A Stochastic Stope Design and an Integrated Stochastic Optimization of Stope Design and Long-Term Underground Mining Production Scheduling for Sublevel Open Stoping Mining Operations

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Contribution of Authors

This section states the contribution of the co-authors of the papers that comprise the present thesis. All the work presented herein has the author of this thesis is the primary author. The work was completed with the supervision and advice of his advisor Prof. Roussos Dimitrakopoulos, who is also the co-author on the two papers. Another co-author of the two papers is Prof. Cláudio Pinto, professor at Department of Mining Engineering, Universidade Federal de Minas Gerais, Belo Horizonte, Brazil.


Abstract

Underground mine planning defines the design of technically producible economic material volumes and development openings, the sequence of multiple underground activities, and the material destinations within a mineral deposit throughout the mine’s life, aiming to maximize the net present value (NPV) of the mining operation. Due to the existence of different, site-specific mining methods and the inherent geological, geotechnical, operational, and computational complexities, underground mine planning is commonly performed through a stepwise optimization process in which the stope layout is preliminarily designed. Based on the generated layout, the network design of primary (i.e. ramps or shafts) and secondary (i.e. drifts, crosscuts, raises) developments interconnecting the production areas is conceived, defining the precedence of underground mining activities. This predefined mine design then becomes the core input for the strategic underground mine production scheduling, the only step that accounts for the time value of money. In the last decades, available optimization methods have been focused separately on each of the underground mine planning steps, which do not benefit from the synergies between the planning steps. Additionally, most of the previously mentioned methods are deterministic; that is, they neglect many sources of uncertainty, such as grades, material types, commodity prices, costs, and rock mass properties, throughout the planning process, which has been extensively demonstrated to have a significant impact on the profitability and feasibility of mining operations. Therefore, this thesis proposes stochastic optimization methods for underground mines by employing a sublevel open stoping mining method and ultimately attempts to integrate the mine design and production scheduling into a single optimization framework.

The first part of this thesis presents a stochastic optimization method of stope design for sublevel open stoping operations along with the commonly used sequential underground mine planning framework. A set of geostatistical simulations is used to quantify the variability and uncertainty of grades within the mineral deposit. The proposed method aims to maximize the undiscounted profit while capitalizing on the upside potential in terms of recoverable metal of the generated stope layout. It also has the flexibility to define practical production levels and accounts for the development costs of potential production levels and stopes to overcome the assumption of equally accessible stopes of available stope design tools. Therefore, the optimization process only selects
a potential level or stope if it is sufficiently profitable to pay for the associated development cost. The application of this stochastic approach at an underground gold mine achieved a 40% higher undiscounted profit and 21% recoverable metal when benchmarked against the Mineable Shape Optimizer (MSO), an industry-standard deterministic stope design software tool. The proposed method selects profitable stoping areas that cannot be identified by the deterministic approaches that rely on a single estimated orebody model, which is a smooth representation of grades within a mineral deposit.

The second part of this thesis presents an integrated stochastic optimization of stope design and long-term underground mine production scheduling, also applied to sublevel open stoping, by extending the method proposed in the first part of this thesis and integrating time-dependent development costs and production targets. The mathematical model seeks to maximize the NPV from the scheduled stopes, as well as to minimize the shaft, drifts and crosscuts development costs and maintenance costs to keep the levels in operation while managing the risk of failing to meet yearly productions targets. Therefore, an optimal underground mine design is yielded as an output from the optimized production schedule. Besides the geological uncertainty, the method also opens new avenues to account for time-dependent sources of uncertainty that cannot be incorporated into the stope design optimization in the sequential underground mine planning framework. A case study at an underground gold mine demonstrates that the proposed method generates more selective stopes and physically different production levels, which correspond to an 11% higher NPV and a shorter life-of-mine by two years, as compared to the sequential optimization framework, in which the stope design is optimized using the method presented in the first part of this thesis, followed by the optimization of the mine production scheduling.

Future research may consider developing stochastic optimization models for underground mining methods, such as cut-and-fill, room-and-pillars, sublevel caving among others, which have different layout, operational and geotechnical constraints. In addition, the proposed integrated optimization framework enables the incorporation of the underground mine planning process into the simultaneous optimization of mining complexes, which are composed by multiple open-pit and underground operations, processing facilities, and stockpile options.
**Resumé**

La planification des mines souterraines étudie la conception des volumes techniquement exploitables de matériaux économiques et des ouvertures de développement, ainsi que la séquence des multiples activités souterraines et les destinations des matériaux dans un gisement minéral tout au long de la vie de la mine, dans le but de maximiser la valeur présente nette (VPN) de l'exploitation minière. En raison de l'existence de différentes méthodes d’exploitation souterraines qui sont spécifiques à chaque mine, et des complexités géologiques, géotechniques, opérationnelles et informatiques inhérentes, la planification des mines souterraines est généralement réalisée par un processus d'optimisation séquentiel dans lequel la disposition des chantiers est conçue au préalable. Sur la base du plan généré, le réseau de développements primaires (rampes et puits) et secondaires (galeries, coupes transversales, et levées) interconnectant les zones de production est conçu, ce qui définit la priorité des activités minières souterraines. Cette conception de mine prédéfinie devient alors l'élément central de la programmation stratégique de la production minière souterraine, et la seule étape qui tient compte de la valeur temporelle de l'argent. Au cours des dernières décennies, les méthodes d'optimisation disponibles ont été axées séparément sur chacune des étapes de planification des mines souterraines, ce qui ne permet pas de tirer parti des synergies entre les étapes de planification. En outre, la plupart des méthodes mentionnées précédemment sont déterministes, c'est-à-dire qu'elles négligent de nombreuses sources d'incertitude, telles que les teneurs, les types de matériaux, les prix des matières premières, les coûts et les propriétés géomécanique, tout au long du processus de planification, ce qui a été largement démontré comme ayant un impact significatif sur la rentabilité et la faisabilité des opérations minières. Par conséquent, cette thèse propose des méthodes d'optimisation stochastiques pour les mines souterraines en utilisant la méthode d'exploitation minière de sous-niveau stoppés vides, et ultimement vise à intégrer la conception de la mine et sa programmation de la production dans un seul cadre d'optimisation.

La première partie de cette thèse présente une méthode d'optimisation stochastique de la conception et disposition de chantiers, pour les opérations d'abattage par sous-niveaux vidés, dans le cadre de planification séquentielle des mines souterraines couramment utilisé. Un ensemble de simulations géostatistiques est utilisé pour quantifier la variabilité et l'incertitude des teneurs dans...
le gisement minéral. La méthode proposée vise à maximiser le profit non actualisé tout en tenant compte du potentiel de hausse en termes de métal récupérable par l’assortissement de chantiers généré. Elle offre également la flexibilité nécessaire pour définir des niveaux de production pratiques et tient compte des coûts de développement de ces niveaux de production et des chantiers potentiels pour surmonter l'hypothèse des méthodes de conception de chantiers couramment disponibles sur laquelle les chantiers sont également accessibles dans un gisement. Par conséquent, le processus d'optimisation ne sélectionne un niveau ou un chantier potentiel que s'il est suffisamment rentable pour payer les coûts de développement associés. L'application de cette approche stochastique à une mine d'or souterraine a permis d'obtenir un profit non actualisé supérieur de 40 %, et 21 % de plus de métal récupérable par rapport à l'outil Mineable Shape Optimizer (MSO), un outil logiciel de conception déterministique de chantiers standard de l'industrie. La méthode proposée sélectionne des zones d'arrêt rentables qui ne peuvent pas être identifiées par les approches déterministiques qui s'appuient sur un unique modèle de gisement minéral estimé, qui est une représentation lisse des teneurs.

La deuxième partie de cette thèse présente une optimisation stochastique intégrée de la conception des chantiers et de la programmation de la production des mines souterraines à long terme, également appliquée à l’abattage par sous-niveaux, en étendant la méthode proposée dans la première partie de cette thèse et en intégrant des coûts de développement et des objectifs de production dépendant du temps. Le modèle mathématique cherche à maximiser la VPN des chantiers programmés, ainsi qu'à minimiser les coûts de développement des puits, des galeries longitudinales ou transversales et les coûts pour maintenir les niveaux en exploitation tout en gérant le risque de ne pas atteindre les objectifs de production annuelle. Par conséquent, une conception optimale de la mine souterraine est produite en tant que résultat du programme de production optimisé. En plus de l'incertitude géologique, la méthode ouvre également de nouvelles voies pour prendre en compte les sources d'incertitude dépendant du temps qui ne peuvent pas être incorporées dans l'optimisation de la conception chantiers dans le cadre de la planification séquentielle du minage souterrain. Une étude de cas dans une mine d'or souterraine démontre que la méthode proposée génère des chantiers plus sélectifs et des niveaux de production physiquement différents, ce qui correspond à une VPN supérieure de 11 % et à une durée de vie de la mine plus courte de deux ans, par rapport à l'optimisation séquentielle de la planification minières, dans
laquelle la conception des chantiers est optimisée à l'aide de la méthode présentée dans la première partie de cette thèse, suivie de l'optimisation de la programmation de la production minière.

Les recherches futures pourraient envisager de développer des modèles d'optimisation stochastique pour d'autres méthodes d'exploitation minière souterraine, telles que coupe et remblai, chambres et les piliers, sous-niveaux foudroyés, parmi d'autres, qui présentent des contraintes différentes en matière d'aménagement, d'exploitation et de géotechnique. En plus, le cadre d'optimisation intégré proposé dans ce mémoire permet d'incorporer le processus de planification des mines souterraines dans l'optimisation simultanée des complexes miniers, qui sont composés de plusieurs mines à ciel ouvert et souterraines, d'installations de traitement et d'options de stockage.
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Chapter 1 – Introduction and Literature Review

1.1. Introduction

Underground mine planning encompasses various interrelated decisions and optimization components aiming to meet production targets, satisfy a series of operational and geotechnical constraints and maximize the underground mine’s net present value (NPV). The NPV is considered the standard financial criterion given its ability to incorporate prices, costs, resources, operational rates, and time value of money, weighing earlier and later decisions over the life-of-mine (LOM) (Lane 1964, 1988; Darling 2011; King 2011, 2018; Hustrulid et al. 2013). Historically, this planning process has been divided into the sequential optimization of stope design, development network design, and mine production schedule (Topal 1998, 2003; Alford et al. 2007; Musingwini 2016; Kumral and Sari 2019). The stope design is first optimized, defining extraction material volumes to maximize the undiscounted profit (Hustrulid and Bullock 2001; Pakalnis and Hughes 2011; Erdogan et al. 2017; Nhleko et al. 2018). Then, the development network, which consists of interconnected routes, such as shafts, declines, drifts, and crosscuts, allowing to conduct the extracted ore from the stopes to the surface, is designed aiming to minimize both development and haulage undiscounted costs (Brazil et al. 2003, 2008; Brazil and Thomas 2007). Finally, based on the two previous designs, the LOM production scheduling step determines the sequencing of production, development, ventilation, exploration, and other activities so as to maximize the mining project’s NPV (Trout 1995; Topal 1998, 2003; O’Sullivan et al. 2015; Sotoudeh et al. 2020).

This stepwise optimization was a necessary simplification due to the diversity of existing underground mining methods and the related conceptual and computational complexities (Alford et al. 2007; O’Sullivan et al. 2015). However, for open-pit mining, it has long been shown that an optimal mine design should be yielded as an output of the long-term mine production schedule and not the opposite (Kim 1967; Johnson 1968; Ramani 1970; Gershon 1983; Ramazan and Dimitrakopoulos 2004a; Osanloo et al. 2008; Newman et al. 2010). Like open-pit mining, the same outcome is expected in underground mining (Little et al. 2013; Copland and Nehring 2016). The misaligned optimization objectives prevent capturing the synergies between the three involved
planning steps resulting in reduced potential profits and moving costs elsewhere in the mine plan. Furthermore, the solution of a planning step is predicated by the previous step, which is very often obtained by a non-optimal heuristic method, leading to a global underground mine plan that highly departs from an optimal NPV (Smith and O’Rourke 2005; Little et al. 2013).

Substantial uncertainties are associated with resources, prices, costs, and productivities, which are the drivers of the mining project’s NPV, among which the material supply uncertainty is the principal source of technical risk (Vallée 2000; Baker and Giacomo 2001; Rendu 2017). However, conventional mine planning relies on estimated orebody models, which are a smooth representation of the mineral deposit, misrepresenting the spatial distribution and variability of grades, material types, and geometallurgical attributes (Goovaerts 1997). The risks related to deterministic mine plans in open-pit (Ravenscroft 1992; Dowd 1994, 1997; Dimitrakopoulos et al. 2002) and underground settings (Myers et al. 2007; Tavchandjian et al. 2007; Dimitrakopoulos and Grieco 2009; Jewbali et al. 2015) have been extensively investigated in the technical literature. Conversely, stochastic mine planning allows assessment and integration of uncertainty into the planning process (Grieco and Dimitrakopoulos 2007; Dimitrakopoulos 2011; Carpentier et al. 2016) through the use of geostatistically simulated orebody models, which quantify the orebody uncertainty and variability (Journel and Huijbregts 1978; David 1988; Goovaerts 1997; Remy et al. 2009; Mustapha and Dimitrakopoulos 2010a; Chilès and Delfiner 2012; Mariethoz and Caers 2015), as well as other simulation methods modeling other sources of uncertainty (Dirkx et al. 2018; Saliba and Dimitrakopoulos 2019; Both and Dimitrakopoulos 2020). Stochastic frameworks generate mine production schedules with higher NPV, extended mine designs, and risk profiles that satisfy production targets than deterministic mine plans.

Previous work attempted to integrate the optimization of stope design and underground mine production schedule into a single model capturing the interdependencies between both components through the direct maximization of NPV (Little et al. 2011, 2013; Copland and Nehring 2016; Foroughi et al. 2019). However, such methods are limited since they ignore any uncertainty source, leading to questionable forecasts, as explained previously. On the other hand, available stochastic underground mining optimization methods follow the stepwise framework and are focused on either the stope design (Grieco and Dimitrakopoulos 2007; Villalba Matamoros and Kumral 2018;
Wilson 2020) or the mine production scheduling (Carpentier et al. 2016; Dirkx et al. 2018; Sepúlveda et al. 2018; Huang et al. 2020; Nesbitt et al. 2021). Thus, this observation establishes a need for simultaneous stochastic optimization of stope design and underground mine production schedule under grade uncertainty.

This chapter reviews the technical literature related to underground mine planning and modelling of supply uncertainty. Section 1.2 reviews the sublevel stoping mining method system and its variants, including the sublevel open stoping mining method, which is the focus of the proposed optimization methods of the current thesis. Section 1.3 covers the deterministic stepwise underground mine planning framework, which comprises stope design optimization methods and underground mine production scheduling optimization methods. Section 1.4 presents deterministic methods attempting to integrate the stope design and production schedule into a single optimization model. Section 1.5 reviews the stochastic mine planning under uncertainty. First, the need for modeling geological uncertainty is explained, then geostatistical simulation methods are presented. Subsequently, the state-of-art of stochastic optimization in open pit mining is revisited. Finally, existing stochastic optimization methods in underground mine planning are discussed. Section 1.6 outlines this thesis’ goal and objectives, and Section 1.7 outlines the remainder of this work.

1.2. Sublevel Open Stoping Underground Mining Method

1.2.1. Definitions

Sublevel open stoping is one of the established underground mining methods. Due to specific mineralization and rock mass characteristics, several mining method variants are encountered in real underground mines that often employ combinations of methods to adapt to the heterogeneous characteristics of mineral deposits (Bullock and Hustrulid, 2001; Hamrin, 2001; Hartman and Mutmansky, 2002; Bullock, 2011). The classification of traditional underground mining methods varies according to the authors (Bullock and Hustrulid, 2001; Hamrin, 2001; Hartman and Mutmansky, 2002; Bullock, 2011), but such methods are usually categorized into caving and stoping methods. Caving implies the controlled collapse of the orebody under the absence of
support of overlaying rock mass, together with gravity and rock stress. In contrast, stoping is characterized by excavating stable voids of variable shapes and dimensions. Stoping methods are usually sub-divided based on their degree of geotechnical support into two classes: unsupported (or self-supported) and supported methods (Hartman and Mutmansky 2002; Adler and Thompson 2011; Carter 2011). The stability of unsupported methods relies on the strength of openings’ walls and pillars, not requiring any major artificial support system. Although eventual backfilling, cable bolts and other small artificial supports might be used, the stresses remain essentially carried by the rock mass. Conversely, major artificial support systems, such as rock fill and cemented tailings, ensure the ground control and stability of supported mining methods.

Qualitative and quantitative ranking systems and numerical modeling methods are proposed for underground mining method selection (Laubscher 1981, 1990; Nicholas 1981; Hartman and Mutmansky 2002; Carter 2011). These methods are out of the scope of the current thesis, but they rely on two primary selection drivers. The first driver is the style of the mineralization, which comprises geometric characteristics, such as shape, orientation, the relation between length, width, and thickness, and the spatial distribution of the valuable mineral, for instance, disseminated or in veins. The second driver is the strength of both the host and the mineralized rock masses. Therefore, the optimization of stope design and mine production scheduling for a sublevel open stoping operation, which is the current thesis’ focus, is performed once a crucial decision is already undertaken, i.e., this mining method has been selected from the technical and economic standpoints.

1.2.2. Sublevel Stopping Underground Mining Methods

Sublevel stoping consists of multiple self-supported mining method variants (Bullock and Hustrulid 2001; Hamrin 2001; Hartman and Mutmansky 2002; Bullock 2011; Pakalnis and Hughes 2011; Villaescusua 2014). Regarding the style of the mineralization, these variants are broadly applied to large massive or tabular and steeply dipping deposits; that is, the inclination of the footwall exceeds the angle of repose of the blasted ore. The spatial distribution of the valuable mineral often defines regular ore boundaries. From a geomechanical perspective, amenable mineral deposits have stable hanging-wall and footwall rocks in both ore and host rocks. High
initial capital investment before production is required, consisting of a significant amount of development. The sublevel stoping variants are characterized by relatively lower operating costs and lower selectivity than supported mining methods while operationally more costly and much more selective than caving methods. If waste zones cannot be incorporated as planned pillars or separated by the required regular stope boundaries, waste pockets cannot be separated from the blasted ore, limiting the sublevel stoping methods’ selectivity. Moreover, high mechanization levels and high production rates for both drilling and loading equipment are usually observed in mines applying this system.

Sublevel stoping comprises the following variants: sublevel open stoping, long-hole stoping, vertical crater retreat (VCR), vein mining, and shrinkage stoping. Some authors classify the sublevel stoping variants as non-entry methods, which means that staff does not have access to the open void of a stope, and the operation is entirely carried out at the drilling and haulage drifts, crosscuts, and raises (Hartman and Mutmansky 2002; Pakalnis and Hughes 2011; Villaescusa 2014). Hence, shrinkage stoping may be classified as a separated mining method. However, it is not uncommon to include this last variant within the sublevel stoping system since the mineralization style and rock masses' strengths are generally the same (Bullock and Hustrulid 2001; Bullock 2011). These variants are mainly distinguished by their layout of drifts and their pattern of blast holes.

1.2.3. Sublevel Stoping Variants

Sublevel open stoping (Pakalnis and Hughes 2011; Villaescusa 2014) mining method (Fig. 1.1) is the focus of this thesis. A steeply dipping orebody is vertically split into production levels containing stopes. In this case, a stope is characterized by verticalized economic extraction blocks with heights exceeding the straight blasting drilling limit, justified by geotechnical studies. Thus, a bottom haulage drift is developed, often connected to multiple crosscuts accessing the draw points, where the blasted ore is mucked. Above this drift, at various heights governed by the drilling length, drilling levels are developed within the stope, where the blast hole drilling is carried out. Stopes are usually separated by transverse (rib) and longitudinal pillars from other stopes. According to the strike direction and depending on the orebody’s thickness, two basic stoping
configurations are possible: longitudinal (for narrow orebodies) or transverse (for thick orebodies). The development starts by opening a draw point and a horizontal undercut at the bottom of the stope to facilitate the fragmented ore’s draw. A slot (initial vertical opening) is then extracted, on a side or in the center of the stope, encompassing the entire stope’s thickness along with the transverse or longitudinal orientation. Once the stot’s free face is created, the production evolves by sequentially blasting multiple rings (fans of blast holes) up to the stope’s depletion. The volumes between the drilling levels are extracted following an overhand or underhand sequence. In the underhand sequence, extraction of the lower drilling volumes precedes those above, requiring permanent rib pillars between stopes to minimize dilution. While in the overhand sequence, the upper volumes between sublevels are extracted in advance. The broken ore can be mucked from the lower drilling drift rather than from a draw point. This sequence requires backfilling in order to provide a working floor as the extraction proceeds upward.

The other sublevel stopping variants are presented herein since the stope layout is generally very similar to the sublevel open stoping. The layout of the long-hole stoping mining method (Pakalnis and Hughes 2011), also named blast-hole stoping (Bullock and Hustrulid 2001), big-hole stoping (Hamrin 2001; Atlas Copco 2007), or single-lift stoping (Villaescusa 2014), is presented in Fig. 1.2. Essentially, this method diverges from the sublevel open stoping by developing a single drilling drift at the top of the stope, thus eliminating intermediary sublevels. Larger diameter and longer blast holes are used to increase the method’s productivity. However, the risk of damaging the stope’s walls due to more explosive charges must be carefully evaluated. The vertical crater retreat (VCR) stoping (Fig. 1.3) has the same drifts’ layout of long-hole stoping. Its main difference comes from abandoning the vertical slot and verticalized fans of blastholes defining rings. In the VCR method, vertical parallel and long blastholes are drilled downward from the top drilling drift. Concentrated spherical explosive charges are placed at the bottom of the blastholes. The first blasted layer, thus, defines a flat free face replacing the vertical slot. Subsequent blasts are carried out by refilling the blastholes and extracting horizontal slices upwards. A longer initial drilling stage is needed, but it is performed only once, unlike in the long-hole stoping, which is, thus, an advantage of the method. The complex VCR charging also must be well mastered to avoid damaging the surrounding rock. The vein mining variant (Fig. 1.5) is tailored for narrow-vein orebodies. A vertical raise is developed on the footwall in middle position of the stope, covering
its entire height. From this raise, horizontal fans of blastholes are drilled covering the stope’s plan area, and an Alimak platform offers the working floor for the drilling equipment. After drilling, the fans are blasted upwards. The blasted ore can be either left within the stope until it is fully blasted or be gradually mucked in the draw-points.

As mentioned previously, shrinkage stoping (Fig. 1.4) is sometimes considered a separated underground mining method since it is an entry, labor-intensive, and hazardous method. Like the VCR, shrinkage stoping is an overhand method in which ore is excavated in horizontal slices from bottom to top. Part of the blasted ore is left in the stope, providing a work floor for the miners, who enter into the open stope to drill the blast holes of the next slice upwards using manual equipment, leading to the safety drawback of this method. Due to ore swelling, about 30-50% of broken ore must be muck from the stope draw points to provide sufficient working space. The remaining material remains within the stope as additional support for the stope. However, this method is still considered a self-supported method since the stope might remain open once the complete extraction of broken ore occurs. In dealing with massive of very thick orebodies, the sublevel stoping variants might accommodate the extraction of adjacent stoping blocks not separated by pillars (Fig. 1.6). In such cases, primary stopes are extracted first, while secondary stopes work as pillars. Once the backfilling of empty primary stopes is performed, the extraction of secondary ones starts. The sequence of extraction, in this case, must limit the exposure of multiple faces of fill mass. In addition, a diaphragm ring may be left unmined between backfilled primary stope and the extracting secondary stopes to prevent backfill mass failure.
Figure 1.1: Sublevel open stoping mining method (Source: Atlas Copco 2007).

Figure 1.2: Long-hole stoping mining method (Source: Atlas Copco 2007).
Figure 1.3: Vertical crater retreat (VCR) stoping mining method (Source: Atlas Copco 2007).

Figure 1.4: Shrinkage stoping mining method (Source: Atlas Copco 2007).
Figure 1.5: Vein mining method (Source: Hustrulid and Bullock 2001).

Figure 1.6: Sublevel open stoping mining method in a massive orebody. (a) Plan view and (b) three-dimensional view (Source: Villaescusa 2014).
1.3. Conventional Stepwise Underground Mine Planning Framework

In line with the sequential structure of underground mine planning, this section covers available deterministic methods, starting with approaches focused on stope design, followed by methods for optimizing underground mine production scheduling. Although this thesis is focused on sublevel open stoping, the methods presented in this section refer to stope design and scheduling approaches also applied to other underground mining methods.

1.3.1. Stope Design Methods

Existing stope design optimization methods are distinguished by their dimensionality (two or three-dimensional designs), their optimization criteria, such as the maximization of the undiscounted profit or metal content, the integration of more realistic geotechnical and operational constraints, and finally, if the output consists of simply an economic stopping boundary (Alford 1995; Ataee-Pour 2004; Nikbin, Ataee-pour, Shahriar, Pourrahimian, et al. 2018) or a stope layout (Topal and Sens 2010; Sandanayake et al. 2015a) with unified stoping volumes (Erdogan et al. 2017; Nhleko et al. 2018).

The Floating Stope Algorithm (Alford 1995) is a three-dimensional stope boundary algorithm aiming to maximize one of the following objectives: ore tonnage, metal grade, metal content, or economic value above a specified cut-off grade. Two envelopes are generated by floating a fixed three-dimensional stope shape on the orebody model. The inner envelope is composed of blocks above the cut-off grade, and the outer envelope has all possible stope positions that satisfy a minimum head grade. Such boundaries shall be used only as a guide for the mine planner to define mineable shapes manually. A critical limitation of this method is the multiple counting of high-grade blocks. For instance, two stopes sharing a high-grade block might be individually economic; however, their union might not be profitable. The Multiple Pass Floating Stope algorithm (Cawrse 2001) generates nested stope boundaries by employing the sensitivity analysis on user-defined ranges of parameters, such as head grade, cut-off grade, and maximum waste inclusion. The nested envelopes provide a more informed guide for the post manual stope shape design.
The Maximum Value Neighborhood (MVN) algorithm (Ataee-Pour 2004) overcomes the multiple counting of high-grade blocks. The algorithm flags all blocks with positive economic value. At a starting flagged block, the values of all possible neighborhoods, generated by a fixed stope size, are computed. The neighborhood of maximum value is selected, and the blocks within this neighborhood are removed from the next searching steps. This process repeats until all positive value blocks are evaluated. A multiple-pass MVN algorithm (Ataee-Pour 2003) is proposed to improve the original version. In a first pass, the MVN algorithm generates an initial stope boundary. A second pass checks the possibility of including ore blocks, which have not been included in the initial boundary. Finally, another pass checks the possibility of excluding waste blocks from the final stoping outline. The output boundary, however, is profoundly affected by the examination order since blocks evaluated earlier are more likely to be included in the solution. The manual intervention is still required to define mineable unified stope volumes within the output boundary.

The algorithm of Topal and Sens (2010) addresses the overlapping stopes issue and also considers variable stope sizes. All possible shapes are floated on the input block model. Subsequently, the algorithm greedily selects stopes from the generated list in decreasing order of a user-defined selection criterion: the stope profit, the stope profit per square meter, or the stope profit per mining time. Once a stope is selected, all its overlapping stopes are eliminated from the list. Consequently, the method suffers from the absence of a more accurate analysis to define combinations of non-overlapping stopes that might yield a higher economic value. For instance, two smaller stopes might have a higher total economic value than a larger stope overlapping them.

Sandanayake et al. (2015a; 2015b) propose a stope design method that integrates pillar considerations, levels’ definition, and a less sensitive to the selection objective or starting point compared to the previous approaches. Like the previous method, the algorithm also floats a set of possible shapes on the block model and eliminates stopes with negative economic value. The overlaps of each remaining stope are evaluated with pillar widths apart, and a set of non-overlapping stopes is generated. Multiple sets of non-overlapping stopes are generated up to a maximum specified limit. The intersections of those sets are evaluated, and the algorithm then retains a unique set with the highest economic value. This more sophisticated fashion to overcome
the overlapping issue provides a layout with higher economic value (Erdogan et al. 2017) than the previously mentioned algorithms.

Villalba Matamoros and Kumral (2017) propose a heuristic method to maximize the undiscounted profit from stopes while minimizing the internal dilution in the final layout. Low-grade blocks within a stope have a reduced recovery in the objective function. The stopes are built by grouping slices of a specified number of blocks in height and width. The slides’ average grade must be greater than a desirable cut-off grade. However, some low-grade slides can become part of a stope if high-grade border slides sandwich them. The stope’s average also must be greater than the specified cut-off grade. This method can efficiently control the internal dilution. Nonetheless, its heuristic search may generate a solution easily trapped in a local optimal. A genetic algorithm is further used to improve the quality of the generated solution under grade uncertainty (Villalba Matamoros and Kumral 2018). This improvement will be discussed in Section 1.5.4.

Besides floating and selecting rectangular shapes on the input block model, Bai et al. (2013) proposed a network flow method for a stope boundary surrounding a central raise, a vertical and cylindrical opening driven upward from one level to a higher level or the surface (Hamrin 2001). The method converts the block model from a cartesian grid to a cylindrical grid, centered at the raise location. The arcs linking the blocks in the network are defined based on the minimum stope width considerations along the horizontal direction and hanging-wall and footwall slope angles in the vertical direction. Although the network flow-based methods provide a fast solution, this method is only amenable to mineralized lenses extracted from a unique and central raise. This approach is further extended to heuristically define a stope boundary around multiple raises adapting to curvilinear orebodies (Bai et al. 2014). Nelis et al. (2016) integrate additional vertical convexity constraints needed due to geotechnical limitations of the original method. However, the method can only be applied to specific orebodies with mineralized lenses extracted from raises.

The Mineable Shape Optimizer (MSO) (Alford and Hall 2009; Alford Mining Systems 2016) is an industry-standard software for stope design that offers different optimization frameworks. For instance, in its Slice Method (Alford and Hall 2009), the input model is discretized into a regular grid defined by the stope's height and strike length, forming transversal tubes. Each tube is sliced
following initial geometric parameters or control wireframes. Seed shapes are then formed by grouping high-grade slices based on a specified cut-off grade and allowable stope widths so as to maximize each tube's economic value. Finally, these seed shapes are annealed to satisfy fine-tuning shape parameters, such as stope’s strike and dip tolerance ranges, and eliminate corners between adjacent stopes. The method is highly dependent on geological control parameters or wireframes, which are highly uncertain (Bárdossy and Fodor 2001; Osterholt and Dimitrakopoulos 2018). Moreover, it requires an upfront cut-off grade that is not adequately known in the early stages of the underground mine planning process.

1.3.1.1. Need for Providing Production Levels

The sublevel stoping mining method requires production levels where the haulage drifts and crosscuts are developed connecting the stopes to ore passes or ramp access points, and different approaches are proposed to build level-based stope designs. An optimum level spacing must be defined based on involved costs and geotechnical constraints unless specific geotechnical conditions need to take precedence in defining the location of some sill pillars and levels. The development costs tend to decrease as the level spacing increases since fewer haulage drifts will be developed. On the other hand, the operating costs tend to increase for higher spacings due to increased ventilation, drainage, and material handling costs (Hartman and Mutmansky 2002). Once a typical level spacing is decided, some approaches are proposed to provide a level-based stope layout. The most straightforward available approach consists of splitting, top-down or bottom-up, the orebody block model into consecutive layers of blocks given a fixed level height, and stope layout optimization subproblems are carried out within the generated levels. This level splitting process is observed in Sandanayake et al. (2015a; 2015b), Villalba Matamoros and Kumral (2017), and in the Slice Method of MSO (Alford Mining Systems 2016) with the eventual search of different starting points for this consecutive level definition process. However, in these methods, the optimization process does not have the flexibility to define variable separations and locations of production levels to outline low-grade zones or reduce the related development costs, which impact the stope layout’s profitability.

Sari and Kumral (2020) propose a heuristic level determination method. First, the economic values of all mining blocks with the same vertical coordinate are summed up. Second, leading and trailing
block layers in the orebody model having negative economic value are trimmed out. Finally, the remaining vertical coordinates are subjected to several runs of level partitioning, from a maximum up to a minimum possible level spacings. The combination of levels with maximum economic value is retained, and the stope layout is optimized constrained by the generated levels in a subsequent step. Once again, the development cost of opening multiple levels is not considered in this current method. Sari (2018) proposes another heuristic level determination method that computes block scores by summing the grades of all surrounding blocks up to a specified stope height. Therefore, high-grade zones would have blocks with higher associated scores. Like the previous method, the scores of blocks at the same vertical coordinate are summed, reducing the problem to a one-dimensional string of values. Binary decision variables control the level opening at a given vertical coordinate with an incurred fixed level selection cost. The method, thus, tries to define flexible locations and heights of production levels while minimizing the development costs. Even though these methods provide a flexible way to determine the production levels, they consist of a pre-processing step before optimizing the stope layout. Therefore, they neglect the spatial connectivity of high-grade zones, which controls the definition of stoping volumes, and the distances of potential stopes from the access point, which dictates the required development of haulage drifts. An optimal definition of production levels shall be optimized simultaneously with the stope layout, accounting for the locations and economic values of the potential stopes within a potential level.

1.3.1.2. Integration of Development Costs and Stope Accessibility

Most available stope optimization methods do not consider the development costs and variable level-based mining costs to determine the economic value of stopes neglecting the inherent interdependencies between each stope's economic potential and its accessibility/remoteness. Some works tried to integrate the stope design and development aspects. The network flow-based method proposed by Bai et al. (2013) imposes a maximum allowable distance of a block from a central raise and a horizontal required mining width, limiting the inclusion of remote blocks in the final stope boundary. This method, however, does not directly integrate development costs. Ding et al. (2004) propose a heuristic approach that computes the development costs associated with the selected stopes and levels for an underground mine accessed through a primary shaft with equally spaced horizontal levels. The development costs are distributed among the selected stopes to
determine their economic values. Subsequently, some marginal grade intermediate stopes that were not included in the layout are reconsidered since high-grade selected stopes already pay the development cost. The algorithm iterates up to a convergence of the final stope layout. Hou et al. (2019) implement a mixed-integer programming formulation with recursive (non-linear) vertical and horizontal development cost constraints for stope design optimization. This method simultaneously maximizes the stopes’ profit and minimizes the total development costs for an underground mine with the same development network configuration as the previous method. The method assumes a tabular orebody with transverse stope width coinciding with the known orebody thickness, simplifying the optimization to a two-dimensional problem. A proposed genetic algorithm solves the non-linear model. The two last approaches provide a more contiguous stope layout around the shaft location to minimize the associated drifts’ development costs. Both approaches rely on fixed locations of levels and can only activate or deactivate a potential level. However, there is no flexibility to determine the more profitable locations of levels.

1.3.2. Underground Mine Production Scheduling Optimization Methods

The optimization of underground mine production scheduling defines the sequencing of different mining activities under geotechnical, operational, and marketing constraints. Production activities extract ore from stopes and are associated with the revenue from the contained metal, mining, and processing costs. In contrast, the non-production activities consist of development, backfilling, support, ventilation, and exploration task to provide access to the production areas, or to attend safety requirements and other general purposes, incurring costs. Unlike open-pit mining, underground mining activities are characterized by widely variable material volumes and time spans and complex mining method-specific precedence structures (Topal, 1998, 2003; O’Sullivan et al., 2015; Sotoudeh et al., 2020).

The resolution of deemed activities and decision-making defines three timeframes generally used in underground mine planning (Topal, 1998; Campeau and Gamache, 2020). At a high level, the strategic or long-term planning usually focuses on defining the starting and ending production times of different mining zones and the main accesses’ development over the mine’s life on yearly periods aiming to maximize the project’s discounted cashflows. The tactical or mid-term planning
has finer resolution resources allocation, such as equipment fleet and crews, and activities’ time fidelity constraints. These plans cover a horizon of a few years with monthly or quarterly periods. At the higher resolution level, short-term or operational planning incorporates equipment fleet dispatch decisions and aims to reach the production targets defined in the coarser planning stages. However, due to the heterogeneous activities durations, different resources’ time basis, and complex precedence rules, some methods for underground mine scheduling try to integrate shorter and longer terms decisions and have no clear definition of planning timeframes (O’Sullivan and Newman, 2014; King et al., 2017; Brickey et al., 2020).

In the conventional sequential planning framework, the available production scheduling methods have two preceding steps: the underground mine design (stope design and development network layout) and the task creation step. This last step subdivides the design into activities with related duration, lag times, logical precedence, and adjacency requirements (Brickey et al., 2020). Various objectives and mine-specific constraints are encountered in the available long-term scheduling optimization models depending on the type of metal (precious or base metal) and whether there are long-term sales contracts. For instance, some methods seek to minimize the period-to-period fluctuations in ore tonnage (Williams et al., 1973), minimize fixed and variable extraction costs (Winkler, 1996), minimize the surplus and shortage deviations from the ore types targets (Kuchta et al., 2004; Martinez and Newman, 2011) or maximize the discounted extracted metal (Carlyle and Eaves, 2001; O’Sullivan and Newman, 2014, 2015). However, the maximization of NPV is the standard financial measure used in strategic mine planning (King 2011, 2018) and the most commonly implemented objective (Trout, 1995; Smith et al., 2003; Sarin and West-Hansen, 2005; Nehring and Topal, 2007; King et al., 2017).

Early developments of underground mine production scheduling models consist of linear programming (LP) models. The model introduced by Williams et al. (1973) for the sublevel open stoping mining method assumes an underground mine composed of horizontal levels homogenously mined from a shaft position up to the orebody’s extremities. Upper levels are always mined in advance with respect to the lower levels. The model aims to minimize the period-to-period fluctuations in ore tonnage. However, LP models are unable to capture discrete decisions, and their assumption of homogenous exploitation misrepresents the mining selectivity. The MIP
model proposed by Trout (1995) has binary decision variables controlling the starting and progression times of extraction, void period, and backfilling of stopes. The mathematical formulation aims to maximize the project’s NPV, subject to constraints related to production targets, intra-stopes mining sequence, and stope adjacency. Its main limitation comes from the fact that the development costs and timing are not directly incorporated. Conversely, time fidelity constraints are set as user-defined extraction and backfilling earliest and latest starting and ending times based on logical sequencing. This model is further extended in Nehring and Topal (2007), where additional constraints limiting the multiple exposures of fill masses in adjacent stopes are added.

Further developments introduce additional operational components and mining method-specific precedence constraints. Carlyle and Eaves (2001) propose a MIP model, which includes binary decision variables controlling the ramps’ and drifts’ development and the exploration activities. Exploration sampling is performed before production to assess the grade and thickness of veins for the deemed cut-and-fill operations. The long-term sublevel caving production scheduling model of Kuchta et al. (2004) aims to minimize the surplus and shortage deviations from the different ore types targets while accounting for load-haul-dump (LHD) equipment fleet allocation. The deterministic scheduling model proposed by Sarin and West-Hansen (2005) aims to maximize the NPV associated with scheduled panels produced using different equipment groups while minimizing the deviations from specified production tonnages and quality targets. This model is tailored for a coal mine that combines long-wall, room-and-pillar, and retreat mine, which require distinct equipment for production, support, and haulage. In the deemed mining methods, the equipment has low displacement flexibility, and hence, its allocation is crucial for the long-term plan. Pillar removal constraints are integrated into the model proposed by O’Sullivan and Newman (2014) for an underground mine employing a combination of room and pillar, long hole stoping, and drifts and fill mining methods. Brickey (2015) partitions the input mine layout into ventilation domains. Resource constraints guarantee that the airflow capacity of each domain is not exceeded, given the estimated required airflow to complete each activity. This model is applied to an underground mine combining different mining methods, such as sublevel stoping variants and cut and fill.
Different activity aggregation and solution methods are proposed to deal with the combinatorial nature of long-term underground mine production scheduling problems, that is, the exhaustive search of all possible solutions within the discrete solution domain is not tractable. Besides the allowable activities’ starting and ending times constraints introduced by Trout (1995), Little et al. (2008) introduce the principles of natural commencement and sequence of intra-stopes activities to reduce the number of binary variables for some intractable instances. King et al. (2017) propose an activity aggregation fashion for long-term schedules while satisfying the needed activity precedence structure of higher resolution activities. Sarin and West-Hansen (2005) implement Bender’s decomposition for solving their previously discussed model. Since the objective function of the method developed by Martinez and Newman (2011) aims to minimize the surplus and shortage deviations from various ore types targets, a decomposition-based heuristic is proposed that solves the subproblem associated with each ore type separately and progressively appends additional global constraints to obtain feasible integer solution. Another heuristic decomposition method is found in O’Sullivan and Newman (2014, 2015), in which the production activities are sorted in decreasing order of grade. Subsequently, a subproblem formed by only high-grade activities is solved for the entire time horizon. Once the solution is obtained, the related decision variables are fixed, and lower-grade activities are allowed to be scheduled. The algorithm iterates until all grade classes are included. Brickey (2015) deals with large scheduling instances using a solver that employs Bienstock and Zuckerberg’s (2010) algorithm to solve the problem’s linear relaxation and the TopoSort rounding heuristic (Chicoisne et al. 2012) to obtain a feasible integer solution. A comprehensive review of long-term underground mine production scheduling methods is provided by Sotoudeh et al. (2020).

1.4. Integrated Optimization of Stope Design and Underground Mine Production Scheduling

Underground mining is considered proportionally more capital extensive than open-pit mining due to its needed infrastructure of accesses, ventilation, and more costly unitary operations. Moreover,
in most underground mining methods, material left unmined, from economic or operational standpoints, may not be recoverable later during mine’s life. Once the underground infrastructure is installed, it is difficult and costly to be changed. Therefore, the definition of economic extraction envelopes is crucial for underground mines (Elkington et al., 2010; Nelson, 2011). In addition, the profitability of a stope depends not exclusively on its metal content. Its distance from access points or other high-grade lodes, and more importantly, the timing of extraction interfere with its potential profit, which gives rise to a circular problem between involved underground mine planning steps: the stope layouts, development network, and production scheduling.

The conventional stepwise underground mine planning framework neglects the close interdependencies between such steps. For instance, available stope design optimization methods rely on the assumption of equally accessible stopes and ignore the effect of the time value of money (Ding et al. 2004; Nhleko et al. 2018). Proposed methods for development network optimization aim to minimize the combination of undiscounted development and haulage costs. The former is proportional to the length of development, and the latter is proportional to the product of the length by the total tonnage hauled (Brazil et al. 2003, 2008; Brazil and Thomas 2007), ignoring the discounting effect. Therefore, the maximization of NPV is hindered during the final scheduling optimization (Smith and O’Rourke 2005; Little et al. 2013).

Some frameworks are proposed to capture the synergies between the underground mine planning steps by iteratively running multiple commercial software packages. Basically, an initial cut-off grade is selected to delineate the production areas in the stope design step. A development network layout is then designed upon the stope layout. Subsequently, production rates are defined, allowing the optimization of the life-of-mine production scheduling. This process is repeated for user-defined ranges of cut-off grades and production rates specified for the entire mine or conveniently partitioned mining zones. A candidate tuple of cut-off grade, production rate, and NPV can be selected graphically by the mine planner. Poniewierski et al. (2003), Elkington et al. (2010), and Bootsma et al. (2018) propose similar frameworks but using different commercial software tools. On the other hand, Whittle (2015) and King and Newman (2018) propose MIP formulations to optimize the LOM production schedules of underground mines consisting of multiple mining zones with associated input commercial software-generated stope designs and development.
network layouts. The MIP models are run for multiple combinations of per-zone cut-off grades. Finally, the best-performing scenario in terms of NPV is selected. Whittle (2015) employs a genetic algorithm to exploit the solution space of combinations of per-zone cut-off grades, while King and Newman (2018) use the algorithm of Bienstock and Zuckerberg (2010) to obtain the solution for the LP relaxation and the TopoSort rounding heuristic algorithm (Chicoisne et al. 2012) to obtain feasible integer solutions. These methods are flexible since they are suitable for mines combining different mining methods. However, they still rely on previously optimized stope designs, very often generated by commercial software heuristic algorithms; thus, not providing a truly integrated optimization of the underground mining plan. Additionally, they provide a single cut-off grade for the mine or for each mining zone rather than a time-varied cut-off grade that would maximize the NPV along the lines of Lane’s theory (Lane 1964, 1988).

Few methods attempt to optimize the stope design and production scheduling simultaneously. All available integrated methods handle single mining method mines employing sublevel stoping operations. The method proposed by Little et al. (2011, 2013) generates a set of potential overlapping stopes by floating an assortment of stope sizes on the input block model. The average grade and economic value of stopes are computed, and negatively valued stopes are excluded. The preprocessing step also maps for each remaining potential stope the following sets: the set of overlapping stopes that share at least one block with it; the set of adjacent stopes that share one lateral boundary with the middle stope; the set of offset stopes consisting of stopes with precisely the same size and lie directly above and below the centered stope; and, finally, the set of extraction level stopes. This last set assumes that adjacent stopes at different base vertical coordinates can be accessed through the same production level. The proposed MIP aims to maximize the NPV under non-overlapping, adjacency, horizontal offset, and draw points constraints to provide a practical layout and schedule. Resource and target constraints, such as ore handling, backfilling, and metal production, are also considered. Applying this method at a hypothetical gold deposit generated a different design and production schedule with a higher associated NPV than a benchmark sequential optimization approach of stope design followed by the scheduling. Results highlight the benefits of the integrated underground mine planning. However, this method relies on the assumption of equally accessible stopes since no development costs and timing nor extraction precedence constraints are incorporated.
A critical arising question for an integrated optimization approach is how to account for development costs and timing reasonably. Since the stope layout is still not materialized, it is impossible to determine a detailed development network layout with the exact positions of declines, drifts and crosscuts to determine the development costs and an extraction logic. Nevertheless, some methods try to incorporate development aspects into the integrated optimization approach. Copland and Nehring (2016) extend the previous formulation by adding decision variables to control production levels’ opening sequence. Therefore, in the preprocessing step, the orebody model is split into extraction levels indexed from top to bottom. The stopes are, then, mapped within those levels. Each level is associated with a discounted incremental opening cost from the immediately above level. Precedence constraints ensure the top-down development of levels from the surface, which might be suitable for mines accessed through ramps. Linking constraints state that the stope production must start only after the level access is built. However, the method ignores the horizontal development cost and timing and an annual advancement rate to define the final schedule.

The integrated approach proposed by Foroughi et al. (2019) consists of an IP formulation with two weighted objectives: the maximization of NPV, which is related to the underground mine production schedule, and the maximization of recovered metal, related to stope design. The preprocessing step generates sets of levels and stopes, similarly to Copland and Nehring’s approach. A simplified development layout consists of various drifts connecting the most centralized potential stope of a level to a primary shaft. Precedence constraints are defined to control the shaft-shaft, shaft-drift, drift-drift, drift-stope, and stope-stope extraction. Furthermore, the model is able to define development rate constraints for the shaft and drifts. The assumption of a time-dependent shaft development is an unrealistic assumption of the method. Again, the lack of development considerations within a level neglects the time-value of development costs and may imply the early scheduling of remote stopes. Finally, different designs and schedules are generated by varying the weight associated with each component of the objective function, which contradicts the principal objective of maximizing NPV.
1.5. Stochastic Mine Planning

1.5.1. Need for Modeling Geological Uncertainty

The previously discussed methods for underground mine planning are deterministic approaches based on a single estimated input orebody model, which is conventionally assumed to be the best representation of spatially distributed geological attributes of interest in a mineral deposit. The estimated orebody models are traditionally generated by Kriging, which provides the minimum-error-variance linear unbiased estimates (David 1977; Journel and Huijbregts 1978; Isaaks and Srivastava 1989; Cressie 1990; Journel 1990; Goovaerts 1997; Chilès and Delfiner 2012). However, the limitations of estimated models are well-understood in the technical literature. Regardless of the estimation method used, these models generate a smooth representation of the mineral deposit that misrepresents the proportions of high and low-end grades, the statistics of conditioning data, and the underlying structural patterns of the variable of interest within the mineral deposit. This characteristic is called the smoothing effect (David 1977, 1988; Journel and Huijbregts 1978; Goovaerts 1997; Chilès and Delfiner 2012). In addition, non-linear transfer functions are involved in all mine planning methods. Hence, an average-type spatial representation of the input attribute of interest, as the estimated model, does not generate an average assessment of the project expectations. This non-linear propagation of errors is particularly problematic in mine planning due to its sensitivity to the connectivity of extreme values (Journel and Alabert 1989; Gómez-Hernández and Wen 1994; Dimitrakopoulos 1996, 2011; Dimitrakopoulos et al. 2002; Qureshi and Dimitrakopoulos 2004; Dimitrakopoulos and Godoy 2014).

Furthermore, deterministic mine planning approaches fail to integrate the inherent geological uncertainty and variability into the optimization process. The material supply uncertainty has shown to be the major contributor of technical risk of not meeting mining projects’ production forecasts, especially in early operation years, when the cash flows must recover the capital investments, thus affecting the actual mining projects' profitability (Vallée 2000; Baker and Giacomo 2001; Rendu 2017). The geological uncertainty comes from the fact that limited knowledge, obtained from very sparsely drilled exploration data, is available to estimate the attribute of interest at any unsampled location (Goovaerts 1997; Rossi and Deutsch 2014).
Stochastic simulations (Journel and Huijbregts 1978; David 1988; Isaaks 1990; Goovaerts 1998; Remy et al. 2009; Mustapha and Dimitrakopoulos 2010a; Mariethoz and Caers 2015), which are discussed in the next section, are used to assess the risk of deterministically generated mine designs and production schedules. Many authors have reported misleading forecasts in open-pit mining in terms of tonnages, grades, and cashflows of conventional mine production schedules (Ravenscroft 1992; Dowd 1994, 1997; Dimitrakopoulos et al. 2002; Dimitrakopoulos 2011). More recently, stochastic simulations are also used to assess the risk associated with deterministic underground mining resources and reserves delineation. For instance, Myers et al. (2007) conclude that the deterministic stope layout methods fail to evaluate alternative stoping geometries and locations since they rely on a single estimated model. Dimitrakopoulos and Grieco (2009) show that the deterministic stope design methods could not capture the upside potential and downside risk of generated layouts in the presence of grade uncertainty. The risk assessment of a deterministically generated stope design performed by Jewbali et al. (2015) highlights that the used conventional stope design method overestimated the number of high-grade stopes, impacting the stope layout’s undiscounted profits and metal content. Finally, the case studies of Tavchandjian et al. (2007) show that, since the estimated orebody models misrepresent the spatial connectivity of high-grades, the internal dilution of production envelopes is also misrepresented, which misguides the selection of a proper underground mining method.

1.5.2. Geostatistical Simulation Methods for Modeling Geological Uncertainty

In geostatistics, a mineral deposit is modelled based on the concept of random field or random function (RF), where the value of an attribute of interest, such as grades, densities, material types, and so on, at any location within the deposit, is considered as the outcome of a random variable. The ensemble of random variables at all possible locations constitutes the RF. Its multivariate probability density function entirely describes an RF (Journel 1974; David 1977; Journel and Huijbregts 1978; David 1988; Isaaks and Srivastava 1989; Isaaks 1990; Goovaerts 1997; Remy et al. 2009; Mustapha and Dimitrakopoulos 2010a; Chilès and Delfiner 2012; Mariethoz and Caers 2015).
A stochastic simulation relies on random sampling from multivariate conditional probability distributions of the related RF to generate multiple equally probable realizations of the mineral deposit. Each map of geostatistical simulations typically: honors the sample data at their locations; reproduces the underlying histograms and spatial statistics, including high-order statistics, if more complex spatial features are incorporated into the simulation process (Journel 1974; Journel and Huijbregts 1978; Goovaerts 1997; Remy et al. 2009; Mustapha and Dimitrakopoulos 2011; Mariethoz and Caers 2015; Gómez-Hernández and Srivastava 2021).

Let $\mathbf{Z}(\mathbf{u}) = \{Z(\mathbf{u}_i), i = 1, ..., N\}$ be a stationary and ergodic RF at $N$ spatial locations $\mathbf{u}_i$ within the domain $D$, defined in $\mathbb{R}^n$, delimiting a mineral deposit, and let $d_n = \{Z(\mathbf{u}_j), j = 1, ..., n\}$ be the set of $n$ available exploration data. The sequential simulation approach (Johnson 1987; Journel and Alabert 1989; Journel 1994; Goovaerts 1997; Gómez-Hernández and Srivastava 2021) relies on the principle of decomposing the multivariate probability density function $f_Z(\mathbf{u}_1, ..., \mathbf{u}_N, z_1, ..., z_N|\Lambda_0)$ of the RF into a product of univariate posterior distributions by recursively applying the definition of conditional probability, as shown in the following equation:

$$f_Z(\mathbf{u}_1, ..., \mathbf{u}_N, z_1, ..., z_N|\Lambda_0) = f_{Z_1}(\mathbf{u}_1; z_1|\Lambda_0) \times ... \times f_{Z_N}(\mathbf{u}_N; z_N|\Lambda_{N-1}) = \prod_{i=1}^{N} f_{Z_i}(\mathbf{u}_i; z_i|\Lambda_{i-1}) \quad (1.1)$$

where $\Lambda_{i-1}$ corresponds to the set of conditioning data of a node $\mathbf{u}_i$ to be simulated. Hence, for the first node $\mathbf{u}_1$, the conditioning data $\Lambda_0 = d_n$ denotes the available exploration data. Once the value $Z(\mathbf{u}_i)$ is simulated, it is included into the set of conditioning data for the subsequent nodes to be simulated, that is, $\Lambda_i = \{\mathbf{u}_i; z_i, \Lambda_{i-1}\}$. This decomposition is not unique since it depends on the ordering defined by the random path visiting the locations $\mathbf{u}_i, i = 1, ..., N$. The steps of the sequential simulation approach are as follows: (a) define a random path through all nodes to be simulated; (b) at each node, build the cumulative conditional probability density function $f_{Z_i}(\mathbf{u}_i; z_i|\Lambda_{i-1})$ given the original data and all previously simulated nodes, that is $\Lambda_{i-1}$; (c) draw a realization from this obtained distribution and add the simulated value to conditioning data; (d) the previous steps are repeated until all nodes are simulated. The sequential simulation approach requires that each univariate conditional distribution is fully known.
The sequential Gaussian simulation (SGS) method (Journel and Alabert 1989; Isaaks 1990; Journel 1994; Goovaerts 1997), following the sequential simulation approach, is the first Gaussian-based simulation method. This method assumes a multi-Gaussian RF model, and hence, each univariate conditional distribution is Gaussian. The Gaussian distribution is a continuous probability distribution that is fully characterized by its covariance function since all its high order cumulants beyond the first two (the mean and variance) are equal to zero. These properties make the univariate conditional distribution easily parametrized since the conditional mean and variance. SGS requires an initial transformation of the input data to a Gaussian space, where the variogram model is inferred from the transformed conditional data. At each visited location in the random path of the sequential simulation mode, a Kriging system formed by the neighboring points is solved to obtain the conditional mean and variance, and a value is drawn from the generated cCDF. Once all nodes are simulated, a back transformation is performed from Gaussian to the original space.

In dealing with multiple correlated variables, Verly (1993) extends the SGS to co-simulate all variables. This method, however, requires the cumbersome inference of cross-covariances and becomes impractical for applications with more than two variables. The simulation method via principal component analysis (PCA) (David 1988; Goovaerts 1993, 1997) performs the non-spatial decorrelation variance-covariance matrix of regionalized variables only at lag zero and proceeds the individual simulation of each decorrelated component using SGS, overcoming the disadvantages of the co-simulation algorithm. Nonetheless, the assumption that regionalized variables remain decorrelated for lags other than zero ignores spatial correlation within the data. To overcome this limitation, the minimum/maximum autocorrelation factors (MAF) simulation algorithm (Switzer and Green 1984; Desbarats and Dimitrakopoulos 2000) performs an additional PCA spatial decorrelation of the variance-covariance matrix for a tested short lag distance Δ, which allows the independent simulation of each MAF factor, via SGS, while carrying out the spatial correlations between the multiple regionalized variables.

Since SGS is implemented in point-support, the same support of the drill hole data, the method becomes computationally expensive and memory demanding for large orebody models containing
millions of blocks discretized into multiple nodes to be simulated (Godoy 2003; Dimitrakopoulos and Luo 2004; Benndorf and Dimitrakopoulos 2007; Boucher and Dimitrakopoulos 2009). The generalized sequential Gaussian simulation (GSGS) (Dimitrakopoulos and Luo 2004) capitalizes on two observations to improve the efficiency of the SGS. First, the screening effect that closest conditional data have over the data at larger distances, and second, adjacent nodes share an overlapping neighborhood. Thus, GSGS defines a random path visiting groups of adjacent nodes and simultaneously simulates each group using the LU conditional simulation method (Alabert 1987; Davis 1987) rather than the node-by-node fashion of the SGS. However, GSGS remains memory-demanding due to the storage of all simulated nodes in point-support.

The direct block simulation algorithm (DBSIM) (Godoy 2003; Boucher and Dimitrakopoulos 2009) improves the memory handling of GSGS by retaining only the simulated values in block-support once the average of the block’s discretizing nodes is computed. The algorithm assumes an RF in point-support, a regularized RF in block-support, and that these two RFs are jointly Gaussian. Hence, their joint distribution is wholly characterized by the related cross-covariance function. Therefore, block discretizing nodes are concurrently simulated using the LU method based on conditioning data in both point and block support. Godoy (2003) and Benndorf and Dimitrakopoulos (2018) discuss the improvements of DBSIM in terms of computational efficiency and high-grades’ connectivity, while Boucher and Dimitrakopoulos (2009) propose the DBMAFSIM algorithm that combines MAF and DBSIM to simulate multiple correlated variables on the block support.

All methods previously mentioned are based on the Gaussian assumption of the related RFs. Thus, they can be conveniently parametrized based on the first order (the means) and second-order (the covariances-cross-covariances). However, the Gaussian-based methods require the initial Gaussian transformation of the input data. In addition, the second-order simulation methods are unable to satisfactorily model complex non-linear and non-Gaussian geological patterns existing in mineral deposits and reservoirs (Journel and Alabert 1989; Journel 2007). Since Gaussian RFs only retain the covariance function from the data, they are known to maximize the entropy, that is, the spatial disorder in terms of connectivity of high values (Journel and Alabert 1989; Journel and Deutsch 1993). Due to the non-linearity of the transfer functions of mining planning optimization
methods, the maximum entropy of the Gaussian RF model does not entail maximum entropy of the space of response in the presence of uncertainty, which highlights the need for more informed RF models beyond the first and second-order statistics, to be used during the mine planning process. These characteristics of Gaussian RFs unfavorably impact production schedules and related forecasts (Journel and Deutsch 1993; de Carvalho and Dimitrakopoulos 2019).

The multiple-point statistics (MPS) simulation methods (Guardiano and Srivastava 1993; Journel 1993, 2003, 2005; Journel and Zhang 2006; Remy et al. 2009; Mariethoz and Caers 2015) focus on incorporating high-order statistics to model complex non-linear and non-Gaussian geological patterns on the sequential simulation approach, without making distributional assumptions. These methods emerge from petroleum applications (Guardiano and Srivastava 1993; Journel 1993), where conditioning data is substantially sparser than at mining applications. MPS methods use training images (TI) to express the average non-linear spatial patterns of the attributes. The required multiple-point statistics are borrowed from the TIs and are exported to the simulation grid following the sequential simulation framework (Strebelle 2002; Journel 2003; Gómez-Hernández and Srivastava 2021). Developed MPS algorithms are distinguished into two types: pixel-based and pattern-based. In the pixel-based algorithms (Guardiano and Srivastava 1993; Journel 1993, 2003; Strebelle 2002; Mariethoz et al. 2010), the expression of a multivariate random function (Eq. 1.1) is experimentally approximated by searching a TI for replicates of the same data-event pattern and using the stored data-event frequencies to infer the local conditional probabilities to simulate a node, rather than random sampling from conditional probability distributions. Whereas pattern-based methods (Zhang et al. 2006; Arpat and Caers 2007; Honarkhah and Caers 2010; Mahmud et al. 2014; Mustapha et al. 2014; Chatterjee et al. 2016) build a database of TI’s patterns and use a similarity function to select the most similar pattern to be pasted in the simulation grid. Hence, the decomposition of the multivariate probability distribution of the RF of Eq. 1.1 is also experimentally approximated by conditional probabilities of blocks of nodes \( \{B(v_1), B(v_2), ..., B(v_{N_b})\} \), with respective centroids \( \{v_1, v_2, ..., v_N\} \), defining the pasted patterns upon the initial nodes set \( \{u_1, u_2, ..., u_N\} \) (Gómez-Hernández and Srivastava 2021), rather than randomly sampling from the local probability distributions.
for $\Lambda_i = \{B(\mathbf{v}_i), \Lambda_{i-1}\}$, where $\Lambda_{i-1}$ corresponds to the initial available exploration data $d_n$, the previously simulated blocks of nodes $B(\mathbf{v}_1), B(\mathbf{v}_2), ..., B(\mathbf{v}_{i-1})$, and the current simulated block $B(\mathbf{v}_i)$. MPS algorithms employ different filters, data structures, clustering techniques to handle the patterns’ database. The main observed limitations of MPS algorithms are: (a) the lack of a mathematical formalism due to the experimental replacement of conditional probability distributions; (b) high-order statistics are partially and indirectly considered; (c) finally, TI-driven realizations are shown to generate conflicts between the statistics of the generated realization and exploration data (Goodfellow et al. 2012; Mahmud et al. 2014; Osterholt and Dimitrakopoulos 2018).

High-order spatial cumulants are defined as spatial connectivity measures proposed to model complex, non-linear, and non-Gaussian geological patterns and can be inferred from conditional data using spatial templates, just like the variograms (Dimitrakopoulos et al. 2010; Mustapha and Dimitrakopoulos 2010b). The high-order simulation algorithms (HOSIM) (Mustapha and Dimitrakopoulos 2010a, 2011; Minniakhmetov and Dimitrakopoulos 2017a; Yao et al. 2018; de Carvalho et al. 2019; Yao et al. 2020; Minniakhmetov and Dimitrakopoulos 2021; Yao et al. 2021a; b) utilize the high-order spatial cumulants to approximate the conditional probability distribution at a node to be simulated through the sequential simulation paradigm. HOSIM utilizes TIs for incorporating additional information into data. In decomposition of the multivariate conditional probability distribution of the RF (Eq. 1.1), at a node to be simulated, the conditional probability density function (cpdf) is approximated by a weighted series of orthonormal polynomials, such as Legendre (Mustapha and Dimitrakopoulos 2010a, 2011), Laguerre (Mustapha and Dimitrakopoulos 2010c), and Legendre-like splines (Minniakhmetov et al. 2018; Minniakhmetov and Dimitrakopoulos 2021), whose weights are inferred from both the available data and TIs. For instance, in the case of Legendre polynomials, the approximation of the pdf at the first node to be simulated $\mathbf{u}_1$ is as follows:
\begin{equation}
    f_Z(u_1, z_1 | \Lambda_0) = \frac{1}{\int_B f(Z(u))dZ(u_1)} \sum_{i_1=0}^{\infty} \cdots \sum_{i_{n-1}=0}^{\infty} \sum_{i_n=0}^{\infty} L_{i_1, \ldots, i_{n-1}, i_n} \bar{P}_m(Z(u_1))
\end{equation}

where \( \bar{P}_m \) are normalized Legendre polynomials of order \( m \), and the weights \( L_{i_1, \ldots, i_{n-1}, i_n} \) are proven to be a function of the high-order spatial cumulants inferred primarily from available data and secondarily from the TIs. HOSIM does rely on any distributional assumption nor requires any data transformation of the attribute of interest. Additionally, by definition, the higher-order spatial cumulants are combinations of lower-order cumulants. As a result, the approach is data-driven since it incorporates the lower-order spatial statistics from data, while high order is derived from both data and the TI. Consequently, the HOSIM realizations honor the lower-order statistics of conditioning data, which is not observed in MPS realizations (Mustapha et al. 2011; Goodfellow et al. 2012).

In later developments, Minniakhmetov et al. (2018) propose using Legendre-like splines to overcome the instabilities in the cpdf approximations generated by the series Legendre polynomials. In dealing with multiple spatially correlated variables, Minniakhmetov and Dimitrakopoulos (2017) employ the diagonal domination condition of high-order cumulants for spatial decorrelation of attributes. This method, unlike the MAF, does not require a multi-Gaussian distribution assumption of the regionalized variables. Yao et al. (2018) simplify the calculation of high-order spatial cumulants and the polynomial approximation by using a single function without the need for explicitly computing cumulants. De Carvalho et al. (2019) propose a direct block HOSIM simulation method by introducing the multiple-support spatial template. Minniakhmetov and Dimitrakopoulos propose two data-driven, high-order methods for joint simulation of multiple categorical variables and Minniakhmetov and Dimitrakopoulos (2021) based on high-order spatial indicator moments, the equivalent of an indicator variograms in two-point statistics. A recursive B-spline approximation algorithm is used to estimate such indicator moments. Finally, de Carvalho and Dimitrakopoulos (2019) explore the effects of using HOSIM realizations in the mining complex production scheduling optimization. This work shows that incorporating non-linear spatial connectivity of high-grades results in more informed LOM production schedules.
1.5.3. Stochastic Optimization in Open Pit Mine Planning

As discussed in Section 1.5.1, conventional mine planning frameworks, which rely on a single estimated orebody model, produce misleading forecasts in open-pit mining regarding tonnages, grades, and cashflows (Dowd 1994; Vallée 2000; Dimitrakopoulos et al. 2002). Once these limitations were understood, stochastic frameworks are developed, aiming to incorporate different sources of uncertainty into the mine planning process. Although the focus of this thesis is underground mine planning, the state-of-art stochastic optimization methods for open-pit mining are reviewed in this section since they introduce essential components that are further implemented or might be potentially integrated into underground mine planning analogs.

Dimitrakopoulos et al. (2007) propose a maximum upside potential/minimum downside risk approach that allows incorporating geological uncertainty into the decision-making process. The approach consists of generating conventional open-pit mine production schedules for each simulated orebody realization, carrying out risk analysis on each schedule, and selecting the best scenario based on its performance on key project indicators (KPI) and a minimum acceptable return (MAR) on investment. However, this framework does not generate an optimal schedule under material supply uncertainty since it simply selects a best-performing LOM production schedule or mine design from a small set of possible scenarios based on user-defined KPIs. It has a cumbersome step of generating multiple deterministic schedules for each simulation, and the mine planner subjectively selects the final design based on the risk quantification of the defined KPIs and the setup MAR.

The probabilistic linear programming formulation of Dimitrakopoulos and Ramazan (2004) incorporates grade uncertainty through the probabilities of each block to have its grade within the desired interval. The model penalizes blocks having lower associated probability. The concept of geological risk discounting (GRD) is introduced, which seeks to postpone the extraction of riskier areas to later periods. The probabilistic models are limited since they only partially incorporate the information available in the set of orebody simulations: First, only local uncertainty is incorporated through summarized probabilities rather than the spatial uncertainty. Second, since the derived
blocks’ probabilities and expected grades are used, the optimization process is not fully informed by the grade variability.

A multi-stage mine production scheduling framework under grade uncertainty is developed by Godoy and Dimitrakopoulos (2004). At the first step, the stable solution domain (SSD) in terms of cumulative ore and waste tonnages for the best and worst scheduling scenarios are derived for each simulation using a conventional long-term mine planning commercial software. An LP model then optimizes mining rates based on this SSD. Subsequently, conventional mine production schedules are optimized for each simulation, given the optimal mining rates. The generated set of mining sequences is finally inputted into a simulated annealing-based (Kirkpatrick et al. 1983) combinatorial optimization process that aims to minimize the yearly deviations from ore and waste production targets. This step combines the input schedules to generate a single production schedule while incorporating spatial grade uncertainty and variability. The application of this framework at the Fimiston open-pit gold mine generated a substantially higher NPV when compared to a benchmark deterministic schedule. Albor Consuegra and Dimitrakopoulos (2009) study the effect of the number of simulations in this approach. The study concludes that over ten simulations, the solution converges due to the substantial change of support associated with the grouping from hundreds to thousands of blocks scheduled for a given production period. The shortcoming of this method is the time-consuming need for multiple deterministic input schedules.

Two-stages stochastic integer programming (SIP) (Birge and Louveaux 2011; King and Wallace 2012) is a classic mathematical programming formulation defined as follows:

\[
\begin{align*}
\min z &= c^T x + E_{\xi} \left[ \min q(\omega)^T y(\omega) \right] \\
\text{s. t. } Ax &= b \\
T(\omega)x + Wy(\omega) &= h(\omega) \\
x &\geq 0, y(\omega) \geq 0
\end{align*}
\]  

(1.4) \hspace{1cm} (1.5) \hspace{1cm} (1.6) \hspace{1cm} (1.7)

Where \( z \) is the objective function value, \( x \) denotes the vector of first-stage decision variables and \( y(\omega) \) denotes the vector of second-stage decision variables, with some variables in \( x \) or \( y(\omega) \) defined as integer variables. The objective function (Eq. 1.4) of this formulation has a first-stage
component $c^T x$, with the corresponding vector of objective coefficients $c$, and second-stage component $E_{\xi} [\min q(\omega)^T y(\omega)]$, where $q(\omega)$ is the vector of stochastic objective coefficients associated with variables $y(\omega)$ for each random event $\omega$. This second component of Eq. 1.4 represents the mean taken over all realizations of random events $\omega \in \Omega$. Equation 1.5 represents the constraints associated solely with the first-stage variables $x$ with the respective matrix of coefficients $A$ and right-hand side vector $b$, while Eq. 1.3 represents the stochastic constraints setting the relations between the two decision stages with the matrices of coefficients $T(\omega)$ and $W$, respectively associated with $x$ and $y(\omega)$, and the random vector $h(\omega)$. The vector $\xi^T(\omega) = [q(\omega)^T, T(\omega)^T, h(\omega)^T]$ represents the second-stage data vector, whose components are also random variables. Equation 1.5 are constraints exclusively related to first-stage decisions $x$. Once a random event $\omega$ is realized, it influences the random variables of $\xi^T(\omega)$ and optimal adaptative decisions $y(\omega)$ (or $y(\omega, x)$) are obtained with prior knowledge of the first-stage decisions $x$. Therefore, decisions $y(\omega)$ differ for different realizations $\omega$. Although this basic mathematical formulation is linear, nonlinear objectives and constraints can also be considered (Birge and Louveaux 2011).

Dimitrakopoulos and Ramazan (2008) translate this mathematical formulation (Eq 4-7) to the context of open-pit mine production scheduling problems. The random events $\omega \in \Omega$ correspond to scenarios of sources of uncertainty, and $x$ are the extraction sequence decision variables with associated feasible mining operation constraints of Eq. 1.5, such as slope constraints. The first-stage component $c^T x$ represents the total NPV from the production schedule to be maximized, while the second-stage component $E_{\xi} [\min q(\omega)^T y(\omega)]$ represents the expected cost associated with the risk of not meeting production targets to be minimized. In Eq. 1.6, the term $T(\omega)x$ corresponds to the tonnes and grades produced, while $Wy(\omega)$ defines the risk defined in terms of deviations $y(\omega)$ from the production targets matrix $h(\omega)$. The concept of the value of the stochastic solution (VSS) is presented by Birge and Louveaux (2011), consisting of the difference between the stochastic programming value solution (ESS) and the deterministic expected value solution (EDS), that is, $VSS = ESS - EDS$. The VSS represents the gain of integrating distributions on uncertain outcomes or the cost of ignoring the uncertainty. It has been proven that VSS is always non-negative (Birge and Louveaux 2011). Hence, Dimitrakopoulos and Ramazan
(2008) demonstrate how the VSS evaluates the performance of stochastic mine designs and production schedules compared to the deterministic mine plans.

A two-stage SIP model applied to mine production scheduling is first introduced by Ramazan and Dimitrakopoulos (2005). The model aims to maximize the project’s NPV while minimizing the deviations from production targets of ore tonnage, metal tonnage, and requirements for processing head grade. The risk management of not meeting production targets is performed through the previously mentioned geologic risk discount (GRD) (Ramazan and Dimitrakopoulos 2004b). This model is initially applied to a two-dimensional gold mineral deposit. The integration of a stockpile option with decision variables for stockpiling blocks and reclaiming material to be processed and an application to a real open-pit gold mine are presented by Ramazan and Dimitrakopoulos (2013). Grade requirement constraints for multiple elements and smoothing constraints are other proposed extensions (Benndorf and Dimitrakopoulos 2013). The applications of this model to various mines highlight its ability to generate schedules with higher NPV, higher recovered metal, and larger pit limits while providing feasible production profiles in terms of tonnages and grades (Benndorf and Dimitrakopoulos 2013; Ramazan and Dimitrakopoulos 2013; Leite and Dimitrakopoulos 2014). Nonetheless, this method relies on the economic value of blocks, which assumes that each block is individually processed at a processing destination and a predetermined destination policy based on specified cut-off grades. Even though optimized cut-off grades (Lane 1964, 1988; Rendu 2014) can be used as inputs to the model, these cut-off grades are not an output of the optimized production schedule. In addition, the linear modeling of the stockpile is another limitation since it assumes completely homogenized reclaimed material.

A mining complex is a mineral value chain where multiple mines provide different material types that flows and are transformed through different processing streams ending up as marketable products (Pimentel et al. 2010; Montiel and Dimitrakopoulos 2015; Goodfellow and Dimitrakopoulos 2016). The mining industry currently employs several available software tools and optimization methods to optimize these various components of the mining complex separately, leading to less profitable and likely infeasible strategic mining plans (Whittle 2004; King 2007, 2011). However, during the last decades, advancements have been made towards integrating some planning components into, both deterministic or stochastic, joint optimization frameworks, aiming

Two-stage SIP models have also been successfully applied to the simultaneous optimization of mining complexes (Goodfellow and Dimitrakopoulos 2015, 2016; Montiel and Dimitrakopoulos 2015, 2018; Del Castillo and Dimitrakopoulos 2019; Saliba and Dimitrakopoulos 2019; Both and Dimitrakopoulos 2020). Non-linear interactions across mineral value chains are incorporated by eliminating the economic value of blocks and letting the economic value of marketable products dictate the optimization process. Therefore, the optimization process capitalizes on the availability of multiple material types from multiple mines and the setup capacities to determine optimal destination policies under supply and market uncertainties. Optimized block-based (Montiel and Dimitrakopoulos 2017) and cluster-based (Goodfellow and Dimitrakopoulos 2016) destination policies are proposed as alternatives to the limited predefined cut-off destination policy. Various sources of uncertainty, other than supply uncertainty, such as commodity prices (Saliba and Dimitrakopoulos 2019) and equipment performance (Both and Dimitrakopoulos 2020), are also incorporated. Capital investments decision variables (Goodfellow and Dimitrakopoulos 2015; Del Castillo and Dimitrakopoulos 2019), tailings management (Saliba and Dimitrakopoulos 2020), multiple products’ transportation options, and operation modes (Montiel and Dimitrakopoulos 2015) are examples of different mining complex’s components considered in various developments.

Some previously mentioned attempts, either deterministic or stochastic, to simultaneously optimize the strategic production schedules of mining complexes have considered underground mines in the modelled mineral value chains (Hoerger, Bachmann, et al. 1999; Hoerger, Hoffman, et al. 1999; King 1999, 2007; Whittle 2004, 2018; Chanda 2007; Montiel et al. 2015; Montiel and Dimitrakopoulos 2018). Nonetheless, due to the existence of multiple employed mining methods and the complexity of underground mining operations, these approaches rely on previously optimized underground mine designs and schedules. The deemed underground mines are often
simply treated as external sources of material, related to continuous decision variables, to feed the various processing destinations. Therefore, the available methods for simultaneous optimization of mining complexes are unable to optimize underground mine designs and production schedules jointly.

Other stochastic programming variants are also applied to open-pit mine production scheduling problems. The anticipative stochastic model of Menabde et al. (2007) simultaneously optimizes the cut-off grade and the mine production schedule so as to maximize the NPV. However, grade uncertainty is integrated through constraints enforcing that mining and processing capacities are satisfied in an average sense over all simulations. This anticipative model does not provide effective risk management. The multistage stochastic programming proposed by Boland et al. (2008) employs non-anticipativity constraints, ensuring that different decisions amongst the simulations might only be taken if the scenarios are sufficiently distinguishable. This approach generates a tree of multiple schedules, which is not implementable in practice. Therefore, the two-stage SIP formulations remain the most prevalent formulation encountered in the literature.

1.5.4. Optimization of Underground Mine Planning under Uncertainty through the Sequential Optimization Framework

As for deterministic approaches, stochastic optimization methods applied to underground mine planning are relatively new and less advanced than the open-pit mining counterparts due to the existence of several differing underground mining methods (Bullock and Hustrulid 2001; Hamrin 2001; Hartman and Mutmansky 2002) and the conceptual complexity of underground mining operations (Alford et al. 2007; O’Sullivan et al. 2015). Furthermore, existing stochastic underground mining optimization methods follow the stepwise optimization framework (already revisited in this literature review for deterministic approaches in Section 1.3). Thus, they are focused on the optimization of either the stope design or the mine production scheduling. The methods reviewed in this section are not necessarily limited to the sublevel stoping mining method, which is the focus of the current thesis, but they present the benefits of assimilating uncertainty into the underground mine planning process.
A stochastic stope design optimization method under grade uncertainty is first introduced by Grieco and Dimitrakopoulos (2007). This probabilistic MIP formulation is developed for a sublevel stoping operation with an input orebody model discretized into layers of transverse panels, which are, in turn, discretized into blasting rings. A stope consists of selecting adjacent blasting rings within a minimum and maximum allowable width in each panel. Each allowable stope width span is associated with a required pillar size. A blasting ring is represented by its expected grade and a probability of being above an input cut-off grade derived from a set of orebody simulations. The model aims to maximize the recoverable metal, while probabilistic constraints ensure that the combined probability of selected rings is greater than a specified minimum acceptable level of risk. The application of this method at an underground copper mine showed that the most conservative level of risk of 100% did not generate the most profitable design due to eliminating some rings with still high probabilities of being ore. As a result, the minimum level of risk might assist the mine planner in assessing the designs' upside potential in terms of higher recoverable metal and undiscounted profit. This approach, nevertheless, has the same shortcomings as the probabilistic open-pit mining approach of Ramazan and Dimitrakopoulos (2004) discussed in Section 1.5.3. Furthermore, the physical constraints are site-specific and do not consider lateral constraints for stopes in adjacent panels.

A three-step stochastic framework for stope design optimization is proposed by Villalba Matamoros and Kumral (2018), suited for a generic stoping mining method. First, the framework utilizes the heuristic method proposed by Villalba Matamoros and Kumral (2017) and reviewed in Section 1.3 to deterministically determine the stope layout for each simulation in the input set. The ensemble of generated designs maps the uncertainty in stopes locations and sizes. However, due to the possibly substantial dissimilarities between the layouts, their direct use in the genetic algorithm (GA) step would lead to an unfeasible combined layout. Thus, in a second step, the initial stope designs are clustered based on their similarities based on a maximum searching radius. Eventual geotechnical constraints violations in the clustered scenarios are fixed, and a smaller population of stope designs is generated. This population of designs still carries out the uncertainty of stopes’ locations while accounting for geotechnical feasibility. Subsequently, the solution space is then reduced to the set of blocks that belong to at least one stope in the generated population of stope layouts. Finally, the GA combines the resulting designs to maximize the undiscounted profit.
and ensure the single final design's feasibility. Villalba Matamoros (2018) tests this approach to different mining directions, i.e., the direction along with the blocks are grouped into slices. The grouping of slices to form a potential stope is performed perpendicularly. This method is applied to an underground gold mine, and it is compared to the deterministic layout generated by the heuristic approach from the same authors using an estimated model. The stochastic layout overperformed the deterministic heuristic layout (Villalba Matamoros and Kumral 2017) based on an estimated orebody model generating 12% higher profit and 11.6% less internal dilution in terms of low-grade slices within the stopes. An observed limitation is that the GA algorithm receives a reduced uncertainty mapping space due to the clustering step, which precludes the final design's profitability under uncertainty.

Wilson (2020) proposes a heuristic stochastic stope design approach for sublevel open stoping operations under grade uncertainty. The method is an extension of the deterministic approach proposed by Nikbin et al. (2018) and utilizes a heuristic approach that slices the orebody model collapsing the economic value of blocks in a stope section into a single block. A dynamic programming algorithm is then used to optimize one-dimensional strings of values given the allowable stope lengths perpendicular to the slicing direction. The method aims to maximize the economic value of blocks within the stope design overall orebody simulations while minimizing the block’s economic value standard deviation, which is penalized in the objective function. A limitation of the proposed method is the absence of adaptative scenario-dependent decisions. Instead, penalties applied to the variance preclude the optimization process from finding stopes in high-grade zones, especially for positively skewed grade distributions having higher grades associated with higher variances due to the so-called direct proportional effect (David 1977; Journel and Huijbregts 1978; Rossi and Deutsch 2014). As a result, this approach might jeopardize the upside potential in terms of metal tonnage in the presence of uncertainty. In addition, lower variance stope layouts do not entail more profitable layouts.

In summary, the mentioned stochastic stope design optimization methods only partially account for the uncertainty and variability information provided by the input set of simulated orebody model through the use of summarized mining blocks’ probabilities (Grieco and Dimitrakopoulos 2007), a reduced uncertainty mapping of uncertainty in stope locations (Villalba Matamoros and
In addition, from an operations research standpoint, some of those methods are heuristic, generating suboptimal solutions. Therefore, a two-stage SIP for stope design optimization, with recourse decisions adapting to each orebody realization, would represent more effective incorporation of grade uncertainty to provide a risk resilient stope design.

Some stochastic optimization methods for underground mine production scheduling are available in the technical literature. A two-stage SIP model for the optimization of a high-level underground mine production schedule under grade uncertainty is proposed by Carpentier et al. (2016). Rather than an input stope layout, the model receives mineralized lenses located at different mines assumed to be accessed through ramps and horizontal drifts. Each lens is represented by a set of equiprobable discrete grade-tonnage curves derived from a set of orebody simulations and a set of grade bins, which, in turn, represent the potential cut-off grades for a lens. The model, therefore, simultaneously optimizes the mining complex’s production schedule and the lenses’ cut-off grade based on their contained reserves. It is assumed that a single cut-off grade must be selected for the entire lens over the LOM. The proposed objective function aims to maximize the net revenue from the lenses, minimize the development, mining, backfilling, mines’ opening and closure costs, and minimize the penalties of deviating from production targets, such as ore tonnage, metal tonnage, development rate, and waste handling. The proposed method is applied at a nickel mining complex containing five underground mines. Each lens is partially extracted using the cut-and-fill and partially using the long-hole stoping mining method. The stochastic approach generated a two-year shorter life of mine but 22% higher NPV when compared to its deterministic counterpart. Moreover, the risk analysis performed on this deterministic schedule anticipates that the NPV would be overestimated by 47% if the grade uncertainty is neglected, highlighting the benefits of incorporating grade uncertainty into the underground mine production scheduling.

A stochastic optimization framework for the transition from open-pit to underground mining is proposed by MacNeil and Dimitrakopoulos (2017). In this framework, the optimization of the open-pit and underground mine production schedules are performed sequentially, both constrained by a specified crown pillar size and position scenario. Therefore, the embedded two-stage SIP model for the underground mine production schedule optimization is, per se, an example how the
incorporation of grade uncertainty enhances the underground mining profitability. The proposed objective function has the two typical components proposed by Ramazan and Dimitrakopoulos (2005) related to maximizing the NPV and minimizing the risk of not meeting production targets. Annual ore tonnage target, mining capacity, and stopes’ precedence are constraints considered in the model that receives an input stope design and the related stopes’ precedence rules designed using commercial software. Instead of block-basis, the first-stage decision variables control the stopes’ sequence of extraction. The application of this framework at a gold mine, whose transition is being evaluated from open-pit to cut-and-fill operation, highlights that, assuming the best tested transitioning scenario, the stochastic method provides substantial lower deviation from the mill tonnage target after the transition to underground operation, whereas the deterministic analogue is unable to control this risk. Although a combined forecasted NPV is presented, for both operations before and after transition, the NPV generated by the underground schedule is consistently higher than the deterministic analogue. A limitation of this formulation is that development costs and timing related to the main ramp and drifts are not considered in both deterministic and stochastic cases.

Another two-stage SIP model for underground mine production scheduling under grade uncertainty also focused on the cut-and-fill mining method proposed by Huang et al. (2020), which adds the timing, precedence, and costs related to development activities. Besides the inputs of the previous method, the current model also receives a network design consisting of a central shaft and equally and vertically spaced drifts. The model is subjected to precedence constraints between development and production activities, vertical and horizontal stopes’ adjacency constraints, development rate, backfilling, and processing capacity constraints, and, finally, stochastic processing head grade requirements. An underground gold mine is used to exemplify the aspects of the proposed stochastic model, which is benchmarked with a deterministic analog based on a Kriged estimated model. The stochastic method generated a physically different schedule with a somewhat higher NPV and metal tonnages over the life-of-mine while being more consistent in meeting the mill head grade targets.

An underground mine production scheduling optimization method under grade and activities’ duration time uncertainties is proposed by Nesbitt et al. (2021). The authors recognize that the
The proposed model is not a traditional multistage stochastic integer programming (Birge and Louveaux 2011) since the scenario-dependent activities start decision variables, which adapt to the uncertainty in grades and completion times, are conditioned to a time-interval width around scenario-independent activities start decision variables, which define a baseline production schedule. Hence, unlike the multistage stochastic integer program proposed by Boland et al. (2008) discussed in section 1.5.3 that generates a different production schedule in a scenario tree, the single output baseline production schedule can be followed in practice. The model is applied to an underground gold mine employing multiple mining methods. Only five scenarios were used to jointly quantifying the two sources of uncertainty. However, the low number of scenarios used in the case study that should be higher than 10 (Albor Consuegra and Dimitrakopoulos 2009; Montiel and Dimitrakopoulos 2017), the restricted slack time interval used in the case study, the arguable modelling of activities duration uncertainty, and the geological uncertainty considered only for high-grade extraction volumes rather than to the entire orebody model provide limited uncertainty information to the optimization process. As a result, the integration of joint uncertainty did not provide differences in NPV compared to the deterministic benchmarking case, highlighting the need for further investigations of the proposed method.

All previous stochastic stope design and underground mine production scheduling optimization models are focused on either self-supported (sublevel stoping variants) or supported mining methods (cut-and-fill). Hereinafter, the models presented focus on block caving operations, which have conceptually different mine designs, the precedence rules, and the unitary operations from the previously mentioned mining methods. These models are reviewed to illustrate the integration of sources of uncertainty, other than the grade uncertainty, into the underground mine production scheduling process. The two-stage SIP model for the stochastic optimization presented by Dirkx et al. (2018) jointly considers scenarios for grade and geotechnical uncertainties. The latter refers to the production delays due to caving draw-point hang-ups (material clogging the draw-points). Geostatistical simulations quantify the first uncertainty source, while a discrete event simulator based on the tonnage between hang-up distribution is used to generate scenarios for the second type of uncertainty. The case study at an underground copper mine showed that the proposed model can substantially reduce the expected production time delays while ignoring the hang-up uncertainty leads to production forecasts that are very unlikely to be realized in practice.
The block caving production scheduling method proposed by Sepúlveda et al. (2018) jointly integrates grade and geometallurgical uncertainties, the latter in terms of processing recovery. The model has two weighted objectives. The first objective aims to maximize the expected discounted net smelter return (NSR) from the producing draw points, while the second objective is one of the following risk measures computed yearly over the input set of scenarios: minimization of the standard deviation of NSR, maximization of the value at risk for NSR, minimization of the deviations from ore production target. Due to the proposed non-linear uncertainty measures, a genetic algorithm (GA) metaheuristic solver is used to solve the proposed bi-objective models. The final schedule is subjectively selected based on Pareto Front charts of each pair of objectives.

The above underground mine production scheduling methods illustrate the advances in incorporating different sources of uncertainty into the planning process. However, they present some limitations. First, the method of Carpentier et al. (2016), by considering mineralized lenses, relies on the assumption of homogenous lenses’ extraction, neglecting the within-lens spatial distribution of grades. A stope layout under geotechnical and operational constraints must be further designed using the optimized lens’s cut-off grade. Thus, the forecasted tonnages, grades, and cash flows might not be realized. Second, although the approach proposed by Sepúlveda et al. (2018) aims to minimize the technical risk, it is not considered a stochastic integer program that is proven to provide optimal schedules under a set of scenarios describing the deemed uncertainties (Birge and Louveaux 2011). Finally, the models of Dirkx et al. (2018), Huang et al. (2020), and Nesbitt et al. (2021) follow the sequential planning framework and receive preconceived mine and network designs generated by deterministic software tools, thus, ignoring any type of uncertainty, time value of money, and the interaction between mine designs and production schedules as discussed section 1.4. Therefore, the profitability and risk management of such generated schedules are hindered by the upstream steps in the underground mine planning process.
1.6. Goal and Objectives

The goal of the research presented in this thesis is to develop and apply an integrated stochastic optimization method of stope design and long-term mine production scheduling for underground mines employing the sublevel open stoping mining method. The following objectives are set to meet this goal:

- Review the technical literature related to the sublevel open stoping mining method, the deterministic and stochastic approaches for the strategic underground mine planning, the attempts of the integrated optimization of the stope design and the long-term mine production scheduling, and the geostatistical simulation techniques of mineral deposits.
- Develop and implement a stochastic optimization model for the sublevel open stoping design that accounts for grade uncertainty and development costs and flexible selection of production levels and an access option.
- Develop and implement an integrated stochastic optimization model of the sublevel open stoping design and long-term production scheduling by extending the previous model, and assess the advantages of such an integrated approach compared to the sequential underground mine planning framework.
- Summarize the main contributions and conclusions of this thesis and provide suggestions for future research.
1.7. Thesis Outline

This thesis is organized into the following chapters:

Chapter 1 provides a literature review of the sublevel stoping mining method, the available deterministic and stochastic methods for optimizing stope design and underground mine production scheduling, evolving to integrated optimization methods, and the state-of-art of geostatistical simulation of mineral deposits.

Chapter 2 presents a stochastic optimization model for defining the stope design considering flexible level definition, selection of primary access, development costs, and project capacities for sublevel open stoping operations. The proposed method is applied to an underground gold mine, and it is compared to an industry-standard stope design approach highlighting the benefits of the stochastic optimization framework.

Chapter 3 presents a stochastic optimization model for the integrated optimization of stope design and underground mine production scheduling for sublevel open stoping operations. The method is applied to an underground gold mine and benchmarked against the sequential underground mine planning framework to assess the effects of such an integrated approach on the life-of-mine production schedule and related forecasts.

Chapter 4 summarizes the contributions of each paper and overall conclusions, followed by suggestions for future work.
Chapter 2 - Stochastic Stope Design Optimization under Grade Uncertainty and Dynamic Development Costs

2.1. Introduction

Stope design optimization consists of defining underground mineable volumes whose shapes and sizes are dictated by the chosen stoping underground mining method, the geotechnical properties of the rock masses, and grade distribution of the orebody from which economic material will be extracted in order to maximize undiscounted profit (Topal and Sens 2010; Erdogan et al. 2017). In the current industrial practice, underground mine planning follows a sequential framework. An optimized stope design is an input to the development network layout optimization, which defines the topology of interconnected access routes, such as shafts, declines, drifts, and crosscuts. Subsequently, the stopes and network layout are inputs to the production scheduling optimization step, which dictates the underground mining project's net present value (Topal 1998, 2003; Alford et al. 2007; Musingwini 2016; Nhleko et al. 2018; Kumral and Sari 2019). Grade uncertainty is the main contributor of technical risk, affecting a mining projects' viability (Vallée 2000; Rendu 2017). However, it is not included in the conventionally generated stope designs, which fail to integrate the geological uncertainty and variability that affect stope sizes and locations, harming production and financial forecasts (Myers et al. 2007; Tavchandjian et al. 2007; Dimitrakopoulos and Grieco 2009; Jewbali et al. 2015). Therefore, the incorporation of supply uncertainty, in grade and material type, into the stope design optimization allows one to address risk earlier in the planning process (Nhleko et al. 2018).

The sublevel open stoping is a self-supported, non-entry, and flexible underground mining method in which the orebody is vertically split into production levels, that are often separated by horizontal pillars. Within the primary levels, stopes are usually delimited by rib and longitudinal pillars, while regularly spaced blasting drifts are developed, defining the sublevels (Fig. 2.1). The stopes remain empty during their exploitation with eventual post-backfilling (Hamrin 2001; Hustrulid and Bullock 2001; Pakalnis and Hughes 2011; Villaescusa 2014). Some three-dimensional sublevel stoping design optimization methods have been proposed, which incorporate progressively different geotechnical and operational aspects. The Floating Stope algorithm (Alford 1995) and
the Maximum Value Neighborhood algorithm (Ataee-Pour 2004) float a fixed minimum stope size on the input orebody model and define profitable envelopes in which the stopes should be further manually designed. A predefined cut-off grade or the economic value of a block's neighborhood is the respective criteria used to build the profitable boundaries in those methods. Topal and Sens (2010) and Sandanayake et al. (2015a; 2015b) present different methods to select non-overlapping stopes with variable sizes to define a stope layout with unified stopes. Furthermore, Sandanayake et al. (2015a; 2015b) integrate more operational constraints, such as pillar requirements and level allocation.

The heuristic method proposed by Villalba Matamoros and Kumral (2017) maximizes the profit from stopes, which are defined by grouping slices of a fixed number of blocks in height and width and variable length, while minimizing the internal dilution, which consists of slices with a grade that is lower than a specified cut-off grade. The previous methods are based on rectangular three-dimensional stope shapes. A heuristic approach combining a one-dimension dynamic programming algorithm and dimensionality reduction greedy algorithm is proposed by Nikbin, et al. (2018) and generates stope boundaries with higher economic values compared to the approaches of Alford (1995) and Ataee-Pour (2004). The stope overlapping issue and pillar constraints are further integrated by Wilson (2020). The Mineable Shape Optimizer (MSO) (Alford and Hall 2009; Alford Mining Systems 2016) is an industry-standard software for stope design optimization. In its Slice Method, the input model is discretized into a regular grid parallel to the main direction, forming transversal tubes. Each tube is sliced and the slices with grades greater than a specified cut-off grade are grouped to maximize each tube's economic value. However, this method is highly dependent on geological control parameters or wireframes, which are uncertain, and requires an upfront cut-off grade that is not adequately known in the early stages of the underground mine planning process. For a comprehensive review of stope design optimization methods, the reader is directed to Ataee-Pour (2005), Erdogan et al. (2017), and Nhleko et al. (2018).

Some approaches aim to consider the development costs in the stope design optimization, integrating the inherent interdependencies between each stope's economic potential and its accessibility/remoteness. The network flow-based method proposed by Bai et al. (2013) imposes
a maximum allowable distance of a block from a central shaft and requires a horizontal mining width for the accessibility of the farthest blocks in the output stope boundary. Ding et al. (2004) propose an iterative approach that redistributes the development costs among selected stopes, re-evaluating their economic value. Hou et al. (2019) implement a mixed-integer programming (MIP) formulation with recursive development cost constraints, which maximizes the stopes' profit and minimizes the total development costs for a fixed-network underground mine. However, the two last approaches are based on a development network layout with fixed shaft location and predefined levels, which have no flexibility to provide optimized production levels.

The previous methods are deterministic and are based on a single conventionally-estimated orebody model, a smooth representation of the mineral deposit (Goovaerts 1997; Dimitrakopoulos et al. 2002). Geostatistically simulated representations of mineral deposits better represent grade and material type distributions of a deposit, reproducing conditional data statistics and are used to quantify the grade uncertainty and variability (Goovaerts 1997; Boucher and Dimitrakopoulos 2009). Some stochastic stope design optimization methods have been proposed. Grieco and Dimitrakopoulos (2007) present a probabilistic mathematical programming formulation with flexible pillar requirements tailored to the sublevel stoping method. Stochastic simulations of a deposit, discretized into blasting rings, are used to derive the ring's average grade and its probability of being above a predefined cut-off grade. The model aims to maximize the stope design's metal content, given a minimum acceptable risk level. The probabilistic approach, however, uses limited information consisting of summarized probabilities representing the uncertainty and estimated grades. Thus, simulations are not directly integrated. Villalba Matamoros and Kumral (2018) propose a three-step stochastic stope design optimization method for a non-specified stoping method variant. First, the previously mentioned heuristic algorithm of Villalba Matamoros and Kumral (2017) generates a stope design for each input simulation. Subsequently, the designs are clustered based on their similarities while fixing geotechnical constraints violations to obtain a population of risk-resilient designs. Finally, a genetic algorithm combines the resulting designs to maximize the profit and ensure the single final design's feasibility. The previously discussed stochastic approaches do not propose scenario-dependent recourse actions, preventing the methods from adapting to grade uncertainty.
Two-stage stochastic integer programming (SIP) formulations (Birge and Louveaux 2011) were initially introduced in mine planning for long-term production scheduling of a single open-pit mine (Ramazan and Dimitrakopoulos 2005, 2013) and have since been successfully extended to the simultaneous optimization of mining complexes (Montiel and Dimitrakopoulos 2015; Goodfellow and Dimitrakopoulos 2016; Del Castillo and Dimitrakopoulos 2019; Saliba and Dimitrakopoulos 2019). Although fewer applications have been applied to underground settings, recent promising SIP formulations have been developed for long-term underground mine production scheduling employing different mining methods, such as a hybrid cut-and-fill with long-hole (Carpentier et al. 2016), purely cut-and-fill (Huang et al. 2020), and block caving (Dirkx et al. 2018). The applications demonstrate that the stochastic frameworks capitalize on the grade and operational uncertainties to provide physically different schedules with a higher expected net present value, while managing the risk of not achieving production targets when compared to the deterministic approaches.

Unlike the vast majority of available methods, which only aim to maximize the undiscounted profit or recovered metal, a new two-stage SIP formulation for stope design optimization is proposed in this paper. Its objectives are to maximize the undiscounted profit whilst minimizing related development costs, and the economic impact of exceeding project capacities. The risk management is incorporated through scenario-dependent recourse actions given the set of geostatistical simulations of the related orebody. The proposed first-stage decisions account for the physical constraints to provide a unique and mineable sublevel stoping design, which are related to selecting levels, stopes, the location of a main shaft, and the horizontal and vertical development costs associated with opening drifts and the shaft, respectively. The second-stage decisions aim to maximize the recovered metal while managing the risk of not satisfying existing or planned capacities under the grade uncertainty. The model inputs the realizations of the deposit, which has blocks that are flagged with distinct geotechnical zones, a set of potential primary access options, such as different shaft locations, the economic and technical parameters, and a library defining the level spacing, the possible stope shapes and pillar sizes for each geotechnical zone. The output is a stope layout with the best levels and stopes locations, as well as the best shaft location that minimize the development costs of shaft and drifts. The next sections discuss an overview of the method, followed by the two-stage SIP formulation. Subsequently, a case study at an underground
gold mine is presented and benchmarked with the stope layout generated by an industry-standard stope design approach. Conclusions and future work follow.

### 2.2. Method

A proposed stochastic stope design optimization method under grade uncertainty and development costs is presented. The deemed sublevel open stoping mining method (Fig. 2.1) considers horizontal production levels that might be separated by horizontal pillars. The stopes have variable heights within each level, but their access is always aligned to a respective production level and can be apart by rib and longitudinal pillars. This feature facilitates the mineability of the stope layout and the optimization of the detailed development design of hauling and drilling drifts and loading crosscuts connecting the selected stopes in the further steps of the underground mine planning.

This section introduces the required inputs, and the preprocessing steps are discussed, followed by the two-stage SIP's mathematical formulation. The proposed method receives a set of stochastic orebody simulations $s \in S$, which quantify the geological uncertainty and variability in grades and material types of mining blocks $i \in I$. The blocks are flagged with different input geotechnical zones $n \in N$ with respective stope size and pillars requirements. Relevant economic and geotechnical parameters, listed in Table 2.1 to Table 2.6, and a set of potential access options $k \in K$, such as different shaft locations, complete the optimization framework's required inputs.
Figure 2.1: Stope in a sublevel stoping method, showing the sublevels and the typical parallel drilling ring pattern (Source: Bullock and Hustrulid 2001).

<table>
<thead>
<tr>
<th>Table 2.1: List of indices</th>
</tr>
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<tbody>
<tr>
<td><strong>Index</strong></td>
</tr>
<tr>
<td>$i$</td>
</tr>
<tr>
<td>$j$</td>
</tr>
<tr>
<td>$d$</td>
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<td>$n$</td>
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<td>$m$</td>
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<td>$s$</td>
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<tr>
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<table>
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<tr>
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<td>$I_{jl}$</td>
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<tr>
<td>$J$</td>
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<tr>
<td>$J_l$</td>
</tr>
<tr>
<td>$J_{dk}$</td>
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<tr>
<td>$N$</td>
</tr>
<tr>
<td>$L$</td>
</tr>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>$L_n$</td>
</tr>
<tr>
<td>$S$</td>
</tr>
<tr>
<td>$K$</td>
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<td>$D_l$</td>
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<tr>
<td>$M_n$</td>
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### Table 2.3: List of economic and technical parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_{is}$</td>
<td>Tonnage of block $i$ in scenario $s$</td>
</tr>
<tr>
<td>$w_{jl}$</td>
<td>Tonnage of stope $j$ in level $l$, $w_{jl} = \sum_{i \in I_{jl}} w_{is}$ in scenario $s$</td>
</tr>
<tr>
<td>$\theta_{ls}$</td>
<td>Indicator $\theta_{ls} = 1$ if block $i$ is greater than a user-defined cut-off grade is scenario $s$, 0 otherwise.</td>
</tr>
<tr>
<td>$o_{jl}$</td>
<td>Ore tonnage of stope $j$ in level $l$ and scenario $s$, $o_{jl} = \sum_{i \in I_{jl}} \theta_{ls} w_{is}$</td>
</tr>
<tr>
<td>$g_{ls}$</td>
<td>Grade of block $i$ in scenario $s$ in percent metal</td>
</tr>
<tr>
<td>$g_{jl}$</td>
<td>Average grade of ore blocks within stope $j$ in level $l$ scenario $s$ in percent metal, such that $g_{jl} = \frac{\sum_{i \in I_{jl}} \theta_{ls} g_{ls} w_{is}}{o_{jl}}$</td>
</tr>
<tr>
<td>$R$</td>
<td>Processing recovery in percent</td>
</tr>
<tr>
<td>$P$</td>
<td>Metal selling price $$/t</td>
</tr>
<tr>
<td>$C_{l}^{drift}$</td>
<td>Unit horizontal development cost of drifts in $$/m in level $l$</td>
</tr>
<tr>
<td>$C_{k}^{shaft}$</td>
<td>Unit vertical (shaft) development cost associated with access option $k \in K$ in $$/m.</td>
</tr>
<tr>
<td>$C_{l}^{mining}$</td>
<td>Mining cost in $$/t for level $l$.</td>
</tr>
<tr>
<td>$C_{process}$</td>
<td>Processing cost in $$/t.</td>
</tr>
<tr>
<td>$U_{k}^{transport}$</td>
<td>Total transportation (hoisting) capacities of access options $k \in K$ in tons.</td>
</tr>
<tr>
<td>$U^{develop}$</td>
<td>Total horizontal development capacity in m.</td>
</tr>
<tr>
<td>$U^{mill}$</td>
<td>Total milling capacity in tons</td>
</tr>
<tr>
<td>$c_{transport}$</td>
<td>Penalty cost associated with the surplus deviations from the transportation (shaft or ramp) capacity.</td>
</tr>
<tr>
<td>$c_{develop}$</td>
<td>Penalty cost associated with the surplus deviations from the horizontal development capacity.</td>
</tr>
</tbody>
</table>

65
Penalty costs associated with the surplus deviations from the mill tonnage capacity.¹

Penalty cost associated with the shortages from the metal content of the stopes selected in a level compared to the metal content of all blocks $i \in I_l$ (applied to each level $l$).¹

**Table 2.4: List of geometric parameters.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_i, y_i, z_l$</td>
<td>Coordinates of block $i$ in meters</td>
</tr>
<tr>
<td>$\lambda^x, \lambda^y, \lambda^z$</td>
<td>Block dimensions in meters</td>
</tr>
<tr>
<td>$\sigma_j$</td>
<td>Origin block of stope $j$ with coordinates $x_{o_j}, y_{o_j}, z_{o_j}$</td>
</tr>
<tr>
<td>$e_j$</td>
<td>Terminal block of stope $j$ with coordinates $x_{e_j}, y_{e_j}, z_{e_j}$</td>
</tr>
<tr>
<td>$\gamma^x_{mn}, \gamma^y_{mn}, \gamma^z_{mn}$</td>
<td>Stope sizes along direction $x$, $y$, and $z$ in number of blocks for stope shape $m \in M_n$ of geotechnical zone $n \in N$</td>
</tr>
<tr>
<td>$\alpha^z_n$</td>
<td>Fixed level height, in terms of the number of blocks, along $z$ axis for geotechnical zone $n \in N$</td>
</tr>
<tr>
<td>$\beta^z_n$</td>
<td>Shift parameter corresponding to the number of blocks to be shifted above the base coordinates $z^\text{base}_l$ of the most profound level $l = 1$ of zone $n$, such that $\beta^z_n &lt; \alpha^z_n$</td>
</tr>
<tr>
<td>$z^\text{base}_l$</td>
<td>Coordinate of the base of level $l$</td>
</tr>
<tr>
<td>$z^\text{roof}_l$</td>
<td>Coordinate of the roof of level $l$</td>
</tr>
<tr>
<td>$z^\text{surface}_l$</td>
<td>Coordinate of surface (starting point of shaft/ramp).</td>
</tr>
<tr>
<td>$\sigma^\text{rib}_x$</td>
<td>Minimum stand-off pillar size between stopes along axis $x$ in meters for geotechnical zone $n \in N$ (multiple of dimension $\lambda^x$)</td>
</tr>
<tr>
<td>$\sigma^\text{rib}_y$</td>
<td>Minimum stand-off pillar size between stopes along axis $y$ in meters for geotechnical zone $n \in N$ (multiple of dimension $\lambda^y$)</td>
</tr>
<tr>
<td>$\sigma^\text{hill}_z$</td>
<td>Minimum pillar size in number of blocks between levels along axis $z$ in meters for geotechnical zone $n \in N$ (multiple of dimension $\lambda^z$)</td>
</tr>
<tr>
<td>$\delta^\text{shaft}_{lk}$</td>
<td>Vertical distance from the surface to $z^\text{base}_l$ of level $l$ for access option $k \in K$ (shaft)</td>
</tr>
<tr>
<td>$\delta^\text{drift}_{jdtk}$</td>
<td>Horizontal distance (in a drift) from stope $j$ to a potential access point of option $k \in K$ along mining direction $d$ in level $l$</td>
</tr>
</tbody>
</table>

¹ The penalties costs have no unit since the deviations $d_s^\text{transport}$, $d_s^\text{process}$, $d_s^\text{develop}$, and $d_s^\text{metal}$ are already converted to dollar values in Eqs. 2.20-2.23. These penalties are calibrated based on a user-based empirical approach that generally relies on testing the order of magnitude for unit cost violations (Ramazan and Dimitrakopoulos 2005; Benndorf and Dimitrakopoulos 2013; Montiel et al. 2015).
### Table 2.5: Binary decision variables.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_{jl}$</td>
<td>Stope selection decision variable, equal to 1 if stope $j$ in level $l$ is selected, and 0 otherwise</td>
</tr>
<tr>
<td>$z_{lk}$</td>
<td>Level selection decision variable, equal to 1 if level $l$ is selected under access option $k$, and 0 otherwise</td>
</tr>
<tr>
<td>$\omega_k$</td>
<td>Access option selection decision variable, equal to 1 if option $k$ is selected, and 0 otherwise</td>
</tr>
</tbody>
</table>

### Table 2.6: Fractional decision variables.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$VDC_k$</td>
<td>Vertical (shaft) development cost assuming access option $k \in K$</td>
</tr>
<tr>
<td>$HDC_{lk}$</td>
<td>Horizontal development cost (drifts) in level $l \in L$, for access option $k \in K$, along with mining directions $d \in D_{lk}$</td>
</tr>
<tr>
<td>$HDC'_{lk}$</td>
<td>Horizontal development cost (drifts) in level $l \in L$, for access option $k \in K$</td>
</tr>
<tr>
<td>$HD_C_{lk}$</td>
<td>Variable used in the relaxation of the product of variables $HDC'_{lk} \times \omega_k$ for $k \in K$</td>
</tr>
<tr>
<td>$d_s^{\text{transport}}$</td>
<td>Surplus deviation in total transportation capacity in scenario $s$</td>
</tr>
<tr>
<td>$d_s^{\text{transport}}$</td>
<td>Dollar value related to deviation $d_s^{\text{transport}}$</td>
</tr>
<tr>
<td>$d_s^{\text{develop}}$</td>
<td>Surplus deviation of total development capacity in scenario $s$</td>
</tr>
<tr>
<td>$d_s^{\text{develop}}$</td>
<td>Dollar value related to deviation $d_s^{\text{develop}}$</td>
</tr>
<tr>
<td>$d_s^{\text{process}}$</td>
<td>Surplus deviation of total processing capacity in scenario $s$</td>
</tr>
<tr>
<td>$d_s^{\text{process}}$</td>
<td>Dollar value related to deviation $d_s^{\text{process}}$</td>
</tr>
<tr>
<td>$d_{ls}^{\text{metal}}$</td>
<td>Reduction of recoverable metal in the stope design comparing the total metal content in a level $l$ in simulation $s$</td>
</tr>
<tr>
<td>$d_{ls}^{\text{metal}}$</td>
<td>Dollar value related to deviation $d_{ls}^{\text{metal}}$</td>
</tr>
</tbody>
</table>

#### 2.2.1. Stopes and Levels Preprocessing Steps

The proposed approach follows the steps presented in Fig. 2.2, which provide a set $L$ of possible levels, with related sets $\Omega_l$ and $\Lambda_l$ of overlapping and adjacent levels, and potential stopes within
each level $J_l$, with associated sets $\Omega_{jl}$ and $\Lambda_{jl}$ of overlapping and adjacent stopes. The steps will be presented in detail in the following subsections.

2.2.1.1. **Level Splitting**

First, the set of blocks $i \in I_n$ belonging to a defined geotechnical zone $n \in N$ is vertically split, from bottom to top, into a set of layers of blocks whose height (in terms of numbers of blocks along the vertical coordinate) corresponds to the set up level spacing $\alpha^z_n$ of zone $n$. A generated layer of blocks will be a potential production. Once the top of the block model is reached, the level splitting restarts by skipping some blocks in the bottom of the geotechnical zone based on the shift parameter $\beta^z_n$. In the end of this process a set of overlapping potential levels $l \in L_n$ is generated as depicted in Fig. 2.3. The total set potential levels is defined as the union of levels of all geotechnical zones, that is, $L = \bigcup_{n \in N} L_n$. The distances $\delta_{lk}^{shaft}$ (Fig. 2.4) from the surface to levels $l$ along each access option $k \in K$ are determined based on the coordinates of the surface $z^{surface}_k$ and the base of each level $z^{base}_l$. It is assumed that the generated levels are formed by blocks belonging to a unique geotechnical zone.

**Figure 2.2:** Steps of the proposed stochastic stope design optimization.
**Figure 2.3:** Representation of the level splitting step generating the set of levels $L_n$ of geotechnical domain $n$.

**Figure 2.4:** Parametrized distances (in red) considered by the method for an access option $k$, a potential level $l$, two mining directions $d$ and $d'$ and two potential stopes $j$ and $j'$. 
2.2.1.2. Level Overlapping Search

The generated levels overlap with some other levels since they share some layers of blocks. Furthermore, due to geotechnical requirements, a horizontal (sill/crown) pillar with a minimum height $\sigma_n^{sill,z}$ (in terms of the number of blocks in $z$ axis) should be ensured between two selected levels in the output design within each zone $n$. Therefore, the overlaps between levels are mapped based on their relative base and roof coordinates, $z_l^{\text{base}}$ and $z_l^{\text{roof}}$, respectively, and the parameter $\sigma_n^{sill,z}$ as presented in Fig. 2.5a. For instance, levels $l$ and $l'$, such that $l, l' \in L_n$, overlap with each other if $z_l^{\text{base}} \leq z_{l'}^{\text{base}}$ and $(z_l^{\text{roof}} + \sigma_n^{sill,z}) \leq z_{l'}^{\text{base}}$ or if $z_{l'}^{\text{base}} \leq z_l^{\text{base}}$ and $z_{l'}^{\text{roof}} \leq (z_l^{\text{base}} - \sigma_n^{sill,z})$. The level overlapping search generates a set of levels $l' \in \Omega_l$ that overlap with level $l \in L_n$.

![Figure 2.5: a) Representation of the level overlapping search, and b) the level adjacency search.](image)

2.2.1.3. Level Adjacency Search

Depending on the modeled surfaces' spatial configuration that individualizes geotechnical zones, two distinct zones $n$ and $n'$ might coexist for some $z$ coordinates. Therefore, some levels of their correspondent sets $L_n$ and $L_{n'}$ might be adjacent. The level adjacency search (Fig. 2.5b) follows the same conditions mentioned above of the level overlapping search step (Fig. 2.5a). In this instance, the adjacency occurs between levels in distinct zones, i.e., $l \in L_n$ and $l' \in L_{n'} \subset L \setminus L_n$, and generates the subset of levels $l' \in \Lambda_l$ that are adjacent to level $l$. 
2.2.1.4. Stope Search

A library of potential stope shapes is defined for each geotechnical domain \( n \in N \) (Fig. 2.6). Each shape \( m \in M_n \) consists of the number of blocks \( \gamma^x_{mn}, \gamma^y_{mn} \) and \( \gamma^z_{mn} \) along axes \( x, y \) and \( z \). A broad combination of stope shapes improves the output stope design since the optimization process has more flexibility to select potential stopes and may better control dilution. However, geotechnical considerations, due to required stope’s size and shape relationships for stable openings (Potvin 1988; Villaescusa 2014), may limit the possible combinations of stope shapes in terms of number of blocks per stope.

Given the dimensions \( \gamma^x_{mn}, \gamma^y_{mn} \) and \( \gamma^z_{mn} \), the stope shapes \( m \in M_n \) are floated within each of generated layers of mining blocks forming the potential levels \( l \in L_n \). This step produces the set of overlapping potential stopes \( J_l \) within a level \( l \in L_n \). It is assumed that all stopes must lie at the base of its level. Therefore, the stope search step looks for stopes with origin block \( o_j \) coinciding with the bottom of the respective potential level. Once a stope is defined, it is identified by its index \( j \in J_l \) and its starting and ending blocks, respectively, \( o_j \) and \( e_j \) (Fig. 2.7a). During the current step, the stope economic values \( v_{jls} \) for each simulation \( s \in S \) are computed. The stopes whose probability of negative economic value is higher than a threshold (Leite and Dimitrakopoulos 2014) can be eliminated. Finally, depending on the relative position to an access option \( k \), the stopes are flagged to a mining direction \( d \in D_l \), for example, the eastern and western sides of a shaft along the strike, forming the subsets of stopes \( J_{dlk} \). Furthermore, the approximated drift development distances \( \delta_{jdlk}^{drift} \) along the specified mining directions are computed, allowing assessment of the accessibility of a stope (Fig. 2.4).
Figure 2.6: Allowable stope shapes for two geotechnical zones.

Figure 2.7: a) Representation of starting and ending blocks of a stope, and b) the stope overlapping search step in plan-view.
2.2.1.5. Stope Overlapping Search

The proposed method assumes that the generated stope design must satisfy operational and geotechnical constraints in order to be readily used as an input for the subsequent optimization of long-term mine production scheduling (Trout 1995; Topal 2003; Sotoudeh et al. 2020) in commonly used sequential underground mine planning framework. Therefore, stopes non-overlapping constraints avoid the manual post-treatment of a stope boundary with overlapping stopes required by some methods (Alford 1995; Ataee-Pour 2004; Bai et al. 2013; Erdogan et al. 2017; Nikbin, Ataee-pour, Shahriar, and Pourrahimian 2018). Furthermore, due to stability considerations, longitudinal and rib pillars must be placed between stopes. The stope overlapping search is performed based on the minimum pillar sizes, in terms of number of blocks, \( \sigma_{n}^{rib,x} \) and \( \sigma_{n}^{rib,y} \) along axes \( x \) and \( y \) for each zone \( n \in N \). Henceforth, these pillars will be named indistinctly as rib pillars, although in the literature, rib pillars are usually transverse to the strike (Hamrin 2001; Villaescusa 2014). The stopes overlapping conditions are defined given the minimum pillar sizes and the coordinates of the starting \( \sigma_j \) and end \( \sigma_j \) blocks of the stopes. As shown in Fig. 2.7b, a stope \( j' \) and the reference stope \( j \) overlap with each other if \( x_{\sigma_j} \leq x_{\sigma_j} \) and \( (x_{\sigma_j} + \sigma_{n}^{rib,x}) \geq x_{\sigma_j} \), or if \( y_{\sigma_j} \leq y_{\sigma_j} \) and \( (y_{\sigma_j} + \sigma_{n}^{rib,y}) \geq y_{\sigma_j} \). The stope overlapping search, for each pair \( j, j' \in J_l \) and \( l \in L_n \), defines the set of stopes \( j' \in \Omega_{jl} \) that overlap with \( j \).

2.2.1.6. Stope Adjacency Search

Overlaps must also be avoided for stopes in different geotechnical domains if the current level \( l \) has a non-empty adjacency set, i.e., \( \Lambda_l \neq \emptyset \) (Fig. 2.5b). Therefore, a similar overlapping search (Fig. 2.7b) is performed to define the set of stopes \( j' \in \Lambda_{jl} \) that overlap with stope \( j \), such that \( j \in J_l, l \in L_n \) and \( j' \in J_{l'}, l' \in \Lambda_l \).
2.2.2. Mathematical Formulation

Three binary decision variables control the proposed stochastic stope design optimization framework. The access options decision variables $\omega_k \in \{0,1\}$ control whether an access option $k \in K$ is selected or not for stope design. Level selection decision variables $z_{lk} \in \{0,1\}$ control which production levels $l \in L$ are selected for the final design given the decision of access option. The stope selection decision variables $y_{jl} \in \{0,1\}$ define whether a stope $j \in J_l$ in level $l \in L$ is to be extracted. The bold characters $y$ and $z$ refer to the coordinates of blocks, stopes, and levels, whereas the italic characters $y$ and $z$ are used for the decision variables present in the proposed stochastic mathematical programming formulation.

Two types of development costs are integrated into optimization. The vertical development cost $VDC_k$ stands for the excavation and commissioning costs associated with shaft option $k \in K$, while the horizontal development cost $HDC_{dlk}$ relates to the excavation and commissioning of the haulage and drilling drifts to access the selected stopes in level $l \in L$ through mining direction $d \in D_l$. The total $HDC_{lk}$ present in the objective function accounts for the contribution of the development costs overall directions $d \in D_k$.

2.2.2.1. Objective Function

The proposed method aims to maximize the selected stopes’ potential revenue while integrating other components in the objective function of Eq. 2.1. Part I of the objective function represents the maximization of the economic value of the selected stopes. Part II accounts for the minimization of the overall vertical development cost and the sum of the horizontal development costs of all possible levels given the access options $k \in K$. Part III penalizes the resulting economic impact of exceeding the project capacities of transportation, the development of drifts, and processing. These overall capacities are defined based on each component’s yearly capacity and an assumed life of mine horizon for the project at hand. Part IV tries to extract the upside potential of the design given the grade uncertainty by penalizing the economic difference $d_{ls}^{\text{smetal}}$ related to the metal content of all blocks in a level $i \in I_l$ and the recoverable metal within the selected stopes in the design, that is, $j \in J_l$, for $l \in L$. 
\[
\begin{align*}
\text{max} & \quad \frac{1}{|S|} \sum_{s \in S} \sum_{l \in L} \sum_{j \in J} v_{ijs} y_{jl} \\
\text{Part I: Revenue from each stope} \\
& - \sum_{k \in K} (VDC_k + \sum_{l \in L} HDC_{lk}) \\
\text{Part II: Vertical and horizontal development costs} \\
& - \frac{1}{|S|} \sum_{s \in S} (c^{\text{transport}} d_{s}^{\text{transport}} + c^{\text{develop}} d_{s}^{\text{develop}} + c^{\text{process}} d_{s}^{\text{process}}) \\
\text{Part III: Penalties applied to the entire stope design} \\
& - \frac{1}{|S|} \sum_{s \in S} \sum_{l \in L} c^{\text{metal}} d_{ls}^{\text{metal}} \\
\text{Part IV: Penalties applied to each level} \\
\end{align*}
\]

The penalties costs \(c^{\text{transport}}, c^{\text{develop}}, c^{\text{process}}\) and \(c^{\text{metal}}\) are set by a user-based empirical approach that generally relies on testing the order of magnitude for unit cost violations (Ramazan and Dimitrakopoulos 2005, 2013; Benndorf and Dimitrakopoulos 2013). Acknowledging that an underground mining project is constrained by the overall capacities addressed in Part III and the miner planner’s preemption to favor the upside potential of the design in Part IV, the objective function integrates this trade-off into the optimization process to provide an optimal and uncertainty-based stope design.

The proposed method offers the possibility to use or not a cut-off grade. For a specified cut-off grade, the economic value \(v_{ijs}\) of a block \(i \in I_l\) in level \(l \in L\) in simulation \(s \in S\) is defined by Eq. 2.2. It is important to highlight that the proposed method does not require an upfront cut-off grade since waste blocks are penalized with negative economic value for a given scenario. If a cut-off grade is not defined, Eq. 2.2 would have only its upper part. Since the mining blocks can belong to different potential levels \((i \in I_l)\), a single block has multiple economic values \(v_{ijs}\) for multiple simulations and levels. In addition, level-based mining costs \(c_{l}^{\text{mining}}\) accounting for extraction, backfilling, and material handling might reflect increasing costs with depth.
\[
\begin{align*}
\nu_{ils} &= \begin{cases} 
  w_{ils} \left( g_{ils} R P - (C^\text{process} + C_t^\text{mining}) \right), & g_{ils} \geq \text{cut-off} \\
  -w_{ils} C_t^\text{mining}, & \text{otherwise}
\end{cases} \\
\end{align*}
\]

The economic value of a stope \( v_{jls} \), in turn, is determined by the sum of the economic values \( v_{ils} \) of all its blocks \( (i \in I_{jl}) \). Note that Eq. 2.3 assigns higher costs for larger stopes. Considering a family of potential stopes that share a common high-grade block, the larger stopes centered in this block are likely to have more waste blocks, resulting in a reduced value \( v_{jls} \). Hence, the optimizer would opt for a smaller stope containing this high-grade block. Therefore, even though the current decision variables \( y_{jl} \) are in stope support scale, the family of potential stopes that share this high-grade block carries the information on block basis, and consequently the optimization process is able to select the better stope locations and sizes in order to control the dilution.

\[
v_{jls} = \sum_{i \in I_{jl}} v_{ils} \tag{2.3}
\]

### 2.2.2.2. Constraints

This section presents the constraints related to geotechnical requirements, the specified capacities, development costs, and link between different variables.

\[
\sum_{k \in K} \omega_k = 1 \tag{2.4}
\]

\[
z_{lk} \leq \omega_k , \quad \forall \ k \in K, l \in L \tag{2.5}
\]

\[
y_{jl} \leq \sum_{k \in K} z_{lk} , \quad \forall \ l \in L, j \in J_l \tag{2.6}
\]

\[
z_{lk} + z_{l'k} \leq 1 , \quad \forall \ l' \in \Omega_l \subset L_n, l \in L_n \tag{2.7}
\]

\[
y_{jl} + y_{j'l} \leq 1 , \quad \forall \ j' \in \Omega_{jl}, j \in J_l, l \in L \tag{2.8}
\]
Equation 2.4 ensures that only one access option is selected. The linking constraints of Eq. 2.5 state that a level \( l \) can be opened with access option \( k \), only if this access option is selected, while Eq. 2.6 guarantees that a stope \( j \) can be selected if its level \( l \) is selected, and vice-versa. Two overlapping levels, including sill/crown pillar requirements, in a geotechnical zone \( n \in N \) cannot be simultaneously selected in the design (Eq. 2.7) as well as two overlapping stopes, including rib pillar requirements, within the same level and geotechnical zone \( l \in L_n \) (Eq. 2.8). The overlaps must also be avoided between the stopes in the contact between two adjacent levels in distinct geotechnical zones. Such type of overlaps is avoided Eq. 2.9.

\[
\begin{align*}
    &\gamma_{jl} + \gamma_{j'l'} \leq 1 , \quad \forall j' \in A_{jl} , l' \in L \setminus L_n , j \in J_l , l \in L_n \\
\end{align*}
\]  

\( (2.9) \)

The integration of the development costs, a substantial proportion of a stope global cost, aims to address the interdependencies between the stope layout and the development network. The proposed approach has linear constraints that determine overall horizontal development cost per level and vertical development cost for the entire design. The vertical development cost for each access option \( k \in K \) is determined by Eq. 2.10. The distance from the surface \( \delta_{lk}^{shaft} \) and the

\[
\begin{align*}
    &VDC_k \geq \left( \delta_{lk}^{shaft} C_k^{shaft} \right) z_{lk} , \quad \forall l \in L , k \in K \\
\end{align*}
\]  

\( (2.10) \)

\[
\begin{align*}
    &HDC_{dlk}' \geq \left( \delta_{jdlk}^{drift} C_i^{drift} \right) y_{jl} , \quad \forall j \in J_{dlk} , d \in D_l , l \in L , k \in K \\
\end{align*}
\]  

\( (2.11) \)

\[
\begin{align*}
    &HDC_{lk}' \geq \sum_{d \in D_l} HDC_{dlk}' , \quad \forall l \in L , k \in K \\
\end{align*}
\]  

\( (2.12) \)

\[
\begin{align*}
    &HDC_{lk} \leq \mathcal{M} \omega_k , \quad \forall l \in L , k \in K \\
\end{align*}
\]  

\( (2.13) \)

\[
\begin{align*}
    &HDC_{lk} \leq HDC_{lk}' , \quad \forall l \in L , k \in K \\
\end{align*}
\]  

\( (2.14) \)

\[
\begin{align*}
    &HDC_{lk} \geq \mathcal{M} (\omega_k - 1) + HDC_{lk}' , \quad \forall l \in L , k \in K \\
\end{align*}
\]  

\( (2.15) \)

The integration of the development costs, a substantial proportion of a stope global cost, aims to address the interdependencies between the stope layout and the development network. The proposed approach has linear constraints that determine overall horizontal development cost per level and vertical development cost for the entire design. The vertical development cost for each access option \( k \in K \) is determined by Eq. 2.10. The distance from the surface \( \delta_{lk}^{shaft} \) and the
respective unit cost $C_k^{shaft}$ parametrize this cost. Whenever the activation of a variable $z_{lk}$ opens a more profound level, the $VD C_k$ is incremented, entailing in a higher overall cost for the stope design.

Similarly, in Eq. 2.11, the horizontal development cost for each potential level $l \in L$ and access option $k \in K$ along each mining direction $d \in D_l$ depends on the distance from a stope to an access option $\delta_{jlk}^{drift}$ and the unit drift development cost $C_l^{drift}$. The selection of a stope further from the access option in a direction $d$ will raise the variable $HDC_{dlk}$. Therefore, this stope will be selected if it is sufficiently valuable to pay for this additional drift development, avoiding unreasonable costs. The overall drift development cost in a level $HDC_{lk}$ corresponds to the sum over all mining directions (Eq. 2.12). The optimization process balances the revenue from the selected stopes and the drifts development cost at a potential level. Equations 2.12-2.15 link the variables $HDC_{lk}$ with the access option decisions $\omega_k$ and are used to linearize the formulation by avoiding the product $HDC_{lk}^t \omega_k$, which is replaced by a new variable $HDC_{lk}$. In Eqs. 2.13 and 2.15, $\mathcal{M} > 0$ is a sufficiently large constant that activates the constraint in the case $\omega_k = 1$ by imposing a loose upper bound on the horizontal development cost variable $HDC_{lk}$. Indeed, this last variable is minimized in the objective function (Eq. 2.1 – Part II).

$$\sum_{l \in L} \sum_{j \in J_l} (y_{jl} \omega_{jls}) - d_s^{transport} \leq \sum_{k \in K} U_k^{transport} \omega_k, \quad \forall s \in S$$ (2.16)

$$\sum_{l \in L} \sum_{k \in K} (HDC_{lk}) - d^{develop} \leq U^{develop}$$ (2.17)

$$\sum_{l \in L} \sum_{j \in J_l} (y_{jl} \omega_{jls}) - d_s^{process} \leq U^{process}, \quad \forall s \in S$$ (2.18)

Some existing or planned capacities might constrain the underground mining project. Therefore, some overall capacities defined by yearly capacities and the projected life-of-mine are considered in these constraints. The surplus deviations $d_s^{transport}$, $d^{develop}$ and $d_s^{process}$ from the specified
capacities are defined in Eqs. 2.16-2.18. Equation 2.16 defines the overall transportation capacity in tons $U_k^{transport}$ of the selected access option $k \in K$. The development advancement of drifts might also be constrained, and its overall capacity $U^{develop}$ in meters is ensured in Eq. 2.17, where the ratio $\text{HDC}_{lk}/\text{DC}$ reflects the maximum distance to be developed per level assuming access option $k \in K$. The overall processing capacity $U^{process}$ must be satisfied considering the sum of the contained ore tonnage $o_{jls}$ in simulation $s \in S$ (Eq. 2.18).

$$\sum_{l \in I_l} (w_l \theta_{l|ls} g_{l|ls}) - \sum_{l \in J_l} (o_{jls} g_{j|ls} y_{j|l}) - d_{ls}^{metal} \leq 0, \; \forall \; l \in L, s \in S \quad (2.19)$$

Aiming to capture the upside potential of recoverable metal in the stope design given the geological uncertainty, Eq. 2.19 penalizes the reduction in metal content of the stope design in level $l \in L$ compared to the total metal content in that level. Levels that have more potential for profitable stopes are less penalized, as well as the levels that have minor variations in metal content, that is $d_{ls}^{metal}$, among all simulations. Part IV of the objective function acts by maximizing the metal content of the stope design under uncertainty and translates the mine planner’s tendency to foresee the potential stoping areas translated by the magnitude of the penalty $c^{metal}$.

$$d_s^{transport} = C_{i}^{\text{mining}} d_s^{transport}, \quad \forall \; s \in S \quad (2.20)$$

$$d_s^{develop} = C_{i}^{\text{drifts}} d_s^{develop} \quad (2.21)$$

$$d_s^{process} = C_{i}^{\text{process}} d_s^{process}, \quad \forall \; s \in S \quad (2.22)$$

$$d_{ls}^{metal} = P d^{metal}, \quad \forall \; l \in L, s \in S \quad (2.23)$$

The previously defined deviations have a different order of magnitude. For instance, $d_s^{transport}$ and $d_s^{process}$ are defined in terms of tonnage of material mined and processed, $d^{develop}$ in meters of development and $d_{ls}^{metal}$ in metal tonnage. Therefore, in Eqs. 2.20-2.23, the initial deviations
are multiplied by pertinent unit costs, or the metal price, converting them into dollar value deviations, with superscript "$\$". These equations provide a balance between the different components of the objective function that drives the optimization process. Finally, the integrality and non-negativity constraints from Eq. 2.24 to Eq. 2.32 complete the SIP formulation and are as follows:

\[ y_{jl} \in \{0,1\}, \quad \forall j \in J_l, l \in L \]  
\[ (2.24) \]

\[ z_{lk} \in \{0,1\}, \quad \forall l \in L, k \in K \]  
\[ (2.25) \]

\[ \omega_k \in \{0,1\}, \quad \forall k \in K \]  
\[ (2.26) \]

\[ VDC_k \geq 0, \quad \forall k \in K \]  
\[ (2.27) \]

\[ HDC_{lk}, HDC'_{lk} \geq 0, \quad \forall l \in L, k \in K \]  
\[ (2.28) \]

\[ HDC'_{dtk} \geq 0, \quad \forall d \in D_t, l \in L, k \in K \]  
\[ (2.29) \]

\[ d_s^{transport}, d_s^{transport}, d_s^{process}, d_s^{process} \geq 0, \quad \forall s \in S \]  
\[ (2.30) \]

\[ d^{develop}, d^{develop} \geq 0 \]  
\[ (2.31) \]

\[ d_{ls}^{metal}, d_{ls}^{metal} \geq 0, \quad \forall l \in L, s \in S \]  
\[ (2.32) \]
2.3. Case Study – Application at an Underground Gold Mine

In this section, the proposed stochastic stope design optimization, under grade uncertainty and considering development costs, is applied to an underground gold deposit employing the sublevel open stoping mining method. Uncertainty of gold grade is accounted for through a set of 25 simulations generated by sequential gaussian simulation (SGS) method (Goovaerts 1997; Remy et al. 2009) for 107,520 mining blocks of size 10m x 10m x 10m (Fig. 2.8a). The set of simulations was generated and properly validated against the exploration data by the mining company. The dataset is located from 400m to 830m below the processing plant level. Two geotechnical zones (Fig. 2.8b) are delimited by an irregular and inclined separation surface (which provides a more general case than simple distinct horizontal geotechnical zones). The blocks lying above and below this separation surface are flagged accordingly. The cross-sections of Fig. 2.8 emphasize a high-grade region in the top-left portion of the deposit, located in the upper geotechnical zone, and another high-grade region from the center to the right, in the lower geotechnical zone. Table 2.7 summarizes the information about the input underground gold orebody.

Each geotechnical zone has a specified level spacing, horizontal (sill/crown), rib, and longitudinal pillar sizes, as well as a set of stoping shapes given the conditions of the underlying rock masses (Table 2.8). The proposed method is used to select the best primary access amongst three potential shafts. The shafts’ headframes are at the same distance from the orebody’s footwall, which is the rock mass beneath a steeply dipping deposit (Hamrin 2001). However, Shaft 1 is located at the center of the deposit’s strike (axis y), whereas the other two options are -200m and +200m away from Shaft 1 (Fig. 2.9b). The shafts have the same hoisting capacity, although different capacities and more locations could be used. In each level, the main drifts are developed along the strike. Accordingly, two main mining directions \( d \in D_l \) are considered, that is, the northern and southern sides of each shaft option, so as to define the subsets of potential stopes (Fig. 2.4). The overall hoisting, processing and development rate capacities are presented in Table 2.9 and are obtained by multiplying yearly capacities by the expected number of years of the life-of-mine of the operation. Other economic and technical parameters used in the stochastic stope design optimization are also presented in this table.
Given the specified level spacing of 40m and 60m, respectively, for the upper and lower geotechnical zones, 51 overlapping levels are generated within the two geotechnical zones. The upper and lower zones have, respectively, 12 and 15 set up allowable stoping shapes. During the stope search step, no cut-off grade was used, which means that only the upper part of Eq. 2.2 is used to determine the economic value of blocks. A probability threshold of 50% is used to exclude negatively valued stopes generating 31,454 potential stopes (with related overlapping information) within the 51 levels during the preprocessing steps of Fig. 2.2. The proposed SIP model is solved using the CPLEX v.12.8.0 software’s solver engine (IBM ILOG 2017) implemented with C++ language. This instance has 31,535 binary decision variables and 97,107 constraints. Using a standard personal computer with six cores and 32 GB RAM, the preprocessing and optimization steps take less than 4 hours to be solved with less than a 1% relative optimality gap (IBM ILOG 2017).

Table 2.7: Input orebody information.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value/Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metal</td>
<td>gold</td>
</tr>
<tr>
<td>Grade unit</td>
<td>g/t</td>
</tr>
<tr>
<td>Total number of realizations</td>
<td>50</td>
</tr>
<tr>
<td>Number of realizations for optimization</td>
<td>25</td>
</tr>
<tr>
<td>Number of realizations for risk analysis</td>
<td>25</td>
</tr>
<tr>
<td>Depth from processing plant level (surface)</td>
<td>400m – 830m</td>
</tr>
<tr>
<td>Dimension along the orebody strike</td>
<td>800m</td>
</tr>
<tr>
<td>Dimension across the orebody strike</td>
<td>320m</td>
</tr>
<tr>
<td>Orientation of strike</td>
<td>Along axis y</td>
</tr>
<tr>
<td>Average orebody dip</td>
<td>70°</td>
</tr>
<tr>
<td>Number of blocks</td>
<td>107,520</td>
</tr>
<tr>
<td>Block size</td>
<td>10m x 10m x 10m</td>
</tr>
</tbody>
</table>
Figure 2.8: a) One simulated realization of gold grades of the input underground deposit with a block size of 10 m x 10m x 10m, and b) the two input geotechnical zones.

<table>
<thead>
<tr>
<th>Zone</th>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper zone</td>
<td>Number of allowable stope shapes</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>Level spacing</td>
<td>40 m</td>
</tr>
<tr>
<td></td>
<td>Stope heights</td>
<td>20 - 40 m</td>
</tr>
<tr>
<td></td>
<td>Stope widths</td>
<td>20 - 30 m</td>
</tr>
<tr>
<td></td>
<td>Stope lengths</td>
<td>30 - 50 m</td>
</tr>
<tr>
<td></td>
<td>Sill/crown pillar size</td>
<td>10 m</td>
</tr>
<tr>
<td></td>
<td>Rib pillar size</td>
<td>10 m</td>
</tr>
<tr>
<td></td>
<td>Longitudinal pillar size</td>
<td>10 m</td>
</tr>
<tr>
<td>Lower zone</td>
<td>Number of allowable stope shapes</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>Level spacing</td>
<td>60 m</td>
</tr>
<tr>
<td></td>
<td>Stope heights</td>
<td>40 - 60 m</td>
</tr>
<tr>
<td></td>
<td>Stope widths</td>
<td>10 - 20 m</td>
</tr>
<tr>
<td></td>
<td>Stope lengths</td>
<td>30 - 50 m</td>
</tr>
<tr>
<td></td>
<td>Sill/crown pillar size</td>
<td>10 m</td>
</tr>
<tr>
<td></td>
<td>Rib pillar size</td>
<td>10 m</td>
</tr>
<tr>
<td></td>
<td>Longitudinal pillar size</td>
<td>20 m</td>
</tr>
</tbody>
</table>
Table 2.9: Economic and technical parameters used in the proposed stochastic stope design optimization of the underground gold mine.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metal price ($/ozt)</td>
<td>1,200</td>
</tr>
<tr>
<td>Processing recovery (%)</td>
<td>94%</td>
</tr>
<tr>
<td>Mining cost ($/t)</td>
<td>118</td>
</tr>
<tr>
<td>Processing cost ($/t)</td>
<td>20</td>
</tr>
<tr>
<td>Shaft development cost ($/m) for all ( k \in K )</td>
<td>20,000</td>
</tr>
<tr>
<td>Drifts development cost ($/m)</td>
<td>7,000</td>
</tr>
<tr>
<td>Density (t/m(^3))</td>
<td>2.9</td>
</tr>
<tr>
<td>Block tonnage (t)</td>
<td>2,900</td>
</tr>
<tr>
<td>Overall mining (transportation) capacity (Mt)</td>
<td>4.3</td>
</tr>
<tr>
<td>Overall processing capacity (Mt)</td>
<td>4.3</td>
</tr>
<tr>
<td>Overall drift development capacity (m)</td>
<td>3,000</td>
</tr>
<tr>
<td>Number of shaft options</td>
<td>3</td>
</tr>
<tr>
<td>Coordinates of shaft 1 (x, y, z)</td>
<td>123575, 258020, 1200</td>
</tr>
<tr>
<td>Coordinates of shaft 2 (x, y, z)</td>
<td>123575, 258220, 1200</td>
</tr>
<tr>
<td>Coordinates of shaft 3 (x, y, z)</td>
<td>123575, 257820, 1200</td>
</tr>
<tr>
<td>Penalty cost for transportation capacity</td>
<td>1</td>
</tr>
<tr>
<td>Penalty cost for development capacity</td>
<td>10</td>
</tr>
<tr>
<td>Penalty cost for processing capacity</td>
<td>10</td>
</tr>
<tr>
<td>Penalty cost for metal component</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Figure 2.9a presents the stochastic stope layout obtained by the application of the proposed method at the underground gold mine. The positions of the stope layout and the potential shaft locations with respect to the surface level and the input orebody models are shown in Fig. 2.9b. The layout has six selected production levels satisfying the spacings of 40m and 60m for each geotechnical zone, and sill/crown pillar height of at least 10m. The bottom portion of the lower geotechnical zone (red block edges of Fig. 2.9b) is also mineralized and has positively valued stopes. Nevertheless, these stopes are not profitable so as to compensate for the related incremental shaft and drifts’ development cost. Consequently, no stope is selected in this bottom area.

The 46 stopes in the final layout satisfy the allowable minimum and maximum stope sizes and the minimum transversal and rib pillar sizes of each geotechnical zone, as presented in Table 2.8. The stoping shapes have variable heights and are aligned with the respective production levels’ bases. These shapes will be further connected by haulage and drilling drifts and loading crosscuts once the development network is designed. The optimization process selects the most centralized shaft
option (Fig. 2.9b). The profitable stopes of the upper geotechnical zone (blue) are more concentrated towards the negative y-axis direction, and the lower geotechnical zone (red), in turn, has more stopes towards the positive y-axis direction (Fig. 2.9a), which are related to the high-grade lobes circled in Fig. 2.8a. Therefore, the selected shaft balances the associated drifts’ development costs. As for the bottom part of the deposit, eventual stoping areas at the periphery of each level are not selected to minimize the final design's drifts development costs. Finally, shaft depth is defined by the deepest selected level, located at 745 m below the surface (Fig. 2.9b).

Figure 2.9: Stochastic stope design: (a) stopes’ gold grade, and (b) the locations of the surface level, the stope layout (front view), the selected shaft, and the geotechnical zones.

Table 2.10 presents the risk analysis of the generated stope layout, which is defined by the non-exceedance probabilities of 10%, 50%, and 90% (P10, P50, P90, respectively) of key performance indicators (KPIs) (Ravenscroft 1992), such as undiscounted profit, recoverable metal, average grade, and total tonnage. A set containing 25 realizations of block’s gold grade, different from those used for the stope design optimization step, is used to generate the risk profiles. The economic potential of the output layout is 217.3 M$, after the deduction of about 30 M$ for the associated shaft and drifts development costs, with a total ore tonnage of 4.6 Mt, 24.1 tons of recoverable metal at an average gold grade of 5.3 g/t, considering the 50th percentile (P50) of the risk profiles.
The penalty costs used in the objective function (Eq. 2.1) are calibrated by testing different orders of magnitude. The magnitude of the penalty cost associated with the metal reduction component in the objective function expresses the mine planner's tendency to capitalize on the design's upside potential in terms of recovered metal. By increasing the magnitude of such penalty ($c_{metal}$), more stopes and levels are selected, resulting in physically different designs and related forecasts. Therefore, a mine planner would test different orders of magnitude for all penalty costs and select a final layout based on pertinent mining aspects, on the generated risk profiles and on performance indicators such as undiscounted profit and/or recoverable metal. For the current case study, the scenario having the highest undiscounted profit and metal was selected.

### 2.3.1. Comparison with a Conventional Stope Design Approach

The proposed stochastic stope design optimization method is compared to the Mineable Shape Optimizer (MSO), an industry-standard automated stope design tool (Alford Mining Systems 2016), and its so-called Slice Method is chosen since it is broadly used in the mining industry for sublevel stoping design. The MSO implements a deterministic approach, requiring an estimated orebody model as an input, which is a smooth representation of the related orebody. An E-type or average model (Goovaerts 1997; Albor Consuegra and Dimitrakopoulos 2009, 2010; Remy et al. 2009) is used as the estimated model by averaging the set of 50 block-support gold grade simulated realizations of the deposit.

To attain an equitable comparison between the two approaches, a simplified version of the proposed stochastic stope design optimization method is used by removing some features of the original approach that the MSO does not consider. First, since MSO defines the stope layout individually for each geotechnical zone based on the fixed levels, sill pillars, and rib pillars, a
single geotechnical zone library is defined for the entire deposit for both methods. The original stochastic method would simultaneously select the best set of levels for multiple zones, rather than optimizing each zone specifically. Second, the variable stopes’ heights and the stopes’ lengths along the strike of the stochastic optimization method were removed. Instead, a fixed stope length (along y) of 40 m and a fixed stope height equal to the level spacing of 60 m are defined to coincide, respectively, with the spacings used by MSO. Capacity constraints and development costs are not considered for the stochastic approach since those components are not incorporated by MSO.

An MSO slice framework oriented along the orebody’s strike direction (axis y) is selected, defining a regular grid of stopes’ height and length, and the required user-defined sill and transverse pillars’ thicknesses and locations. The objective of maximizing the total layout’s economic value is defined using the same economic parameters, such as metal price, recovery, mining, and processing costs, shown in Table 2.9. The MSO software tool provides sophisticated stope shape parameters to fit in the orebody’s footwall/hang-wall strike and dip. Nonetheless, only cuboid shapes are allowed for a comparison with the proposed method. Orebody control wireframes are not used to provide a pure block value-based stope layout since such contours are also uncertain and are based on subjective geologic interpretations (Bárdossy and Fodor 2001; Osterholt and Dimitrakopoulos 2018). The parameters used on MSO are presented in Table 2.11.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value/Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimization method</td>
<td>Slice Method</td>
</tr>
<tr>
<td>Optimization objective</td>
<td>Maximize total value</td>
</tr>
<tr>
<td>Stope shape framework</td>
<td>YZ vertical</td>
</tr>
<tr>
<td>Level spacing (m)</td>
<td>60 m (along axis z)</td>
</tr>
<tr>
<td>Section spacing (m)</td>
<td>40 m (along axis y)</td>
</tr>
<tr>
<td>Slice width (m)</td>
<td>10 m (along axis x)</td>
</tr>
<tr>
<td>Minimum stope width (m)</td>
<td>10 m (along axis x)</td>
</tr>
<tr>
<td>Maximum stope width (m)</td>
<td>30 m (along axis x)</td>
</tr>
<tr>
<td>Sill pillar size (m)</td>
<td>10 m (along axis z)</td>
</tr>
<tr>
<td>Transversal pillar size (m)</td>
<td>10 m (along axis y)</td>
</tr>
<tr>
<td>Longitudinal pillar size (m)</td>
<td>20 m (along axis x)</td>
</tr>
<tr>
<td>Fixed stope dip</td>
<td>90°</td>
</tr>
<tr>
<td>Fixed stope strike</td>
<td>0° (aligned to axis y)</td>
</tr>
<tr>
<td>Number of vertical sub-shapes</td>
<td>3 (sublevels of 20m high)</td>
</tr>
</tbody>
</table>
Figure 2.10 depicts the comparison between the two generated stope layouts and stresses the stochastic optimization approach's essential strengths. Since the smooth input estimated model of MSO misrepresents the spatial connectivity and variability of high grades, a more contiguous stope layout is generated (Fig. 2.10b), as compared to the proposed stochastic approach (Fig. 2.10a). MSO is unable to define stoping areas in the border of some levels. The proposed stochastic approach also has the flexibility to allocate profitable levels and rib pillars, which are required inputs for MSO.

The forecasts of metal tonnage, undiscounted profit, gold grade, and ore tonnage (assuming that all material inside the stopes is ore) of the stope layouts generated by the proposed stochastic method and MSO are presented in Fig. 2.11, where gray dots represent the resultant risk profiles generated by a set of 25 simulations, and the red diamonds represent related 10th, 50th, and 90th percentiles. The black squares represent forecasts generated by MSO. The proposed stochastic method outperforms the MSO (compared to the prompt MSO report), both in terms of recoverable metal and undiscounted profit by 21% and 40%, respectively, highlighting the inherent limits of deterministic stope designs in determining profitable stoping locations and sizes. Moreover, other features of the original stochastic stope design method, such as variable stope length and height, and the integration of development costs and project capacities, are not accounted for by MSO and were not included in the current comparison.

**Figure 2.10:** Comparison of stope designs: (a) simplified stochastic approach and (b) slice method of MSO. The unfilled outlines of (b) correspond to the stochastic stope design of (a).
Figure 2.11: Forecasts stope layouts generated by the proposed stochastic optimization approach (risk analysis) and by the Slice Method of MSO (report generated by MSO): (a) Metal tonnage, (b) undiscounted profit, (c) average gold grade, and (d) ore tonnage.

2.4. Conclusions

A two-stage stochastic integer programming (SIP) model for the stope design optimization of underground mines employing the sublevel open stoping mining method was proposed. Multiple equiprobable simulated orebody models are used to quantify the grade uncertainty and variability, to provide a risk-resilient stope layout that capitalizes on the mineral deposit’s upside potential. The proposed mathematical formulation seeks to maximize the undiscounted profit from the selected levels and stopes, while minimizing the associated development costs of the shaft and drifts, as well as the economic impacts of exceeding the project’s capacities while considering different geomechanics zones. Unlike the conventional stope layout approaches, the proposed approach accounts for uncertainty and variability of grades in the mineral deposit, stopes’ accessibility/remoteness, and the capacities that affect stoping sizes, locations, and profitability. A set of possible primary shaft locations, an assortment of potential production levels, stoping sizes
and positions, as well as associated distances levels-to-surface and stopes-to-access options are the required inputs for the model. The output of the proposed SIP is a mineable stochastic stope design comprising an optimal combination of horizontal production levels separated by required sill/crown pillar heights, unified stopes with variable height, length, and width satisfying rib and longitudinal pillars requirements, and the best shaft location, which minimizes the layout’s vertical and horizontal development costs.

The practical aspects of the SIP model were shown in a case study at an underground gold mine. A set of geostatistical simulations of gold grades, three possible shaft headframe locations, two irregularly separated geotechnical zones with defined multiple allowable stoping shapes, level heights, and pillar sizes, and the overall hoisting, processing, and development rate capacities were the integrated components of the stope layout optimization process. A comparison of the proposed method with the Mineable Shape Optimizer (MSO), a deterministic industry-standard automated stope design tool, was performed. The results highlighted two advantages of the proposed approach. First, the incorporation of grade uncertainty into the optimization process allows one to define some stoping volumes that are not identified by deterministic methods that rely on an estimated orebody model, which misrepresents the connectivity of high grades within the deposit. A second advantage is the proposed method’s flexibility to define optimal production levels and transverse pillars, which are inputs for MSO. The proposed method produced a layout with significantly higher recoverable metal and undiscounted profits (21% and 40%, respectively). It is worth underlining that, in this comparison, some advanced components of the proposed method, such as incorporating the development costs, variable stopes’ lengths along the orebody’s strike and heights, and economic impacts of exceeding the project’s capacities, were not considered since the conventional stope design approach does not incorporate these aspects.

Further extensions of the current method might implement different metaheuristic solvers to make larger instances tractable and incorporate non-linear components into the optimization process. The integration of development costs opens new avenues for simultaneous stochastic optimization of the stope design and the mine production scheduling into a single model. Such an integrated model might be compounded in the optimization of mining complexes accounting for multiple mines, processing destinations, and marketable products in later developments.
Chapter 3 - Integrated Stochastic Optimization of Stope Design and Long-Term Underground Mine Production Scheduling

3.1. Introduction

Underground mine planning is traditionally performed through a sequential process (Alford et al. 2007; Musingwini 2016; Nhleko et al. 2018; Kumral and Sari 2019). A stope layout is first conceived, often based on an initial cut-off grade, to delineate the stoping areas given the geometric, geotechnical, and operational constraints of the selected mining method in order to maximize undiscounted profits (Hamrin 2001; Hartman and Mutmansky 2002; Erdogan et al. 2017; Nhleko et al. 2018). Subsequently, a detailed development network layout of shafts, drifts, crosscuts, and declines is designed based on maneuverability and safety considerations so as to minimize undiscounted development and haulage costs (Brazil et al., 2003, 2008; Brazil and Thomas 2007). The two generated designs allow the further optimization of the life-of-mine (LOM) production schedule, which defines the sequence of production and development activities, seeking to maximize the net present value (NPV) subject to complex activities precedence structure, market, and capacities constraints (Trout 1995; Topal 1998, 2003; Sotoudeh et al. 2020).

Nevertheless, this stepwise process with misaligned objectives precludes the profitability of the output design and schedule, and cannot capture the intrinsic interdependencies of the related steps. Ideally, the long-term production schedule should yield the stope boundary, not the other way around, since pre-optimized mine designs do not account for the time value of revenues and costs (Kim 1967; Johnson 1968; Gershon 1983; Little et al. 2011). In addition, conventional underground mining plans are based on a single estimated orebody model and, hence, do not integrate geological uncertainty and variability into the planning process (Vallée 2000; Dimitrakopoulos et al. 2002; Dimitrakopoulos 2011; Rendu 2017), which adversely affects on underground mine designs, schedules, and related forecasts (Myers et al. 2007; Tavchandjian et al. 2007; Dimitrakopoulos and Grieco 2009; Jewbali et al. 2015).

Due to the underlying complexity and computational limitations, progress has been made separately for the stope design and production scheduling steps. Various deterministic stope design methods have been proposed, adding progressively more stope shapes, as well as geotechnical and
operational constraints (Alford 1995; Ataee-Pour 2004; Alford and Hall 2009; Topal and Sens 2010; Bai et al. 2013; Sandanayake et al. 2015b; Alford Mining Systems 2016; Sari and Kumral 2020a). Fewer stochastic approaches (Grieco and Dimitrakopoulos 2007; Villalba Matamoros and Kumral 2018; Faria et al. 2021) have attempted to integrate the geological uncertainty and variability into the stope design optimization process. The mentioned stope design methods provide the main input for the underground mine production scheduling optimization step. Available scheduling optimization methods are also deterministic (Trout 1995; Topal 1998, 2003; Carlyle and Eaves 2001; Smith et al. 2003; Kuchta et al. 2004; Sarin and West-Hansen 2005; Nehring and Topal 2007; Little and Topal 2011; Fava et al. 2013; O'Sullivan and Newman 2014; Brickey 2015; Zhang et al. 2017; King et al. 2017) or stochastic approaches (Carpentier et al. 2016; Dirkx et al. 2018; Sepúlveda et al. 2018; Huang et al. 2020; Nesbitt et al. 2021). The high-resolution schedules and related forecasts generated by these methods are, however, constrained by previously generated mine designs for the underground mining method(s) considered.

The sublevel stoping mining method is characterized by a set of horizontal production levels containing verticalized extraction volumes called stopes, which are accessed by multiple drifts defining the sublevels (Hamrin 2001; Hustrulid and Bullock 2001; Hartman and Mutmansky 2002; Pakalnis and Hughes 2011; Villaescusa 2014). Some integrated optimization methods of sublevel open stoping design and LOM production schedules have been proposed to overcome the limitations of the commonly used sequential underground mine planning framework in order to maximize the NPV. A critical question for such methods is how to reasonably account for the costs and timing of developments since the stope layout is still not materialized. The first attempt introduced by Little et al. (2011, 2013) generates a design and sequence driven only by the economic values of the stopes, neglecting development aspects. An initial set of overlapping stopes is generated, then the proposed MIP model schedules stopes over the mine’s life under non-overlapping, adjacency, horizontal offset, and draw points constraints to provide a practical layout and schedule. Per-period ore handling, backfilling, and metal production capacities are also considered. Copland and Nehring (2016) extend the previous formulation by adding decision variables to control a top-down levels’ opening sequence with an associated discounted level opening cost. Foroughi et al. (2019) propose an MIP formulation with two weighted objectives: the maximization of NPV, which is related to the mine production schedule, and the maximization
of recovered metal, related to stope design. Equally spaced fixed production levels are accessed through a shaft and by a transverse drift connecting the shaft to the most centralized potential stope in a level. Precedence constraints that control the development of the shaft and drifts, and the stope production are also considered. Thus, the last two approaches ignore the necessary intra-level developments along and across the orebody’s strike for a tabular mineral deposit or along any mining direction for disseminated mineralization. The development of drifts is incorporated in the MIP model of Hou et al. (2019), which is tailored to a stratiform deposit with a fixed input network of shafts and longitudinal drifts. This method, however, only optimizes a two-dimensional schedule and design along the orebody’s strike by constraining the stopes’ transverse widths to the known orebody’s thickness. These integrated approaches generate physically different designs and schedules with higher NPV when benchmarked against traditionally used sequential optimization approaches, highlighting the benefits of the joint optimization of sublevel open stoping mining planning. The limitations of such proposed integrated models are as follows: the inexistent or overly simplistic modeling of developments, which is critical for the accessibility and consequent profitability of the stopes; the fixed input production levels, and the unrealistic time-dependent shaft’s development; and finally, these methods are deterministic, failing to integrate any uncertainty source, either geological or economic, into the optimization process, which leads to misleading designs and forecasts.

A two-stage stochastic integer program (SIP) (Birge and Louveaux 2011) is proposed to jointly optimize the sublevel open stoping design and the related long-term production scheduling. The method extends the stochastic stope design optimization method proposed by Faria et al. (2021) by integrating time-dependent stopes and development decisions, and annual production targets. The proposed integrated optimization is generalizable to accommodate various sources of uncertainty. Grade uncertainty and variability are incorporated into the optimization process through a set of equiprobable stochastic simulations of the orebody (Goovaerts 1997; Gómez-Hernández and Srivastava 2021). The model aims to maximize the NPV, coming from the scheduled stopes, development costs associated with the shaft and needed drifts and crosscuts, as well as from the fixed costs to keep levels in operation while minimizing the risk of not meeting mine production targets. Risk management is accomplished by the concept of geological risk discounting (GRD) (Dimitrakopoulou and Ramazan 2004) that has been proven to manage the
related risk improving the long-term scheduling results for open-pit (Ramazan and Dimitrakopoulos 2005, 2013; Montiel and Dimitrakopoulos 2015; Goodfellow and Dimitrakopoulos 2016) and underground operations (Carpentier et al. 2016; Dirkx et al. 2018; Huang et al. 2020).

Assuming an underground mine with different geotechnical zones and that is accessed through a primary shaft, the proposed integrated model has the flexibility to determine an optimal combination of production levels, satisfying sill pillar requirements, and an optimal within-level layout and sequencing of stopes with variable heights and sizes, separated by longitudinal and transversal pillars. The vertical development associated with the shaft and the horizontal developments, comprising the drifts and crosscuts, along with and across specified mining directions, are accounted for to provide a mineable schedule. A single processing plant and no stockpile option are assumed. The generated schedule accounts for the starting time of the stopes’ production, assuming a natural and continued sequence of intra-stopes activities, such as blasting, mucking, and eventual backfilling (Little et al. 2008). Hence, the generated schedule might be input to the further optimization of a detailed development network and higher resolution scheduling optimization. The subsequent sections are as follows. First, the proposed method is outlined. Then, an application at an underground gold mine and related results are presented, along with a comparison with a sequential stochastic optimization framework of the stope design followed by the scheduling. Finally, conclusions and future developments are presented.

3.2. Method

The integrated stochastic optimization of stope design and long-term underground mine production under grade uncertainty for the sublevel open stoping mining method is presented in this section. The first requirement for a mineable stope layout is the definition of production levels satisfying the level spacing and the sill pillar height dictated by geotechnical considerations for each geotechnical zone $n \in N$. Therefore, a set of potential horizontal levels $l \in L_n$, for all $n \in N$, at multiple depths needs to be provided, allowing the optimization process to select an optimal subset
of unified levels. Assuming primary shaft access, the definition of potential levels with known depths $\delta_{l}^{shaft}$ facilitates modeling the shaft development cost.

The second requirement is a feasible layout of stopes within each level. The stopes are separated by transversal and longitudinal pillars, whose minimum sizes are also dictated by stability considerations. In addition, assuming distinct geotechnical zones delimited by irregular separation surfaces, pillars also separate stopes near the contact of eventual adjacent zones. This case is a generalization of horizontally separated geotechnical zones. The stopes can have variable heights and sizes, aiming to handle the planned dilution, but with bases always anchored on the correspondent level, allowing for equipment maneuverability. Similarly, a set of potential stopes $j \in J_l$, generated given an assortment of allowable stoping shapes $m \in M_n$, is enumerated within each potential level $l \in L_n$. Thus, the model has the flexibility to select an optimal layout of non-overlapping stopes within the selected optimal level. The distances stopes-to-shafts $\delta_{jl}^{drift-shaft}$ along with specified mining directions $d \in D_l$ and crosscut distances $\delta_{jcl}^{crosscut}$ across such directions are required inputs and are essential to model the intra-level development costs in the output schedule. Once a stope is scheduled for production at a given time, the associated excavation of drift and crosscuts depends on the developments performed at previous periods. For instance, if this stope is midway between the shaft and a previously produced stope, no additional development is necessary. Otherwise, the incremental drift section will be developed from the furthest stope produced so far in its path to the shaft.

The underground mine operation at hand might be constrained by some existing or planned capacities or targets. The yearly shaft’s hoisting capacity, the drifts’ development advancement rate, the processing plant capacity, and grade requirements defined by multiple elements are considered in the method presented. The preprocessing steps detailed in Faria et al. (2021) are briefly revisited in the next section, followed by the proposed two-stage SIP formulation.
3.2.1. Steps

Based on the previous considerations for the stope layout and production scheduling, the proposed method receives the following inputs. (a) A set of scenarios \( s \in S \) quantifying the sources of uncertainty considered. Grade uncertainty and variability are incorporated through a set of geostatistical simulations of one or multiple correlated elements and material types within the mineral deposit. Other sources of uncertainty, such as the simulated time-series of metal prices, could be jointly considered in the set \( S \). (b) The multiple geotechnical zones \( n \in N \) with the correspondent subsets of flagged mining blocks \( i \in I_n \subset I \). (c) A library of parameters specifying the level spacing, sill/crown pillar height, and allowable stoping shapes \( m \in M_n \) for each geotechnical zone (Fig. 3.2). Finally, (d) the economic and technical parameters needed for the proposed method listed in Tables 1 to 3.

![Figure 3.1: Steps of the proposed optimization method (Source: Faria et al. 2021).](image)

![Figure 3.2: Examples of allowable stope shapes \( m \) for different geotechnical zones.](image)
Figure 3.3: Parametrized distances $\delta_{l}^{\text{shaft}}$, $\delta_{jd}^{\text{drift-shaft}}$ (in red), and $\delta_{jcl}^{\text{crosscut}}$ (in green) considered by the method, a potential level $l$, two mining directions $d$ and $d'$, two crosscuts $c$ and $c'$ and two potential stopes $j$ and $j'$.

Table 3.1: List of indices.

<table>
<thead>
<tr>
<th>Index</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i$</td>
<td>Block index</td>
</tr>
<tr>
<td>$j$</td>
<td>Stope index</td>
</tr>
<tr>
<td>$d$</td>
<td>Mining direction index</td>
</tr>
<tr>
<td>$c$</td>
<td>Crosscut index</td>
</tr>
<tr>
<td>$l$</td>
<td>Production level index</td>
</tr>
<tr>
<td>$n$</td>
<td>Geotechnical zone index</td>
</tr>
<tr>
<td>$m$</td>
<td>Stope shape index</td>
</tr>
<tr>
<td>$s$</td>
<td>Index of a scenario quantifying the considered sources of uncertainty</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Element (metal) index</td>
</tr>
<tr>
<td>$t$</td>
<td>Production period index</td>
</tr>
</tbody>
</table>

Table 3.2: List of sets.

<table>
<thead>
<tr>
<th>Set</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I$</td>
<td>Set of blocks $i$ in the entire orebody model</td>
</tr>
<tr>
<td>$I_n$</td>
<td>Set of blocks $i$ in zone $n$</td>
</tr>
<tr>
<td>$I_l$</td>
<td>Set of blocks $i$ in level $l$</td>
</tr>
<tr>
<td>$I_{jl}$</td>
<td>Set of blocks $i$ in stope $j$ of level $l$</td>
</tr>
<tr>
<td>Parameter</td>
<td>Definition</td>
</tr>
<tr>
<td>-----------</td>
<td>------------</td>
</tr>
<tr>
<td>$J$</td>
<td>Set of all potential stopes $j$</td>
</tr>
<tr>
<td>$J_l$</td>
<td>Set of potential stopes $j$ in a level $l$</td>
</tr>
<tr>
<td>$J_{dl}$</td>
<td>Set of potential stopes in mining direction $d$ in level $l$</td>
</tr>
<tr>
<td>$J_{cl}$</td>
<td>Set of potential stopes in crosscut $c$ in level $l$</td>
</tr>
<tr>
<td>$N$</td>
<td>Set of disjoint geotechnical zones $n$</td>
</tr>
<tr>
<td>$L$</td>
<td>Set of all possible levels $l$</td>
</tr>
<tr>
<td>$L_n$</td>
<td>Set of potential levels $l$ of geotechnical zone $n$</td>
</tr>
<tr>
<td>$S$</td>
<td>Set of geological uncertainty scenarios $s$</td>
</tr>
<tr>
<td>$T$</td>
<td>Set of periods $t$ of the predefined LOM</td>
</tr>
<tr>
<td>$E$</td>
<td>Set of elements $\epsilon$</td>
</tr>
<tr>
<td>$\Omega_l$</td>
<td>Set of levels $l'$ that overlap with level $l$, such that $l, l' \in L_n$ and $n \in N$</td>
</tr>
<tr>
<td>$\Omega_{jl}$</td>
<td>Set of stopes $j'$ that overlap with stope $j$, such that $j, j' \in J_l$, in level $l \in L_n$, $n \in N$</td>
</tr>
<tr>
<td>$\Lambda_l$</td>
<td>Set of levels $l'$ of other geotechnical zones that are adjacent to level $l \in L_n$ such that $l' \in \Lambda_l \subset L\setminus L_n$</td>
</tr>
<tr>
<td>$\Lambda_{jl}$</td>
<td>Set of stopes $j'$ in levels $l' \in \Lambda_l$ that overlap with stope $j \in J_l$ in level $l \in L_n$</td>
</tr>
<tr>
<td>$D_l$</td>
<td>Set of mining directions $d$ in level $l$</td>
</tr>
<tr>
<td>$C_l$</td>
<td>Set of potential crosscuts $c$ in level $l$</td>
</tr>
<tr>
<td>$M_n$</td>
<td>Set of stope shapes $m$ of geotechnical zone $n \in N$</td>
</tr>
</tbody>
</table>

**Table 3.3: List of economic and technical parameters.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_{is}$</td>
<td>Tonnage of block $i$ in scenario $s$</td>
</tr>
<tr>
<td>$w_{jl}$</td>
<td>Tonnage of stope $j$ in level $l$, $w_{jl} = \sum_{i \in J_l} w_{is}$ in scenario $s$</td>
</tr>
<tr>
<td>$\theta_{is}$</td>
<td>Indicator $\theta_{is} = 1$ if block $i$ has the grade of the main element greater than a user-defined cut-off grade is scenario $s$, 0 otherwise.</td>
</tr>
<tr>
<td>$o_{jls}$</td>
<td>Ore tonnage of stope $j$ in level $l$ and scenario $s$, $o_{jls} = \sum_{i \in J_l} \theta_{is} w_{is}$</td>
</tr>
<tr>
<td>$g_{i\epsilon s}$</td>
<td>Grade of element $\epsilon$ in block $i$, in scenario $s$ (in percent metal)</td>
</tr>
<tr>
<td>$g_{j\epsilon ls}$</td>
<td>Average grade of element $\epsilon$ of ore blocks within stope $j$ in level $l$ scenario $s$ (in percent metal), such that $g_{j\epsilon ls} = \sum_{i \in J_l} \theta_{is} g_{i\epsilon ls} w_{is} / o_{jls}$.</td>
</tr>
<tr>
<td>$R$</td>
<td>Processing recovery in percent</td>
</tr>
<tr>
<td>$P$</td>
<td>Metal selling price $$/t$</td>
</tr>
<tr>
<td>$v_{is}$</td>
<td>Economic value of block $i$ in level $l$ in scenario $s$</td>
</tr>
<tr>
<td>$v_{jl}$</td>
<td>Economic value of stope $j$ in level $l$ in scenario $s$</td>
</tr>
<tr>
<td>$\delta_{i}^{shaft}$</td>
<td>Vertical distance from the surface to the base of level $l$</td>
</tr>
<tr>
<td>$\delta_{jdl}^{drift-shaft}$</td>
<td>Horizontal distance in a drift from stope $j$ to the shaft access point along mining direction $d$ in level $l$</td>
</tr>
<tr>
<td>( \delta_{jcl} )</td>
<td>Horizontal distance in a crosscut ( c ) from stope ( j ) to the drift position in the footwall</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>( C_{l}^{drift} )</td>
<td>Unit horizontal development cost of drifts in $/m in level ( l )</td>
</tr>
<tr>
<td>( C_{shaft} )</td>
<td>Unit shaft development in $/m.</td>
</tr>
<tr>
<td>( C_{l}^{mining} )</td>
<td>Mining cost in $/t for level ( l ).</td>
</tr>
<tr>
<td>( C_{process} )</td>
<td>Processing cost in $/t.</td>
</tr>
<tr>
<td>( C_{fixed} )</td>
<td>Fixed cost incurred for keeping a production level in operation ($/level*year).</td>
</tr>
<tr>
<td>( f^{economic}_t )</td>
<td>Economic discount factor for period ( t ) given an economic discount rate</td>
</tr>
<tr>
<td>( f^{geologic}_t )</td>
<td>Geologic discount factor for period ( t ) given a geologic discount rate</td>
</tr>
<tr>
<td>( U_{shaft}^{t} )</td>
<td>Hoisting capacity of the shaft in period ( t ) (tons/year).</td>
</tr>
<tr>
<td>( U_{develop}^t )</td>
<td>Development capacity in period ( t ) (m/year).</td>
</tr>
<tr>
<td>( U_{process}^t )</td>
<td>Processing capacity in period ( t ) (tons/year).</td>
</tr>
<tr>
<td>( U_{grade}^{max,et} )</td>
<td>Maximum target of element ( \varepsilon ) in period ( t ) (% of element ( \varepsilon )/year).</td>
</tr>
<tr>
<td>( L_{grade}^{min,et} )</td>
<td>Minimum target of element ( \varepsilon ) in period ( t ) (% of element ( \varepsilon )/year).</td>
</tr>
<tr>
<td>( c_{process} )</td>
<td>Penalty costs associated with the surplus deviations from the mill tonnage capacity ($/tons of ore).</td>
</tr>
<tr>
<td>( c_{\varepsilon}^{+} )</td>
<td>Penalty cost associated with surplus deviations from the maximum grade requirement for element ( \varepsilon ) ($/tons of metal ( \varepsilon )).</td>
</tr>
<tr>
<td>( c_{\varepsilon}^{-} )</td>
<td>Penalty cost associated with shortage deviations from the minimum grade requirement for element ( \varepsilon ) ($/tons of metal ( \varepsilon )).</td>
</tr>
</tbody>
</table>

Table 3.4: Decision variables of the formulation for primary access through a shaft.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_{jlt} )</td>
<td>Stope selection decision variable, equal to 1 if stope ( j ) in level ( l ) is selected in period ( t ), and 0 otherwise</td>
</tr>
<tr>
<td>( z_l )</td>
<td>Level selection decision variable, equal to 1 if level ( l ) is selected, and 0 otherwise</td>
</tr>
<tr>
<td>( VDC )</td>
<td>Vertical (shaft) development cost</td>
</tr>
<tr>
<td>( HDC_{dlt} )</td>
<td>Drift’s development cost in level ( l \in L ), along with mining directions ( d \in D_l ) in period ( t )</td>
</tr>
<tr>
<td>( HDC_{dlt}^{*} )</td>
<td>Effective drift’s development cost in level ( l \in L ), along with mining directions ( d \in D_l ) in period ( t )</td>
</tr>
<tr>
<td>( CDC_{dlt} )</td>
<td>Crosscut’s development cost in level ( l \in L ), along with crosscut ( c \in C_l ) in period ( t )</td>
</tr>
<tr>
<td>( CDC_{dlt}^{*} )</td>
<td>Effective crosscut’s development cost in level ( l \in L ), along with crosscut ( c \in C_l ) in period ( t )</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$F_{lt}$</td>
<td>Fixed cost related to keeping a level $l$ in operation in period $t$</td>
</tr>
<tr>
<td>$d_{ts}^{\text{process}}$</td>
<td>Surplus deviation of total processing capacity, in period $t$, and scenario $s$</td>
</tr>
<tr>
<td>$d_{ets}^+$</td>
<td>Surplus deviation from the maximum grade target of element $\epsilon$, in period $t$, and scenario $s$</td>
</tr>
<tr>
<td>$d_{ets}^-$</td>
<td>Shortage deviation from the maximum grade target of element $\epsilon$, in period $t$, and scenario $s$</td>
</tr>
</tbody>
</table>

The steps of the proposed method are presented in Fig. 3.1. In Step 1, the set of blocks $i \in I_n$ belonging to each geotechnical zone $n \in N$ is split, from bottom to top, into the first set of possible production levels based on the level height. Once the top of the block model is reached, the level splitting restarts by skipping some blocks at the bottom of the geotechnical zone. This Step generates the set of overlapping levels $l \in L_n$, such that $L = \bigcup_{n \in N} L_n$. It is assumed that the generated levels are formed by blocks belonging to a unique geotechnical zone. The distance $\delta_l^{\text{shaft}}$ from the base of a level to the surface (Fig. 3.3) is also computed in the current step.

The generated levels overlap with some other levels since they share some layers of mining blocks. Furthermore, a minimum sill pillar size must be ensured between the two output design levels due to geotechnical requirements. Step 2 maps the overlaps between levels considering the sill pillar size, generating a set of levels $l' \in \Omega_l$ that overlap with level $l \in L_n$. Since the surface separating distinct geotechnical zones $n \in N$ might be irregular or near vertical, two levels $l$ and $l'$ belonging, respectively, to different geotechnical zones $n$ and $n'$ might be adjacent, that is, $l$ might have border blocks or a sill pillar area adjacent to border blocks of $l'$. Therefore, in Step 3, the sets of levels $l' \in \Lambda_l \subset L \setminus L_n$ that are adjacent to levels $l \in L_n$ are mapped. Unlike the overlapping levels, it is assumed that two adjacent levels can be selected in the final design. However, the stopes that are close to such adjacent levels’ boundaries are not allowed to overlap with each other.

A library of stoping shapes $m \in M_n$ is defined for each geotechnical domain $n \in N$ based on geotechnical considerations and equipment maneuverability requirements (Fig. 3.2). Each shape consists of the number of blocks along axes $x$, $y$, and $z$. A broader combination of stope shapes improves the output stope design since the optimization process has more flexibility to select potential stopes with fewer waste blocks. In Step 4, the allowable shapes are floated within the generated levels, producing the set of potential stopes $l \in J_l$. Once a stope is evaluated, the distance
stope-to-shaft $\delta_{jdt}^{\text{shaft}}$, along a mining direction $d \in D_l$ (Fig. 3.3) is evaluated. Across the mining directions, the orebody is split into equally spaced regions defining a set of potential stopes $j \in J_{cl}$ within potential crosscuts $c \in C_l$. The distances stope-to-drift $\delta_{jcl}^{\text{crosscut}}$, along a crosscut $c \in C_l$ (Fig. 3.3) are also computed. The stopes’ positions dictate the stopes belonging to the subsets $J_{dt}$ and $J_{cl}$. Finally, the stope economic values $\nu_{jls}$ for each simulation $s \in S$ are also determined. A probability threshold (Leite and Dimitrakopoulos 2014) can be used to trim out stopes with negative economic value in more than a portion of the input number of simulations $|S|$. Due to stability considerations, longitudinal and rib pillars may be placed between stopes. Therefore, in Step 5, the overlaps between stopes are mapped based on the specified pillar sizes defining the set of stopes $j' \in \Omega_{jt}$ that overlap with $j \in J_l$ for $l \in L_n$. Similarly, in Step 6, the overlaps between stopes lying in adjacent levels are mapped to generate the set of stopes $j' \in \Lambda_{jl}$ that overlap with stope $j$, such that $j \in J_l$, $l \in L_n$, and $j' \in J_{l'}$, $l' \in \Lambda_l$ if $\Lambda_l \neq \emptyset$. For more details concerning the preprocessing steps, the reader is directed to Faria et al. (2021).

3.2.2. Mathematical Formulation

The proposed integrated optimization approach of stope design and mine production schedule assumes that the levels are accessed through a shaft, and it is controlled by two binary decision variables (Table 4). Level selection decision variables $z_l \in \{0,1\}$ control which production levels $l \in L$ are selected. These variables are time-independent since the shaft is assumed to be completely developed from the beginning of the LOM. The stope selection decision variables $y_{jlt} \in \{0,1\}$ define whether a stope $j \in J_l$ in level $l \in L$ is to be extracted in period $t$. The sequencing of stopes productions is reasonably collapsed into the decision variables $y_{jlt}$ assuming a natural and continued sequence of intra-stopes activities.

The vertical development cost continuous decision variable $VDC$ stands for the excavation and preparation costs of the shaft, and it is driven by the deepest selected level in the final schedule (Fig. 3.4). Two types of decision variables are used to model the drifts’ and crosscuts’ development costs in each level. Variables $HDC_{dt}$ and $CDC_{cct}$ are driven by the furthest stope selected in a mining direction $d$ or a crosscut $c$ of level $l$ in a period $t$ while the effective development cost
decision variables $HDC_{dlt}^*$ and $CDC_{clt}^*$ accounts for the cumulative developments performed along direction $d$, or a crosscut $c$, up to period $t$ (Fig. 3.5). The fixed cost per level decision variables $F_{lt}$ are related to the maintenance cost, mainly due to ventilation and support, incurred to keep a level under operation in a period. The scenario-dependent decision variables $d_{ts}^{\text{process}}$ refer to surplus deviations from the yearly processing capacity, while $d_{ets}^+$ and $d_{ets}^-$ represent, respectively, the surplus and shortage deviations from maximum and minimum processing grade requirements defined for multiple elements $\varepsilon \in E$.

The objective function (Eq. 3.1) is composed of four parts. Part I represents the maximization of the discounted net revenue of the scheduled stopes. The minimization of the shaft development cost and the sum of the drifts’ development costs of all possible levels comes in Part II, while the minimization of the fixed level operation cost is accounted for in Part III. The horizontal development costs are discounted in time, aiming to postpone developments for later periods. The fixed level operation costs are undiscounted in order to reduce the number of levels concurrently under operation per period. Thus, Part I, II, and III seek to maximize the mining project’s NPV, which is the typical optimization criterion for strategic mine planning given its ability to balance earlier and later decisions over the life-of-mine (LOM) by accounting for the time value of money (Lane 1964, 1988; Darling 2011; King 2011, 2018; Hustrulid et al. 2013). The risk management component of Part IV penalizes the resulting economic impact of deviating from production.
targets, such as the processing capacity and grade requirements for multiple elements $\varepsilon \in E$. This component aims to defer the extraction of riskier stoping areas while adjusting the risk profiles of the output schedule and design to the production targets. The penalties costs $c^{process}, c^{+}_\varepsilon$, and $c^{-}_\varepsilon$, for all $\varepsilon \in E$, are time varied using a geological discount factor $f_{t}^{geologic}$. These penalty costs are calibrated by testing different orders of magnitude (Ramazan and Dimitrakopoulos 2005, 2013; Benndorf and Dimitrakopoulos 2013).

\[ v_{ils} = w_{ls} \left( g_{il1s}RP - (C^{process} + C^{mining}_l) \right), \quad \forall i \in l_{jl}, j \in f_l, l \in L, s \in S \quad (3.2) \]

\[ v_{jls} = \sum_{i \in f_{jl}} v_{ils}, \quad \forall j \in f_l, l \in L, s \in S \quad (3.3) \]

The proposed method considers a mine to have only one possible processing destination with no stockpiling option. Therefore, low-grade blocks within a stope are considered as planned dilution. The economic value of a block $v_{ils}$ of a block $i \in I_l$ in level $l \in L$ in simulation $s \in S$ is defined by Eq. 3.2, which considers the simulated grade $g_{il1s}$ of the valuable metal indexed by $\varepsilon = 1$. The deleterious elements $\varepsilon = E\setminus\{1\}$ are considered in the grade requirement constraints of Eqs. 3.17 and 3.18. A level-based mining cost $C^{mining}_l$ is considered, reflecting the increased ventilation, support, and other operational costs with mine depth. Stopes with a higher number of waste blocks have higher associated mining and processing costs and lower economic value $v_{jls}$ (Eq. 3.3). As a result, the formulation has the ability to select the best stope size and position amongst the family of potential stopes that have a shared high-grade block, aiming to minimize the planned dilution in the design. Equation 3.2 can be easily generalized to accommodate economic value coming from multiple valuable metals. The model can be generalized to account for a simulated time-series of commodity prices. In this case, the fixed metal price $P$ is replaced by the values $P_{st} \forall s \in S, t \in T$ and the economic values become time-dependent, i.e., $v_{ilst}$ and $v_{jlst}$ (with subscript $t$).

The proposed mathematical formulation is subject to several physical, operational, and linking constraints that are as follows:

\[ y_{jlt} \leq z_l, \quad \forall j \in f_l, l \in L, t \in T \quad (3.4) \]
\[ z_l + z_{l'} \leq 1 , \quad \forall \ l' \in \Omega_l \subset L_n , l \in L_n \] (3.5)

\[ \sum_{t \in T} y_{jlt} + \sum_{t \in T} y_{j'lt} \leq 1 , \quad \forall \ j' \in \Omega_{jl} , j \in J_l , l \in L \] (3.6)

\[ \sum_{t \in T} y_{jlt} + \sum_{t \in T} y_{j'lt'} \leq 1 , \quad \forall \ j'' \in \Lambda_{jl} , l' \in \Lambda_l \subset L \setminus L_n , j \in J_l , l \in L_n \] (3.7)

\[ \sum_{t \in T} y_{jlt} \leq 1 , \quad \forall \ j \in J_l , l \in L \] (3.8)

\[ F_{lt} \geq C^{fixed}_{jl} y_{jlt} , \quad \forall \ j \in J_l , l \in L , t \in T \] (3.9)

The linking constraints of Eq. 3.4 ensure that a stope \( j \) can be scheduled to period \( t \) if its correspondent level is selected and vice-versa. Equation 3.5 represents the levels of non-overlapping constraints, including the sill pillar requirements, in a geotechnical zone \( n \in N \). The constraints of Eq. 3.6 avoid scheduling overlapping stopes, while considering rib pillar sizes, within the same level and geotechnical zone \( l \in L_n \) during the LOM. Similar constraints are defined for overlapping stopes belonging to adjacent levels near irregular boundaries between two geotechnical zones (Eq. 3.7). These constraints would not be necessary if the zones were elevation-delimited rather than separated by an irregular separation surface. The reserve constraints (Eq. 3.8) guarantee that a stope is scheduled at most once during the LOM. A fixed operation cost per period is incurred whenever the extraction of a stope is scheduled in a level (Eq. 3.9).

\[ VDC \geq (\delta_l^{shaft} C_l^{shaft}) z_l , \quad \forall \ l \in L \] (3.10)

\[ HDC_{dlt} \geq (\delta_{jdl}^{shaft} c_l^{drift}) y_{jlt} , \quad \forall \ j \in J_{dl} , d \in D_l , l \in L , t \in T \] (3.11)

\[ HDC_{dt1}^* = HDC_{dl1} , \quad \forall \ j \in J_{dl} , d \in D_l , l \in L , t = 1 \] (3.12)
\[ HDC_{dlt}^* = \max \left\{ 0, \left[ HDC_{dlt} - \sum_{t'=1}^{t-1} HDC_{dlt'}^* \right] \right\}, \quad \forall \; j \in J_{dlt}, d \in D_l, l \in L, t > 1 \quad (3.13) \]

\[ CDC_{clt} \geq \left( \delta_{jcl}^{crosscut} C_l^{drift} \right) y_{jlt}, \quad \forall \; j \in J_{clt}, c \in C_l, l \in L, t \in T \quad (3.14) \]

\[ CDC_{cl1} = CDC_{clt}, \quad \forall \; j \in J_{clt}, c \in C_l, l \in L, t = 1 \quad (3.15) \]

\[ CDC_{clt}^* = \max \left\{ 0, \left[ CDC_{clt} - \sum_{t'=1}^{t-1} CDC_{clt'}^* \right] \right\}, \quad \forall \; j \in J_{clt}, c \in C_l, l \in L, t > 1 \quad (3.16) \]

The development costs are accounted for through the linearized constraints of Eqs. 3.10-3.13. The total shaft development cost is driven by the most profound level selected in the final design (Fig. 3.4), and the constraints of Eq. 3.10 control it. Although there is a required time for a shaft to be excavated, it is assumed to be first developed prior to the beginning of LOM, with corresponding undiscounted cost \( V_{DC} \). The decision variables \( HDC_{dlt} \) and \( CDC_{dlt} \) along with a mining direction \( d \), or a crosscut \( c \), at level \( l \) are incremented whenever a further stope is scheduled (Eqs. 3.11 and 3.14). However, the effective drift’s \( HDC_{dlt}^* \) or crosscut’s \( CDC_{clt}^* \) development cost depends on the cumulative effective developments completed so far, Eqs. 3.12, 3.13, 3.15, and 3.16. Therefore, if the current furthest scheduled stope in period \( t \) is midway between the access point and a previously selected stope, the current schedule stope does not contribute to the effective cost, represented in Fig. 3.5. Otherwise, the incremental drift section will be developed from the furthest stope produced in its path to the shaft. The same relationship is defined for the crosscuts. Variables \( HDC_{dlt}^* \) and \( CDC_{clt}^* \) are those actually minimized in the objective function (Eq. 3.1).

\[ \sum_{l \in L} \sum_{j \in J_l} y_{jlt} w_{jls} \leq U_{tshaft}, \quad \forall \; t \in T, s \in S \quad (3.17) \]

\[ \sum_{l \in L} \sum_{d \in D_l} HDC_{dlt}^* + \sum_{l \in L} \sum_{c \in C_l} CDC_{clt}^* \leq U_t^{develop}, \quad \forall \; t \in T \quad (3.18) \]
\[
\sum_{l \in L} \sum_{j \in J_l} (y_{jlt} o_{jls}) - d_{ts}^{\text{process}} \leq U_t^{\text{process}}, \quad \forall \ t \in T, s \in S \quad (3.19)
\]

\[
\sum_{l \in L} \sum_{j \in J_l} (g_{jles} - U_{et}^{\text{grade}}) o_{jls} y_{jlt} - d_{ets}^+ \leq 0 \quad \forall \ \varepsilon \in E, t \in T, s \in S \quad (3.20)
\]

\[
\sum_{l \in L} \sum_{j \in J_l} (g_{jles} - L_{et}^{\text{grade}}) o_{jls} y_{jlt} + d_{ets}^- \geq 0 \quad \forall \ \varepsilon \in E, t \in T, s \in S \quad (3.21)
\]

The existing or planned capacities or targets are defined through Eqs. 3.17-3.21. The hard constraints of Eqs. 3.17 and 3.18 define the yearly hoisting and development advancement capacities given \( U_t^{\text{shaft}} \) and \( U_t^{\text{develop}} \), respectively. The ratios \( HDC_{dt}^*/C_{dl}^{\text{drift}} \) and \( CDC_{cl}^*/C_{cl}^{drift} \) reflect the per-period distance developed per level. The yearly processing capacity \( U_t^{\text{process}} \) must be satisfied considering the sum of the contained ore tonnage \( o_{jls} \) of the scheduled stopes in simulation \( s \in S \). Equation 3.19 defines the related deviations \( d_{ts}^{\text{process}} \). If multiple elements \( \varepsilon \in E \), including deleterious ones, are critical to defining the stope design, the grade constraints (Eqs. 3.20 and 3.21) allow for computing the scenario-dependent deviations from the minimum and maximum grade requirements \( d_{ets}^+ \) and \( d_{ets}^- \). Finally, the following integrality and non-negativity constraints from Eq. 3.22 to Eq. 3.29 complete the two-stages SIP formulation:

\[
y_{jlt} \in \{0,1\}, \quad \forall \ j \in J, l \in L, t \in T \quad (3.22)
\]

\[
z_l \in \{0,1\}, \quad \forall \ l \in L \quad (3.23)
\]

\[
VDC \geq 0 \quad (3.24)
\]

\[
F_{lt} \geq 0, \quad \forall \ l \in L, t \in T \quad (3.25)
\]

\[
HDC_{dt}, HDC_{dt}^* \geq 0, \quad \forall \ d \in D, l \in L, t \in T \quad (3.26)
\]

\[
CDC_{cl}, CDC_{cl}^* \geq 0, \quad \forall \ c \in C, l \in L, t \in T \quad (3.27)
\]
\[ d_{ts}^{\text{process}} \geq 0, \quad \forall \ t \in T, s \in S \] (3.28)

\[ d_{ets}^+, d_{ets}^- \geq 0, \quad \forall \ \varepsilon \in E, t \in T, s \in S \] (3.29)

**Figure 3.4:** Example of vertical development costs decision variable VDC according to the selected levels during a three-year LOM.

**Figure 3.5:** Three-period example of drifts’ development costs decision variables \( HDC_{dlt} \) and \( HDC_{dlt}^* \) along mining direction \( d \) in level \( l \), according to a shaft position.
3.3. Case Study – Application at an Underground Gold Mine

The proposed method for integrating the sublevel open stoping design and production scheduling is applied to an underground gold deposit. The mineralization is located from 400m to 830m below the surface level, and a set of 25 stochastic simulations of gold grades on a regular grid containing 107,520 mining blocks of size 10m x 10m x 10m is used to describe the mineral deposit and incorporate the grade uncertainty and variability into the optimization process (Fig. 3.6a). All blocks are assumed to have the same tonnage in all simulations. Although the model is generalizable to account for multiple sources of uncertainty, only the uncertainty in gold grades is considered in the present application. Table 5 summarizes the information about the input underground gold orebody. The orebody’s strike is oriented along axis y, and there exist two high-grade lodes (marked in red) that can be visualized in the cross-sections of Fig. 3.6a: one in the top-left portion and another high-grade region from the center to the right.

Two geotechnical zones delimited by an irregular and inclined separation surface are considered (Fig. 3.6b). The main distinction between the two zones is the level spacing, which is 40 m for the upper geotechnical zone and 60 m for the lower zone, which leads to different allowable stope shapes and pillar requirements due to geotechnical considerations. In the current sublevel open stoping operation, the production levels are assumed to be separated by sill pillars of 10 m high, equal to the height of one block, and the stopes are separated by different rib and longitudinal pillars sizes, which are presented in Table 6, showing the user-defined parameters of each geotechnical zone.

The mine is accessed through a primary shaft located at the center of the deposit’s strike (axis y). Its headframe is 60 m away from the edge of the orebody model, which is the assumed footwall of an eventual production level starting at the very top of the deposit. A yearly hoisting capacity limits the shaft operation. In each level, the main drifts are developed from the shaft position following the strike direction. Two main mining directions are considered, namely, the northern and southern sides of the shaft (towards the negative and positive y-axis, respectively). Across the strike, the orebody is equally split into 80m to define potential crosscuts in each level. A yearly maximum
development rate is defined in meters, assuming a typical drifts section, to limit the advancement of drifts. This development rate does not limit the shaft’s advancement since this access is assumed to be developed entirely by the beginning of the LOM. The underground mining operation is also constrained by a yearly processing capacity and a lower bound for the mill head grade, as shown in Table 7 that presents the economic and technical parameters used in the optimization process.

Table 3.2: Input orebody information.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value/Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metal grade</td>
<td>gold</td>
</tr>
<tr>
<td>Grade unit</td>
<td>g/t</td>
</tr>
<tr>
<td>Total number of realizations</td>
<td>50</td>
</tr>
<tr>
<td>Number of realizations for optimization</td>
<td>25</td>
</tr>
<tr>
<td>Number of realizations for risk analysis</td>
<td>25</td>
</tr>
<tr>
<td>Depth from processing plant level (surface)</td>
<td>400m – 830m</td>
</tr>
<tr>
<td>Dimension across the orebody strike</td>
<td>320m</td>
</tr>
<tr>
<td>Orientation of strike</td>
<td>Along axis y</td>
</tr>
<tr>
<td>Average orebody dip</td>
<td>70°</td>
</tr>
<tr>
<td>Number of blocks</td>
<td>107,520</td>
</tr>
<tr>
<td>Block size</td>
<td>10m x 10m x 10m</td>
</tr>
</tbody>
</table>

Table 3.3: Information about the geotechnical zones.

<table>
<thead>
<tr>
<th>Zone</th>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper zone</td>
<td>Number of allowable shapes</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>Level spacing</td>
<td>40 m</td>
</tr>
<tr>
<td></td>
<td>Stope heights</td>
<td>20 - 40 m</td>
</tr>
<tr>
<td></td>
<td>Stope widths</td>
<td>20 - 30 m</td>
</tr>
<tr>
<td></td>
<td>Stope lengths</td>
<td>30 - 50 m</td>
</tr>
<tr>
<td></td>
<td>Sill pillar size</td>
<td>10 m</td>
</tr>
<tr>
<td></td>
<td>Rib pillar size</td>
<td>10 m</td>
</tr>
<tr>
<td></td>
<td>Longitudinal pillar size</td>
<td>10 m</td>
</tr>
<tr>
<td>Lower zone</td>
<td>Number of allowable shapes</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>Level spacing</td>
<td>60 m</td>
</tr>
<tr>
<td></td>
<td>Stope heights</td>
<td>40 - 60 m</td>
</tr>
<tr>
<td></td>
<td>Stope widths</td>
<td>10 - 20 m</td>
</tr>
<tr>
<td></td>
<td>Stope lengths</td>
<td>30 - 50 m</td>
</tr>
<tr>
<td></td>
<td>Sill pillar size</td>
<td>10 m</td>
</tr>
<tr>
<td></td>
<td>Rib pillar size</td>
<td>10 m</td>
</tr>
<tr>
<td></td>
<td>Longitudinal pillar size</td>
<td>20 m</td>
</tr>
</tbody>
</table>
Table 3.4: Economic and technical parameters used in the proposed stochastic integrated optimization at the underground gold mine.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metal price ($/ozt)</td>
<td>1,200</td>
</tr>
<tr>
<td>Economic discount rate</td>
<td>10%</td>
</tr>
<tr>
<td>Geologic discount rate</td>
<td>10%</td>
</tr>
<tr>
<td>Processing recovery (%)</td>
<td>94%</td>
</tr>
<tr>
<td>Mining cost ($/t)</td>
<td>118</td>
</tr>
<tr>
<td>Processing cost ($/t)</td>
<td>20</td>
</tr>
<tr>
<td>Shaft development cost ($/m)</td>
<td>20,000</td>
</tr>
<tr>
<td>Drifts development cost ($/m)</td>
<td>7,000</td>
</tr>
<tr>
<td>Density (t/m³)</td>
<td>2.9</td>
</tr>
<tr>
<td>Block tonnage (t)</td>
<td>2,900</td>
</tr>
<tr>
<td>Hoisting capacity (kt/y)</td>
<td>500</td>
</tr>
<tr>
<td>Processing capacity (kt/y)</td>
<td>500</td>
</tr>
<tr>
<td>Drift development capacity (m/y)</td>
<td>1,000</td>
</tr>
<tr>
<td>Minimum gold mill head grade (g/t of Au/y)</td>
<td>4.5</td>
</tr>
<tr>
<td>Penalty cost for processing capacity ($/t of ore)</td>
<td>10</td>
</tr>
<tr>
<td>Penalty cost for minimum Au mill head grade ($/t of Au)</td>
<td>10</td>
</tr>
</tbody>
</table>

Figure 3.6: (a) One geostatistical realization of gold grades of the input underground deposit with a block size of 10 m x 10m x 10m, and (b) the two input geotechnical zones.

The level splitting process (Step 1 of Fig. 3.1) generated 22 potential levels in the upper geotechnical zone and 28 levels in the lower zone, given the setup levels spacing and skipping one block along the z-axis during each pass of the current step. In Step 3, 9,685 and 21,769 potential
stopes were generated in the upper and lower geotechnical zones, respectively, totaling 31,454 stopes based on a probability threshold of 50% to eliminate stopes with potentially negative economic value. The overlaps between levels and stopes were also evaluated in the proposed Steps 2, 3, 5, and 6 of Fig. 3.1.

The output of the integrated stochastic optimization of the stope design and long-term underground mine production schedule is displayed in Fig. 3.7. The integrated stochastic stope design and schedule have five selected production levels, which satisfy the sill pillar heights regarding the physical aspects. The total number of selected stopes is 39. The stope layout satisfies the allowable minimum and maximum stope sizes and the minimum transversal and rib pillar sizes of each geotechnical zone within each level. In addition, the stopes have variable heights and are always aligned with the respective level’s base. This configuration facilitates the mineability of a further detailed network layout of transportation and drilling drifts, and loading crosscuts that are to be designed upon the selected levels and stopes based on higher resolution operational and maneuverability constraints.

Figure 3.8 presents the risk analysis (Ravenscroft 1992) in terms of the non-exceedance probabilities of 10%, 50%, and 90% for NPV, cumulative recovered metal, ore tonnage, mill gold head grade, and cumulative drifts’ development costs generated by the proposed approach, represented by the continued and dashed black lines. This figure features additional information that will be discussed in the subsequent section. The output high-level stopes’ extraction sequence has a life-of-mine of eight years with a correspondent NPV of 134.8 M$ (Fig. 3.8a) and about 20t of recovered gold (Fig. 3.8e). The deepest selected level is 755 m deep from the surface, leading to a required shaft’s development cost of 15.1 M$ (Figs. 7 and 8b). In the first year of operation, the production levels at depths of 455 and 595 m are simultaneously opened to reach high-grade stopes (Fig. 3.7 and Fig. 3.8d). These stopes are sufficiently profitable to pay for the initial drifts’ development cost from the shaft’s access points and the fixed level’s operating cost, as well as to produce the highest discounted cash flows of the LOM (Fig. 3.8b). In the subsequent periods, the drifts are extended, and lower grade stopes are extracted. This decreasing trend of gold mill head grade during the LOM (Lane 1964, 1988) reflects the proposed method’s ability to maximize the NPV while managing the risk of not satisfying the specified minimum head grade (Fig. 3.8d). The
method can also respect the various project’s capacities, with the hoisting capacity (red line of Fig. 3.8c) being the bottleneck of the current project. In general, two levels are concurrently in operation per period due to the incurred fixed cost in the third component of the objective function (Eq. 3.1). If the yearly fixed cost per level is set to zero, a more year-by-year scattered sequence would be obtained. The proposed method was implemented and solved using C++ language on a Visual Studio 15 with CPLEX v.12.8.0 environment. This instance, thus, results in 314,591 binary decision variables and 758,344 constraints.

**Figure 3.7:** Output of the integrated stochastic optimization of stope design and mine production schedule applied to an underground gold mine: (a) stope sequence of extraction, and (b) the stopes’ average gold grade. The surface level and the shaft position are represented in the figures.
3.3.1. Comparison to a Stepwise Stochastic Optimization of Stope Design and Subsequent Mine Production Scheduling

In this section, the proposed method is benchmarked against a commonly used stepwise underground mine planning framework, in which the stope design is previously optimized, followed by the optimization of the long-term production schedule. In the first mine planning step,
assuming the same underground gold mine and the same technical and economic parameters, a stope design is optimized under grade uncertainty based on the method proposed by Faria et al. (2021). This model considers the same assumptions related to the sublevel open stoping mining method discussed in Section 3.2. In the second planning step, the generated stope layout is subsequently an input for the proposed integrated stochastic optimization method (Eqs. 3.1-3.25). However, the level’s non-overlapping constraints (Eq. 3.5) and the stopes’ non-overlapping constraints of Eqs. 3.6 and 3.7 naturally hold since the unified levels and stopes of the input layout already satisfy such non-overlapping constraints.

The comparison between the designs and schedules generated by the sequential (Fig. 3.9a) and integrated (Fig. 3.9b) optimization frameworks is presented. As presented in this figure, the resulting stope designs and schedules are physically different. Different production levels and stoping volumes, with related average gold grades (Fig. 3.10), were selected by the two approaches. Additionally, the sequence of extraction of stopes and the life-of-mine (LOM) are also different. In the stepwise framework (Fig. 3.9a), the outline formed by the unfilled block edges depicts the stopes from the stochastic stope design optimization step, which aims to maximize the undiscounted profit. Some of these stopes, mainly in the borders of some levels, were not scheduled in the subsequent scheduling optimization step that seeks to maximize the NPV accounting for the time value of money. The outline formed by unfilled block edges of Fig. 3.9b, in turn, represents the superimposed schedule of the sequential approach shown in Fig. 3.9a, highlighting the physical differences of outputs generated by the two approaches.
**Figure 3.9:** Comparison of stochastic long-term underground mine production schedules: (a) sequential optimization of stope design and production scheduling (the unfilled blocks correspond to stopes in the stope design that are not mined in the production schedule); (b) proposed integrated approach (the unfilled blocks correspond to stopes of the schedule generated in (a)).

**Figure 3.10:** Comparison of average gold grades of stopes selected in the schedules of (a) sequential optimization of stope design and production scheduling (the unfilled blocks correspond to stopes in the stope design that are not mined in the production schedule); (b) proposed integrated approach (the unfilled blocks correspond to stopes of the schedule generated in (a)).

The comparison of risk profiles for the schedules generated by the integrated and the stepwise approaches are depicted, respectively, in black and green lines in Fig. 3.8. The schedule generated by the integrated approach achieves 11% higher overall NPV, which corresponds to a difference
of 13.2 M$ (Fig. 3.8a), as compared to the schedule generated by the stepwise approach. The related LOM of eight years is two years shorter than the LOM of the schedule produced by the stepwise approach. The difference in NPV is even more pronounced in the first year of operation with a 40%, or 12.0 M$, higher NPV, providing a faster payback time. Regarding the cumulative metal (Fig. 3.8e), the integrated framework recovers 16% lower metal tonnage than the benchmark case. Both schedules have the shaft’s and cumulative drifts’ development costs of about 15.0 M$ (year zero of Fig. 3.8b) and 11.5 M$ (Fig. 3.8f), respectively. Therefore, the integrated approach has better flexibility to define more selective stoping volumes (Figs. 8d and 10) while accounting for developments so as to provide an optimal NPV. These results highlight that pre-optimized stope design in the sequential approach constrains the LOM production schedule’s optimality, reducing the profitability of the underground mining operation at hand.

3.4. Conclusions

The new two-stage stochastic integer program formulation was proposed for the integrated optimization of stope design and long-term underground mine production schedule under geological uncertainty, for underground mines employing the sublevel open stoping mining method and that are accessed through a primary shaft. The method seeks to maximize the NPV related to revenues from scheduled stopes after subtracting the mining, processing, per-period costs to maintain each level in operation, as well as the development costs while managing the risk of not meeting production targets. A mineable stope design and an underground mine production schedule are produced with an optimized set of horizontal production levels separated by required sill/crown pillar requirements and non-overlapping stopes with variable heights, length, and width with necessary rib and longitudinal pillar requirements. Such geotechnical and operational constraints, as well as the incorporation of shaft, drifts, and crosscuts developments are crucial for sublevel open stoping operations, and make the output layout design and schedule suitable for further higher resolution optimization of the network development design and production schedule.

The application of the proposed method at an underground gold mine produced a physically different sublevel open stoping design and production schedule when compared to the design and
schedule generated by the stepwise stochastic optimization approach that optimizes sequentially the stope design followed by the production schedule. The proposed method achieved a 40% higher NPV in the first year, as well as an 11% higher final NPV and a life-of-mine of eight years, which is two years shorter than the schedule generated by a stepwise approach. The results confirm that more profitable underground mine plans are obtained when the synergies of design and planning areas are jointly optimized.

This integrated approach also provides new avenues to address underground mine planning optimization by directly incorporating multiple uncertainty sources. Although the geological uncertainty can be accounted for in each optimization step of the commonly used sequential underground mine planning framework, the preliminary optimization of stope design fails to integrate time-dependent uncertainties. As a result, the investigation of the effect of some sources of uncertainty, as in commodity prices or grades of multiple elements, is straightforward for the proposed integrated method. The incorporation of other more complex sources of uncertainty, such as geotechnical and operational costs, require future research. The proposed formulation might be extended to account for multiple processing destinations and stockpile options, and can be slightly tailored to mines that are accessed through declines. In addition, further developments would seek to incorporate the proposed integrated underground mine planning approach into the simultaneous optimization of mining complexes, containing multiple open pit and underground mines and processing facilities, through elaborated metaheuristic solvers.
Chapter 4 – Conclusions and Future Research

4.1. General Conclusions

The current industry practice for strategic underground mine planning relies on a stepwise planning process. The stope layout is first designed, followed by the development network design of hauling and service openings connecting the economic and feasible material volumes. Subsequently, the underground mine production scheduling is optimized so as to maximize the life-of-mine net present value (NPV). It has been shown that sequential mine planning approaches lead to suboptimal solutions substantially impacting the profitability of a mine. Additionally, only a few methods for each of the above-mentioned underground mine planning steps attempt to integrate material supply uncertainty through stochastic optimization models. This observation has motivated the two stochastic integer program formulations for mines employing the sublevel open stoping mining method presented in this thesis. The first model focuses solely on the stope design optimization under grade uncertainty. The second aims to jointly optimize the stope design and production schedule.

The first method presented in this thesis, in Chapter 2, refers to stochastic stope design optimization. The method uses geostatistical simulations and vertical and horizontal development distances to integrate, respectively, grade variability and uncertainty and development costs into the designing process. The proposed objective function seeks to maximize the undiscounted profit from selected production levels and stopes. At the same time, the related undiscounted shaft and drifts development costs are minimized, while the upside potential in terms of recoverable metal of the generated stope layout is maximized. The correspondent case study consists of an underground gold mine having two geotechnical domains with different stope sizes, pillars requirements, level spacing, and three potential shaft locations. The output stope layout has well-defined and mineable production levels and stopes separated by pillars and a selected shaft location driven by the high-grade zones within the mineral deposit. Another stope design was also performed using the Mineable Shape Optimizer (MSO), the deterministic industry-standard stope design software tool. The proposed method outperformed the deterministic counterpart by 40% higher undiscounted profit and 21% recoverable metal. These results highlight how the smooth
estimated orebody models used as an input for deterministic approaches misguide the definition of stope location and size.

The second proposed method detailed in Chapter 3 extends the previously proposed stochastic stope design towards the goal of this thesis, which is to develop and apply an integrated stochastic optimization method of sublevel open stoping design and long-term mine production scheduling. The mathematical model aims to maximize the NPV from the scheduled stopes and minimize the shaft, drifts and crosscuts development costs and level-based operating costs while managing the risk of not meeting yearly production targets. The proposed integrated stochastic optimization is applied at an underground gold mine providing an optimal and mineable stope layout as an output of the life-of-mine production scheduling optimization. This generated schedule is compared to a sequential stochastic optimization approach, in which the stope design is obtained with the method from Chapter 2, followed by optimizing long-term mine production scheduling. By capitalizing on the inherent synergies for mine design and schedule, the integrated approach produced a schedule with a meaningful higher NPV of 11% and a two-year shorter life-of-mine compared to the sequential framework. These results demonstrate how additional value can be added to mining operations by jointly optimizing the involved steps of the underground mine planning.

4.2. Recommendations for Future Research

Future work in integrated stochastic optimization of sublevel open stoping design and long-term production scheduling can incorporate various sources of uncertainty and additional mining operational aspects as well as improve the solution method. First, as explained in the third chapter of this thesis, the integration of uncertainty in metal prices is straightforward and should be further investigated. Other sources of uncertainty, either spatial, such as geotechnical and geometallurgical, or temporal, such as seismicity and operational costs, highly impact long-term underground mine production schedules and should be considered. Second, multiple processing destinations, stockpiles, and backfilling options are existing features in some underground mines and should also be an extension of the proposed integrated method. Finally, dealing with large underground mine datasets and adding more components and sources of uncertainty lead to an increased number of joint-uncertainty scenarios and millions of related integer decision variables.
The implementation of efficient solvers, combining decomposition and machine learning techniques as well as meta-, mat- and hyper-heuristic algorithms, thus, becomes imperative for the tractability of such large and non-linear mathematical models, as already successfully implemented for complex open-pit mining cases.

Several different underground mine methods, such as cut-and-fill, room-and-pillars, sublevel caving, among others, are employed separately or combined with other methods in real-world underground mines. These methods have a different layout, unitary operations, activities precedence, and geotechnical constraints. Thus, specific mathematical formulations for jointly optimizing mine design and long-term mine production scheduling are still necessary for other mining methods, besides the sublevel open stoping, which was the focus of this thesis. Ultimately, the integrated stochastic approach developed for a single mining method should be coupled with models related to other mining methods, allowing the optimization for underground mines employing combined types of operation.

The simultaneous stochastic optimization of mining complexes has been shown to unlock substantial value for operating mineral value chains containing multiple mines, stockpiles, multi-metal processing facilities, and waste/tailings disposal units. To date, when underground mines are considered, they are not fully simultaneously optimized with the other components of the mining complex, and the mine design and the extraction sequence are required inputs for further optimization of the long-term production schedule of the entire mineral value chain. Additional developments should focus on incorporating the proposed integrated stochastic underground mine planning process into the simultaneous optimization of mining complexes. In doing so, the synergies between open-pit and underground mines and other facilities will be captured, given the uncertainty on market and material supply.

Finally, the time and location for transitioning from open-pit to underground mining remain an area of increasing interest. Available transition approaches also rely on the commonly used sequential optimization of underground mine planning. Multiple input underground mine designs are usually generated for possible transition depths to determine the economic value of underground mining blocks or for the underground mine production scheduling part of the
transition optimization problem. Again, the proposed integrated stochastic method opens new avenues for optimizing the transition timing and depth by enforcing physical constraints to avoid overlapping open-pit and underground mining operations based on crown pillar requirements.
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