Novel optimization models for surface and underground mine planning

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Abstract

Mine planning and optimization affect efficiency, profitability and productivity of operations significantly. Low commodity prices, high resource degredation maintenance costs and high fixed infrastructure costs necessitate the use of optimal decision making tools for mining companies to make profit. All mines have different characteristics and planning phases. In this research, different optimization problems that suit various mining techniques and planning stages are studied. In essential, there are two types of mining: surface mining and underground mining. Surface mining operations are generally long-term because overburden must be removed to access the profitable orebody. This requires strategic long-term planning at the feasibility stage. The first publication in the scope of this research focuses on long-term surface mine planning with environmental considerations. The provided solution optimizes the problem using mixed integer linear programming (MILP). When operation starts and bench sectors are mined on a daily basis, the need for short term planning arises. The second publication addresses the dig-limit optimization problem, which is an important part of short-term planning. With the proposed MILP optimization method, the ore–waste boundaries are delineated with the equipment size constraints. Although underground mining also starts with exploration and resource estimation/simulation stages, the problems that need to be addressed are very different from surface mining techniques and it has its own unique challenges. Special focus is given to the sub-

level stoping underground mining technique. Stope optimization is a complex problem, comprised of two sub-problems: stope layout optimization and stope sequencing. MILP formulations of stope layout optimization are impractical because of the large size of the problem. In the third and fourth manuscripts, two different heuristic stope layout optimization algorithms are presented where the former uses a clustering heuristic to identify stopes with high grade concentration and the latter uses a greedy heuristic based on dynamic programming to solve the sub-problems and explore the promising stope combinations. Fifth manuscript tailors the greedy heuristic algorithm to polymetallic mines with pillars. Both heuristic approaches are shown to be near-optimal through comparing with developed novel MILP formulations case studies in smaller problem instances. When the stope layout is finalized, the sequence can be optimized to yield the optimal project value. In the sixth and final manuscript within the scope of this research, the stope sequencing problem is formulated in MILP. To account for risk emerging from geological uncertainties, chance constrained programming is implemented. This approach maximizes the expected net present value of the operation while minimizing the deviations from the expected value due to ore grade uncertainty. It focuses the search on a unique direction based on the specified desired project risk level.

Résumé

La planification et l'optimisation des mines affectent de manière significative l'efficacité, la rentabilité et la productivité des opérations. La faiblesse des prix des produits de base, les coûts élevés de maintenance de la dégradation des ressources et les coûts élevés des infrastructures fixes nécessitent l'utilisation d'outils de prise de décision optimaux permettant aux sociétés minières d'avoir un profit plus important. Toutes les mines ont des caractéristiques et des phases de planification différentes. Dans cette recherche, différents problèmes d'optimisation adaptés à diverses techniques d'exploitation minière et à différentes étapes de planification sont étudiés. Essentiellement, il existe deux types de mines : les mines à ciel ouvert et les mines souterraines. Les opérations d'extraction en surface sont généralement à long terme, car les morts-terrains doivent être enlevés pour accéder au gisement rentable. Cela nécessite une planification stratégique à long terme au stade de la faisabilité. La première publication dans le cadre de cette recherche se concentre sur la planification à long terme des mines de surface avec des considérations environnementales. La solution fournie optimise le problème en utilisant la programmation linéaire mixte (MILP). Lorsque l'exploitation commence et que les secteurs de référence sont exploités quotidiennement, le besoin d'une planification à court terme se présente. La deuxième publication aborde le problème de l'optimisation de la limite du creusage, qui constitue une partie importante de la planification à court terme. Avec la

méthode d'optimisation MILP proposée, les limites de la pratique du minerai et des déchets sont délimitées par les contraintes de taille de l'équipement. Bien que l'exploitation souterraine commence également par les étapes d'exploration et d'estimation/simulation des ressources, les problèmes à résoudre sont très différents des techniques d'exploitation en surface et présentent des défis uniques. Une attention particulière est accordée à la technique d'extraction souterraine par le sous-niveau. L'optimisation des chantiers est un problème complexe, composé de deux sous-problèmes : l'optimisation de la disposition des chantiers et le séquençage des chantiers. Les formulations MILP de l'optimisation de la disposition du chantier sont peu pratiques en raison de l'ampleur du problème. Dans les troisième et quatrième manuscrits, deux algorithmes d'optimisation heuristique de la disposition des chantiers sont présentés. Le premier utilise une heuristique de regroupement pour identifier les arrêts à forte concentration et le second utilise une heuristique gloutonne basée sur la programmation dynamique pour résoudre les problèmes et explore les combinaisons de chantiers prometteurs. Le cinquième manuscrit adapte l'algorithme heuristique gloutonne aux mines poly-métalliques avec des piliers. Les deux approches heuristiques se sont avérées presque optimales en comparant avec de nouvelles études de cas de formulations MILP développées dans des cas plus petits. Lorsque la disposition du chantier est finalisée, la séquence peut être optimisée pour obtenir la valeur optimale du projet. Dans le sixième et dernier manuscrit, dans le cadre de cette recherche, le problème du séquençage du chantier est formulé dans MILP. Pour prendre en compte les risques découlant des incertitudes géologiques, un algorithme contraint par le hasard est mis en œuvre. Cette approche maximise la valeur actuelle nette de l'opération tout en minimisant les écarts par rapport à la valeur attendue en raison de l'incertitude de la teneur en minerai. Il concentre la recherche sur une direction unique basée sur le niveau de risque de projet souhaité.

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Contribution of Authors

The PhD candidate Yuksel Asli Sari is the primary author of all manuscripts contained within this thesis. The candidate's supervisor Professor Mustafa Kumral is the other contributor of all manuscripts.

Sari, Y. A., & Kumral, M. (2018). A landfill based approach to surface mine design. *Journal of Central South University, 25*(1), 159-168.

Sari, Y. A., & Kumral, M. (2018). Dig-limits optimization through mixed-integer linear programming in open-pit mines. *Journal of the Operational Research Society, 69*(2), 171-182.

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Sari, Y. A., & Kumral, M. (2018). Risk-based stope sequencing optimization for underground mines through chance-constrained programming. *Under review.*

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Abbreviations

CCP Chance-constrained programming

FGC Feasibility grade control

MILP Mixed integer linear programming

NPV Net present value

SA Simulated annealing

SMU Selective mining units

SOS2 Type-two special ordered sets

WIS Weighted interval scheduling

PDM Probabilistic decision making

1 Introduction

1.1 Overview

Mining operations need to make many long-term, medium-term and short-term strategic decisions. These decisions influence the profitability drastically because mines have very high initial and operational costs. If not managed well, the companies might profit less or have losses. Although traditional approaches to mine management exist, with advancement of computer technology it is possible to optimize the outcomes. Therefore, computerized decision aid tools must be coupled with the fundamental knowledge to obtain best results.

Mining comprises of complex problems with many constraints and uncertainties. In order to perform optimization on a computer, the problem should be defined and communicated clearly. Omitting or simplifying the complexities often renders the results impractical. The problems are communicated through mathematical models and algorithms. This thesis consists of manuscripts that solve mine planning problems through computerized techniques.

Mine planning aims to maximize the net present value (NPV) of the mine by determining the portion of the orebody to be extracted and the time of extraction of each sub-portion. Meanwhile, all mine stability and production requirement constraints should be satisfied. All mine planning approaches commence by obtaining samples from the orebody through drill or blast holes. By analyzing these samples, information regarding the ore grade can be obtained. To facilitate the following computations, the mine deposit is conceptually divided into fragments called mining blocks. Mining blocks are typically rectangular and of equal dimensions throughout. From the obtained samples, the entire orebody is estimated or simulated. As a result, an estimated/simulated grade is assigned to each mining block. Majority of the mine planning techniques considers mining blocks are decision variables.

Mine planning problems can be considered in two distinct categories: surface mine planning and underground mine planning depending on the mining technique. Surface mine planning problem consists of determination of mining blocks to extract and their routing to the appropriate destination as well as the extraction time. Possible routes of a block are ore processing plants, stockpiles or waste dumps. Surface mine planning can be further categorized to long-term planning, medium-term and short-term planning. Long-term planning refers to designing the plan of the entire deposit over the mine life. The information is obtained from drill holes and the mining blocks are relatively large. The main concerns in long-term planning include considering the access constraints of blocks and meeting the production targets. Medium-term planning makes decisions in mine-specific management/directory level. It is concerned with putting long-term plans in practice. Short-term planning refers to designing a plan that usually spans several days or weeks. More information is obtained from blast-holes and thus it is less error prone and more reliable. Mining block size used for short-term planning is smaller and therefore a block is called selective mining unit (SMU). SMUs allow more definition in terms of grade variation throughout the orebody and thus permit more precise planning. Short-term mine extraction is constrained by equipment size. Consecutive SMUs that matches the size of an excavator arm must be routed to the same destination. Determination of ore – waste boundaries on a bench sector is called dig-limit optimization.

In underground mining, highly selective production methods exist where everything that is mined is processed. Thus, it contains slightly different constraints depending on the mining technique used. Optimization is particularly important for underground mines. The prominent reasons that make optimization crucial for the planning section of the underground mine extraction project are the following: (1) Underground mining has a high fixed infrastructure cost especially compared to open pit mining. However, open pit mining requires significant extraction of waste whereas underground mining can be much more selective and can avoid extracting the waste material if planned accordingly. (2) Underground mines have resource degradation problem, which can be defined as the deterioration of environment, working conditions, slopes and openings in a mine over time. In time, operation costs increase inevitably because of this phenomenon. Planning accordingly to finance continuous development is a clever strategy because if dynamic improvement is ignored, the operation sustainability can be jeopardized. (3) Commodity prices have been decreasing and a significant recovery is not expected soon. With lower profit per mined ore, it is very important to make decisions rationally. This is not only important for mining industry but also for Canada's economy, which is resource based. Mining industry is particularly important for Canadian economy as 57% of worldwide public mining enterprises are listed on the Toronto Stock Exchange (TSX) and TSX-Venture Exchanges.

In this thesis, the primary focus in underground mine planning is optimizing the sublevel stoping method. Various methods of optimizing the stope layout is explored. From the obtained stope layout plan, stope sequencing optimization is performed.

1.2 Research Motivation

Mine planning is recognized as a significant value added process to mining operations. Mining companies have a mine planning unit in their organization and there mine software industry is very competitive. However, due to problem complexity and required computational time, the current optimization tools used in the industry has room to improve. Furthermore, new challenges in mining (e.g., environmental requirements, short-term quality requirements) and developments in operations research area bring new opportunities. This research proposes to incorporate new challenges or new methods of application into several mine optimization problems. The overall research objective is to develop novel computerized optimization approaches for various mine planning problems and demonstrate the extension of existing optimization methods to suit specific mine needs.

1.3 Research Objectives

1. Include internal dumping option into surface mining block sequencing in MILP,

2. Propose a new dig-limit optimization model using MILP for short-term planning,

3. Develop a fast and efficient heuristic for stope layout optimization for sub-level stoping method,

4. Investigate a cost effective method for selecting sublevel locations,

5. Add pillar requirements into stope layout optimization,

6. Explore a stope limit formulation for stoping methods,

7. Formulate the stope sequencing problem in MILP and attempt to relax the problem constraints for obtaining faster solutions,

8. Model grade uncertainty in stope sequencing problem through chance-constrained programming.

1.4 Scope of Research

The main objective of this thesis is to develop novel mine planning approaches that perform layout planning and/or sequencing for various mining problems from the available financial, production, block grade and other mine specific information. This includes long-term and short-term surface mine planning and underground stope layout planning and optimization. The optimization process aims to add value to the operation, which can be economic value, environmental compliance, better adaptation to equipment limitations and other external conditions or safety-oriented design.

The optimization models mainly focus on optimizing the economic returns of the operation given the available information while considering mine stability and production target constraints. For surface mines, mine stability is achieved through maintaining the slope design and for sublevel stoping mines, through settling the stope and pillar sizes provided by a mining or geotechnical engineer. The optimized resulting plan should also be revised by an engineer before the application. Due to uncertainty in mining operations, the actual results may be different than expected results.

1.5 Original Contributions

In the authored manuscripts, different mine planning optimizations are performed through computerized techniques. In the first manuscript, a new MILP formulation was introduced that optimizes the planning and scheduling of surface mines while landfilling the waste to the areas of the mine that are fully excavated. This way, production voids created in early years of mining are used for waste landfilling in late years of production. In the model, in addition to external dumping, a landfilling option within same pit is proposed for mine design optimization. The formulation consists of maximization of the NPV of the mining project under the constraints of access, landfill waste handling, mining and processing capacities. With the proposed approach, material handling costs decrease while the environmental compliance increases due to less external waste quantity. The second manuscript optimizes the dig-limits, which is a part of short-term planning. With this paper, a MILP formulation was introduced for the first time for dig-limit optimization. Furthermore, the dig-limit optimization problem was shown to be NP-hard. The approach was compared to a ore–waste boundaries drawn manually and shown to exceed in profit. The proposed approach is practical and has potential to increase the value of operation.

The rest of the manuscripts focus on underground mine planning. The third part of the research produced a new heuristic stope layout optimization algorithm. The proposed algorithm identifies ore-concentrated regions of the deposit and prioritizes their extraction through a heuristic clustering technique. The size of the cluster and the search related parameters of the heuristic is defined by the user. This approach is able to generate near-optimal stope layouts in a computationally effective manner.

Following this research, an alternative approach of stope layout planning was proposed in the fourth manuscript. In this approach, a novel greedy heuristic based on dynamic programming is used. The algorithm identifies the recurring subproblems and memoizes their results to decrease the solution time. The only heuristic parameter was introduced to further decrease the solution time and limit the memory usage. It is optional and for smaller problems, the heuristic can be lifted and the approach can be used as an exact method. The fifth manuscript extended this approach to poly-metallic mines with pillars. Additionally, a sublevel determination algorithm was proposed. Furthermore, a MILP formulation that finds the ultimate stope limits was introduced.

Finally, in the last manuscript, a new MILP formulation was developed to solve the stope sequencing problem that accounts for ore grade uncertainty. Chance-constrained programming was used to transform the problem into a multi-objective optimization which maximizes the NPV while minimizing the variation in the NPV caused by the grade uncertainty. The two objectives are balanced depending on the risk level.

1.6 Thesis outline

This thesis is organized as follows: Section 2 summarizes the literature on surface mine planning with waste management, dig-limit optimization, stope layout planning and sequencing. Section 3 proposes a block sequencing formulation including internal dumping option in surface mining operations, Section 4 proposes a ore-waste delineation formulation for bench sectors with equipment size constraints, Section 5 introduces a heuristic clustering algorithm that solves the stope layout planning problem. Section 6 proposes an exact method for solving the stope layout problem with an option to introduce a heuristic parameter for obtaining faster solutions and Section 7 extends the usage of this algorithm to mines with different characteristics. Section 8 develops a MILP formulation for stope sequence optimization that takes the stope layout produced in any of the proposed methods and sequences the stopes while minimizing the grade uncertainty.

2 Literature Review

In this section, mine planning literature that scopes the optimization of surface mines incorporating waste management, dig-limit optimization, stope layout and sequencing optimization is summarized. More detailed literature review related to each topic can be found in their correspondent section.

2.1 Waste Management in Surface Mine Planning

2.1.1 Surface Mine Planning

In mine planning practice, firstly, a 3D block model is created, and each block is estimated or simulated through an appropriate geostatistical technique. A block containing sufficient valuable metal is then classified as ore and otherwise it is classified as waste. This classification is made on the basis of a cut-off grade reflecting minimum metal to be extracted in such a way as to pay-off operation costs of a block. If the grade of a mining block is above the cut-off grade (ore), it is sent to a processing plant, and if not (waste), it is sent to an external waste dump or to a stockpile.

Surface mine planning can be divided into three decision making problems: (1) cut-off grade determination or ore – waste discrimination (where to send the blocks to be produced), (2) block sequencing (when to produce these blocks) and (3) determining production rates (Sari and Kumral, 2016). The division of mine planning optimization into sub-problems increases computational efficiency. In most cases, the production rates are determined previously and the ore - waste discrimination is determined based on the cut-off grade. Then, the blocks are sequenced. However, this may reduce the value of project because (i) cut-off (ore – waste discrimination) cannot be independent of time and (ii) capacity utilization may be reduced due to idle capacity (Kumral and Sari, 2017). Identifying blocks as ore or waste in advance of the optimization may delay or prevent the access to the rich areas of a deposit due to capacity constraints whereas simultaneous optimization of ore–waste discrimination and block sequencing enhances the search space such that a more effective search may be carried out with increased number of decision variables.

Surface mine planning techniques can be grouped under three main stream of approaches: heuristic, meta-heuristic and exact approaches. The most well-known methods are summarized in this section.

Heuristic Approaches

Floating cones Introduced by Carlson et al. (1966), after obtaining the economic value equivalents of each block, a cone is formed for each positive valued block that includes all the overlying blocks. If the overall value of the cone is positive, the cone is determined to be mined. All the blocks within the cone is eliminated from the system and this process is repeated until there are no more positive valued blocks in the block model.

Korobov algorithm Similarly to the floating cones method, a cone from each positive block is drawn but then the block's value is added to the negative blocks in the cone. In the end, if the cone value is positive, it is extracted. Otherwise, the algorithm continues with the next positive valued block until there are not any positive blocks (David et al., 1974).

Ranked positional weight This is another cone based approach introduced by Gershon (1987) except in this algorithm, the cones are inverted. An inverse cone is drawn from each block and a heuristic score based on the block value and the underlying block values is assigned because the removal of the current block will lead the way to the underlying blocks. Then the extraction of blocks are determined based on their scores.

Meta-heuristic Approaches

Simulated annealing The simulated annealing optimization method is applied to determine the extraction period and the destination of the blocks. This is initially accomplished by Kumral and Dowd (2005). An initial solution is generated and iteratively this solution is randomized and the new solutions are accepted according to Metropolis criterion (Metropolis et al., 2004; Kirkpatrick et al., 1983; Černỳ, 1985).

Genetic algorithms The genetic algorithms are applied to mine planning problem by coding random pits as genes and evaluating the fitness of the individuals using the NPV (Clement and Vagenas, 1994; Denby and Schofield, 1994, 1995).

Exact approaches

Lerchs-Grossman algorithm In this approach, the block model is converted to a graph where each block is a node and it is connected by arcs to the overlying nodes. The algorithm works by assigning labels to each arc as strong/weak based on the economic value and plus/minus based on the direction, and each strong branch is iteratively connected to overlying weak branches. This process continues until there are not any more overlying weak branches on strong branches. In the end, all strong branches are included within the final pit (Lerchs and Grossman, 1964).

Maximum flow approach This approach also converts the block model to a graph by converting blocks to nodes. Each block represented by nodes is connected to their overlying blocks by edges. Additionally, the positive valued blocks are connected to source and the other blocks are connected to the sink. After this conversion, the problem is solved by any maximum flow algorithm Johnson (1968).

Dynamic programming approach Koenigsberg (1982) applied dynamic programming for the first time to a three dimensional block model to obtain the ultimate pit limits, later modified by Wright and Weiss (1989).

The algorithm starts by updating the block values to the cumulative sum of the above blocks. Then, starting from the top left corner of the block model the values of blocks are updated identifying the highest value among the upper left, left and lower left blocks. In the end, the maximum value in the first row is backtraced and this delineation denotes the ultimate pit limits.

Mixed integer linear programming model Kumral (2012) introduced the MILP formulation for simultaneous optimization of extraction sequencing and ore - waste discrimination. The formulation maximizes the NPV while incorporating access, mining and production capacity constraints.

2.1.2 Waste Rock Management

Increasing environmental concerns emphasize the importance of mine waste handling. Solid waste management regarding surface mining was discussed by Deshpande and Shekdar (2005) and Deng et al. (2015). The effects of contaminants were detailed and improvement strategies were given. Que et al. (2015) investigated 16 project characteristics to assess socio-political risks affecting stakeholder and community engagements on mining projects on the basis of six demographic factors. Adibi and Ataee-pour (2015) addressed to incorporate sustainability into ultimate pit limit problem as a part of mine planning optimization. Mine waste can be handled through waste rock treatment or incorporating landfilling to the mining operations.

Current mine waste handling approaches focus on treating waste rocks such that environmental effects are minimized. Levis et al. (2013); Li et al. (2014) developed a model for the idea introduced by Williams et al. (2006) where the blocks having the potential to create acid water generation were encapsulated by safe, non-reactive blocks. They reported that the optimization models had an advantage in terms of solution time, truck utilization and cost saving. Although this method is a fair precaution, it is a much better approach to minimize the risk by reducing the quantity of waste rocks to be dumped as much as possible. Lu and Cai (2012) reviewed the management of solid wastes in mining and recommended new utilization strategies. Pimentel et al. (2016) provided a comprehensive review of mining and environment interaction including mine waste management.

2.1.3 Landfilling

Surface mining operations based on mechanical excavation with horizontally shaped deposits provide an opportunity to consider landfilling where waste rocks are disposed into previously emptied production areas of the pit. Landfilling has various advantages; (1) it alleviates environmental problems such as acid mine drainage which is treatable but costly and requires energy and chemicals that result in additional environmental impacts (Hengen et al., 2014; Zuo et al., 2013). Landfilling approaches this problem by reducing the external waste amount in the first place, solving the majority of the problem before its occurrence. (2) Transportation cost being almost half of total mining cost (Thompson and Visser, 2003), makes landfilling option a reasonable approach, which is especially applicable to horizontal or multi-mine operations. (3) In addition to mitigation of environment problems; transportation, road maintenance and safety costs can be also reduced (Li et al., 2015). (4) Furthermore, mine closure and rehabilitation costs will also be reduced because mining voids are filled.

Zaitseva et al. (2007) explored the applicability of internal disposal for flat-dipping and inclined bedded deposits from points of geometry and dip view. Zuckerberg et al. (2007) proposed an approach to use internal dumping for multiple pit operations. This research allows internal disposal to implement for even one pit as long as the pit extension is horizontal. In this approach, when the production in a pit is completed, this pit serves as dumping location for the material coming from other pit. Panov et al. (2011) focused on geotechnical and slope stability aspects of internal disposal. Sakantsev and Cheskidov (2014) addressed internal disposal in steep and deep deposits. They investigated relationship between access road and associated costs. Kalantari et al. (2013) investigated the relationship between long-term mine plans and the final composite tailings produced downstream such that random parameters were incorporated.

2.2 Dig-limit Optimization

In daily surface mining extraction operations, short-term plans are formed to delineate ore and waste SMUs. False qualifications of the units causes dilution or ore loss. Short-term plans need to make sure consecutive SMUs' ore or waste decision comprises of at least the number of SMUs as the equipment size allows while maximizing the profit. This optimization process is especially important if metal(s) are very valuable and dilution/loss is significant.

Contrary to long-term planning, previous research on dig-limit optimization is relatively limited. One of the first ideas is originated by Allard et al. (1994), who pointed out a need of connectivity index such that the value of an SMU depends on its location (surrounding SMUs defined in a frame) as well as its grade. They pointed out that it is important to observe the ore proportion (above cut-off) of the reserve as well as the number of connected components of ore clusters and their size distributions.

Richmond (2002) applied four different risk models based on utility functions (exponential utility function) and portfolio theory dominance models (mean–variance, mean-downside risk and stochastic dominance) to decrease the financial risk of local ore selection. A new mine planning approach with grade control strategy for ore-waste discrimination is proposed by Kumral (2015) that does not use cut-off grades and minimizes the loss associated with misclassification of SMUs. The resulting non-linear model is solved by successive mixed integer programming. However, this classification can be further enhanced for equipment size considerations and to reduce dilution/loss by adding dig-limit constraints.

Dig-limit optimization methods can be classified as heuristic and meta-heuristic approaches.

2.2.1 Heuristic approaches

Floating limits approach Richmond and Beasley (2004) developed a dig-limit algorithm inspired from the floating cone algorithm. They attempted to adapt the floating cone algorithm in two dimensions by floating a circle that represents the dig-line constraint. The algorithm floated the circle in the SMU model and if average grade of SMUs within the circle were above the cut-off grade, the area within the circle was flagged as ore and the perimeter of the circle was extended such that it would include the outward SMUs. Then, the process was repeated until the average grade within the circle fell below the cut-off grade. This is a clever approach in the sense that it both accounts for the dig-line constraint and tries to minimize the ore-waste boundaries hence the dilution/loss will be decreased. Moreover, they adapted their approach to work with different scenarios such as multiple ore types and different strategies and objective functions than net present value (NPV) maximization.

Heuristic pay-off approach Richmond (2004) proposed a local heuristic search algorithm that incorporates dig-limit considerations into open pit mine planning to minimize ore loss and mining dilution by using a pay-off function per block. However, greedy heuristic search methods and does not guarantee optimality.

Feasibility grade control algorithm Wilde and Deutsch (2015) also proposed a greedy algorithm called Feasibility Grade Control (FGC) that takes an initial plan and attempts to optimize the profit iteratively by re-arranging the form that blocks are accumulated into units. In addition to greedy search drawbacks, FGC requires an initial dig-limit solution by the user.

Hierarchical clustering approach Tabesh and Askari-Nasab (2013) tried a different approach to solve the problem. They attempted to use hierarchical clustering to form mineable polygons that are homogenous in grades and rock types by calculating similarity indices for blocks. Although providing useful guidance to the engineer, the approach itself cannot create practical ore-waste boundaries, as the shapes of the clusters were not in control.

2.2.2 Metaheuristic approaches

Approaches using simulated annealing A non-greedy search using the simulated annealing approach is proposed by Norrena and Deutsch (2000). They sought a balance of "accepting dilution" and "wasting ore" in order to maximize profit while satisfying the equipment constraints.

Norrena and Deutsch (2002) developed a simulated annealing approach that maximizes the profit and penalizes smaller angles of operation. Then, they conducted a contest of manual dig-limit determination and compared the results with their computerized method. The method compared well to the outcomes of the contestants.

Another simulated annealing based approach was suggested by Isaaks et al. (2014) where digline misclassifications are evaluated through loss functions constrained by dig-limits constraints. Simulated annealing is a solution space search algorithm that unlike greedy algorithms, moves towards non-improving solutions with a certain probability. This probability is determined by a temperature parameter T: where T is higher, the acceptance rate is higher. In the beginning, T is selected high to explore the solution space but as the algorithm progresses, T is decreased to have a higher chance of moving towards improving solutions. Although simulated annealing is claimed to reach the true optimal solution if T is selected high and decreased slowly enough (van Laarhoven and Aarts, 1987), in practice it generates near-optimal solutions.

Approaches using genetic algorithms Ruiseco et al. (2016) used genetic algorithms to solve the dig-limit optimization problem. Ruiseco and Kumral (2016) later extended the algorithm to handle multi-rock types, multi-process, and multi-metal cases. Genetic algorithms also share the same weaknesses although practically perform better than simulated annealing.

2.3 Stope Optimization

The sublevel stoping method (Figure 2.1) is a commonly used unsupported method used for orebodies with regular boundaries. Some portions of the orebody are selected to be extracted satisfying constraints such as minimum size and the distance between the portions according to the safety measures.

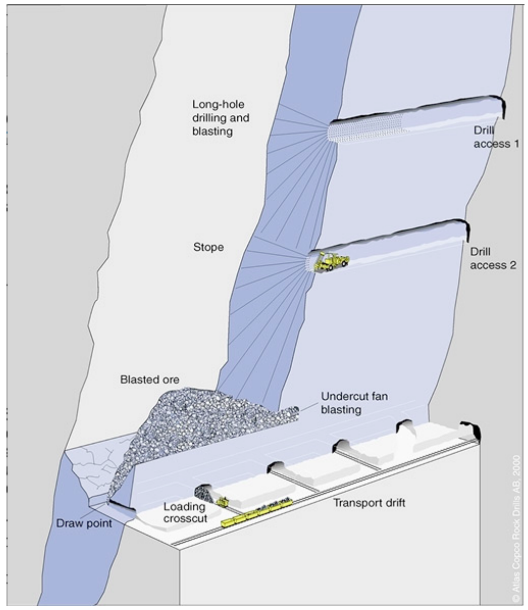


Figure 2.1 – Sublevel stoping (Atlas Copco, 2011)

These portions, which are usually large, vertical pipes of rock, are called stopes. First, access roads at various levels and drawpoints at the bottom of the stope are drilled. From the access levels, the rock is blasted and the crushed, collapsed rock is recovered from the drawpoints. After extracting is finished, stopes are backfilled to provide support and stability (Newman et al., 2010).

Stope optimization problem, also known as stope limit or boundaries optimization problem, has been solved using different methods (Ataee-Pour, 2005). These methods can be categorized in three groups as rigorous algorithms, heuristic algorithms and approaches using geologic models. Rigorous algorithms and heuristic algorithms use the economic model. In other words, rather than using the block grades directly, they use the economic value calculated by using the block grade and costs and revenues estimated at the time of extraction.

2.3.1 Rigorous algorithms

Rigorous approaches are based on mathematical reasoning.

Dynamic programming In underground mine optimization, dynamic programming has been used for designing layout of block caving (Riddle, 1977) and stope optimization (Jalali and Ataee-pour, 2004). This approach is a modification of the algorithm by Johnson and Sharp (1971) for optimizing surface mine pit limits. Instead of taking into consideration of the slope constraints as in open pit mining, a variation of r is allowed in draw control. The algorithm provides optimal 2D solutions, which are then combined into 3D. However, this combination causes a loss in the optimality in addition to arising possible stope constraint violations. Moreover, the optimal solutions do not include footwall regions for support. Footwall regions are introduced later heuristically, which adds to the loss of optimality.

Branch and bound technique To optimize starting and ending points at each row of blocks, branch and bound technique was introduced by Ovanic and Young (1995). Two piecewise linear cumulative functions at each row, representing the physical location of starting and ending points are declared and the problem is solved using a mixed integer programming (MIP) approach. This approach is known as SOS2 (Type-Two Special Ordered Sets), also called as separate programming, allows at most two adjacent ordered set of variables to be non-zero. This approach performs a stope geometry optimization, allowing partial blocks to be included in the optimal stope, which permits the stope boundaries to form an irregular shape. Although row by row this approach is optimal, this does not guarantee overall optimality.

Maximum flow approach Bai et al. (2013) followed a very different approach where the model is defined on a cylindrical coordinate around the initial vertical raise. In order to convert the model into a graph, blocks are converted to nodes and a source and sink are added. The optimizer decides on the best location and height for the raise and then solves the converted maximum flow problem (Picard, 1976). A case study has shown improvement over the floating stope algorithm. Having a vertical raise limits the optimality in cases where the orebody is inclined, results in including too much waste. Also, the study is presented on a small deposit that contains only one stope. For larger deposits more stopes and raises will be required which will complicate the problem.

Probabilistic mixed integer programming Grieco and Dimitrakopoulos (2007) developed a mixed integer programming model that accounts for uncertainty of grade data and plans according to a pre-defined risk level. Instead of the widely-used block model to represent the grade distribution, panels are used and the planning is performed on rings, which is the combination of adjacent panels. The optimization is performed through exploring how many rings each stope will contain within a minimum and maximum stope size range. The drawback of this method is that rings must be pre-defined in location and size to be considered in the model which means if the rings are too large, small sized valuable sections of the orebody may be ignored and not all locations are considered. Along with the possible long running time with large sized deposits, the ring based model will not be optimal because of the reasons given above.

2.3.2 Heuristic algorithms

Floating stope algorithm This method is implemented by the *DATAMINE* mining software package is analogous to the Moving cones method used for the optimization of surface mines. The term is derived by the simulation of floating a stope shape through the orebody to find higher grade concentrated regions (Alford, 1995; Alford et al., 2007). The economic value of each block is calculated after determining the cut-off grade to discriminate ore and waste blocks. The algorithm aims to minimize waste and based on the ore quality restrictions defined by the user it maximizes the grade if the contraint is maximizing the grade or the metal if the constraint is minimizing the dilution. Potential stope with minimum sizes based on geological requirements is floated throughout mineral deposit. If the stope grade averages above the cut-off grade, it is recorded. Optimal boundaries are foreseen to be in between two boundaries: (1) the union of all recorded stopes and (2) the union of best grade stopes. The problem with this method arises when two found stopes overlap. Although considered separately these two stopes may contain high grade regions and may have a high economic value, jointly considered the overall model may not be optimal. A multiple pass floating stope process (MPSFP) has been developed by Carwse (2001) to address the shortcomings of the floating stope algorithm. MPSFP accepts more inputs (such as maximum waste amount) that allows more envelopes to be generated such that the mining engineer is guided with more information when designing the stope layout.

Maximum value neighborhood algorithm This heuristic approach is based on locating best neighborhoods of blocks. After the minimum stope size has been taken as an input, the stope/block ratio SBR is found as in Equation 2.1, then rounded off to the next decimal to find the order of neighborhood O_{nb}.

$$SBR = \frac{\text{minimum stope size}}{\text{fixed block size}} \tag{2.1}$$

O_{nb} yields the number of sequential blocks to be mined in a stope. Among all possibilities found for each block, the neighborhood with maximum net value is included in the final stope. This algorithm is also reported to resolve the problem of overlapping stopes (Ataee-pour, 1997; Ataee-Pour, 2004). This is also not an optimal solution yielding approach as the results change according to the starting point.

Preference based profit maximization approach Sens and Topal (2008) developed an algorithm that produces a stope layout design based on the user defined preference (i.e., maximizing the stope profit or stope profit per square meter or stope profit divided by its total mining time). Among possibilities highest valued stopes are selected, then stopes containing common blocks are eliminated. This process is repeated until there are no more possible stopes left. All combinations of all the stope layouts cannot be evaluated with this approach because the selection is done according to the user's preference. Topal and Sens (2010) address this issue by developing an algorithm starts by creating all the possible stopes from smallest to largest size and saving all the positive stopes in a list. Then, an envelope is created for each individual stope and the economical values contained in the envelope are summed and saved. The stopes are then selected according to the user defined criteria.

Semiautomatic stope design In semiautomatic stope design, after setting up 3D stope ring grids filtering out the rings that do not contain ore, multiple stope shapes are generated using various cut-off grades. Then, the rest of the design is completed manually, using the guidance (Wang and Webber, 2012).

Stope size variation approach Sandanayake et al. (2015b) developed a heuristic algorithm that incorporates the stope size variation. The algorithm aggregates the mining blocks into possible set of stopes then modifying the the attributes of stopes. 10.7% improvement over the maximum value neighborhood algorithm has been reported.

2.3.3 Approaches using geologic models

All of the above models use the economic model as their base for the optimization. In this section however, the models that are using the geologic model directly are summarized.

Octree division approach Cheimanoff et al. (1989) has developed an algorithm that performs octree space division recursively to eliminate blocks based on their calculated economic values. The 3D model is divided into two equal parts in each dimension resulting in eight subvolumes. When divided, one of the three scenarios can happen: (1) If a subvolume is non-valuable (does not contain ore), it can be disregarded and excluded from the final stope layout. (2) If a subvolume is valuable throughout or has the minimum stope dimensions, it can be included in the final plan. (3) If a subvolume is neither completely ore or waste and larger than the minimum stope size, it will be divided in 8 subvolumes. This process repeats until there are no more subvolumes to evaluate. Because the partial stopes are not allowed during the optimization and the algorithm works by equal division of volumes, stope locations are checked only where the minimum stope dimension is a proper divisor. This causes more waste being included than the ultimate layout, yielding a non-optimal solution.

Downstream geostatistical approach Originally introduced by Deraisme et al. (1984) to plan an underground uranium mine and later reviewed by Deraisme and de Fouquet (1984), downstream geostatistical approach is focused on minimizing the dilution. In other approaches, after collecting sample data from drilling and blasting, linear kriging is used to simulate the orebody. However, this results in a smoothed distribution of the data. As underground mining with stoping technique is a selective procedure and involves ore blending, smoothing complicates and misdirects the optimization process. Instead, using probabilistic methods, grade variability is reproduced and sample values are reproduced at data points. The optimization is performed on 2D sections of the model. In addition to the minimum size of the stope constraint, an extra constraint of slope angle 45° is added and two outlines are drawn; a largest and smallest outline containing the economic optimum. Dynamic programming approach is used to find an optimal solution between the two outlines. At the end of this process, as the constraints may be violated, a final change is done manually. This is a very complex approach and manual final touch on the plan is a divergence from the optimality.

2.4 Stope Scheduling

Stope scheduling or stope sequencing problem is more complex than stope layout optimization problem because timing is incorporated into the optimization. The objective is to determine the production times of stopes and maximize the net present value. Usually, because of the time value of money, more valuable stopes are forced to be produced earlier unless constraints are violated.

MIP production scheduling Chanda (1990) has introduced the mixed integer programming method for the scheduling of underground mines. Trout (1995) developed one of the early models that focuses on maximizing the net present value while satisfying the constraints such as stope extraction capacity, stope backfilling demands, minimum metal quantity, hoisting capacity and stope geometry relationships. The quantity of ore and backfill variables are represented in continuous variables which allows them to have non-integer values. A small case study of a representative data set comprising 55 stopes from the Mount Isa mine is presented to demonstrate the efficiency of the method. Due to limited computational resources, the program was terminated prior to the proof of optimality. However, compared to a manually generated schedule, 23% improvement in the net present value of the project has been observed. Further works on this approach include the research of Nehring and Topal (2007), where an additional constraint regarding limitation of multiple fillmass exposures. Little (2007) takes this improved model and reduce the number variables following the logic later presented at Nehring et al. (2010) that suggests to combine the development, drilling and backfilling phases using the concepts of natural sequence and natural commencement which reduces the number of binary decision variables by a factor of five (fifth variable is the backfilled-completed state). Little et al.

(2008) apply the new mixed integer programming model on a small conceptual study, resulting in the same production schedule, yet 80% decrease in the number of binary variables and 92% improvement in the overall solution time. Sarin and West-Hansen (2005) developed a model that maximizes NPV of a coal mine and obtains desired coal quality. Binary variables are assigned to equipment to be used in each section and time period. The quality and the production volume of the coal are tracked by continuous variables. Constraints of the model include smoothing quality and production levels and setting maximum number of sections that can be mined at a time. Benders' decomposition, which is a technique in mathematical programming to solve very large problems that exhibit a special block structure usually found in stochastic problems is applied to solve the problem. Terblanche and Bley (2015) aimed to find a balance between reducing the resolution to smooth the grade data and maintaining enough detail to easily discretize between valuable and non-valuable portions of the deposit. This improved the profitability through selective mining. Although mixed integer programming methods yield optimality, the drawback of using this technique is as with all linear programming applications, as the problem size grows solving time increases exponentially. In a real application, this approach will take a long time. However, it can be combined with heuristic techniques to speed up the overall process.

MIP combined with heuristic techniques O'Sullivan and Newman (2014) combined MIP with a heuristic that will add value to the schedule such as shift the metal production forward in schedule and reduction of waste mining and backfilling delays. O'Sullivan and Newman (2015) used aggregation and optimization-based decomposition heuristic to speed up the optimization. Overall, the process was 98% faster.

Simulated Annealing Manchuk (2008) applied the simulated annealing (SA) approach to stope sequencing problem. Perturbations were accepted if either there is an increase in the NPV or there is a probability that the sequence leading to a more optimal one in the future perturbations. The general flow of the approach is the following: (1) Initially, a feasible, sub-optimal schedule is taken as an input by the program. (2) A stope from the panel of the current schedule is picked randomly. (3) A list of all feasible stopes is created and randomly swapped one of the feasible stopes with the chosen stope in the previous step. (4) If the new order is feasible, the NPV of the updated schedule is calculated. (5) If there is an improvement, the solution is accepted as the new current solution. Otherwise, the solution is accepted with a small probability to allow a better search of the solution space. The results of this algorithm was compared to a logic-driven approach called probabilistic decision making (PDM) approach. In this approach, stope properties such as stope profit, time required to extract stope, costs associated with stope are considered and used to calculate a value P, which is the probability of being a good decision to mine a stope, then gradient descent is performed to test and update the sequencing order. The results have shown that although PDM performs slightly better with smaller problems, as the complexity increased, the random approach, SA performed considerably better.

Additionally, Poniewierski (2005) studied the relationship between stope size, production rate and infrastructure requirements. His findings include that shrinking the stope size lowers the average production rate per stope but increases the per tonne operating cost. He concludes that if shaft, mill and smelter capacities are not reached, it may be beneficial to decrease the stope size to increase the overall production rate. However, for a mine with high fixed and initial capital costs, increasing the stope size may compensate for the high per tonne operating costs. These findings support the notion of economies of scales.

3 A landfill based approach to surface mine design

3.1 Abstract

Surface mining operations extract a large quantity of waste material, which is generally disposed into a dump area. This waste can cause a series of environmental problems ranging from landscape deterioration to acidic water generation and water pollution. Therefore, mine waste management is a significant task in mining operations. As known, in strip mining, the overburden is not transported to waste dumps but disposed directly into adjacent strips which was mined out. This concept can be adapted for mine planning of relatively horizontal deposits through a mixed integer programming (MIP) model. The main idea behind this paper is that, in one pit, production voids created in early year of mining are used for waste landfilling in late years of production. In other words, in addition to external dumping, a landfilling option within same pit is proposed for mine design optimization. The problem is formulated as maximization of the net present value (NPV) of the mining project under the constraints of access, landfill waste handling, mining and processing capacities. A case study using a data set was carried out to see the performance of the proposed approach. The findings showed that this approach could be used in waste management incorporating a landfilling option into mine planning. As a result, (1) material handling costs

decreases, and (2) environmental compliance increases due to less external waste quantity.

3.2 Introduction

Surface mining operations are managed through various management units such as mine operation, geo-technique, planning, environment and maintenance units. Environmental management unit focuses on landscape and environmental protection, compliance and "social licence to operate". These activities require measurement, analysing, testing, monitoring, case studies, literature review and interview processes with the locals and authorities to succeed environmental objectives (Prno and Slocombe, 2014). As a major part of environmental management, mine waste management deals with extraction, hauling, dumping, rehabilitation, analysing and monitoring of waste materials. Essential engineering issues in mine waste handling are landscape degradation; accounting for ground conditions; closure and rehabilitation planning; possibility of acid water generation; and soil and water pollution. As such, mine waste management comprises complex tasks and requires expertise in geology, hydrogeology, soil science, geotechnical, mining and environmental engineering. Therefore, reduction in waste quantities can directly facilitate mine waste management. In this context, landfilling has a strong potential to lessen the magnitude of environmental problems and high costs associated with external dumping. Landfilling is used in environmental sciences to dispose various waste materials (Winterstetter et al., 2015). Even though landfilling is long-known alternative in surface operations such as strip mining, it has not been mathematically formulated for mine planning practices where only one pit is operated. Horizontally extending deposits allow mine planner to use previously produced areas in the same pit for landfilling. This approach can be extended to the surface mining methods based on mechanical excavation. In this context, landfilling is investigated as a part of mine planning optimization.

In mining industry, the wastes can be classified into four groups (Parameswaran, 2005): (i) overburden of soil and/or rock that is extracted to access valuable material; (ii) waste rock whose grade is below the cut-off grade (sub-grade material), (iii) process tailing that is extracted in mineral concentration; and (iv) the contaminated waste that is generated by heap or dump leaching. In general practice, the materials of the first two groups are directly disposed into dump areas.

Current mine waste handling approaches focus on treating waste rocks such that environmental effects are minimized. Levis et al. (2013); Li et al. (2014) developed a model for the idea introduced by Williams et al. (2006) where the blocks having the potential to create acid water generation were encapsulated by safe, non-reactive blocks. They reported that the optimization models had an advantage in terms of solution time, truck utilization and cost saving. Although this method is a fair precaution, it is a much better approach to minimize the risk by reducing the quantity of waste rocks to be dumped as much as possible. Moreover, this may be combined with the above mentioned method for the best practice. Surface mining operations based on mechanical excavation with horizontally shaped deposits provide an opportunity to consider landfilling where waste rocks are disposed into previously emptied production areas of the pit. Landfilling has various advantages; (1) it alleviates environmental problems such as acid mine drainage which is treatable but costly and requires energy and chemicals that result in additional environmental impacts (Hengen et al., 2014; Zuo et al., 2013). Landfilling approaches this problem by reducing the external waste amount in the first place, solving the majority of the problem before its occurrence. (2) Transportation cost being almost half of total mining cost (Thompson and Visser, 2003), makes landfilling option a reasonable approach, which is especially applicable to horizontal or multi-mine operations. (3) In addition to mitigation of environment problems; transportation, road maintenance and safety costs can be also reduced (Li et al., 2015). (4) Furthermore, mine closure and rehabilitation costs will also be reduced because mining voids are filled.

In mine planning practice, firstly, a 3D block model is created, and each block is estimated or simulated through an appropriate geostatistical technique. A block containing sufficient valuable metal is then classified as ore and otherwise it is classified as waste. This classification is made on the basis of a cut-off grade reflecting minimum metal to be extracted in such a way as to pay-off operation costs of a block. In traditional approach, there are two possible destinations; if the grade of a mining block is above the cut-off grade (ore), it is sent to the processing plant, and if not (waste), it is sent to an external waste dump. In this paper, landfilling is added to mine production scheduling as a new destination. This addition inevitably increases the problem complexity from decision-making point of view. Mine planning can be divided into two decision making problems: (1) cut-off grade determination or ore – waste discrimination (where to send the blocks to be produced) and (2) block sequencing (when to produce these blocks) (Sari and Kumral, 2016). The division of mine planning optimization into two sub-problems increases computational efficiency. However, this may reduce the value of project because (i) cut-off (ore – waste discrimination) cannot be independent of time and (ii) capacity utilization may be reduced due to idle capacity (Kumral and Sari, 2017). Identifying blocks as ore or waste in advance of the optimization may delay or prevent the access to the rich areas of a deposit due to capacity constraints whereas simultaneous optimization of ore–waste discrimination and block sequencing enhances the search space such that a more effective search may be carried out with increased number of decision variables. In scenarios where there are multiple waste options, it is not meaningful to use a cut-off grade which discriminates ore and waste. Therefore, a priori cut-off grade in the traditional approach is not used in this research. Ore – waste discrimination is also incorporated into the optimization process. In other words, block destinations are formulated as decision variables to be solved by MIP. Furthermore, landfilling is restricted by void availability at the time of dumping whereas external dumping has more flexible dumping conditions.

Dumping location and capacity requirements of external dumping are not as restrictive as void availability of landfilling. Void availability for landfilling increases the complexity in such a way as to grow the number of decision variables and constraints. To be able to place a block to a landfill location, underling nine blocks of that location, where slope angles are 45°, should be filled previously. In this research, there are two critical assumptions that (1) the produced material is fully landfilled to the previously created void. In other words, it is assumed that swelling is negligible. In practice, when material is excavated, its in-situ volume expands. Depending upon material characteristics, swelling factor associated with volume expansion ranges between 10–60%. In hard rocks, this factor is 30–45% (Bohnet and Kunze, 1990). This is a reasonable assumption because mining equipment can compress the material. The other assumption is that (2) bottom of a landfilled area cannot be mined at a later period. Because of this, mining operation will not be extended beyond ultimate pit limits. Therefore, mine management should be certain about mine life and extensions.

Solid waste management regarding surface mining was discussed by Deshpande and Shekdar (2005) and Deng et al. (2015). The effects of contaminants were detailed and improvement strategies were given. Zaitseva et al. (2007) explored the applicability of internal disposal for flat-dipping and inclined bedded deposits from points of geometry and dip view. Zuckerberg et al. (2007) proposed an approach to use internal dumping for multiple pit operations. This research allows internal disposal to implement for even one pit as long as the pit extension is horizontal. In this approach, when the production in a pit is completed, this pit serves as dumping location for the material coming from other pit. Panov et al. (2011) focused on geotechnical and slope stability aspects of internal disposal. Sakantsev and Cheskidov (2014) addressed internal disposal in steep and deep deposits. They investigated relationship between access road and associated costs. Kalantari et al. (2013) investigated the relationship between long-term mine plans and the final composite tailings produced downstream such that random parameters were incorporated. Lu and Cai (2012) reviewed the management of solid wastes in mining and recommended new utilization strategies. Pimentel et al. (2016) provided a comprehensive review of mining and environment interaction including mine waste management. Que et al. (2015) investigated 16 project characteristics to assess socio-political risks affecting stakeholder and community engagements on mining projects on the basis of six demographic factors. Adibi and Ataee-pour (2015) addressed to incorporate sustainability into ultimate pit limit problem as a part of mine planning optimization.

In this paper, internal waste option is incorporated into mine planning optimization problem. Landfilling is well known in strip mining systems. However, the proposed approach takes a further step and landfilling is formulated as a part of mathematical optimization model. After the description of the problem and literature review are presented in Section 3.2, the optimization model as a MIP (mixed integer programming) problem is developed in Section 3.3. A case study is demonstrated, and the findings and discussion are provided in Section 3.4. Finally, conclusions and future work recommendations are given in Section 3.5.

3.3 Model development

In this research, waste management through landfilling is integrated into mine planning problem such that material handling costs are reduced. In inclined or vertical deposits, this option will have a limited manoeuvring room. For a block to be produced, in our formulation, there are three possible destinations: a mineral processing plant, an external dump or landfill. Since landfilling is less costly than external dumping, the produced waste blocks are forced by the model to destine to internal disposal as long as landfill constraints are met. This formulation raises the number of decision variables. A more challenging issue is the upsurge of the number of constraints. As known, slope constraint is the condition that overlying nine blocks should be produced to access the block located at the apex of upward cone. Slope angle is governed by changing block sizes in orebody modelling stage. Similar to slope (predecessor or access) constraint used to access a block, there is a landfilling constraint for each waste block. The landfilling constraint is to landfill a block, underlying nine blocks should be either within ultimate pit limits or landfilled in the current/previous periods. The landfilling constraint is basically the inverse of the slope (access) constraint. Figure 3.1 shows these constraints. To produce the block located at the apex of the cone in this figure, a downward cone is created and all the blocks within this cone

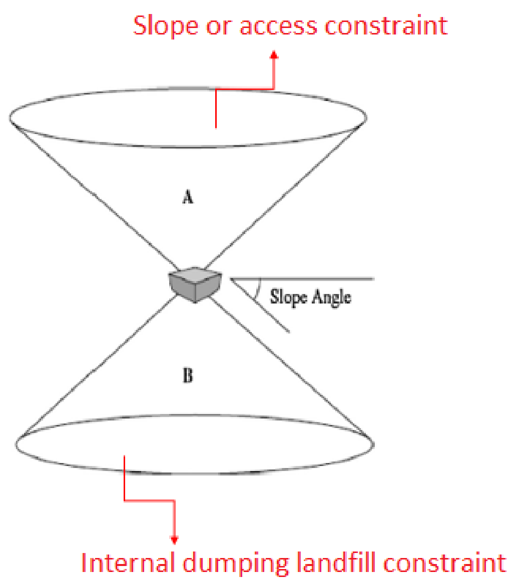


Figure 3.1 – Illustration of access and landfill constraints

should be produced earlier. To landfill the same location, an upward cone is created and all the blocks within this cone should be either landfilled earlier or be within the ultimate pit limits.

The formulation is solved using MIP, which is an exact method that is utilized in various applications (Okoye et al., 2015). The notation used in this formulation, the parameter values in the case study, the indexes and variables are defined as below:

Table 3.1 – Parameters and their values for the case study

Parameters	
N	*Total number of blocks*
X	*Number of blocks in X direction*
Y	*Number of blocks in Y direction*
Z	*Number of blocks in Z direction*
x, y, z	*Block dimensions*
s	*Material density*
P	*Number of periods (in years)*
r	*Recovery*
d	*Discount rate*
m	*Block mass*
p	*Metal price*
C_m	*Mining cost*
C_p	*Mineral Processing cost*
D_i	*Landfill cost*
D_e	*External dump cost*
E_m	*Maximum mining capacity per period*
E_p	*Maximum mineral processing capacity per period*
F_m	*Minimum mine production per period*
F_p	*Minimum mineral processing feed per period*
InV	*Initial investment*

Table 3.2 – Indices and sets

Indices	Sets
t	*Period* $(1, \ldots, P)$
i	*Block index* $(1, \ldots, N)$
j	*Overlying block index* $(1, \ldots, 9)$
k	*Underlying block index* $(1, \ldots, 9)$
l	*Block index for blocks at the mine boundaries below the surface*

Table 3.3 – Binary decision variables

variable	0 (at time t)	1 (at time t)
$\boldsymbol{x(t,i)}$	block i not extracted	block i extracted
$\boldsymbol{y(t,i)}$	block i not sent to mill	block i sent to mill
$\boldsymbol{v(t,i)}$	block i not sent to landfill	block i sent to landfill
$\boldsymbol{s(t,i)}$	block i not sent to external dump	block i sent to external dump
$\boldsymbol{z(t,i)}$	not disposed landfill to location i	disposed landfill to location i

Maximize:

$$\sum_i \sum_t \frac{1}{(1+d)^t} \times \left[\left(\frac{A \times g_i}{100} - E \right) \times y\,(t,i) - B \times x\,(t,i) - C \times v(t,i) - D \times s(t,i) \right] \tag{3.1}$$

Where A $= r \times \ m \times \ p$

B $= C_m \times m$

C $= D_i \times m$

D $= D_e \times m$

E $= C_p \times m$

g_i is the grade of block i

Subject to: I. Time and location constraints:

a. A block can only be extracted once.

$$\sum_{t} x(t,i) \leq 1 \qquad \forall i \tag{3.2}$$

b. A location can be disposed landfill only once.

$$\sum_{t} z(t,i) \leq 1 \qquad \forall i \tag{3.3}$$

c. If a block is extracted, it should be sent to one of the following destinations: mill, landfill or external dump.

$$\left[\sum_{w=1}^{t} y(t,i) + \sum_{w=1}^{t} s(t,i) + \sum_{w=1}^{t} v(t,i) = \sum_{w=1}^{t} x(t,i)\right] \qquad \forall i,t \tag{3.4}$$

d. In order to use landfill option, the void should have been previously created by extraction of the block in this location.

$$z\left(t,i\right) \leq \sum_{w=2}^{t} x\left(w-1,i\right) \qquad \forall i,t \tag{3.5}$$

II. Capacity constraints:

e. Mined block mass should be below mining capacity per period. Mining capacity should be compatible with equipment fleet.

$$\sum_{i} x(t,i) \times m \leq F_m \qquad \forall t \tag{3.6}$$

f. Mined block mass should be above minimum mine production limit per period.

$$\sum_i x(t,i) \times m \geq \ E_m \qquad \forall t \tag{3.7}$$

g. Block mass sent to mill should be below mineral processing capacity per period. Since these capacities are installed before, the ore to be produced cannot exceed.

$$\sum_i y(t,i) \times m \leq \ F_m \qquad \forall t \tag{3.8}$$

h. Block mass sent to mill should be above minimum mineral process feed per period.

$$\sum_i y(t,i) \times m \geq \ E_m \qquad \forall t \tag{3.9}$$

III. Access constraints:

i. Blocks at the mine boundary cannot be extracted because it is assumed that the boundaries cannot be extended.

$$x\,(t,l) = 0 \qquad \forall\ t,l \tag{3.10}$$

j. All overlying blocks must be extracted at the present or earlier periods to allow mining at the current block.

$$\sum_{w=1}^{t} x\left(w,i\right) \leq \sum_{w=1}^{t} x\left(w,j\right) \qquad \forall t,i,j \tag{3.11}$$

k. One of the two conditions should be true in order to be able to landfill for each underlying block; (1) the underlying should also have been landfilled previously or in the same period, or (2) the underlying should not be extracted during mine life.

$$\left[\sum_{w=1}^{t} z(w,i) \geq z\left(w,k\right) \oplus \sum_{t} x\left(t,k\right) = 0\right] \qquad \forall t,k,i \tag{3.12}$$

Expressing conditional constraints is not possible directly in linear programming, the problem is solved using the big M method (Bazaraa et al., 2011) which defines a very large number M and a new binary variable $p(i)$ such that feasibility and optimality are combined. In the end, the above constraint is converted to following constraints:

$$\sum_{w=1}^{t} x\left(w,k\right) - M_1 \times p\left(i\right) \leq 0 \qquad \forall\ t,k,i \tag{3.13}$$

$$\sum_{w=1}^{t} z\left(w,i\right) + M_2 \times p\left(i\right) \geq 0 \qquad \forall\ t,k,i \tag{3.14}$$

$$\sum_{w=1}^{t} x\left(w,k\right)+M_{3}\times\left(1-p\left(i\right)\right)\geq 1 \qquad \forall\ t,k,i \tag{3.15}$$

$$\sum_{w=1}^{t} z\left(w,i\right)-M_{4}\times\left(1-p\left(i\right)\right)\leq\sum_{w=1}^{t} z\left(w,k\right) \qquad \forall\ t,k,i \tag{3.16}$$

IV. Binary constraints:

l. The decision variables can take the value of either 0 or 1

$$y(t,i),x(t,i),s(t,i),v\left(t,i\right),p\left(t,i\right)\ \text{and}\ z\left(t,i\right)\in\{0,1\} \qquad \forall\ t,i \tag{3.17}$$

3.4 Case Study

To demonstrate the performance of the proposed model, a case study was carried out on a deposit extending horizontally. The data were based on an old copper mine, which was mined out long time ago. The operation was implemented in one pit throughout mine life. Using drill-hole data of this deposit, a block model was created and estimated through ordinary kriging. Figure 3.2 shows the 3D shape framing of the orebody. This extension facilitates the use of landfilling. The voids created in early years of the production can be filled in subsequent years of the production. The total number of the blocks are 25,000 (100 (EW) x 25 (NS) x 10 (Vertical)). All parameter values

Table 3.4 – Parameter values for case study

Parameter	**Value**
Total number of blocks	25,000
Number of blocks in X direction	100
Number of blocks in Y direction	25
Number of blocks in Z direction	10
Block dimensions	10m x 10m x 10m
Bulk density	5 tonne/m^3
Number of periods (in years)	5
Recovery	90%
Discount rate	10%
Block mass	5,000 tonnes
Metal price	4,500 $/tonne
Mining cost	5 $/tonne
Mineral Processing cost	15 $/tonne
Landfill cost	1.5 $/tonne
External dump cost	4 $/tonne
Slope angle in all directions	45°
Maximum mining capacity per period	15,000,000 tonnes/year
Maximum mineral processing capacity per period	7,500,000 tonnes/year
Initial investment	$ 2,000,000,000

used in this case study has been given in Table 3.4.

For the optimization model given in Section 3.3, the objective function and constraints were created and this model was then submitted to an optimization tool to generate the solution. The outputs of the optimization have been thoroughly tested and verified that all constraints are satisfied. The number of decision variables is 1,875,000 and the number of constraints is 2,932,314. As can be seen, the number of constraints is much larger than the number of decision variables because slope and landfill constraints lead to swift increase in the problem size.

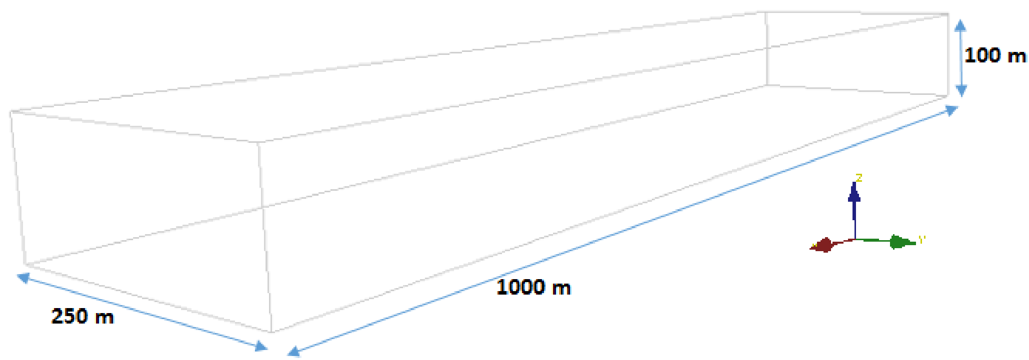


Figure 3.2 – Extension of mineral deposit

Figures 3.3–3.6 demonstrate the results of the case study where each colour corresponds to a year of the operation. Figure 3.3 illustrates the landfill feasibility on a randomly selected cross-section. As can be observed from the cross-section, according to the optimized model, although landfilling takes place each year after the first year, the voids created by producing the east side of the orebody are filled massively (876 out of 910) in Year 5. If the horizontal extension of the orebody was greater, massive backfill may have started in earlier periods. Slope and landfill angle is 450. For different angles, block size should be changed accordingly. Slope angle also affects the feasibility of approach. As slope angle decreases, possibility for horizontal extension grows.

Figure 3.4 shows randomly selected cross-sections of the production plan taken in x- and y-direction.

The annual present values are provided in Table 3.5. Total project life is 6 years (5 operations and one investment year). Initial investment of

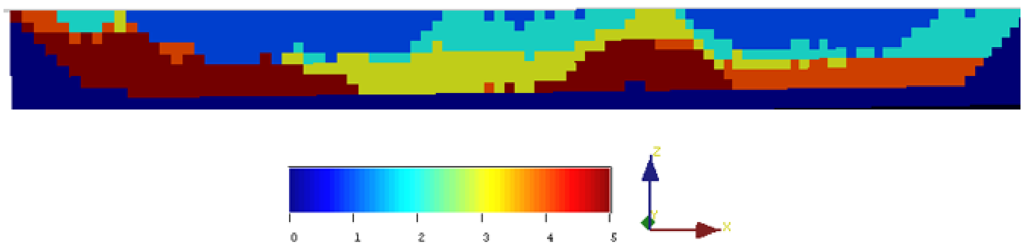
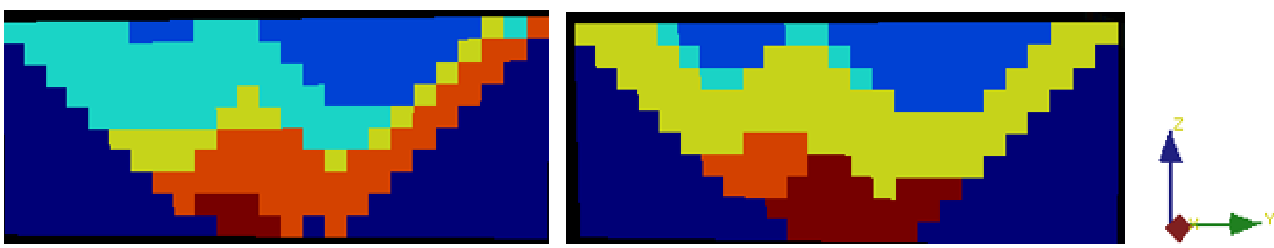
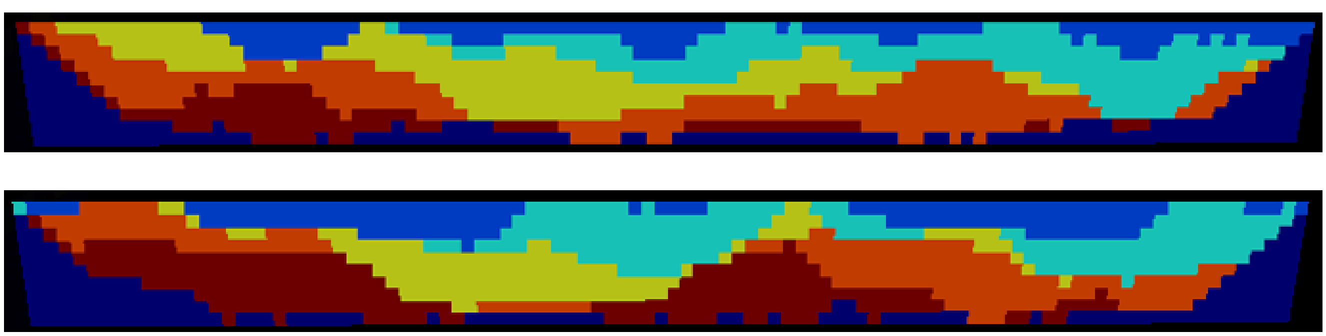


Figure 3.3 – Illustration of landfill feasibility on a cross-section (on 7th Slice of y-direction)



(a) The cross-sections (on 79th and 30th Slices of x-direction)



(b) The cross-sections (on 15th and 10th Slices of y-direction)

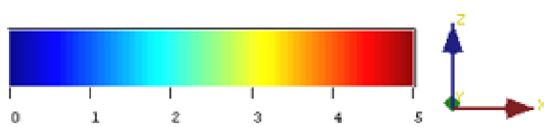


Figure 3.4 – Randomly selected sections in different directions of production plan

Table 3.5 – Summary of production scheduling

Year	Number of Blocks				NPV
	Processing	External dump	Landfill	Total	
Year 1	1,500	1,500	0	3,000	2,035,704,225.0
Year 2	1,500	1,498	1	2,999	1,103,320,511.4
Year 3	1,500	1,478	21	2,999	790,226,497.9
Year 4	1,500	1,434	12	2,946	565,018,782.9
Year 5	1,500	8	876	2,384	349,131,659.4
Investment (Year 0)					(2,000,000,000.0)
Total	7,500	5,918	910	14,328	$2,843,401,676.5

the project is $2,000,000,000. In this case, the net present value (NPV) of the project becomes 4,843,401,676.5 - 2,000,000,000 = $2,843,401,676.5. Throughout the project, a total of 14,328 blocks are produced. 7,500; 5,918 and 910 blocks are sent to mineral processing plant, external dump and landfill, respectively. As can be seen from the parameters, processing capacity is 7,500,000 tonnes per year and each block mass is 5,000 tonnes. A summary of production plan in annual base is given in Table 3.5. Therefore, maximum number of blocks to be produced in a given year is 7,500,000/5,000 = 1,500 blocks. Given that total number of blocks produced during the project is 7,500 and project production life is 5 years, the production rate in upper bound is fulfilled (7,500/5 = 1,500). The difference between the costs of landfill and external dumping is $2.5; when projected to the present, the gain from using landfilling is $7,820,000, which is an approximately 7% decrease in the transportation costs.

For the case study, the optimization matrix is created in ZIMPL (Zuse

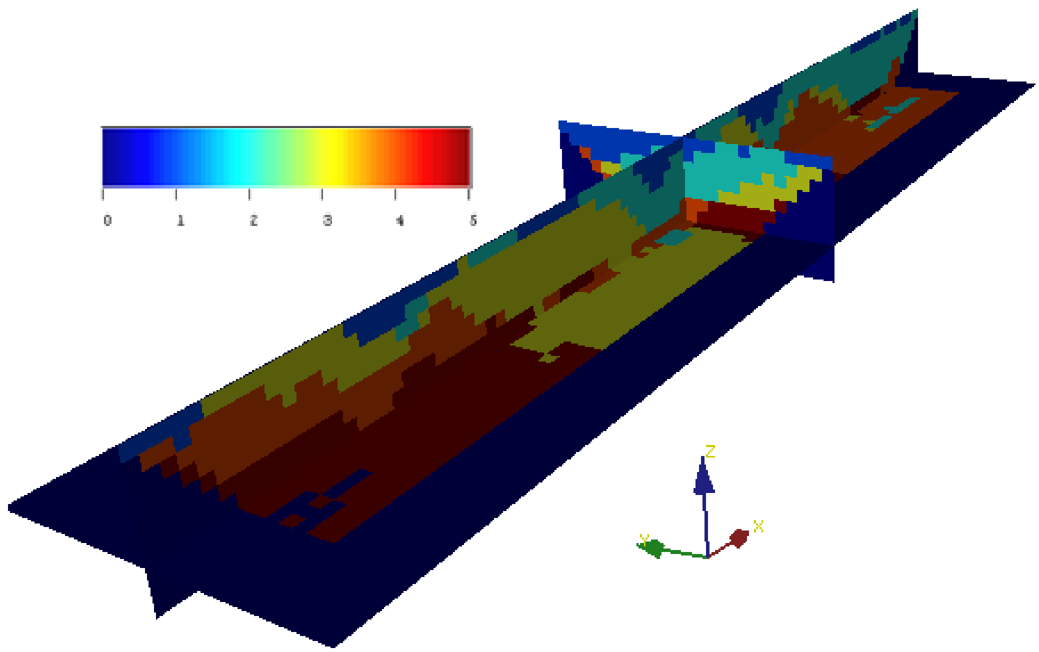


Figure 3.5 – 3D image of production schedule (x = 52, y = 13 and z = 3)

Institut Mathematical Programming Language (Koch, 2004)) and the problem is then solved using IBM ILOG CPLEX Optimization Studio. A Dell Precision T3610 with Intel® Xeon® CPU E5-1620 v2 and 16.0 GB RAM was used and running time was 29.4 hours. The gap for optimality is 0.01%. Figure 3.5 provides a randomly selected 3D view of the production plan generated by the optimization process to give an idea about the evolution of the mining operation.

Figure 3.6 shows the production scheduling plan for a randomly selected cross-section in terms of block destinations and the production periods. In Figure 3.6 (a), the blocks to be sent to mineral processing with production periods are shown. Each colour of the legend represents a period. Figure 3.6

(b) illustrates the blocks to be sent to external waste dump. As can be seen Figure 3.6 (a) and (b), as mining advances to deeper zones, the number of blocks to be sent to external waste dump decreases. There are two reasons for this: (i) more valuable blocks are located in deeper areas, thus the number of block sent to processing increase and (ii) waste blocks are dumped internally because the voids are available for landfilling. 6(c) and (d) show the landfilling from two perspectives on various cross-sections: In Figure 3.6(c), the blocks to be removed for landfilling and in Figure 3.6 (d), the blocks to be located for landfilling are shown. As can be recognized, landfilling is realized towards the end of project. All figures providing the views of the production plan are generated using SGeMS (Stanford Geostatistical Modeling Software).

In addition to cost reduction effect of landfilling associated with shorter transportation distance, mine closure costs will also be reduced. Therefore, the contribution of landfilling is beyond the financial contribution illustrated herein. Furthermore, environmental risk such as acid water generation, and mine rehabilitation and closure costs may be also lowered. As the amount of external dumping is reduced, the area to be rehabilitated will be also smaller. Likewise, landfilling will reduce the size of void created by mining. These, to some extent, will facilitate mine closure and rehabilitation at the end of mining operation. In addition, since waste material will be disposed into its host area in landfilling, the possibility of acid water generation is low because rock characteristics and hydrogeology did not allow this previously.

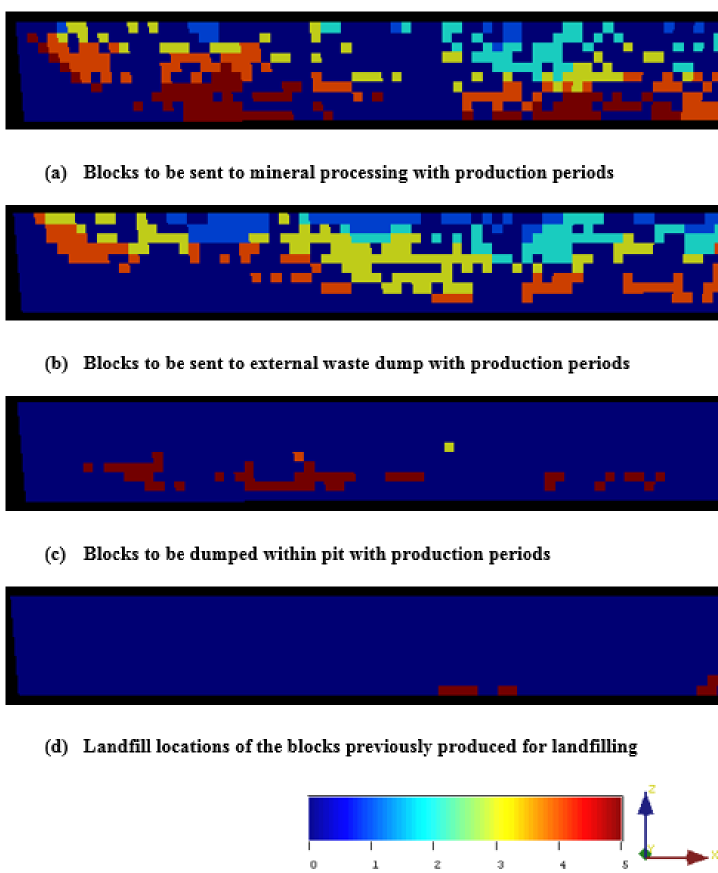


Figure 3.6 – Destination map of ore and waste blocks on various cross-sections of y-direction (each colour represents production or landfill periods)

3.5 Conclusions and Future Works

As stringent environmental regulations put pressure on mining operations, innovative production methods need to be found. The paper proposes a mining production approach that production and landfilling can be carried out in same pit. It presents an environmental friendly waste management optimization approach to be used in surface mining operations based on mechanical excavation. Landfill option is used in strip mining and the proposed approach makes it more mathematical and formal. The proposed approach will reduce mine closure time and costs as well as increasing environmental compliance. Furthermore, in cases where (1) external dumping costs are high, (2) the distance between pit and external dump is long, (3) the capacity of external dump is limited and (4) the deposits are more extended horizontally, landfilling will contribute to increase the NPV of the project. The main challenge in the optimization process is the increase in the problem size. Since this formulation requires additional decision variables, the number of decision variables increases significantly. The number of constraints also grows significantly due to newly introduced landfilling constraint. The approach is tested on a case study where during the project life, a total of 4,550,000 tonnes waste is disposed inside the pit. In other words, almost 13% of a total of 6,828 (910 / (5,918 + 910)) blocks is dumped inside the pit. This reduces waste management and transportation costs. The study showed that optimization based waste management model recommended in this paper can be used to increase the project's NPV and has a potential to enhance operation efficiency in terms of sustainability. Apart from lowering the transportation costs, potential environmental risks are also reduced. As mine life and mine size increase, the contribution of landfilling in cost reduction will also increase. In the future, the research will be extended such that the effect of swelling factor will be incorporated. The approaches that increase computational efficiency associated with the number of decision variables and constraints should be explored. For this reason, various aggregate-disaggregate, decomposition and clustering approaches will be investigated. The contaminant restrictions can be added to the optimization process through encapsulation of pollutant blocks. Finally, the research can be extended to probabilistic optimization model that considers random characteristics of parameters.

3.6 Chapter Conclusion

Although surface mine planning is well studied in the literature, landfilling has not been applied extensively. The contribution of this section is that surface mine planning is extended to a special case, in which landfilling option is incorporated. This approach is especially useful when internal dumping poses possible environmental problems. The limitations of this approach is that the pit limits cannot be extended beyond the optimization limits because of the dumped landfill. However, for horizontally extending, non-deep orebodies this method is practical. For deeper orebodies, it might be practical to set a higher ore price to account for possible increases in the future, thus reaching a deeper pit before starting landfilling.

This work has shown that the mining industry can adapt the current MILP formulations for general problems in the literature for their specific needs. The following section presents a MILP formulation of the dig-limit optimization problem which is encountered frequently in surface mines but for which limited number of approaches are present. For this problem, an extension was not possible hence a completely new approach was developed.

4 Dig limit optimization through mixed integer linear programming in open pit mines

4.1 Abstract

As a type of general layout problems, dig-limit optimization focuses on generating the ore-waste boundaries of a bench sector in an open pit mining operation. Typically, blast holes are dense; therefore, selective mining units (SMUs) are small, which is not compatible with loading equipment. Loader cannot select ore–waste boundaries of SMUs because the arm of the excavator is generally longer than SMU sizes. Therefore, clusters of SMUs being compatible with loader movements need to be formed. In this paper, the dig-limit optimization problem is shown to be NP-hard and formulated to maximize profit to be obtained from a mining sector such that ore and waste clusters corresponding to mine excavator movements are considered and solved by mixed integer linear programming. To see the efficiency of the proposed approach, a case study is conducted on seven sectors of a bench in a gold mine. The results showed that the approach is practical and has potential to increase the value of operation. The resulting average economic value of seven sectors is \$129,060. Additionally, optimal design of one bench solved by the model is compared to a manual design of a mining engineer and a deviation of 6.4% have been observed.

4.2 Introduction

Mine planning has been recognized as a value creative process for a long time in mineral industries. Owing to advances in optimization, hardware and software technologies, mine planning is now a standard process and mining companies have formed planning units. In line with this, a rapidly growing mine planning software industry has emerged. Similar to other engineering projects, mine planning is implemented in three different levels:

1. Long-term (Strategic) mine planning: Long term planning takes a picture of a project and generate a net present value through sequencing and destining material in a block-by-block fashion under access constraints (Kumral, 2013; Souza et al., 2010). Long-term mine planning helps decision makers to contemplate if the project is viable at corporate management level. This plan can be seen as a guide of the operation and gives an idea on how the project can evolve over the time (Osanloo et al., 2008). Long-term planning is based on block models where each block is estimated or simulated using the limited drill-hole information. Since drill-hole information is sparse, block sizes are generally large. If an indicator-based estimation or simulation method is used, ore and waste quantities within a block can be estimated or simulated but this does not say anything about how ore and waste are distributed within block. There is a strong relationship between block size and selectivity. As the block size increases, selectivity is reduced (Jara et al., 2006). In short, even though it provides very valuable information, long term plans are impractical because any process (e.g. fragmentation, hauling,

dewatering and processing) in mining cycle is not considered.

2. Medium-term (Tactical) mine planning: This is used for decision making in mine-specific management/directory level. In this stage, specific operations of mine production cycle are linked to long-term plans, which are based on a block economic model using block grades, prices, costs and recoveries. Medium term planning addresses the planning process in relation to drilling, blasting, hauling and dewatering. In other words, mining operations are synchronized with long-term planning. In this fashion, medium-term planning puts long-term plans into practice (Frimpong et al., 1998; Kear, 2006).

3. Short-term (Operational) mine planning: When operation starts and bench sectors are mined on a daily basis, actual data are obtained through blast holes rather than previously referenced drill holes. Blast hole data provide more precise information concerning the bench grade (Kawalec, 2004). Consequently, the long term plan proves to be impractical which paves way to the need of short term planning. Blast hole data can be assigned to volumes through extending each blast hole to half of hole spacing, creating selective mining units (SMU). SMUs are smaller than blocks, allowing more definition in terms of grade variation throughout the orebody and thus permitting more precise planning (Assibey-Bonsu and Krige, 1999). These plans are usually created manually by the mining geologist on a daily basis possibly diverging from optimality. Decisions regarding short-term planning are made by a superintendent or senior engineers. Additionally, production rates or processing batch capacities, quality requirements, equipment utilization, and

recovery and throughput of mineral processing are managed by short-term planning. In other words, this type of planning puts mining operation into practice (Smith, 1998).

Even though ore – waste discrimination based on SMU grades is highly valuable, this cannot be directly used in practice (Ruiseco and Kumral, 2016). Short term plans need to make sure consecutive SMUs' ore or waste decision comprises of at least the number of SMUs as the equipment size allows. The component of the short term planning that focuses on determination of ore – waste boundaries on a bench sector is called dig-limit optimization. This optimization process is especially important if metal(s) are very valuable and dilution/loss is significant. Dilution can be defined as waste contamination within an ore SMU due to blasting and results in low processing recovery. Dilution decreases the value of operation. In contrast to dilution, loss is defined as ore entry within a waste SMU, and leads to opportunity cost. Using information obtained from blast holes, SMUs may be flagged as ore or waste. However, this will not be practical because (1) excavator's arm will be longer than SMU length in any direction, and (2) dilution/loss will cause changes of SMU grades. Due to blast hole drills spacing, the volume of a SMU is much smaller than a planning block used in long-term planning. For example, if there is 5 m between blast-holes, the surface of an SMU will be 5 x 5 m. On the other hand, surface of an average planning block is approximately 20 x 20 m. Given large shovel arms and dilution/loss associated with blasting, classification of small SMUs as ore and waste based on in-situ grades will not be meaningful.

As well as the practicality, dig limit optimization assists dilution and loss management. (Jara et al., 2006) stated that as the block size gets smaller, the contact perimeter between ore blocks and waste blocks becomes larger, increasing the percentage of dilution. Consequently, as blast hole spacing decreases, fragmentation efficiency increases but dilution/loss also increases. This creates an engineering decision making problem seeking a trade-off between losses associated with fragmentation and dilution/loss. Large blast hole spacing leads to large particle size raising transportation and communition costs. On the other hand, small blast hole spacing leads to ore losses and dilution, and increase in blasting costs. However, this problem is beyond the aim of this research. Nevertheless, dig limit optimization assists to reduce ore losses and dilution in small blast hole pattern. The number of ore – waste contacts based on applying a cut-off grade only is much more than that based on determining dig limits. As such, dig limit optimization will decrease monetary loss associated with dilution/loss. This has vital importance in case where metal to be recovered is of high value and the operation experiences high dilution. Figure 4.1 illustrates effects of dilution/loss on a bench sector. Upper left subfigure shows in-situ grades of SMUs. As known, depending upon the magnitude of blasting, rocks will have inter-blocks movements in form of flying rock. As heterogeneity of rock characteristics increases, this issue will be more potent. The highest and lowest grade areas are given in claret and in navy blue, respectively. The other subfigures summarize the

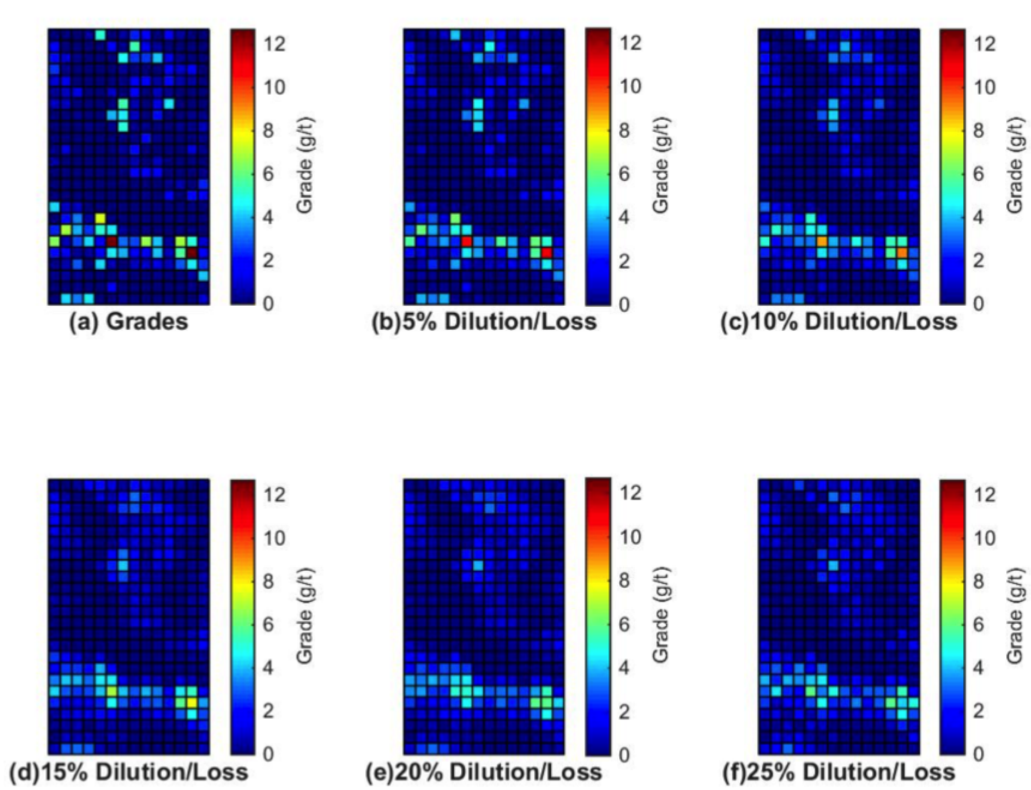


Figure 4.1 – Change of in-situ grades due to dilution and loss

effect of loss and dilution from 5% to 25% of loss/dilution in each direction. As can be seen in the figure, in-situ grades change significantly.

As loss and dilution increase, a clear smoothing effect is observed (Figure 4.2). In other words, the grades of high grade SMUs areas decrease and the grades of low grade SMUs increase. Smoothing makes ore and waste boundaries more apparent. This phenomenon gives important clues for short-term mine planning: (1) A model based on in-situ grades will not be meaningful due to loss/dilution; (2) Given that prediction of loss and dilution is very difficult, in-situ grades cannot fully govern ore – waste decisions; (3)

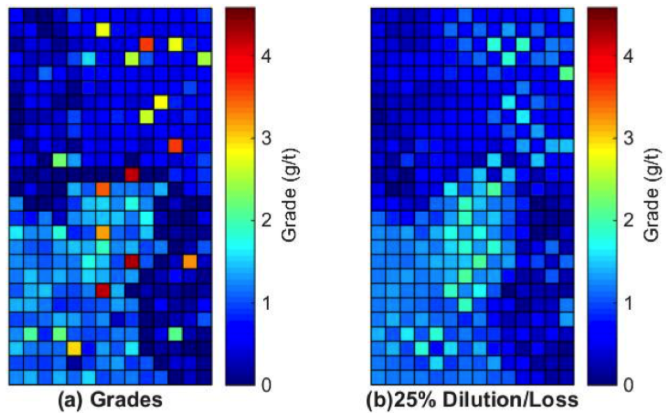


Figure 4.2 – Smoothing of in-situ grades owing to 25% of dilution/loss at each direction

Since loss and dilution is not fully predictable, the best strategy is to reduce the effects of loss and dilution. In this scope, ore and waste clusters can be defined in such a way as to allow equipment movements. In this paper, this process is formulated as an optimization problem and solved by mixed integer linear programming (MILP).

It is also worth noting that as can be seen from below right picture of Figure 4.1, as dilution/loss increases, ore and waste boundaries can be observed bluntly. The areas in navy blue are waste and the rest is ore (see legend). This can be very helpful for the mine geologist, who will draw dig limits manually. Nevertheless, the optimality cannot be guaranteed. As the experience of engineers increases, dig limits can approach the optimality. However, given that dig limits are determined daily, computerized dig limit optimization will save significant time of mine decision makers. The solution of this problem through optimization tools will be helpful the mine management in

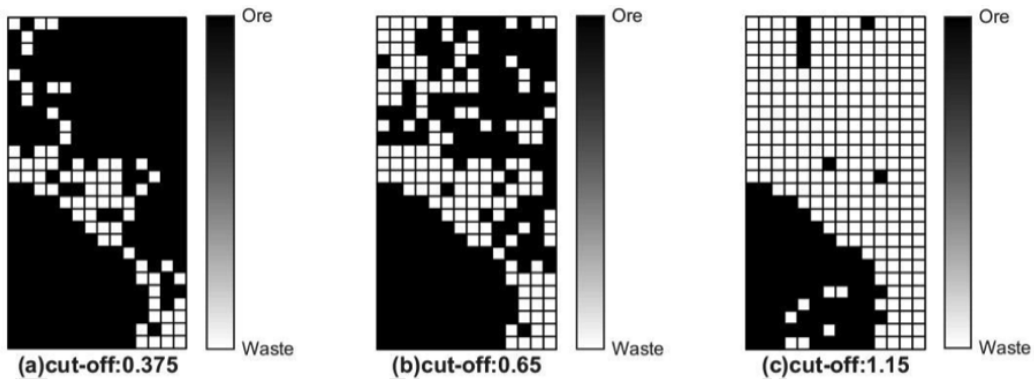


Figure 4.3 – The effect of different cut-off grades on ore and waste clusters

any case.

Figure 4.3 indicates the effect of cut-off grade on loss/dilution. At lower cut-off grades, since the number of ore SMUs is high, dilution will be the more dominant problem. At higher cut-off grades, the size of waste cluster will grow. Ore SMUs within waste cluster will be observed increasingly and this will result in loss problem.

The originality of this paper rests on formulating the dig-limit problem and solving it through an exact method for the first time. Due to complicated dependency relations between the variables, all research to this date have attempted to solve this problem though heuristic methods, greedy search methods or metaheuristics. The approach proposed in this research is to solve this problem through MILP. Given that this is manually implemented by mining geologist on daily basis, the proposed approach has a potential to enhance the profitability of mining operation. The paper is organized as follows: After the problem is described (Section 4.2) and the literature is reviewed (Section 4.3), the optimization model is given in Section 4.4. A weekly production including seven bench sectors is demonstrated through a case study on a gold deposit in Section 4.5. Finally, the conclusions are drawn, pros and cons of the proposed approach are discussed, and future direction of a new research is provided.

4.3 Literature review

Contrary to long-term planning, previous research on dig-limit optimization is relatively limited. One of the first ideas is originated by Allard et al. (1994), who pointed out a need of connectivity index such that the value of an SMU depends on its location (surrounding SMUs defined in a frame) as well as its grade. They pointed out that it is important to observe the ore proportion (above cut-off) of the reserve as well as the number of connected components of ore clusters and their size distributions. Richmond (2002) applied four different risk models based on utility functions (exponential utility function) and portfolio theory dominance models (mean–variance, mean-downside risk and stochastic dominance) to decrease the financial risk of local ore selection. A new mine planning approach with grade control strategy for ore-waste discrimination is proposed by Kumral (2015) that does not use cut-off grades and minimizes the loss associated with misclassification of SMUs. The resulting non-linear model is solved by successive mixed integer programming. However, this classification can be further enhanced for equipment size considerations and to reduce dilution/loss by adding dig-limit constraints. Richmond and Beasley (2004) developed a dig-limit algorithm inspired from the floating cone algorithm. They attempted to adapt the floating cone algorithm in two dimensions by floating a circle that represents the dig-line constraint. The algorithm floated the circle in the SMU model and if average grade of SMUs within the circle were above the cut-off grade, the area within the circle was flagged as ore and the perimeter of the circle was extended such that it would include the outward SMUs. Then, the process was repeated until the average grade within the circle fell below the cut-off grade. This is a clever approach in the sense that it both accounts for the dig-line constraint and tries to minimize the ore-waste boundaries hence the dilution/loss will be decreased. Moreover, they adapted their approach to work with different scenarios such as multiple ore types and different strategies and objective functions than net present value (NPV) maximization. Richmond (2004) proposed a local heuristic search algorithm that incorporates dig-limit considerations into open pit mine planning to minimize ore loss and mining dilution by using a pay-off function per block. However, greedy heuristic search methods and does not guarantee optimality. Wilde and Deutsch (2015) also proposed a greedy algorithm called Feasibility Grade Control (FGC) that takes an initial plan and attempts to optimize the profit iteratively by re-arranging the form that blocks are accumulated into units. In addition to greedy search drawbacks, FGC requires an initial dig-limit solution by the user.

A non-greedy search using the simulated annealing approach is proposed by Norrena and Deutsch (2000). They sought a balance of "accepting dilution" and "wasting ore" in order to maximize profit while satisfying the equipment constraints. Norrena and Deutsch (2002) developed a simulated annealing approach that maximizes the profit and penalizes smaller angles of operation. Then, they conducted a contest of manual dig-limit determination and compared the results with their computerized method. The method compared well to the outcomes of the contestants. Another simulated annealing based approach was suggested by Isaaks et al. (2014) where digline misclassifications are evaluated through loss functions constrained by dig-limits constraints. Simulated annealing is a solution space search algorithm that unlike greedy algorithms, moves towards non-improving solutions with a certain probability. This probability is determined by a temperature parameter T: where T is higher, the acceptance rate is higher. In the beginning, T is selected high to explore the solution space but as the algorithm progresses, T is decreased to have a higher chance of moving towards improving solutions. Although simulated annealing is claimed to reach the true optimal solution if T is selected high and decreased slowly enough (van Laarhoven and Aarts, 1987), in practice it generates near-optimal solutions. There also have been genetic algorithm approaches to solve the problem (Ruiseco et al., 2016; Ruiseco and Kumral, 2016). Genetic algorithms also share the same weaknesses although practically performs better than simulated annealing.

Tabesh and Askari-Nasab (2013) tried a different approach to solve the problem. They attempted to use hierarchical clustering to form mineable polygons that are homogenous in grades and rock types by calculating similarity indices for blocks. Although providing useful guidance to the engineer, the approach itself cannot create practical ore-waste boundaries, as the shapes of the clusters were not in control. A summary of all mentioned approaches as well as their results are provided in Table 4.1. As can be clearly observed, none of the results are optimal except the approach by Kumral (2015) which does not solve the problem of dig-limits. In this paper, a MILP model is proposed that solves the dig-limit problem optimally.

4.4 Model Formulation

Halfway of blast hole drill space on a bench is extended to each direction of blast holes. Thus, SMUs are created. The actual grade of each blast hole is assigned to its SMU. This model is submitted to optimization process.

As can be seen from Figure 4.4, a frame is moved throughout the bench sector. All SMUs within a frame is flagged with same identification as ore or waste. Red circles show the radius of maximum reachable arm of an excavator located in the center of the circle. This radius is then extended to appropriate shape.

If the equipment size were not a constraint in our problem, then the reasonable approach would be to divert SMUs with grades above the cut-off grade (ore SMUs) to an ore processing plant and SMUs below cut-off grade (waste SMUs) to a waste dump. Due to mine equipment size, several consecutive SMUs have to be mined and destined together. In other words,

Table 4.1 – A summary of the literature review

Authors	Approach	Results	Optimal
Kumral (2015)	minimize the loss associated with misclassification of SMUs using MILP	grade control realized successfully, but dig-limit constraints are disregarded	yes
Richmond and Beasley (2004)	floating circle: a heuristic approach	works with multiple ore types and scenarios	no
Richmond (2004)	local search algorithm	incorporates dig-limit constraints	no
Wilde and Deutsch (2015)	feasibility grade control: a greedy search approach	requires an initial solution, optimizes the profit iteratively	no
Norrena and Deutsch (2000)	simulated annealing	incorporates dig-limit constraints	near-optimal
Norrena and Deutsch (2002)	simulated annealing and penalizing small angles	performs well against manual designs	near- optimal
Isaaks et al. (2014)	simulated annealing	solves problem respecting the dig-limit constraints	near-optimal
Ruiseco and Kumral (2016)	genetic algorithms	solves problem respecting the dig-limit constraints	near-optimal
Ruiseco et al. (2016)	genetic algorithms	also works with multiple rock types, processes and metals	near-optimal
Tabesh and Askari-Nasab (2013)	hierarchical clustering	cluster shapes not controlled	no

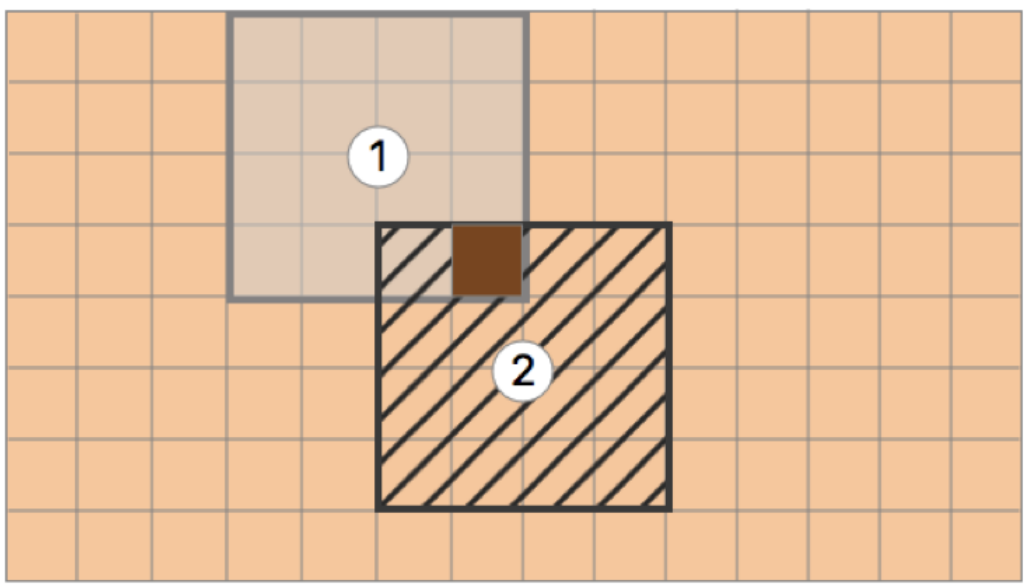


Figure 4.4 – Possible frames an SMU can belong to when the dig-limit width corresponds to 4×4 SMUs

$n \times n$ adjacent SMUs, which correspond to size of equipment, all need to be flagged as ore or waste.

The intuition behind this approach is that if we define equipment dimensions of $n \times n$ size as a frame, every SMU should belong in a frame where all SMU are ore or waste. In the literature, frame is sometimes called “moving windows”. An SMU can be positioned anywhere in a frame. Consequently, with $n \times n$ frame dimensions we can construct $n \times n = n^2$ probable frames, over the moving window, containing the specified SMU at the same time. An example of moving window with frame size 2×2 is demonstrated in Figure 4.5. A frame is called a valid frame if all of its SMU are ore or waste. In our problem, an SMU can belong in more than one frame but it should be

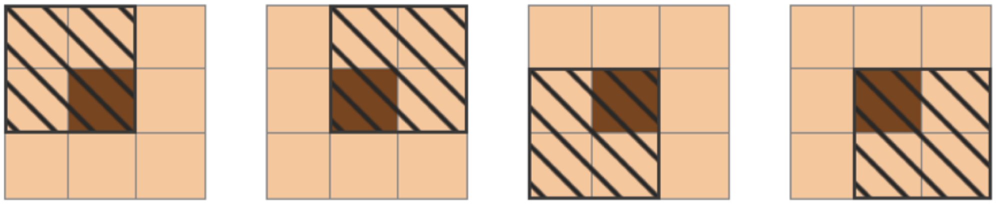


Figure 4.5 – All possible frames the SMU in the middle can belong to are illustrated where the frame dimensions are 2×2

Table 4.2 – Parameters in the model

Parameters	
X	Number of SMU in X direction
Y	Number of SMU in Y direction
m	SMU mass (tonne)
p	Metal price (g/ton)
r	Recovery (%)
C_m	Mining cost (\$/tonne)
C_p	Mineral Processing cost (\$/tonne)
n	Equipment width and length in terms of SMU
$g_{i,j}$	Grade of SMU at (i,j)

positioned in at least one valid frame. Although equipment size limits the search space, the overall objective is to maximize the revenue of the bench sector while satisfying the equipment size constraints.

Optimization model is defined as follows:

Maximize:

$$\sum_i \sum_j m\left[x_{i,j} g_{i,j} p\, r - x_{i,j} C_p - C_m\right] \tag{4.1}$$

Subject to:

Table 4.3 – Indices and their corresponding sets in the model

Indices	Sets
i	SMU index in X direction $(1, \ldots, X)$
j	SMU index in Y direction $(1, \ldots, Y)$
f_x	Frame index in X direction $(1, \ldots, n)$
f_y	Frame index in Y direction $(1, \ldots, n)$
α	Offset index in X direction in a frame $(0, \ldots, n-1)$
β	Offset index in Y direction in a frame $(0, \ldots, n-1)$

Table 4.4 – Decision Variables and their boundaries in the model

Decision variables		
Variable	Boundaries	Explanation
$X_{i,j}$	[0, 1]	1 if SMU at (i, j) is sent to process plant, 0 if disposed
t_{i,j,f_x,f_y}	[-1, n^2]	Total of x values inside a frame
v_{i,j,f_x,f_y}	[0, 1]	1 if a valid frame, 0 if not

Frame constraints:

For each possible frame where SMU at (i, j) may belong, total of xi,j inside a frame is equal to decision variable t_{i,j,f_x,f_y},

$$t_{i,j,fx,fy} = \sum_{\alpha} \sum_{\beta} x_{i-fx+\alpha, j-fy+\beta} \qquad \forall\ i, j, fx, fy \tag{4.2}$$

where $i - fx + n \leq\ X,\ j - fy + n \leq\ Y,\ i - fx \geq 0,\ j - fy \geq 0$

Decision variable t_{i,j,f_x,f_y} is converted to v_{i,j,f_x,f_y} by testing if the frame is valid.

$$v_{i,j,fx,fy} = \begin{cases} 1, & t_{i,j,f_x,f_y} = 0 \ \mid\ t_{i,j,f_x,f_y} = n \\ 0, & \text{otherwise} \end{cases} \tag{4.3}$$

Each SMU should have at least one valid frame.

$$\sum_{f_x}\sum_{f_y} v_{i,j,fx,fy} \geq 1 \qquad \forall\ i,j \tag{4.4}$$

Corner case handling:

As the corner SMUs belong in incomplete frames, these frames need to be disregarded and excluded from the valid frame computation.

$$t_{i,j,f_x,f_y} = -1 \qquad \forall\ i,j,f_y \tag{4.5}$$

where $i - f_x + n > X$ or $i - f_x < \ 0$ and

$$t_{i,j,f_x,f_y} = -1 \qquad \forall\ i,j,f_x \tag{4.6}$$

where $j - f_y + n > Y$ or $j - f_y < \ 0$.

Dig-limits problem is similar to layout problems in operational research such as cutting stock problem (Silva et al., 2014; Song and Bennell, 2014) and facility layout problem (Bernardi and Anjos, 2013) as it focuses on finding an optimal configuration of a set of rectangular facilities (its minimum is defined by equipment arm length) and placing them into a layout. Decision problem version of the dig-limits problem is NP-complete. This can be proved by reducing the knapsack problem to dig-limits problem. Knapsack problem is defined as (Bakirli et al., 2014):

Given a set $S = \{a_1, \ldots, a_n\}$ of non-negative integers, and an integer K, decide if there is a subset $P \subseteq S$ such that $\sum_{a_i \in P} = K$.

Feasibility version of the dig-limits problem can be defined as: A rectangular bench sector with dimensions $i \times j$ is composed of a layout of SMUs of equal sizes where $G = \{g_{1,1}, \ldots, g_{i,j}\}$ corresponds to grade of each SMU. Label each SMU as ore or waste such that each category should have at least $w \times \ w$ consecutive SMUs throughout the sector. Decide if there is a labeling for each $X = \{x_{1,1}, \ldots, x_{i,j}\}$ such that $\sum_{\gamma_i \in V} \gamma_i = PV$ where $V = \{f(g_{1,1}, x_{1,1)}, \ldots, f(g_{i,j}, x_{i,j})\}$, PV is the present value of the sector and $f(\cdot)$ is the function of economic value calculation given a grade and labeling: $m\,[x_{i,j} g_{i,j} p\ r - x_{i,j} C_p - C_m]$.

Before the reduction, it is important to note that each set of $w \times w$ consecutive SMUs of the four corners of a feasible rectangular sector will always have consistent labeling. Following this, all the border squares with size $w \times w$ will have consistent labeling. Also, a sector with size $iw \times \ jw$ has $2i + 2j - 4$ border squares.

Assume we have a black-box algorithm to solve the feasibility dig-limits problem. Given an instance of the knapsack problem $S = a_1, \ldots, a_n$ a sector can be constructed with size $iw \times \ jw$ such that $2i + 2j - 4 \geq \ n$. The parameters related to the sector will be $p = 1$, $r = 1$, $C_m = 0$ and $C_p = 0$. The corner SMUs of n border squares will be assigned a grade $b_{1,1} = a_1$, $b_{1,w+1} = a_2$,$\ldots$, $b_{i-w+1,j-w+1} = a_n$ such that $f(b_{1,1}, \text{ore}) = a_1$, $f(b_{1,w+1}, \text{ore}) = a_2$,$\ldots$,$f(b_{i-w+1,j-w+1}) = a_n$. All other SMUs in the border squares will be assigned a grade of 0. As the mining cost of the sector $C_m = 0$, for all blocks

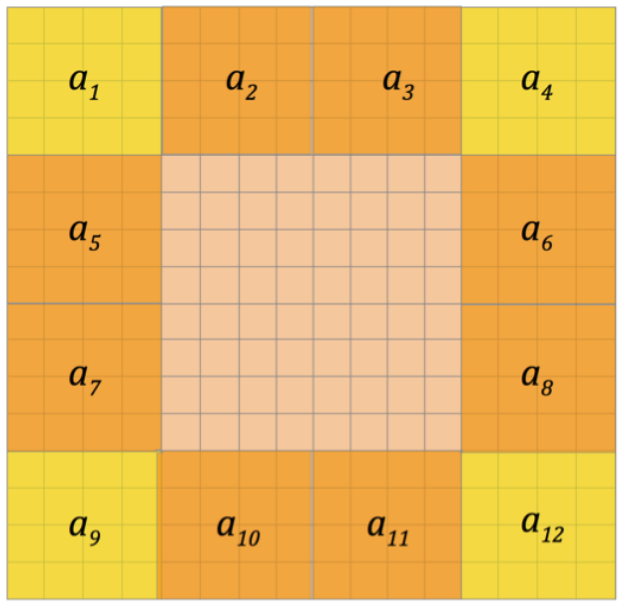


Figure 4.6 – Sample transformation from knapsack problem to dig-limits decision problem

$f(b, \text{waste}) = 0$. An example transformation for a knapsack problem with 12 items can be visualized in Figure 4.6.

This reduction can clearly be done in polynomial time. When the black-box algorithm is called to solve the formed dig-limits problem, the resulting ore-waste decision will correspond to subset P of the knapsack problem. In other words, dig-limits decision problem is at least as hard as the knapsack problem. Given a feasible sector ore-waste discrimination plan, the verification $\sum_{\gamma_i \in V} \gamma_i = PV$ can also be completed in polynomial time, proving this problem is in NP. It can be concluded that the dig-limits optimization problem is NP-hard (non-deterministic polynomial-time hard).

Table 4.5 – Parameters for all sectors

p	\$40/ gr
C_m	\$15/ tonne
C_p	\$15/tonne
r	100%
m	100 tonnes
n	4 SMUs

4.5 Case Study

To demonstrate the performance of the proposed approach, a case study was carried out on one-week production including seven sectors of a bench in a gold deposit. The standard procedure in this mine is to drill blast-holes on the planned sector. The samples taken from blast holes are sent to the laboratory to assay grades. Before production starts on a sector, a mining geologist determines dig limits manually. After the sector is blasted and dilution is incorporated into block identification, manually defined ore clusters within the sector are flagged to destine to mineral processing plant. The sizes of ore clusters should be large enough to allow equipment movements. Manual determination of dig-limits can be subjective and may undervalue the operation. All parameters used in the case study is given in Table 4.5. To show effect of dilution, recovery is taken as 100%. Otherwise, the quantification of ore losses associated with blasting will be more difficult. In this mine, to have the required rock fragmentation, blast hole spacing is selected as 4 m approximately. The excavator needs at least 4 x 4 block ore and waste clusters.

The results of dig limit determination in seven consecutive sectors are provided as below.

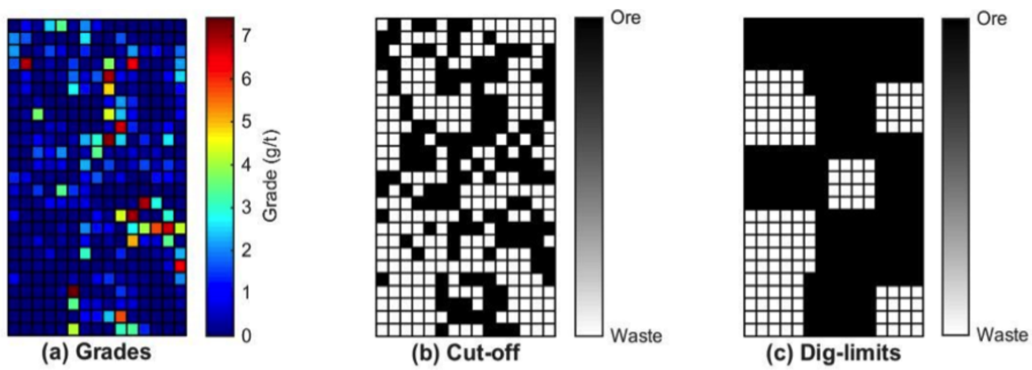


Figure 4.7 – (a) Grade distribution, (b) ore and waste discrimination after cut-off applied, (c) ore – waste discrimination after dig limit optimization (Sector 1)

Sector 1 Sector 1 (Figure 4.7) has 375 SMUs (25 x 15). Ore and waste SMUs are shown by black and white, respectively. In this sector, reddish SMUs are distributed throughout the sector.

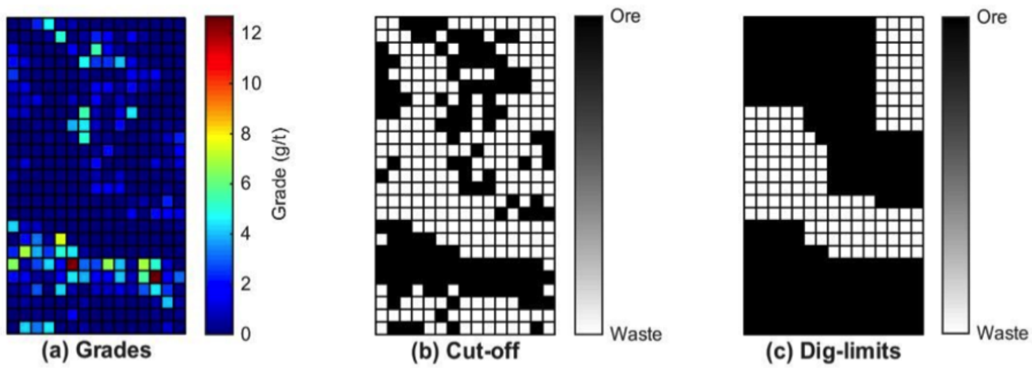


Figure 4.8 – (a) Grade distribution, (b) ore and waste discrimination after cut-off applied, (c) ore – waste discrimination after dig limit optimization (Sector 2)

Sector 2 Sector 2 (Figure 4.8) comprises very high grade ore (please see that the highest grade about 12 g/t). Even though the number of high grade SMUs are low, they have sufficient metal to support surrounding relatively low grade material. Thus, the effect of dilution and loss can be minimized. In the southern part of the sector, there are big waste clusters. However, in north – south direction, three waste SMUs are laid down at maximum. Given that equipment arm needs at least four SMUs. There would be two possibilities: (1) three ore SMUs northing this waste cluster will be waste, (2) a part of this waste cluster will be ore. As can be seen from the optimum limits, this waste cluster is assessed as ore.

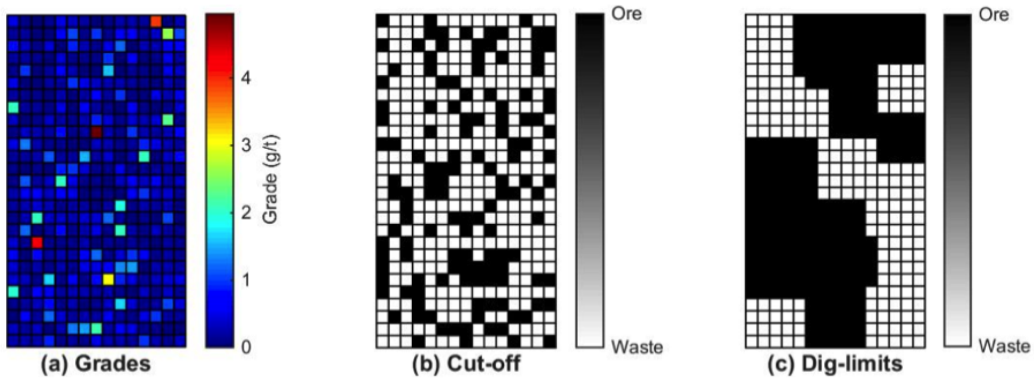


Figure 4.9 – (a) Grade distribution, (b) ore and waste discrimination after cut-off applied, (c) ore – waste discrimination after dig limit optimization (Sector 3)

Sector 3 Sector 3 (Figure 4.9) contains moderate or low ore material. The highest grade in this sector is about 5 g/t. Also, ore is more evenly distributed. Unlike Sector 2, there are no large ore clusters. Therefore, this case would be very difficult for the engineer, who will draw dig limits manually. The proposed approach has significant potential to improve this type of sectors.

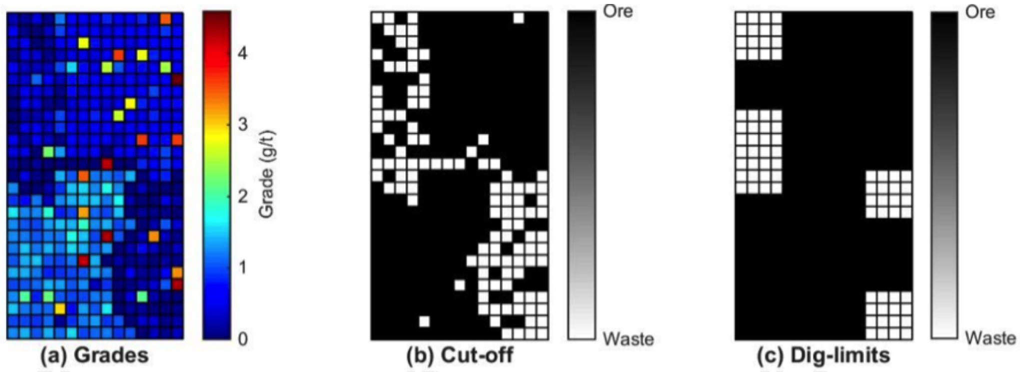


Figure 4.10 – (a) Grade distribution, (b) ore and waste discrimination after cut-off applied, (c) ore – waste discrimination after dig limit optimization (Sector 4)

Sector 4 Sector 4 (Figure 4.10) contains low grade material. There are large ore clusters within the sector. The highest grade is about 4.5 g/t. However, ore and waste boundaries are clearer. This is a relatively easy sector for the engineer.

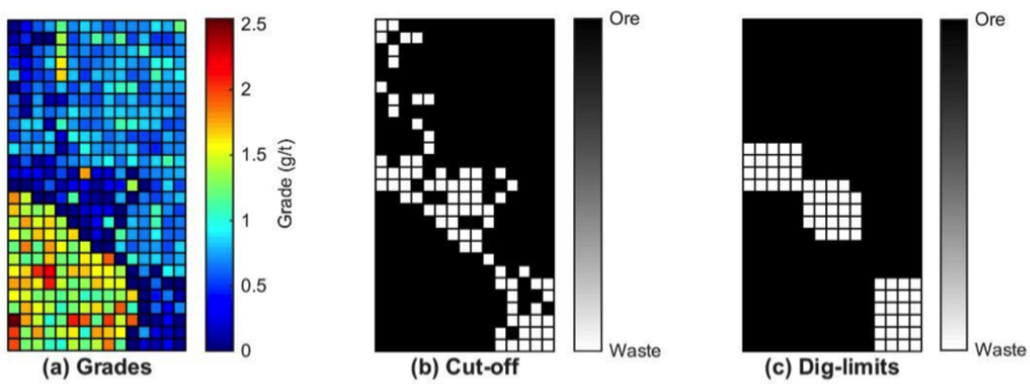


Figure 4.11 – (a) Grade distribution, (b) ore and waste discrimination after cut-off applied, (c) ore – waste discrimination after dig limit optimization (Sector 5)

Sector 5 Sector 5 (Figure 4.11) is a sector having mostly low grade ore. Unlike the sectors having high grade material, ore SMUs within waste cluster (southeast of the sector) are included to waste clusters.

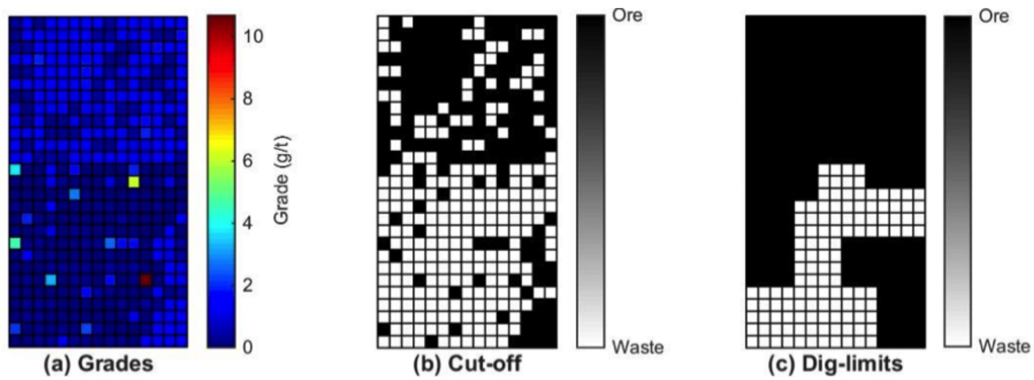


Figure 4.12 – (a) Grade distribution, (b) ore and waste discrimination after cut-off applied, (c) ore – waste discrimination after dig limit optimization (Sector 6)

Sector 6 As a consequence of sector 6 (Figure 4.12) having high grade material, ore and waste boundaries are quite clear. Given that cut-off grade is low, owing to green blocks on the west, waste material surrounding these blocks are included in ore cluster.

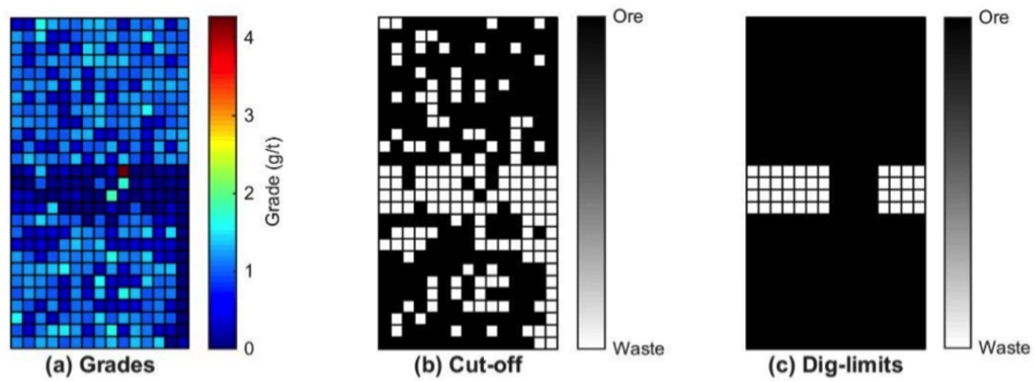


Figure 4.13 – (a) Grade distribution, (b) ore and waste discrimination after cut-off applied, (c) ore – waste discrimination after dig limit optimization (Sector 7)

Sector 7 In Sector 7 (Figure 4.13), grades are distributed higher in the upper and lower sections whereas the middle section has very low grades except a few very high grades in the center. This clearly reflected to the dig-limits design as only the center of the middle section is flagged as ore and the rest is flagged as waste.

The resulting economical values of each sector after the dig-limits optimal design is performed are summarized in Table 4.6. In accordance with the grade mappings of the sectors, sectors with higher average grades resulted in higher profits. As can be observed from the table, all sectors except Sector 3 have positive values. The reason for this negativity is the overall lower grades in this sector. In order to access underlying sectors, this sector has to be extracted and the only decision that can be made to reduce loss is to send the parcels to process or to waste dump. In this case, the model searches for the minimum loss because a profit is not feasible while satisfying

Table 4.6 – Parameters for all sectors

Sector	Economical Value
1	\$202,303
2	\$365,188
3	\$-164,708
4	\$129,348
5	\$137,200
6	\$92,112
7	\$141,980

the dig-limit constraints.

For comparison, a mining engineer has been asked to draw dig limits manually, given the grades and the cut-off map of Sector 2 (Figure 4.8 (a) and (b)). The resulting manual design and its comparison to optimal dig limits from Sector 2 are given in Figure 4.14. Even though manual limits managed to capture main patterns, it is still not optimal as the model optimization output yielded \$365,190 and the manual sector design yielded \$341,640 (6.5% below the optimal value).

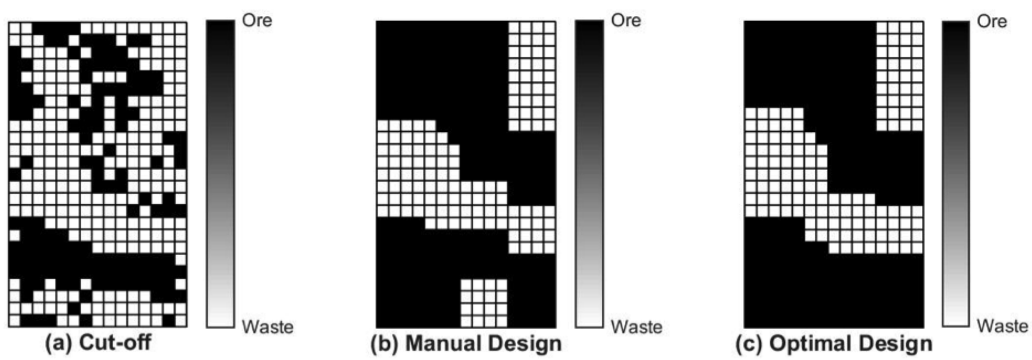


Figure 4.14 – Manual and optimal dig-limits on a sector. Manual design yielded the value of \$341,640 whereas optimum design yields \$365,190.

To summarize, the cut-off grade, ore-waste distribution within the sector and the grade range will affect the speed of optimization process. Due to dilution/loss, in addition to an SMU itself, the surrounding SMUs should be taken into consideration. Figure 4.15 illustrates the dilution effect on free ore-waste selection based on cut-off grades by comparing it to the same effect on selection based on dig-limit constraints. This graph is obtained by averaging the deviations of all 7 sectors. Clearly, dig-limits optimization is affected by dilution much less than free selection. Moreover, at around 10% dilution, dig-limit constraints optimization yields higher mine values.

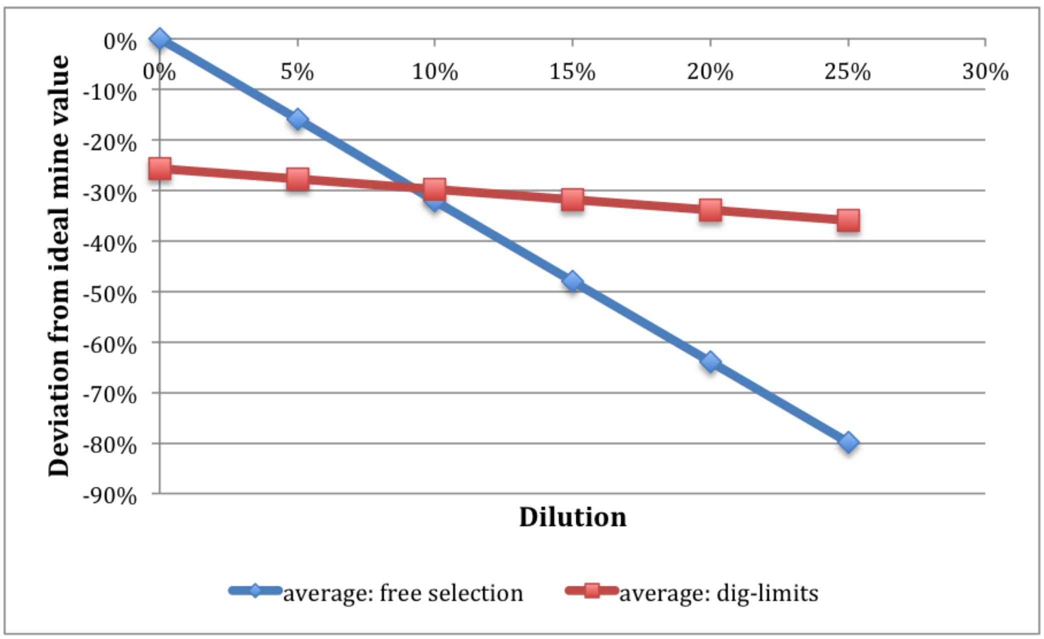


Figure 4.15 – The effect of dilution on the mine value in two cases: (1) free selection based on the cut-off grade, (2) selection based on dig-limit constraints

4.6 Conclusions

Recent low commodity prices and cost increase attributed to growing technical challenges force mining companies to increase performance and efficiency of operations. In this paper, as a part of short-term mine planning, optimal dig limits are determined by MILP model such that practical open pit operations are fulfilled in sense to be compatible with maneuvering capability of the excavator and minimize dilution/loss as opposed to free selection of ore and waste SMUs based on the cut-off grade. The problem has been proved to be NP-Hard and it is formulated as maximization of the profit to be obtained from a sector such that all SMUs are fallen into a frame defined as ore or waste. The size of a frame is defined by the distance that excavator's arm can extend. In addition to this, dilution and loss are decreased because due to clustering structure of the dig-limit constraints, the number of ore – waste contacts are reduced. The proposed approach is especially useful for mining operations where the grade distribution throughout the orebody is highly heterogeneous and dilution is significant. The resulting short-term plans are optimal which will most probably provide better output than manually drawn plans. Even though the professional may be able to produce optimal plans, this computerized approach will still prove useful by saving the professional's time.

At the moment, the biggest challenge is computing time to solve the problem. Future research should focus on reducing problem size. If one develops iterative or heuristics approaches to address the problem, the proposed model can be used as a reference to test the performance of these approaches, which cannot guarantee the optimality. In our study, our objective function aims to maximize the economic value of the sectors. Alternative objectives may consist of minimizing the deviations from an expected ore grade, minimizing the dilution, best satisfying processing plant capacity, maximizing profit given a level of risk or minimizing the number of clusters such that transportation costs are minimized. The research will be also extended to incorporate controllable blasting option that ore and waste patches are fragmented separately. Finally, multiple metals and multiple processing options will be included in the formulation.

4.7 Chapter Conclusion

In this publication, an optimization method that potentially increases the project value is introduced. In addition to satisfying the equipment constraints, it decreases the effect of dilution which is in practice more than 10% many cases. Aside from guaranteeing the optimal solution unlike manual solutions, it saves time and resources for the engineers. This approach can easily be extended to poly-metallic deposits. As mine benches are not composed of many SMUs, the problem contains less decision variables than the surface mine planning problem. Consequently, it can be solved much faster. MILP suits the dig-limit optimization problem very well.

Stope layout optimization, on the other hand is more complicated than the dig-limits problem. First of all, it contains more decision variables (more blocks). Also, the three-dimensional nature of the stope layout optimization problem increases the number of constraints drastically. Therefore, for stope layout planning, MILP is impractical as it would take too much time and memory. When exact methods cannot handle the size of a problem, usually heuristic methods are used. In the next section, a heuristic clustering technique that identifies ore-concentrated regions of the deposit and prioritizes their extraction is proposed.

5 A new heuristic approach to stope layout optimization for the sublevel stoping method in underground mines

5.1 Abstract

Underground mining operations require high operation costs. When metal prices decrease, production sustainability is jeopardized due to high costs. Therefore, the mining management focuses on the practices that increase the efficiency of operations. One way to manage this is to invest in mine planning practices. Stope layout optimization as a part of underground mine planning aims to find orebody portion in form of production volumes called stopes to maximize profit under roadway and stope dimension constraints. This paper proposes a novel approach based on identifying ore-concentrated regions of the deposit and prioritizing their extraction through a heuristic clustering approach. The proposed heuristic was compared with an exact method through a small instance. The heuristics produced almost the same results in a very short time. Finally, using a larger data set, a case study was carried out. This approach generates the near-optimal stope layouts in a computationally effective manner.

5.2 Introduction

Historically, surface mines had constituted the majority of the worldwide mining operations. The main reasons were that deposits closer to the surface were discovered earlier than deeper deposits and surface extraction techniques usually have less safety concerns compared to underground mining techniques. Underground mining has complicated engineering considerations such as rock stress calculations and air distribution through appropriate ventilation. Also, surface mine operations allow higher ore-waste selectivity and it takes advantage of economies of scale, decreasing extraction costs. However, at present, this trend is moving towards underground mining because of the following reasons: (1) A lot of the deposits near the surface have been depleted, (2) overburden waste rock in surface mines is much less or does not exist in underground mining; lower stripping ratio generates higher profits as it lowers the mining and waste handling costs. (3) Another consequence of this is that overall, less material is extracted and all extracted material is produced, therefore environmentally, underground mines cause less impact than surface mines. Mining projects are already very risky due to high uncertainty related to grade distribution and volatile commodity prices (Sauvageau and Kumral, 2017), emphasizing the importance of operational optimization. Although underground mining is considered as it is governed by rock mechanics and there is no room for optimization, the recent prevalence of underground mining techniques emphasizes the importance of computer-aided tools for planning and layout optimization for maximizing the profit and minimizing the environmental impact.

Sublevel stoping is an unsupported underground mining technique that is typically used when the orebody is modular, steep, thick and large in size (Hartman and Mutmansky, 2002). Additionally, the rock substance strength should be medium to strong (Nicholas, 1981). Orebody is accessed through underground access roads from the shaft called levels and in between levels, rectangular extraction areas known as stopes are determined and accessed through sublevels. When the development is completed, first, the stope is drilled from several access points and blasting takes place. The comminuted rock collapses to the bottom of the stope, where it is carried from the draw points to the shaft by the haulage trucks. The minimum and maximum stope length, width and height are determined according to the rock characteristics by the geological engineer. Sublevel stoping is composed of two main problems: stope layout planning and stope sequencing. Stope layout planning is concerned about positioning the sublevels and stopes as well as deciding stope dimensions in such a way that the profit is maximized. Stope sequencing aims to decide on the ordering of the mining of the stopes considering the mine stability, equipment transportation and net present value maximization.

The majority of the current computerized techniques approach the stope layout planning problem by partitioning the orebody into a block model where each block is estimated/simulated an ore grade based on the samples obtained from the drill holes. Then, the grades are converted to economic values using the parameters such as ore price, mining and processing costs. This conversion facilitates the evaluation of the blocks because the profitability of a possible extraction can directly be recognized. The blocks are selected to be extracted such that the profit is maximized and the stability constraints are not violated. These constraints are explained in detail in the formal problem definition section.

In this paper, we introduce a heuristic algorithm that is inspired by the practical approach implemented by mining engineers to plan the stope layout. The originality of this paper is two-fold: (1) A new formulation of the stope layout optimization problem is proposed, and (2) this problem is solved by a new three-stage approach that forms a block model, places the sublevels, then the stopes using clustering heuristics.

This paper is organized as follows: in the next section, different approaches to stope layout planning are discussed. In Section 5.4, the stope layout optimization problem is defined mathematically with a new formulation. In Section 5.5, the proposed heuristic is presented in detail and the corresponding mathematical model is given. In Section 5.6, the approach is tested with a case study and a comparison to the mixed-integer programming model formulation is provided. Finally, in Section 5.7, findings are summarized and conclusions are given.

5.3 Literature Review

Approaches to stope layout planning are mainly composed of three categories: exact methods, approaches using geologic models and heuristic algorithms (Ataee-Pour, 2005). The exact methods are based on mathematical approaches, geologic models approach the problem three-dimensionally and work with block grades directly rather than converting the grades into economic values, and heuristic algorithms provide a fast but non-optimal solution.

Underground mining problem is more difficult than surface mine planning having the same amount of decision variables but more constraints. Exact methods ideally yield optimal results but practically they are not able to handle large deposits as they either run out of memory or simply take a very long time to solve which is impractical unless heuristic approximations are made. Typically a block model consists of thousands to millions of blocks. Generally, to deal with the large number of decision variables, exact methods are modified such that they are faster but non-optimal. Jalali and Ataee-pour (2004) presented a dynamic programming approach for vein type ore bodies based on the modification of algorithm by Johnson and Sharp (1971) for the open-pit layout optimization. Instead of taking into consideration the slope constraints as in open pit mining, a maximum variation of the elevation of both the floor and the ceiling from one column to the next is allowed for draw control. The algorithm provides 2-D solutions by combining column economic values that are perpendicular to the vein direction. Ovanic and Young (1995) introduced the branch and bound technique to optimize starting and ending points at each row of blocks. Two piecewise linear cumulative functions at each row, representing the physical location of starting and ending points are declared and the problem is solved using a mixed integer programming (MIP) approach. This approach, known as SOS2 (Type-Two Special Ordered Sets), also called as separate programming, allows at most two adjacent ordered set of variables to be non-zero. Although row by row this approach is optimal, this does not guarantee overall optimality. Bai et al. (2013) followed a very different approach where the model is defined on a cylindrical coordinate around the initial vertical raise. Blocks are converted into nodes and a source and a sink are added to the model, then solved by the maximum flow approach. Having a vertical raise limits the optimality in cases where the orebody is inclined, results in including too much waste, and in cases where the deposit is larger and more than raise will be required.

Approaches using geologic models consist of octree division approach and downstream geostatistical approach. Octree division approach (Cheimanoff et al., 1989) recursively divides the 3-D model into two equal parts in each dimension resulting in eight subvolumes and includes the subvolume in the final stope layout if it is valuable throughout. Because partial stopes are not allowed during the optimization and the algorithm works by equal division of volumes, stope locations are checked only where the minimum stope dimension is a proper divisor. Downstream geostatistical approach (Deraisme et al., 1984) uses dynamic programming on the 2-D sections of the drilling and blasting data to minimize the dilution.

Heuristic methods, being fast and practical, are most commonly used in the industry. Alford (1995); Alford et al. (2007) proposed an algorithm that floats a potential stope with minimum sizes through the block model. All economical stopes are included in the final stope layout plan. However, the problem arises when two stopes overlap. Also, different results are obtained depending on the starting point of the floating process. Similarly, Ataee-pour (1997); Ataee-Pour (2004) examines the neighborhood of each block in sequence and among all possibilities, the neighborhood with maximum economic value is included in the final stope. Topal and Sens (2010) proposed a preference based profit maximization approach that creates a list of all possible stopes and chooses from them according to the user preference. Wang and Webber (2012) implemented a two-stage approach where the rings that do not contain ore are filtered out and the design is completed manually. Sandanayake et al. (2015b) developed an algorithm that incorporates stope size variation by aggregating the mining blocks into possible set of stopes then modifying the attributes of stopes.

5.4 Formal Problem Definition

Ore deposits are conceptually divided into uniform rectangular grids that are called mining blocks. In a deposit, each potential stope has a certain economic value as it contains a unique set of blocks where each block has a predicted ore grade. Stope layout planning problem is concerned about placing non-overlapping three dimensional stopes in a deposit within constrained dimensions such that the economic value of the deposit is maximized. An additional constraint arises from the construction of the access roads below and above stopes known as sublevels: stopes should be vertically aligned.

Throughout the paper, four mathematical models are presented that share the notation given in Table 5.1. In addition to the shared list of notations, the notation unique to each particular mathematical model is presented following the model.

Table 5.1 – List of notations regarding the mathematical models

Notation	**Explanation**
X	Set of blocks in X direction
Y	Set of blocks in Y direction
Z	Set of blocks in Z direction
g_i	Grade of metal i within a block
R_i	Mill recovery of metal i within a block
F_{eq_i}	Equivalent factor of metal i with regards to the primary metal
p_i	Price of metal i
C_l	Cost of establishing a sublevel
M	The set of metal contained in the deposit
l	Number of layers used for calculating block score
α	Number of sublevel combinations to be considered
β	Number of stope combinations to be considered
$s_{x,y,z}$	Score of the block at coordinates x, y, z
γ_z	Score of the level z
x_a	Minimum number of blocks in a stope in X direction
x_b	Maximum number of blocks in a stope in X direction
y_a	Minimum number of blocks in a stope in Y direction
y_b	Maximum number of blocks in a stope in Y direction
z_a	Minimum number of blocks in a stope in Z direction
z_b	Maximum number of blocks in a stope in Z direction

A new formulation of the stope layout planning problem is presented as below:

Maximize:

$$\sum_{i,j,k}\sum_{\chi=x_a}^{x_b}\sum_{\psi=y_a}^{y_b} y(i,j,k,\chi,\psi,\theta)\times\sum_{q_x=0}^{\chi-1}\sum_{q_y=0}^{\psi-1}\sum_{q_z=0}^{\theta-1} v(i+q_x,j+q_y,k+q_z) \tag{5.1}$$
$$\text{where } i+q_x-1\leq X, j+q_y-1\leq Y, k+q_z-1\leq Z$$

Subject to:

$$\sum_{\chi=x_a}^{x_b}\sum_{\psi=y_a}^{y_b} y(i,j,k,\chi,\psi,\theta)\leq 1, \qquad \forall i,j,k \tag{5.2}$$

$$\sum_{\substack{\chi=x_a\\ \chi\neq 0}}^{x_b}\sum_{\substack{\psi=y_a\\ \psi\neq 0}}^{y_b} y(i+q_x,j+q_y,k,\chi,\psi,\theta)+\sum_{\chi=x_a}^{x_b}\sum_{\psi=y_a}^{y_b} y(i,j,k,\chi,\psi,\theta)\leq 1,$$
$$\forall i,j,k \text{ and } \forall q_x\in\{-x_b+1,x_b-1\}, q_y\in\{-y_b+1,y_b-1\} \tag{5.3}$$
$$\text{where } 0<i+q_x+\chi-1\leq X, 0<j+q_y+\psi-1\leq Y$$

$$\sum_{\chi=x_a}^{x_b} \sum_{\psi=y_a}^{y_b} \sum_{\theta=z_a}^{z_b} \left(\sum_{q_z=z_b}^{-1} y(I_1, I_2, k+q_z, \chi, \psi, \theta) + y(i, j, k, \chi, \psi, \theta) \right) \leq 1$$
$$\forall i, j, k, \forall I_1 \in \{1, X\}, I_2 \in \{1, Y\} \tag{5.4}$$
$$\text{where } I_1 + \chi - 1 \leq X, I_2 + \psi - 1 \leq Y, k < k + q_z + \theta \leq Z$$

In this model, X, Y, Z are number of blocks in X, Y and Z directions respectively, i, j, k are sets of starting coordinates of all valid stopes in the block model, θ is the stope height that is determined previously, q_x and q_y represent offset from the starting coordinates, $y(i, j, k, \chi, \psi, \theta)$ is the decision variable that determines the extraction of stope at the coordinate i, j, k with the sizes χ, ψ, θ, and $v(i, j, k)$ is the economic value of the block at coordinates i, j, k. The objective function in Equation 5.1 maximizes the total economic value of the stopes that are decided to be extracted by the model. Equation 5.2 ensures that only one size can be accepted per stope. Equation 5.3 checks for overlapping stopes in X-Y directions and only allows one of the overlapping stopes to be selected. The additional constraint for preventing overlapping stopes vertically is given in Equation 5.4. However, this constraint is different from the X-Y direction overlap constraint given in Equation 5.3 because the overlap should be avoided not only directly on the stope but also throughout the Z level. As explained earlier, this is important for stable sublevel formation. In this model, sublevel construction cost is assumed to be included in the mining cost. In common practice, due to high costs of building sublevels, inter-distance of sublevels is kept as large as possible and constant within each geological domain. In this case, z_a can be set equal to z_b to simplify the problem.

The stope layout design problem resembles 3D container loading problem (Zhao et al., 2016; Zhu et al., 2017) and it is most similar to capacitated clustering problem (CCP) in operations research where a specified number of clusters are formed from a set of elements with certain weights. Total cluster weight is restrained within a lower and an upper limit. Each pair of elements has a predefined benefit that contributes to the objective value only if the pair is in the same cluster and the objective is to maximize the overall benefit (Osman and Christofides, 1994). CCP can be transformed into stope layout design problem by representing possible stopes as elements with weights of one and setting the maximum weight constraint as infinite and minimum weight constraint as zero. Additionally, the number of clusters should be equal to the number of possible stopes, the predefined benefit of a pair of elements should be assigned in proportion to their sum of economic values or be infinitely negative if two stopes intersect or align in an overlapping fashion on Z axis. When the CCP is solved using these inputs, stopes in the cluster that provide the highest benefit are to be extracted. The reasoning behind this is that stopes that can be selected together (not overlapping or intersecting) will be forced in the same cluster because as the overall benefit increases when the number of pairs increases. If two stopes overlap or intersect, their benefit is negative. Thus they will not be in the same clusters.

As there are as many clusters as stopes, these stopes will be distributed in different clusters. Also, the way the weight capacities are set, allow clusters to have from zero to infinite stopes, not limiting the number of stopes in the mine plan.

5.5 Heuristic methodology

When the linear model of a problem contains an excessive number of variables and constraints, the solving time is impractically long, heuristic methods are preferred (Park and Seo, 2017; Shyshou et al., 2012). As can be observed from the previous section, the vertical alignment condition increases the number of constraints drastically. However, it can be anticipated that with regards to the minimum and maximum stope height constraints, there are relatively few possible combinations of sublevels. Hence, a heuristic that segregates sublevel design and stope layout design is proposed. When sublevel design is carried out previously, the alignment constraints can be eliminated (decreasing the number of constraints) and the problem is divided into smaller problems (decreasing the number of variables). This is realized with the aid of a clustering heuristic in which the value of a block depends not only on its grade but also on the grades of neighboring blocks. The heuristic aims to identify the high ore concentrated sections of the deposit and extract these volumes as stopes, taking into account the structural feasibility of a mine. With this information, first, sublevels are determined, followed by the decision of stopes in between the sublevels. The summary of the three-stage heuristic approach

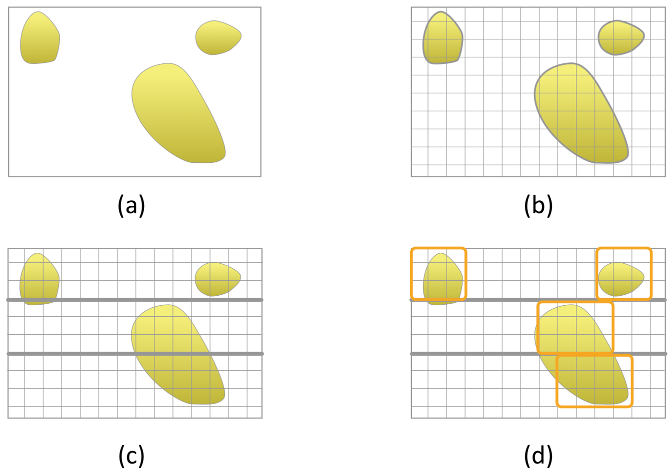


Figure 5.1 – Three-stage summary of the clustering heuristic approach. (a) Initial view of the deposit where the shaded areas denote ore-concentrated regions. (b) Preparation of the model by obtaining the block model and generating the block scores and detecting clusters. (c) Sublevel design through score ranking and selecting the best combination. (d) Stope layout design level by level.

is provided in Figure 5.1. Table 5.1 provides the notation used in this section.

5.5.1 Preparing the model

The input consists of a block model with grades. The identification of concentrated sections is realized through the analysis of the block grades. Each block is assigned a score according to the grade of a given block and the grade of the surrounding blocks. Each set of surrounding blocks of a given shape is called a layer. If only the immediate blocks that are adjacent to the given block are considered, the depth of the surrounding layer becomes 1.

The number of surrounding layers can be increased by considering the next set of blocks adjacent to the previous layer and their grades are also added, optionally multiplying by a discount factor at each increasing level. This is demonstrated in Figure 5.2, in which three layers are framed. This way, the heuristic mimics the clustering approaches. With this heuristic score, each block contains information about its grade and its strategic location. The depth of the layer surrounding the block is customizable in the program. The number of layers is closely related to stope size. If the depth is set to a feasible stope size, each block will contain the heuristic score for the stope where the block is centered. If there is more than one desired metal in the block, secondary metals are converted in terms of the first metal by using the equivalent grade Equation 5.5 and the grades are converted by using Equation 5.6. Observe that for the first metal, $F_{eq_1} = 1$, resulting in $g_{eq} = g_1 + \sum_i g_i \times F_{eq_i}$ where $i \in m$ and $i \neq 1$.

$$F_{eq_i} = \frac{p_i \times R_i}{p_1 \times R_1}, \qquad \forall i, i \in m \tag{5.5}$$

$$g_{eq} = g_i \times F_{eq_i}, \qquad \forall i, i \in m \tag{5.6}$$

Block scores are calculated according to Equation 5.7. After the block scores have been calculated, the block scores on each level of the deposit are

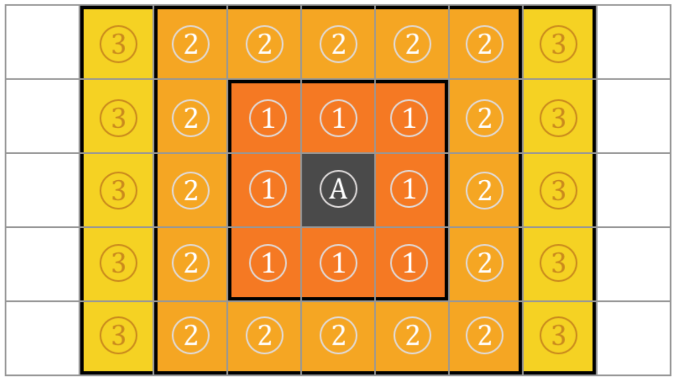


Figure 5.2 – An example of layers surrounding block A in a two dimensional the block model with the size 9x5. In this case, where the depth of layers (l) is 3, to calculate the block A's score, the grades of all the layers are added to the grade of A, optionally multiplying by a discount factor at each level. In a three dimensional block model, all surrounding blocks in each direction are included in a layer.

computed by adding the scores of the blocks on the corresponding level as given in Equation 5.8 and saved as sublevel candidates.

$$s_{x,y,z} = \sum_{i=-l}^{l} \sum_{j=-l}^{l} \sum_{k=-l}^{l} g_{eq_{x+i,y+j,z+k}}, \qquad \forall x \in X, y \in Y, z \in Z \tag{5.7}$$

$$\gamma_z = \sum\nolimits_x \sum\nolimits_y s_{x,y,z}, \qquad \forall x \in X, y \in Y, z \in Z \tag{5.8}$$

5.5.2 Sublevel design

The selection of sublevels is a significant stage in the planning as it influences the succeeding decisions. Practically, sublevels are selected where the ore concentration is high such that the stopes will also have high average grades. Considering that sublevels are the roads of access, and the sublevels are also extracted, sublevels are generally selected in ore concentrated regions as it paves way to more profitable potential stopes. This heuristic is inspired by this practical approach and attempted to optimize it by making use of computational tools. In the current stage, it is important that the height of the sublevels should satisfy the minimum stope heights constraint.

Mathematically, sublevel design can be expressed as follows:

Maximize:

$$\sum_{k=1}^{Z-z_a} \sum_{\theta=z_a}^{z_b} \sum_{\delta=k}^{k+\theta} \left(y(k,\theta) \times \sum_{i=1}^{X} \sum_{j=1}^{Y} v(i,j,\delta) \right) - \sum_{k=1}^{Z-z_a} \sum_{\theta=z_a}^{z_b} y(k,\theta) \times C_l \quad \text{where } k+\theta \leq Z \tag{5.9}$$

Subject to:

$$\sum_{\theta=z_a}^{z_b} y(k,\theta) \leq 1, \qquad \forall k \in \{1, Z-z_a\} \tag{5.10}$$

$$\sum_{\delta=k}^{k+\theta}\sum_{h=z_a}^{z_b} y(\delta,h) - \sum_{\lambda=z_a}^{\theta-1} y(k,\lambda) \leq 1, \qquad \forall k \in \{1, Z-z_a\}, \theta \in \{z_a, z_b\} \quad \text{where } k+2\theta \leq Z \tag{5.11}$$

In Equations 5.9-5.11, k represents the starting block of a stope in Z direction, θ represents the height of the stope, $y(k,\theta)$ is the decision variable that selects a sublevel and the maximum heights of stopes accessed from that sublevel, and $v(i,j,\delta)$ is the economic value of the block at coordinates i,j,δ. The objective function at Equation 5.9 maximizes the total value of the sublevel by summing the economic values of each block within the range of the height of the sublevel. Equation 5.10 expresses that only one size of maximum stope height can be accepted below each sublevel. Equation 5.11 ensures that if a sublevel with a certain stope height is selected, an overlapping level cannot be selected. As a result of this stage, a combination of sublevels will be output. The best scoring solution of the candidate sublevel sets is chosen. This stage of the approach resembles the layer building heuristics for 3D container loading problem (Zhao et al., 2016). As mentioned in Section 5.4, due to high costs of building sublevels an assumption may be made to simplify the problem: inter-distance of sublevels may be kept as large as possible and constant within each geological domain. For this reason, the mathematical model can be simplified as following:

Maximize:

$$\sum_{k=1}^{Z-z_b}\sum_{\delta=k}^{k+z_b}\left(y(k)\times\sum_{i=1}^{X}\sum_{j=1}^{Y}v(i,j,\delta)\right)-\sum_{k=1}^{Z-z_b}y(k)\times C_l \quad \text{where } k+z_b \leq Z \tag{5.12}$$

Subject to:

$$\sum_{\delta=k}^{k+z_b} y(\delta) \leq 1, \qquad \forall k \in \{1, Z-z_b\} \quad \text{where } k+2z_b \leq Z \tag{5.13}$$

The heuristic algorithm that selects the sublevels is designed as follows:

1. Only the levels that can possibly satisfy the stope height constraints are selected as candidates to speed up the search. The non-satisfying levels are eliminated.
2. The remaining levels are ranked according to their scores γ_z.
3. A candidate solution is created by taking the first level in the ranked list, then adding the following levels in the list as long as the candidate solution is feasible. If the addition of a level makes the candidate solution infeasible, the next level is added until the list is exhausted. The feasibility is tested by verifying each level has at least the minimum stope height.
4. The overall score of the candidate solution is calculated by averaging the scores of levels in the solution and multiplying by the number of

levels that can be accessed through the sublevels.

5. If this is the first calculated score, or it is the highest score so far, it is stored as the current best solution. Otherwise, the candidate solution is deleted.

6. The first level in the ranked list is deleted. If there are no more levels in the list or the number of formed level combinations is equal to α, the algorithm is terminated. Otherwise, the algorithm returns to Step 3.

5.5.3 Stope layout design

Given the sublevels with the above approach or manually, the stopes are planned. In mixed integer linear programming terms, stope layout design between sublevels can be expressed as follows:

Maximize:

$$\sum_{i,j,k}\sum_{\chi=x_a}^{x_b}\sum_{\psi=y_a}^{y_b} y(i,j,k,\chi,\psi,\theta) \times \sum_{q_x=0}^{\chi-1}\sum_{q_y=0}^{\psi-1}\sum_{q_z=0}^{\theta-1} v(i+q_x, j+q_y, k+q_z) \tag{5.14}$$

$$\text{where } i+q_x-1 \leq X, j+q_y-1 \leq Y, Z_1 < k+q_z \leq Z_2+1$$

Subject to:

$$\sum_{\chi=x_a}^{x_b} \sum_{\psi=y_a}^{y_b} y(i,j,k,\chi,\psi,\theta) \leq 1, \qquad \forall i,j,k \tag{5.15}$$

$$\begin{gathered}\sum_{\substack{\chi=x_a \\ \chi\neq 0}}^{x_b} \sum_{\substack{\psi=y_a \\ \psi\neq 0}}^{y_b} y(i+q_x, j+q_y, k, \chi, \psi, \theta) + \sum_{\chi=x_a}^{x_b} \sum_{\psi=y_a}^{y_b} y(i,j,k,\chi,\psi,\theta) \leq 1, \\ \forall i,j,k \text{ and } \forall q_x \in \{-x_b+1, x_b-1\}, q_y \in \{-y_b+1, y_b-1\} \\ \text{where } 0 < i+q_x+\chi-1 \leq X, 0 < j+q_y+\psi-1 \leq Y\end{gathered} \tag{5.16}$$

In this model, i,j,k are sets of starting coordinates of all valid stopes in between sublevels, Z_1 and Z_2 are beginning and ending coordinates of the stopes respectively in Z direction, θ is the stope height that is determined previously, q_x and q_y represent offset from the starting coordinates, $y(i,j,k,\chi,\psi,\theta)$ is the decision variable that determines the extraction of stope at the coordinate i,j,k with the sizes χ,ψ,θ, and $v(i,j,k)$ is the economic value of the block at coordinates i,j,k. The objective function in Equation 5.14 maximizes the total economic value of the stopes that are decided to be extracted by the model. Equation 5.15 ensures that only one size can be accepted per stope. Equation 5.16 checks for overlapping stopes in X-Y directions and only allows one of the overlapping stopes to be selected.

This is also realized through a similar but iterative and metaheuristic-like approach. Each level is considered separately and the stopes at a level are decided according to the following procedure:

1. The height of the stopes is settled by observing the level height. If it is within acceptable limits, stope heights are set to the level height. Otherwise, the maximum stope height is set as the current stope heights.

2. A list of blocks in the level is established and sorted according to their scores $s_{x,y,z}$, from high to low.

3. Similarly to the sublevel selecting algorithm, a candidate stope combination solution is created by taking the first block of the ranked list. All stope size combinations are evaluated for their economic value and feasibility. The feasibility test consists of testing if the stope is out of block model bounds and if the stope overlaps with another already selected stope. Within the set of stopes that are feasible, the stope that yields the highest economic value is selected. As long as the feasibility constraints are sustained, the addition takes place. This process is continued until the list is exhausted.

4. The overall economic value of the stope combination is calculated by adding the economic value of each stope in the combination.

5. If the economic value of the combination is positive and it is the highest economic value so far, it is stored as the current best combination. Otherwise, the combination is deleted.

6. The first block of the ranked list is deleted. If there are no more blocks in the list or the number of formed stope combinations is equal to β, the algorithm advances to next step. Otherwise, the algorithm returns to Step 3.

7. This is the metaheuristic step. The most previous iteration's economic value is compared to the most recent iteration. If there is an improvement, the ordering of most recent iteration is kept. Otherwise, the ordering is kept with a probability. This probability decreases as the number of iterations increase. If there is no improvement for 10 iterations, the algorithm terminates.

8. A random change is made to the ordering of the list and the algorithm returns to Step 3.

In this stage of design, although block scores $s_{x,y,z}$ are used when forming the priority list, the final decision is made according to the economic value of the design. This is preferred because the overall objective is to optimize the mine profit. This process may be repeated with different sublevels to start with. As the number of sublevel combinations is limited due to stope height constraints, the number of repetitions will also be low and the result will be closer to optimal.

This strategy can be very effective when the ore concentrated regions are in clusters and unevenly dispersed throughout the deposit as it conveniently prioritizes ore-rich areas, which is common in most deposits. If this is the case, the upper elements in the list will be dominantly greater than the subsequent elements, increasing the possibility that the right combination will contain the upper elements. Another advantage of this approach is that given its modular structure, sublevels may be defined by an engineer or if there is an existing development it can directly be defined in the program and the sublevel design stage above can be omitted.

5.6 Case studies

5.6.1 Case study 1

The heuristic approach has been implemented in C++ and tested on an underground poly-metallic gold-copper mine with 125,000 blocks. The dataset contains two grades for each block (gold and copper) that are the averages of multiple realizations generated by sequential Gaussian simulation. The mining operation plans one mineral processing plant and one waste dump. The parameters regarding the project are given in Table 5.2.

The economic value of an extracted block is calculated according to Equation 5.17 where g_m is grade, p_m is price, R_m is the recovery of mineral m, C_p is processing cost, C_r is mining cost and t is tonnage. The tonnage is calculated by multiplying the specific gravity by the block volume. It is assumed that all mined blocks are processed and value of non-extracted blocks are zero. The resultant plan can be visualized using SGeMS (Remy et al., 2009) in Figure 5.3. Each stope is illustrated with a color that corresponds

Table 5.2 – List of parameters used in the case study 1

Value	Parameter
50, 50, 50	Dimensions of mine in X, Y, Z directions (in blocks)
10, 10, 10	Dimensions of each block in X, Y, Z directions (in m)
30	Mining cost (\$/tonne)
10	Mineral processing cost (\$/tonne)
3	Density (tonne/m^3)
2	Number of metals to be sold (Au and Cu)
40, 4.1	Ore price (Au, \$/gr and Cu, \$/lb respectively)
0.9, 0.75	Recovery (Au and Cu respectively)
30, 30, 30	Minimum frame size in X, Y, Z directions (in m)
70, 70, 70	Maximum frame size in X, Y, Z directions (in m)

to the average grade within the stope. Representative sublevels are shown in gray color. The heuristic approach was able to successfully identify the ore-concentrated areas in the deposit and generate a stope layout plan. It can be observed from the figure that the minimum average stope Au grade is 0.376 g/tonne, which can be considered as the stope cut-off grade for this operation.

$$v(i,j,k) = \left(\sum_{m \in M} g_m p_m R_m - C_p - C_r \right) \times t \tag{5.17}$$

Figure 5.4 demonstrates the effect of program related parameters α, β and l (definitions given in Table 5.1) to the overall profit of the mine. To observe this effect, the program was run multiple times with different program parameter values. α and β were tested in the range 5-20 and l was tested in the range 3-4. Inspecting the figure, it can be inferred that in this case

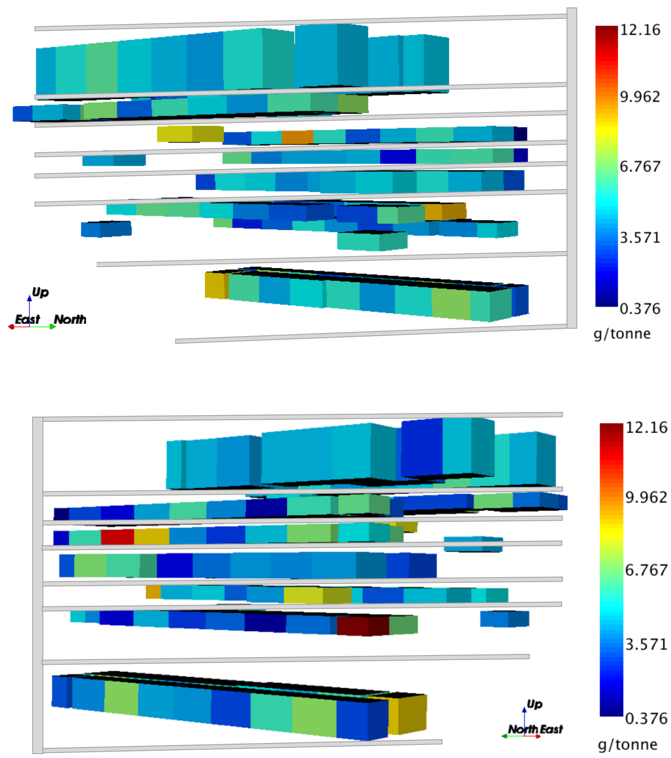


Figure 5.3 – Images from different perspectives of the resultant plan using the presented heuristic approach. The color of each stope corresponds to the average equivalent Au grade within the stope. (Case study 1)

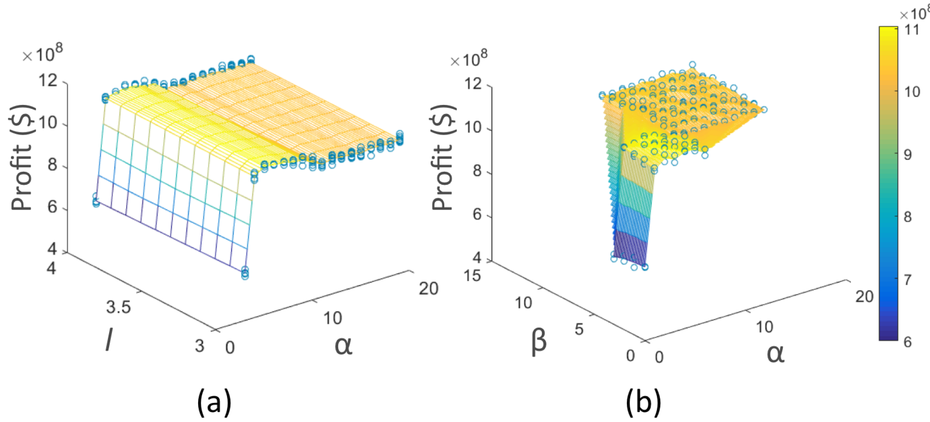


Figure 5.4 – (a) The influence of score and sublevel parameters to the resultant mine economic value (b) The influence of stope and sublevel parameters to the resultant mine economic value (Case study 1)

study, β influenced the profit the most and l did not have an effect. Also, increasing α and β increased the profit until they reach about 10. Above this value, the profit reached a plateau. This indicates the heuristic approach is successful in ranking the more promising sublevels and stopes before unfavorable possibilities. The average running time of the program for this case was 15 minutes 42 seconds on a MacBook Pro 2015 with 2.7 GHz Intel Core i5 processor and 16 GB memory. To decrease ore dilution and loss, the block size can be reduced. However, this would increase the number of decision variables, hence the solution time.

To further evaluate the performance of the heuristic approach, case study 1 has been re-run on a portion of the same deposit with dimensions 15x15x15 blocks and maximum frame size as 50, 50, 50 meters. The linear programming model has been formulated in Zimpl (Koch, 2006) and the same case has also been solved with the linear programming model using CPLEX to compare the outcomes. The optimal mine value obtained by the linear program was \$ 382,037,496 and by the heuristic approach was \$ 377,561,000 which is a difference of 0.1%. On the other hand, the runtime of CPLEX was 61 hours 23 minutes on Dell Precision T3610 workspace with Intel Xeon E5-1620 3.70 GHz processor whereas the heuristic approach took 2 minutes 4 seconds with the same computer used in the previous case study.

5.6.2 Case study 2

A second case study was carried out to test a slightly modified version of the heuristic algorithm. In this version, three changes were made: (1) between sublevel distance was kept constant, (2) sublevel building cost has been added, (3) internal waste has been allowed to plan diluted stopes as opposed to typical mining stopes. Instead of assuming the entire volume of the stope will be extracted, it is presumed blasting can be adjusted such that internal waste can be left on the roof and floor of the stope. In addition, a small block size is used to decrease dilution/loss.

This algorithm is tested on an underground gold mine with 134,400 blocks. The dataset contains a gold grade for each block that are the averages of multiple realizations generated by sequential Gaussian simulation. The mining operation plans one mineral processing plant and one waste dump. The parameters regarding the project are given in Table 5.3. The economic value of an extracted block is calculated in a fashion very similar to that

Table 5.3 – List of parameters used in the case study 2

Value	Parameter
56, 100, 24	Dimensions of mine in X, Y, Z directions (in blocks)
3, 3, 3	Dimensions of each block in X, Y, Z directions (in m)
10	Mining cost (\$/tonne)
25,000	Sublevel build cost (\$/meter)
10	Mineral processing cost (\$/tonne)
3	Specific gravity (tonne/m^3)
1	Number of metals to be sold
40	Ore price (\$/gr)
0.9	Recovery
30, 30, 15	Minimum frame size in X, Y, Z directions (in m)
40, 40, 35	Maximum frame size in X, Y, Z directions (in m)

in the previous case study. The only difference between the calculations is in case study 2, the mining cost does not incorporate sublevel build cost and sublevel build cost is extracted from the economic value of the mine in proportion to its length.

The running time of the program for this case was 96 minutes 17 seconds on a MacBook Pro 2015 with 2.7 GHz Intel Core i5 processor and 16 GB memory. The resultant plan for this case is visualized from two different perspectives using SGeMS (Remy et al., 2009) in Figure 5.5 where each stope is illustrated by the color that corresponds to its average grade and representative sublevels are shown in gray. It can be observed that sublevel distance constraint was respected and internal waste was allowed by the program. High sublevel building cost clearly forced the program to choose as few sublevels as possible that covers access to valuable sections in the mine. Although sublevel generation was faster due to equal distance enforcement,

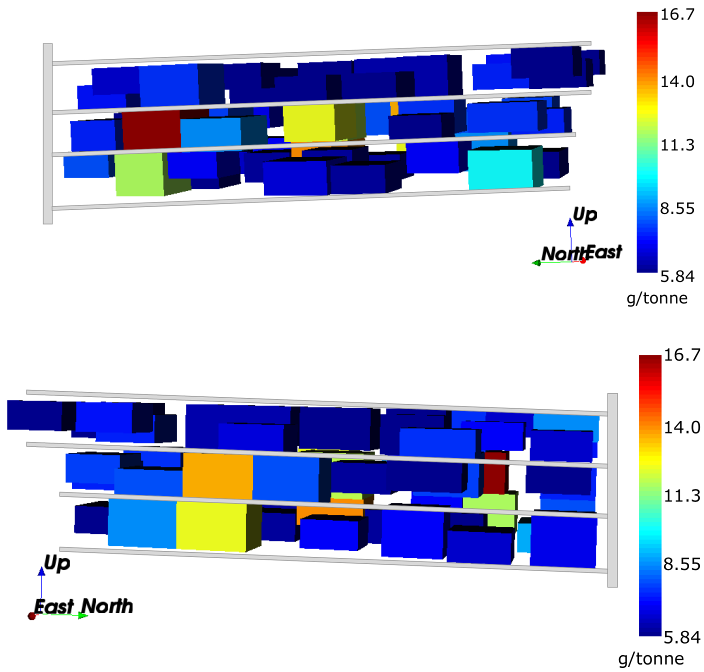


Figure 5.5 – Images from different perspectives of the resultant plan using the presented modified heuristic approach. The color of each stope corresponds to the average equivalent Au grade within the stope. (Case study 2)

internal waste option increased the solution time because the number of possibilities had increased.

5.7 Conclusion

In this paper, a new heuristic method has been developed to solve the stope layout problem. This method assigns scores to each block based on the grade of the block and the surrounding blocks. This scoring approach is used as a clustering heuristic to easily detect the ore concentrated areas in a deposit. At this point, the approach has been broken down to two stages: selecting where the sublevels will be built and selecting the stopes in between sublevels.

The case study demonstrated that the approach is working well without violating any constraints. Also, the program parameters have been shown to converge. In other words, the parameters can be set empirically and be increased until the profit does not improve. Further investigation of the approach involved comparison of the results to an exact method. To achieve this, the problem was formulated as a mixed-integer linear program model and a similar case has been solved both with linear program solver and proposed heuristic clustering approach. The results have shown that the mine profits generated by both approaches were very similar but the heuristic approach reached this result much faster.

The advantages of the proposed approach are that it (1) follows the engineering practices, (2) generates fast, comparable results to optimal and (3) if there was previous development of sublevels in the mine, they can be provided manually in owing to the modular structure of the approach. The comparison to the linear programming model produced promising results and conducting more case studies will help improve the heuristic in future work. Also, the authors intend to extend this approach to stope sequencing.

5.8 Chapter Conclusion

In this section, a fast, practical method is proposed to solve the stope layout planning problem with variable stope dimensions to allow higher selectivity and decrease mining costs. However, the algorithm comprises of several parameters that must be set by the user. In an industrial setting, for non-experienced or non-technical users this might be difficult. Therefore, in the following section, an alternative greedy heuristic method based on dynamic programming is proposed. In the alternative method, there is only one heuristic parameter that needs to be set and this parameter depends on time and computational resources needed to solve the problem. In other words, it is set easily and the larger heuristic will always give a better result. However, it does not perform a random search and might potentially find the optimal answer later than the method proposed in this section.

6 Sublevel stope layout planning through a greedy heuristic approach based on dynamic programming

6.1 Abstract

Sublevel stoping is one of the most widely used mining methods in underground mines. Mines that use sublevel stoping can potentially increase their profit by optimizing the layout plan. Sublevel stope layout planning is a complex problem and larger problem sizes cannot practically be solved using exact methods. Large problem sizes with up to hundreds of thousands variables are common in mine planning. The complexity of the problem is demonstrated by showing that it is a special case of independent set problem, which is an NP-hard problem. To solve the sublevel stope layout planning problem, we propose a novel greedy heuristic approach based on dynamic programming. This approach identifies the recurring subproblems and memoizes their results to decrease the solution time. The heuristic is introduced to further decrease the solution time and limit the memory usage. It is optional and for smaller problems, the heuristic can be lifted and the approach can be used as an exact method. A case study is presented to demonstrate the performance of the approach. The results show that that the stope layout plan is able to capture the valuable regions of the orebody well. The algorithm is able provide a fast and feasible solution.

6.2 Introduction

It gets harder and harder to find orebodies close to the surface. Therefore, underground mining becomes more favorable. Increasing worldwide environmental awareness also favours underground mining because it generates much less waste quantities, and causes less disturbance to the vicinity by noise and dust.

Sublevel stoping is one of the most commonly used methods in underground mines. In this method, first, the orebody is reached from the side by building an inclined ramp for the equipment called declines and a shaft from the surface (Anjomshoa et al., 2013). Once the ramp reaches the bottom of the orebody, horizontal access roads called sublevels are built from the ramp within a certain distance from each other. In between the sublevels, the orebody is mined in stopes, which are large rooms that are drilled and blasted. At the end of blasting, the fragmented rock is loaded from the drawpoints at the lower sublevel and hauled to the shaft from where is transported to the surface.

The locations of the stopes and sublevels influence the profit obtained from the mine and can be optimized. Open pit mine planning and optimization is well studied in the literature. Underground mine planning is relatively new and there is a strong opportunity for improvement through underground mine planning. In current mining practice, stope layout plans mostly aim to extract all deposit within reach within rock stress constraints. This approach is not always optimal and causes dilution (inclusion of waste rock in the mined material). Additionally, not every section of the orebody is profitable because the revenue to be obtained from low grade sections may not pay off mining and mineral processing costs. The stope layout optimization methods take the deposit model and geotechnical information related to the deposit such as minimum and maximum stope sizes and performs an economical optimization while respecting the stability constraints.

The main contribution of this paper is the presentation of a new greedy heuristic approach based on dynamic programming to solve the stope layout planning problem. The proposed approach is fast and able to perform in large instances. The paper is organized as follows: the next section defines the stope layout planning problem. Section 3 provides the literature review for the problem. In Section 4, the methodology is explained. To show the efficiency of the approach, a case study is given in Section 5. Lastly, the discussion and conclusion are provided in Section 6.

6.3 Problem Definition

To evaluate a potential deposit, drill samples are conducted in disperse locations. The deposit is conceptually divided into rectangular sub-volumes called blocks. A typical block is 5 to 30 meters long on each side. The samples are evaluated using estimation or simulation procedures such as kriging (Cressie, 1990) and conditional simulation (Menabde et al., 2018). At the end of this procedure, each block is assigned a grade that represents the amount of ore it contains (Kumral, 2011, 2012). The three-dimensional array that contains the grade value for each block is called the block model. In this section, the problem of planning the stope layout from the input of a block model is described. The nomenclature for this and subsequent sections is given in Table 6.1.

Table 6.1 – List of notations for the mathematical models and methodology

Indices and sets
$j \in T$: set of stopes j
$k \in \tilde{T}_j$: set of stopes that overlap directly in X-Y direction and not leveling in Z direction with stope j
Parameters
v_j: economic value of stope j when processed
S_x: minimum stope dimensions in X direction in blocks
S_y: minimum stope dimensions in Y direction in blocks
X: number of blocks in X direction
Y: number of blocks in Y direction
p: price (\$)
g_j: grade of mineral m of block j
R: recovery (%)
C_m: mining cost (\$)
C_p: processing cost (\$)
t: tonnage of a block
Decision variables
y_j: 1 if stope j is in the final stope layout design, 0 otherwise

The block model of grades can be converted into an economic model using the formula:

$$v_j = [p \times g_j \times R - C_m - C_p] \times t \tag{6.1}$$

The block economic model contains the revenue that would be obtained for each block in case the block is decided to be extracted. If a block is not extracted, the revenue from that block would be zero. A block is extracted if a stope containing the block is extracted. In the literature, most approaches perform the optimization block economic model and enforce stope constraints. In this paper, we take a different approach and convert the block economic model to stope economic model. The conversion involves creating possible stopes at each location. The summary of the conversion is given in Figure 6.1. The possible stopes are referred by their starting point (top left point) and their size in X, Y and Z directions. In the end of this conversion, the possible stopes become the decision variables rather than the blocks.

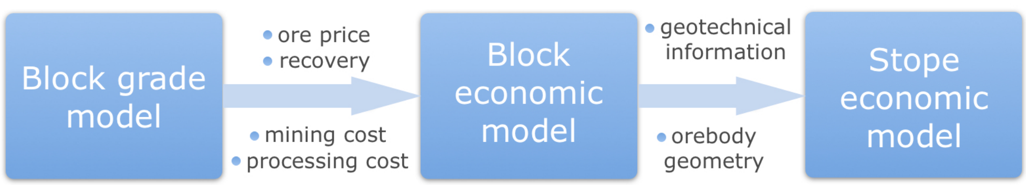


Figure 6.1 – The conversion of the block model to stope economic model

The advantages of this conversion are that (1) repeated calculations for stope economic values are avoided and (2) as the stope economic values are calculated in the beginning, possible stopes with non-positive economic values are determined are excluded from the model. A possible stope with a non-positive economic value will never be extracted in any case. This preprocessing step reduces the search space drastically in most cases.

The stope layout problem can be defined as follows:

Maximize:

$$\sum_{j \in T} y_j \times v_j \tag{6.2}$$

Subject to:

$$\sum_{k \in \tilde{T}_j} y_k + y_j \leq 1 \qquad \forall j \in T \tag{6.3}$$

The objective given in Equation 6.2 is to maximize the economic value obtained from mining the stopes. Economic value of stope j, v_j, is obtained from summing the block economic values v_i that are contained in the stope j. Hence, the stope economic value is a function of stope average grade, ore price, recovery, and mining and mineral processing costs. If the costs exceed the revenue for a given stope, that stope will have a negative economic value and will not be selected as the objective is maximization. The constraint at Equation 6.3 ensures that if a stope is overlapping with other potential stopes, only one of them will be extracted. The overlapping is defined as having at least one block in the same coordinates in X, Y and Z directions or having blocks in the same Z coordinates and also containing blocks with different Z coordinates, thus not leveling in Z direction (Figure 6.2).

Weighted interval scheduling (WIS) problem bears similarity to the stope layout planning problem. In WIS, all jobs have a starting and finishing time and overlapping of jobs within a time frame are not allowed. If in the given

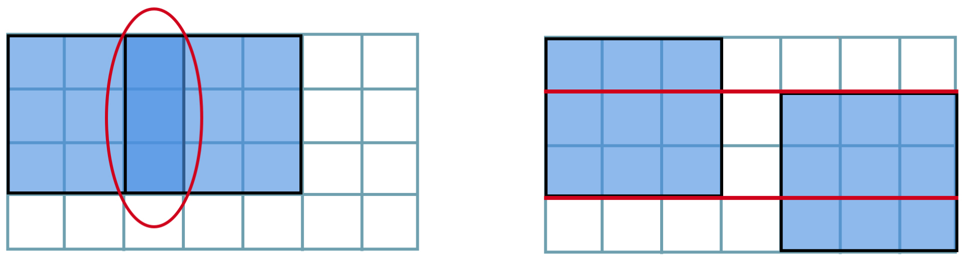


Figure 6.2 – Stope overlapping by sharing blocks (left) or by having blocks in the same and other Z coordinates (right)

jobs, there is such an overlap, at most one of them can be selected. The overall objective is to maximize the weights of the selected jobs. WIS has a very efficient dynamic programming solution where it sorts the jobs by their starting times and compares the weight of the job added to cumulative value of the previous disjoint job to the cumulative value of the previous overlapping job (Afshin Mansouri and Aktas, 2016; Kovalyov et al., 2007; Ng et al., 2014). Stope layout planning problem also bears the overlapping constraint and the economic value maximization objective. However, while the weighted interval scheduling problem is one dimensional, the stope layout planning problem is three dimensional. Multiple dimensional nature of the stope layout planning problem prevents sorting of the stopes in the same sense as WIS. This makes the problem significantly harder.

The stope layout problem is a special case of weighted independent set problem. Weighted independent set problem aims to find the set of nodes that yield the maximum total weight where no two of the nodes in the set are adjacent that are not adjacent in a graph (Lovász, 1994; Sakai et al., 2003).

If stopes are converted to vertices with the weight of the stope economic value and all the overlapping stopes are connected with an edge, a weighted independent set problem is formed. The decision version of the weighted independent set problem (whether a combination can exceed a given weight) is NP-complete and the maximization problem is NP-hard.

The underground stope layout design problem solved by an exact method takes a very long time because of the number of variables and constraints. As the number of blocks increase, the number of variables increase linearly but the number of constraints increase exponentially because each stope may or may not be included in the final design and has overlapping stopes in the amount of the number of blocks it contains multiplied by number of stopes that overlaps vertically Current MILP formulations can only handle a few thousand blocks in a reasonable amount of time. Considering that typically there are at least several hundred thousand blocks in a block model, linear programming is currently impractical to solve the problem. This limitation presents the need for an alternative approach such as heuristics or exact methods that simplify the problem (such as the approaches by Pourrahimian (2013); Nezhadshahmohammad et al. (2018) for block caving). Therefore, a heuristic approach is proposed in this research.

6.4 Literature review

Current approaches to stope layout planning problem include exact and heuristic methods. A block model can consist of thousands to millions of blocks. With the number of constraints that take in to account all the overlaps, the problem becomes unmanageable. As the problem size is too large for exact methods, they generally attempt to simplify the problem by solving parts of it, then combining the parts. This usually involves solving the problem in two dimensions and combining them to three dimensions. Ovanic and Young (1995) used a special type of MILP SOS2 (Type-Two Special Ordered Sets) that allows at most two adjacent ordered set of variables to be non-zero to optimize starting and ending points at each row of blocks. The row by row obtained solutions are then combined. Bai et al. (2013) adapted the maximum flow approach by first changing the rectangular block model to a cylindrical coordinate around the initial vertical raise. The raise is centered at the orebody. The blocks are converted to nodes, source and sink are added. The connection between nodes are made based on the stope size. The placement of the initial raise influences the results. Bai et al. (2014) then developed a new heuristic method for larger deposits with multiple raises.

Alford (1995) presented the floating stope algorithm analogous to floating cone method for open-pit mines. The algorithm floats a stope with minimum size throughout the deposit and includes all stopes with positive economic values. This is a practical solution unless included stopes overlap. Ataeepour (1997) introduced the greedy best neighborhood approach that favors the stopes with higher economic values among overlapping stopes. The problem with this approach is it does not consider combinations and may produce empty spaces in the plan, causing ore loss. Topal and Sens (2010) proposed a user preference based approach where the maximization can be done with other parameters such as stope profit per square meter or stope profit divided by its total mining time. Cheimanoff et al. (1989) proposed the octree division algorithm where the deposit is repeatedly divided in two in each direction until the minimum stope size is reached and it is included in the final layout design if it has a positive economic value. This method may work well depending on the orebody shape. Sandanayake et al. (2015b) aggregated mining blocks into stopes then varying the attribute of the stopes. Villalba Matamoros and Kumral (2017) developed a heuristic that work with multiple mining sectors, variable stope dimensions and that considers internal dilution. A meta-heuristic method is proposed by Villalba Matamoros and Kumral (2018b) uses genetic algorithms for stope layout optimization under uncertainty.

6.4.1 Preparing the model

In sublevel stoping technique, access roads called sublevels are built that partitions the deposit horizontally. The locations of the sublevels can be determined using the approach by Sari and Kumral (2018) or manually. When the sublevels are decided, the height of the stopes are set. As a result, the blocks in the Z direction can be aggregated by adding up the economic values and the problem becomes two-dimensional. The objective of this paper is to provide an approach that decides on the stopes to be extracted between each sublevel. After the sublevels are determined, for each section between sublevels, the stope layout planning algorithm is run separately. The algorithm for stope layout planning is as follows:

1. The maximum number of stopes that the deposit portion can contain is calculated with the following formula:

$$n_{s-max} = \frac{\lceil X \rceil}{S_x} \times \frac{\lceil Y \rceil}{S_y}$$

2. The maximum number of stope set combinations α_{max} is set.

3. The number of stopes n_s is set to the maximum number of stopes (n_{s-max}).

4. Starting from the upper left side, each block location is assigned an index.

5. The stope set S_n is formed for the first time through S_n formation routine. The largest index i in the list is found and all stopes with indices greater than i is added to the memoization list M . The size of combinations at this stage is $M_{size} = 1$.

6. A new memoization list N is created with $N_{size} = M_{size} + 1$. The number of stope set combinations α is set to zero.

7. The stope s_i with largest index i in the set S_n is found. For each element m in the memoization list, s_i is added to the combination in

turn. If the combination is feasible and $\alpha \leq \alpha_{max}$, it is added to the list N and α is incremented by 1. If the combination is feasible and $\alpha > \alpha_{max}$, the stope set combination in N with the least economic value is dropped and the new combination is added to the list N instead. S_n is updated according to S_n update routine. This step is repeated until S_n update routine returns null.

8. The list N is assigned to M and then, N is deleted. Steps 5, 6 and 7 are repeated with the new memoization list M until $M_{size} = n_s$.

9. The combination with the highest economic value is found in the memoization list. This is saved as the highest stope combination.

10. n_s is decreased by 1. If $n_s = 0$, the algorithm is terminated. Otherwise, the algorithm is continued from step 3.

S_n formation routine (Figure 6.3):

1. The stope on the upper left side of the model, which is the first index ($i = 0$), is added to S_n.

2. If the size of S_n is n_s, return S_n. Otherwise, find the stope with the smallest index, larger than i that does not overlap with any stope in S_n and add it to S_n. Repeat this step.

S_n update routine (Figure 6.4):

1. Find the largest index i in the current S_n.
2. Delete index i from S_n.
3. Find the smallest index j, such that $i > j$ and the stope at index j does not overlap any stope in S_n. If j is found, add the stope at j to S_n and return S_n. Otherwise, return null.

Memoization list:

The memoization list is a collection of memoization structures. Each structure has the following properties:

- The stopes contained in the structure
- The total economic value of the stopes contained in the structure
- An instance of the block model where the blocks occupied by the stopes in the structure are distinguished.

The proposed algorithm identifies the subproblems, which are subsets of stope combinations. Forming stope combinations is expensive because both the total economic value must be calculated and overlap conditions must be tested for each pair of stopes. The memoization list allows the re-usage of the subproblems thus transferring the computational burden to computer memory. The recursive nature and reuse of the subproblems of the proposed

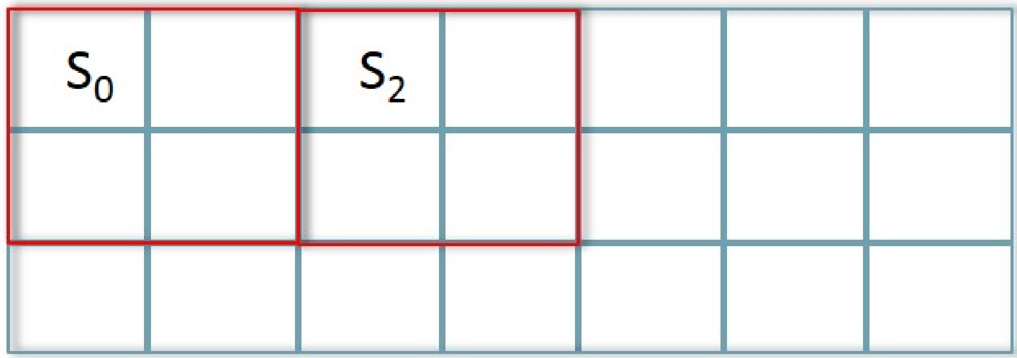


Figure 6.3 – S_n formation routine shown on a block model of 7x3 with 2 stopes. All possible stopes that are not overlapping with S_0 and S_2 with indices greater than 2 will be added to the memoization list

approach is based on dynamic programming concepts. Keeping the best α_{max} solutions is the greedy component of the approach. Placing the stopes in the beginning and forming the subproblems in the remaining space structures the approach and minimizes the search space. If the heuristic parameter α_{max} is removed, and all the combinations are saved instead, this approach becomes exact rather than heuristic.

The block model and the occupancy pattern of the structure is represented using the bitset class template of C++. Each bit represents a block in the model and if the block is occupied by a stope in the structure, the bit is set to 1, otherwise the bit is set to 0. This unique representation speeds up the test of feasibility in case of adding a new stope to the structure and updating the occupancy pattern when a new stope is added. At step 5 of the algorithm, an occupancy pattern o_i is created for the new stope s_i. For the test of feasibility, a simple bitwise AND operation is performed on the

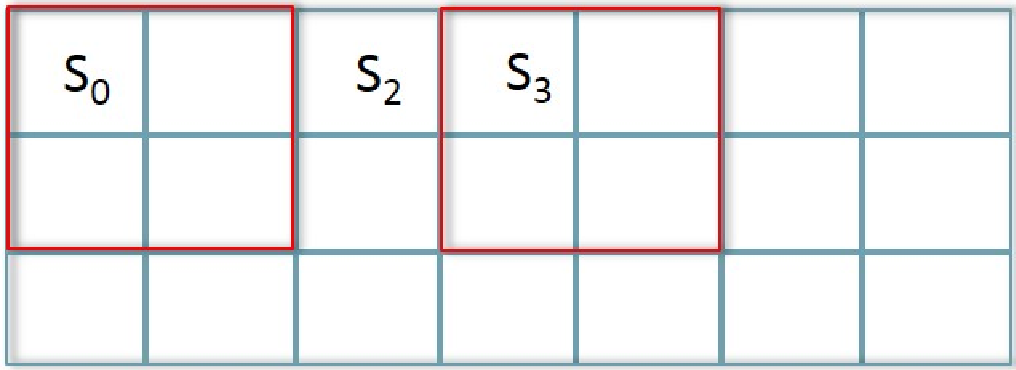


Figure 6.4 – S_n update routine shown on the same block model in Figure 6.3. The stope that was previously in S_2 is update to S_3

occupancy pattern of the memoization structure and o_i (Figure 6.5). If any bit of the resulting bitset is set, it is concluded that there is an overlap. Otherwise, the stope is added to the structure and the occupancy pattern is updated by performing a bitwise OR operation on the structure bitset and o_i (Figure 6.6).

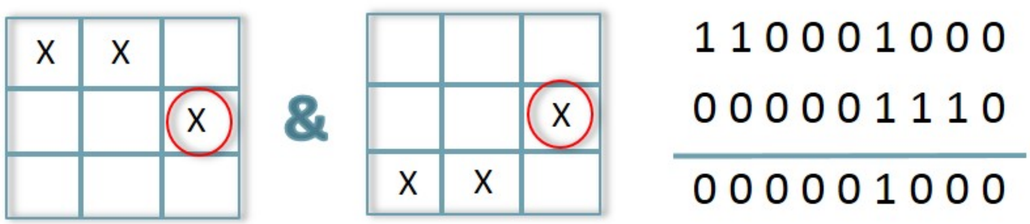


Figure 6.5 – An example bitwise AND operation performed on two stopes (on the left). The resulting occupancy pattern (on the right) has a set bit, which means there is an overlap

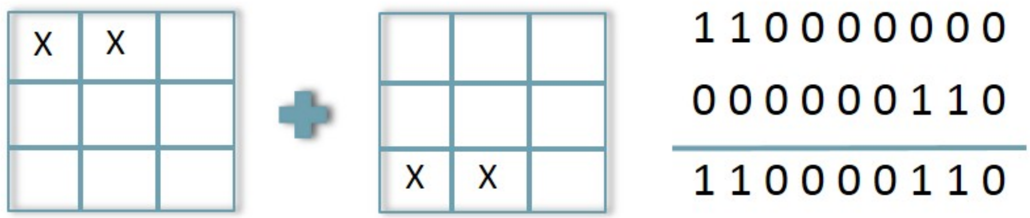


Figure 6.6 – An example bitwise OR operation performed on two stopes (on the left). The resulting occupancy pattern (on the right) has the resulting occupied block locations set

6.5 Case study

To demonstrate the performance of the proposed method, a computer program was written in C++ that implements the described algorithm. The case study is performed on an underground nickel mine with 64,638 blocks, where each block is 10x10x10 m in size. The block grades were generated using sequential Gaussian simulation. Orebody was simulated 20 times and their average was used as input data in the optimization process. The parameters related to the case study can be found in Table 6.2.

It can be observed from the table the stope height is given between 20 and 40 m. As the stope height is determined based on blocks and the block height is 10 m, possible stope heights are 20, 30 and 40 m.

The view of the deposit from different angles can be seen in Figures 6.7, 6.8 and 6.9 (a). The visualizations of the deposit generated using SGeMS (Remy et al., 2009). It can be observed that the orebody is steeply inclined. The distribution of the ore grades is attributed to the legend on the left. The

Table 6.2 – Parameter values for the case study

Parameter	**Value**
Price	\$20,000/ tonne
Mining cost	\$5/ tonne
Processing cost	\$23/tonne
Recovery	85%
Initial Investment	\$500,000,000
Block dimensions	10 m x 10 m x 10 m
Number of blocks in X, Y and Z directions	57 x 54 x 21
Minimum stope size in X, Y and Z directions	30 m x 40 m x 20 m
Maximum stope size in X, Y and Z directions	30 m x 40 m x 40 m
Maximum number of stope set combinations (α_{max})	500,000

grades are highly variant across the orebody and the non-colored areas in the deposit have a grade of zero.

The computer program was run with the given parameters and the block model as inputs and the results for stope layout plan were generated. The program execution took 6,303 seconds on a MacBook Pro 2015 computer with 2.7 GHz Intel Core i5 CPU (dual-core) and 16 GB RAM.

Table 6.3 – Resulting economic values of the stope layout plan

Between sublevels	**Economic value**
0 – 4	\$165,859,000
4 – 8	\$560,988,000
8 – 12	\$788,565,000
12 – 16	\$696,342,000
16 – 20	\$205,535,000
Total	**\$2,417,289,000**

The generated stope layout plan is composed of 106 stopes in total on 6 sublevels and 5 sets of stopes in between the sublevels. As the sublevel plan-

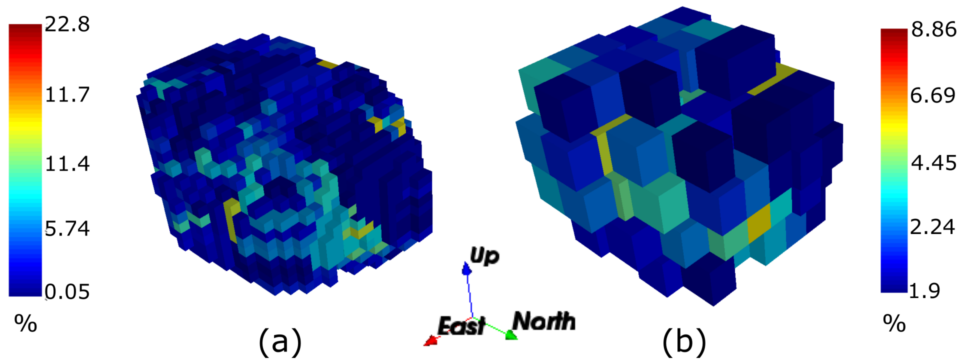


Figure 6.7 – Visualization of the deposit from north-east – south-west direction: (a) image of the block model, (b) image of the stope layouts after planning

ning is done previously, the development costs are not included in this part of the optimization. The economic value obtained from each level is given in Table 6.3. The index of the sublevels is given based on the block number in the vertical direction. It can be observed that the highest economic value is between sublevels 8 – 12 because the highest-grade ore is located between these sublevels. Also, because it is the central level, the planned dilution from including neighboring waste material is minimized. The profit that will be obtained from mining this deposit is \$2,417,289,000 - \$500,000,000 = \$1,917,289,000. It is important to note that this value is not the net present value and depending on the mining schedule obtained by stope sequencing, net present value can be calculated.

The resultant stope layout plan can be inspected in Figures 6.7, 6.8 and 6.9 (b). The legend for the stope layout plan visualizations are located on the right and each color represents the average grade in that stope. Com-

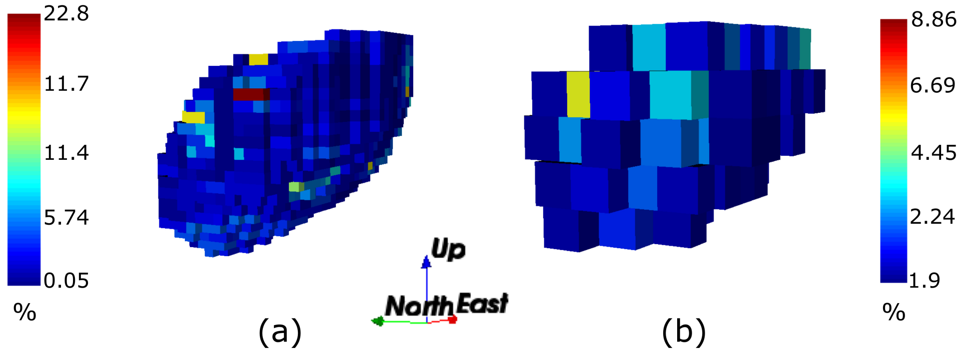


Figure 6.8 – Visualization of the deposit from east-west direction: (a) image of the block model, (b) image of the stope layouts after planning

paratively with the ore resources in (a) of each figure, it can be inferred that the orebody shape has been captured well by the stope layout. It can also be noted that the legend has a smaller range of grade variations on the stope layout than the blocks. This is because the grades of blocks contained in a stope are averaged. As it can also be observed in Table 6.4, this creates a smoothing effect on the grades.

Table 6.4 – Data summary related to the case study

Statistic	**Value**
Number of blocks	64,638
Number of stopes	106
Maximum grade in the block model	22.8%
Minimum grade in the block model	0
Standard deviation of the grade in the block model	1.13%
Maximum average stope grade	8.86%
Minimum average stope grade	0.2%
Standard deviation in the average stope grade	1.9%

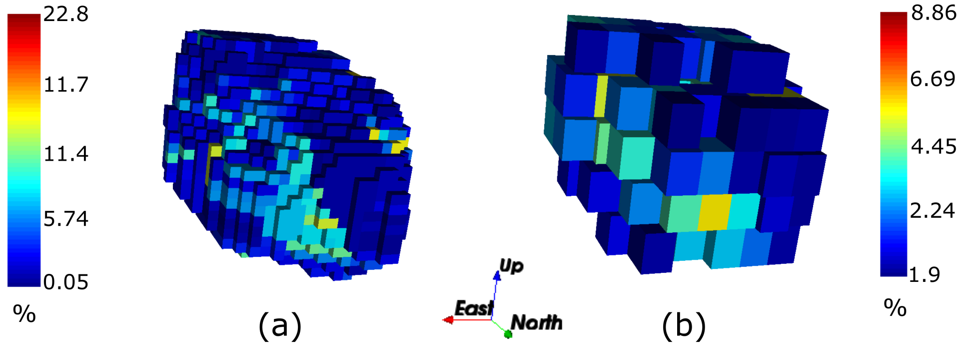


Figure 6.9 – Visualization of the deposit from north-south direction: (a) image of the block model, (b) image of the stope layouts after planning

6.5.1 Discussion and conclusions

In this paper, a new heuristic stope layout optimization method is introduced. This method is based on dynamic programming. A stope layout plan is a set of selected stopes. Even though the complete stope set is different in each stope layout plan, subsets of these stopes are shared among different stope layout plans. The proposed approach takes advantage of the fact that while producing different stope layout plans, the same subset of stopes are used many times. Thus, instead of testing the feasibility and calculating the economic value of the same set of stopes repeatedly, it saves the set in the memory. To this set, other possible stopes are added in turns. Each newly formed set is also saved in the memory. Finally, the set with the highest economic value is picked for the stope layout plan. This approach shifts the computational burden to computer memory. In smaller instances of the problem the computer is able to handle this well. However, as the problem grows, it causes the computer to run out of memory. This problem is solved by the heuristic. Instead of saving all the possible sets, only a portion of the sets is kept. When the number of saved sets is kept relatively large, it can produce results close to optimality. As an alternative, this number can be unlimited and when the computer memory is insufficient, the sets can be saved to file. In other words, when the number of saved sets is used this method is a heuristic and otherwise it is an exact method. Two main preprocessing steps are suggested in this paper to help speed up the execution time: (1) conversion of the block model to stope economic model and (2) dividing the stope layout problem in two: sublevel determination and stope layout design. The first step allows determining the stopes with negative economic value. Thus, they are taken out from the model before the main algorithm, speeding up the search and decreasing the memory requirements. Generally, the recoverable reserve consists of only a portion of the resource, thus many potential stopes can be eliminated, increasing the importance of this step. The second step allows using the method of choice for sublevel determination and stope layout design. Also, the problem size is shrunk by allowing to design between each sublevel independently. Along with the preprocessing steps, the utilization of the bitset class template for feasibility tests and additions to sets contributes to the acceleration of the program execution. The results from the case study show that the stope layout plan is able to capture the valuable regions of the orebody well. The algorithm is able provide a fast and feasible solution. For the future work, it will be compared to other methods to assess the optimality of the result. More mining elements will be incorporated into the plan such as pillars and equipment constrains. Furthermore, this method will be combined with stope production scheduling.

6.6 Chapter Conclusion

This research provided an alternative optimization method to the previous section. This method has more ease of use to non-technical users and does not need experimentation with parameters. On the other hand, it performs a structured search. In this sense, it might potentially take a longer time to find the optimal solution than the randomized search. Conversely, it might take a shorter time due to the unpredictability of the randomization. The other shortcoming of the approach in this form is that the whole orebody is aimed to be extracted without leaving any pillars after the operation completes. The lack of this option limits the usage of this approach in mines with less rigid rock characteristics. In the following section, the greedy heuristic approach is extended to operate with poly-metallic mines with pillars. Also, a methodology for sublevel determination is proposed. Additionally, a MILP formulation that finds the ultimate stope limits is introduced and the results of a case study is compared to the results of the greedy heuristic approach.

7 A planning approach for poly-metallic mines using sublevel stoping technique with pillars

7.1 Abstract

Sublevel stoping technique requires planning of development and infrastructure, stope layout and stope sequencing. In this paper, a sequential approach is proposed to solve the sublevel determination problem which is a part of developmental and infrastructure planning and the stope layout planning problem for poly-metallic sublevel stoping mining with pillars. An algorithm is proposed for the sublevel determination that focuses on minimizing the developmental costs while maintaining access to the profitable portions of the orebody. The output of this algorithm is then submitted to the proposed stope layout planning approach. This approach aims to provide the stope layout plan for the orebody between each consequent pair of sublevels. This is achieved through iteratively generating combinations of stopes, where, at each iteration, the combination length is increased by one stope. The best combinations of stopes are saved in the memory. The number of combinations to save is limited by an input heuristic parameter. Additionally, a new mixed integer linear programming formulation for determining the ultimate stope limits is introduced for benchmarking purposes. A case study has been conducted on a copper-molybdenum mine to demonstrate the proposed ap-

proaches. The results have shown that all constraints regarding stope and pillar dimensions are respected. Furthermore, the output of stope layout plan is within the optimal mining limits, which confirms the validity of the approach.

7.2 Introduction

Sublevel stoping technique is an underground mining technique that requires interaction between geology, mine planning and rock mechanics (Villaescusa, 1998). Planning in sublevel stoping technique consists of the interconnected problems of development and infrastructure, stope layout, and sequencing. Development and infrastructure planning consists of deciding on sublevel, access ramp and shaft locations. Stope layout planning determines the locations of stopes and sequencing determines the order the stopes that will be mined. The interconnected nature of these problems requires that ideally these problems should be solved simultaneously. However, this would increase the search space enormously. Thus, they are solved in stages; first the development and infrastructure is settled, followed by the stope layout plan and sequencing. The profit optimization in planning in underground mines are very important due to high mining costs (Ben-Awuah et al., 2016).

The focus of this paper rests on sublevel determination and stope layout planning. Sublevel height range is determined by equipment and geotechnical properties. Decision of sublevel heights and locations is an engineering problem. To build a stope layout plan, incoming information from drill hole samples are simulated to model the grade distribution on a block by block basis (Mohammadi et al., 2012). From the obtained block model, a stope layout plan is constructed while respecting the geotechnical constraints which are stope and pillar dimensions. In computational point of view, stope layout planning is a combinatorial optimization problem with overlap constraints. Generally, the expected profit of the layout plan is maximized where either the mining blocks or possible stope shapes are decision variables. Stope shapes should respect the given range of stope sizes in each direction. The number of overlap constraints increase as a multiple of the number of possible stopes, which makes the problem very complex in larger problem instances and takes a long time to solve.

In this paper, we solve the sublevel determination and stope layout planning problems. Sublevels are determined with a new approach that minimizes the development costs. For stope layout planning, two alternative approaches are proposed. In the first approach, we introduce a new mixed integer linear programming (MILP) formulation that simplifies the problem and provides a valuable guideline for planning. Instead of finding each stope that will be extracted, this formulation finds the ultimate stope limits, delineating the orebody with the shape of at least the minimum stope size. The assignment of the delineated area to stopes can be done manually or through another algorithm. Alternatively, the output of this approach can be used as a guideline to construct or validate the actual plan. In the second approach, a greedy heuristic solution approach is proposed to solve the stope layout design problem with pillars.

7.3 Literature review

Three main types of approaches exist for stope layout planning in the literature that are exact methods, heuristic methods and metaheuristic methods. Due to the size and complexity of the problem, the majority of the approaches are heuristic methods (Nhleko et al., 2018).

Exact methods solve the stope layout planning problem optimally. Ovanic and Young (1995) introduced the branch and bound technique that optimizes starting and ending points of each row of blocks using MILP. It allows partial blocks to be included in the optimal stope layout, permitting irregular shapes in blocks. Deraisme et al. (1984) presented the downstream geostatistical approach which is an application of geostatistics where instead of using kriging to simulate the grades, which smooths the distribution of the data, probabilistic methods are used to reproduce grade variability. Dynamic programming is used to optimize 2D sections of the model. Bai et al. (2013) defined a vertical raise at the aligned with the orebody and generated a cylindrical coordinate around the initial vertical raise. In this coordinate, the blocks are converted into nodes. Adding a source and sink, the problem is solved with the maximum flow approach (Picard, 1976). Sari and Kumral (2018a) introduced a new formulation of MILP that simultaneously optimizes the stope layout and sublevel positioning.

Heuristic methods aim to solve the problem in a fast, possibly sub-optimal but effective fashion. Floating stope algorithm simulates the floatation of a stope shape with minimum size throughout the orebody and pick the stopes with the positive economic values. Problem arises when two positive stopes overlap. Ataee-pour (1997) developed the maximum value neighborhood algorithm that calculates the stope/block ratio and among all possibilities found for each block, the neighborhood with maximum net value is included in the final stope layout. The same problem with stope overlap is also present in this approach. Villalba Matamoros and Kumral (2017) proposed a heuristic approach that accounts for stope dimensions and manages dilution. Topal and Sens (2010) presented a preference-based profit maximization approach which can be maximizing the stope profit, stope profit per square meter or stope profit divided by it total mining time. Among possible stopes, highest valued stopes are selected. Stope size variation approach considers stopes with different sizes by aggregating the blocks into possible set of stopes then modifying the attributes of these stopes. 10.7% improvement over the maximum value neighborhood algorithm has been reported (Sandanayake et al., 2015b,a). Cheimanoff et al. (1989) introduced the octree division approach that recursively performs octree space division until they reach the minimum stope size and selects the stopes based on their calculated economic values. This is a heuristic approach because stope locations are checked only where the minimum stope dimension is a proper divisor, thus not yielding an optimal result.

Finally, Villalba Matamoros and Kumral (2018b) proposed a three-stage stochastic optimization model using genetic algorithms to perform stope layout optimization under grade uncertainty. Villalba Matamoros and Kumral (2018a) then conducted a research about calibrating the parameters of genetic algorithm for best results.

7.4 Methodology

In this paper, three methods are presented. The first method solves the sublevel determination problem. Then, the stope layout problem is solved in a simplified fashion with MILP in the second method. The third method is a heuristic approach that solves the stope layout design problem without simplifications and with pillar constraints. The nomenclature for all methods is given in Table 7.1.

Table 7.1 – Nomenclature for methodology

Indices and sets
$m \in M$: set of valuable metals within rock m
$i \in B$: set of blocks i
$s \in \tilde{S}_i$: set of stopes s with minimum dimensions that contain block i
$j \in T$: set of stopes j
$l \in \tilde{T}_j$: set of stopes l that overlap non-linearly with stope j
$n \in \tilde{B}_j$: set of blocks that are contained in stope j
$l \in \ddot{T}_j$: set of stopes l that overlap with stope j
$d \in \ddot{T}_d$: set of pillars that overlap with stope j
$c \in \tilde{P}_j$: set of pillars surrounding stope j
Parameters
v_i : economic value of block i when processed
v_j : economic value of stope j when processed
S_x : minimum stope dimensions in X direction in blocks
S_y : minimum stope dimensions in Y direction in blocks
S_z : minimum stope dimensions in Z direction in blocks
$\bar{S}_z$: maximum stope dimensions in Z direction in blocks
p_m : price of metal m
$g_{m,i}$: grade of valuable metal m of block i
R_m : processing recovery of metal m
C_m : mining cost
C_p : mineral processing cost
r_m : refining cost
t : tonnage
P_x : minimum pillar dimensions in X direction in blocks
P_y : minimum pillar dimensions in Y direction in blocks
Decision variables
x_i : 1 if block i is in the final stope limits design, 0 otherwise
y_j: 1 if stope j is in the final stope layout design, 0 otherwise
z_k: 1 if pillar k is in the final stope layout design, 0 otherwise

7.4.1 Sublevel determination

In sublevel stoping technique, access roads called sublevels are built that partitions the deposit horizontally. The cost of sublevel construction is very high (Stebbins and Schumacher, 2001). Therefore, for cost minimization and production volume maximization, sublevel heights need to be maximized within stability constraints.

The algorithm to determine the sublevels is given below:

1. To determine the starting and ending points of the orebody or the vein, the potential economic value of each row of blocks is calculated by evaluating the potential economic values of all blocks in EW–NS directions at every vertical block coordinate in the block model. If at the leading and trailing vertical levels, there are not any blocks with a positive potential economic value, those levels are trimmed from the planning model. The remaining number of levels in the vertical direction is recorded as N_z.

2. The minimum and maximum stope heights (S_z and $\bar{S}_z$) are read from the input.

3. If $\bar{S}_z$ is a proper divisor of N_z, the number of sublevels is $\frac{N_z}{\bar{S}_z} + 1$. The sublevels are placed from the first vertical level of the orebody and with the distance of $\bar{S}_z$ from each other.

4. Otherwise, first the number of sublevels is determined by $\frac{N_z}{\bar{S}_z} + 2$. In

the beginning, $\frac{N_z}{\bar{S_z}} + 1$ sublevels are placed from the first vertical level of the orebody and with the distance of $\bar{S_z}$ from each other. Each sublevel height is decreased by 1 block height until another sublevel can be added with at least minimum height.

The locations of the sublevels can be determined using this approach or manually. The advantage of separating the sublevel determination and stope optimization steps is that these steps become independent and any combination of methods can be used as needed for the optimization.

7.4.2 Ultimate stope limits

Due to the large number of constraints in the stope layout problem, larger problem instances cannot be solved by current MILP models. Large problem instances are common in mine planning. However, MILP is still used underground mine planning with speeding attempts such as simplifying the problem or clustering blocks (Nezhadshahmohammad et al., 2018; Pourrahimian, 2013). A new simplified MILP formulation is proposed in this section. This formulation is a novel approach to the stope layout planning problem and it is analogous to the ultimate pit limits approach. Ultimate pit limits approach simplifies the problem by delineating profitable and non-profitable mineable sections of the orebody instead of finding each pit, which is a larger sized problem (Kumral, 2011, 2012). Similarly, this formulation delineates profitable mineable sections, where a mineable section has at least the size of a stope with minimum dimensions instead of including or excluding each stope particularly. The objective is to maximize the economic value of the mine by deciding on the blocks to be extracted. However, each selected block should be a part of a stope frame that is either extracted or not extracted. If this is not done, irregular shapes will emerge and forming stopes from the resulting limits will not be feasible. The MILP model is given as:

Maximize:

$$\sum_{i \in B} x_i \times v_i \tag{7.1}$$

Subject to:

$$y_j + y_l \leq 1 \qquad \forall j \in T, l \in \tilde{T}_j \tag{7.2}$$

$$\sum_{s \in \tilde{S}_i} y_s \geq x_i \qquad \forall i \in B \tag{7.3}$$

$$\sum_{n \in B_j} x_n \geq y_j \times S_x \times S_y \times S_z \qquad \forall j \in T \tag{7.4}$$

Where:

$$v_i = \sum_{m \in M} [(p_m - r_m) \times g_{m,i} \times R_m - C_m - Cp] \times t \tag{7.5}$$

The objective in Equation 7.1 maximizes the profit obtained from the sale of the extracted material. The constraint 7.2 ensures that two non-linearly overlapping stopes will not be extracted at the same time. This allows the limits to extend width and lengthwise, and vertically only. The constraints 7.3 and 7.4 yield the connection between block and stope decision variables. The constraint 7.3 expresses that a block cannot be extracted if it does not exist in at least one of the extracted stopes. The constraint 7.4 ensures that if a stope is extracted, all blocks contained in that stope should be extracted. Equation 7.5 expresses the profit obtained from a block, which is the revenue of producing and selling the metal subtracted by the mining and processing costs. A given block profit is only incorporated in the objective function if the block is to be extracted (in other words, x_i is 1). If the block is not extracted, the value obtained will be zero.

Two stopes are defined to be non-linearly overlapping stopes constraint if they are at the same location in less than n-1 dimensions where n is the dimension the stopes are defined in. In other words, two stopes can only differ in one dimension to be linearly overlapping. Examples of linear and non-linear overlapping are provided in Figure 7.1. Allowing linear overlap decreases the number of constraints and permits the area to grow in a rectangular shape only. It eliminates the need to choose stopes one by one, but instead defines a rectangular area where the stopes will be fit in.

This model can be further simplified by selecting the sublevels prior to the optimization and evaluating areas between each sublevels individually.

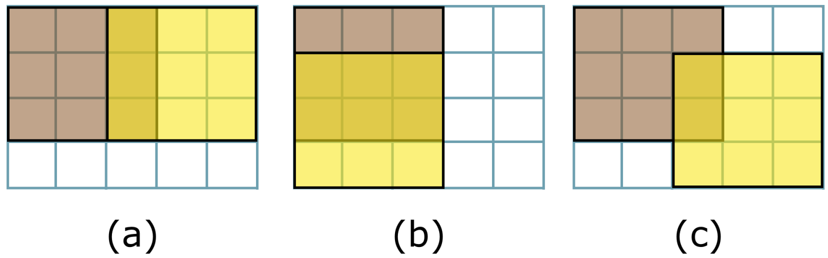


Figure 7.1 – Overlap examples of two stopes. (a) and (b) are examples of linear overlapping while (c) is an example of non-linear overlapping

When the sublevels are selected the stope heights are set. Therefore, the model can be converted to two dimensions by aggregating the blocks in the Z dimension. This can be achieved simply by summing the profit of blocks in Z dimension. As a result, the above proposed formulation for 3 dimensions can be used only by modifying the constraint in Equation 7.4 to Equation 7.6:

$$\sum_{n \in B_j} x_n \geq y_j \times S_x \times S_y \qquad \forall j \in T \tag{7.6}$$

This model outputs the stope limits to provide an insight to the orebody while respecting the stope shapes and where the mineable area is located overall. However, it does not provide the information on which stope will be extracted. This can be done manually after the limits are set. Alternatively, it can be used as guidance or a benchmarking tool for measuring the validity of heuristic algorithms. The proposed algorithm for stope layout problem is given in the following section.

7.4.3 Heuristic greedy algorithm

After the sublevels are determined using the proposed algorithm in Section 7.4.1, for each segment between sublevels, the stope layout planning algorithm is run separately. The proposed stope optimization algorithm can be seen as an extension of the method described in Sari and Kumral (2018c) to poly-metallic mines with pillars. The method presented in this paper can handle more than one metal and pillar requirements. Pillars are portion of the orebody around stopes that are not mined for stability purposes (Guo et al., 2016; Wang et al., 2017). When the rock is relatively less rigid, pillars might be needed. The proposed approach is able to leave pillars in each side of stopes. The pillar size is specified in the beginning and when the pillar size is zero, the algorithm will select stopes without pillars. After the determination of sublevels, the segment between each pair sublevels is converted to two dimensions by aggregating the blocks in the vertical direction, similarly to the MILP model and planned consecutively. The stope dimensions depend on rock and orebody characteristics and must be specified by a geotechnical engineer to ensure mine stability and avoid faults (Li et al., 2018; Sainoki and Mitri, 2017). The proposed algorithm is given as follows:

1. An empty memoization list L_1 is created that is a structure composed of stope chains of length $M = 1$. In addition to the stope chains, the overall profit obtained by mining that set of stopes is saved for each memoization item.

2. The first feasible solution is created by populating the area with stopes and pillars of minimum size starting from the upper left block, where between each stope, a pillar is placed in each direction as illustrated in Figure 7.2. Last placed stope index is recorded as n. All the possible stopes located beyond the stope n-1 are added to the memoization list.

3. n is updated as $n \leftarrow n - 1$. If $n < 0$, the algorithm is terminated and the stope chain with the highest profit is returned. Otherwise, a new empty memoization list L_2 with the stope chains of length M+1 and with the size α is created.

4. The stope at index n is tested for feasibility with all chains in L_1. The feasible chains are added to L_2 with the addition of the stope at index n. If the L_2 is full, the stope chain with the lowest potential profit is dropped. The stope at index n is moved to the next location and addition to the memoization list L_2 is continued. This step is repeated the stope at index n passes through the first stope at the index n+1.

5. Update L_1 as $L_1 \leftarrow L_2$ Return to step 3.

During these calculations, the possible stopes with a non-positive profit are discarded. Moreover, when testing for feasibility, in addition to checking the stope overlap constraint, pillars with minimum size are checked to be located between each pair of stopes. The model solved in the algorithm is given in Equations 7.7, 7.8 and 7.9. The heuristic component of this

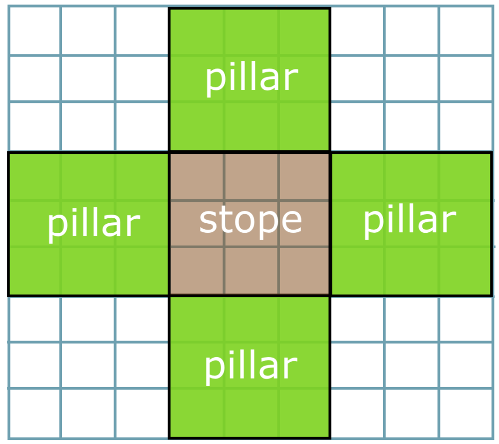


Figure 7.2 – An example stope surrounded by pillars

algorithm is the maximum size of the L_2 list, which is α. If the size of L_2 is set infinite, then the algorithm returns the exact, optimal profit. However, with large datasets, this might take more time and the computer may run out of memory. Therefore, a suitable value for α must be selected, depending on the available computational resources and time.

Maximize:

$$\sum_{j \in T} y_j \times v_i \tag{7.7}$$

Subject to:

$$y_j + y_l \leq 1 \qquad \forall j \in T, l \in \ddot{T}_j \tag{7.8}$$

$$y_j + z_d \leq 1 \qquad \forall j \in T, d \in \ddot{T}_d \tag{7.9}$$

$$y_j \le z_c \qquad \forall i \in B, c \in \tilde{P}_j \tag{7.10}$$

In this formulation, the objective is to maximize the profit obtained from mined stopes. Equation 7.8 ensures overlapping stopes cannot be selected at the same time and Equation 7.9 ensures overlapping stopes and pillars cannot be selected. Equation 7.10 requires that if a stope is selected, all the pillars should also be selected.

7.5 Case Study

To demonstrate the efficiency of the proposed approaches, a case study of a poly-metallic copper and molybdenum deposit is conducted. The case contains a large dataset of 595,056 (98x132x46) blocks where each block has the dimensions of 5x5x5 m. The block model is created using sequential Gaussian simulation. The parameters related to the case study are given in Table 7.2.

The proposed sublevel determination and heuristic stope layout planning approaches are implemented in C++ programming language. Also, the proposed ultimate stope limits formulation for 2–dimensions is implemented using Zimpl programming language (Koch, 2005) and then solved using CPLEX. The sublevel determination and heuristic stope layout planning implementations are tested on a MacBook Pro 2015 computer with 2.7

Table 7.2 – Parameters related to the case study

Parameter	**Value**
Deposit size	98 x 132 x 46 blocks
Block size	5 x 5 x 5 m
Ore prices	\$5,500/tonne and \$15,500/tonne
Mining cost	\$10/tonne
Mineral processing cost	\$23/tonne
Recoveries	85% and 75%
Refining cost	\$500/tonne
Minimum stope size	30 x 40 x 20 m
Maximum stope size	30 x 40 x 40 m
Minimum pillar size	30 x 20 m
Heuristic component (α)	50,000

GHz Intel Core i5 CPU (dual-core) and 16 GB RAM. The execution time took 11,927 seconds. The sublevel determination algorithm decided on the sublevel locations: 0–8–16–24–31–38–45 on the Z dimension. The heuristic component (α) that decides on the L_2 list size for the heuristic stope layout planning approach is set to 50,000. The resultant expected profit obtained given by the stope layout plan for each section between consequent sublevels are given in Table 7.3.

The resulting plan of the heuristic stope layout planning approach is compared to the stope limits generated by the MILP approach on the same dataset and parameters. The comparative images of the deposit and the results are given in Figures 7.3–7.6. In each figure, (a) is the copper grade image of the deposit (in g/tonne), (b) is the molybdenum grade image of the deposit (in g/tonne), (c) is the resulting stope layout plan where there

Table 7.3 – The resulting plan generated by the heuristic stope layout planning approach

Orebody between sublevel locations on Z dimension (in blocks)	**Profit**
0–8	$ 13,819,500
8–16	$ 8,480,150
16–24	$ 10,758,500
24–31	$ 7,360,450
31–38	$ 8,906,530
38–45	$ 7,576,310
Totoal	**$ 43,081,940**

colors correspond to the average copper grade of stopes (in g/tonne) and (d) is the ultimate stope limit model output where only the colored areas are within the limits. As can be observed in Figures 7.3–7.6 the resulting plan is within the ultimate stope limits. In both approaches, it can be noted that the high-grade areas are included in the extraction area. It can be clearly seen that stope overlap constraints and the pillar constraints are satisfied by the heuristic stope layout planning approach. Additionally, it can be seen that the MILP approach generated at least minimum stope sized limits.

It is also important to note that the (1) minimum average stope grade is higher than the minimum copper grade and (2) the maximum average stope grade is lower than the maximum copper grade. The reason behind the first observation is that only the areas with positive expected profit are extracted, which are the higher graded portions. The reason that observation (2) is occurring is the smoothing effect of averaging all the grades in a stope.

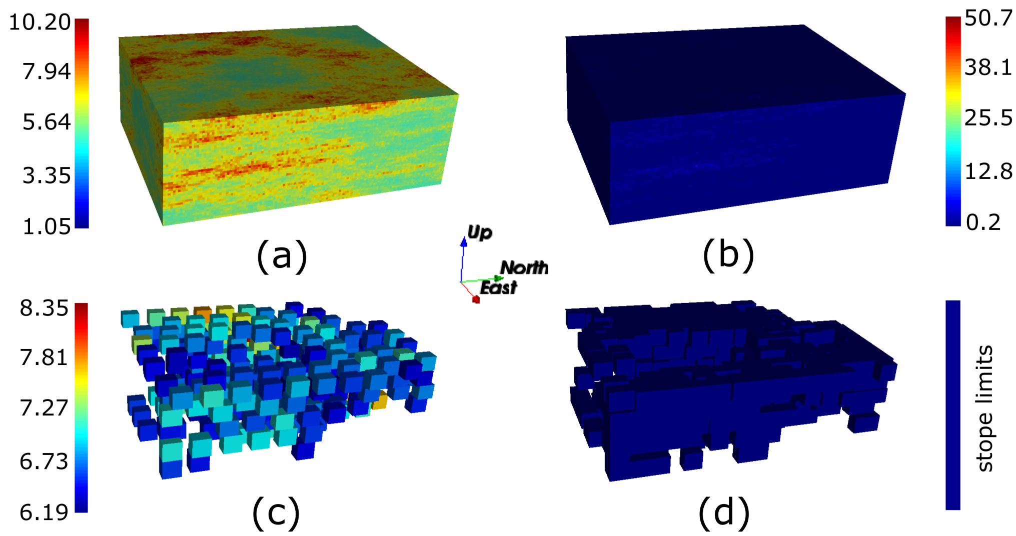


Figure 7.3 – Image capture from the eastern side of the deposit

7.6 Conclusions

In this paper, a new method of sublevel determination and a stope layout planning approach for poly-metallic underground mines that use the sublevel stoping technique with pillars is proposed. The sublevels are determined by an algorithm that aims to minimize the number of sublevels due to their high building cost. This is achieved through taking the highest acceptable stope height into account and decreasing the height if needed to be able to access the mineable reserve. The stope layout planning takes the sublevels as an input and generates a stope layout plan for the portion of the deposit between each sublevel pairs. The stope layout planning algorithm is a greedy heuristic algorithm that is able to incorporate pillars into the plan.

The proposed stope layout planning algorithm identifies the sub-problems

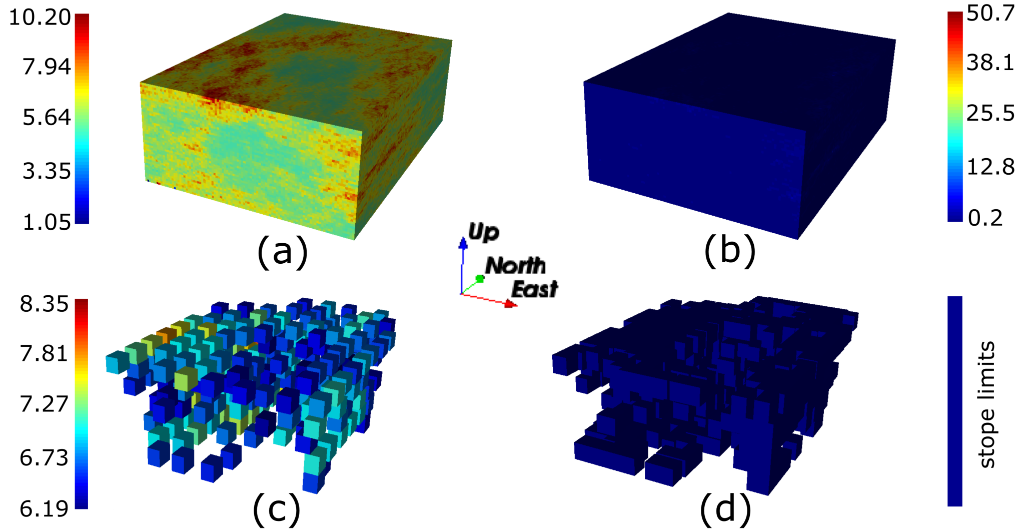


Figure 7.4 – Image capture from the south-eastern side of the deposit

by starting from pairs of combinations of stopes and extending the list iteratively. Heuristic parameter is used to limit the number of combinations to keep in memory. When the number of combinations exceeds the heuristic parameter, the combination with the lowest expected profit is dropped, which is the greedy aspect of the algorithm. The heuristic parameter helps speed up the search and limits memory use. If this parameter is set to infinite, the proposed method becomes an exact method.

Another contribution of this paper is the ultimate stope limits MILP model. Instead of finding the stopes that will be produced, it finds the general frame where the orebody is located while having the minimum size of a stope at each corner. The stopes can then be placed inside the generated frame manually. It is analogical to ultimate pit limits in open pit mine planning. It

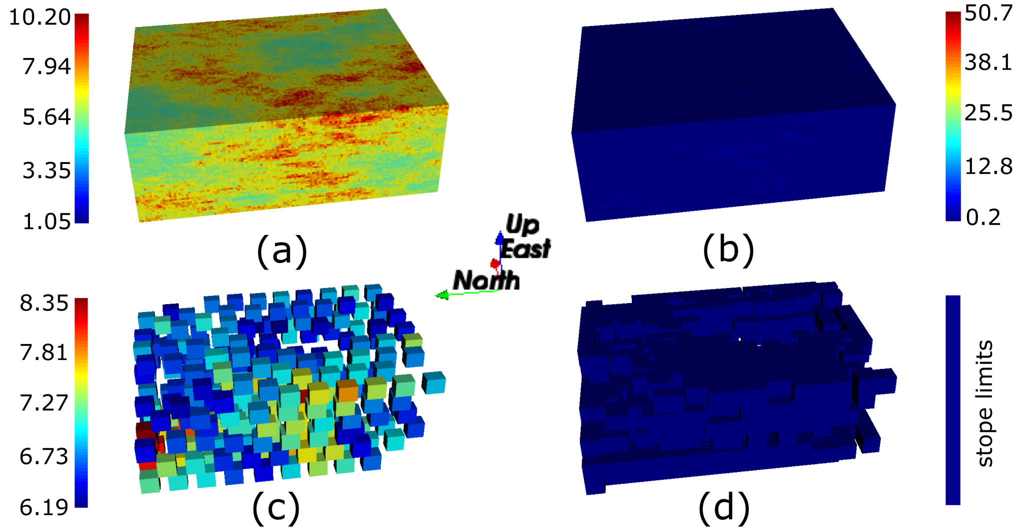


Figure 7.5 – Image capture from the western side of the deposit

simplifies the model by allowing linear overlaps and only including non-linear overlap constraints. This results in rectangular growth, where the minimum rectangle is the minimum stope size.

A case study has been conducted to compare the outputs of the proposed stope layout planning algorithm and the MILP model. The results have shown that the generated stope layout plan is inside the ultimate stope limits. Also, compared with the deposit grade images, the valuable portions are orebody is captured with the proposed plan. The results confirm that the constraints for the stope and pillar dimensions in the layout planning algorithm and the minimum stope dimensions in the MILP model are respected.

The algorithms are able to generate fast and near-optimal results. The future work for this research includes comparing the results to the results

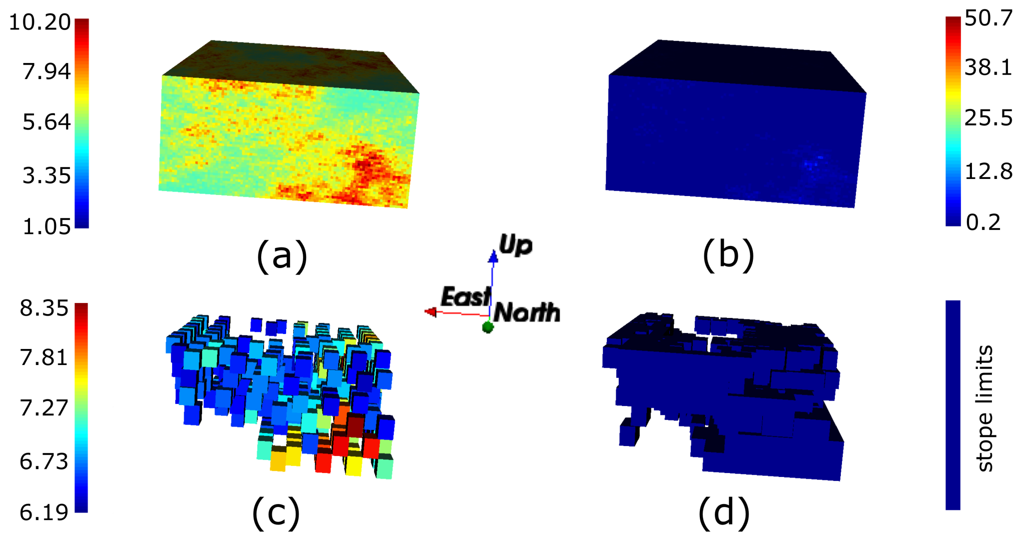


Figure 7.6 – Image capture from the northern side of the deposit

of other algorithms for benchmarking purposes. Also, a stope sequencing approach will be developed that will accept the generated stopes as an input.

7.7 Chapter Conclusion

As a result of this extension to the greedy heuristic method, the approach has become applicable and practical to more underground mines. The comparison of the case study to ultimate stope limits MILP formulation has shown that the proposed approach works very well. In the next section, which is the final component of the thesis, the stope sequencing problem with ore grade uncertainties is explored. After determining the stope layout plan, sequencing has less number of variables. As the problem size is relatively smaller, a MILP formulation is proposed.

8 Risk-based stope sequencing optimization for underground mines through chance-constrained programming

8.1 Abstract

Underground mining requires extensive development and involves high initial and operational costs. Hence, mine planning that accounts for uncertainty is crucial for underground mining operations. Sublevel stoping consists of mine stability, cost minimization and production requirements constraints. The most critical uncertainty for sublevel stope sequencing arises from sparse data of ore grades. A new mixed integer linear program is proposed that accounts for net present value uncertainty using chance constrained programming. A case study was conducted with varying risk levels and sequences were generated. It was shown that the expected NPV increased in higher risk levels.

8.2 Introduction

Sublevel stoping is one of the commonly used underground mining techniques. In this technique, the equipment is transported to the underground by constructing ramps and from the ramps, access roads called sublevels are built to access the orebody. The orebody is mined by drilling and blasting rectangular structures resembling rooms called stopes. Fragmented ore is then hauled from the drawpoints below the stopes and transported to the surface. To maintain mine stability, neighboring stopes cannot be mined at the same time. Moreover, once the stope has been mined, the void created is filled with waste material called backfill. Backfill increases stability but it is not as strong as the mined rock. Thus, after backfilling, a stope should only be exposed on one side (Little et al., 2013). In addition to stability requirements, there are operational/mill requirements such as production capacity and backfill capacity.

Planning problems regarding underground mining using stoping methods can be classified as stope layout and stope sequencing optimization. The stope layout optimization concentrates on finding the most profitable part of mineral deposit under the constraint of feasible stope size imposed by geotechnical requirements. This problem can seen as the equivalent of ultimate limit problem in open pit mining. On the other hand, the stope sequencing focuses on finding the sequence maximizing net present value of underground mining venture under constraints of capacity, and filling and curing stope.

In practice, the stope layout is determined first. The blocks which are out of the layout are removed from the data set. Then, the layout is submitted to the sequencing process. In fact, this successive solution process leads to sub-optimality and these two problems should be solved simultaneously (Kumral, 2012). However, the problem size is large and simultaneous solution is almost impossible due to long time required to solve the problem. Although determining stope layout in advance decreases the number of variables, the complex structure of underground mines (Yilmaz, 2018; Vallejos et al., 2018), hence the number of constraints make stope scheduling a difficult problem to solve. As the number of stopes increases, the problem grows exponentially. Hence, a computational method is needed to solve the stope sequencing problem that accounts for uncertainties.

In mining engineering, many planning decisions are made in the medium of sparse available data. Mining operations comprises of many uncertainties, making risk management crucial at the feasibility stage. These uncertainties mainly arise from limited information concerning the orebody and fluctuation of the future financial parameters. Mine exploration involves drilling the rock and taking samples, which are then sent for laboratory assessment. As a result of this assessment, a grade is assigned to the sample according to the contained ratio of mineral. This procedure is carried out to gain insight to the potential economic value of the orebody. However, drilling is an expensive process. Thus, only a limited number of samples are collected. The remaining portion of the orebody is estimated or simulated from the collected samples, resulting in uncertain grades. The financial uncertainties consist of volatility in the ore prices and mining and mineral processing costs.

Risk management is particularly important in underground mines, in which the operations are smaller compared to surface mines. Due to economies of scale, smaller equipment and operations result in higher cost per mined material unit. Also, as a result of deeper mining and the required infrastructure for ore access and mine stability, mining costs are usually much higher in underground mines compared to surface mines. Moreover, in underground mines the income is delayed until the initial development is complete.

In this paper, we propose a new mixed integer linear programming (MILP) formulation that performs stope sequencing taking account of the uncertainties. The incorporation of uncertainty management is achieved through a multi-objective optimization where the net present value is aimed to be maximized and uncertainty is minimized. The problem is considered at different risk levels and how to choose the suitable sequencing plan is explored. A case study has been conducted to demonstrate the approach.

8.3 Literature Review

MILP is the most commonly used method for stope sequencing optimization in the literature. Chanda (1990) has introduced the mixed integer programming method for the scheduling of underground mines. Trout (1995) developed one of the early models that focuses on maximizing the net present value while satisfying the constraints such as stope extraction capacity, stope backfilling demands, minimum metal quantity, hoisting capacity and stope geometry relationships. The quantity of ore and backfill variables are represented in continuous variables which allows them to have non-integer values. A small case study of a representative data set comprising 55 stopes from the Mount Isa mine is presented to demonstrate the efficiency of the method. Due to limited computational resources, the program was terminated prior to the proof of optimality. However, compared to a manually generated schedule, 23% improvement in the net present value of the project has been observed.

Further works on this approach include the research of Nehring and Topal (2007), where an additional constraint was introduced regarding limitation of multiple fillmass exposures. Little (2007) took this improved model and reduced the number variables following the logic later presented in Nehring et al. (2010) that suggests combining the development, drilling and backfilling phases using the concepts of natural sequence and natural commencement which reduces the number of binary decision variables by a factor of five. Little et al. (2008) applied the new mixed integer programming model on a small conceptual study, resulting in the same production schedule, yet 80% decrease in the number of binary variables and 92% improvement in the overall solution time. Sarin and West-Hansen (2005) developed a model that maximizes NPV of a coal mine and obtains desired coal quality. Binary variables are assigned to equipment to be used in each section and period. The quality and the production volume of the coal are tracked by continuous variables. Constraints of the model include smoothing quality and production levels and setting maximum number of sections that can be mined at a time. Benders' decomposition, which is a technique in mathematical programming to solve very large problems that exhibit a special block structure usually found in stochastic problems is applied to solve the problem. Terblanche and Bley (2015) aimed to find a balance between reducing the resolution to smooth the grade data and maintaining enough detail to easily discretize between valuable and non-valuable portions of the deposit. This improved the profitability through selective mining. Although mixed integer programming methods yield optimality, the drawback of using this technique is as with all linear programming applications, as the problem size grows solving time increases exponentially. In a real application, this approach will take a long time. However, it can be combined with heuristic techniques to speed up the overall process.

An alternative to exact methods was introduced by Manchuk (2008), who applied the simulated annealing (SA) approach to stope sequencing problem. Perturbations were accepted if either there is an increase in the NPV or there is a probability that the sequence leading to a more optimal one in the future perturbations. The general flow of the approach is the following: (1) Initially, a feasible, sub-optimal schedule is taken as an input by the program. (2) A stope from the panel of the current schedule is picked randomly. (3) A list of all feasible stopes is created and randomly swapped one of the feasible stopes with the chosen stope in the previous step. (4) If the new order is feasible, the NPV of the updated schedule is calculated. (5) If there is an improvement, the solution is accepted as the new current solution. Otherwise, the solution is accepted with a small probability to allow a better search of the solution space. The results of this algorithm were compared to a logic-driven approach called probabilistic decision making (PDM) approach. In this approach, stope properties such as stope profit, time required to extract stope, costs associated with stope are considered and used to calculate a value P, which is the probability of being a good decision to mine a stope, then gradient descent is performed to test and update the sequencing order. The results have shown that although PDM performs slightly better with smaller problems, as the complexity increased, the random approach, SA performed considerably better.

Extensive research has been conducted to decrease uncertainty in open pit mines (Kumral, 2010, 2011, 2015; Amankwah et al., 2013). Majority of these approaches integrate uncertain programming to MILP formulations. However, uncertainty is still to be explored in stope sequencing applications.

8.4 Model formulation

Stope layout plan can be generated using a variety of methods (Villalba Matamoros and Kumral, 2017, 2018b; Sandanayake et al., 2015b; Bai et al., 2013; Sari and Kumral, 2018a,b,c; Erdogan et al., 2017). After the plan is settled, the stopes must be sequenced for a production plan. Because of the discounting effect, it is more profitable to prioritize stopes with higher economic values. Profit is obtained from stopes by selling the produced material and by subtracting the costs as given in Equation 8.1. However, sublevel stoping mining method has constraints that prevents free sequencing of stopes. In this section, stope sequencing is mathematically formulated first deterministically and then using probabilistic programming. The notation concerning

Table 8.1 – The list of notations regarding the MILP model

Indices and sets
$s \in S$: set of stopes s
$p \in P$: set of periods p
$\hat{s} \in A_s$: set of stopes that are adjacent to stope s in the X–Y plane
$\bar{s} \in B_s$: set of stopes that are vertically aligned with stope s
$\tilde{s} \in D_s$: set of stopes that are located on different levels than stope s
$\hat{p} \in H_t$: set of periods that are earlier than period p
Parameters
y: ore price
g_s: ore grade of stope s
R: ore recovery
G_m: mining cost
G_p: mineral processing cost
d: discount rate per period
T_s: tonnage of stope s
Q_s: volume of stope s
C_h: extraction capacity per period
C_l: minimum extraction requirement per period
B : backfill capacity per period
Decision variable
x_{sp} : 1 if stope s is extracted at period p, 0 otherwise

this section is given in Table 8.1.

$$v_s = [g_s \times y \times R - G_m - G_p] \times T_s \tag{8.1}$$

8.4.1 Stope sequencing model

The mathematical formulation of the stope sequencing problem is given as follows:

Maximize:

$$\sum_{p \in P} \sum_{s \in S} \frac{v_s}{(1+d)^{p-1}} \times x_{sp} \tag{8.2}$$

Subject to:

$$\sum_{p \in P} x_{sp} \leq 1 \qquad \forall s \in S \tag{8.3}$$

$$x_{sp} + x_{\hat{s}p} \leq 1 \qquad \forall s \in S, \forall \hat{s} \in A_s, \forall p \in P \tag{8.4}$$

$$x_{sp} + \sum_{\bar{s} \in B_s} x_{\bar{s}p} \leq 1 \qquad \forall s \in S, \forall p \in P \tag{8.5}$$

$$\sum_{\hat{p}\in H_t} x_{s\hat{p}} + \sum_{\hat{s}\in A_s} x_{\hat{s}p} \leq 2 \qquad \forall s \in S, \forall p \in P \tag{8.6}$$

$$x_{sp} + x_{\tilde{s}p} \leq 1 \qquad \forall s \in S, \forall \tilde{s} \in D_s, \forall p \in P \tag{8.7}$$

$$\sum_{s\in S} T_s \times x_{sp} \leq C_h \qquad \forall p \in P \tag{8.8}$$

$$\sum_{s\in S} T_s \times x_{sp} \geq C_l \qquad \forall p \in P \tag{8.9}$$

$$\sum_{s\in S} Q_s \times x_{sp} \leq B \qquad \forall p \in P \tag{8.10}$$

The objective given in Equation 8.2 aims to maximize the NPV of the sublevel stoping operation. Equation 8.3 expresses the mathematical constraint that a stope can only be extracted once during the mining operation. Equations 8.4–8.6 express mine stability constraints. Equation 8.4 prohibits adjacent stopes in the X–Y plane to be extracted at the same period while

Equation 8.5 disallows simultaneous extraction of stopes that are aligned in the Z direction. Once a stope is mined and backfilled, although strengthened, it will not provide the same support as the intact rock. For this reason, only one adjacent stope of a backfilled stope can be mined at the same time as formulated in Equation 8.6.

Equation 8.7 requires that only stopes located on the same level can be mined in the same period. This constraint ensures time and costs are conserved by minimizing the equipment transport duration. Equations 8.8–8.10 convey the production requirements. Equation 8.8 ensures the production is below the capacity C_h per period while Equation 8.9 enforces the minimum production requirement. Finally, Equation 8.10 maintains the used backfill material below the capacity per period.

The simplified version of the stope sequencing problem without capacities and the constraints in Equations 8.6 and 8.7 bares similarities to traveling salesman problem with adjacent and vertically aligning stopes having infinite distances between them to prohibit simultaneous extraction. The distances between stopes would be inversely proportional to the stope profit and would have to be increased with the ordering. Therefore, the stope sequencing problem is at least as difficult as the traveling salesman problem.

8.4.2 Stope sequencing model with risk management

To manage the uncertainties caused by sparse grade information, chance-constrained programming (CCP) that is introduced by Charnes and Cooper

(1959) is used. CCP models the stochastic constraints such that they will hold above the confidence or reliability level α provided by the decision maker by converting them to their deterministic components (Liu, 2009). In our problem, each stope has multiple average grades generated by Gaussian sequential simulation. These grades are independent and identically distributed with normal distribution. As the stope NPV is a function of grade, it also follows the normal distribution. Following the formulation that was developed by Shih and Frey (1995) and later used by Kumral and Sari (2017) for open pit mines, chance constrained objective function can be expressed as:

Maximize e:

Subject to:

$$Pr\left\{\sum_{p\in P}\sum_{s\in S} V_{sp}x_{sp} \geq e\right\} \geq \alpha \tag{8.11}$$

where

$$V_{sp} = \frac{v_s}{(1+d)^{p-1}} \tag{8.12}$$

The expected value $E(V_{sp})$ and variance $VAR(V_{sp})$ of NPV of each stope can be calculated from the values obtained from different simulations, where the set of simulations is denoted by K.

$$E(V_{sp}) = \mu_{sp} = [E(V_{spk})] \qquad k \in K \tag{8.13}$$

$$VAR(V_{sp}) = \sigma_{sp}^2 = COV(V_{spj}, V_{spk}) \qquad j, k \in K \tag{8.14}$$

A random variable r that follows a normal distribution can be standardized to $Z(\alpha)$ at a given risk tolerance α as follows:

$$Z(\alpha) = \frac{r - \mu_r}{\sigma_r} \tag{8.15}$$

where $\Phi_Z[Z(\alpha)] = \alpha$ and $\Phi_Z(\cdot)$ is the standard normal distribution function. Then, the probability of r can be expressed as:

$$Pr\{r \geq \mu_r + Z(\alpha)\sigma_r\} = \alpha \tag{8.16}$$

We can apply the same steps to our random variable V_{sp} and obtain (Stancu-Minasian, 1984):

$$\sum_{p \in P} \sum_{s \in S} \mu_{sp} x_{sp} - \Phi_Z^{-1}(\alpha) \left(\sum_{p \in P} \sum_{s \in S} \sigma_{sp}^2 x_{sp}^2 \right)^{1/2} \geq e \tag{8.17}$$

As $\sum_{p \in P} \sum_{s \in S} \sigma_{sp} x_{sp} > \left(\sum_{p \in P} \sum_{s \in S} \sigma_{sp}^2 x_{sp}^2 \right)^{0.5}$, Zhu et al. (1994) stated that the constraint becomes more conservative if the above equation is replaced with:

$$\sum_{p\in P}\sum_{s\in S}\mu_{sp}x_{sp} - \Phi_Z^{-1}(\alpha)\sum_{p\in P}\sum_{s\in S}\sigma_{sp}x_{sp} \geq e \tag{8.18}$$

which linearizes the formulation. Finally, as e is maximized, the left hand side of the inequality is also maximized. Therefore, the objective function in Equation 8.2 can be revised as:

Maximize:

$$\sum_{p\in P}\sum_{s\in S}\mu_{sp}x_{sp} - \Phi_Z^{-1}(\alpha)\sum_{p\in P}\sum_{s\in S}\sigma_{sp}x_{sp} \tag{8.19}$$

The remaining of the formulation can be left intact because the NPV is only present in the objective function. The revised formulation in Section 8.4.1 with the above objective is the chance constrained programming model that accounts for the uncertainty of grades. It can be noted that with the proposed revision, the problem a becomes multi-objective optimization that maximizes the expected NPV and minimizes the standard deviation of the NPV with $\Phi_Z^{-1}(\alpha)$ maintaining the balance between the two objectives.

8.5 Case Study

To investigate the proposed approach, a case study has been conducted for a gold mine. Ten block models were generated using sequential Gaussian

Table 8.2 – The list of parameters regarding the case study

Mining Parameters	Value
Number of blocks in X, Y and Z directions	42, 52, 20
Height, width and depth of blocks	5m, 5m, 5m
Minimum stope size in X, Y, Z directions	30m, 40m, 30m
Maximum stope size in X, Y, Z directions	30m, 40m, 40m
Economic parameters	Value
Price	\$ 38/gr
Mining cost	\$ 40/tonne
Mineral processing cost	\$ 22/tonne
Ore recovery	85%
Discount rate per period (2 months)	2%
Operational parameters	Value
Processing capacity per period	90,000 tonnes
Minimum processing requirement per period	20,000 tonnes
Backfill volume utilization capacity per period	50,000 m^3
Mine life	150 periods

simulation. The block models contain $42 \times 52 \times 20 = 43,680$ blocks and every block is 125 cubic meters. The mining related, economic and operational parameters are given in Table 8.2. The stope layout plan for the mine was generated with the method proposed by Sari and Kumral (2018c), which is a greedy heuristic approach based on dynamic programming. In this method, the potential profits of different stope combinations are tested by starting from one stope and iteratively adding more stopes. The most promising results at each iteration are saved in the memory. The maximum number of most promising results to save is governed by a heuristic parameter.

The stope layout was generated using the average of the block models. The execution time took 127 seconds on a MacBook Pro 2015 computer with

2.7 GHz Intel Core i5 CPU (dual-core) and 16 GB RAM. The resulting plan can be viewed in Figures 8.1 and 8.2. In the figures, the colorbar corresponds to the ore grades of the resource in (a) and to the average grade of each stope in (b). The grades in part (a) are more dispersed according to the colormap compared to part (b). This is because the averages of stopes are taken for the stope grade and this results in a grade smoothing effect. In total, 266 stopes have been generated.

The proposed chance-constrained MILP formulation given in Section 8.4.1 and revised in Section 8.4.2 has been implemented using Zimpl (Zuse Institut Mathematical Programming Language)(Koch, 2006). The Zimpl code for the MILP formulation can be found in the Appendix. The generated stope layout plan has then been solved with the formulation using CPLEX for reliability levels of 50%, 65%, 75%, 85% and 95%. The instances were run on Compute Canada Graham computing clusters with 32 processors for each run. All of the programs terminated under 30 minutes with less than 5% deviation from optimality. The results of the optimization are given in Table 8.3 and visualized in Figure 8.3. The objective is the highest in the 50% reliability level and gradually decreases until the 95% reliability level. This is expected as lower risk levels attempt to minimize the standard deviation of the project in addition to maximizing the NPV.

The visualization of the sequencing plan is given in Figure 8.4. In the figure, the colormap corresponds to the extraction sequence of stopes. Comparing with Figures 8.1 and 8.2, it can be observed that at 50% reliability,

Table 8.3 – Stope sequencing optimization results for each reliability level

Reliability level	Objective
50%	457,393,972.64
65%	360,104,921.72
75%	311,161,844.04
85%	233,333,522.88
95%	123,033,189.69

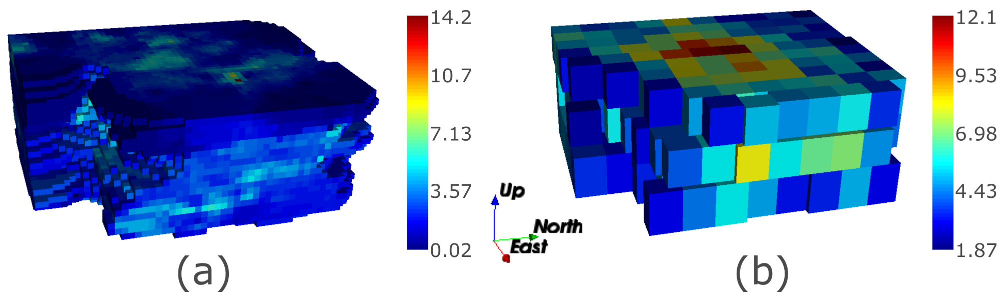


Figure 8.1 – Visualization of (a) the ore grades and (b) the stope layout plan from E–W direction

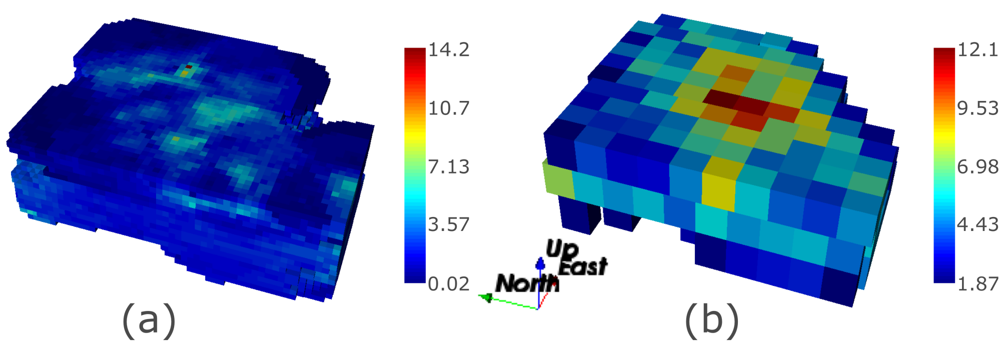


Figure 8.2 – Visualization of (a) the ore grades and (b) the stope layout plan from NE–SW direction

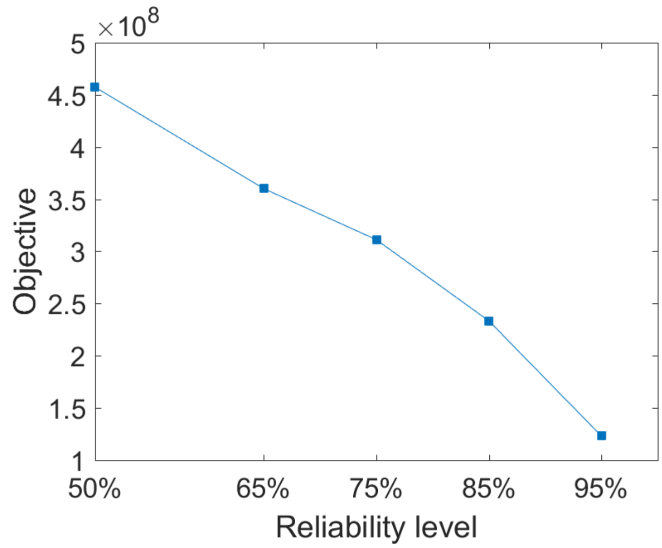


Figure 8.3 – Stope sequencing optimization results for each reliability level

the sequence is most similar to the stope grades color scheme. At 50% reliability, $\Phi_Z^{-1}(\alpha) = 0$ thus the second objective which is the minimization of the standard deviation is eliminated. The objective solely becomes maximizing the NPV with the sublevel stoping constraints. This is equivalent to the deterministic model. Thus, mostly the grade order is preferred as the constraints permit. As the reliability increases, the sequence diverges from the grade order because also the standard deviation is decreased. This remark can be observed more clearly in Figure 8.5 where stope extraction periods are compared to stope profit rankings. As the grade is proportional to profit, the ordering is the same for basing on profit or grade. In Figure 8.5, it can be seen that for lower reliability levels, the stope profit ranking is positively correlated with and proportional to the extraction period. On the other hand,

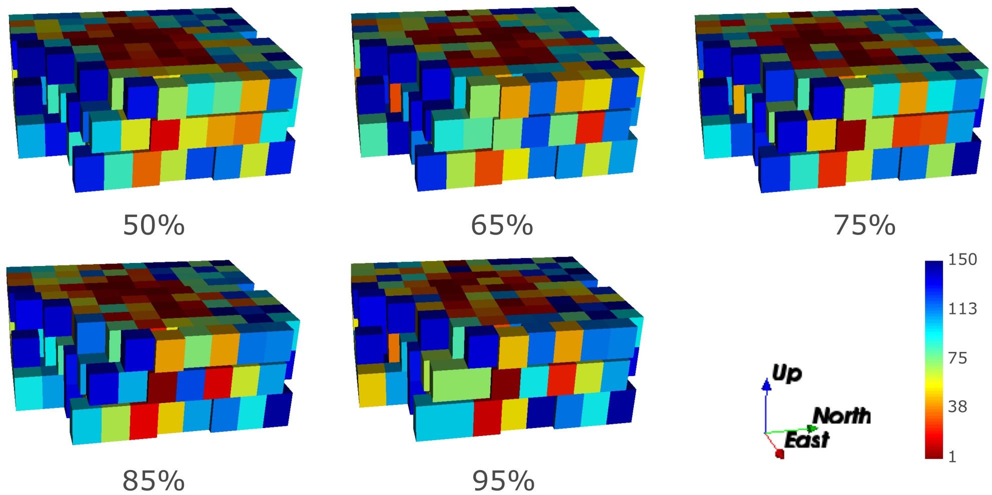


Figure 8.4 – Visualization of the sequencing plan for all reliability levels from E–W direction

as the reliability increases, this correlation decreases and the graphs become more dispersed.

8.6 Discussion

In this paper, a new MILP model for risk-based stope sequencing optimization is introduced. The stope sequencing formulation contains the mathematical constraints; mine stability constraints that disallow simultaneous adjacent and vertically aligning stope extraction, and multiple adjacent stope extraction for backfilled stopes; cost minimization constraints that only allow simultaneous stope extraction on one level and production requirements constraints. Because the drilling samples are sparse, grades of the orebody are simulated from limited data and are uncertain. From the limited number

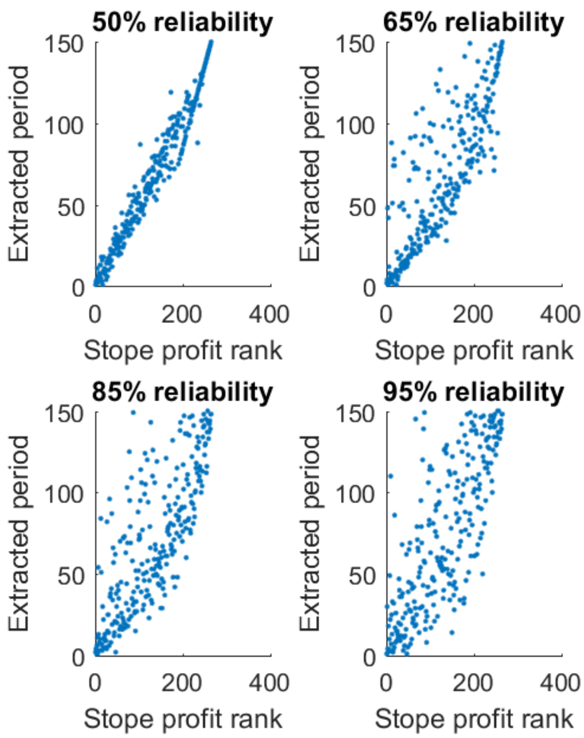


Figure 8.5 – Visualization of stope extraction periods compared to stope profit rankings

of samples, the mine is simulated and a grade is assigned for each block. With Gaussian sequential simulation method, multiple simulations for the grades are generated. The uncertainty is reduced with the risk minimization approach. The risk minimization component of the optimization is realized using CCP. The previous objective of NPV maximization is transformed to a multi-objective maximization problem where one objective is to maximize the mean NPV and the other objective is to maximize the negative standard deviation multiplied by a scalar. The scalar maintains the balance between the two objectives and is selected based on the risk level of the project. Normal distribution is assumed between the simulations and the scalar is chosen according to CCP based on this assumption. It is observed that the scalar grows with higher reliability levels.

As the reliability level increases, the emphasis of optimization shifts to minimizing the standard deviation of the project, thus stabilizing the expected NPV. In other words, as the operational risk taken is decreased by preferring stopes with lower variance over stopes with high variance but also high grade, the expected NPV decreases. Therefore, this causes a trade-off between risk and expected NPV. At 50% reliability, the second part of the multi-objective optimization is set to zero, which is the equivalent to the deterministic model and it provides a comparison to the uncertain models at different reliability levels. Ideally, a sublevel stoping operation management should simulate all risk levels and obtain corresponding stope sequencing plans with their expected NPV. Then, based on the outcomes, they should decide on the risk that they would prefer to take.

The conducted case study has shown that all sublevel stoping constraints are respected. Also, stope sequences are generated based on selected risk levels. As expected, the objective of the projects decreased as the reliability level increased. Another observation from the case study is that at lower reliability levels, stope grade order bares similarity the extraction order. With the minimization of the standard deviation, this order is disrupted. The future work for this research includes incorporating financial elements into the uncertainty modelling and simultaneous optimization of stope layout planning and stope sequencing.

8.7 Chapter Conclusion

This section proposed a MILP model incorporating ore grade uncertainty for stope sequencing. The risk and reliability concepts addressed in this manuscript are complimentary notions where the risk signifies the probability that the project value will be below the expected value. Reliability refers to the opposite probability, which is the actual project value having at least the expected value.

Mine stability is considered through the constraints that specify stope dimensions, disallow simultaneous extraction of adjacent or vertically aligning stopes and multiple exposure of previously backfilled stopes. However, in real practice, depending on the mine characteristics more constraints might be needed. Moreover, during the operation, the rock stress distribution may require modifications to the constraints in time. In these cases, the modifications should be added to the model and the model must be run again. Ideally, the resulting plan should be used as a guidance, reviewed by engineers and must be run regularly with the updated information and constraints.

Appendix: The MILP formulation in Zimpl

```
#Sets
set Stopes := {1 to NO_OF_STOPES};
set Time := {1 to TOTAL_MIN_DUR};
set P := {<i,j> in Stopes*Stopes with i < j};

#Decision variable
var s[Stopes*Time] binary;

#Function definitions
defbool nextX(i,j) := START_X[i]+LEN_X[i] == START_X[j] or
   START_X[j]+LEN_X[j]==START_X[i];
defbool nextY(i,j) := START_Y[i]+LEN_Y[i] == START_Y[j] or
   START_Y[j]+LEN_Y[j]==START_Y[i];
defbool nextZ(i,j) := START_Z[i]+LEN_Z[i] == START_Z[j] or
   START_Z[j]+LEN_Z[j]==START_Z[i];

defbool alignX(i,j) := (START_X[i] <= START_X[j] and
   START_X[i]+LEN_X[i] >= START_X[j]) or (START_X[j] <=
   START_X[i] and START_X[j]+LEN_X[j] >= START_X[i]);
defbool alignY(i,j) := (START_Y[i] <= START_Y[j] and
   START_Y[i]+LEN_Y[i] >= START_Y[j]) or (START_Y[j] <=
   START_Y[i] and START_Y[j]+LEN_Y[j] >= START_Y[i]);
defbool alignZ(i,j) := (START_Z[i] <= START_Z[j] and
   START_Z[i]+LEN_Z[i] >= START_Z[j]) or (START_Z[j] <=
   START_Z[i] and START_Z[j]+LEN_Z[j] >= START_Z[i]);

defbool nextToEachOtherX(i,j) := nextX(i,j) and alignY(i,j)
   and alignZ(i,j);
defbool nextToEachOtherY(i,j) := nextY(i,j) and alignX(i,j)
   and alignZ(i,j);
defbool nextToEachOtherZ(i,j) := nextZ(i,j) and alignY(i,j)
   and alignX(i,j);
```

```
defset adjacentStopesXY(i) := {<j> in Stopes with i < j and
   (nextToEachOtherX(i,j) or nextToEachOtherY(i,j))};
defbool adjacentStopesXYEmpty(i) := adjacentStopesXY(i) == {};
defbool differentSublevels(i,j) := START_Z[i] != START_Z[j];
defbool verticallyAligned(i,j) := alignX(i,j) and alignY(i,j)
   and differentSublevels(i,j);
defset verticalSet(i) := {<j> in Stopes with i < j and
   verticallyAligned(i,j)};
defbool verticallyAlignedEmpty(i) := verticalSet(i) == {};

maximize npv :
    sum <t> in Time: sum <i> in Stopes: (PROFIT_AVG[i]*s[i,t]
       -
       PHI[i]*PROFIT_STD[i]*s[i,t])/((1+DISCOUNT_RATE)^(t-1));

#a stope can only be extracted once
subto extractOnce:
    forall <i> in Stopes do
        (sum <t> in Time : s[i,t] ) <= 1;

#only one of the adjacent stopes is extracted in a given
   period
#only one sublevel is extracted at the same time
subto adjacentStopesAndDifferentSublevels:
forall <i,j> in P do
    forall <t> in Time do
        if (nextToEachOtherX(i,j) or nextToEachOtherY(i,j) or
           nextToEachOtherZ(i,j) or differentSublevels(i,j))
           and not verticallyAligned(i,j)
            then s[i, t] + s[j, t] <= 1
        end;

#only one side of the backfilled stopes should be extracted
   per period
subto backFillAdjacentStopes:
forall <i> in Stopes do
   forall <t> in Time do
      if not adjacentStopesXYEmpty(i) then
```

```
        sum <t1> in {1 to t - 1}: s[i, t1] + sum <j> in
            adjacentStopesXY(i): s[j,t] <= 2 end;

#simulateous extraction of vertically aligned stopes are not
   allowed
subto verticalStopes:
   forall <i> in Stopes do
      forall <t> in Time do
         if not verticallyAlignedEmpty(i) then
            s[i,t] + (sum <j> in Stopes with i < j and
               verticallyAligned(i,j): s[j,t]) <= 1 end;

#production requirements must be met in each period
subto miningCapacity:
   forall <t> in Time do
      sum <i> in Stopes : s[i,t]*TONNAGE[i] <= CAPACITY;

subto lowerProductionLimit:
   forall <t> in Time do
      sum <i> in Stopes : s[i,t]*TONNAGE[i] >= LOWER_PROD;

subto backfillCapacity:
   forall <t> in Time do
      sum<i> in Stopes: s[i,t]*VOL[i] <= BCKFL_CAP;
```

9 Conclusion

9.1 Summary

Mining engineering is comprised of many optimization and decision making problems. Also, there are many uncertainties. Computerized mine planning optimization offers great potential to increase the value of a mining operation. In this thesis, four mine planning optimization problems are presented. These problems are (1) surface mine planning with landfilling option, (2) dig-limit optimization, (3) stope layout planning and (4) stope sequencing. Although these problems are different from each other, the knowledge of optimization techniques enable creating tailored solutions for each different problem and case.

In different manuscripts, optimization with MILP, exact and heuristic methods are presented. The first problem that was solved was the surface mine planning problem with landfilling option. In surface mining operation, block sequencing determines the production order of blocks to maximize net present value of the project. Surface mine planning is a well studied are in the literature. With the contribution of many different researchers, fast MILP models were developed. In the first publication, the landfilling option to surface mine planning was introduced. This optimization model adds economic and environmental compliance value to the operations. In the second publication, a MILP formulation is developed for the first time for dig-limits optimization problem. This formulation allows better adaptation to equipment restrictions and external limitations while optimizing the profit. The optimization is performed on mine benches, thus the problem size is small which made it possible to solve optimally using linear programming.

With decreasing commodity prices, compulsion to continuously finance mine improvements to withstand resource degradation problem and high fixed infrastructure costs, stope optimization is very important as underground mining is selective. In other words, rationalizing decisions is crucial to deal with these challenges and to make profit. MILP and exact methods generally suits well to the problems with less number of decision variables and constraints, which is the reason MILP was used to solve the two-dimensional dig-limits problem. However, the stope layout problem is larger and more difficult. When the problem size is larger, the solution with these methods take too much time and computer memory. When this is the case, heuristic methods can be developed. Well designed heuristic approaches generally provide fast and near-optimal solutions. Two alternate heuristic methods were developed to solve the stope layout optimization problem. The proposed methods address the mine stability concerns by receiving mine-specific inputs to account for geotechnical characteristics such as stope size and pillar size. Comparison to MILP formulations with small datasets has shown that these methods produce near-optimal solutions. Finally, the stope sequencing problem with ore grade uncertainty consideration was solved using MILP with CCP. The proposed model for this problem maximizes the expected mine NPV and minimizes ore grade variability, adding the value of reduced uncertainty. Since the ultimate aim is to provide practical tools for the mining industry, all of the presented approaches solve their corresponding problems in a reasonable amount of time.

This research has shown that although problems in mining operations are challenging, they can be handled or simplified using computerized methods. Furthermore, uncertainties can be decreased by using mathematical methods. Computerized optimization methods can provide assistance to mining engineers and increase the project value.

9.2 Future work

In the future work, future price uncertainty can be incorporated into surface mine planning with landfilling option to ensure that the pit limits will not be extended in the future, dig-limits optimization model can be modified to handle multiple ore processing destinations, parallel programming can be applied to the proposed stope layout planning approaches to accelerate the solution time and stope sequencing can be integrated with a rock stress calculation software that will assess the feasibility of the produced plan. In addition, stochastic optimization approaches can be applied in each problem to provide more robust mine plans.

Underground mine development consisting of mine access network roads and declines can be optimized. Further value can be added to operation through the simultaneous optimization of stope layout planning, stope sequencing and mine development. Also, ventilation, rock mechanics and materials handling aspects can be integrated in stope optimization. The underground mine planning section of the thesis focused specifically on sub-level stoping mining method. However, in the future, the presented approaches can be extended into other underground production methods. Finally, discrete event simulation can be used to conduct an analysis on the sensitivity of the stope sequencing plans in case of possible deviations from the planned timelines.

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