EXPERIMENTS ON WATER HAMMER USING A PIEZO-ELECTRIC PRESSURE INDICATOR

Thesis presented for the Degree of

Master of Engineering

by

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May, 1945

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Contains brief historical summary; outline of theory of water hammer; graphical method of analysis; theory of piezo-electric pressure indicator; results of tests of pressure rise due to instantaneous valve closure; analysis of pressure wave produced by indicator; discussion; bibliography.

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NOMENCLATURE

The symbols given here are those recommended by the Water Hammer Committee (1933).

A = Cross-sectional area of conduit, sq. ft.

e = Thickness of pipe wall, ins.

d = Inside diameter of conduit, ins.

E = Modulus of elasticity of material of pipe wall, lbs./sq. in.

= 29×10^6 lbs./sq. in. for steel.

g = Acceleration due to gravity, ft. per sec. per sec.

k = Volume modulus of water, lbs. per sq. in. = 294,000 lbs. per sq. in.

w = Specific weight of water = 62.5 lbs. per cu. ft.

L = Total length of conduit, ft.

x = Designation as distance from origin or as subscript for conditions at x, distance measured from lower end to point considered.

H_o = Pressure head for steady conditions, ft.

H = Pressure head for surge conditions, ft.

h = Ratio H/H_o.

p = Unit pressure, lbs. per sq. in.

a = Velocity of pressure wave, ft. per sec.

μ = Period of pipe or critical time $= \frac{2L}{a}$.

Q_o = Initial steady flow in conduit prior to closure under head H_o, cu. ft. per sec.

t = Time at instant under consideration.

T = Total time of gate travel.

V_o = Velocity under steady conditions of H_o and Q_o.

V = Velocity in conduit for surge conditions.

v = Ratio of V/V_o.

H_{At} = Head at point A on the pipe, t seconds after water hammer begins, ft.

V_{AT} = Velocity at point A on the pipe, t seconds after water hammer begins, ft. per sec.

$F(t)$ = Variable pressure heights of pressure waves travelling away from point of origin $F(t-\frac{x}{a})$.

$f(t)$ = Variable pressure heights of pressure waves travelling back towards the origin after being reflected from point of relief $f(t+\frac{x}{a})$.

K = Pipe line constant $\frac{aV_o}{2gH_o}$.

B_o = Initial gate opening factor, where $V_o = B_o\sqrt{H_o}$; $B = \frac{A_{gate}}{A} c\sqrt{2g}$ (where A_{gate} = area of orifice; c = coefficient of discharge of particular orifice).

B = Gate opening factor at any time.

τ = Proportion of full gate opening.

BRIEF HISTORICAL INTRODUCTION

When a change in velocity takes the form of a sudden rise or fall in pressure, the accompanying phenomenon is known as water hammer. The earliest studies on water hammer were undoubtedly initiated as a result of experience with water-works distribution systems since these antedate considerably the development of hydraulic-turbine pressure conduits. The water hammer pressure may be of such magnitude as to rupture a pipe and upset the distribution system. The first significant contribution appears to be that of Michaud[1*] in 1878, where the author noted the oscillating character of water hammer and considered the influence of the elasticity of the walls of the conduit and the elasticity of the water as a form of air reservoir of variable capacity.

The most important treatise published at that time was the experimental work of Joukowsky[2] in Moscow in 1898, in which he developed the law of instantaneous water hammer in a water conduit. This is known as the

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* The index numbers refer to the items in the bibliography. There are some two hundred or more papers published on various aspects of water hammer. Only a few papers which give a thorough development of the fundamental theory are mentioned. A supplementary bibliography is also given.

Joukowsky wave formula, and he first established the rate of propogation of this wave and proved that the maximum water hammer was equal to $\frac{av}{g}$. Experimental results indicated that the effect of branched pipes changed this relation and though Joukowsky did not develop the theory on this phase of water hammer, he did state that pressure rises might be experienced equal in magnitude to twice the value determined from his formula for instantaneous water hammer in a simple conduit.

Allievi's[3] work, published as his notes in 1903 and extended in 1913 to include his complete notes I to V, gate the mathematical analysis of water hammer and presented simple charts for the determination of maximum pressure rise for uniform closure in simple conduits. This treatise was translated in many languages and forms the basis for virtually all of our present theory of water hammer.

Mr. N. R. Gibson[4] developed a detailed theory independent of Allievi which gives identical results. Based on Allievi's theory, studies were made which considered other forms of conduits, namely, those having variations in diameter and thickness and also branch pipes and other complications.

Mr. A. H. Gibson[5] made a theoretical and experimental investigation of the rise or fall in pressure in a pipe line caused by the gradual or sudden closing or opening of a valve.

Camichel, Eydoux and Gariel[6] in 1918 conducted experiments in the field and at the University of Toulouse to confirm their analysis of the reflected waves and methods of considering these complicated conditions.

Several authors have attempted to give simple formulas for the determination of maximum water hammer pressure, but these have limited application. Mr. R. S. Quick[7] analyzed in detail the approximate formulas which had been published from time to time and compared the accuracy and limitations of these formulas for various applications.

Because of the large number of articles published and with each author choosing his own symbols, equations developed became involved. The Water Hammer Committee was formed and drew up an approved nomenclature. The Committee invited various engineers to contribute articles to give a more standard approach to the subject. These articles were published in 1933 as a "Symposium on Water Hammer".

The solution of water hammer problems has been simplified within the last twenty years by the development and application of the graphical method. Such men as Kreitner, F. M. Wood, Loewy, L. Bergeron and R. Angus[8, 9, 10] are responsible for the complete analysis of the graphical solution of water hammer problems.

The search for an analytical expression which would include the friction term has gained impetus due to the article of Professor F. M. Wood[11] on the application of operational calculus to water hammer. Following this, Mr. Rich[12] supplemented the work done by Professor Wood by replacing the Heaviside Calculus with the Laplace-Mellin transformation. This latter method is better adapted to problems starting from a steady-state system in motion.

Due to the unsuitability of mechanical indicators to measure water hammer pressures, very little experimental work has been done on this subject. The few field tests that have been made uphold the theory.

In his tests at McGill University, the author used an electronic apparatus for measuring the water hammer pressures.

OUTLINE OF GENERAL THEORY AND CAUSES OF WATER HAMMER

When a closed pipe is filled with a moving liquid (water will be referred to hereafter) the laws governing the changes of pressure and discharge will depend upon the conditions under which the flow occurs. If the flow is steady, the Bernoulli equation may be u sed.

When the motion is unsteady, the Bernoulli equation is no longer applicable. Sometimes during unsteady flow there is a mass movement of the water in the line such as in the case when the water surges back and forth between a reservoir and a tank; such cases require an open tank or an air chamber in the line.

Also, variable motion may be instigated in a fully closed system providing there is anything to start it, the motion being due to the elasticity of the water and pipe. Thus, variable motion may occur in a pipe with a "dead end", due to some change in the system, such as a valve closure, although there may be no delivery of the water through the pipe.

These three cases are important: the first is used to find the size of pipe for a given service; the second case is common in water power plants where surge tanks are necessary to store and restore the water during load changes; the third case is common in all systems.

This paper refers to the latter case of unsteady motion which causes the phenomenon of water hammer.

EFFECT OF ELASTICITY OF PIPE WALLS AND WATER ON WATER HAMMER. If in the system shown in Fig. 1(a) the valve may be made to close instantaneously, the particles of water in immediate proximity to it will be brought to rest. If both the water and the pipe walls were inelastic, then all the particles would be instantaneously brought to rest and the pressure at the gate and all through the pipe would be infinite. That the pressure does not become infinite is due to the compressibility of the water and the elasticity of the pipe walls. To consider the influence of these factors, assume the pipe Fig. 1(a) to be divided into an infinite number of lamina of equal mass.

First Period: When the gate is closed, lamina 1 crowds up against it due to its kinetic energy and is compressed. As lamina 1 is compressed, it distends the pipe wall around due to its increased pressure, Fig. 1(b); meanwhile lamina 2 follows on behind until the compression of lamina 1 is complete. It then suffers retardation and compression, at the same time stretching the pipe wall around it. Other lamina follow in quick succession and the pressure throughout the distended portion of the pipe is then increased at every point by the change of the kinetic energy of the laminae into pressure energy. The laminae being of equal mass, the rise in pressure is uniform at all points and the pressure at any point in the distended portion of the pipe is the original pressure during flow plus the pressure rise. When the last lamina at the inlet has been brought to rest, the mass of water in the pipe is at rest and under excess pressure. If the length of the pipe be L, and t seconds have elapsed between the complete closure of the gate and the compression of the last lamina, then a wave of pressure has swept up the pipe with a velocity "a" equal to $\frac{L}{t}$. With the last lamina brought to rest, the total kinetic energy of the water has been transformed and stored up in the elastic deformation of the water and pipe wall.

Second Period: As soon as the last lamina is compressed, the energy stored in it and in the distended pipe wall will cause it to move out of the pipe. This happens to each lamina in succession until lamina 1 is reached. A wave of reduced pressures has then swept down the pipe, restoring pressures to normal. The pipe wall is no longer distended and the laminae have attained their original velocity but the flow is towards the reservoir.

Third Period: Since the gate is closed, the kinetic energy of lamina 1 will be expended in lowering its pressure below normal. This procedure is repeated by the remaining laminae in quick succession until the water in the pipe is at rest under subnormal pressure.

Fourth Period: With the pressure at the inlet of the pipe below normal, water will now flow into the pipe from the reservoir. The nearest lamina regains its normal pressure and moves with its original velocity towards the gate. The other laminae follow in quick succession until finally all the water is moving toward the gate under normal pressure and with its original velocity.

A cycle of four movements has taken place occupying a time equal to $\frac{4L}{a}$. Other cycles follow, but due to viscuous friction, each one takes place with diminished velocity and the pressure wave gradually dies out.

However, in most cases the friction loss is relatively small, as is the velocity head, and the two terms, if omitted, simplify the problem. The resulting computed pressures will then be higher than the actual pressures, so that the results are conservative. The reservoir level is assumed as constant as its rate of variation will be small compared with the pressure wave velocity.

It is shown subsequently that the magnitude of the pressure wave at the gate is equal to $\frac{a}{g}(V_0 - V)$.

The maximum value of $\frac{av}{g}$ results when V = o; in other words, when the entire velocity is extinguished in the first interval, in which case the pressure rise is $h = \frac{aV_o}{g}$. This shows that the maximum pressure rise is independent of the dimensions of the pipe and of the head, and depends only on the velocity extinguished.

The actual gate movement may be assumed to be made in small steps, each one of which causes a small instantaneous change in the pipe velocity with a corresponding pressure change along the pipe. If the closing time T is less than $\frac{L}{a}$ seconds, the events are shown in Fig. 2(a). The combined series of small instantaneous gate movements produces a wave of sloping front travelling first from the gate. This wave will reach its maximum height before its "toe" reaches point B. When the closure requires $\frac{L}{a}$ seconds, the wave front is shown in Fig. 2(b), the wave front covering the entire length of the pipe; but in case 2(c), in which the closure time exceeds $\frac{L}{a}$ seconds, the returning opposition wave neutralizes some of the positive pressures and the maximum pressure is exerted on the pipe until the point g is above point j. In case 2(d), the maximum pressure is reached but does not remain, as the opposition wave front cancels it; whereas, in case 2(e), the pressure is never as high as in the former cases.

The maximum pressure reached is the same in cases (a) to (d), where $T = \frac{2L}{a}$ (provided the same velocity is extinguished at each step); but the quicker the closure the longer the time will be during which this maximum pressure continues.

For the case in Fig. 2(e), the same maximum pressure is not reached. Also, it may be shown that for linear gate movement, the curvature of the wave front is not great; it is convex to the horizontal axis, as drawn; so that in such cases as those shown in Figs. 2(b), 2(c), 2(d) and 2(e), the total pressure rise has roughly a straight line variation from zero at the reservoir to a maximum at the gate, a fact confirmed by many computations.

This general proposition may then be stated: the maximum pressure rise is produced for closing times equal to, or less than, $\frac{2L}{a}$; this is commonly referred to as sudden or instantaneous closure.

FUNDAMENTAL EQUATIONS FOR VARIABLE FLOW IN CLOSED CONDUITS. From Fig. 4, the general equation of the variable flow in a conduit is given as

$$\frac{\partial p}{\partial x} = \frac{w}{g}\left(R - \frac{\partial^2 x}{\partial t^2}\right) \tag{1}$$

where R denotes an external force acting parallel to the axis of the conduit. For a horizontal position of the conduit, R = o, so that

$$\frac{\partial p}{\partial x} = \frac{w}{g} \cdot \frac{\partial^2 x}{\partial t^2}$$

But $$v = \frac{\partial x}{\partial t} = \frac{dx}{dt}, \quad \therefore \frac{\partial^2 x}{\partial t^2} = \frac{\partial (v)}{\partial t} \tag{2}$$

since x is taken along the axis of the conduit. Writing the complete derivative of v, which is a function of x and t, gives:

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial x} \cdot \frac{dx}{dt}$$

and therefore

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}$$

whence

$$\frac{\partial^2 x}{\partial t^2} = \frac{\partial v}{\partial t} = \frac{dv}{dt} - v\frac{\partial v}{\partial x}$$

and $$\frac{\partial p}{\partial x} = \frac{w}{g}\left(\frac{dv}{dt} - v\frac{\partial v}{\partial x}\right) \tag{3}$$

This is Allievi's Equation I. The variation of v with respect to x, however, is small compared with the variation of v with respect to t, because of the instantaneous character of water hammer. Therefore, this equation may be written as,

$$\frac{\partial p}{\partial x} = \frac{w}{g} \cdot \frac{dv}{dt} = \frac{w}{g} \cdot \frac{\partial v}{\partial t}$$

or, since $\partial p = w\,\partial h$,

$$\frac{\partial v}{\partial t} = g \cdot \frac{\partial h}{\partial x}$$

therefore $$\partial h = \frac{\partial x}{g} \cdot \frac{\partial v}{\partial t} \tag{4}$$

Considering the elemental ring of pipe shown in Fig. 4, the volume of water stored in the length dx during the time dt due to the elasticity of the pipe wall is as follows:

$$E = \frac{\text{Stress in pipe material}}{\text{Change in length per unit length}}$$

For simplicity we may substitute the symbol ∂p for the more exact expression of the increase in pressure with respect to time, i.e., $\frac{\partial p}{\partial t} \cdot dt$. Then the stress in the pipe material due to this force is: $\frac{\partial p \cdot dx \cdot d}{2 \cdot dx \cdot e} = \frac{\partial p \cdot d}{2e}$

The change in length per unit length = $\frac{\partial p \cdot d}{2 \cdot e} \cdot \frac{1}{E}$

Total change in length of pipe wall = $\frac{\partial p \cdot d}{2 \cdot e \cdot E} \cdot \pi d$

The change in radius is therefore: $\frac{\partial p \cdot d^2 \cdot \pi}{4 \cdot e \cdot E \cdot \pi} = \frac{\partial p \cdot d^2}{4 \cdot e \cdot E}$

The change in area is: $\frac{\pi d^2}{4} - \pi\left(\frac{d}{2} + \frac{\partial p d^2}{4 \cdot e \cdot E}\right)^2 = -\frac{\pi d\, \partial p d^2}{4 \cdot e \cdot E} - \pi\left(\frac{\partial p \cdot d^2}{4 \cdot e \cdot E}\right)^2$

The term containing the factor $1/E^2$ may be neglected, so that the change in area may be expressed as: $\frac{\pi d\, \partial p d^2}{4 \cdot e \cdot E}$

The change in volume, due to the application of the force ∂p and to the elasticity of the pipe wall is:

$$\frac{\pi d\, \partial p\, d^2}{4 \cdot e \cdot E} dx = \frac{\pi d^2}{4E} \cdot \frac{d}{e} \frac{\partial p}{\partial t} dt \cdot dx \tag{5}$$

Also, $k = \frac{\partial P}{\text{Change in volume per unit volume}}$

The change in volume due to compressibility is therefore:

$$\frac{\text{Volume}}{k} \cdot \frac{\partial p}{\partial t} \cdot dt = \frac{\pi d^2 dx}{4k} \cdot \frac{\partial p}{\partial t} \cdot dt \tag{6}$$

The total change in volume is then:

$$\frac{\pi d^2}{4e} \cdot \frac{d}{E} \cdot \frac{\partial p}{\partial t} \cdot dt \cdot dx + \frac{\pi d^2 dx}{4k} \cdot \frac{\partial p}{\partial t} \cdot dt \tag{7}$$

This volume may be equal to the difference of the volume of water flowing at the end sections of the length dx during the time dt. Therefore,

$$\frac{\pi d^2}{4E} \cdot \frac{d}{e} \cdot \frac{\partial p}{\partial t} \cdot dt \cdot dx + \frac{\pi d^2 dx}{4k} \cdot \frac{\partial p}{\partial t} \cdot dt = \partial Q \cdot dt = A \frac{\partial v}{\partial x} \cdot dx \cdot dt = \frac{\pi d^2}{4} \cdot \frac{\partial v}{\partial x} \cdot dx \cdot dt$$

$$\frac{\partial v}{\partial x} = \left(\frac{1}{k} + \frac{d}{eE}\right)\frac{\partial p}{\partial t} \qquad (8)$$

This relation is known as Allievi's Equation II. Considering that $\frac{\partial x}{\partial t}$ must be equal to the velocity of the pressure wave, then from (4):

$$h = \frac{av}{g} = \frac{p}{w}$$

and therefore, from this relation and from (8):

$$a = \frac{\partial x}{\partial t} = \frac{\partial v}{\partial p} \cdot \frac{1}{\left(\frac{1}{k} + \frac{d}{eE}\right)} = \frac{g}{aw} \cdot \frac{1}{\left(\frac{1}{k} + \frac{d}{eE}\right)}$$

$$\frac{1}{a^2} = \frac{w}{g}\left(\frac{1}{k} + \frac{d}{eE}\right) \qquad (9)$$

This equation is known as Allievi's Equation III, from which,

$$a = \frac{1}{\sqrt{\frac{w}{g}\left(\frac{1}{k} + \frac{d}{eE}\right)}} \qquad (10)$$

If a is in ft. per sec., w in lbs. per cu. ft., k and E in lbs. per sq. in., and d in ins., then: (See Fig. 5)

$$a = \frac{12}{\sqrt{\frac{w}{g}\left(\frac{1}{k} + \frac{d}{eE}\right)}} \qquad (11)$$

From equations (4), (8), and (10), Allievi's Equation IV may be written as:

$$\frac{\partial v}{\partial t} = g \frac{\partial h}{\partial x}$$

$$\frac{\partial v}{\partial x} = \frac{g}{a^2} \frac{\partial h}{\partial t} \qquad (12)$$

The general integrals of equations (12) are given as:

$$H_t = H_o + F\left(t - \frac{x}{a}\right) + f\left(t + \frac{x}{a}\right)$$

$$V_N = V_o - \frac{g}{a}\left[F\left(t - \frac{x}{a}\right) - f\left(t + \frac{x}{a}\right)\right] \qquad (13)$$

which are Allievi's Equations V and are the basic water hammer equations.

GRAPHICAL SOLUTION OF WATER HAMMER PROBLEMS. A most comprehensive treatment of the graphical solution of water hammer problems is that of Professor R. W. Angus.[8, 9, 10] Therefore, only the fundamental equations are presented here (the reader is referred to the above reference for various applications of the graphical solution).

The graphical analysis of water hammer problems is presented here because it was the author's intention to analyze experimentally the effect of branch lines on water hammer and to compare the results with those obtained from the graphical solution. Considerable work was involved in preparing the charts, and as the theoretical results obtained are very interesting, it is considered not to be out of place to present them as part of this paper. It is hoped that these results may be verified in the near future with a view to studying water hammer in pipe net-works.

Referring to Fig. 6, and from the relation given in equation (4), i.e., $\partial h = \frac{a \partial v}{g}$, which is the rise in pressure due to a part of the velocity, ∂v, being extinguished, the following relations are evident for any given interval:

$$Q = AV = A_{GATE} c \sqrt{2gH}$$

or at any instant $V = \frac{A_{GATE}}{A} c\sqrt{2gH} = B\sqrt{H}$ (14)

also before movement begins

$$V_0 = B_0\sqrt{H_0} \quad (15)$$

Hence $\frac{V}{V_0} = \frac{B}{B_0}\sqrt{\frac{H}{H_0}} = \tau\sqrt{\frac{H}{H_0}}$ where $\tau = \frac{B}{B_0}$ (16)

Taking Allievi's Equations V and adding them gives:

$$H - H_0 = -\frac{a}{g}(V_0 - V) + 2F(t - \tfrac{x}{a}) \quad (17)$$

and by subtracting them: $H - H_0 = \frac{a}{g}(V_0 - V) + 2f(t + \tfrac{x}{a})$ (18)

where H and V are corresponding values for any point x feet from the gate, t seconds after closure begins. Selecting two sections, A and B on the pipe, as shown in Fig. 7(a), equation (17) gives:

$$H_{Bt} - H_{BO} = -\frac{a}{g}(V_{BO} - V_{Bt}) + 2F(t - \tfrac{x}{a}) \quad (19)$$

$$H_{At_1} - H_{AO} = \frac{a}{g}(V_{AO} - V_{At_1}) + 2F(t_1 - \tfrac{x_1}{a}) \quad (19a)$$

Where the pipe is of uniform size throughout, $V_{AO} = V_{BO}$, and if, as is most usual, the velocity heads are small relative to the pressure heads, then $H_{AO} = H_{BO}$.

Again, if a pressure wave takes $\frac{x - x_1}{a}$ seconds to travel from A to B, so that if events for B are reckoned at a time $\frac{x - x_1}{a}$ seconds later than those for A, the very same pressure wave will be under consideration in both cases. But this condition evidently means $t - t_1 = \frac{x - x_1}{a}$ or $t - \frac{x}{a} = t_1 - \frac{x_1}{a}$ or $F(t - \frac{x}{a}) = F(t_1 - \frac{x_1}{a})$. Therefore, with the assumptions made as to times, the subtraction of equation

(19a) from (19) results in:

$$H_{Bt} - H_{At_1} = \frac{a}{g}(V_{Bt} - V_{At_1}) \qquad (20)$$

For the reflected wave, equation (18), when treated in a similar manner, gives (see Fig. 7(b)):

$$H_{Bt_1} - H_{BO} = \frac{a}{g}(V_{BO} - V_{Bt_1}) + 2f(t_1 + \frac{x_1}{a}) \qquad (21)$$

$$H_{At} - H_{AO} = \frac{a}{g}(V_{AO} - V_{At}) + 2f(t + \frac{x}{a}) \qquad (22)$$

For the reasons already stated, times are reckoned so that $t - t_1 = \frac{x_1 - x}{a}$ or $t + \frac{x}{a} = t_1 + \frac{x_1}{a}$, from which (21) and (22) give:

$$H_{At} - H_{Bt_1} = -\frac{a}{g}(V_{At} - V_{Bt_1}) \qquad (23)$$

Any number of points may be studied by the use of the above equations. The equations may also be applied to a system with different pipe diameters, with the single change in the value of a, which is different for different diameters.

It is usually easier to work with the ratios H/H_o and V/V_o instead of H and V. Therefore, dividing equation (20) by H_o there results:

$$\frac{H_{Bt}}{H_o} - \frac{H_{At_1}}{H_o} = \frac{aV_o}{gH_o}\left(\frac{V_{Bt}}{V_o} - \frac{V_{At_1}}{V_o}\right) \qquad (24)$$

The quantity $\frac{aV_o}{2gH_o}$ has been designated as K and is known at the pipe line characteristic. For each pipe and condition of flow, it has a fixed value and is the most important factor in water hammer.

Then writing H/H_o as h and V/V_o as v,* equation (24) gives

$$h_{Bt} - h_{At_1} = \frac{aV_o}{gH_o}(v_{Bt} - v_{At_1}) = 2K(v_{Bt} - v_{At_1}) \qquad (25)$$

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* In a sloping pipe, h is not the true ratio of the pressures. If a section is y feet above the plane of the nozzle or gate, then the true pressure ratio at this point is $(H_{Bt_1} - y)/(H_o - y) = (h_{Bt_1} - y/H_o)/(1 - y/H_o)$

This is the equation of a straight line with a slope equal to $+2K$.

This analysis gives the principle of the graphical solution and the illustrations in Appendix III will show how it is applied.

THEORY OF PRESSURE INDICATOR. The cathode ray oscillograph indicating equipment* consists of five separate parts, namely, the pressure indicator, the amplifier, the oscillograph, the square wave generator and electronic switch, and the frequency oscillator. The pressure indicator, commonly called the pick-up unit (shown in section view, Fig. 8), is screwed into the pipe so that the lower end is exposed to the direct water hammer pressures. Minute voltage impulses, proportional to the pressure on the quartz crystals in the pick-up unit, are carried by cable to the amplifier (for block diagram of electrical apparatus see Fig. 9) where these voltage impulses are magnified up to fifty thousand times, to about 150 volts. The amplifier output is carried over to the oscillograph and registered on the screen by the cathode ray tube. The frequency oscillator and square wave generator impose a time sweep on the screen so that a pressure-time diagram is viewed on the screen.

The pressure pick-up unit is designed to utilize the piezo-electric properties of quartz crystals. The unit consists essentially of two quartz crystals

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* See R.C.A. instruction booklets on indicating apparatus.

separated by an output electrode and maintained under contact between two grounded parts known as the plug and piston. When a pressure is exerted on the piston, thus compressing the two crystals, and electrical charge will appear on the two faces of each crystal due to their piezo-electric properties. The crystals are so polarized that these charges are additive, the crystals thus operating in parallel. The quantity of this charge is a direct measure of the pressure exerted and will vary directly as the pressure varies. The voltage between the center electrode and ground when no current is drawn is proportional to this charge and inversely proportional to the capacity between the two electrodes. This voltage may be applied to the grid of an amplifier tube and its plate current will vary with the pressure applied to the diaphragm. Since it is impossible to have an electric circuit with an infinite resistance and perfect insulation, some of the charge will leak off, causing some error to be introduced, but this error will be small if the rate of leak-off is small with respect to the rate of change of pressure.

The piezo-electric sensitivity constant does not change up to 400° F. The proportions of the quartz crystals used in the pick-up unit are such that their natural frequency is above 1,000,000 cycles per second.

The water hammer pressures produce only minute deformations of the working parts, according to Young's Modulus; there is no actual displacement of any part of the pick-up unit. Since none of the parts move, errors from inertia, common to mechanical indicators, are eliminated.

TESTS CONDUCTED AT McGILL UNIVERSITY

The first water hammer tests were made in February of 1945. It was originally planned to determine the effect of branch lines on the water hammer at the gate, but several problems arose, at first, which had to be solved, so that the time for experimentation was limited.

As the apparatus had not been previously used for the measurement of water hammer pressures, there was no experimental data to act as a guide. It was, therefore, decided to solve the problems that arose from adapting this apparatus to measure water pressures.

DESCRIPTION OF THE APPARATUS. In order to test the foregoing theory, a 2-1/2 in. standard butt weld steel pipe was arranged as shown in Fig. 10. The 30^{o} elbows were used in order that the pipe could be laid on the gallery floor, with as little resistance to flow as possible. The pipe line was supplied from a tank 5 feet in diameter. An inflow valve, at the middle of the tank, governed the discharge in the pipe line. The tank acted as a reservoir.

It was necessary to arrange the inflow pipes so that the already existing pipes for other apparatus would not be disturbed or left disconnected. After passing through 75 feet of the 2-1/2 in. pipe, the flow was directed by a 90° elbow over the edge of the gallery through a large galvanized steel pipe into a measuring tank below the main floor. The discharge was measured by means of a float in the tank attached to a string passing over a pulley to a point riding beside a vertical scale graduated in cu. ft.

The indicator was threaded into a boss welded onto a 12 in. length of 2-1/2 in. pipe. A short section of pipe was used as the indicator, threaded into the boss of the section, could be placed as a unit at any section along the line. The 6 in. allowance between the pick-up unit and the fittings at either end minimized the effect of turbulence at the indicator. The unit was adjusted, in the boss, until the bottom of the piston was flush with the inside wall of the pipe.

The quick-closing gate valve was of the side-pivot type, and was arranged to operate with the automatic timing device shown in Fig. 13. Due to the limited time, it was necessary to choose an apparatus that needed very few alterations in order to suit the problem. An old solenoid was found, attached to a swinging arm, and this was arranged as shown. The solenoid was rewound for a pull of 10 pounds and a 5-amp. current. A hole, drilled through the valve handle close to the valve body, reduced the angular turn of the handle from open to closed position. A heavy string, passed through the hole in the handle to the end of the arm opposite the iron core, acted as a link. When the valve handle was in open position, the lead A made a closed circuit with contact B. When the switch closed, the iron core was pulled into the solenoid, the handle raised to the closed position, and lead A travelled to contact C.

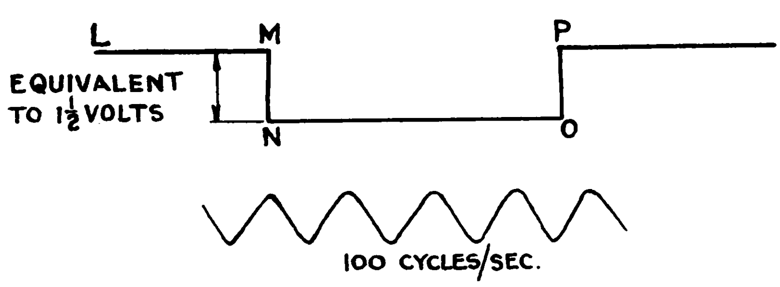

FIGURE 14

Referring to Fig. 14, which was the timing diagram observed on the oscillograph, lead A is in contact with B from L to M. With the switch closed, the iron core is drawn into the solenoid and the contact is broken at B so that the voltage input to the amplifier is zero and the wave drops from M to N and remains at this elevation of zero voltage input until the lead A has travelled across the arc and made contact with C, when the wave will jump from O to P due to the battery voltage input. The time to travel from M to P is measured on the frequency wave of 100 cycles per sec. In attempting to get a quicker gate closure, the switch was accidently left closed and the solenoid burned out. However, readings were obtained directly from the oscillograph screen that indicated that the closure took about 1/20 of a sec.

There was no time to rewind another solenoid, and since it was impossible to obtain a constant gate closure by hand operation, the experimental study of branch lines had to be discarded.

During the first trial run, the waves produced on the screen were of such a varying and fluctuating character as to render the pressure diagram indistinguishable. These extraneous waves were produced by the vibration of the pick-up unit due to the shock of the water hammer blows, that is, the vibrations varied the capacitance of the indicator and lead, which was recorded on the oscillograph. This outside influence was eliminated by attaching the lead securely to a pipe running from the indicator to the amplifier. The 12 in. section of the pipe line, into which the indicator was threaded, was concreted to the floor (see Fig. 12).

Upon examination of the parts of the indicator, it was found that the lower crystal had been chipped.

The chip had wedged itself between the crystal and the piston preventing the retainer from making the unit watertight by holding the diaphragm and rings tightly against the body (see Fig. 11 for array of components). The crystals and insulating washers need to maintain a high electrical resistance of 1000 megohms or more in order that the unit may operate successfully. The parts were chemically cleaned and dried and reassembled.

The next trial run indicated that the above preparations were successful as only the wave due to the water hammer pressures appeared on the screen.

In taking the photographs, the shutter was opened a fraction of a second before the valve was shut, and closed a fraction of a second after the valve, the shutter being open for approximately 1/20 of a sec. The photographs were taken on super XX panchromatic films.

TESTS. With the breakdown of the timing apparatus, it was necessary, in order to achieve some results that would aid future development, to close the valve instantaneously, that is, within the period of the pipe of approximately 1/30 of a sec., by hand operation. An analysis of the pressure wave produced on the oscillograph screen would determine the suitability of this pressure indicator to measure water hammer.

The physical dimensions and constants of the apparatus are as follows:-

$L = 75$ feet.

$d = 2.469$ ins.

$D = 2.875$ ins.

$t = 0.203$ ins.

$A = 4.788$ sq.ins.

$a = 4,410$ ft./sec.

$E = 29,000,000$ lbs./sq.in.

$\frac{2L}{a} = 0.034$ secs.

Sensitivity of indicator* = 3 mv/100 lbs./sq.in.

f = friction factor = 0.02

The tests were run at velocities ranging from 1.96 ft./sec. to 6.05 ft./sec., and photographs taken of each test. The results are shown in Table I.

In order to insure the same operating conditions, the calibration of the screen was made immediately after each test and recorded on Fig. 15. It was found that if the time between the test and the calibration was prolonged, the pressure-time diagram would have shifted, thus altering the calibration under which the test was performed. After each test, the flow was adjusted and allowed to become steady. The discharge was then taken over a period of two minutes in the measuring tank and the velocity calculated from the relation $\frac{Q}{A}$.

Following this, the valve was closed, and the variation in the pressure diagram on the oscillograph was

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* This value was given as 3 mv/100 lbs./sq.in. by the R.C.A. However, it may or may not have changed due to the chipped crystal. This constant was used and the results indicate that it is correct.

recorded on photographs according to the method already outlined.

To analyze the wave form of the pressure-time diagram, test No. 5, Fig. 21, will be used as an example. The horizontal sweep of the pressure diagram is from left to right. The gate started to close at a and the pressure dropped very rapidly to b, indicating a quick smooth closure (pressure rise is below the zero line and pressure drop above). Due to the characteristics of the piezo-electric crystals, their charge began leaking off until the return wave of negative pressure caused the rise to c. But for the characteristics of the crystals, the positive pressure would have remained at elevation bf until the negative wave had reached the gate, when the pressure diagram would have risen to g. However, the magnitude of the pressure at e is the same as that at b, so that the point c should be plotted the distance ec above f in order to calculate the magnitude of the negative pressure. The number of divisions traversed by the line ab is entered in Fig. 15, and from the calibration of test No. 5, the number of millivolts corresponding to the screen divisions is obtained. The value of the screen divisions in millivolts divided by the sensitivity of 3 mv/100 lbs./sq.in. gives the magnitude of the positive pressure in lbs./sq.in. The positive and negative pressures thus obtained are corrected according to the exponential curves given in Fig. 16.

TABLE I

Test Results

Test No. (1)	Fig. No. (2)	Discharge c.f.s. (3)	Velocity f.p.s. (4)	Theoretical Pressure Rise $p = \frac{aV}{g} \cdot \frac{62.5}{144}$ p.s.i (5)	Pressure Rise with Leakage Loss p.s.i. (6)	Critical Time Seconds (7)	Gate-Closing Time Periods (8)	Ratio = $\frac{P}{P'}$ = K' (9)	Pressure Rise with No Leakage Loss p.s.i. (10)
1	17	.065	1.96	115.5	93.5	.034	.43	.81	115
2	18	.092	2.76	163	120	.034	.455	.745	161
3	19	.100	3.01	177.5	130	.034	.50	.735	176
4	20	.120	3.60	212	148	.034	.67	.70	212
5	21	.172	5.17	306	211	.034	.69	.70	302
6	22	.201	6.05	357	233	.034	.87	.65	359

The equations for the exponential pressure drop are derived in Appendix II, and the results for each test plotted, on Fig. 16. The time for the gate closure, i.e., the time from a to b, Fig. 21, is determined from the timing diagram. Fig. 16 is then entered with the ratio of the gate closing time to the full period, to give the ratio of the pressure recorded on the oscillograph to the pressure if no exponential drop had occurred. These values are entered in col. (9) and (8) of Table I. The resulting pressure rise from tests, col. (10), is obtained by dividing col. (6) by col. (9).

Table II gives the detailed calculations for test No. 5. It is seen that the negative pressure drops to absolute zero, as the theoretical negative pressure, which is equal to the positive pressure, is greater than the friction head, velocity head and atmospheric pressure. The negative pressure has not been calculated for the remaining tests as it is difficult to determine the point where the pressure begins to drop due to the exponential factor.

In several of the tests, the gate closure during the first part of the stroke was slow and erratic (see Figs. 18 and 20). However, this had very little effect on the results, as the velocity cut off during the beginning of the stroke was negligible.

As a supplement to the photographic records, a series of bangs was heard in the pipe after the valve was closed. These originated from the multiple recoil or resurge effect of a water column following the instantaneous or approximately instantaneous closure of the valve,* that is, after the compression wave has completed the round trip from the valve to the reservoir and back, the entire column may recoil from the valve, leaving a vacuous space behind and then return to the valve, making a second water hammer blow similar to, though less violent than, the first. The bangs were of continually diminishing intensity, and the time period between successive bangs seemed to diminish also. These were the conclusions drawn by Professor LeConte.

DISCUSSION. The test results in Table I show that the piezo-electric pressure indicator may be used to determine the magnitude of instantaneous water hammer pressures within an accuracy of 1%. The results also check the velocity of the pressure wave, and, in a less rigid manner, the theory presented by Professor LeConte is upheld.

It is unfortunate that the pick-up unit does not record the actual form of the pressure wave. For

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* This phenomenon was discussed by Professor J. N. LeConte in his article, "Experiments and Calculations on the Resurge Phase of Water Hammer". Professor LeConte developed an expression for the time period between successive resurges and compared the results with those obtained from tests.

this reason, the indicator would be unsuitable in determining the effect of friction on the retardation of the pressure wave. This factor has not been examined experimentally and is only included in the analytical and graphical solutions as an approximation.

A new condenser-type indicator is now available which will record the actual form of the pressure wave and may be calibrated statically. This type of indicator will be ideal for friction studies.

It is the author's opinion that the piezo-electric pressure indicator is not ideally suited to the measurement of water hammer pressures because of the exponential pressure leak-off which makes the interpretation of the wave forms difficult and involves the calculations, especially for slow gate closures.

To improve the existing apparatus, a long persistent screen tube should replace the medium persistent tube now in the oscillograph. The long persistent tube will hold the pressure changes on the screen for approximately 1 second, so that more than one wave may be photographed. With this tube and the condenser-type indicator, the friction "die-down" of the pressure wave may be studied and the resurge effect checked.

A smoothly operating valve would greatly improve the apparatus, and with the timing device shown in Fig. 13 arranged with intermediate contacts, an accurate gate-closing-time curve may be plotted. This addition will be necessary if the pressures during slow gate closures are to be studied.

It is hoped that this work may be continued so that more conclusive results may be obtained, especially in the analysis of the following:-

1. The effect of branch lines on water hammer with a view to studying water distribution systems.
2. The effect of friction on the dying-down of the pressure wave.
3. The effect of elbows and bends in the pipe on water hammer.

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APPENDIX I

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APPENDIX II

Determination of Leakage Loss

Let, P^1 = Pressure with no loss.

P = Pressure corresponding to reading on oscillograph screen.

P_2 = Pressure equivalent to the charge that has leaked off the crystals.

t = Time of gate closure, given as a fraction of the period.

T = Period = $1/29.4$ seconds.

With no leakage loss, the rate of change of the pressure with respect to time will be a constant, i.e.,

$$\frac{dP^1}{dt} = C$$

or $$P^1 = CT$$

If the exponential loss of pressure follows the law,

$$P = Ke^{-at}$$

$$\text{then } \frac{dP_2}{dt} = -aKe^{-at} = aP$$

$$\text{also } \frac{dP}{dt} = \frac{dP^1}{dt} - \frac{dP_2}{dt}$$

$$= C - aP$$

$$\frac{dP}{c/a - P} = adt \text{ or } \log\left(\frac{c}{a} - P\right)\Big|_T^o = at\Big|_o^T$$

$$\text{i.e., } \frac{\frac{c}{a}}{\frac{c}{a}-P} = e^{aT} \text{ or } \frac{c}{a}-P = \frac{c}{a}\, e^{-aT}$$

$$\text{i.e., } P = \frac{c}{a}\,(1-e^{-aT})$$

$$P = \frac{P^1}{at}\,(1-e^{-aT})$$

$$\text{let } K^1 = \frac{1-e^{-at}}{at}$$

$$\text{Then } \frac{P}{P^1} = K^1$$

Example: (See Figs. 18 and 20).

From the screen record, the pressure wave drops from 15 to 5 divisions in 1 period, i.e., $e^{-at} = 1/3$ for 1 period, $\frac{1}{\sqrt{3}}$ for 1/2 period and $\frac{1}{\sqrt[4]{3}}$ for 1/4 period.

For closure in 1 full period $= \frac{1}{29.4}$ secs., the theoretical undamped pressure $P^1 = CT$ gives $P^1 = C\times\frac{1}{29.4}$ or $C = 29.4\ P^1$.

The exponential law also gives $e^{-a\times\frac{1}{29.4}} = 1/3$

$$\text{or } \quad a = 30 \log_e 3 = 32.5$$

$$P = \frac{29.4\ P^1}{32.5}\,(1 - 1/3) = .604\ P^1 = K^1P^1,$$

i.e., the pressure indicated on the screen for closure in 1 period is 60.4% of the actual pressure.

The value of K^1 may be found for closure in 1/2 period and 1/4 period. The results are plotted in Fig. 16.

APPENDIX III

Graphical Solution of Water Hammer

The theory has already been given and examples will be shown here as illustrations of the method. The results of the study of the effect of branch lines on the pressure rise at the gate will then be analyzed.

Characteristics of pipe and conditions of flow:-

H_o = 300 ft. L = 1960 ft. d = 24" $\frac{2L}{a}$ = 1 Sec.

V_o = 4.28 ft./sec. a = 3920 ft./sec.

T = 2.5 secs. Stress = 10,000#/sq.in.

NOTE: In all examples, the effect of friction has been excluded and the gate is assumed to close uniformly with time.

Example I (Refer to Fig. 23):

Using only the points of the gate and reservoir, equation (25) may be written as:

$$h_{C.5} - h_{A0} = +2K(v_{C.5} - v_{A0})$$

$$h_{A1.0} - h_{C.5} = -2K(v_{A1.0} - v_{C.5})$$

$$h_{C1.5} - h_{A1.0} = +2K(v_{C1.5} - v_{A1.0})$$

$$h_{A2.0} - h_{C1.5} = -2K(v_{A2.0} - v_{C1.5})$$

$$h_{C2.5} - h_{A2.0} = +2K(v_{C2.5} - v_{A2.0})$$

The chart is drawn using equation (16) which represents a series of parabolas with their vertices at zero. For full gate opening $\tau = 1$ and $\frac{V}{V_o} = \sqrt{\frac{H}{H_o}}$; and after the lapse of 0.5 seconds or 1/5 of the total closing time, $\tau = 0.8$ and $\frac{V}{V_o} = 8\sqrt{\frac{H}{H_o}}$. These parabolas are shown 0.25 seconds apart, or 1/10 of the gate closing time.

The equations given above are represented by two series of parallel straight lines; the third and fifth give lines with a slope whose tangent is +2K, and the remainder have slopes equal to -2K. Conditions at the reservoir are always represented on the horizontal line $\frac{H}{H_o} = 1$. This is true if the reservoir is large and there is no friction in the line. Evidently, the conditions at the gate are always shown on the parabola of the kind described.

Points A_o and $C_{.5}$ are at $V = V_o$ and $H = H_o$, because the conditions at the reservoir end are not changed until 0.5 secs. later than at the gate end, as the pressure wave set up by the gate closure takes 0.5 secs. to travel to the reservoir from the gate. The equations showing the positions of A and C at the different times are given above. In this way the points A and C at the beginnimg and end of each interval are determined and are shown on the diagram. The pressure time curve is shown in Fig. 26.

Example II (Refer to Fig. 24):

The specific case of a branch line of the same length as the main pipe and placed at a distance $\frac{L}{4}$ from the gate will be studied.

The parabolas are drawn as in Example I, and the points of intersection determined from equation (25).

$$h_{A0} - h_{B.125} = -2K(v_{A0} - v_{B.125})$$
$$h_{B.125} - h_{C.5} = -2K(v_{B.125} - v_{C.5})$$
$$h_{B.125} - h_{D.625} = -2K(v_{B.125} - v_{D.625})$$

These equations evidently represent the conditions at zero time, that is, before gate movement begins. After 0.125 seconds, or when the gate is 95% open, the conditions will be as follows:

$$h_{B.25} - h_{A.125} = +2K(v_{B.25} - v_{A.125})$$
$$h_{C.625} - h_{B.25} = +2K(v_{C.625} - v_{B.25})$$
$$h_{D.75} - h_{B.25} = +2K(v_{D.75} - v_{B.25})$$

As D is a "dead end", the velocity at point D will be zero, and, therefore, D will fall along the line $\frac{V}{V_0} = 0$. The position of B on the chart is determined from the conditions at B on the conduit, that is, there must be an equality of flow at the junction B. Since, in this case, the diameters of the pipes are equal, the sum of the velocities flowing towards the junction will be zero. Therefore, in the first phase, when the water is flowing towards C and D, $V_{AB} = V_{BC} - V_{BD}$. The position of B will then be found by determining the difference between V_{AB} and V_{BC}, which should be equal to V_{BD}. This difference is shown on the chart.

The effect of branch lines, at the gate, 1/2 point and 3/4 point, were also studied. The Iso-Pressure chart, Fig. 25, shows the results of these studies and gives the maximum pressure rise at the gate due to a specific length of the branch pipe and placed along the main line at a given distance from the gate.

The following observations may be made:-

(a) The rise in pressure increases as the length of the branch line increases.

(b) The rise in pressure increases as the branch line approaches the gate.

(c) From the general charts and Fig. 26, the maximum rise in pressure takes place in a shorter time after the gate begins to close as the branch line becomes shorter.

Figs. 27 and 28 show the pressure rise for various combinations of branch lines. These problems are solved in the same manner as the case with the single branch line.

This study has been presented here as it was intended to check these theoretical results with actual tests of branch lines. It is hoped that this may be done in the near future.

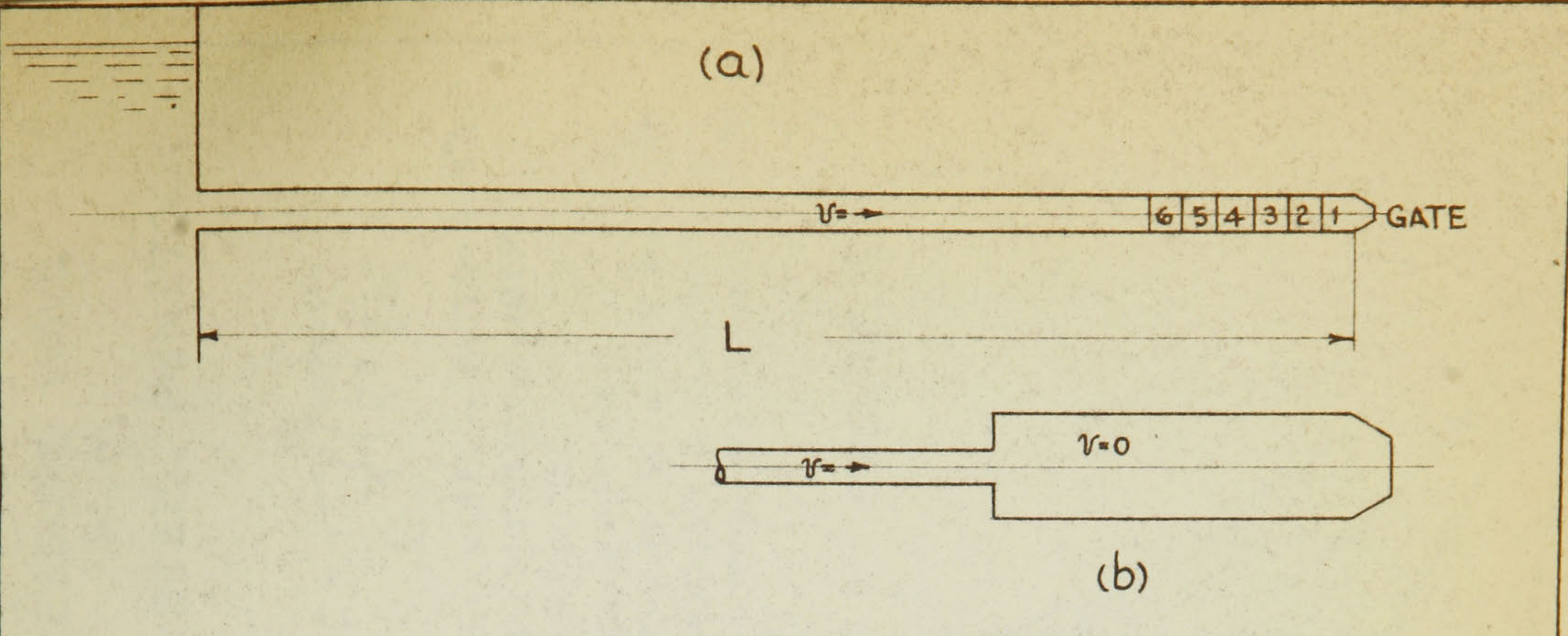


FIGURE 1

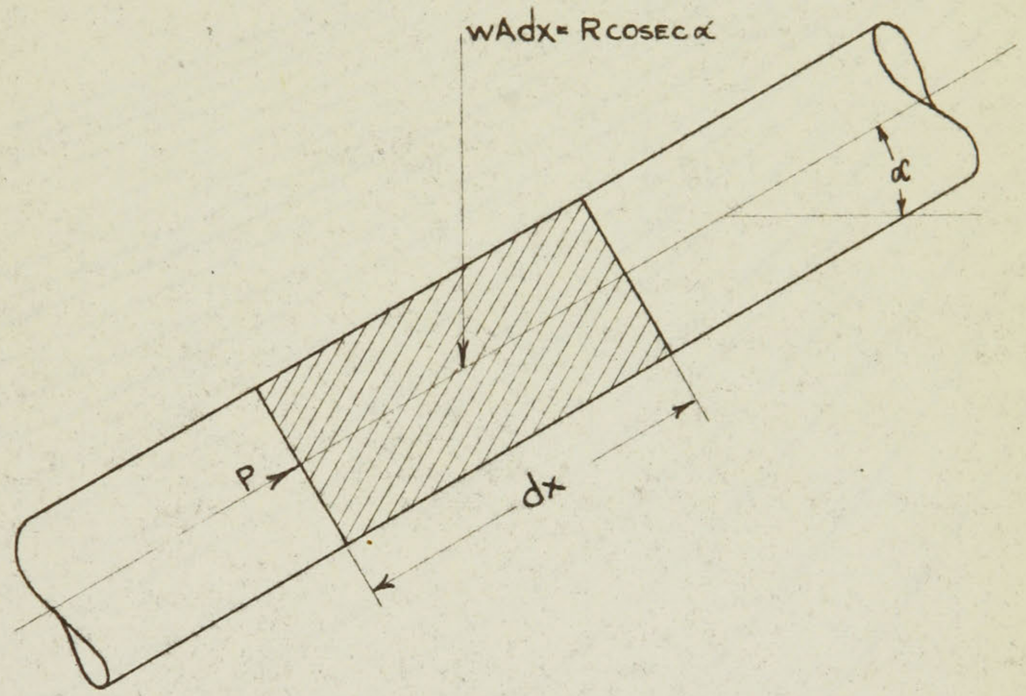
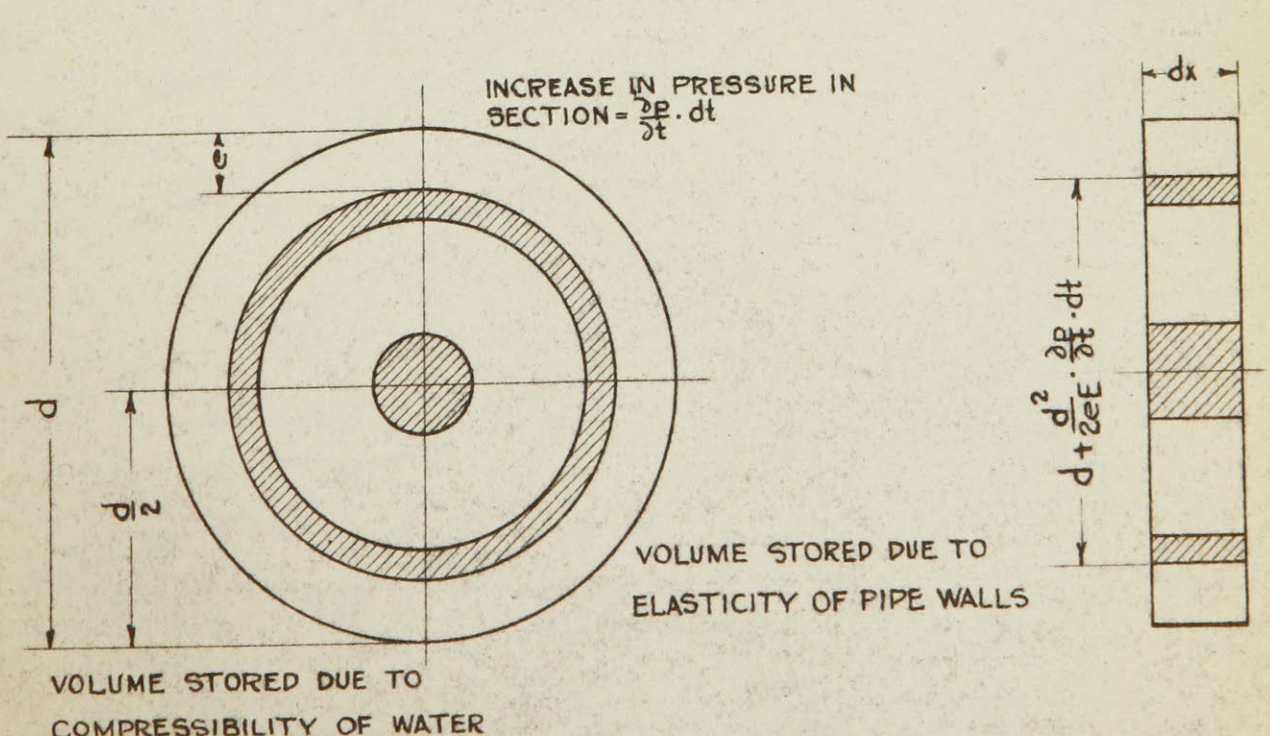
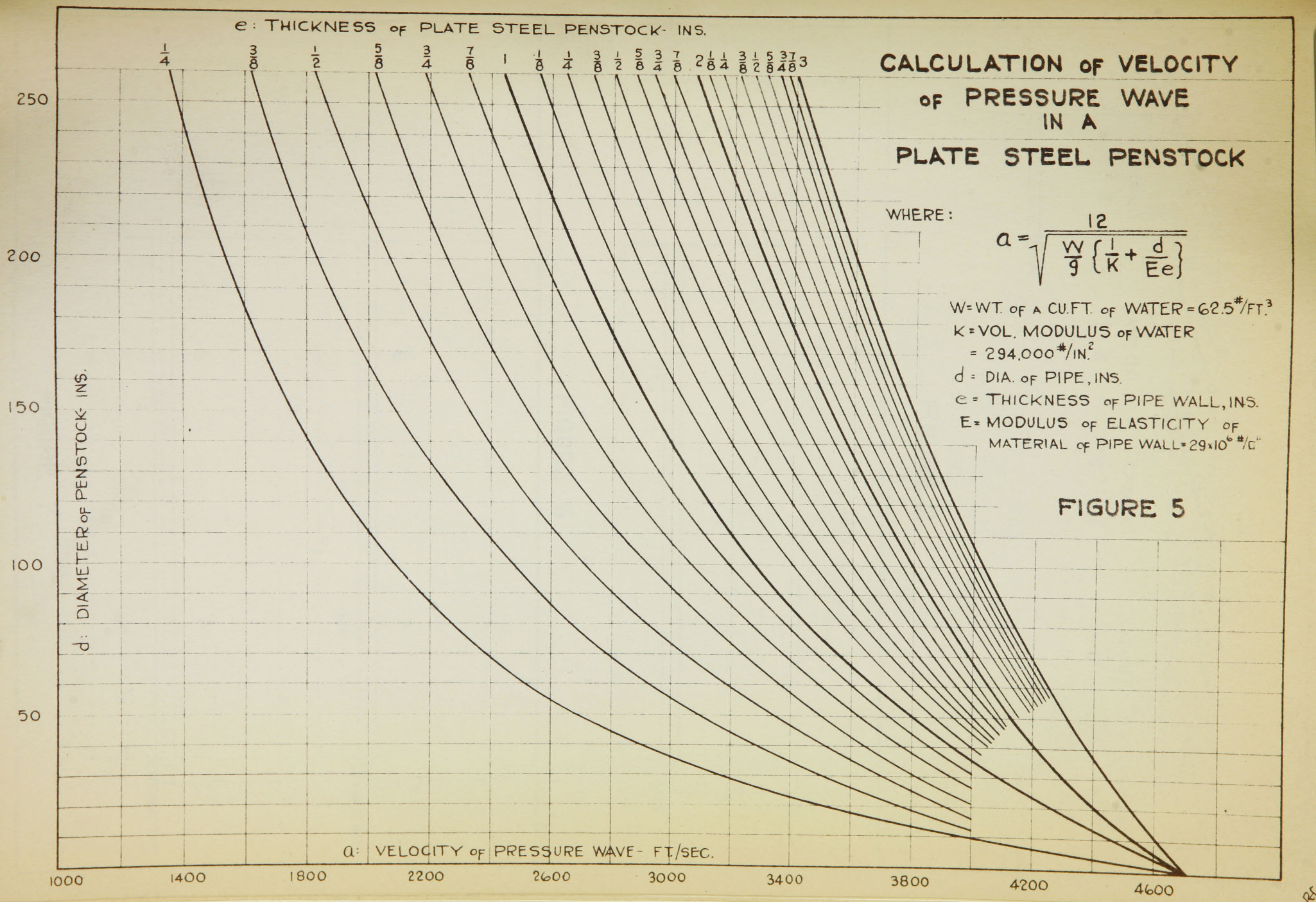


FIGURE 3



DIAGRAMMATIC STUDY OF CHANGES IN VOLUME OF PIPE

FIGURE 4


e: THICKNESS OF PLATE STEEL PENSTOCK- INS.
1/4 3/8 1/2 5/8 3/4 7/8 1 1/8 1/4 3/8 1/2 5/8 3/4 7/8 2 1/8 1/4 3/8 1/2 5/8 3/4 7/8 3
CALCULATION OF VELOCITY OF PRESSURE WAVE IN A PLATE STEEL PENSTOCK
WHERE:
$a = \dfrac{12}{\sqrt{\frac{W}{g}\left\{\frac{1}{K} + \frac{d}{Ee}\right\}}}$
W = WT. OF A CU. FT. OF WATER = 62.5 #/FT.³
K = VOL. MODULUS OF WATER = 294,000 #/IN.²
d = DIA. OF PIPE, INS.
e = THICKNESS OF PIPE WALL, INS.
E = MODULUS OF ELASTICITY OF MATERIAL OF PIPE WALL = 29×10⁶ #/□"
FIGURE 5
d: DIAMETER OF PENSTOCK- INS.
250
200
150
100
50
a: VELOCITY OF PRESSURE WAVE - FT./SEC.
1000
1400
1800
2200
2600
3000
3400
3800
4200
4600

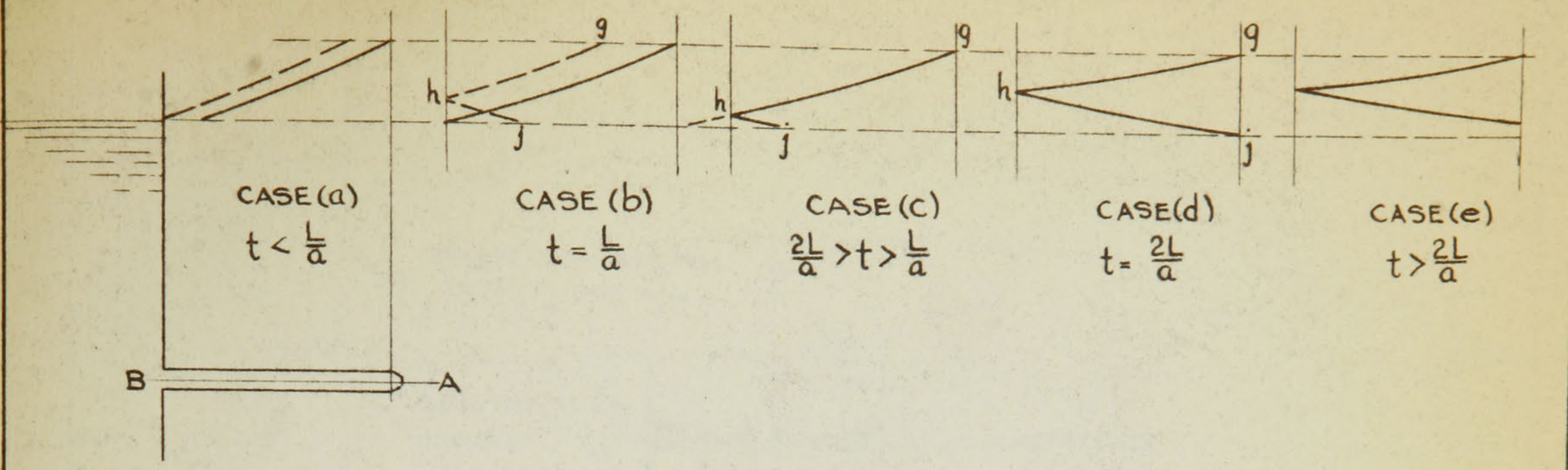


FIGURE 2

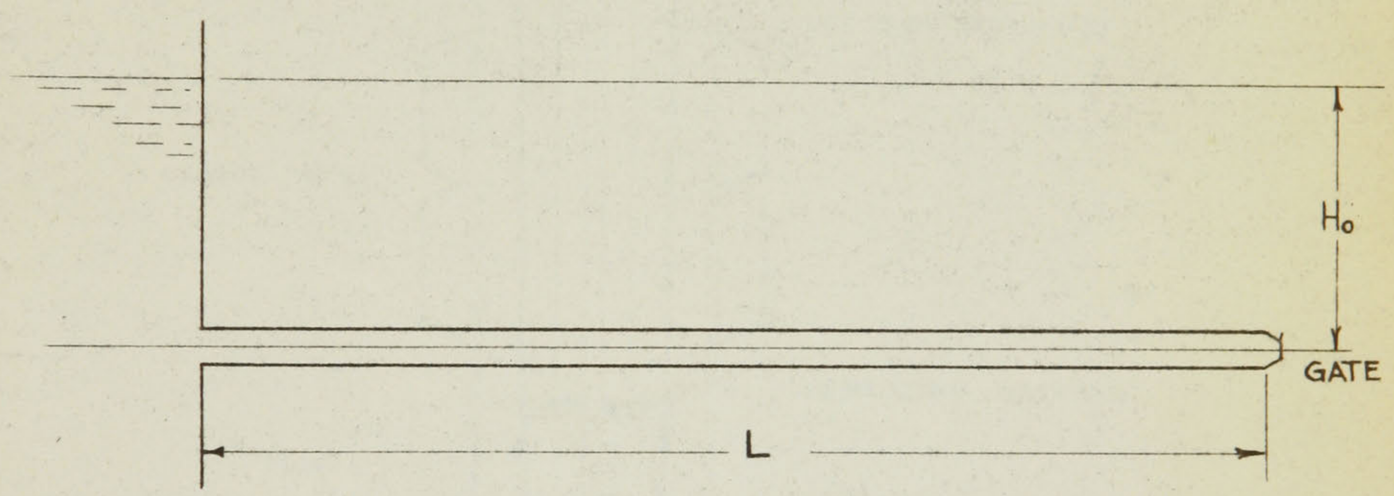


FIGURE 6

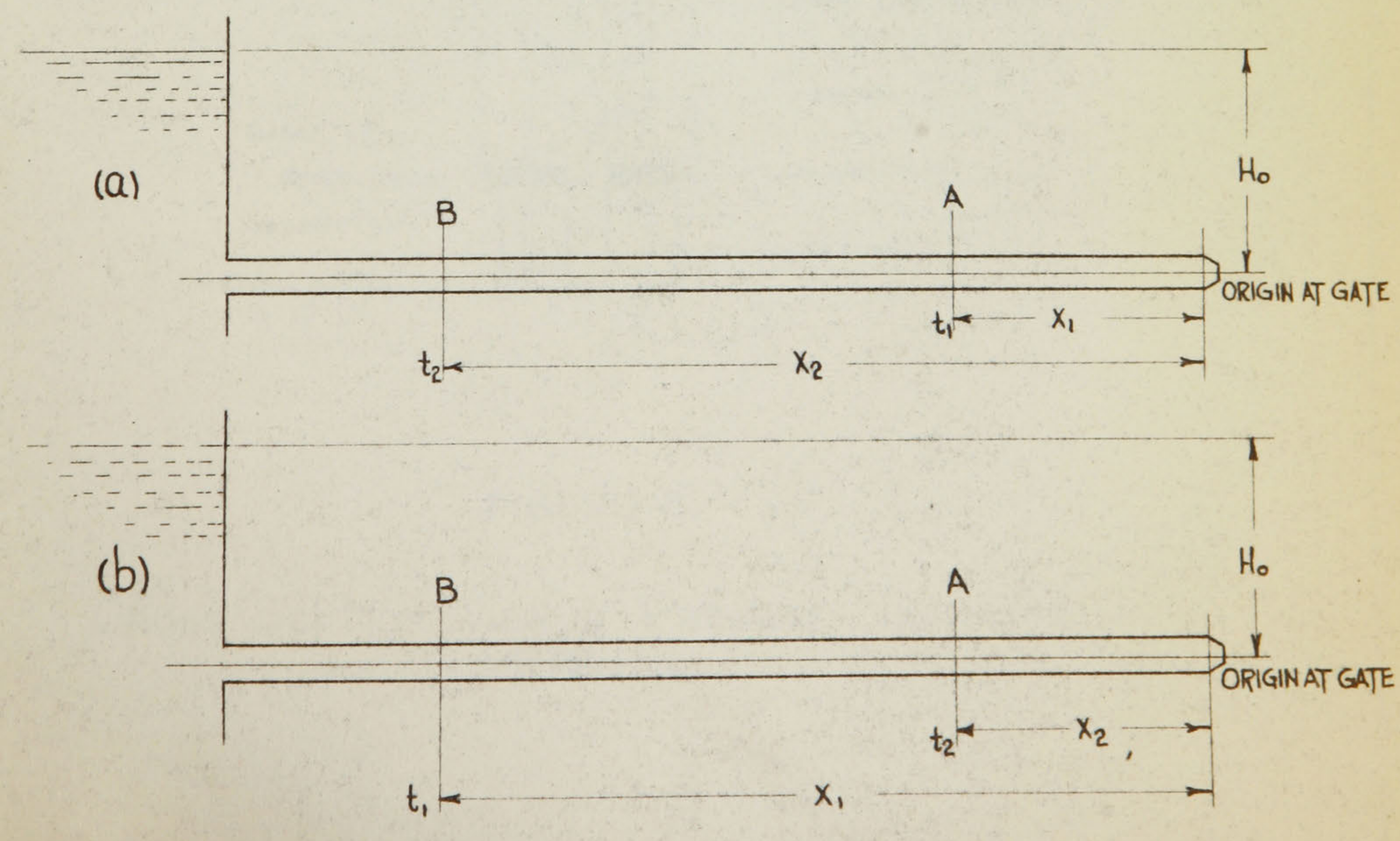


FIGURE 7

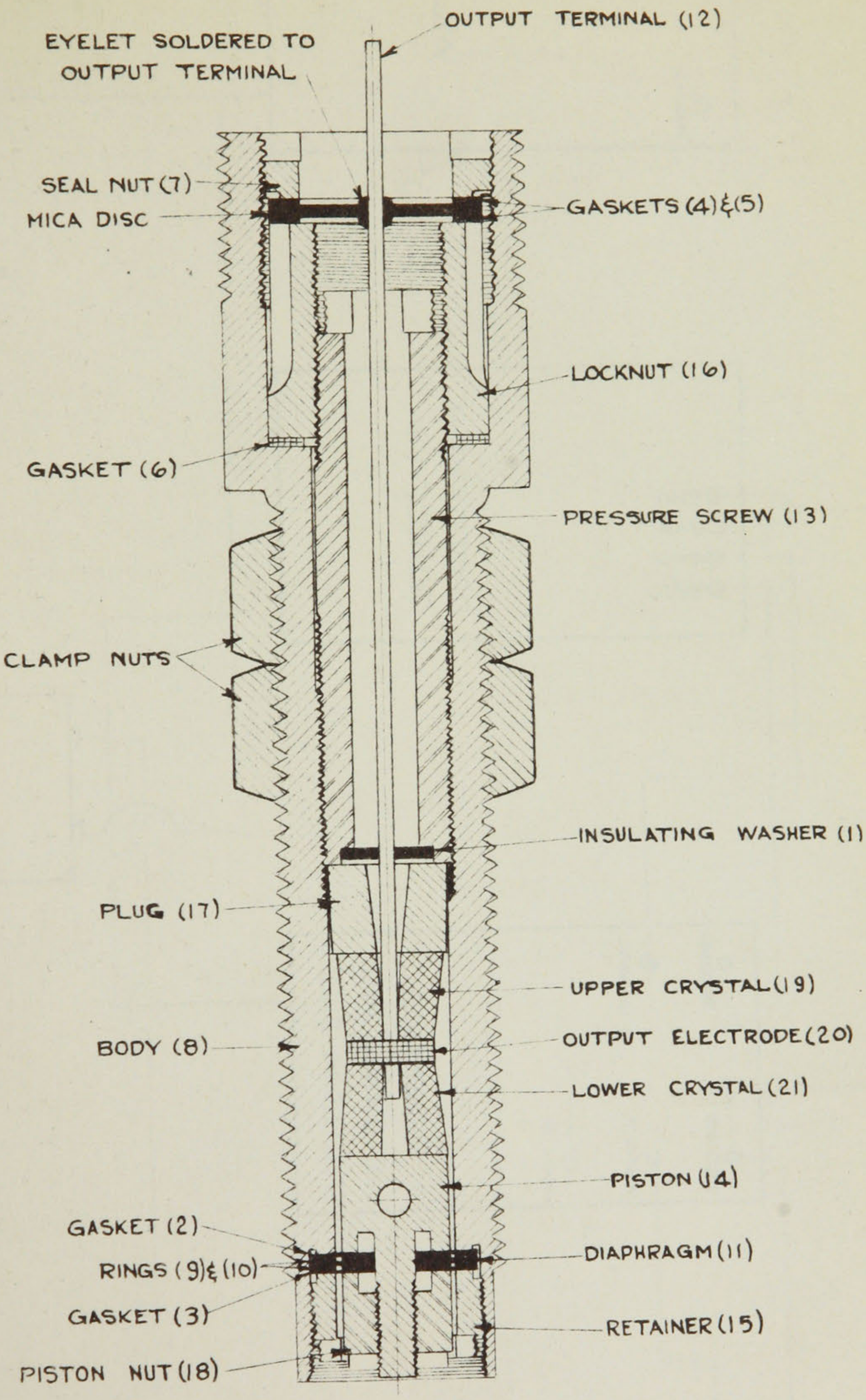


FIGURE 8.

CROSS SECTION VIEW OF PRESSURE INDICATOR
MI-7506A-1

R.H.C.

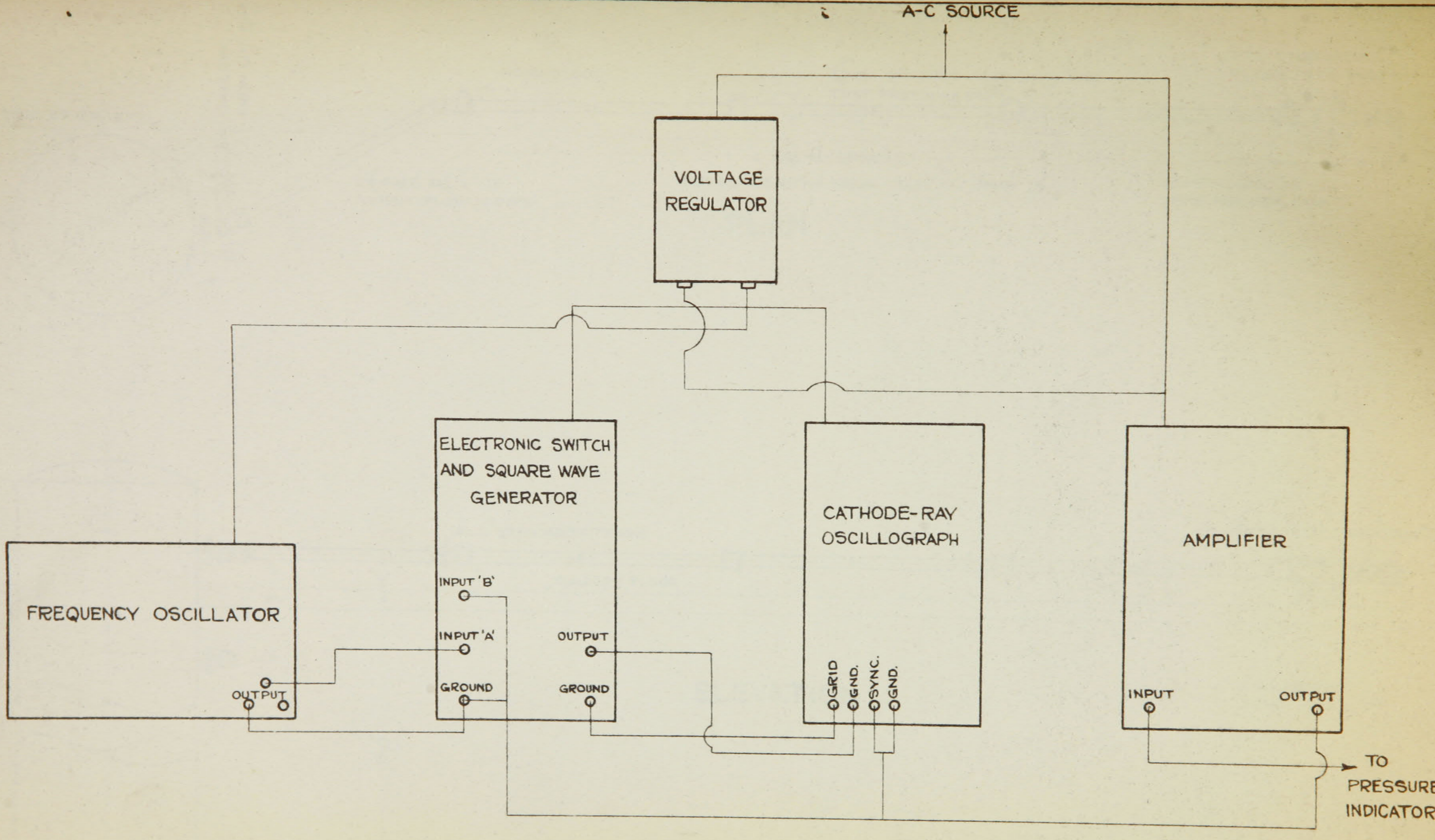


FIGURE 9
BLOCK DIAGRAM OF ELECTRICAL APPARATUS

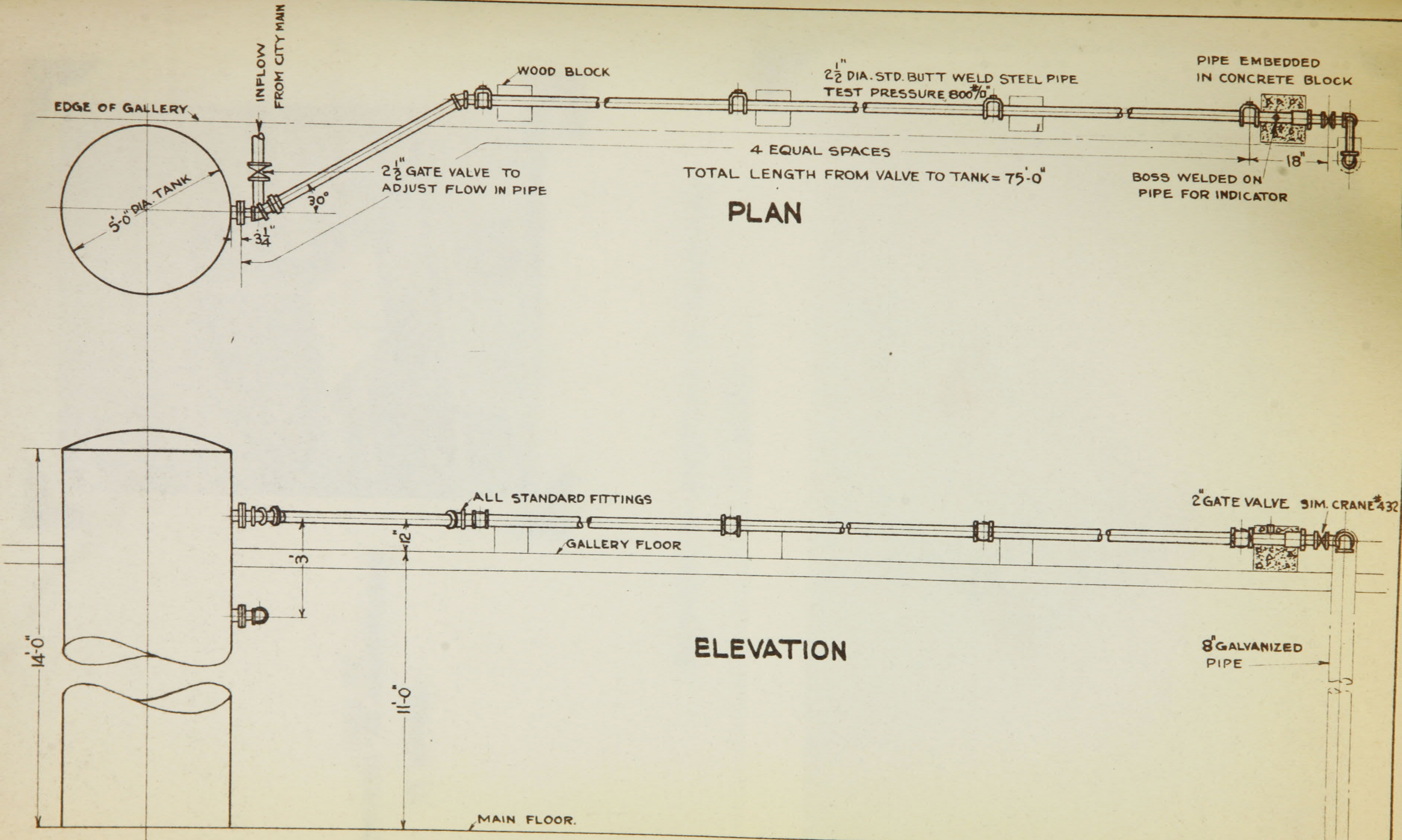


FIGURE 10
GENERAL ARRANGEMENT OF APPARATUS SCALE: $\frac{1}{4}$" = 1 FT.

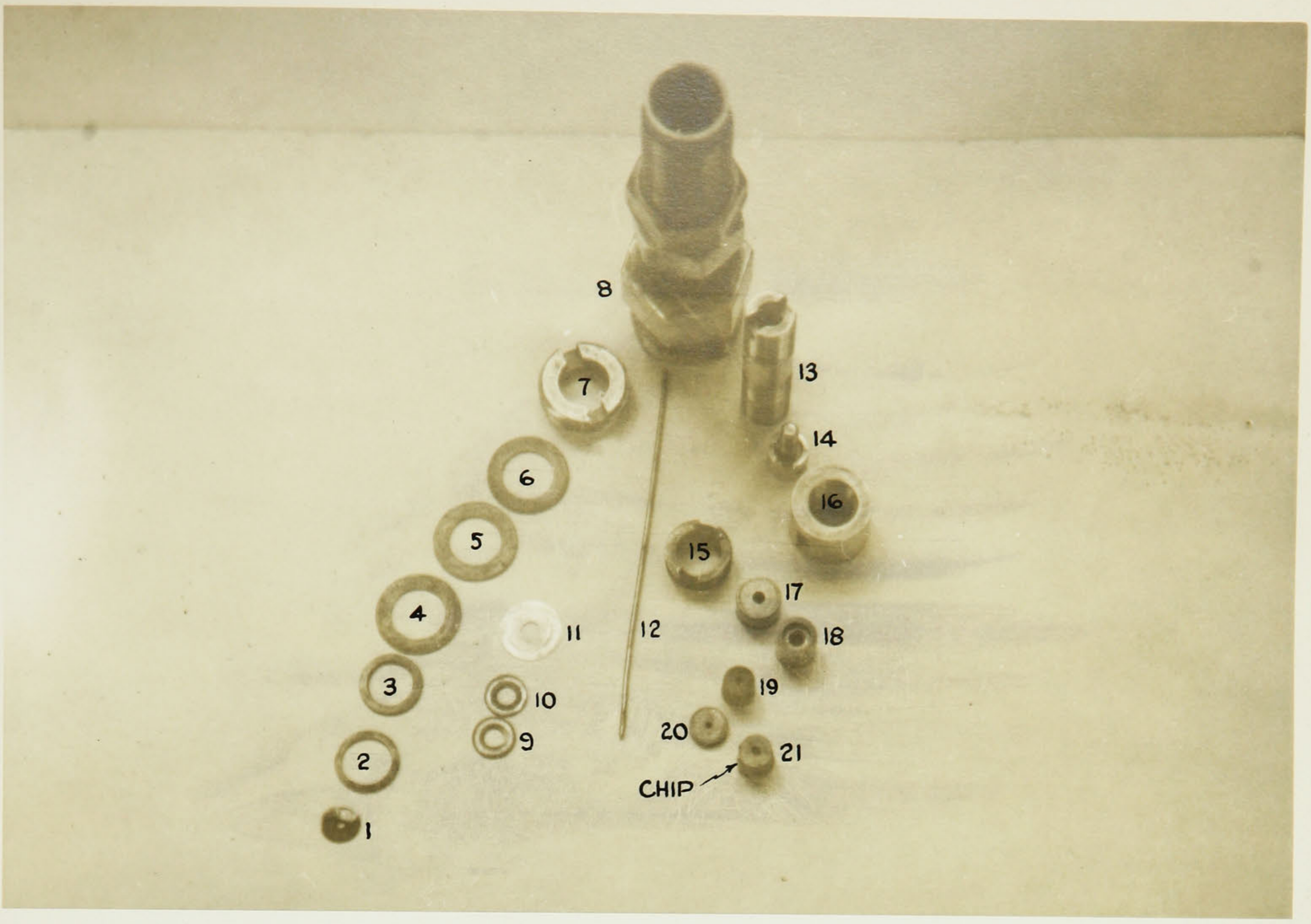


FIGURE 11

ARRANGEMENT OF COMPONENTS OF INDICATOR

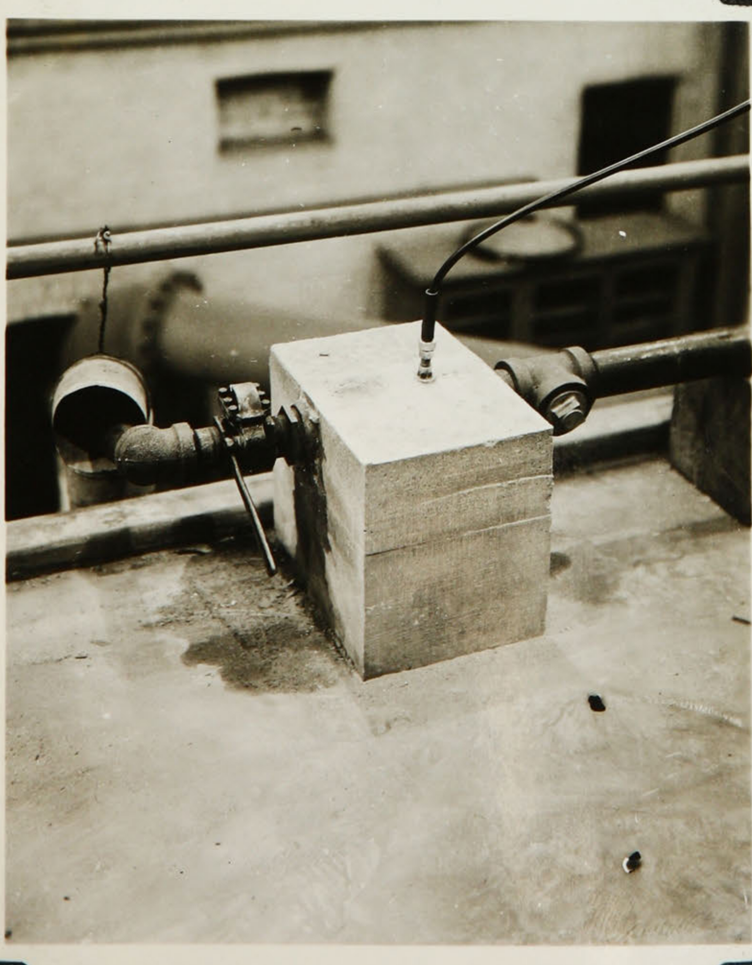


FIGURE 12

ARRANGEMENT OF INDICATOR IN PIPE

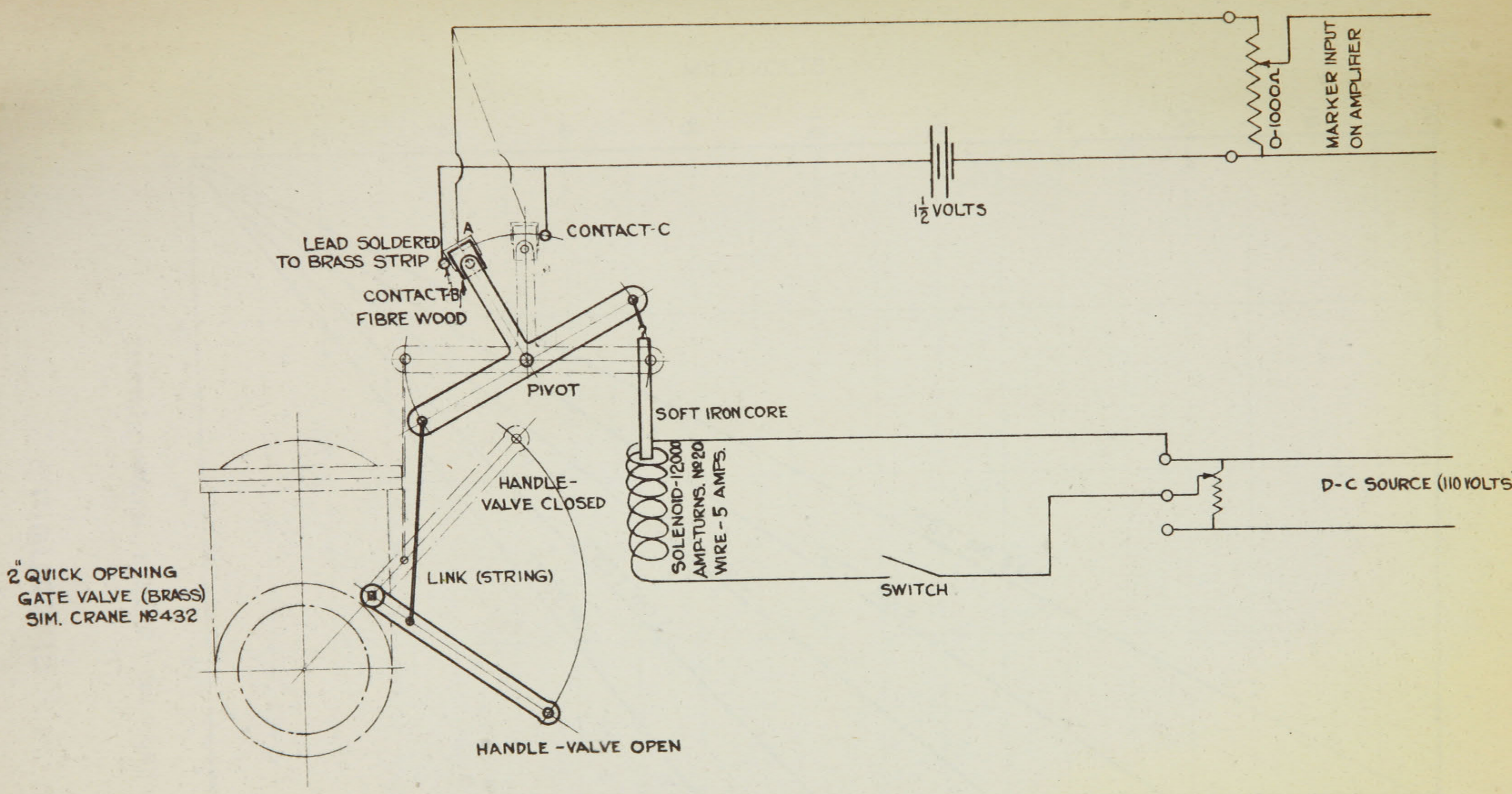


FIGURE 13

GATE TIMING APPARATUS

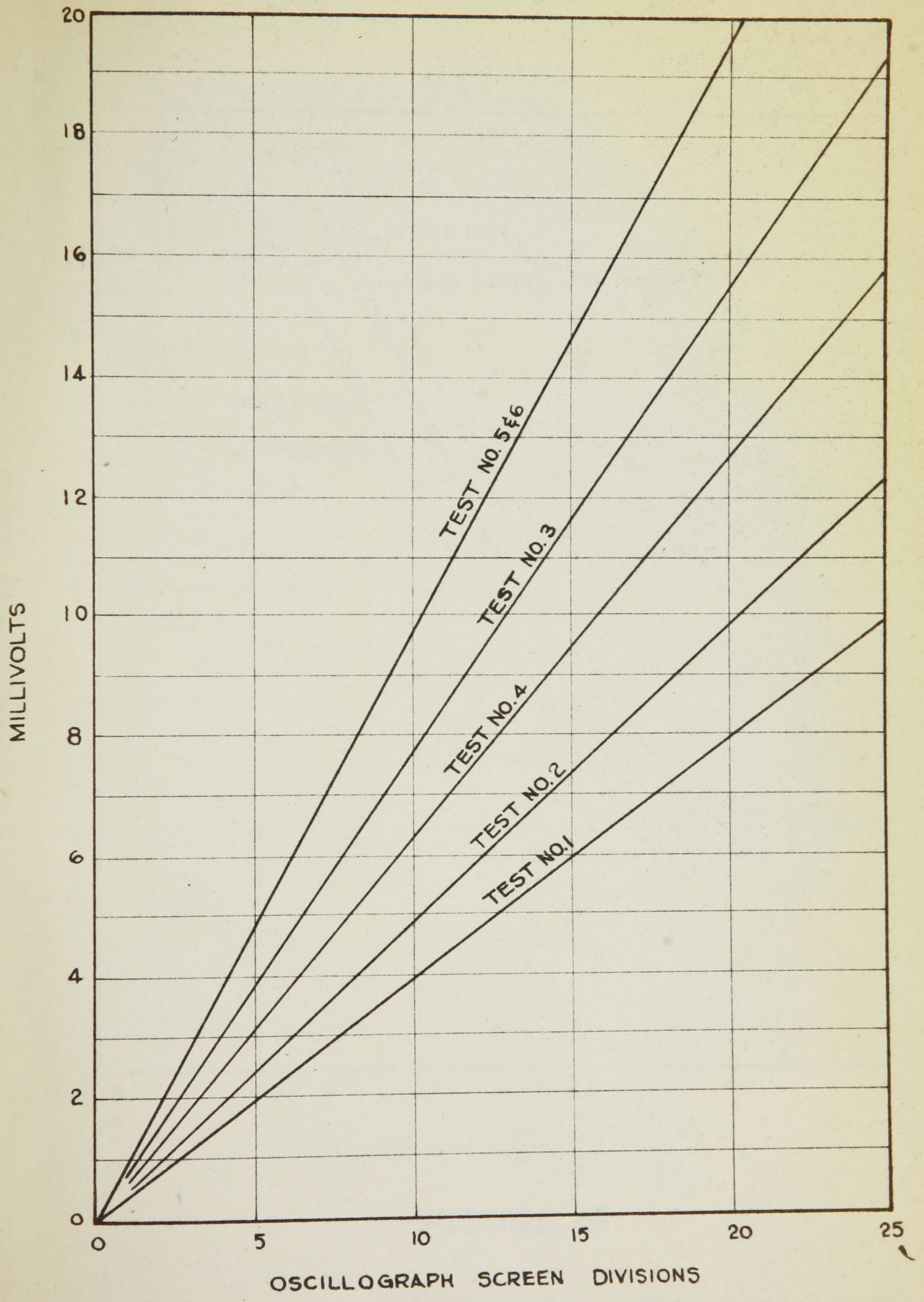


FIGURE 15

CALIBRATION CHART

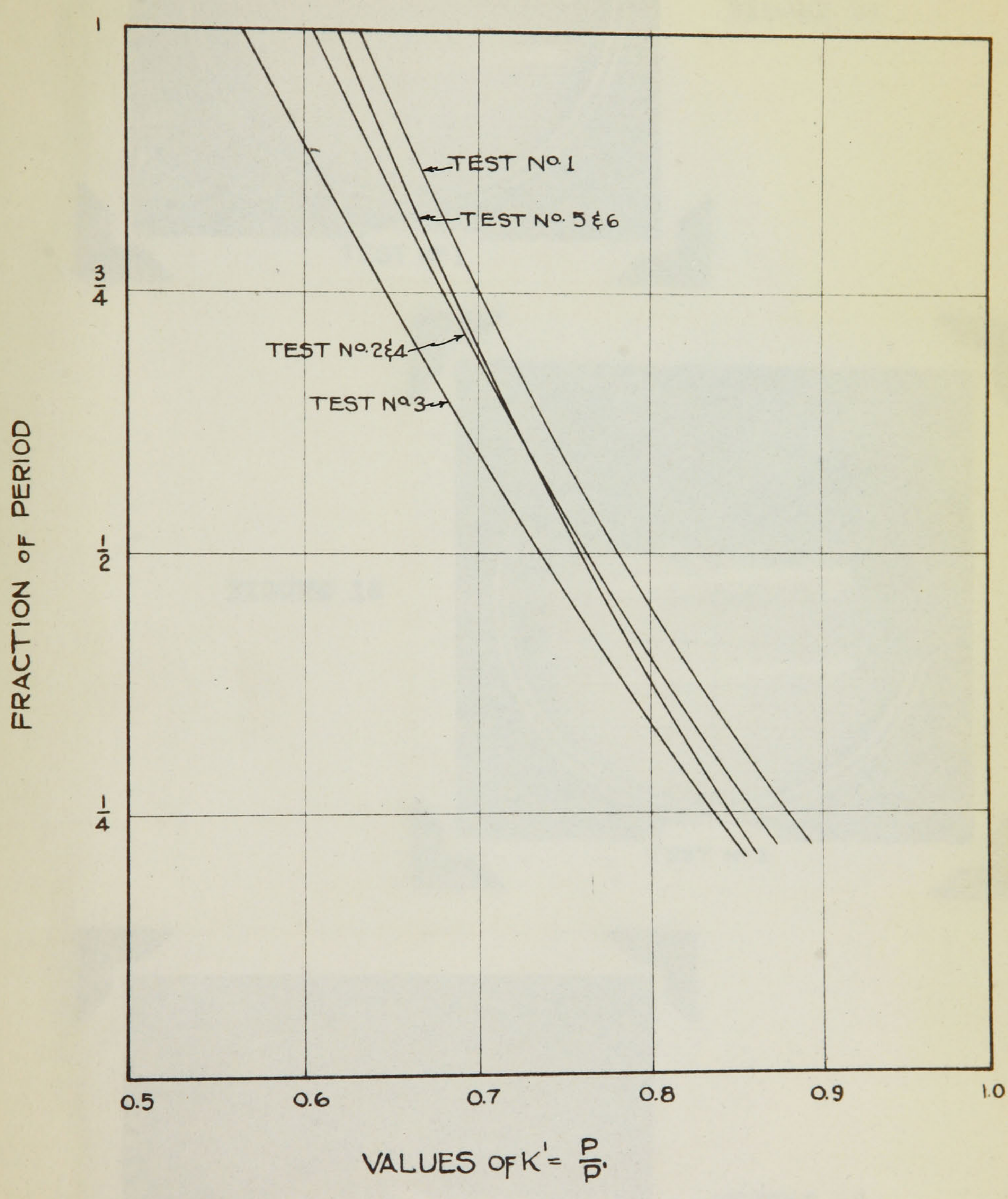


FIGURE 16

CHART SHOWING EXPONENTIAL LEAK-OFF

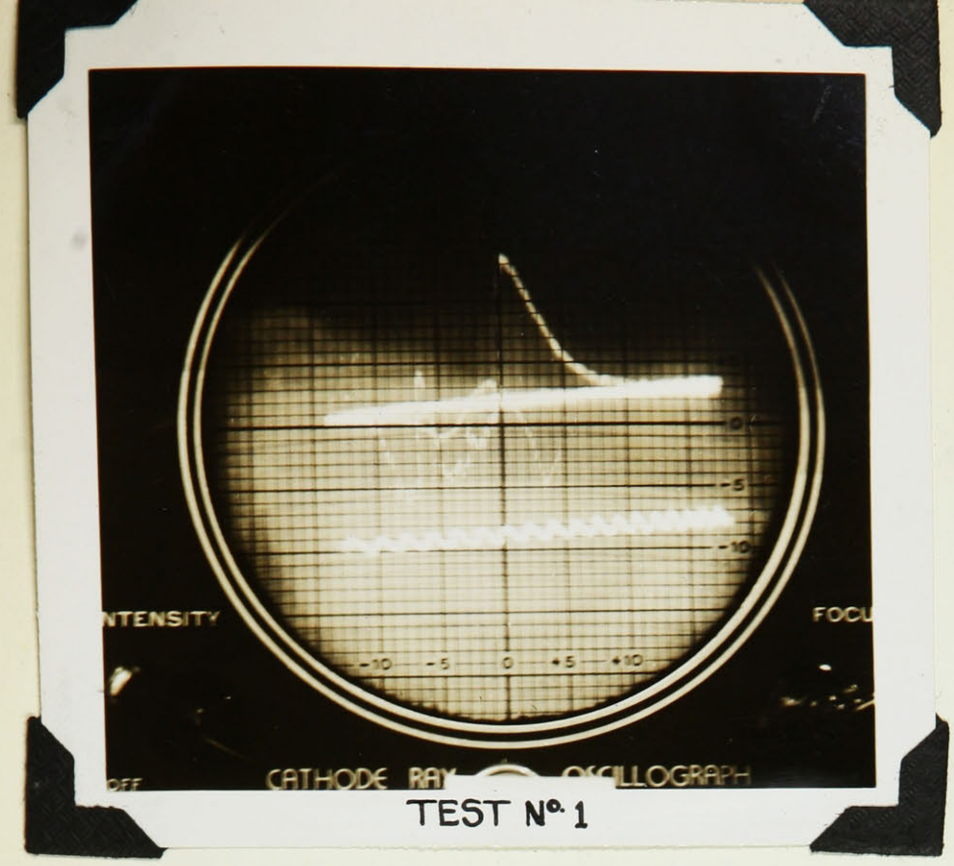
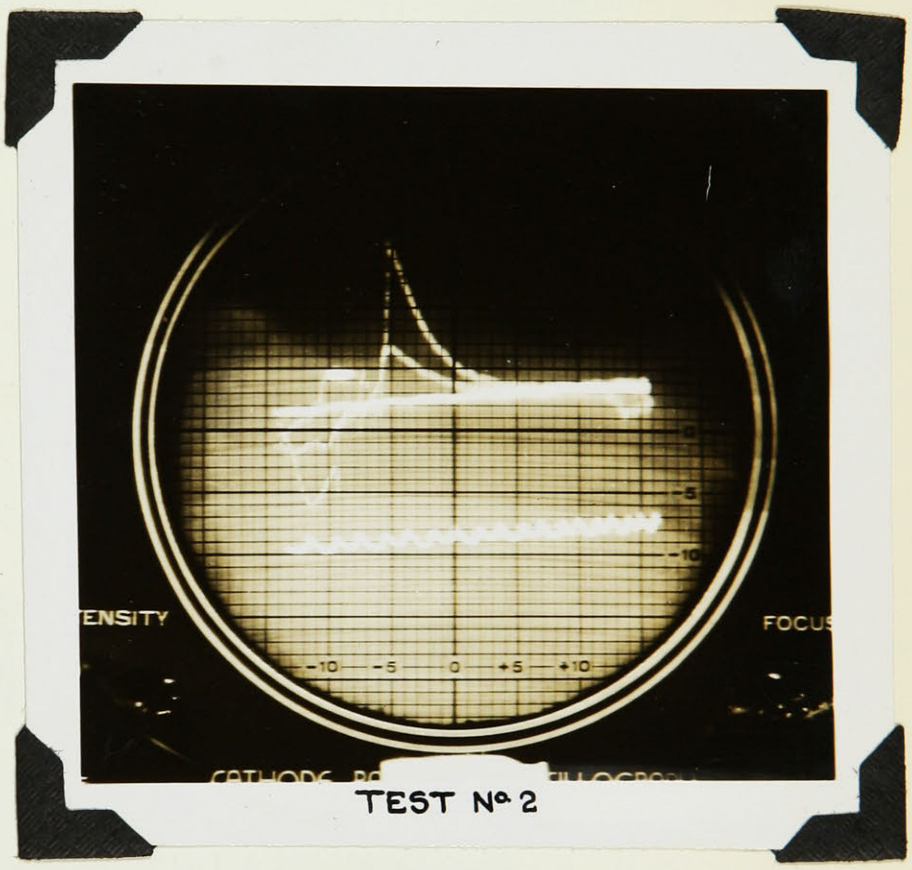


FIGURE 17

FIGURE 18



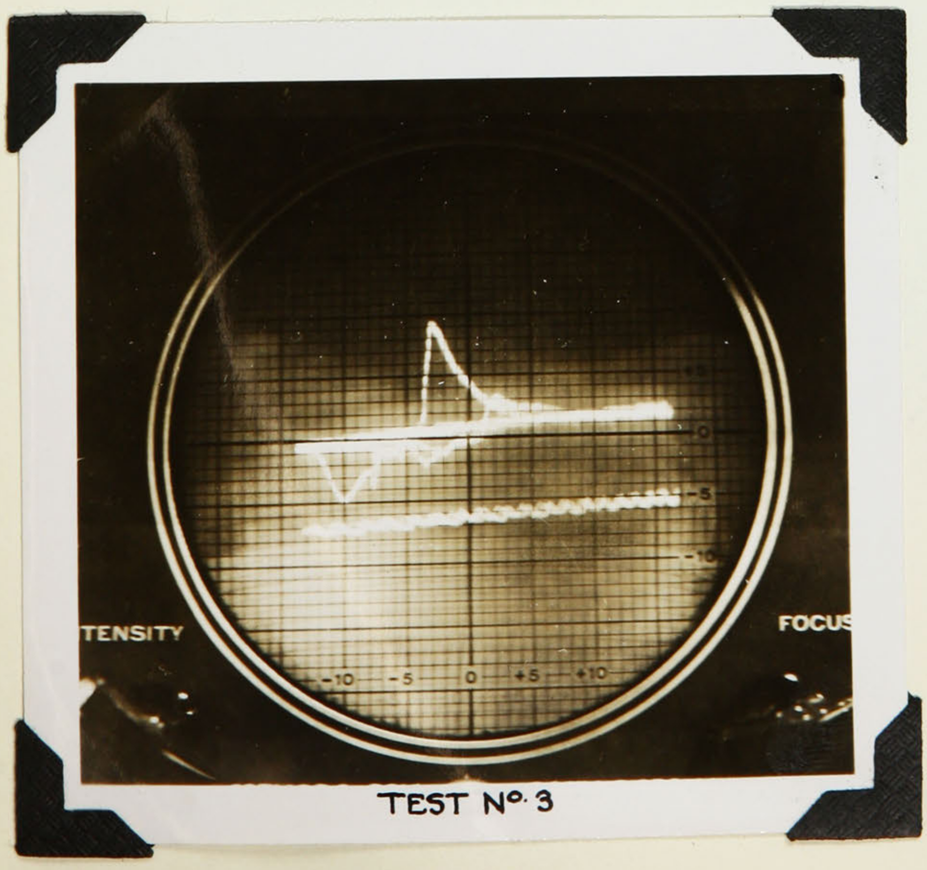


FIGURE 19

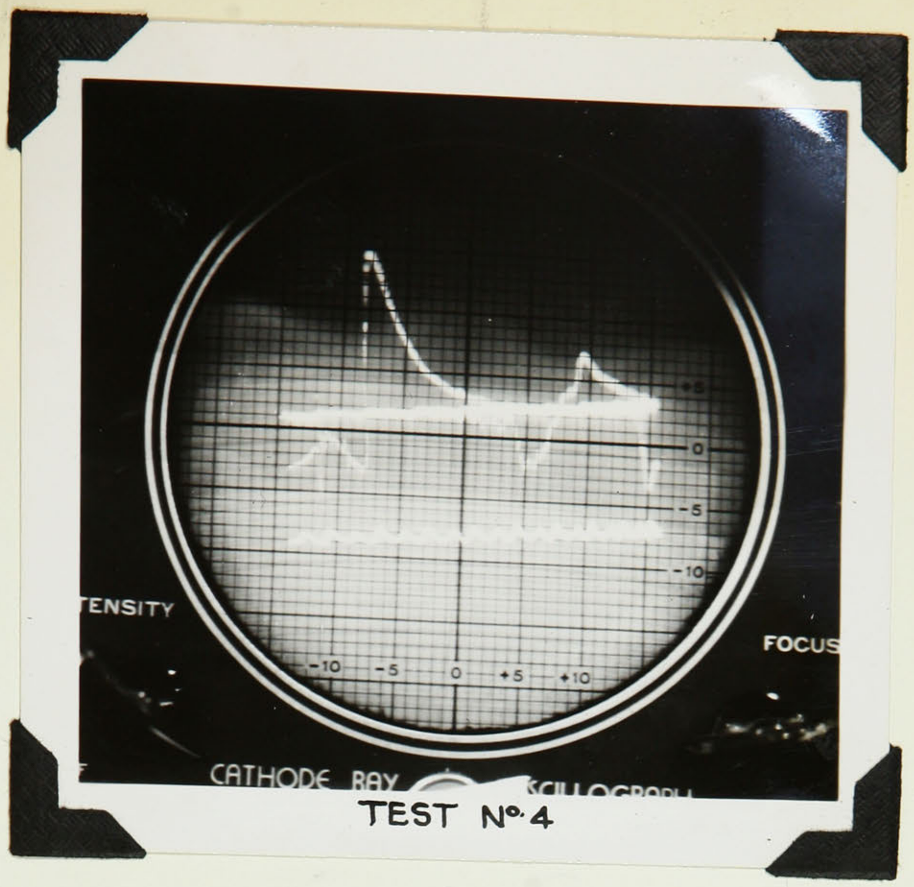


FIGURE 20

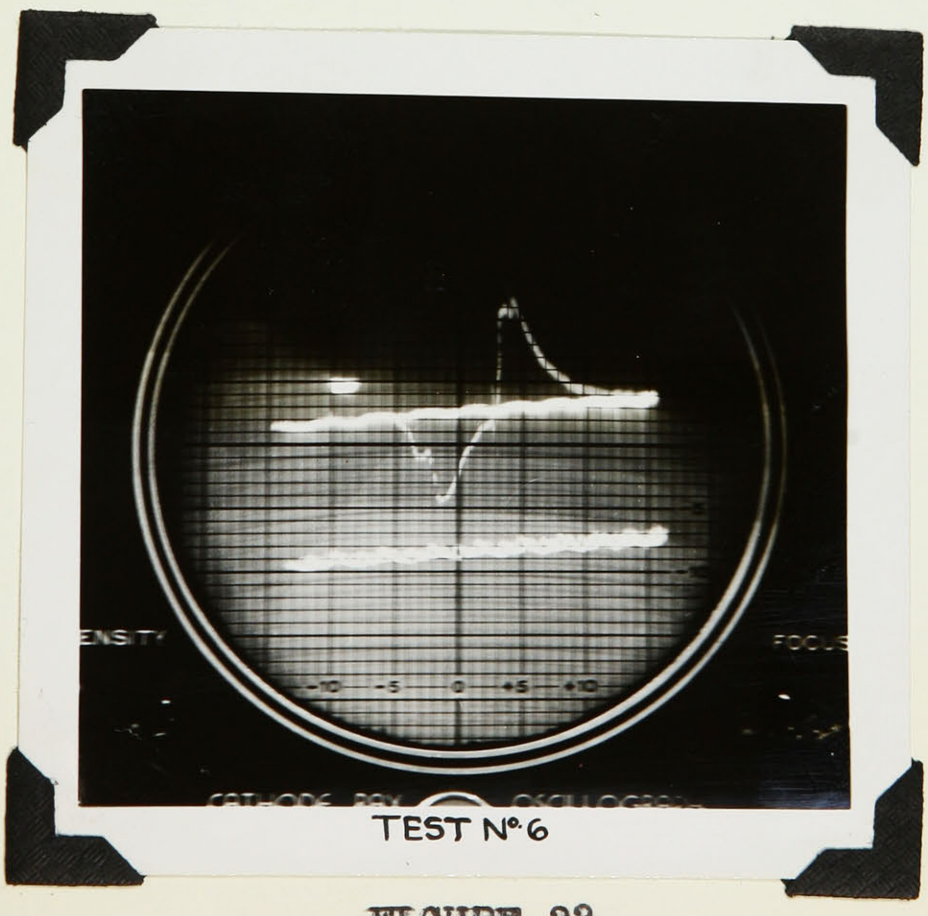


FIGURE 22

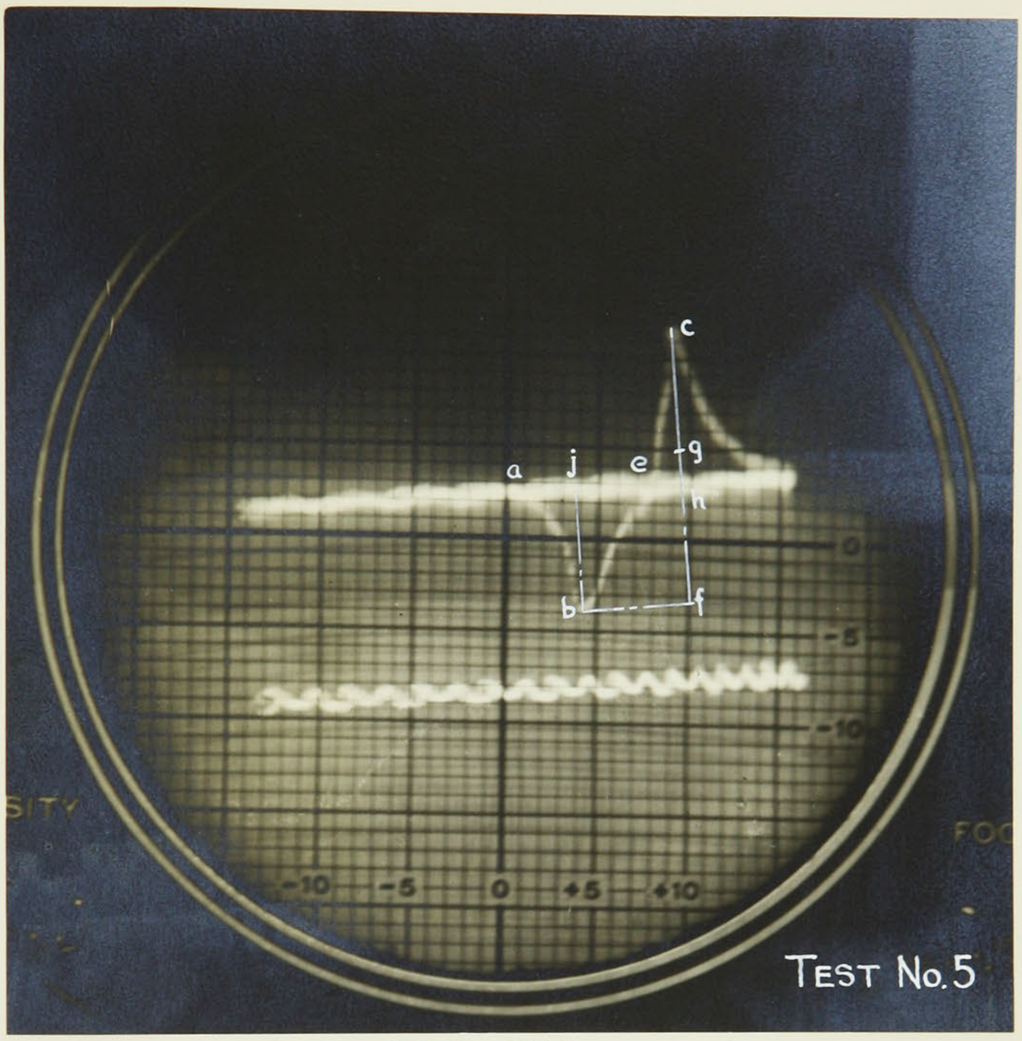


TABLE II

DETAILED CALCULATIONS

POSITIVE PRESSURE

LINE: ab

P: 6.5 DIVISIONS = 6.32 mv. (FIG. 15) = $6.32 \times \frac{100}{3}$ = 211 #/□" = P.

K' = 0.70 (FIG. 16); $P' = \frac{P}{K'}$ = 302 #/□" ; PERIOD: $\frac{2L}{a}$ = .034 SECS.

NEGATIVE PRESSURE

LINE: ec

P: 8.5 DIVISIONS = 8.25 mv. = $8.25 \times \frac{100}{3}$ = 275 #/□" = P

K' = 0.9 ; $P' = \frac{P}{K'}$ = 321 #/□" ; PRESSURE DROP = 19 #/□" BELOW OPER. PRESS.

TOTAL PRESSURE = FRICTION HEAD + VEL. HEAD + ATMOS. PRESS = 3.34 + .47 + 14.7 = 18.45 #/□"

∴ PRESSURE DROPS TO ZERO.

FIGURE 21

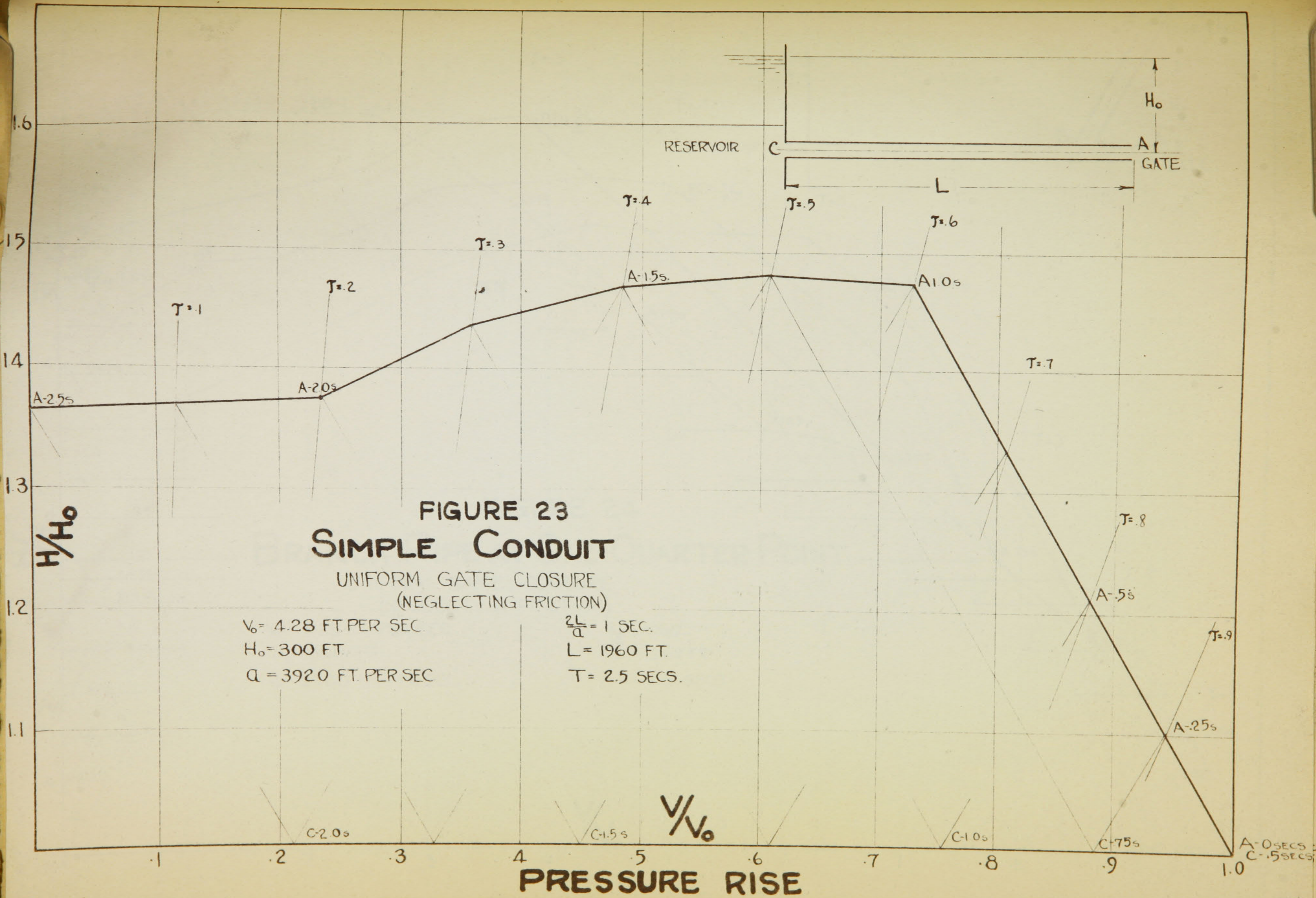


FIGURE 23
SIMPLE CONDUIT
UNIFORM GATE CLOSURE
(NEGLECTING FRICTION)

V_o = 4.28 FT. PER SEC.
H_o = 300 FT.
a = 3920 FT. PER SEC.
$\frac{2L}{a}$ = 1 SEC.
L = 1960 FT.
T = 2.5 SECS.

R-C.

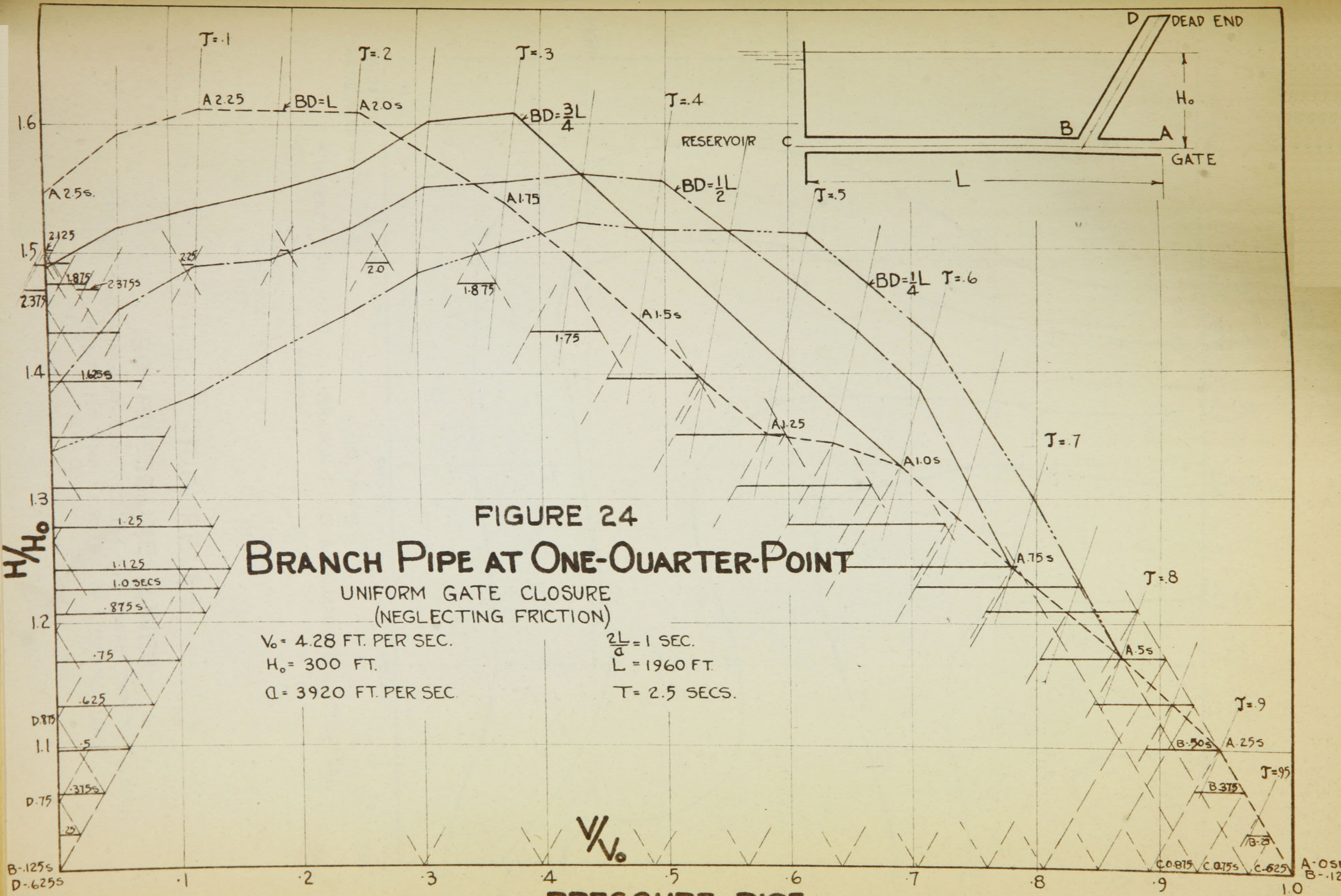


FIGURE 24

BRANCH PIPE AT ONE-QUARTER-POINT

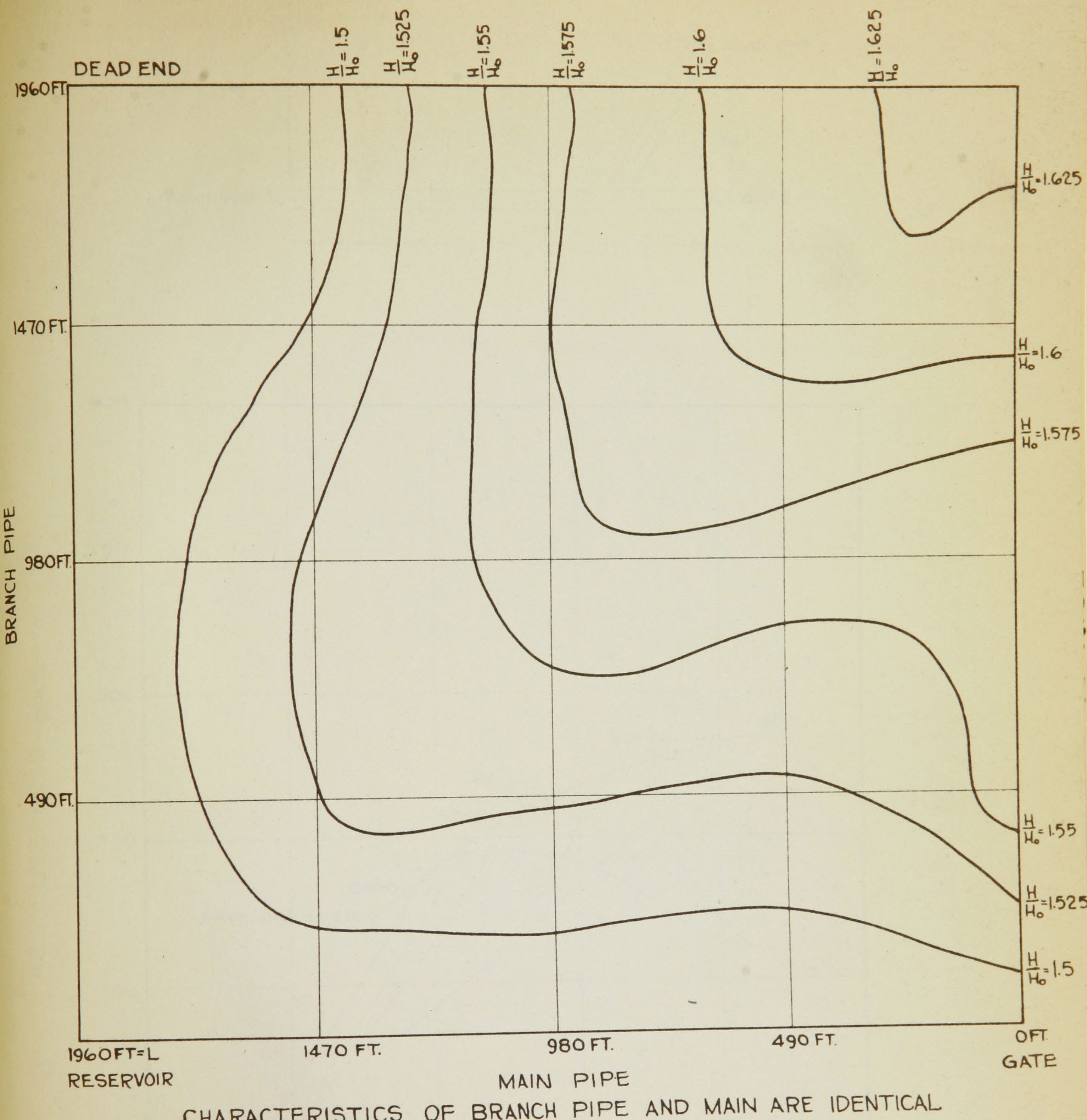
UNIFORM GATE CLOSURE
(NEGLECTING FRICTION)

V_0 = 4.28 FT. PER SEC.
H_0 = 300 FT.
a = 3920 FT. PER SEC.

$\frac{2L}{a}$ = 1 SEC.
L = 1960 FT.
T = 2.5 SECS.

PRESSURE RISE

R.C.



CHARACTERISTICS OF BRANCH PIPE AND MAIN ARE IDENTICAL

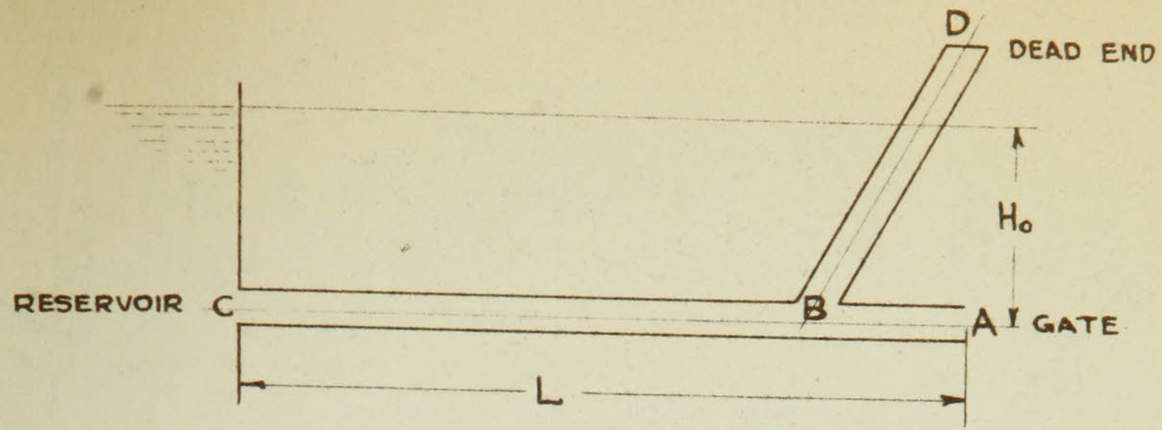
V_o = 4.28 FT. PER SEC.
H_o = 300 FT
a = 3920 FT. PER SEC.
K = .87

$\frac{2L}{a}$ = 1 SEC.
L = 1960 FT.
T = 2.5 SECS.
d = 24 INS.

FIGURE 25

ISO-PRESSURE LINES

CHART SHOWING PRESSURES AT THE GATE DUE TO VARIED POSITIONS AND LENGTHS OF A BRANCH LINE



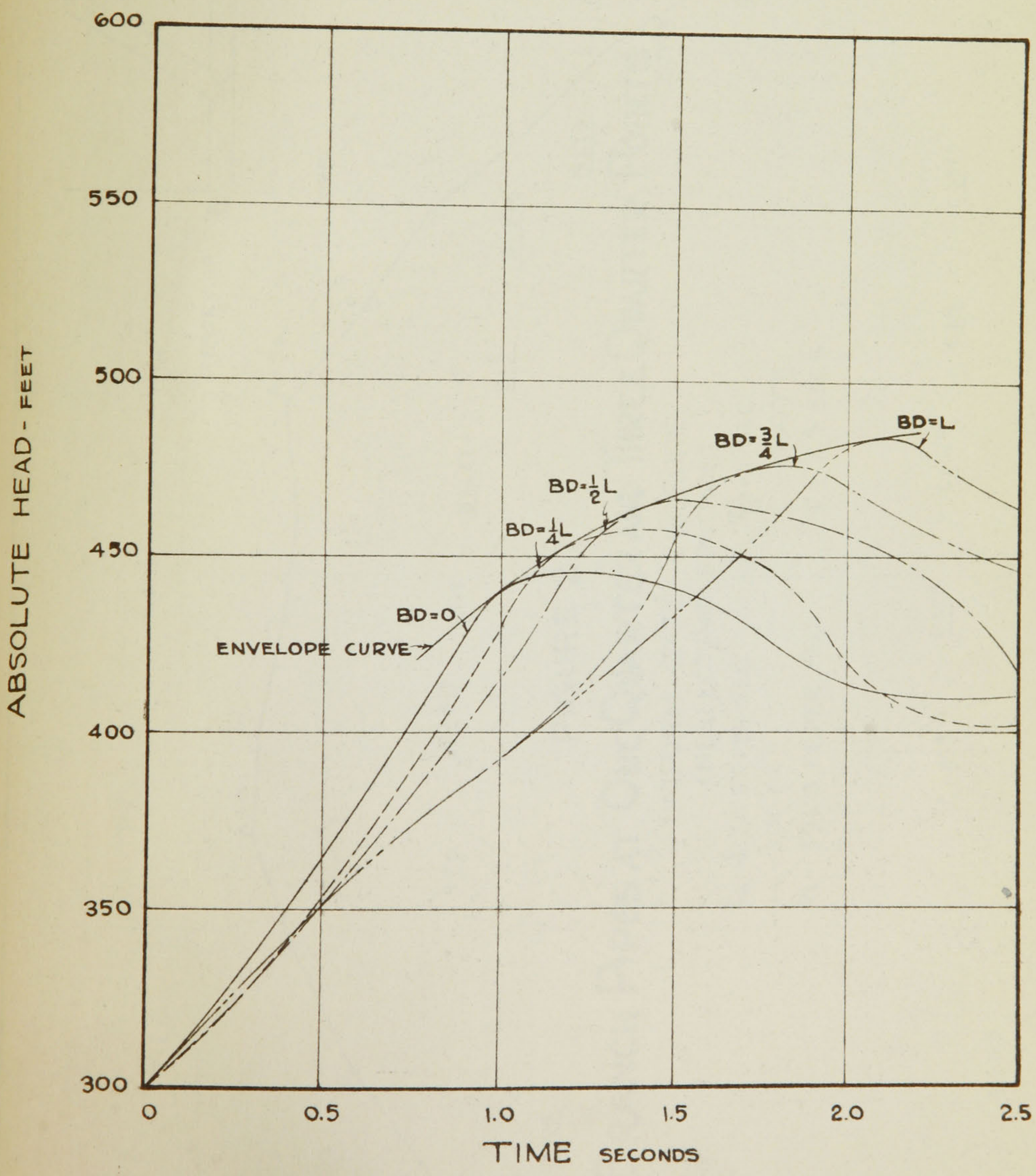


FIGURE 26

PRESSURE-TIME CURVES

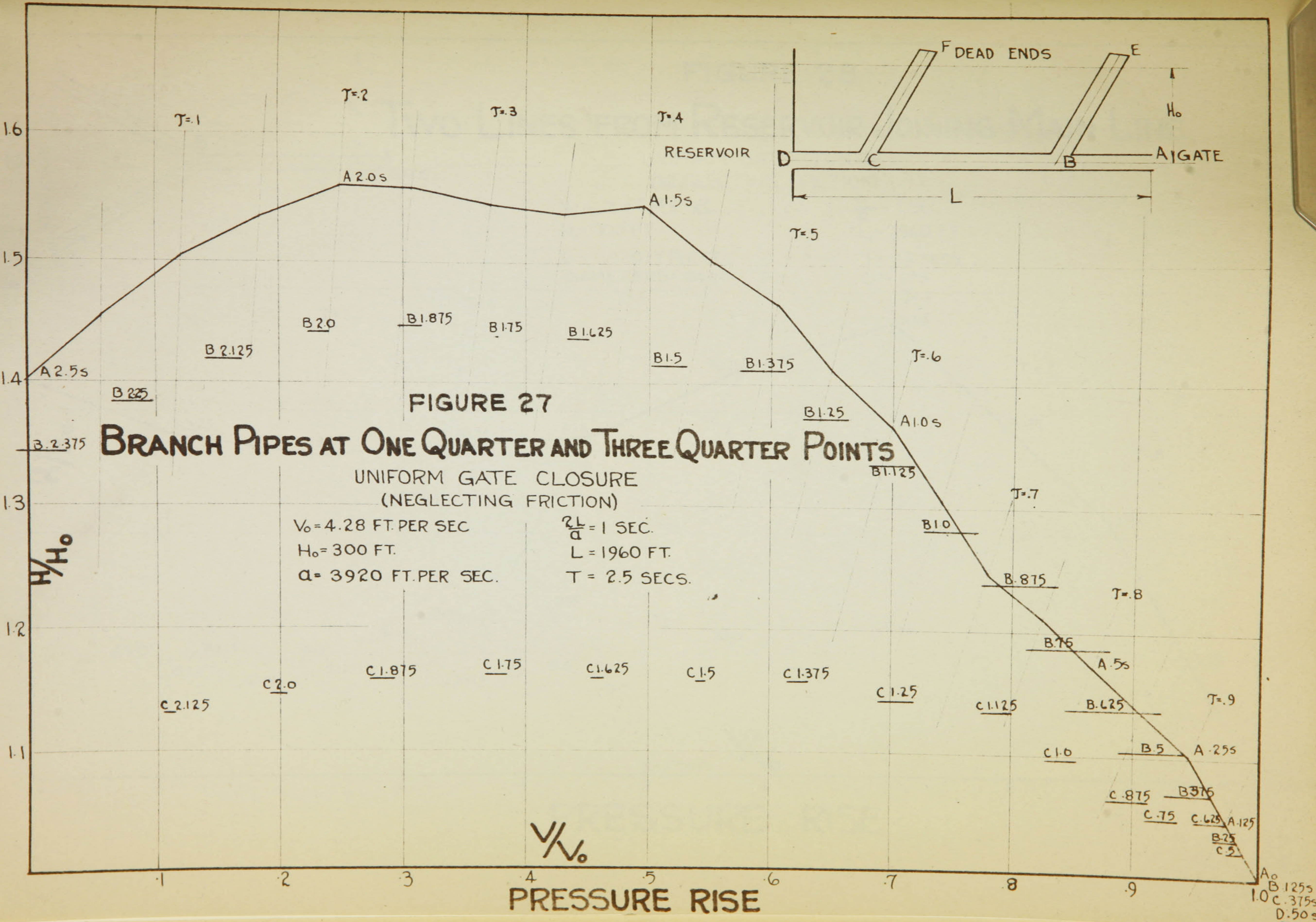


FIGURE 27

BRANCH PIPES AT ONE QUARTER AND THREE QUARTER POINTS

UNIFORM GATE CLOSURE

(NEGLECTING FRICTION)

V_o = 4.28 FT. PER SEC

H_o = 300 FT.

a = 3920 FT. PER SEC.

$\frac{2L}{a}$ = 1 SEC.

L = 1960 FT.

T = 2.5 SECS.

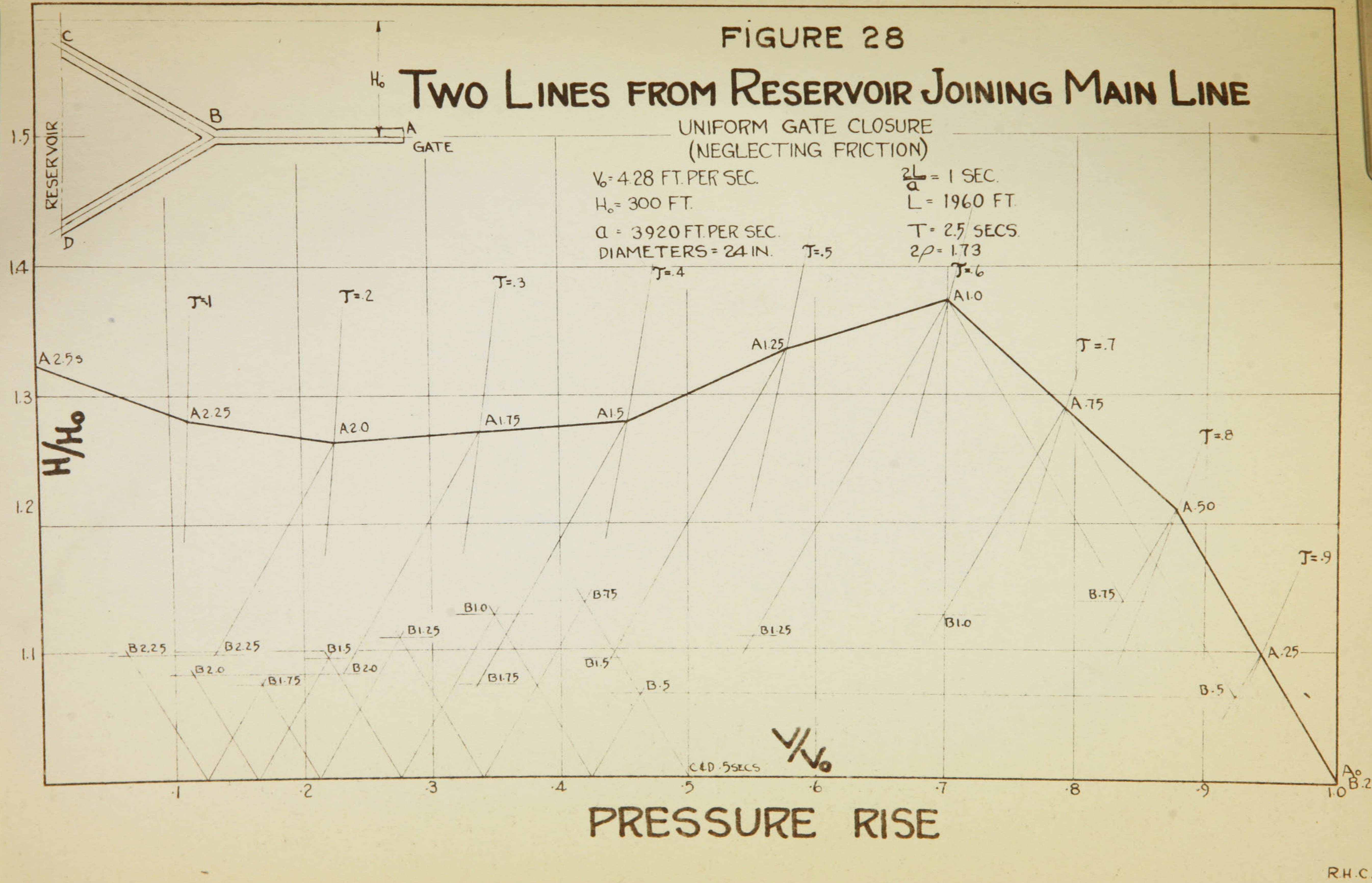


FIGURE 28
Two Lines from Reservoir Joining Main Line