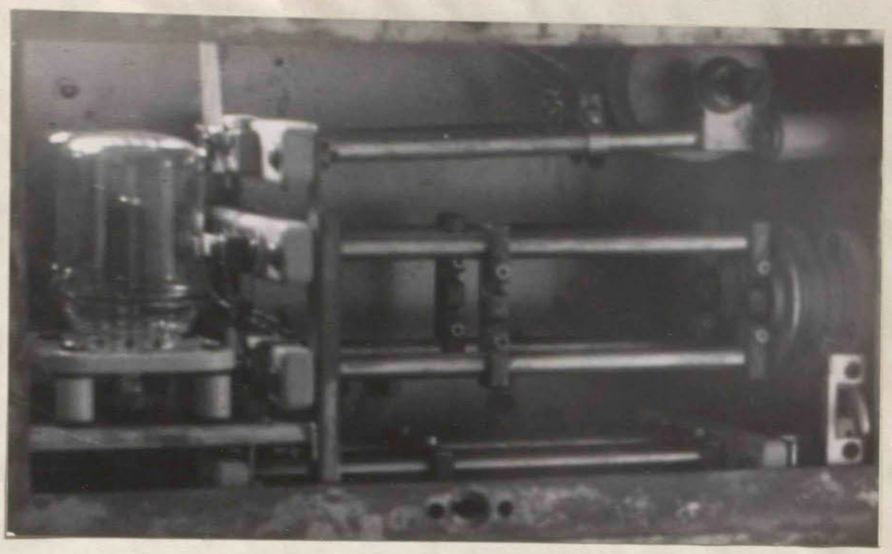


Figure 19

Oscillator used as source of high frequency voltage for the measuring line. The two parallel 826 triodes appear at the left. The bottom lecher bar line is attached to the grids of the 826's. The central line sections are in the cathode circuit, the shorting bars as shown being set for 190 mc. The line at the top is the output coupling section, and the condenser at the right tunes this line to resonance. The moveable clamps on this line feed through balanced cable to the output terminals at the lower left.

line is tuned to quarter-wave resonance with a variable air condenser, and this line is inductively coupled to the cathode circuit of the triodes. Variable taps on the output coupling line lead through balanced cable to the output terminals of the oscillator, at which point the output monitor and the measuring line feeder are connected. The frequency of the oscillator can be varied from about 145 mc. to about 200 mc, the shorting bars being set according to a calibration chart which is provided with the transmitter. Final tuning of the oscillator is accomplished by means of a small air condenser connected across the grid lecher line.

The oscillator frequency was monitored by means of a General Radio 720A frequency meter.

4 - EXPERIMENTAL WORK

4 - 1 Calibration of Measuring Line:

4 - 1.1 Adjustment of Coupling Loop It is essential to the proper functioning of the measuring line that the voltage output and frequency of the oscillator remain constant as the line is tuned through resonance. To achieve this condition the line must represent only a small portion of the load on the oscillator, so that even though the line impedance varies considerably, the overall load impedance seen by the oscillator remains essentially constant. If the coupling between the line and the oscillator is not sufficiently loose, there is a tendency for the oscillator output voltage to drop off when the line becomes resonant, since the maximum line current occurs at this point. Since changes in impedance are being observed in terms of changes in current through the shorting piston, the results can only be interpreted if the input voltage to the line remains constant.

Similarly, if the coupling is not sufficiently loose changes in line impedance reflected back into the oscillator may cause the frequency to shift, and thus invalidate the results.

Either or both of the above effects will result in broadening of the response curve, asymmetry about the resonance point, and possibly the occurrence of double humps, one on either side of the resonance point. It is thought, however, that the phenomenon of double humps in the response curve is due more to unbalanced feed than to overcoupling.

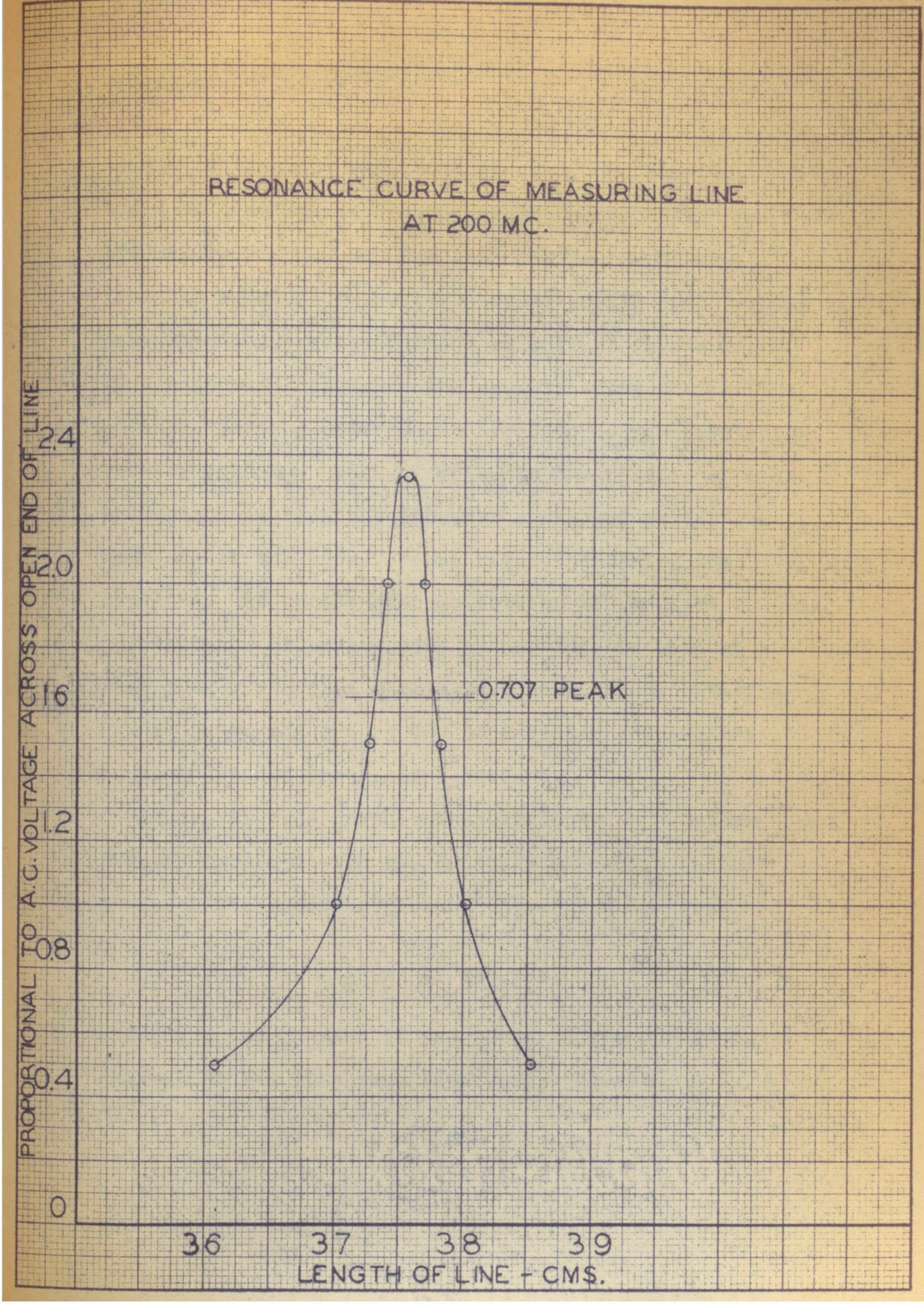
When the feed to the line is unbalanced, in-phase currents are induced in the line conductors, as well as the balanced 180° out-of-phase currents. These in-phase currents give rise to a coaxial mode of propagation using the two line conductors as the centre conductor and the shield as the outer conductor. A slight difference in velocity of propagation between the coaxial and balanced modes results in the line being resonant for one of the modes at a slightly different length from that at which it is resonant for the other mode. The response curve thus has two peaks, and its width at 0.707 point is more than twice what it would be if only one mode were present.

That the above explanation fits the facts would seem to be confirmed by the results of a few simple experiments in which the size and shape of the coupling loop were varied, and the shape of the response curve noted in each case. When the size of the coupling loop was changed, some change in the relative heights of the humps was noted, but the distance or line length between the humps was not affected. Now if the double humps were due to overcoupling, the closer the coupling was made, the farther apart the humps would appear, and this was not the case. On the other hand, if the double humps are due to two separate modes of propagation, changing the shape and orientation of the loop with respect to the line conductors should result in one mode being more pronounced than the other. This proved to be the case, for on distorting the loop slightly, one of the humps became considerably more pronounced than the other, although the separation between them remained constant, and each retained its original position with respect to line length.

To ensure that the coupling between the line and the oscillator would be sufficiently loose, a shielded rubber-covered twin line with high attenuation was used to feed the line. Also, since the coupling loop is located at the shorted end of the line, the impedance presented by the line is always quite small. Without too much difficulty, the loop was positioned in the plane of the line conductors, so as to give a smooth symmetrical response curve. A typical resonance curve taken at 200 mc., with the detector at the open end of the line, and the line self-resonant at the quarter-wave point, is shown in Figure 20. Another resonance curve, at 50 mc., with the line tuned to resonance with a condenser, was presented on an oscilloscope by a method to be discussed later. This curve is shown in Figure 29. They are seen to be of the same general shape, reasonably symmetrical, and free from irregularities such as double humps.

4 - 1.2 Calibration of Revolution Counter The length of the measuring line from the open end to the shorting piston can be read accurately from the revolution counter, only if the equivalent lengths of the polystyrene end support, the detector and the shorting piston, are constant within the frequency range for which the line was designed. To determine if this was the case, and to obtain the amount by which the actual

FIGURE 20


RESONANCE CURVE OF MEASURING LINE
AT 200 MC.
0.707 PEAK
PROPORTIONAL TO A.C. VOLTAGE ACROSS OPEN END OF LINE
2.4
2.0
1.6
1.2
0.8
0.4
0
36
37
38
39
LENGTH OF LINE - CMS.

length of the line must be corrected, the line was tuned to quarter-wave resonance at frequencies from 150 mc. to 200 mc. and the actual line length read from the revolution counter. These actual line lengths were then compared with their equivalent free-space wavelengths to determine the wave velocity on the line, and the length correction to be added to the revolution counter reading. Table 2 presents the results of measurements made with the detector connected across the open end of the line. These figures indicate that within $\pm \frac{1}{2}\%$ on the average between 152 mc. and 200 mc., the velocity of propagation on the line is that of free space; and that within less than 1% for this frequency range the errors due to detector and support loading are not frequency dependent.

Figure 21 is a graphical presentation of similar results obtained with the detector connected in the plane of the shorting piston. Since $f \times \lambda = c$ or

$$\frac{1}{f} = \frac{1}{c} \times \lambda + A$$

where A would be zero if the actual and equivalent lengths of the line were identical. Plotting $\frac{1}{f}$ vs. $\frac{\lambda}{4}$

DETERMINATION OF LINE VELOCITY AND LENGTH CORRECTION

Freq. mc.	Line length	Free Space length	Change in l_1	Change in l_0	$\frac{\Delta l_1}{\Delta l_0}$	Length correction
f	l_1	l_0	Δl_1	Δl_0	%	$l_0 - l_1$
	cm.	cm.	cm.	cm.		cm.
152	45.15	49.34				4.19
160	42.67	46.87	2.48	2.47	100.4	4.20
170	39.96	44.12	2.71	2.75	99.1	4.16
180	37.52	41.67	2.44	2.45	99.6	4.15
190	35.33	39.47	2.19	2.20	99.6	4.14
200	33.38	37.50	1.95	1.97	99.0	4.12

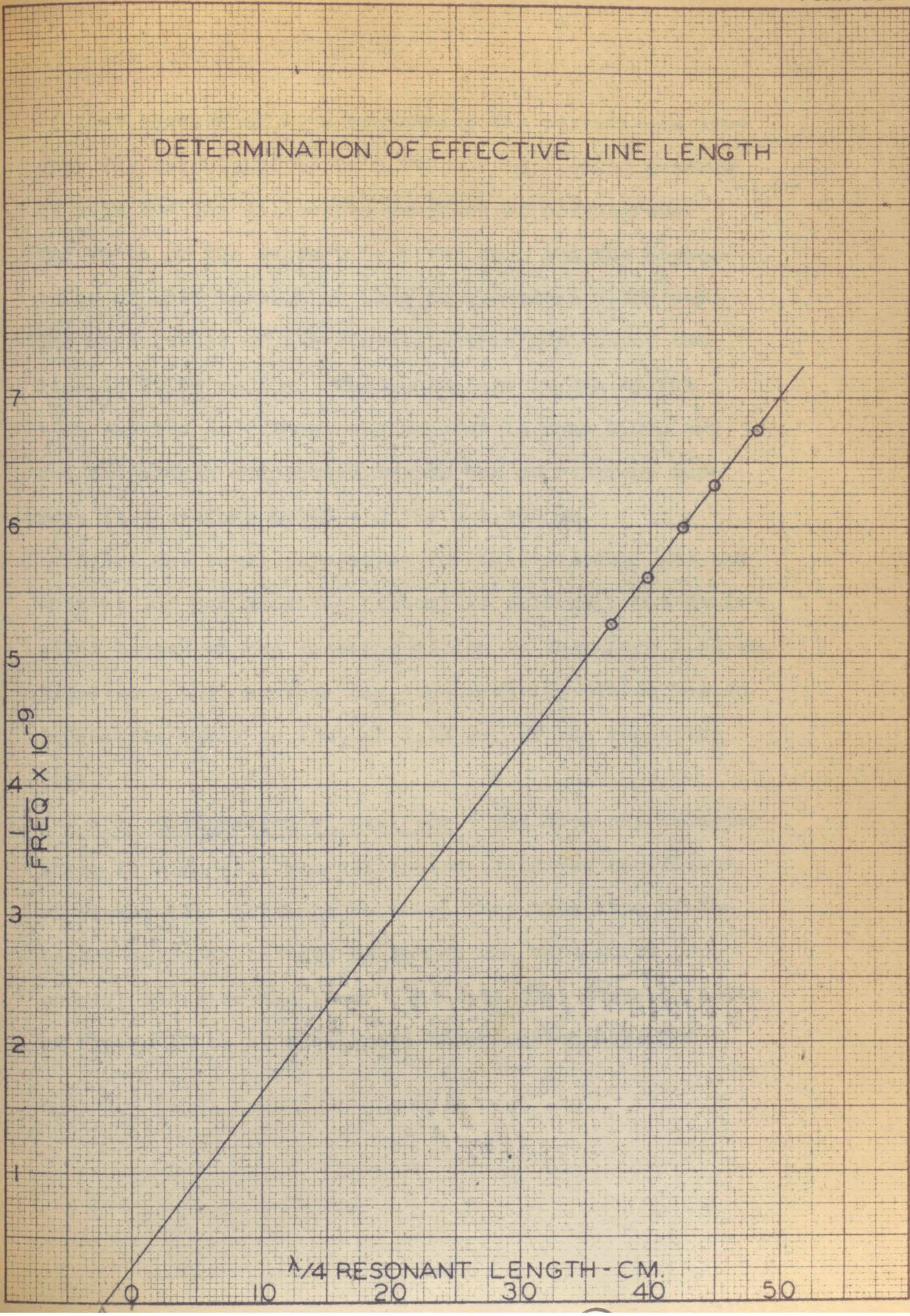
TABLE No. 2

in Figure 21, the negative intercept on the length axis is the difference between the equivalent length of the line and its actual length. It is also seen that the slope of the straight line is $\frac{4}{c}$, from which the velocity on the line is obtained. The fact that the relationship plotted in Figure 21 results in a straight line is proof that the length corrections for the detector, the end support and the shorting piston, are independent of frequency. The revolution counter reading was therefore changed by the amount indicated, and for all subsequent measurements the equivalent length of the line was read directly off the counter.

The length correction with the detector connected across the open end of the line was found to be 4.16 cm. With the detector connected in the plane of the shorting piston the length correction was found to be 2.39 cm. If the detector is assumed to have no effect on the line length in the latter position, the equivalent length of the detector is thus seen to be 1.77 cm.

From the expression for the capacitance per unit length of line given in section 3 - 2.2, the capacitance

FIGURE 21


DETERMINATION OF EFFECTIVE LINE LENGTH
7
6
5
4
3
2
1
$\frac{1}{FREQ} \times 10^{-9}$
λ/4 RESONANT LENGTH - CM.
0
10
20
30
40
50

of that section of line containing the polystyrene support was calculated to be 0.378 uuf/cm., using a value of 2.52 for the dielectric constant of polystyrene. The width of the support is 0.794 cm., and the capacitance of this section of line is therefore 0.300 uuf. The capacitance of the air-dielectric portion of the line is 0.150 uuf/cm. The equivalent length of the line section containing the support is thus 2.000 cm, and the length correction due to the polystyrene support is therefore 2.000 - 0.794 = 1.206 cm.

Since the total correction for the detector, the end support and the shorting piston, is 4.16 cm., the equivalent length of the shorting piston is therefore 1.18 cm. Expressing this another way, the shorting piston has an inductance equal to that of 1.18 cm. of the measuring line.

4 - 1.3 Calibration of Detector The open-end detector shown in Figure 11 was calibrated using the oscillator of Figure 18. It is to be noted that this type of detector responds to both balanced, and unbalanced or coaxial modes on the line. The fact that the oscillator of Figure 18 is of unbalanced construction does not

therefore, affect the calibration of the detector. A General Radio 726A vacuum-tube voltmeter was connected at a junction in the feeder line from the oscillator to the measuring line, and the output of the oscillator varied by means of a Variac in the power line. The measuring line length remained constant for these calibration measurements, and thus the voltmeter reading was directly proportional to the input to the measuring line. The detector output was read on a D.C. vacuum-tube voltmeter, and these values plotted against the A.C. input readings. Calibration curves were taken every 10 mc. from 150 mc. to 200 mc., and in every case the results very closely approximated a linear law of detection. Typical curves are shown in Figures 22 and 23.

The shorting-plane detector shown in Figures 12 and 13 was calibrated using the balanced oscillator of Figure 19, with a detector identical with that previously calibrated connected across the oscillator output. The D.C. output from the shorting-plane detector was too small to be useful, but the oscillator was sufficiently amplitude modulated by its power-supply ripple

FIGURE 22

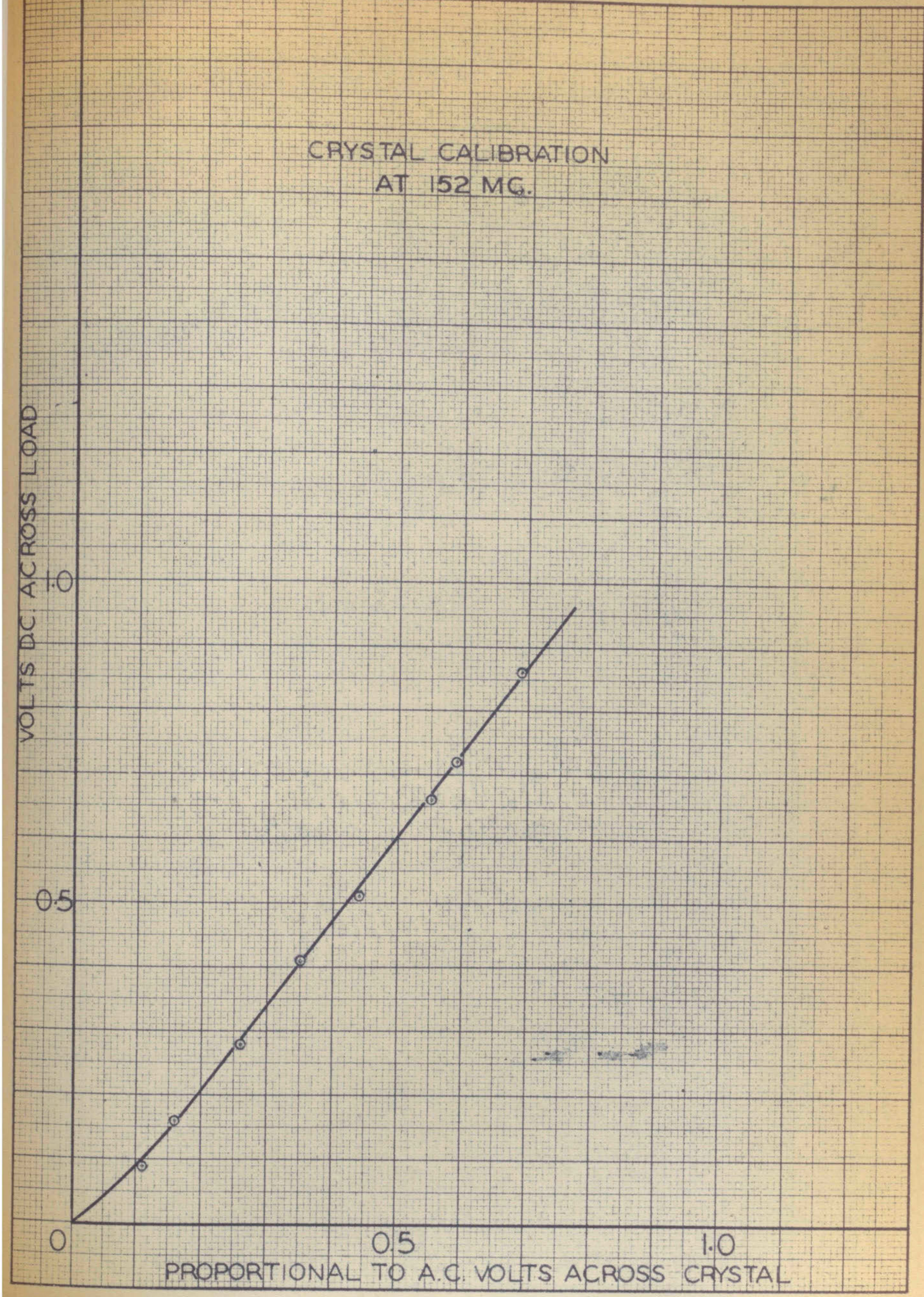
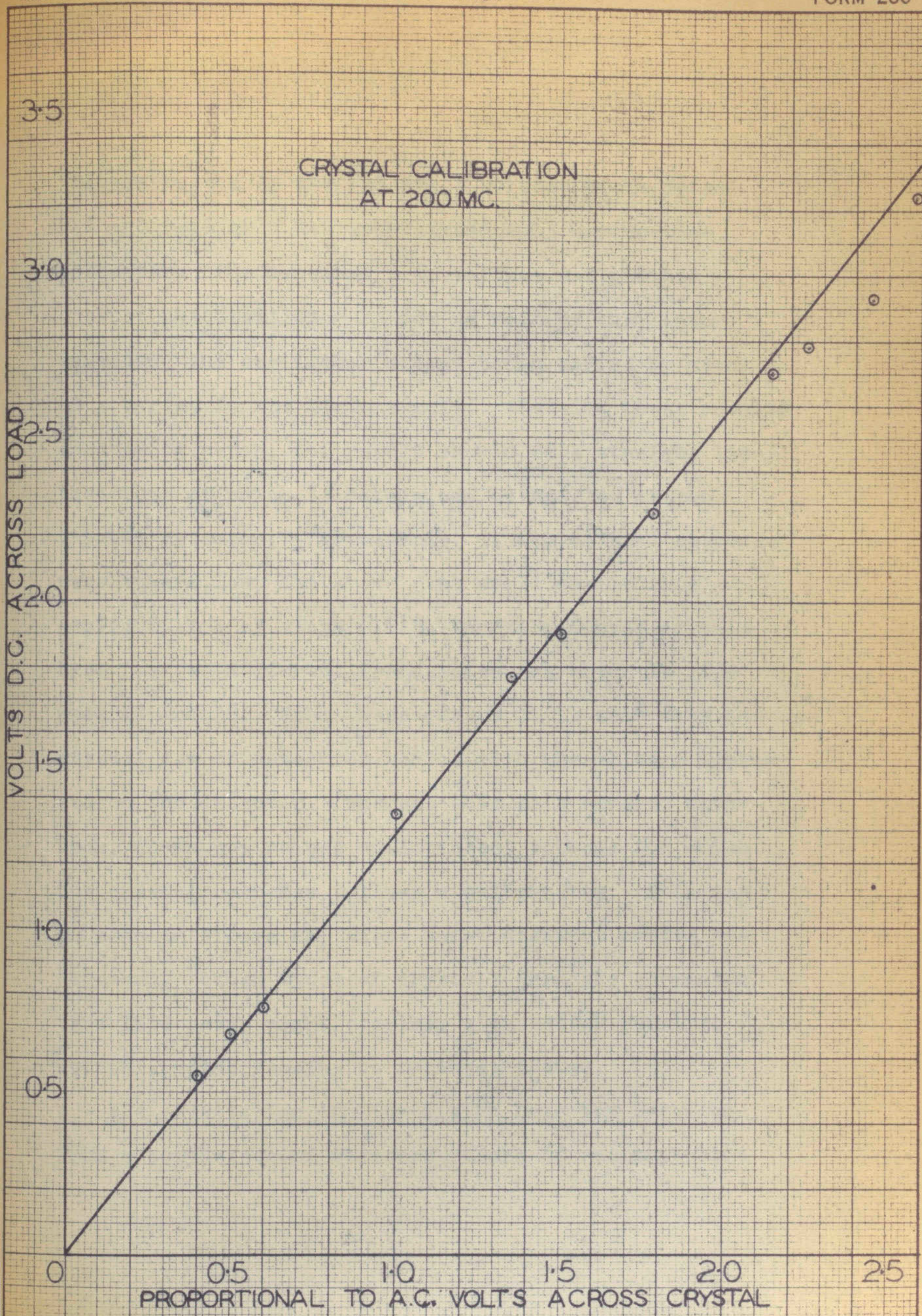

CRYSTAL CALIBRATION
AT 152 MC.
VOLTS DC. ACROSS LOAD
1.0
0.5
0
0.5
1.0
PROPORTIONAL TO A.C. VOLTS ACROSS CRYSTAL

FIGURE 23


CRYSTAL CALIBRATION
AT 200 MC.
VOLTS D.C. ACROSS LOAD
PROPORTIONAL TO A.C. VOLTS ACROSS CRYSTAL
0
0·5
1·0
1·5
2·0
2·5
3·0
3·5

voltage to enable the 120 cycle voltage to be detected and read on a Hewlett-Packard 400A vacuum-tube voltmeter. The detected A.C. voltage was plotted against the oscillator output as read by the linear detector, the oscillator output being varied by means of a Variac in its plate supply. Plotting these results on log-log paper, the result should be a straight line, the slope of which is determined by the law of the crystal detector. For example, if the crystal response were square law, the above plot would result in a straight line of slope 2. Calibration curves of this type for two different frequencies are shown in Figures 24 and 25. From these curves it is seen that the crystal law of detection varies with the voltage impressed on the detector. Whereas this is a serious drawback, there are ranges of impressed voltage for which the crystal response follows a uniform law. This can be seen in Figure 25, where three distinct ranges are apparent. Other types of crystals than the germanium type 1N34 used, might possibly give better results; but this point remains to be investigated.

<u>4 - 1.4 Variation of Line Q with Detector Location</u>

From a comparison of measured values of resonant-line

FIGURE 24

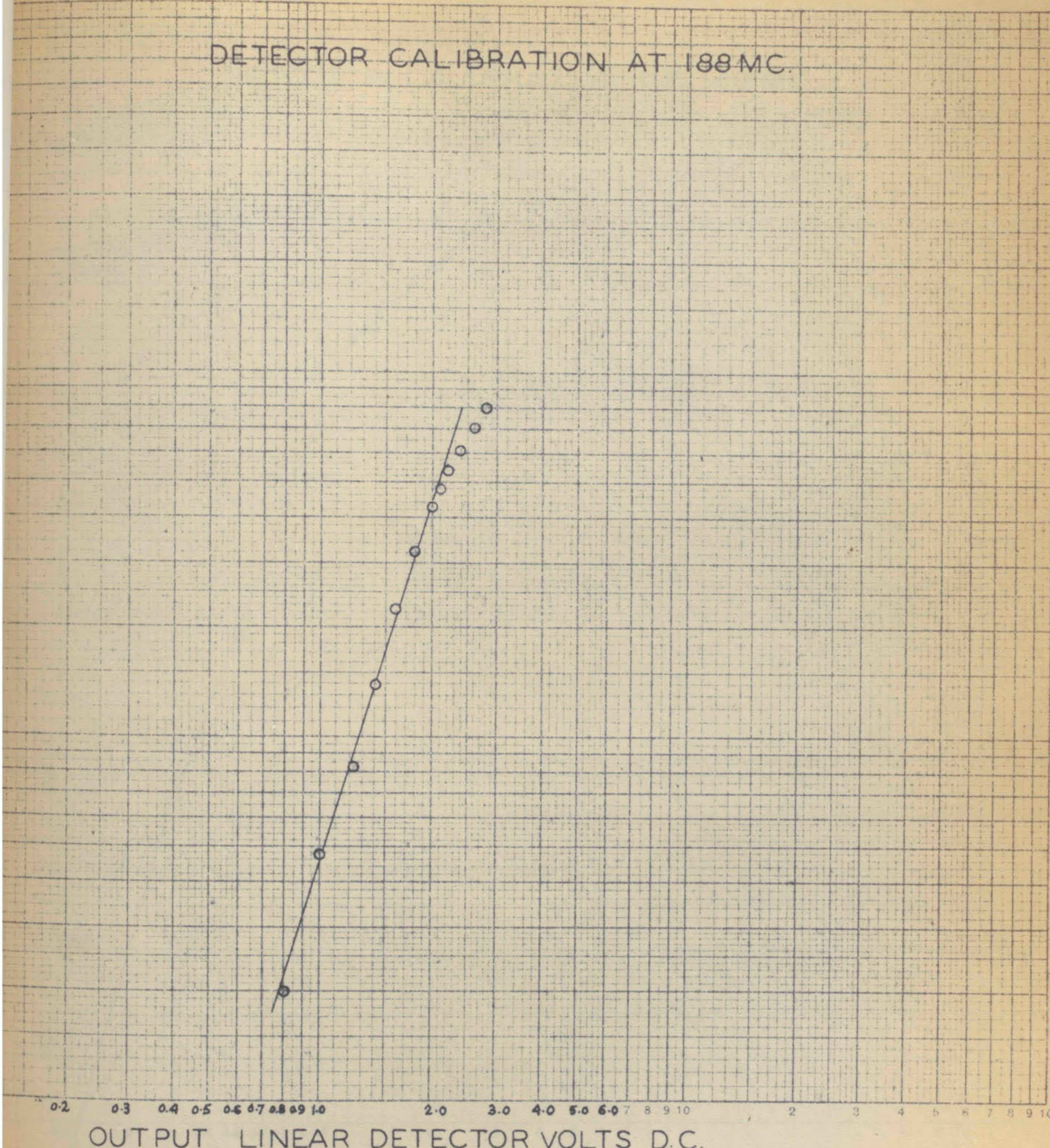
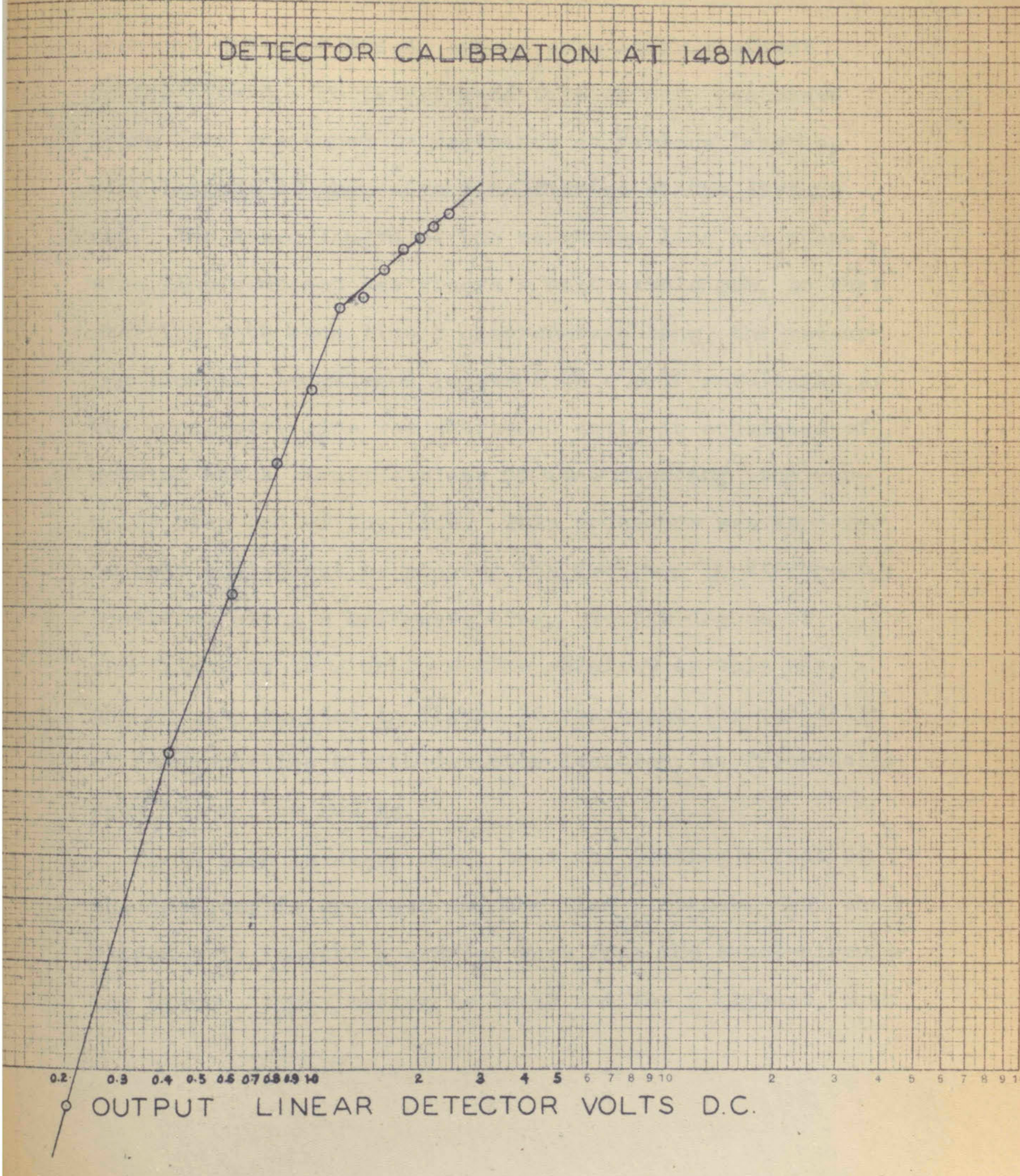
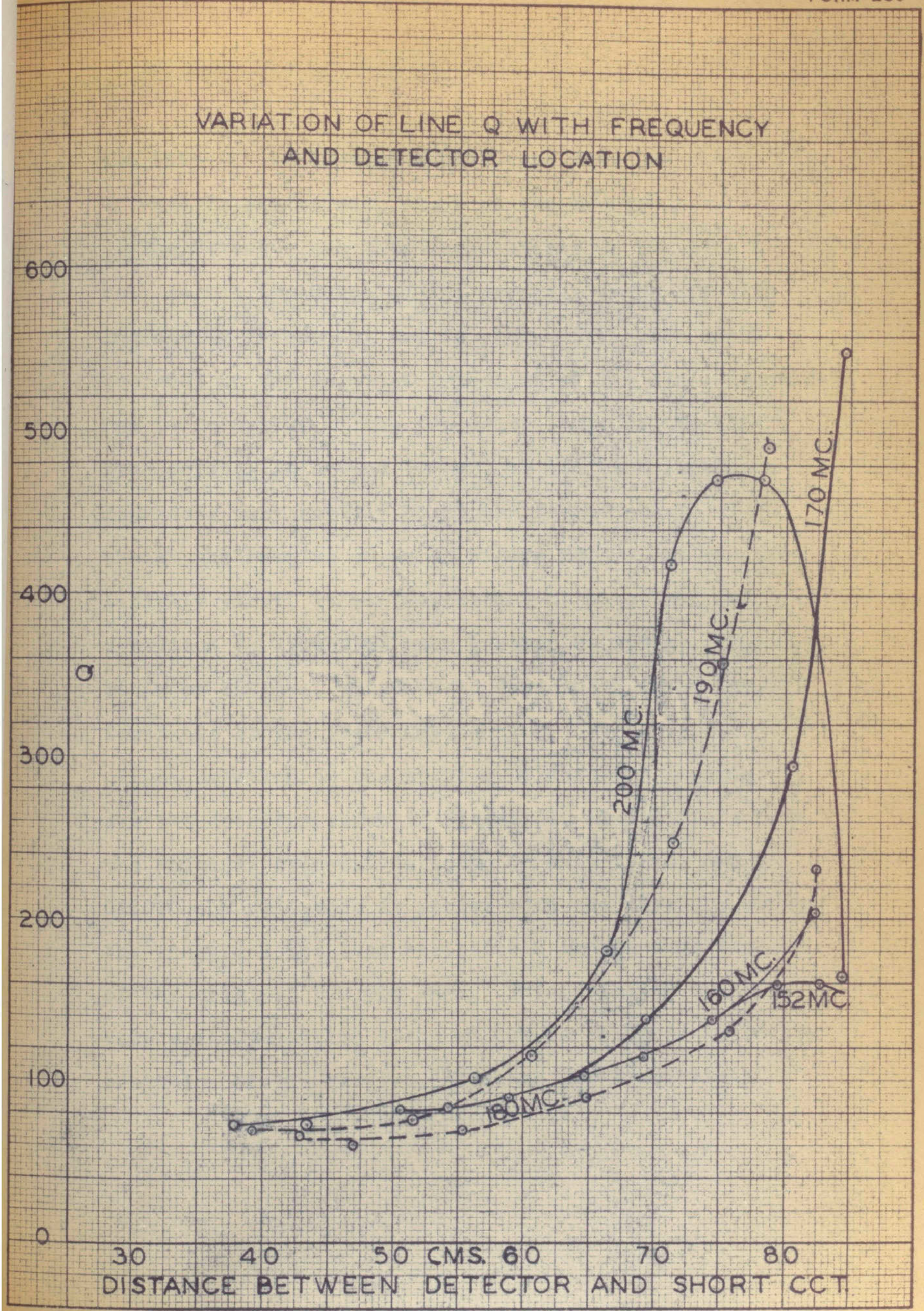

DETECTOR CALIBRATION AT 188 MC.
0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 2.0 3.0 4.0 5.0 6.0 7 8 9 10 2 3 4 5 6 7 8 9 10
OUTPUT LINEAR DETECTOR VOLTS D.C.

FIGURE 25


DETECTOR CALIBRATION AT 148 MC
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
2
3
4
5
6
7
8
9
10
2
3
4
5
6
7
8
9
10
OUTPUT LINEAR DETECTOR VOLTS D.C.

Q with calculated values, it was seen that the original open-end detector had an appreciable loading effect on the line Q. To investigate this effect, the measuring line was tuned to resonance at various lengths, with lengths of open-wire line shorted at the distant end. The total length of the measuring line and open-wire line combined was always a half-wavelength, so that starting with each line a quarter-wave long, the measuring line was lengthened in steps to a half wavelength, as the open -wire line was shortened until it consisted of nothing but a straight piece of wire shorting the normally open end of the line. This procedure was approximately equivalent to placing the detector in different positions along a half-wave line. To the degree of approximation to a uniform line obtained in this way, the results plotted in Figure 26 represent the variation of resonant-line Q with detector location, at frequencies from 150 mc. to 200 mc.

It is seen in every case, that the loading effect of the detector is greatest and the line Q lowest, when the detector is located a quarter wave from the short-circuiting piston. The loading is least and the Q highest, when


VARIATION OF LINE Q WITH FREQUENCY
AND DETECTOR LOCATION
Q
600
500
400
300
200
100
0
30
40
50 CMS. 60
70
80
DISTANCE BETWEEN DETECTOR AND SHORT CCT.
200 MC.
190 MC.
170 MC.
160 MC.
152 MC.
180 MC.

the detector is located a half wave from the short, or what is equivalent, right at the short.

The detector has this large effect on the line Q because the impedance of the crystals drops off with increasing frequency, and is apparently quite small compared to the resonant impedance of the line. According to Cornelius,[136] the frequency limit of the 1N34 type crystal is around 100 mc, as its back impedance decreases very rapidly at higher frequencies. Although the results shown in Figure 26 are not quantitatively exact, it is interesting to note that the variation in line Q is much greater at 200 mc., than it is at 152 mc., which would indicate a higher crystal impedance at the lower frequency.

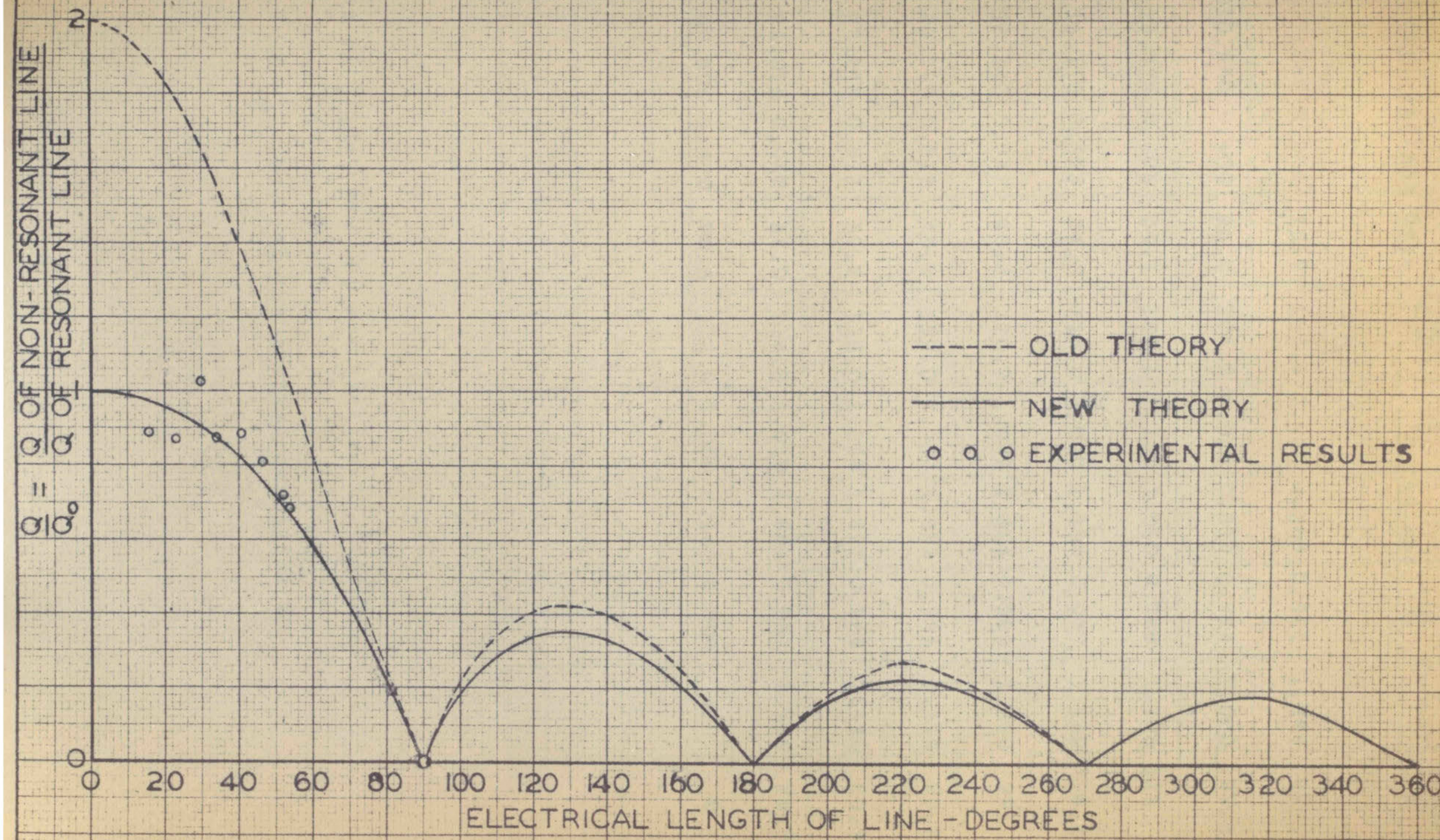
One solution to the problem of detector loading was the detector shown in Figure 11, which differs from the original open-end detector in that it has two 50,000 ohm resistors in series with the crystals across the line, and the load resistor is ten megohms instead of the original 100,000 ohms. This detector operated very satisfactorily, although due to an error in interpretation of the results, this was not immediately obvious at the time. This detector was linear, as was the original detector.

Another solution to the detector loading problem was to move the detector from the open end of the line, and connect it to the shorted end of the line, where, as indicated in Figure 26, its effect on the line Q would be the least. In this position the crystal calibration was somewhat uncertain and variable, as indicated in Figures 24 and 25; but the final answer in this case must await further investigation.

4 - 2 Confirmation of New Theory

4 - 2.1 Experimental Procedure The experimental procedure used in verifying the theoretical conclusions of part 3 - 1, consisted of tuning the measuring line to resonance at various lengths with a variable air condenser connected across the open end of the line. Lengths of silver tubing about 2 cm. long, and 0.3 cm. in diameter were soldered to the condenser terminals and used to connect the condenser to the line. The Q of the condenser, as measured on the Boonton 170A High-frequency Q-meter averaged 1250, and did not depart from this value more than 4% for any setting of the condenser. The condenser had a minimum capacity of 3 uuf., and a maximum capacity of about 40 uuf. The line length for resonance and the change in line length required to drop the response to 0.707 maximum on either side of resonance were recorded. The oscillator frequency was 188 mc., and this was continuously monitored with a General Radio 720A heterodyne frequency meter. When the line was tuned through resonance a slight change in the heterodyne tone was observed, but the shift in frequency was too small to be read on the frequency scale, and it was concluded that such a small change in frequency would have a negligible effect on the accuracy of the results. The oscillator output voltage was also monitored, and remained constant within $\pm$ 1% as the line was tuned.

<u>4 - 2.2 Q of Non-resonant Length of Line</u> The experimental results obtained in the above manner are presented in Table No. 3, along with the calculations necessary to obtain the Q of the non-resonant line. The ratio of the Q of the non-resonant line to the Q of the resonant line is plotted after the manner of Terman[83] in Figure 27. This relationship is plotted for three cases; the dashed line following the theory of Terman, the solid line following the new theory developed in this thesis, and the circles representing the values obtained.experimentally. The agreement between the experimental points and the new theory does NOT appear to be very startling in Figure 27, but this is largely due to the scale used in plotting the curves. This scale was used in order to show the agreement between the old and the new theories for line lengths greater than about 270°. The average



$\frac{Q}{Q_0} = \frac{Q \text{ OF NON-RESONANT LINE}}{Q \text{ OF RESONANT LINE}}$
2
0
0
20
40
60
80
100
120
140
160
180
200
220
240
260
280
300
320
340
360
ELECTRICAL LENGTH OF LINE - DEGREES
------ OLD THEORY
NEW THEORY
o o o EXPERIMENTAL RESULTS

FIGURE 27

LINE TUNED TO RESONANCE, WITH LOSSLESS CONDENSER   Freq. 188mc.

band width $2\Delta l$ cms.	Line Length cms.	radians	deg.	$\frac{1}{2\Delta l}$	$\frac{Q}{Q_o}$	$4\pi \frac{l}{\lambda}$ radians	sine $4\pi\frac{l}{\lambda}$	Sel. Factor	Q_{line} Q/S	$\frac{Q_{line}}{Q_o}$
0.070	7.240	0.284	15.7	103.3	0.924	0.568	0.521	1.045	99	0.884
0.100	10.330	0.406	23.2	103.3	0.924	0.812	0.724	1.060	97.6	0.871
0.105	13.375	0.525	30.1	127.3	1.137	1.050	0.868	1.105	115	1.028
0.135	15.110	0.593	34.0	112.0	1.000	1.186	0.927	1.140	98.2	0.877
0.150	18.140	0.712	40.8	120.9	1.079	1.424	0.989	1.220	99	0.884
0.174	20.790	0.816	46.8	119.0	1.062	1.632	0.998	1.319	90.2	0.806
0.200	23.210	0.911	52.2	116.0	1.036	1.822	0.969	1.440	80.6	0.720
0.210	24.015	0.942	54.0	114.1	1.019	1.884	0.951	1.490	76.6	0.684
0.360	40.305	1.570	90.6	112.0	1.000	3.140	0.000	∞	0	0

TABLE No.

Values of $\frac{Q_{line}}{Q_o}$ vs. βl are plotted in Figure 27.

Values of $\frac{Q}{Q_o}$ vs. βl are plotted in Figure 28.

deviation of the experimental points from the new theory is 6.7%, and the widest deviation 14%. Against this, the average deviation from the old theory is 40.5%, and the maximum deviation is 55%, for the experimental points; and the deviation of the new theory from the old theory is 42.6% on the average, with a maximum deviation of 50%. It should be noted also, that the experimental points are distributed on both sides of the new theoretical curve, whereas they fall consistently below the old theoretical curve. It is concluded, therefore, that the old theory is in error, and that the new expression for the Q of a non-resonant line developed in this thesis is adequate.

4 - 2.3 Q of Resonant Combination of Line and Condenser

The results in this case are also given in Table No.3, as $\frac{1}{2\Delta 1}$, and in the next column the ratio of this Q to the self-resonant Q of the line is given. The results are plotted in Figure 28 for the same three cases as previously, and in this figure the agreement between the experimental points and the new theory shows up more clearly. The average deviation from the new theory is + 2.2%, and the maximum deviation is 14%. The theoret-

FORM 200

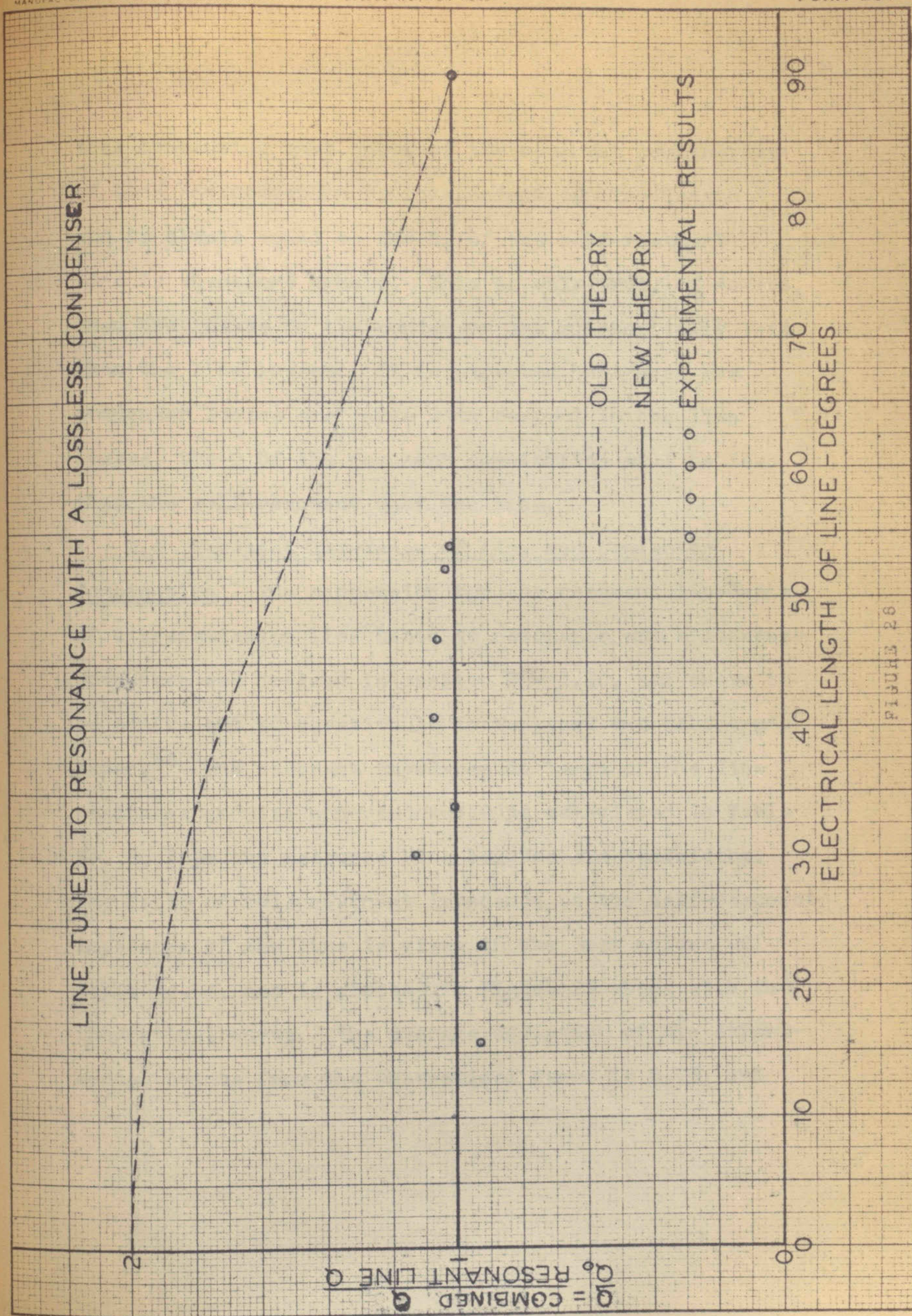


FIGURE 28

ical conclusion that the Q of the resonant combination of line and condenser is independent of line length and is always equal to the Q_0 of the self-resonant line is therefore upheld. This is only true, of course, when the losses in the condenser are substantially less than the line losses. It is expected that when the condenser losses are large with respect to the line losses, the Q of the resonant combination will be less than the self-resonant Q_0 of the line.

4 - 2.4 Resonant Impedance of Line and Condenser Combination The expression for the resonant impedance of a transmission line tuned to resonance with a lossless condenser was derived in section 3 - 1.10, and shown to be equivalent to equation (11) in Nergaard and Salzberg's paper.[56] The resonant impedance in terms of the line reactance and its non-resonant Q is calculated in Table No. 4, from the measured line lengths and bandwidths. The ratio of this resonant impedance to the self-resonant impedance of the line is given in the last column of Table No. 4, and is plotted in Figure 29 along with the theoretical curve. The average deviation of the experimental points from the theoretical curve is 5.7%, and

LINE TUNED TO RESONANCE WITH LOSSLESS CONDENSER Freq. 188mc.

band width $2\Delta l$ cm.	Line length cm.	βl degrees	Z_o ohms	$\tan\beta l$	$Z_o \tan\beta l$	Q_o $\frac{l^o}{2\Delta l}$	S Select factor	Q_{line} Q_o/S	reson. Imped. ohms	$\frac{r}{r_o}$
0.070	7.240	15.7	222	0.281	62.4	103.3	1.045	99	6170	0.196
0.100	10.330	23.2	222	0.394	87.4	103.3	1.060	97.6	8530	0.270
0.105	13.375	30.1	222	0.501	111.2	127.3	1.105	115	12800	0.406
0.135	15.110	34.0	222	0.674	149.5	112.0	1.140	98.2	14690	0.466
0.150	18.140	40.8	222	0.863	191.6	120.9	1.220	99	18980	0.602
0.174	20.790	46.8	222	1.065	236.3	119.0	1.319	90.2	21330	0.677
0.200	23.210	52.2	222	1.289	286.0	116.0	1.440	80.6	23050	0.732
0.210	24.015	54.0	222	1.376	305.5	114.1	1.490	76.6	23400	0.743

When the line is self-resonant, the impedance is calculated from

$$r_o = Z_o \frac{\sin^2\beta l}{\beta\Delta l}$$

to which $Z_o \tan \beta l \; Q_o/S$ reduces in this case.

Δl	$\beta\Delta l$	βl	Z_o	$\sin \beta l$	$\sin^2\beta l$	r_o	$\frac{r_o}{r_o}$
0.180	0.00706	90	222	1.000	1.000	31500	1.000

TABLE No.

Values of $\frac{r}{r_o}$ vs. βl are plotted in Figure 29.

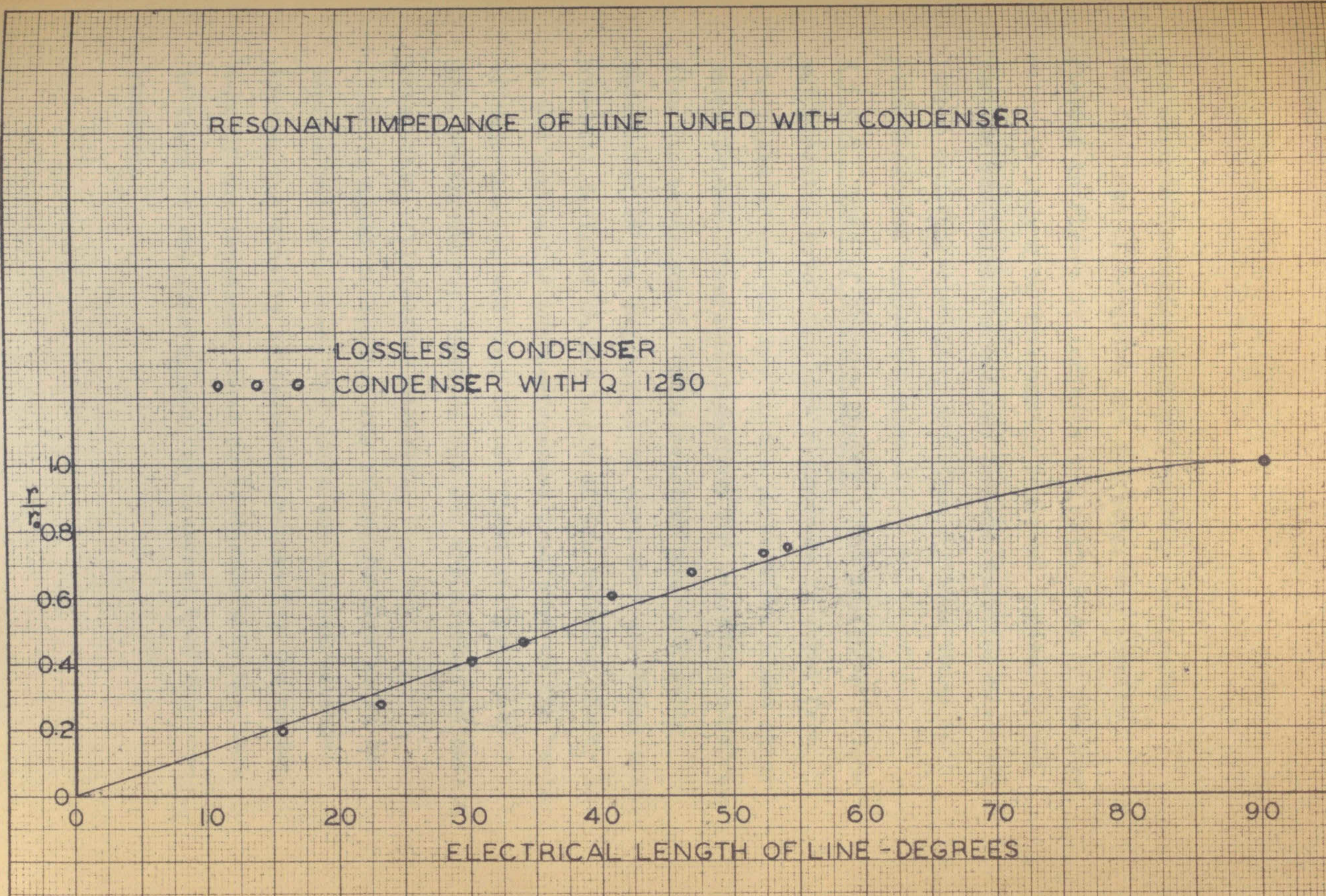

RESONANT IMPEDANCE OF LINE TUNED WITH CONDENSER
LOSSLESS CONDENSER
CONDENSER WITH Q 1250
$\frac{r}{r_0}$
1.0
0.8
0.6
0.4
0.2
0
0
10
20
30
40
50
60
70
80
90
ELECTRICAL LENGTH OF LINE - DEGREES

FIGURE 29

the maximum deviation is 14%. Besides confirming the findings of Nergaard and Salzberg, the results plotted in Figure 29 indicate the usefulness of the non-resonant line Q in determining resonant impedance, and further establish the analogy between parallel-resonant lumped constant circuits, and line sections tuned to resonance with lumped reactances.

4 - 2.5 Uses of the New Theory The design of resonant circuits involving line reactances is moved from the realm of cut and try to the point where close agreement between theoretical designs and practical construction can be expected. This research was originally undertaken at the suggestion of engineers who found that circuits designed on the basis of the existing theory in no way approached the performance predicted on the basis of that theory; and that in the final analysis they were forced to fall back on cut and try methods to obtain reasonably satisfactory results.

It is hoped to develop the theory further so as to include the case where the condenser has appreciable losses; and finally to develop a method for the measurment of any impedance tuned to resonance with the line, in terms of the line Q_o and the measured Q of the combination.

4 - 3 Improvements in Apparatus and Technique

4 - 3.1 Improvements in Accuracy The measuring line described in this thesis has been shown to provide an accuracy of from 2% to 14% when used in the manner outlined previously. The measurement of any impedance, connected to the line as a termination, is possible, using the same general procedure and breaking down the results into R and X components by the methods of Kaufmann or Chipman. The accuracy of measurement could be improved considerably if the changes recommended in the thesis were incorporated in the design of the measuring line. Silver or rhodium plating of the line conductors and the shield, and reduction in size of the polystyrene end support would undoubtedly increase the accuracy of measurements, and it is felt that an accuracy of the order of 1% or better should be attainable.

Although the only measurements theoretically necessary are the length of the line for resonance and the length of the line when the response is down 3db from the maximum, to obtain reasonable accuracy it is common practice to measure a number of points about resonance, and thus obtain the complete response or resonance curve. This procedure is time consuming, especially where a number of measurements must be made, or where measurements are being made continuously, as in production testing of high-frequency cable.

4 - 3.2 Impedance Measurements with an Oscilloscope

A much quicker and more convenient method of making measurements has been devised, but not thoroughly tested due to the lack of suitable equipment. The instruments necessary are a signal generator covering the range of frequencies at which measurements are to be made, and capable of being frequency modulated by an audio signal; and an oscilloscope to the vertical amplifier of which the measuring line detector output is connected. The signal generator is loosely coupled to the measuring line as before, and when the line length is such that the combination of the line and the unknown impedance is resonant at the mid frequency of the F.M. signal, the resonance curve of the circuit is presented on the oscilloscope. The horizontal sweep of the oscilloscope is supplied by the audio signal which is used to frequency modulate the signal generator output. On most F.M. signal generators the frequency swing or bandwidth of the F.M.

signal can be controlled, and read in kilocycles on a meter. The width of the resonance curve at the 3 db points can thus be read off the scope directly, since the length of the horizontal trace is equal to the bandwidth of the F.M. signal. A resonance curve of the measuring line tuned to resonance with a condenser at a frequency of 50 mc. was photographed on a Dumont 208B oscilloscope, and is shown in Figure 30. A Boonton 150A Frequency Modulation Generator was used to feed the line. As the measuring line is only an eighth of a wavelength long at 50 mc., and a higher-frequency F.M. signal generator was not immediately available, no comparison between impedance values measured by the F.M. and Oscilloscope method and the conventional method was made. This method of presenting the information required for impedance measurement on lines, has apparently not been used before, and should prove to be considerably more convenient and time-saving than conventional methods. It is hoped to do further work along this line, and to determine the accuracy obtainable with this method.

4 - 3.3 Resonance Curve Symmetry Check using Oscilloscope

Most expressions for the calculation of impedance

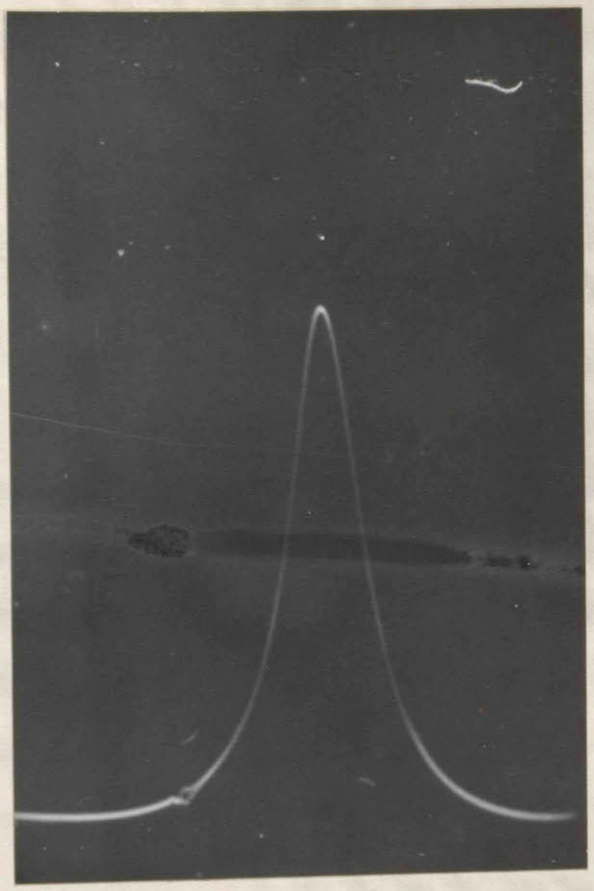


Figure 30

Resonance curve of measuring line tuned to resonance with a condenser, at 50 mc. This curve was obtained by using an F.M. signal generator to feed the line, and displaying the detector output on an oscilloscope.

values from observations made on the response curve of a resonant transmission line, involve the assumption that the resonance curve of the measuring line is symmetrical about the resonance point. This can be checked experimentally by plotting the resonance curve, and then folding the graph along a line perpendicular to the base and passing through the peak of the resonance curve. The two sides of the resonance curve should then appear superimposed.

Using the oscilloscope method of resonance curve presentation, symmetry of the response curve can be checked very simply and conveniently. If the horizontal sweep for the oscilloscope is supplied by the signal generator modulating voltage, two response curves are presented on the scope when the line is tuned to the mid frequency of the F.M. generator output. That is, the generator output swings from a frequency below resonance to a frequency above resonance, and then back again to a frequency below the resonant frequency, while the horizontal sweep of the oscilloscope moves from left to right. If the linear sweep of the scope is used, and the sweep frequency set at **twice** the modulation frequency, the two resonance curves are superimposed, one being traced in the reverse direction with respect to the other. A photograph of such superimposed resonance curves is shown in Figure 32, and it is seen that the degree of symmetry about the resonance point is quite good over most of the curve. The peaks of the two curves are not identical, and possibly there is some connection between this phenomenon and a similar one noticed in plotting resonance curves in the conventional manner. When the shorting piston was moved in one direction, the peak response was slightly higher than when the line was tuned through resonance by moving the piston in the other direction. Whether there is any connection between the two effects or not is unknown, as a satisfactory explanation of the phenomena is lacking. Figures 31 and 33 are photographs of superimposed resonance curves with the line tuned respectively below and above the mid frequency of the F.M. signal.

This method of impedance measurement would be best suited for measurements on impedances of high Q, as in the case of a low Q impedance, the complete resonance curve would not be obtained, and the 3 db points would be difficult to ascertain. The whole method will have to be investigated more thoroughly before any specific

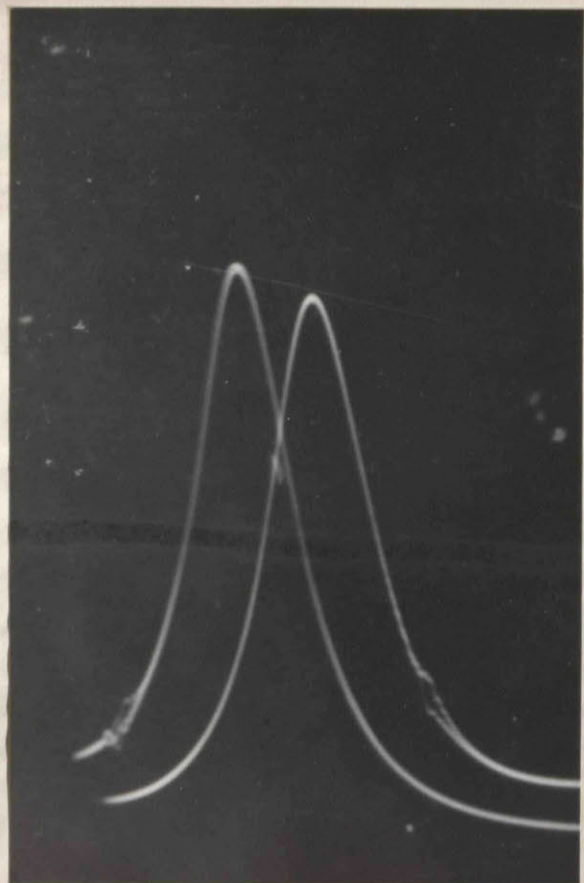


FIGURE 31

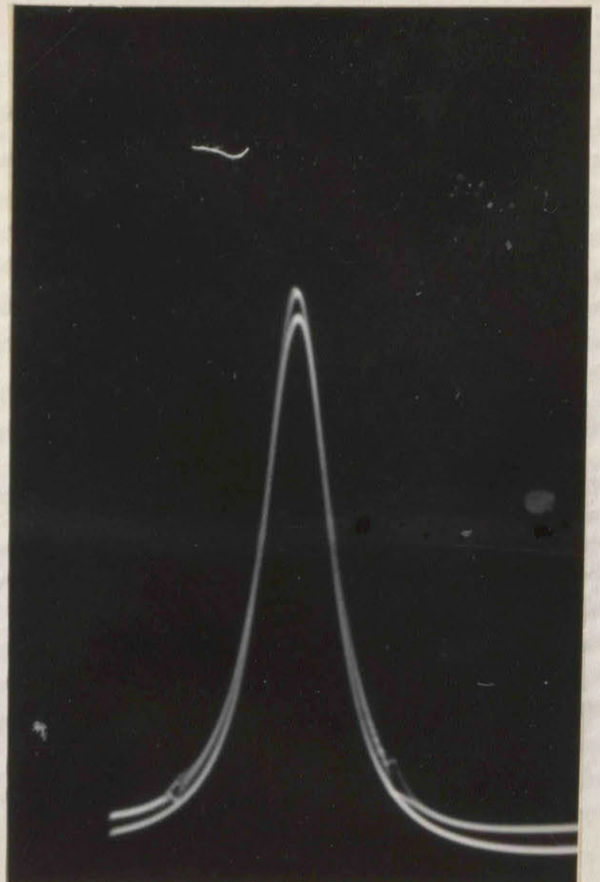


FIGURE 32

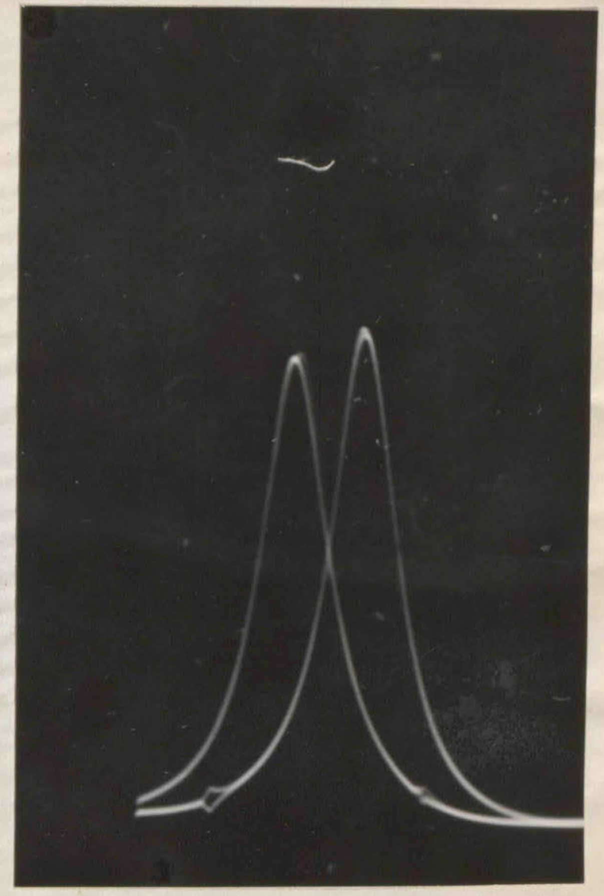


FIGURE 33

claims can be made for it, but it does seem to offer a faster and more convenient method of making impedance measurements than has hitherto been available.

General Summary

Summarizing the results of the research and experimental work involved in writing this thesis, it is suggested that the following has been accomplished:

1. A historical background has been provided, and a comprehensive survey made, of impedance measuring methods.

2. An extensive bibliography on the subject of impedance measurement has been compiled, which, it is hoped, may prove useful to those who wish to gain familiarity with the subject.

3. A working knowledge has been gained, of the problems involved in making impedance measurements at ultra-high frequencies, and of the techniques necessary to circumvent the difficulties which arise.

4. An impedance-measuring line has been designed and built, incorporating a new design of shorting disc and a convenient motor drive and indicating device for positioning the shorting piston. The accuracy obtained with this device was from 2% to 14%, but when the suggested improvements have been incorporated, the accuracy of measurement should be 1% or better. In the frequency

range from 175 mc. to possibly several thousand megacycles, any impedance can be measured. In the range from 90 mc. to 175 mc. any impedance with capacitive reactance can be measured, the measurement of impedances with inductive reactance being limited to values not far removed from Z_o, because of the length of the present line.

5. Errors have been discovered in previous work, and the accepted theory and expressions for the Q of short non-resonant lines, and of transmission line circuits, have been shown to be inaccurate. New expressions have been derived, and these have been verified experimentally.

6. Confirmation of the findings of Nergaard and Salzberg regarding the effect of neglecting the imaginary part of the characteristic impedance on short line resonant impedance calculations has been obtained. A convenient expression for the resonant impedance of a circuit consisting of a line section and a lumped reactance, in terms of the Q of the line section, has been derived; and it has been shown that this expression is analogous to that used in the case of a lumped constant parallel resonant circuit.

7. A quick and convenient method of making bandwidth measurements, and of checking the symmetry of resonance curves, has been demonstrated, using an oscilloscope and a F. M. signal generator.

BIBLIOGRAPHY

Historical References

1. S. H. Christie -

"Experimental determination of the laws of magneto-electric induction."

Phil. Trans. Roy. Soc., Vol. 123, pp. 95-142 (1883).

2. C. Wheatstone -

"An account of several new instruments and processes for determining the constants of a voltaic circuit."

Phil. Trans. Roy. Cos., Vol. 133, pp. 303-327 (1843).

3. C. W. Siemens -

"Collected Scientific Works, Vol 2, pp. 126-127.

4. J. C. Maxwell -

"A dynamical theory of the electromagnetic field."

Phil. Trans. Roy. Soc., Vol. 155, pp. 459-512 (1865).

5. Max Wein -

Ann. der Phys., Vol. 42, pp. 593-621.

6. V. Bjerknes -

"Uber Electrische Resonanz."

Wied. Ann., Vol. 55, p. 121, (May (1895).

7. P. Drude -

Wied. Ann., Vol. 61, p. 466 (1897).

8. R. von Traubenberg and B. Monasch -

"Uber die verwendung kontinuierlicher electromagnetischer schwingungen bei dampfungsmessungen."
Phys. Zeit., Vol. 8, p. 925, Dec. (1907).

9. R. Lindemann -
"Uber dampfungsmessungen mittels ungedampfter electrischer schwingungen."
Verh. der Deutsch. Phys. Ges., Vol. 11, p. 28, Jan.(1909).

10. R. Wuckel -
Ann. der Physik., Vol. 73, p. 427 (1924)

NULL METHODS

11. G. A. Campbell -
"The shielded balance."
Elec. World, Vol. 43, pp. 647-649 (1904).

12. A. C. Bartlett -
"Extension of a property of artificial lines."
Phil. Mag., Vol., 4, pp. 902-907, Nov. (1927).

13. G. Zickner -
"A bridge for the measurement of inductance and capacity."
Wireless Engineer, pp. 280-282, May (1928).

14. J. G. Ferguson -
"Shielding in high frequency measurements."
Bell System Tech. Jour., Vol. 8, p.560, July (1929).

15. W. N. Tuttle -
"Bridged-T and parallel-T null circuits for measurements at radio frequencies."
Proc. I.R.E., p. 23, Jan. (1940).

16. P. M. Honnell -
"Bridged-T measurement of high resistances at radio frequencies."
Proc. I.R.E., p. 88, Feb. (1940).

17. D. B. Sinclair -
"The twin-T ... A new type of null instrument for measuring impedance at frequencies up to 30 megacycles."
Proc. I.R.E., p. 310, July (1940).

18. D. B. Sinclair -
"A radio frequency bridge for impedance measurements from 400 kilocycles to 60 megacycles."
Proc. I.R.E., p. 497, Nov. (1940).

19. G. W. O. Howe -
"The twin-T method of measuring coil constants."
Wireless Eng., pp. 441-442, Nov. (1941).

20. G. W. O. Howe -
"The bridged-T method of measuring the constants of a coil."
Wireless Eng., pp. 135-136, April (1941).

21. R. G. Medhurst -
"High frequency resistance and self capacitance of single layer solenoids."
Wireless Eng. Feb. (1947).

22. T. J. F. Pavlasek -
"Characteristics of composition type resistors."
M. Eng. Thesis, McGill University, May (1948).

RESONANCE METHODS

23. J. H. Dellinger -
"The measurement of radio frequency resistance, phase difference, and decrement."
Proc. I. R. E., p. 27, Feb. (1919).

24. C. N. Weyl and S. Harris -
"A method of measuring at radio frequencies and equivalent series resistance of condensers intended for use in radio receiving circuits."
Proc. I.R.E., p. 109, Feb. (1925).

25. C. T. Burke -
"Substitution method for the determination of resistance of inductors and capacitors at radio frequencies."
Trans. A.I.E.E., Vol. 46, p. 482, (1927).

26. Sylvan Harris -
"An extension of the method of measuring inductances and capacities."
Proc. I.R.E., p. 516-520, March (1929).

27. A. P. Castellain -

The absorption method of capacity and inductance measurement."

Wireless Eng., pp. 81-84, Feb.(1930).

28. Hajime Iinuma -

"A method of measuring the radio frequency resistance of an pscillatory circuit."

Proc. I.R.E., pp. 537-543, March (1930).

29. E. B. Moullin -

"Notes on the distuning method of measuring the high frequency resistance of a circuit."

Wireless Eng., pp. 367-370, July (1930).

30. F. M. Colbrook and R. M. Wilmotte -

"A new method of measurement of resistance and reactance at radio frequencies."

Wireless Eng., pp. 83-84, Feb. (1931).

31. H. Iinuma -

"Resonant impedance and effective series resistance of high frequency parallel resonant circuits."

Proc. I. R. E., p. 467, March (1931).

32. T. C. MacNamara -

"A radio frequency measurement of resistance reactance and impedance."

Wireless Eng., Sept. (1935).

33. R. F. Field and D. B. Sinclair -
"A method for determining the residual inductance and resistance of a variable air condenser at radio frequencies."
Proc. I. R. E., p. 255, Feb. (1936).

34. M. Reed -
"The effect of stray capacities to ground in substitution measurements."
Wireless Eng., May (1936).

35. M. Boella -
"Direct measurement of the loss conductance of condensers at high frequencies."
Proc. I. R. E. p. 421, April (1938).

36. A. Dunford and S. E. Goodall -
"Dielectric loss measurements at radio frequencies."
Wireless Eng., pp. 662-668, Dec. (1938).

37. D. B. Sinclair -
"Parallel resonance methods for precise measurements of high impedance at radio frequencies, and a comparison with the ordinary series resonance methods."
Proc. I. R. E., p. 1466, Dec. (1938).

38. G. W. O. Howe -
"The measurement of the quality of coils."
Wireless Eng. pp. 221-223, May (1939).

39. Carl E. Smith -

"Impedance measuring instrument."

Proc. I. R. E. p. 362, Aug. (1942).

40. V. V. L. Rao -

"The Q meter and its theory."

Proc. I. R. E., p. 502, Nov. (1942).

41. S. Young White -

"Q measurements at U.H.F."

Radio, April (1943).

42. U. Zelbstein -

"Measurements of Q."

Toute La Radio, Vol. 14, pp. 121-123, Mar. and Apr. (1947).

TRANSMISSION LINE METHODS

43. C. Englund -

"Wave limit of oscillations."

Proc. I. R. E., Vol. 15, p. 920. Nov. (1927).

44. J. W. Labus -

"Measurement of resistances and impedances at high frequencies."

Proc. I. R. E. pp, 452-460, March (1931).

45. E. J. Sterba and C. B. Feldman -

"Transmission lines for short-wave radio systems."

Proc. I. R. E., Vol. 20, pp. 1163-1202. July (1932).

46. F. W. Chapman -

"The generation of centimetre waves."

Wireless Eng., Sept. (1932).

47. O. Schmidt -

"Das paralleldrahtsystem als messinstrument in der kurtzwellentechnik."

Hoch. freq. und Elec. akust., Vol. 41, pp. 2-16 Jan. (1933).

48. Ronald King -

"Amplitude characteristics of coupled circuits having distributed constants."

Proc. I. R. E., p. 1142, Aug. (1933).

49. W. L. Barrow -

"Measurement of radio frequency impedance with networks simulating lines."

Proc. I. R. E., p. 807, July (1935).

50. Ronald King -

"Electrical measurements at untra-high frequencies."

Proc. I. R. E., p. 885, Aug. (1935).

51. Leon S. Nergaard -

"Electrical measurements at wavelengths of less than two meters."

52. W. Hempel -

"On the application of a line as a measuring instrument in the region of the decimetre waves."

E. N. T., Vol. 14, p.33, (1937).

53. H. Bruckmann -

H. F. T. Vol. 51, p. 128, (1938).

54. Leon S. Nergaard -

"A survey of ultra-hugh frequency measuremets."

R. C. A. Rev. Vol. 3, pp. 156-197, Oct. (1938).

55. R. A. Chipman -

"A resonance curve method for the absolute measurement of impedance at frequencies of the order of 300 mc/sec."

Journal of Applied Physics, Vol. 10, pp. 27-38, Jan. (1939).

56. L. S. Nergaard and Bernard Salzberg -

"Resonant impedance of Transmission lines."

Proc. I. R. E., p. 579, Sept. (1939).

57. H. Kaufmann -

Hoch. Freq. und Elek. Akus., Vol. 53, p. 61, (1939).

58. F. Hamburger Jr. and C. Frank Miller -

"The measurement of coil reactance in the 100 megacycle region."

Proc. I. R. E., p. 475, Oct. (1940).

59. J. R. Regazzini -

"Transmission lines at ultrahigh frequencies."

A. I. E. E. Journal, pp. 159-167. April (1943).

60. P. J. Kibler -

"A note on impedance measurements at high frequencies with special reference to impedance matching."

Proc. I. R. E., p. 354, June (1944).

61. L. Essen -

"The measurement of balanced and unbalanced impedances at frequencies near mc/s, and its application to the determination of the propagation constants of cables."

Inst. Elect. Eng. Jour., Vol. 91, part iii, June (1944).

62. R. C. Paine -

"Graphical solution of voltage and current distribution and impedance of transmission lines."

Proc. I. R. E., p. 686, Nov. (1944).

63. F. Jones and R. Sear -

"A resonance line method for the measurement of characteristics in the decimetre wave range."

Wireless Eng., pp. 512-520, Nov. (1944).

64. F. Jones and R. Sear -

"Testing high frequency cables."

Wireless Eng., pp. 571-583, Dec. (1944).

65. Willis Jackson -

"High Frequency Transmission Lines."

Methuen and Co. Ltd., London. (Monograph).

66. Discussion on

"The measurement of balanced and unbalanced impedances at frequencies near 500 mc/s, and its application to the determination of the propagation constants of cables."

Jour. I. E. E., Vol. 92, part iii, pp. 218-225, Sept. (1945).

67. J. C. Simmonds -

"Apparatus for measurements on balanced-pair high frequency cables in the range 10-200 mc/s."

Jour. I. E. E., Vol. 92, part iii, pp. 282-286, Dec. (1945).

68. J. M. van Hofweegon -

"The measurement of impedances particularly on decimetre waves."

Phillips Tech. Review, Vol. 8, pp. 16-24, Jan. (1946).

69. D. D. King -

"Impedance measurement on transmission lines."

Proc. I. R. E., pp. 509-514, May (1947).

<u>TRANSMISSION LINES (GENERAL)</u>

70. G. C. Southworth -

"Electrical measurements at ultra-radio frequencies."

Radio Review, Vol 2, pp. 25-30. (1921).

71. Patent - Standard Telephones and Cables - Oct. 1926.
"Handling ultra-high frequencies."
Wireless Eng., p. 234, April (1928).

72. R. G. Allen -
"The establishment of formulae for the self-inductance
of single-turn circuits of various shapes."
Wireless Eng., p. 261, May (1928).

73. Patent - S. G. S. Dicker - June 1929.
"High frequency generators."
Wireless Eng., June, (1929).

74. Patent - H. E. Hollmann - July, 1929.
"Short-wave oscillators."
Wireless Eng., Feb. (1932).

75. Patent - H. E. Hollmann - Aug. 1929.
"Short-wave generators."
Wireless Eng., March (1932).

76. R. L. Smith-Rose and J. S. McPetrie -
"Experimental transmitting and receiving apparatus
for ultra-short waves."
Wireless Eng., pp. 532-542, Oct. (1929).

77. Patent - Marconi's Wireless Telegraph Co. Ltd.,
June 1930.
"High-frequency feedlines."
Wireless Eng., March (1933).

78. Patent - Marconi's Wireless Telegraph Co. Ltd.
and E. W. B. Gill. Aug. 1930.
"Short-wave generators."
Wireless Eng., June (1932).

79. Patent - Marconi's Wireless Telegraph Co. Ltd.
Sept. 1930.
"Short-wave amplifiers."
Wireless Eng., Oct. (1932).

80. Patent - Marconi's Wireless Telegraph Co. Ltd.
and O. S. Puckle.
"High-frequency amplifiers."
Wireless Eng., June (1933).

81. Patent - The M-O Valve Co. Ltd. and
E. C. S. Megaw. Jan. 1932.
"Short-wave generators."
Wireless Eng. Oct. (1933).

82. J. G. Brainerd -
"Inductance at high frequencies and its relation
to the circuit equation."
Proc. I. R. E., p. 395, March (1934).

83. F. E. Terman -
"Resonant Lines in Radio Circuits."
Trans. A. I. E. E. July (1934).

84. L. E. Renkema -

"Transmission lines at very high frequencies."

Trans. A. I. E. E., Vol. 57, pp. 104-107, Feb. (1938)

and Vol. 56, pp. 1002-1011, Aug. (1937).

85. Bernard Salzberg -

"On the optimum length for transmission lines used as circuit elements."

Proc. I. R. E., p. 1561, Dec. (1937).

86. M. Reed -

"Coaxial and balanced transmission lines; some uses at high radio frequencies."

Wireless Eng., pp. 414-422, Aug. (1938).

87. Edmund A. Laport -

"Open-wire radio-frequency transmission lines."

Proc. I. R. E. p. 271, June (1943).

88. C. G. Brennecke -

"Equivalent T and Pi sections for the quarter-wave-length line."

Proc. I. R. E., p. 15, Jan. (1944).

89. F. M. Colebrook -

"Transmission line theory."

Wireless Eng., pp. 167-174. Apr. (1944).

90. A. Bloch -

"Loss-less transmission lines."

Wireless Eng., pp. 161-166, April, (1944).

91. A. Bloch -

"Parallel transmission lines."

Wireless Eng., pp. 280-281, June (1944).

92. G. W. O. Howe -

"The nonreflecting termination of a concentric line."

Wireless Eng., pp. 409-411, Sept. (1944).

93. H. Salinger -

"The quarter-wave step-up transformer."

Proc. I. R. E., p. 553, Sept. (1944).

94. G. W. O. Howe -

"Resonance in quarter-wave lines."

Wireless Eng., pp. 509-511, Nov. (1944).

95. E. N. Dingley Jr. -

"The theory of transmission lines."

Proc. I. R. E., p. 118, Feb. (1945).

96. F. C. Isely -

"A new approach to tunable resonant circuits for the 300-3000 mc. frequency range."

Proc. I. R. E., pp. 1017-1022, Aug. (1948).

REFERENCE TEXTS - R. F. MEASUREMENTS

97. E. B. Moullin -

"The theory and practice of radio-frequency measurements."

98. A. Hund -

"High-frequency measurements."

99. A. Campbell and E. C. Childs -

"The measurement of inductance, capacitance and frequency."

MacMillan and Co. Ltd., London.

100. H. A. Brown -

"Radio-frequency electrical measurements."

101. Various authors -

"Radio at ultra-high frequencies."

R. C. A. Institute Press.

102. W. C. Michaels -

"Advanced electrical measurements."

D. Van Nostrand Co., N. Y.

103. L. Hartshorn -

"Radio-frequency measurements."

John Wiley and Sons, N. Y.

104. J. R. Meagher and H. J. Markley -

"Practical analysis of ultra-high frequency

transmission lines, resonant sections, resonant cavities, wave guides."

MISCELLANEOUS REFERENCES

105. A. E. Kennelly -
"The impedances, angular velocities, and frequencies of oscillating current circuits."
Proc. I. R. E., p. 47, Feb. (1916).

106. A. E. Kennelly and H. A. Affel -
"Skin effect resistance measurements of conductors at radio frequencies up to 100 kc."
Proc. I. R. E., p. 523, Dec. (1916).

107. Carl Englund -
"Note on radio-frequency measurements."
Proc. I. R. E., p. 326, Aug. (1920).

108. Bur. Stds. Sci. paper No. 420 -
"High frequency resistance of inductance coils."
Feb. 24, (1922).

109. J. H. Morecroft -
"Resistance and capacity of coils at radio frequencies."
Proc. I. R. E., p. 261, Aug. (1922).

110. F. W. Grover -
"Formulas and tables for the design and calculation

of single layer coils."

Proc. I. R. E., p. 193, April, (1924).

111. P. O. Pedersen -

"An electrometer method for the measurement of R. F. resistance."

Proc. I. R. E., p. 215, April (1925).

112. W. W. Brown and G. E. Love -

"Designs and efficiencies of large air core inductances."

Proc. I. R. E., p. 755, Dec. (1925).

113. S. Butterworth -

"Effective resistance of inductance coils at radio frequencies."

Wireless Eng., (1926).

114. V. I. Bashenoff -

"Abbreviated method of calculating the inductance of irregular plane polygous of round wire."

Proc. I. R. E., p. 1013, Dec. (1927).

115. H. A. Wheeler -

"Simple inductance formulas for radio coils."

Proc. I. R. E., pp. 1398-1400, Oct. (1928).

116. F. W. Grover -

"The calculation of the inductance of single-layer coils and spirals wound with wire of large cross-section."

Proc. I. R. E., pp. 2053-2063, Nov. (1929).

117. A. J. Palermo and F. W. Grover -
"A study of high-frequency resistance of single-layer coils."
Proc. I. R. E. p. 2041, Dec. (1930).

118. A. T. Starr -
"An aperiodic impedance measuring set."
Wireless Eng., June (1932).

119. J. Steffensen -
"Simplifying the use of the Starr impedance measuring set."
Wireless Eng., Sept. (1932).

120. A . J. Palermo -
"The effect of displacement currents on the high-frequency resistance of circular single-layer coils."
Proc. I. R. E., p. 1807, Nov. (1932).

121. Hideo Seki -
"A new ABAC for single layer coils."
Wireless Eng., Jan. (1933).

122. W. Baggally -
"Resonant functions."
Wireless Eng., Aug. (1935).

123. F. E. Terman -
"Some possibilities for low loss coils."

Proc. I. R. E., p. 1069, Sept. (1935).

124. J. Groszkowski -

"The temperature coefficient of inductance."

Proc. I. R. E., p. 448, April, (1937).

125. F. C. Williams-

"The modulation response and selectivity curves ofa resonant circuit loaded by a diode rectifier."

Wireless Eng., pp. 189-197, April (1938).

126. F. C. Williams -

"The properties of a resonant circuit loaded by a complex diode rectifier."

Wireless Eng., pp. 600-611, Nov. (1938).

127. E. B. Moullin -

"The temperature coefficient of inductances for use in a valve generator."

Proc. I. R. E., p. 1365, Nov. (1936).

128. A. Fairweather and F. C. Williams -

"The variable Q amplifier."

Wireless Eng., pp. 57-66, Feb. (1939).

129. G. L. Pearson -

"Thermistors, their characteristics and uses."

Bell Lab., Record, pp. 106-111, Dec. (1940).

130. L. I. Farren and R. S. Rivlin -

"Stray Capacitances."

Wireless Eng., pp. 313-323, Aug. (1941).
131. R. L. Nielsen -
"Charts for simplifying high-impedance measurements with the R.F. bridge."
Proc. I. R. E., p. 372, July, (1943).
132. S. A. Schelkunoff -
"Representation of impedance functions in terms of resonant frequencies."
Proc. I. R. E., p. 83, Feb. (1944).
133. J. E. Tweedale -
"Thermistors....they perform countless circuit control jobs."
Oscillator, Dec. (1945).
134. J. Walker -
"Thermistors at high frequencies."
Wireless Eng., p. 28, Jan. (1947).
135. W. Rosenberg -
"Thermistors"
Electronics Eng. Vol. 19, pp. 185-187, June (1947).
136. E. C. Cornelius -
"Germanium crystal diodes."
Electronics, pp. 118-123, Feb. (1946).
137. M. Apstein and M. Joffe -
"Tuned circuit design for U. H. F. "

QST. Vol. 30, pp. 13-16, 106, Feb. (1946).

138. L. R. Quarles -

"Transmission lines as resonant circuits."

Communications, Vol. 26, pp. 22, 52, May (1946).

139. F. C. Everett -

"Tuned circuits for U.H.F. and S.H.F. bands."

Communications, Vol. 26, pp. 19-21, 51, June (1946).

140. A. H. Meyerson -

"V.H.F. Coil design."

Communications, Vol. 26, pp. 46-47, 50, June (1946).

141. A. H. M. Arnold,

"The inductance of wires and tubes."

Jour. I. E. E., part 1, Vol. 94, pp. 116-118,

Feb. (1947).

142. M. V. Callendar -

"Q of solenoid coils."

Wireless Eng., Vol. 24, p. 185, June (1947).

143. W. C. Hollis -

"Design of transmission line tank circuits."

Electronics, Vol. 20, pp.130-134, May (1947).

144. A. W. Simon -

Universal coil design."

Radio, Vol. 31, pp. 16-17, Feb. and Mar. (1947).

145. Patrick Conley -

"Impedance Measurements with open-wire lines."

Technical report No. 35, Cruft Laboratory,

Harvard University, Cambridge, Mass.

146. W. L. Everitt -

"Communication Engineering."

McGraw-Hill Book Co.

147. Curtis -

Bell System Tech. Journal, Vol. 15, p. 248, (1936).