A Study of Simultaneous Stochastic Optimization of Open Pit Mining Complexes

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Abstract

Over the last several years advances in the field of mine planning have led to the development of cutting-edge simultaneous stochastic optimization frameworks for mining complexes. The latest methods consider mining operations as a resource-to-market integrated mineral value that transforms raw in-situ materials into sellable products, a mining complex. Simultaneous stochastic optimization frameworks make use of a paradigm shift that considers the value of the sellable products, as opposed to economic block values, to drive the optimization process and capitalize on the synergies between the central, interrelated components of a mining complex. These methods maximize the value of mining operations and manage technical risk by incorporating uncertainty directly into unified optimization formulations. This thesis studies the simultaneous stochastic optimization framework through two real-world case studies, applying the methods and assessing their characteristics and limitations.

The second chapter of this thesis presents an application of a stochastic framework that simultaneously optimizes mining, destination and processing decisions for a multi-pit, multi-processor gold mining complex with challenging geochemical processing constraints. The framework accounts for supply and market uncertainty via stochastic orebody and commodity price simulations as inputs to a unified optimization model. The case study notably assesses the impacts of integrating market uncertainty as input that influences all components of the production schedule. Additionally, cut-off grade decisions are determined by the simultaneous optimization process, considering material variability and operating constraints while reducing the number of *a-priori* decisions to be made. This approach generates solutions that capitalize on the synergies between extraction sequencing, cut-off grade optimization, blending and processing while managing and quantifying risk in strategic plans. Which ultimately leads to more metal production and higher NPVs than traditional methods.

The third chapter applies an extension of the generalized simultaneous stochastic optimization formulation that considers capital expenditure (CapEx) options as part of the life-of-asset planning process. Enabling the case study to consider environmental issues relating to tailings management and model a tailings facility expansion. The application at a multi-element open pit mining complex simultaneously optimizes the extraction sequence, cut-off grades, and downstream decisions from two open-pits with a set of stockpiling options, an autoclave and a tailings storage facility. The project bottleneck is the tailings facility volume because it stores both process tails, and potentially acid-generating waste rock from the mines. Results show that, when given the option, the optimizer chooses to make a significant CapEx investment to expand the tailings storage facility 25% by volume. This expansion allows for a meaningful expansion of both pit limits, 40% by mass, resulting in an extended metal production and revenue generation horizon that yields 14% more gold ounces and a 4% improvement in NPV for the mining complex. The framework provides decision makers with a realistic evaluation of the investment's impact on the mining complex.

Résumé

Au cours des dernières années, les progrès réalisés dans le domaine de la planification minière ont conduit à l'élaboration de modèles d'optimisation stochastique simultanée de pointe pour les complexes miniers. Les méthodes les plus récentes considèrent les activités minières comme une chaîne de valeur allant des ressources minérales au marché financier, transformant les matières brutes in-situ en produits vendables. Les modèles d'optimisation stochastique simultanée utilisent ce paradigme qui prend en compte la valeur des produits, par opposition à la valeur économique d'un bloc seul, pour piloter le processus d'optimisation et capitaliser sur les synergies entre les composants centraux et interdépendants d'un complexe minier. Ces méthodes maximisent la valeur des opérations minières et contrôlent les risques techniques en incorporant directement l'incertitude dans des formulations d'optimisation unifiées. Cette thèse étudie un modèle d'optimisation stochastique simultanée à travers deux études de cas réels, appliquant des méthodes de résolution et en évaluant leurs caractéristiques et leurs limites.

Le deuxième chapitre de cette thèse présente l'application d'un modèle stochastique qui optimise simultanément les décisions d'extraction, de destinations et de traitement pour un complexe aurifère à plusieurs fosses et à plusieurs processeurs soumis à des contraintes de traitement géochimique complexes. Le modèle d'optimisation unifié prend en compte les incertitudes de l'offre et du marché au moyen de simulations stochastiques du gisement et du prix de la marchandise comme données d'entrée. L'étude de cas évalue notamment les effets de l'intégration de l'incertitude du marché sur toutes les composantes du calendrier de production. De plus, les décisions concernant le niveau de la teneur limite sont déterminées par le processus d'optimisation simultanée, en tenant compte de la variabilité des matériaux et les contraintes de fonctionnement, tout en réduisant le nombre de décisions à prendre a-priori. Cette approche génère des solutions qui capitalisent sur les synergies entre la séquence d'extraction, l'optimisation du niveau de la teneur limite, le mélange et le traitement, tout en gérant et quantifiant les risques dans des plans stratégiques. Cela conduit finalement à une plus grande production de métal et à un NPV plus élevé par rapport aux résultats obtenus par des méthodes traditionnelles.

Le troisième chapitre considère une extension du modèle précédent mais qui inclue des options de dépenses en capital (CapEx) dans le processus de planification de la durée de vie des actifs. Cela permet à l'étude de cas de considérer les problèmes environnementaux liés à la gestion des résidus et de modéliser l'agrandissement d'un entrepôt à résidus. L'application à un complexe minier à ciel ouvert multi-éléments optimise simultanément la séquence d'extraction, le niveau de la teneur limite et le flux de matériau provenant de deux fosses avec un ensemble d'options de stockage, un autoclave et une pile de stockage de résidus. Le goulot du projet est le volume de l'entrepôt à résidus car il stocke à la fois les résidus et les résidus potentiellement acidogènes des mines. Les résultats montrent que, lorsqu'il en a la possibilité, l'optimiseur choisit de faire un investissement important en CapEx pour agrandir de 25% en volume le local de stockage des résidus miniers. Cet agrandissement permet une expansion significative des deux limites de la fosse, 40% en masse, ce qui entraîne une augmentation de la production de métal et un allongement de l'horizon de génération de revenus. Cela se traduit par une production de 14% d'onces d'or supplémentaires et une amélioration de 4% du NPV du complexe minier. Le système fournit aux décideurs une évaluation réaliste de l'impact de l'investissement sur le complexe minier.

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Chapter 1 Introduction and Literature Review

1.1 Introduction

Strategic mine planning, also known as life-of-mine or life-of-asset planning, refers to a group of decisions that aim to promote an operation's objective, typically, the maximization of net present value (NPV) and by extension shareholder value (King, 2009). Optimization of strategic mine - plans is a global problem that encompasses the interrelated components of a mining complex and generates life-of-asset production schedules that maximize discounted cash flows and meet production targets, subject to a series of operational constraints. A mining complex is defined as a resource to market integrated mineral value chain that transforms in-situ material into marketable products (Pimentel *et al.*, 2010; Montiel and Dimitrakopoulos, 2015; Goodfellow and Dimitrakopoulos, 2016). Historically, this process has been divided into the step-wise optimization of local value chain components, such as delineation of ultimate pit limits, grouping pushbacks, extraction sequencing, cut-off grades, stockpiling, blending, processing and tailings management (Lerchs and Grossmann, 1965; Johnson, 1968; Gershon, 1983; Whittle, 1989; Dagdelen, 2001; Hochbaum, 2001; Hustrulid et al., 2013). This division of labour is a product of simplifications that were, in some cases, necessary to make strategic mine planning a feasible problem for mathematical modelling techniques such as mixed-integer-programming (MIP). However, the local optimization of individual value chain components, often with misaligned objectives and non-linear transfer functions, leads to suboptimal solutions whose deterioration is compounded by increasing complexity in the mineral value chain (Gershon, 1983; Whittle, 2007; Goodfellow, 2014; Montiel, 2014; Whittle, 2014). This observation establishes a need for strategic planning approaches that can incorporate the optimization of multiple, or ideally all, upstream and downstream components of a mining complex simultaneously.

Mines and mineral deposits are also characterized by a great deal of uncertainty that often complicates the economic viability of mining projects. The most detrimental source of technical risk in mining projects arises from supply uncertainty which comes from mischaracterizing geological orebodies (Baker and Giacomo, 1998; Vallée, 2000; Dimitrakopoulos *et al.*, 2002). Conventional mine planning uses estimated orebody models as inputs to the optimization process, which have been shown to misrepresent the distributions of geological attributes such as grades and material types (David, 1988). The risks associated with using estimated orebody models to represent the in-situ local variability in mineral deposits are well understood in the technical literature (Ravenscroft, 1992; Dowd, 1994; Dimitrakopoulos *et al.*, 2002). The field of geostatistics has developed advanced conditional simulation methods that reproduce geo-spatial statistics of available information and can be used to assess and integrate risk into the mine planning process via stochastic mine planning (Journel and Huijbregts, 1978; Isaaks and Srivastava, 1989; Goovaerts, 1997; Remy *et al.*, 2009; Dimitrakopoulos, 2011; Rossi and Deutsch, 2014). Recent advancements in the field have proposed simultaneous stochastic mine planning frameworks that integrate the related components of mining complexes such as extraction sequences, cut-off grades, processing streams, and transportation alternatives into unified formulations (Montiel and Dimitrakopoulos, 2015; Goodfellow and Dimitrakopoulos, 2016). These approaches capitalize on the synergies and non-linear interactions between various mechanisms in the value chain to generate high-quality solutions and improve project value.

This chapter reviews the technical literature related to strategic mine planning and modelling uncertainty. Section 1.2 covers deterministic approaches to integrate the optimization of mining complexes. Section 1.3 explains of the risk associated with supply and market uncertainty, followed by a review of simulation methods for mineral deposits and commodity prices. Section 1.4 reviews the methods for integrating risk into the mine planning process, initial stochastic mine planning formulations and finally the simultaneous optimization of mining complexes. Section 1.5 outlines the objectives and Section 1.6 outlines the remainder of this work.

1.2 Deterministic approaches to optimizing the mineral value chain

The research and development of 'global optimization' approaches began earnestly in the late 1990s when Urbaez and Dagdelen (1999) were commissioned by Newmont in the late 1990s to develop a simultaneous optimization method for the strategic planning of multiple mines and processing plants along the Carlin Trend. The authors propose a mixed-integer-programming formulation that optimizes the sequencing of material from multiple mines, stockpiles and processing destinations. However, there were issues when scaling the method to handle life-size problems. Hoerger *et al.* (1999) build on the previous work to develop an in-house MIP optimizer that maximizes NPV by modelling the flow of material from mines to stockpiles and processors using pushback sequencing and plant start-ups/shut-downs. The tool is used to capitalize on existing synergies amongst Newmont's vast Nevada operations, and exhibits increased profitability when applied to 50 material sources, eight stockpiles and 60 processing destinations (Hoerger *et al.*, 1999). However, the formulation falls short of actually optimizing extraction sequences as it uses a set of fixed schedules to achieve its objective. There is also a loss of resolution when sequencing is aggregated to the pushback scale that can lead to issues with blending and processing targets. Nonetheless, this work marks an important step in approaching long-term mine planning with a more global view.

Stone *et al.* (2007) outline BHP's mine planning optimization tool, known as Blasor, which also uses a MIP formulation solved by a commercial solver to generate near-optimal ultimate-pit-limits and phase designs for multiple open-pit mines. Blasor aggregates spatially connected and geologically similar blocks to reduce the size of the model and deliver tractable solutions. It starts by optimizing the aggregate extraction sequences and pit limits while accounting for operating constraints. The optimal aggregate extraction sequence is used to generate pushback designs and then finally a panel extraction sequence, where a panel is the intersection of a pushback and a mining bench. The tool is successfully applied at Yandi, BHP's eleven-pit, blended iron-ore mining complex, where it generates feasible, long-term extraction sequences that maximize discounted cash flow (DCF). Zuckerberg *et al.* (2007) outline an extension for Blasor called Blasor-InPitDumping that incorporates the optimization of waste handling. The tool re-fills mined-out pit areas while respecting repose slope constraints and without sterilizing any ore. Zuckerberg *et al.* (2011) describe a deposit specific life-of-mine optimizer called Bodor, developed for BHP's Boddington Bauxite mine in Western Australia. The tool uses a mixed-integer-linear-programming formulation to minimize net present costs, meet blending targets, respect a complex set of environmental and operating constraints. It does so by optimizing the bauxite-pod extraction sequence, fleet size and utilization, and crusher and conveyor infrastructure. A notable limitation is the assumption of homogeneous material within each bauxite pod. Nevertheless, Bodor outperforms a commercially available benchmark, XPAC scheduler (Caccetta and Hill, 2003), reducing the net present costs by 5%.

Chanda (2007) proposes a network linear programming model to optimize production planning from underground and open-pit mines and their associated metallurgical processing facilities. The method aims to find a minimum cost of production and distribution for material flowing from the mines to markets. While the method is successful in modelling material flow through the metallurgical network, the approach uses mine production as an input parameter and does not generate or optimize mining schedules. Whooler (2007) provides an overview of COMET, a commercial strategic mine planning tool, that is not a 'global optimizer' but aims to simultaneously optimize mine production schedules, cut-off grades and mill throughput. COMET uses an iterative algorithm known as 'successive approximation dynamic programming' to optimize successive schedules and operating policies (cut-off grade, process route, throughput, and recovery) to maximize the value of the resource, but it does not guarantee convergence to the optimal schedule. One of COMET's main limitations is the method's suitability for blending operations because it cannot model minimum constraints required to maintain specific material quality ranges.

Whittle (2010) outlines the third iteration of Whittle Consulting's 'Global Optimizer' known as Prober C, building on their previous work in (Whittle, 2007). The commercial tool is designed to globally optimize complex operations consisting of multiple mines, processing streams and blending requirements. Prober aggregates mining blocks into 'parcels' of similar grade and material type which are then further aggregated into 'panels' (typically represent a bench in a pushback). The aggregation reduces the model size by an order of magnitude allowing for more tractable solutions, however, when sequencing the extraction of a fraction of a panel in a period it assumes that the same fraction is extracted from each parcel. This is a reoccurring and notable limitation of methods which rely on the aggregation of material significantly larger than the selective mining unit referred to as a mining block. In order to ensure panel extraction sequences respect slope constraints, Prober only allows the mining of one panel at a time. If a panel represents a bench in a pushback, this implies that a mine can only extract one bench at a time, limiting the potential of the production schedule. Nonetheless, the software allows for a great deal of flexibility in modelling material flow, constraints, costs and revenues through a mining complex including non-linear expressions, cut-off-grades and stockpiling. The algorithm is proprietary, but the solution approach is generally defined: after generating a set of nested pit shells for each mine using an implementation of Lerchs and Grossmann (1965), Prober randomly samples the set of feasible panel extraction sequences, defining the extraction of materials. The algorithm uses linear programming to optimize and evaluate the downstream components. It iteratively tries to improve this solution, searching for a local maximum, then randomly samples another feasible panel extraction sequence and repeats until the top 10 NPV's are within 0.1% of each other. While Prober C is a global optimizer that incorporates many components of the mineral value chain into the framework, it is not a unified model that optimizes all components simultaneously. The Prober 'series' of programs includes several parts, including the nested pit implementation of Lerchs and Grossmann (1965), a local search heuristic, and a linear programming evaluation routine.

Epstein *et al.* (2012) present a method for long-term mine planning for both underground and open-pit mines. The MIP model is a general capacitated multicommodity network flow formulation that aims to integrate several open-pit and underground mines that share multiple processing streams. The method generates a panel extraction sequence as well as a stope extraction sequence for block caving. It accounts for stockpiling and blending by assuming predefined average grades at relevant destinations in order to keep the model linear. The authors outline the application at Codelco's North Division copper mining complex, where the model improves the NPV by 5% over the current benchmark by optimizing each mine individually and a further 3% by integrating both mines. The solution approach overcomes the combinatorial limitations of real-life instances by solving a tight linear relaxation and then using a rounding heuristic to generate an integer solution. Topal and Ramazan (2012) propose a network flow, linear programming model for strategic mine planning and apply it to a case study in Western Australia with more than 100 open- pits and 13 processing streams. The model optimizes extraction capacities, processing capacities, and determines material destination (stockpile or processing stream), improving the NPV by more than 10% over a commercial software benchmark. To ensure linearity in stockpiling components, the authors use a similar strategy to Epstein *et al.* (2012) by using grade bins at each destination to approximate the grade of exit material. While these two approaches (Epstein *et al.*, 2012; Topal and Ramazan, 2012) are not strictly 'global optimizers', they make an important effort to link several value chain components within the long-term mine planning framework.

Dagdelen and Traore (2014) take a global approach to determining the optimal transition depth from a set of open pit mines to an underground mine. They propose an iterative NPV maximization method that utilizes a MILP global production scheduling optimization approach. The method is applied at a case study with six open pits and one underground (long-hole open stope) mine. The iterative approach uses a combination of commercial and in-house software to: generating a series of ultimate pits from the existing mines, fixing a crown pillar design for the underground mine at the depth of the given ultimate pit, setting mining and processing rates, optimizing the LOM production schedules for the combined operations and performing a DCF and NPV analysis. Then the ultimate pit depths are strategically increased, and mining rates are adjusted to favour more open-pit mining, the schedules are reoptimized, and NPV is recalculated. The process repeats until an inflection point in the NPV is located, determining the optimal transition depth. The case study indicates that the approach delivers an economic advantage to independent optimization of the open pit and underground mines.

1.3 Modelling uncertainty

1.3.1 The need for modelling uncertainty

The methods described in Section 1.2 improve on traditional, piece-wise strategic mine planning approaches by incorporating more components into the optimization process. However, they all share a major limitation by failing to account for uncertainty in critical parameters. Deterministic optimizers assume perfect knowledge of input parameters which are inherently uncertainty, such as grades, tonnages, prices and costs. They do not understand uncertainty and therefore generate 'optimal' solutions which do not perform as expected given that they are only optimal or optimized under the assumption of a specific set of parameters. The disregard for uncertainty is a well-known problem in technical literature, many authors have studied and documented the associated risks (Ravenscroft, 1992; Dowd, 1994; Baker and Giacomo, 1998; Benning, 2000; Vallée, 2000; Dimitrakopoulos *et al.*, 2002; Godoy, 2003).

Geological uncertainty is the largest risk for mining projects. (Baker and Giacomo, 1998); Vallée (2000) conducts a sizable survey and notes that over 60% of mines have an average rate of production less than 70% of capacity during their first year of operation when delivering results is critical to establishing investor confidence. Some shortfall can be attributed to production ramp-ups. However, the most significant contributor is a misunderstanding of grades and tonnages. Benning (2000) outlines project financing concerns from a banker's perspective. He highlights "without a doubt [orebody risk] is the single most important characteristic of any resource project…" and that "The over-valuation of the orebody is the single most common cause for failure, or under performance of a mining project."

Conventional, deterministic optimizers use estimated orebody models derived by methods such as ordinary kriging (David *et al.*, 1977; Journel and Huijbregts, 1978; Goovaerts, 1997). However, the field of geostatistics has known for decades that estimation methods do not reproduce geospatial characteristics of in-situ material and misrepresent proportions of metal concentrations within mineral deposits (David *et al.*, 1977; Journel and Huijbregts, 1978; David, 1988; Isaaks and Srivastava, 1989; Goovaerts, 1997; Rossi and Deutsch, 2014). This is due to what is known as the 'smoothing effect' of estimation methods which yields lower variability in histograms and variograms compared to available data (Goovaerts, 1997; Dimitrakopoulos, 1998). Stochastic simulations are used to quantify the impact and then assess the risk of using estimated orebody models and deterministic optimizers, highlighting the unlikelihood of meeting production and cash flow forecasts (Ravenscroft, 1992; Dowd, 1994; Dimitrakopoulos, 1997; Dimitrakopoulos *et al.*, 2002). These impacts are exacerbated by the non-linearity of the mining transfer function as described by Dimitrakopoulos *et al.* (2002). The case study highlights that the NPV forecasted by conventional, deterministic methods has a 5% chance of realization and the median NPV realized by the stochastic simulations is 25% less than forecasted. The conclusion is that average-type inputs do not generate average-type outputs, emphasizing the need for effective modelling of uncertainty and integration into the strategic mine planning process.

1.3.2 Modelling uncertainty in mineral deposits

The importance of modelling geological uncertainty has been established and can not be understated. The inability to accurately estimate the relevant attributes of geological phenomenon demands the use of simulations of reality to characterize the spatial uncertainty of mineral deposits (Journel, 1974). Stochastic simulation and random field models can be used to generate equi-probable realizations of a mineral deposit that respect the available geospatial information, providing a visual and quantitative measure of uncertainty of its attributes of interest (Journel and Huijbregts, 1978; David, 1988; Journel and Alabert, 1989; Goovaerts, 1997; Godoy, 2003; Remy *et al.*, 2009; Rossi and Deutsch, 2014).

Perhaps the most prevalent feature of these methods is the sequential simulation approach which is driven by the decomposition of a multivariate probability density function (pdf) of random fields into products of univariate posterior distributions (Journel and Alabert, 1989; Journel, 1994; Goovaerts, 1997). The most widely used and easily implemented method in mining applications is the sequential Gaussian simulation approach (Journel, 1994; Goovaerts, 1997). Although it is easy to implement the method is computationally expensive and can be time-consuming when simulating deposits large deposits where the number of nodes is on the order of 10^8. Luo (1998) capitalizes on the Lower Upper (LU) decomposition method (Davis, 1987) to generalize the SGS (GSGS) method by simulating neighbourhoods of nodes simultaneously and reduces the computational cost from $O(N^4)$ to $O(Nv_{max}^3)$ where N is the number of nodes and v_{max} is the size of a neighborhood. Dimitrakopoulos and Luo (2004) further improve the computational efficiency by sharing conditioning neighborhoods among a group of adjacent nodes on a grid and using screen effect approximation (SEA) to determine the optimal group and neighborhood sizes.

Though there is a significant improvement in speed from SGS to GSGS, the method remains very costly in terms of memory as it is a point-scale approach. This also requires a change of support post-processing step to generate realizations on the selective mining unit (block) scale. Godoy (2003) provides a step forward from SGS and GSGS with Direct Block Simulation (DBSIM), the method leverages the benefits of both methods and improves memory considerations yielding an efficient and easy to implement algorithm. The method works as follows; the deposit is discretized into groups of nodes, generally by block size. A random path is defined for visiting each block and then each node within a block. Groups of nodes are simulated simultaneously using LU Decomposition and averaging the values of each node, storing only the average value of each block and discarding the individual values. Each block is simulated sequentially, calculating point-point, point-block and block-block covariances. The major improvement in the method comes from discarding the individual simulated values of each node, resulting in substantial memory improvements. Furthermore, the method does not require any post-processing step as the output is directly on the block scale. Benndorf and Dimitrakopoulos (2007) compare the practical aspects of GSGS and DBSIM in terms of accuracy and efficiency on a porphyry copper deposit. The method is extended to simulate multiple correlated variables on the block scale by (Boucher and Dimitrakopoulos, 2009) through the use of minimum and maximum autocorrelation factors (MAF) (Desbarats and Dimitrakopoulos, 2000). The de-correlated variables are simulated independently using the DBSIM framework. Once the nodes within a block are simulated: (1) they are averaged for conditioning the next block, and (2) back-transformed to the Gaussian and then the data space to obtain the average block value, these two processes occur in parallel. The method, known as DBMAFSIM, and its application are described in depth by (Boucher and Dimitrakopoulos, 2009, 2012).

The methods described above are effective and efficient tools for simulating equally probable realizations of real-world mineral deposits that quantify uncertainty and respect univariate and bivariate statistics of the available data. However, they are limited to second-order measures of spatial continuity because they assume the underlying distribution is Gaussian. This assumption is convenient because the distribution can be described by very few parameters, specifically first-order statistics (mean values) and second-order statistics (covariance and variograms). While convenient, it is exceedingly unlikely for natural geological phenomenon to ascribe to this notion, they are known to exhibit non-Gaussian characteristics and complex curvilinear spatial structures (Guardiano and Srivastava, 1993; Dimitrakopoulos *et al.*, 2010), which can not be sufficiently described by second-order statistics. Second-order methods which rely on Gaussian distributions assume that multi-point statistics and connectivity are governed by the same algorithm that simulates the two-point relationships (Remy *et al.*, 2009). Thus they are maximal entropy methods that maximize spatial disorder in higher-order structures and patterns, resulting in a 'salt and pepper' effect amongst extreme values (Journel and Deutsch, 1993) The inability to reproduce these complex patterns connect of extreme values in mineral deposits adversely impacts the optimization of processes made up of non-linear transfer functions such as mine production schedules.

A class of simulation methods, known as multi-point statistic (MPS) methods (Guardiano and Srivastava, 1993; Strebelle, 2002; Journel, 2003, 2005; Toftaker and Tjelmeland, 2013) try to overcome these limitations by using information that is not present in the conditioning data. These methods depart from the random field model to the use of a training image (TI). Training images are used to describe spatial continuity and act as a geological analogue of the variable of interest. Multi-point methods use training images as templates representative of a complex spatial arrangement generally relying on some form of Monte-Carlo sampling of values from the TI to incorporate additional information about the attributes to be simulated. They measure the similarity of a neighbourhood of an unsampled location to that of the TI and assign the unsampled location being simulated the value of the node in the TI with the most similar neighbourhood. It is important to note that MPS methods do not use any spatial information from available hard data, resulting in simulations that are representative of the TI. When the TI and the hard data provide conflicting spatial statistics, MPS methods reproduce those of the TI exposing a limitation of the methods (Dimitrakopoulos *et al.*, 2010). Moreover, it can be challenging to construct appropriate TI's for in-situ mineral deposits when relying on exploration data such as diamond drill-holes, limiting the applicability of MPS methods for greenfield mining projects.

More recently, simulation methods have been developed that extend spatial models beyond second-order statistics, calculating high-order spatial characteristics such as cumulants from hard-data to complement information sampled from the TI (Mustapha and Dimitrakopoulos, 2010, 2011; Minniakhmetov and Dimitrakopoulos, 2017b; Minniakhmetov *et al.*, 2018; Yao *et al.*, 2018). Cumulants can be understood as an higher-order extension to mean (first order) and variance(second order) spatial statistics, composed of moment statistical parameters. Mustapha and Dimitrakopoulos (2010) calculate high-order cumulants and use them to describe the complex geological structures and connectivity. The calculation of cumulants in a sequential framework is computationally expensive, Mustapha and Dimitrakopoulos (2011) improve efficiency by using Legendre polynomials to approximate the underlying conditional probability distribution (cpdf) instead of explicit calculation of the spatial cumulants. However, Legendre polynomials are very unstable at higher-orders, Minniakhmetov *et al.* (2018) propose the use of lower order Legendre-like splines to circumvent stability issues. Yao *et al.* (2018) significantly speeds up and improves the accuracy of High-order simulations by simplifying the approximation of the cpdf without needing to explicitly calculate any spatial moments or cumulants, instead relying on a unified empirical function based on spatial Legendre moments. These advanced simulation methods are incredibly powerful tools for simulating complex geological structures and the connectivity of patterns between extreme values. However they are still performed at the cumbersome point-scale which requires re-blocking to the SMU scale for use in mine scheduling applications. de Carvalho and Dimitrakopoulos (2019) introduces a DBSIM analogue to High-order simulations; the method extends Minniakhmetov and Dimitrakopoulos (2017b) to direct simulation at the block scale, improving computational efficiency. Although the HOSIM frameworks incorporate significant amounts of additional information from hard data, reducing the conflicts between TI and data statistics and improving reproduction of high-order statistics in simulated realizations, the reliance on TI's remains a limiting factor in their overall utility. Minniakhmetov and Dimitrakopoulos (2017a) present a data-driven approach for categorical variables that does not require training images. They are able to obtain the spatial cumulants directly from the hard data by making use of boundary conditions and B-spline functions to calculate and approximate higher order cumulants.

1.3.3 Modelling uncertainty in commodity prices

It is self-evident that commodity prices and the costs necessary to transform them from in-situ resources to sellable products define the economic viability of mining projects. While mining companies can generally exert some measures of control over project costs, it is rare for a mining company to influence metal prices over a long-term time horizon. This means that mining projects are inherently vulnerable to fluctuations in metal prices and that this uncertainty needs to be accounted for in strategic planning.

Metal prices are generally governed by four main forces: supply and demand, regulation by cartels or commodity agreements, negotiation between producers and consumers, fixed prices by a monopoly or oligopoly (Gocht *et al.*, 2012). Which force or combination of forces governs the price formation of a specific commodity depends on the existing marketplace and its characters. However, it is typically accepted that base metal prices are heavily influenced by supply and demand, while precious metals tend to be influenced by investment factors such as interest rates and inflation (Kernot and West, 1991). It follows that specific commodities require different forecasting models.

Despite the substantial amount of econometric research undertaken and a myriad of price forecasting methods available, reliable long-term forecasts remain elusive. Perhaps, for this reason, commodity price forecasting with econometric models is not a common practice within mining companies (Dooley and Lenihan, 2005). It is much more typical for a mining company to run sensitivity analyses with base, upside, and downside case prices than try to integrate metal price forecasts. While accurate and reliable forecasts may be futile in a deterministic sense, the integration and quantification of uncertainty via sets of stochastic price simulations has the potential to improve strategic mine plans. Albor and Dimitrakopoulos (2009) show that due to the volume-variance relationship between SMUs and grade in yearly extraction schedules, 15-20 stochastic simulations of a mineral deposit sufficiently characterize the geological uncertainty in long-term mine plans. However, because commodity prices fluctuate in the temporal space, and require extrapolation instead of interpolation, it is postulated that significantly more stochastic price simulations are necessary (Briggs *et al.*, 2012). Nonetheless, until very recently the incorporation of stochastic price simulations into mine planning has not been feasible. Recent advances in simultaneous stochastic optimization of mining complexes make research into this area a very interesting proposition as there may be a relationship similar to that outlined by (Albor and Dimitrakopoulos, 2009).

Stochastic commodity price forecasts can be categorized as “reduced-form” stochastic models such as those presented by Schwartz (1997), or less parsimonious, structural models of price dynamics based on econometric theory proposed in Pirrong (2011). Reduced-form models are far more ubiquitous in applications due to their simplicity, whereas the structural models are very difficult to parametrize. Schwartz (1997) presents three models for use with different commodities: a one-factor Geometric Brownian Motion (GBM) model, a model that incorporates stochastic interest rates, and a Mean-Reversion (MR) model that accounts for convenience yield.

Castillo and Dimitrakopoulos (2014) incorporate copper price and geological uncertainty into determining ultimate pit limits. They use an MR model proposed by Deng (2000) that incorporates market shocks by random sampling of the Poisson distribution. Bernard *et al.* (2008) test different models including random walk models with generalized autoregressive conditional heteroscedasticity (GARCH), Poisson-based jump-diffusion models with GARCH effects and MR models that incorporate uncertain equilibrium prices. The models are applied to forecast aluminum prices over several time-horizons and find that different components dominate depending on the time-scale.

While it's clear that the success of a forecasting method depends on the commodity and time-scale of interest. The relevant literature indicates that MR GARCH Poisson diffusion models are well suited to base metals and GBR based models work well with precious metals (Kernot and West, 1991; Roche, 1995; Labys *et al.*, 1998; Dooley and Lenihan, 2005; Bernard *et al.*, 2008; Pirrong, 2011; Gocht *et al.*, 2012).

1.4 Strategic mine planning under uncertainty

1.4.1 Incorporating risk in long-term mine planning

The previous sections establish both the need for risk-based mine planning processes that can consider multiple stochastic orebody simulations to value mining projects by managing and minimizing risk appropriately. The first approach proposed by Dimitrakopoulos *et al.* (2007) uses multiple simulated orebody realizations to generate a conventional LOM plan for each simulated realization using Whittle software. The authors quantify the maximum-upside and minimum downside for each design by carrying out a risk analysis using a set of simulations and select a final design that performs best over a set of key project indicators. This is a straight forward approach that improves risk quantification and can be implemented using commercial software tools. However, optimizing a mine plan for one simulation is not ideal because it does not account for the full range of possibilities. The same authors propose another approach, using multiple simulations to build a probabilistic orebody model and a MIP formulation to incorporate geological uncertainty (Ramazan and Dimitrakopoulos, 2004). The simulations are used to encode each block with the probability of specific certain characteristics, such as the desired grade. The concept of 'geological risk discounting' is introduced and the MIP is formulated to maximize the probability of meeting ore tonnage and grade targets and defers blocks with a lower probability of having the desired property to later periods. The formulation also includes schedule smoothing terms which penalize blocks that belong to a pre-defined neighbourhood and are not extracted in the same period. This helps overcome extraction feasibility issues which are typical limitations of other MIP approaches. The method is applied to a nickel-cobalt laterite deposit and generates a feasible schedule that shows an improvement in risk management over a traditional method by scheduling blocks with higher probabilities of having desirable properties earlier in the LOM. Dimitrakopoulos and Grieco (2009) apply a similar probabilistic method to incorporate grade uncertainty and quantify risk, optimize size, location and number of stopes at an underground copper mine in Kidd Creek, Ontario. The limitation of this and other probabilistic methods is that the use of individual block or stope probabilities instead of scenario-based approaches that can incorporate the joint-local uncertainty of combinations of blocks.

Godoy and Dimitrakopoulos (2004) propose the first method that incorporates the joint-local uncertainty using a sequential approach and the simulated annealing algorithm (Kirkpatrick *et al.*, 1983; Geman and Geman, 1984). The multi-step approach uses a combination of techniques to minimize deviation from ore and waste production targets accounting for geological uncertainty. The steps involved are: (1) calculate a stable solution domain of ore and waste extraction over all simulations, (2) calculate optimal mining rates using a mathematical programming formulation, (3) generate an extraction sequence for each simulation using a conventional scheduler and the rates from Step 2, (4) combine the extraction sequences from Step 3 to a single production schedule using the simulated annealing algorithm. Simulated annealing is a combinatorial optimization metaheuristic that iteratively perturbs an initial solution until a stopping criterion is met. In this case, the perturbations involve swapping the period of extraction for a set of mining blocks and evaluating the impact on the deviations from production targets for each scenario. Perturbations are accepted according to the decision rule described by (Metropolis *et al.*, 1953): if they improve the objective function, otherwise, they are accepted or rejected based on a probability function and the annealing temperature. The intermittent acceptance of unfavourable perturbations helps the solution escape local optima while a cooling factor decreases the annealing temperature, allowing the solution to converge. The method is applied to a gold mine in Western Australia, substantially improving the NPV (28%) and reducing the likelihood of deviating from targets (9%) compared to a production schedule generated by a conventional optimizer. Leite and Dimitrakopoulos (2007) apply the same method to a low-grade disseminated copper deposit, reporting similar results concerning improvements in NPV and production targets. Albor and Dimitrakopoulos (2009) study several aspects of the method through a case study on the same copper deposit. Namely the method's sensitivity to the initial extraction sequence, number of extraction sequences and number of simulated orebody realizations. The authors find that after ten extraction sequences the method is not particularly sensitive to the initial sequence or increasing the number of input sequences. Similarly, the method generates a stable solution using approximately 15 simulated realizations of the orebody. This result is attributed to what is known as the volume-variance relationship or support-scale effects. The authors also determine that the deterministic nature of the conventional algorithm for defining pit limits (Lerchs and Grossmann, 1965) cannot provide optimal pit limits in the presence of uncertainty. They propose an alternative approach using the simulated annealing-based method which delineates a 17% larger ultimate pit and improves the project NPV. The method provides an improvement in risk-based approaches but has several limitations: it does not defer risk to later periods, it does not consider material blending constraints, and it only optimizes one component of a mining complex, the extraction sequence.

1.4.2 Stochastic optimization of long-term mine production planning

Stochastic integer programming (Birge and Louveaux, 2011) is a branch of mathematical optimization where at least one variable is uncertain, providing a set of suitable tools for strategic mine planning to incorporate various forms of uncertainty directly. Two-stage stochastic optimization models with recourse are the most prevalent in the technical literature (Ramazan and Dimitrakopoulos, 2007; Dimitrakopoulos, 2011; Benndorf and Dimitrakopoulos, 2013; Ramazan and Dimitrakopoulos, 2013; Rimélé *et al.*, 2018). Where first stage decisions are taken before the revelation of uncertainty and second stage decisions (recourse decisions), which are a function of the outcomes of uncertainty, are taken after. These can also be referred to as scenario-independent (first stage) and scenario-dependent (recourse) decisions. Within the mine planning context, these models allow for a structure that maximizes (or minimizes) an objective while managing technical risk by minimizing deviations from related targets. Risk management is emphasized with the use of geological risk discounting applied to recourse variables which helps to defer risk to later periods as explained in (Dimitrakopoulos and Ramazan, 2004).

The first application of stochastic integer programming (SIP) to the mining context was proposed by Ramazan and Dimitrakopoulos (2007). The formulation aims to maximize the project NPV and minimize deviation from ore, waste, and metal production targets, where the first stage variables are binary extraction variables and the second stage variables are used to measure deviations in each scenario. The method is applied on a two-dimensional test data set with a 3-year LOM and solved with commercial linear optimization software. Ramazan and Dimitrakopoulos (2013) extend the model to include stockpiling and apply a geological risk discount rate to improve technical risk management. The authors test the method on gold deposit, however, to overcome computational limitations they split problem over two-time horizons, first optimizing years 1-4, and then considering years 4-6. Their results indicate a 10% improvement in NPV and substantial reductions in ore target deviations compared with a conventional method. Benndorf and Dimitrakopoulos (2013) propose a similar two-stage SIP model but expand the approach to incorporate blending constraints that allow for ore quality targets in a multi-element deposit and also implement schedule smoothing constraints similar to (Ramazan and Dimitrakopoulos, 2004). They apply the method at the Yandi Central 1 iron ore deposit in Western Australia using 20 simulated orebody models that account for joint-local uncertainty in iron, silica, alumina, phosphorus, and loss on ignition. The group of elements are influential iron ore properties and have a direct impact on performance and beneficiation. Thus a model which can effectively manage and quantify risk relating to their blending targets adds significant project value. The case study experiments with the size of deviation penalties and their effects on schedule dispersion and blending targets, finding that medium penalties ($10 per unit deviation, as opposed to $100 – high, and $1 – low) produced the best results. Rimélé *et al.* (2018) proposes a similar two-stage SIP and expands the model to integrate in-pit waste dumping and applies it at Canadian multi-element iron ore mine. The successful incorporation of in-pit waste disposal improves the environmental performance of the mine, reducing the ex-pit footprint and associated rehabilitation costs while managing risks in blending and ore production targets. The model is too complex to solve simply with commercial optimization software. The authors overcome this by implementing a sliding time window heuristic (Dimitrakopoulos and Ramazan, 2008; Benndorf and Dimitrakopoulos, 2013), iteratively relaxing the binary variables in all but a few consecutive periods, solving and then fixing the solution before repeating over the next 'time window.'

Boland *et al.* (2008) take a different approach to the above, proposing multi-stage SIP without recourse that considers geological uncertainty. The formulation uses posterior-stage variables to consider processing block aggregates and allows both mining and processing decisions to change as uncertainty is revealed. Processing decisions can change in real-time, while mining decisions are subject to a one-year lag in reacting to new information. The most notable difference in the method from other stochastic approaches is that it provides a set of plans for each scenario according to information revealed through advanced extraction. This is achieved by using aggregates with significantly different grade bins and many non-anticaptivity constraints. The former allows for distinction between different scenarios, and the latter ensures that decisions are identical in all scenarios until a point in time that they can be distinguished. The authors test the method against a deterministic equivalent base-case and find that the proposed approach increases the NPV by 3%. The method is an interesting theoretical approach but has several practical limitations. The method aggregates large groups of mining blocks to deal with computational challenges. However, it assumes uniform extraction of aggregates which can lead to misleading processing outputs and it allows partial block extraction which can lead to slope constraint violations. Further, the branching approach that provides a production schedule for each distinguishable scenario assumes that one of the input scenarios will represent reality; this is extremely unlikely. In other words, instead of providing one risk resilient output (a LOM schedule) that performs well over a set of scenarios that characterize uncertainty, it provides multiple outputs that each overfit one of the inputs.

Approaches built on two-stage SIP with recourse models have significantly advanced strategic mine planning practices. Dimitrakopoulos (2011) reviews the methods, advancements and provides additional examples to those listed above. Specifically, the major benefits include the ability to simultaneously maximize NPV while minimizing deviations from production targets, the explicit integration of joint-local uncertainty via simulated orebody realizations, risk management and deferment using geological risk discounting. Nonetheless, there are still limitations to overcome: life-size SIP models with multiple orebody realizations become massive combinatorial optimization problems which are difficult to solve with commercial optimizers, linearizing non-linear functions such as stockpiles and recovery functions to keep models linear and convex is a piecemeal solution that can mislead results, economic block values continue to drive the optimization process limiting the accurate characterization of downstream processes such as blending.

Metaheuristic solution approaches help overcome issues relating to problem size and the need for linear convexity, Section 1.4.3 reviews advancements in simultaneous optimization of mining complexes which addresses limitations relating to downstream processes. Metaheuristic algorithms provide effective strategies to search for solutions where deterministic solution approaches are ineffective, either because the problem is too difficult (NP-hard) or the solution space is too large. They typically combine randomization and local search techniques, characterized by diversification and intensification components. A balanced approach ensures optimality is achievable by a wide exploration of the solution space and eventual convergence on the best solution (Yang, 2010). Although they do not provide certificates of optimality, they often find high-quality solutions in a reasonable amount of time. Godoy and Dimitrakopoulos (2004) introduce the simulated annealing algorithm to stochastic mine planning, the trajectory-based algorithm adapts very well to mine production scheduling problems and is used in several other studies (Leite and Dimitrakopoulos, 2007; Albor and Dimitrakopoulos, 2009; Goodfellow and Dimitrakopoulos, 2013; Montiel and Dimitrakopoulos, 2013). Lamghari and Dimitrakopoulos (2012) propose a diversified Tabu search approach to solve a two-stage SIP formulation for an open pit mine scheduling problem with grade uncertainty. The algorithm generates solutions within 4% of optimality in a fraction of the time taken by commercial optimization software. The authors improve their results over a variety of test cases by applying a Variable Neighborhood Descent metaheuristic to mine production scheduling (Lamghari *et al.*, 2014).

1.4.3 Simultaneous stochastic optimization of mining complexes

The two-stage stochastic integer program (SIP) with fixed recourse (Birge and Louveaux, 2011) addresses limitations associated with conventional mine planning by providing a framework that explicitly accounts of geological uncertainty and manages related technical risk. The adoption of metaheuristic solution approaches meaningfully improves its utility to large-scale applications. Nonetheless, it has been established that the strategic and long-term planning of both operating and new mining projects requires a global approach. One that simultaneously optimizes the interconnected components of mining complexes to capitalize on synergies and maximize project value. The evolution of stochastic mine planning yields simultaneous stochastic optimization of mining complexes (Goodfellow and Dimitrakopoulos, 2015; Montiel and Dimitrakopoulos, 2015; Goodfellow and Dimitrakopoulos, 2016; Montiel *et al.*, 2016; Goodfellow and Dimitrakopoulos, 2017; Montiel and Dimitrakopoulos, 2017; Zhang and Dimitrakopoulos, 2017; Montiel and Dimitrakopoulos, 2018; Del Castillo and Dimitakopoulos, 2019). The state-of-the-art methods address these requirements and overcome many limitations described in the previous sections. By abandoning the practice of characterizing optimization processes via the economic value of blocks, the latest methods connect components and effectively model non-linear interactions across mineral value chains. They allow the value of products generated by the mining complex to dictate the optimization process, considering simulated attributes at the block scale which flow through intermediate destinations and transform into marketable products. This paradigm shift is essential because the concept of a mining block along with its economic value is lost after extraction. There are numerous post-extraction material interactions (blending, stockpiling, processing, transportation) which affect the value of final products. Simultaneous stochastic optimization frameworks explicitly account for various sources of uncertainty and optimize upstream (extraction sequences, destination policies) and downstream (processing, tailings, transportation options) in unified formulations.

Montiel and Dimitrakopoulos (2013) present an initial approach; they model but do not simultaneously optimize a whole mining complex. The method, applied to the Escondida Norte copper mining complex in Chile, considers simulated grades and material types and optimizes part of the mining complex, namely, the extraction sequence while modelling the flow of material through multiple processes. A simulated annealing-based solution approach generates a high-quality solution that compares favourably with a deterministic benchmark. The principal objective is to minimize deviations from specific production and ore quality targets; the solution reduces deviations to 5% compared with 20% in the benchmark. The formulation does not explicitly maximize NPV in the objective function, and it still results in a 4% improvement compared to the benchmark. However, there are misclassification issues with the block-based destination policy because certain processing facilities can only accept specific materials, and as the material type of a block can vary from one simulation to another. Another limitation is that the case study does not include stockpiling options or other downstream components

Montiel and Dimitrakopoulos (2015, 2017, 2018) and Montiel *et al.* (2016) overcome the limitations of the previous model. They propose a framework that optimizes the extraction sequence, destination policy, processing stream decisions as well as operating modes and transportation options (Montiel and Dimitrakopoulos, 2015) and supply from an underground operation (Montiel *et al.*, 2016). The new formulation explicitly maximizes project NPV and minimizes deviations from production targets in the objective function. The simulated annealing-based solution approach iteratively perturbs three decision neighbourhoods: blocks, operating alternatives and transportation systems. The block-based perturbations define the extraction and destination decisions for each block. The destination policy evaluates the overall profitability for each block at each destination considering all scenarios, and then the optimizer chooses the destination as a knapsack problem (Dantzig, 2003). As the optimizer perturbs the solution, it sends the most profitable blocks to their preferred destination, it considers the non-linear interactions with other blocks at each destination and attempts to choose a solution that yields the most overall benefit in the objective function. Operating-based perturbations randomly modify operating modes at processing facilities for a given period; these operating alternatives can impact processing costs, recoveries and capacities due to changes in the grinding circuit. Transportation system perturbations randomly modify the proportions of output material transported from each processing facility a period through each system (e.g. truck or pipeline) and aim to minimize transportation costs and deviation penalties. The algorithm performs a user-defined number of perturbations at leach level, accepting or rejecting changes based on the

(Metropolis *et al.*, 1953) decision rule, lowering the temperature and then cycling through the decision neighbourhoods. The method improves adherence to blending targets, increases the NPV and accounts for existing infrastructure, such as pit-access ramps, at one of the world's most complicated mining complexes, Newmont's Twin Creeks in Nevada (Montiel and Dimitrakopoulos, 2018). The formulation uses a scenario independent, block-based variable to define both extraction and destination decisions which helps reduce the number of integer variables in the problem. However, it can result in material misclassification and does not account for the operational flexibility of mine operators to react to short-term grade control information.

Goodfellow and Dimitrakopoulos (2015, 2016, 2017) propose a generalized, unified simultaneous stochastic optimization framework that provides the basis for the research discussed herein. The formulation is built around a novel two-stage SIP where first-stage decisions optimize extraction sequences and destination policies, and second-stage recourse decisions optimize processing streams. The solution approach comprises a combination of high-level metaheuristics, allowing the framework to tackle large, non-linear problems and include non-additive geo-metallurgical interactions. The framework uses three decision variables to optimize a mining complex: binary extraction sequence variables to define whether a block is extracted in a certain period, binary destination policy variables to define whether a sub-grouping of material is sent to a destination in a certain period, continuous processing stream variables to define the proportion of material sent from one destination to another. Note that the first-stage variables (extraction sequence and destination policy) are scenario-independent while the processing stream variables are adaptive to uncertainty. The model uses attributes to characterize relevant information such as metal quantities or costs, which can quantify uncertainty through joint scenarios (simulation or sampling for various sources of uncertainty) for each attribute. For modelling purposes, attributes are classified as either primary – essential additive variables of interest which can flow through the mining complex such as mass, or quantities of elements; or hereditary – variables which are not necessarily passed from one location to another but are relevant to the optimization model and may be expressed as functions of primary attributes, such as grades, processing costs, revenue from sales. These notions allow for flexible modelling complex (non-) linear interactions across a variety of value chain configurations. The authors propose a new destination policy using scenario-independent variables that avoids misclassification under uncertainty. They extend the robust cut-off grade optimization proposed by Menabde *et al.* (2007) to handle multivariate cases. The destination policy defines where material clusters (sub-groupings based on multivariate distributions of specified attributes) are sent in each period. This approach does not require a destination decision for individual blocks, reducing the number of integer variables. Blocks are classified into clusters using the k-means++ algorithm (Arthur and Vassilvitskii, 2007) which requires a pre-defined number of cluster centroids. The algorithm defines block-membership to a cluster based on the Euclidean distance between a block's simulated attributes and the closest centroid, meaning a block can belong to different clusters in different scenarios. A major advantage of this destination policy is the focus on multivariate attribute distributions, allowing for consideration of deleterious elements, blending and stockpiling and their impacts on the mining complex. However, a drawback is that the number of clusters is user-defined and must be determined empirically. The metaheuristic solution approach combines a modified adaptive multi-neighbourhood simulated annealing-based algorithm with a Particle Swarm Optimization (PSO) (Kennedy, 1995) that is better suited for dealing with continuous variables such as those in the processing stream. The authors apply the method to several case studies (Goodfellow and Dimitrakopoulos, 2016, 2017), a multi-pit, multi-process copper-gold mining complex and a multi-pit, multi-process nickel-laterite operation. Case studies show substantial improvement (>20%) in NPV while satisfying production targets over industry-standard benchmarks. Goodfellow and Dimitrakopoulos (2015) include capital expenditure (CapEx) options into the modelling framework, showcasing the method's flexibility in modelling concepts such as truck and shovel hours to integrate load and haul fleet purchases into the simultaneous optimization process.

Farmer (2017) exploits the generalized nature of the framework to incorporate mining and processing capacity expansion options through the capital expenditure term introduced by (Goodfellow and Dimitrakopoulos, 2015). The adaptation also includes complex revenue calculations such as royalties, metal streams and offtake agreements for a copper-gold mining complex. The author integrates market uncertainty via commodity price simulations in a multi-step process, first, optimizing all components of the mining complex under geological uncertainty, then freezing the first stage decisions and re-optimizing the recourse variables under joint market and geological uncertainty scenarios. Kumar and Dimitrakopoulos (2017) utilize the framework to incorporate non-additive, geo-metallurgical variables at the Escondida mining complex. These attributes are difficult to model in traditional linear formulations, but they have a significant influence on throughput, recovery and processing costs, and their incorporation is important to realistic modelling of mining complexes.

Zhang and Dimitrakopoulos (2017) propose a decomposition method that optimizes production schedules for multiple mines and downstream material flow while accounting for both geological and market uncertainty. The dynamic-material-value-based method optimizes upstream under geological uncertainty and downstream components under market uncertainty. The upstream and downstream optimization is separate, but each component iteratively interacts with the other, providing information feedback until a solution converges. The method utilizes shadow prices in the processing stream to show that ignoring market uncertainty leads to over-investment in strategic assets and underutilized capacities, leading to optimistic long-term profitability forecasts. Zhang and Dimitrakopoulos (2018) propose a two-stage non-linear SIP to integrate forward sales contracts into the optimization framework under market and supply uncertainty. The method highlights the importance of considering dynamic recovery rates when a hedging contract is included.

Del Castillo and Dimitakopoulos (2019) extend the mathematical formulation of a simultaneous stochastic optimization framework (Goodfellow and Dimitrakopoulos, 2016, 2017) to include a dynamic optimization of strategic planning options such as CapEx alternatives and different operating modes. The multi-stage model evaluates the profitability of feasible strategic mine planning options by generating parallel solutions that consider the flexibility of an operation to adapt to new information over the life on a mining complex. A branching mechanism uses a probabilistic decision tree to consider if new investment decisions are profitable over a threshold number of geological scenarios. Thus it maintains design flexibility, resilience to uncertainty and avoids overfitting issues. The formulation employs non-anticaptivity constraints to ensure that decisions are equal in all scenarios until branching is allowed. An application shows that the flexible method improves the NPV of a large mining complex by 10% over the method proposed by (Goodfellow and Dimitrakopoulos, 2016, 2017).

1.5 Goal and Objectives

The goal of the research presented in this thesis is to advance the use of simultaneous stochastic optimization frameworks in strategic mine planning of mining complexes by incorporating more realistic modelling aspects and various sources of uncertainty present in real-world mining complexes. The following objectives are set to meet this goal:

- Review the technical literature related to strategic mine planning, deterministic and stochastic approaches for the optimization of mines and mining complexes. Review the methods developed to simulate mineral deposits and characterize the technical risk associated with geological uncertainty, and review the methods used for commodity price forecasting.
- Incorporate joint market and supply uncertainty directly into the optimization of extraction, destination and processing stream decisions and analyze its effects on production schedules and forecasts. Further, assess the efficacy of the cut-off grade optimization proposed by Goodfellow and Dimitrakopoulos (2016) through application at a case study with challenging blending constraints.
- Incorporate tailings management under environmental constraints as part of the simultaneous stochastic optimization framework with CapEx investment options to evaluate the potential for life-of-asset growth.
- Summarize the main contributions and conclusions of the conducted research and provide suggestions for future study.

1.6 Thesis Outline

The remainder of this dissertation is organized as follows:

- Chapter 2 – Presents an application at a large open-pit mining complex with strict blending constraints that integrates joint uncertainty scenarios and analyzes destination policy and cut-off grade optimization.

- Chapter 3 – Presents an application that incorporates a CapEx investment option that helps manage environmental constraints which bottleneck the multi-element mining complex.
- Chapter 4 – Summarizes the contributions of each paper and overall conclusions, followed by suggestions for future work.

Chapter 2 Simultaneous Stochastic Optimization of an Open Pit Gold Mining Complex with Supply and Market Uncertainty

2.1 Introduction

A mining complex is a resource to market mineral value chain that transforms in-situ materials into valuable products such as concentrate, pellets, bars of precious metals, and others (Montiel and Dimitrakopoulos, 2015). The main components of a mineral value chain are generally: mines, stockpiles, waste dumps, mineral processing facilities, and logistics hubs (ports and railways for example) (Pimentel *et al.*, 2010). Traditionally, components of the mineral value chain are optimized independently, leading to suboptimal solutions which deteriorate substantially as the complexity of the chain increases (Goodfellow and Dimitrakopoulos, 2016, 2017). Simultaneous optimization of a mining complex is an integrated approach where all the components of the chain are optimized simultaneously, leveraging existing synergies towards maximizing the value of an operation (Pimentel *et al.*, 2010; Whittle, 2010).

Development of frameworks that incorporate multiple components of the mineral value chain into the optimization process began in the mid-1990s. Newmont Mining Corporation recognized the opportunity to leverage synergies present in their expansive Nevada operations. This led to the development of an in-house optimizer based on a mixed integer programming (MIP) formulation to maximize discounted cash flows by simultaneously optimizing material movement from a set of open pit and underground mines to multiple destinations (Hoerger et al., 1999). BHP Billiton followed by developing the Blasor mine planning software, which simultaneously optimizes pushback sequences from multiple pits (Stone et al., 2007). The authors present an application at BHP's Yandi mining complex, an eleven pit, blended iron ore joint venture where Blasor is used to maximize the net present value (NPV) while ensuring market tonnage and material quality targets were respected. Blasor can provide tractable solutions by aggregating spatially connected blocks with similar attributes then sequentially generating near-optimal extraction sequences, ultimate pit, phase designs and finally panel (intersections of benches and phases) extraction sequences (Stone *et al.*, 2007). Zuckerberg *et al.* (2007) extend the framework to Blasor-InPitDumping, which incorporates optimized waste handling by utilizing sterile mined-out areas. Whittle (2010) describes a global optimizer, designed to incorporate mining, processing and blending components into the optimization process.

While the methods described above improve past approaches, they all have one or more of the following major limitations: aggregation, which misrepresents mining selectivity, stepwise local optimization of value chain components, and failure to account for the effects of uncertainty present in critical parameters. The main sources of risk in a mining project arise from technical, financial and environmental uncertainty (Dimitrakopoulos *et al.*, 2002; Rendu, 2017). Furthermore, it has been observed that uncertainty in grades and material types is a significant source of technical risk, referred to as supply uncertainty (also geological). The impact of supply uncertainty on a mining project's ability to meet production forecasts is now a well-studied issue (Ravenscroft, 1992; Dowd, 1994). Conventional mine planning methods are deterministic, meaning they use a single estimated orebody model as an input to the optimization process; Hustrulid *et al.* (2013) provide a comprehensive review of conventional open pit mine planning practices. Estimated models are incorrectly assumed to be accurate representations of grades and materials in the ground. Instead, they provide overly smoothed representations of attributes of interest (Goovaerts, 1997). Uncertainty in mineral deposits is incorporated into the optimization process by stochastic optimization frameworks that use sets of equi-probable stochastic simulations (Goovaerts, 1997; Minniakhmetov and Dimitrakopoulos, 2017b; Minniakhmetov *et al.*, 2018) as inputs to stochastic integer programming (SIP) formulations (Ramazan and Dimitrakopoulos, 2007; Birge and Louveaux, 2011; Ramazan and Dimitrakopoulos, 2013). Dimitrakopoulos (2011) provides a review of applications of stochastic optimization in mine planning, noting significant improvements in NPV and metal recovered while managing and reducing technical risk.

Despite the significant influence that market uncertainty, specifically, fluctuations in commodity prices, has on project risk, conventional planning practices assume constant and certain prices. Attempts to overcome this simplification by sensitivity testing different price scenarios a-posteriori are limited in that decisions such as destination policies (cut-off grades), life-of-mine

(LOM), capacities and others are fixed. Past efforts at incorporating the joint supply and market uncertainty into the planning process allow some of these decisions to be made while accounting for the uncertainty. Examples of such efforts include assessing the impacts on phase and ultimate pit designs as well as determining cut-off grade strategies, mining rates and capacities (Asad and Dimitrakopoulos, 2013a; Asad and Dimitrakopoulos, 2013b; Castillo and Dimitrakopoulos, 2014; Kizilkale and Dimitrakopoulos, 2014; Farmer, 2017). Cut-off grade optimization is one of the most important elements in any mining operation, defining the supply of ore and waste material to various destinations throughout the mineral value chain based on economic and technical parameters. It characterizes an operation's destination policy decisions (where to send what material and when). Asad *et al.* (2016) provide a thorough review of cut-off grade optimization (deterministic and stochastic) methods developed for open-pit mining operations. Cut-off grade optimization is conventionally based on Lane's theory on the economic definition of ore (Lane, 1988; Rendu, 2014) which predefines cut-off grades to be used for life-of-mine production scheduling optimization. This is a limited approach based on the consideration of grade-tonnage distributions and capacities while attempting to maximize net present value. However, cut-off grades determined before production scheduling do not account for fluctuations in material availability and quality from one mining period to another and are required because the technologies used to date for production scheduling optimization are unable to generate the optimal cut-off grades as an output of the truly optimal life-of-mine production schedule. This has been the case for several decades and is a limitation addressed by the new digital technologies and simultaneous optimization of cut-off grade policies in conjunction with extraction sequencing and processing stream decisions, proposed in this work and along the resource-to-market mineral value chain.

Recent developments have extended the two-stage SIP models reviewed by Dimitrakopoulos (2011) to the simultaneous stochastic optimization of mining complexes (Montiel and Dimitrakopoulos, 2015; Goodfellow and Dimitrakopoulos, 2016; Montiel *et al.*, 2016; Goodfellow and Dimitrakopoulos, 2017; Montiel and Dimitrakopoulos, 2017, 2018). These state-of-the-art frameworks integrate the optimization of extraction sequences, destination policies, processing streams, operating modes and transportation alternatives for multiple sources and processors simultaneously in a single mathematical formulation. The contributions above move away from linear optimization models to incorporate more realistic stockpiling decisions and non-linear interactions in processing streams. These are enabled by a significant departure from conventional mine optimization practices, such as the economic value of blocks as a driving force in optimization. The practice of valuing material by calculating the economic value of mining blocks pre-extraction, assumes that the concept of independent blocks remains intact as the material is transformed throughout the value chain and is not able to consider changes in the value of material due to blending or other downstream non-linear interactions (Goodfellow and Dimitrakopoulos, 2016). Thus, simultaneous stochastic optimization shifts the focus from optimization with the economic value of blocks to the economic value of products sold at the end of the mineral value chain, removing the need for a-priori cut-off grade optimization. A major benefit of Goodfellow and Dimitrakopoulos (2016) is that the model is highly generalized, allowing for extensions to a large variety of applications. However, it does not incorporate specific transportation alternatives or operating modes as Montiel and Dimitrakopoulos (2015) or supply from an underground mine as Montiel et al. (2016). Farmer (2017) extends the generalized model to include capital expenditure (CapEx) and mining capacity decisions in an application with complex revenue streams such as offtake and streaming agreements. It also aims to integrate market uncertainty but does so in a two-step process that fixes the extraction sequence and optimizes the downstream variables. Del Castillo and Dimitakopoulos (2019) use a dynamic approach to integrate CapEx decisions on the mid-term time horizon. Kumar and Dimitrakopoulos (2019) present another application of Goodfellow and Dimitrakopoulos (2016) with complex geo-metallurgical decisions incorporated into the destination policy at a large copper-gold mining complex.

This work presents an application of the simultaneous stochastic optimization framework from Goodfellow and Dimitrakopoulos (2016) at an intricate Nevada type gold mining complex with strict geochemical blending constraints. Notably, this work explores the effects of joint market and supply uncertainty scenarios by using commodity price simulations as inputs to the optimization model, allowing the simultaneous stochastic optimizer to integrate market uncertainty into all three decision variables. Additionally, this study examines the effectiveness of the simultaneous stochastic optimization framework's cut-off grade decisions by considering the value of downstream products, non-linear blending interactions and the extraction sequence. This replaces the need for *a-priori* cut-off grade optimization using conventional methods and addresses limitations related to determining cut-off grades before production scheduling. Due to the blending requirements in the case study, the proposed approach has the additional benefit of reducing the level of operational complexity in the mining complex by significantly cutting down on the number of material types and stockpiles the operation needs to manage.

The next sections provide a brief description of the optimization model, constraints, and solution approach. Then, a detailed description of the case study, presentation of the results and analysis. Conclusions and future work follow.

2.2 Method

This section describes the adaptation of Goodfellow and Dimitrakopoulos (2016)'s simultaneous stochastic optimization model to the specific application at a large gold mining complex. The general model is configured to accommodate strict geochemical blending constraints related to autoclaving, mineability constraints and market uncertainty.

2.2.1 Definitions and notation

The material in mining complex, $\mathcal{C}$, is extracted from a set of sources (mines), $m \in \mathbb{M}$. Mines are discretized into selective mining units (SMU) known as mining blocks, $\mathscr{b} \in \mathbb{B}_m$, where $\mathbb{B}_m$ denotes the set of mining blocks for a specific mine. The mining cost, $MC_{\mathscr{b},t}$, represents the cost of mining any block, $\mathscr{b} \in \mathbb{B}_m$, in period $t \in \mathbb{T}$. Each block has a set of simulated properties, $a \in \mathbb{A}$, mineralogical (grade) and geochemical (deleterious elements). $\mathbb{S}$, denotes a set of scenarios that quantify the joint uncertainty in grade, geochemical properties, and commodity prices (when applicable). Material is only available for extraction if all predecessors, $\mathbb{O}(\mathscr{b})$, of a block $\mathscr{b}$ are extracted. After extraction, material can be sent from locations $i \in \mathcal{C}$ to several destinations such as stockpiles ($\mathcal{S}$), processors ($\mathcal{P}$), or waste dumps ($\mathcal{D}$). The cost of transporting material property a from a location i in period t is denoted $TC_{i,a,t}$. The amount of a material property a at location i in period t and scenario s is $v_{a,i,t,s}$. Material properties that can be sent from one destination to another and accumulated, such as ounces, belong to the set $p \in \mathbb{P}$, while properties that are calculated such as ounces recovered, or element concentrations belong to the set $h \in \mathbb{H}$. Production targets associated with capacities belong to the set $\mathbb{P}_c$ and those associated with geochemistry belong to $\mathbb{H}_g$. $PC_{i,a,t}$ represents the cost of processing material property a at location i in period t (including refinery charges). $P_{h,t,s}$ Represents the unit selling price of material property h in period t and scenario s. Deviations from a production target associated with property a at location i in period t and scenario s are measured by $d_{i,a,t,s}^{\pm}$, while $c_{i,a,t}^{\pm}$ represents the unit surplus and shortage costs associated with their respective deviations. Mineability targets, enabled by a set of scheduling constraints, ensure the production schedule is feasible in practice. Blocks that lie within a horizontal 'window' around block $\mathcal{b}$ belong to the set $\mathbb{W}_{\mathcal{b}}$, blocks that lie vertically above a block $\mathcal{b}$ are denoted $v \in \mathbb{V}_{\mathcal{b}}$. $d_{\mathcal{b},t}^{smooth}$ Represents the number of blocks in the window $\mathbb{W}_{\mathcal{b}}$ that are mined in a different period from block $\mathcal{b}$ while $d_{\mathcal{b},t}^{sink}$ represents the number of blocks in the set $\mathbb{V}_{\mathcal{b}}$ that are mined in the same period as block $\mathcal{b}$. The penalty costs used to enforce the mineability targets on a per block basis in each period are denoted by $c_{\mathcal{b},t}^{smooth}$ and $c_{\mathcal{b},t}^{sink}$.

2.2.2 Decision variables

There are three types of decision variables that the simultaneous stochastic optimizer can modify to impact the mining complex. Extraction sequence decisions ($x_{\mathcal{b},t} \in \{0,1\}$) define whether a block $\mathcal{b} \in \mathbb{B}_m$, is extracted from mine m in period t. Destination policy decisions ($z_{g,j,t} \in \{0,1\}$) define whether material of grade g is sent to destination j in period t. Processing stream decisions ($y_{i,j,t,s} \in [0,1]$) define what proportion of material is sent from location $i \in S \cup \mathcal{P}$ to destination j in period t and scenario s. The destination policy decisions are derived from the robust cut-off grade policies from (Menabde *et al.*, 2018), where a grade distribution is discretized into bins and the optimizer determines the minimum grade bin from which all bins above are sent to a processor. This policy preserves short-term operational flexibility because a block may fall into a different grade bin from one simulation to another, however, the destination decisions governing a group of bins remain scenario independent and delivers outputs that include optimized cut-off grades.

2.2.3 Objective function

The objective function in (Eq. 1) maximizes the value of the products sold from the mining complex and manages risk by minimizing deviations from targets in the value chain.

$$
\begin{aligned}
\max \frac{1}{\|\mathbb{S}\|} \Bigg\{ \sum_{s\in\mathbb{S}} \underbrace{\sum_{t\in\mathbb{T}} \Bigg\{ \sum_{i\in\mathcal{P}} \sum_{h\in\mathbb{H}} P_{h,t,s} \cdot v_{h,i,t,s}}_{\text{Part I}} \underbrace{- \sum_{i\in\mathcal{P}} \sum_{p\in\mathbb{P}} \left(PC_{i,p,t} + TC_{i,p,t}\right) \cdot v_{p,i,t,s}}_{\text{Part II}} \\
\underbrace{- \sum_{i\in\mathcal{P}} \sum_{p\in\mathbb{P}_c} \left(c^{+}_{i,p,t} \cdot d^{+}_{i,p,t,s} + c^{-}_{i,p,t} \cdot d^{-}_{i,p,t,s}\right)}_{\text{Part III}} \underbrace{- \sum_{i\in\mathcal{P}} \sum_{h\in\mathbb{H}_g} \left(c^{+}_{i,h,t} \cdot d^{+}_{i,h,t,s} + c^{-}_{i,h,t} \cdot d^{-}_{i,h,t,s}\right)}_{\text{Part IV}} \\
\underbrace{- \sum_{m\in\mathbb{M}} \sum_{p\in\mathbb{P}_c} \left(c^{+}_{i,p,t} \cdot d^{+}_{i,p,t,s} + c^{-}_{i,p,t} \cdot d^{-}_{i,p,t,s}\right) \Bigg\} \Bigg\}} _{\text{Part V}} \underbrace{- \sum_{t\in\mathbb{T}} \sum_{m\in\mathbb{M}} \sum_{\mathscr{b}\in\mathbb{B}_m} \left(MC_{\mathscr{b},t} \cdot x_{\mathscr{b},t} + c^{smooth}_{\mathscr{b},t} \cdot d^{smooth}_{\mathscr{b},t}\right)}_{\text{Part VI}} \\
\underbrace{- \sum_{t\in\mathbb{T}} \sum_{m\in\mathbb{M}} \sum_{\mathscr{b}\in\mathbb{B}_m} \sum_{v\in\mathbb{V}_{\mathscr{b}}} c^{sink}_{\mathscr{b},t} \cdot d^{sink}_{\mathscr{b},t,v}}_{\text{Part VII}}
\end{aligned}
\qquad (1)
$$

Part I from Eq. 1 represents the discounted cash flow derived from products sold. Part II represents the processing costs at the various processors and the transportation costs from each location to the destination. Part's III, IV, and V represent the cost of deviating from processing capacity, geochemical, and mining capacity targets, respectively. Part VI represents the mining cost and the cost of deviating from the schedule smoothness constraints. Part VII represents the cost of deviating from schedule sink rate constraints. All penalty costs for deviating from targets are time varied using a geological discount rate, meaning $c^{+}_{i,a,t} = \frac{c^{+}_{i,a,t}}{(1+r)^t}$, where r is the geological risk discount rate, similar to an economic discount rate used in net present value calculations (Dimitrakopoulos and Ramazan, 2004). Penalties are set by a user-based empirical approach that generally relies on the order of magnitude for unit cost violations. For a more discussions on the determination and impacts of penalty costs the reader is directed to (Dimitrakopoulos and Ramazan, 2008; Benndorf and Dimitrakopoulos, 2013; Ramazan and Dimitrakopoulos, 2013).

2.2.4 Constraints

The transformation of sulphide ore material into gold products in a mining complex is a complicated process that can require pressure oxidation as a pre-treatment to conventional gold recovery circuits. This treatment requires the addition of an autoclave to the process flowsheet. An autoclave is a horizontal cylindrical pressure vessel with multiple compartments that require specific physical and metallurgical controls to ensure effective operation (Cole and Rust, 2002). Blending ore material to maintain a feed that respects the autoclaves optimal operating targets is critical to its performance. This requires a set of constraints to measure deviations from the geochemical targets of the autoclave feed which are then penalized in Part IV of the objective function in Eq. 1. The concentrations of sulphide sulphur and carbonate in the feed are carefully monitored for the autoclaving treatment as well as the total amount of acid added. Acidic slurry is often added to help achieve the necessary pH requirements by reducing the carbonate content of the feed. The recovery of gold from the sulphide ore material is dependent on the organic carbon concentrations. Consequently, the organic carbon content is also carefully monitored, and a target concentration is included in the objective function. Eq. 2 and 3 calculate the deviations from upper and lower targets on feed geochemistry which are penalized in the objective function.

$$v_{h,i,t,s} - d^{+}_{h,i,t,s} \leq U_{h,i,t} \ \forall \ h \in \mathbb{H}, i \in \mathbb{P}, t \in \mathbb{T}, s \in \mathbb{S} \tag{2}$$

$$v_{h,i,t,s} + d^{-}_{h,i,t,s} \geq L_{h,i,t} \ \forall \ h \in \mathbb{H}, i \in \mathbb{P}, t \in \mathbb{T}, s \in \mathbb{S} \tag{3}$$

Equations 4 and 5 define the scheduling constraints (smoothing and sink rate) enabling the optimizer to penalize deviations from mineability targets in parts VI and VII in the objective function.

$$|\mathbb{W}_{b}| \cdot x_{b,t} - \sum_{w \in \mathbb{W}_{b}} x_{w,t} \leq d^{smooth}_{b,t} \ \forall \ m \in \mathbb{M}, b \in \mathbb{B}_{m}, t \in \mathbb{T} \tag{4}$$

$$x_{b,t} + x_{v,t} - d^{sink}_{b,t,v} \leq 1 \ \forall \ m \in \mathbb{M}, b \in \mathbb{B}_{m}, v \in \mathbb{V}_{b} \subseteq \mathbb{O}_{b}, t \in \mathbb{T} \tag{5}$$

Recall that the smoothing window $\mathbb{W}_{\mathscr{b}}$ is centered around $\mathscr{b} \in \mathbb{B}_m$ in the same units as block dimensions and that $d_{\mathscr{b},t}^{smooth}$ is used to count the number of blocks scheduled in different periods from $\mathscr{b}$. If a block is mined in period t and it is a member of the smoothing window for its adjacent blocks, who are not also scheduled for extraction, then a penalty is incurred in Part VI of the objective function. The sink rate constraints allow the optimizer to control the number of blocks mining can advance vertically in any period. Recall that the set $\mathbb{V}_{\mathscr{b}}$ contains the block that overlie $\mathscr{b}$, this set can contain at most one block, v, where if block $\mathscr{b} = (\mathscr{b}_i, \mathscr{b}_j, \mathscr{b}_k)$ then block $v = (v_i, v_j, v_{k+SR(\mathscr{b})+1})$ and $SR(\mathscr{b})$ is the sink rate of $\mathscr{b}$. If $x_{\mathscr{b},t} + x_{v,t}$ = 1, meaning both blocks $\mathscr{b}$ and v are mined in the same period, then $d_{\mathscr{b},t,v}^{sink} = 1$, otherwise $d_{\mathscr{b},t,v}^{sink} = 0$, satisfying Eq. 5. It follows that the sum of all $d_{\mathscr{b},t,v}^{sink}$ variables represents the number of sink rate constraint violations in the extraction sequence and incurs a penalty Part VII of the objective function. The model is also subject to: capacity, reserve, slope, destination policy, and processing stream flow constraints detailed in Goodfellow and Dimitrakopoulos (2016).

2.2.5 Solution method

Stochastic modelling allows for integrating various sources of uncertainty into the optimization process; however, this also considerably increases the size and complexity of what is already a challenging combinatorial optimization problem. Solution approaches that involve commercial MIP solvers are often not feasible. Metaheuristics provide a practical alternative for solving these models, and many existing metaheuristics have been successfully adapted to the stochastic optimization of mines and mining complexes (Lamghari and Dimitrakopoulos, 2016). The solution approach uses a combination of metaheuristic algorithms to solve the model and is described by (Goodfellow and Dimitrakopoulos, 2016, 2017).

2.3 Case Study

This section describes and examines the application of the method described herein at a large Nevada style gold mining complex. The results are reported as a probabilistic risk analysis on several key performance indicators (KPIs). In Section 2.4 the cut-off grade optimization component of the simultaneous stochastic optimizer is compared to a base case cut-off grade policy provided by industry partners. In Section 2.5 market uncertainty is integrated into the model and the results are analyzed.

2.3.1 Overview of the mining complex

The material in the mining complex is extracted from two open-pit mines and delivered from a set of nearby operations (referred to as external sources). The material can flow through the mineral value chain to three processing destinations (an autoclave, oxide mill, and oxide leach pad), a waste dump, and a set of stockpiles for each material type. Figure 1 shows the components of the mining complex. Supply uncertainty is incorporated by using a set of multivariate geostatistical simulations provided for each mine. A set of simulations is also generated to capture uncertainty in the delivery of material from external sources. The model comprises several million integer variables and several thousand scenarios for evaluation and optimization.

Pits A and B share a mining fleet which defines an upper bound on their joint mining capacity in each period. The simultaneous optimizer defines the allocation of capacity between the two pits in conjunction with other factors such as material supply and needs of related processing streams in order to maximize project value and minimize deviations from operating targets. However, the mining capacity is not a project bottleneck, meaning it does not materially impact processes such as cut-off grade optimization and as such is not included in the discussion of results. High-grade oxide ore extracted from Pit B is processed at the oxide mill, while a heap leach facility processes lower grade oxide ore material. An autoclave processes sulphide ore extracted from Pit A and delivered from external sources. A non-linear recovery function models gold recovery at each of the processing destinations. At the oxide mill and heap leach pad, the recovery function is dependent only on gold grade in the feed. Recovery at the autoclave is governed by gold grade and organic carbon (OC) content. The autoclave has a strict set of operating requirements for feed geochemistry to effectively treat the sulphide ore material as mentioned in Section 2.2.4. The operating efficacy is dependent on several physical and chemical controls to treat the feed for the next phase of metallurgical gold recovery. The geochemical blending and acid consumption necessitated by the process are of particular interest to this study, especially the ratio of sulphide sulphur (SS) to carbonate (CO_3) concentrations in the feed.

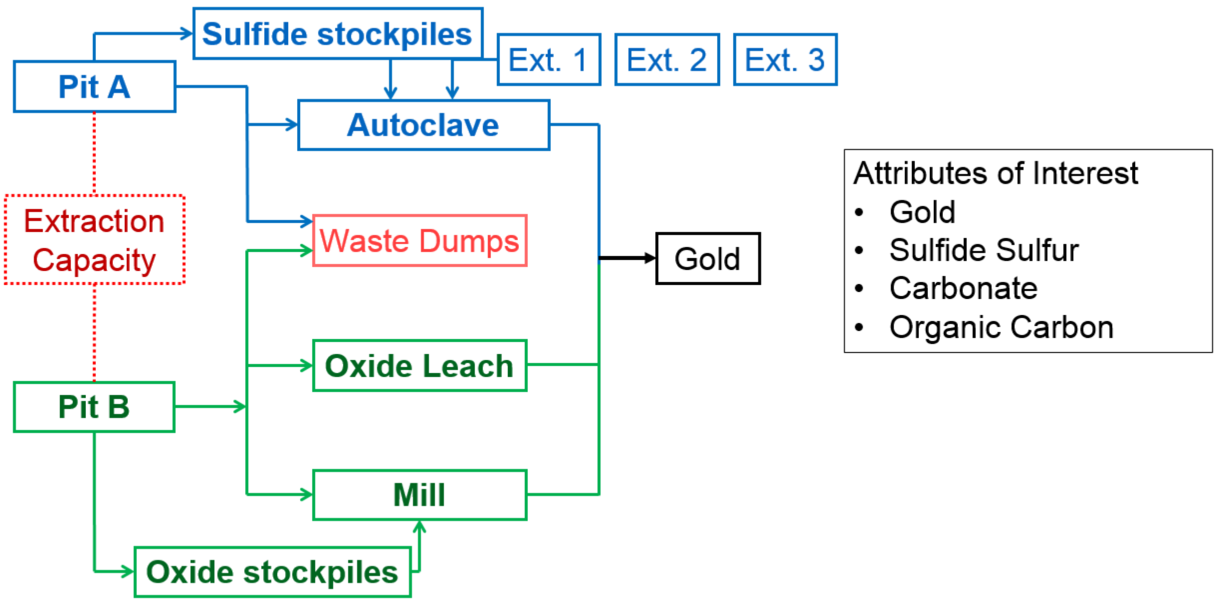

Figure 1: Diagram of the mineral value chain

The sulphide ore material from Pit A is classified into eleven geochemically distinct material types *a-priori* based on SS, CO_3, and OC concentrations. Each material type has its stockpile. The optimizer determines the gold cut-off grade boundaries for each material type, deciding which grade bins are sent to the stockpiles, the autoclave or the waste dump. The material from Pit B is not concerned with the deleterious elements because they do not affect the metallurgical recovery process that governs oxide ore at the mill or the leach pad. The optimizer determines the gold cut-off grade boundaries for sending material to the waste dump, stockpile, leach pad, or mill. Cut-off grade decisions are optimized simultaneously to maximize the net present value considering the extraction sequence, downstream processing and blending decisions and related constraints. This configuration of the mining complex is referred to hereafter as the 'simultaneous case.'

The simultaneous case is compared to the 'base case' configuration of the mining complex. The base case is also a simultaneous stochastic optimization; however, it is constrained by the destination policy and material type configuration provided by the mining complex's conventional cut-off grade optimization procedure. Each of eleven sulphide material types from Pit A is further divided into a maximum of five subgroupings based on gold grades, resulting in forty-five sulphide ore material classifications. Eight of these material types are sent to the autoclave directly, while thirty-seven are sent to distinct stockpiles for future reclamation. Oxide material from Pit B is classified as either waste (<0.0088 Au Oz/ton), low-grade (0.0088 – 0.022 Au Oz/ton), or high-grade (>0.022 Au Oz/Ton). Low-grade material is sent to the leach pad, high-grade material is sent to the oxide mill or stockpiled on an ad-hoc basis.

2.4 Results and comparisons

Figure 2 (a) – (d) presents the risk profiles for discounted cash flow, and ounces of gold recovered, the base case is in blue, and the simultaneous case is in black. The relevant values have been scaled for confidentiality. Figure 2 (a) shows that over the ten years there is a negligible difference in NPV between the two cases (0.16% difference in p50 values). Furthermore, the risk profiles are particularly tight over the first five periods, projecting confident forecasts. Figure 2 (b) shows that both cases deliver stable discounted cash flows (200+ million USD) throughout the life of the operation. Cash flows are comparable in most periods except for period three, where the simultaneous case forecast is significantly higher. Figure 2 (c) and (d) depict gold ounces recovered; they closely mirror the cash flow forecasts. Both optimized plans forecast stable ounce profiles throughout the life of the operation, never delivering less than 300 thousand ounces a year.

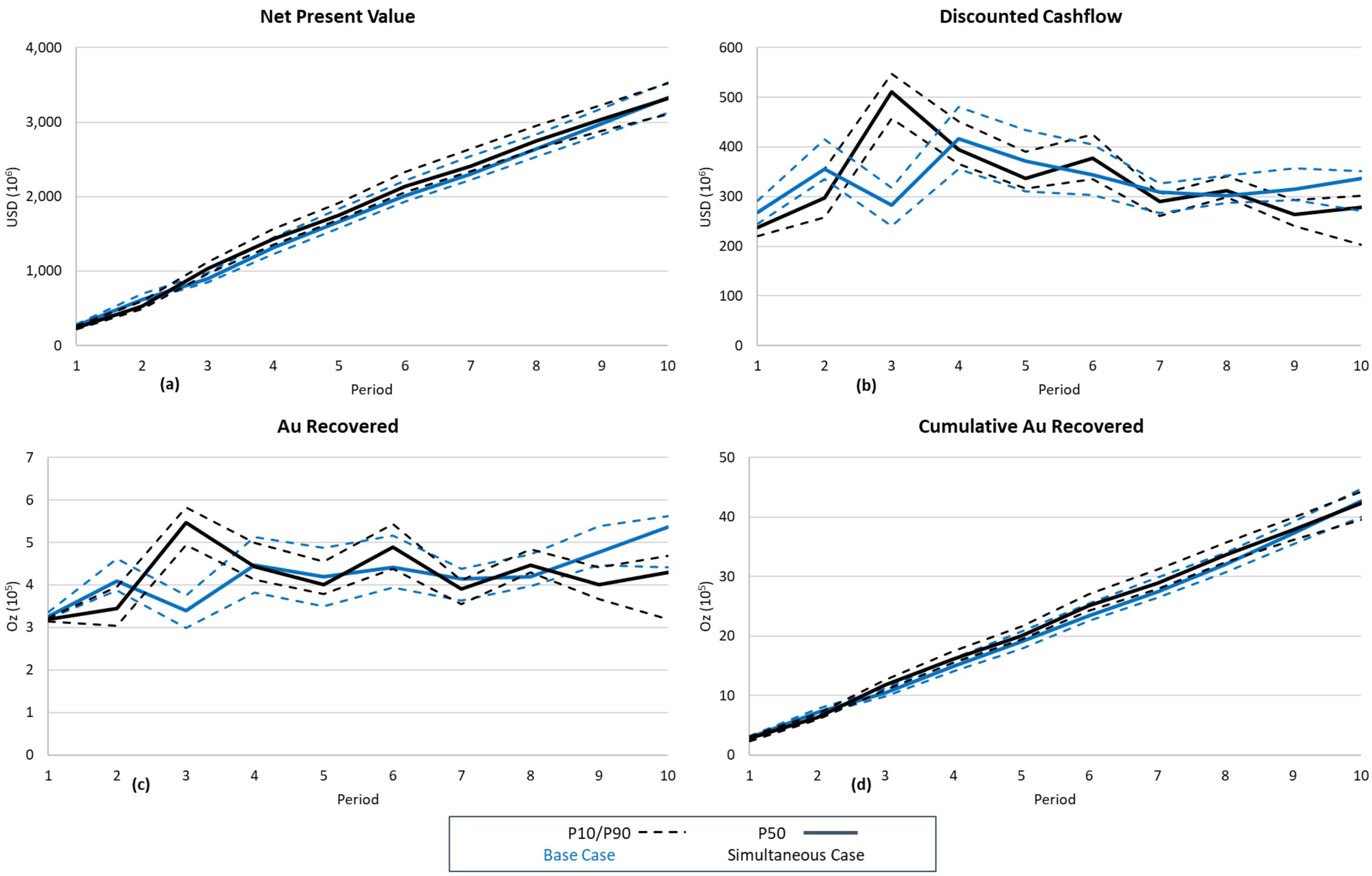

Figure 2: Base case (blue) and simultaneous case (black) cash flow and gold recovered risk profiles

Figure 3 (a) – (c) provides the risk profiles throughput at each processing facility in the mining complex. The base case forecast under delivers autoclave throughput in period 1. Otherwise, both cases respect the capacity targets through the life of the operation. The oxides provide more contrasting differences between the two cases. The flexibility afforded to the optimizer in the simultaneous case enables it to adjust the destination policy decisions to deliver a consistent throughput to the oxide mill. The base case struggles to respect the oxide mill's capacity targets in periods 1, 3-8. It is important to note that the oxide mill capacity violations would negatively impact the base case NPV forecast, meaning the real difference is more pronounced than the

0.16% shown in Fig. 2 (a). Examining the oxide cut off grades and the leach pad throughput provides some insight into these results.

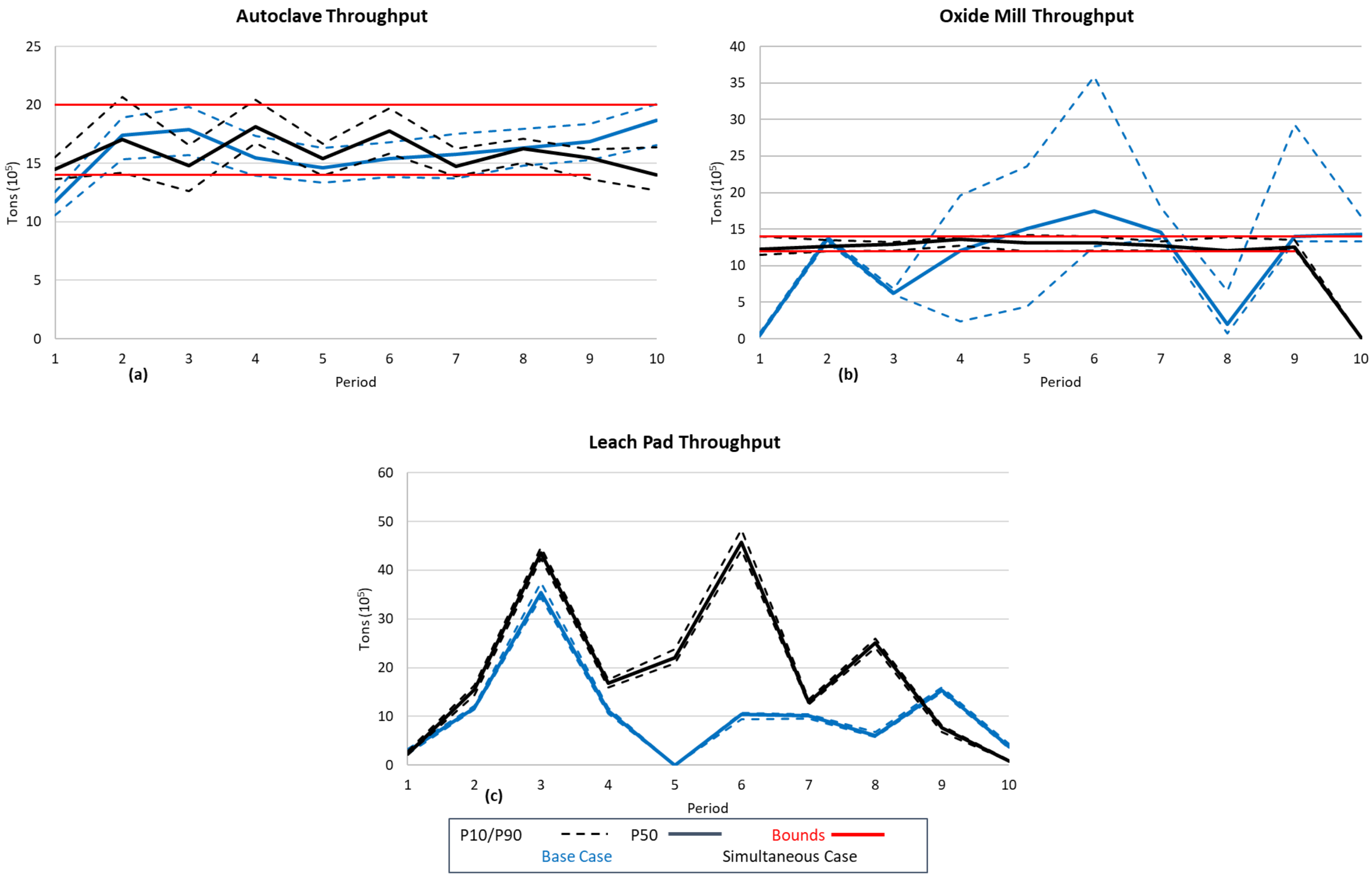

Figure 3: Base case (blue) and simultaneous case (black) throughput forecasts at each processing facility

Figure 4 presents the oxide cut-off grades and Fig. 5 presents the feed grades to both oxide processing facilities. Recall, the base case uses predetermined cut-off grade decisions. The simultaneous case optimizes the cut-off grades in conjunction with the extraction sequence and processing stream decisions. Figure 4 highlights the adjustments made to keep the mill well fed. Cut-offs are raised when there are large quantities of high-grade material available, such as in period 3, and lowered when high-grade materials are scarce such as in period 1. Figure 5 (b) confirms these observations as the trend in feed grade to the oxide mill mirrors the cut-off grade adjustments. It is important to note that just because there is high grade being mined in period

3 of the simultaneous case, it does not mean that the same high-grade material is being mined in the base case, the extraction sequences are not the same. Figure 6 shows that they are very different; in fact, not even the final pit limits are the same. The simultaneous case mines 3% more material than the base case over the ten years.

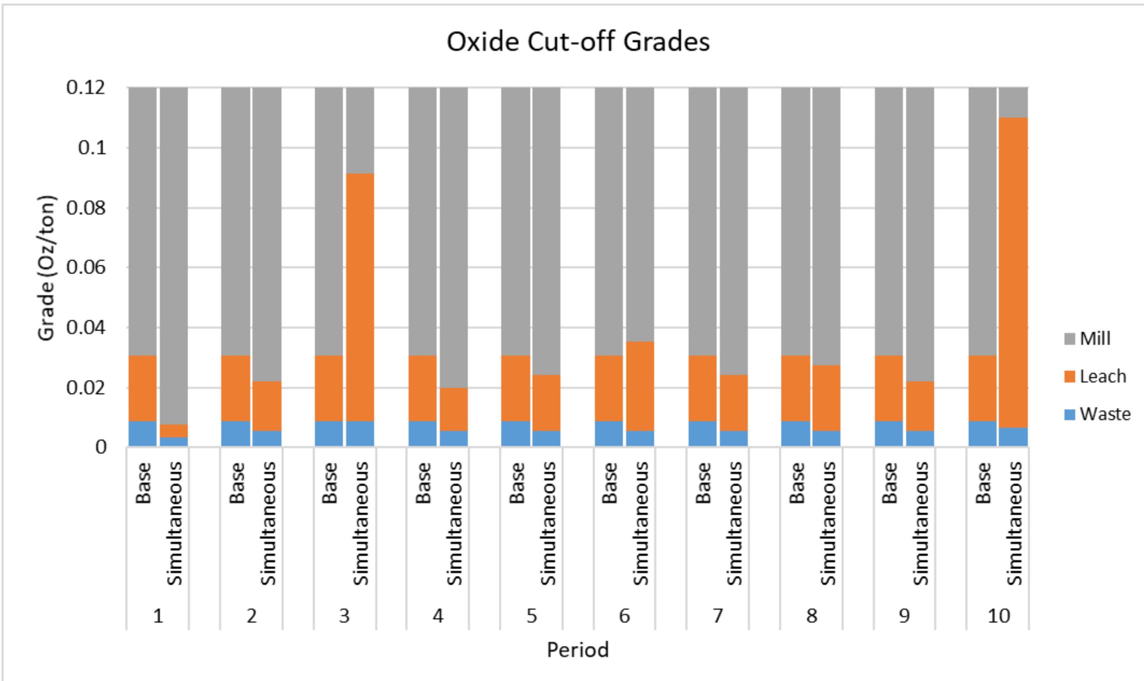

Figure 4: Base case and simultaneous case oxide material cut-off grades

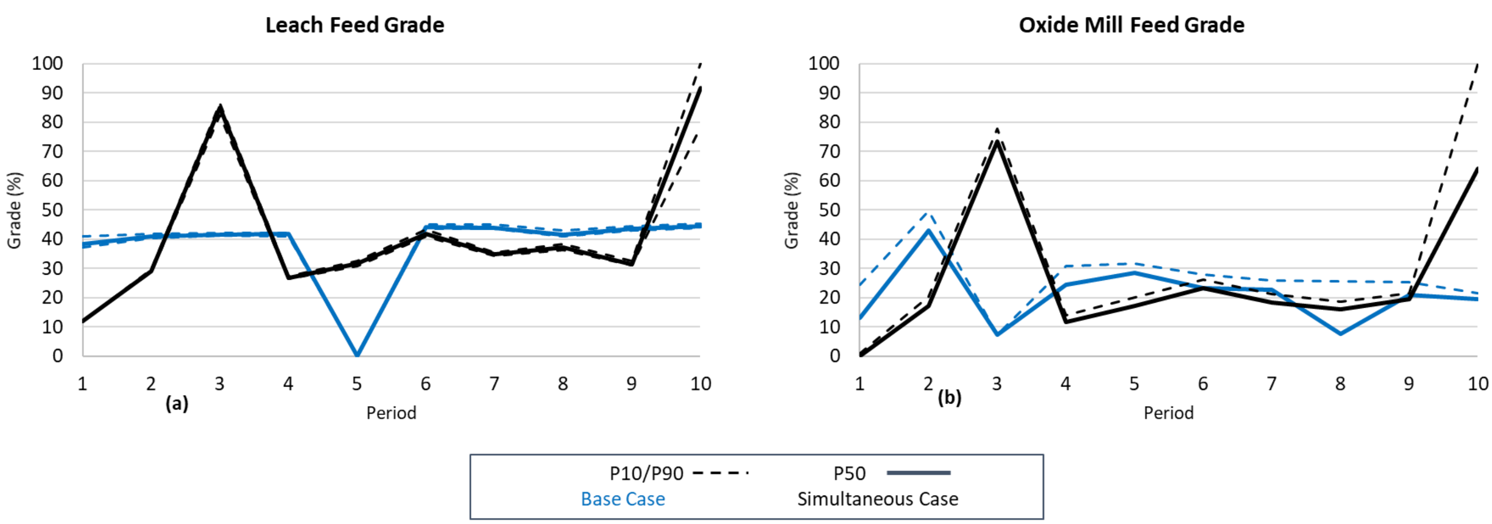

Figure 5: Base case (blue) and simultaneous case (black) oxide mill and leach pad feed grades

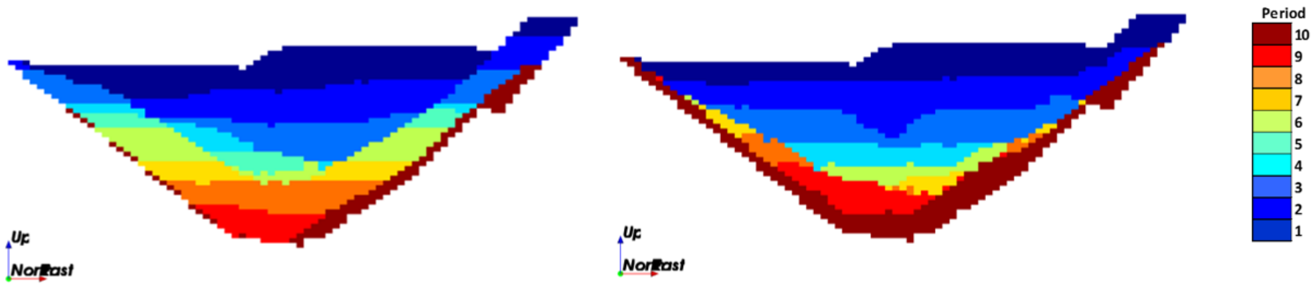

Figure 6: Pit B extraction sequence cross-section: simultaneous case (left) base case (right)

Figure 7 (a) – (e) presents the risk profiles for the blending constraints. Recall the autoclave requires certain levels of acidity in the feed slurry for efficient operation. The most critical blending target is the SS:CO_3 ratio, shown in Fig. 7 (b). When the SS or CO_3 levels fall too low or too high, acid is added to the feed slurry to control the pH. However, acid consumption capacity is strictly limited by regulation. It is evident that other than the SS targets, both cases satisfy the blending constraints in most periods. Although the SS levels do not consistently respect the blending targets, the violations are on the order of fractions of a percentile. The potentially negative impacts are mitigated by the very well controlled SS:CO_3 ratio.

In summary, allowing the simultaneous stochastic optimizer to determine cut-off grade decisions in conjunction with extraction sequence and processing decisions yields several key improvements. The simultaneously optimized cut off decisions perturb the extraction sequence to deliver stable material flows that respect the capacities of the processing facilities. The pre-determined cut-off grades in the base case configuration yield production forecasts that struggle to respect processing capacity targets. While the NPVs of both configurations appear similar, the base case forecasts may be misleading due to these violations. Furthermore, the configuration of the simultaneous case significantly reduces the number of necessary stockpiles on site. Thus, reducing the operational complexity in managing the mineral value chain.

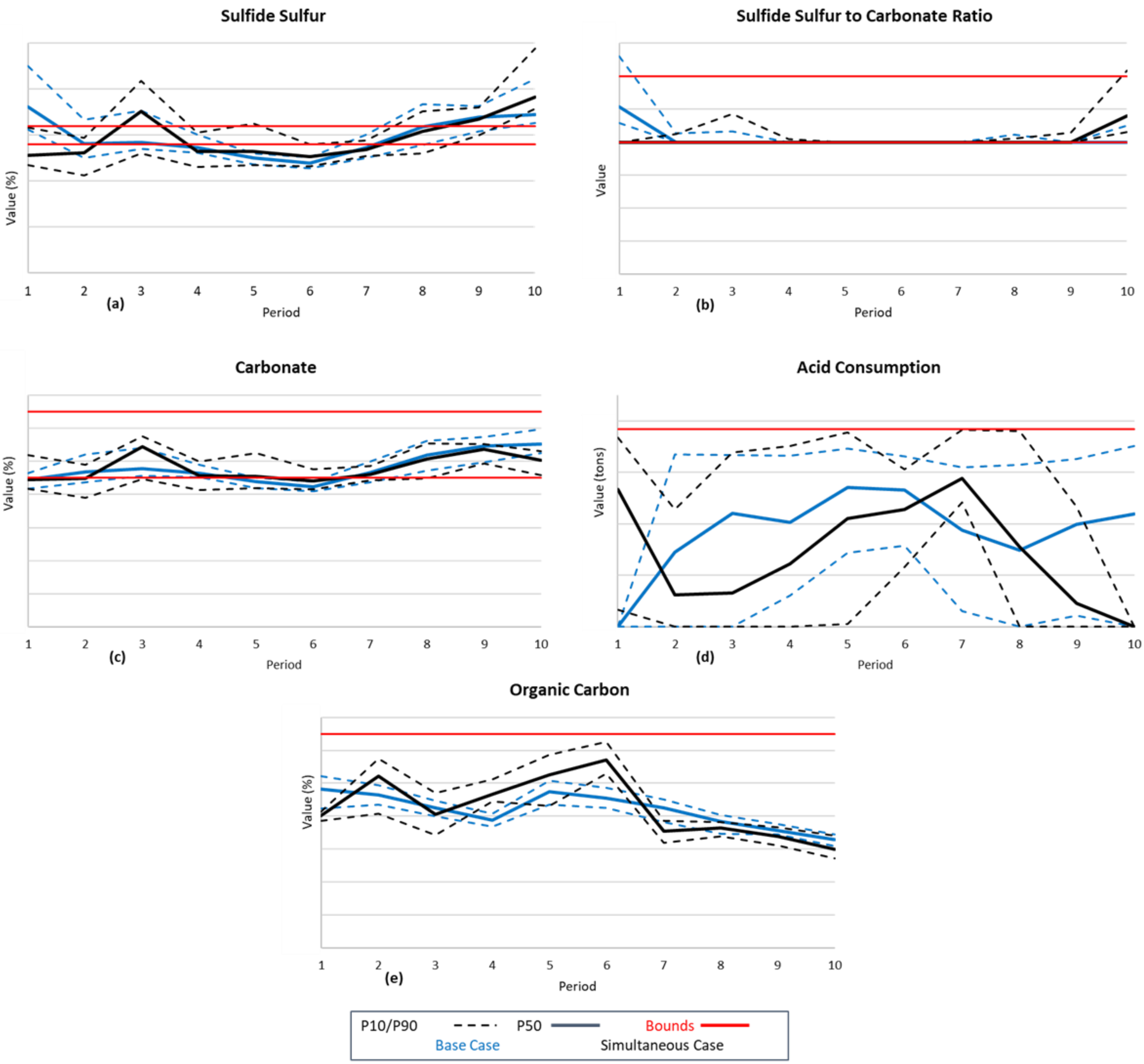

Figure 7: Base case (blue) and simultaneous case (black) blending results

2.5 Incorporating Market Uncertainty

Incorporating market uncertainty through the use of commodity price simulations as inputs to the simultaneous stochastic optimization framework generates a long-term plan that manages and quantifies risk derived from volatile spot markets. Gold prices are simulated using an established model for precious metals, geometric Brownian motion with Poisson jump diffusion (Schwartz, 1997), described by Eq. 6, where S_t is the metal price at time t, W is a Weiner process, β is the size of Poisson jump P. Average annual price drift, η, is the trend component often used in price models where the price is correlated with inflation, σ^2, is average annual price volatility. Figure 8 presents the simulated gold price scenarios used in this case study, the mean of the simulate prices and the constant price used in the comparative case. It is typically accepted that the number of simulations necessary to accurately quantify metal price uncertainty can be on the order of hundreds (Farmer, 2017). Such a number makes the problem intractable as the number of variables is already in the order of $10^{6,}$ and the number of joint uncertainty scenarios is in the thousands. However, the sensitivity of long-term production scheduling for mineral value chains to the number of price scenarios has not been sufficiently explored. For example, Albor and Dimitrakopoulos (2009) show that due to support-scale effects (also known as a volume-variance relationship), fifteen to twenty stochastic orebody simulations are enough to quantify geological uncertainty in long-term mine production scheduling applications. This implies that using more than fifteen simulations as inputs does not change the solution. This application should be considered as a proof-of-concept that it is possible to consider market uncertainty in the simultaneous stochastic optimization framework and that has material differences to the outputs of the process. Further studies must determine the number of commodity price simulations necessary to generate stable solutions and provide accurate quantifications of uncertainty in this context.

$$S_t = S_{t-1} \times \left(\eta \times t - \sigma^2 \times \frac{t}{2} + W + \beta \times P\right) \tag{6}$$

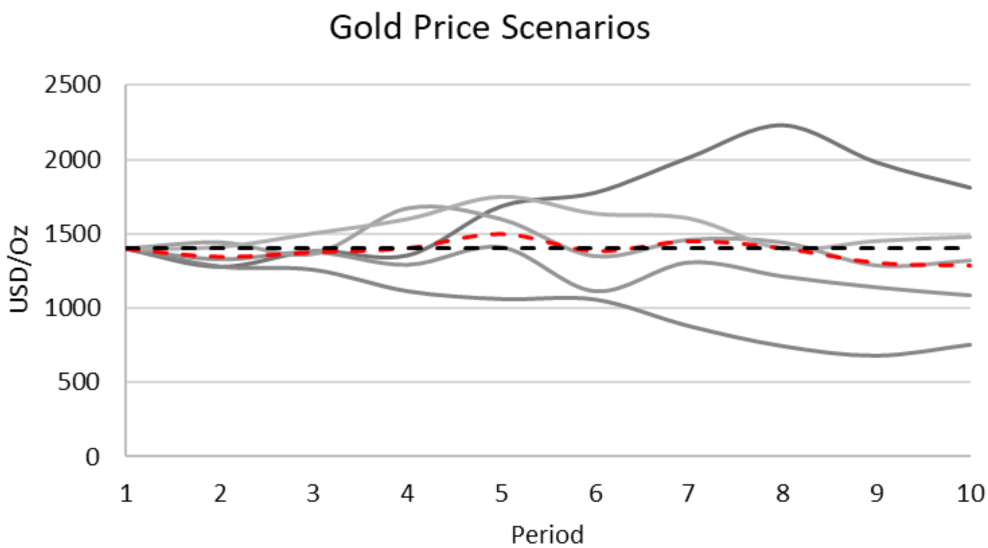

Figure 8: Gold price simulations (grey), mean of simulated prices (red dotted), constant gold price (black dotted)

2.5.1 Risk analysis considering market uncertainty

This section examines the risk profiles of the case study considering joint market and supply uncertainty scenarios. The simultaneous case from Section 2.4, which uses the constant gold price (shown in Figure 8) is included and referred to as the 'supply uncertainty' case to provide a comparison. Figures 9 (a) – (d) present the discounted cash flow and gold recovered risk profiles. The effect of the joint uncertainty scenarios becomes quite evident in the p10 and p90 values after period 4 in Figs. 9 (a) and (b). In all periods, the cash flows are less certain than the case that only considers supply uncertainty. However, through the first four periods the optimizer forecasts cash flows with a reasonable amount of confidence. Examining the gold price scenarios in Fig. 8 provides a reasonable explanation; the fluctuations in gold price simulations begin to vary significantly more as time progresses beyond period 4. However, the forecasts beyond period 4 provide a quantification of risk in projected cash flows. Despite the fluctuating price scenarios in the later periods, the forecasts remain positive throughout the life of the operation, with the p10 value never dropping below 100 million USD and the p50 never dropping below 200 million USD. The joint uncertainty case delivers stable ounce profiles (Fig. 9 (d)) throughout the life of the operation while remaining profitable. In fact, despite the significant exposure to the downside in gold prices, the ounce profile remains comparable to the case that does not consider market uncertainty.

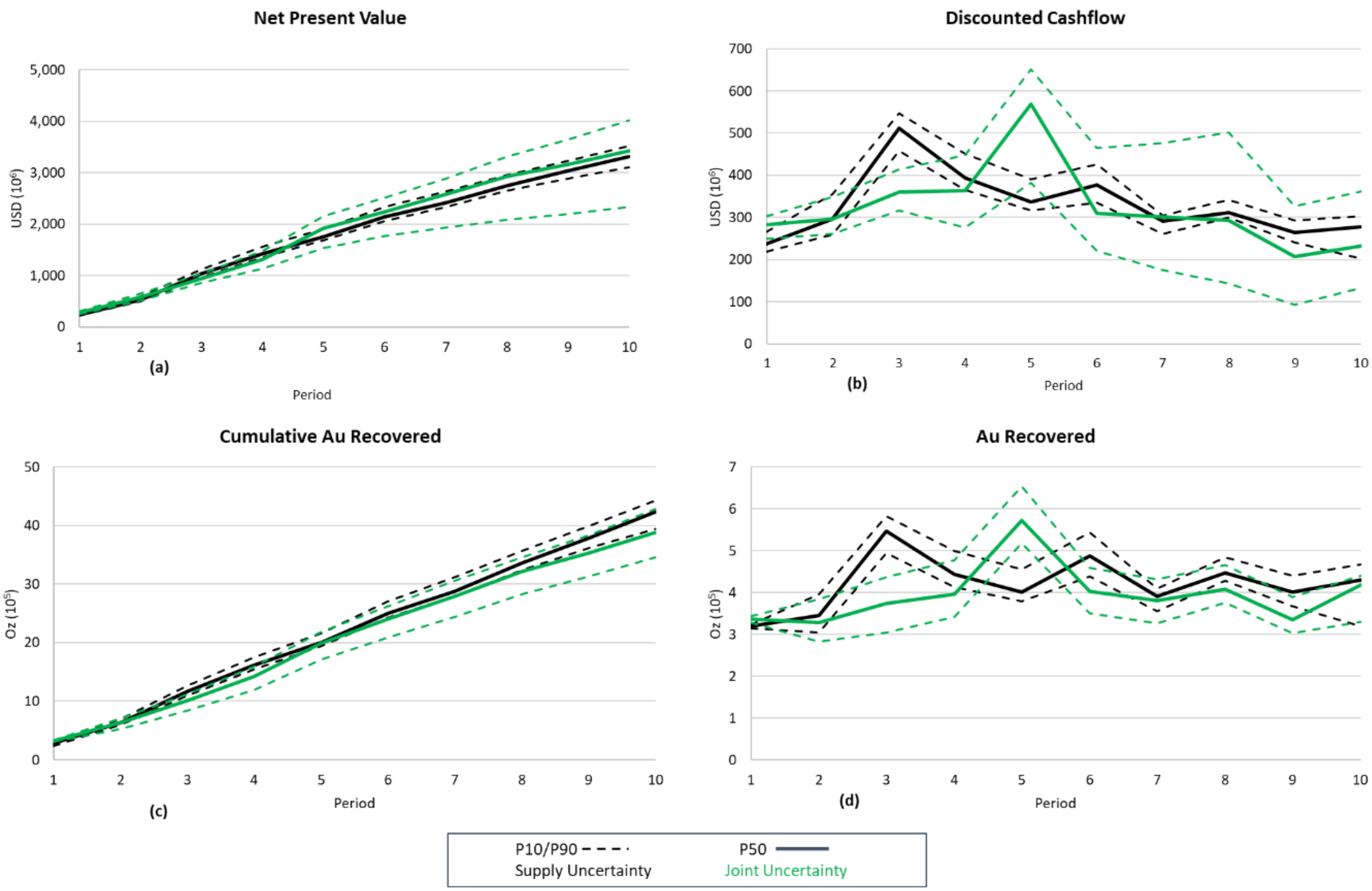

Figure 9: Supply uncertainty (black) and joint uncertainty (green) cash flow and gold recovered risk profiles

Figure 10 (a) – (c) provide the risk profiles for throughput at each processing facility; it is evident that optimizer provides a long-term plan that respects the capacity targets of each processing facility. In Fig. 10 (b) and (c) period five highlights one of the benefits of incorporating market uncertainty into this process. First, note that several price scenarios enjoy an upswing in period 5. The optimizer can take advantage of this upswing and mine more high-grade material, but it does not violate the maximum capacity of the oxide mill. Instead, it utilizes the leach pad to process significantly more tons. This leads to higher cash flows and a larger number of ounces recovered in period 5. Note the relative increase in cut-off grade in period five shown in Fig. 11.

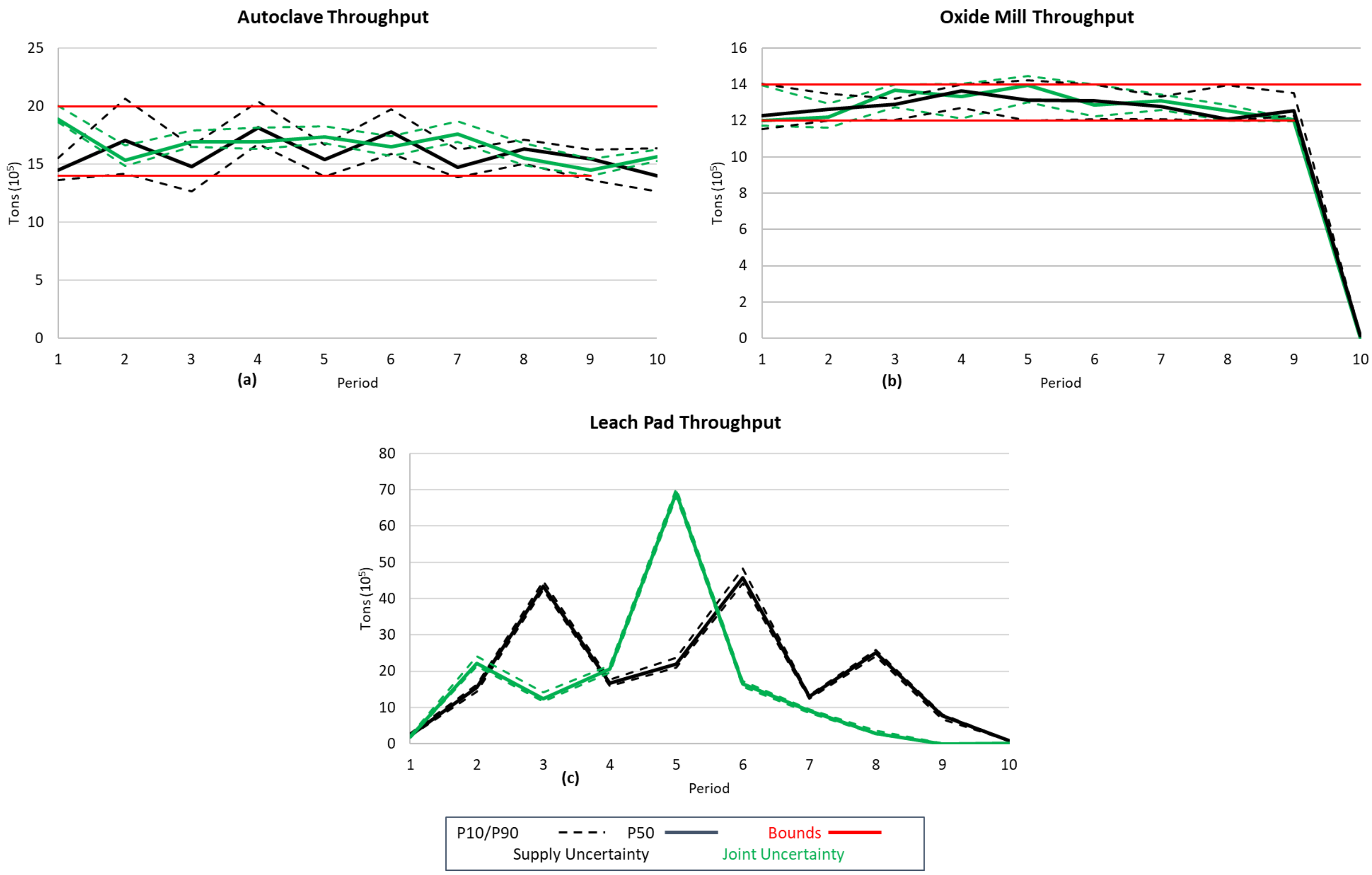

Figure 10: Supply uncertainty (black) and joint uncertainty (green) throughput risk profiles at each processor

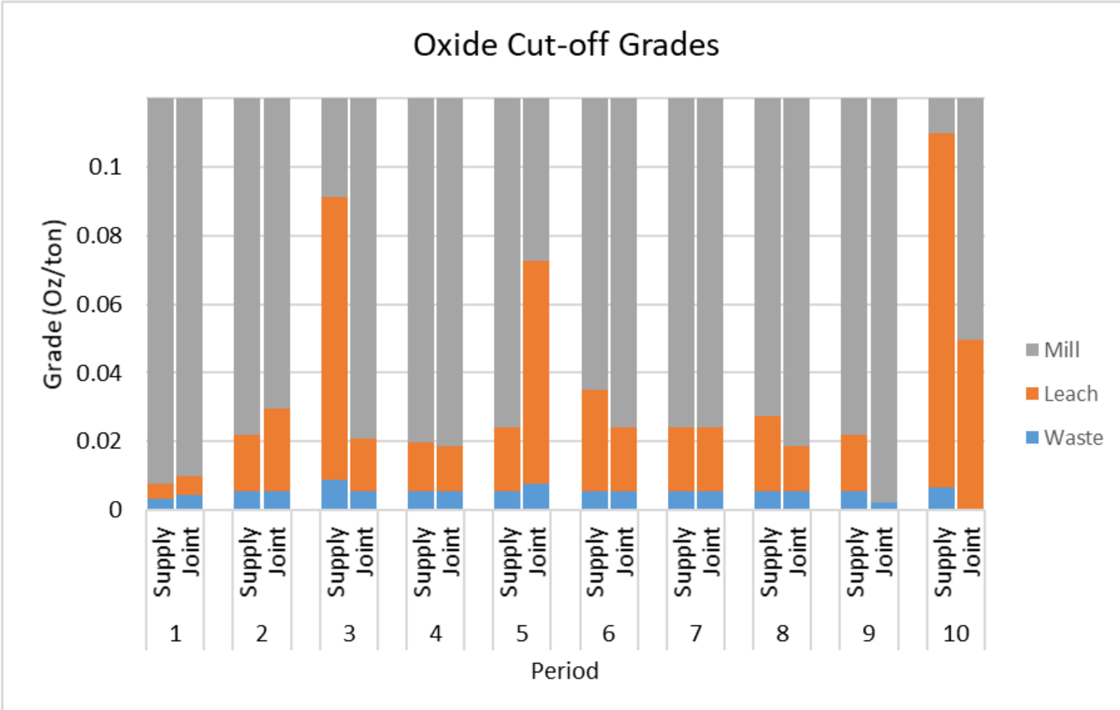

Figure 11: Figure 5: Supply and joint uncertainty oxide material cut off grades

Figure 12 presents the same cross-section of both pit B extraction sequences; the difference in material scheduled for extraction in period 5 is visually clear. Recall, that the extraction sequence is scenario independent. This provides some insight into the differences in material scheduled for extraction in periods 9 and 10 when the downside exposure to price uncertainty becomes more pronounced. Beyond the visual differences in the extraction sequences, the incorporation of joint uncertainty scenarios also leads to differences in the physical boundaries of the open pit mines. The total extracted material mass in the joint uncertainty case is approximately 119 million tons. In the supply uncertainty case, the extracted mass is approximately 174 million tons. Despite the 32% difference in total extracted material, there is only an 8% difference in the p50 values for total recovered ounces. However, this does not translate to an improvement in p50 for the net present value; the joint uncertainty case has a 3% higher p50 net present value. This highlights the ability of the simultaneous stochastic optimizer to capitalize on extra production in elevated price scenarios while managing the impact of risk throughout the life of the operation.

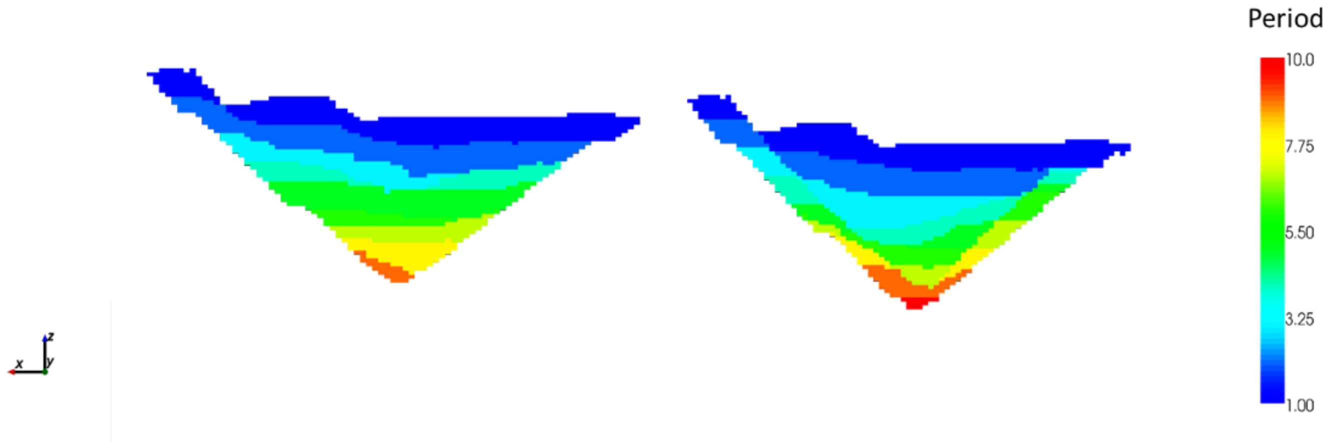

Figure 12: Pit B extraction sequence cross-section. Joint uncertainty (left) and supply uncertainty (right)

2.6 Conclusions

This paper presents an application of the simultaneous stochastic optimization framework at a multi-pit, multi-processor Nevada-type gold mining complex and its strategic mine planning; including extraction sequences, destination policies, and processing stream decisions. The contributions are highlighted by the case study at the mining complex composed of two open pit mines, three external sources of ore delivery, three processing facilities and a series of distinct stockpiles. The mining complex is subject to numerous operating constraints related to capacities and geochemical blending requirements. The application documents the effectiveness of the framework's cut-off grade optimization component by comparing results with a base case model that uses cut-off grades determined *a-priori* by conventional methods derived from Lane's theory (Lane, 1988). It also explores the incorporation of market uncertainty into the simultaneous optimization of a mining complex and effects on all its major aspects. The results show that the proposed approach improves the operation's ability to respect operating capacity targets by optimizing cut-off grades in conjunction with material availability and processing requirements. The approach also allows for a reduction in operating complexity as the simultaneous case utilizes only twelve stockpiles compared to thirty-eight in the base case.

Additionally, the integration of commodity price fluctuations and optimization of the mining complex under joint market and supply uncertainty underscores the flexibility of the framework. The results demonstrate the optimizer's ability to adapt the schedule to mine and process more material during periods of elevated price environments while being more conservative when the downside exposure to prices becomes more pronounced. This highlights the ability of simultaneous stochastic optimizers to generate long-term schedules that manage exposure to commodity price fluctuations and provide accurate quantifications of risk to an operation's strategic decision makers.

As research into more efficient solution approaches develops, the framework will be able to consider larger numbers of joint uncertainty scenarios in reasonable amounts of time. Such advancements would allow for more in-depth sensitivity analyses on the number of commodity price simulations necessary to generate stable and resilient outputs. Future extensions of the method should include the ability to optimize mining capacity and processing rates. Another area of particular interest is the extension of the destination policy to consider multiple attributes into cut-off grade decisions.

Chapter 3 An Application of Simultaneous Stochastic Optimization of an Open-Pit Mining Complex with Tailings Management

3.1 Introduction

Simultaneous stochastic optimization of mining complexes takes an integrated value chain approach to the production scheduling of mining operations. Where a mining complex can include multiple pits or underground mines, stockpiles, processors, waste dumps and tailings facilities that transform in-situ material into economic products across the chain (Pimentel *et al.*, 2010; Montiel and Dimitrakopoulos, 2015; Goodfellow and Dimitrakopoulos, 2016). Traditional optimization of mining projects typically features step-wise approaches that independently optimize various components of the mineral value chain, leading to suboptimal solutions which deteriorate as the complexity of the chain increases (Gershon, 1983; Goodfellow and Dimitrakopoulos, 2016, 2017). Simultaneous optimization capitalizes on the synergies inherent to the interdependent components of a mining complex by optimizing all of them simultaneously to maximize the net present value of a project (Pimentel *et al.*, 2010; Whittle, 2010). This can include capital expenditure (CapEx) investments related to various components in the mining complex which have significant impacts on critical factors such as capacities, operating costs and life of mine. As such, it is necessary to integrate the simultaneous optimization of these components into the evaluation of major strategic decisions to maximize the profitability of a mining venture and manage the risk related to its operation (Montiel and Dimitrakopoulos, 2015; Goodfellow and Dimitrakopoulos, 2016). Previous work on integrating various components of the mineral value chain into the strategic mine planning and optimization framework includes (Hoerger *et al.*, 1999; Chanda, 2007; Stone *et al.*, 2007; Whooler, 2007; Zuckerberg *et al.*, 2007; Whittle, 2010; Zuckerberg *et al.*, 2011)

The methods listed above represent incremental improvements to past approaches. However, they share at least one of several following significant limitations. Aggregation of selective mining units (SMUs) to allow for more tractable solutions which leads to issues with mining selectivity that misrepresent the solution, stepwise and independent optimization of several value chain components, linearization of non-linear transfer functions and or the use of fixed production schedules. Most importantly, the approaches above do not account for uncertainty in critical project parameters such as material supply and variability. They are all deterministic optimization frameworks which use a single, estimated representation of a mineral deposit as an input (Hustrulid *et al.*, 2013). Estimated orebody models provide overly smooth distribution of grades, misrepresenting local variability inherent to geological phenomena (Goovaerts, 1997; Journel, 2005) which ultimately lead to misleading operating forecasts that do not accurately reflect technical risk to mining operations (Ravenscroft, 1992; Dowd, 1994; Dimitrakopoulos *et al.*, 2002).

The value of incorporating supply uncertainty into mine design and production scheduling has been established through many studies on stochastic mine planning (Godoy, 2003; Menabde *et al.*, 2007; Ramazan and Dimitrakopoulos, 2007; Dimitrakopoulos and Ramazan, 2008; Dimitrakopoulos, 2011; Asad and Dimitrakopoulos, 2013b; Benndorf and Dimitrakopoulos, 2013; Ramazan and Dimitrakopoulos, 2013; Groeneveld *et al.*, 2018). Although initial research in stochastic mine planning focused on single open-pit mines recent advances have shifted the focus to mining complexes and address many of the limitations of previous methods. The considerable challenge that previous methods face in effectively tying together the optimization process of upstream and downstream components in the mineral value chain is rooted in the economic valuation of mining blocks. This process assigns block values before optimization of production schedules. However, most mine optimization methods, including those described above, are linear processes that cannot properly account for non-linear interactions that occur after material extraction. The recent paradigm shift that enables the simultaneous stochastic optimization of mining complexes uses the value of sellable products to drive the optimization process (Montiel and Dimitrakopoulos, 2015; Goodfellow and Dimitrakopoulos, 2016; Montiel *et al.*, 2016; Goodfellow and Dimitrakopoulos, 2017; Montiel and Dimitrakopoulos, 2017, 2018). This framework explicitly accounts for supply uncertainty, the time value of money and non-linear material interactions across the value chain thereby maximizing value and managing technical risk. Goodfellow and Dimitrakopoulos (2016) proposes a highly generalized unified formulation allowing for easy adaptation and extension (Farmer, 2017; Goodfellow and Dimitrakopoulos, 2017; Kumar and Dimitrakopoulos, 2019; Del Castillo and Dimitakopoulos, 2019) Goodfellow and Dimitrakopoulos (2015) expands the initial unified model to incorporate CapEx investment decisions and presents a case study where the optimizer can dynamically alter the mining capacity by buying additional load and haul equipment.

This paper presents an application of Goodfellow and Dimitrakopoulos (2015) to evaluate the potential expansion of a tailings storage facility (TSF) within the context of the simultaneous stochastic optimization of a real world, operating open-pit, multi-element-gold mining complex in Central America. The mining complex sources material from two mines and a significant amount of existing stockpile inventory. Gold is extracted from the sulfide ore with copper and silver byproducts. Processing the refractory ore requires an autoclave pretreatment to oxidize sulfide sulfur in the feed to maintain the effectiveness of the recovery circuit. The oxidization precedes a thickening process which makes it possible to remove sulfuric acid and separate the copper-rich base metal liquors from the slurry. Lime is added to oxidized slurry making it possible to recovery silver as well gold during the typical cyanidation and carbon-in-leaching (CIL) process. Throughput at the complex processing plant is highly dependent on the levels of sulfide sulfur in the feed, placing sulfur content of the ore on par with gold grade in terms of importance. The metallurgical recovery and processing costs are highly dependent on a consistently constrained sulfur content.

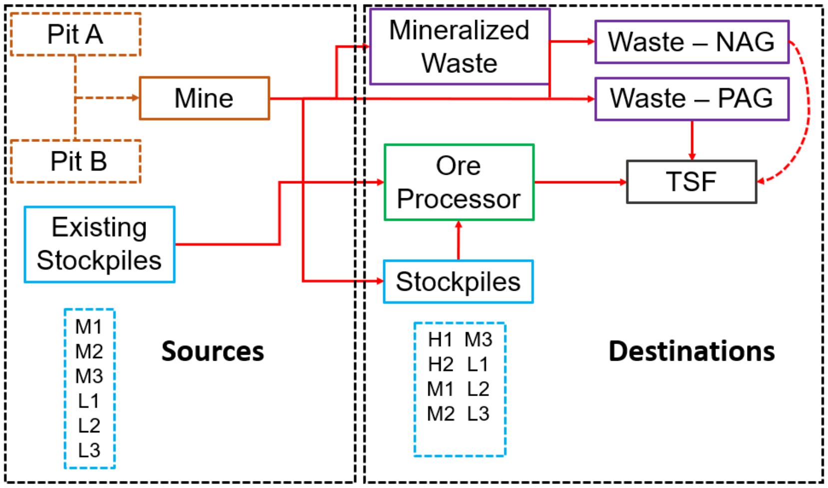

Figure 13: Mineral value chain configuration.

Figure 13 illustrates the configuration of the mining complex. Material is sourced from the mines, comprising two open-pits, and six existing stockpiles with significant inventory. Material can be sent to the ore processing facility, eight stockpiles, a mineralized waste pile, a non-acid generating (NAG) dump or the TSF where PAG waste is handled along with process tails. Roughly 23% of material initially classified as NAG waste in the reserve model is determined to be PAG by short term lab testing and is sent to the TSF; this is accounted for in the model as indicated by the dotted red line in Fig. 13. Material is classified into eight types based on gold and sulfur grades. Supply uncertainty is accounted for via a set of conditionally simulated stochastic realizations (Goovaerts, 1997; Boucher and Dimitrakopoulos, 2009; Remy *et al.*, 2009) of the mineral deposit as inputs to the optimization model. Simulated attributes include gold, copper, silver and sulfur grades. Attributes of the existing stockpile inventory are deterministic figures distributed onto homogenous block models for each stockpile. Studies show that material variability in stockpiles is much higher than what may be expected and that stockpile supply uncertainty can be a significant risk factor (Dirkx and Dimitrakopoulos, 2018). However, the data required for generating simulated stockpile realizations was not available for this case study. Other inputs include economic, operating and geotechnical parameters and constraints, all of which are provided by an industry partner and not specified for confidentiality.

The treatment of potentially acid generating (PAG) waste and process tails generated by the mining operation is a critical bottleneck due to environmental constraints. The mining company believes there is a potential for growth beyond the current mine plan that would require extra tailings capacity. However, given the significant capital expenditure associated with either expansion or development of a new TSF; it is necessary to evaluate the economic feasibility of the option while accounting for supply uncertainty within the simultaneous optimization framework of the current mining complex. The current life-of-mine (LOM) plan, optimized with conventional mine planning software, is focused on the early processing of high-grade ore and disciplined stockpiling of low-grade ore, subject to sulfur blending requirements, to maximize project economics. This plan projects the remaining mine life will last for seven years and that the processing plant will continue to mill stockpiled inventory for eleven years after end-of-mine production. A significant portion of the rock extracted from the mines is PAG and is deposited at the TSF and submerged by tailings to minimize acid rock drainage. However, the volume of the project's remaining reserves significantly exceeds the available TSF capacity, constraining the size of the ultimate pit and the life of the operation.

The remainder of this paper is structured as follows. Section 3.2 summarizes the optimization model and solution approach. Section 3.3 describes the generalities of the case study, prefacing the presentation of the results and describing the different cases under examination. The results include a simultaneous stochastic optimization (SSO) case, a simultaneous stochastic expansion (SSE) case and a comparison to mining complex's current forecasts – referred to as the base case. Conclusions and future work follow.

3.2 Method

This section summarizes the approach of Goodfellow and Dimitrakopoulos (2015) simultaneous stochastic optimization with capital expenditures to the specific application at a large multi-element gold mining complex. The general two-stage stochastic integer programming model is configured to the technical operating constraints and parameters of the case study.

3.2.1 Definitions and notation

A mining complex, $\mathcal{C}$, is composed of mines, $m \in \mathbb{M}$, stockpiles $s \in \mathcal{S}$, and processors, $p \in \mathcal{P}$. For modelling purposes, mines are considered the only sources of material for extraction. Stockpiles can be used to blend, homogenize or store material over time. Generically, a processor is any other destination in the mining complex, typically used to describe locations that transform (mills, leach pads, crushers etc.) or treat material in some manner, but not necessarily. Waste dumps or transportation hubs, for example, are termed processors in the modelling sense. Material is discretized at the mine level into selective mining units (SMUs, also known as mining blocks), $\mathcal{b} \in \mathbb{B}_m$, where $\mathbb{B}_m$ denotes the set of blocks in a mine m. A block has a set of properties of interest, described as attributes. Attributes that are essential material properties, often inputs to the optimization model and can be sent from one location to another in an additive manner (mass, element content) are belong to the set $p \in \mathbb{P}$. Attributes which are properties of interest that can be defined as functions of additive properties (recoveries, grades, pH, etc.) belong to the set $h \in \mathbb{H}$. Simulated properties for each block are sampled from a set $\mathbb{S}$ of equi-probable scenarios which quantify uncertainty in the model. A block is available for extraction if it's predecessors $\mathbb{O}(\mathcal{b})$ are all extracted. After extraction, the concept of a block is discarded, material can be sent to a set of locations $i \in S \cup P$ within the mining complex. State variables are used to keep track of material properties flowing through the various locations. Let $v_{p,i,t,s}$ represent the value of attribute $p \in \mathbb{P}$ at location i in period $t \in \mathbb{T}$ in scenario s and likewise, let $v_{h,t,s}$ represent the value of attribute $h \in \mathbb{H}$. While mining, re-handling, processing, and tailings treatment costs are attributes that belong to the set $\mathbb{H}$, they are explicitly referenced for clarity. Let $MC_{\mathcal{b},t}$, represent the cost of extracting a block, $\mathcal{b} \in \mathbb{B}_m$, in period $t \in \mathbb{T}$. Let $RH_{i,p,t}$, $PC_{i,p,t}$, $TC_{i,p,t}$ represent the costs associated with re-handling, processing and treating tailings attribute p, at location i, and in period t, respectively. Let $p_{h,t}$ denote the unit selling price of attribute h in period t. There are several parameters related to capital expenditure options, in this case only one-time CapEx options are considered, $k \in \mathbb{K}^1 \subseteq \mathbb{K}$. Let $p_{k,t}$ represent the discounted purchase price for CapEx option k, in period t. Let $\mathcal{k}_{k,h}$ denote the per-unit change for a constraint that CapEx option k has on attribute h. Let λ_k and τ_k denote the life and lead time associated with a CapEx option k. The optimization model uses 'soft' constraints to minimize deviations from operating targets. State variables $d^{\pm}_{h,t,s}$ measure deviations from a target for attribute h, at location i, in period t and in scenario s, similar variables measure deviations for p attributes. Deviations from targets are penalized in the objective function by monotonically decreasing time discounted penalty costs, $c^{\pm}_{h,t}$, for unit surplus or shortage of attribute h, in period t. The time varied discounting is known as the geological risk discount (GRD) rate (Dimitrakopoulos and Ramazan, 2004).

3.2.2 Decision variables

There are four types of decision variables defined in the optimization model: extraction sequence, destination policy, processing stream, and capital expenditure decisions.

- $x_{\mathscr{b},t} \in \{0,1\}$ – define whether block $\mathscr{b} \in \mathbb{B}_m$, is extracted in period $t \in \mathbb{T}$
- $z_{g,j,t} \in \{0,1\}$ – define whether material in grade bin $g \in \mathcal{G}$ is sent to destination $j \in S \cup P$ in period $t \in \mathbb{T}$
- $y_{i,j,t,s} \in [0,1]$ – define the proportions of material sent from location $i \in S \cup \mathcal{P}$ to destination $j \in S \cup \mathcal{P}$ in period $t \in \mathbb{T}$ and scenario $s \in \mathbb{S}$
- $w_{k,t} \in \{0,1\}$ – define whether one-time capital expenditure option $k \in \mathbb{K}^1$ is exercised in period $t \in \mathbb{T}$

The extraction sequence, destination policy, and capital expenditure decisions are first-stage decisions that must be made before uncertainty is revealed. The destination policy is a generalized version of the robust binning approach explored by (Menabde *et al.*, 2007). Processing stream decisions and penalties associated with deviation from operating targets are recourse variables, designed to adapt to information as uncertainty is revealed through the optimization process.

3.2.3 Objective function

Equation 7 presents the objective function of the optimization model which maximizes the discounted cash flows from products of the mining complex and minimizes deviations of operating targets.

$$\max \frac{1}{\|\mathbb{S}\|}\left\{\underbrace{\sum_{s\in\mathbb{S}}\sum_{t\in\mathbb{T}}\left\{\sum_{h\in\mathbb{H}} p_{h,t}\cdot v_{h,t,s}\right.}_{\text{Part I}} - \underbrace{\sum_{i\in\mathcal{P}}\left\{\sum_{p\in\mathbb{P}} \left(PC_{i,p,t} + RH_{i,p,t} + TC_{i,p,t}\right)\cdot v_{p,i,t,s}\right.}_{\text{Part II}} - \right.$$

$$\left.\left.\left.\underbrace{\left(c^{+}_{p,t}\cdot d^{+}_{p,t,s} + c^{-}_{p,t}\cdot d^{-}_{p,t,s}\right)}_{\text{Part III}} - \underbrace{\sum_{h\in\mathbb{H}}\left(c^{+}_{h,t}\cdot d^{+}_{h,t,s} + c^{-}_{h,t}\cdot d^{-}_{h,t,s}\right)}_{\text{Part IV}}\right\}\right\}\right\} -$$

$$\sum_{t\in\mathbb{T}}\left\{\underbrace{\sum_{b\in\mathbb{B}_m}\left\{\left(MC_{b,t}\cdot x_{b,t} + c^{smooth}_{b,t}\cdot d^{smooth}_{b,t}\right) - \sum_{v\in\mathbb{V}_b} c^{sink}_{b,t}\cdot d^{sink}_{b,t,v}\right\}}_{\text{Part V}} - \underbrace{\sum_{k\in\mathbb{k}} p_{k,t}\cdot w_{k,t}}_{\text{Part VI}}\right\} \quad (7)$$

Part I represents the discounted revenues from the products sold. Part II represents the cost of the processing, re-handling and treating the tailings of extracted material. Parts III and IV represent the penalty costs for deviating from operating targets related to mining, processing, and tailings treatment activities. Part V represents the cost of mining and the penalties associated with deviating from smoothing and sink rate targets. Part VI represents the costs associated with capital expenditure options.

3.2.4 Constraints

The objective function is subject to a large set of constraints, namely: material flow, mine level attribute calculation, recovery definition, end of year stockpile calculations, one-time CapEx, mine reserve and slope, destination policy, and scheduling constraints. Only select constraints relevant to the case study are described in this work as the rest are thoroughly detailed in (Goodfellow, 2014; Goodfellow and Dimitrakopoulos, 2015). Equations 8-10 define non-additive attribute ($h \in \mathbb{H}$) calculations and deviation constraints.

$$v_{h,t,s} = f_h\left(v_{p,i,t,s}, w_{k,t}\right)\ \forall\, h \in \mathbb{H}, t \in \mathbb{T}, s \in \mathbb{S} \quad (8)$$

$$v_{h,t,s} - d^{+}_{h,t,s} \le U_{h,i,t}\ \forall\, h \in \mathbb{H}, t \in \mathbb{T}, s \in \mathbb{S} \quad (9)$$

$$v_{h,t,s} + d^{-}_{h,t,s} \ge L_{h,i,t}\ \forall\, h \in \mathbb{H}, t \in \mathbb{T}, s \in \mathbb{S} \quad (10)$$

Equation 11 is the one-time CapEx option constraint. Equation 12 models the end-of-year stockpile constraint. The most critical operating constraint in the case study below is related to the maximum capacity of the TSF meaning that it is necessary to account for the accumulating tailings volume year by year. The TSF is modelled as a stockpile to exploit the structure of Eq. 12 which allows $v_{h,t,s}$ to calculate the cumulative tailings volume.

$$\sum_{t\in\mathbb{T}} w_{k,t} \leq 1 \; \forall \, k \in \mathbb{K}^1 \tag{11}$$

$$v_{h,t,s} = v_{p,i,t,s} \cdot \left(1 - \sum_{j\in\mathcal{O}_i} y_{i,j,t,s}\right) \forall \, i \; \in \; S, t \in \mathbb{T}, s \in \mathbb{S} \tag{12}$$

3.2.5 Solution method

The simultaneous stochastic optimization model is solved using a combination of metaheuristic algorithms. While metaheuristics do not guarantee the discovery of optimal solutions, they have been proven to generate high-quality solutions to mining optimization problems (Godoy and Dimitrakopoulos, 2004; Lamghari and Dimitrakopoulos, 2012; Goodfellow and Dimitrakopoulos, 2016; Lamghari and Dimitrakopoulos, 2016; Montiel and Dimitrakopoulos, 2017). The solution approach used in this application uses both simulated annealing and particle swarm optimization (Kirkpatrick *et al.*, 1983; Kennedy, 1995). Perturbing CapEx decision neighborhoods that can modify capacities and production targets can make the search for an optimum solution akin to chasing a moving target. To mitigate this effect, once a CapEx decision neighborhood perturbation is accepted it cannot be perturbed again for a specified number of iterations (Del Castillo and Dimitakopoulos, 2019). In the case study below the number is empirically determined and set to twenty-five thousand iterations.

3.3 Case Study

This section describes the application and results of the method at a real-world multi-pit, multi-product gold mining complex. The results are reported as a probabilistic risk analysis on key project indicators which are compared to the project's operating forecasts (base case) where appropriate comparable data is available. Sections 3.3.2.1 and 3.3.2.2 present and discuss the results of the simultaneous stochastic optimization (SSO) case and the simultaneous stochastic expansion (SSE) case, respectively.

3.3.1 Generalities of the case study

In Section 3.3.2.1 the SSO case considers the mining complex as depicted in Fig. 13 and described thereafter. The base case used for comparison is simply the project's current operating forecasts, generated by a conventional mine planning process with deterministic inputs. This comparison is meant to provide the reader with outputs from two different planning approaches and have a base case that reflects the decision-making reality of mining companies. However, it is important to remember that numerous studies (Ravenscroft, 1992; Dowd, 1994; Dimitrakopoulos *et al.*, 2002) have highlighted the technical risk associated with conventional mine planning forecasts. A major reason for this comes from the 'smoothing' aspect of estimated methods which leads to misrepresented of proportions of high and low-grades in a mineral deposit. The grade-tonnage curves in Fig. 14 of the simulated realizations compared to their averaged model reflects the different representation of grades in the inputs to the stochastic and conventional mine planning approaches.

In Section 3.3.2.2 the SSE case gives the optimizer the flexibility to make a one-time CapEx investment to expand the tailings storage facility capacity by 25% for 200 million USD. There is a two-year lead time between the investment and the realization of extra capacity. The results are compared with the SSO case to evaluate the growth potential of the project.

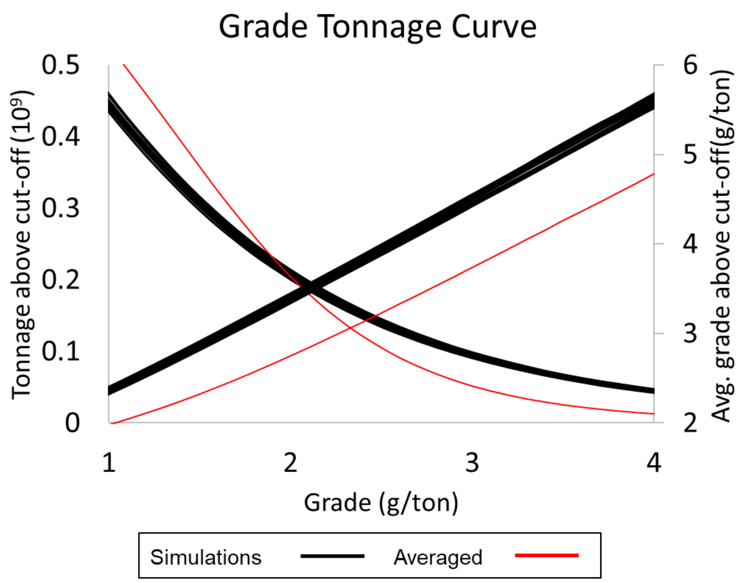

Figure 14: Gold grade tonnage curves of simulations and the average model of the deposit

3.3.2 Results and comparisons

3.3.2.1 Simultaneous stochastic optimization case

Figure 15 presents the SSO and the base case mine tonnage forecasts on an annual and cumulative basis. Mine tonnage is the same in all scenarios because the extraction sequence is scenario independent and density is not a simulated attribute. The SSO plan forecasts a stark divergence from the base case strategy of aggressively mining for the next six years and then reclaiming stockpiles until the tailings storage facility reaches maximum capacity. Conversely, the SSO plan produces a reasonably stable mining rate through the first fifteen years, utilizing both stockpiled inventory and material from the pits to satisfy processing needs. The total tonnage extracted over the life of mine is nine percent less than in the base case, suggesting a more efficient use of resources, particularly, coveted tailings storage facility volume.

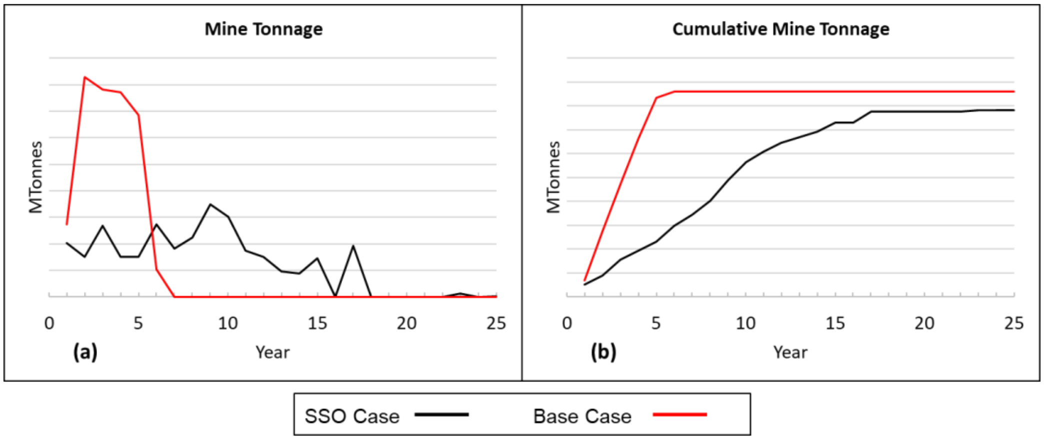

Figure 15: Production forecasts for SSO and base case (a) mine tonnage and (b) cumulative mine tonnage.

Figure 16 shows the SSO case mill throughput forecasts and the P50 values for the proportion of material source in each period, either directly from the two open pits or stockpiled inventory. The mill capacity is almost fully satisfied through the next nineteen periods. The throughput is balanced between mined and stockpiled material through the first seventeen periods. This composition depends on economic and operating factors, such as the quality of material available in the stockpiles, cost, and quality of material available in the pits during those periods. It is important to note the conventional modelling of existing stockpile inventory hinders the effective quantification and management of risk in the throughput forecasts. Forty-five percent of the total processed material comes from the existing stockpile inventory, although the data was not available for this project, simulating existing stockpile inventory would provide more robust forecasts and should be considered in future work.

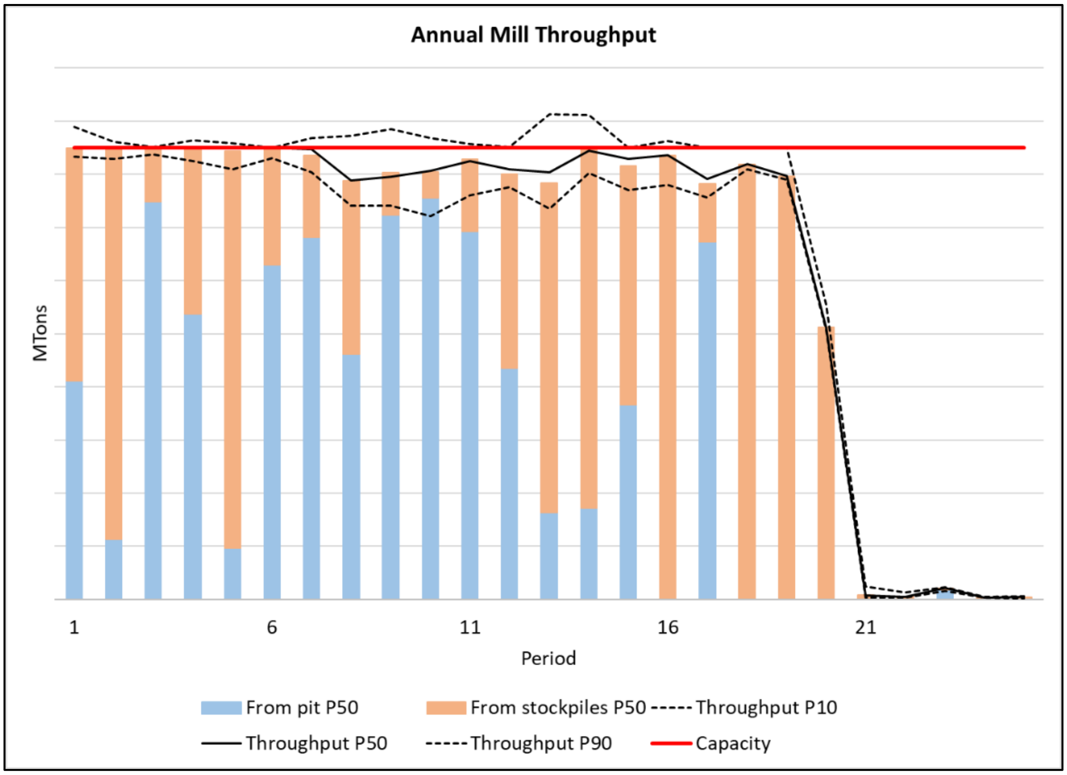

Figure 16: SSO case mill throughput forecasts (black) and material source proportions from the mines (blue bar) and the stockpiles (orange bar)

Figure 17 presents the SSO and base case production forecasts for the project's primary (gold) and secondary (copper and silver) products. Figure 17 (a) illuminates the mine's current strategy, known as 'high-grading.' Where the aggressive mining rate is used to extract and recover as many ounces as possible over first several years by processing high-grade ore and stockpiling medium and low-grade ore for later. The SSO plan also forecasts higher ounce recoveries in the first six years. However, this is not an explicitly stated goal but a consequence of the holistic, simultaneous optimization plan to maximize value over the life of the mining complex. The drastic differences in mining rates over the first six years explain the differences in forecasted ounces over that time horizon. However, as previously noted, the base case forecasts do not account for supply uncertainty and whether they are achievable remains undetermined. The stochastic plan more delivers a stable and consistent ounce profile until the twenty-first year, resulting in a twelve percent cumulative improvement in ounces recovered compared to the base case, shown in Fig. 17 (b).

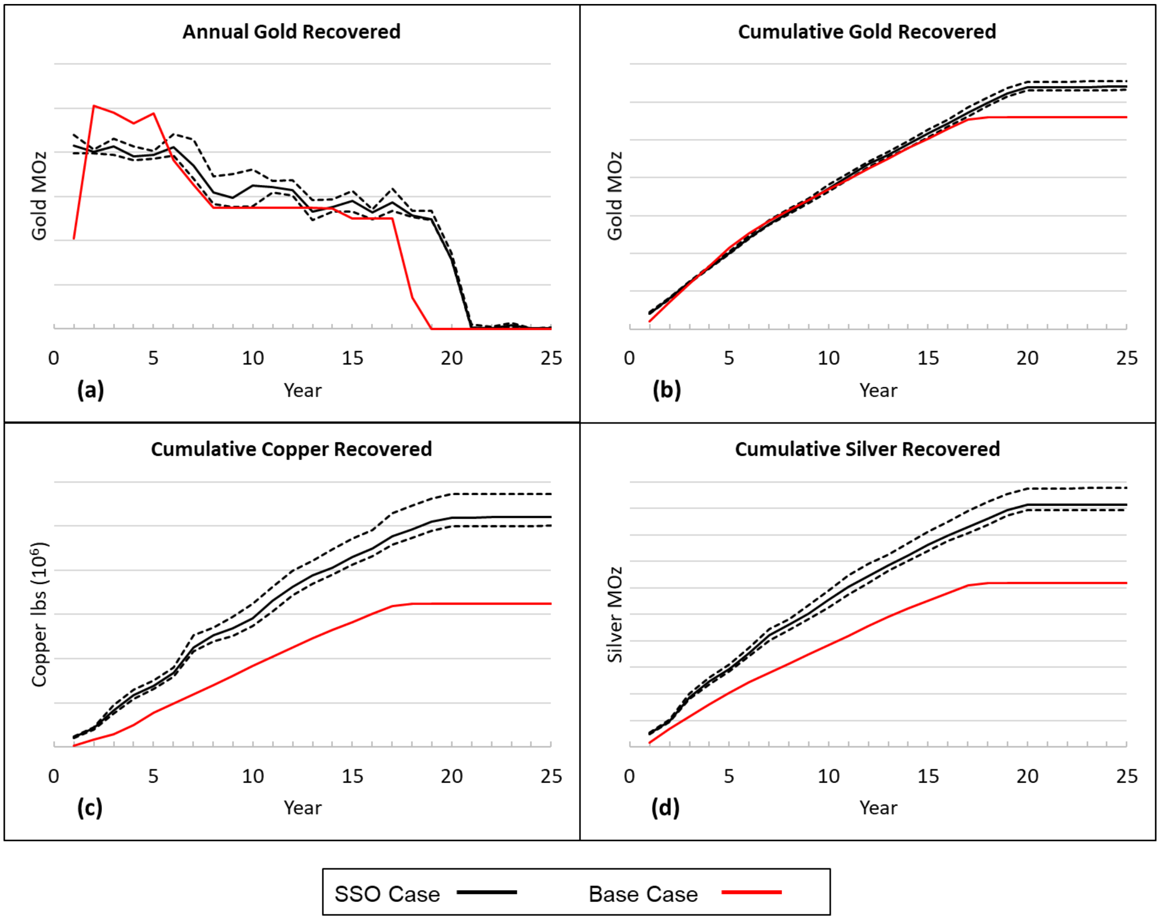

Figure 17: Metal production forecasts. Dotted lines represent P10/P90 of the SSO plan values; solid lines represent P50.

Figure 17 (c) and (d) shows significant differences in secondary metal forecasts, with the SSO plan projecting thirty-eight and thirty-two percent higher copper and silver production than the base case forecast, respectively. Similarly, to the gold production forecasts, the production profiles are stable and resilient throughout the twenty-one years. Figure 18 (a) highlights the mine's early period high-grading strategy. However the SSO plan delivers higher gold grades from year five onwards, mirroring Fig. 17 (a). Figure 18 (b) and (c) show that SSO plan forecasts consistently higher feed grades for the secondary metals. All three feed grade forecasts are consistent with the representations of the grade-tonnage curves, although only the gold grade-tonnage curve is shown in Fig. 14. The feed grade forecasts of all three metals for periods eighteen, nineteen and twenty, where the feed is composed solely of stockpile inventory, reaffirm the need for simulations to quantify the risk associated with this material supply. The feed grades of the secondary metals fluctuate more drastically than the primary metal; it is suggested that this is a consequence of a single-element cut-off grade policy, optimized for gold grades.

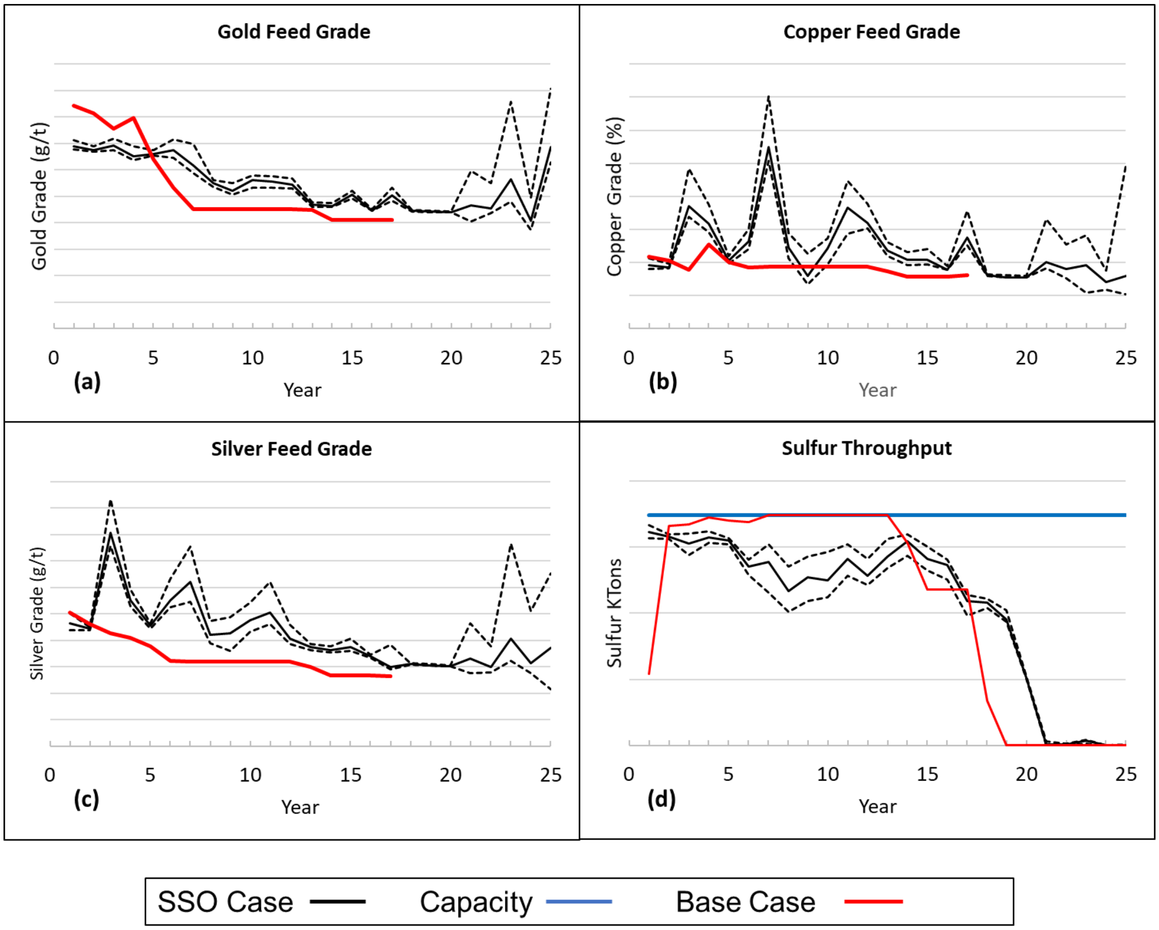

Figure 18: Metal feed grade and sulfur throughput forecasts. Dotted (black) lines represent P10/P90 of the SSO plan values, solid (black) lines represent P50

Figure 18 (d) reports the tons of sulfur processed in each year of the SSO and base case forecasts. Respecting the sulfur processing capacity critical to the metal recovery circuit. Figure 19 presents the cumulative tailings volume and tailings storage facility capacity which is the bottleneck of the system. The impact of the aggressive mining rate forecasted by the base case during the first six years of production on the rate of tailings deposition is evident. The base case forecasts that it will run out of capacity in the tailings storage facility in year 17, whereas the SSO plan runs out in year 19. The more balanced mining and reclamation rates in the SSO plan delays the need for significant CapEx investment to expand the tailings storage facility capacity by two years.

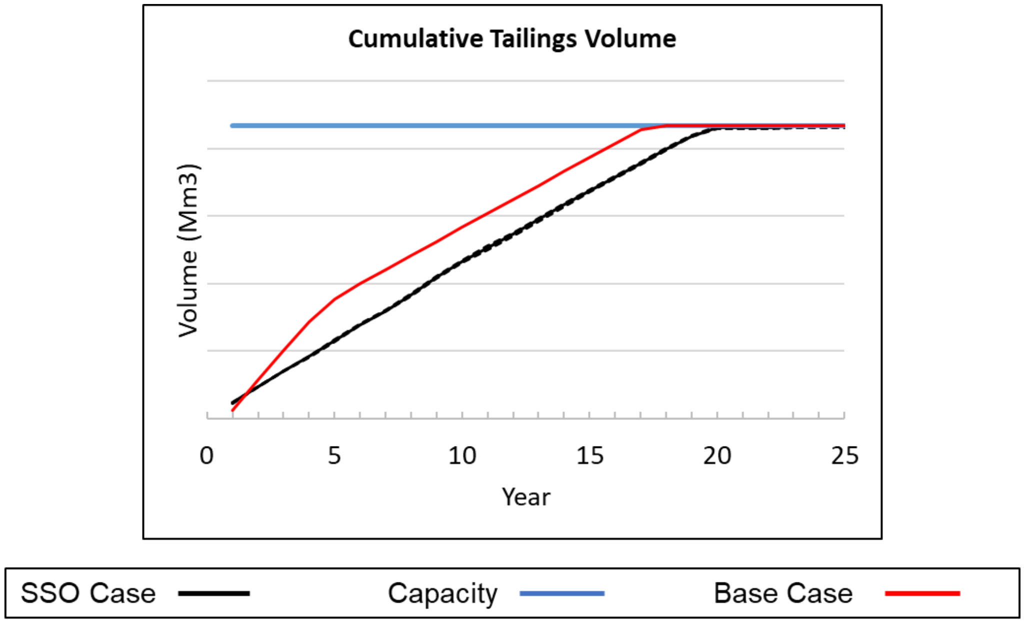

Figure 19: Cumulative tailings volume forecasts. Dotted lines represent P10/P90 of the SSO case values; solid lines represent P50

3.3.2.2 Simultaneous stochastic expansion case

This section examines selected results from the SSE case that highlight the comparison to the SSO case presented in Section 3.3.2.1. Figure 20 provides the tailings volume forecasts for both the SSO and SSE cases. In the SSE case, the extra capacity (25% increase) becomes available in year 19 which is approximately when the SSO case reaches the maximum tailings storage capacity. The rate of deposition increases slightly from years 19 through 22, and the expansion is fully utilized by period 24. As previously mentioned, the expansion has a two-year lead time, meaning the optimizer must exercise the CapEx option in year 17 to realize the extra capacity in year 19. The impact of the investment decision on year 17 discounted cash flow is clear in Fig. 21 (a). Despite the significant investment, the operation remains cashflow positive in all periods of the SSE forecasts, including year 17. Moreover, the expansion allows for significantly higher discounted cash flow forecasts in years 19 – 24, leading to 4% higher P50 values for cumulative discounted cash flow. The financial impact of the expansion is somewhat muted due to discounting over the time horizon. However, the upside effect on gold production, mining, and processing tonnages is substantial.

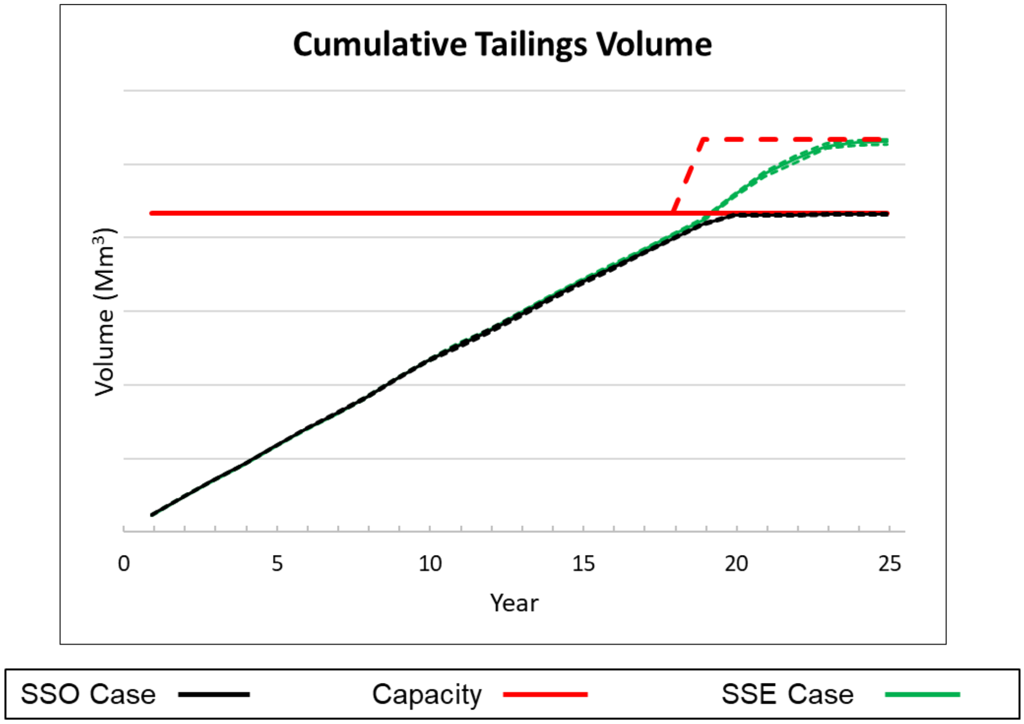

Figure 20: Cumulative tailings volume forecasts. Dotted lines represent P10/P90 of forecasts and the expanded capacity (dotted red)

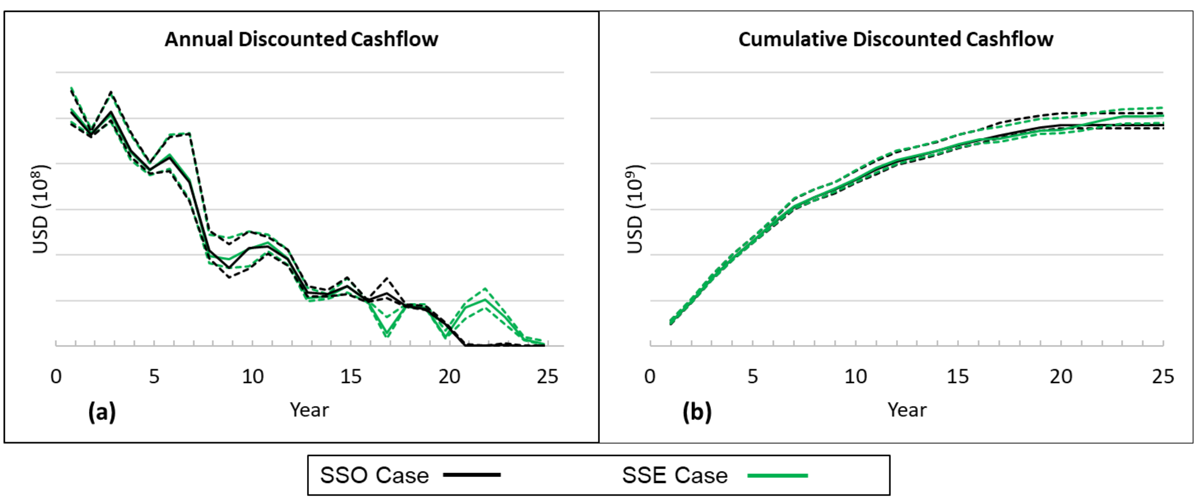

Figure 21: Discounted cash flow forecasts. Dotted lines represent P10/P90, solid lines represent the P50

The impact of the expansion on the gold production profile in Fig. 22 (a) is extremely positive, extending ounce production until year 25 with a significant uptick in years 21 and 22. The expansion allows for a 14% cumulative increase in gold production over the SSO case. Figure 22 (b) shows the increased mining rate over the last years due to the expansion. There are several reasons related to the mill throughput and stockpile inventories and tailings storage capacity, shown in Fig. 23, that explain the significant increase in mining rate after the expansion in year 19. When the tailings storage facility is close to reaching its maximum capacity, the optimizer is discouraged from mining and processing material from the pit because of PAG waste rock, along with process tails fills the little remaining volume. Processing stockpiled inventory only generates process tails, making it more volume efficient in the tailings storage facility; this is evident looking at the mining rate and composition of the mill throughput in years 18 and 19. From year 20 onwards, 44% of processed material comes from stockpiles, compared to only 22% before year 20. The quality of material mined from the pit in the post-expansion periods blends with material from the stockpile to raise feed grades, improving the ounce profile. Figure 24 shows a plan view comparison of both extraction sequences. Note the expanded footprint in the central areas between the two pits which allows the SSE case to access higher quality material at depth in later periods. Thus, the method demonstrates the tailings storage facility expansion yields favorable results: the extra capacity is fully utilized, there is a significant increase in metal production and the operation continues to generate positive cash flows until the end of the time horizon.

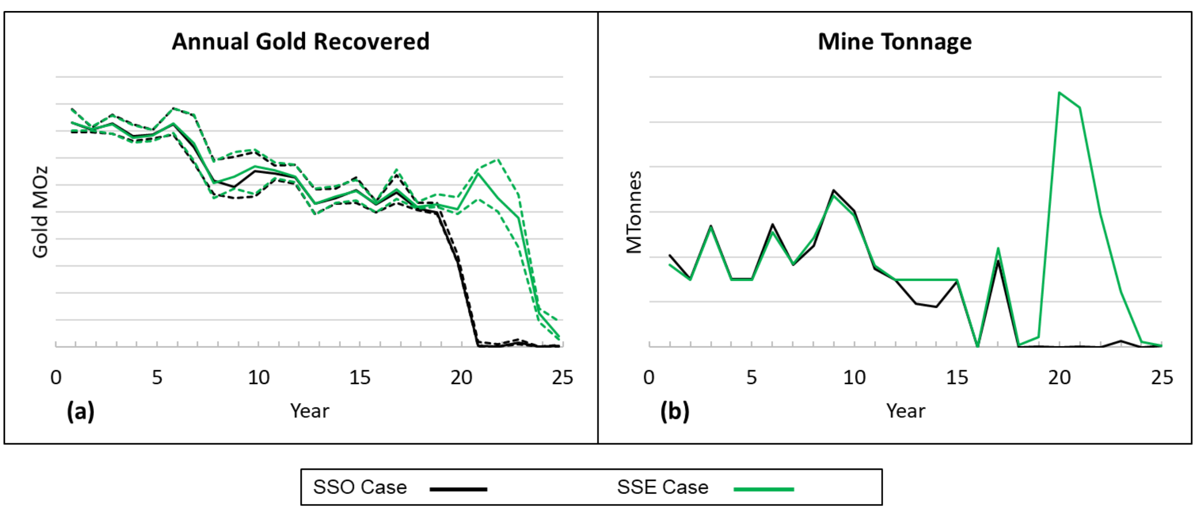

Figure 22: Gold production and mine tonnage forecasts. Dotted lines represent P10/P90

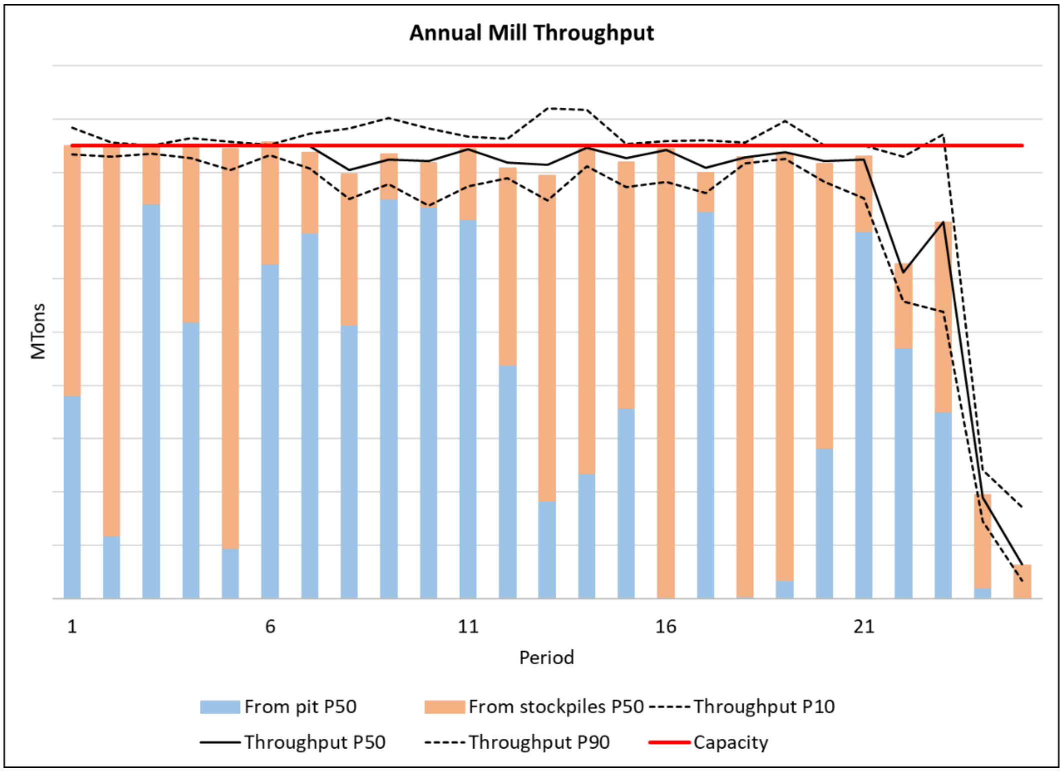

Figure 23: SSE case mill throughput forecasts (black) and material source proportions from the mines (blue bar) and the stockpiles (orange bar)

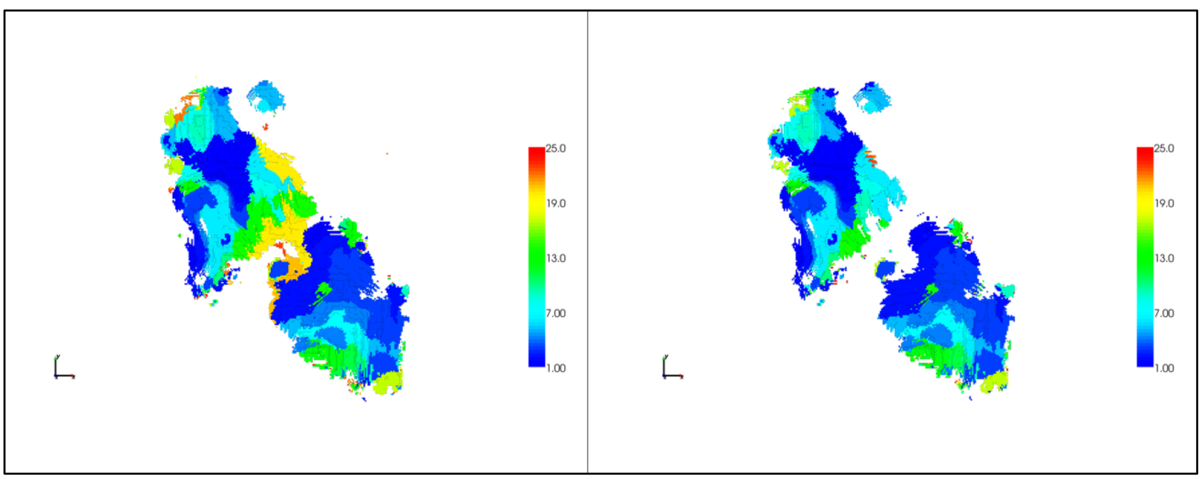

Figure 24: Plan view comparison of the extraction sequences. SSE case (left), SSO case (right)

3.4 Conclusions

This paper presents an application of a simultaneous stochastic optimization framework that manages tailings, acid generating waste rock deposition and evaluates the use of a CapEx option to expand the mining complex's main bottleneck to extend its life. The case study presents the successful optimization central, interrelated components for the long-term production planning of a mining complex via a unified simultaneous optimization model. The method generates an extraction sequence, destination policy, downstream material flows and CapEx investment decisions. The case study is based on a real world-class gold mining complex with copper and silver by-products that comprises multiple pits, an autoclave processing facility to recover metal from the sulfide ore, and a critical tailings storage facility (TSF) to handle the PAG waste products. The environmental constraints surrounding the TSF act as the main bottleneck limiting the potential growth of the mining complex. A TSF expansion (or construction of a new facility) is an extremely capital-intensive undertaking that will cost several hundred million dollars.

This work critically evaluates this potential option while managing technical risk by accounting for supply uncertainty and its effects on each component of the mineral value chain. The results demonstrate that a 25% expansion of the TSF capacity is fully utilized by a significantly enlarged mine footprint, generating 14% more gold ounces and a 4% higher NPV than the simultaneous stochastic (SSO) plan without the expansion. Furthermore, the TSF capacity appears to still be the bottleneck even after the expansion, signalling the potential for the construction of a larger facility and a longer mine life.

The study presented herein also highlights major differences from the mine's current mining and reclamation plan, underscoring the need for risk evaluation testing and offering alternatives to the current, conventional plan. The SSO plan shows real upside in terms of gold, copper and silver production as well as a more efficient use of in-pit and ex-pit resources including the existing TSF. This upside is a result of the framework capitalizing on synergies within the mining complex and management of in-situ supply uncertainty. However, the study identifies the existing stockpile inventory, which makes up 45% of the material processed in the base case plan, as a significant source of supply risk. Stockpile sampling and simulation programs in other studies (Dirkx and Dimitrakopoulos, 2018) show the real extent of material variability in large stockpile inventories, and the grade-tonnage curves in Fig. 14 show the different representations of material proportions in this deposit. These point to a potential misrepresentation of material quality within the existing stockpile inventory and vulnerabilities in the current mine plan which aims to spend 18 years feeding the processing facilities exclusively through stockpile reclamation. Future work should include sampling and simulating existing stockpiles to generate a truly risk-resilient LOM plan. The extensions should consider a larger expansion of the TSF and by extension the mine life as well as the incorporation of the limestone quarrying which provides lime that is critical for the mineral processing circuit.

Chapter 4 Conclusion

4.1 General Conclusions

Advancements in the fields of mine planning, geostatistics and digital computation over the last several decades culminate in simultaneous stochastic optimization frameworks for mining complexes (Montiel and Dimitrakopoulos, 2015; Goodfellow and Dimitrakopoulos, 2016; Montiel *et al.*, 2016; Goodfellow and Dimitrakopoulos, 2017; Del Castillo and Dimitakopoulos, 2019). These methods represent a substantial step-change, address many limitations of previous methods as described in Chapter 1, and further contribute to a paradigm shift in strategic mine planning under uncertainty. This thesis presents research that advances use of these state-of-the-art approaches through two applications at case studies comprising large multi-pit mining complexes. The case studies are designed to explore how various aspects of the framework presented in (Goodfellow and Dimitrakopoulos, 2016, 2017) perform under real-world complexities such as joint-uncertainty scenarios, non-linear blending constraints, multiple non-linear processing streams, stockpiling, tailings management, and CapEx investment options.

The first case study assesses the efficacy of a dynamic cut-off grade optimization strategy that works by conjunction with the simultaneous optimization of extraction sequence and processing stream decisions. The case study results show improvements over traditional approaches (Lane, 1988; Rendu, 2014) in meeting capacity and non-linear blending requirements of multiple processors from several sources of uncertain and highly variable material. Moreover, the simultaneous optimization approach allows for a significant reduction in the number of stockpiles that need to be maintained leading to improvements in operating efficiencies. This case study also incorporates market uncertainty as an input to the mine planning framework that considers joint geological and commodity price uncertainty scenarios. This is the first attempt in technical literature to simultaneously optimize extraction sequence, destination policy and processing stream decisions under both sources of uncertainty. The results highlight the flexibility of the framework to adapt the production schedule to periods of higher and lower commodity prices. Although the solution can be sensitive to the set of price simulations, the framework establishes an approach to effectively quantify the risk to various key project indicators across a set of non-linear transfer functions comprising the mineral value chain.

The highly generalized nature of the simultaneous stochastic optimization framework (Goodfellow and Dimitrakopoulos, 2015, 2016) allows for the modelling of several downstream value chain components. The formulation's flexibility along with the introduction of a CapEx investment option enables the second case study of this thesis to consider environmental issues relating to tailings management and model a tailings facility expansion. The application at a multi-element open pit mining complex in Central America simultaneously optimizes the extraction sequence, cut-off grades, and downstream decisions from two open-pits with a set of stockpiling options, an autoclave and a tailings storage facility. The project bottleneck is the tailings facility volume because it stores both process tails, and potentially acid-generating waste rock from the mines. Results show that, when given the option, the optimizer chooses to make a significant CapEx investment to expand the tailings storage facility 25% by volume. This expansion allows for a meaningful expansion of both pit limits, 40% by mass, resulting in an extended metal production and revenue generation horizon that yields 14% more gold ounces and a 4% improvement in NPV for the mining complex. The framework provides decision makers with a realistic evaluation of the investment's impact on the mining complex. Moreover, comparing the deposit's estimated and simulated grade-tonnage curves as well as stochastic and deterministic forecasts stresses the need for a thorough risk analysis of the existing long-term mine plan.

4.2 Recommendations and Future Work

More intelligent and efficient solution approaches are critical to advancing research in this field. Today's simultaneous stochastic optimization formulations are flexible enough to incorporate various sources of uncertainty and complex interrelated components across mineral value chains. As models become more and more granular, modelling detailed interactions under thousands of joint-uncertainty scenarios with millions of integer variables, solution times can stretch beyond what is practical and feasible for both industrial and research purposes. Moreover, a limitation of the metaheuristic solution algorithms used in this work is the requirement of several user-defined parameters. These parameters are empirically determined for each case, but fine tuning can be a time consuming and subjective process. This drives a need for solution approaches that intelligently explore the exhaustive solution spaces in a more efficient manner such as (Lamghari and Dimitrakopoulos, 2018) which combines machine learning and combinatorial optimization techniques and requires significantly less user-defined parameters. As solution algorithms improve and leverage technological advances such as GPU computing, it will become more viable to generate stable and robust global solutions under various sources of uncertainty such as supply, demand and cost.

Destination policies remain an area of particular interest because single element cut-off grade optimization can be misleading in multi-metal mining complexes with deleterious elements, stockpiles and strict blending demands. Multi-element destination policies such that utilize forms of intelligent material classification such as clustering (Arthur and Vassilvitskii, 2007; Del Castillo and Dimitrakopoulos, 2016) have the potential to add significant value to the optimization framework. In a similar vein, the connection between long-term strategic plans and short-term operating plans is an important area of study to ensure that the notable effort spent to generate optimal long-term mine plans is actualized on an operating basis. More specifically through the use of advanced digital sensors to provide real-time monitoring capabilities and feedback within the system of the mining complex. The incorporation of new data to update reserve models, long-term plans and adapt short term plans (Paduraru and Dimitrakopoulos, 2018; Yüksel *et al.*, 2018; Paduraru and Dimitrakopoulos, 2019) has the potential to unlock meaningful value for operating mining complexes.

References

Albor, F.R., Dimitrakopoulos, R., 2009. Stochastic mine design optimisation based on simulated annealing: pit limits, production schedules, multiple orebody scenarios and sensitivity analysis. Mining Technology 118, 79-90. 10.1179/037178409X12541250836860

Arthur, D., Vassilvitskii, S., 2007. k-means++: The advantages of careful seeding, Proceedings of the eighteenth annual ACM-SIAM symposium on Discrete algorithms. Society for Industrial and Applied Mathematics, pp. 1027-1035.

Asad, M., Dimitrakopoulos, R., 2013a. Implementing a parametric maximum flow algorithm for optimal open pit mine design under uncertain supply and demand. Journal of the Operational Research Society 64, 185-197.

Asad, M.W.A., Dimitrakopoulos, R., 2013b. A heuristic approach to stochastic cutoff grade optimization for open pit mining complexes with multiple processing streams. Resources Policy 38, 591-597.

Asad, M.W.A., Qureshi, M.A., Jang, H., 2016. A review of cut-off grade policy models for open pit mining operations. Resources Policy 49, 142-152. https://doi.org/10.1016/j.resourpol.2016.05.005

Baker, C., Giacomo, S., 1998. Resource and reserves: their uses and abuses by the equity markets, ore reserves and finance: a joint seminar between australasian institute of mining and metallurgy and ASX.

Benndorf, J., Dimitrakopoulos, R., 2007. New efficient methods for conditional simulation of large orebodies. Orebody Modelling and Strategic Mine Planning, Spectrum Series 14, 61-67.

Benndorf, J., Dimitrakopoulos, R., 2013. Stochastic long-term production scheduling of iron ore deposits: integrating joint multi-element geological uncertainty. Journal of Mining Science 49, 68-81.

Benning, I., 2000. Bankers' perspective of mining project finance. Journal of the Southern African Institute of Mining and Metallurgy 100, 145-152.

Bernard, J.T., Khalaf, L., Kichian, M., McMahon, S., 2008. Forecasting commodity prices: GARCH, jumps, and mean reversion. Journal of Forecasting 27, 279-291.

Birge, J.R., Louveaux, F., 2011. Introduction to stochastic programming. Springer Science & Business Media.

Boland, N., Dumitrescu, I., Froyland, G., 2008. A multistage stochastic programming approach to open pit mine production scheduling with uncertain geology. Optimization Online.

Boucher, A., Dimitrakopoulos, R., 2009. Block Simulation of Multiple Correlated Variables. Mathematical Geosciences 41, 215-237. 10.1007/s11004-008-9178-0

Boucher, A., Dimitrakopoulos, R., 2012. Multivariate Block-Support Simulation of the Yandi Iron Ore Deposit, Western Australia. Mathematical Geosciences 44, 449-468. 10.1007/s11004-012-9402-9

Briggs, A.H., Weinstein, M.C., Fenwick, E.A., Karnon, J., Sculpher, M.J., Paltiel, A.D., 2012. Model parameter estimation and uncertainty analysis: a report of the ISPOR-SMDM Modeling Good Research Practices Task Force Working Group–6. Medical decision making 32, 722-732.

Caccetta, L., Hill, S.P., 2003. An application of branch and cut to open pit mine scheduling. Journal of global optimization 27, 349-365.

Castillo, F.D., Dimitrakopoulos, R., 2014. Joint effect of commodity price and geological uncertainty over the life of mine and ultimate pit limit. Mining Technology 123, 207-219.

Chanda, E., 2007. Network linear programming optimisation of an integrated mining and metallurgical complex, In: Dimitrakopoulos, R. (Ed.), Orebody Modelling and Strategic Mine Planning. AusIMM, Spectrum Series 14, Melbourne, pp. 149-155.

Cole, J., Rust, J., 2002. Pressure Oxidation Control Strategies, In: Mular, A.L., Halbe, D.N., Barratt, D.J. (Eds.), Mineral processing plant design, practice and control Proceedings. SME, pp. 2196-2208.

Dagdelen, K., 2001. Open pit optimization-strategies for improving economics of mining projects through mine planning, 17th International Mining Congress and Exhibition of Turkey, pp. 117-121.

Dagdelen, K., Traore, I., 2014. Open pit transition depth determination through global analysis of open pit and underground mine production scheduling, In: Dimitrakopoulos, R. (Ed.), Orebody modelling and strategic mine planning. . AusIMM, Spectrum Series 14, Melbourne, pp. 195-200.

Dantzig, T., 2003. Number the Language of Science (1930). Kessinger Publishing.

David, M., 1988. Handbook of applied advanced geostatistical ore reserve estimation. Elsevier.

David, M., Dagbert, M., Beauchemin, Y., 1977. Statistical analysis in geology: correspondence analysis method. na.

Davis, M.W., 1987. Production of conditional simulations via the LU triangular decomposition of the covariance matrix. Mathematical Geology 19, 91-98.

de Carvalho, J.P., Dimitrakopoulos, R., 2019. High-order Block-support Simulation, Application in the Production Optimization of a Gold Mining Complex and Comparisons. Mathematical Geosciences.

Del Castillo, F., Dimitakopoulos, R., 2019 Dynamically Optimizing the Strategic Plan of Mining Complexes under Supply Uncertainty. Resources Policy. DOI: 10.1016/j.resourpol.2018.11.019

Del Castillo, M.F., Dimitrakopoulos, R., 2016. A multivariate destination policy for geometallurgical variables in mineral value chains using coalition-formation clustering. Resources Policy 50, 322-332.

Deng, S., 2000. Stochastic models of energy commodity prices and their applications: Mean-reversion with jumps and spikes. University of California Energy Institute Berkeley.

Desbarats, A.J., Dimitrakopoulos, R., 2000. Geostatistical Simulation of Regionalized Pore-Size Distributions Using Min/Max Autocorrelation Factors. Mathematical Geology 32, 919-942. 10.1023/a:1007570402430

Dimitrakopoulos, R., 1997. Conditional simulations: tools for modelling uncertainty in open pit optimisation. Optimizing with Whittle, Perth, Whittle Programing Pty. Ltd, 31-42.

Dimitrakopoulos, R., 1998. Conditional simulation algorithms for modelling orebody uncertainty in open pit optimisation. International Journal of Surface Mining, Reclamation and Environment 12, 173-179.

Dimitrakopoulos, R., 2011. Stochastic Optimization for Strategic Mine Planning: A Decade of Developments. Journal of Mining Science 47, 138-150.

Dimitrakopoulos, R., Farrelly, C.T., Godoy, M., 2002. Moving forward from traditional optimization: grade uncertainty and risk effects in open-pit design. Mining Technology 111, 82-88. DOI: 10.1179/mnt.2002.111.1.82

Dimitrakopoulos, R., Grieco, N., 2009. Stope design and geological uncertainty: Quantification of risk in conventional designs and a probabilistic alternative. Journal of Mining Science 45, 152-163. 10.1007/s10913-009-0020-y

Dimitrakopoulos, R., Luo, X., 2004. Generalized Sequential Gaussian Simulation on Group Size ν and Screen-Effect Approximations for Large Field Simulations. Mathematical Geology 36, 567-591. 10.1023/B:MATG.0000037737.11615.df

Dimitrakopoulos, R., Martinez, L., Ramazan, S., 2007. A maximum upside/minimum downside approach to the traditional optimization of open pit mine design. Journal of Mining Science 43, 73-82.

Dimitrakopoulos, R., Mustapha, H., Gloaguen, E., 2010. High-order statistics of spatial random fields: exploring spatial cumulants for modeling complex non-Gaussian and non-linear phenomena. Mathematical Geosciences 42, 65.

Dimitrakopoulos, R., Ramazan, S., 2004. Uncertainty based production scheduling in open pit mining. SME Transactions 316, 106-112.

Dimitrakopoulos, R., Ramazan, S., 2008. Stochastic integer programming for optimising long term production schedules of open pit mines: methods, application and value of stochastic solutions. Mining Technology 117, 155-160.

Dirkx, R., Dimitrakopoulos, R., 2018. Optimizing Infill Drilling Decisions Using Multi-Armed Bandits: Application in a Long-Term, Multi-Element Stockpile. Mathematical Geosciences 50, 35-52. DOI: 10.1007/s11004-017-9695-9

Dooley, G., Lenihan, H., 2005. An assessment of time series methods in metal price forecasting. Resources Policy 30, 208-217.

Dowd, P., 1994. Risk assessment in reserve estimation and open-pit planning. Transactions of the Institution of Mining and Metallurgy (Section A: Mining Industry) 103, A148–A154.

Epstein, R., Goic, M., Weintraub, A., Catalán, J., Santibáñez, P., Urrutia, R., Cancino, R., Gaete, S., Aguayo, A., Caro, F., 2012. Optimizing Long-Term Production Plans in Underground and Open Pit Copper Mines. Operations Research 60, 4-17. 10.2307/41476334

Farmer, I., 2017. Stochastic mining supply chain optimization: a study of integrated capacity decisions and pushback design under uncertainty, M.Eng. Thesis, Mining and Materials Engineering. McGill University, Montreal, QC.

Geman, S., Geman, D., 1984. Stochastic relaxation, Gibbs distributions, and the Bayesian restoration of images. IEEE Transactions on pattern analysis and machine intelligence, 721-741.

Gershon, M.E., 1983. Optimal mine production scheduling: evaluation of large scale mathematical programming approaches. International Journal of Mining Engineering 1, 315-329. DOI: 10.1007/bf00881548

Gocht, W.R., Zantop, H., Eggert, R.G., 2012. International mineral economics: mineral exploration, mine valuation, mineral markets, international mineral policies. Springer Science & Business Media.

Godoy, M., 2003. The Effective Management of Geological Risk in Long-Term Scheduling of Open Pit Mines., School of Engineering. University of Queensland, Brisbane, Australia, p. 256.

Godoy, M., Dimitrakopoulos, R., 2004. Managing risk and waste mining in long-term production scheduling of open-pit mines. SME Transactions 316.

Goodfellow, R., 2014. Unifed modelling and simultaneous optimization of open pit mining complexes with supply uncertainty, Mining and Materials Engineering. McGill University, Montreal, QC, p. 274.

Goodfellow, R., Dimitrakopoulos, R., 2013. Algorithmic integration of geological uncertainty in pushback designs for complex multiprocess open pit mines. Mining Technology 122, 67-77.

Goodfellow, R., Dimitrakopoulos, R., 2015. Stochastic optimization of open pit mining complexes with capital expenditures: application at a copper mining complex, Proceedings, Application of Computers and Operations Research in the Mineral Industry (APCOM). Society for Mining, Metallurgy & Exploration, USA, pp. 657-667.

Goodfellow, R., Dimitrakopoulos, R., 2016. Global optimization of open pit mining complexes with uncertainty. Applied Soft Computing 40, 292-304.

Goodfellow, R., Dimitrakopoulos, R., 2017. Simultaneous Stochastic Optimization of Mining Complexes and Mineral Value Chains. Mathematical Geosciences 49, 341-360. DOI: 10.1007/s11004-017-9680-3

Goovaerts, P., 1997. Geostatistics for natural resources evaluation. Oxford University Press.

Groeneveld, B., Topal, E., Leenders, B., 2018. A new methodology for flexible mine design, Advances in Applied Strategic Mine Planning. Springer, pp. 191-216.

Guardiano, F.B., Srivastava, R.M., 1993. Multivariate geostatistics: beyond bivariate moments, Geostatistics Troia'92. Springer, pp. 133-144.

Hochbaum, D.S., 2001. A new—old algorithm for minimum-cut and maximum-flow in closure graphs. Networks: An International Journal 37, 171-193.

Hoerger, S., Seymour, F., Hoffman, L., 1999. Mine planning at Newmont's Nevada operations. Mining engineering 51, 26-30.

Hustrulid, W.A., Kuchta, M., Martin, R.K., 2013. Open Pit Mine Planning and Design, Two Volume Set & CD-ROM Pack, 3rd edn. CRC Press.

Isaaks, E.H., Srivastava, R.M., 1989. An introduction to applied geostatistics. Oxford university press.

Johnson, T.B., 1968. Optimum open pit mine production scheduling. CALIFORNIA UNIV BERKELEY OPERATIONS RESEARCH CENTER.

Journel, A., Alabert, F., 1989. Non-Gaussian data expansion in the earth sciences. Terra Nova 1, 123-134.

Journel, A.G., 1974. Geostatistics for conditional simulation of ore bodies. Economic Geology 69, 673-687.

Journel, A.G., 1994. Modeling Uncertainty: Some Conceptual Thoughts, In: Dimitrakopoulos, R. (Ed.), Geostatistics for the Next Century: An International Forum in Honour of Michel David's Contribution to Geostatistics, Montreal, 1993. Springer Netherlands, Dordrecht, pp. 30-43.

Journel, A.G., 2003. Multiple-point Geostatistics: A state of the Art. Unpublished Stanford Center for Reservoir Forecasting paper.

Journel, A.G., 2005. Beyond covariance: the advent of multiple-point geostatistics, Geostatistics Banff 2004. Springer, pp. 225-233.

Journel, A.G., Deutsch, C.V., 1993. Entropy and spatial disorder. Mathematical Geology 25, 329-355.

Journel, A.G., Huijbregts, C.J., 1978. Mining geostatistics. Academic press London.

Kennedy, R., 1995. J. and Eberhart, Particle swarm optimization, Proceedings of IEEE International Conference on Neural Networks IV, pages.

Kernot, C., West, D., 1991. Mining Equities: Evaluation and Trading. Woodhead.

King, B., 2009. Optimal Mining Principles. , Orebody modelling and strategic mine planning. AusIMM, Australia.

Kirkpatrick, S., Gelatt, C.D., Vecchi, M.P., 1983. Optimization by simulated annealing. science 220, 671-680.

Kizilkale, A.C., Dimitrakopoulos, R., 2014. Optimizing mining rates under financial uncertainty in global mining complexes. International Journal of Production Economics 158, 359-365.

Kumar, A., Dimitrakopoulos, R., 2017. Introducing geometallurgical constraints into the simultaneous stochastic optimization of mining complexes: Application at a copper-gold mining, In: Dagdelen, K. (Ed.), Application of Computers and Operations Research in the Mineral Industry (APCOM), Colorado, USA.

Kumar, A., Dimitrakopoulos, R., 2019. Application of simultaneous stochastic optimization with geometallurgical decisions at a copper-gold mining complex. Mining Technology. DOI: 10.1080/25726668.2019.1575053

Labys, W.C., Lesourd, J., Badillo, D., 1998. The existence of metal price cycles1. Resources Policy 24, 147-155.

Lamghari, A., Dimitrakopoulos, R., 2012. A diversified Tabu search approach for the open-pit mine production scheduling problem with metal uncertainty. European Journal of Operational Research 222, 642-652.

Lamghari, A., Dimitrakopoulos, R., 2016. Progressive hedging applied as a metaheuristic to schedule production in open-pit mines accounting for reserve uncertainty. European Journal of Operational Research 253, 843-855.

Lamghari, A., Dimitrakopoulos, R., 2018. Hyper-heuristic approaches for strategic mine planning under uncertainty. Computers & Operations Research, In Press. DOI: 10.1016/j.cor.2018.11.010.

Lamghari, A., Dimitrakopoulos, R., Ferland, J.A., 2014. A variable neighbourhood descent algorithm for the open-pit mine production scheduling problem with metal uncertainty. Journal of the Operational Research Society 65, 1305-1314.

Lane, K.F., 1988. The economic definition of ore: cut-off grades in theory and practice. Mining Journal Books.

Leite, A., Dimitrakopoulos, R., 2007. Stochastic optimisation model for open pit mine planning: application and risk analysis at copper deposit. Mining Technology 116, 109-118.

Lerchs, H., Grossmann, I., 1965. Optimum Design of Open Pit Mines, Canadian Institute of Mining Bulletin, 58.

Luo, X., 1998. Spatiotemporal stochastic models for earth science and engineering applications.

Menabde, M., Froyland, G., Stone, P., Yeates, G., 2007. Mining schedule optimisation for conditionally simulated orebodies, In: Dimitrakpolous, R. (Ed.), Orebody modelling and strategic mine planning: Uncertainty and risk management models. AusIMM Spectrum Series.

Menabde, M., Froyland, G., Stone, P., Yeates, G., 2018. Mining Schedule Optimisation for Conditionally Simulated Orebodies, Advances in Applied Strategic Mine Planning. Springer, pp. 91-100.

Metropolis, N., Rosenbluth, A., Rosenbluth, M., Teller, A., Teller, E., 1953. Simulated annealing. Journal of Chemical Physics 21, 1087-1092.

Minniakhmetov, I., Dimitrakopoulos, R., 2017a. A High-Order, Data-Driven Framework for Joint Simulation of Categorical Variables, Geostatistics Valencia 2016. Springer, pp. 287-301.

Minniakhmetov, I., Dimitrakopoulos, R., 2017b. Joint high-order simulation of spatially correlated variables using high-order spatial statistics. Mathematical Geosciences 49, 39-66.

Minniakhmetov, I., Dimitrakopoulos, R., Godoy, M., 2018. High-order spatial simulation using Legendre-like orthogonal splines. Mathematical Geosciences, 1-28.

Montiel, L., 2014. On Globally Optimizing a Mining Complex under Supply Uncertainty: Integrating Components from Deposits to Transportation Systems, Mining and Materials Engineering. McGill University, Montreal, Quebec, p. 181.

Montiel, L., Dimitrakopoulos, R., 2013. Stochastic mine production scheduling with multiple processes: Application at Escondida Norte, Chile. Journal of Mining Science 49, 583-597.

Montiel, L., Dimitrakopoulos, R., 2015. Optimizing mining complexes with multiple processing and transportation alternatives: An uncertainty-based approach. European Journal of Operational Research 247, 166-178.

Montiel, L., Dimitrakopoulos, R., 2017. A heuristic approach for the stochastic optimization of mine production schedules. Journal of Heuristics 23, 397-415.

Montiel, L., Dimitrakopoulos, R., 2018. Simultaneous stochastic optimization of production scheduling at Twin Creeks Mining Complex, Nevada. Mining engineering 70, 12-20.

Montiel, L., Dimitrakopoulos, R., Kawahata, K., 2016. Globally optimising open-pit and underground mining operations under geological uncertainty. Mining Technology 125, 2-14.

Mustapha, H., Dimitrakopoulos, R., 2010. High-order stochastic simulation of complex spatially distributed natural phenomena. Mathematical Geosciences 42, 457-485.

Mustapha, H., Dimitrakopoulos, R., 2011. HOSIM: A high-order stochastic simulation algorithm for generating three-dimensional complex geological patterns. Computers & Geosciences 37, 1242-1253.

Paduraru, C., Dimitrakopoulos, R., 2018. Adaptive policies for short-term material flow optimization in a mining complex. Mining Technology 127, 56-63.

Paduraru, C., Dimitrakopoulos, R., 2019. Responding to new information in a mining complex: fast mechanisms using machine learning. Mining Technology, 1-14. doi.org/10.1080/25726668.2019.1577596

Pimentel, B.S., Mateus, G.R., Almeida, F.A., 2010. Mathematical models for optimizing the global mining supply chain. Intelligent Systems in Operations: Methods, Models and Applications in the Supply Chain, 133-163.

Pirrong, C., 2011. Commodity price dynamics: A structural approach. Cambridge University Press.

Ramazan, S., Dimitrakopoulos, R., 2004. Recent applications of operations research and efficient MIP formulations in open pit mining. SME Transactions 316.

Ramazan, S., Dimitrakopoulos, R., 2007. Stochastic optimisation of long-term production scheduling for open pit mines with a new integer programming formulation. Orebody Modelling and Strategic Mine Planning, Spectrum Series 14, 359-365.

Ramazan, S., Dimitrakopoulos, R., 2013. Production scheduling with uncertain supply: a new solution to the open pit mining problem. Optimization and Engineering 14, 361-380.

Ravenscroft, P., 1992. Risk analysis for mine scheduling by conditional simulation. Transactions of the Institution of Mining and Metallurgy. Section A. Mining Industry 101A, 104-108.

Remy, N., Boucher, A., Wu, J., 2009. Applied geostatistics with SGeMS: a user's guide. Cambridge University Press.

Rendu, J.-M., 2014. Introduction to Cut-off Grade Estimation, 2nd edn. SME, Littleton, USA.

Rendu, J.M., 2017. Risk Management in Evaluating Mineral Deposits. Society for Mining, Metallurgy & Exploration, Denver, Colorado, USA.

Rimélé, M.A., Dimitrakopoulos, R., Gamache, M., 2018. A stochastic optimization method with in-pit waste and tailings disposal for open pit life-of-mine production planning. Resources Policy 57, 112-121.

Roche, J., 1995. Forecasting commodity markets. Probus: London.

Rossi, M.E., Deutsch, C.V., 2014. Mineral resource estimation. Springer Science & Business Media.

Schwartz, E.S., 1997. The stochastic behavior of commodity prices: Implications for Valuation and Hedging. Journal of Finance 52, 923-973.

Stone, P., Froyland, G., Menabde, M., Law, B., Pasyar, R., Monkhouse, P., 2007. BLASOR-Blended Iron Ore Mine Planning Optimization at Yandi. Orebody Modeling and Strategic Mine Planning: Uncertainty and Risk Management Models. The Australian Institute of Mining and Metallurgy, Spectrum Series 14, 285-288.

Strebelle, S., 2002. Conditional simulation of complex geological structures using multiple-point statistics. Mathematical Geology 34, 1-21.

Toftaker, H., Tjelmeland, H., 2013. Construction of binary multi-grid Markov random field prior models from training images. Mathematical Geosciences 45, 383-409.

Topal, E., Ramazan, S., 2012. Strategic mine planning model using network flow model and real case application. International Journal of Mining, Reclamation and Environment 26, 29-37.

Urbaez, E., Dagdelen, K., 1999. Implementation of linear programming model for optimum open pit production scheduling problem. Trans Soc Min Metall Explor 297, 1968-1974.

Vallée, M., 2000. Mineral resource + engineering, economic and legal feasibility = ore reserve. CIM Bulletin 93 (1038), 53-61.

Whittle, G., 2007. Global asset optimisation, Proc. Orebody Modelling and Strategic Mine Planning Symposium.

Whittle, J., 1989. The facts and fallacies of open pit optimization. Whittle Programming Pty Ltd., North Balwyn, Victoria, Australia, 1-7.

Whittle, J., 2010. The global optimizer works–what next, In: Dimitakopoulos, R. (Ed.), Advances in Orebody Modelling and Strategic Mine Planning I. The Australasian Institute of Mining and Metallurgy, Spectrum Series 17, pp. 3-5.

Whittle, J., 2014. Not for the faint-hearted. Orebody modelling and strategy mine planning. The Australian Institute of Mining and Metallurgy, Perth, 3-6.

Whooler, R., 2007. Optimising multiple operating policies for exploring complex resources – An overview of the COMET scheduler, In: Dimitrakopoulos, R. (Ed.), Orebody Modelling and Strategic Mine Planning. AusIMM, Spectrum Series 14, Melbourne, pp. 309-316.

Yang, X.-S., 2010. Nature-inspired metaheuristic algorithms. Luniver press.

Yao, L., Dimitrakopoulos, R., Gamache, M., 2018. A New Computational Model of High-Order Stochastic Simulation Based on Spatial Legendre Moments. Mathematical Geosciences, 1-32.

Yüksel, C., Minnecker, C., Shishvan, M.S., Benndorf, J., Buxton, M., 2018. Value of Information Introduced by a Resource Model Updating Framework. Mathematical Geosciences, 1-19.

Zhang, J., Dimitrakopoulos, R.G., 2017. A dynamic-material-value-based decomposition method for optimizing a mineral value chain with uncertainty. European Journal of Operational Research 258, 617-625.

Zhang, J., Dimitrakopoulos, R.G., 2018. Stochastic optimization for a mineral value chain with nonlinear recovery and forward contracts*. Journal of the Operational Research Society 69, 864-875. 10.1057/s41274-017-0269-5

Zuckerberg, M., Stone, P., Pasyar, R., Mader, E., 2007. Joint ore extraction and in-pit dumping optimization, In: Dimitrakopoulos, R. (Ed.), Orebody Modelling and Strategic Mine Planning, AusIMM Spectrum Series 14, pp. 137-140.

Zuckerberg, M., Van der Riet, J., Malajczuk, W., Stone, P., 2011. Optimal life-of-mine scheduling for a Bauxite mine. Journal of Mining Science 47, 158-165.