String theoretic models of the early universe

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ABSTRACT

This thesis comprises several manuscripts, each exploring aspects of the dynamics of the early Universe. The foundations of the work presented lies in the realm of cosmology, but draws heavily on string theory as a source of guidance. The thesis commences with a motivation for the research and provides an introduction to the contemporary views of Cosmology: following a historical perspective of cosmology, we motivate the inflationary paradigm of Big Bang Cosmology, and introduce several world-views promoted by string theory. The string-motivated models will address shortcomings of cosmologies based on General Relativity and the Standard Model, and will provide a comprehensive, coherent description of the early Universe that is expected to transition to our observed Universe. Two possibilities presented here include String Gas Cosmology (SGC) and the Brane World scenario. We provide an introduction to these two constructions, and subsequently report on the possibility of simultaneously stabilizing the dilaton and moduli fields in SGC stabilization models, a mechanism to solve the overshoot problem of racetrack inflation, and explore the possibility of long-lived relics in warped reheating.

ABRÉGÉ

Cette thèse explore la physique du jeune univers. Nous utilisons la théorie des cordes pour nous aider à comprendre l'époque précédant la nucléosynthèse. Deux manuscrits sont presentés: un concernant la cosmologie de gaz de cordes (string gas cosmology, SGC), et un concernant le scénario d'un univers membrane (brane world, BW). Nous plaçons des contraintes sur la stabilization du moduli et du dilaton dans la construction SGC. Puis, nous étudions lépoque du réchauffement de l'univers dans la construction BW, et nous plaçons des contraintes sur les paramètres fundamentaux de la théorie des cordes.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Contributions of Authors

The manuscripts reproduced in Chapters 2 and 3 are presented in chronological order, but also coincide with an increasing complexity and understanding of research. Chapter 2 is the result of a collaboration with Dr. T. Biswas and Prof. J. Cline. In it, we embedded a gas of strings and branes into the racetrack model of inflation. This work was a natural progression of work accomplished previously with my supervisor, dealing exclusively with moduli and dilaton stabilization in Brane Gas Cosmology [32]. In this colloboratoration I benefitted from the breadth and depth of knowledge provided by the co-authors, and their ideas provided the motivation for incorporating my previous results into this new framework. I was able to verify that a gas of strings could rectify the overshoot problem of the racetrack potential and I authored several sections of the publication.

Chapter 3 is a draft of work soon to be published. The draft to be submitted for publication has been written by the author of this thesis, though a lot of credit is owed to Dr. H. Stoica and Prof. J. Cline for their guidance, explanations, edits, and patience necessary to carry out the calculations presented and to write the paper.

CHAPTER 1
Introduction

The fact that we live at the bottom of a deep gravity well on the surface of a gas-covered planet going around a nuclear fireball 90 million miles away and think this to be normal, is some indication of how skewed our perspective tends to be.—Douglas Adams

1.1 Motivation of the Research

Cosmology is a domain of study probing the fundamentals of our existence, and I am honoured to have been given the opportunity to explore these issues in such a stimulating environment. There are many dichotomies that can be associated with this word, cosmology. In a scientific context (as opposed to philosophic), cosmology explores the origin and structure of the universe as well as its evolution. A further dichotomy exists between theoretical and observational [1] cosmologists and can be roughly divided by the arrow of time in their work. Observationally, we look at objects farther and farther away, up until the surface of last scattering (the cosmic microwave background (CMB)). Since light travels at a finite speed,

[1] Note, I deliberately avoid the use of experimental since there is only one visible universe and, hence, we only have a control—the observable universe. We are not able to recreate the universe to test it under varying conditions, though we can create experiments to observe the universe.

looking at objects far away corresponds to looking at objects farther back in time. Most theorists, however, begin with events before recombination, dealing with issues like singularity-resolution, inflation, reheating, the matter asymmetry, and restrict themselves to models which faithfully reproduce the observable universe. Not surprisingly, the study of our evolving universe involves numerous inter-related parameters which, in turn, control things like the amount of matter and radiation in our visible universe, the commencement of stellar burning, and the possibility of a Big Crunch, to name a few. The difficulty and importance of precisely determining these parameters has prompted the specialization of observational cosmologists, and has motivated numerous experiments to measure the various properties of the observable universe. Unfortunately the wealth of data does not imply that theorists have an easy time interpreting the data and singling out a specific origin of the cosmos. In fact, cosmologies based on General Relativity and the Standard Model (SM) fail to explain many observations. For example, the modern paradigm of inflationary cosmology has promoted a plethora of models and ideas, making the precise identification of the underlying theoretical model quite difficult.

Many models of inflation predict all scales in our visible universe originated at the Planck scale. This is an energy regime well beyond the validity of the Standard Model of particle physics, and is likely controlled by some Theory of Everything (TOE). With this in mind, a high energy theorist researching in cosmology hopes that traits of the underlying theory will be imparted on todays universe. This gives us access to scales of physics not attainable in other experiments, and provides the possibility of constraining fundamental theories. The paradigm of inflation has gained wider acceptance over the last decade, and one immediate effect in the community is the application of string theory towards cosmology.

This thesis presents work following the recent tradition of string theorists: utilizing cosmology to constrain fundamental theories. Our first task is to introduce the current view of the Big Bang and the origin of our universe; with this knowledge we will apply string-motivated models of the early universe with the hopes of ameliorating the current paradigm and constraining the parameters of string theory. A common criticism of this field of study points to the lack of predictions. This is a misrepresentation since numerous predictions are being made, but they are either beyond the reach of current experiments, or data is currently constraining the theory—possibly invalidating it. As a small rebuff to these claims, the discussion starts with a brief history of modern cosmology. This serves to remind the reader that past observations have prompted massive shifts in our understanding—the inflationary paradigm is providing that shift today.

A Historical Perspective

The modern study of cosmology can arguably [2] trace its roots to the Shapley-Curtis debate of the 1920's [1, 2, 3]. Their public debate entitled "The Distance Scale of the universe" saw Heber Doust Curtis of Lick Observatory argue that galaxies such as Andromeda were separate from the Milky Way, while Harlow Shapley of Mt. Wilson Solar Observatory argued that these nebulae were part of our galaxy. The idea that Andromeda was merely some small object resting within our galaxy is now forgotten, but keep in mind that many discoveries had yet to be made: the electron and proton were the only known particles, the source of stellar

[2] Although I reckon anything is arguable, this event marks the first time scientific evidence hinted at the vastness of the universe. The concept of other galaxies and the distance scales involved were quite unfathomable.

energy was still unknown (although we did know that gravitational potential energy and radioactivity were not sufficient), and the theory of general relativity (GR) had just passed its first test after Eddington observed the bending of light during a total eclipse on May 29th, 1919. Numerous other mysteries were unexplained at this time, and in the case of the "Great Debate" both parties ended up being partially correct. Employing Adiaan van Maanen's observations of apparent rotations in spiral galaxies, Shapley argued that the distance to these nebulae would be of the order 10^8 lightyears away -a quantity few astronomers felt comfortable with at the time. Other than placing galaxies within ours, Shapley's view of the Milky Way was more accurate than Curtis's view of a Sun-centred galaxy. Shapley also correctly reported the visible extent of our galaxy to be 20 kpc. The debate was convincingly settled when, in late 1924, Edwin Hubble reported the discovery of Cepheids in M33, M31 and NGC 6822, and used their period-luminosity relationship to confirm their extra-galactic origin. Additionally, his redshift–distance relation allowed the identification of numerous other galaxies through spectroscopic data.

Again, this short diversion is meant to help set the tone of this thesis: the collection of manuscripts subsequently presented deals with topics at the forefront of our understanding. As with the Shapley-Curtis debate, a wealth of information available to us today is subject to interpretation. From a High Energy Theory perspective, however, the amount of data is so large that theoretical proposals must fit an ever-decreasing parameter space. Hence, a proposal which successfully describes the early universe and later evolves to explain today's observations will likely be greeted with much celebration, and scrutiny. In practice, the amount of work required to verify all aspects and applications of a theory is quite substantial and most physicists are proceeding one step at a time. This thesis adopts a similar approach, although embedding the discussion in the context of string theory provides a coherent framework to justify and guide any extensions to the SM and GR. Whereas Shapley's proposal was designed to fit his data, he had no guiding principle to justify the extragalatic nature of his nebulae. In the context of this thesis, we demand that since string theory claims to be the theory of everything, it should provide a unified explanation for the numerous mysteries of our universe. A brief list includes the nature of dark energy, the nature of dark matter, the origin of neutrino mass, an explanation of the weak hierarchy, an explanation for the $SU(3) \times SU(2) \times U(1)$ group structure of the SM, and a realization of inflation.

A Theoretical Perspective

An alternative approach to addressing these issues is through generic model building (or toy models), where the effects of a few assumptions or modifications are characterized. Both approaches have merit, with generic model-building providing a rough guideline for solving the problem at hand. If a particular effect is quite beneficial (for instance, finding a viable model of inflation), one may search for these features in their preferred TOE. A recent example of this interplay comes from the study of extra dimensions, with two ideas laying the foundations for new perspectives on physics. In ref. [5], the weak hierarchy was explained by suggesting spacetime is fundamentally higher-dimensional. By isolating the dependence on the extra dimensions we may identify the $4D$ effective action and relevant $4D$ parameters. For example, given a $4+n$ dimensional product space $\mathcal{R}^4 \times \mathcal{M}_n$ with coordinates x^μ associated with our four dimensions and y^i belonging to n internal directions, the action for Einstein gravity is

$$S = \int d^{4+n}x \sqrt{-\det|G_{MN}|} M^{2+n} R(x^\mu, y^i) \tag{1.1}$$

we isolate the dependence along our directions x^μ

$$
\begin{aligned}
S &= \int d^4x \sqrt{-\det|g_{\mu\nu}|}\left[\int d^ny\sqrt{\det|g_{ab}|}M^{2+n}R(x^\mu) + \right. \\
&\quad \left. \int d^ny\sqrt{\det|g_{ab}|}M^{2+n}R(y^a)\right] \qquad (1.2)
\end{aligned}
$$

With $V_n \equiv \int d^ny\sqrt{\det|g_{ab|}} \simeq L^n$, where L is the size of the internal directions, we identify the $4d$ Einstein action as the first term in (1.2), together with the $4D$ Planck constant

$$
M_{Pl} = V_n M^{4+n} \,. \qquad (1.3)
$$

The expansion in equation (1.2) is based on the assumption of a background described as

$$
\begin{aligned}
ds^2 &= g_{\mu\nu}(x^\mu)dx^\mu dx^\nu + g_{ab}(y^a)dy^a dy^b \\
\Rightarrow R &= g^{\mu\nu}(x)R_{\mu\nu}(x) + g^{ab}(y)R_{ab}(y) \,. \qquad (1.4)
\end{aligned}
$$

The assumption of such background facilitates the easy identification of Newton's constant, though the procedure does generalize to less-trivial backgrounds. The result, in this case, is a theory whose effective $4D$ Planck scale can be extremely high, $M_{pl} \simeq 2 \times 10^{18}$GeV, but whose fundamental scale M is suppressed by the volume of the compact space V_n, and is consequently small. This suppression is greatest for large extra dimensions, but the fact that SM particles don't show signs of extra dimensions suggests that they are confined to a 4-dimensional subspace (called a "3-brane"), while gravity can propagate throughout. In this case, the shortest-scale tests of gravity constrain the size of any extra dimension to $R \leq 44\mu m$ [6, 7], so we have the curious notion that each point along our brane is accompanied by n dimensions which may be larger than the width of a human hair.

Extending this exploration of large extra dimensions, the Randall-Sundrum model of extra dimensions [4] provided an alternative explanation for the weak hierarchy. Their exploration of warped spaces showed that gravity can be mediated at the Planck scale, but physics local to the warped hypersurfaces can be exponentially suppressed (or warped) down to the weak scale. Their model has laid the foundations for warped compactifications in string theory, and has also provided a new mechanism to explain the weak hierarchy. Theory is motivating a lot of new perceptions consistent with our notions of physics; it is all the more amazing when one theory is providing the impetus.

The lesson of the Shapley-Curtis debates hints at the importance of questioning our notions of the universe, and reaffirms the scientific process as an important means for guiding our understanding. This alone, however, does not fully justify research into string theory, even though the scientific method indicates problems with the SM and GR. Instead, new proposals should make unique predictions and provide the possibility of being tested. We have mentioned that string theory makes numerous bold predictions, but it has a tough time making testable predictions. The underlying problem is that string theory naturally exists at the GUT scale ($\simeq 10^{15}$GeV), whereas the best terrestrial experiments reach energies of $\simeq 10^{3}$GeV at accelerators and $\simeq 10^{11}$GeV for cosmic-ray showers (with extremely low luminosity). So, the trouble with string theory is finding situations where stringy signatures may exist. Let us continue our story of modern cosmology where, with the motivation for an inflationary epoch, we'll see that the early universe is a perfect regime for testing string theory. Hopefully the reader will agree that, despite its not being tested, string theory is providing us with novel mechanisms to explain our perception of the universe.

More History

The next most important event in modern cosmology can be traced to the 1964 discovery by Penzias and Wilson of a persistent, homogeneous, and isotropic background radiation. [3] The "noise" of their detector was interpreted by Dicke, Peebles, Roll and Wilkinson [12] as the relic radiation from the Big Bang Nucleosynthesis scenario described by Alpher, Bethe, and Gamow (ABG) [14]. This observation helped establish the application of GR to cosmological scenarios and established the Hot Big Bang model as a viable model for the early universe.[4] As most readers are probably aware, the standard model of cosmology suffers from several key problems. There are numerous reviews of the big bang and its shortcomings (see, for example, [15, 16]), but the following brief listing will serve as our review:

- Horizon Problem: without inflation, scales beyond several degrees in the sky were never in causal contact. This conflicts with the observation that the sky is remarkably homogeneous and isotropic on all scales. Inflation brings all scales into causal contact.

[3] There is an interesting Canadian result often overlooked in the history of the CMB. Over twenty years prior, Dr. Andrew McKellar observed emission lines for diatomic molecules in the interstellar medium. These correspond to rotational modes in the radio regime, and he associated their excitation to some ambient 3K bath [13].

[4] Here "early" refers to the epoch of nucleosynthesis. Assuming a thermal distribution of particles, ABG showed the abundances of light elements are produced in the right proportions. Since nuclear binding energies are several MeV, this corresponds to an era when the universe was $\simeq 10^{-13}$ its current size. This is still much larger than the early universe epoch of inflation or quantum gravity.

- Flatness Problem: the universe can be characterized as either an open ($\Omega > 1$), flat ($\Omega = 1$), or closed ($\Omega < 1$) universe, where the flat solution is unstable. Here, $\Omega = \rho/\rho_{crit}$ is the energy budget of the universe, and $\rho_{crit} = 3H^2/8\pi G$ is the energy density of a flat universe. We observe a flat universe to high accuracy today, so the initial conditions must be extremely fine-tuned to accommodate these observations. Inflation stretches out any intrinsic curvature, thus explaining the initial conditions.
- Monopole Problem: the phase transition from some GUT to the SM should have produced numerous topological defects; however, we do not observe any. If inflation happens after the symmetry breaking, then any defects can be blown outside the horizon.
- Baryon Asymmetry: We live in a matter-dominated universe, but the thermal phase of the Big Bang should have produced equal amounts of matter and anti-matter. Inflation does not provide a simple explanation of this observation, but the Sakharov conditions are known to be necessary conditions for the asymmetry to be produced [8].
- Initial Singularity: evolving the scale factor backwards in time suggests that the universe evolved from a single point. Most models incorporating inflation still suffer from this problem.

Inflation is able to overcome several of the classical problems listed above and, more importantly, it successful predicted a scale-invariant power spectrum, inflation is now considered a key ingredient of the Big Bang model. Developed in the early 1980s by Guth [17] (see also [18] for earlier, related work), this model explained the flatness, monopole, and horizon problems of the universe. More importantly, inflation predicted a scale-invariant power spectrum as the seed for

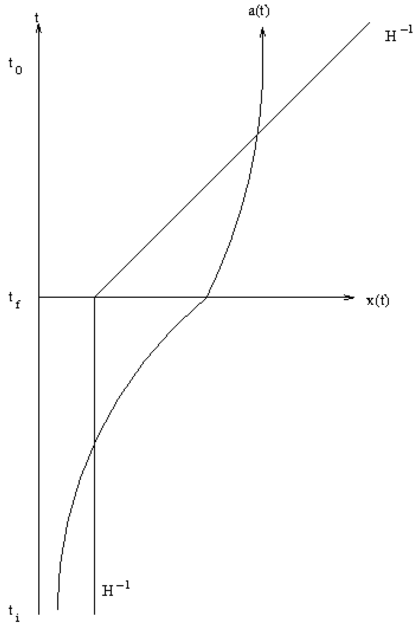

Figure 1–1: Evolution of the Hubble scale and physical modes. During inflation $t_i < t < t_f$, physical modes expand past the horizon, thus freezing-out. Upon transition to a radiation or matter dominated phase $t > t_f$, modes re-enter the horizon since the Hubble scale grows faster than the scale factor.

structure formation [19, 20]; this was later verified by several CMB experiments. A qualitative understanding of this scale-invariance can be seen in Figure 1–1. As seen, physical modes are exponentially stretched and eventually pass the Hubble length H^{-1} during inflation. The Hubble length is (roughly) constant during inflation, so all fluctuations cross the same Hubble scale. If all fluctuations are generated by the same mechanism the amplitude of fluctuations should be the same on all scales; so during inflation, all physical modes cross the same Hubble scale and freeze out with the same amplitude of fluctuation. We observe these fluctuations after reheating, time t_f, when the Hubble scale grows faster than physical modes which consequently re-enter the horizon.

Quantitatively, one can understand this prediction from the linear expansion of the Einstein equations about a homogeneous and isotropic background. For our purposes it suffices to apply Newtonian gravity to a perfect fluid in an expanding background. This approach requires the two parameters ρ and $a(t)$, where the former describes the matter content and $a(t)$ is the scale factor determining the evolution of space. Physical scales $x(t)$ are related to co-moving coordinates q through the relation $x(t) = a(t)q$. Recalling that a body suspended in a homogeneous shell does not experience a net gravitational force, we are prompted to look at fluctuations of the matter field $\rho = \rho_o + \delta\rho$. Newtonian dynamics in a static background results in Poisson's equation

$$\delta\ddot{\rho} = 4\pi G_N \delta\rho \,, \tag{1.5}$$

where G_N is the gravitational constant, and overdots refer to time derivatives. Since gravity lacks negative charges, we see that inhomogeneities are accelerated, so over-densities will experience runaway growth. In an expanding background the analogous equation of motion (in Fourier space) in a vanishing background ($\rho_0 = 0$) becomes

$$\delta\ddot{\rho}_k + 2\left(\frac{\dot{a}}{a}\right)\delta\dot{\rho}_k + \left(\frac{c_s^2}{a^2}k^2\right)\delta\rho_k = 0 \tag{1.6}$$

where $c_s^2 = \frac{dP}{d\rho}$ is the sound speed in the single-component fluid and k is the wavenumber. Equation (1.6) describes, conveniently, the damped harmonic oscillator. During inflation $a(t) = \exp(Ht)$, so the damping is on the Hubble rate, H. Modes within the Hubble length (the horizon) $k^{-1} \ll H^{-1}$ undergo damped oscillation, while modes larger than the Hubble length freeze out. So, modes within the horizon experience damped oscillation until they cross the Hubble scale H^{-1}; subsequent evolution exponentially decays away and the perturbations are frozen in at the Hubble-crossing value. Note, a formal analysis of this system based on general relativity and quantum fields results in a more complicated equation of motion for the behaviour of the fluctuations in equation (1.6), but this simple scenario serves to show the result that the Hubble scale $H \equiv \frac{\dot{a}}{a}$ plays an important role in determining the behaviour of fluctuations. This observation still holds in the more formal setting.

As with most physical phenomena, more information is encoded in fluctuations about the mean than in the mean itself. In cosmology, the mean of the CMB (the relic temperature) gives us a rough estimate of the age of the universe, but anisotropies provide information on the matter content of the universe, the scale of inflation, and several other fundamental parameters. For this reason, the last fifteen years have seen a concerted effort in measuring the CMB on all scales within the horizon; some experiments include efforts from the COBE, MAXIMA-1, Boomerang, and WMAP collaborations [21, 22, 23], plus future experiments such as the South Pole Telescope and Planck satellite [24, 25].

1.1.1 Outline of the Thesis

Our in-exhaustive look at modern cosmology has ended with strong support for the model of inflation coming from the observationally-verified prediction of a flat power spectrum. Circumvention of the flatness and horizon problems places a lower bound on the number of e-foldings during inflation, $N \geq 60$. For instance, to solve the horizon problem the largest scale we observe today (the present horizon H_0^{-1}) must have been smaller than the horizon length during inflation H_I^{-1}. A physical scale today λ_0 will be given by its initial scale λ_i times the amount of intervening expansion

$$\lambda_0 = \lambda_i \left(\frac{a(t_0)}{a(t_f)}\right)\left(\frac{a(t_f)}{a(t_i)}\right) . \tag{1.7}$$

There are two factors contributing to the expansion. The amount during the era of inflation ($t_i \leq t \leq t_f$)

$$\frac{a(t_f)}{a(t_i)} = e^{H_I(t_f - t_i)} = e^N , \tag{1.8}$$

and the amount of expansion during the era of radiation-matter domination ($t_f \leq t \leq t_0$)

$$\frac{a(t_0)}{a(t_f)} = \frac{T_f}{T_0} . \tag{1.9}$$

Here $T_0 \simeq 2.73K$ corresponds to todays CMB temperature and T_f is the temperature at the end of inflation (the reheat temperature). Imposing the condition necessary to solve the horizon problem gives

$$\lambda_i = H_0^{-1} \left(\frac{a(t_f)}{a(t_0)}\right) \left(\frac{a(t_i)}{a(t_f)}\right) = H_0^{-1} \left(\frac{T_0}{T_f}\right) e^{-N} \leq H_I^{-1} \tag{1.10}$$

$$\Rightarrow N \geq \ln\left(\frac{T_0}{H_0}\right) - \ln\left(\frac{T_f}{H_I}\right) \simeq 67 + \ln\left(\frac{H_I}{T_f}\right) . \tag{1.11}$$

Since the scale of inflation is greater than the reheat temperature $H_I > T_f$ the horizon problem is avoided if $N > 67$. Note that the second equality in equation (1.11) assumes reheating occured immediately following inflation. An intermediary phase between inflation and reheating will result in another contribution $\frac{T_f}{T_r}$ to (1.11), though the bound $N \geq 67$ still applies.

A similar calculation shows that modes may originate below the Planck scale $\lambda_i \leq l_p$ and be within our horizon provided

$$\lambda_i = H_0^{-1} \left(\frac{T_0}{T_f}\right) e^{-N} \leq l_p \tag{1.12}$$

$$\Rightarrow N \geq \ln\left(\frac{T_0}{H_0}\right) - \ln\left(l_p T_f\right) \tag{1.13}$$

In this case the logarithmic corrections may be large. For high reheat temperatures $T \simeq 10^{10}$GeV visible modes emerged from the Planck scale in models with $N \geq 85$ e-foldings. Many models of inflation do provide this much inflation, so there is hope that signatures from a quantum theory of gravity may persist to have observational consequences today.

We introduced a few of the motivating factors for the research subsequently presented; these factors are coming from both theoretical and observational domains. As discussed, the theoretical motivation draws on the bold claims of string theory and demands that a TOE should provide a natural explanation for the early universe. Although many mysteries of the universe have known explanations outside the usual constructions of the Standard Model and General Relativity, they may be contrived and poorly motivated. For this reason, this thesis focuses on mechanisms proposed by string theory.

The research presented falls under two broad subjects: String Gas Cosmology [9], and the scenario of brane-antibrane inflation [10, 11]. The next two sections describe the setup of each scenario; they aim to provide a conceptual background in order to access the subsequent chapters. The remaining chapters are reproductions of published, and soon-to-be submitted manuscripts.

1.2 Background Material

1.2.1 String (Brane) Gas Cosmology

The field of String Gas Cosmology (SGC) represents one of the first attempts to embed string theory in a cosmological scenario [9]. The pioneering work of Brandenberger and Vafa (BV) showed how the T-dual nature of string theory may be used to overcome the initial singularity inherent in the Big Bang scenario.

Additionally, they were able to argue why, within the context of string theory, one may expect only three large directions out of the nine predicted.

The possible circumvention of the initial singularity can be easily verified from the SGC equations of motion. They are derived from the low-energy effective action of supergravity; in D dimensions this is [26, 27]

$$S_0 \quad = \quad \frac{1}{2k_D^2}\int d^Dx\sqrt{-G}e^{-2\phi}\left[R+c+4\left(\nabla\phi\right)^2-\frac{1}{12}H^2\right] , \tag{1.14}$$

where c is some constant which vanishes for the critical dimensions $D = 26$ ($D = 10$) for the bosonic (super) string, and $H = dB$ is the antisymmetric tensor field strength, R is the Ricci scalar, ϕ the dilaton, and κ_0 is related to the string length. For $D \leq 10$, $2\kappa_D^2 = \left(2\pi\sqrt{\alpha'}\right)^{D-2} g_s^2\left(2\pi\right)^{-1} = 16\pi G_D$ where $l_s = \sqrt{\alpha'}$ is the string length, g_s is the string coupling, and G_D is the D-dimensional Newton constant. From the form of κ_D it is apparent that Equation (1.14) is the tree-level expansion in α' and the string coupling $g_s = e^{\phi_0}$, where ϕ_0 is the expectation value of the dilaton. This action exhibits a property called scalar factor duality, or T-duality. Under the ansatz

$$ds^2 \quad = \quad -dt^2+\sum_i a_i^2(t)dx_i^2\,,\; a_i\equiv e^{\lambda_i(t)}\,,\; i\in 1,2,\ldots,D-1\,, \tag{1.15}$$

this duality means the configuration obtained through the simultaneous transformations of

$$a_i\rightarrow\tilde{a}_i=a_i^{-1}\,\text{and}\;\phi\rightarrow\tilde{\phi}=\phi-2\sum_i\ln\left(a_i\right)\,, \tag{1.16}$$

is also a solution of the graviton-dilaton system of equations [26].

Scale factor duality is absent in general relativity, but it may play an important role in evading the initial singularity. Recall that the standard Big Bang approaches a singularity as $t \to 0^+$. In the context of string theory one believes that stringy degrees of freedom should become important; in this case it is the dilaton ϕ. Instead of approaching the initial singularity we first approach the string scale. Now, however, physics happening below the string scale $l < l_s$ have a dual description happening on scales above the string scale $\tilde{l} = \frac{l_s^2}{l} > l_s$. We are able to evade the microscopic singularity by describing the system in a dual set of coordinates operating above the string scale.

The second important outcome of ref. [9] was a string-inspired explanation for the dimensionality of spacetime. It provided a dynamical mechanism explaining why only three of the dimensions predicted by string theory may have grown observably large. The starting point is a small, dense and hot universe with all fundamental degrees of freedom near thermal equilibrium—a starting point in analogy with SBB. From the initial state of 9 compact dimensions, BV showed that strings wrapped around these directions act as a confining potential, restricting their growth. Once the network of confining strings evolves, certain directions should lose their winding and they will be free to evolve. As a single string moves, it sweeps out a two-dimensional worldsheet. Two strings will sweep out four spatial directions, but for them to intersect they must share one. Thus, strings will naturally interact in three spatial directions, defining our subspace which subsequently grows large.

Subsequent work in this field has extended the domain of SGC into a near-complete description of the early universe. In ref. [28], the authors exhibit a loitering phase of the universe during which the Hubble radius grows larger than the physical extent of the universe. As a result the brane problem[5] in BGC is solved since branes would be in causal contact and can possibly annihilate. The initial singularity and horizon problems of the SBB scenario are solved without relying on an inflationary phase. Loitering allows for previously causally disconnected regions to communicate; this allows for topological defects to annihilate (the generalized domain wall problem) and for equilibrium to be reached on all scales (the horizon problem). SGC also provides a mechanism for isotropization and for stabilizing the internal directions (the moduli fields) [29, 30, 31, 33]. The first contribution of this thesis falls into the domain of SGC, and investigates the simultaneous stabilization of the dilaton and moduli fields. The works previously cited employed species of momentum modes and winding modes to explain loitering, isotropization and moduli stabilization; but in all cases the dilaton was free to evolve. This poses a problem for late-time dynamics since a varying dilaton leads to a varying Newton's constant, as well as a fifth force. One of my papers [32], together with Prof. J. Cline, shows that the effective potential provided by contributions of winding modes and momentum modes leaves a runaway direction [32]. Fortunately, this outcome was later ameliorated in heterotic string theory using massless string modes (string species carrying both winding and momentum charge) [34]. Chapter 2 is a reproduction of the manuscript [33], a collaboration with Dr. T. Biswas and Prof. J. Cline. It may be seen as an extension of the SGC scenario as it embeds a gas of strings into the racetrack potential for moduli stabilization. In it, we show

[5] The brane problem is a generalization of the domain wall problem in standard cosmology. Extended objects dilute slower than matter or radiation and will quickly come to dominate the energy density of the universe, in conflict with observation. This situation can be avoided if the objects can decay or annihilate.

that wound strings may act to solve the overshoot problem inherent in this model (discussed in the manuscript).

The field of SGC continues to draw considerable interest, and was recently discovered to include a mechanism for producing a flat power spectrum [35, 36]. The importance of this result cannot be overly stressed: CMB observations have verified the existence of a flat primordial power spectrum; however, only cosmic strings and inflation have been shown to produce such conditions. Cosmic strings have subsequently been ruled out as the dominant source of structure formation owing to their differing evolution upon exiting the horizon. The near-scale invariance predicted by SGC predicts a blue spectrum for gravitational waves, giving it a distinctive signature from typical models of inflation. We note however, that this work is not without criticism [37].

SGC continues to be an active field of research and provides a tantalizing view of the early universe. It dynamically predicts the number of large directions, explains the isotropization of space, stabilizes both the dilaton and moduli fields, explains the horizon and defect problems of the SBB, and now provides a mechanism to produce a scale-invariant power spectrum.

1.2.2 Brane World Scenario

The Brane World (BW) scenario is also a popular branch of research attempting to understand our early universe. Following the discovery of D-branes (higher-dimensional objects in strings theory [38]), the authors of [11] described a setup where our spatial dimensions are the surface of a 3-brane and the standard-model particles are interpreted as the endpoints of open strings attached to the brane. Despite the criticism that the stabilization of the internal directions is not achieved through some dynamical mechanism (*i.e.* the compactification is put in by hand and may represent a form of tuning), the BW paradigm provides a robust picture that hints at the explanation of many problems, including the weak hierarchy, the mechanisms of inflation, and the reheating (or preheating) of the universe. Figure 1–2 provides a pictorial view of this situation.

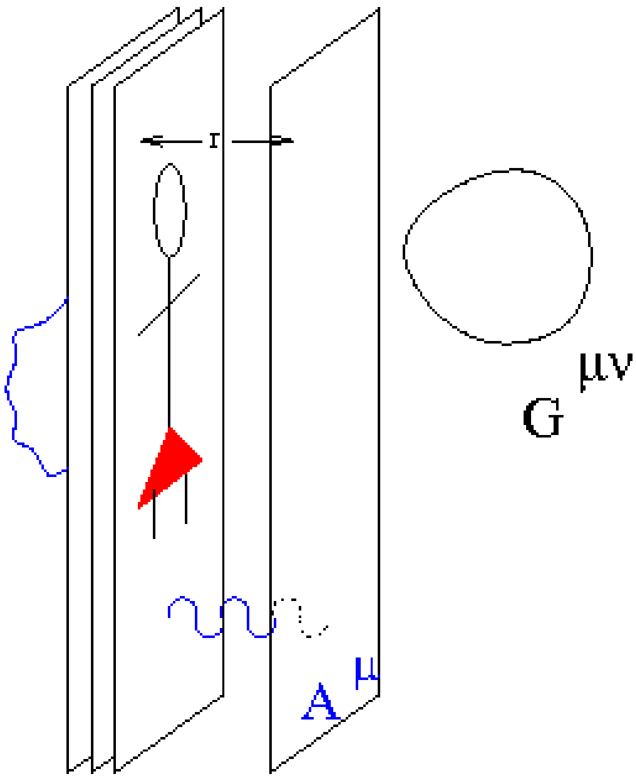

Figure 1–2: A diagram of the Brane World scenario. This generic construction includes a stack of D-branes (left) upon which open strings may end. The branes may be physically separated, in which case open string modes may end on different branes. This is the common setup for Brane inflation, where the brane-separation r behaves as the inflaton. Gravitational modes (right) are closed strings which are free to travel through bulk. Strings may end on different branes even if they are not separated.

The work addressed in Chapter 3 focuses on the process of reheating in the warped BW scenario; we now describe the inflationary mechanism in the brane world scenario to motivate the necessity of warped manifolds. This will give us a more generic, qualitative understanding of the system at hand. As originally pointed out in [10], the interaction energy between a parallel D3-brane and its corresponding antibrane can give rise to inflation in the early universe. The branes feel both a gravitational force (exchange of closed strings) and a gauge force; in the case of parallel branes these forces cancel and the branes feel no net force. For the brane-antibrane system these forces combine to provide an attractive force; in this case the separation r plays the role of the inflaton. As the branes approach each other the inflaton becomes tachyonic[6] at the critical separation $r < 1/M_s$ (the string length), thus ending inflation.

Unfortunately the resultant model of inflation brought in several problems unique to this setup. In particular, to get enough inflation we need the slow-roll parameters to be small; however, one finds that demanding $\eta \propto V''/V \ll 1$ gives inconsistent results. To see this, recall that the inflaton is associated with open string modes stretching between the $D3$ branes. The action is given by the Dirac-Born-Infeld (DBI) action

$$S = -\tau_3 \int d^4x \sqrt{-\det |G_{\mu\nu}|} \tag{1.17}$$

[6] In this context the word tachyonic is employed to indicate that $V''(r) < 0$, indicating that we are not expanding about the true vacuum. This does not imply faster-than-light propagation

where τ_3 is the brane tension and $G_{\mu\nu}$ is the induced metric on the brane

$$G_{\mu\nu} = g_{AB}\frac{\partial X^A}{\partial x^\mu}\frac{\partial X^B}{\partial x^\nu} = \eta^{\mu\nu} + \frac{\partial \phi^I}{\partial x^\mu}\frac{\partial \phi^I}{\partial x^\nu}\,. \tag{1.18}$$

Here, X^μ defines the embedding of the brane worldsheet into physical space, g_{ab} is the worldsheet metric, and ϕ^I are transverse oscillations to the brane. Upon expanding Equation (1.17) to linear order, the DBI action for a fluctuating brane takes the approximate form

$$S = -\tau_3 \int d^4x \left[1 - \frac{1}{2}\left(\frac{\partial \phi}{\partial x}\right)^2 + \cdots\right]\,. \tag{1.19}$$

In the brane-antibrane inflationary scenario, one imagines a $\overline{D}$ starting near a D-brane. Defining the brane-separation as $r^I = \phi^I - \bar{\phi}^I$, where $(\bar{\phi}^I)$ ϕ^I is the fluctuation of the (anti) brane, the Lagrangian obtains terms related to the brane separation

$$\mathcal{L} = -\frac{1}{2}\tau_3(\partial r)^2 - V(r)\,. \tag{1.20}$$

Finally, we recognize the canonically normalized inflaton as

$$\varphi = \sqrt{\tau_3}\, r = \sqrt{\tau_3}\left[\sum_I \left(r^I\right)^2\right]^{1/2}\,, \tag{1.21}$$

and the potential is found to be

$$V(\varphi) = 2\left(\tau_3 - \frac{\kappa_{10}^2\tau_3^4}{\varphi^4}\right)\,,\ \kappa_{10}^2 = M_{10}^{-8} = M_p^{-2}L^6 \tag{1.22}$$

with κ_{10} as the $10D$ Newton constant (related to the $4D$ Planck constant by the compactification volume L^6). For large field values of φ, the potential (1.22) is dominated by the brane tension, and the field may slowly evolve. The slow-roll constraint

$$\eta \equiv M_p^2 \frac{V''(\varphi)}{V} \simeq -\left(\frac{L}{r}\right)^6 \ll 1 \tag{1.23}$$

implies that the brane-antibrane system must be separated by distances (r) larger than the compactification length (L), or $r \gg L$ [39]. Obviously this cannot physically happen. Another drawback of this setup is the assumption of a stabilized volume modulus for the internal directions—the effects of such a mechanism may drastically change the inflaton potential $V(\varphi)$.

Fortunately some of the issues in this string-inflationary model were mended by incorporating background fluxes. In particular, Gidding, Kachru and Polchinksi [40] showed that fluxes in warped compactifications (such as the Klebanov-Strassler (KS) throat [41]), generically stabilize the dilaton and complex structure moduli of type IIB string theory compactified on a 6D Calabi-Yau (CY) manifold. In addition to moduli stabilization, the warped KS throat provides a mechanism for generating large hierarchies similar to the Randall-Sundrum model [4], and the scale of warping is generated from natural values of the quantized flux generating the background.

Repeating the brane-antibrane construction in this warped background, the authors of [43] (collectively known as KKLMMT) showed that the inflaton potential becomes

$$V(\varphi) = \frac{2a_o^4 \tau_3}{1 + a_o^4 \left(\varphi - \varphi_o\right)^{-4}} \simeq 2a_o^4 \tau_3 \left(1 - \frac{a_o^4}{\varphi^4}\right) \tag{1.24}$$

where $a_o^3 = e^{-2\pi K/g_s M} \ll 1$ is the amount of warping, set by the background flux quanta K and M. Now the constraint $\eta \simeq -\left(\frac{L}{r}\right)^6 a_0 \ll 1$ is easily satisfied due to the warp factor a_0. So, embedding the braneworld scenario in a warped background solves the η problem, providing a viable model of inflation in string theory. Embedding the scenario in a warped background also brings about the weak-hierarchy solution previously discussed.

The inflationary story in this model gets more complicated when one considers the dynamics of the overall volume (Kähler) modulus. This is the one modulus which is not stabilized by the fluxes. The interaction of T with the inflaton φ induces a mass for φ of the inflationary scale $m_\varphi^2 \simeq H^2$ and spoils the slow-roll conditions. Numerous solutions have been proposed to overcome the η problem; they generically involve modifying the superpotential W to cancel the unwanted positive contribution of m_φ^2. In ref. [42] it was assumed the Kähler modulus was stabilized by some nonperturbative contribution, and they showed this resulted in a de Sitter vacuum consistent with late-time cosmology. Later, ref. [43] parameterized an additional φ dependence to the superpotential, and showed the inflaton mass can be made sufficiently small for inflation by tuning the new parameter 1 part in 100—now the model can give both early and late-time inflation. Later, the corrections were explicitly calculated within string theory [44] and the corresponding corrections to the F-term potential were calculated by [45]. The result was that the corrections coming explicitly from string theory cannot help with the tuning of the inflaton potential for certain classes of embeddings.

The story of primordial inflation in brane-antibrane configurations of string theory proves to be somewhat tricky, but working models are known to exist. The working example in Ref. [46] envisions a system of two nearby throats together with a brane at the midpoint in the bulk CY. The brane is at an unstable maximum, but the potential may be tuned against the unwanted positive contribution of the Kähler modulus in order to, finally, satisfy $\eta \ll 1$. It should be pointed out

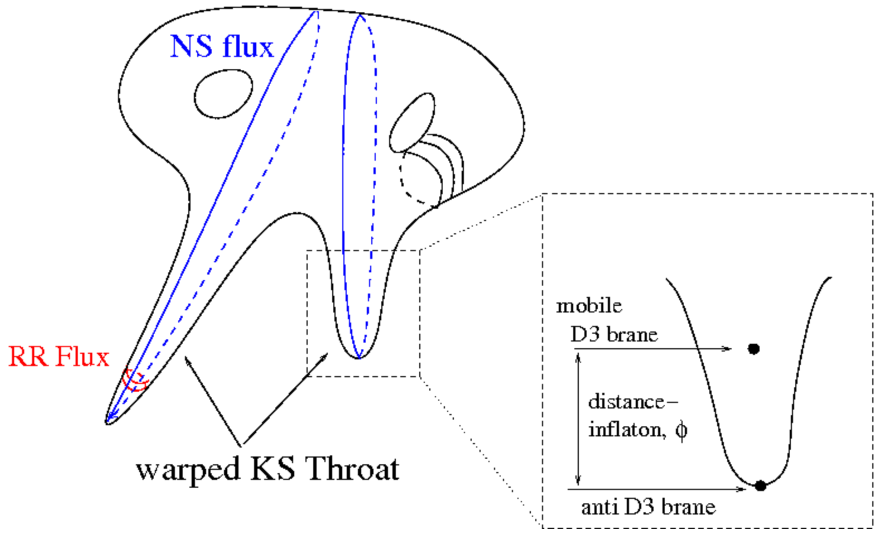

Figure 1–3: The brane inflation construction is based on a background of NS-NS and RR fluxes supporting multiple KS warped-throats. Inflation takes place at a higher scale, so is observationally constrained to shorter throats. A mobile $D3$ falls down the throat and annihilates with a $\overline{D3}$, thus ending inflation. Reheating subsequently happens, although the specifics depend on several factors, including the amount of warping, and the possibility of tunneling to different throats.

that this system does suffer from poorly-motivated initial conditions, and may seem unnatural in its construction. A concise review of the developments in this inflationary scenario is found in ref. [47]; for the purpose of this thesis we have introduced enough background to have a coherent picture of the scenario, and the reader should have a picture of the setup. A pictorial summary is provided in Figure 1–3; here, a generic compactification has a CY-bulk with warped regions caused by NS and RR fluxes in the background. The inflationary scale (set by observation through the COBE normalization) limits the amount of warping, so one imagines a background with numerous warped throats, with inflation happening in one throat and subsequent tunneling to a standard model throat [48].

The process of reheating in the (warped) BW scenario proceeds through a sequence of decays. Chapter 3 deals with the latter phase of this process, and the reader is referred to [49] for a comprehensive review. Regardless of the brane-antibrane construction providing inflation, it ends in a similar matter. So, the results we present on reheating should apply equally to each construction. A clear picture of the reheating process is provided in [49], and is reproduced here in Figure 1–4. Conventional models of inflation characterize reheating through the efficiency of inflaton decay into other degrees of freedom. In the brane-antibrane scenario the initial reheating process is extremely fast and effective, such that all inflaton energy is converted into heavy degrees of freedom in the form of closed strings [50, 51]: this is step 1 of Figure 1–4. This occurs when the inter-brane distance goes below the string length and the inflaton becomes tachyonic, signaling an instability in the system which quickly evolves. Next, the heavy closed strings decay into lighter degrees of freedom in the form of massless gravitational radiation plus its KK excitations (step 2 of Figure 1–4). BBN places constraints on the amount of gravitational radiation; however, this is not a concern in this scenario because the $4D$ coupling to KK modes is exponentially enhanced by the wavefunction overlap [48]. In contrast, the graviton is massless and constant in the bulk and does not contribute exponential warping to the decay vertex, resulting in a smaller branching fraction. As a small added complexity, in multi-throat constructions one must compare the tunnelling rate between throats with the decay rate within a throat.

With light KK states now populated, the corresponding metric fluctuations interact with the remaining $D3$ branes located at the bottom of the throat. Energy is deposited to open string modes on the brane through the KK-modes worldvolume coupling to the DBI term describing the brane (step 3 of Figure 1–4): this interaction primarily excites fluctuations of the brane. The final step to reheat the universe and produce SM particles is the transfer of the open string energy in the form of scalar fluctuations into fermions living on the brane. This interaction may proceed because the gauge theory associated with the brane contains fermions ψ in the adjoint representation. The scalar fluctuations Y interact with these fermions through the tri-linear coupling $\sqrt{g_s}\bar{\psi}Y\psi$ (step 4 of Figure 1–4).

For a detailed description of the reheating sequence, the reader is once again referred to [49]. Additionally, [49] pointed out a potential problem with the reheating sequence that had hitherto been ignored. In the first work exploring the decay of closed strings into SM particles, [48] showed that reheating into massive modes can be quite efficient because the radial wavefunction for massive Kaluza-Klein (KK) modes grows exponentially towards the infrared (IR) end of the throat. This efficient reheating means that inflation can occur in one throat, while the SM brane may reside in another throat. The observation of [49] was that KK modes may be excited not just radially (as in [48]), but also along the internal directions. Owing to the isometries of these directions in the throat, KK modes must decay through angular-momentum conserving vertices. This leaves the possibility of a long-lived relic which corresponds to the lightest state charged along the internal directions (LMCS) since it cannot decay into uncharged, massless states—there is a similar story in supersymmetry involving R-parity and the lightest supersymmetric state. A possible consequence is that new degrees of freedom, beyond the SM, will exist during nucleosynthesis, conflicting with BBN constraints. Chapter 3 quantifies the observations of [49]. We find that reheating can be suppressed if the isometries remain intact; fortunately the throat is attached to a CY region at the tip. The complicated geometry of the CY breaks the isometries in the throat which is expressed as a particular solution, or background shift. The particular solution can absorb the charge of the relic, thus catalyzing the decay. The operator accommodating the decay faces many restrictions since it must absorb the charge of the LMCS, it cannot break 4D Lorentz invariance, the deformation must not ruin the background, and breaking SUSY may be undesirable since it leads to stronger constraints. For the $T^{1,1}$ geometry of the KS throat, we identify a different operator than the one used in ref. [49], and find the lifetime of the possible relic is much less than the estimate therein. Generically, the study may be used to place strong constraints on fundamental parameters of the theory, or face conflicts with BBN and the SUSY-breaking scale.

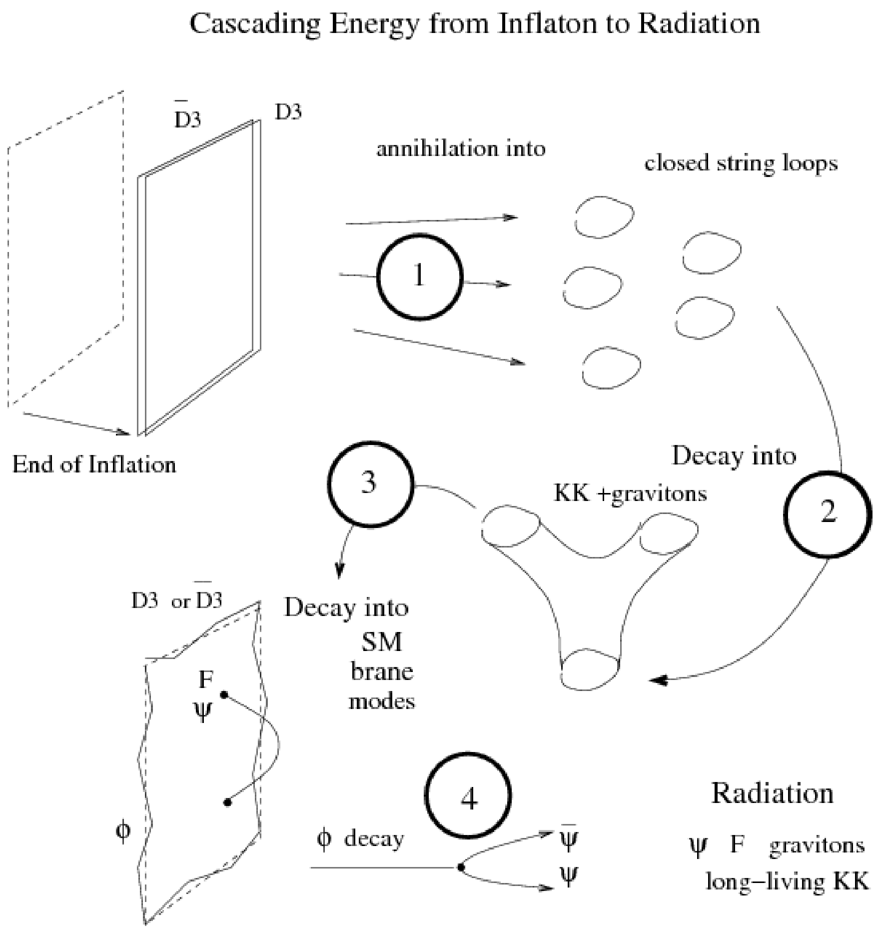

Figure 1–4: Described more thoroughly in the text, this diagram depicts the sequence of decays from the inflaton into radiation in the Brane World scenario. The figure was taken from ref. [49]

CHAPTER 2
Moduli stabilization in brane gas cosmology with superpotentials

Prediction is very difficult, especially if it's about the future.—Niels Bohr

FOREWORD: In this chapter we extend upon a previous publication of this author [32] concerning moduli stabilation in Brane Gas Cosmology. We look at the interplay between a gas of brane and superpotentials arising from flux stabilization scenarios. We find that the gas of branes generates an effective potential with a slowly moving minimum. This acts like a source of friction so that fields rest in the local minimum provided by the superpotential and branes, preventing moduli from overshooting to infinity.

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Abstract

In the context of brane gas cosmology in superstring theory, we show why it is impossible to simultaneously stabilize the dilaton and the radion with a general gas of strings (including massless modes) and D-branes. Although this requires invoking a different mechanism to stabilize these moduli fields, we find that the brane gas can still play a crucial role in the early universe in assisting moduli stabilization. We show that a modest energy density of specific types of brane gas can solve the overshoot problem that typically afflicts potentials arising from gaugino condensation.

2.1 Introduction

A major success of brane gas cosmology (BGC) is the utilization of stringy effects to explain the origin of the hierarchy of dimensions. In the seminal proposal of Brandenberger and Vafa [1], it was argued that in the early universe all directions could fluctuate about the self-dual radius due to the presence of both winding and momentum modes. The argument asserts that strings will generically intersect in (3+1)-dimensional subspaces, so that such a subspace will lose its winding and subsequently expand into the large directions we observe today. This scenario was mathematically realized by Tseytlin and Vafa in the context of dilaton gravity [2], and has since been extended to include the effects of a gas of Dp-branes, where Alexander, Easson, and Brandenberger [3] argued that such a gas would result in a hierarchy of extra dimensions. Namely the original 9-dimensional spatial manifold should decompactify into a hierarchical product space of $\mathcal{T}_4 \times \mathcal{T}_2 \times \mathcal{T}_3$.

Subsequent investigations indicate that wound strings provide a mechanism for isotropization [4] and stabilization [5] of the the compact dimensions, and that the mechanism works on toroidal orbifolds [6]. The framework for these results is usually the low energy effective action of type IIA string theory, where the salient differences from general relativity are a massless dilaton and the dynamics of extra dimensions. These differences lead to the result that negative pressure in the compact directions, due to wound strings, results in contraction, not acceleration. The dilaton is assumed to have no potential other than that which is induced by its coupling to the bulk string frame Lagrangian $e^{-2\phi}(R + (\nabla\phi)^2)$ and possible D-brane sources; most successes of BGC rely on the dynamical running of the dilaton toward weak string coupling, $g_s \ll 1$.

On the other hand one would like to stabilize the dilaton at a value where g_s is still large enough to be consistent with gauge coupling unification [7]. Moreover if g_s becomes too small, the interactions between strings become too weak to allow the annihilation of winding modes in three dimensions, where the space should be allowed to grow [8]. (See also [9] for a discussion similar issues.) Rather there is only a window of finely-tuned initial conditions consistent with three dimensions ultimately growing to be large. A third reason that the dilaton must not continue to roll to arbitrarily small values is the constraint from fifth force experiments and null searches for time variation of physical constants [10]; these preclude the dilaton from continuing to evolve at late times.

For these reasons it is imperative to reconcile brane gas cosmology with the stabilization of both the radion and the dilaton. In [11] it was shown that using just the string winding and momentum modes this is not possible. We therefore first investigate whether by including more general string and brane states one can achieve such a stabilization. (A similar but less general analysis has been done in [12].)

Our result, in the context of superstring theories, is that a general gas of D-branes and strings cannot stabilize both moduli, although they can stabilize one linear combination of them. However, the brane gas can still play an important role in the process of stabilizing both fields, due to the overshoot problem [13]. A much-studied mechanism for stabilizing the radion involves adding racetrack potentials coming from gaugino condensation (and possibly an antibrane [14]). The Minkowski minimum of these potentials is typically separated from a runaway (decompactification) direction by a very small barrier, which would always be overcome by the inertia of the fields if their initial conditions were not finely tuned to be close to the desired minimum. One of our main observations is that a gas of brane winding modes can very robustly solve this problem by slowing down the modulus as it rolls down its steep potential.

Our plan is as follows: in Section 2.2.1 we describe the BGC scenario and motivate the dimensional reduction procedure to obtain a d-dimensional theory of gravity with two scalar fields (the dilaton and radion), with an effective potential coming from the brane gas. In Section 2.2.3 we discuss some of the features of the effective potential; namely, we show that provided the dilaton is stabilized by some other mechanism, branes can stabilize all the extra dimensions. Section 2.3 presents our no-go theorem showing that under the given assumptions, there exists an unstabilized direction in the moduli space of the dilaton and radion no matter what modes are included in the gas of D-branes and strings. In particular we also show that the presence of massless F-string modes do not help in lifting the runaway direction. In section 2.4 we consider the combined effect of the brane gas with a superpotential, such as would arise from gaugino condensation and antibranes, and show that the brane gas can provide a remedy for the overshoot problem. We give our conclusions in section 2.5. Technical details are given in the appendices.

2.2 Effective Brane Gas Cosmology

2.2.1 Supergravity coupled to Strings and Brane Sources

A starting point for BGC is type IIA string theory compactified on a 9-dimensional toroidal background, which may be thought of as the result of compactifying M-theory on S^1. The low-energy bulk effective action of this theory is given by

$$S_{IIa} = \frac{1}{2\kappa^2} \int d^D x \sqrt{-G}\, e^{-2\phi} \left(R + 4G^{MN} \nabla_M \phi \nabla_N \phi - \frac{1}{12} H_{\mu\nu\alpha} H^{\mu\nu\alpha} \right) \,, \qquad (2.1)$$

where G is the determinant of the ten-dimensional background metric $G_{\mu\nu}$, ϕ is the dilaton, H is the field strength corresponding to the bulk antisymmetric tensor field $B_{\mu\nu}$, and κ is the D-dimensional Newton's constant. For simplicity we ignore any flux contributions, and take $H = 0$. We envision this analysis to apply in the late-time era of BGC [1, 3, 6, 15], an epoch where the extra, compact dimensions are expected to be isotropized [4], and winding modes in the large directions have annihilated. Thus, we consider a spacetime consisting of a flat, d-dimensional FRW universe, and an isotropic compact subspace of n extra dimensions

$$\begin{aligned} ds^2 &= G_{MN} dX^M dX^N = g_{\mu\nu} dx^\mu dx^\nu + b^2(t)\, \gamma_{mn} dy^m dy^n & (2.2) \\ &= -dt^2 + a^2(t)\, dx_i dx^i + b^2(t)\, dy_m dy^m, \; i \in \{1, \ldots, d\}, m \in \{1, \ldots, n\} & (2.3) \end{aligned}$$

where y^m are the coordinates of the n extra dimensions. The total action comprises the above bulk action (2.1) and the action of all matter present. Sources are included by adding matter terms for both the strings (ρ_s) and Dp-branes (ρ_p). Owing to the different world-sheet couplings between the dilaton and the branes and strings, the matter action has the form

$$\begin{aligned} S_m &= -\int d^D x \sqrt{-G} \left(\rho_s + e^{-\phi} \rho_p \right) & (2.4) \\ T_{MN} &= -\frac{2}{\sqrt{-G}} \frac{\delta S_m}{\delta G^{MN}} \,. & (2.5) \end{aligned}$$

We continue the construction of late-time BGC by considering separate species of strings and branes, each possibly having excited momentum (in the case of branes also known as "vibrational modes" [16]) in the large or compact subspaces, but having winding modes only along the compact directions. Then one can show (see appendix 2.6) that the stress energy tensor for the strings and branes simplifies to

$$-T^0_0 = \rho_s + e^{-\phi}\rho_p = \sum_i \left[\rho_i e^{-\alpha_i\phi} a^{-d(1+\omega_i)} b^{-n(1+\hat{\omega}_i)}\right] \tag{2.6}$$

$$T^a_b = P\,\delta^a_b = \sum_i \omega_i \left[\rho_i e^{-\alpha_i\phi} a^{-d(1+\omega_i)} b^{-n(1+\hat{\omega}_i)}\right] \delta^a_b \tag{2.7}$$

$$T^m_n = p\,\delta^m_n = \sum_i \hat{\omega}_i \left[\rho_i e^{-\alpha_i\phi} a^{-d(1+\omega_i)} b^{-n(1+\hat{\omega}_i)}\right] \delta^m_n . \tag{2.8}$$

In the preceding expressions the summation is performed over the relevant modes contributing to the gas of strings and branes, P and p being the sum-total pressure along the large and compact directions respectively. $\alpha_i = 0$ for string sources, $\alpha_i = 1$ for brane sources, and ρ_i is the initial energy density for a particular mode, with effective equation of state $p_i = \hat{\omega}_i\rho_i$, $P_i = \omega_i\rho_i$. The values of ω and $\hat{\omega}$ depend on the specific type of mode, dimensionality of the branes and the number of large and extra dimensions (see table 2–1), but the important thing is that all the known modes can be described by these quantities.

Variation of the action (2.1) together with the matter action (2.4) and metric ansatz (2.3) results in the system of equations

$$-d\left(\frac{\dot{a}(t)}{a(t)}\right)^2 - n\left(\frac{\dot{b}(t)}{b(t)}\right)^2 + \dot{\varphi}^2 = e^{\varphi}E \tag{2.9}$$

$$\frac{d}{dt}\left(\frac{\dot{a}(t)}{a(t)}\right) - \dot{\varphi}\frac{\dot{a}(t)}{a(t)} = \tfrac{1}{2}e^{\varphi}P \tag{2.10}$$

$$\frac{d}{dt}\left(\frac{\dot{b}(t)}{b(t)}\right) - \dot{\varphi}\frac{\dot{b}(t)}{b(t)} = \tfrac{1}{2}e^{\varphi}p \tag{2.11}$$

$$\ddot{\varphi} - d\left(\frac{\dot{a}(t)}{a(t)}\right)^2 - n\left(\frac{\dot{b}(t)}{b(t)}\right)^2 = \tfrac{1}{2}e^{\varphi}E\ , \tag{2.12}$$

where we have introduced the shifted dilaton as $\varphi \equiv 2\phi - d\frac{\dot{a}(t)}{a(t)} - n\frac{\dot{b}(t)}{b(t)}$ (recall that $d = 3$ and $n = 6$), and a dot denotes differentiation with respect to time. Eqs. (2.9-2.12) are the string frame, or dilaton-gravity, equations of motion. Equation (2.9) is the 0-0 Einstein equation; notice that in the string frame the kinetic term for the (shifted) dilaton contributes to the energy with apparently the wrong sign—this is due to the nonminimal coupling between the Ricci Scalar and the dilaton. The spatial components of the Einstein equations (2.10-2.11) show that the acceleration of the scale factor is proportional to the pressure, and thus the negative-pressure winding modes lead to contraction—this is the key ingredient of the Brandenberger-Vafa mechanism. Eq. (2.12) is the dilaton equation of motion. Equation (2.9) is not dynamical, but is rather an equation of constraint, which can be used to determine the initial dilaton velocity to be

$$\dot{\varphi} = \pm\sqrt{e^{\varphi}E + d\left(\frac{\dot{a}(t)}{a(t)}\right)^2 + n\left(\frac{\dot{b}(t)}{b(t)}\right)^2}\,. \tag{2.13}$$

It is customary to choose the negative solution since the string coupling $g_s = e^{\phi}$ then evolves toward weak coupling, where a perturbative description is valid. Since all the terms under the square root are positive, the dilaton cannot bounce.

The rolling of the dilaton, although important for the BV mechanism and the stabilization of the moduli fields, may also be deleterious to the BGC scenario. In [8], Easther, Greene, Jackson, and Kabat show that if the dilaton rolls too quickly, winding-mode annihilation may be suppressed, so that dynamical evolution leading to three large spatial dimensions is not favoured. The rolling of the dilaton also implies evolution of volume of the compact space. A conformal transformation on the metric may absorb the ϕ-R coupling term, but this means the Einstein frame scale factors get additional time dependence from $\phi(t)$. This problem has typically been set aside (on the assumption that the dilaton will be stabilized at a later time) in discussions of stabilization of the extra dimensions [5]. However, in order to have a complete and consistent picture in the framework of brane gas cosmology one indeed needs to address the issue of stabilizing the dilaton along with radion stabilization, and this is what we devote the next few subsections to.

The preceding observations also stress the utility of viewing gravity from the point of view of the four-dimensional Einstein frame, which is more intuitive than the 10D string frame. An effective four-dimensional action is achieved by conformally absorbing the dilaton, integrating out the extra dimensions, and performing a second conformal transformation to absorb the scale factor of the extra dimensions. The result is a minimally-coupled theory of BGC, where the original string and brane sources act as an effective potential for both the radion and dilaton fields. This approach was first advocated in [17] to study stabilization of extra dimensions in the presence of hydrodynamical fluids and was used to study string winding and momentum modes in [11]. We now generalize the analysis to include all possible string and brane sources.

2.2.2 Effective Potential

Upon performing dimensional reduction on both the string and brane sources, we obtain general relativity coupled to two scalar fields, the dilaton and radion, with an effective potential coming from the brane gas. As outlined in Appendix 2.6, a string/brane source whose energy density behaves as $\rho = \rho_i e^{-\alpha_i \phi} a^{-d(1+\omega_i)} b^{-n(1+\hat{\omega}_i)}$, with equations of state ω_i and $\hat{\omega}_i$ in the d large and n compact directions respectively, provides an effective potential in $d+1$ dimensions

$$\begin{aligned} V_{\text{eff},i} &= \rho_i \, e^{2\nu_i \psi} \, e^{2\mu_i \varphi} \, \bar{a}^{-d(1+\omega_i)} \\ \nu_i &= \frac{1}{2}\left(-\hat{\omega}_i + \frac{d}{d-1}\left(\omega_i - \frac{1}{d}\right)\right)\sqrt{\frac{(d-1)n}{(d+n-1)}} \\ \mu_i &= \frac{1}{2}\left(-d\omega_i - n\hat{\omega}_i + 1 - \alpha_i \frac{d+n-1}{2}\right)\sqrt{\frac{1}{d+n-1}} \end{aligned} \tag{2.14}$$

This is expressed in terms of the canonically normalized moduli ψ and dilaton φ fields, and the Einstein-frame scale factor $\bar{a}$ of the d large directions. α_i parametrizes string ($\alpha_i = 0$) or brane ($\alpha_i = 1$) contributions, and ρ_i is the initial energy density of the ith component of the brane gas. We work in Planck units, $M_{pl}^{-2} = 8\pi G_N = 1$. The net effective potential will comprise several contributions of the form (2.14), depending on the type of excited modes; Appendix 2.6 discusses the equations of state, and the coefficients μ_i, ν_i for the various string and brane sources and the results are summarized in Table 1.

2.2.3 Radion Stabilization

To understand the effects of string and brane sources in late-time BGC, we now specialize to the case of three large directions ($d = 3$) with winding modes only in the compact dimensions. First suppose that brane sources are not present, so the effective potential for the system is given by contributions from strings alone—this emulates the setup of [1, 5, 11, 15, 18]. Three representative species of strings are considered, namely, W: strings with winding numbers in the compact direction ($\omega = 0$, $\hat{\omega} = -\frac{1}{n}$), M_6: momentum excitations in the compact directions ($\omega = 0$, $\hat{\omega} = \frac{1}{n}$), and M_3: momentum in the large directions ($\omega = \frac{1}{d}$, $\hat{\omega} = 0$). Summing

Table 2–1: A summary of the radion and dilaton couplings in the effective potential due to various species of gas. The spatial background consists of d large and n compact directions, with equations of states ω and $\hat{\omega}$ respectively.

source	$E \propto a^{-d\omega}b^{-n\hat{\omega}}$	ω	$\hat{\omega}$	μ_i	ν_i
			$d=3$		
general string	$a^{-d\omega}b^{-n\hat{\omega}}$	ω	$\hat{\omega}$	$\frac{-3\omega-n\hat{\omega}+1}{\sqrt{4(n+2)}}$	$-\frac{\left(\hat{\omega}+\frac{1}{2}(1-3\omega)\right)\sqrt{n}}{\sqrt{2(n+2)}}$
general brane	$a^{-d\omega}b^{-n\hat{\omega}}$	ω	$\hat{\omega}$	$-\frac{3\omega+n\hat{\omega}+\frac{n}{2}}{\sqrt{4(n+2)}}$	$-\frac{\left(\hat{\omega}+\frac{1}{2}(1-3\omega)\right)\sqrt{n}}{\sqrt{2(n+2)}}$
wound string	a^0b^1	0	$\frac{-1}{n}$	$\frac{1}{\sqrt{n+2}}$	$\frac{1-\frac{n}{2}}{\sqrt{2n(n+2)}}$
wound brane	a^0b^p	0	$\frac{-p}{n}$	$\frac{(p-\frac{n}{2})}{2\sqrt{n+2}}$	$\frac{p-\frac{n}{2}}{\sqrt{2n(n+2)}}$
string momentum	a^0b^{-1}	0	$\frac{1}{n}$	0	$-\sqrt{\frac{n+2}{8n}}$
brane momentum	a^0b^{-1}	0	$\frac{1}{n}$	$-\frac{1}{4}\sqrt{n+2}$	$-\sqrt{\frac{n+2}{8n}}$
			$d=3,\ n=6$		
wound string	a^0b^1	0	$\frac{-1}{6}$	$\frac{1}{\sqrt{8}}$	$-\frac{1}{2\sqrt{6}}$
wound brane	a^0b^p	0	$\frac{-p}{n}$	$\frac{p-3}{2\sqrt{8}}$	$\frac{p-3}{4\sqrt{6}}$
string momentum	a^0b^{-1}	0	$\frac{1}{n}$	0	$\frac{-1}{\sqrt{6}}$
brane momentum	a^0b^{-1}	0	$\frac{1}{n}$	$-\frac{1}{\sqrt{2}}$	$\frac{-1}{\sqrt{6}}$

contributions (2.14), we obtain

$$V_s(\bar{a},\varphi,\psi) \;=\; \rho_W e^{(1-\frac{n}{2})\sqrt{B}\psi} e^{\sqrt{\frac{A}{2}}\varphi}\bar{a}^{-3} + \rho_{M_6} e^{-(1+\frac{n}{2})\sqrt{B}\psi}\bar{a}^{-3} + \rho_{M_3}\bar{a}^{-4}\ , \quad (2.15)$$

where ρ_W, ρ_{M_3}, ρ_{M_6} parametrize the initial energy densities of the three kinds of components, $B = \frac{2}{n(n+2)}$, and $A = \frac{2}{n+2}$. Let us assume that the dilaton has been stabilized by an external potential and consider the effect of the string gases on the unstabilized radion. Taking the dilaton VEV to be $\phi = 0$ and ignoring the M_3 momentum modes, which anyway gets redshifted by the expansion of the universe, the string gas effective potential (2.15) becomes

$$V_s(\bar{a},\psi) \;=\; \bar{a}^{-3}\left[\rho_N e^{(1-\frac{n}{2})\sqrt{B}\psi} + \rho_M e^{-(1+\frac{n}{2})\sqrt{B}\psi}\right]\ . \quad (2.16)$$

Battefeld and Watson point out [11] that this is a stable potential for ψ only if the number of extra dimensions is $n = 1$, in which case it reduces to $V(\bar{a}, \psi = 0) \sim \frac{1}{\bar{a}^3}$. This can be considered a source of dark matter, similar to the string-inspired example of Gubser and Peebles [19]. However, in the case of $n > 2$, [11] points out that the effective potential behaves as $V(\bar{a}, \psi) \sim e^{-a\psi}/\bar{a}^3$, so that the radion also runs away to ∞. Since $n = 6$, one sees that the presence of strings cannot stabilize the dilaton or the radion.

We note that in [20, 21] massless string states were invoked to obtain stabilization of the moduli. However, the former are not present in the type II string (being removed by the GSO projection). Although, they are present at the self-dual radius in the heterotic string,[1] additionally, [21] requires quantized modes of the D-string to achieve complete stabilization of all moduli. It is not clear to us that the D-string can be quantized in the same way as the fundamental string.

Let us therefore consider whether extending the analysis of [11] to the case of general brane sources can solve the problem of moduli stabilization. Consider the contributions to $V_{\rm eff}$ coming from p-branes wrapping the compact dimensions ($\omega = 0$, $\hat{\omega} = -\frac{p}{n}$, denoted $\tilde{N}$), and momentum modes in the compact dimensions ($\omega = 0$, $\hat{\omega} = \frac{1}{n}$, denoted $\tilde{M}$). The net effective potential from equation (2.14) is

$$V_p(\bar{a}, \varphi, \psi) \;=\; \frac{1}{\bar{a}^3}\left[\rho_{\tilde{N}} e^{(p-\frac{n}{2})\sqrt{B}\psi} e^{(p-\frac{n}{2})\sqrt{\frac{A}{2}}\varphi} + \rho_{\tilde{M}} e^{-(1+\frac{n}{2})\sqrt{B}\psi} e^{-(1+\frac{n}{2})\sqrt{\frac{A}{2}}\varphi}\right] \tag{2.17}$$

This scenario is similar to those analyzed in [3, 4, 12, 16, 18]. As we will now explore, the result of including a gas of branes is the improved stability of the radion. Inspection of (2.17) reveals that provided $p > \frac{n}{2}$, all internal directions will

[1] We thank Subodh Patil for discussions on this point.

be stabilized, since there are both rising and falling exponentials depending on ψ:

$$V_p(\bar{a},\psi) \;=\; \rho_{\tilde{N}} e^{(p-\frac{n}{2})\sqrt{B}\psi}\bar{a}^{-3} + \rho_{\tilde{M}} e^{-(1+\frac{n}{2})\sqrt{B}\psi}\bar{a}^{-3} \;. \tag{2.18}$$

where again we have assumed $\varphi = 0$. This has a nontrivial minimum close to $\psi = 0$ provided that $p > \frac{n}{2}$. Since string theory requires $n = 6$, the presence of $(p > 3)$-branes in the compact directions will stabilize the moduli.

However a more detailed analysis may be necessary to realize these stability conditions: According to the heuristic argument of Alexander, Brandenberger, and Easson [3], winding modes will generically intersect in $2p + 1$ dimensions, so that only objects with $p \leq 2$ should remain wound in the 6 compact directions. In this case, the stability requirement will not be satisfied. On the other hand, a quantitative investigation should account for the larger phase space once the $\bar{a}$ directions have grown large, thus decreasing the probability of annihilation, and perhaps leaving some extended objects with $p > \frac{n}{2}$. As well, such an analysis should be carried out within the product space $\mathcal{T}_4 \times \mathcal{T}_2 \times \mathcal{T}_3$ argued for by [3], not the $\mathcal{T}_n \times \mathcal{T}_3$ topology we have considered. We do note though that, as long as the shape moduli is frozen, the exact shape of the tori does not matter and our stability analysis for the volume still applies.

2.3 No-Go Result in Type II String Theory

In the previous section we saw that the radion can be stabilized by a brane gas when the dilaton is assumed to be fixed; similarly one can show the corresponding result when the roles of the dilaton and radion are interchanged. One may naturally wonder whether both of these moduli can be simultaneously stabilized using the most general combination of string and brane sources. As we now show, this is impossible to do with the conventional (winding, momentum or oscillator) string and brane excitations. The argument is made in two steps, starting first with gases where each string or brane has only one kind of excitation ("simple states"), although different species of strings or branes are allowed to co-exist. We then extend the argument to the more general case where individual components of the gas have more than one kind of excitation ("mixed states").

2.3.1 "Simple States"

We consider the situation when the strings/branes have nontrivial wrapping of only some extra dimensions, i.e. it doesn't wrap the large dimensions. They thus appear point-like to 4D observers and redshift like nonrelativistic dust, $\bar{a}^{-3}$, corresponding to $d = 3$, $\omega = 0$ in (2.14). The equations of motion for the radion and the dilaton in the presence of such sources are

$$\ddot{\varphi} + 3H\dot{\varphi} = -\frac{\partial V_{\text{eff}}(\varphi, \psi)}{\partial \varphi} \tag{2.19}$$

$$\ddot{\psi} + 3H\dot{\psi} = -\frac{\partial V_{\text{eff}}(\varphi, \psi)}{\partial \psi} \tag{2.20}$$

with

$$V_{\text{eff}}(\varphi, \psi) = \bar{a}^{-3} \sum_i \rho_i e^{2\mu_i \varphi + 2\nu_i \psi} \tag{2.21}$$

where the sum runs over all possible string and brane states. As summarized in Appendix 2.6, the exponents μ_i, ν_i depend only on the effective equation of state parameter $\hat{\omega}$ along the extra dimensions (2.14), and the coupling exponent α of these states to the dilaton in the string frame:

$$S_{\text{gas},i} = -\int d^D x \; \sqrt{-g}\, e^{-\alpha_i \phi}\, \rho_i\, b^{-n(1+\hat{\omega}_i)}\, a^{-3} \; . \tag{2.22}$$

where $\alpha = 0$, for the fundamental strings since the Nambu-Goto action does not contain any dilaton coupling, while for branes $\alpha = 1$, originating from the dilaton coupling in the DBI action.

After performing the conformal transformations involving the radion and the dilaton (see Appendix 2.6) the above action gives rise to the effective potential (2.14) in the 4D Einstein frame with

$$\mu_i = \frac{1}{2\sqrt{n+2}}\left[1 - \alpha_i - n(\hat{\omega}_i + \frac{\alpha_i}{2})\right]; \quad \nu_i = -\sqrt{\frac{n}{2(n+2)}}\left(\hat{\omega}_i + \frac{1}{2}\right) \tag{2.23}$$

As noted earlier, the value of $\hat{\omega}$ depends upon whether the mode in question has winding, momentum or string oscillations. To analyze the stability of a potential which is a sum over such modes, we will use the technique of [22]: we identify the directions in the φ-ψ plane in which there exists a rising exponential contribution. If such directions are sufficiently numerous, the system is completely stabilized. An exponential of the form $\rho_i e^{2\mu_i\varphi + 2\nu_i\psi}$ rises most steeply along the direction $\cos\theta_i\hat{\varphi} + \sin\theta_i\hat{\psi}$ where $\tan\theta_i = \nu_i/\mu_i$. In the range

$$\theta = (\theta_i - \pi/2, \theta_i + \pi/2) \tag{2.24}$$

there is a rising potential (wall) while along the other half-plane the potential asymptotically falls to zero. Since our potential is a sum of exponentials, it is clear that:

(I) There can be at most a single local minimum and no local maxima.

(II) Such a minimum exists only if the potential grows in any direction away from the minimum. Thus there must exist angles θ_i for which the ranges of angles in (2.24) cover the entire plane.

By looking at the different directions of steepest ascent of the exponentials it is easy to verify whether (II) is satisfied. Curiously, for brane sources ($\alpha_i = 1$) we find a result that is specific to $d = 3$ large dimensions: the direction of steepest ascent is the same (modulo π) for winding, momentum or any other modes. For the winding modes, it is given by

$$\tan\theta = \frac{\nu_i}{\mu_i} = \sqrt{\frac{2}{n}} \quad \Rightarrow \quad \theta = \frac{\pi}{6} + \begin{cases} 0, & p > 3 \\ \pi, & p < 3 \end{cases} \tag{2.25}$$

while for momentum modes

$$\theta = \frac{\pi}{6} \tag{2.26}$$

for all p, as illustrated in figure 2–1. Thus using, say, a gas of 2- and 6-branes in type IIA theory, or a gas of 1 and 5-branes in IIB theory, one could stabilize all directions of the φ-ψ plane except for those orthogonal to the direction of steepest ascents, that is, $\theta = \frac{\pi}{6} \pm \frac{\pi}{2}$ (see figure). Along these directions the potential is flat, and there is a zero mode.

To lift the flat direction, one has to incorporate fundamental string sources, with $\alpha_i = 0$. According to Table 2–1 for F-strings, the angle

$$\tan\theta = \frac{\nu_i}{\mu_i} = \sqrt{\frac{2}{n}}\,\frac{\hat\omega_i + \frac{1}{2}}{\hat\omega_i + \frac{1}{n}} \tag{2.27}$$

depends on $\hat\omega_i$, and so the analysis has to be done separately for each case. For the string momentum, oscillatory and winding modes, $\hat\omega = 1/n,\ 0$ and $-1/n$ respectively. From (2.23) one then finds

$$\theta_{\rm mom} = -\frac{\pi}{2}, \quad \theta_{\rm osc} = -\frac{\pi}{3}, \quad \theta_{\rm wind} = -\frac{\pi}{6} \tag{2.28}$$

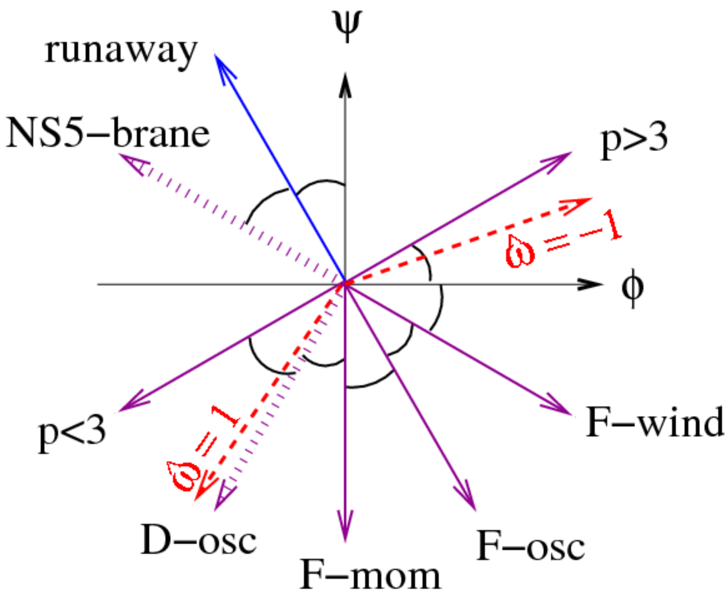

Figure 2–1: Directions of steepest ascent in the ϕ-ψ plane for contributions from different brane gas sources, described in the text. Long dashed lines are for the hypothetical F-string sources with $\hat{\omega} = \pm 1$. All marked angles are 30°. Short dashed lines are for the sources described in section 2.3.3.

All of these modes provide an ascending potential in the direction of $\theta = -\frac{\pi}{3}$, which is the flat direction when only brane sources are present, but not along the opposite direction (see figure 2–1). Thus after including the string modes, there is no longer a zero mode, but there is a runaway direction along $\theta_{\rm run} = \frac{2\pi}{3}$.

In fact we can make an even stronger statement. Suppose that there are other string sources we may be unaware of; nevertheless their equations of state should satisfy the weak energy condition $-1 \le \hat{\omega} \le 1$. Using the Table 1 entries for general string sources and varying $\hat{\omega}$ over this range gives an angle of steepest ascent in the range

$$\pi - \tan^{-1}\frac{3\sqrt{6}}{5} \le \theta \le \tan^{-1}\frac{\sqrt{6}}{7} \quad \rightarrow \quad -124^\circ \lesssim \theta \lesssim 19^\circ \tag{2.29}$$

which again fails to lift the $\theta_{\rm run}$ direction, as shown in figure 2–1.

An alternative understanding of the moduli instability can be directly inferred by a reparametrization of the effective potential (2.14) in terms of new fields $\chi = \sqrt{B}\psi + \sqrt{\frac{A}{2}}\varphi$ and $\eta = \sqrt{\frac{A}{2}}\psi - \sqrt{B}\varphi$. The result for an arbitrary string and brane gas with three large directions and $\omega = 0$ is

$$V_{\text{eff},i}(\alpha = 0) \;=\; \rho_i \, e^{\left(-n(\hat{\omega}+\frac{1}{4})+\frac{1}{2}\right)\chi} \, e^{-(\frac{n}{2}+1)\frac{\eta}{2}} \, \bar{a}^{-3} \text{ (string)} \tag{2.30}$$

$$V_{\text{eff},i}(\alpha = 1) \;=\; \rho_i \, e^{-n(\hat{\omega}_i+\frac{1}{2})\chi} \, \bar{a}^{-3} \text{ (brane)} \tag{2.31}$$

Through a combination of sources it is possible to to stabilize the χ mode; however string sources will only cause η to grow, and a brane gas does not couple to η. Thus $\hat{\eta} = \sqrt{\frac{A}{2}}\hat{\psi} - \sqrt{B}\hat{\varphi}$ is the unstable direction in field space, in terms of the unit vectors $\hat{\varphi}$, $\hat{\psi}$. This direction corresponds to the line

$$\psi = -\sqrt{\frac{A}{2B}}\,\varphi = -\sqrt{\frac{n}{2}}\,\varphi \;, \tag{2.32}$$

which coincides precisely with the principal runaway direction identified previously.

The above result is consistent with ref. [18], which used a perturbation analysis in the BGC scenario to show that the inclusion of branes alone is not enough to stabilize both the dilaton and moduli fields. Thus some other potential is needed to stabilize one of the moduli. Given such a potential, BGC does provide a mechanism of stabilizing the other degree of freedom provided that branes with $p > \frac{n}{2}$ are present.

The factorization of the effective potential is a coincidence of having $d = 3$ large dimensions, as can be seen from the nontrivial dependence on d in eq. (2.14). For scenarios other than $d = 3$, the gas of strings and branes is able to stabilize both fields.

Finally, we note that in the above analysis we did not consider branes or strings which wrap some of the large three dimensions ($\omega < 0$); these do not give any additional leverage for stabilizing the radion.

2.3.2 "Mixed States" and Massless Modes

So far we have only considered states which are purely oscillatory, winding or momentum modes. More generally, strings could have a combination of such excitations. Can such mixed modes help in stabilizing the moduli? The answer, unfortunately is no. For massive modes the reasons are similar to the case of the simple states. The massless modes[2] have to be analyzed separately but they do not alter the conclusions.

In the string frame, the string spectrum is

$$m_F^2 = \frac{m^2}{b^2} + N_{\rm osc} M_s^2 + w^2 b^2 M_s^4 \tag{2.33}$$

where M_s is the string scale and the three terms on the right hand side correspond to the momentum, oscillatory and winding pieces respectively. The source action for the strings (2.22) then becomes

$$S_{\rm str} = \int d^D x \, \sqrt{-g}\, a^{-3}\, b^{-n} \, \sqrt{m_F^2(b) + p^2} \tag{2.34}$$

where p is the momentum along the non-compact directions. After performing the dimensional reduction and conformal redefinitions, as usual we find that the effective potential for the canonical radion and dilaton coming from a gas of such

[2] Although, in the type II theory these are excitations of the tachyon which are removed by the GSO projection, such massless winding states are allowed in the heterotic string theory.

states is given by

$$V_{\mathtt{eff}}(\varphi,\psi) = \rho = nE(\varphi,\psi) \tag{2.35}$$

where n is the number density, and $E(\psi,\phi)$ is the energy of these states which depends on both the dilaton and the radion. Since we already know how the exponents look like for individual momentum, winding and oscillatory modes, it is easy to see that for the more general case the energy is just given by

$$\begin{aligned} E(\psi,\varphi) &= \sqrt{M_s^2[m^2 e^{4(\mu_m\varphi+\nu_m\psi)} + w^2 e^{4(\mu_w\varphi+\nu_w\psi)} + N_{osc}e^{4(\mu_o\varphi+\nu_o\psi)}] + p^2} \\ &\equiv \sqrt{m_F^2 + p^2} \end{aligned} \tag{2.36}$$

with

$$\mu_m = 0\ ;\ \mu_o = \frac{1}{2\sqrt{8}} \text{ and } \mu_w = \frac{1}{\sqrt{8}}$$

and

$$\nu_m = -\frac{1}{\sqrt{6}}\ ;\ \nu_o = \frac{\sqrt{3}}{2\sqrt{8}} \text{ and } \nu_w = -\frac{1}{2\sqrt{6}} \tag{2.37}$$

First let us focus on the massive modes, for which one can ignore the momentum p. The key observation is that since $N_{\rm osc} \geq 0$ for massive modes, the effective potential obtained in the Einstein frame must still satisfy conditions (I) and (II) above in order to have a local minimum. Since again such potentials can have at most one minimum and no maximum, if there exists a minimum, the potential has to keep rising along any direction as one tends towards infinity. Thus to determine whether there is a minimum, it suffices to investigate the behaviour at infinity in the φ-ψ plane. Going far enough toward ∞ along a generic direction, one of the three terms in (2.33) will dominate, and then our previous analysis applies, which assumed the presence of only one term in a given source. In the special direction where b remains constant, no one term dominates, but they all remain proportional to each other, behaving like a single term, so again the previous analysis remains valid.

Next let us focus on the massless modes, for example, the ones considered in [21]. In this case $N_{\rm osc} < 0$; and depending on the winding and momentum quantum numbers one could have $m_F^2 \sim (m/b - wbM_s^2)^2$ leading to a minimum at $b^2 = (m/w)M_s^{-2}$ (which is the self-dual radius when $m/w = 1$) [21]. For these modes one can easily verify that the mass function can be cast as

$$m_F^2(\varphi,\psi) \sim e^{\sqrt{2}\varphi'}\left(e^{\frac{\psi'}{\sqrt{6}}} - e^{-\frac{\psi'}{\sqrt{6}}}\right)^2 \tag{2.38}$$

where

$$\psi' = \frac{\sqrt{3}}{2}\varphi + \frac{1}{2}\psi \tag{2.39}$$

(as one can find by carefully tracing back the conformal transformations) is really the string frame radion and

$$\varphi' = \frac{1}{2}\varphi - \frac{\sqrt{3}}{2}\psi \tag{2.40}$$

is the orthogonal direction. As one can see, the mass (2.38) and the potential have a minimum at $\psi' = 0$ and hence the massless states stabilize the ψ' direction, as argued in [21]. However, ψ' also precisely coincides with the direction that could be fixed just with winding branes. Thus we are still left with the orthogonal runaway direction ($\varphi' \to -\infty$) that we found earlier.

2.3.3 Exotic States

We have seen so far that ordinary D-brane and string states are unable to stabilize both the radion and the dilaton simultaneously. We now briefly discuss how stabilization might be achieved using some less conventional kinds of branes.

One kind of exotic state which has been considered [21] are quantized D-string modes. Whether it is justified to derive these from the Nambu-Goto action like for F-strings seems doubtful, since the D-string is a solitonic object, but for completeness we have derived the exponents corresponding to the different D-string modes[3] . Although oscillator excitations do provide a new direction in field space whose potential has a steep direction, this direction overlaps with ones from other more conventional sources, and do not affect our no-go result. The direction of steepest ascent for the D-string oscillator modes is derived in Appendix 2.6, and is shown in figure 2–1. However if massless D-string modes are also allowed in the string theory spectrum, they can lift the runaway direction in conjunction with other modes, as has been argued in [21].

Another possible source that provides an effective potential is the NS5-brane.[4] Its tension behaves as $T_5^F \propto g_s^{-2}$, so an NS5-brane wrapping the internal manifold corresponds to $\alpha = 2,\ \hat{\omega} = \frac{-5}{n},\ \omega = 0$; this results in the coupling coefficients

$$\mu_5^F = -(\frac{n}{2} - 2)\frac{1}{\sqrt{n+2}} \text{ and } \nu_5^F = (10-n)\sqrt{\frac{2}{n(n+2)}} \tag{2.41}$$

Thus, for the case $n = 6$, the potential rises maximally in the direction $\psi = -\frac{1}{\sqrt{3}}\phi$, corresponding to the angle $\theta_{F5} = 150°$ in figure 2–1. This does not coincide with the runaway direction identified in figure (2–1), but is close enough so that NS5-branes in conjunction with strings or D-branes can stabilize all the moduli.

[3] See Appendix 2.6 for details.

[4] We thank Ali Kaya for pointing this out to us.

2.4 Adding Superpotentials

Since it is not possible to fully stabilize all moduli using the D-brane/string gas, we investigate the dynamics of a system in which the brane gas is present simultaneously with an external stabilization mechanism. A typical potential which could arise in string-motivated supergravity theories is the one which is generated by gaugino condensation in an SU(N) gauge sector. Although it might seem redundant to consider partial modulus stabilization by a brane gas when there is already a potential at zero density, there could actually be several benefits: for example, the brane gas can prevent the problem of the moduli overshooting the desired minimum [13], as we investigate in this section.

2.4.1 Gaugino Condensation Potential

We briefly review the derivation of the nonperturbative gaugino condensate potential in low-energy effective supergravity, starting with the 10 dimensional spacetime which is assumed to be a product of 4D noncompact external spacetime and a 6D compact internal manifold. We limit our present discussion to the dynamics of the radion, $\psi(x)$. A similar discussion should apply for more than one moduli field, but for simplicity we assume that all other moduli (*i.e.*, complex structure and dilaton) have been stabilized. The radion appears in the full metric as

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu + e^{2\psi} g_{mn} dy^m dy^n \tag{2.42}$$

In supergravity, the radion is the real part of a chiral field,

$$T = X + iY \equiv e^{4\psi} + iY \tag{2.43}$$

Dimensional reduction of the supergravity action yields an effective four dimensional theory of gravity coupled to the complex scalar field $T(x)$

$$S = M_p^2 \int d^4x\sqrt{-g}\left[\frac{R}{2} + \mathcal{K}_{T\bar{T}}\,\partial_\mu T\,\partial^\mu \bar{T} - e^{\mathcal{K}}(\mathcal{K}^{T\bar{T}} D_T\mathcal{W}\,\overline{D_T\mathcal{W}} - 3|\mathcal{W}|^2)\right] \quad (2.44)$$

where $\mathcal{K}(T,\bar{T})$ and $\mathcal{W}(T,\bar{T})$ are the Kähler potential and superpotential respectively, while $K_{T\bar{T}}$ is the Kähler metric given by

$$\mathcal{K}_{T\bar{T}} = \frac{\partial^2\mathcal{K}}{\partial T\partial\bar{T}} \quad (2.45)$$

We have also performed a conformal transformation of the four dimensional metric:

$$g_{\mu\nu} \to e^{n\psi} g_{\mu\nu} = e^{6\psi} g_{\mu\nu} \quad (2.46)$$

The kinetic and potential terms for T are computed from $\mathcal{K}(T,\bar{T})$ and $\mathcal{W}(T,\bar{T})$, where the Kähler potential for T is $\mathcal{K} = -3\ln[T+\bar{T}]$ while as in [14] we use the superpotential

$$\mathcal{W} = \mathcal{W}_0 + Ae^{-aT} \quad (2.47)$$

which would be obtained through gaugino condensation in a theory with a simple gauge group. For instance, for $SU(N)$, $a = 2\pi/N$. The constant term W_0 represents the effective superpotential due to any fields that have been fixed already [23], such as the dilaton and complex structure moduli.[5]

[5] Although recent authors have pointed out that a proper construction of the KKLT mechanism includes other non-perturbative contributions to the Kähler potential, we are primarily concerned with addressing the overshoot problem which still exists despite their inclusion [24, 25]. That is, the mechanism proposed still provides an attractor solution despite changes to the form of the Kähler potential.

The scalar-tensor action then reads

$$S = M_p^2 \int d^4x\sqrt{-g}\left[\frac{R}{2} + K - V\right] \tag{2.48}$$

where the kinetic (K) and potential (V) terms are given by

$$K = -3\frac{\partial_\mu T\,\partial^\mu \bar{T}}{|T+\bar{T}|^2} = -12\,\partial_\mu\psi\,\partial^\mu\psi - \frac{3}{4}e^{-8\psi}\,\partial_\mu Y\partial^\mu Y$$

and

$$V = \frac{E}{X^\alpha} + \frac{1}{6X^2}\left[aA^2(aX+3)e^{-2aX} + 3W_0Aae^{-aX}\cos(aY)\right] \tag{2.49}$$

To arrive at the potential (2.49)) we have also included the potential energy coming from an anti-D3 brane (first term) as in [14], which is needed in order to have a nonnegative vacuum energy density. The coefficient E is a function of the tension of the brane T_3 and of the warp factor, if there are warped throats [26] on the Calabi-Yau manifold. The exponent α is either $\alpha = 2$ if the anti-D3 branes are sitting at the end of a warped throat. Otherwise $\alpha = 3$ corresponding to the unwarped region. If a warped region exists, it is energetically preferred.

The imaginary part of the Kähler modulus, the axion Y, has stable minima at $Y = (2n+1)\pi/a$ (assuming $W_0Aa > 0$). We will integrate this field out and focus on the dynamics of the radion, whose kinetic term is[6] $12M_p^2(\partial\psi)^2$, and whose

[6] The kinetic part of the action for the radion can also be derived directly from the Einstein-Hilbert action $S_{10} = \int d^{10}x\sqrt{-\hat{g}}\hat{R}$ contained in the full 10D supergravity action. Using a consistent dimensional reduction ansatz for the metric of the form (2.42) one obtains $S_4 = \int d^4x\sqrt{-g}e^{n\psi}[R - n(n-1)(\partial\psi)^2 + \ldots]$. A conformal transformation precisely of the form (2.46) is needed to convert the action S_4 to Einstein frame, from which one recovers the kinetic piece of the radion given here.

potential becomes

$$V = Ee^{-4\alpha\psi} - \frac{1}{2}\left(W_0 Aae^{-aX} - aA^2e^{-2aX}\right)e^{-8\psi} + \frac{1}{6}a^2A^2e^{-2aX}e^{-4\psi} \tag{2.50}$$

It is convenient to rescale $\psi \rightarrow \psi/\sqrt{24}$ so that the kinetic term is canonically normalized. The action becomes

$$S = M_p^2 \int d^4x\sqrt{-g}\left[\frac{R}{2} - \frac{(\partial\psi)^2}{2}\right.$$
$$\left. - \left(Ee^{-2\alpha_1\psi} - \frac{1}{2}\left(W_0Aae^{-aX} - aA^2e^{-2aX}\right)e^{-2\alpha_2\psi} + \frac{1}{6}a^2A^2e^{-2aX}e^{-2\alpha_3\psi}\right)\right] \tag{2.51}$$

where

$$\alpha_1 = \frac{\alpha}{\sqrt{6}}, \quad \alpha_2 = \frac{2}{\sqrt{6}}, \quad \alpha_3 = \frac{1}{\sqrt{6}}, \qquad X = e^{2\alpha_3\psi} \tag{2.52}$$

The potential has three distinct regions (see figure 2–2, solid curve). (1) For ψ large and negative $V(\psi)$ is dominated by the antibrane contribution, if $\alpha = 3$, or by a combination of the antibrane term and the term proportional to $e^{-2\alpha_2\psi}$ if $\alpha = 2$. In either case, the potential is to a good approximation a pure exponential in ψ, which will be relevant for the analytic solutions we discuss in the next subrection. (2) For $\psi \sim 1$, the different terms in the potential are comparable, creating a minimum at $\psi_{\min}$, followed by a potential barrier at $\psi_{\max}$. (3) For $\psi \gg 1$ the antibrane term again dominates, since the other terms are exponentially suppressed by e^{-aX}.

2.4.2 Attractor solution with brane gases

We now consider the effect of augmenting the vacuum potential in (2.51) with the contribution from a brane gas. Without the brane gas, the dynamics of the radion depend sensitively on the initial conditions. If we start with $\psi > \psi_{\max}$ (the position of the bump in the potential), ψ runs to infinity, where the extra dimensions are decompactified.

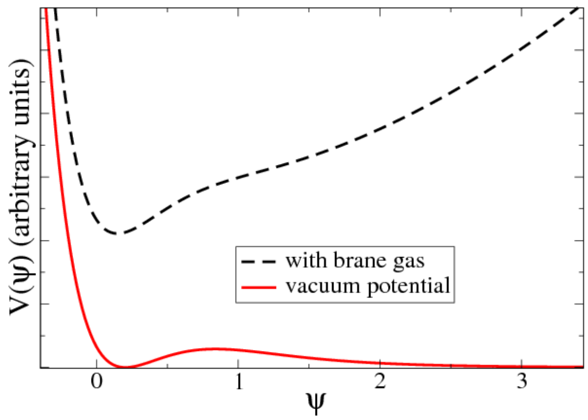

Figure 2–2: Radion potential with vacuum gaugino condensate potential (solid line) and potential at nonzero brane gas density.

Generically one might expect the radion to start closer to the Planck size with $\psi < 0$, so that there is a possibility of reaching the stable minimum at $\psi = \psi_{\rm max}$.

Since the vacuum potential in the region $\psi < 0$ is well approximated by an exponential, the radion quickly reaches the attractor solution discussed in [27]; it tracks the minimum formed between the exponential potential and the rising part of the "brane-gas potential," shown as the dashed line in figure 2–2. This attractor behavior washes out the effect of initial conditions. As long as the attractor is reached before the field has passed the position of the minimum, this will allow ψ to settle into the minimum and avoid the overshoot problem.

Let us recapitulate the details of the attractor solution. The rising part of the brane-gas potential originates from the winding modes of p-branes with $p > 3$. In this region the Friedmann equation and the equation of motion for ψ read (in $M_p = 1$ units)

$$H^2 \cong \tfrac{1}{3}\left(\tfrac{1}{2}\dot{\psi}^2 + Ee^{-2\alpha_1\psi} + \rho_p e^{2\nu_p\psi}\right) \qquad (2.53)$$

with $\rho_p = \rho_p^0 \left(\frac{a}{a_0}\right)^{-3}$ and

$$\ddot{\psi} + 3H\dot{\psi} \cong 2\left(\alpha_1 E e^{-2\alpha_1\psi} + \nu_p \rho_p e^{2\nu_p\psi}\right) \tag{2.54}$$

respectively. The exponents ν_p for p-branes' coupling to the canonically normalized radion were derived in the previous section,

$$2\nu_p = \sqrt{\frac{2}{n(n+2)}}\left(p - \frac{n}{2}\right) = \sqrt{\frac{1}{24}}\,(p-3) \tag{2.55}$$

where $n = 6$ is the number of extra dimensions. This kind of system was studied in [28], where it was shown that there exist tracking solutions in which the energy of the scalar field tracks that of the branes:

$$e^{\psi} = \left[\frac{E}{\rho_p}\left(\frac{\alpha_1(\alpha_1+\nu_p) - 3/4}{\nu_p(\nu_p+\alpha_1) + 3/8}\right)\right]^{\frac{1}{2(\alpha_1+\nu_p)}} \equiv \left[\frac{E}{\rho_p} r\right]^{\frac{1}{2(\alpha_1+\nu_p)}} \tag{2.56}$$

This relation implies that both the potential and kinetic energy of the radion remain proportional to the energy density of the branes,

$$V(\psi) = r^{-1}\rho_p\, e^{2\nu_p\psi} = \frac{8(\nu_p+\alpha_1)^2 - 3}{3(1+r)}K \tag{2.57}$$

The steepest brane-induced effective potential occurs for the maximal value of p, $p = 6$; this provides the greatest resistance to expansion of the internal manifold and will be the most effective case for avoiding the overshoot problem. Eq. (2.57) also shows that for $p = 6$ the ratio of kinetic to potential energy is minimized. For example, if $\alpha = 2$ and $p = 6$ we have $\alpha_1 = \sqrt{2/3}$, $2\nu_6 = \sqrt{3/8}$, leading to $K/V = 12/23$.

In passing we note that such tracking solutions correspond to a power law expansion of the universe

$$a(t) = a_0 \left(\frac{t}{t_0}\right)^{(2/3)(1+\nu_p/\alpha_1)} = a_0 \left(\frac{t}{t_0}\right)^{11/12} \tag{2.58}$$

The universe does not accelerate during this phase. However, as was found in [28], when the analysis is carried out including the dilaton, acceleration can be obtained.

2.4.3 Addressing the Overshoot Problem

The above discussion implies that the overshoot problem will be avoided in the presence of a brane gas so long as the attractor solution can be reached. This means that for a given initial value of ψ, the initial energy density in the brane gas, $\rho_p e^{2\nu_p \psi}$, must be sufficiently large. If not, the brane density is diluted too quickly by the expansion of the universe and the system evolves according to the vacuum potential.

We have confirmed these expectations by numerically integrating the coupled system of Friedmann and radion equations, which we illustrate with a specific example. In the potential (2.49) we consider an antibrane in a warped throat, with $\alpha = 2$. Its tension is tuned to give a Minkowski minimum as shown in figure 2–2, which illustrates the case where $E = 0.00889$, $a = 2.1$, $A = 0.9$, and $W_0 = 0.25$. We first verified that indeed this potential suffers from an overshoot problem, shown in figure 2–3. Starting from an initial condition $\psi \lesssim -0.17$, the field runs away to ∞.

Interestingly, overshooting can be prevented by initial brane densities which are many orders of magnitude smaller than the initial potential energy of the radion. Figure 2–3 shows the evolution starting from exponentially large initial radion potential energy, with $\psi_0 = -100$ and $p = 6$, for several initial brane densities, parametrized by $\zeta = \rho_p e^{2\nu_p \psi}/V_0(\psi_0)$, where V_0 is the potential of the radion alone, excluding the brane gas contribution. The result shows that even for initial brane gas energy densities which are only $10^{-18}\,V_0(\psi_0)$, overshoot can be prevented. For different initial values, the exponent $\log_{10}(\zeta)$ scales linearly with ψ_0. This behavior can be understood analytically, as shown in Appendix D. The minimum required value of ζ is given by

$$\log_{10}\zeta \approx -0.43\left[2\alpha_1\psi_0\left(1-\frac{3-4\nu_p\alpha_1}{4\alpha_1^2}\right)\right] \tag{2.59}$$

The intuitive explanation for this result is that the radion energy initially falls more quickly than that of the brane gas. What counts is not the initial ratio of brane gas to potential energy; rather it is the ratio at the time when ψ is close to its nontrivial minimum. This mechanism has been pointed out in [29] (see also [30, 31]) as a generic way of solving the overshoot problem, using general sources of energy density. Brane gas cosmology provides a concrete setting where this idea can be used advantageously.

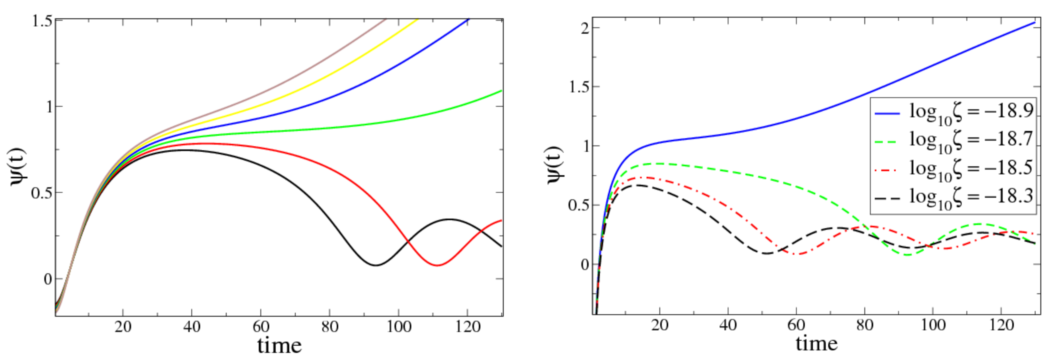

Figure 2–3:
(Left:) Evolution of ψ without brane gas, for several initial values $\psi_0 = -0.15, -0.16, \cdots, -0.2$, illustrating overshoot.
(Right:) Solutions with $\psi_0 = -100$ and different initial densities of brane gas, near the borderline of overshooting.

When the modulus has reached its stable minimum, we are still left with a gas of branes, whose energy density is comparable to the energy density in the scalar fields; otherwise the brane gas would not be effective in slowing the rolling of the modulus. At the bottom of its potential, the scalar field oscillates and and its energy density redshifts as a^{-3} just like the brane gas. The result is a matter dominated universe. We must assume that inflation begins some time after this in order to dilute the branes and reheat the universe. Work on smoothly connecting the modulus stabilization with the beginning of inflation is in progress.

2.5 Conclusions

In this paper we used dimensional reduction to derive the effective action for a gas of strings and p-branes, giving a contribution to the effective potential for the radion and dilaton. In a gas of strings only, this potential could stabilize the radion provided there was only one extra dimension, but not the dilaton. dilaton. Including p-branes allows for the stabilization of either the dilaton or radion if $p > \frac{d}{2}$. However, the brane gas is insufficient for stabilizing both moduli simultaneously, for the type II strings we consider, which have no massless winding modes. Rather, only a linear combination of the moduli can be stabilized by the brane gas.

It thus seems likely that external potentials are needed for modulus stabilization. However the brane gas can still play an interesting role in helping the moduli settle into their typically shallow minima, avoiding the overshoot problem. An attractive feature of this mechanism is that the brane gas can initially be many orders of magnitude smaller in energy density than the potential energy of the moduli and still be effective in slowing the rolling of the moduli, since the brane gas energy redshifts more slowly. There is therefore no need for finely-tuned initial conditions.

In this work we have ignored quantum corrections, as well as higher-derivative corrections to the dilaton gravity action. The first approximation is justified for weak string coupling, $g_s = e^{\phi} << 1$. In this regard, the runaway direction found in Section 2.2 corresponds to $\phi \to -\infty$, showing that quantum corrections cannot lift this flat direction at large field values. Of course it is possible that such corrections could lead to a metastable minimum along the flat direction, which would be a loophole in our no-go result.

Acknowledgments

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Note Added

As this work was being finished, similar results were given in [32]. Our results were presented the week before, at the McGill Workshop on String Gas Cosmology, 30 April 2005.

2.6 Appendices

Appendix 1: Dimensional Reduction

We provide here a brief review of the standard dimensional-reduction procedure, following the procedure of [11, 33]. Our starting point is D-dimensional dilaton-gravity together with a generic contribution of gas. This system is described by

$$
\begin{aligned}
S_{II} &= \frac{1}{2\kappa^2}\int d^D x\sqrt{-G}e^{-2\phi}\left(R + 4G^{MN}\nabla_M\phi\nabla_N\phi - \frac{1}{12}H_{\mu\nu\alpha}H^{\mu\nu\alpha}\right) && (2.60)\\
S_m &= \int d^D x\sqrt{-G}e^{-\alpha\phi}\rho\ , \qquad \rho = \sum_i \rho_i a^{-d(1+\omega_i)}b^{-n(1+\hat{\omega}_i)}\ , && (2.61)
\end{aligned}
$$

for some initial density ρ_i. The dimensional reduction procedure will focus on the string action (2.60); but, by tracking the transformation rules, we can later also reduce the matter components. We obtain an effective theory of BGC by first transforming the string action (2.60) to the Einstein frame through the conformal transformations [34]

$$
\begin{aligned}
G_{MN} \to \tilde{G}_{MN} &= \Omega^2 G_{MN},\ \Omega = e^{-A\phi}\ ,\ A = \frac{2}{D-2}\\
R \to \tilde{R} : R &= e^{-2A\phi}\tilde{R} - 2(D-1)e^{-A\phi}\left(e^{-A\phi}\right)_{;MN}\tilde{G}^{MN}\\
&\quad -(D-1)(D-4)\left(e^{-A\phi}\right)_{;M}\left(e^{-A\phi}\right)_{;N}\tilde{G}^{MN}\\
\phi \to \tilde{\phi} &= \sqrt{2A}\phi && (2.62)
\end{aligned}
$$

to obtain

$$
S \to \tilde{S} = \frac{1}{2\kappa^2}\int d^D x\sqrt{\tilde{G}}\left\{\tilde{R} - \tilde{G}^{MN}\nabla_M\tilde{\phi}\nabla_N\tilde{\phi}\right\}\ , \qquad (2.63)
$$

where $\tilde{\phi}$ is the canonically-normalized dilaton, and we have ignored flux contributions. We dimensionally reduce the action by integrating out the extra dimensions [11, 33]. To perform this last step we consider a string-frame metric of the form (2.3), split into d large directions described by $g_{\mu\nu}$ and n compact directions described by γ_{mn}. For simplicity, we consider the geometry of the extra dimensions to be that of a torus, thus $R[\gamma_{mn}] = 0$. We use the following relations to isolate the scale-factor dependence on the extra-dimensions [11, 33, 34]

$$\sqrt{-\tilde{G}} \;=\; \tilde{b}^n \sqrt{-\tilde{g}} \tag{2.64}$$

$$\tilde{R} = \tilde{R}[\tilde{G}_{MN}] \;=\; \tilde{R}[\tilde{g}_{\mu\nu}] - 2n\tilde{b}^{-1}\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\tilde{\nabla}_\mu\tilde{b} - n(n-1)\tilde{b}^{-2}\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\tilde{b}\tilde{\nabla}_\nu\tilde{b}, \tag{2.65}$$

where, again, $R[\gamma_{mn}] = 0$, n and $\widetilde{b}(x^\mu)$ are the number and scale factor corresponding to the extra dimensions, and $\tilde{g}_{\mu\nu}$ is the metric of the non-compact directions. Since none of the terms in the action depend explicitly on the coordinates from the n extra dimensions, we integrate over these directions to get the low energy effective action of the $d+1$-dimensional theory

$$\begin{aligned} S_{eff} \;=\; \frac{V_n}{2\kappa^2}\int d^{d+1}x\sqrt{-\tilde{g}}\,\Big[&\tilde{b}^n d\tilde{R}[\tilde{g}_{\mu\nu}] - 2n\tilde{b}^{n-1}\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\tilde{\nabla}_\mu\tilde{b} \\ &-n(n-1)\tilde{b}^{n-2}\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\tilde{b}\tilde{\nabla}_\nu\tilde{b} - \tilde{b}^n\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\tilde{\phi}\tilde{\nabla}_\nu\tilde{\phi}\Big]\,, \end{aligned} \tag{2.66}$$

where $V_n \equiv \int d^n y\sqrt{\gamma}$ is the spatial volume of the n extra dimensions under unit scaling ($\tilde{b} = 1$).

A second conformal transformation and field redefinition of the action (2.66) is necessary to obtain the canonical form of the Einstein-Hilbert action. The conformal transformation reuses the identities (2.62) with

$$\bar{g}_{\mu\nu} = \tilde{b}^n\tilde{g}_{\mu\nu} \equiv e^{\sqrt{B}\tilde{\psi}}\tilde{g}_{\mu\nu}\ , \tag{2.67}$$

resulting in

$$S_{eff} \;=\; \frac{V_n}{2\kappa^2}\int d^{d+1}x\sqrt{-\bar{g}}\,\Big(R[\bar{g}_{\mu\nu}] - \bar{g}^{\mu\nu}\bar{\nabla}_\mu\tilde{\psi}\bar{\nabla}_\nu\tilde{\psi} - \bar{g}^{\mu\nu}\bar{\nabla}_\mu\tilde{\phi}\bar{\nabla}_\nu\tilde{\phi}\Big)\ , \tag{2.68}$$

where $B = \frac{d-1}{n(d+n-1)}$. Finally, the system is canonically normalized by identifying the 4D Planck mass as $M_p^2 \equiv \frac{V_n}{\kappa^2}$, and by rescaling the fields as

$$\psi = M_p \tilde{\psi} \ , \ \varphi = M_p \, \tilde{\phi} \tag{2.69}$$

$$\Rightarrow S_{eff} \ = \ \int d^{d+1}x\sqrt{-\bar{g}} \left(\frac{M_p^2}{2} R[\bar{g}_{\mu\nu}] - \frac{1}{2}\bar{g}^{\mu\nu}\bar{\nabla}_\mu\psi\bar{\nabla}_\nu\psi - \frac{1}{2}\bar{g}^{\mu\nu}\bar{\nabla}_\mu\varphi\bar{\nabla}_\nu\varphi \right) \tag{2.70}$$

The net effect of these transformations is to rescale the scale factors and dilaton as

$$\begin{aligned}
\sqrt{-G} \ &\rightarrow \ \sqrt{-\bar{g}}\, e^{D\sqrt{\frac{A}{2}}\frac{\varphi}{M_p}}\, e^{n\sqrt{B}\frac{\psi}{M_p}} \\
a(t) \ &\rightarrow \ \bar{a}(t) = e^{\frac{n}{d-1}\sqrt{B}\frac{\psi}{M_p}}\, e^{-\sqrt{A/2}\frac{\varphi}{M_p}} a(t) \\
b(t) \ &\rightarrow \ \tilde{b}(t) = e^{\sqrt{B}\frac{\psi}{M_p}}\, e^{-\sqrt{A/2}\frac{\varphi}{M_p}} b(t) \\
\phi(t) \ &\rightarrow \ \varphi(t) = \sqrt{2A} M_p\, \phi(t),
\end{aligned} \tag{2.71}$$

Employing the above expressions, we may now express the contribution of a source behaving as

$$\rho \ = \ \rho_i a^{-d(1+\omega_i)} b^{-n(1+\hat{\omega}_i)} \tag{2.72}$$

in the D dimensional string frame, through the effective matter-action

$$\begin{aligned}
S_{eff_m} \ &= \ \int d^{d+1}x\sqrt{-\bar{g}} e^{-\alpha\sqrt{\frac{1}{2A}}\varphi}\bar{\rho} \\
&= \ \int d^{d+1}x\sqrt{-\bar{g}}\rho_i \cdot e^{\left(-\hat{\omega}_i + \frac{d}{d-1}\left(\omega_i - \frac{1}{d}\right)\right)\sqrt{\frac{(d-1)n}{(d+n-1)}}\psi} \\
&\qquad \cdot e^{\left(-d\omega_i - n\hat{\omega}_i + 1 - \alpha_i\frac{d+n-1}{2}\right)\sqrt{\frac{1}{d+n-1}}\varphi}\, \bar{a}^{-d(1+\omega_i)} \\
&\equiv \ \int d^{d+1}x\sqrt{-\bar{g}}\rho_i\, e^{2(\mu_i\varphi + \nu_i\psi)}
\end{aligned} \tag{2.73}$$

with $M_p = 1$. The original theory of dilaton gravity together with string and brane sources can now be interpreted as a theory of Einstein gravity together with sources, plus two scalar fields corresponding to the dilaton (φ) and the moduli field (ψ), this is the action of equation (2.70). As well, the source term (equation 2.61) now acts like an effective potential for the two scalar fields. The inclusion of different excited states will provide different effective potentials, and this freedom can be exploited in the search for a moduli-stabilizing potential.

Appendix 2: Equations of state

In this section we derive the equations of state and the resultant coefficients for the brane-gas effective potential. Using the metric-ansatz (2.3), we derive the gas pressure from the thermodynamic relation.

$$P_a = -\left.\frac{\delta E}{\delta V}\right|_{b=const.} \tag{2.74}$$

The volume is given by $V = \sqrt{-G_s} = a^d b^n$, while energy contributions are generically of the form $E = a^j b^k = \left(a^d\right)^{\frac{j}{d}} \left(b^n\right)^{\frac{k}{n}}$, so that

$$\begin{aligned}
\delta V &= b^n \delta(a^d) + a^d \delta(b^n) & (2.75)\\
\delta E &= \frac{j}{d}\frac{a^j b^k}{a^d}\delta(a^d) + \frac{k}{n}\frac{a^j b^k}{b^n}\delta(b^n) & (2.76)\\
\Rightarrow P_a = -\left.\frac{\delta E}{\delta V}\right|_{b=const} &= -\frac{j}{d}\frac{a^j b^k}{a^d b^n}\frac{\delta(a^d)}{\delta(a^d)} = \omega\frac{E}{V} = \omega\rho & (2.77)\\
\Rightarrow P_b = -\left.\frac{\delta E}{\delta V}\right|_{a=const} &= -\frac{k}{n}\frac{a^j b^k}{a^d b^n}\frac{\delta(b^n)}{\delta(b^n)} = \hat{\omega}\frac{E}{V} = \hat{\omega}\rho\ , & (2.78)
\end{aligned}$$

where we have made the identifications $\omega = -\frac{j}{d}$ and $\hat{\omega} = -\frac{k}{n}$. Thus $E = a^j b^k = a^{-d\omega} b^{-n\hat{\omega}}$. The existence of winding and momentum modes for strings is a well-known result, and is the reason for the T–duality invariant spectrum of closed strings. Finding an embedding with quantized momentum modes of branes is less subtle because the T-dual of a wrapped brane results in a wrapped brane, not a momentum mode. However, we use the embedding described by Kaya [16], which results in momentum modes in the compact direction with energy given by

$$E_n = \frac{\lambda_n}{b(t)} \ , \tag{2.79}$$

where λ_n is an unknown eigenvalue for the n'th momentum mode (we choose $\lambda_n > 0$). The corresponding pressure due to this brane momentum-mode is

$$P_n = \frac{\lambda_n}{b(t)} \ , \tag{2.80}$$

which is a positive quantity.

Appendix 3: D-string oscillator modes

For completeness we consider the naive quantization of D-strings, in case these modes could affect the no-go result for simultaneous stabilization of the dilaton and radion. Ignoring the 2-form gauge field that couples to the D-string, the spectrum of the D-strings looks identical to that of F-strings except for the replacement

$$M_s \longrightarrow M_s' = e^{-\varphi/2} M_s \tag{2.81}$$

The rescaling is again due to the dilaton coupling present in the DBI action for the D-strings. Provided we ignore $N_{\rm osc} < 0$ modes, again it is sufficient to consider only the "pure" modes. A straight forward computation yields the following source actions

$$S_{\rm D,mom} = \int d^D x \ \sqrt{-\hat{g}} a^{-3} b^{-(n+1)} \tag{2.82}$$

$$S_{\rm D,osc} = \int d^D x \ \sqrt{-\hat{g}} e^{-\varphi/2} a^{-3} b^{-n} \tag{2.83}$$

and

$$S_{\rm D,wind} = \int d^D x \ \sqrt{-\hat{g}} e^{-\varphi} a^{-3} b^{-(n-1)} \ . \tag{2.84}$$

The D-string momentum modes looks identical to those of the F-string momentum modes, and yield no new effect. The winding modes are the same as those obtained for D1-branes, which have already been considered. The only qualitatively new contribution comes from the oscillatory D-string modes. Substituting $\alpha = 1/2$ and $\hat{\omega} = 0$ in (2.23) one finds

$$\theta_{\mathrm{D,osc}} = -\frac{\pi}{2} - \frac{\pi}{6} \tag{2.85}$$

Again, this fails to stabilize the runaway direction.

Appendix 4: Solving the overshoot problem

One can analytically estimate of what must be the initial ratio of energy densities in the brane gas and radion in order to solve the overshoot problem. If the initial energy density of branes is much smaller than the potential energy of the radion, the dynamical equations will be given by

$$H^2 \cong \tfrac{1}{3}\left(\tfrac{1}{2}\dot{\psi}^2 + Ee^{-2\alpha_1\psi}\right) \tag{2.86}$$

and

$$\ddot{\psi} + 3H\dot{\psi} \cong 2\alpha_1 E e^{-2\alpha_1\psi} \tag{2.87}$$

The radion rolls freely down the exponential potential and exact solutions are known [35]:

$$a \sim t^{1/2\alpha_1^2} \text{ and } e^{\psi} \sim t^{1/\alpha_1} \sim a^{2\alpha_1} \tag{2.88}$$

Thus the energy densities of the brane and the radion redshift in this non-tracking phase as

$$\rho_p e^{2\nu_p\psi} \sim a^{-(3-4\nu_p\alpha_1)} \text{ while } V(\psi) \sim a^{-4\alpha_1^2} \tag{2.89}$$

Thus as long as

$$3 - 4\nu_p\alpha_1 < 4\alpha_1^2 \tag{2.90}$$

the brane energy density will catch up with the potential energy of the radion. We can calculate when this happens. The ratio of brane energy to radion potential energy is

$$\frac{\rho_p e^{2\nu_p \psi}}{V(\psi)} \sim a^{4\nu_p\alpha_1+4\alpha_1^2-3} \tag{2.91}$$

and we want that this ratio to be $\mathcal{O}(1)$, by the time the radion rolls to the minimum. Hence we need to start with an initial ratio such that

$$\zeta \equiv \frac{\rho_{p0} e^{2\nu_p \psi_0}}{V(\psi_0)} = \left(\frac{a_0}{a_{min}}\right)^{4\nu_p\alpha_1+4\alpha_1^2-3} \tag{2.92}$$

where

$$\frac{V(\psi_0)}{V(\psi_{min})} = e^{-2\alpha_1\psi_0} = \left(\frac{a_0}{a_{min}}\right)^{-4\alpha_1^2} \tag{2.93}$$

From (2.92) and (2.93) we find

$$\zeta = \exp\left[2\alpha_1\psi_0\left(1-\frac{3-4\nu_p\alpha_1}{4\alpha_1^2}\right)\right] \Rightarrow \log_{10}\zeta \approx -0.43\left[2\alpha_1\psi_0\left(1-\frac{3-4\nu_p\alpha_1}{4\alpha_1^2}\right)\right] \tag{2.94}$$

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CHAPTER 3
Warped Reheating

If one could conclude as to the nature of the Creator from a study of his creation it would appear that God has a special fondness for stars and beetles.
— J. B. S. Haldane

FOREWORD: Although the following chapter is a departure from String Gas Cosmology, the research presented also resides in the (very) early universe and attempts to reconcile the framework of string theory with our observable universe. This work-in-progress has yet to be published, but it is worthy of inclusion because the working draft is close to completion. What remains is the explicit evaluation of integrals involving the harmonic expansion of higher-rank objects over a $T^{1,1}$ background. For several cases the exact answer has yet to be found, but the answer is known to be either vanishing or of the string-scale, and we draw several conclusions based on this input.

Abstract

It has been suggested that after brane-antibrane inflation in a Klebanov-Strassler (KS) warped throat, metastable Kaluza-Klein excitations can be formed due to nearly-conserved angular momenta along isometric directions in the throat. If sufficiently long-lived, these relics could conflict with big bang nucleosynthesis or baryogenesis by dominating the energy density of the universe. We make a detailed estimate of the decay rate of such relics using the low energy effective action of type IIB string theory compactified on the throat geometry, with attention to powers of the warp factor. We find that in the KS background it is necessary to turn on SUSY-breaking deformations of the background in order to ensure that the most dangerous relics will decay. However, the decay rate is much larger than the naive guess based on the dimension of the operators which break the angular isometries of the throat. Thus KK relics do not pose a problem for the KS geometry, although they could perhaps do so for other warped compactifications. We derive constraints on the warp factor and the 5D mass of the lightest KK relic in more-general backgrounds; these constraints may come from nucleosynthesis or baryogenesis, and if the decay can only be mediated by a SUSY-breaking operator, one may obtain a constraint on the SUSY-breaking scale.

3.1 Introduction

The success of the inflationary paradigm in providing a natural resolution for the flatness and homogeneity problems of Standard Big Bang cosmology has made inflation an essential part of early-universe cosmology. This success has led to intense efforts in realizing inflation within string theory, resulting in several new scenarios including brane-antibrane inflation, where the interbrane separation plays the role of the inflaton. These constructions provide new possibilities for constraining the parameters of string theory, within compactifications that could be compatible with the Standard Model.

A potential source of new phenomenological constraints, distinct from those arising in generic field theory models of inflation, is the reheating process at the end of inflation. If inflation occurs in one warped throat, while the standard model (SM) is localized in another throat, there can be a difficulty in transferring the energy from brane-antibrane annihilation to the standard model degrees of freedom since the warp factor provides a gravitational potential barrier between the throats. If the barrier cannot be penetrated, "reheating" will be predominantly into invisible gravitons [1]-[3], an unacceptable outcome. In ref. [4] it was argued that the suppression due to the barrier can be counteracted by the enhanced coupling of the Kaluza-Klein (KK) modes to the deeply–warped SM throat; this scenario has been further studied in [5]-[7]. Another interesting possibility is that inflation could deform the SM throat in such a way that its oscillations at the end of inflation efficiently reheat the SM degrees of freedom [8].

A further challenge was recently pointed out in ref. [9], whose authors highlighted the possibility of producing long-lived, heavy KK modes, which could conflict with standard cosmology. In previous studies of reheating in a warped throat, the throat was modelled by a single extra dimension, leading to an AdS_5 geometry. Massive states are strongly peaked in the infrared (IR) region of the throat, so integrating them out by dimensional reduction (DR) resulted in large effective couplings, and hence efficient decay. Ref. [9] emphasized that the actual background solution, the Klebanov-Strassler (KS) solution, contains an additional 5D internal space $\mathcal{M}_5$ with isometries along which non-radial KK excitations can occur. For realistic particle phenomenology $\mathcal{M}_5$ is usually taken as $T^{1,1}$, the Einstein-Sasaski manifold for the group $SU(2) \times SU(2)/U(1)$. These isometries, as we shall review, result in approximately conserved angular momenta which constrain the possible decay channels and result in a long-lived relic corresponding to the lightest "charged" state, *i.e.,* the lightest state with angular momentum in the $T^{1,1}$. We shall refer to this candidate relic as the *lightest massive charged state* (LMCS).

If the KS throat was the entire compactification manifold, the angular isometries would be exact and the LMCS would be stable. However it is necessary to cut off the throat in the ultraviolet (UV) region, joining it to a larger Calabi-Yau (CY) manifold which does not globally preserve the isometries. The process of gluing together the KS throat to the CY thus perturbs the KS geometry in the UV region, and this information propagates down the throat into the IR. We will assume that a mode of the metric which was zero due to the isometry will be sourced in the UV region, and that its radial profile decays exponentially toward the IR region, so that the symmetry breaking is a weak effect in the IR. In the CFT description, via the AdS/CFT correspondence, this corresponds to turning on an irrelevant operator that breaks the symmetry. Of course, if the operator is relevant the symmetry-breaking is strong in the IR, there is no problem of long-lived relics, and the throat geometry is not close to the KS solution.

Since the symmetry-breaking effect is suppressed in the IR, while the radial profile of the LMCS is strongly peaked in the IR, the operators induced in the low-energy effective theory that describe the decay of the LMCS will be suppressed by powers of the warp factor w, which determines the hierarchy of scales between the bottom and top of the throat. If this suppression is too strong, the heavy KK relics are long-lived and can come to dominate the energy density of the universe at unacceptably high temperatures. In particular, they should decay before the era of big bang nucleosynthesis at $T \sim$ 1MeV, at the very least, and most likely also before baryogenesis, since otherwise the entropy produced by their decays will greatly dilute the baryon asymmetry. Assuming that baryogenesis could not have happened later than the electroweak phase transition requires the KK relics to decay at temperatures greater than 100 GeV.

In ref. [9], it was assumed that the suppression of the LMCS decay amplitude was of the form w^p, where $p + 4$ was the dimension of the most relevant charge-violating (but 5D Lorentz and SUSY preserving) operator in the CFT. However, there was no detailed justification for this assumption, and it is not obvious that it should give the same answer as actually computing the decay rate from the effective theory. Our goal in this paper is to make an accurate estimate of the decay rate of the potentially dangerous relics in the $AdS_5 \times T^{1,1}$ type IIB supergravity background. We will find a different result than that of ref. [9], showing that the decay rate is less suppressed than indicated by their estimate. Moreover, we will show that the CFT operator considered in ref. [9] is not sufficient to destabilize the LMCS in the KS background: one must turn on, in addition, an irrelevant SUSY-breaking operator for this purpose.

In the remainder of the paper we examine the constraints on the warp factor and the mass of the LMCS resulting from considerations of BBN and baryogenesis, both for the KS background, and for more general warped compactifications. Details of the analysis specific to the $T^{1,1}$ background are given in the appendices, while more general considerations are given in main body of this text. Appendix 1 provides a comprehensive review of the steps required to analyze the decay of the LMCS. Appendix 2 reviews all possible decay channels of the LMCS involving one background correction to the KS throat; though this section is useful for analysis in other backgrounds because it provides several simple arguments to help identify viable decay channels. Appendix 3 calculates the decay rate for a generic background. Finally, Appendix 4 provides information necessary to evaluate the $T^{1,1}$ harmonics for the possible decay channels. The appendices may be referred to for more detail, but the main text provides enough information to conclude the LMCS of the KS background has a decay rate large enough to avoid BBN and baryogenesis constraints.

3.2 Background Deformations and KK Mode Decay

In this section we will describe in greater detail the origin of the symmetry breaking for the approximate angular isometries, and we will illustrate the approach we are going to take using a simplified toy model. Appendix 1 provides a similar discussion, but with more detail.

The problem of relic angular KK modes is closely related to a moduli problem associated with having an anti-D3 brane $\left(\overline{D3}\right)$ placed at the bottom of the deformed conifold geometry, as one might wish to do in order to uplift the AdS vacuum from Kähler modulus stabilization to dS or Minkowski space, in the manner of KKLT [10]. The energy of the $\overline{D3}$ is minimized at the bottom of the throat, but, as pointed out in ref. [11] the base of the deformed conifold has an $S^2 \times S^3$ topology, whose S^2 shrinks to vanishing size at the location of the $\overline{D3}$. The $\overline{D3}$ can move freely inside the S^3, whose coordinates thus correspond to 3 massless moduli. In order to stabilize these moduli one needs to break the isometries of the S^3.

In ref. [11] this was achieved by considering the dual field theory to the KS background and turning on an irrelevant operator that gives a mass to the fields describing the $\overline{D3}$ position. Precisely the same mechanism can destabilize the would-be angular KK relics since the operator provides a background correction which perturbs the geometry. To analyze this process we choose to work in the gravity side of the gauge/gravity correspondence. Since a renormalization group (RG) flow in the field theory dual corresponds to movement along the radial direction of the AdS space, turning on an operator in the ultraviolet (UV) of the field theory and performing the RG flow corresponds to turning on a source for the bulk classical field dual to that operator and following its effect along the radial direction to the bottom of the throat.

In the AdS background geometry the fields have an exponential dependence on the radial direction, and the profile of the symmetry-breaking perturbation will be related to the warping of the background geometry. This perturbation is sourced by the CY, which generically does not preserve the symmetries of the throat; hence we consider a source in the UV which depends non-trivially on the angular coordinates of the S^3 cycle, and thus breaks the corresponding isometries of the S^3. As a consequence, the KK modes of fields that couple to the source will become unstable since the KK quantum numbers are no longer conserved quantities; this is just the gravitational dual of the mechanism that made the $\overline{D3}$ position moduli massive in [11].

3.2.1 A simplified model

Our basic approach will be to compute the 4D effective Lagrangian for KK relics in the presence of a perturbation to the background geometry. The perturbation leads to symmetry-breaking terms in the effective Lagrangian, including vertices for the decay of the relic. It is useful to illustrate this procedure on a simpler model before tackling the full 10D supergravity theory—a similar, more-comprehensive, description is provided in Appendix 1.

We therefore consider a massless scalar field ϕ in 6D, where one of the compact dimensions corresponds to the angular direction which is an isometry of the unperturbed throat and the other is the radial direction along the AdS. Its Lagrangian is

$$\mathcal{L} = \frac{1}{2}\int d^4x\, dr\, d\theta \sqrt{-g}\, g^{AB}\partial_A\phi\, \partial_B\phi \tag{3.1}$$

and the original throat geometry is described by the line element

$$ds^2 = a^2(r)\, \eta_{\mu\nu}dx^\mu dx^\nu + dr^2 + L^2 d\theta^2 \tag{3.2}$$

between $r = 0$ and $r = r_0$. In Randall-Sundrum coordinates, the warp factor takes the form $a = e^{-kr}$, so $r = 0$ corresponds to the top of the throat (the UV), where it joins to the CY, and $r = r_0$ is the bottom of the throat (the IR). To model the symmetry-breaking effect of the CY, we will assume that (3.2) gets perturbed by the metric functions

$$\Delta ds^2 = \sin(\theta)\left[a^2(r)\alpha(r)dx^2 + \beta(r)dr^2 + L^2\gamma(r)d\theta^2\right] \tag{3.3}$$

corresponding to the lowest KK excitation of the angular direction. These can be thought of as solutions to the vacuum Einstein equations in the throat, sourced by some boundary conditions at the CY, $r = 0$. The linearized Einstein equations have solutions of the form

$$\alpha(r) = \alpha_1 e^{-zkr}, \quad \beta(r) = \beta_1 e^{-zkr}, \quad \gamma(r) = \gamma_1 e^{-zkr} \tag{3.4}$$

where z can take any of the four values

$$z_{\pm\pm} = -2 \pm \sqrt{14 \pm \sqrt{\frac{5}{3}\left(60 - \frac{12}{(kL)^2} - \frac{1}{(kL)^4}\right)}} \tag{3.5}$$

and the relative amplitudes $\alpha_1, \beta_1, \gamma_1$ depend on z.

In the CFT language, these four solutions correspond to two different operators $\mathcal{O}_\pm$ whose dimensions $\Delta_\pm$ are related to $z_{\pm\pm}$ via

$$\Delta_\pm = -z_{-\pm}, \quad 4 - \Delta_\pm = -z_{+\pm}\,. \tag{3.6}$$

In the limit of large kL, the values of the conformal dimensions are

$$\Delta_+ \cong 2 + 2\sqrt{6} \cong 6.9, \quad \Delta_- \cong 4\,. \tag{3.7}$$

As explained in ref. [11], we should turn on an irrelevant symmetry-breaking operator in the CFT, $\mathcal{O}_+$, to produce a perturbation which is weak in the IR. This corresponds to linear combinations of the form

$$\alpha(r) = Ae^{-z_{-+}kr} + Be^{-z_{++}kr} = Ae^{\Delta_+ kr} + Be^{(4-\Delta_+)kr} \tag{3.8}$$

and in the limit $kL \to \infty$, $\beta = -\frac{1}{3}\gamma = \alpha$. The relative sizes of the coefficients A, B are determined by boundary conditions at $r = r_0$, for example $\alpha'(r_0) = 0$, so that $B = \Delta_+/(\Delta_+ - 4)w^{4-2\Delta_+}A$. The two solutions are of comparable magnitude at the bottom of the throat, but the second one quickly comes to dominate as one moves toward $r = 0$, so it is a good approximation to simply take

$$\alpha(r) \cong \alpha_1 e^{(4-\Delta_+)kr} = \alpha_1 e^{-z_{++}kr} \tag{3.9}$$

where α_1 characterizes the magnitude of the symmetry breaking in the UV, and thus could be $\mathcal{O}(1)$.

Next consider the scalar field, which can be expanded in radial (n) and angular (m) KK mode as

$$\phi = \frac{1}{\sqrt{2\pi L}} \sum_{n,m} R_{nm}(r) e^{im\theta} \psi_{nm}(x^\mu)\,. \tag{3.10}$$

In the absence of the metric perturbation, the interactions of the angular excited states with $n = 0$, $m \neq 0$ conserve the total angular momentum, so there is no way for the massive states $\psi_{0,\pm1}$ to decay. These, then, represent the lightest massive charged states (LMCS) in the toy model.

In the perturbed metric, angular momentum is no longer conserved, and we can construct an interaction from the kinetic term for the decay $\psi_{0\pm1} \to \psi_{00} h_{\mu\nu}$, where $h_{\mu\nu}$ is a massless graviton which is a perturbation about the Minkowski metric factor in (3.2): $\eta_{\mu\nu} \to \eta_{\mu\nu} + h_{\mu\nu}/M_p$. The 4D effective interaction is

$$\mathcal{L}_{\rm decay} = h^{\mu\nu}\, \partial_\mu \psi_{0\pm1}\, \partial_\nu \psi_{00}\ \left[-\frac{\alpha_1}{4\pi M_p} \int d\theta \sin\theta e^{\pm i\theta} \int dr\, e^{-(2+z_{++})kr}\, R_{01}(r) R_{00}(r) \right] \tag{3.11}$$

where we note that $R_{00}(r)$ is just a constant. To evaluate the radial integral, we must find the radial wave functions, which obey the equation of motion (eq. 3.55) with 4D mass m_{mn}

$$e^{2kr}\partial_r \left(e^{-4kr} \partial_r R_{nm} \right) - \frac{m^2}{k^2 L^2} e^{-2kr} R_{nm} + m_{nm}^2 R_{nm} = 0\,. \tag{3.12}$$

Defining the warp factor at the bottom of the throat

$$w = e^{-kr_0}\,, \tag{3.13}$$

the solutions have the form

$$\begin{aligned} R_{nm}(r) \;&\simeq\; \frac{w\sqrt{k}e^{2kr}}{J_{\nu_m}(x_{nm})}\left[\, J_{\nu_m}\left(x_{nm}we^{kr}\right) + w^{2\nu_m}Y_{\nu_m}\left(x_{nm}\, we^{kr}\right)\right] \\ \nu_m &= \sqrt{4+(m/kL)^2}\,, \end{aligned} \tag{3.14}$$

where $x_{nm} \sim 1$ is determined by the boundary conditions, and the 4D mass of the excitation is given by $m_{nm} = kwx_{nm}$. The radial behaviour for excited modes is dominated by the J_ν solution, which is strongly peaked in the IR. The zero-mode solution $R_{00} \cong \sqrt{2k}$ is just a constant, whose value is determined by the normalization condition

$$\int dre^{-2kr}R_{nm}^2 = 1\,. \tag{3.15}$$

We can estimate the integral determining the coefficient of the decay-mediating operator in (3.11) using the small- and large-r asymptotics of the Bessel function. Near $r = 0$, the argument of J_ν is exponentially small and $J_\nu(x) \sim (x/2)^\nu/\Gamma(1+\nu)$, while near $r = r_0$, the argument is of order unity. Both behaviours are consistently approximated by $R_{01} \sim w^{1+\nu}\sqrt{k}e^{(2+\nu)kr}$, leading to the estimate

$$\mathcal{L}_{\rm decay} \cong \frac{\alpha_1 w^{1+\nu}}{M_p} h^{\mu\nu}\,\partial_\mu\psi_{0\pm1}\,\partial_\nu\psi_{00}\,. \tag{3.16}$$

In this estimate, the exponential dependence on z_{++} drops out because the integral $\int dre^{(\nu-z_{++})kr}$ converges as $r \to \infty$. In other words, the radial integral is dominated by the UV behaviour of the LMCS, not the IR overlap of the wavefunctions (see Appendix 3 for the generic behaviour of the radial integral).

Let us contrast this with the estimate made in ref. [9]. There it was assumed that the operator $\mathcal{O}_+$ (corresponding to a background correction to the metric) directly mediates the decay in the 4D effective theory, and its degree of irrelevance controlled the amount of warp factor suppression, so that

$$\mathcal{L}_{\rm decay} \sim w^{\Delta_+ - 4} = w^{z_{++}} \,. \tag{3.17}$$

On the other hand our explicit calculation indicates that the warp factor dependence in $\mathcal{L}_{\rm decay} \sim w^{1+\nu}$ is controlled by the mass of the LMCS through ν, not $\mathcal{O}_+$.

3.2.2 Multiple Tadpole Insertions and Mixing

In the full 10D SUGRA model we want to consider, the simple decay process illustrated above will turn out to vanish. This can be overcome by turning on additional symmetry-breaking operators in the CFT. There is no reason not to do so, since all such irrelevant operators are expected to get induced by the lack of symmetry of the CY in the UV region. Here we show how this will allow a decay of the LMCS to two massless states in the toy 6D model.

The basic idea is to generate a mixing of the LMCS with a massless uncharged field which, in turn, is coupled to two other massless fields and thus allows the LMCS to decay. This is illustrated in Fig. 3–1. For example we can obtain mixing of the $\psi_{0\pm1}$ modes of the scalar with the graviphoton $h_{r\mu}$ by turning on a VEV for some higher mode ψ_{0m} which corresponds to another irrelevant operator in the CFT, in addition to the one considered above for the metric, $\mathcal{O}_+$. The equation of motion for the corresponding radial wave function will be the same as (3.12), except with the 4D mass m^2_{mn} set to zero, since the VEV must not have any x^μ

dependence. The solution is

$$R_m^T(r) = Ae^{-z_- kr} + Be^{-z_+ kr} \tag{3.18}$$

where the superscript T indicates that this is a tadpole solution rather than a propagating fluctuation. $z_\pm$ is given by

$$z_\pm = -2 \pm \sqrt{4 + (m/kL)^2} = -2 \pm \nu_m \,, \tag{3.19}$$

where m is the quantized momentum along the θ-direction. In the CFT language this corresponds to an operator with conformal dimension $\Delta = -z_- = 2 + \nu_m$. Although all such operators are marginal in the limit $kL \to \infty$, for any finite kL, they are irrelevant except for $m = 0$. Similarly to the background perturbation (3.8), we impose Neumann boundary conditions at the bottom of the throat, giving $A = \frac{\nu_m - 2}{\nu_m + 2} w^{2\nu_m} B$ so both terms are of the same order near r_0. Again the z_+ solution dominates in the UV, so one can approximate $R_m^T(r) \cong Ae^{-z_+ kr}$ with A of order $M_s\sqrt{k}$, by dimensional analysis. (The factor of M_s takes the place of the 4D wave function ψ which has been set to a constant in the tadpole. Since the physics which breaks the U(1) isometry is in the UV, it is natural to use M_s rather than the warped string scale wM_s.)

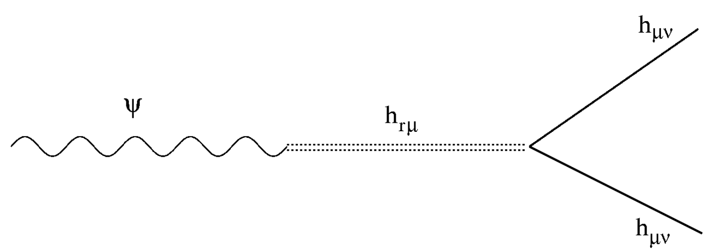

Figure 3–1: Decay of the LMCS (ψ) into massless particles $h_{\mu\nu}$ via mixing with the radial graviphoton $h_{r\mu}$

In this toy model we could turn on a VEV for the LMCS itself, taking $m = 1$, and omit the perturbation (3.3) of the metric which we used previously. However this choice will not work in the 10D model because the LMCS corresponds to a relevant operator in that case. To more closely illustrate the 10D situation, we instead turn on VEVs for the $m = \pm 2$ states of the scalar, in addition to the metric perturbation (3.3). These together with the LMCS have the correct total angular momenta to give a non-vanishing angular integral when dimensionally reducing to 4D.

The Lagrangian (3.1) when dimensionally reduced using the background perturbations (3.3) and (3.18) leads to a mixing between the LMCS and the graviphoton $h_{r\mu}$. Inserting the metric perturbation in the determinant factor $\sqrt{-g}$ and the ϕ perturbation $\delta\phi = e^{\pm 2i\theta} R_2^T(r)$ in $\sqrt{-g}(h^{r\mu}/M_p)\partial_r\phi\partial_\mu\phi$, we obtain

$$\begin{aligned} \mathcal{L}_{\text{mixing}} &= \left(\alpha_1 \int dr\, e^{-(2+z_{++})kr}\, R_{01}\, \partial_r R_2^T\right) [h^{r\mu}\partial_\mu\psi_{0\pm 1}] \\ &\sim w^{1+\nu_1} \frac{M_s}{M_p}\, r_0 k^2\, [h^{r\mu}\partial_\mu\psi_{0\pm 1}] \end{aligned} \tag{3.20}$$

where again the warp factor dependence comes from the normalization of the radial wave function of the LMCS R_{01}. This particular dependence on the warpfactor is a robust result. The radial integral receives UV and IR contributions, but the LMCS is normalized to the IR and the tadpole corrections are normalized to the UV so they contribute, respectively, of order unity in these regions.

The mixing term (3.20) can be combined with a typical gravitational interaction for the graviphoton, like

$$M_p^{-1} h^{\mu\nu}\partial_\alpha h_{\mu r}\partial^\alpha h_{\nu r} \tag{3.21}$$

to obtain an effective interaction

$$\mathcal{L}_{\rm int} \sim w^{1+\nu_1} \frac{M_s}{M_p^2 m_\psi^2}\, r_0 k^2\, h^{\mu\nu}(\partial_\alpha\partial_\mu\psi)\partial^\alpha h_{\nu r} \tag{3.22}$$

The important thing to notice, for our argument, is that the powers of the warp factor come out the same as in the simpler mechanism for decays which gave rise to (3.16)—this despite the increased complexity of tracking two background deformations in order to accommodate the decay.

3.3 KK decay in Type IIB Supergravity

We now want to apply the methods illustrated in the 6D model to the KS geometry sourced by a stack of D3-branes in the 10D theory. The line element is

$$ds^2 = H^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + H^{1/2}\left(dR^2 + R^2 ds^2_{T^{1,1}}\right)\ , \tag{3.23}$$

where

$$H(R) = \frac{27\pi}{4R^4}{\alpha'}^2 g_s M\left[K + g_s M\left(\frac{3}{8\pi} + \frac{3}{2\pi}\ln\left(\frac{R}{R_{max}}\right)\right)\right] \tag{3.24}$$

is in terms of the flux quantum numbers K and M. Far from the tip $R < R_{max}$ we neglect the logarithmic contributions to $H(R)$ and further ignore the small contribution from the second term on the right to obtain the metric of an $AdS_5 \times T^{1,1}$ throat. Using the coordinate transformation $R = k^{-1}e^{-kr}$, the metric becomes

$$ds^2 \;=\; e^{-2kr}\eta_{\mu\nu}dx^\mu dx^\nu + dr^2 + \frac{1}{k^2}ds^2_{T^{1,1}} \tag{3.25}$$

The corresponding low-energy effective theory is type IIB supergravity on an approximate $AdS_5 \times T^{1,1}$ background [14]:

$$\begin{aligned} S_{IIB} \;=\; & \frac{1}{2\kappa_0^2}\int d^{10}x\sqrt{-G}\left\{e^{-2\phi}\left[R+4(\nabla\phi)^2-\frac{1}{12}\left(H^{(3)}\right)^2\right]\right. \\ & \left.-\frac{1}{12}\left(F^{(3)}+A^{(0)}\wedge H^{(3)}\right)^2-\frac{1}{2}\left(dA^{(0)}\right)^2-\frac{1}{480}\left(F^{(5)}\right)^2\right\} \\ & +\frac{1}{4\kappa_0^2}\int\left(A^{(4)}+\frac{1}{2}B^{(2)}\wedge A^{(2)}\right)\wedge F^{(3)}\wedge H^{(3)}\,. \end{aligned} \tag{3.26}$$

As usual, $H^{(3)}$ is the field strength of the NS 2-form $B^{(2)}$, $F^{(n+1)}$ is the field strength for the RR n-form $A^{(n)}$, $2\kappa_0^2 = (2\pi)^7\alpha'^4$, $\sqrt{\alpha'} = l_s = M_s^{-1}$, where M_s is the string scale. In our subsequent analysis, we will refer to several other mass scales. The AdS curvature scale k is determined by the flux quantum numbers M and K through the relation $k^{-4} \equiv \frac{27\pi}{4}\alpha'^2 g_s MK$. The warped string scale, wM_s, is also determined by the fluxes, through $w = e^{-2\pi K/(3g_s M)}$. Finally, the Planck scale is given by $M_p^2 = \frac{2V_6}{g_s^2\kappa_0}$ where V_6 is the compactification volume. From eq. (3.25) it is apparent $V_6 \simeq R_{AdS}^6 = k^{-6}$, so with $g_s < 1$ and $k < M_s$ we find the 4D Planck scale is greater than the string scale.

3.3.1 Identifying the LMCS

The first step is to discover which angular KK excitation, among the many fields that result from dimensionally reducing the action (3.26), is the potentially dangerous relic, the LMCS. Fortunately, the masses and quantum numbers of all the lowest-lying KK excitations in the KS background have been tabulated in refs. [11, 12]. The correspondence between 10D and 5D fields from integrating over the $T^{1,1}$ directions is indicated in Table 3–1, taken from [12].

Table 3–1: The harmonic expansion of the 10D fields. h_{MN} is the 10D metric, A_{MNOP} the 10D four-form, B the complex 0-form, and A_{MN} the 10D complex 2-form. We have not included the NS 2-form. The different polarizations of the fields appear as 5D scalars, vectors, and tensors. (Adapted from ref. [12])

Dim	fields					harmonic
10D 5D	$h_{\mu\nu}(x,y)$ $H_{\mu\nu}(x)$	$h^a_a(x,y)$ $\pi(x)$	$A_{abcd}(x,y)$ $b(x)$	$B(x,y)$ $B(x)$	$A_{\mu\nu}(x,y)$ $a_{\mu\nu}(x)$	$Y(y)$
10D 5D	$h_{a\mu}(x,y)$ $B_\mu(x)$	$A_{\mu abc}(x,y)$ $\phi_\mu(x)$	$A_{\mu a}(x,y)$ $a_\mu(x)$			$Y_a(y)$
10D 5D	$A_{\mu\nu ab}(x,y)$ $b^{\pm}_{\mu\nu}$	$A_{ab}(x,y)$ $a(x)$				
10D 5D	$h_{ab}(x,y)$ $\phi(x)$					$Y_{(ab)}(x,y)$
10D 5D	$\lambda(x,y)$ $\lambda(x)$	$\psi_{(a)}(x,y)$ $\psi^{(L)}(x)$	$\psi_\mu(x,y)$ $\psi_\mu(x)$			$\Xi(y)$
10D 5D	$\psi_a(x,y)$ $\psi^{(T)}(x)$					$\Xi_a(y)$

The corresponding expansions of the fields in terms of scalar ($Y^{\{\nu\}}$), vector ($Y_a^{\{\nu\}}$) and tensor ($Y_{ab}^{\{\nu\}}$) harmonics of the $T^{1,1}$ are given by expressions like

$$h_{\mu\nu}(x,y) = \sum_{\{\nu\}} H^{\{\nu\}}_{\mu\nu}(x) Y^{\{\nu\}}(y) \tag{3.27}$$

$$h_{\mu a}(x,y) = \sum_{\{\nu\}} B^{\{\nu\}}_{\mu}(x) Y^{\{\nu\}}_a(y) \tag{3.28}$$

$$h_{(ab)}(x,y) = \sum_{\{\nu\}} \phi^{\{\nu\}}(x) Y^{\{\nu\}}_{(ab)}(y) \tag{3.29}$$

$$h^a_a(x,y) = \sum_{\{\nu\}} \pi^{\{\nu\}}(x) Y^{\{\nu\}}(y) \tag{3.30}$$

$$A_{abcd}(x,y) = \sum_{\{\nu\}} b^{\{\nu\}} \epsilon_{abcd}{}^{e} \mathcal{D}_e Y^{\{\nu\}} \tag{3.31}$$

$$A_{\mu bcd}(x,y) = \sum_{\{\nu\}} \phi^{\{\nu\}}_{\mu} \epsilon_{bcd}{}^{ef} \mathcal{D}_e Y^{\{\nu\}}_f \,. \tag{3.32}$$

where $x = (x^\mu, r)$, and $\{\nu\} = (j,l,r)$ are the quantum numbers identifying the $T^{1,1} = SU(2) \times SU(2)/U(1)$ representation. j and l are the usual angular momentum quantum numbers corresponding to the two $SU(2)$ factors. Higher-rank fields are given in terms of their dual representation. For the scalar harmonics which will be of most interest to us, $r = 2j_3 = -2l_3$ (where j_3, l_3 are the respective eigenvalues of the T_3 generators of the first and second $SU(2)$'s), and so is restricted to the range $|r| < \min(2j, 2l)$.

Ref. [12] has computed the 5D masses of all the states in the theory as functions of their (j, l, r) quantum numbers, and organized them into supermultiplets of the $N = 1$ 5D SUGRA theory. By going through these results and computing the masses of all particles which have nontrivial (j, l, r) values, we find that the field $b(x)$ in vector multiplet I is the LMCS, with $(j, l, r) \in \{(1, 0, 0), (0, 1, 0)\}$. Thus there are two species of LMCS, depending on whether j or l is nonzero. The 5D masses are defined in terms of the ubiquitously appearing function

$$H_0(j, l, r) = 6\left(j(j + 1) + l(l + 1) - \frac{r^2}{8}\right) \tag{3.33}$$

which takes the value $H_0(1, 0, 0) = 12$ for the LMCS. Its 5D mass is given by

$$m_b^2 = H_0 + 16 - 8\sqrt{H_0 + 4} = -4 \tag{3.34}$$

in units of the AdS curvature, k^2. As is well known, squared masses can be negative on an AdS background without leading to instabilities, down to the Breitenlohner-Freedman bound $m^2 \geq -4$, which is saturated by (3.34). To find the corresponding mass in 4D, one must do the final dimensional reduction on r by solving for the radial wave functions. These are identical to the 6D case (3.14) (assuming that we approximate the throat geometry by AdS$_5$), except for the replacement of the $5D$ mass, m_b^2, in the index of the Bessel function

$$\nu = \sqrt{4 + m_b^2}\,. \tag{3.35}$$

We have carried out this calculation to find the 4D mass as a function of the 5D one, using the boundary conditions of the RS model, as in ref. [16]. The result, shown in Fig. 3–2(a) shows that the 4D LMCS mass is approximately $m_{4D} = 1.7wk$. Fig. 3–2(b) shows this value is quite insensitive to the details of the boundary conditions in the UV (whether they are Neumann, Dirichlet, or mixed), or the method of gluing the throat to the CY. Fig 3–2 extends the result found in ref. [16] from $m_{5d}^2 \geq 0$ to include the Breitenlohner-Freedman bound $m_{5d}^2 \geq -4$. It indicates that finding the LMCS in $5D$ corresponds to finding the $4D$ LMCS as well, i.e., m_{4D}^2 is a monotonically increasing function of m_{5D}^2.. Note, these results are just the lowest state in the radial KK tower.

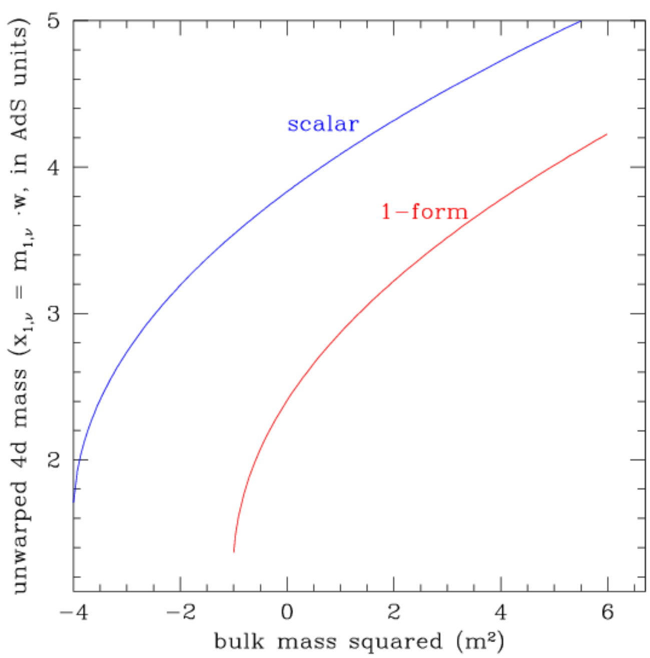

Figure 3–2: (a) Left: 4D mass of the first KK state as a function of the 5D mass (squared). The result is presented for both 5d scalars and vectors. (b) Right: dependence of the 4D mass on the UV boundary conditions, for two different values of the warp factor.

Table 3–2: The lightest states in the KS background: we list the 5D field, its supermultiplet, bulk mass, conformal dimension, and $T^{1,1}$ quantum numbers. Notably, most of the light, charged states correspond to the four form polarized along the $T^{1,1}$, $A^{(4)}_{abcd}$

Properties of states corresponding to relevant operators.

Field	Multiplet	(5d) Mass	Δ	QN's
a	VM III	-4	2	(0,0,0)
a	VM IV	-4	2	(0,0,0)
b	VM I	-4	2	(0,1,0), (1,0,0)
ϕ_3	VM I	-4	2	(0,0,0)
b	VM I	-3	3	(1,1,$\pm$2)
B, ϕ	VM IV	-3	2	(0,0,0)
ϕ_1, ϕ_2	VM I	-3	1	(0,0,0)
ϕ_1, ϕ_2	VM I	-3	3	(1,0,0)
b	VM I	-2.33	3.29	(1,1,0)

Properties of states corresponding to marginal and irrelevant operators.

Field	Multiplet	(5d) Mass	Δ	QN's
$h_{\mu\nu}$	GM	0	4	(0,0,0)
B	GM	0	4	(0,0,0)
ϕ_μ	VM I	0	3	(1,0,0), (0,1,0)
a_2	VM III/IV	0	4	(0,0,0)
b	VM I	0	0	(0,0,0)
ϕ_1/ϕ_2	VM I	0	4	(1,1,$\pm$2)
ϕ_3	VM I	0	4	(1,0,0), (0,1,0)
ϕ_1/ϕ_2	VM I	1.25	4.29	(1,1,0)
b	VM I	1.40	4.32	(2,0,0), (0,2,0)
a_1	VM III/IV	2.79	4.61	(1,0,0), (0,1,0)

Table 3–2 lists the least-massive states in the system; notably several saturate the Breitenlohner-Freedman bound, and the field $b(x)$ from the 4-form is the lightest charged state. Interestingly, most of the lightest charged states come from the 4-form, or the 10D graviton. This table is useful for quick identification of the LMCS and possible irrelevant operators that may be turned on in the background to accomodate the decay.

3.3.2 Interactions of the LMCS

We have seen that in terms of the 10D fields, the LMCS is contained in the RR 4-form $A^{(4)}_{abcd}$ polarized along the internal $T^{1,1}$ directions, resulting in a massive scalar field $b(x^\mu, r)$ from the 5D point of view.[1] Therefore the relevant terms in the action for type IIB supergravity (3.26) are

$$S_{IIB}(A^{(4)}) = \frac{1}{2\kappa_0^2}\int d^{10}x\sqrt{-G}\left[-\frac{1}{240}\left(F^{(5)}\right)^2\right] + \frac{1}{2\kappa_0^2}\int A^{(4)}\wedge F^{(3)}\wedge H^{(3)} \,. \tag{3.36}$$

Let us now try to follow the example of subsection 3.2.1 by turning on the CFT operator used by ref. [11] to stabilize the $\overline{D3}$ moduli. As shown there, this operator corresponds to a KK mode of the warped metric on the $T^{1,1}$ with quantum numbers $(1,1,0)$, call it $\delta g^{(1,1,0)}_{ab}$. In combining this operator with the LMCS in the kinetic term for A_{abcd}, the way to make a $T^{1,1}$ singlet combination which is closest to the 6D example is to choose the two different species of LMCS for the A_{abcd} factors:

$$\mathcal{L}_{\rm decay} = \int dr\, d^5y\sqrt{g_6}\,\delta g^{aa'}_{(1,1,0)}\, h^{\mu\nu}\,\partial_\mu A^{(0,1,0)}_{abcd}\,\partial_\nu A^{(1,0,0)}_{a'b'c'd'}\, g^{bb'} g^{cc'} g^{dd'} \tag{3.37}$$

where y^a are the coordinates of $T^{1,1}$ and $h^{\mu\nu}$ is a massless graviton. Notice that the $T^{1,1}$ quantum numbers of the various factors are such that their product contains a singlet (since j, l are angular momentum quantum numbers for the $SU(2)$ factors, $1\otimes 1 = 0\oplus 1\oplus 2$), so the integral over y^a should not vanish. However, this

[1] Our conventions for indices are: capital Latin letters $\{M, N, \ldots\}$ run over all directions, small Latin letters $\{a, b, \ldots\}$ run over internal directions, small Greek letters $\{\mu, \nu, \ldots\}$ run over the four noncompact dimensions

vertex involves the two different LMCS species, rather than just one of them and a massless mode of A_{abcd}, so it does not provide phase space for the decay of the LMCS.

It is natural then to turn to the Chern-Simons part of the action (3.36) since it contains a single $A^{(4)}$ factor. However, the Chern-Simons action contains terms of the form

$$\int d^5x\, d^5y\, \epsilon^{abcde}\epsilon^{\alpha\beta\gamma\delta\epsilon} A^{(4)}_{abcd}\, \partial_e A^{(2)}_{\alpha\beta}\, \partial_\gamma B^{(2)}_{\delta\epsilon} \tag{3.38}$$

with no appearance at all of the metric. Therefore the operator $\delta g_{ab}^{(1,1,0)}$ cannot induce decay of the LMCS through this term. These two observations prove that we must consider other operators in addition to the one invoked by ref. [11] for the decay of the LMCS to proceed.

There is no *a priori* reason to expect that just one operator should both give masses to the $\overline{D3}$ moduli *and* mediate the decay of the LMCS. We are free to turn on *any* irrelevant operator which breaks the symmetries of $T^{1,1}$ (while preserving other desirable symmetries) since the CY will generically not respect the isometries of $T^{1,1}$. We will use the strategy of subsection 3.2.2, and show that two operators suffice to cause mixing of the LMCS with a massless graviphoton, which can then mediate the decay into massless uncharged particles.

The remainder of this section describes a viable decay channel resulting from the insertion of two operators, while the subsections of Appendix 2 go over all possible vertices involving the LMCS and one symmetry-breaking operator—this is similar to the implementation in ref. [9] which considered the correction due to the single operator $\delta g^{(1,1,0)}$. Several vertices are identified as possible decay channels, but the corresponding $T^{1,1}$ integrals have yet to be evaluated and shown non-vanishing. Some progress towards evaluating these integrals is presented in Appendix 4.

The possible decay channels presented in Appendix 2 will only increase the LMCS decay rate, but since the vertex presented in the remainder of this section (involving background-corrections to both the metric $\delta g^{(1,1,0)}$ and $\delta A^{(2,1,0)}$) already avoids BBN constraints, it is not necessary to evaluate these integrals to avoid the long-lived relic problem. The usefulness of Appendix 2 is to provide the reader with a set of tools to help rule out, or identify, possible decay channels. Guiding principles are considerations of phase space and 4D Lorentz invariance. Whether the angular wavefunctions form a singlet also helps discriminate between viable decay channels.

3.3.3 A viable decay channel

Ideally, one would like to preserve 5D supersymmetry in the process of modeling the effects of the CY. This was a criterion that was used by ref. [11] in identifying $\delta g_{ab}^{(1,1,0)}$ as the most relevant (though still irrelevant) $T^{1,1}$-breaking operator. However, by considering the tables of multiplets in ref. [12] it is easy to see that no Lorentz- and SUSY-invariant operator can help us. We are interested in the fields whose top (highest conformal dimension, E_0) components are scalar fields, which occurs only for the vector multiplets. These have top components corresponding to the fields ϕ, π and a in the harmonic expansions (3.27-3.32). None of these fields appear in the Chern-Simons action, nor do they appear in the expansion of the $A^{(4)}$ field which contains the LMCS. Therefore these operators are all equally incapable as the original $\delta g_{ab}^{(1,1,0)}$ operator of inducing decay of the LMCS. Somewhat surprisingly, we are forced to break either 5D Lorentz symmetry or SUSY to destabilize the LMCS.

To preserve SUSY, one might like to give up 5D Lorentz invariance while maintaining it in 4D by turning on the r component of a vector field. The field ϕ_μ which appears in $A^{(4)}$, eq. (3.32), is a candidate. It is found in the graviton multiplet and gravitino I and III multiplets of [12]; however it is not the top component of these multiplets, and thus its VEV would not preserve SUSY. Rather, the field $B_\mu \sim h_{\mu a}$, eq. (3.28), is the top component. But being a mode of the metric rather than $A^{(4)}$, it is subject to the same arguments as $\delta g_{ab}^{(1,1,0)}$.

We are thus inevitably led to turn on a SUSY-breaking operator. As in the 6D model, it will be sufficient to use a higher KK mode of A_{abcd}, namely $A_{abcd}^{(2,1,0)}$. This together with $\delta g_{ab}^{(1,1,0)}$ has the right quantum numbers to neutralize the charge of the LMCS $b^{(1,0,0)}$, using the representation theory of $SU(2)$. In the same way, $b^{(1,0,0)}$ gets its mixing from the metric perturbation $\delta g_{ab}^{(1,1,0)}$ combined with $A_{abcd}^{(2,1,0)}$. As shown in ref. [12], the conformal dimension of the operators corresponding to the KK modes of A_{abcd} (in vector multiplet I) is given by

$$\Delta = E_0 = \sqrt{H_0 + 4} - 2\,. \tag{3.39}$$

Since the state $A_{abcd}^{(2,1,0)}$ has $H_0 = 48$, its dimension is

$$\Delta_{210} = 4.93 > 4, \tag{3.40}$$

and therefore turning on a small background for this state results in an exponentially decaying perturbation in the throat. For reference, we note that the operator corresponding to the $\delta g_{ab}^{(1,1,0)}$ background used by [11] to give masses to the $\overline{D3}$ moduli is the top component of vector multiplet I whose conformal dimension is $\Delta = E_0 + 2$, hence

$$\Delta_{110} = \sqrt{28} \cong 5.29\,. \tag{3.41}$$

In this way, we obtain mixing between b and the graviphoton $h_{r\mu}$ in close analogy to the 6D model. The angular integrals from the $T^{1,1}$ in the mixing amplitude are proportional to (see Appendix 4 for some guidance in evaluating this integral)

$$\int d\cos\theta_1 d\cos\theta_2\, Y_{1,0,0}\, Y_{1,1,0}\, Y_{2,1,0} \;\propto\; \int d\cos\theta_1 d\cos\theta_2\, F_{\rm even}(\theta_1,\theta_2) \;\neq\; 0\,. \tag{3.42}$$

The integral does not vanish because we are integrating an even function over an even domain, so this proves that the mixing amplitude is allowed by the $T^{1,1}$ group theory. The radial integral is similar to that of the 6D model, eq. (3.20), except z_{++} is replaced by $-4+\Delta_{(1,1,0)}$, and the $U(1)$ quantum number m is replaced by the $T^{1,1}$ quantum numbers (j,l,r):

$$\int dr\, e^{-(\Delta_{110}-2)kr} R_{0(1,0,0)}\, \partial_r R^T_{(2,1,0)} \sim wk\,. \tag{3.43}$$

Here we used that $R^T_{(2,1,0)} \sim e^{(4-\Delta_{210})kr}$, similarly to eqs. (3.18, 3.19), and $R_{0(1,0,0)} \sim w^{1+\nu}\sqrt{k}\, e^{(2+\nu)kr}$ (see above eq. (3.16)), where $\nu \;=\; \sqrt{4+m_b^2} \;=\; 0$ (eq. (3.34)). Moreover since the tadpole breaks SUSY, it should come with a factor of the SUSY-breaking scale $M_{3/2}$ rather than M_s. Following the 6D example, we then estimate the effective interaction induced by LMCS-graviphoton (ψ-$h_{\mu r}$) mixing to be

$$\mathcal{L}_{\rm int} \sim w\,\frac{M_{3/2}}{M_p^2 m_\psi^2}\, r_0 k^2\, h^{\mu\nu}(\partial_\alpha\partial_\mu\psi)\partial^\alpha h_{\nu r}\,. \tag{3.44}$$

Ignoring the moderate hierarchies between M_s, k, M_p and $1/r_0$, and taking $m_\psi \sim wM_s$, we find that the decay rate is of order $\Gamma \;\sim\; w^3 M_{3/2}^2/M_s$, and the decay temperature is of order $T \sim w^{3/2}M_s$. Since the inflationary scale is of order 10^{14} GeV, $w \;\sim\; 10^{-4}$, and $T \;\sim\; 10^{-6}M_{3/2}$. For electroweak baryogenesis to work, the relics should decay before $T \sim 100$ GeV, which leads to a constraint on the

SUSY-breaking scale,

$$M_{3/2} > 10^8 \text{ GeV}\,. \tag{3.45}$$

Most models of string inflation based on the KKLT construction predict an extremely small ratio of tensor to scalar fluctuations r. As was pointed out in [21], if the SUSY breaking scale is $M_{3/2} < 1\text{TeV}$, $r < 10^{-24}$ in these constructions, so a detection of tensor modes would contradict these models of string cosmology. Future observations of the CMB are thought to be limited to the domain $r > 10^{-3}$ [22], but the KKLT construction can breach this domain in the case of superheavy gravitinos, $M_{3/2} > 10^{13}\text{GeV}$. This compromise has important implications for particle phenomenology based on string theory since this is many orders of magnitude greater than the usual gravitino mass range considered by particle phenomenologists. We find that in order for the LMCS to decay before the epoch of baryogenesis we are forced to conclude the SUSY breaking scale is extremely high; so one may expect a large tensor to scalar ratio in these constructions at the cost of a large scale of SUSY breaking.

3.4 Discussions

In this paper we have studied the problem of potentially long-lived KK relics localized in a KS throat following brane-antibrane inflation. We found that accidental symmetries prevent the lightest mass charged state (LMCS) from decaying, even if the angular isometries giving rise to the conserved KK quantum numbers are broken by the Calabi-Yau manifold to which the throat is glued. In ref. [9], it was assumed the isometry-breaking effects of the operator $\delta g_{ab}^{(1,1,0)}$ provided a decay channel for the LMCS; however, this state either doesn't produce a $T^{1,1}$ singlet (so the harmonic-integral vanishes), or it doesn't couple to states with light decay products, so there is no phase space. Instead, for the KS background one must consider deformations of the throat geometry which break not only the isometries but also supersymmetry. Such a term was used to derive the viable interaction in eq. (3.44).

In that estimate, we omitted factors of order unity, like $1/(\Delta_{110} + \Delta_{210} - 8)$. The important point is that even if the $T^{1,1}$-breaking operators have a very high dimension, this only mildly suppresses the strength of the interaction, since one integrates the radial profiles over the length of the throat. The $4D$ effective coupling receives UV and IR contributions, with suppression in the IR controlled by the effects of the CY and the UV controlled by the behaviour of the LMCS (see Appendix 3 for more details). The significant source of suppression is the factor $w^{1+\nu}$, which only depends upon the 5D mass of the decaying particle through $\nu = \sqrt{4 + m_{5D}^2}$. Although it is easy to find high-dimension, massive states for which $w^{1+\nu}$ could be a significant suppression, these will generally have fast, $T^{1,1}$-conserving decays down to the LMCS. Appendix 2 serves to show how hard it is to find viable decay channels, and, in the case of the $T^{1,1}$ background, indicates the necessary isometry-breaking operator also breaks SUSY. Having found a viable channel, we reiterate that regardless of the operator, the decay rate is set by the UV behaviour of the LMCS, not the dimension of the symmetry-breaking operator; thus, decay rates are much greater than in previous estimates.

The decay of the LMCS in other backgrounds should follow a similar pattern since all fields have a kinetic term which couples to a massless graviton. Thus, by inserting background corrections we can produce a mixing vertex like Fig. 3–1, provided that the irrelevant operators combine to form a group singlet. From the vertex presented in eq. (3.43) and the discussions in Appendix 3, we have seen that the decay rate generically behaves as

$$\Gamma \simeq \frac{M_s^3}{M_{pl}^2} w^{2\nu_L+3} \,, \nu_L = \begin{cases} \sqrt{4+m_{5d}^2} & \text{scalar} \\ \sqrt{1+m_{5d}^2} & \text{vector} \end{cases} \,, \tag{3.46}$$

where ν_L depends on the mass of the LMCS m_{5D}, w is the amount of warping, and we have ignored small differences between the AdS, string, and Planck scale. In the case that the LMCS corresponds to a 5D vector field the radial behaviour changes according to the change in ν_L shown—this result is presented in the appendices, but is not relevant to the discussion of the KS background where the LMCS corresponds to a scalar field.

In deriving eq. (3.46), the internal angular integrals enforce momentum conservation and the radial integration of the dimensional reduction procedure determines the overall strength of the decay. If, in some background, the LMCS has $\nu_L > 0$, the decay rate falls below the inflationary scale $M_s w$, and observational consequences may arise. This is not the case for the $T^{1,1}$ background since $\nu_L = 0$; but, generally, demanding decay before the onset of BBN ($\Gamma \geq H_{BBN} \propto T_{BBN}^2 \simeq (1\text{MeV})^2$) rules out theories whose lightest charged state does not satisfy

$$\sqrt{4+m_{5d}^2} \geq \left[\frac{\log\left(M_{pl}T_{BBN}^2/M_s^3\right)}{2\log\left(w\right)} - \frac{3}{2} \right] \,. \tag{3.47}$$

Alternatively, the constraint $\Gamma \geq \Gamma_{BBN}$ can be viewed as a constraint on the warp factor w. Given some LMCS with mass m_{5d}^2, the warp factor must satisfy the bound

$$w \;\; \geq \;\; \left[\frac{H_{BBN}}{M_{pl}} \right]^{1/(2\nu_L+3)} \,, \tag{3.48}$$

so we can place constraints on the inflationary scale, Mw. Fig. (3–3) indicates the allowed parameter space for warping and the bulk mass of the tadpole for different backgrounds resulting from these constraints. We observe that as the inflationary scale decreases (corresponding to increased warping, w), the LMCS decay is further suppressed and we can rule out backgrounds whose LMCS are too massive. The different lines in Fig. 3–3 result from changing the AdS scale. Fig. 3–4 indicates the differences between demanding the decay rate of the LMCS be greater than either the baryogenesis, or the BBN epochs. The baryogenesis epoch is much sooner than the BBN epoch, and this results in tighter constraints on the allowed mass for the LMCS for a given warped throat. For a given background and known 5D mass of the LMCS, one may refer to these figures to determine if the particle may become a long-lived relic, or if it can decay quickly enough.

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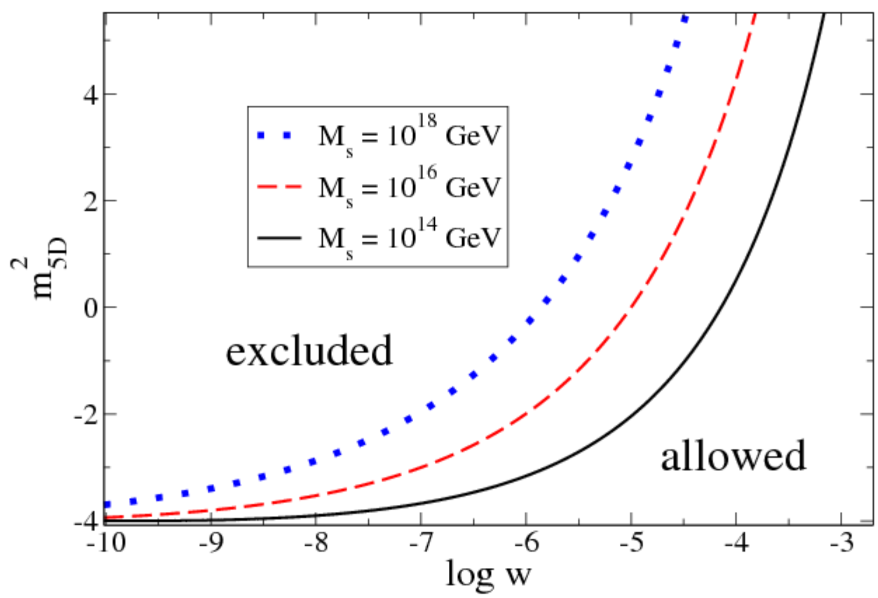

Figure 3–3: The remaining parameter space resulting from the condition that the LMCS decay before the onset of BBN. The different lines correspond to different string scales M.

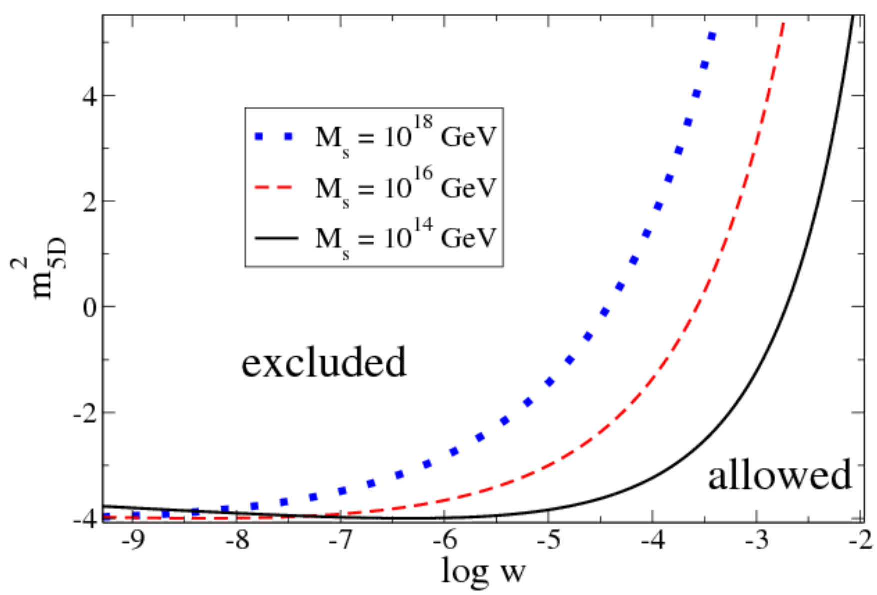

Figure 3–4: The allowed parameter space resulting from the condition that the LMCS decay before the onset of baryogenesis.

3.5 Appendices

These appendices present a detailed investigation into various calculations and results presented in the body of the paper, they are laid out sequentially with the steps required in analyzing the long-lived relic problem. We focus particularly on the $AdS_5 \times T^{1,1}$ KS solution, but aim to point out generalizations to different backgrounds and specify results associated with this background.

To investigate the possibility of long-lived relics in some background, the general procedure will consist, firstly, of identifying the least-massive charged state. In this case, charge refers to a state carrying non-zero momentum along the internal directions. The next step is to identify the various couplings of the field. This can be cumbersome in higher-dimensional theories because the decomposition of vertices into various polarizations over our four spacetime directions can result in numerous possibilities. This step is aided by phase space considerations since the LMCS can only decay into massless particles, as well as Lorentz violations and group theory considerations. Additionally, at least one member of the vertex must encode the isometry-violating information induced by the CY on the finite throat since the isometries are preserved otherwise. Not all states are sourced however, since they must not disrupt the background behaviour and they should preserve 4D Lorentz invariance; this restricts the deformations to states which correspond to irrelevant operators in the dual CFT description (this will impose the constraint that the conformal dimension of the source satisfies $\Delta > 4$). In the warped-reheating scenarios the deformations are induced by propagating the effects of the complicated geometry of the CY in the UV down the finite throat. This source introduces a tadpole correction to the background which catalyzes the charge-violating decay of the LMCS. Finally, we are interested in the resultant 4D vertex so we must dimensionally reduce the system. This will require explicit knowledge of the radial behaviour of the LMCS and the source since these wavefunctions bring in powers of the warpfactor, setting the overall scale of the decay.

Appendix 1: Identifying the LMCS

This section seeks to identify the lightest-mass charged-state (LMCS), plus lays out some of the notational conventions used within the article. We focus on the LMCS because heavier states can shed their charge via couplings to massless gravitons or lighter charged states and thus are not long lived. Thus, the possible long-lived relics corresponds to the lightest charged state. A simple example of this would be a five-dimensional theory with a $U(1)$ isometry group along x^5. Separation of variables gives an internal wavefunction behaving as $e^{i2\pi n x^5}$ with n quantized. In this case the LMCS corresponds to $n = 1$, and any couplings to massless particles will give, upon dimensional reduction, integrals behaving as

$$\int \sqrt{G_{55}}\, dx^5 e^{i2\pi n x^5} \simeq \delta_{n,0}\,, \tag{3.49}$$

indicating that the decay may proceed only in the case $n = 0$, and that the LMCS ($n = 1$) has no direct decay channel. The story is more complicated mathematically for realistic supergravity backgrounds, but the qualitative understanding is the same.

Although the details for this analysis can be applied to other background geometries and other theories, we proceed in the $T^{1,1}$ case. In what follows we tabulate the lowest-mass states for the 5D scalar fields amongst the various multiplets of Ceresole et al. [12]. Starting from the action of 10d type IIB supergravity on $AdS_5 \times T^{1,1}$, they perform the dimensional reduction to five dimensions using the harmonic expansions listed in Table 3–1. The field content includes the complex 0-form and 2-form B and $A_{(2)}$ respectively, the real self-dual four-form $A_{(4)}$, the 10D metric $G_{MN}(x, y)$, and ψ_M and χ are fermionic fields which are zero in this background. We use the conventions that capital Latin letters $\{M, N, \ldots\}$ run over all directions, small Latin letters $\{a, b, \ldots\}$ run over the internal directions $\{y^1, y^2, y^3, y^4, y^5\}$ of $T^{1,1}$, latter-alphabet Greek letters $\{\mu, \nu, \ldots\}$ run over our four dimensions, early-alphabet Greek letters $\{\alpha, \beta, \ldots\}$ run over the AdS with r denoting the radial AdS coordinate. We use the collective coordinates $x \in \{x^0, x^1, x^2, x^3, r\}$, $y \in \{y^1, y^2, y^3, y^4, y^5\}$, and $x^\mu \in \{x^0, x^1, x^2, x^3\}$.

Anticipating the scalar polarization of the 4-form as the LMCS and its important couplings to the graviton, we write out explicitly their harmonic expansions. These are in terms of internal $T^{1,1}$ quantum number $\{\nu\} \in \{(j, l, r)\}$ and polarization tensors ϵ:

$$\begin{aligned}
h_{\mu a}(x, y) &= \sum_{\{\nu\}} B_\mu^{\{\nu\}}(x) Y_a^{\{\nu\}}(y) \\
h_{(ab)}(x, y) &= \sum_{\{\nu\}} \phi^{\{\nu\}}(x) Y_{(ab)}^{\{\nu\}}(y) \\
h_a^a(x, y) &= \sum_{\{\nu\}} \pi^{\{\nu\}}(x) Y^{\{\nu\}}(y) \\
A_{abcd}(x, y) &= \sum_{\{\nu\}} b^{\{\nu\}} \epsilon_{abcd}{}^e \mathcal{D}_e Y^{\{\nu\}} \\
A_{\mu bcd}(x, y) &= \sum_{\{\nu\}} \phi_\mu^{\{\nu\}} \epsilon_{bcd}{}^{ef} \mathcal{D}_e Y_f^{\{\nu\}} \,.
\end{aligned} \tag{3.50}$$

Having defined our notation, we now refer to the tables of Appendix C of Ceresole et al. [12] and compute the KK mass states for the various multiplets; these tables give the mass formula for all $5D$ fields. A list of the lightest states is given in Table 3–2, and we observe that most of the lightest states are uncharged in the $T^{1,1}$, except for the field $b(x)$ in vector multiplet I. This is our possible long-lived relic, corresponding to the 4-form polarized along the $T^{1,1}$ directions, with quantum numbers $(j,l,r) \in \{(1,0,0),(0,1,0)\}$ and $m^2_{5d} = -4$ (in AdS units). We also see that the five lightest charged states correspond to $b(x)$, and the first state corresponding to an irrelevant operator also belongs to the b-field. Another important observation is the limited number of massless modes which, in turn, limits the number of possible decay channels of the LMCS since there is only phase space to decay into massless states.

Appendix 1.1: Background Behaviour

Having identified $b(x)$ as the lightest charged field we now focus on its couplings in the type IIB action in order to determine its decay channels and decay rate. Recalling that this state is the 4-form polarized along the $T^{1,1}$ direction, we rewrite the IIB supergravity action with terms involving $A_{(4)}$. The relevant couplings for this RR field are:

$$S_{IIB}(A_{(4)}) = \frac{1}{2\kappa_0^2}\int d^{10}x\sqrt{-G}\left[-\frac{1}{240}\left(F^{(5)}\right)^2\right] + \frac{1}{2\kappa_0^2}\int A^{(4)} \wedge F^{(3)} \wedge H^{(3)}\,. \tag{3.51}$$

In this expression κ_0 is related to the 10D Newton constant, $F^{(n+1)} = dA^{(n)}$ is the field strength of the RR n-form, and $H^{(3)}$ is the field strength of the NS 2-form $B^{(2)}$. The Chern-Simons term, though providing a coupling between different fields, does not provide a channel to shed the lightest charge since $A^{(2)}$ and $B^{(2)}$ have no massless states. The term could provide a mixing between the RR or NS 2-forms for the terms involving a tadpole shift, but these process are also kinematically ruled out; this leaves the 4-form kinetic term as the only source of interactions.

Since we are interested in the decay of the lightest charged field $b(x)$, we fix one of the 4-forms $A^{(4)}$ to have all indices in the internal directions

$$S_{IIB}(A_{(4)}) = \frac{-1}{480\kappa_0^2}\int d^{10}x\sqrt{-G}G^{A_1B_1}G^{a_2B_2}G^{a_3B_3}G^{a_4B_4}G^{a_5B_5}\partial_{A_1}\underbrace{A_{a_2a_3a_4a_5}}_{LMCS} \cdot\partial_{[B_1}A_{B_2B_3B_4B_5]}\,. \tag{3.52}$$

From a 4D observer's perspective, this term still represents many possible interactions due to the numerous polarizations of the 4-form and the metric; though this can be simplified further. The fact that the background is a product space ($AdS_5 \times T^{1,1}$) means there are no (5D) vector polarizations of the background graviton, and all 5D vector fluctuations are massive in the $T^{1,1}$ background so phase space rules out interactions with 5D graviphotons as decay products. This imparts the general rule that metric indices must always be in the same subspace (A_i, $B_i \in T^{1,1}$, or A_i, $B_i \in AdS_5$), or the graviphoton must be sourced by some deformation, giving a correction to its background behaviour. In the former case, this leaves the following possible interactions:

$$\Rightarrow S_{IIB}(A_{(4)}) = \frac{-1}{480g_s\kappa_0^2}\int d^4x\,dr\,d^5y\sqrt{-|G_{AdS_5}|}\sqrt{|G_{T^{1,1}}|}\,G^{a_2b_2}G^{a_3b_3}G^{a_4b_4}G^{a_5b_5} \cdot\left[G^{a_1b_1}\partial_{a_1}A_{a_2a_3a_4a_5}\partial_{[b_1}A_{b_2b_3b_4b_5]} + G^{rr}\partial_r A_{a_2a_3a_4a_5}\partial_{[r}A_{b_2b_3b_4b_5]} + G^{\mu\nu}\partial_\mu A_{a_2a_3a_4a_5}\partial_\nu A_{b_2b_3b_4b_5}\right]\,. \tag{3.53}$$

Note that the third term does not include the anti-symmetrized terms in order to preserve 4D Lorentz invariance. To complete the dimensional reduction we expand about background values

$$G^{MN} = G^{MN}_{(0)} + h^{MN}\,, \qquad A_{abcd} = A^{(0)}_{abcd} + A^{(1)}_{abcd}\,, \tag{3.54}$$

using the harmonic expansions in eq. (3.50), and isolate the 4D scalar component of the LMCS $b(x) = \hat{b}(x^\mu)R(r)$. Expanding eq. (3.53) to zeroth order and recalling that the background is a product-space facilitates the identification of the (internal) equations of motion and the conditions for a canonically–normalized 4D scalar field in the action (3.53):

$$\begin{aligned}
&\frac{1}{\sqrt{|G_{T^{1,1}}|}}\partial_a\left[\sqrt{|G_{T^{1,1}}|}\,G^{bn}_{(0)}G^{cq}_{(0)}G^{ds}_{(0)}G^{et}_{(0)}G^{ap}_{(0)}\epsilon_{bcde}{}^{f}\epsilon_{nqst}{}^{g}\partial_p\left(\mathcal{D}_f\mathcal{D}_g Y_{\{\nu\}}\right)\right]\\
&= -H_{\{\nu\}}Y_{\{\nu\}}\\
&\Box_r R_n - m_{5d}^2 e^{-4kr}R_n = -m_n^2 e^{-2kr}R_n\\
&\frac{-1}{240}\int d^5y\sqrt{|G_{T^{1,1}}|}\,G^{bn}_{(0)}G^{cq}_{(0)}G^{ds}_{(0)}G^{et}_{(0)}\epsilon_{bcde}{}^{f}\epsilon_{nqst}{}^{g}\mathcal{D}_f Y_{\{\nu\}}\mathcal{D}_g Y_{\{\mu\}} = \delta_{\{\mu\},\{\nu\}}\\
&\int dr\, e^{-2kr}\, R_m R_n = \delta_{m,n}\,.
\end{aligned} \tag{3.55}$$

The first two equations are the equations of motion along the $T^{1,1}$ and radial directions respectively, and the last two equations are normalization and orthogonality conditions. $H_{\{\nu\}}$ comes from the $T^{1,1}$ eigenvalues of the state and relates to the 5D bulk mass-squared m_{5d}^2, and m_n is the 4D mass. The 5D mass was calculated for all the supergravity fields in ref. [12], which for the LMCS $b(x)$ is $H_{\{\nu\}} = 6\left[j(j+1) + l(l+1) - r^2/8\right]$, $m_{5d}^2 = H_{\{\nu\}} + 16 - 8\sqrt{H_{\{\nu\}} + 4}$. $\{\nu\}$ is the set of quantum numbers denoting the charge in the $SU(2)\times SU(2)/U(1)$ isometries of the $T^{1,1}$. Although the $T^{1,1}$ harmonics give a bulk mass to the particles, the radial equation is physically more important since it depends on the warped, or AdS, coordinate r. The radial equation is similar to that of Randall-Sundrum [13] but with a realistic mass spectrum; it results in massive modes (in both the 4D and bulk sense) to be heavily peaked at the bottom of the throat, the IR. The radial solutions for 5D scalars is found to behave as

$$R_n(r) \;=\; \frac{\sqrt{k}\,we^{2kr}}{J_\nu(x_n)}\left[J_\nu\left(x_n we^{kr}\right)+b_{n\nu}Y_\nu\left(x_n we^{kr}\right)\right]\;,\;\nu=\sqrt{4+H_{\{\nu\}}}\;, \quad (3.56)$$

where $w = e^{-kr_c}$ is the warping, and $x_n \equiv \frac{m_n e^{kr_c}}{k} \sim \mathcal{O}(1)$; that is, m_n is at the inflationary scale, while $m_n e^{kr_c}$ is the AdS scale. For massive states, the asymptotic behaviour of the radial solution is dominated by the J_ν solution which peaks in the IR, while both solutions are of the same order in the UV. Through a careful limiting procedure one can recover the massless solution of a constant wavefunction in the bulk, normalized at the string scale. With this semi-qualitative understanding of the radial wavefunction we gain the naive insight that if more massive modes enter a vertex the radial integration is more weighted, so we expect a larger effective 4D coupling between massive states. This may further dampen our optimism since the LMCS can only decay into massless states, but it also confirms our intuition that very massive states can easily decay into several lighter states. This insight was the basis of claims for initial investigations of warped reheating, where only the radial behaviour was tracked [2, 3, 4, 5], and reheating was found to be extremely quick.

As will be seen, there are numerous couplings of the LMCS to 5D vector fields in addition to scalar couplings. Additionally, in other backgrounds the LMCS may correspond to a 5D vector itself. They will turn out to have a slightly different radial behaviour which can effect conclusions meant to apply to scalar fields. For these reasons, we present the dimensional reduction and analysis of spin 1 fields to add to the base of literature dealing with 1-forms, and to remind readers of previous results [20]. Following the notation of Ceresole et al. [12] (eq. 3.51d), we expand the

5D vector fluctuation of the 4-form as

$$A_{\mu abc}(x,r) \;=\; \sum_{\{\nu\}} \phi_\mu^{\{\nu\}}(x) Y_{abc}^{\{\nu\}}(y)\,. \tag{3.57}$$

Note we temporarily use the notation $\alpha\,,\beta\,,\gamma\,,\delta \in \{x^\mu, r\}$ for the AdS_5 directions, while $\mu\,,\nu \in \{x^\mu\}$ denote our 4-brane. From the kinetic term

$$\begin{aligned} S_{kin}(\phi_r) &= \frac{-1}{480\kappa_0^2}\int d^{10}x\sqrt{-G}\,G^{cq}G^{ds}G^{et}\left[G^{\alpha\beta}G^{\gamma\delta}\partial_{[\alpha}A_{\gamma]cde}\partial_{[\beta}A_{\delta]qst}\right] \\ &= \frac{-1}{480\kappa_0^2}\sum_{\{\mu\}\,\{\nu\}}\int d^5x\sqrt{-|G_{AdS_5}|}\,G^{\alpha\beta}G^{\gamma\delta}\partial_{[\alpha}\phi_{\gamma]}^{\{\mu\}}\partial_{[\beta}\phi_{\delta]}^{\{\nu\}} \\ &\quad\cdot\int d^5y\sqrt{|G_{T^{1,1}}|}\,G^{cq}G^{ds}G^{et}Y_{cde}^{\{\mu\}}Y_{qst}^{\{\nu\}} \end{aligned} \tag{3.58}$$

we identify the canonical-normalization constraint

$$\frac{1}{120}\int d^5y\sqrt{|G_{T^{1,1}}|}\,G_{(0)}^{cq}G_{(0)}^{ds}G_{(0)}^{et}Y_{cde}^{\{\nu\}}Y_{qst}^{\{\mu\}} = \delta_{\{\mu\},\{\nu\}} \tag{3.59}$$

which leaves a canonically normalized action for a vector field ϕ_γ:

$$S_{kin}(\phi_r) \;=\; \frac{-1}{4\kappa_0^2}\int d^5x\sqrt{-|G_{AdS_5}|}\,G^{\alpha\beta}G^{\gamma\delta}\partial_{[\alpha}\phi_{\gamma]}\partial_{[\beta}\phi_{\delta]}\,. \tag{3.60}$$

From this point on the discussion may be applied to any 5D vector field, the difference coming from the orthogonality conditions over the internal directions and the bulk mass generated by the harmonics over these directions. Variation of the above, together with a mass term $m_{5d}^2\phi^\alpha\phi_\alpha$ (which comes from derivatives over the $T^{1,1}$ directions) gives

$$\delta S \;=\; -\frac{1}{2\kappa_0^2}\int d^5x\left[-\partial_\alpha\left(\sqrt{-|G_{AdS_5}|}\,G^{\alpha\beta}G^{\gamma\delta}F_{\beta\delta}\right) - m_{5d}^2G^{\gamma\delta}\phi_\delta\right]\delta\phi_\gamma\,, \tag{3.61}$$

where $F_{\beta\delta} = \partial_{[\beta}\phi_{\delta]}$. From the previous equations of motion one finds the radial profile for each component of the 5D vector satisfies Bessel's equation, behaving as

$$R_{1-form}(r) \simeq \sqrt{k}\, we^{kr}\left[J_\nu\left(x_n we^{kr}\right) + b_{n\nu}Y_\nu\left(x_n we^{ky}\right)\right],\ \nu = \sqrt{1+m_{5d}^2}\,. \qquad (3.62)$$

In comparison with the scalar field behaviour of eq. (3.56), the important differences are the single power of the warpfactor in front of the Bessel functions and the different index on the Bessel functions; these differences affect the behaviour at the bottom and top of the throat respectively. In the asymptotic limit the radial wavefunction behaves as $R(r) \propto exp\left(\frac{1}{2}kr\right)$, which is much less peaked than the scalar behaviour $R(r) \propto exp\left(\frac{3}{2}kr\right)$. This may cause some concern because the radial overlap with a 1-form seems suppressed, so one naively expects decays involving 5D vectors to be subdominant with similar decays involving scalars. In the context of turning on a source for this field though, we actually gain an advantage. As will be shown in the next section, the 4D effective vertex receives contributions from the UV and IR regions of the AdS coordinate, r. Background corrections are suppressed in the IR while massive fluctuations are suppressed in the UV, and the scale of the 4D coupling is determined by the dominant contribution. For a vertex involving massive fluctuations, the wavefunction evaluated at the top of the throat behaves as $R(w) \simeq w\sqrt{k}\,J_\nu(w) \sim w^{1+\nu}$ for both scalars and vectors. However since vector fields have $\nu = \sqrt{1+m_{5d}^2}$ and scalars $\nu = \sqrt{4+m_{5d}^2}$, a scalar with the same bulk mass will be more suppressed. So one may conclude that isometry-breaking decays in backgrounds with a vector LMCS decay faster than a background with a scalar LMCS.

Appendix 1.2: Background Affects: introducing a UV source

In the previous section we determined the radial behaviour of 5D scalar and vector fields. We found two Bessel-function solutions that were constrained by a Z_2 symmetry imposed in both the UV and IR, analogous to the RSII setup. Qualitatively, both vector and scalars were peaked in the IR and suppressed in the UV, but scalars were seen to be more peaked and vectors less-suppressed. Although we have belaboured it, this is an important result because the LMCS is forbidden to decay in the $T^{1,1}$ background, but the Calabi-Yau (CY) region induces deformations of the background. As will be shown, these deformations are also suppressed by the radial behaviour of the mode being perturbed, but they also allow for momenta-violating decays since the geometry is being deformed.

To track the Calabi-Yau affects on the AdS background we introduce a source term localized in the UV or CY region that can support the long-lived relic. Owing to the complicated nature of the Calabi-Yau space we should include source terms for all possible modes perturbed by the UV geometry, but we only need to track UV sources which can accommodate the decay of the LMCS. At the level of the action, the presence of the CY introduces a source located at the top of the throat; we track a source $S^{abcd}_{\{\nu\}}$ supporting the b-field with quantum numbers $\{\nu\}$:

$$
\begin{aligned}
S_{\{\nu\}} &= \int d^{10}x\sqrt{-G}\, S^{abcd}_{\{\nu\}} A_{abcd} \\
&= \int d^{10}x\sqrt{-G}\, \underbrace{\tilde{\phi} M_s^4 \delta(r) G^{an}_{(0)} G^{bq}_{(0)} G^{cs}_{(0)} G^{dt}_{(0)} G^{ap}_{(0)} \epsilon_{nqst}{}^{f} \mathcal{D}_f Y_{\{\nu\}}}_{source} \\
&\qquad \cdot \underbrace{\sum_{\{\nu'\}} b_{\{\nu'\}}(x) \epsilon_{abcd}{}^{g} \mathcal{D}_g Y_{\{\nu'\}}(y)}_{field}\ .
\end{aligned}
\tag{3.63}
$$

The source introduces a defect supported in the UV by $\delta(r)$, and carries $T^{1,1}$ charge in the harmonic $Y_{\{\nu\}}$. The strength of the source is parameterized by $\tilde{\phi}$, and is naturally the string scale. The contribution of $\tilde{\phi}$ parametrizes the strength of the UV source and carries a constant profile along our four directions in order to preserve Lorentz invariance. One may propagate the deformation down the throat using Green's function solutions expanded in terms of the radial eigenmodes; or alternatively, one may directly solve the radial EOM (3.55). In the latter case, the source acts like a UV boundary condition, so the correction to massless bosons induced by the finite throat satisfies

$$\Box_r R_{0,\{\nu\}} - m_{5d}^2 e^{-4kr} R_{0,\{\nu\}} \;=\; \tilde{\phi}\delta(r)\,. \tag{3.64}$$

This gives two particular solutions, though one is subdominant over the entire domain so we are left with (the tadpole shift)

$$\Rightarrow R_{0,\{\nu\}}(r) \;=\; \sqrt{k}\tilde{\phi}e^{(2-\nu)kr} = \sqrt{k}\,\tilde{\phi}\,e^{(4-\Delta_{\{\nu\}})kr}\,,\; \nu = \sqrt{4+m_{5d}^2}\,,\; \Delta = \nu + 2\,, \tag{3.65}$$

where Δ is the conformal dimension of the operator. This is one of the more important results of this paper, representing the effect of propagating an isometry-breaking source (parameterized by $\tilde{\phi}$) down the throat. In order to preserve the background solution we demand that this solution decay in the IR, giving the constraint that only sources with conformal dimension $\Delta > 4$ are turned on.[2]

[2] Note that the same procedure indicates that corrections to AdS_5 vectors must satisfy $\Delta > 2$, while even higher-spin objects cannot be sourced in the AdS_5 without ruining Lorentz invariance. In this paper we are only concerned with corrections

Ultimately we are interested in the 4D effective coupling for a vertex which will allow the LMCS to decay. A possible vertex will contain, at least, contributions to the radial integral coming from the LMCS and from the background correction. Integrating over the radial dimension gives the overlap of these wavefunctions; but since the tadpole and LMCS peak in different regimes the effective coupling will typically be small. One may try searching for vertices with more overlap but one is restricted by phase space to massless modes which are constant in the radial direction. So, the best-case scenario should generically be a vertex involving only one LMCS and one insertion of the background correction, other contributions to the radial integral will correspond to massless modes which are constant at the string scale. Before quantifying these statements (see Appendix 3), we first attempt to find a vertex which can form a $T^{1,1}$ singlet involving the LMCS and one insertion of a background correction, and which can proceed kinematically.

Appendix 2: Tadpole Decay—possible decay channels

In this section we consider each coupling of the LMCS, and the resultant 4D coupling coming from the possible vertices in eq. (3.53). The 4D interaction is obtained by integrating out the $T^{1,1}$ and AdS directions to obtain an effective coupling. The explicit evaluation of the internal angular-integrals is left for a separate section because they are often complicated in nature.

Owing to the countably-large number of polarizations of the 5-form field strength, the number of possible decay channels is quite overwhelming. To proceed we break up the search by exhausting the various cases in a methodical fashion;

---

to the scalar background, not sources effecting vectors. The understanding of this radial behaviour is still important for vector fluctuations.

Table 3–3: The conformal dimension for possible sources supporting the tadpole shift. Also listed are the masses of the uncharged state; this is important in phase-space considerations of possible decay channels.

5D field	(j,l,r)	Δ	(j,l,r) of $m_{5D}^2=0$
$b(x)$	(1,0,0)	VMI: 2	(0,0,0)
$b(x)$	(1,1,0)	VMI: 3.29	
$\phi(x)$	(1,0,0)	VMI: ≤ 4	(1,0,0)
$B_\mu(x)$	(1,0,0)	GM: 7	NA
$\phi_\mu(x)$	(1,0,0)	GM: 5	(0,0,0), (1,0,0)
$\pi(x)$	(1,0,0)	VMII: 10	NA
$b^\pm_{\mu\nu}(x)$	(1,0,0)	GM: 6	NA

starting from the action (3.53) we consider each polarization of the second 4-form separately. The search is aided by the harmonic expansion for the various fields given in Table 3–1, and many possibilities may be ruled out by the fact that we can only turn on a source for states corresponding to irrelevant operators. The search is further aided by kinematic considerations, and we will disregard couplings with fluctuations of particles that aren't massless in the $T^{1,1}$ background. The relevant tadpoles and their conformal dimension are listed in Table 3–3. Also listed is the $T^{1,1}$ charge giving rise to a massless bulk state, which shows we can only couple to A_{abcd}, H_{ab}, $A_{\mu bcd}$, and $h_{\mu\nu}$ since other fields don't have massless states. The interaction vertices analyzed here include only one insertion of a background correction. This is similar to the attempts in ref. [9], which assumed the least irrelevant operator preserving SUSY would allow the LMCS to decay. In this section we will see that such a term will not suffice. Furthermore, we will show in the next section that the behaviour of the LMCS sets the decay scale, not the symmetry-breaking operator. This leads to a faster decay rate than previously reported.

Returning to the IIB supergravity with one of the 4-forms to correspond to the LMCS $A_{abcd}(X^M) = \sum_{\{\nu\}} b(x)\epsilon_{abcd}{}^f \mathcal{D}_f Y^{\{\nu\}}(y)$ leaves

$$S_{IIB}(A_{ABCD}) \;=\; \frac{-1}{480\kappa_0^2}\int d^{10}x\sqrt{-G}\left[G^{A_1B_1}G^{a_2B_2}G^{a_3B_3}G^{a_4B_4}G^{a_5B_5}\right. \\ \left.\cdot\partial_{A_1}A_{a_2a_3a_4a_5}\partial_{[B_1}A_{B_2B_3B_4B_5]}\right]\;. \tag{3.66}$$

In the following subsections we go over the possible tadpoles that may be turned on, discussing the origin of the coupling, the possible decay channel, and the resultant angular integral. The sections are divided according to the polarization of the second 4-form in eq. (3.66). The first scenario is described in the greatest detail to acquaint the reader with the possible arguments used to exclude decay channels.

Appendix 2.1: $\{B_2, B_3, B_4, B_5\} \in T^{1,1}$:

$$\sqrt{-G}\left[G^{A_1B_1}G^{a_2b_2}G^{a_3b_3}G^{a_4b_4}G^{a_5b_5}\partial_{A_1}A_{a_2a_3a_4a_5}\partial_{B_1}A_{b_2b_3b_4b_5}\right] \tag{3.67}$$

In this case both 4-forms are polarized along the $T^{1,1}$. The first 4-form is fixed to be the LMCS while the second can be thought of as a sum over all possible states. In order for the decay to proceed one of the fields in eq. (3.66) must be taken as the background correction; this could either be a graviton or the other 4-form.

First let's consider the LMCS decay proceeding through the tadpole of a background metric $G_{a_2b_2}$, so we take all other fields to be massless fluctuations or background values. The 4-form is massless when it is a $T^{1,1}$ singlet, while the massless graviton corresponds to a state singly-charged in one the $SU(2)$'s of $T^{1,1}$,

$G^{ab} = \phi_{\{1,0,0\}}(x) Y^{(ab)}_{\{1,0,0\}}(y)$

$$I \;=\; \frac{-1}{480\kappa_0^2}\int d^{10}x\sqrt{-G}\left[G^{A_1B_1}\underbrace{G^{a_2b_2}}_{tadpole}G^{a_3b_3}G^{a_4b_4}G^{a_5b_5}\partial_{A_1}\underbrace{A_{a_2a_3a_4a_5}}_{LMCS}\partial_{B_1}\underbrace{A_{b_2b_3b_4b_5}}_{(0,0,0)-fluct.}\right]. \tag{3.68}$$

The next step would be to identify the background correction which has nonzero angular overlap; but before finding this state we note this interaction disappears for a simple reason. For kinematic reasons we must take a massless 4-form fluctuation, resulting in a state which is constant in the $T^{1,1}$. The 4-form, however, is expanded in derivatives over the scalar harmonics, so this possibility will always vanish.

This leaves the possibility of turning on a tadpole for the 4-form, and leaving massless fluctuations of the metric. The correction must correspond to an irrelevant operator, so the dominant correction has $T^{1,1}$-charge $\{\nu\} = \{1,0,0\}$, $A^{\{\nu\}}_{abcd} = b^{\{\nu\}}(r)\epsilon_{abcd}{}^{e}\mathcal{D}_e Y^{\{\nu\}}(y)$.

$$I \;=\; \frac{-1}{480\kappa_0^2}\int d^{10}x\sqrt{-G}\left[G^{A_1B_1}\underbrace{G^{a_2b_2}G^{a_3b_3}G^{a_4b_4}G^{a_5b_5}}_{massless-fluct}\partial_{A_1}\underbrace{A_{a_2a_3a_4a_5}}_{LMCS}\partial_{B_1}\underbrace{A_{b_2b_3b_4b_5}}_{tadpole}\right]. \tag{3.69}$$

The expression simplifies by recognizing the tadpole is constant in $\mathcal{M}_4$, so $B_1 \in \{r, T^{1,1}\}$, and four combinations for the $G^{A_1B_1}$ remain:

- $A_1 = r$, $B_1 \in T^{1,1}$ or $B_1 = r$, $A_1 \in T^{1,1}$. In this case we must take $G^{A_1B_1}$ as a fluctuation since it vanishes in the background. This is always a massive fluctuation though, so the decay cannot proceed.
- $A_1 = r$, $B_1 = r$. Then $G^{A_1B_1}$ can be a background value, or a massless fluctuation $G^{rr} = \pi_{\{\nu\}}(x)Y^{(ab)}_{\{\nu\}}$ with $\nu = (j,l,r) = (0,0,0)$.

- $A_1,\, B_1 \in T^{1,1}$. Then $G^{A_1 B_1}$ can be a background value, or a massless fluctuation $G^{a_1 b_1} = \phi_{\{\nu\}}(x) Y^{(a_1 b_1)}_{\{\nu\}}(y)$ with $\nu = (j, l, r) = (1, 0, 0)$.

Thus it remains to check the angular integrals coming from the last two items above, giving a decay into two massless gravitons with the following 5D coupling and angular integral $\hat{F}(\Psi)$:

- 5D coupling:

$$\frac{\sqrt{|G_{AdS}|}}{2\kappa_0^2} G^{rr} \underbrace{\phi^2_{(1,0,0)}}_{(1,0,0)-fluct.} \partial_r \left(\underbrace{b_{(1,0,0)}(x)}_{LMCS} \right) \partial_r \left(\underbrace{b_{(1,1,0)}(r)}_{tadpole} \right) \cdot \hat{F}(\Psi) \tag{3.70}$$

$$\hat{F}(\Psi) \;=\; \int \sqrt{|G_{T^{1,1}}|}\, d^5y\, Y^{(a_2 b_2)}_{(1,0,0)} Y^{(a_3 b_3)}_{(1,0,0)} G^{a_4 b_4} G^{a_5 b_5} \epsilon_{a_2 a_3 a_4 a_5}{}^{f} \mathcal{D}_f Y_{(1,0,0)} \epsilon_{b_2 b_3 b_4 b_5}{}^{f} \mathcal{D}_f Y_{(1,1,0)} \tag{3.71}$$

- 5D coupling:

$$\frac{\sqrt{|G_{AdS}|}}{2\kappa_0^2} \underbrace{\phi^2_{(1,0,0)}}_{(1,0,0)-fluct.} \underbrace{b_{(1,0,0)}(x)}_{LMCS} \underbrace{b_{(1,1,0)}(r)}_{tadpole} \cdot \hat{F}(\Psi) \tag{3.72}$$

$$\begin{aligned} \hat{F}(\Psi) \;=\; & \int \sqrt{|G_{T^{1,1}}|}\, d^5y\, Y^{(a_1 b_1)}_{(1,0,0)} Y^{(a_2 b_2)}_{(1,0,0)} G^{a_3 b_3} G^{a_4 b_4} G^{a_5 b_5} \cdot \\ & \partial_{a_1} \left(\epsilon_{a_2 a_3 a_4 a_5}{}^{f} \mathcal{D}_f Y_{(1,0,0)} \right) \partial_{b_1} \left(\epsilon_{b_2 b_3 b_4 b_5}{}^{f} \mathcal{D}_f Y_{(1,1,0)} \right) . \end{aligned} \tag{3.73}$$

Without explicitly evaluating the above angular integrals $\hat{F}$, we expect these terms to disappear owing to the even/odd behaviour of the integrand, though this needs to be verified. One could also look for overlap with more-irrelevant tadpoles corresponding to states like $\nu = (2, 2, 0)$, but this results in a more massive source and greater suppression factor in the effective coupling. The final step of integrating out the radial behaviour is saved for Section 3.5, but assuming a nonzero overlap of the angular wavefunctions we can read off the 5D coupling from the first line in eqs. (3.71) and (3.73).

Appendix 2.2: $\{B_2\} \in AdS_5\,,\ \{B_3, B_4, B_5\} \in T^{1,1}$:

$$\sqrt{-G}\,\left[G^{A_1B_1}G^{a_2\mu}G^{a_3b_3}G^{a_4b_4}G^{a_5b_5}\partial_{A_1}A_{a_2a_3a_4a_5}\partial_{B_1}A_{\mu b_3b_4b_5}\right] \tag{3.74}$$

In this case the second 4-form is polarized as a 5D vector. An immediate simplification of the above $G^{a_2\mu} = B^\mu(x)Y^{a_2}(y)$ cannot take its background value since it is a product space, resulting in trivial cross-terms otherwise. Furthermore, since this (5D) particle $B^\mu(x)$ is always massive we must use it as the tadpole instead of a fluctuation. With the harmonic expansion

$$A_{\mu abc} \;=\; \sum_{\{\nu\}}\phi^{\{\nu\}}_\mu(x)Y^{\{\nu\}}_{abc}(y) = \sum_{\{\nu\}}\phi^{\{\nu\}}_\mu(x)\epsilon_{abc}{}^{de}\mathcal{D}_dY^{\{\nu\}}_e(y) \tag{3.75}$$

and the vector representation [12]

$$Y^{\{\nu\}}_e \;=\; \left\{\begin{array}{c} Y^{(j,l,r+1)}_{+1} \\ Y^{(j,l,r-1)}_{-1} \\ Y^{(j,l,r-1)}_{+1} \\ Y^{(j,l,r+1)}_{-1} \\ Y^{(j,l,r)}_{0} \end{array}\right\} \tag{3.76}$$

we see that there is, in principle, the possibility of overlap with the LMCS since the $e = 5$ component is the same scalar harmonic as the LMCS. Additionally the second 4-form must be in the massless (though charged) $\nu = (1, 0, 0)$ state since the vector-polarization of the 4-form $A_{\mu abc}$ is constant in the background and massive otherwise. Finally, any scalar metric-fluctuations must carry charge $(1,0,0)$ to be massless in the bulk.

With $G^{a_2\mu} = B^\mu(x) Y^{a_2}_{(1,0,0)}$ and $B^\mu = \tilde{\phi}\epsilon^\mu(r)e^{(1-\nu)kr}$ as a (vector) background correction carrying strength $\tilde{\phi}$, we are left with two possible decay channels (corresponding to $A_1 = B_1 = r$, and $A_1\,,\, B_1 \in T^{1,1}$).

- 5D coupling:

$$\frac{\sqrt{-|G_{AdS_5}|}}{2\kappa_0^2}\, G^{rr}\, \underbrace{B^\mu(r)}_{tadpole}\,\underbrace{\phi_{(1,0,0)}(x)}_{met.-fluct}\,\partial_r\left(\underbrace{b_{(1,0,0)}(x)}_{LMCS}\right)\partial_r\left(\underbrace{\phi^{(1,0,0)}_\mu(x)}_{vect.-fluct.}\right)\cdot \hat{F}(y)$$

$$\hat{F}(y) \;=\; \int d^5y\sqrt{G_{T^{1,1}}}\;\underbrace{Y^{a_2}_{(1,0,0)}}_{tadpole}\,Y^{(a_3b_3)}_{(1,0,0)}\,\underbrace{G^{a_4b_4}G^{a_5b_5}}_{background}\,\epsilon_{a_2a_3a_4a_5}{}^{e}\mathcal{D}_e\,\underbrace{Y_{(1,0,0)}}_{LMCS}\,\epsilon_{b_3b_4b_5}{}^{fg}\mathcal{D}_f\,\underbrace{Y^{(1,0,0)}_g}_{m_{\phi_\mu}=0} \tag{3.77}$$

- 5D coupling:

$$\frac{\sqrt{-|G_{AdS_5}|}}{2\kappa_0^2}\,\underbrace{B^\mu(r)}_{tadpole}\,\underbrace{\phi_{(1,0,0)}(x)}_{met.-fluct}\left(\underbrace{b_{(1,0,0)}(x)}_{LMCS}\right)\left(\underbrace{\phi^{(1,0,0)}_\mu(x)}_{vect.-fluct.}\right)\cdot \hat{F}(y)$$

$$\begin{aligned}\hat{F}(y) \;=\; & \int d^5y\sqrt{G_{T^{1,1}}}\;\underbrace{Y^{a_2}_{(1,0,0)}}_{tadpole}\,Y^{(a_1b_1)}_{(1,0,0)}\,\underbrace{G^{a_3b_3}G^{a_4b_4}G^{a_5b_5}}_{background}\,\partial_{a_1}\left(\epsilon_{a_2a_3a_4a_5}{}^{e}\mathcal{D}_e\;\underbrace{Y_{(1,0,0)}}_{LMCS}\right)\\ & \cdot\partial_{b_1}\left(\epsilon_{b_3b_4b_5}{}^{fg}\mathcal{D}_f\,\underbrace{Y^{(1,0,0)}_g}_{m_{\phi_\mu}=0}\right)\end{aligned} \tag{3.78}$$

To evaluate $\hat{F}(y)$ we need the vector harmonics for $Y^{(1,0,0)}_a$ and the tensor harmonics for $Y^{(1,0,0)}_{(ab)}$. If $\hat{F}(y) \neq 0$ then we have found a viable decay channel. Of all the vertices we have identified this seems like the most-likely candidate.

A possible pitfall for these two interaction terms is that they can give rise to 4D effective vertices of the type:

$$g_{eff} = \int d^4x \sqrt{|\eta|^2} \epsilon^\alpha \phi_\alpha^{(1,0,0)}(x) b_{(1,0,0)}(x) \phi_{(1,0,0)}(x) \,. \tag{3.79}$$

The polarization tensor of the background correction ϵ^α picks a preferred direction in spacetime and breaks Lorentz invariance or 3D rotational invariance. This imposes constraints on the magnitude of the time component of ϵ^α, and can modify the dispersion relation for 4D gravitons [25]. One must be careful to use only the radial polarization of the background ϵ^μ, which also picks out the scalar fluctuation of 5D vector α_μ.

Appendix 2.3: $\{B_2, B_3\} \in AdS_5$, $\{B_4 B_5\} \in T^{1,1}$:

$$\sqrt{-G}\left[G^{A_1B_1}G^{a_2\mu}G^{a_3\nu}G^{a_4b_4}G^{a_5b_5}\partial_{A_1}A_{a_2a_3a_4a_5}\partial_{B_1}A_{\mu\nu b_4b_5}\right] \tag{3.80}$$

We expand the fields as

$$\begin{aligned} A_{\mu\nu cd} &= \sum_{\{\nu\}} b^{\pm}_{\mu\nu}(x) Y^{\{\nu\}}_{cd}(y) \\ G_{a\mu} &= \sum_{\{\nu\}} B^{\{\nu\}}_{\mu}(x) Y^{\{\nu\}}_{a} \,. \end{aligned} \tag{3.81}$$

Employing phase-space considerations and recalling the background is a product space we must have, in this case, both $G^{a_2\mu}$ and $G^{a_3\nu}$ as tadpole insertions. This will bring twice the warpfactor suppression in the decay channel making this a sub-dominant process. Ignoring the kinematics for a moment, we will find a second reason to believe this process does not contribute to the decay width.

If we insert a tadpole for $b^{\pm}_{\mu\nu}(x)$, it is constant in $4D$ to preserve Lorentz-invariance, we are left with $b^{\pm}_{\mu\nu}(x) = b^{\pm}_{\mu\nu}(r)$, so that $A_1 = B_1 = r$ or $A_1\,, B_1 \in T^{1,1}$, and this interaction reduces to

$$\sqrt{-G}\left[G^{A_1B_1}B^{\mu}\epsilon^{a_2}B^{\nu}\epsilon^{a_3}G^{a_4b_4}G^{a_5b_5}\partial_{A_1}\left(b\epsilon_{a_2a_3a_4a_5}{}^{e}\mathcal{D}_eY_{(1,0,0)}\right)\partial_{B_1}\left(\epsilon_{\mu\nu}Y^{(1,0,0)}_{b_4b_5}\right)\right]. \tag{3.82}$$

In this form it is evident that in order to preserve 4D Lorentz invariance we require $\mu = \nu = r$ for the background polarization $\epsilon_{\mu\nu}$, otherwise we pick a preferred direction. However, owing to the antisymmetric nature of the 4-form $\epsilon_{\mu\mu} = 0\,\forall\,\mu$ so this process cannot proceed.

Again, switching on two graviphoton tadpoles instead does not help the situation because $b^{\pm}_{\mu\nu}$ always carries a 5D mass in the $T^{1,1}$ background, so we are still kinematically suppressed.

Appendix 2.4: $\{B_2,\ B_3,\ B_4\} \in AdS_5$, $\{B_5\} \in T^{1,1}$:

$$\sqrt{-G}\left[G^{A_1B_1}G^{a_2\mu}G^{a_3\nu}G^{\rho b_4}G^{a_5b_5}\partial_{A_1}A_{a_2a_3a_4a_5}\partial_{B_1}A_{\mu\nu\rho b_5}\right] \tag{3.83}$$

Now we must turn on at least three graviphoton fluctuations. The arguments of last section apply here again: all graviphotons must be tadpoles to accommodate the LMCS decay, thus introducing a large warpfactor suppression in the IR.

Ignoring phase space considerations and instead taking $a_{\mu\nu\rho}$ as the tadpole introduces a polarization tensor which breaks 4D Lorentz invariance so this channel is not viable as well.

Appendix 2.5: $\{B_2,\ B_3,\ B_4,\ B_5\} \in AdS_5$:

$$\sqrt{-G}\left[G^{A_1B_1}G^{a_2\mu}G^{a_3\nu}G^{\rho b_4}G^{a_5\alpha}\partial_{A_1}A_{a_2a_3a_4a_5}\partial_{B_1}A_{\mu\nu\rho\alpha}\right] \tag{3.84}$$

As with the previous two subsections we must use tadpole fluctuations for each radial graviton, making this a very suppressed decay. Or, we could take $A_{\mu\nu\rho\alpha} = \epsilon_{\mu\nu\rho\alpha}{}^{\delta}\partial_\delta\left(b(x)\right)Y^\nu(y)$ as the tadpole, but this violates 4D Lorentz-invariance and is kinematically forbidden.

Appendix 2.6: Higher-Order in $\sqrt{-\det|G|}$:

At this point all possible interactions to zeroth order in $\det|G|$ have been exhausted. This led to the identification of four possible decay channels expressed in eqs. (3.71, 3.73, 3.77, 3.78). In this section we look at couplings coming in at higher order in $\det|G|$ with the hopes of identifying more possible channels. The tables of Ceresole et al. [12] give the 5D mass spectrum for each polarization of the graviton so we first look at fluctuations from the 5D point of view. In general the expansion can be performed as follows:

$$\begin{aligned}&\det\left(A+\epsilon B\right) = \det A\det\left(\mathbb{1}+\epsilon A^{-1}B\right) = \det A e^{\log\det\left(\mathbb{1}+\epsilon A^{-1}B\right)} = \\ &\det A e^{\mathrm{Tr}\ \log\left(\mathbb{1}+\epsilon A^{-1}B\right)} = \det A e^{\mathrm{Tr}\ \left(\epsilon A^{-1}B-\frac{1}{2}\epsilon^2A^{-1}BA^{-1}B+\ldots\right)} = \\ &\det A\left[1+\epsilon\mathrm{Tr}\ A^{-1}B+\frac{\epsilon^2}{2}\left[\left(\mathrm{Tr}\ A^{-1}B\right)^2-\mathrm{Tr}\ A^{-1}BA^{-1}B\right]+\mathcal{O}\left(\epsilon^3\right)\right].\end{aligned} \tag{3.85}$$

Applying the above result to the metric fluctuations, but with

$$A = \begin{pmatrix} G^{AdS_5}_{\mu\nu} & 0 \\ 0 & G^{T^{1,1}}_{ab}\end{pmatrix}\ ,\ B = \begin{pmatrix} H_{\mu\nu} & H_{\mu a} \\ H_{\mu a} & H_{ab}\end{pmatrix} \tag{3.86}$$

we obtain:

$$\sqrt{-G} = \sqrt{-G_{AdS_5}}\,\sqrt{G_{T^{1,1}}}\,\Big[1 + G^{\mu\nu}_{AdS} H_{\mu\nu} + G^{ab}_{T^{1,1}} H_{ab} + \left[G^{\mu\nu} G^{ab} H_{\mu\nu} H_{ab} - G^{\mu\nu} G^{ab} H_{\mu a} H_{\nu b}\right] + \mathcal{O}\left(\epsilon^3\right)\Big]\ . \tag{3.87}$$

The couplings involving the graviphoton $H_{\mu a}$ and the scalar polarization H_{ab} do not provide couplings not considered in the previous subsections. As before, $H_{\mu a}$ has no massless fluctuations so the last term above is kinematically forbidden. The first-order coupling to $H_{\mu\nu}$ is a new interaction, providing couplings to the 4D graviton and the scalar polarization H_{rr}, which both have massless fluctuations when they carry no internal momenta.

To isolate the 4D couplings we write the fluctuations of the AdS_5 metric as

$$H_{\mu\nu} = \begin{pmatrix} e^{-2kr}\left(\eta_{\mu\nu} + h_{\mu\nu}\right) & 0 \\ 0 & 1 + h_{rr} \end{pmatrix} \tag{3.88}$$

and repeat the expansion (3.85) with

$$A = \begin{pmatrix} e^{-2kr}\eta_{\mu\nu} & 0 \\ 0 & 1 \end{pmatrix}\ ,\ B = \begin{pmatrix} e^{-2kr} h_{\mu\nu} & 0 \\ 0 & h_{rr} \end{pmatrix}\ . \tag{3.89}$$

This particular parametrization is convenient since the zero mode of the graviton is constant along the radial direction. Then we expand the AdS_5 determinant to second order in the fluctuations:

$$\sqrt{-G_{AdS_5}} = e^{-4kr}\left[1 + \eta^{\mu\nu} h_{\mu\nu} + h_{rr} + \frac{1}{2}\left[\left(\eta^{\mu\nu} h_{\mu\nu} + h_{rr}\right)^2 - \eta^{\mu\beta}\eta^{\nu\alpha} h_{\alpha\beta} h_{\mu\nu} - h_{rr}^2\right]\right] \tag{3.90}$$

The only new possibility is the coupling to the 4D graviphoton. We can derive the vertex responsible for the decay of the b-particles into massless gravitons by considering the term in the action providing this coupling:

$$\frac{-1}{480\kappa_0^2}\int d^{10}x\sqrt{-G}\,G^{bn}G^{cq}G^{ds}G^{et}G^{ap}\partial_a \underbrace{A_{bcde}}_{LMCS}\,\partial_{[p}\underbrace{A_{nqst]}}_{tadpole}\,. \qquad (3.91)$$

Choosing one of the A's to be a tadpole and the other to be a fluctuation mode we obtain the KK number violating vertex involving the b-field and two massless 4D gravitons. The factors of the metric that are contracted with the field-strength F_5 are all coming from the $T^{1,1}$ metric and they do not bring any additional powers of the warp factor. We can see this by looking at the full 10D metric, eq.(31) of [18]:

$$ds_{10}^2 = h^{-1/2}(z)\,ds_4^2 + h^{1/2}(z)\left(dz^2 + z^2 ds_{T^{1,1}}^2\right) \qquad (3.92)$$

$$h(z) = b_0 + 4\pi\frac{g_sN + a\,(g_sM)^2\ln(z/z_0) + a\,(g_sM)^2/4}{z^4} \qquad (3.93)$$

In the near-horizon limit we can approximate $h(z) \sim C^4/z^4$ and the metric takes the form:

$$ds_{10}^2 = \frac{z^2}{C^2}ds_4^2 + \frac{C^2}{z^2}dr^2 + C^2 ds_{T^{1,1}}^2 \qquad (3.94)$$

Finally, defining $\frac{C^2}{z^2}dz^2 = dr^2$ the metric can be brought to the usual Randall-Sundrum form. Taking into account that the massless gravitons have a constant profile along the radial direction, the interaction vertex is then:

$$S_{int} = \frac{-1}{480\kappa_0^2}\int\sqrt{-\eta}d^4x\,\hat{b}(x)\,\frac{\eta^{\mu\nu}\eta^{\alpha\beta}-\eta^{\nu\alpha}\eta^{\mu\beta}}{2}h_{\mu\nu}h_{\alpha\beta}\int dre^{-4kr}\,R_{\text{tadpole}}(r)\,R_b(r)$$
$$\cdot\int\sqrt{-g_{T^{1,1}}}dVol_{T^{1,1}}G^{ap}G^{bn}G^{cq}G^{ds}G^{et}\partial_a\epsilon_{bcde}\ \ {}^fD_fY_{(0)}^{(1,1,0)}\partial_p\epsilon_{nqst}\ \ {}^uD_uY_{(0)}^{(1,1,0)} \qquad (3.95)$$

To complete the dimensional reduction we fold the $T^{1,1}$ integral into some unknown function $\hat{F}(y) \simeq \mathcal{O}(1)$, and refer to eq. (3.98) for the radial integration:

$$S_{int} = \frac{-\hat{F}(y)V_6\tilde{\phi}w^{\nu_L+1}}{480\kappa_0^2}\cdot\int d^4x\sqrt{-\eta}\,\hat{b}(x)\frac{\eta^{\mu\nu}\eta^{\alpha\beta}-\eta^{\nu\alpha}\eta^{\mu\beta}}{2}h_{\mu\nu}h_{\alpha\beta}\,. \qquad (3.96)$$

The result is a similar warpfactor dependence to the scalar polarizations of the graviton found in the previous sections. So the conclusion is that higher orders in the determinant expansion due not provide a faster decay channel. This is consistent with the understanding the any overall scaling of the 4D coupling is coming from the LMCS's contribution to the radial integral and that, provided the decay products are massless, they do not bring in any powers of the warpfactor.

Appendix 3: Effective Couplings and Decay Rates

We have seen there is no direct route for the LMCS to decay in the KS throat but, also, a finite throat feels the presence of UV sources. This results in a correction to the background and introduces a tadpole shift for states being sourced. This shift accommodates the decay because it carries internal quantum numbers, the drawback being the introduction of warpfactor suppression in the effective coupling. To utilize the tadpole shift we require, at least, either quartic vertices which are quadratic in $A_{(4)}$ so that couplings to the tadpole result in trilinear vertices, or we require cubic interactions of the LMCS so that couplings to the tadpole will introduce kinetic or mass-term mixing. Referring to the the type IIB action eq. (3.26) (or eq. (3.51)) we see that all fields satisfy the mode-mixing requirement through their kinetic term coupling to the graviton. For this particular setup, then, the quartic interactions will be the only means to shed charge; but this does not preclude the other mechanism from being important in other theories.

We can now go back to the various terms in the action (3.53) and characterize the allowed decays of the charged LMCS to massless gravitons. The possibilities were discovered in Section 3.5 and resulted in four contributions at 0^{th} order in $\sqrt{det|G|}$ (refer to eqs. (3.71, 3.73, 3.77, 3.78)), plus one new vertex coming in at second order in $\sqrt{det|G|}$. Additionally, a vertex involving insertions of two background corrections was discussed in Section 3.3.2 of the main text. Going back to these vertices, we must now separate the radial behaviour from the 4D fields to (finally) obtain the 4D vertex, including the effective coupling. In two cases the decay product is into scalar-polarizations of the graviton and is mediated by the background correction to the 4-form. Two other possibilities are mediated by a correction to 5D vector-polarizations of the graviton and the decay is into gravitons plus a massless mode of the 4-form, polarized as a 5D scalar. The final case results in decay directly to the 4D graviton, and is mediated by the correction induced by a source supporting the 4-form. Coincidentally, these five vertices have the same 4D coupling strength, so we present only one contribution to the decay with below.

This final step involves inserting the radial profile of the LMCS (eq. (3.56)) plus insertions of the background correction (eq. (3.65)). The easiest vertex to evaluate is eq. (3.73), and we proceed by separation of variables into 4D wavefunctions (hatted) and the radial wavefunction as $b_{\{\nu\}}(x) = \hat{b}_{\{\nu\}}(x^\mu) R_{\{\nu\}}(r)$, $\phi_0(x) = \hat{\phi}_0(x^\mu) R_0(r) \propto \hat{\phi}_0(x^\mu)$ (since the massless graviton is constant along the throat)

$$\begin{aligned}\Rightarrow S_{int} &= \frac{-1}{480\kappa_0^2} \int d^4x \sqrt{-|\eta_{\mu\nu}(x)|}\, \hat{b}_{\{\nu\}} \hat{\phi}_0 \hat{\phi}_0 \cdot \int dr\, e^{-4kr}\, R_{\hat{b}} R_{tadpole}(r) \\ &\cdot M_s w^{\nu-1} \int d^5y \sqrt{-|G_{ab}|} G^{bn}_{(0)} G^{cq}_{(0)} G^{ds}_{(0)} G^{(et)}_{(0)} G^{(ap)}_{(0)} \epsilon_{bcde}{}^{f} \epsilon_{nqst}{}^{g} \partial_a \left(\mathcal{D}_f Y_{\hat{b}}\right) \partial_p \left(\mathcal{D}_g Y_{tadpole}\right) \\ &= \frac{w^{\nu_L - 1} \hat{F}(y)}{480\kappa_0^2} \int d^4x \sqrt{-|\eta_{\mu\nu}(x)|}\, \hat{b} \hat{\phi}_0 \hat{\phi}_0 \cdot \int dr e^{-4kr}\, R_{\hat{b}} R_{tadpole}(r) \qquad (3.97)\end{aligned}$$

where $\hat{F}(y)$ is the $\mathcal{O}(1)$ value of the integral over the $T^{1,1}$.

The radial wavefunctions for all scalars (including the b-field and KK gravitons) is given by eq. (3.56), while the background correction for 5D scalars is given by eq. (3.65). Although all the vertices we have identified involve the LMCS plus one tadpole correction, the radial integral $\mathcal{R}$ can easily be generalized for a 10D vertex with N_m massive modes, N_0 massless modes, and N_t tadpoles (with $\Delta > 4$):

$$\begin{aligned}\mathcal{R} &\simeq \int dre^{-4kr}\left[\Pi_{i=1}^{N_m} R_{\{\nu_{L_i}\}}(r)\right] R_1(r)^{N_0}\left[\Pi_{i=1}^{N_t} R_{t_i}(r)\right] \\ &\simeq w^{N_m}\int dre^{-4kr}\left[\Pi_{i=1}^{N_m} e^{2kr} J_{\nu_i}\left(x_n w e^{kr}\right)\right]\left[\Pi_{i=1}^{N_t}\tilde{\phi}_i e^{(4-\Delta_i)kr}\right] \\ &\sim \underbrace{\left[\Pi_{i=1}^{N_t}\tilde{\phi}_i\right]\left[\Pi_{i=1}^{N_m} w^{\nu_{L_i}+1}\right]}_{UV-contrib} + \underbrace{w^{4-N_m}\left[\Pi_{i=1}^{N_t} w^{\Delta_i-4}\right]}_{IR-contrib}\end{aligned} \tag{3.98}$$

The integral is evaluated by splitting the domain into a small-argument behaviour of $J_{\{\nu\}}\left(x_n w e^{kr}\right)$ (the UV) and a large-argument region (the IR), and $\tilde{\phi}$ is the strength of the UV symmetry breaking. With the understanding that massive modes are heavily suppressed in the UV but normalized in the IR, while the tadpole is suppressed in the IR and of the symmetry-breaking strength in the UV, one can make sense of the differing contributions of eq. (3.98). We see that the radial overlap is strongly enhanced in the IR as the number of massive modes entering the vertex is increased. This is why the decay of heavier states is not a problem. For the LMCS however, phase space considerations will rule out the LMCS decay involving more than one massive mode in the vertex, and one is forced to take $N_m = 1$. This fixes the UV contributions of the radial integral. Multiple insertions of background corrections further suppress the IR contributions, but simply add $\mathcal{O}(1)$ effects to the UV through the symmetry-breaking parameter $\tilde{\phi}$.

Returning to the case of $\hat{b}(x^\mu) \to \hat{\phi}_0(x^\mu) + \hat{\phi}_0(x^\mu)$, $N_m = 1$, $N_t = 1$, and the interaction reduces to

$$S_{int} \;=\; \frac{\left[w^{\nu_L+1} + w^{\nu_{tad}+1}\right]\hat{F}(y)\tilde{\phi}V_6}{480\kappa_0^2}\int d^4x\sqrt{-|\eta_{\mu\nu}|}\,\hat{b}\hat{\phi}\hat{\phi}\,. \tag{3.99}$$

For the $T^{1,1}$ background $\nu_L = 0 < \nu_{tad}$ so the UV behaviour dominates over the IR contribution, and we can read off the effective coupling from above. Employing the generic formula for decay into two massless modes ($m_b \to m_0 + m_0$) [17], the relation $(g_s M_{pl})^2 = \frac{V_6}{2\kappa_0^2}$ and recalling $m_b = M_s x_b w$, $x_a \simeq 1$, we find:

$$\Gamma_{m_b \to 0+0} = \frac{1}{16\pi m_b} \frac{(g_s M_{pl})^2 \, w^{2\nu_L+2} \hat{F}(y)^2 \tilde{\phi}^2}{240^2} = \frac{(g_s M_{pl})^2 \, w^{2\nu_L+1} \hat{F}(y)^2 \tilde{\phi}^2}{16\pi \, x_b M_s \, 240^2} \quad (3.100)$$

Recall that $\nu = \sqrt{4 + H_0}$, $H_0 = 6(j(j+1) + l(l+1) - \frac{r^2}{8})$ for the b-field in the $T^{1,1}$ background. The factor 240^2 in the denominator is largely compensated by the 5! polarizations of the b-field in the $T^{1,1}$ in the scattering amplitude and the orthonormality of the $T^{1,1}$ wavefunction. This is the other important result of this work: it indicates that the LMCS decay rate is generically at the scale $M_{pl} w^{2\nu_L+1}$. For the $T^{1,1}$ background $\nu_L = 0$ so we find $\Gamma \simeq M_{pl} w$, which is the inflationary scale. Thus, there is little concern in this step of the reheating process.

To extend this analysis to other backgrounds we keep the conformal dimension $\Delta = \Delta(\nu)$ of the LMCS as a free parameter. Then, demanding $\Gamma_{LMCS} > H_{BBN}$ gives constraints on either w, the scale of inflation, or we can rule out theories whose LMCS is too massive. These ideas are elaborated in the discussions, Section 3.4.

Appendix 4: Evaluating the Angular Integrals

The success of each decay channel requires evaluating the overlap of the $T^{1,1}$ harmonics for the different couplings. In many cases we could not explicitly evaluate the integrals, though we could verify that the vertex forms a group singlet. This section describes the scalar harmonics and provides some general information necessary for evaluating the internal integrals. The scalar harmonics have been calculated in ref. [23, 24] and have the expressions:

$$Y_L(\Psi) = J_{l_1,m_1,R}(\theta_1)\, J_{l_2,m_2,R}(\theta_2)\, e^{\mathbb{i}m_1\phi_1+\mathbb{i}m_1\phi_1} e^{\frac{\mathbb{i}}{2}R\psi}\,, \tag{3.101}$$

where the functions J are, in turn, given by hyper-geometric functions:

$$\begin{aligned} J^{\Upsilon}_{l,m,R}(\theta) &= N^{\Upsilon}_L (\sin\theta)^m \left(\cot\frac{\theta}{2}\right)^{\frac{R}{2}} {}_2F_1\left(-l+m, 1+l+m, 1+m-\frac{R}{2}; \sin^2\frac{\theta}{2}\right) \\ J^{\Omega}_{l,m,R}(\theta) &= N^{\Omega}_L (\sin\theta)^m \left(\cot\frac{\theta}{2}\right)^{\frac{R}{2}} {}_2F_1\left(-l+\frac{R}{2}, 1+l+\frac{R}{2}, 1-m+\frac{R}{2}; \sin^2\frac{\theta}{2}\right) \end{aligned} \tag{3.102}$$

where J^{Υ} is non-singular for $m \geq R/2$ and J^{Ω} is non-singular for $m \leq R/2$. The particular scalar harmonics that come into play for the states we are interested in correspond to $l = 1$, $m = 0$, $R = 0$. The value of m can be inferred following the calculation of ref. [12] where the parameters r and q were defined in terms of the m_1 and m_2 "magnetic" quantum numbers for each of the $SU(2)$ groups. [$r = m_1 - m_2$, $q = m_1 + m_2$]. $q = 0$ for scalars, so for the state $(j, l, r) = (1, 0, 0)$ we have $m_1 = m_2 = 0$ and the scalar harmonic (we also list some other common harmonics)

$$\begin{aligned} Y_{(1,0,0)}(y) &= J_{1,0,0}(\theta_1)\, J_{0,0,0}(\theta_2) = N_L^2 \cos\theta_1 \\ Y_{(0,1,0)}(y) &= N_L^2 \cos\theta_2 \\ Y_{(1,1,0)}(y) &= N_L^2 \cos\theta_1 \cos\theta_2 \\ Y_{(2,1,0)}(y) &= N_L^2 \left[\cos\theta_2 + \frac{3}{2}\cos(2\theta_1+\theta_2) + \frac{3}{2}\cos(2\theta_1+\theta_2)\right]/4\,. \end{aligned} \tag{3.103}$$

To evaluate the various integrals we also require the expression of $\sqrt{g}$ where g is the metric on the T^{11}. If each $SU(2)$ has Euler-angle coordinatization $(\theta_i, \phi_i, \gamma_i)$, and the left-coset acts to mod-out the γ_i's ($\psi = \frac{1}{\sqrt{2}}(\gamma_1 - \gamma_2)$), the metric has the expression $[\psi, \theta_1, \theta_2, \phi_1, \phi_2]$:

$$g_{ab} = \begin{pmatrix} \frac{1}{9} & 0 & 0 & \frac{\cos\theta_1}{9} & \frac{\cos\theta_2}{9} \\ 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 \\ \frac{\cos\theta_1}{9} & 0 & 0 & \frac{1}{9}\left(1+\frac{\sin^2\theta_1}{2}\right) & \frac{\cos\theta_1\cos\theta_2}{9} \\ \frac{\cos\theta_2}{9} & 0 & 0 & \frac{\cos\theta_1\cos\theta_2}{9} & \frac{1}{9}\left(1+\frac{\sin^2\theta_2}{2}\right) \end{pmatrix} \tag{3.104}$$

with the determinant

$$\sqrt{g} = \frac{|\sin\theta_1|\,|\sin\theta_2|}{\sqrt{11664}} , \tag{3.105}$$

and coordinate ranges

$$\theta_i \in (0, 2\pi) \ , \ \beta_i \in (0, \pi) \ , \ \gamma_i \in (0, 4\pi) \ . \tag{3.106}$$

Appendix 4.1: The vector harmonics

Many of the possible decay channels found in Appendix 2 involved vector harmonics. Let us look here in more detail at the vector harmonics for the quantum numbers $(1, 0, 0)$. The expression is:

$$Y_e^{\{(1,0,0)\}} = \begin{Bmatrix} Y_{+1}^{(j,l,r+1)} \\ Y_{-1}^{(j,l,r-1)} \\ Y_{+1}^{(j,l,r-1)} \\ Y_{-1}^{(j,l,r+1)} \\ Y_{0}^{(j,l,r)} \end{Bmatrix} \Rightarrow \begin{Bmatrix} Y_{+1}^{(1,0,+1)} \\ Y_{-1}^{(1,0,-1)} \\ Y_{+1}^{(1,0,-1)} \\ Y_{-1}^{(1,0,+1)} \\ Y_{0}^{(1,0,0)} \end{Bmatrix} \tag{3.107}$$

The relationship between the q, r and the j_3, l_3 quantum number is given in eq. (3.15) of ref. [12]:

$$j_3 = \frac{q+r}{2} \qquad l_3 = \frac{q-r}{2} \, . \tag{3.108}$$

These are also the m_1, m_2 magnetic quantum numbers needed to find the explicit forms of each component of the vector harmonic.

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CHAPTER 4
Conclusions

It requires a very unusual mind to undertake the analysis of the obvious.—Alfred North Whitehead.

Several recent observational results have indicated shortcomings in our views of particle physics and cosmology. Super-Kamiokande and SNO have provided independent evidence for the existence of neutrino mass, observations of type Ia supernovae show a present-day acceleration indicating the presence of dark energy, and CMB experiments give independent evidence for dark matter. These results provide ample motivation to explore models beyond the Standard Model of particle physics. The Standard Model is a beautiful theory, based on two remarkable, simplifying principles of local gauge invariance and renormalizability. Despite these simplifying notions, we now understand that the SM only describes a small percentage of the energy density comprising this universe. Figure 4–1 shows the percent composition of our universe. The small grey area represents particles that behave according to our knowledge of the Standard Model; the rest of the pie chart is dominated by Dark Energy followed by Dark Matter—in both cases we understand their properties, but we do not know their origins.

String theory is providing explanations for many of the problems at hand. In the realm of cosmology, we have seen that string theory is providing complete pictures of the early universe and, in return, cosmology is constraining the parameter space of string theory. This thesis explored two models of the (very) early universe and has shown, directly, how cosmological constraints do place restrictions on these constructions.

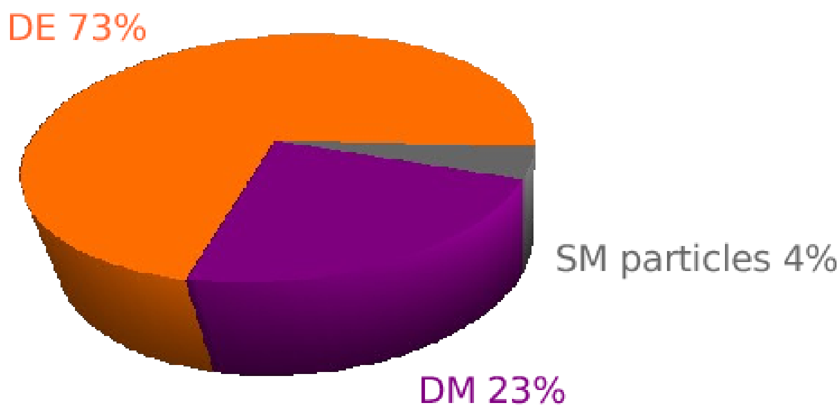

Figure 4–1: A pie-chart representation of the energy budget of the Universe.

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