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Filtration of Liquid Aluminum with Reticulated Ceramic Filters

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June 1994

A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfilment of the requirements of the degree of Doctor of Philosophy

^o C. Tian 1994



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Abstract

Fluid flow and filtration phenomena associated with filtration of liquid aluminum using reticulated ceramic filters were systematically investigated. It was found that fluid flow through this type of filter could be described by Darcy's Law up to a Reynolds number of 8, and the permeability of this type of filter ranged from about $9x10^3$ to $4x10^{-2}$ mm².

Parameters affecting filtration processes during the initial period were identified, some of which could be quantified numerically using a 2-D computational domain. According to these numerical analyses, the clean filter coefficient for this type of filter was linearly dependent on the dimensionless Stokes velocity of the suspended particles, had a -0.96 power dependence on the Peclet number, a -6.93 power dependence on the effective porosity of the filter, and exhibited only a weak dependence on the Reynolds number, in the Darcy velocity regime.

The dynamic behaviour of this type of filter was analyzed theoretically and simulated numerically using newly proposed correlations relating the filter coefficient and the pressure drop to the amount of particles captured within the filter (the specific deposit), and a model describing the morphology of captured particles. The simulated results showed that the filtration efficiency and the pressure drop increased with inlet particle concentration and filtration time; these increases were however, insignificant when the inlet particle concentration was less than 1 ppm for filtration periods of two hours, however, when the inlet concentration (initial and continued) reached 10 ppm, the change became appreciable.

Experimental data, obtained from liquid aluminum filtration tests conducted by the author in both laboratory and industrial settings, compared favourably with the numerical results.

Résumé

Les phénomènes d'écoulement de fluide et de filtration associés à la filtration de l'aluminium liquide à l'aide de filtres réticulaires de céramique ont été étudié systématiquement. L'étude a démontré d'abord que les écoulements de fluide à travers ce type de filtre peuvent être décrit par la loi de Darcy pour des nombres de Reynolds allant jusqu'à 8, et que la perméabilité de ce type de filtre varie de $9x10^{-3}$ à $4x10^{-2}$ mm².

Les paramètres affectant le procédé de filtration pendant la période initiale ont été identifié. Certains d'entre eux ont pu être quantifiés numériquement à l'aide d'un modèle mathématique à 2-D. D'après ces analyses numériques, le coefficient de filtre propre pour ce type de filtre était linéairement dépendant de la vitesse adimentionelle de Stokes des particules en suspension, avait une dépendance de puissance de -0.96 avec le nombre de Peclet, avait une dépendance de puissance de -6.93 avec la porosité effective du filtre, et, démontrait une faible dépendance au nombre de Reynolds, dans le regime de vitesse Darcy.

Le comportement dynamique de ce type de filtre a été analysé théoriquement et simulé numériquement en utilisant des corrélations originales qui lient le coefficient du filtre et la perte de pression à la quantité de particules capturées (les dépôts spécifiques) dans le filtre, ainsi qu'avec un modèle qui décrit la morphologie des particules capturées. Les résultats des simulations ont démontré que l'efficacité de la filtration ainsi que la perte de pression augmentaient avec la concentraton de particules à l'entrée et avec le temps de filtration. Ces augmentations étaient cependant peu importantes lorsque les concentrations de particules à l'entrée étaient de moins de lppm pour des periodes de filtration de deux heures. Par contre, lorsque la concentration (initiale et continue) atteignait 10ppm, le changement devenait appréciable.

Les données expérimentales, obtenues lors de tests de filtration d'aluminium liquide conduits en laboratoire etr en milieu industriel, se comparent favorablement avec les résultats expérimentaux.

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List of Symbols

- a constants.
- a_i coefficients.
- A area.
- b constants.
- c particle concentration; constants.
- c_0 concentration of suspended particles at filter inlet.
- c_L concentration of suspended particles at filter outlet.
- c_a concentration of suspended particles after filter.
- c_b concentration of suspended particles before filter.
- c_i concentration of ith type particles.
- C normalized concentration (c/c_0) ; constants; coefficients.
- C_p discharge coefficient.
- d diameters; constants.
- d_p diameter of suspended particles.
- d_{w} diameter of filter webs.
- d_c diameter of collectors.
- d_a diameter of packed granules or cylinders.
- d_o orifice diameter of LiMCA probes.
- D diffusivity.
- D_{MB} Brownian diffusivity of suspended particles.
- D_i diffusion conductance.
- e east.
- e, removal efficiency of a representative piece of filter.
- e_{a} removal efficiency of a Unit Bed Element.
- *E* filtration efficiency of a filter; east node.

- E_a apparent filtration efficiency of a filter.
- ΔE_{iot} total energy change.
- f friction factor; functions.
- F function.
- F_i dimensionless mass flow rates.
- F forces.
- g gravitational acceleration.
- G signal gain.
- h metal head.
- H Hamaker constant
- k permeability; Boltzmann's constant.
- K kinetic index.
- K_0 kinetic constant.
- K_0 empirical constant.
- K_0 empirical constant.
- *l* thickness of a Unit Bed Element; characteristic length.
- L filter thickness; Lifshitz-Van der Waals constant.
- m empirical constant.
- m_p mass of particles.
- n north.
- N denotes numbers; north node.
- N_G gravitational number, or normalized Stokes velocity $[(\rho_p \rho)gd_p^2/(18\mu V_{st})]$.
- N_R interception parameter (d_p/d_w) .
- *p* hydrodynamic pressure.
- p(t) normalized pressure gradient.
- *P* pressure; dimensionless pressure $(p/\rho V_{sf}^2)$; penetration.
- Para atmosphere pressure.
- Pe Peclet number $(V_{J}d_{v}/D)$.
- P_i defined as F_i/D_i .

- q number of atoms per unit volume of a macroscopic body.
- Q volume flow rate.
- r radius of a sphere.
- r_f radius of a fluid sphere.
- R_{+} residual.
- *Re* Reynolds number $(\rho V_{sf} d_{\mu}/\mu)$.
- $R_{\rm s}$ defined in Figure 62.
- s south.
- S source term; south node.
- S_{\star} surface to volume ratio or specific surface area.
- S_o specific surface area (total surface area per unit volume of a material).
- Sc Schmidt number.
- Sct terbulent Schmidt number.
- Sk Stokes number $(\rho_p d_p^2 V_{st}/18 \mu d_w)$.
- t denotes time.
- T absolute temperature; dimensionless time (tV_{st}/d_{w}) .
- u velocity.
- u_p velocity of suspended particles.
- *u* velocity in *x* direction.
- u_0 characteristic velocity.
- u_{∞} terminal or approaching velocity.
- U dimensionless velocity (u/V_{s}) .
- v velocity in y direction.
- v_i interstitial velocity.
- V velocity.
- V dimensionless velocity (v/V_s).
- V_{sf} superficial velocity.
- V_s volume of solids.
- δV voltage pulse.

- w west.
- W west node.
- x x axes in Cartesian coordinate.
- $X = x/d_y$.
- y y axes in Cartesian coordinate.
- $Y = y/d_w$.
- z z direction in one dimensional Cartesian coordinate.
- α empirical constant; specific deposit.
- α_i specific deposit of ith type of particles.
- B empirical constant.
- δ distance of separation; roughness of filter web surfaces.
- **r** general diffusion coefficient.
- σ surface tension.
- σ_{mp} interfacial tension between melt and particles.
- σ_{mf} interfacial tension between melt and filter.
- Φ denotes general dependent variables.
- μ viscosity.
- ε porosity.
- ε_{*} porosity of agglomerates of captured particles.
- ε_{t} total porosity of filter.
- $\varepsilon_{\rm s}$ effective porosity of filter.
- ε_d dead porosity of filter.
- Idenotes angles formed between filter web surface normal and gravity at point of contact of a particle.
- φ denotes shape factor of particles.
- ρ density of fluid.
- ρ_{p} density of suspended particles.
- λ filter coefficient.

- λ_0 clean filter coefficient.
- η removal efficiency associated with an single collector.
- η_t efficiency of transportation.
- η_{\bullet} efficiency of adhesion.
- τ denotes time; shear stress.
- θ denotes time.
- electrical resistivity.

Introduction

1.1. Impurities in Aluminum Melts and Their Adverse Effects

Harmful impurities in aluminum and its alloy melts, generally speaking, include dissolved hydrogen, alkali trace elements and inclusions. Although hydrogen is unique in that it can dissolve in molten aluminum in significant amounts (Painchaud and Martin, 1989), its solubility drastically decreases upon solidification. As a result, most of the dissolved gas will come out of the melt during solidification, creating voids or pores in the solidified products if the melt is not degassed prior to casting. This process will be aggravated if inclusions also exist, since the dissolved hydrogen nucleates on these inclusions and may sometimes form catastrophically large pockets of gas (Eckert, 1991, see Figure 1). In order to prevent porosity formation in cast products, dissolved hydrogen has to be at least less than 0.15 cm³/100 g (Varley, 1970). However, if a melt is filtered, it may be more tolerant to the amount of dissolved hydrogen. Sigworth (Sigworth, 1993) found a stark difference between filtered and unfiltered samples in the amount of porosity generated. Figure 2 shows the density difference between the filtered and unfiltered samples (Brondyke and Hess 1964).

The commonly encountered inclusions in aluminum melts are oxides, e.g., Al_2O_3 , Al_2O_3 -MgO, etc., aluminum carbides (Al_4C_3), titanium boride (TiB₂), etc.. These particles, apart from being porosity inducers during solidification, can also be detrimental to the mechanical properties of a metal product such as an aircraft part. A hard non-metallic inclusion will act as a stress intensifier while the part containing it is in service

Chapter 1: Introduction

(under stress). This could lead to an initiation of cracking in the intensified stress field around the inclusion, thus impairing the fatigue properties of the part. In addition, these non-metallic particles can cause unacceptable surface defects (Mollard, 1980) and other process problems, thus reducing the yield and consequently increasing costs. Problems associated with inclusions in can sheet and foil manufacturing are rather apparent. According to Szekely (1976), inclusions as small as 5 μ m can be harmful to aluminum foil products.

The alkali elements, namely, sodium and calcium are undesirable in some magnesium-containing alloys since they are believed to cause embrittlement (Ransley and Talbot, 1959) and edge cracking during hot deformation (Yoshida et al, 1985).

1.2. The Available Refining Techniques

Various refining techniques are available for the industries. These include degassing to remove hydrogen; fluxing to remove alkali elements; floatation and sedimentation to remove



Figure 1. Effect of inclusions on the precipitation and distribution of dissolved hydrogen, (Eckert, 1991).

inclusions. Most of the refining processes available commercially are multi-functional. For instance, SNIF (Spinning Nozzle Inert Floatation process) has the function of both inclusion and hydrogen removal, because the inert purging gas not only can capture inclusions (due to collision between gas bubbles and inclusion particles) but can also provide an opportunity for dissolved hydrogen to diffuse out and consequently be carried away when the purging gas finally escapes the melt. Chlorine fluxing has been known to be effective to remove alkali elements (Kulunk and Guthrie, 1992; Kulunk, 1992), hydrogen and inclusions (Tian, Kulunk and Guthrie, 1991). Because of its atmospheric pollution and in-plant corresion problems, chlorine should be used sparingly and alternative methods should be sought whenever possible.

As far as inclusion removal is concerned, the simplest methods would be sedimentation or floatation (Martin et al, 1988; Sztur et al, 1990; Gauckler et al, 1985). By holding the furnace at rest, inclusions can settle down or float up depending on their density. Although the methods are simple to realize, they may not be effective to remove small inclusions since the elimination rate is limited by Stokes' law. For example, the settling velocity of a 5 μ m alumina inclusion



Figure 2. Effect of filtration on the apparent density of aluminum casts (Brondyke and Hess 1964).

in a quiescent bath of liquid aluminum would be about 3 cm/h. In addition, prolonged holding may not be economical because of heat loss and/or reduction in productivity (Dupuis and Dumont, 1993). The effectiveness of in-line refining systems, e.g., SNIF, ALPUR, etc. may also be limited by the short residence time of the metal in the refining units. Because of this, most cast shops use filtration as an alternative or further step to

remove inclusions (Neff, 1990; Dumont et al, 1992).

1.3. Filtration

In-line filtration was introduced into the aluminum industry in the late 1950s (Eckert, 1991). Since then, various filtration systems have been developed. The development of the Ceramic Foam Filter (CFF) or reticulated ceramics in 1974 (Dore and Yarwood, 1976) greatly expanded the application of filtration refining techniques in the aluminum and foundry industries. Today, a majority of the primary aluminum is filtered prior to casting (Apelian and Shivkumar, 1989). Various in-line filtration systems are now in operation in different companies (Hoffman and Soble, 1986). According to Aubrey and Dore (1993), in 1992 eight million metric tons of aluminum were filtered with CFF alone, equivalent to almost 50% of the total production of aluminum in the world. In the meantime, research has been carried out in an attempt to find out the influential parameters upon which removal efficiency of inclusions depends, and the filtration mechanisms involved. Despite these efforts, many questions still remain to be answered, especially for those filtration processes with reticulated ceramic filters.

1.4. Objective of This Study

Since reticulated ceramics are the most widely used filter media in metal processing due to their integrity and excellent adaptability to various existing metallurgical systems, this study will particularly focus on those filtration processes that use this type of filter. The objective of this study is to systematically investigate fundamental aspects of the filtration processes such as the fluid flow characteristics inside the ceramic filter and the influences of the various parameters on filter performances, thus providing guidelines for the design or optimization of filtration systems using reticulated ceramic filters.

1.5. Basic Concepts of Filtration

The objective of this section is to acquaint the reader with the basic concepts and associated terms commonly used in this field, and to inform the reader in which category this study belongs.

1.5.1. Basic Concepts

Filtration is a process of separating dispersed particles or second phases from a dispersing fluid by means of porous media. The dispersing fluid can be a gas or liquid and correspondingly the filtration can be classified into aerosol (if the fluid is a gas), hydrosol (if the fluid is an aqueous liquid) and perhaps 'metsol' (if the fluid is liquid metal, shall we add this word to our vocabulary) filtration. Examples of filtration can be easily found in our day-to-day life. For example, in coffee making, a paper filter is used to separate the solid cafe grains; in a car, an air filter is used to prevent dusts from entering the combustion engine. Filtration has long been an established technology in other science and engineering fields such as aerosol science and chemical engineering where the liquids are gaseous and aqueous at relatively low temperatures. While liquid metal filtration may be different from conventional filtration processes, nevertheless, the traditional concepts and theories of filtration can still be useful in analysing the filtration processes.

1.5.2. Liquid Filtration Modes

Liquid filtration can be classified into three operational modes, i.e., screening, cake and deep bed or depth filtration depending on the specifications of the suspended phase, liquid phase and the filtering medium:

Screening mode. In this case, the filter medium acts as a screen which retains

particles larger than the pore size of the filter medium. The capture mechanism is one of mechanical entrapment.

Cake mode. In this case, the particles are retained by a layer of accumulated particles (cake) on the filter surface. The first layer of the cake is formed according to screening mode, and oncoming particles will be retained virtually by the already retained particles instead of by the filter medium itself. As the cake becomes thicker and thicker, the pressure drop becomes greater and greater until the permeability becomes too low to maintain the operation, and the process stops. This process can only occur when the concentration of suspended particles is very high.

Deep bed mode. In the depth, or deep bed, filtration mode, the particles to be separated are much smaller than the pore size of the filter medium, So that, the separation process takes place within the filter medium itself. The separation mechanisms consist of two parts: first, the particles have to be transported to the internal surfaces of the filter medium according to some mechanisms, and second, they have to be retained there by some forces.

In a real filtration process, however, more than one mode may take part in a single operation. In liquid metal filtration, a majority of the inclusions are much smaller than the pore size of the filter (Apelian and Choi, 1986), therefore, the filtration should be in a depth mode. However, if the melt is dirty, the accumulation of particles inside the filter may quickly fill in the pores and lead to caking formation - change of filtration mode. Some large exogenous inclusions or dross may be captured according to the screening mode because their size may exceed the pore size of the filter.

Figure 3 provides a schematic of the three modes of filtration.

Filtration Theory



Screening filtration



Cake filtration



Deep bed filtration

Figure 3. Schematic of different filtration modes.

1.5.3. General Mechanisms of Depth Filtration

As mentioned before, the capture mechanisms in the depth filtration mode consist of transport and adhesion of particles to the filter's internal surfaces. The transportation mechanisms may include interception, diffusion, inertial impaction, sedimentation due to gravity and hydrodynamic effects. Adhesion mechanisms can be quite complex and are very often difficult to evaluate. In general, whether a particle adheres to the solid filter surfaces or not depends on the resultant forces acting on the particle. These forces can be interfacial forces, van der Waals' forces, coulombic attraction forces, electrical double layer or zeta potential forces, hydrodynamic forces, sintering effects, etc., depending upon a specific filtration system.

1.5.3.1. Transportation Mechanism

Interception. When the fluid flow is laminar across a solid body, e.g., a filter web, the streamlines will diverge to flow around it and re-converge behind it. If the particles carried by the fluid have the same density then, in the absence of other than hydrodynamic forces, they would move with the streamlines. If a particle in the fluid is in such a position that the distance from its centre to the surface of the filter during its passage through the latter is less than the collision diameter, i.e., $d_p/2$, it would contact the surface of the filter and be intercepted.

Brownian Diffusion. Small particles suspended in the fluid will experience a random diffusional movement due to collisional energy transfer by the molecules of the suspending fluid. This process is termed Brownian diffusion, with a diffusion coefficient D_{ME} given by Einstein's (1905) equation:

$$D_{MB} = \frac{kT}{3\pi\mu d_p} \tag{1}$$

where k is the Boltzmann's constant, T, the absolute temperature, μ , the viscosity of the

fluid, and d_p , particle diameter.

Turbulent Diffusion. If the flow is turbulent, a turbulent diffusivity will be added to the Brownian diffusivity to yield effective diffusivity, Γ_e ,

$$\Gamma_e = \frac{\mu}{Sc} + \frac{\mu_r}{Sc_r}$$
(2)

where Sc, Sc, are the Schmidt and turbulent Schmidt numbers respectively, and μ_t the turbulent viscosity.

Inertial Impaction. Inertial impaction occurs when a particle has sufficient inertia to maintain its trajectory as the flow streamlines diverge around the filter media. Inertial impaction is related to a Stokes number, *Sk*, according to Tien (1989), by:

$$Sk = \frac{\rho_p d_p^2 u_0}{18 \mu d_c} \tag{3}$$

where ρ_p is the density of the particles, d_p , d_c is the diameter of the particles and collector respectively and u_0 is a characteristic velocity.

Sedimentation. A particle may leave the streamline due to gravitational forces and contact the filter surfaces while the fluid flows around the filter. The significance of this effect can be characterized by a dimensionless group, N_G , which is defined as the ratio of the Stokes sedimentation velocity to the approach velocity, u_{m} , of a particle:

$$N_{G} = \frac{(\rho_{p} - \rho)d_{p}^{2}g}{18\mu u_{-}}$$
(4)

where g is the gravitational acceleration. Other quantities are the same as defined before.

Hydrodynamic Effects. When a particle is subjected to a shear gradient, i.e., in the boundary layer, it may experience unbalanced hydrodynamic forces such as form and friction drags. As a result, it may cross the fluid streamlines and contact the filter surfaces (Vasseur, 1973). In addition, at higher Reynolds number, flow reversals (wakes) may occur behind the solid filter webs and inclusions may be brought to filter surface in this manner.

1.5.3.2. Adhesion Mechanisms

As mentioned earlier, once a particle is transported to a collector, whether it can adhere to the surface or not depends on the force balance. If the attraction forces are greater than the repulsive or dislodging forces, the particle will be retained by the collector, otherwise, it will be torn away from the collector surfaces.

If η denotes the collection efficiency of a single collector, η_c denotes the collision efficiency, which is the sum of all collisions due to different mechanisms, η_a denotes the attachment or adhesion efficiency, then:

$$\eta = \eta_c \eta_c \tag{5}$$

Adhesion phenomena are so common to us that we take it for granted without questioning why. For instance, things can get dirty in air due to dusts adhesion to them. Another common observation is that when something is wet, the adhesive forces become stronger. A simple experiment will immediately convince you, use your hand to touch some power such as sands or soil, when it is dry, you may only observe some tiny dusts on your hand, however, when your hand is wet, you will see large particles will stick to your hand. Although such phenomena are common, a rigorous theoretical explanation may not be easy to achieve.

1.5.4. Scope of Study

This study will focus mainly on those phenomena, such as fluid flow and filtration processes, associated with depth or deep bed filtration mode. It aims to identify and, wherever possible, to quantify theoretically or experimentally, the various influential parameters (or mechanisms) involved in liquid metal filtration using reticulated ceramic filters.

1.6. Structure of This Thesis

This thesis is composed of six chapters. The first chapter, as seen, gives an introduction to the basic concepts of filtration as well as to the structure of the thesis; the second chapter covers the water modelling investigation of fluid flow characteristics through reticulated ceramic filters; the third chapter presents the numerical investigations of the initial filtration processes; the fourth chapter presents theoretical analyses on the dynamic behaviour of liquid metal filtration; the fifth chapter presents the experimental work of liquid aluminum filtration; while the sixth chapter summarizes the conclusions of this study.

Fluid Flow Characteristics

2.1. Introduction

Filtration processes are closely related to fluid flow characteristics within porous media. Therefore, in order to gain a fundamental understanding of filtration processes through reticulated ceramic filters, the fluid flow characteristics through this type of filter must first be understood. To the author's knowledge, no systematic investigation with respect to this type of filters, which is different to *common* porous media to which all the literature refers, has been carried out.

This chapter, therefore, presents an investigation of fluid flow characteristics for reticulated ceramic filters, along with an introduction of general concepts associated with porous media and a critical literature review on fluid flow through porous media. The experiments were carried out by introducing water through these filters thereby determining the permeability of these filters. The permeability thus determined was comparable with those calculated from the Kozeny-Carman equation.

2.2. Porous Media

2.2.1. General Concepts of Porous Media

Porous Media The term used in the aforegoing text, is defined as solid bodies that contain *pores*. Intuitively, *pores* are void spaces, their sizes are thought to be
intermediate between *caverns* and *molecular interstices*. Pores in a porous media may be interconnected or non-interconnected. Only an interconnected pore system can allow fluid to flow through. Therefore, a porous media can be a paper, cloth, beds of sands, ceramic bodies, etc., in short, anything with interconnecting holes within it.

Porosity The geometry of the pores in a porous media can be regular such as encountered in monolithic filters, or irregular, such as beds of sand. A general quantity characterizing porous media is termed as *porosity*, which is defined as the ratio of void volume to total volume and can be expressed as:

$$\varepsilon = 1 - \frac{\rho_b}{\rho} \tag{6}$$

where ε denotes porosity, ρ_b and ρ are the apparent (or bulk) density and the absolute density of the porous material itself, respectively.

The porosity given in equation (6) represents the total porosity. In order to obtain the *effective* porosity (those that contribute to fluid flow), other methods are needed. One of the methods would be to immerse the known bulk volume of porous media into a liquid, and by measuring the difference of the liquid volume, the effective porosity can be known.

Specific Surface Area Another important parameter of a porous media is the specific surface area or surface to volume ratio, S_v , which is defined as the surface area of the pores per unit bulk volume of porous media. Specific surface can be measured in many ways such as gas adsorption, optical techniques, fluid flow methods, etc..

Pore Size Distribution Very often the sizes of pores in a porous medium are, except regular pore filters, not uniform. Instead, they vary over a certain size range that can usually be characterised by a normal distribution. Figure 4 shows measured pore size distributions of reticulated ceramic filters (Black and Morris, 1992).

2.2.2. Characteristics of Reticulated Ceramic Filters

The reticulated ceramic filter, also known as a Ceramic Foam Filter, was first developed in 1974 (Dore and Yarwood, 1976). Since then, this type of filter has been used to clean (filter) liquid metals, especially aluminum.

One of the major characteristics of this type of filter, compared to other types of filter, is its high porosity (normally in the range of 80%-90% for liquid metal filtration). Because of this, this type of filters will impose very little resistance to fluid flow and consequently a high productivity would result. The structure of this type of filter can be imagined as webs or struts joined together in three



Figure 4. Pore size distribution of reticulated ceramic filters (Black and Morris, 1992).

dimensions, forming a reticulated network. The webs are cylindrical in nature with an internal hollow formed as a result of evaporation of the organic precursor during firing of the ceramics. Figure 5 shows (a) the typical structure, (b) a cross section, of this type of filter.

Reticulated ceramic filters are manufactured by coating an organic precursor with ceramic slurry. The coated precursor is then fired at desired temperatures. During firing, the organic precursor evaporates and the ceramic coating forms a rigid ceramic body. The coating process is performed by dipping the precursor into the ceramic slurry, and then the soaked precursor is squeezed to remove excess ceramic slurry. Very often, the



Figure 5. (a) Typical structure, (b) a cross section, of reticulate ceramic filters.

excess slurry can not be removed completely, leaving some pores 'blocked'. The percentage of blocked pores increases as the pore size gets finer.

The pore size of the reticulated ceramic filters is measured as pores per linear inch (ppi), and can vary from 3 to 100 ppi. The nominal pore size commonly used for liquid metal filtration ranges from 10 to 40 ppi. The average web thickness of this type of filters is typically 200 to 500 μ m for aluminum filtration. Because of their very low thermal mass, these filters have excellent thermal shock resistance, which is an important parameter in molten metal filtration.

2.3. Critical Literature Review on Fluid Flow through Porous Media

Due to the vast amount of literature in this field, it is too lengthy to include a comprehensive literature review in this thesis. Therefore, only a critical review mostly relevant to the present studies, is given below. A broader review can be found in the books by Carman (1956), Bear (1972), Scheidegger (1974), Dullien (1979).

The general equations governing fluid flow are the Continuity and Navier-Stokes equations. They can be written in the form of vectors as follows (Guthrie, 1989):

$$\frac{\partial \rho}{\partial t} + \nabla (\rho V) = 0 \tag{7}$$

$$\frac{\partial}{\partial t}(\rho V) = -[\nabla \cdot \rho VV] - \nabla P - [\nabla \cdot \tau] + \rho g \qquad (8)$$

where ∇ is a gradient operator, $\nabla \cdot$ is a divergence operator.

For an incompressible Newtonian fluid of constant viscosity, equation (8) becomes:

$$\rho \frac{DV}{Dt} = -\nabla P + \mu \nabla^2 V + \rho g \tag{9}$$

where D is the substantial derivative operator.

2.3.1. Darcy's Law

The macroscopic empirical equation governing fluid flow through porous media is, at low flow rates, given by Darcy's law:

$$Q = -\frac{kA}{\mu}\frac{\Delta P}{L} \tag{10}$$

where A is the total cross sectional area perpendicular to the flow direction, P is the pressure, L is filter thickness measured in the flow direction. k is the permeability of the porous medium. Equation (10) can be rewritten as:

$$V_{\rm sf} = \frac{Q}{A} = -\frac{k}{\mu} \frac{\Delta P}{L} \tag{11}$$

where V_{sf} is the superficial velocity. Equation (11) is applicable to situations where the flow is one dimensional and perpendicular to the direction of gravity. For other situations, a more general form of equation should be used (Richardson, 1961):

$$V_{if} = -\frac{k}{\mu} (\nabla P + \rho g)$$
(12)

Equation (12) is applicable to isotropic porous media. For anisotropic media, the values of permeability in different directions are, in general, different. A detailed discussion can be found in the article given by Rice et al (1970)

The value of the permeability k depends on the physical properties of a given porous medium such as porosity, specific surface area, tortuosity, etc.. Numerous theoretical and empirical models and experimental investigations have been made in an attempt to develop a correlation between permeability and the geometrical properties of the porous media. Today, the widely accepted correlation is, despite its criticism, the Kozeny-Carman equation developed by Kozeny (1927) and later on modified by Carman (1937):

$$k = \frac{\varepsilon^3}{5S_0^2(1-\varepsilon)^2}$$
(13)

Inserting equation (13) into Darcy's equation, one obtains:

$$\frac{\Delta P}{L} = \frac{5S_0^2(1-\varepsilon)^2}{\varepsilon^3} \mu V_{q}$$
(14)

where S_0 is the Carman's specific surface exposed to the fluid per specific unit volume of solid (not bulk volume) and is related to S_0 , through the following equation:

$$S_{\nu} = S_0(1-\varepsilon) \tag{15}$$

Other porosity functions different from Kozeny-Carman equation proposed by various authors can be found in the papers given by Rumpt and Gupte (1971), Dullien (1975) and Knauf and Doshi (1986).

The Kozeny-Carman equation was developed based on hydraulic radius theory. Other models used include capillaric, statistical, network, drag theory, etc. (statistical and network models are not reviewed here, but interested readers are referred to Dullien's book, 1975). The development of the hydraulic radius theory is based on the fact that the permeability of a porous media has a dimension of a length squared. Such a length may be called the hydraulic radius of the porous media. Kozeny's theory assumes that the porous media consist of channels of various cross-sections, but of definite length, which is a better representation than the capillary model. Kozeny (1927) assumed, in the derivation of his equation, that the flow was normal to the cross-section of the channels with no tangential component. This has been criticized by Scheidegger (1974) for not allowing for convergent and divergent flow which may occur in porous media. Other assumptions common to hydraulic radius theory, according to Carman (1941,1948), include: all pores are open to flow; pores are distributed at random and are reasonably uniform in size; porosity is not too high; and diffusion and surface effects are absent.

Interestingly, very similar equations to the Kozeny-Carman equation were also developed but based on completely different approaches: the Ergun and Orning (1949) and Scheidegger (1974) equations were based on straight capillaric models, while Irmay (1958) based his equation on averaged Navier-Stokes equations.

In the capillaric model, the porous media is modelled as some kind of capillaric tubes for which an analytical solution of fluid flow is readily obtained (i.e. the Hagen-Poiseuille equation). In drag theory, the resistance of the porous media to flow is thought to be due to the drag of the fluid on the solid surfaces of the porous media. Drag theory was found to be more successful for highly porous media such as packed fibres. The general theoretical approach in this case is to idealize the porous medium as a matrix of rods which are arranged in a certain periodic pattern such as into a square or staggered array. In the idealized array, a 'unit cell' is chosen so that the flow in the whole array is just a repeat, or reappearance, of flow within the unit cell. The Stokes equation is then solved analytically within the unit cell, by assuming a non-slip condition on the surfaces of the rod and a zero velocity gradient (Happel, 1959) or zero vorticity (Kuwabara, 1959) at the boundary of the unit cell. In this regard, a good review can be found in Jackson and James's paper (1986).

For the case of flow perpendicular to cylinder arrays, Happel (1959) presented the following expression for the permeability, k:

$$k = \frac{r_f^2}{4} \left[\ln \frac{r_f}{r} - \frac{1}{2} \left(\frac{r_f^4 - r^4}{r_f^4 + r^4} \right) \right]$$
(16)

where r is the radius of the cylinder and r_r is the radius of the fluid envelope.

2.3.2. Non-Darcy Flow

As mentioned earlier, Darcy's law is valid at low flow rates, i.e., when the inertial forces are negligible in comparison with viscous forces. At high flow rates, Darcy's law breaks down. Many efforts have been expended on finding a critical Reynolds number beyond which Darcy's law becomes invalid. However, owing to the uncertainty or ambiguity of a characteristic length in the determination of the Reynolds number for, as well as the complexity of, porous media, a universal criterion is not found. A great discrepancy regarding the *critical* Reynolds number for the transition from Darcy to non-Darcy flow has been reported by many investigators, values varying from 0.1 (Nielsen 1951) to 75 (Plain and Morrison 1954). Nevertheless, non-Darcian flow for a given porous medium is not difficult to judge, by simply plotting the flow rate or superficial velocity versus the pressure gradient: If a straight line passing the origin results, the flow is Darcian, otherwise, it is non-Darcian.

Heuristic correlations between pressure drop and flow rate for non-Darcian flow regime can be obtained by modifying Darcy's equation with the inclusion of a second order term in velocity, as suggested by Forchheimer (1901):

$$\frac{\Delta P}{L} = aV_{sf} + bV_{sf}^2 \tag{17}$$

where a, b are constants, depending on a specific system.

Another type of heuristic equation takes the following form (Missbch, 1937:

$$\frac{\Delta P}{L} = V_{sf}^{\pi} \tag{18}$$

where m ranges between 1 and 2.

Actually, it was Osborne Reynolds (1900) who first formulated the pressure drop through granular bed as being the sum of two terms, one proportional to the first power of the fluid velocity, the other to the product of the density of fluid and the second power of its velocity:

$$\frac{\Delta P}{L} = a V_{sf} + \beta \rho V_{sf}^2$$
(19)

where a and β are system dependent factors.

The Reynolds equation was further developed by Ergun and Orning (1949) who, by adding a kinetic energy loss term to Poiseuille equation, developed an equation as (see Appendix I):

$$\frac{\Delta P}{L} = 2C_1 \frac{(1-\epsilon)^2}{\epsilon^3} S_0^2 \mu V_{sf} + \frac{C_2}{8} \frac{(1-\epsilon)}{\epsilon^3} \rho S_0 V_{sf}^2$$
(20)

where C_1 and C_2 are correction factors, other terms being the same as defined before.

Hitchcock (1926) found, on the basis of theoretical grounds, that the value of C_1 approximately equals $(\pi/2)^2$, giving a value of 4.935 for $2C_1$, which is very close to 5 - the Carman constant in the Kozeny-Carman equation.

Later on, Ergun (1952) examined the experimental results obtained by flowing CO_2 , N_2 , CH_4 and H_2 gases through packed beds of spheres, sand, and pulverized coke, arriving at the following equation:

$$\frac{\Delta P}{L} = 150 \frac{(1-\varepsilon)^2}{\varepsilon^3} \frac{\mu V_{sf}}{d_s^2} + 1.75 \frac{(1-\varepsilon)}{\varepsilon^3} \frac{\rho V_{sf}^2}{d_s^2}$$
(21)

where d_r is the diameter of the packed granules.

Macdonald et al (1979) found, after comparing the Ergun equation with the experimental data obtained by Rumpt and Gupta (1971), that the values of the two numerical constants are not definitive. They recommended that, for smooth particles, the values of the two numerical constants should be taken as 180 and 1.8 rather than 150 and 1.75.

Ergun claimed that equation (21) was applicable for all types of flow, because in the limiting case of vanishing superficial velocity, the kinetic term becomes negligible, thereby reducing to Darcy's law; as flow rate increases to an upper limiting condition, the viscous term becomes negligible, and the flow becomes fully turbulent. Although it seems that the Ergun equation has been well established both theoretically and experimentally, there is a slight difference regarding the applicability from Forchheimer equation which has been customarily regarded as being applicable to non-Darcy flow, rather than to all types of flow (see more recent work by Fand et al, 1987). That is perhaps one of the possible reasons why the equation for non-Darcy flow is known as Forchheimer formula rather than Reynolds formula, despite he was the first to put forward the two term equation. The difference, however, has been overlooked, and sometimes, it causes confusions.

Irmay (1958), using statistical methods, averaged the Navier-Stokes equation over homogeneous isotropic porous media, and found that the Forchheimer (or Ergun) formula was theoretically correct. The same conclusion was reached later on by Ahmed and Sunada (1969) who averaged the Navier-Stokes equation and accounted for turbulent fluctuations, and by Slattery (1972), Dullien and Azzam (1973), who used volume averaging techniques.

The transition from Darcian to non-Darcian flow has been customarily believed to be caused by turbulence. It has been argued (Scheidegger 1960) that the breakdown of Darcy's law may be simply due to the growing significance of inertial forces compared to viscous forces, as the flow rate increases. The incidence of turbulence should evidently occur at much higher Reynolds numbers. This was in deed observed by Fand et al (1987) who found, by flowing water through a packed bed of soda-lime glass spheres of 2, 3, and 4 mm diameter, that "Forchheimer" flow starts at a Reynolds number equal to 5, while turbulent flow occurred at 120. According to these authors, fluid flow through porous media over the entire velocity domain should consist of six regimes, i.e., pre-Darcian, Darcian, transition from Darcian to Forchheimer, Forchheimer, transition from Forchheimer to turbulent and turbulent. The existence of a pre-Darcy regime is, according to Bear (1972), attributed to the non-Newtonian behaviour of fluids and the fact that the streaming potential generated by the flow, particularly in fine-grained media, can produce small countercurrents along the pore walls in a direction opposite to that of the main flow. In this region, there exists a finite value of the pressure gradient, below which the superficial velocity is zero. However, there is no precise information concerning the magnitude of the Reynolds number at which the pre-Darcy region starts, though Fand et al (1987) suggested a value of 10^{5} , based on published data on Darcy flow. In the same study, Fand et al (1987) found that in Forchheimer region the experimental data could be presented by the Ergun equation with the two constants taken as 182 and 1.92. In the turbulent region, the Ergun equation was still valid, except that the two constants now attained values of 225 and 1.61 respectively. They further found that the transition from one flow region to another is not an abrupt point but rather a region. They argued however, that due to the difficulties in characterizing these regions mathematically, such regions can be approximated as artificial points, incurring negligible errors as far as most engineering applications are concerned.

2.3.3. The friction factor-Reynolds number correlation

The Darcy and Forchheimer type of equations can be casted in dimensionless form, i.e., the friction factor-Reynolds number correlation, f = f(Re). The friction factor, f, for porous media is generally defined as (Richardson, 1961):

$$f = \frac{\Delta P}{2\rho V_{sf}^2 L}$$
(22)

where l is the characteristic length of the porous media.

Due to the difficulties in identifying the characteristic length in porous media, there is no universal definition for it. Consequently, various definitions have been used by different authors. For backed beds of spheres, l is usually taken as the diameter of sphere or some sort of mean diameter if the spheres are non-uniform in size. Other definitions used include: equivalent diameter for beds of non-spherical particulates such as sands; the reciprocal of specific surface area (Blake 1922); hydraulic radius (Geiger and Poirier 1973); square root of permeability (Ward 1964, Spielman and Goren 1968, Beavers and Sparrow 1969); the ratio of the constant associated with the kinetic term in the Forchheimer equation with the exclusion of the density to the constant associated with the viscous term with the exclusion of the viscosity (Ahmed and Sunada 1969, Macdonald et al 1979, Polat 1989), i.e., β/α :

$$\frac{\Delta P}{L} = \alpha \mu V_{sf} + \beta \rho V_{sf}^2$$
(23)

(α , in fact, is the reciprocal of the permeability in Darcy's equation, i.e., $\alpha = 1/k$).

Another type of dimensionless expression for the Forchheimer equation was suggested by Geertsma (1974):

$$Da = 1 + Re \tag{24}$$

where Da is the Darcy number, defined as:

$$Da = \frac{\Delta P}{L \alpha \mu V_{sf}}$$
(25)

and Re is defined as:

$$Re = \frac{\beta \rho V_{sf}}{\alpha \mu}$$
(26)

2.4. Experimental Specifications

The main objective of the present set of experiments was to characterize the fluid flow conditions through these type of filters - namely, (a) to see under what conditions Darcy's law holds, (b) to determine the value of permeability, (c) to determine the transition point from the Darcy to the Forchheimer domain, (d) to evaluate the flow in Forchheimer regime.

The experimental set-up was shown in Figure 6. Tap water was introduced through a water tank into a horizontal plexiglass cylinder of 45 mm inside diameter. The reticulated ceramic filters to be tested were located about 45 cm downstream inside the cylinder. The inlet of the cylinder was tapped by 11°. The flow rate could be controlled by adjusting the water level in the tank and outlet tap. When the flow was stabilized, the flow rate was read from a flow meter attached to the outlet valve. The flow meter was calibrated by using a timed sampling method, namely, collecting water from the outlet of the flow meter for a timed period of time, by weighing the sampling time from a stop watch, the flow rate can be determined. The pressure drops across the ceramic filter at each flow rate were measured by directly reading the difference of the water level in

terms of millimetres displayed on two small (3 mm diameter) vertical plexiglass tubes (piezometers) located immediately before and after the filter.

2.4.1. Determination of Porosity

The total and effective porosity of the filters tested were determined by using *density* and *imbibition* methods respectively. In the density method, the bulk density, ρ_b , of filter was determined by measuring the total weight and volume of a given filter. If the density of the material making up the filter, ρ , is known, the total porosity ε_t should be:



Figure 6. Schematic illustration of the experimental set-up for the water modelling experiments.

$$\varepsilon_{t} = 1 - \frac{\rho_{b}}{\rho}$$
 (27)

The filters tested include 92%-alumina and zirconia filters. the density of alumina was taken as 3800 kg/m^3 and that of zirconia was taken as 5800 kg/m^3 in the determination of the total porosity of these filters.

In the imbibition method, the filter was immersed into a volumetric flasks of water, when the filter was completely saturated with water, the volume displayed by the filter was measured. This volume is the total volume of solid and voids which are not accessible by water, V_s . By knowing the bulk (total) volume of the filter, V_r , the effective porosity, ε_r , can be determined:

$$\varepsilon_e = 1 - \frac{V_s}{V_t} \tag{28}$$

The difference between the total porosity and the effective porosity gives the *dead* porosity, ε_d . The dead porosity was found to be in the range of about 1 - 6% for the filters tested. This is due to the inner hollow of the filter webs which is not permeable by water. Since it is the effective porosity that is the primary concern in the study of fluid flow and filtration, for the reason of convenience, hereinafter, it will be denoted by ε without a subscript.

It should be noted that in the imbibition method, the ceramic webs were assumed to be impermeable by water. If the assumption does not hold for some ceramics, a slightly higher value would result for the effective porosity. On the other hand, if the filter can not be completely caturated or if some air bubbles are trapped inside the filter during imbibition, then the value of the effective porosity thus determined may be slightly lower than what it should be.

Alternatively, the effective porosity can also be determined by using Archimedes method. The author did a cross check and found the discrepancy between the values of effective porosity determined by the two methods was less than one percent.

Table I lists the porosities of the filters tested in the water flow experiments.

filter type	nominal	bulk	total	effective	dead
	size	density	porosity	porosity	porosity
	(ppi)	(g/cc)			
zirconia	30	0.66	0.89	0.85	0.04
zirconia	30	0.67	0.88	0.85	0.03
zirconia	45	0.82	0.86	0.80	0.06
alumina	30	0.39	0.9	0.89	0.01
alumina	30	0.39	0.9	0.89	0.01

Table I. Measured porosities of alumina and zirconia filters used in the water flow experiments.

2.4.2. Determination of Specific Surface Area

The specific surface area or surface to volume ratio were determined by using quantitative optical stereology method described by Underwood (1970):

$$S_v = 2P_L \tag{29}$$

where P_L is the number of points intercepting the filter webs per unit length of test lines, performed on a polished cross section of the ceramic filters. The test lines were applied randomly or systematically over the entire microstructure until a sufficient number of intersections had been counted. The magnification of a given microstructure was taken into account when the value of P_L was determined.

The specific surface area per unit volume of solid, S_0 , was found to be about 16.27 mm⁻¹ for 30 ppi zirconia filter, and 15.36 mm⁻¹ for 45 ppi zirconia filter.

2.5. Experimental Results and Discussion

2.5.1. Fluid Flow Characteristics

It was found that the flow of water through the reticulated ceramic filters obeyed Darcy's law over a certain range of velocities. As flow rate increases, Darcy's law becomes invalid (see Figure 7), since the pressure drop no longer increased linearly with flow rate, indicating that inertial forces became more predominant. Figure 8 shows the fluid flow behaviour of different filters. As expected, the pressure gradient decreased with increase in filter porosity. The transition from the Darcian to non-Darcian (Forchheimer) domain occurred at about 0.015 m/s for 30 ppi alumina filter (89% of effective porosity), 0.028 m/s for 30 ppi zirconia filter (85% of effective porosity) and 0.02 m/s for 45 ppi zirconia filter (80% of effective porosity). At high porosity, the flow may tend to be turbulent, which is perhaps why the transition within the filter of 89% effective porosity took place at a lower superficial velocity than that with 85% effective porosity even though both were of the same nominal pore size. On the other hand, as the pore size becomes finer, the percentage of blocked cells becomes higher. This would cause the local interstitial velocity near the blocked cells to become higher than the averaged value given by the Dupuit-Forchheimer assumption for a given superficial velocity. That may be one of the possible reasons why the transition within the 45 ppi filter took place at a lower superficial velocity than within the 30 ppi zirconia filter.

2.5.2. Determination of Permeability

The permeability of these filters can be obtained from Darcy's law, i.e., from the straight lines in Figure 8. If the viscosity of water at 18° C is taken as 1.05×10^{3} kg/m.s., the permeability of the filters of 80%, 85% and 89% effective porosity was found to be about 0.009375, 0.019811, and 0.040385 mm² respectively (i.e., 9375, 19811 and 40385 Darcy, if one Darcy was taken as 10^{6} mm²).



Figure 7. Measured pressure drop as a function of superficial velocity.



Figure 8. Measured pressure gradient as a function of superficial velocity, of different porosity filters.

2.5.3. Validation of Kozeny-Carman Equation

Substituting the permeability obtained for different filters from the water flow experiments and the corresponding effective porosity into the Kozeny-Carman equation (13), one obtains the values for S_0 , being 16.52, 16.60 and 16.99 mm⁻¹ for 45, 30 ppi zirconia filters and 30 ppi alumina filters respectively. Comparing these values with those determined with quantitative scereology method, one finds the discrepancy to be in the range of two to seven percent, indicating that Kozeny-Carman equation could be used to estimate the permeability and thus the pressure drop within the Darcy domain, provided the effective porosity and the specific surface area for a given filter are known. Figure 9 shows a comparison of permeability obtained from water flow experiments and those calculated from Kozeny-Carman equation.

It should be noted that one of the assumptions of the Kozeny-Carman equation is that the porosity should not be too high, in general, not higher than 0.8. According to Sullivan (1941), however, who studied the flow parallel to a bundle of fibres, no serious error would result from use of the Kozeny-Carman equation until the porosity exceeds 0.9. This is in accord with the findings of this study. The porosity limitation to the Kozeny-Carman equation was based on the argument that the Kozeny theory is based on flow in conduits. At high porosity such as in fibre filters whose porosity is normally greater than 0.95, this condition may break down, and flow may be better described by flow around objects. The structure of the reticulated ceramic filters may lie in between, similar to that of fibre filters in that it consists of webs or struts, in the meantime, it may bear the characteristics of flow in conduits due to its blocked pores.

2.5.4. Characteristics of Non-Darcy Flow

Regression performed on the experimental data for the non-Darcian domain

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Figure 9. Permeability of tested filters as a function of porosity, a comparison between experimentally determined and calculated from Kozeny-Carman equation.

showed that the pressure gradient increased with a 1.7 - 1.9th power dependence on the superficial velocity, which can be well fitted to equation (18). White (1935) found a value of 1.8 for the power of superficial velocity by flowing dry air through packed towers.

If equation (20) is rewritten as:

$$\frac{\Delta P}{LV_{sf}} = a' + b'V_{sf}$$
(30)

a', b' being apparently constants for a given system, a straight line yields, with a' being the intercept (as V_{r} , approaches zero), b' being the slope. If Darcy's law is rewritten in the same fashion, the plot of $\Delta P/LV_{r}$ against superficial velocity should yield a horizontal line in the Darcian domain, which contradicts equation (30), unless the Darcian domain diminishes to zero. Actually, the reasoning made by Ergun, i.e., that as the superficial velocity approaches zero, the Ergun equation reduces to Darcy's law, implies a virtual non-existence of the Darcian domain. However, the contradiction can be practically reconciled if the Darcian domain, for some systems, is really small, or if the slope is very small. Figure 10 shows such plots according to equation (30). It can be seen that a plateau at low flow rates is clearly noticeable (see also Fand et al's work, 1987) for the 45 ppi zirconia filter (with 80% effective porosity). While for the 30 ppi alumina filter (with an effective porosity of 89%), the plateau is not obvious. However, it seems that it is merged with the straight line of the non-Darcy regime which has a small slope. Up to this point, one may tend to conclude that Ergun equation should be used in principle for non-Darcy flow, being within the same domain for application as the Forchheimer equation.

Figure 11 gives a plot of $\Delta P/LV_{sf}$ against the superficial velocity in the non-Darcy regime. As can be seen, the Forchheimer equation is indeed valid for the reticulated ceramic filters.



Figure 10. Pressure drop as a function of superficial velocity, plotted according to Eq.30.



Figure 11. Measured pressure drop as a function of superficial velocity in non-Darcy regime.

2.5.5. Friction Factor - Reynolds Number Correlation for Reticulated Ceramic Filters

If the square root of permeability is chosen for the characteristic length and the friction factor and Reynolds numbers are defined as:

$$f = \frac{\Delta P}{\rho V_{sf}^2} \frac{\sqrt{k}}{L} ; \qquad Re = \frac{\rho V_{sf} \sqrt{k}}{\mu}$$
(31)

then the Forchheimer equation becomes:

$$f = \frac{1}{Re} + \beta \sqrt{k}$$
 (32)

This equation indicates that in Darcy domain the friction factor is simply the reciprocal of Reynolds number. The experimental data obtained in both Darcy and non-Darcy domains plotted according to Equation (32) are shown in Figure 12. As seen, Equation (32) is indeed a valid expression for the reticulated ceramic filters tested.



Figure 12. Friction factor as a function of Reynolds number, where f and Re are defined in Eq.(31).

2.5. Summary

1. According to the water flow experiments, Darcy's law holds valid for a certain superficial velocity domain.

2. The permeability of the commonly used reticulated ceramic filters for liquid metal filtration was ranged from 9,000 to about 40,000 Darcy.

3. The Kozeny-Carman equation can be used as an approximation to estimate the permeability of this type of filter in the Darcian domain.

4. Non-Darcian flow in this type of filter can be characterised by the Forchheimer formula.

Chapter 3

A Numerical Analysis of the Initial Process of Filtration

3.1. Introduction

This chapter presents a numerical investigation of fluid flow and the processes of initial filtration associated with reticulated ceramic filters. It begins with a brief introductory literature review which will provide the reader with some background in this field. A novel approach for modelling fluid flow and filtration phenomena in highly porous media is described and predicted results are presented.

3.2. Introductory Literature Review

3.2.1. Modelling of Fluid Flow in Porous Media

Filtration is intrinsically related to fluid flow through porous media. The latter must, therefore, first be understood before one can proceed to analyze filtration phenomena. In a broader sense, mathematical modelling of fluid flow in porous media may include all theoretical and numerical modelling other than the phenomenological approach where the porous medium is treated as a 'black box'. This category can be divided into macroscopic modelling where only the mean geometrical properties of the porous media are concerned such as the average porosity and specific surface area as in the Kozeny-Carman equation (as discussed in the previous chapter), and microscopic modelling, where an individual filter grain is modeled as a specified geometry such as in Happel's (Happel, 1958) sphere-in-cell model. Microscopic modelling can be further subdivided into analytical and numerical categories. In the former, the flow field around the filter grain is obtained through analytical solution of the Stokes equation; in the latter, it is obtained by numerical solutions to the Navier-Stokes or Stokes equations.

3.2.1.1. Analytical microscopic approach

In the analytical microscopic approach, porous media are modeled as an assemblage of some form of simple geometrical objects (filter grains or channels) such as a sphere, cylinder, capillary tube, or constricted tube. In Happel's (1958) and Kuwabara's (1959) spherical model, the porous media are represented as collections of identical cells containing a solid sphere enveloped within a fluid sphere. The flow field around the solid sphere was determined analytically by solving the Stokes equation with a zero shear or a zero vorticity boundary condition on the surface of the fluid sphere. Neale and Nader (1973, 1974) embedded Happel and Kuwabara's unit cell model into an 'exterior' homogeneous porous media and the boundary condition at the interface was determined based on the solution of Brinkman's (1947) equation for the latter. The cylindrical model is more suitable for fibrous porous media. In this model, the porous media are modeled as cylinders arranged in a certain regular pattern such as a square or triangle array. The cylinder array can be positioned either parallel or perpendicular to the flow direction. The flow field within the cylinder array has been studied by many investigators. The simplest approaches were given by Happel (1959) and Kuwabara (1959). They obtained the flow field around a single cylinder by employing a similar model and boundary conditions as those used in their spherical model. Sparrow and Loeffler (1959) solved the Stokes equation for parallel flows in square and triangular arrays using a power series technique. Hasimoto (1959) calculated using a Fourier series the drag force exerted on cylinders in a square array perpendicular to the flow. Spielman and Goren (1968) applied Brinkman's model (1947) for flow through unconsolidated porous media to fibrous media for four different orientation distributions with respect to the flow direction. Drummond and Tahir (1984) solved the Stokes equation for flow both parallel and perpendicular to the axes of cylinders in square, rectangular, triangular and hexagonal arrays. For the case of flow parallel to the axes of cylinders, they concluded that at very high porosity, the drag force per unit length on these cylinders was almost independent of their geometric arrangement and dependent only on porosity. These authors further concluded that the drag for transverse flow was twice the drag for longitudinal flow at equal porosities, confirming Cox's finding (1970). In another more recent work, Neira and Payatakes (1978) obtained a collocation solution for creeping flow through periodically constricted tubes.

Recognising the limitations of the mono-disperse cylinder model for a real mixed fibrous media where the diameter of the fibres may differ significantly, Ethier (1991) proposed a model consisting of a coarse fibre component embedded into a homogeneous fine fibre matrix. The flow field was analyzed using the Brinkman equation.

3.2.1.2. Numerical Microscopic Approach

The models used in this approach are the same as those employed in the analytical microscopic approach. A typical flow field is obtained by solving the Navier-Stokes equations or Stokes equation numerically. Griffin and Meisen (1973) solved the Navier-Stokes equations for the Kuwabara cell model for a zero vorticity outer boundary condition. Masliyah (1973) solved the Navier-Stokes equation for flow past banks of elliptical cylinders up to a Reynolds number 50. Payatakes et al (1973) solved the full Navier-Stokes equations for the constricted tube model. Gordon (1978) solved the Navier-Stokes equations for laminar flow through triangular array of parallel cylinders. Sangani and Acrivos (1982) solved the Stokes equation for creeping flow past a square and a hexagonal array of cylinders using a least-square technique. In their filtration studies in electrically enhanced fibrous filters, Henry and Ariman (1986) obtained numerical solutions for the flow field using Gauss-Sidel iteration scheme. Fardi and Liu

(1992) studied the flow field around a rectangular fibre for a staggered array, in terms of primitive variables (i.e., velocity and pressure), with a finite difference control volume method.

3.2.1.3. Other modelling approaches

Another type of modelling approach for fluid flow in porous media has been proposed by both Johansen and Anderson (1990), and Du Plessis and Masliyah (1991) recently. In this approach, fluid flow through porous media is modeled by inclusion of additional terms in the momentum equation to represent the drag effect created by the presence of the porous media. The additional terms can be evaluated based on hydrodynamic drag theory according to certain geometric models. In the momentum equation proposed, Johansen and Anderson (1990) also include a term to take into account the drag effect caused by inclusions deposited as the filter ages during filtration. The success of this type of modelling depends solely on the proper description of such drag terms.

3.2.2. Modelling of Depth Filtration Processes

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Approaches used in deep bed filtration studies can, in general, be divided into two broad categories: (a) macroscopic or phenomenological and (b) microscopic or theoretical. In the phenomenological approach, the dynamic behaviour of deep bed filters are described by a set of partial differential equations (see Chapter 4), and the filter media are characterized by empirical parameters, values of which have to be determined experimentally. The main drawback of the this approach is that it offers no insight into the capture mechanisms involved. In the microscopic approach, however, the porous media are modeled as consisting of individual collectors of certain geometrical shape such as a sphere, cylinder, capillary or constricted tubes, the same as those discussed in the aforegoing section. Filtration processes and associated parameters can be studied by examining fluid flow and particle transport associated with a single or unit collector. The macroscopic approach will be discussed further in the next Chapter. The following discussion will mainly focus on the microscopic approach:

Once the flow field associated with the single or unit collector was obtained, analytically or numerically as discussed in the aforgoing sections, the transport equation governing the movement of suspended particles can either be solved analytically or numerically. There are two types of basic equations governing the transport of particles in a fluid: one is based on the principle of conservation, the other is based on the equation of motion. In the former case, particles are regarded as 'point masses' and no geometric details are concerned. The transport of these particles, in this case, can be treated in a similar way to those of mass transfer. In the latter approach, the particle is treated as an object suspended in a fluid, and all forces experienced by it have to be examined.

3.2.2.1. Studies based on conservation equation

The equation governing the transport of particles suspended in an incompressible fluid may, on the basis of a materials balance, be written as:

$$\frac{\partial c}{\partial t} + u_p \nabla c = D \nabla^2 c \tag{33}$$

where c, u_p and D represent, concentration, velocity and diffusivity respectively, of the suspended particles. If the fluid is stationary, Eq.(33) becomes a type of Fokker-Planck equation (Goldberg, et al, 1978), the velocity of the particles in this case being equal to the settling velocity and, if the particles are small enough to invoke Stokes law, then, values of u_p are simply equal to their Stokes velocity. On the other hand, if the fluid is moving and the Stokes velocity is very small in comparison to the velocity of the moving fluid (e.g., when the density of particles is very close to that of the fluid, or when the size of the particles is in the micron ranges), the velocity of the particles can be

approximated by the velocity of fluid, i.e., the particles follow streamlines in such circumstances, then, Eq.(33) reduces to the convection-diffusion type of equation governing convective and diffusive mass transfer phenomena.

In studies into the deposition of particles due to diffusion and/or sedimentation, Eq.(33) has been tackled by many authors, either analytically or numerically, for different but relatively simple geometrical models, e.g., Langmuir (1942), Chen (1955), Levich (1962), Fuchs (1964), Pich (1966), Stechkina and Fuchs (1966), Yao et al (1971), Yeh and Liu (1974), Goldberg et al (1978), Goldberg (1981), Yu and Rajaram (1978), Taulbee (1978), Ingham (1981), Dietz (1981), Fardi and Liu (1992) and Shimada et al (1993). As far as the influence of *Pe* on collection efficiency is concerned, the general consensus (finding) seems to be that a single collector efficiency can be approximated as an algebraic sum of $aPe^{-2/3}$ and bPe^{-1} for creeping flow, $cPe^{-2/3}$ when Pe > 1, and $dPe^{-1/2}$ for potential flow, although the associated coefficients, *a*, *b*, *c*, *d* may differ from author to author.

3.2.2.2. Studies based on trajectory analyses

The trajectory approach was first introduced by Yao (1968) into deep bed filtration studies. In this approach, the trajectories of the suspended particles are analyzed by solving the trajectory equations - equation of motion. If a suspended particle is able to contact a filter grain as it moves past the latter, then it is assumed to be collected. The filtration efficiency can be obtained by determining a *limiting* trajectory - a trajectory beyond which no particles are able to make contact with the filter grain.

Trajectory analyses are based on the equation of motion as dictated by Newton's Second law:

$$\frac{d}{dt}(m_p u_p) = \sum F \tag{34}$$

where m_p and u_p denote the mass and velocity vector of the particle respectively, F denotes the force vector, which may include gravitational and buoyancy forces, the drag imposed by the fluid, at A forces associated with the so-called 'added mass' and Basset terms. The added mass term arises due to the fact that acceleration of a particle in a fluid causes accelerations in fluid immediately adjacent to it due to drag effects. The Basset term, also known as the Basset 'history' term, attempts to account for the dependence of the instantaneous drag on the state of development of fluid motion around the particle which may have accelerated or decelerated in the past instants (see Guthrie et al 1975). Setting out the various forces in mathematical terms for a spherical particle of diameter, d_p , Eq.(34) reads:

$$\frac{\pi}{6}d_{p}^{3}\rho_{p}\frac{du_{p}}{dt} = \frac{\pi}{6}d_{p}^{3}(\rho_{p}-\rho)g + C_{D}\frac{\pi}{8}d_{p}^{2}\rho(u-u_{p})|u-u_{p}| + C_{A}\frac{\pi}{6}d_{p}^{3}\rho\frac{d}{dt}(u-u_{p}) + \frac{C_{B}}{4}d_{p}^{2}\sqrt{\pi\rho\mu}\int_{0}^{t}\frac{d(u-u_{p})}{d\tau}\frac{d\tau}{\sqrt{t-\tau}}$$
(35)

where ρ , μ and u are the density, viscosity and velocity vector of the fluid, ρ_p is the density of the particles, C_D , C_A and C_B are, respectively, the drag, added mass and history coefficients, t, τ denote time. This equation has been proved experimentally by Guthrie et al (1975) to be a valid expression for subsurface motion of spherical objects in a fluid. According to these authors, the classical values of C_A and C_B given by Landau and Lifshitz (1959) and Bassett (1888), i.e., 0.5 and 6.0 respectively, are surprisingly reliable for predicting the motion of falling spheres into a stagnant fluid. For particle deposition studies in deep bed filtration where both the particles and the relative velocity between the particles and fluid are very small, according to Tien (1989), both added mass and Bassett terms are insignificant and hence can be ignored. Furthermore, the drag forces can be approximated from Stokes law, along with corrections for hydrodynamic retardation effects when the particle moves in the immediate vicinity of the solid surfaces of the filter grain. Another implication by invoking Stokes law, is that the moving particle assumes zero acceleration. In other words, the inertial forces (given by the left hand term in Eq. (35)), vanishes.

The possibility of applying trajectory calculations to deep bed filtration was first suggested by O'Melia and Stumm (1967). This idea was first implemented by Yao (1968), who based his calculation on an isolated sphere model, around which the flow was "creeping". Spielman and Goren (1970) included the hydrodynamic and short range London-Van der Waals forces into the trajectory equation for a particle in the immediate vicinity of a large collector. Spielman and FitzPatrick (1973), using the same approach, analyzed the capture of non-Brownian particles from suspensions flowing around spherical, rotating disc and cylindrical collectors. Their theoretical results were checked experimentally by flowing latex particles through beds of packed glass beads; poor agreement was found (FitsPatrick and Spielman 1973). Pavatakes et al (1973) proposed a constricted tube model which they claimed be more realistic than the capillary cylindrical tube model, since the latter could not reflect the converging-diverging character of flows in real packed beds. These authors, based on their constricted tube model, developed a particle trajectory equation which included not only the hydrodynamic and London-Van der Waals forces, but also the electrokinetic forces caused by surface charges (Payatakes et al 1974). Rajagopalan and Tien (1976, 1977) made similar trajectory analyses based on Happel's and single sphere collector models. Beizaie and Tien (1980) analyzed aerosol deposition on a single spherical collector with three-dimensional trajectory calculations. Pendse and Tien (1982a, 1982b) calculated aerosol trajectories based on the constricted tube model. Henry and Ariman (1986) and Tousi et al (1991) included the electrophoretic forces resulting from an external electric field in the trajectory equation.

3.3. Description of Problem

3.3.1. Filtration mode

Since inclusions in liquid metals are usually much smaller than the pore size (or interspaces between the webs) of the ceramic filters (as shown in Figure 13), the

filtration of liquid metals should, in general, operate in depth mode. Cake mode may also be possible for some circumstances when, for example, the melts are too dirty or the size of the inclusions are large. With increased refining technology in liquid metal treatments, the former rarely occurs in most plants.



3.3.2. Identification of Influential F Parameters

Figure 13. The relative size of inclusions and reticulate ceramic filters.

As mentioned in Chapter 1, the capture of particles in deep bed mode includes two steps: transportation to the vicinity of the internal filter surfaces and adhesion thereafter. Hence the possible variables affecting the filtration efficiency may accordingly be grouped into two groups, one associated with transport processes, the other with adhesion processes. If η_r and η_a denote, respectively, the efficiency or probability of transport and adhesion processes associated with a single filter web, then the removal efficiency of a representative piece of filter, e_r , should be:

$$e_r = f(\eta_t \eta_o) \tag{36}$$

The possible variables affecting e_r may include:

$$\boldsymbol{e}_{r} = f(\boldsymbol{\varepsilon}, \boldsymbol{L}, \boldsymbol{d}_{w^{o}} \boldsymbol{V}_{sr}, \boldsymbol{\mu}, \boldsymbol{\rho}, \boldsymbol{\rho}_{p}, \boldsymbol{d}_{p}, \boldsymbol{D}, \boldsymbol{g}, \boldsymbol{H}, \boldsymbol{\sigma}_{mp}, \boldsymbol{\sigma}_{mr}, \boldsymbol{\delta}, \boldsymbol{\varphi}, \boldsymbol{\hat{v}})$$
(37)

where ε , L and d_{w} denote, respectively, the effective porosity, thickness and average web diameter, of the ceramic filter; V_{sf} is the superficial velocity; μ and ρ the melt viscosity and density; ρ_{p} the equivalent particle diameter; D diffusivity of the particle; g
gravitational constant; *H* the Hamaker constant between the particle and filter material; σ_{mp} and σ_{mf} the interfacial tension between the melt and particle and between the melt and filter respectively; δ the roughness of the filter web surface; φ denotes the shape factor or the orientation parameter at point of contact, of the particles; ϑ is the angle formed by the gravity vector and the surface normal at point of contact.

Eq.(37) may not exhaust all the possible variables. For example, hydrodynamic retardation or lubrication effects are not accounted for. Surface charges and chemical reactions between the filter and particles are assumed to be absent. For situations where 100% adhesion can be assumed, the removal efficiency may only depend on the variables associated with the transport processes. If one assumes the short-range surface forces (e.g., Van der Waals) to have negligible contribution to particle transport processes, the last six variables in parenthesis on the right side of Eq.(37) characterising adhesion processes, would vanish. A dimensional analysis for the rest would lead to the following dimensionless correlation (see Appendix II):

$$\boldsymbol{e}_{r} = f\left(\boldsymbol{\varepsilon}, \frac{L}{d_{w}}, \frac{d_{p}}{d_{w}}, \frac{\rho V_{st} d_{w}}{\mu}, \frac{V_{st} d_{w}}{D}, \frac{(\rho_{p} - \rho) d_{p}^{2} g}{\mu V_{st}}, \frac{d_{p}^{2} \rho_{p} V_{st}}{\mu d_{w}}\right)$$
(38)

Ison and Ives (1969) obtained a similar correlation, Unfortunately, they omitted an important parameter, the porosity. The physical significance of these dimensionless groups can easily be identified; L/d_w is the dimensionless filter thickness; d_p/d_w is an interception parameter and is often denoted in the literature by N_R , characterising the role of relative sizes of the particle and the filter web; $\rho V_{sf} d_w/\mu$ the Reynolds number denoted by Re, characterising the fluid flow conditions around a filter web; $V_{sf} d_w/D$ is the Peclet number denoted by Pe, characterising the relative importance of convective and Einstein diffusion; and the last two groups correspond to the gravitational and Stokes numbers. These can be denoted by N_c and Sk respectively.

The task now is to quantify how each of these dimensionless groups affects the filtration efficiency. Principally, this can be done, either experimentally or theoretically, by varying one group at a time while keeping the others constant. Practically, however, the experimental approach may not be realistic since in a real filtration process, different mechanisms may act simultaneously and it may be difficult to isolate them. Therefore, a theoretical approach, based on numerical analysis, may be the only solution to this complex problem, while reserving experimental data only as a means for verification.

3.4. A Model for Reticulated Ceramic Filters

3.4.1. Characteristics of Filter Structure and Associated Flow Field

The reticulated ceramic filters are essentially, as already discussed in the previous chapter, composed of struts or webs of similar dimensions joined in a three dimensional network. The average dimension of the individual webs along their axial direction are larger than (at least twice) that along their radial direction, so that the flow field around an individual web should, in principle, be roughly two dimensional. However, the flow field around the junctures must be of a three dimensional character. From the former point of view, the structure of these filters bear some similarity to those of fibrous filters. The blocked cells resulting from the coating process, as mentioned in Chapter 2, would add further complications in characterising the structures of this type of filter. It should be pointed out that it is incorrect to regard the principle structure of these filters as consisting of quasispherical cells connected to each other by their openings (see Conti and Netter, 1992), which may only be a valid approximation for those blocked areas.

3.4.2. The New Modelling Approach

Ideally, the study of fluid flow and transportation (filtration) phenomena within porous media should be based on the exact geometric structure of the porous media. Unfortunately, such an approach is impractical owing to the complicated, and relatively undefinable nature of a real porous medium. It may be possible, however, with the availability of computer imaging techniques and immense computing capacity, to obtain the exact geometric structure of a given porous medium and to then numerically solve the related transport equations. Such an approach was not attempted in this study, although this temptation inspired the author to embark on the new modelling approach to filtration now discoursed.

To date, therefore, fundamental studies of filtration phenomena in porous media have mainly made use of single or unit collector models. Because of the simple geometrical configuration of these models, are results derived therefrom only approximate the actual phenomena at best, especially when the actual geometric structure bears little resemblance to the simple geometry assumed by the models. Furthermore, the formulation relating the filtration efficiency of a single or unit cell collector to that of the entire filter requires further assumptions. Being aware of these weakness of the existing models, the author proposed a Direct Geometric Approximative (DGA) method which attempts to approximate the geometric properties of the real filter by constructing a 'model filter' to represent a 'piece' of the real filter. This model filter (piece) is sufficiently macroscopic for it to characterise the mean geometric properties of the real filter. As such, transport phenomena therein are representative of those in the whole filter. It is, in the meantime, still microscopic enough to allow direct utilization of the general governing equations, i.e., the Navier-Stokes equations and conservation equation of suspended particles, for the analyses of fluid flow and filtration phenomena taking place inside the real filter. Since the spatial volume occupied by the webs of the reticulated ceramic filter is higher than that occupied by the junctures, a 2-D model, consisting of a triangular (staggered) array of obstacles (see Figure 14, b), was employed as a first approximation. The shape of the obstacles was chosen so as to be easily represented by a Cartesian rectangular coordinate system. (In principle, the configuration of the obstacles should be so constructed that it resembles the real filter structure.

Unfortunately, due to complicated structure of the real filter, as mentioned above, it is unrealistic to rigorously rationalize the model's configuration. This frustration may be eased a little if one keeps in mind the fact that the ultimate objective of the model was to approximate the filtration rate of the real filter, and that any reasonable configuration might be employed in the model as long as these objectives were realized). The porosity of the model filter can be varied by changing the number density of obstacles so as to accommodate the porosity of the real filter. The governing equations can be numerically solved over the whole domain of the model filter, instead of over an individual collector (see Figure 14 for the solution domain, the shadowed area, of different models). In this way, the pressure drop and filtration efficiency can be directly obtained by checking the values at the inlet and outlet or any point downstream at a certain filter 'hickness, thereby avoiding the assumptions necessarily required for the boundary conditions of an unit cell (e.g., zero stress in Happel's model and zero vorticity in Kuwabara's model) and the assumptions needed in relating the individual collector efficiency to that of the whole filter in the single or unit collector modelling approach. In this regards, this model is, in the author's opinion, one step closer to the actual phenomena and therefore should be regarded as a step forward in the modelling of porous media.

3.4.3. Comparison with Other Models

As seen from Figure 14, in single collector modelling, the filter media are assumed to be composed of independent grains such that the flow field around each filter grain is the same as if it were in a fluid of infinite extent, in other words, the influence of neighbour filter grains is ignored. In the unit cell model, the effect of neighbouring filter grain is accounted for by an assumed boundary condition on the outer boundary of the cell. Furthermore, the cell has been idealized with respect to the actual situation (as shown in the Figure 14, in a 2-D domain the actual cell should be a rectangle rather than a cycle, or in a 3-D domain it should be a cube instead of a sphere) in order to ease the mathematical analysis. The concept of unit collector or Unit Bed Elements (UBE) was



Figure 14. Solution domain of different models.

first proposed by Payatakes et al (1973). These authors assumed that a homogeneous, randomly packed medium can be considered to consist of a number of UBE connected in series. Each UBE contains an assembly of collectors of specified geometry and size or size distribution or both. The efficiency of each UBE can be determined based on the efficiency of the individual collectors contained therein. The relation between the single or individual collector efficiency, η , and that of unit collector, e_{μ} , varies depending on the models. η is defined as the ratio of the particles collector normal to the flow direction per unit time. While e_{μ} is simply the ratio of the sum (linear addition) of collectors rate of all individual collectors contained within the unit collector or unit filter media divided by the total number of particles flowing through the latter. For spherical collectors, the filter coefficient, λ , is related to η by the following equations (Tien, 1989):

$$\lambda = \frac{3(1-\varepsilon)^{\frac{1}{3}}}{2d_{c}}\eta$$
(39)

for Happel and Kuwabara's model

$$\lambda = \frac{3(1-\varepsilon)}{2d_c}\eta \tag{40}$$

for Brinkman's model

$$\lambda = \frac{3(1-\varepsilon)}{2d_c} \frac{u_{\star}}{V_{sf}} \eta$$
(41)

for single or isolated sphere model, where u_{∞} and V_{sf} denote approaching and superficial velocity respectively, the relation of which, according to Tien (1989), is ambiguous. Other assumptions made in the derivation of these above equations is indicated in the following derivation for circular cylinder collectors.



Figure 15. (a) a unit volume, (b) a UBE, of packed parallel circular cylinders.

Likewise, for parallel circular cylinders placed perpendicular to the flow direction, the rate of particle collection per unit volume of the filter media is assumed to be equal to the rate of collection of individual collectors times the number of individual collectors contained in a unit volume of filter media (see Figure 15, a). For the Happel and Kuwabara model:

$$\frac{\partial \alpha}{\partial t} = V_{s} d_{f} \eta \frac{1-\varepsilon}{\frac{\pi}{4} d_{g}^{2}}$$
(42)

where α is the specific deposit, defined as the volume of collected particles per unit volume of filter media, d_f is the diameter of the fluid envelope concentrical with the cylindrical collector of a diameter d_g . Since the porosity, $\varepsilon = 1 - (d_g/d_f)^2$, by using the macroscopic equation, Eq.(90):

$$\frac{\partial c}{\partial z} = -\frac{4}{\pi} \frac{(1-\varepsilon)^{1/2}}{d_g} \eta c$$
 (43)

integration, assuming η independent of z (this assumption only holds for initial period of filtration):

$$\frac{c}{c_0} = e^{-\frac{4}{\pi} \frac{(1-\epsilon)^{1/2}}{d_g} \eta I}$$
(44)

where *l* is the arbitrary filter thickness. Compared with the Logarithmic Law for filters:

$$\lambda = \frac{4}{\pi} \frac{(1-\varepsilon)^{1/2}}{d_g} \eta$$
 (45)

Eq.(45) can also be obtained based on the UBE approach (see Figure 15, b). In this case, the filtration efficiency of a UBE is given as:

$$e_{\mu} = \frac{V_{g} d_{f} c \eta \frac{4(1-\varepsilon)l}{\pi d_{g}^{2}}}{V_{g} c}$$
(46)

where *l* is very small (the same order of magnitude of a filter grain, according to Payatakes et al, 1973) and consequently $e_{\mu} < 1$. Invoking the Logarithmic Law, one has:

$$\lambda = \frac{1}{l} \ln \frac{1}{1 - e_u}$$

$$- \frac{e_u}{l}$$
(47)

i.e., $e_{\pi} = \lambda l$, then Eq.(46) approximates to Eq.(45).

Similarly, for the Brinkman's model, one has:

$$\lambda = \frac{4(1-\varepsilon)}{\pi d_g} \eta \tag{48}$$

and for the isolated collector model

$$\lambda = \frac{4(1-\varepsilon)}{\pi d_g} \frac{u_{\infty}}{V_{sf}} \eta$$
(49)

Once e_* is known, the overall filtration efficiency of the whole filter can be found by the following equation (Tien, 1989):

$$E = 1 - \prod_{i=1}^{N} (1 - e_{x_i})$$
 (50)

where N is the number of UBE contained in the whole filter media. Its value is the ratio of the filter thickness, L, to l. In the initial period, when the deposition is small and its effect on particle capture mechanism is negligible, the value of e_* of all UBE would be the same, then Eq.(50) becomes:

$$E = 1 - e^{-\lambda L} \tag{51}$$

By comparison, in the author's proposed (direct geometric approximative or DGA approach) method, rather than evaluating the collection efficiency of an individual filter grain, the filtration efficiency of a representative piece of filter (model filter) is obtained directly by checking the particle concentration at the inlet and outlet, thereby eliminating all the assumptions just outlined.

3.5. The Governing Equations

At steady state, the general governing equations in dimensionless form for incompressible isothermal Newtonian fluid flow can be expressed as: Continuity:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0$$
 (52)

U-momentum:

$$\frac{\partial(U^2)}{\partial X} + \frac{\partial(UV)}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re} \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right)$$
(53)

V-momentum:

$$\frac{\partial(UV)}{\partial X} + \frac{\partial(V^2)}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{Re} \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right)$$
(54)

The general equation governing the transport of particles based on conservation of mass can, for steady state, be written as:

$$U\frac{\partial C}{\partial X} + (V + N_G)\frac{\partial C}{\partial Y} = \frac{1}{Pe} \left(\frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2}\right)$$
(55)

where U and V are dimensionless velocities in the X and Y directions respectively $(U=u/V_{sp}, V=v/V_{sl}, X=x/d_w, Y=y/d_w; P$: the dimensionless pressure $(=p/\rho V_{sp}^2)$, where p is the hydrodynamic pressure); C: the normalized concentration of particles $(=c/c_0, c_0)$ is the inlet concentration in terms of volume fraction). Other variables are the same as those defined previously.

As mentioned before, the model is sufficiently microscopic to allow direct application of general governing equations. This can be justified in that the actual dimensions of the model 'filter' are only a few millimetres, all the control volume elements employed during computations can be adjusted so as to contain only one phase either liquid or solid (filter webs). The control volumes containing solid inside the computational domain can be treated with special techniques so as to render velocities therein zero. This can, for instance, be done by designating an ultra-high value for that control volume's viscosity in solving the discretized forms of the momentum equations. In other words, the solid can be viewed as being a special 'liquid' of essentially infinite viscosity. As such, the general equations governing the flow remain valid for the whole domain of the model filter.

Eq.(55) represents an extension of the Fokker-Planck (see Goldberg, 1981) type of equation from a stationary to a moving fluid and is similar to convection-diffusion type equations. The difference between the two lies in that the former is based on the particle velocity while the latter is based on the fluid velocity. Equations similar to the former have been used in aerosol filtration (see Yen, 1974, and Ingham, 1981). This type of equation is based on a Eulerian frame of reference and can be solved numerically once the flow field is known. It should be noted that analytical solutions to this type of equation are only possible for some simple geometrical domains (see Goldberg, 1981, and Goldberg et al 1978) with certain simplifications. The deviation of particles from the streamlines of the fluid due to gravitational or buoyancy forces was incorporated by including a normalized Stokes velocity, or the gravitational number, N_c , for the particles. This implies that the relative motions of suspended particles in liquids obey Stokes law, which is a reasonable assumption for commonly encountered inclusions in liquid metals. Joo (1989) and Joo and Guthrie (1993) were the first to apply this type of equation to liquid metal systems for their studies of inclusion float-out from metallurgical tundishes.

An equation equivalent to Eq.(55) was proposed by Yao et al (1971), which took the following form:

$$\frac{\partial c}{\partial t} + u \nabla c = D \nabla^2 c + \left(1 - \frac{\rho}{\rho_p}\right) \frac{m_p g}{3\pi \mu d_p} \frac{\partial c}{\partial z}$$
(56)

where u is the fluid velocity vector, z is the coordinate parallel the gravity, m_p is the

mass of the suspended particle. It can be noted that if the particle is taken as a sphere, Eq.(56) becomes identical with Eq.(55).

It should be realized that Eq.(55) may not be a rigorous description for particle motion when the particles are at the immediate vicinity of a solid surface since the particles in this situation would experience a hydrodynamic retardation due to lubrication effect, which has not been accounted for in this equation.

3.6. Numerical Solution Procedure

3.6.1. Discretization of Differential Equations

In order to solve numerically, with a digital computer, the differential equations, i.e., Eq.(52)-(55), over the computational domain given by the model filter, one has to first discretize these equations to form a set of linear algebraic equations wherein the unknowns in question at certain discrete points are related. The discretization can be accomplished in many ways. The method employed in this study is based on finite-difference control-volume formulation as outlined by Patankar (1980). The essence of this method is to divide the calculation domain into a finite number of non-overlapping control volumes, over each of which the differential equations are integrated. The values of the unknowns in question within each control volume are thought to reside at one point (grid point) contained within (see Figure 16). The values of the dependent variables needed at the interface of the control volume during integration can be obtained by interpolation.

Casting the dimensionless differential equations into a more general form as:

$$\frac{\partial \Phi}{\partial T} + \nabla (U \Phi) = \nabla (\Gamma \nabla \Phi) + S^{(57)}$$

where ϕ denotes the dependent variable such as U, V and C; T denotes the dimensionless time $(=tV_y/d_w)$; Γ denotes the general diffusion coefficient (its value equals, for the dimensionless momentum equations, 1/Re, and for the conservation equation of species, 1/Pe); S denotes the source term. One may have noticed that

the pressure gradient terms in the



Figure 16. A typical control volume grid.

momentum equations were not explicitly shown in the above equation, let's for the time being think they were buried in the source term. Integrating this equation along with the continuity equation over one control volume, one obtains the general discretized form of equation (detailed procedure have been given by Patankar 1980 and Guthrie 1992). For a 2-D domain, the implicit discretization equation can be written as:

$$a_P \phi_P = a_E \phi_E + a_W \phi_W + a_N \phi_N + a_S \phi_S + b \tag{58}$$

where

$$a_{E} = D_{e}A(|P_{e}|) + [-F_{e}, 0]$$

$$a_{W} = D_{v}A(|P_{w}|) + [F_{w}, 0]$$

$$a_{N} = D_{R}A(|P_{n}|) + [-F_{n}, 0]$$

$$a_{S} = D_{S}A(|P_{s}|) + [F_{s}, 0]$$

$$a_{P}^{0} = \frac{\Delta X \Delta Y}{\Delta T}$$

$$b = S_{C}\Delta X \Delta Y + a_{P}^{0} \Phi_{P}^{0}$$

$$a_{P} = a_{E} + a_{W} + a_{N} + a_{S} + a_{P}^{0}$$

$$-S_{w}\Delta X \Delta Y$$
(59)

where [] denote the greatest of, F_e , F_w , F_s , F_s , denote the dimensionless mass flow rate

across the four control volume faces respectively, i.e.:

$$F_e = U_e \Delta Y, \qquad F_w = U_w \Delta Y, \qquad F_\pi = V_\pi \Delta X, \qquad F_s = V_s \Delta X$$
(60)

and the corresponding diffusion conductance, D_i :

$$D_{e} = \frac{\Gamma_{e} \Delta Y}{(\delta X)_{e}}, \qquad D_{w} = \frac{\Gamma_{w} \Delta Y}{(\delta X)_{w}}$$

$$D_{\pi} = \frac{\Gamma_{\pi} \Delta X}{(\delta Y)_{\pi}}, \qquad D_{s} = \frac{\Gamma_{s} \Delta X}{(\delta Y)_{s}}$$
(61)

and P_i :

$$P_{\epsilon} = \frac{F_{\epsilon}}{D_{\epsilon}}, \qquad P_{w} = \frac{F_{w}}{D_{w}}, \qquad P_{\pi} = \frac{F_{\pi}}{D_{\pi}}, \qquad P_{s} = \frac{F_{s}}{D_{s}}$$
(62)

where the subscripts e, w, n, s denote, respectively, east, west, north and south faces of the control volume surrounding grid point P, and E, W, N, S denote its four adjacent neighbouring grid points as shown in Figure 16; the superscript O denotes the known values at the previous time step (for steady state situation, the terms associated with the O superscript would vanish from the above equations); and $A(|P_i|)$ is a scheme function which is given in the following Table (Patankar, 1980).

The Power Law scheme was adopted for this study since it better approximates the exact $\phi \sim X$ profile, for convection-diffusion problems, between grid points for a relatively larger variation in the absolute value of P_i .

	 _	_					
			N.				
_					-	 	

Table III. Scheme function

Scheme	$A(P_i)$
Central difference Upwind Hybrid Power law Exponential	$1-0.5 P_i $ 1 $[0, 1-0.5 P_i]$ $[0, (1-0.1 P_i)^5]$ $ P_i /(exp P_i -1)$

The source term S was linearly discretized as
$$S_c + S_p \phi_p$$
. In order to avoid

possible divergence during the iterative computation, S_P should be always assigned a negative value (a positive value may render a_P negative when the values of a_{nb} are positive, and this would lead to an unrealistic physical situation, for example, an increase in ϕ at grid point P when a_P is negative would result in a decrease in its adjacent neighbour points when a_{nb} are positive).

3.6.2. Solution Algorithm

Application of the 2-D implicit discretized equation to a number of $m \times n$ grid points will yield a number of $m \times n$ (excluding the boundary) linear algebraic equations. This set of equations can be solved either directly or iteratively. Direct method is more expensive since it requires large amount of computer storage and time. Moreover, direct method may not be suitable for solving non-linear problems. With the iterative method, one starts from a guessed field of ϕ and solve the equation only for one point (e.g., Gauss-Seidel method) or for one line of points at a time, until the whole domain is visited, then treat the solution as an improved field of ϕ , solve the equation again; repeat the procedure until a converged solution is reached. In this study, the line by line method was employed since it offers a faster convergence than the point-by-point method. The linear algebraic equations in this case were solved with the TDMA (Tri-Diagonal-Matrix Algorithm) method.

The pressure gradient was treated as a source term when the discretized momentum equations are solved. Unfortunately, the pressure field can not be known before the velocity field is known. The two fields are dependent on each other and are related through the continuity equation, therefore, unless a correct pressure field is obtained, the continuity equation will not be satisfied. Thus the pressure and velocity fields have to be solved simultaneously by an iterative method of appropriate algorithms, i.e., solving one at a time and using the obtained tentative field as a known value when the other field is solved. In order to avoid absurd numerical solutions (see Patankar 1980), the pressure field has to be solved at different grid points from those for the velocity field. The grids for the pressure and velocity are placed in an alternative fashion such that the pressure difference between two grids represents the driving force for determining the velocity whose grid point lies between (see Figure 17). This type of gridding is known as 'staggered' grids. Integrating the Umomentum equation over the control



Figure 17. Staggered grids showing the scaler (backward slash) and vector (forward slash) control volumes.

volume shown in Figure 17 (the forward slash volume element), one obtains:

$$U_{w} = \frac{\sum a_{w} U_{w} + b}{a_{w}} + (P_{w} - P_{p}) \frac{A_{w}}{a_{w}}$$
(63)

where subscript *nb* denotes neighbours, A_w denotes area upon which the pressure gradient acts. Similar equations can be written for other control volumes and V-momentum equations. Substitution of the discretized momentum equation into the discretized continuity equation yields the discretized pressure-correction or pressure equation depending on whether the iterative procedure starts with a guessed pressure field or a guessed velocity field. If the procedure starts with a guessed pressure field, the momentum equation is solved first. Based on the obtained velocity field, the continuity equation is checked over each control volume to see whether the mass is conserved, if not, then the pressure correction equation is solved to correct the pressure field as well as the velocity field. With the corrected pressure field, the momentum equation is solved again. This procedure continues until the continuity equation is satisfied, and then the pressure and velocity fields reach a converged solution. This algorithm is known as SIMPLE (Semi-Implicit Method for Pressure-Linked Equations, see Patankar 1980). If the procedure starts with a guessed velocity field, the pressure equation will be solved first and then the momentum equation. If the continuity equation is not satisfied, the pressure-correction equation is solved, but this time, only to correct the velocity (not the pressure) field. This reversed procedure is referred to as SIMPLER (SIMPLE Revised) algorithm and is believed to do a better job than SIMPLE. (For other related algorithms see Guthrie, 1992).

In order to avoid divergence during the iterative solution of the algebraic equations, it is often necessary to slow down (for the line-by-line method) changes from iteration to iteration, by a process referred to as *underrelaxation*.

3.6.3. Computer Coding

The computer code was generated in FORTRAN language and the main structure was based on METFLO-2D as given by Guthrie (1992). In the program, the computational domain was first gridded according to gridding method 'B'. This method of gridding is convenient for handling abrupt changes in domain properties and boundary conditions, since in this case the control volume boundaries are drawn first and the grid points then filled into the control volume. As such, it is easy to locate the boundaries at the interfaces of different phases (domains). This situation is not so easy to handle with gridding method 'A', wherein the scalar grid points are first set out and the control volume faces are then drawn at the middle of two adjacent grid points. After gridding, the variables are properly initialized and initial conditions are provided. Then the pressure equation is solved first, followed by U and V-momentum equations with appropriate boundary conditions and interpolation schemes. Next, the pressure correction equation is solved to correct the velocities. Based on the corrected velocity field, a check for convergence is performed. If convergence has not been reached, the program loops back to re-solve the pressure equation again, and then the U and V momentum equations. The loop breaks only when the convergence criteria have been met, and in that case, the program turns to solve the particle conservation equation. The program stops after the convergence criterion for particle conservation equation is met.

The present computer program has ignored the likelihood of interference of suspended particles on the flow field. This assumption may be justified since the concentration of inclusions in liquid metals is usually very low in terms of volume fraction.

The convergence criteria can be set in two ways: one is based on the literal meaning of 'convergence', i.e., the values from one iteration to another are no longer changing or oscillating, converged; the other is based on mass balance (continuity equation). Hence, the criteria in the former case can be set to be the largest differences between two iterations in the whole computational domain being less than a given small number; while in the latter case, the criteria can be set to be the sum of residuals of the whole domain being less than a given small number, i.e.

...

$$\sum_{i=1}^{N} R_{\phi} < 5 \tag{64}$$

and

$$R_{\bullet} = a_p \phi_p - (\sum a_{ab} \phi_{ab} + b)$$
(65)

where N is the total number of grid points in the whole domain, δ is a given small number.

3.7. Prediction of the Flow Field

The flow field within the model filter (i.e., the staggered array of obstacles) was

obtained by solving, using the numerical techniques mentioned above, the Navier-Stokes equation with non-slip conditions on the solid surfaces of the filter webs (obstacles). The non-slip condition, usually valid when the fluid is liquid since in this case the Knudsen number is vanishingly small, was realized technically by assigning a ultra-high value of viscosity to the control volume occupied by an obstacle. A flat velocity profile was employed at the inlet of the calculation domain, while vanishing velocity gradients were assumed at the outlet. The porosity of the model filter was adjusted by changing the number density of obstacles therein. Figure 18 shows an example of calculated flow and pressure fields. It can be seen that the pressure drops are caused mainly by the filter webs.

3.7.1. Pressure Drop as a Function of Reynolds Number

Figure 19 shows an example of predicted pressure drop in terms of friction factor as a function of Reynolds number. It can be seen that the friction factor is inversely proportional to the Reynolds number when the value of which is less than about 10 (where the Reynolds number is based on the average web diameter). Writing Darcy's law into dimensionless form as:

$$f = \frac{d_w^2}{k} \frac{1}{Re}$$
(66)

where f_{1} the friction factor, is defined as:

$$f = \frac{\Delta P}{\rho V_{sf}^2} \frac{d_w}{L}$$
(67)

it is clear that the flow inside the model filter under this condition belongs to the Darcy domain.

Figure 20 shows the comparison of the triction factor as a function of Reynolds



number for different porosities, between those predicted and that determined from the water flow experiments. It can be seen that the two are roughly comparable when the Reynolds numbers is sufficiently low for a given porosity. The fact that as Reynolds number increases, the flow inside the reticulated filters becomes non-Darcian type, which the calculation failed to predict, indicates that the present filter model may only valid for low Reynolds numbers, the upper limit of which, according to Figure 20, may be set to about 8 when the average web diameter is taken as the characteristic length. This limitation, however, does not jeopardize the cause of this study, for, and as will be seen later, it is in fact meritorious to operate depth filtration at low flow rates, i.e., within the Darcy velocity regime.

3.7.2. Permeability as a Function of Porosity

The permeability of the filters can be obtained from the slopes shown in Figure 20. Figure 21 shows the comparison of permeability between predicted and experimentally determined, as a function of porosity. It can be seen that the model overpredicted the permeability of the real filter at low porosities, whilst it under-predicted it at high porosities. This may be partly due to the irregularities in structure, such as blocked cells which increase as porosity decreases, and partly due to the three dimensional structure of the reticulate webs (web orientation), both of which were not taken into account in the model filter. The blocked cells would inevitably increase the resistance to the flow, hence resulting in a lower permeability. The orientation of the webs with respect to the flow direction and the inhomogeneity of microstructure of filters may also affect the value of the permeability, e.g., according to studies of fluid flow in fibrous filter, the permeability for situations where the fibres are placed parallel with the flow direction is much higher than (almost twice) that when the fibres are placed perpendicular to the flow direction (Drummond and Tahir, 1984). When the structure of the filter is not homogenous, the flow may channel through the filter from areas with the least resistance rather than through the whole cross section. This would also lead to a



Figure 19. Predicted friction factor as a function of Reynolds number, porosity: 92%.



Figure 20. Comparison of friction factor between predicted and experimentally determined.



Figure 21. Comparison of permeability as a function of porosity, between predicted and experimentally determined.

higher permeability than otherwise a homogenous filter. The effect of inhomogeneity on permeability has already observed by Kirsch and Fuchs (1967) in their experiments of flow through parallel fibres, and by Yu and Soong (1975) in their theoretical studies. Despite the discrepancy, it should, in the author's opinion, be regarded as highly surprising for such a simple model to give prediction of permeability so close to those of real filters. One may speculate whether this is merely a fortuitous result. As a matter of fact, this seemingly fortuitous outcome has its theoretical roots. Many theoretical studies in highly porous fibrous media have shown that the permeability of regularly arranged structures is a functional form of only two parameters (see Jackson and James 1986), i.e., the porosity and fibre diameter:

$$\frac{k}{d_w^2} = f(\varepsilon) \tag{68}$$

the exact functional form depending on fibre arrangement. This is precisely what the model filter of this study manifests.

3.8. Simulation of Initial Filtration Processes

To simulate the initial filtration processes, the particle transport equation, Eq. (55), was solved, based on the flow field obtained earlier. The normalized Stokes velocity or the gravitational number, N_G , was, after assuming the particles be of a spherical shape, calculated based on Stokes law:

$$N_{G} = \frac{(\rho_{p} - \rho)d_{p}^{2}g}{18\mu V_{sf}}$$
(69)

The dispersion coefficient. D, needed in determining the value of Peclet number was, in the absence of turbulence, estimated from the Stokes-Einstein (Einstein, 1905) equation: where k is Boltzmann's constant, T denotes absolute temperature.

$$D = \frac{kT}{3\pi\mu d_p} \tag{70}$$

The hydrodynamic retardation effect (a particle will experience a higher drag force than that given by Stokes law as it moves near a solid surface) was not incorporated into the governing equations. Studies (Rajagopalan, 1976) based on the Lagrangian approach have shown that the inclusion of this effect requires the incorporation of Van der Waals forces, otherwise, the extra viscous barrier created by the hydrodynamic effect would prevent a particle from actually contacting the surface of a collector (i.e., a capture is impossible)! Other external forces such as those from electromagnetic fields were assumed to be absent.

Boundary conditions of zero concentration were applied to the filter web surfaces. That is to say, the particles were assumed to be instantaneously captured once they had been transported to the surfaces of the filter webs. This, in effect, attributes a sink condition to filter web locations, thereby simulating actual filtration effects taking place in the real filter. Since the main mean geometric properties of the real filter were reflected in the computational domain, it was hoped that the particle capture rate computed in the simulation would hopefully approximate actual filtration rates. The dimensionless boundary condition set for inclusion concentration density at the inlet was unity, while at the outlet, an inclusion concentration density gradient equal to zero was imposed.

Figure 22 shows an example of computed inclusion concentration fields. The concentration ratio in the liquid phase as a function of filter thickness was obtained directly. The effects of captured particles inside the filter on the flow field, as well as on the mechanism of adhesion, were not considered. As such, the present computations should represent the behaviour of a clean filter during its initial stages of filtration.



Figure 22. An example of predicted iso-particle-concentration (c/c_0) contours within a model filter. The values for the concentration at the filter web locations (which are technically represented by nodal points) are zero.

3.8.1. Effect of Filter Thickness on Filtration Efficiency

Figure 23 shows predicted concentration profiles as a function of filter thickness. A regression analysis showed that the concentration ratio or the penetration followed the logarithmic law, as first observed by Iwasaki (1937) in his water filtration with sand beds. With this clarification of the influence of filter thickness, one can rewrite Eq. (38) in terms of a filter coefficient:

$$\lambda_0 d_w = f(\varepsilon, Re, Pe, N_G, Sk, N_R)$$
(71)

Inertial effects, characterized by the Stokes number (Sk), are normally in terms of filtration studies insignificant when the suspending fluid is a liquid, and were therefore not considered. The interception parameter, N_R , characterising the effect of the relative sizes of the particle and filter web, can be treated as a boundary condition so as to account for the fact that a particle would contact a filter web, once the distance from its centre to the surface of the web is equal to, or less than, the particle radius. This parameter was, however, not investigated in this study, because the grid sizes employed were not fine enough to take it into account.

The effects of other dimensionless groups are discussed in the following sections. Figure 24 - Figure 27 show the dependence of the concentration ratio as a function of filter thickness on Peclet, Reynolds, and gravitational (normalized Stokes) numbers, as well as filter porosity. (More predicted results for the cases of 87% and 92% porosities are presented in Appendix III). Figure 28 - Figure 30 show filter coefficients predicted for clean filters as a function of these dimensionless variables. The dependence of the clean filter coefficient, λ_0 , on an individual dimensionless variable was found to be:



Figure 23. Predicted concentration ratio as a function of dimensionless filter thickness.

$$\lambda_0 d_w \propto P e^{-0.96} \tag{72}$$

$$\lambda_0 d_{\rm w} \propto \varepsilon^{-6.93} \tag{73}$$

$$\lambda_0 d_w \propto |N_G| \tag{74}$$

Careful inspection of the above results revealed that there is no simple functional form, rigorously speaking, for the dimensionless clean filter coefficient. For engineering purposes however, the following approximate expression deduced from the above results may be used:

$$\lambda_0 d_w = e^{-6.93} (0.169353 P e^{-0.96} + 0.033126 |N_G|)$$
(75)

3.8.2. Effect of Peclet Number on Filtration Efficiency

As seen from Figure 24, the predicted filtration efficiency decreases as the Peclet number increases. Quantitatively, as shown in Eq. (72), the clean filter coefficient was predicted to have a - 0.96 power dependence on the Peclet number. It should be noted that the effect of Peclet number is only important for submicron inclusions, i.e., when the diffusivity of the inclusions, D, is not too small as compared to the product $V_{d'}d_w$. For larger inclusions, its effect becomes negligible. For instance, for a 1 μ m particle in a 700°C aluminum melt, its Brownian diffusivity, D, according to Eq.(70), is about 1.21x10⁻¹² m²/s if the viscosity of liquid aluminum is taken as 0.001175 kg/m·s. This corresponds to a Peclet number of 2.5x10⁶ if the superficial velocity and the filter web's diameter are taken as 0.01 m/s and 0.3 mm respectively. As far as the metal quality is concerned, the adverse effects of inclusions arise mainly from those that are much larger than 1 μ m; for most applications, small, or submicron, particles create little harm to a metal's properties. Therefore, the effect of Peclet number should not be the main concern



Figure 24. Predicted effect of Peclet number on penetration, as a function of filter thickness.



Figure 25. Predicted effect of Reynolds number on penetration as a function of filter thickness.



Figure 26. Predicted effect of the gravitational number on penetration as a function of filter thickness.



Figure 27. Predicted effect of porosity on penetration as a function of filter thickness.



Figure 28. Predicted filter coefficient as a function of Peclet number for various filter porosities.



Figure 29. Predicted filter coefficient as a function of gravitational number.


Figure 30. Predicted filter coefficient as a function of porosity.

in liquid metal filtrations.

3.8.3. Effect of the Effective Porosity

The predicted effect of filter porosity on filtration efficiency was shown in Figure 27. As one might anticipate, filtration efficiency increases as the effective porosity of the filter decreases. The clean filter coefficient, as seen from Eq.(73), was found to have a - 6.93 power dependence on ε .

3.8.4. Effect of the Gravitational Number

Predicted filtration efficiency, as seen from Figure 26, increases with the gravitational number or the normalized Stokes velocity. Quantitatively, as seen from Eq.(74), the clean filter coefficient was found to be linearly proportional to N_G . This may be explained in terms of heavier particles having a greater tendency to deviate from the streamlines to contact the surfaces of the filter webs when the flow diverges around the filter webs if the flow direction is the same



Figure 31. Schematic illustration of particle capture mechanisms due to the gravitational number.

as that of gravity. In a more general sense, if one stipulates that N_G is positive when its direction is the same as that of the fluid flow and negative if opposite, one can state that when $N_G>0$, the particles will tend to deviate from the streamlines and move towards the filter webs as the flow diverges around the filter webs, while when $N_G<0$, the particles would do this as the flow converges around filter webs (Figure 31 schematically

illustrates this statement). Since 100% adhesion is assumed in the simulation, a higher rate of transportation of particles to the surfaces of the filter webs would translate into a higher removal _fficiency.

3.8.5. Effect of Reynolds Number

According to the water flow experiments, the Revnolds number, based on the average web diameter, should not exceed 8 for the reticulated filters commonly used in melt filtration in order to stay within the Darcy domain. The predicted influence of Reynolds number on the concentration profile in the Darcy domain was found to be weak (a slight increase in filtration efficiency with Re, as seen from Figure 25). This is because in the Darcy domain, the flow is virtually creeping. In his studies based on the Lagrangian approach, Payatakes (1973) reached a similar conclusion. When filtration is operated outside the Darcy domain, however, especially when the flow inside the filter becomes turbulent, an increase in Reynolds number would enhance particle transportation processes since the effective diffusivity of the particles would be greatly increased due to added turbulent diffusivity. On the other hand, an increase in Reynolds number increases drag forces. This will have an adverse effect on filtration efficiency, because particles may not be able to deposit on the surfaces of the filter webs, even if they are able to make a contact with them. Similarly, partic'es already captured, may be scoured off by the increased drag forces, causing a 'release'. In practice, therefore, filtration should only be operated at low flow rates (low Reynolds number), in order to minimize particle re-entrainment, unless particle adhesion to webs is particularly strong, as in the case for filtering liquid steel to remove alumina particles.

As mentioned earlier, an actual deep bed filtration process can be very complicated. The effects of the various parameters may take place simultaneously. In addition, as captured particles accumulate inside the filter, the internal geometry of the filter, together with its surface properties will change. Such geometrical changes would decrease the filter's effective porosity (a favourable effect). However, this would cause an increase in local interstitial velocities (if the superficial velocity is kept constant, $v_i = V_{st}/\varepsilon$), and by consequence, increase drag forces (an unfavourable effect). The change in surface properties may also be favourable or unfavourable. Therefore, any analytical or numerical analyses can only serve as a general guideline for an actual filtration operation. This is by no mean to devalue the importance of theoretical analyses in providing the insights needed for improving a filter's performance.

3.8.6. Effect of Dimensional Variables

The influence of the dimensional variables on filtration efficiency can be easily recognized by rewriting equations (72)-(74) in their dimensional form:

$$\lambda_0 \propto \frac{1}{d_w} \left(\frac{kT}{3\pi \mu d_p V_{sf} d_w} \right)^{0.96}$$
(76)

$$\lambda_0 \propto \frac{1}{d_w \varepsilon^{6.93}} \tag{77}$$

$$\lambda_0 \propto \frac{1}{d_w} \frac{(\rho_p - \rho) d_p^2 g}{18 \mu V_{sf}}$$
(78)

For Brownian particles, diffusion is dominant, the smaller the particle size, superficial velocity and web diameter, the higher the filtration efficiency. For non-Brownian particles, however, gravitational sedimentation or floatation becomes dominant. Therefore, the greater the particle size and density difference, and the smaller the superficial velocity, the higher the filtration efficiency. As far as the filter parameters are concerned, the filtration efficiency increases as the effective porosity and filter web diameter decrease, and as the filter thickness increases.

In designing filters, however, one should also take aging effects into consideration. Studies (Ives, 1960, and Tien, 1989) have shown that particle deposition inside the filter follows an exponential decay from the inlet to the outlet. This may lead to clogging in the inlet region of the filter as filtration proceeds, leaving the rest insufficiently used. In addition, practical considerations, such as filter priming, minimal pressure drop requirement, filter strength, etc., and most importantly, operational costs, must also enter the designer's formula.

The importance of the present findings embodied in equations (72)-(74) should perhaps be appreciated not only for further metal filtration studies, but also for fluid flow and mass transfer studies in porous media at large. The approach employed in the present study may also be used in the design of filters, catalytic reactors and other engineering systems associated with consolidated, highly porous, structures.

3.8.7. Comparison with Other Studies

In their trajectory analyses, Rajagopalan and Tien (1976) and Tien (1989) developed an approximate expression for the dimensionless clean filter coefficient as a function of various independent dimensionless variables based on Happel's sphere-in-cell model (1958). The results obtained in the present study are roughly in line with their earlier findings, despite the very different modelling approach as employed: they approximated (1) a power dependence of 1.2 on N_G , by assuming that the collection efficiency of a unit collector was much less than 1; (2) a power dependence varying from -7.33 to -10.25 on filter porosity over the range of porosities of interest to this study. (Those authors presented it in terms of a porosity function and the author regressed it into a power function for the sake of comparison, see Figure 32).



Figure 32. Porosity dependence of clean filter coefficient, a comparison with other study.

According to Herzig et al's (1970) quotation, Hall (1957) had suggested that the clean filter coefficient be linearly proportional to the normalized Stokes velocity. Studies based on single collector modelling showed that the removal efficiency, η , associated with an individual collector, due to diffusion alone, was equal to the algebraic sum of aPe^{-1} and $bPe^{-2/3}$ for creeping flow (where *a*, *b* are constants, see Stechkina and Fuchs, 1966, and Ingham, 1981). η , due to gravity alone, was predicted to be linearly proportional to N_G (see Rajagopalan and Tien, 1977). This is in accord with the results of this study (since the filter coefficient, λ , was shown to depend on η raised to the first power (see Yao, Habibian and O'Melia, 1971, and O'Melia and Stumm, 1967).

Experimental observations made by Yao (1968) and Ison and Ives (1969) in their water filtration studies seemed to confirm the significant role of N_G . Ison and Ives observed that the dimensionless filter coefficient was almost linearly dependent on the normalized Stokes velocity for the filtration of kaolinite particles entrained in tap water passing through randomly packed beds of nearly mono-sized glass beads.

3.9. Grid Size Consideration

Grid size very often affects numerical results, especially for situations where steep gradients exist. This is true when the physical geometry can be modelled exactly, in other words, when the physical configuration is known precisely. In this case, the absolute values of variables of interest can be obtained numerically only when the grid size employed is sufficiently small as to capture rapid changes in the variables at certain locations such as at a solid-liquid interface, a cold-hot boundary or other places exhibiting abrupt changes within the domain. In the present study, however, the exact local geometry of reticulated filters is not known. As such, absolute values of the variables of interest can not numerically be obtained based on the above principle. Since the numerical calculation is not based on the exact physical geometry, instead, it is based on a model which bears the main average geometrical properties of the real filters, which is a necessary, not a sufficient condition as far as the filtration process is concerned. The only way to reach the goal is, in this case, perhaps by trial and error, i.e., by changing the grid size (for a give model) to see which grid size best approximates the absolute values of the interested variables obtained experimentally. For a parametric study, however, it is the trend rather than the absolute value which is the primary concern, a model which embodies the average geometrical properties of the real filter should be sufficient for this purpose.

Figure 33 - Figure 34 show the effect of grid size on the predicted clean filter coefficient as a function of Peclet and Gravitational numbers respectively. As seen, the absolute value changed but the trend was not affected (the power of Pe and N_G did not change).

It should be noted that the comparison of predicted pressure drops with that obtained from the water flow experiments shown in Figure 20 and the approximate expression for the dimensionless clean filter coefficient, Eq.(75), were based on the case of one SCV (scaler control volume) occupied by the filter web. As will be further seen in next chapter, among the grid sizes studied in this modelling, the case of one SCV occupied by the filter web actually approximates the filtration phenomena of the real filters better than other cases.



Figure 33. Effect of grid size on clean filter coefficient as a function of Peclet number.

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Figure 34. Effect of grid size on clean filter coefficient as a function of gravitational number.

3.10. Summary

In this chapter, the primary parameters influencing the efficiency of initial filtration of liquid metal through reticulated ceramic filters were identified, after analysing the physical phenomena involved. Some of these were subsequently quantified by numerically solving the Navier-Stokes and particle conservation equations governing inclusion transport in liquid metals. A 2-D computational domain or a model bearing the main average geometric properties, i.e., the effective porosity and average equivalent filter web diameter of the real filters, was used for this purpose. According to the numerical simulation, the clean filter coefficient for this type of filter had a linear dependence on the dimensionless Stokes velocity of the suspended particles, a -0.96 power dependence on the Peclet number, a -6.93 power dependence on the effective porosity of the filter, and a weak dependence on the Reynolds number, in the Darcy velocity regime.

Predicted pressure drops were found to be roughly comparable with data obtained from water flow experiments up to a maximum Reynolds number of 8 (the characteristic length for Re is based on the average diameter of the filter webs). The predicted influences of various dimensionless groups on the initial filtration efficiency were found to be in line with results from other theoretical studies.

Chapter 4

Simulating the Dynamic Behaviour of Filters

4.1. Introduction

In the previous chapter, the initial filtration processes associated with reticulated ceramic filters were analyzed numerically. In this chapter, discussions will be made on filter performance following the initial period.

Filtration processes associated with reticulated filters are bound to be transient, that is to say the filtration processes, as well as the parameters pertaining to them, change with time. This is because the particles captured inside the filter alter the filter's properties - a process that has been referred to as 'filter aging'. By definition, a filter starts to age as soon as particles start to accumulate inside it. During this period, the filter's internal structure changes as a result of the continuous accumulation of captured particles as filtration proceeds. These changes will inevitably cause some of the independent variables listed in Eq.(37) to change. The most apparent one is the porosity, which would decrease with time provided there is no release of inclusions. The specific surface area, or equivalent filter web diameter, (the two are related, either one can be chosen for an independent variable) also change. Depending on how the particles are captured (or deposit morphology), the effect can be different, e.g., if the particles deposit in a uniform fashion onto the filter webs, causing an increase in d_w , this would, as seen from the numerical analyses presented in the previous chapter, have an adverse effect on removal efficiency. Alternatively, if the particles are captured in a dendritic or clustering

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fashion, a favourable filtering effect would result since the diameter of the dendritic clusters, acting as new 'filter webs' capturing oncoming particles, would usually be much smaller than d_w . The change in interfacial characteristics can also be favourable or unfavourable. As particles accumulate (or porosity decreases), flow resistance increases, causing pressure drops to rise together with interstitial velocities, if the flow rate through the filter is maintained constant. As a result, the drag exerted on either the contacting particles or the particles already captured, will increase. This will, in the former case, tend to lead to a lower adhesion efficiency, i.e., the particles may not be able to adhere to filter web's surface even if they are able to first make contact with them owing to increased drag; in the latter case, captured particles may be dragged away if drag forces exceed adhesion. The particles scoured from one location may be recaptured in other locations further down the filter or, worse still, be released into filtered liquid metal.

The actual phenomena happening during the aging period can therefore be extremely complicated. A comprehensive method capable of predicting the dynamic behaviour of these filters over their entire operational spans may, rigorously speaking, be out of reach. This is mainly due to the stochastic nature of particle deposition processes within the filter, which leads to uncertainties over the morphology of the deposits formed by the captured particles as well as its evolution during progressive deposition. However, one may attempt to make such complex phenomena amenable to theoretical analysis by simplifying the actual processes on the basis of some hypothetical models. Such attempts have been made by a few of researchers in studies of deep bed filtration in other engineering fields. For example, Tien et al (1979), described the aging process as consisting of two main stages: In the first stage, they assumed that: particles uniformly adhere to the filter grains to form a smooth layer outside the latter; this process continues until the amount of captured particles reach a certain (transitional) value. In the second stage, they assumed that: the deposition process is dominated by the formation of particle aggregates and their re-entrainment and re-deposition into various narrow pores, leading to blocking of certain parts of the filter bed; and (b) this process

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continues until the reduced area available for flow leads to a sufficient increase in interstitial velocity that the bed becomes non-retentive. They argued that in the first stage, the consequence of deposition was only to decrease porosity and to increase the filter grain's diameter. They supposed that in the second stage, only the interstitial velocity continued to increase, all other parameters remaining unaffected. Based on these assumptions, they evaluated filter performance for the two corresponding stages, and developed some correlations for the filter coefficient and the pressure drop to describe the dynamic behaviour of a filter bed. Meanwhile Chiang and Tien (1985) studied two limiting situations for the aging period. The hypotheses employed in the first limiting situation was similar to those made by Tien et al (1979) for their first stage. In the second limiting situation, they assumed the captured particles form rigid aggregates or dendrites which act as additional collectors. These authors examined filter performance in terms of filtration efficiency and pressure drop as a function of the amount of captured particles inside the filter utilizing the trajectory analysis technique based on the constricted tube filter model proposed by Payatakes et al (1973). In their analysis for the second situation, the movements of the particles were treated as being stochastic, the particles once captured on the filter grain surface were treated as part of the filter grain which collects oncoming particles. The pressure drop was obtained by evaluating the total drag imposed on the filter grain as well as on the captured particles. This was based on the method described by Pendse and Turian (1981). To verify their theoretical analyses, these authors flew lycopodium and ragweed particles of a size of 26 μ m suspended in water through a packed bed of glass beads of 505 μ m diameter. Their experimental results disagreed with their theoretical analyses. The filtration efficiency obtained experimentally was found to be higher than that predicted for the first limiting case and lower than that for second situation. In view of this, these authors proposed an empirical correlation combining both experimental and theoretical results. The dendritic model was also studied by O'Melia and Ali (1978), who proposed a rate of particle collection based on both the filter grain and captured particles.

4.2. Effect of Captured Particles on Filtration Efficiency

The most important issues of interest in the aging period are of course the change in filtration efficiency and pressure drop as a function of time. In other words, how the captured particles affect filtration efficiency and pressure drop. The former directly concerns the quality of the metal filtered, while the latter determines whether a filtration process is still operable without metal overflowing the launders.

One may analyze the former by evaluating the change in the filter coefficient during the aging period, in conjunction with macroscopic mass balance equations describing the filtration process, based on certain assumptions.

4.2.1. Change in Filter Coefficient

The filter coefficient during the initial period was, as seen from the previous chapter, treated as being a constant with respect to time and filter thickness for filters of isotropic structure. During the aging period however, the filter coefficient would change with both time and location due to the effect of captured particles inside the filter, as just discussed. The change may be evaluated based on the result obtained for the initial period. One may write:

$$\lambda_{t,\tau} = \lambda_0 + \Delta \lambda_{\tau} \tag{79}$$

where t, z denote time and location in the main flow direction. $\Delta \lambda_z$ is given as:

$$\Delta \lambda_{z} = \int_{t-\delta t}^{t} d\lambda \tag{80}$$

 $d\lambda$ can, in principle, be evaluated by examining the changes in its independent variables listed in Eq.(37). The variables which may possibly change may include the effective filter porosity, ε ; the filter web diameter, d_{ω} ; the Hamaker constant, H (in the initial

period, particles directly adhere to the filter web surfaces, therefore, the Hamaker constant between the particle and filter material would be in effect, while during the aging period, the particles, in addition to adhering to the filter web surfaces, may also adhere to particles already captured, so that the Hamaker constant between particles should also in effect); the roughness of the filter web's surface, δ ; the orientation at point of contact, φ ; and contacting angle, ϑ ; etc. Other variables such as the superficial velocity, the properties of the melt and or the particles are unlikely to be affected during the aging period. However, the above analysis may not exhaust all possibilities. Once again, if one assumes that the adhesion efficiency or probability during the aging period is 100 percent, or if one neglects all the changes except those in ε and d_{w} , then:

$$d\lambda = \frac{\partial \lambda}{\partial \varepsilon} d\varepsilon + \frac{\partial \lambda}{\partial d_w} d(d_w)$$
(81)

If one starts with a clean filter and proceeds with a small time step, δt , so that $\lambda_{t,x}$ can be found by knowing $\lambda_{t,x,x}$ obtained from the previous time step, then $\lambda_{t,x}$ may, based on the approximate Eq.(75), be given as:

$$\lambda_{t,z} = \lambda_{t-\delta t,z} + \int_{t-\delta t}^{t} \frac{\partial \lambda}{\partial e} de + \int_{t-\delta t}^{t} \frac{\partial \lambda}{\partial d_{w}} d_{w}$$

$$= \lambda_{t-\delta t,z} + \frac{b}{d_{w}^{t-\delta t,z}} \left(\frac{1}{e^{6.93}_{t,z}} - \frac{1}{e^{6.93}_{t-\delta t,z}} \right)$$

$$+ \frac{1}{e^{6.93}_{t-\delta t,z}} \left\{ c_{1} \left[\left(\frac{1}{d_{w}^{t,z}} \right)^{1.96} - \left(\frac{1}{d_{w}^{t-\delta t,z}} \right)^{1.96} \right] + c_{2} \left(\frac{1}{d_{w}^{t,z}} - \frac{1}{d_{w}^{t-\delta t,z}} \right) \right\}$$
(82)

where $\varepsilon_{r,z}$, $\varepsilon_{r,x,z}$, d_w^{t} and $d_w^{t,x}$ denote respectively the effective porosity and equivalent filter web diameter at time t and t- δt at location z. The symbols b, c_1 and c_2 represent constants which may be evaluated from the approximate Eq (75).

Apparently, ε_{Lz} is directly related to the specific deposit, α , i.e., the volume

fraction of the particles captured inside the filter:

$$\boldsymbol{\varepsilon}_{tz} = \boldsymbol{\varepsilon}_0 - \frac{\boldsymbol{\alpha}_{t,z}}{1 - \boldsymbol{\varepsilon}_a}; \qquad \boldsymbol{\varepsilon}_{t-\delta t,z} = \boldsymbol{\varepsilon}_0 - \frac{\boldsymbol{\alpha}_{t-\delta t,z}}{1 - \boldsymbol{\varepsilon}_a}$$
(83)

where ε_0 and ε_a are respectively the effective porosity of the clean filter and the porosity of the agglomerates of the captured particles inside the filter.

If one assumes that the captured particles adhere uniformly to filter webs, then it can be readily shown that d_w^{iz} is related to ε_{iz} by the following equation:

$$d_{w}^{t,z} = d_{w}^{0} \sqrt{\frac{1-\varepsilon_{t,z}}{1-\varepsilon_{0}}} = d_{w}^{0} \sqrt{1+\frac{\alpha_{t,z}}{(1-\varepsilon_{0})(1-\varepsilon_{a})}}$$
(84)

where d_{x}^{0} is the average equivalent diameter of the filter webs when the filter is clean, i.e., when t=0. It can be seen that, in such circumstances, $\lambda_{t,z}$ depends on a single variable, $\alpha_{t,z}$, the specific deposit. An averaged value of $\alpha_{t,z}$ (or an index to it) over a given filter thickness may be obtained experimentally from the pressure drop which can be monitored during filtration. If no release occurs, $\alpha_{t,z}$ should be always greater than $\alpha_{t,d,z}$, while if the measured values show the opposite, then some of the particles retained must have been subsequently released, i.e., when:

$$\frac{\partial \alpha}{\partial t} = \lim_{t \to 0} \frac{\overline{\alpha_t} - \overline{\alpha_{t-\delta t}}}{\delta t} < 0$$
(85)

where the overlined quantities denote an averaged value over a given filter thickness.

If one starts with a clean filter, or if one always uses the clean filter as a reference, then, substituting Eq.(83) and Eq.(84) into Eq.(82), one has:

$$\lambda_{t,z} = \lambda_{0} + \frac{b}{d_{w}^{0}} \left[\frac{1}{\left(\varepsilon_{0} - \frac{\alpha_{t,z}}{1 - \varepsilon_{a}}\right)^{6.93}} - \frac{1}{\varepsilon_{0}^{6.93}} \right] + \frac{c_{1}}{\varepsilon_{0}^{6.93}} \left[\frac{1}{\left(\frac{1}{d_{w}^{0}} \sqrt{1 + \frac{\alpha_{t,z}}{(1 - \varepsilon_{0})(1 - \varepsilon_{a})}} \right)^{1.96}} - \left(\frac{1}{d_{w}^{0}}\right)^{1.96} \right] (86) + \frac{c_{2}}{\varepsilon_{0}^{6.93}} \left[\frac{1}{d_{w}^{0}} \sqrt{1 + \frac{\alpha_{t,z}}{(1 - \varepsilon_{0})(1 - \varepsilon_{a})}} - \frac{1}{d_{w}^{0}} \right]$$

where the subscript or superscript 0 denotes quantities associated with the clean filter.

For non-Brownian particles, c_1 is vanishingly small, and can therefore be neglected (c_1 is proportional to the term associated with the Peclet number, while c_2 is proportional to the term associated with the gravitational number, in Eq.(75)). The constants b and c_2 can, in this case, be set approximately equal to 0.033 | N_G | according to Eq.(75). Eq.(86) then becomes:

$$\lambda_{tz} = \lambda_0 + \frac{b}{d_w^0} \left[\frac{1}{\left(\epsilon_0 - \frac{\alpha_{tz}}{1 - \epsilon_a} \right)^{6.93}} - \frac{1}{\epsilon_0^{6.93}} \right] + \frac{c_2}{\epsilon_0^{6.93}} \left[\frac{1}{d_w^0 \sqrt{1 + \frac{\alpha_{tz}}{(1 - \epsilon_0)(1 - \epsilon_a)}}} - \frac{1}{d_w^0} \right] (87)$$

Eq.(87) is based on a "smooth coating" model due to the assumption made in Eq.(84).

If one assumes that the accumulation of captured particles only affects the effective porosity and that the average filter web diameter remain unaffected (a "only porosity change" model), then the second term on the right hand side of Eq.(87) vanishes.

$$\lambda_{tz} = \lambda_0 + \frac{b}{d_w^0} \left[\frac{1}{\left(\varepsilon_0 - \frac{\alpha_{tz}}{1 - \varepsilon_a}\right)^{6.93}} - \frac{1}{\varepsilon_0^{6.93}} \right]$$
(88)

It may be instructive to note that empirical expressions for $\lambda_{t,z}$ proposed by various authors in the field of Chemical Engineering, as summarized by Herzig et al (1970) and Tien (1989), all took α as the main variable.

4.2.2. The Macroscopic Equation

For a suspension flowing through a deep bed filter, the macroscopic mass balance equation for a one dimensional case can be easily shown as (see Herzig et al, 1970):

$$\frac{\partial(\varepsilon c + \alpha)}{\partial t} + V_{sf} \frac{\partial c}{\partial z} = 0$$
 (89)

where c is the concentration in the liquid phase. Diffusional, as well as gravitational or buoyant effects, were not considered in this equation.

When $\alpha \ge \varepsilon c$, or if one introduces a corrected time (see Tien, 1989 or Appendix IV), Eq.(89) becomes, or transforms into:

$$\frac{\partial \alpha}{\partial t} + V_{sf} \frac{\partial c}{\partial z} = 0$$
 (90)

Combining the Logarithmic Law with Eq. (90), one has:

$$\frac{\partial \alpha}{\partial t} = V_{sf} \lambda c \tag{91}$$

The initial boundary conditions for Eq.(90) and Eq.(91) are obviously:

$$c = c_{in} \qquad at \quad z = 0, \quad t > 0$$

$$c = 0, \quad \alpha = 0 \qquad for \quad z \ge 0, \quad t \le 0$$
(92)

These two hyperbolic partial differential equations may be solved simultaneously using certain techniques, once the functional form for the filter coefficient is known. An alternative method has been demonstrated by Herzig et al (1970) and by Tien (1989), who showed that these two partial differential equations were equivalent to a pair of two ordinary differential equations and that can consequently be solved sequentially. The latter approach was employed in the present study in order to simulate the dynamic behaviour of reticulated ceramic filters.

Applying Eq.(91) at z=0, then:

$$\left(\frac{\partial \alpha_{in}}{\partial t}\right)_{z=0} = V_{zf} \lambda(\alpha_{in}) c_{in}$$
(93)

with initial condition

$$\alpha_{in} = 0 \quad at \quad t = 0 \tag{94}$$

where the subscript *in* denotes quantities at the inlet, i.e., at z=0. One may note that in this case, c_{\perp} is simply a constant, or a known quantity.

Tien (1989) showed that

$$\left(\frac{\partial \alpha}{\partial z}\right)_{t} = -\lambda \alpha \tag{95}$$

with initial boundary conditions

$$\alpha = \alpha_{in} \quad at \quad z = 0 \tag{96}$$

As seen, Eq.(93) and Eq.(95) are essentially ordinary differential equations. α_{in} , the initial boundary condition for Eq.(95), can be obtained by first solving Eq.(93). As such, the value of α as a function of z for a given time can be obtained by solving Eq.(95). Once the value of α as a function of time and location is known, the concentration profile in the liquid phase may be obtained from the logarithmic law, i.e.:

$$\left(\frac{\partial c}{\partial z}\right)_{t} = -\lambda(\alpha)c \qquad (97)$$

the value of $\lambda(\alpha)$ can be obtained based on the correlations presented in the previous section. Alternatively, it can be simultaneously solved in conjunction with Eq.(95), using the Runge-Kutta methods.

To simulate the distribution and evolution of the specific deposit inside the filter, and its effect on the concentration profile in the liquid phase, and thus on the filtration efficiency, during the aging period, Eq.(93) to Eq.(97) were solved numerically using a fourth order Runge-Kutta method (see Appendix V). The functional form for the filter coefficient was taken as Eq.(87) or (88), with the constants b, c_2 being set equal to $0.033 | N_G |$. As will be seen in the next chapter, this value, which is based on Eq.(75), may only be applicable for non-Brownian, non-neutrally-buoyant, particles. Simulations were conducted based on the 'only porosity change' assumption, i.e., Eq.(88) as well as the 'smooth coating' model, Eq.(87). The simulated results presented below are based on the following conditions: effective filter porosity: 83%; average filter web diameter: 0.3 mm; gravitational number: 0.1; clean filter coefficient, λ_0 : 0.0411 mm⁻¹.

4.2.3. Distribution of Captured Particles Inside the Filter

When the filter is relatively clean, the specific deposit was found to follow an



Figure 35. Predicted distribution of specific deposit inside the filter.



Figure 36. Distribution of captured particles inside the filter after two hours of filtration.

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exponential decay from the inlet to outlet (see Figure 35). This is not surprising, since as seen from Eq.(95), when the amount of captured particles is small, the value of λ is almost the same as that of λ_0 (a constant). Integration of this equation gives an exponential correlation relating α and z. This phenomenon has been observed in deep bed filtration studies in other engineering field (e.g., Ives 1960 and Tien 1989).

As filtration continues, the effect of captured particles becomes more and more important, especially in the inlet region. Because of the autocatalytic character of this process (i.e., the higher the α_{Lz} , the higher the λ_{Lz} , which in turn, causes α_{Lz} to be even higher), more particles would deposit in the inlet region as the filter ages, causing a deviation from the exponential regression (see Figure 36). This phenomenon was also observed in O'Melia and Ali's study (1978). Figure 37 - Figure 39 show the evolution in the distribution of specific deposit inside the filter; while Figure 40 - Figure 41 show the change in the specific deposit at different locations, as a function of filtration time.

4.2.4. Possibility of Caking and Filter Clogging

As this process continues, the particles captured would, if their bonding forces are strong enough to prevent them from being dislodged by the increased drag caused by the increased interstitial velocity, eventually bridge up all the openings (pores) of the inlet region, causing the filtration to change mode, from one of depth to cake filtration. This would be accompanied by the occurrence of a rapid increase in pressure drop and a drastic decrease in flow rate provided the captured inclusion agglomerates are permeable for a given allowable head limit. If not, filter clogging would occur. This would bring the filtration operation to an end. In other engineering fields, suggestions have been made to use multi-layer filters with the coarsest layer facing the inlet (Ives 1971), in order to effectively use the capturing capacity of a deep bed filter.



Figure 37. Distribution of captured particles inside the filter as a function of time (inlet concentration 0.1 ppm).







Figure 39. Distribution of captured particles inside the filter as a function of time (inlet concentration: 10 ppm).



Figure 40. Change in specific deposit as a function of filtration time at different locations.



Figure 41. Change in specific deposit as a function of filtration time at different locations.

4.2.5. Possibility of Particle Release

If the bonding forces between the captured particles and the filter, or between the captured particles themselves, are exceeded by the increased drag forces as mentioned above, some of the captured particles can be dislodged, so that then:

$$\frac{\partial \alpha}{\partial t} = \lim \frac{\alpha_{t,z} - \alpha_{t-\delta t,z}}{\delta t} < 0; \quad i.e., \ \alpha_{t,z} < \alpha_{t-\delta t,z}$$
(98)

since in this case, the volume fraction of captured particles inside the filter at time t is less than that at $t-\delta t$, some of the captured particles must have been dislodged. This process was not simulated however, since to do so, one needs to know the specific conditions under which the captured particles would be released. Arbitrary empirical formulae serve, in the author's opinion, no more than a qualitative description of the physical phenomenon itself.

4.2.6. Effect of Melt Cleanliness

The simulation showed that the specific deposit at a given location increases with inlet particle concentration in the melt (see Figure 42). As seen, the rate of increase at the filter inlet becomes appreciable as the inlet concentration reaches 10 ppm (volume fraction).

4.2.7. Change in Filtration Efficiency as a Function of Time and Melt Cleanness

Figure 43 - Figure 45 show the concentration profiles in the liquid phase as a function of time and inlet particle concentration in the melt. As seen, the concentration profiles were shifted downwards as filtration proceeds. However, the change was found to be insignificant provided the inlet particle concentration was less than 1 ppm over a



Figure 42. Effect of inlet particle concentration on the specific deposit as a function of filtration time.

filtration period of two hours. On the other hand, when the inlet particle concentration reached 10 ppm, the change became significant. Such a change in concentration profile as a function of filtration time has been observed experimentally by Mehter et al (1970) in their filtration of aqueous suspensions through a granular bed of carbon. Figure 46 shows the change in filtration efficiency of a 38 mm thick filter as a function of time and inlet particle concentration. As seen, the filtration efficiency increases with time, but the increase was insignificant when the inlet particle concentration is less than 1 ppm, while when the particle concentration reaches 10 ppm, the filtration efficiency increases appreciably (of course, this is only true when no release occurs).

4.2.8. 'Smooth Coating Model' vs 'Only Porosity Change' Model

The results presented in the above sections were based on the 'only porosity change' model. The results based on the 'smooth coating' model showed similar trends. Figure 47 - Figure 51 show the specific deposit and the filtration efficiency as a function of time, filter thickness and inlet particle concentration, based on the two models. As seen, the 'smooth coating' model slightly underestimates the specific deposit at the filter inlet (see Figure 47) when the inlet concentration is less than 1 ppm, while when the inlet particle concentration reached 10 ppm, the difference becomes noticeable (see Figure 48). However, the opposite was observed at the filter outlet (i.e., the 'smooth coating' model slightly overestimates the specific deposit compared to 'only porosity change' model, see Figure 49 - Figure 50). As far as the filtration efficiency is concerned, the 'smooth coating' model gives a lower predicted value than the 'only porosity change' model (see Figure 51). The reason for this can be seen from Eq.(87), i.e., as the filter web becomes thicker, the filter coefficient becomes lower and so does the filtration efficiency. By assuming 'only porosity change' the filter web remained unaffected during the aging period, one in effect assumes that the captured particles form extra 'filter webs', similar to the assumption of forming dendrites, the only difference between the two lies in that the former has a bigger diameter than the latter (see the



Figure 43. Evolution in concentration profile inside the filter during filter aging period (inlet concentration: 0.1 ppm).

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Figure 44. Evolution in concentration profile inside the filter during filter aging period (inlet concentration: 1 ppm).



Figure 45. Evolution in concentration profile inside the filter during filter aging period (inlet concentration: 10 ppm).



Figure 46. Change in filtration efficiency as a function of filtration time for different inlet particle concentrations.


Figure 47. Specific deposit at filter inlet as a function of time, comparing 'smooth coating' and 'only porosity change' models, (inlet concentration: 1 ppm).



Figure 48. Specific deposit at filter inlet as a function of time, comparing 'smooth coating' and 'only porosity change' models, (inlet concentration: 10 ppm).



Figure 49. Specific deposit as a function of filter thickness after 90 minutes of filtration, comparing 'smooth coating' and 'only porosity change' models, (inlet concentration: 1 ppm).



Figure 50. Specific deposit as a function of filter thickness after 90 minutes of filtration, comparing 'smooth coating' and 'only porosity change' models, (inlet concentration: 10 ppm).



Figure 51. Change in filtration efficiency as a function of time, comparing 'smooth coating' and 'only porosity change' models, (inlet concentration: 10 ppm).

schematic illustration in Figure 52). As such, the 'only porosity change' model would underestimate the filtration efficiency compared to the dendritic model, while it would give a higher prediction than the 'smooth coating' model, lying in between the two extreme cases.

Observations made on the spent filters used in aluminum filtration showed that the particles were captured inside the filter in the form of agglomerates (see Figure 74 in next chapter), indicating that neither the 'smooth coating' nor the dendritic model is appropriate for this situation. In other words, the 'only porosity change' model may be more appropriate for the representation of actual phenomena happening inside the filter during filtration of liquid aluminum. The dendritic model may be more suitable for liquid steel filtration when the inclusions are solid alumina particles entrained within the liquid steel, since these particles sinter to the filter web's surfaces or to the alumina particles already captured (Ali, et al, 1985; Tian, 1990), thus forming dendrites.



Figure 52. Schematic representation of different models for the morphology of captured particles.

4.3. Effect of Captured Particles on Pressure Drop

4.3.1. Correlation Between Pressure Drop and Specific Deposit

The pressure drop across the filter as a function of time and filter thickness during the aging period can be obtained by integrating the pressure gradient from the inlet to the outlet of the filter (Tien et al, 1979):

$$\Delta p = -(p_{out} - p_{in}) = -\int_0^L \frac{\partial p}{\partial z} dz$$
 (99)

During the initial period, i.e., when the filter is clean, the pressure gradient should be uniform throughout an isotropic filter. Eq.(99) can be written as:

$$\Delta p = -\left(\frac{\partial p}{\partial z}\right)_0 \int_0^L p(t) dz \qquad (100)$$

where

$$p(t) = \frac{\frac{\partial p}{\partial z}}{\left(\frac{\partial P}{\partial z}\right)_0}$$
(101)

where $(\partial p/\partial z)_c$ denotes the uniform pressure gradient of the clean filter (a constant with respect to time and location), while $\partial p/\partial z$ or p(t) is changing with respect to both time and location during the aging period as a result of the accumulation of particles within the filter. As can be seen from the analyses in the previous sections, p(t) essentially depends on the specific deposit, α_{tz} .

According to the water flow experiments presented in Chapter 2, the initial

pressure gradient across the reticulated ceramic filters can be estimated from Darcy's law, the permeability being approximated by the Carman-Kozeny equation. If one assumes that in the aging period, Darcy's law and the Carman-Kozeny equation still hold, for situations where the flow rate is maintained constant, p(t) may be approximated by the following equation:

$$p(s) = \left(\frac{S_{tz}}{S_0}\right)^2 \left(\frac{1-\varepsilon_{tz}}{1-\varepsilon_0}\right)^2 \left(\frac{\varepsilon_{tz}}{\varepsilon_0}\right)^{-3}$$
(102)

Assuming that $c = 4/d_w$, then substituting Eq.(84) and Eq.(83) into Eq.(102), one finds that p(t) is related to $\alpha_{t,z}$ by the following equation:

$$p(t) = \left[1 + \frac{\alpha_{tz}}{(1 - \varepsilon_0)(1 - \varepsilon_o)}\right] \left[1 - \frac{\alpha_{tz}}{\varepsilon_0(1 - \varepsilon_o)}\right]^{-3}$$
(103)

This equation corresponds to the 'smooth coating' model (due to the assumption made for Eq.(84)). With the 'only porosity change' model, one has:

$$p(t) = \left[1 + \frac{\alpha_{tz}}{(1 - \varepsilon_0)(1 - \varepsilon_o)}\right]^2 \left[1 - \frac{\alpha_{tz}}{\varepsilon_0(1 - \varepsilon_o)}\right]^{-3}$$
(104)

As seen, the 'smooth coating' model would underestimate the pressure drop comparing with the 'only porosity change' model. It was also shown in the studies made by Tien et al (1979) for packed granular beds, that the former would grossly underestimate the pressure drop. Numerous empirical expressions proposed for p(t) in other engineering fields can be found in Tien's book (1989).

Once the value of p(t) as a function of time and location is known, the pr⁻ssure

drop at a given time and filter thickness can be found from Eq.(100) once the initial pressure gradient is known. If one would like to know the relative, rather than the absolute value, then:

$$\frac{(\Delta p)_{t,L}}{(\Delta p)_{0,L}} = \frac{\int_0^L p(t) dz}{L}$$
(105)

4.3.2. Simulated Results

The effect of captured particles was simulated based on Eq.(104), where the value of $\alpha_{t,z}$ was taken from the previous sections based on the 'only porosity change' model. The simulated results, based on conditions of 83% effective filter porosity, a 0.3 mm filter web diameter, a gravitational number of 0.1, and a clean filter coefficient of 0.0411 mm⁻¹, were shown in Figure 53 - Figure 59. As seen from Figure 53, the pressure gradient in the aging period is no longer constant within the filter, decreasing from filter inlet to outlet; the change is small when the inlet concentration is less than 1 ppm for a filtration period of 90 minutes, while at 10 ppm, the change becomes appreciable. As seen from Figure 54 - Figure 56, the pressure gradient constantly increases with time, while the rate of increase drops from filter inlet to outlet. The rate of increase at a give location increases with inlet concentration (see Figure 57 - Figure 59). When the inlet concentration reaches 10 ppm, the pressure gradient at the filter inlet increases rapidly with time, after 90 minutes of filtration. However, when the inlet concentration is 0.1 ppm, the change in pressure gradient at the filter inlet is less than two percent for the first two hours of filtration.

Figure 60 - Figure 61 provide a comparison of the results obtained based on the two different models, i.e., the 'smooth coating' and the 'only porosity change' models. As seen, the 'smooth coating' model underestimates the pressure drop compared to the 'only porosity change' model.



Figure 53. Distribution of normalized pressure gradient, p(t), inside the filter after 90 minutes of filtration for different inlet concentration levels.







Figure 55. Change in normalized pressure gradient, p(t), at filter inlet, middle and outlet, as a function of time (inlet concentration: 1 ppm).



Figure 56. Change in normalized pressure gradient, p(t), at filter inlet, middle and outlet, as a function of time (inlet concentration: 10 ppm).



Figure 57. Effect of inlet particle concentration on the evolution of normalized pressure gradient at the filter inlet.



Figure 58. Effect of inlet particle concentration on the evolution of normalized pressure gradient at the middle of the filter.



Figure 59. Effect of inlet particle concentration on the evolution of normalized pressure gradient at filter outlet.



Figure 60. Change of normalized pressure gradient, p(t), with time at filter inlet, a comparison between 'smooth coating' and 'only porosity change' models (inlet concentration: 0.1 ppm).



Figure 61. Change of normalized pressure gradient, p(t), with time at filter inlet, a comparison between 'smooth coating' and 'only porosity change' models (inlet concentration: 10 ppm).

4.4. Valid Scope of Simulated Results

The simulated results presented above are expected to be theoretically valid under the assumptions made in the analyses. Two of the general assumptions were the probability of adhesion being 100 percent and no occurrance of particle release. The latter may only hold before the filter reaches its 'ripened' state. When the particle load to the filter exceeds a critical value, as mentioned earlier, the increased drag may cause the adhesion efficiency to continuously drop with filtration time; and/or cause the captured particles to be released from the filter into the filtered metai. Under these circumstances, the evaluation of filtration efficiency is not as obvious as during the preripened period. To facilitate such analyses, one may define an apparent filtration efficiency as:

$$E_a = 1 - \frac{\text{particles escaping the filter} + \text{particles released}}{\text{particles entering the filter}}$$
(106)

which differs from the filtration efficiency commonly referred to by taking into account the particles released.

The amount of particles escaped apparently depends on the product (considering a single filter web) of transportation efficiency, η_t , and adhesion efficiency, η_a . If the increased drag only cause the adhesion efficiency to decrease (no net release), then η_t would still increase with time due to continuous particle deposition (even though the deposition rate is lower now) while η_a would decrease with time. The resultant effect on filtration efficiency depends on the relative magnitude of these two factors (*E* may increase, decrease or level-off). If the increased drag not only causes the adhesion efficiency to decrease but also cause a net release, then both η_t and η_a decrease and consequently E_a decreases. Net particle release may occur gradually or abruptly. In the latter case, E_a may even become negative. No attempts were made in this study to quantify these phenomena associated with the post-ripened period. (In practice, an optimum design should terminate the filtration process before the filter reaches its ripened state).

4.5. Summary

The dynamic behaviour of liquid metal filtration with reticulated ceramic filters prior to filter 'ripening' were analyzed theoretically by examining the changes in filter coefficient as a function of its independent variables during the filter aging period. A new correlation based on the 'only porosity change' model was proposed to describe the dynamic behaviour of liquid aluminum filtration.

The specific deposit (the volume fraction of captured particles inside the filter) and its effect on filtration efficiency were simulated based on macroscopic mass balance equations. It was found that in the initial period, or when the melt is cleaner, the specific deposit follows an exponential decay from filter inlet to outlet. As filtration proceeds, however, more particles would be captured in the inlet region, exhibiting an autocatalytic behaviour. The filtration efficiency was found to increase with filtration time as well as inlet particle concentration: the increase was however, negligible when the particle concentration reached 10 ppm, the increase was predicted to be appreciable (particle release being disallowed).

The pressure drop was analyzed by assuming that Darcy's law and the Carman-Kozeny equation were valid during the aging period. A new correlation relating the normalized pressure gradient to the specific deposit was proposed. The simulated results based on a newly proposed model describing the morphology of the captured particles, i.e., the 'only porosity change' model, showed that pressure gradients change from filter inlet to outlet, with the highest gradients at the filter inlet. For all cases, the pressure gradient, when no release occurs, increases with time as well as with inlet particle concentration. The rate of change decreases from filter inlet to outlet. These changes are negligibly small for a two hour period of filtration when the particle concentration is lower than 1 ppm. However, when the particle concentration reaches 10 ppm, these changes become appreciable.

Chapter 5

Experimental Work

5.1. Introduction

This chapter presents the experimental part of liquid aluminum filtration with reticulated ceramic filters. The objectives of the experiments carried out at both laboratory and industrial scales were to verify the theoretical analyses of the initial filtration processes presented in Chapter 3; and to study filtration phenomena during the aging period. The chapter starts with a brief review of previous work; and then a description of experimental studies, followed by a discussion of the results obtained.

5.2. Previous Work in Aluminum Filtration

A number of investigations have been made of aluminum filtration. Frisvold and Engh (1991) have analyzed theoretically the retention of non-wetting particles retention and obtained an $e_{\overline{x}\overline{y}r}$ ression for the forces pressing a spherical particle towards a flat filter surface (see Figure 62):

$$F = \frac{d\Delta E_{iot}}{d\delta} = -\pi R_s^2 (P_{com} + \rho gh) - 2\pi R_s \sigma \qquad (107)$$

where ΔE_{sor} is the total energy change, P_{son} is atmosphere pressure, σ is the melt surface tension, h is the metal-static head. After comparing the forces obtained from equation (107) with the drag force on the spherical particle give by O'Neill (1968), they concluded that the drag is several orders of magnitude smaller than the force pressing the particle towards the filter.

Tian (1990) has found in steel filtration that non-wetting alumina particles can physically contact the filter surface (liquid metal can withdraw from the interface due to non-wetting effects) and consequently bonding by sintering can take place.

Apelian and Mutharasan (1980) have developed a kinetic model for inclusion capture during initial filtration period (i.e., for clean filters):



filter surface

Figure 62. Particle adhesion model (Frisvold and Engh, 1991).

$$\frac{c_L}{c_0} = e^{-\frac{K_0}{V_{sf}}L}$$
(108)

where c_0 and c_L denote the inclusion concentration at inlet and outlet respectively, L is the filter thickness, K_0 is a kinetic parameter coefficient, related to K'_0 and K''_0 :

$$K_0 = K_0' + K_0'' V_{sf}$$
(109)

where K'_0 and K''_0 are empirical constants which were evaluated experimentally in their studies (Mutharasan et al, 1981). Consequently, if the removal efficiency of the filter media is defined as:

$$E = \frac{c_0 - c_L}{c_0}$$
(110)

then,

$$E = 1 - e^{-\frac{K_0}{V_q}L}$$
(111)

Frisvold (1990) reached a similar expression for aging filters (not valid for clean filters):

$$E = 1 - e^{-\lambda L} \tag{112}$$

where λ is given by:

$$\lambda = \eta \frac{A_s}{V_s} \frac{(1-\varepsilon)}{\varepsilon} b$$
 (113)

where A_{j} is the total surface area exposed to the fluid. V_{j} is the total volume of solid, b is the ratio between projected area in the flow direction and total surface area. Other terms are the same as defined before.

Eckert et al (1985) investigated cake mode filtration of aluminum alloys and found that the pressure drop across the cake increased with time, while across the filter medium itself, it decreased with time.

Gauckler et al (1985) found that inclusions larger than 100 μ m can be removed by 30 ppi filter under cast house conditions. About 80% removal was observed in filtering high Mg-containing alloys of aluminum.

Strom et al (1992) found that the inclusions were trapped mainly near the top surface of reticulated filters. Bathen et al (1988), using a sandwich filter of 20 and 40 ppi, found that the pronounced removal of inclusions from aluminum melts took place at the inlet and at the interface region between the 20 and 40 ppi sections. Frisvold et al (1990) found that the presence of chlorides improved the removal of carbides.

Butcher and Rogers (1990) found that their finer filters offered a higher efficiency of inclusion removal. Conti and Netter (1992) and Netter and Conti (1986) applied the trajectory technique to analyze particle removal for open-pore structure filters, where the structure of the ceramic foam was modeled as consisting of quasispherical cells connected to each other at their openings.

Castillejos et al (1992) tested a type of ceramic filter manufactured based on the negatives of packed beds called CEFILPB (CEramic FIlters of Lost Packed Bed) and found comparable removal efficiency with ceramic foam filters. Kakimoto et al (1993) compared the filtration efficiency of CFF with that of rigid ceramic tube filters (RMF) and found that the latter was superior to the former. Eichenmiller et al (1994) discussed the characteristics of bonded particle filters (BPF) and showed that this type of filter offered a higher removal efficiency than CFF.

Other noticeable investigations include those of Dore and Yarwood (1977), Farber and Rupp (1979), Kemppinen and Schnittgrund (1979), Eckert et al (1984), Apelian et al (1985), Frisvold et al (1990), Sane et al (1991), Hollar et al (1991), Keegan and McCollum (1992), Desmoulins (1992), Aubrey and Dore (1993), Dore (1990), Frisvold et al (1992), etc..

5.3. Experimental Specifications

5.3.1. Experimental Set-Ups

5.3.1.1. Laboratory Scale

The present laboratory experimental set-up, as shown schematically in Figure 63, consists of a graphite crucible, a filter holder and a resistance furnace. The dimensions of the graphite crucible measured 102 mm outside diameter, 150 mm high, and 4 mm



fumace crucible (alumina)

Figure 63. Schematics of experimental set-up for laboratory scale experiments.

thick in its side wall. The filter holder, which was screwed on the centre bottom of the graphite crucible, houses a reticulated alumina filter of 25.4 mm diameter with a thickness varying from 12.7 to 38.1 mm. The resistance furnace was equipped with an alumina crucible which measured about 140 mm internal diameter and 320 mm deep. During the experiments, the graphite crucible, along with a mounted filter, was made to sink freely into a liquid aluminum bath. The sinking speed of the graphite-filter assembly, or the superficial velocity through the filter, was controlled by adjusting the weight of a steel ring hung on the lower end of the filter holder. Figure 64 shows the graphite crucible, filter holders, used reticulated ceramic filters and two steel rings used in the experiments.



Figure 64. A photograph showing the graphite crucible, filter holders, used filters, and two steel rings.

5.3.1.2. Industrial Scale

The industrial experiments were carried out in an aluminum cast shop. The casting assembly consists of two holding furnaces, two in-line filtering bowls, an on-line refining system (Alpur), and a DC (Direct Chill) caster (as shown in Figure 65). The experiments were conducted in one of the filtering bowls which was located between the holding furnaces and the Alpur system. A reticulated alumina filter with a cross section of $432x432 \text{ mm}^2$ and a thickness of 38.1 mm or 50.8 mm was mounted inside the filtering bowl. The maximum depth of the filtering bowl above the top surface of the mounted filter was about 254 mm.



Figure 65. Schematic illustration of experimental set-up for the industrial tests.

5.3.2. Aluminum Alloys Filtered

The laboratory filtration experiments were made with commercial pure aluminum with a purity of about 99.7%, while for the industrial filtration tests, the alloy 3003 (with a composition: Si: 0.20% - 0.35%, Fe: 0.60%, Mn: 1.10%, Cu: 0.12% - 0.14%) was used.

5.3.3. Reticulated Filters

The reticulated ceramic filters used in this study were fully sintered 92% alumina filters supplied by Hi-Tech Ceramics, Inc. Studies by Strom, et al, 1992 have shown that this type of filter has a better quality than phosphate bonded filters in terms of mechanical strength, thermal shock and creep resistances. Their micro-structural characteristics have already been described in previous chapters. The nominal pore size of this type of filter is, as mentioned in previous chapters, designated as ppi - pores per linear inch. For a given designation, however, the actual pore size as well as its size distribution, may vary from vendor to vendor depending on the processes used. Table IV lists the typical average pore size and web thickness of 30, 40 and 50 ppi filters.

nominal size (ppi)	ave. pore dia. (µm)	ave. web thick (µm)
30	915	315
40	702	219
50	593	193

Table IV. Average pore diameter and web thickness of sintered 92%-alumina filters (Strom et al, 1992).

The porosity of this type of filter is affected by their bulk densities, a higher bulk

density resulting in a lower effective porosity for a given pore size designation. The author measured the effective porosity of this type of filter with an imbibition method (Dullien 1979), i.e., immersing a known bulk volume (25.4 mm dia. and 38.1 mm high) filter into a known volume of water. By measuring the volume of water displaced by the filter, one obtains the filter's effective porosity (see also Chapter 2). The effective porosity thus determined as a function of the bulk density of 30 and 40 ppi filters are plotted in Figure 66.

5.3.4. The LiMCA Equipment

The LiMCA (Liquid Metal Cleanliness Analyzer) equipment, first developed at McGill University (Doutre and Guthrie 1985) and now available for aluminum industries (Martin and Painchaud 1994), is an on-line detector which simultaneously records the number and size of non-metallic inclusions in liquid metals. It mainly consists of a probe, a vacuum system and several electronic signal processing units.

5.3.4.1. Functional Principle of the LiMCA probe

The principle of LiMCA is based on a resistive pulse technique, the same as that of the Coulter counter (Coulter, 1956). Figure 67 shows the schematics of the LiMCA probe. The probe consists of a sampling glass tube of KIMAX brand borosilicate culture (with a diameter of about 25 mm and a length of about 250 mm) with a torch-made small orifice (e.g. 300 μ m) located at its lower end; two electrodes (made of mild steel) with one located inside and the other outside the tube; and a vacuum or argon supply system on to which the glass tube is to be screw-sealed with a rubber O-ring. During operation, the probe is dipped in the melt and liquid metal is drawn into the glass tube through the small orifice by an applied vacuum, thereby completing a circuit in which a constant DC current of 50 to 60 amps is to be maintained. When a non-metallic inclusion flowing along with the liquid metal passes through the orifice, the electrical resistance increases



Figure 66. Measured effective porosity of alumina filters as a function of their bulk densities.

in the circuit, thereby creating a voltage pulse (see Figure 68) which lasts approximately the time for the inclusion to cross the orifice. The magnitude of the voltage pulse is related, to the first approximation, to the size of the non-conductive inclusion through the following equation (Doutre 1984):

$$\delta V = \frac{4I\varrho d_p^3}{\pi d_o^4} f\left(\frac{d_p}{d_o}\right)^{(114)}$$

where q is the electrical resistivity of the liquid metal, d_p and d_o are respectively the diameters of the nonconductive particle and the orifice, and $f(d_p/d_o)$ is a correction factor,



Figure 67. Schematics of the LiMCA probe.

given by the following expression (DeBlois et al 1977):

$$f\left(\frac{d_p}{d_o}\right) = \frac{1}{1 - 0.8 \left(\frac{d_p}{d_o}\right)^3}$$
(115)

With the above correction, Eq.(114) remains, according to Doutre (1984), valid up to a d_p/d_o ratio of 0.8 with an error of less than 1%.

By registering the number and magnitude of the voltage pulses generated by the LiMCA probe with appropriate electronic equipment, and knowing the flow rate through the orifice, the inclusion concentration in the melts can be determined.



Figure 68. A typical voltage pulse generated by LiMCA.

The volumetric flow rate, Q, through the orifice was estimated based on the Bernoulli's equation (Doutre 1984):

$$Q = C_{p} \frac{\pi d_{o}^{2}}{4} \left(\frac{2(\Delta p + \rho g h)}{\rho} \right)^{\frac{1}{2}}$$
(116)

where C_D is a discharge coefficient, ρ the melt density, Δp the vacuum, h is the metal head above the orifice, g the gravitational acceleration constant. Since the metal head above the orifice changes during sampling, h was usually taken as half the depth of the submergence of the orifice. Its influence however, is negligibly small compared to the dominating vacuum term. The discharge coefficient was found to be 0.97 (Doutre 1984).

5.3.4.2. Signal Processing

The voltage pulses, or electronic signals, generated by the LiMCA probe are processed via a number of interconnected modules (Dallaire 1991), as shown in Figure 69. Due to the extended experimental operations involved, the signals generated were not processed on-line. Rather, they were recorded onto a Digital Audio Tape (DAT) during the experiments and then analyzed off-line, later on.

Equipped with low pass and high pass filters and a 5A22N differential amplifier module, the Tektronix 5223 Digitizing Oscilloscope, in addition to monitoring signals, filters the signals generated during the experiments to remove both the high frequency (>10 kHz) noise and low frequency (<100 Hz) DC component and then pre-amplifies them by 1000 times. The signals, from the output of the scope, were then recorded onto a digital audio tape with a TEAC RD-T101 PCM data recorder equipped with a TEAC TZ-310FA input conditioner module which allowed the bandwidth and voltage range of the input signals to be adjusted. In the mean time, messages describing the experimental conditions and other parameters such as the vacuum, current, orifice size, etc. needed



Figure 69. LiMCA signal processing modules.

in data processing were also recorded through a built-in microphone, connected to a microphone channel in the recorder.

When the recorded signals were analyzed off-line, they were first fed, via a McGill AC-coupling adapter to remove the offset voltage introduced by the POSITION control of the 5A22N differential amplifier module, into a Bomen modified TN-1246 Pulse Sampler, where the pre-amplified voltage pulses were detected and logarithmically amplified according to the following equation (as specified in the operator's manual):

$$\delta V' = \frac{10}{3} [\log(G \cdot \delta V) + 2]$$
 (117)

where, G denotes the pre-amplification factor or a gain which is usually set to 1000 for liquid metals. The logarithmic output, $\delta V'$, together with a STROBE signal generated by the Pulse Sampler are then fed into a TN-7200 Multi-Channel Analyzer (MCA), where the number of voltage pulses (the signals) were counted according to their magnitudes. The function of the STROBE signal was to ensure that the peak magnitude of the pulse was measured by the MCA. The input voltage range of the MCA ranged from 0 to 8 volts. If 512 channels are selected, then the channel into which the counts for a given voltage pulse, i.e., a given particle size, are stored is given by the following equation:

Channel =
$$512 \frac{\delta V'}{8}$$
 (118)
= $\frac{640}{3} [\log(G \cdot \delta V) + 2]$

By registering the counts of voltage pulses in each channel, the numbers and sizes of the non-metallic inclusions in the sampled volume of metal could be determined. This process is referred to as data acquisition and is program-controlled via a personal computer connected to the MCA with a RS-232 serial port.
5.3.4.3. System Calibration

It has been observed that some of the modules, especially the Pulse Sampler, do not function perfectly. Therefore, some deviations from the theoretical relation, Eq.(118), often occur when the LiMCA signals are processed. In order to avoid this, the system must be calibrated very often (as a matter of fact, it was calibrated prior to each experiment). The calibration was carried out before the start of each experiment with a Bomen calibrator which, when activated, generates 96 trains of 8 known amplitude pulses (see Table V). This was done by sending the known amplitude pulses generated by the calibrator via the two electrodes of the LiMCA probe. The calibration signals were recorded together with the real LiMCA signals from the experiment. When these signals were processed, the calibration signals were first analyzed, thereby generating an empirical correlation relating the channel number to the voltage pulses, of the form (Dallaire 1991):

$$Channel = 64 \left[m \log(G \cdot \delta V_{c}) + b \right]$$
(119)

where δV_c denotes the calibration voltage pulse, *m*, *b* were empirical constants which were determined by checking the channel numbers into which any two of the eight calibration pulses fell, i.e.,

$$m = \frac{Ch2 - Ch1}{\delta V_{c_1}}$$
(120)
$$(120)$$

and

$$b = \frac{Chl \log(G \cdot \delta V_{c_1}) - Ch2 \log(G \cdot \delta V_{c_1})}{\delta V_{c_1}}$$
(121)
$$64 \log \frac{\delta V_{c_1}}{\delta V_{c_1}}$$

The LiMCA signals were then analyzed based on the calibrated correlation. Table V lists

the standard amplitudes of the eight calibration pulses and their corresponding MCA voltages and channel numbers calculated from Eq.(117) and Eq.(118), and Figure 70 shows the actual calibration signals obtained based on Eq.(119) - (121).

Pulse	Pulse	МСА	MCA
No	amplitude	voltage (V)	channel
	(μV)		
1	1280	7.024	450
2	640	6.021	385
3	320	5.017	321
4	160	4.014	257
5	80	3.010	193
6	40	2.007	128
7	20	1.003	64
8	10	0	0

Table V. Standard calibration signals and their theoretical MCA channel numbers.

5.3.5. Experimental Procedures

5.3.5.1. Laboratory Scale

A typical experimental procedure in the lab started with the preparation of a melt of 8 kg pure aluminum in the resistance furnace. When the temperature reached 700°C, the melt was blown with air for a few seconds through a glass tube to render it 'dirtier'.



Figure 70. Calibration signals and their corresponding channel numbers.

Then the inclusion concentration of the melt was checked with the LiMCA equipment which had already been calibrated and prepared. The signals were displayed on the Tektronix 5223 Digitizing Oscilloscope, where they were filtered and amplified. The output of the oscilloscope was connected to the TEAC RD-T101 PCM data recorder where the signals were recorded onto a digital audio tape. The level of vacuum used to draw liquid aluminum into the LiMCA probe was around 130 mmHg. When the liquid metal inside the probe reached the melt surface (the probe dipped into the melt about 130 mm), the vacuum was cut out and the probe was flushed with argon gas and the liquid metal was expelled from the probe. After sampling was finished, the LiMCA probe was taken out of the melt, and the filter assembly along with the graphite crucible, after being preheated in another resistance furnace or with a torch, was dropped into the melt. The filter was primed by pushing the filter assembly deep down into the melt almost to the limit of the height of the graphite crucible. Once primed, the filter assembly was allowed to sink freely (due to the added weight) into the melt (the metal drained up-wards through the filter into the graphite crucible). The sinking speed of the filter assembly was timed and recorded with a video camera against a steel ruler which was coated with some enamel spray in order to reduce its light reflectivity. When the filtered metal amounted to about 100 mm high inside the graphite crucible (or when an equilibrium established), the LiMCA probe was dipped into it, and sampling was made to check the inclusion concentrations. The recorded data were processed off-line according to the procedures described in the previous sections and the filtration efficiency was determined by comparing inclusion concentrations before and after the filtration:

$$E = \frac{c_b - c_a}{c_b} \times 100\%$$
 (122)

where c_b , c_a denote inclusion concentration before and after filtration in terms of numbers per kg of metal or volume fraction.

5.3.5.2. Industrial Trials

The typical procedure of the industrial trials conducted in an aluminum cast shop started with mounting a reticulated ceramic filter into the filtering bowl. The filter was then preheated with a gas torch from underneath until it became glowing red, then the liquid aluminum was discharged from the holding furnaces and introduced through the launder into the filtering bowl. The filter was primed after a certain head above the filter had been reached. Once primed, the head above the filter dropped rapidly, and the flow in the down side launder began to increase, and finally the system stabilizes. The head loss, or the pressure drop, across the filter was determined by reading the metal levels before and after the filter against the graduated side walls of the filtering bowl. In the meantime, the LiMCA equipment was being prepared. The major difficulty encountered was the electronic noise, coming from various machineries in the cast shop, which had to be eliminated or minimized in order to obtain acceptable LiMCA signals. The sampling procedure of the LiMCA probe was the same as that described for the laboratory experiments. The sampling was conducted consecutively before and after the filter by physically moving the LiMCA probe back and forth throughout the cast in whole cast usually lasted about 80 minutes or longer). Fortunately, the tube withstood any thermal shock associated with their rapid withdrawal from, and sudden immersion, into the melt at the downstream, or upstream locations. The data, along with the experimental specifications, were recorded onto digital audio tapes for off-line analyses. The flow rate, and thus the superficial velocity through the filter, could be calculated according to the casting speed which was automatically monitored in the cast shop.

5.4. Some Experimental Observations

5.4.1. Filter Priming

In this study, three pore size filters (30, 40 and 50 ppi) were tested in the

industrial trials. The typical priming time (as soon as the metal was seen to penetrate the filter pores and appear downstream to the filter) ranged from 7 to 8 seconds for the 30 and 40 ppi filters after being preheated, up to about 765 - 795°C. The priming time for the 50 ppi filter lasted 13 seconds (only one was tested) and it was primed just before the liquid metal was about to overflow the filtering bowl (i.e., at about 10 inch metal head).

5.4.2. Typical Results Obtained with LiMCA

Figure 71 - Figure 73 show some typical results in terms of numbers per kg of liquid metal as well as volume fraction obtained from LiMCA measurements. As seen, the concentration of inclusions in the liquid aluminum discharged from the holding furnaces are not homogeneous, some decreasing with time, some increasing with time, depending on perhaps the history of liquid metal treatments in the holding furnaces. As seen, the 40 ppi filter exhibited a higher removal efficiency than the 30 ppi filter.

5.4.3. Inclusions in the Alloys Tested

The non-metallic inclusions observed in these melts were, according to Green (1992), the senior metallographic technician in the cast shop, predominately (over 70%) aluminum carbides (Al₄C₃). Aluminum diboride (AlB₂), aluminum borocarbide (Al₄C₄B), Al₂O₃, MgO, spinel and TiB₂ were also observed. The sizes of these inclusions ranged from the sub-micron for individual particles, e.g., TiB₂, to a few hundred microns, for oxides films.

Aluminum carbides originate if the carbon side linings in aluminum cells are exposed to molten aluminum or electrolyte bath during the electrolytic reduction process (Ødegård et al, 1987). To prevent the formation of these inclusions, the carbon lining must be covered by a frozen layer of electrolyte (Gudbrandsen, 1992). The oxides that were present in the melt may originate from several sources such as direct oxidation,



Figure 71. Inclusion concentration in the melt, before and after filtration with a 30 ppi filter.



Figure 72. Inclusion concentration in the melt, before and after filtration with a 40 ppi filter.



Figure 73. Inclusion concentration in the melt in terms of volume fraction, before and after filtration.

refractory erosion, alloy addition, etc. For remelt or recycled aluminum melts, a higher inclusion population would be expected due to the presence of oxidized surfaces, dirt or debris, paint residues, or other unwanted impurities. Spinel (e.g., MgO \cdot Al₂O₃) usually exists in clusters, while the aluminum oxides resulted from direct surface oxidation usually appear in the form of films, or tangled agglomerates.

Titanium diboride (as produced in grain refining alloys is a desirable nucleation agent and is usually artificially introduced into the melt in order to obtain a finer grain size during solidification.

5.4.3. Observation of Spent Filters

The spent filters were cut and polished and examined with optical or electron microscopes to observe the inclusions captured inside. Figure 74 - Figure 76 show some of the typical micrographs. As seen, the particles did not adhere uniformly onto the filter webs. Rather, they were captured in the form of clusters, or agglomerates, at some preferred locations. Another observation is that the particles were captured mainly in the upper region of the web, facing the flow stream.

5.4.4. Problems Associated with Laboratory Scale Experiments

The above observations are based on industrial tests. Some difficulties were encountered in carrying out filtration tests at a laboratory scale. These were:

(a). Capacity limitation: due to the limitation of lab furnace, the load to the filter is limited and therefore the dynamic behaviour could not be investigated without dirty the melt to a certain degree. Over dirty melt may cause other concerns about the hydrodynamic interactions among the particles, e.g., excessive agglomeration, non-Newtonianization, etc..



Figure 74. A micrograph showing the inclusions captured within the filter.



Figure 75. A micrograph showing the inclusions captured within the filter.

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(b). Priming difficulty: it was found that the 30 ppi filter (38 mm thick), even though preheated up to 700°C in another resistance furnace, was difficult to be primed as a result of chilling during transfer from the preheating furnace to the top of the liquid metal bath. The torch preheating was found to be not effective either.

(c). Varying flow rate: with the free sinking approach, the flow rate could not be maintained constant (see Figure 77) and a mean value had to be estimated in order for the filtration process to be evaluated.

(d). Melt contamination and reoxidation: due to the small amount of liquid metal filtered, melt contamination (c: to dusts present inside the filter and the crucible) and reoxidation (due to break-up of the liquid aluminum by the reticulate ceramic filter during priming) may become significant enough to interfere with the analysis.

(e). Effect of settling: due to the sequential operation procedure, the time delay between the LiMCA sampling before and after filtration may introduce an error in the determination of filtration efficiency because of the settling effect, especially for larger particles.

(f). Oxidation of graphite crucible: it was found that preheating the graphite crucible in a resistance furnace leads to considerable oxidation of it. As a result, the crucible became no longer usable after a couple of times of experiments, a new one had to be made. Because of this, a steel can was tested, but it turned out unsuitable. Crucibles made of ceramic materials would give a too high superficial velocity if it was let sink freely into the metal bath. In view of the above, the data obtained from the laboratory scale experiments may only be regarded as a rough approximation.



Figure 77. Measured superficial velocity as a function of filtration time.

5.5. Spot Checking the Predicted Results

The predicted results were spot checked against experimental data (since it is extremely difficult, if not impossible, to completely verify the theoretical analyses experimentally). The predicted results for a clean filter were spot checked against data available from the industrial trials corresponding to the early stages of filtration (about 10 minutes of filtration, or about 18 grams of liquid aluminum filtered per square millimetre of filter area). A check was also made against data gathered from the laboratory experiments. In this case, LiMCA measurements of filtered metal were made when approximately two kg of liquid aluminum had been passed, corresponding about 4 grams (if the filter was fully primed) per square millimetre of filter area. It was hoped that at this stage, the effect of captured inclusions inside the filter would be negligible and the data would approximate those of a clean filter.

The predicted dynamic behaviour of this type of filter was spot checked against data from the industrial trials only. Due to the limited capacity of the laboratory furnace as mentioned earlier, it was impractical to check the dynamic behaviour at the laboratory scale that was available to the author.

5.5.1. Initial Period

The experimental data used to spot check the predicted results were based on 30 ppi filters with an average effective porosity of about 83%. The average diameter of the filter webs was about 0.3 mm. If the average web diameter is taken as the characteristic length, the average superficial velocity as about 10 mm/s and the density and viscosity of liquid aluminum at 700°C as 2350 kg·m⁻³ and 1.175X10⁻³ kg·s⁻¹·m⁻¹ respectively, then the Reynolds number should be about 6. The value of the concentration ratio, c/c_o , corresponds to alumina inclusions with sizes around 50 μ m in liquid aluminum. Under these conditions, the gravitational number, N_G , is about minus 0.1 for the laboratory

experimental configuration, and 0.1 for the industrial case (if the density of the alumina aggregates is taken as 3300 kg·m⁻³).

Figure 78 presents a comparison between the predicted results and the experimental data obtained from the laboratory scale experiments. As seen, the experimental data fluctuated and exhibited a higher removal efficiency than predicted as the filter became thicker. The two data points obtained using lower porosity filters (89%) also showed a higher removal efficiency than predicted. These discrepancies may be attributed to the inaccurate control of the filtration processes as mentioned previously. Nevertheless, the general trend of the experimental data was roughly consistent with the prediction. It can also be noted that the removal efficiency of lower porosity (83%) filters was indeed higher than those of higher porosity (89%) filters, which is consistent with prediction.

Figure 79 provided a comparison of predicted results against the data from the industrial filtration trials. Table VII lists the values of the filter coefficients obtained from the industrial experiments as well as that numerically predicted. As seen, surprisingly good agreement was obtained given the fact that the interception parameter and the hydrodynamic retardation effects were not considered during the simulation. This perhaps indicates that effects of the two parameters may have cancelled each other, since omission of the former would result in an under-prediction of the filtration efficiency, while omission of the latter would lead to an over-prediction of λ_0 , a phenomenon already pointed out by Rajagopalan and Tien (1977). Nevertheless, whether the two would cancel completely under the conditions alluded to above requires further investigation. In the light of this, the approximate equation (75) deduced in Chapter 3 may not be valid for non-Brownian particles whose density is the same as that of the melt ($N_G = 0$) since, in this case, the hydrodynamic force and torque exerted on the particles vanish (Goldman et al, 1967), the hydrodynamic retardation effect would be greatly reduced (this effect can be active up to a distance of five particle diameters away from



Figure 78. Concentration ratio as a function of filter thickness, a comparison between predicted and that experimentally determined (laboratory scale).



Figure 79. Concentration ratio as a function of filter thickness, a comparison between predicted and that experimentally determined (industrial scale).

the solid surfaces, Rajagopalan and Tien, 1977), while the effect of direct interception would remain unchanged. For such situations, Eq.(75) may be modified somehow to take into account the interception effect.

It should be noted that experimental data from the industrial trials correspond to the earliest that were available for representing the filter behaviour during initial stages of filtration (one complete experiment usually lasted about 80 minutes or longer). The first digit of the sampling time shown in Table VII corresponds to the start of sampling ahead of the filter, while the second digit corresponds to the start of sampling after the filter (it is impossible to sample immediately after the start of filtration, as a certain time period was needed for flow stabilization and for the introduction of the LiMCA probes into the melt).

Table VI. Comparison of predicted filter coefficients with those experimentally determined.

No. of experiments	c/c _o size:40-60 µm	filter thickness (mm)	λ (mm ⁻¹)	Sampling time (min.)	V _{sf} (mm/s)
1	0.1776	38.1	0.0454	9-17	11
2	0.1096	50.8	0.0435	11-17	11
3	0.1877	38.1	0.0439	9-19	11
4	0.22	38.1	0.0397	9-21	12
Numerical result (under the same conditions as above experiments, $N_0=0.1$)			0.0411	for clean filte	ers, Re=6.0
Difference factor: averaged value of λ from experiments over predicted value			$0.0430 \div 0.0411 = 1.05$		

The experimental data showed that the removal efficiency for smaller particles (e.g., $20 - 30 \mu m$) was much higher than that predicted (see Figure 71). This may be due to potential agglomeration effects, which were not considered in the prediction, taking place in the filtering bowl (since there is a time for the liquid metal to reach the filter inlet after the LiMCA sampling which took place at the top surface of filtering bowl), especially at the region immediately prior to the filter inlet where the flow field changes rapidly. In addition, oxide films captured within the filter may have acted as effective 'collectors' for smaller particles. This effect however, was not modelled in the prediction. That is perhaps another possible explanation for the observed higher removal efficiency of smaller particles.

5.5.2. Aging Period

The predictions made in Chapter 4 for the aging period were based on some specific conditions. In a real melt, many types of particles exist and the sizes of each type of inclusions vary (not mono-sized). The specific deposit determined from the spent filter must be the sum of all captured particles of all types and sizes. The change in pressure drop and filtration efficiency for a given filter thickness must also be the result of the total effects of all captured particles. To spot check predicted results against experimental data, one should, strictly speaking, do predictions for each type and each size of all the inclusions, and then somehow (e.g., some type of mean) find their total effect. To write the macroscopic equation for one type of inclusions, one has:

$$V_{sf} \frac{\partial c_i}{\partial z} + \frac{\partial \alpha_i}{\partial t} = 0$$
 (123)

$$\frac{\partial \alpha_i}{\partial t} = V_{sf} \overline{\lambda(\alpha_{ij})} c_i$$
 (124)

where the subscripts i, j denote respectively type and size class of inclusions. The overlined quantities denote mean values over the different size classes of ith type inclusion.

$$\Delta p_i = \left(\frac{\partial p}{\partial z}\right)_{t=0} \int_0^L \overline{\mathcal{P}_i(t)} \, dz \tag{125}$$

The total values of the concerned variables may be found by summing up the values of all different types of inclusions present in a given system. Such a procedure may be tedious and sometimes impossible due to a lack of specific information regarding a particular type of inclusions. Moreover, some quantities may not be accurately measured experimentally, e.g., the number of inclusions smaller than 20 μ m cannot be reliably determined by the current LiMCA equipment due to interferences from electronic noises. This renders a more rigorous comparison worthless. Because of these reasons, the rigorous approach was not pursued, rather a comparison between theoretically predicted values and those determined experimentally was made, based on a simplified approach with estimated values for those quantities which could not be measured accurately.

5.5.2.1. The Specific Deposit

Figure 80 provides a comparison of the specific deposit, predicted and determined experimentally. There the concentration of inclusions has been estimated as 10 ppm. Prediction was based on $N_{\sigma}=0.1$, instead of based on some type of mean value over the whole range of all types and sizes of the inclusions. The experimental data for the specific deposit were determined from a polished cross section of a spent 30 ppi filter with the LECO 2005 image analysis system incorporated in the optical scope in the metallurgical department. Ideally, the experimental data should be based on statistical values obtained from the examination of large numbers of cross sections. However, due to the difficulties in obtaining a satisfactory polishing quality for such a size of spent ceramic filter (1.5" thick), such a ideal approach was too costly and was therefore not pursued. Nevertheless, the results, obtained on the basis of simplified approaches for the predicted and experimentally determined, are indeed comparable. One should, however, be aware that data obtained with the image analysis tend to over-represent the actual



Figure 80. Distribution of specific deposit inside the filter, a comparison between predicted and experimentally determined.

value due to the polish defects and voids as well as the enlargement around hard particles in a relatively soft aluminum matrix when the cross section was polished, all of which will be counted as inclusions since they could not be distinguished from the actual inclusions during the analysis. On the other hand, the values for the gravitational number (as 0.1) and inclusion concentration (as 10 ppm) used in the prediction may also overrepresent the actual ones.

Similar distribution profiles of captured particles inside spent filters used for filtering liquid aluminum melts were also observed by Bathen and Engh (1988), Frisvold (1990) and Castillejos et al (1992).

5.5.2.2. The Filtration Efficiency

Figure 81 gives a comparison of filtration efficiency as a function of filtration time between that predicted and that determined experimentally for a 30 ppi filter. The experimentally determined filtration efficiency was based on the volume fraction of inclusions greater than 20 μ m. It should be noted however that the change in filtration efficiency must be affected by all captured inclusions. The volume fraction of inclusions greater than 20 μ m was found to be less than 1 (0.1 - 0.9) ppm in the melts tested. The actual concentration of all inclusions in the unfiltered melts needed in the theoretical prediction for purpose of comparison, was however not known. According to Strom et al (1992), the inclusion concentration determined with LAIS (Liquid Aluminum Inclusion Sampling), which is capable of detecting inclusions smaller than 5 μ m, was 10 to 55 times higher than that determined with LiMCA equipment, indicating that the majority of the inclusions in these melts were smaller than 20 μ m. This is consistent with metallographical observations (see Figure 82). Based on this information, the inlet concentration in the prediction was taken as 10 ppm as an approximation. The gravitational number was again taken as 0.1 for simplification instead of following the rigorous procedure as mentioned in the previous sections. Under these conditions, the



Figure 81. Change in filtration efficiency during filter aging period, a comparison between predicted and experimentally determined.

filtration efficiency predicted and that determined experimentally were more or less comparable.

5.5.2.3. The Pressure Drop

The minimum head drop measured as shown in Figure 65 ranged from 3 to 6 mm of liquid aluminum, much higher than that determined with the water flow experiments for the same superficial velocity. This may be attributed to the fact that the head drop read as such was actually the total head losses, i.e., the losses due to the sudden expansion as the flow entered the filtering bowl from the launder and the contraction as the flow violently exited the filtering bowl, as well as losses due to the filter. In addition, the slight inclination of the launder may also have attributed to the observed higher values. From these points of view, the pressure drop across the filter could not measured directly from the given experimental set-up. A direct measurement would require pressure sensors to be located immediately before and after the filter in the liquid aluminum. Such a configuration would be undoubtedly difficult to realize in practice. The head drops measured from the experiments may be treated as a sort of index, which, nevertheless, still reflect the basic characteristics of the dynamic behaviour of an aging filter.

Figure 83 shows a comparison of the normalized pressure drop, across a 30 ppi filter with a thickness of 38.1 mm and an effective porosity of 83%, between predicted and measured. The predicted result was based on Eq.(105), where the integral of p(t) was performed according to the Trapezoid rule. The value of p(t) as a function of filtration time and location was obtained from Eq.(104), with the predicted value of $\alpha_{t,z}$ as described in the section 5.5.2.1. As seen from Figure 83, under the conditions described above, the model seems to under-predict the change in pressure drop comparing with measured 'head loss index' across the filter, but the trend more or less consistent. It would make no point to adjust the correlations proposed to suit the experimental data



Figure 82. A scanning electron micrograph showing the particles captured within the spent filter.



Figure 83. Change in pressure drop during filter aging period, a comparison between predicted and experimentally determined.

since, for the reason mentioned above, the actual pressure drop across the filter was not known. In addition, the head loss measurement was a rather rough estimate, reading the level of liquid metal against a graduation marked on the walls (with a 1/4 inch of resolution) being difficult. This casts further uncertainties in the determination of real pressure drops across the filter.

As seen from Figure 83, the measured pressure drop index showed some fluctuations during the course of filtration. This may provide some insight into the real dynamic behaviour of the filter, the decline in head loss at a given time may indicate a partial release (although it may also be caused by flow fluctuations), which were not considered in the predictions. The changes in filtration efficiency and head drop of the 30 and 40 ppi filters as a function of filtration time are shown in Figure 84. It should be pointed out that the filtration efficiency determined by LiMCA under this circumstance (when partial release occurs) would be the apparent filtration efficiency (since the released particle would be counted as particles not captured coming from the liquid metal will they were actually dislodged from the captured particles retained previously). The true dynamic change of filtration efficiency should be somehow reflected in the pressure drop. If one defines an index to reflect this as:

$$K = \frac{\overline{\alpha_t}}{V_{st}c_{is}}$$
(126)

where the over-lined α_i is defined in the following equation,

$$\frac{\int_{0}^{L} p(t) dz}{L} = \left[1 + \frac{\alpha_{t}}{(1 - \varepsilon_{0})(1 - \varepsilon_{o})}\right]^{2} \left[1 - \frac{\alpha_{t}}{\varepsilon_{0}(1 - \varepsilon_{o})}\right]^{-3}$$
(127)

in conjunction with Eq.(105). By measuring the pressure drop across a given filter thickness as a function of time, the value of the over-lined α_{t} can be obtained. Figure 85 shows a plot of K as a function of time for 30 and 40 ppi filters, where the value of c_{in} as a function of time was determined by LiMCA.



Figure 84. Measured pressure drop index and filtration efficiency as a function of filtration time.



Figure 85. Dynamic index versus filtration time.

5.6. Hypotheses on Particle Capture Processes within Filter

As seen from previous chapters, present theoretical analyses have been based on an assumption of 100 percent adhesion efficiency. In reality however, this assumption may not hold for all types and sizes of inclusions commonly encountered in liquid aluminum melts. Even for the same type and size class of inclusions, it may hold for some locations while it may not hold for other locations inside the filter (in fact, this is reflected in Eq.(37)). As a matter of fact, the model of 'only porosity change' proposed in Chapter 4 implicitly reflects, to some extent, the latter phenomenon. Due to the difficulties in obtaining exact microscopic geometric information of real filters and information of their physical and chemical properties as well as that of various inclusions in liquid aluminum melts, there is no way to know for certain the exact microscopic dynamic processes of inclusion capture inside these types of filters. Consequently, the analyses made so far may be only treated as general trends rather than bare facts. For the same reason, what is to be presented below may be seen as being speculative in nature.

5.6.1. Proposed Picture of Particle Capture Process

Microscopic observations made on the spent filters revealed that particles were captured in the form of clusters or agglomerates in some preferred locations inside the filter, rather than by uniform adhesion to every filter web, which would surely have been the case, if flows are uniform and, if the adhesion efficiency were 100 percent. Based on these observations, one may picture the inclusion capture process as follows: at the start of filtration, i.e., when the filter is clean, inclusions adhere to filter web surfaces, wherever the condition is met, to become part of the filter media. Incoming particles may, in addition to adhering to available web surfaces, successively adhere to accumulated particles to form a growing cluster. As the deposited cluster grows bigger

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and bigger, if the same process is taking place at adjacent filter webs and the gaps between the webs are not excessive, these clusters may bridge together and secure themselves. The process can therefore be referred to as *bridging*. If the gap between adjacent webs is too great, however, or if the local interstitial velocity is too high for some locations, the clusters may be torn away by the increasing hydrodynamic drag forces imposed by the fluid, on the growing clusters (the drag is proportional to the size of the cluster in Stokes flow regime). If the pore sizes are finer, the possibility for the deposited particles to bridge and thus secure themselves before reaching their critical size to be torn away would be higher. This may partly explain the better performance of 40 ppi filters that was observed in the aging period. The dislodged particles may be recaptured further downstream in some other locations wherever the conditions are met. That may explain why sometimes more particles were found in the lower part of the spent filter than expected from the theoretical analysis (Frisvold 1990). As seen from Figure 86, this phenomenon was more evident for finer filters because in this case (a) the interstitial velocity at the inlet region would increase faster due to the more effective capture of particles and (b) the greater likelihood of recapture in the lower regions of the finer filters. It is also possible, however, that these dislodged inclusions may escape the filter and recenter the liquid metal. As particles accumulate inside the filter to a certain extent, the increased interstitial velocity reaches a magnitude at which the dislodging rate become noticeable, and the removal efficiency then begins to deteriorate (decline).

Given this, a filtration process should be so designed that when the process reaches such a situation (or ripened state), the filter should be changed or the filtration process should be terminated in order to guarantee the required quality of the filtered metal. If the assumption of 100 per cent adhesion holds, the filtration efficiency, as well as the pressure drop across the filter, would continuously increase with time until the filter becomes, (if particle loading is high enough), clogged. This phenomenon was observed (Tian 1990) for steel filtration, where alumina inclusions within the melt passing through the filter, bonded themselves with filter web surfaces, or alumina particles already adhered thereto, by sintering. In such cases, hydrodynamic drag forces are ineffective in dislodging adhering agglomerates.

5.6.2. Adhesion Mechanisms

It is evident from the micrographs of the spent filters that inclusions do adhere to, or can be captured by filter webs. It is not clear, however, what forces can be attributed to the capture or adhesion of these inclusions. This is primarily due to the difficulties in quantifying the true state of contact: the physical and chemical nature of the inclusions and the filter webs such as the microscopic surface structure, the actual contact area; as well as the possible interactions such as sintering, deformation, diffusive mixing, mutual dissolution and alloying, ionic exchange and thus chemical bonding, etc. It is even not clear whether these captured particles physically contact the filter webs, or each other, in the agglomerates. Nevertheless, one may speculate on the nature of the forces possibly involved, as well as the variables influencing their strength, based on the present knowledge gathered so far.

5.6.2.1. Interfacial Tensions

Since adhesion is a surface phenomenon, interfacial energies should play a role. If the interfacial energy (tension) between the particles and the melt is high (or nonwetting), elimination of part of the interface when a particle is very close to a filter web or to another particle already captured may reduce the total energy of the system. Alumina particles are non-wetting with respect to liquid aluminum. As such, since the filters are mainly made of alumina, the interfacial tension should play a positive role in the removal of alumina inclusions. Likewise, if the inclusions are wetting with respect to liquid aluminum, e.g., aluminum carbides (Pederson, 1984). the interfacial tension may play a negative role in the removal of these inclusions since the draining of liquid metal from between them would be more difficult (not favoured thermodynamically). In



Figure 86. Distribution of captured particles inside the spent filters.

this case, actual contact between the wetting particles has to rely on other forces such as Van der Waals, electrostatic or collisional energies.

5.6.2.2. Van der Waals Forces

When a particle is in close contact with, or in an immediate vicinity of, a filter web or particles already captured, Van der Waals forces (with a maximum effective range of 500 Å, Ives, 1971), may come into play. Van der Waals forces, i.e., forces between molecules possessing dipoles or quadrupoles, or molecules polarized as a result of induction by the static dipole or quadrupole fields, and non-polar London-Van der Waals dispersion forces, may be theoretically quantified for some ideal systems, for instance, for two perfect spheres (Corn, 1966):

$$F = \frac{H}{12\delta^2} \left(\frac{d_1 d_2}{d_1 + d_2} \right)$$
(128)

where H is a Hamaker constant, d_1 , d_2 are the diameters of the two spheres, δ is the distance of separation.

For the case of a perfect sphere contacting an ideal flat surface, Eq.(128) becomes:

$$F = \frac{Hd_1}{12\delta^2} \tag{129}$$

since $d_2 > d_1$.

The Hamaker constant has been historically evaluated, primarily by Hamaker (1937), based on a microscopic approach as: $c\pi^2 q^2$, where q is the number of atoms per unit volume of a macroscopic body, c is a constant specific to the interacting molecules. This approach, assuming the forces between individual atoms or molecules being linearly additive, may only be used as a crude estimation for the Hamaker constants for non-
absorbing materials, it may not be applicable for metals (Osborne-Lec, 1988). Another more satisfactory macroscopic approach (first developed by Lifshitz, 1956) for evaluating Van der Waals forces between two macroscopic bodies is based on the dielectric spectral properties of the materials, and is believed to be suitable for all materials. With this approach, the Van der Waals forces between a spherical particle and a flat surface can be expressed as (Bowling, 1988):

$$F = \frac{Ld}{16\pi\delta^2}$$
(130)

where L is so-called Lifshitz-Van der Waals constant, which has to be evaluated by integrating the imaginary portion of the complex dielectric function or the absorptivity over the complete frequency range, of a given material (Osborne-Lee, 1988). It can be seen that, by comparing Eq. (130) with Eq. (129), one has, for this particular system:

$$H = \frac{3}{4\pi}L$$
 (131)

The Hamaker constant for adhesion of two dislike materials may be estimated from their individual Hamaker constants for the case of cohesion, as (Ranade et al, 1988):

$$H_{12} = \sqrt{H_{11}H_{22}} \tag{132}$$

where the subscripts denote the pair of dislike or like materials in consideration. In the case of two dislike materials interacting through a medium, the Hamaker constant can be estimated by the following correlation (Ranade et al, 1988 and Taniguchi and Kikuchi, 1992):

$$\begin{aligned} H_{132} &= H_{12} + H_{33} - H_{13} - H_{23} \\ &= \left(\sqrt{H_{11}} - \sqrt{H_{33}}\right) \left(\sqrt{H_{22}} - \sqrt{H_{33}}\right) \end{aligned}$$
(133)

where H_{132} denotes the Hamaker constant between material 1 and 2 in the presence of a

medium denoted as 3. For interactions between two bodies of the same material through a medium 3:

$$H_{131} = \left(\sqrt{H_{11}} - \sqrt{H_{33}}\right)^2 \tag{134}$$

It can be seen that the interaction force between two bodies would be smaller in the presence of a medium than that otherwise in a vacuum or a transparent medium. If the Hamaker constant for alumina is taken as 15.5×10^{20} J (Taniguchi and Kikuchi, 1992), and that of aluminum at 1000° K as 34.05×10^{20} J (Osborne-Lee, 1988), then the Hamaker constant between the alumina filter and alumina inclusions may be approximated, according to Eq.(134), as 3.603×10^{20} J.

The above expressions for the Hamaker constant are valid for situations where the separation is much smaller than the characteristic absorption wavelengths of the media (Osborne-Lee, 1988). At larger separations, a retardation factor has to be introduced into these correlations, otherwise, Van der Waals forces would be overestimated.

As a rough estimation, treating the adhesion between a spherical alumina particle in liquid aluminum and the alumina filter as the case of a perfect sphere contacting a flat surface, for a 10 μ m particle at a separation of 50 Å, the Van der Waals forces are, according to Eq.(129), 1.201x10⁻⁹N. The corresponding drag force, if estimated from the Stokes equation with a velocity of 0.01 m/s, would be 1.107x10⁻⁹N, which is almost the same as that of Van der Waals forces. The resultant gravitational and buoyancy force would be, if the density of the particle and the liquid aluminum taken as 3300 and 2350 kg/m³ respectively, 4.875x10⁻¹²N. It follows then that in order for a 10 μ m alumina particle to be retained by Van der Waals forces alone to the alumina filter web's surfaces, it has to be very close to the latter (less than 50 Å). However, at such a closeness, the liquid aluminum in between may withdraw due to non-wetting factors, creating a vacuum in the contact region which may then be occupied by dissolved gases or other impurities such as salts diffused thereto from the bulk (Frisvold, 1990); and in this case, both surface tension effects and pressure attributing to adhesion, the adhesion forces would, according to Frisvold (1990), be several orders of magnitude greater than the drag forces.

The above estimation is based on an ideal situation. In reality however, the adhesion forces depend on also, aside from the factors mentioned above, the microscopic surface structures of the particles and filter webs at true state of contact, which is generally unknown. For example, the true contact area may be several orders of magnitude lower than the geometrically apparent one, owing to surface asperities (Bowden and Tabor, 1954); the surfaces in the contact region may also be deformed, thus increasing the contact area; irregularly shaped particles may re-orientate at points of contact, thus altering the contact area, etc.; all these factors would affect adhesion.

5.6.2.3. Electrostatic Forces

Electrostatic forces, resulting from surface charges of the contact materials, may also play a role in adhesion. These forces may come from two types of charges: one is referred to as 'bulk excess charges' on the surfaces, which create a coulombic attraction known as an electrostatic image force; the other is known as the double layer charges or the contact potential created as a result of electron or ion exchanges between two contact surfaces que to their differences in local energy state. Little information is available for surface charges in liquid metal systems, where the solid (i.e., the particles or filter webs) surfaces are buried in a sea of freely moving electrons. According to some studies in other fields of research, the electrostatic forces only become important for particles greater than 50 μ m, for smaller particles Van der Waals forces predominate (Bowling, 1988).

5.6.2.4. Chemical Bonding or Sintering

When surfaces are in intimate contact, chemical reactions between contacting materials may take place. Depending on the nature of the reaction, the newly formed substance may help to bond the two surfaces together. Also, at high temperatures, sintering may occur. It has been observed (Tian, 1990) that alumina particles in liquid steel filtration did sinter to the filter webs and/or to the alumina particles already captured inside the filtered. In liquid aluminum filtration, however, neither chemical bonding nor sintering have been evidently observed, in fact, the frequently observed particle release (Dupuis et al, 1993) may indicate that the possibility of the occurrence of the two is, or if occur, their effects are, minimal.

5.6.2.5. Hydrodynamic Lubrication (Retardation) Effects

When a solid particle is moving in the close vicinity of a stationary solid surface, the drag force experienced by the particle in a viscous fluid would be higher than that given by Stokes law. This phenomenon is generally referred to as 'hydrodynamic retardation'. Physically, this phenomenon is essentially caused by increased shear resistances as a result of non-slip conditions as the gap narrows. Lubrication effects would make the relative movements of the two solid particles more difficult as they closely approach each other (Ambari et al, 1986 and MaLysa et al, 1986). That is perhaps the reason for the viscosity of a fluid to increase as the content of solid particles increases. Further, when the concentration of solids reaches a certain level, the fluid or suspension becomes non-Newtonian.

According to Maude (1961) and Brenner (1961), when a sphere is moving normally towards a flat surface, the drag force can be expressed as:

$$F = 3\pi\mu d_{\mu}u_{\mu}f(\delta) \tag{135}$$

where δ is the gap between the sphere surface and the flat surface. At lubrication limit, i.e., $\delta \rightarrow 0$, $f(\delta) = 1/\delta$, the drag approach infinity.

For situations where the sphere is moving parallel to a solid flat surface, the drag may attain a finite value (Spielman and Goren, 1970) as the gap diminishes. This phenomenon has been observed experimentally by Ambari et al (1984).

It should be recognized that the hydrodynamic retardation effect in the direction normal to the solid surface would hinder particles from making contact with the latter, it is therefore an unwanted effect for filtration purpose. Its effect in the tangential direction would help adhesion, since the increased drag would make the particle movement relative to the filter web's surfaces more difficult. The drag here refers to those forces imposed by the fluid in the gap region as a result of relative movement between the particle and filter web, and should not be confused with the drag imposed by the moving bulk fluid, which acts as a dislodging force trying to drag the particle away.

Along these lines, the captured particle agglomerates may be viewed as being local non-Newtonian elements of fluid of much higher apparent viscosity than the molten metal. These non-Newtonian fluid elements most probably belong to Bingham plastics (Wikinson 1960). As this type of fluid has a definite yield stress, it would not flow until the yield stress was exceeded. This may explain how clusters sustain themselves across the pores even if they may not actually be in direct physical contact with each other (this may be especially true for the capture of wetting Al_4C_3 particles as observed in the examination of the spent filters). Once drag forces acting upon these clusters (non-Newtonian fluid elements) exceed their apparent yield stress, the particle clusters would be dislodged (partially or entirely). External forces such as flow perturbations and surges

may also cause the yield stress to be exceeded and result in a release of particles from a filter. Such a phenomenon has already been observed and reported by Doutre and Guthrie (1985), where physical tapping of a filter with a hammer to help lower the head of metal above it, resulted in the release of a shower of inclusions. Similarly, they observed that a surge in liquid metal head during D.C. casting operations also lead to

5.7. Unresolved Issues

dramatic releases of inclusions from these filters.

Some phenomena observed in liquid aluminum filtration with reticulated ceramic filters seem to be unexpected. The typical peculiar phenomenon is that as the inclusion concentration of incoming metal became significantly lower as filtration proceeded, the inclusion concentration after the filter failed to respond accordingly, as expected from the theoretical analyses. Instead, it maintained a more or less constant level (see the lower diagram in Figure 73). According to established theories, the removal efficiency of a clean filter, for a given type of particles, should have no dependence on the particle concentration in the fluid. In the aging period however, the filter's internal structure changes as a result of the accumulation of captured particles inside the filter, which in turn affects the removal efficiency of the filter. Since this effect is a reflection of the change in internal structure of the aging filter and the rate of this change has a dependence on inlet particle concentration, removal efficiencies do have a dependence on inlet particle concentration in the aging period, as discussed in Chapter 4. (In reality however, the filtration efficiency may drop at certain points due to partial release of captured particles. Since the released particles can not be excluded in the determination of the filtration efficiency, the apparent filtration efficiency may exhibit temporarily lower values than the initial value of the clean filter for the same type of particles. This does not seem to be the case here since particle release would usually exhibit a peak value in particle concentration of the filtered metal).

The peculiar phenomenon mentioned above may, however, be explained if one of the following situations occur:

(a) the filter reached its ripened state where the adhesion efficiency starts to decrease and/or particle releases begin to occur shortly after the start of filtration process. This would only be possible if the melt was dirty enough at the beginning to ripen the filter.

(b) the type and the nature of the particles in the melt encountered in the later period are quite different from those encountered during the initial period of filtration; the difference must be so great that the adhesion probability of particles encountered later on are much lower than that encountered at the initial period. Such a case would only be possible if all the aluminum oxides, which are expected to have a high adhesion probability, concentrated in the top layers of the holding furnaces; while the wetting aluminum carbides, which are expected to have a lower adhesion efficiency, were distributed at the lower region (since the history of the liquid aluminum treatments of the melt concerned are unknown to the author, no arguments can be made here).

(c) the particles only adhered to the surfaces of the filter webs and not to that of the captured particles, the surfaces available decreased as filtration proceeded, similar to a monolayer absorption process.

Otherwise, if non of the above apply, the decline in filtration efficiency (lower than its initial value) as inlet particle concentration decreases seems to suggest a dependence of filtration efficiency on inlet particle concentration even the filter is clean, which is in contradiction to established theory.

Other possible reasons for the levelling-off of the particle concentration of filtered metal may be associated with the LiMCA sampling locations. The sampling before the filter was conducted in the filtering bowl where the flow was very slow, if not stationary; while the sampling after the filter were made close to the exit mouth-hole in the launder where the flow was violently turbulent and which might abrade some of the refractory linings in the mouth-hole area and thus be detected by the LiMCA probe. And/or whether the orientation of orifice of the LiMCA probe in such a highly turbulent stream have an effect on sampling, especially for detecting different shaped inclusions, remains unknown. Until all these possible factors are to be investigated and clarified, no answer seems to be possible to explain this observed peculiar phenomenon.

5.8. Summary

Experimental data obtained from filtration of liquid aluminum in both industrial and laboratory scale environments were used to spot check the numerical analysis of initial filtration processes, and a general consistency was revealed. An evaluation of the distribution of captured particles (specific deposit) inside the 30 ppi spent filter used in the industrial trials showed a comparable result with that predicted theoretically. Inspection of the dynamic behaviour of this type of filtration based on the industrial trials, namely, changes in filtration efficiency and the pressure drops as a function of filtration time, exhibited inconclusive results, some being in accord with the theoretical analyses, others not, the clarification of which requires further investigations.

The model describing the change in pressure drop during the aging period seemed to underestimate the actual pressure drop, but as far as the trend is concerned, the model seemed to be in accord with measured values.

Chapter 6

Conclusions to the Thesis

6.1. Summary

The work presented in this thesis constitutes a systematic investigation of liquid aluminum filtration using reticulated ceramic filters, covering fluid flow characteristics, initial filtration processes, as well as their transient dynamic behaviour involved. It was concluded from the water flow experiments that the flow through this type of filter obeys Darcy's law at low flow rates, the upper limit beyond which Darcy's law becomes invalid varied depending on the specifications of a given filter, the highest Reynolds number based on the average web diameter should not exceed 8 in order to stay within the Darcy domain for the filters tested. The permeability of this type of filter was measured to be in the range of 9 to 40 thousand units of Darcy. It was found that Carman-Kozeny equation can be used as an estimation for the permeability of this type of filter with tolerable errors from engineering point of view. The flow in the non-Darcy regime can be adequately described by the Forchheimer type of equation.

A mathematical model was developed to evaluate the main parameters affecting the initial period of filtration of liquid aluminum through reticulate ceramic filters. In the model, the filter was modeled with a 2-D domain filled with a staggered array of obstacles within which the mean macroscopic geometric properties of the real filter were embodied. The equations governing fluid flow and particle transport, namely the Navier-Stokes equations and the mass conservation equation, were solved numerically, and the main influential parameters identified after examination of the physical phenomena were evaluated. According to the numerical simulation, the clean filter coefficient for this type of filter had a linear dependence on the dimensionless Stokes velocity of the suspended particles, a -0.96 power dependence on the Peclet number, a -6.93 power dependence on the effective porosity of the filter, and a weak dependence on the Reynolds number, in the Darcy velocity regime.

The dynamic behaviour of this type of filter was analyzed theoretically. In conjunction with the results for the initial filtration phenomena obtained from the numerical simulations, two correlations, one relating the filter coefficient, the other the pressure drop, to the specific deposit, were proposed. Based on these correlations, the macroscopic equations governing deep bed filtration processes were solved numerically. Dynamic behaviour (namely, the distribution as well as evolution of the specific deposit inside the filter and its influence on filtration efficiency and pressure drop), was simulated once a model describing the morphology of captured particles (i.e., the 'only porosity change' model) had been proposed. The simulated results showed that a deep bed filtration process possesses an autocatalytic character, i.e., the more particle captured the more effective it becomes. Consequently, the filtration efficiency and the pressure drop were found to increase with inlet particle concentration and filtration time. The influence was found to be insignificant when the inlet particle concentration was less than 1 ppm for a filtration period of two hours. However, when the inlet concentration reached 10 ppm, there was an appreciable change both in filtration efficiency and in pressure drop, and this could be simulated by invoking a 100 percent adhesion efficiency.

To verify the models proposed, liquid aluminum filtration tests were conducted both in laboratory and in industrial scales. The initial filtration efficiency predicted was spot checked against experimental data, and an accord was revealed. Examination of the amount and distribution of captured particles inside a spent 30 ppi filter showed a comparable result with that predicted. As far as the change in filtration efficiency is concerned, the experimental data exhibited an inconsistency, some being in accord with the theoretical analysis, some not. A comparison of pressure drops between an experimentally measured and predicted situation, showed that the model seemed to give an underestimation, but the trend seemed to be consistent.

6.2. Claims to Originality

The fluid flow characteristics through this type of filter in both Darcy and non-Darcy regimes has been evaluated for the first time. The order of magnitude of flow rate at which the transition form Darcian to non-Darcian takes place for filters commonly used in liquid metal filtration has for the first time been determined.

A novel model for the behaviour of reticulated ceramic filters in capturing inclusions was constructed and tested. The predicted results based on this model was showed to be comparable with results from other models traditionally employed in deep bed filtration. The predicted initial filtration efficiency was in accord with the experimental data obtained from filtering liquid aluminum.

Based on the model proposed, the filter coefficient of reticulate ceramic filters has, for the first time, been directly evaluated numerically. An approximate expression for estimating the value of the filter coefficient for non-neutrally buoyant particles was developed.

A new correlation relating the filter coefficient to the specific deposit was proposed and tested against experimental data, and fair consistence was demonstrated.

A new correlation relating the pressure drop in the filter aging period to a specific deposit was proposed. A spot check against experimental data showed the correlation may be used to predict the trend rather than the absolute value.

A new model was proposed to describe the morphology of the captured particles within the filter, the employment of which, along with other correlations proposed, demonstrated a partial success.

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The volume flow rate, Q, through a horizontal circular, is given by Hagen-Poiseuille equation:

$$Q = \frac{\Delta p}{L} \frac{\pi d_t^4}{128\mu} \tag{1}$$

where d_i and L denote, respectively, the diameter and the length of the tube; ∇p is the pressure drop. (This equation is valid for laminar steady-state flow of incompressible fluid in circular tubes where a fully parabolic velocity field can be developed).

Rearrange Eq.(1), one has:

$$\frac{\Delta p}{L} = \frac{32}{d_t^2} \,\mu \,\overline{\mu} \tag{2}$$

where the over-lined u denotes the average velocity through a circular tube.

If one assumes that a packed bed is equivalent to a group of parallel and equalsized tubes, such that their total internal surface and void volume are, respectively, equal to the specific surface area and the porosity of the packed bed (Kozeny assumption), then, for non-Darcy flow, adding a kinetic energy loss term to Eq.(2), one has:

$$\frac{\Delta p}{L} = \frac{32}{d_{\star}^2} \mu \bar{u} + \frac{1}{2d_{t}} \rho \bar{u}^2$$
(3)

and

$$S_{0} = \frac{NL\pi d_{t}}{\frac{L\pi D^{2}}{4}(1-\varepsilon)}$$

$$N\pi d_{t}^{2} = \pi D^{2}\varepsilon$$

$$\overline{u} = \frac{V_{sf}}{\varepsilon}$$
(4)

thus

$$d_t = \frac{4\varepsilon}{(1-\varepsilon)} \frac{1}{S_0}$$
(5)

where D and L denote, respectively, the diameter and the length of the packed bed, N is the number of the tubes contained therein.

Substituting Eq. (5) into Eq. (3), and adding two correction factors, C_1 and C_2 , one obtains:

$$\frac{\Delta P}{L} = 2C_1 \frac{(1-\varepsilon)^2}{\varepsilon^3} S_0^2 \mu V_{sf} + \frac{C_2}{8} \frac{(1-\varepsilon)}{\varepsilon^3} \rho S_0 V_{sf}^2$$
(6)

Appendix II

The dimensionless correlation relating the filtration efficiency to its independent variables, i.e., Eq.(38) presented in Chapter 3, was obtained from a dimensional analysis of Eq.(37) after discarding the last 6 variables associated with adhesion, i.e.

$$f(\boldsymbol{e}_{p},\boldsymbol{\varepsilon},\boldsymbol{L},\boldsymbol{d}_{w},\boldsymbol{V}_{st},\boldsymbol{\mu},\boldsymbol{\rho},\boldsymbol{\rho}_{p},\boldsymbol{d}_{p},\boldsymbol{D},\boldsymbol{g}) = 0 \tag{1}$$

by introducing dimensionless variables (π groups) constructed according to the following

$$\pi = (d_w)^{k!} (L)^{k2} (d_p)^{k3} (\rho)^{k4} (\rho_p)^{k5} (\mu)^{k6} (V_{sy})^{k7} (g)^{k8} (D)^{k9} (\varepsilon)^{k10} (e_p)^{k11}$$
(2)

the value of exponents k1...k11 should be so chosen that π is dimensionless. Obviously, the last two variables are already dimensionless, i.e., k10 = k11 = 1, forming two π groups. The value of k1...k9 can be determined from the matrix formed by the coefficients of k's according to their dimensions in terms of Mass, Length and Time (MLT):

	kl d.	1⁄2 L	k3 d,	k4 ρ	ю Р,	k6 μ	k7 V₅f	k8 l g D	9
M	0	0	0	1	1	1	0	0	0
L	1	1	1	-3	-3	-1	1	1 2	2
Τ	0	0	0	0	0	-1	-1	-2 -	1

The largest non-zero determinant of the above matrix has an order of three, hence, the matrix has a rank of 3. The number of dimensionless groups involved is equal to the number of the variables minus the rank of the dimensional matrix, therefore, there are six dimensionless groups to be determined from the values of k1...k9.

Since the system is undetermined (the number of unknowns is larger than the

number of equations), many solutions are possible, e.g., one can write:

$$k6 = -k5 - k4$$

$$k5 = \frac{1}{3}(k1 + k2 + k3 - 3k4 - k6 + k7 + k8 + 2k9)$$
(3)

$$k7 = -k6 - 2k8 - k9$$

by assigning some arbitrary values to the k's in the right side of the above equations, one could determine the six π groups, e.g.

•	kl	k2	k3	k 4	ୟ	kб	k7	K 8	ю	
•	d _w	L	d_{p}	ρ	ρ _ρ	μ	V_{sf}	8	D	
π1	0	0	0	1	-1	0	0	0	0	
π2	1	0	0	0	1	-1	1	0	0	(4
π3	0	1	0	0	1	-1	1	0	0	
π	0	0	1	0	1	-1	1	0	0	
π	0	0	0	0	-1	1	-3	1	0	
π	0	0	0	0	1	-1	0	0	1	
• •										

i.e.,

$$\pi_{1} = \frac{\rho}{\rho_{\mu}} \qquad \pi_{2} = \frac{d_{w}\rho_{p}V_{sf}}{\mu} \qquad \pi_{3} = \frac{L\rho_{p}V_{sf}}{\mu} \qquad (5)$$
$$\pi_{4} = \frac{d_{p}\rho_{p}V_{sf}}{\mu} \qquad \pi_{5} = \frac{\mu g}{\rho_{p}V_{sf}^{3}} \qquad \pi_{6} = \frac{\rho D}{\mu}$$

which can be rearranged to form the desired groups, for instance,

•

$$\frac{\pi_3}{\pi_2} = \frac{L}{d_w}; \quad \frac{\pi_4}{\pi_2} = \frac{d_p}{d_w} \quad (N_R); \quad \pi_1 \pi_2 = \frac{\rho d_w V_{sf}}{\mu} \quad (Re)$$

$$\frac{\pi_1 \pi_2}{\pi_6} = \frac{d_w V_{sf}}{D} \quad (Pe); \quad \frac{\pi_4^2}{\pi_2} = \frac{d_p^2 \rho_p V_{sf}}{\mu d_w} \quad (Sk) \quad (6)$$

$$\pi_4^2 \pi_5 - \pi_4^2 \pi_5 \pi_1 = \frac{(\rho_p - \rho) d_p^2 g}{\mu V_{sf}} \quad (N_G)$$

therefore,

$$\boldsymbol{e}_{r} = f\left(\boldsymbol{\varepsilon}, \frac{L}{d_{w}}, \frac{d_{p}}{d_{w}}, \frac{\rho V_{sf} d_{w}}{\mu}, \frac{V_{sf} d_{w}}{D}, \frac{(\rho_{p} - \rho) d_{p}^{2} g}{\mu V_{sf}}, \frac{d_{p}^{2} \rho_{p} V_{sf}}{\mu d_{w}}\right)$$
(7)

Appendix III

More predicted results from Chapter 3 are given in this appendix.

3






















Appendix IV

The macroscopic equation, based on mass balance of suspended particles flowing through a deep bed, takes, after neglecting the diffusion effect, following form:

$$V_{sf}\frac{\partial c}{\partial z} + \frac{\partial (\varepsilon c)}{\partial t} + \frac{\partial \alpha}{\partial t} = 0$$
 (1)

by introducing a corrected time, θ ,

$$\Theta = t - \int_0^z \frac{\varepsilon}{V_{sf}} dz \tag{2}$$

and let $z = \xi$, then transform Eq.(1) in terms of two new variables,

$$V_{\mathfrak{F}}\left(\frac{\partial c}{\partial \xi}\frac{\partial \xi}{\partial z}+\frac{\partial c}{\partial \theta}\frac{\partial \theta}{\partial z}\right)+\left(\frac{\partial (\varepsilon c)}{\partial \theta}\frac{\partial \theta}{\partial t}+\frac{\partial (\varepsilon c)}{\partial \xi}\frac{\partial \xi}{\partial t}\right)+\left(\frac{\partial \alpha}{\partial \theta}\frac{\partial \theta}{\partial t}+\frac{\partial \alpha}{\partial \xi}\frac{\partial \xi}{\partial t}\right)=0$$

since

$$\frac{\partial \xi}{\partial z} = \frac{\partial \theta}{\partial t} = 1; \qquad \frac{\partial \xi}{\partial t} = 0$$

then

$$V_{sf}\frac{\partial c}{\partial \xi} + c\frac{\partial \varepsilon}{\partial \theta} + \frac{\partial \alpha}{\partial \theta} = 0$$

since

$$\varepsilon = \varepsilon_0 - \frac{\alpha}{1 - \varepsilon_a}$$

then

$$V_{sf}\frac{\partial c}{\partial \xi} + \frac{\partial \alpha}{\partial \theta} \left(1 - \frac{c}{1 - \varepsilon_a}\right) = 0$$

since $1 > c/(1-\varepsilon)$, then

$$V_{sf}\frac{\partial c}{\partial \xi} + \frac{\partial \alpha}{\partial \theta} = 0$$
 (3)

This equation takes the same form as Eq.(1) if its second term in left side is neglected, i.e.,

$$V_{\text{sf}}\frac{\partial c}{\partial z} + \frac{\partial \alpha}{\partial t} = 0$$
 (4)

The difference between Eq.(3) and Eq.(4) lies in that the former rides on a suspension element flowing through a deep bed filter, while the latter sees a fixed element in the filter. The magnitude of the difference is negligibly small once the filter is primed, therefore, the two equations are used identically for most filtration processes.

Eq.(93) and Eq.(95) presented in Chapter 4 are essentially ordinary differential equations, i.e., after substitution of Eq.(88) for the 'only porosity change' model (or Eq.(87) for the 'smooth coating' model) for λ ,

$$\frac{d\alpha_{in}}{dt} = \left\{ \lambda_0 + \frac{b}{d_w^0} \left[\frac{1}{\left(\varepsilon_0 - \frac{\alpha_{in}}{1 - \varepsilon_a} \right)^{6.93}} - \frac{1}{\varepsilon_0^{6.93}} \right] \right\} V_{sf} c_{in}$$
(1)

with initial condition

$$\alpha_{in} = 0 \quad at \quad t = 0 \tag{2}$$

and

$$\frac{d\alpha}{dz} = -\left\{\lambda_0 + \frac{b}{d_w^0} \left[\frac{1}{\left(\epsilon_0 - \frac{\alpha}{1 - \epsilon_a}\right)^{6.93}} - \frac{1}{\epsilon_0^{6.93}}\right]\right\}\alpha$$
(3)

with initial condition

$$\alpha = \alpha_{in} \qquad at \quad z = 0 \tag{4}$$

This two equations can be numerically solved with the fourth-order Runge-Kutta method, i.e., writing Eq.(1) into

$$\frac{d\alpha_{in}}{dt} = f(\alpha_{in}) \tag{5}$$

then

$$\alpha_{i\pi}^{i+1} = \alpha_{i\pi}^{i} + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$
(6)

where

$$k_{1} = \delta t f(\alpha_{in}^{i})$$

$$k_{2} = \delta t f\left(\alpha_{in}^{i} + \frac{k_{1}}{2}\right)$$

$$k_{3} = \delta t f\left(\alpha_{in}^{i} + \frac{k_{2}}{2}\right)$$

$$k_{4} = \delta t f(\alpha_{in}^{i} + k_{3})$$
(7)

$$\frac{d\alpha}{dz} = g(\alpha) \tag{8}$$

then

$$\alpha^{i+1} = \alpha^{i} + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$
(9)

where

$$k_{1} = \delta z g(\alpha^{i})$$

$$k_{2} = \delta z g\left(\alpha^{i} + \frac{k_{1}}{2}\right)$$

$$k_{3} = \delta z g\left(\alpha^{i} + \frac{k_{2}}{2}\right)$$

$$k_{4} = \delta z g(\alpha^{i} + k_{3})$$
(10)

Similarly, Eq.(97) can be written as

$$\frac{dc}{dz} = - \left\{ \lambda_0 + \frac{b}{d_w^0} \left[\frac{1}{\left(\varepsilon_0 - \frac{\alpha}{1 - \varepsilon_a} \right)^{6.93}} - \frac{1}{\varepsilon_0^{6.93}} \right] \right\} c = F(c, \alpha)$$
(11)

with initial condition

 $\alpha = \alpha_{is}; \quad c = c_{is} \qquad at: \quad z = 0 \qquad (12)$

this equation can be simultaneously solved in conjunction with Eq.(3), i.e.,

$$c^{i+1} = c^{i} + \frac{1}{6}(q_1 + 2q_2 + 2q_3 + q_4)$$
(13)

where

C-----

$$k_{1} = \delta z F(c^{i}, \alpha^{i})$$

$$k_{2} = \delta z F\left(c^{i} + \frac{q_{1}}{2}, \alpha^{i} + \frac{k_{1}}{2}\right)$$

$$k_{3} = \delta z F\left(c^{i} + \frac{q_{2}}{2}, \alpha^{i} + \frac{k_{2}}{2}\right)$$

$$k_{4} = \delta z F(c^{i} + q_{3}, \alpha^{i} + k_{2})$$
(14)

where the values of α^i and ks are given in Eq.(10).

The computer programs are listed below.

C-----THIS PROGRAM CALCULATE THE SPECIFIC DEPOSIT AS A FUNCTION C OF TIME AT THE FILTER INLET BASED ON 'ONLY POROSITY CHANGE' C MODEL. C------

PROGRAM SIGMA0

IMPLICIT DOUBLE PRECISION(A-H,O-Z)

```
F(SIGM)=US*CIN*(AMD0+B*(1/(EPS0-SIGM/EPA)**6.93-1/EPS0**6.93)/DW)
OPEN(9,FILE='C:\TIAN\FAGE\SG.T',STATUS='OLD',FORM='FORMATTED')
C----CONSTANTS cgs system----
    US = 1.0
    CIN = 1.E-7
     AMD0=0.411
    SIGM = 0.0
     B=0.033126*0.1
     DW = 0.03
    EPS0=0.83
      EPA = 0.3
c-----time control in seconds-----
    T=0.
     DELT=10.
     TP = 60.
      TMAX=7200.
     N=0
c-----initial values----
   WRITE(9,6)T,SIGM
3
   N=N+1
   AK1 = DELT * F(SIGM)
   AK2 = DELT*F(SIGM+0.5*AK1)
    AK3=DELT*F(SIGM+0.5*AK2)
    AK4 = DELT * F(SIGM + AK3)
    SIGM = SIGM + (AK1 + 2.0 * AK2 + 2.0 * AK3 + AK4)/6.0
     T=T+DELT
     IF(T.EQ.TP) THEN
     TP=TP+60.
    WRITE(*,6)T,SIGM
    WRITE(9,6)T,SIGM
     ENDIF
    IF(T.LT.TMAX) GO TO 3
6
     FORMAT(2E15.7)
      STOP
      END
```

C----THIS PROGRAM CALCULATE THE SPECIFIC DEPOSIT AS A FUNCTION

C OF FILTER THICKNESS AT A GIVEN TIME BASED ON 'ONLY POROSITY C CHANGE' MODEL.		
PROGRAM SZ		
IMPLICIT DOUBLE PRECISION(A-H,O-Z) DIMENSION SZ0(200),T(200)		
F(SIGM) = -SIGM*(AMD0 + B*(1/(EPS0-SIGM/EPA)**6.93-1/EPS0**6.93)/DW)		
OPEN(8,FILE='C:\TIAN\FAGE\SG.T',STATUS='OLD',FORM='FORMATTED') OPEN(9,FILE='C:\TIAN\FAGE\SG.Z',STATUS='OLD',FORM='FORMATTED') CCONSTANTS_CGS_SUSTER		
NI-121		
$N_1 = 121$		
DO[1] = 1, NI		
$1 \qquad \text{READ}(8,0)1(1),520(1)$		
AMD0=0.411		
SIGM = SZO(2)		
B=0.033126*0.1		
DW=0.03		
EPS0=0.83		
EPA=0.3		
cfilter thickness (cm)		
Z=0.		
DELZ=0.1		
ZP=0.2		
7MAX=3.8		
N=0		
Cinitial values		
WRITE(9.6)Z.SIGM		
3 N = N + 1		
AK1 = DELZ F(SIGM)		
AK2 = DELZ * F(SIGM + 0.5 * AK1)		
AK3 = DELZ * E(SIGM + 0.5 * AK2)		
AK4 = DFL Z * F(SIGM + AK3)		
SIGM=SIGM+(AK1+2.0*AK2+2.0*AK3+AK4)/6.0 Z=Z+DELZ IF(Z.EQ.ZP) THEN		
ZP=ZP+0.1		

WRITE(*,6)Z,SIGM WRITE(9,6)Z,SIGM ENDIF IF(Z.LT.ZMAX) GO TO 3 6 FORMAT(2E15.7) STOP END C-THIS PROGRAM CALCULATE THE SPECIFIC DEPOSIT AND THE CONCENTRATION AS A FUNCTION OF FILTER THICKNESS at a given time С based on 'only porosity change' model. С C----**PROGRAM SZ** C-----IMPLICIT DOUBLE PRECISION(A-H,O-Z) DIMENSION T(130), SZ0(130) c----functions-----F(SIGM) = -SIGM*(AMD0 + B*(1/(EPS0-SIGM/EPA)**6.93-1/EPS0**6.93)/DW) $G(C,SIGM) = -C^{*}(AMD0 + B^{*}(1/(EPS0-SIGM/EPA))^{*6.93-1}/(EPS0^{*6.93})/DW)$ OPEN(8,FILE='C:\TIAN\FAGE\SG.T',STATUS='OLD',FORM='FORMATTED') OPEN(9,FILE='C:\TIAN\FAGE\SG.C',STATUS='OLD',FORM='FORMATTED') C----CONSTANTS cgs system---NI=121 DO 1 I=1.NI READ(8,7)T(I),SZ0(I) 1 CONTINUE CIN = 1.E-7AMD0=0.411 SIGM = SZO(2)B=0.033126*0.1 DW = 0.03EPS0 = 0.83EPA = 0.3C=CIN c-----FILTER THICKNESS (cm)-----Z=0. DELZ=0.1ZP=0.2ZMAX = 5.1

N=0
cinitial values
WETTER 617 SIGN C
$ = \frac{1}{2} $
AK1 = DELZ*F(SIGM)
$AK^2 = DEL Z^*F(SIGM + 0.5^*AK^1)$
$AK3 = DELZ^*F(SIGM + 0.5^*AK2)$
$AK4 = DEL Z^*F(SIGM + AK3)$
$O1 = DELZ^*G(C,SIGM)$
O2 = DELZ = O(C + 0.5 = O1, SIGM + 0.5 = AK1)
O3 = DELZ*G(C+0.5*O2,SIGM+0.5*AK2)
O4 = DELZ G(C + Q3, SIGM + AK3)
SIGM = SIGM + (AK1 + 2.0 * AK2 + 2.0 * AK3 + AK4)/6.0
C = C + (Q1 + 2.0 + Q2 + 2.0 + Q3 + Q4)/6.0
Z=Z+DELZ
IF(Z.EQ.ZP) THEN
ZP = ZP + 0.2
WRITE(*,6)Z,SIGM,C
WRITE(9,6)Z,SIGM,C
ENDIF
IF(Z.LT.ZMAX) GO TO 3
6 FORMAT(3E15.7)
7 FORMAT(2E15.7)
STOP
END
C=====================================
CTHIS PROGRAM CALCULATE THE SPECIFIC DEPOSIT AS A FUNCTION
C OF TIME AT THE FILTER INLET BASED ON 'SMOOTH COATING MODEL'.
PROGRAM SIGMAU

F(SIGM) = US*CIN*(AMD0+B*((1/(EPS0-SIGM/EPA))**6.93-1/EPS0**6.93)/DW1 + (1/(DW*(1+SIGM/((1-EPS0)*EPA)))**0.5)-1/DW)/EPS0**6.93)) Appendix V

C-----THIS PROGRAM CALCULATE THE SPECIFIC DEPOSIT AS A FUNCTION

C OF FILTER THICKNESS AT A GIVEN TIME BASED ON 'SMOOTH COATING C MODEL. C-----PROGRAM SZ C-----IMPLICIT DOUBLE PRECISION(A-H, G-Z) DIMENSION SZ0(200),T(200) c-----functions-----F(SIGM) = -SIGM*(AMD0 + B*(1/(EPS0-SIGM/EPA)**6.93-1/EPS0**6.93)/DW+B*(1/(DW*(1+SIGM/((1-EPS0)*EPA))**0.5)-1/DW)/EPS0**6.93) 1 OPEN(8,FILE='C:\TIAN\FAGE\SG.T',STATUS='OLD',FORM='FORMATTED') OPEN(9,FILE='C:\TIAN\FAGE\SG.Z',STATUS='OLD',FORM='FORMATTED') C----CONSTANTS cgs system--NI=121 DO 1 I = 1.NIREAD(8,6)T(I),SZ0(I) 1 AMD0 = 0.411SIGM = SZO(2)B=0.033126*0.1 DW = 0.03EPS0 = 0.83EPA = 0.3c-----filter thickness (cm)-----Z=0. DELZ=0.1ZP=0.2ZMAX = 3.8N=0 c-----initial values-----WRITE(9,6)Z,SIGM 3 N=N+1AK1 = DELZ * F(SIGM)AK2=DELZ*F(SIGM+0.5*AK1) AK3 = DELZ * F(SIGM + 0.5 * AK2)AK4 = DELZ + F(SIGM + AK3)SIGM=SIGM+(AK1+2.0*AK2+2.0*AK3+AK4)/6.0 Z=Z+DELZIF(Z.EQ.ZP) THEN



```
IF(Z.LT.ZMAX) GO TO 3
6
    FORMAT(2E15.7)
     STOP
     END
C----THIS PROGRAM CALCULATE THE SPECIFIC DEPOSIT AND THE
С
   CONCENTRATION AS A FUNCTION OF FILTER THICKNESS AT A GIVEN
С
    TIME BASED ON 'SMOOTH COATING MODEL'
C---
    PROGRAM SZ
C----
    IMPLICIT DOUBLE PRECISION(A-H,O-Z)
    DIMENSION T(130), SZ0(130)
c----functions--
   F(SIGM) = -SIGM^{*}(AMD0 + B^{*}(1/(EPS0 - SIGM/EPA)^{**6.93} - 1/EPS0^{**6.93})/DW
         +B*(1/(DW*(1+SIGM/((1-EPS0)*EPA))**0.5)-1/DW)/EPS0**6.93)
   1
   G(C,SIGM) = -C^{*}(AMD0 + B^{*}(1/(EPS0-SIGM/EPA)^{**6.93-1}/EPS0^{**6.93})/DW
         +B*(1/(DW*(1+SIGM/((1-EPS0)*EPA))**0.5)-1/DW)/EPS0**6.93)
   1
OPEN(8,FILE='C:\TIAN\FAGE\SG.T',STATUS='OLD',FORM='FORMATTED')
OPEN(9,FILE='C:\TIAN\FAGE\SG.C',STATUS='OLD',FORM='FORMATTED')
C----CONSTANTS cgs system--
    NI = 121
     DO 1 I=1,NI
     READ(8,7)T(I),SZ0(I)
1
     CONTINUE
     CIN=1.E-7
     AMD0 = 0.411
     SIGM = SZO(2)
     B=0.033126*0.1
     DW=0.03
     EPS0 = 0.83
     EPA = 0.3
     C=CIN
    ---FILTER THICKNESS (cm)----
C----
     Z=0.
```

```
Appendix V
```

ZP = ZP + 0.1

ENDIF

WRITE(*,6)Z,SIGM WRITE(9,6)Z,SIGM $Appendix \ V$

P

	DELZ=0.1 ZP=0.2 ZMAX=5.1
C	N=U
C	
	WRITE(9,6)Z,SIGM,C
3	N=N+1
	AK! = DELZ * F(SIGM)
	AK2 = DELZ*F(SIGM+0.5*AK1)
	AK3=DELZ*F(SIGM+0.5*AK2)
	AK4 = DELZ*F(SIGM + AK3)
	$O1 = DELZ^*G(C,SIGM)$
	$Q2 = DELZ^*G(C+0.5^*Q1,SIGM+0.5^*AK1)$
	$Q_3 = DELZ * G(C + 0.5 * Q_2, SIGM + 0.5 * AK2)$
	Q'+=DELZ*G(C+Q3,SIGM+AK3)
	SIGM = SIGM + (AK1 + 2.0*AK2 + 2.0*AK3 + AK4)/6.0
	C=C+(Q1+2.0*Q2+2.0*Q3+Q4)/6.0
	Z=Z+DELZ
	IF(Z.EO.ZP) THEN
	ZP=ZP+0.2
	WRITE(*,6)Z,SIGM,C
	WRITE(9,6)Z,SIGM,C
	ENDIF
	IF(Z.LT.ZMAX) GO TO 3
6	FORMAT(3E15.7)
7	FORMAT(2E15.7)
	STOP
	END
C =	=================
