DYADIC INTERACTION IN AN ELEMENTARY SCHOOL COMPUTER LAB CLASSROOM, AND THE LEARNING OF LOGO GEOMETRY CONCEPTS

by

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ADDENDA

The abbreviation D/A has been used to represent "disagreement" in the transcription selection excerpts throughout the paper.

The term "individualization" in Table 2 and Appendix A has been used as a short-hand reference to situations in which the project topic was self-selected by the participant, and intervention on the part of the Logo teacher or researcher was contingent upon the participant's individual needs.

ERRATA

Due to changes in text in the final stages of printing, the following errors in pagination are to be noted: there are two "page 109" pages, the page prior to p. 460 is not numbered, and page numbers 430 and 488 have not been used.

In each instance in which the phrase "mathematical and geometric concepts" appears, please substitute "arithmetical and geometric concepts." The latter phrase is more appropriate since mathematics includes geometry.

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ABSTRACT

This qualitative naturalistic research study investigated peer interaction and its relationship to the learning of Logo geometry concepts within an elementary school computer lab classroom environment. The work of four focal children (10-11 years old) and their partners was analyzed; thus there was a total of ten children in all. The study looked at (1) the kind of working relationships which existed between the partners, (2) the verbal strategies used by the partners during their mathematics disagreements, and (3) the ways in which the talk between the partners and the strategies they used both contributed to their learning, and reflected their learning, of the geometry concepts (with an emphasis upon the aspect of angular rotation).

The findings revealed that all but one of the dyadic (and triadic) partnerships were collaborative and symmetrical: the children took turns giving information and explanation. (In one case the relationship was didactic and asymmetrical.) At times the information was incorrect. The number of explanations given was very small. A range of peer teaching skills was in evidence. There was no correspondence found between the rate of use of higher level strategies (which included talk supported by reasons), and the child's general Logo achievement. However, the verbal strategies used during disagreements did reveal the children's modes of working and thinking. Seven of the ten children worked in an algorithmic way; the other three seemed to understand what they were doing when they worked with angular rotations. The study was able to report on aspects concerning the children's understanding of angular rotation in the early stages of Logo learning, as well as on the sociocognitive aspect concerning the effects that children working together can have upon their learning of Logo geometry concepts.

RÉSUMÉ

Cette recherche qualitative porte sur l'interaction entre pairs ainsi que l'effet de cette interaction sur l'apprentissage de concepts géométriques avec Logo. L'environnement dans lequel s'est deroulé cette étude était un laboratoire d'ordinateurs situe dans une école élémentaire. Le travail de quatre sujets (âgés de 10 et 11 ans) et de leurs partenaires fut analysé. L'étude a envisagé: (1) les types de relations de travail qui existaient entre les partenaires; (2) les stratégies verbales utilisées par les partenaires lorsque des mésententes concernant les concepts mathématiques se présentaient; et (3) la manière dont les échanges entre les partenaires et les stratégies utilisées contribuèrent à, et réflétèrent, l'apprentissage des concepts mathématiques (avec une emphase sur la notion des rotations angulaires).

Les résultats montrent que toutes les relations dyadiques et triadiques (sauf une) entre partenaires sont empreintes de collaboration et sont symétriques: les enfants donnent de l'information et des explications chacun leur tour. La quantité d'explication donnée était très petite. Parfois l'information et les explications étaient incorrectes. Une certaine abilité à enseigner chez les enfants fut mise en évidence. Il n'y avait aucune correspondence entre le taux et l'usage de stratégies de haut niveau (qui inclut l'échange supporté par le raisonnement) et les résultats de l'enfant dans l'environnement général de Logo. Toutefois, les stratégies verbales utilisées lors des mésententes révélèrent le mode de travail et de pensée de l'enfant. Sept des dix enfants ont travaillé sur la base d'algorithmes; les trois autres semblaient savoir ce qu'ils faisaient lorsqu'ils travaillaient avec les rotations angulaires. L'étude fut à même de donner de l'information concernant la compréhension des rotations angulaires lors des premiers stades de l'apprentissage de Logo, ainsi que les aspects sociocognitifs du travail en petit groupe qui affectent les enfants dans l'apprentissage de concepts géométriques de Logo.

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REVIEW OF THE LITERATURE

The study has looked at children's talk as they worked in pairs in the elementary school Logo microcomputer graphics environment, in order to investigate what the talk could tell us about the children's learning of Logo geometric and arithmetical concepts. Cazden (1986), in her cogent review of the research on "Classroom discourse," suggested that the aspects of peer interaction, talk, and cognition warranted investigation; it is these three aspects which have been dealt with in this study. Cazden's (1986) review indicated a need for continued study of peer interaction as it leads to learning, particularly in the area of process-oriented research in naturalistic settings. It was this need that the present study aimed to fulfill.

Opportunities to investigate the workings of the peer culture within the context of the classroom are limited (Cooper, Marquis & Edward, 1986), for in most school settings the "official world of the teacher's agenda" directs the interactions (Cazden, 1986, p. 451). In a section subtitled "And what about computers," Cazden (1986) suggested that researchers should avail themselves of a new locus of peer activity, the computer terminal. Referring to the work most often cited in the literature in relation to Logo and social interaction, namely the studies by Hawkins, et al. (1982; Hawkins, 1983), Cazden stated that microcomputer work areas "may . . . become a significant site for research on peer interaction, a new context for the exploration of how the social and the cognitive connect" (1986, p. 451).

Language unites the social and the cognitive (National Institute of Education, cited by Cazden, 1986, p. 432) (Note 1). For the purpose of the review of the literature for this study, the three areas which are in truth inextricably interconnected, namely social interaction, talk, and cognition, have been divided so that one can inspect what past research has to say about each of these areas, and can suggest what this study can contribute to the domain. The review of the literature will address the rationale for the choice of topics within the broader areas: why *talk* (not "discourse," not "semiotics"), why *peer talk* (when much of classroom research has dwelt on teacher-student talk), why *peer pairs* (dyadic interaction as opposed to small group interaction, same-age as opposed to older-younger), why *microcomputers*, why *Logo*, why *the concept of angle* within the Logo microcomputer graphics context.

Social Interaction

Peer interaction within the Logo environment must be considered within the larger framework of research in group processes in general, and then more specifically vis-a-vis research in peer interaction and its relationship to learning.

Small group processes, and the "group of two"

There is reference within the literature on small group processes to the unique dynamics of the "group of two" (Bales, 1950; Hare, 1962; Watzlawick, et al., 1967). Hare (1962) noted that a delicate "balance of power" existed and a unique negotiation of roles, where one member gravitates "toward a more active role . . . while the other tends toward a more passive role and holds the power of veto" (p. 241) (Note 2). The finding that peers, both adults (Watzlawick, et al., 1967; Hare, 1962) and children, (Youniss, 1980) negotiate their roles and will maintain a relationship providing that conditions of reciprocity and fairness are perceived to exist have had implications in regard to the viability and durability of peer dyadic relationships in this study. There would have been no opportunity to observe extensive interaction nor the learning which might result therefrom if the pairs could not "get along" as partners.

In the microcomputer environment, Cox and Berger (1981) stated that two, at most three, children, constituted the best size of group for work at a microcomputer. Dickson and Vereen (1979) on the other hand insisted that two is the optimal size, since with three or more children "turntaking" is more likely to be unsuccessful. Cox and Berger (1981), in their non-Logo based study, made mention of the importance of the comparative abilities of the members of the group (dyads and triads). Cox and Berger (1981) found that one degree of difference in achievement between the partners (i.e. A to B, B to C) resulted in a smooth working relationship; students "who differed by two or more grades--such as A to C or A to D--tended to quarrel. . . [There was] friction and verbal abuse, and in some cases the less able student was ignored" (1981, p. 29). The Logo studies which deal with social interaction have reflected both children's comments (Hawkins, 1983; Carmichael, et al., 1985) and researcher's reflections (Hoyles & Sutherland, 1986a) concerning both the positive and negative aspects of working in pairs. This study looked at change rates in partnerships, at the achievement levels of the respective partners, and at the size of the group, in order to see how these aspects affected the interaction between the children.

Peer work groups

The literature on peer work groups has endeavored to define the different settings (Cooper, Marquis & Edward, 1986; Stodolsky, 1984, see Note 3) as well as to investigate what the peer group can contribute to a peer's learning (Wilkinson, 1982; Webb, 1982a; Peterson, et al., 1984). Stodolsky (1984), for example, has stressed that the lack of distinction between settings has led to "confusion in interpreting studies on the effects of experimental group-work settings" (1984, p. 114). Moreover, Stodolsky (1984) insisted that despite what the classroom teacher and the researcher identify as the dynamics in the peer workplace, it is necessary to look closely in order to determine the internal workings which are actually in operation.

Two methodological approaches to looking at group processes: Process-product and sociolinguistic

Cazden's (1986) research review on classroom discourse gives an excellent historical overview of the two paradigms which have been used in the study of group processes-- the process-product approach and the sociolinguistics approach. If one needed to generalize very broadly in order to distinguish the two research traditions, one would say that in the process-product research tradition (Flanders, 1970; Dunkin & Biddle, 1974; Webb, 1982a), talk is analyzed via category systems which were established a priori, with no reference to the actual utterances of the teacher and students, and with the aim of attempting to determine which teaching processes are effective in relation to desired outcomes such as "measurable" student achievement. Sociolinguistic research may also include frequency counts, but these are done a posteriori, using categories which have arisen from within the context of the situation itself. The analysis of the data is qualitative and descriptive. There is currently a growing interest in the sociolinguistic mode of research in education, the beginnings of which can be traced to the publication by Cazden, John and Hymes (1972) of their book Functions of language in the classroom; it has attracted classroom researchers from a number of disciplines such as linguistics, psychology, anthropology, and sociology. The sociolinguistic research tradition tries to describe or define the teaching and learning process (Koehler, cited by Cazden, 1986, p. 432); the evaluation of measurable outcomes does not occupy as prominent a focus in sociolinguistic research as it does in process-product research.

In the area of peer interaction, Webb's (1982a, 1982b, 1984, 1985) work and that of Peterson, Wilkinson, Spinelli and Swing (1984) are relevant. In studies of peer interaction in small groups, Webb (1982a, 1982b, 1984, 1985) found a positive correlation between receiving answers (with explanations) and learning, and between giving explanations and learning; and a negative correlation between the number of questions asked and achievement. Webb's research was done with high school level children, using a process-product approach.

Peterson, Wilkinson, Spinelli and Swing (1984) used both sociolinguistic and process-product (Webb's) approaches in looking at giving help, receiving help, and small group processes in the elementary school setting (grades 2 and 3). Peterson, et al. (1984) found as did Webb, that a positive relationship existed between giving help and achievement; but contrary to Webb's findings (1984) there was a negative relationship between receiving pupil-given explanations and achievement. Peterson, et al. (1984) attributed this finding to a lack in development in cognitive and social skills of the population studied, due to their age (seven and eight years old? Their ages are not given.). Findings from the studies of Cooper, et al. (1984; 1986) suggested that the skill of giving effective explanations emerges in the middle years (grades four to seven, ages nine to twelve).

Webb and Kenderski (1984, p. 167) stated that only an above-average student at the high school can provide effective explanations, but then later in the same article they qualified this remark by stating that the level of difficulty of the subject matter was an important variable.

Logo concepts are challenging, as Papert's (1980) use of the terms "mindstorms" and "powerful ideas" in the title of his book Mindstorms would suggest. In this study it was considered important to investigate (a) what constituted an "effective explanation" and whether an elementary school student could give a partner meaningful explanations, or whether only a teacher's explanations were effective; and (b) whether the more able person in the dyad (with "more able" definitely a relative term) tended "to become the 'explainer,' regardless of his or her absolute ability" (Webb, 1984, p. 168).

Social interaction in the Logo setting

Despite a note of fear in the media about the possibly anti-social ramifications of a situation in which children would cease to interact with others in favor of communication with a machine, many of the reports to date which deal with microcomputers in or out of the classroom setting refer to the positive interpersonal interactive dimension of the medium (Dickson & Vereen, 1979; Borgh & Dickson, 1986; Levin & Kareev, 1980).

Within the domain of Logo-related research, there are three main works which touch upon the aspect of interaction within the Logo setting. One source is the research done by Hoyles and Sutherland (1985; 1986), another a research report by Hawkins, Sheingold, Gearhart and Berger (1982), and the third a broad survey report on the Creative Uses Project conducted in 13 classrooms in the Ontario public school system (Carmichael, Burnett, Higginson, Moore & Pollard, 1985). Many of the comments and observations within the afore-mentioned reports evoked research questions worthy of investigation.

The Creative Uses Project (Carmichael, et al., 1985) report, though so vast as to preclude an in-depth analysis of any one area, does serve a need in that it gives a broad overview of many issues. One of the seven research aspects touched upon in the report is that of teacher-student and student-student interaction. In noting participating teachers' comments about social interaction, the researchers stated:

> There was unanimous agreement among teachers participating in the study that the computer can be a powerful facilitator of social interaction... As one of the teachers pointed out:
>
> > the most neglected variable in education, that of student-student interaction, can be used in a positive way. Children enjoy learning from the teacher and from each other. In order to get anything accomplished, cooperation is necessary.
>
> (1985, p.70)

The present study explored what the Creative Uses Project was able only to touch upon; it looked in depth at the ways in which student-student interaction could lead to the learning of the Logo concepts.

Hoyles and Sutherland (1986a) reported recently that:

> despite marked variation between the patterns of interaction between pupil pairs, instances for each pair have been recorded when collaborative exchanges have: provided challenging ideas for projects, kept the project going in the face of 'obstacles,' changed the level of representation of the work (conceptual to concrete or vice versa), provoked reflection on the process within a procedure to predict its outcome.
> (1986a, p. 354).

They also have found that "pupil pairs tend to have implicitly negotiated individual dominance for particular aspects of the activity. This negotiation of dominance has impeded individual acquisition of particular understandings in some cases" (Hoyles & Sutherland, 1986a, p. 354). There will be further references to the work of the Hoyles and Sutherland research team in subsequent parts of the review of the literature for this study, since Hoyles and Sutherland are the only researchers to date who have dealt with the three main issues to be explored in this study, namely peer interaction, talk, and the learning of mathematical ideas via Logo.

Although there are a number of articles by Hawkins and her colleagues which are worthy of note (Hawkins, 1983; Sheingold, et al., 1983), the work most often cited in the literature concerning the aspect of social outcomes related to computers in the school setting is the article by Hawkins, Sheingold, Gearhart and Berger (1982) entitled "Microcomputers in school: Impact on the social life in elementary classrooms." The article by Hawkins, et al. (1982) reported an increase in the amount of talk the children engaged in while at the computers as compared with the amount of talk which occurred when they were engaged in other child-centred activities. However, what this study insists is at issue is not whether there is a quantitative increase in the amount of talk, or increased interaction, but rather WHAT is it that the students are talking about, and in what ways is the talk and increased interaction contributing to their learning. Indeed, this study dealt with the very question which Hawkins, et al. (1982) considered to be a vital one, one which they themselves specified that they were not able to broach within the parameters of the 1982 study, namely that of the effects that children working together in the microcomputer graphics (Logo) environment can have upon

their learning.

Talk

Why "talk"?

"Talk is intricately related to the learning process" (Adams, 1984); talk has a powerful effect on learning (Martin, 1987) (Note 4). The premise in education is that talk is essential for student learning, but it is usually the teacher's talk with the children that is considered essential. Classroom research and practice still by and large reflects a "transmission" model (Watson & Young, 1986) of teaching and learning, where the teacher is the source of knowledge and expertise, the child an empty vessel into which the knowledge should be poured. Talk between children in the classroom setting has been regarded as "noise in the instructional system" (Cazden, 1986, p. 448).

As part of a larger movement towards a holistic approach to language (Goodman & Goodman, 1981; Edelsky, 1987; Barnes, Britton, & Rosen, 1971), current changes in thinking about talk reflect a change in orientation toward all facets of communication: writing, speaking, listening, and reading. It is Britton's (1980) contention that one makes meaning at the very point of utterance. Talk is not thought spoken aloud (Vygotsky, 1962; Note 5). One can learn about what one knows as one talks (Note 6).

Although the body of research on peer talk is small, just a shadow by comparison to the explosive interest in writing that work by Britton (1970), Graves (1982), and Murray (1980) has fostered over the last fifteen years, there is currently an increased stress in both research and curriculum issues upon talk and upon learning through talk (Genishi, 1984; Gouvernement du Québec New Language Arts Program, 1985). The best-known proponents of the essential role of peer talk in learning are from England: Barnes (1976), Martin (1976, 1987), and Adams (1984). Barnes (1976) has shown how, if given the opportunity, children can "talk themselves into understanding" (Barnes, quoted by Watson & Young, 1986, p. 126).

The term *talk* was a deliberate choice on the part of the researcher, and was selected in order to reflect the active, informal, student-centered dimension of the study. Various other terms, some broader and more general, others seemingly similar, were considered and were found to be unsuitable. There is, for example, the very

broad, global, exotic term *semiotics*, which denotes sign systems of which the language communication system is only a part. If one considers the subset, the language communication system, one notes that it is comprised of nonverbal behavior, paralanguage, kinesics and proxemics, as well as language; all the aspects are vital, but could not be given adequate attention within the parameters of this study; however, consideration was given within the study to the nonverbal behavior as it related to the talk. Terms which are seemingly similar to *talk* are *discourse*, or *speech*, but neither term is synonymous with *talk*. *Discourse* has the connotation of "planned, controlled" presentation; and perhaps due to the fact that the Sinclair and Coulthard (1975) *discourse analysis* work focused on teacher-centered teacher-student interaction, the term *discourse* in educational research may trigger an association with teacher-controlled discourse. Even the term *speech* is too formal; it does not reflect the more relaxed nature of *talk*. For although speech can be used in a general sense, it is also used to represent the formal rhetorical stance. *Talk* was therefore the focal term chosen. Talk, as Dillon (1984) pointed out, is a verb; it connotes active involvement. Dillon (1984) stressed the role that talk should play in children's learning:

> "Talk" . . . prompts images of pairs and small groups in the classroom, actively involved in shaping a project or problem . . . challenging, clarifying, defining, supporting, questioning, wondering. Sometimes the teacher is present as a member of the group, but often not . . . . The emphasis is on meaning-making, transforming new information to make sense of it and bringing to the surface things known only implicitly.
> (1984, p.115)

In two recent articles, teachers and researchers have looked specifically to see how the students were attempting to "make meaning" of concepts in content subjects such as history (Berry, 1984) and science (Mayher & Brause, in press, cited by Cazden, 1986), in the former case by tracing the talk of one student throughout a number of peer group discussions, in the latter case by tracing the students' appropriate or confused use of new science vocabulary during various instructional contexts. Focus on the content subject area as it is connected to the learning represents a rarely-used

research orientation, for in research on "classroom discourse" (Cazden, 1986) the content itself usually serves merely as a backdrop whereby the researcher can look at other facets of the interaction. This study looked to see how elementary school age children were attempting to make sense of mathematics content subject matter.

Talk within the Logo setting

The assumption of talk, both teacher talk and peer talk, as a necessary component for learning seems to underlie most of the articles which deal with the social dimension of Logo (Dickson & Vereen, 1979). The Creative Uses study (1985) did touch specifically upon the aspect of talking and listening, when it noted that one of the objectives of student learning is to develop "vocabulary that enables [the child] to name, describe, reason, explain and use qualitative words as she or he plays, observes, manipulates, creates, and experiments with stimulating materials" (1985, p. 317). In their comments on the evolution of the children's talk together, the Creative Uses Project researchers stated that:

> In working towards a common goal, students working in pairs became more sensitive to a need for expressing their ideas clearly and for critically listening to others' ideas. This became particularly evident when disagreements arose.
> (Carmichael, et al., 1985, p. 317)

Hawkins (1983), in her study Learning Logo together: The social context voiced the commonly held notion that talk, and that disagreement within talk, serve as an impetus for cognitive growth:

> Cognitive skills are enhanced. Children learn better or differently. This claim is frequently grounded in Vygotsky's ideas; it also appears in recent Piagetian research. Conflicts in viewpoints can invoke cognitive change, particularly in children transitional between stages.

> Finally, there is the functional notion that requiring children to verbally formulate information improves their understanding of the material.
> (1983, p. 42)

Hawkins' comments serve as a springboard for consideration of aspects of (a) talking aloud as an aid to learning, (b) the role of disagreements vis-à-vis learning, and (c) Vygotsky's view of peer interaction as complementary, as compared to Piaget's view of it as conflictual.

Talking aloud

Webb (1982a) and Price (1982) have both referred to the important role that talking aloud plays in learning. Webb (1982a), in a thorough review of the literature on cooperative learning, discussed a number of studies that point to the importance of the act of verbalizing material, particularly when the verbalizing involves trying to explain something to someone else. Webb (1982a) cited a 1962 study by Gagné and Smith which found that "vocalizing during practice of a problem-solving task produced better end performance than did non-vocalizing." Webb (1982a) stressed that it is not vocalization alone which is the mechanism responsible for increased achievement among active participants in group work; the age or nature of the audience is important. Students in the Durling and Schick study (1976, cited by Webb, 1982a) performed better when vocalizing to a peer than to an adult, because they assumed that the adult already knew the concept whereas the peer was a learner and the vocalizing student then had the opportunity to "teach." Price (1982) cited a 1979 study by Grover which showed how verbal processes played an important role in four-year-old children's development of geometric and spatial concepts (1982, p. 269).

Of interest in this study was the extent to which talking aloud at the microcomputer work station could aid the children's learning of geometric concepts. Were there instances of peer teaching such as Hawkins, et al. (1982) and Kull (1986) had found? Hawkins et al., (1982) stated that in their study episodes of peer teaching were rare. Hawkins et al., (1982) defined peer teaching as "an asymmetrical interaction between or among children, in which one [played] the instructional role" (Hawkins, et al., 1982, p. 364). An episode was coded as peer teaching if there were at least "three

turns of talk, of which two were taken by a 'teacher'" (1982, p. 364). Although the scope of the study did not permit in-depth consideration of the question, it was nevertheless deemed important to attend to the issue and definition of peer instruction. One needed of course to distinguish between the full gamut of skills that an experienced adult Logo teacher brings to the interaction (Zack, 1986a) and the narrower range of skills that a peer teacher exhibits (Forman & Cazden, 1986; Rogoff & Ellis, 1986; Cooper, Marquis & Edward, 1986). The characteristics of peer teaching within the Logo setting were to be defined. Mere counts of "turns of talk" were not considered sufficient.

Peer conflict

A large proportion of the recent Piagetian research to which Hawkins (1983, p. 42) must have been referring in the above-mentioned quote (there is no appended reference list in the report) has been summarized by Bell, Grossen and Perret-Clermont (1985), in an article entitled "Sociocognitive conflict and intellectual growth." The studies mentioned therein (Doise & Mugny, 1981; Glachan & Light, 1982; Ames & Murray, 1982; Perret-Clermont, 1980) explored the developmental consequences of peer conflict between children aged four to seven years old; the pre-test, experimental interaction session, post-test design featured various Piagetian conservation tasks. The results of the studies suggested that "under certain circumstances, subjects who [participated] in the interaction session [displayed] more cognitive progress on the post-test than subjects in a control group who [did] not interact with a peer" (Bell, et al., 1985, p. 43). Bell, Grossen and Perret-Clermont (1985) stressed certain essential qualifiers without which cognitive growth via conflict could not occur. The child had to have attained a minimum level of task mastery, a level that allowed the child to understand the problem and to "play an active part in the discussion and confrontation." Active participation is essential. Mugny and others (1979) "found that subjects who systematically resolved conflict by accepting the contradictory responses of their partner (an adult in this case) did not demonstrate any cognitive progress" (Bell, et al., 1985, p. 47). Bell, Grossen and Perret-Clermont stated that the research at this point suggests that "the important element in sociocognitive conflict is the conflictual aspect of the interaction and not necessarily its resolution" (Bell, et al., 1985, p. 46).

The Genevan researchers, Perret-Clermont, Doise, Mugny and others, used an experimental design, the training study, that does not allow direct examination of either

the social or cognitive processes. Average group gains are reported, not individual scores, and thus, as Forman and Kraker (1985) have pointed out, "the fact that some peer interactions may be more facilitative of cognitive restructuring than others is obscured" (Forman & Kraker, 1985, p. 28). Forman and Kraker (1985) stressed that a change in methodology from an experimental paradigm to an observational one would be necessary in order to conduct analyses of how peer interaction can affect either "logical invention" or "logical verification" (1985, p. 28).

Peer scaffolding

Vygotsky's theory of learning proposes a complementary interaction. Vygotsky would stress the "active discussion"; the confrontation within the discussion would constitute only one of a number of catalysts for change. Vygotsky postulated a "zone of proximal development" which is

> the distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with peers.
> (Vygotsky, 1978, p. 86)

The term "scaffolding" is a metaphor originated by Wood, Bruner, and Ross (1976), to describe the ideal role of the teacher; the metaphor has since then been elaborated by other researchers (for example Greenfield, 1984, see Note 7). Scaffolding represents the contribution that the adult or able peer can make to a child's learning. Most of the research studies to date which deal with the notion of scaffolding have dealt with adult-child interaction (Wood, Bruner & Ross, 1976; Wertsch, 1979; Palincsar, 1986) and adult-adult interaction (Childs & Greenfield, 1982; Burton, Brown & Fischer, 1984).

Very few studies have been conducted with the focus upon scaffolding within peer relations. The very definition of scaffolding vis-a-vis peer interaction needs to be expanded. In adult-child interaction, the relationship is asymmetric, since the adult possesses the knowledge which the child is attempting to learn (Note 6). Child-child interaction may be asymmetrical, "knower-doer," (Cooper, et al., 1986) or symmetrical,

"doer-doer" (Cooper, et al., 1986). In their work with sixty-nine children ranging in age from five to twelve years old, who comprised the entire enrollment of a Montessori elementary school that actively encouraged peer interaction as a means of accomplishing school learning, Cooper, Marquis and Edward (1986) found that the

> collaborative (doer-doer) and teaching (knower-doer) functions [evolved] across early and middle childhood, with younger children typically engaging in more symmetrical collaboration and more demonstration and announcement forms of teaching. With increased metacognitive skill, older elementary classmates . . . [engaged] in more elaborate and extended arguments, [offered] more effective orientations, and more progressively adjusted guidance (that is, scaffolding) as teachers.
> (1986, p. 294)

Forman and Cazden (1985), in their description of the Forman (1981) doctoral study on fifth grade children's collaboration in problem solving, posited a "symmetrical reciprocal model of peer assistance":

> [Neither] partner can be seen as objectively "more capable," but partners may assume separate but complementary social roles. One child may perform an observing, guiding and correcting role while the other performs the task procedures. This observing partner seems to provide some of the same kinds of assistance that has been called *scaffolding* by Wood, Bruner and Ross (1976). Such support from an observing partner seems to enable the two collaborators to solve problems together before they are capable of solving the same problems alone.
> (1985, p. 341)

Emihovich, Miller and Clare (1985), in their Logo study of two preschool pairs, found that there was an asymmetrical relationship between the two partners, in which one child acquired the concepts more quickly than the partner, and with or without the

teacher's help would direct the partner's actions. It was always the same child who was dominant.

Hoyles and Sutherland (1986a), in a short paper on "Peer interaction in the Logo environment," showed how one of their pairs (Janet and Sally, age 13), in talking through their disagreement about how to proceed with the use of ratios in their figure, "achieved collaboratively a sophistication in their use of variable that they had not thought to use individually" (1986a, p. 357). There is a need to consider as well whether the collaborative situation might have any negative consequences. Hoyles and Sutherland (1986a) cautioned that "it is important not to ignore the possibility that collaborative work (especially over an extended period of time) might impede individual acquisition of particular domains of knowledge and skills as pupils come to rely on their peers to achieve particular goals" (1986a, p. 354). Indeed in another paper featuring the same pair, Sally and Janet, Hoyles and Sutherland (1986b) showed that the fact that Sally was the dominant influence in any discussion on turn and orientation (i. e. angle rotation), impeded the development of Janet's understanding. "It was only when collaboration between the pair broke down because of an argument that Janet made any progress at all" (1986, p. 55).

The term "scaffolding" presumes a supportive, complementary relationship. But what happens when the structure is not there? There is a void in the literature with respect to investigating the way that children at times withhold support from each other (Hood, McDermott & Cole, 1980; Zack, 1986c). This study attempted to touch upon negative as well as positive consequences of the collaborative situation.

It was the finding of Hoyles and Sutherland (1986a) that the benefits of collaboration outweighed the negative aspects. With the exception of certain instances, such as the case of Janet just mentioned above, when Janet learned as a consequence of being obliged to manage on her own, a noncollaborative stance between partners would impede learning. In this study, the ways in which the children did, and did not, work well together in dyads were investigated. As a result of data already analyzed involving the same child subjects as in this study (Zack, 1986b; Zack, 1986a), it was anticipated that occasions of noncollaboration would be outnumbered by instances of collaboration.

Talk and the negotiation of understanding

Can the course of the children's talk, in particular the course of their disagreements in the Logo environment, give us an indication of the individual child's understanding of a Logo concept? Is it indeed true, as the Creative Uses Project (1985) suggested, that "when disagreements [arise], students [become] more sensitive to a need for expressing their views clearly and for critically listening to others' ideas"? (Carmichael, et al., 1985, p. 317). These questions were deemed worthy of close investigation. Just how do children working with their partners proceed when a disagreement over a Logo move occurs? What strategies do they use? The research done by Eisenberg and Garvey (1981) and Genishi and Di Paolo (1982) in their studies of verbal conflicts among preschool children provided a set of categories which could be used in the analysis of cognitive disagreements among elementary school children. The categories at Level I reflect a closed, ego-centered stance (*insistence, challenge, denial, insult, use of physical force, mitigation/aggravation*), while the categories at Level II reflect openness and an awareness of the other person's point of view (*appeal to authority, request for explanation, supporting argument, counter proposal, compromise*) (Eisenberg & Garvey, 1981; Genishi & Di Paolo, 1982). Within the Logo setting, it was expected that a move toward a more frequent use of Level II strategies would be correlated with an emergent mastery of the Logo concepts.

Hoyles, Sutherland and Evans (1985), in their study of peer interaction in a Logo environment, found that there was an increased level of supported and elaborated talk in the second year of the pairs' work together in Logo, as opposed to that of the first year. It seemed reasonable to assume that an increase in the amount of supported talk would reflect the students' increase in control over the Logo learning, as well as their increased familiarity with each other. The Hoyles, Sutherland and Evans study extended over the entire school year every year, and the partnerships were maintained over the three-year span of the study. (The study was completed in 1987, and a book based on the findings is due to be published in early 1989 (Hoyles and Sutherland, in press).) The question in this study was whether such an increase in supported talk would also occur in situations in which the amount of Logo time was shorter in terms of accumulated class time, that is, less than a full year-- in this study for example, the children met for 12 sessions per year--, and in which the partnerships were more fluid, as is the case in most naturalistic school studies such as the Creative Uses Project study,

and as was the case in this study.

The aim in this study was to look closely at the children's talk, in particular their disagreements, in order to determine what the talk could tell us (a) about the children's learning strategies, and (b) about the progress in their learning. One needed also to distinguish, if possible, between talk which aided the child's own learning, and talk which was intended to aid a partner's learning.

Cognition

Logo and Mathematics: Why Logo?

In a 1985 inaugural lecture at the University of London Institute of Education, Hoyles indicated that school mathematics had become a "subject distinct in itself," estranged from commonsense knowledge (Lave, 1984; Gay & Cole, 1967), indeed foreign even to the realm of mathematics (Hoyles, 1985, p. 25; Note 9). She suggested that within a Logo programming environment "it might be possible to shift the focus of activity in school away from learning the formal properties of symbolic codes and syntax to the discovery of their semantic meaning" (Hoyles, 1985, p. 28).

Logo was designed by Seymour Papert and his colleagues (Fuerzig, et al., 1969) as an environment for exploring mathematical and geometric concepts (Note 10). With its adaptation for the microcomputer, Logo virtually exploded into the public domain in the late 1970's. Although Logo has had its share of detractors and nay-sayers (Pea, undated; Tetenbaum & Mulkeen, 1984; Note 11), and there has been an acknowledgement of difficulties within the Logo environment voiced by researchers who have been working in the area since the early 1980's (Higginson, 1984, Leron, 1985; Hillel, 1985), the Logo graphics microcomputer environment does have potential as a context for learning mathematical and geometric concepts (Noss, 1985; Hillel, 1984, 1985; Hoyles, 1985), provided that one is not expecting to see cognitive gains quickly, and provided that one does not expect the results to be concrete, conclusive and measurable chiefly by external criteria (Note 12).

Learning Logo geometry concepts

In order to trace the development in the child's understanding of a Logo geometry concept, one must (a) choose the concept, (b) see what the research literature

has to say about children's understanding of the concept, and (c) determine how one would measure the child's understanding of the concept. The researcher decided to limit the investigation to only one concept since the amount of data in an observational, qualitative study is considerable, and a multi-dimensional look at the data was required. The concept of angle was chosen due to the potential it held for extensive investigation (Zack, 1986b). Since there is currently much research activity in the area, as will be elaborated upon below, the findings from the other studies provided comparative and supportive sources of information for this study. The concept of angle served as a thematic thread whereby the researcher could concentrate on one strand of the Logo mathematics content subject area, and thereby observe what the children learned and how they learned it. One can never know fully what the children understand; one can only investigate what the children do, say, and can explain, and therefrom attempt to describe and interpret what the child seems to "know."

The concept of angle

There is an existing body of research literature which deals with the child's notion of angle. Research studies which have dealt with children's learning of geometry within the regular classroom curriculum (Close, 1981; Fielker, 1979; Wallrabenstein, 1973; Robinson, 1976; Zykova, 1969) have pinpointed certain difficulties which children have vis-à-vis their understanding of angle. One example would be the difficulty with the identification of right, acute, and obtuse angles in orientations other than the "standard" upright one. It was essential to keep note of the observations which had been documented in the literature to date, and to see whether the same difficulties were reflected within the Logo environment.

The advent of Logo onto the popular scene is recent. There are comments in the literature which reflect conflicting views of what the children know about angle in the early phases of their Logo work. At one extreme, adult observers who are not mathematicians look at children's screen products which are often formulaic designs (for example, REPEAT 4 [FD 50 RT 90] will result in a square) and assume therefrom that the children "know" about right angles and squares. At the other extreme, as Hillel (1986) said recently, it is easier to say what the children do not know about angle in the early stages than what they do know. Although research into the acquisition of mathematics concepts by means of Logo is at this point just emerging, there are studies

currently in progress or recently completed which are investigating the elementary and junior high school child's perception of angle (Kieran, 1986; Kieran-Sauvé & Taurisson, 1985; Allaire, 1985; Zack, 1985b; Hoyles, Sutherland & Evans, 1985; Noss, 1985; Hillel, 1984--elementary level; Ludwig, 1986; Mendelson, 1988--junior high school level). Findings from these studies suggest that there is a need to continue examining closely what children actually know about angle in the Logo environment, rather than what some researchers think the children know (Zack, 1986b). **(Please note: Appendix CC, which constitutes an addendum to this section, should be inserted here.)**

A recent study by this researcher, reported in a paper entitled "The ten to eleven year-old students' notion of right angle in Logo geometry" (Zack, 1986b), suggested that many children have an algorithmic, (that is, "rote") command of the use of certain angles, and that they are not yet at the stage of what Skemp (1976; 1979) and Herscovics and Bergeron (1983) call "relational understanding," that is, "knowing what to do and why" (Zack, 1986b). If one used Table 1, Part A below (Zack, 1986b), and placed the twenty-three children in the Zack study at their respective levels of understanding of angle, all but five would be found to be working somewhere within the lowest level.

In the afore-mentioned paper (Zack, 1986b), Table 1, Part A served to display the terms used by various mathematicians (Van Hiele, in Fuys, et al., 1984; Hoffer, 1981; Skemp, 1976; Dienes, 1970; Herscovics & Bergeron, 1984; Allaire, 1985) to denote the levels of understanding through which one would pass when learning a new concept. These levels are not to be interpreted as stages of mental operations. The children do not progress up through the levels en block as they learn various concepts in geometry. For instance, in Logo geometry, a child will very likely more quickly grasp the relational aspect of length than that of angle, and move from a "groping" exploratory mode into an analytical mode when working with length, while at that same time will still be exploring and working toward a relational understanding of the aspect of angle turn. The van Hieles (in Fuys, et al., 1984), in their study of the child and young adult's acquisition of Euclidean geometry concepts, stated that what is implicit at one level becomes explicit at the next level; researchers who have used the van Hiele model of understanding with high school students (Usiskin, 1982; Fuys, Geddes and Tischler, 1985) have confirmed the finding regarding implicit/explicit knowledge. In the Kamii and De Vries (1978) sequence in Table 1, Part B, knowing how one produced an effect when acting upon an object, which in this case would be the Logo turtle

Table 1

Models of Understanding and Models of Language Use in Mathematics

Models of Understanding

						Models of Language Use	
High ↑	Level 4 Level 3	Rigor Deduction	Formal	Symbolization	Formalization		Demotic	Axiomatic-Deductive
	Level 2	Ordering	Intuitive	Abstraction	Abstraction		Metanymic-Analogic	Technical-Symbolic
	Level 1	Analysis	Relational		Procedural	4. explaining causes 3. becoming aware of how one produced the effect	Metaphoric	Intuitive
↓ Low	Level 0	Recognition	Instrumental	Play	Perceptual (Intuitif)	2. acting upon an object to produce a desired effect 1. acting upon an object to see how it reacts	Hieroglyphic	Ethno-Mathematical
	Van Hiele (1957)	Hoffer (1981)	Skemp (1976)	Dienes (1970)	Herscovics & Bergeron (1984) Allaire (1985)	Kamii & DeVries (1978)	Frye (1972)	Kieren (1987)

Part A

Part B

cursor, would precede being able to explain causes.

Language use in mathematics

The right hand part of the chart (Table 1, Part B) endeavors to suggest how a model of language use would be tied in to a model of understanding. In a recent presentation entitled "Language use in mathematics," Kieren (1987, January) proposed two frameworks, one used to explore Logo as a programming language and its relation to level of functioning within the Logo environment, the second to define language use in mathematics.

Logo as a programming language

The name LOGO, after the Greek *logos* meaning "word," was expressly chosen by the creators of the Logo programming language to reflect the word-oriented nature of the design. "The name LOGO was chosen for the new language to suggest the fact that it is primarily symbolic and only secondarily quantitative" (Papert, 1980, p. 211). Learners of the programming language can see the correspondence between the word and the resultant action; FD (FORWARD) 20 moves the turtle cursor forward twenty turtle paces. Error messages are given in what the creators of Logo felt were "user-friendly" terms-- helpful, and positive in tone (Note 13).

Kieren has proposed (1984, pp. 14-15; Kieren, 1987) that the nature of Logo as a programming language has ties with what Frye (1982) has delineated as the historical development of language. Frye (1982) suggested that in its beginnings, language was used only at a *hieroglyphic* level (not to be confused with Egyptian graphic symbols); the word is the object, "this is that" (Frye, 1982, p. 7). Frye stressed the awesome power and magical qualities that ancient peoples attributed to the word. At the next, *metaphoric* level, the level of logical thought, the word can be separated from its symbol. Language is decontextualized; the word represents the object, "this is put for that" (Frye, 1982, p. 7). Only the first two levels of Frye's hierarchy are relevant to the age of the children in the study (aged eight to twelve years old).

Kieren's analogy between level of functioning in Logo and Frye's theory of the genesis of language is a useful one. The IMMEDIATE or DIRECT mode in Logo represents "an immanent use of language." The keyboard command is typed in and produces an immediate action on the screen. "A command word . . . is seen as directly

related to a robotic action or as immanently calling such action" (Kieren, 1984, p. 14). The DELAYED or EDIT mode, Frye's *metaphoric* level, is one step removed from immediate action. Children who can work with relational understanding in the delayed mode can revise and create in the Logo "editor" because they can manipulate the imagery in their minds.

Informal and formal use of language in mathematics

Kieren (1987) proposed a hierarchy which moves from the level of concrete, ego-centered, intimate language ("ethno-mathematical") to progressively more formalized, more abstract levels of language, until at the fourth "axiomatic-deductive" level, there is only symbol and no language. Of the four levels in Table 1, Part B-- ethno-mathematical, intuitive, technical-symbolic, axiomatic-deductive-- the first two were pertinent to the age level of the children in this study. Ethno-mathematical language is not formal mathematical language; children use their own words. For example, when talking about fractions, they would use terms such as "a piece and a bit." The child may use ethno-mathematical language and may at the same time be thinking in a relational way. At Kieren's "intuitive" level, the terms are formal mathematical terms, such as a "third, and a quarter of a quarter." The child could only be said to be functioning at Kieren's "intuitive" level of language if the child used the formal words in a "relational" way, not in a rote way.

In the school setting of this study, the Logo teacher does not use conventional classroom geometry terms such as "degrees" or "angles." The researcher found that the connection between the input to RIGHT/LEFT turns in Logo geometry and the notion of degrees, is not an easy one for the children to make, is not as evident as it would seem to the adult eye to be (Zack, 1986b). Yet at least five of the twenty-three children in the Zack study were working with an implicit notion of the connection between degrees and the input to RIGHT and LEFT turn in the Logo microworld environment (Zack, 1986b). What are the terms which the children use as they speak about rotational turns in both their own words and in Logo-ese? Do the children move from using ethno-mathematical "a little bit more" type language to a more precise, but not yet school formal language? Does a more precise use of language coincide with a more advanced stage of control over the concept of Logo geometry angular rotation?

It would be true to Papert's vision to have the children learn mathematics from a personal, "ego-syntonic" (Papert, 1980; Note 14) point of view, and to be introduced to the formal mathematics language only after the conceptual framework has been established. Papert's aim is to have the children "be mathematicians" rather than learn school based mathematics (Papert, 1972). Burnett (1981) noted that Vygotsky (1962)

> distinguishes between "scientific" and "spontaneous" concepts, pointing to the critically different role that language plays in their respective formation. Spontaneous concepts, which evolve naturally through experience, are supplemented by being made verbally explicit whereas scientific concepts (i. e. school-centered) begin with verbal definitions and move towards concrete manifestations. The concurrent development of both types of concepts mutually support and affect each other as the individual develops.
> (Burnett, 1981, p. 6)

Although not likely to be observed within the time-frame of this study, it was nevertheless considered important to try to trace whether, when and how the two systems came together; and indeed to acknowledge that for some children, the two would never meet (L. Butler-Kisber, personal communication, February, 1987). Perhaps, after children had had extensive opportunity to explore the mathematics in the Logo environment, to "learn mathematics actively, by means of [a] process that involves the reconstruction or reinvention of facts and rules" (Groen, 1978), the Logo teacher would then help those who had not associated the two to make connections between school classroom geometry and Logo geometry, so that each could gain new meaning from the other.

Logo learning in the naturalistic setting of this study

Via his Logo "microworlds", Papert (1980) has aimed to create a natural setting for the exploration of geometric concepts. In the school setting of this study, the learning environment and the pattern of teacher intervention (Zack, 1986a) reflected what Papert (1980) had envisioned in Mindstorms. The introductory paragraph by Jaggar (1985) for a book entitled Observing the language learner is particularly

appropriate for this section on language and mathematics learning, and at this point, on environments which promote learning. First the paragraph is cited in its original form from page 1 of the book; then the paragraph is cited again, but the second time there are substitutions (*the italicized words*) which render it a most effective description of the school environment of this study.

> This book is about children and their language, and about teachers and their roles in developing children's language. It is based on the belief that a teacher's task is not to "teach" children language but, rather to create an environment which will allow language learning to occur naturally. This is an environment where children use language to talk to each other, pose questions, solve real problems, and learn about the world while the teacher works with the students listening to what they say, watching what they do, guiding and promoting their learning.
> (Jaggar & Smith-Burke, 1985, p.1)

> *The Logo setting of this study* is based on the belief that a teacher's task is not to "teach" children *mathematics* but, rather, to create an environment which will allow *geometry* learning to occur naturally. This is an environment where children *working in pairs* use language to talk to each other, pose questions, solve *Logo* problems, and learn about the *Logo micro*-world while the teacher works with the students listening to what they say, watching what they do, guiding and promoting their learning.

The Logo teacher plays an essential role in the learning. To suggest that learning will arise solely out of child-initated exploration and talk is nonsense. Due to space constraints, little has been said in this review of the literature about the teacher's role in the Logo setting, but the information has been detailed elsewhere (Zack, 1986a; Shapiro, 1985).

It is within a Logo setting such as the one described above, that this researcher-observer had the opportunity to observe as the children were in the very

process of attempting to make meaning of the concept of angle. The environment has been set up by the Logo teacher, Monica Shapiro, with a view to maximizing opportunities for collaboration between partners, and for communication among all the child participants; it is Monica's expressed belief (Zack, 1988b) that as the children investigate the Logo microworld, they are learning from each other as well as from her. While the collaborative aspect of learning was not the primary focus in this paper, it represented a vital component of the study. A basic assumption underlying the study was that "learning Logo together" in pairs was different from "learning Logo alone." For example, when working in pairs, one might need to support one's stance to a partner, and might need to ask questions in order to attempt to better comprehend a partner's point of view; this in turn might lead a child to greater understanding of the concept at hand.

The children did say more about what they were doing, and why, when they were working in pairs than when they worked alone, side by side with their peers. When one considers only the data collected for this project, the talk between partners which was available for study was certainly more plentiful and more informative than the talk which had been collected upon those occasions when the children were working alone. As they talked, the adult listener could hear what the children were able to say about the concept, while at the same time those very speakers might be bringing to the surface what was only implicit before, might be talking "themselves into understanding" (Barnes, cited by Watson & Young, 1986). But the adult had to attend carefully, for it was no easy task to make sense of the children's talk as they themselves tried to make sense of the concept.

Statement of the problem

The topic chosen for study was that of the the children's learning of angle in Logo. One facet of the study looked at what the children did at the microcomputer, and what it had been determined via pre- and post-interviews and via analysis of their work, that they could be said to "know" about Logo angle (Zack, 1986b). This component has been dealt with in other studies (Kieran, 1986; Hoyles and Sutherland, 1986b; Noss, 1985; Mendelson, 1988; Hillel, 1984). The main feature of this study which differentiates it from other studies done to date is the extent to which it endeavored to investigate the talk between partners, and the relationship of the talk to the learning

of the concept of angle. The fact that the researcher's role was that of non-participant observer in the elementary classroom is the second main distinguishing feature. The researcher's aim was to derive a more complete picture of the child's developing construct of angle within a naturalistic setting. Informed by the findings of the analysis (of the same children) which looked first at the child's notion of angle (Zack, 1986b), the focus of this study was on the child's talk as it reflected (a) the child's understanding of the concept, and (b) the child's ability, and need, to convey an understanding of the concept to a partner. The research questions which guided the collection of data related to talk in the 1984-1986 years were as follows:

1. What is the nature of the talk about angle in the initial stages of the Logo experience?

2. When children work with partners, is there a change in their talk and interaction strategy as they gain experience in Logo?

3. What role do discussion and disagreement over a Logo move play in the children's learning of a Logo concept?

The analysis of the data for this study concentrated primarily upon the third broad research question above, namely the dyadic disagreements, and discussion, which pertained to the learning of the concept of angular rotation. In addition to the need to investigate the role that disagreements between partners might play in cognitive growth, as proposed by the writer in the first part of this literature review, the theme of disagreements served as a thread to weave together related and varied components contained within the extensive amount of data gathered during the course of this longitudinal study. The study analyzed what disagreements, and agreements, could tell us about the child's understanding of angle, the child's communicative strategies, learning strategies, teaching strategies, and strategies of social interaction. The general research question, and the more specific research questions, were:

GENERAL: What role do disagreements over a Logo move, and the discussion associated with the disagreements, play in the children's learning of a Logo

mathematics and geometry concept?

SPECIFIC:

CAN THE PATTERN OF THE DISAGREEMENTS--their frequency, duration, resolution, and the category of mathematical idea--GIVE ANY INDICATION OF THE CHILD'S UNDERSTANDING OF LOGO ANGLE?

DOES THE KIND OF RELATIONSHIP THAT EXISTS BETWEEN THE PARTNERS INFLUENCE THE LEARNING? IF YES, IN WHAT WAYS IS THE LEARNING FACILITATED OR HINDERED?

WHICH VERBAL STRATEGIES DOES THE CHILD USE IN THE COURSE OF A DISAGREEMENT? CAN THE STRATEGIES CONTRIBUTE TO THE CHILD'S LEARNING, AND TO THE LEARNING OF THE CHILD'S PARTNER?

Does the child use supported talk? Are the supporting arguments used, correct? Are they clearly stated? Do the supporting arguments clarify the mathematical theorem? Does the degree to which talk is supported increase over time?

Do the verbal strategies used by each participant in a dyad during a disagreement reflect each child's understanding of angle? That is, is the child who has a good grasp of Logo angle more likely to use more complex strategies than a child who does not have a good grasp of Logo angle?

During disagreements, are there instances of "asking for help" or for explanations, from a partner, from the teacher, or from a neighboring peer? If yes, what can those instances tell us about the child's understanding of angle, and the child's learning strategies?

During disagreements, does the child give help or explanations? Can the instances of the child "giving help" or giving explanations tell us about the child's understanding of angle, about the child's ability to explain, and about the child's

teaching strategies. If so, what can they indicate?

ARE SOME OF THE VERBAL STRATEGIES WHICH ARE USED DURING DISAGREEMENTS MORE EFFECTIVE IN LEADING TO LEARNING THAN ARE OTHERS? IF SO, WHICH ARE THEY?

Significance of the Research

There is a need to investigate the child's concept of angle in Logo geometry; it is our contention that the construct of the concept of angle in Logo is not as straightforward as it seems to many parents and teachers to be. This study will complement the recent studies which comprise the newly emerging nucleus of research in the area of the children's understanding of the concept of angle in Logo geometry (see Table 2).

There are four ways in which this study will make a contribution to knowledge. First, this study has devoted all of its attention to the Logo learning of children who are in the early stages of the learning. The studies of Hoyles, Sutherland and Evans (1985), Noss (1985), and Solomon (1976) have touched upon the beginning learning of the geometry concepts, and these studies offer important comparative data for the study at hand. However, only Hillel (1984) and Kieran (1986) have concentrated solely upon the beginning phases of learning about Logo geometry.

Second, in this study, the Logo software package reflects the software which Papert and his colleagues designed; the Logo Computer Systems group (LCSI) then produced the software package for the Apple computer company. The Apple Logo software used in this setting has not been modified. This is not to suggest either that Logo ought not to be modified, or that there ought to be changes; rather, the fact that the standard version was used is mentioned in order to point out that the children in the study were working with the version of Logo which is most widely known. Although no survey of Logo use in schools has to our knowledge been done at the time of the writing of this paper, it is our impression that it is the original Logo version, or the updated Logo II version, which is most prevalent in the Montreal and Quebec schools, as well as elsewhere in Canada (and perhaps other parts of the world as well). Hence the difficulties experienced by the children in this naturalistic setting may indeed reflect

Table 2

A Summary of Recent Research Studies in Logo and Mathematics

The Study	Specific mathematics agenda	Logo software changes	Instruction: Individualized instruction	Instruction: Tasks assigned	Instruction: No tasks assigned	Instruction: Researcher intervention	Age of children	Group size	Time periods	Total hours	Location: Homeroom classroom	Location: School computer lab	Location: University research lab
(1) Papert, Di Sessa, Watt and Weir (1978)	no	no	●	●		●	(12-13 year olds) Grade 6	8	40 60 90 mins	25		●	
(2) Hillel (1984)	yes	yes	●	●		●	(8-9 year olds) Grade 3	10 (5 pairs)	60 mins	12			●
(3) Noss (1985)	yes	no	●			●	(i) 8-11 year olds	115	60 mins	30	●		
							(ii) 10 year olds	8 (4 pairs)	60 mins	15			
(4) Hoyles, Sutherland and Evans (1985)	yes	no	●	●		●	Year 1 11-12 year olds	14 (7pairs)	app 60 mins	40	●		
							Year 2 12-13 year olds	8 (4 pairs)	app 60 mins	40			
							Year 3 13-14 year olds	8 (4 pairs)	app 60 mins	40			
(5) Kieran (1986)	no	yes	●		●		9-10 year olds Grade 4	14	30-60 mins	25	●		
	no	no			●	●	9-10 year olds Grade 4	5	60 mins	35			●
	yes	yes		●		●	11-12 year olds Grade 6	12	60 mins	22			●
(6) Zack (1986d)	no	no	●		●		Year 2 (10-11 yr olds) Grade 5	23 (in pairs/triads)	60 mins	12		●	
(7) Ludwig (1986)	yes	no		●			(12-13 year olds) Grade 7	10 (5 pairs)	80 mins	21		●	
(8) Barfurth (1987)	yes	yes		●		●	(9-10 year olds) Grade 4	6 (3 pairs)	75 mins	15			●
(9) Kynigos (1987)	yes	yes		●		●	12 year olds	6 (3 pairs)	90-120 mins	6		●	
(10) Gurtner (1987)	yes	yes		●			12 year olds	6	60 mins	24			●
(11) Hoyles and Noss (1987)	yes	yes		●			13-14 year olds	7		2			●
(12) Mendelson (1987)	yes	no		●			12- 3 year olds Grade 7	8	45 mins	12		●	

hurdles faced by children in other school (and home) settings (see Note 15). [1]

Third, this research study is unique in the sense that the Logo teacher has not assigned tasks, nor has the researcher assigned tasks. The researcher has not intervened in the interaction within the computer lab classroom. The findings about the children's learning in the setting of this study may provide contrasts and perhaps parallels to the findings in (1) situations where tasks have been assigned, to some extent (Hoyles, Sutherland & Evans, 1985; Noss, 1985; Hillel, 1984), or to a great extent (Hillel, Gurtner & Kieran, 1988), in (2) research settings (Hillel, 1984; Hillel, Gurtner & Kieran, 1988), and in (3) school settings where researchers are involved in the ongoing Logo and mathematics curriculum (Hoyles, Sutherland & Evans, 1985; Noss, 1985). If we can gain greater insight into the children's understanding of angle, this insight may inform classroom practice so as to enable the Logo teacher, and the mathematics teacher, to look more closely and critically at the children's work, and to adapt his or her intervention with the children accordingly.

Fourth, the study speaks to the vital need expressed by Hawkins, Sheingold, Gearhart and Berger (1982) and Cazden (1986), namely the need to investigate how peers working together in a microcomputer setting might be contributing to the learning of the concepts. It is expected that the research findings will have implications for the Logo curriculum in schools, and for the school mathematics curriculum.

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[1]Table 2 represents a summary of recent research studies in Logo and mathematics education. Reference to the table will reveal how certain of the components differ from those in this study, factors such as whether the Logo software was changed, whether there was a mathematics agenda as primary focus, whether tasks were or were not assigned, and whether there was active researcher involvement, as well as the duration and the site of the Logo sessions. (An elaborated version of this table is available in Appendix A.)

METHODOLOGY

The Research Design

The study aimed to look at the process of the elementary school child's making meaning of the concept of angle within the Logo microcomputer graphics environment. The questions posed by the study necessitated a qualitative, observational, longitudinal research design. It was not possible to investigate the 10 to 11 year-old child's construct of knowledge by means of a quantitative, experimental, large group, cross-sectional, snapshot, or pencil-and-paper test mode. A pretest-posttest, or pencil-and-paper design was not suitable, for researchers such as Leron (1985), for example, have noted that such tests cannot measure what the child knows about Logo:

> [The] evaluation tools used are, for the most part, inappropriate for the processes which are important in Logo learning; thus they may fail to demonstrate viable learning even when such learning has occurred. For example, most students we have worked with seem to have gained some sort of vague, partial understanding of many powerful ideas. This kind of learning is very valuable, indeed a necessary intermediate step toward a more complete understanding, but it is unlikely to be detected by standard evaluations, where lack of complete understanding is often rated as complete lack of understanding.
> (Leron, 1985, p. 32)

The qualitative approach chosen for this study, and the data collection tools as well, are consistent with that of recent research studies in Logo and mathematics education, as will be seen upon reference to the table in Appendix A.

Evaluation was based upon the "child as her own control" model. Level of understanding and progress in understanding were measured as the result of a composite picture-- what the child did, said, and could explain, to her peers, to her teacher, to the interviewer.

The analysis of the children's talk which is the focus of the present study was based upon a part of the data which had been collected over a period of five years (1983-1988), that is, for four years in an elementary school setting (Grades 3 to 6, when the children were aged 8 to 12 years), and for one year (1987-1988) in the high school

which is affiliated with the elementary school research site (Grade 7, when the children were aged 12 to 13 years). This study dealt almost exclusively with the Grade 5 data, 1985-1986, when the children were aged 10 to 11 years. Prior to this study, and concurrent with the work done for this study, the data were analyzed in diverse ways which in turn provided findings and further questions for the present study (see Table 3).

In regard to the present study, it was anticipated that the disagreements of four Grade 5 pairs of children would add another dimension to the emerging picture of the children's concept of angle in Logo geometry. The analysis of the verbal strategies used in the disagreements was expected to add to the growing body of information concerning how these children were able to make sense of the angular rotation; it was also expected that the analysis would add information concerning the ways in which the children were, or were not, able to contribute to each other's learning.

The Setting and the Subjects

The project was conducted in a semi-private multilingual Jewish day school, where the curriculum covers English, French and Hebrew studies (Note 16). The population of the school is somewhat heterogeneous-- mostly middle-class urban and suburban, some upper-class, some lower middle-class urban; the greater percentage of the student population is Aschenazi (Jews of European descent; English is the mother tongue), and there is a smaller percentage (approximately 15%) of Sephardi Jews (Jews of Spanish descent, or who emigrated from areas in the Middle East; in most cases, French is the mother tongue). The subjects chosen for the analysis, for this study and for the other phases mentioned above, were all English-speaking Aschenazi children from middle-class homes.

Each year, approximately twenty-four children, grouped by homeroom classroom, and split into two groups of approximately twelve in each group, attended the computer sessions in the school computer lab classroom. There were a number of Logo teachers who taught in the school, none of whom were part of the regular school teaching staff. In terms of importance vis-a-vis contribution to this study, Monica was the most vital teacher figure; Sharon, Wendy, and Don worked with some of the study subjects at varying points in time, and references to their involvement will at times be made.

Table 3

Papers and Analyses Done to Date Within the Project

Topic	Size of Sample	Names of Subjects
Focus on disagreements	Two children Case study Grade 4	Amy and Carol (Zack, 1985)
Children's conception of right angle	Entire Grade 5 class & specific pairs	Amy and Leanne Carol and partners Lilly and Rina Russ, Jon & Eric (Zack, 1986b)
Children's conception of angle	Two children Case study Grade 5 - Grade 7	Lilly and Rina (Zack, 1988a)
Pattern of Logo teacher's intervention	Three pairs	Amy and Leanne Lilly and Rina Russ, Jon and Eric (Zack, 1986a)
Pattern of classmate peer interaction	Entire Grade 5 class	(Zack, 1988b)

In 1985-1986, when the study subjects were in Grade 5, the computer sessions for the groups observed took place from October to January, for approximately one hour per session, approximately once a week for 12 weeks; the same time frame was in place for the Grade 4, 6, and 7 computer sessions as well. The group's composition was determined by the French teachers of the respective homeroom classes, for it was during French class time that the computer sessions took place. As has been mentioned, the children were divided into two groups. In 1985-1986, the decision of the French teacher of homeroom 210, Mme L., was to group the 23 children as homogeneously as possible according to their working level in French, so that while one group of children was attending the computer class, the French teacher would work with a "homogeneous" subset, that is, half of the class, in the homeroom classroom. Group A contained 11 children from homeroom 210, and attended the computer session from 10:10 a.m. to 11:10 a.m.; Group B contained 12 children from homeroom 210, and attended the computer session from 11:10 a.m. to 12:10 p.m. in the school computer lab classroom.

One month prior to the first Logo class each year, a permission letter addressed to parents and children was sent to all the children in the upper elementary levels (Appendix B). None of the children in the two classes videotaped in 1984-1985 and in 1985-1986 denied participation consent.

The physical organization of the computer lab classroom, and the classroom routine

The children learned their Logo within the school setting, in a computer lab classroom outside of their homeroom classroom (Note 17). The fact that they were in a separate location, that there was a time slot reserved for the Logo work, that the time was limited to the approximately 60 minutes minutes a week within the school time, that there was an expert Logo teacher designated to teach them, and that the children were free to interact with the teacher and with each other during this time, were all aspects of the physical organization and the classroom routine which were expected to have an influence upon the learning which occurred within the setting.

The computer laboratory was a former "homeroom" classroom on the main level of the school (see Figure 1 and Figure 2). The classroom housed six Appie IIe computers. A printer (Epson MX-80) was attached to one of the computers (computer #6), and was used when a pair had completed a project and was ready to run off a hard copy. (When a group was ready to print a hard copy of their project, the pair using

computer #6 changed work stations with the pair awaiting use of the computer printer.) Two tables pushed together in the center of the class near the door, with chairs around them, were the location for the whole group introductory and end-of-session discussions with the Logo teacher, Monica. Once the whole group introductory part of the lesson was completed, the children moved the chairs over to the computers in order to begin their work at the work station.

The whole group introductory part of the Logo class was teacher-directed. The introductory part lasted an average of 20 minutes during the first 6 sessions in 1985-1986, and an average of 10 minutes during the latter 6 sessions. The large group summation part of the lesson at the end of the class (consisting of the return to the tables in the center and a discussion) lasted for approximately 5 minutes. Thus the children had about 30 minutes (during the first six sessions), then about 40-45 minutes of work time at the computer station during the last six sessions. Their total time in the computer lab was 55 to 60 minutes, that is 20 (10) minutes for the introductory part, 30 (40-45) for the work station part in pairs, and 5 for the summation part. During the 30-40 minutes of student-centered computer work time, for the most part Monica interacted with the children while they were seated at their computer. At times, Monica brought a pair or small group of children over to the blackboard, or into the center of the floor for a demonstration, or back to the large center tables for discussion or presentation.

The children worked in pairs, and at times in triads. The children chose their own partners, but the pairings tended to be flexible throughout the twelve sessions; some pairings remained stable throughout the sessions, others changed a number of times during the Logo 12 week block. The children chose their own projects. There was only one across-class task assigned by the teacher; it was an evaluation task whereby Monica aimed to assess the children's ability to use procedures and subprocedures; the task was assigned on the last day of the 1985-1986 Logo sessions for this group (January 29, 1986).

The children were required to work together and to decide on a project together, to draw a pencil drawing of their prospective picture in their notebook, and to record their commands in their notebooks as they worked (Note 18). The child's computer notebook contained the following: a written record of the commands, planned drawings (and erased and redrawn drawings), a glued-in computer printout of the

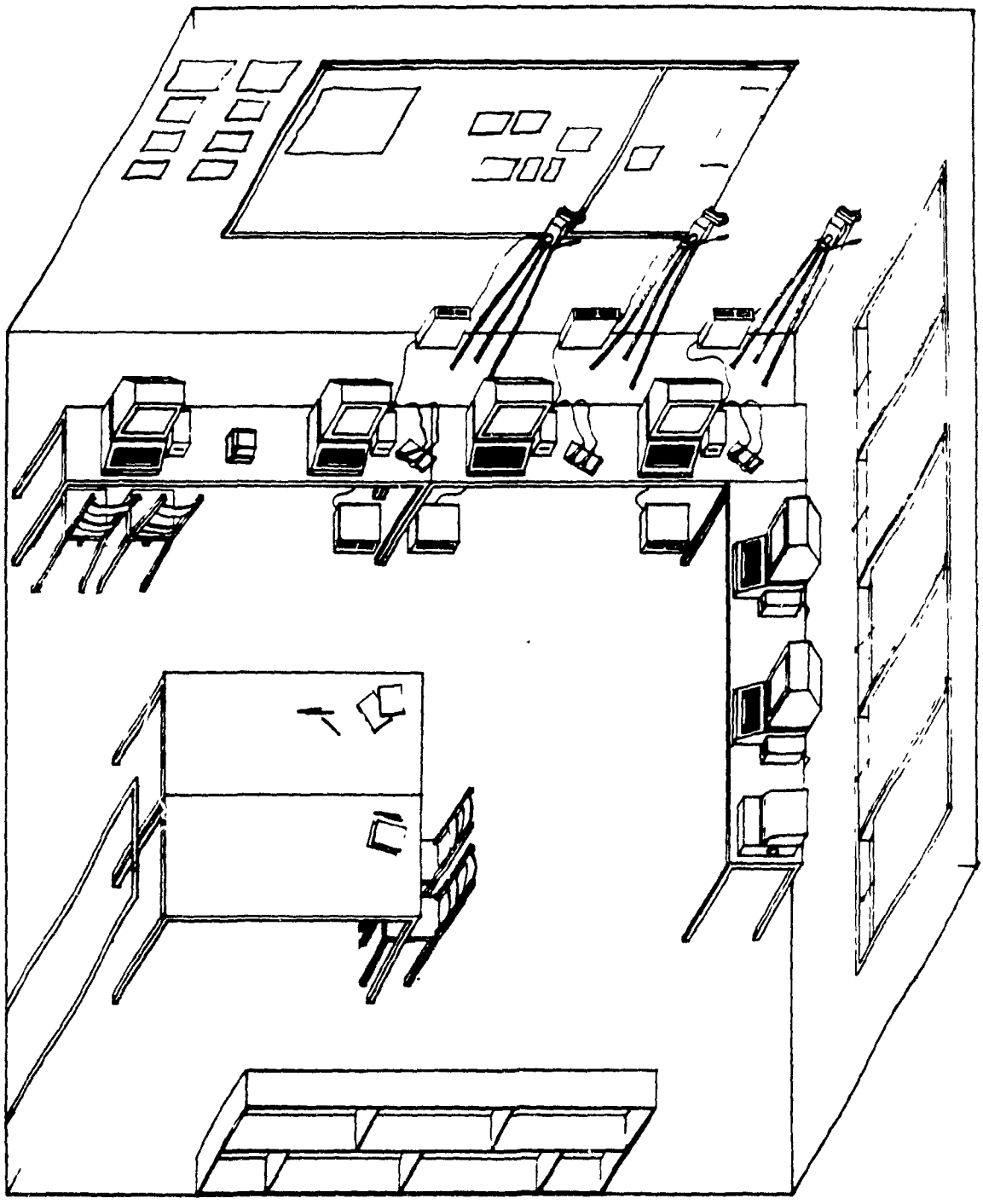


Figure 1. Axonometric drawing of the computer classroom

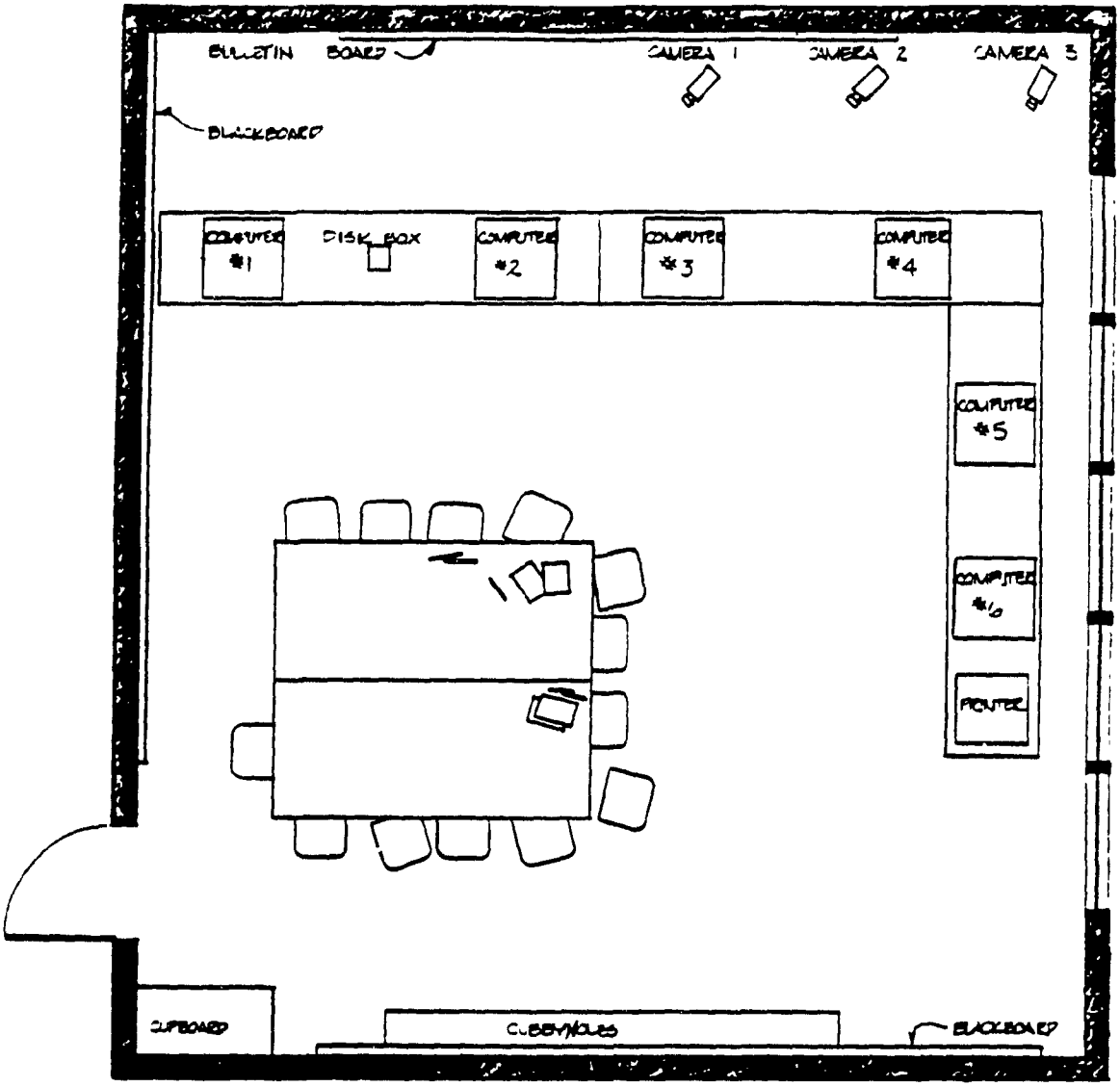


Figure 2. Floor map of the computer classroom

graphic design done by the child and her partner, and various teacher-prepared stencil sheets which provided syntax instructions for "loading," "saving" and using the editor control keys (see Appendix C for samples of children's notebook pages). Several Logo researchers have suggested the use of paper and pencil in planning (Brown, 1973, p. 23) and also in recording the activities (Brown, 1973; Solomon, 1976, p. 14; Papert, et al., 1979, p. 13; Barfurth, 1987, p. 11). As will be shown in the Findings chapter, the ways in which the children in this study used their notebook entries gave insight into the degree to which they did or did not understand what they were doing.

The study subjects: Who they were and how they were chosen

From among the 1985-1986 Grade 5 homeroom 210 class members, the specific children chosen for an in-depth analysis for the present study were: Amy, Carol, Leanne, Lilly, Rina, Russ, Jon, Eric, as well as Shauna and Doug who were Carol and Amy's partners. *Please note that all the children's names have been changed to provide anonymity.* The means by which the choices of subjects were made will now be presented. In the 1984-1985 year of the study, Amy and Carol (then in their Grade 4 year), and 6 other pairs were chosen at random, and were videotaped. Mainly as a result of the fact that of all the pairs videotaped in the 1984-1985 year, only the videotaped data of Carol and Amy's work was complete, the first study and report centred around Carol and Amy (Zack, 1985). When the study continued into the 1985-1986 year, it was therefore Carol and Amy who were chosen for the longitudinal follow-up. In 1985-1986, these two youngsters were together in the same homeroom, room 210, in Grade 5. However, although Amy and Carol were in the first, the 10:10 a.m.- 11:10 a.m., Logo time slot together in 1985-1986, they chose not to work together as partners as they had done in Grade Four. Leanne, Shauna, and Doug were at various times Carol and Amy's partners during the 1985-1986 session. Amy and Carol, and their partners, sat at computer stations #2 and #4 respectively during the first session.

Lilly, Rina, Russ, Jon and Eric were part of the same homeroom 210 classroom which included Carol, Amy, Leanne, Shauna and Doug, but they were part of Group B, the second, the 11:10 a.m. - 12:10 p.m., Logo group. Lilly and Rina remained partners throughout the twelve sessions. Jon, Eric and Russ worked together, in pairs or triads; Jon and Eric worked together for 7 sessions, Jon and Russ for 1 session, and

Jon, Eric and Russ for 4 sessions. These children were chosen due to the fact that during the first Logo session of the year (October 16, 1985), Lilly and Rina, and Jon and Eric, sat down at computer #2 and #4 respectively.

All the children in Carol and Amy's homeroom, room 210, were the subjects of the report on "The ten to eleven year-old students' notion of right angle in Logo geometry" (Zack, 1986b). That study revealed that these children were at various levels of achievement in relation to their understanding of right angle and also of angle generally. Upon consulting Table 4 (Zack, 1986b), it will be noted, progressing from most advanced on the left to least advanced on the right, that Russ, Jon and Eric were at the high level of achievement, Lilly and Rina at the middle level, and Shauna, Carol, Amy, Leanne and Doug were at the low level of achievement, respectively, in descending order. If one had to identify a focal child in each pair or triad (as it was necessary to do in certain instances), one would identify Jon, Lilly, Carol and Amy as the focal children. For the sake of clarity and ease of reference, the letters H., M., and L. will henceforth be affixed to the children's names throughout the study to indicate their level of achievement (see Table 5). It was felt that the fact that the achievement of the children vis-à-vis the concept of angle varied, might result in observable differences in their talk together when the talk aspect of the videotapes was analyzed for the purposes of the present study.

Table 4

Achievement Grid From the Study of the Ten to Eleven Year-old Child's Notion of Angular Rotation and of Right Angle in Logo Geometry (Zack 1986b), Based on All 23 Children in Homeroom 210

	Highly Adept					Moderately Adept							Least Adept										
	Russ H	Jon H	Michael H	Eric H	David H	Gail M	Stuart M	Lilly M	Matthew M	Tamara M	Adina M	Rina M	Shauna L	Joey L	Carol L	Mark L	Dina L	Amy L	Leanne L	Saul L	Raymonde L	Don L	Doug L
90	++	++	++	++	+	++	+	+ -	++			+ -											
45 first interview	++	++		++		++	++		++														
45 second interview	++	++	++	++		++	++	++	++					++									
180	++	++	++	++	++				+		+	+			+	+				+			+
360	++	++	++	++	++	++	++	+		+		+			+	+		+		+		+	
270	++	++	++		++																		
135	++	++	++	++	++	+ 3x45	+ 3x45	+ 90+45	+ 3x45			+			+								
Do you think a right angle is the same as right 90 in Logo?	no	no	yes & no	no	no	yes & no	yes	yes		yes	yes		yes & no			yes							
Use of 90 in Logo work	++	++	++	++	+	+	+	+ -	+	+	+	+ -	+	+	+	+ -	-		+ -	+	+ -	+	+
Use of SETH	++				+		+																
Paper Test																							
*Circle the right angles			+		+			+		+									+				
*Circle the 90 angles	+	+	+	+	+	+		+		+	+	+	+	+		+	+			+	+	+	+
Lateralization problem					◇	▽					▽			▽			▽		◁▽	▽			▽
Relational understanding of 90	yes	yes	yes																				
Allaire (1985) level	2	2	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

KEY

[++] evidence of consistent correct use in Logo work during sessions, in across-class tasks, in interviews Can give support via verbal instructions

[+] consistent correct use over a short period, and/ or during interview only

[+ -] inconsistent use over a short period

[-] no evidence of correct use

Note.
The children's names have been changed to assure anonymity

Table 5

Names, Achievement Levels and Partnerships of the Grade 5 Participants

	Partnerships			
Achievement Level	Focal Child	Partner(s)		
Highly Adept	Jon H.	Eric H.	Russ H.	
Moderately Adept	Lilly M.	Rina M.		
Least Adept	Carol L.	Leanne L.	Shauna L.	Doug L.
	Amy L.	Leanne L.	Shauna L.	

Individual child profiles

The child profiles descriptions which follow were based in part upon the comments drawn from the various classroom teachers who were interviewed, and in part upon the researcher's observations, the Logo teacher's observations, and the results of the study findings. The researcher did not refer to the school records, or to any achievement test results. The teachers' comments concerning the children's achievement were based upon their recollections; they were not based upon the standardized test scores which were available in the school files, nor upon specific school grades achieved (see Note 19). Nine Grade 6 teachers were interviewed by the researcher in November and December, 1987. Fourteen children were discussed, but only ten profiles will be presented, since these ten children are the ones represented in this study.

Each child in the school interacted with at least three homeroom teachers each year, since it was a multilingual school setting in which the children (almost all native English speaking) learned English, French, and Hebrew. Some of the curriculum was taught in languages other than English. For example, a mathematics concept would first be introduced by the English teacher, but then a good deal of the mathematics curriculum, especially the geometry which is pertinent to this study, would be taught in French by the classroom French teacher. Thus, for the interviews, a minimum of three classroom teachers commented upon each child; in most cases, there were comments as well by teachers who had either taught the child in previous years, or who taught the child in the Grade 6 year but was other than one of their three homeroom teachers (for example, since the children were re-grouped for mathematics, the child's mathematics teacher was asked to comment on the child's achievement in that domain).

For their regular school mathematics curriculum, the 69 children from three homeroom classes were regrouped, that is "streamed," into four "homogeneous" mathematics groups; group 1 was made up of the children who were considered the "highest" achievers, and at the other end, group 4 was made up of the "lowest" achieving children. Mrs. S. mentioned that there were 23 children in group 1; information on mathematics group size was not gathered, but the researcher estimates that group 2 had approximately 20, and groups 3 and 4 had approximately 13 children each.

The teachers were asked to comment upon the child's academic achievement, work habits, and social interaction with peers in the classroom (Appendix D). In relation to the child's academic achievement, the classroom teachers were asked to

comment upon their recollections of (1) the child's general achievement in their classroom, (2) the child's general achievement in the language arts component (thinking, speaking, reading and writing), (3) the child's achievement in the mathematics component, and (4) the child's achievement in various subject areas such as social studies and science, and/or in the specific language subject area which was that teacher's domain, such as French or Hebrew. In regard to work habits and other personal attributes, the teachers were asked to comment (1) on the child's pattern of work in teacher-centered lessons, in student-centered sessions (that is, when the child worked alone) and in small peer work groups, (2) on the child's dependence upon, or independence of, the teacher or peers when involved in a task, and (3) on the child's sense of confidence and self-esteem. In regard to the child's relationship with peers, the teachers were asked to comment upon whether they considered the child a *leader* or a *follower*, and whether the child interacted well with peers.

The interviews were audiotaped, and the researcher made brief notes during the interview. A minimum of three teachers, and a maximum of six teachers, commented on each child. The comments of the various teachers concerning each study subject were then compiled; the summaries will be presented below. The child profiles will be presented in order of achievement rating as determined in the Logo project on the child's notion of angle (Zack, 1986b). The description will deal first with the child's profile within the "regular" homeroom setting, and then within the Logo computer lab classroom setting. A summary in chart form of the written descriptions which follow is available in Table 6.

Table 6

Summary of the Child Profiles: Achievement, Work Habits and Peer Relationships of the Ten Study Subjects

	ACHIEVEMENT				WORK HABITS				RELATIONSHIP WITH PEERS	
	Homeroom general achievement	Math achievement	Math group placement *	Logo achievement	Involvement		Independent or dependent		Gets along with peers	Leader/ follower
					CLASS	LOGO	CLASS	LOGO [b]		
Russ H.	high	high	1	high	not consistent	yes	independent	independent	yes	leader
Jon H.	high	high	1	high	yes	yes	independent		yes	both
Eric H.	middle	low	3	high	not consistent	dyad - yes triad - no	dependent	independent	yes	follower
Lilly M.	middle	high	1	middle	yes	yes	independent		yes	follower
Rina M.	middle	low	3	middle	yes	yes	dependent		yes	follower
Shauna L.	low	low	3	low	not consistent	yes	independent		not always	follower (Logo, leader)
Carol L.	low	low	3	low	poor	not consistent	dependent		not always	neither
Amy L.	low	low	4	low	poor	yes	dependent		not always	follower
Leanne L.	low	low	3	low	yes	yes	dependent		yes	follower
Doug L.	low	low	3	low	poor	yes	dependent		no (Logo - yes)	neither

* There were four "streamed" homogeneous mathematics groups, group 1 contained the highest achievers, group 4 the lowest

[b] In the Logo setting in the early stages, all the children needed to call for help (see Table 7). Russ and Eric seemed particularly independent under these circumstances

Child profiles: The children's academic achievement, work habits, and pattern of social interaction with teachers and peers

PROFILE #1: RUSS H. (Age 9;10 in Oct. 1985)

In the homeroom classroom setting:

In terms of general achievement, the five teachers felt that Russ was an above average and bright student, but that due to the fact that he applied himself only when he wished to do so, his achievement was not what it should have been. One teacher (Mrs. B.) felt that Russ "did the minimum." Two teachers (Mrs. C., Hebrew, and Mrs. B., English) stated that Russ's effort was contingent upon whether he liked the teacher and/or the topic. In mathematics, Russ was in group 1, the top mathematics group; his teacher, Mrs. S., rated him as being in the top third of the 23 children in her group. She found him to be extremely bright and quick to understand new concepts.

In regard to his work habits, however, Mrs. S., his mathematics teacher, noted that Russ had problems in organizing and in keeping track of his work, and in remembering to complete his homework assignments. She mentioned that Russ was heavily committed to an afterschool hockey program, and to piano. Mrs. W. (Hebrew) as well commented on Russ's absent-mindedness and lack of organization. Mrs. W. said that Russ was like the proverbial "absent-minded professor." Mrs. W. related that upon one occasion, while Russ was ascending the stage for a piano recital, she recalled wondering whether he had remembered to bring his sheet music. In class, he was not able "to find the paper [he needed] on his desk because the entire desk was covered with papers." In teacher-centered lessons, and in student-centered independent work, his pattern of participation varied. While working in small work groups, one teacher found that at times Russ's attention would wander; he tended to let others do the work (Mrs. B). Another teacher stated that Russ was involved when in small work groups (Mme K., French).

As for the personal attributes, in terms of independence or dependence, the teachers felt that Russ was decidedly independent; one stated that he was ensconced "in his own world." The teachers felt that Russ had a "great" sense of confidence. One teacher commented that she thought Russ was sensitive inside, although he gave one the impression of "coolness." Russ marched to his own drum. One of the teachers (Mrs.

B.) remarked that Russ was not easily perturbed, and noted that perhaps he was far better off being easy-going and relaxed; it seemed from Mrs. B.'s remark that Russ's air of nonchalance may have been interpreted as a sign of detachment and lack of caring, and may have concerned some of the teachers. Two of the teachers felt that Russ could be both a *leader* and a *follower*; two of the teachers stated that he could not be a follower. All of them qualified their statements in some way. Mrs. B. said that in terms of knowledge Russ could be a leader, but that he would then let all the other members of the group do the work; he would have some difficulty being a follower because "he's too bright, he would constantly question the leader." Mrs. C. said that Russ "could be both kinds of a leader, a good leader, and a negative leader." Mme T. said that Russ could be a "negative leader, not a good model." Russ was named by two teachers (Mme K., Grade 6, and Mme T., Grade 5) as someone to whom the children would turn for help (Mme T. specified help in mathematics) if the classroom teacher were not present in the classroom.

In the Logo computer lab classroom setting:

Russ was considered a high achiever as the result of the study of the children's notion of right angle, and of angle generally (Zack, 1986b). The results of the analysis done for this study, and the analysis of other facets such as peer classmate intervention (Zack, 1988b), revealed that Russ had exceptional attributes. Russ was seen by both the researcher and the children to be expert in the mathematics aspects, and in the syntax and programming aspects (see Appendices M, N, and O). His approaches to problem solving were varied and appropriate (see Findings section on "Strategies used . . . in attempting to solve problems"). In the Logo classroom, Russ was a self-starter. Given a challenge and an introduction (actually re-introduction) to the concept of variable, Russ was able to use variables, and to work with proportional measure, in order to create a "stickman" of varied sizes. Monica, the Logo teacher, was unaware that in Russ's case, the idea of variable had been introduced previously by another teacher, Sharon, in Grade 4. However, the concept of variable is a difficult one for the elementary school child, and Russ seemed to be very comfortable with it. The fact that Jon H., and Michael H., both bright children, had difficulty with the notion of variable, indicated that even within the group of high achieving children, there was a discrepancy in being able to grasp this concept. Russ was atypical; he showed an exceptional

mastery of the Logo microworld. The designation of "class expert" was justified in Russ's case.

Russ's work habits in the Logo classroom indicated that he was interested and challenged by the Logo microworld; Russ was consistently attentive and engrossed in the work. He was independent. Russ rarely called for teacher intervention. Indeed, the findings indicated that when Russ joined Jon and Eric, the calls for teacher intervention dropped as compared to the time when Jon and Eric worked as a dyad. Interestingly, there was an increase in teacher-initiated teacher interventions once Russ joined Eric and Jon, since Monica seemed to attempt to oblige Russ to work at the organization of his written procedures, namely to subdivide them into smaller units (see Note 33; also Zack, 1986a).

In terms of his interaction with his peers, Russ got along well with his various partners. He worked with Michael H. for 5 sessions; they seemed to part ways due to Michael's concern about keeping up with, and about not understanding, Russ's work with variables for his "stickman" procedures. Russ worked with Jon and Eric as a triad for 4 sessions. There was no friction between the partners and the interaction went very smoothly; however, Eric did not remain an active participant in the work while in the triad situation. Russ was definitely the leader, and active contributor, in his various partnerships.

PROFILE #2: JON H. (age 10;3 in October 1985)

In the homeroom classroom setting:

In terms of general achievement, four of the five teachers interviewed felt that Jon was "above average"; one teacher (Mrs. B., Grade 6) felt he was an "average" achiever. In regard to the mathematics area, one teacher (Mrs. B., Grade 6) stated that Jon's strength is mathematics. In Grade 6, he was a member of the top mathematics group, group 1. The mathematics teacher of group 1, Mrs. S., said she would consider Jon would be among the middle third in the group 1 math group of 23 children. Mrs. S. found that at the beginning of the year, when Jon sat at the back of the class, he was distracted and she therefore changed his seat. In the latter half of the school year, she felt that Jon worked well, and that he "flourished."

With the exception of the above comment concerning Jon's distractibility during the mathematics classes in the fall of 1986, all the other teachers' comments indicated that Jon was an attentive, hard-working, conscientious student; he was involved and responsible during teacher-centered lessons, individual student-centered work, and small work group sessions. He was an independent student; the teachers felt that when he did ask for help, the call was appropriate and justified. One of the teachers said that Jon was a *follower*; two of the teachers said that Jon could be both a *leader* and a *follower*,. One (Mrs. B., Grade 6) of the latter two teachers qualified her statement concerning Jon's being a leader by stating that "he wanted to be a leader, and for the group members to be servants." Mrs. B.'s statement proved of interest when during the Logo analysis phase, the researcher noted the extent to which Eric H. was Jon's "go-fer." In relation to calls for teacher intervention, for example, Jon would tell Eric to go get Monica, as opposed to eliciting the help himself. It is of interest to note that Eric quite regularly resisted appealing to Monica for help; Eric seemed to want them to attempt to solve the problem on their own.

In the Logo computer lab classroom setting:

Jon H.'s achievement in terms of the mathematics concepts in Logo was judged to be high (Zack, 1986b). He understood what he was doing, and was confident in tackling problems. The findings showed that in instances where the material was new, for the most part Jon was able to follow and understand it (see Appendices M and S). However, in two instances during which he might have been a purveyor of information, Jon was either not able to explain it to his partner, Eric H., or was too impatient to do so (see Selections #9, and #14). Jon was able to answer analytically during the clinical part of the interview (May 28, 1987). Jon was not adept in the syntax or the programming aspects of the Logo environment, however, and here Eric played a dominant, helpful role in their partnership. As has been mentioned above, Jon seemed to want to call upon Monica more often than did either of his partners, Eric H. or Russ (H.). However, since the total number of calls for teacher intervention for the high achieving group was half the number of that for the middle and low groups (see Table 7), one cannot consider Jon excessively dependent. In regard to his relationship with his peers, Jon and his partners worked well together during the Logo sessions.

Table 7

Calls for Teacher Intervention

	TYPE OF CALL				TOTALS
Groups	Plan & Programming	Syntax	Class Rules	Mathematics Concepts	
Jon H, Eric H., and Russ H.	1	8	13	3	25
Lilly M. and Rina M.	-	22	21	3	46
Carol and partners	2	24	23	6	55
Amy and partners	2	23	25	4	54

Note: The average number of calls for intervention for the middle and low groups was 52. The number of calls for the high group was half of that, that is, 25.

PROFILE #3: ERIC H. (age 10;2 in October 1985)

In the homeroom classroom setting:

Four of the five teachers who commented upon Eric's achievement stated that he was an "average" student; these four teachers had taught Eric in Grade 5 and Grade 6. The fifth teacher (French), who had taught Eric in Grades 1, 3 and 4, remembered him as a good student, and mature and responsible. Thus there was not as much consensus in Eric's case as in most other instances. His Grade 6 teacher (English subjects) felt that Eric expressed his ideas well in oral form, but had difficulty with written expression; he was not creative. He showed interest in the content area subjects such as social studies and science. In mathematics in Grade 6, Eric was a member of group 3; he had been moved from group 4 (the lowest group) up to group 3 during the early part of the year in Grade 6.

In regard to his work habits, Eric was found by one teacher (Grade 6, French) to be attentive, and by another teacher to be inconsistent in his involvement. The latter teacher, Mrs. B (Grade 6, English, social studies and science teacher) found that Eric listened at times, but that she had to ascertain that he stayed involved during teacher-centered lessons. In student-centered lessons, that is, when working alone, Eric had difficulty concentrating (Mrs. B., Grade 6). One teacher (Mrs. B.) felt that Eric did not have the creative ideas to contribute when in small work groups; she felt he was a follower. Two teachers (Mrs. T. and Mrs. K.), however, found that Eric worked well and was involved while in small work groups. Eric's Grade 6 Hebrew teacher commented that Eric and Russ worked together closely whenever possible (for example, in small work groups). Four of the teachers found that Eric was dependent; one teacher found that he was easily distracted (Mrs. B, Grade 6). Four of the teachers felt that Eric was lacking in self-confidence. Mrs. B. commented that Eric was "nice, sensitive, and sensitive to other children in the class."

In the Logo computer lab setting:

Eric was judged to be one of the 5 high achievers in the Logo geometry component. The analysis done for the project on angle (Zack, 1986b) revealed that Eric had a good understanding of some of the aspects of angle and distance. However, when the in-depth analysis of the videotaped data for this study was done, it became clear

that there were instances during which Eric could not follow the train of thought of his partner Jon H. (see Selections #9 and #14) or of the teacher (Selection #14, and Appendix S). In Eric's case, there occurred the greatest discrepancy between the classroom assessment of math ability, and the Logo assessment. In view of some of this study's findings, it is possible that the Logo assessment of Eric's achievement which resulted from the project on angle (Zack, 1986b) might have been a bit inflated, and that Eric should have been considered to be functioning at the middle level; but in the Logo assessment he would not have been rated as functioning at the low level, which rating would however have corresponded to his placement in the school mathematics program, that is, group 3 of 4 groups.

Another area of contrast to classroom performace in Eric's case was the aspect of dependence. Four of his homeroom classroom teachers felt that Eric was dependent. Yet in the Logo classroom Eric consistently opposed Jon's efforts to call for Monica's assistance when he and Jon were partners. While Eric was partners with Jon in a dyad relationship, Eric was intent and completely involved in their project work. Eric did however display a marked pattern of distractibility when he became a member of a triad with Jon H. and Russ H. (see Appendix P).

PROFILE #4: LILLY M. (age 10;4 in October 1985)

In the homeroom classroom setting:

In terms of general achievement, three of the five teachers felt that Lilly was an "above average" student, and two of the teachers felt she was an "average student." In terms of her mathematics achievement, in Grade 6, Lilly was in group 1, the highest of the mathematics groups. Her mathematics teacher, Mrs. S., assessed that Lilly was in the bottom third of the group of 23 in group 1.

In relation to work habits, her teachers considered Lilly a serious student, who was very well motivated, diligent, "neat, precise and organized," involved and attentive during teacher-centered, individual, and small work group sessions. Three of the teachers felt that Lilly was an independent student; one teacher (Mrs. C., Hebrew, Grade 6) felt that Lilly's calls for the teacher's help were appropriate and justified. Two of the teachers found that Lilly did depend on their input more than they had expected she would. As far as personal attributes were concerned, one teacher noted that Lilly

was mature, responsible, and had a good sense of humor. Two of her Grade 6 teachers noted that Lilly had become self-conscious about her height (she was taller than all the children in her class), and that her sense of confidence seemed to be diminished due to her self-consciousness. Three of the teachers stated that they felt that Lilly was a *follower* rather than a *leader*; one teacher felt that Lilly could be both a *leader* and a good *follower*. One of the teachers (Mme T., Grade 5), named Lilly as one of the three children to whom the members of the class would turn to for help if the teacher were out of the homeroom; the other two children named by the teacher were Russ H. and Michael (H).

In the Logo computer lab classroom setting:

Lilly's achievement in relation to the mathematics aspects of the Logo environment was in the middle range. She seemed to have a good grasp of Logo angle, and the progress in her learning about Logo angle proceeded more quickly than that of her partner Rina M. (see Zack, 1988a in Appendix W). The findings showed that during the 1985-1986 Logo session, Lilly learned about 90 as a special input. During the February 1986 clinical interview, Lilly was able to distinguish the cause of an error, namely the cause of the jagged line in one of Lilly and Rina's projects; the cause of the error was not easily evident to Michael H. or even to Russ (H). During the May 1987 interview, with prompting, Lilly was able to use an analytic approach to determining the size of an angle (Appendix Y). Rina, on the other hand, could do neither, that is, Rina could neither distinguish the cause of the error nor use an analytic approach.

Lilly had difficulty with the syntax and the programming aspects. In relation to problem solving approaches, Lilly and Rina's problem solving approach relied upon access to, and dependency upon, their book note records. However, in comparison to her partner Rina, Lilly at least recognized that a one-to-one correspondence between the entries in the two books was not required.

Lilly and Rina worked well together as partners. At times they disagreed, in several instances vehemently; but there were no instances of breakdown of communication (as was the case (1) in the Amy and Carol partnership, Grade 4, (2) in the Amy and Leanne partnership, Grade 5, and (3) in the Amy, Carol and Leanne threesome, Grade 5). Lilly and Rina were partners throughout the twelve Logo sessions in 1985-1986.

PROFILE #5: RINA M. (age 10;6 in October 1985)

In the homeroom classroom setting:

Three of the four teachers who commented upon Rina's general achievement felt that she was "average," and one of the three said "average, maybe above average" (Mme T., Grade 5); one teacher (Mrs. B., Grade 6) felt that Rina was a "low average" student. In terms of mathematics achievement, the teacher who taught Rina in Grade 3 and Grade 4 felt she was a good mathematics student, but recalled that she had had difficulty with metric measurement. Her Grade 6 teacher (Mrs. B.) stated that while Rina worked very hard, she "relied totally on memory," and did not have an understanding of the concepts." Rina was a member of the group 3 (out of 4) mathematics groups.

Three of the teachers commented that Rina worked very hard. "Things don't come easily to her. She's a worrier" (Mrs. C., Hebrew). "She had to work very hard at getting a decent mark" (Mrs. B., English, social studies and science). Mme T. (Grade 5) said that Rina put in a consistent effort, and had good work habits.

In terms of Rina's personal attributes, three of the teachers felt that Rina was decidedly dependent: "She asked questions constantly to check," (Mrs. B); "The teacher had to keep reassuring her" (Mrs. C., Hebrew); "She wanted to be sure" (Mme T., Grade 5). Mrs. B. and Mrs. C. both felt that Rina lacked self-confidence. Mrs. C. mentioned that Rina would begin to ask questions before the teacher was finished speaking. "She was so busy asking questions about what she might not understand that she didn't listen." In responding to the question of Rina's interaction with her peers, whether she was a *leader* or a *follower*, or popular, one teacher felt that Rina could be both a leader and a follower (Mme T., Grade 5); a second teacher said she could be both, however her command of Hebrew was not good enough, and so she felt Rina would have been a follower (Mrs. C., Hebrew); and the third teacher (Mrs. B.) said that Rina wanted to be a leader, but would be a follower if there were a strong leader. Mrs. B. did add that in small work groups, Rina could at times be very persistent in insisting that the work be done "her way." One teacher felt that Rina was well adjusted socially (Mme G., Grade 3, 4). Another teacher (Mrs. C.) felt that while Rina had an enthusiastic "effervescent" way about her, she could at times be arrogant and short-tempered with her peers.

In the Logo computer lab classroom setting:

Rina's achievement in the mathematics aspect of the Logo program was in the middle, but low middle, range. She did not make as steady a progress in the aspects of learning about angle as did her partner Lilly (M.). (See profile on Lilly above). Rina had difficulty with the syntax aspects of the Logo microworld.

There were two ways in which Rina's way of working in the Logo classroom reflected what the teachers had observed in the homeroom classroom setting. Mrs. B. had noted that Rina relied upon memory, and that her understanding of the material was not good. There was evidence on Rina's part of almost absolute reliance upon teacher-given sources. One example of this reliance upon rote occurred when Rina depended totally upon the stencil instruction sheet which Monica had given the children, without understanding it. In two instances, Rina relied upon the use of the instructions, but followed the wrong set of instructions. Because she did not understand why the sequence was as it was, Rina used the "load" instead of the "save" instructions, and thereby erased the work done to date upon the computer.

The second example of Rina's dependence upon rote was Rina's absolute reliance upon the notebook records which she had recorded. As will be shown in the Findings chapter, Rina used the notebook source in an algorithmic way, as a "recipe." Rina and Lilly's main approach to solving problems was to reconcile their notebook entries, and to re-enter the commands. Due to the difficulties that Rina and her partner Lilly encountered, and perhaps due to their inability to solve the problems, it was noted during the analysis that halfway through the 1985-1986 Logo sessions, Rina adopted a "slow but sure" philosophy, in that she resolved to make note of every move that she and Lilly had made. Rina's predisposition to being careful, and to relying upon rote learning in the Logo classroom, reflected her mode of working in the regular classroom. In relation to the issue of dependence upon the teacher, however, there was not an excessive number of calls for teacher intervention in the Logo classroom on Rina and her partner Lilly's part, as one might expect that there might be in light of the homeroom teachers' comments above concerning Rina's dependency upon the teacher (see Table 7).

PROFILE #6: SHAUNA L. (age 10;11 in October, 1985)

In the homeroom classroom setting:

The three teachers who commented upon Shauna's general achievement felt that it was low. However, all three of these teachers stressed that they felt that Shauna's achievement was poor because her effort was inconsistent, and that when she tried she "did well." The teacher's reflections upon Shauna's achievement in mathematics echoed the aforementioned statements. Mme G. (Grades 3 and 4) said that Shauna's mathematics achievement was low "not because she didn't have the ability, but because she wasn't interested in school." In the Grade 6 mathematics groupings, Shauna was in group 3 (of 4 groups); her mathematics teacher, Mrs. D., stated that Shauna had a good grasp of the concepts, and could do well if she listened and attended consistently; she stated that Shauna was "underachieving." Mme T. (Grade 5) recalled that Shauna's achievement in mathematics was "not bad"; she mentioned that Shauna's work in geometry was good.

In regard to her work habits, two of the teachers commented that Shauna had poor work habits. Mme T. (Grade 5) said that Shauna did not concentrate and was "distractible," did not listen attentively in teacher-centered lessons, and could work well in small work groups if she were interested, but could also become distracted during small work group sessions, or during individualized work. Mrs. D. (Grade 6) stated that Shauna did not hand in her homework, and was preoccupied with social activities.

In terms of her interaction with her peers, Mrs. D. felt that Shauna wanted to be a leader, but could be a negative leader, an instigator. Mme T. felt that Shauna was a *leader* in the social sphere, but a *follower* in the academic domain; Mme T. felt that Shauna was not popular in the sense of having many friends. Mrs. W. mentioned the conflict between Shauna, Carol L., and Amy L. that had gone on during the Grade 5 year (see Profile: Amy, below).

In the Logo computer lab classroom setting:

Shauna's achievement was rated in the low level, but was at the high end of the low range, as a result of the analysis of the children's notion of right angle (Zack, 1986b; Table 4). However, the analysis done for this study offered new insights. As will be shown in the Findings chapter, while Shauna had not been correct in some of her

answers during the clinical interview (October, 1985; February, 1986), and while her work when viewed on the videotapes revealed that she was still unsure of some of the concepts which other children had mastered, such as the input of 90, or the amount of input for a total turn (360) or half turn (180), nevertheless Shauna had a better feel for the geometry microworld than did some of the children whose answers were correct (see Findings section on "One child's use of proportional measure," and Appendix L).

Shauna was consistently attentive and intent on her work during the four sessions in which she was videotaped. There were two instances (recorded by the researcher in the observation notes) during which Shauna was seen to digress for short periods of time (tussling with a peer in one case, and drawing on the chalkboard in another case); however, Shauna did not exhibit the distractibility of which her homeroom teachers spoke. Shauna was videotaped while she was partners with Amy L. (for 3 sessions), and with Carol L. (for 1 session). In terms of her interaction with her peers, there were no overt conflicts between Shauna and her partners. However, Shauna did have five different partners over the 12 period Logo session, more partners than any of the other study subjects.

PROFILE #7: CAROL L. (age 10,2 in October, 1985)

In the homeroom classroom setting:

Carol's general achievement was low. In the mathematics area, her achievement was below average; in Grade 6, she was placed in math group 3, of four groups. Her Grade 6 mathematics teacher, Mrs. D., found that Carol had difficulty mastering certain concepts (but Mrs. D. did not name any specific concepts). In teacher-centered lessons, Carol was said to be slower to complete her work than was the majority of the class. Three of the teachers felt that with a consistent effort, Carol could achieve better results. Her work habits were poor. One teacher, Mrs. G., who had taught Carol in Grades 1 and 3, recalled having suggested to the family that Carol should not stay in the multilingual school system. The family, however, chose to have Carol stay in the school, but did not help Carol by providing tutoring help or other support as was the case in other instances when children needed additional guidance (either academic or emotional, or both). Carol was quite dependent upon teacher feedback. She needed reassurance often. One teacher (Mrs. S., Grade 4) commented that Carol "[asked]

questions which [had] no relevance to what [was] happening." Her self esteem was not good. She was "young" and immature, and "sometimes silly about things" (Mrs. S., Grade 4). Carol's relationship with her peers was not good. She had severe altercations with some of the children (see Amy L.'s profile). In the classroom setting, she was not a *leader*, and she would not follow. During a small group activity, she might often choose to do nothing.

In the Logo computer lab setting:

In terms of achievement, Carol was placed at the low level, but she had a better grasp of some of the mathematics aspects than did some of her peers who were placed at the low level. She retained the notion of 90 as a special input for turn from the Grade 4 to the Grade 5 year (see Appendix Q), and seemed to follow more of what Monica explained than was the case with her partner Amy in Grade 4 (Appendix Q and Selection #19). Carol had difficulties with the syntax and with the programming aspects, as did the majority of the 23 Grade 5 children in homeroom 210. Carol concentrated well upon her work during the Logo sessions, and was not often distracted. Carol and her partners' pattern of request for teacher intervention was comparable to that of the other low level and middle level achievers (see Table 7).

Carol seemed to fare quite well within the Logo setting. However, her self image concerning her grasp of computers was not good. During the interview of October 23, 1985, Carol stated twice (once at the start, and again at the end of the interview): "I'm not good at computers" (see Note 20). In terms of her relationship with her peers during the Logo time, Carol was at times a difficult partner with whom to work. She had difficulty maintaining her partnerships (Note 21).

PROFILE #8: AMY L. (age 10;9 in October, 1985)

In the homeroom classroom setting:

The teachers felt that Amy's general achievement was low. Amy's profile is somewhat different from that of the other study subjects in that the teachers noted there was a growth and change perceived during the Grade 6 year. Thus, Amy's profile is different in the Grade 6 year, as opposed to the profile of Grades 1 through 5.

Amy's Grade 6 English and Hebrew teachers both commented that midway through the Grade 6 year, they noted an improvement in Amy's achievement. She had become an avid reader, and perhaps as a result, her confidence improved overall. She seemed to concentrate for longer time periods, and paid attention to the organization of her work (materials, homework, etc.). Her Grade 6 Hebrew teacher felt that the change might have been due to a growth in maturity. She seemed to be more independent; she worked more autonomously during student centered work, and in cooperative work groups.

The present study deals primarily with the Grade 5 year (1985-1986). Amy's achievement in the grades prior to Grade 6 was considered by four of her former classroom teachers to have been "very weak," and "below average." Amy's work habits and general attitude, and her interaction with her peers, in the classroom during the years prior to Grade 6, were not positive. She was disorganized, easily distracted, was dependent upon the teacher during teacher-centered lessons and often called upon the teacher for help and assurance. She lacked self-confidence. Even in Grade 6, when she gained in confidence, Amy was considered by her Grade 6 homeroom English teacher to be a *follower*, rather than a leader; that same Grade 6 teacher noted that Amy did not always want to cooperate and contribute when in a peer work group.

In regard to the years prior to Grade 6 (Grade 5 especially), two of the teachers noted that she had "difficulty getting along" with some of the children. One of the teachers, Mrs. W., commented upon one specific case of friction between Amy and two of her peers; all three children are study subjects. Mrs. W. recalled that there had been an extended feud between Amy, Carol L. and Shauna L. during the Grade 5 year; it was not clear, however, at what point during the school year the feud began. This constituted new information for the researcher; it may possibly explain why Amy and Carol did not work together as partners when in Grade 5, although they were in the same Logo session time slot together, and it may help to explain the rivalry between Amy and Carol who vied with each other in Grade 5 to have Leanne L. as a partner. Both Amy and Carol worked with Shauna at various times in Grade 5, for 3 and 1 Logo session(s) respectively, but the partnerships were not maintained. The interviews with the classroom teachers gave greater insight into the ongoing in-and-out-of-school social dynamics between these three children.

In regard to Amy's mathematics achievement throughout all the elementary school years, four teachers stated that Amy was extremely weak in that area of the program. In Grade 6, Amy was in the lowest of the streamed mathematics groups, that is, in group 4. Her Grade 6 teacher (Mrs. S) found that Amy had never grasped some of the basics such as the times tables, and how to do "borrowing"; her Grade 4 mathematics teacher (Mrs. G.) also commented that she had noted wide gaps in Amy's learning of the mathematics concepts, and attributed it to the fact that Amy's effort and attention were not consistent. This study's findings supported the teacher's observations that Amy had difficulty with certain basic computational aspects (see two examples, in Selection #32 and Appendix Q).

In the Logo computer lab classroom setting:

In the Logo setting, Amy's general achievement was low. She had difficulty with all the features of the Logo environment, the mathematics aspects (both the geometry and the arithmetic components), the syntax aspects (working the machine, such as loading, saving, and working in the editor) and the programming aspects. Her problem solving approach was to clear the screen and re-enter the commands again, and again; Amy did not attempt to stop and to try to determine what may have caused the error (see Note 23). In the Grade 4 analysis of the data, Amy's dependence upon Monica, the Logo teacher, for help was more marked than in the Grade 5 data. In Grade 5, the middle and low achieving groups had approximately the same number of calls for teacher intervention (see Table 7).

As far as relationships with her peers was concerned, there was one instance of negative, non-constructive behavior on Amy's part in her interactions with her partner, Leanne L., in Grade 5; at that time Amy was deliberately obstructionist. Leanne L. at one point indicated that Amy was sometimes belittling in her manner toward her (see Selection #18). For the most part, however, Amy was generally positive in her interactions with her partners.

PROFILE #9: LEANNE L. (age 10;6 in October, 1985)

In the homeroom classroom setting:

Four of the five teachers stated that Leanne's general achievement in the classroom was very weak; only one teacher (Mrs. T., Grade 5) felt that Leanne was an "average" student. All the teachers agreed that Leanne made a consistent effort and that school was "hard for her (Mrs. K.)." In terms of her mathematics achievement, the teacher who taught Leanne in Grade 3, 4 and 5 (Mme G.) felt that Leanne was a low achiever in mathematics: "[Leanne] was very, very low in math. She [had] difficulty with any new math [concept], even simple, that [was] presented." However, in the Grade 6 school year, Leanne was in the second to lowest, not the lowest, mathematics group, and maintained her place there; she was in group 3 (Mrs. D.'s group) of the four streamed groups, and managed to stay in that group because she was systematic in her work; one teacher (Mrs. S.) commented that Leanne relied on rote methods in her mathematics.

As far as her general work habits were concerned, the teachers found that Leanne worked quietly during teacher-centered lessons, and "tried very hard." It is of note that four of the five teachers stated that during teacher-centered lessons, Leanne sat silently, and would not ask questions, and that even if she did not understand she would not ask. Leanne worked quietly when expected to work independently during student-centered lessons. The teachers felt that she did participate (again, quietly) when part of a small work group, and that she would ask peers for help. The teachers had difficulty with the researcher question concerning dependence versus independence. They felt that Leanne was insecure and shy, and lacking in self-confidence, and that despite the fact that she would not ask the teacher for help, she was dependent upon teacher input.

In terms of her relationship with her peers, the teachers found that Leanne was a *follower*. She was meek and soft-spoken, and did not have many friends. Three of the teachers named one child as Leanne's "best friend"; this child, Denise (H.), was Leanne's Logo partner in the Grade 4 Logo session (and the pair was observed by the researcher), but Denise was not in homeroom 210 in 1985-1986. None of the teachers seemed to be aware that there was a "friendship" (and rivalry) between Leanne, Amy and Carol. One teacher (Mrs. K., Hebrew) felt that Amy L. and Leanne would not be good partners; she felt that Leanne and Denise (H.) would be good partners because

Denise is bright and a good worker and would serve as a model, and would be able to help Leanne.

In the Logo computer lab classroom setting:

Leanne's achievement in the mathematics concept aspect of the Logo program was low (Zack, 1986b). Leanne had difficulty with the mathematics concepts, the syntax, the programming and the problem solving aspects of the Logo environment. In many ways Leanne's difficulties and frustrations were similar to those experienced by Amy L. (see Profile #8 on Amy). Leanne did work diligently, but she was often at a loss because there were so many aspects which were a mystery to her. It is very likely that in her Grade 4 partnership with Denise H., Denise took the lead and performed many of the functions and tasks, and that Leanne depended heavily upon Denise. In the partnership with Carol L., and then with Amy L. (who both vied for Leanne as a partner), there was no one to assume the role of "expert," and so the Grade 5 partners, who were closer to each other in ability than were the Grade 4 partners Leanne L. and Denise H., struggled along together.

In terms of personal attributes, in both the Leanne-Amy (6 sessions), and in the Leanne-Carol (4 sessions) partnerships, and in the Leanne-Amy-Carol threesome relationship (2 sessions) in 1985-1986, Leanne was the follower and the more (most) acquiescent partner. However, this is not to suggest that she was meek or passive in the Logo setting. Leanne was an active partner, and did when needed voice her input forcefully. Leanne is by nature a very sweet and sensitive child, and in one instance when Amy had become obstructionist, Leanne did realize that she (Leanne) would have to become conciliatory in order to break the deadlock (Zack, 1986c). The issue of dependence and independence, and calls for teacher intervention, proved different in the Logo as compared to the classroom setting. Leanne and her Grade 5 partners' pattern of calls for teacher assistance was similar to that of the other groups at the low and middle level (Table 7). The fact that Leanne felt comfortable in calling upon Monica, the Logo teacher's, help contrasted strikingly with the homeroom classroom teachers' observations that Leanne "wouldn't ask for anything, wouldn't say anything, even if she didn't understand" (Mme G., Grade 3, 4 and 6) (see Note 27).

PROFILE #10: DOUG L. (age 10;6 in October, 1985)

In the homeroom classroom setting:

In terms of general achievement, Doug was considered a low achiever by the 4 teachers who commented. He did poorly in reading, and had difficulty with his writing, but had creative ideas. In the mathematics part of the school program, Doug was a member of group 3 (out of 4 groups); he was considered by his Grade 4 mathematics teacher, Mrs. B., to have been a weak math student. Mrs. D., his Grade 6 mathematics teacher, said that Doug had difficulty understanding new concepts. In the other content areas (science, social studies, and language study), Doug was considered "low average." One teacher commented that he was interested in politics and history.

Doug's work habits were depicted as follows: he had a short attention span, was inconsistent in his work pattern, was *extremely* dependent upon teacher feedback during teacher-centered lessons, felt uncomfortable working alone in student-centered lessons, and was easily distracted during small work group sessions. He completed his work in order to be done with it; "quick and dirty" was how one teacher described his effort to finish an assigned task or project. One teacher commented that she felt that Doug had a very low self esteem, and then went on to state that the issue was more complex than that; it seemed to her (Mme T., Grade 5) that Doug thought he was clever, but that he felt other people did not appreciate the fact that he was clever.

When asked about Doug's relationship with his peers, one teacher commented that he was neither a *leader* nor a *follower*; one teacher stated that Doug was a "loner." When one teacher, Mrs. W., was shown the listing of the Logo child pairs by the researcher, and was asked to comment upon her perception of the compatibility of the partners who had worked together, Mrs. W., stated that she did not know with whom Doug would be "good partners."

In the Logo computer lab classroom setting:

Doug was a low achiever in terms of the mathematics component of the Logo program (Zack, 1986b); he was rated last of the 23 Grade 5 children (see Table 4). Doug knew and used 90 as an input to RT and LT, but it is vital to note that 90 was the only input he knew. When asked about angles, either in reference to classroom geometry, or to Logo geometry, during all three clinical interview sessions (October,

1985, February, 1986, and May 1987), it became clear that Doug knew very little about angles.

Doug had difficulty with the syntax and the programming aspects of the Logo microworld as well as with the geometry component. When he was partners with Carol L. for 4 sessions, it seemed that Doug depended upon Carol for the syntax. Doug called upon Monica for help, but was no more dependent than the other low and middle level achievers.

Doug's work habits and his pride in his product within the Logo classroom provided a striking contrast to the pattern which was in evidence in the homeroom classroom. During the four sessions of work with Carol, Doug concentrated on his work intently. He drew a camel in his notebook as a goal, and during his sessions with Carol (who was not overly enthusiastic about the project, or about being partners with Doug) kept his focus directed toward the completion upon the Logo screen of the "camel" design. At one point, Doug strenuously refused to compromise on his goal, but then saw that he and Carol would have to alter it a bit (one hump instead of two) if they were to finish it. Doug showed great pride in his work. When his "camel" design was enlarged by the researcher for the purposes of discussion during the third clinical interview (May 24, 1987), Doug smiled to see the product again; after the interview, he asked if he could keep it.

Table 8

Ages of the Study Subjects in the Years During Which They Were Observed and/or Videotaped

Dates of the Logo Sessions	Oct. 82 - Nov. 82	Jan 84 - Mar. 84	Oct. 84 - Jan 85	Oct. 85 - Jan. 86	Oct. 86 - Jan 87	Nov 87 - Jan. 88
Grade	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7
Age of child in	Oct. 1982	Jan. 1984	Oct. 1984	Oct. 1985	Oct. 1986	Nov 1987
Russ H.				9; 10	10, 10	
Jon H.				10; 3	11; 3	
Eric H.				10; 2	11; 2	
Lilly M.				10; 4	11; 4	12; 5
Rina M.				10; 6	11; 6	12; 7
Shauna L.	7; 11[a]			10; 11	11; 11	
Carol L.		8; 5	9; 2	10; 2	11; 2	
Amy L.			9; 9	10; 9	11; 9	
Leanne L.				10; 6	11; 6	
Doug L.				10; 6	11; 6	

[a] 7; 11 represents 7 years, 11 months of age.

Note: In 1982, Shauna was part of another Logo project, in a University computer lab research setting, not in the regular school setting.

Table 9

Partnership, Videotaping and Attendance Record of the Study Subjects

		FOCAL CHILDREN AND THEiR PARTNERS				
Session	Date	Jon H. & partners	Lilly M. & Rina M.	Carol L. & partners	Amy L. & partners	Absences
1	Oct. 16, 1985	Jon H. & Eric H.	Lilly M. & Rina M.	Carol L. & Leanne L.	Amy L. & Shauna L	
2	Oct. 23, 1985	Jon H. & Eric H.	Lilly M. & Rina M	Carol L & Leanne L.	Amy L. & Shauna L	
3	Oct. 30, 1985	Jon H. & Eric H.	Lilly M. & Rina M	Carol L. & Leanne L.	Amy L. & Shauna L.	
4	Nov. 6, 1985	Jon H. & Eric H.	Lilly M. & Rina M.	Carol L. & Leanne L	Amy L	
5	Nov. 13, 1985	Jon H. & Eric H.	Lilly M. & Rina M	Carol L., Leanne L. & Amy L		
6	Nov. 20, 1985	Jon H. & Eric H.	Lilly M. & Rina M.	Carol L. & Shauna L.	Amy L & Leanne L.	
7	Dec. 4, 1985	Jon H. & Russ H.	Lilly M. & Rina M.	Carol L. & Doug L.	Amy L & Leanne L.	Eric H.
8	Dec. 11, 1985	Jon H , Eric H & Russ H.	Lilly M & Rina M	Carol L. & Doug L.	Amy L. & Leanne L.	
9	Dec. 18, 1985	Jon H., Eric H & Russ H	Lilly M. & Rina M	Carol L. & Doug L.	Amy L. & Leanne L	
10	Jan. 8, 1986	Jon H., Eric H & Russ H	Lilly M & Rina M	Carol L. & Doug L	Amy L. & Leanne L.	
11	Jan. 15, 1986	Jon H., Eric H. & Russ H.	Lilly M. & Rina M	Carol L., Leanne L. & Amy L.		
12	Jan. 29, 1986	Jon H., & Eric H.	Rina M.	Carol L.	Amy L. & Leanne L.	Lilly M. Russ H.

Note: The shaded sections indicate sessions which were not videotaped.

The Logo teacher

Monica Shapiro has taught the Logo graphics microcomputer language to children from Grade 3 through Grade 6 in the school setting of the study for the past six years. She also teaches the introductory and advanced Logo courses, as well as other technology courses, to adults in McGill University Faculty of Education's Technology Certificate Program. Monica has been actively involved in issues concerning Logo and teacher education (Shapiro, 1985). Since Monica is exceptionally well-informed and secure concerning the Logo programming language, and is also a wonderfully able and sensitive teacher, she cannot be considered representative of the "average" Logo teacher.

In the elementary school computer lab classroom, Monica's pattern of teaching was to encourage the children to choose their own projects, and their own partners; she intervened according to need, whether the need on the part of the children was academic or social. The nature of the learning environment which Monica provided was touched upon briefly previously, in the Review of the Literature section. At the core of the activity was the child. The drive to learn emanated from the child. The child's inquiry was nurtured by the teacher who served the multiple functions which Barfurth (1987) has identified, those of *guide, facilitator, provoker, demonstrator* and *participating observer* (Barfurth, 1987, p. 38).

Monica's philosophy, and aims for the Logo program

The section which follows will present Monica's aims for the Logo program. Monica and the researcher had had many informal discussions over the period of 3 years (1983-1986) in which they were involved in the research project together. As the result of an in-depth (3 1/2 hour) interview with Monica, held September 18, 1987, the researcher gained yet further insight into Monica's philosophy. Monica responded to interviewer questions dealing with diverse aspects, such as (1) her aims for the Logo program, (2) her objectives in having the children work together in pairs, and in encouraging interaction between all the children in the computer class, (3) her role as a Logo teacher in this setting, (4) the degree of structure she had provided, and (5) the connections which the children made and did not make between Logo geometry and school geometry (Appendix E). Monica's reflections upon these questions gave insight into her philosophy; her remarks will be shared in brief below.

Monica's aims for the Logo computer program

Monica's overall aims for the Logo program touched upon both the cognitive and the social aspects. We will deal first with the cognitive, and then with the social, dimension.

Monica denoted three main, general cognitive objectives during the interview. One, Monica envisioned that the children would discover new ideas while they "worked the gears" of the Logo geometry microworld-- a reference to Papert's own personal eureka-like discovery at a young age of how the gears worked (Papert, 1980).

Two, Monica expected that the children would be learning about geometry, but not in a standard way; they would first get the feel of the turns, and thereby understand the concept of angle spontaneously and implicitly. Monica's reflections were reminiscent of Vygotsky's (1962) comments on spontaneous versus scientific concepts (as quoted by Burnett, 1981) and of Groen's (1978) belief in an active learning process whereby the child makes meaning by re-inventing or reconstructing a rule; both Vygotsky (Burnett, 1981) and Groen (1978) were cited earlier in the Review of the Literature section. Monica felt that the children would later be able to fit the Logo geometry learning to what they were doing content-wise in the official school mathematics curriculum.

> I wanted them to just have experience with what an angle is, but way before (pause)- I didn't even want to tell them that is an angle. I wanted then when they would come to angles in Grade 8 or 7. . . I would [hope] that the teachers, then when they do angles, would be able to latch on to our gears, which was our Logo. I think that [the children would] have to digest that turning and play a lot with the turtle before I would want to give them the word *degree* and *angle*.

Three, Monica's aim was that the children would progress at their own rate, and would "make their own" whatever knowledge they did absorb, no matter how small a bit it might be. In giving instruction, Monica interacted with a pair when they were at the work station; but Monica was seen to consider them as individuals and also to work with them as individuals even whilst they were in pairs. Concerning the aspect of autonomy and individualization, Monica touched upon the affective domain as well as upon the cognitive domain when she said:

> The [aim was] very definitely that the kids could progress really on their own, at their own rate. . . . One of the other aims of the Logo program . . . is this autonomy thing. I wanted whatever they can learn, whatever they do, to be able to be done without me. So that even if they learn less. . . they might only know five little-, this much,- Let them feel the joy, that 'I can do it'. . . . So that was an aim for me. It was individual, meaning kids moving at a rate [at which] they were comfortable and enjoying their learning, and also kids being able to do what they can do on their own.

It is important at this point to highlight the fact that although Monica indicated that she expected that the children would be learning geometry ideas, the mathematics part of the Logo curriculum, while a vital feature, was not the sole or even the primary focus in her program; the computer literacy aspect, and the student interaction component, were also named by her as important considerations. This factor must be kept in mind in regard to the research findings relating to the students' learning of mathematics concepts. For in this study, it was the researcher who chose to investigate the mathematics learning within the setting. Monica's Logo curriculum was clearly not driven by a mathematics agenda. A specific mathematics agenda was however in place in most of the studies outlined in the comparative chart of recent Logo research studies (Table 2).

We will now deal first with the social, and then with the socio-cognitive, components of Monica's objectives. As a matter of interest, it must be pointed out that although children worked in pairs in six of the twelve studies listed on the comparative chart, that is six excluding this study (see Table 2), in only one of the studies, namely the Hoyles, Sutherland and Evans (1985) study, was there an explicit rationale given for the fact that the children would work in pairs and an expressed belief in the socio-cognitive benefits of working in pairs (Hoyles, Sutherland & Evans, 1985, p. 5). In regard to this study, Monica's answer in response to the second interview question concerning her aims in having the children work together in pairs in the Logo computer lab classroom made it clear that the social dimension was considered by Monica to be as vital as the cognitive aspect. Monica considered Logo as much a vehicle for learning how to interact with a partner, as a vehicle for learning geometry concepts. "That's why I like Logo, you see. I like that aspect of it. . . . It's 50-50 It's that aspect [social

interaction, and learning life skills of communicating and getting along and negotiating with a partner], and then the other aspect is the gears and the building blocks."

In regard to the socio-cognitive dimension, Monica felt that the children could learn as much from each other as from the teacher. Monica used the phrase "Ask three before you ask me" in order to encourage the children to make use of other "authorities" in the class who could share their expertise. During the interview, an interesting discussion between Monica and the researcher ensued concerning the extent to which the classmates were and were not able to be of help to those seeking help. The findings concerning the analysis of peer classmate intervention, as well as Monica's reflections upon the findings, have been presented elsewhere (Zack, 1988b in Appendix X). Monica was surprised to hear (1) that the children did call upon each other for help more often than she had thought, but that (2) the helpers were as likely to be incorrect as to be correct when responding. After contemplating the latter comment, Monica gave evidence again of her commitment to the value of peer collaboration when she asserted firmly her belief that even if the chances were only 50% that a peer helper might be right, nevertheless part of the life skill of interacting with other people in a relatively new learning setting is that of distinguishing "the good help from the bad help."

As a final note, it is of import to relate that Monica's interaction with the children reflected the expert work of a teacher who herself vividly remembered the confusion and the newness entailed in the learning of Logo. Indeed, Monica the Logo teacher and Vicki the Logo researcher had met while learning to use Logo, and to use a computer, under Groen's tutelage, in 1982. Whereas in other classroom instruction, the teacher is decades removed from the role of learner of elementary level material, in this instance both the teacher and the researcher were able to relate in a far more immediate way to the children's learning and to certain of their difficulties than would otherwise have been the case.

The researcher

The researcher had been involved in the school setting of the study prior to the research project, in the role of McGill University supervisor of preservice teachers, for 7 years. Her acquaintance with the school staff undoubtedly helped to facilitate her entry into the school setting in the role of researcher, and enabled her to continue in

the school on a longitudinal basis.

The researcher was a non-participant observer. During the 1985-1986 year of the project, she bore the sole responsibility for the note-taking, and the working of the equipment. The researcher attempted to avoid any intervention in the children's work so as not to affect the normal course of teacher-student and student-student interaction. The presence of the researcher is always a factor in altering to some degree the natural workings of the class. After an initial period of curiosity and "playing to the camera," the study subjects seemed for the most part to ignore the camera videotape equipment and to carry on with their work. It was felt that the intrusion factor was minimal, and that the study captured as true-to-life a learning situation in this specific setting as was possible under research conditions.

Data Collection

The data sources, and the tools used for data collection

In order to construct as complete a picture of the child's understanding as possible, it was necessary to compile and analyze data from varied sources; no one medium could suffice. We will shortly touch upon the notion of *triangulation*, that is the use of multiple data sources (as well as multiple methods and more than one researcher), as a means of enhancing the validity of the research findings. First, however, we will list what the various tools for data collection were, and will indicate very briefly the level of detail of the transcription work. Details concerning the research questions, and the way in which the analysis of the data was done, will follow in the next section.

Sources of the Data:

- Videotape records of the Logo sessions--

  (a) the camera videotape which captured the face to face interaction of the children (see Figure 3)

  (b) the videotape of the Logo computer work (see Figure 4)

  Thus there were 2 videotapes made of each pair per session. The sessions were taped from the beginning to the end of the class, including the whole class interaction with the teacher which took place at the start and at the

end of the class. The portion of the tape which represented the child-child interaction at the computer work station (30-45 minutes) was later transcribed (see below).

- Review notes of the videotaped sessions done by the researcher after the sessions

- Researcher observation notes done during the sessions (but no extensive field notes were subsequently written up).

- The children's notebook records (Appendix C)

- Interviews with the students

  (a) Beginning of the Grade 5 session (October, 1985), with all the 23 students in the Grade 5 class; cognitive tasks administered (audiotaped) (Appendix F)

  (b) End of the Grade 5 session (February and March, 1986), with all the 23 students in the Grade 5 class; cognitive tasks administered (videotaped) (Appendix F)[2]

  (c) End of the Grade 6 year (May and June, 1987), with 56 of the 69 Grade 6 students in the school; cognitive tasks administered, as well as elicitation of student impressions concerning aspects of the Logo sessions such as the collaborative element, the time frame, and the teacher's role (audiotaped; videotape of the cognitive tasks portion) (Appendix G, Appendix H)

- Interviews with the Logo teacher, Monica

  (a) Teacher-researcher informal discussion transpired after most of the Logo sessions. Monica also shared the brief comments she had written in her journal concerning the children's work.

  (b) Long 3-hour interview with Monica to discuss her philosophy of teaching Logo, the children's progress, and her reactions to student

[2]The results of the Grade 5 interviews (October, 1985, and February and March, 1986) have been reported at length elsewhere (Zack, 1986b). The most relevant features of the Grade 5 interviews will be referred to in the main Findings section of this study.

interview and tape comments (September 8, 1987) (Appendix E)

- Interviews with 9 of the 11 classroom teachers who taught the 23 children in Grade 5, and in Grade 6, in order to aid in the development of a child profile for each of the 10 study subjects (September to December, 1987) (Appendix D)

- Cognitive tasks administered (during the interviews only)

- Stimulated student recall during interview #3 (Grade 6, May and June, 1987), using portions of student's own videotape, pre-selected by the researcher

- Transcription of the portion of the session which represented the partners' work time at the computer (30-45 minutes of videotape) was done by the researcher. Each of the twelve sessions videotaped (with the exception of 3 which were not videotaped due to lack of equipment), representing each of the focal children and her or his partner, was transcribed (See Table 9 for the record of videotaped sessions). Forty-three sessions were transcribed. In the transcription, the researcher attempted to display

  (a) original text of speech, indicating pauses, and pitch changes
  (b) description of most salient nonverbal behaviors (keypress intention and motion, direction of eye gaze, gestures, facial expression) that co-occurred with transcribed speech
  (c) description (when pertinent) of location of speakers in space, relative to each other and to the computer; particular note was made of teacher movement and of peer classmate movement into the frame of the camera videotape of face-to-face pair interaction (see Figure 3)

  The transcript conventions which were used are outlined in Appendix I.

All the materials were collected, reviewed, summarized, transcribed (entire videotapes, parts of interviews), and indexed (informally) for reference so that instances of the occurrence of "angular rotation," and of verbal and nonverbal behavior related



Figure 3. Videotape of camera view of a dyad

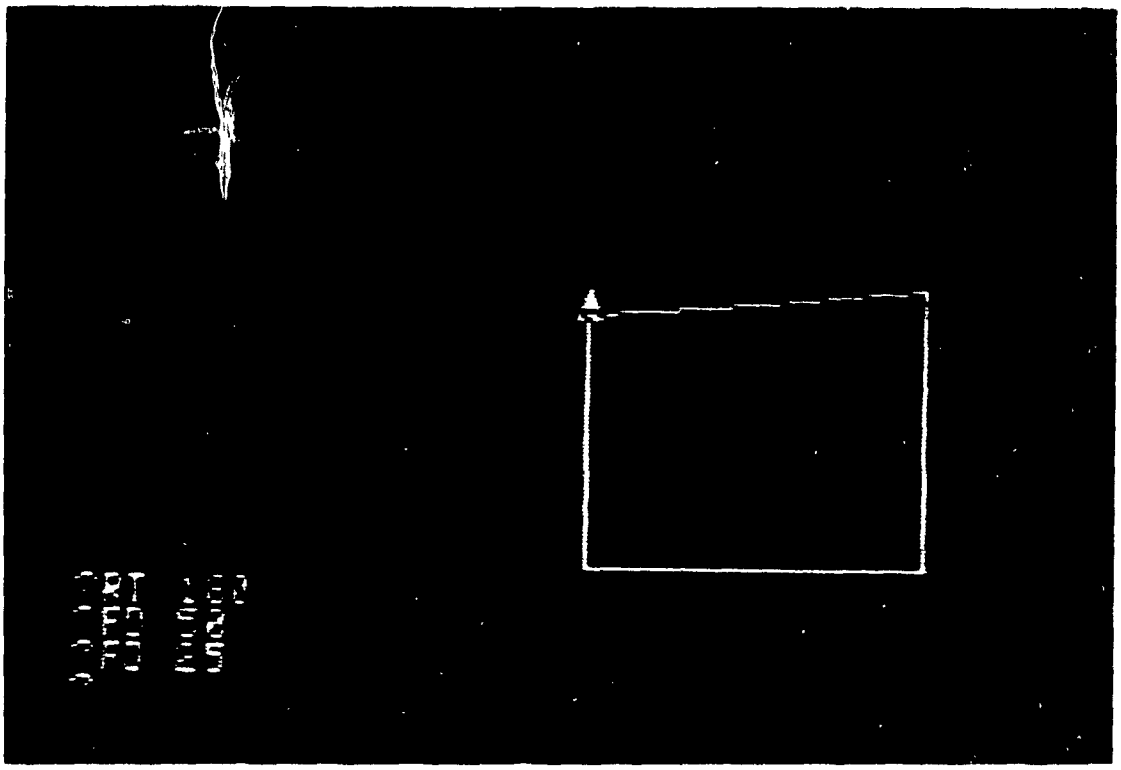


Figure 4. Videotape of computer monitor screen view of the Logo work

to the concept, could be retrieved. This data was of course intended later to serve as material for a fine grain analysis of how the use of verbal and other strategies during disagreements could reflect the children's understanding of the notion of angular rotation in Logo geometry.

Data triangulation

There was an effort made to determine points of convergence and nonconvergence among accounts and sources. What Mathison (1988, p. 14) has referred to as a "data triangulation method" was used in this study in relation to data concerning (a) the interpretation of selected Logo episodes, (b) the evaluation of the children's achievement, and (c) the impressions concerning various aspects of the Logo sessions such as the collaborative component, and the role of the teacher. The researcher's, the Logo teacher's, the classroom teachers', and the children's perceptions were all taken into consideration. Thus, the "data triangulation" method (Mathison, 1988, p. 14), which involves the use of several data sources, played a role in this study. The "investigator triangulation" method (Mathison, 1988, p. 14), which requires the involvement of more than one investigator in the research, could not however be used, and the lack thereof represented a methodological shortcoming in this study (see METHODOLOGY section on "Interrater correspondence check").

Triangulation is considered "a strategy for increasing the validity of evaluation and research findings" (Mathison, 1988, p. 13). Of note is Mathison's (1988) stress that the strategy of *triangulation* does not lead to convergence (1988, p. 13), nor does it eliminate bias (1988, p. 14): "[Triangulation] results in convergent, inconsistent and contradictory evidence that must be rendered sensible by the researcher" (1988, p. 13).

The recording equipment

Both videotape and audiotape equipment was used in the study. The audiotape equipment used was a Sony TC 110-B cassette tape recorder, with an external microphone attached (and a power supply cord). The audiotape equipment was used for the interview sessions, and rendered excellent sound quality.

The videotape equipment used was as follows: 2 Panasonic WV 3180-6X video cameras, 4 Panasonic NV 8420 port-o-pac videotape recording machines, 4 local microphones (2 per dyad), 4 NVB-59 battery adapters, and two tripods for the cameras.

During four of the sessions, another set of equipment was available for a third computer work station, and it was set up at computer #3; it was a camera/videotape unit (that is, a video camera and video recorder unit in one, in this case a General Electric 1-CVM-6060-E Camcorder), a tripod, 2 local microphones, and a Panasonic WV 3180-6X port-o-pac recording machine for the videotape of the computer work. (See Figure 1 for an aerial-type axonometric view of the classroom equipment set-up). As was mentioned above, for each of the twelve sessions, there was a camera videotape made of the children's face-to-face and face-to-computer interaction, in order to track the verbal and non-verbal behavior of the dyad, and the within-frame classroom interactional activities (see Figure 3). As was mentioned previously as well, there was also a videotape "dump" recording made of the children's Logo work on the screen (see Figure 4), with its own accompanying soundtrack. The local microphones attached to both the Panasonic port-o-pac machines (i.e. to each camera, and each video dump) allowed for a clear sound track centred on the pair in question. When Monica entered the frame to interact with a pair, the camera and local microphone were able to capture very clearly Monica's intervention with the study subjects. However, when Monica took the children to another part of the classroom, although one of the two or three cameras was likely to capture the visual record, the sound quality was poor due to the other activity in the classroom.

The use of videotape as a data collection tool was particularly valuable. The videotape equipment served to capture the events at the computer work station, although admittedly these events were only a microcosm of the activity which went on in the room. The use of videotape made the data retrievable. The researcher was able to return to the videotaped (and audiotaped) data as often as was needed during the analysis phase of the study. As has been noted above, selected portions of the videotapes were used with the ten study subjects during the Grade 6 interviews (May-June, 1987), in order to elicit the child-as-informant's perspective concerning the Logo episodes, and the socio-cognitive aspect of the interaction. In addition, the researcher used the videotapes to reanalyze and reinterpret the data in various ways; diverse aspects investigated were, for example (a) the Logo teacher's intervention with the children (Zack, 1986a), (b) the Logo teacher's use of metaphor to enhance cognitive and social knowledge (Zack, 1987a), and (c) the role of nonverbal messages in social interaction (Zack, 1987b).

Data Analysis

The components which were analyzed within the talk were as follows: (a) the kind of verbal strategies used during the disagreements; (b) the level of formality of the mathematics talk; (c) the mathematics strategy used; and (d) the Logo programming mode (draw mode or edit mode) within which the talk and work occurred.

The primary focus of the study was on the verbal strategies used during the disagreements. The next section of the methodology chapter will deal in detail with the aspect of the verbal strategies; it will review the research questions in order to focus again upon what the study aimed to do, and then will elaborate upon the procedure used in the analysis of the disagreements, in order to relate how the data analysis was done.

Dyadic disagreements and their relationship to Logo learning

The analysis of the videotapes, and of the other data, was done with a view to answering the following research questions:

GENERAL: What role do disagreements over a Logo move, and the discussion associated with the disagreements, play in the children's learning of Logo mathematics and geometry concepts?

SPECIFIC:

I CAN THE PATTERN OF THE DISAGREEMENTS--their frequency, duration, resolution, and the category of mathematical idea--GIVE ANY INDICATION OF THE CHILD'S UNDERSTANDING OF LOGO ANGLE?

II DOES THE KIND OF RELATIONSHIP THAT EXISTS BETWEEN THE PARTNERS INFLUENCE THE LEARNING? IF YES, IN WHAT WAYS IS THE LEARNING FACILITATED OR HINDERED?

III WHICH VERBAL STRATEGIES DOES THE CHILD USE IN THE COURSE OF A DISAGREEMENT? CAN THE STRATEGIES CONTRIBUTE TO THE CHILD'S LEARNING, AND TO THE

LEARNING OF THE CHILD'S PARTNER?
ARE SOME OF THE VERBAL STRATEGIES WHICH ARE USED DURING DISAGREEMENTS MORE EFFECTIVE IN LEADING TO LEARNING THAN ARE OTHERS? IF SO, WHICH ARE THEY?

The rationale for the consideration of each of the components, and an elaboration of the categories or the criteria which were used, will be presented below.

Research question #1: The pattern of the disagreements

The first component which was analyzed was that of the pattern of the disagreements. The focus was on the disagreements which related to the geometry and arithmetic aspects of the Logo learning, aspects such as distance, rotational turn, and computation, as well as iteration and variable if the last two (which are more difficult concepts) did appear in the work of the children. The aim was to determine whether there was a relationship between any of the aspects related to the disagreements, such as their frequency, their duration, their type (that is, the mathematical idea), and their resolution or nonresolution on the one hand, and the children's achievement vis-a-vis their understanding of Logo angle and of other mathematics and geometry concepts on the other. For example, did the children who were more adept at the Logo geometry work argue more or less frequently, did they sustain longer or conduct shorter disagreements, resolve more or fewer disagreements, on more difficult or less difficult mathematics concepts, than their peers who were not as adept?

Research question #2: The working relationships between the partners

Hawkins, et al. (1982) and Carden (1986) have stressed the need for research concerning the effects that peers working together in the microcomputer setting can have upon the children's learning. It was deemed necessary, therefore, to look at the kind of working relationships which existed between the partners, and to attempt to determine the extent to which the existing pairings either promoted or inhibited learning. Thus the second main component which was dealt with was that of the working relationships between the partners.

DOES THE KIND OF RELATIONSHIP THAT EXISTS BETWEEN THE PARTNERS INFLUENCE THE LEARNING? IF YES, IN WHAT WAYS IS THE LEARNING FACILITATED OR HINDERED?

Cooper, Marquis and Edward's (1986) definition of peer cognitive role relationships was used in the investigation. Cooper, et al. (1986) have stated that peer interaction may be *doer-doer*, in which case both children are exploring and neither is in the position of information giver, or it may be *knower-doer*, in which case one child possesses the knowledge which the other partner needs. In *collaborative, symmetrical* interaction, the children share the power of directing the interaction more equally (Cooper, et al., 1986, p. 282) than would be the case in *didactic, asymmetrical* interaction, where one child knows the information, and assumes the role of information-giver. Whether the cognitive relationships between the partners in this study were *doer-doer* or *knower-doer* or a combination of both was an aspect to be explored.

One would expect that learning might be facilitated in cases where the partner demonstrated, or used supporting talk to elaborate or to explain. One would expect that learning would not be advanced as a consequence of peer interaction in cases where the partner was not able to explain, was not able to explain clearly, or was unwilling to discuss, explain or elaborate. Thus the main component that the study analyzed was that of the verbal strategies that were actually used by the partners during the course of a disagreement, with a view to investigating their relation to the learning.

Research question #3: Verbal strategies used during disagreements

The third component of the investigation was focal, and dealt with the verbal and nonverbal strategies used by the children during the disagreements. The investigation dealt first with the inquiry concerning which strategies were used; then it dealt with how the verbal strategies (1) might contribute to learning and (2) might reflect the children's understanding of Logo geometry.

The two levels of verbal and nonverbal strategies, and the categories within the levels, were based upon the work done by Eisenberg and Garvey (1981) and Genishi and Di Paolo (1982). During the analysis phase of the 1984-1985 study on the

disagreements between Carol L. and Amy L. (Zack, 1985), the researcher found it necessary to add a number of additionai entries. The original Eisenberg and Garvey (1981), and Genishi and Di Paolo (1982) categories are listed in Appendix AA. The categories and definitions which were used in this study will be presented shortly.

The Level I strategies are statements or actions with no accompanying elaboration (for example, *insistence, denial, challenge, action taken*). The Level II strategies are statements which elicit elaboration (for example, (a) *request for explanation, request for information*; and (b) *call for teacher or for peer intervention*), or which furnish elaboration (for example, (c) *supporting argument, demonstration, focusing, reference to recorded notes, reference to prior knowledge, reference to computer screen verdict, and conditional test hypothesis*; and post disagreement discussion). A disagreement which featured the use of only Level I strategies was a *simple argument*. A disagreement which featured the use of both Level I and Level II strategies was termed a *complex* disagreement. The broad research question which guided this part of the investigation was:

> WHICH VERBAL STRATEGIES DOES THE CHILD USE IN THE COURSE OF A DISAGREEMENT? CAN THE STRATEGIES CONTRIBUTE TO THE CHILD'S LEARNING, AND TO THE LEARNING OF THE CHILD'S PARTNER? ARE SOME OF THE VERBAL STRATEGIES WHICH ARE USED DURING DISAGREEMENTS MORE EFFECTIVE IN LEADING TO LEARNING THAN ARE OTHERS? IF SO, WHICH ARE THEY?

The Level II strategies section could be subdivided into three main parts. These three parts were: (a) requests for explanation or information, (b) request for help from teacher or peer, and (c) instances of supported talk. The aim was to pay particular attention to the children's use of the categories within these three sections, for it was felt that the use of these approaches might be conducive to learning. It was thought that the use of Level I strategies alone, that is *simple arguments*, would not be as conducive to learning via interaction as would *complex arguments*, since no elaboration transpired when *simple arguments* were used. It was not sufficient of course to merely identify the number of *simple* as compared to *complex* arguments used; one needed to investigate

what it was that the children said. Thus first the quantitative, then the qualitative aspect, needed to be covered.

In the pilot study which dealt with the disagreements between Carol and Amy concerning Logo moves (Zack, 1985), it was found that the two did not use many higher level (Level II) strategies. In looking at the disagreements within the talk of these two, Carol L. and Amy L. as they worked with new partners in the 1985-1986 year, the researcher aimed to see whether there would be a rise in the use of Level II strategies as the children gained more experience in Logo, as was the case in the Hoyles, Sutherland and Evans (1985) study. In regard to all the pairs, it was considered vital to see whether there was a relationship between the incidence of use of higher level arguments and the child's grasp of the concept of angle in Logo. There were thus a number of subsidiary research questions which guided the investigation into these just-noted, and other, diverse aspects of the use of verbal strategies. The subsidiary research questions were as follows:

> Does the child use supported talk? Are the supporting arguments used, correct? Are they clearly stated? Do the supporting arguments clarify the mathematical theorem? Does the degree to which talk is supported increase over time?
>
> Do the verbal strategies used by each participant in a dyad during a disagreement reflect each child's understanding of angle? That is, is the child who has a good grasp of Logo angle more likely to use more higher level strategies than a child who does not have a good grasp of Logo angle?
>
> During disagreements, are there instances of "asking for help" or for explanations, from a partner, from the teacher, or from a neighboring peer? If yes, what can those instances tell us about the child's understanding of angle, and the child's learning strategies?
>
> During disagreements, does the child give help or explanations? Can the instances of the child "giving help" or giving explanations tell us

about the child's understanding of angle, about the child's ability to explain, and about the child's teaching strategies. If so, what can they indicate?

It was determined that particular attention would be paid to the occurrence of both solicited and spontaneous explanations due to Webb's (1982a; Webb & Kenderski, 1984) stress on the important role that peer explanations play in learning. More detail about the facet of *explanations* will follow in the part of the chapter which deals with the qualitative analysis of the data.

The analysis procedure for identifying the disagreements, and coding the episodes

The following section will outline how the disagreements were identified, the categories applied, and the manual coding and frequency distribution count done. The researcher returned to the videotapes and to the transcriptions a number of times during the data analysis phase.

Identification of the disagreements themselves

The Eisenberg and Garvey (1981) definitions and conventions were used for the identification of the adversative episode itself. A disagreement episode was defined as the interaction which grew out of an opposition to a suggested action by a child. "The negating responses or oppositions include refusals, . . . denials, and objections. Thus, an adversative episode is a sequence which begins with an opposition and ends with a resolution or dissipation of conflict" (Eisenberg & Garvey, 1981, p. 150). The following conventions were used in the examples, following Eisenberg and Garvey (1981, p. 151): The suggested action, an event antecedent to the opposition, was numbered 0, the opposition statement was numbered 1, and all subsequent responses and counter responses were numbered sequentially; events immediately preceding the antecedent event, or following the agreement, or not pertaining to the specific topic of the disagreement, were unnumbered. Each numbered line represented a change of speaker in a dyadic interaction.

Example #1A

Jon and Eric, October 16, 1985, D/A # 4, p. 6.

		CODING	
Jon H.:	Awright, now let's write [he means type], (pause) no (pointing to the screen) now we have to make a right turn (finger on the keyboard key).	0.	[PROPOSER]
Eric H.:	I know, I know (in a high-pitched voice).		
Jon H.:	Right turn ninety.		
Eric H.:	That's a left turn, Jon.	1.	[OPPOSER]
Jon H.:	Right, (point to the right on the screen), that way.		
Eric H.:	(Keys) (pause) Left.		
Jon H.:	Right.		

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The child who made the initial opposition was considered the *opposer*; the child who suggested the action was then the *proposer*.

For each mathematics concept disagreement, the researcher distinguished which mathematics concept was being deliberated at the time of the episode. In Example #1A above, the children were arguing about *lateralization*, that is, whether they should turn the turtle left or right. The list of the various mathematics concepts which were the topics of the disagreements is available in Appendix K.

One can see all the components of the analysis of the disagreement in the elaboration of the aforementioned transcript, in Example #1B below. The suggested action was coded as either supported or unsupported; if supported, then the specific Level II category used was noted. The statement(s) made by both the proposer and the opposer were categorized as correct or as incorrect wherever it was possible to evaluate the statement in such a way. The researcher then determined which verbal strategies

were being used by the children in each of the turns of talk, and recorded the verbal strategies (and later, the corresponding code number) upon the transcript page. Whether the disagreement was resolved or not was noted.

Example #1B-- an elaboration upon Example #1A
Jon and Eric, October 16, 1985, D/A # 4, p. 6.
Disagreement concerning lateralization

		CODING	
Jon H.:	Awright, now let's write [he means type], (pause) no (pointing to the screen) now we have to make a right turn (finger on the keyboard key).	0.	SUGGESTED ACTION (UNSUPPORTED) (INCORRECT)
Eric H.:	I know, I know (in a high-pitched voice).		
Jon H.:	Right turn ninety.		
Eric H.:	That's a left turn, Jon.	1.	Counter/contradiction (CORRECT)
Jon H.:	Right, (point to the right on the screen), that way.	2.	Insistence, supporting argument, focusing (INCORRECT)
Eric H.:	(Keys) (pause) Left.	3.	Insistence
Jon H.:	Right.	4.	Insistence
Eric H.:	It's a left, because (stands up and turns himself to the left) it's going down (points down) and it's turning like that (indicates right with his finger on the screen)	5.	Insistence supporting argument focusing demonstration (CORRECT)
Jon H.:	Oh- (pause) O. K., (?we'll solve this).	6.	Conditional test hypothisis
Eric H.:	(stands and keys) Left ninety?		
Jon H.:	Ya.		

CODING

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Jon H.:	(keys -L -T 90)		silence computer gives verdict
	(They look at the screen)		acceptance [RESOLUTION]
Eric H.:	(?I) Told you (smiles, then sits down).	7.	Post disagreement comment

A return to the videodump videotapes (see Figure 4) was necessary in the evaluation of the correctness or incorrectness of a move. A return to the camera videotape was at times necessary in order to verify intonations, nonverbal facial expressions, and important kinesic movements such as the child's focusing with finger or hand upon the screen.

Once the verbal categories and all the other necessary information were written on the transcript page, and checked against the videodump tape, the entries for each disagreement were coded upon a sheet; one sheet was used per disagreement (see Appendix J, coding sheet). The frequency distributions were then calculated manually. It was expected that the data in the coding sheets would serve to give information concerning certain of the cognitive and the social aspects. The researcher tallied the correct versus incorrect answers given by each child, tallied the number of times the child was a proposer versus an opposer in the partnership, and the number of times the child accepted and resolved a disagreement. It was felt that the ratio of correct versus incorrect answers might give insight into the child's understanding of the Logo geometry; the ratio of proposed to opposed moves, and the number of times the child accepted a partner's arguments and thereby resolved a disagreement might give insight into both the cognitive and the social dimensions. The coding sheet data supplied information sought in line with certain of the research questions, such as which were the strategies used by the various children, what was the ratio of use of Level I to Level II strategies, did the more adept child use more Level II strategies--which served to support their position or to elicit information-- than did the less adept child, and did the degree to which talk was supported increase over time (see Research questions).

There was information which could not be culled from the coding sheets alone, such as whether the child's acceptance was due to persuasion on her partner's part, and if the partner was persuaded to accept, which verbal or nonverbal strategies might have led the partner to be persuaded; a return to the source transcripts was necessary in order to determine what may have triggered the persuasion.

There were other, vital aspects which could not be answered by the quantitative information contained in the coding sheets. These aspects dealt with the qualitative dimension, the HOW-- How were the strategies used by the partners to convince and to inform?, and How did the use of the strategies reflect the children's understanding of Logo geometry? We will deal shortly with the qualitative component of the data analysis. But first, the Level I and Level II strategies and their definitions will be listed, and then following that, the aspect of interrater correspondence will be presented.

The Level I and the Level II strategies: The categories used and their definitions

Listed below are the definitions for the utterances and nonverbal moves which were coded in the disagreement episodes. The first and second large sections below represent the Level I and Level II categories sections, respectively. Reference has been made to the definitions used by Eisenberg and Garvey (1981), by Genishi and Di Paolo (1982), and others (for example, Labov & Fanshel, 1977) where applicable. In some cases, the original Eisenberg and Garvey (1981), or Genishi and Di Paolo (1982) definition has been used. Abbreviations have been employed in the interest of conciseness: E & G for Eisenberg and Garvey, and G & Di P for Genishi and Di Paolo. Additional comments by the researcher are included in square brackets [ ].

Please note that the order of the categories listed below corresponds to the original coding sheet (Appendix J). (Coding sheet categories which are not verbal strategies have also been included, such as the correct/incorrect ratings.) All the categories and code numbers used on the coding sheets during the analysis have been cited below. During the data analysis stage, some categories were added a posteriori (for example, *post disagreement discussion*, hence its position on the coding sheet in a spot where it could be fitted in, namely 26 and 27, rather under the Level II section where it belonged); some of the category names were changed slightly (for example, *Appeal to recorded authority--peer* became *Reference to recorded notes*); and some of the verbal strategies listed here were never used by the children during the mathematics

concepts disagreements (for example, *Appeal to recorded authority--teacher*), or were eliminated (for example, *reason*).

LEVEL I VERBAL STRATEGIES

These categories feature utterances which have no accompanying elaboration

1 *Insistence*: Repeat of previous utterance without (or with) a change in wording and with no new information added. [E & G: Support for a previous utterance that adds no new information]

2 *Insistence- Action taken*: Repeat of previous utterance without/with a change in wording and with no new information added; action corresponding to the utterance is taken.

3 *Action taken*: Action is taken, no utterance is made.

4 *Challenge*: [G & diP] A calling into question of the other's utterance.

5 *Denial/Resistance*: [G & diP, *Denial*] A negation of the previous utterance that begins with a negative form and may precede a contradiction, for example "No, it's not."

6 *Insult*: An affront.

7 *Refusal*: A statement which indicates that the child will not accept or consent; without supporting argument. [G & diP: A negative form used in response to a request.]

8 *Threat*: A declaration of intention to punish, hurt, retaliate. [G & DiP: A statement of intention to retaliate.]

9 *Ignores*. No response to previous utterance is made. [A. Intention to ignore the utterance is implied OR B. It is possible that the previous utterance was not heard.]

10 *I--Dismisses suggestion*: A statement is made dismissing suggestion of the other child.

11 *I--puts "on hold"*: A statement is made implying or demanding that the other child should desist from action.

12 *Physical force*: A nonverbal move is made to physically prevent the other child from action.

13 *Silence*: No utterance is made.

14 *Anger*: A statement or nonverbal move is made expressing anger, or extreme annoyance. [This category was also used to code instances of sulking.]

15 *Mitigation*: [E & G] Paraphrase of previous request (or utterance) that increases in 'politeness'. [Labov & Fanshel, 1977: Paraphrase of previous utterance that increases in positive tone and in the attempt to please/placate the other.]

16 *Aggravation*: [E & G] Paraphrase of previous request (or utterance) that decreases in 'politeness'. [Labov & Fanshel, 1977: Paraphrase of previous utterance that increases in hostility or negative tone, and increases in the attempt to aggravate the other.]

17 *Counter/Contradiction*: An alternative proposal, without supporting argument. [G & diP: *Counter*--An assertion that opposes the other's utterance.] [E & G: *Contradiction*-- An alternative proposal.]

18 *Counter-suggestion*: A statement that opposes the speaker's own previous utterance.

19 *Prediction*: A statement that predicts the move of the turtle prior to taking action.

20 *Evaluation*: A statement which evaluates the 'goodness of fit' of the move vis-a-vis the goal after taking action.

21 Statement unclear, ambiguous

22 Statement inaudible

23 Statement incomplete

24 Statement not relevant

[The following categories, namely 26a, 26b, 27a, and 27b, were added a posteriori during the analysis. They should have been placed in the Level II section on the coding sheet, but due to lack of room on the right hand side of the coding sheet, they were placed with the Level I categories.]

26a *Post D/A (post disagreement) follow-up*: Verbal or nonverbal followup to the disagreement just resolved.

\+ Positive: Confirmation-- nod, smile, voice stress

\- Negative: "I told you so"

Other: Expresses surprise; and so on

26b *Post D/A openness for discussion*: A statement which reflects openness for discussion dwelling on the content of the disagreement just resolved.

27a *Post D/A comment*: A (brief) comment on the content of the disagreement just resolved.

27b *Post D/A elaboration*: A statement of elaboration on the content of the disagreement just ended.

28 Not scoreable: The utterance cannot be scored because none of the categories apply, and the utterance did not merit the creation of a new category.

29 **CORRECT:** The Level I utterance is correct; coded in cases where correctness/incorrectness can be determined.

30 INCORRECT: The Level II utterance is incorrect; coded in cases where correctness/incorrectness can be determined.

LEVEL II VERBAL STRATEGIES

These categories feature utterances which elicit elaboration (40, 41, 42, 47, 48) or which furnish elaboration (49-63)

40 *Request for Explanation*: A statement of request for explanation. The soliciting party would be expecting the solicitee to respond in detail concerning the issue at hand, for example a step-by-step description of how to do a task.

41 *Request for Information*: A statement of request for information. The soliciting party would be expecting a direct, short answer. Detail or elaboration concerning the "how" or "why" is not expected.

42 *Request for Clarification*: A statement of request for clarification, usually of the other's previous utterance.

43 *Request for a Hearing*: A statement of request that the presentation of one's position be heard.

44 *Request fulfilled*: The request-- for *explanation* or *information* or *clarification* or for a *hearing*--is fufilled by the solicitee.

45 *Request not fulfilled*: The request--for *explanation* or *information* or *clarification* or for a *hearing*--is not fulfilled by the solicitee.

46 *Cannot fulfill Request*: The solicitee is unable to fulfill the request.

47 *Appeal to teacher* [Originally, *Appeal to Authority-- teacher.*]: [G & diP, p. 56] A reference to an authority's rules or previous utterance OR an utterance directed to an authority, in this case the teacher. [Note that Genishi and di Paolo (1982) rank *Appeal to authority* within Level I (see Appendix AA). In this study, *Appeal to teacher* and *Appeal to peer* were considered by the researcher to be Level II strategies, since they potentially could either represent a type of supported statement or constitute an attempt to seek help or elaboration.]

48 *Appeal to peer* [Originally, *Appeal to Authority-- peer.*]: A reference to a peer authority's rules or previous utterance OR an utterance directed to a peer.

49 *Appeal to recorded authority--teacher*: A reference to an authority's rules or written statement, in this case a teacher's written record, for example, stencil information sheets. [This category was not used during the mathematics concepts disagreements.]

50 *Reference to recorded notes* [Originally, *Appeal to recorded authority-- peer*]: A reference to a previously written statement, in this case the child's own, or another peer's, recorded statements, usually recorded in a notebook).

51 *Appeal to prior knowledge--Supported*: A reference to one's own prior knowledge (school classroom-based, computer classroom-based, world knowledge), with supporting argument.

52 *Appeal to prior knowledge--UNsupported*: A reference to one's prior knowledge (school classroom-based, computer classroom-based, world knowledge), without supporting argument.

53 *Appeal to personal construct*: A reference to one's personal (i.e. non-standard) construct as a supporting argument. [This category was not used during the mathematics concepts disagreements.]

54 *Appeal to computer experience*: A reference to one's prior experience with computers as a supporting argument.

55 *Reference to computer screen verdict* [Originally, *Appeal to--computer gives verdict*]: A reference to the computer as the authority, since it has "given the verdict."

56 *Other-directed*: The action taken, or the resolution of the disagreement, is other-directed, by the teacher, or by another peer.

57 *Reason*: Use of this category was cancelled for reasons noted below.

58 *Supporting argument*: [G & diP] A fact that adds new information to previous utterance.
[Originally, it was thought that *Supporting argument* would be coded when the statement explained HOW one would go about doing the action, or how one had done the action, and that *Reason* would be coded when the statement explained WHY the utterance was made. The attempted differentiation between *Reason* and *Supporting argument* proved untenable. Thus in the end, only *Supporting argument* was used as a category.]

59 *Focusing*: Use of (nonverbal) body movement and/or voice stress to add emphasis to the utterance; and/or use of body movement to trace upon, and focus attention upon, a specific portion of the computer screen.

60 *Demonstration*: An utterance, or combination of utterances, which describes and explains, with the help of specimens.

61 *Conditional test hypothesis*: A statement of a hypothesis, with a suggestion to test via computer to see if the hypothesis is correct.

62 *Counter proposal*: [E & G use term *Counter*] An alternative proposal, with a supporting argument given.
NOTE: See *Counter/Contradiction* above.

63 *Compromise*: [G & diP] An alternative proposal that implies give and take or sharing or compromise on each, or one, of the partner's part.

64 *Acceptance*: [G & diP] An agreement to comply. [The agreement to comply may be overt or implicit. For example (a) overt acceptance would be represented by an expression such as "Okay."; (b) "What do we do next?" implies tacit acceptance of partner's prior utterance, and the resolution of the disagreement.]

65 RESOLUTION: Coded when there is resolution of the disagreement; however, the resolution is not counted as a category within the verbal strategies sections. Resolution is arrived at via (a) acceptance by one of the partners, or (b) computer giving the verdict, and both partners accept.

66 CORRECT: The Level II utterance is correct; coded in cases where correctness/incorrectness can be determined.

67 INCORRECT: The Level II utterance is incorrect; coded in cases where correctness/incorrectness can be determined.

The interrater correspondence check

Jeannine Lewis, who had been involved in the 1984-1985 year of the study (as non-participant observer in the classroom, and videotape equipment expert) had coded the disagreements for the 1984-1985 analysis featuring Carol L. and Amy L. when they were partners in the Grade 4 Logo session. In January, 1988, the researcher asked Jeannine to code two of the sessions in this study, using the Level I and Level II strategies, in order to provide for a comparative check upon the researcher's coding. (Jeannine was not asked to determine whether the children were correct or incorrect.) The two sessions coded were Lilly M. and Rina M.'s 6th and 7th Logo sessions in 1985, November 20, 1985 and Dec. 4, 1985. There were 16 mathematics concepts disagreement episodes. Interrater correspondence between the researcher's (Vicki Zack) coding and Jeannine's coding for these two sessions was 61%. In 19% of the cases, the similarity between the categories chosen by each rater was close. In 20% of the cases, there were categories coded in one which had not been included by the

other; in general, the researcher had coded more entries than had Jeannine, although in two instances, Jeannine had coded two items which the researcher had missed.

The interrater correspondence in the 1984-1985 part of the project had been higher; at that time the correspondence between raters Vicki and Jeannine was 77% (Zack, 1985). It is possible that the correspondence was higher in 1984-1985 due to the larger number of instances compared. It is more likely, however, that the lower correspondence rate in 1985-1986 was due to the fact that in the 1984-1985 year, Vicki and Jeannine had similar experience with the verbal strategies categories and both did the coding during the first one or two passes through the transcripts, whereas in 1985-1986, Vicki, the primary researcher, had had longer and more extensive experience with the material.

The differences in regard to the 19% of instances which were "close" but not exactly the same, and the 20% of instances which had been missed by either Jeannine or Vicki, were resolved by discussion. The discussion session brought to the fore the desirability of having a co-researcher involved in the project. In this study, it was not possible to arrange to work in a team; the hours required for the analysis (the researcher spent approximately 220 hours doing the coding of the disagreements for the 45 sessions) precluded Jeannine's expert participation in the analysis of all the sessions; finding a new person who was an able researcher with a background in mathematics and Logo and who had the extensive time required proved futile, costs aside. Ideally however, there should be another able person such as Jeannine involved, in order to sharpen the primary researcher's understanding. The task is to come as close as possible to what the child participants had in mind. One endeavors not to impose one's own notions, but bracketing is easier to do in theory than in practice. It is necessary to have another pair of eyes to help one see better, and perhaps to highlight some of the failures in bracketing, or at the very least to present an alternate point of view as basis for discussion. A methodological shortcoming in this study, albeit unavoidable due both to the unavailability of suitable personnel and to the cost factors, was the absence of another researcher. In achieving thesis research, there is a limitation in regard to funding and to time.

Analysis of the *explanations*

Once the quantitative count concerning the verbal strategies was complete, the investigation of the qualitative aspect was to be pursued. The intent was to look first at what the children said during the disagreements; and then to consider how they said it, namely the terminology that they used (ethnomathematical or standard), and how they did the Logo work (working modes: visual or analytic; immediate mode or edit mode programming context). The aim concerning the first component, namely the qualitative investigation of the verbal strategies, was to distinguish (1) how the use of the strategies during the disagreements, especially the Level II strategies, reflected the children's understanding of Logo geometry, and (2) how the use of the verbal strategies might be contributing to the child's learning, and to the learning of the child's partner.

As has been mentioned previously, it had been determined that particular attention would be directed specifically upon the aspect of *explanations*. There would be an effort made to determine the complexity of the level of explanation, and to see whether the explanation episode(s) might be seen to provide "scaffolding" (see Note 1, Greenfield (1984)) for the listening partner, and might be classified as a teaching-type episode. During the course of the analysis, the scope of the aspect of *explanation* grew. It proved necessary to distinguish between different types of explanations, and as a result, Donaldson's (1986) categories were used. Since the degree of detail which was eventually needed had not been anticipated at the outset of the study, Donaldson's (1986) work has not heretofore been referenced in this paper. We will therefore cite now from her work concerning the rationale behind eliciting and giving explanations and its importance to learning, as well as her definitions for the four modes of explanations she identified. First, Donaldson's statement of rationale:

> Why do we give explanations? There are two main possibilities. We may give an explanation to help someone else understand something; alternately, we may give an explanation to demonstrate our own understanding. Similarly, when we ask for an explanation, we may be aiming to extend our own understanding; alternately, we may be aiming to test someone else's understanding. Thus, explanation provides both a means of conveying knowledge and a means of assessing knowledge. For this reason, explanation is central to

> education. The ability to give explanations, the ability to understand explanations, and the ability to seek explanations are important assets for both the teacher and the pupil
> (Donaldson, 1986, p. 1).

Donaldson (1986) then listed four types of questions, and the corresponding four different types of explanations which she termed *empirical, intentional, deductive* and *procedural*:

> - In giving an EMPIRICAL explanation, the speaker is attempting to provide an answer to the *What happened to cause . . .?* version of the *Why* question. . . .
> - The speaker's aim in giving an INTENTIONAL explanation is to answer the *For what purpose . . .?* version of the *Why?* question. . . .
> - When the speaker gives a DEDUCTIVE explanation, his aim is to provide an answer to a *How do you know that . . .?* question. [It] . . . requires an explanation . . . in terms of . . . a rule, or some observable evidence.
> - The speaker's aim in giving a PROCEDURAL explanation is to answer a *How do you do . . .?* question. His task is to outline the steps leading up to a particular goal in such a way that his description would tell the listener what he would have to do in order to achieve the goal. (Donaldson, 1986, p. 4)

Once the children's explanations were identified within the study data, the analysis of the explanations was more meaningful and coherent when one went beyond the general classification of *explanation* to look at the explanations more specifically in relation to the four types: *empirical (causal), intentional, deductive (logical),* and *procedural.*

Analysis of the formality of the mathematics talk

In a recent talk, Kieren (1987) outlined four levels of language use in mathematics--ethnomathematical, intuitive, technical-symbolic, and axiomatic-deductive--, the first two of which are pertinent to the age level of the

children in the present study (see Table 1, Part B). Ethnomathematical language is not standard mathematical language. Children use their own words; for example, when talking about fractions, they will use the words such as "a piece and a bit." At what Kieren (1987) has called the *intuitive* level, the terms used are mathematical in nature; for example, the children will use formal terms such as a "third" and a "quarter of a quarter." Since in the environment of the study, Monica the Logo teacher does not use conventional classroom geometry terms such as "degrees" or "angles," the researcher planned to document the terms which the children used as they spoke in a mixture of their own words and Logo-ese, and to determine whether a more precise language (although it may not necessarily be formal school terminology) was concurrent with a stronger grasp of Logo angle rotation concepts.

Analysis of the mathematics strategies: Levels of thinking, and modes of working used

Skemp (1979) spoke of the *instrumental* and the *relational* levels of thinking in mathematics. A person working at the *instrumental* level uses rote rules, that is "rules without reasons." A person working at the *relational* level knows "what to do and why" (Skemp, 1979, p. 250).

In a recent paper and presentation, Kieran, Hillel, and Erlwanger (1986) defined two modes of working in Logo geometry, the *perceptual* and the *analytic*. Hillel and Kieran (in press) subsequently indicated a preference for the term *visual* instead of *perceptual*, and so in this study the two modes will in the main be referred to as the *visual* and *analytic* modes. They are defined as follows:

> *Perceptual*: Inputs are chosen on the basis of perceptual cues, independent of what has gone on beforehand; rationale for inputs is often expressed by "It looks like . . . "
>
> *Analytical*: Inputs are chosen on the basis of mathematical relationships inherent in the given data and/or from mathematical knowledge; rationale for inputs is often expressed by "Since this . . . is 60 degrees, then . . ."

Specific Angle-Schema:

45-90 schema: Slanted lines have to do with 45° turns; horizontal and vertical, with 90° turns. This schema is a particular case of the perceptual schema.

Protractor schema: Inputs to turns are based on usage of the protractor in "standard position."

(Kieran, Hillel, & Erlwanger, 1986, pp. 156-157)

The researcher added to the above-listed category system the *SETH* (SETHEADING) *schema* which some children use by rote, and others seem to use as navigators do: "*SETH schema*: Inputs to turns are based on the Logo setheading provided by the system."

The aim was to investigate the working mode of the children, that is whether it was the *visual* or *analytic* mode that they used, both while working at their computer station during the Logo sessions, and while engaged in a researcher-assigned task during the one-to-one Grade 6 clinical interview with the researcher (May-June, 1987). The last-mentioned cognitive task, designed to elicit whether the children were using a *visual* or an *analytic* schema, is outlined in the first part of Appendix Z.

The aim was also to explore the level of thinking exhibited by the children, that is, to determine whether it was *instrumental* or *relational*. How well did the children understand what it was they were doing?

Taking note of the Logo programming mode

The programming mode was to be coded as IMMEDIATE (DRAW) MODE or EDIT (DELAYED) MODE. It has been noted by researchers (Hillel, 1986; Leron, 1985) that even experienced child programmers prefer to debug in the immediate mode. A close look at the working and problem solving styles of the children was deemed necessary. Kieren (1987), in a recent talk on "Language use in mathematics," spoke of Frye's (1975) terms *hieroglyphic* and *metaphoric*. Kieren indicated that a learner in the beginning stages relied upon the concrete, and upon the graphic screen image (hence *hieroglyphic*), whereas the learner who had a more secure command of

the concept could conjure up the graphic image in his mind and could work in the abstract, decontextualized, *metaphoric* domain. The child who could work in the EDIT MODE with comfort, without having to check the step-by-step movements of the "turtle cursor" was at a more advanced level than a child who needed the constant image before her or him on the screen (Kieren, 1984; Allaire, 1985).

Limits to Generalizability

The nontraditional school setting of the study, the homogeneous religious affiliation of the student population, as well as the exceptional qualifications of the Logo teacher, will limit the ability to generalize from this study. And yet the findings from analysis already done on the data to date (Zack, 1985; 1986b) indicate that the achievement of the children in this setting, and the various difficulties experienced by the children, may not be atypical.

It is stressed here, and will be emphasized again in the Findings chapter, that the frequency distributions which emerged from the data represented a small number of instances; as a result, seeming trends would have to be subject to further investigation before broad generalizations could be made. However, the in-depth analysis such as that done for this study can and should only be done with small groups; the kind of analysis required makes it impossible to work with large numbers at any time.

Conclusion

It was expected that the findings concerning the various components, namely (a) the pattern of the disagreements; (b) the working relationships between the children, (c) the children's talk while working together, that is, the verbal strategies used (Level I or Level II), (d) the mathematical strategies used (*visual* or *analytic*), (e) the teaching and learning strategies used, (f) the level of language used (ethnomathematical or standard), and (g) the Logo programming mode used, would enable the researcher to gain insight into the question of the children's concept of angle. The aim of the research was to investigate whether the children were able to progress in making sense of the Logo geometry, and whether they were able to contribute to each other's learning-- and if yes, how the children were seen to do so.

FINDINGS AND DISCUSSION

The aim of the study was to investigate the role that disagreements over a Logo move, and the discussion associated with the disagreements, played in the children's learning of a Logo mathematics and geometry concept. The findings regarding this investigation will be divided into three main sections. For ease of reference, the sections will be referred to as PATTERN, COLLABORATION, and SUPPORTED TALK throughout the findings portion of the paper. The reasons for the choice of these specific short title names will be given below.

The first section will deal with the findings related to the PATTERN of the mathematics concepts disagreements-- their frequency, duration, resolution, and the mathematical idea which was the subject of the disagreement (hence the short title PATTERN). The report will deal with the ways in which the pattern of the disagreements gave an indication of the children's understanding of angle.

The second section will deal with the working relationships between the partners. It was found that in all but one instance, the relationship between the children was *collaborative* (Cooper, et al., 1986), in that the child partners worked together to solve problems, sharing and alternating the role of guide and information giver; in only one instance was the relationship *didactic* (Cooper, et al., 1986), since in that case one child knew more than did his partners, and was often the information giver. Therefore the short title chosen to be used for reference to this portion of the findings was COLLABORATION. The report will consider the ways in which the children were able to contribute to each other's learning, and will detail how the deliberations between the partners reflected the children's understanding of angle.

The third and largest section of the findings will deal primarily with the verbal strategies which elicited or furnished elaboration and discussion. The main focus will be on the supported talk of the children, hence the decision to use SUPPORTED TALK as the short title reference. This section of the report will describe how the verbal strategies were used by the children to promote discussion and to effect persuasion. The section will feature what the analysis of the use of the verbal strategies revealed about the children's understanding of Logo angle, and also about the children's strategies for learning about angle.

THE PATTERN OF THE DISAGREEMENTS

The prime focus of the study was on the aspect of the disagreements which dealt with mathematics and geometry concepts. It was of interest, however, to determine how the occurrence of the disagreements concerning mathematics and geometry concepts fit within the framework of the larger picture of all the disagreements which occurred while the children worked together in pairs. The analysis began with a look at all the disagreements, and then concentrated solely upon the disagreements which were related to mathematics concepts. The report on the pattern of the disagreements will likewise begin with an overview of all the disagreements, and then will focus solely upon the mathematics concepts disagreements.

Four main categories of disagreements were identified: MATHEMATICS AND GEOMETRY CONCEPTS, PLAN AND PROGRAMMING, SYNTAX, and CLASS RULES. It was expected that the same attention would be paid to syntax problems as was reflected in the analysis of calls for teacher intervention and peer intervention (Zack, 1988b), where calls for intervention with syntax problems outnumbered calls for intervention with mathematics concepts problems by a ratio of 5:1 (teacher) and 4:1 (neighboring peer). It was clear from that analysis (Zack, 1988b), and from the data as a whole, that the children had great difficulties with the syntax, that is, the mechanical ins and outs of working the microcomputer. It was expected therefore that the majority of the children's disagreements would be related to these trouble spots, to syntax concerns such as loading and saving, editing, and machine grammar (Appendix K). The findings did not support the prediction. The largest number of the children's disagreements did not relate to syntax; the largest number of disagreements had to do with mathematics concept issues, and pl n and programming issues. The disagreements over mathematics and geometry concepts outnumbered the disagreements over syntax by a ratio of approximately 2:1 (see Table 10).

The findings seemed to indicate that the children were comfortable drawing lines and making turns within the Logo graphics setting, when they were not stymied by machine grammar. The findings reflected the fact that the children did not require exact answers for the majority of their moves; rather, they were able to "fiddle around" (Rina, interview, May 1987) and adjust the fit of the turn or line until it "looked all right." In that the environment allowed the children to work through the Logo geometry problems (without for the most part being aware that they were even doing

Table 10

Frequency of Occurrence of the Four Types of Disagreements

Participant Groups	Mathematics Concepts Disagreements	Planning and Programming Disagreements	Syntax Disagreements	Class Rules Disagreements
Jon H., Eric H. & Russ H.[a]	36	38	23	2
Lilly M. & Rina M.	64	59	35	2
Carol L. & partners	20	28[b]	14	-
Amy L. & partners	14	28[b]	2	4
Totals	134	147	74	8

[a] Jon and Eric worked as a dyad for 4 sessions, Jon and Russ as a dyad for 1 session and Jon, Eric and Russ as a triad for 4 sessions.

[b] Six planning/programming disagreements were common to the 'Carol and partners' and 'Amy and partners' subgroups, as Carol L., Amy L. and Leanne L. worked as a triad during two sessions.

any geometry), Papert's objective of providing an environment for independent inquiry and discovery was met.

The children often needed to go to an outside source for help with syntax, for syntax required exact answers. Syntax is conventional and not subject to discovery; hence it made sense to go to an outside source for syntax answers. On the other hand, it also made sense within the environment of this Logo classroom to continue to work independently on the mathematics concepts aspects, and for the most part the children did not call for help from resources outside the dyad. It is entirely possible that the children did not perceive that their moves, or projects, required teacher or neighboring peer intervention. The implications of this perception for their mathematics learning is vital, and will be considered in greater detail later, in the section entitled "SUPPORTED TALK: Solicited teacher intervention."

The lack of perceived need of teacher intervention in the mathematics area and the more pressing need for teacher intervention evident in the syntax area explained the discrepancy between the low number of calls for intervention concerning mathematics and the high number of calls for intervention concerning syntax. It would also explain the discrepancy between the high number of disagreements about mathematics and geometry aspects relative to the low number of disagreements about syntax. The children were inclined to ask for outside help with respect to syntax and were less inclined to work independently on the problem themselves; the children were less inclined to ask for outside help with respect to mathematics problems and were more inclined to work out the Logo geometry and mathematics problems themselves.

The Mathematics and Geometry Concept Disagreements: Their Number, Duration, Type, and Resolution

After consideration was given to the types and relative occurrence of all the disagreements, the analysis then focussed upon the disagreements related to mathematics and geometry concepts, in order to answer the first research question which was:

CAN THE PATTERN OF THE DISAGREEMENTS--their frequency, duration, resolution, and the category of mathematical idea--GIVE ANY INDICATION OF THE CHILD'S UNDERSTANDING OF LOGO ANGLE?

The number, type, duration and resolution of the disagreements were analyzed in order to distinguish whether there was a relationship between the pattern of the disagreements and the children's achievement. For example, in comparison to their less adept peers, did the children who were more adept at the Logo geometry work argue more or less frequently? Did they sustain longer or conduct shorter disagreements? Did they resolve more or fewer disagreements, on more difficult or less difficult mathematics concepts? The children's achievement levels which had been determined via the study which investigated their notion of angle, in particular right angle, were the levels used (Zack, 1986b; Table 4). As a result of the findings in that study (Zack, 1986b; Zack, 1986d), Russ, Jon and Eric were found to be at the high level of achievement; Lilly and Rina were at the average or mid-level; Carol, Amy and Leanne were considered to be at the low end in terms of achievement.[3] The findings indicated that certain aspects of the pattern of the disagreements reflected the children's level of achievement, whereas other aspects did not. There seemed to be a relationship between the children's level of achievement, and the mathematical idea inherent in the disagreement and the resolution of the disagreement; however there seemed to be no relationship between the children's level of achievement, and the frequency and duration of the disagreements. Elaboration of these statements will now follow.

[3]In order to facilitate reference to the child's level of achievement throughout the paper, the letters (H), (M), (L) have been appended to the children's names at the beginning of a paragraph as follows: Russ H., Jon H., Eric H., Lilly M., Rina M., Carol L., Amy L., Leanne (L.). The order in which the children's names have been presented reflects their rank in terms of achievement (Table 4). Whenever two children or more children need to be listed, the order in which the children's names are given will in most instances reflect their relative ability, for example Lilly M. will precede Rina M. when paired, except for instances in which doing so would interfere with the communication of meaning. *Please note, as has been previously stated, that the children's names are pseudonyms.*

The Frequency of the Disagreements

There was no relationship evident between the total number of mathematics and geometry concept disagreements in which the children engaged and the level of achievement of the child. The less adept children, Carol L., Amy L., and Leanne L. and their respective partners had 20, 14, and 14 disagreements respectively; the moderately adept pair, Lilly M. and Rina M., had 64 disagreements; and the most adept children, Jon H. and Eric H., had 29 (Eric H. with Jon H. and Russ H.) and 36 (Jon H. with Eric H. and Russ H.) disagreements respectively (see Figure 5).

It is possible to look at the pattern in Figure 5 and imagine that there is reason behind the low, high, then low curve of the pattern. Perhaps the low achieving children are not reacting to some of the aspects responded to by the middle achieving children because the less adept children have less of a grasp of the Logo mathematics and geometry concepts than do the middle level children. Perhaps the middle level pair are having more disagreements because they are more in tune with some of the concepts than are the low achieving children, but have not mastered some concepts already mastered by the higher achieving children. Hence the higher rate of disagreements for the middle pair, and the lower rate of disagreements for the higher achieving pair.

One can find instances which would support this speculation. There were a number of occasions upon which the less adept children, Carol, Amy and Leanne, did not engage in a disagreement because they had not yet grasped a concept. For example, Amy L. suggested 83, 90, and 93 as inputs to RT when she and Leanne L. were attempting to create a square (the correct input would be RT 90), but Leanne at no point countered with either a suggestion, or even a comment on Amy's arbitrariness (see Selection #2 in the COLLABORATION section). Another example concerned the correction of a erroneous move. Amy L. and Leanne L. had one, and only one, disagreement concerning the correction of turn via the use of the inverse, and both Amy and Leanne were incorrect in their suggestions. They did not continue to engage in disagreements over this aspect, but reverted to a simpler strategy (see Selections #8A and 8B). Lilly M. and Rina M. had three disagreements concerning the correction of a turn via the use of the inverse; both Lilly and Rina were correct in their strategies, although their respective problem solving strategies were different. The content differences between Amy L. and Leanne L.'s work on the one hand and Lilly M.'s and Rina M.'s work on the other warrant closer inspection, and will be investigated later.

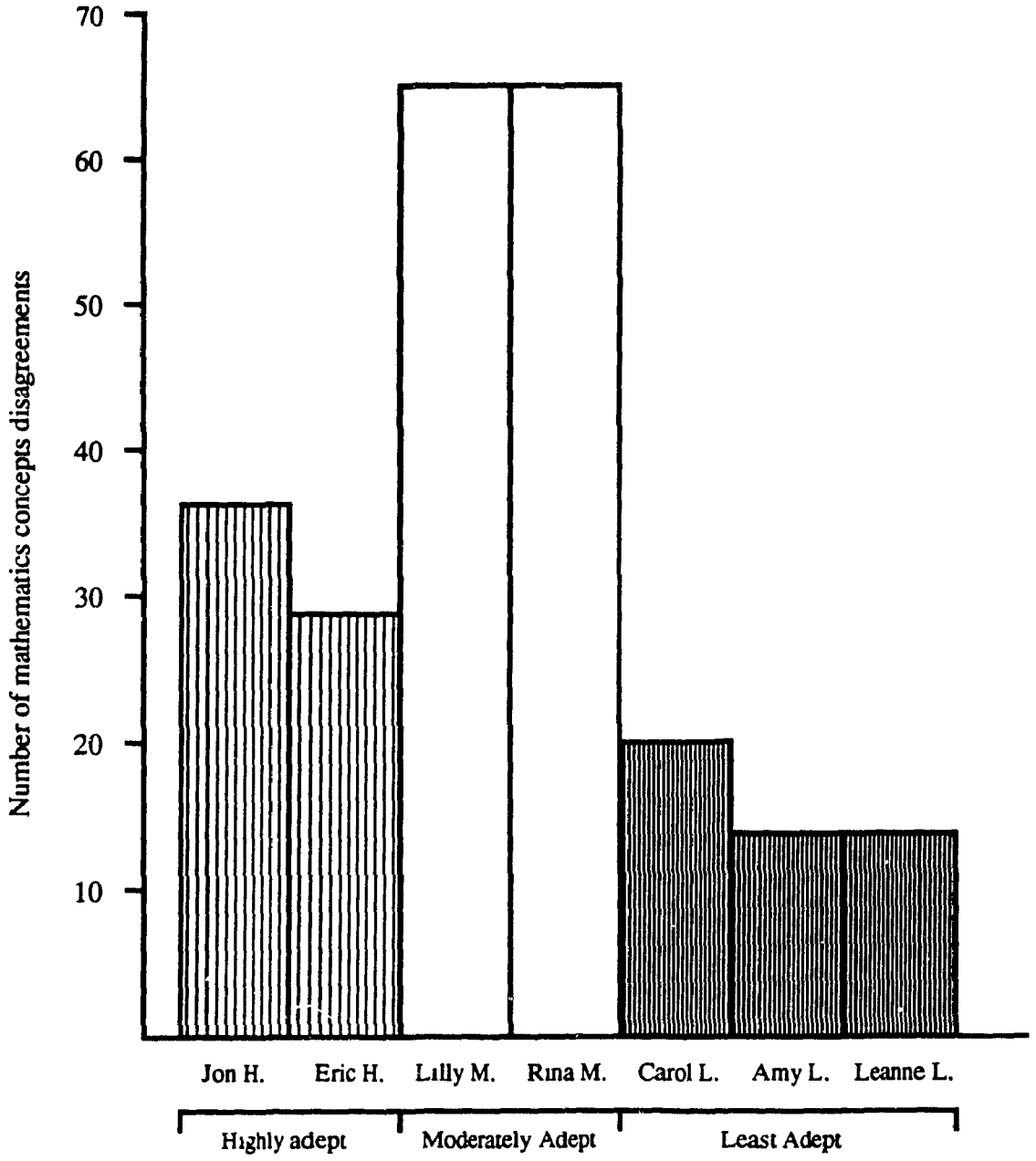


Figure 5. Number of mathematics concepts disagreements engaged in by each participant

Jon H. and Eric H., at the higher level, were not seen to argue about this aspect, but circumstances and the nature of the projects they chose for the most part precluded a need to do so. Russ H. never engaged in disagreements related to correction via use of the inverse because he had mastered the concept and performed the necessary steps without hesitation. Indeed, the number of disagreements seemed to drop when Russ joined Eric and Jon. Russ was in command of many of the Logo concepts which Jon and Eric had not yet grasped. It is possible that the number of mathematics and geometry concept disagreements engaged in by Jon and Eric would have stayed higher had Russ not joined them. It may be that Jon and Eric engaged in fewer disagreements because Russ was working with Jon and Eric at a level which was a comfortable independent working level for Russ. If Russ had worked on his "stick man" (with variables) with a partner who could work at the same very high level, there might then have been a higher number of disagreements between Russ and his partner, for they would then have had diverse new challenges to work through together.

This is all speculation. It is of course possible that it was due to Russ H.'s involvement with Jon H. and Eric H. that the number of disagreements engaged in by Russ, Jon and Eric dropped in the last few sessions. So it seemed to me, until I saw that during the last few weeks of Lilly M. and Rina M.'s Logo sessions the number of disagreements also dropped. And for Lilly and Rina, they dropped by an even greater ratio. The disagreements between Lilly M. and Rina M. dropped from 48 in the first half of the sessions, to 16 in the latter half, a drop of 66%. There were 25 disagreements between Jon H. and Eric H. in the first half of the sessions; the number of disagreements between Jon, Eric and Russ dropped to 10, thus the disagreements dropped by 60% when Russ joined Jon and Eric. While a possible pattern seemed to be emerging such that the rate of disagreements was high at the start and then dropped, a look at Amy L. and Carol L.'s disagreements with their respective partners dispelled that notion. The rate of Amy and Carol's disagreements stayed the same when one compared the first part and the latter half of the sessions. Amy L. and Shauna L. had 7 mathematics concepts disagreements in the first half; Amy L. and Leanne L. had 6 disagreements in the second half of the Logo sessions. Carol L. and her partners Leanne L., and then Shauna L., had 8 disagreements in the first half of the Logo sessions; Carol L. and her partner Doug L. had 10 disagreements in the second half. Thus the pattern of the drop in disagreements in the latter half of the sessions was not

a consistent one. It seemed very possible that given other pairs at the low, medium and high level, the resulting number of disagreements might be different at the low, middle and high end.

The pure quantitative aspect may nevertheless be important. If it is true that verbal conflict promotes learning, then the children who engage in very few verbal disagreements may possibly not be gaining the full benefit of this medium for learning. But the whole issue is certainly more complex than the simple quantitative notion. For one, no matter how few or how many the disagreements, what happens during the disagreements must be important. And it is the larger qualitative question which this study will attempt to address. Secondly, the whole issue is certainly more complex than simply the aspect of disagreements alone. There are many modes of learning, of which engaging in disagreements is but one. It is essential always to keep in mind the larger picture, the role played in this environment by the teacher, by neighboring peers, and by independent inquiry in the learning of the Logo mathematics and geometry concepts. Although this study concentrates upon the disagreements, and upon the specific components within the disagreements that might help contribute to the researcher's understanding of the children's process of learning Logo geometry, it is important constantly to keep touch with the macrocosm of which disagreements is only a part, in order not to lose sight of the relationship of the disagreements to the larger whole.

The Duration of the Disagreements

It seemed worthwhile to investigate whether it was the less adept or the more adept who tended to talk at greater length in order to attempt to solve a problem or to resolve a disagreement. In attempting to predict the pattern, I found that I could not anticipate a possible relationship; I could envision diverse possibilities which were diametrically opposed. A less adept student might need longer to ruminate or to explain (hence longer duration), or might give up or give in more quickly (hence shorter duration). A more adept student might persist longer at the disagreement due to a need to explain, or due to the challenge of solving the problem (hence longer duration), or might be more attuned to the nature of the Logo concept and be able to talk through the difficulty in short order (hence shorter duration).

The findings indicated that there was no relationship between the duration of the disagreements and the level of achievement of the child. When the number of turns of talk were averaged, the mean per child was found to be between 4 and 5 (4.5 to 4.9) turns per disagreement for six of the seven focal children. One child, Amy, had a mean of 6.2 turns per disagreement with her partners. It was a surprise to find the averages so closely clustered (see Table 11).

The findings concerning the range reflected as well the similarity between the pairs. The episodes ranged in length from 1 to 15 turns of talk, with one "outlier" which was a disagreement which lasted 33 turns (Lilly M. and Rina M., Nov. 20, 1985). The distribution of the lengths of the disagreements was highly skewed; 92% of the episodes were 10 turns of talk or less, and 69% of the episodes were 5 exchanges of talk or less. Thus, no differences between the pairs were found either in relation to mean turns of talk per disagreement, or in relation to the range.

The Mathematical Idea Inherent in the Disagreements

The specific mathematical idea underlying the disagreement gave the best indication of the child's understanding of angle, and was the aspect most closely tied to the level of achievement of the child. The main disagreements about angle were related to lateralization, the amount of input to RT or LT, the combining of inputs, and debugging an incorrect turn by using the inverse in two steps, or in one combined step. Other disagreements about turns dealt with (a) choices, and with (b) combined moves; but these disagreements were fewer in number (see Table 12; Appendix K).

Table 11

Mean Turns of Talk per Disagreement per Participant

Participant	Mean Turns of Talk
Shauna L.[b]	7.10
Amy L.	6.21
Leanne L.	4.92
Rina M.[a]	4.84
Lilly M.[a]	4.84
Eric H.	4.82
Carol L.	4.60
Jon H.	4.57
Doug B.[b]	4.11
Russ H.[b]	3.33

[a] Rina M.'s and Lilly M.'s means are the same (4.84) because they only engaged in disagreements with each other.

[b] Shauna L., Doug L. and Russ H. were videotaped for only 4, 4, and 6 sessions respectively.

Table 12

Mathematics Concepts Disagreements: Category by Student Group Frequencies

Student Groups	A lateralization	B input to LT/RT	C combining of input to LT/RT	D inverse LT X —> RT X, RT X	E inverse LT X —> RT 2X	F FD BK	G input to FD/BK	H inverse of FD, BK	I combining of input FD X + FD Y	J combining of input FD X + BK Y	K Turn or Distance	L circle lateralization	M circle (R, L) input	N semi-circle curve	O variable	P evaluation	Q FD (or) semi-circle	R BK (or) BK + CIRCLER	S CIRCLER (or) turn + CIRCLER	T Turns and leaves no trace	U FD X + Y + Z = BK (X + Y + Z)	V Disagreements per pair	
	Turn					Distance					Circler Circle			Turn & Distance									
Jon H., Eric H. & Russ H.	6	4					13			1		2	2		1	4	1			1		1	36
Lilly M. and Rina M	9	14	7	3[a]			17		2		7	1		1			2	1					64
Carol L. and partners		5				1	7				2		3						1		1		20
Amy L. and partners	2	5			1		1					2	2				1						14
Carol L. and Amy L.(G4)[b]	2	3					6				2		8				1						
Total number of disagreements per category	**17**	**28**	**7**		**4**	**1**	**38**		**2**	**1**	**9**	**5**	**7**	**1**	**1**	**4**	**4**	**1**	**1**	**1**	**1**	**1**	

[a] collapsed frequency: categories D and E

[b] Carol and Amy's Grade 4 mathematics disagreements are shown for reference only. They are not included in the totals.

There was a relationship between concepts attempted and strategies used on the one hand, and the level of achievement of the children on the other hand. Some ideas were attempted by some of the children and not by others, or attempted but not successfully accomplished by some of the children. For example, although Amy L. and her partner Shauna L. asked Monica, the teacher, how to make a semi-circle, they did not pursue the task. Only the middle (Lilly M. and Rina M.) and high level (Jon H. and Eric H.) pairs made an attempt at making a semi-circle or curve. Only the high level pair, Jon and Eric, attempted to do the inverse of the curve in order to erase it. Neither pair, Lilly M. and Rina M., or Jon H. and Eric H., was successful in the effort to draw, and to erase (Jon and Eric), the desired curve. Only Jon and Eric (the high level achievers) were introduced to the concept of variable by Monica, the teacher, and disagreed as they struggled to make sense of it. Only the most adept, Russ H., was seen to employ a testing strategy for checking the length of a line in his disagreement with Jon over the input needed [BK ($\underline{X}$ + $\underline{Y}$ + $\underline{Z}$) = FD $\underline{X}$, FD $\underline{Y}$, FD $\underline{Z}$] (see Table 12; Selection #16 in the SUPPORTED TALK section).

The occurrence of diverse mathematical theorems.

Hillel (1984) spoke of spontaneous concepts, and also of mathematical theorems emerging from the Logo experience which relate to relative measure, additive measure, subtractive measure and proportional measure. The theorems identified by Hillel (1984; see Appendix U) manifested themselves in the children's work and deliberations in a way that denoted a hierarchical order of difficulty. The category of mathematical idea which was the subject of the disagreement reflected the various levels of expertise of the children. The children dealt first with the combining of like inputs to displacement moves (FD), and with like inputs to rotational turns (to RT or to LT); and only dealt subsequently, if at all, with combining unlike inputs to displacement moves (FD $\underline{X}$, BK $\underline{Y}$), and with combining unlike inputs to rotational turns (LT $\underline{X}$, RT $\underline{Y}$). Carol L. and Amy L. and their partners, and Lilly M. and Rina M., only disagreed over the combining of like inputs to displacement moves and turns, that is when the turtle cursor continued its movement in the same direction, for example in a displacement--FD $\underline{X}$, FD $\underline{Y}$ = FD ($\underline{X}$ + $\underline{Y}$), or a rotation--RT $\underline{X}$, RT $\underline{Y}$ = RT ($\underline{X}$ + $\underline{Y}$). There were no disagreements on the part of Carol and Amy and their partners, or on the part of Lilly and Rina, over the combining of diverse direction inputs to distance commands, that

is FD $\underline{X}$, BK $\underline{Y}$ = FD ($\underline{X}$ - $\underline{Y}$) where $\underline{Y}$ is smaller. Jon H. and Eric H. did engage in one such disagreement (see Table 12, Category J). Nor were there disagreements between Carol L and Amy L. and their partners, or between Lilly M. and Rina M., or between Jon H. and Eric H. for that matter, over the combining of unlike inputs to angular rotation, that is RT $\underline{X}$, LT $\underline{Y}$ = RT ($\underline{X}$ - $\underline{Y}$) where $\underline{Y}$ is smaller. There was evidence in Russ H.'s work that indicated that he was able to do these moves, but there was no instance of disagreement over this move between Russ H. and his partner Michael H. (one session), or between Russ H. and partners Jon H. and Eric H. (five sessions).

Hillel (1986) has suggested that children progress more quickly with distance concepts than with concept of angular rotation. The finding concerning the appearance of the various concepts, and the finding that certain theorems were in use by the more adept of the children as they engaged in disagreements (and agreements) confirmed Hillel's (1986) observation concerning the disparate pace in the mastery of distance versus rotation concepts. The rotation concepts seemed to be more difficult than the corresponding displacement concepts. And the more difficult concepts were the subject of disagreement and use by the more adept children.

The disparate appearance of certain concepts and strategies.

At times the frequency count did not accurately reflect the relative grasp of the concept by different pairs. Absence of occurrence might mean either mastery of the concept by the children at one extreme, or inability or lack of opportunity to consider the issue at the other extreme. For example, the nature of the children's self-chosen projects might render the concept inapplicable, and therefore one would not see a manifestation of certain kinds of attempts, hence the afore-mentioned lack of opportunity to participate in certain explorations. In between the extremes there could be a range of ways in which the children dealt with an issue.

As was previously mentioned, such was the case with the aspect of correction of an error in rotation. In their project work, Jon H. and Eric H. never needed to adjust for an incorrect turn of other than 90 degrees. Amy L. and Leanne L. had one disagreement over adjustment (1/14= 7%) (Table 12, Category E); Lilly M. and Rina M. had three disagreements over adjustment (3/64 = 5%) (Table 12, Category D & E). A consideration of the content of Lilly and Rina's various disagreements over how

to correct an error in rotation of other than 90 revealed that even though Rina and Lilly continued to argue about it, their grasp of the concept was more secure than was Amy and Leanne's. It is indeed likely that Lilly and Rina continued to argue about their individual approaches because each was secure in touting her own approach. A look at the one disagreement that Amy L. and Leanne L. had revealed that they were unsuccessful in their attempt, and thenceforth abandoned the use of the inverse strategy and resorted to a simpler strategy, namely that of clearing the screen and starting over again (Selection #8A and #8B, in the COLLABORATION section). Thus even when the same concept was dealt with by different pairs, at times the less adept pair dealt with the concept less successfully than did the more adept pair.

The mathematical idea and the manifestation of its use, and the disparate ways in which it was used at times, revealed a relationship with the level of achievement of the children. It proved necessary to look further, to investigate the content of the disagreements, about using the inverse to correct an incorrect turn for example, in order to distinguish the ways in which the pairs achieved different measures of success in the use of a strategy. At times the frequency of occurrence only raised further questions as to what went on within the disagreement itself; the content aspect will be dealt with later in the sections on COLLABORATION and SUPPORTED TALK.

The findings indicated that the mathematics concept category of the disagreement did reflect the level and the pace of the child's learning. The more adept children were having disagreements over the more difficult concepts of which the less adept were not yet aware, or with which they did not attempt to deal.

The mathematics concept category represents a domain of difficulty.

The mathematics concept category of the disagreement was in itself an indicator of difficulty, that is, the mathematical idea represented an area of difficulty for at least one partner. Not all the disagreements engaged in by the high level achievers were related to the more difficult concepts. For example, it was of interest to find that one high achieving child (Jon H.) had difficulty with a concept which one might assume was a relatively easy one to master, namely that of lateralization (that is, the direction in which the turtle turns to make a move, right or left). Jon H. had difficulty with the concept of lateralization notwithstanding the fact that he was simultaneously grappling with some of the more difficult concepts (such as variable) which neither the

moderately adept nor the least adept students ever broached. Jon H. and Eric H. engaged in 6 disagreements concerning lateralization, 6/36 or 17% of Jon's total number of disagreements.

When one looked at the entire Grade 5 class, it was found that lateralization proved a stumbling block for two of the high achieving children, Jon H. and David H., in the class of twenty three children (see Table 4). A number of the members of the class had difficulty with this aspect. The difficulty which Jon had with lateralization was not evident in the initial or post session Grade 5 interviews (whereas David's difficulty was evident in both interviews); Jon's difficulty with lateralization only became evident as a result of the analysis of the disagreements. Thus, the analysis of the disagreements was able to add to an understanding of the difficulties certain of the children had. It is possible that the disagreements engaged in by Jon H. and Eric H. concerning the lateralization aspect were able to contribute to Jon's learning. Here again, one would need to look further, to the actual content of the disagreements over lateralization, to determine whether what went on during the disagreements over lateralization could have contributed to Jon's learning.

The Resolution and Nonresolution of Disagreements

It was expected that the complex disagreements (which included Level I and Level II strategies) were more likely to be resolved than the simple disagreements (which featured only Level I strategies), since complex disagreements contained Level II strategies which supported and represented discussion, elaboration and compromise. This was Genishi and Di Paolo's (1982) finding; this was my own earlier finding (Zack, 1984) which was based on an analysis of all the disagreements (not only mathematics and geometry concept disagreements) engaged in by one pair of the children, Carol L. and Amy L., when they were partners in Grade Four. The finding in this study was that in the case of Carol L., and Amy L., and their respective partners in Grade 5, the simple mathematics and geometry concept disagreements (which featured the Level I strategies only) were as likely to be resolved as were the complex mathematics and geometry disagreements (which featured both Level I and Level II strategies). In the case of the middle (Lilly and Rina) and high level achieving children (Jon, Eric and Russ), the complex disagreements were more likely to be resolved than were the simple disagreements (see Table 13). In each level of achievement (low, medium and high

Table 13

Number and Percentage of Simple, Complex and Complex/Persuasion[a] Disagreements per Student Group

	Disagreements with Resolution				Disagreements without Resolution			Total # of Disagreements
Student Groups	Simple	Complex	Persuasion	TOTAL	Simple	Complex	TOTAL	
Jon H. & Eric H. (Grade 5)	5 = 16%	6 = 19%	17 = 55%	28 = 90%	1 = 3%	2 = 6%	3 = 10%	31
Jon H., Eric H. & Russ H. (Grade 5)	5 = 14%	7 = 20%	17 = 49%	29 = 83%	2 = 6%	4 = 11%	6 = 17%	35
Rina M. & Lilly M. (Grade 5)	3 = 5%	16 = 25%	30 = 47%	49 = 77%	1 = 1%	14 = 22%	15 = 23%	64
Carly L. & partners (Grade 5)	5 = 25%	5 = 25%	5 = 25%	15 = 75%	-- --	5 = 25%	5 = 25%	20
Amy L. & partners (Grade 5)	-- --	7 = 50%	1 = 7%	8 = 57%	1 = 7%	5 = 36%	6 = 43%	14
Amy L. & Carol L. (Grade 4)	3 = 12.5%	3 = 12.5%	9 = 37.5%	15 = 62.5%	2 = 8%	7 = 29%	9 = 37.5%	24

[a] i.e., complex disagreements resolved due to persuasion

level of achievement) there was a greater proportion of disagreements resolved than left unresolved. However, the more adept children seemed to have a slightly greater proportion of resolved over unresolved disagreements than did the less adept.

It seemed at first that the children at the medium and high level of achievement who were incorrect in their stance were more likely to accept their partner's correct argument and were more likely to be persuaded by their partner's argument than were the low achievers (see Table 14). The question which then followed was, why? Were the middle and high level partners better persuaders, or were the middle and high level persuadees more atuned to the Logo workings and therefore more receptive to their partner's argument, or both? However before that question was entertained, recourse to the Grade 4 results of the less adept children, Carol L. and Amy L., refuted the pattern of Grade 5 results which featured a weak performance by Carol L. and Amy L. as persuaders. The Grade 4 analysis of incorrectness and resolution by means of persuasion revealed on Carol L. and Amy L.'s part a quantitative pattern consistent with the middle and high level groups in Grade Five (see Table 15). The findings indicated that it was necessary to look more closely at the differences between Carol L. and Amy L. and their respective partners in Grade 5, and Carol and Amy working together in Grade Four. It was also necessary to investigate further in order to determine whether there were other differences between the children at the various levels, and to look further in seeking to answer the question of how the children were persuaded.

Certain verbal and nonverbal strategies emerged as effective ones in leading to persuasion. The Level II strategies which were seen most often to result in persuasion were *Supporting arguments* --25%, *Focusing* --16%, and *Reference to the computer screen verdict* --16%. There was one strategy used exclusively by one pair, Lilly M. and Rina M., namely *Reference to the student's own recorded notes* --14%. Thus the findings concerning resolution, and resolution due to persuasion, and the findings which revealed that certain supporting arguments played a role in persuasion, suggested that the direction for further investigation ought to be related to the qualitative dimension, to the how's. It was deemed necessary to investigate how these strategies were used by the children to persuade their partners, and how the use of the strategies reflected the children's understanding of Logo geometry. This component will be developed in the section on SUPPORTED TALK.

Table 14

Number of Times the Participant was Incorrect, was Incorrect and Acceptor/ Resolver of the Disagreement, and was Incorrect and Persuaded

Groups	Total Number of Incorrect Responses	a	Number of Incorrect Responses as Acceptor/Resolver	Number of Times Participant Was Incorrect and Persuaded	a	Number of Times the Participant Should Not Have		Number of Times the Participant Accepted for the Sake of Expedience
						Accepted	Been Persuaded To Accept	
Grade 5								
Jon H.	9	(36)	5	3	(5)			
Eric H.	6	(29)	5	4	(5)			
Lilly M.	12	(64)	11	6	(11)			
Rina M.	18	(64)	9	9	(9)			
Carol L.	8	(20)	2			1	1	
Amy L.	7	(14)	1			1		
Leanne L.	4	(14)	2					1

[a] total number of disagreements

Table 15

Number of Times the Participant was Incorrect, was Incorrect and Acceptor/Resolver of the Disagreement, and was Incorrect and Persuaded (Grade 4 Group Included)

Groups	Total Number of Incorrect Responses	[a]	Number of Incorrect Responses as Acceptor/Resolver	Number of Times Participant Was Incorrect and Persuaded	[a]	Number of Times the Participant Should Not Have		Number of Times the Participant Accepted for the Sake of Expedience
						Accepted	Been Persuaded To Accept	
Grade 5								
Jon H.	9	(36)	5	3	(5)			
Eric H.	6	(29)	5	4	(5)			
Lilly M.	12	(64)	11	6	(11)			
Rina M.	18	(64)	9	9	(9)			
Carol L.	8	(20)	2			1	1	
Amy L.	7	(14)	1			1		
Leanne L.	4	(14)	2					1
Grade 4								
Carol L.	4	(24)	2	2	(2)			
Amy L.	11	(24)	6	5	(6)			

[a] total number of disagreements

In summary then, in considering the pattern of the mathematics and geometry disagreements, the preliminary look revealed that the total number of disagreements per pair and their duration bore no relationship to the level of achievement of the children, whereas the mathematical idea inherent in the disagreement, and whether the disagreement was resolved or unresolved, were related to the level of achievement of the children. The findings in turn suggested direction for further investigation. Therefore the study then proceeded with a closer look at the supported talk, especially at the supporting arguments, since the supporting arguments represented 25% of the verbal strategies which when used resulted in persuasion; these findings will be reported in the section on SUPPORTED TALK. It was necessary as well to look at the kind of working relationships which existed between the partners, and to attempt to determine the extent to which the existing pairings either promoted or inhibited learning; these findings will be reported in the next section to follow, the section on COLLABORATION.

WORKING RELATIONSHIPS BETWEEN THE DYADIC PARTNERS

Cooper, Marquis and Edward (1986) have stated that child-child interaction may be *doer-doer* and symmetrical, in which case both children are learning and striving to make sense of the subject matter, or *knower-doer* and asymmetrical, in which case one child possesses the knowledge, and the other child is still striving to learn it. The data were analyzed in order to identify whether the kind of relationship that existed between the partners was *doer-doer*, or *knower-doer*. The investigation also aimed to probe a broader issue which was:

> DOES THE KIND OF RELATIONSHIP THAT EXISTS BETWEEN THE PARTNERS INFLUENCE THE LEARNING? IF YES, IN WHAT WAYS IS THE LEARNING FACILITATED OR HINDERED?

The cognitive aspect of the working relationships will be the prime focus in the section which follows. The social aspect, while vital, will only be given brief mention in this report, toward the end of this COLLABORATION section. The findings revealed that for the most part the interaction between the partners was symmetrical and collaborative. Cooper, et al. (1986) have stated that "in collaborative interaction, the children share the power of directing the interaction more equally [than in didactic interaction], either through alternating or sharing the teaching role, or by not assuming a clear leader-follower pattern" (p. 282). The report will describe ways in which the children explored together and took turns at being the guide to learning, and the ways in which the attempts made to guide and to explain were, and at times were not, successful. The sections dealing with the collaborative aspects are entitled: "Exploring together: Collaborative *doer-doer* relationships," "Taking turns being the *knower*," "The *knower* as guide: The *knower* explains to the *doer*," "A *doer* sees herself in the role of *knower*," "The *knower* knows, but is not effective as guide to understanding."

In the study, there was one single instance of a didactic, asymmetrical relationship; in that relationship, one of the partners, Russ H., knew more about the geometry and syntax aspects of Logo than did his partners, and was a giver of information far more often than a receiver of information. The didactic component will be dealt with in two sections entitled: "A *knower-doer-doer* triad relationship in which

the *knower* is class expert" and "*Knower* functions: Giving information and teaching."

Hawkins, et al. (1982) have stressed the need for investigation of "the consequences of children working together on learning and social skills" (p. 372). The emphasis in this section and in the subsequent section on SUPPORTED TALK will be on the cognitive component, on the "kind of learning that *may* occur as a result of children working together" (italics added; 1982, p. 372). One of the primary aims of the study was to investigate just that question which Hawkins, et al. (1982) have insisted is so vital, namely the ways in which peers working together may be promoting both their own and their partner's learning.

Symmetry in the Cognitive Aspect of the Dyadic Partnerships

The Emihovich, Miller and Clare study (1985) with preschool children, dyads aged 5 to 6 years old, found that it was the same child in the partnership who was consistently the giver of information. In this study, it was expected that the children who had a better general grasp of Logo angle and other mathematics and geometry concepts would play a more dominant role (though not an exclusively dominant role) in each relationship. In order to check whether such a relationship did indeed exist between the pairs, the researcher used the results of the interview data (February and March, 1986, and May and June, 1987), and placed the children in an ordinal rank position (a task which is always difficult since the interview evaluation cannot measure certain understandings) (see Table 4). If one ignored the impossibility of placing children into an absolute ordinal rank position and one could accept for the sake of argument that these were accurate relative achievement gauges, then the children would be ranked as follows in terms of their understanding of angle, beginning with the highest achievement level on the left, and proceeding to the lowest achievement level at the end, at the lower right:

Russ H. Jon H. Michael H. Eric H. Lilly M. Rina M. Shauna L. Carol L. Amy L. Leanne L. Doug L.

Thus, if the more able child in the dyad (and the most able in a triad) would be expected to be more dominant in their relationships with a partner or partners, then one would predict that the child listed on the left below would be expected to be

dominant over the child listed on the right, in the case of disagreements in which statements could be judged correct or incorrect:

	Groups		
Most Adept	Jon H.	and	Eric H.
	Russ H.	and	Jon H. and Eric H.
	Russ H.	and	Michael H.
Moderatley Adept	Lilly M.	and	Rina M.
Least Adept	Carol L.	and	Amy L.
	Carol L.	and	Leanne L.
	Carol L.	and	Doug L.
	Shauna L.	and	Carol L.
	Amy L.	and	Leanne L.
	Shauna L.	and	Amy L.

However when the suggested move statements (both the proposal for a move and the opposition to the suggestion) which could be evaluated as either correct or incorrect were analyzed, it was found that the relationships related to the cognitive aspects were more balanced during the mathematics concepts disagreements than had been expected (see Table 16).

The analysis of the working relationships between the partners revealed that in most instances the roles of *knower* and *doer* (Cooper, et al., 1986) shifted constantly. At this juncture it must be pointed out that in all but one instance (namely Russ H.), the term *knower* is being used in a qualified, relational way, in that the *knower* was often only a small step ahead of the other child; the latter child was then identified as the *doer*. With the exception of the one aforementioned case, the case of Russ H., the term *knower* is not meant to signify one who is expert.

Another aspect which is of significance is the size of the gap between any two of the children. When one looks at the entire class of the twenty-three Grade 5 children (Table 4), one sees that the gap between the afore-mentioned individuals who paired up was, in certain instances, greater than in other instances. In the case of all the

Table 16

Frequency of Correct and Incorrect Statements Made as Proposer or Opposer of a Move

Participant Groups	Correct as Proposer or Opposer	Incorrect as Proposer or Opposer
Jon H.	6	6
Eric H.	7	6
Lilly M.	26	16
Rina M.	20	22
Carol L.	1	2
Amy L.	3	7
Leanne L.	2	6
Russ H. (6) [a]	4	1
Shauna L. (4)	5	2
Doug L. (4)	2	4

Note: In certain instances, it was not possible to evaluate position statements as correct or incorrect; therefore, the totals do not balance.

[a] Russ, Shauna and Doug were videotaped for 6, 4 and 4 sessions respectively.

pairings, however, the children seemed to be suited to each other in terms of achievement levels. Among the participant groups videotaped for this study, there were no instances of pairings across extremely wide gaps such as Cox and Berger (1981) have indicated might present a difficulty, such as matching a high achiever with a low achiever. But the more important question is how the children communicated with each other across the gap. Whether there is a small gap, or a somewhat larger gap in terms of achievement between the partners, the salient issue is: does the *knower* explain and does the *doer* seem to understand? This point will be touched upon in the portion to follow, but the question of explanations and of the role played by explanations will be dealt with more comprehensively in the subsequent section of the findings chapter which will consider the entire body of supported talk in the disagreements.

Doer-doer and *Knower-doer* Relationships

During the disagreements. there were instances at all levels--the highly adept, the moderately adept, and the least adept--of *doer-doer* relationships, that is, times during which both partners were exploring various components of the concept of angle. There was some evidence of *knower-doer* relationships in which one child knew more than the other in relation to one concept, and then in relation to another concept, the roles would change. In these *knower-doer* relationships in which one child knew more than the other, in some cases the *knower* did support the talk, in some instances the *knower* did not support the talk, or did not express the supported argument clearly. The following excerpts from the transcriptions will illustrate examples of each type of situation.

Collaborative Relationships

Exploring together: *Doer-doer* relationships

The excerpts below represent *doer-doer* relationships between the partners; they feature situations in which both children are groping and in some cases, both partners are incorrect. In the first excerpt, Amy L. and Shauna L. sought to complete a 180 (degree) turn, although they were not aware that this was the input that they needed. In the subsequent three excerpts, the pairs (Amy L. and Leanne L., Carol L. and Doug L., Lilly M. and Rina M.) were all trying to make a square corner for their various

projects, and needed an input of 90 (degrees), although it is clear that they did not at that point know that 90 was the "special number" that other members of their class used for making corners. Note that Monica, the Logo teacher, had introduced 90 when the children were in Grade 3. During two of the Logo sessions in Grade 4, Monica worked on the concept of 90 with Amy L. and Carol L. while they were at their computer station (see Selection #19, and Appendix Q). The fact that some of the children were still in the process of "discovering" the concept of 90 (90 degrees in formal language) attested to the complexity of the notion, and to the amount of repetition and continued experience the children need before they "assimilate" (Piaget) a "powerful idea" (Papert).

In the following excerpt, Amy and Shauna had already keyed in RT 10, RT 30, RT 15, for a total turn of RT 55. They then tried to get the turtle cursor turned all the way around (half of a full rotation), so that it would be "facing down."

Selection #1
Amy L. and Shauna L., Oct. 23, 1985, D/A #3, p. 11 & 12

Amy L.:	M-m-m, now it's going to point to there (pointing to computer screen). So five more, five more. [INCORRECT]
Shauna L.:	We need it to be down. We need- (pause) I'll tell you how much. (Pencil points to her notebook.) 'Cause (pointing to the screen) that's exactly half now. (Writes something.) O. K. (keys in 55) [INCORRECT]
Amy L.:	-R -T fifty-five? Is that -R -T fifty-five so far? Wai- Is that -R -T fifty-five?
Shauna L..	Ya. (keys in RT 55) So far. O. K. Look where it is now. (pause) It's right here. So-

Amy L.: So it's perfect.

Shauna L.: No, it's right there (indicating with pencil on the screen).

Amy L.: It's perfect.

Shauna L.: No. (pause) It's not. (pause) We have to push it more so it'll be facing down. (Points to the screen to indicate.)

Amy L.: So how many?

Shauna L.: Let's try, let's try (pause)

Amy L.: Who did -R -T fifty-five? (hands on hip, tone indignant)

[Total turn is now 110]



Shauna L.: Me

Amy L.: (funny look)

Shauna L.: We're supposed to.

Both Amy L. and Shauna L. were incorrect. They were aiming to turn turtle 180 (although they were not aware that this was their goal). They had turned 55 at this point in their work. Amy's suggestion of "five more" was clearly not sufficient. And a turn of 55 certainly did not represent the halfway mark such as Shauna L. thought when she was heard to remark "Cause that's exactly half now", since the total had to be 180. However, this was the first instance in which Shauna was heard to speak of the parts of the total turn in a proportional way. We will see in a subsequent findings section (entitled "Supported talk as a reflection of the child's understanding of Logo angle") how 55, and its multiple 110, became pivotal in Shauna's own unique construct of the Total Turtle Trip theorem (Note 22). The Papert (1980) theorem, in its simplest form, is related to the standard measure of 360 degrees in a full rotation. The Shauna L. theory, as it developed later, was that 110 represented half a rotation, and therefore 220 represented a full (our 360 degree) turn.

In the excerpt below, Selection #2, one sees that Amy L. has also been working with an incorrect notion. Amy L.'s (mis)understanding was due to her not so outlandish but nevertheless incorrect deduction that since a square had equal sides and equal

angles, the inputs for both the sides and the rotational turns of the square that she and Leanne L. were making ought to be the same. As a result, Amy and Leanne have begun by keying FD 83 and RT 83. These inputs were chosen despite the fact that in her own Grade 4- Grade 5 computer notebook that was lying at her side, and which she consulted, Amy had recourse to the recorded commands for the rectangle that she and Carol L. had worked on in Grade 4, with Monica's help (see Selection #20, and Appendix Q; Appendix C). That recorded listing had the notation of FD 109, RT 90, FD 80, RT 90, etc. for the Grade 4 project which Amy and Carol had called "Bullet." The RT 90 command was therefore available for rote use by Amy, but its importance was not appreciated by her. For the square which Amy L. and Leanne L. were attempting to make below, the input to RT ought to have been 90, not 83 or 93.

Selection #2

Amy and Leanne, November 20, 1985, D/A #2, p. 6.



Leanne L.: O. K. (keys in -R -T 83)
[INCORRECT][FD 83]
[RT 83]

Amy L.: NO! Not eighty-three, it's gonna be the same thing.

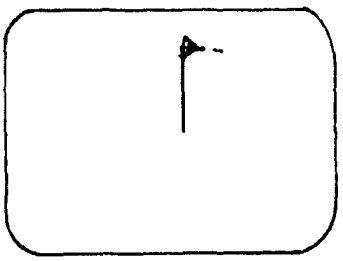
Leanne L.: (looks quizzical)

Amy L.: Ninety-three.
[INCORRECT]

Leanne L.: -R -T ninety-three?

Amy L.: Ya. (pause) -R -T ten now (keying in), -R -T ten. O. K. There. Perfect.

[RT 10]



Notice that Amy L. said "perfect." It has been noted elsewhere by the researcher (Zack, 1988a) that when the children did not have a sense for the size of the rotational turn, they would "fiddle around" (Rina M.'s term, interview May, 1987) until it looked like it was perfect. Before moving the turtle they would often pray aloud that it be perfect,

and in looking at the turtle cursor's end state, would evaluate it as perfect before the next move revealed that it was not perfect. In the above instance, it did not even look perfect. Visually, the end state of turtle's position as a result of the turn of 93 looked to be more askew than the turn of 83 had rendered it. And despite that, Amy L.'s evaluation was that it was perfect. At this point neither Amy L. nor Leanne L. had a concept of 90, even as a "special number" for making square corners.

In the following excerpt, Carol L. and Doug L. attempted to turn and make a square corner in their camel project. They needed an input of 90 to their turn, in this case RT 90 as opposed to LT 90.

Selection #3

Carol L. and Doug L., December 11, 1985, D/A #2, p. 24



Carol L.: (Keying in) -R -T space (looks to the side, at her book), what do we do here? Um, let's try thirty. ----
[INCORRECT]
(Keys) [Should be -R -T 90]

Doug L.: No vay (sic). (They both look at the screen.) We're going the wrong way.
[INCORRECT]

Carol L.: (Agrees, nods, hand out to keep Doug from keying in.) -L -T thirty.
(Keys in) [INCORRECT]
[Carol should not have accepted to do LT.]

In this selection, both were incorrect about the input of 90, and Carol L. was ill-advised by Doug L. to turn left instead of right.

In the following excerpt, Lilly M. and Rina M. attempted to do the second corner of their rectangle which was part of their "ghettoblaster" portable cassette tape recorder project. They needed a turn of 90 but did not know it. Rina M. stated that they needed

to "go half," and then referred to the input previously used in their attempt to do a corner namely 85. Lilly M. at first did not agree that they had previously used 85 as an input, but she then realized that they had.

Selection #4
Lilly and Rina, November 20, 1985, D/A # 3, p. 7.

Rina M.: -R -T (writing), we wanna go half (uses hand motion), we want -R -T, -R -T eighty five. [INCORRECT]



Lilly M.: No, 'cause we were, -R -T eighty (-five, spoken very quietly), right (nods). [INCORRECT]

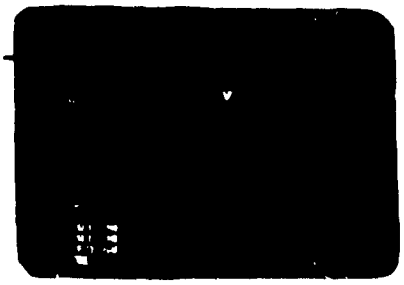
Both partners agreed to use 85. Both talked in terms of the quadrant of the circle (which we know contains 90 degrees), but although they saw the quadrant, as "half" of a half turn in their case, neither Lilly M. nor Rina M. at this point in time appreciated that 90 was the amount needed to complete a quarter turn, not 85.

Taking turns being the *knower* as they explore

The excerpts cited below illustrate *knower-doer* relationships between the partners, but the children take turns assuming the roles of knower and doer. One child knows more than the other during one disagreement, then in a subsequent instance the roles change. In the first selection below, one sees Jon H. insisting incorrectly that the turn ought to be to the right. It has been stated previously in the findings (in the section on mathematical idea) that despite his overall ability with Logo, Jon H. was noted to have difficulty with the lateralization aspect, that is, whether the turtle cursor ought to turn left or right. Eric H. was quite sure about the direction of the turn, and in the instance below, Selection #5, Eric not only told Jon but, as support for his position, also demonstrated to him how the turtle cursor was going to turn. Jon countered with a suggestion that they test it out on the computer (conditional test hypothesis) --and the computer verdict then showed Jon that Eric was correct.

Selection #5

Jon and Eric, October 16, 1985, D/A # 4, p. 6.



Jon H.: Awright, now let's write [he means - - type], (pause) no (pointing to the screen) now we have to make a right turn (finger on the keyboard key).

Eric H.: I know, I know (in a high-pitched voice).



Jon H.: Right turn ninety.

Eric H.: That's a left turn, Jon.

Jon H.: Right, (point to the right on the screen), that way.

Eric H.: (Keys) (pause) Left. - - - -

Jon H.: Right.

Eric H.: It's a left, because (stands up and turns himself to the left) it's going down (points down) and it's turning like that (indicates right with his finger on the screen)

Jon H.: Oh- (pause) O. K., (?we'll solve this).

Eric H.: (stands and keys) Left ninety?

Jon H.: Ya.

.

.

.



Jon H.: (keys -L -T 90)

(They look at the screen) - - - -

Eric H.: (?I) Told you (smiles, then sits down).

Eric's demonstration was particularly interesting in view of Papert's (1980) notion of "body-syntonicity" (See Note 14). Eric used his body and turned in a way that

simulated the turtle's own turn. He showed that from the turtle cursor's vantage point, when turtle is facing down ▽, the turn which looks like a right turn to us ▽→ is a turn to the left for the turtle cursor. Hillel's (1984) observation that the "children 'played turtle' mostly to resolve lateralization problems (left/right)" (1984, p. 11) was confirmed in this study; in this study the only occasions upon which the children "played turtle" were in cases of lateralization difficulties.

The date of the above transcription, October 16, 1985, represents the first of Jon H. and Eric H.'s Logo sessions together. During their Logo time together, Jon H. was incorrect about the needed direction of the Logo turtle cursor's turn in 4 out of the 5 the disagreements about lateralization. However Jon was more in control than was Eric in regard to the amount of rotation needed when turtle turned. In the following excerpt, on October 16, 1985, Jon was aware that the turn for the corner of the T.V. set they were making would be accomplished by using an input of 90. Eric was incorrect when he countered that "ninety is not enough."

Selection #6
Jon and Eric, October 16, 1985, D/A # 2, p. 5.

Jon H.: (Looks at his fingers spread) Right turn (points to the right). (Pause.) Write it. [He means "Type it."] Right turn.



Eric H.: (Keys) (pause) How much? (Looks over to computer #2 on his left.)

Jon H.: Ninety, ninety.
[CORRECT]

Eric H.: (Keys) Ninety is not enough, (looks at the computer screen) but anyway (keys RETURN).
[INCORRECT]
[Eric accepts, and thereby the disagreement is resolved.]
(They look at the screen)



Jon H.: (Looks at Eric.) Ninety. (Jon smiles and nods.)

Thus we were able to see that in the first instance (Selection #5), Eric H. was the *knower* and Jon H. was the *doer*, and in the second instance (Selection #6), Jon H. was the *knower* and Eric H. the *doer*. In these two selections, in each instance the *knower* stressed the information at the end of the disagreement. This possibly can be considered an example of "showing off," "rubbing it in," or "one-upmanship." However, the stress made at the end by the *knower* might also have served for the hearer as a point of emphasis, an important learning underlined. It may indeed have been a little of both.

The *knower* as guide: The *knower* explains to the *doer*

The first three excerpts of the selections which follow immediately below represent a *knower-doer* relationship in which the child who is the *knower*, in this case Lilly M., explains the mathematical idea to her partner, Rina M. The mathematical idea which is presented in all six sample transcription excerpts is that of the adjustment of an erroneous move via the use of the inverse. The samples will illustrate three instances of Lilly's attempt to explain her point of view concerning the correction strategy to Rina. Two other contrasting samples of children working with the same strategy have been chosen; in one instance the children are in a *doer-doer* relationship, in the other instance the children are in a *knower-doer* relationship but the *knower* does not explain the concept to his partner. Thus, the samples illustrate (a) the way in which Lilly M., the *knower*, explained the concept to her partner Rina M.--Selections #7A, 7B and 7C; (b) the way in which two *doers*, Amy L. and Leanne L., who did not yet know the concept of correcting for inverse, struggled to explain their understanding of it to each other, for each had a glimmer of some of the components-- Selections #8A and 8B; and (c) a situation in which Jon H., a *knower* of the concept, did not explain the concept to his partner Eric H.--Selection #9.

It seems appropriate at this point to describe briefly the mathematical idea itself. The correction or "debugging" strategy can be represented by the sequential steps which are listed in Table 17. Jon H. and Russ H. were very adept at the use of this "debugging" strategy; they performed the steps quickly and automatically without

discussion, and only recorded the correct move (step 3) in their record-keeping books. The components which Jon H. and Russ H. found easy were still being explored by the other children. As was mentioned previously in the section on frequency of occurrence of certain mathematical ideas, the correction strategy was attempted by Amy L. and Leanne L. upon one occasion, and Lilly M. and Rina M. at various times, with different measures of success. Lilly's, Rina's, Amy's and Leanne's degree of understanding of the strategy is noted in Table 17. Their arguments in support of their individual strategies for the correction and recording of erroneous turns will be heard in the transcript portions which follow.

In the three following excerpts, Lilly is the *knower* as concerns the concise way to adjust for an incorrect move. The samples from the transcripts illustrate three of the six instances of disagreements between Lilly and Rina concerning this concept of adjustment; the three disagreements cited below were coded as mathematics concepts disagreements, and the three disagreements not cited here were coded as plan and programming --recording-the-steps disagreements. In each selection, Lilly M. attempted to persuade Rina M. that her (Lilly M.'s) correction strategy ought to be used.

Selection #7A
Lilly and Rina, October 23, 1985, D/A #5, p. 6.

Rina M.:	So it's left, left fifteen'. [They had gone RT 15 in error.]
Lilly M.:	'Left fifteen, no, left fifteen plus what you want.
Rina M.:	No, no, no, no, no.
Lilly M.:	'Yes, left thirty, because you wanted it to go fifteen and, but you went fifteen that way so if you turn fifteen and then "you turn-
Rina M.:	"You want to get it straight, you want to get it straight like this (points to the screen).
Lilly M.:	Left fifteen (agrees to Rina's suggestion).

Table 17

Steps Taken in the Correction of an Erroneous Move by Highly, Moderately and Least Adept Users of Logo

Sequence of Steps	Russ H. and Jon H.	Lilly M.	Rina M.	Amy L.	Leanne L.
	1. Student makes the wrong turn.	✓	✓	✓	✓
	2. Student adjusts with inverse AND makes the correct turn.	✓ in one move	✓ in two moves	in two moves; but wrong calculation	
	3. Student records only the correct move.	records all moves	records all moves		✓

Selection #7B

Lilly and Rina, October 23, 1985, D/A #19, p. 17

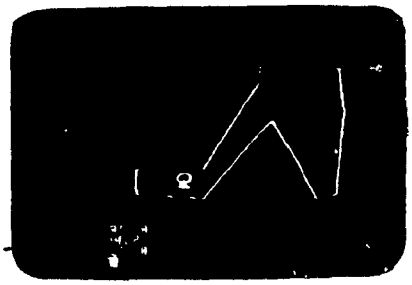


Rina M.: Right ten (keys) [RT 10]

Lilly M.: No, left.

Rina M.: Left ten, left.

Lilly M.: No, left twenty, because we just-, we want to go back and ten



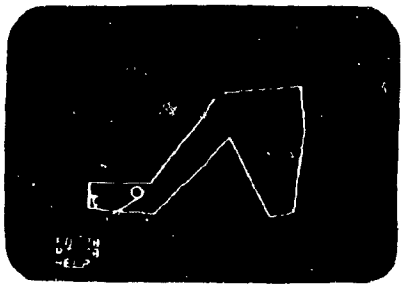
Rina M.: Left twenty, left twenty (keys, [thereby indicating acceptance]) . . .

[LT 20]

It might seem at this point that Rina has accepted that Lilly has a good strategy for doing the correction via a two-in-one step correction. In the subsequent session cited below, Selection #7C (October 30, 1985), however, Rina was not amenable to Lilly's suggestion, and Rina did not counter with her one-step-at-a-time strategy, but rather responded with a suggestion to use an (inexplicable) input of ninety-nine.

Selection #7C

Lilly and Rina, October 30, D/A # 5, p. 17



[RT 70]

Lilly M.: No. It went the wrong way. (Traces on the screen.) So now it's left, left seventy plus what we wanted to do. [Lilly's strategy is RT X --- LT 2X, that is, RT X is adjusted by LT 2X]

Rina M.: (Hand motion, then looks startled) Oh, we're not even writing this down! (Grabs her book, looks at the

screen.)

.
.
.

Rina M.:	(Keying) Left, I'm going R-, left ninety-nine. [INCORRECT for inverse]
Lilly M.:	Left seventy plus what we wanted.
Rina M.:	Na-a-a.
Lilly M.:	Seventy-
Rina M.:	Hold on, left (reaches over to write in her book).
Lilly M.:	Left 'seventy-
Rina M.:	'I'm writing left ninety-nine.
Lilly M.:	Left a hundred and forty, you know why, because (tracing on the screen) because we wanna go back - (pointing to the screen).
Rina M.:	O. K.

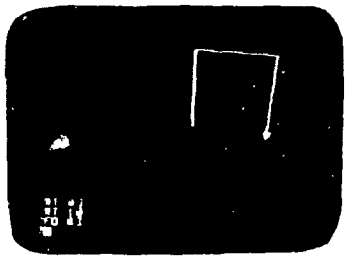
In all the disagreements, it was clear that Lilly M.'s strategy for correction was to adjust for the erroneous move by a two-in-one step move. Since they had in error turned RT 15, RT 10, and RT 70 respectively, and the turns should have been LT 15, LT 10, and LT 70, the adjustment should be LT 30, LT 20, and LT 140 respectively. Rina M.'s strategy in two of the disagreements (Selections #7A and #7B) was to adjust for the incorrect move with its direct inverse first, that is, to "get it straight" back to its former state (via LT 15, LT 10), and then to proceed with the move that should have been done originally (LT 15, LT 10). In one of the disagreements, Lilly accepted to go along with Rina's strategy (Selection #7A), in two of the cases Rina accepted to go along with Lilly's strategy (Selections #7B and #7C).

Rina M. seemed ready to acknowledge that the quicker strategy was the better, more efficient one. But in fact in later sessions Rina was seen to revert to her step-by-step "safer" strategy both in adjusting for the error and in recording the steps.

Lilly M.'s approach was faster and more advanced. Rina's reluctance to adopt Lilly's strategy reflects the fact that a *knower* can persuade or convince a partner to advance only when that partner is ready to take the step. In Vygotskian terms, Rina M. was not prepared to move from her zone of actual development to the proximal zone despite repeated urging by her (in this instance) more able partner, Lilly. Rina was in the end not ready to give up her more methodical, simpler strategy, perhaps because she was not sure that the other strategy was as reliable.

A *doer* sees herself in the role of *knower*

The excerpt below is out of place in the respect that it belongs thematically within the *doer-doer* relationship portion of the paper. However, content-wise, it was considered most pertinent and vital for this section. The transcript portions cited below reveal how the two *doers*, Amy L. and Leanne L., attempted to deal with the "debugging strategy" of correcting for an erroneous turn. They had turned LT 83, instead of the needed RT 83. At first Leanne was ready to clear the screen and begin again, using the simplest of the "debugging" strategies; however, Amy stopped her and proposed to demonstrate a better alternative strategy. Amy saw herself in the role of *knower*, but unfortunately did not have an adequate command of the components of the strategy.



Selection #8A
Amy and Leanne, November 20, 1985, D/A #3, p. 7

[Amy has just suggested that they key in LT 83, and Leanne has done so]

Leanne L.: Eeeeee! (Hand on mouth). Made a boo-boo.

Amy L.: Le-anne!

Leanne L.: Oh well, -C -S!

Amy L.: No-a! (keying in) Leanne, all you have to do is go like this (Erases in her book, leans over to key in) -R -T

space (moves back to her book), what's eighty-three plus eighty-three? Three plus eighty-three, three is six, sixteen, -R -T space one one six (keys in). Return. (pause) --- [INCORRECT]



You see? (points to the screen) Now, you just have to go a little more.

Leanne L.: (Leans forward toward the screen) O. K.

As we see, Amy L. suggested that they needed to turn to the right double the amount--Lilly M.'s strategy, but Lilly would most probably have stated the correct amount. would have stated that RT 166 was needed. An error in calculation on Amy's part resulted in a sum of 116, as opposed to 166. Had Amy been more atuned to this process of correction, the fact that she found that she had to go "a little more" (it is in fact more than a little more, it is 50 more that they needed) would have been a cue that there was an error in the calculation, since the adjustment ought to have brought her back to exactly the spot originally desired. But Amy was not able to detect that, nor was Leanne L. able to offer any counter-argument. Amy could only propose further "patch up" repairs.

In the interaction which followed as the disagreement continued, Amy and Leanne continued to try to adjust for the error in the turn. It almost sounded as if they were each proposing an alternative strategy. The partners were not even aware that they were disagreeing about the use of the diverse essential components of the same process, all parts of which needed to be present. Amy suggested again that they needed to go "a little bit more so it can go straight." Leanne insisted that they needed only to record the correct turn, that is, RT 83, not cognizant of the fact that indeed only the correct turn need be recorded, provided that all the proper adjustments had first been made (See Table 17).

Selection #8B

Amy and Leanne, November 20, 1985, D/A #3, p. 8



Amy L.: O. K. So that was -R -T (pause) one sixteen*, and now-

Leanne L.: *No, no, no

Amy L.: well, we need a little bit more so it can** go straight

Leanne L.: **No, you know, you know why (? because) we, we should write -R -T eighty three, because before we made a boo-boo, that's why we had to go, ***it was -R -T eighty-three (writes in her book)

Amy L.: ***A boo-boo (mimics Leanne's statement) What do you mean? -R -T eighty- No!! ****(? not audible)

Leanne L.: ****The end.

Amy L.: So now we have to do -R -T eighty-three again.



Leanne L.: 'Kay, let me type it. Let me type it.

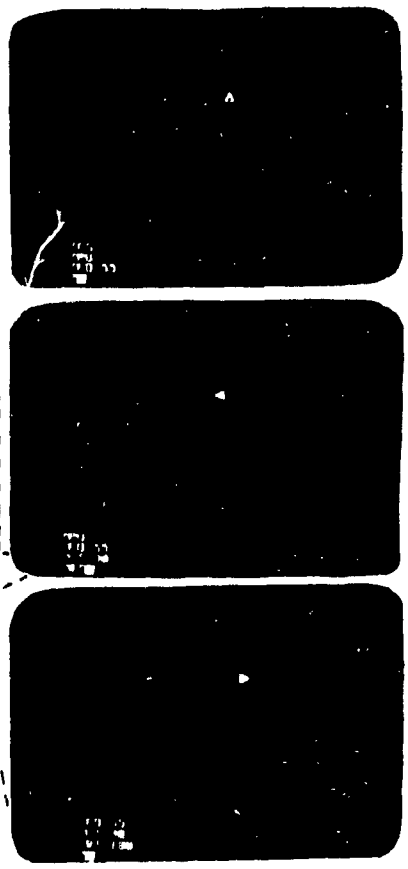
[Note that the single and multiple stars (*, **, ***) indicate occurrences of overlap in the participants' talk.]

The partners proceeded to "patch up" by adding another RT 83 (instead of the required 50), and did not record their original wrong turn of LT 83, which resulted in much grief for them later on in the session, for they could not decipher what had gone wrong. Their eventual extreme frustration with their ur ccessful problem solving attempts may well have contributed to the bitter fight and the breakdown of communication between Amy and Leanne which occurred during the last five minutes of this November 20, 1985 session (Zack, 1986c; Note 23). The disagreement represented by Selections

#8A and #8B was in the end the only instance of disagreement between Amy L. and Leanne L. over the use of the inverse to correct an erroneous turn. In subsequent instances, when they made an incorrect turn, they reverted to the easier strategy of clearing the screen and starting anew, a strategy which Leanne L. had suggested at the outset.

The *knower* knows, but is not effective as a guide to understanding

The first instance presented below is that of a *knower-doer* relationship in which the *knower* does not explain. The excerpt deals with the concept of adjustment for an erroneous turn, as has been the case in the samples just cited above (Selections #7A, #7B, and #7C; Selections #8A and #8B). Jon was a *knower* as far as the concept of adjustment was concerned. In the excerpt below, Jon was seen at first to make a lateralization error in turning LT 90. Jon then self-monitored, and adjusted quickly with RT 180, stating aloud only RT 90. It is clear that Eric had not followed the sequence.



Selection #9A
Jon and Eric, October 16, 1985, D/A #11, p. 14.

Jon H.:	Now we need left turn ninety. [INCORRECT--lateralization] [LT 90]
Eric H.:	(Erasing in his recording book) We have it here.
Jon H.:	(Keys) (pause) No, right-, it's right turn ninety. [RT 180]
Eric H.:	I have left (erases)
Jon H.:	(Keys) (pause) It's left turn ninety after for- [slip of the tongue, Jon should have said "right turn ninety"]
Eric H.:	After right ninety, what is it?

Eric accepted, understood that Jon actually meant "right turn ninety" when he inadvertently stated "it's left turn ninety after for-," and wrote RT 90 in his book. But two statements later Eric felt lost again. Part of Eric's confusion may have been due to the fact that he had also not been able to follow another move just previous to this one, in which Monica, the Logo teacher, nad shown Jon how to combine turtle movements made in opposite directions [FD 105 - BK 60 = FD 45 ; Hillel (1984, p. 23) rule: (FD $\underline{X}$, BK $\underline{Y}$ = FD ($\underline{X}$ - $\underline{Y}$) $\underline{Y}$ < $\underline{X}$] Whatever the cause for the confusion, Eric was in enough of a muddled condition to state:

Selection #9B
Jon and Eric, October 16, 1985, p. 14

Eric H.:	Ugh. Wanna start all over?
Jon H.:	(Erasing in his book) No.
Eric H.:	I don't understand what you're doing.
Jon H.:	Just copy my commands after (writing in his book).
Eric H.:	O. K. Can you gimme the eraser?
.	
.	
.	
Eric H.:	(erases) (pause) (stand up) 'Kay, lemme just copy from you, 'kay?

Although the 90--180 adjustment for an erroneous turn was a move Eric H. was later seen to use without difficulty, it was clear that he could not follow what Jon H. had done in this instance. And as one sees in the transcript, Jon did not explain during the disagreement, nor did Jon explain when Eric elicited an explanation by stating: "I don't understand what you are doing." Jon did not attempt to ask what it might be that Eric had not understand. He merely told him that he (Eric) could copy from his (Jon's) book: "Just copy my commands after."

The absence of explanation on the part of Jon raises the issue of reciprocity. An aspect which in the end was not able to be analyzed within this study was the extent to

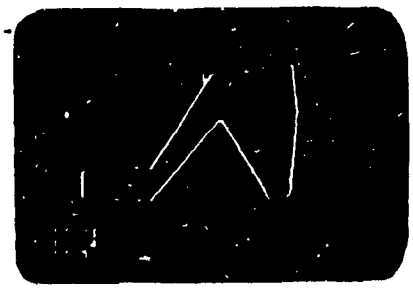
which there was mutuality in the giving of information and explanation by participating partners, due to the low number of instances overall upon which to draw (see Table 26 for example). Thus there could be no pattern detected. However, there is a need for further research regarding how the degree of reciprocity might be related to the socio-cognitive aspect.

The *knower* does not explain clearly

In the example just cited (Selection #9), the *knower* did not explain. The second instance (Selection #10) which follows below is that of a *knower-doer* relationship in which the *knower*, Lilly, did not explain clearly to the *doer*, Rina. Lilly wished to find out where the turtle cursor was pointing. Lilly kept asking Rina in diverse ways where the turtle was: "Where are you?" "How do you know where we're going?", and suggested that they use the HELP routine in order to distinguish the turtle's "heading." Rina wished to go forward; Lilly's point, quite rightly, was that they needed to locate the turtle's heading before they could proceed to decide what the next step ought to be. However, when Lilly finally had to explain "why" they ought to check the heading, her explanation was not clear: "Yes, but we're gonna, if he's, if he's pointing straight on the line, and then we're gonna go straight down, right?"

Selection #10
Lilly and Rina, October 23, 1985, D/A #13, p. 14.

Rina M.:	O. K. Perfect (looking at the screen). ---
Lilly M.:	Not really. Where are you?
Rina M.:	No, we're gonna -P -U, -P -U (pushes Lilly's hand away from the keyboard), pen up.
Lilly M.:	How do you know where we're going?
Rina M.:	We have to bring it down here.
Lilly M.:	Yes, but we're gonna, if he's, if he's pointing straight on the line, and then we're gonna go straight down,



right?

Rina M.: Just press -P -U!

Lilly M.: (Keys) Press -H -E (for HELP routine)

Rina M.: I'm not- I'm never going to work with you again. [Keys -H -E -L -P, thereby accepting Lilly's suggestion.]

If Lilly had added certain qualifiers, her explanation would have been clearer. For example, Lilly might have said something such as the following: "Yes, but we're gonna [check by using the HELP command in order to see] if he's pointing straight on the line, and then [we'll know that] we're gonna go straight down, right?" Rina accepted Lilly's suggestion, albeit begrudgingly; but whether or not Rina understood Lilly's explanation was not able to be determined.

The third instance (Selection #11) is that of a *knower-doer* relationship in which the *knower* (Shauna L.) explained, but the *doer* (Carol L.) did not understand. In order to appreciate the short segment cited below, a brief setting of the scene which preceded it is in order. Shauna had tried to recall the HELP routine (mentioned and used by Lilly and Rina in the selection above) which Monica, the Logo teacher, had shown to Shauna L. and Amy L. upon another occasion (Oct. 23, 1985). Shauna attempted at first to solicit teacher intervention but was unsuccessful. However, a suggestion from Carol herself, and then an echo of that suggestion by a neighboring peer, Saul L. (after an incorrect attempt at intervention by Saul), spurred Shauna on to reinventing the HELP routine herself (Appendix L, pp. 422 to 426). Shauna proceeded to discover that in order to test turtle's heading, one needed first to be sure that his pen was up (PU) so that he left no trace, and then one moved Forward (FD) a certain amount, in this case FD 50, and then Back (BK) the same amount, in this case BK 50, and the Forward-Back motion of the turtle showed where the turtle cursor was pointing (i. e. his heading). Carol L. was not able to follow what Shauna L. did, despite the fact that Carol may well have given Shauna some suggestion of how to proceed when Carol said: "We're still -P -U, so we can go -F -D, and see where he is" (Appendix L, p. 423). We see in the selection below that Carol insisted on recording in her book the FD and BK moves which were used only for the purposes of checking turtle's heading. Shauna tried

to explain that recording those moves was unecessary. It seems the rule that whenever a child was not sure about what had transpired, the child insisted on recording the steps.

Selection #11
Carol L. and Shauna L., November 20, 1985, p. 11, D/A on Plan --recording in book.

Carol L.:	(Leans over forward, reads from the screen, records in her book) (? And I'm at) -B -K fifty.
Shauna L.:	We don't need that! That's just to show us where he is (waving her hands). Forget the -F -D's and -B -K's.
Carol L.:	Well, I did, and I'm gonna write it anyway.

Without the collaboration of Carol and Saul, who gave Shauna suggestions which comprised the first part of the solution, Shauna could not have discovered how the HELP routine worked. This episode (Appendix L) was a prime example of how *doers*, in interacting and discussing together, were able to contribute to the development of a product which no one child could have accomplished on her or his own. Due to the collaboration of her peers, Shauna moved "the distance between the actual development level as determined by independent problem solving" to "the level of proximal development as determined through problem solving . . . in collaboration with peers" (Vygotsky, 1978, p.86). Shauna was seen to move from the role of *doer* to the role of *knower* in the context of this incident. Carol was not able to understand the procedure however, and Shauna, in the role now of *knower*, was not able to explain it to her.

Didactic Relationships

A *knower-doer-doer* triad relationship in which the *knower* is class expert

The particular case of the *knower-doer-doer* relationship which will be described shortly was unique in this study in comparison to the other groups in the following three ways: (a) it was a triad; (b) the group was same sex-all male, whereas the others were same sex-all female, or at one point mixed sex (Carol L. and Doug L.); and (c) the *knower* was an expert. Since the *knower* was an expert, the partner role of *knower* was not shared back and forth by the three participants in the group. The relationship between Russ H. and his partners Jon H. and Eric H. was an asymmetrical one. In only one instance was Russ H. in the *doer-learner* role in relation to his partners, and in that case the partner who gave the information was Jon (H).

Russ H. was judged by the researcher to be very able as a result of the analysis of his work, as a result of the interviews (February-March, 1986; May-June, 1987), and as a result of the analysis of his contributions as peer helper to neighboring peers (Zack, 1988b), and as designated tutor to Jon and Eric (Zack, 1988b; Appendix N). The designation of "expert" is one which both the researcher and his classmates assigned to Russ (Appendix O).

The vital question for consideration here is the extent to which Russ H. was able to connect with his partners within a relationship which was not balanced, in that Russ was himself so able and could complete a project well singlehandedly. Reference to Russ H.'s first partnership, with Michael H. (Oct. 23, 1985; Oct. 30, 1985; Nov. 6, 1985; Nov. 13, 1985; Nov. 20, 1985) suggested the kind of difficulties that working with an expert might entail. Russ and Michael seemed to work well together for the first five sessions. This comment draws only upon impressions garnered from general observations, since only one of the five sessions (Nov. 13, 1985) was videotaped. However, after the fifth session, Russ and Michael ceased working together. Russ and Michael's partnership seemed to have been dissolved due to cognitive-social, not purely social reasons as was the case for the other pairs. In their last session together, on Nov. 20, 1985, Michael was not able to follow Russ's manoeuvres with variables, and subsequently asked Monica, the class teacher, for a change of partners. It was all the more interesting, then, to follow the course of Russ's interaction with Jon H. and Eric (H.).

There were five sessions (Dec. 4, 1985--with Jon alone; Dec. 11, 1985; Dec. 18, 1985; Jan. 8, 1986; Jan. 15, 1986) during which Russ H. worked with Jon H. and Eric H. We proceed next to look briefly at these sessions in order to see the ways in which Russ did indeed interact with Jon and Eric across the gap, and to see how that triadic relationship evolved over the time which Jon, Eric and Russ spent as partners.

As has been mentioned in the findings section concerning the frequency of disagreements among the children, there was a drop in disagreements between Jon H. and Eric H. (and Russ H.) after Russ joined Jon and Eric. It is possible that Russ's expert facility with the mathematics (and syntax, and plan and programming) aspects of Logo resulted in a decrease in the occurrences of disagreements. Since the expert could do much of the difficult work, occasions for conflict about certain aspects decreased. If disagreements might indeed play a role in encouraging discussion about mathematics concepts, and thereby promote learning, then the fact that the rate of disagreements decreased could signal a decline in the active participation, and consequently the learning, of two of the three partners, namely Jon and Eric. The researcher therefore returned to the data in order to see whether Russ had indeed taken control and was doing all the work, or whether there was ongoing active participation between Jon and Eric and Russ, as had been the case between Jon and Eric when the two of them worked together in the first six sessions. The nature of Jon and Eric's interaction with Russ and with each other in the triadic situation differed in two respects: the amount of each child's use of supported talk, and each child's overall involvement in the project work which consisted of drawing a hockey goalie. Jon's involvement will be dealt with first, and then Eric's participation will be discussed.

The findings revealed that Jon remained an active, involved participant when Russ joined Jon and Eric and they became a triad. In looking at both his talk and his overall involvement, one found that Jon held his own, although he did not know as much about Logo as did Russ. For the purpose of comparison, the rate of use of supporting arguments outside of disagreements was used, for there was a larger frequency of occurrences upon which to draw, with the drop in the number of disagreements during the last four sessions. As Table 18 will show, the rate of use of supporting arguments by Jon and Eric during four sessions in which they worked as a dyad, and in which they used supporting arguments, was (surprisingly) equal, that is, 4.5 per session. When Russ H. joined them for 4 sessions, Russ's rate of use was high (5.5

Table 18

Rate of Use of Supporting Statements[a] by Jon H., Eric H. and Russ H. Outside the Mathematics Concepts Disagreements

	Jon	$\bar{X}$ per session	Eric	$\bar{X}$ per session	Russ	$\bar{X}$ per session
4 sessions Jon and Eric as dyad	18	4.5	18	4.5		
4 sessions Jon, Eric and Russ as triad	10	2.5	3	.75	22	5.5

[a] Supporting statements included mathematical justifications and descriptions, as well as other kinds of supported talk (see Table 33).

per session). Jon's rate of use dropped somewhat (4.5 per session with Eric, to 2.5 per session with Russ and Eric), but nevertheless his overall participation remained active. However, the use of supporting arguments by Eric dropped substantially (4.5 per session with Jon, to .75 per session with Jon and Russ), and his participation decreased as well.

Eric H.'s participation in the triad was different from that of Jon. As a result of peer comments, the researcher learned that Russ H. and Eric H. were close friends outside of school, whereas Jon H. and Russ H. were not. One might almost have expected that a strong dyadic affiliation might be struck by Russ and Eric, but this was not how the working relationship evolved. When Russ joined Eric and Jon, the three worked intently together for the first session. There then began a gradual change in Eric's involvement in the work. Eric was seen to become less of an active participant in the Logo work, and in the disagreements and agreements components of the interaction. Whereas there was active participation in the "goalie" project (particularly the hands-on computer part) on the part of Jon and Russ, the same was not true for Eric. Eric's involvement became mainly non-Logo; for example, for almost the entire December 11, 1985 session he immersed himself in doing the goalie pencil drawing for the group's next project. The fact that Eric was at times uncommitted during the Dec. 11, Dec. 18, Jan. 8 and Jan. 15 sessions was reflected in his comments, and in his wandering attention, attempts at distraction and increasingly common absences (Appendix P). Thus at times Jon and Russ maintained a dyadic rather than a triadic relationship. It is not clear whether Eric's participation dropped because (a) Russ joined and took control, and Jon was able to maintain an active role in relation to Russ but Eric could not, or (b) Russ joined and Russ was an expert (Appendix O), and working with an expert might in itself be a unique situation, or (c) because working in a triad is difficult in and of itself (Note 24). Eric H.'s participation may have dropped due to all, two, one, or none of the above reasons. What was clear was that Eric's active involvement in the Logo computer work decreased in the last sessions.

The question of optimal group size was raised in the "Review of the Literature" portion of the paper, and it is considered to have vital implications for learning. Cox and Berger (1981) stated that two, at most three, children constituted the best group size, while Dickson and Vereen (1979) insisted that two was the optimal size, since with three or more children "turntaking" was more likely to be unsuccessful. In this study,

as has already been stressed, the analysis of triadic relationships was limited to the one afore-mentioned group[4], and in that instance one of the threesome was a class expert, which in itself made it a unique situation. Due to the small sample size of one group, and due to that group's unique composition, it is not possible to generalize as to whether a triad group arrangement in the computer classroom setting is or is not to be recommended. Further investigation of the strengths and weaknesses of the threesome as compared to the twosome is warranted. However, at this point and until such time as further investigation on this issue is done, and based upon the admittedly limited observations offered by this study's data, the researcher would side with Dickson and Vereen's (1979) suggestion of two as the optimal size. Due in part to the children's own perceptions as voiced in the interviews of May and June 1986 (Appendix H, Q. 1.3.1), and due to Eric's change of involvement pattern when he moved from a dyadic to a triadic situation, albeit only one instance, the researcher would venture to suggest that the dyadic relationship, rather than the triadic, would be more likely to facilitate children's learning of the Logo concepts.

The other area which warrants further investigation is that of the role in a relationship of the child who is expert. There will be mention made later in this chapter in reference to findings concerning unique features of Russ's explanations and peer teaching skills. However, investigation of "peer as class expert" was limited to one instance in this study. There is a need for further examination of the question of whether or not a child who is expert plays a distinctive type of role in dyadic, triadic, or small group learning.

Knower functions: Giving information, and teaching

Cooper, Marquis and Edward (1986) spoke of *doer-doer* interactions as serving collaborative functions, and *knower-doer* interactions as fulfilling teaching functions (1986, p. 294). It was interesting to note that during the interviews which took place when the subjects were in Grade 6 (May-June, 1987, when 56 of the 69 students in Grade 6 were interviewed), it became evident that the children drew a distinction

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[4]There were two other occasions (November 13, 1985, and Jan. 15, 1986) during which three study participants, Amy L., Carol L., and Leanne L., comprised a triad. Those sessions were not considered for analysis since no Logo project work was done during that time. There was considerable friction between Carol and Amy during one of the sessions (November 13, 1985).

between giving information functions and teaching functions. Although most stated that they did ask their partners, and sometimes their neighboring peers, for help and explanations, and gave help and explanations as well, many of the children's comments suggested that ultimately they felt it was the teacher's role to teach (see Appendix H, Q 2.1). The children interpreted the term "teaching" in a traditional sense. It is thought that the children would probably have identified as "helping" rather than as "teaching" the more informal peer teaching functions --such as focusing, guiding while the peer partner performed the task, modeling (demonstrating with commentary), and giving information -- which were seen to take place in the computer classroom.

In one case the researcher was able to compare one child both in the role of *knower* --information-giver and in the role of *knower* --teacher. A difference in tone and in use of varied teaching skills were detected when that child was placed in the role of peer teacher. The child was Russ H., who in the role of *knower* --information-giver had been observed at various times to give explanations and demonstrations of proof to his partners. The transcript selection of Russ as *knower* --information-giver that proved especially interesting was the occasion upon which he explained to Jon H. how the command SETH (setheading) worked in Logo, in response to a solicited request for explanation on Jon's part (Appendix M). Upon one occasion, Russ H.--who admittedly may well be atypical due to his ability--was specifically assigned to the role of tutor by the teacher. He was asked to help Jon H. and Eric H. with the concept of variable. In that designated tutoring session, the "knower's" peer status changed, the time frame for concentration on the topic became more extensive, and more teaching functions emerged than was the case in the other peer interventions. These teaching skills included: assessing needs, checking on progress, marking critical features, and eliciting critical thinking (Appendix N).

It is evident that much was happening when the children worked together as dyads which made the learning pattern different from the pattern which would have resulted had the children worked alone. Whether or not one considers that for the children the terms teaching, giving information, and helping had differing connotations, there were peer teaching and collaborative functions taking place which contributed in some way to the learning. From seemingly more minor but vital helping functions such as checking and correcting typing errors, to more substantial helping functions such as guiding and demonstrating, and the above-mentioned teaching functions, the children

were making some small or large contribution to the learning of their peers.

The Social Aspect of Working Relationships

In dealing with working relationships, a conscious attempt was made to stay within the cognitive dimension, for the social dimension is an entire issue in itself. But the issues of social dominance and submission in the partnerships deserve some, albeit brief, mention, as does the issue of how the social aspect impinges upon the cognitive aspect of the learning.

Symmetry and asymmetry in the social-cognitive relationships between partners

Before analyzing the data concerning the mathematics concepts disagreements, it was the researcher's impression that the social relationships were more asymmetrical than they ultimately proved to be. Some of the children seemed to be more aggressive than their partners in suggesting moves and ideas, and in insisting that their suggestions ought to be the ones carried out. Jon H. seemed more dominant than Eric H., Amy L. more forceful than Leanne L., Carol L. than Leanne L., and Carol L. than Doug (L). (Lilly M. and Rina M. in Grade 5, and Carol L. and Amy L. in Grade 4, seemed equally argumentative.) The pattern of dominance that the researcher perceived was not reflected in the results of the data analysis of rate of proposer of moves versus opposer of moves, for had it been, the rates of Jon H.'s and Amy L.'s proposed moves would have been much higher than those of their respective partners, Eric H. and Leanne L. for example, and that was not the case. As Table 19 will show, the frequency of proposed actions, and their complement, opposed actions, during the mathematics concepts disagreements, revealed a balanced rather than an asymmetric relationship between the partners.

However, the researcher's observation that certain of the children appeared submissive in relation to their partner's more aggressive behavior seemed to be borne out by the data reflecting the frequency with which certain children were the accepter, and thereby resolved the disagreement. The number of times Eric H. and Leanne L. accepted and effected the resolution seemed disproportionately high in relation to the number of times their partners did so; Eric and Leanne accepted 65% and 78% of the time, respectively (see Table 20). It is possible that a more distinct pattern of social dominance and submissiveness might have appeared if all the disagreements were part

Table 19

Students' Assumption of Proposer and Opposer Roles During Mathematics Disagreements

	Jon H.	Eric H.	Lilly M.	Rina M.	Carol L.	Amy L.	Leanne L.
Proposer	13	12	(17) 26	(30) 37	10	8	3 with Carol 3 with Amy
Opposer	12	13	(30) 37	(17) 26	10	6	5 with Carol 3 with Amy
	25[a]	25	63[b]	63	20[c]	14[d]	14[e]
	[a] 25 disagreements with each other		[b] 63 disagreements with each other		[c] 20 disagreements with her partners (see below)	[d] 14 disagreements with her partners (see below)	[e] 14 disagreements with her partners (see below)

	Breakdown: Amy and her Partners					Breakdown: Carol and her Partners					
	Amy	Amy and Leanne		Amy and Shauna		Carol	Carol and Leanne		Carol and Shauna		Carol and Doug	
Proposer	8	3	3	5	3	10	5	3	0	2	5	5
Opposer	6	3	3	3	5	10	3	5	2	0	5	5

Note: The numbers in parentheses represent the total count after 6 Logo sessions.

Table 20

Number and Percentage of Times Each Participant Was the Acceptor (thereby resolving the agreement)

Participant	Number of Times the Participant Was the Acceptor	Number of Resolved Disagreements	% of Times the Participant Was the Acceptor
Jon H.	9	26	35
Eric H.	17	26	65
Lilly M.	21	46	46
Rina M.	25	46	54
Carol L.	7	14	50
Amy L.	1	9	11
Leanne L.	7	9	78

of the data. In particular, the disagreements related to planning the Logo projects might have rendered pertinent information, for it seemed to the researcher that certain of the children often had the final say in the choice of what project the pair was to make. This impression might prove ungrounded once the disagreements on plan are analyzed; however, this analysis has as yet not been done.

Most of the children represented in the study were able to relate well to each other socially. Amy L. and Carol L. were the exception in that their relationships with their partners were not smooth. When they worked with each other in Grade 4, and with each other and Leanne L. (begrudgingly) in a triad in Grade 5 (two sessions), and when they worked with their various partners, there were instances of negative and insensitive behavior on Amy and Carol's part which were not observed in the other children's behavior. At times, Amy and Carol were seen to be deliberately obstructionist. It is possible that frustration due to difficulty with the cognitive concepts might in turn have led to quarrels of a social nature between the partners (Zack, 1986c; Note 23). On a number of occasions, Monica, the Logo teacher, was obliged to intervene in an effort to repair the communication and the working relationship between the Carol L. and Amy L. and their partners. In particular, there was much evidence of friction in Carol's relationships with her partners (Grade 4, Grade 5 and Grade 6); this pattern seemed to support comments made by the children's classroom teachers about Carol's social difficulties. In cases where there were episodes constituting a complete breakdown of communication, as between Amy L. and Carol L. in Grade 4 (October 30, 1984, session #5), between Carol L., Amy L. and Leanne L. in Grade 5 (November 13, 1985, session #5), and between Amy L. and Leanne L. in Grade 5 (November 20, 1985), there was no learning via interaction (Zack, 1986c; Note 25).

Conclusion

This section looked at various cognitive and social components of the working relationships between the partners. For the most part the relationships were more balanced than had been expected, both in the cognitive and in the social domain. There were situations observed in which the partners were, and others when they were not, able to explain their position to each other. Most of the time the children were actively involved in the activity; at times certain of the children were not actively involved in

the activity, or in a very few cases, obstructed the activity. Learning was expected to be facilitated when both/all children were involved in the activity, and when the partners made an effort to discuss the project, and specifically the mathematics aspects of the project with each other. It was expected that learning would be hindered, or would not be advanced, when a child was not involved or attentive to the ongoing activity, when the partners were not able to explain to each other, were not able to explain clearly, or were unwilling to explain. Sample selections have illustrated examples of each type of instance, but further study is needed. Questions were raised and suggestions for further investigation were made concerning aspects such as (1) mutuality between partners, (2) optimum group size, and (3) the possibly distinctive role of the class computer expert.

The analysis in this section of the findings which dealt with working relationships between the dyadic partners was only able to touch briefly upon certain of the ways in which the children interacted together and discussed their Logo work. It was deemed necessary to delve further, to explore the whole body of the supporting arguments used, and to distinguish the understanding and misunderstanding about Logo geometry that these arguments embodied. The analysis of the verbal strategies that were used in the course of a disagreement was therefore done in order (1) to investigate the content of the support statements, (2) to investigate further how the child's supported talk might contribute to the learning of the Logo mathematics concepts, and (3) to determine in what way these statements might reflect the child's understanding of the mathematics concepts. These findings, and the deliberations concerning these findings, will now be presented.

TALK WHICH PROMOTES AND REFLECTS LEARNING

The overall encompassing aim of this study was to investigate the role that disagreements over a Logo move, and the discussion associated with the disagreements, played in the children's learning of a Logo mathematics and geometry concept. This section will deal with the verbal and non-verbal strategies used within the disagreements themselves. The Level I verbal strategies were statements with no accompanying elaboration (*insistence, challenge, denial, insult, use of physical force*). The Level II verbal strategies were statements which elicited elaboration (*request for explanation*, or *information,* or *clarification*; *call for teacher* or *for peer intervention*) or which furnished elaboration (supported talk: *supporting argument, demonstration, focusing, counter-proposal, reference to recorded notes, reference to prior knowledge (supported), reference to computer screen verdict,* and post-disagreement discussion). This section of the findings chapter will deal with the role of supported talk as both a promoter of learning, and as a means whereby the child's learning is reflected.

The main focus of the report will be upon the Level II strategies, the *higher level* strategies, for it was by means of the Level II statements that the children elicited information, proposed means of further investigation, and discussed their work. Since the aim of the study was to see what the talk could tell us about the child's understanding of Logo angle, and the child's strategies of learning about Logo angle, it was expected that the findings related to the use of the Level II strategies might be particularly informative, and this was indeed found to be the case.

The research questions which guided the investigation of the analysis of the talk aspect of the data were as follows:

> WHICH VERBAL STRATEGIES DOES THE CHILD USE IN THE COURSE OF A DISAGREEMENT? CAN THE STRATEGIES CONTRIBUTE TO THE CHILD'S LEARNING, AND TO THE LEARNING OF THE CHILD'S PARTNER? ARE SOME OF THE VERBAL STRATEGIES WHICH ARE USED DURING DISAGREEMENTS MORE EFFECTIVE IN LEADING TO LEARNING THAN ARE OTHERS? IF SO, WHICH ARE THEY?

The findings will deal first with the quantitative aspect of the children's use of verbal strategies. They will report on the first component of the objectives mentioned above, namely: WHICH VERBAL STRATEGIES DOES THE CHILD USE IN THE COURSE OF A DISAGREEMENT? The first section will also address some of the other subsidiary aspects which can be answered via frequency counts or by means of yes-no responses, such as: (a) Does the child use supported talk?; (b) Do the verbal strategies used by each participant in a dyad during a disagreement reflect each child's level of achievement? That is, is the high achiever more likely to use more complex strategies than a child who has a low level of achievement?; and (c) Does the degree to which talk is supported increase over time?

The next part of this section of the findings will report on the qualitative aspect, namely the results of the content analysis of the verbal strategies used by the children. It will attempt to distinguish how the use of particular verbal strategies can reflect the child's understanding of Logo mathematics and geometry concepts, in particular Logo angle. It will relate how the use of certain of the verbal strategies can reflect the child's learning and helping strategies, as the instances of the child's asking for help or explanations, from a partner, from the teacher, or from a neighboring peer, and the child's giving of help or explanations, are reported.

The main objectives which guided the qualitative analysis of the content comprised the latter component of the above-mentioned objectives:

> CAN THE STRATEGIES CONTRIBUTE TO THE CHILD'S LEARNING, AND TO THE LEARNING OF THE CHILD'S PARTNER? ARE SOME OF THE VERBAL STRATEGIES WHICH ARE USED DURING DISAGREEMENTS MORE EFFECTIVE IN LEADING TO LEARNING THAN ARE OTHERS? IF SO, WHICH ARE THEY?

Three main categories of Level II verbal strategies were investigated and the resulting findings will be reviewed: (a) requests for explanation, (b) requests for help from teacher or peer, and (c) instances of supported talk. Because supporting arguments were found to play a predominant role in the resolution of the disagreements, especially those resolved by means of persuasion, a portion of this section of the findings will be devoted specifically to the *supporting argument* component of the Level

II strategies. The study investigated whether the supporting arguments used were correct, were clearly stated, and whether they clarified the child's point. It was found necessary to distinguish further concerning the clarifications in two ways: first, whether they were solicited or unsolicited, and second, whether or not they elaborated upon the mathematical aspects. Thus the findings will report (a) whether the attempt at clarification was due to a solicited request (and therefore considered an explanation, or attempt at an explanation) or was unsolicited (and therefore considered a justification or attempt at justification); and (b) whether the explanation or justification was a general elaboration, or whether it was related to the mathematical domain, explaining by means of use of causal or logical mathematics-based supporting arguments.

This section of the paper will also report on the findings, related to the disagreements, which concerned the working modes of the children (visual and analytic), and the problem solving and recording methods of the children. It will endeavor to show how the verbal strategies used during the arguments both reflected the child's understanding of the mathematics concepts, and contributed to the learning of the Logo mathematics concepts.

The quantitative aspect of the use of verbal strategies

The analysis of the frequency of occurrence of use of the various verbal strategies both at Level I and at Level II revealed that all the children used a range of verbal strategies, and that all the children used Level II strategies which elicited or reflected supported talk. With one exception, there were no unique instances of use of one particular strategy. This one exception was that of Lilly M. and Rina M.'s *reference to recorded notes*, that is, their reference during disagreements to their own notebook records (see Table 21). This distinctive use of a verbal strategy will be elaborated upon later, in the section entitled "Supported talk as it reflects learning: The child's understanding of Logo angle," since it was found to have much relevance to the children's (Lilly and Rina's) understanding of angular rotation, and to their problem solving methods.

The frequency of occurrence of various verbal strategies used by the children is reported in Table 21, in answer to the first of the research questions for this section, namely: WHICH VERBAL STRATEGIES DOES THE CHILD USE IN THE

	LEVEL I STRATEGIES																			
	Insistence	Insistence—Action taken	Action taken	Challenge	Denial/Resistance	Insult	Refusal	Threat	Ignores	I--Dismisses suggestion	I--puts on "hold"	Physical force	Silence	Anger	Mitigation	Aggravation	Counter/contradiction	Counter-sugge	Predic
Jon H	20	4	3	2	12		1		4	1			1			1	20	5		
Eric H	9		2	2	4				1				1		1		18			
Lilly M.	31	3	2	4	29	2		2	4	2	2	4	4	5		3	30	5	2	
Rina M.	33	12	4	4	34	9	2	1	8	4	2	2		2	2	5	24	4		
Carol L	2		7	1	9	1	2		3			1	1		1	2	15	2	1	
Amy L	10	1	3	3	7	2			1	1				1	2	1	7	1		
Leanne L	6	1	1	3	4				1				1	3	2		4	1		
Carol L Gr 4	10		2	2	5	4			5	1				2		2	14	2		
Amy L Gr 4	10	4	8		7	2			5	3	2				2	1	12	5		

Note Only those children who participated in 10 or more videotaped sessions are in this table Russ,

Table 21

of Use of Level I and Level II Strategies

...uction	...ggestion	Evaluation	TOTAL LEVEL I STRATEGIES	% OF TOTAL USE	LEVEL II STRATEGIES: Request for explanation	Request for information	Request for clarification	Request for a hearing	Request fulfilled	Request not fulfilled	Cannot fulfill request	Appeal to teacher	Appeal to peer	Reference to recorded notes	Reference to prior knowledge—supp.	Reference to prior knowledge—unsupp	Reference to computer experience	Reference to computer screen verdict	Other-directed	Supporting argument	Focusing	Demonstration	Conditional test hypothesis	Counter proposal	Compromise	Post D/A discussion	Post D/A elaboration	TOTAL LEVEL II STRATEGIES	% OF TOTAL USE
5		79	66		1	1			3					1	3			1		13	13		1	1			1	1	40	34
6		44	54		4			1	1	1					1					12	8	1	1	2	3	1	1	37	46
2		136	55		4	4	1		3			1	3	16	6	2		6		29	14		2	9	3	6	2	111	45
3		155	65		1	2	1	1	6					19		3		7		18	11	1	2	2	1	3	4	82	35
2		50	71						1							1	1	2		7	5		3					20	29
4		44	71		2	1			2			1		1						6		1	2	1	1			18	29
		27	73		1		1								1					2	3			2	1			10	27
5		54	72		1		1	1	1			1						4		9	2						1	21	28
1		62	77					1		1	1		1		1		1			8	2			1	1		1	19	23

s, Shauna and Doug are therefore not represented

COURSE OF A DISAGREEMENT? It is clear from the data in Table 21 that all the children used Level II verbal strategies. It was expected that the child who was more adept might be more likely to use complex strategies, that is Level II strategies, and might use them more often, than the child who was not adept. These Level II verbal (and nonverbal) strategies might include *supporting arguments, demonstration, focusing, counter proposal, and appeal to prior knowledge (supported).* As Table 22 will indicate, the data did not show a pattern of relationship between the achievement level of the children as had been determined as a result of the project on the child's notion of angular rotation (Zack, 1986d), and their use of Level II strategies. Had there been such a relationship, the proportional use of Level II strategies would have risen steadily higher as one moved from the less adept (L.) to the average (M.) to the highly adept (H.). It must however be stressed, as always, that the number of instances upon which the averages are based is small, and very few statements deleted or added could have changed the results substantially.

Only the children who were videotaped for at least two thirds of the Logo sessions were included in Table 22. That unfortunately ruled out Russ H., Shauna L., and Doug L ; however, the data which were available concerning the use of Level II strategies by Shauna and Russ were of particular interest, and are therefore represented in Table 23. The indications were that Russ H. and Shauna L. used a high proportion of Level II strategies. In looking at the results as represented in Table 23, the reader is reminded that there were only 6 sessions, 4 and 4 sessions of tape on Russ H., Shauna L., and Doug L. respectively. Had there been more complete data, the final ratios may have been different. But it is nevertheless important to note that the total number of Level II statements by Shauna L. during 4 sessions outnumbered the Level II statements made by Amy L. and Leanne L., and almost equalled Carol L.'s number; Amy, Leanne, and Carol were videotaped for 12 sessions but had mathematics disagreements in only 4, 4, and 6 of them respectively.

Increase in the rate of use of Level II strategies

There was no pattern of increase in the rate of use of the Level II strategies over the twelve sessions on the part of the Grade 5 participants within the September 1985 to January 1986 time span. There was a slight increase of 6% in the proportional use of Level II as opposed to Level I strategies on the part of Amy L., from the Grade 4 data to the Grade 5 data. But the same increase was not seen in Carol L.'s

Table 22

Frequency and Percentage of Participants' Use of Level I and Level II Verbal Strategies

			Level I Strategies		Level II Strategies		
Participant	# of Videotape Sessions	# of Sessions With No Math Concepts Disagreements	*f*	%	*f*	%	Total # of Statements
Jon H.	10	1	79	66	40	34	119
Eric H.	9	1	44	54	37	46	81
Lilly M.	11	3	136	55	11	45	247
Rina M.	11	3	155	65	82	35	237
Carol L.	12	6	50	71	20	29	70
Amy L.	12	8	44	71	18	29	62
Leanne L.	12	8	27	73	10	27	37

Table 23

Frequency and Percentage of Participants' Use of Level I and Level II Verbal Strategies (Russ H., Shauna L. and Doug L. included)

			Level I Strategies		Level II Strategies		
Participant	# of Videotape Sessions	# of Sessions With No Math Concepts Disagreements	*f*	%	*f*	%	Total # of Statements
Jon H.	10	1	79	66	40	34	119
Eric H.	9	1	44	54	37	46	81
Lilly M.	11	3	136	55	11	45	247
Rina M.	11	3	155	65	82	35	237
Carol L.	12	6	50	71	20	29	70
Amy L.	12	8	44	71	18	29	62
Leanne L.	12	8	27	73	10	27	37
Russ H.	6	1	18	50	18	50	36
Shauna L.	4	1	34	52	32	48	66
Doug L.	4	1	28	70	12	30	40

Note: Only the children who were videotaped for at least two thirds of the Logo sessions were included in Table 22. The remaining participants (i e , Russ L , Shauna L. and Doug L) are represented here.

proportional use of the verbal strategies in Grade 5 as compared to Grade 4 (see Table 24).

The question of increase in use of elaborative talk, namely use of Level II strategies, was intended to apply mainly to the longitudinal component of the study; the aim was to look for any change in the use of Level II verbal strategies by Amy L. and Carol L. in Grade 5 as compared to their use of these verbal strategies in Grade Four. As has been stated above, there was no consistent pattern of change. The question was originally posed in the review of the literature because of the Hoyles, Sutherland and Evans (1985) finding that in their study an increased rate of supported talk occurred in the second year of their work with the children. In this study it seems most likely that the length of time at which the children were at work at the Logo (that is, twelve 60-minute sessions, or even twenty-four sessions with an intervening gap of twelve months for Amy L. and Carol L.) was of too short a duration to effect an increase in the supported talk. One must recall that each year of the Hoyles, Sutherland and Evans (1985) study extended over the full school year. The ramifications for learning of the duration of the Logo sessions in this study will be explored in the "Pedagogical implications" portion of this report.

The qualitative analysis of the content of the verbal strategies

The analysis of the content of the verbal strategies used by the partners was done with a view to answering the following question:

> CAN THE STRATEGIES CONTRIBUTE TO THE CHILD'S LEARNING, AND TO THE LEARNING OF THE CHILD'S PARTNER?

It was expected that three main categories of Level II verbal strategies which could in turn result in learning would be used by the children (see Table 25). These would be: (A) requests for explanation, (B) requests for help from teacher or peer, and (C) instances of supported talk. Verbal strategies which requested explanation (elaboration, information, clarification) or requested that one's point of view be heard were coded under the Level II categories *request for explanation, request for information, request for clarification, and request for a hearing*. Verbal strategies which

Table 24

Frequency and Percentage of Grade 4 and 5 Participants' Use of Level I and Level II Verbal Strategies

			Level I Strategies		Level II Strategies		
Participant	# of Videotape Sessions	# of Sessions With No Math Concepts Disagreements	*f*	%	*f*	%	Total # of Statements
Amy L. (Grade 4)	8	1	62	77	19	23	81
Amy L. & Partners (Grade 5)	12	8	44	71	18	29	62
Carol L., (Grade 4)	8	1	54	72	21	28	75
Carol L. & Partners (Grade 5)	12	6	50	71	20	29	70

Table 25

Subcategories Within Three Level II Strategies Sections

Request for explanation *Request for information* *Request for clarification*	*Appeal to teacher* *Appeal to peer*	*Supporting argument* *Demonstration* *Focusing* *Reference to recorded notes* *Reference to prior knowledge (supp.)* *Reference to computer screen verdict* *Reference to computer experience* *Conditional test hypothesis* *Counter proposal*
A. Request for Explanation or Information	B. Appeal to Teacher or Peer	C. Supported Talk

elicited help from the teacher, or neighbouring peer, in seeking to resolve the disagreement or to solve the mathematical problem were coded under Level II categories *Appeal to teacher* and *Appeal to peer.* Verbal strategies which represented support for one's position were coded under Level II categories, for example *supporting argument, demonstration, focusing, reference to recorded notes, reference to prior knowledge, and reference to computer screen verdict.*

It was found that the verbal strategies located within the third section (C) above, namely those representing support for one's stance, figured most prominently in the disagreements. Verbal strategies within section (A), those calling for explanation and information, were not used as frequently within the disagreements as might have been expected. Strategies within the second section (B) above, which represented calls for intervention from the teacher or from a neighboring peer to resolve the disagreement or to help solve the mathematical problem, and that seemed to hold potential for learning, were used very rarely. The reported findings will deal first with sections (A) and (B), that is, with solicited explanations, and solicited teacher and peer interventions which occurred during the disagreements. The report on the results of the analysis of the supported talk, section (C) above, will then follow.

TALK AS IT PROMOTES LEARNING

Solicited Explanations

The research question which guided the investigation into the children's requests for explanations and for help was the following:

> During disagreements, are there instances of "asking for help" or for explanations, from a partner, from the teacher or from a neighboring peer? If yes, what can those instances tell us about the child's understanding of angle, and the child's learning strategies?

The term "asking for help" was found to be an inadequate term when dyadic interaction was the focus, but did serve as a viable umbrella term for the aspect of solicited teacher and outside peer intervention. In all but isolated instances, a child did not ask a partner for help. A partner was expected to help; that seemed to be an implicit part of the contract when dyads worked together. There were, however, a handful of occasions upon which partners refused to cooperate, or actively obstructed the activity. In this study, these instances always involved either Amy L. or Carol L., or both Amy and Carol. In one case, Leanne L. was heard to insist that Amy fulfill her obligation as a partner (Note 26).

The children did ask each other for explanations, and for information and clarification. Donaldson (1986) has stated that "when one asks for an explanation, one may be aiming to extend one's understanding, or one may be aiming to test or challenge someone else's understanding" (1986, p. 1). Since an aim of the study was to investigate what instances of asking for explanations might tell us about the child's understanding of angle, it was deemed necessary not only to look at requests for explanation within the mathematics concepts disagreements, but also to look at other instances of requests for explanations outside the mathematics concepts disagreements themselves. The findings which will be reported first are those which relate to requests for explanation within the mathematics concepts disagreements. Then the results of a look at the requests for explanations outside the

mathematics concepts disagreements will also be given brief attention; these include requests for explanation of an idea related to mathematics or Logo geometry within other disagreements (Plan & Programming and Syntax), as well as requests for explanation outside all disagreements.

Procedural, empirical, intentional and deductive explanations

As will be noted in Table 26, there were not a large number of occurrences of request for explanation, for information or for clarification; the total number of requests was 27. Information or explanation followed in 16 of the instances in response to the request; in 10 of the instances, explanation or information did not follow or were not adequate. Four of the requests for explanation were answered by procedural explanations relating to "how to do . . ." (Donaldson, 1986, p. 4). One was an empirical explanation which answered "what happened to cause . . ." (Donaldson, 1986, p.4). One was an intentional explanation which "explained the aim or intention of the speaker" (Donaldson, 1986, p. 4). Some selections from the transcripts which feature these explanations, and which report as well on sample explanation requests which were not answered, are cited below.

Selection #12
Request for Explanation, Eric H. to Jon H., Oct. 23, 1985, D/A #6, p. 26

Eric H.:	Circle-L.
Jon H.:	No (points to the screen). First it has to be pointing out.
Eric H.:	Why?
Jon H.:	So it could go like this (indicates the motion and direction by tracing on the screen).

Jon H. has given a procedural explanation. Jon's suggestion of turning first and then making a circle was evidence of more control over the movements of the turtle cursor than some of the other children possessed (such as Lilly M., Rina M., Carol L., Amy

Table 26

Frequency and Type of Requests for Elaboration, and of Responses Given, Within Mathematics Disagreements

Participant	Request for Information	Request for Explanation	Request for Clarification	Information Does Not Follow	Explanation Does Not Follow	Information Follows	Explanation Follows	Type of Explanation
Russ H.								
Jon H.	1 (to Eric)					1		
		1 (to Russ)					1	deductive
Eric H.		4 (to Jon)	1		2	1	2	procedural procedural
Lilly M.	4	4		1	2	3	2	empirical intentional
Rina M.	2	1	1		1 (not adequate)	3		
Shauna L.	1 (not clear)	1 (rhetorical)		1				
Carol L.								
Amy L.	1	2				1	2 (Shauna)	procedural procedural
Leanne L.	1	1		1	1			
Doug L.		1			1			
Totals	**10**	**15**	**2**	**3**	**7**	**9**	**7**	

L., Leanne L., Shauna L. and Doug L.) who were always seen to make their moves from the turtle's point of origin.

Selection #13

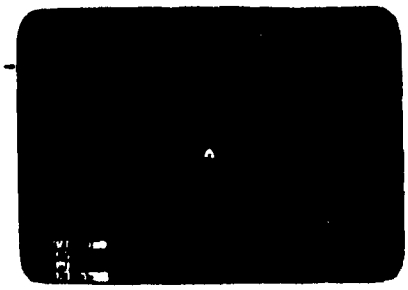
Request for explanation, Lilly M. to Rina M., Oct. 23, 1985, D/A #4, p. 6.

Rina M.:	-R -T. I'm doing right.
Lilly M.:	Why -R -T?
Rina M.:	Just look, I'll show you why.

Rina then proceeded to show and explain her intentions for the next move in the drawing; thus we have an example of an intentional explanation.

Selection #14

Request for clarification , Eric H. to Jon H., explanation is not given, Oct. 16, D/A # 10, p. 13.



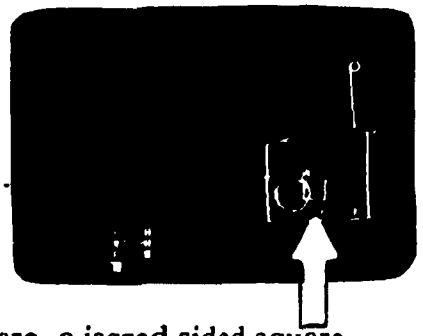
Eric H.:	What?
Jon H.:	(keying) Instead of 105, put 55. [Jon gives only information, no explanation.]
Eric H.:	You mean instead of 60.
Jon H.:	No, instead of, the f- (points to spot in Eric H.'s notebook)
Eric H.:	Oh, O. K.

Monica had just spent time with both Jon and Eric, pointing out that if the turtle cursor goes forward a certain amount, in this case FD 105, and then goes back a certain amount, in this case BK 60, then one needed to subtract the latter from the former and only record the difference in the notebook. Eric had not followed nor understood Monica's point, although she had made a particular point of reiterating it to him. Jon H. might have at this point explained again to Eric H., thus making the teacher-to tutee-to-tu ee kind of teaching bridge of which both Emihovich, Miller and Clare

(1985) and Cooper, Marquis and Edward (1986) have spoken. However Jon did not give Eric the needed explanation, but rather supplied just the information. Jon's choice of action was akin to another instance involving Jon and Eric previously mentioned in the section on COLLABORATION (Selections #9A and 9B). In that episode which took place during the same Logo session (October 16, 1985), Jon H. also offered no explanation, but just told Eric that Eric could copy from Jon's book: "Just copy my commands after."

Selection #15

Request for explanation, Rina M. to Lilly M., Dec. 4, 1985, D/A #5, p. 25. The explanation is not adequate.



Rina M.: But how come it doesn't look like a square?

Lilly M.: (keys) Because it's not s'posed to be!

As one will notice in the drawing, the figure was indeed a square, a jagged sided square turned on its side. Due to the turn that was made at the outset, the square was akilter, but it had all the attributes of a square. Lilly M.'s answer was not adequate nor clear, however it reflected her lack of understanding. It was indeed a square, and by virtue of Lilly's own stated suggestion on the previous transcript page, it was "s'posed to be a square." Lilly's own comment a minute before had been: "We didn't finish it (tracing on the screen). We have to make it like a square." And yet when she finished it she did not recognize that it was a square, most likely because it was a square turned on its side. It may be that the jagged lines confused Lilly. The jagged line often served as a cue to the children that they had done something wrong. In this instance, the machine technology caused the lines to be jagged as opposed to straight (see Appendix Q, p. 252) because the entire shape was tilted, but there was no error. The shape was indeed a square.

The one and only explanation which featured a deductive procedure was given by Russ H. in the selection which follows. In it Russ explained himself "in terms of a rule . . . or other evidence" (Donaldson, 1986, p. 4). Jon H. began by opposing Russ's suggestion of 125, and by proposing that an input of 110 be given to the FD command,

following a logical process of seeking to align the input needed to the corresponding line length previously used (110). The shape was to be rectangular. Jon himself implicitly used the deductive logical process of "if that line opposite was 110, and I wish to make a rectangle, therefore this input ought to be 110." Jon's error lay in the fact that whereas the line length had been 110, it was now 125, but he was not aware of that fact. Jon began by challenging Russ's suggestion of an input of 125, and Russ then demonstrated that 125 was indeed correct.

Selection #16

Jon H. and Russ H., Dec. 4, 1985, D/A #1, p. 11

[N. B. There was no camera videotape made of this session; the one videotape made was of the screen Logo moves.]



Russ H.: It's a hundred and twenty-five.

Jon H.: That's 110, how come (? inaudible)

Russ H.: But Jon it isn't-, Jon:n, just say a hundred and twenty-five, I'll show you, O. K.? (keys) -B -K a hundred and twenty-five.

Jon H.: Ya.

[It seems that Jon is looking at the screen to see the turtle cursor go BK back exactly to the start, and therefore sees that BK 125 equals FD 75, FD 5, FD 20, FD 20, FD 5. In general terms the mathematical theorem would be FD V, FD W, FD X, FD Y, FD Z = BK (V + W + X + Y + Z). Russ H. thus taught a checking strategy, although he did not verbalize it explicitly.]



FD 20
FD 20
FD 5
BK 125

Russ H.: (keys FD 125 to get it back to the spot again) O. K.?

There are several observations which may be made concerning this episode. There were no instances of such a checking strategy being used or shown in relation to angular rotation by any of the children. The example in Selection #16 was related to displacement (length). Russ's check on the length seemed an easy check, and yet none but Russ H. was seen to use it. Russ had used it before, but without explanation. In one instance, Jon observed its use but did not ask about the procedure. One may feel that now that Russ H. has shown this strategy to Jon H., Russ has taught it and Jon has learned it. It would only have been possible to confirm that some learning had taken place concerning this aspect had Jon H. been seen to use this testing strategy, which he was not seen to do within the time frame of this study. There is no doubt however that at that time during that session, the explanation resulted in Jon's becoming aware of the checking strategy.

Requests for explanation and for information within mathematics concepts disagreements as compared to other requests for explanation and information

It was of interest to find that there was a difference in ratio between the requests for explanation and for information within the mathematics concepts disagreements, as compared to those within other disagreements and outside all disagreements. Within the mathematics concepts disagreements the number of requests for explanation and for information were close in number, that is 15 and 10 respectively (17 and 10 if one combines together requests for explanation and requests for clarification, see Table 26). But in tallying the total requests for explanation, and the total requests for information outside the mathematics concepts disagreements, it was found that the total requests for mathematics and geometry information outnumbered the total requests for mathematics and geometry explanations by a ratio of approximately 7:1 (32:7, see Table 27). The children seemed at times more intent on finding out the "what" (the information) than on finding out the "why" and the "how."

An extended procedural explanation (outside of disagreements)

The selection chosen below reflects Donaldson's (1986) view of a request for explanation as an attempt to extend one's understanding. The sample is one of request for explanation outside of all disagreements. It may be of interest to note the beginning

Table 27

Total Requests for Explanation and Information Not Made During Mathematics Concepts Disagreements

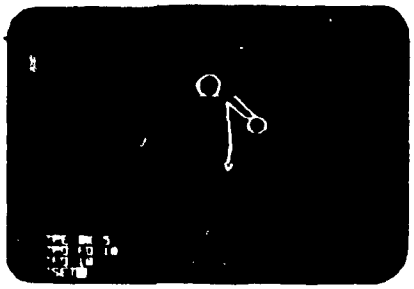
Participant	Requests for Explanation	Requests for Information
Russ H.		
Jon H.	3	9
Eric H.	1	3
Lilly M.	2	8
Rina M.		6
Shauna L.	1	5
Carol L.		
Amy L.		1
Leanne L.		
Total	**7**	**32**

statements of the episode, since in the transcripts it almost seemed as if this were the beginning of a disagreement. It could have been a disagreement had the tone of voice of the participants been different, but when the videotape was checked, the statements definitely appeared not to represent differing suggestions or points of view.

The selection involved Jon H. as he inquired of Russ H. what SETH was. SETH is an acronym for "setheading"; setheading is how adult users of Logo are heard to refer to SETH. In this study the children were always heard to refer to the command as SETH, never setheading, and it may have been for the majority a catchphrase without meaning. Indeed, the extent to which certain of the acronyms and error messages may have bewildered some of the children deserves further study, but investigation of that component must be left to another time. It must be stressed that the explanation episode to follow is atypical, in that Russ H. spent much more time on the explanation than was the rule, as becomes evident when this episode is compared with other sample selections such as the ones just cited. The other selections were part of disagreements. Perhaps the hallmark of explanations or turns within disagreements is their brevity; each child strives to make a point, but the underlying push is to settle the disagreement and to get on with the work. The selection below is unique not only by virtue of the length of time spent, but also due to the various teaching skills that Russ H. exhibited. The ways in which this episode constituted a peer teaching segment have been elaborated in Appendix M.

Selection #17

Jon H. and Russ H. discuss SETH, Dec. 18, 1985, p. 27.



Russ H.:	Now, seth, seth two seventy. (keys) -- [Russ has just completed the curved part of the goalie's body.]
Jon H.:	What is seth?
Russ H.:	It puts (stops keying) the turtle in a-, how do you want. (Points to the screen and traces.) This is zero, and it turns, that's ninety, that's one eighty, that's two seventy, and that's

three sixty.

Jon H.: (giggles)

Russ H.: So you say, uh, seth, eigh-, seth ninety, it'll be facing that way (pointing) (pause), seth one eighty, it'll face that way (points downwards), 'seth

Eric H.: (fiddling with the contrast dial on the T. V. monitor) 'The goalie is a ghost. [Eric is off task.]

Russ H.: two seventy that way (shows), seth three sixty will be that way.

Jon H.: (nods, shows that he feels he understands) O. K.

.
.
.

Russ H.: So seth, seth two seventy (leans over and keys), now watch (points) it's gonna face that way.

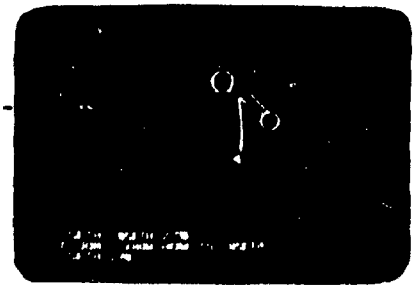
.
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.

Eric H.: You never heard of seth? (looking down)

Russ H.: Now watch, Eric, watch, uh Shawn-, uh, Jon.

Jon H.: Jon. (smiles at the mixup and string of names)

Russ H.: (Keys) Toing! (sound effect for emphasis)



Eric H.: I know about seth.

Russ H.: (to Jon) Now it's perfectly even.

Jon H.:	(nods) Ya.
Russ H.:	(Points to the screen, turns more toward Jon) So that's, you know, if you turn, like hrum (sound effect)
Jon H.:	(?)
Russ H.:	Two hundred and seventy (? times)
Jon H.:	Ya. So you don't have to figure out how much you want it to go.
Russ H.:	That's right.

It was noted that Russ spontaneously exhibited certain vital helping skills which Webb (1987) has listed as important skills that often need to be taught to student helpers: "The helping skills include . . . responding to requests for help, giving explanations of "how" and "why" instead of only giving the answer, checking to make sure the explanations are understood, and providing opportunities for the help-seeker to practice or to paraphrase the explanation" (1987, p. 9). Russ gave an explanation of "how" and "why" to use SETH, and he checked to make sure that his explanations were understood. His explanation elicited from Jon, the requestor, the opportunity to paraphrase the explanation: "Ya. So you don't have to figure out how much to go."

There was one aspect of the use of SETH which Russ H. was not seen to broach in his explanation to Jon. Russ H. used SETH appropriately in his Logo work, in that he never used it purely as an alternate to RT/LT 90, 180, 270, or 360, as did certain of the other children (David H., Michael H., and Stuart M.). In this way, Russ was advanced in his use of the command. Russ used SETH only when he needed to turn turtle to a desired position and it was not possible to ascertain easily what the total rotational turn had been up to that point, as one can see with the curve of the goalie's body pictured above. In the selection, Russ was not heard to explain to Jon the pragmatics of using SETH for this purpose, which was in actuality the most powerful one. Perhaps Russ H. used SETH that way implicitly without even being aware that he used it thus. Jon H. did extend his learning about rotation by means of this SETH episode. In a subsequent session (January 8, 1986) it was observed that Jon was not secure in the use of SETH however, for he attributed an error in the goalie program

to the fact that they had used SETH. During the January 8, 1986 session, Russ did accurately judge that the SETH 180 command was not the source of the problem. It is clear that not only explanation, but experience with working with the notion would be necessary before Jon could include SETH appropriately into his repertoire.

Possible cognitive and social aspects which may have affected the number of requests for explanations

It is significant that in this section on explanations, the selections which have been extensive and particularly informative (the deductive explanation; the extended procedural explanation) have featured the two highest level achievers, Russ H. and Jon (H.). The selections reflect the extent to which the children understand, or do not understand, certain aspects of Logo angle. The attempt to challenge and question one's partner's understanding and point of view is part of the very essence of disagreement. There were however, as was noted, only rare occurrences of deductive arguments (Russ H. and Jon H. concerning the checking strategy, in relation to displacement, not rotation, Selection #16) or arguments presenting alternative points of view (Lilly M. and Rina M. concerning correction for the inverse, section on COLLABORATION, Selections #7A, 7B and 7C). Most of the requests for explanation were seeking procedural, intentional or empirical explanations. Perhaps if the children had had more time and experience with Logo, interaction reflecting divergent points of view, as well as logical arguments supporting one's proposals, might have occurred more frequently.

It was of interest to note that some children made no requests at all for explanation and/or information within mathematics concepts disagreements (Russ H., Carol L., see Table 26), or outside of the mathematics disagreements (Russ H., Carol L., Leanne L., see Table 27). One might mull over the "why." There were requests for explanation from their partners by most of the children, but certainly in comparison to the use of other verbal strategies, the overall number of requests for explanations were relatively infrequent.

The interviews with the children offered some possible reasons for the small numbers of requests for explanations. The interview questions which rendered insight into this issue were phrased as follows:

When your partner tries to explain something to you, does it help you to understand it?

Does your partner explain things to you?

Do you like to explain things to your partner?

(If yes) What kinds of things do you like to explain?

In Logo, can you tell me about a time when your partner helped you to understand something you had never understood before?

Does talking with your partner help you with your Logo work?

What is the difference between the child as explainer and the teacher as explainer?

The children responded in ways that indicated (1) that for some the teacher was the preferred resource person; and (2) that there were at times reasons, both social and cognitive, for choosing not to ask one's partner for an explanation, and for not offering an unsolicited explanation.

For one, it was clear during the interviews that it was the perception of some of the children that the teacher was the source of knowledge, and the children stated that they would sometimes opt to ask the teacher for help and for explanations in many instances, rather than a neighboring peer, or at times even their partner (Appendix H, Question 2.1, 2.3). The question of appeal for teacher intervention, and the resultant findings, will be discussed shortly. The social and cognitive reasons for hesitancy at times to ask for explanations, and the unwillingness to offer unsolicited explanations, were best voiced by Leanne L., and Jon H. respectively during the interviews in May and June, 1987, when they were in Grade 6.

Leanne L.'s comments reflected the fact that there must be a sympathetic accepter of the request in order for the child to be comfortable in making the request for information or for an explanation:

Selection #18

Interview with Leanne L., (Grade 6), May 28, 1987.

I: In Logo, um, did you ever try to explain anything to your partner?

Leanne L.: (pause) No, she mostly knew lots of stuff. (?Well, we-) When we weren't sure, we didn't ask each other, 'cause sometimes I wouldn't- I wasn't gonna ask her [Amy L.] anymore because I found a lot a times when I wasn't sure of something she'd say "Oh, c'mon, you're so stupid" and stuff like that.

I: Ah.

Leanne L.: So mostly I would ask the teacher, or I'd ask somebody else.

I: (pause) And did you do a lot of asking of people on the side?

Leanne L.: We:l:l, sometimes I wasn't sure I'd ask them.

Thus Leanne L. voiced her reason for desisting from asking questions of Amy L.; this may explain in part the paucity of requests issued by Leanne L. (see Tables 26 and 27). The same unreceptive and somewhat negative characteristics which Leanne L. attributed to Amy L. might well have been true for Carol L. who at times denigrated Leanne's efforts when they were partners together. Leanne L.'s comments also reflected the fact that the teacher would be her chief source of recourse (Note 27).

Leanne did not voice any cognitive reasons for not asking Amy L. for information or for explanations. It is the researcher's impression that in many instances Amy L. and Carol L. would not have been able to help. During the interview session with the Grade 6 students (May-June, 1987), a number of the children stated that if they did not know the answer, neither would their partner. This could be a case of the children projecting unto their partners the state of their own knowledge. Whether or not it was indeed true that their partner did not know the answer, the comment may however explain why the children would desist from asking a partner for explanations or for information. Jon H. was unique in his answer, for it was his impression that both he and his partner knew the material, and therefore needed neither to ask for explanations from, nor to give

explanations to, each other.

The response of Jon H. to the interviewer questions concerning explanations gave rise to reflection concerning solicited as opposed to unsolicited explanations, as seen from the viewpoint of the children. More than once Jon H. stressed that he would not welcome an unsolicited explanation; indeed without an expressed need on his part, the explanation would be an insult, since the inference would be that he did not know what he was doing. Thus, it became clear to the researcher that there may be a social convention underlying the children's interactions which directed the give and take. We might therefore expect to encounter explanations as response to solicited requests, but might be less likely to encounter explanations as unsolicited supported arguments. In line with Jon H.'s comment, the researcher will attempt to desist henceforth from using the term "explanation" in its more general, generic sense. Up to this point the term "explanation" has been used in a broad sense, to refer to any attempt by a child to elaborate upon a statement by giving reasons. "Giving reasons" would include supplying justifications for actions, or elaborating in ways other than by means of explanations. The term "explanation" will be reserved for cases of response to a solicited request, in which a child attempts to "make plain or clear; render intelligible; make known in detail" something that is not known or understood (Random House College Dictionary, 1975, p. 466). The distinction between solicited request and the spontaneous offering of reasons will become even more pertinent in the upcoming review of the findings concerning the supporting arguments.

Solicited Teacher Intervention during Episodes of Mathematics Disagreements

The following section will report the findings related to the aspect of requests for teacher and peer help during mathematics concepts disagreements, the as yet untreated dimension of the research question which read:

> During disagreements, are there instances of "asking for help" or for explanations, from a partner, from the teacher or from a neighboring peer? If yes, what can those instances tell us about the child's understanding of angle, and the child's learning strategies?

It was expected that the teacher would play a role in resolving disagreements when children argued over a Logo move dealing with a mathematics concept. It was thought that the children who had been considered by their classroom teachers to be dependent, namely Rina M., Amy L., and Leanne L. (see the child profiles in the METHODOLOGY section) might be especially disposed to seeking such help. The findings reflected an extremely low rate of calls for teacher intervention in resolving disagreements on the part of all the children. Two main episodes will, however, reflect how the teacher was indeed able to contribute to the children's learning, and will also reflect what the children understand about angle. It must be stressed that these meager instances relate only to the aspect of solicited teacher intervention during a mathematics disagreement.

The fact that only two episodes will be presented below does not diminish the significance of Monica's role as teacher in the setting. There were many solicited calls for teacher intervention, and teacher-initiated interventions, concerning syntax (94), plan and programming (8) and class rules (151) problems, as well as mathematics concepts problems (18) outside of the mathematics disagreements. An analysis of the teacher's intervention with the children (Zack, 1986a) in all phases of their learning and interaction with each other has already highlighted the vital contribution of Monica, the Logo teacher. It was somewhat surprising to find so meager a representation of teacher involvement as a resource in the children's mathematics disagreements. The researcher chose to dip into the Grade 4 data dealing with Amy L. and Carol L.'s work together, in part due to a paucity of instances in Grade 5, but more particularly because the Grade 4 instance was especially valuable; it exemplified how Monica nudged Carol and Amy toward learning about the rotational input of 90. The two episodes will illustrate how Monica used the solicited intervention during mathematics disagreements as occasions for teaching about angle.

The first episode (Selection #19) will depict how Monica intervened when the children, Amy L. and Carol L. were exploring the concept of right angle, or input of 90, when in Grade 4. The second episode (Selection #20) will show how Monica responded when the children, Rina M. and Lilly M., were disagreeing about the input to a rotational turn, an input of 135 versus an input of 181 (Grade 5). The second excerpt will also feature one instance of a call for a classmate's intervention (*appeal to peer*) in order to resolve a mathematics disagreement (Zack, 1988b); there were only a total

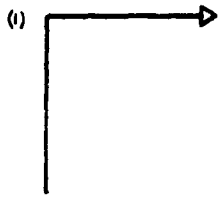
of 3 instances overall of cases in which an *appeal to peer* was made (see Table 21).

Carol L. and Amy L. solicit teacher intervention concerning input sum (for right angle - - 90), in Grade 4.

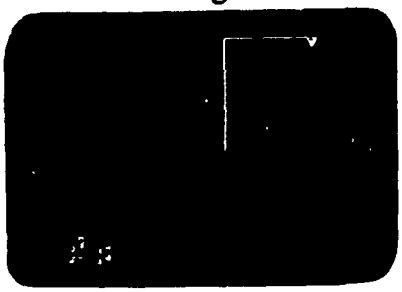
Selection #19

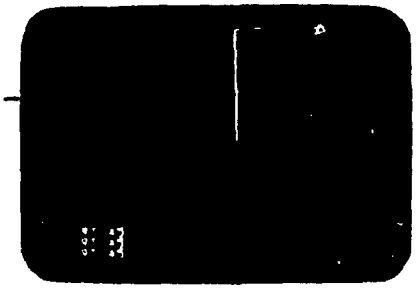
Amy L. and Carol L., Grade 4, October 9, 1984, D/A #4, pp. 16-18

The children's goal was to make a house. They were attempting to make the second turn (which should be an input of 90). Monica had helped Amy and Carol with the first turn (1) during the October 2 session (Appendix Q). It is of interest to see the interaction between Monica and the children during the exploration stage.



[During the first part of the Grade 4 videotaping, only the video of the children's Logo computer work was done; there was no camera videotape of the children's face-to-face interaction. Therefore the transcript which follows does not have notations about the nonverbal component; in a few instances it was not possible to distinguish the voices.]





(A)

Amy L.: -R -T forty-five and forty-five? [INCORRECT]

Carol L.: No, no, no, no [CORRECT]

Amy L.: Let's do like, I think it's -R -T space forty-five return (keys).

Carol L.: Now look what you did (pause). You brought him all the way back here.

Amy L.: We could just make-

Carol L.: -B -K

Amy L.: We could make a triangle. (Pause) [Amy wants to deplan.]

Carol L.: That's dumb. (Pause) Tell Monica. Raise your hand.

Amy L.: Monica!

? : You have to ask three before you ask me. [It is not clear if Amy or another child says this oft heard slogan (cf. Zack, 1988b)]

Carol L.: The answer is 90 for me, not counting this one.

Amy L.: (walks toward Monica and brings Monica back with her.)

Note that in the last statement in part (A), Carol stated what the answer indeed ought to be, that is, 90 (45 + 45, not counting the last turn of 45). In part (C), we will see how long it took for Monica, the teacher, to elicit exactly that answer.

(B) MONICA'S INTERVENTION

Amy L.: Monica, we have a problem. Look, we went too far.

Carol L.: No, because you did forty-five again, and you went all the way over here. [Carol is probably responding "no" in answer to Amy's use of "we," because Carol insists on attributing the move to Amy alone.]

Monica T.: So what did you want to do at that corner?

Carol L.: So you want to write -P -E and then -B -K? [Carol is incorrect since neither Penerase (PE) or going back (BK) are used to correct an angular rotation.]

Monica:	Well, this is for turning. He doesn't back up. You just made him turn like this-
Carol L.:	So (pause)--, -F -D. I mean- (sounds unsure)
Monica:	-F -D will make him go forward, eh?
Carol L.:	Wait, -R -T. [Carol is now back to the mode for rotation, RT or LT, but actually moving him RT something will not be the answer to this problem. They need to turn LT 45 to cancel the last RT 45.]
Amy L.:	Hey, let's make a triangle! [Again, an attempt is made to deplan.]
Monica:	Well, you're trying to make this!
Amy L.:	Right.
Monica:	You should try and make this. That's good. Well, let me ask you one question.
Amy L.:	-L -F. [Amy suggests an acronym for LEFT, namely LF.]
Monica:	-L -T
Amy L.:	-L -T, okay.
Monica:	(chuckling) The (?command) is -L -T, not -L -F.
Carol L.:	Space.
Amy L.:	Space.
Monica:	You always need that [i. e. the space]. What is it you're asking him to do?
Carol L.:	Go like that.
Monica:	A-ha. Right [i. e."You're correct"]. You turn left. -L -T, space-

Amy L.: Forty-five.

Monica: Try it. I don't know. [Monica wants them to see for themselves the result.]

? : (Keys)

Amy L.: Ya!



They have thus completed the inverse, and thereby cancelled the erroneous excess last move of RT 45.

In the segment of the transcript which follows, note Monica's stress on the aspect of "corner." Carol L. previously said at the end of part (A) that her total was 90: "The answer is 90 for me, not counting this one," and that is exactly the input for corner which we see Monica trying so assiduously to elicit from them.

(C)

Monica: O. K. So what was that corner, girls?

Carol L.: Forty-five, so-

Monica: No-no-no, no-no-no, what was that corner?
What is a corner? How much was this corner?

Amy L.: That was forty-five-

Carol L.: Wait, ninety-

Monica: Right. And how much is this corner?

? : Wait, uh-

Monica: Are the corners the same? (Pause) Oh, no-no-no, Car-ol, you don't have track, you haven't written down all the numbers that you have here. All I'm asking you is, how much does he have to turn at this corner?

Carol L.: He has to turn forty-five again.

Monica:	You think it's forty-five?
Carol L.:	He turned forty-five like this.
Monica:	That didn't make it forty-five, Carol. You had a lot of forty- You had how many forty-fives here?
Carol L.:	Three, but these forty, these two forty-fives-
Monica:	You're right
Carol L.:	made it go like this and then she-
Monica:	So how many forty-fives, how many forty-fives do you want him to turn?
Amy L.:	Two.
Carol L.:	Two.
Monica:	O. K. What are two forty-fives?
Carol L.:	Ninety.
Monica:	Good. So that corner is ninety. And-
Carol L.:	I wrote ninety.
Monica:	Terrific! O. K., so what you want to do is turn him right 90. That's very good, Carol, terrific. Now he's ready to go ahead. What do you want him to do now?

The preceding excerpt (especially part C) represents the second long episode during which Monica worked long and hard with Amy and Carol at eliciting that the input of 90 will make a corner-type (right angle) turn. (The first episode occurred on October 2, 1984, the week previous the above-cited one; it is presented in Appendix Q.) It might seem at the end of these interactions with Monica in Grade 4, both in Selection #19, and in Appendix Q, that Carol L. and Amy L. have understood the idea of the input of 90 for a corner of a square or rectangle. As has already been demonstrated by means of selection #2 in the section on COLLABORATION, in Grade 5 Amy did not recall that 90 was a "special" number, despite the fact that she even had her Grade 4 book next to her at her work station, and specifically referred

to it for reference. Carol L. did retain this information, and was seen to use the "special" number, 90, in her project work with her partner Doug L. in Grade 5.

One may also consider that whereas Carol L. and Amy L. called for teacher intervention during Grade 4 work, there were no corresponding calls for teacher intervention with certain of the problems in Grade 5. The incident of Amy L. and Leanne L.'s struggle with their "square" which was related in selections #8A and 8B in the section on COLLABORATION was a case in point. Amy L. and Leanne L. did not ask for help. They even expressed pride over not asking for Monica's help: "Monica, do you realize we didn't even call you once!" It is possible however that their sense of independence was on this occasion a little misplaced. For in the end their frustration with the problem rendered them unable to even continue to work together and to persevere with the problem (see Note 23).

Lilly H. and Rina M. solicit peer and teacher intervention concerning input sum (135,181)

In Selection #20 below, Rina M. and Lilly M. have a disagreement about the sum of the input numbers which represent the most recently completed rotational turn. The disagreement began with Lilly's (angry) request asking for the sum total of the input (for the turn) keyed to that point (line 1); implicit in her suggestion is the proposal that they ought to stop and do the addition. Each then proceeded to add up the inputs. Due to the fact that Lilly perceived that there was a discrepancy in the sums of the ones column (Rina stated 25, while Lilly stated 31, lines 8 and 9), Lilly appealed to Jon H. to do the calculation and went to him with the notebook record (*Appeal to peer*, lines 11 and 13). Rina was at that point still in the midst of adding; it could not be determined by the researcher whether Lilly had completed the entire calculation, or had stopped at the first column and then had gone to see Jon. Rina completed her calculation, and her total sum was 135; Lilly returned from her consultation insisting that the total was 181 (which it was, due to the fact that she had in error included the (LT) 46 with the other addends, and so had Jon). Rina and Lilly next took care to reconcile the entries (line 31), but Lilly still did not see that she had included LT 46 by mistake. Lilly then went to get Monica to intervene, in order to determine who was correct.

Selection #20

Lilly M. and Rina M., November 20, 1985, D/A # 10, pp. 31-35.



0 Rina M.: Now, -R -T five (keys)

1 Lilly M.: One minute! (Yells) HOW MUCH DID YOU GET?

2 Rina M.: (Keys)

3 Lilly M.: WAIT A MINUTE (slaps table), O. K.?

4 Rina M.: (Writes) O. K. This.

5 Lilly M.: -R -T five, six

6 Rina M.: What's twenty, hold it, twenty-three and five, is eight- (holds up her book)

7 Lilly M.: One minute, I'm adding it up •(? inaudible).

8 Rina M.: •Sh-, ten, thirteen, hold on, five, ten, fifteen, twenty, twenty-five.

9 Lilly M.: It's thirty-ONE, thirty-

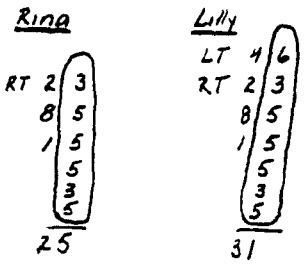
10 Rina M.: Be quiet (hand over Lilly's mouth), I'm not talking to you (counting) four-

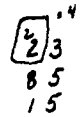
11 Lilly M.: (Yells to Jon H.) Jon! Jon •add this up-

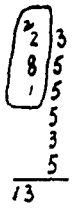
12 Rina M.: •(Using fingers on the table, counting on from four) five, six, seven, eight, nine, ten, eleven, twelve, thirteen-

13 Lilly M.: Jon, add this up, one minute (gets up to give her book to Jon H.)

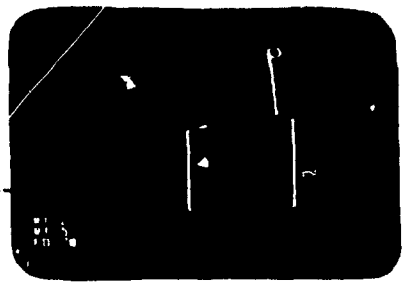
14 Rina M.: Thirteen. (Yells) Oh, I found it, it's a hundred-, I <u>found</u> it. (Pause) I found it, it's right, thirty-, a hundred







and thirty five. (Stands up) It's right a hundred and thirty-five (pause) (writes) Forward seventy (keys, then looks at the screen.) Awright! (Writes)



15 Lilly M.: (Lilly is with Jon H. and Eric H. at their work station computer #1, and is telling Jon the numbers to add.)

16 Rina M.: (Yells to Lilly) What-, are you working with them? I'm going without you. (Keys.)

17 Lilly M.: I don't care. I'm (? waiting). Rina!

18 Rina M.: (Looks at the screen, and is surprised) We'll make just a tape thing (tracing on the screen.)

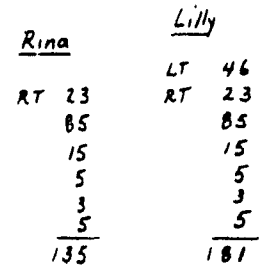
19 Lilly M.: (Moves partway back to computer #2) Could you WAIT! No, we won't. (Goes back to Jon.)



20 Rina M.: We screwed up, so we have to make this a tape thing.

21 Lilly M.: (Lilly comes back.)

22 Lilly M.: How much did you get? How much did you get? (Pause.) A hundred and thirty-five? It's a hundred- , it's a HUNDRED and eighty ON:E!



23 Rina M.: Sorry.

24 Lilly M.: (Erasing)

25 Rina M.: No, it isn't.

26 Lilly M.: Jon added it up.

27 Rina M.: So, just because Jon added it up, Jon added it doesn't mean that he (? inaudible).

28 Lilly M.: (Writing) I'm taking a hundred and eighty-one because-

29 Rina M.: 'Cause you don't know how to add?

30 Lilly M.: Because that's what I got also.

31 Rina M.: (Pause) O. K. Fine. Look. Maybe you have the wrong things [i. e. addends]. I have 23, 85, 15, 5, 2, 5.

32 Lilly M.: That's what I have.

33 Rina M.: 'Kay, five is eight, five, ten, fifteen (pause), fifteen, (? these are) seventeen, ten fifteen, twenty.

34 Lilly M.: (Lilly goes to get Monica. It is possible to discern only some of their talk due to the distance. Lilly is heard to say: "(?inaudible) add it up right, I added it up right, and now she thinks it's not added up right.")

(B) MONICA'S INTERVENTION

(Monica comes over.)

35 Monica T.: Let's make the computer 'add it up

36 Rina M.: 'Hold on, twenty-

37 Monica T.: 'Kay? The computer can do addition.

.

.

.

I want to do some math on the computer.

38 Rina M.: O. K.

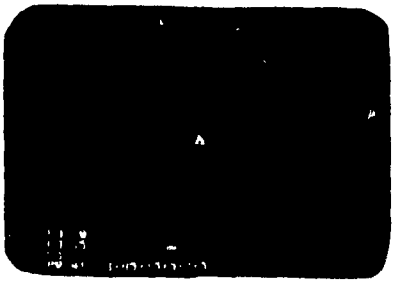
39 Monica T.: Watch (hands up, fingers splayed for attention, establishes set) I want to show you how to do some math, O. K.? (keys) . . . O. K. (keys) we're ---



gonna tell the computer to print, how much (points to Rina) are you adding? Twenty-three, eight, O. K., twenty-three-

.
.
.

40 (Rina and Lilly then proceed to tell Monica the input numbers they have recorded in their books, and Monica shows them how to use the "plus" symbol on the computer keyboard. This section has been omitted.)



41 Monica T.: That's it? (turns to Lilly)

42 Rina M.: Ya (erasing something in her notebook.)

43 Lilly M.: Ya.



44 Monica T.: O. K. You press return (to Rina).

45 Rina M.: (keys)

46 Monica T.: (Points to the screen) And the answer is one thirty-five.

47 Rina M.: (Checks her notebook) I told you I was right (holds up her book).

48 Lilly M.: (open-mouthed and speechless, sits down)

49 Monica T.: Finished.

50 Michael H.: (Michael comes over to tell Monica that "they" are fooling around. Monica leaves with Michael.)

51 Rina M.: O. K.? Are you happy?

Monica showed Lilly M. and Rina M. how to take advantage of the computer technology in order to check a sum. It was perfectly appropriate for Monica to

intervene and seek to resolve the disagreement by means of checking what the correct sum actually was. That seemed to be the mandate given to Monica by Lilly. Lilly wanted to know which one of them had "added it up right" (line 34). Thus it followed that Monica responded directly to their need, and showed them a new tool which they could use. However, at the same time, Monica unwittingly reinforced Lilly and Rina's arithmetical approach to the solution for angular input, for Monica did not introduce any reference to the size of the turn, nor suggest using a "Does it make sense?" strategy. Indeed, neither Lilly nor Rina were heard to cite what their respective sums were. Had they done so, Monica might perhaps have pointed out to them that a turn of 135 is, after all, quite different from a rotation of 181. In Logo, 135 and 180 are frequently occurring and useful, and at times rotely used, signposts, but neither Lilly nor Rina showed evidence of being aware of this (see Notes 28 and 36).

One might hypothesize as to how Monica might have responded had she chosen to pursue the intervention differently, or to extend it further. The following reflections are just that-- hypothetical ruminations. It would diminish the import of the teacher's role were the comments to be interpreted by the reader as a suggestion that more could or should have been said by Monica. The researcher has had the time, and the need, to see the videotape multiple times in order to distinguish what the problems were. Had the researcher been in the teacher's place, she would have been satisfied to have responded as did Monica, that is, to suggest using the computer in order to check the sum. The time in the Logo computer lab classroom is short, with many demands placed upon the teacher's time. Monica did respond directly to Lilly's request, and then retreated to solve another (seemingly social) problem (line 50). Monica was noted in other instances to distinguish errors of inclusion of inappropriate inputs; the fact that such was not the case here may in part reflect some of the difficulties inherent in the problem.

It must be stressed that at this point of their project, it was extremely difficult for both Monica and the children to distinguish the heading and movement of the turtle, and subsequently the amount of rotational turn; it would have been easier had the turtle cursor been in a horizontal or vertical starting position, and turning in an upright position rather than upside down. Hillel and Kieran (in press) have noted children's reliance upon a vertical-horizontal frame of reference. It was difficult for the researcher to recreate the beginning and the subsequent moves of the turtle, as it would have been

difficult for Lilly or Rina or Monica to review the moves, since Rina and Lilly had begun at an oblique position, had turned LT 46, then had adjusted with RT 23 (rather than the direct inverse of RT 46), and then had added on RT 85 (were they aiming for 90?) and subsequently had moved a little bit at a time-- RT 15, 5, 2, 5.

One might hypothesize what might have transpired had Monica, or Rina, asked for a procedural explanation from Lilly, such as: "How did you get that answer of 181?" It might have enabled them to see how it was that Lilly and Jon had arrived at their sum of 181. Monica surely would have seen that the error lay in Lilly's erroneous incorporation of the input 46 (LT 46) into the arithmetical calculation of RT 23 + RT 85 + RT 15 + RT 5 + RT 2 + RT 5. The essential finding which emerged from this episode was the degree to which Lilly and Rina relied upon pure calculation. Lilly derived the total input via an addition algorithm, with no corresponding understanding of the size of the turn. It was likewise for Rina M.; however in this instance Rina happened to get the correct answer, and what was most striking, Rina M. persisted in declaring it correct, even in the face of conflicting evidence to the contrary issued by an able mathematics student, Jon H. (Appendix R). The significance of this algorithmic mode as an indicator of the state of the child's understanding of angle will be addressed later in the section entitled "Supported talk as it reflects learning: The child's understanding of angular rotation in Logo geometry."

As one reflects upon the number of calls for teacher intervention overall, and for teacher intervention in relation to mathematics concepts, it must be stressed that a good part of Monica's teaching was done when she perceived a need. At times when a child or a pair solicited help for one purpose and Monica thought the timing suitable to introduce a pertinent concept unrelated to the solicited intervention, she would do so. Upon occasion, the fact that the information or explanation was unsolicited rendered its impact less strong (Appendix S).

But we need to touch again upon the possible causes for the infrequent calls for teacher intervention in the mathematics domain. As has already been mentioned, it was clear during the interviews with the children that it was a perception of a number of the children that the teacher was the source of knowledge (see Appendix H, Q. 2.1, 2.3). It is even more perplexing, then, to try to determine why there was so little call for teacher intervention during disagreements over mathematics concepts. As has already been mentioned earlier in the findings (PATTERN), this source of intervention

was not accessed often in relation to calls for intervention concerning the mathematics concepts; it was noted in that section that the number of calls in relation to the syntax problems outnumbered the occasions of solicited teacher intervention concerning mathematics problems by a ratio of 5:1.

It seems feasible that the children did not seek help for the mathematics area because it was not a mathematical world they were encountering, but rather a drawing world. We as adults with a school background in Euclidean geometry can look at what the children were doing, and we would think in terms of 90 degree angles, of complementary, supplementary, opposite angles, total turns of 360 degrees, properties of special quadrilaterals (squares, rectangles), triangles and so forth. Most of the children felt that they could work through the problem by testing and exploring and drawing. It may perhaps be for the very same reasons that at times they did not ask each other for explanations. They did not appreciate enough of the workings of Logo geometry to perceive that there was a pattern and a systematic way of approaching the problems. Perhaps the children would only seek explanations from each other and from the teacher and neighboring peers if it were apparent to them that the puzzle was logical. It did indeed seem that most of the children did not perceive the geometric underpinnings inherent in Logo, and therefore did not work in a logical, "analytic" way (Hillel and Kieran, in press) with the information. The lack of requests for logical and causal explanations from each other, and from the teacher and classmates, may well reflect the absence as yet of a perception of an underlying system. This would be consistent with the findings concerning the children's mode of working with angular rotation, and with the problem solving methods and recording methods used by the children. These findings will be dealt with shortly, in the section entitled "Supported talk as it reflects learning: The child's understanding of angle, the visual-algorithmic mode." These varied aspects, namely the lack of requests for explanations and for interventions, the modes of working and problem solving, all reflect the stage of the children's understanding of angle at this point in their Logo experience.

The SUPPORTED TALK Component of the Verbal Strategies

The findings concerning the first two component segments of the Level II strategies, namely (A) requests for explanation and for information, and (B) calls for teacher and for peer intervention have been reported. The following section will deal with supported talk, segment (C) (see Table 28). The main focus will be on the *supporting arguments* within the supported talk segment, but consideration will be given to other verbal strategies in Level II which reflect support for one's statement or position. As represented in Table 28, the verbal and nonverbal strategies representing supported talk include the following categories: *Supporting arguments, Demonstration, Focusing, Reference to recorded notes, Reference to prior knowledge (supported), Reference to computer screen verdict, Reference to computer experience. Conditional test hypothesis* and *Counter proposal.* These categories all represent or include some form of support for one's statement and should be considered the components of the larger, more comprehensive term "supported talk." In the write-up, the afore-mentioned categories will be lumped together for some purposes under the broader term "supported talk," and then at other times will be considered separately in order to allow for finer definition.

The aim of this section is to examine the supported talk in order to see how the use of the verbal strategies can contribute to the child's learning, and to the learning of the child's partner, and also to see how the use of the strategies reflects the child's understanding about Logo angle. To return again briefly to the aims which guided the analysis, the questions were as follows:

CAN THE STRATEGIES CONTRIBUTE TO THE CHILD'S LEARNING, AND TO THE LEARNING OF THE CHILD'S PARTNER?

ARE SOME OF THE VERBAL STRATEGIES WHICH ARE USED DURING DISAGREEMENTS MORE EFFECTIVE IN LEADING TO LEARNING THAN ARE OTHERS? IF SO, WHICH ARE THEY?

Table 28

Subcategories Within the Supported Talk Section of the Level II Strategies

A. Request for Explanation or Information	B. Appeal to Teacher or Peer	C. Supported Talk							
		Supporting argument	*Demonstration*	*Focusing*	*Reference to recorded notes*	*Reference to prior knowledge (supp.)*	*Reference to computer screen verdict*	*Reference to computer experience*	*Conditional test hypothesis*	*Counter proposal*

This findings section will look at the issue of clarification. As had been stated in the review of the literature, it was deemed necessary to look critically at the question of whether the children did indeed clarify themselves during their disagreements, as the authors of the Creative Uses Project (Carmichael, et al., 1985, p. 317) suggested they do; and if so, in what ways did they clarify themselves? This question has already in part been addressed and answered in the affirmative in the findings section on **Explanations**. The children did clarify themselves by means of their explanations. This section will continue with the question, and will look specifically at the supporting arguments in order to deal with the following issues:

Are the supporting arguments clear? Are they correct?
Do the supporting arguments clarify? If yes, how?

The findings will consider whether the supporting arguments clarify only the child's general purposes, or whether they also clarify the mathematics aspect. If yes to the latter, it will distinguish the ways in which the clarification of the mathematics aspect reflects the child's understanding of the arithmetic and geometric components of Logo.

There were two main categories of supporting arguments which emerged, and which are most pertinent to the learning of the mathematics concepts. They have been assigned the terms **Descriptions** and **Mathematical Justifications**. The **Descriptions** were announcements of location, heading, and size of the turn of the turtle cursor. The language used to describe was predominantly ethnomathematical ("a little bit more" "a little bit less"), and computer screen context-referenced ("up here," "we wanna go down there") language. The **Mathematical Justifications** provided causal and logical reasons, the hows and whys, for the turtle cursor's position and heading and size of the turn, and used geometry referents such as attributes of a square or rectangle, and arithmetical referents such as calculation of sums of input, in the supporting argument. The section on these two types of supporting arguments will show how the mathematical justifications, and the descriptions, reflected the children's understanding about Logo geometry.

Since the study is geared toward the investigation of the learning of the mathematics and geometry concepts within Logo geometry, the focus will be directed

mainly upon the supporting arguments which are mathematical justifications. The findings will endeavor to convey how the mathematical justifications play a role in the learning both in cases when they do serve an explanatory function--as some are seen to do, and in cases where they encourage a mathematical way of thinking. The report will look mainly at the supporting arguments within the disagreements, but will also consider supporting arguments outside of the mathematics concepts disagreements, and outside of all disagreements.

In attempting to distinguish which of the verbal strategies might have been more effective than others in leading to learning, the issue of resolution of the disagreements via persuasion will be considered again, and the results of the investigation of the content of the verbal strategies which played a role in persuasion will be reported. It was noted earlier (in the findings section on PATTERNs in the disagreements) that certain of the Level II strategies, such as *focusing, reference to computer screen verdict*, and especially *supporting argument*, played an important role in persuasion. It must be stressed that persuasion is not to be interpreted as synonymous with teaching and learning. This section of the findings will look at the supporting arguments and other verbal strategies used in persuasion in order to consider the ways in which these strategies did seem to play a role in the children's learning. The means by which the children were persuaded to accept a partner's argument did offer clues as to the state of the child's understanding of Logo. The last part of the findings section will consider the ways in which various aspects of the supported talk of all the children gave insight into the children's understanding of Logo geometry.

Supported Talk: The Role of Clarification

The issue of clarification will now be presented. The role played by all the aspects of supported talk during disagreements will be considered. In the section entitled "Supported talk: Persuasion," particular attention will be paid to the diverse strategies subsumed under the title of supported talk. But the report on "Supported talk: The role of clarification" will focus mainly upon the *supporting arguments*, since supporting arguments comprised a large proportion (46%) of the "supported talk" component of the Level II strategies (see Table 29).

The research questions which guided the analysis were:

Table 29

Percentage of Total Level II Verbal Strategies Represented by Request for Explanation and Information, Appeals for Teacher and Peer Intervention and Supported Talk

		f	Total Level II Statements	%
Requests for Explanation and Information	Request for Explanation	13	315	4.1
	Request for Information	8	315	2.8
	Request for Clarification	2	315	0.6
	Total Requests	23	315	7.3
Appeals for Teacher and Peer Intervention	Appeal for Teacher Intervention	2	315	0.6
	Appeal for Peer Intervention	3	315	1.0
	Total Appeals for Intervention	5	315	1.6
Supported Talk	Supporting Arguments (within supporting talk)	86	188	45.7
	Supporting Talk (combined categories)	188	315	59.7

Note: The total number of Level II statements is 315. The total percentage (i.e., 7.3% + 1.6% + 59.7%) does not equal 100% because other Level II strategies have not been included.

CAN THE STRATEGIES CONTRIBUTE TO THE CHILD'S LEARNING, AND TO THE LEARNING OF THE CHILD'S PARTNER?
ARE SOME OF THE VERBAL STRATEGIES WHICH ARE USED DURING DISAGREEMENTS MORE EFFECTIVE IN LEADING TO LEARNING THAN ARE OTHERS? IF SO, WHICH ARE THEY?

The findings will be dealt with under three subsidiary themes, in line with a number of questions previously outlined: (1) Are the supporting arguments clear? Are they correct?; (2) Do the supporting arguments convince? Which ones convince?; and (3) Do the supporting arguments clarify? If yes, how?

For the first of the segments, that is (1) above, only findings which drew upon the data from the mathematics concepts disagreements will be presented. For the second of the sections, that is (2), the disagreements which ended in resolution by persuasion will be considered. The third section, that is (3) above, will look mainly at the supporting arguments within the mathematics concepts disagreements, but in certain cases will also consider supporting arguments outside of the mathematics concepts disagreements, and outside of all disagreements. The aim is to derive as informative a picture as possible of the ways in which the supporting arguments might have contributed to the children's learning and of what the supporting arguments can tell about the children's understanding.

Clarity and correctness of the supporting arguments

The objectives asked: Are the supporting arguments correct? Are the supporting arguments clear? Out of general interest, all the children's statements (not only the supporting arguments) were reviewed with an eye to judging their clarity. It was found that the majority of the children's statements were clear. In total, only 23 statements out of a total of 850 statements, or 3%, were rated as incomplete, unclear, irrelevant or inappropriate. Of these 23 statements, 11 were supporting arguments. It was expected that the children would ask for explanation or clarification in cases of unclear statements; however there was only one occasion of a follow-up request of this kind, and the follow-up request itself was not easy to understand.

It seems to be a truth of human nature that when we interact we often proceed as if we do understand in cases when we do not understand. MacKay (1974) has stated that "in the course of conversation unclear utterances are often allowed to pass. Conversations proceed regularly in the absence of complete understanding" (1974, p. 226). The children may have rendered their own interpretation in cases where the statement was unclear; at times we understand what is meant rather than what is said. They may not have been interested in pursuing the case, or in asking for further elaboration; or they may have been proceeding as we all seem often to do in an ambiguous situation--that is, in most instances, we let it pass.

To return to the issue of the clarity of the supporting arguments at this point, of the 86 supporting arguments in all, 11/86, or 13%, of the supporting arguments were judged not clear. The focus was then directed specifically upon the supporting arguments which were mathematical justifications, for it was felt that these supporting statements would give more insight into the children's understanding than would the arguments which were categorized as descriptions (some of which were unclear due to their context-bound nature; their correctness was not judged). There was a total of 29 mathematical justifications, that is, supporting arguments which used arithmetic or geometry referents such as arithmetic calculations, or references to attributes of geometrical shapes such as a square or rectangle. Of these, only 5/31, or 16%, were not clearly stated; a higher proportion, 9/31, or 29%, were not correct (see Table 30). The two examples below were chosen as illustration of mathematical justifications which were not correct (Selection #21) or were not clearly stated (Selection #22). Others have already been presented elsewhere, as for example Lilly M.'s error of inclusion of LT 46 which rendered the sum 181, not 135 as it should have been (Selection #20); and Amy L.'s error in calculation (83 + 83 = 116), and in her lack of understanding of the working of inverse (Selection #8A and 8B). The two examples cited below served to provide insight into the children's understanding and misunderstanding.

In the first selection, Shauna L. wanted to turn right a half turn (or 180), but she was both confused about how to do so, and about the terminology to use.

Table 30

Correctness and Clarity of Mathematical Justifications
Within Mathematics Disagreements

Participant	Total Number of Math Justifications Within Mathematics Concepts Disagreements	Correctness		Clarity	
		Number Correct	Number Incorrect	Number Clear	Number Unclear
Jon H.	2	1	1	2	
Eric H.	3	2	1	3	
Lilly M.	10	7	3	8	2
Rina M.	4	2	2	4	
Carol L.	2	2	1	1	1
Amy L.	1			1	
Leanne L.					
Russ H.[a]	5	5		4	1
Shauna L.	4	3	1	3	1
Doug L.					
Totals	**31**	**22**	**9**	**26**	**5**

[a] Russ H., Shauna L. and Doug L. were taped for only 6, 4 and 4 sessions respectively

Selection #21

Oct. 23, 1985, D/A #1, pp. 9-10



Shauna L.: And we have to make a little tiny circle for it to go around. We need help with circle. - - - - - - - - - -

Amy L.: No. No. Because if he's facing on forward and he has to go to the side.

Shauna L.: We need him to turn around to be like this (hand motion to the left, to the tune of the movie Jaws) then he'll be facing down and it's half. Now, if he goes around, he'll be (singing and swings her hands upward, indicating the completion of a full turn, a full circle with her hands together). Y-, we have to make half a circle.

What Shauna actually wanted was a half, and then a full rotation, not a circle drawn. It is possible that Shauna was perhaps thinking about a trace such as would be left by turtle if the turtle point could leave a trace. (One other child, Carol L., was observed to confuse turtle's rotation with a line trace that could be left by the turtle's tip if he could leave a trace in turning.) If one went through half a rotation, such as on a clock face, the surface covered by the arm of the clock would be a semi-circle. Amy used a description-supporting argument in saying "he's facing on forward and he's got to go on to the side." Amy later stated correctly that "we're supposed to turn right to make a circle," and Shauna accepted and keyed in RT 10 to start.

In the following very short example, Jon H. and Russ H. were trying to understand why their procedure was not working. Their difficulty was a case of syntax error, in that they had not typed TO PICTURE :SIDE on the top line of their procedure which was to use a variable. Russ at first suggested trying to change the :SIDE within the procedure itself to just :S, but then he stated himself that the change would not result in any different outcome, because Russ knew that the change of name

did not matter.

Selection #22
Russ H. and Jon H., D/A # 4, December 11, 1985, p. 29.

Russ H.:	(Keying) Just try -S.
Jon H.:	No, that wouldn't work.
Russ H.:	Well, it'll be the same thing.

Russ was correct, but not clear; the lack of clarity is, however, in this case most likely due to the difficulty of the concept. Russ meant that using another variable name, such as :S, rather than :SIDE would not correct the difficulty in the program, since it was the same thing; the name would not change anything. It is not clear whether the argument was understood by Jon H. in this instance. It is also not certain why Jon said that :S would not work, since it actually would render the same result. It is possible that Jon might have felt that just one letter would not be sufficiently informative to the computer. Sutherland's (1987) study has shown that many of the children misunderstood the nature of variable names; they felt that a specific name was required, for instance :LENGTH, was necessary for a procedure for rectangle.

Verbal strategies which resulted in persuasion

During disagreements there were strategies which served to convince the listener. The section which follows deals with the aspect of "supported talk" in episodes of disagreements which ended in persuasion. It will look at the ways in which various of the verbal strategies, in particular *supporting arguments*, clarified the child's position. It will consider whether the supported talk which led to persuasion contributed to the children's learning in a way that distinguished it from the other disagreements.

As Table 31 will show, the majority of verbal strategies which led to persuasion are found at the right hand side of the grid, the location for the Level II strategies. The strategies most strongly represented were *supporting arguments*--25%, *focusing*--17%, *reference to computer screen verdict*--17%, and *reference to recorded notes*--14%. The striking occurrence, as has been mentioned before, was the category of *reference to recorded notes*, used exclusively by Lilly M. and Rina M. (with the exception of one

Frequency of Participants' Use o

	LEVEL I STRATEGIES																	
	Insistence	Insistence—Action taken	Action taken	Challenge	Denial/Resistance	Insult	Refusal	Threat	Ignores	I--Dismisses suggestion	I--puts on "hold"	Physical force	Silence	Anger	Mitigation	Aggravation	Counter/contradiction	Counter-suggest
Jon H.																1		
Eric H.																1		
Lilly M.		1																
Rina M	1	1									1					1		
Carol L																1		
Amy L.																		
Leanne L.														1				
Russ H.[a]																		
Shauna L.[a]																		
Doug L [a]																		
TOTALS	1	2									1			1		4		
% of total strategies used in persuasion	1	2									1			1		5		

[a] Russ H , Shauna L and Doug L were videotaped for only 6, 6, and 4 sessions respectively

Table 31

of Level I and Level II Strategies Leading To Persuasion

...estion	Evaluation	TOTAL LEVEL I USE RESULTING IN PERSUASION	LEVEL II STRATEGIES: Request for explanation	Request for information	Request for clarification	Request for a hearing	Request fulfilled	Request not fulfilled	Cannot fulfill request	Appeal to teacher	Appeal to peer	Reference to recorded notes	Reference to prior knowledge—supp.	Reference to prior knowledge—unsupp.	Reference to computer experience	Reference to computer screen verdict	Other-directed	Supporting argument	Focusing	Demonstration	Conditional test hypothesis	Counter proposal	Compromise	TOTAL LEVEL II USE RESULTING IN PERSUASION	LEVEL I AND LEVEL II COMBINED
													2				2	3			1				
										1	2		3				6	2							
					1			1		2	1		6				4	3	1	1					
										9			1				7	4				2			
													1				2	1							
													1							2					
													1							1					
																	1	2							
9				1				1		12	3		15				22	15	3	5	2			79	88
10				1				1		14	3		17				25	17	3	6	2			90	100

instance of use each by Jon H and Amy L., see Table 21). Speculation upon what this can tell us about Lilly and Rina's concept of angle will follow later (in the section entitled "Supported talk as it reflects learning: The child's understanding of angular rotation in Logo geometry").

It had been expected that persuasion would be more likely to occur if a partner used supported talk of some kind. This was indeed found to be the case. However, it is of interest to note that in a number of instances, admittedly very small, persuasion was effected by means of *simple arguments*, that is by use of only Level I strategies. Upon two occasions, a *counter/contradiction* by one child of the input of 90, without additional supporting argument, was sufficient to elicit acceptance. This was seen to work in instances where the listener was aware of the use of 90 as a "special" number for corners. In one case Robert L. persuaded Carol L. to use RT 90 (Dec. 11, 1985, D/A #4, p. 29), and in the second case, Jon H. persuaded Eric H. to use RT 90 (November 13, 1985, D/A #4, p. 25). The mere suggestion of 90 without support was enough to trigger acceptance in cases where the listener needed no further elaboration upon the merits of using the input of 90, since the partners shared an understanding about when 90 ought to be used.

It was expected that the less adept students might be more likely to use the result of the action on the screen as support for a suggested move, or to depend upon it a posteriori, in order to insist that their position had been correct. However all the children were found to refer to the screen result as a final "clincher" as it were, in some of their disagreements; the final end results served as an incontestable verdict. The *reference to computer screen verdict* was seen to be used a number of times (17%), alone or in concert with other supported talk, as a means of effecting persuasion. Another strategy, in this case nonverbal, was seen to be used by all the children, and was almost always used in concert with other strategies in order to clarify a point; that was the *focusing* action of pointing or tracing with finger or hand, which served to locate, underline, and stress a verbal point.

Supporting arguments which led to persuasion comprised 25% of the verbal strategies employed in persuasion (see Table 31). A breakdown of the *supporting arguments* which led to persuasion was done in order to investigate what type of arguments were used, and to distinguish whether the allotment of different types of supporting arguments differed from the ratio of occurrence in the body of all

mathematics concepts disagreements. It was found that 9/22, or 41%, were **mathematical justifications** (based on an arithmetic or geometry referent); 6/22, or 27%, reflected description of heading of the turtle, or used ethnomathematical descriptive terms, and 3/22, or 14%, represented descriptive considerations concerning screen fit, hence a total of 9/22, or 41% were based on **descriptions**; 3/22, or 14%, reflected esthetics or evaluation arguments; and 1/22, or 4%, represented a strategy argument. There were some small differences found between the ratio of occurrences of mathematical justifications and descriptions within the disagreements which ended in persuasion on the one hand, and their occurrence within the entire body of mathematics concepts disagreements on the other. The differences were as follows. In the larger body of all mathematics concepts disagreements, 29/79, or 37%, of the supporting arguments were **mathematical justifications**, while 50/79, or 63%, were **descriptions**. In the persuasion episodes, the mathematical justification type represented a larger proportion of the *supporting arguments* as compared to the description type, that is, 41% were mathematical justifications, and 41% were descriptions. The mathematical justification type of supporting argument was seen to play a slightly greater role in persuasion than in the overall corpus of mathematics concepts disagreements. The ways in which these mathematical justifications played a role in the disagreements, and the ways in which they reflected the child's understanding of Logo geometry, specifically Logo angle, will be dealt with in the next section to follow, entitled "Supporting arguments: How do they clarify?" It was felt that treatment in both sections would be superfluous.

The focus in this section was upon the aspect of persuasion. In what ways might the resolution of conflictual points of view by means of persuasion be seen to lead to new learning? Did the children learn something new during these episodes? Did new knowledge play a role in convincing them to accept their partner's argument? It was found that during these persuasion episodes the children did convince their partners by clarifying, by seeking corroboration (such as referring to the screen verdict), and by reviewing and consolidating information previously covered but not yet learned (such as for example Jon H.'s difficulty with lateralization). Since repetition and access to evidence is important for learning, we may consider that these instances did contribute to learning, in that the acceptance and resolution seemed to be more pronounced in these instances of persuasion than in cases where there was no persuasion, or in cases

where there was not even resolution.

However, the learning was not always subsequently maintained and consolidated. There was one instance of a disagreement which was coded as having been accepted as the result of persuasion, in which it was clear later due to the analysis of subsequent sessions that the accepter was not convinced in the long run of the persuader's point of view. This case was related to selections cited earlier in the section on COLLABORATION. Selections #7B and #7C showed that Rina M. did accept Lilly M.'s supporting argument vis-à-vis the use of the inverse, by means of a two-steps-in-one method, in two of the three cases. This constituted new information for Rina. These two disagreements were coded as persuasion of Rina by Lilly. Lilly's method of using the inverse in a two-in-one step way was superior to Rina's method, and seemed during the October 23, 1985 and the October 30, 1985 sessions to be accepted by Rina. But as the researcher later detected in subsequent Logo sessions, Rina reverted to her slower strategy, opting thereby to use the "slow but sure" route.

As has been mentioned previously, and is being stressed here again, the intent is not to equate persuasion with learning. The strategies used by the children can contribute to the learning of both the speaker and the receiver, but learning is not automatically the consequence of a one-time, or even many times, exposure. The strategies which effected persuasion informed us as to the strategies used most frequently during the disagreements which ended in persuasion. There must be some correspondence between the use of verbal strategies at one end, such as when children hypothesize and test, refer to evidence, appeal to supported prior knowledge, focus with voice and motion as a point of emphasis and in order to support a statement, and learning at the other end; but the relationship is not direct, and is certainly not self-evident. One must however assume that there is likely to be some learning happening. It is at times easier to denote instances when learning has not taken place, or to detect that a child has reverted to a formerly held point of view. Tracing the evolution of the learning is difficult.

Supporting arguments which clarified

The following section will deal with supporting arguments. It will deal with the research question which asked whether the children clarify themselves in their talk, and if yes, how they did so. The broader general underlying question which directed the

investigation was of course:

> CAN THE STRATEGIES CONTRIBUTE TO THE CHILD'S LEARNING, AND TO THE LEARNING OF THE CHILD'S PARTNER?
> ARE SOME OF THE VERBAL STRATEGIES WHICH ARE USED DURING DISAGREEMENTS MORE EFFECTIVE IN LEADING TO LEARNING THAN ARE OTHERS? IF SO, WHICH ARE THEY?

The first aspect to be considered will be that of the two types of supporting arguments which emerged during the analysis, namely the **Mathematical Justifications**, and the **Descriptions**. The children gave reasons for their actions. It was deemed important to distinguish whether the supporting arguments clarified only the child's general purpose, or whether they clarified the mathematics aspect as well. The main focus of the study was after all on the mathematics learning. If the answer to the latter part was that the supporting arguments did indeed clarify the mathematics aspect, then it was necessary to delve more deeply and ask: (1) In what way could the clarification using mathematics terms be seen to contribute to a "geometric way of thinking" between the partners; (2) Did any of the mathematical justifications serve the function of explanation?; and (3) In what way could this clarification be seen to reflect the child's understanding of the geometric components of Logo geometry?

Supporting arguments: Descriptions and Mathematical Justifications

As has already been indicated, there were two main categories of supporting arguments which emerged, and which were considered most pertinent to the learning of the mathematics concepts. They have been assigned the terms **Descriptions** and **Mathematical Justifications**. To reiterate their definitions briefly, the **Descriptions** were announcements of location, heading, and size of the turn of the turtle cursor. The **Mathematical Justifications** presented the causal and logical reasons for the turtle cursor's position and heading and size of the turn, and used geometry referents such as attributes of a square or rectangle and arithmetical referents such as calculation of sums of input in the supporting argument.

Clarification by means of Descriptions

Supporting arguments which were classified as **descriptions** served an important clarification function in regard to the interaction between the partners. For one, using reasons in general was important in fulfilling the social dimension of communication; during the Grade 6 interviews, the children often mentioned that it was imperative to talk together when one worked in pairs, because one needed to discuss ideas and the steps whereby the ideas could be executed. The talk together using descriptive terms met many of the interactional, instrumental, regulatory, informative and even heuristic needs that Halliday (1974) specified that language filled in the life of social man (see Appendix H, Q.3). The descriptions could not however fulfill the need to explain "how," or "what" or "where" in mathematical terms.

Many of the decisions between the partners were determined on the spot, as the result of a screen move following imminently upon an action (Kieren, 1984; Kamii and DeVries, 1978). The way in which this represented an early stage of development of understanding of Logo angle on the part of the children will be elaborated upon in a subsequent section. The descriptions were ontext-bound announcements; the descriptions often did not give additional information which suggested how to proceed. When information was given, the supporting arguments categorized as descriptions again offered the information in context-bound, very general terms ("let's do it a little more," "this way," or "I want it over there") that carried no specific procedural indications of how to accomplish the move. The selections which will follow shortly will illustrate examples of supporting arguments which were descriptions. First, however, the aspect of supporting arguments which were mathematical justifications (since they used mathematics or geometry referents), and what these supporting arguments could and could not do, will be presented.

Clarification by means of **Mathematical Justifications**

The supporting arguments which were classified as mathematical justifications provided elaboration via reference to a standard system of notation. One aspect was the arithmetic system of combining, either summing FD X, FD Y, FD Z = FD (X + Y + Z), or subtracting FD X, BK Y = FD (X - Y), where X is larger. The latter, subtraction, calculation was used by only a few children in the study: Russ H., Jon H., and Eric H.; only Russ H. was seen to use it more than once. It was noted that the children referred to squares and to rectangles as a goal or subgoal in their project work. On only one occasion did the children in this study, Lilly M. and Rina M., attempt a project using triangles, and reference will be made to their Star of David project later (see Selection #41, and Note 31). It was expected that when the children expressed a goal of making a square or rectangle, that they would be keeping in mind the diverse essential attributes of line and angle entailed in the shape. However it became obvious only upon repeated checking that only the lines, not the angles, were referenced. In some cases, the 90 input for the angle was seen to be used, but it was never referenced as an essential component, even in the discussion of the repeat command for making a rectangle (see Selection #29C below; Note 29). Although two of the children, Russ H. and Jon H., made the geometric shapes of squares and rectangles in a way that one was able to infer therefrom that they were aware of the vital attributes, the absence of denotation of the angles by the children was deemed to be an important aspect to pursue and monitor; this aspect deserves further investigation in the future.

The supporting arguments which used arithmetic or geometric referents were more abstract and objective, and less context-bound than were the descriptions (Note 30). As a result of the children's commentary, one was able to visualize some or much of what was happening. The children who used the mathematical justifications talked about causes, or used logical arguments to give suggestions on how to proceed with simple displacements, or with side lengths of squares or rectangles. But as will be pointed out in the selections (see Selection #37), the researcher looked but did not find causal or logical supporting arguments which reflected an analytic (Kieran, Hillel and Erlwanger, 1986) basis of explanation for the turtle's turn. Although it was expected that there would be some instances of causal or logical mathematical reasons expressed in the data for the turtle cursor's position and heading and size of turn, the reasons presented rested for the most part on calculation rather than on analytic bases. What

the children attended to, and how they arrived at the defence of their positions of argument, served of course to give an indication of the child's understanding of angle at that point in time.

The selections below will present examples of these two types of supporting arguments, **Descriptions** and **Mathematical Justifications**. In an endeavor both to cite a number of examples, but also to restrict the length of the section, there will be short anecdotal comments attached to a number, but not all, of the excerpts. For the most part the topic of how the descriptions, and the mathematical justifications, reflected the children's understanding about Logo geometry will be dealt with later in the findings.

Examples of **Descriptions**

The **Descriptions** referred to the turtle cursor's heading and position by means of announcements such as "he's here" and "he's pointing that way." (Turtle was at times anthropomorphized as "he," or neutered and referred to as "it.") The descriptions were context bound, that is, one could not determine where the turtle cursor was without access to the screen, nor could one determine what had preceded the event. The citations below feature descriptions related to the cursor's heading and position; they reflect the children's consideration of "screen fit"; and they show how the children used their own ethnomathematical nonstandard terms, such as "all the way around" for a 180 turn, in describing an amount of turn.

- Descriptions related to the turtle cursor's HEADING

  Selections #23 (#23A, #23B, #23C)

(A)

Lilly M.: Perfect, perfect! He's pointing that way.
(Lilly to Rina, October 23, 1985, p. 15)

(B)

Lilly M.: No, see, I think that's his head now.
(Lilly to Rina, Rina then uses the HELP routine to check the heading, October 3, 1985, D/A #4, p. 17)

(C)

Lilly M.: No, because now he's pointing that way [Lilly is incorrect about the heading].

Rina M.: (Uses the HELP routine)

Lilly M.: Where is he pointing? [Request for information]

Rina M.: Down, like this (tracing on the screen)

(Lilly and Rina, October 23, 1985, D/A #16, p. 15.)

- Descriptions related to turtle's POSITION

Selections #24 (#24A and #24B)

(A)

Shauna L.: He's right there

(Shauna to Carol, Shauna has lost track of where turtle is, November 20, 1985, D/A #2, p. 6)

(B)

Rina M.: See, we're right on the line.

[She is incorrect.]

(Rina to Lilly, they are not on the line although they look as if they are; they have used HELP, but still do not see that their heading is off the line; post disagreement discussion, October 23, 1985, p. 14)

It is very difficult, for children and adults alike, to distinguish where the turtle cursor is pointing. The technology is weak; the turtle's point of heading, in the Apple II Logo version at least, is not distinguishable from the turtle's other two vertices--it is after all a triangular cursor. Adults using Logo are seen to need to check the heading from time to time. When children who are not aware of the size of the turn are moving the cursor around, they are truly at a disadvantage. The repeated use of the HELP aid was evidence of the children's inability to track the turtle's movement, due in part to the technology, but due in greater part to their lack of appreciation of the size of the turns. (Russ H. was observed to use the HELP routine only once, for example, and Jon H. and Eric H. used it infrequently, whereas the children in the middle and less adept groups used it often, provided they were aware of it (see Appendix L, where Shauna L. worked to recreate the HELP routine.)

- Descriptions which are context-bound to the computer screen

There was talk which occurred which could not be interpreted by the researcher without recourse to the computer screen videotape or to the camera videotape (and sometimes not even then).

Selection #25

Doug L.: (Points to the screen) O. K. Let's say it went to here, eh (tracing on the screen), then we have to go down, like this (pause)(tracing on the screen) and then (looks at his book), no, wait a second, we go up.

(Doug to Carol L., January 8, 1986, p. 14)

On a number of occasions, the talk was cryptic, as when Amy L. stated to Leanne L., pointing at the screen: "Forty-five, cause it's, ya." (December 18, 1985, p. 9.) A close investigation of the screen events was necessary to determine what the children meant when they said they needed to "get all the way around"-- in one case "get all the way around" meant a quarter turn (Lilly M. and Rina M., November 20, 1985, p. 6), or when they said they needed to go "all the way around back"-- in one instance "all the way around back" referred to a half turn, our 180 degrees (Shauna L. and Carol L., November 20, 1985, p. 11).

In most instances, nonverbal motions accompanied the content bound descriptive talk.

Selections #26 (#26A and #26B)

(A)

Doug L.: We have to go (points to the direction)

(Doug L. to Carol L., December 11, 1985, D/A #5, p. 30)

(B)

Doug L.: We'll just go down (pointing and tracing on the screen), and then like that again.

(Doug to Carol, note that there is no reference to "how," to the size of the turn, or the length of the displacement, January 8, 1986, D/A #1, p. 17.)

The researcher did find context referenced descriptions which at the same time also gave indications of "how" the children intended to complete the move, although the children did not qualify why they chose the particular displacement of, for example, FD 99, or FD 25. Rina and Lilly were still testing and exploring, hence the input of 99.

Selections #27 (#27A and #27B)

(A)

Rina M.: We want to get him over here (keys) O. K. -F -D ninety-nine.

(Rina to Lilly, October 23, 1985, D/A # 14, p. 14)

(B)

Rina M.: No, forward twenty-five (points to the screen). Look, he's over there.

(Rina to Lilly, October 30, 1985, D/A #6, p. 22.)

- Descriptions that reflect consideration of SCREEN FIT

The following excerpts feature the children's expression of concern about the whole product fitting within the frame of the computer screen; there was no explicit consideration in the discussion, however, of the geometric aspects of displacement and angle which were inherent in the project design.

Selections #28 (#28A, #28B, #28C and #28D)

(A)

Jon H.: No, we gotta go lower.

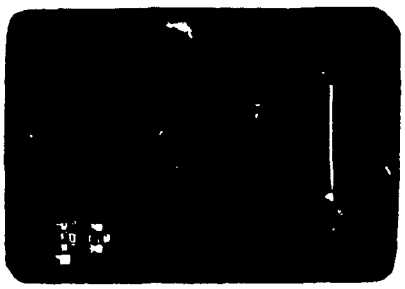
Eric H.: Why?



Jon H.: Because we need room for the antennas.

(Jon and Eric, October 16, 1985, D/A #3, p. 5)

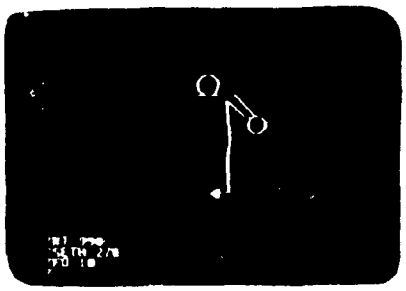
(B)



Jon H.: No, it won't. We have to have room to put this and the inside (?square).

(Jon and Eric, October 16, 1985, D/A #8, pp. 8-9.)

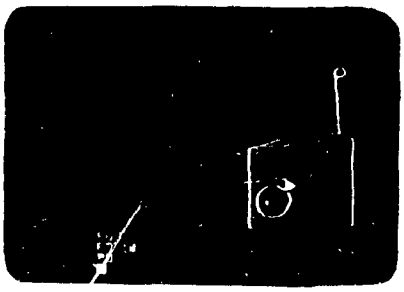
(C)



Russ H.: No, because this circle- we're not gonna have enough room.

(Russ, Jon and Eric, December 18, 1985, D/A on plan, p. 31.)

(D)



Lilly M.: No, -F -D --, it'll be too much, 'cause we need room (points to the screen) for the speaker, don't forget.

(Lilly to Rina, December 4, 1985, D/A #4, p. 21.)

- Descriptions which use ETHNOMATHEMATICAL terms

Ethnomathematical terms relating to moves which would add "a bit more" on to the line or turn were commonplace; they were coded as ethnomathematical when there was no additional qualification of how much more, and why a certain amount was suggested, and when one could not envision what was happening on the screen. For example, Lilly stated: "No, you wanna go a bit more and then we'll make that (tracing on the screen) a bit longer."

Transition to the mathematical justification

The talk which follows directly below was slightly more difficult to categorize, for it represented a transition stage between the ethnomathematical description, and the supporting argument which would be considered a mathematical justification. The children, Lilly M. and Rina M., used terms which were context-bound, and not easily understood at first in relation to standard reference to a 360 degree circle rotation. However, when the talk was examined more closely, it became clear that the children were discussing fractional parts, but using a different frame of reference than what was expected. Lilly and Rina were not in the instances which will be cited below talking about bits of moves, but rather about parts in relation to a whole. The talk which began the discussion concerning the amount of turn contained a descriptive contextual component; Rina questioned whether the sum of 80 was correct for the total turn just completed by saying: "It does -R -T eighty to go all the way around? . . . You sure?" (Nov. 20, 1985, p. 6); she then checked her notebook to find that the sum was actually 85 (see Selection #45, and Appendix Y). It only became clear upon closer consideration that in this case, the ethnomathematical "all the way around" meant a quarter turn. (Shauna L., for example, in another instance, used the phrase "all the way around" to refer to a 180 degree turn, see Selection #35.)

In the following three excerpts, we note that both of the children, Rina and Lilly, used the conventional term "half." It was upon viewing the screen that the researcher understood that they were actually referring to a quarter turn. They were aiming to move turtle 90 degrees, but are as yet unaware that such was their goal. We see in the transcript selections #29A, #29B-1, #29B-2, and #29C, a move toward the standard, for in the last excerpt, although Lilly was still using the term half, she seemed more confident about the 90; she was even seen at one point to use the term "quarter" in relation to the input of 90 (Selection #29B-1 below).

Selections #29 (#29A, #29B-1, #29B-2, #29C)

(A)

Rina M.: "We wanna go half (hand motion), we want -R -T, -R -T eighty-five"

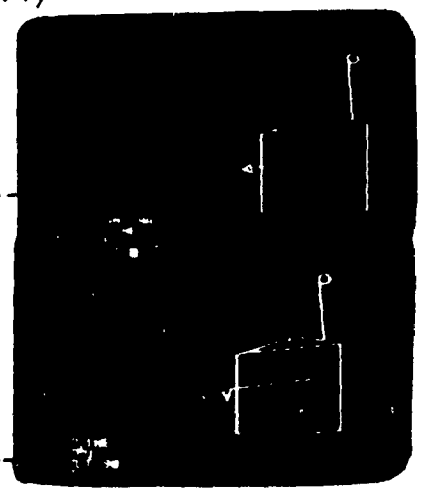


(Rina to Lilly, talking about amount of turn, said half but meant a quarter turn; Rina was incorrect about the input of 85 since at that time she did not yet know about 90; November 20, 1985, D/A #3, p. 7)

(B)

(B-1)

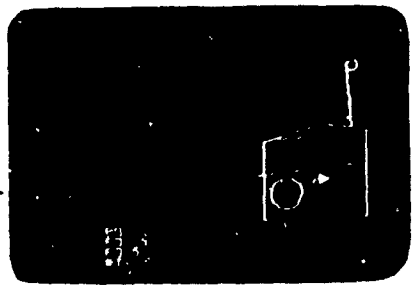
Lilly M.: -L -T 90 (points to the screen) 'cause he's gonna go half -- (pause) . . . I think.



(B-2)

Lilly M.: -L -T another 90 'cause he went a quarter (pointing to the screen). ----

(Lilly to Rina, Lilly used the term "half," which was indeed half of a half turn of 180, that is, a quarter; one thought that Lilly knew about 90 until she was heard to say "I think." We noted that shortly after the first citation (B-1), Lilly did use the term "quarter" in conjunction with the input of 90 (B-2); supporting arguments outside of mathematics concepts disagreements, December 4, 1985, p. 8.)

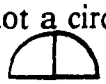


(C)

Rina M.: Right 90 ----

Lilly M.: Because it's half.

(Rina and Lilly, December 4, 1985, p. 23; agreement and supporting argument; this time, Lilly sounded more sure--there was upon this occasion no "I think" qualifier.)

Lilly and Rina were looking at fractional parts; their "whole" however was not a circle, but rather a half circle  . A half of their "whole" was therefore  . There was an indication that Lilly was beginning to see the quadrant element when the

partners needed to turn yet another 90, for the circular rotation was then beginning to take shape  . The colored portion represents the next "90." Lilly and Rina decided in the end that they would not proceed with the last move, and so their discussion of "half" and "quarter" ended for a while with the last excerpt cited above (Selection #29C).

Thus the beginnings of understanding of some aspects of Logo geometry were perceived to emerge. During the November 20 and December 4, 1985 sessions, as the children talked, questioned each other, and supported their statements, one was able to catch a glimpse of the move from the ethnomathematical "all the way around" description to a consideration of various inputs which might correspond to a quarter turn. The aspect of proportional measure was also in evidence, and will be touched upon again later in the findings chapter, in the section entitled "The visual-algorithmic mode: One child's use of proportional measure."

Examples of Mathematical Justifications

The section to follow will demonstrate how the children did at times support their position by use of mathematical justifications. The section dealing with mathematical justifications will be relatively brief due to the fact that much of the rest of report on the findings will concentrate specifically upon an elaboration of the mathematical justifications, namely how the children learned by means of these kinds of supporting arguments, and what the various kinds of mathematical justifications revealed about the children's understanding of Logo geometry.

Mathematical referent

The excerpts which follow feature arithmetic references to calculation, namely subtraction in relation to displacements (Selections #30A and #30B) and addition in relation to inputs for rotational turns (Selections #30C and #30D).

Selections #30 (#30A and #30B)

(A)

Jon H.: Don't--. Write -B -K ninety (writes), now let's try forward . . . twenty

(keys).

Eric H.: (Looking at the screen) Ya. (pause)
So, it's supposed to go back seventy.
(writes) Back seventy.

(Eric H. to Jon H., subtraction algorithm in relation to displacement, post disagreement elaboration. Note that Jon has executed the forward and back moves. Eric has observed, and recorded only the difference; November 6, D/A #1, p. 6)

(B)

Carol L.: No thirty, 'cause then you'll have twenty (pointing to the screen).

(Carol L. to Doug L., subtraction algorithm suggestion in relation to displacement. BK 50 had been keyed but not entered; Carol then suggested deleting the 50 and substituting BK 30, because the difference, BK 20,was the preferred length; agreement, Dec. 11, 1985, p. 14)

Selections #30 (#30C and #30D)

(C)

Lilly M.: A hundred seventy (looking at her book). No, zero, zero, zero, eight and nine is nineteen [error which she then self-corrects], is seventeen.

(Lilly persuaded Rina, addition algorithm in relation to rotational turn. Rina insisted that the sum of the input to LT was 117 and Rina was incorrect; by means of reviewing the addition calculation Lilly persuaded Rina that the sum was 170, not 117. December 4, 1985, D/A #2, pp. 8-9. Discussed again at length later in the findings in Selection #34B)

(D)

Lilly M.: Seventy and ten is eighty.

(Lilly persuaded Rina that the sum of the input to RT was 80, addition algorithm in relation to rotational turn; November 20, 1985, D/A #1, p. 6. Discussed again later in the findings in selection #34A)

When the descriptions and mathematical justifications were classified into sub-types (Appendix T), it was noted that all 7 of Lilly M. and Rina M.'s mathematical justifications related to relational turns relied upon calculation. The children in the study who were at the low level of achievement, namely Carol L., Amy L. and Leanne L., were seen to use calculation of inputs to rotational turns extensively in their work, but this strategy was not reflected in their supporting arguments. In the end the identification of this calculation strategy proved to be an important finding. During the Grade 6 interviews (May-June, 1987), it was found that even some of the children thought to be at the high levels of achievement relied upon calculation as opposed to analytic methods of proof. There was extensive dependence upon calculation in some cases, and complete dependence upon calculation in other cases. This dependence upon arithmetic calculation when working with rotational turns, and the ways in which this reliance reflected a limit to the children's understanding of the size of angular rotation and of the relationships between angles, will be developed in a later section of the findings entitled "Supported talk as it reflects learning: The child's understanding of angle, the visual-algorithmic mode."

Geometry referent

We have already noted geometry references to turns (see Lilly and Rina, Selection # 29).

The following excerpts refer to the attributes of squares and rectangles. One notes that the children attended to the length of the sides; there was no mention of the angular rotation of 90 needed for a square or rectangle.

Selections #31 (#31A, #31B, and #31C)

(A)

Eric H.: You can't do fifteen! . . . Too big! That's not a square.

Jon H.: . . .it's not, it's not gonna be a square [i. e., it's to be a rectangle.]

(Eric and Jon, concerning square and rectangle, commenting on the length of the sides, November 13, 1985, D/A #3, p. 23.)



(B)

Jon H.: No, two ten, two twenty.

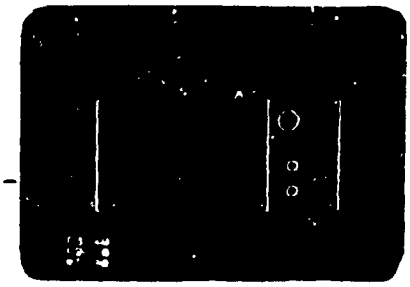
(Jon to Eric, on rectangle; the previous line drawn was 110 --the width; Jon's suggestion for the other line was 220, or twice the length of 110; October 16, 1985, D/A #9, p.9.)



(C)

Jon H.: (Pause) No, (points to the screen and traces) a rectangle goes two, three, two, three (removes his hand). So we can do repeat.

(Jon to Eric, on rectangle, attending to the length of the sides, November 13, 1985, D/A on plan, p. 7)



As concerns other kinds of geometry proof, there was no analytic kind of mathematical justification heard during the sessions which relied upon the child's knowledge of complementary, supplementary angles, opposite angles, or alternate angles along parallel lines such as was elicited from a small number of the children during the interview sessions. Although Russ H. for example was seen to be adept in

the use of the analytic mode during the interview sessions (February-March 1986; May-June, 1987), he did not voice these analytic approaches in his supporting arguments during his Logo work; as will be indicated in Selection #37, it seemed that Russ H. relied mainly on visual means during the Logo sessions. One recalls the observation of the van Hieles (in Fuys, et al., 1984) that a child at first works at a level at which the understanding is implicit, and only later can she or he become explicit about the learning. It is possible that while Russ was able to function at an analytic level in some of his Logo work, he was not yet inclined to be explicit about it. One could argue as well that the use of analytic mathematical justifications could not be expected to be prevalent among the children unless the children were geared toward a mathematical orientation within the Logo setting of the study. The computer lab classroom environment of this study was one wherein the learning of Logo geometry was an aim, but was not the focal objective, as was the case in the Noss (1985), Hoyles, Sutherland and Evans (1985), Hillel (1984), Kieran (1986), Kynigos (1987) and numerous other studies whose findings have provided supporting and contrasting data for this investigation.

There seemed to be a greater frequency of occurrences of mathematical justifications among the moderately adept and highly adept groups (7 and 3.3 per child respectively) than among the less adept groups (1.2 per child) during mathematics concepts disagreements (see Table 32). It seemed at first that it might be the case that the moderately and the highly adept groups would have heard and offered more mathematical justification type supporting arguments, thereby indicating that they were more attuned to a mathematical way of thinking, and were promoting a mathematical way of thinking in their working relationships. But firstly, it was decided that before any such conclusion could be entertained, it would be necessary to investigate the mathematical justifications further; this will be done in the forthcoming sections. In addition, it was seen in the frequency of occurrences of supporting arguments outside of the disagreements that the gap between the high and the low groups grew less distinct, that is 2.7 per child use of mathematical justification by the high level achiever, and 1.6 per child by the low level achiever (see Table 33). Thus the greater frequency of occurrences of use of mathematical justifications among the high level group did not remain consistently high throughout the sessions. The tendency toward a higher use of mathematical justifications by the moderately and highly adept groups is nevertheless

Frequency of Occurrence of Two Types of Supporting Arguments Within Mathematics Concepts Disagreements

	Mathematics Justifications			Descriptions		
Participant	Number per Child	Total per Subgroup	Mean per Child	Number per Child	Total per Subgroup	Mean per Child
Russ H. (6)[a]	5	10	3.3		14	4.7
Jon H.	2			7		
Eric H.	3			7		
Lilly M.	10	14	7	16	22	11
Rina M.	4			6		
Shauna L. (4)[a]	4	6	1.2	4	14	2.8
Carol L.	1			2		
Amy L.	1			4		
Leanne L.						
Doug L. (4)[a]				4		
TOTALS		**30**			**50**	

[a] Russ H., Shauna L. and Doug L. were taped for only 6, 4 and 4 sessions respectively.

Table 33

Frequency of Occurrence of Two Types of Supporting Arguments Outside Mathematics Concepts Disagreements

	Mathematics Justifications			Descriptions		
Participant	Number per Child	Total per Subgroup	Mean per Child	Number per Child	Total per Subgroup	Mean per Child
Russ H. (6)[a]	3	8	2.7	6	18	6
Jon H.	5			5		
Eric H.				7		
Lilly M.	13	19	9.5	12	20	10
Rina M.	6			8		
Shauna L. (4)[a]	4	8	1.6	3	12	2.4
Carol L.	1			1		
Amy L.	3			4		
Leanne L.				3		
Doug L. (4)[a]				1		
TOTALS		**25**			**50**	

[a] Russ H., Shauna L. and Doug L. were taped for only 6, 4 and 4 sessions respectively.

of interest, although larger numbers of samples would be needed before any conclusions could be drawn.

Supporting arguments which clarify via explanations

In the findings section on requests for explanations, there was illustration of how the children did explain to each other, and how the explanations reflected the children's understanding about Logo geometry. As was mentioned toward the end of the section on solicited explanations, at first the researcher had used the notion of the children "giving reasons" generally and interchangeably with the idea of the children "explaining." It was Erlwanger (personal communication, March 1988) who suggested that it may not have been the intention of the child to "explain" when he or she gave reasons for the action or proposed action. While the supporting arguments might fulfill the functions of solicited explanation, it could not be assumed or expected that the supporting arguments would do so. Consequently, as has been stated previously, for the purposes of this investigation the definition of the term "explain" was then limited to instances in which the child strove to "make plain or clear; render intelligible; make known in detail" that "which is not known or understood" (Random House College Dictionary, 1975, p. 466). Subsequent to the meeting with Erlwanger (March, 1988), the researcher decided that all the supporting arguments which used mathematics referents would be classified as mathematical justifications, until such time as further investigation could be done to see whether any could be considered mathematical explanations as well.

Once the analysis of supporting arguments as descriptions and mathematical justifications was completed, the mathematical justifications were then studied again in order to find which of these supporting arguments clarified by means of explanation. The mathematical justifications were expected to fill empirical, procedural, intentional, or deductive purposes, as the explanations presented in the findings section entitled EXPLANATIONS were seen to do, in order to be considered explanations.

The instances cited below represent selections from among the small number of instances in which mathematical justifications (mathematics and geometry referent supporting arguments) did serve as explanations. Supporting arguments within the mathematics concepts disagreements, as well as supporting arguments outside of mathematics concepts disagreements, were surveyed, and the kinds of explanation function they were found to fulfill are presented in Table 34.

Table 34

Mathematical Justifications Which Served an Explanation Function

	Within Mathematics Concepts Disagreements				
Participant	Procedural	Empirical (Causal)	Intentional	Deductive (Logical)	
Lilly M.	4				
Amy L.	1 (incorrect)				
	Outside of Mathematics Concept Disagreements				
Participant	Procedural	Empirical (Causal)	Intentional	Deductive (Logical)	
Jon H.		2 (both incorrect)	1		
Eric H.	1 (incorrect)				
Russ H. (6)	2 (1 unclear)			2 (1 incorrect)	
Shauna L.(4)				1 (incorrect)	
Totals	8	2	1	3	= 14

Notes: This table refers to mathematical justification explanations (both spontaneous and solicited), rather than to all explanations. Table 26 refers to general explanations given in response to solicited requests. There is therefore a difference between the total sum here and the total sum in Table 26.

The ratios of explanations to (a) total mathematical justifications, and (b) total supporting arguments are as follows:

$$\frac{\text{explanations}}{\text{total mathematical justifications}} = \frac{14}{55} = 25\% \qquad \frac{\text{explanations}}{\text{supporting arguments}} = \frac{14}{86} = 16\%$$

Examples of supporting arguments from within the mathematics concepts disagreements which served an explanatory function

The procedural examples from Lilly M. and Amy L. which are listed below have already been cited elsewhere in the findings. Selection #32 will illustrate how Amy L. attempted to explain the way in which the procedure for doing the inverse worked; due to an error in calculation, and a lack of understanding about the process, Amy was not successful in her explanation (see Selection #8A).

Selection #32

Amy L.: Leanne, all you have to do is go like this (leans over to key in) -R -T space (moves back to her book), what's eighty-three plus eighty-three? Three plus eighty-three, three is six, sixteen, -R -T space one one six (keys in). Return. (Pause) [INC] You see? (points to the screen) Now you just have to go a little more.

Lilly's explanations which follow in Selections #33A and #33B below referred to the inverse, and in Selections #34A and #34B to the accuracy of her addition calculations. The transcript excerpts can be found in fuller form in previously presented Selections #7B and #7C. In the brief citations found below, Lilly was heard to explain how the two-in-one step procedure for inverse needed to be done, and why the sums were indeed 85, and 170 respectively.

Selections #33A and #33B

Lilly M.: No, left twenty, because we just- we want it to go back and ten.

(Oct. 23, 1985, D/A # 19, p. 17)

Lilly M.: So now it's left (pause) seventy plus what we wanted to do. . . . Left a hundred and forty, you know why, because we wanted to go back.

(Oct. 30, 1985, D/A #5, pp. 17-18)

Selections #34A and #34B

Lilly M.: Seventy and ten is eighty (points to her book)

(Nov. 20, 1985, D/A #1, p. 6)

Lilly M.: A hundred and seventy (looks at her book). No, zero, zero, zero, eight and nine is nineteen [error, and then corrects herself], is seventeen.

(Dec. 4, 1985, D/A #2, pp. 8-9)

The disagreement episode last mentioned above, in which Lilly M. persuaded Rina M. that the arithmetic calculation of 170 was the correct answer, was of interest because it constituted one of a number of examples of how the children were unaware of the size of the rotational turns. Selection #34B is therefore elaborated below. The disagreement proceeded as follows:

Dec. 4, 1985, D/A #2, pp. 8-9.



Rina M.: (Writes) Seventeen.

Lilly M.: Seventy, (looks toward Rina), seven ty.

Rina M.: (Erases) That's impossible.

Lilly M.: A hundred seventy (looks at her book). No, zero, zero, zero, eight and nine is nineteen [error, and then she corrects herself], is seventeen.

Rina M.: (Then finger counts and checks her book)
(Writing) oh, nine (finger count) (mouths seven) (pause). It's a hundred and seventy.

Note that one seventeen (117) did not even make sense, but the children do not use the "Does it make sense?" test at all. One may look at the addends, 80 + 90. The aware observer may think of the angle turn. A turn of 170 is almost halfway around, as compared to a turn of 117. As we have seen in their disagreement in Selection #20 (November 20, 1985), when Rina calculated 135 and was correct and Lilly calculated 181 and was incorrect, this disagreement, and their reliance upon calculation as method for proof, reflected Lilly M. and Rina M.'s relative unawareness still of the size of the turns. The single instances of focus upon the size of the turn, in the case of the turn of 85 (Selections #29A, #29B, #29C; Selection #45 to come), and of quarter turns, suggested a beginning realization of the size of the rotational turn, at least in reference to the right angle turn; but they remained the only instances.

Examples of supporting arguments outside of the mathematics concepts disagreements which served an explanatory function

As one can see in Table 34, the number of mathematical justifications which are explanations are low, both within the mathematics concepts disagreements and outside of the mathematics concepts disagreements. Of the selections chosen to be cited below, a number feature explanations which were incorrect; one was not clear. They are of interest because of the way both the incorrect and the correct explanations gave insight into the children's understanding, and into the attempts the children made to understand.

Eric H.'s procedural explanation for the "input to CIRCLER 50" was incorrect, but indicated that he was trying to make sense of the relationship between the input of 50 to CIRCLER and the resultant activity on the screen. Eric L. stated (October 23, 1985, p. 20): "That's because he goes around fifty times." The input of 50 actually represents the distance in turtle steps from the center, that is, the size of the radius. It certainly does not represent the number of times that turtle makes a complete circle,

or even the number of turtle steps taken around the circumference of the circle.

In two cases, Jon H. was incorrect in suggesting causes for the error and resulting changed product on the screen. Jon H. stated (Dec. 18, 1985, p. 19): "I don't think he put back twenty. That's why he made it over there."; and then again shortly after, he said (Dec. 18, p. 19): "See (points to the screen), you didn't put BK."

It was of special interest to see how Shauna L. deduced her own unique nonstandard construct about the size of a half turn (our 180 turn was Shauna's 110) and of a full turn (our 360 turn was Shauna's 220), due to errors in calculation made both on October 23, 1985 (Selection #1, with Amy L.), and on November 20, 1985, with Carol (L.). A verification by the researcher of the computer screen videotape of November 20, 1985 revealed that the following commands had been typed: RT 30, RT 40, RT 70, RT 30, RT 10 (see Appendix L, p. 427). The total was 180 and was seen to bring the turtle cursor all the way through a half of a turtle trip (cf. Papert's Total Turtle Trip Theorem). Shauna L. had inadvertently forgotten to record the RT 70, and so her total sum was 110. Her explanation to Carol then proceeded as follows:

Selection #35
November 20, 1985, p.

Shauna L.: I know, get, I know how to get halfway around (points) so if we have to get all the way around back is a hundred and ten, and that's two hundred and twenty. - - - - - -

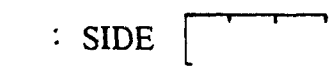


Throughout the five years of observation, the researcher looked particularly carefully for episodes during which children became aware of, and possibly explicit about, certain of the components of turtle Logo geometry. Had Shauna's calculation been correct, she would have spoken about 180, and 360. One would have been able to say: Here was a wonderful sample of how children at one time or another will discover the total turtle trip. There were, however, no such "AHA!" experiences related to the 180, and 360 turn among the children in this study. Some, such as Russ H. and Jon H., worked with these turns in a relational way from the start of the researcher's

observation of them (in Grade 5 in Russ and Jon's case). The other children in the study seemed to continue to explore toward it. The fact that there were so few cases of spontaneous discovery (one concerning Russ H. follows below) emphasizes the fact that discoveries which may seem to us to be just around the corner, may in many instances not be reached despite the fact that consolidation seems so very close. What was of particular importance in Shauna's case was the emergence, in both the videotapes and in the interviews, of a understanding of the proportional relationships between parts of a turn; readers will recall Hillel's (1984) mention of proportional measure (see Appendix U). Although she was not yet working with an appreciation of standard units of measure of turn, Shauna's relating of parts to each other and to the whole in a proportional way was unique among the low achieving children in this study.

Four of the explanations given by Russ H. are of interest; one (procedural) explanation was not clear, another procedural explanation was clear (Selection #36). One seemingly deductive explanation was not deductive (Selection #37). The second deductive explanation represented a rare instance of an "AHA!" experience, in that Russ H. transmitted with excitement to his partners, Jon H. and Eric H., his discovery about the size of the turtle cursor itself (Selection #39).

In the first instance, Russ H. explained the procedure for using a "times" multiplication symbol in the procedure for a project with a variable. The star (*) multiplication sign was needed; by multiplying whatever the input for SIDE would be by three, the result would then be consistent with the ratio relationship that was needed for the design. Russ's explanation was not clear, however, for he stated only: "F-D dots side times three (pause). Because we have to multiply it by three 'cause it's thirty." (Dec. 11, p. 25). He should have elaborated further by saying something about the method of applying proportions consistently when considering the various parts of the design.

: SIDE 

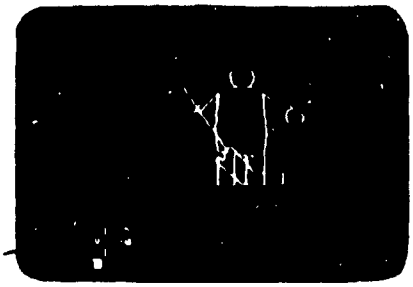
The second procedural explanation was of interest because it related to a syntactic as well as to a mathematics aspect of Logo. Russ H. was heard to point out to Jon H. that there was an essential difference in computer use between the alpha key for the letter O, and the numeric key for the number 0 (which has a stroke through it, unlike the typewriter key for nought).

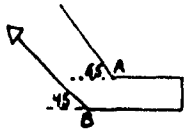
Selection #36
Dec. 18, 1985, p. 34.

Jon H.:	That should do it.
Russ H.:	No it won't (points to the keyboard). The zero has a line through it.

In the following selection, Russ attempted to explain, a posteriori, why the goalie stick was too thick at the bottom. His explanation sounded deductive at first.

Selection #37
Jan. 15, 1986, p. 24.





Russ H.:	Oh, I know why, I know 'exactly why
Jon H.:	'Be-cause
Russ H.:	(pointing to the screen) 'Cause over here we did fifty-five, and here we did forty-five. [Russ then worked backward to erase the line drawn last, and then turned another RT 10 for a total of fifty-five.]

Russ felt that at point A they had turned 55; actually, he had used SETH 270 at point A because he was at that time coming down along the diagonal slope of the hockey stick, and using 270 was easier than figuring out how much he needed to turn at point A. Russ's strategy in this case was actually a visual one (Kieran, Hillel, and Erlwanger, 1986). He determined after the fact that since they had already turned RT 45 at point B, and the resultant corner proved too thick, they ought to erase and turn 55. His explanation might have seemed to be the result of analytic process of identifying opposite angles along parallel lines and transverse lines, had the turn at point A indeed been 55. And yet the turn at point B could still have been based purely on the visual even had the turn at point A been 55. The visual seemed to be the guiding mode in this

instance. For when the researcher asked Russ H. during the interview how he had known what to turn at point A once the goalie stick bottom edge was seen to be too wide, Russ answered as follows (May 29, 1987): "It would look like 45, but we probably tried that and it was too thick, so we went with either 50, or 55, and then 60." It is vital to note that even though Russ was able to work through the problems given during the interview using analytic methods, the analytic method was never heard to be used explicitly during Russ's computer classroom Logo work. This does not mean that he did not use these means implicitly, for his work with rotational turns in the stickman project could have been based in part on analytic methods. However, Russ was never heard to talk through the process with his partners in the way that he was heard to do when responding to the researcher-interviewer questions. For example, Russ stated his awareness of supplementary angles in the Logo environment in response to one of the interviewer questions in the following analytic way:

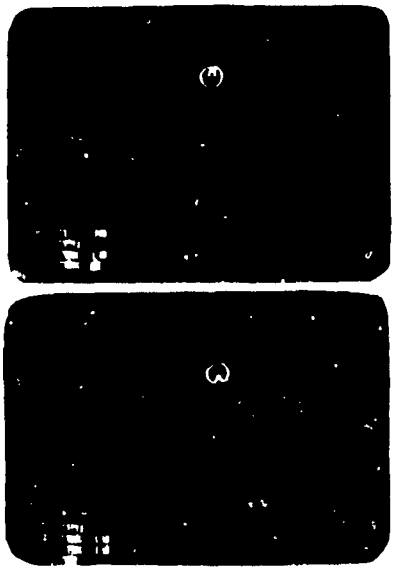
Selection #38
Interview May 29, 1987

Russ H.:	If the turtle was 45 degrees, the (points to) [remaining] angle would be 135 degrees.
I:	Oh, O. K. . . . how do you work it out so quickly?
Russ H.:	This is straight, so that's a hundred and eighty degrees, and I turned 40, if I were to turn 45 degrees, I would have to take it off a hundred and eighty degrees.

It is important to note at this point that Russ H.'s facility with the supplements of 180, and 360, and other angle relationships, and the connections which Russ made between Logo geometry and classroom Euclidean geometry, were far more advanced than that in evidence among the other children in the study, both in terms of the Logo work and the interview data. His responses were advanced when considered in relation to all the

children interviewed; Russ's answers compared favorably with the top 5/56 children, or 9% of the children who were interviewed. (Fifty-six of the 69 Grade Six children were interviewed in May and June 1987.)

In the following deductive explanation, Russ H. was seen to share his discovery about the size of the turtle cursor. The explanation is heavily reliant upon the screen context, and therefore may not be as clear as it seemed when viewed on videotape. One will note as well that Russ generalized in his use of the term "degrees" which in standard measure of course refers to amount of turn. The term "steps" would have been more accurate, but as we have seen in many of the transcript excerpts, the children's wording was often not consistent with standard mathematical terms.



Selection #39
December 18, 1985, p. 6

Russ H::	Aha, -a, I-, I, just figured something out.
Eric H.:	(?)
Russ H.:	Gat-. The turtle is exactly ten degrees. (Pause) (Eric and Jon stare at the screen) (Russ points) I mean, it's, if you go up, that- (points) the bottom would be where the point is.
Jon H.:	Ya.

It was noted elsewhere by the researcher that the measure of displacement via turtle steps in Logo geometry is not standard (Zack, 1986b), whereas the rotational turn is measured via the standard unit, degrees. This of course is not to say that the children see the relationship between the input to RT or LT on the one hand, and degrees which they have been introduced to in their classroom static geometry on the other; the project on the child's notion of angle revealed that many of the children did not make the connection (Zack, 1986d). Perhaps the fact that the turtle step is not tied to a standard measure makes it less likely for the children to look for a connection to a

standard measure for the turns, which are based on standard measures of "degrees." The researcher admits that perhaps this conclusion may be stretching the issue. What is vital in terms of Russ H.'s discovery is that while he had always had a good feel for the size of the displacements, he now had discovered a nonstandard unit of measure whereby he could gauge the size of the turtle's step.

Both the explanations within the mathematics concepts disagreements and outside of the mathematics concepts disagreements were considered. It was observed that Erlwanger's aforementioned caution had been well-advised; there were indeed few mathematical justifications which served as explanations. Even among those cited, it may well be the judgement of the reader that the explanations in some cases lacked clarity, or warranted further elaboration, or did not offer closure. If explanations are expected to "make clear," "render intelligible" and "make known in detail," perhaps certain of the excerpts might not qualify as explanations if strict criteria were to be applied. However, one must be cautious of applying adult criteria when evaluating the quality of children's explanations.

The "mathematical justification-explanations" did have the potential of contributing to the children's learning. They at times offered new information. The new knowledge could in turn later be refuted or consolidated. One would hope that it would not be the inaccurate information that was absorbed. And yet learning incorrect information, and interpreting correct information incorrectly, is an integral part of the learning process. Acquiring knowledge is not only a matter of learning new information; it is also a matter of returning to re-assess and refine the learning one has constructed to date. In addition to constituting potential contributers to the children's learning, the supporting arguments which were explanations did serve to offer glimpses of the nature of the children's understanding, and misunderstanding, of Logo geometry. In the section which now follows, we will move from the limited sphere of one small component of the Level II verbal strategies just discussed, that of supporting arguments which were mathematical justifications which were also explanations, back to the broader component of supported talk again. The aim is to deliberate upon the ways in which the supported talk reflected the children's understanding of Logo geometry, in particular angular rotation.

Supported Talk as It Reflects Learning: The Child's Understanding of Angular Rotation in Logo Geometry

The supported talk contributed to the picture of the child's understanding of Logo geometry that had started to take shape for the researcher as a result of previous analyses of the data (Zack, 1986b; Zack, 1986d; Zack, 1988a). The children's actions, their notebook entries, and their use of Level I strategies as well as of Level II strategies, were all important contributors to the emerging picture of each child's understanding of Logo geometry. It was the analysis of the supported talk component of the Level II strategies which proved to be particularly informative, for it both complemented the findings of previous investigations, and offered new insight into the children's understanding of Logo geometry, specifically the concept of angular rotation.

This part of the report will therefore elaborate upon certain aspects of the supported talk already touched upon previously in the findings, in order to deal more fully with the research question which asked how the talk offered insights into the children's understanding of Logo angle. It will present what the disagreements, and specifically the supported talk within the disagreements, were able to reveal about the children's understanding of rotational turn, and about connections made or not made between the input to RT or LT and the amount of rotational turn; it will deal with what the recording strategies and problem solving strategies used by the children at the low, middle and high levels of achievement revealed about the child's understanding of angle.

Skemp (1979) has spoken about two levels of thinking in mathematics, one the *instrumental*, the other the *relational*; the former involves using rules without reasoning about them, thus termed algorithmic by the researcher; the latter involves "knowing what to do, and why" (1979, p. 250). Kieran, Hillel, and Erlwanger (1986) have denoted, and Hillel and Kieran (in press) have elaborated upon, two modes of working with Logo geometry: the *visual* and the *analytic*. The researcher determined that in these early stages of Logo learning, there was evidence of more than one level of thinking within the visual mode itself, as well as evidence of an analytic mode of approach, amongst the children studied. Within the *visual* mode was found the visual-algorithmic, and the visual-algorithmic --proportional, and the visual-relational.

The report will for the most part be limited to the insights about the child's understanding of angle which were gained as a result of the analysis of the supported talk aspect of the disagreements. The first part of this section of the paper which deals with the visual-algorithmic mode will be much longer than the last part which deals with the visual-relational mode. For one, the majority of the focal children and their partners were found to be working at the visual-algorithmic level: Lilly M., Rina M., Carol L., Amy L., Leanne L., Shauna L., and Doug (L.); three of the children were found to be working at the visual-relational level: Russ H., Jon H., and Eric (H.). Also, the use of the visual-relational mode was not well represented in the supported talk; much of what was gleaned concerning the relational understanding, and the analytic abilities (Russ H. and Jon H.), of the high achievers was gained by means of the analysis of their work and by means of their responses in interview sessions, not as a result of an analysis of their supported talk. Despite the lower rate of supported talk on their part concerning angle, the mode of working of the high achievers will be presented; their work served as a fascinating juxtaposition to that of the other children in the study, and it represented a reflection of their understanding of the aspect of angular rotation in Logo geometry.

The VISUAL-ALGORITHMIC mode

The majority of the children in the study worked only at a visual level. Kieran, Hillel, and Erlwanger (1986) have defined what they have termed the "visual schema," and Hillel and Kieran (in press) subsequently elaborated upon it, stating:

> By a 'visual schema' we refer to Logo constructions of geometric figures where the choice of commands and of inputs to commands is made on visual cues. Rationale for choices is often expressed by "It looks like. . ."
> (Hillel & Kieran, in press)

The decision for each action was dependent upon the screen results. In this study, the researcher found that there were two ways of working within the visual-algorithmic mode. The first was algorithmic due to the child's use of rote arithmetic calculation; the second was also algorithmic but the child recognized relationships between parts as

well. Within the visual-algorithmic mode, the child manipulated discrete inputs. Within the visual-algorithmic--proportional mode, the child perceived that there could be proportional relationships between the inputs to RT/LT; the child would for example halve, or double certain inputs. The high achievers also worked chiefly in the visual mode, but not in an algorithmic way; their work will be dealt with in a later section.

This section will deal with the visual-algorithmic mode. It will draw upon citations and comments already presented in previous parts of the findings, and will elaborate upon them, in order to describe the rote nature of this mode. As Skemp (1979) has stated, people operating at this level are using "rules without reasons" (1979, p. 250). The inputs that the children used, the way the children recorded the inputs, the way they used their records and the ways in which they disagreed about certain actions, gave insight into the nature of their understanding of Logo angle.

The section will deal with the work of the following study subjects: Lilly M., Rina M., Shauna L., Carol L., Amy L., Leanne L., and Doug (L.). The focus in the section for the most part will be upon Lilly M. and Rina M., due to the fact that it was the striking tendancy of these two children to use their book notes in their disagreements which prompted the researcher to do further investigation of the children's reliance upon rote formulaic recipes when they were engaged in attempts at reentering project data and at solving problems and repairing errors. Similarites and differences between the approaches of Lilly and Rina M. on the one hand, and Carol L., Amy L., and Leanne L. on the other, will be indicated whenever possible.

Reference to recorded notebook entries

The researcher observed that Lilly M. and Rina M., as well as Carol L., Amy L., Leanne L., and Doug L., relied heavily upon their recorded notes, more so than did Russ H. and Jon H. (and Eric H. to a lesser degree) for example. Since it was part of the established Logo classroom context, all of the children recorded in and referred to their notebooks during their Grade 5 year. In that way Rina and Lilly were like the rest. But the extent to which Lilly and Rina used their notebook records to support their case during disagreements, and the role that the use of this support played in persuasion, was quite unique. For example, Amy L. and Jon H., did refer to their book notes in the course of their disagreements, but did so only once. None of the other children used specific explicit reference to their notes during their disagreements. But

Lilly and Rina referred 16 and 19 times respectively to their book notes in their disagreements (see Table 21). It is not surprising therefore, that their use of this strategy should stand out. The reference to the written notebook entries played an important role in the disagreements resolved due to persuasion, but of course only in Lilly M. and Rina M.'s case. As was noted in the section on supported talk and persuasion, *reference to recorded notes*, in each case their own recorded notebook entries, was used 14% of the time to effect persuasion, and thus represented a substantial proportion of the strategies which led to persuasion (see Table 31). By virtue of reference to book notes, Lilly was persuaded by Rina twice, and Rina was persuaded by Lilly nine times, to accept and resolve the disagreement. In instances of persuasion, by reference to the recorded book notes the children cited precedent, and correct calculations were used as supporting arguments to clinch the case (Pedersen, personal communication, May 10, 1988). The recourse to their book notes also played an important role in Rina and Lilly's attempts to repair their errors; examples of successes and failure with this strategy will be presented below. The citations selected, and the corresponding comments, will show how dependence upon a calculation strategy (using addition), and reliance on the book notes (1) revealed a lack of understanding of the amount of rotational turn, and (2) reflected a limit to diverse reasoning and logical means of problem solving.

The child's understanding of rotational turn

The extent to which Lilly and Rina used reference to their records in their supported talk gave insight into the state of their understanding of rotational turn. Neither had a feeling for the size of the turn, and they were not yet at point where they could use other (analytic) means to arrive at an answer; so Lilly and Rina were seen to use the entries which they had recorded as the "recipe"-like authority for their projects.

The turns were merely numbers, or arithmetic calculations of sums. The inputs to RT or LT were discrete entities. If seen relationally at all, it was only in terms of "a little bit more, a little bit less." There were only rare occasions upon which the input was seen proportionally in relation to a larger or smaller input, that is half of, or double, one third of, or three times the input at hand. In this study, with the exception of the instance of Shauna L. (as has been indicated previously in Selections #1 and

#35), and the comments made by Lilly M. and Rina M. (see Selections #29A, #29B and #29C), the children who worked at the visual-algorithmic level did not speak of proportional relationships such as "double" or "half" in reference to turns, or in reference to displacements (that is, distances or line lengths). Russ H. and Jon H. were heard to speak so in relation to displacements, but not turns.

Whether the inputs were discrete entities, or were discrete entities combined together, it was also clear in many cases that the input number to RT or LT (for example RT 135), and the corresponding amount of rotational turn were not being linked. As has been seen in examples already cited in which there were disagreements between Lilly M. and Rina M. concerning rotations of 135 versus 181 (Selection #20), and rotations of 117 versus 170 (Selection #34B), as well as instances of errors such as RT 116 --which should have been RT 166-- made by Amy L. and Leanne L. (Selection #8B), the children were not taking into account the amount of turn. The children's attention was focused upon the individual addends, and upon the addition algorithm which produced the combination.

The Star of David project

There was one episode related to Lilly M. and Rina M.'s attempt to make a triangle (as part of what the researcher has referred to previously as the Star of David project) which brought to the fore again the fact that Lilly and Rina were not cognizant of the amount of turn represented by the inputs to RT and LT. A short description of the difficulties with which the partners struggled follows below.

On Dec. 11, 1985, Lilly M. and Rina M. began a project at computer #6 while they awaited an opportunity to return to their regular work station, computer #2. When upon their return to computer #6 Lilly and Rina reentered their commands as listed in their notebooks, they were aghast to find that their project did not "work" as it had just done at computer #6. They tried to determine what it was that had gone wrong, but could not "debug" the error. In seeking Monica's help, the children kept repeating that they did not know why it was not coming out as it had previously. They were bewildered that their procedure needed to be so much longer than the original had been (Note 31). It was only upon consulting their notebook page that the researcher was able to distinguish what had happened; Lilly and Rina had combined the inputs RT 140 and RT 120, and an error in addition had resulted in a sum of RT 160 rather than

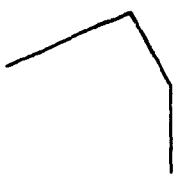
RT 260 (see Appendix C).

Selection #40
Book page

```
TRI
RT  25
FD  45   } FD 80
FD  35   }
RT  40   }
RT  45   } RT 120
RT  35   }
FD  80
LT 120
RT 140   } RT 160 ⇦
RT 120   }
LT  20
FD 80
```

At no time did Lilly M. and Rina M. take note of the discrepancy between the turns that RT 160 and RT 260 represented.

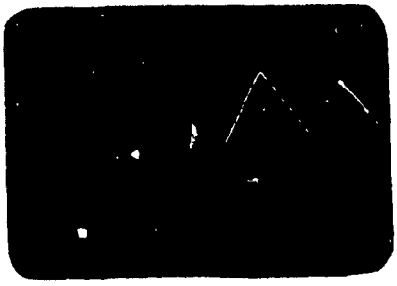
Upon their first attempt to repeat the work that had been done at computer #6, the product which resulted due to their error looked like this:



When first viewing the videotape records, the researcher was at a loss to figure out what the ultimate goal of the project had originally been; it was assumed that the children had been aiming to make a triangle, since the procedure was entitled TRI. The researcher was aided by a comment given at the end of the Dec. 11, 1985 Logo session by Lilly M., who spoke into the microphone specifically to state what it was that she

and Rina had been attempting. Lilly said:

Selection #41
Dec. 11, 1985, pp. 29-30.

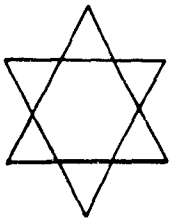


Lilly M.: We're doing mountains now. See, we started off with a star of David, but it didn't work.

.
.
.

Lilly M.: (whispers to Rina) I just talked to her [that is, to Vicki, the researcher].

Due to Lilly's comment, the researcher inferred that the original goal was the Star of David shape:

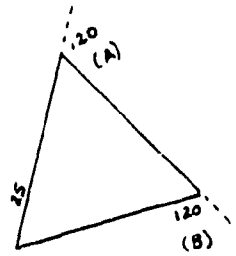


As a result of following the correct commands written in their notebooks, it was also assumed by the researcher that the beginnings of their product at computer #6 originally looked like this:

Selection #42

Listing

```
TRI
RT 25
FD 45 }  80
FD 35 }
RT 40 }
RT 45 }  120  ]  120 (A)
RT 35 }       ]
FD 80
```



LT 120
RT 140 } 160
RT 120 }
LT 20
FD 80

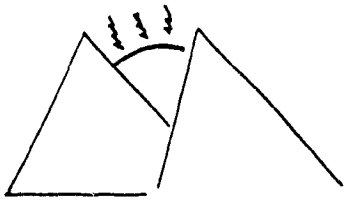
] 120 (8)

An outside turn of 120 is essential for the construction of an equilateral triangle in Logo geometry. However, Lilly M. and Rina M. were completely unaware of the importance of the 120 outside turn which they had accomplished not once, but twice. It is not surprising that they took no notice of the sums of 120, especially in the second case (Selection #42, part (B)). It was difficult even for the researcher to distinguish the RT 120 resulting from LT 120 RT 140 RT 120 LT 20. And the amount of turn had no significance for them.

It is of interest to note that Lilly M. felt she had to explain to the researcher what it was they had been attempting. Lilly obviously felt that there was a great discrepancy between what their goal had been, and what they eventually accomplished at the end of the Dec. 11, 1985 session. The children did not have the means whereby they could achieve their goal which was the Star of David. And so they gave up on that goal, determined what the product present before them on the screen suggested to them pictorially, which was mountains



(Note 32), and continued to work with the product which had emerged due to their errors, and which they titled Mountain Lillyn (for Lilly and Rina):



This was an example of what Noss (1985) has called *de-goaling* and what the researcher has termed *deplanning*. "The [de-goaling] strategy was exploited extensively by children who, in seeing that the turtle's movements did not correspond to those intended, consistently preferred to modify their goal rather than debug their drawings" (Noss, 1985, pp. 138-139). It was seen to be used by all of the children in the study. The children who were in the moderately adept and less adept groups deplanned often, and there was much greater discrepancy between their goal drawing and the screen product, due to the fact that they were in control of fewer Logo concepts than were the high achievers. The high level achievers did change certain aspects of their plans, but the changes tended to be minor as compared to those of the middle and low level groups. For example, Jon H. and Eric H. could not arrive at figuring out how to make the curve required for the antennas for their T. V. project, and so they made straight line antennas. The changes which Jon H., and Eric H. decided to make were far less drastic than the *de-goaling* done by the other groups.

Dependence upon book records

It can be seen from the Star of David episode that the children did not appreciate the amount of rotational turn represented by RT 140 and RT 120, either as discrete entities or as a combined product. Neither the individual entities nor the combined sum evoked any connection to size of turn. Even though in this instance it was an error in calculation which had created the difficulties, so that unknown to them the reference to notebook record strategy and dependence on calculation had actually failed them, one can see why Lilly and Rina needed to depend upon their book records. They had no other recourse. There was such a strong reliance upon the book record on Lilly and Rina's part, and on the part of Shauna L., Carol L., Amy L., Leanne L. and Doug L., here and in other episodes, because there was no understanding of the amount of turn such as was evident in Russ H. and Jon H.'s case. Until the children do develop a feel for the amount of turn, which might then act as a gauge whereby they can assess the sums using a "Does it make sense?" strategy, they have no choice but to "do school" by rote, and to try to be particularly careful in the recording of the commands. It does not pay to take risks if one cannot easily redo.

Jon H. was seen to record carefully, but was able to trouble-shoot with relative ease; he could often locate where in the sequence of commands the wrong move might

have occurred. Russ H. was seen to test and redo without qualms; at one point (Dec. 11, 1985, p. 13), the computer was inadvertently reset and Russ H. lost the work done in the previous (approximately) ten minutes of time. Russ had not saved, and he had not in this instance recorded his most recent commands. Others of the children, even Jon H. and Eric H., would have been deeply perturbed. Russ H. was seen to be completely nonchalant, however; one imagines that it was because he knew he could recreate the product easily, which he proceeded to do without hesitation (Note 33). Had this happened to Lilly or Rina or Amy and Leanne L., they would have been frantic, as indeed they had been upon the number of occasions when they did lose their work (Note 34).

The essential difference between Russ H., Jon H., and Eric H. to a lesser degree on the one hand, and the children represented in this group who used only the visual-algorithmic mode (namely, Lilly M., Rina M., Carol L., Amy L. and Leanne L.) on the other hand, was that Russ, Jon and Eric had a feel for the size of the turn, and for the relationship between certain of the angular rotations. In the case of the majority of the children in the study, there was no systematic basis of comparison yet learned, against which to match the turns. One such gauge would be the use of 90 as a reference point, which some of the children were seen to use during the interview in relation to blackboard work with angles, but were not seen to use in their Logo work (Note 35, Appendix H, Q.5). The children in this group were still working toward learning about 90 during most of the Logo sessions (Zack, 1986b; Zack, 1988a). Unless they have an idea of how big the turns are, Lilly M., Rina M., Shauna L., Carol L., Amy L., Leanne L., and Doug L. have nothing else to rely on other than their carefully recorded, although not always correct, entries. This may to some extent explain why Rina M. for example chose to follow a "safe but sure" rule of thumb. This is possibly why the children could not use an "it does not make sense" criterion when looking at erroneous inputs (Lilly and Rina, inputs of 135 and 181, Selection #20).

There were occasions when Monica, the Logo teacher, emphasized that the children ought to combine inputs. Watt and Watt (1987) have noted that teachers feel that when children have the arithmetic skills at their disposal, such as addition, they should be expected to combine inputs when such combining can be done. It was seen in the instances presented previously (Selections #8A and #8B, Amy L. and Leanne L., sum of 116 instead of 166; Selection #40, Lilly M. and Rina M., the Star of David

project, sum of 160 instead of 260) that errors made in addition can create great difficulties when the children do not have an underlying understanding about the amount of turn which could subsequently help them "debug" the problem. It must be stressed that even when the resulting sums are correct, such combining is evidence only of a pure computation mode at work, when the underlying understanding about rotation is not present. One saw by virtue of some of the disagreements in which the children engaged that the combining was purely rote calculation (for example, Lilly and Rina, Selection #20, 135 versus 181).

The result of a combined sum may look neater than does a string of many entries, but it does not represent understanding unless it is tied to the larger Logo geometry framework. The resulting sum may in some cases even give the impression that the children "know" about 90, or 135, or 60, when the sums happen to be such. In one episode, for example (October 23, 1985), Lilly M. and Rina M. had combined LT 40 and LT 50, and had recorded LT 90. It was thought that perhaps as a result of the combining, Lilly and Rina had realized the significance of the special number, 90. Subsequently, however, when Lilly suggested that they should type LT 90 (when they were re-entering their commands October 30, 1985), Rina M., who was adhering to a "slow but sure" approach, or may have been simply being obstinate, refused to comply.

Selection #43
Oct. 30, D/A #1, p. 14

Lilly M.:	-L -T 40. No!! Erase -L -T 40 because we should have went -L -T 90 (checking her book).
Rina H.:	No, we're not, I'm sorry.

It may appear at times that when the sums which are important signpost numbers such as 45, 90, 135, 180, or 270 appear in the children's notebooks or in their project printouts, that the children "know" about these turns. One has to check carefully to determine whether the children do indeed appreciate that a certain input to RT and LT represents a corresponding amount of turn. When a combining strategy is used, it

may result in a look which is tidier than that rendered by a string of discrete command entries. The sums may meet the teacher's computational demands. When certain key entries are made, such as 90, 135, 180, 45, for inputs to turns, one may surmise that the children understand the significance of the input. One must however be very wary of any unverified assumptions of the children's understanding of the turns themselves, if those assumptions are based only upon the look of the records or printout of the projects. The interview questions posed in February-March, 1986 revealed diverse and prevalent gaps in the children's understanding of the size of the turn, and of its relationship to the mathematics classroom textbook Euclidean static measure of degrees (Zack, 1986b; Note 36).

Beginning to attend to the size of the rotational turn of 90

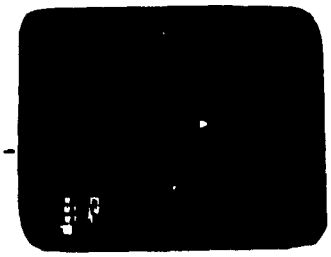
It was stated at the start of the section that for the most part Lilly M. and Rina M. did not have a feel for the size of the turn. Also, they were far from the point where they could use means such as analytic means to approach their problems. There were indications, however that Lilly and Rina were making more, though slight, progress in moving toward a developing understanding of rotational turn than were Carol L., Amy L., Leanne L. and Doug (L). The pace of the learning on Lilly and Rina's part was slow. The rate of the learning between Lilly and Rina was not the same-- nor indeed should it be expected that the pace of the learning of any two people would be parallel. But the slight glimmers of understanding detected in the work and talk and disagreements between Lilly M. and Rina M., outlined below, distinguished them from the less adept children who did not exhibit a perception of some of the same connections.

A discussion (the second selection below, Selection #45) reveals the first instance of attention to the amount of rotational turn represented by an input number. The transcript excerpts cited below represent first the disagreement (Selection #44), and then a post-disagreement discussion concerning the calculation which rendered a sum of 85 (Selection #45).

Selection #44

Lilly M. and Rina M., Nov. 20, 1985, D/A # 1, p. 6

Rina M.:	So hold on, add this up (adding). Seventy, ten and fifty [the fifty is incorrect; it should be five].
Lilly M.:	(Writing in her book) Equals-
Rina M.:	What's -R-?
Lilly M.:	Eighty-five. (Pause) Eighty-five.
Rina M.:	It's im-po-SSIB:LE.
Lilly M.:	Seventy and ten is •eighty- (pointing to book)
Rina M.:	•Oh ya. -R -T, O. K., oh ya (erasing). I went, uh, right fifty, -R -T five.

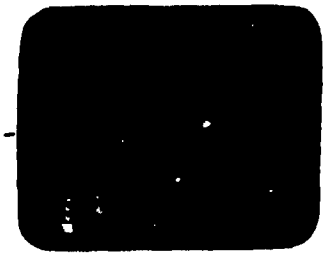


Note that Rina said that she had written "right 50"; once persuaded, she erased it and wrote RT 5. There is quite a difference between 50 and 5 degrees of turn!

Selection #45

The post disagreement discussion, Nov. 20, 1985, p.6.

Rina M.:	So, it's eighty.
Lilly M.:	Look at that, look at that! It's straight! O. K., now •we need-
Rina M.:	•It does -R -T eighty to get all the way around? (points to screen)
Lilly M.:	(silent, looks at Rina)
Rina M.:	You sure? (questioning, looks down at her book) (pause) You have (pencil pointing at her book, looking at the sum) •seventy-
Lilly M.:	•O. K. -F -D



Rina M.: Eighty, (shakes head "no"), it's eighty-five (without looking, moves Lilly's elbow to stop her from keying in) Seventy (pointing in her book), eighty-five.

Lilly M.: Right, it's eighty five. (Nodding) That's right.
[Moves to the next step] (Keys) -F -D forty eight.



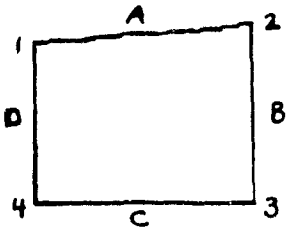
Note how the children stopped and discussed the combined sums, that is, the sum of 80 which was incorrect, and the sum of 85, which was correct in terms of calculation, but did not yet meet the required 90. Most importantly, note how Rina tried to connect the input number of 80, or 85, with the amount of rotational turn, when she said: "It does -R -T eighty to get all the way around?" The selection represents the first deliberation on the part of these two children of the amount that would be needed to fill a quadrant. The input of 85 is not adequate, although at this point visually it "looked" exactly right. One heard Lilly exclaim: "Look at that, look at that. It's straight." The partners soon discovered that it was not "straight," in that the next move (FD 48) resulted in a jagged line (Zack, 1988a). However, the post disagreement discussion represented the first explicit connection that the two made between the quadrant (which should be filled using 90) and the calculation sum of 85.

The aforementioned was one of the many experiential episodes dealing with 90 which occurred during the Logo sessions (see Selections #29A, #29B, #29C; Zack, 1988a). The fact that there was a post-disagreement discussion at all was important in itself. It was noted by the researcher that only the children at the moderate and the highly adept levels engaged in any post-disagreement discussions (see Table 21). In most cases, the comments were short; at times the post disagreement comments were sarcastic comments which stressed that a wrong move had been made on the partner's part; at other times, the post-disagreement comments underlined the learning (see Selection #6: Jon H. and Eric H. concerning 90). What was vital in terms of the post disagreement discussions was that however small the pause in most cases, there was at least at times some pause for deliberation upon the issues entailed in the disagreement.

The fact that the less adept children did not engage in post disagreement discussion may mean that as a result they lack a needed point of emphasis and reflection.

As Lilly and Rina attempted to complete their rectangle, they were still exploring as they did the turns at the other corners of the rectangle mentioned above.





They turned 95 correctly at corner 2, and that was pure coincidence (see Appendix Y); they then used RT 90 for corner 3, and RT 90 for corner 4. Although it might have seemed that their use of 90 for the latter two corners represented a breakthrough, upon analysis it was found that both Lilly and Rina were subsequently still not secure with the notion of 90. The beginning connection between the input number and the corresponding amount of turn was just a beginning. On November 20, and on December 4, 1985, Lilly and Rina were heard to refer explicitly to the size of the turn and the corresponding input number to RT or LT in relation to 90 (see Selections #29A, #29B and #29C; there were however no further explicit references either in relation to 90 or to other inputs. Observations concerning their subsequent exploration of right angle throughout the sessions, and their responses to interview questions, have been presented in the paper entitled "Say it's perfect, then pray it's perfect: The early stages of learning about Logo angle" (Zack, 1988a in Appendix W). Rina progressed more slowly than did Lilly toward an understanding of the concept of right angle.

Although all the children in the group (Lilly M., Rina M., Shauna L., Carol L., Amy L., Leanne L., and Doug L.) were seen to rely upon visual and calculational methods throughout the Logo sessions, Lilly M. and Rina M. were observed to pay some explicit attention, however small, to the relationship between the the input number and the rotational turn. They did in the latter sessions use 90. Carol L. and Doug L. used 90, but seemed to know it by rote. Amy L. and Leanne L. were not seen to use 90 for corners (except during the Jan. 29, 1986 across class task session when they may have been influenced by neighboring peers). Lilly M. and Rina M. responded

more effectively to some of the questions concerning angle during both sets of interviews February-March, 1986; May-June, 1987) than did Carol L., Amy L., Leanne L., and Doug (L.). Perhaps Lilly and Rina's reflection upon the size of the turn, and their discussion concerning the inputs, and their continuing disagreements about angle and the resolution of those disagreements, did play some part in the progress they made in learning about Logo angle, despite the fact that the pace of the learning might seem slow in relation to what other studies have found (Zack, 1988a in Appendix W).

Reference to recorded notes: A strategy for solving problems

Lilly M. and Rina M.'s problem solving strategies were entirely dependent upon their book records. When Rina M. and Lilly M. encountered an error in redoing their project, the chief method by means of which they approached the debugging was to compare the entries each had written in her book; this was first evident during session #6 (November 20, 1985) of the Grade 5 Logo work. Lilly M. was aware that there were at times discrepancies in the number of entries tallied due to the fact that she (Lilly) had combined certain inputs and Rina had not, and Lilly would at times endeavor to show Rina that although their entries did not comply with a one-to-one correspondence, they were nevertheless equivalent. There were occasions upon which Rina M. would refuse to accept Lilly's point, and would insist that the discrepancy in the number of entries listed was due solely to the fact that Lilly M. was sloppy in her recording; the fact that there were indeed occasions when Lilly was sloppy somewhat diminished Lilly's ability to strongly refute that statement. Rina M. began to feel so wary as a result of mishaps whose cause neither Rina nor Lilly could determine due to their lack of knowledge of some Logo essentials, that Rina grew to adopt a "slow but sure philosophy." The first selections below represent instances in which Lilly and Rina argued about reconciling differences, and argued about one-to-one correspondence of entries (Selections #46-49). A subsequent section reflects Rina M.'s rationale for using a "slow but sure" strategy for recording and executing moves (Selections #50 and #51).

Reconciling entries in the record notebooks

The selections below were chosen with a view to illustrating that Rina endeavored to check the entries by looking for a one-to-one correspondence between the entries in the two books, whereas Lilly M. attempted to reconcile the entries by looking for

equivalence as well as for one-to-one correspondence.

In the excerpt which follows directly below (Selection #46), Rina thought that Lilly had copied incorrectly from the screen, but Lilly had combined LT 10 and LT 5, and had recorded LT 15.

Selection #46
November 6, 1985, D/A # 4, p. 14.

Lilly M.:	(Reads) -L -T fif-te:en.
Rina M.:	(Keys) I'm telling you [i. e., it's LT 10].
Lilly M.:	(? Oh, ya, ya, ya?)
Rina M.:	Y- I told you (keys). You didn't copy it right, I did every, no- (checks her book)
.	
.	
.	
Rina M.:	(Keying) -L -T, lemme just do it my way.
Lilly M.:	(Looking at the screen) Oh, well, I wrote -L -T fifteen, O. K.?

In the following selection, Rina asked Lilly to wait while she (Rina) checked the entries one by one.

Selection #47
November 6, 1985, p. 15.

Rina M.:	(Keys) Lemme, O. K., I'm gonna check with you every step (looking at her book and keying).

In the following excerpt, Lilly is recommending to Rina that she should combine some of the entries.

Selection #48
November 20, 1985, D/A on plan, p. 13

Lilly M.:	I added them up, I added a 'lot of them up
Rina M.:	(Holds her notebook up) 'O. K., this is one, (pause), two, three, four, five.
Lilly M.:	I, but I added a lot of them up. Add it-, add all of them you can up
Rina M.:	(Ticks something in her book)
Lilly M.:	and you'll get- Look (points to her book).

In the next selection, Lilly M. tried to show Rina that the reason for the discrepancy between the number of entries they have is that she (Lilly) had combined, and had not recorded entries which were erroneous moves.

Selection #49
December 11, 1985, D/A on Plan (reconciling entries), pp. 17-18.

Rina M.:	. . . You've got everything 'wrong.
Lilly M.:	'Wh-at? No, I (takes her book back) (pause), no, I don't have it ''wrong.
Rina M.:	''Mine is right, I'm telling- (ready to key)
Lilly M.:	Oh yes, because, because (? inaudible), because I added that up, and I didn't write those two down. Look. So (pointing to Rina's book)

(pause) one two.

Rina M.: It doesn't MA:TTER (pushes Lilly's hand away). Why are we counting (?something). It's so stoo-pid.

It is of interest to note that Rina used the argument that it was nonsense to count the entries, when in all other instances she not only insisted upon the count, but also often refused to listen to Lilly's argument that at times the discrepancies they encountered were due to the fact that equivalences had been used.

Strategies used by the low level and the high level children in attempting to solve problems

The children at the low level and the children at the high level of achievement offered contrast to the two at the middle level, Lilly M. and Rina M., for none other than Lilly and Rina used the approach of reconciling entries. The low level achievers used one strategy only, that of re-entering their commands as listed in their notebooks. On one occasion, Amy L. and Leanne L. attempted to work through their problem after a calculation error and failure to record an entry caused them great difficulty (see Selection #8A and 8B, and Note 23). Their attempts at solving the problem during that episode were reviewed very carefully by the researcher; although each accused the other of being sloppy, at no time in their 8 attempts to draw their shape did they try to check their entries side by side, or attempt any other strategy. Amy L. and Leanne L. took turns entering the commands, alternating between sets of records; each relied upon her entries totally. They had only the one approach, that of re-entering the same commands, clearing the screen between times, then trying the same commands again, almost as if the result they wanted would suddenly re-appear. And as has been mentioned, as far as the other children at the least adept level were concerned, there were no efforts made by Carol L., Shauna L., or Doug L. to check their entries or compare entries with their respective partners as Lilly M. and Rina M. were seen to do, or to use any other strategy. It was due to the low and middle achieving children's recourse to their notebook records that the dependence on the rote aspect manifested itself. Had there not been a class routine which required the use of notebooks, the students would have been just as unaware of what was happening, but the fact that

this was indeed the case would not have been as apparent.

Neither Jon H. and Eric H. as a dyad, nor Russ H., Jon H. and Eric H. as a triad, were seen to reconcile entries. When Jon H. and Eric H. worked together, each recorded in his respective notebook. However, none of Jon and Eric's attempts at recreating their product seemed to give them difficulty of the kind the aforementioned subjects experienced, namely the inability to achieve the shape first created, when the subsequent attempt was made: hence there were no comparable data for analysis of problem solving in Jon and Eric's sessions as a dyad. When Russ H., Jon H., and Eric H. worked as a triad, they used a different notetaking system. One of the partners served as a scribe; they took turns filling the role of secretary, although whether the roles were shared equally was not investigated (Note 37). Only the high level achievers in this study were seen to use one set of notebook instructions. Further investigation would be needed to determine whether the tendency to use one set of notes was indeed representative of the high achiever, and if yes, what it might signify.

In the case of the Russ H., Jon H., and Eric H. threesome, the one set of notes served as reference when there was difficulty, as for example in session #10, January 8, 1986, when there was an error in the "goalie" procedure. But during this problem solving episode, Russ H. did not rely solely upon the notes; Russ referred to them, but tried alternative means as well. Perhaps he realized that the notes were not the only place in which to look for the problem. Indeed, in this instance, the cause of the error was a missing entry; in the second step of the "goalie" procedure, the turtle cursor turned RT 90 and then drew the round head; without elaborating upon why it was so, suffice it to say that the missing move of RT 90 was very difficult to distinguish. The RT 90 had not been recorded. As main trouble shooter for the threesome, with Jon an active participant in the attempt, Russ used diverse tactics in order to find the error. Russ broke the program listing into smaller parts, reentered parts in immediate mode, observed the step by step movements on the screen, tried to gauge where in the listing in the editor version the error might have occurred, and verified several seemingly pertinent parts of the procedure. In order to retest parts in immediate mode, Russ asked his partners to read the notebook commands aloud to him, as when for example he said to Jon: "O. K. Read me til -P -U home." At one point, Eric and Jon suggested to Russ that the use of SETH had caused the problem (see Selection #17). Even though Russ stated that he himself did not think so-- "I think it's something different"

(January 8, 1986, p. 5)--, Russ nevertheless verified by changing the input, from SETH 180 to SETH 80 as they suggested; he thereby proved that the error did not lie therein. After fifteen minutes of work at trying to solve the problem, Russ suddenly seemed to recall the RT 90 and checked for it: "Wait! Where's the -R -T 90? (points to the screen editor listing, then looks at the notebook)." It was not clear what sparked the realization, and Russ gave no support to his statement which would enable the researcher, or his partners, to appreciate how he found the error.

Thus the findings showed that the children at the different achievement levels used different strategies. Amy L. and Leanne L. relied upon their entries totally, as did Lilly M. and Rina M. as well. All of the children at the low and middle levels used a method which relied upon precedent, upon stored rote information; they did not employ reasoning. Lilly and Rina did go one small step further and attempted to check that their entries matched, that they had the same amount of entries (Rina M.), or that there was equivalence when the one-to-one correspondence count did not balance (Lilly M.). They were at least discussing the entries, if only at the literal, surface level, as the children at the low level of achievement did not do. However, Lilly and Rina's method was minimally effective, and was not effective at all when reasoning about which inputs might have produced certain effects was needed. What Lilly and Rina could not yet do was to visualize in the abstract in order to picture in their minds what moves the inputs would render upon the screen.

The researcher did not see enough of Jon H. and Eric H. (as a dyad) acting upon a problem to state what their approaches were; it did appear that in his work Jon was able to visualize some of the moves before they were actually carried out on the screen. He, Eric and Russ were "aware of how one produced [certain effects]" (Kamii and DeVries, 1978; see Table 1, Part A, Part B). (Even in their recording methods, Jon's, and Eric's, and of course Russ's, approach reflected a good grasp of the significance of many of the moves; Jon, Eric and Russ did not need to record step-by-step as did the children at the low and middle levels who wrote each and every command, and combined the inputs only after the entries were written on the paper. At times Jon worked rapidly through a number of steps on the computer, giving Eric only the final step as entry for the notebook). In contrast to the low and middle level achievers, as was noted above, Russ was seen to use diverse approaches to solving a problem.

Finally, a note reiterating what prompted the children to try to "debug." The children did not attempt to solve problems (or did not persevere long in their attempts) when their graphic product on the screen did not correspond to their intended goal; they would "deplan" by modifying their goal rather than attempt to debug their product (see Note 32). There were however occasions when the children keyed in what they thought were a listing of commands that would issue again the product they had just made, but the re-entered listing rendered a different graphic product. It was in this instance that upon a number of occasions the children did try to solve the problem.

Rina's "safe but sure" philosophy

Rina began to voice her adherence to a "slow and safe but sure" way of recording her entries in December 1985. This may have been due in part to her personal working style and school habits (see Rina's child profile, METHODOLOGY section). It may also have been due to the fact that errors which could not be corrected had begun to make her more cautious. As has been expounded at length at the beginning of this section, the learning about angular rotation and about other facets of the Logo graphics environment (such as syntax and programming features) was still in its beginning stages. There were so many surprise "bugs" that the way to be surer of the end result was to record all "as is," since any changes had potential for surprise. It is not a wonder that Rina should have grown so suspicious of even such measures as combining, or not recording errors As we have seen (Selection #8A and 8B, Amy and Leanne concerning the inverse 116-166), unless the children are completely sure of what is happening, the omission of an entry, either done deliberately but done when it should not have been, or omitted inadvertently, can wreak havoc with their product. When they did not have the tools whereby to correct, and to follow the commands and envision where the missing move might be (and for that they need to be able to work in the abstract), they could not solve the problem. It has been noted that when the children were not able to follow what had transpired, they were likely to record all the commands (Carol L., Selection #11).

Rina's suggestion cited below, to use a smaller input number in order to be on the safe side, was heard to be suggested by three other children in the study (Amy L., Jon H. and Eric H.), either during the Logo sessions, or during the interviews. Especially in the early sessions, they were concerned with not "going over," perhaps

because they would then subsequently need to delete. They felt that it was much more prudent and efficacious to add on than to have to subtract and/or delete.

Selection #50
October 23, 1985, D/A #23, p. 5-6

Rina M.: You know why (shakes head no). You don't understand. To be on the safe side we have to do a lower number than eighty seven.

In the following excerpts, Rina stated her intention to record all the work, including the mistakes. She was heard to insist that her record was correct because she wrote everything down.

Selections #51 (#51A - #51G)
(A)
Rina M., Dec. 11, 1985, p. 13

Rina M.: Write down all our mistakes.
Lilly M.: (no response)
Rina M.: I'm writing all our mistakes.

(B)
Rina and Lilly, December 11, 1985, D/A on Plan (recording in notebook), p. 12

Rina M.: I put the -P -E here . . .
Lilly M.: Why? It's not necessary! (frowning)
Rina M.: (Pounding the table, yelling) YES! I'm telling you it's gonna work. (normal voice) It's gonna work. You can write down things as long as it turns out good at the end, which it

will.

(C)

Rina M., December 11, 1985, p. 14

Rina M.: O. K. (takes her book) I'm gonna read you over the, the things. And mine is right. I'm telling you (facing Lilly) it's right.

(D)

Rina M., December 11, 1985, p. 18.

Rina M.: Mine is right. I write down everything.

(E)

Rina and Lilly, November 20, 1985, D/A on Plan (recording in notebook), p. 15

Rina M.: See, you're getting confused. You don't, you don't write all of these down! (pause)

Lilly M.: Because it's stoo-pid to write -L -T ninety when it's a mistake, (pause) right?

Rina M.: No, but it still, it still corrects itself.

(F)

Rina M., December 18, 1985, p. 8.

Rina M.: It doesn t matter. I'll write down the mistake. As long as it turns out.

(G)

Rina M., January 8, 1986, p. 4.

> Rina M.: I don't, I don't mind writing my mistakes. At least I know they will turn out somehow.

The support which Rina M. was seen to give for her actions in using small increments, and in recording all her steps, served to give insight into the work habits of many of the children in the study who were at the beginning stages, but who were not as talkative about their methods and motives as Rina was. Rina's caution helps us to understand why some of the children are resistant to combining, or do not see the virtue of combining. One cannot blame these children for not being risk takers, when one sees what a slip such as a non-entry, or a miscalculation, can cost the children in time and frustration. There is a need for further investigation to see whether the recipe-like measures begin to disappear as the children progress in their understanding of the workings of Logo.

The visual-algorithmic mode: One child's reference to proportional relationships

Among the five low level achievers in the study, Shauna L. was the one child who dealt with certain of the inputs in a proportional way. As has been shown elsewhere (Selections #1 and #35), due to a calculation error Shauna interpreted the half and full rotation of a 360 degree turn in nonstandard units of 110 and 220, instead of in the standard measure of 180 and 360. The amounts represented her own unique construct. Despite the nonstandard "unit" of 110, Shauna was nevertheless seeing the turn in terms of the relationship of one part to the whole. At the middle level, Lilly M., and in one instance Rina M., spoke of "half," in relation to the "whole," (see Selections #29A, #29B, #29C), but did not pursue what the subsequent turn totals would measure; had they done so, in their case, in line with the input they used for the first turn (85), they might for example have estimated that 85 + 85 would equal 170, and so on. Hillel (1984) noted that in relation to angular rotation only one pair among the five pairs of the 8 to 9 year-olds in his study used proportional measure, and in that case as in this study, the use was in terms of double or half a certain input (1984, p. 44). Thus the finding in this study supports Hillel's (1984) finding that use of proportional measure is in evidence, but its use by children in the early stages of Logo learning is not yet prevalent.

The fact that Shauna did note the proportional relationships was of interest in that she alone of the low level achieving children did so. One especially striking response which relied heavily upon proportional relationships was given by Shauna in Grade 6 (May 1987), in answer to one of the post-session across-class paper and pencil questions. The researcher-posed question aimed to elicit whether the children saw that the input to RT (or LT) in Logo, for example RT 46, was equal to the measure in degrees of the equivalent pencil and protractor-drawn angle in static geometry, angle ABC = 46° (which of course it is). Shauna had answered the questions correctly when the question was posed in Grade 5 (see Appendix F: the pencil and paper test). When the same question was asked in Grade 6 (with changes made to the specific input amounts), Shauna's responses suggested that she perceived that the relationship between the input to RT on the one hand, and the number of degrees in the same paper drawn angle on the other hand, was a proportional one. That is, she wrote that the input to RT might be half, or double, or equal to, the number of degrees in the paper drawn angle; her responses are duplicated in the right hand column of the

excerpt below:

Blackboard static angle	Logo
46°	RT 92 or 23 or 46
68°	RT 136 or 34 or 68

No other of the written responses by the other 60 Grade 6 children who completed the sheet, or the 23 Grade 5 children including Shauna (February, 1986), was similar to Shauna's Grade 6 answer cited above. Perhaps Shauna actively looked at certain elements in terms of proportional relationships.

Of all the children who were classified at the low level of achievement, Shauna was in many ways especially interesting. The analysis done for the current study was able to shed light upon Shauna's achievement in ways that added to the information that had been gained by means of other projects previously done. When judged on a correct/incorrect scale, as she was in the oral interview in Grade 5 (February-March, 1986), Shauna's achievement level was judged to be low (Zack, 1986b). However, as will be noted upon consultation of the grid resulting from the project on the children's notion of angle (Zack, 1986b; Table 4), she was considered to be at the top of the group of low achieving children. This study gave insight into various aspects of Shauna's strengths. It has already been noted that she, along with Russ H., used a high percentage of Level II strategies during the sessions in which their work was taped, that is, 4 and 6 sessions respectively (see Table 23). Shauna L. and Russ H. also used a high percentage of mathematical justifications when compared to the rest of the children (see Tables 32 and 33). In this study Shauna was found to be correct more often than were her partners during her disagreements with her partners Amy L. (three sessions), and Carol L. (one session). The comparisons with others at the low level in terms of correct and incorrect statements showed the following:

Correct statements					Incorrect statements				
Shauna	Carol	Amy	Leanne	Doug	Shauna	Carol	Amy	Leanne	Doug
5	4	3	2	2	2	9	7	6	4

It was almost 2:1 in favor of correct statements for Shauna L., as opposed to approximately 1:2 on the part of Carol L., Amy L., Leanne L., and Doug (L.). It must be stressed again, as always, however, that the total number of correct and incorrect statements, and the number of disagreements as a whole, was very small, and so no conclusions can be drawn. It would, however, have been an interesting feature to pursue, had one been able to do so.

Shauna seemed to have more of a feel for geometry than was able to be revealed by certain of the interview questions used. Leron (1985) has cautioned that there are many aspects of the children's understanding that cannot be measured by conventional means; his statement of concern has been cited in the "Methodology" chapter of this paper, but is worthy of reiteration at this point:

> [The] evaluation tools used are, for the most part, inappropriate for the processes which are important in Logo learning; thus they may fail to demonstrate viable learning even when such learning has occurred. For example, most students we have worked with seem to have gained some sort of vague, partial understanding of many powerful ideas. This kind of learning is very valuable, indeed a necessary intermediate step toward a more complete understanding, but it is unlikely to be detected by standard evaluations, where lack of complete understanding is often rated as complete lack of understanding.
> (Leron, 1985, p. 32)

Leron's statement is of course pertinent to all the children in the study, but seemed especially applicable in Shauna's case. For example, Shauna was not able to respond correctly to interview questions which probed whether the child had a grasp of certain of the aspects of static geometry, or even of standard dynamic Logo geometry. But Shauna was able to re-discover the HELP routine, a noteworthy accomplishment (see Selection #11 and Appendix L). She alone of the low and middle level group used symmetry as one of her criteria for the evaluation of a product (although she did not

have enough control over the workings of Logo-- both the geometry and the syntax aspects-- to be able to know how to "fix" the asymmetrical product). Thus, along with her feeling for proportional measure, Shauna's work showed other evidence of a sensitivity for the geometric environment of Logo. The analysis done for this study, though very time consuming, resulted in a deeper understanding of Shauna's strengths at this point in her learning.

The VISUAL-RELATIONAL mode

The high level achievers worked in a relational way, that is, they understood what they were doing concerning their moves vis-à-vis angular rotation. At times, they did not voice their strategies in their supporting arguments. They often did the work rather than talked aloud at length about what they were doing. The subsequent analysis of their actions, as well as their talk, served to reveal that they knew why they were doing what they did. In accordance with Skemp's (1979) aforementioned stipulation concerning mathematical thinking at a level above the rote level, then, they "knew what to do and why" (1979, p. 250). In the following section, we will first deal with the visual-relational aspect as it appeared in the work of the high achievers, and then we will deal with evidence of the analytic mode (Hillel & Kieran, in press) which emerged only during the interview sessions in Grade 5 (February-March) and Grade 6 (May-June, 1987). The connections which some of the high achieving children made, and others (middle and low achieving children) did not make, within the Logo geometry environment, and between Logo geometry and classroom Euclidean geometry, will be noted. The importance that the findings concerning such connections hold both for the Logo and the mathematics school curriculum, and for future research, will be discussed in the next chapter of this paper.

Use of the visual-relational mode, and of the analytic mode

It has been pointed out that Russ H., and Jon H. and Eric H. as well, were seen to rely on the visual mode while working with Logo (Selection #37, Russ and the goalie stick), even though during the Grade 6 interviews (May-June, 1987) Russ and Jon, and Eric with prompting, were able to work with angles in an analytic way (see Appendix H, Q. 9). We will return soon to a consideration of why the analytic mode may not have been employed within the Logo experience observed in the study.

There has been ample mention made in various selections, and in the discussion within the findings sections themselves, concerning ways in which the high level achievers, Russ in particular, showed that they understood what they were doing. They, and again Russ especially, used mathematical justifications which revealed an appreciation of the workings of geometry. At times Jon acted but did not explain; at times Russ, Jon and Eric gave explanations both outside of the disagreements and within the disagreements, which showed their understanding. The high achievers were seen to use problem solving and recording strategies which were more advanced than those used by the middle and the low achievers, and which reflected their understanding of what they were doing, in particular Russ.

In the section on the use of the visual-algorithmic mode, it was shown that dependence upon algorithm, namely the use of a calculation strategy (using addition), and reliance upon the book notes, revealed a lack of understanding of the amount of rotational turn, and reflected a limit to diverse reasoning and logical means of problem solving. The high achievers were judged to be using a visual-relational mode because they did exhibit an understanding of amount of rotational turn; as a result of that (and other) understanding they were able to apply diverse strategies in attempting to solve their problems.

In this study, the use of the analytic mode, which featured the use of logic in determining supplementary and complementary relationships between angles, was only in evidence during the interviews, with the exception of Russ H.'s implicit use of it during the Logo sessions. When a child is working in an analytic mode, an ability to work at the relational level is implied; but one can work at the relational level and not yet have attained the analytic level. Kieran, Hillel and Erlwanger (1986), and Hillel and Kieran (in press), defined the "analytic schema" as follows:

> analytical: inputs are chosen on the basis of mathematical relationships inherent in the given data and/or from mathematical knowledge; rationale for inputs is often expressed by "Since this (Vee) is 60 degrees, then . . ."
>
> Kieran, Hillel and Erlwanger (1986)

> By an 'analytic' schema we refer to solutions based on an attempt to look for exact mathematical . . . relations within the geometry of the figure.
>
> Hillel and Kieran (in press)

The analytic mode was seen to be used by the high level achievers (Eric H. with prompting) during the Grade 6 interview. Interestingly, Lilly M. was also seen to use the analytic schema, with prompting, in the Grade 5 interview, and with slight prompting during the Grade 6 interview. This represents an advance for Lilly M.; the same growth in understanding was not seen in Rina M.'s case. The reason that the analytic mode may not have appeared explicitly in the children's supported talk during the Logo sessions may be that the type of project work which the children were doing did not require logic, since they were drawing; they were not constructing using geometric relationships inherent in the figures. They could indeed "fiddle around" as Rina so aptly described her approach (Grade 6 interview; see Appendix Z); they could "deplan." They were not obliged to fulfill tasks which had exact specifications, or which would meet pre-determined goals, as was the case in the Hillel and Kieran (in press) study (Note 38). There was evidence of implicit use of analytic relationship between supplementary angles, and an awareness of opposite angles  , in Russ's work; but during the Logo sessions he never voiced aloud his use of the analytic schema in his work with angular rotations. The van Hieles have observed (in Fuys, et al., 1984) that a child at first works at a level at which the understanding is implicit and can not yet be rendered explicit. Perhaps Russ was not inclined or able to be explicit about it with his partners; or perhaps as will be suggested below, he assumed that his partners were as aware as he was, that is, he projected that his knowledge was also their knowledge. In any case, by the time of the interviews in Grade 6, Russ was able to express his use of the analytic mode to the researcher-interviewer (for example, Selection #38).

Making connections

One must be conscious that when looking at Logo from an adult point of view one can make unfounded assumptions about what the children do indeed "see" and what they "know." The following aspects concerning connections made and not made were seen

to emerge as a result of the elicited explanations (which served as "supported talk") and the discussion which took place between the researcher and the children during the interview sessions, especially the Grade 6 session in May-June, 1987. The connections are important because they deal in yet another way with relational understanding, in other words, with understanding about relationships. Four aspects will be considered: (1) the connection between the input to RT or LT and the corresponding size of the turn; (2) the connection between the input to LT or RT, and the number of degrees it represented given the equivalent angle in school classroom geometry; (3) the general connections that the children felt existed between Logo geometry and classroom geometry; and (4) the fact that those who made the connections mentioned above felt that the connections were apparent, and could not imagine that their classmates did not see them.

First we will consider the question of input to RT or LT, and the corresponding amount of the turn. In a previous section entitled "Beginning to attend to the size of the rotational turn of 90," it was noted that during the Grade 5 Logo sessions, Lilly M. and Rina M. were only becoming aware of the amount of turn represented by an input of 80, 85 and finally 90, when they wished to arrive at a quarter turn. It may well be that when children are suggesting a turn of RT 2, RT 10 or RT 17, these commands may only be synonyms for the term "a little bit." One would need to investigate further to probe what meaning the input number holds for the children. The study has raised questions concerning the children's appreciation of how the input number relates to the amount of turn (see Note 36, on Stuart M.). Russ, and the high achievers, knew about the amount of rotational turn. Russ's response below answers the question at hand, and also anticipates the next point to be raised, namely the relationship between input to LT/RT and the notion of degrees. His answer was given in response to a question by the interviewer which was related to connections between the classroom geometry and Logo geometry:

Post-session interview with Russ H., Grade 6, May 29, 1987.

I: Is there any way that your classroom geometry can help you in working with Logo?

Russ H.: Ya.

I: How?

Russ H.: Well, if you're gonna know all about how (pause), how much, how far the turtle would move, if it turned 80 degrees or 90 degrees, and if you learnt how (?inaudible) 80 degrees or 90 degree angle, you have a better idea (pause) of how the turtle will move when you tell it to go to a certain amount.

It may seem to casual observers that the children make the connection between the amount of input to RT/LT in Logo on the one hand, and degrees in the static Euclidean angle learned in classroom geometry (in the Grade 5 mathematics curriculum) on the other. As a result of the interviews in both Grade 5 and Grade 6, it was clear that it was not as "easy" a connection to make as it might seem to be. In Grade 5, only 8/23, or 35%, of the students responded in a way that reflected understanding of the relationship between the two. A higher number saw the relationship when 56 students were interviewed in Grade 6, that is, 25/56, or 45% (see Appendix H, Q.9). And yet the child who was sure of the connection made it sound so straightforward and simple. The following was Jon H.'s answer given during the Grade 5 interview (Feb-March, 1986):

Post-session interview with Jon H., Grade 5, February 20, 1986.

I: How do you explain to someone what "degrees" mean?

Jon H.: It's like the amount that it moved, or how big it is.

Russ's answer below was not one given in response to the class question which dealt with the input to RT/LT and its relationship to degrees, but rather in response to another question concerning Papert's aim of having children learn about angles as a result of their activities within the Logo environment (see Appendix H, Q. #6):

Post-session interview with Russ H., Grade 6, May 29, 1987.

I: Now, the reason I'm asking you these questions is that Seymour Papert is the fellow who developed Logo and also worked on Logowriter, and what he felt is that children can learn a lot about angles through using Logo. (Pause.) D'you think that they can learn a lot about angles through using Logo?

Russ H.: Uh-huh (agreement, Russ nods).

I: How?

Russ H.: How? Because every turn, every turn that that the turtle would make he's turning in degrees, and an angle has, it's made up of degrees.

I: And what if I told you that there are many youngsters who after years of doing Logo have no idea that when turtle turns it has anything to do with degrees?

Russ H.: They have no idea of degrees or (?inaudible)

[a few seconds of tape was inaudible at this point and therefore precise transcription was not possible, but the essence of what Russ said was that children in Grade 1 and 2 have no idea of degrees because they don't learn about angles in their class work.]

I: O. K., you're talking young ones. I'm talking, if they don't understand, I'm talking youngsters in Grade 4, 5, and 6.

Russ H.: They've probably already learnt about angles.

I: Uh-huh (agreement). (Pause) And?

Russ H.: And so then they would understand how Logo, how Logo is used with angles.

I: Oh, so you think that they would understand 'cause they learnt it in the classroom?

Russ H.: Ya.

The above excerpt was cited in full because it not only reflected the connections between input to RT and LT, and degrees in static Euclidean classroom geometry (and in Logo geometry of course as well, since the turtle turns are measured in degrees) that Russ understood, but also featured Russ's perception that once children had learned about angles in their regular classroom as part of the mathematics curriculum, they were bound to see the connection between Logo geometry and classroom geometry. As a result of the findings from the Grade 5 interviews which had indicated to the researcher that such was not the case (since only 8/23, or 35% did see the relationship), the reviewer tried to propose to Russ that the connection was not so apparent to all children. As was evident here in Russ's response, and in several other instances when high achievers responded to certain questions posed by the interviewer, those who saw the connections-- whether between Logo geometry and school Euclidean geometry, or between the input to RT/LT and degrees-- could not believe that the connections which were so obvious to them were not apparent to their classmates.

A number of questions which were posed during the Grade 6 interview (May-June, 1987) evoked some thought-provoking responses. A few questions were aimed specifically at eliciting whether the children felt that there was a relationship between Logo geometry and school (static Euclidean) geometry. The questions were parased as follows:

May-June, 1987, Grade 6 Interview

Question #6: Seymour Papert is the person who developed Logo. He felt that by using Logo, children could learn about angles. What do you think?
(If no, why?)
(If yes, how?)

Question #7: Could Logo help you with your classroom geometry? (If yes, how? If no, why not?)

Question #8: Could what you do with your classroom geometry help you with your Logo work?

Question #10: How are classroom geometry and Logo geometry the same? How are they different?

As the tables in Appendix H will indicate, a higher percentage of the Grade 6 students felt that there were connections between the two than did not feel that there were connections between Logo and classroom geometry[5]. And yet, several comments which emerged indicated that such connections were not clear cut, even among certain of the high achievers and middle achievers. The comments also reflected the children's perception of the Logo environment. Sean H., for example, in response to the Papert question (Question #6) responded: "That's what he developed it for? I thought he developed it for, enjoyment." Sean insisted that he had learned all about angles from his classroom teacher (his French teacher, since the geometry part of the Grade 4 and Grade 5 mathematics curriculum was taught in French). He stated: "I learned everything [about angles] from my French teacher but . . . nothing [in] Logo, just how to make a picture." When the researcher later probed and asked where and when Sean felt he had learned about 180 (during the analytic question when Sean was asked about what the supplement to a paper-drawn angle would be), Sean stated that without doubt he had learned about 180 in his Logo class, either with Monica or with Sharon as his teacher, he could not remember which; but he definitely remembered learning about 90 + 90 = 180 first in his Logo class (and then in his classroom). Sean seemed to locate the learning of geometry subject matter in the physical space of his school classroom, and to associate the subject matter with explicit, formal teaching methods. Sean stated that "classroom geometry is a lot more (pause), em, it goes into more detail than Logo." There was evidence of two extremes, and the range in between, in the responses of the children. Sean H. at one end of the spectrum felt that Logo was created purely for enjoyment, not for learning; Jay H. at the other extreme felt that through Logo one could learn everything there was to know about geometry. Both of these youngsters, Sean H. and Jay H., were (like Russ H.) exceptionally able in their Logo work (and in their school mathematics work, as was attested to by their classroom teachers), but were not part of the Grade 5 observation group.

A number of the children remarked that they never thought about angles when doing Logo (11/56, or 20%). Lilly M. for example, stated that she didn't think about

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[5]Question #6, Yes--42/56 = 75%; No--8/56 = 14%
Question #7, Yes--37/51 = 73%; No-- 10/51 = 27%
Question #8, Yes--36/55 = 65%; No--8/55 = 14%

angles while doing Logo, she just thought about doing turns. Lilly's response to the Papert question reflected her feeling that in order for learning about angles to occur, one must be made conscious of angles in the Logo environment:

Post-session interview with Lilly M., Grade 6, May 29, 1987.

I:	Papert is the fellow who developed Logo, so that children could learn about angles through using Logo. How do you feel ˙about that?
Lilly M.:	˙(shakes her head "no")
I:	Can you learn a lot about angles ¨(?inaudible)?
Lilly M:	¨No.
I:	No? Why?
Lilly M.:	'Cause we don't learn angles, we just learn like, left turn, right, kinda thing. I don't think, you learn (?inaudible)--, how to turn it [i. e. the computer] on and how to turn it off. How to do circles, that has nothing to do with angles.
I:	Okay. But when you're turning left and right, you feel that has nothing to do with angles?
Lilly M.:	Well it has something to do with angles, but you don't really think about it at all.
I:	Mm, okay, 'cause that's interesting, 'cause he [i. e. Papert] felt that children could learn a lot about it, but I want to hear from everybody their impression, whether they think that children could learn a lot about it. So you (?feel) that they really don't think about angles?
Lilly M.:	If they <u>think</u> about (?it), they could learn about it. We don't think about it, we just think about our turns.

Lilly's comments were echoed by a number of children (11/56 or 20%) who said that they did not think of angles when doing Logo. Logo was seen by some solely as a

drawing environment (13/54, or 24%). One of the questions in the interview was designed to probe the reasons for the complaint that had been issued by a number of the children when they began the year's session, the complaint that they "had to do more Logo." The children were asked to comment on the aforementioned complaint. Responses in answer to that question reflected the perception of a number of the children that Logo was "just drawing" (13/54, or 24% as mentioned above), that Logo was limited in what it could do (7/54, or 13%), and that Logo was not challenging enough. While some of the children felt that the amount of time spent on Logo was too much (34/54, or 63%), others thought it just right (13/54, or 24%); no one thought that they had spent too little time working with Logo (see Appendix H, Q. 12).

A final reflection upon Lilly's comments, and the deliberation they evoked, ought to be noted. One, it is possible that Lilly's perception about her learning may not totally represent what actually transpired. Lilly may herself not have appreciated how much she had learned (see Zack, 1988a in Appendix W). For while it was Lilly's expressed feeling, in the above-cited interview selection, that she did not "learn" about angles, and did not "think" about angles while doing Logo, it was evident from the analysis in Lilly's case that she had indeed progressed in her learning about dynamic angles in Logo geometry. Two, it is possible that Lilly's "model" for angle was the "static," classroom geometry textbook angle, and hence, that Lilly did not find <u>that</u> angle represented in the Logo microworld environment.

In summary, then, a number of vital issues were seen to arise as a result of the interview answers, and of course as a result of the findings concerning the children's Logo work throughout the Logo sessions. One issue relates to the connections made or not made within the Logo environment itself (relational aspects), and between Logo geometry and classroom geometry. The second issue pertains to the comments made by Lilly M. which concern bringing learning to the fore, making it more explicit. And the third relates to the discrepancy between, on the one hand, the researcher's finding that the pace of the learning about geometry had for the most part been slow and that there was so much that the children could yet learn, and on the other hand the general feeling that emerged from the children's answers, namely that the children felt they knew all there was to know about Logo. The interview findings seemed to echo those of Hawkins (1983) who noted that the children thought of the computers as "game devices" (1983, p. 9), and that the computer activity had an "indeterminate status" within

the school setting: "Is it legitimate, accountable work for which individuals are responsible, or is it supplementary, fun activity?" (1983, p. 9).

Now, one need certainly have no difficulty with the children's perception of Logo as an enjoyable environment, as a drawing activity or as a game. There is no incongruity between having fun and learning; considering Logo a "fun activity" does not obviate learning. Some of the children who were seen to learn some of Papert's "powerful ideas" were among the ones who considered the environment a fun one, "not like school at all." It is the "plateau" which represents the threat to further learning, for if the children do not perceive that there is more to learn, they may balk at the thought of continuing. The rationale for continuing must be made evident and meaningful to them. At some point in the Logo agenda, they need to see that there is a geometry framework underlying the Logo microworld environment, and that new discovery and learning will continue to be found within it.

It is also not necessary to make connections between the Logo geometry world and the classroom geometry world; the Logo geometry world can stand alone. However, if the relationships within the Logo microworld itself are not evident or are not emerging, what, if anything, should be done? The three aforementioned issues are closely bound to the more general question of whether Logo should play a role in the elementary school mathematics curriculum; and if yes, what role it should play. We propose that Logo can and should play a role in the curriculum. We will next consider how the information which emerged from the study can inform us as to the direction to take. The pedagogical implications arising from the study findings will be explored as part of the next chapter entitled "Concluding remarks."

Summary of the Main Findings

The study aimed to investigate the the role that disagreements over a Logo move, and the discussion associated with the disagreements, played in the children's learning of Logo geometry concepts. This summary will highlight the main findings, with reference to the research questions which guided the analysis of the data.

The pattern of the disagreements

The first component which was analyzed was that of the pattern of the disagreements.

> CAN THE PATTERN OF THE DISAGREEMENTS--their frequency, duration, resolution, and the category of mathematical idea--GIVE ANY INDICATION OF THE CHILD'S UNDERSTANDING OF LOGO ANGLE?

The objective was to detect whether there was a relationship between any of the above-mentioned aspects, and the children's achievement vis-à-vis their understanding of Logo angle and of other mathematics and geometry concepts. Did the children who were more adept at the Logo geometry work argue more or less frequently, did they sustain longer or conduct shorter disagreements, resolve more or fewer disagreements, on more difficult or less difficult mathematics concepts, than their peers who were not as adept? The findings indicated that certain of the aspects of the disagreements reflected the children's level of achievement, whereas other aspects did not. There seemed to be no relationship between the children's level of achievement, and (a) the frequency and (b) the duration of the disagreements; however, there seemed to be a relationship between the children's level of achievement, and (c) the mathematical idea inherent in the disagreement and (d) the resolution of the disagreement.

The specific mathematical idea underlying the disagreement was the aspect most closely related to the level of achievement of the child. The mathematics concept category of the disagreement was a reflection of the level and the pace of the child's learning. The more adept children were having disagreements over the more difficult concepts of which the less adept were not yet aware, or with which they did not attempt to deal.

In relation to the resolution and nonresolution of disagreements, it was expected that the complex disagreements (which included Level I and Level II strategies) were more likely to be resolved than the simple disagreements (which featured only Level I strategies). The finding in this study was that in the case of the low level achievers, Carol L. and Amy L. and their respective partners in Grade 5, the simple mathematics and geometry concept disagreements were as likely to be resolved as were the complex disagreements. In the case of the middle (Lilly M. and Rina M.) and the high level (Jon H., Eric H., and Russ H.) achievers, the complex disagreements were more likely to be

resolved than were the simple disagreements. In each level of achievement (low, medium and high level of achievement) there was a greater proportion of disagreements resolved than left unresolved. However, the high achievers had a slightly greater proportion of resolved over unresolved disagreements than did the middle and low achievers (see Table 13).

Another issue was that of resolution of disagreements as the result of persuasion of one child by his or her partner. Not all verbal strategies had equal impact in effecting persuasion. The Level II strategies which were seen most often to result in persuasion were *Supporting arguments*, *Focusing*, and *Reference to the computer screen verdict*. One resolution-via-persuasion strategy was used exclusively by one pair, Lilly M. and Rina M., namely *Reference to recorded notes*. The latter finding proved important, since it subsequently led to extensive consideration concerning the children's level of thinking about angle and their mode of working with angle.

The findings in relation to the pattern of the disagreements gave quantitative information on the resolution and nonresolution of the disagreements. What followed next was a consideration of the "how," of the means whereby the children resolved their disagreements. The study took a close look at the talk, and at the supported talk in particular; and the *supporting arguments* within the supported talk were given special attention, since the *supporting arguments* represented 25% of the verbal strategies which when used resulted in persuasion. The study delved as well into the kind of working relationships which existed between the partners as they interacted in disagreement and agreement situations, in an attempt to determine the extent to which the existing pairings either promoted or inhibited learning.

The working relationships between the partners

The second main component dealt with the working relationships between the partners.

DOES THE KIND OF RELATIONSHIP THAT EXISTS BETWEEN THE PARTNERS INFLUENCE THE LEARNING? IF YES, IN WHAT WAYS IS THE LEARNING FACILITATED OR HINDERED?

It had been expected that the greater the general grasp on the part of the child of Logo angle and other mathematics and geometry concepts, the more dominant might be the

role played by the individual in the partnership. However, it was found that the relationships, both cognitive and social, were more balanced than expected. Many relationships at all levels were *collaborative*, *doer-doer* (Cooper, et al., 1986) relationships in which both partners were exploring various components of the concept of angle. Within the same partnerships, there was also evidence of *knower-doer* relationships in which one child knew more than the other; however, in some instances the *knower* did not explain, or did not explain clearly. There was a number of *knower-doer* situations in which one child knew more than the other, and explained the concept. The children took turns, alternating between the role of information-giver, and learner. In the case of all but one partnership, the roles of *knower* and *doer* (Cooper, et al., 1986) were relative, and in a constant state of flux.

The working relationships which involved the class expert, Russ H., represented an exception to this pattern. Russ was consistently the *knower*, and often the information- giver. Thus the relationship was an *asymmetrical* one, and at times was a *didactic* one (Cooper, et al., 1986). The findings concerning Russ's interaction with his two partners, Jon H. and Eric H. (as well as the observations concerning his work for five sessions with Michael H.), gave rise to a consideration of two important issues. One issue was that of optimal group size. It was found that there was change in Eric H.'s involvement when he was part of a triad (Eric, Russ H. and Jon H.) in contrast to being part of a dyad (Eric and Jon); due in part to that finding, and due to the comments of the Grade 6 children interviewed favoring dyadic over the triadic partnerships (May-June, 1987), the researcher's tentative recommendation was that the optimal group size should be "two." The second issue was that of the dynamics of the relationship between partners when an "expert" is one of the group members. Whether Eric's involvement lagged due to his being part of a triad, or due to Russ's being an "expert," or due to both factors or neither factor, could not be determined. However, the issue, both the one of optimal group size, and of the possibly distinctive role an expert plays in a partnership, were considered issues warranting further investigation.

In both the *collaborative* and the *didactic* interactions, there was evidence of peer helping and teaching skills. There was a range of ways in which help was given, and information was transmitted, from the least complex aspect of "other-monitoring" at one extreme to the most involved aspect, "peer teaching," at the other. The children were seen to focus, to guide while the partner performed, to tell the information or give an

explanation, and at times, to demonstrate with accompanying commentary. There were peer teaching skills exhibited by Russ H. which were unique among the children in the study. Although instances were not numerous, Russ not only gave explanations of "how" and "why," but also checked to make sure the explanation was understood, and provided an opportunity for the soliciting partner to paraphrase the explanation.

In the didactic relationship, the child who was expert was able to give correct information and explanation and in that way contribute to a peer's learning. In the collaborative relationships, the back and forth reciprocal turns of help-giving by the children was a way in which the children contributed to each other's learning. In one instance (Shauna L. and two classmates, Selection #11), "doers" were able to work together to solve a problem which one child could not have solved alone (Forman & Cazden, 1985, p. 341). Thus, in both cases, there was evidence of what Cooper, et al. (1986, p. 275) have called "peer scaffolding."

Verbal strategies used by the children: Talk which promotes learning and reflects learning

The third component of the investigation dealt with the verbal and nonverbal strategies used by the children during the disagreements. The Level I strategies were statements or actions with no accompanying elaboration (for example, *insistence, denial, challenge, action taken*). The Level II strategies were statements which elicited elaboration (for example, (a) *request for explanation, request for information*; and (b) *call for teacher or for peer intervention*), or which furnished elaboration (categories within (c) supported talk such as *supporting argument, demonstration, focusing, reference to recorded notes, reference to prior knowledge, reference to computer screen verdict, and conditional test hypothesis*; and post disagreement discussion). The broad research question which guided the investigation was:

> WHICH VERBAL STRATEGIES DOES THE CHILD USE IN THE COURSE OF A DISAGREEMENT? CAN THE STRATEGIES CONTRIBUTE TO THE CHILD'S LEARNING, AND TO THE LEARNING OF THE CHILD'S PARTNER? ARE SOME OF THE VERBAL STRATEGIES WHICH ARE USED DURING DISAGREEMENTS MORE EFFECTIVE IN

LEADING TO LEARNING THAN ARE OTHERS? IF SO, WHICH ARE THEY?

The children used a wide range of both Level I and Level II strategies. It was expected that three main categories of the Level II verbal strategies mentioned above, namely (a) requests for explanation or information, (b) request for help from teacher or peer, and (c) instances of supported talk, would be used by the children, and that these approaches might be conducive to learning. The last category, (c), that of supported talk, was found to be by far the most prominent; approximately two thirds of the statements fell within the range of the supported talk category. And within the supported talk domain itself, the aspect of *supporting argument* was most prominent, since it constituted almost half of the supported talk statements (and almost a third of the total Level II statements). By comparison, there were few requests for explanation and other elaboration, less than 10%, and even fewer calls for teacher or peer help during the disagreements (2%).

One of the aims had been to take special note of the occurrence of explanations, both solicited and spontaneous. Although the total number of solicited explanations within the disagreements was 15, admittedly very small, the 7 responses to solicited requests for explanation within the mathematics disagreements were seen to fulfill procedural, empirical, intentional and deductive functions (Donaldson, 1986). The researcher paid particular attention to occurrences of deductive (that is, logical) explanations. Correct deductive explanations were rare; only two correct deductive explanations were found, both given by Russ H. (solicited explanation, Selection #16; spontaneous explanation, Selection #39).

It proved necessary to distinguish the term "explanation" from the more general idea of justification one associates with the term *supporting argument*. A *supporting argument* was not always an explanation in the sense of making something more intelligible, and more clear. The specification of a distinction between the terms "explanation" and "supporting argument" proved pertinent, for it was subsequently found that only approximately 20% of the supporting arguments could be considered to fulfill the function of explanation (Table 34). And the explanations were not always correct. It may be that since the children were in the early stages of learning and in the process themselves of making meaning of the Logo setting, they may not yet have been able to

give many correct explanations. But answers which are not totally correct are not necessarily not helpful. Perhaps in collaborative settings where children are working together, partial answers do have a role to play, in that they may at times act as a spur to learning, as in the one instance just noted in which a classmate's incorrect, and then incomplete piece of information, and a partner's comment, helped Shauna L. recreate the HELP routine (Selection #11 and Appendix L).

Strategies within the second section (b) above, which represented calls for intervention from the teacher or from a classmate, and that seemed to hold potential for learning, were used very rarely. The fact that the teacher was so rarely called upon during the mathematics concepts disagreements (and was not called upon as often in cases of mathematics concepts difficulties as in cases of syntax difficulties) was considered to be the result of the children's perception of the Logo environment as a drawing world rather than as a Logo geometry world. The children were content to draw in a step by step mode, and they did not often require help in their step by step, fragmented approach activity; they used this approach because they did not as yet perceive the underlying geometric structures inherent in Papert's universe.

As has been noted, strategies located within the third section (c) above, namely those representing support for one's stance, figured most prominently in the disagreements. Among the subsidiary research questions which had guided part of the investigation of this component were the following:

> Does the child use supported talk? Are the supporting arguments correct? Are the supporting arguments clearly stated? Do the supporting arguments clarify the mathematical theorem? If yes, how? Does the degree to which the talk is supported increase over time?

It was found that the children did attempt to clarify. They used a range of strategies subsumed under the title "supported talk" (namely, *supporting argument, demonstration, focusing, reference to recorded notes, reference to prior knowledge, reference to computer screen verdict*, and *conditional test hypothesis*). An analysis of the *supporting argument* category within the supported talk section indicated that most of the supporting arguments were clear; however, a smaller proportion of the supporting arguments were correct (see Table 30). The supporting arguments which were incorrect

served to provide insight into the children's understanding and misunderstanding about Logo angle. The fact that the children at the low and middle levels were unaware of the size of the rotational turn, and that they relied heavily upon calculation to determine the amount of input to RT and LT for their recording and later replication of the turn, emerged in their talk, and was markedly evident in the supporting arguments these children used during their disagreements.

The investigation considered the way in which the supporting arguments might be contributing to the learning of the mathematics and geometry concepts. Two subsidiary categories of *supporting arguments* emerged which were pertinent to the learning of mathematics and geometry concepts; they were assigned the terms **Description** and **Mathematical Justification.** The findings were that all the children used *descriptions* (context-referenced, ego-centered language) as well as ***mathematical justifications*** (geometry and arithmetic referents). But it seemed that the middle level, and high level achievers respectively used more mathematical justifications than did the low achieving children (see Tables 32 and 33).

One of the research questions was aimed at investigating whether there was a relationship between the level of achievement of the child and the rate of use of higher level (Level II) strategies, among them supported talk. It was expected that the child who was more adept might be more likely to use complex strategies, and to use them more often, than the child who was less adept. No such relationship was found.

Another research question asked whether the rate of supported talk increased over time. This question pertained mainly to the two children (Carol L. and Amy L.) whose work was monitored in both Grade 4 and Grade 5, but the supported talk of the Grade 5 children in the study (that is, the 7 children who were videotaped for 10 or more sessions) was analyzed as well. No increase in the amount of supported talk was found either on the part of the Grade 5 subjects, or in the work of Carol who was followed longitudinally. A slight increase of 6% in the proportional use of Level II as opposed to Level I strategies on the part of Amy L., from the Grade 4 to the Grade 5 data, was detected. It was concluded that the amount of time (12 sessions, or 24 sessions with a gap of one year in between) did not allow the children ample time to make substantial gains in their learning of the Logo concepts, and this might be one reason for the lack of increase in the amount of supported talk.

Talk as it reflected the children's learning

The disagreement episodes served as a means whereby the researcher was able to gain a more complete picture of the children's understanding of angle. The goal underlying all the research questions was: What can the talk, and what can the use of the various verbal strategies, tell us about the child's concept of angular rotation in Logo geometry? A focus on the talk during the disagreements added to knowledge already acquired (Zack, 1986b), and offered new perspectives on the children's understanding of Logo geometry in the early stages of learning.

The analysis of the supported talk component of the higher level (Level II) strategies proved to be particularly informative. What the children disagreed about served to signal their areas of difficulty; th- strategies they used to resolve their disagreements reflected their working modes (*visual* or *analytic*) and their levels of understanding (*algorithmic* or *relational*).

The findings showed evidence of more than one level of thinking within the *visual* mode itself, as well as evidence of (rare) use of the *analytic* mode. Within the *visual* mode, low and middle level achievers (7 children) worked at a *visual-algorithmic* level; there was evidence of use of proportional measure on the part of only two of these children (Shauna L., and Lilly M.). The high achievers (3 children) worked at the *visual-relational* level; that is, they knew "what to do and why," exhibiting an understanding of the size of the rotational turns, and of the relationships between the turns (the *relational* aspect), but they also relied upon the *visual* feedback from the screen in order to determine their next move. During the Grade 6 interview session, (May-June, 1987), two of these high achieving children (Jon H. and Russ H.) were able to work in an *analytic* mode; however, during the Logo sessions, only Russ worked (implicitly, not explicitly) in an *analytic* way with the angular rotations.

The rote, algorithmic nature of the work of the low and middle achievers became evident during the course of the analysis. It was in particular the reliance of one pair, Lilly M. and Rina M., upon their notebook entries which prompted the researcher to do further investigation of the children's recording and problem solving strategies vis-à-vis angular rotation.

In relation to the recording strategies, the low and middle achievers attempted to maintain a one-to-one correspondence between the entry upon the screen and the notebook entry. The high achievers for the most part tested, deliberated, and often

combined screen command inputs, and then subsequently recorded them. The children at the low and middle levels depended exclusively upon their notebook entries because they did not yet have a feeling for the size of the angular rotation. Some of the connections not yet made included: (1) the connection between the input to RT or LT and the corresponding size of the turn; and (2) the connection between the input to RT or LT and the number of degrees it represented. It was due to the fact that they did not yet have a feeling for the size of the rotational turn, or for the relationship of the part to the whole geometrical framework, that the children at the low and middle levels resorted to calculation and to uncritical reliance upon recorded information.

The children at the three levels of achievement used different problem solving methods when the need arose to reenter data in order to reproduce a design. The low level achievers kept reentering the data as recorded in their notebook. The middle level children did the same; however, they also used the strategy of reconciling entries. Of the two middle achievers, Rina M. insisted upon a one-to-one correspondence of entries in the two notebooks; Lilly M. did in addition take into account the equivalence of certain entries. The high level achievers' approaches to solving problems, when difficulties with reentering data arose, featured diverse techniques such as subdividing the list of commands, verifying different hypotheses, and reentering part of the data in "immediate (draw) mode" in order to study it more deliberately and carefully. The approaches of the high level achievers reflected their understanding of the aforementioned relationships, that is, they had an understanding of the size of the turns and of the relationships between the angular rotations; they also seemed to be able to visualize the turn in the abstract.

The analysis offered a glimpse into periods of growing awareness. There were indications that Rina and Lilly were beginning to attend to the size of the rotational turn (specifically that of 90 in this case, see Selection #45). Further such connections between the input number and the size of the rotational turn were not voiced explicitly, but may have gone on implicitly on Lilly and Rina's part, for during the post session interviews (February-March, 1985) there was evidence that some progress had been made concerning the turn of 90, particularly on the part of Lilly.

In conclusion, then, the ways in which the children resolved their disagreements, the mathematical idea underlying the disagreement, the explanations and other verbal strategies which they used during their disagreements, and the discussion which

occurred before, during, and after the disagreement, emerged as important indicators of each child's understanding of the mathematics and geometry concept under consideration. There were ways in which the talk contributed to both the speaker's and the listener's learning. The study, set in a naturalistic elementary school computer lab setting, was able to report on some aspects of the question deemed by Hawkins, et al. (1982) and by Cazden (1986) to be a vital one, namely that of the effects that children working together in the microcomputer (Logo) graphics environment can have upon their learning.

CONCLUDING REMARKS

The chapter which follows will deal first with the pedagogical implications which the study findings hold for the elementary school mathematics and computer curriculum. Second, it will consider the possible directions future research in the area might take.

Pedagogical Implications

The findings chapter ended with the following general issues: (1) Should Logo play a role in the elementary school mathematics curriculum?; (2) If yes, what kind of role should Logo play?; and (3) How can the findings which emerged from the study inform as to what directions to take? The answers may hinge on how important a role one considers that Logo should play in the school curriculum. For as part of the computer curriculum in the school setting of the study, Logo can serve as an introduction to programming and to other aspects of computer literacy, in isolation from the regular school mathematics curriculum, as has been the situation to date. Purists may of course argue that Logo can offer children an introduction to intrinsic geometry while removed from the school setting entirely, and insist that it was never intended to be housed within the school setting. Our premise, as has been stated elsewhere, is that Logo has much to offer within the school mathematics curriculum, and that it should play a role therein. It is our belief that the teacher must play a role. The main issues then are what kind of role Logo and the Logo teacher can play.

The teacher's role in helping children make connections

We have mentioned previously the connections made and not made by the children within Logo geometry, and the connections made and not made between Logo geometry and school geometry. We will submit that whereas Logo *intrinsic geometry* (see Glossary) is sufficient unto itself and can indeed be studied in isolation, that relationships between Logo geometry and school geometry ought to be pointed out, as long as Euclidean geometry is part of the school mathematics curriculum (in Quebec, Canada, and in other parts of the world). It is our position that there ought to be a bringing together of the two so that children can make use of whatever knowledge they have accumulated within each domain. Otherwise, except for a few children who make

the connections themselves, the learning remains discrete. Rather than maintain two solitudes, perhaps if the relationship between the two were made more obvious, the learning occurring in one geometry environment would help make the other geometry environment more meaningful. The question of how, and when, brings us to the issue of the teacher's role.

During the researcher-Logo teacher interview on September 18, 1987, Monica commented on the researcher's Grade 6 interview findings concerning the children having made, or not having made, connections between Logo geometry and classroom Euclidean geometry. In her comment Monica stressed the need for the teacher to help the children in their learning. Monica pointed out that a child like Russ H. was a rarity in that he was able to make strides in his learning without much teacher guidance.

Interview with Monica, September 18, 1987.

Monica: Right now, we're leaving the bridge to happen in the [child's] head. . . . And I think that that bridge will happen in only a few. Almost the few that will, you know there are some people that just make those bridges automatically. But that's what we're here for [in] teaching. . . We're basically here to help people make bridges. . .

Monica also came to a realization that in her school setting there should be communication between the Logo teacher and the classroom mathematics teacher, so that certain relationships between the Logo geometry and the classroom geometry could be pointed out to the children. In the excerpt from the interview which follows below, Monica was commenting on the responses of the 56 children to questions #7 and #8 in their Grade 6 interview; the questions #7 and #8 were related to whether they felt Logo had helped them with their classroom geometry, and vice versa (see Appendix H). It should be noted that the responses of the children had not yet been tallied at the time of the researcher-Logo teacher interview.

Interview with Monica, September 18, 1987.

Monica: But you know, but you know something? Their responses to 7 and 8 would be more "yes," would be more positive, if the

> teachers have made some kind of connection. But here . . . I know that the classroom teacher does not know Logo and I know that I have not referred to anything, because I don't know what their math curriculum is. We're leaving 7 and 8, this is really a question which . . . is a very valid question, but I think that if I had made some references and the teacher had made some references, I'm wondering . . . if there would be a few more positives, because, um, (pause) maybe they totally see it as separate, cause that's the way they're being presented.

In the setting of this study the missing relational ties might be happening more slowly or not at all because of the fact that the Logo learning is indeed removed from the regular classroom setting, by virtue of the fact that (1) the learning activity is physically in another place, namely the school "computer lab"; (2) an expert Logo teacher who is however not part of the regular school staff is teaching it, and so the children would be even less likely to connect Monica with regular classroom learning (in some ways a most advantageous situation (see Note 27)); and (3) the connections are not being made by either the Logo teacher or by the classroom mathematics teacher, or by both as Monica suggested she now felt they should be made.

If, as we submit, the connections ought to be made, the important issue is "when." It is vital that both the Piagetian-Papertian elements and the Vygotskian elements be preserved. It is essential that the children be given ample opportunity to explore and to discover, just as Monica encouraged them to do and just as she supported them in their doing. Monica believed in encouraging experiential discovery (Papert, 1980), and gave pertinent information and explanation and specific direction respectively when she deemed appropriate (Wertsch, 1979). The suggestion is not that the pacing of the sessions or the teacher intervention could have been or should have been different. The recommendation, to be elaborated upon below, is that there ought to be more of it (both time, and teacher intervention when needed), perhaps in a more concentrated period of time.

In regard to the role of the teacher in the Logo setting, there is a need first and foremost to dispel the myth that Papert believes that children will learn solely by virtue of interaction with the Logo environment, and that Papert does not believe that

children have to be taught about Logo. Although this notion has not been documented in print, it has certainly been voiced in discussion sessions at conferences, and by persons who have read Papert's (1980) work, Mindstorms. This perception is erroneous. It must be the lock-step standard school curriculum, not teaching, which Papert opposes. It is clear from the Brookline Project work of which Papert was an integral part (Papert, et al., 1979) that not only did the researchers play an active role, but there were various points in the report at which the researchers Papert, Watt, di Sessa and Weir (1979) stressed that the teacher would likely have to be involved if a certain theorem was to be mastered (see Note 22). The aspect lacking from the Brookline Project reports was an indication of the ways in which the teachers and researcher-teachers did in fact intervene with the children. The present study was not able to deal with the topic of teacher intervention except indirectly. Monica's intervention with three groups of children during three of the Logo sessions has been analyzed and presented in a previous paper (Zack, 1986a); however, that paper does not do justice to a most complex domain. The teacher as guide and support is central to the children's learning. There is a need for future research which will investigate the teacher's role in the Logo environment.

In terms of the school geometry curriculum, there are ways in which Logo intrinsic geometry can play a role in broadening the children's understanding of classroom Euclidean geometry. For example, having worked through the various experiences with rotations leading up to an understanding of 90 degrees, a bringing together by the teacher (either the Logo teacher, or the classroom mathematics teacher) might have sparked Lilly M. and Rina M. to a deeper understanding of how 90 fit into the larger framework of angular measure in general. As has been noted elsewhere (Zack, 1988a in Appendix X), Monica expected that the secondary school curriculum would be the suitable place for the explicit invoking of connections. However, as became evident when Lilly M. and Rina M. were observed during their Grade 7 year, such was not the case. It is vital that the Logo-mathematics teacher be cognizant of the fact that Logo intrinsic geometry and Euclidean geometry represent two distinctive geometries. It is essential that the Logo teacher continue to build upon the children's beginning learning, and at times to extend the children's reasoning by means of explicit, teacher directed, contingent connections, if the potential of Logo intrinsic geometry learning is to be realized. Both Euclidean school classroom geometry

and intrinsic Logo geometry can be rendered more meaningful if connections between the two are made.

Pace of the learning

The pace of the learning concerning angular rotation was slower in this project than had been expected, and slower than the pace in evidence in other work such as the Hillel (1984), Noss (1985), and Hoyles, Sutherland and Evans (1985) studies. We will consider first the possible reasons for the slower pace, and then will suggest recommendations for change within the school setting of this study.

It was felt that the discrepancy in the pace between this study and the others mentioned above was due to a number of factors, and the factors were (1) the nature of the program agenda, (2) the fact that the other studies had some assigned tasks as part of the research design, and (3) the time frame.

Let us consider first the program agenda, and the aspect of task assignments (other studies, see Appendix A) versus student self-chosen projects (this study). It must be recalled that the children were not geared toward a mathematical orientation within the Logo setting of this study. The computer lab classroom environment of this study was one wherein the learning of Logo geometry was an aim, but was not the focal objective; mathematics learning was the primary agenda in the Noss (1985), Hoyles, Sutherland and Evans (1985), Hillel (1984), Kieran (1986), Kynigos (1987) and numerous other studies whose findings have provided supporting and contrasting data for this investigation (Appendix A). It would seem that the pace might have been somewhat faster, and the learning of certain mathematics concepts more marked, had the agenda been directed toward mathematics learning.

In this study, there were no tasks assigned by Monica, the teacher, save one evaluation task during the last session (January 29, 1986); the children determined what their projects would be. Task assignments in some of the other studies (Appendix A) obliged the children to attempt certain goal projects which they would otherwise not have tried. The tasks may have directed attention to specific features of import, such as "square corners" for example. This may in turn have affected the pace of the learning of concepts such as right angle. The researcher is not hereby advising that tasks should be assigned to the children; however, if they were included, it should be done sparingly, and care should be taken to assure avoidance of a lock-step curriculum.

In the researcher's estimation, the time frame was a vital factor. In the setting of this study, the children participated in approximately twelve Logo sessions a year, once a week, over a period of approximately 16 weeks, for approximately 60 minutes per session (i. e. about 12 hours), for three years, in Grade 3, Grade 4, and Grade 5. (In Grade 6 they worked with Logowriter, and none of the children used the Logo features within Logowriter.) If one were to calculate the maximum accumulated number of hours of Logo in the elementary school of this study, then the total would be 36. In a Logo study currently in progress (Clements, et al., 1988), the children attend their Logo classes for 30 hours per year, in concentrated periods of three two-week blocks of time (Note 39). The total number of hours per year in the Noss (1985) and Hoyles, Sutherland and Evans (1985) studies were 30 and 40 hours respectively. In this study, the childen met for 12 hours in 1985-1986, one day a week, with some gaps in between due to school or holiday events. The stretch of time between blocks of Logo sessions was at least a year. At the beginning of each of the Logo "twelve week" blocks some review time was required, and of course there was some learning which had not been retained from the previous year. It was for this reason that over the three to four years that these children were "doing Logo," they were still at the beginning stages.

It would seem to be more advantageous to the children to have recourse to their Logo experience of 30-40 hours over a concentrated block of time and during one year, preferably in the Grade 5 year to coincide with the geometry component of their mathematics curriculum of that year, rather than to have it split over three to four years as has been the case to date. However, in the school setting of the present study, it may well be impossible to make such a change. Due to the fact that there is such a heavy demand for time within a curriculum in which the children learn three languages, and learn in three languages (English, French and Hebrew), it may not be possible to "spare" more time for computer learning during a one year span than that which has currently been set aside. If changes were feasible, the recommendations would be that:

1. A longer time frame per year of 20 to 25 weeks is desirable, with greater concentration of sessions within that time frame, for example one-hour classes twice a week, to equal 30-40 hours.
2. A Logo geometry agenda is needed; but the agenda should allow for exploration and attendant, contingent guidance by the Logo-mathematics

teacher, such as has been the hallmark of the program to date.

3. Appropriately timed connections between Logo geometry and classroom geometry should be made by the Logo teacher and the mathematics teacher.

Learning Logo geometry is not easy

The finding that the pace of the learning was slow gave rise to a consideration of possible causes, and resulted in suggestions for possible change. However, it must be stressed that the statement that the pace of the learning was slower than had been expected is not to be interpreted as a statement suggesting that little was learned. To any who may be tempted to summarize the study in this way, may it be suggested that one ought to remember how long it takes for learning to be accomplished. These children learned. Indeed, given the time constraints under which the children and Monica, their Logo teacher, worked, what more learning could one expect? It was not possible to measure all that the children did learn, or to tap emergent learning, or to evaluate in what way this experience may have laid the foundation for future learning.

What is clear is that learning takes time. It is never easy or rapid. A quote about geometry which has been attributed to Euclid seems especially appropriate at this point. When asked by the reigning Egyptian king if there were not some easier way than his to learn geometry, Euclid is said to have replied: "There is no royal road to geometry" (New Century Cyclopedia of Names, Volume 2, p. 1481). The very same might be said of Logo geometry. It may seem to us as adults that Papert's graphics microcomputer language makes learning of geometry easy. Perhaps it is because we already have some basic knowledge, derived from our Euclidean geometry learning, that the Logo geometry seems so straightforward to us as adults. We must never forget that we as adults have a foundation upon which to build, and a context, and a basis of comparison by means of which to systematize what the similarities and differences between Euclidean geometry and Logo geometry might be. However, the children are approaching Logo as a brand new experience. We must endeavor to see it with their eyes. A passage from Dewey (1916, in Archambault, 1964) states expressively the differences between the adult, the educator on the one hand, and the child, the learner on the other hand, and also stresses the role of the teacher in the learning:

> The subject matter of the learner is not, . . . it cannot be, identical with the formulated, the crystallized, and systematized subject matter of the adult. . .
> To the one who is learned, subject matter is extensive, accurately defined, and logically interrelated. To the one who is learning, it is fluid, partial, and connected through his personal occupations. The problem of teaching is to keep the experience of the student moving in the direction of what the expert already knows.
> (Dewey, 1916, in Archambault, 1964, p. 361, p. 363)

The myth prevalent among the public is that Logo makes geometry easy to learn. The researcher has noted that many sources dismiss in short order what this study has shown to be a difficult concept, namely the notion of right angle, and launch into consideration of the complex "challenges," such as for example the writing of procedures with variables. The notion of 90 degress, or the notion that a full circle contains 360 degrees, are difficult concepts. Logo is not an easier way to learn geometry; Logo provides an alternate way to learn geometry. We must take into account the challenge Logo represents for the child, as well as the issue of the fragmentation of the child's knowledge. We must acknowledge both that it takes time for the child to make sense of the Logo geometry world, and that we as adults, teachers and researchers, as well as the child's peers, have a role to play in the child's learning about that world.

Directions for Future Research

There are many avenues to investigate in relation to the issues discussed in this study, some of which have already been presented in the body of the paper, in particular in the "Summary of the Findings" section. The list which follows is by no means exhaustive, but mentions a few of many possibilities.

For one, the aspect of the child's understanding of the aspect of angle, both in Logo geometry and in Euclidean geometry, requires further study. The question of whether changes to the Logo software are recommended or required in regard to studies of the child's concept of angle, ought to be broached as well.

There is a need for continued research of Logo learning in natural school settings, as was the case in this study. However, there is a need for research in schools with

heterogeneous populations; the setting of this study was quite unique. Research should be done in diverse classroom settings, that is, in regular homeroom classrooms where computers are present throughout the school day, as well as in school computer lab environments. Involvement by teachers as researchers, and teachers guided by researchers, is needed; two studies by high school classroom teachers-researchers (Ludwig, 1986; Mendelson, 1988) to date stand alone. As has been mentioned previously in the paper, in-depth investigation of the teacher's role in the Logo setting is absolutely essential. Other aspects which have been noted previously in the paper as warranting further study are (1) the aspect of group size, dyad as compared to triad for example, and (2) the role of the class expert in the classroom. The latter investigation would need to look closely at the expert's relationship with partners and with other classmates as well, and would seek to analyze whether the expert affects the dynamics of a partnership in a unique way.

In relation to the sociocognitive dimension, there is a need to continue to investigate the role that cognitive conflicts between peers plays in their learning, as well as to investigate the general aspect of collaboration between peers and its contribution to learning. In this study, the relationships between the peer partners were found in all but one instance to be symmetrical, in terms of the extent of the children's knowledge. However, the children differed in the degree to which they gave help or explanations, and in the kind of help they gave. There ought to be more investigation of the degree to which some children reciprocate to their partners, while others do not at times, and the ways in which that may affect the learning. The children differed in the degree to which they were able to aid each other's cognitive growth. There is a need to continue to investigate the ways in which the relationships between peers, both the symmetrical and asymmetrical relationships, serve to contribute to the learning of the children involved. As we become more knowledgeable about (1) the difficulties that the children encounter when working with Logo geometry, and geometry in general, (2) the ways in which children can contribute to each other's learning, and (3) the ways in which the teacher contributes to their learning as she guides them while they work in groups, we can become increasingly better equipped to evaluate and enhance the role that peer interaction can play in the learning of Logo geometry concepts, and in learning in general.

NOTES

1. Cazden (1986) quoted from the National Institute of Education (1974) Conference on Studies in Teaching, report of Panel 5 on "Teaching as a linguistic process in a cultural setting," as follows:

> The study of linguistic phenomena in school settings should seek to answer educational questions. We are interested in linguistic forms only insofar as through them we can gain insight into the social events of the classroom and thereby into the understandings which students achieve. Our interest is in the social context of cognition: speech unites the cognitive and the social. The actual (as opposed to the intended) curriculum consists in the meanings enacted by a particular teacher and class. In order to learn, children must use what they already know so as to give meaning to what the teacher presents to them. Speech makes available to reflection the processes by which they relate new knowledge to old. But this possibility depends on the social relationships, the communication system, which the teacher sets up. (Cazden, 1986, p. 432)

Griffin, Cole and Newman (unpublished paper cited by Cazden, 1986, p. 453) cautioned that while they believe strongly that "language unites the social and the cognitive," they believed just as strongly that "we don't have ready made procedures for specifying how this relationship works."

Note that Cazden (1986) also cautioned about the "general difficulty of establishing relationships between language and thought--or *thinking* and *speaking*."[6] She added that we should not be surprised therefore that it is "also difficult to verify this relationship in the classroom" (p. 452). Yet talk, and student artifacts, offer a partial means of access to what might be going on in the

[6] Wertsch (1979) has pointed out that Thinking and speaking would be the more accurate translation from the Russian of Vygotsky's (1962) book title, Thought and language.

cognitive domain. "Speech makes available to reflection the processes by which [the children] relate new knowledge to old" (NIE, cited above). Although the researcher must remain aware that talk is not thought spoken aloud, and much cognitive processing goes on without any formulation in speech, talk and product do provide at least a window into the child's learning. In the final analysis, however, all that the researcher can do is to describe the speech, and interpret the intent and meaning.

2. Hare (1962) spoke of the unique characteristics of the dyad as follows:

> [It] is suggested that unresolved differences appear to be more tolerable in large groups and may be handled by compartmentalization and other similar devices rather than by resolution . . . [Groups] of two appear to show unique characteristics (Becker & Useem, 1942). They tend to have high rates of showing tension, consistently avoid disagreement and antagonism, have high rates of asking for opinion, but avoid giving opinion, and concentrate rather on exchange of information and agreement (or acknowledgement) (Bales & Borgatta, 1955) . . . . [In] groups of two a delicate balance of power exists when there are few group norms regarded as binding except those to which both members currently assent . . . Either member can prevent task completion by disagreement or withdrawal. Both members have to proceed within certain limits to avoid this reaction on the part of the other . . . [The] two-man group may be viewed as having built into it an implicit agreement that the two will stay within spheres on which they can agree. In spite of the delicate balance of power . . . there is a tendency for one member to gravitate toward a more active role and exercise the power of initiative, while the other tends toward a more passive role and holds the power of veto. (1962, pp. 240-241)

3. Stodolsky (1984, p. 114) defined five types of instructional work groups: peer tutoring, helping permitted, helping obligatory, cooperative, completely cooperative.

- *Completely Cooperative*

  Common end or goal
  Common means and activities
  All members expected to interact-contribute, joint product evaluated

- *Cooperative*

  Common end or goal
  Some divided activities or tasks
  All members expected to interact-contribute, joint product evaluated

- *Helping Obligatory*

  Individual goals
  Interaction required, helping from any member to any member
  Each individual evaluated

- *Peer Tutoring*

  Tutee's goals
  Help in one direction from tutor to tutee
  Tutee work evaluated

Stodolsky insisted that one must distinguish between "helping" and "cooperation," and between "helping obligatory" and "helping optional."

Stodolsky also showed how in two research studies (Webb, 1980; Peterson & Janicki, 1979) in which "small groups were formed with the expectation of mutual assistance," that in actuality the "internal workings" of the groups differed from the intended work pattern. "In the research cited the symmetric relations characteristic of a helping-obligatory group were not enacted. Instead the

asymmetric peer tutoring configuration was employed by the children" (Stodolsky, 1984, p. 116).

4. Martin's (1987) statement may seem to be universally held, but as Cazden (1986) has indicated, crediting Michael Stubbs (1976), the comment may be "culture-bound." Cazden (1986) remarked that "many sociolinguistic researchers share the assumption of teachers, that discourse . . . affects the thought processes of each of the participants and thereby the nature of what is learned," but then referred the readers to Stubbs' (1976) observation that "this view of education is culture specific" (1976, p. 104). "Philips (in Cazden, et al., 1972) for example, showed that the Indian groups believe that learning occurs through observation, supervised participation and self-initiated testing. Speech is minimal in this process" (Stubbs, 1976, p. 105).

5. Lee (1985) traced the way that Sapir's work may have influenced Vygotsky, in an article entitled "Origins of Vygotsky's semiotic analysis." In his book Language, Sapir (1949) formulated the thesis that underlies Vygotsky's Thought and language:

> Most people, asked if they can think without speech, would probably answer, "Yes, but it is not easy for me to do so. Still I know it can be done." Language is but a garment! But what if language is not so much a garment as a prepared road or groove? It is, indeed, in the highest degree likely that language is an instrument put to uses lower than the conceptual plane and that thought arises as a refined interpretation of its content. The product grows, in other words, with instrument, and thought may be no more conceivable in its genesis and daily practice without speech, than is mathematical reasoning practical without the lever of an appropriate mathematical symbolism.
>
> (Sapir, 1949, p. 15)

Vygotsky then used a similar metaphor on the creative dialectic between thought and word:

> The structure of speech does not simply mirror the structure of thought; that is why words cannot be put on by thought like a ready-made garment. Thought undergoes many changes as it turns into speech. It does not merely find expression in speech; it finds reality and form. (Vygotsky, 1962, p. 126)

For Sapir, language creates the possiblity of abstract thinking, and language and thought grow in a dialectical fashion (Lee, 1985, p. 79): "We see this complex process of the interaction of language and thought actually taking place under our eyes. The instrument [thought] makes possible the product [speech], the product refines the instrument" (Sapir, 1949, p. 17). What Vygotsky added to Sapir was a developmental account of how language mediates thought.

6. Halliday (1981), in reflecting on how children learn their native language, spoke of "learning language, learning through language, and learning about language"; it is first perhaps a sequential dialectic, but then a totally interrelated one. In the microcomputer environment, one might see the learning of the syntax and semantics of the Logo graphics commands in somewhat the same way. First the child learns the minimal syntax needed to "get on with it," then learns about the underlying workings of the commands so as to control the moves, while all the while using talk and action to learn aspects of the system.

7. Patricia Marks Greenfield (1984) gave the following description of scaffolding:

> The scaffold is a metaphor, originated by Wood, Bruner and Ross (1976), to describe the role of the teacher. This metaphor is the basis for a theoretical model of the teacher in informal education. The scaffold, as it is known in building construction, has five characteristics: it provides a support; it functions as a tool; it extends the range of the worker; it allows the worker to

> accomplish a task not otherwise possible; and it is used selectively to aid the worker when needed . . . These characteristics also define the interactional scaffold provided by the teacher in an informal learning situation . . . [The] teacher structures an interaction by building on what he or she knows the learner can do. Scaffolding thus closes the gap between task requirements and the skill level of the learner, creating what Hunt (1961) called "the match" between the cognitive level of the learner and the characteristics of instruction, or what Brown (1975, 1979) referred to as "headfitting."
> (Greenfield, 1984, p. 118)

8. Harste, Woodward & Burke (1984) have proposed that the notion of scaffolding itself posits a "transmission of knowledge" model, in their terms a "stimulus-response" interaction between the learner and the environment. What Harste, et al. (1984) call their "transactional" model of learning is more in line, they feel, with Halliday's notion of "tracking" (1980, cited by Harste, et al, 1984, p. 96). To quote directly:

> [What] participants do psycholinguistically is "semantically track" each other; this produces psycholinguistic shifts and moves throughout the event. In adult-child conversation, the shifts which adults make in adjusting their language to the child--what Halliday calls 'tracking'--might be more simply viewed as a case of adults reacting to the child as 'conversational informant'; the shifts and moves which the child makes in reacting to the adult, as 'the adult as conversational informant.' . . . Both adult and child are seen as actively structuring the event. (Harste, et al., 1984, p. 96)

What Harste, Woodward and Burke, and Halliday of course, identify as "tracking" seemed to me to be a vital component of my personal definition of scaffolding. But then I looked more closely at excerpts from Bruner (1985) and Greenfield

(1984). I was astounded to find that Bruner had stated that the tutor, at the start of the interactive task, "is consciousness for two" (1985, p. 29). No tutor, no "other" can be a consciousness for two. Interaction assumes two active individuals.

Upon checking one other source, Greenfield (1984), I found that she had stated that "scaffolding [resembled] the concept of shaping from Skinnerian psychology (1938). Both create an environment which reduces both error and failure experiences at the early stages of learning a new skill" (Greenfield, 1984, p. 119). Harste, et al. (1984) countered that the term "scaffolding" assumes that the "adult is in charge, simplifying, manipulating, or structuring the environment for learning. Scaffolding as a term thus pulls attention away from process to the environment and hence implies that language learning is the result of an environment-response bond" (Harste, et al., 1984, p. 96).

I must continue to investigate the issue. My definition of scaffolding is in line with Halliday's term "tracking", but in thinking of the work of Forman and Cazden (1985), and Cooper et al., (1986), I also detect no discrepancy between their view of scaffolding and Halliday's notion of tracking. I need to delve further.

9. Hoyles (1985) referred to the commonsense world knowledge via which everyday commerce takes place with accuracy, as opposed to high error rates when the same problems are given to the same population in a school context (Lave, et al., 1984; see also Carraher, 1988). She also cited Vergnaud's (1982) findings that "even at an early age children come across high level mathematical relationships and are able to apply successfully these 'theorems in action'", but in school are not able to relate to simpler problems (1985, p. 12). She described the estrangement of school mathematics as follows:

> School mathematics can . . . be regarded as a subject distinct in itself, that is, distinct even from mathematics. It has its own ways of working (for example step by step and hierarchical) and even its own concept names and symbols (see Dorfler & McClone, 1985). Thus what is termed by the French mathematical educators a 'didactical transposition' has occurred in which the 'adaptive treatment of mathematical knowledge'

> takes place in order 'to transform it into knowledge to be taught' (Balacheff, 1984, p. 35).
> (Hoyles, 1985, p. 25)

10. The history of the development of the Logo programming language, and a review of many of the research studies are available in the work of Maniatis (1983) and Noss (1985). The National Logo Exchange keeps as up-to-date as possible a listing of all articles, books, and theses published on the subject of Logo. Only the works immediately pertinent to the focus of this study, namely studies relating to the mathematical concepts, will therefore be noted.

11. Emihovich, Miller and Clare (1985) have detailed some of the aspects of disillusionment which set in in some quarters in the years just after the publication of Mindstorms (1980), and speculated that it perhaps occurred because people had such lofty hopes about Logo learning, namely that children would "effortlessly learn deep mathematical concepts," would by exploring Papert's "microworlds experience the full power of their minds in ways that current educational practice could not, and would not match" (1985, p. 1). Emihovich, et al. (1985) listed a few of the many detractors who "rushed to do battle with" what they felt were Papert's (and the world's!) claims for Logo (1985, p. 1). Two of the detractors even called for a moratorium on Logo instruction in the schools (Tetenbaum & Mulkeen, 1984).

    Higginson's (1984) title-- for a paper published in the Preproceedings of the 1984 National Logo Conference. -- "About that rose garden . . .," reminds us most effectively that Papert never 'promised us a rose garden.' Learning does not come quickly, nor should we give up on the exploration of the learning process via Logo so quickly, nor cease to pursue the investigation of some of the true-to-life difficulties which are being documented (Higginson, 1984; Hillel, 1985; Leron, 1985) as one moves from theory to practice.

12. Is it possible that part of the reason for the frustration may be cultural? Adams (1984) has observed that North Americans want results quickly, and want the results to be obvious, and conclusive. Adams, lecturer in education at the

University of Cambridge, England, spoke in an interview in Language Arts of his concern about the misuse of theoretical models. He spoke specifically about models of language, mentioning Britton, Moffett and Halliday, but his concerns apply equally to considerations about Logo.

> I have some concern about the kind of stranglehold which Britton's model seems to have on people here. In many ways, I think it has never been satisfactory and needs refining and expanding. . . [I] think we are lacking the right kind of model of language use for teachers to make use of in the classroom. Britton, Moffett, and Halliday have helped us come a long way toward it, but I believe that any of those three would agree we have a long way to go. Yet I still get the feeling here in North America that there is a view that all that has to be said on this particular issue has been said and we know where we stand. It may be a North American tendency that fashions are taken up, believed in absolutely and firmly, until a disaster occurs and some other fashion takes its place to be believed in totally in its turn and so on. What worries me is that people don't understand how models are supposed to work but rather they reify them and consider them to be concrete reality. We are already seeing textbooks based on particular models . . . Perhaps it's because North American educators tend to think very concretely and want to know where they stand in relation to external criteria. Perhaps it makes one feel safer that way, but that approach may prevent further growth and understanding. (1984, p. 124)

One is reminded as well of the occasion upon which Piaget, on a speaking tour in the United States, was asked how children could be taught the concept of conservation more quickly. Somewhat the same kind of question must have been posed to him a number of times, for Piaget responded that it struck him that North American audiences were preoccupied with "speeding up" the rate of

learning (Duckworth, 1979).

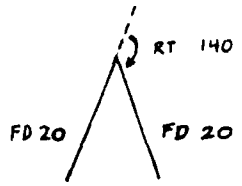
13. Papert, in citing his debt to the field of artificial intelligence (1980, p. 157), might be envisioning how Logo could be looking forward to the day when human language will drive the computer. LISP-processing computer languages work with concatenations of word commands, are interpreted and executed. At this point, despite the fact that there are indeed some products already in the marketplace (for example, the LINK program by Burroughs), the computer human language systems are still in their infancy.

    I have observed that children in the initial stages of Logo learning do not find the error messages helpful; and the initial learning stages are, of course, the point at which help is most needed. For example, the phrase "NOT ENOUGH INPUTS TO . . ." holds no meaning. There were numerous occasions upon which it was noted that the children in this study tried to make sense of the error messages.

14. One of Papert's (1980) concepts is that of "syntonicity"; in Mindstorms he spoke of "body-syntonicity" and "ego-syntonicity." Papert feels that one can understand basic procedures in turtle geometry by using one's body and moving through the motions, by "playing turtle." Kieren (1984) has elaborated upon Papert's (1980, p. 63) brief presentation and has used the concept of angle as an example:

> Papert claims that the simpler aspects of turtle geometry are body syntonic in this way--the "turtle's" action is in sync with a person's action. In claiming that turtle geometry is ego-syntonic, he is claiming that its geometry is in sync with the way in which a person would build up geometric ideas. From a mathematical point of view, Turtle geometry is a local geometry--seen and

done from an internal, personal point of view. Thus, an angle is looked at from the amount of turn used in drawing it, rather than as a product, or a pair of segments separated.



(Kieren, 1984, pp. 16-17; the drawing has been slightly changed.)

I wish to draw attention to the arms of the angle that Kieren draws to make his point about angle and rotation. What became clear in my study of the children's notion of angle (Zack, 1986b) was that the children who were the most comfortable with manipulating angle turns, were the ones most insistent upon the geometry classroom definition of angle as having rays. Thus, it was not an angle unless it had "lines" (Zack, 1986d, pp. 101-102). I proposed that a more accurate term for Logo use would be "angular rotation" (Zack, 1986d, p. 103), not "angle," since it is not an angle, either from a child or adult perspective, unless it has rays. It should not be surprising also that in the early stages most children have difficulty identifying the size of an angular turn, since it is a disembodied "angle" when it is without the guiding delimiters, the rays.

15. This study was conducted in a naturalistic setting, with no changes made to the Logo software and no assigned tasks (with the exception of an across class evaluation-type task during the last Logo class session, January 29, 1986). By contrast to this study, some of the Logo and mathematics studies have, for the purposes of research, appropriately and ingeniously, introduced restrictions within the microworld (Barfurth, 1987; Hillel, Kieran & Erlwanger, 1986; Hillel, Gurtner & Kieran, 1988; Kynigos, 1987) and/or have structured new environments within Logo (Hoyles and Noss, 1987; Clements, et al., 1988), in order to facilitate children's geometry learning, and in order to be able to trace certain features of that learning. If the sessions presented at the last International Conference of the Psychology of Mathematics Education (Hungary, 1988) are an indication of a

trend, then there is a growing interest in studies of particular Logo microworlds which are tailored to promote discovery about specific, related aspects of the geometry world; examples would be Kynigos' (1987; 1988) coordinate geometry, Euclidean geometry and dynamic, intrinsic geometry microworlds; Edwards' (1988) transformational geometry microworld; and Hoyles and Noss' (1987; 1988) parallelogram microworld. The findings from these studies which are currently in progress, or have just ended, promise to add yet another dimension to our knowledge about children's learning of Logo geometry. Whether or not there ought to be changes to the standard Logo software package, and what these changes might be, is an issue which is sure to be debated in the near future. At this point, however, the findings from the studies just mentioned may not be as pertinent as is this study to school settings in which the standard Logo software is being used.

16. Historically, the origin of the term Hebrew "day school" lies in its precursor, the Hebrew "afternoon school"; after the regular school day was finished, Jewish children would travel from the public elementary school to attend Hebrew study classes, for two or three afternoons a week. The Hebrew "day school" system began in Montreal in the mid-1950's. In the post-war years, perhaps in part due to the baby boom, enrolment was sufficient to support an all-day multilingual school system which fulfilled the government curriculum guidelines in the English and French studies, and also provided a curriculum of study in Hebrew language and religion. Thus the school setting would be considered a non-traditional one.

The term "semi-private" is used in order to distinguish the school from the "private" schools in Montreal. The school in this study is considered "semi-private" in that the students paid high fees (approximately $2000.00 per year), which while lower than those of some schools in the Montreal private school system, are nevertheless far higher than in the public school sector where education is free. However, there are no entrance examinations or specific academic performance selection criteria in this "day school"; although within the school setting of the study the children are regrouped for mathematics according to achievement, the average class size for the teacher-directed classes is 24, whereas, for example,in one of the private schools, where the average homeroom size is also 24, the

student to teacher ratio is approximately 12:1 for most of the school day.

One would have to say that the particular characteristic of the parent body within this school setting was the high value which was placed upon education. However, the researcher felt that the level of achievement of the children in this non-traditional setting would not be substantially different than that of any regular public school which draws its student population from a middle and upper class socioeconomic area.

17. A recent non-Logo study (Miller & MacGregor, 1988) compared the uses of microcomputers by Grade 5 students in two environments, namely the elementary school classroom and the elementary school computer lab setting. The researchers found differences in the nature of teacher-student interactions, computer use, the impact of computers on students and student perceptions of the learning environment. Miller and MacGregor (1988) stated that: "Based on the data from the student inventories and the researcher observations, it appears that the computer lab is a more effective learning environment for elementary school students. The lab functions in a more cohesive, single purpose manner, more computer time is allocated per student, and more adult assistance is available in the laboratory setting. . . . [Students] appeared to be very cooperative in helping each other solve problems in the lab environment" (1988, p. 12).

In this study, during the interviews in May-June, 1987, the 56 children interviewed were asked whether they would prefer to have the computer(s) in their homeroom classroom or in the computer lab. Forty of the 56 children, or 71%, preferred to continue having the computer sessions in the lab; only 5 of the 56, or 9%, said they would like to have the computer in their classroom. Certain of the children's concerns echoed some of the Miller and MacGregor (1988) findings. The difficulties that the children mentioned were that if the computers were in the homeroom classroom, (1) they would miss the teacher contact, (2) they would have to miss some of the teacher-presented class work and would have to make it up, and (3) the noise factor would bother them (see Appendix H, Question 11.5).

18. The children's classroom notebooks were important artifacts (see Appendix C). The classroom routine which governed the use and care of the notebooks will therefore be presented briefly. As has been noted in the main text, the children were required to record their commands in their notebooks, and to draw their planned design. Once the children completed a project, they then printed four copies of the graphic picture (at computer #6); the pair signed one copy and posted it on the bulletin board at the back of the room, they each glued a copy of their picture into their notebooks, and they gave one copy to Monica which she then glued into her own journal. The children's computer notebooks were collected at the end of each class, and were stored on the shelves (Figure #2, cubbyholes). Monica insisted that the children leave their notebooks in the computer lab, since this minimized the need to run back to the classroom to retrieve a forgotten notebook. There were some instances in which, despite all efforts, the notebook was not present in the computer lab. However, for the most part the notebooks were maintained over the year, and in some cases over more than one year. This factor in the end proved most helpful for the researcher, since in some instances, the notebook the child was using represented two or more years of work. The use which the child was seen to make of her previous year's work proved enlightening; for example, see Amy L., Selection #2.

19. At the start of the interview with the classroom teachers, the researcher stressed to the teachers that they were being asked to recall their impressions concerning the study subjects, so as to allay or forestall a teacher's concern that she needed to have actual scores. At the time of the interview with the children's Grade 6 classroom teachers (conducted in November and December, 1987), the study subjects had completed elementary school and had moved on to Grade 7, to secondary school, and so the actual school grades achieved by the children were no longer available. The teachers' recollections proved enlightening. In some cases, the recollections concerned interactions with the study subjects at various points during the elementary school years. Some of the teachers had taught certain children for more than one year.

The researcher asked whether the standardized test scores played a role in the teachers' assessment of the children's achievement. The teachers responded

by stating that they neither consulted nor gave any weight to the standardized test scores; one teacher mentioned that she was not interested in even the previous year's performance of the child as outlined in the final school progress report, since she wished to make an independent judgement of the child's performance when that child entered her class; she did say that she consulted the child's school file if the child began to experience difficulty in either the academic or the social domain.

20. Three of the children expressed aloud their uncertainty and lack of confidence about Logo during the individual beginning session interview held in October and November, 1985; Carol L., Leanne L., and Joey (L). each stated that they were not "good" at computers. Klein (1988) has noted that "persons with favorable mathematical self-concept and aspirations tend to have higher levels of ability (skill) and vice versa" (1988, p. 40). Klein referred to two large studies (Husen, 1967; Crosswhite, 1972) as well as to his own research studies in Hungarian schools on smaller samples (Klein, 1971, 1973, 1975, 1977, 1980, 1987) which found that the children's perception about their mathematics ability was correlated with their mathematics achievement. The affective aspect was not pursued in this study. However, it would prove of interest to do so. It would be particularly informative to trace the cases of children whose sense of confidence in the Logo setting was extremely high, but whose achievement in the Logo mathematics area and even in the syntax area where they thought themselves "expert" was not good (Zack, 1988b).

21. In Grade 4, Carol and Amy did stay together as partners despite the fact that they had arguments often, and Carol displayed obstructionist behavior, such as refusing to share information, and turning the color button on the screen and making Amy believe that the machine had been turned off, with the resultant loss of their work. In Grade 5, Carol bossed Leanne L. at times, but valued the partnership with Leanne; they were partners for 4 sessions in Grade 5. Carol did not enjoy her partnership with Shauna L. (1 session), desperately wanted to have her work with Doug L. completed because she wished to not continue as his partner (they were partners for 4 sessions), and with her negative attitude obstructed the

possibility of any work being done during one of the two sessions when Carol, Amy L. and Leanne L. were together as a triad. In Grade 6, Carol worked with her partner Nella L. for only half of the twelve sessions (and they argued quite regularly), and then they went their separate ways.

22. The Total Turtle Trip Theorem refers to the fact that in order to make any polygon, the turtle will move through a 360 degree revolution. The fact that a rotation of 360 degrees will turn the turtle through one full revolution is but one (and perhaps the least difficult) of the aspects of the theorem. The Total Turtle Trip Theorem has been explained by Papert, Watt, di Sessa and Weir (1979) as follows:

> [For] any polygon, the turtle will turn through a total rotation of 360 degrees (p. 5.64) . . . A square is constructed by repeating the same "thing" four times . . . FORWARD something, RIGHT 90. In doing so, the turtle has rotated a total of 360 degrees. *The student also knows that a rotation of 360 degrees will turn the turtle all the way around.* In order to draw a triangle, the turtle will also make a shape in which it turns all the way around, this time in three steps. The amount of rotation needed for each step is thus 360/3 or 120 degrees. The student can now draw a triangle by repeating FORWARD something, RIGHT 120 three times.
> (p. 5.64; emphasis added)

Papert, Watt, di Sessa and Weir (1979) did stress that "the reasoning is far from trivial, and it is often the case that a student and a teacher will work together on this process, rather than a student figuring it out independently" (1979, p. 5.65). Hillel (1984) as well pointed out that the Total Turtle Trip Theorem is more complex than merely a rotation of 360 degrees on a point. Hillel stressed that Total Turtle Trip Theorem "is not going to emerge out of the activities of 8-9 year olds who are still trying to understand the meaning of angle as an input to rotation. On the other hand, the fact that angular measure is periodic (modulo

360) can be within the reach of such an age group" (1984, p. 6). Hence, when referring to Shauna's own unique construct, one is denoting the simplest, but still powerful, idea of one complete revolution (of 360).

23. During the November 20, 1985 session, partners Amy L. and Leanne L. both felt a great deal of frustration in their repeated attempts to solve their problem and to figure out why the eight subsequent attempts at the recreating the first drawing  resulted in so altered a product. (See also Lilly M. and Rina M.'s confusion in relation to their attempts to recreate their triangle, Note 31 below.) Amy and Leanne kept clearing the screen, redoing the steps as recorded (8 times), with one minor change in one of the instances (Attempt #3), and an error of omission in another instance (Attempt #7). They worked at their attempts for 11 minutes without calling for Monica's help. They kept testing out the same design in the immediate mode (also called the draw mode), in exactly the same way, never attempting to redo any part, or to try to detect via a step-by-step slow process just where they had gone awry. One beginner problem solving strategy might have included looking at the screen at each of the steps as they completed the keying in of the command, but they did not do so. Indeed, on their first attempt they did not even look at the screen while keying in, and only noted at the end that the graphic representation on the screen did not match their goal. Another strategy might have entailed redoing everything from scratch when all else failed, thereby ignoring the recorded notebook entries completely. (There were learnings that they had not yet acquired and therefore could not be expected to use. For example, since they did not have a conception of the size of the turn, they therefore could not visualize mentally that there was something missing, that is, the unrecorded LT 83.) Amy and Leanne just kept referring first to one set of recorded instructions (for example Amy's) and then to another set (for example, Leanne's), each insisting that the other had not recorded the steps properly. They did not try to reconcile their entries in order to determine whether there were any discrepancies either in specific entries, or in the number of entries. They kept keying in the record notebook instructions and seemed to be hoping that upon one of the occasions, it would magically turn out right. After 15 minutes of computer class time, 11 minutes of which was spent going over the same ground

again and again, Monica intervened with what was potentially a good suggestion (i. e. "Check your instructions"); however Monica did not realize that the only means Amy L. and Leanne L. knew whereby to check the instructions was to key in again the written instructions recorded in their books, without pause or deliberation. The frustration caused by the cognitive aspect was a key factor in the bitter fight which transpired during the last five minutes of the November 20, 1985 session (See Note 26 below; Zack, 1986c).

24. During the May-June 1987 interviews with 56 of the 69 Grade Six children, the children were asked about advantages and difficulties of working in a threesome (see Appendix H, Question 1.3). Among the most common disadvantages noted were the observations that in a threesome it was more difficult to resolve differences if ideas diverge than would be the case in a dyad; typing time had to be shared three ways; seating was more cramped, and access to computer keyboard and screen was more difficult in a triad and favored the person sitting in the middle.

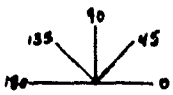
25. Hoyles and Sutherland (1986b) described an episode between two of the children in their study (Sally and Janet) in which a breakdown in communication resulted in a discovery by Janet about an aspect of angle rotation which Sally had already mastered and therefore had always done. Due to the fight, and to Sally's refusal to perform what had become her assigned role, Janet learned something new about angle rotation. In this study there were no such instances of solitary discovery as a result of a child being left to her or his own devices due to a breakdown in communication.

26. The partners who were part of the study were not heard to ask each other for help; they worked on the premise that help would be given whenever it was needed, provided the partner was able to do so (Interviews, May-June, 1986). There was an exception noted in one instance when one partner (Amy L.) actively obstructed the activity, and the other partner (Leanne L.) was obliged to literally beg for help. The episode occurred at the end of the session of November 20, 1985 (see Note 23 above). At the point in the transcript when Leanne said to

Amy: "So, do you MIND telling me?" it might seem for a moment to be an instance of Leanne asking her partner for help, but should be considered rather an instance of insistence by the child being obstructed (Leanne L.) that the partner (Amy L.) was not being complying to the contractual obligation of the partnership. As has been amply outlined in Note 23, Amy L. and Leanne L. were both frustrated by the end of the November 20, 1985 session. At one point, Amy L. grew angry over Leanne L.'s insistence that the procedure name be the one which had been decided upon previously. In retaliation, Amy resisted giving Leanne L. any helpful input concerning a syntax problem, namely how to do the double quotation marks (needed for the EDIT "procedure name command). When Leanne L. turned to ask a neighboring peer, Don L., for help, Amy L. obstructed even that avenue and told Don L. not to help Leanne L. (Zack, 1986c). Leanne L. implored Amy for help and Amy refused to comply.

27. Leanne's stated and observed willingness to ask Monica the Logo teacher for help ran counter to the observations and comments of all of Leanne's regular classroom teachers who were unanimous in feeling that Leanne L. never ventured to ask for any explanation or information in class when she did not understand (See child profile in the METHODOLOGY section). It seemed that the environment that Monica, the Logo teacher, provided, was nonthreatening. Leanne was seen to call for intervention from Monica, but not in reference either to mathematics problems, or in reference to disagreements concerning mathematics concepts as will be seen in the section entitled "Solicited TEACHER INTERVENTION in disagreement episodes." The calls for intervention from both teacher (17 calls) and other classmates (3 calls) on the part of Leanne L. and her partners were exclusively calls for help with syntax problems. (The ratio of calls by Leanne and her partners in relation to the total number of solicited calls for intervention concerning syntax problems by the study subjects was 17/78, or 22% of the calls for teacher intervention, and 3/23, or 13% of the calls for peer intervention.)

28. Kieran, Hillel and Erlwanger (1986) speak of a "protractor schema" which is used by some of the children: "The inputs are based on usage of the protractor in

standard position" (1986, p. 157).  It might seem from the following response made by Russ during the Grade 6 interview (May 29, 1987) that he used a protractor schema, when in fact he did not. His comments evoked connections to a protractor schema due to the important signpost numbers which he was heard to cite: "Because the (?inaudible) degrees would be, start from zero and up to 45 and then 90 and then 135 and then 180." There was evidence that a number of the children did work in a rote way with these special "signpost" numbers (for example, Joey L. used 45 as a special number for making diagonal lines, as is noted below in Note 36); but Russ was not one who operated in a rote way. Russ worked with angles with relational understanding, in that he could see the interplay of parts and of multiples of angular rotations.

29. The input of 90 was used with varying degrees of expertise. Russ H. and Jon H. used it with "relational" (Skemp, 1979) understanding (Zack, 1986d). Eric H. developed an adept mastery of the 90 input during the sessions. In Carol L.'s case and in Doug L.'s case, the input of 90 was not used with understanding about the relationship between 90 and other inputs, and was not always used when it should have been (as for example in instances when the children needed an outside turn of 90). Doug L.'s case was particularly intriguing because the input of 90 was the only input he used, and the only input with which he felt comfortable; Doug was seen to be at a complete loss when questioned during the interview about other inputs for rotational turn (February, 1986).

30. In a recent article, Pelligrini and Galda (1988) spoke of two modes of children's uses of language, the *narrative* and the *paradigmatic*. The *narrative* mode is personal and "typified by temporally motivated language" (1988, p. 183); the *paradigmatic* mode is typified by impersonal and objective language (1988, p. 184). Several interesting parallels may be drawn between Pelligrini and Galda's terms *narrative* and *paradigmatic*, and the **descriptive** and **mathematical justification** categories which emerged in this study. One similarity would be the contrast between the concrete, context-bound, personal nature of one pair, that of *narrative* language (Pelligrini and Galda) and **descriptions** (this study); and the more abstract, more context-free nature of the second pair, that of *paradigmatic*

language (Pelligrini and Galda) and **mathematical justifications** (this study). Another similarity would be the insistence on the part of the authors, Pelligrini and Galda, that the modes are different but complementary, and that one, the *paradigmatic*, ought not to be considered a higher form than the other by virtue of the fact that it is abstract. They stated:

> Generally, the paradigmatic mode has been considered a higher form (e. g., more developed) than the narrative. It may be, as Hymes and Cazden (1980) cogently point out, that whenever two forms of language exist, high/low assignments are made. In the present case, the high/low dichotomy is thought to reflect cognitive sophistication. That is, paradigmatic language, because it is thought to be context free (if such a language variant exists) and causally motivated is said to be more abstract than its context-bound sibling, narrative language. . . .
>
> The position we take is that the two modes are *different* and, in most cases, complementary (Bruner, 1986; Hymes & Cazden, 1980).
>
> (Pelligrini and Galda, 1988, p. 184)

Pelligrini and Galda found that the children moved between use of both types, paradigmatic and narrative, and that the children were more likely to use narrative language in cases of ambiguity than in clearly defined situations (1988, pp. 184-185). The findings in the current study indicated that all children used both descriptions and mathematical justifications. The children who were more adept and at ease in the Logo graphics setting, namely those who were highly adept and moderately adept, seemed to use more mathematical justifications than did the children who were less adept and who were perhaps feeling 'ambiguous' and unsure (see Tables 32 and 33).

31. Lilly and Rina could not understand why the first set of commands which resulted in a triangle when they were at computer #6 comprised a short list, and yet at computer #2, their list was much longer, and the triangle was not at all as symmetrical. The reason the second attempt resulted in a longer listing was that they had to adjust due to the error in the turn (RT 160 instead of RT 260), and also due to erroneous penup and pendown moves. The selections below illustrate the confusion experienced by Rina, Lilly, and Monica, the Logo teacher, as well.

Dec. 11, 1985, p. 15

Rina M.: Lilly, we went from this, we went from this many commands [that is, few] and it worked, to this many commands [that is, more]. Wierd.

.
.
.

Dec. 11, 1985, p. 16

Monica T.: It's much too much though. You better divide it up.
Rina M.: We can't.

Dec. 11, 1985, p. 25.

Rina M.: It stops off 'There at -F-D eighty.
Lilly M.: 'It stops off right here.

.
.
.

But, on that computer, and now we tried it here and it didn't work, and like-

Monica T.: Looking at the book, tracing along the page) Then there was a problem in here (pause).

Rina M.: Then how come it worked *(?inaudible)

Lilly M.: *(?It couldn't, there), because it worked right there (points to computer #6), on that computer.

Monica indicated that the listing of commands was too long; she wanted them to "divide it up." Rina M. and Lilly M. knew that they could not make it shorter; they needed all the steps in order to make their triangle. It was a 'catch-22" (Joseph Heller, 1961). They could not find the error, and therefore the procedure became too long; because the procedure was too long and convoluted, the chances of finding the error, either themselves or with Monica's help, lessened.

32. The selection below illustrates how the children deplanned. The term "deplanning" is a derivative of Noss's term "de-goaling" (Noss, 1985). Noss defined the "de-goaling" strategy as follows:

> The [de-goaling] strategy was exploited extensively by children who, in seeing that the turtle's movements did not correspond to those intended, consistently preferred to modify their goal rather than to debug their drawings. (Noss, 1985, pp. 138-139)

Noss then proceeded to distinguish two categories. "The first category of 'de-goaling' strategy was that which involved the incorporation of errors into the picture, without changing the planned picture in its entirety" (1985, p. 139). This type of de-goaling strategy was used by Lilly M. and Rina M. in their just-completed project, the "ghetto.blaster"; the drawing, though lopsided and not a reflection of their more symmetrical notebook drawing, nevertheless was considered by them to still adequately represent a large portable cassette tape player (see Appendix C). "The second category involved a 'post-hoc' redefinition of what the picture represented, as a result of its failure to look like

the intended end result" (Noss, 1985, p. 139). This "redefinition" is exemplified by what transpired during Lilly and Rina's "star of David" project. Parts of the transcript follow below.

Dec. 11, 1985, pp. 20-21.

Lilly M.:	(Pounds Rina's arm) Told you you went wrong somewhere.
Rina M.:	(Looks in her book) So we did it at -B (pause) (Looks at Lilly, makes a noise) [Rina is incorrect.]
Lilly M.:	(Looking at the screen) What could this look like? What does this look 'like?
Rina M.:	'Ya!
Lilly M.:	Two mountains we can make. (Pause) Mountains! And then we can have a sun.
.	
.	
.	
Lilly M.:	O. K. So let's (?change)-
Rina M.:	O. K. (erasing in her book). Let's change the picture. O. K.

33. In the analysis of the pattern of the Logo teacher, Monica's, intervention with the children (Zack, 1986a), it was noted that in the case of all the children but Russ H., the majority of the instances of teacher intervention were child-elicited. Russ's case was an exception; the teacher was seen to initiate a substantial number of interventions with Russ H. when he was partners with Jon H. and Eric H.; there were few child-elicited calls for intervention once Russ began working with Jon and Eric. To Russ, Monica stressed recording the procedures consistently, keeping the procedures short, and sharing typing time with his partners. Russ at times

forgot his computer notebook, and the rule about having the notebook present in which to record was a strict rule in the Logo computer lab. It was interesting to note how nonchalant Russ was about having his notebook, and about recording, although when he did record he was very organized. When working in a triad, the boys alternated in serving as recorder. There was no insistence upon, or felt need to have, each child record.

In describing Russ and his work habits in the regular classroom, one of Russ's regular classroom teachers commented that he was akin to the absent-minded professor. In giving an example, Mrs. W. (the teacher of the Hebrew subjects) stated that Russ was a superb pianist, but the kind of child that one would worry would not remember to bring his sheet music to his recital. Another teacher, Mrs. B. (the teacher of the English subjects), noted that when he moved from place to place, or did his work, Russ was relaxed and easy-going, almost too relaxed. He did not get frazzled no matter what happened. The fact that Russ did not get flustered when his work was erased may thus in part have been due to his ability to recreate easily, and in part due to his temperament.

34. There were two striking examples of instances when Rina M. and Lilly M., and Amy L. and Leanne L., lost the work done during the session due to syntax problems. Rina and Lilly had great difficulty with the "save" and "load" syntax, and at one point thought that they had lost all the work that they had done during the session. In actual fact, they were not able to "save" due to their not having defined their work in the editor. They had to reenter their commands a number of times due to various syntax errors, and as a result were very frustrated at the slow pace and the need to redo. In Amy L. and Leanne L.'s case, in three repeated attempts to save their work (twice on December 4, and once on Dec. 11, 1985), Amy L. kept turning the computer off. Amy had incorrectly generalized that when the disk was put in (in this case to "save"), the computer had to be turned off as was the case when one loaded the Logo systemmaster. In order to "save" the work in memory, one of course ought not to turn the computer off! Amy and Leanne kept reentering the data; and Amy would turn the machine off, put the 'save' disk in, and attempt to save. Monica thought that the Amy and Leanne were accidentally pressing the reset button. Finally, upon one occasion, Monica was present,

standing beside them, and was able to observe what was actually happening, and then explained to Amy why she must never turn off the computer once the data was entered if she wished to "save" her work on her disk. In both instances, the pairs were extremely frustrated.

35. Hillel (1984) found that 90 was used by the children as an input to RT or LT; however, it is also vital to point out that the researchers introduced 90 to the group of 9 to 10 year old participants during session #3 of the Hillel (1984) study. In this study, 90 was introduced in Grade 3 by the Logo teachers, when the children were 8 to 9 years old, and was mentioned to some small groups or pairs during the Grade 4 sessions, when the children were 9 to 10 years old (for example, Carol L. and Amy L., Selection #19 and Appendix Q). Some of the children retained the information, but the majority in this study were seen to be working toward "discovering" it during their Grade 5 Logo sessions. Hillel (1984) reported that the 9 to 10 year old children in his study demonstrated an understanding of the concept of differentiating acute and obtuse angles (i. e. angles relative to 90) (1984, p. 46). In this study, the researcher could not detect that the children who worked with 90 in a rote way were using 90 as a gauge. Russ H. and Jon H. seemed to be doing so implicitly, yet never spoke of it. During the Grade 6 interviews (May-June, 1987), 14 of the 56 children, or 25%, used 90 as a reference point, in order to defend their position that certain blackboard angles were either acute or obtuse (Appendix H, Question 5).

36. As has been noted in the study of the children's notion of right angle (Zack, 1986d), input numbers such 90 or 45 were seen to be used by certain of the twenty-three Grade Five children in the study for specific purposes, but with no appreciation of the larger geometric picture. During the Grade 5 interview (Feb. 1986), Joey L. spoke of using a 45 as input to RT or LT whenever a diagonal line was needed:

Grade 5 interview with Joey L., Feb. 27, 1986

Joey L.: Forty-five is when he's turning, like diagonally, like that, and 90 he's straight. (Joey shows the turn on the corner of the table with the chocolate "turtle" given to him by the researcher.)

Interviewer: How do you know it [turtle] will face like that?

Joey L.: -R-T 90 is like that, but if you go 45, it will probably face a corner.

Joey expressed the key which some children were using when they needed a 45 degree rotation for their drawings. Kieran (personal communication, May 1986) has noted that the Grade 6 children whom she observed tended to associate 45 with the diagonal dimension in their Logo drawings. In this study, none of the 7 children listed as working at the visual level in this study were seen to use these visual cues.

Stuart M. used both RT and LT 90, or SETH 90 to go "exactly straight," that is along the horizontal or vertical axis. Yet when he was asked during the interview what the turtle cursor would turn to make the same rotation as was being made by the turn between the rays of the right angle (90 degree angle) on the blackboard, Stuart said that the turtle would turn RT 40 or 30. Despite the fact that Stuart used 90 to make corners for squares and rectangles, he had not made the connection between input to RT and LT, and the use of measures of degrees in static Euclidean geometry.

At the end of their Grade 5 year, the twenty three children in homeroom 210 were interviewed individually, and were also asked an across the class pencil and paper question, which attempted to elicit from them whether they detected a connection between the input to RT or LT, and the number of degrees of turn in a drawn blackboard and worksheet set of 3 angles (Appendix F). They had done work in classroom (static) geometry as part of their Grade 5 mathematics curriculum. Fifteen of the 23 children, or 65%, did not seem to make the connection between input to LT and RT on one hand, and the number of degrees

in an angle drawn on paper on the other hand; 8 of the 23, or 35%, had made the connection. When the question was posed again in Grade 6, both on paper and during the interviews, the former homeroom 210 students responded as follows: 12 of the 21, or 57%, did not seem to see the direct connection; 9 of the 21, or 43%, had made the connection. There seemed to have been some change, perhaps due to more experience (although no Logo was done in Grade 6-- other Logowriter features were explored by all the children; and the classroom teachers stressed that they had had no time to do much geometry), or maturation. It was of interest to note that two of the children who answered correctly in Grade 5, namely Shauna L. and Joey L., did not answer correctly in Grade 6. The responses of the Grade 6 group taken as a whole revealed a bit of a different pattern from that of the former homeroom 210 group: 23 of the 55, or 42%, did not see the connection clearly, namely that turtle turn steps were measured in degrees; 29 of the 55, or 53%, did see the connection. The average achievement for the former homeroom 210 children was a little below the average achievement of the group as a whole.

	NO Did not see the connection	YES Did see the connection
Gr. 5	15/23 = 65%	8/23 = 35%
Gr. 6 (former) (HR 210)	12/21 = 57%	9/21 = 43%
Gr. 6	23/55 = 42%	29/55 = 53%

37. Indeed, it may at times be more pragmatic not to share the roles equally, for as one of the children (Jay H.) commented in the May-June 1987 interviews, working together allowed the partners to each contribute to the group what they were able to do best. Jay H. offered an example: "If we have to go fast, the best typer would

type." Russ was at times requested to take over the typing when the work needed to be done in a hurry. However, it did not seem that most of the time the children in the study were willing to relinquish their "due" in deference to the partner who was "better" at the job than they were. During the sessions, it was the researcher's impression that many of the children were emphatic about getting equal time doing various "jobs," especially as far as the typing was concerned.

38. It seemed to this researcher that a setting in which tasks were assigned, as opposed to the setting in this study wherein projects were child-selected and the children often were not even aware of the geometry inherent in the Logo microworld, would be more conducive to the use of analytic approaches, hence the reference to the pre-determined tasks in the Hillel and Kieran study (in press). A recent update concerning some of the findings in the aforementioned research work of Hillel, Kieran and Gurtner was therefore most illuminating. At a recent Psychology of Mathematics Education Conference presentation, Kieran (Hillel, Gurtner, & Kieran, 1988) reported that when given a task with certain specifications, the children were seen to take the specifications into account while working with pencil and paper; but once they entered their program into the computer, and then needed to make changes, they proceeded to de-structure the task and to ignore the essential task specifications. Instead of using analytic means, many of which were evident in their pencil and paper plans, the students attempted to adjust their graphic screen product by means of visual strategies. The consideration of how the medium of the computer may be influencing the strategies that students choose to use is just in the beginning stages. More investigation is needed. Leaving it as a "task versus child-chosen project" dichotomy would certainly not do justice to the complexity of the issue.

39. In the Clements, at al., (1988) study, the children at the Grade 2, Grade 4, and Grade 5 level (in three different elementary school sites) meet with their own classroom teachers in the computer room for 45 minutes each day, five days a week, in blocks of two weeks at a time. The Logo microcomputer environment in the Clements (1988) study has been altered in a number of ways so that comparisons with this study are not able to be made. However, the fact that in

the Clements (1988) study, the classes take place on consecutive days, and that the classroom teachers can use some of the classroom time to continue discussion, and can take the opportunity within the homeroom setting to reinforce certain aspects in pertinent situations, might be aspects to consider when evaluating the conditions within the present study with a view to suggesting alternatives.

Of the aforementioned studies, the Hillel (1984) study was somewhat comparable to the present study (1984), in terms of the time frame (12 weeks) and the age of the children. The findings (which have been documented elsewhere (Zack, 1985)), were that Carol L. and Amy L., admittedly at the low level of achievement, did not achieve the same level of competence in mathematics concepts attainment or in programming skills as was achieved by the Hillel study subjects. The discrepancy in the pace of the learning may have been due to the fact that Carol L. and Amy L. may not have been as able as the Hillel subjects, but may also be due to the fact that there was a mathematics agenda in place in the researcher's approach to the study, and there was some assignment of tasks in the Hillel study. The setting of the Hillel study was a computer lab which was housed at Concordia University (in Montreal, Quebec), in the Mathematics Department; the location may in itself have lent prestige to the enterprise, and may have made the children more serious about the activity than were the children in the present study who considered their time in the lab a "break" from the school routine.

Indications seem to point to the possibility that the learning of Logo geometry concepts could be enhanced if the involvement with Logo were more concentrated in terms of time, and if the teaching staff (both Logo and classroom staff in this case) were to make more obvious what remains a "hidden agenda" for many of the children, namely the mathematics component inherent in the Logo microworld.

REFERENCES

Abraham, J., & Bibby, N. (1988). Mathematics and society: Ethnomathematics and a public educator curriculum. For the Learning of Mathematics, 8, 2, 2-11.

Adams, A. (1984). Talk and learning in the classroom: An interview with Anthony Adams. Dennis Searle, interviewer. Language Arts, 61 (2), 119-124.

Allaire, R. (1985). Le concept d'angle dans un environnement Logo. Presentation at the National Council of Teachers of Mathematics Conference, October 3-5, 1985, Montreal, Quebec.

Ames, G. J., & Murray, F. (1982). When two wrongs make a right: Provoking cognitive change by social conflict. Developmental Psychology, 18 (6), 894-897.

Archambault, R. D. (Ed.) (1964). John Dewey on education: Selected writings. Chicago: The University of Chicago Press.

Bales, R. (1950). Interaction process analysis: A method for study of small group interaction. Addison-Wesley.

Barfurth, M. A. (1987). Learning math through Logo: A procedure-based approach. Unpublished master's thesis, Master in the Teaching of Mathematics, Concordia University, Montreal, Quebec.

Barnes, D. (1976). From communication to curriculum. Harmondsworth, England: Penguin Books.

Barnes, D., Britton, J., & Rosen, H. (1971). Language, the learner, and the school. (rev. ed.) Harmondsworth, England: Penguin Books.

Bell, N., Grossen, M., & Perret-Clermont, A-N. (1985). Sociocognitive conflict and intellectual growth. In M. W. Berkowitz (Ed.), Peer conflict and psychological growth. San Fancisco: Jossey-Bass Inc.

Berry, K. S. (1985). Talking to learn subject matter/Learning subject matter talk. Language Arts, 62 (1), 34-42.

Borgh, K., & Dickson, W. P. (1986). Two preschoolers sharing one computer: Creating prosocial behavior with hardware and software. In P. Campbell & G. Fein (Eds.), Young children and microcomputers. New Jersey: Prentice Hall.

Britton, J. (1970). Language and learning. London: Allan Lane The Penguin Press.

-------- (1980). Shaping at the point of utterance. In A. Freedman & I. Pringle (Eds.), Reinventing the rhetorical tradition. Ottawa, Ontario: The Canadian Council of Teachers of English, Carleton University, Ottawa, Canada K1S 5B6.

Brown, S. I. (1973). Syracuse University Logo Project for 1972: Final report. NSF-TIE-GJ 32222-3, June, 1973. SU/LOGO Project, 313 Link Hall, Syracuse, New York, 13210.

Bruner, J. S. (1985). Vygotsky: A historical and conceptual perspective. In J. V. Wertsch (Ed.), Culture, communication and cognition: Vygotskian perspectives. New York: Cambridge University Press.

-------- (1986). Actual minds, possible worlds. Cambridge: Harvard University Press.

Burnett, D. (1981). Educational psychology and mathematical knowledge: An analysis of two student protocols. Focus on Learning Problems in Mathematics, 3 (2 & 3), 1-167.

Burton, R. R., Brown, J. S., & Fischer, G. (1984) Skiing as a model of instruction. In B. Rogoff & J. Lave (Eds.), Everyday cognition: Its development in social context. Harvard University Press.

Butterworth, G., & Light, P. (Eds.). (1982). Social cognition: Studies of the development of understanding. Chicago: The University of Chicago Press.

Campbell, P., & Fein, G. (Eds.). (1986). Young children and microcomputers. Englewood Cliffs, New Jersey: Prentice Hall.

Carmichael, H., Burnett, J. D., Higginson, W., Moore, B., & Pollard, P. J. (1985). Computers, children and classrooms: A multisite evaluation of the creative use of microcomputers by elementary school children. (Final Report on the Creative Uses Project, to the Ontario Ministry of Education). The Ontario Government Bookstore, Publication Centre, 880 Bay St., 5th Floor, Toronto, Ontario M7A 1N8

Carraher, T. (1988). Street mathematics and school mathematics. Proceedings of the Twelfth International Conference of the International Group for the Psychology of Mathematics Education (PME 12), Hungary, July 20-25, 1988, 1-23.

Cazden, C. B. (1981). Performance before competence: Assistance to child discourse in the zone of proximal development. The Quarterly Newsletter of the Laboratory of Comparative Human Cognition, 3, (1), 5-8.

-------- (1986). Classroom discourse. In M. C. Wittrock (Ed.), Handbook of research on teaching. Third edition. New York: MacMillan Publishing Company.

Cazden C., John, V. P., & Hymes, D. (Eds.). (1972). Functions of language in the classroom. New York: Teachers College Press.

Childs, C. P., & Greenfield, P. M. (1982). Informal modes of learning and teaching: The case of Zinacenteco weaving. In N. Warren (Ed.), Advances in cross-cultural psychology. (Vol. 2) New York: Academic Press.

Church, M. J., & Wright, J. L. (1986). Creative thinking with the microcomputer. In P. C. Campbell & G. Fein (Eds.), Young children and microcomputers. Englewood Cliffs, New Jersey: Prentice Hall.

Clements, D., Battista, M., Beckum, F., Hazlett, M. B., & Hallenbeck, L. (1988). NSF Logo and geometry project. Presentation at the Great Lakes-East Coast Logo Conference, Cleveland, Ohio, May 5-7, 1988. Educational Computer Consortium of Ohio (ECCO), 1123 S. O. M. Center Road, Cleveland, Ohio 44124.

Close, G. S. (1981). Children's understanding of angle at the primary/secondary transfer stage. Unpublished master's thesis, The Department of Mathematical Sciences and Computing, Polytechnic of the South Bank, Borough Road, London SE1 0AA

Cooper, C. R. (1980). Development of collaborative problem solving among preschool children. Developmental Psychology, 16, 433-440.

Cooper, C. R., & Cooper, R. G. (1984). Skill in peer learning discourse: What develops? In S. A. Kuczaj (Ed.), Discourse development: Progress in cognitive development research. New York: Springer-Verlag.

Cooper, C. R., Marquis, A., & Ayers-Lopez, S. (1982). Peer learning in the classroom. In L. C. Wilkinson (Ed.), Communicating in the classroom. New York: Academic Press.

Cooper, C. R., Marquis, A., & Edward, D. (1986). Four perspectives on peer learning among elementary school children. In E. C. Mueller & C. R. Cooper (Eds.), Process and outcome in peer relationships. Toronto: Academic Press.

Cox, D., & Berger, C. F. (1981). Microcomputers are motivating. Science and Children, September 1981.

Dienes, Z. P. (1970). Some basic processes involved in mathematics learning. In R. B. Ashlock & W. L. Herman, Jr. (Eds.), Current research in elementary school

mathematics. Toronto: Collier-MacMillan Canada.

Dillon, D. (1984). Editorial: Talk. Language Arts, 61 (2), 115.

Dickson, W. P., & Vereen, M. A. (1983). Two students at one microcomputer. Theory into Practice, 22, 296-300.

Doise, W., & Mugny, G. (1981). Le développement social de l'intelligence. Paris: Inter-éditions.

Donaldson, Morag L. (1986). Children's Explanations. New York: Cambridge University Press.

Duckworth, E. (1979). Either we're too early and they can't learn it or we're too late and they know it already: The dilemma of "applying Piaget." Harvard Educational Review, 49, (3), 297-312.

Dunkin, M. J., & Biddle, B. J. (1974). The study of teaching. New York: Holt, Rinehart & Winston.

Edelsky, C. (1986). Whole language: Sharpening up, moving up. Presentation at the Springboards '87 Quebec Language Arts Conference, April 9-10, 1987, Montreal, Quebec.

Edwards, L. (1988). Children's learning in a transformation geometry microworld. Proceedings of the Twelfth Annual Conference of the International Group for the Psychology of Mathematics Education (PME 12), Hungary, July 20-25, 1988, 263-270.

Eisenberg, A. R., & Garvey, C. (1981). Children's use of verbal strategies in resolving conflicts. Discourse Processes, 4 (2), 149-170.

Ellis, S., & Rogoff, B. (1986). Problem solving in children's management of instruction. In E. C. Mueller & C. R. Cooper (Eds.), Process and outcome in peer relationships. Academic Press.

Emihovich, C., Miller, G. E., & Clare, V. (1985). Learning Logo: The social context of cognition. (ERIC Document Reproduction Service No. ED 261 662)

Erlwanger, S. H. (1975). Case studies of children's conceptions of mathematics. The Journal of Children's Mathematical Behavior, 1 (3), 157-283.

Feurzig, W., Papert, S., Bloom, M., Grant, R., & Solomon, C. (1969). Programming languages as a conceptual framework for teaching mathematics. (Report 1889). Cambridge, Massachusetts: Bolt, Baranek and Newman.

Fielker, D. S. (1979). Strategies for teaching geometry to younger children. Educational Studies in Mathematics, 10, 85-133.

Flanders, N. (1970). Analyzing teacher behavior. Addison-Wesley.

Forman, E. A. (1981). The role of collaboration in problem-solving in children. Unpublished doctoral dissertation, Harvard University.

Forman, E. A., & Cazden, C. B. (1985). Exploring Vygotskian perspectives in education: The cognitive value of peer interaction. In J. V. Wertsch (Ed.), Culture, communication and cognition: Vygotskian perspectives. New York: Cambridge University Press.

Forman, E. A., & Kraker, M. J. (1985). The social origins of logic: The contributions of Piaget and Vygotsky. In M. W. Berkowitz (Ed.), Peer conflict and psychological growth. San Francisco: Jossey-Bass Inc.

Freudenthal, H. (1973). Mathematics as an educational task. Dordrecht, Holland: Reidel.

Frye, N. (1982). The great code: The bible and literature. New York: Harcourt, Brace, Jovanovich.

Fuys, D., Geddes, D., & Tischler, R. (Eds.) (1984). English translations of selected writings of Dina van Hiele-Geldof and Pierre M. van Hiele. Brooklyn, New York: Brooklyn College, School of Education.

Fuys, D., Geddes, D., & Tischler, R. (1985). An investigation of the van Hiele model of thinking in geometry among adolescents. (Final Report). Brooklyn New York: Brooklyn College School of Education, Brooklyn, New York 11210

Gay, J., & Cole, M. (1967). The new mathematics and an old culture: A study among the Kpelle of Liberia. New York: Holt, Rinehart and Winston.

Genishi, C. (1982). Observational research methods for early childhood education. In B. Spodek (Ed.), Handbook of research in early childhood education. New York: The Free Press.

Genishi, C. (1984). Research currents: What is a context for learning through language. Language Arts, 61 (1), 52-58.

Genishi, C. & Di Paolo, M. (1982). Learning through arrangement in a preschool. In L. C. Wilkinson (Ed.), Communicating in the classroom. New York: Academic Press.

Glachan, M., & Light, P. (1982). Peer interaction and learning: Can two wrongs make a right? In G. Butterworth & P. Light (Eds.), Social cognition: Studies of the development of understanding. Chicago: The University of Chicago Press.

Goodman, Y. (1985). Kidwatching: Observing children in the classroom. In A. Jaggar & M. T. Smith-Burke (Eds.), Observing the language learner. Urbana, Illinois: National Council of Teachers of English.

Goodman, K. S., & Goodman, Y. (1981). A whole-language, comprehensive reading program. A position paper. Program in Language and Literacy, University of Arizona, Tucson, Arizona.

Goodman, K. S., Goodman, Y., & Burke, C. C. (1978). Reading for life: The psycholinguistic base. In E. Hunter-Grundin & H. U. Grundin (Eds.), Reading: Implementing the Bullock report. London: Ward Lock.

Gouvernement du Québec. (1985). Elementary school curriculum guide: English language arts, cycle I, elementary 1,2,3.

Graves, D. (1982). Writing: Teachers and children at work. Heinemann.

Greenfield, P., & Lave, J. (1982). Cognitive aspects of informal education. In D. A. Wagner, & H. W. Stevenson (Eds.), Cultural perspectives on child development. San Francisco: W. H. Freeman and Company.

Greenfield, P. M. (1984). A theory of the teacher in the learning activities of everyday life. In B. Rogoff & J. Lave (Eds.), Everyday cognition: Its development in social context. Harvard University Press.

Groen, G. (1978). The theoretical ideas of Piaget and educational practice. In P. Suppes (Ed.), Impact of research on education: Some case studies. Washington, D. C.: National Academy of Education.

Gurtner, J. L. (1987). Success and understanding while solving geometrical problems in Logo. Proceedings of the Eleventh Annual Conference of the International Group for the Psychology of Mathematics Education (PME 11), Montreal, Canada, July 19-25, 1987, 229-235.

Halliday, M. A. K. (1974). Language and social man. England: School Council Publications.

Halliday, M. A. K. (1981). Three aspects of children's language development: Learning language, learning through language, learning about language. In Y. M. Goodman, M. M. Haussler, & D. S. Strickland (Eds.), Oral and written

language development research: Impact on the schools. Urbana, Illinois: National Council of Teachers of English.

Hamlyn, D. (1982). What exactly is social about the origins of understanding? In G. Butterworth & P. Light (Eds.), Social cognition: Studies of the development of understanding. Chicago: The University of Chicago Press.

Hare, A. P. (1962). Handbook of small group research. New York: The Free Press.

Harste, J. C., Woodward, V. A., & Burke, C. L. (1984). Examining our assumptions: A transactional view of literacy and learning. Research in the Teaching of English, 18 (1), 84-108.

Hawkins, J. (1983). Learning Logo together: The social context. (Technical Report No. 13). New York: Bank Street College of Education.

Hawkins, J., Sheingold, K., Gearhart, M. & Berger, C. (1982). Microcomputers in schools: Impact on the social life of elementary classrooms. Applied Developmental Psychology, 3, 361-373.

Herscovics, N., & Bergeron, J. (1980). The training of teachers in the use of clinical methods. Proceedings of the Fourth International Conference for the Psychology of Mathematics Education, Berkeley, California, August 16-17, 1980, 384-393.

Herscovics, N., & Bergeron, J. C. (1983). Models of understanding. Zentralblatt für Didaktik der Mathematik, 2, 75-83.

Herscovics, N., & Bergeron, J. C. (1984). A constructivist versus a formalist approach in the teaching of mathematics. Proceedings of the Eighth International Conference for the Psychology of Mathematics Education, Sydney, Australia.

Higginson, W. (1984). About that rose garden: Remarks on Logo, learning, children and schools. Pre-proceedings of the 1984 National Logo Conference, Massachusetts Institute of Technology, June 26-29, 1984.

Hillel, J. (1984). Mathematical and programming concepts acquired by children, aged 8-9, in a restricted Logo environment. (Research Report No.1). Department of Mathematics, Concordia University, Montreal, Quebec. NOTE:The research report has been published elsewhere. Look for: Hillel, J. (1985). Mathematical and programming concepts acquired by children aged 8-9 in a restricted LOGO environment. Recherches en Didactique des Mathématiques, 6, (2.3).

-------- (1985). On Logo squares, triangles and houses. For the Learning of Mathematics, 5 (2), 38-45.

-------- (1986). Should Logo be implemented in the upper elementary school curriculum? Opening plenary presentation, Second International Conference for Logo in Mathematics Education, University of London Institute of Education, July 15-18, 1986.

Hillel, J., Gurtner, J. L., & Kieran, C. (1988). Structuring and destructuring a solution: An example of problem solving work with the computer. Proceedings of the Twelfth International Conference of the International Group for the Psychology of Mathematics Education, Hungary, July 20-25, 1988, 402-409.

Hillel, J., & Kieran, C. (in press). Schemas used by 12 year-olds in solving selected turtle geometry tasks. Recherches en Didactique des Mathématiques.

Hillel, J., Kieran, C., Erlwanger, S., & Winer, L. (1986). Turtle geometry without FD and BK: Sabotaging the drawing schema. In B. Harvey (Ed.), Proceedings of the 1986 Logo Conference, Massachusetts Institute of Technology, Boston, MA.

Hoffer, A. (1981, January). Geometry is more than proof. Mathematics Teacher, 11-18.

Hood, L., McDermott, R., & Cole, M. (1980). "Let's try to make it a good day"--Some not so simple ways. Discourse Processes, 3, 155-168.

Hoyles, C. (1985). Culture and computers in the mathematics classroom. An inaugural lecture delivered at the University of London Institute of Education, June 20, 1985. University of London Institue of Education, 20 Bedford Way, London WC1H OAL.

Hoyles, C., & Noss, R. (1987). Seeing what matters: Developing an understanding of parallelogram through a Logo microworld. Proceedings of the Eleventh Annual Conference of the International Group for the Psychology of Mathematics Education (PME 11), Montreal, July 19-25, 1987, 17-23.

Hoyles, C., & Noss, R. (1988). Formalizing intuitive descriptions in a parallelogram Logo microworld. Proceedings of the Twelfth Annual Conference of the International Group for the Psychology of Mathematics Education (PME 12), Hungary, July 20-25, 1988, 417-424.

Hoyles, C., & Noss, R. (in press). Children working in a structured Logo environment: From doing to understanding. Recherches en didactiques des mathématiques.

Hoyles, C., & Sutherland, R. (1986a). Peer interaction in a programming environment. Proceedings of the Tenth Annual Meeting of Psychology of Mathematics Education (PME 10), July 20-25, 1986, London, England, 354-362.

Hoyles, C., & Sutherland, R. (1986b). When 45 equals 60. Proceedings of the Second International Conference for Logo and Mathematics Education, July 15-18, 1986, University of London Institute of Education, 54-63.

Hoyles, C., & Sutherland, R. (in press). Logo mathematics in the classroom. Routledge.

Hoyles, C., Sutherland, R., & Evans, J. (1985). A preliminary investigation of the pupil-centred approach to the learning of Logo in the secondary school mathematics classroom. Logo Maths Project, University of London Institute of Education, 20 Bedford Way, London WC1H OAL.

Hymes, D., & Cazden, C. (1980). Narrative thinking and story-telling rights: A folklorist's clue to a critique of education. In D. Hymes (Ed.), Language in education: Ethnolinguistic essays (pp. 126-138). Washington, D. C.: Center for Applied Linguistics.

Jaggar, A., & Smith-Burke, M. T. (Eds.). (1985). Observing the language learner. Urbana, Illinois: National Council of Teachers of English.

Johnson, D.W., & Johnson, R.T. (1979). Conflict in the classroom: Controversy and learning. Review of Educational Research, 49, 51-70.

Kamii, C., & De Vries, R. (1978). Physical knowledge in preschool education: Implications of Piaget's theory. New Jersey: Prentice Hall.

Kieran, C. (1986). Logo and the notion of angle among fourth and sixth grade children. Proceedings of the Tenth Annual Meeting of Psychology of Mathematics Education (PME 10), July 20-25, 1986, London, England, 99-104.

Kieran, C., Hillel, J., & Erlwanger, S. (1986). Perceptual and analytic schemas in solving structured turtle geometry tasks. Proceedings of the Second International Conference for Logo and Mathematics Education, July 15-18, 1986, University of London Institute of Education, 154-161.

Kieran-Sauvé, C., & Taurisson, A. (1985). Logo et la notion d'angle en quatrième année. Unpublished paper, Université du Québec à Montréal, Montreal, Quebec.

Kieren, T. (1984). Logo in education: What, how, where, why and consequences. Edmonton, Alberta: University of Alberta.

Kieren, T. (1987, January). Language use in mathematics. Visiting lecture, January 26, 1987, Concordia University Department of Mathematics, Montreal, Quebec.

Klein, S. (1988). A look at the affective side of mathematics learning in Hungarian secondary schools. Proceedings of the Twelfth Annual Conference of the International Group for the Psychology of Mathematics Education (PME 12), Hungary, July 20-25, 1988, 24-53.

Kull, J. A. (1986). Learning and Logo. In P. Campbell & G. Fein (Eds.), Young children and microcomputers. New Jersey: Prentice Hall.

Kynigos, C. (1986). Constructing a conceptual pathway from intrinsic to extrinsic geometry. Proceedings of the Second International Conference for Logo and Mathematics Education, July 15-18, 1986, University of London Institute of Education, 64-71.

-------- (1987a). A case study of two 12 year-old children changing turtle's state in the coordinate plane. Proceedings of the Third International Conference for Logo and Mathematics Education, July 14-18, 1987, Concordia University, Montreal, Canada, 86-93.

-------- (1987b). From intrinsic to non-intrinsic geometry. Proceedings of the Eleventh Annual Conference of the International Group for the Psychology of Mathematics Education (PME 11), Montreal, July 19-25, 1987, 43-49.

-------- (1988). Constructing bridges from intrinsic to Cartesian geometry. Proceedings of the Twelfth Annual Conference of the International Group for the Psychology of Mathematics Education (PME 12), Hungary, July 20-25, 1988, 449-456.

Labov, W., & Fanshel, D. (1977). Therapeutic discourse: Psychotherapy as conversation. New York: Academic Press.

Lave, J., Murtaugh, M., & de la Rocha, O. (1984). The dialectic of arithmetic in grocery shopping. In B. Rogoff & J. Lave (Eds.), Everyday cognition: Its development in social context. Cambridge, MA: Harvard University Press.

Lee, B. (1985). Origins of Vygotsky's semiotic analysis. In J. V. Wertsch (Ed.), Culture, communication and cognition: Vygotskian perspectives. New York: Cambridge University Press.

Lehr, F. (1985). ERIC/RCS report: Instructional scaffolding. Language Arts, 62 (6), 667-672.

Leron, U. (1985). Logo today: Vision and Reality. The Computing Teacher. February, 26-32.

Levin, J. A. & Kareev, Y. (1980). Problem solving in everyday situations. The Quarterly Newsletter of the Laboratory of Comparative Human Cognition, 2 (1), 47-52.

Ludwig, S. C. (1986). Indicators of growth within a Logo/motion geometry curriculum environment. Unpublished master's thesis, University of Alberta, Edmonton, Alberta.

MacKay, Robert. (1974). Standardized tests: Objective/objectified measures of competence. In Cicourel, A. V., Jennings, K. H., Jennings, S., Leiter, K. C. W., MacKay, R., Mehan, H., and Roth, D. R., Language use and school performance. New York: Academic Press.

Maniatis, E. G. (1983). An analysis of the differences in problem solving of gifted and non-gifted children using the Logo programming language. Unpublished master's thesis, McGill University, Montreal, Quebec.

Martin, N. (1987). In their own words: The potential of students' talk. Presentation at the Springboards '87 Quebec Language Arts Conference, April 9-10, 1987, Montreal, Quebec.

Martin, N., Williams, P., Wilding, J., Hemmings, S., & Medway, P. (1976). Understanding children talking. Penguin, 1976.

Mathison, S. (1988). Why triangulate? Educational Researcher, 17, (2), 13-17.

McLane, J. B. (1981). Dyadic problem solving: A comparison of child-child and mother-child interaction. Unpublished doctoral dissertation, Northwestern University.

Mendelson, D. (1987). Using Logo as a tool for developing the concept of complementary/supplementary angles in Grade 7. Proceedings of the Third International Conference for Logo and Mathematics Education, July 14-18, 1987, Concordia University, Montreal, Canada.

Mendelson, D. (1988). Using Logo to develop the concept of angle in Secondary I. Unpublished master's thesis, Master in the Teaching of Mathematics, Concordia University, Montreal, Canada.

Miller, S.A., & Brownell, C.A. (1975). Peers, persuasion and Piaget: Dyadic interaction between conservers and non-conservers. Child Development, 46, 992-997.

Miller, D. S., & MacGregor, S. K. (1988). A study of instructional uses of microcomputers in elementary classrooms and computer labs. Paper presented at the annual meeting of the American Educational Research Association, April 5-9, 1988, New Orleans, LA.

Mueller, E. C., & Cooper, C. R. (Eds.). (1986). Process and outcome in peer relationships. Toronto: Academic Press.

Mugny, G., Giroud, J., & Doise,W. (1979). Conflit de centration et progrès cognitif. Bulletin de psychologie, 32, 978-985.

Murray, D. (1980). The feel of writing--and teaching writing. In A. Freedman & I. Pringle, (Eds.), Reinventing the rhetorical tradition. Ottawa, Ontario: The Canadian Council of Teachers of English.

Noss, R. (1985). Creating a mathematical environment through programming: A study of young children learning Logo. Published doctoral dissertation, University of London Institute of Education.

Palincsar, A. S. (1986). The role of dialogue in providing scaffolded instruction. Educational Psychologist, 21(1&2), 73-98.

Papert, S. (1972). Teaching children to be mathematicians versus teaching about mathematics. International Journal of Mathematics, Education, Science and Technology, 3, 249-262.

Papert, S. (1980). Mindstorms: Computers, children and powerful ideas. New York: Basic Books.

Papert, S., Abelson, H., Bamberger, J., di Sessa, A., Weir, S., & Watt, D. (1978). Interim report of the Logo project in the Brookline public schools: An assessment and documentation of a children's computer laboratory. Cambridge, Massachusetts: MIT AI Lab, Logo Memo No. 49.

Papert, S., Watt, D., di Sessa, A., & Weir, S. (1979). Final report of the Brookline Logo Project Part II: Project summary and data analysis. Cambridge, Massachusetts: MIT AI Lab, Logo Memo No. 53.

Pea, R. D. (undated). Logo programming and problem solving. (Technical Report No. 12). New York: Bank Street College of Education.

Pelligrini, A. D., & Galda, L. (1988). The effects of age and context on children's use of narrative language. Research in the Teaching of English, 22, (2), 183-195.

Perret-Clermont, A.-N. (1980). Social interaction and cognitive development in children. London: Academic Press.

Peterson, P. L., Wilkinson, L. C., & Hallinan, M. (Eds.) (1984) The social context of instruction: Group organization and group processes. Toronto: Academic Press.

Peterson, P.L., Wilkinson, L.C., Spinelli, F., & Swing, S. (1984). Merging the process-product and the sociolinguistics paradigms: Research on small-group processes. In P. L. Peterson, et al. (Eds.), The social context of instruction: Group organization and group processes. Academic Press.

Philips, S. (1972). Participant structures and communicative competence: Warm Springs children in community and classroom. In C. Cazden, V. P. John, & D. Hymes (Eds.), Functions of language in the classroom. New York: Teachers College Press.

Piaget, J. (1965). The child's conception of the world. Totawa, N. J.: Littlefield Adams.

Piaget, J. (1977). Developments in mathematical education. (Proceedings of the second international conference on mathematical education). In H. E. Gruber & J. J. Vonèche (Eds.), The essential Piaget. New York: Basic Books. (Originally published 1972)

Price, G. G. (1982). Cognitive learning in early childhood education: Mathematics, science and social studies. In B. Spodek (Ed.), Handbook of research in early childhood education. New York: The Free Press.

Robinson, E. (1976). Mathematical foundations of the development of spatial and geometrical concepts. In J. L. Martin & D. A. Bradbard (Eds.), Space and geometry: Papers from a research workshop. Columbus, Ohio: ERIC Center for Science and Environmental Education.

Rogoff, B., & Gardner, W. (1984). Adult guidance of cognitive development. In B. Rogoff & J. Lave (Eds.), Everyday cognition: Its development in social context. Harvard University Press.

Sacks, H,. Schegloff, E. A., & Jefferson, G. (1978). A simplest systemics for the organization of turn taking for conversation. In J. Schenkein (Ed.), Studies in the organization of conversational interaction. New York: Academic Press.

Sapir, E. (1921, 1949). Language. New York: Harcourt, Brace and World.

School Council Publications (1976). From information to understanding: What children do with new ideas. Ward Lock Educational.

Schenkein, J. (Ed.) (1978). Studies in the organization of conversational interaction. New York: Academic Press.

Shapiro, M. (1985). The teacher's role in the Logo environment. Presentation at the National Logo Conference, MIT, Cambridge, Massachusetts, July 22-25, 1985.

--------- (1986). The teacher, the student, the school: Three perspectives on learning. Unpublished paper, McGill University Faculty of Education, Montreal, Quebec.

Sheingold, K., Kane, J. H., & Endreweit, M. E. (1983). Microcomputer use in schools: Developing a research agenda. Harvard Educational Review, 53, 412-432.

Sinclair, J. McH., & Coulthard, R. M. (1975). Toward an analysis of discourse: The English used by teachers and pupils. London: Oxford University Press.

Skemp, R. L. (1976). Relational understanding and instrumental understanding. Mathematics Teaching, 77, 20-26.

Skemp, R. L. (1979). Goals of learning and qualities of understanding. Proceedings of the Third International Conference for the Psychology of Mathematics Education, University of Warwick, July 9-14, 1979, 250-261.

Slavin, R. (1980). Cooperative learning. Review of Educational Research, 50, 315-342.

Solomon, C. (1976). Problem solving in an anthropomorphic computer culture. Unpublished master's thesis, Graduate School of Arts and Science, Boston University.

Steinberg, Z. D. (1979). Peer teaching discourse: One peer group's negotiation of status and role in a school for the emotionally disturbed. Unpublished doctoral dissertation, Harvard University.

Stodolsky, S. (1984). Frameworks for studying instructional processes in peer work groups. In P. L. Peterson, et al. (Eds.), The social context of instruction. Toronto: Academic Press.

Stubbs, M. (1976). Language, schools and classrooms. London: Methuen.

Sutherland, R. (1986). Developing an understanding of variable in Logo: An illuminative case study. Proceedings of the Third International Conference for Logo and Mathematics Education, July 14-18, 1987, Concordia University, Montreal, Canada, 119-125.

Sutherland, R., & Hoyles, C. (1986). Logo as a context for learning about variable. Proceedings of the Tenth Annual Meeting of Psychology of Mathematics Education (PME 10), London, England, July 20-25, 1986, 301-306.

Tetenbaum, T. J., & Mulkeen, T. A. (1984, November). Logo and the teaching of problem solving: A call for a moratorium. Educational Technology, 16-19.

Usiskin, Z. (1982). Van Hiele levels and achievement in secondary school geometry. (Final Report of the CDASSG Project). Chicago: University of Chicago Press. (ERIC Document Reproduction Service No. ED 220 288)

Vygotsky, L. S. (1962). Thought and language. Cambridge: MIT Press. (Originally published in Russian in 1934)

Vygotsky, L. S. (1978). Mind in society: The development of higher psychological processes. Edited by M. Cole, V. John-Steiner, S. Scribner & E. Souberman. Cambridge, Massachusetts: Harvard University Press.

Wallrabenstein, H. (1973). Development and signification of a geometry test. Educational Studies in Mathematics, 5, 81-89.

Watzlawick, P., Beavin, J. H., & Jackson, D. D. (1967). Pragmatics of human communication: A study of interactional patterns, pathologies, and paradoxes. New York: W. W. Norton.

Watson, K., & Young, B. (1986). Discourse for learning in the classroom. Language Arts, 63 (2), 126-133.

Watt, D. H., & Watt, M. L. (1987). Assessing Logo learning in classrooms. Talk presented at the Fifth Annual Conference on Computers in Education, McGill University, Montreal, Quebec, November 18-20, 1987.

Webb, N. M. (1982a). Student interaction and learning in small groups. Review of Educational Research, 52 (3), 121-145.

-------- (1982b). Group composition, group interaction, and achievement in cooperative small groups. Journal of Educational Psychology, 74, (4), 475-484.

-------- (1983). Predicting learning from student interaction: Defining the interaction variables. Educational Psychologist, 18, 34-42.

-------- (1987). Helping behavior to maximize learning. Paper presented at the Annual Meeting of the American Educational Research Association, Washington, D. C.

Webb, N. M., & Kenderski, C. M. (1984). Student interaction and learning in small-group and whole-class settings. In P. L. Peterson, et al. (Eds.), The social context of instruction: Group organization and group processes. Academic Press.

Wertsch, J. V. (1979). From social interaction to higher psychological processes: A clarification and application of Vygotsky's theory. Human Development, 22, 1-22.

Wilkinson, L. C. (Ed.). (1982). Communicating in the classroom. New York: Academic Press.

Wirszup, I. (1976). Breakthroughs in the psychology of learning and teaching geometry. In J. L. Martin & D. A. Bradbard (Eds.), Space and geometry: Papers from a research workshop. Columbus, Ohio: ERIC Center for Science and Environmental Education.

Wood, D., Bruner, J. S., & Ross, G. (1976). The role of tutoring in problem solving. Journal of Child Psychology and Psychiatry, 17, 89-100.

Youniss, J. (1980). Parents and peers in social development. (A Sullivan-Piaget perspective) Chicago: The University of Chicago Press.

Zack, V. (1985). Dyadic talk, Logo problem solving and teacher intervention: A look at disagreements and their resolution in the interaction of one 9 to 10 year-old dyad. Unpublished paper, McGill University, Faculty of Education, Montreal, Quebec.

-------- (1986a). An analysis of one Logo teacher's intervention with three pairs of ten to eleven year-old children in a school computer lab environment. Unpublished paper, McGill University, Faculty of Education, Montreal, Quebec.

------- (1986b). A clinical-observational study of ten to eleven year-old students' notion of right angle in Logo geometry. Unpublished paper, Department of Mathematics, Concordia University, Montreal, Quebec.

-------- (1986c). The noncollaborative stance and its relationship to learning in the Logo environment. Unpublished paper, McGill University, Faculty of Education, Montreal, Quebec.

-------- (1986d). A study of ten to eleven year-old students' notion of right angle in Logo geometry. Proceedings of the Second International Conference for Logo and Mathematics Education, July 15-18, 1986, University of London Institute of Education, 96-103.

------- (1987a). One Logo teacher's use of metaphor to enhance social and cognitive knowledge. Unpublished paper, McGill University, Faculty of Education, Montreal, Quebec.

------- (1987b). The role of nonverbal messages in social interaction. Unpublished paper, McGill University, Faculty of Education, Montreal, Quebec.

------- (1988a). Say it's perfect, then pray it's perfect: The early stages of learning about Logo angle. Proceedings of the Twelfth Annual Conference of the International Group for the Psychology of Mathematics Education (PME 12), Hungary, July 20-25, 1988, 657-664.

------ (1988b). Ask three before you ask me: A teacher's attempt to increase the role of peer involvement in Logo learning. Proceedings of the North American Chapter of the International group for the Psychology of Mathematics Education (PME/NA 10), Northern Illinois University, De Kalb, Illinois, November 3-5, 1988, 394-400.

Zykova, V. J. (1969). Operating with concepts when solving geometry problems. In J. Kilpatrick & I. Wirszup (Eds.), Soviet studies in the psychology of learning and teaching mathematics. Vol. 1. The learning of mathematics concepts. Stanford, CA: School Mathematics Study Group.

The Study	Specific mathematics agenda	Logo software changes	Instruction: Individualized instruction	Instruction: Tasks assigned	Instruction: No tasks assigned	Instruction: Researcher intervention	Duration of study	
(1) **Papert, Di Sessa, Watt and Weir (1978)**	no	no	●	●		●	7 weeks in 1978	(12-13
(2) **Hillel (1984)**	yes	yes	●	●		●	12 weeks	(8-9 yc
(3) **Noss (1985)**	yes (geometrical concepts and elementary algebraic concepts)	no	●			●	18 months (longitudinal) (Oct 1982 March 1984) (Sep 1983 March 1984)	(i) 8-11 (ii) 10 y
(4) **Hoyles, Sutherland and Evans (1985)** see also Hoyles and Sutherland (1986a 1986b) Sutherland and Hoyles (1986) Sutherland (1987)	yes	no	●	●		●	3 years (longitudinal) Year 1 School A and B 1983 1984 Years 2 and 3 School A 1984 1986	Year 1 Year 2 Year 3
(5) **Kieran (1986)** see also Kieran Hillel and Erlwanger (1986)	A no	yes	●		●			9 10 ye
	B no	no			●	●		9-10 ye
	C yes (notion of angle perceptual and analytic schema)	yes		●		●		11 12 y
(6) **Zack (1986d)**	no	no	●		●		12 weeks (1985 1986 year of longitudinal study)	Year 2
(7) **Ludwig (1986)**	yes (transformational geometry)	no		●			24 weeks	(12-13 y
(8) **Barfurth (1987)**	yes	yes		●		●	12 weeks	(9 10 ye
(9) **Kynigos (1987a, 1987b, 1988)**	yes (intrinsic geometry Euclidean geometry cartesian geometry)	yes		●		●		12 year
(10) **Gurtner (1987)** see also Kieran Hillel and Gurtner (1987) Hillel Gurtner and Kieran (1988)	yes (problem solving geometric problems)	yes		●			24 weeks	12 year
(11) **Hoyles and Noss (1987)** see also follow up study Hoyles and Noss (1988)	yes (concept of a parallelogram)	yes		●				13 14 ye
(12) **Mendelson (1987)**	yes (complementary and supplementary angles)	no		●			20 weeks	(12 13 ye

Appendix A

Recent Logo and Mathematics Research Studies (See Table 2)

Age of children		Prior Logo experience	Group size	Time periods	Total hours	Location: Homeroom classroom	Location: School computer lab	Location: University research lab	Tools used for data collection: Videotape records (computer work)	Audiotape records	Transcriptions	Dribble file	Researcher observation notes	Cognitive tasks administered	Clinical interviews with students	Interviews with teachers	Student journal	Teacher journal	Tests
2-13 year olds)	Grade 6	none	8	40 60, 90 mins	25		●					●	●				●	●	●
9 year olds)	Grade 3	none	10 (5 pairs)	60 mins	12			●	●								●		
8-11 year olds		none	115	60 mins	30	●							●				●		
10 year olds		30 hrs	8 (4 pairs)	60 mins	15					●	●				●			●	●
or 1 11-12 year olds		none	14 (7pairs)	app 60 mins	40	●			●		●		●	●			●		
or 2 12 13 year olds		40 hrs	8 (4 pairs)	app 60 mins	40														
or 3 13-14 year olds		80 hrs	8 (4 pairs)	app 60 mins	40														
0 year olds	Grade 4	none	14	30 60 mins	25	●							●		●				
0 year olds	Grade 4	none	5	60 mins	35			●				●	●						
12 year olds	Grade 6	none	12	60 mins	22			●	●						●				
ir 2 (10-11 yr olds)	Grade 5	20 hrs	23 (in pairs/triads)	60 mins	12		●		●	●	●		●		●	●	●	●	
13 year olds)	Grade 7	15 hrs (intro)	10 (5 pairs)	80 mins	21		●		●				●	●	●		●		●
0 year olds)	Grade 4	none	6 (3 pairs)	75 mins	15			●	●			●	●	●			●		
ear olds		60 hrs	6 (3 pairs)	90 120 mins	6		●		●		●	●	●	●			●		
ear olds		none	6	60 mins	24			●				●					●		
4 year olds		120 hrs	7		2			●				●	●				●		
3 year olds)	Grade 7	8 hrs	8	45 mins	12		●						●	●			●		●

APPENDIX B

LETTER OF INFORMATION, WITH THE CONSENT FORM, SENT TO PARENTS AND CHILDREN OF THE GRADE 5 CLASSES

September 22, 1985.

Dear Children and Parents:

RE: MICROCOMPUTER SESSIONS

This year, Monica Shapiro (Logo teacher) and Vicki Zack (McGill observer) are continuing to observe the learning process which takes place while Logo is being mastered. Vicki will be videotaping some of the Logo computer sessions. We hope to see how the children's discussion, and collaboration in problem-solving, contribute to their learning of the Logo computer language. One pair (or more) will be observed on a weekly basis so that we can gain an idea of how a pair's collaboration evolves over time. The observation and videotaping will only be done with parental and child consent (PLEASE SEE BELOW).

Should you wish any additional information, please do not hesitate to call Vicki Zack at 486-6826. All children and adults involved in the project will have access to all the material gathered, and to the resulting write-up. No references to the children, oral or written, will identify the children or our school.

Cordially yours,

Gita Silver
Principal

Please return by OCTOBER 1, 1985.

I consent to be a member of the group observed.

_____ Child's signature

I/We consent to have our child be a member of the group observed.

_____ Signature of Parent(s)/Guardian

Grade _____ Room _____

APPENDIX C

SAMPLES OF STUDENT NOTEBOOK PAGES

Please Note: The children wrote in pencil. Therefore, the xerox copies are very faint at times. The fact that the work was in pencil also meant that it was often not possible to trace the evolution of a planned project, or certain of the changes in recorded commands, due to erasures.

Included in this appendix are excerpts from the work of Amy L., both from her Grade 4 work and from her Grade 5 work, since they were stored in the same notebook. Pages from the notebooks of other study subjects have been included as well: Eric H. (T. V., goalie), Russ H. (stickman), Rina M. & Lilly M. (shoe, cassette recorder, mountain), Carol L., and Doug L. (camel), Carol L. and Shauna L. (googleyes).

Hilroy

SUBJECT INDEX
INDEX DES SUJETS

CLASS SCHEDULE — HORAIRE

TIME / HEURE									
MON / LUN									
TUE / MAR									
WED / MER									
THU / JEU									
FRI / VEN									
SAT / SAM									

FINAL EXAMINATIONS — EXAMENS FINALS

DAY / JOUR	DATE	TIME / HEURE	PLACE / LIEU	COURSE / COURS

NAME / NOM

TELEPHONE / TÉLÉPHONE

ADDRESS / ADRESSE

SCHOOL / ÉCOLE

CLASS / CLASSE

Hilroy

exercise book-cahier d'exercices

Amy

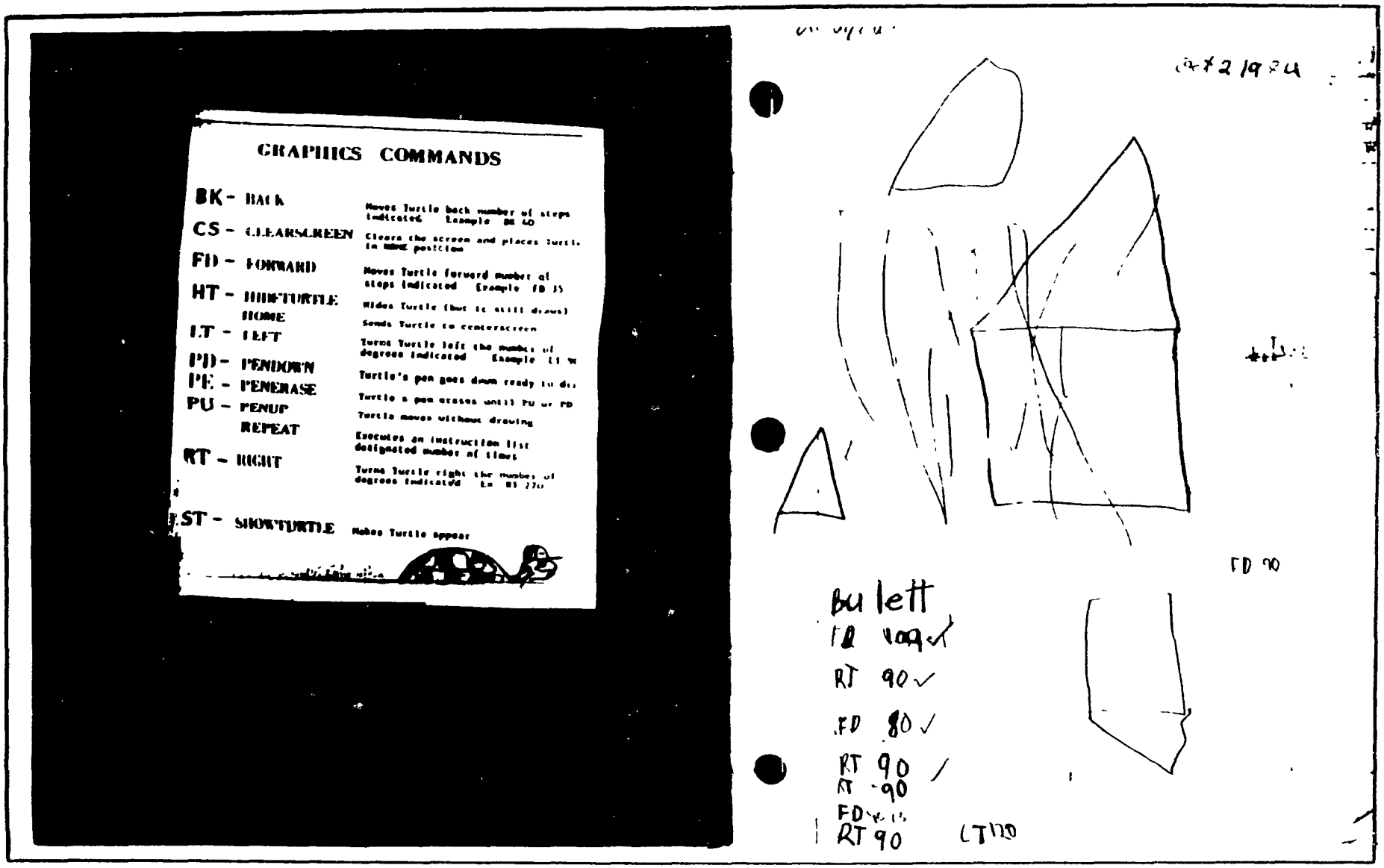
name-nom

subject-sujet

Hilroy

32 Pages
10"/₁₀ x 8"/₁₀ 27.6 x 20.9 cm.
ruled - ligné

Amy's notebook, Gr 4, Oct 2, 1984 Amy and partner Carol planned to make a "house," but were seen to deplan, and made a "bullet" See Selection #19, and Appendix G



GRAPHICS COMMANDS

BK - BACK — Moves Turtle back number of steps indicated Example BK 40

CS - CLEARSCREEN — Clears the screen and places Turtle in HOME position

FD - FORWARD — Moves Turtle forward number of steps indicated Example FD 35

HT - HIDETURTLE — Hides Turtle (but it still draws)

HOME — Sends Turtle to centerscreen

LT - LEFT — Turns Turtle left the number of degrees indicated Example LT 90

PD - PENDOWN — Turtle's pen goes down ready to dr

PE - PENERASE — Turtle s pen erases until PU or PD

PU - PENUP — Turtle moves without drawing

REPEAT — Executes an instruction list designated number of times

RT - RIGHT — Turns Turtle right the number of degrees indicated Ex RT 27

ST - SHOWTURTLE — Makes Turtle appear

Oct 2 1984

FD 90

Bullett
FD 109 ✓
RT 90 ✓
FD 80 ✓
RT 90 /
RT -90
FD
RT 90 LT 90

Note that in Grade 4, Amy recorded FD 109, RT 90, FD 80, RT 90; However, when Amy consults her book in Grade 5 (see Selection #2), the input of 90 for a "square corner" has no special meaning for her

AMY'S NOTEBOOK, Jan 1986 (Grade 5).

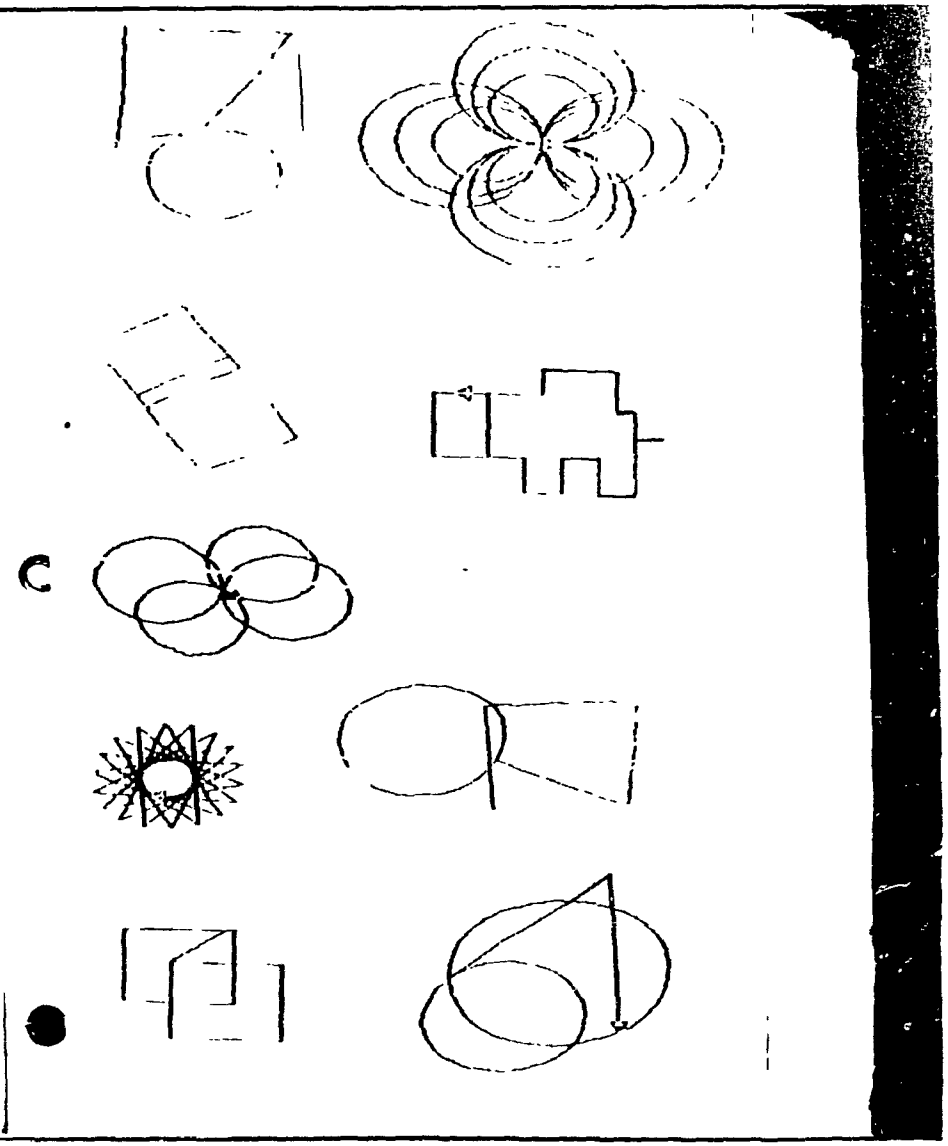
(a) Amy's self-evaluation sheet

NAME: Amy

LOGO PROGRESS CHART

	I can do this all by myself	I need some help doing this	I can teach someone how to do this
1 Boot the Logo Disk to begin working	✓		
2. Use all the Graphic Commands on the mimeo sheet		✓	
3 Use the REPEAT command		✓	
4 Make the Turtle draw a A SQUARE	✓		
B = TRIANGLE		✓	
C = CIRCLE without the bar at the bottom		✓	
D = FILL Grill shape or more than 4 sides	✓		
5 Draw a picture Breaking it into parts PROGRAMS		✓	
6 Use the EDITOR to revise programs			
7 Save programs in a file on a disk		✓	
8 Load my file from the disk into the machine		✓	
Use projects I made "DRAW", TREE & LOGO FOLLOWS		✓	

(b) Collage of products, Homeroom 210, Group A, 10 10 - 11 10.



ERIC'S NOTEBOOK, Gr 5, October 16, 1985

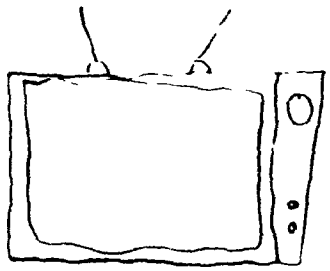
TURTLE COMMANDS

BK 10	turtle moves BACK 10
FD 30	turtle moves FORWARD 30
RT 20	turtle turns to face RIGHT 20
LT 90	turtle turns to face LEFT 90
CS	CLEARS the SCREEN
HT	HIDES the TURTLE
ST	SHOWS the TURTLE
PU	turtle puts it's PEN UP
PD	turtle puts it's PEN DOWN
PE	turtle PUTS an ERASER under itself



October 16 1985

Eric



Big Square

✓

```
PU
FD 55
Rt 90
PD
{FD 110
 Rt 90}   Repeat 2 [FD 110  Rt 90]
{FD 110
 Rt 90}
FD 220
{Rt 90
 FD 110
 Rt 90
 FD 110}  Repeat 2 [Rt 90  FD 110]
```

Eric's notebook drawing of the goalie, done Dec 11, 1985 (Session #8), in preparation for the "goalie" project done by Russ, Jon and Eric as a triad (See Appendix P)

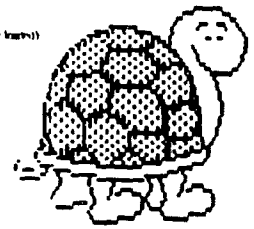
Eric

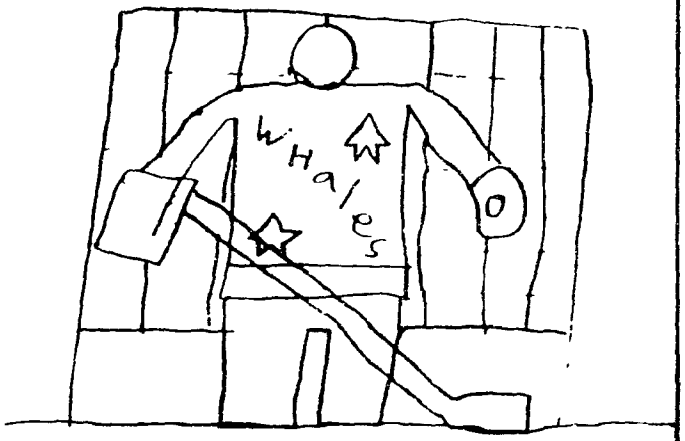
SAVING MY WORK ONTO A DISK

1) Make sure the computer speaks LOGO
2) Put a disk in the drive
3) Type POTS (to see what progs are in saving)
4) Type SAVE FILE NAME (my name and my pic [illegible] with a dot between them)

LOADING MY WORK INTO THE COMPUTER

1) Make sure the computer speaks LOGO
2) Type ERALL (to erase any progs files that are already in the work space)
3) Put a disk in the drive
4) Type CATALOG (to see the name of my file)
5) Type LOAD "FILE NAME
6) Type POTS (to see what progs are I have loaded)



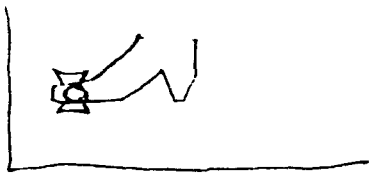


Lilly's notebook, October 16, 1985 (Gr 5) Lilly and Rina's "shoe" project

SHOE

Lilly

October 16 1985



FRONT

PU

FD 57

PD

LT 56
LT 56 } 147
LT 35

RT 90
FD 49
LT 90
RT 90
FD 55
LT 19
RT 23
LT 13
FD 95
LT 55
LT 40 } 49
RT 10
FD 55
LT 53
FD 99
RT 91
RT 23
FD 87
RT 23
RT 46
RT 10 } RT 246
RT 80
RT 55
RT 15
RT 12
FD 50
LT 95

DISK

LT 50
FD 30
RT 15
LT 15
LT 10
FD 25
FD 10
LT 10
LT 50
FD 25
FD 25
FD 25

FD 30
PD
CIRCLE 0
RT 10
LT 20
FD 30
RT 10
LT 140
LT 94
FD 30

HEEL

BULL

LT 23
LT 23) 46
LT 10
LT 5
PD
FD 119
PU
LT 15
LT 25
RT 45
RT 50
LT 50) RT 40
PU

Rina's notebook records on the "geto blaster" project she did with Lilly See Appendix X concerning the RECTANGLE part, and Selection #20 concerning the RT 135 (vs RT 181) turn in the LINE procedure.

Geto Blaster

Rectangle

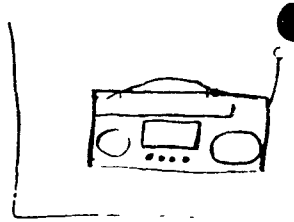
RT 35
RT 35 } RT 85
RT 10
RT 5
FD 48 } FD 96
FD 48
RT 85 } RT 95
RT 10
FD 50
FD 35 } FD 85
LT 90
RT 180
FD 96
LT 90
RT 180
FD 50
FD 25 } FD 75

HANDLE

RT 35 } RT 70
RT 35
FD
RT 10
FD 25
RT 10 } 15
RT 5
FD 25

ANTENA

LT 90 } LT 115
LT 25



RT 5 } RT 15
FD 35 } FD 60
FD 25
CIRCLER 5

LINE

PU
RT 180
FD 96
PD
LT 46
RT 23
RT 85
RT 15
RT 5 } RT 135
RT 2
RT 5
FD 96

SPEAKER

PU
LT 90 } LT 170
LT 80
FD 40
BK 25
FD 10

Geto Blaster

PO
CIRCLER

TAPE

PU
FD 10
FD 15
FD 5 } FD 20
RT 90
FD 20
LT 90
RT 180
FD 20
RT 90
FD 20

SPOOLER

PU
LT 90
RT 180
FD 40
PD
CIRCLER 15

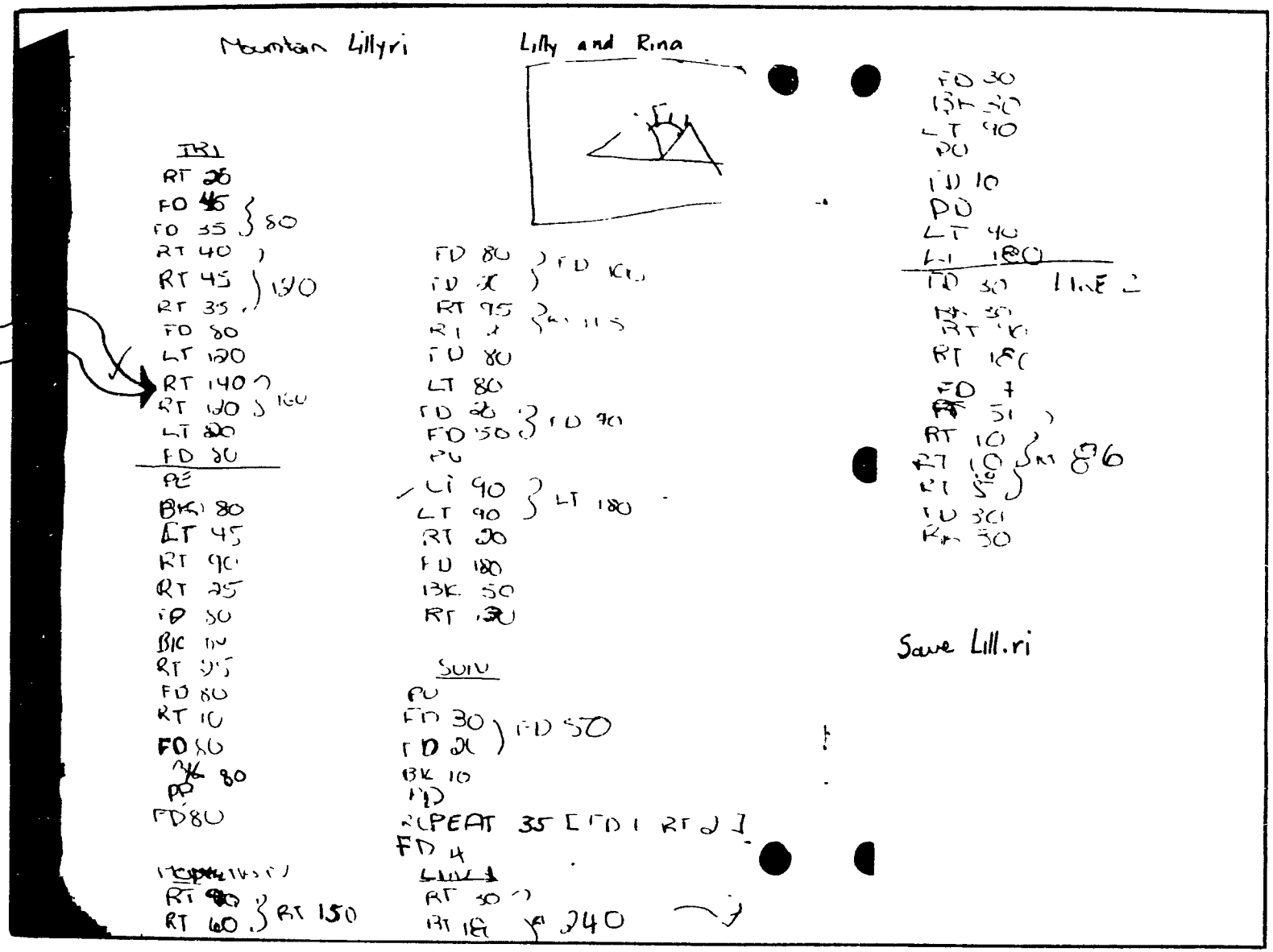
GetoBlaster



Rina and Lilly

saved DISK 1

Lilly's notebook: the Lillyri "mountain" project (Lilly and Rina) See Selecton # 41, and Note 31 concerning RT 140 + RT 120 = RT 160



Mountain Lillyri

IR1
RT 20
FD 45 } 80
FD 35
RT 40)
RT 45 } 130
RT 35)
FD 80
LT 120
RT 140)
RT 120 } 160
LT 90
FD 80

PE
BK 80
LT 45
RT 90
RT 25
FD 50
BK 10
RT 95
FD 80
RT 10
FD 80
PD 80
FD 80

RT 90 } RT 150
RT 60 }

NB

Lilly and Rina

FD 80 } FD 160
FD 80 }
RT 95 } RT 115
RT 20 }
FD 80
LT 80
FD 20 } FD 90
FD 50 }
PU
LT 90 } LT 180
LT 90 }
RT 20
FD 180
BK 50
RT 130

SUN
PU
FD 30 } FD 50
FD 20 }
BK 10
PD
REPEAT 35 [FD 1 RT 2]
FD 4
LINE 1
RT 30)
RT 15 } 240

FD 30
RT 20
LT 90
PD
FD 10
PU
LT 90
LT 180

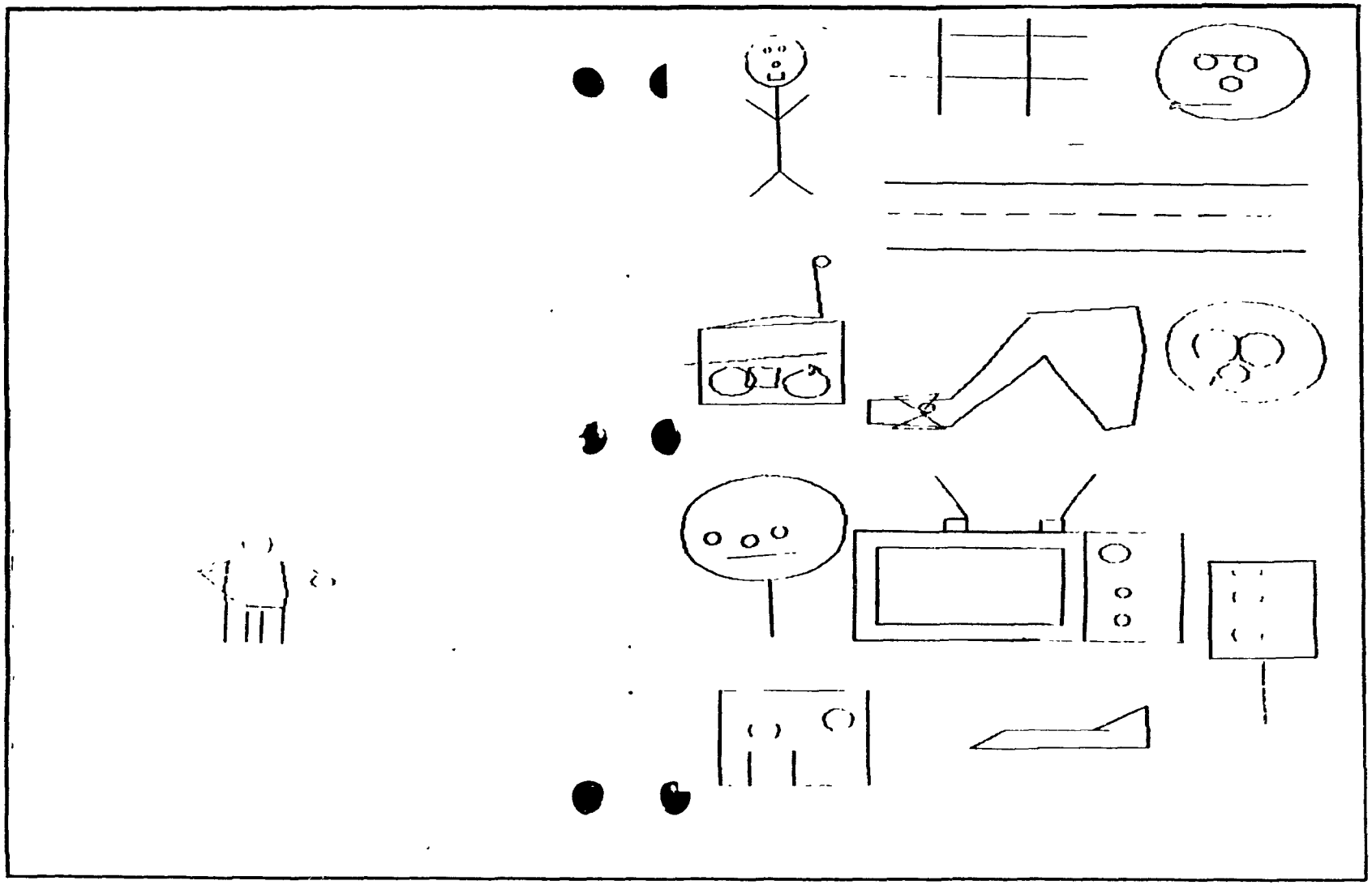
FD 30 LINE 2
RT 30
RT 90
RT 150
FD 7
FD 51)
RT 10
RT 150 } RT 886
RT 150 }
FD 30
RT 30

Save Lill.ri

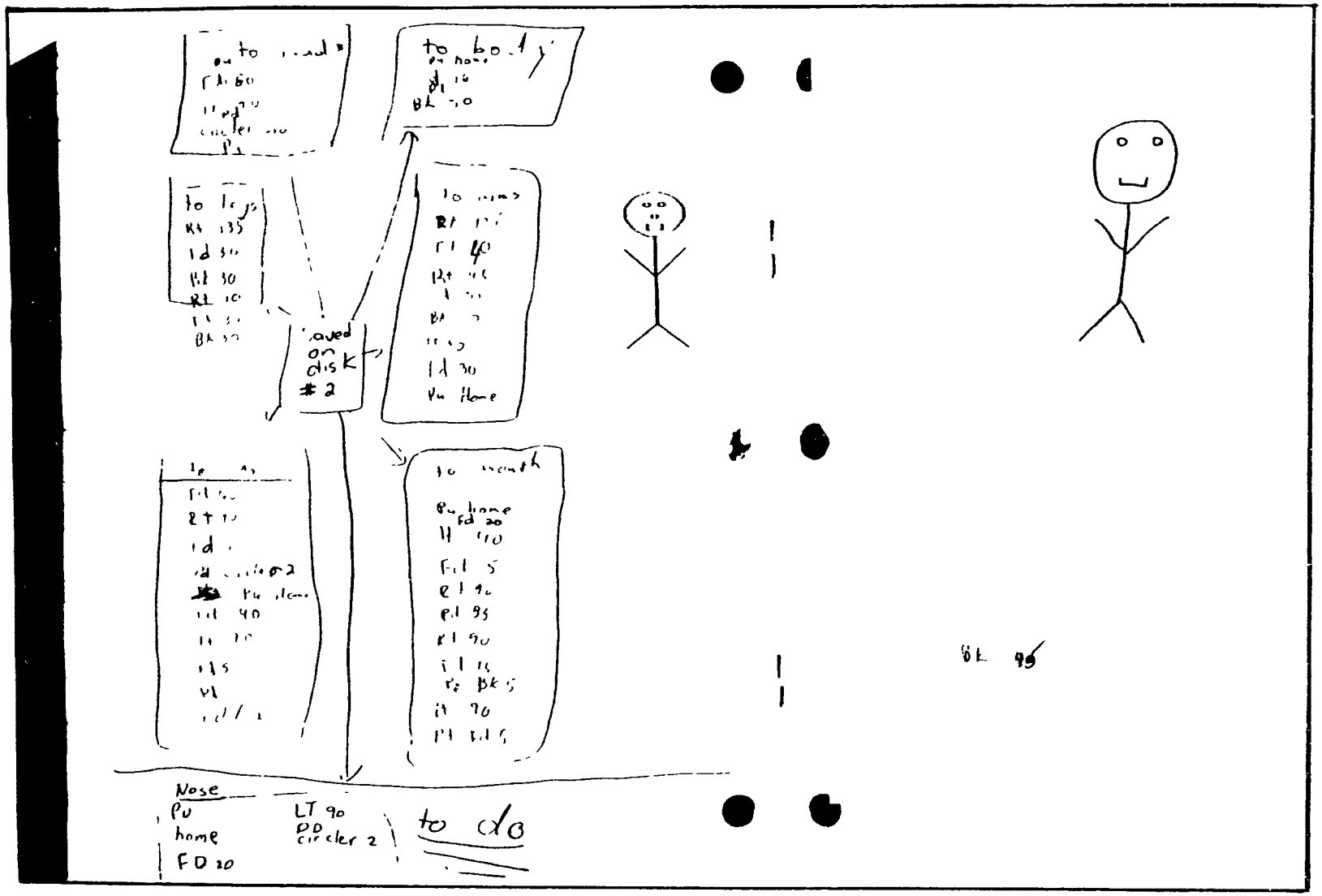
JON'S NOTEBOOK, Jan 1986

(a) Printout of the goalie (project printout incomplete)

(b) Collage of products, Homeroom 210, Group B, 11.10 - 12.10



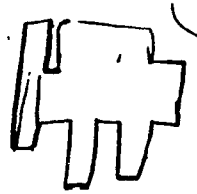
Russ's notebook, Dec 4, 1985, "stickman" project



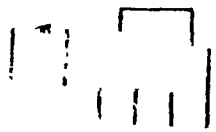
CAROL'S NOTEBOOK Carol and Doug, Dec 4, 11, 18, 1985; Jan 8, 1986 Project "camel"

Carol and Shauna, Nov 20, 1985 Project "googleyes" -- see Appendix L

I Love Computers.



Camel



Head

```
FD 50
LT 90
FD 30
LT 90
FD 50
LT 90
FD 30
```

Leg one

```
FD 30
RT 90
FD 30
RT 90
FD 3
LT 1
FD 30
RT 90
FD 20
```

Leg two

```
RT 90
FD 30
LT 90
FD 20
LT 90
FD 30
FD
```

ass

```
FD
LT 10
FD 15
LT 90
FD 20
```

Hump Dr4

```
LT 90
FD 16
RT 90
LT 90
FD 40
LT 90
FD 20
RT 90
FD 40
```

Disk number 6

Gogely's

circle1	60
circler	60
circle1	50
circler	50
circle1	40
circler	40
circle1	30
circler	30
circle1	20
circler	20
circle1	10
circler	10
circle1	5
circler	5

PU turnkeys

```
RT 30
RT 40
RT 70
FD 50
BK 50
RT 50
FD 50
BK 50
RT 10
Fd 12
RT 5
BK 10
PD
```

APPENDIX D

INTERVIEWS WITH THE HOMEROOM TEACHERS, NOVEMBER-DECEMBER, 1987

Could you comment on each of the following children?

I would like to get a general idea of the child's mode of functioning in the regular, as opposed to the Logo, classroom.

++

Interview with homeroom teacher _____ RE STUDENT:

1. First, please tell me about the child's general achievement in the classroom.

2. Thinking/Speaking achievement:

   Reading/Writing achievement:

   Creativity:

3. Mathematics achievement: GROUP_____

   Comments:

4. Achievement in specific subject areas: (Social studies; Science; French; Hebrew; other)

5. Working style in teacher-centered lessons:

in student-centered lessons:

in cooperative peer work groups:

Sense of dependence/independence:

6. Child's sense of confidence/ self-esteem:

7. Child's relationship with peers:

leader? follower? popular? best friend(s)

8. Any other points which come to mind which you wish to note?

Ask the following questions if time allows:

9. Have a look at the partnerships in the Grade 5 Logo class (1985-86) listed below.

Could you comment on the pairings?

10. Would you like to comment in any way on the possible effects of the Logo program on the children's subsequent functioning in the classroom?

Were there ever any STUDENT comments about Logo in the regular classroom time?

Were there ever any STUDENT comments tying Logo geometry to the textbook geometry material during classroom mathematics?

11. Any general comments you would like to add about the computer program?

****

PAIRINGS:
Carol and Shauna
Carol and Leanne
Carol, Leanne and Amy
Carol and Doug

Amy and Leanne
Amy and Shauna

Tammy and Adina

Lilly and Rina

Jon and Eric
Jon, Eric and Russ

Russ and Michael

APPENDIX E

INTERVIEW WITH MONICA SHAPIRO, LOGO TEACHER, SEPTEMBER 18, 1987

The researcher (V. Zack) asked Monica, the Logo teacher, to elaborate upon certain aspects which the two of them had discussed informally during the course of the project. The interview questions are listed below. The interview lasted 3 1/2 hours.

1. What were your aims for the computer program in general, and your aims for the Logo program specifically?

2. Could you comment upon your objectives vis-a-vis having the children work together in pairs?

3. What do you feel is your function as teacher in the Logo environment?

4. Could you comment on the degree of structure which you as a teacher provided in the Logo environment?

5. Could you elaborate upon your use of the phrase "Ask three before you ask me": its origin, your aim in using it, and to what extent it was successful?

6. We have discussed the connections which the children made, or did not make, between Logo geometry and school geometry. Could you comment?

APPENDIX F

CLINICAL INTERVIEWS GRADE 5, 1985-1986 (INCLUDING COGNITIVE TASKS ASSIGNED)

Included in this appendix are the following:

- Beginning session interview (with 23 students in homeroom 210), Grade 5 year, October-November, 1985

- Post-session interview (with 23 students in homeroom 210), Grade 5 year, February-March, 1986

- Across class task (pencil and paper assignment), Grade 5, March, 1986

- Across class task repeated, with minor changes (pencil and paper assignment), Grade 6, May, 1987

PLAN AND PROCEDURE: THE BEGINNING SESSION GRADE 5 INTERVIEW

The first clinical interview series was done during the first Logo sessions in October and November, 1985. Due to the restricted time schedule in this multi-lingual setting, requests to release children from their regular homeroom classroom work were kept to a minimum. The interviews were conducted on a one-to-one basis in a room adjoining the computer room. The duration of time was approximately 15 minutes per interview, and the children were drawn out for this short period during their assigned computer time. The questions dealt with the degree of turn that "turtle" was turned upon a mock bristleboard computer. The researcher turned the cardboard turtle (Δ) to the right (right 45 degrees) and asked the child (1) HOW DID TURTLE GET TO THIS POSITION? in order to elicit whether the children would indicate that the turtle turned, and to see if the child perceived the degree of the turn (within an error factor of 5-10 degrees); (2) HOW MANY WAYS COULD DIFFERENT CHILDREN GET TURTLE TO DO THIS? in order to test whether the children would list combinations for the number, and or would indicate that the turtle could have turned counter-clockwise, or would include in error other commands other than "turn"; and (3) NOW TURTLE IS IN THIS POSITION. HOW CAN IT GET BACK TO THE WAY IT WAS BEFORE? in order to test the knowledge of the inverse relation, and to detect whether there were children who had an appreciation of the 360 degree rotation. The interviewer probed in order that the child would state as many possiblities as possible, until the child exhausted all suggestions. The interviews were audiotaped, and the tapes were then transcribed.

BEGINNING SESSION INTERVIEW WITH THE 23 GRADE 5 STUDENTS IN HOMEROOM 210, OCTOBER-NOVEMBER, 1985

Hello _____ _____ [identify the child for audiotape transcription purposes]

Let's imagine this [the bristol board model] is a computer screen.

Look at turtle now. [i. e. in starting position]

Close your eyes. [Interviewer moves the triangular "turtle" to RT 45 position]

Now look.

1. How did turtle get to this position?

   Write the command as it would show on the screen.

2. Do you think _____ [name another child in the group] will give the same answer?

   What would _____ say? Why?

3. How many different ways could different children get turtle to do this?

   Any more?

   [Prompt if necessary: If there were 10 computers in the room and all the children had this on the screen, what could the variety of commands be?]

4. Now turtle is in this position. How can it get back to the way it was before?

   Any other ways? [As above, keep prompting until the child exhausts all possibilities]

PLAN AND PROCEDURE: THE POST-SESSION GRADE 5 CLINICAL INTERVIEW QUESTIONS

The end of session interviews (approximately 30 minutes) were done on a one-to-one basis with the 23 Grade 5 children in homeroom 210. The researcher asked that the children be excused from the homeroom classroom in order that they might be interviewed in the computer lab classroom. The child's computer work was videotaped (Figure 4), and another videotape/camera recording followed the child's verbal and non-verbal responses to the interviewer's questions.

In order to investigate the use of right angle within a closed figure, the square was used as a focal point. A relational understanding of square would require the child to: recognize a square; demonstrate a knowledge that a square has 4 equal sides and 4 right angles; recognize the square in its various orientations.

CAN YOU MAKE ONE SQUARE WITH THIS PIPE CLEANER? IMAGINE YOU WERE TELLING SOMEONE ABOUT WHAT A SQUARE IS, SOMEONE WHO HAD NEVER SEEN A SQUARE. WHAT WOULD YOU SAY?

PUT A CIRCLE AROUND ALL THE SHAPES THAT ARE SQUARES.

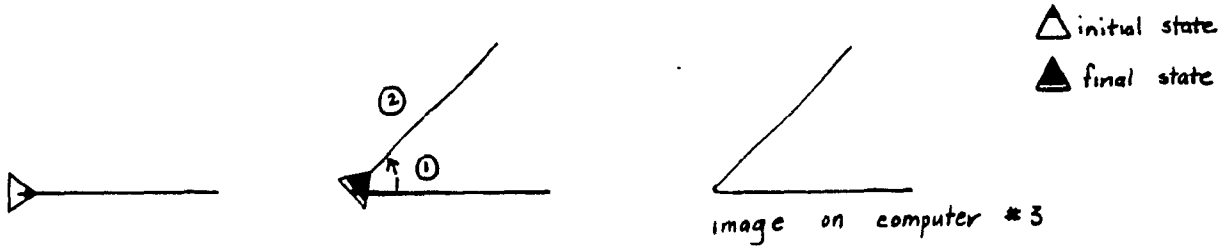
A set of questions were posed to sound out the child's familiarity with school-textbook Euclidean geometry:

TELL ME ALL THAT YOU KNOW ABOUT ANGLES.
DO YOU KNOW WHAT A RIGHT ANGLE IS?
PUT A CIRCLE AROUND ALL THE SHAPES THAT ARE RIGHT ANGLES.

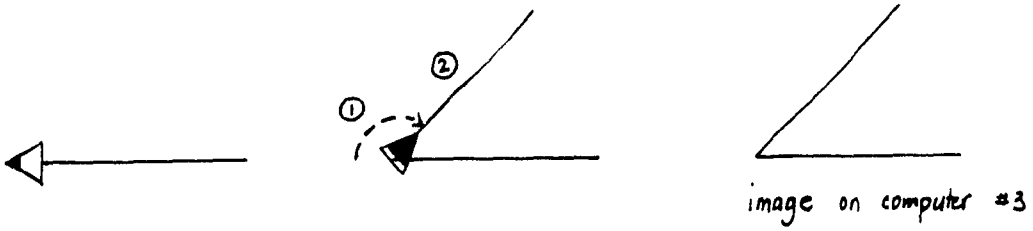
A number of questions were posed in an endeavor to see what level of grasp the child had of the 90 degree, the 45 degree, the 180 degree, the 360 degree and other degree turns on the screen: (1) the interviewer keyed in code "V" (a RIGHT 45) and then subsequently code "Z" (a RIGHT 90 degree turn) and asked the child what the input keyed in might have been (Question #2); (2) the interviewer keyed in a RIGHT 47 turn with a request to the child to visualize where RIGHT 51 would then be (Question 4); the interviewer requested that the child predict where turtle would end up

if RIGHT 180, and then another input would be keyed in, and where turtle would end up if RIGHT or LEFT 360 were keyed in (Questions #5 and #8). Another question endeavored to probe the child's appreciation of the relationship between 90 degrees and 45 degrees, and to detect whether any of the children associate the 45 degree with a diagonal, and the 90 degree with a perpendicular line. The children were also asked why there is no trace left when the turtle has "gone" right or left X, in order to see whether the child could verbalize that the turtle turns, but leaves no trace.

Two questions required the child to construct an angle on one computer congruent with an angle already present (previously prepared by the interviewer) on the computer screen to the right of the interviewer and child. Upon the first attempt, the interviewer began the task in such a way as to require the child to complete the angle with a 45 degree turn to the left (thus making the interior angle), and then to complete the arm by going forward 80 (allowing a margin of 5-10 for each dimension) (Question #3).



Later, the child was required to again construct an angle congruent to the existing one; however this time the interviewer began in such a way as to require the child to turn through an exterior rotation, right 135 degrees, and then to complete the arm by going forward 80 (Question #6).



The interviewer asked where in the physical environment a chocolate turtle (from a box of chocolate Turtles) would have to turn to negotiate a corner, and what the turn would be-- in order to detect whether the child responded with RIGHT or LEFT 90 at

the corners of the table, or the blackboard (Question #9).

In order to attempt to distinguish just what kind of relationship the child detected or did not detect between RIGHT 90 in Logo and the term RIGHT ANGLE in classroom geometry, the child was asked the following question: SOMEONE YOUR AGE HAS SAID THAT RIGHT 90 IS A RIGHT ANGLE, OR IN FRENCH, UN ANGLE DROIT. IS RIGHT 90 A RIGHT ANGLE? WHAT DO YOU THINK? To ascertain that the child knew what the term RIGHT ANGLE referred to, s/he was asked: DO YOU KNOW WHAT A RIGHT ANGLE/UN ANGLE DROIT IS? S/he was shown the represention of right angle in the school mathematics textbook, and then was asked the question again, that is, IS RIGHT 90 A RIGHT ANGLE? WHAT DO YOU THINK? In order to determine whether the child was or was not able to recognize a RIGHT ANGLE and a SQUARE in different orientations, the child was asked to put a circle mark around the right angles, and the square shapes in Question #12.

The post-session interview sheet of questions used by the researcher follows, as does the worksheet containing the pencil and paper task which was assigned to the entire class. Findings which resulted from the analysis of the interviews and the worksheets are reported elsewhere (Zack, 1986b; Zack, 1986d).

APPENDIX F

POST SESSION INTERVIEW WITH GRADE 5 STUDENTS (23 STUDENTS IN HOMEROOM 210), FEBRUARY-MARCH, 1986

Interview with _____ Date [ ]
Time _____-_____

1. Can you do ONE SQUARE with this pipe cleaner?

   Imagine you were telling someone about what a SQUARE is, someone who had never seen a square. What would you say?

2. You remember when we were not using the computer, and I showed you that turtle went from here to here? Let me show it to you on the computer screen. [Key in code V which turns turtle RT 45] WHAT DID TURTLE DO?

   Now if I type in RT something (the something is a number, but it's a secret number) [Key in code Z which turns turtle 90] WHAT DO YOU THINK THE number IS?

   _____

3. Look at the design on computer #3.

   Can you do that design on our computer? I'll start it for you.

   CS
   RT 90
   BK 80

   cs; type in RT 47

4. I just went RT 47.
Imagine that I now did C S, and typed R T 51. Where would turtle be?

[Hand the student a STRAIGHTEDGE to use a indicator]

5. [C S] I am typing in R T 180. Turtle is pointing this way [indicate down] If it went R T _____ [the student's response from question #2, part A] where would turtle end up?

6. Look again at the design on computer #3. I'm going to ask you to do that design on our computer again, but I'll start it differently this time.

CS
LT 90
FD 80

7. I'm going to clear the screen again and go R T 90.

If I wanted to go more than halfway back, what would I key in?

_____

Try it.

[cs]

8. If turtle went R T 360 (or L T 360) where would turtle end up?
SHOW ME.

If we now typed in H T (hideturtle), what would you see?

TRY IT.

Why do you think that happens?

9. If turtle (the chocolate turtle) were to walk along this table's edge [have the child do so], what would s/he have to turn here? [that is, at the corner of the table edge]

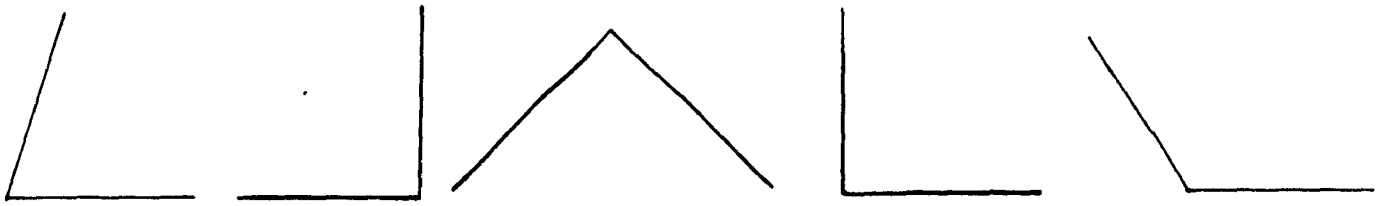
10. I heard someone your age say that RIGHT 90 is a RIGHT ANGLE, or in French, UN ANGLE DROIT.

(Show the page from the Grade 4 or 5 text, Passeport mathématique)
(p.177, p.251)

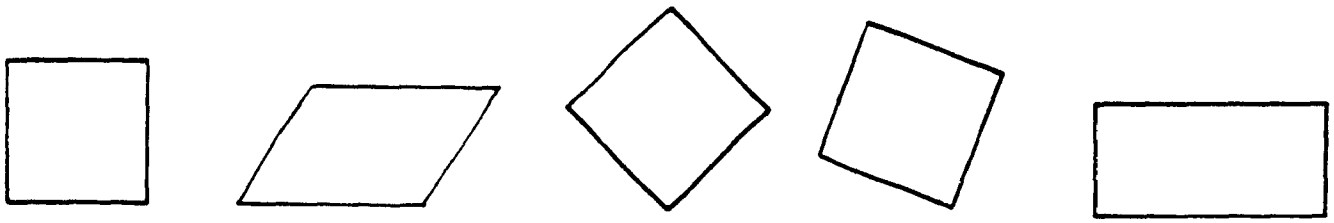
What do you think?

11. Tell me ALL that you know about angles.
Imagine that you are the teacher, and are teaching what an angle is to someone who has never learned about angle. Show me what you would say and do.

12. Put a circle around all the shapes that are RIGHT ANGLES.



Put a circle around all the shapes that are SQUARES.



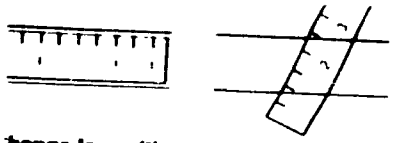
13. When you are working on your program, for instance the
which you and made, and you get to a part that isn't coming out right,
what do you do?

14. Whom in your class would you ask for help if you were having a problem?

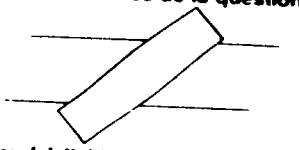
_____

Les lignes parallèles

1 Utilise ta règle pour tracer des droites parallèles qui se croisent



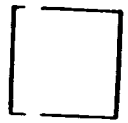
2 Change la position de ta règle. Trace des droites parallèles qui se croisent. Essaie de les faire différentes de celles de la question 1.



3 Trace un carré à l'aide de ta règle et le sommet d'un carré.

Dessine ces figures. Indique les paires de côtés parallèles en les coloriant de la même couleur.

4



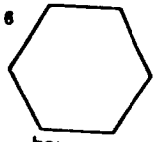
carré

5



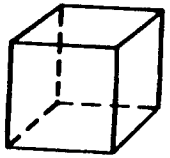
parallélogramme

6



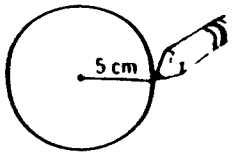
hexagone

7 Trace ce solide.
Trouve les droites parallèles.
Trace-les de la même couleur.



Les angles

1 Trace un cercle à l'aide d'une corde de 5 cm. Découpe-le



2 Plie-le en deux



Trace une ligne le long du pli Décris ce que tu as obtenu



3 Plie-le encore en deux



Trace une ligne le long du pli
Décris ce que tu as obtenu



C'est un sommet spécial ou angle
Ce sommet de carré s'appelle un «angle droit»

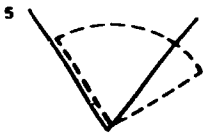


Trouve les angles qui sont plus grands qu'un angle droit.

4



5



6



7



8

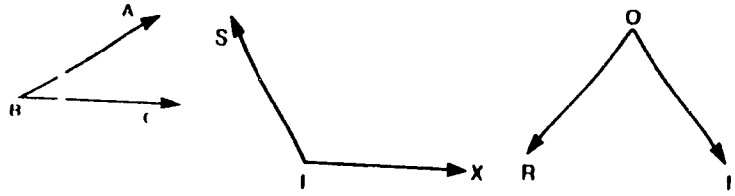


9



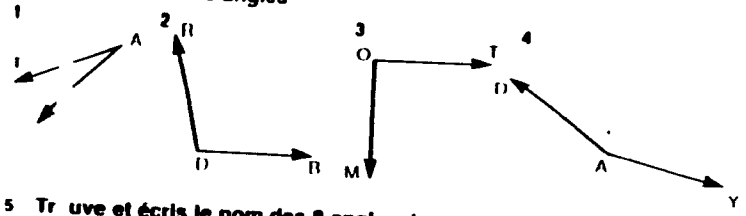
Identifier les angles

Nous nous servons de lettres majuscules pour nommer les angles

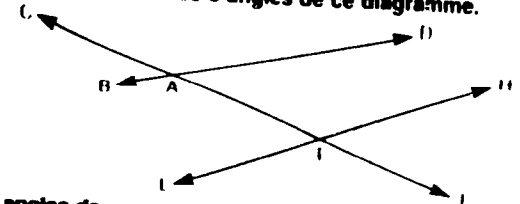


angle ABC ou CBA angle SIX ou XIS angle ROI ou IOR

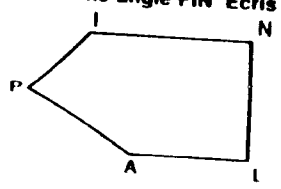
Ecris le nom de ces angles



5 **Trouve et écris le nom des 8 angles de ce diagramme.**

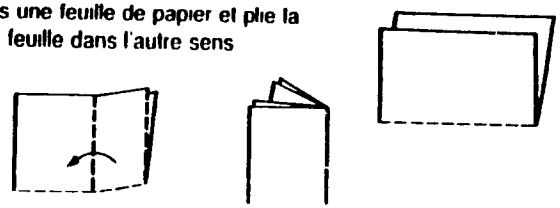


6 **Un des angles de ce pentagone se nomme angle PIN Ecris le nom des quatre autres angles**



Les angles droits

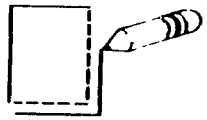
Prends une feuille de papier et plie la
Plie la feuille dans l'autre sens



Le coin ou les deux plis se rencontrent est un angle très special
C est un angle droit

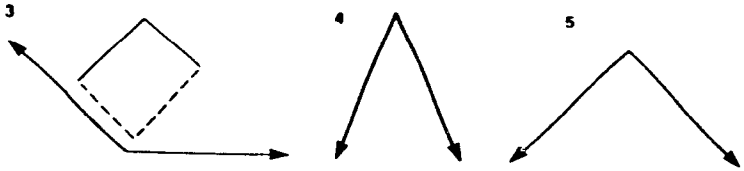
Tu peux utiliser ton coin comme constructeur et vérificateur d angle droit

1 Utilise ton coin pour tracer
des angles droits



2 Les coins de cette page sont des angles droits
Dans la classe trouve 3 autres exemples d angles droits

Indique si, par rapport à un angle droit, chaque angle est plus grand plus petit ou égal Utilise ton coin comme vérificateur d angle droit

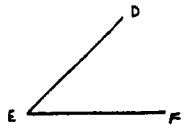


Across class task (pencil and paper assignment), Grade 5, March, 1986

Name _____

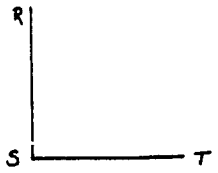
ON THE BLACK BOARD

1. ∠DEF = 45°

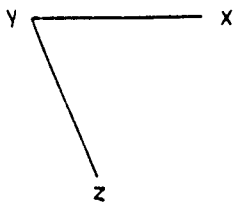


Turtle would turn RT _____

2 ∠RST = 90°

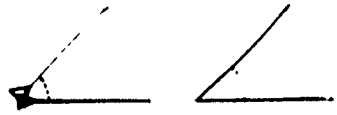


3. ∠XYZ = 68°



ON THE COMPUTER USING LOGO

1 Turtle turns from ◁ to ▷



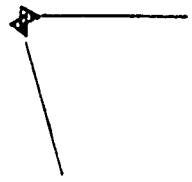
Turtle turned RT _____

2 Turtle turns from △ to ▷



Turtle turned RT _____

3 Turtle turns from ▷ to ◁



Turtle turned RT _____

Name _____

4 Put a circle around all the angles that are 90° angles



5 Look at the angle on the blackboard

Does this angle have anything to do with what you do in Logo?
EXPLAIN your answer

_____

_____

_____

_____

_____

_____

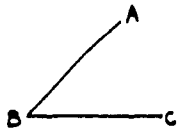
_____

Across class task repeated, with minor changes (pencil and paper assignment), Grade 6, May, 1987

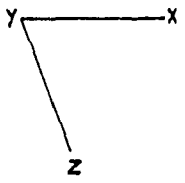
Name _____ May - June 1987

ON THE BLACKBOARD

1. $\measuredangle ABC = 46°$

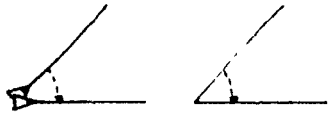


2 $\measuredangle XYZ = 68°$



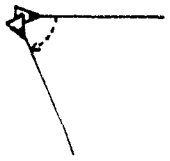
ON THE COMPUTER USING LOGO

1 Turtle turns from ◁ to ▷



Turtle turned RT _____

2 Turtle turns from ▷ to ◁



Turtle turned RT _____

Name _____

Date _____

1. Put a circle around all the angles that are 90° angles



2. Look at the angle on the blackboard.

Does this angle have anything to do with what you do in Logo? _____

EXPLAIN your answer.

_____

_____

_____

_____

_____

APPENDIX G

POST-SESSION INTERVIEW WITH THE GRADE 6 STUDENTS, MAY-JUNE, 1987

ON WORKING WITH A PARTNER

1. Since there are 6 computers in this room and approximately 12 students in each group, everyone is expected to work in PAIRS almost all the time. Let me ask you how you feel about working with a PARTNER.

   What are the good features?

   What are the not-so-good features?

   Have you worked in a group of THREE? Is it different from working in pairs? How is it different?

2. In LOGO, when your partner tries to EXPLAIN something to you, do you find that it HELPS you to understand it? For example, . . . [use an example from the child's own work, if possible.]

   Do you like to EXPLAIN things to your partner?

   What kinds of things do you EXPLAIN?

3. Does TALKING with your partner help you in your LOGO work?

   HOW?

4. In LOGO, can you tell me about a time when your partner HELPED you to understand something that you hadn't understood before?

   WHAT was it?

   Who was the partner?

   What grade was it?
   Was it ever another classmate, not your partner, who helped you understand something in LOGO that you hadn't understood before?

ABOUT PAPERT AND LOGO GEOMETRY

5. Can you explain to me, in your own words, what an ANGLE is?

   Use the blackboard if you like. [Supply blackboard-size and regular-size ruler, protactor, compass.]

6. This is a picture of Seymour Papert, the man who developed LOGO. [Show picture.] Papert thought that children could learn a lot about ANGLES through using LOGO.

   Do you feel that LOGO can help you learn about ANGLES? HOW?

7. Could what you do in LOGO help you with your CLASSROOM GEOMETRY? [Show mathematics textbook pages if necessary.] EXPLAIN your answer.

8. Could what you do in CLASSROOM GEOMETRY help you with your LOGO? EXPLAIN further.

9. If the child is not responding to questions 7 and 8, try probing:

   Use the sheet attached to show how the static angle relates to the angular rotation in geometry. That is, if the Logo turtle went "R-T forty-six," and you measured that turn with a protractor, it would be 46°, forty-six degrees. [Use a protractor.]

   Then ASK questions 7 and 8 again: Can you think now . . .

10. Are there any other ways that you can think of to show how CLASSROOM GEOMETRY and LOGO are the same? --are DIFFERENT?

GENERAL QUESTIONS

11. Would you like to change the computer-class time in any way?

If YOU could change it, what would you CHANGE?

[PROBE:
Amount of time--more, less;
Ratio of children per computer, teacher per group size
Type of software: LOGO, Applewriter, Printshop,
Logowriter, Appleworks,
OTHER (eg. games)
Prefer having computers WITHIN the homeroom
classroom itself?]

ANY OTHER things you'd like to say?

12. I have heard some children in the classes say "Oh, why do more LOGO? Let's do other things." They mention BASIC, or other suggestions.

Why do you think they do not want to do LOGO anymore?

What is your opinion? Is there too much LOGO or not enough? . . .

13. Do you have a computer at home?

What kind is it?

What do you do on your computer at home?

Games? If yes, what kind?
Logo?
Word processing?
OTHER?

14. Is there ANYTHING ELSE you would like to add?

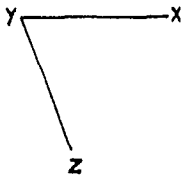
Name _____ May - June 1987

ON THE BLACKBOARD

1. $\measuredangle ABC = 46°$

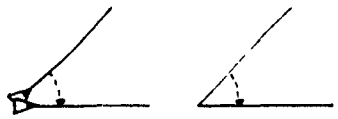


2 $\measuredangle XYZ = 68°$



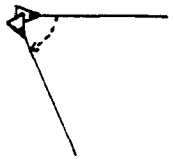
ON THE COMPUTER USING LOGO

1 Turtle turns from ◁ to ▷



Turtle turned RT _____

2 Turtle turns from ▷ to ◁



Turtle turned RT _____

APPENDIX H

RESPONSES TO THE INTERVIEW WITH THE GRADE 6 STUDENTS, MAY-JUNE, 1986.

The interview took place in an empty classroom within the school, and was conducted on a one-to-one basis with 56 of the 69 Grade 6 students. (Of the 69 Grade 6 level students, 3 declined to be interviewed, and there was not sufficient time to interview the remaining ten Grade 6 children.) Each interview was conducted by the researcher (V. Zack) and lasted from 30-40 minutes per child. The entire interview was audiotaped. Responses to questions 5-10 were videotaped. Included among the 56 students interviewed were the 10 study subjects: Russ H., Jon, H., Eric, H., Lilly, M , Rina, M., Shauna, L , Carol, L., Amy, L., Leanne, L., and Doug (L.). The interview tapes were reviewed, analyzed and summarized. Selected portions were transcribed.

The results of the analysis of the Grade 6 children's responses to the May-June, 1986 interview questions are presented in the following (twenty-two) pages of Appendix H.

Appendix H - 2

Question 1
How do you feel about working with a partner?

	frequency	percentage
almost always yes	12/56	21.43
sometimes yes, sometimes no	9/56	16.07
always yes	9/56	16.07
no response	6/56	10.71
prefer to work alone	5/56	8.93

Analysis of "yes" responses

if get along	13/24	54.17
if partners are friends	13/24	54.17
if same level of ability	3/24	12.50
if more advanced than partner	1/24	4.17
if partner is good	1/24	4.17

Appendix H - 3

Question 1.1
What are the good features?

	frequency	percentage
get help	37/56	66.07
get ideas	25/56	44.64
fun	12/56	21.43
better product	6/56	10.71
teach each other	5/56	8.93
finish faster	3/56	5.36
understand how others work	3/56	5.36
job sharing	2/56	3.57
learn to cooperate	2/56	3.57
get to know each other more	1/56	1.78
more economical	1/56	1.78
discuss with each other	1/56	1.78
easier for learning	1/56	1.78

Question 1.2
The not-so-good features:

	frequency	percentage
not get along	46/56	82.14
having to slow down when partner doesn't understand	3/56	5.36
not like the partner	2/56	3.57

Analysis of "not-get-along" responses

partners do not agree	14/46	30.43
argue	8/56	17.39
having to compromise	6/46	13.04
one partner is domineering	5/46	10.87
one partner wants to do something else	4/46	8.69
desire to work alone at times	2/46	4.35
one partner feels left out	1/46	2.17
immature behaviors	1/46	2.17
prefer own ideas	1/46	2.17

Appendix H - 4

Question 1.3
Have you ever worked in a group of three?

	frequency	percentage
yes	32/55	58.18
no	22/55	40.00
almost never	4/55	7.27
almost always	1/55	1.82

Reaction to working in threesome:

prefer never threesome	8/54	14.81
not bad	3/54	5.56
sometimes good, sometimes bad	3/54	5.56
prefer always threesome	0/54	0.00

Appendix H - 5

Question 1.3.1

Is [working in a group of three] different from pair work?

	frequency	percentage
yes	26/54	48.15
no	6/54	11.11
good features		
explain more	2/54	3.70
better product	1/54	1.85
faster	1/54	1.85
division of labour	1/54	1.85
can do a bigger problem	1/54	1.85
not-so-good features:		
difficult to compromise	25/54	46.29
divide keyboard time by 3	11/54	20.37
not enough room at computer	7/54	12.96

Appendix H - 6

Question 2
When your partner tries to explain something to you, does it help you understand it?

	frequency	percentage
yes	36/52	69.23
no	7/52	13.46

Qualifier of "yes" response

learn more from a friend	1/36	2.78
peers speak your own language	1/36	2.78

Qualifier of "no" response

get confused	1/7	14.28
prefer teacher to explain	1/7	14.28

Appendix H - 7

Question 2.1
Does your partner explain things to you?

	frequency	percentage
yes	24/53	45.28
no	15/53	28.30
no response	14/53	26.42

Qualifier of "no" response

it is the teacher's role to explain	17/53	32.08
not enough time	11/15	73.33
the partners are at the same level	11/15	73.33
the receiver (interviewee) is more advanced	7/15	46.67
doesn't like to explain	5/15	33.33
partner is more advanced, goes ahead and does it	2/15	13.33
it is insulting to the receiver	2/15	13.33
explanation must be elicited by peer	1/15	6.67
teacher can explain better	1/15	6.67
partner's explanation can be more confusing	1/15	6.67
partner is not good at explanation	1/15	6.67

Difficulties listed:

I need to be shown , as well as told	4/53	7.55
I cannot remember what the difficulties are	1/53	1.87
too much material, cannot be absorbed	1/53	1.87
material is too difficult	1/53	1.87
I need to do, while partner tells	1/53	1.87

Question 2.2
Do you like to explain things to your partner?

	frequency	percentage
yes	33/55	60.00
no	11/55	20.00
no response	1/55	1.82

Qualifier for "yes" response

child relate to child-child talk more easily than to teacher-child talk	4/54	7.41
explanation given when partner doesn't understand	3/54	5.56
good at explaining	1/54	1.85

Qualifier for "no" response

explanation must be elicited by partner	9/54	16.67
partner (explainee) is more advanced than me	7/54	12.96
the partners are at the same level of ability	5/54	9.26
it is the teacher's role	4/54	7.41
I am not good at explanations	3/54	5.55
doesn't like to explain	2/54	3.70
not enough time	1/54	1.85
it is insulting to the partner	1/54	1.85
like to get on with own work	1/54	1.85
hard to explain things	1/54	1.85

Either partner explains only if:

peer has not understood	9/54	16.67
peer has been absent	5/54	9.26
peer has not listened intently	4/54	7.41
peer cannot remember	1/54	1.85

Question 2.3
What kind of things do you like to explain?

	frequency	percentage
gives an example		
syntactic material	20/53	37.74
Logo content: mathematics	8/53	15.09
Logo content: programming	1/53	13.89
cannot give an example	10/53	18.87

Question 2.3
Difference between child as explainer and teacher as explainer

	frequency	percentage
teacher is more effective at explaining	8/53	15.09
child (peer) is more effective at explaining	4/53	7.55

Question 3
Does talking with your partner help you with your Logo work?

	frequency	percentage
yes	37/56	66.07
no	6/56	10.71
sometimes	4/56	7.14
asks for clarification of the question	4/56	7.14

Analysis of "yes" response:

more fun	2/37	5.40
showing is important	2/37	5.40
easier	1/37	2.70

How?

talk helps because others can give help (heuristic)*	21/56	37.50
talk helps me to understand because others can explain (heuristic)*	16/56	28.57
talk helps the planning (interactional)*	15/56	26.79
talk allows for communication of ideas	13/56	23.21
(informative)*	13/56	23.21
(instructional)*	6/56	10.71
(interactional)*	4/56	7.14
talking aloud helps one organize one's thoughts (heuristic*; Sapir)	11/56	19.64
talk helps because of the running commentary (interactional)*	9/56	16.07
talk allow one to express one's feelings (personal*, interactional)*	4/56	7.14

*note:
instructional ("I want"): satisfying material needs
regulatory ("Do as I tell you"): controlling behaviour of others
interactional ("Me and you"): getting along with other people
personal ("Here I come"): identifying and expressing the self
heuristic ("Tell me why"): exploring the world around
imaginative ("Let's pretend"): creating a world of one's own
informative ("I've got something to tell you"): communicating new information
- Halliday (1974). Language and social man, School Council Publications, p.17

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Question 3.1
Do you ever find that talking aloud about an idea helps you to understand the idea? (Sapir question, 1921, 1949)

	frequency	percentage
no	16/37	43.24
yes	8/37	21.62
gives back the same answer that interviewer fed him/her	7/37	18.92

Question 4
In Logo can you tell me about a time when your partner helped you to understand something you had never understood before?

	frequency	percentage
does feel the neighbouring classmates helped	35/54	64.81
can name the thing learned	26/55	47.27
yes	23/55	41.82
no	20/55	36.36
can name the partner	15/55	27.27
not partner, but teacher	12/55	21.82
not Logo, but Logowriter	9/55	16.36
can name the teacher	7/55	12.72
cannot remember	1/55	1.82

Question 1 to 4
Indicates that expertise with Logo is related to home use of computer

	frequency	percentage
reference to partner	6/54	11.11
reference to self	5/54	9.26
reference to classmate	5/54	9.26
reference to classmate as expert	3/54	5.56

Question 5
Explain what an angle is.

	frequency	percentage
can explain	32/56	57.14
meaningful explanation given	16/56	28.57
unrated	15/56	26.79
cannot explain	9/56	16.07
knows:		
degrees	16/56	28.57
complementary angles	9/56	16.07
supplementary angles	7/56	12.50
relies on:		
complementary (90) in explanation	8/56	14.29
supplementary (180) in explanation	7/56	12.50
protractor comfort level:		
excellent	5/56	8.93
confusion about placement	11/56	19.64
confusion about counting	3/56	5.36
confusion about numbers	2/56	3.57
uses 90 degrees as reference point	14/56	25.00
cannot use 90 degrees as reference even with prompting	1/56	1.79
90 degrees = (called) straight angle	8/56	14.29
uses SETH	3/56	5.36
confusion about:		
outside turn = inside turn	17/56	30.36
number of degrees in full turn	6/56	10.71
right angle	4/56	7.14
uses terms:		
"open angle", "closed angle"	6/56	10.71
"bend in a line"	2/56	3.57
RT __ = __		
yes	23/55	41.81
no	29/55	52.72
unrated (maybe)	3/55	5.45

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Question 6
Seymour Papert is the person who developed Logo.
He felt that by using Logo, children could learn about angles.
What do you think?

	frequency	percentage
yes	42/56	75.00
no	8/56	14.29
never thought of angles while doing Logo	11/56	19.64
see clear relationship between Logo angles and classroom instruction on angle	10/56	17.86
see some relationship between Logo and classroom instruction on angle	1/56	1.79

How does Logo help you learn about angles?

familiarize with concept of angles	8/56	14.29
familiarize with concept of degrees	4/56	7.14
through the use of drawings	2/56	3.57
Logo and angles are the same thing	1/56	1.79

Question 7
Could Logo help you with your classroom geometry?

	frequency	percentage
yes	37/51	72.55
no	10/51	27.03

Analysis of "yes" response

helps learning the concept of angle	10/37	27.03
visual representation of what angle looks like	6/37	16.22
basically the same thing	3/37	8.11
measurement = turn	1/37	2.70

Question 8

Could what you do with your classroom geometry help you with your Logo work?

	frequency	percentage
yes	36/55	65.45
no	8/55	14.55

Analysis of "yes" response

easier to replicate on the computer	8/36	22.22
become more familiar with concept of degree and angle	4/36	11.11
combine learning computer and geometry	1/36	2.78

Question 9

Analytic question: If this part (angle) were 23 (or 41 or other) and you wanted turtle to turn and then go along this line (route, direction), what would turtle have to turn?

	frequency	percentage
perceptual level	47/56	83.93
analytic level	15/56	26.78
with prompting	10/56	17.86
completely	5/56	8.92
partially	3/56	5.36

Question 10
How are classroom geometry and Logo geometry the same?

	frequency	percentage
no (different)	28/43	65.11
different mechanism	9/43	20.93
Logo more automatic	3/43	6.98
Logo more exciting	1/43	2.33
yes (the same)	17/43	39.53
all about angles	7/43	16.28
draw the same thing	3/43	6.98
estimate length	1/43	2.33
atmosphere is different	6/43	13.95

Question 10.1
Which is more exact? Logo or protractor?

	frequency	percentage
Logo	3/5	60.00
protractor	2/5	40.00

Question 11
If you had the chance, would you make changes to the computer classtime?

	frequency	percentage
yes	43/56	76.79
no	4/56	7.14

Question 11.1
Amount of time

	frequency	percentage
more computer time wanted	38/43	88.37
all year	24/43	55.81
half hour more per session	18/43	41.86
more times per week	12/43	27.90
more minutes per session	9/43	20.93
up to 20 sessions	5/43	11.62
half year	2/43	4.65
less computer time wanted	0/43	0.00
amount of teacher introduction time too much	14/43	32.55

Question 11.2
Ratio of computer to children

	frequency	percentage
pairs optional	36/54	66.67
pairs always	20/54	37.03
single always	5/54	9.26

Question 11.3
Number of teachers per 12-person group

	frequency	percentage
2 teachers	20/53	37.74
1 teacher sufficient	16/53	30.19
1 teacher plus assistant	10/53	18.87

Question 11.3.1
Students as teachers

students as teachers: yes	16/52	33.07
students as teachers' aid	1/52	1.92
students as teachers: no	13/52	25.00
teachers know more	8/13	61.50
prefer someone older	4/13	30.77
students would have to give up computer time	4/13	30.77
kids fool around	1/13	7.69

Question 11.4
What is your favourite type of software?

	frequency	percentage
Logowriter	32/56	57.14
Printshop	10/56	17.86
Applewriter	4/56	7.14
Appleworks	1/56	1.78
Logo	0/56	0.00

Question 11.5
Would you prefer having the computers within the homeroom classroom

	frequency	percentage
no	40/56	71.43
yes	5/56	8.93
only in addition to what exists now	3/56	5.36
difficulties mentioned:		
no teacher contact	16/56	1.79
missed work - have to make up	15/56	26.79
noise factor	9/56	16.07

Question 12
Why do some children complain about doing more Logo?

	frequency	percentage
bored of it	48/54	88.89
have computers at home	7/54	12.96

Question 12.1
Child's opinion: Too much Logo or not enough?

	frequency	percentage
too much Logo	34/54	62.96
has had Logo for 3 years	7/54	12.96
has had Logo for a long time	3/54	5.56
has had Logo every year	2/54	3.70
has had Logo for 4 years	1/54	1.85
has had Logo for 6 years	1/54	1.85
amount just right	13/54	24.07
Logo is just drawing	13/54	24.07
Logo is not challenging enough	9/54	16.67
Logo is limited in what it can do	7/54	12.96
not enough Logo	0/54	0.00
suggestions on how to make it more challenging:		
use of other software	18/54	33.33
give students more autonomy	4/54	7.41
teacher give assignment	2/54	3.70
teacher not give assignment	2/54	3.70
more variety	2/54	3.70
teacher give more commands	1/54	1.85

Question 12.2
Child has had previous experience with:

	frequency	percentage
other graphics language	11/54	20.37
Logo	7/54	12.96
where?		
computer course	7/54	12.96
home - parents taught	3/54	5.56
camp	2/54	3.70
home - siblings taught	0/54	0.00

Will Logo help with geometry in high school?

yes	4/54	7.41
no	2/54	3.70

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Question 13
Do you have a computer at home?

	frequency	percentage
yes	27/56	48.21
no	27/56	48.21
is getting one	2/56	3.57
just recently acquired	1/56	1.79

Question 13.1
What kind?

	frequency	percentage
IBM	10/56	17.86
Apple II E	7/56	12.50
Vic 20	7/56	12.50
Apple II C	2/56	3.57
MacIntosh	2/56	3.57
Other	2/56	3.57
Apple Plus	1/56	1.79

Question 13.2
Use of computer

	frequency	percentage
often	12/56	21.43
seldom	4/56	7.14

Kind of use:

word processing	20/56	35.71
games	18/56	32.14
other	11/56	19.64
Logo	4/56	7.14

Question 13.3
Computer literacy

	frequency	percentage
Computer literacy is important	3/56	5.36

Question 14
Additional comments

	frequency	percentage
course work should be marked	1/4	25.00
computer helps you to work faster	1/4	25.00
those who already know Logo should be exempted	1/4	25.00
need good typing to do computer	1/4	25.00

APPENDIX I

EXPLANATION OF TRANSCRIPT NOTATION USED IN THIS STUDY

The following are explanations for the conventions used in the presentation of transcribed data in this paper. A number of the conventions were adopted from the work of Jefferson and her colleagues (Schenkein, 1978). Others were adopted from the work of Steinberg (1979). Still others are a hybrid adaptation whose sources can no longer be traced.

The children's speech is set out vertically. Each utterance or group of utterances by an individual youngster which continues until that child is interrupted is represented sequentially, and is identified as a "turn" of talk (Sacks, Schegloff, & Jefferson, 1978). Unlike most linguistic transcripts, it was not considered essential for the uses of this paper to number the individual turns (with the exception of Selection #20).

Contextual information and nonverbal behavior are specified where necessary and enclosed in round brackets. Comments upon what is transpiring are enclosed within square backets.

The speech is transcribed in standard English orthography. The children's and adult's individual pronunciation is represented as is (for example, *I'm gonna*), and has not been changed to correspond with written convention (where one would see *I am going to* rather than *I'm gonna*). Standard punctuation was used to show exclamations, questions, and so forth.

The following is a list of the symbols used.

(? XXX)	Parentheses and a question mark, are placed around transcripted material about which there is some doubt concerning what was said. The XXX represents the researcher's interpretation of the utterance or the part of the utterance.
(?)	Empty parentheses, with a question mark, point out that sounds could be heard but not deciphered.
( )	Round brackets enclose commentary. The commentary includes contextual information, non-verbal activity, the addressee of the utterance where necessary, and so on.
[ ]	Square parentheses enclose comments.
::	Colons are used to indicate prolongations of syllables. More are used to indicate longer stretching, for example :::::
,	The comma designates a brief pause.
(pause)	Longer pauses are shown in this fashion, both within and between speaker turns. In the source transcripts, the researcher used the following convention: /. . ./; each dot represents roughly one beat. This level of detail, namely number of pulses per pause, was not considered essential for this paper.
-	A hyphen at the end of an uncompleted utterance indicates an abrupt halt or break in the stream of talk, as when someone is uncertain of an answer, or changes the topic abruptly, or is interrupted.
-B -K	A hyphen before the alphabet letter(s) indicates that the speaker is saying the letter names, for example, the child would be saying BEE KAY.
_____	Underlining is used where part of an utterance receives unusually heavy stress.
XXXXX	Capital letters are used where part of an utterance is spoken in a loud voice.
. . .	Three vertical ellipses are used to indicate that part of the original transcript has been omitted, namely intervening turns of talk.

. . . Three horizontal ellipses indicate that an utterance is being reported only in part, with additional speech coming before and/or after the reported fragment, depending on the location of the ellipses.

A star is used when two people speak simultaneously; the star shows the beginning of the overlap in their talk. Two stars (and then three) are used when there is a second or third overlap within adjacent turns of talk.

The left hand margin of the transcript is sometimes used to point to a feature of interest which the analyst has noted within the body of the text. Large dots (bullets) have been used with the intention of facilitating location within a long transcript, of points which the researcher wishes to stress.

APPENDIX J

SAMPLE TRANSCRIPT, WITH EXAMPLES OF DISAGREEMENTS CODED

Lilly and Rina, November 20, 1985, pp. 7 & 8

Mathematics concepts disagreements #3, #4, and #5, and two disagreements on Plan.

		CODING	
			D/A on PLAN
Lilly M.:	Now -L -T, (pointing to. the screen), now we wanna turn to go up, right? And then we're gonna go like that, and then we're gonna go like that, right? (seems to be tracing or pointing to spots on the screen).	O.	Suggested action (supported) focusing
Rina M.:	(Pause) (Looking at the screen)No,theopposite on the screen).	1.	denial/resistance; counter/contradiction;(traces supporting argument, focusing
Lilly M.:	Down? (Pause)	2.	request for clarification
Rina M.:	(Keys)	3.	insistence-action taken
Lilly M.:	O. K. (Hand motion to affirm.) (Pause) So we wanna go right again.	4.	acceptance (RESOLUTION)
			D/A #3 Math Concept (on turn input)
Rina M.:	-R -T (writing), we wanna go half (hand motion to indicate a quarter turn), we want -R -T, -R -T eighty-five.	0.	suggested action (supported), supporting argument (INCORRECT)
Lilly M.:	No, 'cause we were, -R -T eighty (five, stated quietly), right (nods).	1.	denial/resistance; supporting argument (INCORRECT) acceptance (RESOLUTION)
Rina M.:	(Keys)		

Lilly M.:	(points to the screen)	
Rina M.:	(Looks at the screen) Oh! (hand over mouth, keys, Looks at the screen) -R -T, -R -T eighty five. (Keys) (Looks at the screen) (Pauses).	
		D/A #4 Math Concept (on turn input)
	No, it's -R -T ninety. (Writes) -R -T	0. Suggested action (UNSUPPORTED) (INCORRECT)
Lilly M.:	It's <u>not</u> -R -T ninety. Another five.	1. denial/resistance, counter/contradiction; (CORRECT)
Rina M.:	Another 'ten.	2. counter/contradiction; (CORRECT)
Lilly M.:	'Cause if you're gonna do -R -T ninety, you're gonna-	3. supporting argument
Rina M.:	(keying) I'm going -R -T <u>ten</u>.(pause, stops, looks at Lilly) Well? [As in "I told you so."]	4. insistance-action taken, computer gives verdict
Lilly M.:	(shrugs)	5. silence, acceptance (RESOLUTION)
Rina M.:	(writes)	
(page 8)		
Rina M.:	(writing) Now, forward, we can't go forward-, (pointing to the screen), we can't make it the same as this.(Forward fifty (keys).	example of supporting argument-**mathematical justification**-the geometry referent to a rectangle is implicit
Lilly M.:	We have to make it, (?ya-)	
Rina M.:	(Keying) rectangle.	
Lilly M.:	That was forward ninety-SIX. (Looks toward	

Rina).

Rina M.: (Writing) Forward fifty, now we're going another, I'm going another forward (Pause) thirty-five. (Keys)

Lilly M.: (Writes)

Rina M.: (Looking at the screen) Per-fect. (Leans forward) Oh, g-, oh, it's not gonna-, (pause)

L. M. & R. M.: (Watch the screen intently)

Rina M.: 'Kay, the turtle's ju:st (pointing, tracing on the screen) gonna go-. It's gonna be too small, I don't like it. (Turns to Lilly M.) (Long pause) Okay, go left, type.

Lilly M.: Why'd you go -F -D?

Rina M.: (Writes). Left ninety.

Lilly M.: -F -D

Rina M.: (Keys)

Lilly M.: (Moves her head) N-

Rina M.: (Keying) Left ninety. (Looks at the screen) Oh no! (Writes)

Lilly M.: -R -T ninety. Now we want ninety plus ninety is a hundred-

Rina M.: -R -T (Rina is writing)

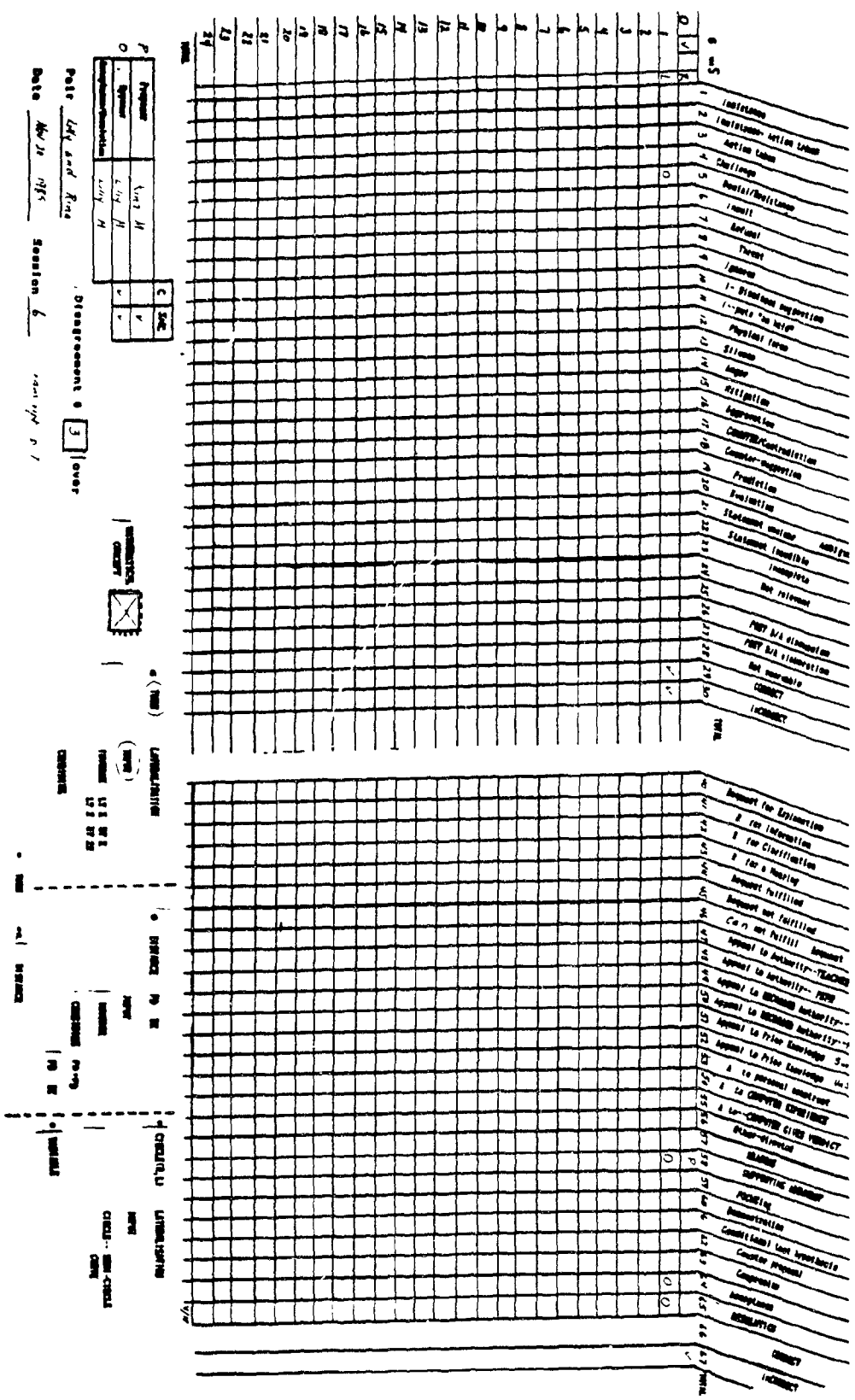
Lilly M.: Ninety plus ninety is, okay, -L -T (getting ready to key), I know

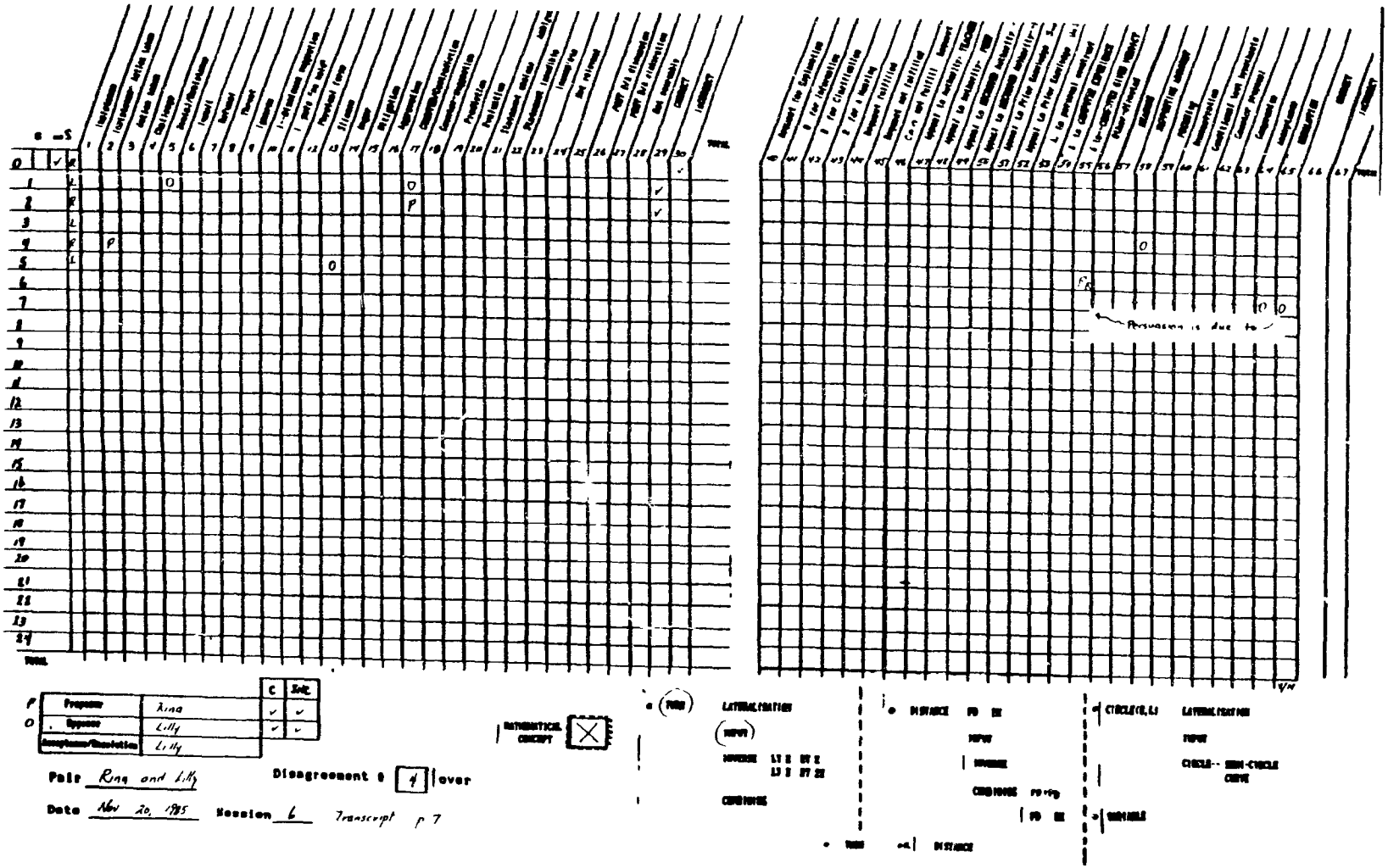
D/A on PLAN (next step turn or DISTANCE)
0. Suggested action (UNSUPPORTED)
1. challenge
2. ignores
3. insistance
4. insistance-action taken
5. denial/resistance
6. insistance-action taken

D/A #5 Math Concept (Lateralization)
0. Suggested action (UNSUPPORTED) (CORRECT)
1. counter/contradiction; supporting argument: prior knowledge

	what I'm doing-		(unsupported) (INCORRECT)
Rina M.:	(Moves Lilly's hand away) No, no, *-R -T what's ninety-	2.	Physical force, denial resistance, insistance (CORRECT)
Lilly M.:	*-L -T a hundred and eighty, -L -T	3.	insistance
Rina M.:	(writing, checking in her book) -R -T a hundred and eighty.	4.	insistance
Lilly M.:	-L -T! We just went -L -T!	5.	insistance, supporting argument (INCORRECT)
Rina M.:	(Keying) -R -T a hundred and eight:y:y	6.	insistance-action taken reference to computer screen verdict
Lilly M.:	(Nodding, looking up at the screen) Oh, ya, -R -T a hundred 'n eighty.	7.	acceptance (RESOLUTION)
Rina M.:	Perfect.		

CODING SHEET

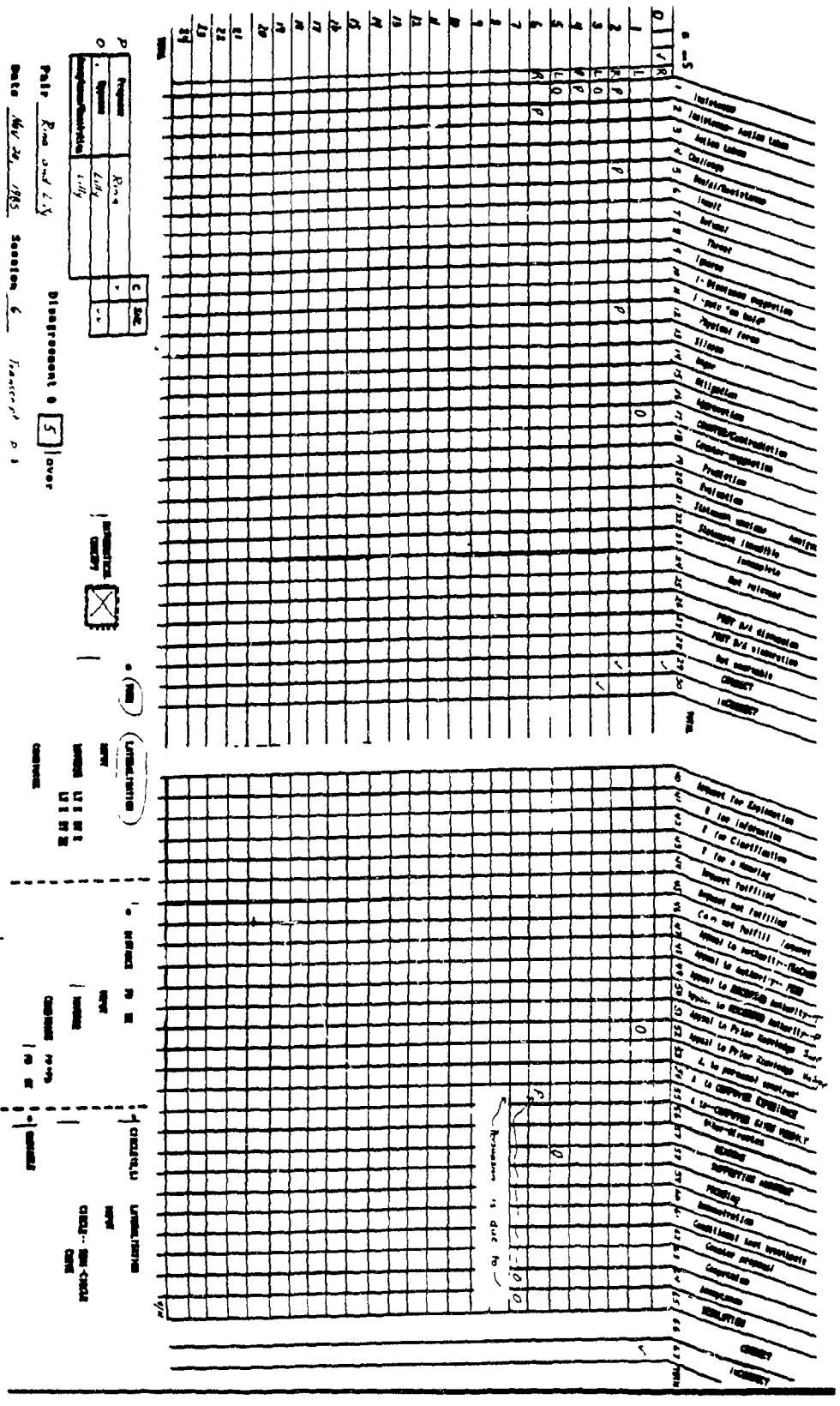




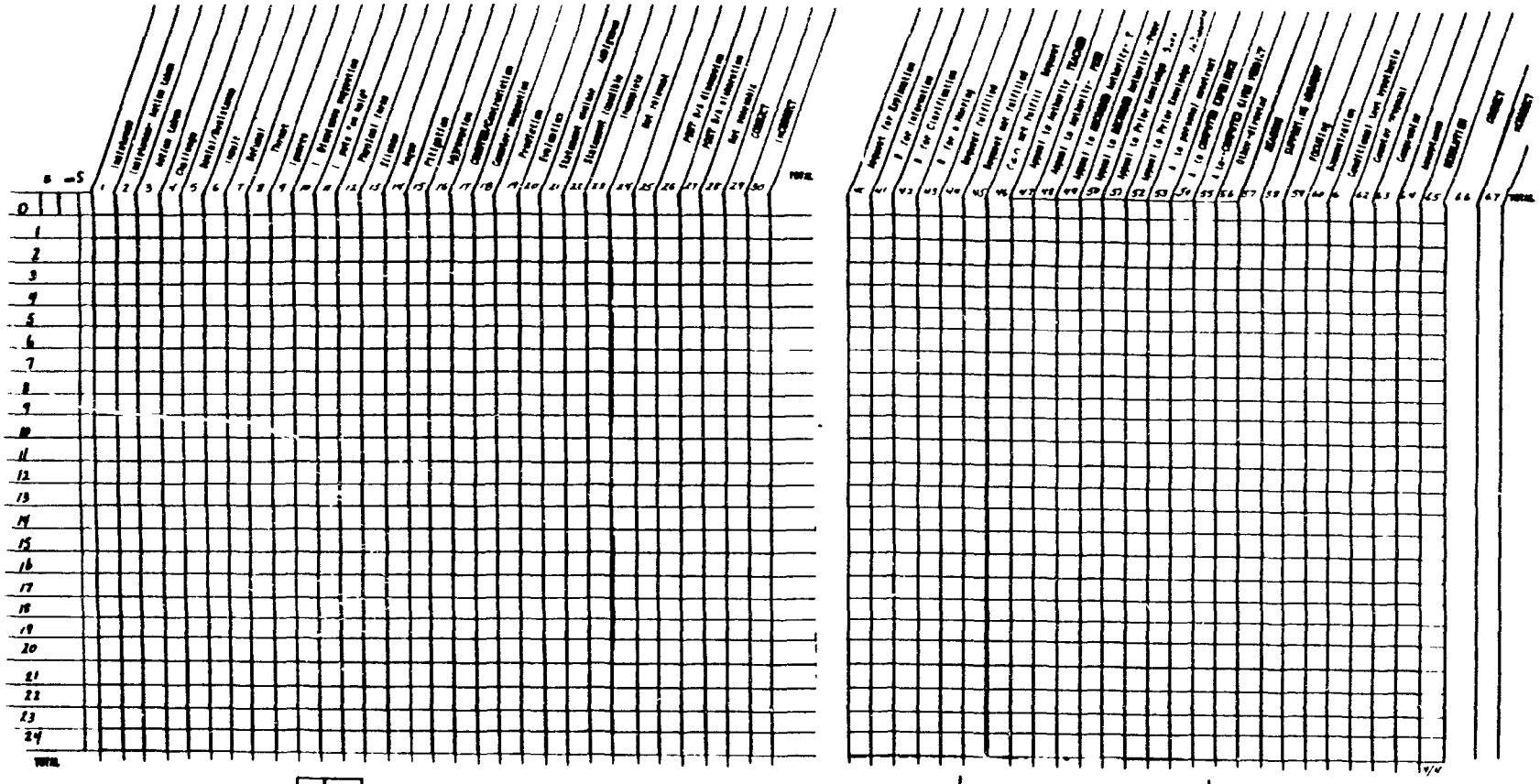
P / O		C	Inf.
P	Proposer: Xing	✓	✓
O	Opposer: Lilly	✓	✓
	Acceptance/Resolution: Lilly		

Pair Ring and Lilly Disagreement 0 [4] over

Date Nov 20, 1985 Session 6 Transcript p 7



CUDING SHEET (BLANK SAMPLE)



	C	INC
Proposer		
Opposer		
Acceptance/Resolution		

Pair _____ Disagreement @ ☐ over

Date _____ Session ____ transcript page ____

MATHEMATICAL CONCEPT ☐

PLAN ☐

SYNTAX ☐

• TURN LATERALISATION

INPUT

INVERSE LT X RT X
LT X RT 2X

COMBINING

• DISTANCE FD BK

INPUT

INVERSE

COMBINING 10+FD
FD BK

• VARIABLE

• CIRCLE(D,L) LATERALISATION

INPUT

CIRCLE SEMI CIRCLE
CURVE

• TURN OR DISTANCE

APPENDIX K

CATEGORIES AND SUBCATEGORIES FOR THE FOUR TYPES OF DISAGREEMENTS: DISAGREEMENTS CONCERNING MATHEMATICS CONCEPTS, PLAN AND PROGRAMMING, SYNTAX, AND CLASS RULES

The following lists will feature the various categories of disagreements which were identified during the analysis. The code numbers listed below were also used in the identification of problems encountered by the children in the various domains, apart from the disagreement episodes. The code numbers were used as well in the analysis of instances of teacher and peer intervention, both spontaneous and elicited. The mathematics concepts disagreements categories have been explained very briefly. The other items listed below under the plan and programming, syntax, and class rules headings have not been explained; the purpose of the appendix was to stress that there was a broad range of issues which the children encountered and argued about, and which often reflected the children's difficulties. The syntax aspect in particular presented a wide variety of problems for the children.

Mathematics Concepts Disagreements (See Table 12)

Turn

1. Lateralization [determining whether to go right or left. LT or RT with an input number turns turtle left or right the corresponding number of degrees. Errors arise when children do not realize that commands to RT/LT are relative to the turtle's state, not the child's own state.]
2. Input to LT or RT [amount of the number input to use, for example, RT 45 or RT 47]
3. Combining of input to LT or RT [adding the input numbers to RT; for example RT 45, RT 45 would equal RT 90 if the addition is done correctly]
4. Doing the inverse of LT X, by using RT X, RT X [This a move to correct an incorrect first move, namely LT X]

5. Doing the inverse of LT X, by using RT2 X [This is a move to correct an incorrect first move in one step rather than in two steps, see Table 17]

Distance

6. FD, BK [whether to go forward or back in terms of a displacement]
7. Input to FD/BK [what the amount of input should be, for example, FD 25 or FD 29]
8. Inverse of FD; BK [whether the move should be made to go the opposite way, usually in order to undo an error in displacement]
9. Combining of input, such as FD X, FD Y [which would result in FD (X + Y) if the addition is correct]
10. Combining of input FD X, BK Y [which would result in FD (X - Y , if X > Y)
11. Turn or distance [whether to turn or to do a displacement move]
12. Circle lateralization [whether to use CIRCLER or CIRCLEL)
13. CIRCLER/CIRCLEL input [what to use as the number input to CIRCLER or CIRCLEL]

    Turn and displacement [when a move requires both a move forward or back, and an angular rotation, such as to create a curve, or a circle from scratch, i. e. without using CIRCLER or CIRCLEL]
14. Semi-circle
15. Curve
16. Variable
17. Evaluation
18. FD, or semi-circle
19. BK, or BK and CIRCLER
20. CIRCLER, or Turn and CIRCLER [whether the correct move is to do a CIRCLER command, or whether to turn turtle first and then do a CIRCLER command]
21. Turtle turns and leaves no trace [whether there is no trace left behind when turtle does a turn-- the answer is that there is no trace]

Disagreements Concerning Plan and Programming

What to make and how to make it

1. What to make and how
2. Initial beginning
3. Next step
4. Closure [whether to continue or to conclude the project]

5A. Size of the project (awareness of extent)

5B. Size of the procedure

6. "Deplan" (see Note 32)
7. Naming the project or parts

How to do it and keep track of it

8. Recording step by step--mistakes included
9. Recording after testing out-- after combining, or after correcting

How to debug, how to test

10. How to proceed-- Programming Procedures
11. How to test out the product-- Testing Procedures
12. Debugging in IMMEDIATE (DRAW) MODE
13. Using clearscreen to erase an error
14. Working backward [to debug an error]
15. Subdividing into parts, and testing
16. Re-entering the commands in IMMEDIATE MODE, and reconciling entries
17. Continuing on, adding on to correct procedures
18. Using PU (to test procedures in IMMEDIATE MODE)
19. Debugging within the EDITOR
20. Order of sequence of procedures (when testing in IMMEDIATE MODE)

20B. Interface problem concerning sequence of procedures

21. Using already defined procedure as a building block
22. Re-editing procedures-- reconciling differences in 2 sets of notes
23. First-time editing of procedures-- reconciling differences in 2 sets of notes

How to proceed when programming the procedure

24. Editing the procedure itself
25. Re-entering in immediate mode, in error, thinking it is the edit mode
26. Re-entering in immediate mode, in order to test [an appropriate testing strategy]
27. Editing superprocedure
28. Using already defined procedure as a building block
29. Re-entering all commands again, not using corresponding procedure which the child(ren) just completed [Error]
30. Evaluation

Disagreements Concerning Syntax

1. Load
2. Save
3. PU, PD
4. PE
5. Editor
6. Control keys
7. Use of control keys in IMMEDIATE MODE (an error in Apple Logo)
8. Delete keys in editor
9. Problems with LOAD [eg. "Was it saved last time?" "Is the *save* or *Logo* disk in the disk drive?"]
10. Title-- which title was it
11. Add .LOGO, or not [.LOGO should not be added, but the catalogue listing shows the filename with .LOGO attached to it]
12. Use quotations marks, only one opening set [eg., "camel]
13. Reading the CATALOG listing and interpreting it [eg., 4 LILLY.RINA.LOGO is interpreted by the children as meaning 4 procedures were saved-- incorrect]
14. Which disk to load first (LOGO or SAVE)
15. Confusion between *file.name* (name by which it was saved on the disk) and *code.name* (name of the Logo procedure)

16. Confusion between notebook name, and the *code.name*
17. Confusion between SAVE and LOAD
17B. Confusion about whether the file has already been loaded in
18. How to run a procedure
19. Use of TO to run a procedure (instead of just the code.name)
20. ER to erase from the workspace (computer: ERALL, or disk: ERASEFILE)
21. Dot or other between two words that should not be separated by a space
22. Logo commands [for example, CS; HOME; child-coined commands such as DW, to get turtle *down*, which does not exist]
23. REPEAT
24A. Mistype [wrong letter, wrong name used, wrong spelling of name, space forgotten, and so forth]
24B. Type [use of ED instead of EDIT is an expedient shortcut]
25. Miscellaneous [eg putting :SIDE onto all appropriate lines in the editor; going into BASIC suddenly, etc.]
26. Printer

Disagreements Concerning Class Rules

Some of the class routine was explicit, and some was implicit. The teacher established and reinforced the rules. The children complied in calling for the teacher to fulfill both the teaching and the administrative functions, for example, to "tick it off" (1), or to "check their progress to date" (3). The teacher initiated certain interventions, such as insisting that the children subdivide their program into smaller parts if the procedure was too long (7), or stressing that they use the proper recording format in their notebook (9). The categories below were most applicable to categories of student calls for teacher intervention, or for teacher-initiated teacher intervention. But in some instances, the children had disagreements concerning these issues, and therefore they are all listed here.

Class Rules, and Teacher Function

1. Teacher ticks the completed section
2. Check that it works

3. Checking progress to date/ Reporting on progress to date
4. Direction maintenance
5. Use the book and test in immediate mode

5B. Use the stencil instructions

6. Ready to go to the printer
7. Sub-dividing large program into smaller parts
8. Defining in the editor; test it after
9. Recording properly (format-wise) in the notebook

9B. Student must have notebook for the computer class

10. Time allotment per class; pacing
11. Negotiating closure (i. e. when a picture is considered finished)
12. Where to sit
13. What is not permitted to be done on the computer (doodling, wrapping)
14. Pairs are expected to work together [social aspect, with teacher as mediator when necessary]
15. Partners share the time at the computer work station, for example, taking turns typing
16. Each partner contributes to the plan
17. Classmate peers work together ["Ask three before me"-- see Appendix X]

17B. Teacher delegates teaching role--> peer--> peer

Teacher Role

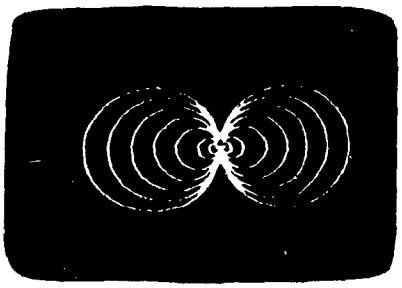
18. Teacher provides (a) comfort, encouragement, praise, (b) frustration control, and (c) assistance
19. Administrative role [has the children put disks back in place, etc.]
20. Teach and guide

APPENDIX L

SHAUNA L. AND THE HELP ROUTINE

Nov. 20, 1985, Session #6, pp. 7-11

Carol L. and Shauna L. were working on a project they called "GOOGLY-EYES." The nested circles as drawn below were a commonly seen product; the circles were drawn with one main computer command, CIRCLER and CIRCLEL, with an input number which dictated the size of the circle.



Shauna and Carol wanted to get the turtle from ▲ to ▼ (180 degrees in formal geometry terms).

Shauna recalled drawing these circles in her introductory Logo computer class in the McGill Faculty of Education library (actually in the microcomputer lab facility of the library), in November 1982. She was heard to mention it to the researcher, since it was the researcher herself who had been Shauna's Logo teacher in 1982 (see Section A in the transcript below). At that time I was the Logo teacher and worked with a small group of 12 children who ranged in age from 8 to 12 years old; Shauna was one of the group, and was 7;11 years old then (see Table 8). Shauna expressed that she felt much more in control of the function of making the nested circles now (that is, in Grade 5).

The reader will note that the interchange with Doug L. reflected a touch of the ongoing social interaction, which maintained a natural tone even though the children expressed here as elsewhere their awareness of the microphone and the camera (see Section B).

Beginning in Section C, Shauna pondered, probed, and persisted in her search for the aid to help them to see where the turtle was pointing. She recalled the aid; she did not remember its name but knew its function. (Monica, the Logo teacher, had mentioned the aid and showed it to Shauna and her partner Amy during the Oct. 23rd class, 1985.) Shauna was referring to the HELP aid which two Grade 6 children (Jonathan and Kevin) had designed and added to the Logo start-up file in 1983-84, with

the assistance of Sharon, another Logo teacher. This addition of HELP to the Logo start-up file used in this school was highly inventive and unique. It was recognized by the schoolchildren themselves as a marvellously useful and original creation. In one case, when a child wanted to praise a peer's creation, (Michael H. praising Robbie's design, Dec. 1984), Michael H. was heard to tell Robbie that the product was as good as the HELP creation.

Although the HELP aid was very simple in structure, in that turtle moved forward, paused and then moved back in place without leaving a trace, and thereby displayed its heading, it was deemed an invaluable aid by most of the children, since the turtle's heading was never clearly distinguishible on the screen due to the equi-dimensions of the equilateral turtle cursor. HELP was most often used by the children who were not in control of the turtle's angular rotation, for they could use it to check their position. What was fascinating to discover in the instance cited below (beginning at Section C), was that unbeknownst to Shauna L., and indeed to the researcher herself until close scrutiny revealed it, Shauna actually recreated the HELP procedure. Shauna attempted to ask for Monica, the Logo teacher's, help only once (see Section C-2). Shauna could not elicit from Saul L., a neighboring peer, the computer command that she was searching for, i. e. HELP. At first, Saul thought Shauna wanted to see the textscreen, and he told her that the answer was CONTROL-T (see Section D). Shauna knew that Control-T was not what she was seeking, and told him that what she wanted made turtle go "zoom" there and back (see Section E). Saul gave Shauna a clue when he keyed in FD 50 which moved turtle forward, but Saul was probably not aware of his function as facilitator (see Section F).

Shauna L. then worked slowly through the very steps which Jonathan and Kevin, the creators of HELP, compiled for the start-up file. Shauna L. tried to tell Carol L. that she (Carol) did not have to record the forwards and backs which just moved turtle forward and back to test where turtle was heading (see Section G). However, because Carol could not follow what Shauna was doing, Carol chose to copy down each one of the steps in her recordkeeping notebook (see Section H)

N.B. In every case where Shauna and Carol's talk is related to figuring out where the turtle is heading, italics are used.

(S = Shauna; C = Carol where used.)

November 20, 1985, pp. 7-11

SECTION A

COMMENTS



(The Logo teacher, Monica, is at the computer station with Carol and Shauna.)

Monica: Carol, (lifts Carol's notebook and encircles Carol) if you (points to instructions on Carol's book page) want to see the turtle draw your program, (withdraws her finger), what do you do?

Shauna: Um, we write 'googlees.

Carol: Type, type 'googlees.

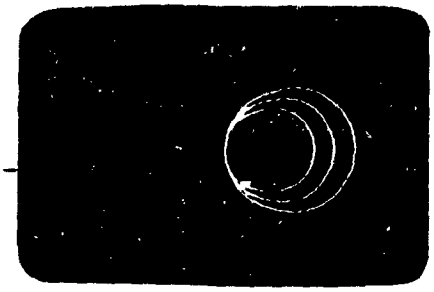
Monica: (Nods) Try it. (Puts Carol's notebook down, and retreats backwards to look at Mark and Don's work.)
(Monica is now gone.)



C: -L-Y-S (reaches over to key in)

S: (Excited) 'Kay!

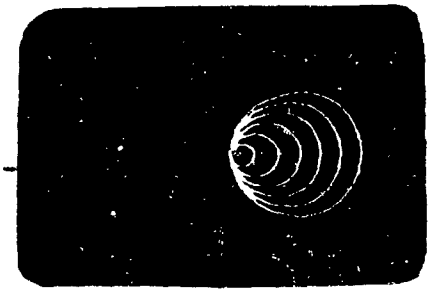
C: We don't even have to do any work! (Pause.) Just watch it (smiling, looking at the screen).



S: Don't you think we should have done it by fives? (tracing on the screen).

C: (Shakes head "no.")

S: It would have taken longer (pause), but it would have been more. Do you think we should go back and do it by fives?



C: NOOO! (smiles)

S: (To Vicki, the researcher, who was her computer teacher in 1982) I remember we did something like this when we were in the McGill 'library. I kept on doing this.

COMMENTS

C: "Ten! Ten, five. (Points to the screen.)

S: (To Vicki, the researcher) It was the only thing I knew how to do.

C: Look. Did you see (Shauna turns toward Carol), it went right, right, like that. (Points to the monitor)

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SECTION B

Doug: (Doug walks over.) Carol, when did you come to school?

C: Now.

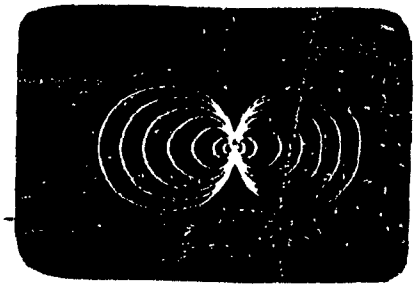
S: (Turns) none of your business.

D: Fuck off. (Turns and walks away.)

S: Watch it, bec- (points) mike's right here. See look, right there.

C: There.

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S: O.K. Now. We have to go down.

C: (Leans over and points) Nose.

S: What are we going to call the nose? (ready to write in her notebook). Wrinkles. We're going to call the nose "wrinkles." (Shauna writes in her book.)

C: (Carol's back is turned to Shauna, and Carol is not writing anything.)

S: (Turning to Carol) We're gonna call the nose "wrinkles," 'kay? 'Cause it's like, it's gonna be more (Carol nods.)

One, two, three.

COMMENTS

C: (Nods) O.K. (said quietly)

S: But we have to go (pause), PENUP.

C: (Playing with the clip cover on the disk drive.)

S: (Keys in) -P -U (pause). 'Kay, write -P -U.

C: (Takes her pencil, leafs through her book to the recording page.)

S: (Writes, then turns toward Carol whose back is to her.) How do we turn around?

C: -B -K, backwards. (Turns only her head, then writes in her book)

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Shauna: O.K. Now we're going to have to turn to the right. (Shows.) (Carol now looks at Shauna, at her hand on the keyboard.) And we're gonna do -R -T space. (Turns to face Carol, and with emphasis in her voice says:) How much should we do? [rhetorical question] Let's try thir-ty. (Keys in.)

C: (Pounds with her pencil, then presses the RETURN key.)

S: Yay!

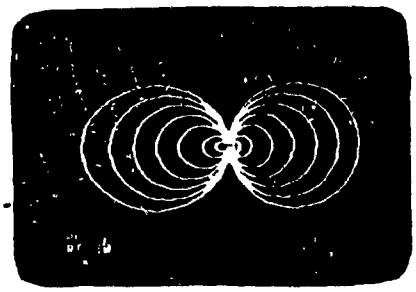
C: (Smiles, leans back) That didn't do us.

SECTION C

Shauna: *Where, where is he?*

C: (Leans over to show her.)

Mark: (Mark L. comes over to tell Shauna



Shauna means "Where is he pointing?" Carol answers literally and points to where the turtle cursor is.

something.)

COMMENTS

S: (Turns) What?

Mark: (? inaudible)

S: (Turns back to her book and begins to write) -R -T thir-ty.

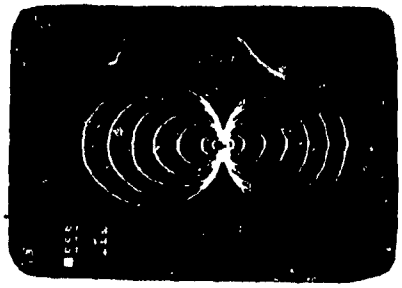
C: Space (keys), now let's try--

Shauna: *No-o. (Pause) Where is he?*

C: (Points at the screen.)

Shauna: *How do we see where he is?*

C: (Pointing to the screen) *He is there (pause), right there.*



S: *He's there? So let's do, R-T* (keys in) (pause). *Now, if he was in the middle (pointing to screen) he'd be perfectly straight now.*

C: (Leans forward, mouth open) We're still-- (to Shauna, who's calling Monica.)

(C-2)

S: Mo-ni-ca!

C: *We're still P-U, so we can go F-D (i.e. forward) and see where he is.*

Carol's suggestion may give Shauna a clue

S: *F-D?*

C: *Ya, 'cause it's still penup. Unless you--*

Shauna: *No, no, there's a thing that you press to see where he is.*

Shauna is thinking of the HELP command but cannot remember its name

SECTION D

Saul L.: (Saul is walking over to their computer station.)

COMMENTS

Saul: Oh, ya, I know, I know the (? inaudible) command for it. Control-T. (Keys in.) (Shauna and Carol stare at the screen.)

Saul gives them an incorrect answer; this is not what Shauna (and Carol) are searching for.

Saul: And if you wanna get out of it, just press Control-S. (He returns to his seat, walking backwards.)

SECTION E

Shauna: (Turns towards Saul) *Nooo! We wanna see where he is.* (Points to the screen.) *He goes, he goes like this, zoooom, back.* (Saul returns to Shauna and Carol's work station.)

Shauna describes what help does

Saul: O.K. Is pen up? Are you sure?

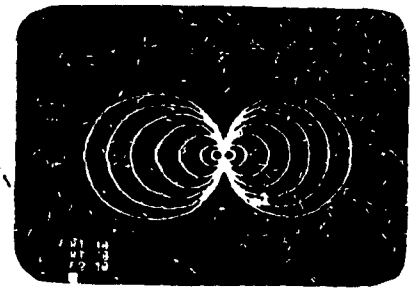
Shauna: Yes.

SECTION F

Saul: (Keys in FD 50.) - - - - - - - - - - -

(Carol is doodling. Saul returns to his seat.)

Saul gives Shauna a clue, but Saul himself does not realize it.



Shauna: (Shauna turns to Carol's back and says) *So he's facing there, right?* (Points to the screen.) (Carol turns to her.) *He's facing right there. Let's go-- B-K.* (Keys in BK 50.) (Pause.) *Now he's right there* (points) *so let's do thirty.*

C: (Turns) No we can't.

S: (Keys in RT 30.)

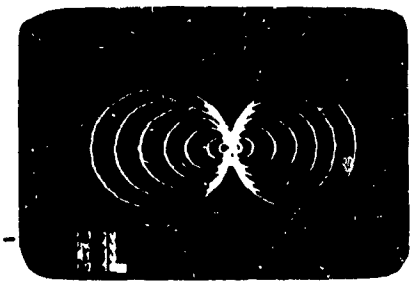
C: (Is doodling, looks over.) He's still off.

Carol is correct. Turtle has now turned 170, needs to be at 180 therefore RT 10 more is needed.

S: No. He's right there. (Pointing)

COMMENTS

C: (Turns away from her doodling) And you have (?to) put pen, pen down if you want him to do anything. (Points as she leans over to indicate on the screen, then turns back to her doodling.)



S: I know. *But where is he? Let's go* (keys in) *-F -D* [i.e. FD 50]. (pause) *He's right there.*

C: Nah. [Not negation, just a friendly mutter as in "You don't say!"]

S: We have to get him like ten, ten, (points at the screen) fifty.

Shauna means RT 10 more

Here Shauna means BK 50

C: B-K ten.

Carol misunderstands

S: (Keys in BK 59 instead of BK 50)

C: (Looks at the disk drive.)

S: No! (Corrects herself on the keyboard, keys in BK 50.)

C: *Now he's back where he started from.*

Carol is correct, as is Shauna

S: No, he isn't, he's right there (points). Now we have to do (keys in RT.)

C: (Turns to write or doodle in her book.)

S: R-T ten, return. (In error, keys in RT10.)

C: (Looks toward the screen.)

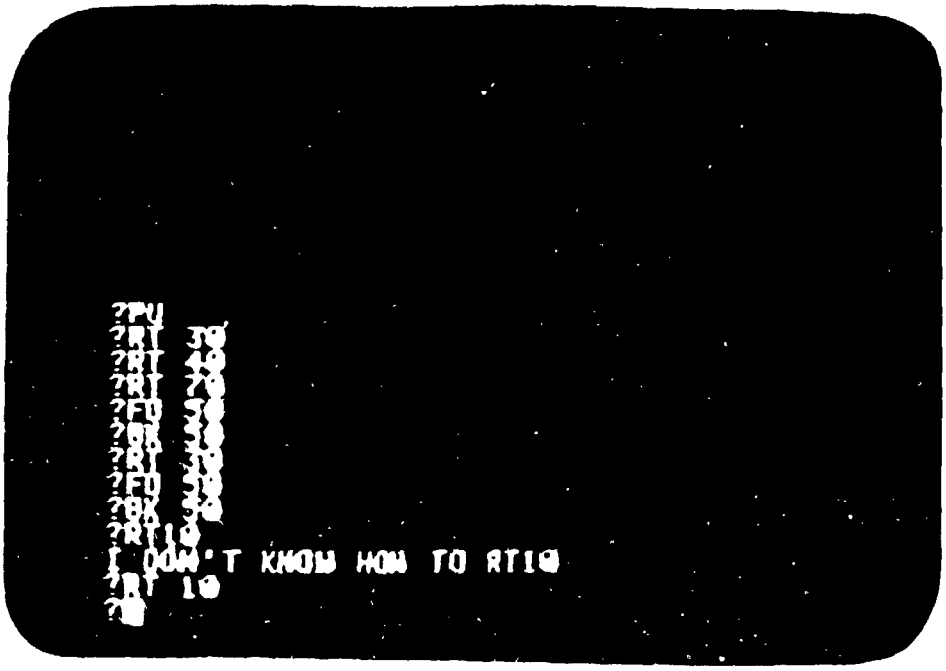
S: A-lu-lu-- R-T 10, R-T 10 (keys in to correct.)

+ RT 10 = 180 = 180°
Now turtle is perfectly placed on the screen.
Turtle has gone from △ to ▼

C: Are you writing down our commands? (While writing, looks towards Shauna, and back to her book.)

		COMMENTS
S:	We did it, we did it!	
C:	(No response.)	
S:	Now I have to press POTS, and see what it is.	Shauna is wrong about POTS
C:	No, we have to press Control-T, to see what we've done. (Leans over, reads from the screen, records.) (Pause.) *I'm at -B -K 50.*	Carol is correct. Carol then states that she has written in her notebook all the commands listed on the screen.
SECTION G		
Shauna:	*You don't <u>need</u> that! That's just to show us where he is.* (Waving her hands.) *Forget the -F -Ds and* -B -Ks--	Shauna is trying to tell Carol that one should eliminate the commands used only for testing out. Carol hasn't grasped the significance of the FD 50 BK 50 moves and as the children seem to do in instances when they are not sure, she records everything. (See the computer screen commands below.)
SECTION H		
C:	*Well, I did, and I'm gonna write it anyway.*	
S:	R-T 30, R-T 40, [Shauna skips RT 70] R-T 30 (?), (pause) R-T 10 (crossing out after she has written, after she has totalled up all the recorded numbers). (Turns towards Carol's back; Carol is still writing.) I know, get, I know how to get halfway around (points) so if we have to get all the way around back is a hundred and ten, and that's (with index finger pokes into Carol's back) two hundred and twenty.	Shauna has totalled the turns, but has missed one addend (RT 70) which would have resulted in the correct sum for half a rotation, that is, 180 degrees. Shauna knows about one complete revolution and knows that it would be double what they have done to date. So she creates her own, albeit inaccurate, algorithm. For Shauna, half a rotation is 110, a full rotation would therefore be 220.
C:	Can you not stab me!	

The computer screen



RT 30		30
RT 40		40
RT 70	omitted	
FD 50 BK 50	= HELP routine	
RT 30		30
FD 50 BK 50	= HELP routine	
RT 10		10
		110

APPENDIX M

RUSS H. AS INFORMATION GIVER TO JON H., CONCERNING SETHEADING (SETH)

As has been noted in the text of the paper, the selection below is unique not only by virtue of the length of time spent, but also due to the various teaching skills that Russ H. exhibited. The ways in which this episode constituted a peer teaching segment have been elaborated below. The various peer teaching skills have been listed on the right.

Selection #17

Jon H. and Russ H. discuss SETH, Dec. 18, 1985, p. 27.

Russ H.:	Now, seth, seth two seventy. (keys) [Russ has just completed the curved part of the goalie's body.]	
Jon H.:	What is seth?	Request for explanation
Russ H.:	It puts (stops keying) the turtle in a-, how do you want. (Points to the screen and traces.) This is zero, and it turns, that's ninety, that's one eighty, that's two seventy, and that's three sixty.	Explanation given - focusing ("points") tracing ("traces") demonstration
Jon H.:	(giggles)	
Russ H.:	So you say, uh, seth, eigh-, seth ninety, it'll be facing that way (pointing) (pause), seth one eighty, it'll face that way (points downwards), seth	Explanation continues focusing focusing
Eric H.:	(fiddling with the contrast dial on the T. V. monitor) The goalie is a ghost. [Eric is off task.]	
Russ H.:	two seventy that way (shows), seth three sixty will be that way.	
Jon H.:	(nods, shows that he feels he	

	understands) O. K.	
.		
.		
.		
Russ H.:	So seth, seth two seventy (leans over and keys), now watch (points) it's gonna face that way.	Russ establishes set (i.e. directs attention to the matter at hand), focuses, anticipates, and locates
.		
.		
.		
Eric H.:	You never heard of seth? (looking down)	
Russ H.:	Now watch, Eric, watch, uh Shawn-, uh, Jon.	
Jon H.:	Jon. (smiles at the mixup and string of names)	
Russ H.:	(Keys) Toing! (sound effect for emphasis)	
Eric H.:	I know about seth.	
Russ H.:	(to Jon) Now it's perfectly even.	Russ emphasizes the geometric aspect; the position of the turtle is perpendicular to its (unseen) base
Jon H.:	(nods) Ya.	
Russ H.:	(Points to the screen, turns more toward Jon) So that's, you know, if you turn, like hrum (sound effect)	
Jon H.:	(?)	
Russ H.:	Two hundred and seventy (? times)	Russ stresses the functionality
Jon H.:	Ya. So you don't have to figure out how much you want it to go.	Jon sees the benefits of using SETH, and comments upon them
Russ H.:	That's right.	

This interchange between Russ H. and Jon H. enabled the researcher to gain insight into the peer teaching skills used by the high achieving children; it thereby added to the larger, more general question concerning the ways in which the children can contribute to each other's learning.

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APPENDIX N

RUSS H., ASSIGNED TO THE ROLE OF TUTOR, TEACHES JON H. AND ERIC H. ABOUT "VARIABLE"

During one session, December 11, 1985, Monica assigned Russ H. to be a tutor to Jon H. and Eric H. concerning the concept of variable. Monica had worked with Jon H. and Eric H. alone at the front of the class for approximately ten minutes at the beginning of the session. Russ had just completed a project which he entitled "stickman" and in which he used variables. Russ demonstrated an awareness of the relationship between the parts, for he used ratio in his procedures. For example, the EYES subprocedure for Russ's stickman was as follows:

```
TO EYES :X
FD :X*4
RT 90
FD :X/2
PD CIRCLER :X/5
PU BK :X/2
LT 90
BK :X*4
FD :X*4
LT 90
FD :X/2
PD
CIRCLER :X/5
END
```

Monica was very explicit both with Russ, and later with Jon H. and Eric H., about what she was expecting from each. Russ was to be the tutor, Jon and Eric were to be the tutees. Monica's designation conferred upon Russ the status of peer teacher.

Monica to Russ

December 11, 1985 p. 8

Monica:	These two are going to come and join you because what I'd like you to do now is, they've just (?learnt) how to do a variable square. I'd like you to show them how you made your stick man.

Russ H.: 'Kay.

Later, Monica spoke with Jon H. and Eric H. and was heard to stipulate to them that Russ was to work with them. The three stood far from the microphones, positioned at the blackboard, so that one could not hear exactly what they were saying. Monica stated something such as: "I want you to go with Russ, he's gonna show you how, because (?inaudible ?a man, he will have all the different sizes)."

It is of interest to note that this is what Russ felt was the function of variables, that they offered someone an opportunity to produce "all the variations." Monica's words seemed to emphasize the same feature, namely that Russ's man "will have all different sizes." The descriptions in Russ's talk with his partners and with visiting peers as they viewed the various stick people of different sizes referenced the figures upon the screen as follows: "He's going smaller," and "It's a mommy and a poppy" (Dec. 11, 1985, p. 11). During all the time that Russ demonstrated the stickman and played with both small and very large input numbers (for example, 1000 which created a maze design), there was no mention made by Russ concerning how the stickman was made, nor how the proportions were derived. Russ seemed to feel that the demonstration alone was sufficient, that variations was what variables entailed. When Monica checked with Russ to monitor the progress they were making, it seemed that Russ felt that demonstration was both appropriate and sufficient.

December 11, 1985, p. 17.

Monica: What I'm trying to do is make sure that they understand-, do they understand how to make a stick man?

Jon H.: (Tapping Russ H. on the shoulder, Stuart M. is a classmate looking on). Make a poppa.

Russ H.: (to Monica) They should. (Chuckles.)

Russ and his partners had made so many variants of the stick people, calling the large ones "poppas," the middle size ones "mommy," and the small ones "baby" that Russ felt

that they ought by that point to understand. Jon and Eric did seem to know that typing in STICK with an input number (provided it was not too large) would make a stick man of different sizes. But Russ did not up to this point explain how to create the procedure.

Monica asked Russ H. (Dec. 11, 1985, p. 18) to continue to work with Jon H. and Eric H., and to maintain a guiding role: "Russ, they will type, but you will help them in their *thinking* please" (emphasis added). We will see below how Russ did play a tutoring role in guiding Jon H. through some of the procedural "how to" steps which Jon needed in order to create a procedure using a variable; Jon's design was a square.

Monica gave Jon H. and Eric H. the assignment to make a square (she called it a box) (Dec. 11, 1985, p. 19); however once Monica left their work station, it became clear when one listened to the tape that Jon and Eric were not making connections between the assignment given and what Monica had demonstrated and discussed with them at the start of the class. Eric and Jon worked at the task together for a while; when Russ joined them, Eric withdrew from active participation (see Appendix P). As Russ helped Jon, Russ showed evidence of more teaching skills than was the case in the other instance in which he explained the function of SETH to Jon (Appendix M). Mention of Russ's peer teaching skills have been placed within some of the square brackets [ ].

Dec. 11, 1985, p. 21.

Jon H.:	Russ, (? I forget). Do we put (? side) up here (pointing to the screen)?
Russ H.:	(Standing) What?
Jon H.:	(Pointing to the screen) D'we put (? side) up here?
Russ H.:	Yep, y-, well, you hafto, ya, at the beginning [guide with commentary] you have to write the dot, the dots, (? side) and then the, dots and the side. There's <u>two-</u> [marking critical features]
Eric H.:	Well-, (?) a colon.

Russ H.: (Encircles Jon) Hey, look, it's repeat. [focusing]

Eric H.: Colon.

Russ H.: (Keys) Whatever it is. (keys) -S -I -D -E? [demonstrates TO BOX :SIDE; performs function with commentary]

Russ did a good job in guiding, demonstrating, and performing the function with commentary. It was not clear whether Russ's comment "whatever it is" referred to the variable name, or to the (:) which Eric called "colon" and which Russ referred to as "dots." If Russ's statement "whatever it is" referred to the variable itself, Russ may have been stating that it did not matter what the variable name used was; it could be "side" or any other name. Russ pointed out the same information in Selection #22 (:S, :SIDE). This statement reflected important understanding on Russ's part. Sutherland (1987) has reported that children think that the variable name has implicit value in and of itself, and would use only a meaningful word such as SIDE for the side of a square, when in fact the letter X, or indeed any nonsense word, would serve the same function. The statement "whatever it is" could be interpreted as referring to Eric's offer of "colon" as the official term for two dots. However, "dots side" would be the correct way to read aloud the designation :SIDE.

Russ moved away, and then came back to check on the progress which Jon and Eric had made. It was evident that he had assumed the role of tutor. Russ was seen to refer implicitly to class rules, and performed the role of the teacher by checking progress, and doing direction maintenance, and comprehension monitoring.

Dec. 11, 1985, p. 24.

Russ H.: Aren't you doing with the variations? [checks progress, direction maintenance; Russ is looking at their work on the screen and could see that they were not using variables REPEAT 4 (FD 20 RT 90)]

December 11, 1985, p. 25.

Russ H.:	Let's see what you have. [checking progress, assessing comprehension] [Note that Russ ignores Eric who is trying to distract, and is concentrating on Jon who is doing the task of the square.]
Russ H.:	Well now (pause), start putting in the variables. [focusing attention]
Jon H.:	Like, how d'you do that?
Russ H.:	Well, it's the same thing. -F -D.
Jon H.:	-F -D ˙ten.
Russ H.:	˙Dots "side." [guiding, correcting]
Jon H.:	-F -D, O. K.
Russ H.:	(Watches carefully) [monitoring]
Jon H.:	Oh, now I understand it then. (Keys)
Russ H.:	'Kay, wait, just erase the (keys) ten there. [monitoring]
Jon H.:	(Mouths) -F -D.
Russ H.:	-F -D, dots SIDE times three. (Pause) Because we have to multiply it by ˙three 'cause that's thirty. [explanation is not clear]
Jon H.:	˙-F -D [Jon may not be able to follow Russ's explanation.]
.	
.	
.	
Russ H.:	(?inaudible) (Keys) Times three. [performing function, demonstrates the (*) sign, marking critical features]
	[There is an error which Russ detects.]

Russ H.: (Points to the screen) Don't you want the line down there?

Jon H.: Ya . . .

.
.
.

Russ H.: O. K., so you gotta turn (tracing on the screen) that there and make it go down until there, and make it go halfway down.

Jon H.: How d'you do that?

Russ H.: Well, *figure* it out. It's the same thing. [Emphasis was added. Monica did direct Russ to help them with their "thinking." Russ challenges Jon to *figure it out* for himself. But in the end, as we shall see below, Russ does the changes himself; "halfway" is derived by doing :SIDE/2.]

Dec. 11, 1985, p. 28

Russ inserts a correction, using proportion:

	The correction
TO BOX :SIDE REPEAT 4 [FD :SIDE RT 90] RT 90 FD :SIDE	RT 90 FD :SIDE/2

Russ H: (keys) Oh, dots are (? oh no) side (keys) divided by this (keys) two, O. K.
[Russ has reverted to the knower role. He is a very good troubleshooter, but does not in this case give a good explanation. Russ has made the needed change himself.]

Although Russ did challenge Jon to think and to work it out, one notes that in the last excerpt Russ has taken control of the action. This move by Russ may in part be due to the difficulty of the content itself. One must keep in mind that the notion of variable is a complex one, and it is possible that although Russ knows how to work with variable implicitly, he is not yet able to explain explicitly, or to continue to guide his tutee toward the correct answer.

APPENDIX O

RUSS H., CLASS COMPUTER EXPERT

A study of the peer intervention that transpired between the children and their classmates in the computer lab (Zack, 1988b) revealed that when partners called upon their classmates for help with a problem, the relationship was a collaborative rather than a didactic one. It was found that the helpers were as often incorrect as they were correct, with one exception. Russ H. was the exception in this study, for he was correct in his interventions in all but one instance. And in that one instance Russ shouted unsolicited help from afar to Lilly and Rina, and did not do a needs assessment as it was his custom to do. The needs assessment which Russ executed in most instances was in itself a unique feature which set him aside from the other peer helpers, and exemplified a teacherly approach.

When the Grade 5 students were asked in the interviews (February-March, 1986) to name the peers from whom they would seek help if they were in difficulty, the children named Russ most often (10 instances). The others cited who had 4 or more nominations were:

GROUP B:	Russ H. (10); Stuart M. (7); Michael H. (6); David H. (5); Jon H. (4)
GROUP A:	Saul L. (4)

Russ was named not only by the children in his own group (Group B which met from 11:10-12:10, 12 children), but also by children in the other group (Group A which met from 10:10-11:10, 11 children) who did not have computer class contact with Russ during the Grade 5 session. Several of the children referred to Russ (and to others) as computer experts. The question posed by the interviewer was: "Whom (from among your classmates) would you ask for help if you needed help?"

Joey L.:	If Russ were in my group I'd ask him 'cause he's a computer whiz.
Michael H.:	Russ 'cause he's the smartest, that's all, that's who I really trust.

Eric H.:	A friend, a person who knows how to use a computer, Russ.
Jon H.:	[Names Russ quickly, then also adds David H. and Stuart M.] Russ, 'cause Russ is really good at computers.
Mark L.:	[Names Russ, although Russ is in the other group-- but Russ and Mark are friends outside of school] Oh, I know, Russ, he's an expert on (?computers).
Matthew M.:	Russ because he knows the most on computers.
David H.:	Someone who knows about computers and who has [had] a computer class . . . maybe Russ 'cause he has a computer, and Michael. No one else.

The findings concerning nominations of class expert are of interest especially in light of the findings of the Hawkins, et al. (1982) study, "Microcomputers in schools: Impact on the social life in elementary school classrooms." In the Hawkins, et al. (1982) study, the children's nominations and the classroom teacher's nominations concurred in selecting two children (10 and 11 year-olds, fifth and sixth graders) as classroom "computer experts." The parameters of the Hawkins, et al. (1982) study did not include an investigation of what actually transpired in relation to the interaction of these two "computer experts" when they did intervene with their peers. The researchers, Hawkins, Sheingold, Gearhart, and Berger (1982), stressed that the results did "not bear directly on the question of the kind of learning that may occur as a result of the children's working together" (1982, p. 372). When the analysis which related to intervention by classmates was done using the data of this study (Zack, 1988b), the findings suggested that one must be careful about the use of the term "computer experts." Although the finding concerning Russ was consistent with the Hawkins, et al. (1982) study, in that the children did nominate as class expert a child who was indeed "expert" in syntax, and programming, and mathematics concepts problems (namely Russ), the other nominations did not correspond to what one would consider expertise. This is not to say that the children who intervened were not at times helpful, but rather that they could not provide the expert help which their peer was expecting. It may indeed constitute an example of Webb's (1984) observation which was cited in the Review of the Literature portion of the paper, namely a case of the most able person in the group (with "most able" definitely a relative term) becoming "the 'explainer,' regardless of his

or her absolute ability" (Webb, 1984, p. 168).

Perhaps expert help is not what the soliciting peer should expect; however that is not the issue at hand. The one caution that one might voice in relation to the Hawkins, et al. (1982) study is that their use of the term "class experts" does connote expertise. The investigation of "neighboring peer" intervention (Zack, 1988b) revealed some of the ways in which the children were able and were not able to help when intervening to assist partners or individuals. At times the help was appropriate, clear, and contributed to helping to solve a problem; it was sometimes given in a didactic, but most often in a collaborative, mode of interaction between giver and receiver. At times the intervention resulted in frustration because the intervening classmate not only could not help, but in some instances made matters worse. Thus, peer nominations which reflect the children's and the teacher's perceptions of who the class helpers are, constitute only one of issues which need to be considered when dealing with the cognitive, and social, aspect of classmate peer intervention, and its relationship to learning. (See also Appendix R)

APPENDIX P

ERIC H.'S ABSENCES AND ATTEMPTED DISTRACTIONS WHILE A MEMBER OF A TRIAD

The pattern of Eric's absences and distractions was of interest since their emergence coincided with his becoming part of a triad, beginning with the December 11, 1985 Logo session. As part of the Jon-Eric dyad (Oct. 16- Nov. 20, 1985), Eric had been actively involved in the work. The camera videorecorder at computer #3, which was used to tape the triad of Russ H., Jon H., and Eric H. during three (Dec. 11, 1985, Jan. 8, and Jan. 15, 1986) of the four sessions which will be described, was not equipped with a timing device. Therefore, it was not possible to measure the amount of time during which Eric was distracted during those sessions, as it was possible to do upon the occasion of the Dec. 18, 1985 session when the videorecorder used at computer #3 had a timing device which registered the time in minutes and seconds (and parts of a second). The transcript pages will be indicated so that the reader can infer passage of time-- when there are no lapses in the talk, one transcript page represents on the average one minute of talk. There are brief notations concerning what was transpiring at the time.

Dec. 11, 1985 (No timer.)

P. 8	Monica asked Russ to teach Jon H. and Eric H. about variable. Russ has not yet joined Jon and Eric in order to teach, or to monitor their work when they use a variable.
P. 12	Eric started to draw the goalie.
PP. 18-25	Monica told Eric and Jon that she would like them to draw a square and line using variables. Monica left. Jon and Eric worked on their task together, trying to recall what Monica had shown them during their approximately 5 minute session alone with her at the blackboard together at the start of the class.

P. 25 Russ came over, Eric showed Russ the picture of the goalie.

P. 25 Jon asked Russ about how to work with the variable, and Russ and Jon interacted concerning the task of making the square while it seemed that Eric was concerned mainly with his drawing of the goalie in his notebook.

PP. 25-32. When Eric's comments were considered in isolation, they were found to deal only with the goalie drawing. Eric took time to chat with his neighbors, Lilly and Rina.

Dec. 18, 1985.

P. 21. For 20 minutes, until p. 21, Eric was actively a part of the project work with Jon and Russ. Then he began to be inattentive, and began sporadically to try to distract Jon and Russ from their work.

P. 21, 22 & 24 Off-task comments

PP. 25-33. (37:52-46:00, 11 minutes) Eric was involved again but was often off-task. He made some extraneous comments; he turned the color dial on the computer monitor to eliminate the color contrast; and he sang and tried to distract Russ and Jon.

P. 28 Eric was off-task. He looked toward Monica to see what Monica was doing, burped, commented on the drawing, made off task comments, spoke with David, a neighboring peer at computer #1, came back and complained to his partners about a stomach cramp.

P. 34-41 (46:29-57:07, 11 minutes) Eric began to type and was more involved.

January 8, 1986. (No timer.)

From p. 27 onward:

P. 27. Eric wandered off for a drink of water.

P. 28. Eric engaged some neighboring peers in conversation.

P. 31. Eric returned, and talked about the picture of the goalie and the goalie skates.

P. 32. Eric sang, pretended that he was an announcer giving the hockey broadcast on radio, using the microphone as prop. Then, Eric sang again. Eric went over to the doorway to greet and converse with one of the other Grade 5 boys who was not in his homeroom.

P. 34. Russ called Eric back to the work station because the goalie body was finished and on the computer screen.

PP. 35, 36 Comments by Eric were off-task.

PP. 38-41 Eric became more involved in the activity of "breaking and entering" in which Jon and Russ were involved.

January 15, 1986. (No timing device.)

PP. 1-1 Eric was involved in the activity.

P. 10. Eric left.

P. 15. Eric came back, and then left again.

P. 19. Eric came through the doorway, Russ and Jon called out to him and Eric came over.

P. 21. Eric criticized the screen drawing of the goalie to which he had contributed very little.

Eric began to evade involvement, complained of a stomache ache, and walked away.

P. 22. Eric returned, and complained of hiccups.

P. 23. Eric stood mid-class and was heard talking to Monica about his hiccups. Monica was heard to remark "You're gonna float away," most likely a reference to his continued exits for water to cure his hiccups.

P. 26. Eric came back into the room, came over to Russ and Jon, saying: "Somebody try to scare me, please" also a reference to his hiccups and his effort to cure them.

P. 28. Eric stayed at the work station (computer #3) with Russ and Jon. He was very quiet. Once, he made an effort to turn the color dial so as to eliminate the color contrast on the screen (hence no image).

An effort was made in this appendix to detail some of the ways in which Eric was distracted and uninvolved at times while working in a triad. It did not seem that any of the other children working in dyads in the study were off task to the same extent during their work. It would require an analysis of the transcriptions representing each focal child and his or her partner before any comparisons could actually be made. However, it was our general impression that the other children were actively involved with their computer task, as was Eric himself when he worked in a dyad. Eric's pattern of distractibility when in a triad was far different from his pattern of working attentively while in a dyad with Jon. However, as has been noted in the body of the paper, it was not able to be determined whether Eric's distractibility was due to his involvement in a threesome, or due to the fact that the new partner, Russ, was a "class expert," or due to both or neither factors.

APPENDIX Q

TEACHER INTERVENTION IN RELATION TO THE CONCEPT OF 90: MONICA WITH CAROL L. AND AMY L. IN GRADE 4

The following episode was chosen to illustrate how Monica helped nudge the children, Carol L. and Amy L., toward a discovery of the importance of the input 90 for making square corners. It took place during their Grade 4 year. This episode dealing with 90, although pivotal for the study of children's learning of the notion of right angle, was not included within the body of the reported findings because it did not occur during a disagreement or as a result of a child-elicited appeal for help during a disagreement. Monica's intervention with the children began during a disagreement concerning lateralization (left/right) (see Section A); Monica was present throughout the disagreement on lateralization, that is, her presence was not the result of a call for help on the part of the children during the disagreement.

The teacher intervention concerning the concept of 90 occurred as a follow-up to the afore-mentioned disagreement on lateralization, and is an excellent example of how Monica took advantage of the appropriate moment to teach Logo geometry concepts which were pertinent to the design the children were attempting to make. In this case the notebook design drawing was a house  . As will be noted, the process of Monica's intervention with them is long and sounds somewhat like a guessing game. What Monica is aiming at is clear to us as adults; one must however keep in mind that the agenda is not evident to the learners. They need many hints in order to arrive at the sum of 90 for the first square corner of their house.

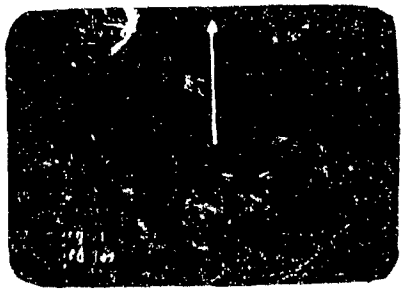
Carol L. and Amy L., Grade 4, October 2, 1984, D/A #1, p. 20

[N. B. There are no nonverbal notations since only the computer screen videotape was made for this session.]

SECTION A

Monica: So there are the two words that make him turn.

Amy L.: So he needs to go left.



Carol L.: Thank you. [Carol seems to be addressing someone on the side].

Amy L.: -L -T.

Monica: You want him to go left or right?

Amy L.: -L -T. [She is incorrect but Monica does not intervene.]

Monica: O. K.

Carol L.: Left is <u>that</u> way.

Monica: Aha. Which way do you want him?

Carol L.: We want him to go <u>right</u>, this is left, this is right.

Amy L.: All right, so -R -T.

Carol L.: Right.

[End of disagreement over lateralization.]

SECTION B

MONICA'S INTERVENTION CONCERNING 90
(transcript pp. 20-23)



Monica: Return. He's now facing up here. Did he turn --- enough? You wanted him to turn this way.

Carol L.: O. K., so you go um-

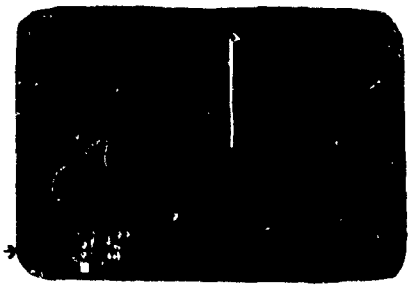
Monica: You'd better write down that you turned him -R -T fifty-six first, 'cause remember how we did this, we're gonna have to know everything he did. Let's try fifty-six. Now what do you want to do now? Did you turn right enough?

Amy L.: No.

Monica: You want to turn right any more, Carol?

Carol L.: Ya.

Monica: How much <u>more</u> do you want him to go?



Carol L.: Space-

Amy L.: Wait, no.

Carol L.: Right space -4 -0.

Monica: O. K. Does that look right? [The total input is (RT) 96].

Carol and Amy: Ya.

[Neither Carol nor Amy perceives that the turtle is not pointing directly to the right ◁▷.]

Monica: So write down right forty [that is, in the notebook].

Carol L.: -R -T, O. K.

Monica: Is he facing the way you want him to face?

? : Yes.

Monica: O. K. Now, what do you want him to do?

Carol L.: Straight.

Amy L.: -R -T.

Carol L.: No, straight.

Monica: Oh, she said straight, do you want him to go straight? (Pause.) Forward, -F -D, now we (?erase)



Amy L.: Oh, can I just write -F -D?

.
.
.

Now, um, Forward -F -D space forty-five.

[The top line is tilted, but the surface is not too jagged.]

Carol L.: What did you do:o?

Monica: Now, what happened?

Carol L.: We'll change it.

Monica: No, there's nothing wrong. It went but you don't like that I think, eh. You wanted it to be (? inaudible) --, so really girls when the machine was facing, when you thought it was facing straight here but really it was facing a little bit more, so when we had him turned right, how much did we have him turn right? (pause) --, altogether there?

Carol L.: Nine six.

Monica: Ninety six looks like it's a little bit-

Amy L.: Ten! . . . A hundred and fifteen [It's not clear where she got this number 115].
[? + 10 = 115]

Monica: Well, you're saying a hundred and fifteen and she's saying ninety-six, you'd better check it.

Carol L.: Ninety-six is right.

Monica: Check it again. (Long pause.) Wait, you're adding all of these, Carol. You're just supposed to be checking what he did for turning right, not how much he went forward, are you Carol?

Carol L.: (Quietly) Ninety-six . . .

Monica: Looks like-

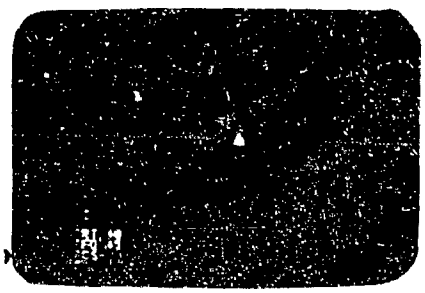
Carol L.: Ninety-six.

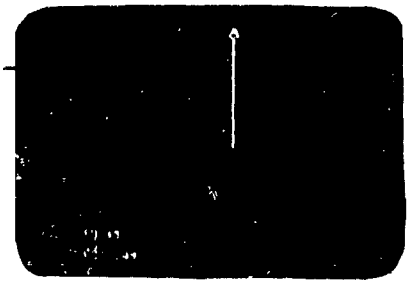
Monica: Aha! Now look what happened with ninety-six girls. Ninety-six made him turn a little bit too much.

Amy L.: So just four more.

[96 + 4 = 100]

Monica: Ninety-six was a bit too much, Amy. Let's clear the screen and start again. And remember that ninety-six is a little bit too much, eh? Let's try again. (Pause) What are the first directions he does?





(They key in -F -D 109)

Monica: Last time we had him turn right ninety-six, but it was a little bit too much. Will you want to try and-

Carol L.: Fifty-six.

Monica: No, ninety-six, together he did ninety-six and forty, right? [Monica says "ninety-six and forty," but meant to say fifty-six and forty]. Ninety-six turned him too much. You want to try a number less than ninety-six.

Carol L.: I know, I know one, I know what. It's-

Amy L.: Where's the paper?

.
.
.

(Amy and Carol get sidetracked and distracted.)

Monica: Now wait, let's do this, Carol. Erase the fifty-six and the forty because together they are-- how much?

Carol L.: Ninety-six.

Monica: Ninety-six. So write in ninety-six. And you too Amy. (Pause) Here. How much did he turn right last time?

Amy L.: Fifty.

Monica: No, he did- (pause) ?

Amy L.: Oh, ninety-six.

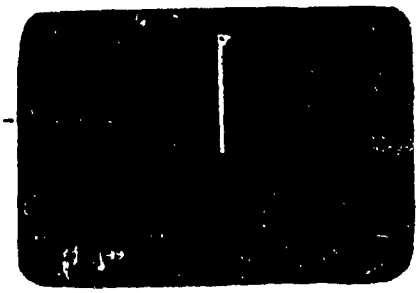
Monica: Right. And this time we're gonna try something-

Carol L.: I know what, I'm trying-

Monica: Carol, what are you gonna try?

Carol L.: I'm gointa try seventy.

Monica: Try it.



Carol L.: (? inaudible)

Monica: O. K. Now he's turned, and he's turned- ---

? : (? inaudible)

Monica: he's facing up here. Is that enough?

Carol L.: No. Eighty. Try again.

Monica: How much <u>more</u> do you want him to turn?

Amy L.: Forty.

Monica: Forty <u>plus</u> what he's turned here?

Amy L.: Ya.
[(70) + 40]

Monica: Don't forget he's already gone seventy.

Amy L.: O. K., twenty.

Monica: Try it, try it.

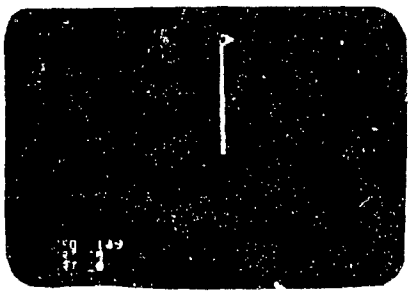
Carol and Amy: -R -T, -R -T

Amy: (Amy keys in a 4, and seems ready to add the 0)

Monica: No, you didn't say <u>that.</u> Amy. You said twenty. Or do you want that? It's up to you. You do whatever you want.

Carol L.: -2 -0.

Monica: You want -2 -0? You can try whatever you like.



Carol L.: Now he's facing down. - - - - -

Monica: Now do you think that's better?

Amy L.: Ya.

Monica: Does it look like he might be right?

Amy L.: No.

Carol L.: Sort of facing down.

Amy L.: No, I think it's right.

Monica: I have no idea. You have to, you're gonna have to experiment.

Carol L.: O. K.

Monica: First of all, let's mark down how much he turned this time. How much did he turn this time?

Amy L.: Seventy.

Monica: This was our last time, Amy. (Pause.) And now, how much did he turn this time?

Carol L.: Ninety.

Monica: Is it ninety? So wait a minute girls. You're gonna-, we know that ninety-six is no good any more, so erase ninety-six, we tried that,

Carol L.: Ninety.

Monica: erase your ninety-six, Amy, right, and instead we're gonna to try this number, which is, how much?

Amy L.: Seventy, eighty, ninety.

Carol L.: Ninety.

Monica: Write down the ninety, Amy, (pause), write it down.

Amy L.: O. K.

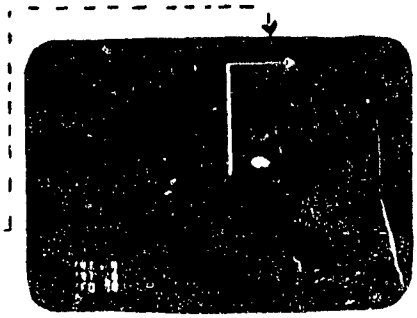
Carol L.: Now I wanna try it.

Monica: Now we're gonna to try it.

Carol L.: Um, um.

Monica: And now what do you want him to do? He's facing the right way and now what do you want him to do?

Carol L.: Space, -F -D fifty -- (pause). It looks a little too skinny.



Monica: Oh, that's the way the lines go when they're going this way [that is, on the horizontal axis], they're very thin. When they're going up, they're a little bit fatter, and when they're going a little bit slanted they jump.

[Monica's comments about the 'slanted' and jagged line are elaborated upon below.]

One will note that Monica had to keep helping the children, in particular Amy L., keep in mind that they were aiming at a number less than 96. It seemed that Amy was confused, since on a number of occasions, she suggested inputs, or additional inputs, which rendered the total greater than 96, for example, ? + 10 = 115, (96) + 4. (70) + 40. When Amy stated the last-mentioned 40, and Monica cautioned that turtle had already turned 70, Amy subsequently suggested 20, but was however beginning to key in RT 4- ; Monica helped Amy attend again to what Amy herself had said. Twenty was keyed in, and so the sum total of the input was 90, that is, 70 + 20. Finally, despite the fact that visually Carol was not sure that the turtle cursor was facing 'straight to the side' but Amy felt it looked right, the children tested it out by proceeding with a lateral move of FD 80. The resulting line was straight with no jagged edges.

The children's experience with the jagged line which resulted from an input of 96 is of interest. Their aim was to have turtle go "straight." The jagged line which resulted from an input which was less than 90 (Lilly M. and Rina M., Grade 5, Zack, 1988a) or from an input that was more than 90, as in the case of the input of 96 in the selection above, was a cue to the children that something had gone wrong, but they were not able to distinguish what had gone wrong. Among the Grade 5 group, the children who did not know that 90 was the input for the rotation, which would in turn render a straight line when turtle was in an upright position △, were evidently perplexed about why the subsequent line was jagged. The line went "da-da-da-da-da," stated Leanne L. as she traced along the line which slanted diagonally (Leanne L. to Amy L., November 20, 1985, p.5); it went "zig-a-zig-a-zig" said Lilly M. (Lilly M. to Rina M , November 20, 1985, p. 9). The researcher has documented some of the deliberations of the chi dren in the study concerning the jagged line, and is in the process of writing a paper entitled "The jagged line: A Logo whodunit." It was intriguing to note that at the end of the excerpt cited above, Monica took time to explain to Carol and Amy that due to the

technology, the computer graphics lines varied in width and smoothness; of particular interest to the researcher, due to the afore-mentioned focus on the "jagged line," was Monica's comment that "when they're going a little bit slanted they jump."

In the Grade 5 year, Carol L. showed evidence of having held on to the information about 90. Amy L., in Grade 5, was still in the process of learning that the input for a square corner was 90, despite the two sessions during which Monica worked with her and with Carol when they were in Grade 4 (above, October 2, 1984, and Selection #19, October 9, 1984). The fact that Amy was not able to appreciate the importance of 90 even as a special number for making corners attested both to the fact that the learning about 90 is not easy, and to the fact that much repetition and diverse kinds of teacher presentation and ample experiential situations are needed for learning.

APPENDIX R

SOLICITED PEER CLASSMATE INTERVENTION: REFLECTIONS UPON ONE CASE-- LILLY M. AND JON H.

The question of which of the peer classmates the children choose to approach when help is needed is an intriguing one. What criteria did the children use? The teacher was often the first choice of call when help was needed, but such was not the case in the episode in question, namely Selection #20, in which Lilly M. asked Jon H. for help in checking the sum of the inputs. Why was Jon H. approached? Was it due to his proximity, availability, approachability, his expertise, or a combination of these, or some other criterion? There was no taping done at Jon and Eric's station that day, and so Jon's availability could not be determined; but Jon was almost always actively engrossed in his Logo work. He did make time for Lilly's task, and thus we know that he did make himself available. Jon H. and Eric H. worked at computer #1 on that day (November 20, 1985), and therefore they were adjacent to Lilly M. and Rina M.'s work area, computer #2. However, there were three other occasions upon which Lilly (still at computer #2 with Rina) specifically asked Jon for help. On those other occasions, Jon was at computer #4, not far but not adjacent; Eric H., and Eric and Russ H., were Jon's partners on those other occasions, but it was Jon whose help was solicited by Lilly.

Lilly appeared to view Jon as approachable, and also seemed to value his expertise; in the case of the appeal for peer help in Selection #20, it was perhaps Jon's ability in mathematics that Lilly deemed an important criterion. Lilly M. and Jon H. were both grouped together in the top (homogeneous) mathematics group in their regular school studies; 23/69 children, or a third of the Grade 6 students, were grouped together in this "top" group.. The classroom mathematics teacher of that group, Mrs. S., considered that Jon was at the middle achievement level of ability within the "top" group of 23 children, Lilly was at the low level of achievement within the "top" group, and Russ was at the high level of achievement. It would have been of interest to ask Lilly why she chose Jon H. as helper, over Russ H. for example, but the data as to solicited peer interventions were only analyzed in January 1988, long after the

interviews with the children took place. The researcher did look back at the interview transcripts and notes. When Lilly was asked during the Grade 6 post-session interview in May-June 1987 whether she felt that other classmates helped her in her work (other than her partner), Lilly said that they had, but could not name specific classmates, nor the information that was offered. Consultation of the notes of the Grade 5 post-session interview (February-March, 1986) revealed that when Lilly was asked to whom she would turn for help if she needed help, her nominations were: Russ H., Michael H., Eric H., and Tammy (M). It is noteworthy that Lilly did not mention Jon H., whom she had consulted 4 times; she did consult Russ and Eric, but only one time each throughout the 12 sessions. Lilly did seem to have a good estimation of her classmates' expertise (observed in the Grade 5, Grade 6, and Grade 7 Logo settings). This was not necessarily true of many of the other children, who during the Logo sessions chose to consult children who could not give them help; and who during the interiews nominated friends, or people they thought were able but were not actually proficient, as possible class helpers.

The Hawkins, Sheingold, Gearhart and Berger (1982) study found that the children's nominations did single out two experts, and that the children's choices corresponded with the teacher's selections. This study found that the children did nominate one computer expert, Russ H. (10 nominations) more often than other of the children (see Appendix O). Two of the children nominated by their peers were in fact weak as concerned minimal criteria regarding helpfulness, let alone expertise (Stuart M. and Saul L.; see Appendix O).

In the Hawkins, et al. (1982) study, it seemed that the technological aspect (i. e. syntax and programming) was the criterion upon which the children's and the teacher's judgement was based (1982, p. 371); it is possible that the criterion may be technological on the part of the children in this study as well. Whereas the researcher was chiefly concerned with the mathematics aspects in this study, it was perhaps the syntax aspect that remained dominant for the children. The Hawkins, et al. (1982) study's emphasis upon the syntax and the programming aspects sparked a renewed consideration of why Lilly M. might not have nominated Jon. Perhaps expertise with microcomputer technology may have been how Lilly interpreted the researcher's question concerning help resources. The researcher posed the question approximately as follows: "Whom would you ask for help if you needed help?" Jon H. was adept in

the mathematics area, but was not adept terms of the machine workings, whereas Russ H. and Eric H. were adept in the syntax area. But Michael H. and Tammy M. were not adept with the syntax and programming aspects; however, it may have been Lilly's perception that Michael and Tammy knew more about the machine workings than they actually did. The videotapes did not capture any instance of Lilly having asked Michael H. or Tammy M. for any help, in relation to syntax or to programming or to mathematics.

To return again to the correspondence between nominations and actual solicitations, as was stated above there was a interesting gap between the persons Lilly M. nominated as class experts or potential helpers, and the one(s) she chiefly approached for help. One would therefore suggest that the children do not always nominate those from whom they solicit aid, and thus one must not equate their nominations with the actual "peer helpers" in the group; the list of those they nominate must be verified against the youngsters actually approached for help. That can only be done by means of consulting the videotape records. Only by determining who was approached, and in what way they were able to help could one truly (1) judge whether the solicitations corresponded with the nominations; (2) determine the expertise of the helper; and (3) delve into how the intervention might have contributed to the soliciting child's learning (Zack, 1988b). This the Hawkins, et al. (1982) study was not able to do. Indeed, in reference to the last-mentioned point, the researchers themselves stated that their results did not "bear directly on the question of the kind of learning that may occur as a result of the children working together" (1982, p. 372).

Appendix S

TEACHER-INITIATED TEACHER INTERVENTION

The Logo teacher, Monica, took advantage of "the moment" to try to teach appropriate material. Monica was exceptionally adept at entering in midstream and understanding what the children needed, or by means of a needs assessment, eliciting what the children needed. There were, understandably, a few instances in which Monica misjudged the children's actual needs. However, far and away the greatest proportion of the teacher's interventions seemed to be helpful. It was the researcher's impression, only partly supported by analysis (Zack, 1986a), that both Monica's teacher-initiated and child-solicited interventions were appropriate, and in most cases well-timed. Such may not have been the children's point of view, however. At times it seemed that unsolicited intervention on the teacher's part was not received with as rapt attention as was solicited intervention.

RT 90, RT 90, RT 90 equals LT 90

I would like to give one example of an unsolicited intervention dealing with angular rotation. During one session (January 8, 1986), Monica took time to point out to Lilly M. and Rina M. that they had just completed two turns which together equalled 270, that is, RT 90, and RT 180. Monica then showed them that these two turns to the right could be accomplished as easily and in shorter order by one turn to the left by means of the use of LT 90.

Monica with Lilly and Rina, January 8, 1986, pp. 12-13

Monica: (Pointing to Lilly's book) But you've got -L -T 90. She's got-, you know what you really have here. Look! Look, look, Rina, look.

Rina M.: -L -T 90, -P -U.

Monica: Look, you know what you really have here, look (pointing to the screen). You've got -R -T 90 and -R -T 180 which really is left 90.

Lilly M.: No, I have -R -T 180.

Monica: (Looks at Lilly's book) Oh so you've got -R -T 90.

Lilly M.: It goes further up (?inaudible). It's further up.

Monica: I'm going to show you something. Put your hands up both of you like this. Put your hands up straight (demonstrates), turn right 90, turn it right 180. [Here is an example of "playing turtle" in relation to lateralization.]

Rina M.: O. K.

Monica: Brings you here, eh? I am where you were. To get to there, what do I really have to do?

Lilly M.: -L -T 90. Left 'ninety.

Rina M.: Left eighty, 'ninety.

Lilly M.: Left ninety.

Monica: Instead of going all the way around, I can just do left ninety, eh?

Lilly M.: Yeah.

Monica: Do you see that? O. K. But we'll leave it the way you had it

Lilly M.: Ya, because (?it's the same).

Monica: because that's the way you thought it-

Note that Monica gave them the choice to proceed as they wished. They could either change the listed commands or retain them as they were. She specifically gave them leave to maintain the status quo in their recording. The fact that Lilly and Rina both left the records as RT 90, RT 180 in their books, without discussion or disagreement between them, could have meant (1) that they were not completely sure of what Monica was saying, (2) that they had half an ear to it (as Lilly seemed to do when one noted her body position and the direction of her gaze), or (3) that they did understand the concept but accepted Monica's offer to leave the work listed as it was "since it was the same," since it reflected their own way of working-- "because that's the way you

thought it [out]," and since they wanted to get on with the next phase.

The fact that Lilly M. and Rina M. did in the end desist from recording the turn as LT 90 is worthy of mention. Solomon (1976) has also noted in the child study portion of her thesis that seven year old Est insisted upon turning RT 90 three times, as opposed to doing a left turn of 90 (1976, p. 7.46). Among the focal children in this study, there were no other examples of either the use of the three turns where one turn would be equivalent, or of the substitution of a single turn of 90 for the triple turn. The aspect of equivalence deserves further investigation. For it might seem to adult eyes that the equivalence is an easy concept to learn; however, this equivalence is by no means as easy a notion as it seems on the surface to be. It is my belief that children do indeed learn this concept, but I would question a conclusion concerning this notion which Church and Wright (1986) have stated in regard to the learning of the five year old children in their study:

> Their discoveries . . . were small but significant in the overall scheme of learning . . . geometric concepts. [They learned that] there are two ways to face in a particular direction, for example, turn left (90 degrees left) or turn right (270 degrees right).
> (1986, p. 149)

Church and Wright (1986) were not inferring that the children were understanding anything about degrees; the bracketed notations are meant to elucidate the adult reader's understanding of the turtle cursor's moves. But Church and Wright (1986) were assuming that the children were able to equate the one quarter turn in one direction with its corresponding three quarter turn in the opposite direction. As Lilly and Rina's reluctance to write LT 90 in place of RT 90, RT 180 indicated, this equivalence is not as easy a concept as it might at first glance appear to be.

Teacher intervention with Jon and Eric

There were many occasions upon which Monica took advantage of the moment to teach material which was most appropriate to what the children were doing, when she thought the children were ready for it. Just a few instances which occurred when Monica intervened with Jon H. and Eric H. during the October 16, 1985 session will

be cited briefly. Jon and Eric called Monica in order that she would "tick off" the portion of work done up to that point. While she was with them, Monica gave them important information concerning (1) the combining of displacement moves done in opposite directions, that is "forward (FD)" and "back (BK)" (FD 105 - BK 60 = FD 45) and (2) the deleting of unneeded commands from their records. As has been noted in Selection #9A, Eric was not able to follow Monica's explanation. He may not have been concentrating because he was busy getting an eraser. (Subsequently it became clear that Eric did not understand what had transpired, and Jon was only heard to tell him what to write, and to indicate the spot on the page without giving an explanation, Selection #9A and #9B.) However, for Jon H., Monica's explanation had meaning; he seemed to understand the procedure, despite the fact that he calculated incorrectly and derived the answer of FD 45. Later during that same session, Jon and Eric again called Monica over so that she could check and "tick off" their work, and Monica took the opportunity to work with them, eliciting the fact tht there was a pattern which corresponded well to the use of the REPEAT command. This learning, which may have been introduced to Jon or Eric previously in the Grade 3 and/or Grade 4 Logo sessions, did remain with them, as was evident by the references which Jon made from time to time concerning using REPEAT (for example, Selections #31B and #31C).

Thus it can be seen that there were many opportunities for teaching which came at times other than calls for teacher intervention concerning a mathematics concept, or calls for intervention within a mathematics concept disagreement --we have seen how rare the latter calls were! Monica provided the "scaffolding." On some occasions the support was timely. In other instances, it was noted that the children did not absorb the new offerings, as for example in the case mentioned above with Eric, and perhaps also with Lilly and Rina; another example would be Amy L. and Monica's intervention concerning the concept of 90, both in Appendix Q, and in Selection #19. Re-introduction, or alternate presentation, and/or more experiential episodes would be needed in order for learning to take place.

APPENDIX T

Table 36

Occurrence Within the Mathematics Concepts Disagreements of Two Types of Supporting Arguments, Mathematical Justifications and Descriptions, Further Subdivided into Subcategories (See Table 32)

	MATHEMATICAL JUSTIFICATIONS											DESCRIPTIONS							
	Arithmetic			Geometry				Other											
	addition	subtraction	inverse	square	rectangle	quadrant = 90	proportional relationship	symmetry	variable	size of turtle step	TOTALS	size	direction	location	heading	screen fit	announcement	uncategorized	TOTALS
Russ H.[a]	1						1		2	1	5								
Jon H.				1	1						2		3			4			7
Eric H.		1		1			1				3	4	1	2					7
Lilly M	4		3	2		1					10	5	6		2		1	2	16
Rina M	3						1				4	5		1					6
Shauna L.[a]			1				2	1			4	1		2		1			4
Carol L.		1									1		1	1					2
Amy L.			1								1	1	1			2			4
Leanne L.																			
Doug L.[a]												1	3						4

[a] Russ H., Shauna L. and Doug L. were videotaped for only 6, 4 and 4 sessions respectively.

Appendix U

HILLEL'S (1984) LIST OF SPONTANEOUS CONCEPTS, AND MATHEMATICAL THEOREMS, WHICH MAY EMERGE FROM LOGO ACTIVITES

Hillel (1984, p. 23) identified certain spontaneous concepts and mathematical theorems which may emerge as the result of working with Logo. The three spontaneous concepts are as follows: (1) that outputs vary directly with inputs so larger inputs produce larger effects; (2) that two successive displacements (rotations) is a displacement (rotation); and (3) displacement FD $\underline{X}$, FD $\underline{Y}$ is longer than FD $\underline{X}$, that is, FD $\underline{X}$, FD $\underline{Y}$ > FD $\underline{X}$ and FD $\underline{X}$, BK $\underline{Y}$ < FD $\underline{X}$, and analogously for rotations.

Theorems noted by Hillel as emerging from the Logo activities are those related to *relative measure, additive measure, subtractive measure,* and *proportional measure.* These are theorems I to IV in the report (Hillel, 1984, p. 23). They are given below.

I *relative measure*

$$\underline{X} < \underline{Y} <=> |FD\ \underline{X}| < |FD\ \underline{Y}|$$
$$\underline{X} < \underline{Y} <=> |RT\ \underline{X}| < |RT\ \underline{Y}|$$

II *additive measure*

$$|FD\ \underline{X}, FD\ \underline{Y}| = |FD\ (\underline{X} + \underline{Y})|$$
$$|RT\ \underline{X}, RT\ \underline{Y}| = |RT\ (\underline{X} + \underline{Y})|$$

III *subtractive measure*

$$|FD\ \underline{X}, BK\ \underline{Y}| = |FD\ (\underline{X} - \underline{Y})| \quad \underline{Y} < \underline{X}$$
$$|RT\ \underline{X}, LT\ \underline{Y}| = |RT\ (\underline{X} - \underline{Y})| \quad \underline{Y} < \underline{X}$$

IV *proportional measure*

$$|FD\ (r\underline{X})| = r\ |FD\ \underline{X}|$$
$$|RT\ (r\underline{X})| = r\ |RT\ \underline{X}|$$

As a *state change* operator, RT also satisfies properties of:

V *periodicity* $\quad RT\ (360 + \underline{X}) = RT\ \underline{X}$

VI *complementarity* of RT and LT $\quad (RT\ 360 - \underline{X}) = LT\ \underline{X}$

APPENDIX V

This appendix contains a copy of the paper:

Zack, V. (1986d). A study of ten to eleven year-old students' notion of right angle in Logo geometry. Proceedings of the Second International Conference for Logo and Mathematics Education, July 15-18, 1986, University of London Institute of Education, 96-103.

A STUDY OF TEN to ELEVEN YEAR-OLD STUDENTS' NOTION OF RIGHT ANGLE IN LOGO GEOMETRY

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Abstract: The clinical-observational study in a naturalistic elementary school computer-room setting investigated 10-11 year-old children's notion of right angle in Logo geometry. Student strategies during the twelve weekly Logo graphics sessions, and student clinical interview data, were analyzed in an endeavor to determine each child's construct of right angle. Findings indicated that the majority of the group of twenty-three children did not have a relational understanding of the concept of right angle.

The focus of the study was on the elementary school child's (10-11 years of age) construction of meaning for the concept of angle, in particular right angle, in Logo geometry. The right angle was selected as a focal point of interest because the assumptions concerning children's relatively speedy supposed mastery of RIGHT/LEFT 90 in Logo need closer scrutiny. Researchers have recorded the use of RIGHT and LEFT 90 by children as young as 6 years of age, (Lawler, 1985; Church and Wright, 1983) but the degree to which children have control over the concept warrants investigation. The 'right angle' would SEEM to be a natural beginning point for children. Children see "corners" in the physical environment around them, and the immediate surroundings provide a context for interviewer-interviewee discussion of angles even with those children who have no formal school-taught conception of angle. Children often attempt Logo drawings which feature squares and rectangles; these attempts allow for a "means-ends" type of analysis in that the researcher can compare the goal with the resultant production (whereas with other angles the child can always "deplan" [Richard Noss' term]).

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One research question was: What is the nature of the strategies the child uses to create the right angle in Logo (or its approximation), and what do those strategies reveal about the stage of his/her construct? In order to look at the level of the child's construct, (psychological analysis), Allaire's (1985) application to Logo of the Herscovics and Bergeron "models of understanding" (1983) was used. For example, a child functioning at the first level, the "niveau intuitif", would be "groping" towards 90, or its approximation; at the second level, the child has control over the use of RIGHT/LEFT 90.

In terms of the mathematical framework, the aim of the study was to look at the child's definition of angle, and the child's perception of angle in the Logo microworld. As Close (1981) has pointed out, there are two most commonly used definitions of angle, one DYNAMIC (i.e. the quantity of the rotation); the other STATIC (i.e. the portion of a plane included between two straight lines). (Close, 1981, p.9) Research question two was: What is the 10-11 year-old child's definition of angle? Does it fit the dynamic, rotation view [Logo], or the static [Euclidean] view most often taught in the mathematics classroom? Is there a synthesis of the two? Does the child make a connection between the '90' in Logo geometry and the 'right angle' in Euclidean geometry?

Research Design : An observational-clinical research design in a naturalistic elementary school computer-room setting was used. The school is a private, multilingual Jewish day school; all participants (12 boys, 11 girls) came from a middle class background. The heterogeneous grade five class of 23 children, split into two groups,

attended twelve 50-minute Logo computer sessions in the room equipped with 8 Apple IIe microcomputers and one printer. The expert Logo teacher, Monica Shapiro, used an individualized approach with the student pairs; the projects were child-selected in all but one case (Monica assigned an across-class task during the last session.) The researcher was a non-participant observer. The data included: the researcher's, teacher's and children's notes; two clinical semi-structured interviews (one at the start, one post-sessions); and in-depth videotape records of the work of five pairs of children (both the Logo work and the camera record of their interaction).

The analysis of the data was done in light of the criterion that mathematically, a relational understanding of 90 degrees in Logo geometry would require the child to: (1) demonstate the correct use of 90 degrees in situations where right/left 90 are required, for example in a design which requires a square, rectangle, stair-way construction; (2) demonstrate the knowledge that 90 degrees creates a corner that has both an inside angle of 90° and an exterior rotation of 90°; (3) demonstrate the knowledge that right angle represents a corner of a plane in the physical environment; (4) recognize a right angle/ a 90 degree turn in its various orientations and transformations. The stress throughout the study was on the analysis of the child's construct of right angle within the natural context of his/her work in Logo. The aim was to investigate the child's strategies, and the degree of support the child could give to his verbalizations about right angle, to his peers, to his teacher, and to the researcher-interviewer.

Some Findings

Due to space constraints, the findings will deal very briefly only with aspects concerning the children's control of the use of right angle in Logo geometry; the perceived relationship between RT/LT 90 in Logo and right angle in Euclidean geometry; and the children's definition of "angle."

The findings suggest that in the group of 23 children observed, the number who have control of the workings of 90 degree rotation in Logo is small. For ease of reference, I have attached a chart on which I have placed the children on a scale (admittedly simplified), from "most advanced" to "least advanced". (Figure 1) In terms of levels of understanding, one child was found to be functioning at Level 3 in relation to Allaire's criteria (Allaire, 1985), four were functioning at Level 2, and eighteen of the children were still at the exploratory "groping" Level 1 in their strategies vis-a-vis turtle rotation. Three of the children (Russell, Jon, Michael) displayed full ease in their work with RIGHT/LEFT 90, and could be said to have a relational understanding of 90 as defined above. Twelve of the 23 children showed consistent appropriate use of 90 in their Logo projects. For nine of these eleven children, the 90 in RT/LT 90 seemed to be the "magic number to make corners"; there was only partial or no understanding of the total picture, i.e. the relationship of 90 to multiples or fractional parts of 90, and to the general set of angles. Six children in the class showed inconsistent use of the right angle turn in their Logo work; however all six children did recognize the static right angle in various orientations on paper. Two of the 23 children (Dina,

Table 1

	Highly Adept					Moderately Adept							Least Adept										
	Russ H	Jon H	Michael H	Eric H	David H	Gail M	Stuart M	Lilly M	Matthew M	Tamara M	Adina M	Rina M	Shauna L	Joey L	Carol L	Mark L	Dina L	Amy L	Leanne L	Saul L	Raymonde L	Don L	Doug L
90	++	++	++	++	+	++	+	+ -	++			+ -											
45 first interview	++	++		+ -		++	++		++														
45 second interview	++	++	++	++		++	++	+ +	++					++									
180	++	++	++	++	++				+		+	+			+	+				+			+
360	++	++	++	++	++	++	++	+		+		+			+	+		+		+		+	
270	++	++	++		++																		
135	++	++	++	++	++	+ 3x45	+ 3x45	+ 90+45	+ 3x45			+			+								
Do you think a right angle is the same as right 90 in Logo?	no	no	yes & no	no	no	yes & no	yes	yes		yes	yes		yes & no			yes							
Use of 90 in Logo work	++	++	++	++	+	+	+	+	+	+	+	+ -	+	+	+	+ -	-	-	+	+	+ -	+	+ -
Use of SETH	++				+		+																
Paper Test																							
*Circle the right angles			+		+			+		+									+				
*Circle the 90 angles	+	+	+	+	+	+		+		+	+	+	+	+		+	+			+	+	+	+
Lateralization problem					◇	▽					▽			▽			▽		◁ ▽	▽			▽
Relational understanding of 90	yes	yes	yes																				
Allaire (1985) level	2	2	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

KEY

[++] evidence of consistent correct use in Logo work during sessions, in across-class tasks, in interviews. Can give support via verbal instructions

[+] consistent correct use over a short period, and/or during interview only

[+ -] inconsistent use over a short period

[-] no evidence of correct use

Note

The children's names have been changed to assure anonymity

Amy) showed no mastery of the notion of 90 even as a tool to make "corners," as evidenced by their performance on Logo projects, and in the across-class task.

One of the aims of the study was to determine the child's definition of RIGHT/LEFT 90 in Logo, i.e. do they see it as an angle? If yes, does it fit the dynamic (rotation) view, the static (Euclidean) view, or is there a synthesis of the two? One post-session interview question which drew some interesting responses was the following: I HEARD SOMEONE YOUR AGE SAY THAT RIGHT 90 IN LOGO IS THE SAME AS A RIGHT ANGLE. WHAT DO YOU THINK? The children had difficulty with the term "right angle" itself, but after the term was clarified, 8 said YES; 4 said NO; 1 said NO/YES, 2 said YES/NO; while the remainder said either yes or no but were unable to support their response. Of the children who said YES, two added the arms of the angle to their rotational move via drawing, and three finger-traced or made a frame with their hands showing the perpendicular and right-horizontal lines. It was a surprise to note that the 4 youngsters who said 'NO, right 90 in Logo is not the same as a right angle because there are no arms', are children at the "most advanced" level (Figure 1).

I HEARD SOMEONE YOUR AGE SAY THAT RIGHT 90 IN LOGO IS THE SAME AS A RIGHT ANGLE. WHAT DO YOU THINK?

- Russell: No...'cause there's no lines to make the angle.
- Jon: No... because it only turns right, it doesn't make an angle.
- Eric: No...It's only a right turn.
  Int: But it doesn't have anything to do with a right angle, right 90?
  Eric: Y-ya.
  Int: How?
  Eric: If there was a box like this, it would be turning there. (indicates corner)

- David: No... 'Cause right ninety turns him just turns him into ninety. It isn't the same as that (points to school maths textbook page).
  Int: How isn't it the same?
  David: Because there are no lines.

- Shauna: (YES/NO) Ya, but not all the way. (pause) It has to have two lines.

The children all seemed to have an understanding of the fact that the turtle turns when it goes RT/LT, but none seemed to suggest that the rotation itself was for them synonymous with the term "angle." In their mind, an angle MUST have rays. Their definition of angle itself seemed wholly static, despite the classroom mathematics textbook's sometimes representation of angle as dynamic.

Areas requiring further study : Two aspects which were touched upon in the study (though not above) and which warrant further investigation, concern angular measure and the degree of turtle's rotation. Do the children see the relationship between the input number to LT/RT and the degree of rotation? For example, two youngsters who were able to work comfortably with 90 in their project work, when shown the static angle on the blackboard/paper and asked "What would turtle turn to make this same angle?", answered that the turtle would turn RT 40 or 30 (Stuart), or RT 93 (Eric), to make the right angle. Do the children see the relationship between the number of degrees in the Euclidean static angle in the mathematics textbook and the input number to RT/LT in Logo? Only one child (Gail) spontaneously voiced an intuition of the possible connection between the two when during the post-session interview, she grabbed the protractor and checked the measure of 45 degrees on the angle on the computer screen, and said, "Ya. Forty-five degrees. Perfect."

Connections that these 10-11 year-old children have made between Logo geometry and school geometry are tenuous at best, and must be further explored. The terms "angle" and "degrees" in reference to the Logo concept of "angle" should be used cautiously, with the realization that the terms may well be inaccurate and irrelevant respectively as components of the children's construct of angle at this age. The findings from this small group indicate that for the children an angle does not exist without the presence of arms. Should we not then be referring to "turtle turn," or "angular rotation" rather than angle in Logo geometry? The aspect of the RT/LT 90 turn in Logo motion geometry requires continued study. The findings from this small group indicate that there are few children who are working with "relational understanding," that is, who "know what to do and why" (Skemp, 1976) vis-a-vis the RT/LT 90 turn in Logo, and vis-a-vis angular rotation generally.

References :

Allaire, R. (1985). Le concept d'angle dans un environnement Logo. Talk presented at National Council of Teachers of Mathematics Conference, Montreal, Quebec, Oct. 3-5, 1985.

Church, M. J. & Wright, J. L. (1986). Creative thinking with the microcomputer. In P. Campbell & G. Fein (Eds.), Young children and microcomputers. Prentice Hall.

Close, G. S. (1981). Children's understanding of angle at the primary/secondary transfer stage. Master's Thesis. The Department of Mathematical Sciences and Computing, Polytechnic of the South Bank, London.

Herscovics, N. & Bergeron, J. C. (1983) Models of understanding. Zentralblatt fur Didaktik der Mathematik.

Lawler, R. W. (1985). Computer experience and cognitive development: A child's learning in a computer culture. John Wiley.

Skemp, R. L. (1976). Relational understanding and instrumental understanding. Mathematics Teaching , 77 , 20-26.

APPENDIX W

This appendix contains a copy of the paper:

Zack, V. (1988a). Say it's perfect, then pray it's perfect: The early stages of learning about Logo angle. Proceedings of the Twelfth Annual Meeting of the Psychology of Mathematics Education Conference (PME 12), Hungary, July 20-25, 1988, 657-664.

SAY IT'S PERFECT, THEN PRAY IT'S PERFECT: THE EARLY STAGES OF LEARNING ABOUT LOGO ANGLE.

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Abstract: The longitudinal naturalistic study has been investigating elementary school children's understanding of angle. Findings indicate that while discoveries about angle are indeed being made, the pace of the learning has been slow. There is a need for more time and continuity in the learning and teaching of Logo, and for more explicit teacher-elicited connections between Logo geometry and school geometry, if Logo is to play a role in the mathematics curriculum.

Within the emerging nucleus of work concerning the learning of the concept of angle in the Logo environment, there are a number of studies which devote attention to the early phases of the learning (Hillel and Erlwanger, 1983; Hillel, 1984; Noss, 1985; Hoyles, Sutherland and Evans, 1985; Kieran, 1986). This paper focuses on the learning about angle which takes place in the early stages of work with Logo-- the first 30 hours or so of Logo learning.

In my study, the exploratory "groping" stage toward the learning of angle has been more prolonged, on the part of some of the children, than I had anticipated at the outset. This longitudinal study in a naturalistic elementary school, and then secondary school, classroom computer laboratory setting has traced some of the children's work in Logo (approximately 12 sessions per year) from grade 3 (8 to 9 year-olds) to grade 7 (12 to 13 year-olds); and yet the majority of the children can be seen, in grade 7, to be still at the exploratory stage of work with angles. The difficulties experienced by the children in this study confirm some of the findings reported by the afore-mentioned researchers, but the pace of the learning has been slower. The discrepancy in pace between this study and those cited above might in part be attributed to differences in setting and curriculum agenda (for example: assigned tasks; time frame; mathematics agenda; staffing by research and mathematics experts). What is certain is that the children have more difficulty with the 'seemingly simple' aspects of Logo than the literature would sometimes have us believe.

Research was supported by a Social Sciences and Humanities Research Council Doctoral Fellowship.

In a previous paper (Zack, 1986), I presented findings concerning the level of attainment vis-à-vis angle of all of the 23 (grade 5, 10 to 11 year-old) children in my study, using as a means of focus their understanding of "right angle." This companion paper will feature two of the children from that class. It will touch upon (a) their difficulty with determining the size of a 90° and other turns; (b) the language the children use to describe turtle's location, heading, amount of turn; (c) their problem solving and recording strategies; and (d) the fact that they do not make connections between Logo geometry and school geometry.

RESEARCH DESIGN

The setting was a private, multilingual Jewish day school. All participants (11 girls, 12 boys) came from a middle class background. In 1985-86, the heterogeneous grade five class of 23 children, split into two groups, attended twelve 50-minute Logo computer sessions in the computer classroom equipped with 6 Apple IIe microcomputers, Apple Logo I software, and one printer. The expert Logo teacher, Monica Shapiro, used an individualized approach with the student pairs. The projects were child selected (exception: Monica assigned an across-class task during the last session). No changes were made to the Logo software (exceptions: use of slowturtle in grade 3; addition of a HELP command to the startup aids to help students check turtle's heading when needed). The researcher was a non-participant observer. An observational-clinical research design was used. The data included: the researcher's, the teacher's, the children's notes; interviews with the Logo teacher; three clinical interviews with the children (one at the start, one at the close of the grade five session, 1985-6, with 23 children; one at the end of the grade six year, May 1987, with 56 children); and in-depth videotape records, transcribed, of the work of five pairs of children (both the Logo work and the camera record of their interaction). Monica, the Logo teacher, wanted the children to learn the Logo theorems via exploration, and via her input when her help was solicited; but the learning of mathematics concepts via Logo was not the primary objective in her agenda.

Of the five pairs whose work was videotaped, transcribed and analyzed, I have chosen the work of one pair to discuss in this paper, that of Lilly and Rina. Their attainment in relation to the rest of the class was average; on the class grid, they will be found in the middle range of the class (Zack, 1986, p. 100).

STRIVING TO MAKE SENSE

The subtitle "Striving to make sense" pertains both to the children's trying to make sense of Logo angle, and to the researcher's trying to make sense of what the children were doing and saying vis-à-vis their turns. Lilly and Rina, and 18 of the other children in the class, used a purely visual (Kieran, Hillel, and Erlwanger, 1986) feedback strategy for determining inputs to RT/LT. I have termed it a context-referenced strategy, for they only made decisions on inputs to RT/LT when in immediate mode, using the screen as contextual reference. Rina and Lilly used their own terms, what Kieren (1987) calls ethno-mathematical language, to describe the turtle's location, and amount of turn (See below, eg. "it's straight," "go half," "go all the way around"). It was only via contextual reference to the videotape that the researcher was able to comprehend how Rina and Lilly's verbal descriptions matched the end result of the back and forth, left and right, exploratory moves that they had made.

PROBLEM SOLVING STRATEGIES AND RECORDING STRATEGIES

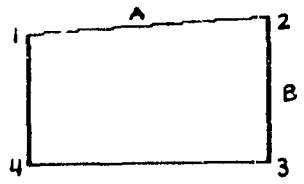
Lilly and Rina used the turtle to navigate (Sylvia Weir) around the page as they drew. Because they did not yet have a sense for the size of the rotational turn, they would "fiddle around" (Rina, May 1987) until it looked like it was perfect. They would then often say "It's perfect," and sometimes pray aloud that it be perfect.

They recorded step by step in their Hilroy book concurrently with their moves. They combined on paper by bracketing in pencil the "live" inputs. (There were no occurrences of the combining of unlike inputs, either FD X + BK Y = FD (X-Y), or, more difficult, RT X + LT Y = RT (X-Y).) Lilly and Rina used the editor as a (hopefully) accurate trace of their immediate mode commands. No debugging was done in the editor. Rina especially subscribed to a "Be safe but sure" motto concerning her Hilroy entries: "I'm not taking any chances (Dec. 4) . . . . I'll write down the mistake. As long as it turns out (Dec. 18)." In checking their Hilroy notebook inputs when attempting to find a bug, they could only resort to reconciling the number and sameness (and for Lilly, the equivalence) of the actual written entries. In trying to resolve an error in angular rotation, they were never heard to use a "Does it make sense?" test, most probably because they did not have a concept of the size of the turn against which to gauge their input.

EXPLORING THE RIGHT ANGLE TURN

I would like to consider more closely the difficulties for Rina and Lilly entailed in what might seem to an observer to be a simple task, namely that of constructing a rectangle. In grade 3 the children had been "given" RT/LT 90 for making corners. And yet when Lilly and Rina embarked upon the first part of their chosen project, the rectangle part of the tape/stereo "ghetto-blaster" they wished to make, they had to work through each corner tu r. The episode below took place midway through their Logo sessions (November 20, 1985--Session #6). I chose it as a focal point of reference because it offered a glimpse at the children's moves, their use of language, their awareness of visual cues which signal error, and their interaction with neighbouring peers vis-à-vis their product.

They arrived at the first turn (marked 1 below) by keying RT 35, RT 35, RT 10, RT 5. Lilly declared: "It's straight." As soon as they proceeded with Line A, they saw that the line was jagged. Their evaluation followed. Lilly said, "It's good"; Rina said, "It's bad"; Lilly countered with: "It doesn't matter." [However, as became clear in subsequent comments, the jagged line did bother them--Rina especially-- very much.] At this point they let it be. Rina expressed surprise at the combined sum--85, and stopped to reconfirm with Lilly that it did indeed take "RT eighty-five" to "get all the way around."



For turn 2, Rina used the information from the previous turn. She stated: "We wanna go half, we want RT 85." [It is only by contextual reference to the screen, and by the fact that one knows that they are aiming for a "corner" that one follows that "all the way around" and "we wanna go half" both refer to a quarter turn.] Rina and Lilly then proceeded to disagree about the input, and it sounded as if they were still disagreeing when they both decided on an input of RT 95, which, with serendipity, was the correct input. The subsequent line, Line B, was straight.

Turns 3 and 4 were the results of inputs of 90; there were lateralisation errors, but both children agreed that 90 was the input, and they used corrections of 180 when needed. One might think that they had now grasped the importance of 90 in making a smooth corner with straight line arms. It was clear however that they had not yet mastered the 90 when one listened to Rina and Lilly's interaction with neighbouring peers (Russ

Michael) which followed shortly after the teacher had come by and "ticked off" that the rectangle part was done.

Russ: Rina, Rina- Shouldn't the line on top be straight?
Rina: Ya, but we didn't do that.
Russ: So why didn't ya do it?
Rina: We didn't do it. (Points to the CRT) It turned out like that.
Russ: Ya, 'cause you did something wrong.
Rina: No, we didn't do 'anything wrong.
Lilly: 'No, we didn't.
Russ: (inaudible)
Rina: (getting back to work, leafing through the pages of her recording book)
'Kay, we have to get into the editor.
Michael: You DID, 'cause you went downwards. (inaudible)
Russ: A line would never be like that.
Rina: You wanna bet it would? It happened '(? to me a couple o' times)
Lilly: 'BIG DEAL! (Now sitting straight, looks at her book)
Rina: O.K. Logo editor. (Looks away from Russ and Michael, signalling the end of their interchange with the boys.)

The jagged line served as a cue to Russ and Michael, as it had served to Lilly and Rina, that something was wrong. But the girls did not respond well to the peer intervention by Russ and Michael. Rina and Lilly's lukewarm reception of Russ and Michael's comments may be due to the fact that (1) the boys were offering an unsolicited,negative evaluation of their product; and (2) Lilly and Rina had just completed the "rectangle" part of their project and were anxious to get on with the next part of their work.

I wondered when reviewing the tapes whether Lilly and Rina had desisted from debugging because they were rushed, or because they were unable to correct. I therefore looked at subsequent tapes closely and noted evidence as late as January 29, 1986 that Lilly was not completely in command of the 90 as input. In the January 29 teacher-assigned across-class task requiring squares, Lilly groped for the input to LT (LT 50, LT 19, LT 11), then suddenly cleared the screen and said sharply: "LT 90!" During the clinical interviews in Feb. 1986, and in May, 1987, I asked Lilly and Rina about the rectangle. When asked (in Feb. 1986) what she could have done differently in making the rectangle, Lilly was able to state that she would have had to turn RT 90 "to make (the line) straight."

Rina, however, in both the February 1986 interview, and in the May 1987 interview, asserted that she did not know how she could have made "that corner even. . . .I don't know why that (i.e. the jagged line) happens."

During the February 1986 (Zack, 1986) and the May 1987 interviews, it was clear that Rina and Lilly had made progress in their understanding of certain aspects of angular rotation, though there were still gaps to be filled. Rina was able to identify a right angle in different orientations (on paper). She was able to use it as a point of reference when needed. For example, in proving that the blackboard angle must be obtuse, she showed how it was more than 90; 14/56 students were seen to use 90 as a reference in this way. Rina was not able to use an analytic (Hillel, Kieran and Erlwanger, 1986) problem-solving approach to determine the supplement for an angle of 175. She stated that she "would fiddle around" until she got where she wanted to be. She was not able to use analytically the classroom geometry information she knew by rote, namely that there are 180° in a straight line. She was also still working toward consolidating the fact that the input to RT/LT is equal to the number of degrees in an angle (Zack, 1986).

Lilly was able, with prompting, to figure out the amount of turn needed for the supplement of a 50° turn. [A total of 10/56 or 17.8% of the students interviewed were able to use an analytic approach with prompting; and 5/56 or 8.9% of the students were able to do so without prompting.] Lilly used a method employed by three of the students who were able to use an analytic approach. She first moved the cardboard turtle through a turn of 90; and then worked within the one remaining quadrant, determining that the complement of the given angle was 40.

IMPLICATIONS FOR THE CURRICULUM--NOW WHAT?

The pace may seem slow, but the learning offers a rich foundation upon which to build. For Lilly and Rina, the right angle and its relationship to other angles would be a fruitful starting point, in view of the time they spent making some sense of it. The findings indicated that Rina and Lilly, and the majority of the children interviewed, had not been able to make connections between Logo geometry and school geometry (Zack, 1986), nor to work analytically within the Logo environment itself; indeed, the fact that they were "drawing" precluded the need to do so. And yet as the project drew to a close in January, 1988, it was clear that major obstacles lay in the way of future progess in the children's learning of angle via Logo, namely: (1) the children's perception about Logo;

(2) the time frame; and (3) WHO was going to help make the connections. The perception of some of the children interviewed in May 1987 was that Logo was "just drawing" (13/54, or 24%); the majority felt that they had learnt everything there was to know, and that they had outgrown Logo by grade 5 or 6. When asked "Do you feel that Logo can help you learn about angles? How?", eleven out of the fifty-six children interviewed, including Lilly, said that they never thought of angles while doing Logo. But certainly a teacher could highlight the vital connections. The question then is when and who. It was the elementary Logo teacher's (Monica's) expectation that the high school curriculum would be the most appropriate stage for the explicit invoking of connections. At the end of the grade 7 Logo component (again approximately 10 sessions in November to January, 1987-88, and the last year of Logo), it was clear that Monica's expectation was not going to be realized. The reasons were, briefly, as follows. The computer teacher spoke of the limited time span he could allot to Logo within the grade seven computer curriculum, he noted the pupil-teacher ratio (1/24), and the desire to cover topics such as "variables" and programming skills. The computer teacher (who had also taught the mathematics grade 7 course) overestimated what the children knew about angles and Logo geometry. The two Grade 7 mathematics teachers interviewed stated that they did not make any connections to Logo geometry in their mathematics classroom; one said this was due in part to her unawareness of what the children had learnt.

If the slow pace of the learning detected in this study reflects the pace in many current Logo school programs, this factor may in part explain the frustration with Logo that Watt and Watt (1987a) have reported teachers are feeling. The Watts (1987b) have cited teachers' complaints that "Logo isn't working" and that "students aren't learning important mathematics and computer science through exploration and discovery." We are in some danger, then, of throwing out the proverbial "baby with the bath water." Findings from this study indicate that there is a need for a more concentrated, continuous time frame for the Logo learning, a need for an underlying but not restricting mathematics agenda, and a need for teacher-elicited explicit connections to be made concerning both the underlying structures of Logo geometry, and the relationship between Logo geometry and school geometry, if Logo is to play the contributing role it can play in the mathematics curriculum.

REFERENCES

Hillel, J. (1984). Mathematical and programming concepts acquired by children, aged 8-9, in a restricted Logo environment. (Research Report No. 1). Department of Mathematics, Concordia University, Montreal, Quebec.

Hillel, J., & Erlwanger, S. (1983). Observations, reflections and questions about children's Logo learning. Unpublished paper, Concordia University, Montreal, Quebec.

Hoyles, C., Sutherland, R., & Evans, J. (1985). A preliminary investigation of the pupil-centered approach to the learning of Logo in the secondary school mathematics classroom. Logo Maths Project, University of London Institute of Education.

Kieran, C. (1986). Logo and the notion of angle among fourth and sixth grade children. Proceedings of the Tenth Annual Meeting of Psychology of Mathematics Education (PME 10), July 20-25, 1986, London, England, 99-104.

Kieran, C., Hillel, J., & Erlwanger, S. (1986). Perceptual and analytical schemas in solving structured turtle-geometry tasks. Proceedings of the Second International Conference for Logo and Mathematics Education, July 15-18, 1986, London, England, 154-161.

Kieren, T. (1987, January). Language use in mathematics. Visiting lecture, January 26, 1987, Concordia University Department of Mathematics, Montreal, Quebec.

Noss, R. (1985). Creating a mathematical environment through programming: A study of young children learning Logo. Published doctoral dissertation, University of London Institute of Education.

Watt, D. H., & Watt, M. L. (1987a). Assessing Logo learning in classrooms. Talk presented at the Fifth Annual Conference on Computers in Education, McGill University, Montreal, Quebec, November 18-20, 1987.

Watt, M. L., & Watt, D. H. (1987b). Exploratory research on critical aspects of Logo learning: A progress report. Education Development Center, 55 Chapel Street, Newton, Massachussetts 02160.

Zack, V. (1986). A study of ten to eleven year-old students' notion of right angle in Logo geometry. Proceedings of the Second International Conference for Logo and Mathematics Education, July 15-18, 1986, University of London Institute of Education, 96-103.

APPENDIX X

This appendix contains a copy of the paper:

Zack, V. (1988b). Ask three before you ask me: A teacher's attempt to increase the role of peer involvement in Logo learning. Proceedings of the North American Chapter of the International Group for the Psychology of Mathematics Education (PME/NA 10), Northern Illinois University, De Kalb, Illinois, November 3-5, 1988, 394-400.

"ASK THREE BEFORE YOU ASK ME": A TEACHER'S ATTEMPT TO INCREASE THE ROLE OF PEER INVOLVEMENT IN LOGO LEARNING

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Abstract: The inquiry looked at the role played by 10-11 year old classmates as they intervened to help partners at their Logo (graphics) work in an elementary school classroom computer lab environment. Attention to the geometry aspect of the Logo learning was focal. Findings suggested that the relationship between the help-givers and the help-receivers was more collaborative than didactic.

Non-Logo (Cooper, Marquis and Edward, 1986; Forman and Cazden, 1985; Webb, 1982) and Logo (Hawkins, et al., 1982) research studies dealing with the social context of learning have raised questions which gave rise to the present inquiry. Cooper, Marquis and Edward (1986) defined two prototypic situations in which children can learn together: the collaborative and the didactic (1986, p. 270). In collaborative, symmetrical, "doer-doer" relationships between peers, the children have an equal amount of information concerning a problem. In asymmetrical, "knower-doer" relationships, one child knows the information, while the other child does not. Forman and Cazden (1985) have reported that support from a partner seemed to "enable two collaborators to solve problems together before they [were] able to solve them alone" (1985, p. 341). Inherent in both the Forman and Cazden (1985) and the Cooper, Marquis and Edward (1986) studies is the Vygotskian notion of the contribution which a peer can make to another peer's learning. A phrase coined by Thyberg (1985), and heard increasingly often within the community of Logo teachers-- "Ask three before me"-- reflects a belief in peer expertise. This inquiry was undertaken in order to investigate the ways in which the peer classmates intervened to help.

Within the naturalistic elementary school computer laboratory setting of the study, the researcher has looked at both the interaction between

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peer partners, and at the interaction between the partners and other classmates which occurred while the ten to eleven year-old children, in self-selected pairs, were learning Logo. The aspect of peer collaboration and peer teaching between partners and its relationship to the learning of Logo geometry represents one component of the researcher's doctoral thesis (Zack, in preparation). However, the focus of this paper is not on the partners but rather on the interaction which transpired when help was needed by a pair, and consequently neighboring peers intervened to help that pair at their work. Particular attention was paid to the requests for help which were related to the learning of geometry and mathematics concepts (Zack, 1986; Zack, 1988).

RESEARCH DESIGN

The setting was a private, multilingual Jewish day school. All participants (11 girls, 12 boys) came from a middle class background. In 1985-86, the heterogeneous grade five class of 23 children, split into two groups, attended twelve 50-minute Logo computer sessions in the computer classroom equipped with 6 Apple IIe microcomputers, Apple Logo I software, and one printer. The expert Logo teacher, Monica Shapiro, used an individualized approach with the student pairs. The projects were child selected. No changes were made to the Logo software (exceptions: use of slowturtle in grade 3; addition of a HELP command to the startup aids to help students check turtle's heading when needed). Monica wanted the children to learn the Logo theorems via exploration, and via her input when her help was solicited; but the learning of mathematics concepts via Logo was not the primary objective in her agenda. The extent to which the children perceived that Logo offered an environment conducive to the learning of dynamic geometry concepts varied, as became clear in the interview sessions.

The researcher was a non-participant observer. An observational-clinical research design was used. The data included: the researcher's, the teacher's, the children's notes; interviews with the Logo teacher; three clinical interviews with the children (one at the start, one at the close of the grade five session, 1985-6, with 23 children; one at the end of the grade six year, May 1987, with 56 children); and in-depth videotape records, transcribed, of the work of 8 children -- 4 focal children and their partners (both the Logo work and the camera record of their

interaction). A separate project was done (Zack, 1986) in order to determine what the children were learning about the Logo geometry concepts, in particular the notion of angular rotation. The researcher's thesis currently in preparation looks at the relationship between the partners and the ways in which the children can contribute to each other's learning of Logo geometry. The present inquiry dealing specifically with intervention by neighboring peers sought to distinguish whether the classmates were contributing to the Logo geometry learning of the 8 children, and if they were, then how were they doing so.

The transcript data representing the twelve weeks of work of four children and their partners were reviewed; instances of solicited and unsolicited peer interventions relating to the 8 children were tallied and analyzed to determine (a) the circumstances in which the students would ask a neighboring peer for help; (b) the frequency with which the various types of problems were referenced, that is, mathematics and geometry concepts, syntax, plan and programming problems; (c) the children's response to solicited versus unsolicited peer interventions; (d) the degree to which the peer help was indeed "helpful"; (e) the instances in which the helping episode could be considered a teaching episode; and (f) the strategies used by the child-teachers (for example, guiding, demonstrating, using commentary, giving meaningful explanations).

FINDINGS

Solicited and unsolicited interventions

The partners almost always attempted to work through a problem themselves first. Once they decided that they needed help, their first reaction tended to be to elicit help from the "expert" source, that is, the Logo teacher, Monica. They would elicit help from neighboring peers when the teacher was busy. That the first choice of call in this setting would be the Logo teacher was not surprising. It is clear from the interview data that some children considered it the teacher's role to teach; the children frequently stressed that the teacher was the expert--and this teacher was indeed an expert in the Logo area. Seven of the twenty-nine solicited peer interventions from classmates were the result of explicit redirections by the teacher. The teacher would when busy direct the petitioner to "ask somebody else," or would use a variant

of Thyberg's phrase: "Ask three before you ask me." Unsolicited offers of help by a peer classmate were on the whole not well received. Offers to help were refused, and stated suggestions by would-be contributors were ignored, in 13 of the 16 unsolicited interventions.

Calls for help with "syntax" problems (those related to working the computer functions such as loading, saving, editing) outnumbered calls for help with mathematics and geometry problems by a ratio of 4 to 1. The learning was considered to be in an early phase (Zack, 1988); consequently the children were still struggling with controlling the machine workings. Whereas the Logo geometry aspect was more forgiving, in that the partners could "fiddle around" with angle and distance and arrive at a somewhat close approximation, the syntax required EXACT specifications; hence perhaps the large proportion of calls both for teacher and peer intervention concerning syntax problems as opposed to geometry and mathematics problems.

Giving help

It was found that the peer interventions were as often incorrect as they were correct, except in the case of one specific child (Russ). The majority of the solicited and unsolicited interventions were executed by children who were themselves still at the exploration stage, and so the relationship between giver of help and receiver of help was on the collaborative (Cooper, et al., 1986) level, rather than on the didactic (Cooper, et al., 1986) level. This finding is consistent with the pattern of working relationships found to exist between the partners themselves in the larger study, that is, the relationships were found to be collaborative ones, except in certain instances where Russ was involved as a partner (Zack, in preparation). This finding in relation to collaboration is also consistent with that of Hawkins, et al. (1982) who reported that there were few peer teaching episodes, and a far larger proportion of collaborative interactions in their study.

In the "knower-doer" relationship, the peer expert knows the information. There was one child, Russ, who was considered by the researcher to be a "computer expert" in that he was in command of the Logo syntax, geometry and programming concepts; his interventions were correct in all but one instance. Three self-proclaimed "computer experts" were seen to emerge; they offered help, but their suggestions were often inappropriate and incorrect.

Peer teaching skills

There was a range in the ways in which help was administered. On six occasions, the peer helper performed the function without commentary, an approach which was not conducive to learning, as the children themselves noted in the interviews. However, in the majority of peer interventions, some teaching skills--such as focusing, guiding while the peer performed, giving information, demonstrating with commentary--were in evidence. In one instance, the above-mentioned "class expert," Russ, was specifically assigned to the role of tutor by the teacher; he was asked to help two students with the mathematics concept of variable. In that designated tutoring session, the "knower's" peer status changed, the time frame became more extensive, and more teaching functions emerged than was the case in the other peer interventions; these teaching skills included--checking on progress, marking critical features, giving explanations, and eliciting critical thinking.

Interventions related to mathematics concepts

The breakdown of the 9 peer interventions related to mathematics and geometry concepts revealed that 4 were concerned with angular rotation, 3 with circle and semi-circle, 1 with variable, and 1 involved a check on the accuracy of a calculation. There were three unsolicited peer evaluation statements related to errors in angular rotation, in which classmates commented on the "crooked" or "jagged" look of another pair's line but gave no suggestion as to what the partners might do to correct the error. The "jagged line" was the result of an input or 85 or other, when 90 should have been keyed in. None of the "onlookers" were heard to offer a suggestion for change. In each case the evaluative comment was ignored or dismissed by the receivers (Zack, 1988). In accordance with Webb's (1982) finding that "receiving help was effective only when given in response to [expressed] student need (1982, p. 425), and Webb's observation that "receiving explanations about the task would not be expected to relate to achievement if the explanations are inaccurate" (1982, p. 427) or if no explanation at all is given (1982, p. 425), one would not expect that in the instances of the unsolicited evaluations, or in instances of incorrect help, learning would result. However, there were indications in this study that Webb's criteria of "correctness" and "explanation" may be more consistent with the didactic than with the collaborative situation; as will be described below in the episode between

Saul and Shauna, a partial suggestion without attendant explanation, when given at the appropriate moment, was seen to spur learning.

Two helpful interventions

There were two helpful interventions, one related to turtle heading, the other related to doing the inverse of a curve. What was most interesting was that neither intervention was complete, nor answered the need fully. But each was nevertheless helpful to the petitioner. In the first case, after a false, incorrect start, Saul, one of the three afore-mentioned self-declared experts, gave Shauna the nudge she needed to discover that she could use FD X BK X when turtle's pen was up, as a tool for checking turtle's heading. Without the collaboration of Saul who gave Shauna the beginnings of an idea of how to proceed toward the solution, Shauna could not have gone on to re-invent the HELP routine for herself. This episode was a prime example of how "doers," in interacting together, were able to contribute to the development of a product which one child could not have accomplished on her or his own (cf. Forman and Cazden's (1985) above-mentioned comment, but in this case in relation to classmates, not to working partners).

In the second case, Jon and Eric sought help from the teacher to enable them to delete a curve by doing the inverse. They could not solve the problem of doing a two step inverse, that is, since they had keyed in REPEAT 30 [FD 5 LT 5], what they needed was REPEAT 30 [RT 5 BK 5] to undo. The teacher redirected them to ask a peer. David, the able Logo user solicited by Jon and Eric, could not help solve the problem of doing the inverse of REPEAT 30 [FD 5 LT 5], so instead he showed them how to circumvent the problem by going HOME and starting again. Thus Jon and Eric did not learn the geometry aspect from David, but they did solve their problem!

The Logo teacher's reflections upon "Ask three before you ask me"

During the researcher-teacher interview (June, 1987), Monica was surprised to learn that the tapes revealed such a large number of peer interventions (48 in all), for it was her impression that the children had rarely asked neighboring peers for help, despite her encouragement to them to do so. She was also surprised to find that a substantial portion of the peer interventions were not correct or helpful. These findings caused her to reflect upon the complex issues underlying the seemingly simple teacher

exhortation to "Ask three before you ask me." One of Monica's suggestions was that despite the fact that the inability to obtain expert help may result in frustration for the soliciting child, nevertheless part of the learning process in this peer-centered collaborative setting entailed developing the skill of distinguishing good help from incorrect help.

Conclusions: The children issue a "call for service" (Marilyn Merritt) with a view to receiving expert and correct help (from a teacher or from an able peer) in order that they might get on with their work. However, in this study the directive to "Ask three before you ask me" was less a matter of "recognizing and promoting peer expertise" (1985, p. 1) than Thyberg expressed in her article. It is, rather, more a matter of the children learning to (1) acknowledge that the child helper may herself need collaboration; (2) develop the skill of differentiating the "good" help from the "bad" help; and (3) persevere at times with the quest until the appropriate help and expertise is found.

REFERENCES

Cooper, C. R., Marquis, A., & Edward, D. (1986). Four perspectives on peer learning among elementary school children. In E. C. Mueller & C. R. Cooper (Eds.), Process and outcome in peer relationships. Toronto: Academic Press.

Forman, E. A., & Cazden, C. (1985). Exploring Vygotskian perspectives in education: The cognitive value of peer interaction. In J. V. Wertsch (Ed.), Culture, communication and cognition: Vygotskian perspectives. New York: Cambridge University Press.

Hawkins, J., Sheingold, K., Gearhart, M., & Berger, C. (1982). Microcomputers in schools: Impact on the social life of elementary school classrooms. Journal of Applied Developmental Psychology, 3, 361-373.

Thyberg, L. F. (1985). Too many questions, not enough me. The National Logo Exchange, 3, 9, 1-3.

Webb, N. M. (1982). Student interaction and learning in small groups. Review of Educational Research, 52, 3, 421-445.

Zack, V. (1988). Say it's perfect, then pray it's perfect: The early stages of learning about Logo angle. Proceedings of the Twelfth Annual Meeting of Psychology of Mathematics Education Conference (PME 12), July 20-25, Hungary, 1988.

Zack, V. (1986). A study of ten to eleven year-old students' notion of right angle in Logo geometry. Proceedings of the Second International Conference for Logo and Mathematics Education, July 15-18, 1986, University of London Institute of Education, 96-103.

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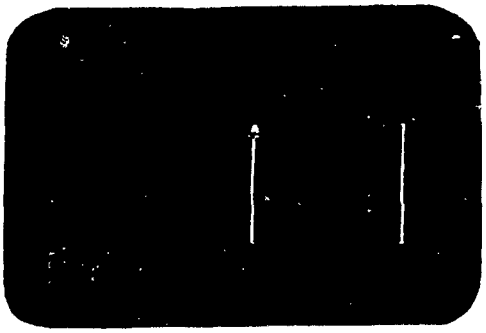
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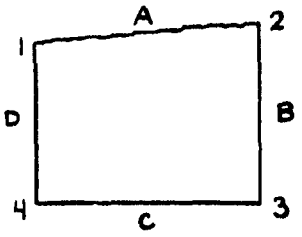
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Appendix Y

LILLY M. AND RINA M.: THE FOUR CORNERS OF ONE RECTANGLE

The rotational turn at corner 1 in the rectangle below was 85. As has been noted, the subsequent line (line A) was jagged.





Lilly and Rina's efforts to construct the other three corners of the rectangle are of interest, since they did manage to make rotational turns which resulted in 3 smooth sides for sides B, C and D of the rectangle. The turns at corner 1 (described in the text of the thesis in relation to Selection #45), and at corner 2 (to be described below), are of particular interest because of the light they shed upon the children's exploration of right angle.

In Selection #Appendix Y-1, we see that in trying to do the turn at corner 2, Rina referred to the amount of turn by using a reference to a quarter turn. Just previously she had referred to the turn in an ethnomathematical way, when she questioned whether the turtle cursor had indeed 'done' 80 to get "all the way around" (Selection #45, Nov. 20, 1985, p. 6). She now spoke in a more standard way in terms of fractional parts. She is heard here to state "we wanna go half"; in this case it is half of a half (180 degrees) turn, or a quarter turn. Rina then referred back to the input amount previously used for the turn at corner 1, namely 85; Lilly is heard first to object (by stating 80), and in almost the same breath is heard to accept the suggestion, when she herself recalled that it was indeed 85 that they had used at corner 1.

Selection # Appendix Y-1

Lilly and Rina, November 20, 1985, D/A # 3, p. 7

Rina M.:	-R -T (writing), we wanna go half (hand motion to indicate a quarter turn), we want -R -T, -R -T eighty-five.
Lilly M.:	No, 'cause we were, -R -T eighty (five, stated quietly)

It is important to note that at turn 2 the input proposed by both Rina and Lilly, whether it be 85 or 80, is not yet the key input of 90. The fact that their first choice of input harkened back to their prior use of 85 at turn 1 is interesting, because building upon previous work done can be a useful strategy; but their choice of 85 did not take into account the fact that a jagged line (line A) had resulted after turn 1, and therefore it is not an effective strategy. However, the very fact that they chose 85 again for turn 2 reflects their lack of understanding about the turn of 90. It is most likely that at this point Lilly and Rina were unable to make the connection between the jagged line and what caused it, namely the turn of 85.

Thus, the partners proceeded first (above) with the input of 85, which Rina keyed in. In Selection #Appendix Y-2 below, Rina is heard to react to the fact that 85 was not sufficient. Then both partners, Lilly and Rina, suggested that they ought to turn 'a little bit more.' Rina suggested 10 (added on to 85), and Lilly suggested 5 (added on to 90). It is important to note that Lilly and Rina have progressed from using just any numbers to using numbers 2, 5 and 10, and their multiples for their input suggestions. It is even more fascinating to discover that the partners were disagreeing without realizing that they were proposing equivalent inputs. Coincidentally, each was proposing the correct input. Rina suggested 85 plus 10, and Lilly suggested 90 plus 5. The subsequent line (line B) was therefore straight. Not only were Lilly and Rina each not aware of the equivalence, but without numerous returns to the videotape, it was also very difficult for the researcher to distinguish that indeed each child was talking about a turn of 95.

Selection #Appendix Y-2

Lilly and Rina, November 20, 1985, D/A #4, p. 7

Rina M.:	(Has keyed in RT 85, and is looking at the screen) Oh! (hand over her mouth, keys, looks at the screen) No, it's -R -T ninety. (Writes) -R -T
Lilly M.:	It's <u>not</u> -R -T ninety. Another five.
Rina M.:	Another 'ten.
Lilly M.:	'Cause if you're gonna do -R -T ninety, you're gonna-
Rina M.:	(keying) I'm going -R -T <u>ten</u>. (pause, stops, looks at Lilly) Well? [As in "I told you so."]

The input chosen for the next two turns, for both the turn at corner 3 and the turn at corner 4, was 90. Why was it suddenly 90? There was no indication of why the choice was 90; there was no comment made about the selection by either Lilly or Rina. It might seem to the casual observer that now the children had proof, that is, screen evidence of the efficacy of 90, and would henceforth be able to apply it readily in subsequent instances. This was not the case. Learning takes time. As has been elaborated elsewhere (Zack, 1988a: Appendix W), both Lilly and Rina moved slowly toward an understanding of right angle.

APPENDIX Z

RINA M. AND OLIVER: FIDDLING AROUND IN THE VISUAL MODE

One of the Grade 6 interview (May-June, 1987) questions was aimed at eliciting whether the children could work in the analytic mode. The interview was conducted on a one-to-one basis with each of the children (56 in all). This appendix will deal with the question itself, and with the responses by Rina and Oliver to the question.

The children were asked to determine what would be the supplement, β , to an angle already drawn by them (for example,) and whose measure was then assigned by the researcher or by the child, (for example, , that is, $\angle$ = 23). Without using the term supplement, what the researcher was indeed asking was: What would the supplement, the outside turn, be? β = ? It is important to note that the size of the inside angle ($\angle$) and of the outside rotation (β), and the orientation of the angle, varied with each child, since the researcher was often using an angle drawn freehand on the blackboard by the child, or an angle which was part of the child's own project design (as for example, Rina's "Shoe" in Selection #App 18-1 below). The question aimed to build upon the children's own products, and was therefore not addressed to all the children in a standard, structured way.

The question as phrased during the interview went approximately as follows (Appendix H, Q. 9):

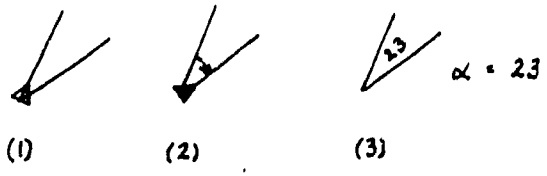
> If this part (angle) were 23 (or 41, or other) and you wanted turtle to turn and then to go along this line (route, direction), what would turtle have to turn?

This part of the interview took place at the blackboard and was videotaped, whereas the major part of the interview was only audiotaped. The question was asked with accompanying motions, and the nature of the child's response dictated the kind of prompting and further questioning which followed. In an attempt to clarify how the question was posed, the question has been broken down into four parts (below), and accompanying motions and elaboration are supplied in the italicized bracketed portions.

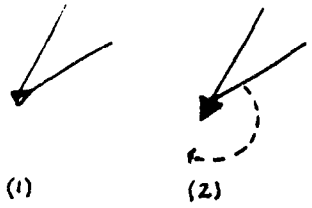
△ initial heading

▲ final heading

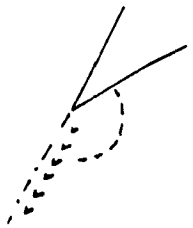
If this part were 23 *(the researcher moved the cardboard piece representing the turtle cursor from its initial heading to its final heading to show the size of the first part)*



and you wanted turtle to turn *(the researcher moved the cardboard turtle cursor in the direction indicated by the dotted curved line)*

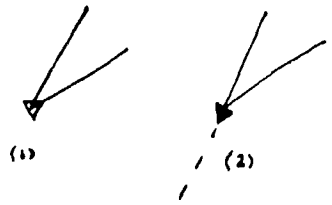


and then go along this line *(the researcher indicated the direction along an imaginary line or an existing line, represented by the - - - -- below)*



what would turtle have to turn?

(The researcher then again showed the initial heading △ , and then the final heading ▲ of the cursor)

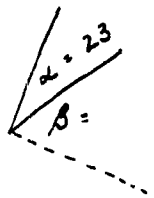


One of the ways a child might have thought out the response if she or he were able to use an analytic approach was the following: "If α = 23, and the total turn is 180°, therefore β = 180 - α, that is, β = 180 - 23. My answer for β = 157.

An interesting analytic strategy which emerged unexpectedly saw the children use the complement rather than the supplement of the given angle.
First they moved turtle through a rotation of 90.



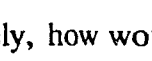
They then considered only the remaining quadrant which contained the angle of 23, and reasoned somewhat as follows: I have turned turtle 90, and so I have 90 left. Ninety minus the given angle will give me my answer, that is 90 - α = β , therefore β = 67. Lilly M. and two others used this analytic strategy of working with the complementary angles (that is, the angles totalling 90) rather than with the supplementary angles (that is, the angles totalling 180).

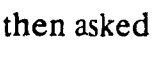
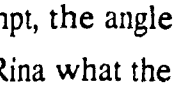
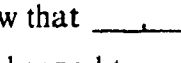
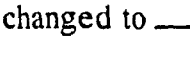


Some of the children (5/56, or 9%) responded readily and correctly to the interview question, and were considered to be using an analytic mode; among them were Russ H. and Jon (H.). Others, (10/56, or 18%), such as Eric H. and Lilly M.,

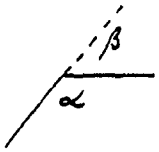
responded as a result of varying degrees of prompting, but were also considered to be able to use the analytic mode. The majority of the 56 children interviewed, 32/56, or 57%, among them Rina M., Carol L., Amy L., Shauna L., Leanne L., and Doug L., were not able work analytically. They either did not know that a straight line contained a turn of 180, or even if they did know the information (either from the classroom geometry curriculum or as a result of Logo) they were not able to apply it.

Of the 32 children who were only able to work in the visual mode, I chose to cite from two interview segments which reflected the children's reliance upon the visual, step by step moves, and upon notebook entries. One segment features part of Rina's response to Question 9 above. The second segment relates part of Oliver's answer to Question 9. Oliver had not been a member of the Grade 5 observation class. The researcher did observe his work both in the Grade 3 initial study, and during the Grade 4 pilot project (Zack, 1984). Oliver seemed to be a high achiever It came as a surprise therefore, to hear him comment that in his Logo work he (like Rina) used the strategy of moving a bit at a time, recording his inputs and then calculating the sum.

The first interview, Selection #Appendix Z-1 below, was with Rina (M.). First, Rina drew her "Shoe" freehand on the blackboard; then she was asked an open-ended question, namely, how would she know what to turn when going from  to make the top line of the "Shoe." Rina talked of how she would 'fiddle around' and keep moving a bit at a time until she managed to get turtle to its destination. The children would only be expected to use the visual mode at this point in the questioning. It was of interest to note that Rina began with an estimate of RT 45. She then referred to her frequent use of the HELP routine, which was used often by the children who had not yet developed a feel for the size of the rotational turn as a tool whereby they could check turtle's heading.

Rina was then asked to estimate what the angle  was. She was seen to have a bit of difficulty, and the difficulty may have been due to the orientation of the angle  . When Rina first attempted to turn the angle upright, she drew  . Upon her second attempt, the angle she drew  was closer to the size of . The researcher then asked Rina what the angle would be, in degrees, and Rina said 175°. Rina stated that she knew that  was a 180° angle, a straight angle, and that therefore  , then changed to  , was 175°.

But when asked what she would turn for β, if she knew that α were 175°, Rina did not apply her knowledge about the 180°, but reverted to the step by step means of "fiddling around."



Selection Appendix Z-1

Interview with Rina M., May 29, 1987 (Question #9: Analytic question)

Rina M.:	(Draws "Shoe" freehand on the blackboard)
I:	Oh, now let's say turtle . . . Let's say turtle is here, you are finished and it's right here
Rina M.:	Mmmm.
I:	Okay (pause). How do you know what to turn?
Rina M.:	Well, I just think it's a matter of figuring, like, let's say you don't want to go over, you'd just go something like right forty-five and I, and I pressed that (? inaudible). I use HELP a lot because it'd tell me like, where it is and if it's at the place where I want it to be, then I'll just go forward wherever.
I:	Oh, okay. Okay. So that's how you would work it out?
Rina M.:	It's, it's, it's fiddling around, just playing around with it, just where-, when I get the right thing. I'll just try to get it right the first time. But if I don't, I just, like, I just, (?inaudible) playing around with it 'til I get it where I want.
I:	Okay. Let me just ask you here, let's say turtle is facing there . . .
Rina M.:	Okay.
I:	Let's say you knew what this [angle] was. Okay. How much do you think









that this would be?

Rina M.: Like this angle?

I: If you measured it as an angle, yeah.

Rina M.: Oh.

I: How would you figure it out in the first place?

Rina M.: Well, okay (takes the chalk). I'll just make (starts to draw an acute angle).



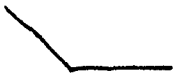
I: Just estimate. You can use your protractor, but just estimate.

Rina M.: (Finishes drawing the angle.) There, that's about the same.



I: Is it the same?

Rina M.: Oh, no, no, no. It's about that (draws).



I: Can you do it upside down? I think it would be easier for you if you just looked at it upside down.

Rina M.: Okay.

I: Can you picture how it would be, upside down?
[The researcher should have said: Can you figure out how much the angle would be while looking at it upside down? Both Rina responds to the intended question, despite the phrasing of the actual question.]

Rina M.: (Cocking her head and looking at the blackboard) Yeah, I guess so.

I: Okay. What do you think it would be, in degrees?

Rina M.: Em, I'd say it would be like, ah, like a (writes on the blackboard) one seventy five degree angle.

175°

I: Okay. And how would you know that's a hundred and seventy five degrees?

Rina M.: Because I know that a one hundred and eighty degree angle is that, is straight (draws on the blackboard ———)

I: Mmmm

Rina M.: and this is about one seventy five degrees (first draws ⌐, then draws ¬).



I: Okay, so would you know what to turn turtle (holding the cardboard turtle cursor at the initial heading, and moving it to the final heading), if you knew that this was a hundred and seventy five degrees (indicates angle ⌐ part of the "Shoe")?



Rina M.: Em, well, if it was facing there, I'd just guess I'd turn it like, like, well I'd just keep (pause) fiddling around you know, I'd just keep fiddling around.

I: You'd just fiddle around until it gets to where you want it to be?

Rina M.: Yeah.

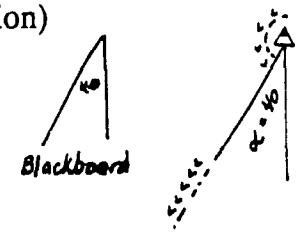
The interview segment with Oliver which dealt with the analytic question was, as with Rina, a long one. Only an excerpt will be presented in Selection #Appendix Z-2. The segment begins just after Oliver has said that 110 would be the supplement to 40; the answer should be 140, which he gives subsequently.

Selection #Appendix Z-2

Interview with Oliver, June 2, 1987 (Question #9, Analytic question)

I: Okay, and would that be enough?

Oliver: Could be a hundred.



I:	If this is forty?
Oliver:	Could be, a hundred and fifteen, er, twenty, maybe a hundred and, forty (he turns at the blackboard to face the researcher, and smiles).
I:	Why a hundred and forty?
Oliver:	Because, 'cause I, I know it's a hundred and eighty to, all the way and then I take away the forty, so it would be a hundred and forty.
I:	Okay. Did you ever use this in your Logo, working your angles?
Oliver:	No.
I:	No? What, what did you think that you would do mainly in Logo? (Pause).
Oliver:	Er, if I wanted to make, like, a design like this
I:	Yeah-
Oliver:	Oh, I just, I just turn until I get what I want
I:	Okay-
Oliver:	And then I'd add it all up when I write it down in my book.

And so, Oliver, with prompting, did figure out that the supplement to 40 was 140, but he insisted that he would not use that strategy when working with Logo. In Logo, "I just . . . turn it until I get what I want . . . and then I'd add it all up when I write it down in my book." In Logo, in the draw mode, the children have no need for analytic strategies.

APPENDIX AA

The Eisenberg and Garvey (1981) and Genishi and Di Paolo (1982)
Levels and Categories of Verbal Strategies

LEVEL I		LEVEL II	
Eisenberg & Garvey	Genishi & Di Paolo	Eisenberg & Garvey	Genishi & Di Paolo
Insistence			Acceptance
	Challenge	Request for explanation	
	Denial / Resistance		
	Insult	Reason	Supporting argument
	Refusal		
	Threat	Conditional directive	
	Ignores	Counter proposal	
	Physical force	Compromise	
Mitigation / Aggravation			
Counter	Contradiction		
	Appeal to authority		

APPENDIX BB

THE VAN HIELE LEVELS OF UNDERSTANDING VIS-A-VIS GEOMETRY

It has proven extremely difficult to obtain a copy of the Van Hiele's work in English, although one is available: Fuys, Geddes, and Tischler, (Eds.) (1984). English translations of selected writings of Dina van Hiele-Geldof and Pierre M. van Hiele. Brooklyn College School of Education.

The following outline of the Van Hiele categories is taken from the work of Wirszup (1976), the chief proponent of their work who made the research of the Van Hiele's known in North America; brief examples from a short article by Hoffer (1981) are also included. The three levels presented here are applicable to the elementary school age children of this study.

Basic Level: This initial level is characterized by the perception of geometric figures in their totality as entities. . . . The children who reason at this level distinguish figures by their shape as a whole. They recognize, for example, a rectangle, a square, and other figures. . . The child can [recognize] the figures by their shapes alone, but he does not recognize the square as a rhombus, or the rhombus as a parallelogram. To him, these figures are still completely distinct.
(Hoffer: For example, . . . a student will recognize a picture of a rectangle but likely will not be aware of many properties of rectangles.)

Level 1: The pupil who has reached level one begins to discern the components of the figures. . . . The properties of the figures are established experimentally; they are described, but not yet formally defined. These properties which the student has established serve as a means of recognizing figures. . . . That a figure is a rectangle means that it has four right angles, that the diagonals are equal, and that the opposite sides are equal. However, these properties are still not connected with one another. For example, the pupil notices that in both the rectangle and the parallelogram of general type the opposite sides are equal to

one another, but he does not yet conclude that a rectangle is a parallelogram.

(Hoffer: A student may realize that the opposite sides and possibly even the diagonals of a rectangle are congruent but will not notice how rectangles relate to squares or right triangles.)

Level 2: Students who have reached this level . . . establish relations among the properties of a figure and among the figures themselves . . . [There] occurs a logical ordering of the properties of a figure and of classes of figures. The pupil is able to discern the possibility of one property following from another, and the role of definition is clarified. . . However at this level the student still does not grasp the meaning of deduction as a whole. . . At the second level, the square is already viewed as a rectangle and as a parallelogram.

(Wirszup, 1976, pp. 77 &78)
(Hoffer, 1981, pp.13 & 14)

THE ALLAIRE LEVELS OF UNDERSTANDING VIS-A-VIS LOGO GEOMETRY

Richard Allaire's four levels are based on the levels of understanding defined by Herscovics and Bergeron (1983). The notes below are based on the talk entitled Le concept d'angle dans un environnement Logo, given by Allaire at the NCTM Conference in Montreal, October 1985, and are unfortunately incomplete and far less comprehensive than a written report would be. The levels are (1) niveau intuitif; (2) niveau mathématization initiale; (3) niveau abstraction; (4) niveau formalization.

LEVEL ONE: INTUITIF

At Level One, the child is cautious, uses small increments, is uncertain about left or right, hesitates, uses gestures as an aid. RIGHT 90 (if indeed RT 90 is the chosen goal) would be made up of smaller increments.

LEVEL TWO: MATHEMATIZATION INITIALE

At Level Two, the child is aware of the rules for creating a specific angle directly, and is aware of the left and right orientations. Use of the inverse becomes almost natural. When necessary, the child adjusts combining two steps in one, as for example if instead of LT 45 the child turned RT 45, he uses LT 90, i.e. by using LT 90, he adjusts by combining the inverse with the next step. The child uses the REPEAT command, and can relate the number value outside the square brackets [ ] and the number of times the move inside the brackets will be done. The child is comfortable with written records of her work, and has the book at her side. At Level Two, the child works much with pencil and paper, in the abstract, and has an image in her head. The child can estimate well, can anticipate the movement (for example, that turtle will turn right), has a good feel for the 45, 60, and 90 degree turns, and for combinations such as 90 + 45 = 135. (Allaire calls this an example of "mathématization primière.") The child knows that the input for a complete turn is 360 degrees.

LEVEL THREE: ABSTRACTION

The child can anticipate far ahead of the action, for example the four or five movements ahead. She can create procedures, and combine procedures (while understanding their nature). The child's use of variable starts to be precise at this stage.

LEVEL FOUR: FORMALIZATION

The child can formulate rules, for example rules governing complementary angles.

APPENDIX CC

ADDENDUM TO THE CHILD'S CONCEPT OF ANGLE SECTION, IN THE REVIEW OF THE LITERATURE CHAPTER

The following section deals with aspects concerning the child's concept of angle which should have been part of the Review of the Literature chapter in the thesis. Dr. J. Dale Burnett, who served as external examiner for the thesis, suggested that "a review of students' mathematical knowledge, particularly angles" (Burnett, personal communication, April 27) ought to have been included. I am indebted to him for that comment, because it made me aware that the aspect which I had described in detail in another paper, entitled "A clinical-observational study of ten to eleven year-old students' notion of right angle in Logo geometry" (Zack, 1986b), was not adequately dealt with in this review, as I should have been. In this appendix, I will confine myself to the consideration of the aspect of mathematical knowledge related to Logo angle, and will deal briefly with (1) a definition of angle, (2) some of the findings from Logo studies concerning the child's concept of angle in Logo, and (3) the criteria used to evaluate the child's knowledge about Logo angle.

As Close (1982) has pointed out, there are two categories into which most definitions of angle fall, one *dynamic* and the other *static*. The *dynamic* category has to do with the amount of turn: "an angle is the amount of rotation necessary to bring one of its rays to the other ray without moving out of the plane" (Kieran, 1986, p. 99). The *static* category depicts angle as part of a plane: "an angle is a part of the plane included between two rays meeting in a point" (Kieran, 1986, p. 99). Logo geometry reflects the dynamic dimension; as Kieren (1984) has said: [An] angle is looked at from the amount of turn used in drawing it, rather than as a product, or a pair of segments separated" (1984, p. 16). Freudenthal (1973) has suggested that in the teaching of school geometry, both the static and the dynamic aspects ought to be presented to children. Consequently, the question that one might pose in investigating the children's perception of angle is whether there is an integration on the child's part between the (basically) static textbook Euclidean definition of angle on the one hand, and the dynamic Logo representation on the other.

Of import to this investigation are the findings related to the children's notion of angle in the early stages of their work with Logo. Hoyles, Sutherland and Evans (1985) reported that some children used input numbers for RIGHT and LEFT based upon a selection of conveniently located keys, such as 77, 99, and so on, or based upon multiples of 30 and 50; hence the extent to which the children understood what the input numbers represented was in question. To an adult using Logo, it is evident that the number keyed in, for example RT (RIGHT) 65, would result in a specific amount of turn on the part of the turtle cursor, namely a turn of 65 degrees to the right. However, findings from two studies have suggested that this was a connection not easily made by the children. One finding which was reported concurrently by Zack (1986d; see Appendix V, p. 469) and Kieran (1986, p. 103) in independent studies was that there was a lack of connection on the part of many of the ten to twelve year-old elementary school children in their studies between the input number and the resulting amount of rotation. Another finding which emerged from the analysis done for the aforementioned Zack project (1986b; Zack, 1986d) was that there was often no awareness of the fact that the input number represented "degrees" of turn.

The criteria used for the analysis of the children's notion of angle in the Zack (1986b) study were the criteria underlying the investigation in this study as well. There was a concentrated focus in that study upon the aspect of *right angle*, or 90 degree angle. So, for example, a child would be considered to have a relational (Skemp, 1979) understanding of 90 degrees in Logo geometry if the child (1) demonstrated the correct use of 90 degrees in situations where RIGHT/LEFT 90 are required, as for example in a design which required a square or rectangle, (2) demonstrated that 90 degrees creates a corner that has both an inside angle of 90^0 and an exterior rotation of up to 270^0, (3) recognized a right angle/a 90 degree turn in its various orientations and transformations, and (4) demonstrated the relationship of 90 to multiples or fractional parts of 90, and to the general set of angles. The criteria were adapted to apply to angles of other sizes as well. It was deemed necessary for this study to investigate whether the children were applying mathematical knowledge which they had learned in the classroom, or in the Logo environment, to solve problems in their Logo work (cf. Kieran, Hillel, & Erlwanger's (1986) "analytic schema").

GLOSSARY

The author's primary purpose in appending the glossary to the paper was to give a brief explanation of some of the terms used in the body of the paper to those readers who are not acquainted with the Logo graphics microcomputer environment. In certain instances, comments concerning the study findings have been included. The writer has acknowledged the source of the definition in all cases, and would refer the reader who needs more background back to some of the cited sources, such as for example, Solomon (1976), Hillel (1984), and Barfurth (1987).

Angle: The *dynamic* and *static* definitions of angle have been presented in Appendix CC, p. 505.

Clinical interview: The most useful short definition of the clinical interview and the clinical method was found in a work by Genishi (1962). Genishi stated that: "Piaget's clinical method (Piaget (1965)) is a special form of interviewing intended to reveal the nature of children's thought. Interviewers employing this method are also observers as they watch what children do" (1962, p. 578).

The clinical interview has been used by researchers in the area of mathematics in order to examine children's thinking, and to diagnose children's difficulties. Herscovics and Bergeron (1980) have defined the "clinical interview" as follows:

> This technique, developed by Piaget, has been specifically conceived to bring to light some evidence regarding the intellectual processes being used. It takes the form of a dialogue in which the interviewer questions his subject on his actions, his hypotheses, his predictions . . . Should intersubject comparisons be desired, standardized or at least semi-standardized versions of the interview are needed: same material, same manipulations, same questions. However, some flexibility must remain allowing the researcher to explore new and interesting avenues or to clarify answers considered ambiguous. The clinical interview lends itself particulary well to the analyses of errors and the evaluation of mental blocks in the construction of a concept or the development of a mathematical skill. It is well known that most errors made by students are not due to their lack of

> attention but to false rules which they have invented and which seem perfectly logical to them (Erlwanger, 1975) . . .
> (1980, pp. 384-393)

Displacement: Hillel (1984) has used the term displacement in reference to the distance moves of the turtle; Hillel's term has been used in this study as well. An alternate term would have been "measure of length," or "amount" of a move forward or back, but the children were often moving the turtle forward or back without being aware of any associated measure, and hence the term "measure" may have been misleading; this was perhaps especially true when there were no lines drawn on the screen and yet turtle moved, as for example when the turtle's pen was up. Hillel has commented on the effect upon the turtle of the commands which direct both the displacement and the rotation movements, in the following way:

> FD (BK) and RT (LT) could be viewed as acting either on the 'turtle' or the 'turtle state'. The first point of view looks at FD as a displacement in a fixed direction (with or without associated line segment, depending on the 'pen' position) and at RT as a rotation about a fixed point. The 'turtle state' point of view is of FD as a change of position and RT as a change of orientation (heading). We are not claiming here that children are conscious of these distinctions; they likely use the two points of view interchangeably, as when using FD to "draw a line" or to "move the turtle from here to there."
> (Hillel, 1984, p. 22)

(See glossary section on turtle *state, position* and *heading*)

Dribble file: "The dribble file records <u>each</u> Logo command that is typed into the computer. This is saved on a designated file on a diskette which can later be used to produce a 'hard copy' (i. e. a printed copy) of the entire section (Barfurth, 1987)." The *dribble file* capacity was not used in this study, since the researcher was at first not aware of it, and once introduced to the dribble file (in the 1987-88 year), was not sufficiently proficient in its use to employ it. Had the dribble file program which contains a graphics capacity been used, it would have served to report the various stages of the children's screen drawings, as for example when the writer wished to show

the graphics which corresponded with the excerpt selections presented in the paper. The dribble file was used as a data collection tool in a number of the Logo and mathematics research studies cited in Appendix A.

EDIT MODE: Logo offers the user two ways in which to execute commands; one is the IMMEDIATE or DRAW or DIRECT MODE; the other is the EDIT or INDIRECT MODE. When the child works in the EDITOR, the sequence of commands is typed, but the execution of the (graphics) commands is delayed until the child leaves the editor. In this study, the children worked most often in the IMMEDIATE MODE; they used the EDIT MODE for defining their *procedures* or strings of commands, because the definition of the lists of commands in the editor was the only means whereby the procedures could subsequently be saved on disk. In this study, none of the children were seen to use the procedures as Papert had envisioned they would, namely as building blocks or modules to be used when making new designs and projects. (See *IMMEDIATE MODE*)

Ethnomathematical: The following (two) definitions of ethnomathematical language are related; however, the first one is more restricted, and the second more general and global. Within the Review of the Literature portion of the paper, the writer relayed Kieren's (1987) definition: "Ethnomathematical language is not formal mathematical language; children use their own words and their own terms." Abraham and Bibby (1988) made mention of D'Ambrosio's (1985) coining of the term *ethnomathematics*, used to refer to "mathematics which is practised among identifiable cultural groups, such as national-tribal societies, labor groups and so on" (1988, p. 3). The broader dimension which considers the cultural context, in particular Third World countries and indigenous native groups, is reflected in the latter definition.

HELP Routine: This addition to the school's LCSI Applelogo software STARTUP AIDS was developed in 1983-84 by two Grade 6 students (Jon and Kevin) with the assistance of one of the Logo teachers, Sharon. The children used this aid in order to determine where the triangular turtle cursor was pointing. With its pen up, the turtle cursor moved forward (FD 20), paused, and then moved back (BK 20) to its starting

point without leaving a trace, thereby displaying its heading, that is, displaying where turtle was pointing.

IMMEDIATE MODE (also **DRAW MODE, DIRECT MODE**): In the IMMEDIATE, or DRAW, MODE (also called in some studies direct-drive mode, or step-by-step mode), the user executes one command at a time. Once the command is keyed in (and the RETURN key pressed), it produces a corresponding action upon the screen. The findings of the Hillel (1984) study were similar to the findings of this study, namely that there was extensive use of the draw mode by the children (Hillel, 1984, p. 13). Hillel noted that this was not surprising,

> since for most purposes, it is the easiest and most efficient mode to use. Its instant feedback feature allows the child to plan, to execute and to 'debug' virtually simultaneously. We argue further that the tendency to use direct-mode for planning ('planning-in-action') is resonant with the 'local' nature of turtle geometry since the turtle deals with space 'a little piece at a time'. Thus executing commands in direct-mode encompasses a range of activities including exploration, planning, verification of programs, [and] adjustment of numerical inputs to commands.
> (Hillel, 1984, p. 13).

Intrinsic geometry: Kynigos (1987a) has defined turtle intrinsic geometry as follows: "Turtle geometry, besides being defined as intrinsic, has a special characteristic; it invites children to identify with the turtle [Solomon (1976) used the term *anthropomorphic*] and thus form a body syntonic thinking 'schema', to drive [turtle] on the screen to make figures and shapes" (1987a, p. 86). The terms *syntonicity*, both ego syntonicity and body syntonicity (see NOTE 14), are related to Papert's (1980) theory that when children "do" turtle geometry, "they use the personal experience of [body] motion to think about the shapes and figures they want to make" (Kynigos, 1987a, p. 86). It is of interest to note that Hillel's (1984) findings indicated that the children identify less with the turtle in terms of body movement than with the turtle as a drawing instrument (1984, p. 27); the findings of this study support those of the Hillel study.

Solomon (1976, section 5.12) and Kynigos (1987a) have both defined the three geometric worlds: turtle geometry, Euclidean geometry, and coordinate geometry. In differentiating the three geometries, Kynigos stated:

> The nature of the geometrical notions underlying turtle geometry is characterized by Papert as intrinsic, i. e. turtle geometry belongs to the family of *differential* (italics added) geometrical systems where growth is described by what happens at the growing tip. This geometry is contrasted to the "logical" [E]uclidean geometry of theorems and proofs and to the "analytical" [C]artesian geometry where changes of state are caused by location descriptions.
> (Kynigos, 1987a, pp. 86-87)

Microworld: Kynigos (1987b, p. 45) cited from work by Hoyles and Noss in which they defined microworlds as follows: "[Microworlds are] Logo-based situations constructed so that the pupil will come up against embedded mathematical ideas in the context of meaningful activity (Hoyles & Noss, in press)." Edwards's (1988, p. 263) definition is slightly more abstract: "A microworld is an instantiation of the central objects and relations of a domain into a concrete form which is accessible to new learners."

Papert's Logo geometry microworld is fuller and more general than those constructed by Kynigos (1986; 1987a; 1987b) or Edwards (1988) or Hoyles and Noss (1987, 1988), in that Papert was envisioning that the child, in working with the Logo "gears," would discover a whole gamut of powerful geometry ideas. For Papert, learning the subject matter of turtle geometry included "concepts involving measurement and estimation of angles and distances, the relations among angles and distances necessary to produce certain well defined shapes such as square, triangle, polygon, star or circle, such general geometric concepts as similarity, scaling and symmetry, etc." (Papert, et al., 1978, p. 11). Note well the "etc." Kynigos used Papert's Logo microworld, but restricted the environment in such a way as to promote exploration of Cartesian geometry (1987a), and Euclidean geometry, and to invoke exploration of the connections between the various geometries (intrinsic or turtle geometry, Cartesian geometry and Euclidean geometry) (Kynigos, 1986, 1987b, 1988). Edwards (1988) constructed a *transformation microworld* using the Logo programming language (although in the Edwards case, another programming language could just as well have

been used in the creation of the software). Thus, whereas Papert's microworld contains the elements for investigating Cartesian geometry, for example, or parallelograms, or even transformations (see Ludwig, 1986), the Kynigos (1987a) *Cartesian microworld*, the Hoyles and Noss (1987, 1988) *parallelogram microworld*, and the Edwards (1988) *tranformation microworld*, respectively, provoke investigation of and concentration upon just those elements.

Primitives: We reiterate here again that LCSI AppleLogo was used throughout the project, on Apple IIe computers. The *primitives* are those commands which are part of the Logo software; they are "given to the children as . . . part of the Logo vocabulary" (Hillel, 1984, p. 4). A Logo procedure created by the teacher or researcher which becomes part of the software would also be considered a primitive (Hillel, 1984, p. 4). On the other hand "any Logo command created by the user we call a *procedure*" (Hillel, 1984, p. 4). The *primitives* which were part of the Logo software used in this study were as follows: ForwarD (FD), BacK (BK), RighT (RT), LefT (LT). PenUp (PU), PenDown (PD), PenErase (PE), ClearScreen (CS), HideTurtle (HT), ShowTurtle (ST), CIRCLER, and CIRCLEL (make a circle to the right, and a circle to the left, respectiveley, given an input number which represents the radius), HOME and REPEAT. In this study, there was one sole case of a Logo procedure which was added by a Logo teacher and two students (Sharon, and Jon and Kevin, 1983-1984) and which subsequently became part of the software startup aids and consequently served the function of a primitive; this was the HELP routine.

The aforementioned primitives were all related to the turtle geometry aspect of the Logo environment. There were other Logo primitive commands related to the operation of the system, such as SAVE, LOAD, CATALOG, EDIT, as well as various CONTROL keys for the purpose of editing.

Procedure: The explanation of *procedure* which follows will relate to the programming aspect. A *procedure* is defined in the EDIT MODE. It consists of a name (i. e. title), and a list of commands. Once defined, a procedure will always do the same thing. A procedure is defined in the EDIT MODE, but can only be run in the IMMEDIATE

MODE. "Once defined, a procedure is used in the same way as a [command or a *primitive*]" (Barfurth, 1987, p. 20).

The glossary definition does not deal with the children's understanding concerning the ways in which procedures work. Whereas the Hillel (1984) study analyzed the children's behavior from the perspective of (1) what use was made of procedures, and (2) what understanding there was of the relation between a procedure and the object which it defines, this study was only able to touch briefly upon the aspect of *procedures*, namely the children's difficulties in using and running and 'debugging' procedures. (See also glossary sections on *EDIT MODE*, and *Primitives*).

Screen fit error: This type of error occurs "when a child's plan does not take into account the limitation of the screen size" (Hillel, 1984, p. 28).

SETH: The command SETH (which stands for setheading), plus an input, automatically sets the turtle cursor at a specific heading. Thus, if the student keys in SETH 90 when the cursor is in an initial heading position at HOME, that is pointing north, the cursor then turns to face east (see below, section on *State of the turtle*).

State of the turtle: The source of the following definition which relates to the *state*, the *position* and the *heading* of the turtle cursor is Solomon (1976, definition 5.2).

> The turtle's state at any time consists of a POSITION and a HEADING. The position is a place in the field [or video screen], the heading is a number between 0 and 360 representing the direction: 0 or 360 means NORTH or straight up; 90 means EAST; 180 means SOUTH or down; . . . [270 means WEST]; etc. In Logo the state is a list of three numbers: XCOR [X coordinate], YCOR [Y coordinate], and HEADING. But for pure turtle geometry we need not worry about how the position is represented."
> (Solomon, 1976, 5.2)

In the body of the paper, when there was a need to trace the *initial* and the *final* state of the turtle, the following symbols were used: △ for the *initial state*, and ▲ for the *final state* (cf. Kieran, Hillel, & Erlwanger, 1986).

Syntax: The way in which the term *syntax* was used for the purpose of this study seems to be unique, and therefore warrants a brief explanation. In this paper, the term has been used in reference to getting the so-called "mechanical" aspects of the microcomputer working, aspects such as "loading," "saving," doing revisions in the editor, and so on. These workings require an exact "syntax," or statement, without which the desired step cannot be carried out. It was noted by the researcher that Brown (1973) had used the term *syntax*, but upon consulting his paper, the researcher learned that although Brown referred to children learning the proper use of the various commands, of spacing properly and of learning how to edit, Browm did not use the term *syntax* to refer to these activities, as has been the case in this paper. Brown used the term *syntax* in an exclusive way, namely to refer to the specific syntax of Logo LIST programming, for example, IFTRUE PRINT "HI," IFFALSE PRINT "BYE" (1973, P. 14).

Syntonicity, also **ego syntonic** and **body syntonic**:
See NOTE 14 in the paper.

Total Turtle Trip Theorem: See NOTE 22 in the paper.