Structural Engineering

SELECTED PAPERS IN STABILITY OF STRUCTURES

Edited by Ghyslaine McClure

Structural Engineering Series Report No.92-2

July 1992

Department of Civil Engineering
and Applied Mechanics

McGill University
Montreal

SELECTED PAPERS IN STABILITY OF STRUCTURES

Edited by
Ghyslaine McClure
Assistant Professor

July 1992

Structural Engineering Series Report No. 92-2
Department of Civil Engineering and Applied Mechanics
McGill University, Montreal, Quebec, Canada

PREFACE

The following three technical papers were selected as the best among thirteen submissions in course 303-605B *Stability of Structures* during the 1992 Winter Term.

I would like to thank the authors for their contribution.

Ghyslaine McClure
Assistant Professor
15 July 1992

CONTENTS

I

The Geometric Matrix for Stress Stiffening and Buckling

by

Huei-twan Chuo

ABSTRACT

The objective of this study is to discuss the applications of the geometric matrix and the accuracy in computing the critical load. Results for several end conditions of column, beam, and frame elements, obtained with the consistent and inconsistent geometric matrices, are presented and discussed.

I. Introduction

Buckling of members or structures may occur as a structural response to membrane forces which act along the axes of members and tangent to plate and shell midsurfaces. When buckling occurs, the membrane strain energy of the member or structure should be converted into the strain energy of bending with no change in externally applied load.

As we consider a slender column, the axial stiffness is much greater than the bending stiffness. Similarly, in a shell structure, the membrane stiffness is orders of magnitude greater than the bending stiffness. Therefore, comparatively large lateral deflections and cross-section rotations are needed to absorb the energy in bending deformation, but only small membrane deformations can store a large amount of this energy.

Thus buckling occurs when the compressive membrane forces are large enough to reduce the bending stiffness to zero. If the membrane forces are tensile rather than compressive, the bending stiffness will be effectively increased. This effect is called stress stiffening. In this study, we are interested in the buckling behavior, since the stress stiffening behavior is only the change of the sign in applying the geometric matrix in a linearized analysis.

We will use the matrix method to study the buckling problem of a member or a structure. The matrix method is a numerical technique that uses matrix algebra to analyze structural systems. The details of the formation of element stiffness matrix and geometric matrix will not be illustrated in this paper, since they can be found in several related texts. We will directly use these matrices to see the accuracy in calculating the critical load of a member or a structure. Several end conditions and different geometric matrices will be used to compare with the exact theoretical solutions and to see the computational efficiency.

2. Geometric matrix

In this section, we will illustrate the relations between buckling behavior and the geometric matrix. As we discussed in the first section, the effects of the **membrane forces can** be accounted for by a matrix $[\mathbf{k_g}]$ that augments the conventional element stiffness matrix $[\mathbf{k}]$. Matrix $[\mathbf{k_g}]$ has several different names, as follow: initial stress stiffness matrix, differential stiffness matrix, geometric stiffness matrix, and stability coefficient matrix. In what follows we give $[\mathbf{k_g}]$ the name geometric stiffness matrix.

We know that the matrix $[\mathbf{k_g}]$ is defined by an element's geometry, displacement field, and state of stress. It is independent of the elastic properties. The member's or structure's geometric matrix, global geometric matrix, $[\mathbf{K_g}]$ may be built by summing overlapping terms of the element matrices $[\mathbf{k_g}]$ in the same way as the conventional stiffness matrix $[\mathbf{K}]$ is built.

Considering a beam column element, if we choose a deflection function, displacement field, as

[1] $\quad \mathbf{y = A+Bx+Cx^2+Dx^3}$

This cubic lateral-displacement field satisfies the conditions of constant shear and linearly varying bending moment that exist in the beam column element. Although this displacement field is not exact when a beam column carries axial load as well as loads that produce bending, accuracy can be obtained by dividing a given beam column into two or more elements. By using the energy approach, we obtain the following equations: (Two-dimentional element and its d.o.f. are shown in Figure1)

[2] $$[Q] = \{ [K] + P\,[K_g] \}\,[D]$$

[3] $$[k] = EI/L^3 \begin{bmatrix} 12 & -6L & -12 & -6L \\ -6L & 4L^2 & 6L & 2L^2 \\ -12 & 6L & 12 & 6L \\ -6L & 2L^2 & 6L & 4L^2 \end{bmatrix} \qquad \text{for } \{d\} = [w_1\ q_1\ w_2\ q_2]$$

[4] $$[k_g] = P/30L \begin{bmatrix} 36 & -3L & -36 & -3L \\ -3L & 4L^2 & 3L & -L^2 \\ -36 & 3L & 36 & 3L \\ -3L & -L^2 & 3L & 4L^2 \end{bmatrix} \qquad \text{for } \{d\} = [w_1\ q_1\ w_2\ q_2]$$

in which [Q] contains the transverse loads that cause bending, [D] contains the corresponding bending deformations, and **P** is the axial load that is positive in tension. The stiffness matrix consists of two parts, **[K]**, the conventional stiffness matrix of a member or a structure subject only to flexure, and **[K_g]**, a matrix which accounts for the effect that the axial load **P** has on the stiffness of the flexural member or structure. **[k]** is the element stiffness matrix, and **[k_g]** is the element geometric matrix (we say this **[k_g]** in Eq [4] is consistent, since it is built from the same shape functions used to build the conventional element stiffness matrix [k], for the inconsistent geometric matrix will be discussed later.).

We can rewrite Eq [2] in the form:

$$[5] \qquad [D] = \{ [K] + P[K_g] \}^{-1} [Q]$$

It is obvious that **[D]** increases without bound for finite values of **[Q]** only when the inverse of the stiffness matrix becomes infinite. That is, the critical load can be found by setting the determinant of the stiffness matrix equal to zero. Several end conditions of columns using this consistent geometric matrix will be discussed later.

Let us consider different displacement fields between the conventional stiffness matrix **[k]** and the geometric matrix **[k_g]**. In this case, the geometric matrix is termed " inconsistent". By using this inconsistent matrix we will obtain a less accurate critical load than that by using the consistent matrix, but computational efficiency will be increased. We will recall from the finite element method to see that the inconsistent matrix is reasonable.

We recall the convergence requirements that if a strain energy expression involves displacement derivatives of order m, the displacement field must provide inter-element continuity of displacement derivatives of order m-1 as the mesh is refined. When we use the energy approach to find the geometric matrix, in the energy integrals, the term [k_g] involves first derivatives of displacements, therefore, the continuity of the displacement field is all that is required. Thus we will be able to determine the critical load for a member or structure by using this inconsistent matrix. For a given accuracy, we should divide the member into more elements than when we use the consistent [k_g].

A very simple inconsistent [k_g] is given below that is based on the two-dimentional bar with a linear displacement shape function.

[6] $$[\mathbf{k_g}] = \mathbf{P/L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \textbf{for } \{\mathbf{d}\} = [w_1 \; q_1 \; w_2 \; q_2]$$

3. Examples

Examples with several end conditions of columns by using one, two or more elements with consistent or inconsistent geometric matrix are discussed. We will use the figure of the stability functions, see Figure 2, which may be obtained from several related texts, to obtain the minimum length of the element required for linear analysis. The results for the critical loads obtained from matrix method are compared with the theoretical values in Table1 and Table 2.

It is interesting to observe that the error of the pinned-pinned case with two elements is the same as that of the fixed-free case with one element, since the pinned-pinned case can be assumed as two elements of the fixed-free case with length of L/2. Therefore, the critical load by the matrix method of the pinned-pinned case is exactly four times as that of the fixed-free case.

Another example for a frame problem is adopted from the assignment #7 of this course, shown in Fig. 3.

4. Discussion of results

From the results in the previous section, we see that results using the consistent geometric matrix are very good if more than two elements are used. If we consider the minimum mesh length required in Fig.2 (these values are listed on the bottom of Table 1), it is obvious that using two elements for any end conditions is quite adequate in order to obtain the critical load within 2.5% error.

We know that a member or structure is well modeled when the elements are compatible and when it is not softened by low-integration rules. Then such formulation yields an upper bound to the magnitude of the correct buckling load. This is evident from the results in Table 1 and 2. If **[k]** and **[k_g]** were based on the displacement fields that include the buckled shape, sine or cosine functions, the matrix method would yield the correct buckling load. The cubic polynomial as we used in Eq [1] still yields a very good result if more than two elements are used.

As we see in Table 2, more elements are required to obtain a good result by using the inconsistent geometric matrix. When we use this inconsistent matrix in Eq [6], some mathematical problems will be encountered. **[k_g]** in Eq [6] is semi-positive definite since it contains all zero values in some rows and columns that will yield zero eigenvalues. Some computer programs solving the eigenproblem can not be used directly since they are based on the Jacobi method that must use symmetric and positive definite matrices. Fortunately, **[k_g]**, Eq [6], is a very simple matrix that make the hand computation applicable within four elements. For more than four elements, some numerical method for finding the smallest or largest eigenvalue can be used (Spencer 1980). Otherwise, we can use an alternative procedure for determining the critical load, which consists of assuming an ever-increasing value of P_{cr} and evaluating the determinant at each step. The critical load then corresponds to the value of P_{cr} for which the determinant reduces to zero.

Table 3 is adopted from the assignment no.7 of the lecture. The matrix method with consistent matrix also yields a very good result, even when only two elements are used in each member (only 1.8% error).

5. Remarks on $[K_g]$

The result from the inconsistent $[k_g]$ is not as accurate as that from the consistent $[k_g]$ for simple column problems. If we want to increase computational efficiency with little loss in accuracy, however simpler $[k_g]$ is sometimes recommended. For a complicated structure, if the accuracy and computational efficiency are both important, then the "best" $[k_g]$ is required. This best geometric matrix is probably intermediate to the consistent one and the simplest possible $[k_g]$ (Cook 1989). Numerical evidence suggests that the computed critical loads are increased when $[k_g]$ is simplified.

We have only considered the two-dimentional columns and frames with axial loading in this paper. Indeed, the geometric matrices have been devised for many buckling problems. For example, for plate buckling (Haskell 1970), torsional and torsional-flexural buckling of prismatic members (Barsoum 1970), tapered bars and plates (Tinawi 1972), three-dimensional beams (Argyris 1979), geometrically nonlinear analysis of beams (Yang 1986), and geometrically nonlinear analysis of thin-walled frames (Conic 1990).

Some special structures have no conventional stiffness matrix, for example, a linkage of pin-connected bars with each link idealized as rigid. Straight cables and flat membranes, which have no bending stiffness with which to resist lateral loads, are problems that can be analyzed by $[\mathbf{K}_\sigma]\{D\} = \{Q\}$, where $\{\mathbf{D}\}$ contains d.o.f.s associated with small lateral deflections (Cook 1989).

6. Conclusions

The finite element method is a numerical procedure for analyzing members and structures, and it is a very powerful tool in analyzing the stability problems of continua, especially, for the complicated structures. For the simple column problems in this paper, we find that a large mesh size , say two elements, can yield a very good result, and similarly, for two-dimensional frame problems, Indeed, each member of a frame structure may be considered as one beam-column element with appropriate end conditions. Therefore, using two elements in each member is quite adequate.

References

J. H. Argyris, O. Hilbert, G. A. Malejannakis, and D. W. Scharpf, 1979, On the Geometric Stiffness of a Beam in Space– a Consistent Virtual Work Approach, *Computer Methods in Applied Mech. and Engg.*, Vol.20

R.S.Barsoum and R. H. Gallagher, 1970, Finite Element Analysis of Torsional and Torsional-Flexural Stability Problems, *Int. J. Numer. Methods Engg.*, Vol.2, No.3

A. Chajes, 1974, *Principles of Structural Stability Theory*, Prentice-Hall, Englewood Cliffs, NJ.

A. Conci and M. Gattass, 1990, Natural Approach for Geometric Non-linear Analysis of Thin-walled Frames, *Int. J. Numer. Methods Engg.*, Vol.30 No.2

R. D. Cook, D. S. Malkus, and M. E. Plesha, 1989, *Concepts and Applications of Finite Element Analysis*, John Wiley & Son, New York, 3rd ed.

W. E. Haskell, 1970, *Geometric Non-linear Analysis of Thin Plates by Finite Elements*, Ph.D. Thesis, University of Massachusetts

R. K. Livesley, 1975, *Matrix Methods of Structural Analysis*

J. S. Przemieniecki, 1968, Discrete-Element Method for Stability Analysis, *Aeronautical J.*, Vol.72, No.12

A. J. M. Spencer, 1980, *Engineering Mathematics*, Van Nostrand Reinhold, New York

R. A. Tinawi, 1972, Anisotropic Tapered Elements Using Displacement Models, *Int. J. Num. Meth. Engg.*, Vol.4, No.4

Y. B. Yang, 1986, Stiffness Matrix for Geometric Non-linear Analysis, *J. ASCE*, V.112, No.4

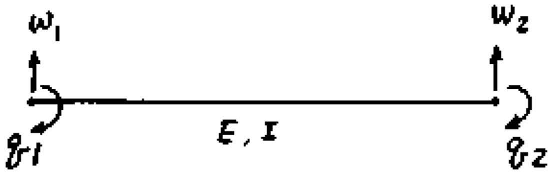


Fig. 1 Two-dimentional element and its d.o.f.s

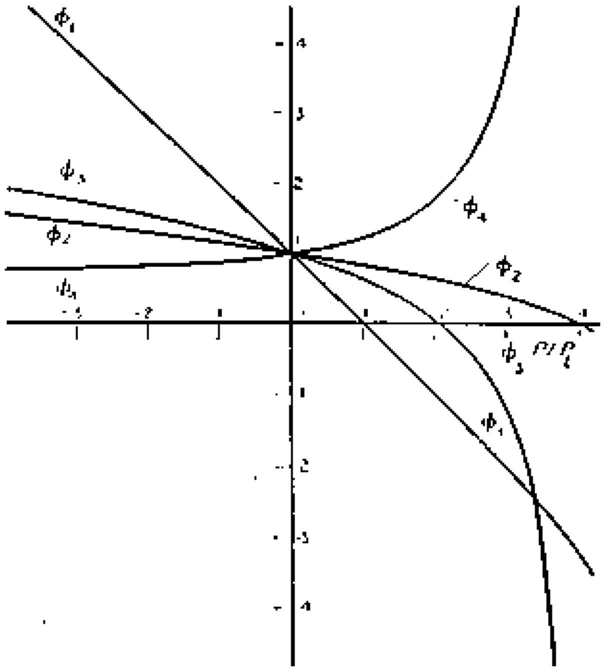


Fig. 2 Stability functions (Livesley, 1975)

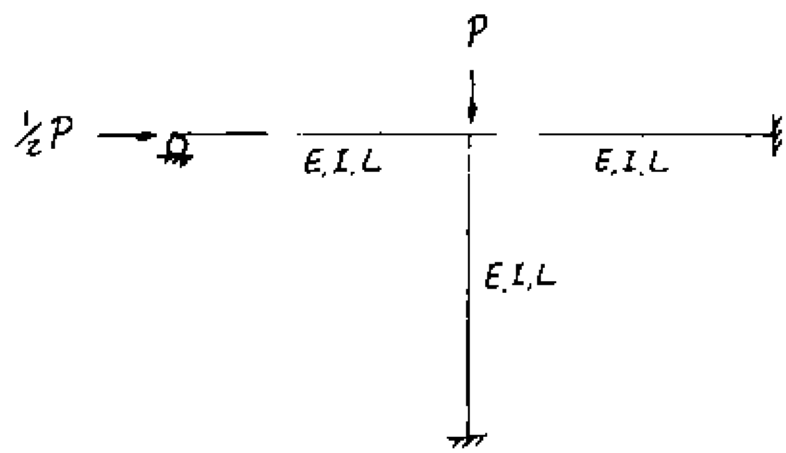


Fig. 3 Stability of a simple two dimentional frame

(Assignment #7 of the lecture)

Table 1 Column stability analysis –
using consistent geometric matrix.

End Condition	pinned-pinned	pinned-fixed	fixed-fixed	fixed-free
Buckled Shape				
Exact P_{cr}	$9.8696\ EI/L^2$	$20.1909\ EI/L^2$	$39.4784\ EI/L^2$	$2.4674\ EI/L^2$
1 element	12.0 (21.6%)*	30.0 (48.5%)	N/A	2.4860 (0.75%)
2 elements	9.9439 (0.75%)	20.7088 (2.57%)	40.0 (1.32%)	2.4687 (0.053%)
4 elements	9.8747 (0.051%)	20.2322 (0.16%)	39.7754 (0.75%)	2.4675 (0.004%)
8 elements	9.8699 (0.003%)	20.1935(0.013%)	39.4986(0.051%)	2.4674 (0.000%)
16 elements	9.8696 (0.000%)	20.1909(0.000%)	39.4797(0.003%)	2.4674 (0.000%)
min. mesh length required	1.0L**	0.699L	0.5L	2.0L

* indicates the value of the error

** min. mesh length is obtained by considering $-1 \le P_{cr} / P_{t} \le 1$ from Fig. 2

Table 2 Column stability analysis –
using inconsistent geometric matrix.

End Condition	pinned-pinned	fixed-free
Exact P_{cr}	9.8696 EI/L^2	2.4674 EI/L^2
1 element	N/A	3.0 (21.59%)
2 elements	12.0 (21.59%)	2.5967 (5.24%)
3 elements	10.8 (9.43%)	2.5243 (2.31%)
4 elements	10.3866 (5.24%)	

Table 3 P_{cr} for the frame problem of Fig.3 –
using consistent geometric matrix.

Exact P_{cr}	23.237 EI/L^2
3 elements	32.066 (38.00%)
6 elements	23.662 (1.83%)
9 elements	23.356 (0.51%)
18 elements	23.257 (0.086%)
24 elements	23.252 (0.065%)

II

Lateral Torsional Buckling of Steel Link Beams

by

Kent A. Harries

ABSTRACT

The development of the theory of lateral-torsional buckling of steel H sections is investigated. Particular attention is paid to the lateral buckling of sections subject to high shear loads, such as those used in link beams in eccentrically braced frames. Current design philosophy and criteria, as specified by CAN/CSA S16.1-M89, will be presented and discussed. The recently suggested, four fold increase in required lateral bracing forces, will also be presented and discussed in the context of link beams in eccentrically braced frames and steel coupling beams in reinforced concrete wall systems.

1. Introduction

Link beams are an integral part of eccentrically braced frame (EBF) structural systems. The link beam is the element designed to dissipate energy under circumstances of extreme lateral load such as in the event of an earthquake. The EBF system is designed so that the link beam will form a plastic hinge and protect the remainder of the structural system, especially the columns, from damage. For short to medium span link beams this hinge will form in shear over the length of the beam, while for longer spans, two flexural hinges will form at the ends of the link beam. This paper will discuss the behaviour and design considerations for short and medium span link beams, often termed 'shear links'. The discussion, however, is largely valid for any length of member. The preferred modes of deformation for two EBF systems with shear links are shown in Fig. 1.

Typically, link beams will not experience high stress levels under service conditions although they will be required to deform inelastically under severe lateral loading. Link beams therefore must be able to exhibit both large ductility levels and energy absorption characteristics

Since the most efficient mode of energy dissipation of a short steel H section is in shear, a link beam must be designed to remain elastic in flexure while attaining its probable capacity, including the effects of strain hardening of the web, in shear. Typically, satisfying this criteria will require a section with a disproportionately heavy flange, such as an S or built-up section. Such a section will be prone to instability of the compression flange at the high stress levels required to yield the web in shear. As the web approaches its ultimate capacity, it will buckle perpendicular to the primary tension field, this will increment the instability in the flange.

Flange instability resulting from the high stress levels and web buckling comes about because the restrained compression flange essentially acts as a column on elastic foundations. At some point the compression forces in the flange, from the compression-tension couple making up the flexural moment applied to the beam, cause the flange to buckle laterally, about its strong axis. The flange is fully supported by the web, which restrains the flange from buckling about its weak axis, until the onset of web

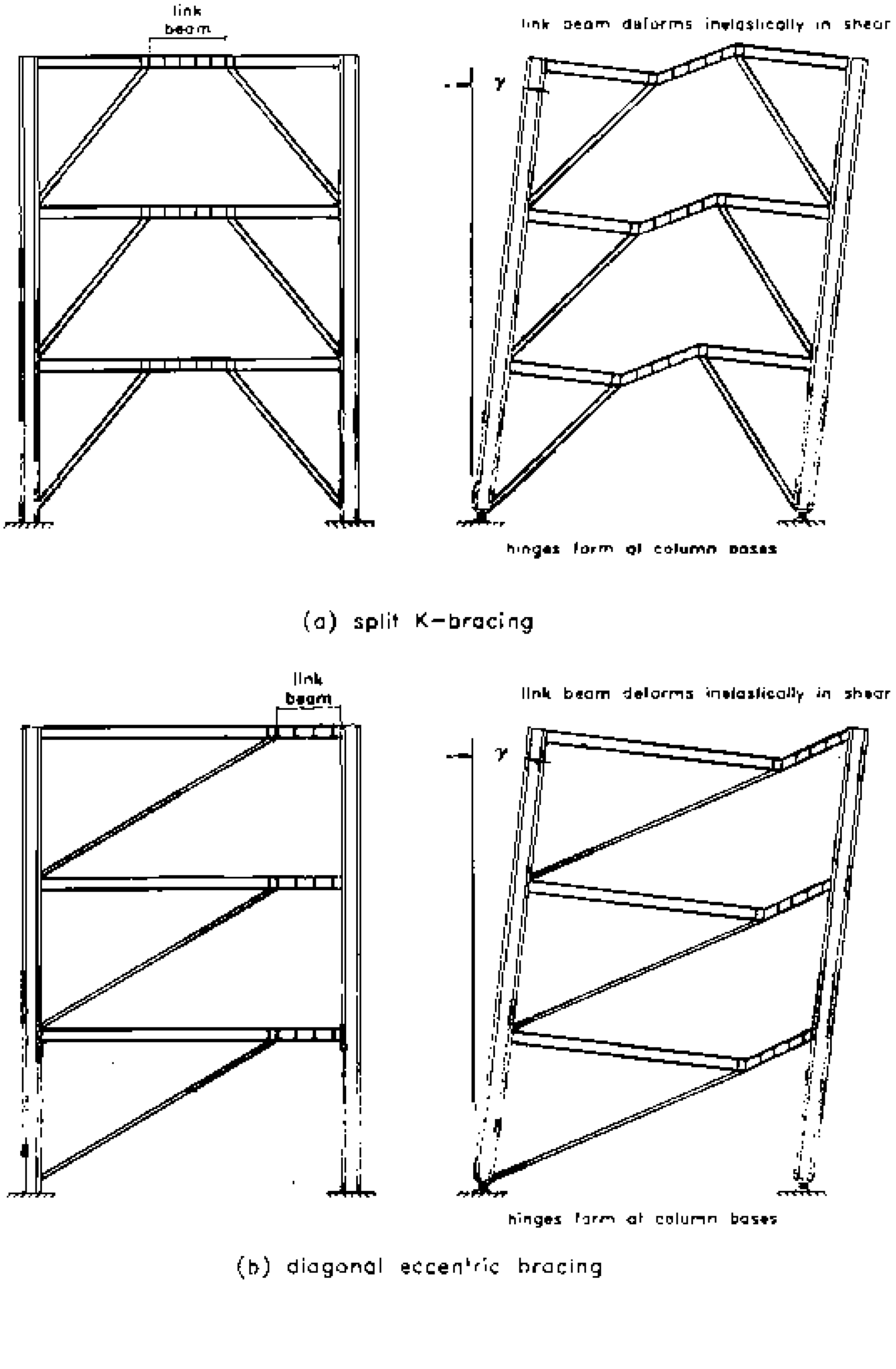


Figure 1 Preferred mode of deformation for EBFs

buckling. Once the web has buckled the flange will experience local instabilities about its weak axis, coinciding with locations of web buckling and high flexural stresses on the section. Experimental results have shown that lateral instability will be the dominant flange instability mechanism for a properly detailed short span member.

2. Lateral Torsional Buckling

Lateral flange instability is properly termed 'lateral torsional buckling'. The torsional mechanism comes about as the compression flange becomes unstable, wanting to buckle laterally, while the tension flange is stable, wanting to remain straight. In this manner a twisting moment is induced.

2.1 Torsional Resistance

When a structural element of non-circular cross section is subjected to a twisting moment, the cross sections of the member may warp out-of-plane. If the member is allowed to warp freely, the applied torque is resisted entirely by St. Venant shearing stresses. This behaviour is called pure or uniform torsion. If the member is restrained from warping freely, the applied torque is resisted by a combination of St Venant shearing stresses and warping resistance. This behaviour results in non-uniform torsion.

For conditions of uniform torsion, the torque, T_u, is related to the angle of twist, ß, by the expression:

$$T_u = GJ\frac{d\beta}{dz}$$

where: G is the shear modulus,

z is the direction along the axis of torsion, and

J is St Venant's torsional constant given by: $\frac{1}{3}\sum b_i t_i^3$ with b and t, the width and height of the components making up the cross section.

For conditions of non-uniform torsion, normal stresses are induced in the longitudinal fibres of the section which cause warping. This warping causes strong axis flexure of the beam flanges. Figure 2

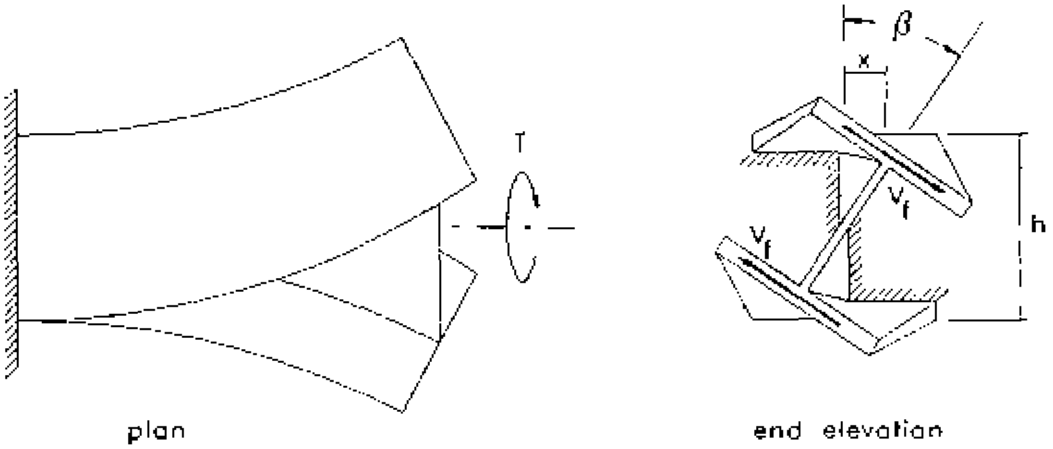


Figure 2 Twisting of an H section restrained against free warping (adapted from Chajes, 1974)

illustrates the twisting of a section restrained against free warping. The warping causes opposite shearing forces in the flanges, V_f, which form a couple over the height, h, of the section. The warping torsion, T_w can therefore be expressed as

$$T_w = V_f h$$

Recognising that V_f can be related to the strong axis bending moment of the flange, M_f, by the expression:

$$V_f = \frac{dM_f}{dz}$$

Letting the lateral displacement of the centreline of the compression flange (see Fig. 2) equal x, the expression for the flange bending becomes

$$M_f = EI_f \frac{d^2x}{dz^2}$$

where I_f is the moment of inertia of the flange about its strong axis.

Substituting $x = \frac{1}{2}\beta h$, the expression for warping torsion becomes:

$$T_w = \frac{EI_f h^2}{2}\frac{d^3\beta}{dz^3}$$

The $\frac{1}{2}I_f h^2$ term is called the warping constant of the section and is given the notation Γ.

The total torsional warping resistance, T, of any section is defined as the sum of the uniform and non-uniform warping torsions:

$$T = GJ\frac{d\beta}{dz} - E\Gamma\frac{d^3\beta}{dz^3}$$

A complete derivation and examples of application of this expression are given in Article 5.2 of Chajes (1974).

2.2 Lateral Torsional Buckling of Beams

Lateral torsional buckling is a combination of twisting and lateral bending brought about by the instability of the compression flange. The first investigations of lateral buckling of H sections were conducted in the early 1900's by Timoshenko. Contemporaries include Winter in the 1940's and Galambos in the 1960's. Lateral buckling calculations can be made using a variety of numerical or analytical means. These calculations, however, are complex and make a number of assumptions that are not entirely consistent with typical *in situ* conditions.

Investigations of lateral torsional buckling have, for the most part, dealt with sections proportioned for normal structural uses where flexure of the member will be the controlling design criteria. Until recently, little investigation has been conducted into the stability response of short span members, i.e. members where shear is the controlling design parameter.

3. Design Code Approaches to Lateral Torsional Buckling of Link Beams

Design philosophies for the resistance to lateral torsional buckling of link beams are derived through a combination of theoretical and empirical studies.

3.1 Research Programmes Involving Shear Links

Research involving link beams in EBFs has been virtually the exclusive domain of the Earthquake Engineering Research Center at the University of California at Berkeley. Through the 1970's and 80's, Egor Popov has supervised numerous research programmes involving link beams in EBFs. The findings of this research is incorporated into most North American steel design codes including the *Uniform Building Code* (UBC), the SEAOC *Recommendations for Lateral Force Requirements* (1988) and CAN/CSA S16.1-M89, *Limit States Design of Steel Structures*, the Canadian steel design code. Although Popov has been very prolific in this area, a few publications merit particular attention as strong summaries of the design philosophies and criteria for link beams, these include Popov's work with Roeder (1977), Malley (1983), Kasai (1986) and Engelhardt (1989). The latter, in particular, summarises the behaviour and design considerations for stability that represent the basis for most new code provisions.

3.2 Requirements for Lateral Stability of Link Beam Flanges in CAN/CSA S16.1-M89

In order for the link beam in an EBF to behave in the desired ductile manner, it must be properly designed and detailed. A properly designed and detailed link beam will not only exhibit excellent energy absorption characteristics, but will also resist lateral instability.

Lateral torsional buckling is inevitable in any beam tested to failure. The aim of the design philosophy is to ensure that the beam attains its maximum capacity before such instability occurs.

The Canadian steel design code, CAN/CSA S16.1-M89 (1989), includes new mandatory provisions for the design and detailing of link beams in EBFs. These provisions, comprising Appendix O: Seismic

Design Requirements for Eccentrically Braced Frames, are based, almost exclusively, on the provisions given by SEAOC (1988), which, in turn, come from the research of Popov.

Clause D1 of Appendix D defines a link beam as "the segment of a beam in an eccentrically braced frame that is designed to yield, either in flexure or shear, prior to yield of other parts of the structure." This clause requires that, for these critical sections, the web must conform to Class 2 requirements and the flange must conform to Class 1 requirements. That is, in order to avoid local buckling of the web, the height, h, and thickness, w, of the web must conform to the requirement[1]:

$$\frac{h}{w} \le \frac{1100}{\sqrt{F_y}} \qquad \S 11.11.1$$

where F_y is the yield strength of the steel.

To avoid local buckling of the flange, the width, b, and the thickness, t, of the flange must conform to the requirement:

$$\frac{b}{2t} \le \frac{145}{\sqrt{F_y}} \qquad \S 11.11.1$$

It should be noted that doubler plates are not permitted to be used for link beam webs and therefore cannot influence the class of the section.

In order to control lateral buckling, the maximum unsupported length of the link beam, L_{cr}, is defined by Clause 27.2.2.1(b) as having to conform to the requirements for plastic design:

$$L_{cr} = \frac{980 r_y}{\sqrt{F_y}} \qquad \S 13.7$$

where r_y is the radius of gyration of the section about it weak axis.

The requirements outlined above are fundamental aspects of structural steel design. The detailing requirements for ductility, new to the design standards, were first adopted by SEAOC (1988) and have

[1] the equation numbers given correspond to the clause numbers of CAN/CSA S16.1-M89.

now been adopted by most North American design codes. These detailing provisions were developed by Popov. Popov's initial provisions are summarised in Engelhardt and Popov (1989) and are the provisions given by CAN/CSA S16.1-M89.

In order to ensure that the section will be able to exhibit its maximum ductility, Clauses D6 to D10 provide guidelines for web stiffeners in the link beam. Because link beams are subject to reversed cyclic loading behaviour all required stiffeners must be full height and welded to both flanges and the web. Full depth stiffeners on both sides of the web are required at the ends of the link beam, these stiffeners must have a combined width of b - 2w and a thickness not less than 0.75w nor 10 mm.

Full depth intermediate stiffeners are required for link beams controlled either by shear or by flexure, were shears are greater than 45% of the factored shear resistance. Intermediate stiffeners need only be provided on one side of the web when the section is less than 650 mm deep, otherwise, stiffeners must be provided on both sides. This requirement helps to ensure the stiffeners themselves do not buckle. Intermediate stiffeners must have a thickness of 10 mm. Stiffener spacing determined in Clause D8, is a function of the expected rotations of the link beam. Following these provisions will result in what appears to be an over-stiffened link beam. It must be realised, however, especially in the case of single sided stiffeners, that the stiffeners will be called upon to stabilise both the flange and the web. In a properly detailed link beam, severe web buckling, perpendicular to the tension field in the web should not cause local instability of the flange. Under circumstances of extreme loading, it has been observed that the stiffeners yield in double curvature flexure before instabilities of the flange are noted (Harries, 1992).

If these design provisions are followed, the link beam will be able to exhibit large ductilities and large amounts of energy absorption. Stable hysteretic behaviour at ductility levels of 8 to 10 times the displacement at yield are commonly reported in link beam tests. Beyond this point, it is often observed that lateral torsional buckling is the primary mode of failure (Engelhardt and Popov, 1989, and Malley and Popov, 1983, among others). Figure 3 shows an example of the typical hysteretic behaviour exhibited by a properly designed and detailed link beam in an eccentrically braced frame.

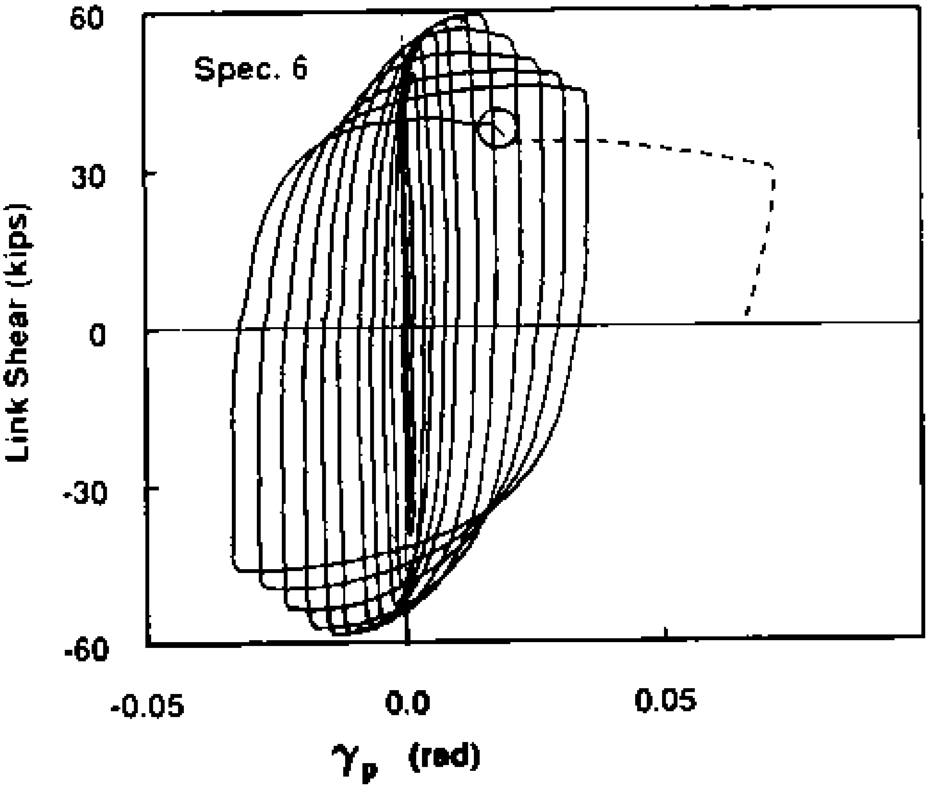


Figure 3 Applied shear versus rotation of a link beam in an eccentrically braced frame, reported by Engelhardt and Popov (1989)

4. Lateral Bracing Requirements for Link Beams

Engelhardt and Popov (1989) reported that in 5 of 14 full scale link beam test specimens, lateral torsional buckling was the primary mode of failure. In a further 3 specimens of the same series, lateral torsional buckling was cited as an 'additional failure mode'. In two earlier series of tests (Reoder and Popov, 1978 and Malley and Popov, 1983), similar results can be seen from post-test photographs, although the effect of lateral torsional buckling on link beam response was not under consideration in these programmes. Furthermore, recently at McGill, the initial stages of lateral torsional buckling were evident in specimens having reinforced concrete walls linked with steel coupling beams (Harries, 1992). In this programme, testing was stopped before full lateral buckling could take place because of constraints of the data acquisition system and safety considerations.

It is clear from these results, that lateral torsional buckling can be the nemesis of a member required to exhibit significant inelastic response. In order to control this mode of failure, Clause D17 of CAN/CSA S16.1-M89 requires that both the top and bottom flanges of a link beam be provided with lateral bracing at intervals no greater than $200/\sqrt{F_y}$. This lateral bracing must be designed to resist 0.015 times the beam flange resistance, $P_{y\,flange}$, which is defined as $b \times t \times F_y$, at the link beam ends and 0.01 times the flange resistance along its length.

These lateral resistance factors are essentially arbitrary and stem from theoretical analyses of strong axis bending of the compression flange. Recent work by Popov and Engelhardt (1989) has suggested that the 1.5% lateral resistance requirement is inadequate for link beams in eccentrically braced frames. Of 14 full scale link beam tests, the observed out-of-plane forces, exerted on the lateral supports at the link beam ends, exceeded $0.015P_{y\,flange}$ in all but one specimen. The values ranged from $0.007P_{y\,flange}$ to $0.22P_{y\,flange}$, the average value over all 14 specimens was $0.071P_{y\,flange}$, with three specimens having observed values exceeding $0.1P_{y\,flange}$. Figure 4 shows the hysteretic response of out-of-plane forces for three of the specimens tested by Engelhardt and Popov. The notations P1 and P2 refer to the out-of-plane forces of the bottom and top flanges, respectively, while the horizontal axis refers to total curvature of the link beam assembly (γ , in Fig. 1). Figure 4(a) shows the response of Specimen 11, the one specimen which remained below the 1.5% lateral resistance requirement. This specimen reached a peak lateral load of $0.007P_{y\,flange}$. Figure 4(b) shows the response of Specimen 1, having a peak out-of-plane load of $0.1P_{y\,flange}$. Specimen 1 shows a progressive increase in the out-of-plane forces with cycling, suggesting a relatively controlled occurrence of lateral torsional buckling. Conversely, Specimen 12 (see Fig. 4(c)) is typical of a sudden lateral buckling failure. The peak out-of-plane load recorded for Specimen 12 was $0.22P_{y\,flange}$. Shown on the right vertical axis of each hysteresis curve is the $0.015P_{y\,flange}$ out-of-plane force requirement. Figure 5 shows two lateral buckling failures observed by Engelhardt and Popov. The severity of this mode of failure is clearly apparent.

From his results, Popov has suggested that large out-of-plane forces are likely to occur at the link beam ends, where large loads are transmitted to braces or columns. Any eccentricity caused by a misalignment in these joint regions will be very significant in causing out-of-plane forces which lead to lateral torsional buckling. Existing misalignments will be further incremented by instabilities in the joint region. Popov has suggested that local buckling of the link beam also contributes to the inducement of large lateral forces. After local buckling, the cross section of the beam is no longer symmetrical. Consequently, the shear centre no longer coincides with the centroid, this, in turn induces further twisting moments on the cross section.

Popov has suggested that the minimum lateral force requirements for link beams be increased a minimum four fold to $0.06P_{y\,flange}$ (see Fig. 4, right vertical axes). Furthermore, he has proposed minimum stiffness requirements for the lateral bracing members, although no quantitative values have yet been determined. Popov emphasises that in order to effective at large applied loads and displacements, a properly designed lateral bracing system should not restrict any in-plane motion of the link beam. The implication of this is that floor slabs should not be considered adequate for lateral bracing of the top flange of link beams in EBFs.

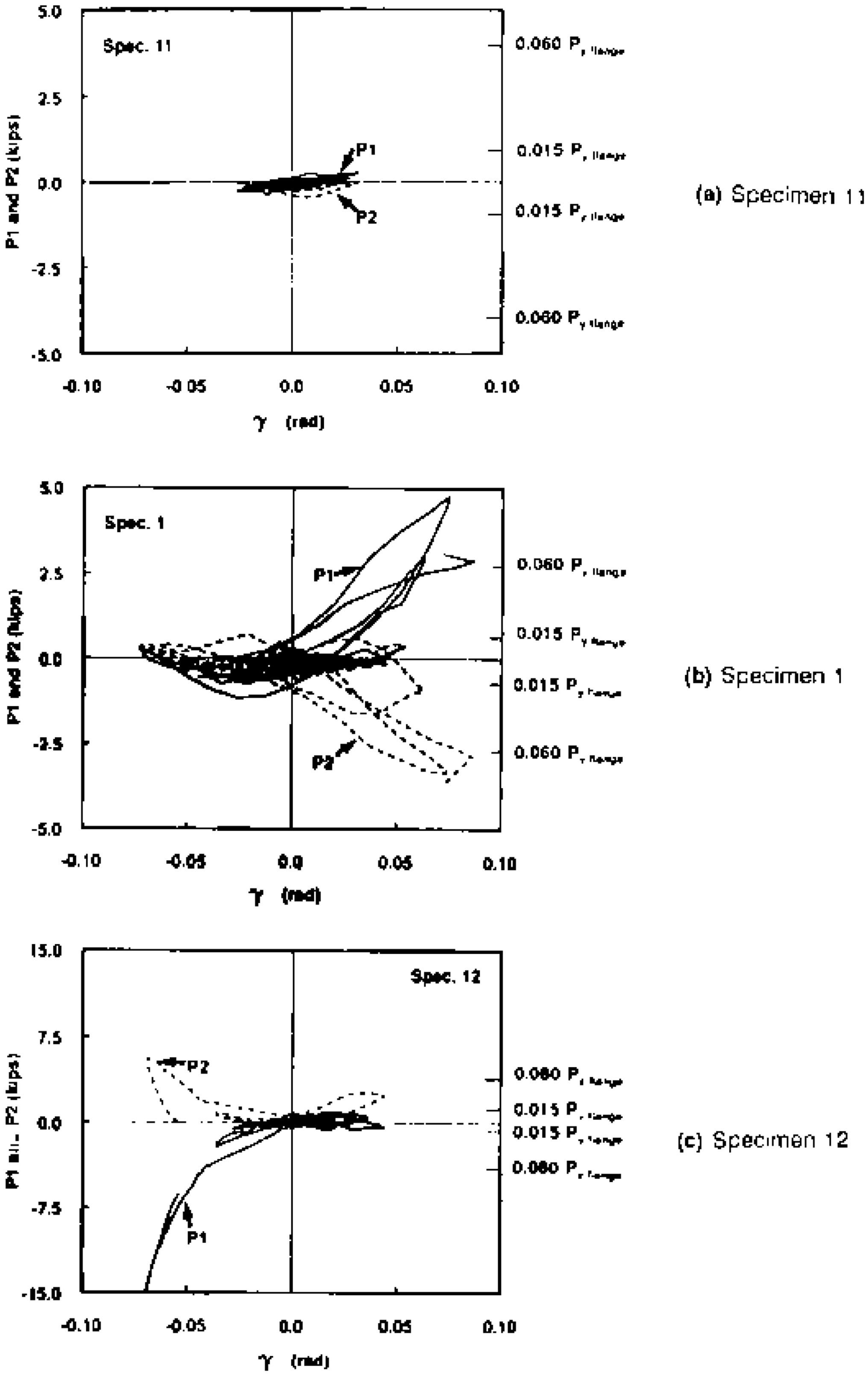


Figure 4 Out of plane forces at link beam ends (from Engelhardt and Popov, 1989)

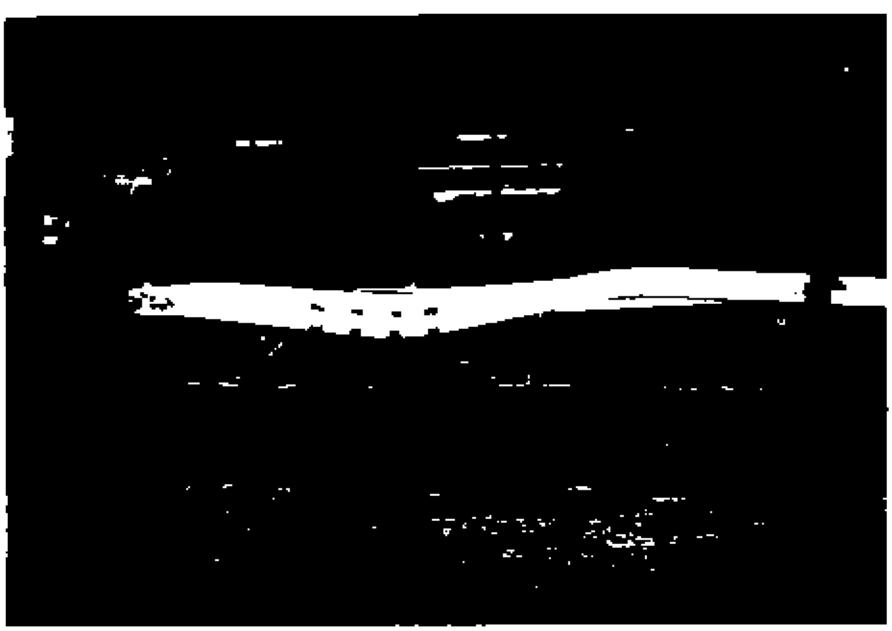


Figure 5 Lateral buckling failures reported by Engelhardt and Popov (1989)

These requirements will, undoubtably, make their way into code provisions in the near future. Although not yet an amendment to CAN/CSA S16.1-M89, the code committees in Canada are suggesting that a minimum value of $0.05P_{y\,flange}$ be adopted for the design of eccentrically braced frames. This value, lower than the 6% suggested in the United States, reflects the difference between the limit states design approach used in Canada and the working stress approach, traditionally used in the United States.

4.1 Application to Steel Coupling Beams in Reinforced Concrete Walls

The use of steel coupling beams, in lieu of diagonally or traditionally reinforced concrete coupling beams, in ductile reinforced concrete wall systems, is currently being investigated at McGill University (Harries et al, 1992 and Harries, 1992). Two coupling beams, tested to 20% reduction from ultimate capacity, began to show signs of lateral torsional buckling. The primary reason for the onset of this buckling was a progressive increase in clear span of the coupling beams brought about by spalling of the concrete. By the end of both tests the coupling beams had exceeded their specified maximum unsupported length of $980r_y/\sqrt{F_y}$, this, together with pronounced web buckling was tending toward large flange instabilities.

The lateral forces set up in coupling beams are significant, not so much in the clear span, as in the embedment region. Large lateral forces impose twisting moments on the walls, a phenomenon not exhibited in properly designed EBF systems. These twisting moments may become significant if they occur in the lower storeys, where the wall may be forming a plastic flexural hinge. Furthermore, significant lateral forces cause spalling of the embedment concrete beyond that caused by the cyclic shear stresses in the coupling beam. This effect will further reduce the effectiveness of the embedment leading to crippling of the coupling beam web in the embedment, an effect that should be avoided (Harries, 1992).

Although empirical values are not available for steel coupled walls, further research programmes will be investigating the out-of-plane forces generated by instability of the coupling beam. It is the author's opinion that values greater than those proposed for the lateral forces induced in EBFs be used in considering the lateral bracing which the reinforced concrete embedment of a steel coupling beam must provide.

5. Conclusions

It is clear that lateral torsional buckling is a primary concern for the designers of link beams in eccentrically braced frames. Lateral torsional buckling can cause sudden catastrophic failures at high load levels. These failures can be triggered by a number of mechanisms such as initial eccentricities, local web or flange buckling or web crippling. Current North American design codes adequately deal with these conditions in the form of provisions for designing and detailing link beams in EBFs.

One aspect of the response of these members which does not seem to be dealt with adequately is the lateral bracing requirements. Provisions requiring lateral bracing resistance equal to 1.5% of the flange resistance have been found to be inadequate. Values exceeding $0.06P_{y\,flange}$ are now being recommended as good design procedure for EBFs. In the case of reinforced concrete walls linked with steel coupling beams, lateral support provided by the reinforced concrete embedment must be checked and should certainly exceed $0.06P_{y\,flange}$. Furthermore, the lateral support provided by the reinforced concrete wall must take into account the effects that progressive cover spalling will have on the embedment region and on the embedment and clear span lengths.

One aspect of design which, although mentioned in the design codes, is not adequately emphasised is the requirement that *both* flanges be braced against lateral instability. The critical loads in this type of member will likely be seismically induced, therefore the reversed cyclic nature of the loading must be taken into account. Both flanges will be in compression during the reversed cyclic time history.

Another design aspect, not adequately emphasised (in fact, relegated to a footnote in the SEAOC (1988) provisions) is that the lateral support provided by a floor slab or other topping is *NOT* adequate lateral support for the top flange of link beams in EBFs designed to resist seismic load. Floor slabs, due to their nature, tend to restrict the in-plane as well as out-of-plane motion of the link beam. Since the link beam is the critical member in the horizontal load resisting system, the slab should not be designed to accept lateral forces from the link beam, rather it should be designed to 'go along for the ride'. Typically, if the link beam is experiencing large deformations, the floor slab will be far beyond being able to resist anything beyond its own dead load anyway.

Despite the stringent design and detailing requirements, most of which are aimed at providing stability, for link beams in EBFs, the eccentrically braced frame structural system is an exceptionally efficient system for resisting extreme horizontal loads due to earthquakes. Properly designed and detailed, the link beams in the system should dissipate large amounts of energy and protect the overall structure from catastrophic damage. The critical design consideration is to provide adequate lateral stability in order that sudden lateral torsional buckling failures can be avoided. There is a great deal of research currently being carried out to describe the lateral response of link beams. Popov has even suggested (Engelhardt and Popov, 1989) that lateral bracing forces may not even be related to beam flange strength. For the moment, however, design must be carried out with the best data we have available.

References

CISC, 1985, Handbook of Steel Construction, Canadian Institute of Steel Construction, Willowdale, Ont.

CSA, 1989, CAN/CSA-S16.1-M89, Limit States Design for Steel Structures, Canadian Standards Association, Rexdale, Ont.

SEAOC, 1988, Recommended Lateral Force Requirements, Seismology Committee, Structural Engineers Association of California (SEAOC).

Chajes, A., 1974, Principles of Structural Stability Theory, Prentice Hall, 336 pp.

Engelhardt, M.D. and Popov, E.P., 1989, Behaviour of Long Links in Eccentrically Braced Frames, Earthquake Engineering Research Center, Berkeley, Report No. UCB/EERC-89/01, 406 pp.

Galambos, T.V., 1963, *Inelastic lateral Buckling of Beams*, Journal of the Structural Division, ASCE, Vol. 89, No ST5, 1963.

Harries, K.A., 1992, Seismic Response of Steel Beams Coupling Concrete Walls, Master's Thesis, McGill University, May, 1992

Harries, K.A, Cook, W.D., Redwood, R.G. and Mitchell, D., (1992) *Concrete walls coupled by ductile steel link beams*, to be published in: Proceedings, Tenth World Earthquake Engineering Conference, Madrid, July, 1992.

Kasai, K. and Popov, E.P., 1986, A Study of Seismically Resistant Eccentrically Braced Frames, Earthquake Engineering Research Center, Berkeley, Report No. UCB/EERC-86/01, 215 pp.

Malley, J.O. and Popov, E.P., 1983a, Design of Links and Beams to Column Connections for Eccentrically Braced Steel Frames, Earthquake Engineering Research Center, Berkeley, Report No. UCB/EERC-83/03, 62 pp.

Malley, J.O. and Popov, E.P., 1983b, Design Considerations for Shear Links in Eccentrically Braced Frames, Earthquake Engineering Research Center, Berkeley, Report No. UCB/EERC-83/24, 117 pp.

Redwood, R.G., 1991 *Seismic Provisions of CAN/CSA-S16.1-M89*, Seismic Design Seminar Series, Montréal, April 1991, 36pp.

Roeder, C.W. and Popov, E.P., 1978, *Eccentrically Braced Steel Frames for Earthquakes*, Journal of the Structural Division, ASCE, Vol. 104, No. ST3, March 1978, pp 391-411.

Stafford-Smith, B., 1990, Analysis and Design of High-Rise Structures, (course notes for 303-611B, Analysis and Design of High Rise Buildings, Winter 1991, McGill University.).

III

The Human Knee Joint as a Structural Stability Problem

by

Sol Anibal Lorenzo

ABSTRACT

Approaching a problem in Biomechanics from a Structural Engineering viewpoint requires considerable change of focus. Analysing a problem in structural stability, we usually assume boundary conditions for the joints and concentrate on the instability behaviour of the members. However, when we begin to treat the stability of the knee joint analytically, we realize that the structural members - the bones - are stable for all but the most extreme overloads. The problem is therefore to understand the way in which biological mechanisms act to provide stability to the complex and vulnerable geometry of the joint.

Idealizing the joint with models of varying complexity allows us to reach an understanding of its stability behaviour for different positions and loading conditions. This understanding is the basis for treating damaged joints through therapy, surgery, or in the most extreme cases prosthetic replacement.

1. Introduction

"From a consideration of the construction of the knee-joint it would at first sight appear to be one of the least secure of any of the joints in the body. It is formed between the two longest bones, and therefore the leverage which can be brought to bear upon it is considerable; the articular surfaces are but ill adapted to each other, and the range and variety of motion which it enjoys is great. All these circumstances tend to render the articulation very insecure; but, nevertheless, on account of the very powerful ligaments which bind the bones together, the joint is one of the strongest in the body." *(Gray's Anatomy)* The stability of the human knee is, like many other biological phenomena, a remarkable piece of engineering. A synovial joint like the knee is an articulation between two bones, with a gap in between for the lubricating synovial fluid. The contact surfaces of the bones are made up of articular cartilage, and in the knee there are extra cartilaginous structures called menisci in between the bones which serve to distribute load and allow extra mobility. The bones, the femur above and the tibia below, are bound together by strong collagen structures, ligaments, which through action in tension provide the joint its structural stability. The whole is enclosed in a ligamentous capsule. Figure 1-5 show important anatomical features of the joint.

Analysing the knee joint from the standpoint of structural stability helps us to appreciate the function of the different structures in resisting the loads to which the joint is regularly subjected. For instance, if the ligaments are non-functional, the joint will tend to buckle laterally with the slightest axial load, as a pin-ended column with a hinge in the middle. To understand the structural stability of the knee-joint, it is appropriate to consider different "modes" of stability failure under a given loading condition. These modes are a

Fig. 1.—Right knee-joint. Anterior view.

Fig. 2.—Right knee-joint. Posterior view.

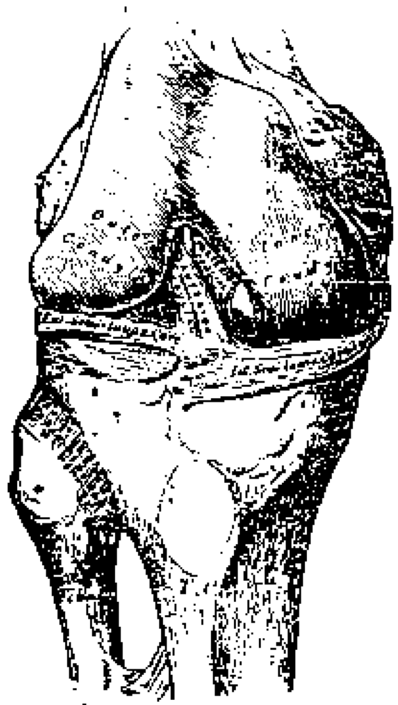


Fig. 3.—Right knee-joint. Showing internal ligaments.

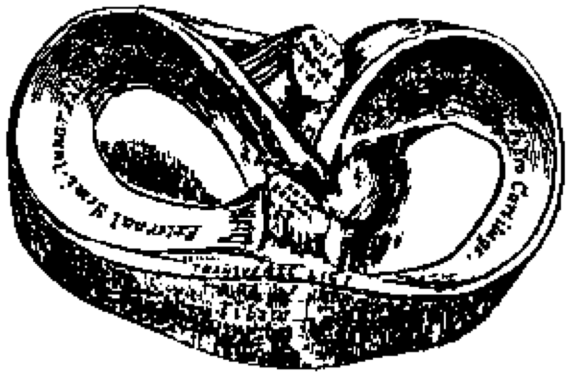


Fig. 4 — Head of tibia, with semilunar cartilages, etc. Seen from above. Right side.

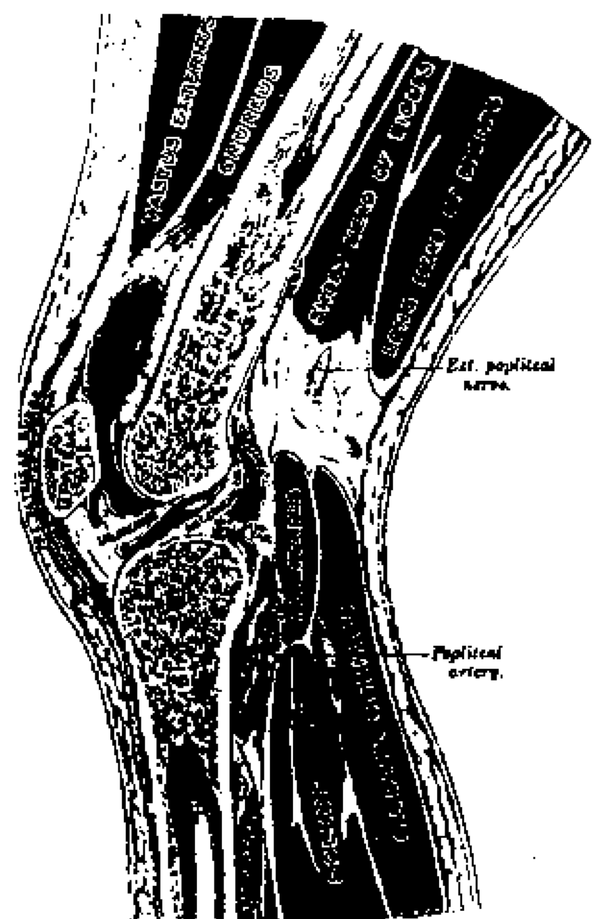


Fig. 5 — Longitudinal section through the middle of the right knee-joint. (After Braune.)

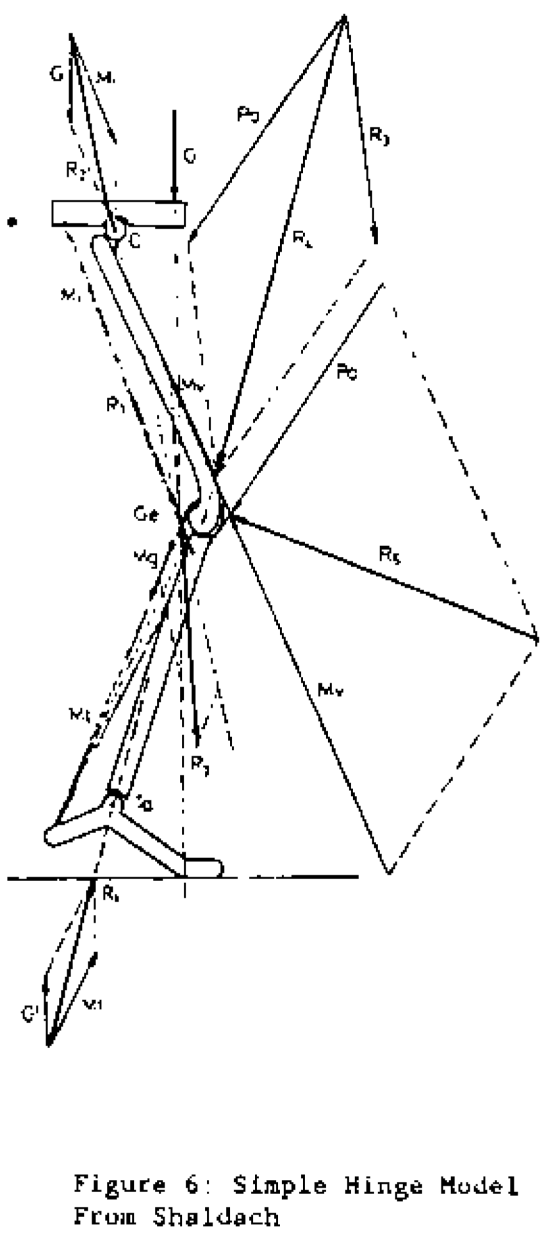
Figures 1-5 from <u>Gray's Anatomy</u>

shear-like translation, the anterior-posterior or drawer movement; a bending or pivoting in the lateral direction, known as varus-valgus displacement, and torsional rotation inwards or outwards. Lateral translation is also conceivable, but the varus-valgus mode can be said to usually control over it. Potential instability causing loads applied in the plane containing the axes of the major bones are effectively resisted by the large muscle groups acting on the knee, a very important feature in the knee's response to dynamic load.

The goals of this paper are very modest: to discuss the biological means of providing structural stability in various geometric configurations to a joint that is by definition a mechanism, and one with a wide range of motion. The approach will be to discuss some idealizations or models of the knee, to present the properties of the materials involved, and finally to discuss an analytical approach to the problem. By way of conclusion, the importance of such an analysis will be discussed with reference to its application in the field of prosthesis design.

2. Idealization of the Joint

A first attempt at understanding the mechanics of the joint would likely begin with a simple hinge (Figure 6). This model gives the joint one degree of freedom in rotation, corresponding to the principal actions of the joint in flexion and extension. However, examination of the joint reveals that its nature is much more complex, as discussed in the anatomical texts. In addition to flexion and extension, the joint allows some inwards and outwards rotation when partially flexed. The axis about which flexion and extension take place is also not necessarily normal to the axes of the members. More importantly, the movements of flexion and extension do not take place in a simple, hinge-like



Stressing of the skeleton of the lower limb in the sagittal plane. C : centre of the hip joint, G : body weight, Ge : centre of the knee joint, Mg : Mm. gastrocnemii, M_I : ischiocrural muscles, M_t : triceps surae, Mv : Mm.vasti, Pa : Lig.patellae R_1 : resultant of G and M_t, R_2 : resultant of G and M_I, R_3 : resultant of R_1 and Mg, R_4 : resultant of R_3 and Pa, R_5 : resultant of M and Pa, Ta : centre of the ankle joint (11)

Figure 6: Simple Hinge Model From Shaldach

manner but have a certain amount of conjunct gliding and rotation. The parts and areas of the main articular surfaces which are in contact change, as do the axis and radius of motion, with the degree of flexion. In extreme flexion, the posterior portion of the articular surfaces are in contact. As the leg is brought forward into extension, the upper portion of the tibia glides forward over the condyles of the femur, accompanied by a shift in the axis of rotation. Therefore in the extended position, it is the anterior portion of the articular surfaces which make up the contact area. This gliding action serves to allow the knee the great range of motion it enjoys; if the joint were to act like a simple hinge, soft tissues from the enclosing joint capsule would be trapped between the bones during flexion, causing pain and restricting motion.

An important aspect of these secondary movements of the knee is referred to as the "screw-home" mechanism in the biomechanics literature. As the knee reaches the last 15-20 degrees of extension, there is a conjunct rotation of the tibia outwards about its own longitudinal axis, or of the femur inwards about this vertical axis. This is mainly due to the relative lengths of the members, and one of its main effects is to put all the ligaments of the joint in tension. The movement is guided by the menisci (semilunar cartilages in Figure 4), which are themselves compressed during extension of the knee as the articular surfaces come into greater contact. In the position of full extension, the ligaments are maximally stretched. In fact, there must be a force applied to maintain the joint in this position: this can be the force of the extensor muscles themselves, or the reaction arising from an asymmetrical standing position in which the body's center of gravity passes in front of the joint. If no force is present, the ligaments will simply spring the joint out of full extension. This position, while stressful to the joint, is the position of greatest stability. The tightness of the ligaments and the large contact surface can provide stability without the aid of the muscles.

The hinge idealization does not represent these complex motions adequately. Analysis using the hinge idealization would require that an equivalent stiffness be derived in the degrees of freedom corresponding to varus-valgus, drawer, and rotational motion. Using these stiffnesses a critical load for the stability of the joint could then be found for any degree of flexion and applied load. This demands significant analytical or experimental work to derive a model which does not provide the desired representation of the stability behaviour of the knee. Nonetheless, the hinge model is very practical and was the basis, with the required stiffnesses based on medical experience, for early prosthesis design The model is actually very useful in calculating the forces in the extension and flexion muscles, and the important Joint Reaction Force, through basic statics It gives a meaningful estimate of the forces the joint undergoes during normal activity.

Figure 7 shows how a more detailed two-dimensional model could be useful. This model takes into consideration the irregular shape of the articular surfaces, and the actual location of the insertions of ligaments and tendons into the bone structure. It allows some representation of the function of the menisci in distributing load, and an estimate of the contact stresses can be made. Most importantly, by representing the gliding motion of the knee and the changing axis of rotation in at least one direction, it can provide a meaningful understanding of the motion of the knee. This two dimensional model is the basis of papers arguing that the knee works as a "closed kinematic chain", a mechanical arrangement in which the length of the members (ligaments in this case) determine the path of their connections to the other members. (Shaldach,1976)

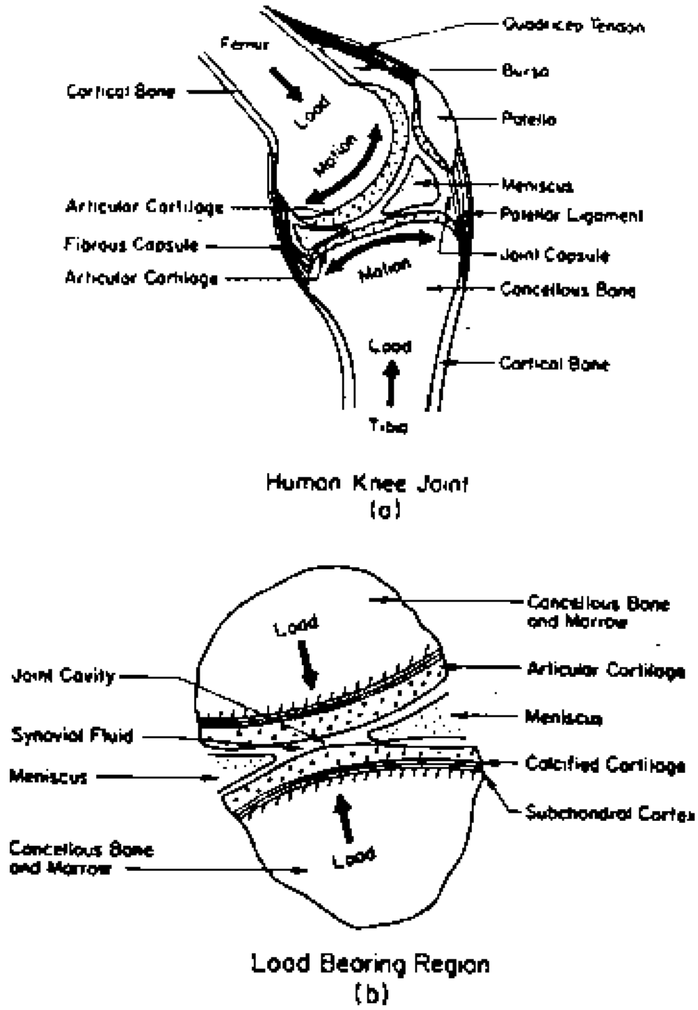


FIG. 7 (*a*) Schematic representation of the human knee joint, showing salient features which are important in its mechanical function. (*b*) Enlargement of the region of load bearing, showing the articular cartilage, calcified cartilage, subchondral cortex, and cancellous bone. The calcified cartilage and subchondral cortex function as a shell, providing backing to the articular cartilage, and are supported by the cancellous bone. The cancellous bone–bone marrow system provides the shock-absorbing mechanism for the diarthrodial joint.

Figure 7 from Skalak and Chien

3. Loading

The knee joint is one of the most highly stressed in the body. Throughout its normal range of motion, it is subjected to high axial compressive loads, great tensile forces in the ligaments and tendons, and possibly bending stresses in the bones as well. As they arise from the kinematics of the human body, these loads are by definition dynamic and repetitive. They can include reversal of stress, impact, and vibrations.

The forces arising in the joint can be estimated through equilibrium in its principal planes. The problem is complex due to the number of muscles which act on the knee. Furthermore, most of these muscles act on other joints as well, so their tension is not determined uniquely by the configuration of the knee. The dynamic effects can be accounted for by considering the inertia force of the body, but this is not very convenient as the acceleration of the limbs is difficult to calculate. Considerable research has been done using an experimental technique involving filming subjects performing various activities on a force platform.(Dowson and Wright, 1981) A time history of the force exerted on the ground is obtained, which is then compared to the position of the limbs. The force and geometry at any moment in time are used to calculate the Joint Reaction Force.

From these analyses, there is a general consensus as to the forces which the knee undergoes. These are estimated as three to five times body weight during walking, about ten times body weight during running, and more than twenty times body weight during jumping. These values are expressed as equivalent static loads. This is quite appropriate when we take into account the energy absorbing characteristics of cartilage and ligament tissue, and the damping effect on the joint of the surrounding synovial capsule and soft tissue. The great increase in joint force with increased mobility is of course due to the inertia force. To assure stability during such dynamic loading, a person either consciously or instinctively seeks to have the inertia force act in such a way that it can be resisted by the large muscle groups rather than by the ligaments alone. This involves the orientation of the leg at the moment of foot strike so that the axes of the leg bones and the vector of the inertia force lie in the same plane. The same load which can be controlled by the muscles in the plane of flexion and extension can be dangerously destabilizing if applied in a transverse or rotatory manner, such as when turning rapidly. Torn ligaments can often be the result.

4. Materials

4.1 Generalities

The human structure is composed of long compression members balanced by active tension members and bound together by passive tensile connections. Bone is an anisotropic, surprisingly ductile material, with high strength in both compression and tension. Tendons and especially ligaments have high tensile strength, and ligaments in particular have a unique stress-strain relationship which allows them to absorb considerable energy before rupture. The material properties are well suited to the structural function of the different members, and bone especially can change its properties during life. They have been determined by many researchers over the years using both destructive testing of samples (not *in vivo*!!) and ultrasound testing.

4.2 Bone and Cartilage

Bone is a fiber-matrix type composite material with properties well-suited to severe combined and dynamic loading conditions. A detailed discussion of the nature and properties of human bone is far beyond the scope of this paper, but it is appropriate to note that there are two principal types of bone, cancellous and compact, or cortical, bone. Cancellous bone is a spongy, flexible material which inhabits the interior of many of the skeletal members. It has very low strength and stiffness, but it always appears with compact bone, which is the dense, strong and stiff load-bearing material of the skeleton. Tables 1 and 2 show some important material properties of compact bone. Furthermore, in bones such as the femur and the tibia, the arrangement of compact bone near the surface with cancellous bone inside is well-suited to a structural member. In such an arrangement, resistance to bending, torsion, and other loading is provided entirely by the compact bone due to its much greater stiffness.

Since the high strength of compact bone is provided by its mineral fibers, there are several different subclassifications which differentiate between the orientation of these fibers. The great advantage of an anisotropic composite material is that its fibers can be arranged so that in one direction it is much stronger than an isotropic material of the same composition could be. This excellence is paid for by weakness in the other directions, but in bone large stresses usually only occur in the axial direction. In a case where large forces are normally applied in a transverse direction, as in a muscle insertion, the bone is locally altered so that the strong fibers lay in line with the muscle pull, smoothly transitioning over a short distance so as to align themselves with the long axis.

Examining the stress-strain curves for bone (Figure 8), we see behaviour somewhat similar to that of familiar structural materials. There is a straight, elastic portion, a rounded yield region, and most importantly a large plastic region before rupture. The response terminates in a brittle fracture, but the overall behaviour is ductile: more energy is absorbed in the plastic zone than

TABLE 1 The yield stress, ultimate stress, and ultimate strain for human bone tissue

Notation for yield stress*	Yield stress,† MPa	Ultimate stress,† MPa	Ultimate strain or rotation‡
σ_{3}^{+}	115 (11.2) [202]	133 (14.1) [202]	0.0293 (0.0094) [202]
σ_{3}^{-}	182 (14.4) [19]	195 (19.6) [114]	0.0220 (0.0057) [114]
σ_{30}^{+}	....	100 (10.4) [23]	0.0198 (0.0083) [23]
σ_{30}^{-}	....	173 (13.8) [5]	0.0280 (0.0052) [5]
σ_{10}^{+}	....	61 (12.2) [19]	0.0069 (0.0022) [19]
σ_{10}^{-}	....	133 (15.0) [7]	0.0311 (0.0010) [7]
σ_{90}^{+}	....	51 (10.1) [31]	0.0072 (0.0016) [31]
σ_{90}^{-}	121 (9.2) [3]	133 (16.7) [13]	0.0462 (0.0260) [13]
τ	54 (5.2) [19]	69 (4.4) [31]	0.3299 (0.0890) [19]

Source: Summarized and condensed from Reilly and Burstein (1975) and Cezayirlioglu et al. (1985).

* See text for explanation.

† The first number is the yield or ultimate stress in megapascals. The number in parentheses is the standard deviation, and the number in brackets is the number of specimens.

‡ The first number is the ultimate strain or the ultimate rotation in radians, the number in parentheses is the standard deviation, and the number in brackets is the number of specimens.

TABLE 2 Technical constants for human bone (values in gigapascals)*

Group	Reilly and Burstein (1975)	Yoon and Katz (1976)	Knets and Malmeisters (1977)	Ashman et al. (1983)
Bone	Femur	Femur	Tibia	Femur
Symmetry	TI	TI	OrTH.	OrTH.
Method†	M	U	M	U
E_1	11.5	18.8	6.91	12.0
E_2	11.5	18.8	8.51	13.4
E_3	17.0	27.4	18.4	20.0
G_{12}	3.6‡	7.17	2.41	4.53
G_{13}	3.3	8.71	3.56	5.61
G_{23}	3.3	8.71	4.91	6.23
ν_{12}	0.58	0.312	0.49	0.376
ν_{13}	0.31‡	0.193	0.12	0.222
ν_{23}	0.31‡	0.193	0.14	0.235
ν_{21}	0.58	0.312	0.62	0.422
ν_{31}	0.46	0.281	0.32	0.371
ν_{32}	0.46	0.281	0.31	0.350

* The 3 direction is coincident with the long axis of the bone; the 1 and 2 directions are radial and circumferential, respectively.

† M = standard machine testing; U = ultrasound.

‡ Not measured.

Tables 1 and 2 from Skalak and Chien.

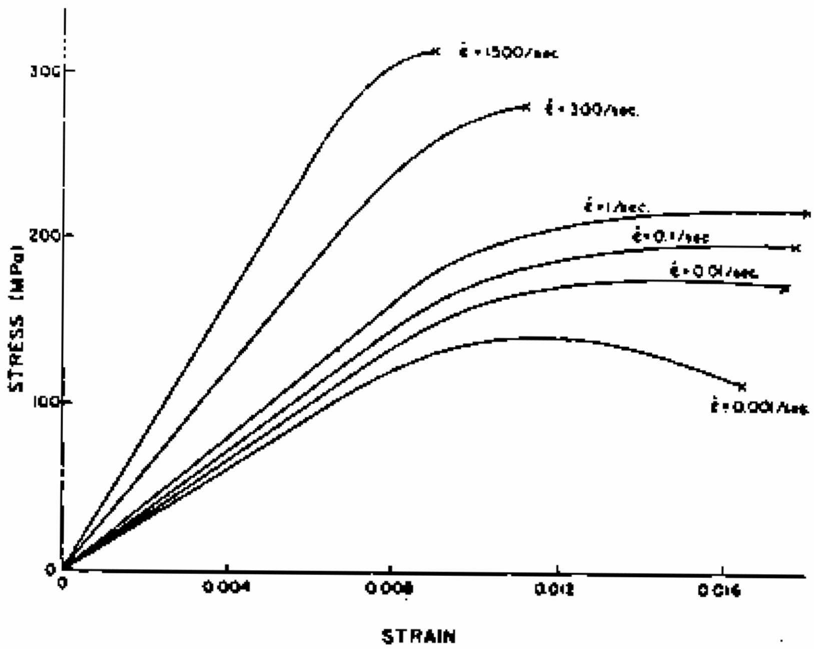


FIG. 8 Illustration of the strain rate dependence of the stress-strain curve. (*Redrawn from McElhaney, 1966.*)

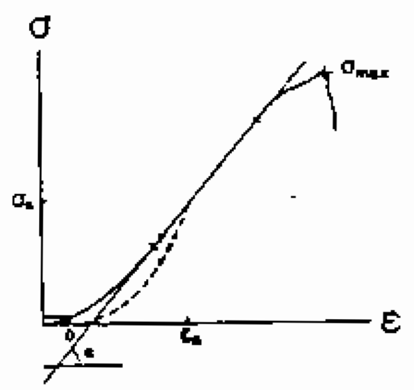


FIG. 9 Stress (σ)–strain (ε) curve for parallel-fibered collagenous tissue until failure (solid line); tan α is elastic stiffness. The beginning and end of the linear segment are marked with bars. The dashed curve is for unloading, if the cycle is reversed at the point (σ_a, ε_a).

Figures 8 and 9 from Skalak and Chien.

the elastic. This is important physiologically, as bone which has yielded but not fractured will be repaired by the body. Furthermore, bone exhibits visco-elastic behaviour in that its stress-strain curve changes with the strain rate. The effect of the variation is to increase the stiffness, strength and energy absorbance with increasing strain rate. The physiological strain rate for high impact activities such as running has been estimated at .01 per second. Thus in the physiological range, greater loads tend to be applied at higher strain rates taking advantage of an important material property of bone.

The properties of the articular cartilage are less well-known, due to their complexity and the difficulties of working with the material experimentally. Essentially, the material serves as a bearing between the articular surfaces, distributing the load and absorbing energy.(Skalak et al, 1987)

4.3 Ligaments

The ligaments and tendons of the knee have an unusual stress-strain curve. The curve starts out shallow, steepens into a straight-line portion, and then reaches rupture rather suddenly; see Figure 9. Physiologically, the ligament itself rarely breaks, but rather it comes apart from the bone. Therefore the entire straight line portion of any ligament in tension can be counted upon to absorb energy from an impact. Clinical works (Evans, 1986) often refer to ligaments as having an "elastic phase" and a "non-elastic phase". This does not refer to yield of the ligament, but to the amount of resistance an examiner feels in producing passive movements on the knee. The former is the transition from a lax ligament to the first portion of the stress-strain curve described above. The latter is the region of the curve in which the examiner meets quickly increasing resistance to the passive movements; the steep linear portion of the curve.

5. Stability of the Joint

5.1 Physical Instability Phenomena

A stability failure can be defined as a reduction in the load bearing capacity of a member arising from its geometric configuration rather than from yielding of its material. Damage to material, i.e. yielding of a steel column or tearing of a ligament, may occur, but it is more a consequence of the continued application of load to the deformed geometry than a necessary result of the initial stability failure. In a structure supporting dead and live loads, this type of damage almost inevitably follows after instability in a member: redundant load paths redistribute some load after failure, but this is a passive process, usually not enough to prevent permanent damage to the member. In the human body, however, neurological receptors in the joint capsule contain reflexive impulses which can act to prevent damage to the components of the joint. They act by inhibiting function of the muscles which support the load causing the initial instability. In the case of the knee joint, this means the leg muscles, and the effect may be to fall down, to slip, or to drop a weight. The cause is that the body "prefers" to fall down than to risk damage to tendon, ligament, or bone. Thus in some cases, instabilities at the knee are actually protective mechanisms. For example, a healthy knee joint moves in a path strictly controlled by the tightness of the ligaments, with virtually no side-to-side movement. Lax ligaments allow the articular surfaces to slide about randomly on one another. In this way sudden, intense stresses can occur, which may cause the receptors to fire, inhibiting all muscle activity and resulting in a momentary instability under no external load.

It is possible to look at a knee joint under load as a problem in structural stability. We know from previous discussion that the two-dimensional or hinge models are effective in finding muscle and joint reaction forces for any given load and degree of flexion. Therefore, a load which can be supported by the joint mechanism at one configuration may cause instability upon an attempted change of position. This would be the type of instability mentioned above: the nervous system would inhibit motor activity, effectively causing the members which stabilize the mechanism to vanish, and thereby relieving the joint of a potentially damaging load. This type of instability is related to an equivalent static analysis of the joint, but its essential nature is beyond the grasp of a structural stability analysis. Also, this approach only deals with instabilities in the direction of the primary motions of the knee, which are not the most dangerous ones from a stability standpoint. We need an approach which considers the potential for instabilities in the varus-valgus, drawer, and rotatory modes.

One possiblity would be to use an energy approach to the problem. For any given degree of flexion, we would analyze the geometry to determine the state of the ligaments, the articular contact area, etc. We would then suppose a unit deflection as usual, and for each of the potential modes of instability compare the strain energy in the deflected configuration to the energy of the stable geometry. Knowing the material properties of bone, cartilage, and ligaments allows us to calculate the strain energy. The various modes would be coupled only by the activation of the resistances of the same ligaments; i.e. they would be uncoupled in terms of the analysis.

This is the approach implicitly taken by Crowninshield et al in their Analytical Model of the Knee. They used a finite elements (energy-based by definition) model to represent the contribution of the various ligaments to joint stability. They achieved this by neglecting the contribution of the muscles (very appropriate since they verified their results with tests on cadaveric limbs) and

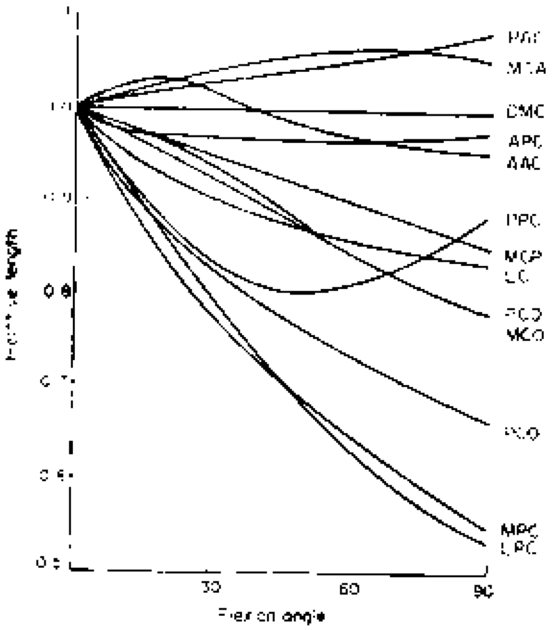


Fig. 10 Relative ligament length as predicted by the analytical model.

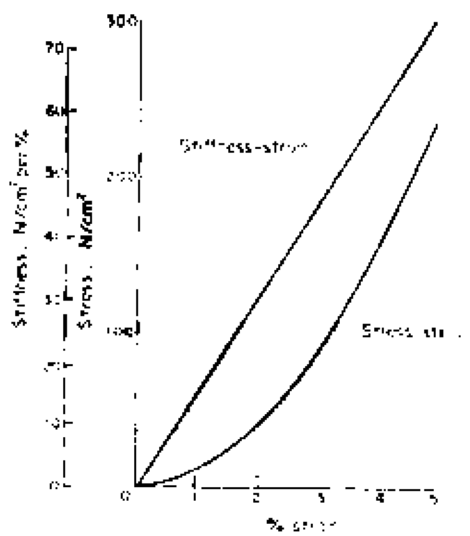


Fig. 11 Stiffness-strain and stress-strain relationships of ligament tissue.

Figures 10-11 from Crowninshield et al. Note that the ligament model the authors used only resists tension within 5% of ultimate length. Thus the strain % in Fig. 11 is from 96 to 100% of ultimate.

representing the cruciate, collateral, and capsular ligaments by thirteen elements. Figures 10 and 11 show important features of the ligament modelling they used. They then evaluated the stiffness of the model at flexion angles between 0 and 90 degrees in the three aforementioned modes of displacement, obtaining good agreement with experimental results. Such a model is of great value in understanding the stability behaviour of the knee. Crowninshield et al used it to obtain quantitative values for the importance of the particular ligaments in resisting the various modes of instability (known from anatomical and clinical experience.) It could therefore be used to determine which is the controlling mode of failure for several loadings and positions of the joint.

Actually, the stability of the knee joint is ensured by the combined action of the muscles, the ligaments, and the articular surfaces. The muscles are of particular importance, since they can compensate for deficiencies in the other components. For instance, a knee with torn or lax ligaments is usually given quadriceps exercises; these exercises do nothing for the ligaments, but the strengthened muscles will play an increased role in joint stability. However, this increased role is still dependent on the orientation of the foot strike as discussed in section 3.. No matter powerful the muscle becomes, its contribution to rotational stability is minimal.

5.2 Role of Menisci and Articular Surfaces

The menisci are important structures for maintaining joint stability. They cradle the condyles of the femur during application of any compressive load. This serves to provide lateral support against drawer or lateral instability. Their shape allows the tibial contact area to be up to three times greater, decreasing the contact stress by up to seven times.(McBride and Reid, 1988) It is the menisci which allow active rotation of the flexed knee, and they guide the screw-home motion which brings the knee into its position of greatest stability.

5.3 Role of Ligaments

The ligaments bear the burden of controlling the motion of the knee, of absorbing energy from dangerous lateral loads, and of preventing the knee from buckling in a lateral or rotatory manner. As stated before, each ligament's contribution to the overall stability of the joint is controlled by its geometry, with the sites of its attachments to the femur and tibia being the most important. Figure 12 is from Crowninshield et al, being a summary of their findings for the relative importance of the major ligaments in the three main instability modes. Table 4 is a summary of important physiological information determining the contribution to joint stability of some of the major ligaments.

6. Injuries

In the knee, instability causes injury, and injury causes instability. Specific medical attention to injuries of the knee related to its function seems to be a recent phenomenon; *Gray's* states "...dislocation from traumatism is of very rare occurrence." This is very likely a result of the great leap in athletic performance in the last several decades. The quick turning motions required in modern sports, combined with the increasing physical power of the high visibility professionals, have led to great advances in the understanding of the mechanics of the joint through treatment of trauma.

Figure 12 and Table 4 can serve to predict the reduced stability of the knee after injury to any particular ligament. Clinical examinations have sets of passive motions which, when imposed on the knee, reveal through excessive flexibility in a given movement damage to a particular ligament. For instance, damage to the Anterior Cruciate Ligament leads to instability in anterior

Table 3 Coordinates of ligament attachment sites (mm). Area is given relative to that of the anterior fibers of the medial collateral

		Attachment sites						
		Femoral			Tibial			Relative
Ligament elements	Abbrev.	Xf	Yf	Zf	Xt	Yt	Zt	area
Lateral Collateral	LC	35	10	27	[illegible]	25	−35	0.5
Post Med Coll	MCP	−35	[illegible]	30	−20	[illegible]	−50	0.5
Ant Med Coll	MCA	−35	−7	30	−20	−7	−40	1.0
Med Coll Oblique	MCO	−35	1	[illegible]	35	30	[illegible]	0.5
Deep Med Coll	DMC	−35	0	27	−35	0	−50	0.5
Ant Post Cruciate	APC	−5	−3	[illegible]	−3	25	−5	[illegible]
Post Post Cruc	PPC	−5	17	20	[illegible]	25	−5	0.5
Ant Fib Ant Cruc	AAC	7	8	25	−7	−5	0	0.5
Post Fib Ant Cruc	PAC	5	4	25	0	−2	0	0.5
Lat Post Cap	LPC	25	25	30	25	25	−30	0.4
Med Post Cap	MPC	−25	25	30	−25	25	−30	0.4
Post Cap Oblique	PCO	25	25	30	−25	25	−30	0.1
	PCO	−25	25	30	25	25	−30	0.1

Table 3 from Crowninshield et al. It is a list of the ligament elements they used. Some ligaments were represented by more than one element.

TABLE 4
Stability Behavior of Major Ligaments

Ligament	Anatomical Description (After *Gray's Anatomy*)	Stability Function (After Crowninshield et al, 1980 and Evans, 1986)
Posterior Cruciate	Largest cross-sectional area. Almost axially aligned between tibia and femur.	It is the main stabilizer of the knee in drawer motions.
Anterior Cruciate	Crosses the posterior cruciate; obliquely attatched between tibia and femur.	Provides stability in anterior drawer and internal rotation.
Medial Collateral Anterior	The attatchment between the patella and the tibia; the end of the extensor arm.	The width of this ligament provides varus stability.
Lateral Collateral	The long, vertically aligned exterior ligament.	Provides most of the varus stability.
Posterior Capsular	Posterior portion of the band of ligaments enclosing the joint.	Provides significant external rotational stability.

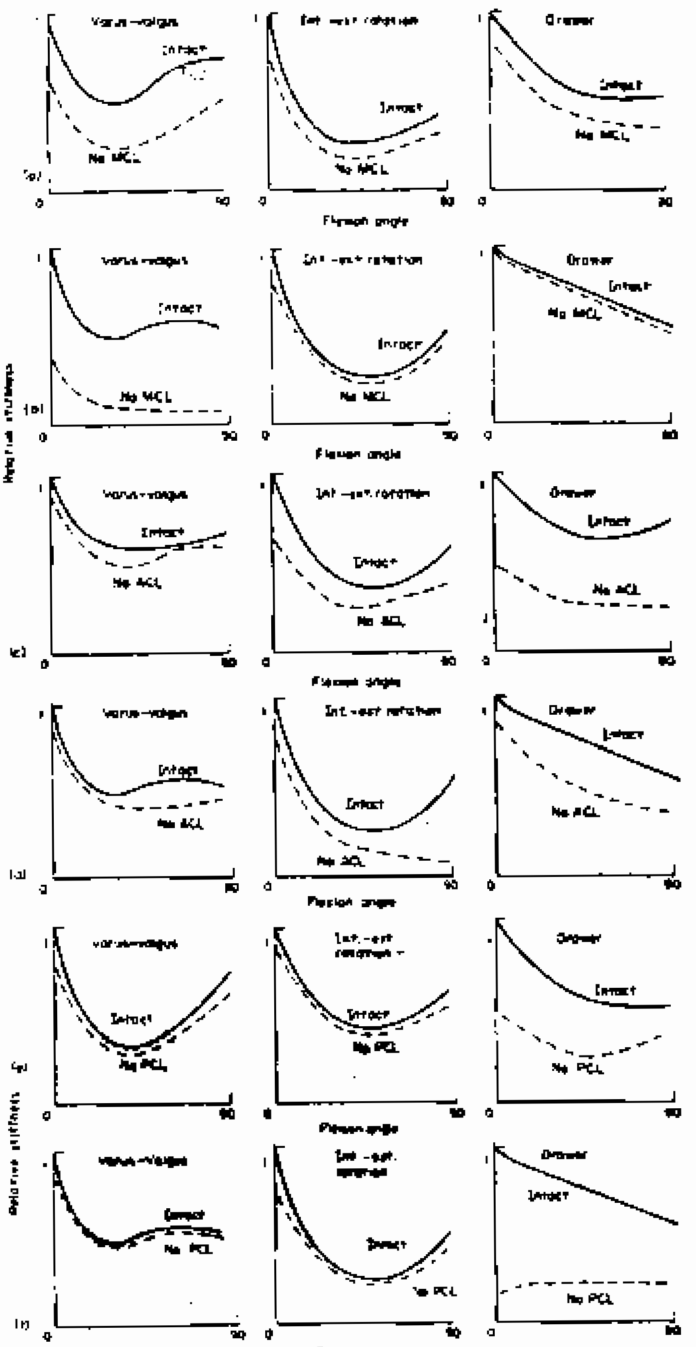


Fig 12 A comparison of the theoretical and experimental eliminations of ligament elements. (a) Actual stiffness after transection of medial collateral. (b) Predicted stiffness after transection of medial collateral. (c) Actual stiffness after transection of anterior cruciate. (d) Predicted stiffness after transection of anterior cruciate. (e) Actual stiffness after transection of posterior cruciate. (f) Predicted stiffness after transection of posterior cruciate.

Figure 12 from Croninshield et al.

translation and internal rotation. This might be adequately treated by providing an orthosis (brace) which fits the leg tightly, with straps to limit translation and rotation in the weak directions. (Kosiuk 1990)

7. **Prostheses**

The field of modern biomechanics has been linked on the one hand to the increased interest in joint and muscle behaviour of sports medicine, and on the other to the advance in surgical techniques to the point where complete joint replacement is a feasible option in treatment of extreme joint degeneration, such as in advanced osteoarthritis. In the latter case, the need is for prosthetics which meet the dual requirements of stability and mobility. In the field of total knee replacement, there are two basic solutions: a hinge type joint or an "inter-condylar" joint.

The first hinge type prostheses were based on the assumption that it would be impossible to reproduce the complex motions of the knee with a mechanical device. The solution is to implant a simple hinge, sacrificing the natural motions. The advantage of this solution is that a single degree of freedom hinge will provide great stability in all directions. Unfortunately, many hinge type implants develop problems at the interface of prosthesis and bone. This is because the hinge has virtually no energy absorbtion features. Therefore, the energy which would be absorbed by the ligaments in a normal knee is transmitted to the bone interface, eventually causing loosening of the prosthesis. To minimize this effect, the bone/metal interface surface is increased, requiring greater removal of bone, which may have detrimental effects.

In patients where some ligaments can be preserved, a more modern solution is the intercondylar type of implant. Damaged joint surfaces are replaced by metal condyles on the femur and a plastic bearing pad on the tibial plateau; the structure of the joint is conserved, as the prosthesis requires little anchorage. The philosophy of this solution is to reproduce the motions of the knee as closely as possible, with stability provided by the remaining ligaments combined with the closely fitting articular surfaces of the artificial joint. These same features provide some energy absorbance, contributing to the important medical advantage of requiring minimal removal of healthy bone.

8. Summary

A structural stability approach to the geometry of the healthy knee is a good way of analysing the response of the joint to various load conditions. The biological structures which provide stability *in vivo* to the healthy knee are extremely complex to model, and much more so to reproduce artificially in the manufacture of functional joint prostheses. However, a complex, biologically accurate model is justified, as the results of a structural stability analysis such as load-deflection response can help us understand the injuries often seen in the knee. Likewise, a stability analysis is essential when determining the adequacy of prostheses, of orthoses, or of other methods for repairing a dysfunctional joint.

REFERENCES

Gray's Anatomy, fifteenth edition, Bounty Books 1977

Evans, Philip The Knee Joint, Churchill Livingstone 1986

Skalak, Richard and Chien, Shu ed. Handbook of Bioengineering, McGraw-Hill 1987

Shaldach, M. ed. Advances in Artificial Hip and Knee Joint Technology Springer-Verlag 1976

Dowson, D. and Wright, V. ed. An Introduction to the Biomechanics of Joints and Joint Replacement, Mechanical Engineering Publications LTD 1981

Kostuk, Monica Quantitative Analysis of Knee Appliances in ACL Deficient Knees MS Thesis, Dept. of Physical Education, McGill U. 1990

Crowninshield, R., Pope, M. and Johnson, R. An Analytical Model of the Knee Journal of Biomechanics, 1976, Vol. 9, pp 397-405

McBride, I. and Reid, J. Biomechanical Considerations of the Menisci of the Knee, Canadian Journal of Sports Science, 1988, Vol. 13 no.4 pp 175-187