THE DESIGN OF MULTI-CONDUCTOR UNBALANCED TRANSMISSION LINES FOR BROADCAST FREQUENCIES

J.L.Marshall

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McGill University

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The Design of Multi-Conductor Unbalanced Transmission Lines for Broadcast Frequencies

The present day necessity of installing multi-tower antenna arrays for many transmitters in the broadcast band (540 kc. to 1600 kc.) in order to prescribe the pattern of their radiated fields, brings with it the need for longer transmission lines. The transmission efficiency of a line should, of course, be a maximum compatible with economical installation and good engineering practice, since thie portion of the system carries the most expensive energy of a transmitter plant. Furthermore, in Canada and the U.S. at least, government regulations limit the amount of power that can be lost in the external circuits i.e. power-dividing network, transmission line, and antenna-matching networks. Therefore, where two or more antennas are used requiring an equal number of antenna-matching circuits and a considerable length of transmission line, the losses in both of these components must be kept small in order to satisfy the regulations.

These conditions draw attention to the relative merits of the two common types of line, the concentric tube and the open-wire. The characteristics, performance, and cost of concentric lines are well known, since they have been used for many years, and their various sizes have been standardized.

In recent years, however, multi-conductor open-wire lines have been gaining favour. There are two factors which make it possible to approach the transmission efficiency of a concentric line, with an open-wire type. First, the characteristic impedance can be made larger, by greater separation of the conductors. This produces lower attenuation, for any given conductor resistance. Secondly, a number of grounded conductors can be disposed around the live conductor to ensure that a large proportion of the return current flows in them, and not in the earth. If this is not done, losses will be incurred by the earth currents, and to a smaller degree, radiation. Such losses can be reduced by using balanced lines in which the currents in the "go" and "return" conductors are equal and opposite. However, balanced lines are seldom used now in broadcast practice, for the following reasons:-

a) The transmitters are generally designed with unbalanced output stages which leads to economy of components and circuit simplicity.
b) The same advantages as in (a) apply to the design of the antenna-matching circuits, and power dividing networks.
c) To obtain perfect electrical balance often requires critical circuit adjustments. If perfect balance is not maintained,

the unbalance current will produce losses, due to earth currents and extra radiation.

In short-wave broadcasting, on the other hand, balanced lines are normally employed, since the antennas are generally of the doublet type, and impedance matching at the antenna is simplified by the use of line sections.

In the case which forms the subject of this paper, it was required to provide transmission lines for two fifty-kilowatt transmitters and one of ten kilowatts. Two of these transmitters were to have two-tower antenna arrays. Consideration was given to the use of a ten-wire line for the following reasons:-

a) The losses could be made low enough to meet government regulations, even at transmitter locations where the earth conductivity was low.

b) The line is exposed and therefore permits easy inspection and clearing of faults.

c) The line will withstand high voltage surges without breaking down.

d) The line can be installed by any electrical contractor without specially skilled personnel.

e) The maintenance of an open open-wire line is negligible, where-as a pressurized concentric line does entail some cost for maintenance.

HISTORICAL NOTE ON TRANSMISSION LINES

The early open-wire transmission lines were single wires with ground return. This type of line has a high transmission and radiation loss. The two-conductor balanced line came into popular use next. The transmission efficiency of this line is high if a good electrical balance is maintained, and the line is widely used, particularly in short-wave broadcasting. A four-wire balanced line for short-wave transmitters came into use in Great Britain during the last war (1939-1945)[1]. A secondary use for the extra wires was the adjustment of impedance, at any point in the line. This was done by reducing the spacing between each pair of wires of the same polarity, over a short length of the line. Such adjustments, which introduce reactance of one sign or the other, are used in short-wave practice to compensate for impedance irregularities caused by supporting poles or bends in the line. An eight-wire line[2] has also been used, and for similar reasons. It is actually a balanced four-wire line in duplicate, arranged so that one set of four wires can be rotated with respect to the other set, thus changing the line impedance.

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1. McLean R.C. and Bolt F.D. "The Design and Use of Radio-Frequency Open-Wire Transmission Lines and Switch Gear for Broadcasting Systems" Jour. I.E.E. Vol. 93, Part 111, No. 23, May/46.

2. Christiansen W.N. and Guy J.A. "An Eight-Wire Transmission Line for Impedance Transformation" Amalgamated Wireless (Australasia) Technical Review, Vol. 7, No. 3, 1947.

Let us return to the unbalanced open-wire transmission lines used in the frequency range, 540 kc to 1600 kc, or "broadcast" band, as it is called on this continent. In this field, multi-conductor lines have been in use for about fifteen years. A four-wire, and later a six wire type were put into service[3]. Also, during the last war (1939-1945) a twelve-wire line was installed in Great Britain[4] to carry 800 kw. of radio-frequency power. This line had four live conductors, to handle the large amount of power, and eight grounded wires.

HISTORICAL NOTE ON TRANSMISSION LINE THEORY

Two of the notable pioneers in transmission line theory were Kelvin and Heaviside[5], the latter in particular. The principles were developed from Maxwell's Equations which can be applied equally well to either free waves, as from an antenna, or guided waves along a transmission line. The familiar basic transmission line equations are:-

$$\left(L\frac{\partial}{\partial t} + R\right)I = -\frac{\partial}{\partial x}V \tag{1}$$

$$\left(C\frac{\partial}{\partial t} + G\right)V = -\frac{\partial}{\partial x}I \tag{2}$$

where x is the distance along the line and the other symbols are standard. These can be formed into differential equations whose solutions are:-

3. Brown G.H. "Characteristics of Unbalanced Overhead Transmission Lines" Broadcast News, May 1941.

4. Footnote 1, Loc. cit.

5. O. Heaviside. "Electromagnetic Theory", Vol. 1.

$$I = Ae^{-\gamma x} - Be^{\gamma x} \tag{3}$$

$$V = Z_o A e^{-\gamma x} - Z_o B e^{\gamma x} \tag{4}$$

the symbol γ, is the propagation constant; Zo, is the characteristic impedance of the line, and A and B are constants. Equations (3) and (4) each contain a direct, and a reflected wave.

The basic transmission line equations yield highly accurate results from a practical point of view and most of the design work to be described in this thesis is based upon them. However, it was recognized by Heaviside that the equations did not satisfy all the conditions of electromagnetic theory when rigorously applied to a transmission line. During the past thirty years there has been considerable elucidation and refinement of transmission line theory, including analyses of wire line transmission in strict accordance with electromagnetic theory. It would seem appropriate here to briefly sketch the progress that has been made.

Equations (1) and (2) can be generalized for a line of n parallel wires[6] thus:-

$$\sum_{Z_o=1}^{n} Z_{jZ_o} I_{Z_o} = -\frac{\partial}{\partial x} V_j \qquad (j = 1, 2 \text{-----} n) \tag{5}$$

$$\sum_{Z_o=1}^{n} Y_{jZ_o} V_{Z_o} = -\frac{\partial}{\partial x} I_j \qquad (j = 1, 2 \text{-----} n) \tag{6}$$

the general solutions of which are:-

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6. Carson J.R. "The Present Status of Wire Transmission Theory and some of Its Outstanding Problems" Bell Sys. Tech. Jour. Vol. Vll, April 1928.

$$I_j = \Sigma A_{jz_o} e^{-\gamma_{z_o} x} - B_{jz_o} e^{\gamma_{z_o} x} \tag{7}$$

$$V_j = \Sigma Z_{ojz_o} A_{jz_o} e^{-\gamma_{z_o} x} + Z_{ojz_o} B_{jz_o} e^{\gamma_{z_o} x} \tag{8}$$

which represent n direct and n reflected waves, each with a different propagation constant, thereby differing from the basic transmission line equations.

Even in the general case these equations are approximate, (yet accurate to a high degree) because the characteristic impedance, Zo, and the propagation constant, γ , are only approximately calculable from the geometry of the line and its electrical constants. Furthermore, a purely plane transverse principal wave is assumed, propagated with the velocity of light. But in a line with conductors of finite resistance, the field is not exactly transverse since there must be a small component of the electric potential in the direction of the line. By starting out with Maxwell's equations, and making no assumptions as to the magnitude of the propagation constant, Carson[7] has shown that a rigorous solution of wire-line transmission reveals properties additional to those given by the generalized form of the basic equations. Besides the n modes of propagation mentioned above, an infinity of other modes of propagation appear, which are called "complimentary" waves. These differ from the principal

7. Footnote 6, loc. cit.

wave in that they are not quasi-plane, and are rapidly attenuated along the line, being of significance only near the terminals. The complimentary waves contribute to the total electromagnetic field of the transmission line, and produce radiation. Working from the antenna theory point of view, Schelkunoff[8], in an analogous way, produced expressions for complimentary waves, which are identified with the ends of an antenna.

The well known hyperbolic form of the basic transmission line equations has been extended by King[9] to what is called the "completely hyperbolic solution". In this form the over-all effect of multiple reflections at the two ends of the line are expressed as two quantities (instead of by the usual series), which can be added directly to the attenuation and phase shift of the line itself.

In the design work to be dealt with in this thesis, use will be made of the fact that the potential on any conductor of a line can be expressed as a function of the charges (or currents) in all the conductors, and their geometrical arrangement with respect to each other, and the ground plane. Both rigorous[10,11] and approximate[12] statements of this method have been made. It is also pertinent to the problem at hand to note that multi-con-

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8. Schelkunoff, S.A. "Principal and Complimentary Waves in Antennas" Proc. I.R.E. Vol. 34, pp. 23P-32P Jan. 1946

9. King, Ronald "Transmission Line Theory and Its Applications" Journ. App. Physics, Vol. 14, Nov. 1943

10. Carson, J.R., "Rigorous and Approximate Theories of Electrical Transmission Along Wires" Bell Sys. Tech. Journ. Vol. Vll, pp. 11-25, Jan. 1928.

11. King, R.W.P. "Electromagnetic Engineering" Vol.1, McGraw Hill 1945 pp. 472

12. Woodruff, L.F. "Electric Power Transmission", John Wiley and Sons, 1938, pp.74.

ductor transmission lines of all practical configurations, satisfy the basic transmission line equations if a pure transverse mode of propagation is assumed. This has been demonstrated by Frankel[13], who analyzed various line configurations using electromagnetic field theory, and produced equations of the basic form as given by our equations (1) and (2).

From a practical standpoint, radiation from a multi-conductor unbalanced line of four or more conductors is small enough to be neglected in most cases. Nevertheless, in order to make a complete assessment of the losses, some account of it will be taken here. It was found by Carson[14] that radiation from a uniform line was due entirely to the complimentary waves, but the amount of radiated energy could best be determined by calculating the distant field due to the principal wave. His paper included an explanation of this apparent paradox. The method of calculating the field was the vector potential method of Lorentz. Pistolkors[15] also applied this method and his results have been used by others[16, 17]. The theory is more directly applicable to balanced than to unbalanced lines, but results can be approximated for the latter. By employing the Poynting Vector method Brown[18] has also produced values for the radiation from unbalanced lines.

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13. Frankel, S. "Equation of Generalized Transmission Line", Electrical Communication, Vol. 23, No. 3, September 1946.
14. Carson, J.R. "The Guided and Radiated Energy in Wire Transmission" Trans. A.I.E.E. 1924.
15. Pistolkors, A.A. "The Radiation Resistance of Beam Antennas" Proc. I.R.E. Vol. 17, No. 3, March 1929.
16. Sterba E.J. and Feldman C.B. "Transmission Lines for Short-Wave Radio Systems" Proc. I.R.E. Vol. 20, No. 7, July 1932.
17. Whitmer, Robert. "Radiation Resistance of Concentric Conductor Transmission Lines", Proc. I.R.E. Vol. 21, No.9, Sept. 1933.
18. Footnote 3, Loc. cit.

To illustrate the method of design, let us consider a two-wire unbalanced line with conductors of radius ρ, spacing b, and for which the effect of the earth is neglected. See figure 1. Assuming that there is a charge, Q, on the live conductor, it will produce an electric induction density:-

$$D = \frac{Q}{2\pi r \times 1}$$ per unit length at distance, r, from it. And the electric intensity at the same distance is

$$E = \frac{Q}{2\pi \varepsilon r \times 1} \qquad (9)$$

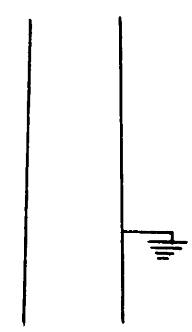

Fig. 1.

per unit length where $\varepsilon = \frac{1}{36\pi} \times 10^{-9}$ farads per meter, is the dielectric constant of free space.

But $E = -\frac{dV}{dr}$ is the voltage gradient.

The voltage on the live conductor with respect to the grounded conductor is, of course, the amount of work done in bringing unit charge from the grounded conductor up to it, i.e.,

$$V = \int_{\rho}^{b} -\frac{dV}{dr} = \frac{Q}{2\pi\varepsilon}\int_{\rho}^{b} \frac{1}{r}\, dr$$

$$= \frac{Q}{2\pi\varepsilon} \log_e \frac{b}{\rho}$$

Inserting the numerical value of E gives:-

$$V = 180 \times 10^8 \, Q \, \log_e \frac{b}{\rho} \quad \text{volts} \qquad (10)$$

if Q is in Coulombs per unit length. Equation (10) can be expressed as

$$V = 60 \times 3 \times 10^8 \, Q \, \log_e \frac{b}{\rho} = 60 \, cQ \, \log_e \frac{b}{\rho} \qquad (11)$$

where c, is the velocity of propagation in free space, and therefore a constant. From the basic transmission line equations we know that $Z_o = \sqrt{\frac{L}{C}}$

or very nearly so, for low-loss lines at radio frequencies.

Also $c = \frac{1}{\sqrt{LC}}$ so that $Z_o = \frac{1}{cC}$ (12)

The characteristic impedance can now be found from equation (11), for we know that $\frac{1}{C} = \frac{V}{Q}$

and therefore $Z_o = \frac{V}{cQ}$

Then, from equation (11) $Z_o = 60 \log_e \frac{b}{\rho}$

and changing from natural logarithms to base 10:-

$$Z_o = 138 \log \frac{b}{\rho} \qquad (13)$$

which is a familiar expression for an unbalanced line free from earth effects i.e., a concentric line.

Considering, now, a balanced two-wire line, if one conductor bears a charge, Q, the other will have a charge of, -Q. The work done in bringing unit charge from one conductor to the other will, therefore, be twice that for the unbalanced line, and as a result the potential difference between the two conductors will be doubled, i.e.,

$$V = 276 \; c \; Q \log \frac{b}{e}$$

$$\text{or} \quad Z_0 = 276 \log \frac{b}{e} \qquad (14)$$

Equation (14) will be recognized, too, as the expression for the characteristic impedance of a balanced line.

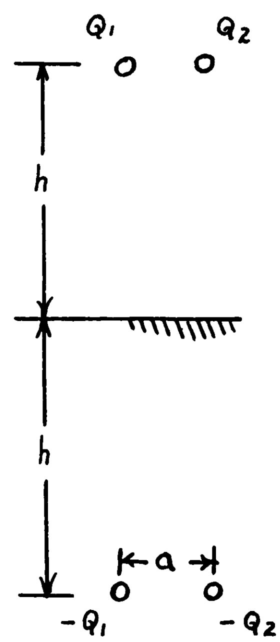

Fig. 2

Effect of the Earth

Now, let us assume that a two-wire line runs parallel to the earth at a height, h, above it, as sketched in Fig. 2. The charges on the two conductors are Q_1 and Q_2. By the well known image theorem, each charge will have on image charge a distance, h, below the surface of the earth, these image charges being $-Q_1$ and $-Q_2$. From the foregoing the voltage on conductor No. 1, is

$$V_1 = 138\, c \left(Q_1 \log \frac{1}{e} + Q_2 \log \frac{1}{a} - Q_1 \log \frac{1}{2h} - Q_2 \log \frac{1}{\sqrt{(2h)^2 + a^2}} \right) \qquad (15)$$

where the term $\sqrt{(2h)^2+a^2}$ is the distance from conductor No. 1, to the image charge $-Q_2$

Grouping terms of equation (15) the voltage is:-

$$V_1 = 138\,c\left(Q_1 \log \frac{2h}{\rho} + Q_2 \log \frac{\sqrt{(2h)^2+a^2}}{a}\right) \tag{16}$$

The conductor spacing, a, is normally very small compared to, 2h, so that with negligible error we can write:-

$$V_1 = 138\,c\left(Q_1 \log \frac{2h}{\rho} + Q_2 \log \frac{2h}{a}\right) \tag{17}$$

Similarly the voltage of conductor No. 2, may be written:-

$$V_2 = 138\,c\left(Q_2 \log \frac{2h}{\rho} + Q_1 \log \frac{2h}{a}\right) \tag{18}$$

For an unbalanced line one of the conductors is at ground potential. Calling wire No. 2 the grounded conductor, then $V_2 = 0$ or $Q_2 \log \frac{2h}{\rho} + Q_1 \log \frac{2h}{a} = 0$

Then the ratio of the charge Q_2 to charge Q_1 is:-

$$\frac{Q_2}{Q_1} = -\frac{\log \frac{2h}{a}}{\log \frac{2h}{\rho}} \tag{19}$$

Let, h, be 12 ft. or 144 inches; a, 10 inches; and ρ, 0.081 inch, which is the radius of #6 B. & S. gauge wire. Putting these values in equation (19) gives $\frac{Q_2}{Q_1} = -.411.$

But current is the rate of flow of charge, i.e. $I = cQ$, where c is the velocity of propagation. So the ratio of the ground-wire current to the current in the live wire,

$$\frac{I_{gw}}{I_{lw}} \text{ is } -.411.$$

The remainder of the return current must flow back through the earth, i.e.

$$\frac{I_{gd}}{I_{lw}} = -.589$$

From equation (17) for conductor No. 1 the characteristic impedance can be found as follows:-

$$Z_o = \frac{1}{cC} = \frac{V}{cQ} = 138\left(\log\frac{2h}{\rho} - 0.411\log\frac{2h}{a}\right)$$

$$= 138(3.55 - 0.6) = 407.5 \text{ ohms}$$

If the effect of the earth could be neglected, as could be done for a concentric line, the impedance given by the usual expression would be $Zo = 138 \log \frac{a}{\rho} = 288.6$ ohms.

At the risk of repetition a three-wire line will be worked out before proceeding to a ten-wire line, but this time the transmission losses will be determined. By choosing a three-wire system the method of treating multi-conductor lines will be made clear without introducing long equations. The line is represented by Fig. 3. Conductors 1 and 3 are grounded, hence they are at zero potential, and their charges are equal. The voltage on the live conductor is:-

$$V_2 = 138c\left[Q_2 \log \frac{2h}{\rho} + Q_1 \log\left(\frac{2h}{a}\right)^2\right] \qquad (20)$$

and on either grounded conductor the voltage is:-

$$V_1 = 138c\left[Q_1 \log \frac{(2h)^2}{2a\rho} + Q_2 \log \frac{2h}{a}\right] = 0$$

Let ρ = .081 in., h = 144 in., and a = 1.825 in.

Then $V_1 = 5.447\,Q_1 + 2.198\,Q_2 = 0$ which gives the ratio $\frac{Q_1}{Q_2} = \frac{2.198}{5.447} = -.404$ since there are two conductors at charge Q_1 the ratio of the total ground-wire charge to the charge on the live wire is

$$\frac{2Q_1}{Q_2} = -.808$$ and this is equal to $\frac{I_{gw}}{I_{lw}}$

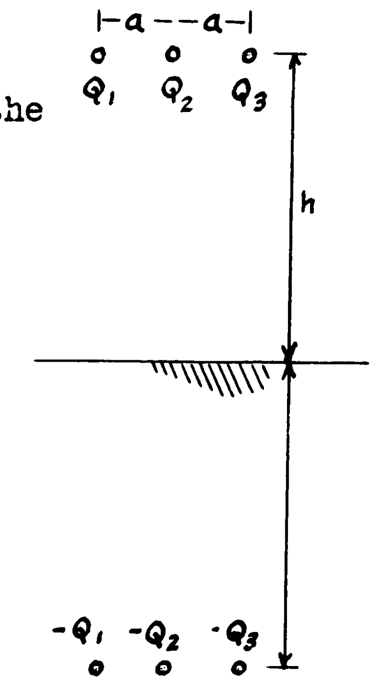

Fig. 3

This ratio will be called k. From equation (20) the characteristic impedance can be found:-

$$Z_0 = \frac{V_2}{cQ_2} = 138\left[\log \frac{2h}{\rho} + \frac{Q_1}{Q_2} \log\left(\frac{2h}{a}\right)^2\right]$$

$$= 138\left(3.550 - .404\,(4.396)\right) = 245.3 \text{ ohms}$$

COPPER LOSSES

In calculating the attenuation due to the resistance of the conductors, the resistance at the operating frequency must be used. It is therefore necessary to consider the depth of penetration of the current which is expressed by

$$\delta = \sqrt{\frac{2}{\mu \omega \sigma}}$$ meters, where μ is the permeability of free space and is equal to $4\pi \times 10^{-7}$ henries per meter.

The conductivity, σ, is taken as 97 percent of the conductivity of standard annealed copper, which is an average value for medium hard copper. Accordingly, σ is 5.74 X 10^7 mhos per meter cube. The expression for skin depth can be simplified to:

$$S = \frac{\sqrt{10^7}}{2\pi\sqrt{f\sigma}} \quad \text{meters.} \tag{21}$$

The resistance of a cylindrical conductor is $R = \frac{1}{2\pi\rho S\sigma}$ ohms per meter, where ρ is its radius in meters. Inserting the expression for skin depth from (21) gives:-

$$R = \sqrt{\frac{f}{10^7}} \; \frac{1}{\rho\sqrt{\sigma}} \quad \text{ohms per meter.}$$

If the unit of frequency is changed to megacycles; the radius to inches, and the length to one foot, R is $\frac{\sqrt{f_{mc.}}}{1995\,\rho}$ ohms per foot. Using this value of resistance the attenuation of the line can be written[19]:-

$$\alpha_c = \frac{2.17\sqrt{f_{mc.}}}{\rho\, Z_o}\left[\frac{1}{m} + \frac{1}{n}\left(\frac{I_{gw}}{I_{lw}}\right)^2\right] \quad \text{db. per 1000 ft.} \tag{22}$$

where m and n are the number of live conductors and grounded conductors respectively. Substituting the data for the three-wire line in equation (22), and assuming a frequency of 1.6 mc., the attenuation due to the copper losses is:-

$$\alpha_c = \frac{2.17\sqrt{1.6}}{.081(245.3)}\left[\frac{1}{1} + \frac{1}{2}(-.808)^2\right] = 0.184 \quad \text{db. per 1000 ft.}$$

19. See Appendix 1.

EARTH CURRENT LOSSES

The unbalance current in the transmission line, which is equal to the current returning through the earth, can be regarded, for analytical purposes, as a current flowing in a single-conductor transmission line of the same mean height. This fact and the distribution of current density in the earth directly below the line were determined by Brown[20], who derived the following formula for the attenuation of the earth current:-

$$\alpha_e = \frac{13.720}{Z_0\, h_{ft.}}\left[1 - |k|\right]^2 \sqrt{\frac{10^{-13} f_{mc}}{\sigma_{e.m.u.}}} \text{ db. per 1000 ft.} \qquad (23)$$

Inserting the necessary quantities for the three-wire line in equation (23), and assuming an earth conductivity of 40×10^{-15} e.m.u. results in:-

$$\alpha_e = \frac{13\,720}{(245.3)\,12}\,(.192)^2 \sqrt{\frac{10^{-13}\, 1.6}{40 \times 10^{-15}}} = 0.344 \text{ db. per 1000 ft.}$$

RADIATION LOSSES:

There are two ways of approaching the radiation problem, the Poynting - vector method and the induced e.m.f. method. Radiation due to the unbalance current can be treated by the Poynting vector method in accordance with antenna theory. It can also be approximated by regarding the unbalance current and the return current in the earth as forming a balanced line where the conductors are separated by a distance, 2h, i.e: the distance

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20. See Appendix 11.

between the conductor and its image. In addition, it would seem reasonable to regard the current which flows in the grounded conductors as one side of a balanced line, with the live wires carrying an equal current as the other side. On that assumption the radiation due to both these factors could then be superimposed. Heretofore it seems that only the unbalance current has been considered in calculating radiation, from an unbalanced line.

To apply the Poynting-vector method, the field strength[21] at a point in space is first determined. It is:-

$$F = 37.25\,\frac{2\pi h}{\lambda}\,I_{gd}\cos\theta\,\frac{\sqrt{-2\cdot\cos\left[\frac{2\pi L}{\lambda}\left(1 - \sin\theta\cos\phi\right)\right]+2}}{(1-\sin\theta\cos\phi)} \qquad (24)$$

where θ, is the angle measured from the zenith and ϕ, is the horizontal angle measured from the transmission line, and L is the length of the line. Taking the sum of the Poynting-vectors over the surface of a hemisphere, for a length of line, $L = \frac{\lambda}{2}$ gives:-

$$P_{rad} = 3500\ I_{gd}^{2}\left(\frac{h}{\lambda}\right)^{2} \text{ watts}$$

and for $L = \lambda$

$$P_{rad} = 5250\ I_{gd}^{2}\left(\frac{h}{\lambda}\right)^{2} \text{ watts}$$

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21. Footnote 3, Loc. cit. and see Appendix 111.

To calculate the radiation from the three-wire lines, assume that the current supplied to the line is 10 amperes. Then the unbalance current will be $.192 \times 10 = 1.92$ amperes. And the power radiated from the line due to the unbalance current at a frequency of 1.6 mc. is:-

$$3500\,(1.92)^2 \frac{(12)^2}{(615)^2} = 4.9 \text{ watts},$$

The value of the current I_{gw} would be $.808 \times 10 = 8.08$. From a radiation point of view this may be considered as a balanced current in the line. The expression[22] giving the radiation loss from a balanced line is:-

$$P_{rad} = 160\, I^2 \left(\frac{\pi a}{\lambda}\right)^2$$ where a, is the spacing between conductors. In the three-wire line the current I_{gw} would be divided between the two grounded conductors, therefore, the radiated power due to it would be half that for a two-wire line, i.e.,

$$P_{rad} = 80\,(8.08)^2\,(.00079)^2 = .003 \text{ watts}$$

The power into the line would be $I^2 Z_0$ or

$$10^2 \times 245.3 = 24{,}530 \text{ watts}$$

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22. Footnote 16 loc. cit.

DESIGN OF TEN-WIRE LINE

In the particular problem of designing a multi-conductor unbalanced line for high power transmitters it was decided to start by determining the characteristic impedance and losses of a six-wire line for three sizes of conductor, namely #4, #6 and #8, B & S gauge. The same procedure was then followed for an eight-wire line and a ten-wire line. These lines are sketched in Figures 4, 5 and 6 respectively. The spacing of the conductors was decided upon from a knowledge of existing six-wire lines; the required characteristic impedance; and from preliminary calculations. By using two live conductors at the center of the line, it is possible to maintain a wide spacing between conductors without making the characteristic impedance too high. An impedance range of 175 to 250 ohms is common for the output of broadcast transmitters and for antenna-matching units. The second live conductor also provides a safety factor, and permits a higher voltage on the line before corona starts.

The dimensions of the ten-wire line are shown in Figure 6. The "messenger" cable and the conductors supported by it will be discussed later. The grounded conductors of the transmission line proper are equally spaced on the circumference of a circle whose center is mid-way between the two live conductors.

For the initial design calculations a diameter of 15 inches for this circle was used. In the line as installed this diameter was 14 7/16 inches due to the manner of clamping the grounded conductors to a steel ring. This value will be used in the calculations which follow. However, the tabulated data and charts in the section "Calculated Results" are for a line of 15 inches diameter.

With reference to Fig. 6, the charges will be numbered to correspond to the numbering of the conductors. The mean height of the line is 12 ft. and this value can be used for calculating the effect of the image charges with negligible error.

Applying the method previously described the voltage on either live conductor may be written:-

$$V_1 = 138c\left[Q_1 \log\frac{2h}{e} + Q_2 \log\frac{2h}{b} + Q_3 \log\frac{2h}{y} + Q_4 \log\frac{2h}{z} + Q_8 \log\frac{2h}{y} + Q_7 \log\frac{2h}{z} + Q_5 \log\frac{2h}{p} + Q_6 \log\frac{2h}{p} + Q_9 \log\frac{2h}{x} + Q_{10} \log\frac{2h}{x}\right] \tag{25}$$

It is clear from the symmetry of the line that the charges on the two live conductors are equal, i.e., $Q_1 = Q_2$. For the same reason $Q_3 = Q_4 = Q_7 = Q_8$ and $Q_5 = Q_6 = Q_9 = Q_{10}$

Using these equalities and grouping terms:-

$$V_1 = 138c\left[Q_1 \log\frac{(2h)^2}{eb} + Q_3 \log\frac{(2h)^4}{y^2z^2} + Q_5 \log\frac{(2h)^4}{p^2x^2}\right] \tag{26}$$

The voltage on conductor 3, or on any of the other three wires bearing the same charge is:-

$$V_3 = 138c\left[Q_3 \log\frac{(2h)^4}{\rho ade} + Q_1 \log\frac{(2h)^2}{yz} + Q_5 \log\frac{(2h)^4}{ac^2d}\right] = 0 \qquad (27)$$

and for any conductor bearing a charge equal to Q_5 the voltage is:-

$$V_5 = 138c\left[Q_5 \log\frac{(2h)^4}{\rho ade} + Q_1 \log\frac{(2h)^2}{xp} + Q_3 \log\frac{(2h)^4}{ac^2d}\right] = 0 \qquad (28)$$

The calculations will be made for a conductor-size of #6, B & S gauge, i.e., the radius ρ = .081 inch. By putting the required values in equation (27) it becomes:-

$$V_3 = 7.90356\,Q_3 + 3.19466\,Q_1 + 5.95809\,Q_5 = 0 \qquad (29)$$

and equation (28) becomes

$$V_5 = 5.95809\,Q_3 + 3.21178\,Q_1 + 7.90356\,Q_5 = 0 \qquad (30)$$

To facilitate calculating the ratio of ground wire charges to live wire charges, equations (29) and (30) will be expressed as:-

$$V_3 = 7.90356\,\frac{Q_3}{Q_1} + 5.95809\,\frac{Q_5}{Q_1} = -3.19466 \qquad (31)$$

$$V_5 = 5.95809\,\frac{Q_3}{Q_1} + 7.90356\,\frac{Q_5}{Q_1} = -3.21178 \qquad (32)$$

From these two equations the two ratios $\frac{Q_3}{Q_1}$ & $\frac{Q_5}{Q_1}$ can be found.

They are:-

$$\frac{Q_3}{Q_1} = -0.22668$$

$$\frac{Q_5}{Q_1} = -0.23548$$

Since there are twice as many grounded conductors at charge Q_3, and at charge Q_5, as there are live conductors at charge Q_1, the ratio of the total ground-wire charge to the total live-wire charge, k, is

$$\frac{2\,(Q_3 + Q_5)}{Q_1} = -\,0.92434 \qquad (33)$$

It is now possible to find the characteristic impedance from equation (26) which, by inserting values, becomes:-

$$V_1 = 138c \quad 5.61236\,Q_1 + 6.38932\,Q_3 + 6.42356\,Q_5 \qquad (34)$$

Since there are two wires at charge Q_1, the capacity between these wires and the other conductors is $C = \frac{2Q_1}{V_1}$.

The characteristic impedance is: $Z_o = \frac{1}{cC} = \frac{V_1}{2Q_1 c}$.

Then using equation (34) :.

$$Z_o = 69\left[5.61236 + 6.38932\,\frac{Q_3}{Q_1} + 6.42356\,\frac{Q_5}{Q_1}\right]$$

Putting in the values obtained for the charge ratios $\frac{Q_3}{Q_1}$ & $\frac{Q_5}{Q_1}$

$$Z_o = 69\,[2.64850] = 182.74670 \text{ ohms.}$$

or to an accuracy of practical interest, $Z_0 = 182.7$ ohms

The attenuation due to copper losses can now be computed from the formula previously given:-

$$\alpha_c = \frac{2.17\sqrt{f_{mc.}}}{e\ Z_o}\left(\frac{1}{m} + \frac{1}{n}\frac{k^2}{\ }\right) \quad \text{db. per 1000 feet}$$

Choosing a frequency of 1.6 mc. the attenuation is

$$\alpha_c = \frac{2.17\sqrt{1.6}}{.081 \times 182.7467}\left(\frac{1}{2} + \frac{-.92434}{8}^{2}\right) = .1127/\text{db. per 1000 ft.}$$

The attenuation of the earth current is calculable from equation (23), which is

$$\alpha_e = \frac{13,720}{Z_o\ h_{ft.}}\left(\frac{I_{gd}}{I_{1w}}\right)^2 \sqrt{\frac{10^{-13} f_{mc.}}{\delta_{e.m.u.}}} \quad \text{db/1000 ft.}$$

The ratio $\dfrac{I_{gd}}{I_{1w}} = \dfrac{1-|k|}{1} = .07566$

At a frequency of 1.6 mc., and for a ground conductivity of, say, 40×10^{-15} e.m.u.

$$\alpha_e = \frac{13,720}{182.7467 \times 12}(.07566)^2\sqrt{\frac{10^{-13}\ 1.6}{40 \times 10^{-15}}} = .0717 \text{ db./1000 ft.}$$

It will be noted that the frequency and earth conductivity chosen, represent about the worst conditions, as far as losses are concerned, likely to be encountered. It will be of interest to compare these two loss components with those for the three-wire line, previously worked out, using the same values of frequency, earth conductivity, and conductor size.

	Three-Wire Line	Ten-Wire Line
α_c db./1000 ft.	0.184	0.113
α_e db./1000 ft.	0.344	0.072
α TOTAL	0.528	0.185

On a percentage basis there would be a loss of 12.90 percent of the transmitted power in a 1000 ft. of the three-wire line compared 4.4 percent in the same length of ten-wire line.

The characteristic impedance and the losses are affected by the height of the line, as will be apparent from the voltage equations. As the height, h, is increased, the effect of the earth diminishes, so that the characteristic impedance tends toward the value for a concentric line, and the earth-return current decreases. For the 10-wire line of #6, B & S gauge wire, rough calculations show that, for h = 18 feet, the ratio k would be -0.934; Z_o = 204 ohms, and the combined earth current and copper losses are 0.118 db. per 1000 feet. Corresponding figures for h = 12 ft., are: k = -.924; Z_o = 182.7 ohms, and the combined losses are 0.185 db. per 1000 ft.

Ten-Wire Line with Messenger Group

On the ten-wire line as installed, a messenger cable of wire rope was supported on the same poles, at a distance of three feet below the center of the transmission line. This messenger cable was used to support several conductors for auxiliary circuits, namely:-

1. Lead-covered cable, 7/8", in diameter, containing several pairs for audio frequency and telephone circuits.

1 Lead-covered cable, 1 1/8", in diameter containing four, #4 wires, to supply A.C. power for tower lighting etc.

1 or 2 Radio-frequency leads of either 3/8" copper co-axial line or type RG-12U flexible co-axial cable, for supplying sampling current from the towers to a phase monitor in the transmitter building. These, of course, were only used for installations having a two-tower array.

The messenger cable group would be expected to have some effect on the impedance of the line since a small part of the return current would flow in it. Furthermore, this portion of the current, if large enough, could induce some radio-frequency voltage in the cables supported by the messenger cable. Consequently it is of some interest to ascertain the effects produced by the cable group. This was estimated by considering them as additional grounded conductors, since the messenger cable and the outer metallic sheaths of the other cables are at gound potential. The mean distance between the messenger group and the transmission line proper was taken as 38 inches. The height of the messenger group above the surface of the ground is, then, 106 inches or .736 h. Its image is a similar distance below the surface. Calling the charge on each cable, Q_{11}, the mean distance from the transmission line proper to the image charge, $-Q_{11}$, is 1.736 h. And the Q_{11} charges are separated from their images by $2 \times .736\, h = 1.472\, h$.

Their effect on the voltage of one of the live wires, for example, were included in the voltage equation for that conductor, i.e.

$$V_1 = 138c \left[Q_1 \log \frac{(2h)^2}{eb} + Q_3 \log \frac{(2h)^4}{y^2 z^2} + Q_5 \log \frac{(2h)^4}{p^2 x^2} + Q_{11} \frac{(1.736h)^5}{38^5} \right] \qquad (35)$$

The equations for V_3 and V_5 were similarly modified. Furthermore an equation for the voltage on any conductor in the messenger group was written:-

$$V_{11} = 138c \left[Q_{11} \log \frac{(1.473h)^5}{\rho_1 D_2 D_3 D_4 D_5} + Q_1 \log \frac{(1.736h)^2}{38^2} + Q_3 \log \frac{(1.736h)^4}{38^4} + Q_5 \log \frac{(1.736h)^4}{38^4} \right] = 0$$

where ρ_1, is the mean radius of the cables and D_2, D_3, D_4, D_5, are the diameters of the other cables in the group; the assumption being that all the cables are touching each other. It is clear that this method gives only approximate results, but the magnitude of the effect of the cables is so small, that it would hardly be justifiable to calculate the coefficient of Q_{11} for the individual cables and then take the average.

From the equations for V_3, V_5 and V_{11} the ratio of the ground wire charges to the live wire charges was determined, with the following results:-

$$Q_3 = -.22333\ Q_1$$

$$Q_5 = -.23215\ Q_1$$

$$Q_{11} = -.01017\ Q_1$$

The ratio of the number of wires at charge Q_3 and at charge Q_5, to the number at charge Q_1 is 2. The ratio of the wires at charge Q_{11} to those at charge Q_1 is $\frac{5}{2}$ or 2.5.

$$\text{Therefore, } k = \frac{2Q_3 + 2Q_5 + 2.5\,Q_{11}}{Q_1} = -.9364$$

The characteristic impedance, obtained by dividing equation (35) by $2Q_1c$ is:-

$$Z_o = 182.643 \text{ ohms,}$$

which is only a fraction of an ohm lower than for the line without the messenger group.

In the formula for copper losses:-

$$\alpha_c = \frac{2.17\sqrt{f_{mc.}}}{\rho\, Z_o}\left[\frac{1}{m} + \frac{1}{n}\left(\frac{I_{gw}}{I_{1w}}\right)^2\right] \text{ db. per 1000 feet}$$

it is assumed that I_{gw} is equally divided amongst the n grounded conductors. Since the cables of the messenger group carry much smaller currents than the grounded conductors of the line, they can-not be added directly to make a new value for n. In Appendix lV the effect of the cables in the messenger group, carrying small currents which total 5 X .01017 = .05085 I_{1w}, is calculated to be approximately equal to $\frac{1}{10}$ of the effect of one of the grounded conductors proper. Accordingly n is made 8.1 instead of 8. Then the copper loss becomes

$$\frac{2.17\sqrt{1.6}}{.081 \times 182.643}\left[\frac{1}{2} + \frac{1}{8.1}(.936)^2\right] = .11301 \text{ db. per 1000 ft.}$$

Earth Current Losses:-

Recalling the expression for earth current attenuation:-

$$\alpha_e = \frac{13{,}720}{Z_o\, h_{ft}}\left(\frac{I_{gd}}{I_{1w}}\right)^2\sqrt{\frac{10^{-13} f_{mc}}{\sigma_{e.m.u.}}} \text{ db./1000 feet}$$

this loss amounts to:-

$$\frac{13{,}720}{182.643 \times 12}(.0636)^2\sqrt{\frac{10^{-13} \times 1.6}{40 \times 10^{-15}}} = .05065 \text{ db. per 1000 ft.}$$

The combined loss $\alpha_c + \alpha_e$ = .16426 db. per 1000 ft. This loss is 12 percent less than the loss of the line without the messenger cable.

Induction in Messenger Cable Group

We can now consider the effect on the cables themselves, of the small fraction of return current which flows in each of them. Since $Q_{11} = -.01017\ Q_1$, it follows that each of the cable sheaths carries approximately one percent of the total current in the line. In terms of power this amounts to

$$I^2 Z_o = .0001 \times 182.643 = .01826 \text{ watts},$$

for each ampere of line current.

For a transmitter power of 50 kilowatts the line current would be:-

$$\sqrt{\frac{P}{Z_o}} = \frac{50000}{182.6} = 16.6 \text{ amperes}$$

The corresponding average power in the sheath of one of the cables would be 16.6 X .01826 = 0.32 watts. In decibels above a reference of one milliwatt, the power level would be $10 \log \frac{.32}{.001} = 25$ db. This power level would not induce voltages sufficient to cause interference in the audio-frequency circuits within one of the lead-covered cables.

Radiation Losses

The current, I_{gd}, which is equal to $I_{1w} - I_{gw}$, will cause radiation since it is an "unbalance" current in the line. It is equivalent to an equal current[23] flowing in a single conductor at the same mean height as the transmission line. On this assumption the expression[24] for radiated power, from a length of line equal to one-half wave length, is:-

$$3500\ I_{gd}^{2}\left(\frac{h}{\lambda}\right)^{2}$$

In the ten-wire line, carrying 50 kilowatts,

$$I_{gd} = (1-0.924) \times 16.6 = 1.26 \text{ amperes}$$

Considering a frequency of 1.6 megacycles, for which $\frac{\lambda}{2}$ is 307.5 feet, the power radiated is:-

$$3500\ (1.26)^{2}\ \left(\frac{12}{615}\right)^{2} \approx 2.1 \text{ watts}$$

This value is for the line without the messenger cable group. The radiation from the line with the messenger group would be less, since the value of I_{gd} is smaller.

As mentioned previously, the "balanced" currents, i.e. currents equal to I_{gw} flowing in the "go" and "return" conductors will contribute to radiation. However, this loss is extremely small. Supposing, for example, the line were a

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23. 24. See Appendix 111.

two-conductor balanced line carrying a current equal to I_{gw}, the radiated power is expressed as:-[25,26]

$$160\ I_{gw}^2 \left(\frac{\pi a}{\lambda}\right)^2$$

where a, is the spacing between conductors. Putting a = 7.25 in., which is the average spacing in the 10-wire line, the value of radiated power, also at 1.6 mc., is

$$160 \times (15.3)^2 (.0031)^2 = 0.36 \text{ watts}$$

The radiation from the 10-wire line under the same conditions would be less than the above, because the currents are divided amongst several conductors, and the grounded conductors being on all sides of the live conductors, would tend to cancel the fields of each other.

For comparison we may consider the radiation loss from the 6-wire line of #6, B & S gauge wire, having approximately the same outer diameter and conductor-spacing as the ten-wire line. Under the same operating conditions as given above the radiation due to "unbalance" current would be:-

$$3500\ (3.06)^2 \frac{(12)^2}{(615)^2} = 12.4 \text{ watts}$$

for 307.5 feet of line. The impedance of the six-wire line is 231 ohms, and the ratio $\frac{I_{gw}}{I_{lw}}$ is -.792.

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25. Footnote 15 loc. cit

26. Footnote 16, loc. cit.

It is evident that radiation from the 10-wire line, can be neglected from a loss standpoint, and that very little trouble should be experienced due to the mutual impedance between the transmission lines, where they run close to each other to feed multi-tower arrays.

Owing to the smallness of the radiation loss it was not included in the tabulated data and charts.

Calculated Results

The calculated results for the 6-wire, 8-wire and 10-wire lines are plotted in charts #1 to #9. Curves are shown for the three sizes of wire #4, #6 and #8, B & S gauge. The following characteristics will be noted:-

1. There is a marked difference in earth current losses between the three types of line and a comparatively small difference in copper losses. See charts #1 and #2.
2. With regard to the combined Earth Current and Copper losses, there is a large gain in efficiency by using the 8-wire line instead of the 6-wire type, and a smaller gain by using the 10-wire system instead of the 8-wire line. See Chart #3 to Chart #9.

FIGURE # 4

TITLE SIX-WIRE UNBALANCED TRANSMISSION LINE

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DATE APRIL 12/49 ISSUE

SCALE 1/2" & 3" = 1'-0" BY J. L. M.

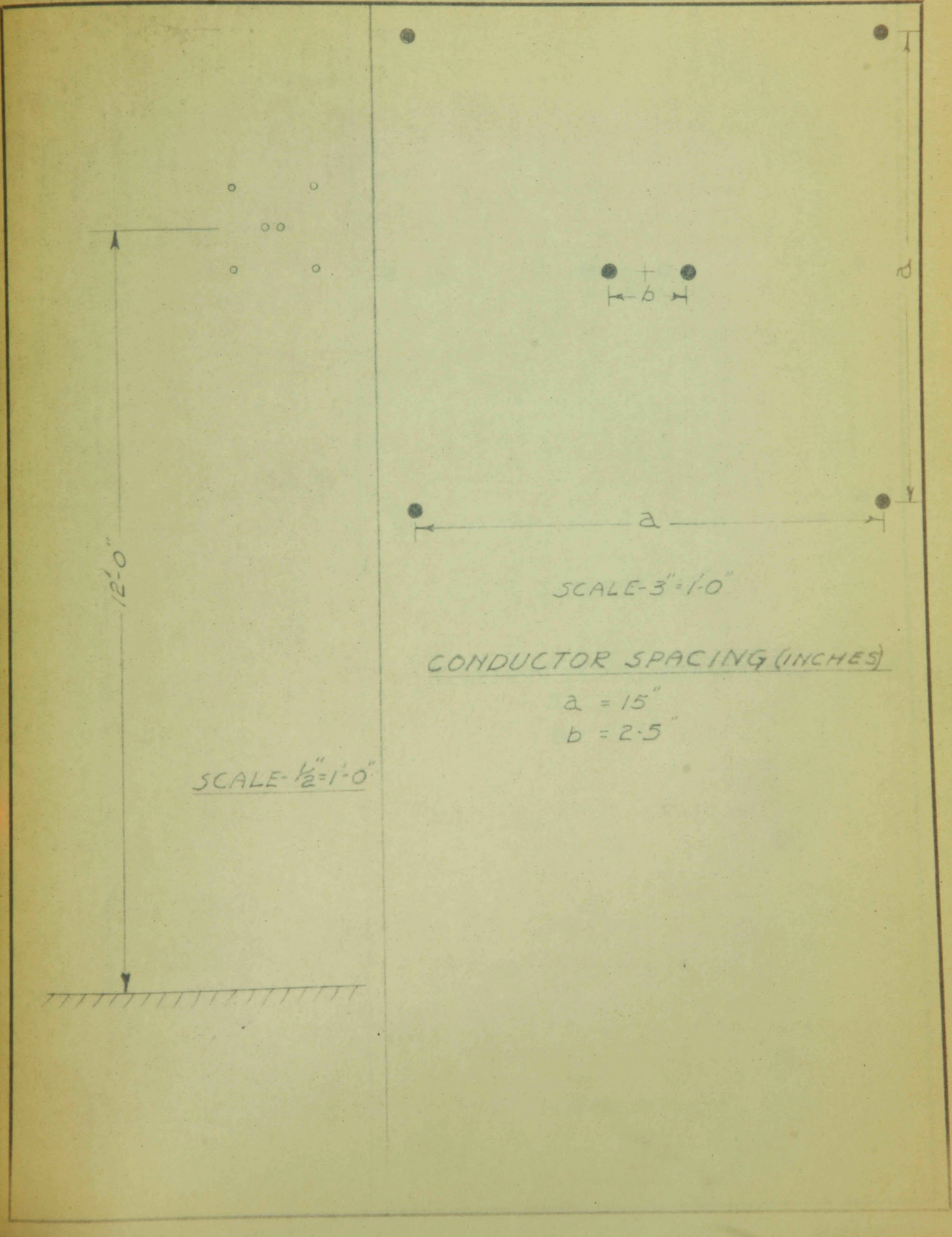


FIGURE #5

TITLE EIGHT-WIRE UNBALANCED TRANSMISSION LINE

DWG NO. Page 37

DATE APRIL 12/49 ISSUE

SCALE ½" & 3"=1'-0" BY J.L.M.

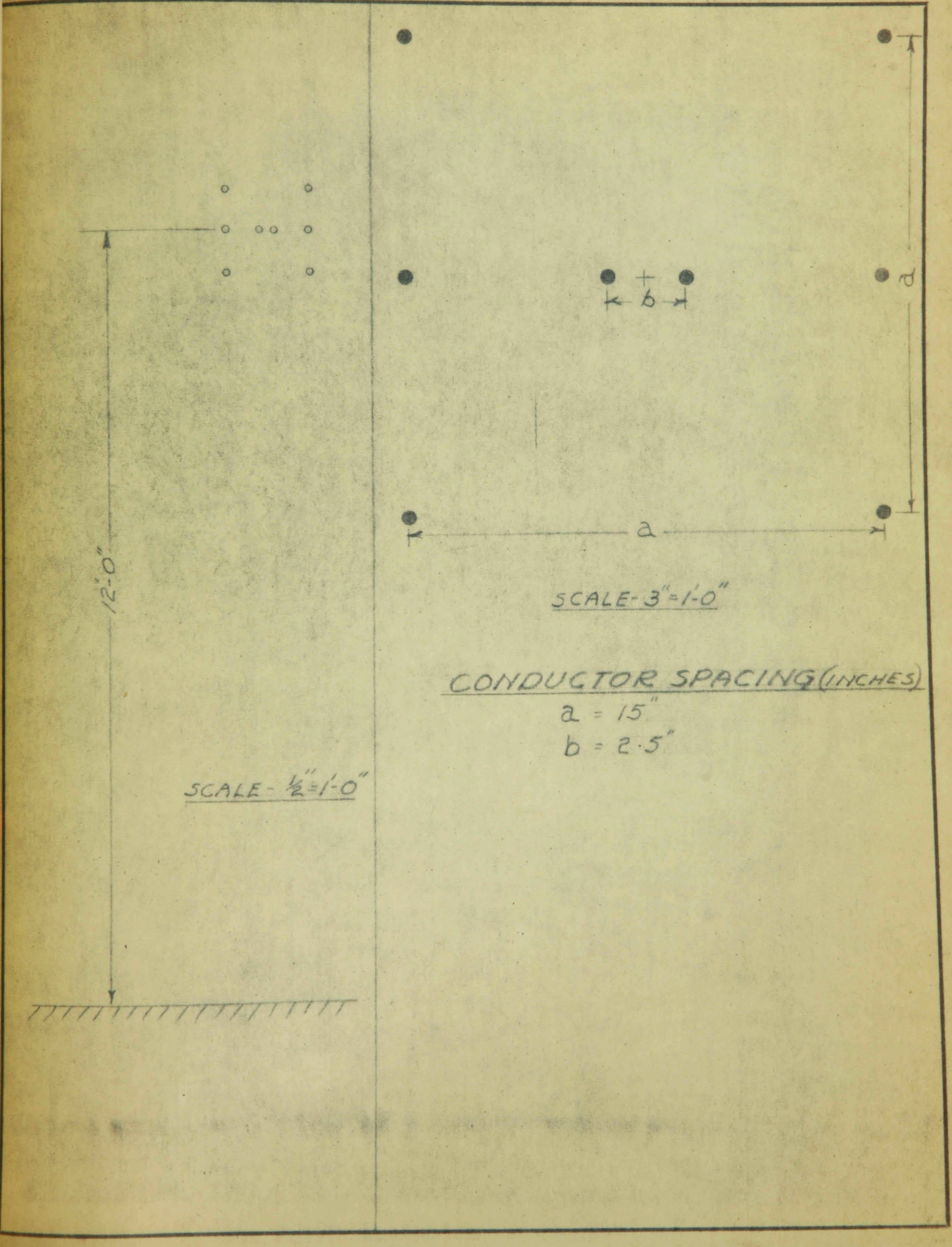


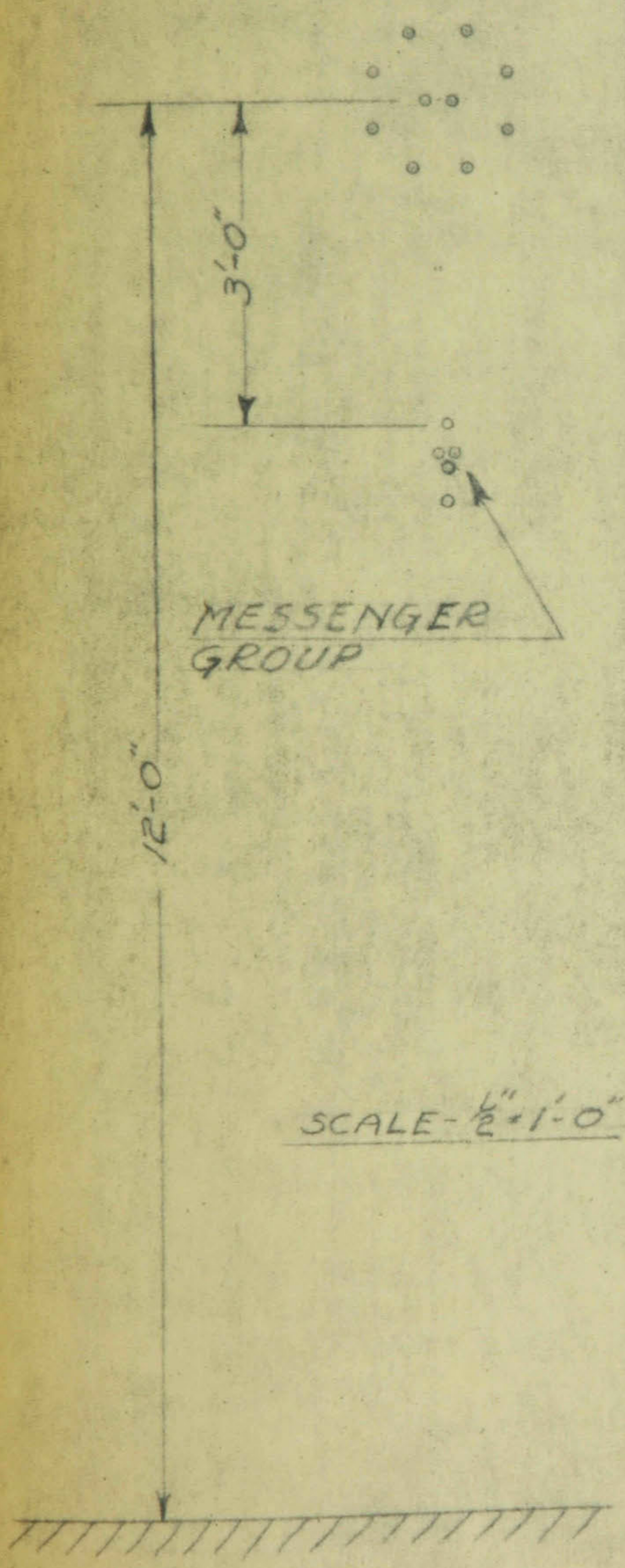
FIGURE #6

TITLE TEN-WIRE UNBALANCED TRANSMISSION LINE

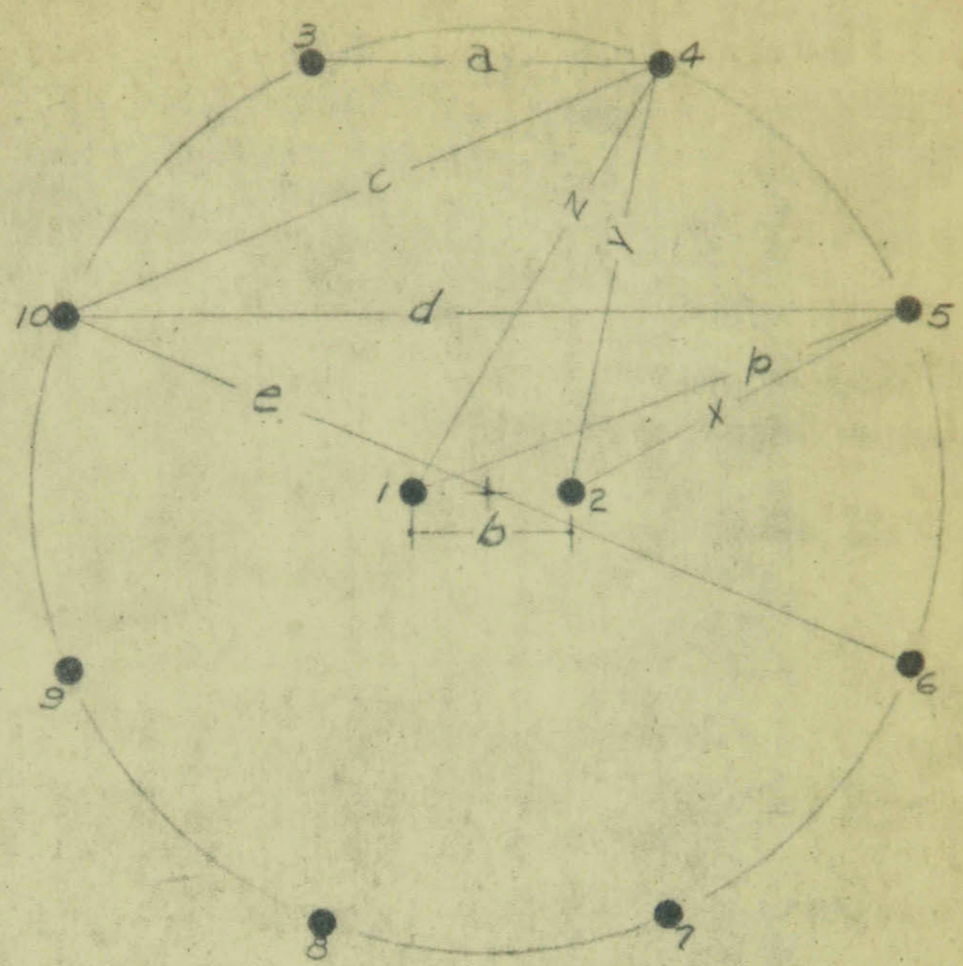
DWG NO. Page 38

DATE APRIL 12/49 ISSUE

SCALE 1/2" & 3" = 1'-0" BY J.L.M.



SCALE - 1/2" = 1'-0"



SCALE - 3" = 1'-0"

CONDUCTOR SPACINGS (INCHES)

a = 5.531
b = 2.500
c = 10.156
d = 13.281
e = 14.4375 (DIAMETER)
p = 8.380
x = 6.078
y = 6.810
z = 7.780

NOTE:

TABULATED DATA AND CHARTS ARE FOR LINE WITH DIAMETER, e = 15 IN. AND NO MESSENGER GROUP.

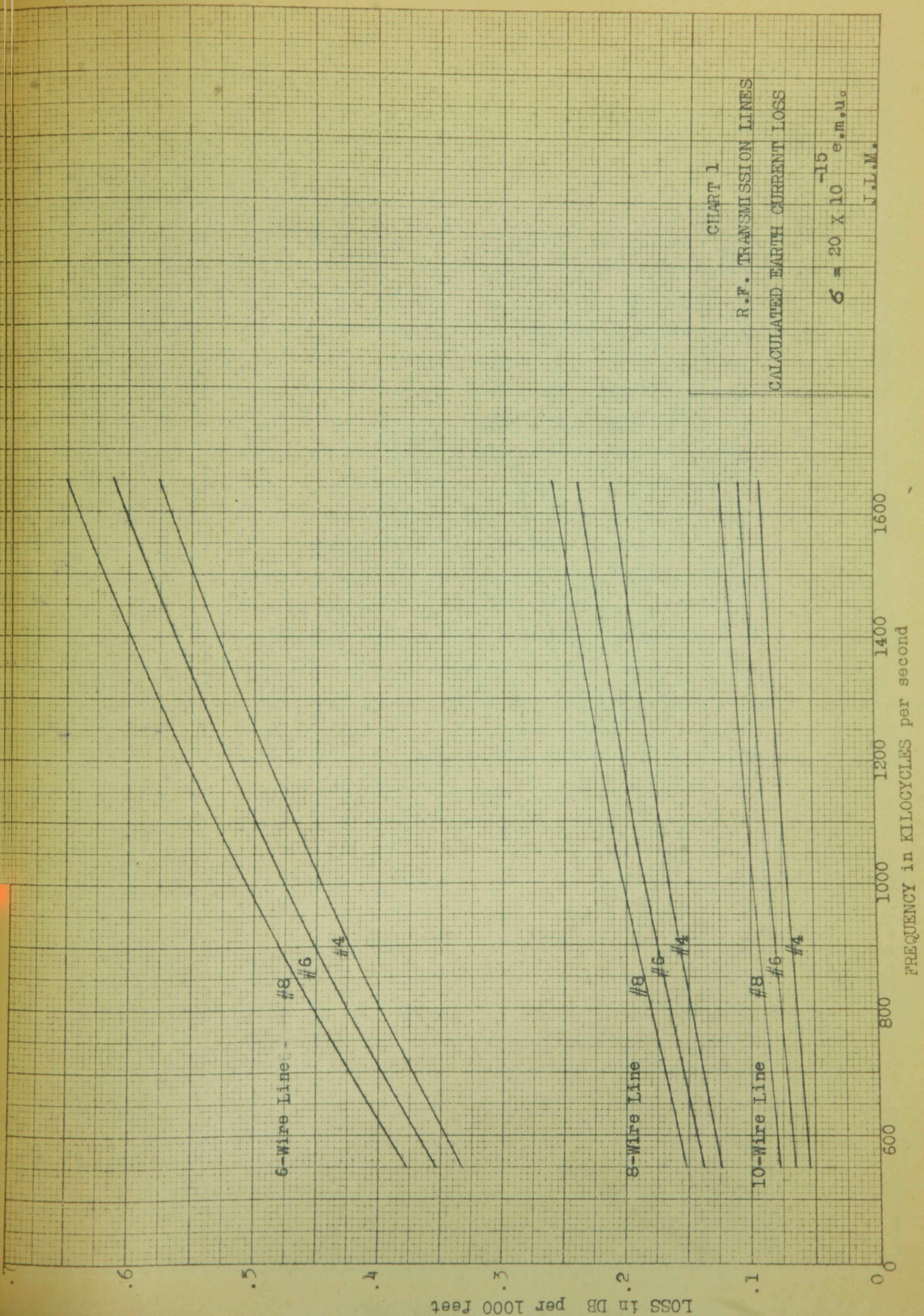


CHART 1
R.F. TRANSMISSION LINES
CALCULATED EARTH CURRENT LOSS
$\sigma = 20 \times 10^{-15}$ e.m.u.
J.L.M.
6-Wire Line
8-Wire Line
10-Wire Line
#8
#6
#4
LOSS in DB per 1000 feet
FREQUENCY in KILOCYCLES per second
0
.1
.2
.3
.4
.5
.6
.7
600
800
1000
1200
1400
1600

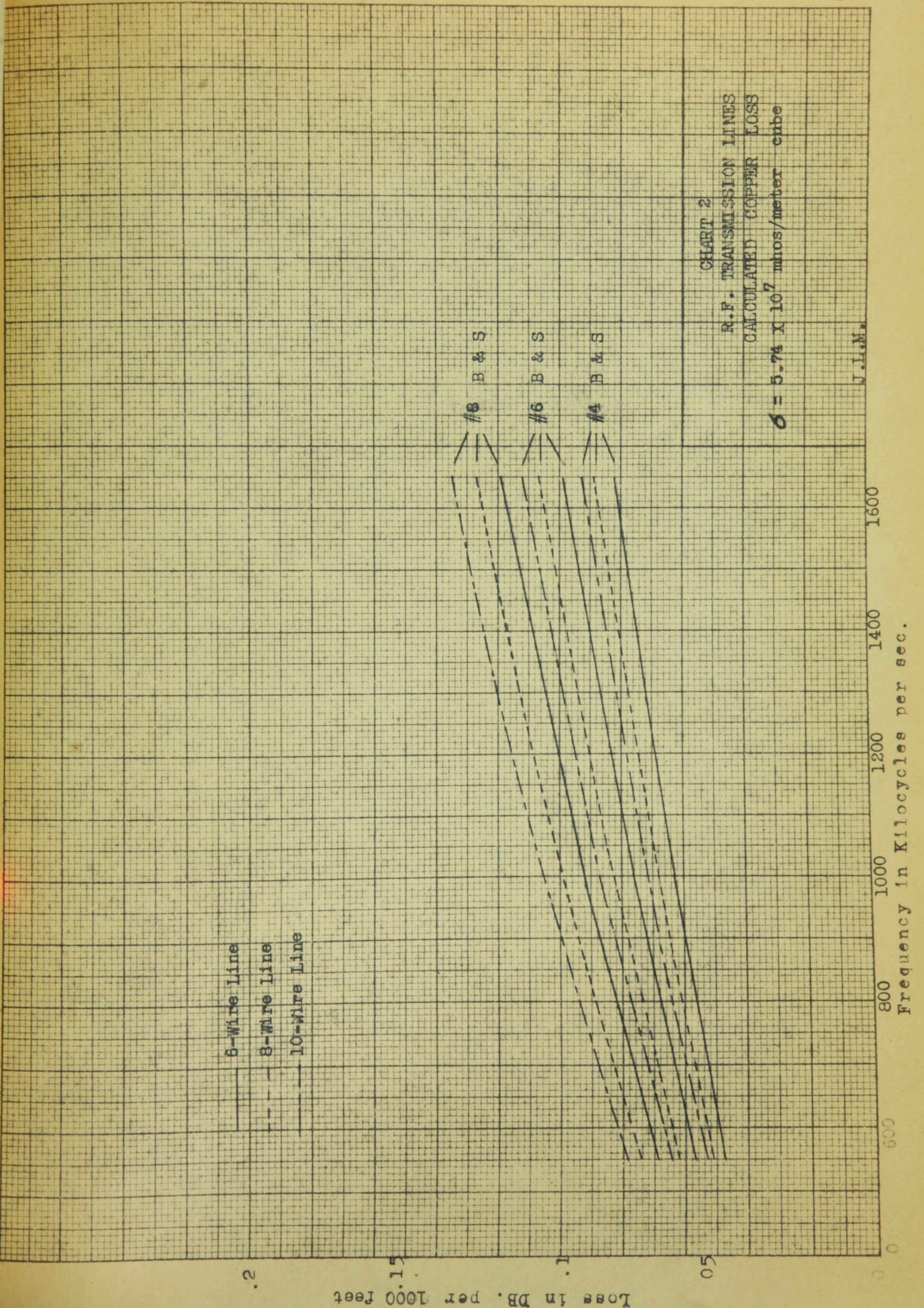


CHART 2
R.F. TRANSMISSION LINES
CALCULATED COPPER LOSS
σ = 5.74 X 10^7 mhos/meter cube
J.L.M.
#8 B & S
#6 B & S
#4 B & S
6-Wire Line
8-Wire Line
10-Wire Line
0
600
800
1000
1200
1400
1600
Frequency in Kilocycles per sec.
.05
.1
.15
.2
Loss in DB. per 1000 feet

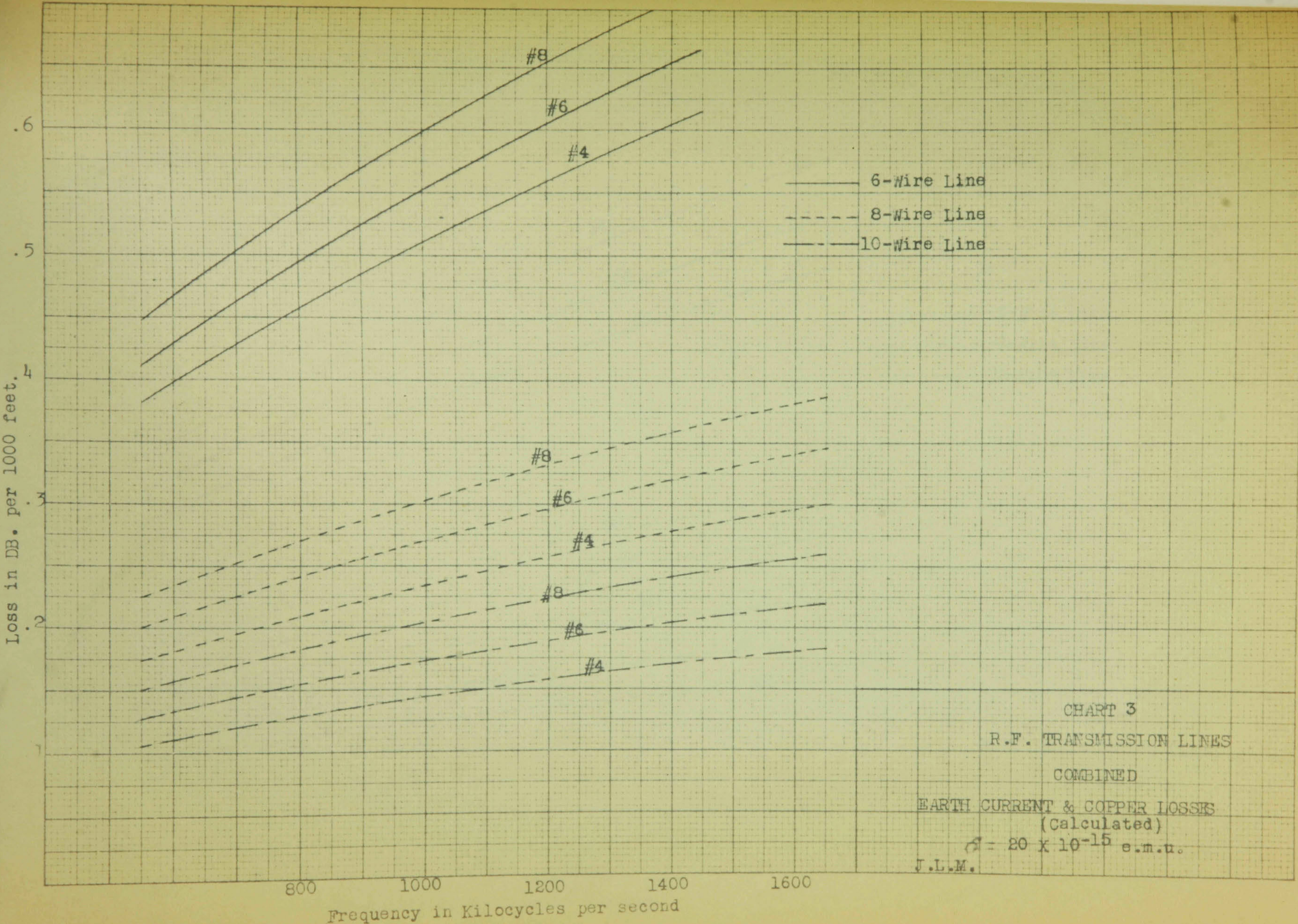
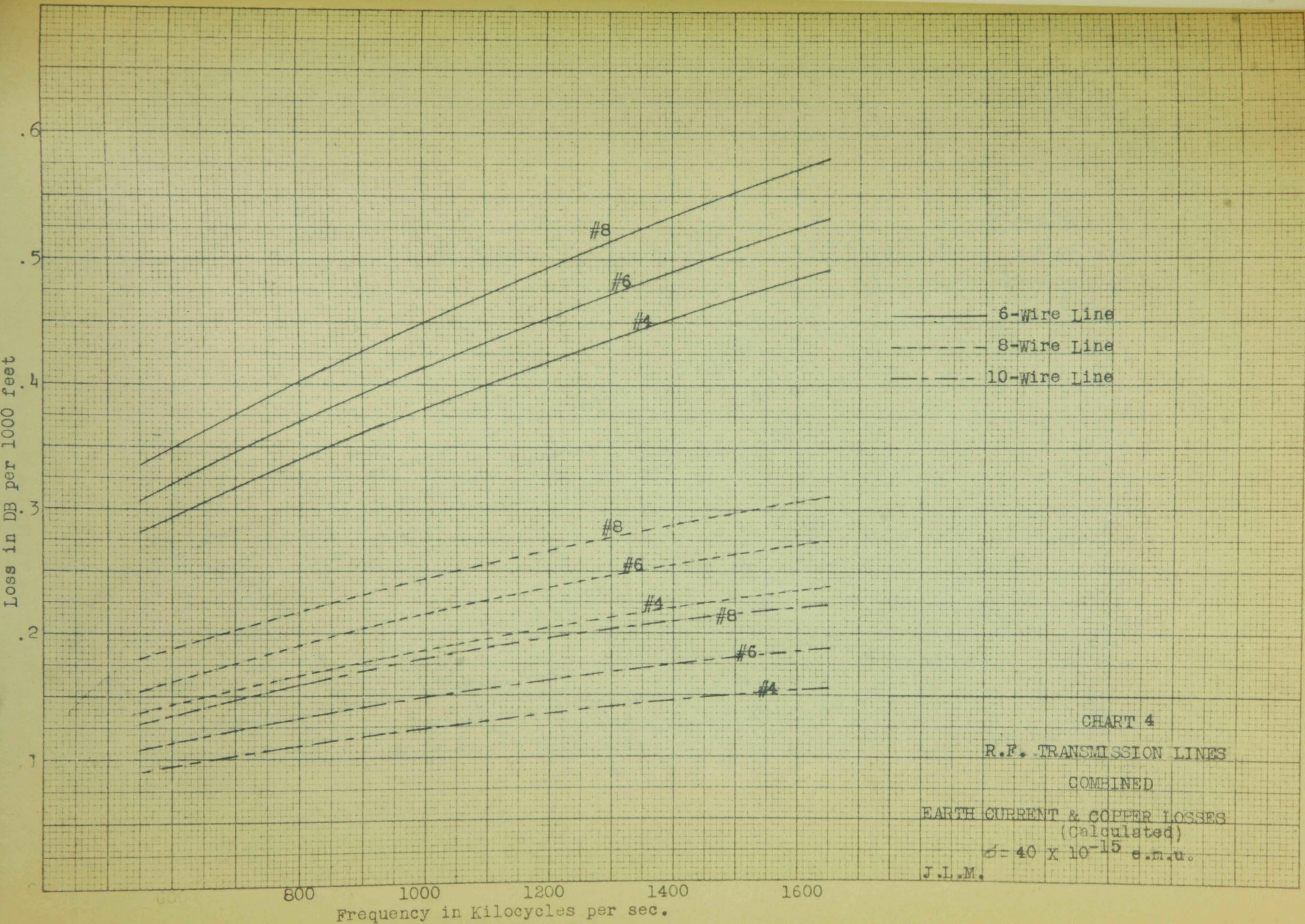


CHART 3
R.F. TRANSMISSION LINES
COMBINED
EARTH CURRENT & COPPER LOSSES
(Calculated)
$\sigma = 20 \times 10^{-15}$ e.m.u.
J.L.M.
6-Wire Line
8-Wire Line
10-Wire Line
#8
#6
#4
#8
#6
#4
#8
#6
#4
.6
.5
4
.3
.2
.1
800
1000
1200
1400
1600
Loss in DB. per 1000 feet.
Frequency in Kilocycles per second



.6
.5
.4
.3
.2
.1
Loss in DB per 1000 feet
800
1000
1200
1400
1600
Frequency in Kilocycles per sec.
#8
#6
#4
#8
#6
#4
#8
#6
#4
6-Wire Line
8-Wire Line
10-Wire Line
CHART 4
R.F. TRANSMISSION LINES
COMBINED
EARTH CURRENT & COPPER LOSSES
(Calculated)
σ = 40 X 10-15 e.m.u.
J.L.M.

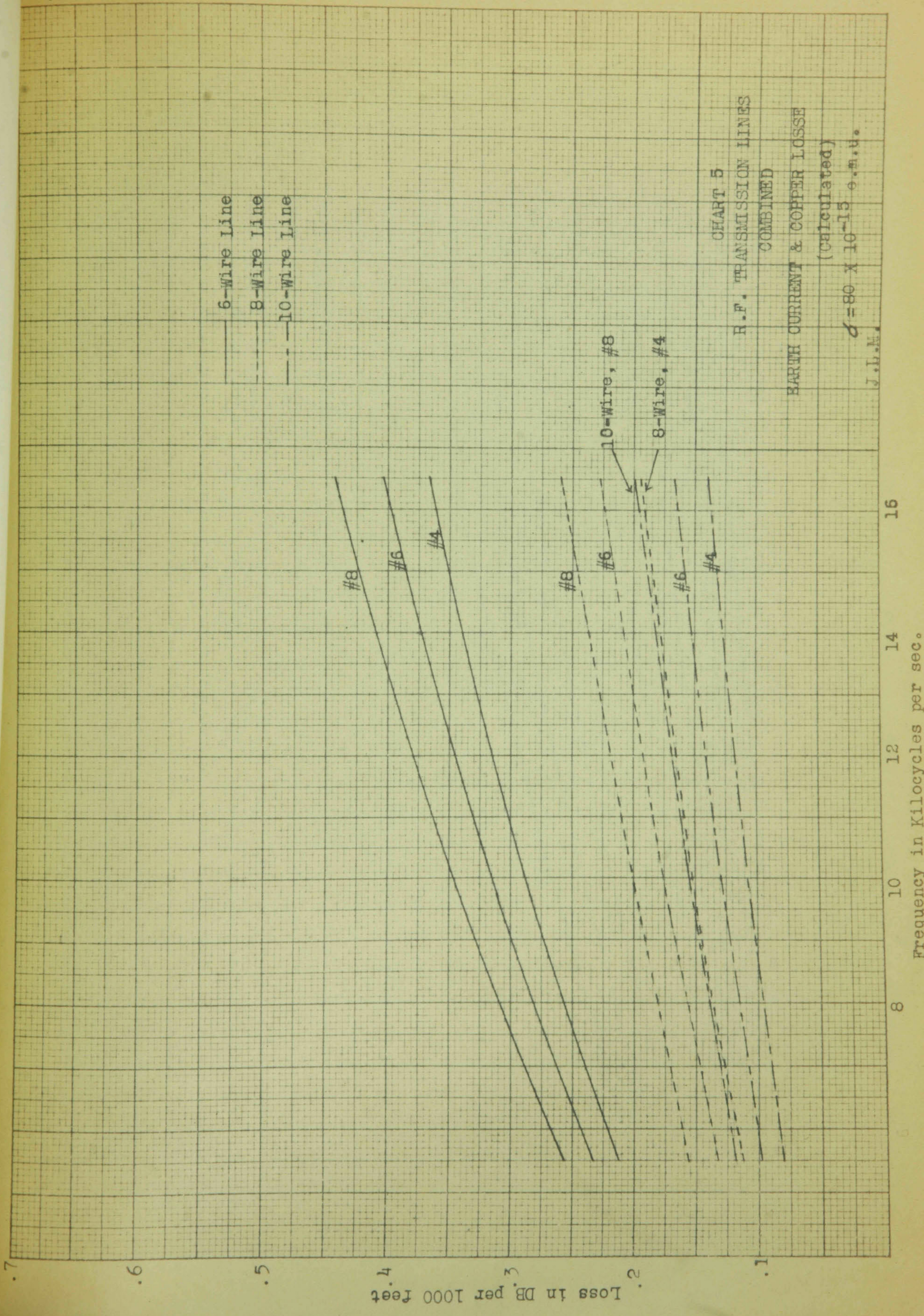


CHART 5
R.F. TRANSMISSION LINES
COMBINED
EARTH CURRENT & COPPER LOSSE
(Calculated)
σ = 80 X 10^-15 e.m.u.
J.L.M.
6-Wire Line
8-Wire Line
10-Wire Line
10-Wire, #8
8-Wire, #4
#8
#6
#4
8
10
12
14
16
Frequency in Kilocycles per sec.
.1
.2
.3
.4
.5
.6
.7
Loss in DB per 1000 feet

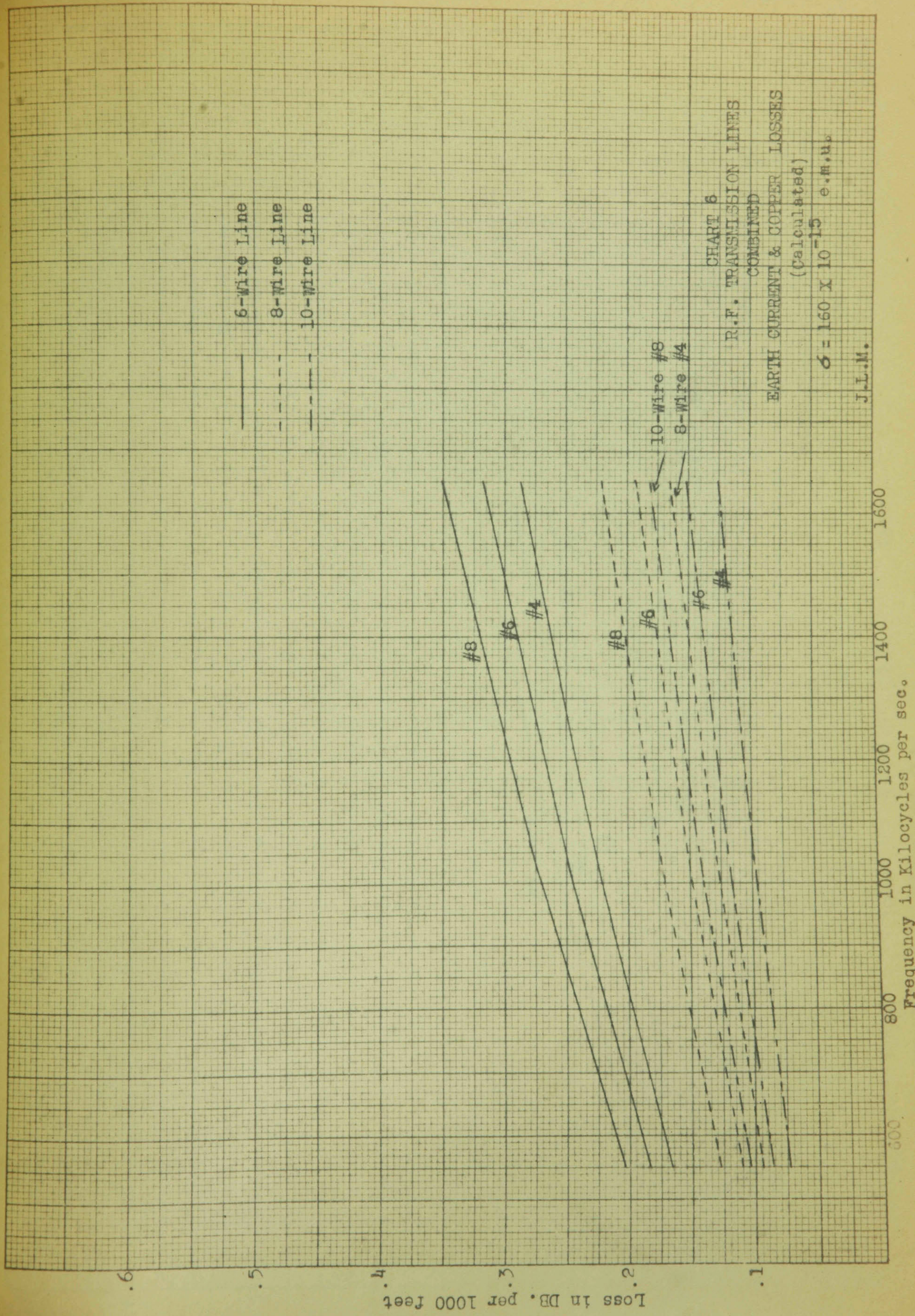


CHART 6
R.F. TRANSMISSION LINES
COMBINED
EARTH CURRENT & COPPER LOSSES
(Calculated)
$\sigma = 160 \times 10^{-15}$ e.m.u.
J.L.M.
6-Wire Line
8-Wire Line
10-Wire Line
10-Wire #8
8-Wire #4
#8
#6
#4
#8
#6
#6
#4
600
800
1000
1200
1400
1600
Frequency in Kilocycles per sec.
.1
.2
.3
.4
.5
.6
Loss in DB. per 1000 feet

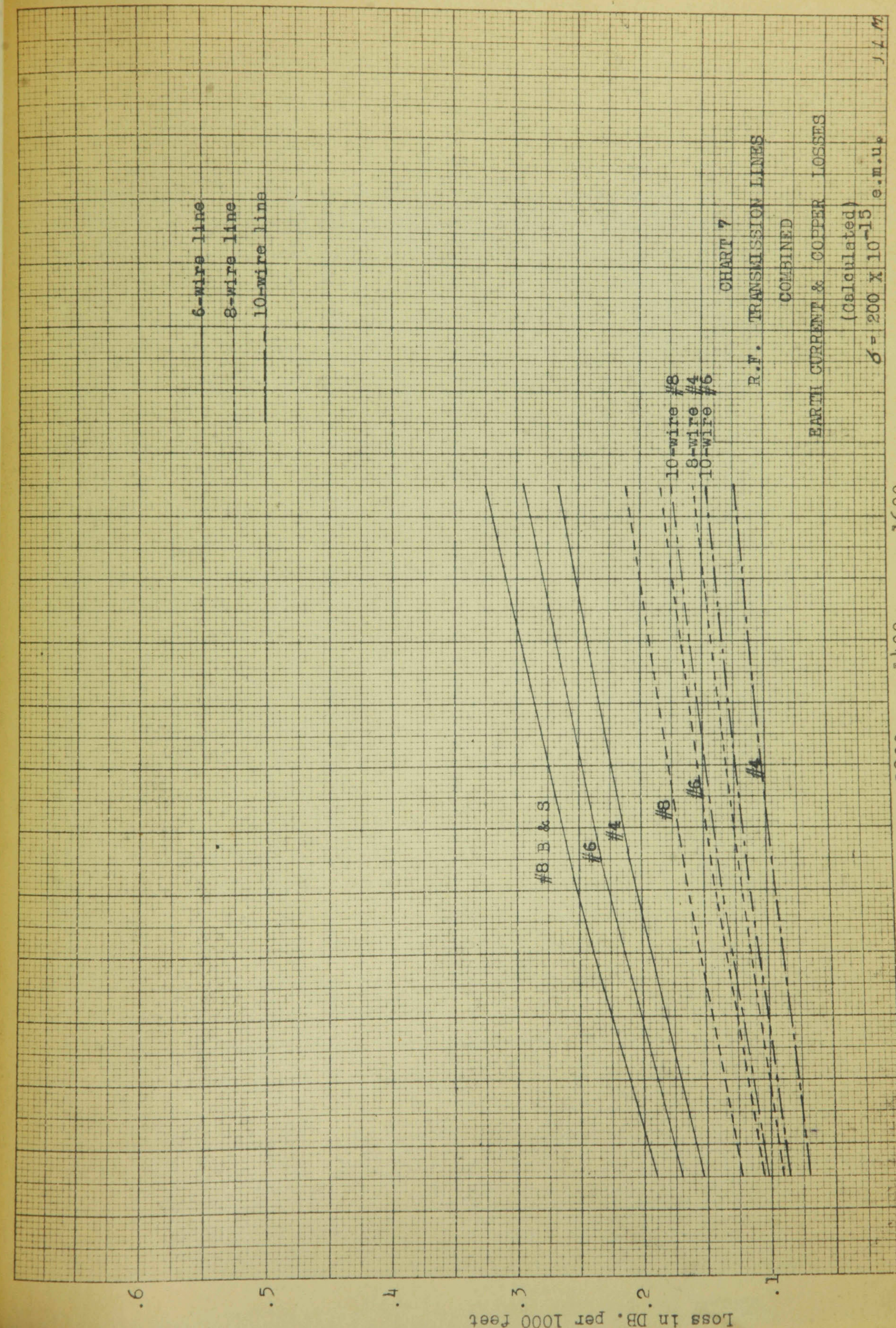


CHART 7
R.F. TRANSMISSION LINES
COMBINED
EARTH CURRENT & COPPER LOSSES
(Calculated)
σ = 200 x 10^-15 e.m.u.
6-wire line
8-wire line
10-wire line
#8 B & S
#6
#4
#8
#6
#4
10-wire #8
8-wire #4
10-wire #6
600
800
1000
1200
1400
1600
Frequency in Kilocycles per sec.
.1
.2
.3
.4
.5
.6
Loss in DB. per 1000 feet
JLM

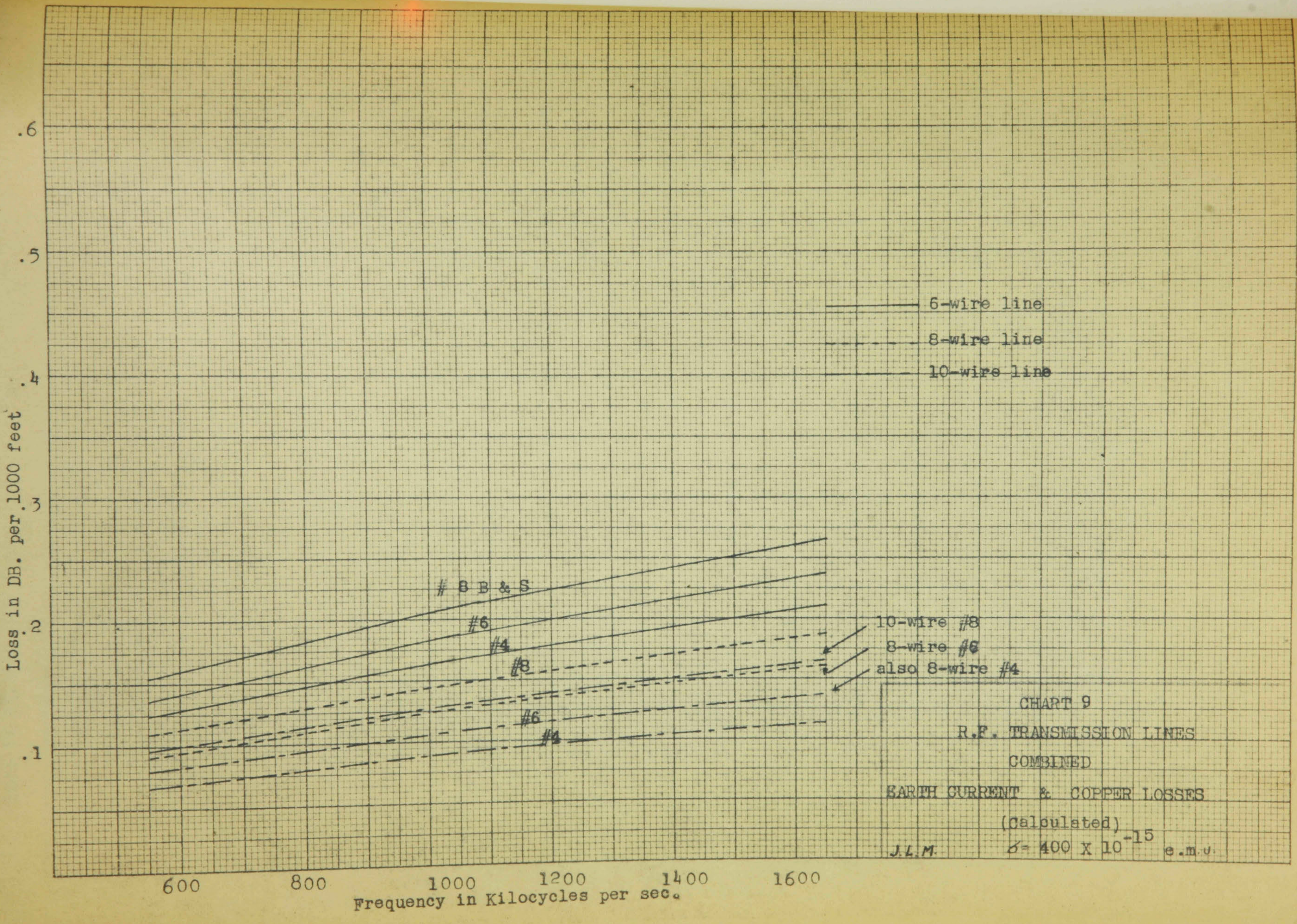


CHART 9
R.F. TRANSMISSION LINES
COMBINED
EARTH CURRENT & COPPER LOSSES
(Calculated)
σ = 400 X 10^-15 e.m.u
J.L.M.
6-wire line
8-wire line
10-wire line
8 B & S
#6
#4
#8
#6
#4
10-wire #8
8-wire #6
also 8-wire #4
Loss in DB. per 1000 feet
.1
.2
.3
.4
.5
.6
600
800
1000
1200
1400
1600
Frequency in Kilocycles per sec.

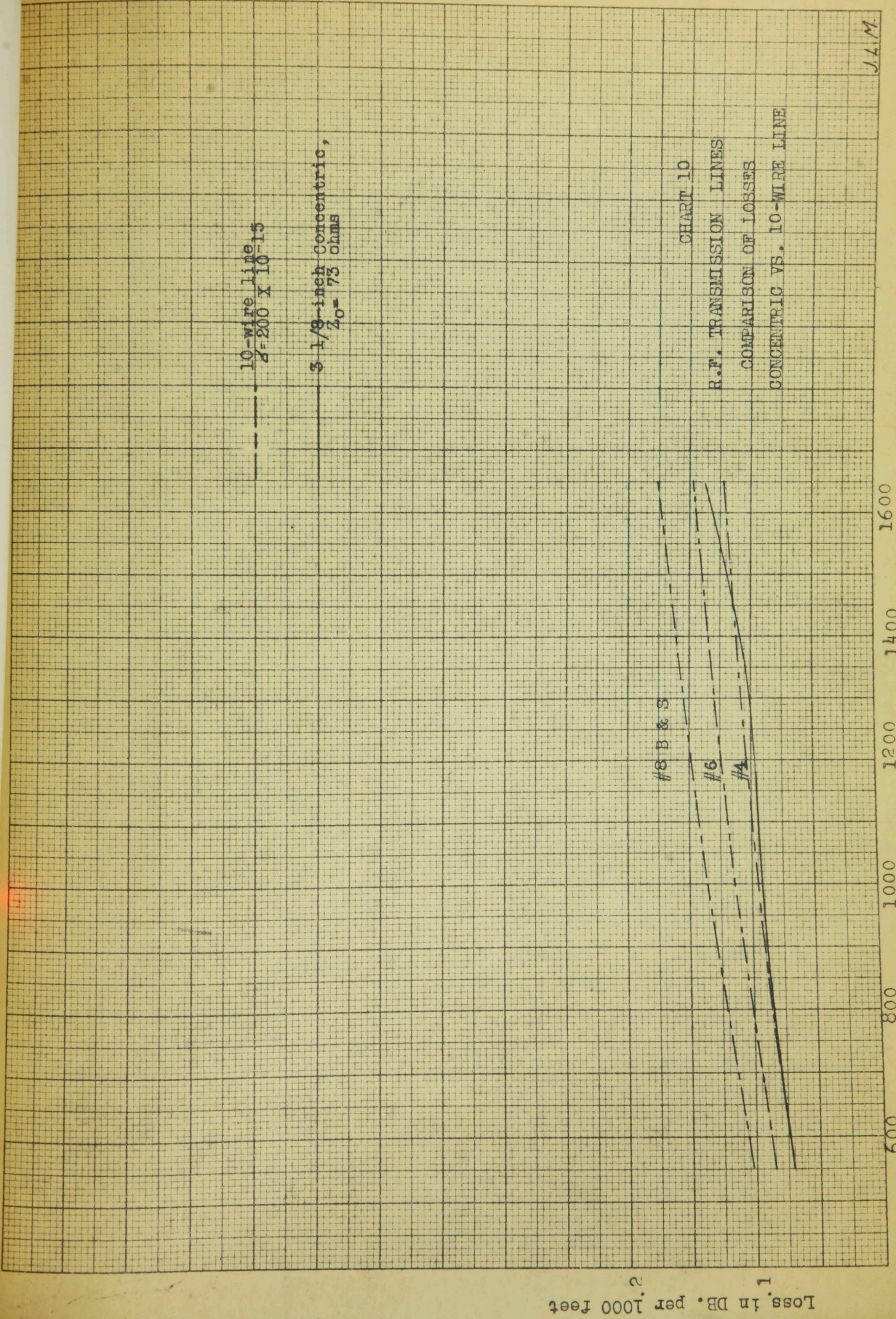


CHART 10
R.F. TRANSMISSION LINES
COMPARISON OF LOSSES
CONCENTRIC VS. 10-WIRE LINE
10-wire line, Z=200 X 10-15
3 1/8-inch concentric, Zo= 73 ohms
#8 B & S
#6
#4
600
800
1000
1200
1400
1600
Frequency in Kilocycles per sec.
.2
.1
Loss in DB. per 1000 feet
J.L.M.

[redacted]

TITLE

[redacted]

DWG. No.

DATE APRIL 1/49 ISSUE

SCALE BY J. L. M.

6 - Wire Line - Earth Current
& Copper Losses, DB. per 1000 Ft.

Wire Size	Z_0	$\sigma \times 10^{-15}$	600 Kc. α_c	α_e	$\alpha_{TOT.}$	800 Kc. α_c	α_e	$\alpha_{TOT.}$	1000 Kc. α_c	α_e	$\alpha_{TOT.}$	1200 Kc. α_c	α_e	$\alpha_{TOT.}$	1400 Kc. α_c	α_e	α_{TOT}	1600 Kc. α_c	α_e	α_{TOT}
No. 4	222	20	.3461	.0490	.3951	.3996	.0566	.4562	.4468	.0633	.5101	.4895	.0693	.5388	.5287	.0749	.6036	.5651	.0801	.6452
		40	.2447	"	.2937	.2826	"	.3392	.3160	"	.3793	.3461	"	.4154	.3748	"	.4497	.3997	"	.4798
		60	.1998	"	.2488	.2507	"	.2873	.2580	"	.3213	.2826	"	.3519	.3060	"	.3809	.3263	"	.4064
		80	.1731	"	.2221	.1998	"	.2564	.2234	"	.2867	.2447	"	.3140	.2647	"	.3396	.2826	"	.3627
		120	.1413	"	.1903	.1632	"	.2198	.1824	"	.2457	.1998	"	.2691	.2161	"	.2910	.2307	"	.3108
		160	.1222	"	.1712	.1411	"	.1977	.1580	"	.2213	.1730	"	.2423	.1872	"	.2621	.2000	"	.2801
		200	.1092	"	.1582	.1262	"	.1828	.1413	"	.2046	.1549	"	.2242	.1675	"	.2424	.1789	"	.2590
		300	.0893	"	.1383	.1030	"	.1596	.1152	"	.1785	.1262	"	.1955	.1368	"	.2117	.1460	"	.2261
		400	.0774	"	.1264	.0894	"	.1460	.0999	"	.1632	.1095	"	.1788	.1184	"	.1933	.1264	"	.2065
No. 6	231	20	.3686	.0590	.4276	.4256	.0681	.4937	.4759	.0762	.5521	.5213	.0834	.6047	.5631	.0901	.6532	.6020	.0964	.6984
		40	.2602	"	.3192	.3006	"	.3687	.3365	"	.4127	.3685	"	.4519	.3990	"	.4891	.4260	"	.5224
		60	.2125	"	.2715	.2438	"	.3119	.2748	"	.3510	.3015	"	.3849	.3260	"	.4161	.3480	"	.4444
		80	.1841	"	.2431	.2127	"	.2808	.2380	"	.3142	.2605	"	.3439	.2820	"	.3721	.3013	"	.3977
		120	.1505	"	.2095	.1738	"	.2419	.1943	"	.2705	.2126	"	.2960	.2302	"	.3203	.2459	"	.3423
		160	.1304	"	.1894	.1506	"	.2187	.1683	"	.2445	.1842	"	.2676	.1995	"	.2896	.2130	"	.3094
		200	.1164	"	.1754	.1345	"	.2026	.1505	"	.2267	.1649	"	.2483	.1782	"	.2683	.1902	"	.2866
		300	.0950	"	.1540	.1098	"	.1779	.1229	"	.1991	.1345	"	.2179	.1457	"	.2358	.1555	"	.2519
		400	.0824	"	.1414	.0951	"	.1632	.1064	"	.1826	.1166	"	.2000	.1260	"	.2161	.1348	"	.2312
No. 8	240	20	.3921	.0712	.4633	.4528	.0822	.5350	.5062	.0919	.5981	.5546	.1007	.6532	.5990	.1087	.7077	.6403	.1162	.7565
		40	.2772	"	.3484	.3200	"	.4022	.3580	"	.4499	.3920	"	.4927	.4249	"	.5336	.4530	"	.5692
		60	.2262	"	.2974	.2617	"	.3439	.2923	"	.3842	.3202	"	.4209	.3470	"	.4557	.3701	"	.4863
		80	.1960	"	.2672	.2261	"	.3083	.2531	"	.3450	.2770	"	.3777	.3000	"	.4087	.3201	"	.4363
		120	.1600	"	.2312	.1849	"	.2671	.2067	"	.2986	.2262	"	.3269	.2450	"	.3537	.2618	"	.3780
		160	.1388	"	.2100	.1601	"	.2423	.1790	"	.2709	.1960	"	.2967	.2120	"	.3207	.2263	"	.3425
		200	.1240	"	.1952	.1432	"	.2254	.1601	"	.2520	.1753	"	.2760	.1898	"	.2985	.2025	"	.3187
		300	.1012	"	.1724	.1181	"	.2003	.1306	"	.2225	.1430	"	.2437	.1549	"	.2636	.1654	"	.2816
		400	.0877	"	.1589	.1012	"	.1834	.1132	"	.2051	.1240	"	.2247	.1342	"	.2429	.1434	"	.2596

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TITLE

DWG. No.

DATE APRIL 1/49 ISSUE

SCALE BY J.L.M

8-Wire Line. Earth Current & Copper Losses,
DB. per 1000 ft.

Wire Size	Z_0	$\sigma \times 10^{-15}$	600 kc α_c	600 kc α_e	600 kc α_{TOT}	800 kc α_c	800 kc α_e	800 kc α_{TOT}	1000 kc α_c	1000 kc α_e	1000 kc α_{TOT}	1200 kc α_c	1200 kc α_e	1200 kc α_{TOT}	1400 kc α_c	1400 kc α_e	1400 kc α_{TOT}	1600 kc α_c	1600 kc α_e	1600 kc α_{TOT}
No.4	197	20	.1284	.0527	.1811	.1482	.0609	.2091	.1657	.0681	.2338	.1815	.0745	.2560	.1961	.0805	.2766	.2096	.0868	.2964
		40	.0908	"	.1435	.1048	"	.1657	.1172	"	.1853	.1284	"	.2029	.1390	"	.2195	.1482	"	.2350
		60	.0741	"	.1268	.0855	"	.1464	.0957	"	.1638	.1047	"	.1792	.1132	"	.1937	.1209	"	.2077
		80	.0642	"	.1169	.0742	"	.1351	.0829	"	.1510	.0909	"	.1654	.0983	"	.1788	.1049	"	.1917
		120	.0523	"	.1050	.0605	"	.1214	.0676	"	.1357	.0740	"	.1485	.0801	"	.1606	.0855	"	.1723
		160	.0454	"	.0981	.0524	"	.1133	.0586	"	.1267	.0642	"	.1387	.0695	"	.1500	.0741	"	.1609
		200	.0406	"	.0933	.0469	"	.1078	.0524	"	.1205	.0575	"	.1320	.0622	"	.1427	.0664	"	.1532
		300	.0331	"	.0858	.0382	"	.0991	.0428	"	.1109	.0468	"	.1213	.0507	"	.1312	.0547	"	.1415
		400	.0287	"	.0814	.0332	"	.0941	.0371	"	.1052	.0406	"	.1151	.0439	"	.1244	.0474	"	.1342
No.6	204	20	.1445	.0640	.2085	.1668	.0738	.2406	.1865	.0825	.2690	.2043	.0904	.2947	.2207	.0976	.3183	.2359	.1044	.3403
		40	.1021	"	.1661	.1180	"	.1918	.1319	"	.2144	.1445	"	.2349	.1562	"	.2538	.1670	"	.2714
		60	.0835	"	.1475	.0965	"	.1703	.1077	"	.1902	.1180	"	.2084	.1278	"	.2254	.1362	"	.2406
		80	.0723	"	.1363	.0835	"	.1573	.0933	"	.1758	.1021	"	.1925	.1105	"	.2081	.1180	"	.2224
		120	.0590	"	.1230	.0681	"	.1419	.0762	"	.1587	.0835	"	.1739	.0903	"	.1879	.0964	"	.2008
		160	.0511	"	.1151	.0590	"	.1328	.0660	"	.1485	.0723	"	.1627	.0782	"	.1758	.0835	"	.1879
		200	.0457	"	.1097	.0528	"	.1266	.0590	"	.1415	.0646	"	.1550	.0700	"	.1676	.0747	"	.1791
		300	.0372	"	.1012	.0431	"	.1169	.0481	"	.1306	.0528	"	.1432	.0571	"	.1547	.0609	"	.1653
		400	.0323	"	.0963	.0373	"	.1111	.0417	"	.1242	.0458	"	.1362	.0495	"	.1471	.0528	"	.1572
No.8	213	20	.1571	.0770	.2341	.1814	.0890	.2704	.2028	.0990	.3018	.2222	.1085	.3307	.2400	.1172	.3572	.2566	.1253	.3819
		40	.1110	"	.1880	.1282	"	.2172	.1434	"	.2424	.1570	"	.2655	.1700	"	.2872	.1815	"	.3068
		60	.0908	"	.1678	.1049	"	.1939	.1171	"	.2161	.1284	"	.2369	.1389	"	.2561	.1482	"	.2735
		80	.0785	"	.1555	.0908	"	.1798	.1014	"	.2004	.1111	"	.2196	.1202	"	.2374	.1284	"	.2537
		120	.0642	"	.1412	.0741	"	.1631	.0828	"	.1818	.0908	"	.1993	.0982	"	.2154	.1048	"	.2301
		160	.0555	"	.1325	.0641	"	.1531	.0717	"	.1707	.0785	"	.1870	.0850	"	.2022	.0907	"	.2160
		200	.0497	"	.1267	.0574	"	.1464	.0642	"	.1632	.0703	"	.1788	.0760	"	.1932	.0812	"	.2065
		300	.0406	"	.1176	.0468	"	.1358	.0523	"	.1513	.0575	"	.1660	.0620	"	.1792	.0662	"	.1915
		400	.0352	"	.1122	.0405	"	.1295	.0454	"	.1444	.0498	"	.1583	.0537	"	.1709	.0574	"	.1827

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DWG. No.
DATE APRIL 1/40 ISSUE
SCALE BY J.L.M.

10-Wire Lines. Earth Current & Copper Losses, DB. per 1000 ft.

Wire Size	Z_o	$\delta \times 10^{-15}$	600 kc α_c	600 kc α_e	600 kc $\alpha_{TOT.}$	800 kc α_c	800 kc α_e	800 kc $\alpha_{TOT.}$	1000 kc α_c	1000 kc α_e	1000 kc $\alpha_{TOT.}$	1200 kc α_c	1200 kc α_e	1200 kc $\alpha_{TOT.}$	1400 kc α_c	1400 kc α_e	1400 kc $\alpha_{TOT.}$	1600 kc α_c	1600 kc α_e	1600 kc $\alpha_{TOT.}$
No.4	180	20	.0555	.0556	.1111	.0640	.0642	.1282	.0716	.0717	.1433	.0784	.0786	.1570	.0847	.0849	.1696	.0906	.0907	.1812
		40	.0392	.0556	.0948	.0453	.0640	.1093	.0506	.0717	.1223	.0555	.0786	.1341	.0600	.0849	.1449	.0640	.0907	.1547
		60	.0320	"	.0876	.0370	"	.1010	.0413	"	.1130	.0453	"	.1239	.0490	"	.1339	.0523	"	.1430
		80	.0278	"	.0834	.0320	"	.0960	.0358	"	v .1075	.0392	"	.1178	.0425	"	.1274	.0453	"	.1360
		120	.0226	"	.0782	.0262	"	.0902	.0292	"	.1009	.0320	"	.1106	.0347	"	.1195	.0370	"	.1277
		160	.0196	"	.0752	.0226	"	.0866	.0253	"	.0970	.0277	"	.1063	.0300	"	.1149	.0320	"	.1227
		200	.0176	"	.0732	.0203	"	.0843	.0226	"	.0943	.0248	"	.1034	.0279	"	.1128	.0287	"	.1194
		300	.0143	"	.0699	.0165	"	.0805	.0185	"	.0902	.0202	"	.0988	.0219	"	.1068	.0234	"	.1141
		400	.0124	"	.0680	.0143	"	.0783	.0161	"	.0878	.0175	"	.0961	.0190	"	.1039	.0203	"	.1110
No.6	187	20	.0659	.0672	.1331	.0761	.0776	.1537	.0851	.0868	.1719	.0932	.0950	.1882	.1007	.1027	.2034	.1076	.1098	.2174
		40	.0466	.0672	.1138	.0539	.0776	.1315	.0602	.0868	.1470	.0660	.0950	.1610	.0714	.1027	.1741	.0762	.1098	.1860
		60	.0380	"	.1052	.0440	"	.1216	.0491	"	.1359	.0538	"	.1488	.0593	"	.1620	.0622	"	.1720
		80	.0330	"	.1002	.0380	"	.1156	.0426	"	.1294	.0466	"	.1416	.0505	"	.1532	.0539	"	.1637
		120	.0269	"	.0941	.0311	"	.1087	.0347	"	.1215	.0381	"	.1331	.0412	"	.1439	.0440	"	.1538
		160	.0232	"	.0904	.0268	"	.1044	.0301	"	.1169	.0329	"	.1279	.0356	"	.1383	.0380	"	.1478
		200	.0208	"	.0880	.0241	"	.1017	.0269	"	.1138	.0294	"	.1244	.0319	"	.1346	.0340	"	.1438
		300	.0170	"	.0842	.0196	"	.0972	.0220	"	.1088	.0240	"	.1190	.0261	"	.1288	.0278	"	.1376
		400	.0147	"	.0819	.0170	"	.0946	.0191	"	.1059	.0209	"	.1159	.0226	"	.1253	.0241	"	.1339
No.8	194	20	.0755	.0815	.1570	.0872	.0941	.1814	.0975	.1052	.2027	.1067	.1153	.2220	.1154	.1245	.2399	.1231	.1331	.2562
		40	.0534	"	.1349	.0616	"	.1557	.0690	"	.1741	.0755	"	.1908	.0817	"	.2062	.0873	"	.2204
		60	.0436	"	.1251	.0504	"	.1445	.0563	"	.1615	.0616	"	.1769	.0667	"	.1912	.0712	"	.2043
		80	.0378	"	.1193	.0436	"	.1377	.0487	"	.1539	.0534	"	.1687	.0577	"	.1822	.0617	"	.1948
		120	.0308	"	.1123	.0356	"	.1297	.0398	"	.1450	.0436	"	.1589	.0472	"	.1717	.0504	"	.1835
		160	.0267	"	.1082	.0308	"	.1249	.0345	"	.1397	.0378	"	.1531	.0408	"	.1653	.0436	"	.1767
		200	.0238	"	.1053	.0276	"	.1217	.0308	"	.1360	.0338	"	.1491	.0366	"	.1611	.0390	"	.1721
		300	.0195	"	.1010	.0225	"	.1166	.0252	"	.1304	.0276	"	.1429	.0298	"	.1543	.0318	"	.1649
		400	.0169	"	.0984	.0195	"	.1136	.0218	"	.1270	.0239	"	.1392	.0259	"	.1504	.0276	"	.1607

3. The difference between the efficiencies of the lines decreases as the ground conductivity increases. The losses of the 8-wire and 10-wire lines, in particular, become quite close to each other. For a ground conductivity of 80 X 10^{-15} e.m.u. the 8-wire line of #4 conductors shows a slightly lower loss than the 10-wire line of #8 conductors; and this tendency increases with increasing earth conductivity. See Charts #3 to #9.

4. Under good conditions of earth conductivity the losses of the ten-wire line of #6, B & S gauge approach those of the concentric line. This is illustrated by Chart #10, where a value for of 200 X 10^{-15} e.m.u. was used.

Measured Results

Impedance measurements were made on the three ten-wire lines, using a General Radio type 916-A radio-frequency bridge. A substitution method was used, whereby known values of resistance, above and below the calculated value of characteristic impedance, were connected across the line terminals. For each value an impedance measurement, looking into the line, was made at some frequency in the broadcast band. These measurements were repeated at a second frequency considerably displaced in the band from the first. For one value of resistance the bridge reading would be exactly the value of the known terminating resistance, at both frequencies. This value was taken to be the characteristic impedance. The values found for the three lines consistently showed a resistive component of 177 ohms, with varying small reactive terms. The resistive term was, therefore, 2.7 percent lower than the calculated figure. This discrepancy may be accounted for by extra capacity in the line, due to the circular steel ring on each pole, the insulators, and to the end plates on which the grounded conductors were terminated by means of turnbuckles.

Measurements of the open- and short-circuit impedances, in order to determine the characteristic impedance met with little success. It is difficult to obtain a true short circuit condition, and, owing to the end-plates previously mentioned, it is doubtful whether a true open-circuit condition could exist.

With regard to power loss, it is difficult to make accurate direct measurements in the field. The radio-frequency ammeters used to measure normal line current and antenna current are large-range instruments, on which it is difficult to read small differences with accuracy. Furthermore they cannot be conveniently inserted into an installed line at other than their normal locations. One ammeter is normally connected at the output of the transmitter and another in series with the antenna itself. Therefore, when a power dividing network is used between the transmitter and two transmission lines, there is added difficulty in measuring the loss in the lines alone. As for using a voltmeter, these are not readily available in large ranges, and of high accuracy, at radio frequencies.

Current measurements were made in one case where the transmitter fed one line directly. At the far-end another ammeter was placed in series with antenna-matching circuit which terminates the line in its characteristic impedance, or very nearly so.

For a line 430 ft. long the loss was found to be approximately 600 watts. The earth conductivity at the site in question was very high, beingestimated at 400×10^{-15} e.m.u. The operating frequency was 990 kc. The calculated losses based on these values was 526 watts. Apart from a small loss due to radiation, the remainder of the discrepancy is attributable to leakage conductance losses.

Corona and Flash-Over

For parallel wires there is a critical ratio between conductor spacing and conductor radius, for which the corona voltage and flash-over voltage have the same value. This ratio,

$$\frac{s}{r} \text{ is } \doteq 30$$

For a ratio just below this value, corona and flash-over may occur together, when the applied voltage is sufficiently high. And for values of $\frac{s}{r} < 5.85$ the only kind of breakdown is flash-over. For values of $\frac{s}{r} > 30$ corona appears first, and as the applied voltage is further increased, flash-over eventually takes place.

For the ten-wire line, described herein, the ratio $\frac{s}{r}$ is 75 for the smallest spacing between any two conductors of opposite polarity. Hence corona would occur first, under high voltage conditions. The voltage at which corona begins, called the "disruptive critical corona" voltage[27], is, for an air dielectric,

$$e_d = g_d \frac{r\left(\frac{s}{2r} - 1\right)}{\sqrt{\left(\frac{s}{2r}\right)^2 - 1}} \cosh^{-1} \frac{s}{2r} \text{ kv. to neutral} \qquad (36)$$

where g_d, the disruptive voltage gradient is

$$30\left[1 + \frac{0.3}{\sqrt{r}} \times \frac{1}{(1 + 230r^2)}\right] \text{ kv. per cm.}$$

and r is the radius in cm.

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27. Peek F.W. "Dielectric Phenomena in High Voltage Engineering" 3rd Ed. 1929 - McGraw Hill, New York.

For the ten-wire line

and g_d = 32 kv. per cm.

e_d from expression (36) is 28 kv.

Since this value is for one side of the line and neutral, the disruptive voltage between conductors of a balanced line would be twice this value.

When conductors at the same potential are located close together the corona voltage is greater than for a single conductor. For the two center conductors of the ten-wire line, which are spaced 2.5 inches apart, the increase in corona voltage would be approximately 40%. This would make the effective value of disruptive critical voltage for the line,

28 X 1.4 = 39 kv maximum. If the conductors are wet the corona voltage is reduced by approximately 50%.

Other factors which reduce the corona voltage are dirt, oxidization, and roughness on the wires.

The flash-over voltage is given by the expression

$$e_s = g_s \, r \log_e \frac{s}{r} \quad \text{kv. to neutral}$$

where $g_s = 30\left(1 + \frac{.01}{\sqrt{r}} \frac{s}{r}\right)$ kv. per cm. is the maximum voltage gradient. For the ten-wire unbalanced line g_s is 80 kv per cm., and e_s is therefore 71 kv. maximum.

Power Handling Capacity

The operating voltage on the ten-wire line, for fifty kilowatts transmitter, is 3000. For an operating voltage of say 9400, the line would carry 500 kilowatts. The corresponding current would be approximately 27 amperes in each of the live conductors, which would be well within their heat-dissipation rating.

Mechanical Features

The mechanical design of the ten-wire line, which was largely done by other members of the C.B.C. engineering staff, was similar to that of the usual open-wire radio-frequency line, except for some special hardware.

The mechanical features are illustrated by the photographs on pages 59 and 60.

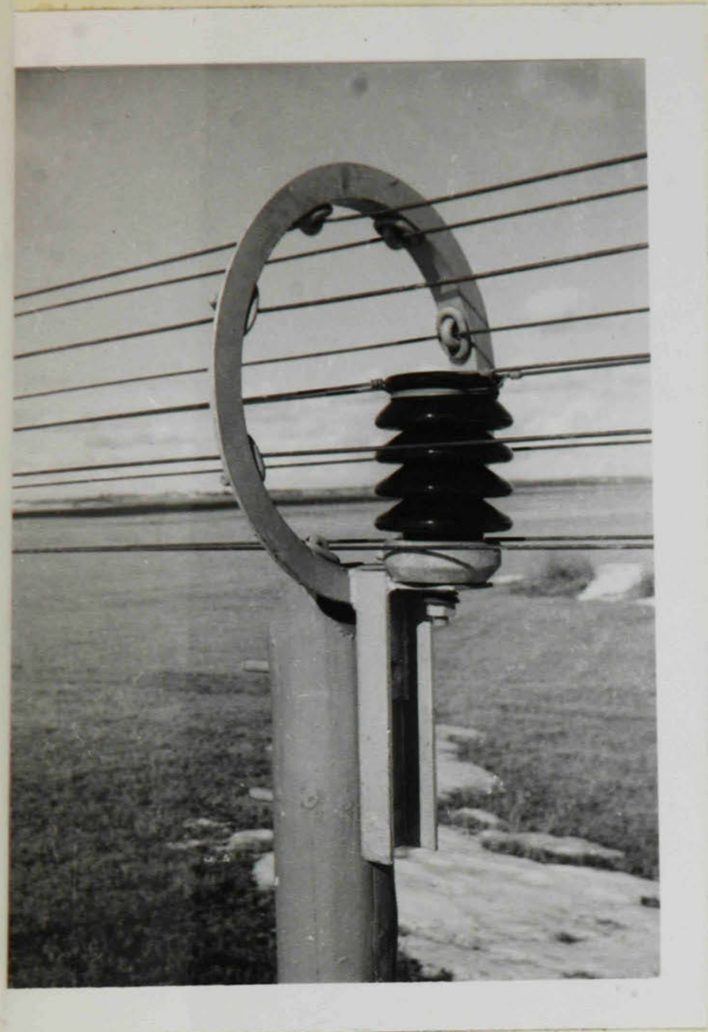
The conductors were of #6, B & S gauge copperweld wire of 40% conductivity. At broadcast frequencies the current is concentrated in the outer copper layer of the wire due to skin effect. This wire has a breaking load of 2430 lbs. and weighs 72.8 lbs. per 1000 ft. Comparable figures for #6 hard-drawn copper wire, are 1280 lbs. and 79.5 lbs. respectively. Tensioning of the wires was done in accordance with available stringing charts.

At two of the installations the ten-wire line was extended into the transmitter building to a point above the transmitter terminals. At the antenna tuning houses, a two-conductor lead-in was used. At the other installation a three-conductor lead-in line was used at both the transmitter building and the antenna tuning houses. In this respect the concentric line has the advantage in that it can be easily made continuous to the equipment terminals within a building.

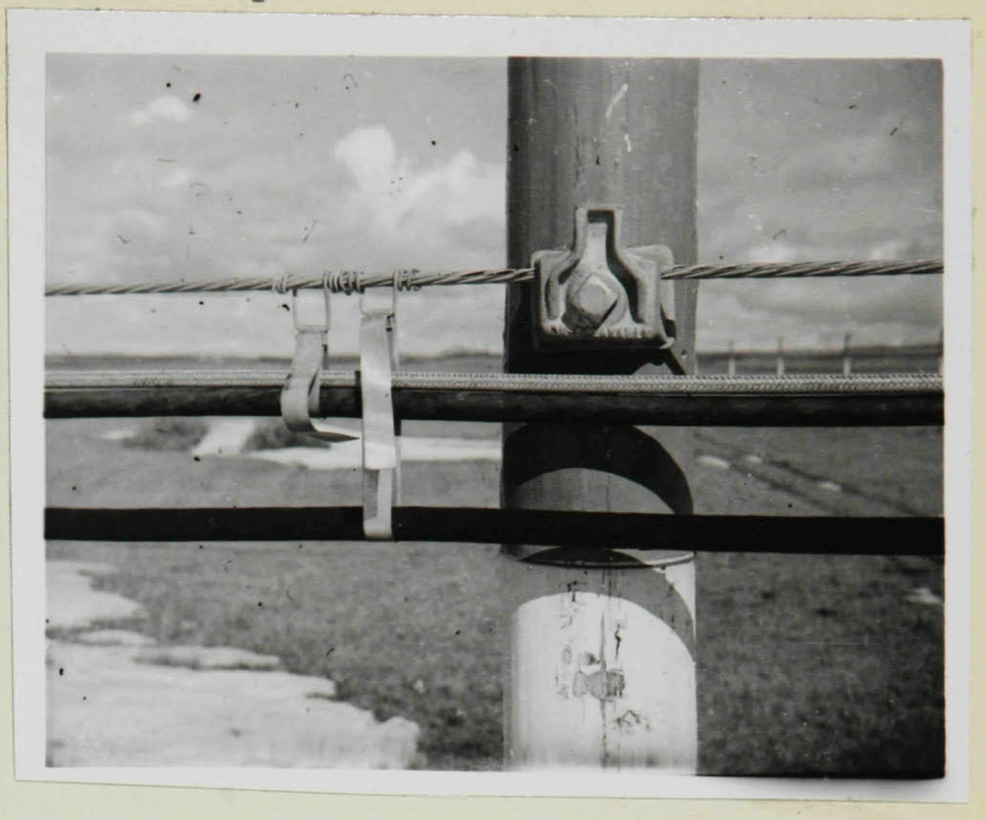
The outer ring of conductors of the ten-wire line was bonded to the antenna ground system at each pole, i.e., every 35 feet. This tends to maintain the outer conductors at ground potential, and reduces the amount of radiation.

It was found necessary to put a grounding ring (of #6 B & S gauge copperweld wire) around the outer conductors of the line at each end to make the characteristic impedance agree with the calculated value.

Photographs of Ten-Wire Transmission Line with Messenger-Cable Group



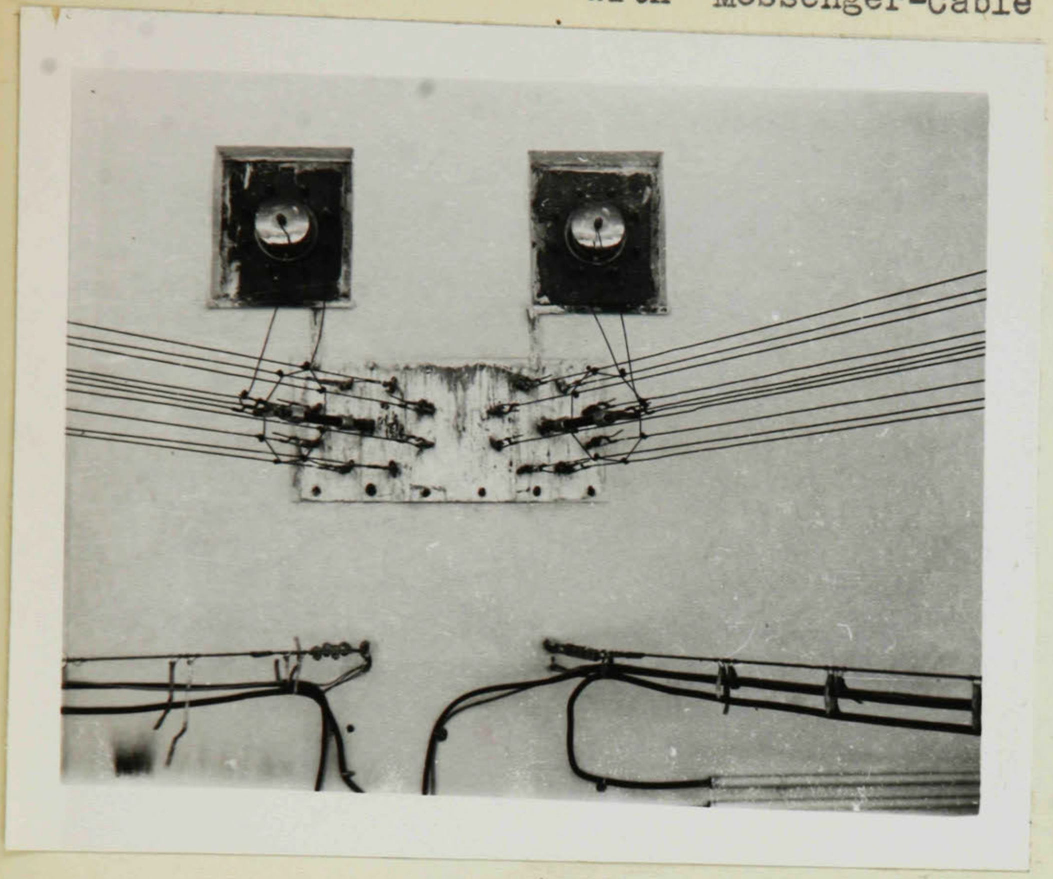
Pole-Top Assembly. Two live wires are bound to the insulator. The eight grounded wires are clamped to the steel mounting ring.



2. Typical messenger-cable group. The cables, from top to bottom are:- messenger cable, co-axial cable for R-F sampling current, audio-frequency cable and power cable.

ver-all view of 10-wire line, ooking toward antenna tuning ouse.

Photographs of Ten-Wire Transmission Line
with Messenger-Cable Group



Termination of two 10-wire lines on transmitter building. Note the bonding rings around the grounded conductors.

Termination of ten-wire line at an antenna tuning house.

Costs

The average cost of installing the ten-wire lines, including the messenger cable group, was $8.10 per foot. This is approximately ten percent less than the cost of installing a 3 1/8-inch concentric line alone.

Conclusion

The performance of the ten-wire unbalanced transmission line agreed fairly well with the calculated values; the measured losses being approximately twenty percent higher than the design figures.

The presence of a messenger cable group, supported on the transmission-line poles, has little effect on the characteristic impedance, but reduces the total loss of the line.

For optimum conditions of earth conductivity and frequency the losses of the ten-wire line of #6 B & S gauge, are comparable to those of a 3 1/8-inch diameter concentric line. Under the most unfavourable conditions the losses are about 50 percent higher than those of the concentric line. Radiation losses from the ten-wire line are negligible and induction in other lines nearby would also be slight.

Increasing the height of an open-wire line, increases its characteristic impedance and substantially reduces the losses. However, the cost of line construction increases with increasing height.

Open-wire lines, in general, will withstand very high voltage surges. The flashover voltage for the ten-wire line is 70 kv. maximum. This is approximately the same as the theoretical value for a 3 1/8-inch concentric line. However, the safe operating voltage of the latter is usually assumed to be less than for a comparable open-wire line.

Open-wire line construction can be adequately done by ordinary electrical contractors. The cost of installing the ten-wire line with messenger group, is about 90% of the installation cost of a 3 1/8-inch concentric line alone.

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Appendix 1

Attenuation Due to Copper Losses

If I_{lw} flows in m parallel conductors and I_{gw} in n parallel conductors, all wires being the same in size and material, the I^2R loss is $R\left(\frac{I_{lw}^2}{m} + \frac{I_{gw}^2}{n}\right)$ per unit length of line, where R is the resistance per unit length. As shown on page 18, $R = \frac{\sqrt{f_{mc.}}}{1995\,\rho_{in.}}$ ohms per foot.

Then the loss in power per unit length of line

$$\frac{dp}{dl} = \frac{\sqrt{f_{mc.}}}{1995\rho}\left(\frac{I_{1w}^2}{m} + \frac{I_{gw}^2}{n}\right) \qquad \text{watts}$$

The amount of power at any point in the line is

$P \doteq P_o^{\,-2\alpha l}$ where P_o is the initial power, α, the attenuation constant in nepers, and l, the length of line.

Then $-\frac{dP}{dl} = -2\alpha P_o^{\,-2\alpha l}$, the decrement in power along the line.

and $\alpha = \frac{1}{2P}\,\frac{dP}{dl}$

Hence, from the foregoing:-

$$\frac{1}{2P}\,\frac{\sqrt{f_{mc.}}}{1995\rho_{in.}}\left(\frac{I_{1w}^2}{m} + \frac{I_{gw}^2}{n}\right) \qquad \text{nepers per foot.}$$

The power, P, can be taken as $I_{1w}^2 Z_o$ for short lengths of line, with negligible error. Using this value, and evaluating the loss in db. per 1000 ft. of line we get[28]

$$= \frac{8.68\,\sqrt{f_{mc.}}\,1000}{2\,\rho_{in.}\,I_{1w}^2\,Z_o\,1995}\left(\frac{I_{1w}^2}{m} + \frac{I_{gw}^2}{n}\right)$$

$$= \frac{2.17\sqrt{f_{mc.}}}{Z_o\,\rho_{in.}}\left|\frac{1}{m} + \frac{1}{n}\left(\frac{I_{gw}}{I_{1w}}\right)^2\right|$$

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28. Footnote 3, Loc. Cit.

APPENDIX 11

Attenuation Due to Earth Current Losses

The current, I_{gd}, was proven by Brown[29] to be equivalent to an equal current flowing in a single conductor at the same height above ground as the transmission line. Figure 7 represents the equivalent transmission line.

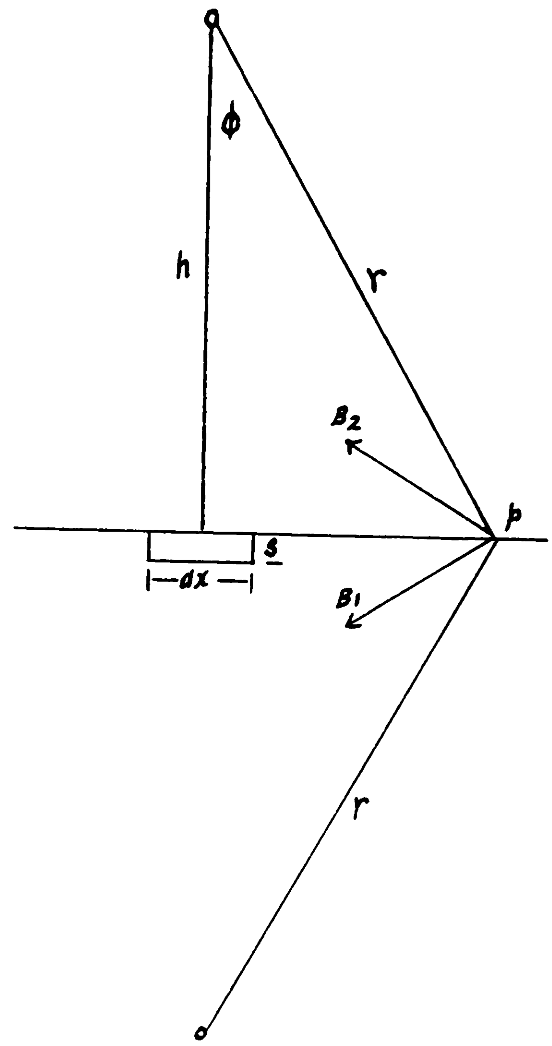

Fig. 7

The proof is as follows:-

Point p, on the surface of the earth is a distance x from the line at right angles to it. The magnetic flux density vector at p, due to the line current is

$$B_1 = \frac{\mu I_{gd}}{2\pi r}$$

An equal flux density B_2, is caused by the image current. The vector sum of these is $B = 2 B_1 \cos\phi = 2 \frac{h}{r} B_1$

which is parallel to the earth's surface. The actual earth current is concentrated near the surface. In the small cross-sectional area of unit width shown directly below the line, the current density is:-

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29. Footnote 3, loc. cit.

Summing up all the earth current gives:-

$$\int_{x=-\infty}^{x=\infty} I\,dx \int_{x=0}^{x=\infty} \frac{dx}{h^2+x^2} = \frac{2}{\pi} h I_{gd} \left[\frac{1}{h} \tan^{-1} \frac{x}{h}\right]_{x=0}^{x=\infty} = I_{gd}$$

which proves the initial assumption.

The resistance of the soil depends upon the depth of current penetration, which is given by:- $S = \dfrac{1}{2\pi\sqrt{f\sigma 10^{-9}}}$ cm.

where f is in c/s., and σ is in mhos per centimeter cube.

The resistance of the small cross-sectional area, of length dl in the direction of the line is $\Delta R = \dfrac{dl}{\sigma S \; dx}$

and the power lost in this small volume is $I^2 \dfrac{dl}{\sigma S \; dx}$

By integrating this, so as to include all the ground current, the power dissipated in a length dl is

$$\Delta P = I_{gd}^2 \frac{dl}{h} \sqrt{\frac{10^{-9} f}{\sigma}}$$

Expressing the attenuation as in Appendix 1:-

$\alpha = \dfrac{1}{2} \dfrac{1}{P} \dfrac{dp}{dl}$. Inserting values for P and dP gives:-

$$\alpha = \frac{1}{2} \frac{1}{I_{lw}^2 Z_o} \frac{I_{gd}^2}{h} \sqrt{\frac{10^{-9} f}{\sigma}} \qquad \text{nepers}$$

per unit length, where the linear dimensions are in centimeters. By changing to practical units of: h in feet, σ in e.m.u. f in mc./sec., and α in decibels, the attenuation per

1000 feet of line becomes:-

$$\alpha_e = \frac{13,720}{Z_o\, h_{ft.}} \left(\frac{I_{gd}}{I_{1w}}\right)^2 \sqrt{\frac{10^{-13} f_{mc.}}{\sigma_{e.m.u.}}}$$

APPENDIX 111

Radiation

In Appendix 11 it was proven that the net current in the line, I_{gd}, may be considered as flowing in a single conductor parallel to the earth, of the same height as the transmission line. For such a case it has been shown by Aharoni[30] that the vertical currents which flow in the generator and the terminating impedance also contribute to the total field. The expression for this field, is

$$E = \frac{60 j k h I_o e^{-jkx}\left(1 - e^{-jkL(1 - \sin\theta\cos\phi)}\right)}{x(1 - \sin\theta\cos\phi)} \tag{37}$$

where k = $\frac{2}{\pi\lambda}$

L = is the length of the line

x = is a distance along the line

θ = angle measured from the zenith

ϕ = horizontal angle measured from the direction of the line.

The bracketed terms in the numerator of equation

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30. J. Aharoni, "Antennae" Oxford, Clarendom Press 1946

(37) can be changed to the form:-

$$1-\cos\frac{2\pi L}{\lambda}(1-\sin\theta\cos\phi)+j\sin\frac{2\pi L}{\lambda}(1-\sin\theta\cos\phi)$$
$$=\sqrt{-2\cos\left[\frac{2\pi L}{\lambda}(1-\sin\theta\cos\phi)\right]+2}\,.$$

Using this term in equation (37) it becomes:-

$$E_{\theta,\phi}=60j\left(\frac{2\pi h}{\lambda}\right)I_o e^{-j\frac{2\pi x}{\lambda}}\sqrt{\frac{-2\cos\left[\frac{2\pi L}{\lambda}(1-\sin\theta\cos\phi)\right]+2}{x(1-\sin\theta\cos\phi)}}$$

If the net line current is taken as $I_{gd}\cos\theta$, and the expression changed to give the field strength in milli-volts per meter at one mile, we have

$$E_{\theta,\phi}=37.25\,\frac{2\pi h}{\lambda}\,I_{gd}\cos\theta\,\sqrt{\frac{-2\cos\frac{2\pi L}{\lambda}\left[(1-\sin\theta\cos\phi)\right]+2}{(1-\sin\theta\cos\phi)}} \tag{38}$$

This vector expression is also used by Brown[31]. If a line length of $\lambda/2$ or 180° is used, and the Poynting vector, expression (38) is integrated over the surface of a large hemisphere the power radiated is found to be

$$3500\ I_{gd}^{2}\ \frac{(h)^2}{(\lambda)^2}\qquad \text{watts}$$

For $L=\lambda$ or 360°, the power radiated is

$$5250\ I_{gd}^{2}\ \frac{(h)^2}{(\lambda)^2}\qquad \text{watts}$$

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31. Footnote 3 loc. cit.

APPENDIX IV

Effect of Messenger Cable Group

To include the messenger group in the calculation of attenuation an approximation was made as follows:- The resistance of a cylindrical conductor at radio frequencies is proportional to $\frac{1}{\rho\sqrt{\sigma}}$. For copper, σ was arbitrarily taken as unity so that the factor $\frac{1}{\rho\sqrt{\sigma}}$ copper $= \frac{1}{.1875}$ for either of the co-axial lines of radius .1875 inch. This factor was found for the other cables by using the proper radius, and value of relative conductivity, in each case, thus:-

$$\frac{1}{\rho\sqrt{\sigma}} \qquad \frac{1}{.1755\sqrt{\frac{1}{30}}} \qquad \frac{1}{.032} \qquad \text{(steel)}$$

$$\frac{1}{\rho\sqrt{\sigma}} \qquad \frac{1}{.5\,\frac{1}{12.7}} \qquad \frac{1}{.14} \qquad \text{(lead)}$$

Here the average radius of the two lead-covered cables is 0.5 inch. These factors were added for the five cables and the average taken. It is $\frac{1}{.089}$, which represents an "equivalent" copper conductor of .089 inch radius.

Since $Q_{11} = -0.01017\, Q_1$ it follows that the five cables carry $5 \times .01017 = .05085$ of the total return current.

Suppose each of the "equivalent" conductors carries 1/5 of the total current in the messenger group. Then the power lost in conductor resistance would be proportional to

$\left(\frac{1}{5}\right)^2 \times 5. = .2$ units If we replace the 5 conductors by one, then the current to produce the same power loss $\sqrt{\frac{.2}{1}} = .45$ of the total current. Therefore, we can postulate a single conductor of .089 inch radius carrying a current of $.45 \times .05085$ .02282 of the actual total current. Let us consider the equivalent conductor as being #6 B & S gauge, since $.089 \doteq .081.$

The average current carried by each of the grounded conductors bearing charge Q_3 and Q_5 is $.22774\ I_{1w}$ Then the "equivalent" conductor carries

$$\frac{.02282}{.22774} = .1005$$

of the current carried by the "average" of the grounded conductors of the line. Therefore, in the formula

$$\alpha_c = \frac{2.17\ f_{mc}}{e\ Z_o}\left(\frac{1}{m} + \frac{k^2}{n}\right)$$

we can write 8.1 for n instead of 8, the number of grounded conductors proper.

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