ANALYTICAL AND EXPERIMENTAL STUDIES OF SOIL CUTTING

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SUMMARY:

Comparisons are made between predicted values obtained from the application of the limit solution developed, with laboratory measurements from this study for cutting blades in sand and with those given elsewhere. This approach can successfully predict measurements so long as the physical requirements conform closely to the analytical model.

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R.N. Yong[1] and C.K. Chen[2]

PREFACE

Various methods of analysis are attempted to provide for evaluation of the effective thrust of blades in soil.

Comparisons, using the laboratory measured values obtained from this study, are made between the predicted values obtained from the application of the limit solution (developed in the study) with those given elsewhere.

INTRODUCTION

In the study of moving rigid blades in soils, of the various methods attempted to provide for evaluation of the effective thrust of the blade in soil, the most convenient one appears to combine the measurement of forces acting on the blade (as input information) together with the strength properties of the soil. Thus, while a separate measure of the two individual elements (i.e. rigid blade and soil) is made, the final form of analysis attempts to provide a rational link between the two.

The three methods of analyses which appear feasible in application to the study of the moving blade problem are, (1) methods relying on dimensional analysis, e.g. [4], [5]*, (2) rigid body statical solution techniques, and (3) plasticity techniques such as those using a form of the Prandtl solution, specialized for bearing stability, e.g. [2], [3]. In applying the first technique, using laboratory model tests, a required assumption is that soil properties can be adequately scaled. The difficulties surrounding the scaling of soil, or the derivation of appropriate scaling laws for soils, are most complex, [8]. In addition to the problems of separate examination of component parts, one must recognize that the problem of a moving rigid blade in soil is an interacting phenomenon and, therefore, should be considered as an interdependent system. The behavior of the soil mass in the front of the moving blade is conditioned by not only the geometry of the blade itself, but also by the driving forces associated with the blade. In turn, the progress of the blade in the soil is controlled or influenced by the manner in which the soil will deform, the boundary conditions,

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* Numbers in square brackets [ ] refer to references.

and constitutive makeup of the soil itself.

In this study, the problem of the moving blade in soil is examined in terms of soil-blade inter-relationships. The soil in the front of the moving blade is in a state of limit equilibrium, and the boundary conditions are specified in part by conditions imposed by the moving blade. Thus, the stability of the soil mass in the face of the driving force produced by the blade is examined in relation to the interdependent limit equilibrium problem. The type of analysis that would arise utilizes the method of characteristics as a solution technique. Momentum relationships and inertia terms can be incorporated in the analysis following certain simplifications based upon quasi-static relationships. With the addition of velocity terms, the analysis can handle a wide variety of problems of moving blades in soils. In the present study, attention is focussed on predominantly frictional soils where cohesion C is small (or zero). The case involving highly cohesive soil will be examined later. The results from the associated laboratory tests performed in this study are compared with the analysis, and reported results taken from studies performed by other individuals are also compared with the limit equilibrium approach.

THEORETICAL CONSIDERATIONS

The method employed here for analysis relies on the principle of limit equilibrium where the stability of the soil mass is controlled, or affected by the moving blade. The problem is reduced to two-dimensional considerations without any loss of generality by assuming plane strain conditions. For a generalised case where the velocity and inertia terms are to be considered, it is possible to analyse the situation in terms of velocity fields associated with the dynamic conditions, and prescribe the appropriate governing equations. The resultant formulations become most complex, and the mathematics arising therefrom do not allow for ready solutions.

A simple alternative method for the generalised analysis requiring the super-position of inertia and velocity terms on to a quasi-static solution can be used. Since the dynamic forces are small in relation to the yield stress of the soil, it can be assumed that these forces do not affect the associated development of the stress and velocity fields in the quasi-static condition. Thus, the associated stress field is a velocity field which describes the resultant flow. Using a similar approach, Yong and Japp [7], have shown that constitutive relations for soil dynamical behavior for accelerations developed in clay soils of less than 100 g's, can be adequately written.

The quasi-static limit equilibrium case for a fully frictional material (sand) where cohesion C=0 has been examined previously by Yong et al, [6]. In the following, the generalised case for a cohesion-friction material, (i.e. C-ϕ mixed soil material) is examined, as an extension of the previous C=0 material analysis, [6]. The differences here involve the additional complexity of cohesion in soil. These can be specialized to a fully frictional soil by taking C=0 at any stage of the theoretical development.

YIELD CONDITIONS

The diagram shown in Figure 1 is a schematic representation of a blade moving in soil. The soil mass is assumed to be isotropic, homogeneous and incompressible, and obeys the Mohr-Coulomb Yield Condition:

$$(\sigma_r - \sigma_\theta)^2 + 4\tau_{r\theta}^2 = (\sigma_r + \sigma_\theta + 2H)^2 \sin^2\phi \qquad (1)$$

where $H = C \cot \phi$
C = cohesion
ϕ = angle of internal friction
σ_r = radial stress
σ_θ = tangential stress
$\tau_{r\theta}$ = shear stress

Figure 2 shows the polar coordinate system used for the above notation. The angle of friction δ between the blade and soil is assumed to be constant.

In terms of polar coordinates, the Mohr-Coulomb Yield Condition for a mixed soil (C-ϕ) material can be expressed as follows:

$$\sigma_r = \sigma(1 + \sin\phi \cos 2\psi') + c\cos\phi\cos 2\psi'$$
$$\sigma_\theta = \sigma(1 - \sin\phi\cos 2\psi') - c\cos\phi\cos 2\psi'$$
$$\tau_{r\theta} = \sigma\sin\phi\sin 2\psi' + c\cos\phi\sin 2\psi' \qquad (2)$$

where $\sigma = \frac{\sigma_r + \sigma_\theta}{2} = \frac{\sigma_1 + \sigma_2}{2}$ = mean stress

EQUATIONS OF EQUILIBRIUM

The partial differential equations of equilibrium are as follows:

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\sigma_r - \sigma_\theta}{r} = \rho g \cos\theta$$
$$\frac{\partial \tau_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_\theta}{\partial \theta} + \frac{2\tau_{r\theta}}{r} = -\rho g \sin\theta \qquad (3)$$

With the use of a similarity solution technique, the following are assumed:

$$\psi' = \psi'(\theta)$$
$$\sigma = \rho g r s(\theta) \qquad (4)$$

where ρg = density
r = radial distance
$s(\theta)$ = stress function

Then substitution of equations (2) and (4) into equations (3) will provide the following equations:

$$\frac{d\psi'}{d\theta} = \frac{\cos\theta - \sin\phi\cos(2\psi'+\theta) - s\cos^2\phi}{2\left(s\sin\phi + \frac{c}{\rho g r}\cos\phi\right)(\cos 2\psi' - \sin\phi)} - 1$$
$$\frac{ds}{d\theta} = \frac{s\sin 2\psi' - \sin(2\psi'+\theta)}{\cos 2\psi' - \sin\phi} \qquad (5)$$

It is seen that for C=0, equations (5) are those shown by Yong et al [8] as:

$$\frac{d\psi'}{d\theta} = \frac{\cos\theta - \sin\phi\cos(2\psi'+\theta) - s\cos^2\phi}{2s\sin\phi(\cos 2\psi' - \sin\phi)} - 1$$
$$\frac{ds}{d\theta} = \frac{s\sin 2\psi' - \sin(2\psi'+\theta)}{\cos 2\psi' - \sin\phi}$$

BOUNDARY CONDITIONS

Referring to Figure 3, it can be seen that the stressed soil in front of the blade is divided into two regions; Region I being the simple passive Rankine Zone in which the major principal stress is in the horizontal direction, and Region II being the Radial Shear Zone.

The boundary conditions in Region I for both a fully frictional or a mixed C-ϕ soil are given as follows:

$$S = \frac{\cos\Theta}{1-\sin\phi} \quad , \quad \psi' = \pi/2 - \Theta \tag{6}$$

where $\Theta = \pi/4 + \phi/2$

At the blade, the boundary condition can be established by the following relationship:

$$\tau_{r\theta} = \sigma_\theta \tan\delta \quad \text{, whence}$$
$$\psi' = 0.5[\pi - \delta - \arcsin\frac{\sin\delta}{\sin\phi}] \tag{7}$$

It now remains to establish the stress function $S(\theta)$, and $\psi'(\theta)$ (6) in relation to the angle Θ . This is done by utilising the boundary equations (6) and (7) and equations (5), and the 4th Order Runge-Kutta method. With these results, having noted that $\psi'(\theta)$ does influence the value of $S(\theta)$, the stresses or forces acting on the surface of the moving blade may be computed from equations (2) and (4).

The results of the computations for $\psi(\theta)$ and $S(\theta)$ for a material with $\phi=38^o$ and C=0 and other values of C=5 and 10 psi. are presented in Figures 4 through 8. It is significant to note that while the presence of cohesion C materially affects $\psi'(\theta)$, the stress fraction $S(\theta)$ remains relatively unaffected - so long as ϕ is sufficiently large. With decreasing values of ϕ and increasing values of C, it is seen that $S(\theta)$ will become more sensitive to the presence of C.

SLIP LINES

If reference is made to equations (5) it can be shown that the denominators of the right-hand side of these equations vanish when :

$$\psi' = \pm\mu$$

where $2\mu = \frac{\pi}{2} - \phi$

The numerators may vanish simultaneously, or they may be different from zero.

The two sets of slip lines obtained are:

$$r_{\theta+} = r_\beta \exp\int_\beta^\theta \cot(\psi' + \mu)\,d\theta \quad = \text{I characteristic} \quad (8)$$

$$r_{\theta-} = r_\beta \exp\int_\beta^\theta \cot(\psi' - \mu)\,d\theta \quad = \text{II characteristic} \quad (9)$$

where β = blade angle
r_β = contact length of blade-soil interface

This is illustrated on the Mohr-Coulomb diagram shown in Figure 9.

The results for the computation of the I-characteristic (i.e. failure surface in the physical body) ($r_\theta+$) for low values of cohesion C and ϕ=38° are shown in Figure 10 for a blade angle of 30°. Comparisons between the I-characteristic and actual failure surfaces for the tests conducted show that the surfaces and the characteristics do coincide for all values of β, ϕ and C considered.

DYNAMICAL CONSIDERATIONS

By comparing the theoretical development with that of Yong et al [6], for the quasi-static case, it has been shown experimentally that the failure surfaces, computed slip lines and characteristics are the same for both a ϕ and C-ϕ material, so long as limit equilibrium is generated throughout the stressed soil. The major difference in calculating the normal and shear stresses for these two types of material, is that for a C-ϕ material the term containing C has to be algebraically added to the values obtained for these stresses for a C-ϕ material, [refer to equation (2)].

To obtain the total forces, the forces arising from inertia have to be computed, and added to the forces obtained for the quasi-static case. Three possibilities are suggested:

1. The soil is assumed to act as a fluid with the blade considered as an inclined surface moving through the fluid.

   Since the blade is inclined at an angle β to the vertical, the absolute velocity of the soil (acting as a fluid and moving along the blade) would be V sin β, if friction of the soil along the blade is neglected.

   The change in velocity of the soil is therefore:

$$V(1 - \sin\beta) \qquad (10)$$

The weight of the soil reaching the blade per second would be:

$$Db\rho g$$

where D = depth of cut
b = width of blade
v = velocity of blade
ρ g = density of soil

Therefore, the change of momentum per second, or the force:

$$F_H = \frac{Db\rho g}{g} \quad v^2 \;(1 - \sin\beta) \qquad (11)$$

$$F_V = \frac{Db\rho g}{g} \quad v^2 \cos\beta \qquad (12)$$

2. A second possibility considered is to assume that the soil acts as a rigid mass within the failure surface, and moves with the blade, and with the same velocity in the direction of the motion of the blade.

If W is the weight of the soil within the failure surface, the momentum of the failed soil mass is $\frac{W}{g}$ v. If the acceleration of the blade is $\frac{dv}{dt}$ the horizontal force is $\frac{W}{g}\frac{dv}{dt}$, there being no vertical force since there is no vertical component to the horizontal velocity.

3. A third possibility that can be considered is to assume that each particle of soil acquires a velocity in the direction of the failure plane as the blade moves through the soil. The failure plane is assumed to make an angle of $(\pi/4 - \phi/2)$ with the horizontal. Figure 11 shows the velocity diagram for this case. If W is the weight of the soil within the failure surface, and U is the assumed velocity of the soil along the blade, it can be shown that if the acceleration is known the horizontal and vertical forces are:

$$F_H = \frac{W}{g}\left(1 - \frac{\sin(\pi/4 - \phi/2)\sin\beta}{\sin(\pi/4 + \beta + \phi/2)}\right)\frac{dv}{dt} \qquad (13)$$

$$F_V = \frac{W}{g}\left(\frac{\sin(\pi/4 - \phi/2)\cos\beta}{\sin(\pi/4 + \beta + \phi/2)}\right)\frac{dv}{dt} \qquad (14)$$

The first method is not considered in this study as evidence shows that the soil does not act in this manner. The second method does not have a vertical component of force. [When observing a test with a slow-moving blade, the upward movement of the failing soil mass in front of the blade can always be seen].

The third system of velocities is considered to be most suitable, as in observing low velocity it was noticed that for a small movement of the blade the soil mass within the failing surface moved upward during failure.

In order to calculate the acceleration for the experiments, it can be assumed that the blade accelerates from zero initial velocity to its final velocity v when the soil fails. It is, therefore, necessary to find the time of failure, and from this the acceleration.

For the associated experiments, tests conducted at the Caterpillar Tractor Company, at speeds of from 0.25 ft/sec. to 1 ft/sec., the inertial effect was found to be very small. It was thus possible to conclude that the force at failure was to all intents and purposes equal to the quasi-static force.

EXPERIMENTATION

The test system used to conduct the moving blade study consisted of a glass-sided box which allowed for visualisation of the grid markings placed on the soil, (see Figure 12). The material used was a silica sand with a frictional angle of $\phi=38^\circ$, and a friction angle between sand and aluminum blade of 29°. In general, the material was maintained at a constant dry density of 100 lb. per cubic foot.

The aluminum blade used to simulate the cutting blade measured 4 inches wide by 5 inches long. The blade angle with regard to the vertical varied from 0° to 40°, and the depth of cut was either $4\frac{1}{2}$ inches or $3\frac{1}{2}$ inches. The horizontal speed of the blade was varied for each series of tests.

Two experimental conditions were maintained for measurement of forces acting on the blade: (1) measurement of horizontal force at a constant depth of embeddment, and (2) measurement of horizontal force at a constant vertical pressure acting on the blade. Only the first series need be reported on since these tests constitute a more complex set of experimentation, and if the limit equilibrium solution is to hold, the more critical case for examination would be the variable vertical and resultant variable horizontal force. Typical horizontal force-displacement diagrams (draft force) are shown in Figure 13. The first peak represents the major or maximum force development at first failure, and the subsequent perturbations which generally show higher values, correspond to the subsequent development of secondary shear planes. The higher values are in general attributable to the accumulation of surcharge at the face of the moving blade. Examination of all the horizontal force-displacement diagrams shows that with a smaller depth of cut, this increased surcharge effect can contribute a force increase as much as one-half of the first peak force. This effect will be discussed at a later stage.

The analysis developed in this study refers only to the first initiation of shear failure (as a condition of limit equilibrium) and pays no attention to the surcharging effect. Thus, for comparative reasons, both from this study and with other test results, the first peak will be taken as a basis for examination of this theory. It must be mentioned here that it is not always possible to obtain information on first peaks in many reported studies for comparative information. It is expected that the perturbating asymptotic value may be the one generally used, since this in all probability represents a steady state condition. In that regard, the limit equilibrium approach to the analysis of the problem would in general under-estimate the total draft or horizontal force in view of the absence of the surcharging term in the analysis.

DISCUSSION OF RESULTS

In associated experimental tests conducted at the Caterpillar Tractor Company soil bin facility, with different experimental constraints, it is seen that the measured and the predicted or computed values for both the horizontal and vertical forces appear to agree closely. (See Figure 14). The method for computation of the theoretical or expected vertical and horizontal forces for the quasi-static condition (assuming no inertia and velocity terms) has been reported previously [6]. The stress functions S_a and S_b at points A and B (refer to Figures 4 through 8) for the particular value of blade inclination, β, and as a function of ϕ and C are obtained from Figures 4 and 5. Correspondingly the values for ψ'_A, ψ'_B are also obtained from Figures 6 through 8. From the Figures, the stress function can now be used to calculate the mean stress σ given in equation 4. This value can be further examined in terms of the vertical and horizontal components of forces acting on the blade.

In order to ascertain the validity of the method used for prediction or computation of the forces acting on the blade, the theoretical development allows for an examination of the failure surface generated upon development of the first peak stress. This utilises equation (8) and the values for ψ' . With the glass-sided box and high-speed photography, it is possible therefore to examine the actual failure surface generated upon development of first peak stress, and compare with the predicted values. It has been shown by Yong et al [6], that the correspondence between computed and measured failure surfaces do agree very closely.

In addition to the above comparison of predicted and measured failure surface for a check on the theory, a further computational procedure may be used to establish the validity and applicability of the theory. Recalling that the limit equilibrium approach requires the admissibility of the Rankine condition, the simplest considerations would show that the horizontal force or the horizontal component of the total force acting on the blade would be proportional to the square of the depth of cut. This is obtained from the following relationship:

$$F_H = \frac{1}{2}\gamma d^2 K \qquad (15)$$

where d = depth of cut
γ - soil density
K = coefficient of stress transmissibility

Assuming that the coefficient of transmissibility K and the densities are constant, it becomes evident that F_H **varies** in direct proportion to d^2. In Figure 15 the measured values for horizontal forces with regard to blade angle inclination variation are plotted for the two depths of cut. As seen, the forces developed for the 4½ inches embedded depth of cut, d, are higher than the 3½ inches depth. Thus, in theory:

$$\frac{F_{H(3\frac{1}{2})}}{F_{H(4\frac{1}{2})}} = \frac{(3.5)^2}{(4.5)^2} = 0.605 \qquad (16)$$

In actual fact it is seen that:

$$\frac{F_{H(3\frac{1}{2})}}{F_{H(4\frac{1}{2})}} = 0.610, \qquad \frac{F_{V(3\frac{1}{2})}}{F_{V(4\frac{1}{2})}} = 0.620 \tag{17}$$

The Rankine condition is thus satisfied, and since this is a necessary requirement for the development of the limit equilibrium approach to the problem, the viability of the analytical technique used is assured.

The analytical methods suggested by Reece, [3], Osman, [2] and Hettiaratchi et al [1] for computing the theoretical forces may be examined, using the test results obtained from this study as a base for comparison. In Figure 16, these comparisons are shown together with the present limit equilibrium approach (labelled as "McGill") computations for values of forces acting on the blade. It is seen that the predicted horizontal forces using the other methods do not agree with the values obtained from this present study. However, there appears to be a fair degree of correspondence in prediction of vertical forces with Reece's method, for blade inclinations (to the vertical) up to 20°.

The limit equilibrium analysis has been applied to predict Osman's test results (Figure 17). In the other comparisons shown, the predicted vertical values are also given, although no vertical force values have been reported. It is apparent that the limit equilibrium approach is sufficiently successful in predicting Osman's test results. It is perhaps instructive to note that while the Osman [2] method of prediction is sufficiently valid for his own study, his method of analysis does not allow for a reasonable prediction of the test results obtained in this study (McGill), as shown in Figure 16. On the other hand, it is seen that the limit equilibrium approach used for the McGill study can predict both the McGill test results and Osman's results, [2].

In Figure 18 a comparison of the predicted values from the limit equilibrium approach and quasi-static conditions is made with those reported by Wismer and Forth [4]. This comparison has been separated from the previous ones in that the velocity of 19 inches per second reported in this study are sufficiently high, and provides a basis for examination using either the quasi-static condition, or the "dynamic" approach. In the same Figure the theoretically computed quasi-static values are shown together with the "dynamic" solution, and compared with those reported by Wismer and Forth [4]. If the assumption is made that the quasi-static condition ignores the inertia effect etc., the "dynamic" approach shown in the development of the theory which has been adopted, shows that the addition to the force calculations is less than 2% for the velocities used. Thus, a closer agreement is seen to be established with the reported results.

CONCLUSIONS

The results from this study show that for conditions where limit equilibrium in the soil is approached, and for the purposes of predicting first failure in the soil under the action of a moving blade, the analytical technique developed, using the method of characteristics for solution, shows a good correspondence between computed and measured values. Comparisons with reported values from other studies show the applicability of this method of approach to the solution of the problem. The fundamental point in consideration is that conditions for and relating to instability in soil is a function of the forces imposed by the moving blade. Thus, the examination of problems of soil cutting should account for inter-dependencies established between the movement of the blade and the soil constitutive relationships.

ACKNOWLEDGMENTS

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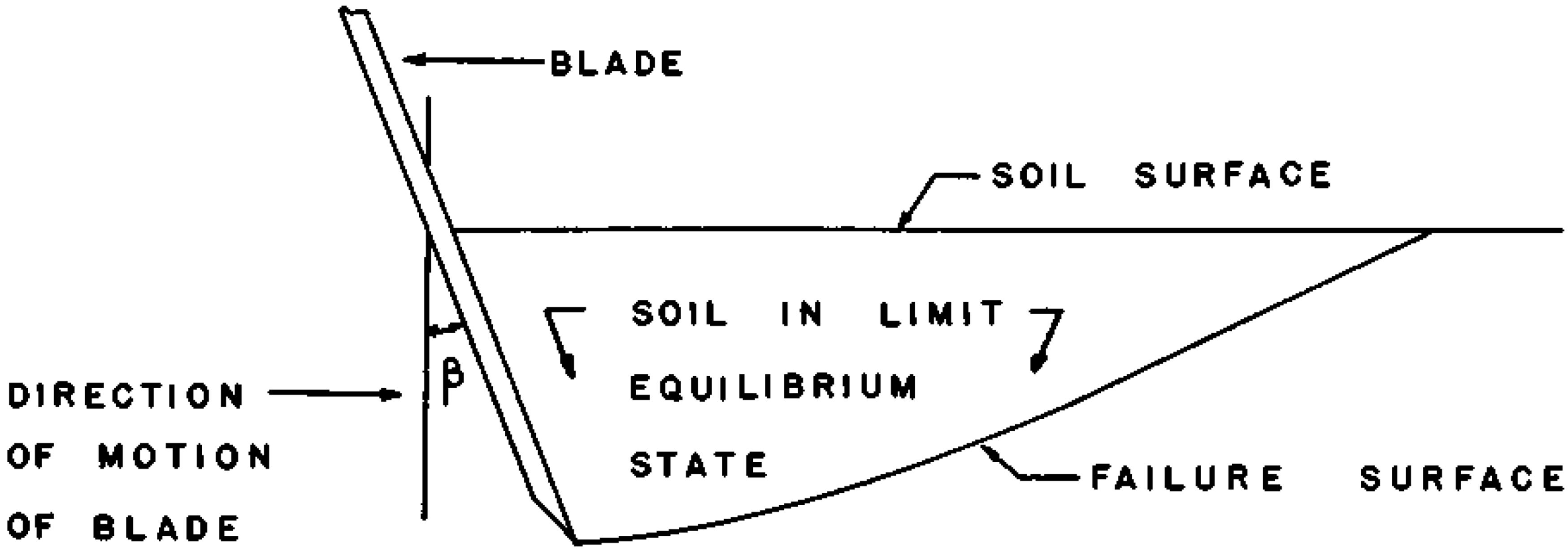

FIG. 1 SCHEMATIC REPRESENTATION OF MOVING BLADE AND SOIL CUTTING

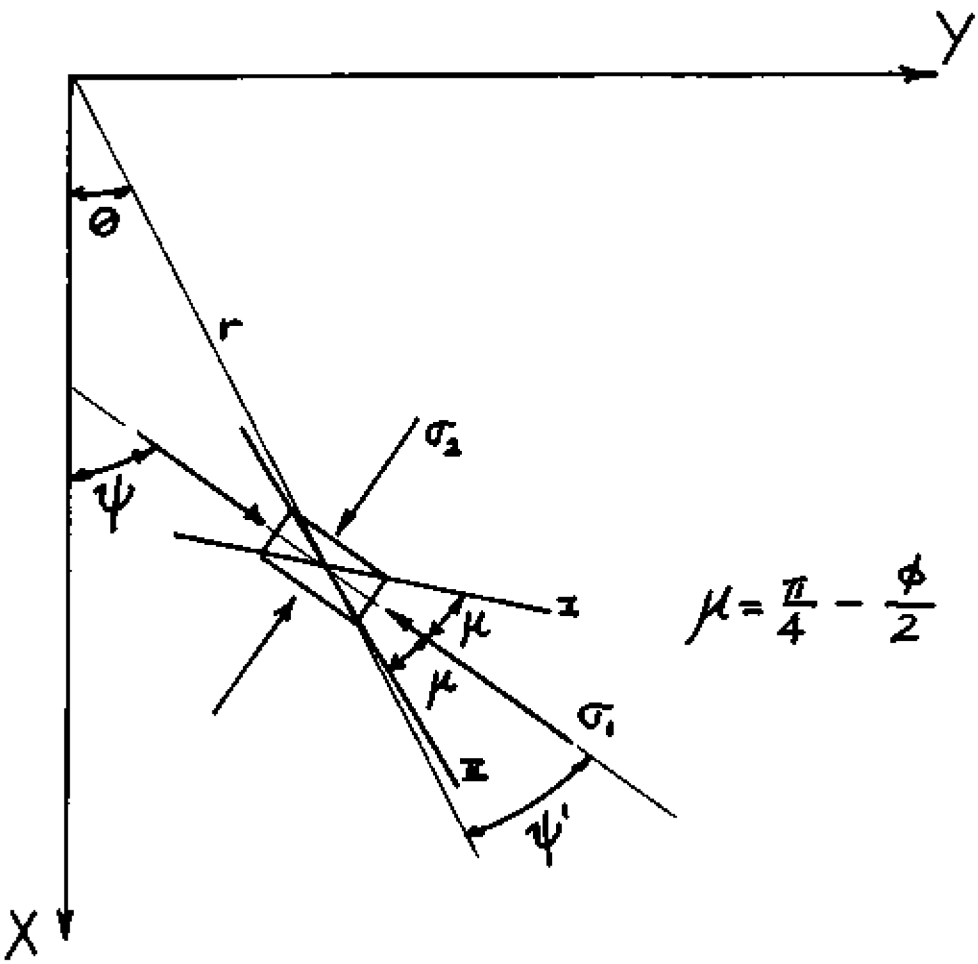

FIG. 2A COORDINATE AXES

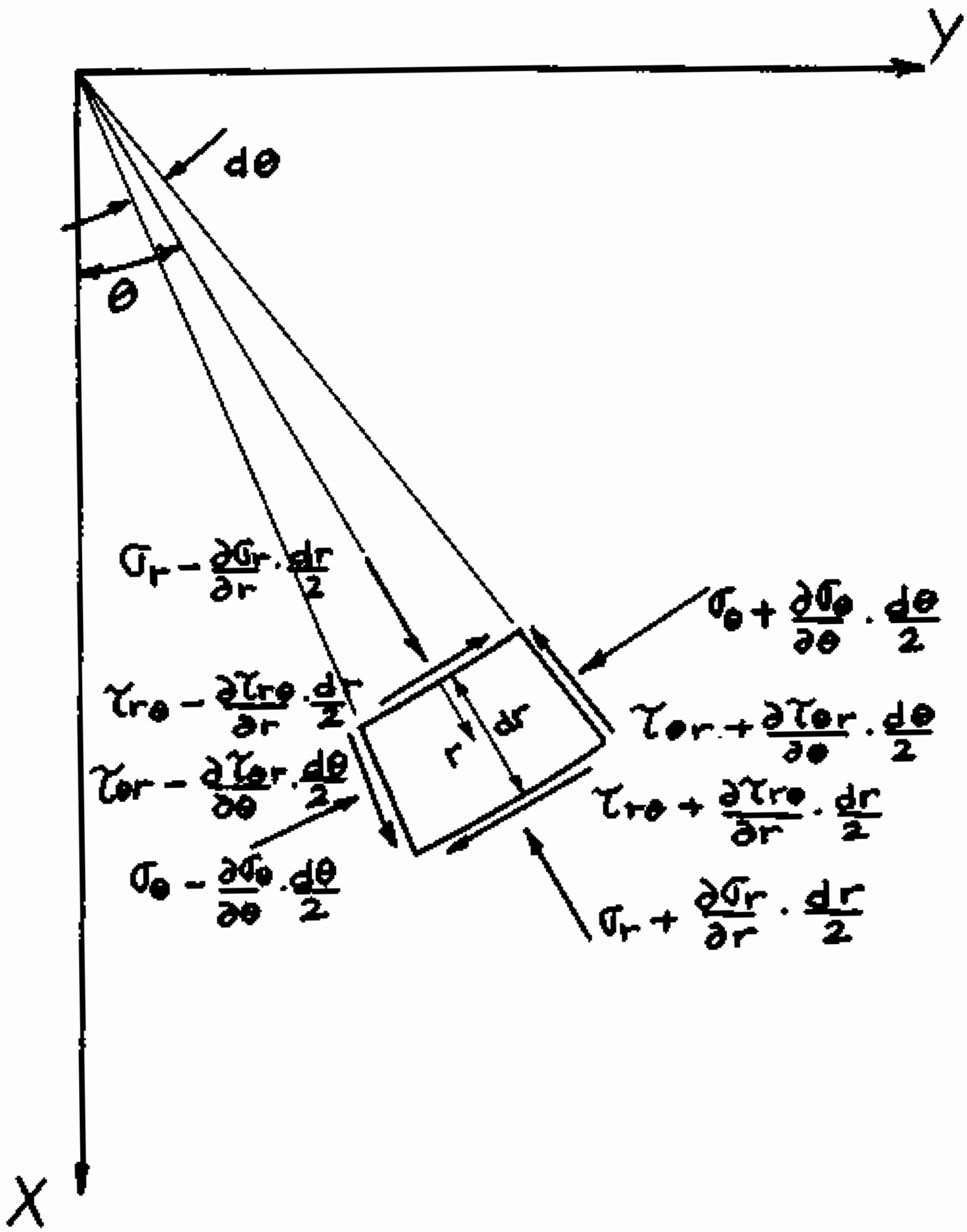

FIG. 2B STRESSES ON ELEMENT

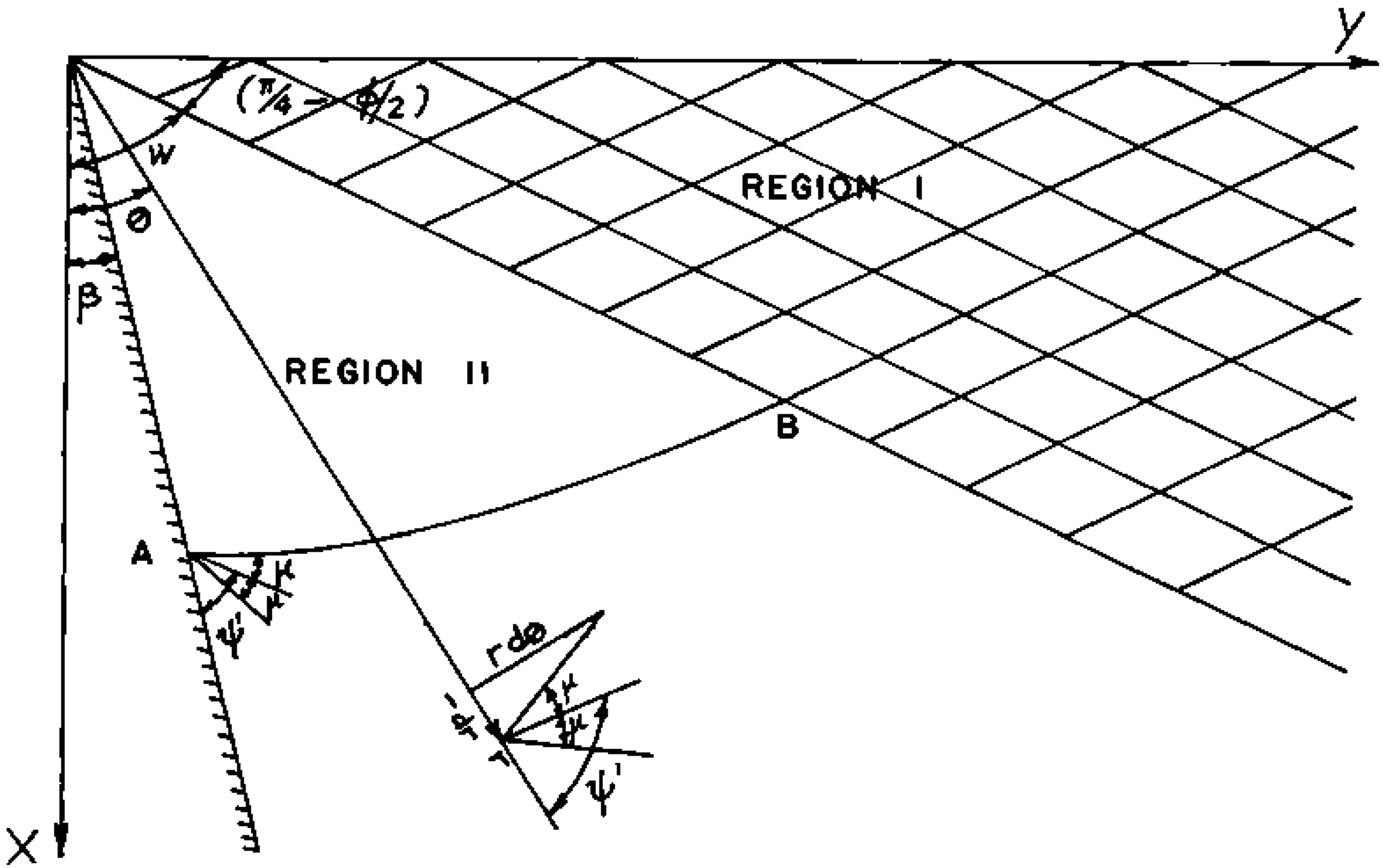

FIG. 3 ANALYTICAL MODEL FOR DESCRIBING BOUNDARY CONDITIONS

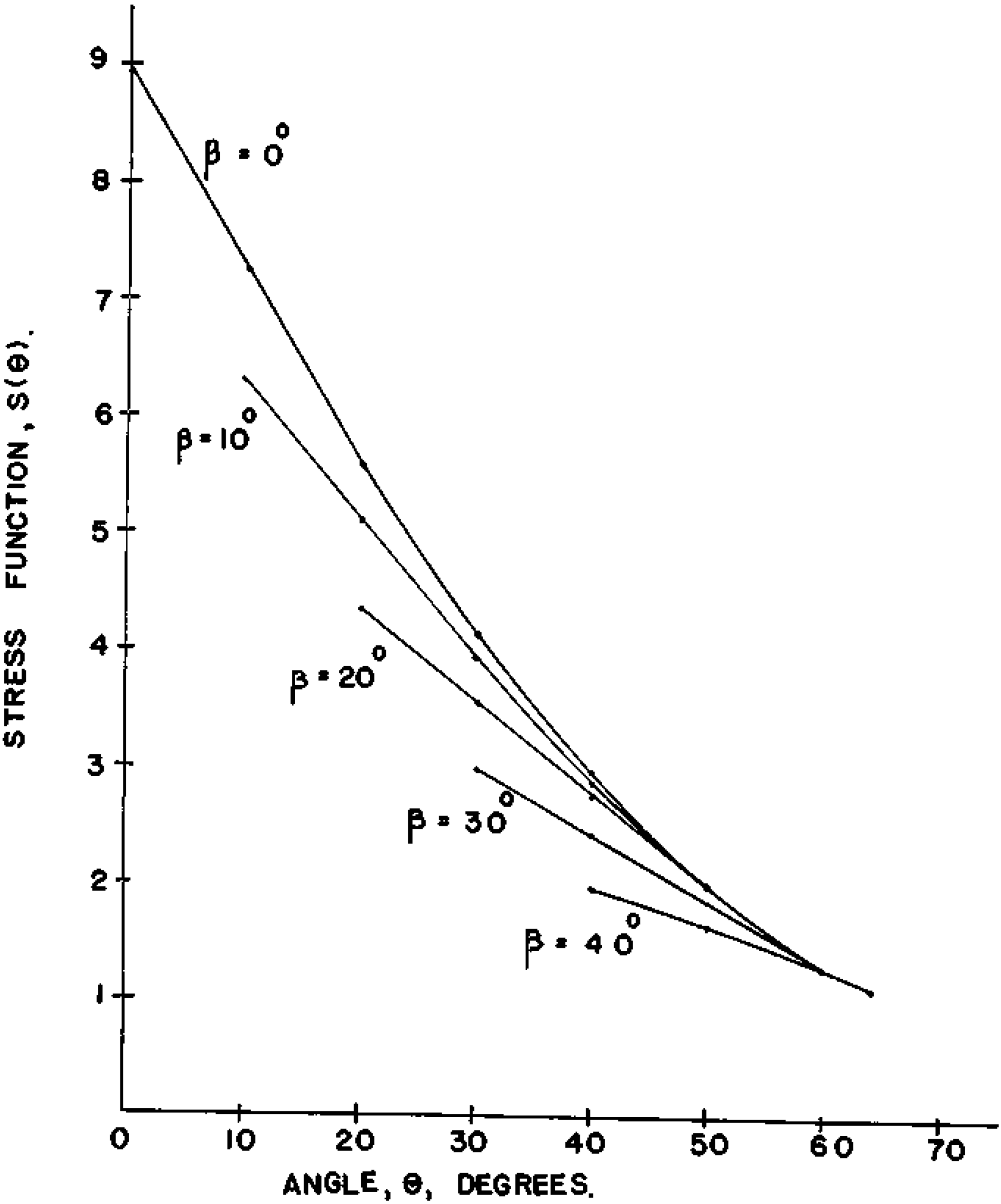

FIG. 4 STRESS FUNCTION VS. θ.

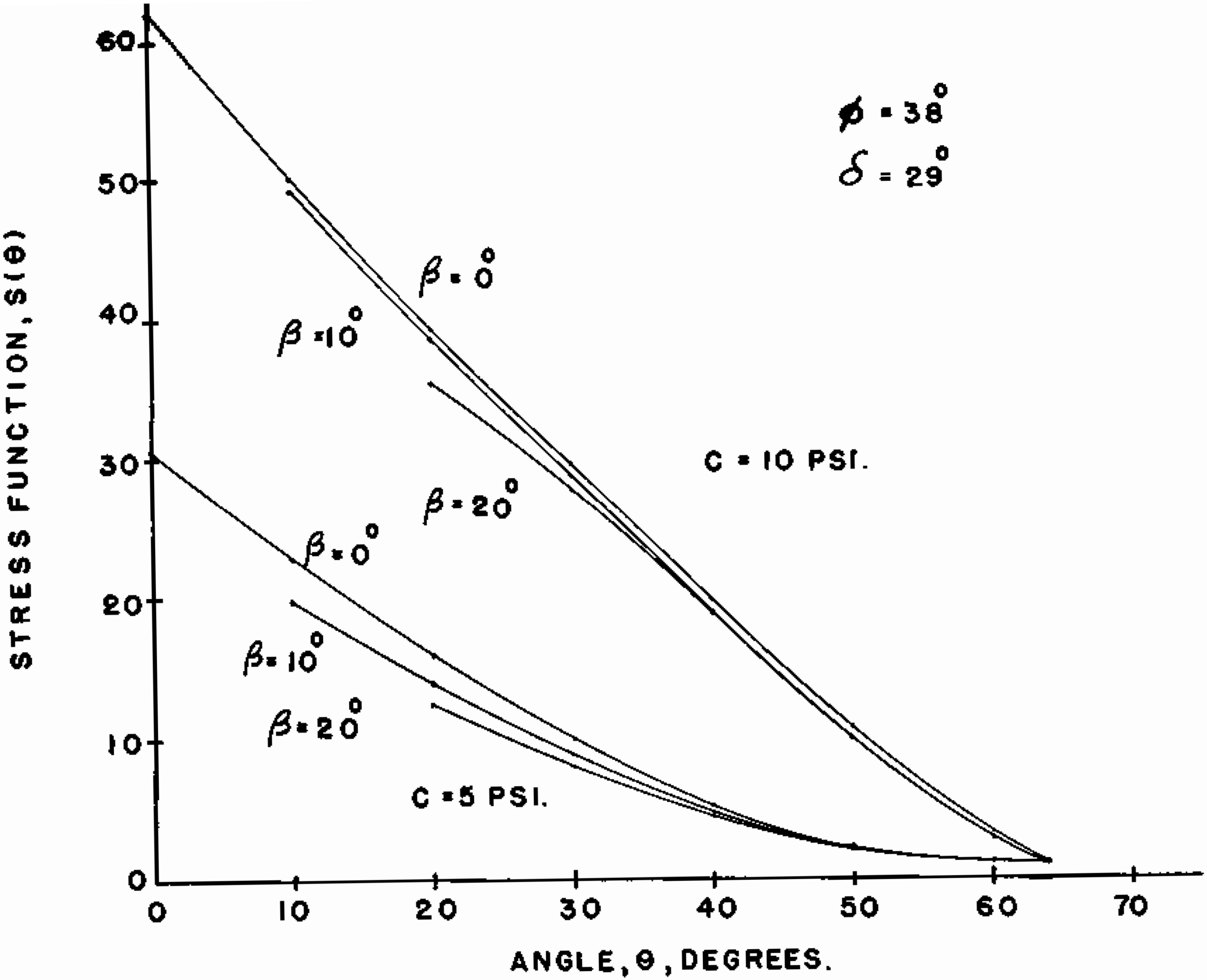

FIG. 5 STRESS FUNCTION VS. θ.

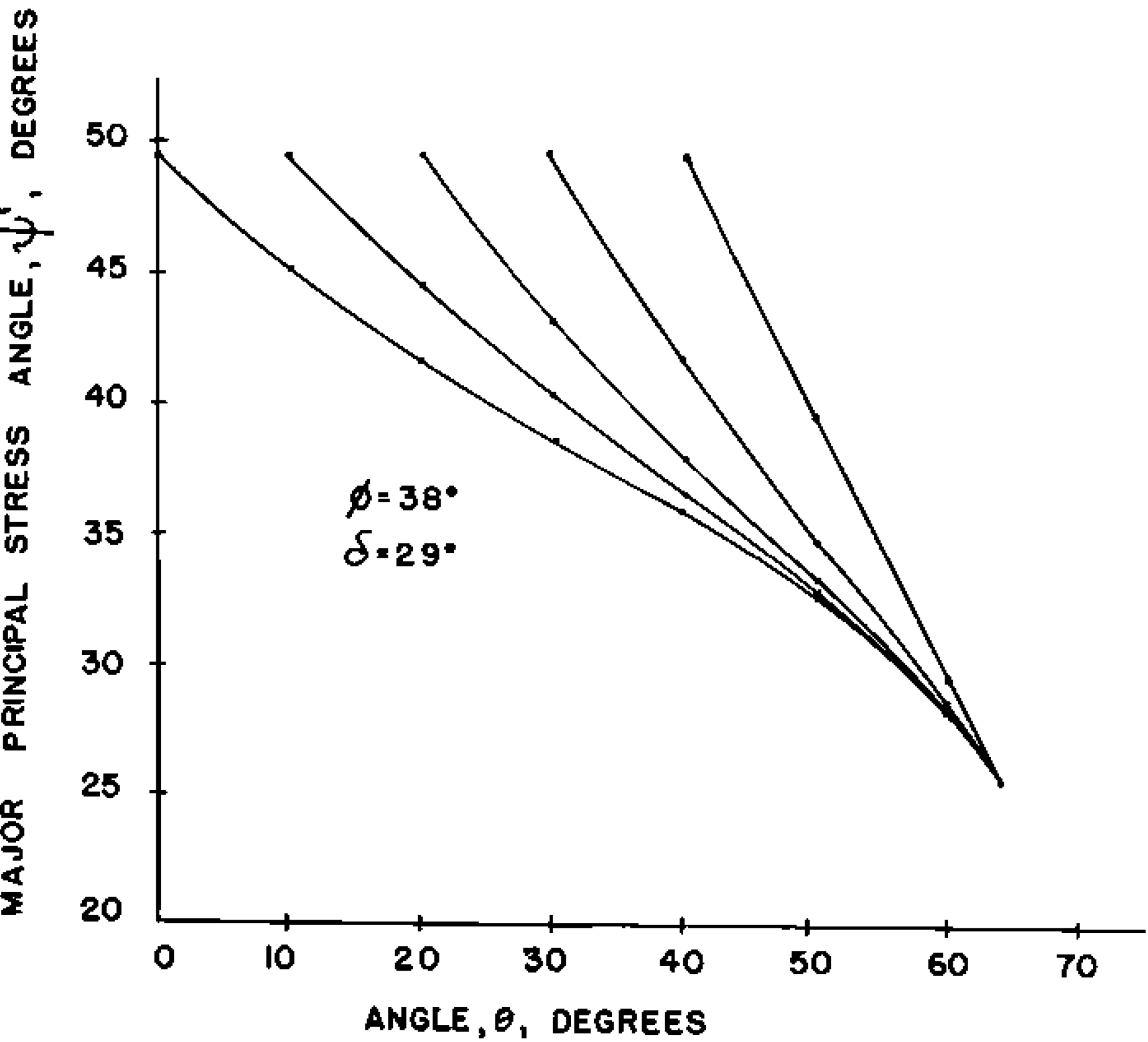

FIG. 6 MAJOR PRINCIPAL STRESS ANGLE VS. θ

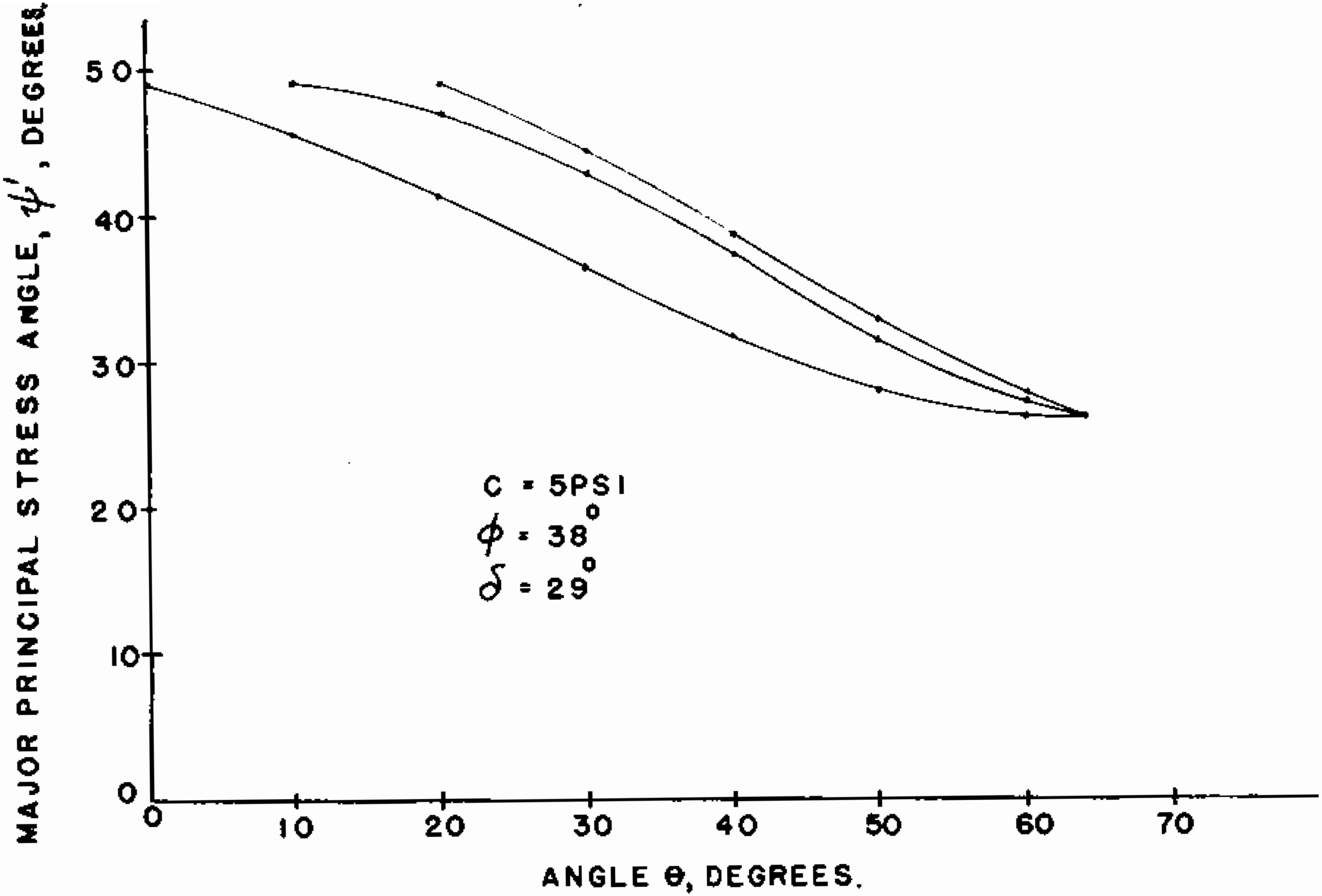

FIG. 7 MAJOR PRINCIPAL STRESS ANGLE VS. θ.

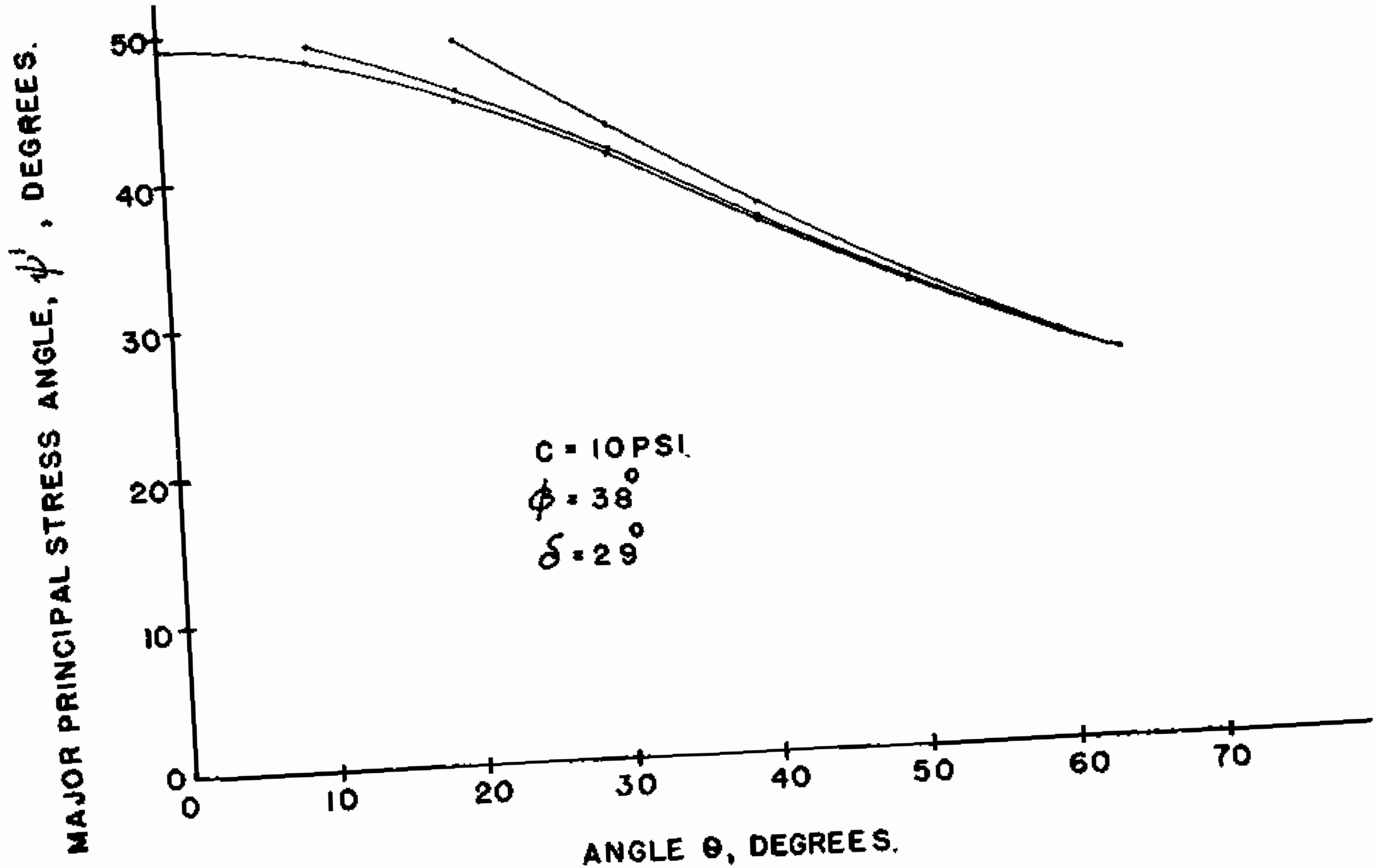

FIG. 8 MAJOR PRINCIPAL STRESS ANGLE VS. θ.

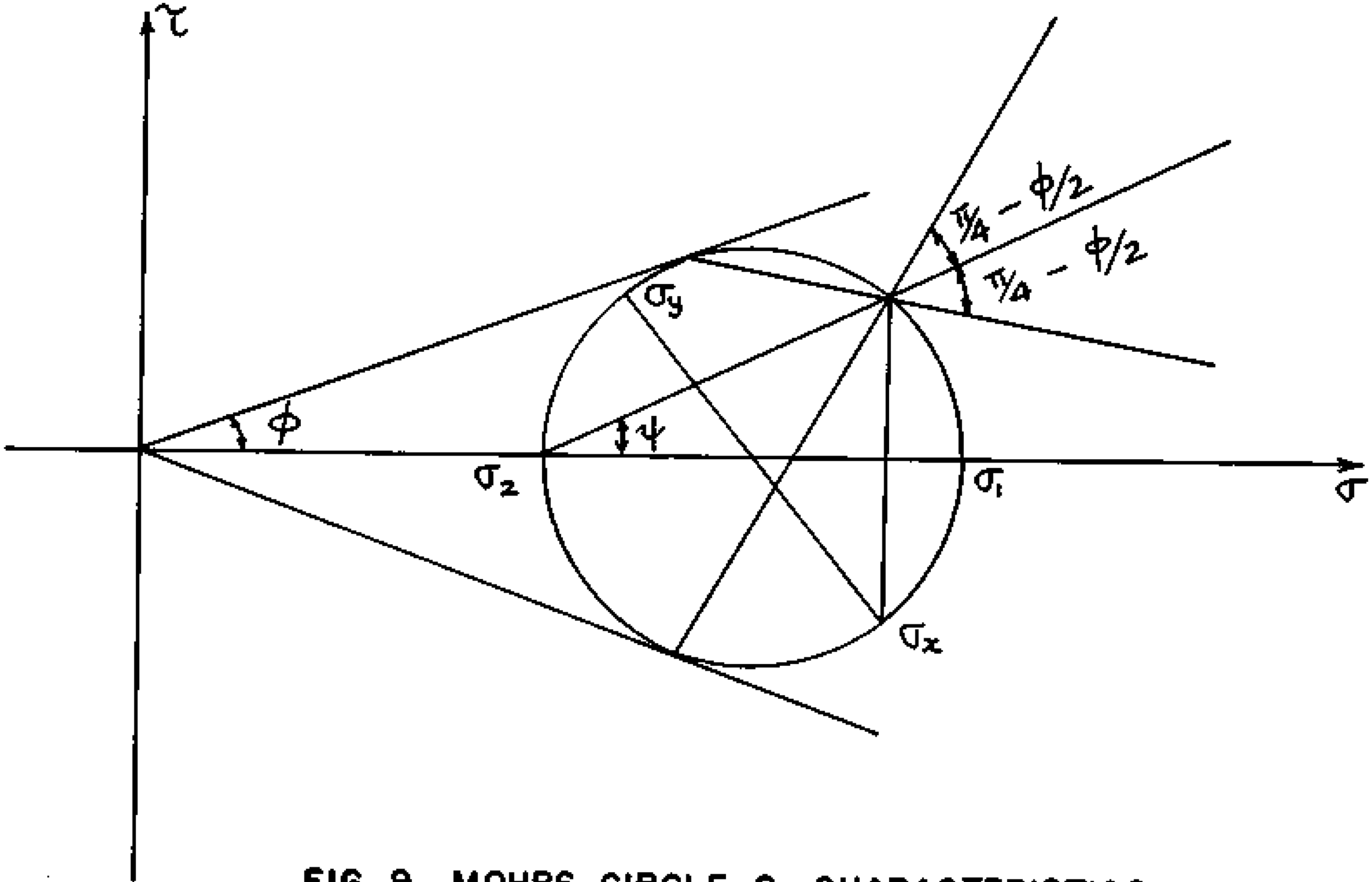

FIG. 9 MOHRS CIRCLE & CHARACTERISTICS

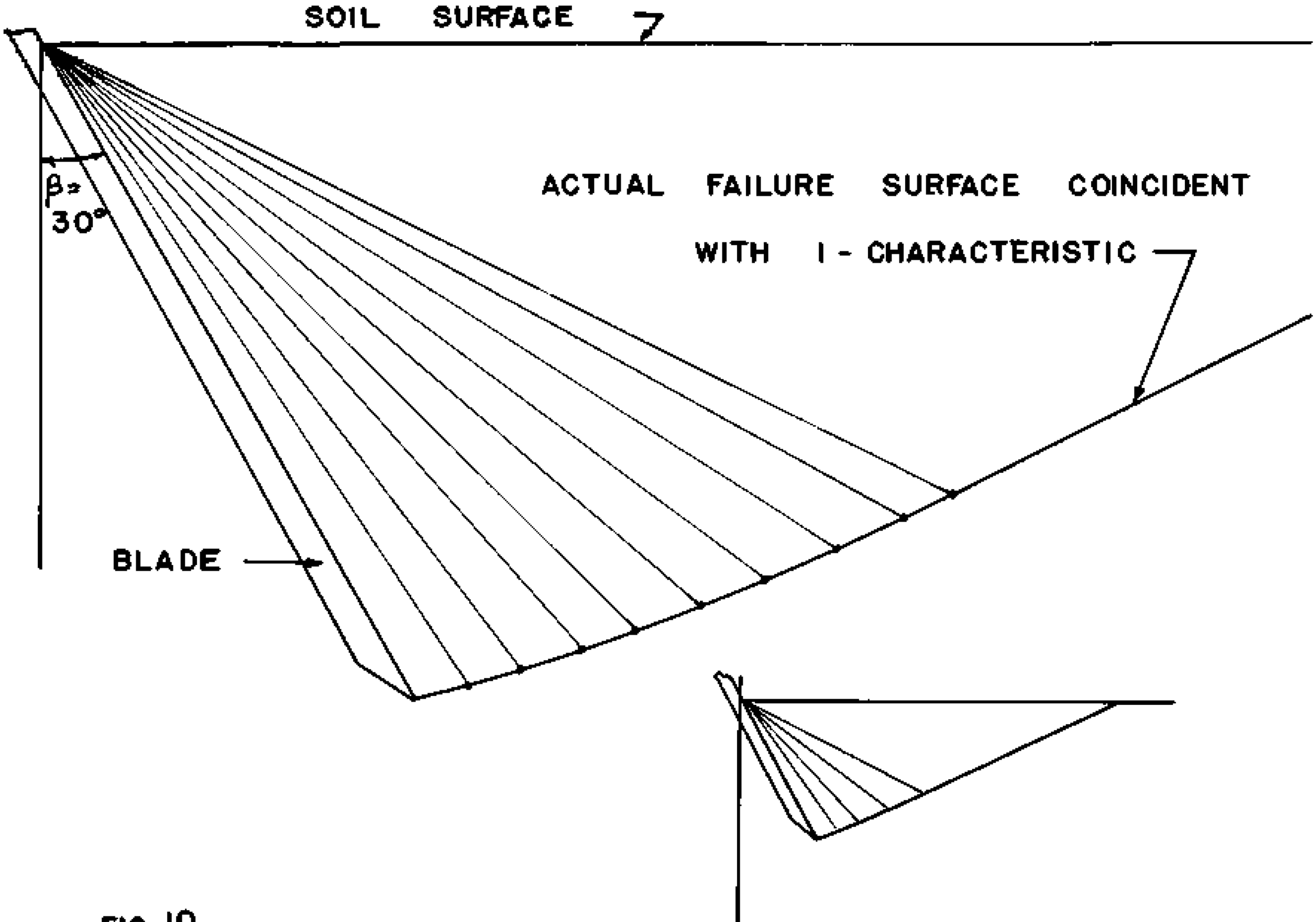

FIG. 10
PREDICTION OF I - CHARACTERISTIC FOR CASE I ($\beta = 30°$)

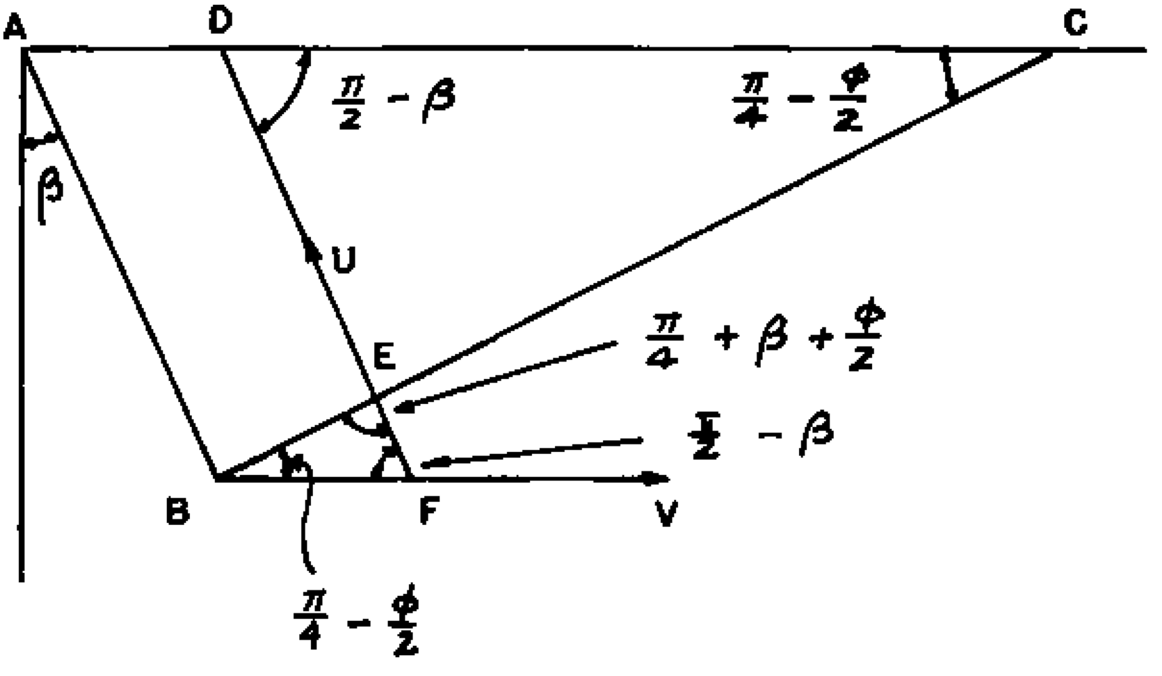

FIG. II VELOCITY DIAGRAM.

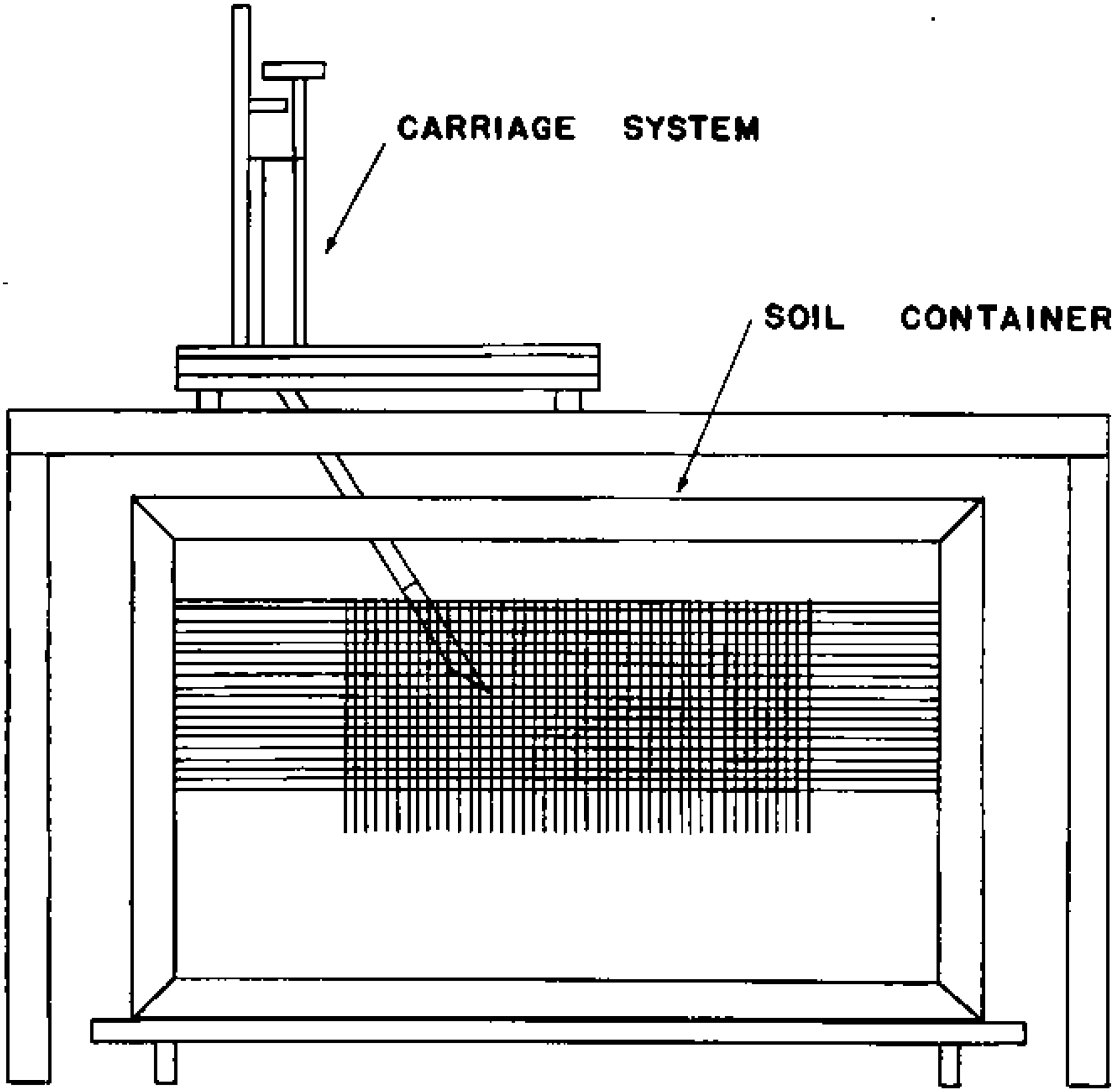

FIG. 12 TEST EQUIPMENT - SIDE VIEW

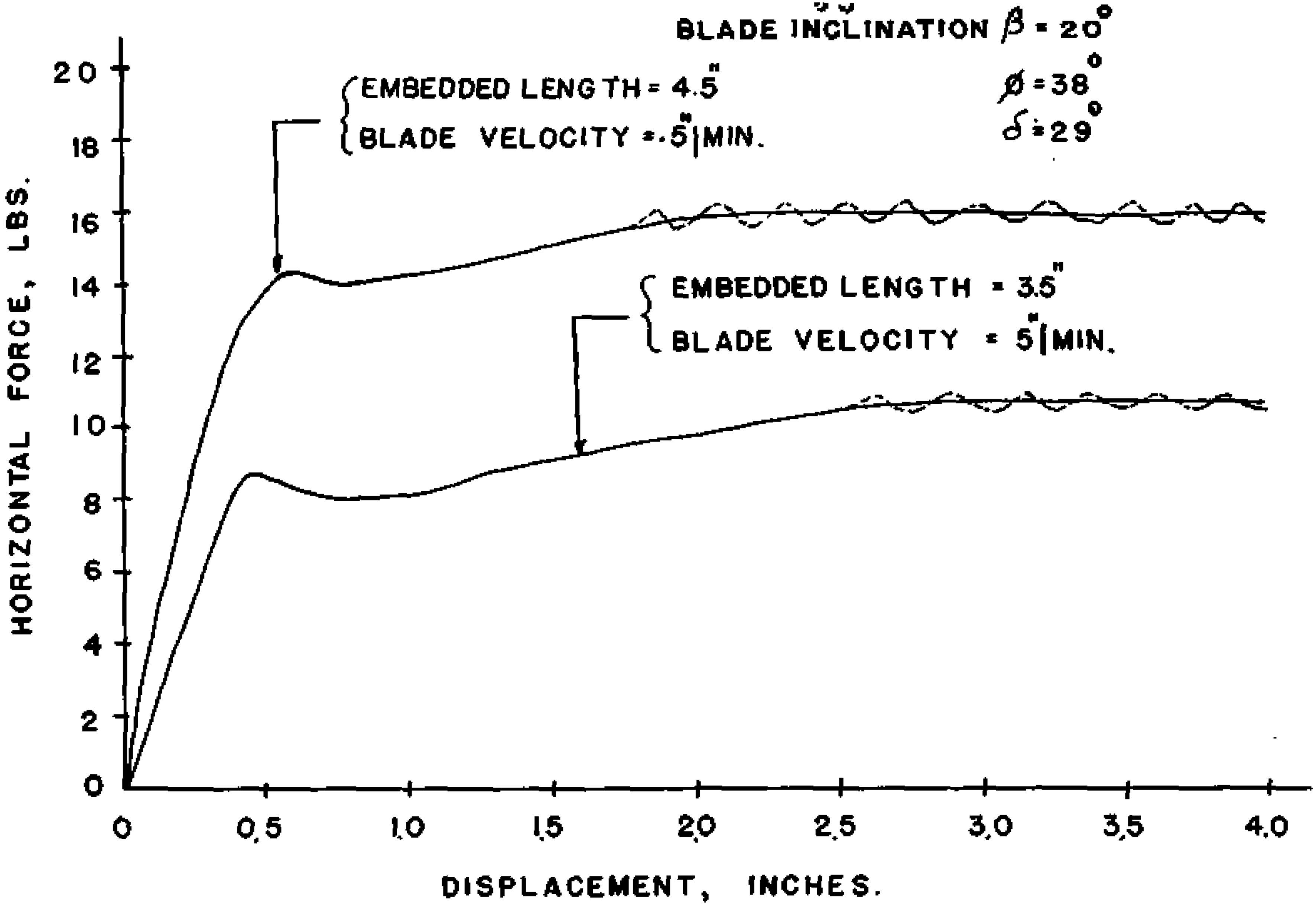

FIG. 13 HORIZONTAL FORCE VS DISPLACEMENT.

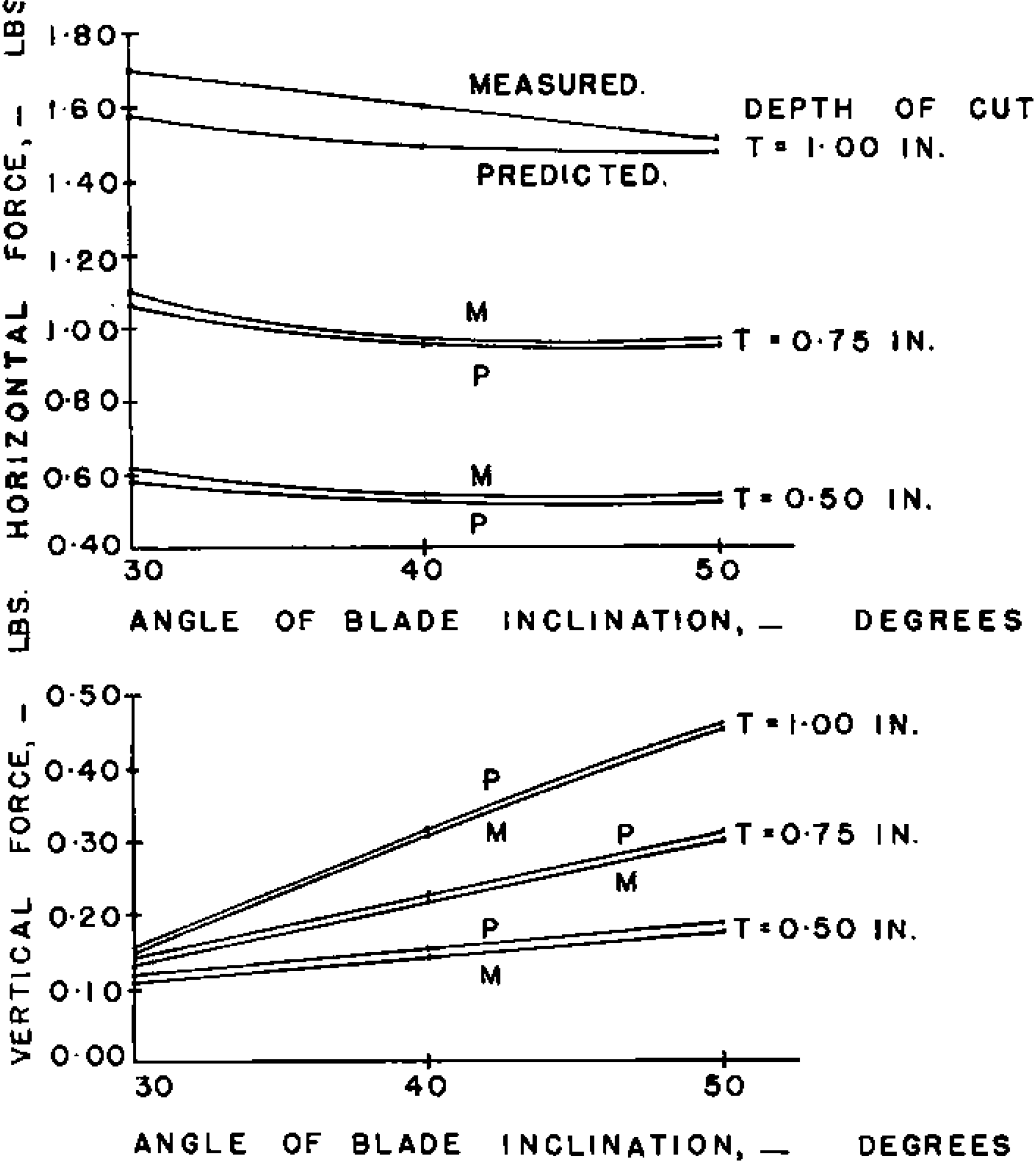

FIG.14

PLOTS OF PREDICTED AND MEASURED HORIZONTAL AND VERTICAL FORCES VS. ANGLE OF BLADE INCLINATION FOR CATERPILLAR TESTS.

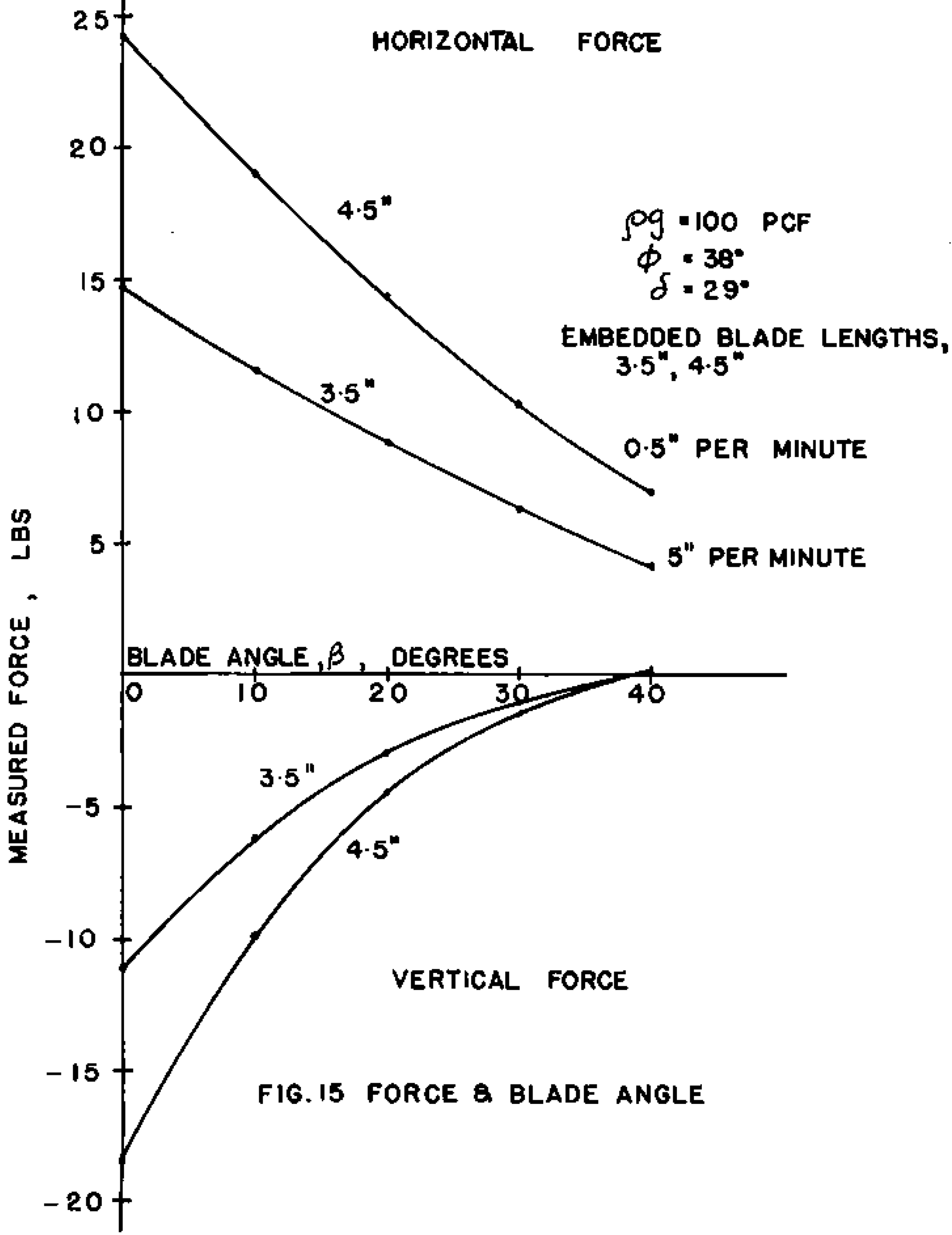

FIG. 15 FORCE & BLADE ANGLE

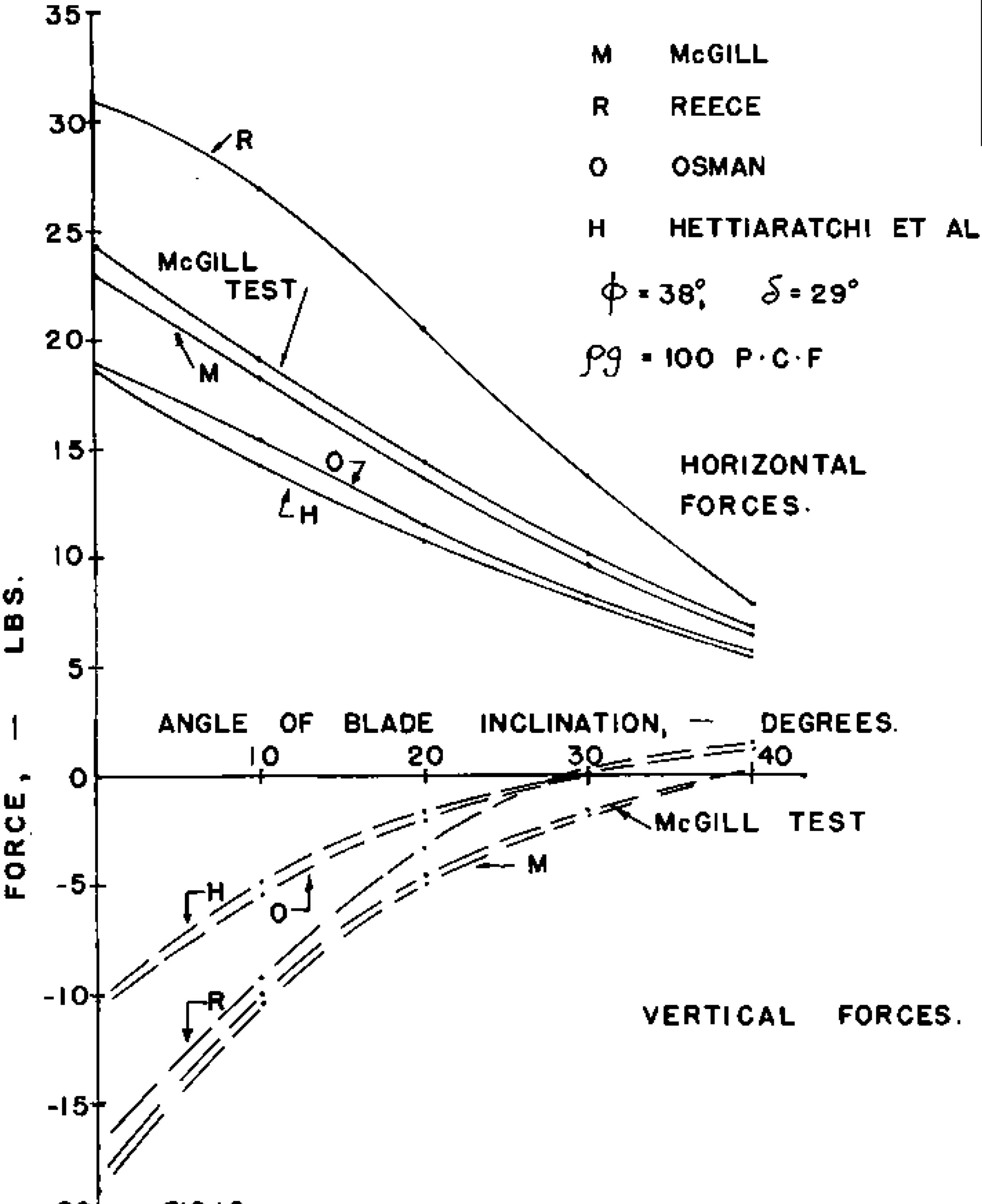

FIG.16

PLOTS OF HORIZONTAL AND VERTICAL FORCES VS. ANGLE OF BLADE INCLINATION FROM McGILL TESTS; AND McGILL, REECE, OSMAN, AND HETTIARATCHI ET AL. METHODS OF COMPUTATION

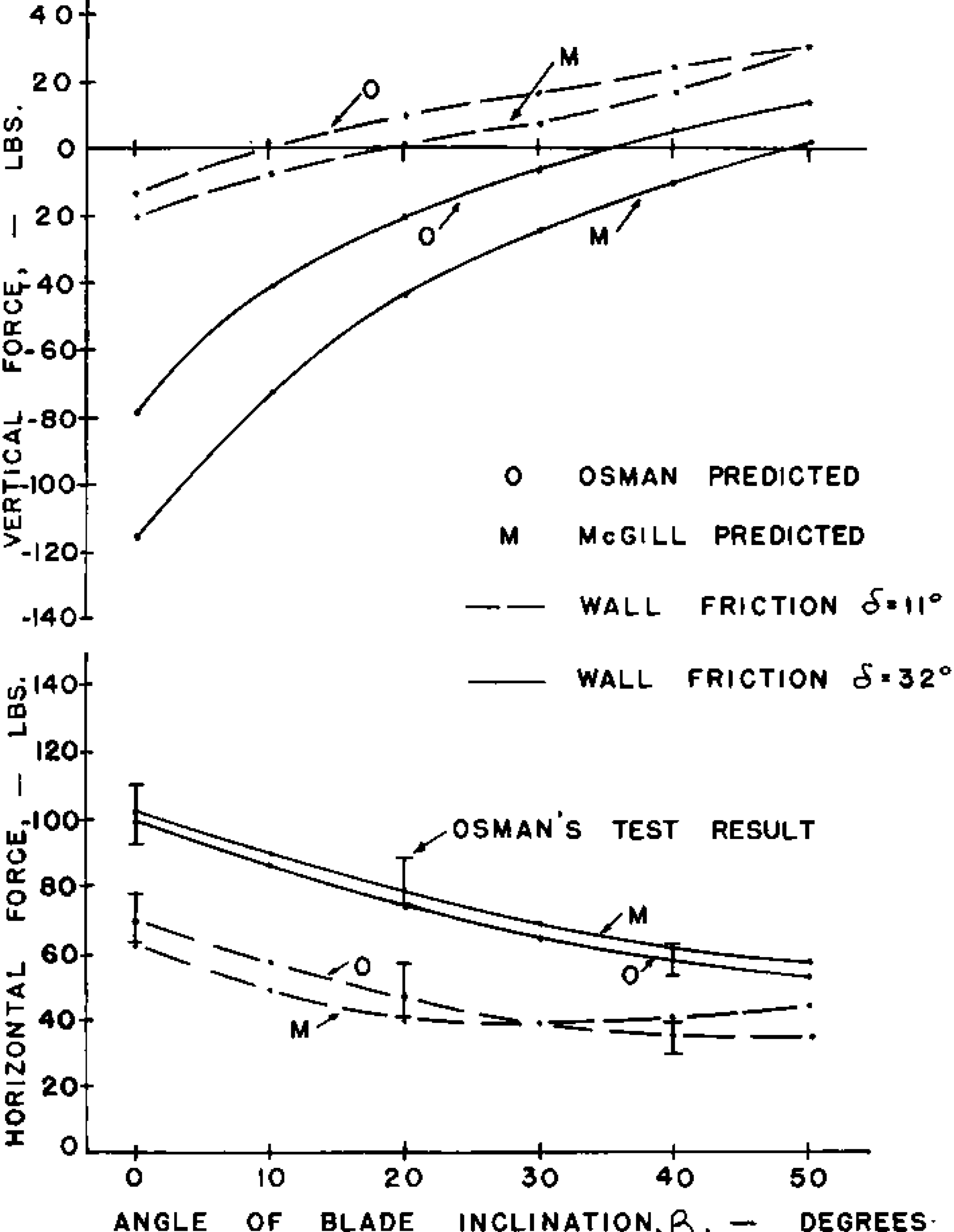

FIG. 17

COMPARISON OF THE McGILL AND OSMAN METHODS OF FORCE COMPUTATION FOR OSMAN'S TESTS.

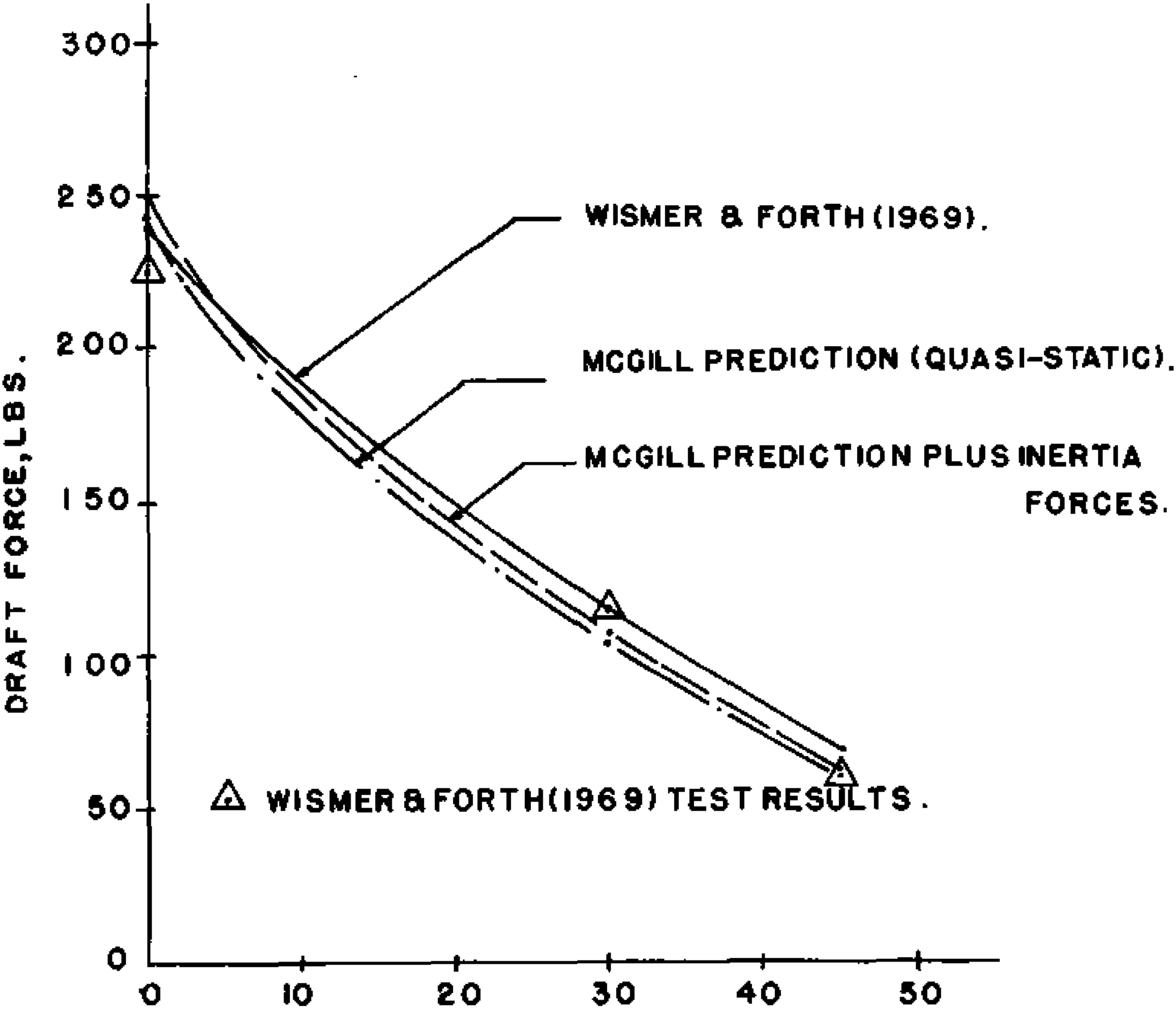

FIG. 18 ANGLE OF BLADE INCLINATION, (β) DEGREES.