Maximality in the Semantics of Modified Numerals

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Abstract

This thesis develops a new theory of the semantics of modified numerals—a central topic in current linguistic research. The theory arises from a detailed investigation of a new paradigm in the interpretation of modified numerals. Specifically, numerical expressions like *less than n* and *between m and n* sometimes convey maximality, other times minimality, and still other times neither, depending on their linguistic environment. For instance, the sentence *Between five and ten guests arrived late* indicates that the *maximum* (or total) number of guests who arrived late is between five and ten, whereas the sentence *Between 20 and 30 potatoes can fill that sack* indicates that the *minimum* number of potatoes that can fill that sack is between 20 and 30. Conversely, the sentence *Between three and five students lifted the piano together* conveys neither maximality nor minimality: it is compatible with, say, a group of two students or a group of seven students having lifted the piano together, just as long as at least one group of three to five students did so too.

Building on previous works that deal with similar 'flips' between maximality and minimality (Beck and Rullmann 1999; von Fintel, Fox, and Iatridou 2014), I propose that the lexical semantics of certain numeral modifiers involves an 'informativity'-based maximality component, where the ordering of numbers that maximality operates on is based on how informative they are relative to some property of numbers. The crux of the theory is that, for a number to be maximally informative, sometimes it must be the largest, other times the smallest, and still other times it need not be either. This move to maximal informativity results in a theory that captures the full range of data discussed in the thesis.

Along the way, I also propose and examine in detail two fairly standard (but ultimately unsuccessful) analyses of a subset of the paradigmatic data (maximal and non-maximal readings), where maximality is of the 'standard' kind, i.e. based on the natural ordering of numbers. The availability of maximal and non-maximal readings is captured either by flexible scope (on one account) or by the optional presence of the maximality component (on the other account). Both theories, however, face overgeneration problems, and neither theory is able to derive genuinely 'minimal' readings.

Résumé

Cette thèse propose une nouvelle théorie de la sémantique des expressions numériques modifiées—un sujet central de la recherche récente en linguistique. Cette théorie est le résultat d'une étude détaillée d'un nouveau paradigme d'interprétation des expressions numériques modifiées. Plus précisément, les expressions numériques telles que *moins de n* et *entre m et n* servent parfois à exprimer un minimum, un maximum, ou aucun des deux, en fonction de leur environnement linguistique. Par exemple, la phrase *Entre cinq et dix invités sont arrivés en retard* sert à affirmer que le nombre *maximal* (ou total) d'invités qui sont arrivés en retard se trouve entre cinq et dix, alors que la phrase *Entre 20 et 30 patates peuvent remplir ce sac* signifie que le nombre *minimal* de patates nécessaires à remplir le sac se trouve entre 20 et 30. À l'inverse, la phrase *Entre trois et cinq étudiants ont soulevé le piano ensemble* ne sert à exprimer ni un maximum ni un minimum. Cette phrase est compatible avec une situation où un groupe de deux ou de sept étudiants ont soulevé un piano ensemble, tant qu'un groupe de trois à cinq étudiants l'a fait aussi.

En me basant sur des travaux antérieurs portant sur de telles alternances de sens entre l'expression d'un maximum et un minimum (Beck and Rullmann 1999; von Fintel, Fox, and Iatridou 2014), je propose que certains modificateurs d'expressions numériques possèdent dans leur sémantique lexicale une composante de maximisation « d'informativité », selon laquelle l'ordonnancement des nombres pertinent à la maximisation est basé sur la valeur d'information de ces nombres relativement à une propriété des nombres. Le cœur de cette théorie est que parfois, pour qu'un nombre soit maximalement informatif, il doit être le plus élevé alors que dans d'autres circonstances, il doit être le moins élevé, et parfois il ne doit être aucun des deux. Ce passage vers la maximisation du contenu informatif donne lieu à une théorie qui rend compte de toute l'étendue des données de cette thèse.

Au passage, je proposerai et examinerai en détail deux analyses relativement communes (mais ultimement infructueuses) d'un sous-ensemble des données du paradigme des alternances maximales/non-maximales, où la maximalité est du type « standard », c'est-à-dire basée sur l'ordre naturel des nombres. La disponibilité des lectures maximales et non-maximales est expliquée soit par la flexibilité de la portée (selon une analyse) ou par la présence optionnelle d'une composante de maximalité (selon une autre). Toutefois, les deux théories font face à des problèmes de surgénération et aucune des deux n'arrive à produire des lectures minimales adéquates.

Acknowledgments

I am extremely fortunate for having had the opportunity to do a Ph.D. in linguistics at McGill University. As an undergraduate student at Loyola University Chicago, I studied mathematics and classics and developed an interest in the formal study of grammar; however, LUC had no linguistics department or program (or even classes), so I had very little idea what modern linguistic theory was about, or how to even approach it. Luckily, I discovered that McGill's linguistics department offered a 'qualifying year program', intended to get prospective graduate students up to speed in linguistic theory before they apply for the actual graduate program. Thankfully, I was accepted to the QY program, arrived in Montreal, and immediately felt right at home, both academically (what I was learning) and in terms of the community. Before I knew it, I enrolled in the Ph.D. program, and now here I am. I have a lot of people to thank for helping me get this far.

First and foremost, I thank Bernhard Schwarz for his unparalleled supervision, not only throughout this dissertation, but also during my first evaluation paper, and more generally as someone I could always talk to, about semantics or otherwise. Having Bernhard as a co-supervisor made my dissertation journey about as pleasant as anyone could ever hope for a dissertation journey to be. No matter how muddled my thoughts about something were, he was always able to pinpoint the underlying issue, absorb and reflect on it, and reformulate it in clearer and more concise terms—all in a matter of seconds, usually. At many meetings, I arrived with no thoughts at all, only to leave an hour later with more than a dozen. When it came to my writing, Bernhard's speed and diligence in giving detailed comments knows no comparison; if I sent him a draft of something, I would get the draft back, marked up with comments, the very next morning, like clockwork. I could not have asked for a more dedicated supervisor.

I am also extremely indebted to Luis Alonso-Ovalle, my other co-supervisor. Luis was also lightning fast at providing detailed and critical comments on everything I wrote, and I secretly wonder whether he and Bernhard had a contest to see who could reply first. In addition, Luis's encyclopedic knowledge of semantic literature is unmatched: no matter what issue I was struggling with, Luis had a half dozen references to suggest and a whole book to lend me. His ability to synthesize my ideas and connect them with deeper trends in semantic theory was remarkable, and something I aspire to be able to do myself in the future. One last (but not least) thing I admire and appreciate about Luis is his positivity: he manages to balance honest criticism with sincere encouragement in a way that pushes you to be better without feeling helpless. His upbeat messages, no matter how small or simple, often had enormous impacts, especially towards the end.

Benjamin Spector deserves special thanks for his intellectual contributions to this thesis. When I discovered that Benjamin was working on similar issues as me, and coming up with a similar theory, I immediately felt worried. However, Benjamin kindly proposed that we join forces, and our collaboration over the last few months has made this thesis much better than it would have been otherwise—it was like having a third co-supervisor, really. (Ironically, in the end, I argue against both of our original theories.) The number of ideas that are due to Benjamin (and/or to colleagues he has discussed this work with, most notably Emmanuel Chemla and Philippe Schlenker) is large. I have done my best to signal these ideas both in the Preface and throughout the main text.

I thank Michael Wagner for his help, especially in the very early stages of this thesis, when I thought I might work on the focus sensitivity of certain modified numerals. Although the final thesis diverged quite a bit from that topic, I am grateful for the help that Michael gave me and has given me throughout the years. Discussing linguistics with Michael—be it semantics, syntax, or phonology, or any other area—is always a pleasurable and valuable experience.

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I thank Morgan Sonderegger for his supervision, in various capacities, over the last two years, from supervising my second evaluation paper (and teaching me about finite state transducers) to giving me advice about academia, jobs, how to write abstracts, how to act in an interview, and much, much more.

I thank Junko Shimoyama for introducing me to degree constructions during my very first year as a Ph.D. student, in a seminar on comparative constructions, and for talking with me about all kinds of interesting syntactic and semantic phenomena over the years, from Japanese to Italian.

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Preface

The main discoveries and generalizations presented in the first part of this thesis (chapter 1) were made independently by Benjamin Spector (Spector 2014) and me (Buccola 2015). After finding out that we were working on similar problems, and proposing similar solutions, we decided to collaborate, which resulted in a joint paper (Buccola and Spector 2015). A large part of this thesis derives from that collaboration, and I would like here to give Benjamin Spector credit for the portions that he is (sometimes solely) responsible for. Of course, any errors in this thesis are purely my own, and the ideas I ultimately defend may (and probably do) differ from those that Spector would defend. (In particular, I argue here in favor of one particular account of the puzzle, whereas in Buccola and Spector 2015 we remain fairly agnostic about which account fares best.)

Chapter 1, which presents the basic puzzle and lays the groundwork for the thesis, and chapter 2, which presents the first account (of four) of the puzzle, are expanded and revised versions of what appear in Buccola 2015; they are also close to what is presented in Buccola and Spector 2015.

Chapter 3 presents three predictions for the theory developed in chapter 2. The first prediction (section 3.2) was first described in Buccola 2015. The second prediction is due to Benjamin Spector. The third was made jointly. All three also appear in Buccola and Spector 2015.

Chapter 4 develops an idea by Benjamin Spector (Spector 2014) and is a (very) slightly revised version of what appears in Buccola and Spector 2015.

Chapter 5 is almost totally brand new. Parts of this material were presented at Sinn und Bedeutung 20 and will appear as Buccola (in prep.). Notably, the puzzle that arises for *between* (section 5.6) appears in Buccola and Spector 2015.

Chapter 6 is a heavily updated and more in-depth version of what appears in Buccola and Spector 2015. The original, underlying idea is due to Philippe Schlenker.

Chapter 7 is a revised and expanded version of what appears in Buccola and Spector 2015.

Note to the reader. The PDF version of this thesis contains hyperlinks (though they are not in color). Click on chapter numbers, section numbers, footnote numbers, example numbers, years (for citations), etc. to jump to the appropriate chapter, section, etc.

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Chapter 1

Introduction

The goal of this thesis is to solve a previously unnoticed puzzle concerning the interpretation of so-called *modified numerals*.[1, 2] The puzzle is this: certain modified numerals, like *less than five* and *between two and four*, seem to mean different things (and, in particular, to have different monotonicity properties) depending on the types of nominal and verbal predicates they combine with. When they combine with predicates that license downward inferences on their arguments, i.e. inferences from groups to subgroups (typically, distributive predicates, like *frowned* and *smiled*, as in *Less than five students frowned*), they convey a kind of upper bound (and are non-upward-monotone), whereas when they combine with predicates that do not license downward inferences (typically, collective predicates, like *surrounded the castle* and *lifted the piano together*, as in *Less than five students lifted the piano together*), they do not convey the same kind of upper bound (and are upward monotone). And while it is easy to formulate a lexical entry for such modified numerals that works for each of the two individual cases, it does not seem possible

[1]Examples of modified numerals include *more than three*, *less than five*, *between two and four*, *up to ten*, and *at most seven*. The expression that combines with the numeral (e.g. *more than*, *up to*, *at least*) is called the *numeral modifier*. The recent literature on modified numerals is quite large and includes, but is not limited to: Krifka 1999; Geurts and Nouwen 2007; Büring 2008; Nouwen 2010; Schwarz, Buccola, and Hamilton 2012; Rett 2014; Kennedy 2015. Nouwen 2010 in particular has become an influential paper on the description and classification of a large number of numeral modifiers.

[2]The examples in this thesis systematically have *less than* rather than *fewer than*, even when the modified numeral combines with a count noun (*less than five students*). Readers who find *fewer than* more natural in such contexts are welcome to mentally replace *less than* by *fewer than*.

to formulate a single entry that works for both cases. Ultimately, I argue that such a formulation *is* possible, but only once a more complete battery of data is brought to bear on the puzzle. This chapter presents the initial, core data that drives this thesis, describes the puzzle in detail, and lists the main objectives of the thesis. It is organized as follows. Section 1.1 introduces the initial data to be accounted for and describes basic puzzle in an informal way. Section 1.2 provides some necessary background in terms of the plural semantics framework that I will assume and presents the puzzle in a more explicit way. Finally, section 1.3 provides a roadmap of the thesis.

1.1 The first and basic puzzle

The main intuitive difference between expressions like *more than three*, on the one hand, and *less than five* and *between two and four*, on the other hand, is that the latter, but not the former, convey some kind of an upper bound. For example, whereas (1) is intuitively consistent with ten students having frowned, (2) and (3) are not. Rather, they both convey an upper bound of four on the number of students who (may have) frowned. By contrast, (1) conveys no upper bound at all; any number of students (above 3) may have frowned.

(1) More than three students frowned.

(2) Less than five students frowned.

(3) Between two and four students frowned.

Put differently, (2) and (3) both intuitively entail (4), while (1) does not.

(4) It is not the case that five or more students frowned.

Let us call such readings for (2) and (3) *upper-bounded* readings, and note that no other reading for (2) or (3) is intuitively available. Furthermore, note that (2) is consistent with no students having frowned; that is, whereas (1) and (3) both entail that at least some student(s) frowned, (2) does not. To be sure, (2) may trigger the implicature that some student(s) frowned, but this should not be viewed as part of the literal meaning of the sentence: for example, (5) is true even if there were some mornings when no students at all frowned.

(5) Every morning last week, less than five students frowned.

Now consider (6a), in which a *collective* interpretation of *lifted the piano* is forced by the expression *together*. Intuitively, (6a) does *not* entail (6b): in a context where, say, three semantics students lifted the piano together, and seven phonology students lifted the piano together, (6a) is true in virtue of the semanticists' feat of piano lifting, while (6b) is false in virtue of the phonologists' feat of piano lifting.[3]

(6) a. Less than five students lifted the piano together.
 b. It is not the case that five or more students lifted the piano together.

Moreover, (6a), unlike (2), seems to have an existential entailment: if no students at all lifted the piano (only three professors did, say), then (6a) is judged false.[4] In addition, this existential inference seems to really be an entailment, and not an implicature: (7) is false if there was at least one morning where no students lifted the piano.[5]

(7) Every morning before school last week, less than five students lifted the piano

[3]This reading can often be made even more salient and natural by adding the word *surprisingly*: intuitively, what is surprising is that (a group of) so few students managed to lift the piano together (namely, the three semanticists), regardless of whether other (larger groups of) students also lifted the piano together (e.g. the phonologists). For example, if I expect that it takes at least five students to lift the piano, and it turns out that three students lift it together, then (6a) is both felicitous and true.

[4]Again, if I expect that it takes at least five students to lift the piano, and it turns out that *no* (group of) students at all lift it, then (6a) is false.

[5]This also explains why a sentence like *Less than five babies lifted the piano together* feels false, not true, in most contexts. Compare with *Less than five babies were drinking at the bar*, which feels true (though of course very misleading), not false, in most contexts.

together.

A good paraphrase of what (6a) means is 'A group of less than five students lifted (managed to lift) the piano together', which makes it clear (i) that other, larger groups of students may also have lifted the piano together, and (ii) that at least some (group of) student(s) lifted the piano together. Let us call this reading a *non-upper-bounded, existential* reading.[6]

Similarly, consider (8a), in which a *cumulative* (or *co-dependent*) interpretation of *drank more than ten beers* is forced by the expression *between them*. (8a) does not intuitively entail (8b): in a context where, say, three semantics students drank fifteen beers between them, and seven phonology students drank only five beers between them, (8a) is true in virtue of the semanticists' feat of beer drinking, while (8b) is false in virtue of the whole group of ten students having drunk twenty beers between them.[7]

(8) a. Between two and four students drank more than ten beers between them.

b. It is not the case that five or more students drank more than ten beers between them.

Again, a good paraphrase of what (8a) means is 'A group of between two and four students drank (managed to drink) more than ten beers between them', which makes it clear that other, larger groups of students may also have drunk more than ten beers between them. It also makes it clear that (8a) has an existential entailment, but this is already quite clear and unsurprising in (8a) given the contribution of the lower-bounding numeral *two* in *between two and four*.[8]

[6]Whether (6a) also has an upper-bounded reading is unclear; I will address this question in chapter 3. The important point now is that a non-upper-bounded, existential reading is available.

[7]Assume that the semantics students and phonology students are disjoint sets of students. Or just suppose that there were, in total, five or more students who drank, in total, more than ten beers between them.

[8]Once again, though, as with (6a), the analogous sentence with *less than* (*Less than five students drank more than ten beers between them*) has an existential entailment, viz. that at least one group of less than five students drank more than ten beers between them.

Thus, (6a) and (8a) both have *non*-upper-bounded readings. On the basis of these readings, and the paraphrases that I have given, it seems natural to entertain the following lexical entries for *less than five* and *between two and four*, where x ranges over groups (or *sums*) of individuals and $\textbf{card}(x)$ is the number of atomic parts of x (Link 1983).[9]

(9) a. $[\![\text{less than five}]\!] = \lambda P_{et} . \lambda Q_{et} . \exists x[\textbf{card}(x) < 5 \wedge P(x) \wedge Q(x)]$

b. $[\![\text{between two and four}]\!] = \lambda P_{et} . \lambda Q_{et} . \exists x[2 \leq \textbf{card}(x) \leq 4 \wedge P(x) \wedge Q(x)]$.

As I will show in section 1.2, however, lexical entries of this kind have disastrous consequences for sentences like (2) and (3)—they lead to a well-known problem commonly called Van Benthem's problem (first described in Van Benthem 1986).

For sentences (2) and (3), lexical entries along the following lines would work. These entries are familiar from Generalized Quantifier Theory (GQT; Barwise and Cooper 1981), but updated to fit with the plural semantics framework that I will introduce in section 1.2 (see, e.g., Winter 2001, and the references therein).[10]

(10) a. $[\![\text{less than five}]\!] = \lambda P_{et} . \lambda Q_{et} . \neg\exists x[\textbf{card}(x) \geq 5 \wedge P(x) \wedge Q(x)]$

b. $[\![\text{between two and four}]\!] = \lambda P_{et} . \lambda Q_{et} . \exists x[\textbf{card}(x) \geq 2 \wedge P(x) \wedge Q(x)]$
$\wedge \neg\exists x[\textbf{card}(x) > 4 \wedge P(x) \wedge Q(x)]$

These entries, however, would incorrectly predict that (6a) and (8a) entail (6b) and (8b), respectively.

[9] I will properly introduce these notions, and more, in section 1.2.

[10] To make the connection with GQT even clearer, the entries in (10) could be written equivalently using a maximality operator, as shown below. Since I will ultimately adopt the view that modified numerals denote generalized quantifiers over degrees, rather than GQT-style determiners, I will say nothing further about these entries. I will, however, have much more to say about maximality (and maximality operators).

(i) a. $[\![\text{less than five}]\!] = \lambda P_{et} . \lambda Q_{et} . \max(\lambda n . \exists x[\textbf{card}(x) = n \wedge P(x) \wedge Q(x)]) < 5$
(cf. GQT: $[\![\text{less than five}]\!] = \lambda X . \lambda Y . |X \cap Y| < 5$)

b. $[\![\text{between two and four}]\!] = \lambda P_{et} . \lambda Q_{et} . 2 \leq \max(\lambda n . \exists x[\textbf{card}(x) \wedge P(x) \wedge Q(x)]) \leq 4$
(cf. GQT: $[\![\text{between two and four}]\!] = \lambda X . \lambda Y . 2 \leq |X \cap Y| \leq 4$)

We thus face a somewhat surprising situation, where the very same expression (*less than five* or *between two and four*) seems to be interpreted differently depending on its linguistic environment, and while it is easy to formulate lexical entries that work for each of the two individual cases, it is not straightforward to formulate a single entry that works for both cases.

Let me also point that viewing the puzzle in terms of lexical ambiguity, while *ad hoc*, makes several correct predictions beyond just perceived truth conditions. For example, the entry given in (10a) makes *less than five* downward monotone on both its restrictor (subject NP argument) and nuclear scope (VP argument). As a result, if this entry is used, then we expect *less than five* to license downward inferences (that is, inferences from groups to subgroups) in both its restrictor and its nuclear scope, and to license weak negative polarity items (NPIs) in both its restrictor and its nuclear scope (Ladusaw 1979). By contrast, the entry in (9a) treats *less than five* as an existential quantifier, hence makes it upward monotone on both its restrictor and nuclear scope. So if this entry is used, then we expect *less than five* to license upward inferences in both its restrictor and its nuclear scope, and thus to *not* license weak NPIs either in its restrictor or in its nuclear scope. The following contrasts regarding inference patterns and NPI licensing are therefore in line with truth-conditional judgments.

(11) a. Less than five people bought a piano.
  $\Rightarrow$ Less than five people bought a grand piano.
  ($\not\Rightarrow$ Less than five people bought a musical instrument.)
b. Less than five people bought any piano(s).

(12) a. Less than five people lifted a piano together.
  $\Rightarrow$ Less than five people lifted a musical instrument together.
  ($\not\Rightarrow$ Less than five people lifted a grand piano together.)
b. *Less than five people lifted any piano(s) together.

In (11), the subject combines with a distributive predicate, and if we use the lexical entry in (10a), then the nuclear scope of *less than five* is a downward entailing environment; thus, downward inferences are licensed, as is the occurence of the NPI *any*. In (12), however, the subject combines with a collective predicate, and if the entry in (9a) is used, then the nuclear scope of *less than five* is an upward entailing environment; thus, upward inferences are licensed, and the occurrence of *any* is not licensed.

Of course, it is again mysterious why we need two distinct rules of interpretation for such modified numerals, one for each case. These contrasts in inference patterns and acceptability judgments thus illustrate the very same puzzle as the one that was previously highlighted in terms of truth-conditional intuitions.

The puzzle, then, is how to derive upper-bounded, non-existential readings for sentences where *less than n NP* or *between m and n NP* combines with a distributive predicate like *frowned*, while still being able to derive non-upper-bounded, existential readings for sentences where they combine with non-distributive predicates like *lifted the piano together* and *drank more than ten beers between them*. As I hope to convince the reader, the solution to this puzzle is not straightforward. I will eventually settle on a theory that I think is right, but getting there will require that we first look at a number of inadequate theories, go down several misleading paths, and consider more and more puzzling data.

Before continuing, however, I should say a few words about other 'upper-bounding' numeral modifiers, such as *at most*, *up to*, and *exactly*. Interestingly, these numeral modifiers appear not to pattern like *less than* and *between*: even in non-distributive contexts, they seem to have only upper-bounded readings. For example, in a context where one group of three semantics students lifted the piano and another group of seven phonology students lifted the piano, (13a) feels only marginally true and felicitous—and (13b) and (13c) feel even less so.

(13) a. Exactly three students lifted the piano together.

b. At most three students lifted the piano together.

c. Up to three students lifted the piano together.

That *less than* and *between* should behave differently from *at most* and *up to* when it comes to this paradigm may perhaps not come as a big surprise, given the recent evidence that the former numeral modifiers are part of a different class than the latter: according to Nouwen (2010), *less than* and *between* are 'class A' numeral modifiers, while *at most* and *up to* are 'class B' numeral modifiers, the distinction having to do with whether or not the modified numeral in question gives rise to ignorance inferences. Whether this class distinction is in some way relevant to, or responsible for, the observations made in this thesis is a question I leave for future research. Given that *at most*, *up to*, and *exactly* do not participate in the kinds of inferences that I am interested in, from here on I will ignore such numeral modifiers and concentrate exclusively on *less than* and *between*, leaving a more exhaustive analysis for future research.

1.2 An adjectival theory of numerals

I will now present a simple theory of numerals and plurals, often called the *adjectival theory* of numerals,[11] and show how it leads to some well-known problems for expressions like *less than five* and *between two and four*. Although this theory ultimately fails to account for all of the data, it nevertheless frames the puzzle described above in a more explicit way and forms the backdrop against which the main theories of this thesis are elaborated.

1.2.1 Technical background and conventions

The semantic framework that I adopt is largely that of Heim and Kratzer 1998, in which syntactic structures called Logical Forms (LFs) are mapped to various kinds of semantic

[11] Landman (2004) traces the adjectival theory back to Bartsch 1973, Verkuyl 1981, and Link 1987. Szabolcsi (2010) writes that it probably originates with Milsark 1977 and Verkuyl 1981.

values (denotations) by the semantic component of the grammar. I will depart slightly from Heim and Kratzer 1998 by using less English and more logical expressions (of some appropriate logic) as a metalanguage. The reasons for this choice are mainly brevity and clarity. In particular, I do not assume any indirect translation process of the kind found, e.g., in Montague 1973; rather, I assume that English expressions directly denote model-theoretic objects, with logical notation being used to conveniently stand for such objects. For example, the denotation of *students* relative to a model M and variable assignment g, notated by $[\![\text{students}]\!]^{M,g}$, will be written as $\lambda x_e \,.\, \mathbf{students}(x)$, rather than $\lambda x_e \,.\, x$ are students (or that function f from individuals to truth values such that $f(x) = 1$ just in case x are students). I will also consistently omit superscripts on denotation brackets when no confusion arises.

Following Link 1983 and subsequent work, let us assume that our domain of individuals, D_e, includes both ordinary individuals like John (j), Mary (m), this table (t), and that chair (c), as well as *sums* of individuals, such as John and Mary ($j \sqcup m$), Mary and that chair ($m \sqcup c$), and so on.[12] We will take the sum formation operation, $\sqcup$, to be associative,[13] commutative,[14] and idempotent.[15] D_e is closed under sum formation[16] and is partially ordered by the *part of* relation, $\sqsubseteq$, induced by the sum formation operation.[17] The *proper part of* relation, $\sqsubset$, is defined as usual.[18] An *atom* is an individual with no proper subpart,[19] and the *cardinality* of a sum x, notated by $\mathbf{card}(x)$, is the number of atoms that are part of x.[20] Except where indicated (see chapter 6), I take the standard view that there is no null (empty) individual, i.e. there is no individual x such that $\mathbf{card}(x) = 0$.

[12] I use the terms *group*, *plurality*, and *sum* interchangeably, with no theoretical distinction between them.
[13] $\forall x, y, z[(x \sqcup y) \sqcup z = x \sqcup (y \sqcup z)]$. (This means that sums have a 'flat' structure.)
[14] $\forall x, y[x \sqcup y = y \sqcup x]$.
[15] $\forall x[x \sqcup x = x]$.
[16] $\forall x, y[x, y \in D_e \rightarrow x \sqcup y \in D_e]$.
[17] $\forall x, y[x \sqsubseteq y \leftrightarrow x \sqcup y = y]$.
[18] $\forall x, y[x \sqsubset y \leftrightarrow x \sqsubseteq y \wedge x \neq y]$.
[19] $\forall x[\mathbf{atom}(x) \leftrightarrow \forall y[y \sqsubseteq x \rightarrow y = x]]$. Note: In chapter 6, I will entertain the possibility of adding to the domain of individuals D_e a *null individual*, which will be part of every other individual, but will not itself be an atom. This move will therefore require that the notion of *atom* be redefined (see chapter 6, footnote 24).
[20] $\forall x[\mathbf{card}(x) = |\{y : y \sqsubseteq x \wedge \mathbf{atom}(y)\}|]$

For a more detailed overview, see Champollion and Krifka 2015 and Nouwen 2015.

In addition to the ordinary types e (for individuals) and t (for truth values) (and, in chapter 6, s for possible worlds), I also assume a type d for degrees. Since this thesis is concerned just with modified numeral constructions, I take the domain of degrees, D_d, to simply be the set of natural numbers $\mathbb{N} = \{0, 1, 2, \ldots\}$; that is, I identify degrees with numbers (hence also use the words 'degree' and 'number' interchangeably). D_d will (eventually) serve both as the domain for denotations of numerals, e.g. *three* denotes 3, and as the range of the cardinality operator, **card**, e.g. $\textbf{card}(x) = 3$. Note that even though there is no empty sum, i.e. the range of **card** does not include 0 (except in one part of chapter 6), 0 is still a degree in D_d; thus, a metalanguage formula like $\exists n\phi$ is verified if $\phi[n \mapsto 0]$ (the result of substituting free occurrences of n in ϕ with 0) is true.

For typographical convenience, the types of functions are written $(\sigma\tau)$ (rather than, say, $\langle\sigma, \tau\rangle$), and I always drop the outermost parentheses. Subscripts on (occurrences of) variables indicate the type of the variable, e.g. x_e has type e ($x \in D_e$). If $f_{\sigma t}$ is a boolean function (for some type σ), and x_σ is in the domain of f, then for convenience I write $f(x)$ for $f(x) = 1$, and $\neg f(x)$ for $f(x) = 0$. I also talk interchangeably about boolean functions and the sets they characterize, so that, for example, $P_{dt} = \varnothing$ means $\{n_d : P(n)\} = \varnothing$, and $P_{et} \subseteq Q_{et}$ means $\{x_e : P(x)\} \subseteq \{x_e : Q(x)\}$, and so on. Finally, I assume that $\rightarrow$ and $\leftrightarrow$ bind less tightly than other logical operators ($\wedge$, $\vee$, ...), so that, e.g., $p \wedge q \rightarrow r$ always means the same thing as $[p \wedge q] \rightarrow r$, never $p \wedge [q \rightarrow r]$.

1.2.2 Bare numerals, distributivity, and plurality

Consider sentence (14), the meaning of which can be paraphrased as, 'There is a group x such that x are Canadians, x are students, and x attended.'

(14) Canadian students attended.

One way to analyze (14) is to take *Canadian* and *students* to each be a predicate of plural individuals (they denote sets of sums of individuals), which intersect to form the complex property of being a sum whose atomic parts are both Canadian and students.[21]

(15) a. $[\![\text{Canadian}]\!] = \lambda x_e \,.\, \mathbf{Canadian}(x)$

b. $[\![\text{students}]\!] = \lambda x_e \,.\, \mathbf{students}(x)$

c. $[\![\text{Canadian students}]\!] = \lambda x_e \,.\, \mathbf{Canadian}(x) \wedge \mathbf{students}(x)$

Attended likewise denotes a set of sums (all those sums whose atomic parts attended). Finally, let us assume that a silent existential determiner, $\varnothing_\exists$, akin to *some*, connects everything up: $\varnothing_\exists$ combines with *Canadian students* and *attended* and says that the two sets they denote have an individual in common.[22]

(16) $[\![\varnothing_\exists]\!] = \lambda P_{et} \,.\, \lambda Q_{et} \,.\, \exists x[P(x) \wedge Q(x)]$

On this analysis, (14) is true iff there is a group of individuals who are Canadian and are students and attended, as desired. An appropriate LF, and the derived truth conditions, are shown in (17).

(17) a. Canadian students attended.

b. [$\varnothing_\exists$ [Canadian students]] attended

c. $\exists x[\mathbf{Canadian}(x) \wedge \mathbf{students}(x) \wedge \mathbf{attended}(x)]$

Now consider (18).

[21]For concreteness, I assume the following rule of *Predicate Modification* (Heim and Kratzer 1998).

(i) **Predicate Modification**
If the daughters of α are β and γ, and β and γ both have type *et*, then $[\![\alpha]\!] = \lambda x_e \,.\, [\![\beta]\!](x) \wedge [\![\gamma]\!](x)$.

[22]Assuming a null determiner in the analysis of bare plurals and (numerical) indefinites is fairly common (see Krifka et al. 1995 for discussion). We could also assume a global rule of existential closure, as in Heim 1982. The choice is, as far as I can tell, immaterial to the cases we will be concerned with.

(18) Three students frowned.

It seems natural to try and analyze (18) analogously to (14) by taking *three* to be a predicate of pluralities (just like the adjective *Canadian*): it denotes the set of all sums with three atomic parts, as shown in (19).[23]

(19) $[\![\text{three}]\!] = \lambda x_e \,.\, \mathbf{card}(x) = 3$

We then intersect it with *students* to form the complex property of being a three-membered plurality of students. *Frowned* (like *attended*) is a predicate of pluralities, and the silent determiner connects everything up as before. (Examples of this kind of setup can be found in Link 1987 and Krifka 1999.)

(20) a. Three students frowned.
b. [$\varnothing_\exists$ [three students]] frowned
c. $\exists x[\mathbf{card}(x) = 3 \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]$

The sentence then winds up meaning that a group of three students frowned.

It is important to note that (20c) does not entail any upper bound on the number of students who frowned. Thus, for example, (20a) is predicted to be consistent with, and in fact entailed by, ten students having frowned. The reason is that, if there is a plurality of ten students who frowned, then there is necessarily also a plurality of nine, eight, . . . , three, two, and one student(s) who frowned: for example, just pick any three (distinct) students among the plurality of ten students who frowned, and those three are a plurality of three students who frowned.

The more precise reason why (a group of) ten students frowing entails that (a group of) three students frowned is because *students* and *frowned* are both distributive predicates.[24]

[23]There is, of course, an alternative route, which is to assume that *three* is a determiner, rather than adjectival, as is done in GQT.

[24]In the terminology of Krifka 1989, they refer divisively, or have 'divisive reference'.

For our purposes, distributivity can be defined as in (21).[25, 26] Thus, *frowned*, for example, is distributive because if, say, Ann, Bill, and Carol frowned (**frowned**$(a \sqcup b \sqcup c)$), then it intuitively follows that Ann and Bill frowned (**frowned**$(a \sqcup b)$), that Ann and Carol frowned (**frowned**$(a \sqcup c)$), that Carol frowned (**frowned**(c)), and so on. And the same holds for *students*.

(21) **Distributivity**

P_{et} is *distributive* iff $\forall x, y[P(x) \wedge y \sqsubseteq x \rightarrow P(y)]$.

(... iff, if P is true of a sum x, then P is true of every subpart of x.)

This ensures that the argument in (22) is valid. Thus, on the analysis in (20), (18) is predicted to mean that at least three students frowned.

(22)

	$\exists x[\mathbf{card}(x) = 10 \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]$	(Ten students frowned.)
	$\forall x, y[\mathbf{students}(x) \wedge y \sqsubseteq x \rightarrow \mathbf{students}(y)]$	(*students* is distributive.)
	$\forall x, y[\mathbf{frowned}(x) \wedge y \sqsubseteq x \rightarrow \mathbf{frowned}(y)]$	(*frowned* is distributive.)
$\vDash$	$\exists x[\mathbf{card}(x) = 3 \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]$	(Three students frowned.)

It is well known, however, that sentences like (18) are most often interpreted as implying an upper bound, i.e. *three* often seems to mean ‘exactly three’, not ‘at least

[25]For convenience, I will say that an (object-language) expression like *frowned* is ‘distributive’ to mean that its extension is distributive in the sense of (21). The same holds, *mutatis mutandis*, for cumulative expressions below.

[26]Strictly speaking, the definition in (21) is too weak to capture the intended notion. For example, we can imagine a world (or model) in which Ann lifted the piano (by herself), Bill lifted the piano (by himself), Ann and Bill also lifted the piano together, and no one else lifted it. Relative to this world, the extension of *lifted the piano together*—call it P—is distributive according to (21) because $P = \{a \sqcup b, a, b\}$. However, intuitively, *lifted the piano together* should count as non-distributive. The problem, of course, is that we are only considering the extension of *lifted the piano together* in one particular world, in which it so happens that the extension has the property in (21). The difference between *lifted the piano together*, on the one hand, and *frowned*, on the other hand, is that, even though there may be worlds (models) in which the extension of *lifted the piano together* has the property in (21), its extension relative to many other worlds does *not* have that property; by contrast, the extension of *frowned* has this property in *every* world. In chapter 6, I will define the (related) notion of a downward scalar degree predicate (and other types of scalarity) in a way that takes into account extensions across worlds. For the present purpose, however, the definition in (21) ought to suffice. I thank Alan Bale for bringing this issue to my attention.

three'. One approach to capturing this intuition consists in viewing this stronger meaning as the result of pragmatic strengthening, i.e. as a kind of scalar implicature that results from the fact that numerals form a scale (e.g. via Gricean reasoning). There are, however, many reasons why such an account cannot be the full story. In particular, the 'strong' (or 'exactly', or 'two-sided') meaning of numerals tends to be easily accessible even in syntactic environments where standard scalar items tend to lose their strengthened meaning (see, e.g., Horn 2006, Geurts 2006, Spector 2013 for a survey, and Kennedy 2015 for a recent proposal). There is also psycholinguistic evidence (Musolino 2004; Papafragou and Musolino 2003; Marty, Chemla, and Spector 2013) suggesting that the strong meaning of numerals is acquired and processed differently from the strengthened meaning of scalar items. The consensus view seems to be that numerals, in some way or another, are ambiguous between the 'at least' (or 'one-sided') reading that I am assuming here and a stronger reading that implies an upper bound. At this point, however, I only consider the reading derived in (20), but I return to this issue in chapter 4.

Now, based on our assumptions so far, when a numerical phrase like *three students* combines instead with a non-distributive predicate, like *lifted the piano* on its collective interpretation, as in (23), then the numeral is predicted to get an 'exactly three' interpretation—in a certain sense. What (23) is predicted to mean is that a group of three students lifted the piano together. It is not expected to be verified by a group of, say, ten students having lifted the piano together, but it is still consistent with such a scenario.[27]

(23) a. Three students lifted the piano.

b. [$\varnothing_\exists$ [three students]] [lifted the piano]

c. $\exists x[\textbf{card}(x) = 3 \wedge \textbf{students}(x) \wedge \textbf{lifted}(x)]$

[27] For simplicity, I analyze transitive predicates like *lifted the piano* and *surrounded the castle* as 1-place predicates rather than 2-place predicates.

More precisely, *lifted the piano*, on its collective interpretation, lacks the distributivity property that we saw with *frowned*: if, say, Ann, Bill, and Carol (collectively) lifted the piano ($\mathbf{lifted}(a \sqcup b \sqcup c)$), it does not necessarily follow that Ann and Bill lifted the piano ($\mathbf{lifted}(a \sqcup b)$). Thus, the following argument is *not* valid.

(24) $\exists x[\mathbf{card}(x) = 10 \wedge \mathbf{students}(x) \wedge \mathbf{lifted}(x)]$ (Ten students lifted the piano.)
$\forall x, y[\mathbf{students}(x) \wedge y \sqsubseteq x \rightarrow \mathbf{students}(y)]$ (*students* is distributive.)
$\nvDash$ $\exists y[\mathbf{card}(y) = 3 \wedge \mathbf{students}(y) \wedge \mathbf{lifted}(y)]$ (Three students lifted the piano.)

These predictions do indeed accord with judgments about the meaning of (23).

1.2.3 Modified numerals, cumulativity, and Van Benthem's problem

A straightforward extension of the adjectival theory of bare numerals to modified numerals might maintain that modified numerals, just like bare numerals, are predicates of pluralities, as in (25), (26), and (27) below. On this view, the numeral modifier simply changes the relation between the numeral and the sum's cardinality: whereas the bare numeral *three* has =, the modified numeral *more than three* has >, *less than five* has <, and so on. As before, the null determiner combines the nominal and verbal predicates.

(25) $[\![\text{more than three}]\!] = \lambda x_e \,.\, \mathbf{card}(x) > 3$

(26) $[\![\text{less than five}]\!] = \lambda x_e \,.\, \mathbf{card}(x) < 5$

(27) $[\![\text{between two and four}]\!] = \lambda x_e \,.\, 2 \leq \mathbf{card}(x) \leq 4$

This approach works fine for expressions like *more than three*, as (28) illustrates. The truth conditions simply say that a group of more than three students frowned, which seems quite correct.

(28) a. More than three students frowned.

b. [$\varnothing_\exists$ [[more than three] students]] frowned

c. $\exists x[\mathbf{card}(x) > 3 \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]$

However, we run into serious trouble with expressions like *less than five* and *between two and four*. Consider (29), which is predicted to be true iff a group of less than five students frowned.[28]

(29) a. Less than five students frowned.

b. [$\varnothing_\exists$ [[less than five] students]] frowned

c. $\exists x[\mathbf{card}(x) < 5 \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]$

There are two problems with the truth conditions in (29c). First, since we are assuming that there is no null individual (i.e. every sum has a cardinality of at least 1), (29c) entails that at least one student frowned;[29] however, as we have seen, (29a), unlike (28a), is judged true even if no students frowned (it intuitively has no existential entailment). Call this the *existential entailment problem*.

Second, (29c) (just like the bare numeral case) does not entail any upper bound. Thus, for example, (29a) is predicted to be consistent with, and in fact (just like in the bare numeral case) entailed by, ten students having frowned, as the argument in (30) illustrates.[30] And yet, as we have seen, (29a) is judged false if five or more students frowned (it intuitively entails an upper bound). This latter problem was first pointed out

[28]These are the same truth conditions that we would arrive at by using the lexical entry in (9a) from section 1.1, where *less than five* is treated as a kind of existential determiner, rather than as a predicate of individuals.

[29]In fact, if we assume that the plural noun *students* only has sums of cardinality 2 or more in its extension, then (29a) is expected to entail that at least *two* students frowned. Following Hoeksema 1983, Van Eijk 1983, Krifka 1989, Sauerland 2003, Spector 2007, and Zweig 2009, among others, I assume that plural nouns include atomic individuals in their extension. One reason in support of this view is that, if *students* had only sums of cardinality 2 or more in its extension, then a sentence like *No students passed the test* would incorrectly be predicted to be true if exactly one student passed.

[30]I am assuming in (30) that *students* and *frowned* both have atomic individuals in their extensions (see footnote 29), but the argument would be valid even if the latter only had sums of cardinality 2 or more in their extensions, since $2 < 5$.

by Van Benthem (1986) and has since become known as *Van Benthem's problem*.[31]

(30)
$\exists x[\mathbf{card}(x) = 10 \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]$ (Ten students frowned.)
$\forall x, y[\mathbf{students}(x) \wedge y \sqsubseteq x \rightarrow \mathbf{students}(y)]$ (*students* is distributive.)
$\forall x, y[\mathbf{frowned}(x) \wedge y \sqsubseteq x \rightarrow \mathbf{frowned}(y)]$ (*frowned* is distributive.)
$\vDash \exists x[\mathbf{card}(x) = 1 \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]$ (One student frowned.)
$\vDash \exists x[\mathbf{card}(x) < 5 \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]$ (Less than five students frowned.)

What these two problems (the existential entailment problem and Van Benthem's problem) together amount to is that (29a) should have the reading 'at least one student frowned', which of course it does not.

A similar problem arises with (31a), which, under the LF in (31b), is predicted to have the truth conditions in (31c),[32] which are equivalent to (31d).

(31)
a. Between two and four students frowned.
b. [$\varnothing_\exists$ [[between two and four] students]] frowned
c. $\exists x[2 \leq \mathbf{card}(x) \leq 4 \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]$
d. $\exists x[\mathbf{card}(x) \geq 2 \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]$

In this case, an existential entailment is predicted, but this is not problematic, since (31a) surely does entail (31d). However, as with (29a), the predicted reading for (31a) ('at least two students frowned') fails to entail an upper bound, which is again an instance of Van Benthem's problem.

[31] Marty, Chemla, and Spector (2015) present experimental evidence suggesting that, although not detectable introspectively, this reading does influence speakers' behavior in truth-value judgment tasks. I set aside why this might be and henceforth assume that such readings are simply unavailable, hence should be explained as such.

[32] Again, these are the same truth conditions that we would arrive at by using the lexical entry in (9b) from section 1.1, where *between two and four* is treated as simple existential determiner.

Returning to (29a), repeated in (32a), its actual, attested truth conditions are better represented as in (32b), or equivalently, (32c).[33, 34]

(32) a. Less than five students frowned.

b. $\neg\exists x[\mathbf{card}(x) \geq 5 \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]$

c. $\max(\lambda n \,.\, \exists x[\mathbf{card}(x) = n \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]) < 5$

According to these truth conditions, no plurality x of students who frowned has five or more atomic parts. Now, this, in and of itself, does not exclude there being, say, 100 distinct pluralities of students who frowned, each with cardinality less than 5—in which case, there would be many more than five frowning students. For example, suppose we had the following:

$$\begin{array}{lll} & \mathbf{students}(a \sqcup b) & \wedge \quad \mathbf{frowned}(a \sqcup b) \\ \wedge & \mathbf{students}(c \sqcup d) & \wedge \quad \mathbf{frowned}(c \sqcup d) \\ \wedge & \mathbf{students}(e \sqcup f) & \wedge \quad \mathbf{frowned}(e \sqcup f) \\ \wedge & \ldots & \end{array}$$

(with $a \neq b \neq c \ldots$). Since *students* and *frowned* are both distributive, this would incorrectly entail there being five or more students who frowned:

[33] In a completely parallel way, the truth conditions we need for (31a) are represented by the following two (equivalent) formulas:

(i) a. $\exists x[\mathbf{card}(x) \geq 2 \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)] \wedge \neg\exists x[\mathbf{card}(x) > 4 \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]$

b. $2 \leq \max(\lambda n \,.\, \exists x[\mathbf{card}(x) = n \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]) \leq 4$

These truth conditions, and those in (32), are what we would derive if we used the GQT-style lexical entries in (10) in section 1.1.

[34] We must assume that $\max(\varnothing) = 0$ to allow (32b) to be true in a scenario where no students frowned, i.e. to avoid an existential entailment problem. See chapter 2 (section 2.3.3) for a detailed discussion.

$$\begin{aligned} & \textbf{students}(a) \wedge \textbf{frowned}(a) \\ \wedge\ & \textbf{students}(b) \wedge \textbf{frowned}(b) \\ \wedge\ & \textbf{students}(c) \wedge \textbf{frowned}(c) \\ \wedge\ & \ldots \end{aligned}$$

To be sure, (29a) is not judged to be consistent with five or more students having frowned, as we have seen. However, such a state of affairs is in fact impossible, because *students* and *frown* are not only distributive, they are also *cumulative*,[35] as defined in (33): for example, if Ann frowned ($\textbf{frowned}(a)$) and Bill frowned ($\textbf{frowned}(b)$), then it follows that Ann and Bill frowned ($\textbf{frowned}(a \sqcup b)$).[36]

(33) **Cumulativity**

P_{et} is cumulative iff $\forall x, y[P(x) \wedge P(y) \rightarrow P(x \sqcup y)]$.

(... iff, if P is true of two sums x and y, then P is true of the sum of x and y.)

The cumulative nature of *students* and *frowned* ensures that the scenario just described is *not* consistent with the truth conditions in (32). As illustrated in (34), if, say, three distinct pairs of students frowned, then by cumulativity, (a group of) six students frowned, which contradicts the formulas in (32).

(34)
$$\begin{aligned} & \textbf{students}(a \sqcup b) \wedge \textbf{frowned}(a \sqcup b) \\ & \textbf{students}(c \sqcup d) \wedge \textbf{frowned}(c \sqcup d) \\ & \textbf{students}(e \sqcup f) \wedge \textbf{frowned}(e \sqcup f) \\ \vDash\ & \textbf{students}(a \sqcup b \sqcup c \sqcup d \sqcup e \sqcup f) \wedge \textbf{frowned}(a \sqcup b \sqcup c \sqcup d \sqcup e \sqcup f) \end{aligned}$$

[35] In the terminology of Krifka 1989, they have 'cumulative reference'.

[36] Cumulativity, so defined, is a property of 1-place predicates. In chapter 3, we will also encounter the so-called 'cumulative reading' (or co-dependent reading) of sentences involving *transitive* (2-place) predicates and plural arguments. There, cumulativity (being now a property of 2-place predicates) will have a somewhat different sense.

In sum, since *students* and *frowned* are both cumulative, the formulas in (32) correctly amount to saying that either zero, one, . . . , or four students frowned, and no more than that.

These two properties—distributivity and cumulativity—follow immediately if we assume that the only way in which a predicate like *frowned* can be applied to non-atomic individuals is by pluralizing a more primitive version of *frowned* that is defined only for atoms. On this view, we start from a predicate *frowned*, which has only atoms in its extension, and a pluralizing operator, usually notated by * (and called the *star operator*), turns this predicate into one that is defined for both atomic and non-atomic individuals, by closing its extension under the sum operation (see Link 1983 and subsequent work; see also chapter 6).

(35) Let E be a set of individuals (atomic or non-atomic). Then the *closure under sum* of E, written $E^{\sqcup}$, is the smallest set F such that:

a. $E \subseteq F$, and

b. $\forall x, y[x \in F \wedge y \in F \rightarrow x \sqcup y \in F]$.

($E^{\sqcup}$ is the set of all sums that can be formed by summing up members of E.)

(36) $[\![*]\!] = \lambda P_{et} \,.\, \lambda x_e \,.\, x \in \{y : P(y)\}^{\sqcup}$

($[\![{*}\alpha]\!]$ is the set of all sums that can be formed by summing up members of $[\![\alpha]\!]$.)

When *frowned* combines with an expression that denotes a non-atomic individual, it first needs to be pluralized, i.e. parsed as **frowned*, and (the denotation of) **frowned* is now both distributive in the sense of (21) and cumulative in the sense of (33).[37]

Having established that the truth conditions in (32) are what we want for the sentence in (29), i.e. our original (2), (plus the assumption that *students* and *frowned* are both

[37] When the star operator applies to an expression that already has non-atomic individuals in its extension, such as the collective predicate *surrounded the castle* or the mixed predicate *lifted the piano*, the resulting predicate is cumulative, but not distributive.

cumulative), the problem is that there is no way to arrive at those truth conditions under the assumptions made so far: none of the lexical entries, compositional rules, or predicate properties involve any maximality operator or negation. This example thus illustrates that we need to introduce, in some way or another, a maximality component into the semantics of modified numeral constructions to explain the upper-bound facts associated with sentences like (2) and (3).

1.2.4 Non-distributive predicates and Van Benthem's non-problem

When we move to non-distributive predicates, however, Van Benthem's problem and the existential entailment problem (in the case of *less than*) both become non-problems for the adjectival theory—a point which, to my knowledge, has never been made in the literature. That is, for sentences like (37a), an LF analogous to (29b), as in (37b), delivers what seem to be the intuitively correct truth conditions.

(37) a. Less than five soldiers surrounded the castle.

b. [$\varnothing_{\exists}$ [[less than five] soldiers]] [surrounded the castle]

c. $\exists x[\mathbf{card}(x) < 5 \wedge \mathbf{soldiers}(x) \wedge \mathbf{surrounded}(x)]$

First, as I have argued, a sentence like (37a) is intuitively consistent with groups of five or more soldiers having surrounded the castle, as predicted by (37c).[38] Second, (37a) intuitively entails that at least some group of soldiers surrounded the castle. That is, (37a)

[38]To inject a bit of variety into my examples, I have momentarily switched from students lifting the piano (cf. (6a)) to soldiers surrounding the castle, taking for granted that both sentences ought to receive the same (non-upper-bounded, existential) analysis. However, several speakers that I have consulted feel that (37a) conveys an upper bound. My response is that, in many (perhaps even most) conceivable scenarios, only one group of soldiers surrounds a particular castle. In such cases, if we learn that (a group of) less than five soldiers surrounded the castle, then we infer that no other group of soldiers (hence, no group of five or more soldiers) surrounded it. However, this upper-bound inference is due more to context than to *less than*. Notice, for instance, that we make a similar inference with an analogous sentence containing a bare numeral, such as *Four soldiers surrounded the castle*. One reason that a sentence like (6a) (*Less than five students lifted the piano*) more clearly has no such upper-bound inference is, I think, simply that in this case it is quite easy to imagine scenarios where multiple groups of students lift the piano (e.g. piano-lifting competitions).

is judged false in a scenario where no soldiers at all surrounded the castle. This entailment is captured by the existential statement in (37c), given our ontological assumption that there is no null individual (i.e. that every individual x has at least one atom as part).

In other words, the existential entailment problem and Van Benthem's problem are actually not problems at all for sentences with a non-distributive predicate. Quite the opposite: they are precisely what is needed.

Furthermore, as expected on the adjectival theory, there is, as far as I can tell, no reading of (37a) that has (38) as its truth conditions. This hypothetical reading can be paraphrased as 'There is no group of five or more soldiers who surrounded the castle.' (I will have more to say about this hypothetical reading in chapter 3.)

(38) $\max(\lambda n\,.\,\exists x[\mathbf{card}(x) = n \wedge \mathbf{soldiers}(x) \wedge \mathbf{surrounded}(x)]) < 5$

1.2.5 Alternative theories

We have seen that, on the one hand, the adjectival theory does quite well in capturing the fact that sentences in which modified numerals like *less than five* and *between two and four* combine with non-distributive predicates are interpreted existentially, with no upper bound. On the other hand, when such modified numerals combine with distributive predicates, the reading predicted by the adjectival theory is intuitively unavailable (the predicted reading involves an existential entailment problem and Van Benthem's problem), and the adjectival theory is unable to capture the intuitively available, upper-bounded reading of such sentences.

Many alternative theories have been proposed to solve or avoid the two problems faced by the adjectival theory. All of them involve maximality in some way, shape, or form, and all of them exclusively derive upper-bounded readings for sentences with *less than* and *between*—something they would of course consider a feature (not a bug), given that the non-upper-bounded data introduced in this thesis have not been in the purview of such theories. I will quickly present two such theories, both of which involve innovations that I will make use of in chapter 6.

Hackl (2000) proposes that *less than five* and *between two and four* denote generalized quantifiers over degrees, which encode a maximality component (or negative component), as in (39) below, and that existential quantification over individuals is contributed by a silent determiner, ⟨*many*⟩, which is parameterized for degrees, as shown in (40). (⟨*many*⟩ is just like our $\varnothing_\exists$, except it takes a degree as its first argument.)

(39) a. $[\![\text{less than five}]\!] = \lambda P_{dt} \,.\, \max(P) < 5$

b. $[\![\text{between two and four}]\!] = \lambda P_{dt} \,.\, 2 \leq \max(P) \leq 4$

(40) $[\![\langle many \rangle]\!] = \lambda n_d \,.\, \lambda P_{et} \,.\, \lambda Q_{et} \,.\, \exists x[\mathbf{card}(x) = n \wedge P(x) \wedge Q(x)]$

The schema in (41) illustrates a derivation involving *less than five*. The reason that only upper-bounded readings are ever derived is that, for type reasons, *less than five* must raise (creating a predicate of degrees, with a numeral trace n of type d), while ⟨*many*⟩ stays low (to combine directly with n), with the result that the maximality operator always applies above the individual quantifier.

(41) a. Less than five NP VP.

b. [less than five] [λn [[[n ⟨*many*⟩] NP] VP]]

c. $\max(\lambda n \,.\, \exists x[\mathbf{card}(x) = n \wedge [\![\text{NP}]\!](x) \wedge [\![\text{VP}]\!](x)]) < 5$

Landman (2004), by contrast, tries to salvage the adjectival theory by proposing an operation of 'maximalization'. The idea is that modified numerals have the adjectival entries in (25), (26), and (27), and that modified numerals like *less than five* and *between two and four* (but not bare numerals like *three*) are specified for a feature requiring that maximalization apply.[39] Omitting details, the upshot is that basic sentences with *less than*

[39] Landman (2004) specifically discusses *at least three*, *at most three*, and *exactly three*, but presumably *less than five* and *between two and four* have the relevant feature as well.

five (and similar for *between two and four*) are always assigned truth conditions like in (42), where $\bigsqcup P$ is the largest (maximal) sum of individuals having property P.

(42) $\textbf{card}(\bigsqcup(\lambda x_e\,.\,[\![\text{NP}]\!](x) \wedge [\![\text{VP}]\!](x))) < 5$

Crucially, to avoid any existential entailment problem, Landman (2004) assumes the existence of a null individual, which is in the extension of pluralized NP and VP predicates. Thus, for example, if no students frowned, then the only individual that is in the extension of both *students* and *frowned* is the null individual, whose cardinality is zero (hence, less than five). Despite their cosmetic differences, then, (41c) and (42) are in fact truth conditionally equivalent, within their respective frameworks.

The problem with these two theories (and others like them[40]), however, in light of the data with non-distributive predicates, is that they are (by design) unable to derive non-upper-bounded, existential readings, which are required for sentences like (6a) (*Less than five students lifted the piano together*) and (8a) (*Between two and four students drank more than ten beers between them*).

1.2.6 Summary of the first puzzle

Let us refer to readings (truth conditions) of the form

(43) $\exists x[\textbf{card}(x) < n \wedge P(x)]$

as non-upper-bounded (or non-maximal) readings with an existential entailment. They entail that at least some group x has property P (e.g. being students who frowned, or being soldiers who surrounded the castle), and in general, they are consistent with other

[40]See also, for example, Krifka 1999, in which *less than* and *between* are defined in such a way that they only ever convey upper bounds (e.g. *Between two and four students frowned* excludes alternative propositions of the form 'n students frowned' for all $n > 4$).

sums larger than x having property P.[41]

Let us likewise refer to readings (truth conditions) of the form

(44) $$\max(\lambda m\,.\,\exists x[\mathbf{card}(x) = m \wedge P(x)]) < n$$

as upper-bounded (or maximal) readings with no existential entailment.[42] They entail an upper bound on the number of individuals with property P (whenever P is cumulative), and in general, they are consistent with no individuals having property P.

The facts laid out in this chapter suggest that any theory of modified numerals needs to be able to generate both upper-bounded, non-existential readings and existential, non-upper-bounded readings. However, it needs to do so in such a way that only upper-bounded readings get assigned to some types of sentences (e.g. those with distributive predicates), while only existential readings get assigned to other types of sentences (e.g. those with certain types of non-distributive predicates). Put differently, when it comes to sentences with *less than* and *between*, the presence of maximality is, in some sense, variable, and the puzzle is how to explain this variability. Before I give a roadmap of the rest of the thesis, let me first take care of some semantic housekeeping.

1.2.7 Semantic housekeeping

To be able to derive the meanings of expressions like *more than three*, *less than five*, and *between two and four* compositionally from the meanings of *more than*, *less than*, and *between*, and from the meanings of *two*, *three*, and so on (that is, to be able to specify categorematically the meanings of *more than*, *less than*, and *between*), we need to alter the

[41] I say in general because P might, for example, be rigidly true of at most one sum, in which case, if P is true of a sum x, then there is no sum y distinct from x (hence, no sum y larger than x) which P is true of. An example of such a predicate is *ate that whole pizza together*, since one and only one group can ever eat a particular pizza in its entirety. But in cases like this (*Less than five students ate that whole pizza together*), the upper bound clearly comes from that particular property of the predicate, not from some upper-bounding component separate from the predicate.

[42] Henceforth, for consistency, and to facilitate comparison among different theories, I will stick to using a maximality operator (which I will define explicitly in various ways as we go along), rather than negation, to represent upper-bounded readings.

adjectival lexical entry for *three* in (19). *Three* cannot simply denote a type *et* predicate of pluralities because the numeral modifier needs access to the '3' part of the meaning of *three*. Thus, we let *three* simply denote the number (degree) 3, and similar for *five* and other numerals. On this basis, it is easy to define some plausible denotations for *less than* (and other numeral modifiers). Some examples are given below.[43]

(45) $[\![\text{three}]\!] = 3$

(46) $[\![\text{less than}]\!] =$

a. $\lambda n_d \,.\, \lambda x_e \,.\, \mathbf{card}(x) < n$

b. $\lambda n_d \,.\, \lambda P_{dt} \,.\, \exists k[k < n \wedge P(k)]$

c. $\lambda n_d \,.\, \lambda P_{dt} \,.\, \exists k[k < n \wedge \max(P) = k]$

To derive the type *et* predicate that we originally posited as the meaning of *three*, I assume a silent syntactic operator, `isCard`, which maps a degree-denoting expression (a numeral or numerical variable) n to a predicate of pluralities, namely that predicate which characterizes the set of pluralities whose cardinality is n. The intersective use of *three* is now derived syntactically (at LF) as `isCard` *three*, which I write as $three_{\texttt{isCard}}$ for convenience.[44, 45]

(47) $[\![\texttt{isCard}]\!] = \lambda n_d \,.\, \lambda x_e \,.\, \mathbf{card}(x) = n$

(48) $[\![n_{\texttt{isCard}}]\!] = [\![\texttt{isCard}\ n]\!] = \lambda x_e \,.\, \mathbf{card}(x) = [\![n]\!]$

[43] Of course, the choice between different possibilities has to be made not only on the basis of the behavior of modified numerals, but also as a part of a more general theory of comparative constructions. A number of works on comparatives assume that comparative quantifiers have a lexical entry that includes a maximality component, along the lines of (46c) (see, e.g., Heim 2000 and, especially, Hackl 2000 and the references therein). Some of these accounts also assume that *less* should be decomposed into *little* + *-er* and that the maximality component is introduced by *-er* (Rullmann 1995; Heim 2006).

[44] `isCard` is essentially a Bresnan-style *many* (Bresnan 1973). My choice of name is simply to mnemonically contrast `isCard` from another syntactic operator to appear later (`isMax` in chapter 4 and chapter 7).

[45] Note that the way I have chosen to implement the 'shift' of a numeral from a degree to a predicate of individuals is arbitrary. For example, I could also have posited a semantic typeshifting operation (Partee 1987). For convenience, I will loosely say that n '(type)shifts' to $n_{\texttt{isCard}}$ (or to an intersective type), even though my particular implementation is not, strictly speaking, a typeshifting rule.

1.3 Roadmap of the thesis

The rest of this thesis is organized as follows.

- Chapter 2 proposes that existential readings should be generated by the grammar across the board, but that certain existential readings are then filtered out by a pragmatic blocking mechanism. It also explains how upper-bounded readings can be derived, by introducing the notion of maximality, which is taken to be lexically encoded into the semantics of numeral modifiers like *less than* and *between*. The variable presence of maximality is explained in terms of scope interaction between modified numerals (which encode maximality) and $\varnothing_\exists$ (which encodes existential quantification over individuals).
- Chapter 3 discusses three major predictions made by the theory developed in chapter 2. The first prediction is that a subclass of collective predicates that license 'weak' downward inferences (such as *gather* and *hold hands*) ought to pattern like distributive predicates (*frown*) in having only upper-bounded interpretations with *less than* and *between*. The second prediction is that we expect an asymmetry in how modified numerals are interpreted with cumulative transitive predicates, depending on whether they occur in the subject position or the object position of the predicate (because those positions, I will argue, license different kinds of inferences). These first two predictions are indeed borne out. The final prediction, which is not borne out, is that sentences with non-distributive (e.g. collective) predicates ought to be genuinely ambiguous between an upper-bounded reading and a non-upper-bounded reading.
- Chapter 4 explores the idea that maximality should be 'severed' from the meanings of numeral modifiers. That is, modified numerals simply make existential statements about degrees, with no reference to maximality; maximality is then viewed as a separate and optional operator (which operates on numerical variables, i.e. the traces

created by modified numerals). On this view, the variable presence of maximality is explained in terms of the optional application of the maximality operator. I show that this this theory is roughly on a par with that of chapter 2 as far as data discussed so far go.

- Chapter 5 presents new data from the domain of genericity in an attempt to tease apart the two theories elaborated in chapter 2 and chapter 4. The data include sentences like *Between three and five people can carry that piano upstairs*, which has a rather unexpected 'minimal' reading: 'The minimum number *n* such that (any group of) *n* people can carry that piano upstairs is between three and five.' Surprisingly, neither theory is able to derive this minimal reading.
- Chapter 6 builds on the previous chapter by developing an ambitious theory that accounts for the entirety of the data presented. The theory involves moving from a 'standard' notion of maximality, which is based on the natural ordering of degrees, to an 'informativity-based' (or logical) notion of maximality, where the ordering of degrees is based on how informative (logically strong) they are relative to some property of degrees (Beck and Rullmann 1999; von Fintel, Fox, and Iatridou 2014). The central idea is that, for a number to be maximally informative, sometimes it must be the largest, other times the smallest, and still other times it need not be either.
- Chapter 7 considers, but ultimately rejects, the idea of 'severing' (in the sense of chapter 4) logical maximality from the meanings of numeral modifiers, concluding that the theory presented in chapter 6 is the most favorable theory.
- Chapter 8 provides a recap and discusses several open issues.

Chapter 2

Lexical maximality and scope ambiguity

2.1 Overview

Last chapter, I presented data showing that any adequate theory of modified numerals must be able to generate both upper-bounded, non-existential readings and non-upper-bounded, existential readings of sentences with modified numerals like *less than five* and *between two and four*, but it must do so in such a way that some sentences are only assigned one type of reading, while other sentences are only assigned the other type. In this chapter, I develop a proposal based on the following idea: both upper-bounded and non-upper-bounded readings are grammatically generated across the board, but non-upper-bounded readings are sometimes unattested because they violate a pragmatic constraint against certain types of 'weak' readings. In other words, I propose that the grammar overgenerates, and that grammatical overgeneration is reigned in by a pragmatic blocking mechanism. The method of overgeneration is based on scope ambiguity. (In chapter 4, I will consider a different method.) It relies on the flexible scope orderings of two key ingredients: a maximality component (taken to be part of the lexical meanings of *less than* and *between*) and existential quantification over individuals (taken to be a silent determiner, $\varnothing_\exists$, separate from *less than* and *between*). Since maximality is assumed here to be part of the lexical semantics of numeral modifiers, I refer to this theory as *Lexical Maximality* (LMax, for short). I first explain the rationale behind pragmatic blocking, and then I present the LMax system that generates both types of readings.

2.2 Pragmatic blocking

Consider again (2), repeated below as (49), and the unattested existential reading generated for it by the adjectival theory. We have already seen that this reading incurs both an existential entailment problem and Van Benthem's problem, but I would like to probe this issue further.

(49) a. Less than five students frowned.

b. [$\varnothing_\exists$ [[less than five] students]] frowned

c. $\exists x[\mathbf{card}(x) < 5 \land \mathbf{students}(x) \land \mathbf{frowned}(x)]$

Due to the distributivity of *students* and *frowned*,

(50) $\exists x[\mathbf{card}(x) < 5 \land \mathbf{students}(x) \land \mathbf{frowned}(x)]$

is in fact equivalent to just

(51) $\exists x[\mathbf{students}(x) \land \mathbf{frowned}(x)]$

The entailment from (50) to (51) follows simply from conjunction elimination. For the reverse entailment, suppose that z is a sum verifying (51). Then, since *students* and *frowned* are distributive, each atomic part of z is a sum with cardinality 1 who is a student who frowned, which verifies (50). More precisely, the argument in (52) is valid.

(52) $\exists x[\mathbf{students}(x) \land \mathbf{frowned}(x)]$

$\vDash$ $\exists x[\mathbf{card}(x) = 1 \land \mathbf{students}(x) \land \mathbf{frowned}(x)]$

$\vDash \quad \exists x[\mathbf{card}(x) < 5 \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]$

Thus, despite the presence of *less than five*, the LF in (49b) expresses an extremely weak proposition: it simply says that at least one student frowned.[1] One way to think about the weakness of (49b) is that the numeral *five* effectively does no semantic work: the same truth conditions would be derived if *five* were replaced, say, by *four* or by *six*.[2]

Similarly, the unattested existential reading for (3) generated by the adjectival theory is weak in a completely parallel way.

(53) a. Between two and four students frowned.

b. [$\varnothing_\exists$ [[between two and four] students]] frowned

c. $\exists x[2 \leq \mathbf{card}(x) \leq 4 \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]$

In this case,

(54) $\exists x[2 \leq \mathbf{card}(x) \leq 4 \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]$

is equivalent to just

(55) $\exists x[\mathbf{card}(x) \geq 2 \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]$

by similar reasoning as before. Thus, *between two and four* is interpreted as 'at least two', so that the numeral *four* does no semantic work: the same truth conditions would be derived if *four* were replaced, say, by *three* or by *five*.

[1] Nevertheless, this existential reading is not, strictly speaking, weaker than the upper-bounded reading ('No group of five or more students frowned') since, if no students frowned, then the upper-bounded reading is true while the existential reading is false.

[2] I have assumed here that plural expressions like *students* contain atoms in their extension (Krifka 1999, Sauerland, Anderssen, and Yatsushiro 2005; see also footnote 29 of chapter 1), but this assumption is not crucial. If the extensions of plural expressions contain only non-atomic sums, then $\exists x[\mathbf{students}(x) \wedge \mathbf{frowned}(x)]$ entails $\exists x[\mathbf{card}(x) = 2 \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]$, which again entails (29c), and the semantic vacuity of *five* still follows.

Given these observations, I would like to put forth the idea that the LFs in (49b) and (53b) are unavailable precisely *because* the numerals *five* and *four*, respectively, have no semantic import. More generally, it seems plausible that the adjectival theory is in fact on the right track: it derives exactly the right reading for sentences with non-distributive predicates, and the weak readings derived for sentences with distributive predicates are ruled out on pragmatic grounds.[3] Thus, existential readings should not be abolished across the board, as the maximality-based theories described in section 1.2.5 (Hackl 2000; Landman 2004) would have it. Rather, they are generated but sometimes blocked because they violate a pragmatic constraint against using an expression like *less than five* or *between two and four* when *five* (resp. *four*) makes no semantic contribution.[4] One way to formulate such a constraint is given in (56).[5]

(56) **Pragmatic economy constraint**

An LF ϕ containing a numeral n may not, for any m distinct from n, be truth conditionally equivalent to $\phi[n \mapsto m]$ (the result of substituting m for n in ϕ).

Given the argumentation above, this pragmatic constraint filters out the unattested, non-upper-bounded readings with existential entailments in distributive cases.

Importantly, it correctly does *not* filter out those same, attested readings in non-distributive (e.g. collective) cases. Consider, for example, the truth conditions in (57c) for (57a). Since *surrounded the castle* is non-distributive, the conjunct $\mathbf{card}(x) < 5$ actually has semantic import and cannot be dropped. In other words, the numeral *five* in (57b) actually does semantic work: if we replaced *five* with *four*, we would get stronger truth

[3] Link (1997) has maintained that distributivity is, in a sense, more derivative than non-distributivity, as evidenced by the fact that typologically distributivity tends to be overtly marked. Thus, it is perhaps not surprising that the adjectival theory needs to say something special for distributive predicates.

[4] Of course, it remains to explain why upper-bounded readings sometimes do arise; this is the topic of section 2.3.

[5] As formulated, with specific reference to numerals, this constraint certainly cannot be a general grammatical principle. I will briefly discuss other ways of formulating the constraint at the end of this section.

conditions, and if we replaced *five* with *six*, we would get weaker truth conditions. Hence, this reading is not blocked by the constraint in (56).

(57) a. Less than five soldiers surrounded the castle.

b. [$\varnothing_\exists$ [[less than five] soldiers]] [surrounded the castle]

c. $\exists x[\mathbf{card}(x) < 4 \wedge \mathbf{soldiers}(x) \wedge \mathbf{surrounded}(x)]$

More generally, when P is non-distributive, then

(58) $\exists x[\mathbf{card}(x) < n \wedge P(x)]$

is *not* equivalent to the following, for any $m \neq n$:

(59) $\exists x[\mathbf{card}(x) < m \wedge P(x)]$

because there is no entailment relation at all between the m and n alternatives when P is non-distributive. And it is also not equivalent to

(60) $\exists x P(x)$

because P may only be true of pluralities with cardinalities greater than n.

As it stands, then, the adjectival theory generates just the right reading for *less than* and *between* sentences with non-distributive predicates, and the unavailability of the reading it generates for *less than* and *between* sentences with distributive predicates is due to a violation of a pragmatic constraint like (56). In the next section, I show how to extend the adjectival theory to also generate upper-bounded readings.

But first, let me say a few words about the formulation of the constraint in (56). My original hope was to formulate a more general constraint that would be easier to motivate in terms of Gricean maxims. A natural idea would be to interpret the maxim of *manner* as implying that a given LF is ruled out (infelicitous) if it is truth-conditionally equivalent to some syntactically simpler LF. On this view, a sentence of the form *Between m and n NP VP*, for example, could not be assigned an LF which expresses the proposition 'at least *m* NP VP', because this proposition could be expressed by a simpler LF (one with a bare numeral, say: *m NP VP*). While appealing, such a principle does not make clear predictions short of an explicit complexity metric (see Katzir 2007 for an attempt to define such a metric). Moreover, unless it were more narrowly formulated to refer just to numerical expressions, it might also rule out as infelicitous certain felicitous sentences: for instance, the sentence *I read a book that was very interesting* would be ruled out because it is truth-conditionally equivalent to and more complex than *I read a very interesting book*.

Furthermore, we might have considered a weaker version of our condition, where ϕ would be ruled out if, for *every m* distinct from n, ϕ is truth conditionally equivalent to $\phi[n \mapsto m]$. This would arguably be a more direct implementation of the intuition that what is wrong with the relevant LFs is that some numeral is in some sense doing no semantic work. However, with such a weaker constraint, we might fail to exclude the existential reading for (53a) (*Between two and four students frowned*), which is equivalent to *At least two students frowned*, since replacing *four* with, say, *one* in (53b) would make the resulting sentence (i.e. *Between two and one students frowned*) either contradictory or equivalent to *At least one student frowned*, depending on how exactly *between* is treated.

2.3 Generating readings

The system that I propose for generating both existential and upper-bounded readings is a combination of the adjectival theory of numerals and the degree-based maximality theory of modified numerals. Bare numerals denote degrees but can be interpreted adjectivally by combining with `isCard` (see (47)), while modified numerals denote generalized quantifiers over degrees (just as in Hackl 2000). Sums are existentially quantified over by the silent existential determiner, $\varnothing_\exists$ (so there is no parameterized determiner ⟨*many*⟩ as in Hackl

2000; see section 1.2.5). The crux of the system is that modified numerals, which encode maximality, can interact scopally with $\varnothing_{\exists}$ to derive the two different types of readings. Since maximality is part of the lexical meanings of numeral modifiers, I refer to this theory as *Lexical Maximality* (LMax, for short). I first present the lexical entries and define the notion of maximality that I assume. I then show how the system works using *between*. Finally, I turn to *less than*, which requires that we slightly revise the definition of maximality.

2.3.1 Lexical entries and maximality

The numeral modifiers we have been interested now have the following lexical entries, where $\max_{\leq}$ is a relation (defined below) between a number and a set of numbers, and $>$ and $\leq$ both refer to the natural ordering over numbers.

(61) $[\![\text{more than}]\!] = \lambda n_d \,.\, \lambda P_{dt} \,.\, \exists k[k > n \wedge \max_{\leq}(P)(k)]$

(62) $[\![\text{less than}]\!] = \lambda n_d \,.\, \lambda P_{dt} \,.\, \exists k[k < n \wedge \max_{\leq}(P)(k)]$

(63) $[\![\text{between}]\!] = \lambda m_d \,.\, \lambda n_d \,.\, \lambda P_{dt} \,.\, \exists k[m \leq k \leq n \wedge \max_{\leq}(P)(k)]$

The maximality component, max, is viewed here as a relation between a partially ordered set P (of numbers) and a number n.[6] A precise definition of max (for an arbitrary partial ordering $\leq$) is given in (64).[7]

[6]Recall that a partial order is a binary relation $\leq$ over a set P which is reflexive, antisymmetric, and transitive. That is, for all x, y, and z in P: (i) $x \leq x$ (reflexivity), (ii) if $x \leq y$ and $y \leq x$, then $x = y$ (antisymmetry), and (iii) if $x \leq y$ and $y \leq z$, then $x \leq z$ (transitivity).

[7]Nothing really hinges at this point on the use of a max operator at all; it simply makes exposition and comparison of theories easier. We could instead have, for example:

(i) $[\![\text{less than}]\!] = \lambda n_d \,.\, \lambda P_{dt} \,.\, \neg\exists m[m \geq n \wedge P(m)]$

Notice also that, if a degree predicate P, ordered by the natural ordering over degrees, has no greatest element (e.g. it is an interval open at the upper end), then the entry in (62) makes $[\![\text{less than}]\!](n)(P)$ return false for every n (as does the entry in (i)). That is, maximality failure leads to falsity, rather than, say, undefinedness. That maximality failure should be handled in this way is potentially supported by a

(64) For any predicate of degrees P_{dt}, any partial ordering $\leq$ over P, and any degree n, $\max_{\leq}(P)(n)$ iff $P(n) \wedge \neg\exists m[P(m) \wedge m > n]$.

In plain English, a number n is a maximal element of P just in case n is in P and there is no number m in P greater than n. Now, depending on the ordering, a number n need not be the unique greatest element of P in order to count as a maximal element of P.[8] However, the ordering we are interested in here is the natural ordering of numbers, which is total.[9] Thus, if a number n is maximal in P, it is necessarily the unique greatest element of P. For this reason, we can make use of the following convenient notation:

(65) If $\leq$ is a total order over P, then we may write $\max_{\leq}(P)$ to denote the unique n in P such that that $\max_{\leq}(P)(n)$ is true.

In addition, if the ordering under consideration is obvious, then I will omit the subscript on max. Until chapter 6, the ordering will always be the natural one over numbers, so I will always omit the subscript until then. Given these notational conventions, our lexical entries can, for now, be written as follows (which corresponds more closely to what is seen in the literature; see, e.g., von Stechow 1984, Rullmann 1995, Kennedy 1997, Heim 2000, among many others).[10]

sentence like (ii), which is intuitively false, rather than a case of presupposition failure. The issue of maximality failure will arise several more times in this thesis, but in those cases the exact way that it is handled will turn out to not be all that important.

(ii) Less than five prime numbers are odd.

[8]This will become important later on, in chapter 6, where I switch from the natural ordering of numbers to an informativity-based ordering.

[9]A total order is a partial order $\leq$ over a set P in which the condition of reflexivity ($x \leq x$, for all x in P) is replaced by the stronger condition of totality: for all x and y in P, $x \leq y$ or $y \leq x$.

[10]I could have defined max more specifically as picking out the unique largest number in a set (relative to the natural ordering of numbers), thus arriving at these simpler-looking entries right away. The reason for defining max in the more general way that I did is because the more general version will be required in chapter 6; thus, defining max in this way will facilitate development of another theory (and hence comparison between theories) later on.

(66) $[\![\text{more than}]\!] = \lambda n_d \,.\, \lambda P_{dt} \,.\, \exists k[k > n \wedge \max_{\leq}(P)(k)]$
$= \lambda n_d \,.\, \lambda P_{dt} \,.\, \max(P) > n$

(67) $[\![\text{less than}]\!] = \lambda n_d \,.\, \lambda P_{dt} \,.\, \exists k[k < n \wedge \max_{\leq}(P)(k)]$
$= \lambda n_d \,.\, \lambda P_{dt} \,.\, \max(P) < n$

(68) $[\![\text{between}]\!] = \lambda m_d \,.\, \lambda n_d \,.\, \lambda P_{dt} \,.\, \exists k[m \leq k \leq n \wedge \max_{\leq}(P)(k)]$
$= \lambda m_d \,.\, \lambda n_d \,.\, \lambda P_{dt} \,.\, m \leq \max(P) \leq n$

With all this in place, we now have the following meanings for *more than three*, *less than five*, and *between two and four*.

(69) a. $[\![\text{more than three}]\!] = \lambda P_{dt} \,.\, \max(P) > 3$
b. $[\![\text{less than five}]\!] = \lambda P_{dt} \,.\, \max(P) < 5$
c. $[\![\text{between two and four}]\!] = \lambda P_{dt} \,.\, 2 \leq \max(P) \leq 4$

2.3.2 Scope ambiguity

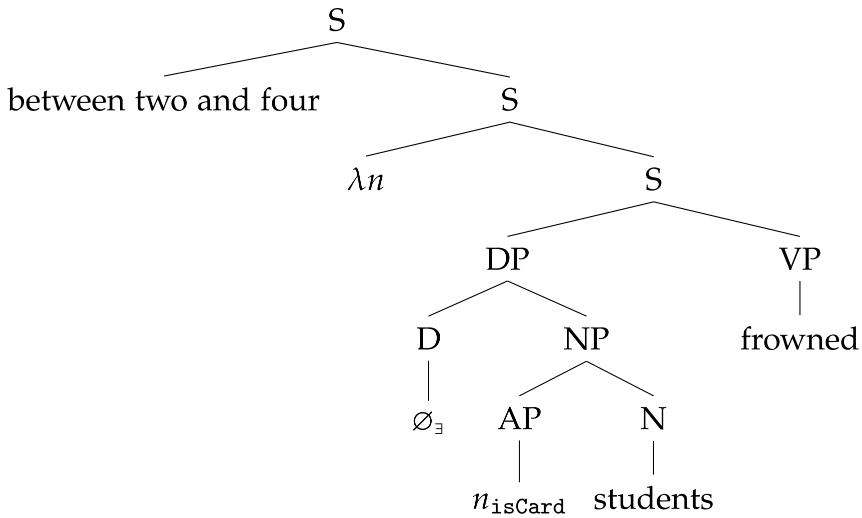
I will now illustrate how the two types of readings (upper-bounded readings and existential readings) can be generated, using *between two and four* as my example modified numeral. I assume that, as a generalized quantifier over degrees, *between two and four* can move; in fact, it must move, for type reasons. For example, consider (3), repeated in (70).

(70) Between two and four students frowned.

There is no way for *between two and four*, whose denotation has type $(dt)t$, to combine with an expression like *students*, whose denotation has type *et*. As a result, it is uninterpretable *in situ* and must therefore move (QR). One movement possibility is illustrated in the LF in (71), where *between two and four* moves to take scope over $\varnothing_{\exists}$. More precisely, it QRs and adjoins to the S node of the sentence, creating a degree predicate in its scope. Its trace, n, is then shifted to $n_{\texttt{isCard}}$, making it of the appropriate type (*et*) to combine with *students* by intersection (see chapter 1, footnote 21).[11] The reading thus derived is the (attested) upper-bounded reading, which says that the maximum (total) number of students that frowned is two, three, or four.

(71) **Adjoining to S**

a.



b. [between two and four] [λn [[$\varnothing_\exists$ [$n_{\texttt{isCard}}$ students]] frowned]]

c. $2 \leq \max(\lambda n\,.\,\exists x[\mathbf{card}(x) = n \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]) \leq 4$

In principle, however, *between two and four* could move but remain below $\varnothing_\exists$. More precisely, it could quantify into AP or into NP. One way to guarantee this possibility is to assume that APs and NPs, like VPs, have an internal subject position that can be abstracted over. For example, APs have an internal subject position filled with a phonologically null (and semantically vacuous[12]) item PRO (Heim and Kratzer 1998), as in (72a). PRO may optionally QR, creating an abstraction over a variable x, as in (72b). Now, *between two and four* can quantify into AP, meaning QR just below λx, resulting in a

[11]I write the binding operator and trace as λn and n, respectively, rather than with indices, e.g. as 1 (or λ_1) and t_1. The latter is perhaps more standard (see, e.g., Heim and Kratzer 1998), but the former makes it easier to see immediately what sort of object is being abstracted over (here, a numeral). When the object is an individual, I write the binder as λx (λy, etc.) and the trace as x (y, etc.). I am thus overloading the use of λ, x, n, etc. in the interest of making the correspondence between LFs and meanings more perspicuous.

[12]PRO is semantically vacuous in the sense that it has no denotation. It can, however, move, creating a semantically non-vacuous trace.

predicate of pluralities with between two and four atomic parts, as in (72c).

(72) a. $[_{\text{AP}}$ PRO $[_{\text{A}'}$ between two and four$]]$

b. $[_{\text{AP}}$ PRO $[_{\text{AP}}\ \lambda x\ [_{\text{AP}}\ x\ [_{\text{A}'}$ between two and four$]]]]$

c. $[_{\text{AP}}$ PRO $[_{\text{AP}}\ \lambda x\ [_{\text{AP}}\ [_{\text{DegP}}$ between two and four$]\ [_{\text{AP}}\ \lambda n\ [_{\text{AP}}\ x\ [_{\text{A}'}\ [_{\text{A}}\ n_{\texttt{isCard}}]]]]]]]$

If *between two and four* QRs above PRO, then it does not matter whether PRO also QRs or instead remains *in situ* because the two LFs in (73) are logically equivalent.[13]

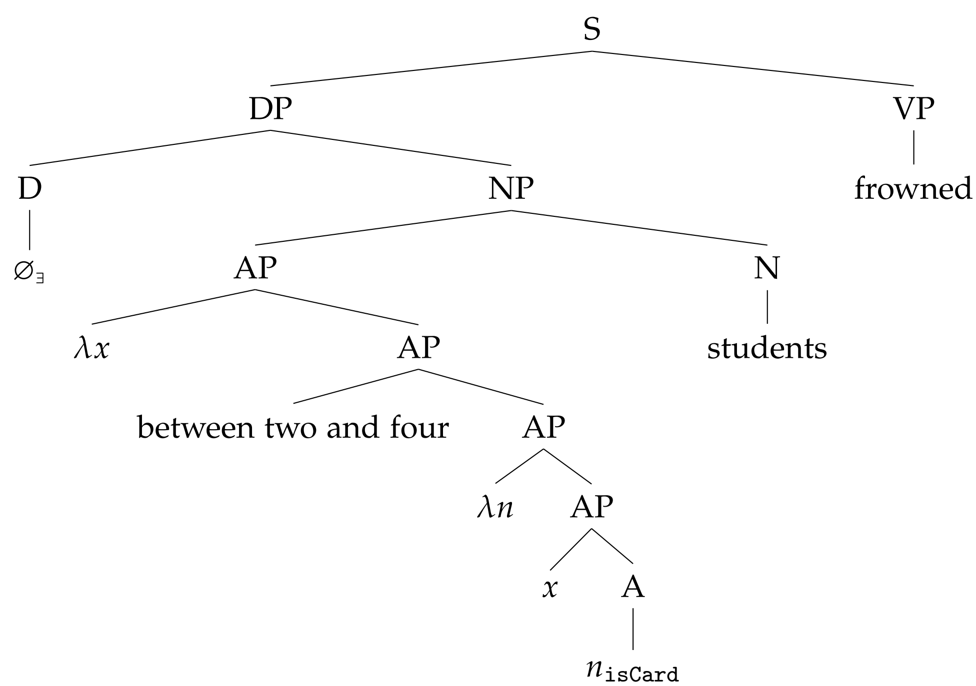
(73) a. $[_{\text{AP}}$ PRO $[_{\text{AP}}\ \lambda x\ [_{\text{AP}}\ x\ [_{\text{A}'}\ [_{\text{A}}\ n_{\texttt{isCard}}]]]]]$

b. $[_{\text{AP}}$ PRO $[_{\text{A}'}\ [_{\text{A}}\ n_{\texttt{isCard}}]]]$

The following example illustrates *between two and four* quantifying into AP. (I omit PRO here, and henceforth, for simplicity.)

(74) **Quantifying into AP**

a.



[13] Technically, the AP written '$n_{\texttt{isCard}}$' in (70) should be considered short-hand for (73b).

b. [$\varnothing_\exists$ [[λx [[between two and four] [λn [x $n_{\texttt{isCard}}$]]]] students]] frowned

c. $\exists x[2 \leq \max(\lambda n\,.\,\mathbf{card}(x) = n) \leq 4 \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]$

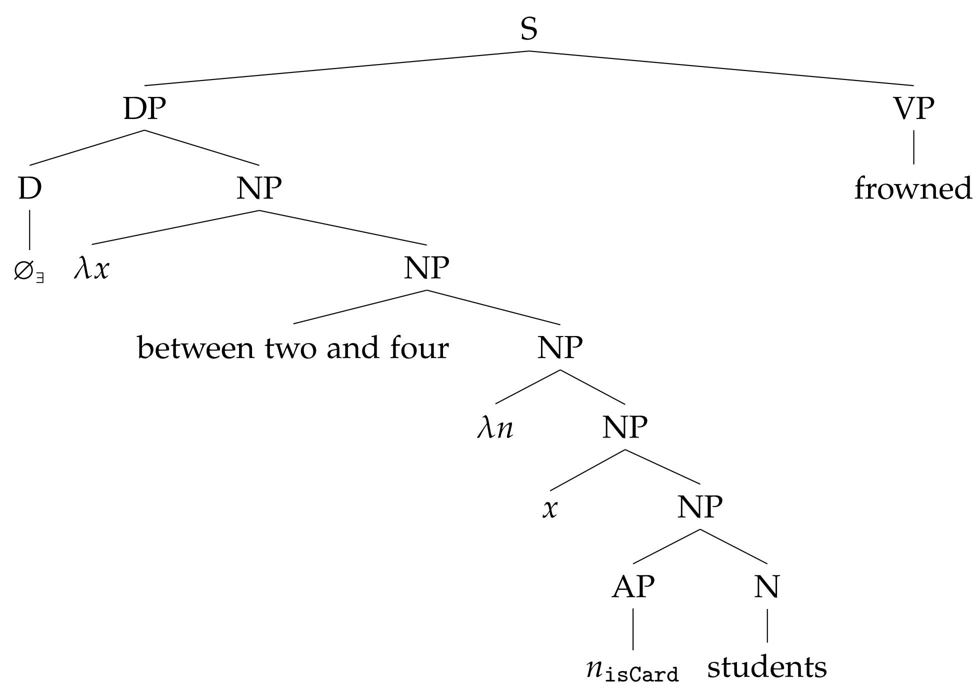
$\equiv \exists x[2 \leq \mathbf{card}(x) \leq 4 \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]$

When *between two and four* quantifies into AP, it creates an AP denoting the property of being a plurality x with two to four atomic parts. This property is then intersected with the property of being a plurality of students, resulting in the complex property of being a plurality with two to four atomic parts, each of which is a student. The empty determiner combines this complex property with the property of being a plurality of individuals who frowned, resulting in the reading that there is a plurality of students who frowned, containing two to four atomic members. This, of course, is the non-upper-bounded, existential reading, which amounts to 'At least two students frowned.' This LF is therefore ruled out by the pragmatic economy constraint in (56).

Quantifying into NP turns out to yield the exact same reading.

(75) **Quantifying into NP**

a.



b. [$\varnothing_\exists$ [λx [[between two and four] [λn [x [$n_{\texttt{isCard}}$ students]]]]]] frowned

c. $\exists x[2 \leq \max(\lambda n\,.\,\mathbf{card}(x) = n \wedge \mathbf{students}(x)) \leq 4 \wedge \mathbf{frowned}(x)]$

$\equiv \exists x[2 \leq \mathbf{card}(x) \leq 4 \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]$

When *between two and four* quantifies into NP, it creates a complex property of being a plurality of students with between two and four atomic parts. This property (the highest NP node in (75)) is equivalent to the NP node in (74) (which was the intersection of the AP and N properties). Thus, the same complex property is fed to the null determiner in both cases. Hence, the ultimate truth conditions are equivalent.[14] As a result, this LF is also excluded by the pragmatic constraint in (56).

We see, then, that when *between two and four* QRs above $\varnothing_\exists$, we get an upper-bounded reading, which is exactly what is needed in distributive cases (cf. (71)). And when *between two and four* QRs below $\varnothing_\exists$ (be it into AP or into NP), we derive an existential reading, which we do not want for distributive cases (cf. (74) and (75)). In those cases, however, the relevant LFs are correctly filtered out by the pragmatic economy constraint in (56).

Existential readings (LFs) are not, of course, ruled out when the relevant modified numerals combine with non-distributive predicates. Thus, (76a) is assigned the right reading under an LF like (76b).

(76) a. Between five and ten soldiers surrounded the castle.

b. [$\varnothing_\exists$ [[λx [[between five and ten] [λn [x $n_{\texttt{isCard}}$]]]] soldiers]] [surrounded . . .]

c. $\exists x[5 \leq \max(\lambda n\,.\,\mathbf{card}(x) = n) \leq 10 \wedge \mathbf{soldiers}(x) \wedge \mathbf{surrounded}(x)]$

$\equiv \exists x[5 \leq \mathbf{card}(x) \leq 10 \wedge \mathbf{soldiers}(x) \wedge \mathbf{surrounded}(x)]$

[14]This is indeed the case in general, thanks to the equivalence in (i). This equivalence holds because of the fact that, for every sum x, we have $\max(\lambda n\,.\,\mathbf{card}(x) = n) = \mathbf{card}(x)$, given that every sum has exactly one cardinality.

(i) $\exists x[2 \leq \max(\lambda n\,.\,\mathbf{card}(x) = n \wedge [\![\text{NP}]\!](x)) \leq 4 \wedge [\![\text{VP}]\!](x)]$ (Quantifying into NP)
$\equiv \exists x[2 \leq \max(\lambda n\,.\,\mathbf{card}(x) = n) \leq 4 \wedge [\![\text{NP}]\!](x) \wedge [\![\text{VP}]\!](x)]$ (Quantifying into AP)
$\equiv \exists x[2 \leq \mathbf{card}(x) \leq 4 \wedge [\![\text{NP}]\!](x) \wedge [\![\text{VP}]\!](x)]$

Here, the 'absence' of maximality—that is, the reason why we can interpret (76a) as not conveying any upper bound, despite the presence of *between*—is explained the fact that $\max(\lambda n \,.\, \mathbf{card}(x) = n)$ is equivalent to $\mathbf{card}(x)$ for all x (see footnote 14).

However, we do predict a genuine ambiguity for such cases. For example, (77a) could also receive the parse in (77b), where now maximality is non-vacuous.

(77) a. Between five and ten soldiers surrounded the castle.

b. [between five and ten] [λn [[$\varnothing_\exists$ [$n_{\texttt{isCard}}$ soldiers]] [surrounded the castle]]]

c. $5 \leq \max(\lambda n \,.\, \exists x[\mathbf{card}(x) = n \wedge \mathbf{soldiers}(x) \wedge \mathbf{surrounded}(x)]) \leq 10$

Unlike existential readings in distributive contexts, this reading is not blocked by the constraint in (56): replacing *ten* with *nine* results in a stronger reading, while replacing it with *eleven* results in a weaker reading; similarly, replacing *five* with *four* results in a weaker reading, while replacing it with *six* results in a stronger reading.

Now, is such a reading available? If it were, then the sentence would be judged false if, say, one group of eight soldiers surrounded the castle, while another group of twenty soldiers surrounded the castle. As mentioned in chapter 1, it is not clear that such a reading exists. I return to this point in chapter 3.

2.3.3 *Less than* and empty degree predicates

Before concluding this chapter, we need to make an important revision to our definition of the maximality component (max). The reason has to do with what LMax predicts when *less than five* combines with a degree predicate whose extension is empty.

Suppose that no students frowned. Then, following the discussion in chapter 1, we want a sentence like (2), repeated in (78a), to be true.

(78) a. Less than five students frowned.

b. [less than five] [λn [[$\varnothing_\exists$ [$n_{\texttt{isCard}}$ students]] frowned]]

Now, what does LMax currently deliver for this sentence under the LF in (78b)? Well, the statement

(79) [$\varnothing_\exists$ [$n_{\texttt{isCard}}$ students]] frowned

is false for every number n, including 0: since we are assuming there is no null individual, i.e. no sum x such that $\textbf{card}(x) = 0$, then the cardinality predicate $0_{\texttt{isCard}}$ returns false for every x. Thus, the degree predicate

(80) λn [[$\varnothing_\exists$ [$n_{\texttt{isCard}}$ students]] frowned]

is empty. As a result, there is no maximal element of this degree predicate, and so (78a) winds up being false. (The existential statement over numbers contributed by *less than*, 'There is a number k which is maximal . . . ', is false.)

To resolve this problem, we can simply stipulate that 0 counts as maximal, relative to a partially ordered set P, if P is empty. That is, we can redefine max as follows.

(81) For any predicate of degrees P_{dt}, any partial ordering $\leq$ over P, and any degree n,

$$\max_{\leq}(P)(n) = \begin{cases} 1 & \text{if } P(n) \wedge \neg\exists m[P(m) \wedge m > n] \\ 1 & \text{if } n = 0 \wedge \neg\exists m P(m) \\ 0 & \text{otherwise} \end{cases}$$

This formulation is the same as before, except that it ensures that $\max(\varnothing) = 0$. Thus, (78a) correctly winds up meaning that zero, one, . . . , or four students frowned, and no more than that.

This move also has a welcome consequence for the special case of *between zero and n*: a sentence like *Between zero and five students frowned* is now correctly predicted to be true if no students frowned.

2.4 Summary

In this chapter, I proposed a way to capture the fact that sentences in which a modified numeral like *less than five* or *between two and four* combines with a distributive predicate like *frowned* have upper-bounded, non-existential readings, whereas sentences in which they combine with a non-distributive predicate like *lifted the piano together* or *surrounded the castle* have existential readings. The idea is that the intrinsic meaning of (the relevant) modified numerals involves a maximality component and that modified numerals can interact scopally with a silent existential determiner ($\varnothing_\exists$), thus giving rise to both upper-bounded (or maximal) readings and non-upper-bounded, existential readings, depending on whether the modified numeral scopes above or below $\varnothing_\exists$. When scoping below $\varnothing_\exists$ gives rise to a kind of weak reading (i.e. when the modified numeral makes no semantic contribution), this reading is blocked by a pragmatic economy constraint (cf. (56)). Thus, the interaction between the generative component of the grammar and a pragmatic blocking mechanism explains why a sentence like *Less than five students lifted that piano together* has a non-upper-bounded, existential reading, while *Less than five students frowned* does not. In the next chapter, I turn discuss three predictions that this account makes.

Chapter 3

Three predictions

3.1 Overview

In chapter 2, I presented a fairly standard account (Lexical Maximality, or LMax) of how both upper-bounded readings and existential readings can be generated for sentences involving modified numerals like *less than five* and *between two and four*. In a nutshell, modified numerals denote generalized quantifiers over degrees, which lexically encode a maximality component and which can move to take scope either below or above a silent existential determiner ($\varnothing_\exists$). Scoping below $\varnothing_\exists$ gives rise to existential readings, while scoping above $\varnothing_\exists$ gives rise to upper-bounded readings. In addition, a pragmatic blocking mechanism regulates, in certain cases, where the modified numeral may land: when landing below $\varnothing_\exists$ destroys the semantic contribution of the modified numeral (e.g. in distributive contexts), that LF is ruled out. In this chapter, I explore three predictions that such an account makes. The first two predictions relate to the pragmatic blocking mechanism; they have to do with cases where we observe 'downward inferences' (inferences from groups to subgroups) beyond just the typical case of a distributive predicate like *frown*—hence, cases where the constraint (correctly) blocks non-upper-bounded readings. Specifically, section 3.2 investigates a class of predicates which are collective but which nevertheless license a kind of downward inference, while section 3.3 explains a predicted asymmetry that arises in the interpretation of cumulative transitive predicates. The third prediction has more to do with the grammatical system of LMax; namely, section 3.4 revisits the upper-bounded readings generated by LMax in non-distributive contexts. I argue that these readings seem to be unavailable, which therefore presents a problem for the LMax account.

3.2 Prediction 1: Collective predicates with downward inferences

To recap so far, we've seen that a sentence like (82), with the distributive predicate *frowned*, has an upper-bounded reading with no existential entailment, which can be paraphrased as, 'The maximum number of students who frowned, if any, is less than five.' It does not have the non-upper-bounded, existential reading, 'A group of less than five students frowned.' By contrast, (83), with the collective predicate *lifted the piano*, does have the non-upper-bounded, existential reading, 'A group of less than five students lifted the piano.'

(82) Less than five students frowned.

(83) Less than five students lifted the piano.

Now consider (84), with the collective predicate *gathered*. Intuitively, (84) works just like (82), and not like (83): (84) is true iff the maximum number of students who gathered is less than five. That is, if (a group of) five or more students gathered, then (84) is false.

(84) Less than five students gathered.

I argue that the pragmatic constraint proposed in chapter 2 (see (56)) actually predicts this. The argument relies on the fact that collective predicates like *gather*, despite being collective, do in fact allow a kind of weak downward inference. Suppose that Ann, Bill, Carol, and Dan gathered. Then we cannot conclude that, say, Ann gathered, for *Ann gathered* is an unacceptable sentence, and it is unclear what it would mean for a single individual to gather. However, intuitively, we *can* conclude that Ann, Bill, and Carol gathered; that Ann, Bill, and Dan gathered; that Ann and Bill gathered; and so on. In other words, although *gather* does not distribute all the way down to the atoms of a plurality, it does seem to distribute down to (at least some of) the non-atomic subparts of a plurality. Thus, if 1000 students gathered, then it seems natural to conclude that 999, . . . , 2 students gathered.[1]

What is important, as far as our pragmatic constraint is concerned, is that if a group x gathered, then we can always find some subgroup y of x such that $\mathbf{card}(y) = 2$ and y gathered. Let us call this property *weak distributivity*.[2]

(85) **Weak distributivity**
P is weakly distributive iff
$\forall x[P(x) \wedge \mathbf{card}(x) \geq 2 \rightarrow \exists y[y \sqsubseteq x \wedge \mathbf{card}(y) = 2 \wedge P(y)]]$.
(. . . iff, if P is true of a non-atomic plurality x, then P is true of some subpart of x with two atomic parts.)

Now, the inference from *Ten students gathered* to *Two students gathered* is predicted to be valid, as shown in (86). As argued above, this appears to be a welcome prediction.

(86) $\exists x[\mathbf{card}(x) = 10 \wedge \mathbf{students}(x) \wedge \mathbf{gathered}(x)]$ (Ten students gathered.)
$\forall x, y[\mathbf{students}(x) \wedge y \sqsubseteq x \rightarrow \mathbf{students}(y)]$ (*students* is (strongly) distr.)

[1] At least, such an inference seems as natural to me as the inference from *1000 students attended* to *999, . . . , 2 students attended*.

[2] On this definition, every (strongly) distributive predicate (in the sense of (21)) is weakly distributive, but not vice versa.

$$\forall x[\mathbf{gathered}(x) \wedge \mathbf{card}(x) \geq 2 \rightarrow \exists y[y \sqsubseteq x \wedge \mathbf{card}(y) = 2 \wedge \mathbf{gathered}(y)]]$$

(*gathered* is weakly distr.)

$$\vDash \quad \exists y[\mathbf{card}(y) = 2 \wedge \mathbf{students}(y) \wedge \mathbf{gathered}(y)]$$

(Two students gathered.)

What we would like now is for our pragmatic economy constraint in (56) to block any LF of (84) that corresponds to the existential, non-upper-bounded reading represented in (87).

(87) $\exists x[\mathbf{card}(x) < 5 \wedge \mathbf{students}(x) \wedge \mathbf{gathered}(x)]$

If the weak distributivity of *gathered* is taken into consideration, then (87) is equivalent to (88).

(88) $\exists x[\mathbf{students}(x) \wedge \mathbf{gathered}(x)]$

The entailment from (87) to (88) follows from conjunction elimination. The reverse entailment follows because if z is a plurality of students who gathered (which verifies (88)), then, since *gathered* has only non-atomic sums in its extension, $\mathbf{card}(z) \geq 2$, and so by weak distributivity of *gathered* (and of *students*, which is also strongly distributive), there is a subpart z' of z with cardinality $2 < 5$ who are students who gathered, which verifies (87).

What this reasoning illustrates is that the numeral *five* (in an LF of the kind in question) has no semantic import. The same truth conditions would be derived (by similar reasoning) if *five* were replaced, say, by *four* or by *six*. Thus, this reading is ruled out by our pragmatic constraint, leaving only the upper-bounded reading.

Other collective predicates that seem to work like *gather*, i.e. that are at least weakly distributive in the sense of (85), are *fit into the elevator*,[3] *be neighbors*, *hold hands*, *know*

[3] It should be noted that *fit*, despite being a collective predicate, in fact seems to be strongly distributive, in the sense of (21): if a group x fit into the elevator, then every part of x, including the atoms of x, also fit.

each other, and *be similar*. As expected, the sentences in (89) only have upper-bounded readings.

(89) a. Less than five students fit into the elevator.[4]

b. Less than five students are neighbors.

c. Less than five students were holding hands.

d. Less than five students know each other.

e. Less than five students are similar.

3.3 Prediction 2: An asymmetry with cumulative readings

Let us return to our example in (8a), repeated below as (90).[5]

(90) Between two and four students drank over ten beers between them.

This is an instance of the so-called *cumulative* (or *co-dependent*) reading of transitive predicates, which is forced here by the phrase *between them*. The important point here is that on the cumulative reading of *drank*, there is no downward inference regarding the subject argument. That is, (91a) below does not entail (91b):

(91) a. Ann, Bill, and Carol drank over twenty beers between them.

b. Ann and Bill drank over twenty beers between them.

Given the absence of any downward inference, the subject of such a cumulative predicate will behave just as if it were the subject of a collective predicate that does not allow any downward inference. As a result, the existential, non-upper-bounded reading of (90) will

[4]This intended reading of this sentence is an episodic one. For discussion of the *characterizing* (i.e. generic) reading, where *fit* is interpreted as *can fit*, see section 5.5.1.

[5]The asymmetry described in this subsection is due to Benjamin Spector.

(correctly) not be blocked by our pragmatic economy constraint.

Now, let us see what we predict if a modified numeral of the relevant sort (i.e. *less than n* or *between m and n*) occurs in the object position of a cumulative predicate. The predictions we make depend on the specific treatment of cumulative predicates. We will assume (contrary to what is often assumed) that the object of a cumulative predicate licenses downward inferences,[6] so that we expect an upper-bounded reading to be forced for modified numerals in object position.

Let us first consider the following simple sentence:

(92) These twenty boys danced with those ten girls.

On the cumulative reading, (92) is standardly analyzed as meaning that every one of these twenty boys danced with at least one of those ten girls, and that for every one of those ten girls, at least one of these twenty boys danced with her. One way to derive this reading is to analyze the verb *dance* as $dance_{cumul}$, which has a cumulative meaning defined in terms of a primitive lexical entry for *dance* that is only defined for atoms.[7]

(93) $[\![\text{dance}_{\text{cumul}}]\!]$

$$= \lambda Y_e \,.\, \lambda X_e \,.\, \forall x[\mathbf{atom}(x) \wedge x \sqsubseteq X \rightarrow \exists y[\mathbf{atom}(y) \wedge y \sqsubseteq Y \wedge [\![\text{dance}]\!](y)(x)]] \wedge \forall y[\mathbf{atom}(y) \wedge y \sqsubseteq Y \rightarrow \exists x[\mathbf{atom}(x) \wedge x \sqsubseteq X \wedge [\![\text{dance}]\!](y)(x)]]$$

On such a view, there is no downward inference either on the subject side or on the object side, and as a result we would expect no difference for modified numerals depending on whether they occur in subject or in object position.

[6] This observation has also been independently made by Viola Schmitt (Schmitt 2015) and to some extent (in my view anyway) by Seth Cable (class notes on Krifka 1999, available at `http://people.umass.edu/scable/LING720-FA10/Handouts/Krifka-1999.pdf`).

[7] The actual analysis of how cumulative readings arise is unimportant here. The purpose of providing lexical entries is just to be explicit about the meanings of the readings under discussion and how they interact with the pragmatic blocking constraint in (56).

However, there is some evidence that (93) leads to truth conditions that are too strong. This can easily be seen if we use another type of predicate:

(94) These twenty chickens laid those ten eggs between them.

On the basis of a lexical entry such as (93) for lay_{cumul}, (94) would be predicted to entail that every one of the twenty chickens laid one of the ten eggs, which entails that some eggs were laid by two different chickens—an impossible state of affairs. That is, (94) would be a contextual contradiction. However, we observe that it is not. Rather, it simply entails that each of the ten eggs was laid by one of the twenty chickens. This suggests a much weaker lexical entry:

(95) $[\![\text{lay}_{\text{cumul}}]\!] = \lambda Y_e\,.\,\lambda X_e\,.\,\forall y[\textbf{atom}(y) \wedge y \sqsubseteq Y \rightarrow \exists x[\textbf{atom}(x) \wedge x \sqsubseteq X \wedge [\![\text{lay}]\!](y)(x)]]$

Now, on such a view, the cumulative reading of *lay* does actually license a downward inference on the object side. Informally, X *laid* Y now means that every atomic part of Y was laid by an atomic part of X. So suppose that X *laid* Y is true and that Y' is a proper subpart of Y. Since every atomic member of Y' is also an atomic member of Y, every atomic member of Y' must have been laid by an atomic part of X, and therefore X *laid* Y' is true as well.

Given this, it is now predicted that our pragmatic economy constraint will block the existential, non-upper-bounded reading of a modified numeral in object position, i.e. force an upper-bounded reading. This is exactly what we observe: (96a) is interpreted as being equivalent to (96b).

(96) a. These chickens laid less than ten eggs between them.
b. It is not the case that these chickens laid ten or more eggs between them.

We also have an explanation for the following contrast:

(97) a. Less than ten chickens laid more than twenty eggs between them.
$\rightsquigarrow$ Non-upper-bounded reading for the subject: compatible with twenty chickens having laid more than twenty eggs between them.

b. More than ten chickens laid less than twenty eggs between them.
$\rightsquigarrow$ Upper-bounded reading for the object: entails that there is a group of more than ten chickens who did not lay twenty or more eggs between them.[8]

3.4 Prediction 3: Upper-bounded readings in non-distributive contexts

So far, we are able to account for the fact that, in certain environments, modified numerals can only give rise to an upper-bounded reading. The gist of the explanation is that the other reading (namely the existential, non-upper-bounded reading) is ruled out by our pragmatic economy constraint in certain environments. We do, however, predict a genuine ambiguity in cases where the constraint does not rule out the existential, non-upper-bounded reading. Consider now (6a), repeated below as (98).

(98) Less than five students lifted the piano together.

In chapter 1, I argued that sentences like (98) can *only* receive the existential, non-upper-bounded reading, and I observed that replacing *the piano* with the NPI *any piano(s)* leads to a deviant sentence, which seems to add further support that the upper-bounded reading is unavailable. This, however, is not expected if (98) is ambiguous, as we currently predict,

[8] I have in mind here an LF where *more than ten* scopes above *less than twenty*, and both degree phrases scope out of their respective DPs (hence, take scope over their respective silent determiners). It is unclear to me whether an LF where *less than twenty* scopes above *more than ten* is also available. (Such an LF would seem to violate Kennedy's generalization, which states that if the scope of a quantificational DP (e.g. *more than ten chickens*) contains the trace of a degree phrase (e.g. the trace left by *less than twenty*), then it must also contain the degree phrase itself; see Kennedy 1997 and Heim 2000.)

between the existential and the upper-bounded readings. Under the parse given in (99a), the sentence should be able to be interpreted as in (99b), which amounts to saying that no group of five or more students lifted the piano (and is consistent with no group of students at all having lifted the piano).

(99) a. [less than five] [λn [$\varnothing_\exists$ [$n_{\mathtt{isCard}}$ students] [lifted the piano]]]

b. $\max(\lambda n \,.\, \exists x[\mathbf{card}(x) = n \wedge \mathbf{students}(x) \wedge \mathbf{lifted}(x)]) < 5$

Somehow, we need to rule out this reading, but doing so does not seem possible based solely on the pragmatic constraint in (56); hence, LMax seems to overgenerate. In chapter 6, I will develop a theory where a non-existential, upper-bounded interpretation is simply not generated at all when the modified numeral occurs in a position that does not license a downward inference. I will therefore take that theory to be superior to LMax as far as these data go.

Before turning to this alternative theory, however, I would like to discuss a case where the ambiguity predicted by the LMax account has been argued to be detectable for at least some speakers. The relevant examples involve the predicate *form a stable coalition*.[9] Consider the following sentences:

(100) a. I will despair of politics only if less than 50 MPs form a stable coalition.

b. Unfortunately, less than 50 MPs formed a stable coalition.

c. When less than 350 MPs form a stable coalition, there cannot be a stable government.

(101) Surprisingly, less than 50 MPs formed a stable coalition and these MPs were able to counterbalance the much larger coalition that 360 other MPs formed at the same time.

[9]The following data are due to Philippe Schlenker (p.c. with Benjamin Spector).

The intended readings for (100a), (100b) and (100c) are, respectively, the following:

(102) a. I will despair of politics only if no coalition with 50 or more MPs is formed.
b. Unfortunately, there was no coalition formed by 50 or more MPs.
c. When no coalition with 350 or more MPs is formed, there cannot be a stable government.

In these three cases, the intended reading is one where the modified numeral receives the upper-bounded interpretation. To the extent that these readings are available for the sentences in (100), it appears that a modified numeral that combines with *form a coalition* can give rise to an upper-bounded interpretation, despite the fact that *form a coalition* does not seem to license downward inferences. (If a group of MPs formed a stable coalition, it does not follow that any proper subgroup did.)

In the case of (101), however, an upper-bounded reading would lead to a contextual contradiction, and the only possible reading is the existential, non-upper-bounded one, paraphrased in (103).

(103) Surprisingly, there was a coalition formed which contained less than 50 MPs, and the members of this coalition were able to counterbalance the much larger coalition formed by 360 other MPs.

If both types of readings are available, we have evidence for a genuine ambiguity. Let me note, however, that judgments are far from uniform: only some speakers appear to accept the interpretations of the sentences in (100) suggested in (102). Furthermore, even for those speakers who access these readings (e.g. Benjamin Spector, p.c.), they seem to be possible only with some predicates. This might suggest that the relevant predicates are themselves ambiguous between a meaning that licenses downward inferences (contrary to appearances) and a meaning that does not. In that case, it seems better to have a theory that derives upper-bounded readings only with predicates that license downward inferences, rather than a theory (like LMax) that derives upper-bounded readings across the board.

3.5 Summary

In this chapter, I presented three predictions made by the LMax account. The first prediction is that *less than n* and *between m and n* in the subject position of 'weakly distributive' predicates (e.g. *gather*) are only interpreted as conveying an upper bound. The second prediction is that when it comes to cumulatively interpreted transitive predicates, there is an asymmetry in how these modified numerals are interpreted: in subject position, they do not convey an upper bound, while in object position they do. I showed that these two predictions are indeed borne out. The third and final prediction is that sentences where modified numerals combine with a collective predicate ought to be ambiguous between an upper-bounded reading and an existential reading. I argued that, on the whole, this prediction seems to be wrong; thus, LMax overgenerates here. In the next chapter, I develop a proposal that was independently made by Benjamin Spector, in which maximality is 'severed' from the semantics of modified numerals. This theory, I show, makes exactly the same predictions as LMax does regarding the core data discussed so far (including those in this chapter). However, it differs from LMax in that it also generates so-called 'split-scope' readings, where an operator intervenes between the modified numeral and the maximality component.

Chapter 4

Severing maximality from numeral modifiers

4.1 Overview

In chapter 2, I proposed a theory, Lexical Maximality (LMax), that generates both upper-bounded readings and existential readings of sentences with *less than* and *between*. LMax does this by positing a maximality component as part of the lexical meanings of *less than* and *between* and allowing modified numerals to interact scopally with a silent existential determiner ($\varnothing_\exists$). This system grammatically overgenerates readings, but the grammatical overgeneration is reigned in (at least in part) by a pragmatic blocking mechanism that filters out readings in certain cases. In this chapter, following a proposal by Benjamin Spector (Spector 2014), I develop a slightly different method of overgeneration, in which the maximality component is viewed instead as an autonomous, and in principle optional, operation, separate from numeral modifiers. For this reason, I call this theory *Separate Maximality* (or SMax, for short). SMax is capable of generating even more LFs (for the relevant sentences) than LMax. For the core sentences we have looked at so far, the extra LFs generated by SMax turn out not to be logically distinct from those already generated by LMax. However, when we turn to so-called 'split-scope' data, SMax indeed generates readings that LMax does not, hence makes slightly different predictions than LMax.

The maximization operation proposed in this chapter is inspired by the sort of operation that may be responsible for upper-bounded interpretations of bare numerals (see the discussion in section 1.2.2). I therefore start in section 4.2 by showing how the operation is applied in the bare numeral case. Afterwards, in section 4.3, I move to modified numerals. Finally, section 4.4 discusses the very subtle split-scope data that potentially tease apart LMax and SMax.

4.2 Maximality and bare numerals

We build on our 'semantic housekeeping' (see section 1.2.7) by assuming not only the operator `isCard`, repeated in (104), but also an operator `isMax`, given in (105). Whereas `isCard` maps a type *d* numeral to a type *et* predicate of individuals, `isMax` maps a numeral to a type $(dt)t$ generalized quantifier over degrees. Note that the metalanguage relation max used in (105) is the same as the (revised) one that I defined in (81) in chapter 2 (see section 2.3.3), and recall that we are interested here in the natural ordering over numbers, hence the notational simplifications.[1] In the same way that I write $n_{\texttt{isCard}}$ for `isCard` n, I will write $n_{\texttt{isMax}}$ for `isMax` n.

(104) $[\![\texttt{isCard}]\!] = \lambda n_d \,.\, \lambda x_e \,.\, \mathbf{card}(x) = n$

(105) $[\![\texttt{isMax}]\!] = \lambda n_d \,.\, \lambda P_{dt} \,.\, \max_{\leq}(P)(n)$
$= \lambda n_d \,.\, \lambda P_{dt} \,.\, \max(P) = n$

(106) $[\![n_{\texttt{isMax}}]\!] = [\![\texttt{isMax}\ n]\!] = \lambda P_{dt} \,.\, \max_{\leq}(P)([\![n]\!])$
$= \lambda P_{dt} \,.\, \max(P) = [\![n]\!]$

[1] Recall also that, under the revised version of max in (81), if P is empty, then the maximum of P is defined to be 0. In this way, we can explain why *Less than five students frowned* is true even if no students frowned. See section 2.3.3 for discussion.

Consider now example (107) and the LF in (108a). In this LF, *three* has shifted to *three*$_{\texttt{isCard}}$, the intersective meaning of *three*, and the truth conditions are derived exactly as we already saw in (20) in section 1.2.2, under the adjectival theory. The sentence ends up meaning that at least three students frowned.

(107) Three students frowned.

(108) a. [$\varnothing_\exists$ [three$_{\texttt{isCard}}$ students]] frowned

b. $\exists x[\mathbf{card}(x) = 3 \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]$

But now consider the LF in (109a). In this LF, *three* has shifted to *three*$_{\texttt{isMax}}$, which (for type reasons) must move. In this case, it raises above the null determiner, creating a degree predicate in its scope, with its trace shifted to an intersective numerical variable. This derives the upper-bounded truth conditions in (109b), which state that the maximum number of students who frowned is three, i.e. that exactly three students frowned.

(109) a. three$_{\texttt{isMax}}$ [λn [[$\varnothing_\exists$ [$n_{\texttt{isCard}}$ students]] frowned]]

b. $\max(\lambda n\,.\,\exists x[\mathbf{card}(x) = n \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]) = 3$

This approach to the interpretation of bare numerals is close to a number of recent proposals (see, e.g., Geurts 2006; Kennedy 2015) in which various typeshifting operations account for the different attested readings of numerals. It is particularly close to Kennedy 2015, in which $n_{\texttt{isMax}}$ is taken to be the basic meaning of a numeral n. The analysis in (109) is exactly what Kennedy (2015) proposes for the interpretation of bare numerals.

At this point, I should note that it is also possible to derive the non-upper-bounded reading of numerals in distributive contexts even on the basis of $n_{\texttt{isMax}}$, by making sure that $n_{\texttt{isMax}}$ occurs in the scope of the null existential quantifier (following the logic of Van Benthem's problem).[2]

[2]Notice that we again (see chapter 2, footnote 14) rely on the equivalence between $\max(\lambda n\,.\,\mathbf{card}(x) = n)$ and $\mathbf{card}(x)$, which in turn ensures the equivalence between *three*$_{\texttt{isMax}}$ *[λn [x $n_{\texttt{isCard}}$]]* and *x three*$_{\texttt{isCard}}$.

(110) a. $[\varnothing_\exists\ [[\lambda x\ [\text{three}_{\texttt{isMax}}\ [\lambda n\ [x\ n_{\texttt{isCard}}]]]]\ \text{students}]]\ \text{frowned}$

b. $\exists x[\max(\lambda n\,.\,\mathbf{card}(x) = n) = 3 \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]$

$\equiv \exists x[\mathbf{card}(x) = 3 \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]$

We now turn to how this theory treats numeral modifiers like *less than* and *between*.

4.3 Maximality and modified numerals

Since we are severing maximality from the lexical meanings of *less than* and *between*,[3] the new entries for the latter, given in (111) and (112), look weaker than what we saw in chapter 2. There, *less than five* (see (62)), for example, combined with a predicate of degrees P and returned true iff $\max(P) < 5$, i.e. iff either P is true of some $k < 5$ and moreover k is the maximum of P, or else P is empty. Now, we require only that P be true of some $k < 5$, and not that k also be the maximum of P; the maximality part will come separately, via the operator `isMax` in (105).[4]

(111) $[\![\text{less than}]\!] = \lambda n_d\,.\,\lambda P_{dt}\,.\,\exists k[k < n \wedge P(k)]$

(112) $[\![\text{between}]\!] = \lambda m_d\,.\,\lambda n_d\,.\,\lambda P_{dt}\,.\,\exists k[m \leq k \leq n \wedge P(k)]$

First consider (113). In the given LF, *less than five* scopes above the empty existential determiner, $\varnothing_\exists$, creating a degree predicate in its scope, with its trace shifted to an intersective denotation ($n_{\texttt{isCard}}$). Due to the distributivity of *frowned*, the derived truth conditions are weak existential truth conditions, which amount to saying that at least one student frowned. At this point, we invoke the pragmatic constraint in (56) from chapter 2 to block this LF.[5]

[3]Put differently, we are assuming that the upper-bounded interpretations of *less than n* and *between m and n* have the same source as the upper-bounded interpretations of bare numerals, viz. `isMax`.

[4]The new meaning for *less than* in (111) is not strictly weaker than the one in (62) from chapter 2, since the latter, but not the former returns true when the degree predicate is empty.

[5]Of course, this sort of LF is exactly right for the analogous sentence with a non-distributive predicate, such as *lifted the piano together* or *drank more than ten beers between them*, and that LF will not be blocked.

(113) a. Less than five students frowned.

b. [less than five] [λn [[$\varnothing_\exists$ [$n_{\texttt{isCard}}$ students]] frowned]]

c. $\exists n[n < 5 \wedge \exists x[\mathbf{card}(x) = n \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]]$

$\equiv \exists x[\mathbf{card}(x) < 5 \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]$

Now consider (114). The given LF involves two movements of *less than five*. It first raises above $\varnothing_\exists$, creating a degree predicate in its scope, with its trace shifted to an intersective denotation ($m_{\texttt{isCard}}$), as is familiar by now. It then raises a second time, creating a second degree predicate, with its second trace shifted to an `isMax` denotation ($n_{\texttt{isMax}}$). The resulting truth conditions are upper-bounded with no existential entailment, i.e. precisely the truth conditions we want for a sentence like this with the distributive predicate *frowned*.

(114) a. Less than five students frowned.

b. [less than five] [λn [$n_{\texttt{isMax}}$ [λm [[$\varnothing_\exists$ [$m_{\texttt{isCard}}$ students]] frowned]]]]

c. $\exists n[n < 5 \wedge \max(\lambda m . \exists x[\mathbf{card}(x) = m \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]) = n]$

$\equiv \max(\lambda m . \exists x[\mathbf{card}(x) = m \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]) < 5$

SMax can thus generate the same set of truth conditions as the previous theory; however, it does so not by scope ambiguity, but rather by optional application of maximization.

Of course, in principle nothing we have assumed so far prevents *less than five* from scoping below the empty existential determiner in this theory as well. (115) illustrates this case with no maximization, and (116) with maximization.[6] It turns out that both LFs derive the exact same, weak, existential truth conditions as were already derived in (113).[7]

(115) a. Less than five students frowned.

[6]The LFs in (115) and (116) both involve *less than five* quantifying into AP. Quantifying into NP is also possible but, as discussed in section 2.3.2 (see especially footnote 14), quantifying into AP and quantifying into NP yield logically equivalent LFs.

[7]Again, these LFs would be suitable for the analogous sentence with a non-distributive predicate, such as *lifted the piano together* or *drank more than ten beers between them*, and would not be blocked.

b. $[\varnothing_{\exists}\ [[\lambda x\ [[\text{less than five}]\ [\lambda n\ [x\ n_{\texttt{isCard}}]]]]\ \text{students}]]\ \text{frowned}$

c. $\exists x[\exists n[n < 5 \wedge \mathbf{card}(x) = n] \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]$

$\equiv \exists x[\mathbf{card}(x) < 5 \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]$

(116) a. Less than five students frowned.

b. $[\varnothing_{\exists}\ [[\lambda x\ [[\text{less than five}]\ [\lambda n\ [n_{\texttt{isMax}}\ [\lambda m\ [x\ m_{\texttt{isCard}}]]]]]]\ \text{students}]]\ \text{frowned}$

c. $\exists x[\exists n[n < 5 \wedge \max(\lambda m\,.\,\mathbf{card}(x) = m) = n] \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]$

$\equiv \exists x[\max(\lambda m\,.\,\mathbf{card}(x) = m) < 5 \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]$

$\equiv \exists x[\mathbf{card}(x) < 5 \wedge \mathbf{students}(x) \wedge \mathbf{frowned}(x)]$

The reason that the same truth conditions are derived is twofold. First, the lexical meaning of *less than* on this theory is a simple existential statement (about degrees); thus, the existential quantifier (over degrees) contributed by *less than* and the existential quantifier (over individuals) contributed by the silent determiner *commute*, meaning that there is no scope dependency between *less than five* and $\varnothing_{\exists}$; thus, (113b) and (115b) are equivalent. Second, when max ($n_{\texttt{isMax}}$) takes lowest scope, it has no semantic effect, due to the by now familiar equivalence between $\max(\lambda n\,.\,\mathbf{card}(x) = n)$ and $\mathbf{card}(x)$ (cf. the LF in (110a); see also chapter 2, footnote 14); thus, (115b) and (116b) are equivalent. It follows that (113b), (115b), and (116b) are all equivalent.

The upshot is that SMax can derive non-upper-bounded, existential truth conditions (which are required for non-distributive cases, not shown here) in several ways, as in (113b), (115b), and (116b), and it can derive upper-bounded truth conditions as well (which are required for distributive cases) with the scope order *less than five* $> n_{\texttt{isMax}} > \varnothing_{\exists}$, as in (114b). And the same pragmatic constraint we posited in chapter 2 for the LMax account can be invoked to block the unattested LFs in (113b), (115b), and (116b).

Of course, the very same predictions made by the LMax account, discussed in chapter 3, are also made the SMax account, since the two accounts derive the same LFs in those cases (up to logical equivalence) and rely on the same pragmatic blocking constraint. Thus, under SMax, *gather*-type collective predicates are correctly predicted to give rise only to upper-bounded readings with modified numerals like *less than n* and *between m and n* (see section 3.2), and the asymmetry concerning the interpretation of modified numerals in the subject vs. the object position of cumulative transitive predicates is also correctly predicted (see section 3.3). However, just as with LMax, SMax incorrectly predicts upper-bounded readings (in addition to existential readings) of modified numerals that combine with a collective predicate, as shown below, where the modified numeral takes wide scope and maximization applies in its scope, above the silent existential determiner.

(117) a. Less than five students lifted the piano together.

b. [less than five] [λn [$n_{\texttt{isMax}}$ [λm [[$\varnothing_{\exists}$ [$m_{\texttt{isCard}}$ students]] [lifted ...]]]]]

c. $\exists n[n < 5 \wedge \max(\lambda m\,.\,\exists x[\mathbf{card}(x) = m \wedge \mathbf{students}(x) \wedge \mathbf{lifted}(x)]) = n]$

$\equiv \max(\lambda m\,.\,\exists x[\mathbf{card}(x) = m \wedge \mathbf{students}(x) \wedge \mathbf{lifted}(x)]) < 5$

As discussed in section 3.4, this reading, which amounts to 'no group of five or more students lifted the piano', seems to be unavailable; hence, SMax, like LMax, overgenerates readings in non-distributive contexts. Nevertheless, let us set aside this issue and ask the question, Is there any reason to prefer SMax over LMax? In the next section, I discuss some data that bear on this question, but I conclude that judgments are too delicate to conclude one way or another which theory fares best. Nevertheless, the discussion sets the stage for the next chapter, where I discuss data that, at least on first appearances, seem to favor SMax over LMax.

4.4 Split scope: evidence for severing maximality?

A key difference between the LMax account and the SMax account is the following. On the LMax account, the maximality component is part of the very meaning of modified numerals, while on the SMax account, it is part of the meaning of the numerical variable bound by the modified numeral phrase. It follows that while the SMax account can generate all the readings that the LMax account can, the reverse is not true. Let me first illustrate this point in an abstract and schematic way. On the SMax account, LFs of the following type are in principle available:

(118) [less than ten] [λn [Op [$n_{\texttt{isMax}}$ [λm [...[$\varnothing_{\exists}$ [$m_{\texttt{isCard}}$ ...]] ...]]]]]

The crucial point here is that between *less than ten* and $n_{\texttt{isMax}}$, a 'scope-splitting' operator, *Op*, intervenes. Such an LF is unavailable on the LMax account: if *less than ten* takes widest scope, then so does the maximality component, because this component is part of the meaning of the modified numeral. We can observe the relevance of this point by considering now a real sentence, where *Op* is instantiated by the universal modal *have to*.[8]

(119) I have to take less than ten pills.

Let us schematize the surface form of this sentence as follows, where $\Box$ is a necessity operator standing for *have to*.

(120) $\Box$ [I take less than ten pills]

On the LMax account, the numerical phrase *less than ten* has to move to combine with a predicate of degrees, and (120) is ambiguous depending on where exactly *less than ten*

[8] It might be tempting to think that we can instantiate *Op* by a quantificational DP, rather than a modal, as in the LF in (ib). The idea would be that SMax, but not LMax, predicts that sentence (ia), under the LF in (ib), ought to have a reading like the one in (ic), which can be paraphrased as, 'There is a number $n < 5$ such that every student read *exactly* n books.' Intuitively, such a reading is *not* available, which would thus constitute evidence against SMax, in favor of LMax. However, the LF in (ib) violates a well-known generalization known as Kennedy's generalization, which states that if the scope of a quantificational DP (here, *every student*) contains the trace of a degree phrase (here, the trace left by *less than five*), then it must also contain the degree phrase itself (see Kennedy 1997 and Heim 2000). Thus, the unavailability of such a reading could be explained, under SMax, as simply violating Kennedy's generalization.

(i) a. Every student read less than five books.
b. [less than five] [λn [[every student] [λx [$n_{\texttt{isMax}}$ [λm [[$\varnothing_{\exists}$ [$m_{\texttt{isCard}}$ books]] [λy [x [read y]]]]]]]]]
c. $\exists n[n < 5 \wedge \forall x[\textbf{student}(x) \rightarrow \max(\lambda m \,.\, \exists y[\textbf{books}(y) \wedge \textbf{read}(y)(x)]) = n]]$

lands. I assume here that the numerical phrase cannot be interpreted lower than the silent determiner, $\varnothing_\exists$, due to our pragmatic blocking mechanism.[9] There are then two possible LFs, depending on whether *less than ten* scopes below or above the necessity modal, as illustrated in (121):

(121) a. Low-scope for *less than ten*:

(i) $\Box$ [[less than ten] [λn [[$\varnothing_\exists$ [$n_{\texttt{isCard}}$ pills]] [λx [I take x]]]]]

(ii) $\Box[\max(\lambda n\,.\,\exists x[\mathbf{card}(x) = n \wedge \mathbf{pills}(x) \wedge \mathbf{take}(x)([\![\text{I}]\!])]) < 10]$

$\rightsquigarrow$ 'In every permissible world, the number of pills I take is smaller than ten.'

b. Wide-scope for *less than ten*:

(i) [less than ten] [λn [$\Box$ [[$\varnothing_\exists$ [$n_{\texttt{isCard}}$ pills]] [λx [I take x]]]]]

(ii) $\max(\lambda n\,.\,\Box[\exists x[\mathbf{card}(x) = n \wedge \mathbf{pills}(x) \wedge \mathbf{take}(x)([\![\text{I}]\!])]]) < 10$

$\rightsquigarrow$ 'The number n such that I have to take n pills and don't have to take more than n pills is smaller than ten.'

On the low-scope reading, (120) simply states that I am forbidden to take ten or more pills. On the wide-scope reading, the sentence states that the minimal required number of pills I have to take is smaller than ten—on this reading the sentence is compatible with a situation where I am allowed to take 100 pills.[10]

Now, on the SMax account, both of these readings are generated (depending on whether maximization applies or not). However, a third ('split-scope') reading is predicted to be available, whose LF is given in (122a):

[9] This point relies on the fact that *take* is distributive on its object argument. In other words, the predicate *λx [I take x]* is distributive: if I take x, then I take every part of x.

[10] While such a reading is not pragmatically plausible in this case, structurally identical sentences clearly license the wide-scope reading. Consider, for instance, (i), inspired by Heim 2000:

(i) Fortunately, we have to write less than ten pages.
Intended: 'Fortunately, the minimal required length of the paper is less than ten pages.'

(122) a. [less than ten] [λn [$\Box$ [$n_{\texttt{isMax}}$ [λm [[$\varnothing_{\exists}$ [$m_{\texttt{isCard}}$ pills]] [λx [I take x]]]]]]]

b. $\exists n[n < 10 \wedge \Box[\max(\lambda m\,.\,\exists x[\mathbf{card}(x) = m \wedge \mathbf{pills}(x) \wedge \mathbf{take}(x)([\![\mathrm{I}]\!])]) = n]]$

c. 'There is a number n, smaller than ten, such that I have to take *exactly n* pills.'

The resulting reading states that there is a certain number, smaller than ten, such that I have to take *exactly* that number of pills. Now, does this reading exist? At an intuitive level, we seem to access this reading in the following type of discourse:

(123) I visited the doctor yesterday, and I don't remember everything he told me. He prescribed a certain medication. I have to take less than ten pills, but I don't remember how many exactly.

One can certainly understand (123) as implying that there is a number n smaller than ten such that I have to take exactly n pills. This, however, is not sufficient to establish that the LF in (122a) is really available. The reason is the following. In normal situations, doctors prescribe a specific number of pills. Suppose that this information is taken for granted by the speaker and addressee of (123). Now, assume that *I have to take less than ten pills* is parsed as in (121a). Then it simply means that in every permissible world the number of pills I take is smaller than ten (the 'low-scope' reading). While this low-scope reading is strictly weaker than the reading corresponding to (122a), together with the contextual knowledge that there is a number n such that I have to take exactly n pills, it turns out to be contextually equivalent to (122a). That is, if, in every permissible world, I take less than ten pills (meaning of (121a)), and if, furthermore, I take the same number of pills in every permissible world (contextual information), then it must be the case that there is a specific number n, smaller than ten, such that in every permissible world I take exactly n pills (meaning of (122a)).

The problem we face here is that the type of reading we are trying to probe, viz. (122a), is not very plausible in general. For instance, if we replace *take less than ten pills* with *solve less than ten problems*, the corresponding reading would imply that there is a specific number n such that I have to solve n problems and am forbidden to solve more, which is a highly unlikely state of affairs. But the kind of contexts that make readings corresponding to (122a) plausible also tend to make this reading contextually equivalent to the low-scope reading (corresponding to (121a)).[11]

One potential way around this problem is to construct an example where the entailment relations between putative readings are reversed. This we can do by replacing the upward-entailing modal *have to* with the downward-entailing modal *be forbidden to*. And of course we have to create an appropriate context. Here is one attempt.[12]

(124) a. Context: Peter belongs to a weird cult, in which a certain number (maybe 13) is viewed as evil, and it is absolutely forbidden (on pain of death, say) to invite exactly that number of people. Any other number is OK.

b. Peter is forbidden to invite between 10 and 20 people, but I don't remember the exact number.

Now, the intended (split-scope) reading is, 'There is a number n between 10 and 20 such that Peter is forbidden to invite exactly n people.' This reading is true, say, if Peter can invite any number of people distinct from 13 but is forbidden to invite exactly 13 people. Importantly, in such a situation, the low-scope reading ('It is forbidden for Peter to invite 10, 11, 12, ... people') is false since Peter can invite 12 people, for instance. As for the potential wide-scope reading ('There is a number n between 10 and 20 such that n is the

[11]One could argue that the sluice (*I don't remember how many exactly*) independently forces, for syntactic reasons, a wide-scope construal of its antecedent, *less than ten pills*, which would be consistent with Romero 1998 and Johnson 2001. Yet the whole argument would then depend on a specific syntactic analysis of sluicing, which might be open to challenges.

[12]I am now switching from *less than ten* to *between ten and twenty* because, for some reason, the relevant reading is slightly more accessible for me with *between* than with *less than*.

maximum of the extension of λn *[Peter is forbidden to invite n or more people]*'), it should be either false or a presupposition failure, depending on how maximality is failure is handled in the definition of max, simply because either the degree predicate is empty (Peter is allowed to invite any number of people), or if it is non-empty, then it has no maximal element (if Peter is forbidden to invite n or more people, then he is forbidden to invite m or more people for all $m > n$; see also chapter 2, footnote 7).

So, to the extent that (124b) could be considered appropriate, it would provide evidence for a parse of the following sort:

(125) a. [between 10 and 20] [λn [forbidden [$n_{\texttt{isMax}}$ [λm [[$\varnothing_{\exists}$ [$m_{\texttt{isCard}}$ people]] [λx [Peter invites x]]]]]]]

b. 'There is a number n, between 10 and 20, such that Peter is forbidden to invite *exactly* n people.'

Now, the question is, is this reading intuitively available? We are dealing here with very delicate judgments for which introspective intuitions might be unreliable. My personal feeling is that this reading is marginal at best, and that the analogous reading for the analogous sentence with *less than* (*Peter is forbidden to invite less than ten people* $\rightsquigarrow$ 'There is a number $n < 10$ such that Peter is forbidden to invite exactly n people') is even less accessible. Nevertheless, Benjamin Spector (p.c.) reports that he and the informants he has consulted seem to be able to access such readings (in French). I therefore conclude for now (in the absence of a more detailed investigation) that split scope data do not provide clear evidence in either direction regarding the choice between LMax and SMax.

4.5 Summary

In this chapter, I developed a second theory (proposed independently by Benjamin Spector; see Spector 2014) to account for the core data introduced in chapter 1. This theory (SMax), like the theory developed in chapter 2 (LMax), is based on the idea that sentences with modified numerals are ambiguous between two readings and that certain readings are sometimes pragmatically excluded. SMax differs from LMax in the way that that ambiguity arises: both accounts posit that a maximality component is at work in modified numeral constructions, but they differ in the nature of this maximality component. LMax takes maximality to be part of the lexical meaning of modified numerals and then relies on scope ambiguity between modified numerals and the silent existential determiner, whereas SMax takes maximality to be a separate (non-lexical) component which may apply optionally. Setting aside split-scope data, the two theories make exactly the same predictions, due crucially to the fact that, on the SMax account, there is no scope dependency between modified numerals and the silent existential determiner (they are both existential quantifiers, which commute). To distinguish the two accounts, it therefore seems necessary to find a case where, on the SMax account, there *is* a scope dependency. To that end, I move now to data from the generic domain, where numerals and modified numerals receive a quasi-universal, rather than existential interpretation.

Chapter 5

The second puzzle: evidence from genericity

5.1 Overview

So far, I have introduced two accounts of why modified numerals like *less than five* and *between two and four* are sometimes interpreted as upper-bounded, other times as existential, depending on whether they occur with a distributive or a non-distributive predicate. Both theories posit a maximality component to derive upper-bounded readings, and both theories rely on ambiguity, coupled with pragmatic blocking, to explain the range of readings we observe. Their main difference is in the source of maximality: the Lexical Maximality (LMax) account locates maximality within the intrinsic, lexical meaning of numeral modifiers, whereas the Separate Maximality (SMax) account takes maximality to be a non-lexical and optional component, separate from the meaning of numeral modifiers (in particular, part of the meaning of bare numerals and numerical variables). As discussed in the last chapter (ignoring split scope data), SMax generates exactly the same readings LMax does, despite generating a greater number of LFs for a given modified numeral sentence. The reason is because, under SMax, modified numerals are treated as simple existential quantifiers over degrees, and so there is no scope dependency at all between modified numerals and the silent existential determiner. If, of course, we can find a case where there *is* such scope dependency, then we might be able to tease apart LMax and SMax once and for all. In this chapter, I consider data from the generic domain, where instead of a silent existential determiner, we have a silent generic operator that contributes quasi-universal quantification; hence, SMax is able to derive a reading that LMax is not.

The chapter is organized as follows. Section 5.2 provides a very brief introduction to genericity and the assumptions I make concerning the structure of characterizing (or generalizing) statements (Krifka et al. 1995), i.e. generalizations about (groups of) individuals, such as *Birds fly*. Section 5.3 introduces and analyzes data involving bare numerals in generalizing sentences, such as *Three people can carry that piano upstairs*. Section 5.4 turns to modified numerals in generalizing sentences, focusing in particular on *less than*. I show that the adjectival theory of numerals and LMax both fail to generate the most salient reading of a sentence like *Less than five students can lift this piano*, viz. 'There is a number $n < 5$ such that, in general, any group of n people can carry that piano upstairs.' The adjectival theory and LMax both generate readings that are unattested, and they fail to generate the attested reading just described (i.e. they both overgenerate and undergenerate). By contrast, SMax is able to capture the relevant reading, meaning that it only overgenerates (for *less than*). Section 5.5 explains how, perhaps surprisingly, the pragmatic blocking mechanism that was posited in the existential domain can also cut down the grammatical overgeneration we observe in the generic domain, which seems to make SMax come out on top. Finally, section 5.6 turns to modified numerals with *between* in generalizing sentences and shows that, in fact, SMax undergenerates here. Its ability to capture the right reading for generalizing sentences with *less than* does not carry over to those same sentences with *between*: for instance, *Between three and five people can carry that piano upstairs* can be interpreted as, 'The *minimum* number n such that, in general, any group of *n* people can carry that piano upstairs is between three and five,' which SMax cannot capture. This discussion, however, sets the stage for the theory I develop in chapter 6, which *is* able to capture such a reading, and which is the theory I ultimately argue in favor of.

5.2 Characterizing sentences and quasi-universal interpretations

To set the stage, consider a run-of-the-mill generic, or characterizing/generalizing (Krifka et al. 1995), sentence, such as (128a).[1]

(126) Birds fly.

For the purposes of this thesis, I will adopt the rather simple-minded view that, on its generalizing reading, (126) means, roughly, that in general (with some exceptions), all birds fly, or all typical birds fly. To capture this reading, I assume a generic operator, $\varnothing_{\text{Gen}}$, given in (127), which quantifies over all 'typical', or 'normal', individuals and is therefore a kind of restricted universal, notated in the metalanguage by $\forall_{\text{Gen}}$. The expression '$\forall_{\text{Gen}} x[P(x) \rightarrow Q(x)]$' is to be read as, 'All typical P's are Q's,' or 'In general, any x in P is also in Q.' I also make the simplifying assumption that $\varnothing_{\text{Gen}}$ quantifies over typical individuals relative to the actual world.[2]

[1] In this thesis, I will not be able to do any real justice to the vast literature on genericity. The reader is referred to Carlson 1980, Schubert and Pelletier 1987, and Krifka et al. 1995, and the references therein, for a more detailed discussion. In addition, I follow Krifka et al. 1995 in calling sentences like (126) *characterizing* (or generalizing) sentences, or even just generic sentences. Since I am only interested in this particular subclass of generically interpreted sentences, no confusion should arise by using the more general term.

[2] One inadequacy of this simple-minded view of the generic operator (i.e. that it simply quantifies over a subset of birds in the actual world) is that it cannot capture the intuition that, if Fido were a bird (but he is not), then he would fly. However, intensionality in this regard plays no role in the discussion to come. What is important for our purposes is just that the generic operator contributes non-existential force. Therefore, I will stick to the simpler, albeit ultimately inadequate extensional version presented here. For a survey of a number of more elaborate proposals, see Krifka et al. 1995.

(127) $[\![\varnothing_{\text{Gen}}]\!] = \lambda P_{et} \,.\, \lambda Q_{et} \,.\, \forall_{\text{Gen}} x[P(x) \rightarrow Q(x)]$

In chapter 1, I assumed that bare plurals and numerical indefinites with existential force contain a silent existential determiner at LF, $\varnothing_{\exists}$. For ease of exposition, and to facilitate comparison of data and analyses, I assume much the same thing in the generic domain: bare plurals and numerical indefinites with generic force contain $\varnothing_{\text{Gen}}$ at LF. The idea, then, which is very much in the spirit of Kamp 1981 and Heim 1982, is that a sentence with a bare plural or numerical indefinite can be interpreted either existentially or generically, depending on the quantificational force of the bare plural or indefinite, which is contributed either by $\varnothing_{\exists}$ or by $\varnothing_{\text{Gen}}$. The intended, generalizing reading of (126) is thus represented by the LF in (128b) and the truth conditions in (128c), which state that, in general, if x is a group of birds, then x flies.[3]

(128) a. Birds fly.
b. [$\varnothing_{\text{Gen}}$ birds] fly
c. $\forall_{\text{Gen}} x[\mathbf{birds}(x) \rightarrow \mathbf{fly}(x)]$

On this analysis of generics, the following argument is correctly predicted to be valid, assuming that all (typical) cardinals are considered to be typical birds.

(129) Birds fly.
Cardinals are birds.
⇒ Cardinals fly.

If an argument of the above form is judged invalid, then I take that to be because the subject term of the second premise is an exception to the generalization, i.e. is atypical. For example, the following argument is intuitively invalid because penguins are not typical birds.

[3] And since *fly* is distributive, this entails that every typical individual (atomic) bird flies.

(130) Birds fly.

Penguins are birds.

$\not\Rightarrow$ Penguins fly.

There are, of course, many well-known problems for such a simple-minded approach to genericity. First of all, I have not made precise exactly how $\varnothing_{\text{Gen}}$ works, in particular how exceptions are allowed for and (relatedly) how exactly the metalanguage symbol $\forall_{\text{Gen}}$ is interpreted.[4] Second, and perhaps more troublingly, not all generically interpreted sentences even have such strong, quasi-universal readings, as the following examples from Schubert and Pelletier (1987) illustrate.

(131) a. Snakes are reptiles.

b. Telephone books are thick books.

c. Guppies give live birth.

d. Italians are good skiers.

e. Frenchmen eat horsemeat.

f. Unicorns have one horn.

As Schubert and Pelletier (1987) state (with example numbers suitably changed):

> Obviously, we understand the truth of (131a)–(131f) as calling for different relative numbers of instances of the subject terms satisfying the predicate term. In (131a) it is all; in (131b) most; in (131c) some subset of the females (= less than half); in (131d) some small percentage, but a greater percentage than other countries; in (131e), quite possibly a very small percentage—somehow, from the vantage point of North America, the mere fact of its happening at all is striking; and in (131f) no unicorns have one horn.

[4] See Krifka et al. 1995 for a survey of a number of different proposals.

Nevertheless, I would like to abstract away from these problems and make the following point: Whatever the right *general* analysis of generics is, and however it is precisely formalized, as long as it involves some kind of universal (or simply non-existential) quantification, at some level, even with exceptions, it will pose serious challenges both for the (naive) adjectival theory of modified numerals (see section 1.2) and for a theory like LMax. This point will hopefully be made clearer by the end of section 5.5.

5.3 Characterizing sentences with bare numerals

Consider sentence (132).[5]

(132) Three people can carry that piano upstairs.

This sentence has a number of different readings, depending on a number of different factors, including, but not limited to: whether the predicate *carry that piano upstairs* is interpreted distributively or collectively; whether the modal *can* is interpreted as indicating permission, ability, etc.; whether the sentence is interpreted existentially or generically; and so on. For example, on its 'existential, distributive' reading, with *can* indicating ability, (132) states that there is a group of three people, each of whom is able to carry that piano. On its 'existential, collective' reading, with *can* indicating ability, (132) states that there is a group of three people who, collectively (together, as a group), are able to carry that piano.

The reading we will be concerned with in this section is the 'generic, collective' one, with *can* indicating ability. Before I state what I think is a good paraphrase of that reading,

[5] What I present here and in section 5.4 is, as far as I know, the first analysis of generic indefinites with numerals and modified numerals. Link (1987) discusses the sentence *Three men can lift the piano*, and even paraphrases its meaning the same way I will, as 'Any three men can lift the piano.' However, he does not provide any real analysis of such sentences (his main concern is finding evidence for genuine quantification over plural individuals), nor does he talk at all about modified numerals.

let me say that, as my starting point, I take the following argument to be valid, assuming (as before) that the subject terms in the second premise do not constitute an exception to the generalization in the first premise. In other words, the argument is only judged invalid if Ann, Bill, and/or Carol are exceptions to the rule, e.g. if Ann is a small child, or if Bill has a broken arm.

(133) Three people can carry that piano upstairs.
Ann, Bill, and Carol are people.
⇒ Ann, Bill, and Carol can carry that piano upstairs.

With this is mind, I submit that the analysis in (134) correctly captures the meaning of (132), on its generic reading.

(134) a. Three people can carry that piano upstairs.
b. [$\varnothing_{\text{Gen}}$ [three$_{\texttt{isCard}}$ people]] [can carry that piano upstairs]
c. $\forall_{\text{Gen}} x[\mathbf{card}(x) = 3 \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{carry}(x)]$

The LF in (134b) is isomorphic to that of an existentially interpreted sentence like *Three students frowned* (cf. (20b)), up to the choice of silent determiner ($\varnothing_{\text{Gen}}$ vs. $\varnothing_{\exists}$) and predicates. The symbol $\Diamond$ represents ability modality; $\Diamond\phi$ can of course be taken to be short-hand for some more fine-grained formula involving existential quantification over accessible possible worlds. The formula in (134c) can be paraphrased as, 'In general, any group of three people can carry that piano upstairs.'[6] Or in possible-worlds-talk, 'In general, if x is a group of three people, then there is a world w consistent with x's abilities in the actual world such that x carries that piano in w.'

[6]This is the precisely the way that Link (1987) describes sentences with generically interpreted numerical noun phrases: for example, *Two tires can be used* means the same thing as *Any two tires can be used*. Link (1987) takes this as potential evidence in favor of an adjectival theory of numerals: the 'flip' between existential force (*some three men*) vs. quasi-universal force (*any three men*) is due to the contribution of either a silent existential determiner or a silent universal determiner. As I will show, however, just like in the existential domain, interpretations of modified numerals in the generic domain lead to serious challenges for the adjectival theory.

It might be tempting to think that the generic reading is better represented by a formula in which the modal takes widest scope and the propositional argument of the modal has existential force, as in (135).

(135) $\Diamond\exists x[\mathbf{card}(x) = 3 \wedge \mathbf{people}(x) \wedge \mathbf{carry}(x)]$

This formula can be paraphrased as, 'There is an accessible world w such that, in w, there is a group of three people who carry that piano upstairs.' One problem with this representation is that it is too weak to capture the validity of the argument in (133): it simply states that, in *some* accessible world, *some* group of three people carry that piano upstairs, from which it does not follow that Ann, Bill, and Carol in particular can do so.[7]

Another problem with the representation in (135) is that it is consistent with the formula in (136b), which would presumably be the representation of (136a) (assuming that negation can scope below the modal).

(136) a. Three people cannot carry that piano upstairs.

b. $\Diamond\neg\exists x[\mathbf{card}(x) \wedge \mathbf{people}(x) \wedge \mathbf{carry}(x)]$

Thus, (132) would be predicted to be consistent with (136a), which does not seem right.

Similarly, the generic reading cannot be represented by a formula in which existential quantification takes widest scope, as in (137).

(137) $\exists x[\mathbf{card}(x) = 3 \wedge \mathbf{people}(x) \wedge \Diamond\mathbf{carry}(x)]$

Although this formula does represent one reading of (132), it is (like (135)) too weak to represent the generic reading of (132) under discussion. More concretely, (137) represents

[7]Moreover, it seems unlikely that the generic reading could arise as a combination of this weak reading together with some kind of pragmatic strengthening, along the following lines: if (in a world w) there is a group of three people who can carry that piano upstairs, then (in w) any other group of three people can also carry that piano upstairs. The reason this is unlikely is that such an inference is intuitively invalid: just because, say, Dave, Eliza, and Fred can carry that piano upstairs, it does not follow that Ann, Bill, and Carol can do so too (perhaps Dave, Eliza, and Fred are just extraordinarily strong).

a reading of (132) that is salient in a context such as the following. Suppose that (in the actual world) there are three extraordinarily strong people: Dave, Eliza, and Fred. Suppose furthermore that these three people can, in fact, carry the piano upstairs (as a group), but that no other group of people can. In particular, Ann, Bill, and Carol cannot (even as a group) carry the piano upstairs. Then sentence (132) is predicted to be true on the reading represented by (137) (and my intuitions agree). However, this reading is not the generic one. If the generic reading were true, then it would entail that Ann, Bill, and Carol could carry the piano upstairs, which, by hypothesis, is false.[8]

Finally, note that NPIs are licensed in the restrictor of generically interpreted *three*, as (138) illustrates, which is fully expected on the analysis in (134c) (*any* occurs in a downward-entailing environment, just like the restrictor of *every*), but not on the analyses in (135) or in (137) (*any* occurs in an upward-entailing environment).

(138) Three people with any experience in the moving business can carry that piano upstairs.

In sum, by 'generic reading' of sentences with an ability modal *can*, I mean a reading where quantification over (plural) individuals, in the form of quasi-universal quantification, takes scope over the modal.

5.4 Characterizing sentences with modified numerals

We now turn to characterizing sentences with modified numerals. For now, I focus just on *less than five*, saving discussion of *between three and five* for section 5.6 because it introduces some important complications.

The analysis sketched above for generic sentences with bare numerals raises challenges when we move to modified numerals like *less than five*. The sentence under discussion is

[8] And, as described in footnote 7, appealing to pragmatic strengthening appears to be of no use here.

given in (139).

(139) Less than five people can carry that piano upstairs.

I claim that (139) has a generic reading that can be paraphrased as in (140), and formalized as in (141).

(140) There is a number n less than five such that, in general, any group of n people can carry that piano upstairs.

(141) $\exists n[n < 5 \wedge \forall_{\text{Gen}} x[\mathbf{card}(x) = n \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{carry}(x)]]$

Just like in the case where *less than five* combines with a collective predicate, here too there is, puzzlingly, no notion of maximality at play, and moreover, here too we get a lower-bound entailment: (139) intuitively entails that at least some number of people can carry that piano upstairs. That is, if no people at all can carry that piano upstairs—it is too heavy for any number of people to carry it, say—then (139) is intuitively false. This also explains why sentences like *Less than five babies can carry that piano upstairs* and *Less than ten eggs are sufficient to build a house* feel false, not true, in most contexts.[9]

Now, I should say that this lower-bound entailment is not really represented by the formula in (141); in fact, if we took $\forall_{\text{Gen}}$ to work similarly to $\forall$, then (141) would seem to express a tautology: $\forall_{\text{Gen}} x[\mathbf{card}(x) = 0 \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{carry}(x)]$ seems universally true, since $\mathbf{card}(x) = 0$ is false for all x; and since $0 < 5$, this makes (141) universally true. However, since I am not committing myself to any specific analysis of how exactly $\varnothing_{\text{Gen}}$ is interpreted, I ignore this issue for now (it will reappear in chapter 6, though, where I attempt to resolve this complication). For now, the important point will be that SMax can generate the reading in (141), which seems to be on the right track, while LMax and the adjectival theory do not generate anything close to the salient reading of (139).

[9]I thank Bernhard Schwarz and Benjamin Spector for coming up with these sentences, respectively.

I will first describe the problems that this sentence creates for the (naive) adjectival theory, and why it cannot derive the right reading of (139).[10] Afterwards, I will describe the problems that arise for a theory like LMax that encodes maximality into the meaning of *less than five*. Finally, I will show that SMax, which severs maximality from numeral modifiers, *does* generate the right reading. All of the theories discussed will be shown to generate other readings, too. In section 5.5, I will argue that those readings are judged to be unavailable but that, perhaps unexpectedly, they can be ruled out by the pragmatic blocking mechanism that was proposed for independent reasons in chapter 2.

5.4.1 The adjectival theory of modified numerals

Let us first consider the sorts of (generic) readings derivable within the adjectival theory of modified numerals. On the adjectival theory, *less than five* simply denotes a predicate of individuals, and hence does not take scope. Thus, there is no scope dependency between the empty generic determiner and *less than five*, and so we derive only one generic reading, illustrated below. This reading can be paraphrased as, 'In general, any group of less than five people can carry that piano upstairs.' I will refer to this reading as the *strong generic reading*.

(142) a. Less than five people can carry that piano upstairs.

b. [$\varnothing_{\text{Gen}}$ [[less than five] people]] [can carry that piano upstairs]

c. $\forall_{\text{Gen}} x[\textbf{card}(x) < 5 \wedge \textbf{people}(x) \rightarrow \Diamond\textbf{carry}(x)]$

Clearly, (142c) is stronger than (141): (142c) entails, for example, that in general, any *single* person can carry the piano upstairs. Thus, the following argument is expected to be valid, assuming that Ann counts as a typical instance of a plurality of people with less than five

[10] Since we have already ruled out the (naive) adjectival theory as an adequate theory of modified numerals, this discussion is not strictly necessary. However, the problems it faces in the generic domain are surprisingly similar to the ones it faces in the existential domain, and so it is instructive to compare the two situations. Furthermore, it provides a nice segue into the discussion of the problems faced by LMax.

atomic parts.[11]

(143) Less than five people can carry that piano upstairs.
Ann is a person.
⇒ Ann can carry that piano upstairs.

My judgment, however, is that the above argument is invalid on the intended generic reading of (139). Moreover, the intuitive invalidity of the argument has nothing to do with Ann potentially being an exception to the generalization, because the judgment is the same no matter who Ann is replaced by. Nor is plurality at issue here (i.e. the assumption that plural expressions may contain atomic individuals in their extension; see chapter 1, footnote 29), since the following argument is likewise expected to be valid, yet intuitively is not valid, even if Ann are Bill are not exceptions to the generalization.

(144) Less than five people can carry that piano upstairs.
Ann and Bill are people.
⇒ Ann and Bill can carry that piano upstairs.

Thus, the one reading of (139) derived by the adjectival theory does not correspond to the most salient reading of the sentence. I believe, in fact, that the reading it does derive is not available at all, but I defer further discussion until section 5.5.

5.4.2 Lexical maximality (LMax)

Let us now turn to the scopally flexible, lexical maximality theory from chapter 2 (LMax). I will show that, while LMax derives more generic readings than the adjectival theory, it still fails to derive the right reading paraphrased in (140). Moreover, note that because LMax is generatively more powerful than both Landman's (2004) revised adjectival theory

[11] Recall that I am assuming that plural expressions like *people* may indeed contain atomic individuals, like Ann, in their extension (see chapter 1, footnote 29).

and Hackl's (2000) theory with a silent determiner ⟨*many*⟩ parameterized for degrees (see section 1.2.5), it follows that these latter theories also fail to derive the right reading.

Since *less than five* denotes a generalized quantifier over degrees, and hence takes scope, there are two scope orders to consider. First, *less than five* can scope above the silent generic determiner, thus deriving a sort of upper-bounded, generic reading, which can be paraphrased as, 'The maximum number *n* such that, in general, any group of *n* people can carry that piano upstairs is less than five.' This is the same reading that Landman (2004) (using individual maximization) and Hackl (2000) (using the parameterized determiner ⟨*many*⟩) would derive. I will refer to this reading as the *upper-bounded generic reading*.

(145) a. Less than five people can carry that piano upstairs.

b. [less than five] [λn [[$\varnothing_{\text{Gen}}$ [$n_{\texttt{isCard}}$ people]] [can carry that piano upstairs]]]

c. $\max(\lambda n\,.\,\forall_{\text{Gen}} x[\mathbf{card}(x) = n \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{carry}(x)]) < 5$

This reading is clearly different from the one we are after. For example, it entails that it is not the case that (any typical group of) five people can carry that piano upstairs. By contrast, the reading we are after is consistent with (any typical group of) five people being able to carry that piano upstairs. (In fact, as I will argue in section 5.5, the salient reading of the sentence entails that any typical group of five people *can* carry that piano upstairs.)

Second, *less than five* can scope below the generic determiner, by quantifying into AP (shown below) or NP (not shown, but logically equivalent; see chapter 2, footnote 14), thus deriving the same, strong generic reading that the adjectival theory derives.

(146) a. Less than five people can carry that piano upstairs.

b. [$\varnothing_{\text{Gen}}$ [[λx [[less than five] [λn [x $n_{\texttt{isCard}}$]]]] people]] [can carry . . .]

c. $\forall_{\text{Gen}} x[\max(\lambda n\,.\,\mathbf{card}(x) = n) < 5 \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{carry}(x)]$

$\equiv \forall_{\text{Gen}} x[\mathbf{card}(x) < 5 \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{carry}(x)]$

Again, I believe that such a reading is not available for (139), but I defer further discussion to section 5.5.

The main point for now is that the reading in (140), which I take to be the most salient (and probably only) generic reading of (139), is not derivable by LMax.

5.4.3 Separate Maximality (SMax)

Let us finally turn to the optional, separate maximality theory from chapter 4 (SMax). I will show that SMax derives not only the same two readings as LMax, but also a third reading: the reading paraphrased in (140) and formalized in (141).

First, the upper-bounded generic reading is derived when *less than five* takes widest scope, with maximization applying in its scope but above the generic determiner.

(147) a. Less than five people can carry that piano upstairs.

b. [less than five] [λn [n_{max} [λm [[$\varnothing_{\text{Gen}}$ [$m_{\texttt{isCard}}$ people]] [can carry ...]]]]]

c. $\exists n[n < 5 \wedge \max(\lambda m\,.\,\forall_{\text{Gen}} x[\mathbf{card}(x) = m \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{carry}(x)]) = n]$

$\equiv \max(\lambda m\,.\,\forall_{\text{Gen}} x[\mathbf{card}(x) = m \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{carry}(x)]) < 5$

Second, the strong generic reading is derived when *less than five* scopes below the generic determiner. In this case, it does not matter if maximality applies, since its application is vacuous (just like in the case where *less than five* scopes below the existential determiner; see chapter 4).

(148) a. Less than five people can carry that piano upstairs.

b. [$\varnothing_{\text{Gen}}$ [[λx [[less than five] [λn [x $n_{\texttt{isCard}}$]]]] people]] [can carry ...]

c. $\forall_{\text{Gen}} x[\exists n[n < 5 \wedge \mathbf{card}(x) = n] \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{carry}(x)]$

$\equiv \forall_{\text{Gen}} x[\mathbf{card}(x) < 5 \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{carry}(x)]$

(149) a. Less than five people can carry that piano upstairs.

b. $[\varnothing_{\text{Gen}}\ [[\lambda x\ [[\text{less than 5}]\ [\lambda n\ [n_{\max}\ [\lambda m\ [x\ m_{\texttt{isCard}}]]]]]]\ \text{people}]]\ [\text{can carry} \ldots]$

c. $\forall_{\text{Gen}} x[\exists n[n < 5 \wedge \max(\lambda m\,.\,\mathbf{card}(x) = m) = n] \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{carry}(x)]$

$\equiv \forall_{\text{Gen}} x[\exists n[n < 5 \wedge \mathbf{card}(x) = n] \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{carry}(x)]$

$\equiv \forall_{\text{Gen}} x[\mathbf{card}(x) < 5 \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{carry}(x)]$

Lastly, the reading we have been after, in which existential quantification over degrees takes scope over generic quantification over individuals, is derived when *less than five* scopes above the generic determiner, and either maximization does not apply, or it applies vacuously below the generic determiner.[12]

(150) a. Less than five people can carry that piano upstairs.

b. $[\text{less than five}]\ [\lambda n\ [[\varnothing_{\text{Gen}}[n_{\texttt{isCard}}\ \text{people}]]\ [\text{can carry that piano upstairs}]]]$

c. $\exists n[n < 5 \wedge \forall_{\text{Gen}} x[\mathbf{card}(x) = n \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{carry}(x)]$

(151) a. Less than five people can carry that piano upstairs.

b. $[\text{less than 5}]\ [\lambda n\ [[\varnothing_{\text{Gen}}[[\lambda x\ [n_{\max}\ [\lambda m\ [x\ m_{\texttt{isCard}}]]]]\ \text{people}]]\ [\text{can carry} \ldots]]]$

c. $\exists n[n < 5 \wedge \forall_{\text{Gen}} x[\max(\lambda m\,.\,\mathbf{card}(x) = m) = n \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{carry}(x)]]$

$\equiv \exists n[n < 5 \wedge \forall_{\text{Gen}} x[\mathbf{card}(x) = n \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{carry}(x)]]$

Crucially, the reason why SMax derives more readings than LMax in the generic domain (as opposed to the existential domain, where LMax and SMax are on a par) is that the existential degree quantifier contributed by *less than five* and the quasi-universal individual quantifier contributed by the generic determiner do *not* commute. More formally, in the

[12] In footnote 8 of chapter 4, I noted that for a sentence like *Every student read less than five books*, an LF with the scope ordering *less than five* > *every student* > $n_{\texttt{isMax}}$ yields a reading that is intuitively unavailable, but that the unavailability of such an LF can be explained as a violation of Kennedy's generalization (Kennedy 1997; Heim 2000), which states that if the scope of a quantificational DP (e.g. *every student*) contains the trace of a degree phrase (e.g. the trace of *less than five*), then it must contain the degree phrase itself. In the present context (on either LF), the scope of $\varnothing_{\text{Gen}}$ contains the trace of *less than five*, but does not contain *less than five* itself. However, it seems to me that this is not a violation of Kennedy's generalization (at least as formulated in Heim 2000) because $\varnothing_{\text{Gen}}$, unlike *every student*, is not a quantificational DP. Thus, *less than five* can move across $\varnothing_{\text{Gen}}$ in precisely the same way that it can move across $\varnothing_{\exists}$.

existential domain, we have

$$\exists n[P(n) \wedge \exists x[Q(x) \wedge R(x,n)]]$$
$$\equiv$$
$$\exists x[Q(x) \wedge \exists n[P(n) \wedge R(x,n)]]$$

where R denotes some relation between x and n, e.g. $\mathbf{card}(x) = n$. However, in the generic domain, it is not in general the case that

$$\exists n[P(n) \wedge \forall_{\text{Gen}} x[R(x,n) \wedge Q(x) \rightarrow S(x)]]$$
$$\equiv$$
$$\forall_{\text{Gen}} x[\exists n[P(n) \wedge R(x,n)] \wedge Q(x) \rightarrow S(x)]$$

5.5 Applying the pragmatic account to generic cases

In the previous section I showed that, among the theories of modified numerals discussed (so far) in this thesis, the only theory that successfully captures the intended generic reading of (139) (*Less than five people can carry that piano upstairs* $\rightsquigarrow$ 'There is a number $n < 5$ such that any group of n people can carry that piano upstairs') is the theory where maximality is severed from *less than* (SMax). This is because the intended reading—call it the *intermediate generic reading*[13]—involves only existential degree quantification and generic individual quantification; it does not involve maximality. On SMax, existential degree quantification is contributed by *less than*, generic quantification is contributed by the generic determiner, and maximization need not apply at all (or can apply vacuously, below the degree quantifier and the generic determiner).

However, in the same way that SMax overgenerates in the existential domain, it also seems to overgenerate in the generic domain. In this section, I will illustrate a couple

[13] As opposed to the upper-bounded generic reading and the strong universal generic reading.

of readings that I believe SMax overgenerates and explain how, perhaps unexpectedly, the pragmatic constraint from chapter 2, which was already independently proposed to rule out unattested readings in the existential domain, can also be applied to rule out unattested readings in the generic domain.

Consider again sentence (139), repeated in (152), with *can* still indicating ability. The question we are addressing is whether (152) has all the readings in (153), or rather just (153a).

(152) Less than five people can carry that piano upstairs.

(153) a. There is a number $n < 5$ such that, in general, any group of n people can carry that piano upstairs. (*intermediate reading*)

b. In general, any group of less than five people can carry that piano upstairs. (*strong universal reading*)

c. The maximum number n such that, in general, any group of n people can carry that piano is less than five. (*upper-bounded reading*)

I wish to answer that question as follows: the extent to which (153b) and (153c) are unavailable depends on the extent to which the following inference is valid.

(154) n people can carry that piano upstairs.

$\Rightarrow$ $n+1$ people can carry that piano upstairs.

If *can* is interpreted as something like 'is physically able to', then this inference is intuitively valid: the more people who partake in the piano carrying, the easier the carrying gets.[14] If that is the case, then, borrowing some terminology from Beck and

[14]One might object that there are conceivable scenarios (worlds, or models) in which, say, any group of 5 to 10 people can carry that piano upstairs, but no group of more than 10 people can do so—simply because, say, 11 people cannot actually fit around the piano. My response would be that, since, on the intended reading, *can* indicates ability (e.g. physical ability, or strength), it is still literally true that 11, 12, . . . people 'can' carry that piano upstairs—they have the physical strength (ability)—to do it, even though there are circumstances preventing them from ever actually doing it. Put differently, it is simultaneously true both

Rullmann 1999, the degree predicate

(155) λn [[$\varnothing_{\text{Gen}}$ [$n_{\texttt{isCard}}$ people]] [can carry that piano upstairs]]

is *upward scalar*; or rather, it denotes a function of type *dt* which is upward scalar in the following sense.

(156) **Upward scalarity**

P_{dt} is upward scalar iff $\forall m, n[P(n) \wedge m > n \rightarrow P(m)]$.

Let us assume that the predicate in (155) is indeed upward scalar.[15] In other words, we make the following assumption:

(157) For any numbers *m* and *n*, if $m < n$, then:

$$\forall_{\text{Gen}} x[\mathbf{card}(x) = m \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{carry}(x)]$$
$$\rightarrow \forall_{\text{Gen}} x[\mathbf{card}(x) = n \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{carry}(x)]$$

Now consider the formula in (158), which represents the strong universal reading (153b).

(158) $\forall_{\text{Gen}} x[\mathbf{card}(x) < 5 \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{carry}(x)]$

Because of the quasi-universal quantifier, $\varnothing_{\text{Gen}}$, this formula entails that, in general, any *single* person can carry the piano upstairs (possibly with exceptions):

(159) $\forall_{\text{Gen}} x[\mathbf{card}(x) = 1 \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{carry}(x)]$

that 11 people 'can' carry that piano upstairs (in view of their physical ability) and that 11 people 'cannot' carry that piano upstairs (in view of the dimensions of the piano), depending on how exactly *can/cannot* is interpreted. This argument is directly related to the discussion in Kratzer 1981 concerning German *können*: Kratzer argues that a sentence like *Ich kann nicht Posaune spielen* ('I cannot play the trombone') means different things depending on the situation in which it is uttered (maybe I cannot play it because I don't know how, or because I have asthma and can hardly breathe, or because my trombone sank with the ship I had been on, and so on).

[15]The upward scalarity of (155) is undoubtedly tied to the fact that *can carry that piano* licenses inferences from groups to supergroups on its subject argument: If Ann can carry that piano upstairs, then it intuitively follows that Ann and Bill can (together) carry that piano upstairs.

And this latter formula, together with the assumption in (157), entails that, in general, any group of people *at all* can carry the piano upstairs (possibly with exceptions):

(160) $\forall_{\text{Gen}} x[\textbf{people}(x) \rightarrow \Diamond\textbf{carry}(x)]$

This widescope generic reading, together with our assumption, therefore amounts to an extremely strong, quasi-universal reading. This is strikingly reminiscent of how in the existential domain, the widescope existential reading amounts to an extremely weak existential reading.

Moreover, in the same way that in the existential case the numeral *five* was shown to play no semantic role,[16] so too does *five* play no semantic role in the present case: the same truth conditions would be derived if *five* were replaced by *four*, *six*, etc. Crucially, this is so no matter how the exceptions to the generalization are formalized, and no matter how many exceptions there are, as long as the exceptions remain the same across the board. Thus, if the assumption in (157) holds, then this widescope generic reading is predicted, on a pragmatic blocking account like SMax, to be ruled out.[17]

Now consider the formula in (161), which represents the upper-bounded reading (153c). Let us also continue to make the assumption in (157).

(161) $\max(\lambda n \,.\, \forall_{\text{Gen}} x[\textbf{card}(x) = n \wedge \textbf{people}(x) \rightarrow \Diamond\textbf{carry}(x)]) < 5$

The set of degrees to which max applies above can be described as the set of numbers n such that, in general, any group of n people can carry the piano upstairs. Suppose that some number m is in that set. Then by assumption (157), so is every number greater than

[16] For example, *Less than five students frowned* ends up meaning that at least one student frowned (see chapter 1 and chapter 2).

[17] Actually, the constraint in (56) makes reference to 'truth-conditional' equivalence between LFs. The equivalences that I have just discussed in the generic domain, and others that I will discuss momentarily, arise due in part to the assumption in (157), which is arguably part of contextual (world) knowledge, and not a part of the literal, truth-conditional meaning of anything in the expression *can carry the piano upstairs*. I leave the question open to what extent the constraint in (56) should be sensitive to such encyclopedic knowledge about the world (or to meaning postulates).

m. In other words, if the set to which max applies is non-empty, then it has no maximum, and so the sentence is predicted to be either false or a presupposition failure, depending on how maximality failure is handled, which I ignore here (see chapter 2, footnote 7). The important point is that the only way for the sentence to ever be true is if the set to which max applies is empty (because $\max(\varnothing) = 0 < 5$; see section 2.3.3). This same result obtains if we replace the numeral *five* with, say, *four* or *six*. As a result, we see that, once again, if assumption (157) holds, then, just like with reading (153b), reading (153c) is ruled out by our pragmatic constraint.

In sum, if the assumption in (157) is valid—which it intuitively seems to be—and if SMax together with the pragmatic constraint in (56) are on the right track, then the sentence *Less than five people can carry that piano upstairs* is predicted to have just one reading: (153a). This prediction indeed conforms to my judgments and those of the informants I have consulted.

5.5.1 A prediction regarding *can fit into that elevator*

A clear prediction now arises: if we switch from *can carry that piano upstairs* to a different predicate for which an assumption analogous to (157) is intuitively *invalid*—for example, if we consider a degree predicate which is downward scalar, as defined in (162), rather than upward scalar—then an upper-bounded generic reading and a universal generic reading ought to be available.

(162) **Downward scalarity**

P_{dt} is downward scalar iff $\forall m, n[[P(n) \wedge m < n] \rightarrow P(m)]$.

One such predicate is *can fit into that elevator*: it is intuitively *not* the case that, if n people can fit into that elevator, then so can $n+1$. Rather, the reverse inference is valid: if n

people can fit, then so can $n-1$.[18] In other words, the degree predicate in (163) appears to be downward scalar.

(163) λn [[$\varnothing_{\text{Gen}}$ [$n_{\texttt{isCard}}$ people]] [can fit into that elevator]]

Let us assume that this is the case. That is, we make the following assumption.

(164) For any numbers m and n, if $m < n$, then:

$\forall_{\text{Gen}} x[\mathbf{card}(x) = n \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{carry}(x)]$

$\forall_{\text{Gen}} x[\mathbf{card}(x) = m \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{carry}(x)]$

Consider now sentence (165).

(165) Less than five people can fit into that elevator.

This sentence indeed appears to have the upper-bounded generic reading in (166), which can be paraphrased as, 'It is not the case that five or more people can fit into that elevator.' That this reading is available is perfectly expected, since in this case we do not run into any maximality failure: if the set to which max applies in (166) contains a number n, then it also contains every number less than n, but not necessarily any number greater than n. Thus, replacing *five* (in the relevant LF) by *four* yields a stronger reading, and replacing it with *six* yields a weaker reading. Thus, this reading is (correctly) not ruled out.

(166) $\max(\lambda n\,.\,\forall_{\text{Gen}} x[\mathbf{card}(x) = n \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{fit}(x)]) < 5$

The strong universal reading, given in (167), is likewise not ruled out: replacing *five* (in the relevant LF) with *four* yields a weaker reading, and replacing it with *six* yields a stronger reading.

[18]The validity of this inference is undoubtedly related to the fact that, if Ann, Bill, and Carol can fit, then so can Ann and Bill (without Carol), and so can Bill (by himself), and so on.

(167) $\forall_{\text{Gen}} x[\mathbf{card}(x) < 5 \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{fit}(x)]$

Now, it is unclear to me whether such a reading for (165) is indeed available. If it is not, then SMax can simply be altered so that modified numerals may not scope below silent determiners. (Recall that in the existential domain, allowing modified numerals to scope below $\varnothing_\exists$ was unnecessary but also harmless, because there is no scope dependency between modified numerals and $\varnothing_\exists$.) One way to do this would be to make use of the innovation in Hackl 2000 (see section 1.2.5), replacing $\varnothing_\exists$ and $\varnothing_{\text{Gen}}$ by two versions of ⟨*many*⟩—both parameterized for degrees, but differing in the type of quantification they contribute (existential vs. quasi-universal).[19]

Finally, consider the hypothetical 'intermediate' reading for (165) given in (168), which is analogous to the attested reading that we wanted to derive for (139) (*Less than five people can carry that piano upstairs*).

(168) $\exists n[n < 5 \wedge \forall_{\text{Gen}} x[\mathbf{card}(x) = n \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{fit}(x)]]$

Given the assumption in (164), this reading is in fact equivalent to (169).

(169) $\exists n \forall_{\text{Gen}} x[\mathbf{card}(x) = n \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{fit}(x)]$

The entailment from (168) to (169) follows from conjunction elimination. The reverse entailment follows because, if z is a number verifying (169), then either $z < 5$, which verifies (168), or $z \geq 5$, in which case, by the assumption in (164), every $z' < z$ is such that any group of z' people can fit into that elevator; thus, there is a z' (e.g. 1) verifying (168). As a result, this reading is excluded by our pragmatic constraint. In fact, the logic of

[19]More precisely, SMax could posit the following, where $\langle many_\exists \rangle$ is the same as Hackl's ⟨*many*⟩ (see also section 6.2.5 for a more detailed discussion):

(i) a. $[\![\langle many_\exists \rangle]\!] = \lambda n_d . \lambda P_{et} . \lambda Q_{et} . \exists x[\mathbf{card}(x) = n \wedge P(x) \wedge Q(x)]$
b. $[\![\langle many_{\text{Gen}} \rangle]\!] = \lambda n_d . \lambda P_{et} . \lambda Q_{et} . \forall_{\text{Gen}} x[\mathbf{card}(x) = n \wedge P(x) \rightarrow Q(x)]$

this result is exactly analogous to the logic of Van Benthem's problem in the existential domain (see chapter 1).[20] Intuitively, the reading in (168) is indeed unavailable, just as the weak existential reading in distributive contexts is also unavailable.

To recap, when we move to a predicate like *can fit*, which licenses a different kind of inference than *can carry* (viz. downward rather than upward inferences), then upper-bounded generic readings suddenly become available, while 'intermediate' readings (those of the form 'There is a number $n < 5$ such that ...') become unavailable, and SMax is readily able to explain these facts.

Interim summary

It appears, then, that SMax has an advantage over LMax: it derives the most salient reading of (139) (*Less than five people can carry that piano upstairs*), and the additional readings it derives are correctly ruled out by the pragmatic constraint in (56); moreover, it manages to simultaneously derive exactly the right reading for (165) (*Less than five people can fit into that elevator*). As we will see in the next section, however, the former advantage turns out to be just a fortunate accident for SMax: when we move to the analog of (139) with *between*, SMax is unable to generate the right reading. This discussion will set the stage for the proposal I develop in the next chapter, and the one I ultimately defend.

5.6 A puzzle with *between*

In the last couple sections, I have shown that SMax (in contrast to LMax and the adjectival theory) is able to generate a certain intuitively available generic reading for (170), namely (171a). I have also shown that the unattested readings for (170) that SMax grammatically overgenerates, namely (171b) and (171c), are ruled out by the pragmatic constraint in (56).

[20]This because a degree predicate like λn *[[$\varnothing_\exists$ [$n_{\texttt{isCard}}$ students]] frowned]* is likewise downward scalar, in the sense of (162).

(170) Less than five people can carry that piano upstairs.

(171) a. There is a number $n < 5$ such that, in general, any group of n people can carry that piano upstairs. (*intermediate reading*)

b. The maximum number n such that, in general, any group of n people can carry that piano is less than five. (*upper-bounded reading*)

c. In general, any group of less than five people can carry that piano upstairs. (*strong universal reading*)

In this section, I show that, quite surprisingly, SMax fails to generate *any* generic readings for (172), the *between* analog of (170), despite the fact that there is an intuitively available generic reading of (172), to be discussed below. More precisely, I show that, while SMax initially (i.e. grammatically) generates all three readings in (173), the pragmatic constraint in this case actually rules out all three readings, including (173a).[21]

(172) Between three and five people can carry that piano upstairs.

(173) a. There is a number n, with $3 \leq n \leq 5$, such that, in general, any group of n people can carry that piano upstairs. (*intermediate reading*)

b. The maximum number n such that, in general, any group of n people can carry that piano is three, four, or five. (*upper-bounded reading*)

c. In general, any group of three, four, or five people can carry that piano upstairs. (*strong universal reading*)

I will first show why all three readings are initially generated by SMax but then blocked by the pragmatic constraint. I will then discuss a generic reading of (172) that I think *is* available, and how we might go about capturing it.

[21] That the reading in (173a) derived by SMax is actually too weak (in contrast to the analogous reading for the *less than five* sentence) was first pointed out to me by Benjamin Spector (p.c.).

5.6.1 Why SMax undergenerates

For the following arguments, I continue to make the assumption in (157) that the degree predicate

(174) λn [[$\varnothing_{\text{Gen}}$ [$n_{\texttt{isCard}}$ people]] [can carry that piano upstairs]]

is upward scalar.[22]

Intermediate reading

First, the derivation in (175) illustrates how SMax generates the 'intermediate' reading of (172) paraphrased in (173a).

(175) a. Between three and five people can carry that piano upstairs.

b. [between three and five] [λn [[$\varnothing_{\text{Gen}}$ [$n_{\texttt{isCard}}$ people]] [can carry ...]]]

c. $\exists n[3 \leq n \leq 5 \wedge \forall_{\text{Gen}} x[\mathbf{card}(x) = n \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{carry}(x)]]$

Due to our assumption in (157), the formula in (175c) is actually equivalent to:

(176) $\exists n[n \leq 5 \wedge \forall_{\text{Gen}} x[\mathbf{card}(x) = n \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{carry}(x)]]$

The entailment from (175c) to (176) is straightforward. The reverse entailment holds for the following reason. Suppose that z is a number verifying (176). If $z \geq 3$, then (175c) is verified. If $z < 3$, then by our assumption in (157), every $z' > z$ is such that any group of z' people can carry that piano upstairs, which means there is a z' verifying (175c).

What this argument shows is that the numeral *three* in (172) does no semantic work: we would derive the same truth conditions if *three* were replaced, say, by *two* or by *one*.[23]

[22]Thus, the predictions described here for *can carry that piano upstairs* do not hold for *can fit into that elevator*, which I leave aside, since it is the former that is troublesome for the SMax account.

[23]Incidentally, the derived truth conditions are equivalent to those of the 'intermediate' reading of *Less than six people can carry that piano upstairs*.

Hence, this reading is ruled out.

Upper-bounded reading

Second, the derivation in (177) illustrates how SMax generates the upper-bounded reading of (172) paraphrased in (173b).

(177) a. Between three and five people can carry that piano upstairs.

b. [between 3 and 5] [λn [n_{max} [λm [[$\varnothing_{\text{Gen}}$ [$m_{\texttt{isCard}}$ people]] [can carry ...]]]]]

c. $3 \leq \max(\lambda n \,.\, \forall_{\text{Gen}} x[\mathbf{card}(x) = n \land \mathbf{people}(x) \rightarrow \Diamond \mathbf{carry}(x)]) \leq 5$

Consider the set to which max applies, and suppose that it contains some number *m*. Then by our assumption, it also contains every number greater than *m*, and hence has no maximum, let alone a maximum less than or equal to 5. But if the set to which max applies is empty, then by the definition of max, the maximum is 0 (see section 2.3.3), which is not greater than or equal to 3. Thus, the derived reading is a contradiction. Assuming that contradiction is a source of unacceptability (Gajewski 2003), this alone might rule out such a reading. Alternatively, we can once again appeal to our pragmatic constraint: replacing *three* by another numeral (e.g. *two* or *one*) and/or replacing *five* by another numeral (e.g. *six* or *seven*) would again derive a contradiction; hence, the numerals play no semantic role, and so this reading is ruled out.

Strong universal reading

Finally, the derivation in (178) illustrates how SMax generates the strong universal reading of (172) paraphrased in (173c).

(178) a. Between three and five people can carry that piano upstairs.

b. [$\varnothing_{\text{Gen}}$ [[λx [[between three and five] [λn [x $n_{\texttt{isCard}}$]]]] people]] [can carry ...]

c. $\forall_{\text{Gen}} x[3 \leq \max(\lambda n \,.\, \mathbf{card}(x) = n) \leq 5 \land \mathbf{people}(x) \rightarrow \Diamond \mathbf{carry}(x)]$

$$\equiv \forall_{\text{Gen}} x[3 \leq \mathbf{card}(x) \leq 5 \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{carry}(x)]$$

Due to our assumption, the formula in (178c) is equivalent to:

(179) $\forall_{\text{Gen}} x[\mathbf{card}(x) \geq 3 \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{carry}(x)]$

which in turn is equivalent to:

(180) $\forall_{\text{Gen}} x[\mathbf{card}(x) = 3 \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{carry}(x)]$

We see, then, that the numeral *five* does no semantic work: we would derive the same truth conditions if *five* were replaced, say, by *six* or by *seven*. Thus, this reading is ruled out.

To recap, when *between three and five* takes widest scope, without maximization (the intermediate reading), then the numeral *three* is semantically vacuous. When *between three and five* takes widest scope, with maximization (the upper-bounded reading), then we derive a contradiction, and thus both numerals are *ipso facto* semantically vacuous. And when *between three and five* scopes below the generic determiner (the strong universal reading), the numeral *five* is semantically vacuous. All three readings are thus ruled out.

5.6.2 Minimal readings

Intuitively, is there any generic reading available for (172), repeated below in (181)?

(181) Between three and five people can carry that piano upstairs.

First, consider the following scenario: the piano is so heavy that it takes at least six people to carry it upstairs. In this scenario, (181) is clearly judged false.

Now consider a different scenario: the piano is light enough that two people can carry it upstairs. In this scenario, (181) once again is judged false (according to my judgments and those of my informants).

If, however, the scenario is such that, say, four people can carry that piano upstairs, then (181) is true.

Thus, it seems that the reading (181) expresses is the following: it takes at least three people to carry that piano upstairs, but it does not take more than five people to do it. Another way of stating this reading is as follows:

(182) The *minimum* number n such that, in general, any group of n people can carry that piano upstairs is three, four, or five.

This formulation is almost exactly the same as the one paraphrased in (173b), except that, quite surprisingly, 'maximum' has been replaced by 'minimum'. What it says is that for some number n such that $3 \leq n \leq 5$, it holds that n people can carry that piano upstairs (i.e. n people are sufficient), and no *less* than n people can carry that piano upstairs (i.e. n people are necessary). In other words, it (correctly) entails that two people (or one person) cannot, in general, carry that piano,[24] and it (correctly) entails that it does not take more than five people to carry that piano (although five or more certainly could do it, as well).

To formally represent this reading, we can define a minimality operator, min, modeled on our definition for max (see chapter 2), as follows.

(183) For any set P_{dt}, any partial ordering $\leq$ over P, and any number n, $\min_{\leq}(P)(n)$ iff $P(n) \wedge \neg\exists m[P(m) \wedge m < n]$.

[24] This really seems to be an entailment and not an implicature. For example, consider a context in which a school with 50 classrooms of students wishes to have a pizza day. In this context, my informants tell me (and I agree) that the following sentence is false if there are some classrooms of students that can be fed by just one or two pizzas. (Of course, there is in principle another reading that says that three to five pizzas can feed the whole school, which of course is implausible in the given context.)

(i) Between three and five pizzas can (are sufficient to) feed every classroom of students.

Then the relevant reading can be represented as follows (where, as usual, min operates on the natural ordering of numbers).

(184) $3 \leq \min(\lambda n \,.\, \forall_{\text{Gen}} x[\mathbf{card}(x) = n \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{carry}(x)]) \leq 5$

Before considering how to generate such a reading, let me first show why this reading is not blocked by the constraint in (56), i.e. why both numerals have semantic import.

Consider the set to which min applies, and suppose that m is in that set. Then by our assumption in (157), we can conclude that $m+1$ is in that set, but we cannot conclude that $m-1$ is in that set. Thus, the set can have a minimum. What (184) conveys is that this minimum is at least three and at most five. It should be clear that, if *three* were replaced by *two*, we would derive a weaker statement, and if *three* were replaced by *four*, we would derive a stronger statement. Similarly, if *five* were replaced by *four*, we would derive a stronger statement, and if *five* were replaced by *six*, we would derive a weaker statement. In sum, both numerals are semantically non-trivial here; hence, the constraint in (56) does not block this reading.

Let me also point out that, even though SMax was already shown to be able to derive the intuitively available ('intermediate') generic reading for (185), formalized in (186a), the formula in (186b) is actually equivalent to (186a).[25] The entailment from (186a) to (186b) follows because, if z is a number verifying (186a), then either z is the minimum of the set to which min applies in (186b) (which verifies (186b) since $z < 5$), or z is not the minimum, in which case there is some $z' < z$ which is the minimum (which again verifies (186b) since $z' < z < 5$). The reverse entailment is more straightforward: if z is the minimum that verifies (186b), then z also verifies (185a).

(185) Less than five people can carry that piano upstairs.

[25] Unlike with max (see section 2.3.3), we probably have to assume that $\min(\varnothing)$ is undefined, rather than being equal to 0, in order to capture the fact that (185) licenses a lower-bound inference, viz. that at least some number of people can carry that piano upstairs. I leave this detail aside since, ultimately, I do no wish to argue for an account involving any minimality component, but the issue will arise again in chapter 6.

(186) a. $\exists n[n < 5 \wedge \forall_{\text{Gen}} x[\mathbf{card}(x) = n \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{carry}(x)]$

b. $\min(\lambda n\,.\,\forall_{\text{Gen}} x[\mathbf{card}(x) = n \wedge \mathbf{people}(x) \rightarrow \Diamond\mathbf{carry}(x)]) < 5$

The problem can thus be stated as follows. In the existential domain, we need a maximality component in order to express upper-bounded readings (at least with predicates that license downward inferences, such as *frown* and *gather*; see chapter 1 and chapter 3), and in the generic domain, we need a minimality component in order to express lower-bounded readings (at least with predicates that license upward inferences, such as *can carry that piano upstairs*). The fact that SMax is able to generate the right lower-bounded reading in the case of (185) (*Less than five people can carry that piano upstairs*) seems to just be an accident: in that case, the effect of minimality is obscured by the semantic contribution of *less than*. SMax's ability to capture that reading does not generalize to the analogous lower-bounded reading of (181) (*Between three and five people can carry that piano upstairs*).

5.7 Summary

In comparing the LMax and SMax approaches, I argued that it was necessary to find sentences with modified numerals in which, on the SMax account, the existential quantifier of the modified numeral does not commute with the existential force of the nominal in which it is contained. This led, in this chapter, to some new and interesting data from the generic domain that seemed, at first sight, to favor SMax over LMax: SMax, but not LMax, is able to generate the right reading for a sentence like *Less than five people can carry that piano upstairs*, viz. 'There is a number $n < 5$ such that, in general, any group of n people can carry that piano upstairs.'

Surprisingly, however, once we moved to the analogous sentence with *between three and five*, SMax was unable to generate the right reading. What we discovered (and which was obscured in the *less than* example) is that these sentences in fact have lower-bounded, or 'minimal' readings. In other words, quite unexpectedly, a modified numeral like *between three and five* sometimes conveys maximality, sometimes even minimality, and still other times neither.

To be more precise, the predicates

(187) a. λn [[$\varnothing_{\exists}$ [$n_{\texttt{isCard}}$ students]] frowned]

b. λn [[$\varnothing_{\text{Gen}}$ [$n_{\texttt{isCard}}$ people]] [can fit into that elevator]]

are downward scalar because, as argued in chapter 1, if, say, ten students frowned, then so did nine, eight, and so on, and, as argued in section 5.5.1 of this chapter, if, say, ten people can fit into that elevator, then so can nine, eight, and so on. When *between three and five* combines with this sort of predicate, we get an upper-bounded, or 'maximal' reading: *Between three and five students frowned*, for instance, entails that not more than five students frowned, and *Between three and five people can fit into that elevator* entails that not more than five people can fit.

The predicates

(188) a. λn [[$\varnothing_{\exists}$ [$n_{\texttt{isCard}}$ soldiers]] [surrounded the castle]]

b. λn [[$\varnothing_{\exists}$ [$n_{\texttt{isCard}}$ students]] [lifted the piano together]]

are neither downward scalar nor upward scalar (i.e. they are non-scalar). When, say, *between five and ten* combines with this sort of predicate, we get a non-upper-bounded (but also non-lower-bounded) reading: for instance, *Between five and ten soldiers surrounded the castle* does not entail that more than ten soldiers *didn't* surround the castle, nor does it entail that less than five soldiers *didn't* surround the castle. If, say, three different groups of soldiers surrounded the castle—one group of 4, one group of 7, and one group of 12—then the sentence is true (see footnote 38, chapter 1).

Finally, when, say, *between three and five* combines with an upward scalar predicate, then, as we saw in this chapter, we get a lower-bounded, or 'minimal' reading: *Between three and five people can carry that piano upstairs* entails that it is not the case that (any group of) less than three people can do it.

At this point, we face a very puzzling situation—where modified numerals sometimes convey maximality, other times minimality, and still other times neither—and the situation for both SMax and LMax seems quite bleak. However, it turns out that this puzzling array of facts leads quite naturally to very elegant theory of modified numerals, which I turn to in the next chapter.

Chapter 6

Informativity-based maximality

6.1 Overview

At the end of the previous chapter, I argued that a sentence like (189a), where *between three and five boys* combines with a predicate that licenses upward inferences (*can lift this piano*), has a reading that can be paraphrased as, 'The minimum number n such that, in general, any group of n boys can lift this piano is between three and five,' formalized as in (189b).[1]

(189) a. Between three and five boys can lift this piano.

b. $3 \leq \min(\lambda n\,.\,\forall_{\text{Gen}} x[\mathbf{card}(x) = n \wedge \mathbf{boys}(x) \rightarrow \Diamond\mathbf{lift}(x)]) \leq 5$

This 'minimal' reading is quite unexpected in light of the other sorts of readings explored in this thesis, which have involved either some kind of maximality component or simply a lack of maximality (and of minimality) altogether.

For example, when *between three and five students* combines with a predicate that licenses downward inferences, such as the distributive predicate *smiled*, as in (190a), the reading we get is upper-bounded (or 'maximal'): 'The maximum number n such that n

[1] Recall that the modal *can*, under the intended reading of the sentence, indicates ability (and not, say, permission); see also footnote 14 of chapter 5.

students smiled is between three and five.'[2]

(190) a. Between three and five students smiled.

b. $3 \leq \max(\lambda n\,.\,\exists x[\mathbf{card}(x) = n \wedge \mathbf{students}(x) \wedge \mathbf{smiled}(x)]) \leq 5$

And when *between five and ten soldiers* combines with a predicate that licenses neither downward nor upward inferences, such as the collective predicate *surrounded the castle*, as in (191a), we get a non-maximal, non-minimal (i.e. basic existential) reading: 'A group of between five and ten soldiers surrounded the castle.'

(191) a. Between five and ten soldiers surrounded the castle.

b. $\exists x[5 \leq \mathbf{card}(x) \leq 10 \wedge \mathbf{soldiers}(x) \wedge \mathbf{surrounded}(x)]$

We thus face a surprising state of affairs: modified numerals of the form *between m and n* (and *less than n*) sometimes convey maximality, other times minimality, and still other times neither—depending on the types of predicates they combine with. Up until now, our typology of readings was missing these 'minimal' readings. The main goal of this chapter is to develop an account of how and when all three readings arise.

Let me now sketch the core desideratum of this chapter in an abstract, schematic way, focusing for now just on *between five and ten*. Assume that we can generate LFs of the kind given in (192), where the modified numeral combines with a predicate of degrees (and $\varnothing$ stands ambiguously for $\varnothing_{\exists}$ or $\varnothing_{\text{Gen}}$).

(192) [between five and ten] [λn [[$\varnothing$ [$n_{\texttt{isCard}}$ NP]] VP]]

Now, let P be the denotation of the degree predicate *λn [[$\varnothing$ [$n_{\texttt{isCard}}$ NP]] VP]*. We can then describe the puzzle as follows, using the 'scalarity' terminology introduced in the last chapter (see (156) and (162)):

[2]The same could be said about the case where *between three and five people* combines with *can fit into that elevator* in a characterizing sentence (see section 5.5.1); however, I ignore this case henceforth.

- When P is downward scalar, *between five and ten* conveys maximality, i.e. gives rise to an upper-bounded reading.
- When P is upward scalar, *between five and ten* conveys minimality, i.e. gives rise to a lower-bounded reading.
- When P is non-scalar, *between five and ten* conveys neither maximality nor minimality, i.e. gives rise to a basic existential reading.

What we would like to do, therefore, is to provide a lexical entry for *between five and ten* that is somehow sensitive to the scalarity of P, so that *between five and ten* conveys maximality, minimality, or neither, depending on whether P is downward scalar (DS), upward scalar (US), or non-scalar (NS), respectively. One straightforward way to do this is to define for our metalanguage a special relation $\mathcal{R}$ between degree predicates and degrees, which, depending on the scalarity of the degree predicate it combines with, corresponds to a maximality operator, a minimality operator, or neither (i.e. a simple membership relation).

(193) $[\![\text{between five and ten}]\!] = \lambda P_{dt} \,.\, \exists k[5 \leq k \leq 10 \wedge \mathcal{R}(P)(k)]$

(194) $$\mathcal{R}(P_{dt})(k_d) = \begin{cases} 1 & \text{if } \mathbf{DS}(P) \wedge \max(P) = k \\ 1 & \text{if } \mathbf{US}(P) \wedge \min(P) = k \\ 1 & \text{if } \mathbf{NS}(P) \wedge P(k) \\ 0 & \text{otherwise} \end{cases}$$

With this in place, if P is downward scalar, then $[\![\text{between five and ten}]\!](P)$ is true iff $5 \leq \max(P) \leq 10$; if P is upward scalar, then it is true iff $5 \leq \min(P) \leq 10$; and if P is non-scalar, then it is true iff P holds of some k between 5 and 10.

Now, an analysis along these lines is rather *ad hoc* and unsatisfactory if we simply define and use such a relation by brute force. For example, the correspondences in $\mathcal{R}$ between downward scalarity and maximality, between upward scalarity and minimality, and between non-scalarity and lack of maximality/minimality are hard-coded directly into $\mathcal{R}$. We seem to be missing a generalization by doing that. It would be better instead for these correspondences to fall out naturally from the underlying concept that $\mathcal{R}$ represents. But what concept might $\mathcal{R}$ represent?

It turns out that a similar 'flip' between maximality and minimality has appeared elsewhere, such as in the semantics of degree questions (Beck and Rullmann 1999) and in the semantics of definite descriptions (von Fintel, Fox, and Iatridou 2014; see also Schlenker 2012). To make theoretical sense of this flip, these authors have proposed that we move from a 'standard' notion of maximality, where a degree is maximal in P if it is the largest degree in P (i.e. maximal with respect to the natural ordering of degrees), to an 'informativity'-based notion of maximality, where a degree is maximal in P if it is, in some sense, 'maximally informative' (with respect to P). The crucial insight of these works is that the 'maximally informative' degree is sometimes the largest, other times the smallest, depending on the nature of the degree predicate. Thus, a natural candidate for what $\mathcal{R}$ might be (or represent) is (some version of) the notion of maximal informativity.

In addition, an analysis of *between* along these lines would have a potential beneficial effect beyond just solving the undergeneration problem pointed out in the last chapter: since it would be able to generate exactly the right readings using only LFs of the kind in (192), it would not need to rely on scope ambiguity or optional maximization. This in turn means that the analysis would not overgenerate, hence would not have to rely on any pragmatic blocking mechanism.

In this chapter, I will develop a proposal based on this move to maximal informativity.[3] The relation $\mathcal{R}$ above will be instantiated by a maximality relation $\max_{\leq_{\mathcal{P}}}$ that operates not on the natural ordering of degrees ($\leq$), but rather on an informativity-based ordering

[3]The account that I develop in this chapter is due originally to Philippe Schlenker, who suggested it to Benjamin Spector, who in turn described a version of it in Spector 2014. Schlenker's original intention was, I believe, to capture the basic contrast between upper-bounded readings with distributive predicates vs. non-upper-bounded readings with collective predicates. As far as I am aware, Schlenker did not apply his proposal to the data with upward scalar degree predicates that I will discuss in this chapter.

of degrees ($\leq_{\mathcal{P}}$) determined by some property of degrees ($\mathcal{P}$). This relation will hold between a set of degrees D and a degree n just in case n is, in a sense to be made precise, a 'maximal' element of D relative to the informativity-based ordering $\leq_{\mathcal{P}}$. In practice, the informativity-based ordering will be the one induced by (the intension of) the degree predicate that a modified numeral combines with, with the important result that the scalarity of the degree predicate determines the type of ordering and hence what counts as maximal. I will show that this proposal does a very good job of predicting the contrast between upper-bounded ('maximal') readings in downward scalar contexts, lower-bounded ('minimal') readings in upward scalar contexts, and unbounded (non-maximal, non-minimal) readings in non-scalar contexts. However, it also faces a serious challenge having to do with what is predicted when a modified numeral like *less than five* combines with a degree predicate whose extension is empty: the proposal seems unable to cope with the empirical fact that when *less than five* combines with an upward scalar or non-scalar degree predicate, it licenses a lower-bound inference, but when it combines with a downward scalar degree predicate, it does not. I offer a couple ways to handle this problem.

The rest of this chapter is organized as follows. In section 6.2, I present the account and show how it naturally explains the contrasts just mentioned, using modified numerals with *between* to illustrate. In section 6.3, I discuss the aforementioned complication that arises for modified numerals with *less than* under this account, and how this complication might be resolved. Section 6.4 summarizes the results.

6.2 Maximal informativity

The proposal I will develop is based on a fairly simple idea: A sentence of the form *Between five and ten NP VP* means that there is a number k, with $5 \leq k \leq 10$, such that k is a 'maximally informative' member of the set characterized by the degree predicate λn *[n*

NP VP]. Whether a number counts as maximally informative will depend on the scalarity of the degree predicate involved: sometimes, it must be the largest number of which the degree predicate holds; other times, the smallest number; and still other times, it need not be either.

On this account, the notions of informativity and scalarity are intimately related. (Their exact relationship will become clear as we go along.) In order to formalize both notions precisely, it is necessary that we now be able talk about *propositions*, i.e. functions from possible worlds to truth values, and to view degree predicates as representing *properties* of degrees, i.e. functions from worlds to (characteristic functions of) sets of degrees. Being able to talk about propositions will allow us to talk about informativity content (cashed out in terms of logical strength), and being able to talk about properties of degrees will allow us to talk about scalarity.

I will first explain how scalarity can be defined in terms of the *intensions* of degree predicates. Doing so will also allow me to informally describe what it means for one number to be 'more informative' than another number relative to (the intension of) a degree predicate. Afterwards, I define informativity (of degrees) more precisely as a partial ordering over a set of degrees induced by some property of degrees. Having discussed this informativity-based partial ordering, I then describe the notion of a 'maximally informative' degree, and from there how *between* incorporates such an informativity-based maximality component.

6.2.1 Types of scalarity

Consider the degree predicate in (195), which is a typical example of a 'downward scalar' degree predicate.

(195) λn [[$\varnothing_{\exists}$ [$n_{\texttt{isCard}}$ students]] smiled]

In a world w, its extension is a predicate of degrees P with type dt: it denotes the set of all numbers n such that n (or more) students smiled in w. Up to now, we have called this predicate downward scalar in the (rather informal) sense that if n is in its extension, then so is every number smaller than n (down to, and including, 1). That is, $P(n)$ implies $P(m)$ whenever $m < n$ (see (162)). But the deeper point is that this is the case in every world, not just some particular world: every world w is such that if n students smiled in w, then $n-1$ students smiled in w, $n-2$ students smiled in w, and so on. By shifting our attention from the *extension* of a degree predicate in a specific world (i.e. a set of degrees) to the *intension* of a degree predicate (i.e. a property of degrees), we can express this fact.

The *intension* of (195) is a function $\mathcal{P}$ of type $s(dt)$, with s being the type of possible worlds: it maps a world w to the extension of the degree predicate in w, i.e. to the set of numbers n such that n students smiled in w. For example, $\mathcal{P}(w) = P$, where P and w are as above.[4] Downward scalarity, as described above, can now be defined as follows.[5]

(196) **Downward scalarity**

A property of degrees $\mathcal{P}_{s(dt)}$ is *downward scalar* iff, for any two (non-zero) numbers m and n, if $m < n$, then $\lambda v_s \,.\, \mathcal{P}(v)(n) \subseteq \lambda v_s \,.\, \mathcal{P}(v)(m)$.

[4] I use the stylized (calligraphy) letter $\mathcal{P}$ for the intension of a degree predicate, i.e. a function of type $s(dt)$, while normal P (or D) stands for the extension of a degree predicate in some world, i.e. a function of type dt (a set of degrees).

[5] The three definitions for the three types of scalarity will all have a non-zero condition on m and n. The reason for this is rather technical and can be ignored, but for the sake of precision, I will explain the reasons in footnotes as we go along. The reason for the non-zero condition in the case of downward scalarity is because without it a predicate like (195) would not be downward scalar. Here's why: 0 is less than, say, 10, but the set of worlds in which a sum of 10 students smiled is *not* a subset of the set of worlds in which a sum of 0 students smiled, simply because the latter set of worlds is the empty set. This is because there is no world at all in which a sum of zero students smiled, simply because of our ontological assumption that there is no sum with cardinality zero, i.e. no sum x such that $\mathbf{card}(x) = 0$, hence no sum x such that the cardinality predicate $0_{\texttt{isCard}}$ is true of x. (See also the discussion in section 6.3.)

Note that the non-zero condition here is sufficient only for degree predicates involving a 'strongly' distributive predicate like *smile*. In the case of a 'weakly' (or semi-) distributive predicate like *gather* (see section 3.2), we would have the stronger condition 'not equal to zero or one', otherwise a degree predicate like (195) but with *gather* would fail to count as downward scalar, since the set of worlds where, say, ten students gathered is not a subset of the set of worlds where one student gathered (the latter set is empty because in no world is it the case that a single individual gathered). Thus, to be precise, we should really define a *family* of types of downward scalarity (down to 1, down to 2, etc.), and then say that $\mathcal{P}$ is downward scalar just in case it is one of those types.

In plain English, $\mathcal{P}$ is downward scalar iff, whenever $m < n$, then the proposition that $\mathcal{P}$ holds of n entails the proposition that $\mathcal{P}$ holds of m. For example, (195) (or, more precisely, the intension of (195)) is downward scalar because the proposition that, say, ten students smiled entails the proposition that nine students smiled. That is, every world in which ten students smiled is a world in which nine students smiled. The downward scalarity of (195) is of course due crucially to the fact that *smiled* licenses downward inferences (it is distributive).

Now, the idea that I am going to capitalize on is this: when $\mathcal{P}$ is downward scalar, larger numbers are 'more informative' (relative to $\mathcal{P}$) than smaller numbers because propositions involving $\mathcal{P}$ and larger numbers entail propositions involving $\mathcal{P}$ and smaller numbers. For example, 10 is more informative than 9 relative to (the intension of) the degree predicate in (195) because the proposition that ten students smiled is more informative than (entails) the proposition that nine students smiled.

Let us now turn to upward scalarity, which can be defined similarly, except the direction of entailment is reversed.[6]

(197) **Upward scalarity**

A property of degrees $\mathcal{P}_{s(dt)}$ is upward scalar iff, for any two (non-zero) numbers m and n, if $m < n$, then $\lambda v_s . \mathcal{P}(v)(m) \subseteq \lambda v_s . \mathcal{P}(v)(n)$.

In plain English, $\mathcal{P}$ is upward scalar iff, whenever $m < n$, then the proposition that $\mathcal{P}$ holds of m entails the proposition that $\mathcal{P}$ holds of n. For example, the degree predicate in (198) is upward scalar because the proposition that any group of ten boys can lift this piano entails the proposition that any group of eleven boys can lift it. That is, every world

[6] In this case, the non-zero condition is necessary because without it the predicate in (198) would not be upward scalar: 0 is less than, say, 10, but the set of worlds in which any sum of zero boys can lift this piano is not a subset of the set of worlds in which any sum of ten boys can lift this piano, simply because the former set corresponds to the set of *all* worlds. This because every world is (trivially) such that any sum of zero boys can lift this piano, simply because there is no sum of zero boys, due to our ontological assumption that there is no x such that $\mathbf{card}(x) = 0$.

in which any group of ten boys can lift this piano is a world in which any group of eleven boys can lift it.[7] In this case, relative to (the intension of) (198), the number 10 is more informative than 11 (but less informative than 9).

(198) λn [[$\varnothing_{\text{Gen}}$ [$n_{\texttt{isCard}}$ boys]] [can lift this piano]]

Finally, we turn to non-scalarity, which I will define as follows.[8, 9]

(199) **Non-scalarity**

A property of degrees $\mathcal{P}_{s(dt)}$ is non-scalar iff there are no two distinct (non-zero) numbers m and n such that $\lambda v_s \,.\, \mathcal{P}(v)(m) \subseteq \lambda v_s \,.\, \mathcal{P}(v)(n)$.

For example, (200) is non-scalar because for no two distinct numbers m and n is it the case that the proposition that m soldiers surrounded the castle entails the proposition that n soldiers surrounded the castle. For example, there are worlds in which ten soldiers surrounded the castle, but nine did not, and eleven did not. Thus, in this case, relative to

[7]One might object that there are conceivable worlds in which, say, any group of 5 to 10 boys can lift the piano, but no group of 11 or more boys can lift it—simply because, say, 11 boys cannot actually fit around the piano. My response (see footnote 14 of chapter 5) would be that, since *can* indicates ability (e.g. physical ability, or strength), it is still literally true that 11, 12, . . . boys 'can' lift the piano—they have the physical strength (ability) to do it—even though there are circumstances preventing them from ever actually doing it. Put differently, it is simultaneously true both that 11 boys 'can' lift this piano (in view of their physical ability) and that 11 people 'cannot' lift this piano (in view of the dimensions of the piano), depending on how exactly *can/cannot* is interpreted. This argument is directly related to the discussion in Kratzer 1981 concerning German *können*: Kratzer argues that a sentence like *Ich kann nicht Posaune spielen* ('I cannot play the trombone') means different things depending on the situation in which it is uttered (maybe I cannot play it because I don't know how, or because I have asthma and can hardly breathe, or because my trombone sank with the ship I had been on, and so on).

[8]The reason for the non-zero condition here is because without it a predicate like (200) would fail to be non-scalar. This is because the set of worlds in which a sum of zero soldiers surrounded the castle corresponds to the empty set (due once again to our ontological assumption that there is no x such that $\mathbf{card}(x) = 0$), hence is a subset of the set of worlds in which, say, ten soldiers surrounded the castle.

[9]Note that this definition of non-scalarity is stronger than the one (used in chapter 5) which defines non-scalarity as simply being neither downward nor upward scalar. The stronger notion—which perhaps could more appropriately be called *anti-scalarity*—will allow me to list a number of important facts about the interaction of scalarity and maximality (see Facts 1–3 below). As far as natural language goes, I don't see any harm in using the strong definition since I doubt there are predicates in natural language that are neither downward scalar, nor upward scalar, nor anti-scalar. They would be predicates which, for example, make 10 more informative than 5, but not more informative than 7. (In other words, I assume that non-scalarity in the weak sense implies non-scalarity in the strong sense, i.e. anti-scalarity, as far as natural language predicates go.)

(the intension of) (200), the number 10 is neither more nor less informative than 9 or 11.

(200) λn [[$\varnothing_\exists$ [$n_{\texttt{isCard}}$ soldiers]] [surrounded the castle]]

6.2.2 An informativity-based ordering of degrees

Now that I have explained informally what it means for a number to be 'more informative' than another number, relative to a property of degrees (i.e. the intension of a degree predicate), I will now define what it means for a number to be 'maximally informative'. This notion of 'maximally informative' will be a prominent part of the semantics of numeral modifiers like *between*.

First, we define a new type of relation on numbers, which makes precise the notion of 'at least as informative as' relative to some property of degrees.

(201) Let $\mathcal{P}_{s(dt)}$ be a property of degrees. Then for any two numbers m and n, $m \leq_{\mathcal{P}} n$ iff $\lambda v_s \,.\, \mathcal{P}(v)(n) \subseteq \lambda v_s \,.\, \mathcal{P}(v)(m)$.

In plain English, n is at least as informative as m relative to $\mathcal{P}$ iff the proposition that $\mathcal{P}$ holds of n entails the proposition that $\mathcal{P}$ holds of m. Put differently, if you plug n into $\mathcal{P}$, then the proposition you get is at least as informative (at least as logically strong as) the proposition you get by plugging m into $\mathcal{P}$.

Now, for any set D of degrees, $\leq_{\mathcal{P}}$ determines (for some $\mathcal{P}$) a partial order over D: the elements of D are partially ordered in terms of their informativity relative to $\mathcal{P}$.[10] Recall now the definition of max in (64) from chapter 2, repeated below.[11]

(202) For any degree predicate P_{dt}, any partial ordering $\leq$ over P, and any degree n, $\text{max}_{\leq}(P)(n)$ iff $P(n) \wedge \neg\exists m[P(m) \wedge m > n]$.

[10]See chapter 2, footnote 6 for the definition of a partial order.

[11]For now, I am using the first version of max, which, unlike the revised version in (81) from section 2.3.3, does not stipulate that 0 counts as maximal if the set to which max applies is empty. I will consider this alternative definition in section 6.3.

The idea now is that modified numeral constructions (in particular, numeral modifiers like *between*) will involve a maximality component that operates on an informativity-based ordering of degrees. That is, they will involve $\max_{\leq_{\mathcal{P}}}$ for some property of degrees $\mathcal{P}$—specifically, the intension of the degree predicate that the modified numeral combines with—rather than $\max_{\leq}$ (where $\leq$ is the natural ordering of numbers). In this case, plugging in $\leq_{\mathcal{P}}$ for the ordering relation in (202), we get:

(203) $\max_{\leq_{\mathcal{P}}}(P)(n)$ iff $P(n) \wedge \neg\exists m[P(m) \wedge m >_{\mathcal{P}} n]$

In plain English, n is a $\mathcal{P}$-maximal element of P just in case n is in P and there is no number m in P which is more informative (relative to $\mathcal{P}$) than n. Since informativity is defined in (201) in terms of logical entailment, we can restate this as follows: n is a $\mathcal{P}$-maximal element of P just in case n is in P and there is no number m in P such that the proposition that $\mathcal{P}$ holds of m asymmetrically entails the proposition that $\mathcal{P}$ holds of n.[12]

Note that the notion of maximal informativity that I use here is quite *weak*. For a number n to be maximal in P relative to an informativity-based ordering $\leq_{\mathcal{P}}$, it is not necessary that n be the unique most informative number, i.e. more informative than every other number. If for some other number m, m is in P and n is not more informative than m, then n can still be maximally informative, so long as m is not more informative than n, i.e. neither number is more informative than the other. For this reason, it is perfectly possible in principle for two numbers m and n (or even every number in a set) to count as maximal.[13] This will be the case whenever the informativity-based ordering is induced by a non-scalar degree predicate, since in that case no number is more informative than any other number.

[12] Since informativity is cashed out here in terms of logical strength, I use the terms 'informativity-based maximality' and 'logical maximality' interchangeably; they contrast with the 'standard maximality' of earlier chapters.

[13] In this respect, the notion of maximal informativity that I use here is weaker than what is found in some other works that rely on maximal informativity (e.g. Dayal 1996; Rullmann 1995; Beck and Rullmann 1999; Fox and Hackl 2006; Abrusán 2007; Abrusán and Spector 2011; von Fintel, Fox, and Iatridou 2014).

More generally, let us note the following useful facts. If $\mathcal{P}$ is downward scalar, then $\leq_{\mathcal{P}}$ corresponds to $\leq$ (the natural ordering) because larger numbers are more informative than smaller numbers relative to $\mathcal{P}$. If $\mathcal{P}$ is upward scalar, then $\leq_{\mathcal{P}}$ corresponds to $\geq$ (the reverse of the natural ordering) because smaller numbers are more informative than larger numbers relative to $\mathcal{P}$. And if $\mathcal{P}$ is non-scalar, then $\leq_{\mathcal{P}}$ corresponds to $=$ (the identity relation) because no number is more informative than any other number relative to $\mathcal{P}$. We can list these facts succinctly as follows, including what $\max_{\leq_{\mathcal{P}}}$ amounts to relative to the given ordering.

(204) **Fact 1:** If $\mathcal{P}$ is downward scalar, then for any two (non-zero) degrees m and n, $m \leq_{\mathcal{P}} n$ iff $m \leq n$. Hence, for any set of degrees D and any degree n, $\max_{\leq_{\mathcal{P}}}(D)(n)$ iff $\max_{\leq}(D) = n$.

(205) **Fact 2:** If $\mathcal{P}$ is upward scalar, then for any two (non-zero) degrees m and n, $m \leq_{\mathcal{P}} n$ iff $n \leq m$. Hence, for any set of degrees D and any degree n, $\max_{\leq_{\mathcal{P}}}(D)(n)$ iff $\min_{\leq}(D) = n$.[14]

(206) **Fact 3:** If $\mathcal{P}$ is non-scalar, then for any two (non-zero) degrees m and n, $m \leq_{\mathcal{P}} n$ iff $m = n$. Hence, for any set of degrees D and any degree n, $\max_{\leq_{\mathcal{P}}}(D)(n)$ iff $n \in D$.

We now see very clearly how moving from the natural ordering of degrees to an informativity-based ordering allows our maximality component to sometimes act as a (standard) maximality operator, other times as a (standard) minimality operator, and still other times as neither (basically, as a membership relation). Thus, we seem to have successfully derived the desired relation described (as $\mathcal{R}$) in (194).

[14]See (183) in chapter 5 for a definition of the minimality operator.

6.2.3 *Between* and maximal informativity

Having developed the notion of a partial ordering based on informativity, we now turn to the semantics of the numeral modifier *between*. The idea here is that the lexical entry for *between* is exactly the same as it is in the LMax account, except that the underlying ordering that the maximality component operates on is based on informativity, rather than on the natural ordering of numbers.[15]

(207) $[\![\text{between}]\!]^w = \lambda m_d \,.\, \lambda n_d \,.\, \lambda \mathcal{P}_{s(dt)} \,.\, \exists k[m \leq k \leq n \wedge \max_{\leq_\mathcal{P}}(\mathcal{P}(w))(k)]$

For example, *between five and ten* has the following semantics.

(208) $[\![\text{between five and ten}]\!]^w = \lambda \mathcal{P}_{s(dt)} \,.\, \exists k[5 \leq k \leq 10 \wedge \max_{\leq_\mathcal{P}}(\mathcal{P}(w))(k)]$

In plain English, $[\![\text{between five and ten}]\!]^w(\mathcal{P}_{s(dt)})$ means that there is a number k, with $5 \leq k \leq 10$, such that k is a maximally informative member of the set $\mathcal{P}(w)$ (relative to $\mathcal{P}$). Now, given the three facts above, we seem to have derived exactly what we want:

(209) **Fact 4:** If $\mathcal{P}$ is downward scalar, then:

$[\![\text{between five and ten}]\!]^w(\mathcal{P}) = 1$ iff $\exists k[5 \leq k \leq 10 \wedge \max_{\leq}(\mathcal{P}(w)) = k]$.

(210) **Fact 5:** If $\mathcal{P}$ is upward scalar, then:

$[\![\text{between five and ten}]\!]^w(\mathcal{P}) = 1$ iff $\exists k[5 \leq k \leq 10 \wedge \min_{\leq}(\mathcal{P}(w)) = k]$.

(211) **Fact 6:** If $\mathcal{P}$ is non-scalar, then:

$[\![\text{between five and ten}]\!]^w(\mathcal{P}) = 1$ iff $\exists k[5 \leq k \leq 10 \wedge \mathcal{P}(w)(k)]$.

[15] Compare the entry in (207) with the LMax entry in (63): the main difference is that the maximality component in (63) involves the natural ordering $\leq$, while the maximality component in (207) involves the informativity-based ordering induced by the intension of the degree predicate ($\leq_\mathcal{P}$). (Another difference, of course, is that (63) involves a predicate of degrees, while (207) involves a property of degrees, since a property of degrees is required to define the informativity-based ordering that the maximality component operates on.)

We will see *between* in action in a moment, and why these facts hold. But first, I need to explain a few details about the intensional syntax-semantics framework that I am assuming.

I am now treating modified numerals as *intensional operators*, because they need to have access to the intension of their argument so that the informativity-based ordering that the maximality component operates on can be defined. I assume here an intensional system in which denotations are relativized to a world parameter (which occurs as a superscript to the right of the denotation brackets). Furthermore, predicates, generalized quantifiers, etc. retain purely extensional types, but we now need two rules of functional application: the standard one and the one that Heim and Kratzer (1998) call *Intensional Functional Application*. Our two rules of functional application are therefore the following.

(212) **Functional application (FA)**

If the daughters of α are β and γ, with $[\![\beta]\!]^w$ of type $\sigma\tau$ and $[\![\gamma]\!]^w$ of type σ, then $[\![\alpha]\!]^w = [\![\beta]\!]^w([\![\gamma]\!]^w)$.

(213) **Intensional functional application (IFA)**

If the daughters of α are β and γ, with $[\![\beta]\!]^w$ of type $s(\sigma\tau)$ and $[\![\gamma]\!]^w$ of type σ, then $[\![\alpha]\!]^w = [\![\beta]\!]^w(\lambda v_s \,.\, [\![\gamma]\!]^v)$.

With this in place, we can keep the very same lexical entries as before for predicates, numerals, the cardinality operator `isCard`, and the silent existential ($\varnothing_{\exists}$) and generic ($\varnothing_{\text{Gen}}$) determiners. We just need to add a world parameter to the interpretation function. For example:[16]

(214) a. $[\![\text{students}]\!]^w = \lambda x_e \,.\, \textbf{students}(w)(x)$

$[\![\text{smiled}]\!]^w = \lambda x_e \,.\, \textbf{smiled}(w)(x)$

[16]Note that **students**$(w)(x)$ should be read as, 'x is a plurality each of whose atomic parts is a student in w' (since *students* is distributive), **surrounded**$(w)(x)$ should be read as, 'x (is a plurality who) surrounded the castle in w', and so on.

$[\![\text{surrounded the castle}]\!]^w = \lambda x_e \,.\, \textbf{surrounded}(w)(x)$

b. $[\![\text{three}]\!]^w = 3$

c. $[\![\texttt{isCard}]\!]^w = \lambda n_d \,.\, \lambda x_e \,.\, \textbf{card}(x) = n$

$[\![n_{\texttt{isCard}}]\!]^w = [\![\texttt{isCard}\ n]\!]^w = \lambda x_e \,.\, \textbf{card}(x) = [\![n]\!]^w$

d. $[\![\varnothing_\exists]\!]^w = \lambda P_{et} \,.\, \lambda Q_{et} \,.\, \exists x[P(x) \wedge Q(x)]$

$[\![\varnothing_{\text{Gen}}]\!]^w = \lambda P_{et} \,.\, \lambda Q_{et} \,.\, \forall_{\text{Gen}} x[P(x) \rightarrow Q(x)]$

As I will now show, focusing on *between five and ten*, the lexical entry in (207) captures the basic contrast between downward scalar, upward scalar, and non-scalar degree predicates.

Downward scalar case

Consider first what we get for a sentence like (215), under the given LF, with the distributive predicates *students* and *smiled*.

(215) a. Between five and ten students smiled.

b. [between five and ten] [λn [[$\varnothing_\exists$ [$n_{\texttt{isCard}}$ students]] smiled]]

Between five and ten combines syntactically with a predicate of degrees, whose semantic type is dt, namely:

(216) λn [[$\varnothing_\exists$ [$n_{\texttt{isCard}}$ students]] smiled]

In a world w, this expression denotes (the characteristic function of) the set of numbers n such that n students smiled in w. But because the meaning of *between five and ten* requires an argument of type $s(dt)$, IFA has to be used, and so the argument of $[\![\text{between five and ten}]\!]^w$ is the following:

(217) $\lambda v_s \,.\, [\![\lambda n\ [[\varnothing_\exists\ [n_{\texttt{isCard}}\ \text{students}]]\ \text{smiled}]]\!]^v$

Call this function $\mathcal{P}$. Then what we wind up getting is the following:

(218) $[\![(215)]\!]^w = 1$ iff $\exists k[5 \leq k \leq 10 \land \max_{\leq_\mathcal{P}}(\mathcal{P}(w))(k)]$

That is, (215) is true in w just in case there is a number k, with $5 \leq k \leq 10$, such that the following two conditions hold:[17]

(219) a. $\mathcal{P}(w)(k)$. (That is, k students smiled in w.)

b. $\neg\exists k'[\mathcal{P}(w)(k') \land k' >_\mathcal{P} k]$. (That is, there is no k' such that the proposition that k' students smiled is both true in w and asymmetrically entails the proposition that k students smiled.)

Now, because $\mathcal{P}$ is downward scalar, the informativity-based ordering induced by $\mathcal{P}$ is equivalent to the natural ordering on numbers. That is, the following two statements are equivalent for any two (non-zero) numbers m and n (see Fact 1 above):[18]

(220) a. $m >_\mathcal{P} n$. (The proposition that m students smiled asymmetrically entails the proposition that n students smiled.)

b. $m > n$.

Thanks to this equivalence, we can restate (219) as follows:

(221) a. k students smiled in w.

b. There is no k' such that k' students smiled in w and $k' > k$.

[17]Henceforth, for ease of exposition, I write meanings in plain English instead of logical notation. However, since I continue to give explicit LFs for the sentences under discussion, the reader should (with the lexical entries above) be able to derive the more 'logically explicit' version if necessary.

[18]See chapter 1, example (22), for the proof that if ten students smiled (in w), then three students smiled (in w). This easily generalizes so that for any two numbers m and n such that $m > n > 0$, the proposition that m students smiled asymmetrically entails the proposition that n students smiled.

So, for k to be maximal relative to $\mathcal{P}$, k should simply be the *largest* number which $\mathcal{P}$ is true of in w. As a result, we have the following (see Fact 4 above):

(222) (215) is true in w just in case the largest number k such that k students smiled in w is between five and ten.

This is precisely the upper-bounded ('maximal') reading we want to derive.

Of course, this result will generalize to all predicates which, like *smile*, license downward inferences with respect to their argument. In all such cases, we get exactly the same result as the one that follows from a lexical entry for *between* in which the maximality component operates on the natural ordering of numbers (cf. the entry for *between* given in (63)), because the natural ordering ($\leq$) and the informativity-based ordering ($\leq_{\mathcal{P}}$) happen to be equivalent in such cases (see Facts 1 and 4).

Upward scalar case

Let us now turn to a sentence with a predicate that licenses upward inferences, such as *can lift this piano*, in the context of a generalizing sentence (see chapter 5).

(223) a. Between five and ten boys can lift this piano.

b. [between five and ten] [λn [[$\varnothing_{\text{Gen}}$ [$n_{\texttt{isCard}}$ boys]] [can lift this piano]]]

The compositional steps in the calculation of the meaning of (223) are the same as in the previous case. So the sentence ends up true in a world w just in case there is a number k, with $5 \leq k \leq 10$, such that the following two conditions hold:

(224) a. In general, any group of k boys can lift this piano in w.

b. There is no k' such that the proposition that, in general, any group of k' boys can lift this piano is both true in w and asymmetrically entails the proposition that, in general, any group of k boys can lift this piano.

Now, because the degree predicate

(225) λn [[$\varnothing_{\text{Gen}}$ [$n_{\texttt{isCard}}$ boys]] [can lift this piano]]

is upward scalar (see chapter 5), the following two statements are equivalent for any two (non-zero) numbers *m* and *n* (see Fact 2):

(226) a. The proposition that, in general, any group of *m* boys can lift this piano asymmetrically entails the proposition that, in general, any group of *n* boys can lift this piano.

b. $m < n$.

Thanks to this equivalence, we can restate (224) as follows:

(227) a. In general, any group of *k* boys can lift this piano.

b. There is no k' such that $k' < k$ and k' boys can lift this piano.

So, in this case, for *k* to be maximal, it should be the *smallest* number of which the degree predicate is true in *w*. As a result, we have the following (see Fact 5):

(228) (223) is true in *w* just in case the smallest number *k* such that *k* boys can lift this piano in *w* is between five and ten.

Once again, this is exactly the lower-bounded (or 'minimal') reading we want to derive for such cases. This result will generalize to all predicates which, like *can lift this piano*, license upward inferences with respect to their argument. In all such cases, we get exactly the same result as what would follow from a lexical entry for between involving a minimality operator, because the natural ordering and the informativity-based ordering happen to be *opposites* in such cases (see Facts 2 and 5).

Non-scalar case

Let us finally turn to a typical non-distributive predicate like *surround the castle*:

(229) a. Between five and ten soldiers surrounded the castle.

b. [between five and ten] [λn [[$\varnothing_\exists$ [$n_{\texttt{isCard}}$ soldiers]] [surrounded the castle]]]

Again, the compositional steps in the calculation of the meaning of (229) are the same as in the previous two cases. So the sentence ends up true in a world w just in case there is a number k, with $5 \leq k \leq 10$, which meets the following two conditions:

(230) a. k soldiers surrounded the castle in w.

b. There is no k' such that the proposition that k' soldiers surrounded the castle is both true in w and asymmetrically entails the proposition that k soldiers surrounded the castle.

In this case, it turns out that the second clause, (230b), is trivially true, simply because the degree predicate here is non-scalar: for any two distinct numbers m and n, there is no entailment relation in either direction between the following two propositions:

(231) a. m soldiers surrounded the castle.

b. n soldiers surrounded the castle.

As a result, we have the following:

(232) (229) is true in w just in case there is a number k, with $5 \leq k \leq 10$, such that k soldiers surrounded the castle in w.

This is precisely the non-upper-bounded ('non-maximal', 'non-minimal') reading we wish to derive for such cases. This result will of course generalize to all predicates which, like *surrounded the castle*, license neither downward nor upward inferences with respect to their argument. In all such cases, we get exactly the same result as the one we get from not applying maximality (of any kind) at all, because the informativity-based ordering happens to correspond to the identity relation, and hence maximal informativity corresponds simply to set membership (see Facts 3 and 6).

We see, then, that this account, which I will dub *Lexical Informativity-based Maximality*, or LMax_{inf} for short (since the maximality component is lexically encoded into the numeral modifier and operates on an informativity-based ordering), is able to capture all three attested readings for all three types of sentences with *between five and ten*.

6.2.4 Four predictions revisited

Now that we've seen how LMax_{inf} captures the three basic, attested readings of sentences with *between five and ten*, let us briefly revisit the three predictions that were discussed in chapter 3, as well as the prediction discussed in section 5.5.1, to see how LMax_{inf} fares. I will argue that LMax_{inf} makes all the same correct predictions as LMax and SMax (section 3.2, section 3.3, section 5.5.1), while also correctly *not* making the wrong prediction that LMax and SMax both make (section 3.4).

The first prediction (section 3.2) is that a sentence like *Between 50 and 100 protesters gathered*, with the collective but 'weakly' distributive predicate *gather*, has only an upper-bounded (maximal) reading. On the LMax_{inf} account, this follows because the predicate

(233) λn [[$\varnothing_{\exists}$ [$n_{\texttt{isCard}}$ protesters]] gathered]

is downward scalar (down to 2): if 100 protesters gathered, then so did 99, ..., 2 (see section 3.2 for discussion), and so as a result, larger numerals are more informative than smaller ones, and so *gather* (as well as *fit*, *be neighbors*, *hold hands*, and so on) will work just like *smile* as far as the relevant modified numeral constructions go.

The second prediction (section 3.3) is that an asymmetry arises with cumulative transitive predicates, depending on whether a modified numeral like *between five and ten* occurs as the subject argument or as the object argument of the transitive predicate. For example, in a sentence like *Between 5 and 10 chickens laid more than 20 eggs*, the modified numeral *between 5 and 10* is interpreted 'non-maximally' (the sentence is consistent with more than 10 chickens laying more than 20 eggs), whereas in a sentence like *More than 20 chickens laid between 5 and 10 eggs*, the modified numeral *between 5 and 10* is interpreted maximally (the sentence entails that there is a group of more than 20 chickens who failed to lay more than 10 eggs between them). This asymmetry is predicted on the LMax_{inf} account because the following two predicates have different scalarities (I ignore here internal structure of the degree predicates for simplicity):

(234) a. λn [n chickens laid more than 20 eggs] Upward scalar

b. λn [more than 20 chickens laid n eggs] Downward scalar

For example, suppose there are 50 chickens in total. Then, if 5 of them laid 30 eggs between them, then it is true (though arguably misleading, depending on context) to say that 6, . . . , 50 of them laid 30 eggs between them. Conversely, if a group of 30 chickens laid only 10 eggs between them, then it is true (but again misleading) to say that 30 chickens laid 9, . . . , 1 egg(s) between them.

Crucially, the LMax_{inf} account, unlike LMax and SMax, correctly does *not* derive any upper-bounded reading in the case of non-scalar (collective) predicates (see section 3.4). That is, a sentence like *Between 5 and 10 soldiers surrounded the castle* is correctly predicted to not have the reading, 'A group of 5 to 10 soldiers surrounded the castle, and no group of more than 10 soldiers surrounded the castle.' This is because, as we have seen, the degree predicate

(235) λn [[$\varnothing_{\exists}$ [$n_{\texttt{isCard}}$ soldiers]] [surrounded the castle]]

is non-scalar, which means that maximal informativity does not amount to (standard) maximality, and so on the $LMax_{inf}$ account, there is no way that (standard) maximality could ever arise in the interpretation of such a sentence.

Finally, the $LMax_{inf}$ account correctly predicts the contrast in interpretation between a generalizing sentence like *Between 5 and 10 people can fit into that elevator*, which has an upper-bounded reading (see section 5.5.1), and a generalizing sentence like *Between 3 and 5 boys can lift that piano*, which has a lower-bounded reading. The important point here is that the two degree predicates in (236) have different scalarities.

(236) a. λn [n people can fit into that elevator] Downward scalar

b. λn [n boys can lift that piano] Upward scalar

Since (236a) is downward scalar, maximal informativity corresponds to standard maximality, and we get an upper-bounded (maximal) reading; and since (236b) is upward scalar, maximal informativity corresponds to minimality, and we get a lower-bounded (minimal) reading.

In sum, $LMax_{inf}$ seems to do a very good job so far: it generates the intuitively correct truth conditions for all three types of sentences with *between*; it correctly makes the same two predictions regarding 'weakly' distributive predicates and cumulative transitive predicates that LMax and SMax do; unlike LMax and SMax, it also correctly does not predict upper-bounded readings in collective contexts; and it correctly predicts that generalizing sentences sometimes have upper-bounded readings, sometimes lower-bounded readings. I now present another argument in favor of $LMax_{inf}$ over LMax and SMax, which is that $LMax_{inf}$ does not seem to need to appeal to any pragmatic blocking mechanism.

6.2.5 No need for pragmatic blocking?

We have just seen that LMax_{inf} is able to derive the right readings for sentences with *between five and ten*, regardless of the type of predicate it combines with; that is, it solves the undergeneration problem associated with *between* that was pointed out at the end of the last chapter and again at the beginning of this chapter. What's more, it can derive these readings using just one type of LF: namely, one where the modified numeral scopes above the silent determiner ($\varnothing_{\exists}$ or $\varnothing_{\text{Gen}}$) and takes as its argument the intension of the derived degree predicate, as schematized in (192), repeated below.

(237) [between five and ten] [λn [[$\varnothing$ [$n_{\texttt{isCard}}$ NP]] VP]]

What this means is that LMax_{inf} does not need to rely on scope ambiguity or on optional application of maximization to derive attested readings.

That being said, nothing I have assumed so far prevents the modified numeral from scoping below the silent determiner, just like it can (and sometimes must) do on the LMax account. For example, LFs like the following are also predicted to be available on the LMax_{inf} account:

(238) [$\varnothing$ [[λx [[between five and ten] [λn [x $n_{\texttt{isCard}}$]]]] NP]] VP

It turns out that the readings thus derived are exactly the same as the ones derived under identical LFs on the LMax account. The reason is because, just like with LMax, the maximality component has no semantic effect: for any ordering $\leq$ (informativity-based or otherwise) and any sum x, we have $\max_{\leq}(\lambda n\,.\,\mathbf{card}(x) = n)(k)$ just in case $\mathbf{card}(x) = k$, simply because the set to which $\max_{\leq}$ applies is the singleton set containing the cardinality of x (because every sum x has exactly one cardinality). As a result, if LMax_{inf} were to allow the same scope flexibility that LMax has, then it would generate exactly the same problematic readings that LMax does, such as weak, non-upper-bounded readings in distributive contexts (Van Benthem's problem), and would thus need to appeal to pragmatic blocking (see chapter 2 for discussion).

Given that LMax_{inf} (unlike LMax) does not appear to require scope ambiguity to generate the right readings, and that allowing scope ambiguity would only seem to force LMax_{inf} to adopt a pragmatic blocking mechanism to handle the associated overgeneration, it is worth considering whether there is a way to force modified numerals to always scope above silent determiners, i.e. to generate only LFs like (237), not (238).

We have in fact already seen one way to do this: Hackl's (2000) idea that the silent determiner is parameterized for degrees (see chapter 1, section 1.2.5). In chapter 2, I argued *against* such a view, because in the context of LMax, it would prevent the sort of scope ambiguity that LMax relies on. In the present context, however, Hackl's (2000) innovation seems to be just what we need.

The idea is that our silent determiners now have the following lexical entries (see also footnote 19 of chapter 5).

(239) a. $[\![\varnothing_{\exists}]\!]^w = \lambda n_d \,.\, \lambda P_{et} \,.\, \lambda Q_{et} \,.\, \exists x[\mathbf{card}(x) = n \wedge P(x) \wedge Q(x)]$

b. $[\![\varnothing_{\text{Gen}}]\!]^w = \lambda n_d \,.\, \lambda P_{et} \,.\, \lambda Q_{et} \,.\, \forall_{\text{Gen}} x[\mathbf{card}(x) = n \wedge P(x) \rightarrow Q(x)]$

Now, we will have LFs like the following, where *between five and ten* raises, creating a predicate of degrees in which the degree trace n combines directly with the silent determiner (i.e. n is not shifted to the intersective type $n_{\texttt{isCard}}$).

(240) [between five and ten] [λn [[[$\varnothing$ n] NP] VP]]

Crucially, it is not possible for *between five and ten* to scope below the silent determiner, simply because the determiner needs to combine with an expression of type d, e.g. the degree trace created by movement of *between five and ten*. That is, an LF of the kind in (238), repeated below, is uninterpretable ($\varnothing$, which now requires an argument of type d, is combining with an expression of type *et*, namely the intersection of the numerical AP and the NP):

(241) [∅ [[λx [[between five and ten] [λn [x $n_{\texttt{isCard}}$]]]] NP]] VP

Thus, if we adopt the lexical entries for the determiners in (239), then LMax_{inf} is able not only to generate the readings we want for *between* sentences, but also to do so without any unwanted ambiguity, hence without any appeal to a pragmatic blocking mechanism.

While this move seems quite desirable at this point, I will give some potential independent evidence in the next chapter, based on the infelicity of sentences with *more than* and upward scalar predicates, that LMax_{inf} may in fact need to be supplemented with some kind of a pragmatic blocking mechanism. If that is true, then it may not really matter if we adopt the entries in (239) or the original ones: if we adopt the original ones, then the same blocking mechanism that LMax_{inf} may need for sentences with *more than* and upward scalar predicates might be able to be used to block the unattested readings generated by scope ambiguity. I will also discuss this issue in more detail at the end of the thesis, in chapter 8.

The important point now is that LMax_{inf} is our best candidate yet. If we adopt the entries in (239), then it predicts no ambiguity (that is, it avoids Van Benthem's problem in distributive contexts, and it also does not derive upper-bounded readings in non-distributive contexts), and it derives the intuitively correct truth conditions regardless of whether the modified numeral combines with a downward, upward, or non-scalar predicate. However, we will now see that a number of complications arise when we move from *between* to *less than*. The main problem has to do with cases where the relevant predicates (e.g. *smile*, *surround the castle*, and *can lift the piano*) have empty extensions. I will first present these complications, and then I will sketch some potential ways to resolve them.

6.3 *Less than* and empty degree predicates

Let us now turn to the case of *less than*, which has the lexical entry in (242), modeled on the one for *between* in (207).

(242) $[\![\text{less than}]\!]^w = \lambda n_d \,.\, \lambda \mathcal{P}_{s(dt)} \,.\, \exists k[k < n \wedge \max_{\leq_{\mathcal{P}}}(\mathcal{P}(w))(k)]$

Now suppose that, in a world w, no students at all smiled. Then, as discussed in chapter 1 and chapter 2, we want a sentence like (243) to come out true in w.

(243) a. Less than three students smiled.

b. [less than three] [λn [[$\varnothing_\exists$ [$n_{\texttt{isCard}}$ students]] smiled]]

What does LMax_{inf} deliver for this case? Based on the lexical entry for *less than* in (242) and the definition of max in (202), the sentence is predicted to be true in a world w just in case there is a number k, with $k < 3$, such that k is in the extension of the degree predicate

(244) λn [[$\varnothing_\exists$ [$n_{\texttt{isCard}}$ students]] smiled]

in w (call it $\mathcal{P}(w)$ for short), and k is maximally informative relative to $\mathcal{P}$. Given our assumptions, however, if no students smiled in w, then $\mathcal{P}(w)$ is empty. This is because of our ontological assumption that there is no individual x such that $\mathbf{card}(x) = 0$, hence no individual x such that the cardinality predicate $0_{\texttt{isCard}}$ holds of x: an individual always has at least one atomic member as part. As a result of $\mathcal{P}(w)$ being empty, *no* number k in $\mathcal{P}(w)$ is maximally informative, not even 0, and so the sentence is predicted to be false.

This unwelcome result is in fact exactly the same as the result we encountered on the LMax account by using the definition of max in (64) (see chapter 2, especially section 2.3.3).[19] So, just as we did in chapter 2 (section 2.3.3), we could again stipulate

[19] This is not surprising since, again, $\max_{\leq}$ and $\max_{\leq_{\mathcal{P}}}$ are equivalent when $\mathcal{P}$ is downward scalar (see Fact 1).

that if a set D is empty, then the number 0 counts as maximal in D. We would then have the following definition for max, which is precisely the one we settled on for LMax and SMax (cf. (81)).

(245) For any degree predicate P_{dt}, any partial order $\leq$ over P, and any degree n,

$$\max_{\leq}(P)(n) = \begin{cases} 1 & \text{if } P(n) \wedge \neg\exists m[P(m) \wedge m > n] \\ 1 & \text{if } n = 0 \wedge \neg\exists m P(m) \\ 0 & \text{otherwise} \end{cases}$$

Using this definition for max in the lexical entry for *less than* in (242), we now correctly predict that (243) is true in w if no students smiled in w: in this case, there is no m such that $\mathcal{P}(w)(m)$, which means there *is* a $k < 3$ such that $\max_{\leq_{\mathcal{P}}}(\mathcal{P}(w))(k)$, namely $k = 0$, because $\max_{\leq_{\mathcal{P}}}(\mathcal{P}(w))(0)$ is true here.

However, this move now has an undesirable outcome for non-distributive predicates. Sentence (246), for example, is predicted to be true in a world where no soldiers surrounded the castle.

(246) a. Less than ten soldiers surrounded the castle.

b. [less than ten] [λn [[$\varnothing_{\exists}$ [$n_{\texttt{isCard}}$ soldiers]] [surrounded the castle]]]

That is, (246) is predicted to be true just in case either no soldiers surrounded the castle or some group of less than ten soldiers did (which is compatible with another, larger group of soldiers having done so too). However, as I argued in chapter 1, (246) licenses an existential inference, viz. that some soldier(s) surrounded the castle.

Now, we have seen that LMax faces a very similar problem: under the LF in (246), LMax derives an upper-bounded ('maximal') reading, which I argued (in section 3.4) is unattested but nevertheless not blocked. The way that LMax generates the attested, non-maximal, existential reading is by scoping the modified numeral below the silent determiner. Thus, while LMax is able to generate the right reading (via narrow scope of the modified numeral), it incorrectly predicts a genuine ambiguity where there is none.

Similarly, the LMax_{inf} account actually *could* derive the existential reading if we simply allowed *less than ten* to scope below $\varnothing_\exists$ (see section 6.2.5). However, we would still *also* derive an unattested, non-existential reading. That is, just like with LMax, we would incorrectly predict a genuine ambiguity where there is none.

Moreover, we now derive the wrong reading for a sentence like (247): it is predicted to be true in a world where no (number of) boys can lift the piano (it's far too heavy, say).

(247) a. Less than five boys can lift this piano.

b. [less than five] [λn [[$\varnothing_{\text{Gen}}$ [$n_{\texttt{isCard}}$ boys]] [can lift this piano]]]

That is, (247) is predicted to be true if either no boys can lift this piano or some number of boys less than five can lift it. However, as I argued in chapter 5, (247) licenses a lower-bound inference, viz. that some number of boys can lift this piano.[20] In this case, and unlike in the previous case of (246), appealing to scope ambiguity would not help to derive the right reading: scoping *less than five* below $\varnothing_{\text{Gen}}$ would result in the reading, 'In general, any group of less than five boys can lift this piano,' which is strictly stronger than, and hence does not correspond to, the most salient reading of (247) under discussion here. (As we saw in chapter 5, LMax generates this reading too, which is then blocked by the pragmatic economy constraint.)

We thus face the following dilemma: the data in (243) (with *smiled*) strongly suggests that we modify our definition of max along the lines of (245), while the data in (246) (with *surrounded the castle*) and, especially, (247) (with *can lift this piano*) strongly suggest we retain the definition in (202). It is unclear to me that the modified version of max in (245) can ever be made to work in a way that derives the lower-bound entailments we want in

[20] This is why, for example, the sentences *Less than five babies can lift this piano* and *Less than ten eggs are sufficient to build a house* feel false, not true.

upward scalar cases. However, it does seem possible, at least in principle, to make the version of max in (202) work in such a way that lower-bound entailments are *not* derived in downward scalar cases. In the next section, I sketch how that might work based on the introduction of a null individual into the semantic ontology and some additional stipulations regarding the denotations of distributive and non-distributive predicates. Afterwards, I discuss an alternative route, in which max is redefined in an even more fine-grained way than in (245).

6.3.1 Introducing the null individual

We may try to solve the dilemma just observed by reverting back to the original definition of max in (202) and introducing a *null individual* into our semantic ontology. I sketch here what I take to be a conceivable approach to the problem, but point out that there are non-trivial holes to be filled in for such an approach to work (beyond the fact that it is also quite stipulative).

First, instead of assuming that the set of pluralities in our ontology is a join-semilattice, we now take it to be a complete lattice, i.e. as having a bottom element, which we call the *null individual*, notated by **0**. (See Landman 2004 for another proposal that incorporates the null individual into the ontology.) The null individual is a part of every individual,[21] and the sum consisting of the null individual and any individual x is simply x.[22] Crucially, **0** does not have any part distinct from itself,[23] but is also not itself an atomic individual,[24] so that we have $\mathbf{card}(\mathbf{0}) = 0$.[25]

[21] $\forall x[\mathbf{0} \sqsubseteq x]$.

[22] $\forall x[\mathbf{0} \sqcup x = x]$.

[23] $\forall x[x \sqsubseteq \mathbf{0} \rightarrow x = \mathbf{0}]$.

[24] The notion of *atom* (see footnote 19, chapter 1) must be redefined so that an atom is a non-null individual whose only proper subpart is the null individual: $\forall x[\mathbf{atom}(x) \leftrightarrow x \neq \mathbf{0} \wedge \forall y[y \sqsubseteq x \rightarrow [y = x \vee y = \mathbf{0}]]]$.

[25] The set of pluralities is now isomorphic to the power set of the set of atoms, where the null individual ($\mathbf{0}$) is identified with the empty set ($\varnothing$), an atomic individual is identified with a singleton set, and the *part of* relation ($\sqsubseteq$) is identified with set-theoretic inclusion ($\subseteq$). This is because the power set of a set defines a complete lattice, where the ordering (*part of*) relation is set-theoretic inclusion ($\subseteq$), the *join* of two sets is their union ($\cup$), and the *meet* of two sets is their intersection ($\cap$).

Second, we now assume that predicates that license downward inferences on their arguments, such as *smile* and *gather*, necessarily include the null individual in their extension (i.e. the null individual is in their extension in every world), and that predicates that do not license downward inferences on their argument, such as *surround the castle* and *can lift this piano* (in a generalizing sentence), never have the null individual in their extension.

Finally, pluralized nouns, such as *students* and *soldiers*, but not singular nouns, also necessarily have the null individual in their extension.

As I will discuss shortly, it is not obvious how to derive all of these outcomes in a way that is not entirely *ad hoc* and thus reduces the conceptual appeal of this version of the LMax_{inf} account. But let us put this aside for the moment.

We also need to modify the lexical entries of quantifiers like *some* and *no*.

(248) a. $[\![\text{some}]\!]^w = \lambda P_{et} . \lambda Q_{et} . \exists x[x \neq \mathbf{0} \wedge P(x) \wedge Q(x)]$

b. $[\![\text{no}]\!]^w = \lambda P_{et} . \lambda Q_{et} . \neg\exists x[x \neq \mathbf{0} \wedge P(x) \wedge Q(x)]$

With all this in place, keeping exactly the same lexical entries as before for numerals, numeral modifiers, `isCard`, $\varnothing_{\exists}$, and $\varnothing_{\text{Gen}}$, we now get what we want. Consider again (243), repeated in (249).

(249) a. Less than three students smiled.

b. [less than three] [λn [[$\varnothing_{\exists}$ [$n_{\texttt{isCard}}$ students]] smiled]]

Now, if no students smiled in w, then the following expression is true in w

(250) [$\varnothing_{\exists}$ [$0_{\texttt{isCard}}$ students]] smiled

because $\mathbf{0}$ is, *ex hypothesi*, in the extensions of both *students* and *smiled* in w. Therefore, 0 has the property (call it $\mathcal{P}$) denoted by

(251) $[\lambda n\ [[\varnothing_{\exists}\ [n_{\texttt{isCard}}$ students]] smiled]]

in w, i.e. $\mathcal{P}(w)(0)$ is true. Finally, 0 is the only number in $\mathcal{P}(w)$, and is thus a maximal element of $\mathcal{P}(w)$ relative to $\mathcal{P}$, which thereby makes the whole sentence true because $0 < 3$.

Importantly, introducing the null individual does not disrupt cases where a non-zero number of students smiled. If, say, two students smiled in w, then $\mathcal{P}(w)$ would be $\{0,1,2\}$, and the sentence would again be true, but this time because 2 is a maximal element of $\mathcal{P}(w)$ (the others are not), and $2 < 3$. And if, say, ten students smiled in w, then $\mathcal{P}(w)$ would be the interval $[0,10]$; in this case, the sentence would be false because none of the numbers less than 3 in that interval are maximal (only 10 is $\mathcal{P}$-maximal in $[0,10]$, but $10 \nless 3$). As a result, (249) is correctly predicted to mean that zero, one, or two students smiled, and no more than that.

By contrast, things will be different for a case like (246), repeated in (252).

(252) a. Less than ten soldiers surrounded the castle.

b. [less than ten] $[\lambda n\ [[\varnothing_{\exists}\ [n_{\texttt{isCard}}$ soldiers]] [surrounded the castle]]]

If no (group of) soldiers surrounded the castle in w, then, since **0** is, *ex hypothesi*, not in the extension of *surrounded the castle* in w, the following expression is false in w

(253) $[\varnothing_{\exists}\ [0_{\texttt{isCard}}$ soldiers]] [surrounded the castle]

and so no number has the property (call it $\mathcal{P}$) denoted by

(254) $\lambda n\ [[\varnothing_{\exists}\ [n_{\texttt{isCard}}$ soldiers]] [surrounded the castle]]

in w, i.e. $\mathcal{P}(w) = \varnothing$. As a result, no number is maximal in $\mathcal{P}(w)$, and so the existential statement introduced by *less than ten* is false.

Similarly for (247), repeated in (255).

(255) a. Less than five boys can lift this piano.

b. [less than five] [λn [[$\varnothing_{\text{Gen}}$ [$n_{\texttt{isCard}}$ boys]] [can lift this piano]]]

If no number of boys can lift this piano in w, then, since **0** is, *ex hypothesi*, not in the extension of *can lift this piano* in w, the following expression is false in w

(256) [$\varnothing_{\text{Gen}}$ [$0_{\texttt{isCard}}$ boys]] [can lift this piano]

and so no number has the property denoted by

(257) λn [[$\varnothing_{\text{Gen}}$ [$n_{\texttt{isCard}}$ boys]] [can lift this piano]]

in w. As a result, the existential statement introduced by *less than ten* is false.

6.3.2 Shortcomings of the null individual approach

While an approach based on the introduction of a null individual may seem like the beginning of a solution, it is important to see that there is no straightforward way to derive what we want by minimally modifying a standard plural semantics framework. Let us for the moment ignore the fact that certain collective predicates like *gather* and *fit* license downward inferences and hence pattern like distributive predicates as far as modified numeral constructions go (see section 3.2); that is, let us focus for now just on 'pure' distributive predicates like *smile*. Then, on the one hand, we need an operation which, starting from a 'basic' denotation for *smile*, which includes only atomic individuals, returns the distributive (in the sense of (21)[26]) predicate *smile*′, whose denotation now includes the null individual and every plurality whose atomic parts are all in the extension of *smile*. On the other hand, we need to *prevent* the null individual from entering the

[26]According to (21), P_{et} is distributive iff $\forall x, y[P(x) \wedge y \sqsubseteq x \rightarrow P(y)]$.

denotations of predicates like *surround the castle* and (in generic contexts) *can lift this piano*.

It is clear that the so-called *star operator* (Link 1983), which is responsible for making predicates like *students* and *smiled* both distributive in the sense of (21) and cumulative in the sense of (33)[27] (see chapter 1), cannot be the operator that is also responsible for introducing the null individual into their extensions. The reason is because nothing would prevent the star operator from also applying to predicates like *surrounded the castle*. In fact, presumably the star operator *should* (and does) apply to such predicates, because they too have cumulative reference: if the soldiers from army #1 surrounded the castle, and the soldiers from a different army #2 surrounded the castle, then it seems true to say that the soldiers from armies #1 and #2 surrounded the castle, which suggests that a sentences like (252), repeated in (258), involves a starred version of *surrounded the castle*. If the star operator were to introduce the null individual, then (258) would incorrectly be predicted to mean that either no (group of) soldiers surrounded the castle, or some group of less than ten did (just like on the LMax_{inf} account with the revised version of max in (245)).

(258) Less than ten soldiers surrounded the castle.

Now, maybe the null individual is introduced by a special distributivity operator, Δ, which applies to a predicate like *smiled* and returns the set of all sums each of whose atoms smiled, as well as the null individual.

(259) $[\![\Delta]\!]^w = \lambda P_{et} \, . \, \lambda x_e \, . \, x = \mathbf{0} \vee \forall y[y \sqsubseteq x \wedge \mathbf{atom}(y) \rightarrow P(y)]$

Then the expression *smiled* in (249), repeated in (260), would be parsed as Δ *smiled* (and *students* would be Δ *student*), which would be distributive and also include the null individual; hence, (260) would correctly be assigned an upper-bounded reading with no lower-bound entailment.

[27] According to (33), P_{et} is cumulative iff $\forall x, y[P(x) \wedge P(y) \rightarrow P(x \sqcup y)]$.

(260) Less than three students smiled.

Crucially, if Δ were to apply to *surrounded the castle*, which necessarily (in all worlds) includes only non-atomic sums in its extension, then it would return the singleton set containing the null individual: $\{\mathbf{0}\}$. As a result, a sentence like (258) would end up as a tautology. Conversely, a sentence like *Between five and ten soldiers surrounded the castle* (with Δ applying to the verb phrase) would end up as a contradiction. Presumably, such readings could simply be ruled out on the grounds that they are either tautologies or contradictions (Gajewski 2003), and so the only available reading for (258) would be one where Δ does not apply. Alternatively, we could formulate a more syntactic account of the distribution of Δ, e.g. by assuming that predicates like *smiled*, but not *surrounded the castle*, are specified for a feature requiring that they occur in the scope of Δ.

The problem with an approach involving a distributivity operator becomes clear when we recall now that the collective predicates *gather*, *fit*, and all the others discussed in section 3.2 pattern like *smile* when it comes to the relevant modified numeral constructions (we only get maximal readings). Applying Δ to *gather*, for example, could never work: presumably, *gather* (just like *surrounded the castle*) necessarily includes only non-atomic sums in its extension, and so applying Δ to it would always result in the degenerate set $\{\mathbf{0}\}$, which in turn results in a tautology in the case of a sentence like *Less than 50 protesters gathered* (and a contradiction in the case of a sentence like *Between 50 and 100 protesters gathered*.)

In sum, there does not seem to be any straightforward way of guaranteeing that the null individual is in the extension of predicates like *smile* and *gather*, while also preventing it from being in the extension of predicates like *surrounded the castle* and *can lift this piano*, short of stipulating that that's how things are.

6.3.3 A more fine-grained definition of maximality

Instead of introducing a null individual, one other thing we could do to try and solve the dilemma (and the last attempt I give here) is to redefine max in an even more fine-grained way. What we want is for 0 to count as maximal, relative to a set D, only when D is both empty *and* the extension of a downward scalar degree predicate. Of course, max, as we currently view it, takes only two basic arguments: a partially ordered set D and a number n. It has no access to the scalarity of the degree predicate which is responsible for the partial order. What we can do, however, is define instead an *intensional* maximality component, which I will write imax, that *does* have access to the scalarity of the degree property that it combines with. In a nutshell, imax will take as arguments a degree property $\mathcal{P}$, a world w, and a number n and say that n is maximal in $\mathcal{P}(w)$ relative to the informativity-based ordering induced by $\mathcal{P}$ iff certain conditions are met.

I first define imax in a way that makes it completely analogous to the first (i.e. unrevised) definition of max given in (202), which I repeat below for ease of comparison. The only difference is that imax is specifically informativity-based, whereas max is generalized to any partial ordering.

(261) **Non-intensional max**

For any degree predicate P_{dt}, any partial ordering $\leq$ over P, and any degree n, $\text{max}_{\leq}(P)(n)$ iff $P(n) \wedge \neg\exists m[P(m) \wedge m > n]$.

(262) **Intensional max (to be revised)**

For any degree property $\mathcal{P}_{s(dt)}$, any world w, and any degree n, $\text{imax}(P)(w)(n)$ iff $P(w)(n) \wedge \neg\exists m[P(w)(m) \wedge m >_{\mathcal{P}} n]$.

This gives us the following equivalence:

(263) For any degree property $\mathcal{P}_{s(dt)}$, world w, and number n, $\text{max}_{\leq_{\mathcal{P}}}(\mathcal{P}(w))(n)$ iff

imax($\mathcal{P}$)(w)(n).

As a result, this version of imax will work just as well as max when it comes to *between* sentences, but will also inherit the same problems as max when it comes to *less than* sentences.

However, we can now slightly revise imax to make a special case for 0 when $\mathcal{P}(w)$ is empty *and* $\mathcal{P}$ is downward scalar (as defined in (196)).

(264) **Intensional max (final version)**

For any degree property $\mathcal{P}_{s(dt)}$, any world w, and any degree n,

$$\text{imax}(\mathcal{P})(w)(n) = \begin{cases} 1 & \text{if } \mathcal{P}(w)(n) \wedge \neg\exists m[\mathcal{P}(w)(m) \wedge m >_{\mathcal{P}} n] \\ 1 & \text{if } n = 0 \wedge \textbf{DS}(\mathcal{P}) \wedge \neg\exists m \mathcal{P}(m) \\ 0 & \text{otherwise} \end{cases}$$

We must also revise our lexical entries for *less than* and *between*.[28]

(265) $[\![\text{less than}]\!]^w = \lambda n_d \,.\, \lambda \mathcal{P}_{s(dt)} \,.\, \exists k[k < n \wedge \text{imax}(\mathcal{P})(w)(k)]$

(266) $[\![\text{between}]\!]^w = \lambda m_d \,.\, \lambda n_d \,.\, \lambda \mathcal{P}_{s(dt)} \,.\, \exists k[m \leq k \leq n \wedge \text{imax}(\mathcal{P})(w)(k)]$

With all this in place, imax will behave precisely as we want no matter whether the predicate is downward scalar, upward scalar, or non-scalar.

Now, this solution to the dilemma may look quite stipulative upon first glance. However, it seems to me that it is no less stipulative than the original revision we had to make for max in the context of the LMax account (cf. (81), chapter 2). In other words, to whatever extent this trick is *ad hoc*, it is no more so than the trick we already employed:

[28] One might think it unnecessary to revise the entry for *between*, but there are a few reasons to do so: (i) to maintain uniformity between *less than* and *between*, (ii) to cut down on the number of operations our semantics must refer to (just imax instead of imax and max), and perhaps most importantly, (iii) to handle the special case of *between zero and n*: for instance, *Between zero and five students smiled* is true even if no students smiled.

for some puzzling reason, the grammar simply makes a special case for 0 when it comes to certain types of *less than* sentences. Given this fact, and given that LMax_{inf} has so many other merits, I conclude that the 'zero problem' for LMax_{inf} is no reason to abandon LMax_{inf}, and that LMax_{inf} is indeed the best theory we have seen so far.

6.4 Summary

In this chapter, I developed an account of modified numerals, LMax_{inf}, in which expressions like *between* and *less than* lexically encode a maximality component that operates on an informativity-based ordering of degrees (induced by the intension of the degree predicate that the modified numeral combines with), rather than on the natural ordering of degrees. I showed that LMax_{inf} does a very good job at capturing the fact that predicates that license downward inferences lead to 'maximal' (upper-bounded) readings, predicates that license upward inferences lead to 'minimal' (lower-bounded) readings, and predicates that license neither downward nor upward inferences lead to non-maximal, non-minimal readings.

Moreover, LMax_{inf} is able to generate such readings using just one type of LF, i.e. without resorting to scope ambiguity and without positing any separate, optional maximization operation. Thus, the account does not seem to overgenerate. This means, in particular, that LMax_{inf}, unlike both LMax and SMax, does not generate non-maximal readings in distributive contexts (which LMax and SMax do generate and must explain by appealing to a pragmatic economy constraint), nor does it generate maximal readings in collective contexts (which LMax and SMax do generate and which the pragmatic economy constraint does not block).

All that being said, LMax_{inf} faces a serious challenge when it comes to *less than* and degree predicates with an empty extension. Without some extra stipulations, it seems unable to capture the fact that we sometimes get lower-bound entailments (in upward scalar and non-scalar contexts), and other times do not (in downward scalar contexts). I gave a couple ways this complication might be resolved, including one in particular (refining the definition of what counts as maximal) that seems to work and which, I argued, is no less stipulative than what we already had to do for LMax and SMax.

In sum, despite this one shortcoming, LMax_{inf} fares much better than both LMax and SMax.

Chapter 7

Against severing informativity-based maximality

7.1 Overview

In the last chapter, I developed a theory (Lexical Informativity-based Maximality, or LMax_{inf}) based on the idea that numeral modifiers like *less than* and *between* lexically encode a maximality component that operates an informativity-based ordering of degrees, rather than on the natural ordering of degrees. I showed that LMax_{inf} has a large number of advantages over the other two accounts discussed so far (LMax and SMax) and is, therefore, the best candidate on the table for a theory of numeral modifiers like *less than* and *between*. However, in the same way that we had the choice of severing 'standard' maximality from the lexical meanings of numeral modifiers (i.e. in the move from LMax to SMax), we now have the choice of severing maximal informativity from the lexical meanings of numeral modifiers. That is, there is a fourth theory to consider, which I will call *Separate Informativity-based Maximality* (SMax_{inf}, for short). Investigating SMax_{inf} is important because it could lead to three possible outcomes: (i) SMax_{inf} fares strictly better than LMax_{inf}, (ii) SMax_{inf} fares strictly worse than LMax_{inf}, and (iii) SMax_{inf} and

LMax_{inf} are on a par (either they make the same predictions or each has some advantages over the other). I will argue, based on data involving *more than* and upward scalar degree predicates (*can lift this piano*) that SMax_{inf} fares strictly worse than LMax_{inf}, and that LMax_{inf} is therefore the most successful and favorable theory.

7.2 Severing maximal informativity from numeral modifiers

Much like a theory based on standard maximality can assume either that the maximality component is or is not part of the intrinsic, lexical meaning of numeral modifiers, the same choice arises for a theory based on maximal informativity. Instead of locating the maximality component in the meaning of numeral modifiers, there is an alternative where it is part of the meaning of numerical variables. In this section, I spell out what exactly such a theory might look like, which I call SMax_{inf}.

We can proceed in a way analogous to what we did in chapter 4 for SMax, where we posited an operator `isMax`, except that now the partial ordering that the maximality component of `isMax` operates on is informativity-based. For this reason, we need to make the operator intensional, i.e. its second argument is now the *intension* of a degree predicate, which will be responsible for determining the informativity-based ordering.[1]

(267) $[\![\texttt{isMax}]\!]^w = \lambda n_d \,.\, \lambda \mathcal{P}_{s(dt)} \,.\, \max_{\leq_{\mathcal{P}}}(\mathcal{P}(w))(n)$

As usual, we write $n_{\texttt{isMax}}$ for `isMax` n:

[1] As usual, there is a choice here between a version of max that makes a special stipulation about 0 and empty degree predicates and a version that does not (see chapter 2, section 2.3.3, and especially the last chapter, section 6.3). The choice here is not inconsequential: the same problems that plague LMax_{inf} regarding *less than* and empty extensions of degree predicates also plague SMax_{inf}. For concreteness, we could assume that `isMax` encodes the fine-grained version imax from section 6.3.3, since that version seems to work without problems (and without introducing a null individual). However, since this chapter focuses mainly on *between* and *more than*, I remain agnostic about which version of max is operative, and how those issues are dealt with, and simply use our normal max, with an informativity-based ordering.

(268) $[\![n_{\texttt{isMax}}]\!]^w = [\![\texttt{isMax}\ n]\!]^w = \lambda\mathcal{P}_{s(dt)}\,.\, \max_{\leq_\mathcal{P}}(\mathcal{P}(w))([\![n]\!]^w)$

Numerical variables of the form $n_{\texttt{isMax}}$ are thus treated as intensional operators, which can combine with expressions of type dt thanks to the rule of intensional function application introduced in the last chapter (see (213)).

As for numeral modifiers, we now revert back to basic, existential lexical entries, just as we did in chapter 4 (see (111) and (112)).

(269) $[\![\text{less than}]\!]^w = \lambda n_d\,.\,\lambda P_{dt}\,.\,\exists k[k < n \wedge P(k)]$

(270) $[\![\text{between}]\!]^w = \lambda m_d\,.\,\lambda n_d\,.\,\lambda P_{dt}\,.\,\exists k[m \leq k \leq n \wedge P(k)]$

With all this in place, the following three sentences are assigned the right truth conditions under the LFs provided.[2]

(271) a. Between five and ten students smiled.

b. [between 5 and 10] [λn [$n_{\texttt{isMax}}$ [λm [[$\varnothing_\exists$ [$m_{\texttt{isCard}}$ students]] smiled]]]]

(272) a. Between five and ten boys can lift this piano.

b. [between 5 and 10] [λn [$n_{\texttt{isMax}}$ [λm [[$\varnothing_{\text{Gen}}$ [$m_{\texttt{isCard}}$ boys]] [can lift . . .]]]]]

(273) a. Between five and ten soldiers surrounded the castle.

b. [bw. 5 and 10] [λn [$n_{\texttt{isMax}}$ [λm [[$\varnothing_\exists$ [$m_{\texttt{isCard}}$ soldiers]] [surrounded . . .]]]]]

These three LFs are identical up to the choice of predicate and choice of silent determiner, but of course differ in the scalarity of their innermost degree predicate. Let us now think about the outermost degree predicates, one by one.

First, in (271), the degree predicate

(274) λn [$n_{\texttt{isMax}}$ [λm [[$\varnothing_\exists$ [$m_{\texttt{isCard}}$ students]] smiled]]]

[2]The corresponding sentences with *less than* are also assigned the right truth conditions, putting aside the special case where the relevant degree predicates have an empty extension (see footnote 1).

denotes the property that a number *n* has if *n* students smiled and the proposition that *n* students smiled is not entailed by any true proposition of the same form. The only *n* that can have such a property is the *largest* (i.e. total) number of students who smiled. As in chapter 6, the fact that the (innermost) degree predicate is downward scalar is crucial to getting this result (because of the role played by entailment). As a result, the sentence means that the total number of students who smiled is between five and ten.

In (272), with the predicate *can lift this piano*, the degree predicate

(275) λn [$n_{\texttt{isMax}}$ [λm [[$\varnothing_{\text{Gen}}$ [$m_{\texttt{isCard}}$ boys]] [can lift this piano]]]]

denotes the property that a number *n* has if *n* boys can lift this piano and the proposition that *n* boys can lift this piano is not entailed by any true proposition of the same form. In this case, the only *n* that can have such a property is the *smallest* number of boys who can lift this piano. As before, the fact that the (innermost) degree predicate is upward scalar is crucial to getting this result. As a result, the sentence means that the smallest number of boys who can lift this piano is between five and ten.

Finally, in (273), with the collective predicate *surrounded the castle*, the condition that cares about entailment is vacuous (because the (innermost) degree predicate non-scalar), and so we get the basic existential, non-upper-bounded meaning that a group of five to ten soldiers surrounded the castle.

Now, in addition to these LFs, there are three more LFs available, where `isMax` is not used. These LFs, and their associated truth conditions, are exactly the same as the ones that SMax generates by not applying maximization (see chapter 4). They always give rise to non-upper-bounded readings. The pragmatic economy constraint, which SMax_{inf} (like SMax) clearly must appeal to, rules out these readings whenever necessary, e.g. in distributive contexts (Van Benthem's problem; see chapter 2), and in the case of *between* plus an upward scalar predicate (see chapter 5).

Now, are there reasons to prefer this approach, SMax_{inf}, over the LMax_{inf} approach?

The differences in predictions between LMax and SMax that I discussed in section 4.4, which pertained to cases where SMax predicts additional scope ambiguities ('split scope'), carry over to the comparison between LMax_{inf} and SMax_{inf}. However, as I argued there, introspective judgments about those cases are, in my view, too subtle to conclusively say one way or another whether maximality ought to be severed.

That being said, an interesting question that immediately arises is whether there are split-scope data that would provide evidence in favor of SMax_{inf} over SMax (and hence also over LMax_{inf}). Such data would involve LFs with the following kind of scope order:

between 10 and 20 > Op > `isMax` > upward scalar predicate

On the SMax_{inf} account, we would expect readings like, 'There is a number n, with $10 \leq n \leq 20$, such that it is required/forbidden (etc.) that n be the *minimum* number such that . . . ' However, the intervening operator apparently cannot be deontic, because (i) upward scalar predicates seem to always involve an ability modal *can* (or *sufficient*), and (ii) it makes no sense to impose a requirement/prohibition on someone or something's *ability* to do something: *??Between 10 and 20 boys are forbidden to be able to lift this piano together*, *??Between 5 and 10 eggs are forbidden to be sufficient to make an omelet for these guests*. Moreover, even if we found such data, I suspect that introspective judgments would be even less reliable than in the cases we looked at in section 4.4. For these reasons, I am skeptical that split-scope data can be used to support the SMax_{inf} account.

So it seems to me that SMax_{inf} not only fails to have any clear advantage over LMax_{inf}, it also inherits the overgeneration problems of SMax, which means it must appeal to pragmatic blocking. For these reasons alone, it seems best not to adopt SMax_{inf}. However, in the next section, I will give one last definitive argument against SMax_{inf}, which involves the interpretation of *more than* in upward scalar contexts.

7.3 Upward scalar predicates and the case of *more than*

Consider the following sentences.

(276) a. More than five boys can lift this piano.

b. More than ten eggs are sufficient to bake a cake for all these guests.

c. You can pass this class with a final grade of more than 60%.

On the SMax_{inf} account, the following LFs should be available:[3]

(277) a. [more than five] [λn [$n_{\texttt{isMax}}$ [λm [m boys can lift this piano]]]]

b. [more than ten] [λn [$n_{\texttt{isMax}}$ [λm [m eggs are sufficient ...]]]]

c. [more than 40] [λn [$n_{\texttt{isMax}}$ [λm [you can pass this class with m%]]]]

Let us assume the following lexical entry for *more than*, modeled on *less than*, but with < replaced by >.

(278) $[\![\text{more than}]\!]^w = \lambda n_d \,.\, \lambda P_{dt} \,.\, \exists k[k > n \wedge P(k)]$

Now, assuming that the degree predicates in these LFs are all upward scalar, the predicted readings for these sentences (under these LFs) are the following.

(279) a. The smallest number k such that k boys can lift this piano is greater than five.

b. The smallest number k such that k eggs are sufficient to bake a cake for all these guests is greater than ten.

c. The smallest number k such that you can pass this class with a final grade of k% is more than 40.

[3] For simplicity, I am now glossing over the internal structure of the innermost degree predicate. The important point here is simply that these degree predicates are all upward scalar.

These readings, of course, express perfectly reasonable (that is, contingent) propositions, and they are not ruled out by our pragmatic economy constraint (which, to repeat, SMax_{inf} must retain). However, these sentences do not appear to have these readings. For example, (276a) does not seem to entail that five (or fewer) boys *cannot* lift this piano; (276b) does not seem to entail that ten (or fewer) eggs are *insufficient*; and (276c) does not seem to entail that you *cannot* pass with a 60% (or less).

In this case, LMax_{inf} fares better than SMax_{inf}. This is because nothing forces the LMax_{inf} approach to provide parallel lexical entries to *less than* and *more than*. Thus, while on the LMax_{inf} account, the lexical entry for *less than* must involve maximal informativity (see chapter 6), the entry for *more than* need not involve maximality at all. That is, LMax_{inf} could entertain the following lexical entry for *more than*:

(280) $[\![\text{more than}]\!] = \lambda n_d \,.\, \lambda P_{dt} \,.\, \exists k[k > n \wedge P(k)]$

This, of course, is the exact same entry we just saw for SMax_{inf}, but the point now is that LMax_{inf}, unlike SMax_{inf}, does not have a separate maximality component available, which means that the LFs above are unavailable, leaving only the following LFs, which involve no maximality.

(281) a. [more than five] [λn [n boys can lift this piano]]

b. [more than ten] [λn [n eggs are sufficient ...]]

c. [more than 40] [λn [you can pass this class with n%]]

A little bit of reasoning will show that these LFs correspond to the following very weak readings.

(282) a. There is a number k such that k boys can lift this piano.

b. There is a number k such that k eggs are sufficient.

c. There is a number k such that you can pass this class with k%.

Let me use (281a) and (282a) to illustrate. The LF in (281a) is true in a world w just in case there is a number k, with $k > 5$, such that, in general, any group of k boys can lift this piano in w. Suppose now that there is some number m such that, in general, any group of m boys can lift this piano. Then, whether or not m is less than or greater than five, it follows that, for any $n > m$, n boys can also lift this piano, because the degree predicate *λn [n boys can lift this piano]* is upward scalar; therefore, it follows that for some $k > 5$, k boys can lift this piano. In other words, the sentence simply means that for some number k, k boys can lift this piano.

Now, it is unclear to me whether these sentences really have these weak readings, or if they just sound deviant altogether. If they have these weak readings, then LMax_{inf} is clearly in good shape. If not, then LMax_{inf} could avail itself of our pragmatic economy (which, up to now, it did not seem to need). It could block the weak reading under the LF in (281a) because replacing *five* with any other numeral (say, *four* or *six*) does not alter the predicted truth conditions. Thus, LMax_{inf}, supplemented with the pragmatic economy constraint, would have an explanation for the unacceptability of this sentence.

Likewise, it should be noted that both LMax and SMax predict these sentences to be deviant. Under LMax, either the LFs in (281) suffer from maximality failure, or, if the lexical entry in (280) is chosen, the pragmatic economy constraint is violated. Under SMax, we use the lexical entry in (280), and then there is a choice between the LFs in (281), which violate our pragmatic economy constraint, and the LFs in (277), which again result in maximality failure.

The take-home message here is that SMax_{inf} very clearly overgenerates. Whether or not the weak reading that LMax_{inf} generates is available seems to be much less of a problem; hence, I take these data to provide quite convincing support for LMax_{inf} over SMax_{inf}.

Importantly, I should also point out that the entry for *more than* in (280) also works exactly as we need for acceptable sentences like *More than three students smiled* and *More* *than ten soldiers surrounded the castle* on the LMax_{inf} approach (as well as LMax and SMax).

7.4 Summary

In this chapter, I presented the fourth and final theory of modified numerals to be discussed in this thesis (SMax_{inf}). It involves severing the maximality component, now viewed as maximal informativity, from the lexical meaning of numeral modifiers like *less than* and *between*. I argued that this move does not have any clear advantages and, moreover, that it has two major disadvantages. First, just like the SMax account, SMax_{inf} grammatically overgenerates readings (e.g. by not applying maximization in downward scalar contexts, thus leading to Van Benthem's problem all over again) and hence, unlike LMax_{inf}, must rely on a pragmatic blocking mechanism to filter out unattested readings. Second, it predicts a 'minimal' reading for sentences where *more than* combines with an upward scalar degree predicate—a reading which I argued to be unavailable. LMax_{inf} fares better in this regard because it can be made to parse such sentences without any maximal informativity component at all. The upshot, then, is that, of the four theories discussed in this thesis, LMax_{inf} comes out on top.

Chapter 8

Conclusion

8.1 The long and winding road

In this thesis, I presented an array of new and puzzling data indicating that the interpretation of modified numerals like *less than five* and *between two and four* is, on the one hand, extremely variable, yet on the other hand, also entirely predictable from the scalarity of the property of degrees they combine with. The road to an adequate theory of such modified numerals was long and involved a number of important but quite misleading paths, some of which I would like to highlight.

I started this thesis, in chapter 1, by pointing out the following contrast: when *less than five* combines with a distributive predicate, as in (283a), we get an upper-bounded (or maximal), non-existential reading, whereas when it combines with a collective predicate, as in (283b), we get an existential, non-upper-bounded reading.

(283) a. Less than five students smiled.
b. Less than five students lifted the piano.

This puzzle led us, in chapter 2, to believe that the meanings of numeral modifiers like *less than* and *between* involve some kind of maximality component, and that modified numerals may interact scopally with a silent existential determiner ($\varnothing_\exists$): when the modified numeral scopes above $\varnothing_\exists$, we derive maximal readings, and when it scopes below $\varnothing_\exists$, we derive existential readings. This theory, dubbed LMax, therefore seemed to solve the initial undergeneration problem, but it also then overgenerated, so we posited a pragmatic blocking mechanism to rule out certain unattested readings—but not all. For example, (283b) was incorrectly predicted to be ambiguous between an existential, non-upper-bounded reading (which is available) and an upper-bounded, non-existential reading (which is unavailable).

In chapter 4, I presented an alternative theory, SMax, in which maximality is 'severed' from the meanings of numeral modifiers. Setting aside the subtle 'split-scope' data (section 4.4), SMax was shown to derive the exact same readings as LMax for the initial, core data. The reason was that it made no difference whether a modified numeral scoped above or below $\varnothing_\exists$, because the existential quantifier (over degrees) contributed by the modified numeral would commute with the existential quantifier (over individuals) contributed by $\varnothing_\exists$.

This then led us, in chapter 5, to search for new data that would distinguish SMax from LMax, namely data that lacked such commutativity. Here, we first looked at generically interpreted sentences with *less than*, such as (284a), which I argued involve a silent generic determiner, $\varnothing_{\text{Gen}}$, which contributes quasi-universal quantification (hence, under SMax does not commute with modified numerals).

(284) a. Less than five people can carry that piano upstairs.
b. There is a number $n < 5$ such that, in general, any group of n people can carry that piano upstairs.

Indeed, it turned out that SMax was able to derive an available reading for (284a), given in (284b), which LMax was unable to derive. In particular, the scope trick that LMax relied on in the existential domain did not generalize to the generic domain: scoping the modified numeral below the generic determiner yielded a reading that was strictly stronger than the salient reading we were after. Thus, the fact that LMax worked well for the initial data was really just a fortunate accident.

However, once we moved to the analogous sentence with *between*, given in (285a), we discovered that the optional maximization trick that SMax relied on several times previously did not generalize: whether maximization applied or not, the derived reading was blocked. Worse yet, SMax was completely unable to derive the salient reading of (285a), given in (285b).

(285) a. Between three and five people can carry that piano upstairs.

b. The minimum number *n* such that, in general, any group of *n* people can carry that piano upstairs is between three and five.

Thus, the fact the SMax was able to derive the right readings in previous cases also seemed to just be a fortunate accident.

At this point, although we had ruled out both competing theories, we now had a new, more complete typology of readings: maximal, minimal, and neither. This revelation led, in chapter 6, to move from a 'standard' notion of maximality (where the ordering that maximality operates on is the natural one over degrees) to an 'informativity-based' notion of maximality. Despite some complications arising with *less than* and empty degree predicates (section 6.3), the theory that I developed, LMax_{inf}, is able to capture the full range of data *and* to do so without generating unwanted ambiguities, hence without needing to resort to any pragmatic blocking. (I also showed, in chapter 7, that severing maximal informativity from modified numerals leads to several problems of overgeneration and is therefore not to be preferred.)

8.2 Main lessons learned

One of the main lessons that we learn is that there is, in a sense, no truly ‘non-upward-monotone’ quantifiers, at least as far as comparative quantifiers go. For example, a quantifier like *less than five* is not, strictly speaking, downward monotone, but rather simply creates downward-entailing environments under extremely particular (though admittedly common) circumstances: namely, when it combines with a downward scalar degree predicate. When *less than five* combines with an upward scalar or non-scalar predicate, then it does not create a downward-entailing environment.

The crucial ingredient that makes a degree predicate downward scalar appears to be that the (nominal and verbal) predicates license downward inferences, i.e. inferences from groups to subgroups. This is the case not only with typical, ‘pure’ distributive predicates like *smile*, but also with certain non-distributive predicates like *gather* (section 3.2) and *(can) fit into that elevator*, regardless of whether the sentence has existential force (*Less than five students smiled/gathered*) or generic force (*Less than five people can fit into that elevator*). Thus, the ability of, say, *less than five* to create downward-entailing environments, license NPIs, and give rise to maximal readings is not tied specifically to distributivity, as we may have thought at the outset of chapter 1, nor to existential force, as we may have thought at the outset of chapter 5, but rather to the more general notion of downward scalarity, which crosscuts the distributive/collective and existential/generic distinctions.

Another lesson we learn, mentioned already above, is that to capture the full range of facts, it is not enough to count on variable scope or the variable presence of maximality: we need to move to an informativity-based notion of maximality. Thus, we find quite unexpected reference to informativity in (at least some) modified numerals.

Finally, based on the arguments I gave in favor of LMax_{inf} and against SMax_{inf}, it appears that reference to maximal informativity is quite highly lexically controlled.

8.3 The distribution of maximal informativity

As just mentioned, one thing we learn is that reference to maximal informativity is lexically controlled. First, there does not appear to be a separate, informativity-based maximality component available to the grammar. Second, some numeral modifiers, including *less than* and *between*, lexically encode this reference to informativity, while others, including *more than*, do not.

This view of things opens up several questions. For example, there is now a puzzle about why there is asymmetry between *more than*, which does not refer to maximal informativity, and *less than*, which does. If, as some have argued (see, e.g., Hackl 2000), the maximality component associated with comparative quantifiers is ultimately derived from a comparative morpheme (*-er*), and if *more* and *less* (or *fewer*) are both analyzed as being decomposed into an adjective (*many*, *little*/*few*) plus a comparative morpheme (*-er*), then we would expect *more than* and *less than* to not only both refer to maximality, but to refer to the same kind of maximality. Thus, we would not expect the asymmetry that we seem to have found evidence for.

Along the same lines, we might also expect maximal informativity to appear in full comparative constructions, where maximality is contributed by a comparative morpheme. For example, a sentence like (286a) is standardly analyzed in a way that derives the meaning paraphrased in (286b).

(286) a. Fewer students than professors smiled.

b. The *maximum* (total) number of students who smiled is less than the *maximum* (total) number of professors who smiled.

If maximality here were maximal informativity, then we might expect a sentence like (287a), on a generalizing reading, to have the meaning paraphrased in (287b). This, in fact, seems quite right, according to my intuitions.

(287) a. Fewer students than professors can lift this piano.

b. The *minimum* number of students who can lift this piano is less than the *minimum* number of professors who can lift it (i.e. it takes fewer students than professors to lift it).

And, if *more than* really is different from *less/fewer than* in terms of reference to maximality, then the analogous sentence with *more than*, given in (288a), should not have the reading in (288b)—again, this seems quite right.

(288) a. More students than professors can lift this piano.

b. The *minimum* number of students who can lift this piano is more than the *minimum* number of professors who can lift it (i.e. it takes more students than professors to lift it).

But the question still stands what the source of the asymmetry between *more* and *less/fewer* is in terms of their reference—or not—to maximal informativity.

8.4 The status of pragmatic blocking

Let me conclude with some brief remarks on the pragmatic blocking constraint proposed in chapter 2. The theory that I ultimately defend in this thesis, LMax_{inf}, was shown to not require any kind of blocking mechanism *if* there is a way to prevent a modified numeral from ever scoping below the silent determiner ($\varnothing_{\exists}$ or $\varnothing_{\text{Gen}}$) that heads the DP that contains it. One way to achieve this is simply to adopt Hackl's (2000) innovation and assume that $\varnothing_{\exists}$ and $\varnothing_{\text{Gen}}$ are parameterized for degrees (see section 1.2.5 and section 6.2.5). This move is perfectly valid and seems to work quite well for English. However, we might then expect to find other languages that are different from English in having regular silent determiners, which are not parameterized for degrees. Assuming that LMax_{inf}, as a general grammatical account, applies to those languages as well, and that LMax_{inf} does not have any pragmatic blocking resource, then we would expect that in those languages, the equivalent of (283a) (*Less than five students smiled*) should be genuinely ambiguous between a maximal reading and an existential reading. It seems implausible that such a language exists, but of course one would need to thoroughly investigate before making that claim.

If, however, no language like that exists, then this is a puzzling cross-linguistic generalization that needs explaining. One possible explanation is that, from a diachronic view, there really is a pragmatic constraint like the one proposed in chapter 2 that is responsible for hard-wiring (so to speak) the kind of syntax-semantics (e.g. Hackl-style determiners parameterized for degrees) that we are led to for English, which constrains what scope modified numerals are allowed to take.

In other words, if indeed no language displays the kind of ambiguity described above, then, even if we can formally describe those languages without resorting to a pragmatic blocking mechanism, it seems plausible that it was a pragmatic blocking mechanism that made those languages become that way in the first place.

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