Distribution network Optimal Reconfiguration

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ABSTRACT

This thesis reports on research conducted on the Optimal Reconfiguration (OR) of distribution networks using Mixed Integer Linear Programming (MILP). At the operational hourly level, for a set of predicted bus loads, OR seeks the optimum on/off position of line section switches, shunt capacitors and distributed generators so that the distribution network is radial and operates at minimum loss. At the planning level, OR seeks the optimum placement of line switches and shunt capacitors so that, over the long-term, losses will be minimized. The main steps and outcomes of this research are (i) the development of a simplified single-phase distribution network model for Optimal Reconfiguration; (ii) the development of a linear DC load flow model with line and device switching variables accounting for both active and reactive power flows; (iii) the development of an algorithm HYPER which finds the minimum loss on/off status of existing line switches, shunt capacitors and distributed generators; (iv) the extension of HYPER to find the optimum (minimum loss) placement of switches, capacitors and distributed generators; (v) the representation of losses via supporting hyperplanes enabling the full linearization of the OR problem, which can then be solved using efficient and commercially available MILP solvers like CPLEX.

KEYWORDS

Distribution Network, Optimal Reconfiguration, OR, Loss minimization, Mixed-Integer Linear Programming, MILP, Operations research, Linear network model, DC load flow, Supporting hyperplanes, Real time optimization, Switch, Capacitor and Distributed Generator placement, Power Systems Operations and Planning.

RÉSUMÉ

Ce mémoire de thèse rend compte des produits d'activités de recherche menée relativement à la Reconfiguration Optimale (RO) de réseaux de distribution par Programmation Linéaire en Variables Mixtes (PLVM). Dans un contexte de conduite de réseau, la RO s'applique à déterminer l'état ouvert/fermé optimal des interrupteurs, disjoncteurs, condensateurs et producteurs distribués, avec objectif d'opérer à un niveau de pertes minimum un réseau de distribution radial. La RO s'applique également, dans un contexte de planification, à identifier l'emplacement optimal sur le réseau d'interrupteurs, disjoncteurs et condensateurs visant le maintien, sur le long terme, des pertes à un niveau minimum. Les principaux résultats de cette recherche sont: (i) le développement d'un modèle unifilaire simplifié de réseau de distribution pour la Reconfiguration Optimale; (ii) le développement d'un modèle d'écoulement de puissance linéaire avec variables contrôlant l'état des lignes, adapté autant pour l'écoulement de puissance actif que réactif; (iii) le développement de l'algorithme HYPER capable d'identifier l'état ouvert/fermé optimal (minimum de pertes) des interrupteurs, disjoncteurs, condensateurs et producteurs distribués; (iv) une extension de l'algorithme HYPER permettant de déterminer l'emplacement optimal (minimum de pertes) d'interrupteurs, disjoncteurs, condensateurs et producteurs distribués; (v) la représentation des pertes via hyperplans-porteurs permettant la linéarisation complète du problème RO et sa résolution par l'emploi de solveurs PLVM performants et commercialement disponibles tels que CPLEX.

MOTS CLÉS

Réseau de distribution, Reconfiguration Optimale, RO, Minimisation des pertes, Programmation Linéaire en Variables Mixtes, PLVM, Recherche opérationnelle, Modèle de réseau linéaire, Écoulement de puissance linéaire, Hyperplans-porteurs, Optimisation temps réel, Interrupteur, Disjoncteur, Condensateur, Producteur privé, Exploitation, Conduite, Planification.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

PART I | INTRODUCTION

I.1 Research motivation

I.1.1 Summary

Ordinarily, the primary goal in planning and operating medium voltage distribution networks consists of assuring that the electricity service is reliable and of quality (frequency and voltage close to their nominal levels); less emphasis is placed in attempting to maximize the efficiency of delivery, that is, the flow of power from substation to consumer.

In general, some attempts are made to reduce heat losses on medium voltage circuits, but often these initiatives are locally instigated and aimed at solving particular issues exclusive to specific parts of the system. As of today, only moderate efforts are deployed to systematically consider the efficiency aspects of the broad electric distribution network, even though practices are gradually evolving in that regard.

Optimal control of distribution networks is a research field that has gained increased attention in recent years stimulated by the industry's need for a more efficient grid; the so-called smart grid. And so, as utilities are seeking leaner operations through sustained utilization of automated equipment, Optimal Reconfiguration (OR) of distribution feeders is emerging as a technically and economically sound option.

The sense of OR can be understood as follows: Switches are traditionally intended exclusively for protection purposes, for example, to clear faults for protecting the integrity of equipment, or to isolate line sections for protecting workers during scheduled maintenance. Moving forward however, OR suggests the utilization of switches during normal operation to route the transit of power at minimum loss.

I.1.2 Expected research outcomes

The principal goal of this thesis is to develop an algorithm that solves the minimum loss OR problem through a scheme based on Mixed Integer Linear Programming (MILP) and Supporting Hyperplanes (SHP).

We first show how to linearize the power flow model through a distribution network. Then we detail how, with successive additions of supporting hyperplanes, we converge to the minimum loss OR solution. We also demonstrate how the solution algorithm can be used both in the context of planning and operation. Finally, observations regarding the practicality and implementation of the proposed approach are discussed.

I.2 Literature review

I.2.1 Existing approaches to Optimal Reconfiguration

Several methods and their refinements have been used to solve the OR problem since its original formulation. Two surveys were produced, one in 1994 [16] and the other in 2003 [17], highlighting common approaches used until then. We present here a review that considers the above developments together with more up to date ones.

Most researchers point to the branch and bound technique presented by Merlin and Back [1] as being the first brick on the wall. It was followed by the work of Ross *et al* [18] who proposed adaptations based on the use of performance indices and specific branch exchanges. These ideas were then also notably developed later by Cinvalar *et al*.[19], Shirmohammadi and Hong [20], Borozan *et al*.[21], Baran and Wu [22] and Liu *et al*. [23].

Subsequent works based on Simulated Annealing (SA), Genetic Algorithms (GA) and Ant Colony (ACO) where respectively initiated by Chiang and Jean-Jumeau [11], Nara *et al.*[24] and Ahuja and Pahwa [25].

More recently, several other methods have been proposed, including brute force [26], particle swarm optimization [27], ranking indices, fuzzy logic, Bender's decomposition [28], and MILP with an equivalent loss function [29].

The table next page lists and briefly describes the existing approaches.

	Description
MILP	Network, load flow, constraints and objective function are all linear. Continuous and integer variables are used.
Branch exchange	Initially assumes all switches closed (meshed network) in a non-linear model. They are then opened one by one, following a heuristic (e.g.: starting with the branch with the lowest current).
Brute force	Enumeration of all possible solutions.
Benders decomposition	Decomposition of the problem in layers, then resolved, and solutions cross-tested.
Simulated annealing	Artificial intelligence. Definition of a *configuration space*, *set of feasible moves*, *cost function*, *cooling schedule*.
Genetic algorithms	Artificial intelligence. Inspired by genetics natural selection and the evolutionary process. Population based search points.
Tabu	Artificial intelligence. Mimics the memory process. Use TABU *lists*.
Artificial neural networks	Artificial intelligence. Based on brain structure: neurons with links (weighted).
Ant colony	Artificial intelligence. Progressive path construction. Amongst other things, uses *pheromone values* and *transition probabilities*.
Particle swarm	Artificial intelligence. Emulates the behaviour of a bird flock. Random particles velocity iteratively updated
Fuzzy logic	Rules based on historical or other type of data. Used typically in conjunction with artificial intelligence methods.

Table 1 – Existing OR approaches

I.3 Research objectives

I.3.1 Summary

This research defines a novel approach to OR using Mixed Integer Linear Programming (MILP) and Supporting Hyperplanes (SHP). The approach takes advantage of both the convexity of the system loss function [30] and of the efficiency provided by commercial MILP optimization packages such as CPLEX. The main features of this study are:

- Optimal solution of the global problem, through MILP iterations;
- Calculation of losses due to both active and reactive power flow;
- Simultaneous optimization of the on/off switch status of:
  - Lines
  - Capacitor banks
  - Distributed generation
- Operation and planning applications.

The approach described in this thesis stands out by its relative ease of implementation, guaranteed optimality and feasibility, broad range of applications and consideration of practical concerns.

I.3.2 Applications and benefits of Optimal Reconfiguration

I.3.2.1 Short-term Operation

Optimal Reconfiguration, and in particular the approach presented in this research, can be used in operation to:

- Determine the on/off state of equipment to minimize losses;
- Maintain voltages at all nodes within limits;
- Minimize the number of switching operations over a period of time.

I.3.2.2 Long-term Planning

Complementarily, for planning, Optimal Reconfiguration will:

- Locate the position of switches;
- Locate the position and define the size of capacitors;
- Locate the best point of connection for distributed generators (whenever possible);
- Satisfy the short-term operational goals.

For both short-term operation and long-term planning, loss reduction through optimal reconfiguration translates into costs reduction, utilization factor increase and capital expenditures' deferral.

I.3.3 Level of activity in Optimal Reconfiguration

I.3.3.1 Mapping of prominent papers

The following diagram shows the lineage of prominent papers addressing the problem of optimal reconfiguration of distribution networks to minimize loss reduction. Based on this analysis, we see that [1], [2], [3] and [11] have had the greatest impact.

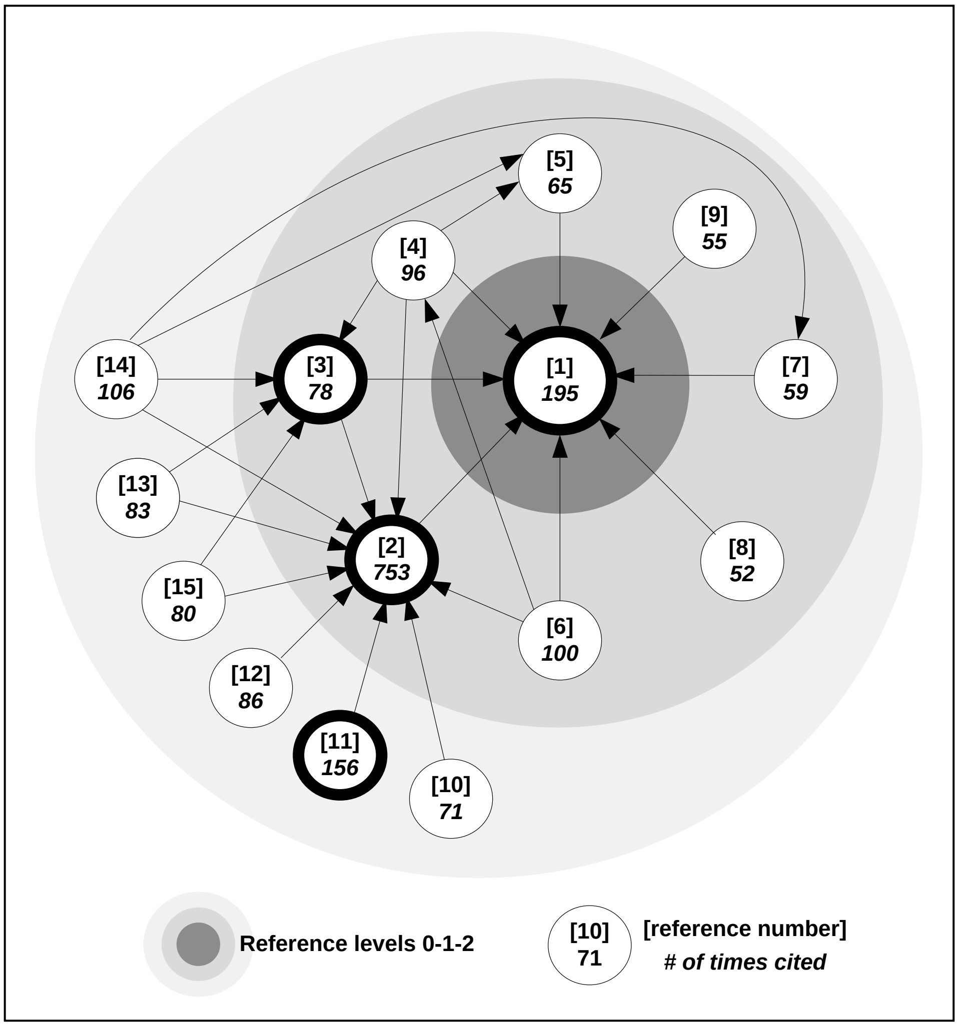

Figure 1 – Prominent papers reference map

I.3.3.2 Number of publications

As another measure of the level of activity in the field of OR, the next figure shows the quantity of papers published since the original work from Merlin & Back in 1975 [1].

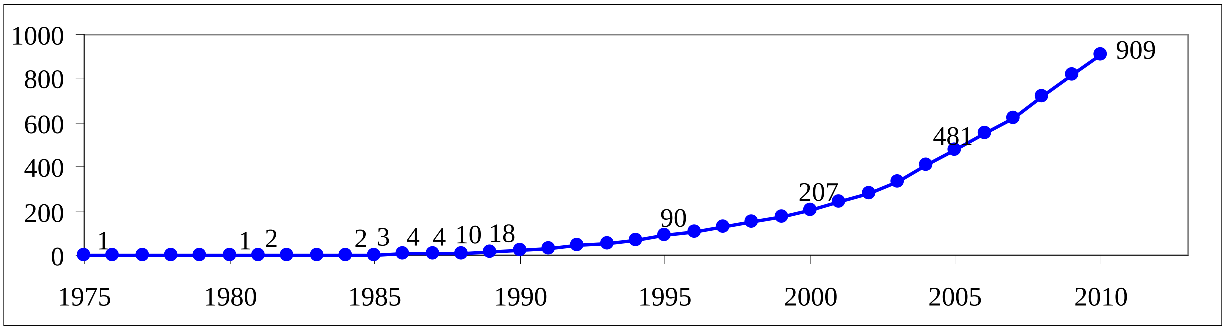

Figure 2 – Cumulative quantity of articles published relating to OR

I.4 Thesis organization

Part I	**Introduction** *Research motivation and objectives, litterature review*
Part II	**Network model** *Development of a simplified single-phase distribution network model*
Part III	**Linear active and reactive load flow** *Development of a DC load flow model with line switching variables*
Part IV	**Problem formulation** *Definition of the objective function and constraints*
Part V	**Solution algorithm based on MILP and SHPs** *Development of HYPER*
Part VI	**Operational applications** *Utilisation of HYPER to minimize losses*
Part VII	**Planning applications** *Utilisation of HYPER to position equipments*
Part VIII	**Conclusion** *Summary and practical implementation*

Table 2 – Thesis organization

PART II | NETWORK MODEL

II.1 Physical network

A typical North American distribution circuit is three phase, Y-grounded, unbalanced, non-transposed and radial. These attributes, characteristic of networks where loads are geographically spread out, require less capital expenditure and facilitate detection of line to ground faults.

The traditional approach for utilities to enhance efficiency is to design feeders to operate at higher nominal voltages, select higher conductor gauges and make more extensive use of automated capacitors and voltage regulators. Feeders can be overhead conductors or underground cables, depending on the density and type of the service area (urban, semi-urban or rural). Some distributed generation can also be present.

Figure 3 shows an actual distribution network with multiple lines fed by a common substation.

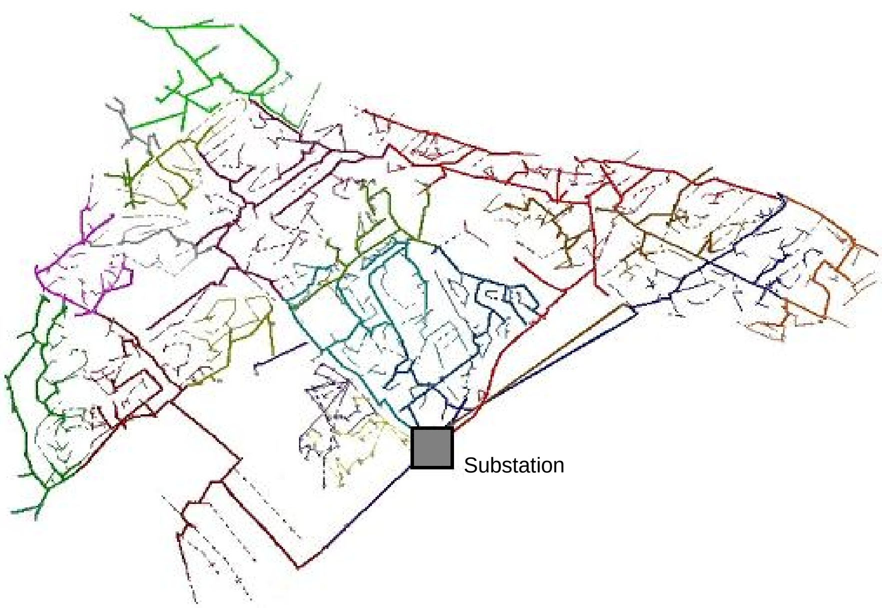

Figure 3 – Multiple feeders distribution network – every color is a feeder

II.2 Typical equipment

The proper operation of a distribution network necessitates an adequate number of different types of equipment, well positioned, properly sized and regularly maintained. This is addressed at the planning level and has to be carried out with scrutiny seeing that distribution equipment has a long life expectancy and its purchase and installation costs are relatively onerous.

The typical equipment encountered in electric distribution networks is listed in Table 3 with a description of its primary purpose.

		Purpose			
Class	**Equipment**	**Distribution**	**Protection**	**Monitoring**	**Efficiency**
Transformer	Power transformer	X			
	Voltage regulator	X			
	Voltage sensor			X	
	Current sensor			X	
Switch	Circuit breaker[1]	X	X		
	Interrupter	X	X		
	Disconnector	X	X		
Compensation	Capacitor				X
	Inductance		X		

Table 3 – Typical electric distribution equipment

A distribution network is usually comprised of thousands of electrical nodes or buses, each typically having one or more of the above listed equipment as well as some load.

[1] Also commonly referred to as a recloser

II.3 Simplified single-phase network model for Optimal Reconfiguration

The 3-phase diagram in Figure 4 illustrates how power is carried at Medium Voltage (MV) from the substation to consumption areas, and then distributed at Low Voltage (LV) for consumer use.

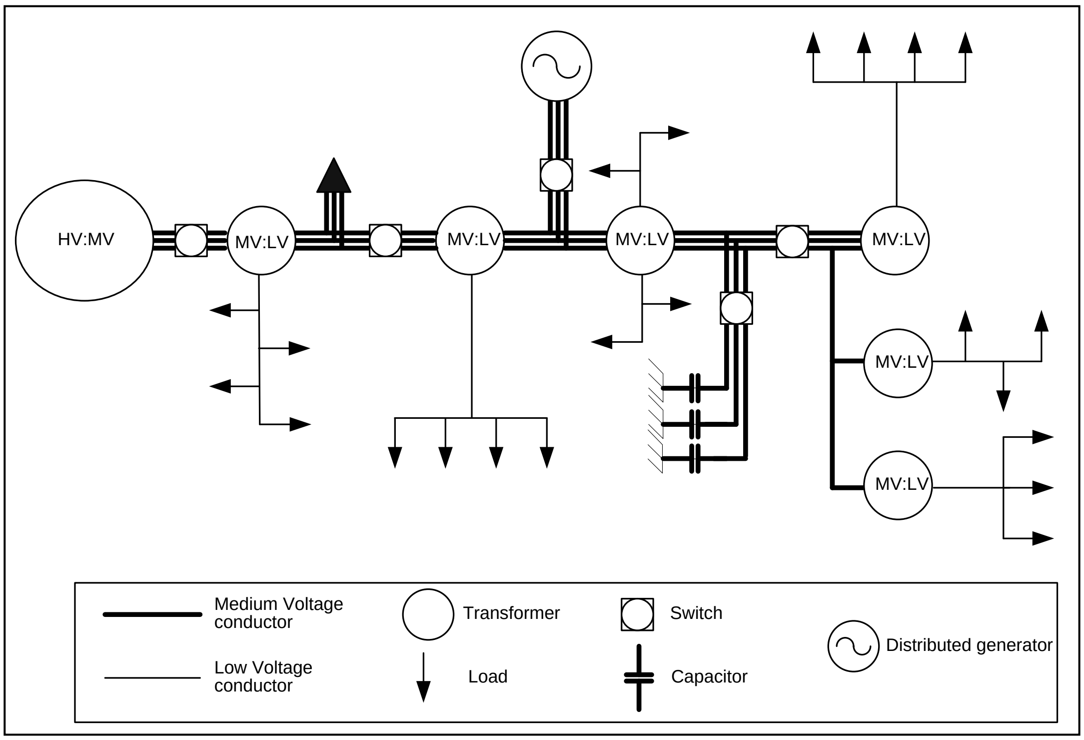

Figure 4 – Example of 3-phase distribution network model

The analysis in this thesis is able to accommodate all characteristics common in typical distribution systems:

- Y-grounded radial network;
- Feeders lateral branches (unbalanced loads);
- Non-transposition;
- Switchable capacitors.

We also consider the presence of switchable distributed generators.

However, to address the problem of OR, such a detailed 3-phase characterization of the network and all its constituents is unnecessary. Since most networks are equipped with only a few automated switches, feeders comprising multiple branches and individual loads may be represented by an equivalent branch and aggregated load.

In addition, we simplify the 3-phase network model by using a single-phase equivalent, assuming that aggregated loads and line impedances are well balanced. This is a fair hypothesis, as this is exactly how planners design feeders so as to maximize the utilisation of conductor capacity and reduce losses.

An illustration of such a simplified single-phase network, including switches, used in the remainder of the analysis, is shown in Figure 5.

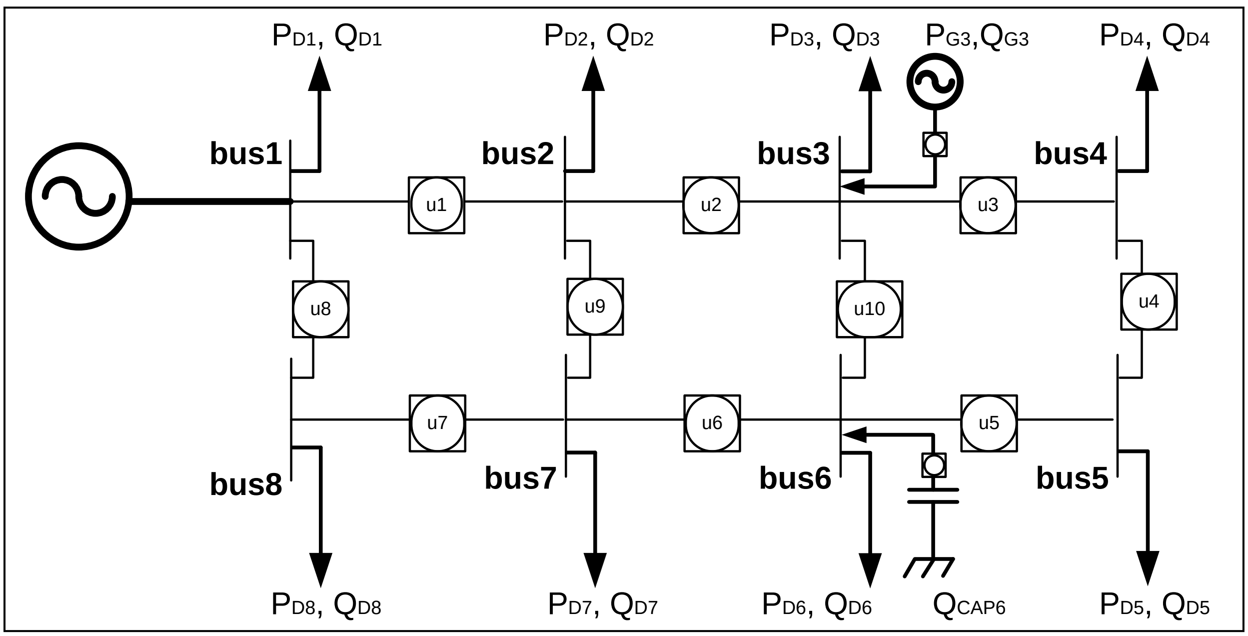

Figure 5 – Single-phase simplified network model with switches

In a single-phase simplified network, at each bus i the net injections, P_i and Q_i, are found from the real and reactive power demands, P_{Di} and Q_{Di}, to which a reactive generation, $Q_{CAPi} = \left(V_i^0\right)^2 B_{CAPi}$ is added if the bus has a switchable capacitor, and to which P_{Gi} and Q_{Gi} are also added if the bus has a private producer. Thus,

$$\begin{aligned} P_i &= P_{Gi} - P_{Di} \\ Q_i &= Q_{CAPi} + Q_{Gi} - Q_{Di} \end{aligned} \tag{2.1}$$

Each overhead line ℓ connecting buses i and j has a resistance r_{ij} and a reactance x_{ij}. When considering overhead lines, we deliberately omit line susceptances to ground since their impact is negligible. Susceptances are however important when considering underground cables.

Without loss of generality, the bus at the distribution substation supplying power to the network, denoted as bus 1, is the so-called slack bus supplying the necessary distribution losses, as well as playing the role of reference bus with a phase angle of zero. In addition, the voltage magnitude at bus 1 is regulated to a value in the neighbourhood of 1 pu.

PART III LINEAR ACTIVE AND REACTIVE LOAD FLOW

III.1 Motivation

For purposes of OR, we make use of the well-known DC load flow [31] to describe the relationship between real power and phase angles, and that between reactive power and bus voltage magnitudes. This decoupled linear load flow modeling will be shown to be sufficiently accurate as well as enabling the use of a linear solver to solve the OR problem.

III.2 Active DC load flow

The basic DC load flow assumptions are: (i) line shunt capacitances are neglected; (ii) line series reactances are much larger than the corresponding resistances; (iii) bus voltages are near nominal; (iv) voltage phase angle differences across line sections are small. The numerical comparisons in PART VI show that the linearized DC load flow model yields results comparable to those from the full AC load flow model.

Recall that the DC load flow model is of the form,

$$P = B\delta \tag{3.1}$$

where P is the vector of net (generation minus demand) real power bus injections at the nb buses and δ is the vector of the nb bus voltage phase angles. Note that, without loss of generality, the phase angle at the distribution substation, taken as bus 1, is the reference with $\delta_1 = 0$. The network susceptance matrix B in terms of the switch positions u is defined by,

$$B = A(u)\, diag(b)\, A(u)^T \tag{3.2}$$

In (3.2), b is the vector of elements equal to 1 over the series reactance for all $n\ell$ line sections, $diag(b)$ is a matrix with the vector b along the diagonal and zeros elsewhere, while $A(u)$ is the network incidence matrix of dimension $nb \times n\ell$ expressed as a function of the switching vector u representing the open/closed or 0/1 status of all line switches. It readily follows that,

$$A(u) = A\ diag(u) \tag{3.3}$$

where A is constant and corresponds to the network incidence matrix when all switches are closed.

Combining the above equations, we obtain what we term the *DC load flow model with line switching variables,*

$$\begin{aligned} P &= Adiag(u)\,diag(b)\,diag(u)\,A^T\delta \\ &= Adiab(ub)\,A^T\delta \end{aligned} \tag{3.4}$$

In (3.4), we have used the property that since u_i is a 0/1 variable, then $u_i = u_i^2$. In addition, we have defined a new vector of binary variables of dimension $n\ell$, ub, whose elements are defined by $ub_i = u_i b_i$ and are either $ub_i = b_i$ or $ub_i = 0$, depending on the whether u_i is zero or one. Note that the switching variables, u, represent decision variables in the optimum reconfiguration problem, as are the phase angle variables, δ.

Although the DC load flow equations contain nonlinearities of the form $u_i\delta_j$, the appendix shows how products of a 0/1 binary variable with a continuous variable can be uniquely expressed by an equivalent set of linear inequalities. This property is important since it allows the model to retain its linear property, thus allowing the solution of the OR problem through efficient commercially available MILP solvers.

Defining now $\theta = A^T \delta$ as the vector of dimension $n\ell$ representing the angle differences across all line sections, the DC load flow model with switching variables can be expressed in the form,

$$P = A \, diag\left(ub\right)\theta \tag{3.5}$$

As shown below, this more compact active power DC load flow model in terms of angle differences is especially useful when characterizing the distribution network loss function by its supporting hyperplanes.

III.3 Reactive DC load flow

The assumptions behind the DC load flow also serve to find an approximate linear relation between reactive power injections and bus voltage magnitudes. Thus, defining Q as the vector of bus net reactive power injections, a similar set of equations relating Q to the vector of bus voltage magnitudes V can be found,

$$Q = A \, diag\left(ub\right) A^T \, V \tag{3.6}$$

By defining the vector of bus voltage magnitude differences across the lines as $W = A^T V$, the above equation takes the form,

$$Q = Adiag(ub)W \tag{3.7}$$

Recall that the voltage magnitude at the distribution substation is controlled by the utility within a narrow range near one per unit. The consequence of this boundary condition together with the condition that $\delta_1 = 0$ is that, all along the distribution network, all phase angles are near zero and all voltage magnitudes are near 1 pu, as assumed by the DC load flow model.

PART IV FORMULATION OF OPTIMAL RECONFIGURATION PROBLEM

IV.1 OR Problem Formulation

Having established the network model and load flow equations, including line switching variables, the OR problem is now formulated as a minimization with the line losses as the objective function. This minimum is sought over a set of decision variables subject to a number of constraints, the decision variables being the vectors u, V and δ. The constraints are described in the section below.

IV.2 OR Constraints

IV.2.1 Network connectivity

The connectivity between buses is dependent on the state of the switches u and is denoted by the matrix $A\,diag(u)$.

Now, from circuit theory we know that the number of independent loops in a connected network equals the difference between the number of lines and the number of buses, n_b, plus one. Assuming that each line ℓ has a switch whose on/off status is denoted by the binary variable u_ℓ, then the number of lines through which power flows is equal to $\sum_\ell u_\ell$. In addition, since distribution circuits are operated radially, there are zero independent loops, so that a necessary condition on the switching variables u is therefore,

$$\sum_\ell u_\ell - n_b + 1 = 0 \tag{4.1}$$

In addition to (4.1), another necessary and sufficient condition guaranteeing connectivity of the network (needed to avoid situations where the network is radial but not connected) is that by injecting 1 MW into any bus i, 1 MW can be extracted from any other bus j. This condition need not however be explicitly imposed since it is guaranteed if the matrix $Adiab\left(ub\right)A^T$ is non-singular, a condition which is ensured by the DC load flow equations in (3.4). The only exception to this is when the net injection at some bus is identically zero. This is however a very rare, if not impossible, case seeing that it would correspond to a distribution bus with no real or reactive load and no generation.

IV.2.2 Load flow equations

The flow of power must satisfy the laws of Kirchhoff and Ohm, here expressed via the linear mathematical model derived previously,

$$P = A\,diag\left(ub\right)\theta \tag{4.2}$$

$$Q = A\,diag\left(ub\right)W \tag{4.3}$$

In these equations, the vectors P and Q are the net real and reactive power injections defined by: (i) the known constant demands at load buses; (ii) the known generation at buses with independent generators; (iii) the generation at the slack bus 1 supplying all loads.

IV.2.3 Limits on decision variables

The distribution network variables must lie within certain ranges so as to respect the equipment nominal operating values, thus assuring a proper response and preserving their life expectancy. Thus, the bus voltage magnitude limits must satisfy,

$$V_i^{\min} < V < V_i^{\max} \tag{4.4}$$

In addition, limits could be imposed on the line current magnitudes expressed in linear form in terms of the angles, δ.

IV.2.4 Reference voltage and slack bus injections

We assume that the source at bus 1 is voltage regulated and typically near 1 pu. More generally however the source voltage magnitude could be defined as another decision variable with which to minimize losses further. Thus,

$$V_1 = E \approx 1 \tag{4.5}$$

One consequence of (4.2) is that the real and reactive power generations at the slack bus 1 are dependent on the bus demands, the power injections at the capacitor buses, and the power injections at the private producer buses. From the DC load flow approximation (neglecting real and reactive losses), it follows that,

$$P_1 = -\sum_{i \neq 1} P_i \tag{4.6}$$

$$Q_1 = -\sum_{i \neq 1} Q_i \tag{4.7}$$

where P_i and Q_i are respectively the net real and reactive power injections at bus i defined by (2.1).

IV.3 Objective function

The objective function to be minimized within the constrained search space is the network real loss, P_{loss}. Without linear approximations, this function is given by,

$$P_{loss} = \frac{1}{2} \sum_{i=1}^{nb} \sum_{\substack{j=1 \\ j \neq i}}^{nb} G_{ij} u_{\ell} \left[V_i^2 + V_j^2 - 2V_i V_j \cos\left(\delta_i - \delta_j\right) \right] \tag{4.8}$$

where u_ℓ is the binary switching variable for line ℓ, connecting buses i and j. If such a line does not exist then the corresponding G_{ij} is zero. If such a line exists but it has no switch then the corresponding switching variable is set to one.

Invoking the condition that the line phase angle and voltage magnitude differences are small, the δ_i being near zero and the V_i near 1 pu, we decouple the loss equation and distinguish losses due to active and reactive power flows as follows:

$$\begin{aligned} P_{loss} &= \frac{1}{2}\sum_{i=1}^{nb}\sum_{\substack{j=1 \\ j\neq i}}^{nb} G_{ij}u_\ell\left[V_i^2+V_j^2-2V_iV_j\cos\left(\delta_i-\delta_j\right)\right] \\ &\approx \frac{1}{2}\sum_{i=1}^{nb}\sum_{\substack{j=1 \\ j\neq i}}^{nb} G_{ij}u_\ell\left[V_i^2+V_j^2-2V_iV_j\left(1-\frac{\left(\delta_i-\delta_j\right)^2}{2}\right)\right] \\ &= \frac{1}{2}\sum_{i=1}^{nb}\sum_{\substack{j=1 \\ j\neq i}}^{nb} V_iV_jG_{ij}u_\ell\left(\delta_i-\delta_j\right)^2+\frac{1}{2}\sum_{i=1}^{nb}\sum_{\substack{j=1 \\ j\neq i}}^{nb} G_{ij}u_\ell\left(V_i-V_j\right)^2 \end{aligned} \tag{4.9}$$

where we approximated the cosine terms by a second order Taylor series. Then, we can split the loss into two terms,

$$P_{loss} = P_{loss}^{active} + P_{loss}^{reactive} \tag{4.10}$$

where the first term is obtained by setting all voltage magnitudes to one per unit,

$$P_{loss}^{active} = \frac{1}{2}\sum_{i=1}^{nb}\sum_{\substack{j=1 \\ j\neq i}}^{nb} G_{ij}u_{ij}\left(\delta_i-\delta_j\right)^2 = \sum_{\ell}\frac{G_\ell}{2}u_\ell\theta_\ell^2 \tag{4.11}$$

while the loss component due to reactive power flows is,

$$P_{loss}^{reactive} = \frac{1}{2}\sum_{i=1}^{nb}\sum_{\substack{j=1 \\ j\neq i}}^{nb} G_{ij}u_{ij}\left(V_i-V_j\right)^2 = \sum_{\ell}\frac{G_\ell}{2}u_\ell W_\ell^2 \tag{4.12}$$

PART V HYPER – SOLUTION OF OPTIMAL RECONFIGURATION PROBLEM BASED ON MILP AND SUPPORTING HYPERPLANES

V.1 Summary

Whereas the OR constraints are fully linear (recall, as shown in the appendix, that products of binary variables and continuous variables can be replaced by an equivalent set of linear inequalities), the loss function being minimized is quadratic in the vectors θ and W. In this part V, we show how to approximate this quadratic function by a set of linear inequalities known as supporting hyperplanes (SHPs), set which is iteratively updated as shown below. For each updated set of supporting hyperplanes, the minimum loss solution is obtained using the MILP solver CPLEX in GAMS.

This approach to OR based on MILP and supporting hyperplanes is referred to as HYPER.

V.2 Development of HYPER

We begin the process by guessing the vector of on/off switches, an initial guess denoted by u^0 (for example, a common configuration of the switches leading to a radial network). Based on this guess of the switching vector, the solution of the corresponding load flow equations yield unique vectors θ and W, denoted here as θ^0 and W^0 (recall that the net power injections are known at all buses of the distribution network except at the source or slack bus).

With initial guesses of the phase angles and voltage magnitudes' differences, θ^0 and W^0, we next find the initial set of SHPs. Being a function of the continuous variables, θ, P_{loss}^{active} is now expanded around θ^0 in terms of $\Delta\theta$, where $\theta = \theta^0 + \Delta\theta$. Then,

$$\begin{aligned} P_{loss}^{active} &= \frac{1}{2}\sum_{\ell} u_{\ell} G_{\ell} \left(\theta_{\ell}^0 + \Delta\theta_{\ell}\right)^2 \\ &= \frac{1}{2}\sum_{\ell} u_{\ell} G_{\ell} \left(\theta_{\ell}^0\right)^2 + \sum_{\ell} u_{\ell} G_{\ell} \theta_{\ell}^0 \Delta\theta_{\ell} + \frac{1}{2}\sum_{\ell} u_{\ell} G_{\ell} \left(\Delta\theta_{\ell}\right)^2 \end{aligned} \tag{5.1}$$

However, since the term $\frac{1}{2}\sum_{\ell} u_{\ell} G_{\ell} \left(\Delta\theta_{\ell}\right)^2 \geq 0$ for any u and θ, it follows that,

$$P_{loss}^{active} \geq \frac{1}{2}\sum_{\ell} u_{\ell} G_{\ell} \left(\theta_{\ell}^0\right)^2 + \sum_{\ell} u_{\ell} G_{\ell} \theta_{\ell}^0 \Delta\theta_{\ell} \tag{5.2}$$

The above inequality defines a supporting hyperplane (SHP) for the loss function P_{loss}^{active}, in other words, a linear lower bound in terms of the decision variables u and θ. This lower bound can be seen as the unique hyperplane tangent to the nonlinear function P_{loss}^{active} at the expansion point characterized by u^0, θ^0 and W^0, hence the name "supporting". Note that the trivial condition that $P_{loss}^{active} \geq 0$ also represents a SHP.

For the same expansion point, θ^0 and W^0, we also calculate a SHP for $P_{loss}^{reactive}$, yielding,

$$P_{loss}^{reactive} \geq \frac{1}{2}\sum_{\ell} u_{\ell} G_{\ell} \left(W_{\ell}^0\right)^2 + \sum_{\ell} u_{\ell} G_{\ell} W_{\ell}^0 \Delta W_{\ell} \tag{5.3}$$

Having these two initial SHPs, we now solve the minimum loss MILP problem for a new set of switching variables u^1 subject to all previously stated constraints, as well as subject to (5.2) and (5.3). Next, given the updated set of switching variables, u^1, the DC load flow (4.2) and (4.3) is solved for new values of angles and voltages[2], θ^1 and W^1. These new expansion points define two new SHPs, which are added to the previously found ones as additional stricter necessary conditions for the next MILP step. This process is repeated, stopping when the solution changes by a sufficiently small amount. At each iteration, the accumulation of new and previous SHPs gradually improves the characterization of the nonlinear loss function.

V.3 Implementation flow chart

The iterative approach using hyperplanes was coded in GAMS based on the program flowchart described in Figure 6 below.

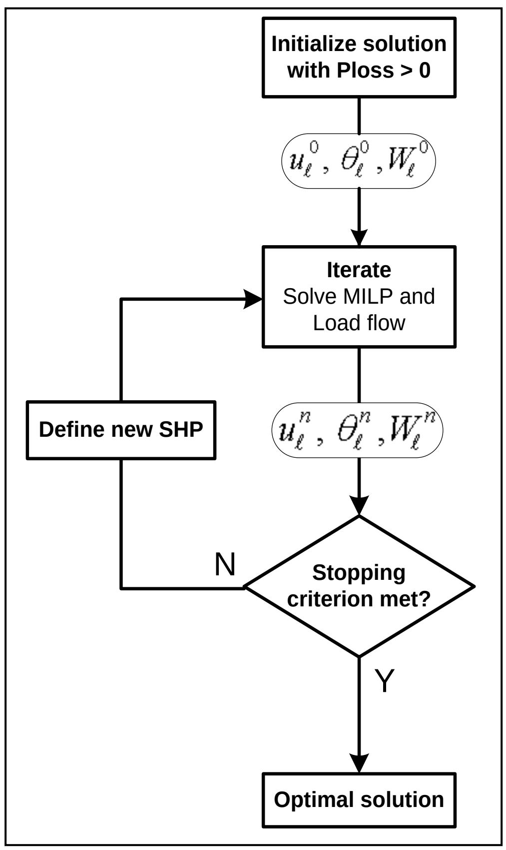

Figure 6 – Hyper flowchart

[2] The updating of the phase angles and voltages for a known u can be done via a nonlinear AC load flow for greater accuracy at the expense of increased computation. Here, however, we used the DC load flow with satisfactory results.

V.4 Graphical interpretation of Supporting Hyperplanes

To understand how the successive addition of hyperplanes leads to a converging optimal solution, observe the following illustrative figure where we iteratively represent the convex loss function through additional hyperplanes.

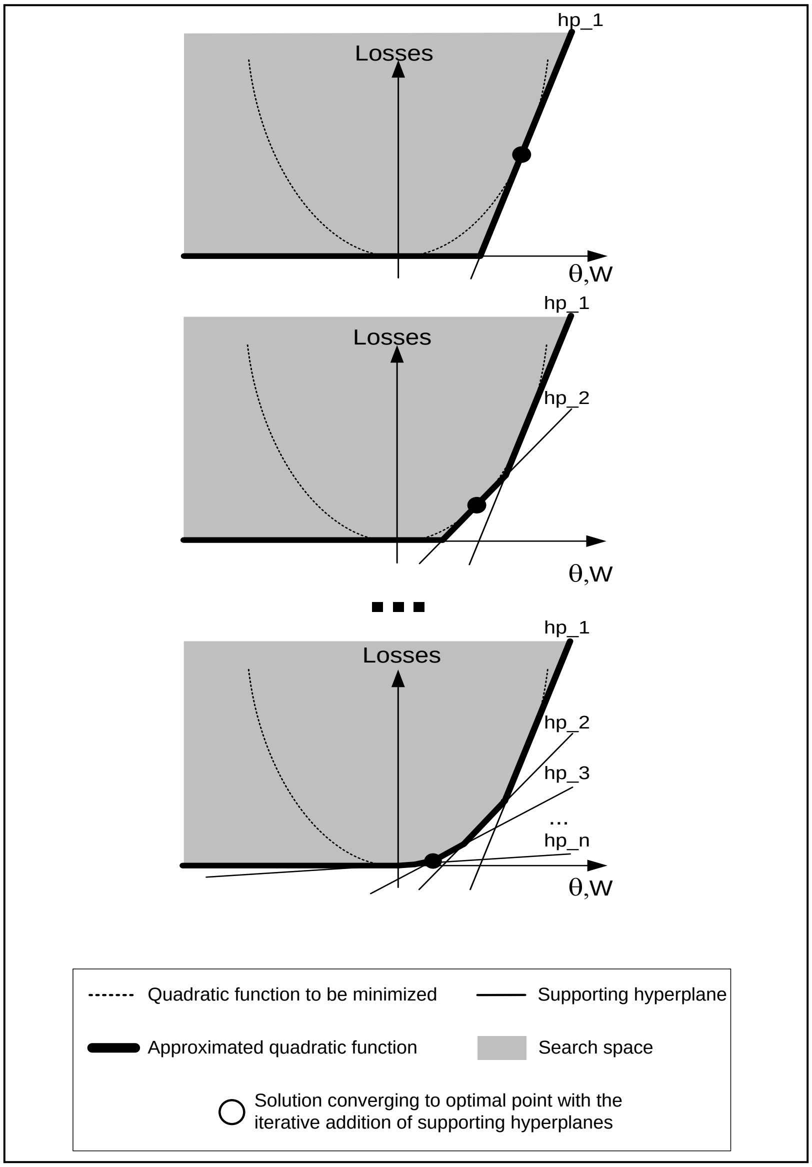

Figure 7 – Graphical interpretation of the SHP approach

PART VI | OPERATIONAL APPLICATIONS OF HYPER

VI.1 Presentation

The operation of a distribution network refers to the set of real time actions that are taken to maintain a continuously reliable and efficient delivery of electricity. Operating a network implies being able to cope with demand changes, availability of generation capacity, outages and scheduled maintenance.

During the course of normal operations, some aspects need immediate – almost instantaneous – attention, while others have to be analyzed and acted upon over a relatively long period of time. Energy efficiency falls into the latter category.

In the context of loss minimization, the predominant phenomena to account for are variations in active and reactive load levels. For example, as we move from day to night time or winter to fall, the nature and magnitude of the electric consumption changes require the network reconfiguration to adapt accordingly in order to maintain security and optimality.

We now test and validate the proposed solution algorithm HYPER by:

1. Showing how HYPER converges to the optimal operating point after a few SHP iterations;
2. Comparing the results to an AC load flow;
3. Examining the characteristics of the method and its results.

VI.2 Test case

VI.2.1 Network data

We consider a typical distribution network constructed with ten 4/77Al conductors, each of 3 km in length, with impedance values as follows:

	R (Ω/km)	X (Ω/km)
linesec001	0.122	0.395
linesec002	0.122	0.395
linesec003	0.122	0.395
linesec004	0.122	0.395
linesec005	0.122	0.395
linesec006	0.122	0.395
linesec007	0.122	0.395
linesec008	0.122	0.395
linesec009	0.122	0.395
linesec010	0.122	0.395

Table 4 – Network data

The network, shown in Figure 4, has a line connectivity defined by Table 5.

	from bus	to bus
linesec001	1	2
linesec002	2	3
linesec003	3	4
linesec004	4	5
linesec005	5	6
linesec006	6	7
linesec007	7	8
linesec008	8	1
linesec009	2	7
linesec010	3	6

Table 5 – Line connectivity

The base quantities for transforming to the pu domain are as follows:

$S_{3\phi}$ (MVA)	$V_{l\text{-}l}$ (kV)
21	24.94

Table 6 – Base quantities

We assume that there is a switch in every line section, so that the incidence matrix $A(u)$ in terms of the switching variables is given by,

$$A(u)=\begin{bmatrix} u_1 & 0 & 0 & 0 & 0 & 0 & 0 & -u_8 & 0 & 0 \\ -u_1 & u_2 & 0 & 0 & 0 & 0 & 0 & 0 & u_9 & 0 \\ 0 & -u_2 & u_3 & 0 & 0 & 0 & 0 & 0 & 0 & u_{10} \\ 0 & 0 & -u_3 & u_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -u_4 & u_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -u_5 & u_6 & 0 & 0 & 0 & -u_{10} \\ 0 & 0 & 0 & 0 & 0 & -u_6 & u_7 & 0 & -u_9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -u_7 & u_8 & 0 & 0 \end{bmatrix} \tag{6.1}$$

VI.2.2 Bus data

To illustrate the functioning of HYPER and the results that can be obtained, we first take the case of the following uniform load distribution.

	P_{Di} (pu)	Q_{Di} (pu)
bus002	1.000	0.330
bus003	1.000	0.330
bus004	1.000	0.330
bus005	1.000	0.330
bus006	1.000	0.330
bus007	1.000	0.330
bus008	1.000	0.330

Table 7 – Test case with uniform load distribution

In addition to these loads, bus 6 has a capacitor with value $j0.33$ pu while bus 3 has a private producer with fixed generation $0.66 + j0.2178$ pu. Both the capacitor and the private producer have an associated switching variable u_{CAP} and u_{DG} which are additional binary decision variables optimized by HYPER.

VI.2.3 Results from HYPER

VI.2.3.1 Optimal reconfiguration

Running HYPER, we obtain the following minimum loss network configuration, together with the status of the switching variables for the capacitor and the private producer, both of which are 'on'. This optimal reconfiguration generates losses of 9.68% and is obtained after 12 iterations (that is, with 12 SHPs).

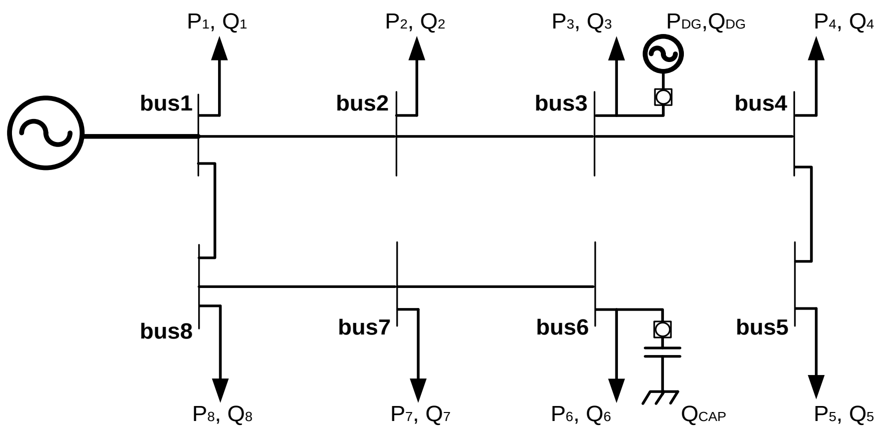

Figure 8 – Optimal network configuration

The tabulated results are as follows:

	u
linesec001	1
linesec002	1
linesec003	1
linesec004	1
linesec005	0
linesec006	1
linesec007	1
linesec008	1
linesec009	0
linesec010	0

Table 8 – Optimal line switch status

uDG	uCAP
1	1

Table 9 – Optimal capacitor and private producer switch status

VI.2.3.2 Iterative update of switching states

Table 10 shows the vector of switching variables u as new hyperplanes are added during the iterations. The optimal solution is reached at the 12th iteration even though we added 15 SHPs to demonstrate that convergence was reached.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
u(linesec001)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
u(linesec002)	1	1	1	1	1	1	1	1	0	1	0	1	1	1	1
u(linesec003)	1	1	0	1	1	1	0	1	1	0	1	1	1	1	1
u(linesec004)	1	0	1	0	1	0	1	1	1	1	1	1	1	1	1
u(linesec005)	1	1	1	1	0	1	1	0	1	1	1	0	0	0	0
u(linesec006)	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1
u(linesec007)	1	0	1	1	1	1	1	1	1	0	0	1	1	1	1
u(linesec008)	0	1	0	0	1	1	1	1	1	1	1	1	1	1	1
u(linesec009)	0	1	1	1	0	0	0	0	0	1	1	0	0	0	0
u(linesec010)	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
uDG	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
uCAP	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Table 10 – Vectors u, u_{DG} and u_{CAP} as new hyperplanes are added

VI.2.3.3 Iterative addition of hyperplanes

In Figure 9 we plot the SHPs for every one of the 10 line sections:

- The red curve is the exact quadratic loss term $P_{loss,\ell}^{active}$ as a function of θ;
- The blue lines are the corresponding supporting hyperplanes;
- The black dot signals the operating point – the optimum. There are none for line sections 5, 9 and 10 since their switch is open in the optimal solution;
- There is a maximum of 12 different hyperplanes per plot, corresponding to the number of necessary iterations needed to converge to the optimal solution.

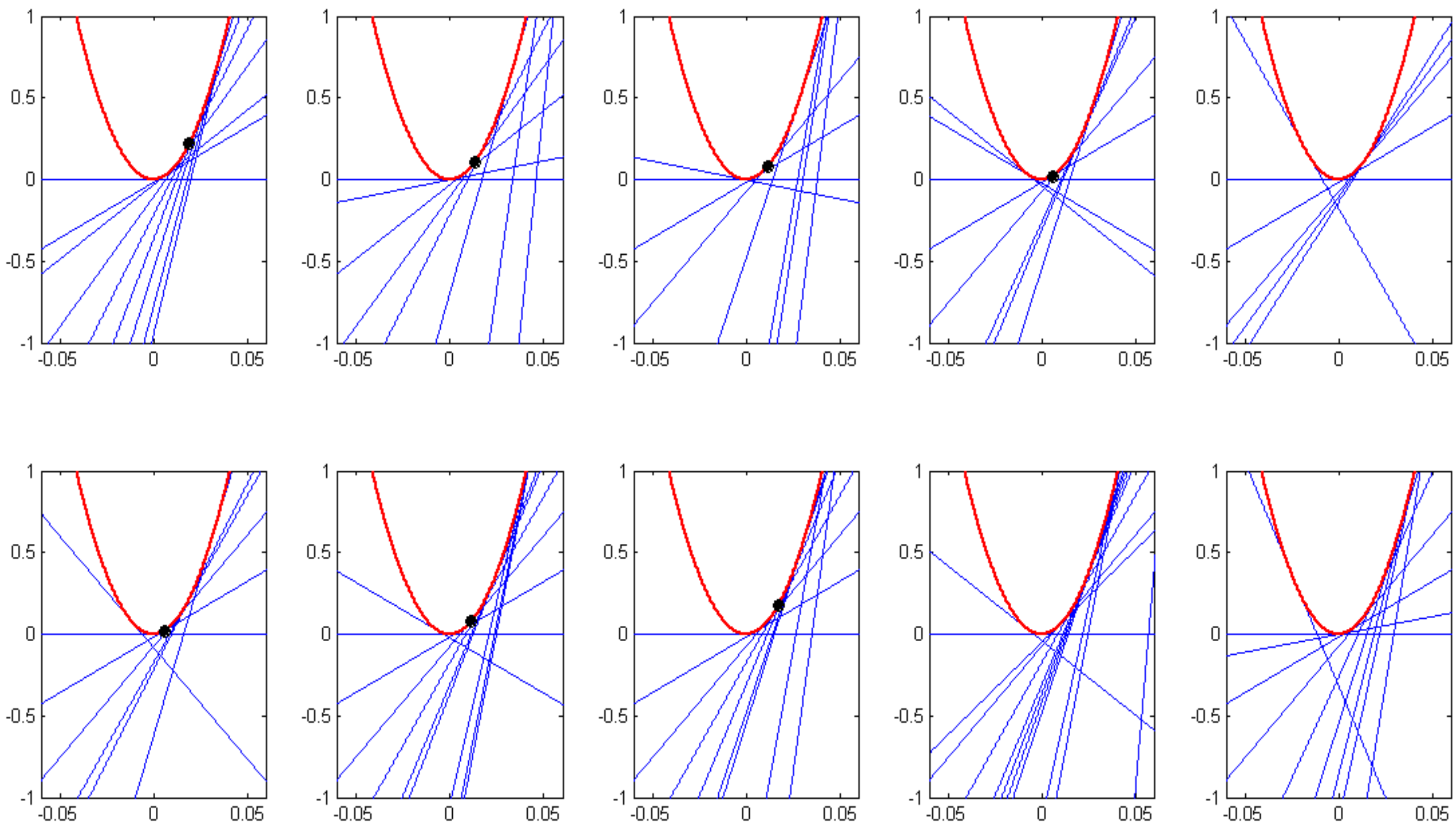

Figure 9 – Iterative addition of hyperplanes – active power

A similar result is obtained for $P_{loss,\ell}^{reactive}$ and is shown in Figure 10. There is no reactive power flow in line section 6 because bus 6 is at the end of a radial path and consumes zero net reactive power, resulting from the turning 'on' of the capacitor.

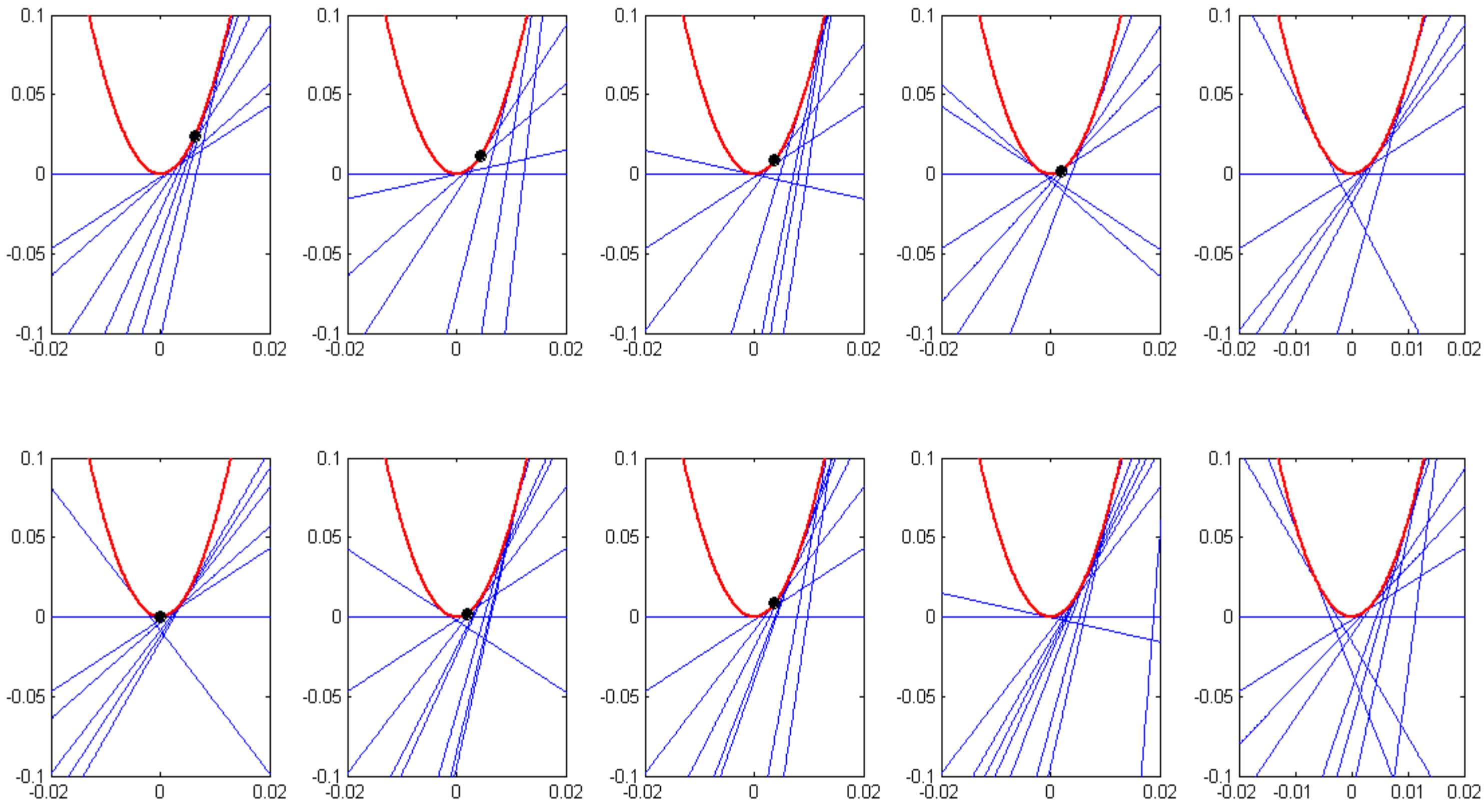

Figure 10 – Iterative addition of hyperplanes – reactive power

VI.2.3.4 Loss analysis

In Figure 11, the curve in blue shows how, through the addition of supporting hyperplanes, the loss converges at the optimal reconfiguration. The initial estimation of losses by HYPER is zero, however this value increases monotonically as the number of SHPs increases. Note also the accompanying green curve showing how the losses found from the AC load flow compare with the orange curve showing the loss found from the DC load flow. The DC load flow is more optimistic yielding somewhat lower losses.

Lastly, observe that although the minimum point is reached at the 8th iteration, four more hyperplanes are needed before the solution converges to the optimum.

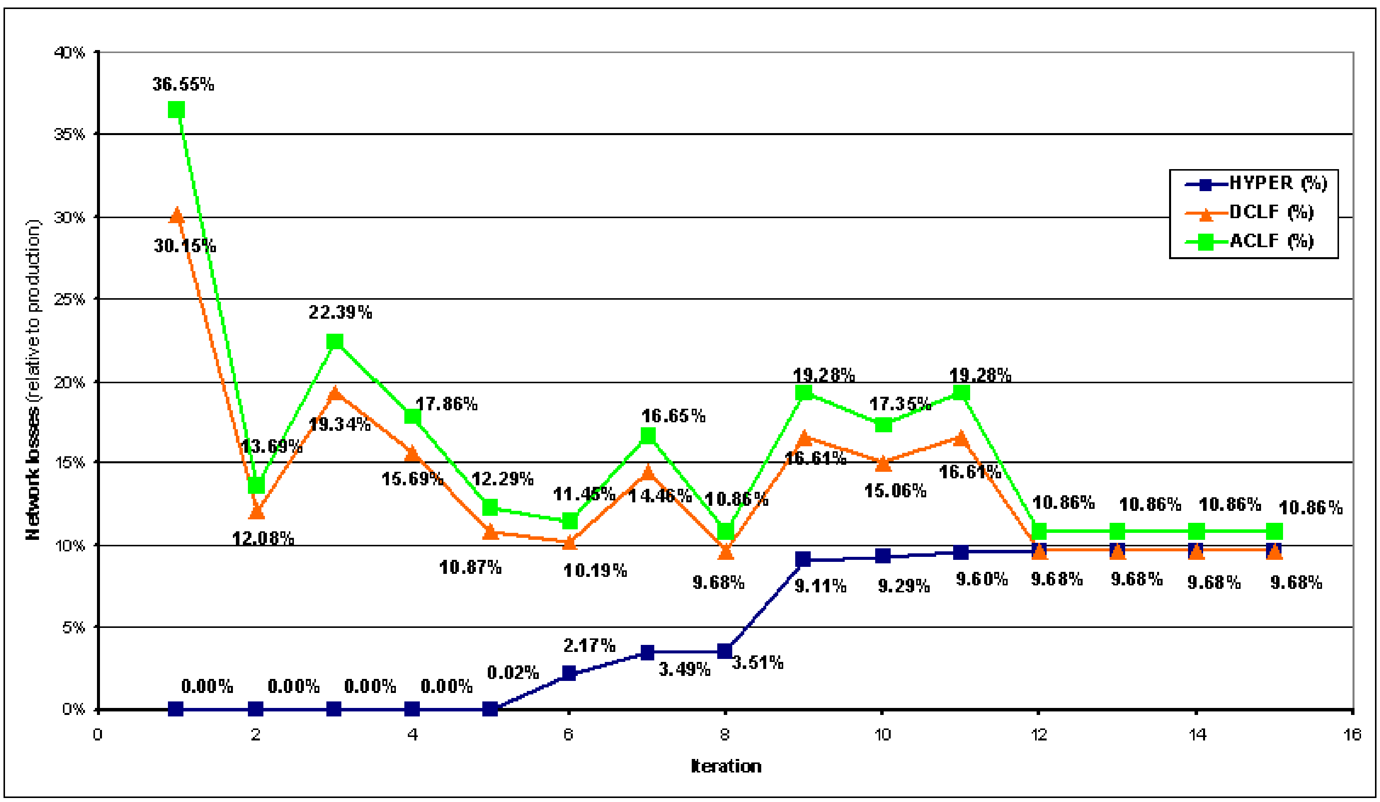

Figure 11 – Convergence of network losses as more SHPs are added

Figure 12 plots the AC load flow losses of every line section versus iteration. It tells us about the compromise that must be made at the detriment of a line section and in favour of another, in order to globally minimize losses.

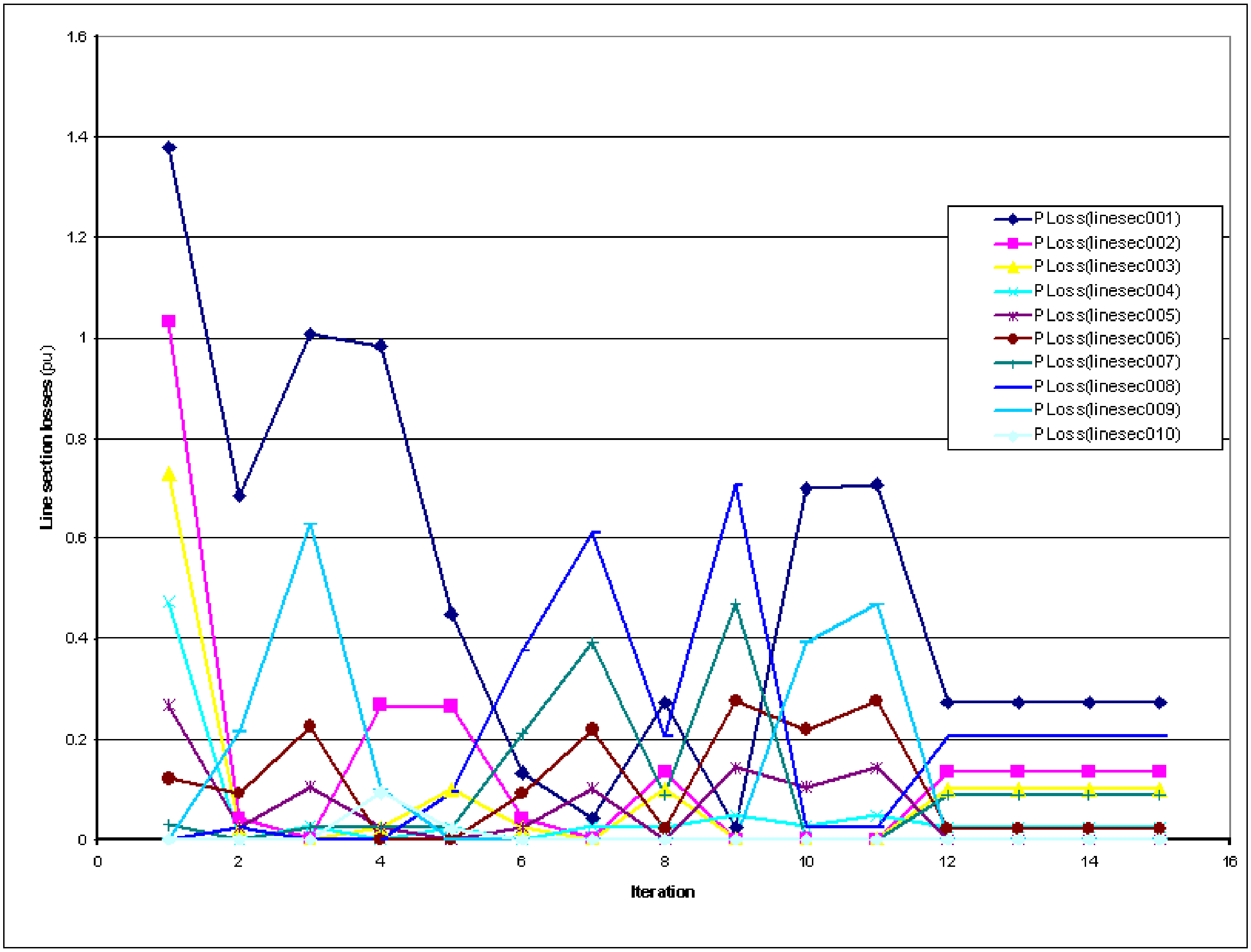

Figure 12 – Line section losses (ACLF) – globally minimized after 12 iterations

Figure 13 indicates the relative contribution of each line section to the total network losses at the optimal solution. It is clear that the segments directly connected to the source at bus 1, line section 1 and line section 8, generate most of the losses since they carry the total production. As we move closer to the termination of the radial branches, line section 4 and line section 6, the current carried decreases and so do the losses.

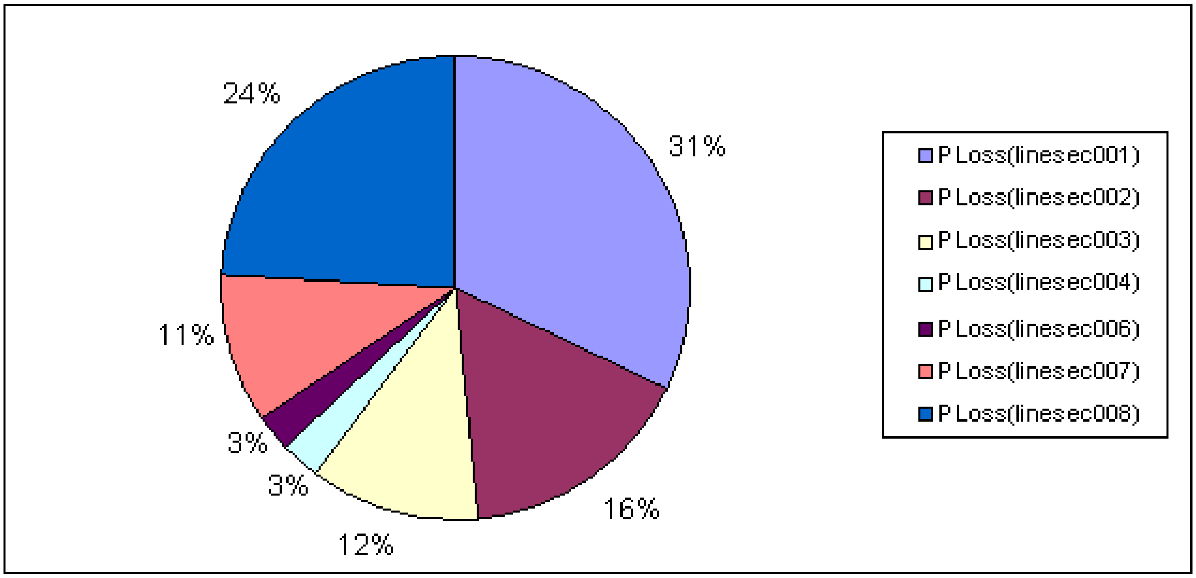

Figure 13 – Relative contribution of each line section to the total network losses

VI.2.3.5 Optimal voltage magnitudes and angles

The following tables show the voltage magnitudes and phase angles obtained from the optimal reconfiguration from both a DC and an AC load flow. Generally, the linear load flow tends to estimate higher levels of voltage magnitudes. Yet, despite some differences between the linear and non-linear voltage calculations, based on Figure 11, these differences do not play a crucial role in the evaluation of losses, which is the principal objective function in OR.

	HYPER/DCLF (pu)	ACLF (pu)
bus001	1.000	1.000
bus002	0.994	0.987
bus003	0.989	0.979
bus004	0.986	0.971
bus005	0.984	0.967
bus006	0.994	0.984
bus007	0.994	0.985
bus008	0.996	0.991

Table 11 – Voltage magnitudes – HYPER/DCLF vs. ACLF

	HYPER/DCLF (rad)	ACLF (rad)
bus001	0.000	0.000
bus002	-0.019	-0.0175
bus003	-0.032	-0.0300
bus004	-0.044	-0.041
bus005	-0.050	-0.046
bus006	-0.034	-0.033
bus007	-0.029	-0.027
bus008	-0.017	-0.016

Table 12 – Voltage angles – HYPER/DCLF vs. ACLF

VI.2.3.6 Voltage profile following OR

From Figure 14, we see that the optimal configuration produced by HYPER, represented by the green curve, tries to flatten the voltage profile by distributing the uniform loads evenly between two branches. We can see this tendency by comparing the curve in magenta representing the voltage profile for the first iteration with the green curves representing the final voltage profile.

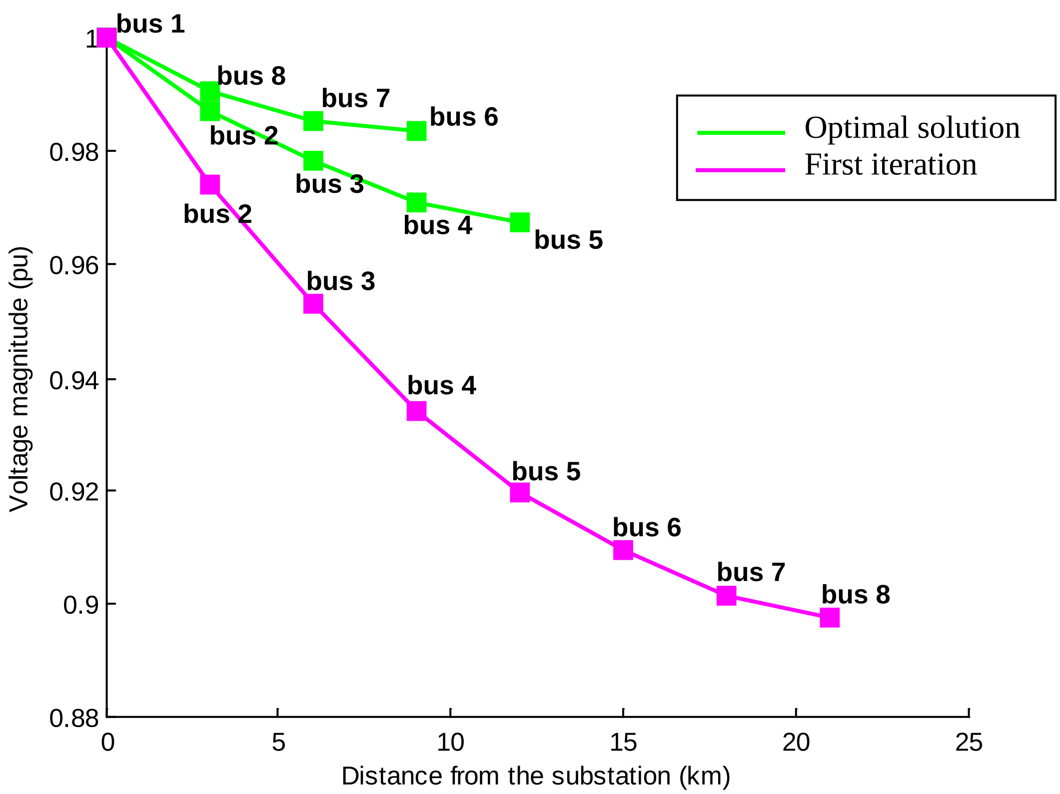

Figure 14 – Voltage magnitude profile comparison (iteration #1 vs. #12)

This behaviour can be explained by the fact that since heat losses in a line section are equal to $r_\ell I_\ell^2$, HYPER attempts to minimize individual I_ℓ currents flowing across line sections ℓ in order to minimize global network losses, $P_{loss} = \sum_{\ell=1}^{n\ell} r_\ell I_\ell^2$. This translates into a voltage drop reduction between buses.

In addition, as shown by the 2 set of curves in green and blue in Figure 15, the voltage magnitude profile is even flatter with the capacitor and distributed generator ‘on’.

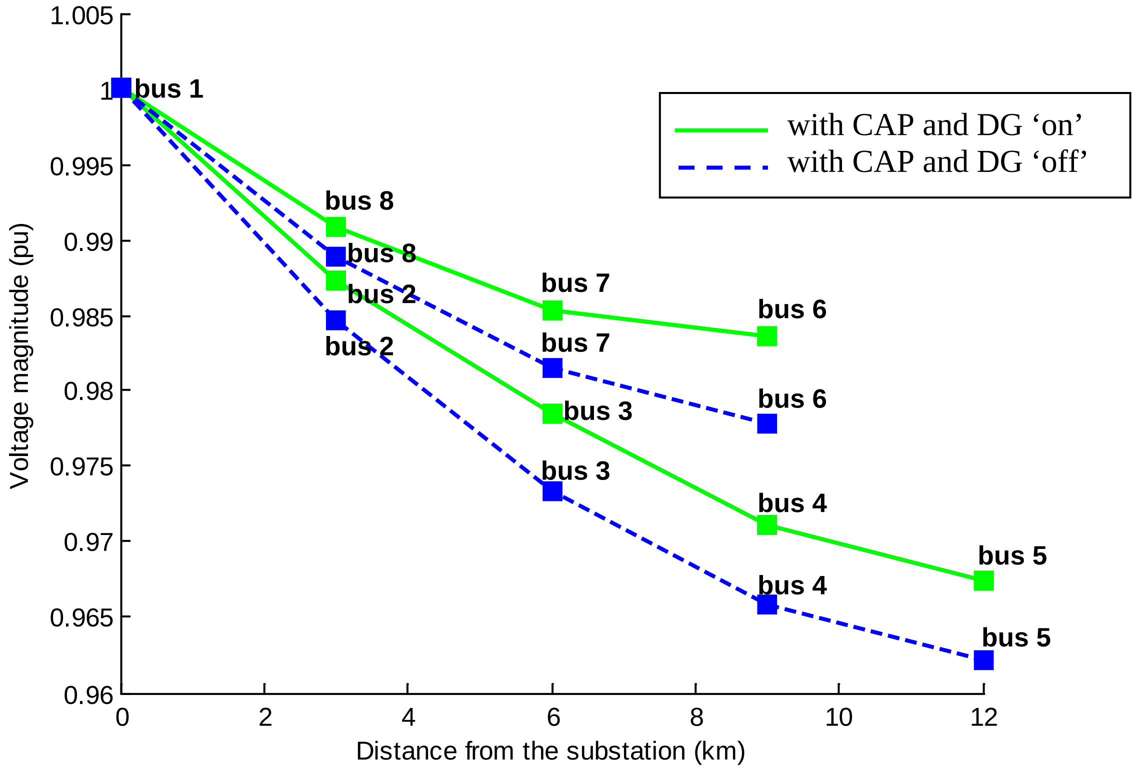

Figure 15 – Voltage magnitude profile for optimal configuration

From the previous two figures, we draw the conclusions that, in general, HYPER attempts to:

- Minimize the current in every line section ℓ;
- Reduce the length traveled by the current to reach loads at branch terminations, meaning it favours a branched distribution (green curve in Figure 14) over an un-branched distribution (magenta curve in Figure 14)
- Bring bigger loads electrically closer to generation buses.

VI.2.3.7 Computational effort

Although in this case we needed 12 iterations to converge, corresponding to the addition of 12 SHPs, the computational effort is worth it. If we were to solve this case by full enumeration of all possible configurations, we would have to consider 56 feasible radial paths, plus the possible participation of a capacitor and private producer, which brings the total number of possible reconfigurations to 224. For larger networks, the full enumeration solution would yield a very high number of feasible paths. HYPER, in contrast, offers a systematic approach to identifying the optimal reconfiguration solution based on efficient MILP solvers such as CPLEX.

Figure 16 shows the computation time required in this test case for each iteration of HYPER running CPLEX on GAMS with the operating system Windows XP pro. The material used is a standard computer with a 3GHz processor chip and 2GB of RAM. The best fitting curve through this set of data points is a straight line, whose equation and correlation factor are indicated on the graph. Clearly, the computation time increases proportionally with the addition of SHPs.

In general, computation time should not be a major issue, since HYPER needs to be run only occasionally whenever the load changes significantly. In addition, dedicated computers could be used to run HYPER either at the Distribution Command Center or even at the substation.

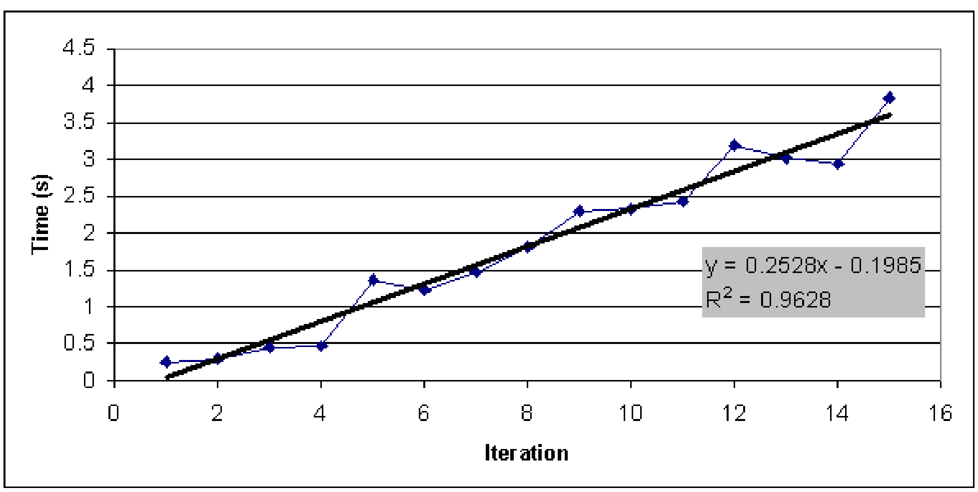

Figure 16 – Computation time required for each iteration

VI.2.4 Additional commentary

VI.2.4.1 On the importance of the reactive power equations

The load flow model used by HYPER decouples the effects of active power flow on voltage magnitudes and the effects of reactive power flow on voltage phase angles, thus allowing us to build a linear model. However, the price of decoupling is some load flow inaccuracy. Normally the flow of active power would affect voltage magnitudes more perceptively but our model is unable to fully reproduce that. We can nonetheless state that such inaccuracies did not introduce important errors in terms of identifying the optimal configuration, as can be seen by comparing the curves of Figure 11. Voltage differences between linear and non-linear load flows are also reported in Table 11 and Table 12.

The effects of coupling are more perceptible for networks where the X/R ratio is close to unity, because such a case invalidates one of DC load flow assumptions. Additionally, coupling effects will also be sensed when the reactive load level is very high and the line sections are very long.

VI.2.4.2 On the usefulness of distributed resources

The algorithm HYPER identifies quickly in the iterative process that the capacitor and the distributed generator advantageously offset the power consumption at their own bus. We see from Table 10, which tracks the changes of switches state across iterations, that the presence of distributed resources is deemed preferable very early.

We also show in Figure 15 that the voltage profile is improved by the presence of the capacitor and the distributed generator. The injection of power at these buses clearly relieves the network by effectively reducing its loading.

VI.2.4.3 On the stopping criterion

The stopping criterion has for purpose to limit the number of iterations before claiming the optimality of the solution. We proposed earlier as a general and typical condition to stop adding supporting hyperplanes when the change in losses is very small. However, as can be seen from Figure 11, without loss of precision, we could use two other criteria:

- Stop when the loss level has not changed for a certain number of iterations;
- Stop when the loss level computed by HYPER equals that of the DC load flow.

VI.3 Three additional test cases

VI.3.1 Non-uniform load distributions

Intuitively, when one bus has a big load compared to the loads at other buses, the optimum network reconfiguration should call for a shortening of the distance between that load and the generation bus. In contrast, we would not expect smaller loads to be close to the generation bus. Nonetheless, because of the network constraints and the complexity of the loss function, such general expectations may not necessarily apply.

To illustrate, consider two examples of non-uniform load distribution, as shown in Figure 17 and Figure 18, tested under HYPER.

In Figure 17, after running HYPER, the optimum configuration is given by $u = [1 \quad 1 \quad 1 \quad 0 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0]$. We see that, being the largest, the load at bus 2 is connected to the single source at bus 1 through the line section equipped with the switch CB1. Other possible configurations that could also feed the load at bus 2 are excluded by HYPER as these would lead to higher losses. In Figure 18, the optimum reconfiguration given by HYPER, $u = [1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0]$, calls for the largest load at bus 8 to be connected to the single source through switch CB8.

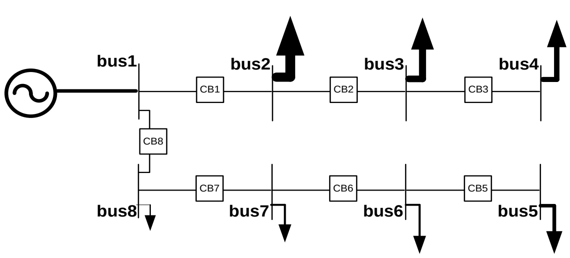

Figure 17 – Non-uniform load distribution

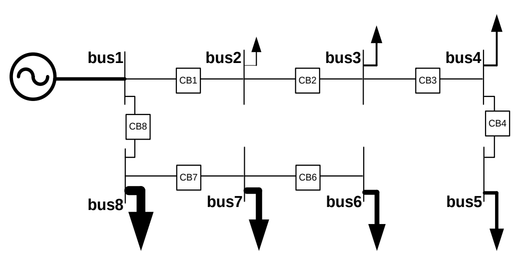

Figure 18 – Non-uniform load distribution

VI.3.2 Line sections with different lengths

Since the longer the line section, the higher the impedance and the higher the losses, intuitively, the optimum reconfiguration seeks the radial path of least resistance that can feed every load. Thus, a short line equipped with a switch may likely be closed most of the time. Conversely, a long line with a switch will have a tendency to remain open because current flowing through it will generate more losses. Once again, however, these principles apply only within the feasibility conditions restricting the number of paths.

To illustrate observe the 3-bus, 3-line network in Figure 19. Assuming that $P_1 = P_2$ and that the lines are made of the same conductor type, HYPER calls for $u = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$, opening CB3. If however $P_1 >> P_2$ then HYPER calls for $u = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$ so as to reduce the losses through line 1 and in so doing, reduce the overall losses. This is so since the increased resistance of line 3 is more than made up by the lower current through that line.

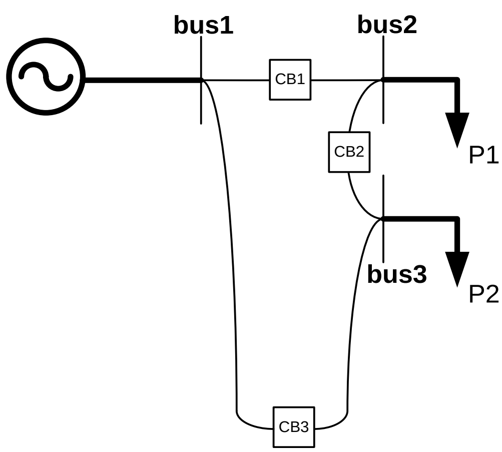

Figure 19 – 3 bus network

VI.3.3 Parallel paths

When two buses are connected by multiple paths as shown in Figure 20, HYPER, as expected, closes the path with the smallest resistance.

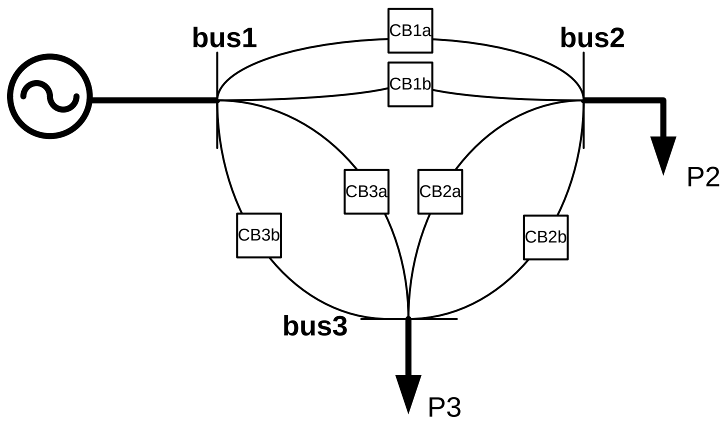

Figure 20 – Network with parallel paths

VI.4 Extended commentary

VI.4.1 Load sensitivity to voltage

Load sensitivity to voltage is frequently modelled by,

$$P_{Di} = k_P P_{Di}^0 + k_I P_{Di}^0 \left(\frac{V_i}{V_i^0} \right) + k_Z P_{Di}^0 \left(\frac{V_i}{V_i^0} \right)^2 \qquad (6.2)$$

$$Q_{Di} = k_P Q_{Di}^0 + k_I Q_{Di}^0 \left(\frac{V_i}{V_i^0} \right) + k_Z Q_{Di}^0 \left(\frac{V_i}{V_i^0} \right)^2 \qquad (6.3)$$

where P_{Di}^0 and Q_{Di}^0 are respectively the active and reactive load demands at bus i for the nominal voltage V_i^0, and the k's are known coefficients.

The linearization of these two equations is achieved by replacing V_i^2 by a first order Taylor series approximation around $V_i^0 = 1$ with $\Delta V_i = V_i - V_i^0$, which is 99.7% accurate for $0.95 < V_i < 1.05$,

$$P_{Di} = P_{Di}^0 \left(k_P + k_I V_i + k_Z \left(1 + 2\Delta V_i \right) \right) \qquad (6.4)$$

$$Q_{Di} = Q_{Di}^0 \left(k_P + k_I V_i + k_Z \left(1 + 2\Delta V_i \right) \right) \qquad (6.5)$$

VI.4.2 OR using mixed-integer nonlinear solvers

The optimal reconfiguration problem can be formulated as a non-linear mixed-integer problem without DC load flow approximations and SHPs. However, the state of the art of non-linear mixed-integer solvers, such as MINLP in GAMS, is such that neither convergence nor optimality is guaranteed. Our experience with nonlinear mixed-integer programs is that they are untrustworthy, while MILP is trustworthy and reliable.

PART VII PLANNING APPLICATIONS OF HYPER

VII.1 Presentation

The adequate planning of a distribution network should lead to a high probability of uninterrupted service delivery during normal operating conditions and under contingencies. Such planning requires conducting several types of analysis, including the optimal positioning of equipment.

Besides the application discussed in previous chapters to a more efficient short-term network operation, HYPER can also be used in planning how to locate remotely controlled switches and capacitors. Similarly, HYPER can identify the most suitable bus to connect a private producer, all optimum analyses with a significant long-term economic impact.

In this section we therefore present an adaptation of HYPER that examines how to position the following remotely controlled equipment:

- Switches;
- Capacitors;
- Distributed generators.

VII.2 Optimal placement problems

VII.2.1 Optimal placement of switches

VII.2.1.1 Motivation

One important consideration in the optimal placement of equipment is to consider the different load levels that occur during a long time interval. Generally speaking, if we assume that a line has a switch and that, for all load levels, HYPER calls for that switch to be 'on', then we deduce that there is no need for a switch in that particular line. However this simplistic approach may not be correct if the switch turns out to be 'on' only part of the time.

A more systematic and complete approach optimizes the location of the switches and their on/off operation over a long time interval, typically a year. We begin by defining a predicted load duration curve for the network injections,

$$\left\{\left(P^t, Q^t, \Delta^t\right); t = 1, 2, \ldots, T\right\} \tag{7.1}$$

where $\left(P^t, Q^t\right)$ are the vectors of net real and reactive bus injections predicted to occur over time interval t of duration Δ^t .

VII.2.1.2 Resolution technique

Suppose we want to optimally locate $ns \le n\ell$ switches in a distribution network, where $n\ell$ is the number of lines. For each line ℓ, we now define a new 0/1 binary variable, w_ℓ, to represent whether or not there should be a switch in that line. Thus,

$$w_\ell = \begin{cases} 1; \text{ if a switch is needed in line } \ell \\ 0; \text{ if a switch is not needed in line } \ell \end{cases} \tag{7.2}$$

Clearly, these binary variables must satisfy,

$$ns \geq \sum_{\ell} w_{\ell} \tag{7.3}$$

Also, we assume that the variables w_{ℓ} are time invariant, that is, that the presence of a switch is either needed in line ℓ for the entire time horizon or it is not needed at all.

Next, we define a 0/1 binary variable, u_{ℓ}^{t}, which represents the state of the switch in line ℓ during time interval t of duration Δ^{t}, that is,

$$u_{\ell}^{t} = \begin{cases} 1; & \text{if the switch at line } \ell \text{ is closed during period t} \\ 0; & \text{if the switch at line } \ell \text{ is open during period t} \end{cases} \tag{7.4}$$

Since the value of u_{ℓ}^{t} is only relevant if w_{ℓ} is equal to 1, we define the binary variable v_{ℓ}^{t} as,

$$v_{\ell}^{t} = u_{\ell}^{t} w_{\ell} + \left(1 - w_{\ell}\right) \tag{7.5}$$

which says that,

$$v_{\ell}^{t} = \begin{cases} u_{\ell}^{t}; \text{ if } w_{\ell} = 1 \\ 1\ ; \text{ if } w_{\ell} = 0 \end{cases} \tag{7.6}$$

Note that the product of binary variables $u_{\ell}^{t} w_{\ell}$ can be exactly replaced by a binary variable x_{ℓ}^{t} plus the linear constraints, $x_{\ell}^{t} \leq u_{\ell}^{t}$, $x_{\ell}^{t} \leq w_{\ell}$, and $x_{\ell}^{t} \geq w_{\ell} + u_{\ell}^{t} - 1$.

We can now define a vector bv^{t} whose elements are $b_{\ell} v_{\ell}^{t}$ and, as in (3.4) , the linear load flow becomes,

$$P^{t} = Adiab\left(bv^{t}\right) A^{T} \delta^{t} \tag{7.7}$$

In addition, from (3.6),

$$Q^t = A\ diag\left(bv^t\right) A^T\ V^t \tag{7.8}$$

If for a particular reason a line section ℓ cannot be equipped with a switch, for example because of environmental constraints, the following constraint is added,

$$\begin{aligned} w_\ell &= 0 \\ v_\ell^t &= 1 \end{aligned} \tag{7.9}$$

To the above, we can also add a constraint to consider a limitation on the number of available switches for optimal placement,

$$0 < ns < ns^{\max} \tag{7.10}$$

Of course, one has to be careful in limiting the number of switches lest there be no feasible solution through which all loads can be served by at least one path. In a practical network where a planner is looking at installing switches, that condition is obvious and already accounted for. One way of avoiding this concern, is by replacing the hard inequality (7.10) by a term proportional to ns in the objective function representing the cost of the ns switches.

Finally, if the number of switching operations per year is restricted to limit wear-and-term, for each line ℓ we impose,

$$\sum_t v_\ell^t \le N_s^{\max} \tag{7.11}$$

The basic objective function being minimized now is the energy lost through losses across all T time intervals in the planning time horizon, that is,

$$\sum_{t=1}^{T} \Delta^t \sum_\ell \frac{G_\ell}{2} v_\ell^t \left(\theta_\ell^t\right)^2 \tag{7.12}$$

where $\sum_{\ell} \frac{G_\ell}{2} v_\ell^t \left(\theta_\ell^t\right)^2$ are the losses during time interval t of duration Δ^t (see equation (4.11)). A similar term loss term is obtained from the voltage magnitude differences as per equation (4.12).

This multi-period optimization problem is solved using HYPER considering all constraints as well as iteratively adding supporting hyperplanes as before. The optimal solution locates ns switches on nl line sections and the vectors w and u^t characterize the optimal solution.

VII.2.2 Capacitor optimal placement

The optimal placement of a capacitor is completely analogous to that of optimum switch placement. We begin by defining two new sets of binary variables, one set consisting of one binary variable per bus (rather than one per line) to define the presence or not of a capacitor. The second set consists of one binary variable per time interval to denote whether in that time interval the capacitor is switched 'on' or 'off'.

The size of an added capacitor, Q_{Ci}, can be fixed in advance or be of variable size. In the latter case the following constraint is needed,

$$0 \le Q_{Ci}^{\min} < Q_{Ci} < Q_{Ci}^{\max} \tag{7.13}$$

VII.2.3 Distributed generator optimal placement

The optimal location of a distributed generator is practically limited to a small search space. Producers select their site based on geographical and economical factors, which have generally little to do with the distance that separates them with the closest distribution feeder. However, if there are some degrees of freedom as to the optimal connection of distributed generators, the application of HYPER in this context is analogous in all aspects to the capacitor placement problem.

PART VIII CONCLUSIONS

VIII.1 Thesis summary

This research has led to the development of a solution algorithm for minimizing losses in a distribution network via the on/off operation of its line switches. HYPER, as the algorithm is called, iteratively adds supporting hyperplanes to represent the non-linear quadratic loss function, thus enabling the linearization of the optimal reconfiguration problem. In this form, Optimal Reconfiguration can be solved using efficient and commercially available MILP solvers like CPLEX. The main characteristics of HYPER are that it guarantees optimality and feasibility, it is relatively easy to implement, it has a broad range of applications, and it considers practical concerns.

The five key outcomes of this work can be summarized as:

1. The development of a *simplified single-phase network model for Optimal Reconfiguration*, including constraints;
2. The development of a *DC load flow model with line switching variables*, accounting for both active and reactive power flow;
3. The representation of losses via supporting hyperplanes enabling the full linearization of the Optimal Reconfiguration problem;
4. The development of the *solution algorithm HYPER* which permits the optimal control (minimum loss) of switches, capacitors and distributed generators on a distribution network;
5. The extension of the utilization of HYPER to planning, for the optimal placement (minimum loss) of switches, capacitors and distributed generators.

VIII.2 Five key research outcomes

VIII.2.1 Simplified single-phase network model for Optimal Reconfiguration

A typical North American distribution circuit is three phase, Y-grounded, unbalanced, non-transposed and radial. However, to address the problem of Optimal Reconfiguration, such a detailed three phase characterization of the network and all its constituents is unnecessary. Most feeders being equipped with only a few automated switches, we can reduce its representation originally comprised of multiple branches and individual loads to an equivalent one with a smaller number of nodes and aggregated loads. Under this assumption we developed a *Simplified single-phase network model for Optimal Reconfiguration*.

VIII.2.2 DC load flow model with line switching variables

For purposes of Optimal Reconfiguration, we adapted the well-known DC load flow to describe the relationship between real power and phase angles, and that between reactive power and bus voltage magnitudes. We developed an accurate linear model to estimate losses that we call *DC load flow model with line switching variables*.

VIII.2.3 HYPER for operations

We demonstrated for operations how HYPER can be used to optimize the state of switches, capacitors and distributed generators in order to minimize losses. The algorithm is capable of handling all constraints, including voltage limits and radial distribution, and finds a solution that reconfigures the network so as to generate minimum losses.

VIII.2.4 Representation of losses via supporting hyperplanes

Whereas the Optimal Reconfiguration constraints are fully linear, the loss function being minimized is quadratic. To deal with this obstacle preventing us from otherwise having direct recourse to MILP, we developed HYPER. The idea behind this technique is to approximate the quadratic loss function by a set of linear inequalities known as supporting hyperplanes (SHP), set which is iteratively updated as shown in Figure 21 The progressive addition of SHPs produces an optimal solution after a few iterations.

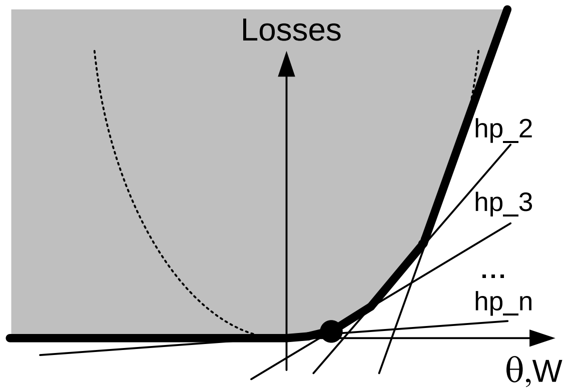

Figure 21 – Representation of losses via supporting hyperplanes

VIII.2.5 HYPER for planning

The principles behind HYPER were extended to the planning of a distribution network. In this context, the algorithm is used to optimally locate switches, capacitors and distributed generators. It considers the different load levels that could be expected during an entire year in order to decide where the equipment should be placed. The technique is analogous in all essential aspects to that used in operations.

VIII.3 Summary of the test cases

We demonstrated the utilisation of HYPER to optimally reconfigure a distribution network by controlling the state of switches, capacitors and distributed generators. After the addition of a few supporting hyperplanes, one per iteration, the algorithm converges to an optimal solution where losses are minimized.

We confirmed that Optimal Reconfiguration yields significant loss reductions, from both the perspective of the global network and the individual line sections. We also showed voltage profiles from different iterations, including with and without the presence of capacitors and distributed generators, to illustrate that at optimum voltages are closer to 1 pu.

We validated our approach, formulated as a MILP problem, by showing that the loss calculations with our linear network and load flow models were very close to loss calculations made with a non-linear load flow.

We finally showed that HYPER is able to deal with extreme situations, including when loads are concentrated in one region, when line sections have very different lengths and when there exists multiple paths between two buses.

VIII.4 Implementing Optimal Reconfiguration at a utility

VIII.4.1 Operations

The Optimal Reconfiguration of a network enables maximum efficiency in the distribution of electricity. It is sound economically because it reduces losses and can defer capital expenditures. It is also sound technically because it enhances the utilisation of assets while maintaining the network within limits.

Most utilities that operate automated equipment have a real time Distribution Management System (DMS) at their Distribution Command Center (DCC). It is typically connected with the data & measurement acquisition system (SCADA), the Geographic Inventory System (GIS) and other relevant systems which allow the proper operation of the distribution network.

Some of the basic applications of a DMS are crew management, power restoration and scheduled maintenance. Some of the advanced applications, i.e. the ones that rely on a real time load flow, are Volt Var Control (VVC) and automated fault detection. As a logical addition in this latter category, Optimal Reconfiguration for loss minimization via HYPER should be envisioned.

VIII.4.2 Planning

The significance of planning applications comes with the recognition that equipment life expectancy is long and that the cost of relocation is very high. In distribution, software is used by engineers to ensure that load growth is sustainable and that it does not bring the network outside its operational limits. In this context HYPER can help solve planning problems through a systematic approach beyond standard methods based on heuristics.

PART IX | REFERENCES

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PART X | APPENDIX

X.1 Expressing binary-continuous variable products as linear inequalities

Any product of a 0/1 binary variable u with a bounded continuous variable x, $y = ux$, is equivalent to four linear inequalities,

$$ux^{\min} \le y \le ux^{\max} \tag{10.1}$$

$$-(1-u)L \le y - x \le (1-u)L \tag{10.2}$$

where L is a very large positive number.