SYNTHESIS OF UNEQUALLY SPACED AND NON-UNIFORMLY EXCITED LINEAR ANTENNA ARRAYS

by
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ABSTRACT

This thesis deals with the synthesis of unequally spaced linear arrays with nonuniform current distributions. It has been established in the literature that allowing freedom in the element positions yields an additional design parameter whereby the specified results can be achieved more efficiently. The synthesis process presented here is such that desired a radiation pattern, as specified a priori, is achieved by determining unequal element spacings as well as nonuniform element excitations. A two step iterative numerical process is presented whereby such linear arrays can be synthesized. Examples are presented which validate the method in terms of known cases and demonstrate it through an "open-ended" case.

ABSTRAIT

Cette these est sur le sujet de synthese des antennes lineaire dont l'espacement des elements est variables et aussi la distribution de l'intensite des courants est non uniforme. Il ete etabli dans la litterature qu'en permettant du liberte dans les positions des elements, on recoit une parametre de calculation additionelle avec laquelle on peut obtenir des resultats plus efficaces. Le proces de synthese present er ceci est tel que la patterne de radiation desire comme il est specifier en avance, est obtenu en permettant des espacements des elements non uniforme et aussi des courants non egales. La methode presente est un proces numerique a deux etapes par lequel tel antennes lineaires peut etre synthetiser. Des examples sont presenter qui verifies la methode en utilisant des resultats deja connue et qui demontre la methode en presentant des resultats unique.

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1. INTRODUCTION

The subject of this thesis is the synthesis of linear antenna arrays with non-uniform current excitations as well as unequally spaced elements. The complex excitations and the positions of the array antenna elements are to be determined such that a specified radiation pattern is achieved.

The element spacings of unequally spaced arrays provide an additional design parameter which, along with the amplitude and phasing of the element current, are used to control the radiation pattern of the system. In arrays with equally spaced elements each element can be thought of as having one degree of freedom, that is, its complex excitation. However, for the case of arrays with arbitrarily distributed elements another degree of freedom is added to each element, namely its position along the axis of the array. By taking advantage of this additional degree of freedom in the synthesis process it should be expected that arrays falling into this latter category would need, in general, fewer elements to achieve the same performance than their equally spaced counterparts. Or, if it is decided that a fixed number of elements are to be used, better results should be obtained when taking into consideration the freedom in element positions. These advantages will be reiterated once again when discussing the results obtained in the synthesis procedure presented in this thesis.

The analysis of an array antenna, of course, presumes that the geometry and excitation are specified and hence the radiation pattern can be determined. In the inverse process of synthesis, the specification of the radiation pattern is used to invoke the array configuration. Synthesis techniques generally assume either a particular geometry or a certain current distribution. In this thesis, however, a mathematical and a numerical process is presented allowing for control of both the geometry and excitation.

A form of optimization thus results such that if in a particular case the number of elements is fixed then an optimum pattern is obtained for that number of elements. Conversely, if the pattern is to be optimized, this can be achieved by utilizing the least number of elements by determining the most advantageous spacing geometry.

The object of this thesis is to establish the opportunity and need for such a synthesis technique, to present the mathematical basis of its formulation, to produce the algorithms and software for its implementation and to demonstrate the utility by presenting a series of examples.

Thus, Chapter Two contains a historical review and a summary of currently available synthesis methods relevant to the objectives of this work. It is noted that in approaching this problem, other workers have demonstrated that controlling element spacing as well as element excitation can substantially improve the effectiveness and economy of linear arrays. The need for further improvement and for an optimization of such designs thus forms the basis for the present work.

Chapter Three presents the mathematical basis for the numerical processes developed in the current work. The algebraic formulation is developed first. The second part of the chapter outlines the manner in which the inversion of the problem can be carried out. This thus provides the foundation for the numerical algorithms and the consequent software needed to execute the synthesis procedure.

Chapter Four describes the key work of this thesis, namely the numerical techniques and the software which has been developed for this synthesis process. First the structure of the process is established and the necessary algorithm described. Next the software required is summarized and the tests, performed to determine the convergence properties, are described. Despite the synthesis process complexity, it is shown that it can be accommodated on an enhanced IBM PC XT level computer.

Chapter Five contains a series of results obtained by using the synthesis procedure. First are presented results for known cases which are also predictable analytically and these serve as a validation of the process. Results are then presented for an 'open ended' case in which the required pattern is specified and the solutions to it obtained. The effectiveness of the synthesis procedure is thus demonstrated.

Chapter Six summarizes the work and results of this thesis, discusses the contributions made and suggests further work which has been made possible as a consequence.

In summary, this thesis presents a process for the synthesis of linear antenna arrays in which optimization is achieved by controlling both the array geometry (inter-element spacing) and excitation. The principal contributions are the concept of the method and its mathematical formulations, the development of the necessary numerical process and its implementation and finally its validation and practical demonstration by synthesizing a number of cases.

2. HISTORICAL REVIEW

This chapter is a survey of previous relevant work on linear array synthesis. The subject of linear arrays received first major attention well before the Second World War. At that early stage, however, the approach was largely analytical or experimental. Schelkunoff and others developed a general theory for arrays for determining radiation patterns. Early synthesis techniques such as that by Dolph [16], were based on this theory and are well described in the literature. Therefore the survey presented here concentrates on work on synthesis techniques since the early 1960's. The survey is categorized into three sections. The first is about equally spaced linear arrays where only the complex element excitations are allowed to vary. The second contains work on unequally spaced linear arrays where the current distribution is assumed to be uniform. The third and final category is about the cases in which both the element spacing and the complex excitations are allowed to vary. Tables 2.1 - 2.3 , located at the end of this chapter, are listings of the literature considered and their appropriate classifications. These can give the reader a ready comparative overview of the type of research carried out so far and of its direction.

2.1 Equi-Spaced Linear Arrays

Initially most of the work carried out on linear arrays was concentrated on those having equal spacing. Much theoretical and analytical work was done to arrive at formulations of the radiation pattern function. These pattern functions in turn were examined and methods were proposed for the solution of the element excitations that would yield either an optimum beam (optimum in the sense of lowest sidelobe levels for a specified beamwidth or vice versa) or one closest to a specified pattern.

Dolph, [16], used the minimal property of Chebyshev polynomials to determine the element currents that produce array patterns yielding equi-level sidelobes that are at the lowest level for a particular beamwidth. Ma, [36], considered various synthesis methods using three different expressions to describe the desired pattern. First, he used the Bernstein polynomial to model the desired power pattern. Equating this to the array polynomial yielded a solution for the element currents. Secondly, he used interpolation formulae such as Lagrange and the trigonometric formulae to represent the radiation pattern. Lastly, Legendre polynomials were used to approximate the array pattern.

Stutzman, [74], introduced an iterative sampling procedure to achieve the desired radiation pattern. At each iteration a series of correction patterns were added to the array pattern of that iteration in an attempt to bring it closer to the desired pattern. The correction pattern is a function of increments to the original element currents thus enabling their solution. Convergence is demonstrated by examples. Inagaki and Nagai, [26], used circuit theory to arrive at an integral expression relating the current distribution to the radiation pattern. Sirnov, [68], applied the regularization principle to determine the phase distribution of an equally spaced linear array, using the criterion that the deviation of the resulting pattern from the desired one is minimized. The deviation is measured in terms of the L_2^{II} norm (see Appendix 1).

Synthesis methods described as "beam shaping", have been presented by some authors, [45],[83],[19],[12], and have similar principles as those described above. "Beam shaping", however, implies more stringent requirements because arbitrary patterns rather than simple "rectangular" pencil beams are required. Salahos, [56], introduced a synthesis method in which Chebyshev polynomials are used to model the desired pattern.

2.2 Unequally Spaced Linear Arrays with Uniform Excitation

Arrays with unequal element spacing are much more difficult to deal with. They introduce nonlinearities into the formulae used for radiation pattern calculations and thus make the synthesis problem more complex. In spite of this, however, they have been found to have definite advantages. King, Packard, and Thomas, [32], undertook a study which compared unequal spacing with equal spacing. The results obtained pointed favourably towards unequal element positioning. Brown, [8], indicated that a practical alternative to current tapering is nonuniform spacing.

One method of overcoming the difficulties brought about by nonlinearities is to choose an initial equi-spaced position vector for the array, assume this vector is to be perturbed and then solve for the perturbation vector. Additional assumptions that the perturbation is small lead to simplifications in the method which in turn make the solution possible. Harrington, [24], and Hodjat and Hovanessian, [25], utilize iterative perturbation techniques to reduce sidelobe levels. Baklanov, Pokrovsky and Surdutovich, [3], have also used a perturbation technique for the spacing. The element positions are deviated by an amount dx_k . By fixing a priori the desired sidelobe levels, these deviations are solved. They also give a brief outline of a method using Chebyshev currents with unequal element spacing. Schuman and Strait, [64] have introduced a method with constraints placed upon the relative positions. Ishimaru and others [13],[27],[28], use Poisson's summation formula to model the radiation pattern enabling them to solve for the position vector that will yield the desired pattern. Ishimaru and Chen, [27], further use Anger functions to model the system. Thomas,[77], makes the nulls of the desired pattern correspond to the nulls of an equivalent Chebyshev pattern. The resulting non- linear system of equations is solved using a least-square optimization technique based on methods of Gauss and

Lavenberg. Several authors, [50],[69],[7], have used the method of dynamic programming. It is a systematic search procedure that utilizes computers to find the optimum solution. It was found, however, that the results although favourable are not truly optimal. Lau and Wegrowicz,[34], used Rosenbrock's algorithm in a direct search method. A performance function is established such that the search is for minimum sidelobe levels. Patel, [46], used a direct search method. Here the optimum value of the element positions are obtained by successive three-point parabolic minimization along a specified direction. Tantaratana, [76], uses a l_p^{II} norm, along with the Fletcher and Powell method of minimization.

In all cases it was found that by allowing freedom in the element positions, comparable results were obtained in relation to the case of equal spacing with variable current distribution, and better results in comparison with the case with both equal spacing and equal current distribution.

2.3 Unequal Spacing and Non-uniform Current Distribution

Evidently the next step is to investigate the case in which both the element currents and the element positions are allowed to vary. This gives each element more degrees of freedom and is shown to give better results.

Unz, [80], formulated a method for nonuniform arrays based upon the eigenvalue method. The radiation pattern of the array is assumed to be of the form:

$$F(u) = \sum_{l=0}^{L} A_l \cos(ux_l)$$

where, A_l is the current excitation of the l^{th} element

x_l is the element distance from the center, measured in half wavelengths

$u = \pi\sin\Theta$, where Θ is the angle normal to the array axis.

Consider the following integral equation

$$I(x_l, x_m) = \int_{-\pi}^{\pi} \cos(ux_l)\cos(ux_m)\, du$$

By setting this integral to zero and evaluating, using trigonometric identities, one obtains the relationship for the eigenvalues

$$(x_l\pi)\tan(x_l\pi) = (x_m\pi)\tan(x_m\pi)$$

For these eigenvalues, the integral equation will have orthogonality properties. Using these properties along with the radiation pattern expression yields the following:

$$A_l = \frac{1}{\pi}\left[1 + \frac{\sin 2x_l\pi}{2x_l\pi}\right] \int_{-\pi}^{\pi} F(u)\cos(ux_l)\, du$$

Thus in using the above two equations to determine the element spacing and excitations, one could obtain a linear array which would approximate a required radiation pattern. However, depending upon the desired function $F(u)$, the resulting pattern may be only a crude approximation. In addition, the desired pattern is specified only in terms of a desired beamwidth and a specific sidelobe level. Therefore the results are not always satisfactory. Sanzgiri and Butler, [60], similarly utilized the eigenvalue method, subject, however, to a constraint placed on the sidelobe levels. This method has been shown to give results that are comparable to those obtained using the Dolph-Chebyshev technique. In addition, although the method can be applied to unequally spaced elements, it cannot be used to determine the spacing itself.

Popovkin and Scherbakov, [51], and Maffett and Curtz, [39], expressed the desired radiation pattern in terms of a Fourier-Stieltjes integral. From this an orthogonal polynomial resulted whose roots correspond to the element coordinates. The results obtained indicate that unequal spacing of the elements is better than equal spacing in terms of achieving a closer fit to the desired pattern. The method, however, can become very complex depending on the desired radiation patterns. Perini and Idselis, [48], and Butler and Unz, [10], used the method of steepest descent in an iterative procedure. Initial position and current vectors were first chosen and then new ones computed by moving against the gradient.

Goad and Stutzman, [22], used a two step iterative procedure based upon perturbations to firstly, the position vector and secondly, the current vector. An equally spaced array was used as the initial approximation. Since only the synthesis of symmetric radiation patterns was considered, the array factor is given by,

$$F^0(u) = 2\sum_{n=1}^{\frac{N}{2}} I_n \cos[ku(n-½)d]$$

where, I_n are the element currents

N is the number of elements

d is the element spacing

The difference between the improved pattern to be obtained and the initial pattern is taken. Imposing the condition that Δd be small and using also a trigonometric identity to approximate the cosine term, the following is achieved:

$$F^{1}(u)-F^{0}(u) = -2\sum_{n=1}^{\frac{N}{2}}{}' ku\Delta d_n - I_n \sin[ku[(n-½)d]]$$

where, Δd_n are the unknown element perturbations for the first iteration. The method used here gives the best solution in the discrete linear norm, l_2 (See Appendix 1). Ideally one would want to set $F^{1}(u)$ equal to the desired pattern $F_d(u)$. These are equated at sample points u_m and the process is generalized to the p^{th} iteration.

$$w(u_m)[F_d(u_m)-F^{p-1}(u_m)] = -2ku_m \sum_{n=1}^{\frac{N}{2}} I_n \Delta d_n^p \sin[ku_m[(n-½)d+\Delta d_n^1+\Delta d_n^2+ \cdots +\Delta d_n^{p-1}]]$$

$\frac{N}{2}$ sample points were chosen resulting in a square matrix equation to solve for the Δd . A similar process was applied to determine the element current perturbations. The resulting pattern will not equal exactly the desired pattern at the sample points since an approximation to linearize was made. Also, there are limitations in taking a discrete linear norm as a measure of the deviation since its accuracy is highly dependent on the number of points considered. Convergence of the process was shown with numerical results.

Abramovich and Sverdlik, [1], Ma and Walters, [37], and Schjaer-Jacobsen and Madsen, [65], utilize an iterative synthesis technique based on the minimax method which applies the concept of minimizing the maximum deviation using approximation theory. This deviation is defined more explicitly as the l_∞^{II} norm which is a discrete linear norm. The latter two methods do not explicitly determine the excitations but rather start from a Dolph-Chebyshev distribution. The Schjaer-Jacobsen procedure is described as follows: If $P_D = P_D(\psi)$ is the desired pattern and $P = P(x, \psi)$ is the obtained pattern, where x is the n-dimensional vector of design parameters, then $f(x)$ is the m-dimensional vector of residuals such that

$$f_j(x) = w_j[P(x,\psi_j) - P_D(\psi_j)] \quad , \quad j=1, \ldots, m$$

The problem is to minimize the maximum error $F(x)$ where

$$f(x) = \left|f(x)\right| = \max_{1 \leq j \leq m} \left|f_j(m)\right|$$

The technique used here is iterative and is based on successive linear approximations to the non-linear residuals. At the k^{th} stage of the process, define

$$F_k(h) \equiv \left|f(x_k) + \beta_k(h)\right|$$

where β_k is an approximation to the derivative matrix

$$\left\{\frac{\partial f_j}{\partial x_i}\right\}.$$

The increments h are then found by solving the constrained linear minimax problem'

$$F_k(h_k) = \min_{|h| \leq \lambda k} [F_k(h)]$$

The bound λ_k can be adjusted during the iterations.

Another procedure that has commonly been employed, [53],[41],[4], is the least squares method. Redlich, [53], uses a two step iterative procedure in which the L_2^{II} norm is minimized. Murthy and Kumar, [41], minimize either the L_2^I or L_∞^I norm. Balakrishnan et al, [4], minimize the l_p^I norm subject to constraints imposed on either the current distribution or the spacing. Redlich's, [53], two step process uses one step to compute the optimum currents and a second to compute the optimum spacings. The array factor is given by

$$F(\Theta) = A_0 + \sum_{n}^{N} A_n \cos(S_n \sin\Theta) + B_n \sin(S_n \sin\Theta)$$

where A_n , B_n are the element excitations

S_n are the element positions

N is the number of pairs of elements

The problem is to solve for A_n , B_n and S_n while minimizing the squared error expressed as a L_2^{II} norm,

$$E = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (P_s - F)^2 d\Theta$$

where P_s is the specified pattern. In the first step, the excitations A_n and B_n are solved by setting the partial derivatives of E, with respect to A_j and B_j , to zero.

The partial derivative of E with respect to S_j would lead to transcendental equations. To avoid this, it is assumed the spacing is perturbed by δ_n i.e., $S_n = S_n + \delta_n$. Assuming in addition that the δ_n are small, the corrected array factor is

$$F_2(\Theta) = F(\Theta) + \sum_n^N \delta_n \sin\Theta[-A_n \sin(S_n \sin\Theta) + B_n \cos(S_n \sin\Theta)]$$

δ_n are determined by solving

$$\frac{\partial(E_2^2)}{\partial \delta_j} = 0 \quad , \quad j = 1, 2, \cdots, N$$

where E_2 is the error using F_2 .

The results obtained show improvement over the Chebyshev array. However, no information is given regarding convergence of the process or of the uniqueness of the solution. In addition, the theory is applicable only to symmetric arrays.

Schjaer-Jacobsen, [65], uses a method similar to the minimax method. Here, however, the parameters to be optimized, namely the element positions or excitations, are subject to tolerances. This means that the worst desired case is given for the required pattern in terms of the maximum allowable sidelobe level. Wei et al, [86], introduce two methods whose objectives are to optimize with respect to high directivity and to low sidelobe levels. The first uses matrix theory and the second uses the iterative sampling method. For larger arrays the matrix calculations require high computation times, making the method impractical. For the iterative sampling method it is necessary to choose the initial geometry of the array in terms of its radiation pattern. The success of the method is dependent on how close this initial pattern is to the desired one. Yur'ev and Goncharova, [88], synthesize nonequidistant arrays by minimizing the noise level in the visible region which also ensures minimum mean sidelobe level. The results indicate, though, that the efficiency is limited. Miller and Goodman, [40], use Prony's method to solve the inverse problem. This is a two step procedure where the roots of a characteristic equation determine the positions of the elements. A disadvantage to this procedure is that complex (i.e., non-real) source locations may be obtained.

The following tables are a summary of the results of key papers about synthesis. The tables show the Authors, the method used and the essential results obtained.

Table 2.1 is for equally spaced arrays,

Table 2.2 is for unequally spaced arrays with uniform currents.

Table 2.3 is for unequally spaced arrays with non-uniform current distribution.

The following chapter describes the mathematical basis of the synthesis process that is proposed by this thesis.

Table 2.1 Literature summary of work on synthesis of equally spaced arrays

AUTHORS [Biblio ref]	METHOD	RESULTS
C.L. Dolph [16] 1946	Chebyscheff polynomials	improvement over equal current distribution
W.L. Stutzman [74] 1971	iterative sampling method	improvements over Woodward-Lawson
N. Inagaki & K. Nagai [26] 1971	circuit theory is used to solve for the applied voltages. Dipoles only	comparible to Dolph-Chebysheff
I.V. Sirnov [68] 1974	for a given amplitude distribution the phase is determined using regularization	decrease in the first sidelobe level
V.A. Obukhovets D.M. Sazonov [45] 1978	Lagrange multipliers	no examples presented
A. Chakraborty B.N. Das & G.S. Sanyal [12] 1982	method of stationary phase is used to determine the phase distribution	results are compared to those of a continuous source
E.Van Lil & A.Van de Capelle [83] 1983	modification of a regularization method	obtained pattern is a close approximation of desired pattern
R.S. Elliot & G.J. Stern [19] 1984	beam is shaped by shifting roots on the Schelkunoff unit circle	better than the Woodward synthesis method

Table 2.2 Literature summary on synthesis of unequally spaced arrays with uniform current distribution

AUTHORS [Biblio ref]	METHOD	RESULTS
D.D. King R.F. Packard & R.K. Thomas [32] 1960	a study of several methods involving unequal spacing	advantages to unequal element spacings
R.F. Harrington [24] 1961	a perturbation technique	sidelobe reduction; better than equally spaced arrays
F.W. Brown [8] 1962	points out that an alternative to current tapering is nonuniform spacing	
Ye. V. Baklonov V.L. Pokrovsky G.I.Surdutovich [3] 1962	perturbation of the position vector	reduced sidelobe levels
M.G. Andreasan [2] 1962	numerical methods; for arrays with large inter-element spacing	improvement over Dolph-Chebyshef ie. fewer elements for same results
A. Ishimaru [27] 1962	uses Poisson's formula to transform radiation pattern to that of	reduction of first sidelobe level
M.I. Skolnik G. Nemhauser & J.W. Sherman [69] 1964	dynamic programming	not optimal but favourable
A. Ishimaru & Y.S. Chen [28] 1965	Poisson's formula and Anger fncs are used to model rad. patt.; applicable to large arrays	reduced number of elements
Y.L. Chow [13] 1965	Poisson's formula with exponential spacing assumed	reduction in grating plateau
H.K. Schuman & B.J. Strait [64] 1968	assumes the element positions deviate by dx	sidelobe reduction

Table 2.2 Literature summary on synthesis of unequally spaced arrays with uniform current distribution **(Continued)**

AUTHORS [Biblio ref]	METHOD	RESULTS
R.P. Dooley [17] 1972	a numerical search technique	long run time
J.P. Basart [6] 1974	conditions are placed on the positions result-ing in simultan-eous equations	does not give optimum results
V.I. Popovkin & A.V. Mamorin [50] 1974	dynammic programming	positive
D.T. Thomas [77] 1976	nulls of the desired pattern are made to correspond with Chebysheff	comparible to Dolph-Chebysheff patterns
A. Kumar & P.K. Murthy [33] 1977	a perturbation technique using the Lp norm and mini-max as error criteria	sidelobe reduction; better than equal element spacings
F. Hodjat & A. Hovanessian [25] 1978	a perturbation technique a continous source	reduction in number of elements and sidelobe levels
M.G. Sarma & G.S. Sanyal [67] 1979	Anger functions; applicable to large arrays	reduced number of elements
D.C. Patel [47] 1982	l1 norm is error criteria; a direct search method	reduced sidelobes; better than equal spacing
S. Tantaratana [76] 1984	Lp norm is error criteria	sidelobe reduction
K. Wolfenstetter R. Hornung [87] 1984	mini-max error criterion	

Table 2.3 Literature summary of unequally spaced arrays with nonuniform current distribution.

AUTHORS & [Biblio ref]	METHOD	RESULTS
H. Unz [80] 1966	by considering an orthogonal int-egral and its eigenvalues	an algorithm only - no results
M.T. Ma & L.C. Walters [37] 1966	mini-max error criterion	better than Dolph-Chebysheff
A.L. Maffett & T.B. Curtz [39] 1967	Fourier-Stieltjes transform is applied to the desired pattern function	better than uniform arrays; practical limitations to method for larger arrays
J.K. Butler & H. Unz [10] 1967	method of steepest descent	little difference in gain from uniform array
W.A. Sandrin C.R. Glatt & D.S. Hague [59] 1969	direct search technique	desired sidelobe level is shown to be acheived
S.M. Sanzgiri & J.K. Butler [60] 1971	eigenvalue method	comparible to Taylor's method
V.I. Popovkin & G.I. Scherbakov [51] 1971	element positions correspond to roots of a poly	shortening of array length in comparison to equi-spaced array
J. Perini & M. Idselis [48] 1971	method of steepest descent is used to minimize error	to a certain extent beamshaping is possible
R.W. Redlich [53] 1973	2 step iterative procedure; for symmetric arrays only	improvements over Chebysheff array
Y.I. Abramovich M.B. Sverdlik [1] 1974	mini-max error criterion only - no results	presentation of method

Table 2.3 Literature summary of unequally spaced arrays with nonuniform current distribution. **(Continued)**

AUTHORS [Biblio ref]	METHOD	RESULTS
H. Schjaer-Jacobsen & K. Madsen [65] 1976	mini-max error criterion	comparible to Dolph-Chebysheff
P.K. Murthy & A. Kumar [41] 1976	L2 or L norm is used as error criterion	results obtained are better than those for equally spaced arrays
P.M. Russo [55] 1977	an algorithm using mean-square error criterion	algorithm presented only
N. Balakrishnan P.K. Murthy & S. Ramakrishna [4] 1979	least-squares method where error crtierion is the lp norm	results obtained are better than those for equally spaced arrays
S.D. Goad & W.L. Stutzman [22] 1979	2 step iterative perturbation technique; error is measured as a l1 norm	better than Woodward-Lawson
H. Schjaer-Jacobsen [66] 1980	mini-max error criterion	reduction in sidelobe levels
W. Wei, H. Jingxi & H. Shiming [86] 1983	iterative sampling method	reduction in sidelobe levels
E.K. Miller & D.M. Goodman [40] 1983	Prony's method, a procedure for estimating the paraeters of a sum of exponentials	reduction in SLL, however there is the possibility of the source positions being complex

3. GENERALIZED SOLUTION AND REGULARIZATION METHOD

This chapter presents the mathematical basis of the method used in the thesis for the synthesis of an unequally spaced linear array. The synthesis problem considered in this thesis is the design of a linear array whose desired directional function is specified a priori. For some given functions this may not be achievable, however, the objective is to seek an approximation which is as close as possible. There are two design parameters involved, namely, the unequal element spacing and the nonuniform complex current distribution. This problem, as for other problems in antenna synthesis leads to an inverse problem which consists of the inversion of the cause effect relationship.

The solution to the problem is constructed in two stages. The philosophy of this is expressed by L. Wegrowicz in [85]. The first stage is the direct problem. This consists of the mathematical analysis and algebraization of the problem. The second stage consists of the inverse problem and consists of formulation of the algorithm for the inversion of the overdetermined system of equations obtained by algebraization.

This inverse problem belongs to the class of ill-posed problems and is not directly solvable since theory does not exist for design. Therefore, a direct problem is first formulated for which the exact solution is known. Following the solution of the direct problem, however, there do exist some remaining unknowns. Solving for these gives the solution of the entire inverse problem.

The direct problem which consists of formulation and algebraization of the problem is outlined in Section 3.1. The second stage is the inverse problem. This involves the construction of a rational algorithm for the inversion of the overdetermined system of equations obtained by algebraic formulation, The method used is the generalized matrix inverse method, or if needed, Tikhonov's method of regularization.

This second step is outlined in Section 3.2.

3.1 Algebraic Formulation

In order to find an effective solution to the resulting inverse problem it is necessary to algebrize the corresponding direct problem. To achieve this, assume that the sources are located in a region V, that they are independent of the x - coordinate and are a one dimensional array along the z - axis. Assume also that their time dependence is e^{-jwt} and that their current density is of the form $J = I_x \Phi(y) f(z)$, i.e., that it is separable. Note that in engineering practice the convention used is e^{+jwt}. However the $(-jwt)$ convention usually used in mathematical literature will be followed. This does not affect the theory or results.

For discrete sources which are located along the z axis, the current density is assumed to be

$$J = f(z) = \sum_{n=1}^{N} c_n \delta(z - z_{0n}) \qquad --- \quad (3.1)$$

where N is the number of sources and z_{0n} and c_n denote respectively the position and the complex excitation of the n^{th} source.

From Equation (7) of [83] the field at the point of observation P excited by the sources from V may be represented as follows:

$$4\pi E_x(P) = j\omega\mu I_x \int\int_V \Phi(y) f(z) G(Q,P)\, dv \qquad --- \quad (3.2)$$

Note that due to the x coordinate independence of the source distribution, what is a volume integration over V becomes a double integration only.

$G(Q,P)$ is the two dimensional Green's function and is required to be chosen such that it firstly, satisfies the desired singularity at the source locations $R = 0$ and secondly, satisfies the boundary condition at infinity. That is,

$$G(Q,P) \rightarrow -2\, ln(kR) \qquad ; \quad R \rightarrow 0$$

$$\rightarrow \left(\frac{2\pi}{kR}\right)^{1/2} e^{j(kR+\frac{\pi}{4})} \qquad ; \quad R \rightarrow \infty$$

The following relation satisfies the required conditions:

$$G(Q,P) = j\pi H_0^{(1)}(kR) \qquad --- \quad (3.3)$$

where $R = (y^2+(z-z_{0n})^2)^{1/2}$

$H_0^{(1)}(kR)$ is the zero order Hankel function of the first kind.

Substituting Equation 3.3 into Equation 3.2 yields

$$E_z(P) = \frac{-\omega\mu I_z}{4} \iint_V \Phi(y) f(z) H_0^{(1)}(kR)\, dv \qquad --- \quad (3.4)$$

$\Phi(y)$ can be dropped from both sides yielding,

$$E_z(r,\Phi) = \frac{-\omega\mu I_z}{4} \int_Z f(z) H_0^{(1)}(kR)\, dz \qquad --- \quad (3.5)$$

The addition theorem for Bessel functions is,

$$H_0^{(1)}(kr) = \sum_{m=0}^{\infty} \epsilon_m H_m^{(1)}(kr) J_m(kz_0)\cos(m\Phi) \qquad \text{--- (3.6)}$$

where $|kr| > |kz_0|$

J_m is Bessel's function of order m .

ϵ_m is the Neumann factor and is defined as follows:

$$\epsilon_m = 1 \quad , m = 0$$

$$= 2 \quad , m \geq 1 \qquad \text{--- (3.7)}$$

For large r, i.e. in the far field

$$H_m^{(1)}(kr) \approx \left[\frac{2}{\pi kr}\right]^{1/2} e^{j(kr-\frac{\pi}{4})} e^{-jm\frac{\pi}{2}} \qquad \text{--- (3.8)}$$

Substituting Equations 3.8 and 3.6 into 3.5 we get

$$E_z(r,\Phi) = \frac{-\omega\mu I_z}{4} \int_z f(z_0) \sum_{m=0}^{\infty} \epsilon_m \left(\frac{2}{\pi kr}\right)^{1/2} e^{j(kr-\frac{\pi}{4})} \cdot e^{\frac{-jm\pi}{2}} J_m(kz_0)\cos(m\Phi)\, dz_0 \qquad \text{--- (3.9)}$$

Noting that the field can be written as

$$E_z(r,\Phi) = f_1(\Phi) \cdot f_2(r) \; ,$$

one can eliminate the functions of r from both sides as just scaling factors. Therefore,

$$E_z(\Phi) = \int_z f(z_0) \sum_{m=0}^{\infty} \epsilon_m e^{\frac{-jm\pi}{2}} J_m(kz_0)\cos(m\Phi)\, dz_0 \qquad \text{--- (3.10)}$$

Due to the uniform convergence of Equation 3.6, the order of the integration and the summation may be interchanged,

$$E_z(\Phi) = \sum_{m=0}^{\infty} \epsilon_m e^{\frac{-jm\pi}{2}} \int_z f(z_0) J_m(kz_0)\cos(m\Phi)\, dz_0 \qquad \text{--- (3.11)}$$

Equation 3.11 can be rewritten as,

$$E_z(\Phi) = \sum_{m=0}^{\infty} (-j)^m A_m \cos(m\Phi) \qquad \text{--- (3.12)}$$

where

$$A_m = \epsilon_m \int_z f(z_0) \cdot J_m(kz_0)\, dz_0 \qquad \text{--- (3.13)}$$

Using the $f(z_0)$ of Equation 3.1,

$$A_m = \epsilon_m \sum_n c_n J_m(kz_{0n}) \qquad \text{--- (3.14)}$$

The radiation pattern as depicted in Equation 3.12 is written in the same form as a Fourier series. $E_z(\Phi)$ is the desired radiation pattern and can itself be expressed as a Fourier series.

$$F(\Phi) = \sum_{m=0}^{\infty} a_m \cos(m\Phi) \qquad \text{--- (3.15)}$$

where a_m are the Fourier coefficients of the pattern. Equating 3.15

with 3.12 gives the following relation:

$$a_m = (-j)^m A_m \quad , \qquad m = 0,1,2,\cdots ,$$

or,

$$a_m = (-j)^m \epsilon_m \sum_n c_n J_m(kz_{0n}) \qquad --- \quad (3.16)$$

Rewriting Equation 3.16 in matrix form gives the following system of equations:

$$A = K \cdot C \qquad --- \quad (3.17)$$

where A is a column vector containing the complex Fourier coefficients of the desired pattern.

K is a rectangular matrix such that the elements are

$$k_{mn} = (-j)^m \epsilon_m J_m(kz_{0n})$$

C is a column vector containing the element complex excitations.

Equation 3.17 is the exact solution to the direct problem. The problem is well posed. Hence, given the matrix K and the vector C, the determination of A is straightforward and yields exact results. In addition, the convergence of the solution is assured. Consequently, this equation is used as a base for further considerations.

3.2 The Inverse Problem

The matrix relation presented in 3.17 represents the system to be solved for the synthesis problem. Thus 3.17 and 3.15 together form the algorithm for the solution of the inverse problem. The array characteristics such as the properties of the sources and element positions are contained in the matrix K which is explicitly expressed in terms of Bessel functions while the array current distributions are exhibited in the vector C. For the inverse problem, the desired radiation pattern in terms of its complex Fourier coefficients is given in vector A.

Algebraic formulation of the problem led to the system of equations depicted in Equation 3.18. The dimension of vector A is dependent upon the number of terms, M, maintained in the Fourier series of the desired pattern. This dimension M is in turn chosen such that the resulting function is satisfactory. Due to the linear independence of the Bessel functions, K is a maximal rank matrix. The number of columns N in K as well as the dimension of vector C is determined by the chosen number of elements in the system. The number of rows in K is infinite thereby inhibiting the process of its inversion. Consequently, for the purposes of computation, a matrix K_1 is formed from K by dropping the rows with indices $m > M_1$, where $M_1 \geq M$. The elements of K exhibit asymptotic behavior. Due to this asymptotic behavior one can always find M_1 such that the contribution of the neglected part is arbitrarily small. Hence for all practical engineering purposes the system is finite. In the actual calculations, a choice must be made of the point at which matrix K is truncated to form K_1. This may require several trials to reduce the truncation error to an acceptable level.

The type of sources and the initial geometry of the array are required to be chosen and hence the matrix K is considered to be known. The inverse problem consists of knowing a priori the directional function $F(\Phi)$ in terms of its Fourier coefficients in matrix A and determining both the excitation matrix C and the position matrix Z. However, the directional function has a nonlinear dependence on the element spacing. This leads to a procedure that involves a two step iterative process. In the first step of a given iteration the position vector is kept constant while the optimized current vector is determined. This is the linear part of the problem. In the second step of the iteration the current vector is maintained constant. Due to the nonlinear dependence on the position vector, there are no available direct methods for its solution. Therefore a linearization approach is applied and the Newton-Raphson-Kantorowich method is first applied. The system is then rewritten maintaining only the linear terms. This thus enables a solution of the position vector increments.

The element type and the initial geometry of the array being already chosen, the first step involves the solution of the system depicted in Equation 3.17. An exact solution of this system exists only in the special case when the given directional function $F(\Phi)$ can be expressed exactly as a sum of N partial functions F_n where the F_n correspond to the columns of K. Note that K is always a maximal rank matrix and hence it is always nonsingular. The system is overdetermined and has only one generalised inverse solution which will minimize the quadratic deviation. This solution is,

$$C = [K^{*}K]^{-1} \cdot K^{*}A \qquad \text{---} \quad (3.18)$$

where K^{*} is the transpose conjugate of K. The quadratic deviation that is minimized is

$$|| A - K \cdot C ||_2 \qquad \text{---} \quad (3.19)$$

The solutions for element excitations in 3.18 however may yield values which would lead to superdirectivity (i.e. exceptionally high maximum current to minimum current ratios) which is undesirable. There is no protection against superdirectivity. If, however, the solution is found to display such properties, the regularized approach is used. According to

Tikhonov [73], in order to find a regularized solution to Equation 3.17 it is sufficient to find a vector C that minimizes the following functional:

$$m^{\alpha}(C) = ||K \cdot C - A||_2 + \alpha ||C||_2 \qquad --- \quad (3.20)$$

This minimum corresponds to:

$$C = [K^* K + \alpha I]^{-1} K^* A \qquad --- \quad (3.21)$$

In the second step, which constitutes the determination of the element position increments, the position vector, Z, is perturbed. This increment vector is denoted by ΔZ. The Newton- Raphson-Kantorowich method is first applied to the system for linearization. An assumption of this method is that ΔZ be small. Equation 3.17 is then rewritten, expanding the matrix K with respect to the powers of ΔZ. Retaining only the linear term, we get

$$A = K|_{\Delta Z=0} \cdot C + J[K]_{\Delta Z=0} \cdot D(C_n) \cdot \Delta Z \qquad --- \quad (3.22)$$

where J[K] is the derivative matrix of K such that its elements are described by:

$$j_{mn} = \frac{dk_{mn}}{dz_n}$$

and D denotes the diagonal matrix.

The generalized inverse approach is once again applied on the following:

$$J[K] \cdot D(C_n) \cdot \Delta Z = A - K \cdot C$$

If it is seen that the results are not converging, i.e. if the residual of a given iteration is not less than the residual of the previous iteration, only then is regularization applied. In this case the L_2 norm that is minimized is,

$$m^{\alpha}(\Delta Z) = \| JK \cdot D(C_n)\Delta Z - A - KC \|_2 + \beta \| \Delta Z \|_2 \qquad \text{--- (3.23)}$$

where β is the regularization parameter. The minimum corresponds to the solution of

$$[JK \cdot D(C_n)]^* [JK \cdot D(C_n)]\Delta Z + \beta \Delta Z = [JK \cdot D(C_n)]^* [A - KC] \qquad \text{--- (3.24)}$$

The generalized solution is

$$\Delta Z = [JK \cdot D(C_n)]^* [JK \cdot D(C_n)] + \beta I^{-1} [JK \cdot D(C_n)]^* [A - KC] \qquad \text{--- (3.25)}$$

If Δ_i characterizes the residual in the right hand side of Equation 3.17 after the ith iteration, then if, as i increases, the error Δ_i goes to zero, the solution is converging. It is not proven that the two step process does converge. A mathematical proof of the convergence would seem difficult due to the complex nature of the expressions. It is, however, known that the linear part gives the exact solution. In addition, it is anticipated that by keeping the increments small enough, the non-linear part can always remain within the bounds of convergence. For any practical problem convergence can be verified numerically. It should be expected that the solution converges to a local minimum since due to the non-linear nature of the equations even the notion of a global minimum cannot be applied. A different initial vector Z will probably give rise to convergence to another local minimum. However, if the results obtained at a possible local minimum are satisfactory enough one can stop at this point.

In this chapter, the mathematical basis for the problem and the algebraic formulation was presented along with a rational algorithm for the inversion of the overdetermined system that resulted.

In the chapter that follows, the software implementation of the above described mathematical process is presented.

4. SOFTWARE IMPLEMENTATION

This chapter details one of the major contributions of this thesis. Described here are the numerical techniques and the software which has been developed for the synthesis process. First, a generalized block diagram is presented to outline the basic structure and logic of the program. Secondly, this is followed by a description of some of the major software components. Thirdly, the verification of the convergence of the process is outlined in the presentation of the various tests performed throughout the execution of the program. Finally, the options available in the logic of the program execution are outlined.

4.1 Generalized Block Diagram

The mathematical process described in the preceding chapter is implemented using Fortran V on an IBM XT personal computer. Figure 4.1 depicts a generalized block diagram of the program. As mentioned in Chapter 3, the first iteration computes only element excitations since the given initial positions of the elements are used. In all subsequent iterations both new position and new excitation values are computed for each element. Upon completion of the desired number of iterations, the results are printed. A much more detailed and explicit flowchart is included in Appendix 2. This includes all the paths and computations undertaken.

The inputs to the program are:

1. number of elements in the array
2. an initial position vector, Z
3. the desired pattern in terms of its Fourier coefficients, A_n

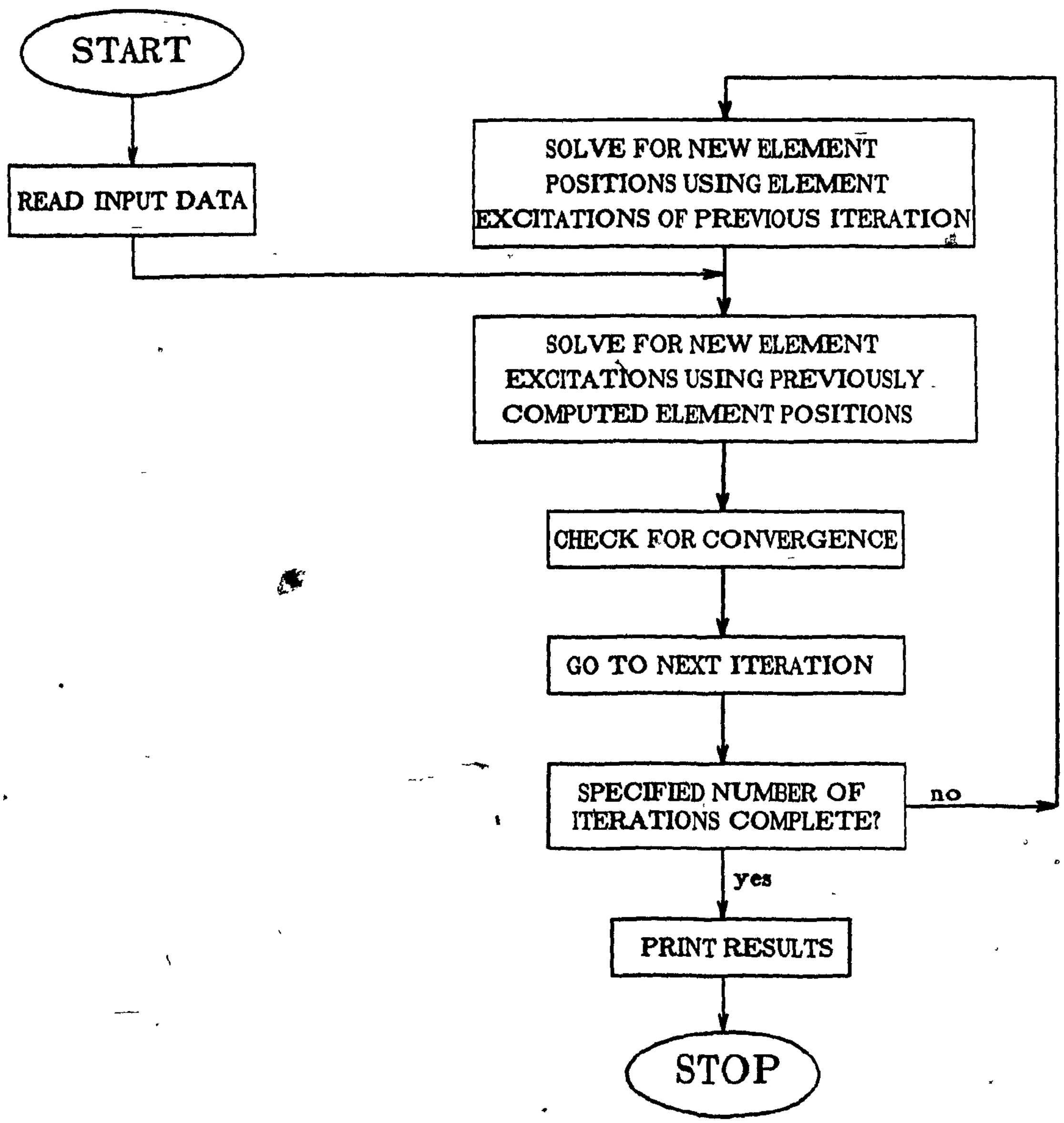


FIGURE 4.1: GENERALIZED BLOCK DIAGRAM

The first input, the position vector, contains also the choice for the number of elements in the array depicted indirectly by the dimension of the vector. Given the position vector, Z, the matrix K is computed using Bessel functions such that an element $k_{mn} = (-j)^m \epsilon_m J_m(kz_{0n})$. The first iteration is half an iteration in the sense that only the complex current distribution is determined. For all subsequent iterations first, the change in the position vector, ΔZ, is solved assuming the excitations arrived at during the previous iteration. Second, maintaining these new positions, new excitations are computed. Immediately following each of the above computations, the residual, Δ, is calculated to verify convergence. As mentioned in Section 3.2, $\Delta_i < \Delta_{i-1}$ implies convergence of the solution and the program continues on to the next iteration. Otherwise, the regularization procedure is introduced in a local loop so that the parameter α or β, whichever is applicable, is made non zero. This regularization parameter of a particular iteration increases in a loop until the convergence criteria are satisfied.

4.2 Solution of an Overdetermined System of Equations

The system to be solved is of the form

$$A = K \cdot C \qquad --- \quad (4.1)$$

where A and K are given and C is to be determined. The dimension of vector A is M. The rectangular matrix K is of dimension M_1 x N where M_1 is finite and greater than M. C is a vector of length N. In general M is greater than N and hence the system is overdetermined. Matrix K is, however, of maximal rank. Consequently, the matrix $[K^*K]$, which is of dimension N x N, is always invertible.

As seen in Section 3.2 the least squares solution is

$$C = [K^*K]^{-1} K^*A \qquad --- \quad (4.2)$$

Computationally, however, the frequently ill-conditioned nature of equations of this form can be the cause of large errors. Therefore,it is generally not advisable to use this expression directly to compute C. A better way to find the vector C which minimizes $||K \cdot C - A||_2$ is by performing a QR decomposition [87] of matrix K. In theory, the values of C obtained using Equation 4.2 or by using a QR decomposition approach should be the same.

The QR decomposition of K is

$$K = Q \cdot R$$

where Q is a M1 x M1 unitary matrix (i.e. $Q^* Q = I$) and R is an N x N upper triangular nonsingular matrix. Therefore, (4.1) can be written as,

$$Q^* A = Q^* KC$$

or,

$$Q^* A = RC$$

Since R is upper triangular, this is solved by backward substitution.

4.3 Bessel Functions

Matrix K requires the determination of Bessel functions of various orders and arguments. The subroutine used to compute these functions is based upon that by D.J. Sookne, [69].

4.4 Complex Matrices

All of the matrices involved are complex except for the position vectors Z and ΔZ. For simplicity in computation the complex matrices are written as partitioned matrices which are real. In this way all complex computations are eliminated. A complex matrix K is modelled as

$$\left[\begin{array}{c|c} \mathrm{Re}K & -ImK \\ \hline \mathrm{Im}K & \mathrm{Re}K \end{array}\right]$$

A complex vector C is modelled as

$$\left[\begin{array}{c} \mathrm{Re}\,C \\ \hline \mathrm{Im}\,C \end{array}\right]$$

4.5 Testing for Convergence

There are four consecutive tests that are executed throughout the process of a given iteration (except for the first which has only one test) to ensure the solution is converging.

4.5.1 Factor Test Although there is no analytical proof for the convergence of the process, convergence is highly probable provided the position increments, ΔZ, at each iteration are small. The value *Factor* is a limit placed on the size of the norm of ΔZ. The Factor test is the first test of the iteration and is performed immediately following the determination of ΔZ. If the test is passed, i.e., if $||\Delta Z|| < Factor$, the iteration continues. If the test is not passed then the regularization parameter β is activated in a loop and ΔZ is recomputed using regularization.

4.5.2 Linear Intermediate Test This test is performed after the Factor test using the new element positions. The residual $\Delta_{int.lin}$ is given as

$$\Delta_{int\,lin}^{(p+1)} = ||A_{res}^{(p)} - JK(Z^{(p)})\cdot D(C^{(p)})\cdot\Delta Z^{(p+1)}||$$

where p is the previous iteration number

$p+1$ is the current iteration number

$$A_{res}^{(p)} = A - K[Z^{(p)}]\cdot C^{(p)}$$

$||x||$ is the l_2 norm

Δ_{fin} is the residual measured at the end of the previous iteration. If $\Delta_{int.lin}^{(p+1)} < \Delta_{fin}^{(p)}$ the test is passed and the iteration continues. If the test is not passed then the regularization method is used and ΔZ is recomputed.

4.5.3 Nonlinear Intermediate Test This test is performed after the linear intermediate test. Matrix K is first recomputed using the new position vector $Z^{(p+1)}$. $\Delta_{n.l.int}^{(p+1)}$ is given by,

$$\Delta_{n.l.int}^{(p+1)} = ||A - K(Z^{(p+1)})\cdot C^{(p)}||$$

Once again the residual at this point is compared to $\Delta_{fin}^{(p)}$. If $\Delta_{n.l.int}^{(p+1)} < \Delta_{fin}^{(p)}$ then the test is passed and the iteration continues on to compute the new current excitations. If the test is not passed then ΔZ is recomputed using the method of regularization. If regularization has already been activated through a previous test of the same iteration, (i.e., if $\beta > 0$) then Δ Z is recomputed while imposing a stronger regularization parameter. (β is increased).

4.5.4 Final Test The final test for convergence is performed after the new excitations, C, are computed. $\Delta_{fin}^{(p+1)}$ is given by,

$$\Delta_{fin}^{(p+1)} = ||A - K(Z^{(p+1)})\cdot C^{(p+1)}||$$

If $\Delta_{fin}^{(p+1)} < \Delta_{fin}^{(p)}$ then the test is passed and the next iteration is initiated. If the test is not passed, ΔZ is recomputed using regularization. If regularization has already been used in the current iteration, a stronger β is imposed

4.5.5 Test for Superdirectivity This test is optional and can be activated when desired through a logical switch. It is used when the element excitations are found to display superdirective properties. If such is the case, then an additional regularization parameter, α , is introduced and regularization is used for the solution of the element excitations.

4.6 Software Structure and Logical Switches

The detailed structure of the software is shown in the Appendix 2 flowcharts. A major aspect fo the software structure is that of the logical switches which allow a variety of options to be chosen subject to the progress of the computation. Since the switches form a crucial aspect of the computational strategy, their purpose is described here.

There are seven logical switches in the program. These are denoted in the detailed flowchart by the numbered triangles. Each switch gives an option in the strategy of the computations, and an explanation of each switch is given below:

(1) In computing the norm of the position increment vector, ΔZ , there is an option of using either the l_1 or the l_2 norm. There will be no major difference in the results obtained by either. However the l_2 norm does impose a stronger restriction on the increment size.

(2) This switch comes into effect only if the Factor test does not pass. In this case, ΔZ can either be recomputed using regularization or the vector ΔZ can be modified directly either by proportional decreasing or by direct limiting.

(3) This switch also plays a role only when the Factor test fails. In addition, it is used only when regularization is not chosen in switch 2. The option here is the method with which to modify ΔZ . It can be modified either by reducing those increments ΔZ_i which are too large or by decreasing the entire vector proportionally.

(4, 5 & 6) These switches occur after each of the last three tests for convergence and are used only upon failure of the tests for convergence. Each presents the option of using or not using regularization.

(7) This switch occurs at the beginning of an iteration and once again presents a choice of using either regularization to control the size of ΔZ or the parameter Factor. Accordingly, the appropriate parameters are initialized.

It is useful to comment that the above described operations and their associated software were developed for and executed on a readily available enhanced IBM PC XT level computer system. The actual system used contained two 360k floppy disk drives and 640 kB of working memory. The program listing is stored on one diskette with a second diskette used for storage of the compiled and linked version.

The software structure as given in Appendix 2 is described by five sub flowcharts which indicate in great detail the internal structure of the program. This includes all the subroutines which were developed.

In conclusion, the software is now in a state that is ready for use by others but detailed comments and more precise documentation would be needed to make the software available for application by professional designers.

The following chapter validates the numerical process that has been developed and demonstrates it by a numerical example.

5. RESULTS AND DISCUSSION

In this chapter the effectiveness of the synthesis process is validated and demonstrated. The validation is carried out by using the process to determine array parameters for well known cases determined independently. The process is then demonstrated by choosing array pattern specifications for a practical application.

The following cases were used:

For validation:

1. The pattern of a twenty element equally spaced array, determined by the Woodward-Lawson method, was used as the specified pattern.
2. The pattern of a twenty element array based on the Dolph - Chebyshef method was used as the specified pattern.

For demonstration:

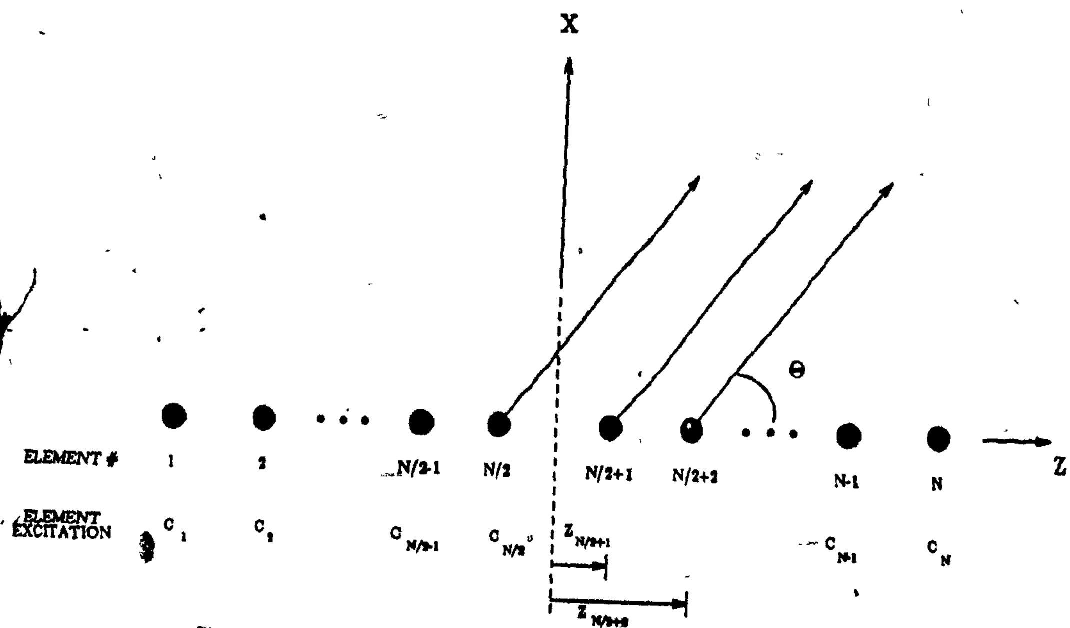
1. The beam of a hypothetical satellite system is used as the specified pattern

Figure 5.1 depicts the geometry of a linear array with non-uniformly spaced elements.

5.1 Case I

As a first case the pattern of a twenty element equally spaced array, whose excitations were determined by the Woodward - Lawson method of synthesis, was used as the desired pattern. The number of elements actually used in the process was then reduced to sixteen by using the method developed in this thesis. This is a substantial decrease in the number of elements thus reiterating the advantages of allowing for control of the element spacings.

Figure 5.1 : Geometry of the linear array.



The dimensions M and M_1 were chosen to be 30 and 45 respectively. Neither of the regularization parameters, α or β needed to be introduced for this particular example. Figure 5.2 shows the Woodward - Lawson pattern (desired pattern) along with the pattern obtained after the first iteration. Figure 5.3 shows the desired pattern along with the pattern obtained after 20 iterations. Figure 5.4 is a comparison after 75 iterations. It can be clearly seen that in allowing for unequal element spacing, the same results are achievable with a twenty percent reduction in the number of elements. Figure 5.5 is a plot of Δ_{fin} versus the iteration number, where Δ_{fin}, as explained in Chapter 4, is a measure of the residual between the desired and the achieved pattern. The monotonically decreasing nature of this function is an indication of convergence of the results. Table 5.1 gives the initial and final geometry and initial and final current distributions. Note that all element positions are in terms of λ, the wavelength of operation. Also, for this case, the element excitations are co-phasor.

5.2 Case II

The Dolph - Chebysheff synthesis method is applicable to equally spaced linear arrays. It is based on the Chebysheff polynomials and yields optimum beamwidth - side lobe level performance. That is, for a specified beamwidth the side lobe level is as low as possible or vice versa, for a specified side lobe level the beamwidth would be as narrow as possible.

To further illustrate the benefits of using both element excitations and element spacings as design variables, a Dolph - Chebysheff array pattern was used as the desired pattern. The pattern of a twenty element array designed for -30 dB side lobes was input as the desired pattern. This pattern is depicted in Figure 5.6. Superimposed upon this pattern is the pattern obtained after 15 iterations using 16 elements instead of twenty. The dimensions M and M_1 were chosen to be 32 and 48 respectively. Once again this example was executed without the need of activating the regularization parameters α and β. Figure 5.7 is a plot of Δ_{ftn} versus the iteration number. Its monotonically decreasing characteristic is an indication of the convergence of the process.

This case again yielded a twenty percent reduction in the number of elements over an equally spaced array for virtually identical directional functions. It is obvious that using this method of synthesis and allowing unequal element spacings, yields more efficient results.

5.3 Case III

As a more practical example, a beam for a conceivable spacecraft array for remote sensing synthetic aperture radar was used as the design objective. The antenna is required to provide a shaped beam in the elevation plane with a narrow azimuth beamwidth. The plan is to use 32 elements, equally spaced with an inter-element spacing of 0.82λ. The 0.82λ element spacing is the largest allowable before the appearance of grating lobes.

Using the method described in this thesis, with 32 elements, the results are depicted in Figures 5.8 and 5.9. Table 5.3 shows the element current distributions and positions. The dimensions M and M_1 were chosen to be 62 and 92 respectively.

To further demonstrate the advantages of using unequal spacing for the elements, 28 elements were used instead of 32. These results are shown in Figures 5.10 and 5.11. M and M_1 in this case were 60 and 90 respectively. For these cases neither α nor β were activated.

Plots for Δ_{ftn} versus iteration number for both of the above examples are depicted in Figures 5.12 and 5.13. Their monotonically decreasing nature is an indication of the convergence of the process. Tables 5.3 and 5.4 gives the initial and final geometry as well as the initial and final element excitations for both of the above examples.

Figure 5.2 : The pattern of a 20 element Woodward-Lawson array as the desired pattern (curve A) and the first iteration using only 16 elements (curve B).

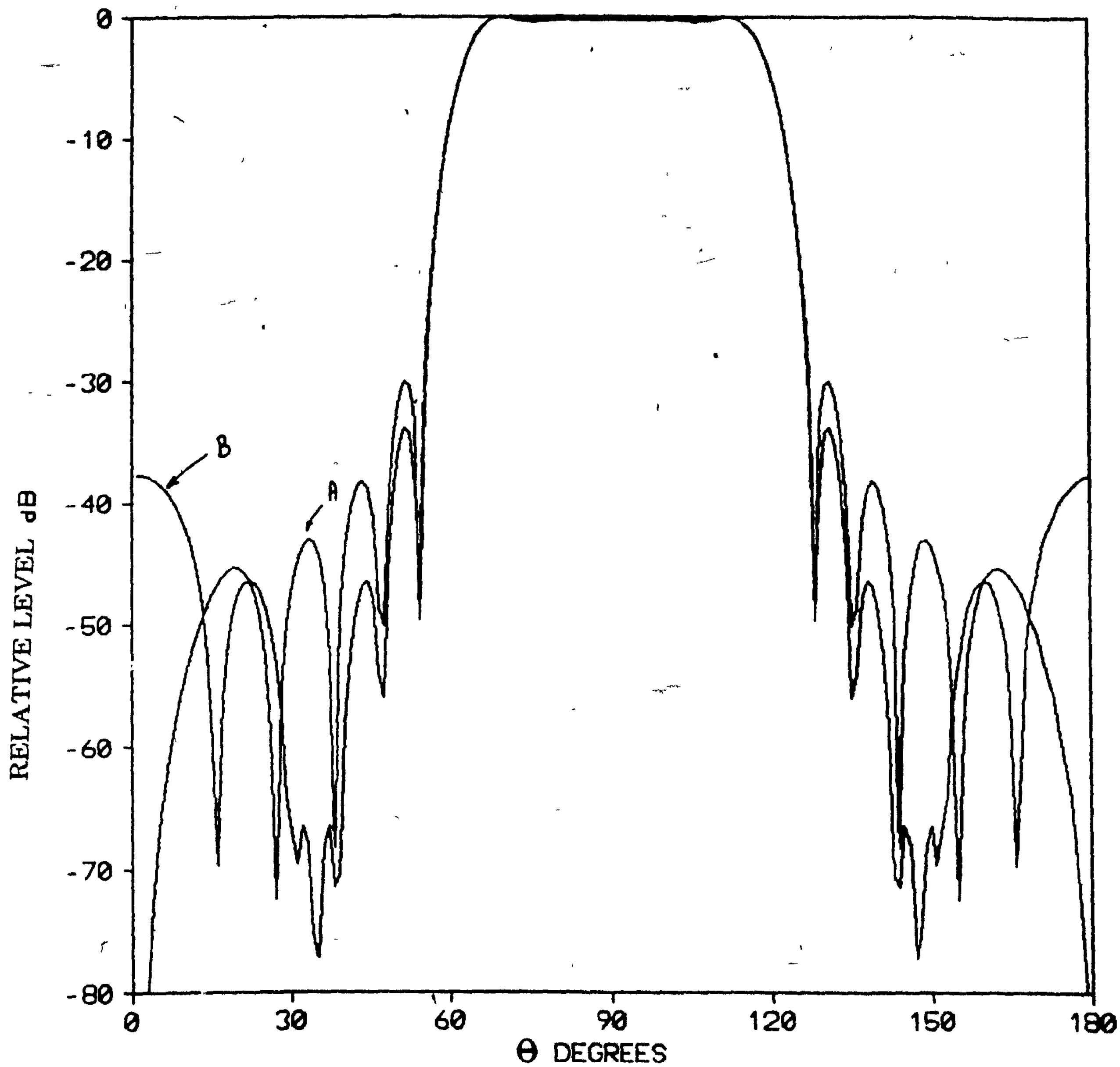


Figure 5.3 : The pattern of a 20 element Woodward-Lawson array as the desired pattern (curve A) and the 20th iteration using only 16 elements (curve B).

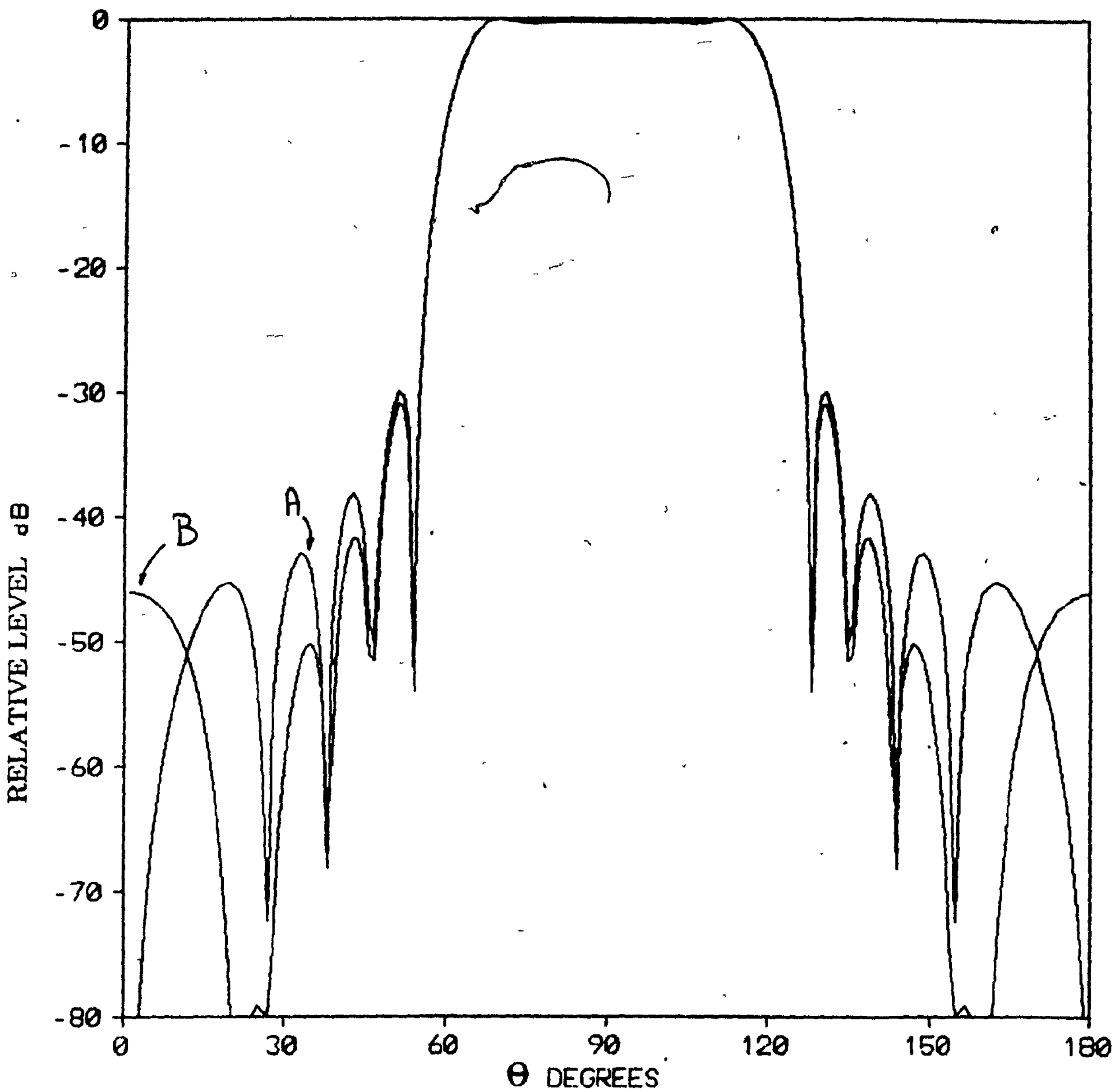


Figure 5.4 : The pattern of a 20 element Woodward-Lawson array as the desired pattern (curve A) and the 75th iteration using only 16 elements (curve B).

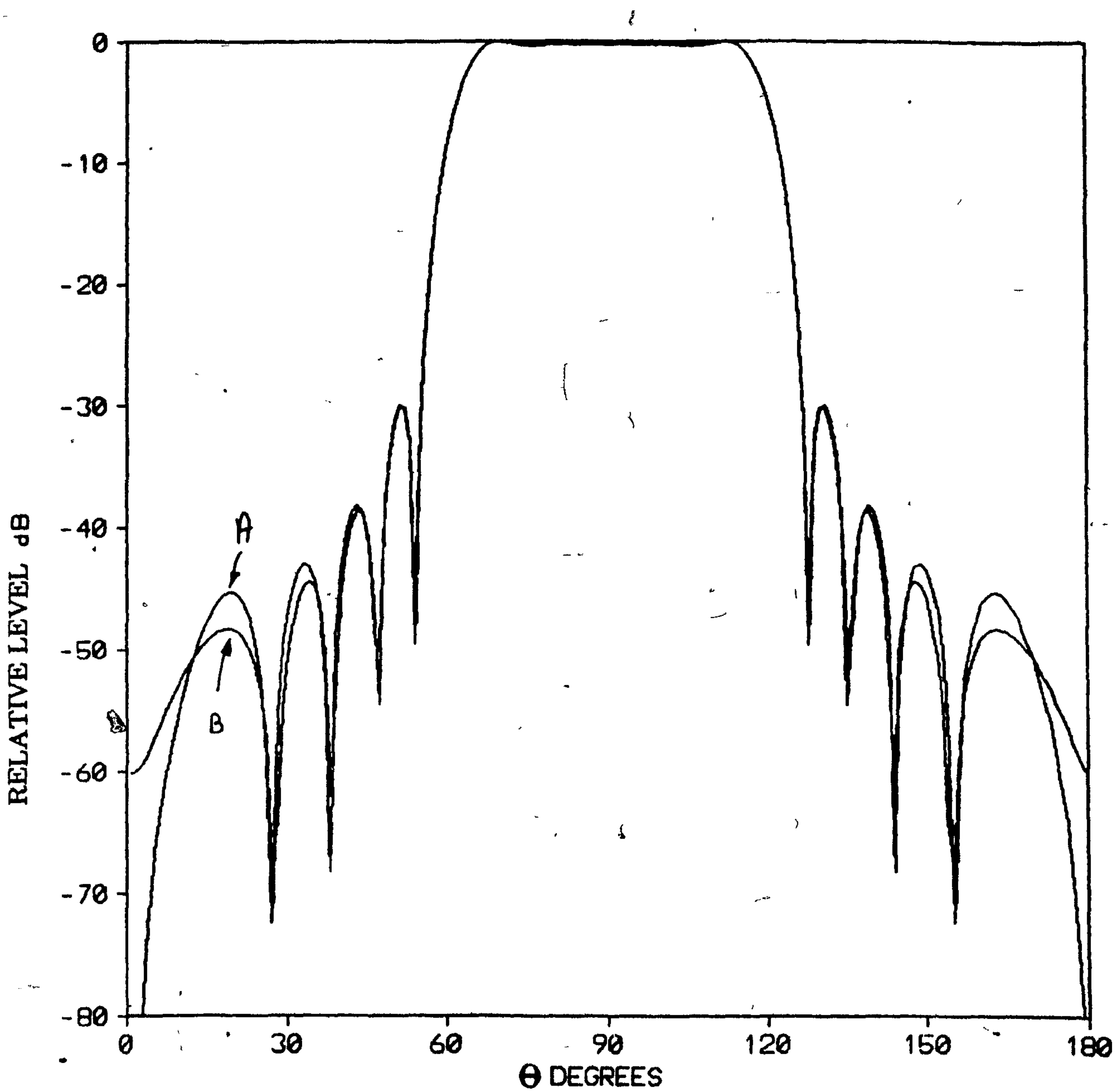


Table 5.1 Specifications of the 16 element array used to achieve pattern of 20 element Woodward-Lawson array

Note: The element excitations are relative and that the units are amperes. Element positions are in terms of $k \cdot z_n$ where, $k=2\pi/\lambda$

ELEMENT NUMBER	ELEMENT EXCITATIONS (amps)			ELEMENT POSITIONS (kz_n)		
	ITERATION NUMBER			ITERATION NUMBER		
	1	20	75	1	20	75
1	0.109	0.102	0.096	-28.27	-27.88	-27.59
2	-0.080	-0.086	-0.094	-24.50	-24.40	-24.27
3	-0.277	-0.271	-0.264	-20.73	-20.71	-20.68
4	0.424	0.422	0.419	-16.96	-16.93	-16.89
5	0.237	0.237	0.233	-13.19	-13.23	-13.36
6	-1.172	-1.172	-1.163	-9.42	-9.43	-9.43
7	0.632	0.632	0.626	-5.65	-5.65	-5.64
8	5.141	5.139	5.140	-1.88	-1.89	-1.89
9	5.141	5.139	5.140	1.88	1.89	1.89
10	0.632	0.632	0.626	5.65	5.65	5.64
11	-1.172	-1.172	-1.163	9.42	9.43	9.43
12	0.237	0.237	0.233	13.19	13.23	13.36
13	0.424	0.422	0.419	16.96	16.93	16.89
14	0.277	-0.271	-0.264	20.73	20.71	20.68
15	-0.080	-0.086	-0.094	24.50	24.40	24.27
16	0.109	0.102	0.096	28.27	27.88	27.59

Figure 5.5: A plot of Δ versus the iteration number.

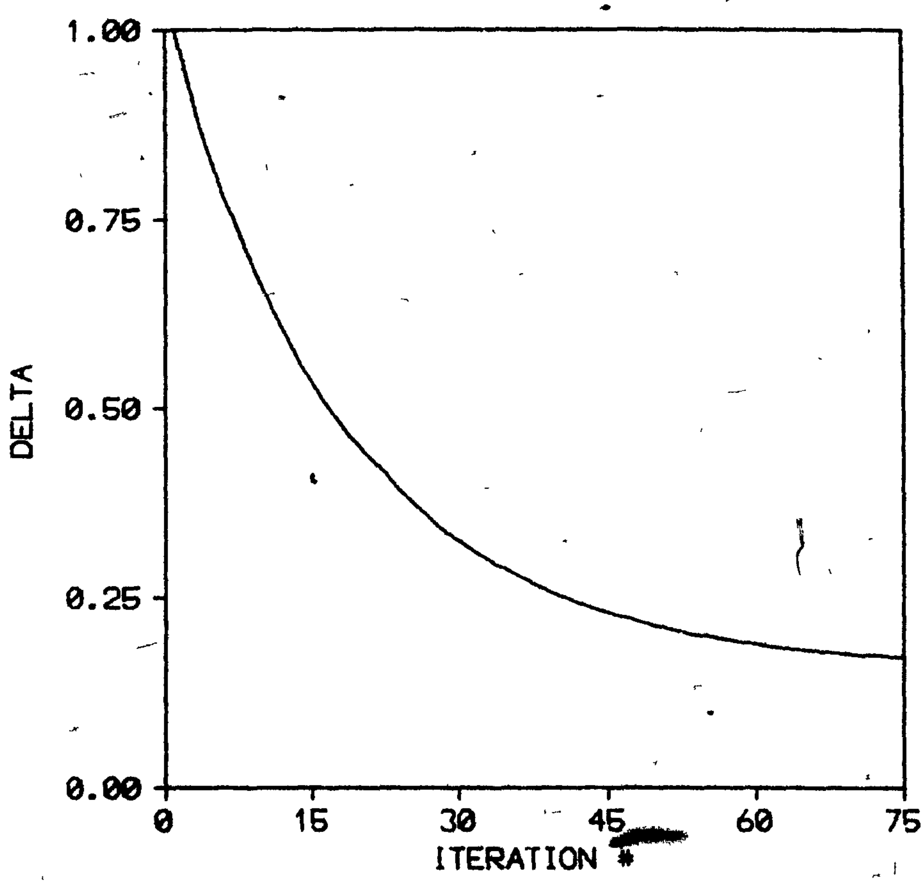


Figure 5.6: The pattern of a 20 element Chebycheff array with -30 dB sidelobe levels as the desired pattern (curve A) and the achieved pattern after 15 iterations using only 16 elements (curve B). It should be noted that the two patterns are virtually indistinguishable.

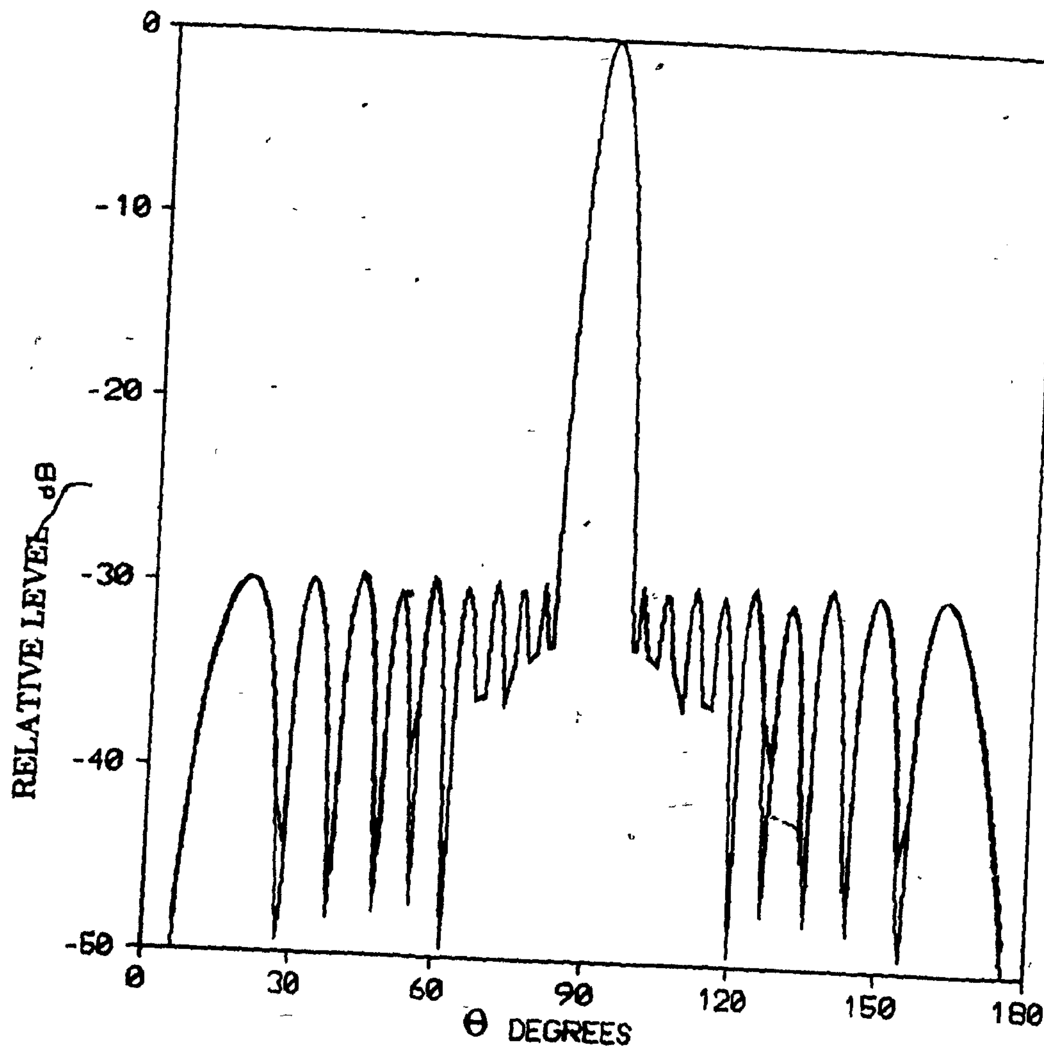


Table 5.2 Description of 16 element array used to achieve the radiation pattern of a 20 element uniformly spaced Chebysheff array with sidelobe levels of -30 dB.

ELEMENT NUMBER	ELEMENT EXCITATIONS (amps)		ELEMENT POSITIONS (kz_n)	
	ITERATION NUMBER		ITERATION NUMBER	
	1	15	1	15
1	1.100	1.096	-4.750	-4.740
2	1.191	1.195	-4.117	-4.120
3	1.753	1.740	-3.483	-3.484
4	2.239	2.253	-2.850	-2.849
5	2.798	2.793	-2.217	-2.217
6	3.253	3.244	-1.583	-1.583
7	3.569	3.586	-0.950	-0.949
8	3.771	3.750	-0.317	-0.317
9	3.771	3.750	0.317	0.317
10	3.569	3.586	0.950	0.949
11	3.253	3.244	1.583	1.583
12	2.798	2.793	2.217	2.217
13	2.239	2.253	2.850	2.849
14	1.753	1.740	3.483	3.484
15	1.191	1.195	4.117	4.120
16	1.100	1.096	4.750	4.740

Figure 5.7: Plot of Δ versus the iteration number for the 16 element array used to achieve the pattern of a 20 element Chebycheff array.

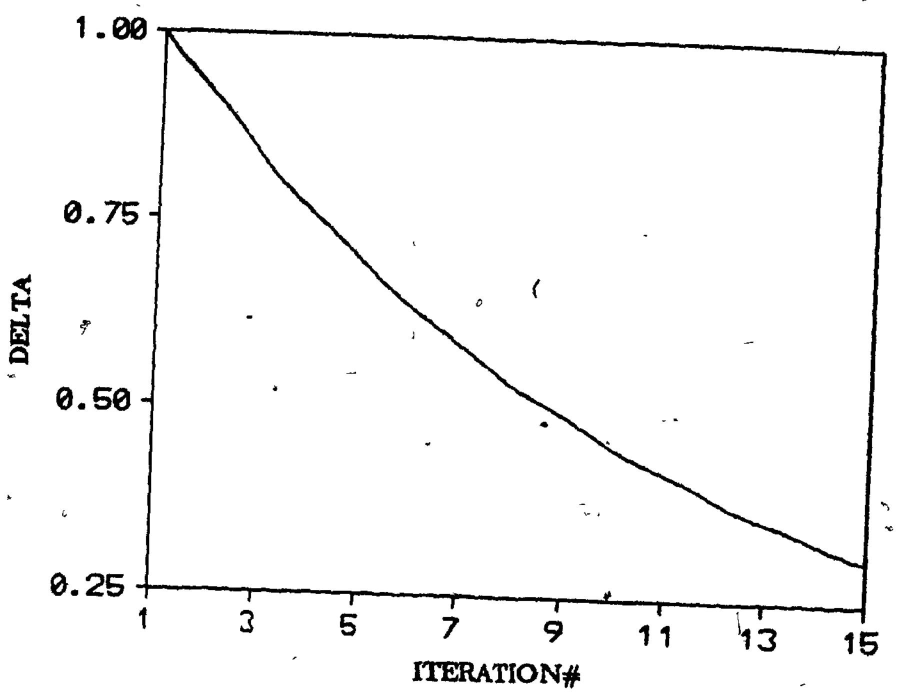


Figure 5.8 : The desired pattern of a hypothetical satellite beam (curve A) and the pattern achieved after the 1st iteration using 32 elements (curve B).

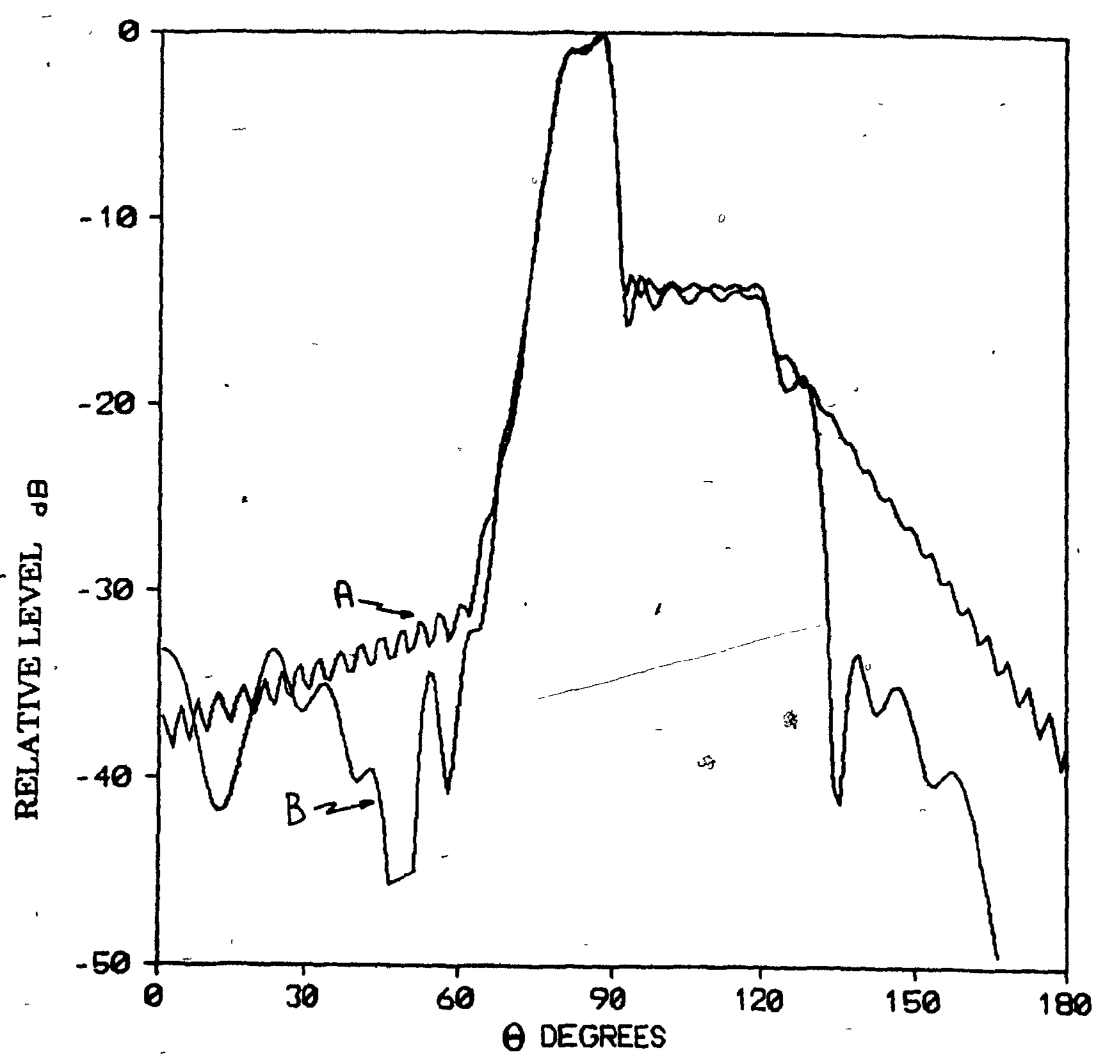


Figure 5.9: The desired pattern of a hypothetical satellite beam (curve A) and the achieved pattern after 20 iterations using 32 elements (curve B).

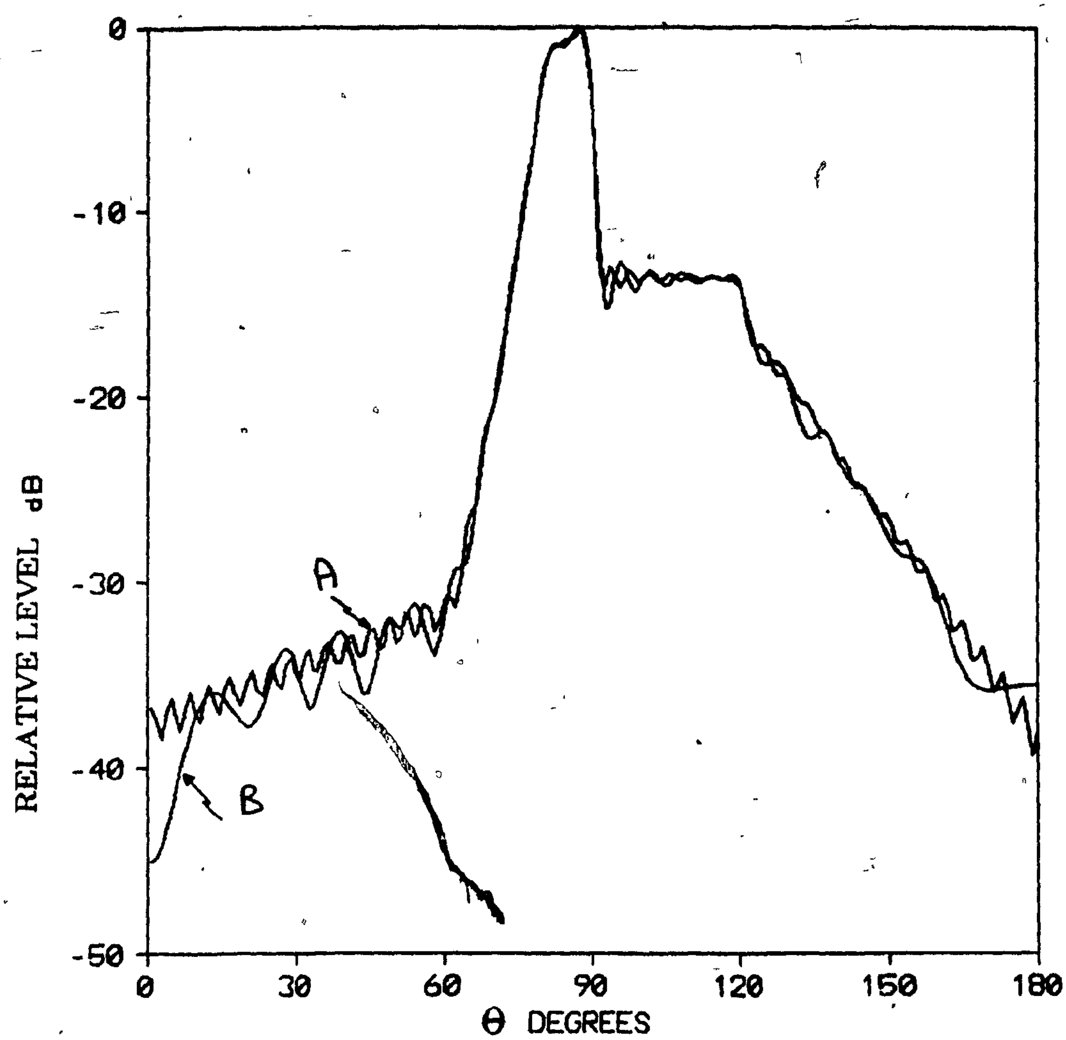


Figure 5.10: Plots of the desired pattern for a hypothetical satellite beam (curve A) and the achieved pattern after the 1st iteration using only 28 elements (curve B).

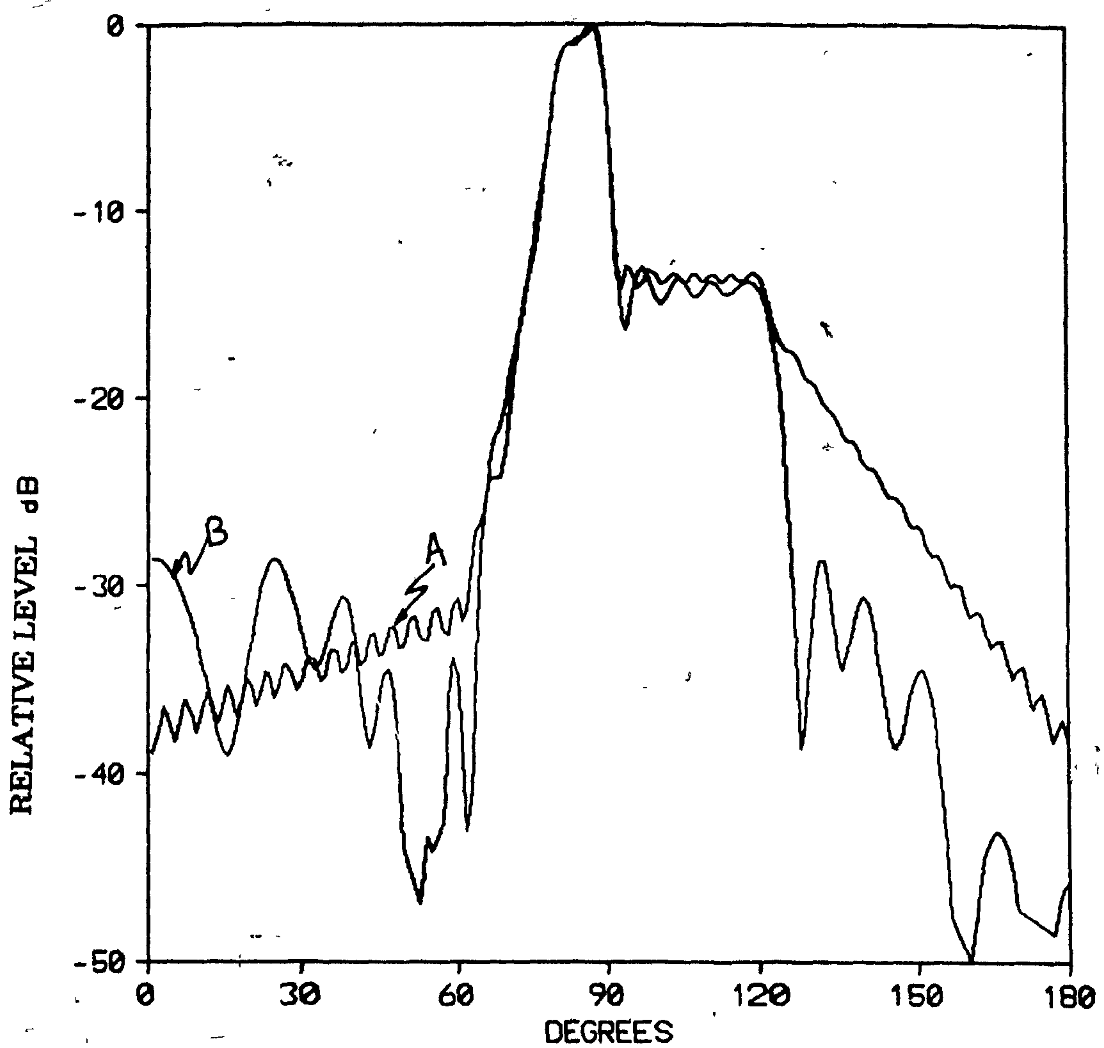


Figure 5.11: Plots of the desired pattern for a hypothetical satellite beam (curve A) and the achieved pattern after 15 iterations using only 28 elements (curve B).

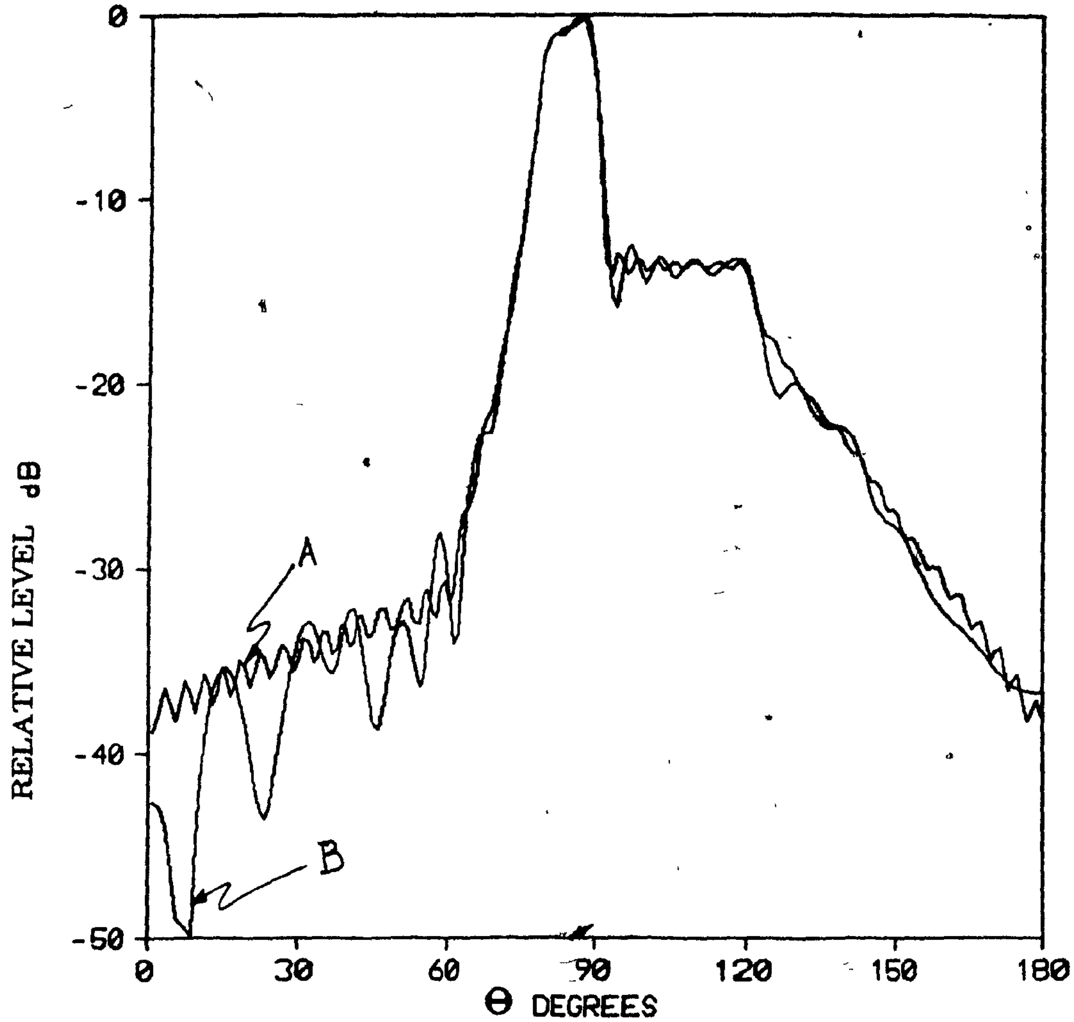


Table 5.3 Description of the 32 element array used to achieve the pattern of a hypothetical satellite beam

ELEMENT NUMBER	ELEMENT EXCITATIONS (amps) ITERATION NUMBER 1 mag.	phase	ITERATION NUMBER 20 mag.	phase	ELEMENT POSITIONS (kz_n) ITERATION NUMBER 1	20
1	0.541	212	0.512	212	-9.300	-9.337
2	0.737	206	0.711	206	-8.700	-8.713
3	0.829	237	0.904	237	-8.100	-8.102
4	1.175	233	1.081	233	-7.500	-7.503
5	0.930	245	1 001	245	-6.900	-6.903
6	1.193	268	1 168	268	-6.300	-6 298
7	1.047	256	1.011	256	-5 700	-5 691
8	0.707	271	0.735	271	-5.100	-5.133
9	0.905	238	0.851	238	-4.500	-4 510
10	1.531	223	1.587	223	-3.900	-3.887
11	2.729	234	2.696	234	-3.300	-3.305
12	3.860	244	3 815	244	-2.700	-2.700
13	5.955	273	6.004	273	-2.100	-2.104
14	8.052	288	7.970	288	-1.500	-1.495
15	8.516	331	8.577	331	-0.900	-0 914
16	17.517	1	18.027	1	-0.300	-0.288
17	17.517	359	18.027	359	0 300	0.288
18	8.516	29	8.577	29	0.900	0.914
19	8 052	72	7.970	72	1.500	1 495
20	5.955	87	6 004	87	2.100	2 104
21	3.860	116	3 815	116	2.700	2.700
22	2.729	126	2.696	126	3 300	3.305
23	1.531	137	1.587	137	3 900	3.887
24	0.905	122	0.851	122	4.500	4.510
25	0.707	89	0 735	89	5.100	5.133
26	1.047	104	1.011	104	5.700	5.691
27	1.193	92	1 168	92	6.300	6.298
28	0.930	113	1.001	113	6.90Q	6.903
29	1.175	127	1.081	127	7.500	7.503
30	0.829	123	0.904	123	8.100	8.102
31	0.737	154	0.711	154	8.700	8.713
32	0.541	148	0.512	148	9.300	9.337

Table 5.4 Description of the array used to achieve the pattern of a hypothetical satellite beam using 28 elements instead of 32 elements

ELEMENT NUMBER	ELEMENT EXCITATIONS (amps)				ELEMENT POSITIONS (kz_n)	
	ITERATION NUMBER				ITERATION NUMBER	
	1		20		1	20
	mag.	phase	mag.	phase		
1	0.650	203	0 556	203	-8.640	-8.645
2	0.926	234	1.045	234	-8.000	-8.026
3	1.286	240	1.271	240	-7.360	-7.356
4	1.010	245	0.973	245	-6.720	-6.704
5	1.196	274	1.228	274	-6.080	-6.107
6	1 186	260	1.117	260	-5.440	-5 442
7	0.580	232	0.589	232	-4.800	-4.743
8	1.162	243	1.188	243	-4.160	-4 192
9	2.751	226	2.677	226	-3.520	-3.533
10	3 404	237	3 539	237	-2.880	-2.868
11	6.009	270	5.846	270	-2.240	-2.244
12	8.158	283	8.181	283	-1.600	-1.596
13	9 057	327	9.038	327	-0.960	-0.977
14	17.961	360	18.678	360	-0.320	-0.305
15	17.961	0	18 678	0	0 320	0 305
16	9 057	33	9.038	33	0.960	0.977
17	8.158	77	8.181	77	1.600	1.596
18	6 009	90	5.846	90	2.240	2.244
19	3.404	123	3 539	123	2.880	2.868
20	2 751	134	2 677	134	3 520	3.533
21	1.162	117	1.188	117	4.160	4 192
22	0.580	128	0.589	128	4 800	4.743
23	1.186	99	1 117	99	5.440	5.442
24	1.196	86	1.228	86	6 080	6.107
25	1 010	115	0.973	115	6.720	6.704
26	1.286	119	1 271	119	7.360	7.356
27	0.926	126	1.045	126	8.000	8.026
28	0 650	157	0.556	157	8.640	8.645

Figure 5.12: Plot of Δ versus Iteration Number for 32 element hypothetical satellite array.

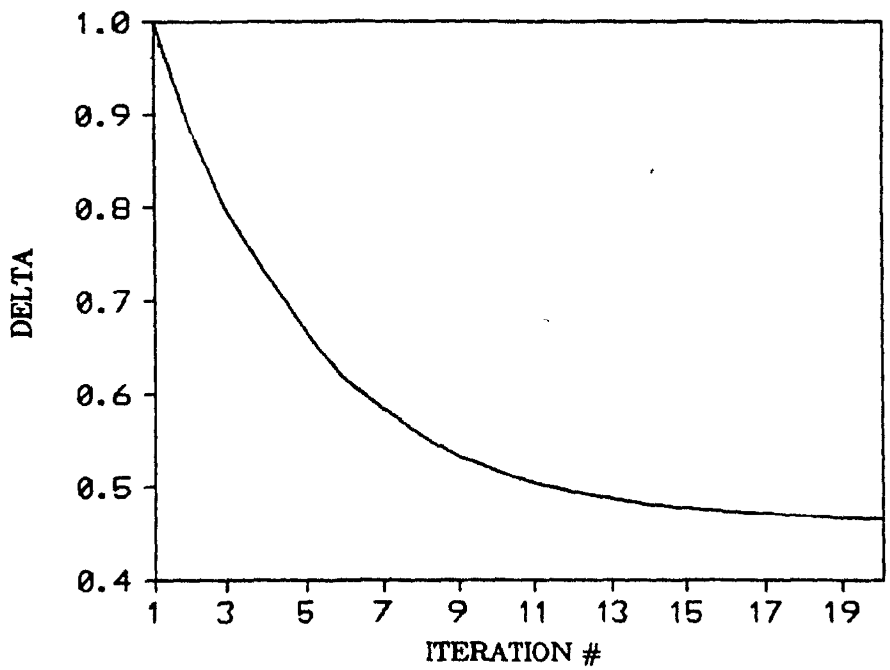
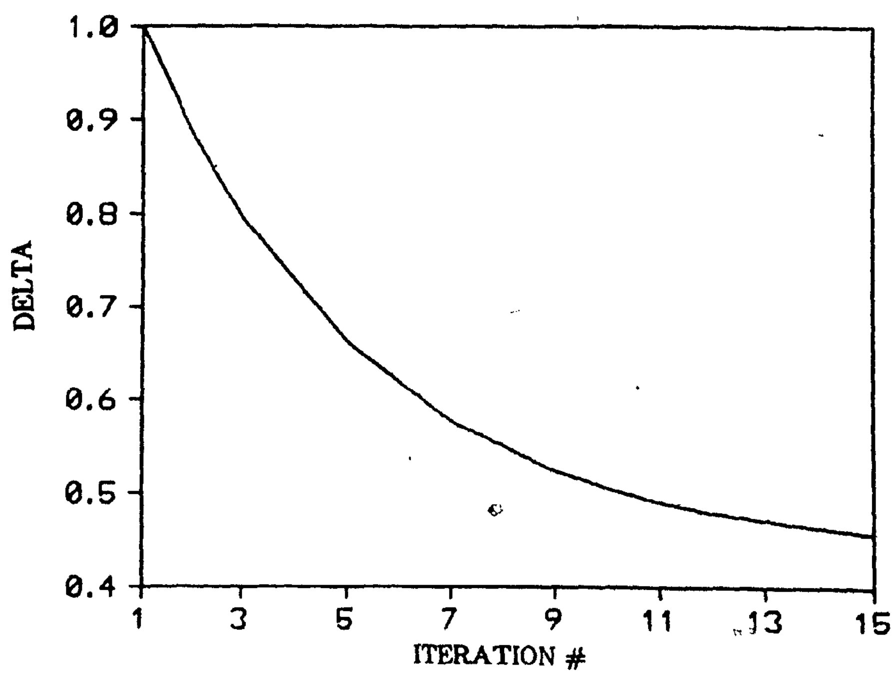


Figure 5.13 : Plot of Δ versus Iteration Number for 28 element hypothetical satellite array.



6. CONCLUSIONS

This thesis has dealt with the problem of radiation pattern synthesis of linear antenna arrays taking into consideration both the element spacing and element excitation.

In the chapter dealing with the current level of development of array synthesis techniques it was shown that while this problem has been addressed in the past, it is nevertheless one meriting further examination for a fuller exploitation of the advantages of obtaining more effective designs through control of both the spacing and excitation.

The objective of the thesis, thus, has been to develop a synthesis process which would yield improvements over previously known techniques. The results demonstrate that this has been achieved.

It has already been demonstrated by previous workers that combined spacing and excitation control yields considerable advantages in array design over the classical uniform spacing techniques such as those of Shelkunoff and Dolph. However, such existing "combined" techniques have various limitations as has been indicated in Chapter Two. For example, these limitations include constraints on the patterns which can be acheived or on the array geometry which can be used. The results of the present work indicate that improvements have been achieved over previous methods.

For example, Unz, [43], presents a method where the desired pattern can be specified only in terms of its beamwidth and sidelobe level. The structure of Redlich's, [53], and Goad and Stutzman's, [49], process is similar to that presented here, however, it only allows for the synthesis of symmetric arrays. In addition, no information is given regarding convergence of the process.

As described in Chapter Three, the present process is based on and benefits from the application of the Generalized Inverse approach and the Tikhonov Regularization Method. It is these methods and their application to inversion that has made the synthesis procedure always possible.

The synthesis process, the implementation of the synthesis process as a structured numerical algorithm and its associated software, form the principal contribution of this work and this has been described in Chapter Four. The entire synthesis process is structured as an integral entity which can be readily adapted to a software package for use by practising designers.

The various software components as well as the total process have been validated and demonstrated. The process is operational and has been used to calculate a number of test cases as described in Chapter Five. Two of these were chosen as a validation by synthesizing arrays of two well known cases determined independently. The third case was chosen as an "open ended" one to demonstrate a practical application.

In the two validation cases it is clearly shown that by using non-uniform element spacing the same patterns can be achieved with approximately 20% fewer elements than with uniform spacing for the number of elements used in these two cases. In addition, the convergence of the process has been clearly demonstrated.

The techniques of others have traditionally been implemented on mainframe computer systems. As yet, no indication appears in the literature of such techniques - where both unequal element spacing and non-uniform current distribution are allowed - being implemented on a PC. Here, however, from the onset the synthesis process has been designed and developed successfully for such a readily accessible system and hence its conversion to a commercial, modular, user friendly package can be easily implemented if desired.

In conclusion, this thesis has developed an array synthesis process whose effectiveness and advantages have been clearly demonstrated.

This now provides the possibility of examining the extension of such a process to more general array cases. For example, extension to non - linear (eg. curved, circular, planar) arrays would be highly desirable. Additionally, in the light of present day needs, the requirements of conformal arrays and of adaptive arrays would bear examination.

It should be noted that further intensive use and application of the existing process should be undertaken and should, in addition to its practical utility, establish more clearly the range and bounds of the benefits achievable.

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Definitions of Norms

If in a real linear space X, the norm of x (denoted by $||x||$) satifies:

$$(i) \quad ||0|| = 0\,, \qquad ||x|| > 0 \text{ if } x \neq 0$$

$$(ii) \quad ||x_1 + x_2|| \leq ||x_1|| + ||x_2|| \quad \text{for } all\ x_i\,,\ x_i \epsilon X$$

$$(iii) \quad ||ax|| = |a| \cdot ||x|| \quad \text{for } all\ a \in R\,,\ x \in X$$

then x is a normed linear space. Some special normed linear spaces are defined as follows:

$$L_p^I \equiv \int_y |X|^{1/p}\, dy$$

$$L_p^{II} \equiv \int_y |X|^p\, dy$$

$$L_p^{III} \equiv \left[\int_y |X|^r\, ay\right]^{1/p}$$

These are continuous norms where it is required that X be known continuously over y. Conversely there are discrete linear norms.

$$l_p^I \equiv \sum_i |x_i|^{1/p}$$

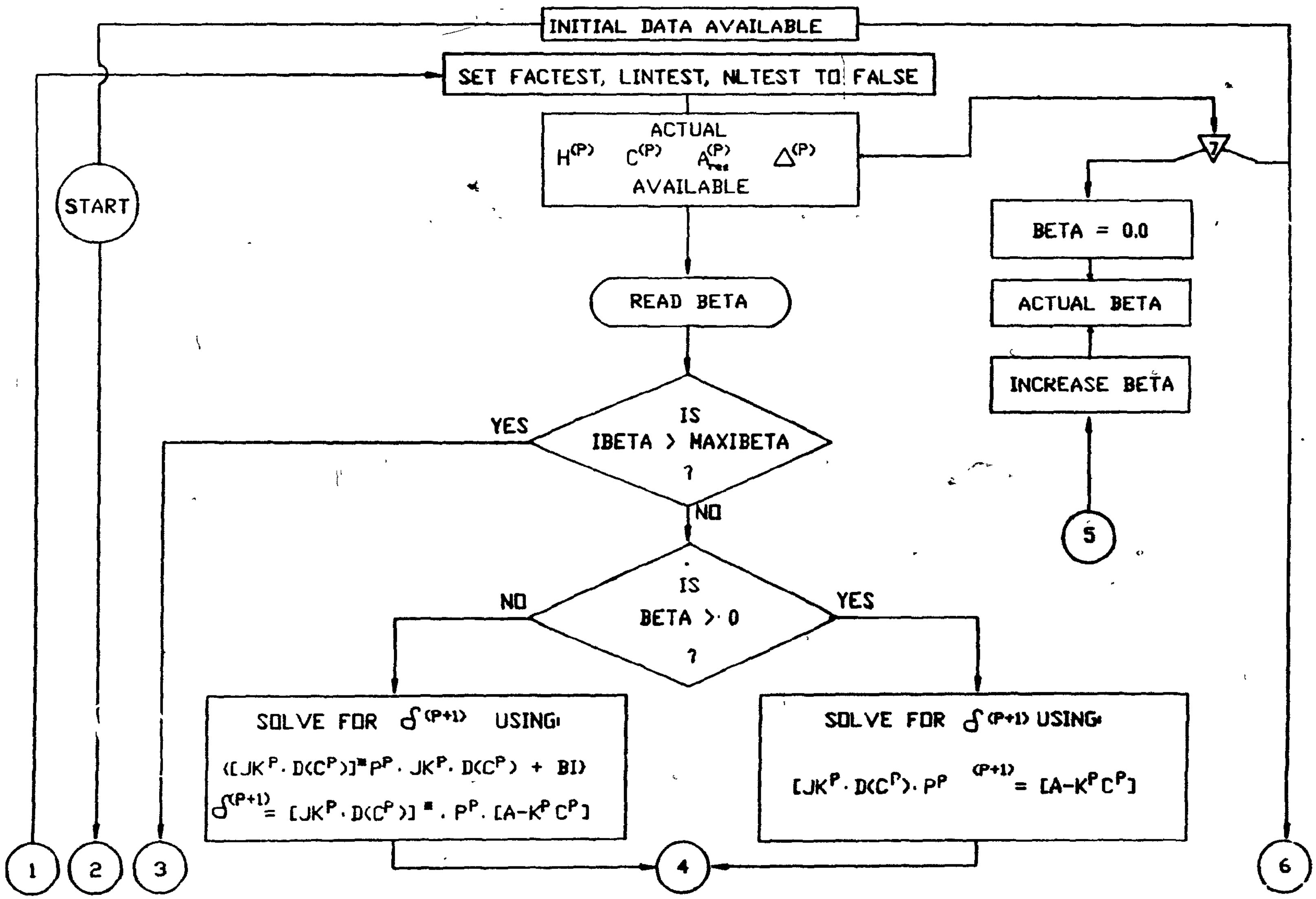
$$l_p^{II} \equiv \sum_i |x_i|^p$$

$$\hat{l}_p^{III} \equiv \left[\sum_i |x_i|^p\right]^{1/p}$$

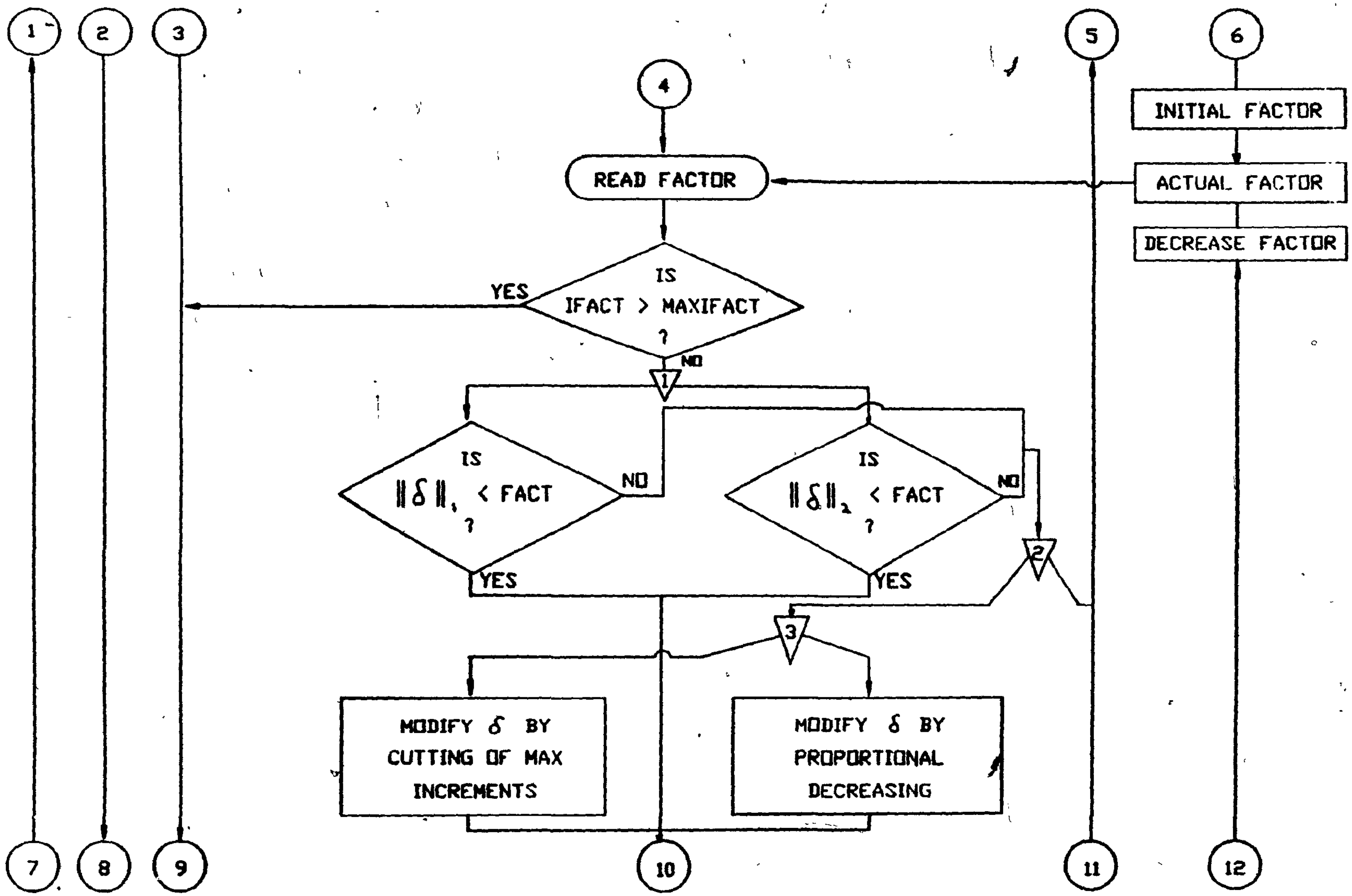
Program Flowchart

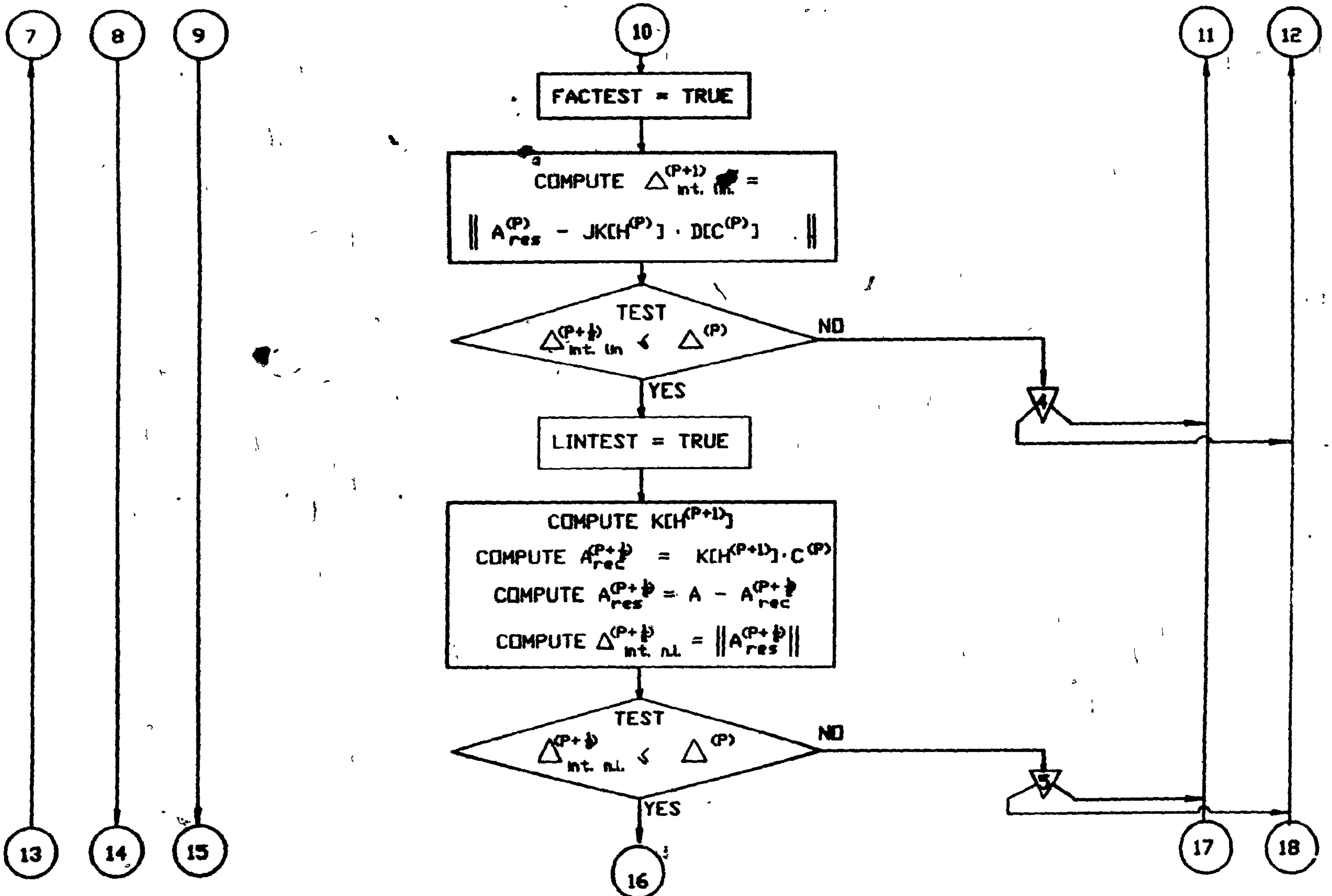
The next five pages contain a detailed flowchart of the program. The following is an explanation of the variables:

p	iteration number
$H^{(p)}$	element positions in the p^{th} iteration
$C^{(p)}$	element excitations in the p^{th} iteration
$\delta^{(p)}$	element position increments in the p^{th} iteration
A	Fourier coefficients of the desired pattern
$\Delta^{(p)}$	residual in the p^{th} iteration

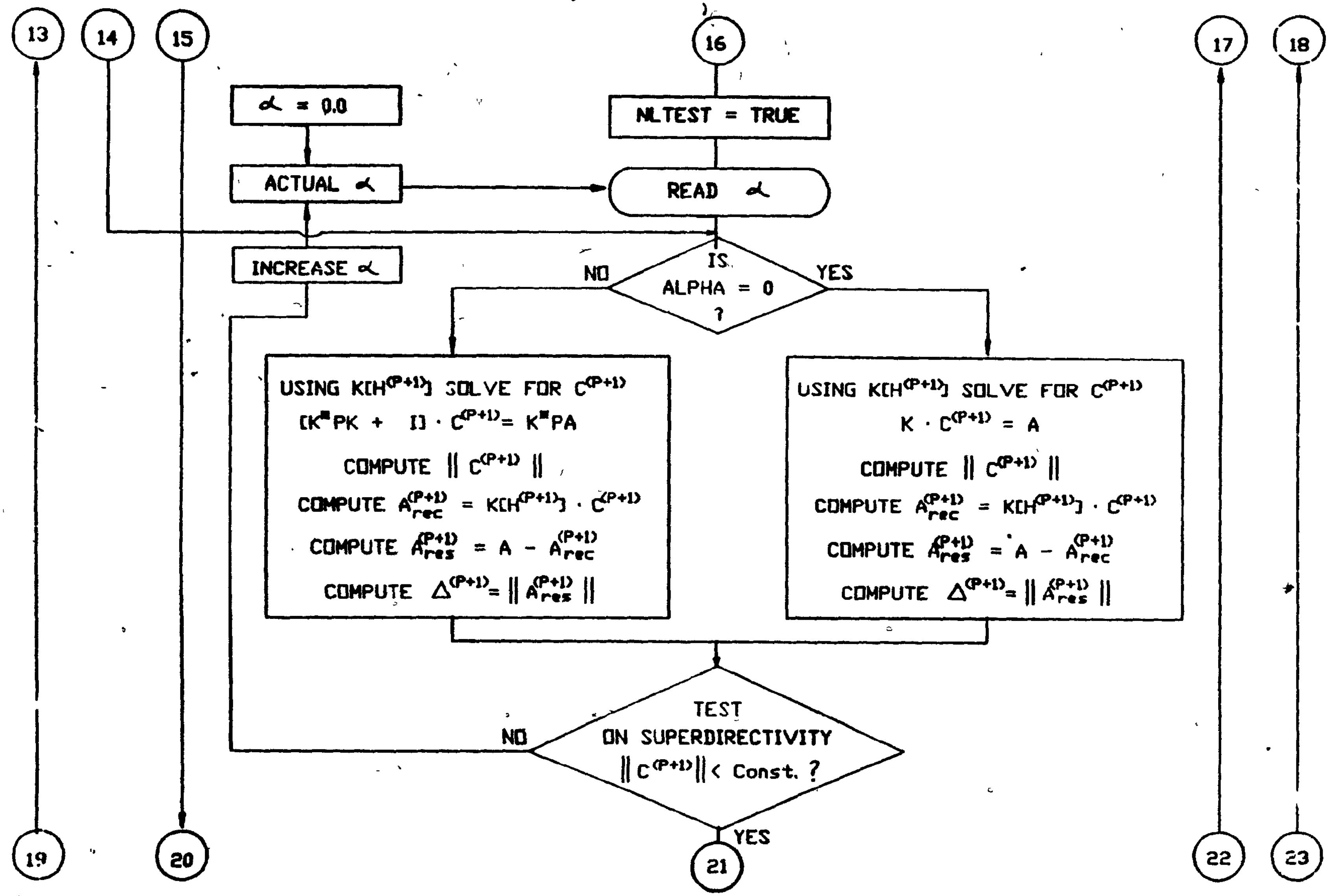

INITIAL DATA AVAILABLE
SET FACTEST, LINTEST, NLTEST TO FALSE
ACTUAL
$H^{(P)}$ $C^{(P)}$ $A^{(P)}_{req}$ $\Delta^{(P)}$
AVAILABLE
START
READ BETA
IS IBETA > MAXIBETA ?
YES
NO
IS BETA > 0 ?
NO
YES
SOLVE FOR $\delta^{(P+1)}$ USING:
$([JK^P \cdot D(C^P)]^* P^P \cdot JK^P \cdot D(C^P) + BI)$
$\delta^{(P+1)} = [JK^P \cdot D(C^P)]^* \cdot P^P \cdot [A-K^P C^P]$
SOLVE FOR $\delta^{(P+1)}$ USING:
$[JK^P \cdot D(C^P) \cdot P^P$ $^{(P+1)} = [A-K^P C^P]$
BETA = 0.0
ACTUAL BETA
INCREASE BETA
7
5
1
2
3
4
6

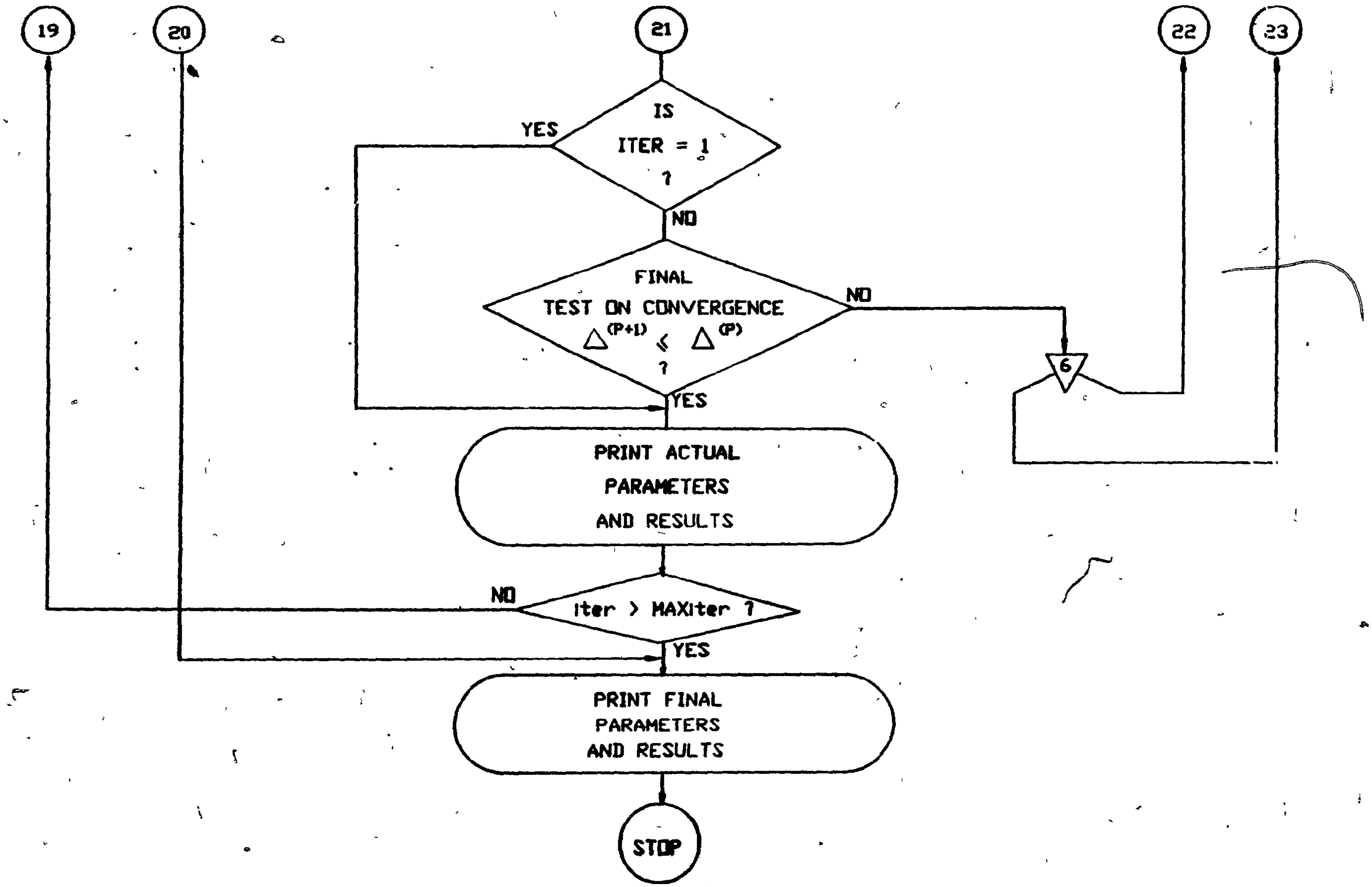
APPENDIX 2-2


1
2
3
4
5
6
INITIAL FACTOR
ACTUAL FACTOR
DECREASE FACTOR
READ FACTOR
IS IFACT > MAXIFACT ?
YES
NO
1
IS ‖δ‖₁ < FACT ?
NO
YES
IS ‖δ‖₂ < FACT ?
NO
YES
2
3
MODIFY δ BY CUTTING OF MAX INCREMENTS
MODIFY δ BY PROPORTIONAL DECREASING
7
8
9
10
11
12


7
8
9
10
11
12
FACTEST = TRUE
COMPUTE $\Delta^{(P+1)}_{int. (n}$ =
$\| A^{(P)}_{res} - JK[H^{(P)}] \cdot D[C^{(P)}] \|$
TEST
$\Delta^{(P+\frac{1}{2})}_{int. ln} \leq \Delta^{(P)}$
NO
YES
4
LINTEST = TRUE
COMPUTE $K[H^{(P+1)}]$
COMPUTE $A^{(P+\frac{1}{2})}_{rec} = K[H^{(P+1)}] \cdot C^{(P)}$
COMPUTE $A^{(P+\frac{1}{2})}_{res} = A - A^{(P+\frac{1}{2})}_{rec}$
COMPUTE $\Delta^{(P+\frac{1}{2})}_{int. n.l.} = \| A^{(P+\frac{1}{2})}_{res} \|$
TEST
$\Delta^{(P+\frac{1}{2})}_{int. n.l.} \leq \Delta^{(P)}$
NO
YES
5
13
14
15
16
17
18

APPENDIX 2-4


13
14
15
16
17
18
α = 0.0
ACTUAL α
INCREASE α
NLTEST = TRUE
READ α
IS ALPHA = 0 ?
NO
YES
USING K[H(P+1)] SOLVE FOR C(P+1)
[K*PK + I] · C(P+1) = K*PA
COMPUTE ‖C(P+1)‖
COMPUTE A(P+1)rec = K[H(P+1)] · C(P+1)
COMPUTE A(P+1)res = A − A(P+1)rec
COMPUTE Δ(P+1) = ‖A(P+1)res‖
USING K[H(P+1)] SOLVE FOR C(P+1)
K · C(P+1) = A
COMPUTE ‖C(P+1)‖
COMPUTE A(P+1)rec = K[H(P+1)] · C(P+1)
COMPUTE A(P+1)res = A − A(P+1)rec
COMPUTE Δ(P+1) = ‖A(P+1)res‖
TEST ON SUPERDIRECTIVITY
‖C(P+1)‖ < Const. ?
NO
YES
19
20
21
22
23



19
20
21
22
23
IS
ITER = 1
?
YES
NO
FINAL
TEST ON CONVERGENCE
$\Delta^{(P+1)} \leq \Delta^{(P)}$
?
NO
YES
6
PRINT ACTUAL
PARAMETERS
AND RESULTS
NO
Iter > MAXiter ?
YES
PRINT FINAL
PARAMETERS
AND RESULTS
STOP

APPENDIX 2-6