Dig-Limit Optimization in Open Pit Mines through Genetic Algorithms

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Thesis Prepared for the Degree of

MASTERS OF ENGINEERING

McGill University

May 2016

Abstract

Dig-limit optimization is an operational decision-making problem that significantly affects the value of open-pit mining operations. Dig-limits, traditionally drawn by hand, classify practical ore and waste boundaries suiting equipment sizes in a bench. An optimization approach based on genetic algorithms (GAs) was developed to approximate optimal dig-limits on a bench, given grade control data, equipment constraints, and processing and mining costs. The GA proved to be both robust and flexible after testing multiple cases with a wide variety of applications and levels of difficulty. The success of the application of GAs to the dig-limit problem is outlined in two separate studies; a third study was carried out to show the flexibility of the GA, and its potential applications in equipment sizing.

Abstrait

L'optimisation de la limite du creusage est un problème de prise de décisions opérationnelles qui affecte considérablement la valeur de l'exploitation de mines à ciel ouvert. Les limites de creusage - qui sont habituellement dessinées à la main - classifient les limites de la pratique du minerai et des déchets convenant à la taille des équipements sur un niveau. Une approche d'optimisation basée sur des algorithmes génétiques a été développée dans un but d'approximation des limites optimales du creusage sur un niveau, en tenant compte des données de contrôle de teneur, des contraintes des équipements ainsi que des coûts du traitement et de l'exploitation minière. Les algorithmes génétiques se sont avérés robustes et flexibles suite a des tests sur de nombreux cas ayant une grande variété d'applications et niveaux de difficulté. Le succès de l'application des algorithmes génétiques par rapport au problème de la limite du creusage a été présenté dans deux études différentes; une troisième étude a été menée afin de montrer la flexibilité des algorithmes génétiques, ainsi que leurs applications potentielles au dimensionnement des équipements.

Acknowledgements

I would like to thank Professor Mustafa Kumral for his invaluable guidance and understanding.

This thesis would not have been possible without the unyielding support of my grandmother Cecilia Ruiseco. I would like to thank my mother Maria Cecilia Ruiseco, who has made great sacrifices for my education. My father, Mauricio Ramirez, who taught me that Engineering is fun. I would like to give my most sincere thanks to Sheelah McCarthy, who has been an immense source of emotional and professional support. I would especially like to thank my grandfather, Juan Manuel Ruiseco, without him I would never have become an engineer.

Francisco Albor has guided me immensely throughout my professional development. He has taught me that operability is the guiding light for all developments in Engineering.

Contributions of Authors

The author of this thesis is Julian Ramirez Ruiseco. Dr. Kumral was the supervisor of the author's Master of Engineering Degree, and co-authored "Optimizing ore – waste dig-limits as a part of operational mine planning through genetic algorithms", which has been published in the Natural Resources Research Journal.

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1. Introduction

1.1 Problem Statement

Generating optimal dig-limits is a key factor in maximizing value for operating mines. Dig-limit generation being the definition of material destination zones in operational mine planning. Considerable research has been devoted to this topic over the past 25 years. As high-grade deposits have been depleted, operating/production costs and operational complexity have increased substantially. Whereas in the past, a mine could have two destinations with few metals, newer mines can have many destinations, each of which could comprise a large number of physical and chemical factors. Therefore, dig-limits are becoming more complex. Furthermore, the impact of misclassification has become more comprehensive: sending material to the incorrect destination could negatively affect the entire chemical process.

Dig-limit generation methods fall into five general categories: hand-drawn, linear, heuristic, clustering, and meta-heuristic. These methods are traditionally carried out on grade control block models, which must be carefully generated in order to derive effective dig-limits.

Traditionally, dig-limits have been generated using hand-drawn methods, which are highly subjective, do not optimize the profit directly, and tend to be highly inconsistent. Formulating dig-limits autonomously and optimally is a high-priority task for the mining industry. The genetic algorithm (GA) approach seeks to generate viable, mineable, and near-optimal dig-limits with a minimum of human intervention.

1.2 Research Objectives

This research aims to develop a new methodology for dig-limit generation using GAs. The algorithm optimizes the profitability of the deposit with the losses expected from deviation of an optimal mining layout. Tackling the dig-limit problem with GAs is expected to maintain suitable performance for highly complex problems that would be "NP-Hard" (i.e., at least as hard as any nondeterministic polynomial problem) using linear methods. By quantifying the profitability of each block in terms of processing, mining destination costs, and geological considerations, dig-limits are optimized directly on the profitability instead of current indirect methods such as minimization of ore-loss.

1.3 Originality and Success

GAs have never been used to directly optimize the dig-limit problem. Existing methodologies either do not accurately generate diggable limits, do not directly optimize profit, or are NP-Hard for complex deposits. This application of the GA is original because of the formulation of the clustering problem. The cross method guarantees effective quantification of deviation from diggable shapes. Furthermore, the formulation of a GA for a 2-dimensional spatial problems is in itself new to the mining industry.

The resulting algorithm was shown to significantly outperform current industry methodologies, while exhibiting sufficiently low run times for the implementation of this tool in the industry. Due to ease of use and the relative flexibility of the meta-heuristic input parameters, the algorithm was shown to be effective for application beyond mine planning, such as fleet selectivity sizing.

1.4 Social Impacts and Economic Benefits

The mining industry has shifted considerably in the last 15 years:

1) Higher complexity tools are being used for optimization and planning.
2) Deposits are becoming more complex and difficult to plan.
3) Formulation of mining problems using linear methods is increasingly becoming NP-Hard.
4) Strategic planning has largely embraced meta-heuristic optimization algorithms.

However, as strategic methods have become more complex, short-term planning methodology has gone largely unchanged. The planning burden lies squarely on the shoulders of the pit geologists, who are often ill-prepared to generate optimal and effective dig-limits. Application of this research is expected to improve the profitability of mines and place the dig-limit planning burden onto meta-heuristic algorithms, leaving pit geologists more time to plan adherence in the pit, while improving the overall quality of the plans.

1.5 Thesis Organization

Chapter 1 defines the problem and identifies original contributions to the field.

Chapter 2 reviews existing methodologies and research for the short-term mining and dig-limit problem.

Chapter 3 details the methodology carried out by the GA and the methodology used to code the program.

Chapter 4 demonstrates how the algorithm outperforms traditional hand-drawn methods.

Chapter 5 demonstrates the flexibility of the algorithm by carrying out a 5 destination case with the use of a mining direction.

Chapter 6 demonstrates the value of the GA dig-limit tool for quantifying the profitability differences between selectivity sizing in a mining operation with grade control data.

Chapter 7 concludes the thesis by outlining the advantages of applying this methodology.

2. Literature Review

2.1 Contextualization of Dig-Limit Problem

Mining operations are planned at three levels: long-term (strategic), medium-term (tactical) and short-term (operational). Long-term mine planning produces a "big picture" of the project at the managerial level. It helps to understand (a) whether the project is profitable, (b) how the project will evolve over time, and (c) what macro-economic sensitivities affect the project. Long-term planning aims to maximize the net present value (NPV) of a mining venture. Medium-term planning focuses on how the objectives of a long-term plan can be managed and explores how to maximize adherence to the long-term mining plan—given more complex objectives and more detailed information. For example, specific objectives related to marketing, production, or maintenance must be managed as medium-term plans. Short-term planning manages day-to-day performance of mining operation. Achieving targets defined by medium-term planning to a large extent depends upon short-term planning. In other words, short-term planning puts into practice the medium-term plan. Figure 1 summarizes the fundamental characteristics of long- to short-term mine planning at a typical mine.

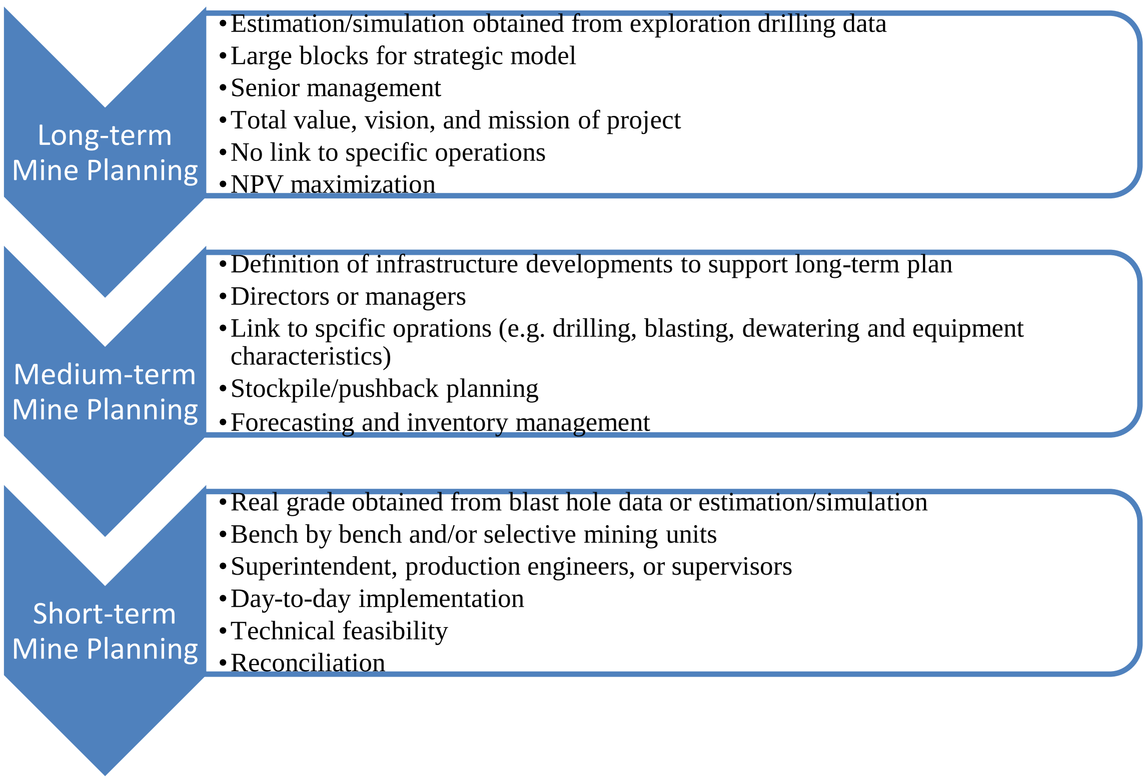


Figure 1: Fundamentals of mine planning

Long-term planning is based on long-term market forecasts and exploration drill holes. Medium-term planning uses higher detail forecasts and also takes advantage of historic operational knowledge in mine plan generation. Short-term planning uses blast-hole grade control information, which vastly improves understanding of the lithology of specific blasts, and must also work around the day-to-day difficulties related to mine operation.

Although long-term planning aims to maximize NPV, it does not address how other activities (e.g., drilling, blasting, loading, and hauling) are realized; the NPV of long-term planning reflects a vague picture of the project. Furthermore, market or orebody characteristics may compel different objectives such as head grade control, maximizing equipment and capacity utilization, or minimizing grade fluctuation, possible losses by misclassification, dilution, or risk. If these objectives are accomplished, NPV will be maximized.

In this scope, dig-limit optimization is an operational planning component because it is implemented daily on the grade control model. At the long-term planning level, an orebody is defined by a block model and the production time of each block is determined. These blocks are known as planning blocks. The information is generated by geostatistical estimation or simulation, based on exploration data. However, production is carried out on a bench-by-bench basis, focusing on a grade control block model derived from blast hole, and grade control drilling. During the operational stage, block-by-block production is impossible. Production units/clusters are much smaller than planning blocks and are known as selective mining units (SMUs). Another method not considered in this approach is to generate contour-line dig-limits from raw blast-hole data.

During the operational stage, ore-waste classification based on planning blocks is not meaningful because mining equipment is large and cannot manage the production on a block-by-block basis, and even if a bench is mined on an SMU basis, the SMUs within the operation radius of equipment cannot be discerned. Significant dilution and loss problems will occur associated with low equipment selectivity. When a bench is blasted, all materials will be mixed such that blast-hole information is no longer meaningful. However, methods exist to apply blast movements to block models, thereby improving dig-limit definition.

For the purpose of this study, it is assumed that input SMU models have had such transformations applied. Considering equipment selectivity, some SMUs identified as ore will be inevitably destined to waste dump and vice versa. For example, a high-grade SMU surrounded by four waste SMUs can be destined to waste dump because of low selectivity of equipment and high dilution associated with blasting.

Figure 2 illustrates dig-limit problem: blue and red SMUs represent waste and ore, respectively, according to a cutoff grade (COG). As can be seen from this figure, there are waste patches within ore clusters in the south and ore patches within waste clusters in the north. Figure 2 represents the post-blast-movement block model. Had this blast-movement not been applied this ore, waste classification would not be meaningful because of mixing of materials. Therefore, in practice, a mining geologist determines practical ore-waste boundaries within a bench or blasting area.

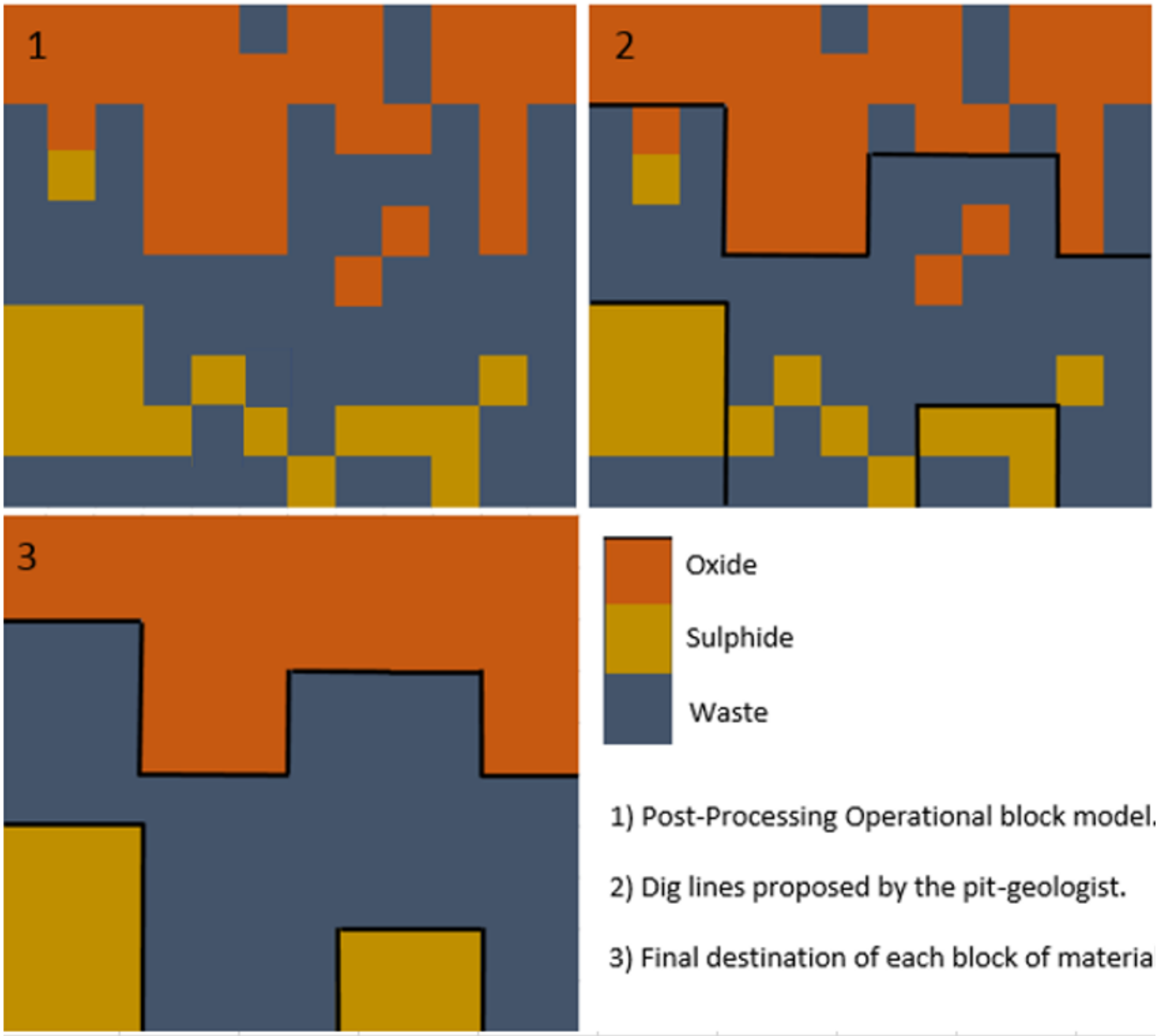


Figure 2: Illustration of ore-waste discrimination based on (a) cutoff grade block model (b) dig-limit optimization and (c) a clustering size of 2 x 2 m blocks

Dig-limits such as those shown in Figure 2 can be determined depending upon equipment maneuvering capability, and dilution and losses associated with drilling, blast design, and SMU grades. Thus, ore and waste contacts are reduced, thereby minimizing ore losses and dilution. In the case of separate ore and waste blasts, the dilution can be further minimized. In summary, in addition to grade or economic value of an SMU, grades or economic values of neighbouring SMUs should be also considered.

The interactions among medium-term planning, short-term planning, grade control modelling, and dig-limit generation must be clarified. Medium-term planning hands off a set of mining pushbacks, which defines how much material to remove from each level and zone of a mine. Short-term planning then generates a set of plans that include blast planning, equipment allocation, drainage, electric line movement (for electrohydraulic fleets), stockpile management, and blast drilling. During the blast drilling campaign, the material removed from the blast hole is sampled. These samples are then sent for laboratory analysis to determine various pertinent chemical and physical parameters. These parameters are then reported to short term planning, where they are compiled with the global position of the blast hole from which the sample was taken to create blast hole assays. These assays are then used via geostatistical methods (e.g., inverse power distance) to estimate the pertinent parameters for each block. This "grade control block-model" is then manipulated to reflect blast movement and reported to pit geology.

Mining dig-limits drawn by hand are generated sequentially. As limits are defined and packets of material identified, the rest of the hand-drawn dig-limit is predisposed to follow the shape of earlier decisions. As a result—depending on the initial location of drawing and the initial best guess of the pit geologist—a dig-limit will differ each time it is drawn. Given such a high level of subjectivity, hand-drawn dig-limits are not repeatable and tend to be highly variable. The differing clustering techniques innate to each pit geologist will result in reconciliation issues caused by inconsistent dilution.

In light of the limits above, the tedious nature of developing hand-drawn dig-limits, and the difficulty of properly exploring alternative solutions, this thesis designed an automatic dig-limit generator. SMU-by-SMU analysis of destination is an insufficiently refined solution, because the resulting dig-limit will not be operational. Therefore, the aim of this thesis is to generate dig-limits by taking into account the effective selectivity of the fleet for efficient mining of SMUs, while maximizing profitability and ensuring the results are more reproducible than hand-drawn methods.

2.2 Other Dig-Limit Optimizing Approaches

Early methods of dig-limit design generated mining polygons directly from drill hole information, without carrying out a geostatistic estimation or simulation of the blast bench. Norrena and Deutsch (2001) also emphasized that geometric approaches such as triangulation or polygons have limitations: they ignore economics, uncertainty, and equipment size. Verly (2005) reviewed grade control modelling methods, focusing on the superiority of grade control block models for dig-limit design. By establishing that block models offer superior understanding of the nature of a deposit, the generation of dig-limits on these block models can be identified as best practice.

The quality of a dig-limit has been quantified using multiple approaches, such as enhanced metal recovery, which focuses on maximizing the amount of metal recoverable given the various hard constraints of the mine's mineral processing and trucking capacity; minimum loss, which quantifies the amount of value in ore not processed by the mine; and maximum profit, which directly calculates the value of a dig-limit (Glacken, 1997; Isaaks, 1990; Richmond & Beasley, 2004b; Srivastava R.M, 1994). These methods only target the economic value of the destination of a perimeter (one piece of a dig-limit), and do not quantify the actual mineability of the proposed dig-limit design. However, they serve as the basis for dig-limit optimization tools. The method of quantifying deviation from diggability differs among solutions.

Most dig-limit quantification methods are based on the concept of the marginal COG. Between each process and destination, there is a mineral content at which it becomes more profitable to assign a block to a different destination. Take for example the COG between waste and ore for a two-destination plant. A specific COG can be calculated and is termed the marginal COG between these two destinations (equation 1).

$$prz_{\mathrm{c}} - c_{\mathrm{m}} - c_{\mathrm{p}} = -c_{\mathrm{m}} \Leftrightarrow z_{\mathrm{c}} = \frac{c_{\mathrm{p}}}{pr}$$

Equation 1: Marginal cutoff grade equation (Verly, 2005).

Where

- p: price
- r: recovery factor
- c_m: mining cost per tonne
- c_p: Processing cost per tonne

Other methods of deriving the optimal block destination involve the use of simulated block models and profit/loss functions. These functions optimize the destination based on the grade distribution curve of each block, and fall into two subsets: waste (equation 2) and ore (equation 3). Various applications of these equations serve as the basis for interpreting dig-limits. The loss functions quantify the value loss of improperly defining the destination of a block. Both of these equations would be applied to a given block, and the destination that yields the lowest loss would be selected as the optimal destination.

$$g(z) = \begin{cases} (-c_{\mathrm{m}}) - (prz - c_{\mathrm{m}} - c_{\mathrm{p}}), & z \le z_{\mathrm{c}} \\ (prz - c_{\mathrm{m}} - c_{\mathrm{p}}) - (prz - c_{\mathrm{m}} - c_{\mathrm{p}}), & z > z_{\mathrm{c}}, \end{cases}$$

$$L_{\mathrm{ORE}} = E[g(Z)] = E[i(Z, z_{\mathrm{c}}) \times (-prZ + c_{\mathrm{p}})].$$

Equation 2: Loss function for a block with a destination of waste (Verly, 2005).

$$g(z) = \begin{cases} (-c_{\mathrm{m}}) - (-c_{\mathrm{m}}), & z \leq z_{\mathrm{c}} \\ (prz - c_{\mathrm{m}} - c_{\mathrm{p}}) - (-c_{\mathrm{m}}), & z > z_{\mathrm{c}}, \end{cases}$$
$$L_{\mathrm{WST}} = E[g(Z)] = E[(1 - i(Z, z_{\mathrm{c}})) \times (prZ - c_{\mathrm{p}})].$$

Equation 3:Loss function for a block with a destination of ore (Verly, 2005).

Profit functions are similar to the loss functions; however, they target the profit directly (e.g., equations 4 and 5). These equations directly calculate the profitability of each destination for a block, and form a logical basis for the grade control process.

$$g(z) = prz - c_{\mathrm{m}} - c_{\mathrm{p}},$$
$$P_{\mathrm{ORE}} = E[g(Z)] = E[prZ - c_{\mathrm{m}} - c_{\mathrm{p}}]$$

Equation 4: Profit function for a block with a destination of ore (Verly, 2005).

$$g(z) = -c_{\mathrm{m}},$$
$$P_{\mathrm{WST}} = E[g(Z)] = E[-c_{\mathrm{m}}] = -c_{\mathrm{m}}.$$

Equation 5: Profit for a function with a destination of waste (Verly, 2005).

All dig-limits are based on the size of the grade-control block model blocks, and the selectivity of the equipment used. The minimum SMU block size is a function of the drilling density and the selectivity is a function of the equipment size. Jara, Couble, Emery, Magri, and Ortiz (2006) analyzed the effects of different support sizes on mine planning, dilution, and equipment selection. As equipment becomes larger, the cost per tonne of material decreases; however, the ability to select smaller parcels of material to send to specific destinations is reduced. Larger equipment will incur lower mining costs per tonne; however larger equipment will be forced to misclassify more material to generate mineable limits.

Accurately understanding mineralogy during grade control drilling contributes greatly to dilution and recovery. Dominy, Platten, Xie, and Minnitt (2010) investigated the effects of sampling on grade control. They emphasized the importance of (i) ore mineralogy and (ii) ore particle deportment, size, and distribution for an effective grade control campaign. These data are inputs to the grade control block model, the quality of which depends upon the drilling data quality. Similarly, Abzalov, Menzel, Wlasenko, and Phillips (2010) remarked that different data qualities and the spatial distributions of samples led to grade control errors. Thus all data used during grade control block model generation must be from a dataset of the same quality. Dominy and Platten (2012) pointed out that effective geological mapping and sampling increase performance of grade control, such that dilution is minimized and process recovery is maximized. In summary, in order to derive a representative grade control block model, sampling must be carefully managed using the same equipment and method for each borehole. The dataset derived from the boreholes must contain all data regarding grades, mineral types, zones, and geology.

Traditional optimization methods have attempted to describe the short-term dig-limit problem in terms of a linear optimization; however (Gonzalez, 1982) stipulated that the methodology is NP-Hard using existing technology. Multiple authors have recently formulated and solved the problem using mixed integer programming (MIP). (Weintraub, Pereira, & Schultz, 2008) proposed a two-stage mixed MIP process. The first stage identified groups of similar blocks and the second stage maximized the aggregation factor to guarantee mineability. However, mathematical comparisons to traditional methods were not provided. (Tabesh & Askari-Nasab, 2011) formulated the problem using mixed integer linear programming (MILP) to define hierarchichal clusters, and included multi-period optimization. The algorithm exploits similarity indexes to define the hierarchy of the optimal cluster for each block; the similarity parameters involve location, grade, rock type, and the desired mining cut shape. A Tabu search is then used to post-process and improve the solution. (Ben-Awuah & Askari-Nasab, 2011) solve the dig-limit problem using Mixed Integer Goal programming, whereby the goal equations act as hard constraints for the tonnage in each period. (Yavarzadeh, Abodallheisharif, & Neishabouri, 2014) implemented a MILP method that integrated mining direction and multi-period optimization. The method was based on the determination of "free faces", which defined those blocks that were accessible by the mine. Furthermore, the approach guarantees tonnage and grade targets for each period. The algorithm did not attempt to solve the clustering problem.

Heuristic methods are custom algorithms that generate an approximate solution. (Busnach, Mehrez, & Sinuany-Stern, 1985) generated a heuristic algorithm that used binary classification and block-by-block clustering punishments to approximate a solution. (Gershon, 1983) proposed a similar methodology involving the ability to partially mine blocks, and to use predefined boundaries as input to optimization. Allard, Armstrong, and Kleingeld (1994) proposed the use of morphological operators to find feasible dig-limits. Thus, a connectivity index could be established and diggable ore components could be found through simulation, combined with an appropriate morphological operator. Image processing was indicated as a research area to solve the dig-limit problem. In this case study, the authors noted that the two-morphological case generated better cleaning power than geostatistical simulations and was more applicable.

Richmond and Beasley (2004b) developed a multi-objective model (i.e., maximize pay-off and minimize financial risk) governed by a weighting factor and solved by a local search heuristic for open pit optimization, such that ore losses and mining dilution are incorporated. In a separate study, Richmond and Beasley (2004a) generated a 2-dimensional floating cone algorithm that searched the solution by overlaying a search ellipsoid onto the block model. This search ellipsoid was defined based upon the dimensions of the mining equipment, and would delimit whether a specific window of material would improve profit. This methodology essentially redefined the dig-limit problem as a 2-dimensional mining pushback problem, using a well-known heuristic search algorithm that guaranteed mineability. This approach appeared to be based on a 2-dimensional case of the Lerchs-Grossman algorithm. Clustering falls into two primary categories. Partitional clustering splits a large number of blocks into increasingly smaller mineable clusters. Hirearchical clustering begins with individual blocks, and creates clusters that increase in size until a steady state solution is reached.

Tabesh & Askari-Nasab, 2013 applied an agglomerative hierarchical clustering algorithm, which creates clusters out of individual blocks, and larger clusters out of these clusters. Each iteration checks against a set of tonnage, period, and shape constraints when deciding whether to merge clusters. Weintraub et al. (2008) generated aggregates of similar blocks, and then optimized the destinations using MIP across the clusters. This method is of particular interest because of the high quality solutions generated, and the high speed at which clusters are interpreted. By generating clusters that are smaller than the desired dig-limit size, the algorithm can quickly search through these destinations using practically sized components, thereby reducing the search size for the MIP to a non-NP-Hard level. K-means clustering commonly forms the basis of the agglomerative hierarchical clustering algorithms; an explanation of the methodology for a spatial model was carried out by Alsabti, Ranka, and Singh (1997). Furthermore, this approach laid the groundwork for the implementation of cluster sizing for the k-means algorithm, which is used extensively for the dig-limit problem.

Meta-heuristic algorithms carry out an approximation of a solution using some variation of a decision algorithm. The primary difference between heuristics and meta-heuristics is the acceptance methodology for changes. Meta-heuristic algorithms quantify the value of a change, and based upon the relative improvement, assign a probability of acceptance. This probability can be non-0 for negative changes in value, which allows the procedure to accept changes that worsen the current solution, but allow the algorithm to search the solution space more fully.

Norrena K (2001) pointed out that obtaining a smoothed image of a bench through consecutive implementation of erosion and dilation would not be sufficient because the ore value is ignored. The author proposed formulating the dig-limit problem as an optimization problem, solved by simulated annealing (SA). The primary problem with the use of meta-heuristics like SA is to find an initial solution and method-specific parameters. Wilde (2015) proposed the feasibility grade control method, whereby the problem is expressed as profit maximization calculated as the sum of economic values of ore and waste SMUs. Wilde uses a predefined set of acceptable mining shapes that the algorithm uses to generate an optimal solution by plugging in these shapes and evaluating the relative value of the change. Whereas previous authors generated dig-limits using full-sized blocks that had a destination for the entire volume, Chad T. Neufeld (2003) defined dig-limits as a polygon, the shape of which does not have to conform to the regularized grid of the grade control block model. Using SA to manipulate the x and y positions of the points that make up the dig-limits, it is possible to generate a mineable and optimal dig-limit that can take specific portions of the target blocks. Furthermore, this method allows for definition of smoother edges than those generated by other methods. The primary drawback of this method is the necessity for valid initial solutions that define the search ellipsoid of the algorithm, and must be of high quality and sufficiently varied to adequately optimize the dig-limit. This methodology would work well as a post-processing step for one of the algorithms described earlier.

Shaw, Khosrowshahi, Richmond, McKevitt, and Godoy (2009) generated a methodology to implement a stochastic optimization method for generating dig-limits. The heuristic method used is standard and has been outlined above. The novel portion of the interpretation was the inclusion of simulations into the process. By generating a separate dig-limit for each simulation of the grade-control block model, the authors could then generate a single, combined dig-limit. The authors also discussed the risk analysis advantages of such an approach, which could generate distribution curves for profit, grades, and physical parameters for each destination. The paper does not go so far as to propose a full stochastic optimization, and there is no mention of run times. The expectation is that since the dig-limits must be generated at least once for each simulation, then combined, the methodology could be lengthy.

Recent advances have been made in the formulation of the dig-limit problem. For example Kumral (2015) formulated grade control as a quality control problem and determined material destinations through loss minimization. Isaaks (2015) added a minimum width constraint into dig-limit optimization. In this approach, SMUs are clustered so as to be compatible with equipment digging capability. Thus, the risk associated with misclassification through dig-limit design is minimized. The SMUs identified as ore and waste through a grade control strategy and their grades are also used as input data. Grade control and dig-limit strategies can vary, depending on ore material heterogeneity and grade, commodity value, and the mining tradition of the enterprise.

2.3 Equipment Sizing

Equipment sizing is an intrinsic facet of the dig-limit problem. All literature reviewed thus far assumes a SMU size and clustering size. These constants, which form the basis of the grade control model, and the dig-line geometry are defined from the moment the mine is opened. SMUs have nothing to do with the size of the equipment, they are sized as a function of the geology and drill density of the deposit. The best practices scenario for sizing a SMU is to try various block sizes, and to fill these with grades using the nearest neighbor from drillhole data. The variograms of the SMU grades should be identical to those of the input drill holes. Finding the correct SMU size is a matter of trial and error, and should focus on attempting to find the smallest possible regularized size that will replicate the geostatistical distribution of the deposit. The clustering size is reflective of the equipment used; each shovel has a certain boom length and bucket size. These two parameters along with the class of shovel (i.e., retro, frontal, hydraulic, line, electro-hydraulic) will define the working size that a shovel must have to operate at capacity.

Best practices as (Bozorgebrahimi, Hall, & Blackwell, 2003) take into account the desired productivity of the deposit and various parameters related to the maintenance and downtimes of the equipment by size. Dilution is considered regarding the bench height; however, no analysis is carried out to define the impact of equipment size on the generation of dig-lines. Bozorgebrahimi et al. (2003) generated this line using a methodology such as best destination, and re-blocked the initial deposit into larger SMUs based upon the equipment used Bozorgebrahimi, Hall, and Morin (2005); (Samanta, Sarkar, & Mukherjee, 2002) studied the parameters affecting equipment sizing extensively. These parameters are summarized in Figure 3

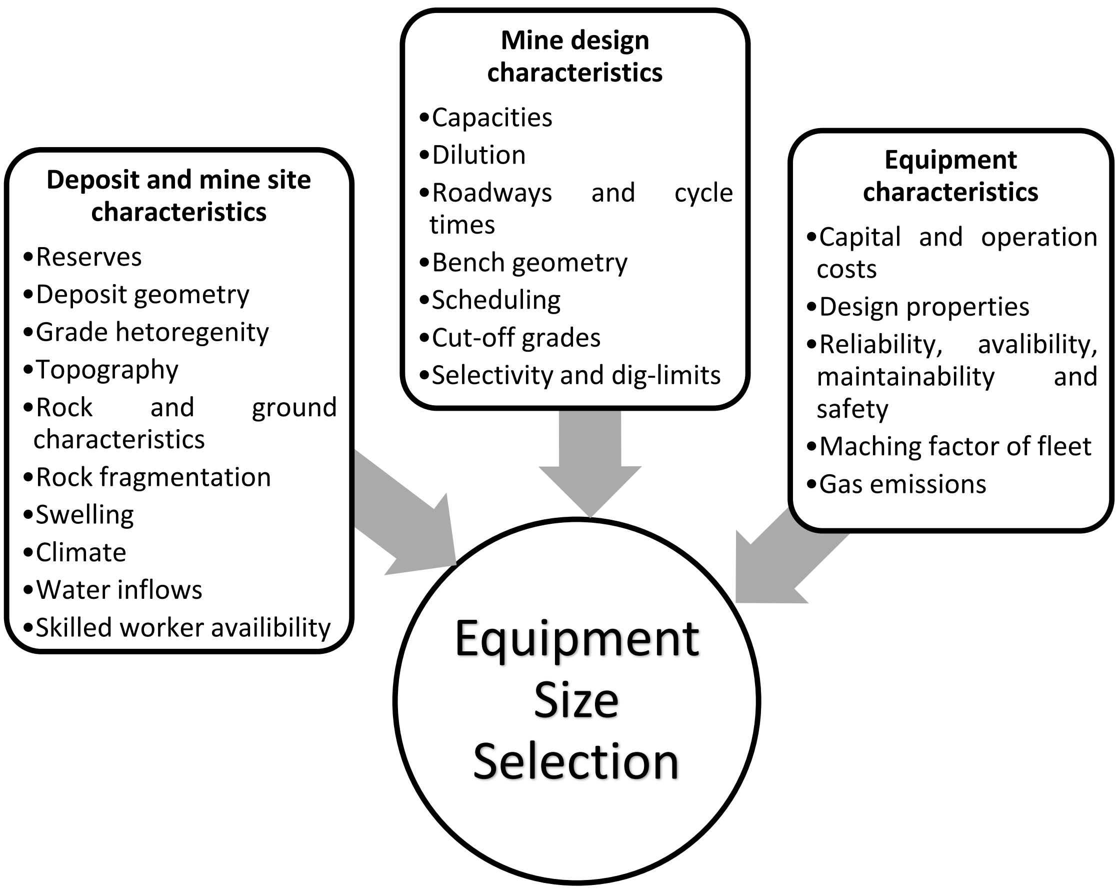


Figure 3: Variables of importance for equipment sizing

Bascetin (2004) included all factors related to pricing and productivity within a series of eigenvectors, to adequately optimize the exact equipment size that yielded the lowest cost per tonne. Extensive considerations included ramp angles, shovel and truck selection, maintenance, and useful life.

Equipment selection methodology revolves primarily around the efficient and effective movement of blocks. Equipment is chosen based on the most profitable size considering a block-by-block dilution calculation. The impact of equipment size on internal dig-limits has not been a topic of extensive research.

2.4 Genetic Algorithms

A GA follows the principles of evolution and survival of the fittest to develop solutions for complex, non-linear, and multi-objective problems (Murata & Ishibuchi, 1995). The GA starts with a set of premade or random solutions to a problem, and then ranks these solutions based on how well they solve the objective function, also known as "fitness". In this research, fitness refers to the profit of a feasible dig-limit in a bench minus a mineability deviation parameter. It will "breed" solutions together, randomly combining elements from two solutions. Solutions with greater fitness resulting from the mixing of the genes of the parents have an improved breeding probability. The breeding creates a new set of solutions called a "population". In other words, two solutions are perturbed or reconfigured, thereby generating a new solution. The generation of new feasible dig-limits through perturbation is made by genetic operators. This "generation cycle" continues until an objective has been reached or a predefined number of generations have elapsed. A GA searches the possible solution space by manipulating how children are created and survive to breed. The search is widened further by introducing genetic mutations, which occur randomly in the generation of children. If the mutations increase the fitness, the probability of survival improves, and the possibility of the new shape being disseminated throughout the rest of the population increases (Albuquerque & Mazza, 2001).

Like all meta-heuristic searches, a GA may get caught in local maxima. When this happens in a GA, it is referred to as an early or premature convergence

(Chatterjee & Bhattacherjee, 2011; Pandey, Chaudhary, & Mehrotra, 2014; Xu, Zhang, Zeng, & Chan, 2015). Avoiding local maxima is handled by adjusting the mutation rate and creating a predator that is less selective about killing lower fitness solutions. This approach reduces the pressures homogenizing the population, allowing for longer and more complete searches of the solution set (Thierens & Goldberg, 1994). There are several structural ways to avoid early convergence in GAs (Poli, 2001; Whitley, Rana, & Heckendorn, 1999): (a) the use of a varied initial population; (b) introduction of mutations to populations; and (c) splitting the population into sub-populations that are rarely bred together. For the purpose of this study, only a single population was considered. The GA employed high mutation probabilities, varied initial solutions, and careful calibration of the predator strength to minimize premature convergence.

This thesis presents a dig-limit optimization approach based on GA using MATLAB. In the next chapter, the methodology is introduced. Then, application of the algorithm on a nickel deposit consisting of a single level is demonstrated. Finally, the flexibility of the algorithm is tested by applying various useful features, such as mining direction, and multi-element, multi-destination, and selectivity sizing to show the robustness of the approach.

3. Methodology

A formulation of the GA methodology for dig-limit optimization has been implemented specifically for the mining industry.

3.1 Mining Context

In the formulation developed in this research, each feasible dig-limit is treated as a “chromosome” or “solution” and each SMU is treated as a “gene”. Each SMU contains information on nickel grade and destination. The initial destinations for the first generation are a mix of totally random solutions and optimal destination considering free selection. This combination of first-generation input dig-limits defines a clear direction of optimization, while maintaining a large search radius for the algorithm. The destinations will be bred and mutated until the value is optimized and the corresponding mineable dig-limit is created.

3.2 Steps

The algorithm developed follows the following steps (Figure 4).

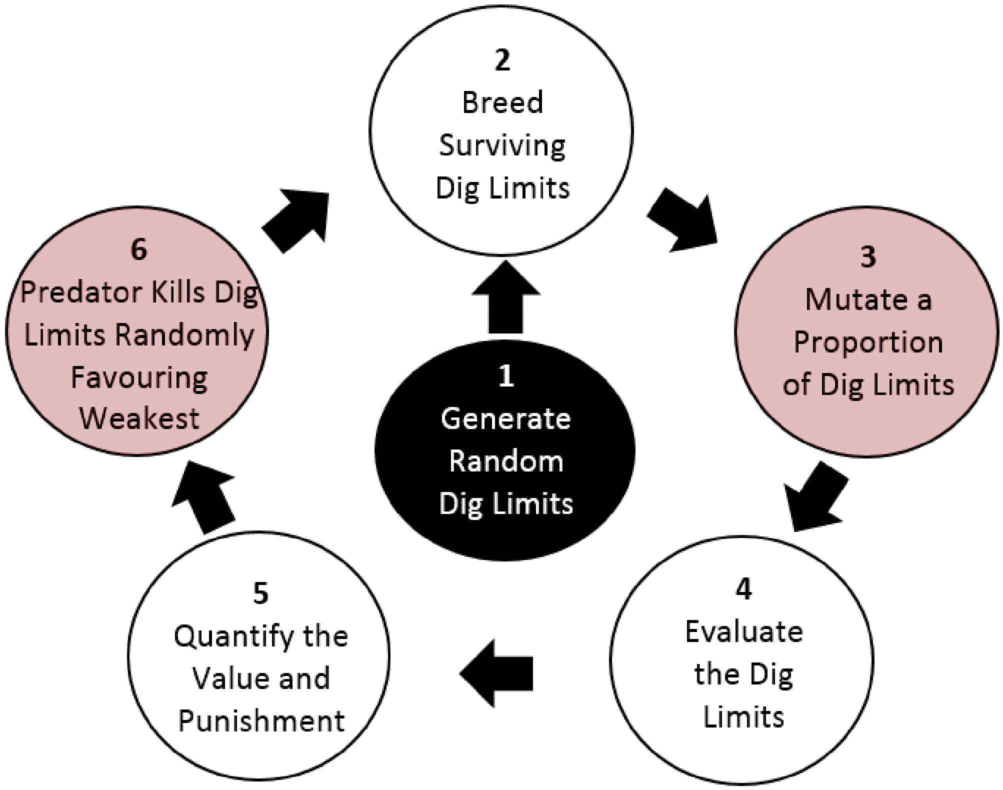


Figure 4: GA loop used in dig-limits optimization

1. The first generation population is randomly generated, consisting of a set of feasible dig-limits. These are generated autonomously and do not require intervention by the user.
2. To contextualize the following description, each dig-limit is a "parent" and each parent is composed of SMU destinations or "genes". Breeding combines two random parents to make a new child solution. The genes are selected at random from either parent solution, with a 0.5 probability per gene to be drawn from each parent. There is no mechanism to guarantee that an equal proportion of genes will be contributed by each parent. The

breeding process in this case will triple the new population, creating three children for each parent.

3. A child solution is a new dig-limit created from the SMU destinations of the two selected parent solutions. The inserted mutation will follow the clustering size to avoid the creation of unfavorable changes, as interpreted by the clustering size of the mining equipment. The child schedules in early generations are very likely to receive a mutation, whereas those of later generations are not. The mutation randomly selects a position to insert a line of blocks equal in width or height to that of the clustering size. In this case, a mutation can be either be 3x1 blocks, or 1x3 blocks. Since the mutation respects clustering size and all blocks are assigned the same randomly chosen destination, the probability of generating valid mutations is increased.
4. The "preparer" function analyzes the neighbours of each block and assigns a clustering deviation based on the respect given to the clustering size. Perimeter clusters that do not have sufficient widths are assigned an incremental punishment, which is later transformed into fitness deviation by the quantifier.
5. The quantifier asses the economic value of the block, as well as clustering deviation. By reducing both optimization targets to a single number, a single variable predator can be used to select the surviving members of a

schedule. This single value is the fitness, which is the profitability of the dig-limit minus the diggability deviations.

6. The predator uses a weighted roulette wheel selection method. The set of feasible dig-limits are ranked from best to worst fitness, where each inferior schedule is slightly more likely to be selected than its predecessor. A selective predator will have a strength increase between each successive evolution, thereby killing schedules of lower fitness more consistently. A less selective predator increases the probability less drastically, killing schedules with lesser discrimination. The predator strength begins as a low value, and is linearly increased over the course of the GA. This guarantees a degree of flexibility at the beginning of the meta-heuristic search and eventually forcing convergence.

3.3 Fitness (Objective) Function

To evaluate the fitness of each dig-limit, the profit from each block is computed and penalties from clustering problems are then applied. The clustering problems are quantified as the deviation from the correct clustering size. A general clustering approach is illustrated in Figure 5. It is important to understand that using the 4-direction clustering method should not yield any deviation penalty unless a dig-limit specifically violates clustering constraints. This directional check is difficult to implement with linear methods, since the clustering penalty generated in each direction is a function of whether the clustering is respected in the opposite direction. This reason and those related to run times justify avoiding exact methods to formulate this optimization problem.

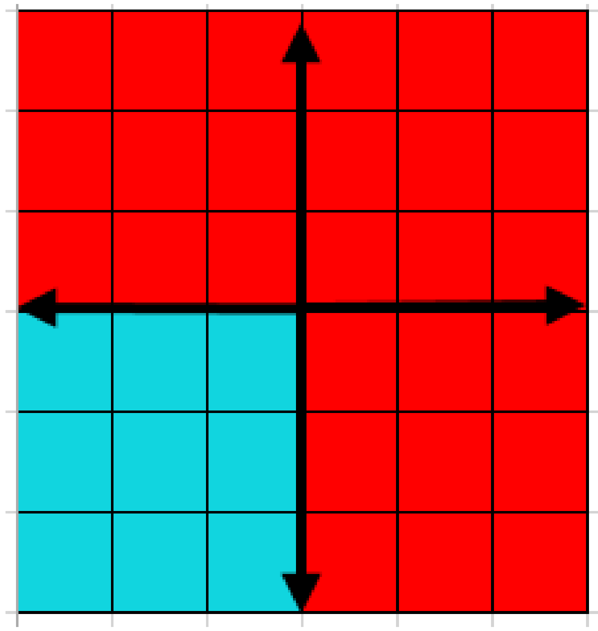


Figure 5: Corner clustering case. 3 x 3 m blocks

3.4 Optimization Model in Linear Form

The general formulation of the dig-limit optimization is calculated by the following equations, presented as an approximation of the genetic method. The objective is to maximize profit under the constraint that SMUs falling into directional search radius should be sent to the same destination, and thus

$$\sum_{x=1}^{x\_max} \sum_{y=1}^{y\_max} \sum_{d=1}^{D} P_{xyd}\, b_{xyd}$$

Subject to:

a. All SMUs must be produced and sent to a destination:

$$\sum_{d=1}^{D} b_{xyd} = 1 \qquad x = 1, \dots x\_max \text{ and } \; y = 1, \dots \; y\_max$$

b. SMUs located northwest of the SMU under consideration are sent to the same location:

$$b_{xyd} = b_{(x-k)(y-l)d}$$

$$d = 1, \dots, D; \; x = 1, \dots, x\_max; \; y = 1, \dots, y\_max; \; k = 1, \dots, XDir \text{ and } l = 1, \dots, YDir$$

c. SMUs located northeast of the SMU under consideration are sent to the same location:

$$b_{xyd} = b_{(x-k)(y+l)d}$$

$$d = 1, \dots, D; \; x = 1, \dots, x\_max; \; y = 1, \dots, y\_max; \; k = 1, \dots, XDir \text{ and } l = 1, \dots, YDir$$

d. SMUs located southwest of the SMU under consideration are sent to the same location:

$$b_{xyd} = b_{(x+k)(y-l)d}$$

$$d = 1, \dots, D; \; x = 1, \dots, x\_max; \; y = 1, \dots, y\_max; \; k = 1, \dots, XDir \text{ and } l = 1, \dots, YDir$$

e. SMUs located southeast of the SMU under consideration are sent to the same location:

$$b_{xyd} = b_{(x+k)(y+l)d}$$

$$d = 1, \dots, D; \; x = 1, \dots, x\_max; \; y = 1, \dots, y\_max; \; k = 1, \dots, XDir \text{ and } l = 1, \dots, YDir$$

f. Binary variable:

$$b_{xyd} = \begin{cases} 1 & if\ SMU\ located\ in\ x, y\ is\ sent\ to\ destination\ d \\ 0 & otherwise \end{cases}$$

where *P* is profit obtained by sending the SMU positioned at *x* and *y* coordinates to destination *d*; *D* is the number of destinations; *x_max* is the number of SMUs in *x*-direction of the bench; *y_max* is the number of SMUs in *y*-direction of the bench; *XDir* is the number of SMUs corresponding to the equipment operation radius in *x*-direction; and *YDir* is the number of SMUs corresponding to the equipment operation radius in the *y*-direction. This directional check constraint is added into the objective function in the form of a clustering penalty, which is basically Lagrengian parameterization. This is used to reduce computational time and facilitate the GA formulation.

3.5 Formulation

This GA formulation includes a Lagrangian multiplier for clustering penalty. Given this application of the GA, the predator operates solely on the fitness, which therefore includes the clustering deviation parameter as part of the fitness optimization function.

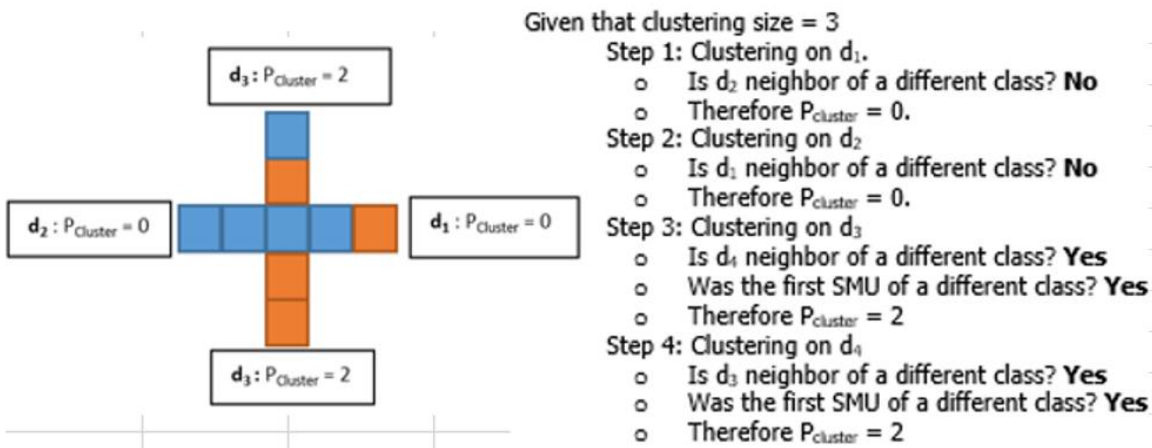
$$\boldsymbol{MAX}(\boldsymbol{Dig} - \boldsymbol{Limit\ Fitness}) = \sum_{x=1}^{x\_max} \sum_{y=1}^{y\_max} SMU\_Revenue_{x,y} - SMU\_Process\_Cost_{x,y} - Clustering\_Penalty_{x,y}$$

$$\boldsymbol{SMU\_Revenue}_{x,y} = Ni_{recoverable\ x,y} * Price$$

$$\boldsymbol{SMU\_Processing\_Cost_{x,y}} = f(x)_{x,y} = \begin{cases} 0, & destination_{x,y} = Waste \\ P_{processing}, & destination_{x,y} = Ore \end{cases}$$

$$\boldsymbol{Clustering\_Penalty_{x,y}} = \sum_{d_1}^{d_4} \sum_{1}^{Cluster\_size} f(x)_{x,y} = \begin{cases} SMU_{x,y} == SMU_{target}, & x = 0 \\ SMU_{x,y} \,!= SMU_{target}, & x = 1 \end{cases}$$

- The clustering calculation is only carried out in the opposite direction from neighboring SMUs of a different class.
- *d* represents the potential check directions defined as north, east, south, and west.
- $d_{n=1\rightarrow 4}$
  - d_1= SMUs on the x+ direction
  - d_2= SMUs on the x– direction
  - d_3= SMUs on the y+ direction
  - d_4= SMUs on the y– direction
- When iterating from 1 to cluster size, if a value returns as 1, all further values in that direction are also 1.
- Example:



The formulation of the fitness function outlined above is highly flexible. The algorithm can optimize any number of destinations for any number of optimization grades and parameters. Given that the profit of a block is a function of its type, destination, and grades, an equation can be made to represent the potential values of the block. These values form the basis of the optimization algorithm; in fact the values could be pre-calculated to further optimize the running time of the GA. Furthermore, the search distances $d_{1..4}$ are variable, which represents the capability to define a mining direction and apply different mining geometry. Tonnage targets are easily implemented by manipulating the equations within the quantification function: the tonnage of blocks to each destination can simply be summed while calculating the other variables, and then punished if the tonnages deviate from targets.

4 Case Study I: Dig-limit optimization through GA

A case study was carried out to demonstrate the performance of dig-limit optimization by the GA relative to the hand-drawn method.

4.1 Economic Model

The GA was run for 25 iterations using the following input parameters. All dollar values are Canadian dollars.

- Mineral Value = $1.333/lb.
- SMU Revenue = recoverable nickel × mineral value.
  - The recoverable nickel amount is the post-recovery quantity of nickel in each SMU. This was specified to obscure the data source.
- SMU processing cost = $30 (for ore) and $0 (for waste).
- Mining cost is set to 0 for this case study, but is flexible and variable within the algorithm.
- Recovery = the recovery applied to SMU grades. In other words, grades are recoverable grades.
- Clustering size = 3 x 3 SMUs
- Clustering penalty (Lagrangian multiplier) = $8 per unit of deviation from regularized clusters. This value should be very high to avoid any unfeasible solution to guarantee diggability at all points, without making it the only optimization parameter of note.
- Predator strength: constant strength of 1.01
- Mutation probability: linearly scaled from 0.2 to 0 by generation.

The parameters attributed to the GA are the following.

- <u>Cluster size:</u> Clusters are measured by the number of SMUs of the same destination in one direction. The directions are the four directly neighboring SMUs in plan view.

- Population size: Determining the size of population requires careful balancing. Too small a population could guide the GA to poor solutions from an insufficient search radius, whereas too large a population will expend too much computing power on each generation, yielding impractically long runtimes. There must be enough members in the population to provide diverse chromosomes while maintaining reasonable computation times.
    - Population sizing: 600 parents. These breed up to 1,800 children before being culled by the predator.
- Free selection solutions: Inserting free selection solutions to the initial population improve runtimes. However, too many will narrow the searchable solution set and result in convergence to local-maxima. A total of 5% of the initial population was used as free selection solutions.
- Predator strength: This is the discrimination level at which the predator selects the weaker solutions for death. It is set at a value of 1.01.
- Mutation probability: A high mutation probability will cause more mutations to occur in the solutions. This will increase the amount of the solution set explored, but will make it more difficult for generations to converge on a solution. Exploring the solution set early is ideal in a GA. However, in later generations as the final solution is being refined, the mutations are ideal. To achieve this, a downward linear scaling mutation probability was used.
    - Mutation probability selection: A mutation line was selected through trial and error. The mutation linearly decreases from 0.2 to 0 at generation 700.

A nickel bench was used to test the performance of the GA. There are two destinations (processing and waste dump) for a single ore type. It was assumed that no stockpiling or blending was being carried out. Each destination type has an associated production cost and recovery for each destination. The GA was run on a dataset of 3,150 SMUs, the grade of which can be seen in Figure 6.

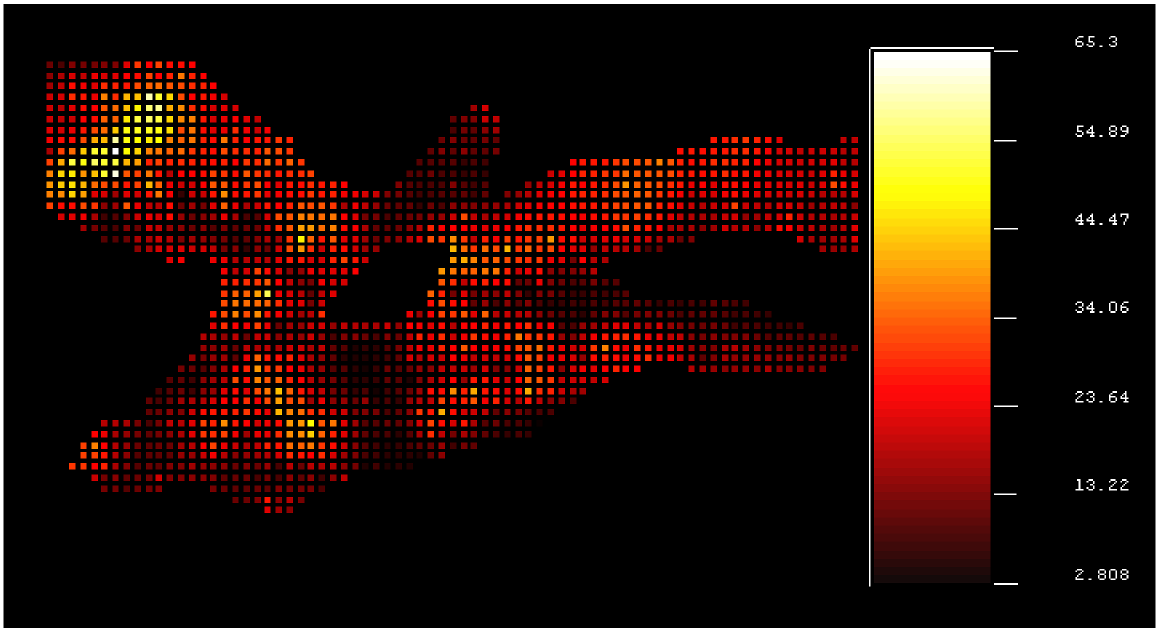


Figure 6: Ni grades of grade control block model

Given grades that range from 1 to 65 lb of nickel per tonne and a processing cost per tonne of $30, the COG is derived as follows.

$$COG = \frac{P_{Processing}}{P_{Mineral}} = \frac{\$30}{\$1.333} = 22.51\ lb/tonne$$

Figure 7 displays the COG classification of each SMU, which can also be interpreted as the free-selection destinations. Since Ni grades are recoverable, it is not necessary to use the recovery.

Figure 7: Cutoff grade model

4.2 Results

A set of 25 solutions was generated using the GA, and compared to describe the sensitivity of the solutions generated. The optimal dig-limit generated can be found in Figure 14.

From comparison to the COG model, it appears that the dig-limits closely followed the expected limits based on COG boundaries, while respecting mining clustering constraints. Some clustering deviations occurred along the perimeter of the bench: these are unavoidable and are caused by the geometry of the deposit. Typically, only internal clustering can be completely avoided, whereas border clustering can only be minimized.

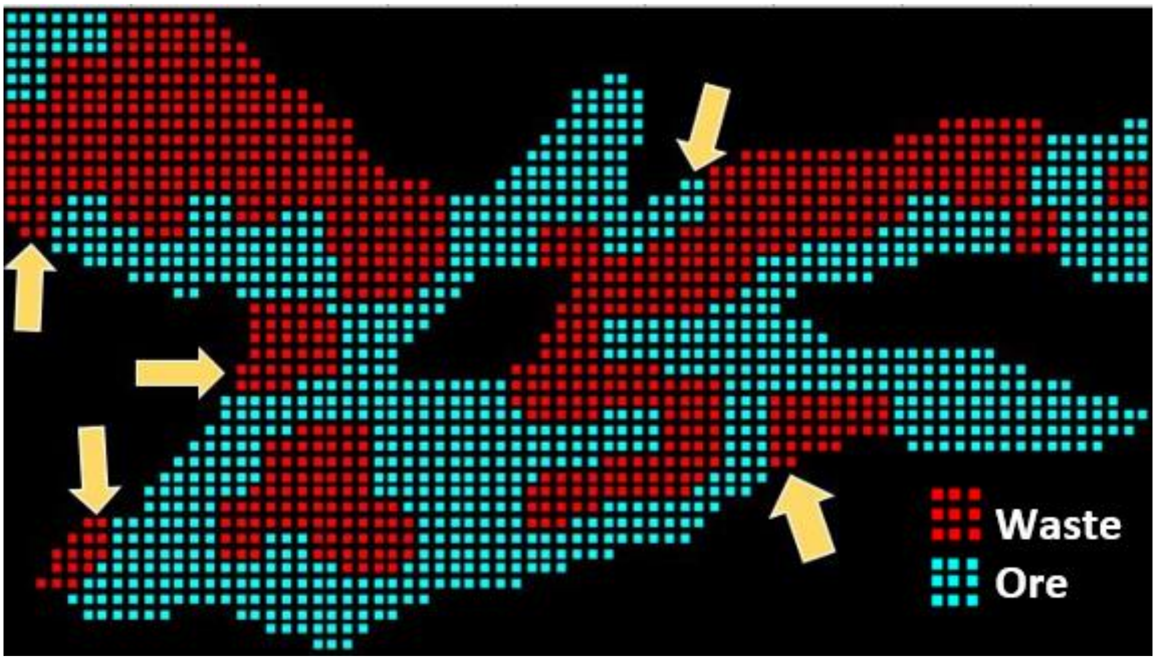


Figure 8: Best dig-limit found by genetic algorithm

4.3 Solution Paths of Multiple Iterations

The development of the dig-limits over 400 generations can be seen in Figure 9, which displays the most fit dig-limit per generation. The dig-limit from the first generation is the free selection solution from the COG model. While profitable, this dig-limit is not operational because of severe clustering issues. Generation 100 shows that internal SMUs between waste and ore zones have been homogenized, but clustering issues exist on perimeter SMUs between different destinations. By generation 200, almost all of the clustering issues on SMUs between classes have been eliminated. At generation 300, clustering issues have been resolved, but when compared to the free selection solution, there are some zones where misclassification occurs. By generation 400, the full discrimination of waste between ore SMUs in zone A has been correctly identified (Figure 9).

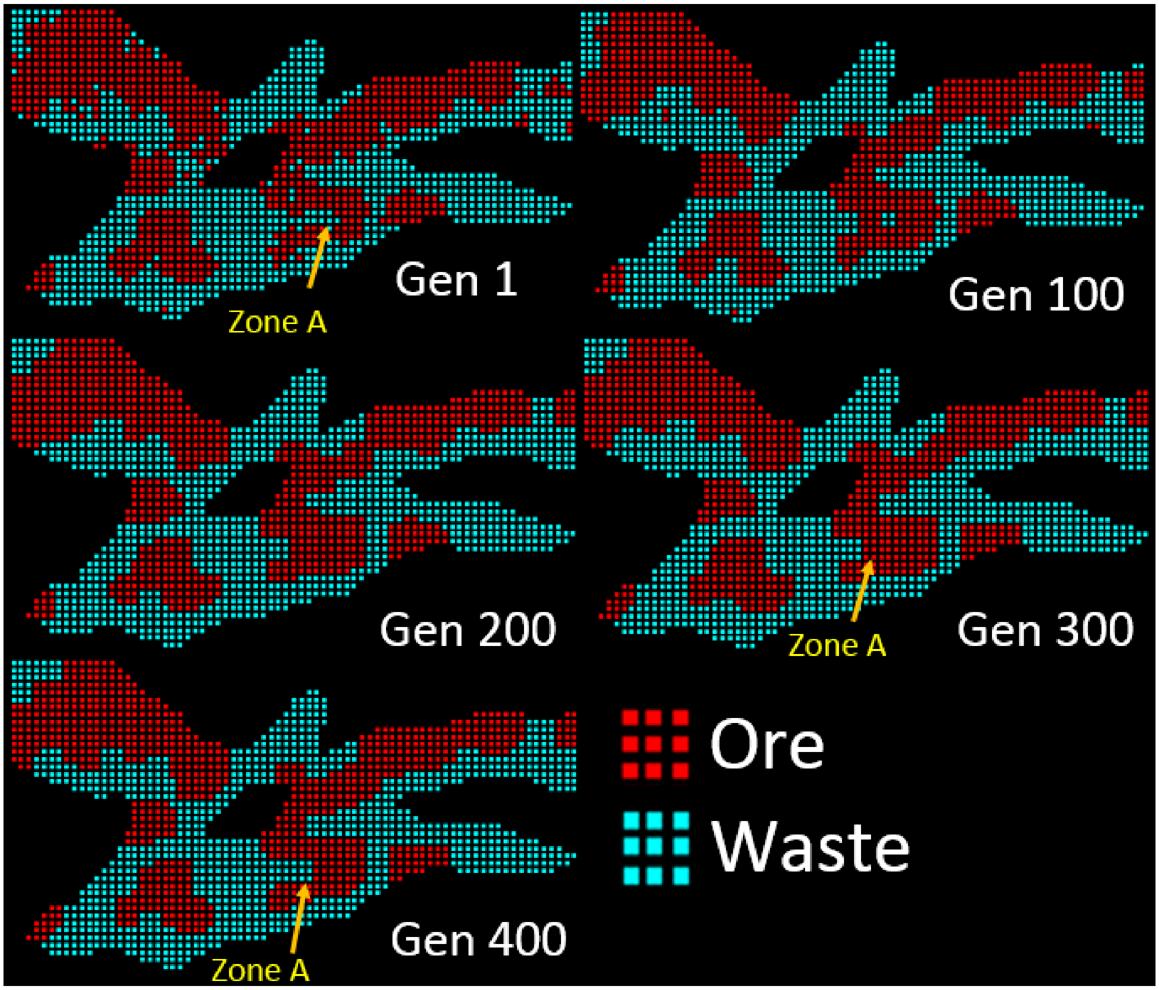


Figure 9: Evolution of the solution

Further analysis of generational fitness and profit development can be seen in Figure 10, which shows the best solutions per generation. Profits generated by feasible solutions are generally lower than profits COG-based solutions, because a solution obeying model constraints is more important. Once reasonable boundaries have been selected within the population, the optimal solution is then found by optimizing material recovery, while maintaining diggable limits. The result of this process can be identified as the rebound in profit between generations 100 and 400. Furthermore, a comparison between generations 200 and 400 within Figure

9 shows that—while generation 200 exhibits strong clustering—the highest value dig-limit boundary has not been located.

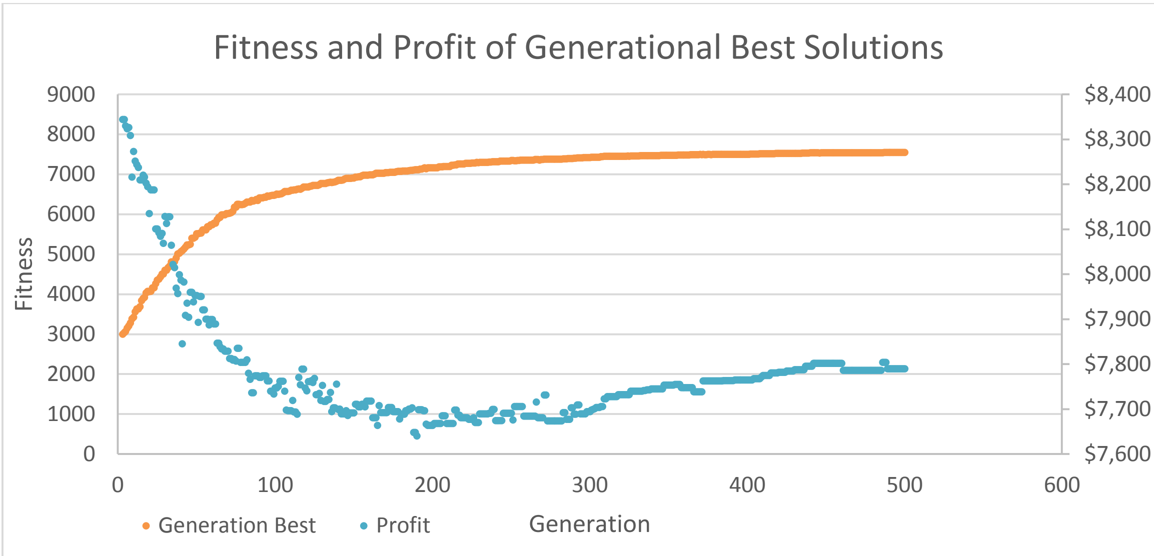


Figure 10: Fitness progression

4.4 Sensitivity Analysis

Statistical analysis of 25 runs carried out to model the distribution of the solution fitness showed that the results of the GA are reliable (Figure 11). The range of the fitness value is small and weighted towards the upper limit. This shows that, although the GA process does not always reproduce the same solution, it approximates the optimal solution with a high degree of consistency.

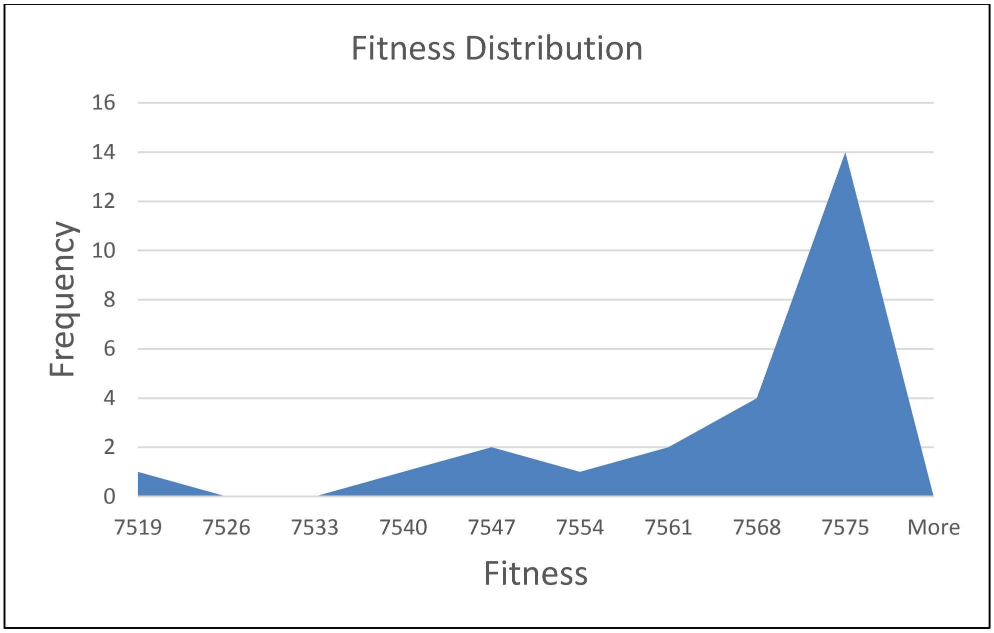


Figure 11: Statistical distribution of 25 solutions

To further explore the variability between the dig-limits, the worst, median, and best solutions were visualized in Figure 12, Figure 14, and Figure 8, respectively. The differences among these solutions appear to be in two zones, identified as zones 2 and 3 in Figure 12. The best solution closely follows the median solution for zone 2 and 3, but differs in zone 1. Each zone is well clustered; therefore the variation in fitness results from profitability considerations. Hence, these questionable zones are local-maxima, which have trapped the solution due to clustering considerations.

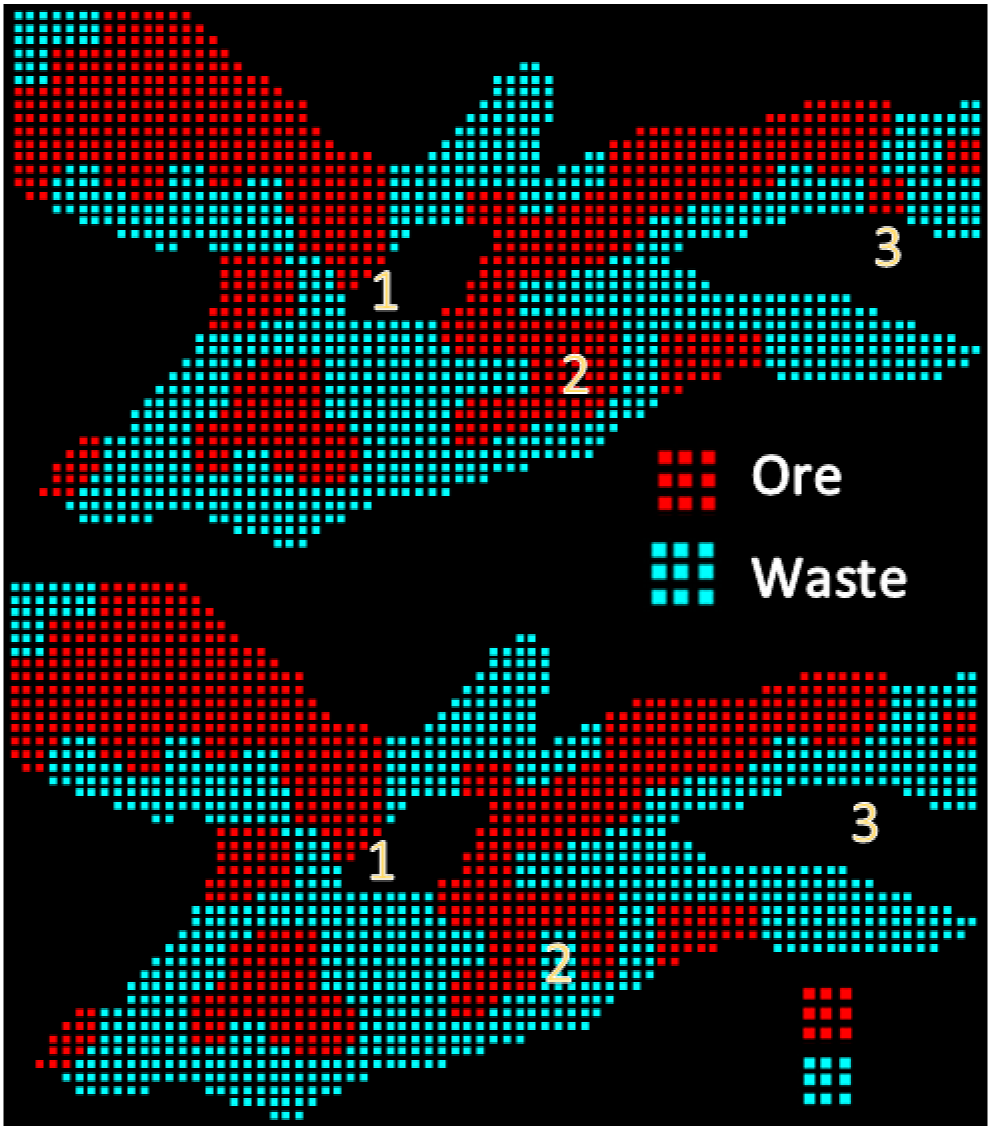


Figure 12: Median (top) and worst (bottom) GA dig-limit from 25 runs

4.5 Comparison to Hand-Drawn Results

A dig-limit was hand-drawn to compare with GA dig-limits. A comparison of benchmarks between worst, median, and best GA dig-limits generated by both methods shows that the GA outperforms the hand-drawn method (Table 1).

Table 1: Profitability analysis between four models of dig-limits

Results	Hand-drawn	Worst	Median	Best
Fitness	6,832.00	7,518.36	7,568.67	7,575.49
Profit	7,832	7,854	7,825	7,832
Number of SMUs Misclassified to Waste	65	80	87	89
Number of SMUs Misclassified to Ore	65	66	71	69
Number of Ore SMUs	798	774	772	768
Average Ore Grade	29.87	30.12	30.11	30.16

A key factor to understand from the analysis in Table 1 is that hand drawing dig-limits focuses on optimizing the proper SMU by SMU classification. Indeed, based on COG classification, the hand-drawn dig-limit sends more SMUs to the correct location. However, by viewing profit and fitness, the GA is clearly able to reproduce similar profits while creating more diggable limits. Visually comparing the optimal dig-limit generated with GA (Figure 14) with the dig-limit drawn by hand (Figure 13) indicates that the GA dig-limit is smoother, and contains far fewer clustering deviations. Particular improvement can be seen in the bench perimeter, where the GA minimized deviation. By comparison, hand drawing focused primarily on internal boundaries.

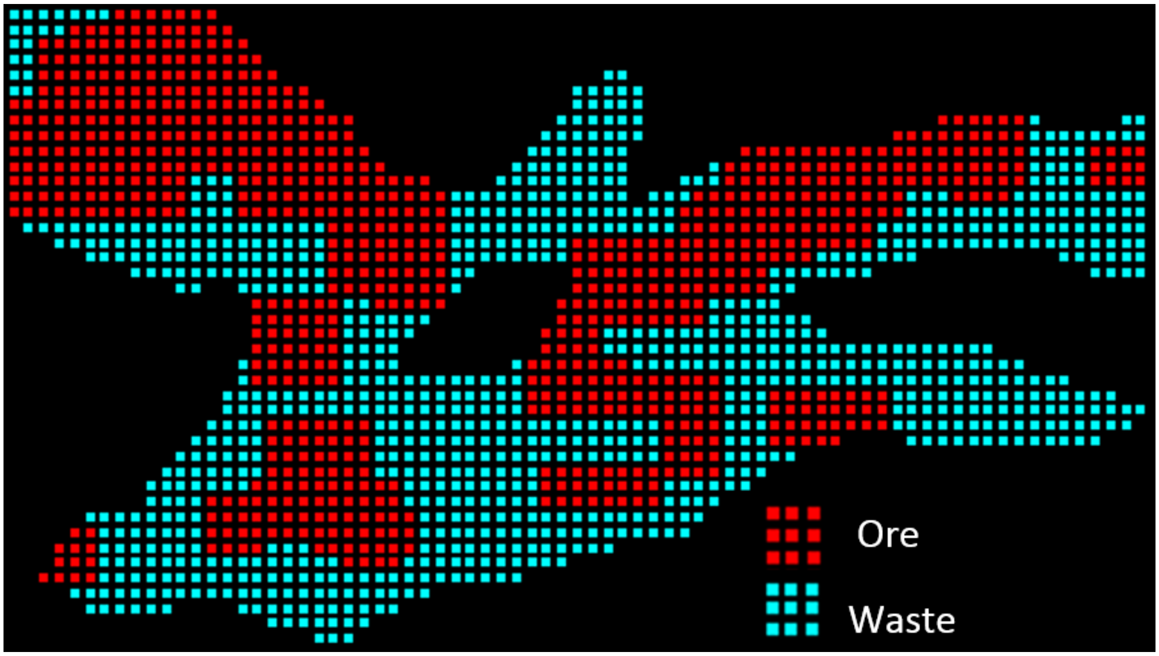


Figure 13: Hand-drawn dig-limit

Figure 14: Best GA dig-limit

4.6 Discussion

In this chapter, a dig-limit optimization based on a GA was presented such that near optimal dig-limits were created from random and automatically generated initial solutions. The methodology outlined eliminates the necessity for the user to hand draw initial solutions for the algorithm. The GA generated mineable ore and waste boundaries by focusing on SMU clustering and generating practical dig-limits. It did this while maximizing profitability within the dig-limits and improving the average grade of mined material to the ore stream. The GA outperformed the hand-drawn dig-limits. With additional constraints not considered here—such as grade targeting or geological risk—the GA is likely to outperform hand-drawn limits, although run-times are expected to lengthen.

5. Case Study II: Directional Mining Dig-Limits with Sub-Blocking

A key flexibility necessary from a dig-limit generation perspective is the ability to define mining directions. A mining dig-limit with a direction is defined as one where the geometry is larger perpendicular to the mining direction, than parallel to it (Figure 15). Shovels operate perpendicular to the mining direction, carrying out “pushbacks” within the digging zone. As the shovels near the end of one dig-limit, they either move forward, double back, or move into new material with a new destination. Mining direction flexibility is of particular importance when using frontal and electrohydraulic shovels, because of the increased ease with which these shovels move parallel to the mining wall. This case study seeks to guarantee that the GA works as expected in a highly variable, poly-destination, multi-element deposit, while respecting the mining direction.

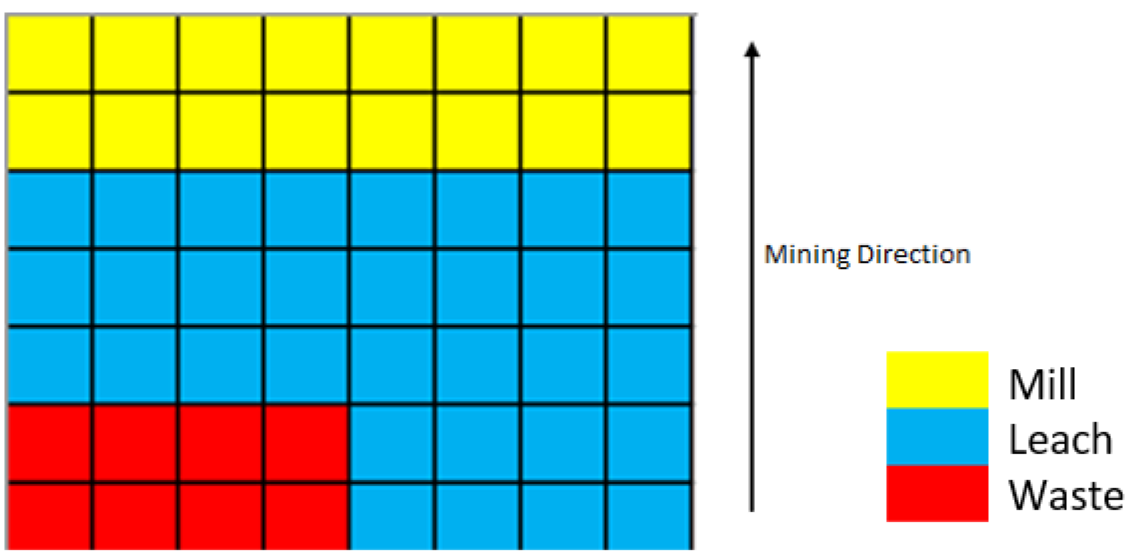


Figure 15: Mining direction dig-limit geometry

5.1 Economic Model

The case study deposit is an iron mine with five destinations and the target grades outlined in Table 2. Each of the five potential destinations have strict and complex grade target constraints, making this a high difficulty dig-limit to draw by hand. Table 3 outlines the specific mining parameters that must be respected. The grade-control block model comprises 5 x 5 x 5m blocks. Therefore, clustering applied to this deposit will be 5 x 2 blocks, with the greater length perpendicular to the mining direction (south–north).

Table 2: Target grades

Material Type	Fe(%)	SiO2(%)	Al2O3(%)
Direct-shipping iron ore (DSO)	55	<14	2
SP1 - High Silica	>40	>15	<2.5
SP2 - High Alumina	>45	<10	>4
SP3 - Low Grade	30	>20	3
Waste	-	-	-

Table 3: Mining parameters

Equipment Size	100 t on Trucks
Bench Height	10 m
Minimum Mining Width	25 m
Material Destinations (5)	DSO SP1 – High Silica SP2 – High Al2O3 SP3 – Low Grade Waste Dump
Quality Targets	*Based on Economic Cut Off and Quality Constraints*

The grade control block model was generated through a set of sample drill-holes (Figure 16) overlaid upon the GA-generated dig-limits. These drill holes came from an operational iron mine: the information was provided directly from the operation and reflects a highly realistic scenario, based on an existing deposit and machinery. The grade control model details can be found in Figure 17, to Figure 20. The complexity of the grade distribution is of particular importance for this deposit because of the sheer number of possible destinations and the importance of not deviating from targets. Any stockpile deviations can result in massive losses to the mine and could harm client relationships. Therefore, the highest priority optimization factor is maintaining chemical targets.

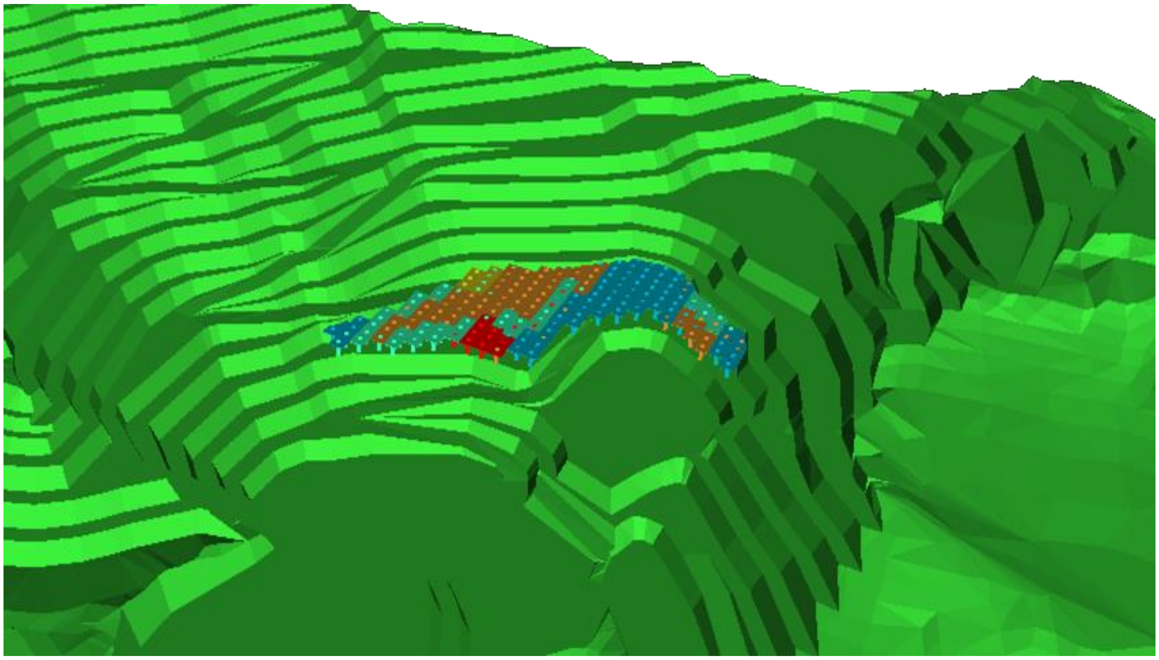


Figure 16: General view of the target blast

Figure 17 displays the iron grade distribution within the bench. There are three high-grade iron pockets, with medium- and low-grade iron situated between them. The high variability of the grades, ranging from 66.9 to 6.3% iron, adds considerable complexity to the issue. Classifying purely by best destination, while giving priority to the ore clusters, could result in a low-grade block being taken. If a block with 6.3% iron content is mined, the total profit for that cluster will be reduced severely.

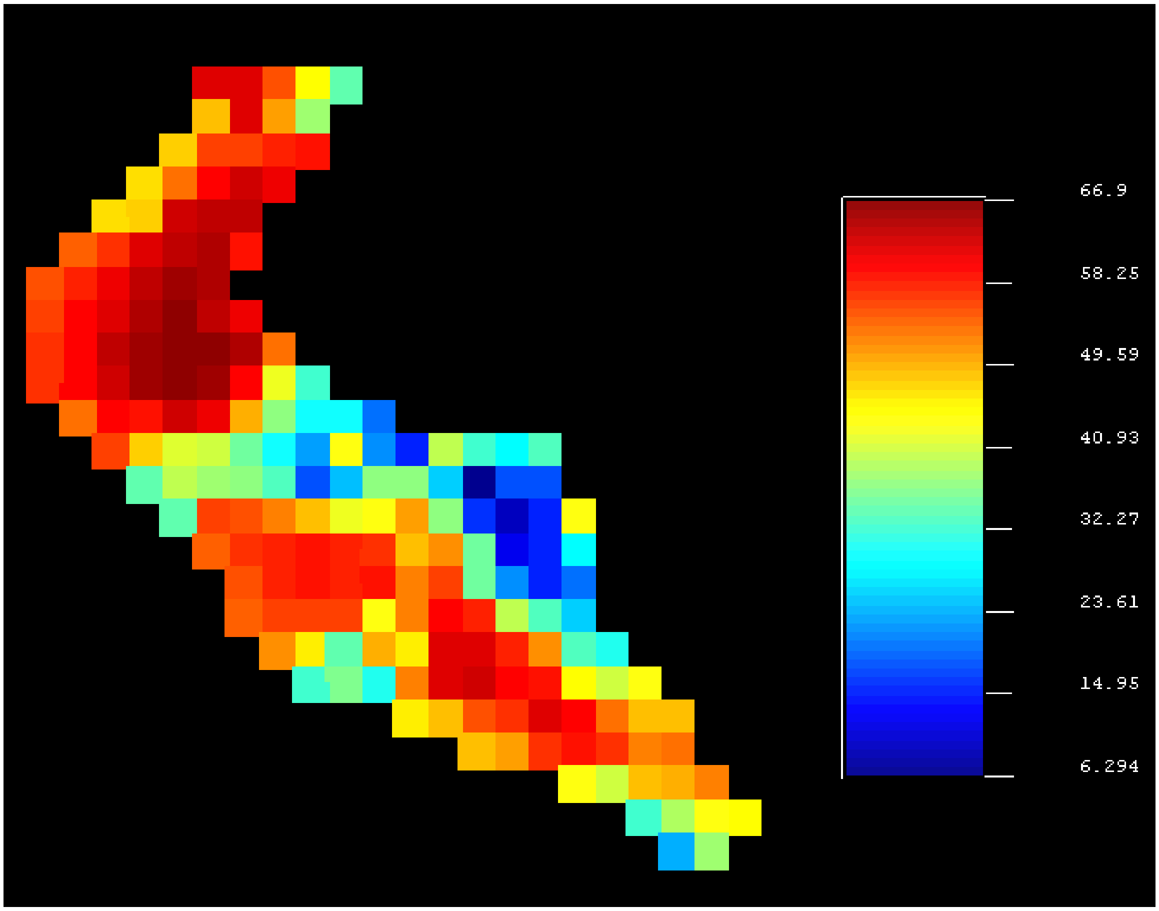


Figure 17: Fe grade distribution within the blast

The silica grade distribution is relatively homogeneous within the interior of the deposit, with only some high silica blocks in the very center (Figure 18). However, the perimeter of the bench contains multiple high silica blocks. The silica content must be carefully controlled, particularly to the direct-shipping iron ore (DSO) option. It appears that low silica grade blocks have above average iron grades. Therefore, many of the high silica blocks will be classified as waste.

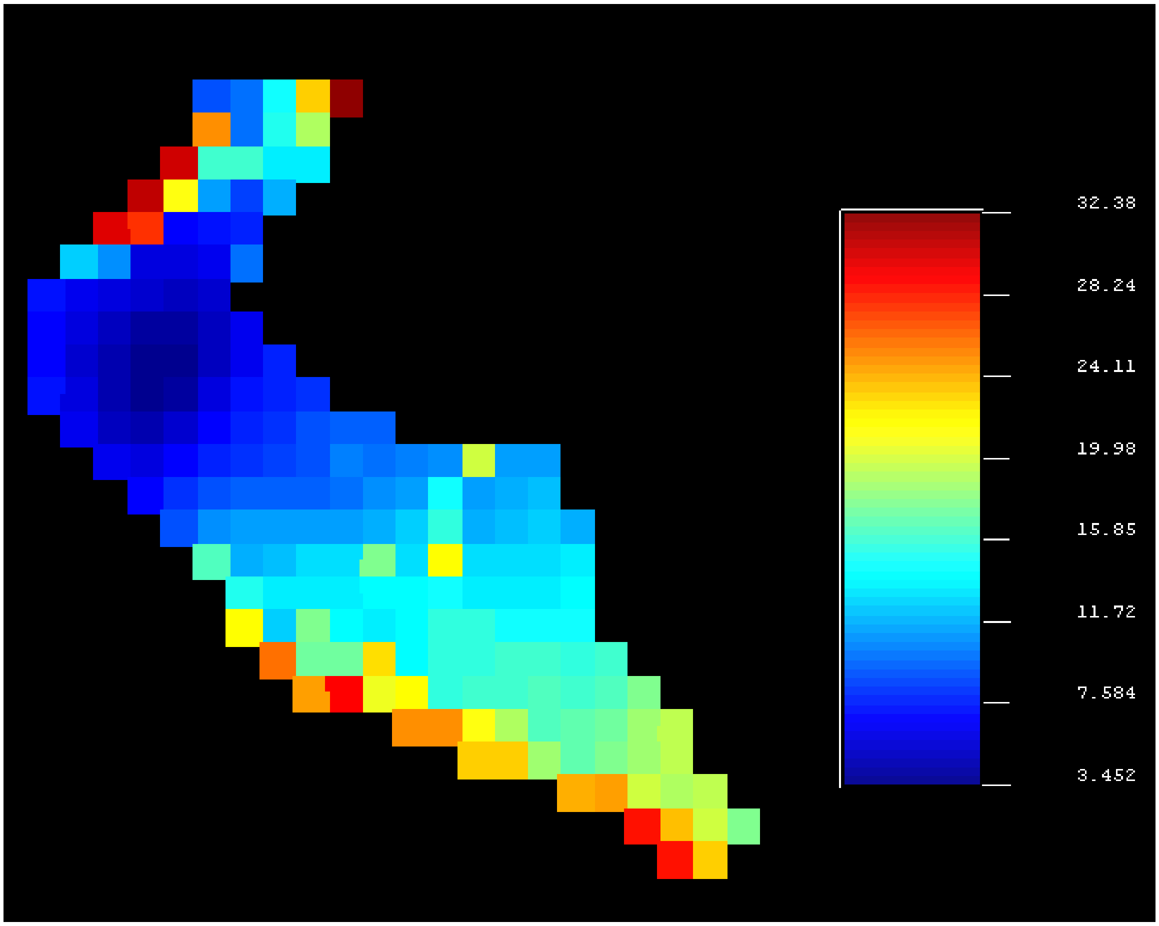


Figure 18: SiO_2 grade distribution within the blast

The AlO_2 grade is inversely related to iron grade (Figure 19). This is particularly apparent in the low-grade blocks that divide the center of the bench. Alumina blocks may be sent to the high alumina stockpile, but are considerably less valuable than the DSO and high silica blocks. Therefore, the alumina sent to each of the stockpiles must be maximized, without exceeding targets. The rest of the material must be sent to the alumina stockpile.

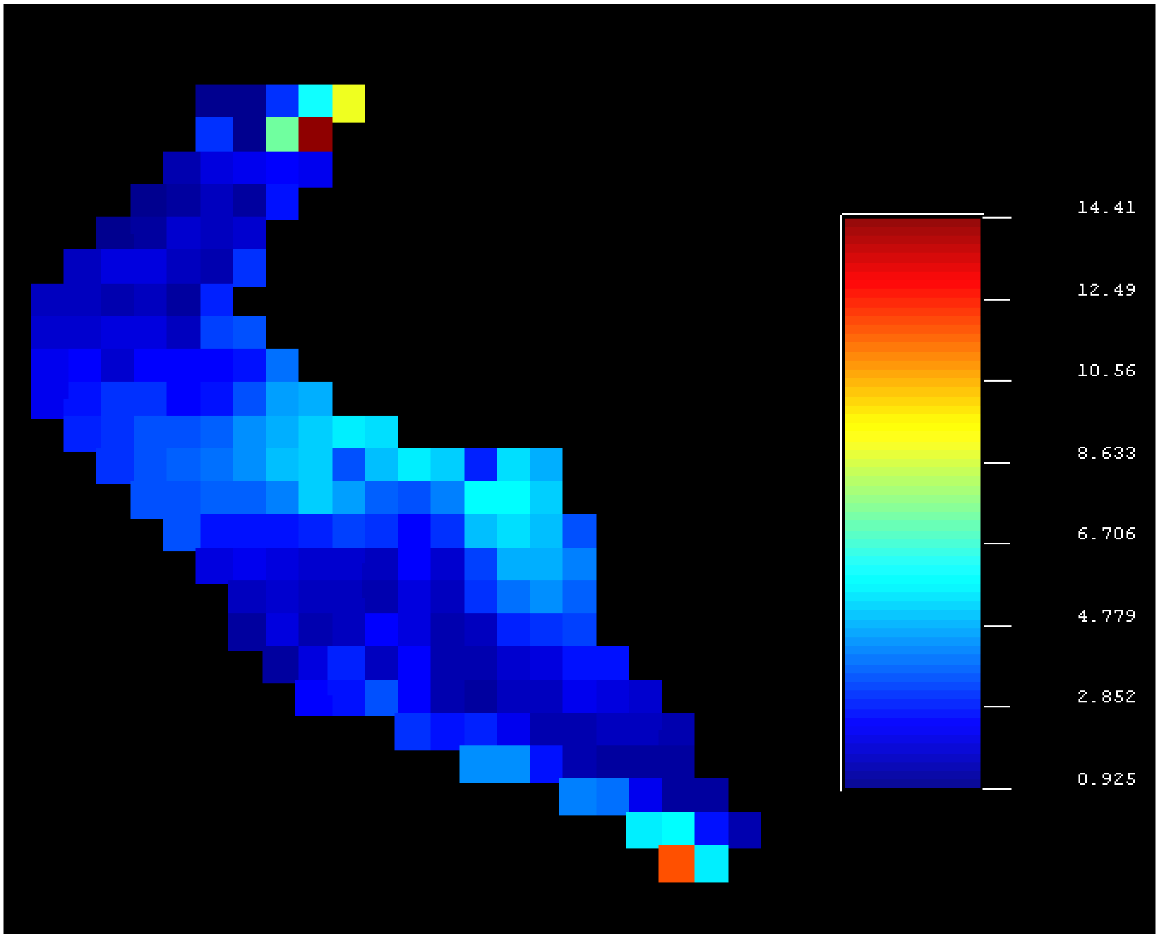


Figure 19: Al_2O_3 grade distribution within the blast

Figure 20 displays how partial blocks are handled by the algorithm. The block model that is fed into the GA must be regularized and maintain full-sized blocks. However, some of the north-west blocks have partial tonnages. This is reflected directly in the density, instead of maintaining a fill factor. These wall blocks must be mined with care to maintain the integrity and design expected from the pit wall. Furthermore, high silica blocks are very high density, while DSO blocks tend to be less dense.

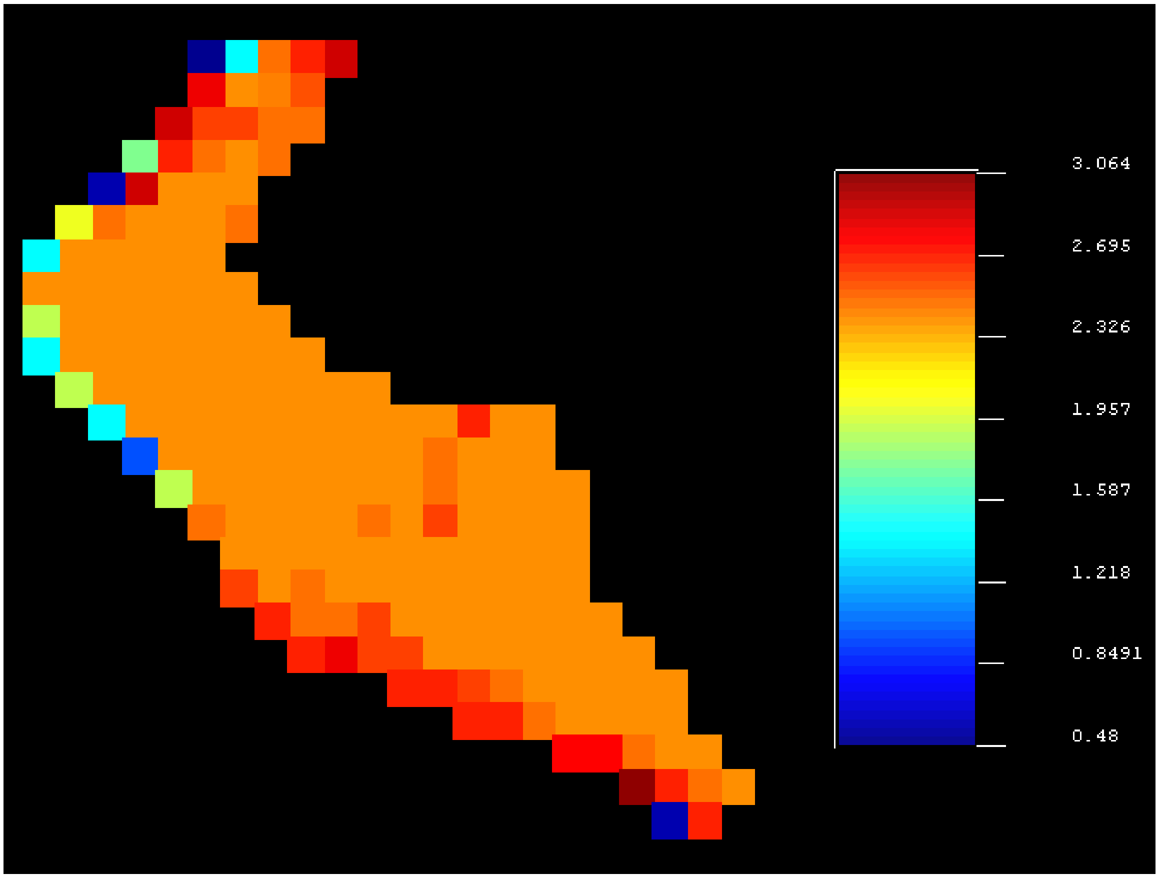


Figure 20: Density distribution across the input bench.

Once the grade-control block model was generated and the grades were determined, it was possible to evaluate the value of each destination based on the grades. The profit of each block was calculated as shown in Equation 6. The deviation punishment factor was applied on a per tonne basis, this factor ensures that the GA respects the chemical constraints of the destination of each block. This factor must be scaled sufficiently high that the blocks are not sent to the wrong location, but sufficiently low to ensure that the GA retains some flexibility when misclassifying will improve the recovery by a large margin. Please note that there is no processing cost associated with the profit equation. This mine is a DSO Iron mine, and therefore only incurs mining costs which include all costs related to development, blasting, mining, trucking, handling, shipping to port, and sale.

Equation 6: Profit of an iron block

$$Profit = Volume * Density * (Value_{(by\ type)} - Cost_{mining} - 500 * dev(Al, SiO_2, Fe))$$

Table 4 displays the market value of each tonne of material for each destination. Clearly the DSO commands a much higher value due to its desirable physical and chemical properties. Thus, if no deviation penalty was applied, the optimizer would simply assign all ore to the DSO destination.

Table 4: Price per tonne by destination

Material Type	Value (USD/Tonne Fe)
DSO	56
SP1 - High Silica	42
SP2 - High Alumina	28
SP3 - Low Grade	14
Waste	0

5.2 Meta-Heuristic Genetic Algorithm Parameters

A complete explanation of the GA parameters can be found in Section 4.2.

- <u>Cluster Size:</u> 5 x 2 with longer side perpendicular to the mining direction.
- <u>Population Size:</u> 1500 parents breeding 4500 children
- <u>Free Selection Solutions</u>: 80%

- Predator Strength: 1.005 scaling up to 1.01
- Mutation Probability: .2 scaling down to 0.

5.3 Preliminary Results

Simple inspection of the free selection optimal destination of each block (Figure 21) can lead the reader to conclude that this particular dig-limit is difficult to draw, due to the waste pocket layout and the apparent –45-degree angle of the grade continuity. Furthermore, the level itself is quite narrow, making it difficult to define where to place the destination boundaries, while still respecting the defined equipment clustering size.

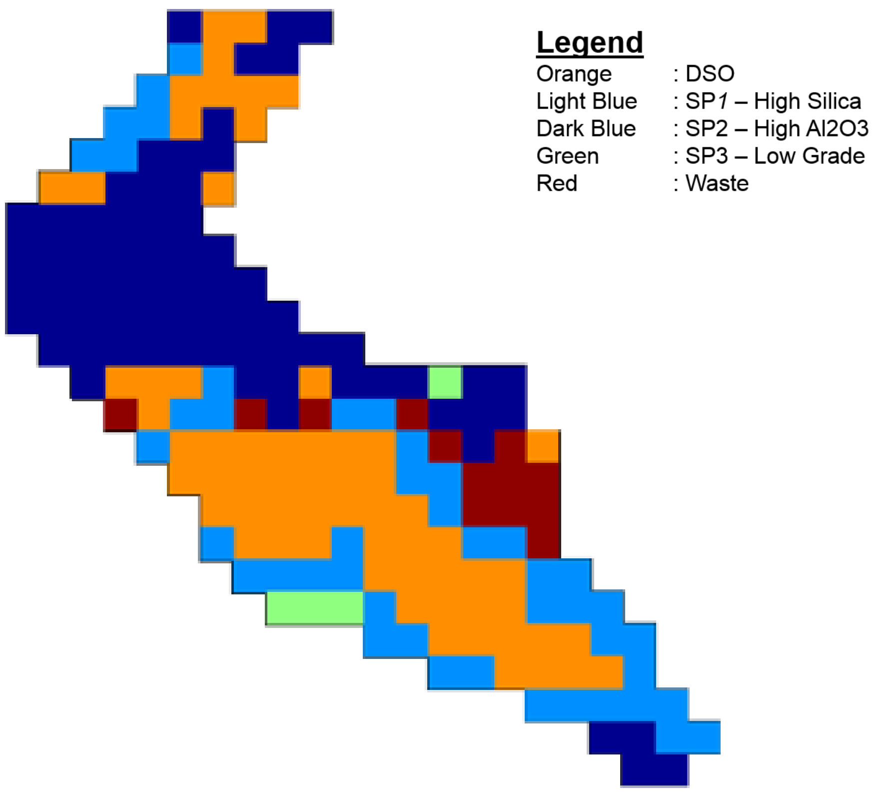


Figure 21: Free selection optimal destination

Figure 22 displays the hand-drawn limit generated by a pit geologist. Please consider zones 1 and 2. Zone 1 did not adequately respect the clustering size, generating block clusters of 4 rather than 5. From inspection of the free selection block model, it is easy to determine why the pit geologist defined this boundary: the clustering deviation is small and the geometry of that zone of the deposit lends itself well to this boundary location. Zone 2 is also of interest because the pit geologist chose to assign as many blocks as possible to the correct destination (high silica). However, multiple waste blocks that were of low Fe grade were included in this dig-limit, as seen in Figure 17. In general, this dig-limit is highly representative of the "correct colors in correct baskets" methodology.

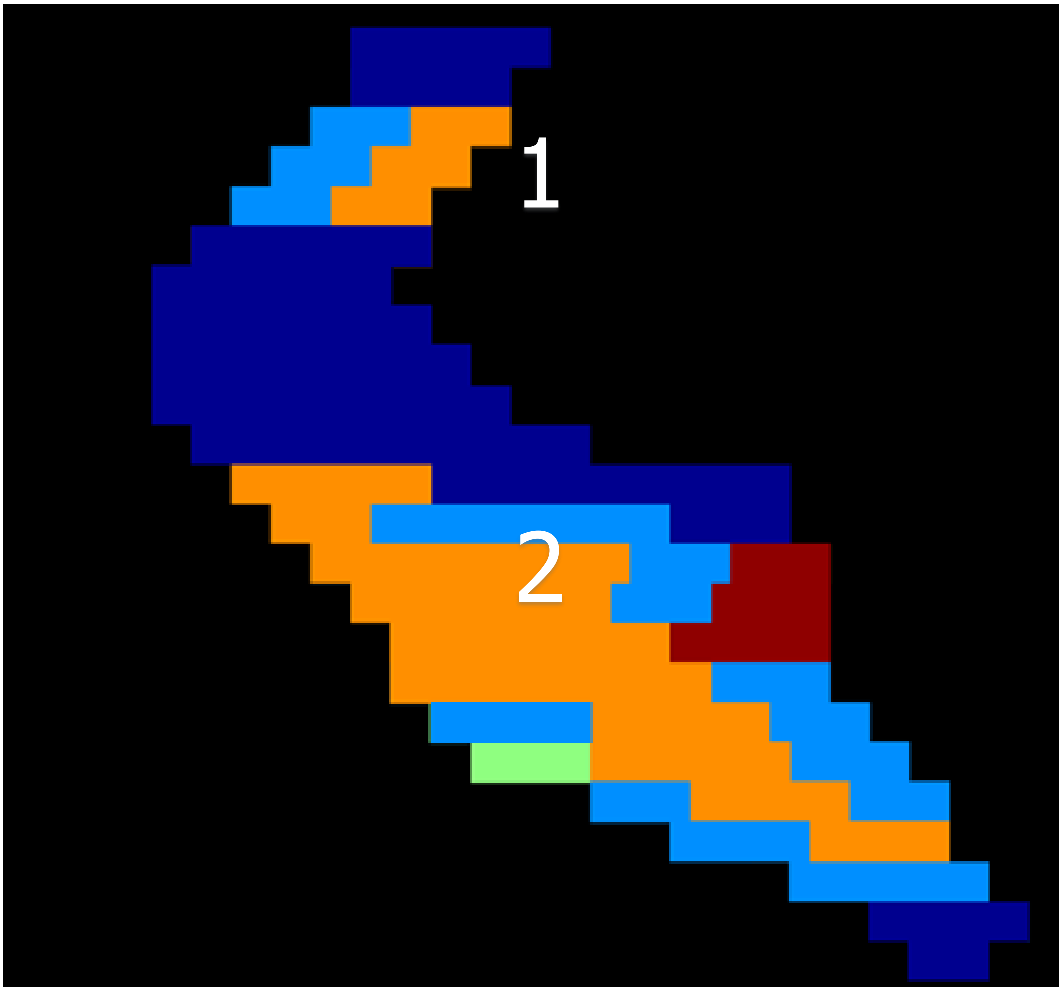


Figure 22: Hand drawn dig-limit

The dig-limits generated by the GA (Figure 23) are largely similar to those shown in Figure 22. However, considerable differences exist in zones 1 and 2. In zone 1, where the pit geologist assumed that incurring the mining deviation penalty was justified, the GA instead chose to respect the mining widths completely. In zone 2, the GA completely departed from any possible decision that could be inferred from the optimal destination of each block. Note in Figure 21 that zone 2 is a combination of waste and high-silica; however, when clustering, the algorithm defined the zone as low-grade ore. This decision eluded the geologist drawing the comparative dig-limit and makes logical sense. The GA deviated from the target geometry less often, generated lower complexity dig-limits, and made decisions based on mathematics and optimal solutions, as opposed to the color classification method displayed by the hand-drawn solution. In a production environment, the GA would require fewer flags to denote the dig-limit and would be considerably easier to mine while classifying strategically rather than tactically.

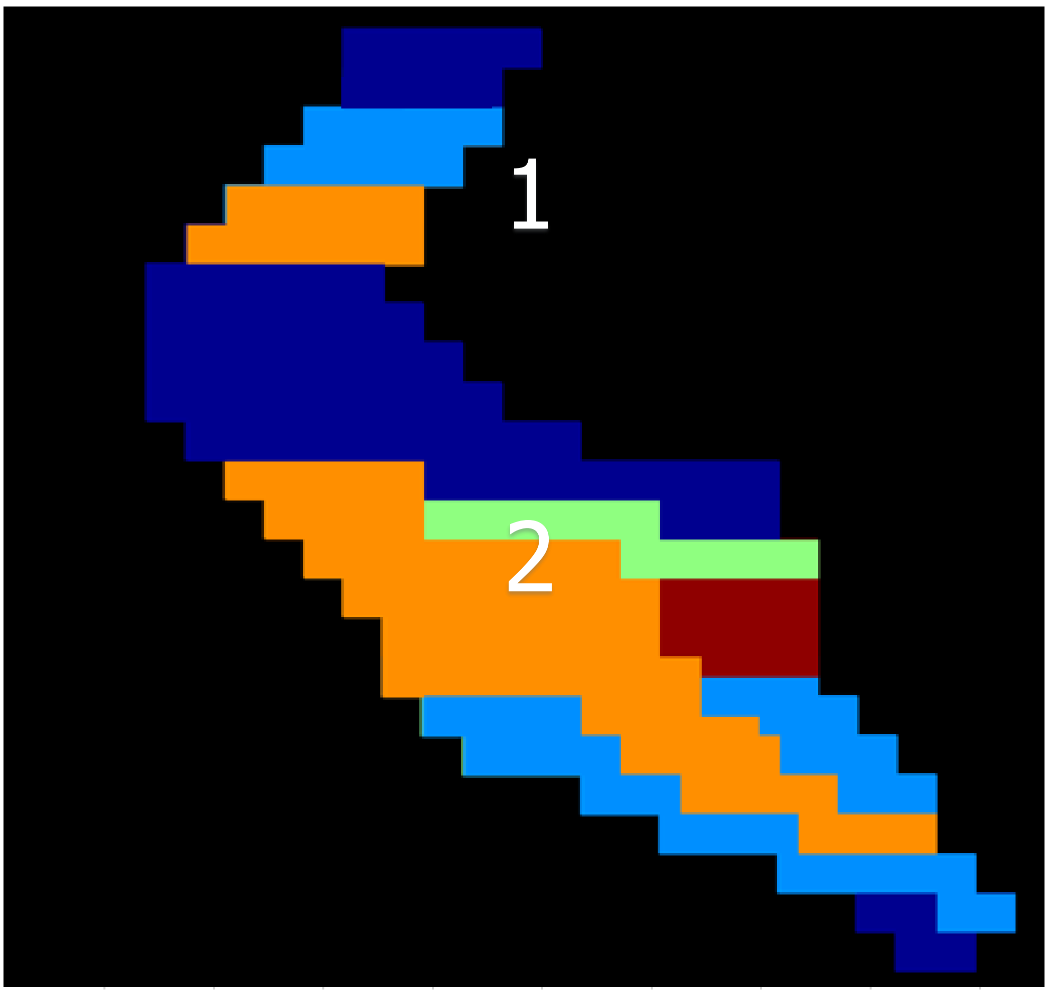


Figure 23: Genetic algorithm

It should be noted that the hand-drawn solution will nearly always outperform the GA when comparing the resulting grades per destination, and the number of misclassified blocks (Table 5 and Table 6). However, the purpose of dig-limits is to generate the most profitable and mineable design, while adhering to the geometric and chemical constraints imposed by the mining equipment and processing facilities used by the mine. The GA outperformed the hand-drawn limits by approximately 2.5% in terms of profitability (Table 7).

As expected, the hand-drawn perimeter tends to more closely replicate the grades from the free selection solution, particularly the material sent to the waste dump (

Table 5). These deviations are due to the profit-driven optimization method. The purpose of dig-limits is not to replicate free selection, but to generate the most profitable perimeter, while respecting all constraints.

Table 5: Grade parameter comparison

Destination Grades(%)	Mineral	Free Selection	Hand-Drawn	Genetic Algorithm
DSO	Fe	55.07	54.5	54.04
	Al	2.13	2.12	2.11
	SiO_2	13.83	13.61	13.4
SP1 - High Silica	Fe	44.11	42.34	47.53
	Al	2.53	2.82	2.36
	SiO_2	19.04	17.76	19.54
SP2 - High Alumina	Fe	49.11	49.43	48.53
	Al	3.85	3.71	3.95
	SiO_2	9.17	10.13	9.85
SP3 - Low Grade	Fe	32.64	39.79	26.94
	Al	2.86	2.35	4.31
	SiO_2	23.74	22.72	22.84
Waste Dump	Fe	21.42	22.11	24.45
	Al	4.3	4.24	3.88
	SiO_2	12.66	13.15	13.65

Table 6 confirms the known tendencies of the algorithm. More blocks are misclassified by the GA than the hand-drawn approach.

Table 6: Number of misclassified blocks

Misclassified (# of Blocks)	Free Selection	Hand Drawn	Genetic Algorithm
DSO	0	54	59
SP1 - High Silica	0	58	66
SP2 - High Alumina	0	4	16
SP3 - Low Grade	0	28	40
Waste	0	24	26

The GA outperformed the hand-drawn algorithm significantly in terms of profit (Table 7). By optimizing grade mixing and the layout of mining boundaries, the GA will always outperform the hand-drawn case, particularly if there are time limits placed upon the pit geologist, or if the bench is of high complexity such as the one outlined in this case study.

Table 7: Economic results of hand-drawn vs. GA methods

Result	Cutoff Grade	Hand Drawn	Genetic Algorithm
Profit	3.15 M	2.8 M	2.9M
Percent Recovered	100%	90.80%	93%

5.4 Discussion

This case study built upon the methods outlined in Chapter 4: a similar methodology and algorithm was used. However, the case study considered five destinations and three grades for optimization and the algorithm tested directional mining design. In general, the case study demonstrates the feasibility of the GA for defining dig-limits for highly complex problems. The GA outperformed the hand-drawn method in speed, flexibility, diggability, and profitability. The case study conclusively shows that meta-heuristics can indeed be used for the dig-limit problem, and should be considered for large-scale application in the mining industry.

6. Case Study III: Selectivity Sizing for multi-destination, multi-rock and multi-metal deposits using genetic algorithms

As outlined in Chapter 2.3, the dilution to equipment size relationship used to optimize the equipment size is based solely on the bench height. No existing publications attempt to relate the clustering size of the dig-limits to the possible profit derived from the deposit. The case study presented here will show that the correlation between profitability and selectivity size is non-linear. Furthermore, size selectivity appears to have threshold clustering sizes, above which the relative profitability severely drops. Therefore, this case study will show that the dig-limit problem should be considered during equipment sizing and is of considerable importance regarding the ultimate profitability and operability of a mining operation. In order to standardize the process and ensure that the dig-limits are adequately drawn and consistent, a GA was used to generate all dig-limits.

6.1 Economic Model

Three potential destinations and two types of ore contain two economic minerals (Table 8). The sulphides are refractory and cannot be processed using leaching. The gold in the sulphide is not recoverable when sent to the mill. Leaching will only recover copper for both mineral types. This highly complex economic model is of particular interest: the ability to adequately discriminate between the ore types and waste will often be the difference between recovering nothing and generating a high degree of profitability. Thus adequate selectivity is of particular importance, since the misclassification of blocks can generate extreme losses, or exclude blocks with high profit.

Table 8: Economic model for the baseline case (4 x 4 clusters)

Material	Selling Price				
Gold	37.71 $/g				
Copper	4578 $/tonne				
	Detail	Leach	Mill	Waste	Units
All Materials	Mining Cost	9	10	8	$/tonne
Processing Details	Detail	Mill	Leach	Waste	Units
Oxide	Base Processing Cost	6	2.55	0	$/tonne
	Cost Increase per Au gram	0	0	0	$/g
	Cost Increase per Cu tonne	180	190	0	$/tonne
	Recovery Au	0.8	0	0	fraction
	Recovery Cu	4.4*Cu (%)	0.65	0	fraction
Processing Cost	Detail	Mill	Leach	Waste	Units
Sulphide	Base Processing Cost	3	2.55	0	$/tonne
	Cost Increase per Au gram	0	0	0	$/g
	Cost Increase per Cu tonne	190	190	0	$/tonne
	Recovery Au	0	0	0	fraction
	Recovery Cu	0.65	0	0	fraction

A high degree of variability is evident between oxide and sulphide boundaries (Figure 24). Due to the disorganized quality of the sulphide oxide boundaries, hand drawing the dig-limits is highly difficult. Furthermore, the degree of selectivity necessary to classify this material by type can be roughly 20 or 30 m (2 or 3 blocks). This case study will show that this visual inspection differs greatly from the optimal sizing necessary to derive maximum profit from this deposit.

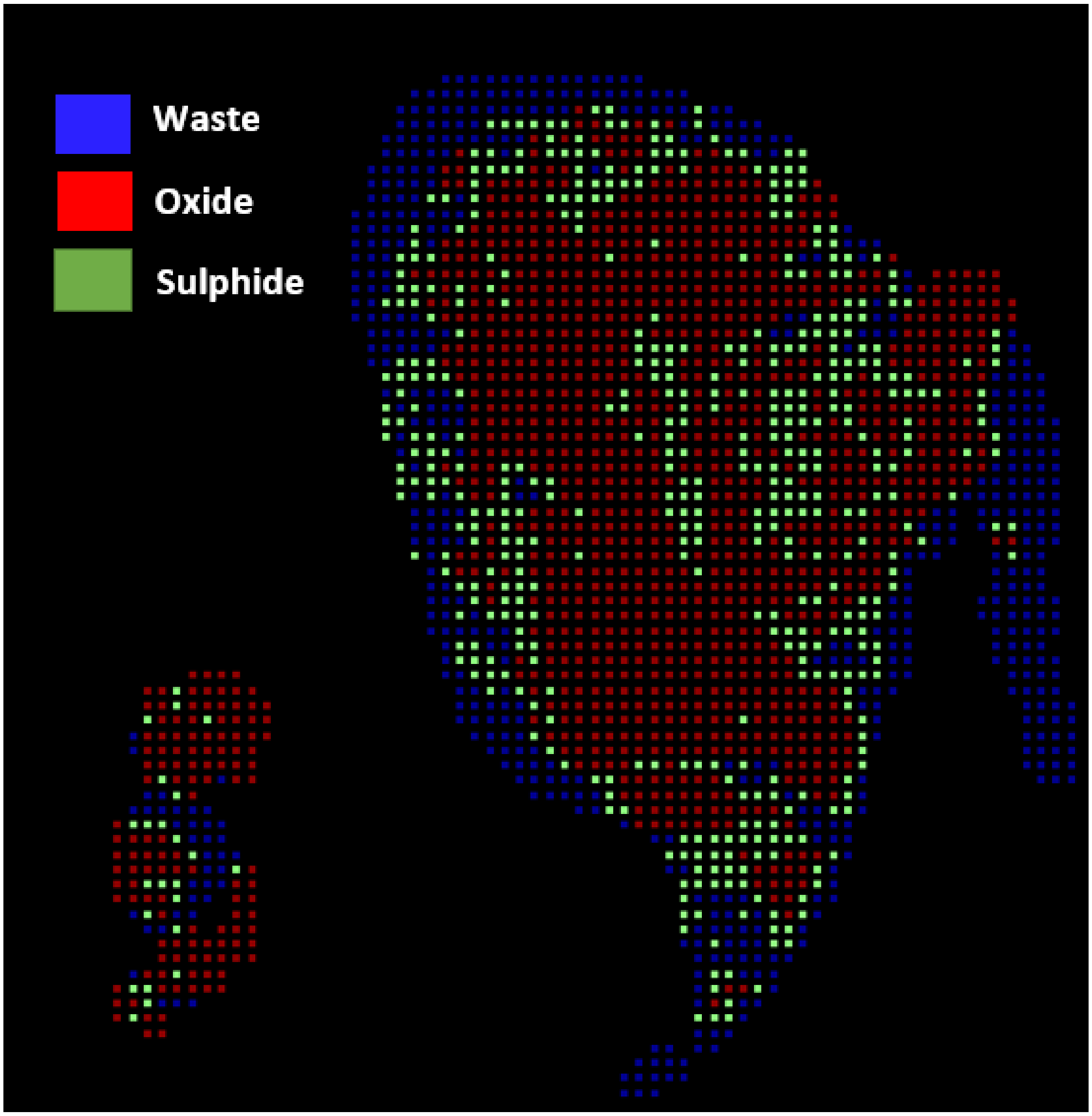


Figure 24: Material type (Waste, Oxide,Sulphide) distribution within the bench

The oxides appear to hold a greater quantity of gold than the sulphides (Figure 25). Furthermore, the gold grade is primarily contained in two pockets in the north and south portions of the deposit. These pockets have high-grade centers and exhibit decreasing grades moving away from the center. Large areas of the deposit have near 0 grade, indicating large variability and strong boundaries for the material type.

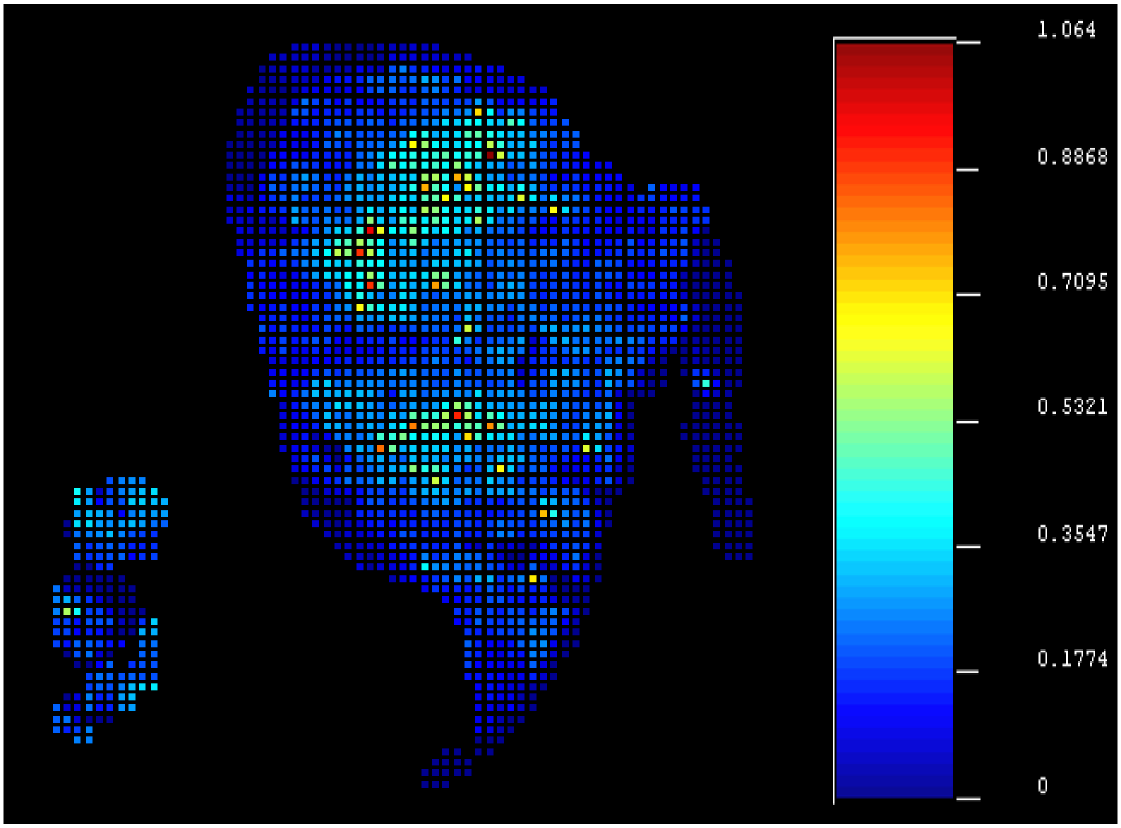


Figure 25: Au grade distribution within the bench

Copper does not appear to be significantly related to either sulphide or oxide (Figure 26); however, the highest grade pockets in this bench are oxide blocks. The waste blocks have low copper and gold grades. There appears to be little correlation between gold and copper grades in this bench. Furthermore, the continuity of the mineralization is fairly clear at a near north–south angle. Some high-grade blocks can be found in near the center of the mineralization. Note the large sections of the bench with near 0 copper grades.

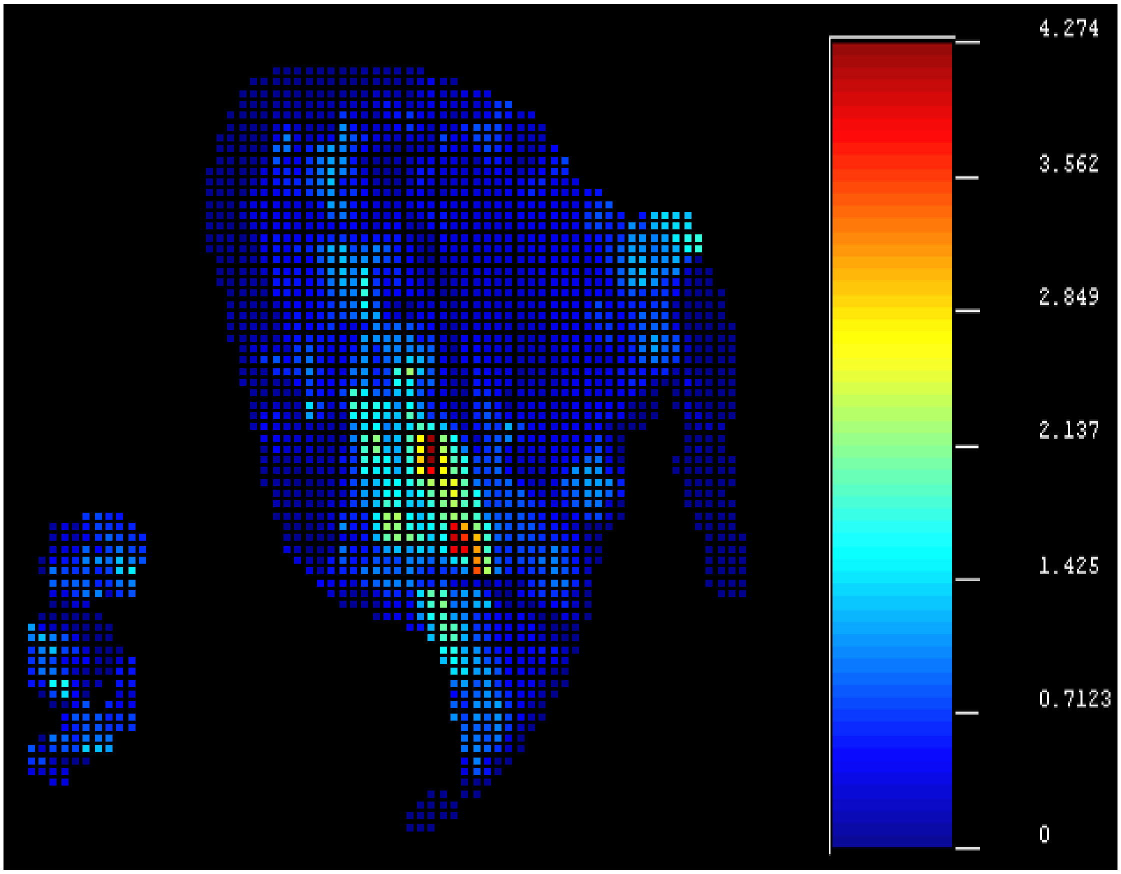


Figure 26: Cu grade distribution within the bench

In summary, the gold and copper grades for this deposit do not appear to be correlated. Copper oxides of high grade are ideally processed using leaching, whereas high-grade gold concentrations are ideally processed with a mill. Low-grade oxides are optimally processed using leaching. Sulphides of moderate and high grades are best sent to the mill, while low-grade sulphides should be classified as waste. Large portions of the deposit have near 0 gold and copper grades, which must be adequately classified to incur minimal processing losses.

6.2 Preliminary Results

Dig-limits were generated at various selectivity sizes using an automated dig-limit algorithm, which guaranteed clustering and maximized profitability. The purpose of this execution was to determine the profitability at each selectivity size, while respecting mining geometry. The goal was to accurately quantify the impact of mining equipment on the profitability of a mine, while considering the difficulties of adhering to the required geometry.

As equipment size grew, the complexity of blasting, planning, and hauling decreased. The mining costs at each selectivity size therefore decreased geometrically (Figure 27). The mine was originally designed to use equipment with a selectivity size of 40 m. The mining costs for each block were scaled with a factor of approximately 0.75, with some variation due to the variable number of blocks to each destination (Table 9).

Table 9: Profit and mining cost vs. cluster size

Cluster Size	Mining Cost ($)	Pre_scaling Mining Cost ($)	Relative Price	Post Scaling Profit ($)
10	15,668,250	19,284,000	7	26,317,050
20	4,265,556	19,195,000	2	28,807,944
30	3,204,000	19,224,000	1.5	29,240,200
40	2,385,750	19,086,000	1.13	29,260,350
50	1,782,094	19,009,000	0.84	29,727,506
60	1,347,680	19,167,000	0.63	30,307,520
70	1,016,402	19,274,000	0.47	30,355,298
80	760,245	19,222,000	0.36	29,555,655
90	550,873	18,571,000	0.27	24,470,977

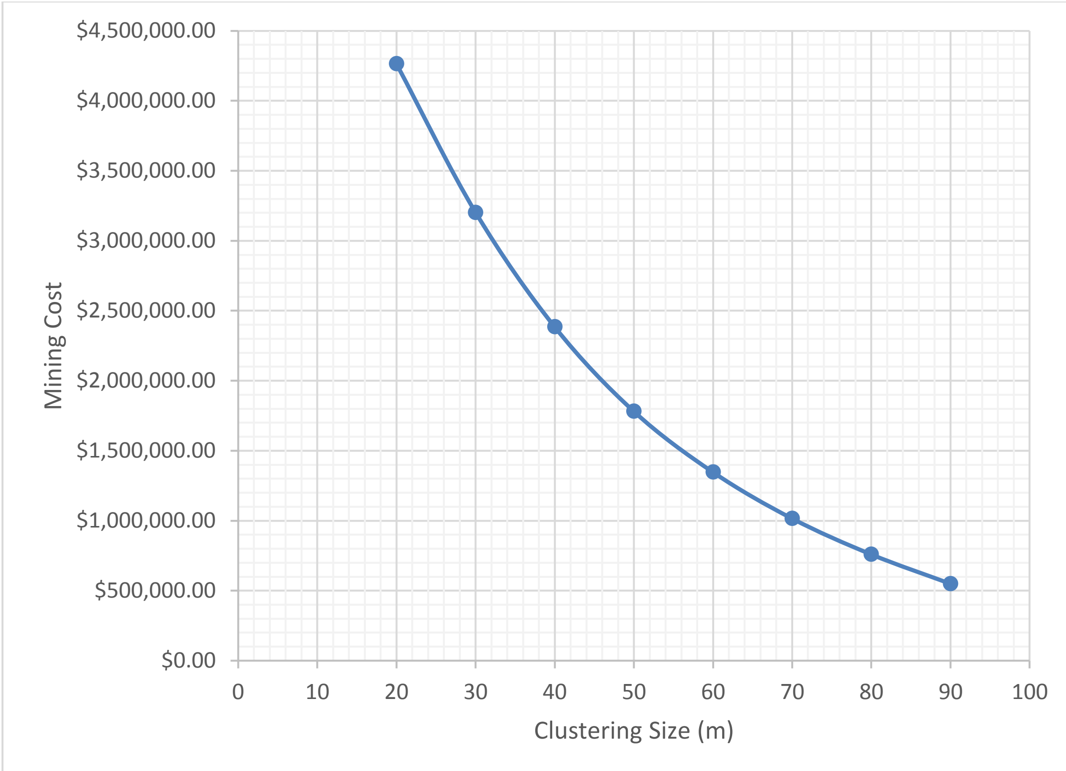


Figure 27: Mining cost vs clustering size

The number of blocks sent to the mill decreased as selectivity size increased (Figure 28). Since the cost of milling material is much higher, and the relative downsides also more severe regarding the processing of sulphides, it makes economic sense to blend the mill materials into the leach destination. Furthermore, at approximately a clustering size of 80, the capability of discriminating between waste and leach blocks became severely impaired. In terms of replicating the optimal number of blocks per destination as shown by the free selection solution, the original clustering size chosen by the mining staff best replicated the results. Lastly, there were no constraints regarding the tonnage to each destination, this could be implemented if necessary.

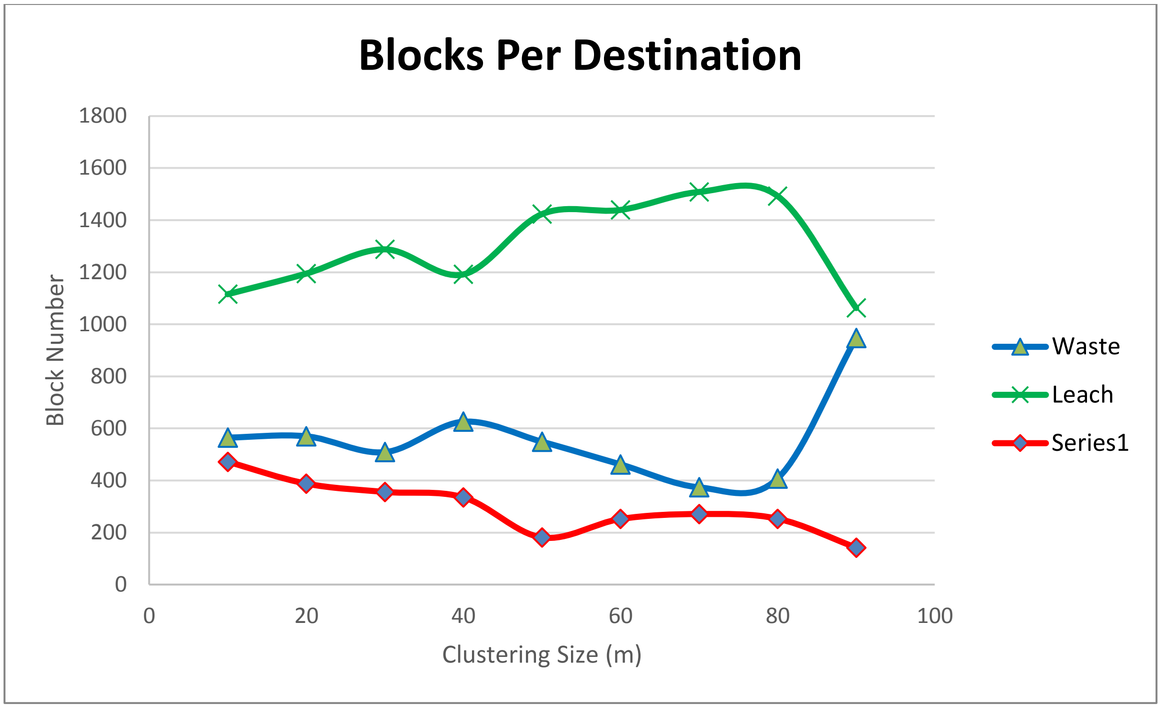


Figure 28: Blocks per destination by clustering size

Given the various selectivity sizes and mining costs, it became possible to execute the GA such that the dig-limits were optimized using the aforementioned inputs. The resulting profits at each selectivity size are outlined in Figure 29. Upsizing the equipment from a selectivity of 10 m to a selectivity of 70 m considerably improved profits. However, increasing the selectivity size past 70 m caused a steep decline in profit, reducing potential earnings by approximately $0.5m USD compared to the selectivity size and equipment chosen by the owner. It is clear that mining equipment can only be upscaled to a certain point before the loss in selectivity begins to outweigh the cost benefits of larger equipment. The profitability did not change appreciably between selectivity sizes of 3 x 3 blocks (30mx30m) and 4 x 4 blocks (40mx40m). The selectivity sizing reached a soft breakpoint, where the geometry of the deposit did not lend itself well to the 40 x

40 m selectivity sizing (Figure 28). This is particularly worrisome given that the mine in question was designed to make use of this equipment size. At the 40 x 40m selectivity size, a disproportionate number of blocks are sent to waste; many of these blocks were removed from the leach destination.

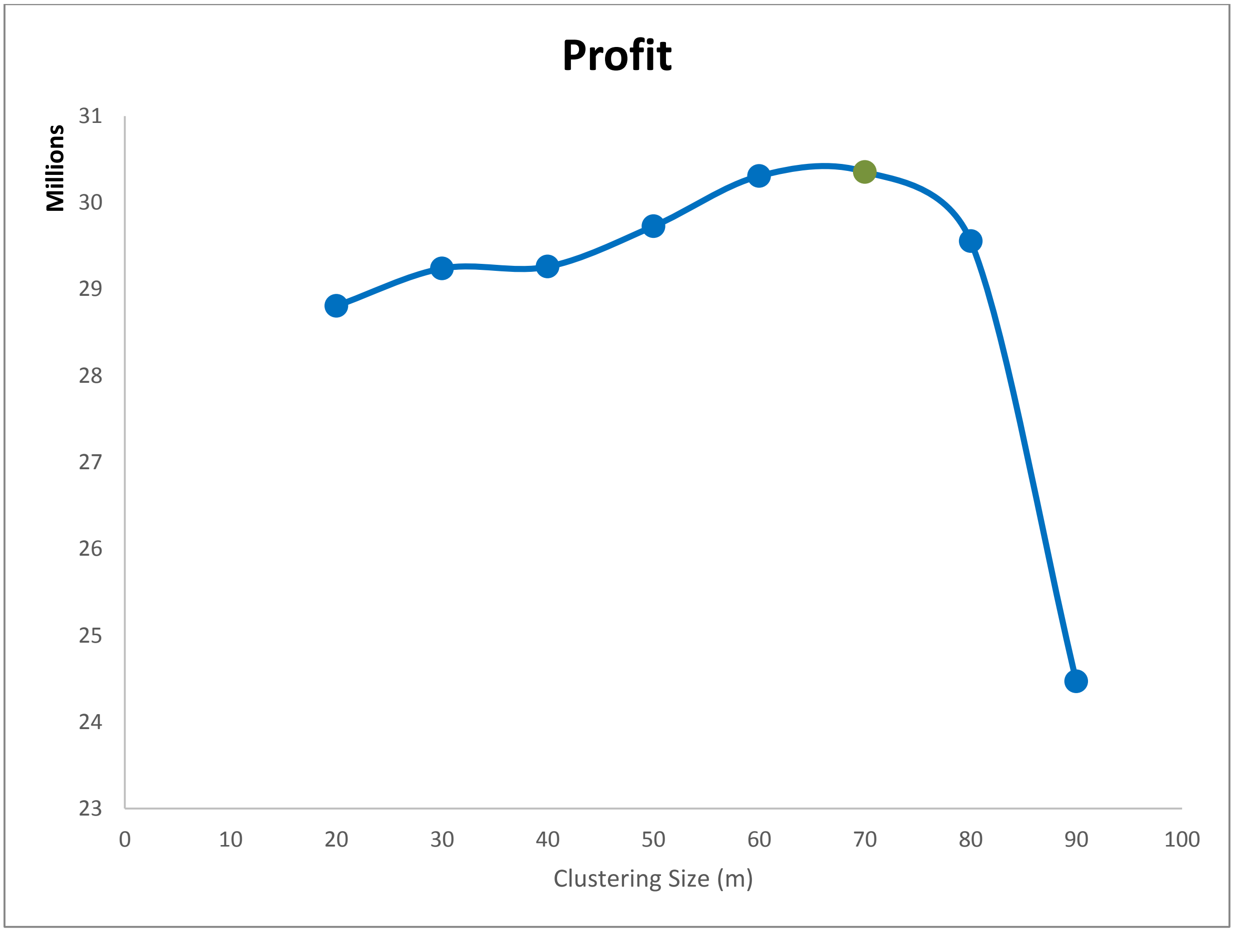


Figure 29: Profit vs. clustering size

6.3 Comparison Between Clustering Sizes

When clustering size is manipulated, the overall geometry and strategy can change significantly. This is critically important when considering the operability of a mine plan. If similar financial results can be obtained while significantly reducing the complexity of the solution, the lower difficulty solution is preferable. This notion is of particular importance when comparing the various dig-limits generated by manipulating the selectivity size. In addition to lowering per tonne operational costs involved in direct mining activities (blasting, loading, hauling, dumping), indirect costs are also significantly reduced. For example, fleet management complexity is reduced due to the use of a smaller fleet for the same output. Alternatively, a similarly sized fleet with higher throughput could be used, which could increase the NPV because of discounting. The smaller surface area of material boundaries between destinations considerably reduces the number of dig-limit flags that must be placed in the bench.

The free selection dig-limit is presented in Figure 30. As mentioned previously, the optimal block destination occurs in small block clusters (1 to 4 blocks) and can be classified as highly irregular.

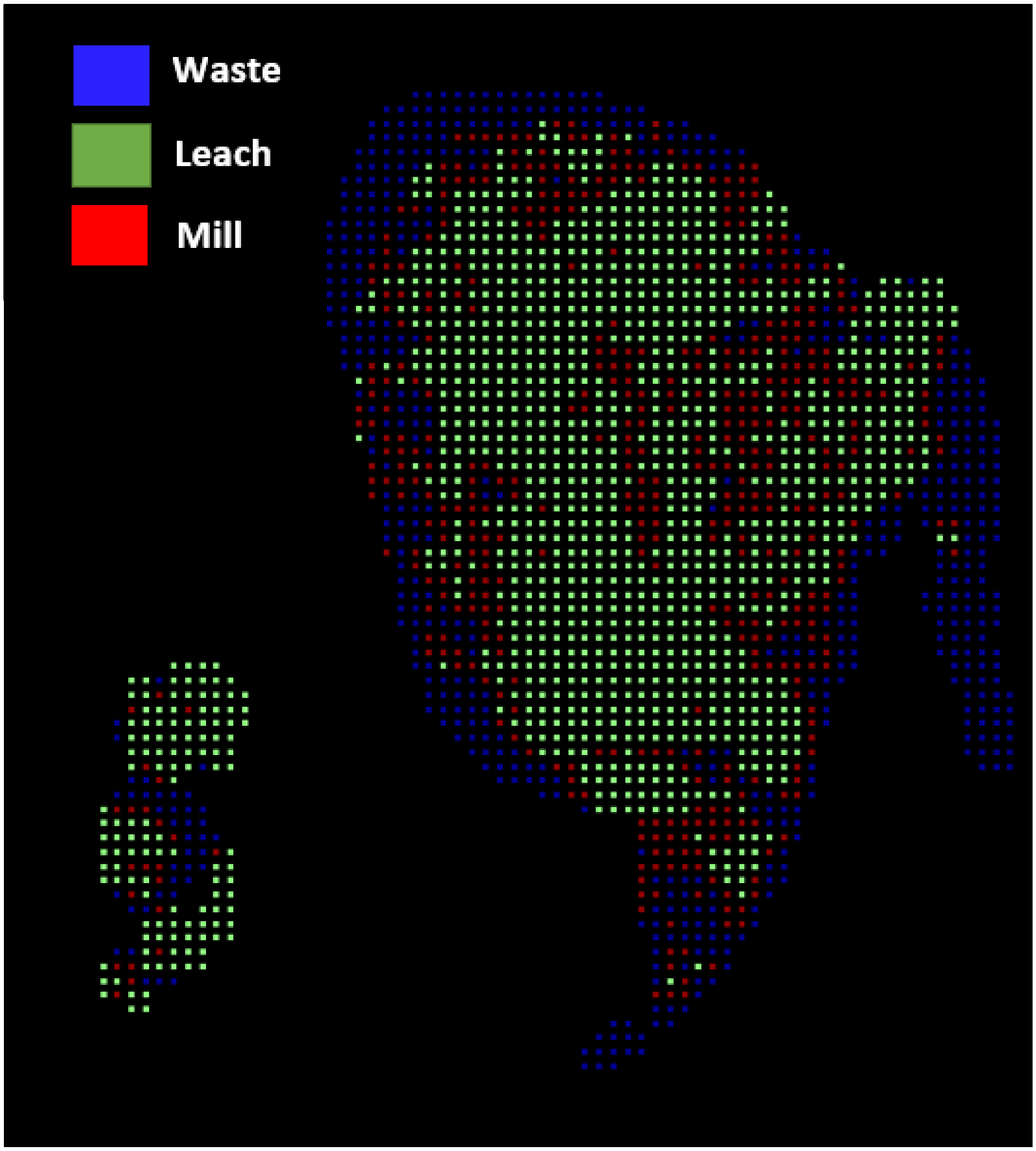


Figure 30: Free selection dig-limits

Within a dig-limit generated by using a clustering size of 20 x 20 m, some deviations are observed due to the clustering deviation chosen (Figure 31). The highly selective nature of this selectivity size means that the punishment applied to the deviation was outweighed by the improvement of deviating from mining geometry in several portions of the deposit.

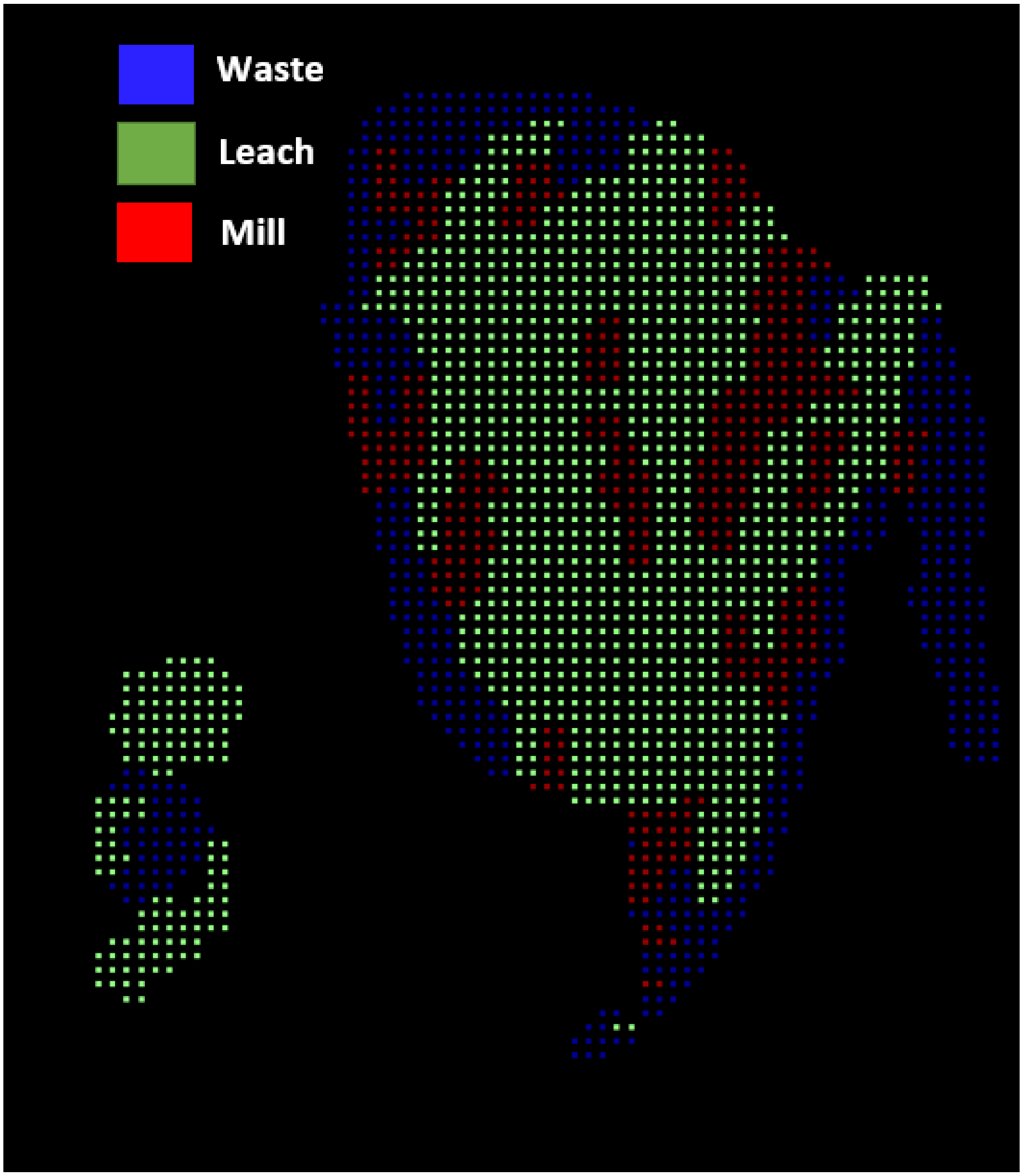


Figure 31: 20 x 20 m clustering dig-limits

In the 30 m clustering case, there are no clustering deviations and the dig-limit is by and large a reflection of the free selection case (Figure 32). Note the reduction of mining limits compared to the 20 m selectivity dig-limit. This tendency will continue as the selectivity size is increased.

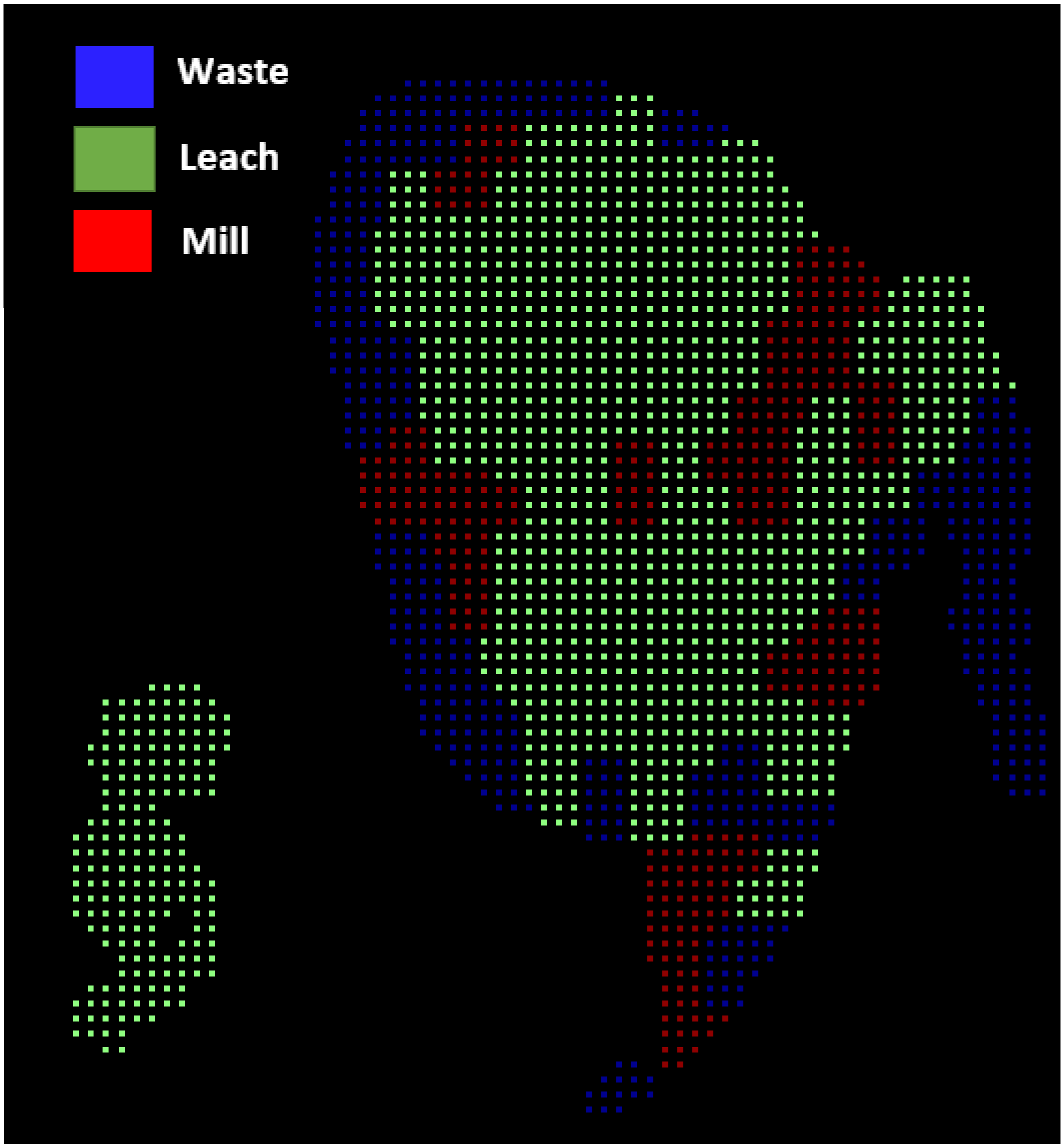


Figure 32: 30 x 30 m clustering dig-limits

The 40 m clustering case differs considerably from other dig-limits in the upper right quadrant of the deposit (Figure 33). The selectivity is sufficiently small to allow for the collection of all smaller ore packets, though sufficiently unselective to allow the deposit to differentiate out the waste packets, as was done by the 30 m selectivity. There is now a grand total of 12 mining packets, down from 15 in the 30 m selectivity case.

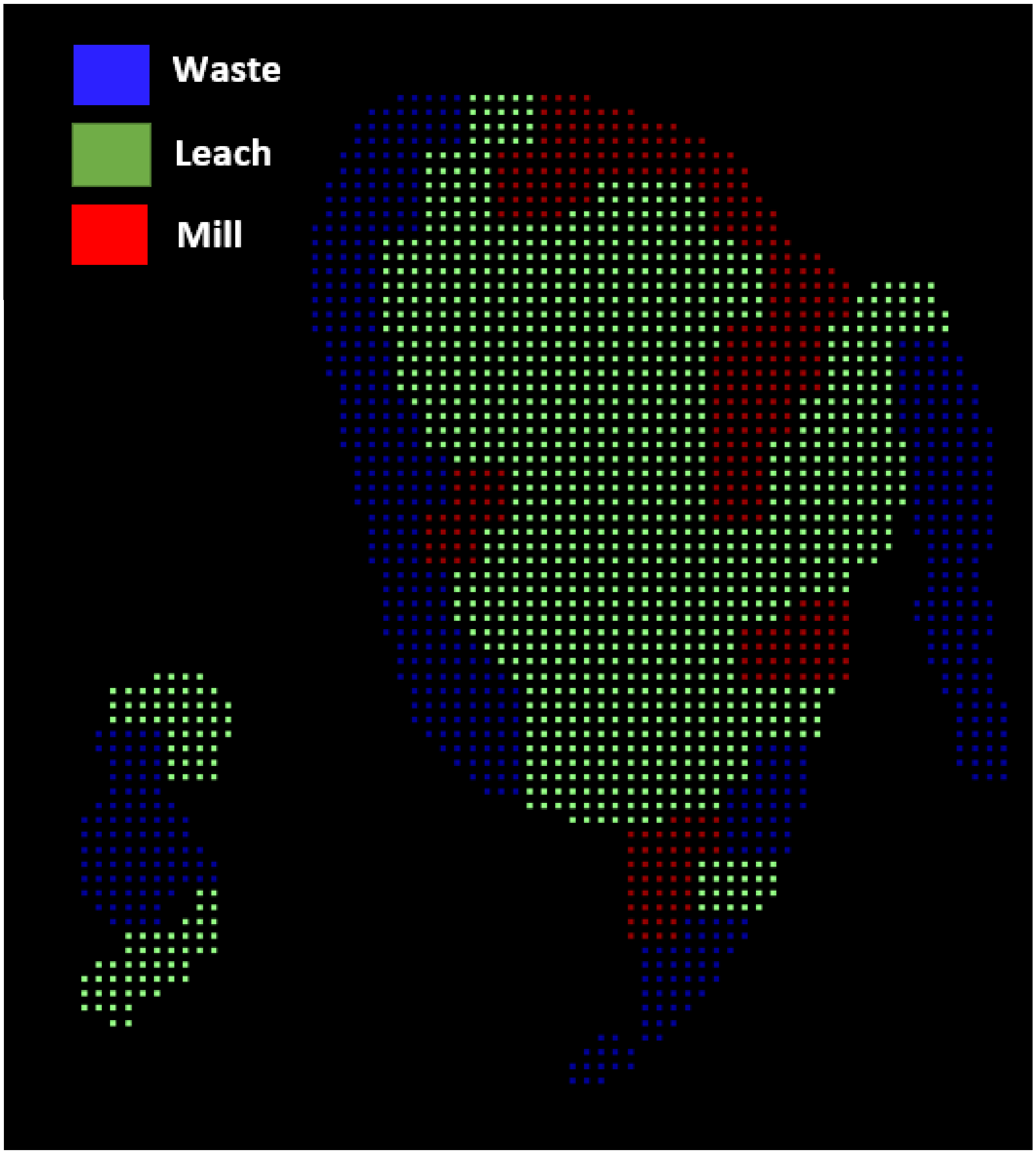


Figure 33: 40 x 40 m clustering dig-limits

Previously identified trends continue with the 50 m clustering size (Figure 34). With only 10 mining packets in total, the solution has become simple compared to the lower selectivity dig-limits. At this point, block mixing is becoming a major factor. Mathematical optimization allows for this to be done in a way that maintains and indeed improves profit compared to the 40 m selectivity.

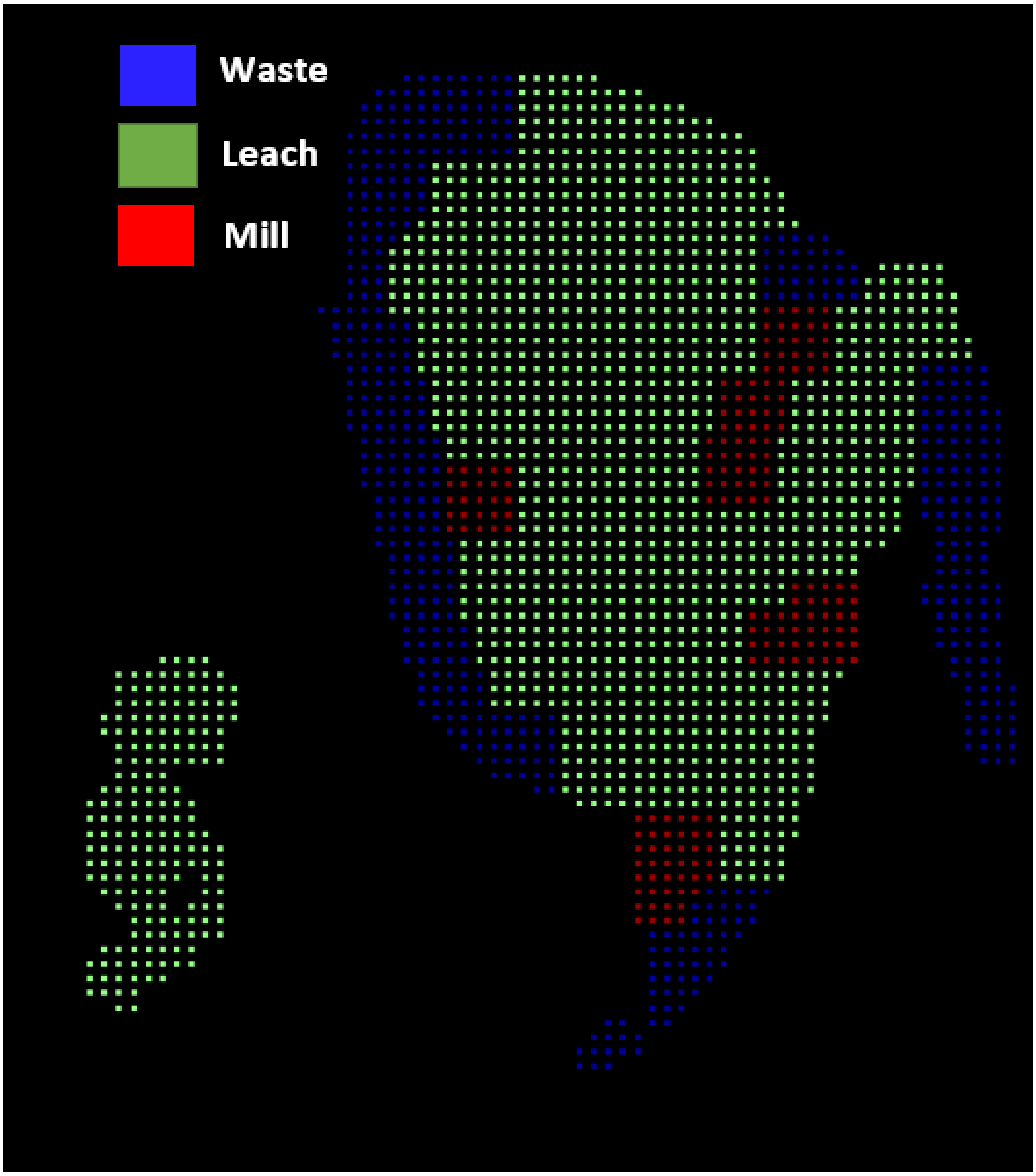


Figure 34: 50 x 50 m clustering dig-limits

For the 60 x 60 m dig-limits—maintaining 10 mining packets—the primary differences involve the removal of the waste packet in the upper right quadrant (Figure 35). Furthermore, as the dig-limits become larger, more waste from zone 1 will need to be sent to leaching.

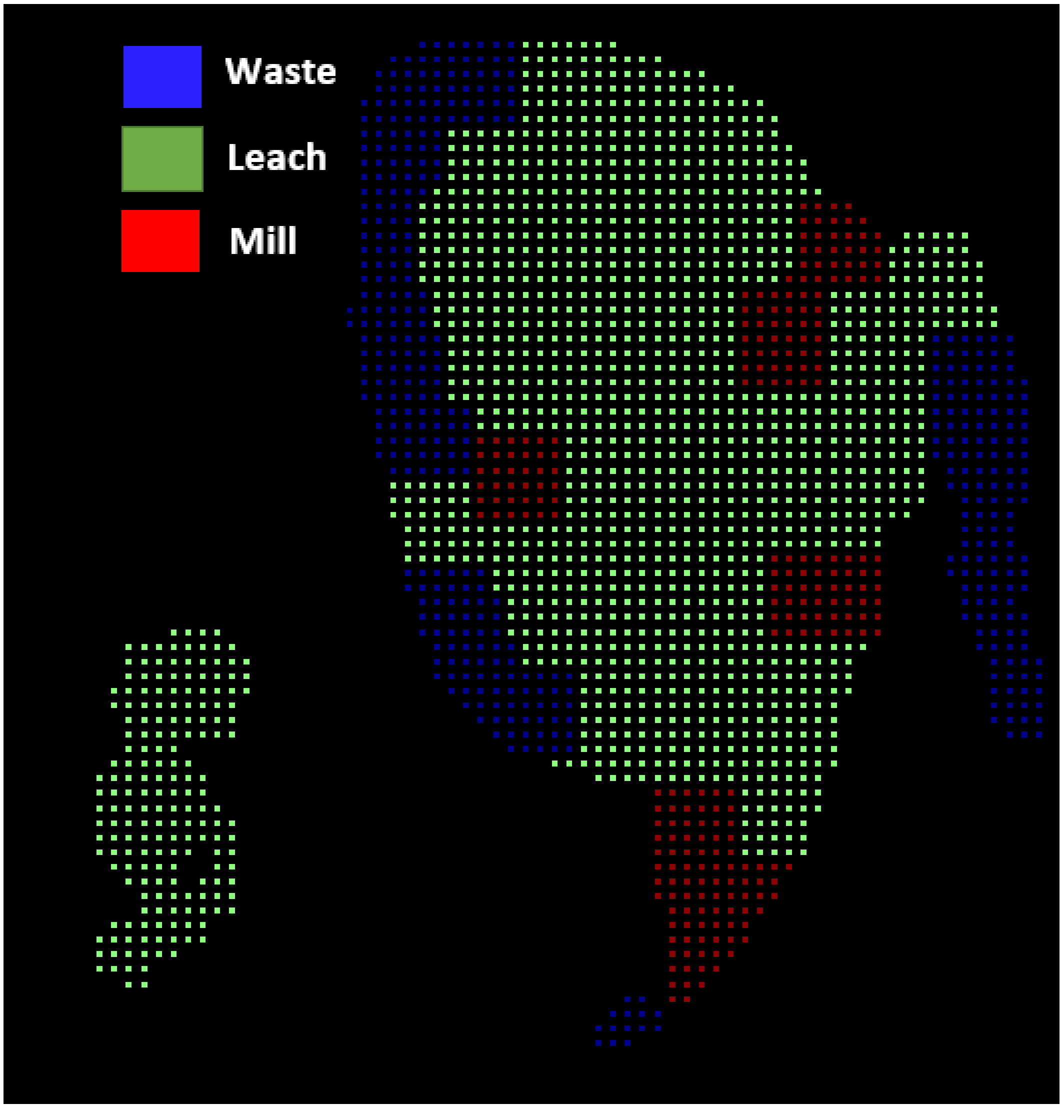


Figure 35: 60 x 60 m clustering dig-limits

The 70 m selectivity dig-limit is far more profitable than smaller selectivity sizes (Figure 36). Most of the interior and upper portions of the bench do not resemble the free-selection model. However, it is clear from the profit curves that the cost reductions associated with selectivity size increases outweigh the misclassification errors.

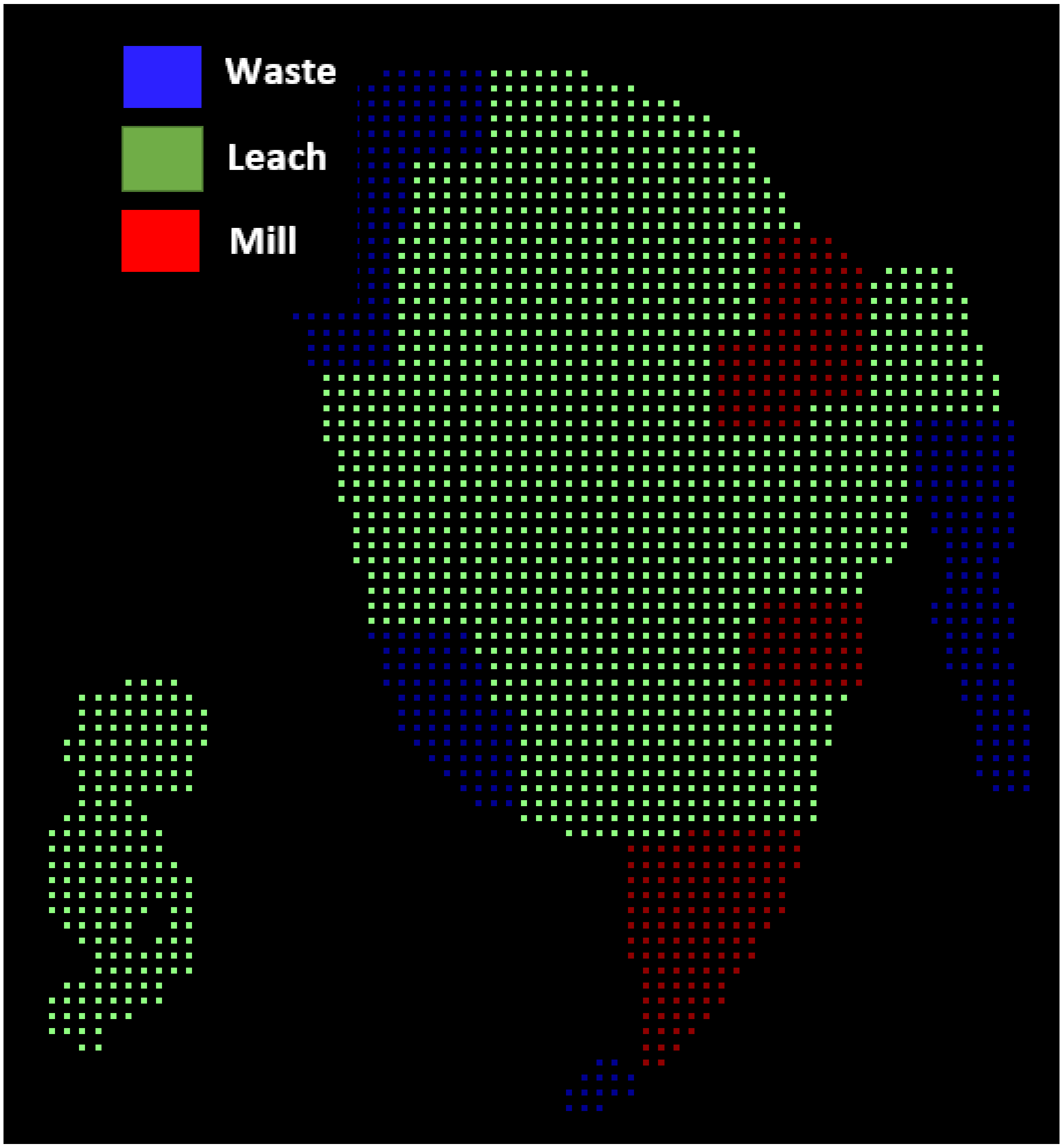


Figure 36: 70 x 70 m clustering dig-limits

All selectivity has been removed from the entire south side of the deposit in the 80 m and 90 m selectivity sizes (Figure 37 and Figure 38, respectively). Furthermore, the waste packets on the left are now well into the sulphide material. A great deal of mill material is being classified as leach. Overall the dig-limits are becoming very simple, but too much selectivity has been lost.

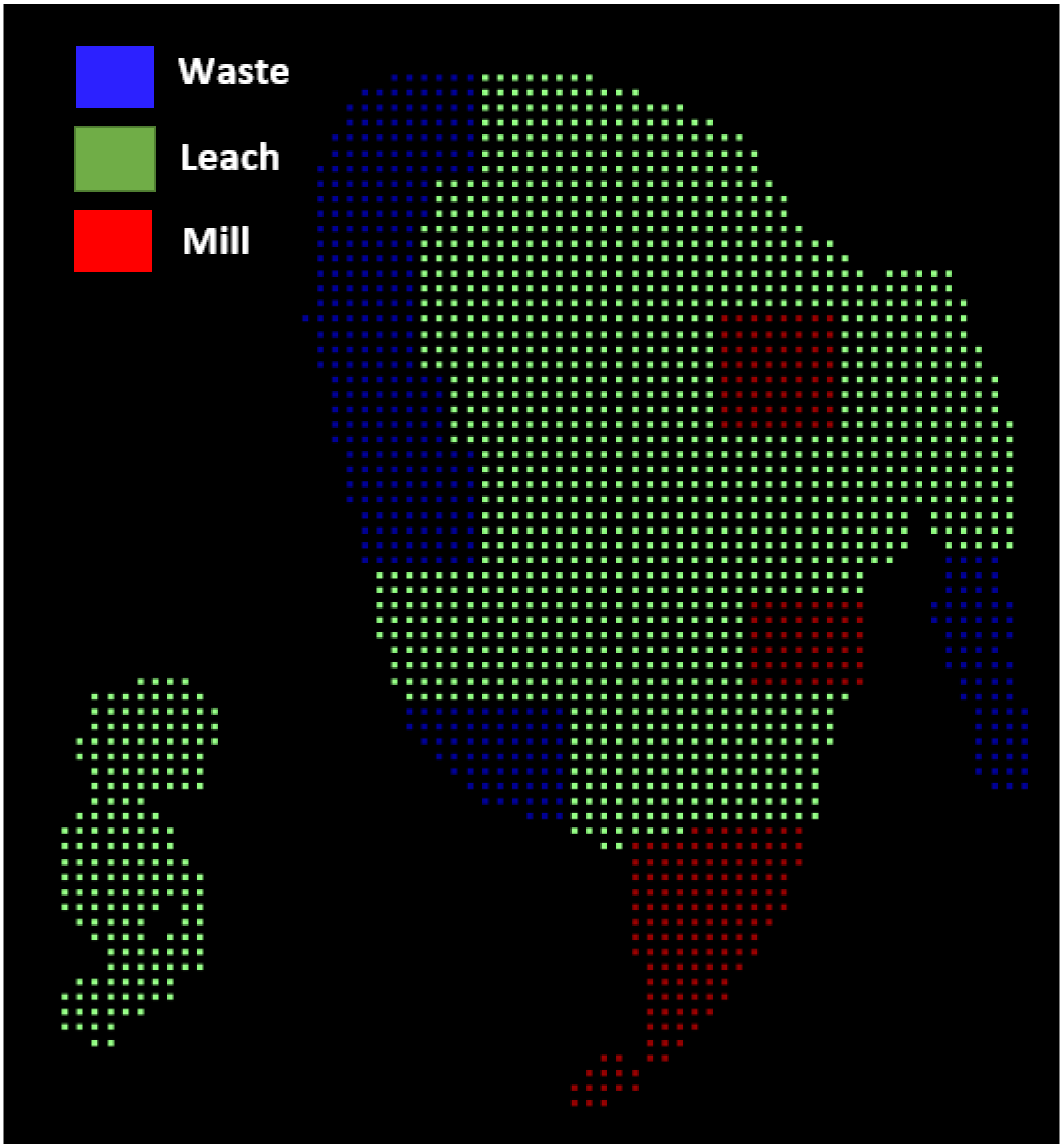


Figure 37: 80 x 80 m clustering dig-limits

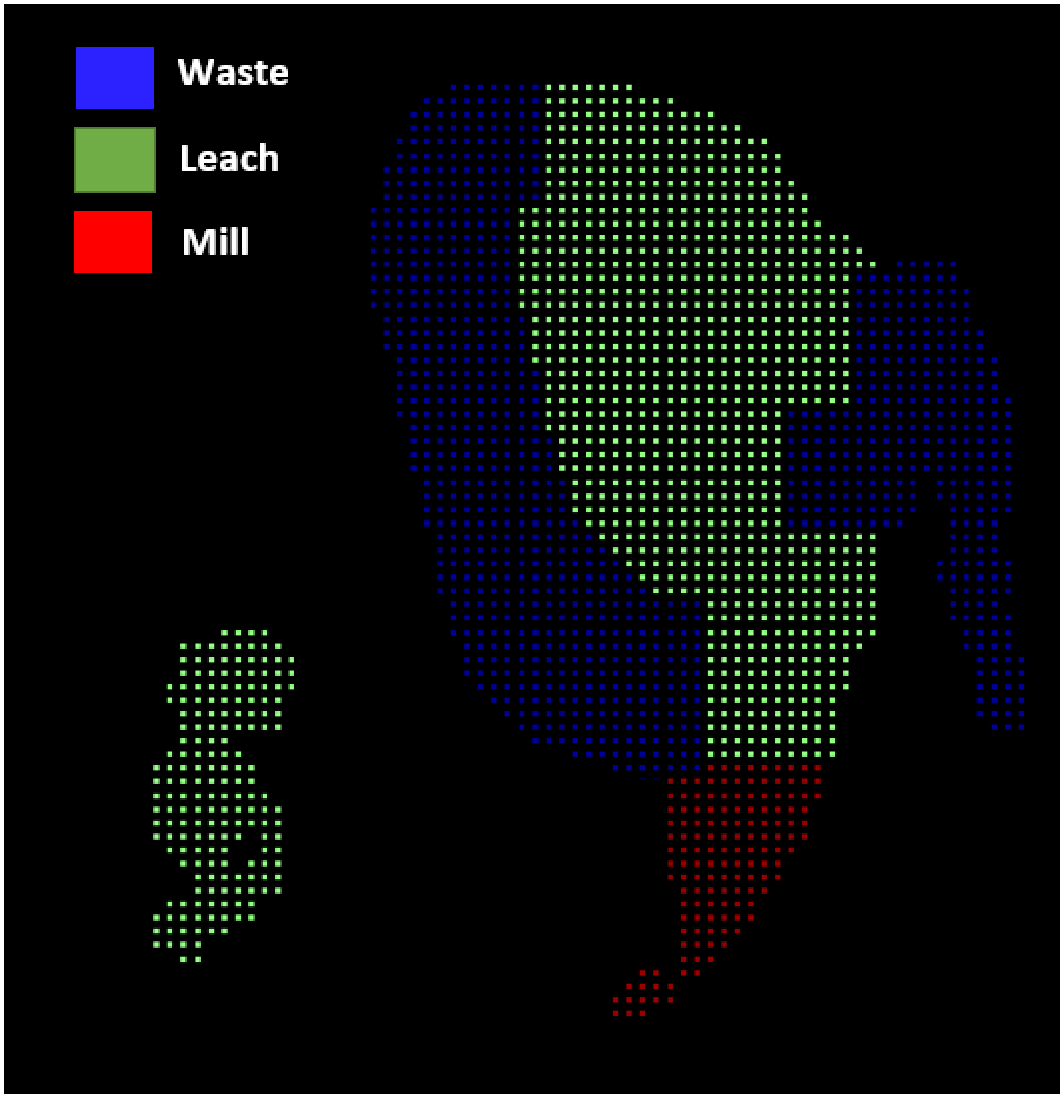


Figure 38: 90 x 90 m clustering dig-limits

6.4 Discussion

The selectivity sizing problem is complicated by the time-intensive nature of generating potential dig-limits for each scenario. This case study outlines a methodology for testing variable selectivity sizes, and presents the results that could be derived from these tests. It is clear that the selectivity sizing to profitability relationships are non-linear, and appear to have severe breakpoints.

For the purpose of this study, the selectivity size of 70 m was chosen. The size yielded only 9 digging zones, considerably reducing both the operational and mining complexity. Furthermore, the profitability was improved compared to the selectivity size chosen by the mine.

Several assumptions were made when calculating the cost reductions from upsizing the mining equipment, specifically the cost reductions were estimated to be constant for each size reduction. In addition, the bench height was not modified to reflect the equipment size changes. Future work could focus on integrating these changes into the model and further developing the modified cost models to better reflect the costs associated with each equipment selectivity size. Finally, directional mining limits—as outlined in the previous case study—could be implemented.

7. Conclusion and recommendations

Generation of mining dig-limits has been an integral part of open pit mining since the advent of modern blasting techniques. Though many aspects of mining and mine planning have improved, dig-limit generation techniques have gone largely unchanged. Due to the ever-increasing complexity of new deposits, mineral processing and mining approaches have likewise become more complex. Thus, modern dig-limits require consideration of more information to identify the best classification for an ever increasing number of destinations. The hand-drawn method of dig-limit generation continues to become increasingly inadequate: better solutions methods are needed. Improvements in the fields of computer science and non-linear optimization have provided a solution for the mining industry. Employing such techniques will result in more consistent, easier to operate, and ultimately more profitable mines.

This thesis outlined the generation, use, and potential applications of an automated dig-limit generation algorithm using GAs. The research conducted thus far indicates the efficacy and quality of the solutions generated. Furthermore, the dig-limits generated by the GAs are both more profitable and more operational to those drawn by hand.

GAs are well suited to the dig-limit problem, due to its sheer magnitude. Linear alternatives that guarantee optimality have been shown to be NP-Hard. Other meta-heuristic methods applied to the dig-limit problem present poor results, require too much computation time, or require high-quality initial solutions. The methodology outlined in this thesis does not exhibit these shortcomings, and performs well.

The high flexibility of the proposed GA approach allows for multiple forms of constraints. Global constraints can be applied during the quantification step of each iteration. Indeed, case studies were carried out to demonstrate that this approach can handle multiple destinations, multiple properties, mining direction, variable clustering size, and grade mixing.

Future work on this topic could involve the integration of post-processing methods using meta-heuristic tools such as SA to account for the mining of multiple benches at once and tackle the problem of grade mixing. Other applications could include generation of strategic mine plans, which would require the optimization of potentially millions of blocks. Thus, future improvements could also include the reimplementation of the algorithm in a compute optimized environment, running on C++. The potential for cloud computing for this method is highly attractive, particularly if the code is formulated for the use of Graphics Processing Unit optimized computing. Ultimately, GAs are a guided brute force approach: the more computing power applied to solve the problem, the better will be the ultimate solutions.

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