# THE QUARK AND GLUON DAMPING RATES IN HIGH-TEMPERATURE QCD

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# ABSTRACT

The dominant term in the damping rate for quarks and transverse gluons at high momentum ($\mathbf{p} >> gT$) is calculated within the framework of perturbative QCD at finite temperature. It is shown that the damping rate, $\gamma$, takes the form $\gamma = cg^2T \log(1/g)$ with $c = N/4\pi$ for transverse gluons and $c = (N^2-1)/(8\pi N)$ for quarks where N is the number of colours, g is the coupling constant, and T is the temperature. The sign and the gauge invariance of $\gamma$ are easily verifiable due to the simplicity of the argument. This result agrees with the more complicated (unpublished) calculations of Pisarski et al. but disagrees with those of Lebedev and Smilga.

# RESUME

Le terme dominant dans le taux d'amortage des quarks et des gluons transverses à hautes impulsions (p>>gT) est calculé à l'aide de la théorie perturbative chromodynamique quantique à température finie. Il est démontré que le taux d'amortage, $\gamma$, est de la forme $\gamma = cg^2T \log(1/g)$ avec $c = N/4\pi$ pour les gluons transverses et $c = (N^2-1)/(8\pi N)$ pour les quarks, où N est le nombre de couleurs, g est la constante de couplage, et T est la température. Le signe et l'invariance de jauge de $\gamma$ sont aisément vérifiables grâce à la simplicité de l'argumentation. Ceci est en complet accord avec les calculs plus complexes (non-publiés) de Pisarski et ses collègues, mais contradit ceux de Lebedev et Smilga.

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# INTRODUCTION

The accepted theory of the strong interactions is called quantum chromodynamics (QCD). In QCD, the interactions are generated by a non-Abelian SU(3) gauge theory of coloured quarks and gluons that are permanently confined to colour singlet hadronic bound states. A very interesting feature of QCD is its high energy behaviour known as asymptotic freedom. As one probes short distance scales (high energy transfers) the coupling strength g decreases to produce an almost noninteracting system [1]. This behaviour makes the use of perturbation theory possible, since at high energies (short distances) the coupling will be small enough so that the terms involving large powers of g may be neglected.

The success of perturbative QCD at short distances has been very encouraging. The approximate scaling observed in the deep inelastic scattering experiments of leptons off hadrons can be explained using asymptotic freedom. Deviations at high energy from this scaling have been predicted by QCD and are consistent with the observed scaling, given the large error in these measurements [2]. Other predictions include the narrow width of charmonium and the existence of quark and gluon jets. QCD is also consistent with much of the phenomenology of the strong interactions such as the symmetry patterns of the hadrons.

Now that we possess a theory of strong interactions it is natural to try to test the theory in different environments, in particular at high temperatures. The predictions of high temperature QCD could be tested in at least three new domains. First, there may exist significant high temperature effects within neutron stars where the density is considerably greater than nuclear density. The second possibility is in heavy ion collisions at very high energy per nucleon, in which states of high density and temperature might be formed. Finally the standard cosmological models allow one to extrapolate back to when the universe was at a temperature comparable to nucleon rest energies in units where $c=\hbar=k=1$. It is hoped that high temperature QCD might provide some predictions concerning the evolution of the universe. Thus new insights into the nature of matter at very high temperature and density might be gained by studying QCD at high temperature.

There is evidence that at sufficiently high temperatures QCD loses confinement, and a quark-gluon plasma screening all colour-electric flux is formed [3]. There are several theoretical reasons for studying the gluon plasma. Early studies of gauge theories at finite temperature were carried out within the framework of QED [4], and it would be interesting to compare the photon plasma to the gluon plasma because QCD, unlike QED, includes self interaction of the gauge fields. Furthermore, by first attempting the calculation of some simple amplitudes, the rich mathematical structure of QCD should be exposed and the techniques required to deal with any difficulties that may arise can be developed. This is important because in a more complicated calculation the mathematical difficulties could be overwhelming and might obscure important concepts that need to be addressed in hot QCD. The goal of this thesis is to determine the dominant contribution to the quark and gluon damping rates within this plasma, at momenta much greater than gT (where g is the coupling constant and T is the temperature).

The plasma parameters that we wish to calculate are the plasma frequency $\omega(\mathbf{p})$, and the damping constant $\gamma(\mathbf{p})$. The plasma frequency is the energy of the propagating quark or gluon, and the damping constant is the inverse lifetime of the quark or gluon. The gluons and quarks obtain effective lifetimes within the QCD plasma because they can be absorbed into the ambient heat bath. Both these parameters are determined by the position of the pole of the gluon propagator or zero of the inverse propagator, $E(\mathbf{p})=\omega(\mathbf{p}) - i\gamma(\mathbf{p})$, in the complex energy plane. The plasma frequency at zero momentum ($\mathbf{p}=0$) has been computed by many authors [5] and found to be $\omega=g\sqrt{NT/3}$ (neglecting quark contributions), where g is the coupling constant, N is the number of colours, and T is the temperature. This result is recalculated here as an exercise and to serve as a check on our methods. The damping rate (for gluons in the static limit) arises at order $g^2$, however, the results that have been obtained in the literature are not consistent with one another [6]. In some cases the damping is found to be negative indicating an instability of the plasma and other calculations indicate that the damping rate is a gauge dependent quantity. Obviously these calculations cannot be taken seriously since all physical quantities are gauge independent and there is no reason to believe that the quark-gluon plasma is thermodynamically unstable. The problem that is common to all these one loop calculations is that they are all incomplete in the following way.

In zero temperature QCD, an order $g^{2n}$ computation requires the evaluation of all n loop Feynman diagrams, however, this is not the case at finite temperature. At finite temperature an infinite number of diagrams can contribute to any order in g. The reason for this complication is that the infrared divergences are more severe in hot QCD than in zero temperature QCD. The source of these severe divergences can be attributed to the behaviour of the Bose-Einstein distribution function $n(q)=(e^{q/T}-1)^{-1}$

for small q. At low momentum $n(q) \approx T/q$, thus quantities can diverge as powers of the infrared cutoff rather than logarithmically as they do at zero temperature. Notice that $n(q) \approx e^{-q/T}$ for $q >> T$ ensures there will not be any new ultraviolet divergences beyond those at zero temperature thus the zero temperature renormalization scheme will suffice. In particular, to compute the gluon damping rate at zero momentum to order $g^2$ requires more than a one loop calculation [7]. A resummation is required to include all higher loop diagrams that contribute to order $g^2$.

A method for resumming all the relevant diagrams has been developed by Braaten and Pisarski [7]. In their analysis, they show that it is necessary to distinguish between hard momenta (of order T) and soft momenta (of order gT). Ordinary perturbation theory (bare propagators and vertices) applies at hard momenta, but over soft momenta, effective propagators and vertices are required. They have also proved that to leading order in g, in this effective perturbation expansion, the quark and gluon damping rates are gauge invariant and positive within the Coulomb and covariant gauges. Kobes, Kunstatter, and Rebhan then showed that within an even larger class of gauges the damping rate is independent of the choice of gauge [8]. The value obtained by Braaten and Pisarski [9] for the gluon damping rate at zero momentum is $\gamma(\mathbf{p}=0)=+a\, g^2NT/24\pi$ where the constant a was determined numerically to be $a \approx 6.63538$ . An analytical expression for the damping rate could not be obtained due to the complexity of the resummation.

The validity of this resummation is currently the subject of some debate [10], however, one can compute the dominant contributions to the quark and gluon damping rates at high momenta ($\mathbf{p}>>gT$) without employing a full resummation. The question is how can one compute the dominant contribution to the damping rate when perturbation theory is failing? The answer to this question is best summarized by Steven Weinberg. "When it is infrared effects that invalidate perturbation theory, the introduction of a floating cut-off may not restore perturbation theory, but it does allow us to say useful things about the infrared effects themselves" [11]. This is precisely what is required. Perturbation theory fails for QCD at finite temperature due to the infrared behaviour of the theory at momenta less than or equal to $g^2T$. The introduction of a cut-off $\lambda > g^2T$ allows us to use perturbation theory above the scale of the cutoff. It is then a matter of studying the behaviour of the computed quantites with respect to the infrared cut-off. We must also realize that like any physical quantity, the damping rate cannot be a function of the cut-off. Thus the contribution to the damping rate from the nonperturbative region (momenta less than $\lambda$) must precisely cancel the cutoff dependence from the perturbative part of the calculation. This property is sufficient to determine the dominant part of the result. These considerations allow us to determine the most dominant contributions to the damping rate for energetic quarks and gluons in hot QCD.

# FINITE TEMPERATURE FIELD THEORY

The purpose of the following sections is to review some important aspects of finite temperature field theory that are required in this investigation. It is assumed that the reader has a knowledge of both field theory and statisical physics. For readers who would like a more rigourous treatment of the following material, references [12,13,14] are recommended.

## 2.1 Quantum Statisical Mechanics

There are three types of ensembles that one usually considers in equilibrium statistical mechanics. They are the micro-canonical ensemble, the canonical ensemble, and the grand canonical ensemble.The micro-canonical ensemble is used to describe a system with fixed energy E and a fixed volume V. The canonical ensemble is used to describe a system in contact with a heat reservoir at temperature T. The system is free to exchange energy with the reservoir thus only T and V are constant. In the grand canonical ensenble the system is free to exchange particles and energy with the heat bath therefore T, V and possible chemical potentials $\mu_i$ are the fixed variables.

The ensemble that is best suited for describing the quark-gluon plasma is the grand canonical ensemble. We are assuming the plasma is at equilibrium and that quarks and gluons can be emitted from or absorbed into the ambient heat bath. In order to start our description of this plasma we must introduce an important function in thermodynamics, the partition function.

The grand canonical partition function is given by

$$Z=\sum_N \sum_j e^{-\beta(Ej-\mu N)} \tag{2.1.1}$$

where $\beta=1/kT$, k is the Boltzman constant and Ej is the energy of the $j^{th}$ N-particle state. The partition function can be written in a more compact form

$$Z=Tr(\rho) \tag{2.1.2}$$

in which $\rho$ is the statistical density matrix given by

$$\rho=e^{-\beta(\mathbf{H}-\mu_i \mathbf{N}_i)} . \tag{2.1.3}$$

In this expression **H** and **N** are the Hamiltonian and conserved number operators respectively (sum over i is implied). In relativistic QCD the difference between the number of quarks and antiquarks is a conserved quantity, not the quark or antiquark number seperately. The reason why this is an important function in thermodynamics is that all other standard thermodynamic properties can be determined from it. For example, the pressure, particle number, entropy and energy are related to the partition function by

$$P=T\, \partial \ln Z/\partial V,$$
$$N_i=T\, \partial \ln Z/\partial \mu_i,$$
$$S=\partial(T\ln Z)/\partial T,$$
$$E=-PV\ +TS+\mu_i N_i . \tag{2.1.4}$$

A useful relation is that the ensemble average of any operator O is given by

$$<O>=\frac{Tr(\rho O)}{Tr(\rho)} . \tag{2.1.5}$$

Now that we have established the importance of the partition function, we must find a method for evaluating the partition function for a given system. In order to evaluate the partition function in a nonrelativistic system, equation (2.1.1) tells us we must determine the complete set of N-particle quantum states and corresponding energies. In principle this requires one to solve Schrodinger's equation for an N-particle system. If the particles are non-interacting (ideal gas approximation) then one can reduce the problem to solving a one-particle Schrodinger equation [15]. If the particles are interacting then one would need the solution to the N-particle Schrodinger equation. It is a well known fact that one cannot solve a general two-body quantum mechanics problem, therefore it is useless to even consider solving a system with $\approx 10^{23}$ particles. What is needed is an alternative method for calculating the partition function in systems with weakly interacting particles and in relativistic systems where the particle number is not conserved.

## 2.2 Functional Integral Representation of the Partition Funtion

From quantum mechanics we know that the transition amplitude for going from a state $|\phi_a\rangle$ at time t=0 to some other state $|\phi_b\rangle$ after a time $t=t_f$ is given by $\langle\phi_b| e^{-iHt_f} |\phi_a\rangle$ . For equilibrium statistical mechanics, we are interested in the case when the system returns to its original state after a time $t_f$. We also know that the transition amplitude can be expressed as a path integral [16] as

$$\langle\phi_a| e^{-iHt_f} |\phi_a\rangle = \int[d\pi] \int_{\phi_a}^{\phi_a}[d\phi] \exp(i \int_0^{t_f} dt \int d^3x\, L(\phi,\pi)) . \qquad (2.2.1)$$

The symbols $[d\phi]$ and $[d\pi]$ denote functional integration over the field $\phi(\mathbf{r},t)$ and the conjugate momenta $\pi(\mathbf{r},t)$, L is the Lagrangian of the system and the limits of the $\phi$ integration are chosen such that $\phi(\mathbf{r},0)=\phi_a(\mathbf{r})$ and $\phi(\mathbf{r},t_f)=\phi_a(\mathbf{r})$. Note that the partition funtion Z can be reexpressed as

$$Z=\int d\phi \, \langle \phi \,|\, e^{-\beta(H-\mu_i N_i)} \,|\phi\rangle \,. \tag{2.2.2}$$

Notice that this expression for Z is very similar to that for the transition amplitude given by equation (2.2.1). In fact, Z can be expressed as a functional integral over the fields and the conjugate momenta by switching to imaginary-time ($\tau$=it) and making use of (2.2.1) [12]. The limits of integration on $\tau$ become 0 and $\beta$. Thus we may write the partition function as

$$Z=N' \int_{\text{periodic}} [d\phi] \exp\left( \int_0^\beta d\tau \int d^3x \, L\right) \tag{2.2.3}$$

where N' is a normalization constant and the integration is over imaginary-time. The term "periodic" means that the field is constrained so that $\phi(\mathbf{r},0)=\phi(\mathbf{r},\beta)$ for bosons and that $\phi(\mathbf{r},0)= -\phi(\mathbf{r},\beta)$ for fermions. The antiperiodicity of the fermions can be understood by recalling that a fermionic wavefunction changes sign if a rotation of $2\pi$ is made about some axis. Both the angle $\theta$ and imaginary time $\tau$ are defined on compact intervals, thus the fermionic wavefunction changes sign if $\tau$ is varied from 0 to $\beta$. If we now define the argument of the exponential to be the action S, the partition function can be written in a very compact and useful form.

$$Z=N' \int_{\text{periodic}} [d\phi] \;\; e^{S} \,. \tag{2.2.4}$$

The functional integral in (2.2.4) can be evaluated in closed form if the Lagrangian in S does not contain terms which are more than quadratic in the fields. This situation is very similar to that encountered in section (2.1). Recall that we could evaluate the partition function for the ideal gas case (no interaction), but not for the interacting system. The difference is that we can now make the approximation that the interaction is very weak and expand the partition function in powers of the interaction. To perform this expansion first decompose the action into its "free" and "interaction" parts.

$$S=S_O+S_I \tag{2.2.5}$$

Now substitute (2.2.5) into (2.2.4) and expand $e^{S_I}$ in a power series. To obtain the free energy, take the logarithm of both sides and obtain

$$\ln Z = \ln(N'\int[d\phi]\, e^{S_O}) + \ln\left( 1 + \frac{\sum_{n=1}^{\infty} \frac{1}{n!} \int[d\phi]\, e^{S_O}\, S_I^n}{\int[d\phi]\, e^{S_O}} \right) . \tag{2.2.6}$$

This can be rewritten as

$$\ln Z = \ln Z_O + \ln Z_I \tag{2.2.7}$$

which explicitly separates the interaction contribution from the ideal gas contribution.

The quantity that actually needs to be computed within $\ln Z_I$ is $S_I$ raised to an arbitrary power and averaged over the ideal gas ensemble represented by $S_O$. This can be seen by comparing $\ln Z_I$ to equation (2.1.5).

$$\langle S_I^n \rangle_O = \frac{\int[d\phi]\, e^{S_O}\, S_I^n}{\int[d\phi]\, e^{S_O}} \tag{2.2.8}$$

Equation (2.2.8) should look extremely familiar to those acquainted with quantum field theory at zero temperature. Proceeding as in regular field theory, we can develop diagrammatic techniques for representing the terms in the expansion defined by (2.2.6). The finite temperature Green's functions defined by equation (2.2.8) are, in general, not the connected Green's functions, since they include completely disjoint pieces. We can, however, define the connected Green's functions in a very elegant way related to the generating functional for the Green's functions. If Z[J ] is the generating functional for the Green's functions and W[J ] is the generating functional for the connected Green's functions then they are related by [16,17]

$$Z[J] = e^{iW[J]} \quad . \tag{2.2.9}$$

Notice that W[J ] is proportional to the logarithm of Z[J ]. Using these facts, one can show that to compute $\ln Z_I$ , we must sum only the connected diagrams. Given the Lagrangian for any theory, we can now evaluate $\ln Z_I$ and determine the thermodynamic properties of the system. In fact many studies have been performed in both QED and QCD where quantities such as the pressure of a photon-electron or quark-gluon gas have been computed [12,18].

Although $\ln Z_I$ and the partition function will not be explicitly evaluated in this project, they are closely related to a function that we must evaluate, namely, the finite temperature propagator.

## 2.3 The Finite Temperature Propagator

In field theory at zero temperature, the propagator in position space is defined by

$$D(\mathbf{r}_1,t_1;\mathbf{r}_2,t_2) = \langle 0 | T[\phi(\mathbf{r}_1,t_1)\,\phi(\mathbf{r}_2,t_2)] | 0 \rangle \tag{2.3.1}$$

where T is the time ordering operator and $|0\rangle$ is the vacuum state. Similarly, using equation (2.1.5), the finite temperature propagator is defined by

$$D(\mathbf{r}_1,\tau_1;\mathbf{r}_2,\tau_2) = Z^{-1}\,\mathrm{Tr}\{\rho\, T[\phi(\mathbf{r}_1,\tau_1)\,\phi(\mathbf{r}_2,\tau_2)]\} \tag{2.3.2}$$

where T is now the imaginary-time ordering operator. For bosons the imaginary time ordering operator is given by

$$T[\phi(\tau_1)\,\phi(\tau_2)] = \phi(\tau_1)\,\phi(\tau_2)\,\theta(\tau_1-\tau_2) + \phi(\tau_2)\,\phi(\tau_1)\,\theta(\tau_2-\tau_1), \qquad (2.3.3)$$

and for fermions the imaginary-time ordering operator is given by

$$T[\phi(\tau_1)\,\phi(\tau_2)] = \phi(\tau_1)\,\phi(\tau_2)\,\theta(\tau_1-\tau_2) - \phi(\tau_2)\,\phi(\tau_1)\,\theta(\tau_2-\tau_1). \qquad (2.3.4)$$

Using the fact that T commutes with $\rho$, and the cyclic property of the trace operation, it is easy to show that

$$\mathbf{D}(\mathbf{r}_1,\tau;\,\mathbf{r}_2,0) = \mathbf{D}(\mathbf{r}_1,\tau;\,\mathbf{r}_2,\beta) \qquad (2.3.5)$$

for bosons, and that

$$\mathbf{D}(\mathbf{r}_1,\tau;\,\mathbf{r}_2,0) = -\,\mathbf{D}(\mathbf{r}_1,\tau;\,\mathbf{r}_2,\beta) \qquad (2.3.6)$$

for the fermions. This implies that $\phi(0)=\phi(\beta)$ for the bosons and that $\phi(0)= -\,\phi(\beta)$ for the fermions which agrees with our "periodic" limits for the $\phi$ integration in the partition function.

The field can be expanded as

$$\phi(\mathbf{r},\tau) = \sqrt{(\beta/V)}\,\sum_{n=-\infty}^{\infty}\sum_{\mathbf{p}} e^{i(\mathbf{p}\cdot\mathbf{r}+\omega_n\tau)}\,\phi(n,\mathbf{p})\,. \qquad (2.3.7)$$

Since $\phi(0)=\phi(\beta)$ for bosons, this implies that $\omega_n=2\pi nT$ for the bosons. The constraint that $\phi(0)=-\phi(\beta)$ for the fermions implies that $\omega_n=(2n+1)\pi T$ for the fermions. Since this analysis is being performed in imaginary-time, the zeroeth component of the Minkowski-signature momentum four-vector is given by $2\pi i nT$ for bosons and $(2n+1)\pi iT$ for the fermions.

The real-time finite temperature propagator can be obtained from the imaginary-time finite temperature propagator by analytic continuation. The relationships between the real-time propagator and the imaginary-time propagator are given by [12,19]

$$\mathbf{D}^R(\omega,\mathbf{p}) = \mathbf{D}(i\omega_n \to \omega + i\varepsilon,\mathbf{p}) \quad \varepsilon\to 0^+\,, \qquad (2.3.8)$$

$$\mathbf{D}^A(\omega,\mathbf{p}) = \mathbf{D}(i\omega_n \to \omega - i\varepsilon,\mathbf{p}) \quad \varepsilon\to 0^+\,, \qquad (2.3.9)$$

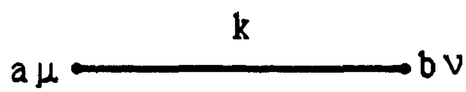
with $D^R$ representing the "retarded" real-time propagator and $D^A$ representing the "advanced" real-time propagator. The real-time thermal propagators are continuous functions of energy ω, and are useful for determining physical properties such as the plasma parameters. To calculate the plasma parameters, the analytic continuation given by (2.3.8) is used. The reason for using the retarded Green's function is explained in the chapter on plasma oscillations.

## 2.4 Feynman Rules for QCD at Finite Temperature

We use the Feynman rules in the covariant gauges found in reference [16]. These rules will be used in all subsequent calculations and all notation and conventions are completely explained in the appendix.

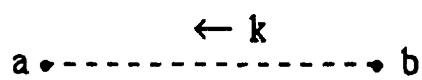
**Propagators**

gluons



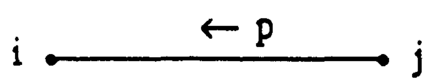

$\delta_{ab} \frac{d_{\mu\nu}(k)}{k^2}$

ghosts



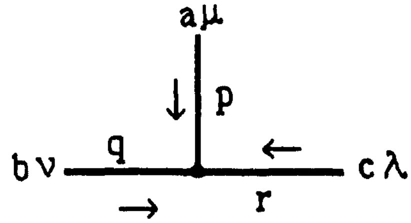

$\delta_{ab} \frac{-1}{k^2}$

quarks



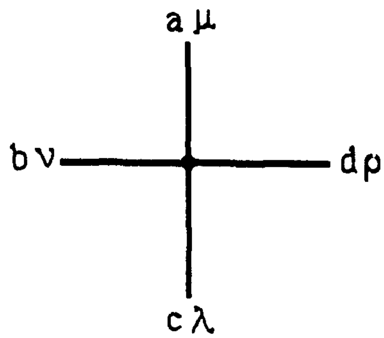

$\delta_{ij} \frac{1}{m-\not{p}}$

**Vertices**

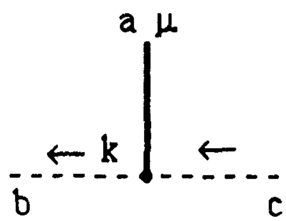



3-gluon

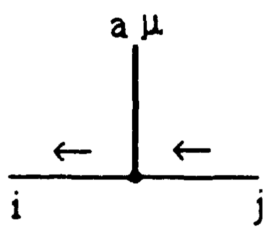
$-igf^{abc}\, V_{\mu\nu\lambda}(p,q,r)$




4-gluon $-g^2 W^{abcd}_{\mu\nu\lambda\rho}$




gluon-ghost $-igf^{abc}\, k_\mu$




gluon-quark $g\gamma_\mu T^a_{ij}$

Where the following abbreviations have been used.

$$d_{\mu\nu}(k) = g_{\mu\nu} - (1-\alpha)\, k_\mu k_\nu / k^2$$

$$V_{\mu\nu\lambda}(p,q,r) = (p-q)_\lambda g_{\mu\nu} + (q-r)_\mu g_{\nu\lambda} + (r-p)_\nu g_{\lambda\mu}$$

$$W^{abcd}_{\mu\nu\lambda\rho} = (f^{ac.bd} - f^{ad.cb}) g_{\mu\nu} g_{\lambda\rho} + (f^{ab.cd} - f^{ad.bc}) g_{\mu\lambda} g_{\nu\rho} + (f^{ac.db} - f^{ab.cd}) g_{\mu\rho} g_{\lambda\nu}$$

The expression $f^{ab.cd}$ denotes the following combination

$f^{ab.cd} = f^{abx} f^{cdx}$ (summation over x implied).

The SU(3) generators of QCD are represented by $T^a$ (a=1,2,...,8) and generate a closed algebra,

$$[T^a,T^b] = if^{abc}\,T^c\ ,$$

with $f^{abc}$ the structure constants. Some useful identities are

$$f^{acd} f^{bcd} = \delta_{ab} N \quad \text{(for SU(N))}\ ,$$

$$f^{ade} f^{bef} f^{cfd} = N/2\ f^{abc}\ ,$$

$$T^a_{il}\, T^a_{lj} = \delta_{ij}\, C_F \quad \text{where } C_F=(N^2-1)/2N\ .$$

The above rules are identical to those at zero temperature. The only difference in the Feynman rules is with the loop integration. At zero temperature we usually associate the following integral for every loop in a diagram.

$$\int \frac{d^4k}{(2\pi)^4 i}$$

The integral is over the loop momentum and there is an extra factor of -1 for each fermion loop and ghost loop within the diagram. Since we are using the imaginary-time formalism, the zeroeth component of all momentum four-vectors is discrete and imaginary. Thus the loop integration is replaced with what is known as a thermal sum.

$$T \sum_{n=-\infty}^{\infty} \int \frac{d^3k}{(2\pi)^3}$$

The sum over n is for the zeroeth component of k, where $k_0=2\pi inT$ for bosons and $k_0=(2n+1)\pi iT$ for fermions. When evaluating a Feynman diagram with loops, one must also remember to include the symmetry factor to correct for overcounting after the above rules have been applied.

Methods for determining symmetry factors can be found in references [16,17].

## 2.5 Evaluation of Thermal Sums

A standard method for evaluating the thermal sums is contour integration. Suppose the thermal sum (or frequency sum) that we want to evaluate is of the form

$$T \sum_{n=-\infty}^{\infty} f(p_0) \qquad (2.5.1)$$

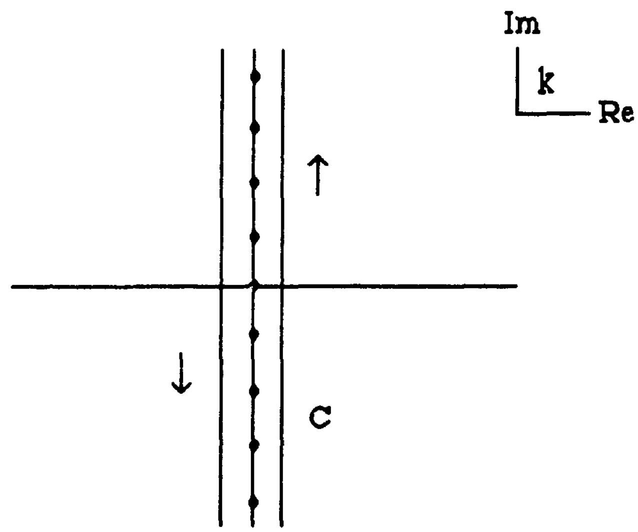
where f means any function of $p_0$. To evaluate the sum as a contour integral we need a function that has poles at $p_0$ with a residue of one and is everywhere else analytic and bounded. Assuming that f(k) is regular along the imaginary axis, we can write

$$T \sum_{n=-\infty}^{\infty} f(p_0) = \int_C \frac{dk}{2\pi i} f(k)\, g(k) \qquad (2.5.2)$$

where $g(k)=1/(\exp(k/T)-1)$ for bosons and $g(k)=-1/(\exp(k/T)+1)$ for fermions. The contour is shown in the following figure and encloses the poles in a counter-clockwise direction. If the integral is split into two line integrals from $-i\infty+\varepsilon$ to $i\infty+\varepsilon$ and from $+i\infty-\varepsilon$ to $-i\infty-\varepsilon$ and substituting $k \to -k$ in the second integral, one finds [20]

$$T \sum_{n=-\infty}^{\infty} f(p_0) = \frac{1}{2\pi i} \{ \int_{-i\infty}^{i\infty} dk\, f(k) \pm \int_{-i\infty+\varepsilon}^{i\infty+\varepsilon} dk \frac{f(k)+f(-k)}{\exp(k/T)\mp 1} \}, \qquad (2.5.3)$$

the upper sign is for bosons and the lower sign is for fermions. Notice how the thermal sum naturally splits into a temperature dependent piece (matter part) and a temperature independent piece (vacuum part). All the ultraviolet divergences associated with zero temperature field theory are found in the vacuum piece. If the singularities of f(k) are simple poles off the imaginary axis, then we can close the contours with infinite semi-circles and evaluate the frequency sum by using Cauchy's theorem.

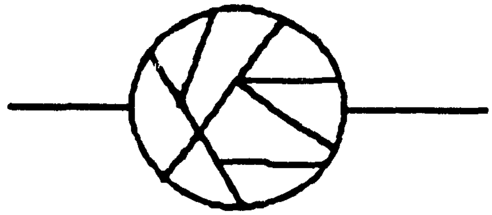



Note: The singularities in the above diagram represent a thermal sum for a boson. If the sum was for a fermion, the poles would be shifted along the imaginary axis. In either case, the contour remains the same.

## 2.6 Infrared Behaviour of QCD at Finite Temperature

Due to the complexity of the Feynman rules for QCD, the evaluation of any diagram can involve a considerable amount of algebra. To study the infrared behaviour of the theory, the Feynman rules will be greatly simplified so that simple power-counting arguments can be used. The diagrams that will be studied are the corrections to the propagator or gluon self-energy diagrams. These diagrams have been chosen since it is the self-energy correction that determines the position of the pole in the full propagator from which the plasma parameters can be obtained.

Consider the following general, L-loop self energy diagram.



The above diagram has V vertices, L loops and P propagators (internal lines only). The total number of vertices is given by

$$V = V_3 + V_4 \ , \tag{2.6.1}$$

where $V_3$ is the number of 3-leg vertices and $V_4$ is the number of 4-leg vertices. The total number of internal lines P is given by

$$P = (3V_3 + 4V_4 - 2)/2 \tag{2.6.2}$$

by using the "conservation of ends". We also have the relation

$$V + L - P = X. \tag{2.6.3}$$

where X is the Euler number which has the value 1 on a disk. Substituting (2.6.1) into (2.6.3) and multiplying (2.6.2) by two, we obtain the following

$$V_3 + V_4 + L - P = 1 \ , \tag{2.6.4}$$

$$2P = 3V_3 + 4V_4 - 2 \ . \tag{2.6.5}$$

The general self-energy diagram will be of the schematic form

$$\Pi = \{T \sum_{n=-\infty}^{\infty} \int d^3k \}^L \left(\frac{1}{k^2}\right)^P k^{V_3} g^{(V_3+2V_4)} \ . \tag{2.6.6}$$

Notice that there is a thermal sum for each loop and a factor of $1/k^2$ for each propagator in the diagram. A factor of g and k has been included for every 3-leg vertex and a factor of $g^2$ for each 4-leg vertex.

Since we are interested in the infrared behaviour of the diagram, we only need to consider the n=0 part of the sum since all other terms will be suppressed. Using equations (2.6.4) and (2.6.5) in (2.6.6), Π can be simplified to

$$\Pi = g^{2L} T^{L} \int_{\lambda}^{\infty} k^{1-L} dk \ . \tag{2.6.7}$$

For example, (2.6.7) indicates that a L=1 loop diagram has a self energy correction $\Pi \equiv g^2 T(T-\lambda)$. The upper limit should be taken to be T because in a complete computation the Bose-Einstein factor will prevent ultraviolet divergences. The infrared cutoff $\lambda$ can be set to zero thus $\Pi \equiv g^2 T^2$. This indicates that the self-energy contribution is suppressed by a factor of $g^2$ compared to the tree-level contribution.

The infrared behaviour of the self-energy is actually worse than equation (2.6.7) indicates. Factors of external momentum P can be factored out of the integral in (2.6.7) because the 3-leg vertices are functions of momentum flowing through all of their legs. If we factor out $P^2$ we can compare the self-energy contribution to the tree-level result which is proportional to $P^2$. Doing so we find

$$\Pi = P^2 (g^2 T/\lambda)^L \quad , \tag{2.6.8}$$

where $\lambda$ is the infrared cutoff. Clearly, higher loop diagrams can contribute on the same order as tree-level ones if $\lambda$ is on the order of $g^2T$. Therefore in order to trust perturbation theory we must choose the infrared cutoff such that $\lambda \gg g^2T$. The contribution of wavelengths $k \leq g^2T$ cannot be computed using perturbation theory, and unless some assumptions are made concerning the low frequency behaviour of the plasma, there is currently no method for calculating their contribution.

# PLASMA OSCILLATIONS

The purpose of this chapter is to outline the method used to determine the quark and gluon plasma parameters. Expressions relating the plasma frequency and the damping rate to the self-energy are derived. These expressions are used to determine the plasma frequency for static gluons, and the damping rates for energetic quarks and gluons in the following chapters.

## 3.1 Linear Response Analysis

The basic equation of linear response theory is

$$\delta\langle\phi(\mathbf{r},t)\rangle = \int_{-\infty}^{\infty} dt' \int d^3\mathbf{r}'\ J(\mathbf{r}',t')\ D^R(\mathbf{r},t\ ;\ \mathbf{r}',t') \qquad (3.1.1)$$

where $\delta\langle\phi(\mathbf{r},t)\rangle$ is the variation of the ensemble average of the field in response to a small perturbation from an external source J [12,13]. The field $\phi$ is coupled to an external source J via the equation

$$\delta L_{ext}(t) = \int d^3\mathbf{r}'\ J(\mathbf{r}',t)\ \phi(\mathbf{r}',t)\ . \qquad (3.1.2)$$

It has been assumed that the change in the ensemble average is small, so the linear response approximation remains valid. Taking the Fourier transform of equation (3.1.1) one obtains

$$\delta\langle\Phi(\mathbf{q},\omega)\rangle = \mathbf{J}(\mathbf{q},\omega)\ \mathbf{D}^{\mathbf{R}}(\mathbf{q},\omega) \qquad (3.1.3)$$

which has a very simple form. The change in the ensemble average of the field is given by the product of the external source and the retarded propagator in frequency-momentum space. The physical modes of the plasma are given by the positions of the singularities in the retarded propagator [12,13]. Since we are working in the covariant gauge, there is one unphysical degree of freedom in the retarded propagator. To determine the plasma parameters, we must "project out" only the poles corresponding to physical degrees of freedom. The positions of these physical "plasmon poles" are gauge invariant [8]. In our analysis we will instead determine the positions of the zeroes of the inverse propagator which must be identical to the positions of the poles.

### 3.2 Physical Modes of the Gluon Plasma

In order to determine the physical modes of the gluon plasma, we must find the position of the zeroes of the full inverse gluon propagator. The most general symmetric tensor that we can construct with two indices $\mu\nu$ to represent the full inverse gluon propagator is

$$D^{-1}_{\mu\nu} = A' g_{\mu\nu} + B' k_\mu k_\nu + C' \eta_\mu n_\nu + E'(k_\mu n_\nu + \eta_\mu k_\nu) . \qquad (3.2.1)$$

The full propagator is constructed from the metric $g_{\mu\nu}$=diag(1,-1,-1,-1), the four-momentum $k_\mu$, a four-vector $n_\mu$=(1,0,0,0) representing the rest frame of the plasma and some undetermined scalar functions A', B', C' and E'. Although this expression is correct, it is not very useful for algebraic manipulations. The full inverse propagator can be expressed in the following more useful form.

$$D^{-1}_{\mu\nu} = AP^{T}_{\mu\nu} + BP^{L}_{\mu\nu} + CS_{\mu\nu} + EL_{\mu\nu} \qquad (3.2.3)$$

$P^{T}_{\mu\nu}$ is constructed such that it is transverse to both the four-vector k and the three-vector **k** and is defined in the following manner.

$$P^{T}_{\mu\nu} = \{P^{T}_{00}=0,\ P^{T}_{0i} = P^{T}_{i0}=0,\ P^{T}_{ij}= g_{ij}+ k_i k_j /\mathbf{k}^2\} \qquad (3.2.4)$$

$P^{T}_{\mu\nu}$ is constructed to "project out" the two, physical transverse modes.The projection operator $P^{L}_{\mu\nu}$ is constructed such that it is also transverse to the four-vector k but it is longitudinal to the three-vector **k**. It is defined in the following manner.

$$P^{L}_{\mu\nu} = g_{\mu\nu} - k_\mu k_\nu/k^2 - P^{T}_{\mu\nu} \tag{3.2.5}$$

$P^{L}_{\mu\nu}$ is constructed to "project out" the single, physical longitudinal mode. The operator $S_{\mu\nu}$ is given by the expression

$$S_{\mu\nu} = (k_\mu u_\nu + u_\mu k_\nu) \ , \tag{3.2.6}$$

where $u_\mu = (\eta_\mu - k_0 k_\mu/k^2)$ . The final operator is simply $L_{\mu\nu} = k_\mu k_\nu/k^2$. Now introduce the following four-vectors.

$$k^\mu=(\omega,0,0,|\mathbf{k}|)$$

$$n^\mu=(1,0,0,0)$$

$$i^\mu=(0,|\mathbf{k}|,0,0)$$

$$j^\mu=(0,0,|\mathbf{k}|,0)$$

$$m^\mu=(|\mathbf{k}|,0,0,\omega) \tag{3.2.7}$$

To determine the condition for the transverse modes in the plasma construct a spacelike vector transverse to k and transverse to **k**. Such a vector is given by $V^\mu=ai^\mu+bj^\mu$, where a and b are arbitrary constants. Now evaluate $D^{-1}$ V and determine the condition that it be a zero eigenvector. We find that $D^{-1}$ V = A V with A from equation (3.2.3) , thus the zero eigenvalue condition for the transverse modes is A=0.

Similarly, to determine the longitudinal modes in the plasma, construct a vector transverse to k and longitudinal to **k**. Let this vector be $W^\mu=ak^\mu+bm^\mu$. Using the same procedure as above, we find that the zero eigenvalue condition for the longitudinal modes is

$$B + C^2\mathbf{k}^2/E = 0 \ . \tag{3.2.8}$$

It is now a matter of determining the scalar functions A,B,C and E. The gluon self-energy $\Pi_{\mu\nu}$ is defined through the Schwinger-Dyson equation

$$D^{-1}_{\mu\nu} = D^{-1}_{(o)\mu\nu} - \Pi_{\mu\nu} \tag{3.2.9}$$

with $D^{-1}_{(o)\mu\nu}$ being the bare inverse gluon propagator. The bare inverse gluon propagator in the covariant gauge is given by

$$D^{-1}_{(o)\mu\nu}(k) = k^2( g_{\mu\nu} - k_\mu k_\nu/k^2 + \frac{1}{\alpha} k_\mu k_\nu/k^2 ) . \qquad (3.2.10)$$

Thus we can express the functions A, B, C and E in terms of the gluon self energy. Solving for A as a function for the gluon self-energy, and setting A=0, we find that the transverse modes satisfy the condition

$$k^2 = -\frac{1}{2} (\Pi_{ii} - \frac{\mathbf{k}^i\mathbf{k}^j}{\mathbf{k}^2} \Pi_{ij}) \ . \qquad (3.2.11)$$

The condition for the longitudinal modes is somewhat more complicated. If the self-energy satisfies $k_\mu k_\nu \Pi^{\mu\nu}=0$ (as it does at one loop), then the expression can be simplified and reduces to

$$k^2 = ( \Pi_{oo} - \frac{\mathbf{k}^i\mathbf{k}^j}{\mathbf{k}^2} \Pi_{ij}) \ . \qquad (3.2.12)$$

The conditions for the transverse and longitudianal modes derived above agree with those of reference [21]. In order to obtain the plasma frequency and the damping constant assume that the solution of equations (3.2.11) and (3.2.12) has the form, $k_o = \omega - i\gamma$, with $\gamma << \omega$. If we rewrite the right hand side of equations (3.2.11) and (3.2.12) in the general form $F( k_o,\mathbf{k})$, then the plasma frequency is obtained from

$$\omega_p^2 = \mathrm{Re}\ \{F( k_o,\mathbf{k})\} + \mathbf{k}^2 \qquad (3.2.13)$$

and the damping constant will be given by

$$\gamma = \frac{-1}{2\omega} \mathrm{Im}\ \{F( k_o,\mathbf{k})\} \ . \qquad (3.2.14)$$

For the cases we are considering, $F( k_o,\mathbf{k})$ is accurate to order $g^2$. To solve for $\omega^2$ and $\gamma$ to order $g^2$, we must substitute the zeroeth order approximation of $k_o$ into $F( k_o,\mathbf{k})$. In the limit $|\mathbf{k}| \rightarrow 0$, $k_o=0$ is used and in the limit $|\mathbf{k}|>>gT$, $k_o=\omega_p=|\mathbf{k}|$ is used.

In this study we shall only consider the transverse modes for the following reasons. In the static limit ($\mathbf{k}\rightarrow 0$), the transverse and longitudinal modes are degenerate due to rotational invariance, therefore only one mode needs to be computed. In the high momentum limit (**k**>>gT), the residue of the longitudinal mode is exponentially damped [22], thus only the transverse mode survives.

### 3.3 Dispersion Law for Quarks within the Quark-Gluon Plasma

To determine the dispersion relation for a quark travelling through the quark-gluon plasma, the position of the zero of the retarded inverse quark propagator must be determined. Proceeding in a manner similar to the gluon case, the dispersion relation for quarks is defined by the condition

$$\det [\not{p} + \Sigma] = 0 \tag{3.3.1}$$

where $\Sigma$ represents the quark self-energy and the quark mass has been neglected. Using the fact that the self-energy has the form

$$\Sigma = \gamma_0 \, \Sigma_0 - \frac{\boldsymbol{\gamma}\cdot\mathbf{p}}{\mathbf{p}} \Sigma_3 \tag{3.3.2}$$

equation (3.3.1) can be solved and the following condition is obtained.

$$p_0 = -\Sigma_0 \mp (\mathbf{p} + \Sigma_3) \tag{3.3.3}$$

Since we are only interested in the damping rate of the quarks, we can take the imaginary part of equation (3.3.3) and write

$$\gamma = \mathrm{Im} \, \{\Sigma_0 \pm \Sigma_3\} \, . \tag{3.3.4}$$

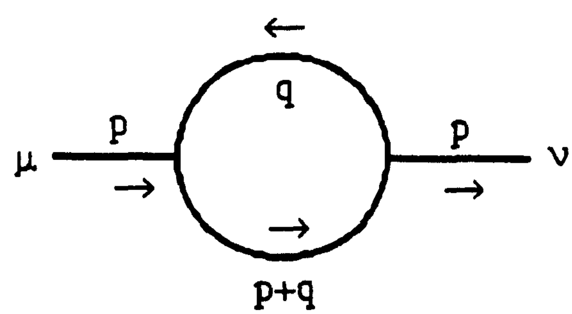
Expression (3.3.4) for the quark damping rate agrees with that derived in reference [23]. Notice that there are two possible modes given in (3.3.4). The upper sign refers to a collective excitation ("longitudinal mode") with chirality equal to minus the helicity that is special to light fermions in an ultrarelativistic plasma [7]. The lower sign refers to the standard excitation called the "transverse" mode with chirality equal to helicity. The two modes are degenerate at **p**=0, however, at high momentum (**p**>>gT) the residue of the collective excitiation ("longitudinal mode") vanishes exponentially [24]. Thus only the "transverse mode" for the quarks will be considered.

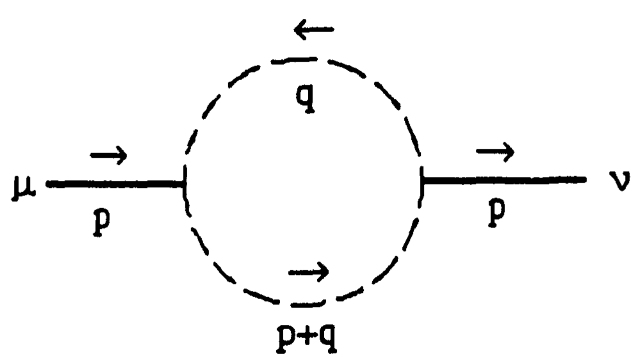
# GLUON PLASMA PARAMETERS IN THE ONE-LOOP APPROXIMATION

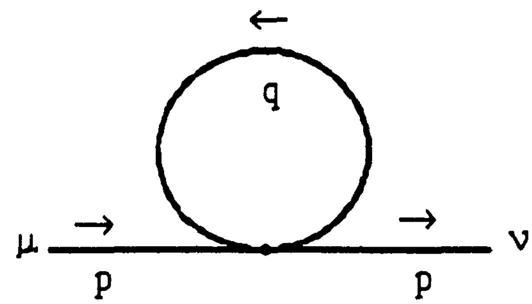
In this chapter, the gluon plasma frequency (in the static limit) will be computed to order $g^2$. We will also show that the gluon damping rate is not computable in this approximation.

## 4.1 Gluon Plasma Frequency in the Static Limit

To compute the gluon plasma frequency to order $g^2$, the one-loop gluon self-energy must be determined. In the covariant gauges, there are three (neglecting the quark-loop) one-loop diagrams that contribute to order $g^2$. The three diagrams to be evaluated are the gluon-loop, the ghost-loop and the tadpole diagram. The three diagrams are shown below.






Using the Feynman rules of section 2.4, the contributions from each diagram are listed below. The contribution from the gluon loop is given by

$$\Pi^{G}_{\mu\nu}(p) = \frac{g^2N}{2} T \sum_{n=-\infty}^{\infty} \int \frac{d^3q}{(2\pi)^3} \frac{1}{q^2(q+p)^2} \left[ A_{\mu\nu} + (1-\alpha) B_{\mu\nu} + (1-\alpha)^2 C_{\mu\nu} \right] \tag{4.1.1}$$

where $A_{\mu\nu}$ , $B_{\mu\nu}$ and $C_{\mu\nu}$ are defined in the following way.

$$A_{\mu\nu} = (2q^2+2pq+5p^2)g_{\mu\nu} + 10q_\mu q_\nu + 5(q_\mu p_\nu + p_\mu q_\nu) - 2p_\mu p_\nu$$

$$B_{\mu\nu} = -\frac{(q^2+2pq)^2}{q^2} g_{\mu\nu} + \frac{(q^2+2pq-p^2)}{q^2} q_\mu q_\nu + \frac{(q^2+3pq)}{q^2} (q_\mu p_\nu + p_\mu q_\nu) - p_\mu p_\nu$$
$$+ (q \to q+p,\ p \to -p)$$

$$C_{\mu\nu} = \frac{(p^2 q_\mu - pq p_\mu)(p^2 q_\nu - pq p_\nu)}{(q^2(q+p)^2)}$$

The contribution for the ghost-loop (Faddeev-Popov) is given by

$$\Pi^{FP}_{\mu\nu}(p) = -g^2NT \sum_{n=-\infty}^{\infty} \int \frac{d^3q}{(2\pi)^3} \frac{(p_\mu q_\nu + q_\mu q_\nu)}{q^2(q+p)^2} . \tag{4.1.2}$$

The final contribution from the tadpole diagram is given by

$$\Pi^{TP}_{\mu\nu}(p) = g^2NT \sum_{n=-\infty}^{\infty} \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{(-3g_{\mu\nu} + (1-\alpha)(g_{\mu\nu} - q_\mu q_\nu/q^2))}{q^2} \quad . \qquad (4.1.3)$$

In all of the above sums, $q_0 = 2\pi inT$. Although the ghost fields obey fermionic statistics, they are periodic rather than antiperiodic in imaginary time. This can be understood by recalling that the Faddeev-Popov determinant represents a variation with respect to infinitesimal gauge transformations and the gauge transformations must be periodic since they are proportional to the periodic gauge fields.

The complete expression for the gluon self-energy at one-loop (neglecting the quark-loop) is

$$\Pi_{\mu\nu}(p) = \Pi^{G}_{\mu\nu}(p) + \Pi^{FP}_{\mu\nu}(p) + \Pi^{TP}_{\mu\nu}(p) \quad . \qquad (4.1.4)$$

To determine the plasma frequency for transverse gluons in the static limit, expression (4.1.4) must be substituted into equation (3.2.11). All terms proportional to $\mathbf{p}$ and $\mathbf{p}^2$ may be neglected because we are interested in the limit $\mathbf{p} \to 0$. Terms proportional to $p_0$ and $p^2$ may also be neglected because gluons are massless at tree-level. Making the above substitutions and simplifications, we find that the terms proportional to the gauge parameter $\alpha$ cancel, and we are left with (see Appendix B)

$$k^2 = \frac{-1}{2}\Big( 4g^2NT \sum_{n=-\infty}^{\infty} \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{1}{q^2} + 4g^2NT \sum_{n=-\infty}^{\infty} \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{\mathbf{q}^2}{q^4}$$
$$- 4g^2NT \sum_{n=-\infty}^{\infty} \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{\mathbf{q}^2 \cos^2(\theta)}{q^4} \Big) , \qquad (4.1.5)$$

where $\theta$ is the angle between $\mathbf{p}$ and $\mathbf{q}$. The thermal sums are evaluated using equation (2.5.3) and the plasma frequency is determined using

(3.2.13). The plasma frequency for transverse gluons in the static limit then works out to be

$$\omega_p^2 = \frac{1}{9} g^2 N T^2 \quad . \tag{4.1.6}$$

The longitudinal plasma frequency in the static limit can be computed in a similar fashion by using equation (3.2.12). At zero momentum, the transverse and longitudinal plasma frequencies must be the same due to rotational invariance. Thus the calculation of the longitudinal mode can serve to test the consistency of our calculations.

Substituting expression (4.1.4) into the condition for the longitudinal mode (3.2.12), taking the same limits as before and simplifying, one obtains (see Appendix B)

$$k^2 = \Big(- 4g^2NT \sum_{n=-\infty}^{\infty} \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{1}{q^2} + 4g^2NT \sum_{n=-\infty}^{\infty} \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{q_0^2}{q^4}$$

$$- 4g^2NT \sum_{n=-\infty}^{\infty} \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{\mathbf{q}^2 \cos^2(\theta)}{q^4} \Big). \tag{4.1.7}$$

After performing the thermal sums, and calculating the plasma frequency using (3.2.13), we find

$$\omega_p^2 = \frac{1}{9} g^2 N T^2 \tag{4.1.8}$$

as expected. The quark contribution to the gluon plasma frequency is easily determined by considering the quark-loop contribution to the gluon self-energy. By adding the quark-loop contribution of the plasma frequency to equation (4.1.8), one obtains the well known result

$$\omega_p^2 = \frac{1}{9} g^2 T^2 \,(N + N_f/2)\,, \tag{4.1.9}$$

where $N_f$ is the number of massless quarks in the fundamental representation.

### 4.2 The Gluon Damping Rate in the Static Limit

As shown in chapter three, the damping rate for gluons in the static limit can be determined from equation (3.2.14) which is

$$\gamma = \frac{-1}{2\omega} \mathrm{Im}\ \{F(k_o,\mathbf{k})\} \qquad (4.2.1)$$

with $k_o$=0. The function $F(k_o,\mathbf{k})$ has been evaluated for both longitudinal and transverse gluons and found to be a real function. Thus there is no damping constant at the one-loop level. The reason why no imaginary parts are generated is that all terms proportional to $p^2$ have been neglected. Some authors have kept these terms in order to generate the imaginary part, however, this makes the calculation inconsistent. The one-loop approximation can be trusted up to order $g^2$, but a simple analysis shows this is not sufficient to determine the damping constant at zero momentum. If we assume $k_o = \omega - i\gamma$, where $\omega \approx gT$ and $\gamma \approx g^2T$, then from equations (3.2.11) and (3.2.12) we obtain

$$F(k_o,\mathbf{k}) \equiv k^2 \cong \omega^2 - 2i\omega\gamma \equiv ag^2T^2 - bg^3T^2 \ . \qquad (4.2.2)$$

To determine the damping rate at zero momentum, $F(k_o,\mathbf{k})$ must be accurate to order $g^3$ which is not the case with a one-loop calculation. Multi-loop diagrams are required to calculated the damping constant accurately. The problems encountered in the early attempts to calculate the damping constant are not surprising. The appearance of negative and gauge dependent damping is simply an indication of an incomplete and inconsistent calculation. A deeper reason for not expecting damping at the one-loop level is due to unitarity. The damping rate (inverse lifetime) is generated by the decay of one massless gluon into two. This reaction is kinematically forbidden thus the one-loop damping rate should be zero. As already discussed in the introduction, to determine the damping rate at zero momentum, a resummation of multi-loop graphs is needed, however, there exist some physical quantities that can be computed without employing a complete resummation. To demonstrate this, the dominant contribution to the damping rate for both transverse gluons and quarks at high momenta is calculated in the next chapter.

# THE QUARK AND GLUON DAMPING RATES AT HIGH MOMENTUM

The damping rate for transverse gluons and quarks with momenta $\mathbf{p} \gg gT$ is given by $\gamma(\mathbf{p}) = cg^2 \log(1/g)T + O(g^2T)$. In this chapter, the dominant contribution (the logarithmic term) will be calculated for both transverse gluons and quarks. It is assumed that the temperature T is the largest energy scale in the problem and that the coupling constant g is very small.

## 5.1 Damping of Energetic Transverse Gluons

To determine the dominant behaviour of gluons at high momentum, it is useful to divide the self-energy into two parts

$$\Pi_{\mu\nu}(E,\mathbf{p}) = \Pi^{\text{soft}}_{\mu\nu}(E,\mathbf{p},\lambda) + \Pi^{\text{hard}}_{\mu\nu}(E,\mathbf{p},\lambda)\,, \tag{5.1.1}$$

where all loop integrations in $\Pi^{\text{hard}}_{\mu\nu}(E,\mathbf{p},\lambda)$ are cut off in the infrared at $\lambda$. If $\lambda$ is chosen large enough, then $\Pi^{\text{hard}}_{\mu\nu}(E,\mathbf{p},\lambda)$ is calculable using the loop expansion. In contrast, $\Pi^{\text{soft}}_{\mu\nu}(E,\mathbf{p},\lambda)$ is not calculable using the loop expansion because it involves integration over all momenta less than the infrared cutoff and therefore must be obtained by other means. An important issue to be addressed is how much any desired quantity depends on the largely undetermined $\Pi^{\text{soft}}_{\mu\nu}(E,\mathbf{p},\lambda)$. For the case of the $g^2\log(g)$ terms, they can be determined with no knowledge of $\Pi^{\text{soft}}_{\mu\nu}(E,\mathbf{p},\lambda)$ because they are completely determined by the infrared divergent part of $\Pi^{\text{hard}}_{\mu\nu}(E,\mathbf{p},\lambda)$.

If the infrared cutoff $\lambda$ is chosen such that $\lambda \gg gT$, then the lowest-order contribution to $\Pi^{\text{hard}}_{\mu\nu}(E,\mathbf{p},\lambda)$ arises at order $g^2$ due to the one-loop graphs described in chapter four. If, however, the infrared cutoff is chosen to satisfy $g^2T \ll \lambda \ll gT$ then multi-loop graphs can contribute to the same order in g as do the one-loop graphs. This behaviour is due to a subclass of diagrams that are even more infrared singular than indicated by our simple power-counting argument in chapter two. The dangerous graphs are those such as the "ring" graphs within which multiple self-energy insertions are made along a single internal line [12]. In reference [7] Braaten and Pisarski argue that these last contributions may be resummed by dressing all "soft" lines (lines carryimg momenta less than or of order gT) by the calculable contributions of "hard thermal loops". After resumming these higher-loop contributions, $\Pi_{\mu\nu}^{hard}(E,\mathbf{p},\lambda)$ is given by the same one-loop diagrams as before, with the proviso that each propagator and vertex is to be replaced with an "effective" resummed propagator or vertex. These "effective" propagators and vertices are relatively complicated functions and can make the algebraic manipulations very cumbersome, however, at hard momenta the resummed propagators and vertices agree with the usual bare propagators and vertices. Since we are interested in computing the damping rate at "hard" momenta, all the vertices in the one-loop graph will carry hard momenta and therefore the bare vertices can be used. One of the propagators must also carry "hard" momenta and therefore will not need to be dressed.

To compute the damping rate for transverse gluons with momenta $\mathbf{p}$~T>>gT, we must consider the function (3.2.11)

$$F^{hard}(T,\mathbf{p},\lambda) = -\frac{1}{2}\,(\Pi_{ii}^{hard} - \frac{\mathbf{p}^i\mathbf{p}^j}{\mathbf{p}^2}\,\Pi_{ij}^{hard}) \qquad (5.1.2)$$

where $\lambda$ is chosen such that $g^2T \ll \lambda \ll gT$. As is established in more detail below, the contribution of $\Pi_{\mu\nu}^{hard}$ to $F^{hard}$ diverges logarithmically with the present cutoff. We can therefore write (5.1.2) as

$$F^{hard}(T,\mathbf{p},\lambda) = g^2\,[\,A\log(\lambda/\mu_{hard}) + B + O(\lambda/T)\,] + O(g^3) \quad (5.1.3)$$

with A and B being purely functions of $\mathbf{p}$ and T. The constant $\mu_{hard}$ is a calculable energy scale of the high-frequency part of the theory which is of the order gT. As already shown in chapter four, gluons acquire an effective mass $m = \omega_p(\mathbf{p}=0) = \frac{1}{3} gT\sqrt{(N + N_f/2)}$ at finite temperature.

In order to extract information about

$$F(T,\mathbf{p}) = F^{soft}(T,\mathbf{p},\lambda) + F^{hard}(T,\mathbf{p},\lambda) \tag{5.1.4}$$

from equation (5.1.2) it is necessary to place a constraint on $F^{soft}(T,\mathbf{p},\lambda)$. The only property that is required of $F^{soft}(T,\mathbf{p},\lambda)$ is that its $\lambda$ dependence must cancel that of $F^{hard}(T,\mathbf{p},\lambda)$. This must be true since the infrared cutoff is introduced artificially and has no physical significance. This implies that

$$\lambda\frac{\partial F^{soft}}{\partial\lambda} = -\lambda\frac{\partial F^{hard}}{\partial\lambda} \tag{5.1.5}$$

and therefore $F^{soft}(T,\mathbf{p},\lambda)$ must have the form

$$F^{soft}(T,\mathbf{p},\lambda) = g^2 [\, A\log(\mu_{soft}/\lambda) + X + O(\lambda/T)\, ] + O(g^3)\,. \tag{5.1.6}$$

Where $\mu_{soft}$ is the largest mass scale present in the soft part of the problem. $\mu_{soft}$ is taken to be of the order $g^2T$, because this is the scale at which perturbation theory fails and is the scale at which the damping rate appears. To the best of our knowledge there is no other physics between the scales gT and $g^2T$ to set the scale of $\mu_{soft}$. Notice that A is the same function as in equation (5.1.3). Adding equations (5.1.3) and (5.1.6) gives

$$F(T,\mathbf{p}) = -Ag^2 \log(\frac{\mu_{hard}}{\mu_{soft}}) + g^2(B+X) + O(g^3)\,. \tag{5.1.7}$$

Expression (5.1.7) can be written as

$$F(T,\mathbf{p}) = -Ag^2 \log(\frac{1}{g}) + O(g^2) \tag{5.1.8}$$

which determines the coefficient of the $g^2\log(g)$ term completely in terms of the calculable coefficient A.

The task is now to determine the coefficient A. Since A is determined by the infrared divergent part of $F^{hard}(T,\mathbf{p},\lambda)$, it is necessary to find the most divergent part of $F^{hard}(T,\mathbf{p},\lambda)$. There are three Feynman diagrams that contribute to the self energy at one-loop. The most divergent diagram is the graph with a gluon loop. The tadpole and ghost-loop diagrams are not as divergent as the gluon loop due to the Feynman rules. The tadpole diagram has no external momentum dependence and therefore the thermal sum is not infrared divergent. The ghost-loop diagram can produce terms that have at most one power of external momentum mutiplying the thermal sum. The gluon-loop can produce terms that have $p^2$ or $\mathbf{p}^2$ multiplying the thermal sums, thus these are the most divergent thermal sums. Only the terms with $\mathbf{p}^2$ need to be considered since $p^2=0$ for the external gluon line. Futhermore, the complete thermal sums do not have to be evaluated because the most divegent part of the sum is due to the n=0 term. As already mentioned, only one propagator needs to be "dressed" since hard momentum flows through all other lines. Therefore we must multiply the self energy by a factor of two to account for the two possible ways of routing the "soft" momenta.

The expression for the gluon-loop contribution to the gluon self-energy is given by

$$\Pi^{G}_{\mu\nu}(p)=\frac{g^2N}{2}T\sum_{n=-\infty}^{\infty}\int\frac{d^3q}{(2\pi)^3}V_{\mu\lambda\rho}(p,-p-q,q)D^{\lambda\kappa}(p+q)D^{*\rho\sigma}(q)V_{\nu\sigma\kappa}(-p,-q,p+q) \tag{5.1.9}$$

where V is the bare three-gluon vertex, D is the bare gluon propagator and D* is the "dressed" gluon propagator. The dressed gluon propagator can be obtained by inverting

$$D^{*-1}_{\mu\nu}=D^{-1}_{(o)\mu\nu}-\Pi_{\mu\nu} \tag{5.1.10}$$

where $\Pi_{\mu\nu}$ is given by equation (4.1.4). Since only n=0 contributes to the infrared divergent part, the resummed propagator has the form (see Appendix C)

$$D^{*\rho\sigma}(q)=\frac{1}{q^2}[g^{\rho\sigma}-\frac{m^2\delta^{\rho\sigma}}{(m^2-q^2)}] \tag{5.1.11}$$

with $\delta^{\rho\sigma}$= diag(0,-1,-1,-1). Performing the contractions in expression (5.1.9) yields

$$\Pi_{\mu\nu}(p) = \frac{g^2N}{2} T \sum_{n=-\infty}^{\infty} \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{1}{q^2(q+p)^2} [A_{\mu\nu} - \frac{m^2 B_{\mu\nu}}{(m^2-q^2)} + (1-\alpha)(C_{\mu\nu} + \frac{m^2 E_{\mu\nu}}{(m^2-q^2)})] \tag{5.1.12}$$

with

$$A_{\mu\nu} = (2q^2+2pq+5p^2)g_{\mu\nu} + 10q_\mu q_\nu + 5(q_\mu p_\nu + p_\mu q_\nu) - 2p_\mu p_\nu$$

$$B_{\mu\nu} = -(4\mathbf{p}^2+4\mathbf{p}\cdot\mathbf{q}+\mathbf{q}^2)g_{\mu\nu} + (q-p)^2\delta_{\mu\nu} - 3p_\mu \mathbf{p}_\nu - 3p_\mu \mathbf{q}_\nu - 3q_\mu \mathbf{q}_\nu - 3\mathbf{q}_\mu q_\nu - 3\mathbf{p}_\mu p_\nu - 3\mathbf{q}_\mu p_\nu$$

$$C_{\mu\nu} = -\frac{(q^2+2pq)^2}{q^2} g_{\mu\nu} + \frac{(q^2+2pq-p^2)}{q^2} q_\mu q_\nu + \frac{(q^2+3pq)}{q^2}(q_\mu p_\nu + p_\mu q_\nu) - p_\mu p_\nu$$

$$E_{\mu\nu} = \{(q^2-p^2)^2\delta_{\mu\nu} + (q^2-p^2)(p_\mu \mathbf{p}_\nu - q_\mu \mathbf{q}_\nu + \mathbf{p}_\mu p_\nu - \mathbf{q}_\mu q_\nu) + \mathbf{p}\cdot\mathbf{q}(q_\mu p_\nu + p_\mu q_\nu) - \mathbf{p}^2 p_\mu p_\nu - (2\mathbf{p}^2+3\mathbf{p}\cdot\mathbf{q}+2\mathbf{q}^2)q_\mu q_\nu\}/\{(q+p)^2\}$$

The damping rate for transverse gluons is determined by substituting equation (5.1.12) into (5.1.2). Taking the n=0 part of the sum, multiplying the self energy by two, and keeping only terms proportional to $\mathbf{p}^2$ we obtain

$$F_{\rm div}^{\rm hard}(T,\mathbf{p},\lambda) = \frac{-g^2NTm^2\mathbf{p}}{2\pi^2} \int_\lambda^\infty d\mathbf{q} \frac{1}{\mathbf{q}(m^2+\mathbf{q}^2)} \log(\frac{\mathbf{q}-2\mathbf{p}+i\varepsilon}{\mathbf{q}+2\mathbf{p}})$$

$$= +\frac{ig^2NT\mathbf{p}}{2\pi} \log(\lambda/m)\,. \tag{5.1.13}$$

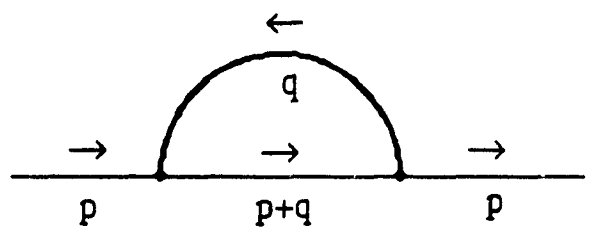
We can set $m=\mu_{\rm hard}$ and read $A=ig^2NT\mathbf{p}/2\pi$ . To evaluate the damping constant, substitute (5.1.13) into (3.2.14) and use the lowest-order mass shell condition $\omega=\mathbf{p}$. Doing so, we obtain our final result

$$\gamma(\mathbf{p}) = +\frac{g^2NT}{4\pi} \log(1/g) + O(g^2)\,. \tag{5.1.14}$$

Notice that the real part of F is infrared finite so only the damping rate acquires a $g^2\log(g)$ contribution. There are several aspects of this calculation that bear emphasis and will be discussed in the conclusions.

## 5.2 Damping of Energetic Quarks

The technique used to isolate the $g^2\log(g)$ term in the gluon damping rate can be repeated for quarks. The quark self-energy can be determined from the following Feynman graph.




Using the Feynman rules of section two and the "dressed" gluon propagator given by equation (5.1.11), the self-energy of a massless quark is

$$\Sigma(p)= T\sum_{n=-\infty}^{\infty}\int\frac{d^3\mathbf{q}}{(2\pi)^3}\, g\gamma_\mu T^a_{ik}\frac{\delta_{kn}}{(-\not{p}-\not{q})}\, g\gamma_\nu T^b_{nj}\delta_{ab}D^{*\mu\nu}(q)\,. \qquad (5.2.1)$$

The diagram in which the gluon line carries "hard" momenta is not as divergent as the above graph and can be neglected. This is a result of the fact that the quarks are antiperiodic in imaginary time. Notice that $(p_0+q_0)=(2n+1)\pi iT$ is proportional to T for n=0, whereas $q_0=2n\pi iT$ equals zero for n=0. From this it follows that the diagram with "soft" gluon momenta is the most divergent. Performing the contractions, keeping the n=0 part of the sum, and doing the angular integral, equation (5.2.1) reduces to

$$\Sigma(p)=\frac{g^2C_FT}{8\pi^2\mathbf{p}}\int_\lambda^\infty d\mathbf{q}\,\frac{(m^2\gamma_0p_0+m^2\gamma\cdot\mathbf{p}-2\mathbf{q}^2\not{p})}{\mathbf{q}(m^2+\mathbf{q}^2)}\log\left(\frac{\mathbf{q}-2\mathbf{p}+i\varepsilon}{\mathbf{q}+2\mathbf{p}}\right). \qquad (5.2.2)$$

Since we are interested in computing the damping rate, only the imaginary part of equation (5.2.2) is required. Recalling equation (3.3.2) and using expression (5.2.2) we find

$$\mathrm{Im}\{\Sigma_0\} = \frac{g^2 C_F T}{8\pi} \int_{\lambda}^{2p} dq \, \frac{(m^2 - 2q^2)}{q(m^2 + q^2)} \tag{5.2.3a}$$

$$\mathrm{Im}\{\Sigma_3\} = \frac{-g^2 C_F T}{8\pi} \int_{\lambda}^{2p} dq \, \frac{(m^2 + 2q^2)}{q(m^2 + q^2)} \quad . \tag{5.2.3b}$$

The damping rate for "transverse" quarks is given by equation (3.3.4) which is

$$\gamma = \mathrm{Im}\ \{\Sigma_0 - \Sigma_3\}\ . \tag{5.2.4}$$

Substituting equations (5.2.3a) and (5.2.3b) into equation (5.2.4) and performing the integration, the coefficient of the $g^2\log(g)$ term is found to be

$$A = \frac{g^2 C_F T}{4\pi} \quad . \tag{5.2.5}$$

Thus the dominant term in the "transverse" quark damping rate at high momentum is

$$\gamma(\mathbf{p}) = + \frac{g^2 C_F T}{4\pi} \log(1/g)\ . \tag{5.2.6}$$

# CONCLUSIONS

There are several aspects of this calculation that need to be emphasized. (i) First of all, the dominant behaviour of the damping rate for both the quarks and gluons is independent of the gauge parameter $\alpha$. All the terms that depend on the gauge parameter contribute only an infrared-finite amount. (ii) The sign of the damping rate $\gamma$ is positive in both cases. Thus the quark and gluon plasma oscillations are stable. (iii) In order to determine the subleading $O(g^2)$ contributions, knowledge of the coefficients B of equation (5.1.3) and X of equation (5.1.6) is required. Although the coefficient B is calculable using the complete resummation formalism of Pisarski et al., the coefficient X is not and can only at present be determined by making assumptions about the behaviour of the plasma in the low-frequency regime $\lambda \approx g^2T$. Thus the coefficient B need not be gauge-independent or positive, although calculations by Pisarski et al. indicate that it is. Only the sum of the coefficients B and X need be gauge-independent and positive. (iv) Since it is a logarithmic infrared-divergence that is responsibe for the logarithmic dependence on g, its coefficent is insensitive to the details of how the cutoff is implemented. Thus the discrepancy between the results of Lebedev and Smilga [25] and our results which agree with Pisarski et al., is due to the choice of the scale of $\mu_{soft}$. We assumed $\mu_{soft}$ to be of the order $g^2T$ because this is the scale at which perturbation theory fails and that the damping rate appears. The results of Lebedev and Smilga would require that $\mu_{soft}$ is of the order $g^{4/3}T$. At present we do not understand what physics should choose this scale of $g^{4/3}T$. (v) Finally, the infrared-divergent term in $F^{hard}$ is explicitly proportional to $m^2$ (see equation 5.1.13) thus it only receives contributions from two-loop and higher graphs that serve to dress the soft gluon propagator. Also the imaginary part arises only from the self energy of the internal lines which carry soft loop momenta $q < gT$ since the mass m may be neglected for large loop momenta. This agrees with what is expected physically from unitarity given the constraints of energy and momentum conservation in the plasma.

Thus we conclude that for some quantities in which infrared divergences in the perturbative expansion introduce a logarithmic dependence on the gauge coupling constant g, it is possible to very simply identify the dominant contributions. The simplicity of this method allows one to check the more complete and involved calculations employing the complete resummation formalism.

# APPENDIX A
# NOTATION AND CONVENTIONS

Throughout this thesis we use the natural units $c=\hbar=k_B=1$, where c is the velocity of light, $\hbar=h/2\pi$ where h is Planck's constant, and where $k_B$ is the Boltzman constant. Our metric in the Minkowski space $\{x^\mu:\mu=0,1,2,3\}$ is given by $g^{\mu\nu}$ with $g^{\mu\nu}=g_{\mu\nu}=$ diag(1,-1,-1,-1). The contravariant vectors of the space-time coordinate and energy-momentum are given by $x^\mu=(t,\mathbf{r})$ and $p^\mu=(E,\mathbf{p})$ where t is the time coordinate, **r** is the space coordinate, E is the energy, and **p** is the momentum. The covariant vectors are given by $x_\mu=g_{\mu\nu}\,x^\nu=(t,-\mathbf{r})$ and $p_\mu=g_{\mu\nu}\,p^\nu=(E,-\mathbf{p})$. All four-vectors are represented by plain text characters (ie. p) and all three vectors are represented by bold characters (ie. **p**).

The Dirac gamma matrices satisfy the anticommutation relation $\{\gamma^\mu,\gamma^\nu\}=2g^{\mu\nu}$. We use the following representation of the gamma matrices

$$\gamma^0=\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \gamma^i=\begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

where $\sigma^i$ are the Pauli matrices given by

$$\sigma^1=\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^2=\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma^3=\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

From these definitions it follows that $(\gamma^0)^2=1$ and $(\gamma^i)^2=-1$.

# APPENDIX B

## EVALUATION OF THE GLUON PLASMA FREQUENCY IN THE STATIC LIMIT

To determine the gluon plasma frequency in the static limit, $\Pi_{ii}$, $\Pi_{00}$, and $\frac{\mathbf{p}^i\mathbf{p}^j}{\mathbf{p}^2}\Pi_{ij}$ must be evaluated in the limit $\mathbf{p}\rightarrow 0$. Terms proportional to p may be neglected because tree-level gluons are massless. In this approximation we find:

$$\Pi_{ii}^{G} = g^2NT\sum_{n=-\infty}^{\infty}\int\frac{d^3\mathbf{q}}{(2\pi)^3}\left[\frac{-3}{q^2}+\frac{5\mathbf{q}^2}{q^4}+(1-\alpha)\left(\frac{3}{q^2}+\frac{\mathbf{q}^2}{q^4}\right)\right]$$

$$\Pi_{ii}^{TP} = g^2NT\sum_{n=-\infty}^{\infty}\int\frac{d^3\mathbf{q}}{(2\pi)^3}\left[\frac{9}{q^2}+(1-\alpha)\left(\frac{-3}{q^2}-\frac{\mathbf{q}^2}{q^4}\right)\right]$$

$$\Pi_{ii}^{FP} = -g^2NT\sum_{n=-\infty}^{\infty}\int\frac{d^3\mathbf{q}}{(2\pi)^3}\left[\frac{\mathbf{q}^2}{q^4}\right].$$

$\Pi_{ii}$ is obtained by summing the above contributions. In doing so, we find

$$\Pi_{ii} = g^2NT\sum_{n=-\infty}^{\infty}\int\frac{d^3\mathbf{q}}{(2\pi)^3}\left[\frac{6}{q^2}+\frac{4\mathbf{q}^2}{q^4}\right].$$

Taking the same limits as before, we can evaluate $\frac{\mathbf{p}^i\mathbf{p}^j}{\mathbf{p}^2}\Pi_{ij}$ as follows.

$$\frac{\mathbf{p}^i\mathbf{p}^j}{\mathbf{p}^2}\Pi_{ij}^{G} = g^2NT\sum_{n=-\infty}^{\infty}\int\frac{d^3\mathbf{q}}{(2\pi)^3}\left[\frac{-1}{q^2}+\frac{5\mathbf{q}^2\cos^2(\theta)}{q^4}+(1-\alpha)\left(\frac{1}{q^2}+\frac{\mathbf{q}^2\cos^2(\theta)}{q^4}\right)\right]$$

$$\frac{\mathbf{p}^i\mathbf{p}^j}{\mathbf{p}^2}\Pi_{ij}^{TP} = g^2NT\sum_{n=-\infty}^{\infty}\int\frac{d^3\mathbf{q}}{(2\pi)^3}\left[\frac{3}{q^2}+(1-\alpha)\left(\frac{-1}{q^2}-\frac{\mathbf{q}^2\cos^2(\theta)}{q^4}\right)\right]$$

$$\frac{\mathbf{p}^i\mathbf{p}^j}{\mathbf{p}^2}\Pi_{ij}^{FP} = -g^2NT\sum_{n=-\infty}^{\infty}\int\frac{d^3\mathbf{q}}{(2\pi)^3}\left[\frac{\mathbf{q}^2\cos^2(\theta)}{q^4}\right].$$

Taking the sum of the above terms we find

$$\frac{p^i p^j}{p^2}\Pi_{ij} = g^2NT\sum_{n=-\infty}^{\infty}\int\frac{d^3q}{(2\pi)^3}\left[\frac{2}{q^2} + \frac{4q^2\cos^2(\theta)}{q^4}\right].$$

Substituting the expressions for $\frac{p^i p^j}{p^2}\Pi_{ij}$ and $\Pi_{ii}$ into equation (3.2.11), we recover equation (4.1.5) from which we can determine the gluon plasma frequency in the static limit for the transverse modes.

The thermal sums in equation (4.1.5) can be evaluated using the technique described in chapter 2. Performing the thermal sums, and subtracting the infinite vacuum contribution, we find

$$T\sum_{n=-\infty}^{\infty}\int\frac{d^3q}{(2\pi)^3}\left[\frac{1}{q^2}\right] = \frac{-T^2}{12}$$

$$T\sum_{n=-\infty}^{\infty}\int\frac{d^3q}{(2\pi)^3}\left[\frac{q^2}{q^4}\right] = \frac{T^2}{24}$$

$$T\sum_{n=-\infty}^{\infty}\int\frac{d^3q}{(2\pi)^3}\left[\frac{q^2\cos^2(\theta)}{q^4}\right] = \frac{T^2}{72}.$$

Using the above thermal sums in equation (4.1.5), the gluon plasma frequency $\omega_p^2 = \frac{1}{9}g^2NT^2$ is obtained.

To determine the gluon plasma frequency in the static limit for the longitudinal mode, $\Pi_{00}$ must be evaluated in the limit $\mathbf{p}\rightarrow 0$.

$$\Pi_{00}^{G} = g^2NT\sum_{n=-\infty}^{\infty}\int\frac{d^3q}{(2\pi)^3}\left[\frac{1}{q^2} + \frac{5q_0^2}{q^4} + (1-\alpha)\left(\frac{-1}{q^2} + \frac{q_0^2}{q^4}\right)\right]$$

$$\Pi_{00}^{TP} = g^2NT\sum_{n=-\infty}^{\infty}\int\frac{d^3q}{(2\pi)^3}\left[\frac{-3}{q^2} + (1-\alpha)\left(\frac{1}{q^2} - \frac{q_0^2}{q^4}\right)\right]$$

$$\Pi_{00}^{FP} = -g^2NT\sum_{n=-\infty}^{\infty}\int\frac{d^3q}{(2\pi)^3}\left[\frac{q_0^2}{q^4}\right]$$

Summing the above expressions and substituting the values for the thermal sums, we find that $\Pi_{00} = 0$. We can now evaluate the longitudinal gluon plasma frequency in the static limit. Substituting $\Pi_{00}$ and $\frac{\mathbf{p}^i\mathbf{p}^j}{\mathbf{p}^2}\,\Pi_{ij}$ into equation (3.2.12), the longitudinal plasma frequency in the static limit is found to be $\omega_p^2 = \frac{1}{9}\,g^2NT^2$ .

# APPENDIX C

## EVALUATION OF THE GLUON DAMPING RATE AT HIGH MOMENTUM

The resummed covariant gluon propagator is determined by inverting equation (3.2.9)

$$D^{-1}_{\mu\nu} = D^{-1}_{(o)\mu\nu} - \Pi_{\mu\nu}$$

where $D^{-1}_{(o)\mu\nu}$ is the bare inverse gluon propagator, and $\Pi_{\mu\nu}$ is the gluon self-energy. Since we are interested in the most divergent contribution to the gluon damping rate, only the low frequency limit of $D_{\mu\nu}$ is required. Using equation (4.1.4) for the gluon self-energy, the zero momentum limit of $\Pi_{\mu\nu}$ is determined to be

$$\Pi_{\mu\nu} = m^2\delta_{\mu\nu}$$

with $\delta_{\mu\nu}$ = diag(0,-1,-1,-1). Using equation (3.2.10) for the bare inverse gluon propagator and keeping only the n=0 contribution, we find

$$D^{-1}_{(o)\mu\nu} = -\mathbf{k}^2 g_{\mu\nu} + A\mathbf{k}_\mu \mathbf{k}_\nu$$

with $A = (\frac{1}{\alpha} - 1)$. Using the above expressions for $D^{-1}_{(o)\mu\nu}$ and $\Pi_{\mu\nu}$ in equation (3.2.9) and inverting, we find the resummed gluon propagator to be

$$D_{\mu\nu} = \frac{-1}{\mathbf{k}^2} g_{\mu\nu} + \frac{(\frac{1}{\alpha} - 1)\mathbf{k}_\mu \mathbf{k}_\nu}{(m^2+\mathbf{k}^2)(m^2+\mathbf{k}^2/\alpha)} - \frac{m^2\delta_{\mu\nu}}{(m^2+\mathbf{k}^2)\mathbf{k}^2} .$$

Keeping only the most divergent terms in $D_{\mu\nu}$ , and since only the n=0 terms contribute, $D_{\mu\nu}$ may be expressed as

$$D^{*\mu\nu}(k) = \frac{1}{k^2}[g^{\mu\nu} - \frac{m^2\delta^{\mu\nu}}{(m^2-k^2)}]$$

which agrees with equation (5.1.11).

To determine the most divergent term in the gluon self-energy which contributes to the damping rate of transverse gluons, note the following facts. To determine the transverse gluon modes, we must apply equation (3.2.11)

$$p^2 = -\frac{1}{2}\left(\Pi_{ii} - \frac{\mathbf{p}^i\mathbf{p}^j}{\mathbf{p}^2}\Pi_{ij}\right)$$

to the gluon self-energy. The most divergent integrals in equation (5.1.12) will have factors of $\mathbf{p}^2$ or $p_\mu p_\nu$ in the $A_{\mu\nu}$ , $B_{\mu\nu}$ , $C_{\mu\nu}$ , and $E_{\mu\nu}$ terms. Equation (3.2.11) will "kill" any terms proportional to $p_\mu p_\nu$ , thus only terms with $\mathbf{p}^2$ are important. Since we are trying to determine a damping rate, only the imaginary contribution of the integral is required and must be of order $g^2$. Imposing these constraints on equation (5.1.12), we find that the most divergent term contributing to the damping rate at order $g^2$ is in $B_{\mu\nu}$ and is given by $-(4\mathbf{p}^2)g_{\mu\nu}$ . Keeping only this term in equation (5.1.12) and applying equation (3.2.11) to determine the transverse modes, we obtain equation (5.1.13) after performing the angular integration.

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