

New Dig-Limits Optimization Models Incorporating Blast Movement and Grade Uncertainties with Spatial Entropy Analysis in Open Pit Mines

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Abstract

The optimization of dig-limits is an important step in grade control and short-term mine planning due to its crucial role in enhancing the profitability and efficiency of open-pit mining operations by reducing ore loss and dilution. This thesis, through its three journal papers, investigates the challenges posed by blast movement and spatial heterogeneity in ore and waste classification through accounting for these factors when determining optimal dig-limits. The first study introduces a methodology to quantify uncertainty in blast movement and its impact on dig-limits optimization. Using measurement data from blast movement monitoring balls and multivariate distributions fitted via D-vine copula, blast movement realizations are generated through Monte Carlo simulation. A mixed-integer linear programming model then determines optimal dig-limits, resulting in a probabilistic ore probability map. The case study reveals that neglecting blast movement can lead to a significant overestimation of expected profit.

The second study emphasizes the importance of managing ore loss and dilution, which significantly impacts the economic, environmental, and technical outcomes of mining operations. By calculating the newly proposed global and local spatial entropy indices, the study assesses the influence of spatial heterogeneity on ore loss and dilution. High global spatial entropy index values correlate with increased ore loss and dilution, indicating potential profit reduction. The local spatial entropy index is used to identify areas that are suitable for installing blast monitoring balls to reduce material misclassification due to blast movement. Case studies demonstrate the relationship between spatial entropy, cut-off grades, blast movement, and profit, highlighting the necessity for controlled blasting in specific bench sections to mitigate profit reduction.

The third study presents an integrated workflow for optimizing dig-limits under grade and blast movement uncertainties. By incorporating multivariate geostatistical simulation workflows,

including Projection Pursuit Multivariate Transformation and Sequential Gaussian Simulation, the study captures spatial uncertainties in grade distribution and blast movement. The Maximum Expected Profit method is utilized to determine optimal material destinations, enhancing mining profitability. The newly proposed risk-based dig-limits optimization model accommodates mining equipment selectivity, irregular bench shapes, and varying orebody orientations, ensuring reliable post-blast material classifications and improved project profitability and operational efficiency. A case study on a porphyry copper deposit illustrates the significant impact of blast movement on ore loss and dilution, underscoring the need for accurate blast movement modeling in grade control procedures and presenting the outcomes of the integrated workflow proposed in this study.

Through these studies, this thesis provides comprehensive methodologies to address blast movement and spatial heterogeneity challenges in dig-limits optimization, ultimately contributing to more profitable and efficient open-pit mining operations.

Résumé

L'optimisation des limites d'excavation est une étape importante dans le contrôle de la teneur et la planification à court terme des mines en raison de son rôle crucial dans l'amélioration de la rentabilité et de l'efficacité des opérations minières à ciel ouvert, en réduisant la perte de minerai et la dilution. Cette thèse, à travers ses trois articles de journal, examine les défis posés par les mouvements de la masse dynamitée et l'hétérogénéité spatiale dans le cadre de la classification des zones de minerai et de stériles, en tenant compte de ces facteurs lors de la détermination des limites d'excavation optimales. La première étude présente une méthodologie pour quantifier l'incertitude des mouvements de la masse dynamitée et son impact sur l'optimisation des limites d'excavation. En utilisant des données de mesure provenant de balles de surveillance du mouvement de la masse dynamitée et des distributions multivariées ajustées via l'utilisation de copules D-vine, des réalisations de mouvement de la masse dynamitée sont générées par simulation de Monte Carlo. Un modèle de programmation linéaire mixte en nombres entiers détermine ensuite les limites d'excavation optimales, ce qui donne une carte de la probabilité de présence de minerai. L'étude de cas révèle que négliger le mouvement de la masse dynamitée peut conduire à une surestimation significative du profit attendu.

La deuxième étude met l'accent sur l'importance de la gestion des pertes de minerai et de la dilution, qui impactent de manière significative les résultats économiques, environnementaux et techniques des opérations minières. En calculant les nouveaux indices globaux et locaux d'entropie spatiale proposés, l'étude évalue l'influence de l'hétérogénéité spatiale sur les pertes de minerai et la dilution. Des valeurs élevées de l'indice global d'entropie spatiale sont corrélées à une augmentation des pertes de minerai et de la dilution, indiquant une réduction potentielle du profit. L'indice local d'entropie spatiale est utilisé pour identifier les zones adaptées à l'installation de

balles de surveillance du mouvement de la masse dynamitée afin de réduire la mauvaise classification des matériaux due aux mouvements. Les études de cas illustrent les relations entre l'entropie spatiale, les teneurs de coupure, le mouvement de la masse dynamitée et le profit, soulignant la nécessité d'un dynamitage contrôlé dans des sections spécifiques des bancs miniers pour atténuer la réduction des profits.

La troisième étude présente un flux de travail intégré pour optimiser les limites d'excavation en tenant compte des incertitudes de teneur et de mouvement de la masse dynamitée. En incorporant des flux de travail de simulation géostatistique multivariée, incluant l'utilisation de la transformation multivariée par poursuite de projection et la simulation séquentielle gaussienne, l'étude capture les incertitudes spatiales dans la distribution des teneurs et le mouvement de la masse dynamitée. La méthode de l'espérance de profit maximum est utilisée pour déterminer les destinations optimales des matériaux, améliorant ainsi la rentabilité d'exploitation. Le nouveau modèle d'optimisation des limites d'excavation basé sur les risques prend en compte la sélectivité des équipements miniers, les formes irrégulières des bancs miniers et les orientations variées des corps minéralisés, assurant des classifications fiables des matériaux après dynamitage et améliorant la rentabilité et l'efficacité opérationnelle du projet. Une étude de cas sur un gisement de cuivre porphyrique illustre l'impact significatif du mouvement de la masse dynamitée sur les pertes de minerai et la dilution, soulignant la nécessité d'un modèle précis de mouvement de la masse dynamitée dans les procédures de contrôle de la teneur et présentant les résultats du flux de travail intégré proposé dans cette étude.

À travers ces études, cette thèse fournit des méthodologies complètes pour relever les défis liés au mouvement de la masse dynamitée et à l'hétérogénéité spatiale dans l'optimisation des limites d'excavation, contribuant ainsi à des opérations minières à ciel ouvert plus rentables et efficaces.

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Contributions of Authors

The author of this thesis is the primary author for all manuscripts contained within. Professor Mustafa Kumral is the supervisor of the author's Ph.D. and included as a co-author in all of the following articles:

Chapter 3 – Hmoud, S., & Kumral, M. (2022). Effect of blast movement uncertainty on dig-limits optimization in open-pit mines. *Natural Resources Research*, 31(1), 163-178. <https://doi.org/10.1007/s11053-021-09998-z>

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List of Abbreviations

ABC	Artificial bee colony algorithm
AHP	Analytical hierarchy process
ANN	Artificial neural network
AUC	Area under the receiver operating characteristics curve
BMM	Blast movement monitoring
CAE	Convolutional auto-encoder network
CNN	Convolutional neural network
CS	Cuckoo search algorithm
CV	Coefficient of variation
FGC	Feasibility grade control
GA	Genetic algorithm
GIS	Geographic information system
GSEI	Global spatial entropy index
GSII	Global spatial information index
GWO	Gray wolf optimizer
KDE	Kernel density estimate
LISA	Local indicators of spatial associations
LSEI	Local spatial entropy index
LSII	Local spatial information index
MAF	Minimum/maximum autocorrelation factors
MCS	Monte Carlo simulation
MEL	Minimum expected loss

MEP	Maximum expected profit
MIP	Mixed integer programming
OK	Ordinary Kriging
PBD	Position-based dynamics
PPMT	Projection pursuit multivariate transformation
RC	Reverse circulation
RF	Random forest
SGS	Sequential Gaussian simulation
SMU	Selective mining unit
SVR	Support vector regression

Chapter 1: Introduction

1.1 OVERVIEW

Ore-waste classification is an essential part of grade control and short-term mine planning that aims to maximize profit from mining operations by sending blasted materials to their optimal destinations and minimizing ore loss and dilution. The ore-waste classification should also consider a number of operational factors such as equipment selectivity and blast movement to delineate ore from waste. This classification involves creating a block model with a block size equal or smaller than the selective mining unit size that shows the spatial distribution of ore and waste zones within an open-pit bench. Due to the small size of these blocks, they are grouped into larger, spatially coherent clusters called dig-limits, which are mined by large mining equipment. The block model generated to classify ore and waste is normally referred to as grade control block model and it is used as the base for short-term mine planning. Misclassifying selective mining units can lead to significant ore losses and dilution, especially when ore and waste are visually indistinguishable, as is often the case with precious metal deposits. Blast movements further complicate accurate ore-waste boundary determination, adding uncertainty that can result in financial losses. Therefore, accurate modeling of blast movement is crucial for determining optimal dig-limits.

Current methods for determining dig-limits often rely on the expertise of grade control geologists who manually digitize ore and waste polygons, guided by rock types, cut-off grades, and economic factors. However, this manual process is subjective, time-consuming, and suboptimal. Various heuristic and metaheuristic algorithms, such as simulated annealing, genetic algorithms, and greedy searches, have been proposed to address these limitations, but they often fail to guarantee

optimal results and require careful parameter selection. The same applies to spatial clustering methods, which attempt to identify spatially coherent blocks sharing the same characteristics but still suffer from subjectivity in selecting hyperparameters. Alternatively, mixed-integer programming has been used to determine dig-limits but it suffers from high computational costs.

Blast movement significantly impacts ore loss and dilution, yet many dig-limits optimization techniques fail to account for it. Accurate modeling of blast movements is essential for practical and profitable mining operations. Approaches to modeling blast movements include multi-physical simulations, physical field measurements to measure blast movement distance and direction, and machine learning techniques. Each of these approaches has its challenges, such as high computational cost and the need for good number of physical field measurements to track the movement of blasted materials within the blasted bench section.

To reduce ore loss and dilution, various strategies can be employed, including improved blasting designs, accurate orebody characterization, and optimized grade control procedures. Understanding ore heterogeneity can help assess the degree of expected ore loss and dilution pre-blast. The same concept can be used post-blast to assess the changes in ore loss and dilution due to blast movement. In fact, developing customized indices to measure the spatial heterogeneity of ore and waste on bench sections at the scale of selective mining unit is not explored in literature and requires further studying.

The implementation of grade control procedures that integrate blast movement modeling and dig-limits optimization is vital for minimizing ore loss and dilution. Effective grade control practices ensure accurate short-term production schedules, and this leads to maximize profitability. Deterministic estimation methods such as Ordinary Kriging and Inverse Distance are commonly used for grade control models, but they can introduce bias and suboptimal decisions when it comes

to determine optimal blasted material destinations. Geostatistical simulation techniques offer a better alternative by producing equiprobable grade values at every location in the bench section, and these simulated values will generate optimal destinations of material under uncertainty that also account for asymmetric profit functions when metallurgical recovery and grade show nonlinear relationship.

Uncertainty in blast movement, originating from factors like inconsistent blast designs and rock mass heterogeneity, must be captured and managed to improve grade control and dig-limits optimization. Although blast movement uncertainty is considered an important contributor to understanding better ore loss and dilution uncertainties, it has never been incorporated in dig-limits optimization to generate the optimal destination of blasted material under uncertainty. Aside from blast movement uncertainty, grade uncertainty is also considered one of the main sources of uncertainty that has significant impact on material classification and dig-limits optimization especially when dealing with erratically distributed orebodies.

This thesis proposes an integrated approach that combines grade and blast movement uncertainties to determine optimal dig-limits. Blast movement uncertainty is modeled on flitch-by-flitch basis as well as block-by-block basis to capture uncertainty in differential blast movement. The effect of blast movement uncertainty on dig-limits is investigated in this thesis. The proposed methodology extends to propose customized spatial entropy indices to describe ore and waste heterogeneity with an aim to understand the relationship between spatial entropy and cut-off grade, blast movement, and dig-limits optimization outcomes, and how these factors impact project profitability. This thesis is developed to enhance ore recovery, reduce dilution and ore loss, and improve the overall efficiency of open-pit mining operations.

1.2 RESEARCH OBJECTIVES

The primary aim of this PhD thesis is to advance the field of grade control and short-term mine planning by addressing the complexities introduced by blast movement and spatial heterogeneity of ore and waste on dig-limits optimization outcomes such as ore loss and dilution. This research focuses on developing innovative methods and workflows to determine optimal dig-limits that improve ore recovery, reduce waste, and maximize profit.

Outlined below are the proposed objectives to reach this target:

- Develop an integrated approach that combines grade and blast movement uncertainties to determine optimal dig-limits.
- Quantify the risk associated with blast movement and evaluate its effects on profit, dilution, and ore losses in bench sections.
- Develop customized spatial entropy indices for describing the spatial heterogeneity of ore and waste at the selective mining unit scale within bench sections and use them to understand better ore loss and dilution.
- Quantify the relationship between cut-off grade, spatial entropy, and the running time and outcomes of dig-limits optimization.
- Model differential blast movement and capture uncertainty on a block-by-block basis.
- Improve the dig-limits optimization model by accounting for multi-destinations, irregular bench shapes, higher shovel selectivity near bench free faces, and various orebody orientations while ensuring that the outcomes of the dig-limits optimization remain optimal.

1.3 ORIGINAL CONTRIBUTIONS

The following points represent the original contributions of this PhD thesis:

- Investigating the impact of blast movement uncertainty on dig-limits optimization outcomes such as profit, dilution, and ore loss.
- Developing new spatial entropy indices that describe the spatial heterogeneity of ore and waste at the selective mining unit scale within bench sections and use them to understand better how pre-blast and post-blast ore heterogeneity impact ore loss and dilution.
- Proposing an integrated workflow that considers grade and blast movement uncertainties in determining optimal dig-limits. It also models stochastically differential blast movement on block-by-block basis to generate more accurate representations of actual blast movement.
- Improved the dig-limits optimization model to account for multi-destinations, irregular bench shapes, enhanced shovel selectivity near bench free faces, and various orebody orientations. These improvements make the optimization model more robust and adaptable to real-world mining conditions, thereby increasing operational efficiency and effectiveness.

1.4 THESIS OUTLINE

The outline of this thesis provides a comprehensive understanding of dig-limit optimization in open-pit mining, emphasizing the challenges and solutions related to blast movement and spatial heterogeneity. The detailed outline is presented below.

Chapter 1 provides a general background on grade control operations and their importance in open-pit mining. It introduces key concepts such as dig-limits optimization, blast movement,

material classification, and spatial entropy, explaining their roles in effective grade control. Additionally, this chapter outlines the research motivation, objectives, and original contributions of the thesis.

Chapter 2 provides comprehensive information related to grade control procedures that include, geostatistical modeling of grade control blocks models, modeling blast movement, dig-limits optimization, material classification, and spatial entropy and its application in mining.

Chapter 3 introduces a methodology for optimizing dig-limits in mine planning by accounting for blast movement, which traditional pre-blast dig-limits often ignore. Blast movement causes ore loss and dilution, leading to financial losses. The proposed method uses blast movement data from monitoring balls and applies statistical modeling and Monte Carlo simulation to predict these movements. A mixed-integer linear programming model optimizes dig-limits based on these predictions, producing an ore probability map. A case study shows that including blast movement in dig-limit calculations significantly reduces profit overestimation and identifies high-risk areas for misallocation.

Chapter 4 explores the impact of spatial heterogeneity on ore loss and dilution in open-pit mining, which significantly affects the economic and operational success of mining projects. It focuses on uncontrollable factors by applying the concept of spatial entropy to measure heterogeneity at the scale of selective mining units. The study uses global spatial entropy to evaluate the potential for ore loss and dilution and local spatial entropy to guide the placement of blast movement monitoring balls. High global spatial entropy values indicate a greater risk of ore loss and dilution, thus reducing profit. The research investigates how changes in cut-off grades, blast movement, and dig-limit optimization affect spatial entropy and profit. The results highlight the importance of

controlled blasting in specific bench sections to manage ore loss and dilution, revealing an exponential increase in profit reduction with higher global spatial entropy.

Chapter 5 presents a comprehensive workflow for optimizing dig-limits in open-pit mining, considering the uncertainties of grade distribution and blast movement. The methodology integrates these uncertainties to improve material classification and destination optimization, aiming to minimize ore loss and dilution. It uses multivariate geostatistical simulation, including Projection Pursuit Multivariate Transformation and Sequential Gaussian Simulation, to model blast movement accurately. The Maximum Expected Profit method determines the optimal destinations for materials, enhancing overall mining profitability. A case study on a porphyry copper deposit demonstrates the significant impact of blast movement on ore loss and dilution, highlighting the necessity of accurate blast movement modeling. The workflow ensures reliable post-blast material classifications, reducing suboptimal decisions and improving both profitability and operational efficiency.

Chapter 6 concludes the research and indicates the future avenues of the research.

Chapter 2: Literature Review

In this section, the literature on the grade control process is presented. This review covers data collection in grade control, the creation of grade control models using geostatistics, open pit blasting, blast movement modeling, dig-limit optimization, material classification that discriminates between various ore and waste types, and spatial entropy, which describes the degree of spatial heterogeneity in the orebody and how it might be used to quantify ore loss and dilution. Detailed reviews of each topic are provided in their respective sections.

2.1 GRADE CONTROL

Grade control operations are an essential part of open pit mining that directly impacts the economic viability of mining operations. It involves the accurate classification of mined material into different material types, such as ore and waste, based on their economic value. The primary goal is to ensure that high-grade ore is sent to the mill for processing, while low-grade material is designated as waste and directed to the dump. This classification process relies heavily on assays of drill samples taken from blastholes or infill drilling reverse circulation (RC) holes, which are then used to estimate the grade distribution within the orebody. Effective grade control minimizes dilution (the contamination of ore with waste) and ore loss (the misclassification of ore as waste), thereby optimizing the overall economic returns of the mining operation (Hmoud and Kumral, 2022; 2023; Rossi and Deutsch, 2013).

In open pit mining, the accuracy of grade control is influenced by several factors, including the geological complexity of the orebody, the quality of the sampling data, and the effectiveness of the interpolation techniques used. In open pit mines, controlling ore grade relies on sampling from blastholes or additional infill drilling (Abzalov, 2016). Grade control drilling commonly uses RC

or open hole percussion drill rigs (Abzalov et al. 2010). These methods are favored due to the limited time available for drilling and the need to collect large volumes of representative samples. Mine logistics constraints demand low-cost drilling techniques with high drilling rates that do not compromise sample quality. Auger drilling can also be employed for grade control in soft ground conditions, enabling the use of auger drilling (Abzalov and Bower 2014). An alternative approach is direct sampling from mine faces or using shallow trenches and winzes. Although this method was extensively used in the past, it has largely been replaced by grade control drilling. The high density of grade control data distribution enables the most accurate determination of ore body contacts and delineation of internal waste contours.

According to Rossi and Deustch (2013), a number of methods have been proposed to construct grade control block models: (1) conventional methods; (2) kriging-based methods; (3) simulation-based methods. Conventional methods used for grade control include blast hole averaging, inverse distance methods, and nearest-neighbor-based methods. Kriging-based methods that are typically used in constructing grade control models are Ordinary Kriging and Indicator Kriging. Kriging methods provide estimates that minimize estimation variance and produces unbiased estimate. Simulation-based methods aim to optimally select ore and waste according to different optimality criteria. It is also valuable when dealing with several material types and destinations.

2.2 GEOSTATISTICAL MODELING OF GRADE CONTROL BLOCK MODELS

Kriged-based methods and simulation-based methods are part of the geostatistical modeling approaches that can be applied to construct grade control block models. To understand these approaches, a brief literature review on the origin and development of geostatistics is provided.

Krige (1951) was a pioneer in the statistical valuation of mineral deposits. Matheron (1963) developed the theory of regionalized variables and unbiased spatial estimation, naming it Kriging in honor of Danie Krige for his contributions to mineral resource valuation. Kriging theory is well documented in literature and its application in mining is discussed by a number of authors (David, 1977; Isaaks & Srivastava, 1989; Journel & Huijbregts, 1978).

Kriging remains a cornerstone of geostatistical approaches in grade control due to its ability to provide unbiased estimates with minimum variance. Ordinary Kriging and Simple Kriging have been developed to address different aspects of grade estimation. Each Kriging method has its strengths and is suited to specific types of data and geological conditions. Simple kriging assumes that the mean is known and stationary (Rossi and Deutsch, 2014; Isaaks and Srivastava, 1989). Ordinary Kriging constrains the weights to samples in the search range to sum to 1 to remove the need for the global known mean (Rossi and Deutsch, 2014; Isaaks and Srivastava, 1989).

Indicator Kriging (Journel & Huijbregts, 1978) is another form of Kriging that convert grade into binary variable based on a cut-off and interpolate the binary variable to predict probability of exceeding cut-off grade and it is used as well to delineate ore from waste in grade control models (Abzalov, 2016).

In addition to these Simple, Ordinary, and Indicator Kriging methods, other multivariate geostatistical techniques, such as Cokriging (Isaaks and Srivastava, 1989), have been employed to enhance grade control. Cokriging models variables of interest while honoring their spatial relationships (Rossi and Deutsch, 2014).

Geostatistical simulation is a method used to model spatial uncertainty in ore grades, allowing for the assessment of economic consequences for decisions made under uncertainty (Deutsch et al.,

2000; Glacken, 1996; Isaaks, 1991; Neufeld et al., 2005; Richmond, 2003). Techniques such as Sequential Gaussian Simulation (SGS) (Isaaks, 1991) capture the spatial uncertainty of grade distribution, enabling mining engineers and grade control geologists to better understand the risks of ore loss and dilution and develop more accurate grade control block models that reproduce grade histograms and variograms. Other methods such as Turning Bands (Journel, 1974), and Partial Differential Equations Simulation (Lindgren et al., 2011) are used as well to generate realizations of grade distribution. However, SGS is the most widely used method in the mining and petroleum industries.

SGS models spatial uncertainty by generating multiple realizations of a continuous variable such as ore grade. This method involves transforming the ore grade data into a normal distribution, then sequentially simulating values at unsampled locations based on the spatial structure summarized by a variogram model. Each location is simulated by drawing from a Gaussian distribution conditioned on both the original data and previously simulated values to maintain spatial dependencies. Finally, the simulated values are back-transformed to their original units and validated to ensure the quality of the simulation. This process results in a suite of equally probable spatial distributions, providing a detailed picture of grade variability within the bench and allowing for the assessment of risks and uncertainties in grade control decisions (Isaaks, 1991; Deutsch & Journel, 1998; Goovaerts, 1997; Chilès & Delfiner, 1999).

2.3 OPEN PIT BLASTING

Open pit blasting is a critical operation in surface mining, involving the controlled use of explosives to break rock for extraction. The design and execution of blasts in open pit mines significantly impact safety, efficiency, and economic performance. This section provides an

overview of open pit blast design, the blasting mechanism, and various blasting techniques, including cast blasting, pre-splitting, and cushion blasting, drawing on well-established literature.

The design of an open pit blast is a complex process involving careful planning to achieve the desired fragmentation while minimizing adverse effects such as ground vibrations, fly rock, and air overpressure. Key elements of blast design include the determination of blast patterns, hole diameters, burden, spacing, explosive type, and the amount of explosive charge per hole.

Blast patterns are typically designed based on the geology of the rock mass, the required fragmentation, and the desired bench configuration. Burden and spacing are critical parameters influencing the blast's effectiveness. Burden refers to the distance from the blast hole to the free face, while spacing is the distance between adjacent blast holes. A well-designed burden and spacing ensure that the explosive energy is effectively transmitted through the rock, resulting in uniform fragmentation (Jimeno et al., 1995). Figure 2.1 presents a schematic of a bench section, illustrating an example of blast design and highlighting the key terms used in describing the design.

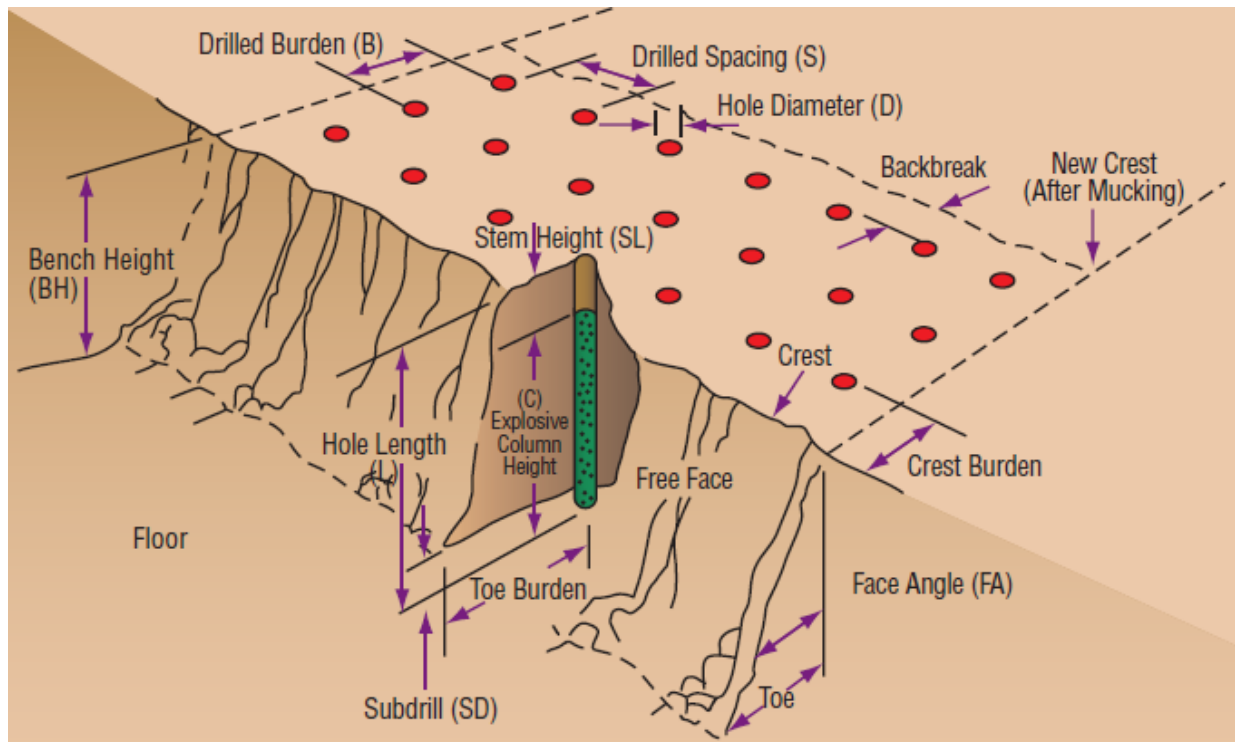


Figure 2.1: Example of bench section layout showing blast design (from Nobel, 2010).

The explosive charge is determined based on the rock's characteristics, such as its density, strength, and elasticity. Modern blasting techniques often involve the use of electronic detonators, which allow for precise timing of the blast sequence, optimizing fragmentation and reducing environmental impacts (Persson et al., 1994).

The blasting mechanism involves the rapid release of energy from the explosive material, generating a shock wave that propagates through the rock mass. This shock wave causes the rock to fracture, leading to fragmentation. The blast's effectiveness is influenced by several factors, including the type of explosive used, the initiation sequence, and the rock's properties (Persson et al., 1994).

When an explosive is detonated, it produces high-pressure gases that expand rapidly, generating a shock wave that travels at supersonic speed through the rock. This wave induces compressive and

tensile stresses in the rock, leading to crack formation. The cracks propagate through the rock, causing it to break into smaller fragments. The fragment size is determined by the explosive's energy, the blast holes' spacing, and the rock's natural fracture system (Langefors & Kihlström, 1978).

Controlled blasting techniques, such as pre-splitting, cushion blasting, and trim blasting, are often employed to manage the extent of fragmentation and reduce damage to the surrounding rock. These techniques involve creating a series of closely spaced blast holes that are detonated in a specific sequence to create a smooth final wall in the pit, minimizing overbreak and enhancing pit wall stability (Ash, 1980).

Pre-splitting is a controlled blasting technique used to create a clean separation between the final pit wall and the rest of the rock mass. This method involves drilling a row of closely spaced holes along the final excavation line, which are then lightly charged and detonated before the main blast. The purpose of pre-splitting is to induce a controlled crack in the rock, reducing the risk of overbreak and ensuring a stable pit wall (Hustrulid, 1999).

Pre-splitting is particularly useful in situations where wall stability is critical, such as in steeply dipping rock formations or when dealing with highly fractured rock masses. This technique minimizes damage to the final wall, preserving its structural integrity and reducing the likelihood of slope failures (Chiappetta, 1990).

Cushion blasting, also known as smooth blasting, is another controlled blasting technique used to achieve a smooth final wall in an open pit. This method involves placing a series of lightly charged blast holes along the final excavation boundary, with the holes spaced more closely together than

in conventional blasting. The light charges produce minimal fracturing, reducing the risk of overbreak and creating a smoother final surface (Persson et al., 1994).

Cushion blasting is often used in conjunction with other blasting techniques, such as pre-splitting, to enhance wall stability and reduce the potential for backbreak. The smooth walls created by cushion blasting are beneficial in maintaining long-term pit wall stability, particularly in operations where the final wall will be exposed for extended periods (Chiappetta, 1990).

Trim blasting, also known as perimeter blasting, is similar to cushion blasting but is used primarily for trimming the final pit walls. It involves the use of smaller explosive charges and closer hole spacing to minimize overbreak and achieve a clean wall surface. Trim blasting is typically conducted as a secondary operation after the main blast to refine the pit wall and ensure it meets the desired design specifications (Ash, 1980).

Trim blasting is particularly useful in finalizing the geometry of the pit wall and is often used in combination with other techniques like pre-splitting and cushion blasting. This method helps to ensure that the final wall is stable and free from loose rock, which could pose a safety hazard during mining operations (Wyllie & Mah, 2004).

Another important blasting technique that is widely used in coal mining that aims to move a large volume of overburden material into a pre-determined area without the need for additional handling. This method effectively reduces the need for costly material handling equipment and increases overall mining efficiency (Wyllie & Mah, 2004).

In cast blasting, the design is tailored to maximize the horizontal displacement of the blasted material, known as "casting." The blast design typically involves larger hole diameters, increased burden and spacing, and higher explosive charges compared to conventional blasts. The timing

sequence is also critical in cast blasting, as it determines the direction and extent of material displacement (Wyllie & Mah, 2004).

The primary advantage of cast blasting is its ability to reduce the overall cost of stripping by minimizing the need for draglines, shovels, or trucks to move the material. By casting a significant portion of the overburden directly into the spoil pile, the method streamlines the process, leading to faster overburden removal and improved production rates (Dyno Nobel, 2020).

2.4 BLAST MOVEMENT

The movement of blasted material significantly affects grade control accuracy. When a blast is initiated, the explosive energy fractures the rock and displaces it, causing movement of the material. This movement can result in the mixing of ore and waste, leading to dilution and ore loss if not properly accounted for. Understanding and managing blast movement is essential for maintaining the integrity of grade control.

One key aspect to manage blast movement in order to reduce ore loss and dilution is to model it. Hmoud and Kumral (2023) categorized blast movement modeling approaches into four main types: (1) multi-physical simulations, (2) heuristic methods, (3) machine learning models, and (4) stochastic methods. Each of these approaches has unique strengths and limitations, and their applicability depends on various factors, including the complexity of the geological conditions and the availability of computational resources.

2.4.1 Multi-Physical Simulations

Multi-physical simulations integrate various physical processes to model blast dynamics. These simulations consider factors such as explosive energy, rock mass properties, and blast design parameters to predict the movement of blasted material.

One of the primary challenges in using multi-physical simulations to model blast movement in open pit mines is the difficulty in accurately gathering all necessary input parameters due to the complexity of the simulation. This process often requires extensive computational time, which is impractical given the frequency of blasting operations. Additionally, the variability of rock mass properties and blast conditions adds to the complexity, making it challenging to develop accurate models without extensive field data. However, with the advancement of cloud and parallel computing, solving these complex simulations in a short time is applicable.

Zou and Jun (2021) investigated the rock fragmentation movement during bench blasting, emphasizing its importance alongside blast fragmentation. They aimed to understand the shape of the muckpile post-blast. Traditional methods for modeling rock fragmentation movement are force-based and computationally intensive. In this study, they extended the position-based dynamics (PBD) method to simulate rock fragmentation movement in production blasting, describing rock mass displacement within a rigid-body dynamics framework. The model discretizes the rock mass volume into small, irregularly shaped blocks using a Voronoi algorithm, assigning velocity to each block based on blast energy. The PBD method then simulates the movement, collision, and landing of the blocks. A practical bench blasting case was simulated to validate the PBD method, successfully reproducing and analyzing the movement of rock fragmentation to form the final muckpile.

Yu et al. (2022) investigated blast movement, a major cause of ore loss and dilution in mines that rely on pre-blast ore boundaries for shovel loading. The study used a simulation approach combining the finite element method for blast loading calculations and the bonded-particle method for bench blasting simulation. They obtained micro-mechanical parameters, equivalent blast loading, and wave velocity, then analyzed particle velocity distribution, accumulation process, and

blast movement characteristics for single-hole and four-hole bench blasting. The study also examined the effects of burden and bench height on blast movement. Using colorful rock strips to model the bench, they observed that rock strips far from the bench face exhibited a fold-back stacking phenomenon, with blast movement distance increasing from the bench floor to the blasthole bottom, peaking at the bottom of the stemming section. Rock strips near the bench face showed a forward flapping state due to natural accumulation under gravity. The study found that blast movement became more pronounced with increasing bench height, decreasing burden, and decreasing distance from the bench face. The numerical results were validated using maximum throw distance calculations, blast movement measurements, and actual muckpile profiles.

Gharib et al. (2017) explores the flow characteristics and wear prediction of Herschel-Bulkley non-Newtonian fluids, specifically focusing on paste backfill as it flows through pipe elbows. The Herschel-Bulkley model, which characterizes non-Newtonian fluids that exhibit both solid-like and fluid-like behaviors depending on shear stress, is employed to analyze how flow dynamics and pipe geometry influence both the wear and efficiency of material transport. The research highlights the importance of understanding fluid rheology and optimizing pipe designs in industrial settings such as mining, where materials like paste backfill are commonly used. By predicting wear patterns and flow behavior, the study provides valuable insights into prolonging pipeline lifespan and improving overall operational efficiency.

This approach to modeling non-Newtonian fluids and predicting wear can be extended to the context of blast modeling in multiphysics simulations, particularly in understanding the behavior of detonation gases and rock fragmentation. Blasting phenomena involve complex interactions between high-pressure gases and fractured rock, where the expansion of gases and flow of fragmented material through rock voids can be conceptualized using principles from fluid

dynamics. Similar to the erosion and wear predictions in the study, wear and damage to rock structures post-blast can be modeled using analogous techniques. The integration of multiphysics models combining fluid dynamics, solid mechanics, and thermal effects can help predict rock breakage patterns, gas flow behavior, and long-term wear on mine structures, offering a comprehensive framework for optimizing blasting operations in mining.

Bharathan et al. (2019) investigates the pressure loss and friction factor in non-Newtonian mine paste backfill, using a combination of modeling, loop tests, and mine field data to comprehensively assess the flow behavior of these complex materials. This research is particularly relevant in underground mining, where paste backfill—a non-Newtonian material—must be transported through pipelines. The authors employed empirical models and field tests to understand how factors like flow rate, pipe diameter, and rheological properties of the paste affect pressure loss and friction factor. Their findings provide critical insights into optimizing pipeline design and operational efficiency in paste backfill applications, offering practical solutions for minimizing energy consumption and wear in mine infrastructure.

The modeling approach used in this study shares strong parallels with techniques applied in multiphysics simulations of blasting, where gas expansion and debris flow interact with fractured rock in complex, non-linear ways. In both cases, understanding the flow behavior of non-Newtonian materials, whether paste backfill or detonation gases, can inform more accurate predictions of pressure losses, material wear, and system performance. The integration of empirical models with field data, as demonstrated in this research, highlights the importance of combining theoretical and practical approaches in mining operations. Extending these methods to blasting phenomena can help in predicting post-blast material flow and optimizing mine design to enhance both safety and operational efficiency.

2.4.2 Heuristic methods

Heuristic methods provide quicker sub-optimal solutions for modeling blast movement by combining empirical rules with physical field measurements, such as post-blast topography. These methods are often used when quick, approximate solutions are needed, and they can be implemented with relatively low computational resources. However, they lack guaranteed optimality and do not fully capture the uncertainty in blast movement predictions.

Isaaks et al. (2014b) addressed the issue of blast movement in mining, which contributes to ore loss and dilution. They discussed the use of blast movement monitoring (BMM) devices, which track material movement during blasting. These devices provide vectors showing the magnitude and direction of movement. The paper presented a method to model post-blast muck piles, accounting for both displacement and internal dilution using simulated annealing. They re-blocked the pre-blast ore control block model into smaller sub-blocks, displaced them using simulated BMM vectors, and then aggregated these into new ore control model blocks. This approach allowed for the design of new dig lines based on the updated ore grades. A case study was included to demonstrate the method's effectiveness.

Vasylchuk and Deutsch (2019b) developed an optimization algorithm to model blast movement in 3-D using pre- and post-blast topography grids and direct blast movement measurements. The problem is framed as an optimization assignment problem and solved with a heuristic algorithm for approximate solutions within a reasonable time. The paper details the objective function and optimization process and provides an example demonstrating the modeling steps. The realistic blast movement model enhances grade control by improving the positioning of dig lines and accurately determining the correct destination for mined rock, making it suitable for mapping pre-

blast grades, categories, expected profit, or other information onto the post-blast muckpile configuration.

2.4.3 Machine learning models

Machine learning models use historical data to predict blast movements. These models can analyze large datasets to identify patterns and correlations that may not be apparent through traditional methods. When current geological and blast design conditions mirror the historical data used in training, machine learning models can provide accurate predictions. While machine learning models are effective in predicting blast movements under consistent geological conditions, they may not yield reliable predictions for deposits with varying characteristics and blast designs. Additionally, these models often do not capture and quantify the uncertainty in blast movement predictions, which can limit their applicability in risk-sensitive mining operations.

Yu et al. (2019) focused on the indirect and accurate determination of blast-induced rock movement to reduce ore loss, dilution, and environmental impact. The study aimed to predict rock movement at the Husab Uranium Mine in Namibia, Coeur Rochester Mine in the USA, and Phoenix Mine in the USA, proposing three new hybrid models using a genetic algorithm (GA), artificial bee colony algorithm (ABC), cuckoo search algorithm (CS), and support vector regression (SVR). These models, GA-SVR, ABC-SVR, and CS-SVR, utilized eight typical blasting parameters as input variables and horizontal blast-induced rock movement as the output variable. The models' predictive performances were assessed using three metrics: the correlation coefficient, mean square error, and variance account for. The results demonstrated that the GA-SVR, ABC-SVR, and CS-SVR models provided satisfactory predictions of rock movement, with the GA-SVR model outperforming the GWO-SVR, CS-SVR, and artificial neural network (ANN) models in both predictive performance and calculation speed.

Yu et al. (2020) investigated the distribution law and prediction of blast-induced rock movement to address the issue of ore loss and dilution when using a pre-blast ore boundary for shovel guidance. Due to difficulties in measuring the post-blast ore boundary, the study applied a blast movement monitoring system to collect accurate data during four blast movement trials. Statistical analysis, an ANN model, a random forest (RF) model, and a gray wolf optimizer algorithm–support vector regression (GWO-SVR) model were used to analyze the collected data. The results indicated that horizontal, vertical, and 3D movements first increased and then decreased, with maximum displacement occurring near the top of the charging section. A good linear relationship was found between horizontal and 3D movements, suggesting that horizontal movement can be used to guide shoveling and reduce ore loss and dilution. Additionally, the GWO-SVR model outperformed the ANN and RF models in accuracy. The study concluded that blast-induced rock movement can be controlled by adjusting the burden and spacing and reducing power factor variables during mining.

Yu et al. (2024) proposed a multilayer dig-limit approach to address ore and profit losses caused by rock fragment movement during blasting in complex-orebody hard rock open-pit mines. This method integrates blast movement into dig-limit optimization using machine learning and heuristic algorithms. The study predicts blast movement distances and directions, determines the post-blast ore boundary, and calculates an optimized dig-limit for maximum profit. A case study showed that this approach improves ore recovery and economic profit compared to manual methods. The study also found that reducing equipment size, increasing the number of layers, and decreasing the powder factor can further minimize ore and profit losses.

2.4.4 Stochastic methods

Stochastic modeling techniques utilize data from physical field measurements of blasted benches to create representative multivariate statistical distributions. These distributions capture the uncertainty in blast movements, particularly near ore zones, and help identify areas at high risk of material misclassification.

Hmoud and Kumral (2022) investigated the uncertainty in blast movement using D-vine Copula and Monte Carlo simulation on a flitch-by-flitch basis and integrate that with dig-limits optimization to understand the effect of blast movement uncertainty on dig-limits realizations. They used field data from blast movement monitoring balls to model blast movements. A mixed-integer linear programming model determined optimal dig-limits for economic block models adjusted with these realizations. Their case study compared scenarios with and without considering blast movement, revealing that ignoring blast movement can overestimate expected profit by 65.3%. The study also identified high-risk areas for ore and waste misallocation post-blasting.

2.5 MATERIAL-TYPE CLASSIFICATION

Material-type classification in mining is a critical task that involves determining the optimal destination for blasted material based on a production plan. This classification ensures that ore is sent to the processing plant while waste is sent to the waste dump, optimizing the economic return of mining operations. Under conditions of grade uncertainty, several methods have been proposed in the literature to determine the optimal material type: (1) average simulation, (2) minimum expected loss (MEL), (3) maximum expected profit (MEP), and (4) MEL and MEP with risk coefficients. This literature review examines these methods in detail, highlighting their applications, strengths, and limitations.

The average simulation method assigns the destination of a block in a mine bench based on the average of the simulated grade values within that block. This method is closely related to ordinary kriging, as both approaches use the mean of grade values to estimate the economic potential of a block. However, average simulation is particularly advantageous when dealing with skewed grade distributions, as it can better reproduce the histograms of such distributions and provide more representative results that accurately capture high-grade values (Verly, 2005).

The MEL method determines the optimal destination for a block by calculating the expected costs associated with assigning the block to all possible destinations, then selecting the destination that minimizes the expected loss across all possible scenarios (Verly, 2005; Isaaks, 1991; Srivastava, 1987; Vasylchuk & Deutsch, 2018).

The MEP method determines the optimal destination of a block by calculating the expected profits associated with assigning the block to all possible destinations, then selecting the destination that maximizes the expected profit (Glacken, 1996; Deutsch et al., 2000; Neufeld et al., 2005). The mining cost to the calculation of the MEP in which a lost opportunity cost is added in case material is classified falsely as waste (Deutsch et al., 2000).

Neufeld et al. (2005) applied the MEP with different costs of mining ore and waste. Positive and negative profit values were calculated based on a cut-off grade in which positive profit value occurs when grade exceeds the cut-off grade. The cost of processing waste is used to scale negative profit values. When the calculated profit values are positive, then material will be classified as ore, otherwise the material is classified as waste.

Glacken (1996) modified the MEL and proposed coefficients for underestimation and overestimation decisions that are chosen based on the mining operation and its way of treating waste material as ore and sending ore to the waste dump.

Dimitrakopoulos and Godoy (2014) added risk coefficients to both MEL and MEP methods to ensure that specific operational requirements are met, such as increasing the likelihood of sending ore with some chance of being waste to the processing plant or reducing the chance of sending waste to the waste dump.

The addition of risk coefficients allows for more flexible and tailored classification decisions that account for the specific risk tolerance and operational objectives of a mining operation. These coefficients adjust the profit and loss functions to reflect the desired risk profile, ensuring that classification decisions are aligned with the overall strategic goals of the mine (Dimitrakopoulos & Godoy, 2014).

Applying risk coefficients requires continuous updating, as operational policies may change throughout the life cycle of the mine. This dynamic nature can make the application of risk coefficients challenging in practice, requiring robust data management and real-time decision-making capabilities (Dimitrakopoulos & Godoy, 2014).

Vasylichuk and Deutsch (2018) proposed a modification to the MEL method in which if ore and waste decisions were correct then loss is assigned a value of zero. Otherwise, the loss is calculated as the difference between the cut-off grade and the grade of the variable at the block. They also proposed that underestimation and overestimation coefficient can be added to the calculation as per Glacken (1996).

In scenarios with a linear profit function and no specific risk coefficients, average simulation, MEL, and MEP methods tend to provide similar results. However, when dealing with non-linear profit functions, such as when the recovery model is not a linear function of grade, the MEP method is generally considered the best approach for determining optimal destinations under uncertainty (Verly, 2005).

2.6 DIG-LIMITS OPTIMIZATION

In daily open-pit mining operations, short-term plans are essential for delineating ore and waste SMUs. Misclassification of these units can lead to dilution or ore loss. These plans must ensure that consecutive SMUs' ore or waste decisions align with the equipment size while maximizing profit. This optimization is crucial, especially when dealing with valuable metals and significant dilution or loss.

Hmoud and Kumral (2023) categorized dig-limits optimization methods into five main categories: (1) exact methods, (2) heuristic methods, (3) metaheuristic methods, (4) hybrid methods, and (5) spatial clustering methods.

2.6.1 Exact methods

Exact methods, such as mixed-integer programming (MIP), ensure optimal solutions for dig-limits optimization. These methods involve formulating the optimization problem as a mathematical model and solving it using exact algorithms. MIP is a powerful optimization technique that can generate optimal dig-limits by considering various constraints and objectives. However, MIP suffers from long processing times, especially when the moving window that enforces equipment selectivity is large. Despite these challenges, advancements in parallel computing and high-performance computers have made it increasingly feasible to use MIP for generating optimal dig-limits.

Kumral (2015) developed a decision-making tool to minimize losses from misclassification of mining parcels, which relies on estimated or simulated values from sparse data. The research converted a non-linear optimization problem into a MIP problem and solved it using a standard MIP optimization engine. A case study with gold and silver variables demonstrated the approach's effectiveness compared to conventional grade control methods. The study provided a practical solution for grade quality control, enhancing profit and operational efficiency, and introduced the use of target grades instead of cut-off grades, addressing common challenges in mining operations.

Sari and Kumral (2017) developed a method to optimize dig-limits in open-pit mining by creating ore-waste boundaries that loaders can handle effectively. They formulated this problem as mixed-integer linear programming problem with an objective function that aims to maximize profit while considering equipment selectivity. A case study on seven bench sectors in a gold mine showed the method's practicality and potential to increase operational value. The optimized design deviated by only 6.4% from a manual design by a mining engineer, demonstrating its efficiency.

Hmoud and Kumral (2022) adopted Sari and Kumral (2017) MIP dig-limit optimization model after coding it in Python to assess the impact of blast movement uncertainty of dig-limits optimization outcomes. They conclude that blast movement uncertainty has significant impact on dig-limits optimization outcomes that can result in ore loss and dilution that leads to significant profit reduction.

Nelis et al. (2022) introduced a new approach to address the mining cut definition problem, incorporating geometallurgical interactions in short-term mine planning for open-pit operations. The process involves aggregating blocks into clusters that are extracted and processed as single units, considering operational constraints of loading equipment selectivity and aiming to maximize operational objectives. Utilizing mixed integer programming and a model inspired by column

generation, their approach defines decision variables directly on the set of all feasible cuts. This eliminates the need for linear approximations of geometallurgical behavior, allowing the use of any nonlinear function. An industry-sized dataset demonstrated that the model could be solved in a reasonable time, with results showing that nonlinear recovery functions significantly influence destination policy and expected profit. The study concluded that the traditional free selection policy based on cutoff grade is suboptimal when geometallurgical interactions are taken into account.

Nelis and Morales (2022) addressed the challenge faced by short-term open pit planners in designing feasible production schedules that meet processing, mining, and operational constraints while maximizing profit or total metal produced. These schedules must align with long-term production plans and incorporate new blasthole sampling data. Traditionally, this task has been performed with limited optimization tools, risking suboptimal results. The authors proposed an optimization model that simultaneously tackles operational and scheduling issues by defining mining cut configurations and production schedules. The model uses representative SMUs as potential locations for mining cuts, assigning each SMU to a specific location. Tested with a real case study, the model successfully generated mining cuts and extraction sequences that fulfilled all constraints, including access restrictions due to ramp locations on each bench. The design captured most of the profit, providing a valuable guide for short-term mine planners.

Faraj (2024) emphasizes the need for accurately classifying material types before mining, as current manual practices for drawing ore/waste dig limits are subjective and lead to significant dilution and ore loss. Using two weeks of production data from a homogeneous Cu porphyry and a heterogeneous Manto-type Cu deposit, the study compares the variability in hand-drawn dig limits by 20 professionals with optimal ore-waste delineation. The author adopted Sari and Kumral

(2017) dig-limits optimization model in this paper. Results show that manual dig-limits result in profits ranging by 3.7% and 5.9% for homogeneous and heterogeneous deposits, respectively, whereas optimal delineation increases profits by 1.9% and 7.0%. The main drivers of ore loss and dilution are identified as natural variability, diggability, and selectivity. The study recommends integrating dig limit optimization algorithms to reduce subjectivity and improve profitability.

2.6.2 Heuristic methods

Heuristic methods, like exact methods, aim to optimally separate ore from waste to maximize operational efficiency and profit while respecting equipment selectivity. Unlike exact methods, heuristic approaches generate near-optimal solutions without guaranteeing absolute optimality. However, their strength lies in producing these solutions within a reasonable timeframe, making them highly efficient and practical for real-world applications.

Richmond and Beasley (2004) proposed greedy local search heuristics to estimate financially efficient dig-lines in mining operations. These heuristics use a floating circle-based perturbation mechanism to generate alternate dig-lines while ensuring mining equipment constraints are met. The financial payoffs for these alternate dig-lines are evaluated using a mean-downside risk efficiency model, based on distributions from multiple conditional simulations. Computational results indicated that equipment constraints significantly increase financial risk for a given expected payoff and that the heuristic efficient frontier is sensitive to the mining strategy. The study found that mining waste material at transitions before mining ore material minimizes upfront operating costs, maximizes profit for a given financial risk level, and frees milling capacity.

Wilde and Deutsch (2015) addressed the challenge of predicting recoverable reserves before mining begins, a crucial aspect of ore reserve estimation. They proposed a new method called Feasibility Grade Control (FGC), which automates the process and is applicable during feasibility

studies. FGC eliminates the need for the time-consuming creation of initial dig limits while establishing dilution and ore loss. The study demonstrated that the results of FGC are nearly identical to traditional methods but are more realistic and efficient, saving significant professional time.

Vasylchuk and Deutsch (2019a) introduced an algorithm to optimize the classification of surface mine material while considering excavating constraints. Using high-resolution expected profit models, the algorithm generates classification maps tailored to site-specific rectangular excavating constraints. Traditional analytical solutions are ineffective for this optimization problem, so the authors developed a practical heuristic algorithm that quickly determines the optimal material destination under realistic constraints. The optimization process is fast, achieving up to 98–99% of the total expected profit compared to free selection. This algorithm is a practical tool for short-term grade control and managing multiple realizations in long-term resource estimation.

2.6.3 Metaheuristic methods

Norrena and Deutsch (2000) proposed an optimization approach for determining dig limits in grade control that considers mineral grades, economic costs, and mining equipment selectivity. The authors used simulated annealing to optimize dig limits, combining constraints of maximum profitability and equipment capabilities. They recommended geostatistical techniques to map expected profits and account for grade uncertainty. The optimization balances dilution and ore waste to maximize profit, incorporating an equipment curve to quantify mining equipment limitations. This approach demonstrated effective results and highlighted areas for future improvement.

Isaaks et al. (2014a) highlighted that traditional grade control methods focus on blasthole sampling and ore control block model (OCM) grade estimation, often neglecting dig-limits design.

Misclassification of block grades, leading to financial loss, is commonly addressed using maximum profit selection (MPS). However, dig-limits design can also misclassify OCM blocks, even if initially classified correctly by MPS, as blocks are rarely mineable individually by ore type. The authors proposed an algorithm for constrained optimum dig-limits, which evaluates misclassifications through loss functions or MPS, minimizing financial losses while adhering to minimum mining width constraints. A case study demonstrated the benefits of this approach.

Ruiseco (2016) developed a GA approach to approximate optimal dig-limits, considering grade control data, equipment constraints, and processing and mining costs. The GA demonstrated robustness and flexibility across various cases and complexities. The paper outlines the success of the GA application in two separate studies and further explores its flexibility and potential applications in equipment sizing in a third study.

Ruiseco et al. (2016) introduced an optimization approach using a GA to approximate optimal dig-limits on a bench, considering grade control data, equipment constraints, and processing and mining costs. A case study was conducted on a sample disseminated nickel bench in a two-destination, single ore-type deposit. The results demonstrated that the GA-based approach was effective for dig-limit optimization, outperforming traditional hand-drawn methods.

Ruiseco and Kumral (2017) examined the impact of varying equipment sizes on practical dig-limits in a highly variable bench with gold and copper in a sulfide/oxide deposit. They used a GA-based dig-limits optimization in this study. While current equipment sizing methods consider block dilution, they do not address the effects of selectivity changes. The study revealed that the relationship between selectivity, profit, and equipment size is nonlinear, with significant break points occurring when using insufficiently selective equipment. The proposed technique provides

a method for sizing mine equipment in complex deposits, considering the interplay between productivity and grade distribution.

Williams et al. (2021) investigated using a CNN to evaluate the clustering of SMUs, which were optimized using a GA for mine bench dig limits. These limits categorize SMUs into waste and processing categories, ensuring feasible and profitable extraction. While the GA method is effective, it is time-consuming and costly. The study aimed to reduce the high computational costs by employing deep learning to assess the quality of GA-computed dig limits, significantly cutting down the time, which can account for up to 70% of total computation time. A case study with a mine bench containing multiple destinations and 420 SMUs demonstrated that the CNN accurately predicted clustering quality and reduced computation time by 3900%, making the process more efficient for short-term mine planning.

2.6.4 Hybrid methods

The hybrid approach leverages the strengths of both exact optimization methods, such as MIP, and near-optimal methods, such as metaheuristic algorithms. Initially, the exact method explores the solution space, and after a certain number of iterations, it stops to provide an initial solution. This solution is then refined by the metaheuristic methods, which start with an acceptable solution and aim to find a near-optimal solution in a shorter time frame. This strategy efficiently combines the precision of exact methods with the speed of metaheuristics to deliver practical and timely solutions.

Deutsch (2017) proposed a hybrid approach by combining branch and bound with simulated annealing to determine the dig-limits. In this approach, when the branch and bound need many iterations, simulated annealing takes part resulting in sub-optimal solutions. This proposed method

could solve big computational problems in a relatively short time. However, they do not guarantee optimality, and they require careful selection for input parameters to obtain near-optimal results.

2.6.5 Spatial clustering methods

Spatial clustering offers an alternative method for generating dig-limits that effectively separate different materials. This approach is commonly used to create production schedules by dividing open-pit benches into clusters and grouping blocks with similar characteristics to form dig-limits. However, these methods do not necessarily find optimal dig-limits for distinguishing ore from waste. Additionally, they require careful selection of input clustering parameters, such as the desired number of clusters, making the resulting solution highly subjective.

Tabesh and Askari-Nasab (2011) addressed the challenge of using exact optimization methods for open pit production scheduling due to the large size of real mining problems, which makes them intractable. The paper aimed to develop, implement, verify, and validate a clustering algorithm for block aggregation to aid in production scheduling. This algorithm aggregates blocks into selective mining units based on a similarity index defined by rock types, ore grades, and distances between blocks. A two-stage clustering approach, utilizing an agglomerative hierarchical algorithm and tabu search, was developed and tested. A case study on an iron ore life-of-mine production schedule validated the algorithm, demonstrating that the size and shape of aggregated blocks can affect the project's net present value by 10-15% and significantly impact the practicality of the generated long-term production schedules.

Tabesh and Askari-Nasab (2013) focused on the multiple stages in mining operations where engineers draw polygons to guide operations. Traditionally drawn by hand based on the engineer's expertise, these polygons can benefit from automatic procedures to enhance quality and reduce effort. Long-term planning requires large polygons for mining cuts, while short-term planning

needs mineable shapes that are homogenous in grades and rock types to accurately estimate material quality and dilution. The mining direction also influences the desired polygon shapes. To address these issues, the authors introduced a clustering algorithm with shape control that can generate guidelines for both long-term and short-term planning by adjusting its parameters. The algorithm was tested on two small datasets, and its performance was further evaluated on a real gold deposit using different mining strategies. The evaluation considered homogeneity of grade, rock types, determined destinations, and run times, demonstrating the algorithm's effectiveness.

Tabesh and Askari-Naseb (2019) highlighted the importance of incorporating geological uncertainty into mine production planning. While advances in mathematical models and heuristic approaches have improved the ability to address simpler scheduling problems, more complex instances that incorporate uncertainty remain challenging. Aggregating blocks can reduce solution times. This paper presents four variations of the agglomerative hierarchical clustering algorithm: one based on deterministic estimates and three using a possible worlds approach that leverages geostatistical realizations to form aggregates considering geological properties and uncertainties. Case studies demonstrated that uncertainty-based algorithms create aggregates less susceptible to uncertainties, while the proposed algorithm produces controlled-size, minable shapes.

Salman et al. (2021) explored clustering approaches used to group similar objects for enhanced analysis and decision-making, specifically focusing on short-term planning in open-pit mines. In this context, clustering aims to aggregate blocks based on attributes like geochemical grades, rock types, and geometallurgical parameters, while adhering to constraints such as cluster shape, size, mining direction alignment, destination, and rock type homogeneity. This method helps reduce the computational cost of optimizing short-term mine plans. Previous studies have not fully addressed mining-specific constraints in clustering. The paper introduces a novel block clustering heuristic

that honors a comprehensive set of mining block aggregation requirements and constraints, including clustering adjacent blocks, achieving high destination homogeneity, controlled cluster size, consistency with mining direction, and creating mineable shapes with homogeneous rock types. The algorithm's application on two datasets demonstrated its efficiency in generating reasonable block clusters while satisfying the predefined constraints.

2.7 SPATIAL ENTROPY

The mining industry constantly seeks to minimize ore loss and dilution, which are critical factors impacting operational efficiency and profitability. A key aspect influencing these factors is the spatial heterogeneity of the orebody at the selective mining unit (SMU) scale. Spatial heterogeneity refers to the variation in grade distribution within the orebody, which directly affects the distinction between ore and waste. This review delves into the concept of entropy, its application in measuring spatial heterogeneity, and various methodologies proposed over the years to account for spatial data in entropy calculations.

The concept of entropy, originating in thermodynamics, has been adapted for various scientific fields to measure the degree of disorder or heterogeneity in a system. Shannon (1948) introduced entropy in information theory as a measure of the information content in a signal. Shannon's entropy quantifies the expected value of the information contained in a message. In the context of mining and geospatial data, Shannon's entropy can be adapted to measure the heterogeneity of grade distributions within an orebody, providing a way to quantify the disorder and variability in the spatial distribution of ore and waste.

Entropy has been widely applied in various scientific disciplines. For example, in ecology, Patil & Taillie (1982) used entropy to measure biodiversity and the distribution of species within an

ecosystem. In their paper, an intrinsic diversity ordering of communities is defined and shown to be equivalent to stochastic ordering. This provides a mean of comparing the diversity of different communities. It also developed the concept of the sensitivity of a diversity index to rare species which helps in understanding how changes in a rare species may affect over diversity in a community.

Similarly, in hydrology and hydrogeology, Aghakouchak (2014) explored the integration of entropy theory and copulas to enhance probability inference in hydrology and climatology. The paper reviews and categorizes existing entropy-copula models into three main groups based on their structures, providing a numerical example to demonstrate their formulation and implementation. It discusses the potential applications of these models in hydrology and climatology, including an example application to flood frequency analysis. By combining entropy and copula concepts, the study leverages the strengths of both methods to better describe the probability distribution and dependence structure of multiple dependent variables.

Butera et al. (2018) investigated the impact of transmissivity fluctuations in heterogeneous porous media, treating transmissivity as a spatial stochastic variable. These fluctuations induce stochasticity in groundwater velocity and transport features. The study emphasizes the importance of understanding the relationships between variables characterizing flow and transport, noting that while linear relationships are easier to manage, the interdependence of these variables becomes increasingly nonlinear with greater heterogeneity. The work highlights the significance of nonlinear linkages, proposing information theory tools to detect their presence. By comparing the cross-covariance function and mutual information, the study contrasts the amount of linear and nonlinear linkage. To avoid analytical approximations, data from Monte Carlo simulations of

heterogeneous transmissivity fields were analyzed, revealing that nonlinear components can be significant even when cross-covariance values are nil.

Manchuk et al. (2021) assessed groundwater quality and geochemical changes in the Athabasca region of Alberta, Canada, using data from over 5,000 wells collected from the 1950s to 2014. They developed bootstrap techniques to detect changes, finding increased anomalies in Cl, total dissolved solids, B, and naphthenic acids, and decreased SO₄ anomalies in the McMurray formation between 2003 and 2008. The study revealed sampling biases in several formations, suggesting the need for expanded sampling. It identified wells for continued observation based on entropy and relative magnitude of time series, focusing on wells with low measurements and low entropy near active industry lease boundaries.

Singh (1997) examined the applications of entropy theory in hydrology and water resources. The paper reviews recent contributions of entropy in hydrological modeling and water resources, highlighting the usefulness and versatility of the entropy concept in these fields. The author reflects on both the strengths and limitations of the entropy concept and concludes with comments on its implications for developing countries.

The application of entropy in geology is also well-documented in literature. Freizi et al. (2017) introduced a new hybrid method for assigning evidential weights in mineral potential mapping by combining the Analytical Hierarchy Process (AHP) with Shannon Entropy weighting. They applied this approach to a case study for porphyry-Cu potential mapping in Markazi Province, Iran. Geo-datasets were gathered, and evidence layers were generated for integration using the TOPSIS method with the combined AHP–Shannon Entropy weighting. The resulting mineral potential map was evaluated through field checking and chemical analysis of samples, revealing two outcrops with evidence of a porphyry system in areas with high potential values. There was

also a strong correlation between high potential values and the copper content of field samples, demonstrating the effectiveness of the AHP–Shannon Entropy weighting method for mineral potential mapping.

Liu et al. (2021) investigated the Xiadian orogenic gold deposit in the Jiaodong Peninsula, Eastern China, which is associated with the Zhaoping detachment fault. Through field investigation, 3D modeling, spatial analysis, and prospectivity modeling using a multi-layer perceptron deep neural network, they explored the relationship between gold mineralization and structural deformation. The deep neural network with multilayer perceptron and cross-entropy loss is calculated and minimized to train the models. They found that gold mineralization primarily occurs in convex segments of the fault footwall within 150 meters of the fault buffer, where the dip is gentle. High-grade sulfide vein ore is typically located in steeply dipping fracture-fill areas distal to the fault. Their study identified six potential gold targets and demonstrated the effectiveness of 3D prospectivity modeling in understanding the orogenic gold spatial distribution and structural controls.

Zhang et al. (2021) investigated the use of convolutional neural networks (CNN) for mineral prospectivity mapping by integrating multi-geoinformation with unsupervised convolutional auto-encoder networks (CAE). They employed cross-entropy to fit the CNN model. Two CAE networks were built to identify high-error patches of a tif image, representing prospective areas, and low-error patches as non-prospective. By adjusting the CAE architecture, training epochs, and evidence map combinations, reliable results were achieved. The area under the receiver operating characteristic curve (AUC) indicated that high reconstructed errors matched prospective areas. These were then used in CNN modeling, showing a strong spatial correlation with known gold deposits. The results, including training loss, accuracy, and favorability maps, were comparable to

previous studies, demonstrating the method's feasibility. This approach of using CAE to extract spatial signatures for CNN learning shows promise for mineral prospectivity mapping.

However, traditional entropy measures do not account for the spatial location of data points, treating datasets with identical probability mass functions but different spatial configurations as having the same entropy. This limitation led to the development of spatial entropy indices, which incorporate spatial location to provide a more accurate measure of heterogeneity.

Theil (1972) introduced a spatial entropy measure to quantify inequality or diversity in spatial distributions. In the context of social and administrative sciences, spatial entropy quantifies the unpredictability or heterogeneity of a given distribution at a certain location, such as income distribution, demographic characteristics, or resource allocation for certain cities or countries. Higher entropy indicates greater diversity and randomness, reflecting a more even distribution across categories, whereas lower entropy suggests more predictability and concentration in certain categories.

Batty (1974) addressed a significant issue in information theory concerning the derivation of a continuous measure of entropy from the discrete measure. While many analysts have noted the incompleteness of Shannon's treatment of this problem, few have reworked his analysis. Batty proposed a new measure of discrete entropy that explicitly incorporates interval size, termed spatial entropy, which is fundamental to geography. This measure was first demonstrated through its application to one- and two-dimensional aggregation problems. Batty then explored the implications of this statistic for Wilson's entropy-maximizing method, reinterpreted Theil's aggregation statistic in spatial terms, and suggested heuristics for designing real and idealized spatial systems where entropy is maximized.

Batty (1976) introduced an approach to measuring locational phenomena within a spatial hierarchy using entropy statistics. The paper derives several statistics suitable for studying spatial aggregation, which are then decomposed at different levels of the spatial hierarchy using decomposition principles initially applied by Theil. These decomposition statistics are compared with variance analysis methods used by Moellering and Tobler (1972) and the spatial entropy measure suggested by Curry (1971). The application of these statistics is illustrated with data from the Reading subregion and New York City. The paper concludes by analyzing the potential role of entropy and information in addressing problems related to equal-area zoning.

Batty (2010) examined the use of entropy measures in geographical analysis, focusing on the concept of human systems in equilibrium. He discussed how entropy maximizing, similar to equilibrium statistical mechanics, provides a framework for location and interaction models, as popularized by Wilson (1970). Batty reviewed two extensions: introducing "spatial entropy" to account for spatial variation and decomposing/nesting entropy to capture variations at different scales. He identified gaps in substantive interpretations of entropy for various geographical shapes and sizes and the dynamics of generating probability distributions, such as power laws in complex systems. Batty called for further research to link entropy maximizing, entropy measure interpretations, and equilibrium distribution dynamics, emphasizing the need for new entropy measures that illustrate how equilibrium spatial distributions result from dynamic processes reaching a steady state.

Anselin (1995) addressed the need for new exploratory data analysis techniques in geographic information systems (GIS) to focus on the spatial aspects of data. He introduced a general class of local indicators of spatial association (LISA) to identify local patterns of spatial association. LISA statistics decompose global indicators, such as Moran's I, into the contributions of individual

observations. These statistics serve two main purposes: identifying local pockets of nonstationarity or hot spots, similar to the G_i and G^* statistics of Getis and Ord (1992) and assessing the influence of individual locations on the global statistics and identifying outliers. Anselin evaluated the properties of a LISA statistic through the local Moran, applying it to the spatial pattern of conflict in different countries and in several Monte Carlo simulations.

Karlström and Ceccato (2000) developed a new measure of spatial association called the S statistics, based on information theory by defining a spatially weighted entropy measure that considers spatial configuration. The S statistics have an intuitive interpretation and fulfill the expected properties of an entropy measure. This global measure of spatial association can be decomposed into LISA. The new measure was tested using employment data from the culture sector in Stockholm County and compared with existing global and local spatial association measures. The S statistics demonstrated similarities to Moran's I and Getis-Ord G_i statistics, with the local S_i statistics showing significant spatial association akin to the G_i statistic but with the advantage of aggregation to a global measure. The S statistics can also be extended to bivariate distributions, and the commonly used Bayesian empirical approach can be interpreted as a Kullback-Leibler divergence measure. One advantage of the S statistics is that they select only the most robust clusters, eliminating smaller clusters that might inflate the global measure.

O'Neill et al. (1988) developed three indices of landscape pattern using information theory and fractal geometry to quantify spatial patterns that correlate with ecological processes. Using digitized maps, these indices were calculated for 94 quadrangles covering most of the eastern United States. The study demonstrated that the indices are reasonably independent of each other and effectively capture major features of landscape patterns. One of the indices, the fractal dimension, was found to correlate with the degree of human manipulation of the landscape. This

research provides valuable tools for quantifying and understanding the spatial patterns in landscape ecology.

Claramunt (2005) conducted a preliminary study to explore a representation of spatial information diversity that supports the analysis of spatial structures. This approach is based on Shannon's information theory, which offers a quantitative method for evaluating diversity in one-dimensional spaces. Claramunt introduced a measure of spatial diversity, extending Shannon's measure and incorporating principles from the First Law of Geography. The study examined and illustrated the properties and unique aspects of spatial diversity compared to conventional measures. Relationships between spatial diversity and notions of order and cohesion were also discussed. The potential for combining spatial diversity with other diversity measures for ecological, social, and economic studies was highlighted, although further validation and experimentation are needed.

Li and Claramunt (2006) introduced a spatial decision tree to address the limitations of conventional decision trees when applied to geographical datasets affected by spatial autocorrelation. Conventional decision trees often underperform in these scenarios because they do not account for spatial distribution. The proposed spatial decision tree incorporates a spatial diversity coefficient that measures the spatial entropy of a geo-referenced dataset, extending the conventional notion of entropy to include spatial autocorrelation phenomena. This integration results in a classification process tailored to geographical data. The potential of this approach was demonstrated through a case study focused on classifying an agricultural dataset in China.

Li and Reynolds (1993) addressed the limitations of the contagion index proposed by O'Neill et al. (1988), which was designed to quantify spatial patterns of landscapes but proved to be insensitive to changes in spatial patterns. They introduced a new contagion index that corrects a mathematical error in the original formulation. The error was identified and corrected

mathematically. Both the original and new contagion indices were then evaluated against simulated landscapes to demonstrate the improved sensitivity and accuracy of the new index in quantifying spatial patterns.

Parresol and Edwards (2014) emphasized the importance of studying spatial patterns of landscapes to quantify human impact, predict wildlife effects, and describe landscape features. A robust landscape index should measure both the composition and configuration of landscape diversity. They critiqued the use of relative contagion indices in landscape ecology, demonstrating through simulated landscapes that these indices are mathematically untenable. To address this, they developed a new entropy contagion index (Γ). They derived the distributional properties of Γ , showing it to be asymptotically unbiased, consistent, and normally distributed. A variance formula for Γ was also derived using the delta method. As an application, they analyzed the patterns and changes in forest types across four soil-geologic landform strata on the 80,000 ha Savannah River Site in South Carolina, USA. Using one-way analysis of variance for hypothesis testing of contagion among strata, they provided insights for managers to meet structural objectives based on differences in contagion across the strata.

Riitters et al. (1996) investigated the landscape contagion index, which measures the degree of clumping of attributes on raster maps by computing the frequencies of adjacent attribute pairs. They highlighted that subtle differences in tabulating attribute adjacencies can affect the applicability of the standard index formula, leading to potentially incomparable published index values. The paper derived formulas for the contagion index that accommodate different methods of tabulating attribute adjacencies, both with and without preserving the order of pixels in pairs and using two methods of determining pixel adjacency. When pixel order is preserved, the standard formula is obtained. Without preserving pixel order, a new formula is derived due to the reduced

number of possible attribute adjacency states. Additionally, they found that estimated contagion is smaller when each pixel pair is counted twice instead of once, as this double-counting makes the attribute adjacency matrix symmetric across the main diagonal.

Leibovici (2009) addresses the challenge of identifying geographical patterns by analyzing the spatial configuration and distribution of events, objects, or their attributes. Traditional measures, like Shannon entropy, shed light on data organization but fall short in describing spatial organization. The paper advances spatial entropy by focusing on the co-occurrence of categories at multiple orders, treating adjacency as a second-order co-occurrence with zero collocation distance. Leibovici introduces a spatial entropy measure that incorporates multivariate data with covariates, allowing for flexible spatial interaction models between attributes. By using a multivariate multinomial distribution of collocations, this approach assesses interactions through an entropy formula, where the collocation distance serves as a scale factor for spatial organization analysis.

Leibovici et al. (2014) examined how to characterize the distribution of spatially observed entities by their geometries and attributes using Shannon entropy, which is widely applied in fields like ecology, regional sciences, epidemiology, and image analysis. They extended entropy measures by incorporating spatial patterns from topological and metric properties, using distance-ratios and co-occurrences of observed classes. The research introduced new indices and explored their applications as global and local indices in spatio-temporal domains. By employing a multiplicative space-time integration approach at macro and micro levels, they developed spatio-temporal entropy indices that combine co-occurrence and distance-ratio methods. This framework supports spatio-temporal clustering, providing a structured perspective on the distribution of class instances. The methodology was demonstrated with simulated data evolutions of three classes over seven-

time stamps, and preliminary results were discussed for a study of conflicting maritime activities in the Bay of Brest, exploring spatio-temporal patterns of a categorical variable representing conflicts between maritime activities.

In geostatistics, Journel and Deutsch (1993) investigated the relationship between maximum prior entropy and response distribution. They introduced a global spatial entropy measure that calculates entropy at various lag distances for a set of discrete variables, providing a comprehensive measure of spatial entropy. However, this measure was not designed to quantify entropy at a local level, such as SMU scale. Their method calculates entropy for all possible pairs separated by specific lag distances within a defined area of interest. To quantify local spatial entropy at specific locations and account only for relevant pairs, their spatial entropy formulation needs modification. This adjustment would enable the calculation of both global and local spatial entropy, enhancing its applicability to more localized analyses.

Altieri et al. (2017) explored the concept of entropy, originally introduced in information theory and popularized through Shannon's formula to measure heterogeneity among observations. They focused on incorporating spatial components into entropy measures, addressing the limitation that traditional indices are computed based on a chosen distance. By extending Shannon's entropy for categorical variables to include space as a second dimension, the study investigated the properties of residual entropy and mutual information. This approach allowed the extension of univariate entropy measures to bivariate distributions, ensuring a well-defined probabilistic meaning for all components. Consequently, a spatial entropy measure satisfying the additivity property was developed, with global residual entropy being the sum of partial entropies based on different distance classes. Mutual information, which measures the information added by the inclusion of

space, also demonstrated additivity. A comparative study is presented to compare the new indices to Leibovici indices and Shannon Entropy.

Altieri et al. (2018) examined the use of entropy in applied sciences to measure the heterogeneity of observations, focusing on recent attempts to develop entropy measures for spatial data to capture the influence of space on variable outcomes. They identified limitations in these developments, such as the computation of indices conditional on a single distance and the lack of additivity between local and global spatial measures. The study reviewed recent univariate distribution-based approaches and introduced a new method linked to bivariate distributions. Firstly, they decomposed Shannon's entropy into spatial mutual information, which accounts for the role of space in determining variable outcomes, and spatial global residual entropy, which summarizes the remaining heterogeneity of the variable. Both terms satisfy the additivity property, being sums of partial entropies measured at different distance classes. The proposed indices were applied to measure the spatial entropy of a marked point pattern of rainforest tree species, demonstrating that the new entropy measures are more informative and address a broader set of questions than existing literature proposals.

Altieri et al. (2019) addressed the inefficiency in urban diffusion, a significant concern for biologists, urban specialists, planners, and statisticians in both developed and developing countries. Urban sprawl, often identified as chaotic urban expansion, is linked to the concept of entropy. The paper employed a rigorous spatial entropy-based approach to measure urban sprawl associated with the diffusion of metropolitan cities. To assess the effectiveness of the measures, a comparative study was conducted on archetypical urban scenarios, followed by the application of the measures to quantify the degree of disorder in the urban expansion of three European cities.

The results were easily interpretable and could be used as absolute measures of urban sprawl or for comparative analysis over space and time.

Altieri et al. (2021) reviewed a selection of spatial entropy indices, including some recent ones, designed to handle spatial data on variables with a finite number of categories. While the paper focuses on biodiversity data, the methods are applicable to other environmental phenomena. They introduced the new R package *SpatEntropy*, which computes spatial entropy measures, extending traditional entropy measures to their spatial versions. This package works with both areal and point data. The paper includes a practical section where the package is applied to two types of environmental data: tree biodiversity and urban expansion. The results demonstrate that *SpatEntropy* is a user-friendly and valuable tool for researchers and practitioners working with spatial entropy measures.

Zhang et al. (2020) explored the formation mechanisms of vegetation patterns, key self-organized structures in ecological systems, traditionally attributed to dynamic bifurcations. They examined how statistical indicators, specifically Shannon entropy and the contagion index, influence the regularity of these patterns. Unlike previous studies that used randomly generated patterns or vegetation maps, they simulated the formation of vegetation patterns using a discrete vegetation-sand model under different bifurcation scenarios. The corresponding Shannon entropy and contagion index of these simulated patterns were calculated using modified formulas. The study revealed that the variation trends of Shannon entropy and the contagion index are closely linked to the formation stages of vegetation patterns. Additionally, the different final values of these indicators in various patterns can help determine the dominant bifurcation when both bifurcations are present.

Hmoud and Kumral (2023) emphasized the crucial role of effective management of ore loss and dilution for successful grade control and short-term mine planning, given their significant economic, environmental, and technical impacts on open-pit mining operations. The paper applies the spatial entropy concept to capture ore heterogeneity at the scale of selective mining units. They use global spatial entropy to assess the impact of spatial heterogeneity between ore and waste blocks on ore loss and dilution, while local spatial entropy guides the allocation of blast movement monitoring balls pre-blast. High global spatial entropy values indicate a higher potential for ore loss and dilution, which reduces profit. The study also explores the relationships between spatial entropy, cut-off grades, blast movement, dig-limits optimization model running time, and profit through various case studies. Results indicate that changes in cut-off grade and blast movement significantly affect spatial entropy post-blasting, increasing ore loss, dilution, and profit reduction, thus highlighting the need for controlled blasting in specific bench sections. Additionally, they demonstrate an exponential increase in profit reduction due to ore loss and dilution with rising global spatial entropy.

Chapter 3: Effect of Blast Movement Uncertainty on Dig-Limits Optimization in Open Pit Mines

3.1 ABSTRACT

The determination of dig-limits is one of the most critical steps in grade control and short-term mine planning. Dig-limit optimization aims to identify blasted materials to their optimal destinations while honoring equipment selectivity. The dig-limits determined in the pre-blast stage are not operational in the post-blast stage due to blast movement. Based on the blast design configuration and rock characteristics, blasted materials will move in certain directions. The magnitude of blast movement in those directions varies across bench levels called flitches. Dig-limits without incorporating blast movement can cause ore losses and dilution, leading to severe financial losses. In this paper, a new methodology is proposed for quantifying uncertainty in blast movement and assessing the impact on dig-limits optimization. Blast movements are modeled by using measurement data obtained from blast movement monitoring balls that are installed in blast holes. The multivariate distributions for measured blast movements across flitches are fitted using D-vine copula and blast movement realizations generated using Monte Carlo simulation. A mixed-integer programming model is used to determine the optimal dig-limits for all economic block models corrected and adjusted with blast movements realizations. An ore probability map is generated showing locations of ore and waste blocks in a probabilistic fashion. A case study for demonstrating the proposed methodology is presented. In this case study, two scenarios are investigated; the first scenario incorporates blast movement in determining dig-limits, while the second scenario discards blast movement effect on dig-limits. The result of this comparison shows that discarding the blast movement when determining dig-limits can lead to over-estimating the

expected profit by 65.3% on average when compared with the other scenario that incorporates blast movement. The post-blasting ore and waste areas with a high-risk of being misallocated by the dig-limits are identified.

3.2 INTRODUCTION

Ore-waste classification is an essential part of grade control and short-term mine planning. It aims to maximize profit from short-term mining operations by sending the blasted materials to their optimal destinations and reducing ore losses and dilution while honoring the selectivity of mining equipment. A block model is generated to understand the spatial distribution of ore and waste zones within a bench. This model is a simplified representation of the bench, and it consists of a number of small computer-generated blocks called selective mining units (SMUs). SMUs are the smallest volumes of materials on which ore-waste classification decisions are made (Sinclair & Blackwell, 2006). These SMUs are too small to be mined by themselves. Therefore, SMUs are grouped together into spatially coherent clusters called dig-limits that can be mined by large mining equipment. The misclassification of SMUs can cause severe ore losses and dilution issues that lead to significant financial losses. This misclassification problem becomes even more evident when ore cannot be visually distinguished from waste during mining operations, such as in most (if not all) gold deposits. During bench blasting, ore and waste blasted rocks are moved by the blast in various directions and distances throughout the blasted section of the bench. It becomes difficult to quantify the distance of this blast movement accurately as a consequence of the variations of geologic and rock properties. These variations may add uncertainty in ore and waste boundary exactness, resulting in financial losses. Therefore, assessing blast movement uncertainty is considered a crucial predecessor step before determining optimal dig-limits. If dig-limits are

determined pre-blast, they will have a limited or no operational use post-blast due to blast movements.

Most of the current methods applied to determine dig-limits are based on the experience of the grade control geologists who manually digitize ore and waste polygons on a bench-by-bench basis. Their judgment on the shape of dig-limits is guided by rock types and cut-off grade that varies according to deposit type, ore recoveries, and other economic factors such as commodity price and operational costs. However, this manual determination of dig-limits suffers from several limitations: (1) it is subjective, (2) it takes a relatively long time to manually digitize dig-limits, (3) it does not produce optimal dig-limits that maximizes profit, (4) it does not correctly account for the differential blast movements within a bench among flitches, and (5) it does not account for the unavoidable uncertainty in modeling blast movement. These limitations may result in profit losses caused by dilution and ore loss.

To overcome these limitations in solving the dig-limits problem, various methods have been proposed in the literature. Most of these proposed methods rely on heuristics and metaheuristics algorithms such as simulated annealing (Deutsch, 2017; Isaaks, Treloar, et al., 2014; Norrena & Deutsch, 2000), genetic algorithms (Ruiseco, 2016; Ruiseco & Kumral, 2017; Ruiseco et al., 2016; Williams et al., 2021), heuristics (Richmond & Beasley, 2004; Vasylichuk & Deutsch, 2018, 2019a), greedy searches (Wilde & Deutsch, 2015), or they adopted a hybrid approach such as combining branch and bound with simulated annealing to determine the dig-limits (Deutsch, 2017). In this approach, when the branch and bound need many iterations, simulated annealing takes part resulting in sub-optimal solutions. These techniques could solve big computational problems in a relatively short time. However, they do not guarantee optimality, and they require careful selection for input parameters to obtain near-optimal results.

Another way of solving dig-limits problem could be spatial clustering. Tabesh and Askari-Nasab (2011) proposed a hierarchical clustering algorithm for generating dig-limits based on a similarity index that includes distance between blocks, rock types, destinations, metal grade, and other attributes to produce spatially coherent clusters or dig-limits. The generated clusters are refined in a later stage using a Tabu search. However, this post-processing step reduces cluster homogeneity and increases ore loss and dilution. Tabesh and Askari-Nasab (2013) proposed another spatial hierarchical clustering algorithm to define spatially coherent groups of blocks used at different stages of mine planning. Their algorithm accounts for mining shapes and the direction of mining when creating these clusters. Tabesh and Askari-Nasab (2019) extended their proposed spatial clustering algorithm to account for geological uncertainty and proved that it could be implemented on real-size block models and generate clusters within a reasonable processing time. Salman et al. (2021) explored a block clustering algorithm based on the K-means clustering algorithm to define dig-limits. This algorithm aggregates blocks with a similar grade, rock types, spatial proximity and generates clusters with controllable sizes used as dig-limits.

These clustering approaches summarized above are used to produce quick mine schedules by dividing open-pit benches into a number of clusters and grouping blocks with the same characteristics together to form dig-limits. However, these approaches do not find optimal dig-limits when forming their clusters to separate ore from waste. They also require careful selection for input clustering parameters, such as the desired number of clusters, making the solution very subjective. Furthermore, the dig-limits optimization problem is formulated as a mixed-integer programming (MIP) problem (Kumral, 2015; Nelis & Morales, 2021; Sari & Kumral, 2018). These models guarantee optimality, but they usually require a longer time to find the optimal solutions.

Many dig-limits optimization techniques do not account for blast movement when determining optimal dig-limits. Therefore, the determination of dig-limits ignoring blast movements will not be practical (Engmann et al., 2013; Thornton et al., 2005). The possible dilution associated with blast movement can be minimized if blast movement is considered appropriately (Yennamani, 2010; Zhang, 1994). As a result of incorporating blast movement in determining dig-limits, ore recovery can increase because mined materials are sent to their optimal destinations (Gilbride, 1995; Harris, 1997; Taylor & Firth, 2003).

Modeling blast movements throughout the bench section is an essential step for determining optimal dig-limits. There are three approaches for modeling blast movements in literature: (1) by conducting a multi-physical simulation of blast movements (Zou & Jun, 2021), (2) by modeling blast movements using physical field measurements (Isaaks et al., 2014; Vasylichuk & Deutsch, 2019b; Yu et al., 2021), and (3) by the machine learning techniques on the previous blast movement information to predict future blast movement in the mine (Yu et al., 2020; Yu et al., 2021). Physical field measurements will also be required for validating multi-physics simulation models of blast movement. Furthermore, these measurements may be used for machine learning applications for training purposes. If the mechanism behind the blast movement distance is not well-quantified, there will be high ore losses and dilution levels. Given that blasting is a relatively cheap process in mining, the operations tend to overbreak rock, leading to turbulent movements. In other words, the benefit acquired by small particle size through blasting can be lost due to loss and dilution.

No matter what approach is used for modeling blast movements, there will be inevitable uncertainty associated with the direction and distance of the movement. Rosa and Thornton (2011) remarked that blast movements are uncertain because of inconsistencies of blast design and the rock mass characteristics. They emphasized the fact that accurate calculation of the movement is

not possible if there are no detailed physical field measurements. Moreover, there would be inevitable human factor issues and geologic fluctuations. As a matter of fact, no method precisely guarantees predictions for blast movements. When physical field measurements are not available, evaluating the risk associated with rock movement uncertainty will be essential to quantify dilution and loss.

Based on previous research conducted on modeling blast movement, the uncertainty in blast movement remains unstudied. For this reason, this research is proposed to assess the effect of blast movement uncertainty on dig limit optimization while honoring the selectivity of mining equipment. The originality of this paper is two-fold: (i) a combined approach of blast movements and the dig-limit optimization, and (ii) the quantification of risk attributed to blast movement in bench sections such that the effects of blast movement uncertainty on profit, dilution, and ore losses are measured.

The paper was organized as follows. The proposed methodology is elaborated in the next section, followed by a case study demonstrating the importance of incorporating blast movement in dig-limits optimization. Finally, the conclusions and future work are provided.

3.3 METHODOLOGY

The methods in this paper are based on combined approaches: MIP, the simulation of the blast movements through Monte-Carlo simulations (MCS) using the D-vine copula, and statistical analysis. Figure 3.1 summarizes the methodology used in this research.

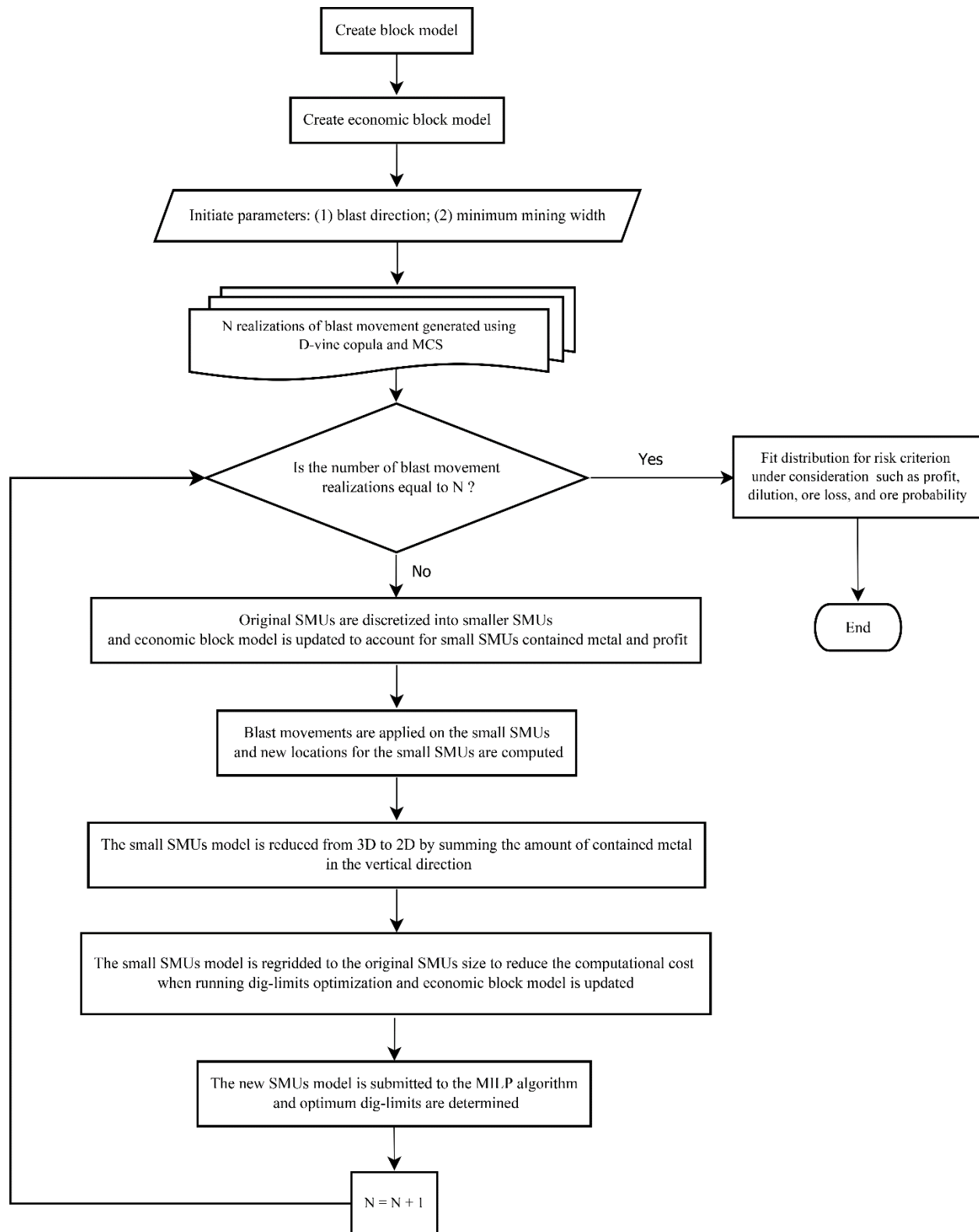


Figure 3.1: Flow chart summarizing the proposed methodology

A grade control block model is formed using the exploration drill holes and blast holes resource database. The estimation methods such as Ordinary Kriging can be used to estimate ore grades and produce a grade control block model. Ordinary Kriging (OK) is one of the most widely used interpolation techniques. In the context of resource estimation, the OK estimates the grade at an SMU location by using a variogram model and grade data in the neighborhood of the estimation location. The variogram model provides OK with covariance values used to assign weights for the neighboring grade data to estimate the unknown grade value at an SMU location. This process is repeated until all SMUs in the grade control block model are estimated. The most significant advantage of using OK is that it is the best linear unbiased estimation technique compared to other techniques such as inverse distance. This grade control block model should cover the investigated section of the bench that needs to be blasted. When creating the grade control block model, the number of SMUs in the vertical direction should equal the number of bench levels called flitches. This grade control block model is transformed into an economic block model by applying various parameters, including cost, price, recovery, and metal quantity, to all SMUs. The principal input parameters used in generating an economic block model are operational costs, metal price, and ore recovery. These parameters are used to calculate a break-even cut-off grade ($COG_{B.E.}$) and compute the profit. $COG_{B.E.}$ is calculated using Equation 3.1.

$$COG_{B.E.} = \frac{C_p + C_m}{p * r} \quad (3.1)$$

Where C_p is the processing cost per tonne, C_m is the mining cost per tonne, p is the metal price, and r is the ore recovery. $COG_{B.E.}$ is used to distinguish ore SMUs from waste SMUs because it

includes the cost of mining which is vital to calculate the profit of all SMUs in the economic block model. The profit obtained from an ore SMU is calculated through Eq 3.2.

$$P_{i,j} = p * m_{i,j} * r - t * (C_p + C_m) \quad (3.2)$$

Where $P_{i,j}$ is the profit of SMU located in i and j on a bench if it is assessed as ore, $m_{i,j}$ is the contained metal of SMU i, j , and t is SMU tonnage. If the mined material is considered waste based on $COG_{B.E.}$, then the profit generated from mining this waste SMU is calculated using Eq 3.3

$$W_{i,j} = -t * C_m \quad (3.3)$$

Where $W_{i,j}$ is the extraction cost of SMU located in i and j on a bench if it is assessed as waste. After generating an economic block model, two initial input parameters must be specified: (1) blast movement direction and distance, and (2) minimum mining width. The direction and magnitude of blast movement are estimated from physical field measurements using blast movement monitor (BMM) balls. Minimum mining width must be specified based on the selectivity of the equipment used for mining the bench.

The extent of horizontal blast movement is different in each flitch because of the blast design and rock characteristics. In general, most explosives are placed in the mid-holes. This placement results in the blasted rocks in the middle flitches of the bench moving further than the upper and lower flitches. These differential movements generate a D-like shape structure prior to settling a new location. Figure 3.2 illustrates a typical D-like shape of blast movements. The number of flitches

depends on the blasting design and SMU sizes used in the estimation stage. Typically, the direction of blast movement is parallel to the initiation direction of the blast (Gilbride, 1995).

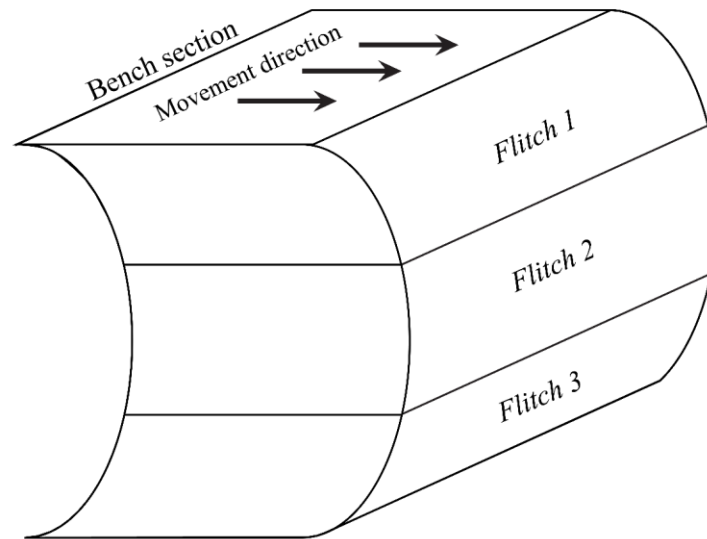


Figure 3.2: Schematic diagram illustrating the expected D-like shape of a blasted bench with three flitches.

Even though the use of BMM balls in determining the magnitude of blast movement is considered one of the best and the most reliable sources of blast movement information, the measurements of blast movement obtained from these balls remain uncertain because (1) BMM balls do not cover the entire blasted section of the bench, (2) it is difficult to determine the locations of the BMM balls post-blasting, and (3) the blast movement distances associated with the balls varies even within the same flitch. For this reason, the uncertainty in modeling blast movement needs to be quantified before determining any dig-limits because it significantly impacts ore loss, dilution, and, subsequently, mine profit.

After bench blasting, the new locations of the BMM balls are determined, and they are used to calculate the magnitude and the overall direction of blast movement. To help modeling blast movement across bench flitches, blast movement data collected from BMM balls are grouped together based on the bench flitch located within. Then, a multivariate statistical distribution is fitted to the blast movement measurements across these flitches. This multivariate distribution honors the existing correlations between flitch movements. Applying MCS to the fitted multivariate distribution of blast movement measurements, several blast movement realizations are generated. These realizations yield the expected D-like shape from blast movements. There is always a chance to produce unexpected shapes because of issues emerging from the fluctuations in blast design, rock and geologic features that affect the magnitude of blast movement.

In this paper, the multivariate relationships between blast movements of bench flitches are modeled using vine copula (D-vine), a subclass of regular vine copula (R-vine), which was proposed by Aas et al. (2009). This method preserves the multivariate associations between the blast movements of bench flitches using bivariate copulas and a nested set of trees. Figure 3.3 shows an example of D-vine copula trees for the blast movement measurements of three flitches where F_1 is blast movement for Flitch1, F_2 is blast movement for Flitch2, and F_3 is blast movement for Flitch3. F_1F_2 and F_2F_3 are the bivariate blast movement copula distributions between Flitch1-Flitch2 and Flitch2-Flitch3. $F_1F_3|F_2$ is the Flitch1 and flitch3 bivariate copula distribution given Flitch2 blast movement. Bivariate copulas are functions that describe the dependency among two one-dimensional distributions. If both marginal distributions and copula are known, then the entire bivariate distribution of those two one-dimensional distributions is also known. Following Sklar's theorem (Sklar, 1959), if $H(x,y)$ is a two-dimensional distribution function with marginal

distribution functions $F(x)$ and $G(y)$, then there exists a copula C that can be calculated as in Eq 3.4.

$$H(x, y) = C(F(x), G(y)) \quad \forall x, y \text{ in } R \quad (3.4)$$

When applying copulas to blast movement scenarios $F(x)$ and $G(y)$, which are two continuous marginal distributions, they will represent blast movement magnitudes in two bench flitches. Based on Sklar's theorem, if both marginal distributions are continuous, then a unique copula must exist.

Copulas are used for modeling complex multivariate relationships. They have been applied to solve a limited number of mining-related problems (Ardian & Kumral, 2020; Singh et al., 2021). In this paper, the way modeling blast movement across flitches is different from the abovementioned references. The D-vine copula is selected instead of the canonical vine copula (C-vine). This difference lies in capturing blast movement correlation between the flitches. This decision of selecting a D-vine copula is made due to the nature of the problem in which all flitch movements are equally important. No flitch movement controls all other flitch movements when rocks are blasted. The C-vine copula is only used in fitting multivariate distributions when one variable controls several other variables. This situation does not exist in the blast movement problem.

A number of well-known bivariate copula distributions can be used to fit bivariate relationships of blast movements between flitches such as Gaussian and Archimedean copulas. The Gaussian copula is used to model linear bivariate relationships. On the other hand, Archimedean copulas are used to model non-linear or heteroscedastic bivariate relationships. Clayton, Frank, and Gumbel are good examples of Archimedean copulas. These bivariate Archimedean distributions have a

single parameter that controls the degree of dependence (θ). The parameter θ of the Archimedean copula is the leading and only parameter used in constructing a bivariate Archimedean distribution, and it indicates the association between random variables. The parameter θ can be derived from Kendall's Tau correlation coefficient due to its ability to disentangle the marginal distribution effect, which is suitable for non-parametric methods (Chemen & Teilly, 1999; Frees & Valdez, 1998). Kendall's Tau correlation coefficient is computed using Equation 3.5.

Kendall correlation coefficient

$$= \frac{2(n_{\text{concordant}} - n_{\text{discordant}})}{n(n-1)} \quad (3.5)$$

Where $n_{\text{concordant}}$ is the number of concordant pairs, $n_{\text{discordant}}$ is the number of discordant pairs, and n is the sample size. For Clayton, Frank, and Gumbel copulas, the value of θ can be calculated by Equations 3.6, 3.7, and 3.8, respectively (Genest & Favre, 2007).

$$\theta = \frac{2\tau}{(1-\tau)} \quad (3.6)$$

$$\theta = \frac{1}{(1-\tau)} \quad (3.7)$$

$$\frac{[Y_1(\theta) - 1]}{\theta} = \frac{(1-\tau)}{4} \quad (3.8)$$

Where $Y_1(\theta)$ is the Debye's model that can be estimated using Equation 3.9 (Genest & Favre, 2007).

$$Y_k(\theta) = \frac{1}{\theta} \int_0^\theta \frac{t}{e^t - 1} dt \quad (3.9)$$

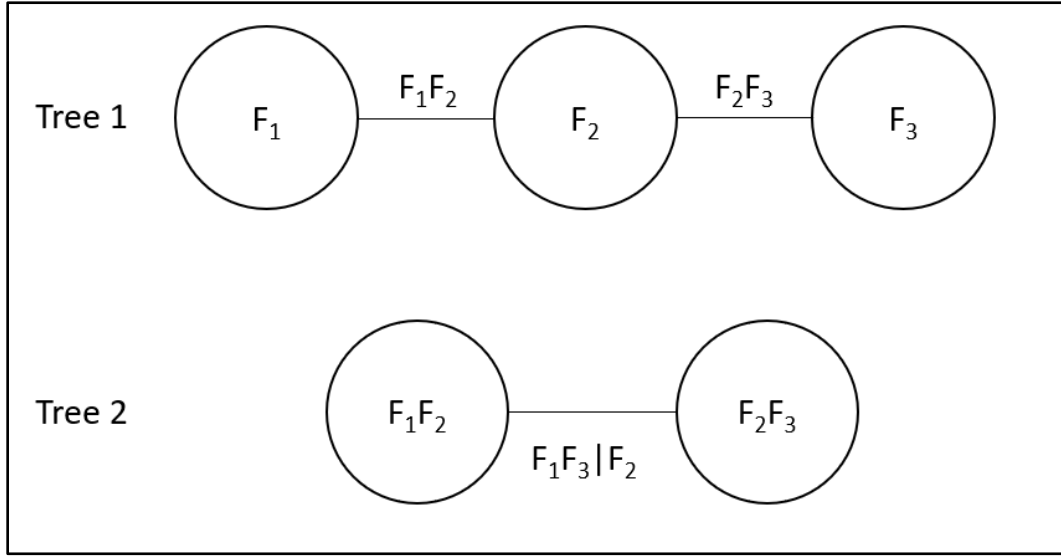


Figure 3.3: Example of D-vine copula trees for three flitches blast movements.

For modeling Archimedean-type copulas (e.g., Clayton, Gumbel, and Frank), Nelsen (2007) provided the models as given in Equations 3.10, 3.11, and 3.12, respectively.

$$C = [\max(u^{-\theta} + v^{-\theta} - 1, 0)]^{-\frac{1}{\theta}} \quad (3.10)$$

$$C = e^{\left(-[(-\ln u)^\theta + (-\ln v)^\theta]^{\frac{1}{\theta}}\right)} \quad (3.11)$$

$$C = -\frac{1}{\theta} \ln \left(1 + \frac{(e^{(-\theta u)} - 1)(e^{(-\theta v)} - 1)}{e^{(-\theta)} - 1} \right) \quad (3.12)$$

where C is the bivariate copula function. In order to estimate copula, the original values of the two one-dimensional distributions are converted to u and v . Then, u and v follow a uniform distribution with $[0,1]$. Examples of Gaussian and Archimedean copulas are presented in Figure 3.4.

Most phenomena in nature exhibit non-linear or heteroscedastic behavior. The linearity assumption is typically considered to simplify most real-life problems. Therefore, the Clayton model is highly recommended for modeling blast movements data across flitches where blast movement correlations between these flitches are stronger near blast holes. They get weaker and weaker when the distance becomes larger from the nearby blast holes.

The proposed methodology can deal with blast movements in any direction. First, the azimuth angle of the overall blast movement direction is initiated as an input parameter. Then, all pre-blast blocks will be moved to the direction specified. Blast movement simulations generated by D-vine copula multivariate distribution are reflected on the block monetary values, including the geological properties of the deposit, grade, metal quantity, and block profits. Later, the SMUs within the block model are further divided into smaller sizes. Thus, small SMU movements can be captured, and the accuracy of dig-limit optimization is increased. In the end, a post-blast block model is yielded to submit to dig-limit optimization module.

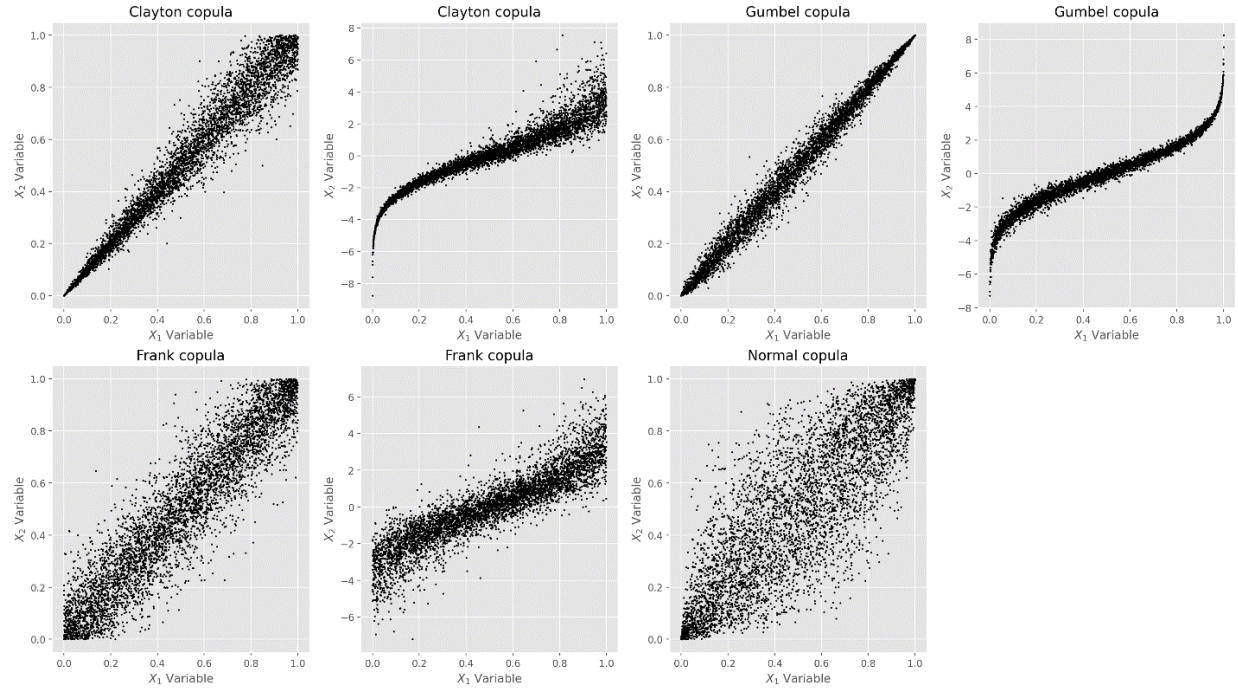


Figure 3.4: Examples of widely used bivariate copulas distributions.

Before running dig-limits optimization, the 3D post-blast economic block models are transformed into 2D economic block models. In doing so, the x and y coordinates of their centroids are kept the same, and the contained metal values of all SMUs are added such that a 2D model is obtained. This step reduces the size of the dig-limits optimization problem significantly. This step will not have an impact on the final dig-limits because dig-limits are drawn on 2D maps, and they are not determined on a flitch-by-flitch basis. Another important step to further reduce the size of dig-limits optimization processing time is that of re-gridding the small SMUs. In this step, SMUs are re-gridded back to their original SMU size, and the total amount of contained metal and profit are calculated for these re-gridded SMUs. This re-gridding step may slightly increase ore loss and dilution applied to the SMU model. As a result of this step, 2D post-blast economic block models are generated that contain the amount of contained metal at original SMU support. Finally, those re-gridded 2D economic block models are used as an input to the dig-limits optimization model to

generate a number of dig-limits realizations corresponding to the number of post-blast 2D economic block models.

The formulation of dig-limit optimization used in this research is based on the MIP model developed by Sari and Kumral (2018). This model was re-coded using Python 3.8 and run on the CPLEX solver (IBM, 2021). The objective function is to maximize the profit of the blasted section of the bench while satisfying equipment selectivity constraints. To describe this approach in simple words, equipment dimensions are defined as an $n \times n$ frame where n represents the number of SMUs in x and y directions. Every SMU should be assigned to a frame in which all SMUs in it are either ore SMUs or waste SMUs. A frame is called a valid frame if all SMUs inside it are either ore or waste. If an SMU belongs to more than one frame, it should be assigned at least one valid frame. The dig-limits optimization model is formulated as:

$$\text{Maximize } \sum_{i=1}^{D_x} \sum_{j=1}^{D_y} [x_{i,j} P_{i,j} + (1 - x_{i,j}) W_{i,j}] \quad (3.13)$$

Where $x_{i,j}$ is a binary decision variable at i,j SMU, and $P_{i,j}$ is the economic value of SMU i,j when mined as ore, $W_{i,j}$ is the economic value of SMU i,j when mined as waste. The i and j are SMU indices in x and y directions, respectively.

Subject to

$$b_{i,j,f_a,f_b} = \sum_{\gamma=0}^{n-1} \sum_{\delta=0}^{n-1} x_{i-f_a+\gamma, j-f_b+\delta} \quad (3.14)$$

$$f_a = 0, \dots, n-1; i = 1, \dots, D_x; f_b = 0, \dots, n-1; j = 1, \dots, D_y$$

Where γ and δ are the frame index in x and y directions, respectively, b_{i,j,f_a,f_b} is a decision variable that represents the total number of SMUs (x_{ij}) at (i, j) locations that belong to a frame where

$$\begin{aligned}
i - f_a + n &\leq D_x, \\
j - f_b + n &\leq D_y, \\
i - f_a &\geq 0, \\
j - f_b &\geq 0
\end{aligned} \tag{3.15}$$

D_x and D_y are the number of SMUs in the x -direction and y -direction, respectively, n is the minimum mining width in SMU terms, f_a and f_b are frame indices in x and y directions, respectively.

The decision variable b_{i,j,f_a,f_b} is transformed a new binary decision variable, v_{i,j,f_a,f_b} , controlling if a frame is valid:

$$v_{i,j,f_a,f_b} = \begin{cases} 1, & \text{if } b_{i,j,f_a,f_b} = 0 \text{ or } b_{i,j,f_a,f_b} = n^2 \\ 0, & \text{otherwise} \end{cases} \tag{3.16}$$

Where v_{i,j,f_a,f_b} is a valid frame. This constraint ensures that an SMU is assigned at least to one valid frame

$$\sum_{f_a=0}^{n-1} \sum_{f_b=0}^{n-1} v_{i,j,f_a,f_b} \geq 1 \quad i = 1, \dots, D_x; \quad j = 1, \dots, D_y \tag{3.17}$$

To remove incomplete frames at the corners, the following constraints are added.

$$b_{i,j,f_a,f_b} = -1 \quad i = 1, \dots, D_x; \quad j = 1, \dots, D_y; \quad f_b = 0, \dots, n-1 \tag{3.18}$$

Where

$$i - f_a + n > D_x \text{ or } i - f_a < 0 \tag{3.19}$$

$$b_{i,j,f_a,f_b} = -1 \quad i = 1, \dots, D_x; j = 1, \dots, D_y; f_a = 0, \dots, n-1 \quad (3.20)$$

Where

$$j - f_b + n > D_y \text{ or } j - f_b < 0 \quad (3.21)$$

After running dig-limits optimization using all 2D economic block models generated from the N blast movement realizations, N number of dig-limits realizations are generated. Those N realizations of dig-limits are used to calculate the probability of having ore and waste for all SMUs. Furthermore, uncertainty in ore loss and dilution amounts can be quantified, and profit distribution can be generated for the blasted section of the bench.

3.4 CASE STUDY

In this section, a case study demonstrates the impact of the blast movement uncertainty on dig-limits optimization. First, a 3D grade control block model is generated from blasting hole data at a porphyry gold mine. The block model definition is summarized in Table 3.1. Next, this block model is used to generate an economic block model applying a cut-off grade based on economic and metallurgical input parameters listed in Table 3.2.

Table 3.1: Block model properties.

Item	Unit	X	Y	Z
Block size	(m)	5	5	5
Number of blocks in a direction		40	40	3
Minimum centroids	(m)	2.5	2.5	2.5
Maximum centroids	(m)	197.5	197.5	12.5
Azimuth	(degree)	0		
Dip	(degree)	0		

Table 3.2: Parameter values used for cut-off grade and block economics values.

Item	Unit	Value
Gold price	(\$/gram)	60
Mining cost	(\$/tonne)	5
Processing cost	(\$/tonne)	20
Ore recovery	(%)	70
Rock bulk density	(tonne/m ³)	2.65

The section of the bench under consideration is partitioned into three flitches. Each flitch has a thickness of 5 m. Blast movement data are simulated from 51 BMM ball field measurements, including the three flitches at 17 locations. In other words, each blast hole has three BMM balls. The movement direction is North-East (45° azimuth) toward the two free faces of the bench on the

north and east sides. The direction of movement is ascertained by comparing the pre-blast locations of BMM balls against their post-blast locations. The blast movements data were best fitted to triangular distributions, which are widely used in engineering simulations. It is a helpful distribution because it has the fixed minimum and maximum values, unlike the normal distribution and other distributions where their minimum and maximum values go to positive and negative infinity. The selection of a distribution extending to both infinities (e.g., normal distribution) might generate unrepresentative results. In this case, a truncated normal distribution could have been used. These distributions are used to fit Clayton bivariate copulas and D-vine multivariate copulas between flitches blast movements. MCS is then used to generate 1,000 realizations of blast movements from the fitted D-vine copula distribution to check the reproduction of multivariate relationships between flitch movements.

Figure 3.5 shows the results of simulating blast movement realizations through MCS using a D-vine copula where the lines represent the fitted triangular distributions for the three flitches. In Figure 3.5, n is the number of realizations, μ_x is the mean of the simulated realizations, X_{50} is the median of the simulated realizations, σ is the standard deviation, CV is the coefficient of variation, X_{min} is the minimum value, and X_{max} is the maximum value. Figure 3.6 shows if the multivariate relationships between blast movements are agreed upon after simulating blast movement realizations from the D-vine copula. Comparing the observed movements from BMM balls and the simulated blast movements realizations using MCS is presented in Figure 3.7. These movements were applied to the centroids of the blocks in flitches at depths of 2.5 m, 7.5 m, and 12.5 m. The sound reproduction of the multivariate input relationships between BMM measurements and simulated realizations is observed in Figure 3.6 and Figure 3.7.

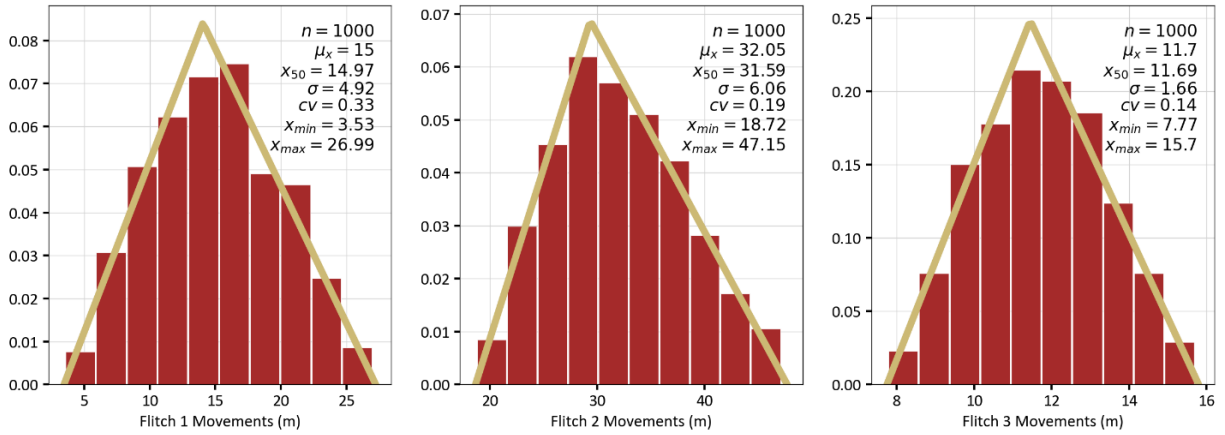


Figure 3.5: Checking the reproduction of univariate blast movement distributions after simulation.

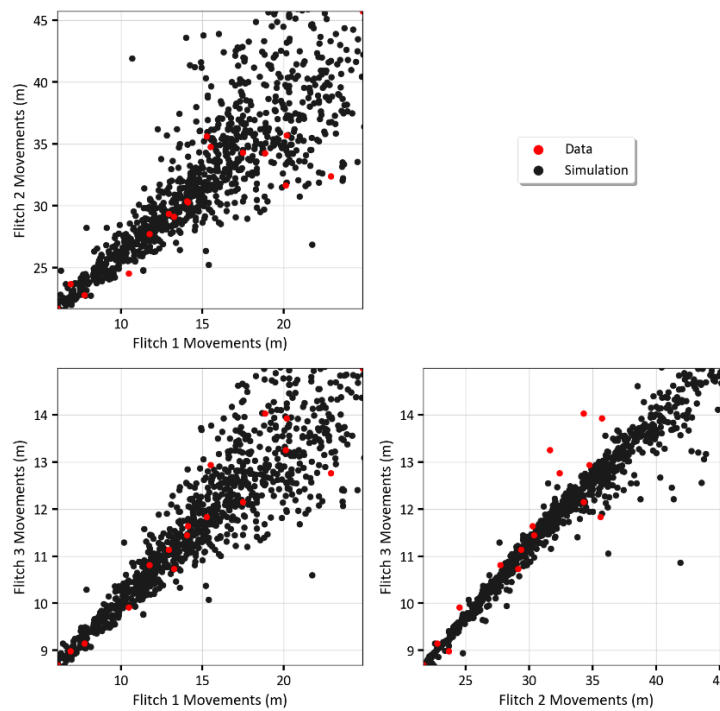


Figure 3.6: Checking the reproduction of multivariate relationships between blast movement data.

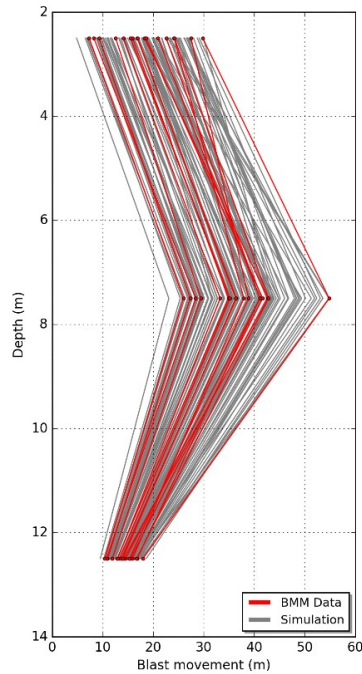


Figure 3.7: Checking the reproduction of input blast movement measurements with simulated realizations.

One hundred blast movement realizations are submitted to the dig-limits optimization model to determine the optimal destinations for the SMUs while honoring equipment selectivity. A minimum mining width of 10 m by 10 m is selected to represent the selectivity of the shovel's bucket used to mine the section of the bench. The optimal dig-limits for the 100 blast movement realizations are produced and then used to generate the ore probability map. Figure 3.8 summarizes the results of the proposed methodology, starting from identifying pre-blast destinations, simulating blast movement realizations, identifying post-blast destinations, applying dig-limits optimization of post-blast 2D economic block models and generating post-blast dig-limits realizations, and finally producing the ore probability map.

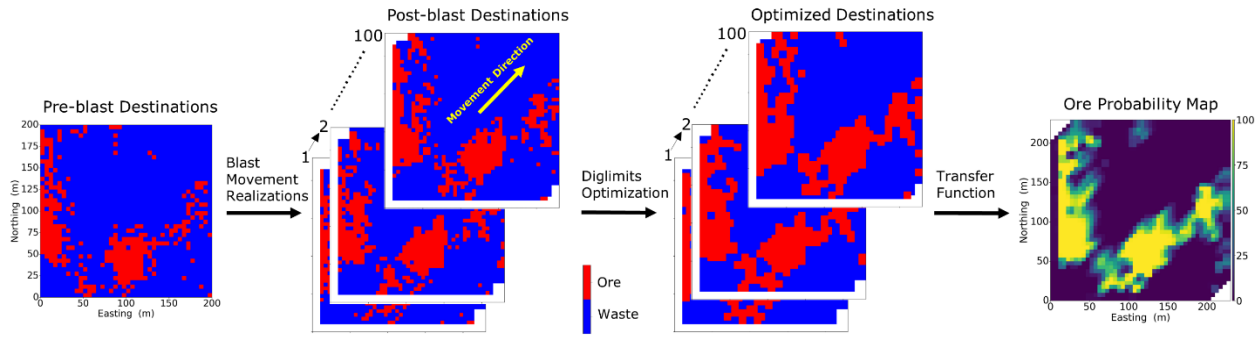


Figure 3.8: Summary of the proposed methodology.

As seen in Figure 3.8, the ore probability map was generated. In the map, the areas shown in yellow and navy blues are certain ore and waste areas, respectively. The greenish areas might be either ore or waste, and they will be referred to as risk areas in this paper. These risk areas need special attention when determining the best destination for their materials. This ore probability map can be used to determine areas in which a careful grade control sampling program should be implemented. This map helps optimize the number of grade control samples so that grade control geologists can design a sampling program that focuses on a more detailed sampling of high-risk areas. As a result of producing an ore probability map, risk areas are identified (Figure 3.9). This ore risk map can be used to reduce the number of check samples taken during grade control, and this will result in reducing the cost and the time needed in assaying certain ore and waste areas; thus, increasing the profit from mining this section of the bench. It is apparent that the spatial distribution of risky areas relies on three factors: (1) the degree of heterogeneity of the mineralization, (2) the difference in the magnitude of blast movement across flitches, and (3) the cut-off grade. A low degree of heterogeneity will reduce the number of risky areas in the blasted section of the bench. A high cut-off grade generates more isolated ore areas than a low cut-off

grade, increasing the risk areas. Having a significant difference in blast movement magnitude across flitches increases the dilution and ore losses, which also increases the risk areas.

Additionally, the best locations for installing BMM balls can be determined using ore risk maps. In fact, BMM balls are known for their high costs and their intermediate recovery rate after blasting. Therefore, identifying high-risk areas before blasting brings more valuable information on the magnitude and direction of blast movement at those uncertain locations and can reduce the extra cost attributed to installing BMM balls at low-risk areas. To generate blast movements realizations and produce ore risk maps, grade control geologists can utilize their knowledge and experience from previous blasting operations conducted on benches with similar geological features and blast designs. This pre-blast study will enable grade control geologists to generate ore risk maps and determine high-risk areas within the bench that are considered ideal locations for installing BMM balls.

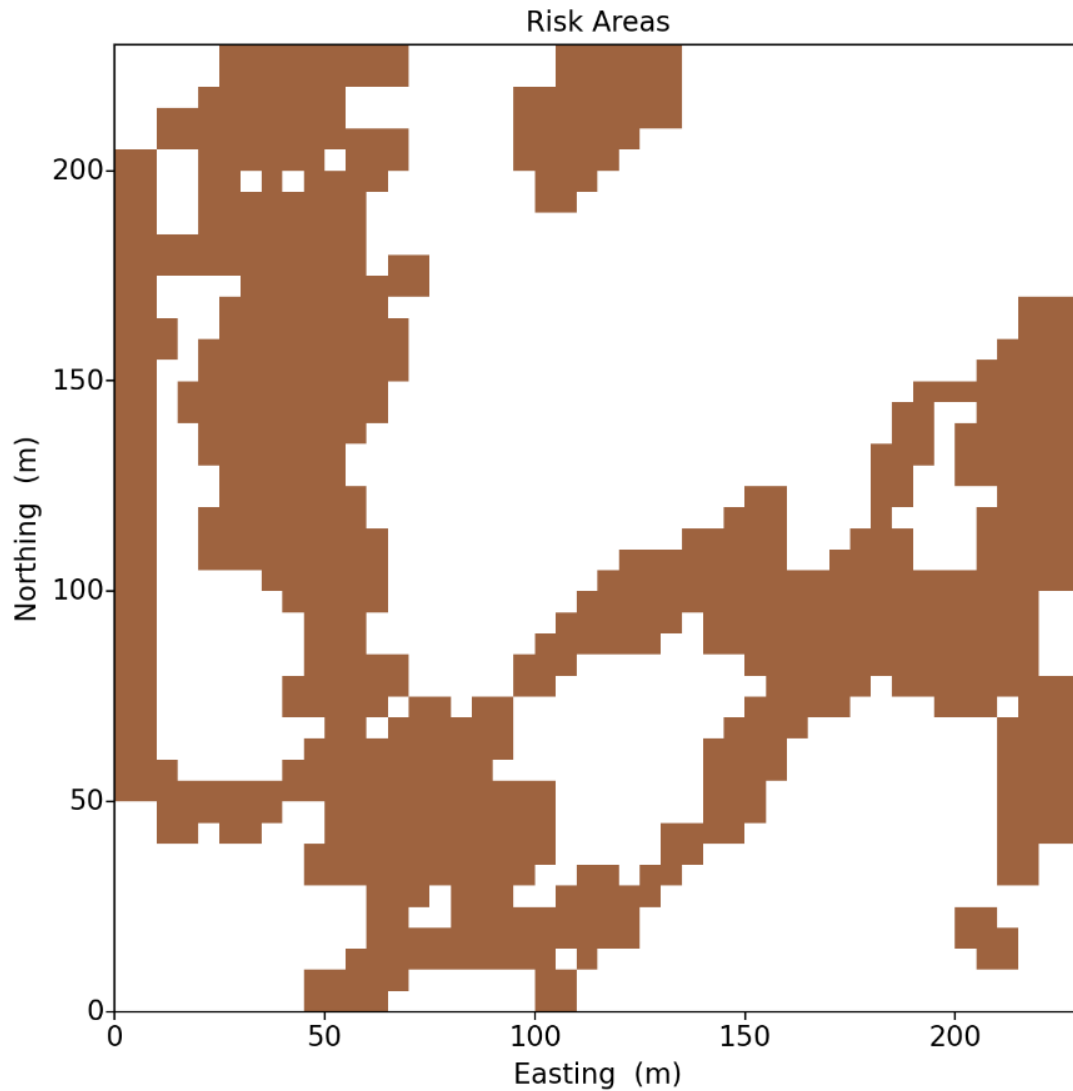


Figure 3.9: The areas with high-risk probability in determining ore-waste selective mining units (shown in brown).

To show the effect of ignoring blast movement impact on dig-limits optimization, another scenario is investigated where the same proposed dig-limits optimization model is applied on the same grade control block model using the same input parameters but without considering blast movement. Figure 3.10 shows the results of this scenario where original ore and waste SMUs are compared to their optimal pre-blast destinations after applying dig-limits optimization.

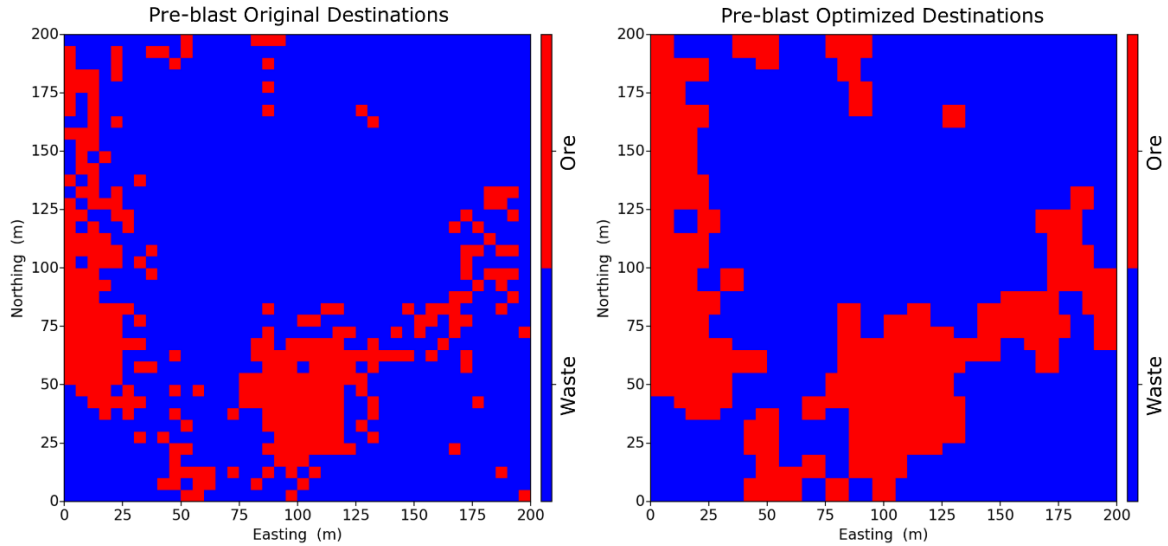


Figure 3.10: Map of the original selective mining units' destinations on the left and the optimized pre-blast units' destinations on the right.

The results of running dig-limit optimization on a pre-blast economic block model show that equipment selectivity increases the amount of ore loss by 3.2% and dilution by 35.4%. Neglecting this amount of ore loss and dilution will lead to an over-estimate of the profit from approximately \$2.36 Million to \$2.80 Million (15.7% profit over-estimation).

In this case study, the equipment selectivity is considered relatively low when it is compared to the size of SMUs. The low equipment selectivity (1) increases dilution by adding extra waste to the mined ore, and (2) increases ore losses by mining small portions of the ore as waste while mining the surrounding waste and then sending it to the waste dump. Consequently, the low equipment selectivity decreases the profit obtained from mining. The problem of profit decrease is further accelerated when the effect of blast movements is ignored in the dig-limit optimization. To understand the effect of neglecting blast movements when determining optimum dig-limits, the profit values calculated for each optimized dig-limits realization are compared with the single

profit value obtained when no blast movements are considered. The comparison result shows that the profit obtained from single optimum dig-limits that ignored blast movements is over-estimated by 65.3% when compared to the expected profit value where blast movements are incorporated into the process. Figure 3.11-a denotes the distribution of profit realizations when blast movements are considered in determining dig-limits. Figure 3.11-b presents the distribution of profit over-estimation caused by neglecting the effect of blast movements.

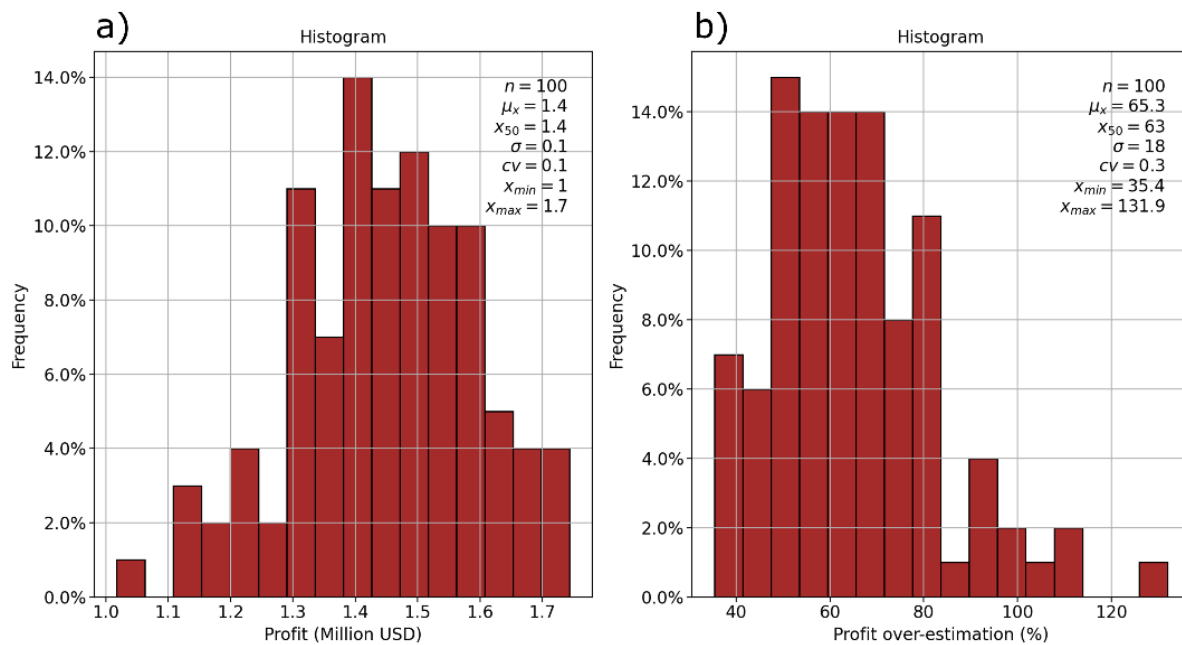


Figure 3.11: (a) Histogram of profit distribution; (b) Histogram of profit over-estimation.

Modeling blast movement prior to running dig-limits optimization has a significant impact on resulting dig-limits. The results obtained from this study should not be generalized for all mineral deposits because mineral deposits vary in the spatial distribution of their mineralization, cut-off grade, and blast designs. However, this study shows the importance of incorporating blast movement when determining optimal dig-limits to generate a more realistic assessment of the

expected profit. In addition, the reconciliation programs at the mine should include blast movements when tracking ore and waste at stockpiles and the process plant.

3.5 CONCLUSION AND FUTURE WORK

Quantifying risk in the dig-limits optimization to account for the unavoidable uncertainty in blast movement prediction is important for grade control and short-term mine planning. The paper's contribution lies on quantifying the uncertainty attributed to blast movements on the optimal dig-limits through a copula-based simulation approach. Physical field measurements for blast movement obtained from BMM balls are used to furnish the multivariate blast movement distributions for a number of flitches within benches using a D-vine copula. The realizations of blast movements are generated through MCS after fitting a D-vine copula distribution to the blast movements of flitches. The MIP algorithm has been utilized to find the optimal dig-limits for all blast movements realizations. The ore probability map is generated showing locations of ore and waste in a probabilistic fashion. The expected profit from a modeled bench was over-estimated by 65.3% on average when blast movement was disregarded in comparison to the expected profit modeled from the same bench when the blast movement realizations are incorporated in the proposed workflow. High-risk ore and waste areas post-blasting are identified from the ore probability map. Due care should be taken when mining these uncertain (high-risk) areas.

Future work will focus on adding geological uncertainty in conjunction with the blast movement uncertainty to produce optimal dig-limits. Further, the proposed methodology will be extended to cover more than two destinations. At the same time, local variations in the direction of blast movement will be modeled and added to the proposed workflow. With the advanced development in parallel computing, aggregation/disaggregation, and decomposition methods such as nested Bender decompositions, the solutions that can be generated in an acceptable time will be explored.

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3.7 NEXT STEPS

After examining the impact of blast movement uncertainty on dig-limits optimization outcomes, this research will now focus on understanding how blast movement affects ore heterogeneity post-blasting. To achieve this, both global and local spatial entropy indices, calculated at a scale equivalent to the selective mining unit, are proposed. These indices will be used to investigate the relationship between blast movement horizontal distance, ore heterogeneity, ore loss, dilution, and the resulting profit reduction through a series of case studies. The outcomes of this study will provide insights into how controlled blasting can mitigate the increase in post-blast spatial entropy, thereby reducing ore loss and dilution in specific bench sections and increase mining operations profitability.

Chapter 4: Spatial Entropy for Quantifying Ore Loss and Dilution in Open Pit Mines

4.1 ABSTRACT

Effective management of ore loss and dilution is essential for successful grade control and short-term mine planning due to their significant impact on the economic, environmental, and technical aspects of open-pit mining operations. Factors influencing ore loss and dilution fall into two categories: (i) controllable factors like mine equipment selectivity and blast design and (ii) uncontrollable factors such as spatial heterogeneity of ore and waste blocks on a bench. This paper focuses on the second category by applying spatial entropy concept to capture heterogeneity at the scale of selective mining units. In this paper, global spatial entropy is used to assess the impact of spatial heterogeneity between ore and waste blocks on the magnitude of ore loss and dilution, while the local spatial entropy can guide the allocation of blast movement monitoring balls pre-blast. High values of the global spatial entropy indicate increasing potential of ore loss and dilution, which reduce profit. Furthermore, the study investigates the relationship between spatial entropy, cut-off grades, blast movement, dig-limits optimization model running time, and profit through a number of case studies. The results show that changes in cut-off grade and blast movement can significantly affect spatial entropy post-blasting and increase ore loss, dilution, and profit reduction, revealing the need for controlled blasting at specific bench sections. Additionally, the results demonstrate an exponential increase in profit reduction due to ore loss and dilution with a rising global spatial entropy.

4.2 INTRODUCTION

Controlling ore loss and dilution is a vital issue in grade control and short-term mine planning due to its significant impact on the profitability of mining projects. Dilution refers to mixing waste material with the ore that will be sent to the processing plant. Ore loss is defined as ore material not separated from the waste and sent to the waste dump. Ore loss and dilution are classified as planned and unplanned. Shovel size and capability, to a certain degree, govern planned ore loss and dilution. On the other hand, uncertainty in blast movement also causes unplanned ore loss and dilution, which are rather difficult to control by grade control geologists and short-term mine planning engineers. In addition to shovel size and blast movement, the misclassification of ore and waste significantly contributes to ore loss and dilution. This misclassification occurs due to the limitations of estimation and simulation methods, data scarcity, assaying errors, and grade control methods. Consequently, the mined material is sent to the wrong destination.

The consequences of ore loss and dilution are quite distinct. Ore loss is an opportunity cost, where the potential profit that could be gained from processing a block is missed. Dilution represents a cash cost where a block that does not contain sufficient metal to generate a profit is still processed. Dilution has economic, environmental, and technical impacts. It increases operational costs, energy consumption, and tailing quantities (Zarshenas & Saeedi, 2016).

Reducing dilution in mining operations can reduce the project's carbon footprint, material handling costs, and the unnecessary processing of waste materials. This can be achieved through improved blasting designs, equipment sizing, more accurate orebody characterization, and improved grade control procedures (Zarshenas & Saeedi, 2016; Dowd & Dare-Bryan, 2018)

Several factors determine the level of ore loss and dilution in any mining operation, including the shape of the orebody, the spatial distribution of ore and waste zones within the orebody, equipment selectivity, blast movement, and the selected mining method (Marinin et al., 2021). Some of these factors, such as the shape of the orebody and the spatial distribution of ore and waste zones, may vary locally within the same deposit, leading to varying degrees of ore loss and dilution within the same mine. While these locally varying factors are typically considered uncontrollable, equipment selectivity, blast design, and the selected mining method are regarded as controllable factors that can be controlled to minimize ore loss and dilution; thus, reducing material misclassification.

It is impossible to eliminate ore loss and dilution entirely. Therefore, this paper employs the concept of spatial entropy to help assess the magnitude of potential ore loss and dilution pre-blasting. Local analysis of spatial entropy can also guide the selection of areas within the bench section that require careful blast movement monitoring to reduce ore-waste misclassification. This research uses a cut-off grade to discriminate blocks into ore and waste and calculate global and local spatial entropy indices at specific bench sections. These indices can be used as an early warning system alerting grade control geologists, short-term mine planning engineers, and blasting engineers to identify possible ore loss and dilution problems in a bench section. To reduce the negative impact of ore loss and dilution, the following processes can be optimized:

- (1) Dig-limits delineation: Implement a solution for dig-limits that effectively reduces ore loss and dilution that honors equipment selectivity.
- (2) Blast design: Develop a blasting design strategy that mitigates the increase in ore loss and dilution due to blast movement.

The degree of acceptable ore loss and dilution may vary from one mine site to another (Pakalnis et al., 1996). Therefore, it is crucial for mining operations to minimize ore loss and dilution to an acceptable degree, if possible, to enhance the profitability of the project.

Dig-limits optimization aims to quantify, control, and reduce the amount of planned ore loss and dilution in open pit mines. These dig-limits establish boundaries that separate ore from waste, ensuring that the blasted materials are sent to their optimal destinations to maximize profits while honouring equipment selectivity and accounting for blast movement (Hmoud & Kumral, 2022). Numerous methods have been proposed in the literature to address the dig-limits problem. These methods can be classified into five groups: (1) exact methods (e.g., mixed-integer programming (MIP) (Hmoud & Kumral, 2022; Kumral, 2015; Nelis et al., 2022; Nelis & Morales, 2022; Sari & Kumral, 2017)); (2) heuristics (Richmond & Beasley, 2004; Vasylchuk & Deutsch, 2019a; Wilde & Deutsch, 2015); (3) metaheuristics (e.g., simulated annealing (Isaaks et al., 2014a; Isaaks et al., 2014b; Norrena & Deutsch, 2000; van Duijvenbode & Shishvan, 2022); genetic algorithms (Ruiseco, 2016; Ruiseco & Kumral, 2017; Ruiseco et al., 2016; Williams et al., 2021); (4) hybrid approach combining MIP and metaheuristics (Deutsch, 2017); and (5) spatial clustering (Salman et al., 2021; Tabesh & Askari-Nasab, 2011, 2013; Tabesh & Askari-Nasab, 2019).

MIP guarantees optimal solutions. However, these solutions require high computational cost and long processing times to reach optimality. The required time to solve dig-limit problem is case-specific and highly dependent on entropy. With the advancements in high-performance computing, using MIP to generate reasonably fast optimal results is achievable if entropy is low. Otherwise, near-optimal solutions for the dig-limits problem are obtained using heuristics and metaheuristics algorithms. These algorithms are relatively fast when compared to MIP. However, the results of these methods are highly sensitive to the selected input parameters. Therefore, hyperparameter

tuning is essential for achieving acceptable results with these algorithms. However, since the solution to this problem is unknown in advance, tuning the hyperparameters becomes a challenging task.

Deutsch (2017) combined MIP and metaheuristics methods in a workflow. This hybrid approach ensures reasonable starting solutions for the MIP from the initial solutions of the non-optimal MIP to increase the chances of reaching an acceptable near-optimal solution for the dig-limits problem within a reasonable time. The MIP in this approach is forced to stop at a pre-specified gap value or after a certain time to generate non-optimal but acceptable solutions for the metaheuristics. Like heuristic and metaheuristic algorithms, this approach is also prone to non-optimal solutions and is sensitive to input parameters, limiting its ability to deliver guaranteed optimal results consistently. Nevertheless, near-optimality is guaranteed.

Additionally, spatial clustering techniques are used to solve the dig-limits problem (Tabesh & Askari-Nasab, 2011, 2013; Tabesh & Askari-Nasab, 2019). These approaches rely on generating dig-limits based on hierarchical or k-means clustering, which is based on the distance between blocks, rock types, material destinations, metal grade and the shape of the mining cuts as input to generate spatial clusters that delineate dig-limits. However, a refinement stage may be needed to homogenize the spatial cluster to form coherent zones, and this step increases the ore loss and dilution. The advantage of these approaches is in their ability to generate dig-limits in a fast time, but they do not guarantee optimality. Additionally, selecting input clustering parameters, such as the desired number of clusters, requires careful consideration. This subjectivity introduces uncertainty that must be captured and managed to ensure the outcomes of the spatial clustering algorithms are meaningful and reliable.

Blast movement is another critical factor that affects ore loss and dilution. However, most dig-limits optimization techniques and workflows do not account for blast movement when determining optimal dig-limits. Therefore, any mathematically optimized dig-limits that ignore blast movements are not operationally optimal (Engmann et al., 2013; Hmoud & Kumral, 2022; Thornton et al., 2005). Incorporating blast movement into dig-limit optimization process will also reduce misclassification (Zhang, 1994; Yennamani, 2010). Furthermore, monitoring blast movements can increase ore recoveries and controlling head grade in mineral processing (Gilbride, 1995; Harris, 1997; Taylor & Firth, 2003).

Modeling blast movements prior to running any dig-limits optimization algorithm is essential to capture ore and waste zones post-blasting. There are four main approaches for modeling blast movements in the literature: (1) the multi-physical simulation of blast movements that requires physical field measurements to validate the multi-physical model (Yu et al., 2022; Zou & Jun, 2020); (2) modeling blast movements using heuristics and physical field measurements (Isaaks et al., 2014a; Isaaks et al., 2014b; Vasylchuk & Deutsch, 2019b); (3) training machine learning algorithms on blast movement information obtained from previously blasted benches (Yu et al., 2020; Yu et al., 2019); and (4) stochastic modeling of blast movement using Monte Carlo simulation and physical field measurements obtained from blast movement monitoring (BMM) balls (Hmoud & Kumral, 2022).

The main limitation of using a multi-physics simulation to model blast movement is the challenge of accurately collecting all necessary input parameters. Due to the complex nature of the simulation, it can be difficult to ensure that all relevant parameters are included and accounted for in order to generate acceptable blast movement models. Additionally, the current multi-physical simulators require significant computational time to generate a blast movement model. The

importance of generating a blast movement model in a short time is essential when considering the nature of the blast operation that is performed a few times a week in a mine. Blast movement modeling using heuristics and physical field measurements such as post-blast topography surfaces is a faster solution to model blast movement. However, heuristics do not guarantee optimality, and how much they deviate from optimality is unknown. Moreover, not all mining operations perform post-blast topography surveys of their blasted bench sections before delineating dig-limits. Applying machine learning predictive modeling to predict future blast movement from historical blast information is also another potential solution for modeling blast movement. It may generate an acceptable prediction of blast movement when the geology and the blast design of the bench to be blasted are similar to some of the historical blast information that was used to train the blast movement predictive models. However, every deposit has unique geological characteristics and a degree of heterogeneity that vary within the same deposit at different zones. Therefore, training a machine learning algorithm to predict blast movement based on historical blast data for deposits with different geology and blast designs may not be accurate enough, especially when uncertainty in the predicted movement is not captured and managed.

Uncertainty in the blast movement can be attributed to the inconsistencies in blast design, the heterogeneity of blasted rock mass, the incomplete knowledge of geology, and human factors. It is highly unlikely to get accurate models of blast movements using only theoretical modeling approaches without obtaining physical field measurements before blasting any bench section (La Rosa & Thornton, 2011). As a result, using stochastic modeling techniques that use blast movement data obtained from physical field measurements coming from the same blasted bench is considered a reasonable solution for this problem (Hmoud & Kumral, 2022). However, achieving adequate coverage of these physical field measurements in bench sections, including all flitches, is

necessary. This helps construct representative multivariate statistical distributions encompassing blast movement uncertainty, particularly around ore zones. It also aids in identifying high-risk areas of material misclassification, demanding meticulous grade control sampling protocols.

As emphasized before, the magnitude of ore loss and dilution depends on the degree of spatial heterogeneity within the orebody at the selective mining unit (SMU) scale. The degree of spatial heterogeneity of ore and waste is influenced by the spatial distribution of grade and the cut-off grade that separates ore from waste. As an attempt to measure the degree of spatial heterogeneity between ore and waste, the concept of entropy is used. Entropy is a concept that originated first in thermodynamics to describe the state of gases or fluids within a system and the distribution probability of molecules. The special use of entropy was adapted from information theory by Claude E. Shannon as a measure of information contained in a given signal (Shannon, 1948). When computing the disorder of a system, also known as heterogeneity in more geological terminology, for a set of units (e.g., locations) with a finite number of categories (e.g., mined material destinations), a standard way to assess this heterogeneity is to compute entropy. Over the years, the Shannon entropy has been widely used in many applied sciences, such as ecology and geography (Hoeting et al., 2000; Patil & Taillie, 1982), hydrology and hydrogeology (Aghakouchak, 2014; Butera et al., 2018; Manchuk et al., 2021; Singh, 1997) and geology (Feizi et al., 2017; Liu et al., 2021; Zhang et al., 2021).

All previously mentioned applications of entropy do not consider the spatial location of data. Therefore, datasets with the same probability mass functions, but different spatial configurations will have the same entropy values. However, the need to have a measure of entropy that accounts for the spatial location of data emerged over the years. These spatial entropy indices are calculated only on categorical variables, which make them suitable for measuring the heterogeneity of ore

and waste zones within the orebody. Theil (1972) introduced the concept of spatial entropy to measure the level of inequality or diversity in the distribution of variables across space. Batty (1974, 1976, 2010) extended Theil (1972) and developed a spatial entropy measure that is restricted only to one category at a time by applying partitioning for the spatial system to define a representative number of subareas to conduct further spatial analysis and measure spatial entropy for these unequal space partitions of these subareas. Karlström and Ceccato (2000) modified Batty (1974, 1976) to satisfy the property of additivity to ensure that all local spatial entropy measures sum to the exact global spatial entropy measure following the idea of Local Indices of Spatial Association (LISA) proposed by Anselin (1995). O'Neill et al. (1988) developed spatial entropy indices that quantify landscape heterogeneity to discriminate among major landscape types. They also proposed the contagion index that measures the extent to which locations with the same categorical variable are spatially clustered. Claramunt (2005) and Li and Claramunt (2006) proposed a measure of spatial diversity that accounts for the spatial configuration as a weight factor in the Shannon entropy. This weight factor is the ratio of the average distance between pairs of a particular category (intra-distance) to the average distance of those pairs with different categories (extra-distance). Leibovici (2009) and Leibovici et al. (2014) extended the work of O'Neill et al. (1988) and Claramunt (2005) to account for the multivariate distribution of co-occurrences at various distances. A number of contagion indices are proposed in literature based on the spatial entropy concept (Li & Reynolds, 1993; Parresol & Edwards, 2014; Riitters et al., 1996; Zhang et al., 2020).

Altieri et al. (2017, 2018, 2019, 2021) proposed a spatial entropy measure extending beyond two categories and simultaneously considering multiple distance ranges. In their approach, the total entropy measure (akin to Shannon's entropy) is broken down into spatial mutual information,

revealing how categorical data cluster in space, and spatial global residual entropy, summarizing the variability in the variable's spatial distribution. However, mutual information can be compromised by weak spatial correlation between distant categories, potentially yielding misleading low values. This means that spatial mutual information might seem insignificant even if certain categories cluster strongly in close proximity. Mutual information and spatial global residual entropy can be further divided into distinct components describing specific distance intervals. Despite these enhancements, the partial spatial residual entropy, capturing spatial heterogeneity at short distances, cannot quantify entropy at precise locations. Nonetheless, it can provide a summary of heterogeneity between pairs within short distance intervals across the area of interest.

In geostatistics, Journel and Deutsch (1993) explored the relationship between the maximum prior entropy and the response distribution. They introduced a global spatial entropy measure that calculates entropy at various lag distances for a set of discrete variables. Their spatial entropy measure provided a global measure of spatial entropy and it was not aimed to quantify entropy at a local level (i.e., SMUs). Additionally, their spatial entropy calculates entropy for all possible pairs separated by a certain number of lag distances within a certain area of interest. Their spatial entropy formulation needs to be modified to quantify local spatial entropy at certain locations with the surrounding SMUs and only account for relevant pairs when calculating spatial entropy both globally and locally.

Our research customizes the spatial entropy formulation to account for the scale of equipment selectivity when measuring global and local spatial entropy. This adjustment aims to enhance the accuracy of assessing ore loss and dilution on both global and local scales. Furthermore, it calculates local spatial entropy measure by considering only the blocks surrounding every block

within the block model that are located inside all possible SMUs around that block. This approach provides a more refined assessment of entropy at each location within the block model by considering only relevant pairs and removing irrelevant pairs from the calculation. The number of blocks used are determined by the size of the SMU.

It is also important to note that the optimization of blast design should consider the impact of orebody heterogeneity resulting from the spatial distribution of ore and waste blocks within the bench sections, because it may cause issues related to ore loss, dilution, and material misclassification resulting in significant profit reduction (Dowd & Dare-Bryan, 2018).

Based on previous research on estimating ore loss and dilution, the relationships among orebody spatial heterogeneity, cut-off grade, blast movement, and dig-limits optimization running time remain unstudied. This paper builds upon Hmoud and Kumral's (2022) study which investigated the incorporation of blast movement uncertainty into dig-limits optimization workflow. The originality of this paper is four-fold: (1) applying customized spatial entropy indices to describe the spatial heterogeneity of ore and waste at the SMU scale within bench sections; (2) quantifying the relationship among cut-off grade, spatial entropy, and dig-limits optimization running time and outcomes; (3) investigating the impact of blast movement on spatial entropy and profit reduction; and (4) showing how spatial entropy can be used to guide blast design to influence control on the magnitude of ore loss and dilution.

The organization of this paper is as follows: The proposed methodology is elaborated upon in the next section. Next, case studies that demonstrate the application of spatial entropy to understand the impact of cut-off grade and blast movement on the level of ore loss and dilution are presented. These case studies also explore how spatial entropy can explain the variability in dig-limits

optimization running time. Finally, the results are discussed, conclusions are drawn, and avenues for future research are proposed.

4.3 METHODOLOGY

This section offers a comprehensive overview of the spatial entropy concept and demonstrates how it was adapted to quantify spatial heterogeneity both globally and locally at the SMU scale. To demonstrate the proposed methodology, a block model, which is a numerical representation of an orebody, is generated. The blocks constituting the block model are typically half the size of the SMU, aiming for a finer resolution of the block model that allows for a more accurate representation of the deposit's heterogeneity. Spatial heterogeneity between ore and waste within a bench section can be explored by applying a cut-off grade to the blocks that have grade estimates. This process reveals how the degree of heterogeneity can vary locally, contingent on factors like the type of mineralization, location within the mineral system, and the applied cut-off grade. Figure 4.1 displays three distinct scenarios of spatial heterogeneity of ore and waste blocks within a bench section.

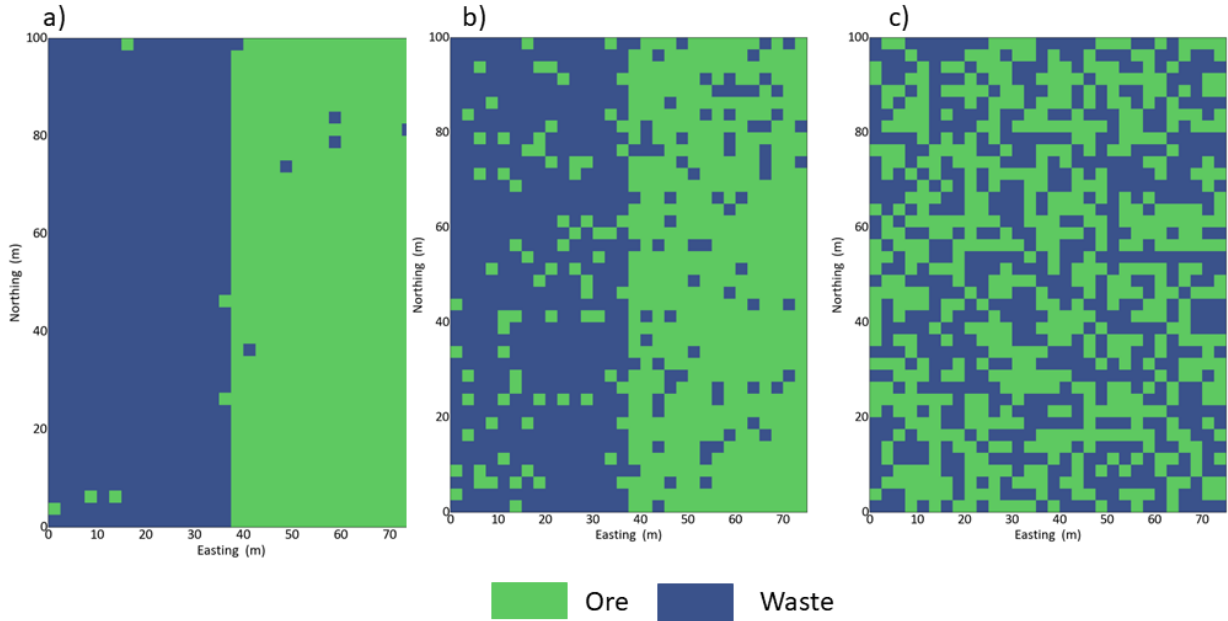


Figure 4.1: Three simulated benches showing different degrees of spatial heterogeneity between ore and waste blocks. a) low degree of spatial heterogeneity; b) medium degree of spatial heterogeneity; and c) high degree of spatial heterogeneity.

To explain the concept of entropy, the following notation is adopted. Let $I = \llbracket 1, N_{east} \rrbracket, N_{east} \in \mathbb{N}$, which represents the set of block indexes in the east direction. Similarly, let $J = \llbracket 1, N_{north} \rrbracket, N_{north} \in \mathbb{N}$, denotes the set of block indexes in the north direction. The 2-dimensional grade control block model consists of the set of blocks $B = I \times J = \{b_{i,j} = (i,j), \forall i \in I, \forall j \in J\}$. Each block $b_{i,j} \in B$ is associated with the event of having a specific block category $c_{i,j} \in \Omega$, where $\Omega = \{\omega_0, \omega_1\}$ is the set of possible outcome categories for a given block representing waste and ore, respectively. A discrete random variable $X: \Omega \rightarrow \{0,1\}$ is defined to model block category such that $X: \omega \in \Omega \mapsto X(\omega) \in \{0,1\}$ donates the occurrence of either waste or ore category, respectively. The probability of a block having the ore or waste category is calculated as

$$\mathbb{P}(X = k) = \mathbb{P}_X(k) = \frac{\text{card}(f^{-1}(\{\omega_k\}))}{\text{card}(B)}, \forall k \in \{0,1\}, \text{ where } f: B \rightarrow \Omega \text{ maps each block to its}$$

associated category realization and *card* refers to cardinality which describes the number of items in a given set. Given $\mathbb{P}(X = 1)$, the probability of having a waste block $\mathbb{P}(X = 0)$ can be determined as $\mathbb{P}(X = 0) = 1 - \mathbb{P}(X = 1)$. According to Cover and Thomas (2006), Shannon's entropy for two categories can be defined as

$$H_X(X) = \mathbb{P}(X = 0) * \log_2(\mathbb{P}(X = 0)^{-1}) + \mathbb{P}(X = 1) * \log_2(\mathbb{P}(X = 1)^{-1}) \in [0,1] \quad (4.1)$$

To better explain how Shannon's entropy concept is tailored to capture the spatial heterogeneity of ore and waste blocks within a bench section at the SMU scale, the following steps are taken. First, a frame $W_{i,j} = \{b_{i+k,j+l}, \forall k \in \llbracket -1,1 \rrbracket, \forall i \in I, \forall j \in J\}$ is defined. This frame surrounds $b_{i,j}$ and includes all blocks that might be combined with $b_{i,j}$ to form an SMU. For example, when the SMU size is 2-block by 2-block, the blocks that might be considered part of the SMU surrounding $b_{i,j}$ will form a 3-block by 3-block frame in which $b_{i,j}$ is at the centroid of this frame. $P_{i,j}, \forall i \in I, \forall j \in J$ represents the set of admissible pairs inside the frame $W_{i,j}$ and is donated by $P_{i,j} = \{(b_{i,j}, b_{i+k,j+l}), \forall (k,l) \in \llbracket -1,1 \rrbracket^2 \setminus (0,0)\}$. The set of possible pairs of block category outcomes is denoted by $\Omega' = \{\omega'_0, \omega'_1\}$, where ω'_0 and ω'_1 correspond to the events of having pairs of blocks with different and same categories, respectively. To obtain the associated category realization for each pair, the application $g_{i,j}$ is used and defined as $g_{i,j}: P_{i,j} \rightarrow \Omega'$ and

$$g_{i,j}: (b, b') \mapsto \begin{cases} \omega'_0, & \text{if } f(b) \neq f(b') \\ \omega'_1, & \text{otherwise} \end{cases}. \text{ This application defines pairs of blocks with both}$$

different and identical categories and utilizes this information to calculate their probabilities in order to derive the local spatial entropy index.

A discrete random variable $Z_{i,j}, \forall i \in I, \forall j \in J, Z_{i,j}: \Omega' \rightarrow \{0,1\}, Z_{i,j}: \omega \in \Omega' \mapsto Z_{i,j}(\omega) \in \{0,1\}$ is represented to correspond to the pairs of block categories within the frame $W_{i,j}$. This discrete

random variable is used to measure local spatial entropy at $b_{i,j}$ inside the frame $W_{i,j}$. The probability of having pairs of the same or different categories is calculated using $\mathbb{P}(Z_{i,j} = k') = \mathbb{P}_{Z_{i,j}}(k') = \frac{\text{card}(g_{i,j}^{-1}(\{\omega_{k'}\}))}{\text{card}(P_{i,j})}, \forall k' \in \{0,1\}$. The probability of pairs with different categories can be calculated by subtracting the probability of pairs with the same categories from 1, and it equals $\mathbb{P}(Z_{i,j} = 0) = 1 - \mathbb{P}(Z_{i,j} = 1)$. Figure 4.2 depicts a schematic describing how $\mathbb{P}_{Z_{i,j}}$ is calculated around $b_{i,j}$ that is surrounded by a frame $W_{i,j}$.

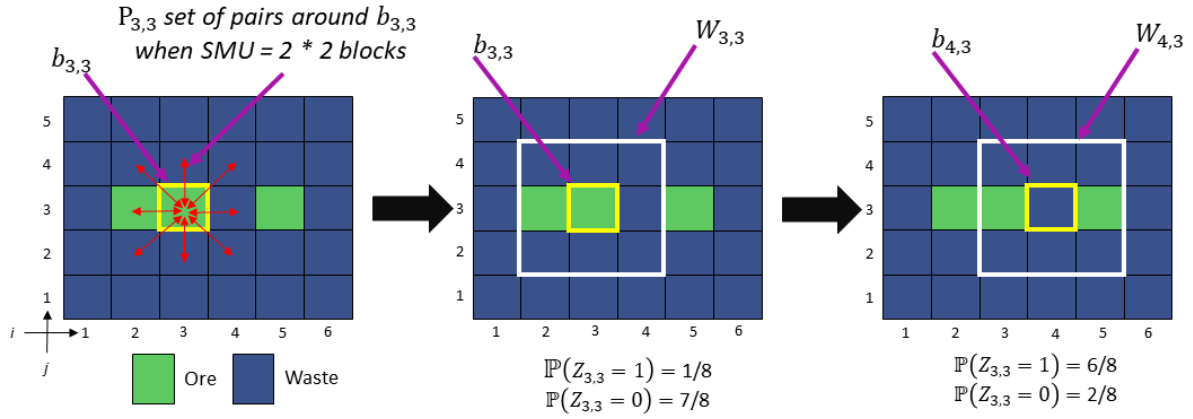


Figure 4.2: Schematic describing the calculation of local spatial entropy probabilities within known SMU size.

To define the Local Spatial Entropy Index (LSEI) as a measure of local entropy for each block, the following equation is proposed:

$$h_{i,j}(Z_{i,j}) = \mathbb{P}(Z_{i,j} = 0) * \log_2 \left(\mathbb{P}(Z_{i,j} = 0)^{-1} \right) + \mathbb{P}(Z_{i,j} = 1) * \log_2 \left(\mathbb{P}(Z_{i,j} = 1)^{-1} \right) \quad (4.2)$$

$$\in [0,1], \forall i \in I, \forall j \in J$$

Where $h_{i,j}(Z_{i,j})$ represents LSEI at $b_{i,j}$. The following equation is proposed to measure Local Spatial Information Index (LSII) at each block:

$$i_{i,j}(Z_{i,j}) = 1 - h_{i,j} \in [0,1] \quad (4.3)$$

Where $i_{i,j}(Z_{i,j})$ represents LSII at $b_{i,j}$. To calculate the Global Spatial Entropy Index (GSEI) for the entire bench section, the expected value of all Local Spatial Entropy Index (LSEI) values is computed. The GSEI is defined as:

$$H = \sum_{i=1}^{N_{east}} \sum_{j=1}^{N_{north}} \frac{h_{Z_{i,j}}(Z_{i,j})}{N_{east} * N_{north}} \in [0,1] \quad (4.4)$$

Where H represents GSEI. Moreover, the Global Spatial Information Index (GSII) is calculated by computing the expected value of LSII, and it is defined as:

$$I = \sum_{i=1}^{N_{east}} \sum_{j=1}^{N_{north}} \frac{i_{Z_{i,j}}(Z_{i,j})}{N_{east} * N_{north}} = 1 - H \in [0,1] \quad (4.5)$$

Where I represents GSII. The GSII index reflects the degree of spatial continuity or clustering between ore and waste blocks. The impact of spatial entropy on ore loss, dilution, dig-limits running time, and profit is investigated using the derived spatial entropy indices that take into consideration equipment selectivity, cut-off grade, and blast movement. This study provides a comprehensive evaluation of the potential ore loss and dilution related to specific bench sections and identifies opportunities to control ore loss and dilution using the concept of spatial entropy.

This paper adopts dig-limits optimization incorporating blast movement methodology from Hmoud and Kumral (2022). Additionally, it uses the concept of spatial entropy to quantify ore and waste spatial heterogeneity within selective mining units. Spatial entropy has a significant impact on ore loss, dilution, and ultimately, mining profitability. The dig-limits optimization model itself was developed by Sari and Kumral (2017) and re-coded using Python 3.8, running on the CPLEX

solver (IBM, 2021). The dig-limits optimization problem is modeled as a MIP model, aiming to maximize the profit from the blasted section of the bench while adhering to equipment selectivity constraints. Equipment selectivity is represented in a 2-block by 2-block configuration in both the east and north directions within a grade control block model. Blast movement scenarios are applied to the grade control block model, and spatial entropy is calculated before and after blasting to understand the impact of spatial entropy, cut-off grade, and blast movement on ore loss, dilution, dig-limits optimization running time, and profit. The following section presents several case studies and discusses the applied workflow.

4.4 CASE STUDIES

This section presents a series of case studies that evaluate the effectiveness of global and local spatial entropy indices in assessing the extent of profit reduction resulting from dilution and ore loss across various levels of spatial heterogeneity, cut-off grades, and blast movement scenarios. These indices were applied to a simulated gold mine bench section. The conditional simulation model utilized gold grade values obtained from an actual mine operation in North America, with the mine name withheld due to data confidentiality. The simulated block model mimics the grade control block model at that bench section.

Using simulated benches in this study is essential because real-world data may not capture the required range of spatial heterogeneity levels required for the study. The study encompasses a spectrum from low to high levels of spatial heterogeneity, and using simulated benches ensures that each of these simulated benches contains the same amount of contained metals for accurate comparison purposes.

The subsequent sections provide detailed information on the data generation process for each study, along with their corresponding findings.

4.4.1 Local and global spatial entropy indices

A bench section with simulated gold grade values was constructed to assess the adequacy of the proposed local and global spatial entropy indices. The bench was simulated using unconditional sequential Gaussian simulation (Deutsch & Journel, 1998), and the simulated Gaussian values on the bench were transformed into gold grades using actual gold grade values from a real mine. The unconditional simulation aimed to generate one realization with the lowest level of spatial heterogeneity, where ore was situated on the eastern side of the bench while waste was on the western side. The block size of the simulated grade control block model was set at half the size of the SMU. This configuration served as the base case for assessing spatial heterogeneity. The parameters regarding the grade control block model definition are given in Table 4.1. A cut-off grade of 0.45 g/t was applied to differentiate between ore and waste. Economic and metallurgical parameters for calculating the cut-off grade are summarized in Table 4.2.

The same information used in Table 4.2 was employed to construct an economic block model specifically for dig-limits optimization purposes.

Table 4.1: Grade Control block model definition.

Item	Unit	X	Y	Z
Block size	(m)	2.5	2.5	15
Number of blocks in a direction		40	30	1
Minimum centroids	(m)	1.25	1.25	7.5
Maximum centroids	(m)	98.75	73.75	7.5
Azimuth	(degree)	0		
Dip	(degree)	0		

Table 4.2: Parameter values used for cut-off grade calculation and block economics values.

Item	Unit	Value
Gold price	(\$/gram)	53.1
Mining cost	(\$/tonne)	3.0
Processing cost	(\$/tonne)	16.7
Ore recovery	(%)	81.9
Rock bulk density	(tonne/ m ³)	2.7

After generating the base case grade control block model, blocks were shuffled to generate various degrees of spatial heterogeneity while ensuring that the total amount of gold within the bench remained constant, and the only difference was the spatial distribution of grade values. Figure 4.3

describes generating the base case (i.e., the lowest), low, medium, and high spatial heterogeneity grade control block models resulting from the shuffling process.

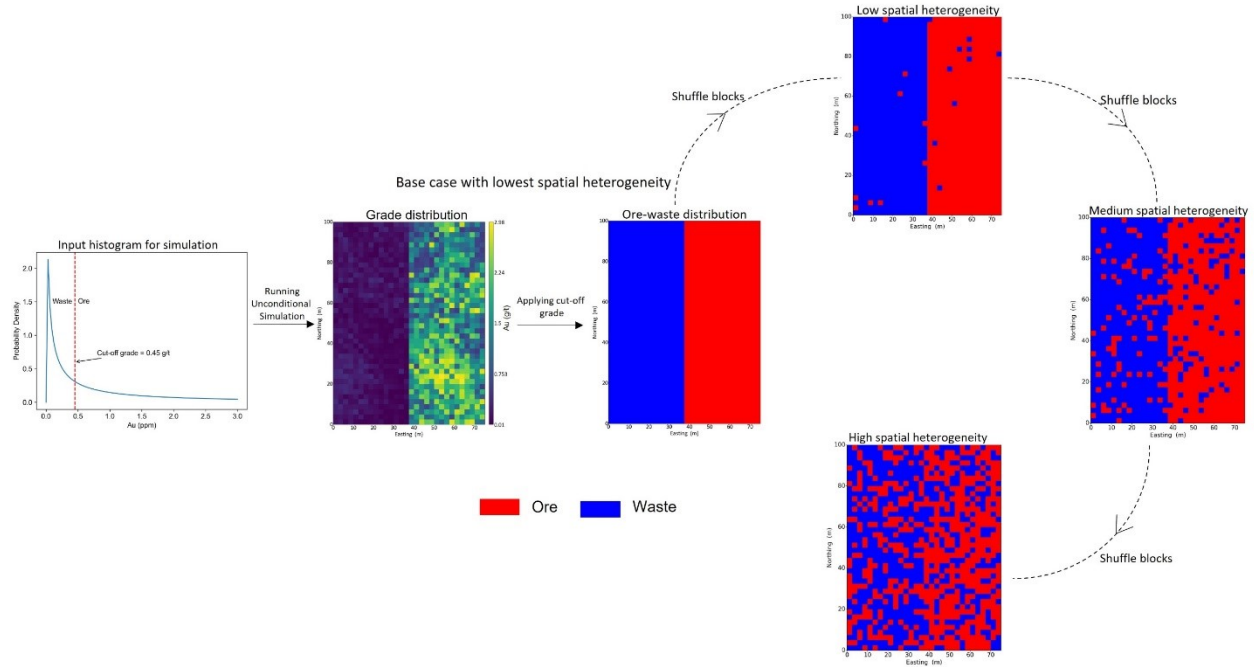


Figure 4.3: Summary of the proposed methodology to generate grade control block models with various spatial heterogeneity levels.

The proposed GSEI, GSII, and LSEI were calculated for the low, medium, and high spatial heterogeneity scenarios. The result of this comparison is summarized in Figure 4.4. As shown in Figure 4.4a-c, the three spatial heterogeneity scenarios have the same Shannon's entropy with a value of 0.5. However, the GSEI calculated for each scenario properly reflects the degree of spatial heterogeneity. The GSII, on the other hand, provides a good measure of spatial continuity or clustering. These two indices provide reasonable global measures of spatial heterogeneity.

To obtain an accurate assessment of the level of spatial heterogeneity at a specific location, the LSEI can be used. The calculated LSEI results for the three spatial heterogeneity scenarios are

shown in Figure 4.4d-f. These calculations were performed using a frame size of 3-blocks by 3-blocks, representing an SMU size of 5m.

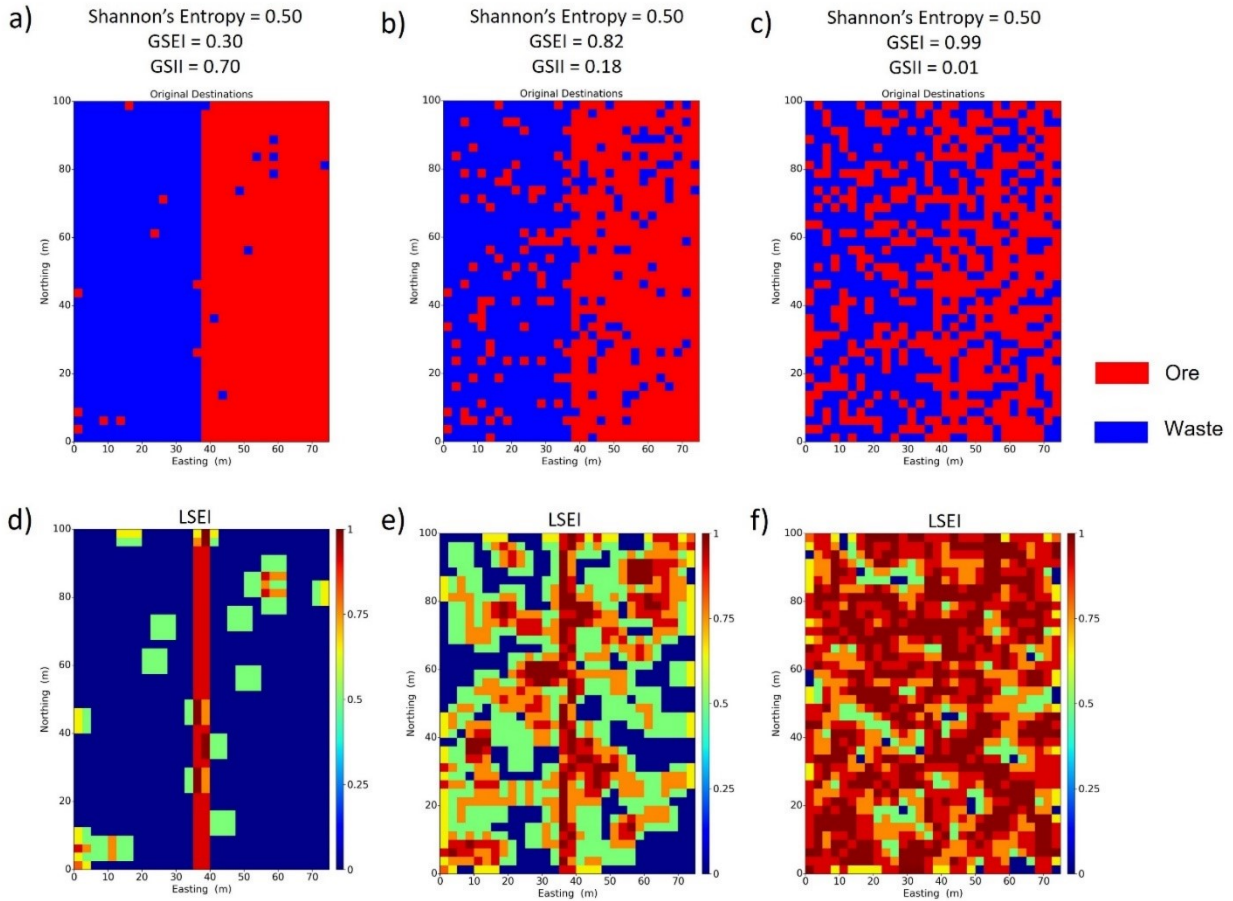


Figure 4.4: Comparison between Shannon's entropy, GSEI, GSII and LSEI for three bench sections with low, medium, and high spatial heterogeneity levels.

The optimal allocation of BMM balls is an essential aspect of reducing ore loss and dilution. A cost-benefit analysis conducted by Loeb and Thornton (2014) demonstrates that increasing the number of BMM balls, up to 100 and 250 balls in their case study, would effectively reduce ore-waste loss and dilution while remaining cost-effective. However, in practical implementation, installing many BMM balls becomes impractical due to the time required for installing and recovering BMM balls in each blast, which can result in operational delays. Therefore,

incorporating information obtained from the LSEI maps, in conjunction with a cost-benefit analysis of installing BMM balls at high LSEI locations, can guide the optimization of the installation process. Focusing on locations with high LSEI values improves the allocation decision of BMM balls, leading to a higher return on investment.

4.4.2 Impact of spatial entropy on ore loss and dilution

As can be seen from the previous discussions, the proposed indices for measuring the level of global and local spatial heterogeneity informed dig-limit optimization and blast movement monitoring. In this section, the relationship between GSEI and ore loss and dilution was further investigated. This study used previously simulated bench sections with varying GSEI values, representing different degrees of spatial heterogeneity. Profit reduction was used as a proxy to represent the overall percentage of ore loss and dilution post-blasting. The profit reduction was calculated for each scenario after applying dig-limits optimization, taking equipment selectivity into account to ensure that the profit values closely align with what can be achieved during mining operations. The findings of this experiment are summarized in Figure 4.5.

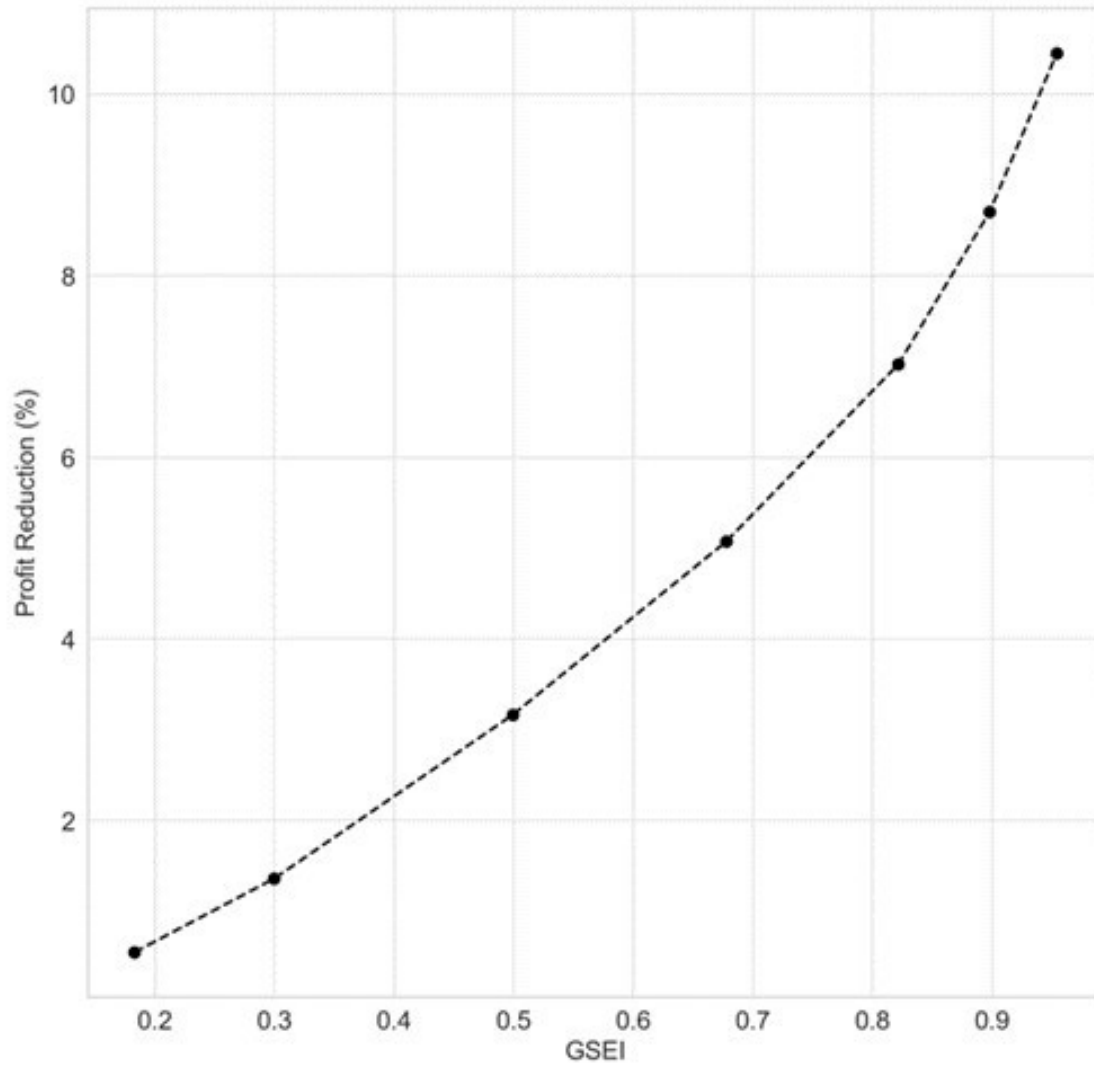


Figure 4.5: Relationship between GSEI and profit reduction calculated after applying dig-limits.

The result of this study indicates that as GSEI increases, ore loss and dilution also increase, leading to reduced profits. The relationship between GSEI and profit reduction exhibits exponential behaviour. It was noticed in this study that increasing the GSEI led to an increase in dig-limits optimization running time. Therefore, the next section will investigate this relationship in more detail.

4.4.3 Impact of spatial entropy of dig-limits optimization performance

Running dig-limits optimization models can be slow in some instances, particularly when employing an exact method to generate optimal dig-limits. In this exercise, the dig-limits optimization model is applied to several simulated benches, each exhibiting varying degrees of spatial heterogeneity. The runtime of the model, along with the GSEI, is measured for each scenario. The dig-limits optimization model was executed on a laptop equipped with an Intel(R) Core(TM) i7-7700HQ CPU running at 2.81GHz, a 64-bit operating system, and 32 GB of RAM. The results of this experiment are illustrated in Figure 4.6.

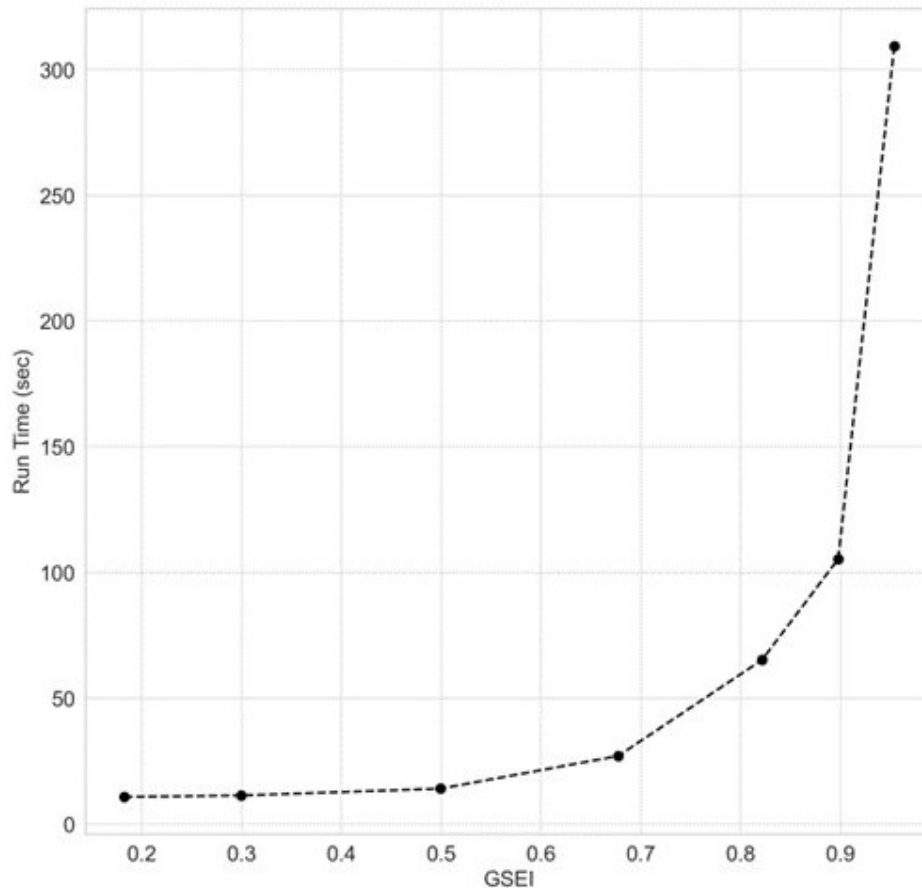


Figure 4.6: Relationship between GSEI and dig-limits optimization running time.

While the runtime can be considered reasonable for regular bench section sizes, it should be noted that geological, chemical, and physical controls over orebodies result in some degree of spatial structure. Consequently, most deposits have lower spatial entropy values than those expected for randomly distributed noise, and this helps to manage the runtime of the optimization process.

However, the assessment of ore loss and dilution may change over the life of mine due to changing cut-off grades because of changing commodity prices and mining and processing costs. The next section will investigate these relationships.

4.4.4 Impact of cut-off grade on spatial entropy and dig-limits optimization performance

To assess the impact of cut-off grade on spatial entropy and dig-limits optimization runtime, the base case grade control block model with the lowest GSEI was subjected to several scenarios of cut-off grade values, ranging from 0.25 g/t to 1.5 g/t. The model had ore and waste separated in the middle with a 0.45 g/t cut-off grade. GSEI was calculated for each scenario, and the dig-limits optimization model was executed for all scenarios to measure its runtime. Figure 4.7 presents the results of this experiment.

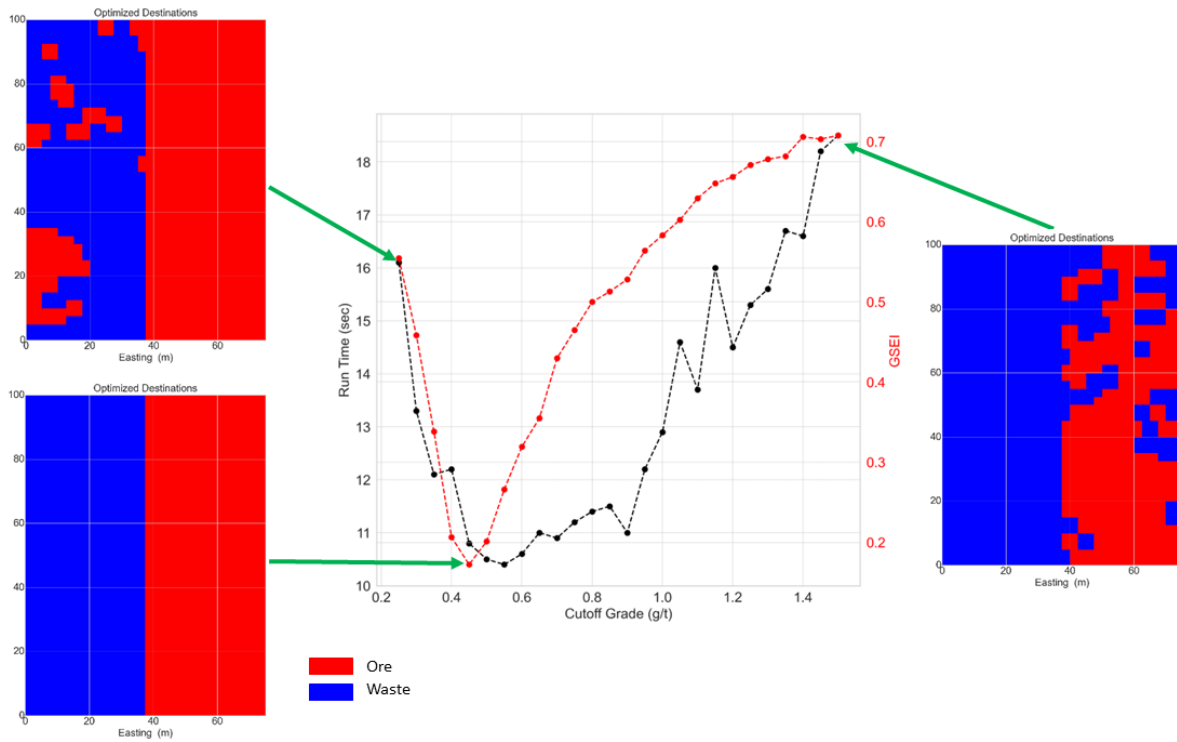


Figure 4.7: Results of running dig-limits optimization model on various scenarios of cut-off grade values on GSEI and running time.

The results of this experiment reveal that the scenario with the lowest GSEI value, where ore and waste blocks are entirely separated, is associated with the shortest running time. However, when the cut-off grade value is either below or above 0.45 g/t, both running time and GSEI increase significantly.

The assessment of spatial heterogeneity should always be updated following the change in any economic and metallurgical parameters that impact the cut-off grade. The aim is to have a block model with bench sections that are assigned GSEI values to assess the expected level of dilution and ore loss and mitigate any potential increase in them by optimizing the blast design to reduce them while taking into consideration the cost of applying these designs in actual operations and prioritize those sections that should be given due care.

4.4.5 Impact of blast movement on ore loss and dilution

It is obvious that the more blast movement we have in our blasts, the higher the ore loss and dilution occur. However, it is also worth exploring the magnitude of expected ore loss and dilution pre-blasting to help design blasts. This section will discuss the relationship between blast movement and ore loss and dilution through several possible scenarios of blast movement that may occur on a bench section.

This study simulated blast movement scenarios with varying degrees of movement. These simulated blast movement scenarios were applied to the base case bench section. This base case bench section, which is 15m thick, is divided into five flitches, each with a thickness of 3m and has one-fifth of the contained metal. Figure 4.8 shows a schematic representation of the bench section with flitches. Figure 4.9 shows the simulated blast movement scenarios that range from low blast movement to high blast movement. The blast movement magnitudes were chosen to cover all possible movements, including an extreme movement with 35m to ensure that the results cover a wide range of potential movements in actual mining operations.

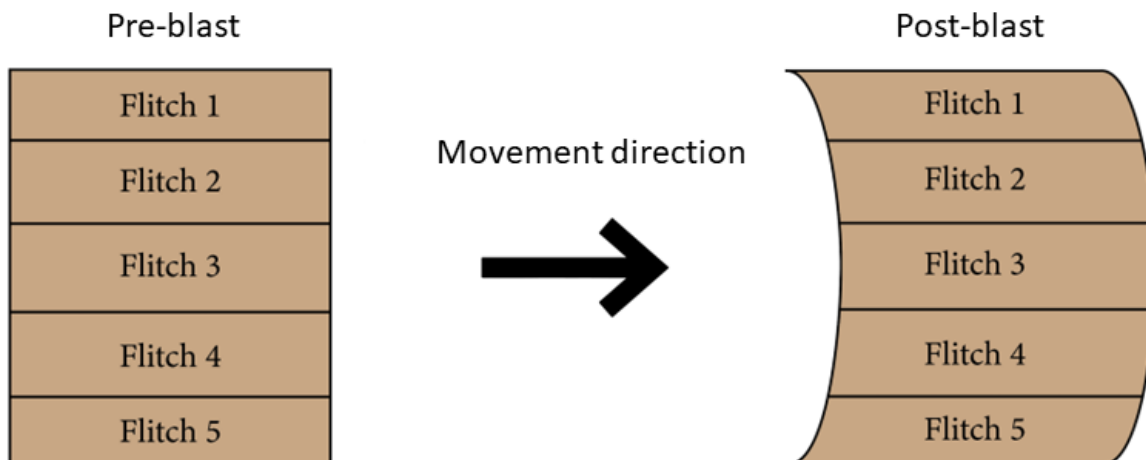


Figure 4.8: Schematic showing the shape of the bench section with the flitches.

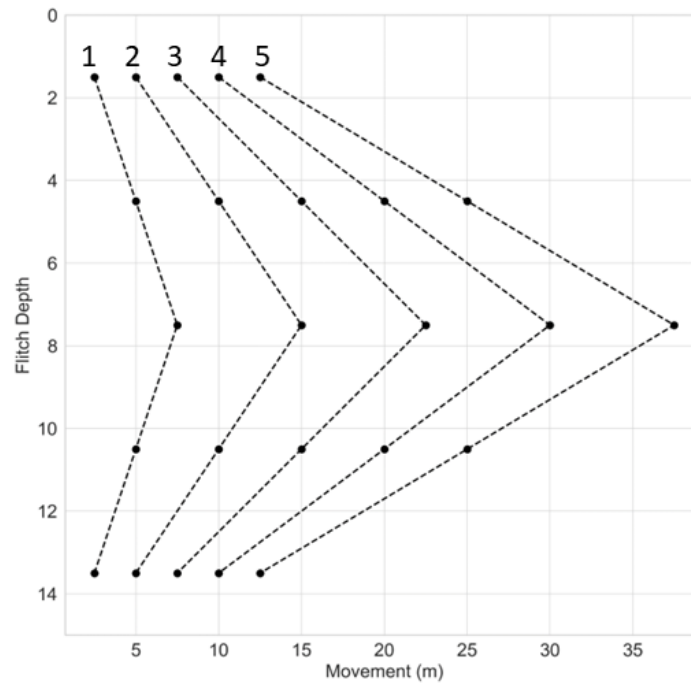


Figure 4.9: Magnitude of simulated blast movement for each flitch where the points in black represent the centroid of each flitch in each scenario.

The simulated blast movement scenarios play a critical role in determining the impact of blast movement control on the project economics and how this relates to the original orebody heterogeneity and cut-off grade. Through these blast movement simulations, this study can assess how controlling blast movement can improve project profitability and how this varies across different levels of spatial heterogeneity and cut-off grades.

This study aims to analyze the effect of blast movement on ore loss and dilution, beginning with a fixed spatial distribution of ore and waste blocks in the block model with a known spatial entropy index. This analysis is proposed as a preliminary step towards examining the impact of varying degrees of spatial entropy and blast movement on ore loss, dilution, and profit in the following section.

To achieve this, five different scenarios of blast movements, ranging from low to high blast movement, were applied to the block model with a direction towards the east. These blast movement scenarios were discussed earlier in the data preparation section. Figure 4.10 shows the final location of ore and waste blocks after blasting for each scenario.

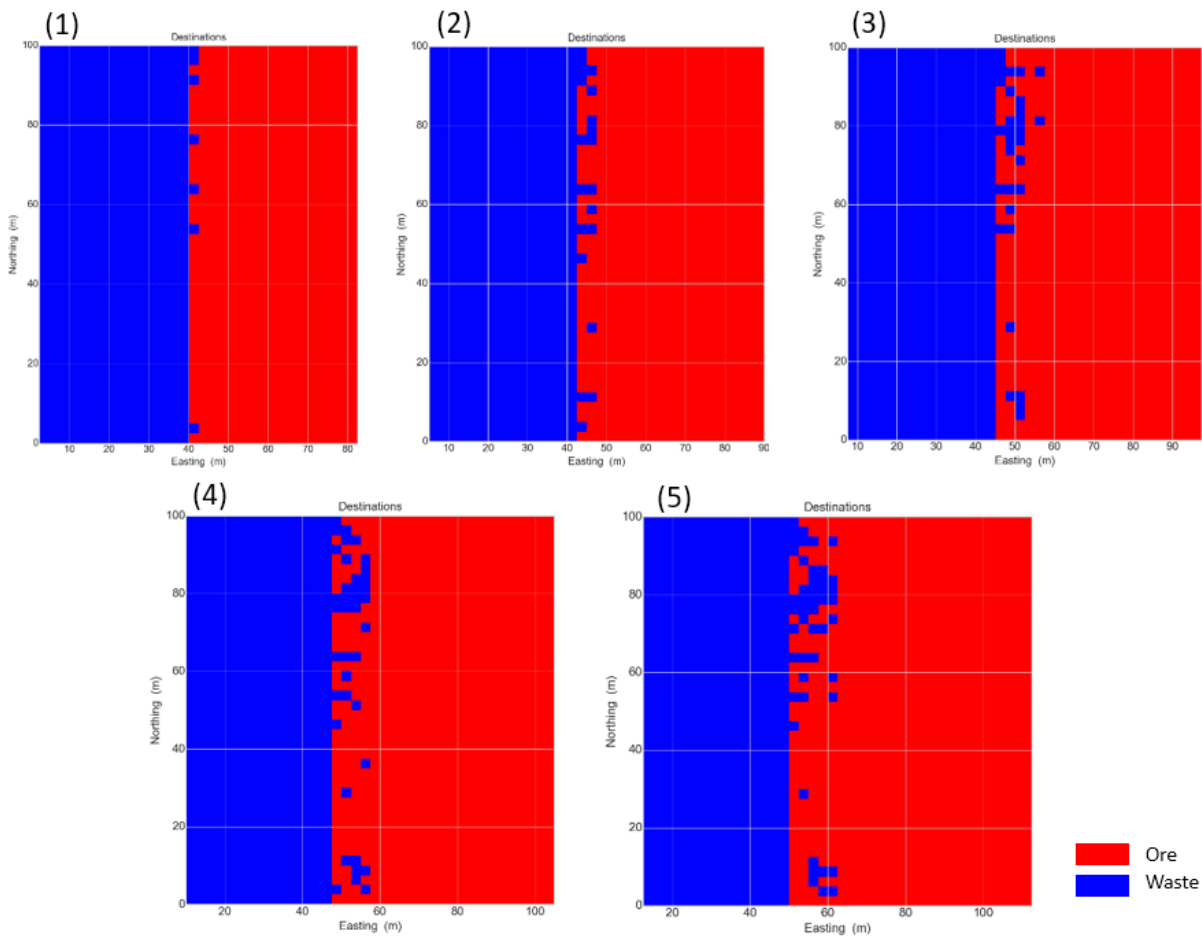


Figure 4.10: Scenarios of ore and waste distribution post-blast based on blast movement scenarios applied to the block model with a blast movement toward the east; scenario 1 has the lowest blast movement, while scenario 5 has the highest blast movement.

The dig-limits optimization model was implemented on these five blast scenarios (ranging from low to high movement), and the profit reduction percentages obtained after executing the dig-limits optimization for each scenario are summarized in Figure 4.11.

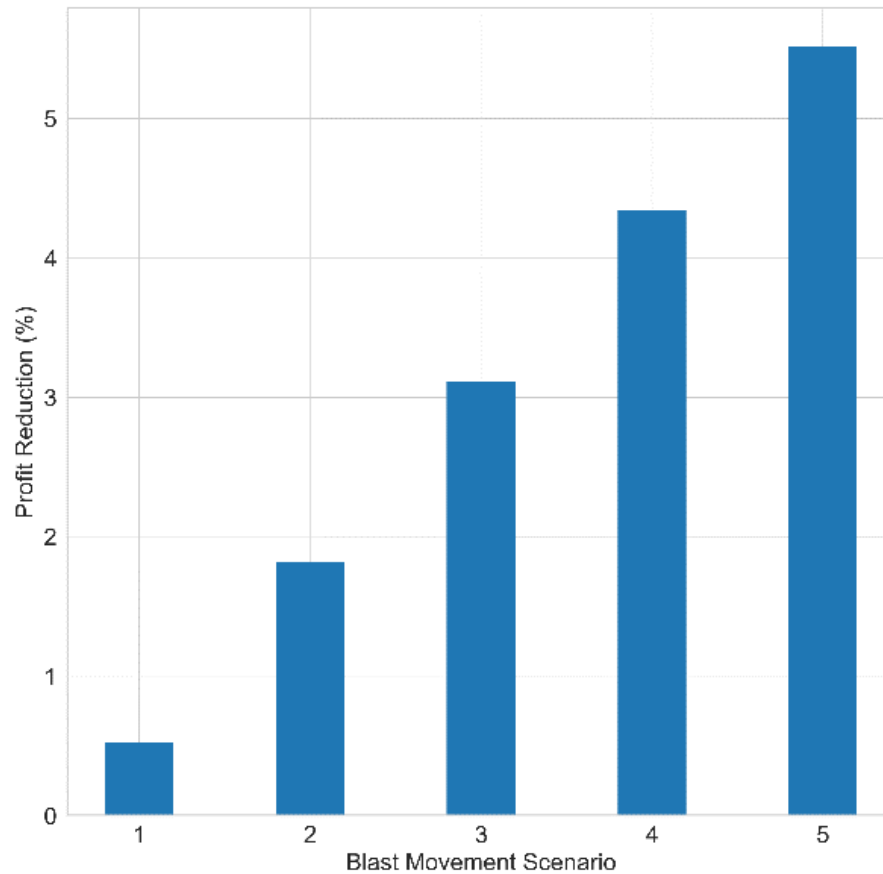


Figure 4.11: Profit reduction caused by the five blast movement scenarios after applying dig-limits optimization.

The study's results show that increasing the magnitude of blast movement leads to a reduction in profit due to the resulting increase in ore loss and dilution caused by the movement when spatial entropy pre-blast is kept constant. The profit reduction resulting from ore loss and dilution is attributed to post-blast spatial entropy.

Blast engineers who work with this information to design better blasts or at least to have an idea of the magnitude of expected dilution and ore loss before blasting and communicate this to the short-term mine planning engineers and mineral processing engineers. Also, by knowing the expected magnitude of post-blast movement in advance, the LSEI can highlight areas requiring installing BMM balls. The same study can be conducted on various types of mineral deposits that have various shapes and spatial continuities of ore and waste within the benches.

4.4.6 Controlling blast movement for profit maximization in mining operations

For this study, seven grade control block models were generated, each with varying levels of spatial heterogeneity (ranging from low to high GSEI), while maintaining their total contained metal. Each of these block models had five blast movement scenarios applied, as discussed in the previous section, ranging from low to high blast movement. Consequently, 35 scenarios of block models with different levels of spatial heterogeneity and blast movement were created. The dig-limits optimization model was applied to all scenarios to measure the amount of profit reduction resulting from post-blast spatial entropy index impact on the magnitude of ore loss and dilution. Figure 4.12 shows the relationship between pre-blast GSEI and post-blast profit reduction for all blast movement scenarios obtained after running dig-limits optimization model.

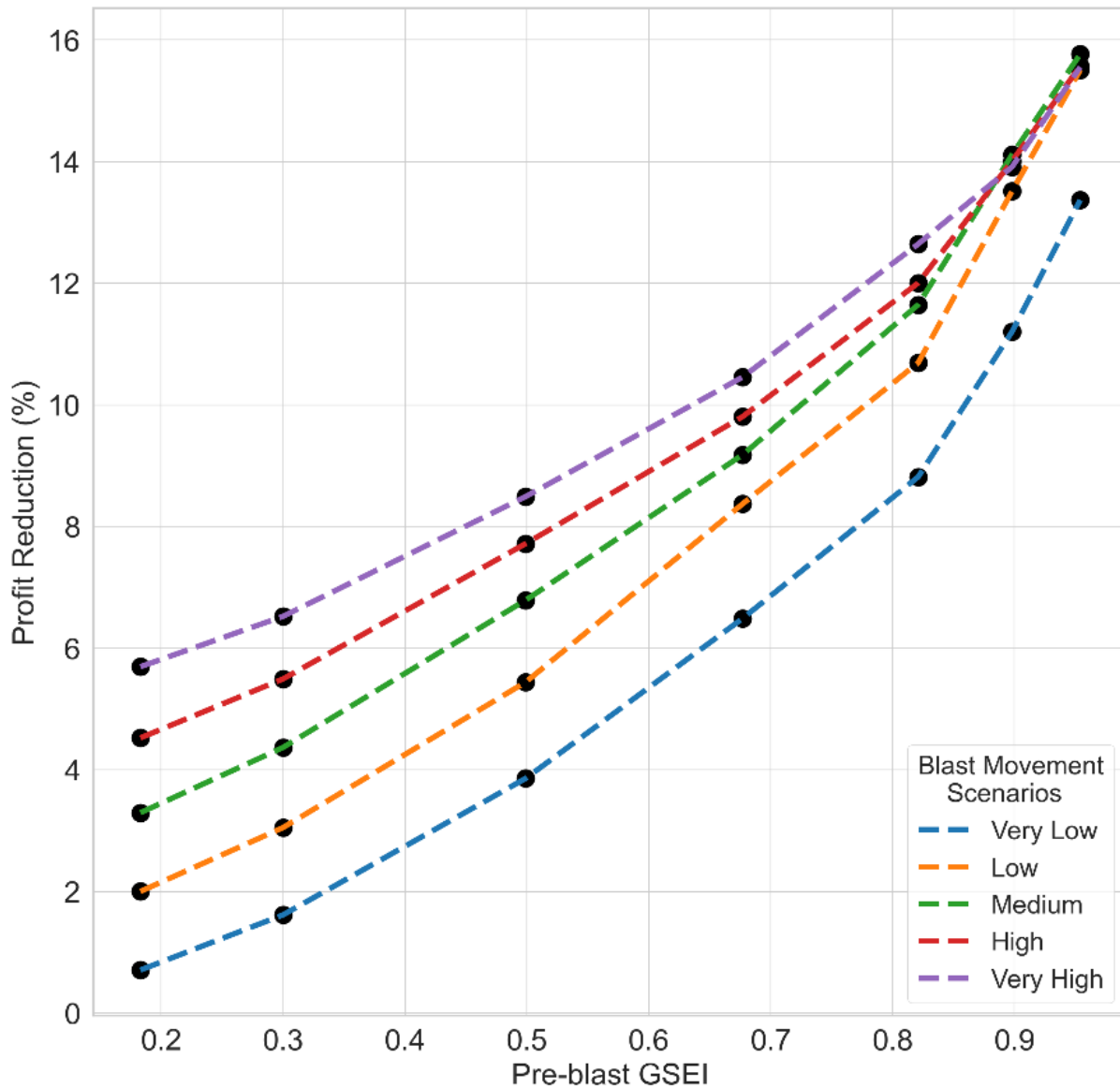


Figure 4.12: Relationships between profit reduction, pre-blast GSEI and blast movement scenarios

The study results indicate that profit reduction due to ore loss and dilution increases with increasing pre-blast GSEI. Additionally, while maintaining the same pre-blast GSEI, a higher magnitude of blast movement results in greater profit reduction due to increased mixing between ore and waste blocks, leading to increased ore loss and dilution. However, when pre-blast GSEI exceeds 0.9, the

effect of blast movement on profit reduction is negligible. By implementing controlled blasting, blast engineers can ensure that the profit reduction due to ore loss and dilution remains below a designated threshold set by their mining companies. For example, if the accepted profit reduction due to ore loss and dilution is 10%, then a pre-blast GSEI value of 0.5 always falls below this threshold, irrespective of the magnitude of blast movement. Furthermore, choosing a blast design that minimizes blast movement can reduce profit reduction from 8.4% to 4%. However, if the pre-blast GSEI value is 0.8, the only way to achieve a profit reduction value below 10% is by controlling the blast movement and selecting a blast design with low blast movement.

4.5 DISCUSSION

Spatial entropy plays a crucial role in identifying bench sections that need customized blasting designs to minimize blast movement and guide the allocation of BMM balls within the bench, resulting in improved profitability through reduced ore loss and dilution.

To optimize blasting in mining operations, it is recommended to apply an indicator variable that shows which bench sections require careful blasting designs. This indicator variable relies on GSEI and blast movement information obtained from various blast designs. The threshold value of GSEI in which controlled blasting should be implemented in that bench section to meet ore loss and dilution requirements will vary from mine to another. Such sections should be identified in advance, and the indicator variable should be updated during the life-of-mine plan in response to changes in commodity price, mining and processing costs, and ore recoveries. Controlled blasting techniques, such as cushion or pre-splitting techniques, can be considered. However, a cost-benefit analysis should be conducted before implementing any controlled blasting design to justify the extra cost of implementing such designs during mining operations. Some bench sections may require controlled blasting methods due to the significant impacts of changes in cut-off grade on

GSEI. This approach enables blasting engineers to revisit their blast designs and methods when warranted in order to help reduce ore loss and dilution and improve mining operations' efficiency. Conversely, this proposed method helps the blasting engineer to identify where the cost and effort required to execute a particular controlled blasting design do not justify the potential gain.

The study results further reinforce the significance of understanding the proposed GSEI in achieving profit maximization in mining operations through implementing controlled blasting techniques on the problematic bench sections where high levels of ore loss and dilution are expected. Applying the proposed approach can lead to more efficient mining operations, resulting in cost reductions and improved profitability.

4.6 CONCLUSION AND FUTURE WORK

This paper employs Shannon's entropy principle to quantify global and local spatial heterogeneity within ore and waste regions in open pit mining. Our tailored spatial entropy indices precisely measure SMU-scale heterogeneity, offering crucial insights into potential ore loss and dilution. Case studies demonstrate the indices' efficacy in assessing spatial heterogeneity across various bench sections, informing blast engineers about expected ore loss and dilution while guiding BMM ball allocation.

The study found that the computational time required to solve the dig-limits optimization problem also increases exponentially with rising spatial entropy, but the overall computational time remains acceptable when the grade control block size is equal to half the size of SMU. The scenario with the lowest spatial entropy value, where ore and waste blocks are entirely separated, is associated with the shortest running time. However, running time and spatial entropy increase significantly when the cut-off grade exceeds certain limits that vary from bench to bench.

The study also demonstrated that an increase in the magnitude of blast movement leads to more profit reduction due to the resulting increase in ore loss and dilution. However, controlling blast movement can improve project profitability, particularly at different levels of spatial heterogeneity and cut-off grades.

Future work could focus on understanding the economics of blast design and when to apply special blast designs that reduce blast movement. The formulation of spatial entropy in this paper can also be extended to more than two categories (i.e., more than ore and waste). A holistic optimization approach that aims to maximize mining operation profitability by taking in consideration various scenarios of blast movement, fragmentation size, cost of loading, hauling, and crushing could be investigated.

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4.8 NEXT STEPS

This chapter introduced the concept of spatial entropy as a method to assess ore loss and dilution in open-pit mines. By comparing post-blast spatial entropy to pre-blast spatial entropy, we gain an understanding of the impact of blast movement on ore loss and dilution at the selective mining unit scale. Blast movement remains a key factor affecting ore loss and dilution, and from the previous chapter, it is evident that modeling blast movement without considering uncertainty can result in suboptimal decisions regarding material types and dig-limits. The next chapter will focus on incorporating sources of uncertainty, such as grade uncertainty and blast movement uncertainty, into material classification post-blasting and dig-limits optimization. It will also aim to improve the current dig-limits optimization model to account for multiple destinations, irregular bench section shapes, and different orebody orientations. A case study demonstrating the new model and its improvements will be presented.

Chapter 5: Risk-based Optimization of Post-blast Dig-limits Incorporating Blast Movement and Grade Uncertainties with Multiple Destinations in Open-pit Mines

5.1 ABSTRACT

Dig-limits optimization is one of the most important steps in the grade control process at open-pit mines. It aims to send blasted materials to their optimal destinations to maximize the profitability of mining projects. Grade and blast movement are key uncertainties affecting the optimal determination of dig-limits. This paper presents an integrated workflow for optimizing dig-limits under grade and blast movement uncertainties. The proposed methodology incorporates these uncertainties into the grade control process to enhance material classification and destination optimization, thereby minimizing ore loss and dilution. A multivariate geostatistical simulation workflow is developed to capture spatial uncertainties in grade distribution and blast movement distance and direction. Through applying Projection Pursuit Multivariate Transformation and Sequential Gaussian Simulation for modeling blast movement distances at all locations and flitches within the bench section, the anticipated D-like shape from blasting is reproduced, and uncertainty is quantified. The Maximum Expected Profit method effectively determines optimal material destinations under uncertainty improving overall mining profitability. The proposed risk-based dig-limits optimization model honors mining equipment selectivity, irregular bench shapes, and varying orebody orientations, resulting in operational and economically viable dig-limits. A case study on a porphyry copper deposit demonstrated the significant impact of blast movement on ore loss and dilution, emphasizing the need for accurate blast movement modeling and its integration

into grade control procedures. By accounting for differential blast movement, the proposed workflow ensures reliable post-blast material classifications, reducing suboptimal decisions, thus, improving project profitability and operational efficiency.

5.2 INTRODUCTION

Grade control is one of the most important operations in open-pit mines. It aims to classify blasted materials into several material types and send them to their optimal destinations, thereby minimizing ore loss and dilution, and maximizing the profitability of these mines (Abzalov, 2016). Grade control operations include: (1) drilling blast holes or, in certain cases, reverse circulation (RC) holes; (2) sampling from drilled core or cuttings; (3) assaying the collected samples; (4) estimating grades on a grade control block model; (5) classifying rocks into material types (e.g., ore, waste, oxides, sulfides, etc.); (6) modeling blast movement; and (7) setting optimized post-blast dig-limits.

The accuracy of grade control operations is influenced by a number of factors, including sampling error, bias in grade estimation, selectivity requirement, and blast movement (Dimitrakopoulos and Godoy, 2014). Uncertainty in blast movement also leads to unplanned ore loss and dilution challenges that grade control geologists and short-term mine planning engineers find difficult to manage. Blast movement uncertainty is not typically integrated into grade control procedures and the decision-making process.

The need for an integrated grade control workflow that accounts for both grade and blast movement uncertainties has become increasingly important in the mining industry. As orebodies become more complex and operational constraints evolve, there is a growing need to enhance traditional grade control methods to further optimize dig-limits. Incorporating these uncertainties

into the grade control process ensures that both ore recovery and economic outcomes are optimized, reducing the risk of costly ore losses and unanticipated dilution. Such an integrated approach not only improves the accuracy of short-term mine planning but also enhances the overall operational efficiency by changing static grade control models into dynamic models that accounts to real mining conditions from blast movement and equipment selectivity. The development of a robust, uncertainty-informed workflow is critical for the modern mining industry, where precise and timely decisions have a direct impact on mine profitability and sustainability.

This paper focuses on considering grade and blast movement uncertainties to determine optimal dig-limits with more than two destinations under equipment selectivity constraint. By optimizing dig-limits while considering these uncertainties, ore loss and dilution can be reduced, which in turn will improve the overall profitability of the project.

The originality of this paper is four-fold: (1) develop an integrated workflow that considers grade and blast movement uncertainties in determining optimal post-blast dig-limits; (2) modeling differential blast movement and capturing the uncertainty in the movement on a block-by-block basis; (3) determining material types under grade and blast movement uncertainties; (4) improving the dig-limits optimization model to consider multi-destinations, irregular bench shapes, the option of shovels being more selective at zones close to the free face side of the bench, and considering various orebody orientations.

The organization of this paper is as follows: The next section provides a general background on the key topics relevant to this study. Following that, the proposed methodology is described in detail. A case study is then presented to illustrate the application of the newly developed integrated approach for determining dig-limits under both grade and blast movement uncertainties. Finally,

the paper presents and discusses the results, draws conclusions, and suggests directions for future research.

5.3 BACKGROUND

5.3.1 Grade control models

The accuracy of the grade control model has a significant impact on ore–waste classification on a bench. To improve the accuracy of grade control procedures and models, several critical aspects must be taken into account: (1) Grade uncertainty should be quantified at the bench scale, while ensuring that grade realizations are unbiased (Verly, 2005); (2) Material-type classification must consider the movement of blasted materials (Hmoud & Kumral, 2022, 2023, Vasylchuk and Deutsch, 2019); (3) Blasted materials should be accurately classified into various types, taking into account grade uncertainty and the non-linear and asymmetric profit functions (Srivastava, 1987; Isaaks, 2001; Verly, 2005); and (4) The optimal destinations for blasted materials should be determined at the selective mining unit (SMU) scale, which is controlled by equipment selectivity (Hmoud & Kumral, 2022, 2023). The outcomes of effective grade control practices enable short-term planning engineers to develop more accurate short-term production schedules, thereby maximizing mill utilization and project profit.

One key step in grade control procedures is to generate grade control models. Linear estimation methods such as Ordinary Kriging and Inverse Distance are widely used to build grade control models because of their simplicity (Dimitrakopoulos & Godoy, 2014; Godoy, Dimitrakopoulos, & Costa, 2001). The issue with using linear estimation techniques to determine material types and destinations based on cut-off grades is that these techniques assume equal penalties for underestimation and overestimation of profit. This assumption becomes particularly problematic when dealing with complex, non-linear, and asymmetric profit functions (i.e., losses from sending

ore to waste dump do not equal losses from sending waste to processing plant), and this behavior of profit functions leads to biased and suboptimal decisions (Srivastava, 1987; Isaaks, Treloar, & Elenbaas, 2014; Vasylchuk & Deutsch, 2016).

Geostatistical simulation techniques can overcome the issue of bias in predicting profit. Geostatistical simulation techniques (Alabert, 1987; Isaaks, 1990) produces a range of equiprobable grade values at all locations within a mining bench section that is planned for blasting. Sequential Gaussian Simulation (SGS) is a widely used spatial simulation technique that generates equiprobable realizations of a spatially distributed variable, such as ore grade in a mineral deposit (Journel & Deutsch, 1992). It works by sequentially simulating values at unsampled locations, conditioning each simulation on both the original sample data and the previously simulated values. This process ensures that the simulation preserves both the declustered histogram of the input data and the spatial continuity model, which is captured by a variogram model that measures spatial dissimilarity of the variable over distance. In case of simulating polymetallic mineral deposits, projection pursuit multivariate transformation (PPMT) is one of the methods used to simulate multiple elements while preserving their non-linear multivariate relationships (Barnett et al., 2014).

Grade uncertainty is the result of incomplete information of the geology at a scale equal or smaller than the SMU size. The simulated grade values can be used to assess better the profit of any grade control decision (e.g., sending low-grade material to the waste dump, stockpile, or mill). Moreover, geostatistical simulation facilitates the determination of profit when using asymmetric or non-linear profit functions, allowing for more accurate and precise decisions in grade control. Consequently, this ensures that the blasted materials are sent to their optimal destinations. The profit function can be calculated as well for multiple elements, and this requires using multivariate

geostatistical simulation methods. Vasylchuk (2018) proposed a multivariate simulation workflow with locally varying anisotropy to accurately predict grade uncertainty in the presence of multiple grade variables that go into the profit function calculation with an internal validation check to ensure reasonable automatic results.

5.3.2 Material-type classification

After quantifying grade uncertainty, material type classification is normally carried out to enable determining the right destination for blasted material based on the production plan. A number of methods were proposed in the literature to determine the optimal material-type under grade uncertainty: (1) average simulation (Verly, 2005); (2) minimum expected loss (MEL) (Isaaks, 1990; Vasylchuk and Deutsch, 2018); (3) maximum expected profit (MEP) (Glacken, 1996, Deutsch et al, 2000, Neufeld et al, 2005), (4) MEL and MEP with risk coefficients (Glacken, 1996, Dimitrakopoulos and Godoy, 2014).

The average simulation method assigns the destination of a block in a mine bench based on the average of the simulated values within that block based on a cut-off grade. The average of the simulated grade values is equivalent to the ordinary kriging estimate; therefore, the final destinations determined using the average simulation and ordinary kriging are expected to be very similar as well when domains are stationary. The model of average simulation values is known as the E-Type model. However, it is noted that average simulation method has the capability to reproduce the histograms of skewed distribution better, therefore it may generate more representative results that capture the high grades and translate that into profit more accurately (Verly, 2005). The MEL method determines the optimal destination for a block by calculating the expected costs associated with assigning the block to all possible destinations. It then selects the destination that minimizes the expected loss across all possible destinations. MEP method

determines the optimal destination of a block by calculating the expected profits associated with assigning the block to all possible destinations, then selecting the destination that maximizes the expected profit across all possible destinations. Risk coefficients can also be added to profit and loss functions to ensure that some specific mine requirements are met, such as increasing the chance of sending waste to the processing plant or reducing the chance of sending waste to the waste dump. The choice of coefficient values depends on the operations; some operations are willing to take some risk and send waste that has some chance of being ore to the processing plant and others may prefer to send ore with some chance of being waste to the waste dump. Applying these coefficients requires continuous updating because these policies may change during the life cycle of the mine and this might make applying these risk coefficient hard in mining operations. In the presence of a linear profit function with no clear coefficients to apply to profit and loss functions, average simulation, the MEL and the MEP methods provide similar results. However, when the profit function is nonlinear such as when the recovery model is not a linear function of grade, the MEP method becomes the best approach for determining optimal destinations under uncertainty (Verly, 2005).

5.3.3 Blast movement

Another important source of uncertainty that is normally not accounted for is coupled with blast movements. Hmoud and Kumral (2023) showed that blast movements affect the results of ore loss and dilution, thus impacting dig-limits outcomes. Dig-limit optimization formulation that ignores blast movements results in unexpected loss and dilution (Engmann et al., 2013; Hmoud & Kumral, 2022; Thornton et al., 2005). Integrating blast movement in the dig-limits optimization process will also reduce material misclassification (Zhang, 1994; Yennamani, 2010). Additionally, using blast movement monitoring (BMM) balls can enhance ore recovery and control head grade in

mineral processing plants (Harris, 1997; Gilbride, 1995; Taylor & Firth, 2003). Hmoud and Kumral (2023) grouped blast movement modeling approaches into four categories: (1) the multi-physical simulation (Yu et al., 2022; Zou & Jun, 2020); (2) heuristics methods (Isaaks et al., 2014a; Isaaks et al., 2014b; Vasylchuk & Deutsch, 2019b); (3) machine learning models (Yu et al., 2019, Yu et al., 2020; Yu et al., 2024); and (4) stochastic methods (Hmoud & Kumral, 2022, 2023).

The primary challenge in using multi-physics simulations to model blast movement in open pit mines is the difficulty in accurately gathering all necessary input parameters due to the complexity of the simulation. This process often requires extensive computational time, which is impractical given the frequency of blasting operations. Alternatively, heuristic methods combined with physical field measurements like post-blast topography, provide quicker solutions, though they lack guaranteed optimality and they do not capture uncertainty in the predicted movements. Additionally, not all mining operations conduct post-blast topography surveys, which is a requirement for setting dig-limits accurately under these methods. Using machine learning to predict blast movements based on historical data can be effective when current geological and blast design conditions mirror the historical data used in training. However, these methods might not yield reliable predictions for deposits with varying geological characteristics and blast designs, particularly if these models do not capture and quantify uncertainty in blast movement predictions.

Uncertainty in blast movement arises from various factors such as (1) imperfections in blast designs, (2) the heterogeneity of the rock mass, (3) incomplete geological knowledge, and (4) human error. Theoretical blast movement modeling approaches alone are unlikely to yield accurate models of blast movements without physical field measurements taken before blasting any bench section (La Rosa & Thornton, 2011). Stochastic modeling techniques that utilize data from physical measurements of the specific blasted bench offer a viable solution (Hmoud & Kumral,

2022, 2023). However, comprehensive coverage of these measurements across bench sections, including all flitches, is required. This comprehensive data collection enables the creation of representative multivariate statistical distributions that capture the uncertainty in blast movements, particularly near ore zones, and helps identify areas with high-risk of material misclassification that require careful grade control sampling protocols. The current stochastic blast movement technique proposed by Hmoud and Kumral (2022) applied movements on a flitch-by-flitch basis without considering differential blast movement within the same flitch. Differential blast movement refers to the modeling of blasted materials on a block-by-block basis, where each block has its own distinct movement distance and direction. To the authors' knowledge, no research has been conducted on modeling and capturing the uncertainty in differential blast movement.

5.3.4 Dig-limits optimization

While incorporating blast movement models into grade control is considered important, optimizing dig-limits is another important component of good grade control models. The methodology for defining dig-limits has evolved significantly in recent decades, transitioning from manual drawing to sophisticated automated processes utilizing mathematical algorithms. Manual drawing, being subjective and difficult to revise, fails to adequately minimize ore loss and dilution, and does not consider the selectivity of mining equipment (Faraj, 2024). The objective of optimizing dig-limits is to measure, manage, and reduce the anticipated ore loss and dilution within open-pit mining operations. These limits define the separation between ore and waste, directing blasted materials to their most profitable destinations, thereby enhancing revenue while considering equipment selectivity and blast movement (Hmoud & Kumral, 2022). Hmoud and Kumral (2023) grouped dig-limits optimization into five main categories: (1) exact methods, such as mixed-integer programming (MIP) (Hmoud & Kumral, 2022; Kumral, 2015; Nelis et al., 2022; Nelis & Morales,

2022; Sari & Kumral, 2017); (2) heuristic algorithms (Richmond & Beasley, 2004; Vasylchuk & Deutsch, 2019a; Wilde & Deutsch, 2015); (3) metaheuristic techniques, including simulated annealing (Isaaks et al., 2014a; Isaaks et al., 2014b; Norrena & Deutsch, 2000; van Duijvenbode & Shishvan, 2022) and genetic algorithms (Ruiseco, 2016; Ruiseco & Kumral, 2017; Ruiseco et al., 2016; Williams et al., 2021); (4) hybrid models that combine MIP with metaheuristics (Deutsch, 2017); and (5) methods based on spatial clustering (Salman et al., 2021; Tabesh & Askari-Nasab, 2011, 2013; Tabesh & Askari-Nasab, 2019).

Exact methods such as MIP ensure optimality, but they require long processing times, especially when the moving window that enforces equipment selectivity is large. However, with advancements in parallel computing and high-performance computers, using MIP to generate optimal dig-limits has become achievable. Heuristics, metaheuristics, and hybrid methods provide near-optimal solutions when the problem size increases, offering faster solutions. One challenge with these methods is the optimal selection of hyperparameters, which requires fine-tuning. The same issues appear when using spatial clustering methods to determine dig-limits. However, unlike heuristics, metaheuristics, and hybrid methods, near-optimal solutions are not guaranteed with spatial clustering methods.

5.3.5 Fully integrated grade control workflow

Integrating all grade control steps into one workflow, from estimating and simulating grade on a bench, to classifying blasted materials into several material types, and modeling blast movement, to finally optimizing dig-limits, is important for any grade control model to be considered practical. A good example of this integrated approach can be found in Vasylchuk (2018), where the author highlighted the importance of improving grade predictions, considering uncertainty in material classification, accounting for blast movement deterministically using a heuristic algorithm,

optimizing material destinations at a scale smaller than the SMU size by applying a heuristic dig-limits optimization model to determine the final destinations for materials considering equipment selectivity. These improvements aimed to minimize ore loss and dilution, ultimately enhancing the overall efficiency of mining operations. However, blast movement uncertainty was not considered in the proposed workflow. Moreover, a heuristic method for modeling dig-limits is used that does not ensure optimality.

Despite previous research on grade control procedures, the uncertainty in blast movement and its implications for determining optimal material types and dig-limits have not been studied. Additionally, existing models for determining optimal dig-limits using MIP fail to address irregular bench shapes, various mining directions, and increased selectivity for shovels in zones near the bench's free face. This paper aims to address these challenges by proposing a new solution. Building on the work of Hmoud and Kumral (2022, 2023), this study takes a step forward by incorporating blast movement and grade uncertainties in determining optimal dig-limits, while also addressing the gaps and issues identified in previous research.

5.4 METHODOLOGY

The proposed integrated workflow in this paper for modeling dig-limits under blast movement and grade uncertainties is divided into six main steps: (1) data gathering and processing; (2) geostatistical modeling of grade and blast movement uncertainties; (3) generation of post-blast grade realizations; (4) converting 3D post-blast block models to 2D block models; (5) post-blast material-type classification under uncertainty; (6) dig-limits optimization. Figure 5.1 presents a

flowchart that summarizes the proposed workflow. Details on each of these steps are discussed in the next subsections.

5.4.1 Data gathering and processing

In the proposed workflow, two sources of information are required to model blast movement and grade uncertainties to optimize dig-limits: (1) blast hole or RC drilling data, and (2) BMM balls measurement data. Blast holes provide valuable information related to grade at very short sample spacings which are used to generate grade control models using geostatistical techniques. To enhance the accuracy of the drilling data used to generate grade control models, RC drilling is sometimes performed, wherein samples from multiple depths within the bench are collected and sent to a laboratory for assay analysis. In addition to grade, other details such as rock type can be recorded for each sample, and that helps in the development of high -accuracy geostatistical grade control models.

The BMM balls measurement data contain the pre-blast and post-blast locations of BMM balls. This information enables grade control geologists to track materials after blasting and provides insights into the movement distance and direction of blasted materials at various flitches and locations within the bench section. This data will be used to model differential blast movement on a block-by-block basis, where each block in the pre-blast grade control model has distributions of possible blast movement distances and directions. These blast movement distances and directions distributions can then be used to generate post-blast realizations of grade distribution within the bench section.

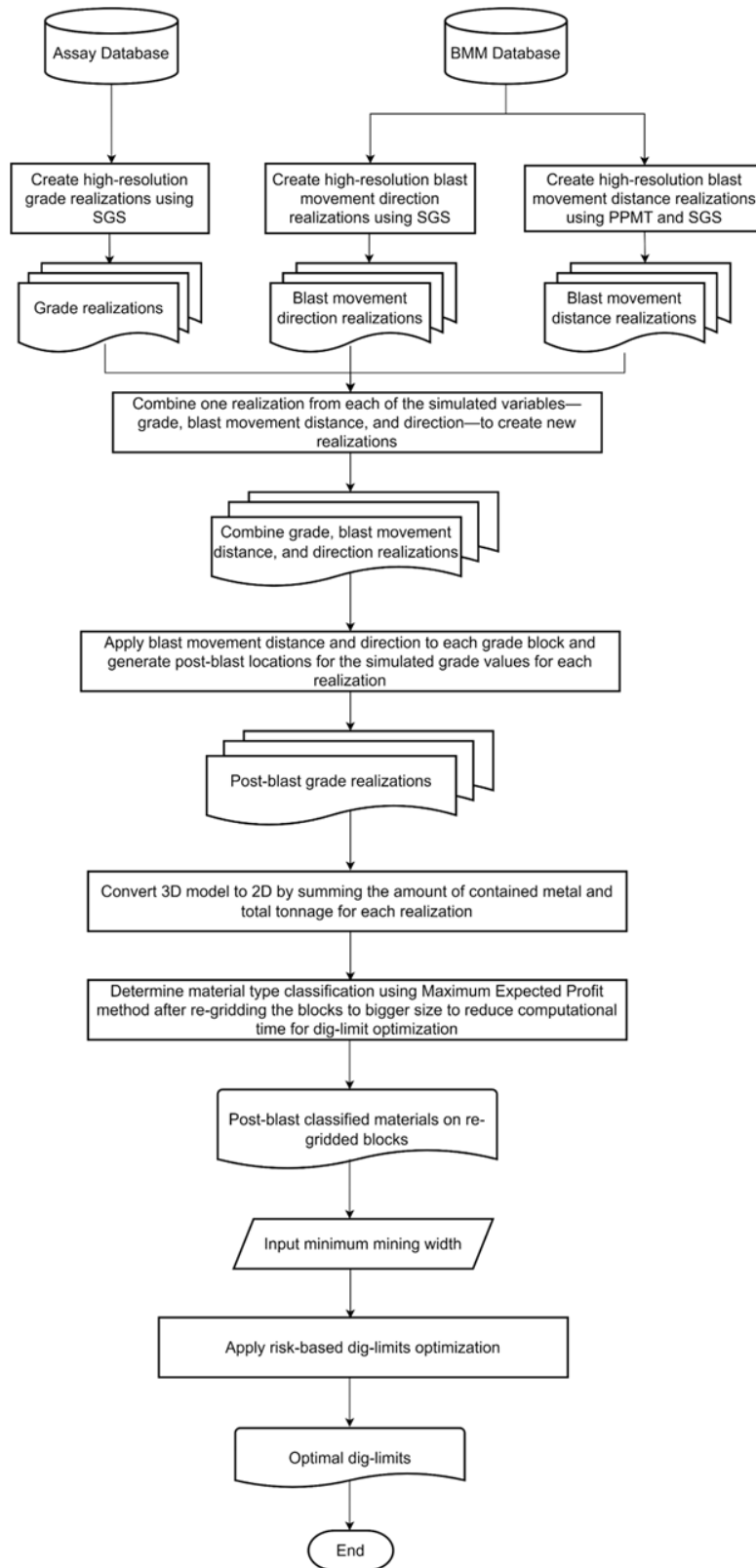


Figure 5.1: Flowchart with the proposed methodology.

5.4.2 Geostatistical modeling of grade and blast movement uncertainties

Several methods have been proposed to model spatial uncertainty, such as SGS (Journel & Deutsch, 1992; Goovaerts, 1997; Chiles & Delfiner, 1999), Turning Bands (Journel, 1974), and Partial Differential Equations Simulation (Lindgren et al., 2011). Among these, SGS is the most widely used method in the mining and petroleum industries.

SGS models spatial uncertainty by generating multiple realizations of a continuous variable. This method involves transforming data to a normal distribution, then sequentially simulating values at unsampled locations based on the spatial structure summarized by the variogram. Each location is simulated by drawing from a Gaussian distribution conditioned on both original data and previously simulated values to maintain spatial dependencies. Finally, the simulated values are back-transformed to their original units and validated to ensure the quality of the simulation. This process results in a suite of equally probable spatial distributions, providing a detailed picture of grade variability within the bench and allowing for the assessment of risks and uncertainties in grade control decisions.

In the presence of multi-element deposits, multivariate simulation techniques can build geostatistical realizations that reflect the uncertainty in grade values for those elements while respecting their correlations. Methods such as co-simulation using linear model of coregionalization (LMC) (Journel & Huijbregts, 1978) help model these relationships. However, co-simulation using LMC operates under the assumption that all relationships are multivariate Gaussian, which impacts outcomes when nonlinear relationships exist in the data. Additionally, fitting a large number of direct and cross variograms is challenging, as they must be modeled to ensure positive definite results when solving the normal equation matrices to estimate the

conditional mean and variance for the simulation and to draw realizations from that distribution using Monte Carlo methods.

An alternative approach involves decorrelating the variables before simulating them independently and then back-transforming them to reproduce the original correlations. Examples include minimum/maximum autocorrelation factors (MAF) (Desbarats & Dimitrakopoulos, 2000) and PPMT (Barnett et al., 2014). PPMT begins by applying a normal score transformation to each variable, followed by data sphering to reduce correlations and standardize variance. The method then searches for the least Gaussian projection to identify non-normal features across all projections. Each variable is simulated independently using SGS. The final step involves a back-transformation, which uses recorded tables from the Gaussian mapping to transform the non-Gaussian distributions back into their original units while maintaining the spatial relationships between data points. This approach avoids the need for modeling cross-variograms and allows for the accurate simulation of complex, correlated data. Due to its ability of modeling non-linear complex data, the PPMT method is used as part of the simulation workflow proposed in this paper.

Modeling blast movement at all locations within the blasted bench section requires modeling blast movement distances and directions. PPMT accompanied with SGS is used to model multivariate relationships between blast movement distances that vary from one location to another within the bench section and between flitches to produce the D-like shape resulting from explosives installed in the middle of the bench, causing the middle part to move further relative to the top and bottom flitches. Horizontal distance values obtained from BMM balls for each flitch represent a continuous variable, and PPMT helps model these variables while respecting their correlations. Figure 5.2 shows the main steps for generating simulated data using PPMT.

To smooth the simulated values when back-transformed, a kernel density estimate (KDE) (Parzen, 1962) can fit the sparse movement data. Values from the fitted distribution can then be sampled using a Gibbs Sampler (Geman & Geman, 1984). PPMT uses these simulated values to back-transform the fitted KDE and smooth the simulated values. This step is not implemented in the proposed workflow.

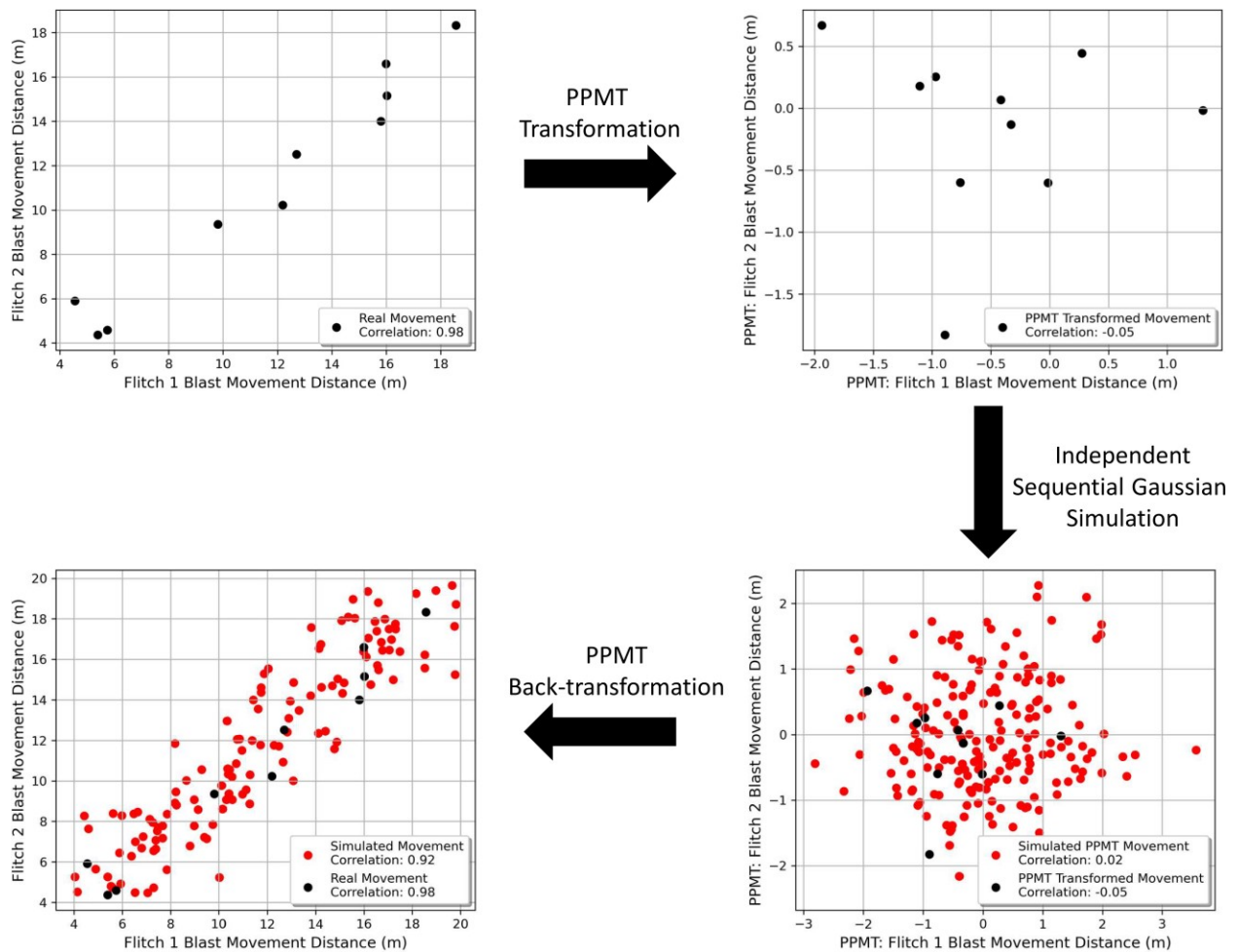


Figure 5.2: Decorrelation workflow using PPMT and Sequential Gaussian Simulation for modeling blast movement distances between Flitch 1 and Flitch 2 as an example.

Another important component of blast movement is the direction of the movement. The direction can be determined by comparing the pre-blast and post-blast locations of BMM balls to calculate

the azimuth of movement. Alternatively, a direction perpendicular to the detonation lines, going in the opposite direction of the blast sequence, can also be used to infer the movement direction. When BMM balls show relatively small variations in directions, blast movement direction uncertainty might not be significant, and a fixed direction of movement can be applied. However, when the directions of movement vary significantly due to the blast design or unexpected geological features that impact the movement direction, then modeling stochastically blast movement direction using SGS will be a useful exercise and should be part of the proposed integrated workflow.

Azimuth values obtained from BMM measurements can be divided into two components: a sine component and a cosine component, which are calculated as follows:

$$\theta = \tan^{-1} \left(\frac{\sin(\theta)}{\cos(\theta)} \right) \text{mod } 360 \quad (5.1)$$

In this equation, θ represents the azimuth of the movement in radians, and the mod 360 ensures that the back-transformed angles remain between 0 and 360°. These components can be simulated using PPMT in case of strong correlation. The simulated components are then used to calculate the azimuth at every location on a block model. Increasing the number of BMM balls helps in improving the modeling of variograms in the presence of sparse or limited data, as well as reducing uncertainty in blast movement models.

5.4.3 Generation of post-blast grade realizations

After stochastically modeling grade and blast movement distances and directions on a block-by-block basis using geostatistical simulation, the pre-blast grade blocks are relocated to their post-blast positions based on the realizations of blast movement distances and directions. To achieve this, each grade realization is randomly associated with a corresponding blast movement distance

and direction realization. It is important to note that grade, blast movement direction, and distance are uncorrelated at any location. Therefore, it is appropriate to combine realizations, one from each variable, to facilitate generating post-blast grade realizations.

In each of these post-blast realizations, the post-blast block locations are calculated as follows:

$$U_{postblast} = U_{preblast} + (V_{component} * D) \quad (5.2)$$

where $U_{postblast} = (X_{postblast}, Y_{postblast})$ represents the coordinates of the block centroid after blast, and $U_{preblast} = (X_{preblast}, Y_{preblast})$ represents the coordinates of the block centroid before blasting. The vector $V_{direction} = (X_{component}, Y_{component})$ corresponds to the unit direction vector of the blast movement, where $X_{component}$ and $Y_{component}$ are the components of the direction in the X - and Y - axes, respectively, and D is the horizontal movement distance.

By applying this method on a flitch-by-flitch basis, each block within the flitch is moved to its post-blast location in each realization while accounting to the differential blast movement that varies depending on the location of the block within the bench section and the flitch. Figure 5.3 illustrates the concept of differential blast movement, where the movement is longer near the free face area in the direction of the blast and shorter near the adjacent non-blasted bench section in the opposite direction.

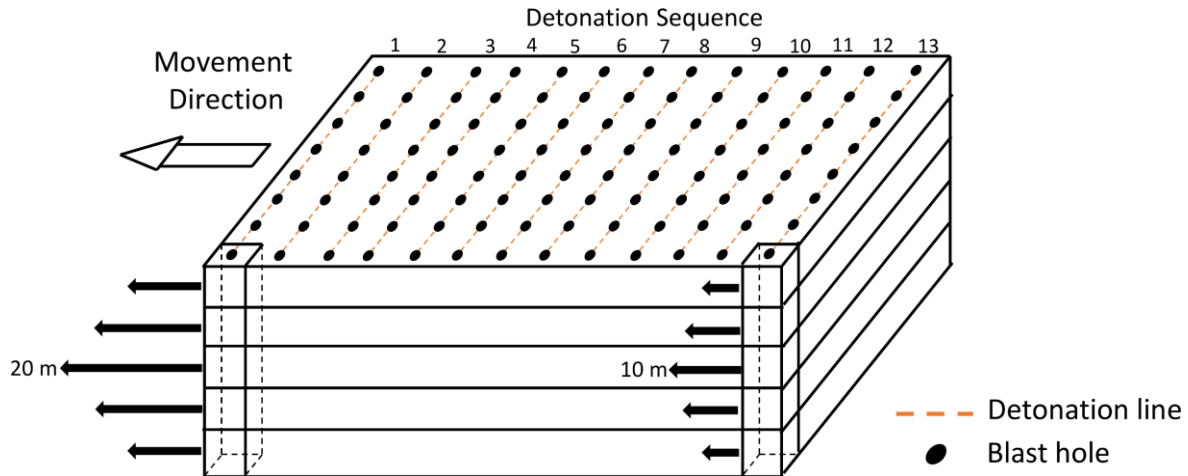


Figure 5.3: Schematic demonstrating the differential blast movement in the blasted section of the bench.

The outcomes of this step are realizations of grade values at post-blast locations. Essentially, this means generating post-blast grade control block models that reflect the uncertainty in grade and the associated blast movement distances and directions. These realizations will be used to quantify grade uncertainty at all locations within the blasted section of the bench.

Since this simulation is purely geostatistical and does not account for physical interactions between blocks, some blocks may end up sharing the same post-blast location. However, this will not impact the dig-limits optimization, which is treated as a 2D problem and is unaffected by changes in elevation. Consequently, the total contained metal and tonnage at each X and Y location will remain unchanged.

Blocks near adjacent bench sections that are not yet blasted can be assigned a fixed direction to prevent blasted blocks from ending up on top of the non-blasted areas. The extent to which these blocks will have a fixed direction can be determined by understanding the maximum influence distance of the blast, as measured from the BMM balls.

5.4.4 Converting 3D post-blast block models to 2D block models

Before running classifying material into types under uncertainty and running the risk-based dig-limits optimization, the 3D post-blast grade block models are converted into 2D block models. In this process, the x and y coordinates of the centroids remain unchanged, while the metal content from all SMUs with the same x and y coordinates is aggregated to create 2D block models that represent post-blast grade realizations. This conversion simplifies the dig-limits optimization problem significantly and does not affect the final dig-limits because the optimization operates on the entire bench rather than a flitch-by-flitch basis.

To further reduce the optimization processing time, smaller blocks in the X and Y directions are re-gridded to form bigger blocks with a size equal to one-half or one-third of the SMU size. The total metal content, tonnage, and grade are recalculated for the re-gridded blocks. While this re-gridding may slightly increase ore loss and dilution, it generates 2D post-blast models that maintain the metal content at the original SMU scale. These 2D block models will be used next to determine classify materials into types under uncertainty and determine optimal risk-based dig-limits.

5.4.5 Post-blast material-type classification under uncertainty

The MEP method (Glacken, 1996, Deutsch et al, 2000, Neufeld et al, 2005) is used in this workflow because of its ability to find the optimal destination under uncertainty, regardless of whether the profit function is linear or non-linear. At each block $b_{i,j} \forall i \in \{1, \dots, X\}, j \in \{1, \dots, Y\}$ inside the bench. $k_{i,j}$ is an indicator variable to identify blocks that are blasted within the bench (in-bench) and separate them from air blocks and adjacent non-blasted bench sections (out-of-bench) following this expression:

$$k_{i,j} = \begin{cases} 1, & \text{block } b_{i,j} \text{ is in the blasted bench} \\ 0, & \text{otherwise} \end{cases} \quad (5.3)$$

Where $k_{i,j}$ is the indicator that identifies whether block $b_{i,j}$ is in the blasted section of the bench or not. An Air block is a block that has zero chance of having blasted material moved to it.

There are $l \in L$ number of simulated grade values $z_l \in Z$. Profit for sending a block $b_{i,j}$ to $d \in D$ of destinations for all grade realization are calculated as follows:

$$\begin{aligned} P_d(b_{i,j}) &= E \{P_d(b_{i,j}, Z)\}, \forall Z = \{z_1, \dots, z_l\}, d = \{1, \dots, D\}, i = \{1, \dots, X\}, j \\ &= \{1, \dots, Y\} \text{ if } k_{i,j} = 1 \end{aligned} \quad (5.4)$$

The optimal destination of block $b_{i,j}$ is the destination that has the maximum profit compared to all other profits calculated for other destinations for this block, and it is expressed as:

$$d_{\text{optimal}}(b_{i,j}) = \arg \max_{d \in \{1, \dots, D\}} P_d(b_{i,j}) \quad \forall i \in \{1, \dots, X\}, \quad j \in \{1, \dots, Y\} \text{ if } k_{i,j} = 1 \quad (5.5)$$

The optimal destinations for these blocks are determined post-blast, prior to running the dig-limits optimization model. These destinations are then used within the dig-limits optimization model to assign expected profit values for sending materials to all possible destinations. Figure 5.4a shows a 2D post-blast grade control model with blocks assigned to their optimal destinations using MEP method. Figure 5.4b shows the in-bench and out-of-bench blocks based on the $k_{i,j}$ indicator.

5.4.6 Risk-based dig-limits optimization model

Before running the dig-limits optimization model to identify the final optimal destinations of blocks that honor account for equipment selectivity, a pre-processing step should be applied to identify blocks that do not meet the minimum mining width constraint. These blocks can be either (1) confined between other blocks on unmined bench sections or (2) located at the free face, where

more selective mining can be performed using the same shovel or a smaller dozer. The optimal destinations for these problematic blocks have already been determined using the MEP method during the material classification step.

Equipment selectivity in this context is represented by a moving frame around each block in the grade control block model. This moving frame encompasses a certain number of blocks in the x-direction n_x and y-direction n_y , reflecting the selectivity of the mining equipment and the SMU size. A block is considered problematic if none of the frames around it meet the in-bench condition, which requires at least one frame to have all its blocks within the blasted bench section.

Blocks at the free face zone that do not meet the minimum mining width requirement will be flagged as problematic because they can be mined more selectively using a shovel or a dozer. Additionally, blocks that are surrounded by non-blasted adjacent bench sections and are smaller than the minimum mining width requirement will also be flagged as problematic and described as confined blocks. These blocks will be processed as confined blocks and can be mined later once the adjacent bench section is blasted.

Problematic blocks are removed from the dig-limits optimization model and will be added back later, after identifying the optimal destinations of blocks that meet the minimum mining width requirement. For example, as shown in Figure 5.4c, the two types of problematic blocks described above are identified in this bench section: (1) one block is confined between blocks from a non-blasted bench section and is less than the minimum mining width of 2x2 blocks, and (2) two blocks in the free face area have a minimum mining width less than 2x2 blocks. Figure 5.4d shows the processed destinations of blocks before running the dig-limits optimization. Table 5.1 describes the indices used in defining the pre-processing step variables and the dig-limits optimization model.

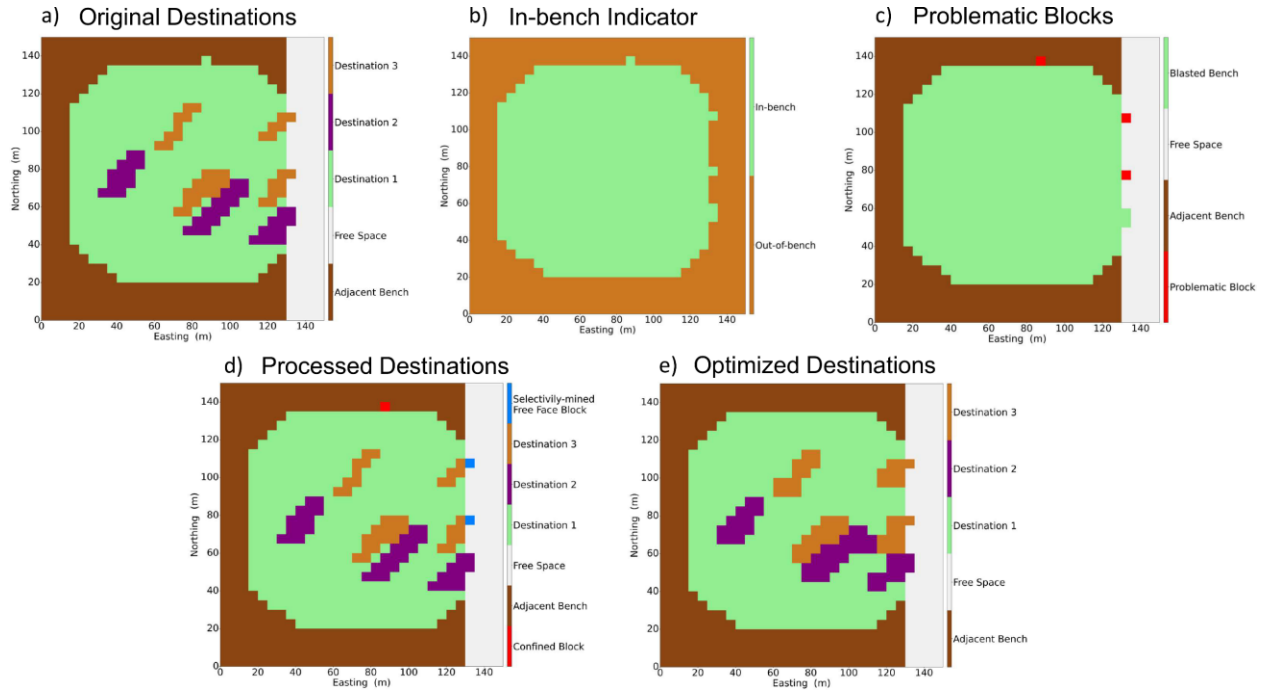


Figure 5.4: Plan views of a post-blast bench section showing pre-processing steps to generate optimal dig-limits with a minimum width of 2x2 blocks.

Table 5.1: Description of pre-processing and dig-limits optimization model indices

Index	Description	Sets
i	Block index in the X direction	$i \in \{1, \dots, X\}$
j	Block index in the Y direction	$j \in \{1, \dots, Y\}$
d	Destination index	$d \in \{1, \dots, D\}$
f_x	Frame index in the X direction	$f_x \in \{1, \dots, n_x\}$
f_y	Frame index in the Y direction	$f_y \in \{1, \dots, n_y\}$
α	Frame offset index in the X direction	$\alpha \in \{1, \dots, n_x\}$
β	Frame offset index in the Y direction	$\beta \in \{1, \dots, n_y\}$

Mathematically, the confined and free face blocks are identified by first defining the following indicators:

$$A_{i,j} = \begin{cases} 1, & \text{Air} \\ 0, & \text{Otherwise} \end{cases}, \quad \forall i,j \quad (5.6)$$

In which $A_{i,j}$ is an indicator variable that represent blocks of air, and

$$AB_{i,j} = \begin{cases} 1, & \text{Adjacent bench section} \\ 0, & \text{Otherwise} \end{cases}, \quad \forall i,j \quad (5.7)$$

Where $AB_{i,j}$ represents the blocks adjacent to the blasted bench section. For identifying free face blocks, the following indicator can be calculated as

$$FF_{i,j} = \begin{cases} 1, & \text{if } \sum_{\alpha}^{f_x} \sum_{\beta}^{f_y} A_{i-f_x+\alpha, j-f_y+\beta} > 0 \text{ and } k_{i,j} = 1 \\ 0, & \text{Otherwise} \end{cases}, \quad \forall i,j \quad (5.8)$$

Where $FF_{i,j}$ is an indicator for a free face block that can me mined in a more selective way using a dozer and front load equipment at block $b_{i,j}$ within the blasted section of the bench. For determining confined blocks, another indicator variable is calculated as follows:

$$CB_{i,j} = \begin{cases} 1, & \text{if } \sum_{\alpha}^{f_x} \sum_{\beta}^{f_y} AB_{i-f_x+\alpha, j-f_y+\beta} > 0 \text{ and } k_{i,j} = 1 \\ 0, & \text{Otherwise} \end{cases}, \quad \forall i,j \quad (5.9)$$

Where $CB_{i,j}$ represents an indicator variable for confined blocks at $b_{i,j}$ within the blasted bench section. Indicators $FF_{i,j}$ and $CB_{i,j}$ are then used to remove problematic blocks before running the risk-based dig-limits optimization model to ensure that the minimum mining width constraint is met knowing that these blocks are added back for any economical assessment and destination identification.

A new formulation for the dig-limits optimization model is used in this research. The new risk-based dig-limits optimization model allows for processing outcomes of the MEP method, working with more than two destinations, irregular bench shapes, and various orebody orientations and mining directions. The objective function is to maximize the expected profit of mining these blocks within the bench while satisfying equipment selectivity constraint. Description of model decision variables is found in Table 5.2.

Table 5.2: Description of dig-limits optimization model decision variables.

Decision Variable	Type	Values	Description
$X_{i,j,d}$	Binary	$\{0,1\}$	1 if material is sent to destination d and 0 otherwise
t_{i,j,d,f_x,f_y}	Integer	$\llbracket -1, n_x * n_y \rrbracket$	Total of X values inside a frame for destination d
v_{i,j,d,f_x,f_y}	Binary	$\{0,1\}$	1 if a valid frame for destination d and 0 otherwise

In this model, a frame is considered valid if all blocks in the frame have the same destination. Each block within a bench should be assigned to at least one valid frame. The model aims to find the frames that maximize profit and meet the minimum mining width constraint. The dig-limits optimization model is formulated as follows:

$$\text{Maximize } \sum_{i=1}^X \sum_{j=1}^Y \sum_{d=1}^D X_{i,j,d} P_{i,j,d}, \quad \forall k_{i,j} = 1 \quad (5.10)$$

Where $k_{i,j}$ is the indicator that identifies whether block $b_{i,j}$ is in the blasted section of the bench or not. $P_{i,j,d}$ is the profit value of sending block $b_{i,j}$ block represented by $X_{i,j,d}$ to destination d for all blocks with expected profit values inside the blasted section of the bench.

Subject to:

$$t_{i,j,d,f_x,f_y} = \begin{cases} \sum_{\alpha}^{f_x} \sum_{\beta}^{f_y} X_{i-f_x+\alpha, j-f_y+\beta} \quad \forall i, j, d, & \text{if } \sum_{\alpha}^{f_x} \sum_{\beta}^{f_y} k_{i-f_x+\alpha, j-f_y+\beta} = n_x * n_y \\ -1, & \text{if } \sum_{\alpha}^{f_x} \sum_{\beta}^{f_y} k_{i-f_x+\alpha, j-f_y+\beta} < n_x * n_y \end{cases} \quad (5.11)$$

This represents a constraint on the model to ensure that only complete frames are included in the search for the optimal solution. All incomplete frames are disregarded. To test the validity of the frame, in which all blocks within the frame are sent to the same destination, the following indicator variable is introduced:

$$v_{i,j,d,f_x,f_y} = \begin{cases} 1, & \text{if } t_{i,j,d,f_x,f_y} = 0 \mid t_{i,j,d,f_x,f_y} = n_x * n_y \\ 0, & \text{otherwise} \end{cases} \quad (5.12)$$

The following constraint ensures that only frames with blocks with same destinations are considered as valid frames:

$$\sum_{f_x=1}^{n_x} \sum_{f_y=1}^{n_y} v_{i,j,d,f_x,f_y} \geq 1, \quad \forall i, j, d \quad (5.13)$$

And to ensure that every block is sent to one destination only, the following constraint is introduced:

$$\sum_{d=1}^D X_{i,j,d} = 1, \quad \forall i, j \text{ if } k_{i,j} = 1 \quad (5.14)$$

After finding optimal destinations of blocks considering equipment selectivity, problematic blocks with their optimal destinations from MEP step are added back to the bench section and actual final profit is calculated from sending all blasted blocks to their final optimal destinations. Figure 5.4e shows the result of applying the dig-limit optimization model with 2 blocks by 2 blocks minimum mining width to find optimal destinations.

To account for the direction of mining when optimizing dig-limits, a rotation matrix can be applied to the bench before the dig-limits optimization pre-processing step. This aligns the direction of mining with the principal directions (north-south and east-west). The dig-limits optimization model can then be executed on the rotated blocks, and the results can be visualized in the original

coordinate system afterward. This step ensures that the optimization frames align with the mining direction.

5.5 CASE STUDY

This section presents a case study demonstrating the proposed integrated workflow for optimizing dig-limits under grade and blast movement uncertainties. First, a 3D grade control block model with a resolution of 0.5m X 0.5m X 3m was generated from the blast hole resource database at a porphyry copper mine. The block model definition is summarized in Table 5.3. The generated grade control block model is including extra air blocks to accommodate the muck pile shape post blasting. Fifty BMM ball field measurements were simulated to cover the five flitches at ten locations within the bench section. The simulated BMM measurements preserve honor the correlation between flitches in terms of distance and generate the D-like shape anticipated from the blast. Figure 5.5 provides a plan view of the simulated bench section layout. As shown in Figure 5, 168 blast holes and 10 monitoring holes, which only had BMM balls installed without explosives, were used in this blast. The bench was divided into five flitches, with 5 BMM balls at each monitoring hole covering the five flitches. The flitches are numbered from top to bottom, 1 to 5. The overall movement direction was towards the west, with minor deviations. The direction of movement was determined by comparing the pre-blast and post-blast locations of the BMM balls.

Table 5.3: Block model properties

Item	Unit	X	Y	Z
Block size	(m)	0.5	0.5	3
Number of blocks in a direction		350	240	20
Minimum centroids	(m)	-43.75	-18.75	1.5
Maximum centroids	(m)	131.25	101.25	61.5
Azimuth	(degree)	0		
Dip	(degree)	0		

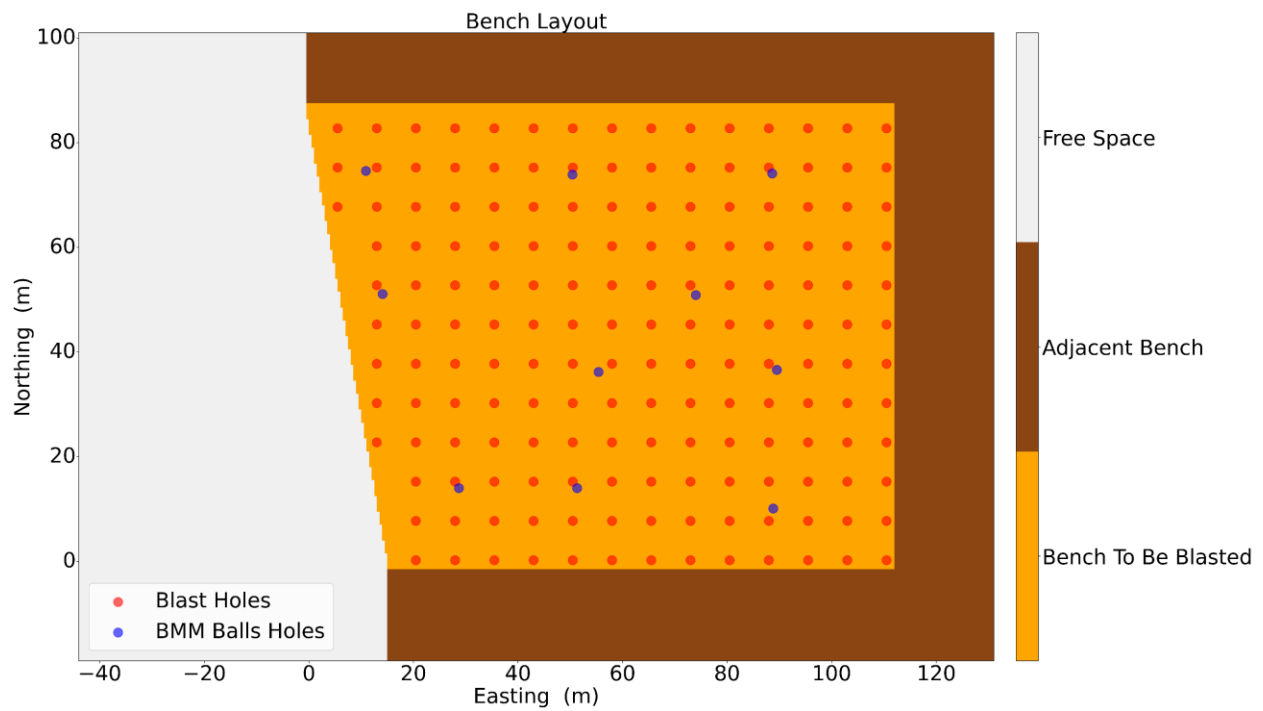


Figure 5.5: Bench layout showing blast holes and BMM balls holes location.

SGS was used to simulate 50 realizations of copper grade from the blast holes. These realizations were validated by checking histogram and variogram reproduction, which showed good reproduction of input statistics. Additionally, fifty realizations of blast movement distances were simulated using PPMT and SGS, ensuring that relationships between flitch movements at every BMM location were respected. Blast movement distances were validated through histogram and variogram reproduction. Furthermore, the blast movement distances simulation was validated by comparing the bivariate distributions of movements between all flitches in one of the realizations to the actual blast movement distances obtained from BMM balls. The results of this comparison are summarized in Figure 5.6.

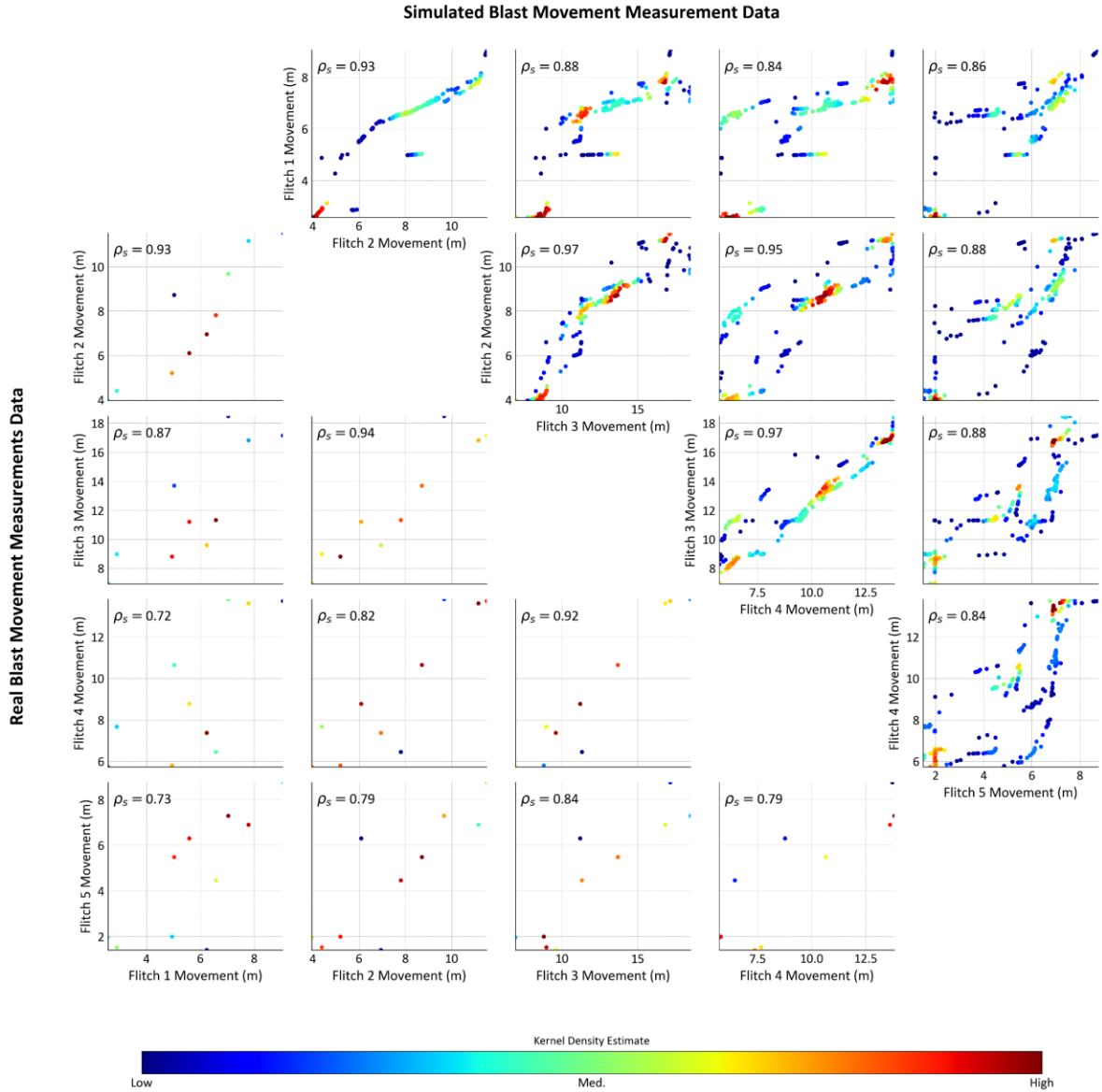


Figure 5.6: Testing correlation reproduction between flitch movements in the simulated data against the real BMM data.

The outcomes of the flitch movement distance simulation show good reproduction of multivariate relationships compared to actual data, as evidenced by the shape of the distribution and the reproduction of similar Spearman correlations ρ_s (Spearman, 1961). It is also noted that some correlations might be impacted due to the non-smooth histograms of the real BMM measurement

data during the simulation back-transformation process. Histogram smoothing can be applied to overcome this challenge. Overall, highly correlated flitch movement distances were reproduced. If movements were simulated independently, most correlations would be close to zero, and the D-like shape resulting from having explosives in the middle part of the bench, which pushes rocks further, would not be achievable.

The direction of movement is simulated using SGS, with each location's movement direction divided into sine and cosine components. The sine and cosine components were simulated using SGS and PPMT, then Equation 5.1 was used to calculate to derive the azimuth values for each realization. Minor deviations from the 270° azimuth were observed in the data. The decision to simulate movement direction is made to demonstrate the full integrated workflow. Direction simulation is only recommended when significant changes in movement direction occur due to blast design or unforeseen factors. Blast movement direction realizations were post-processed to prevent blocks from moving over the adjacent bench sections. This post-processing step involved assigning a fixed direction value of 270° azimuth to blocks within 20m of the north and south bench section borders, which is consistent with the maximum movement distance observed from the BMM balls in the middle flitch.

As a result of the simulation step, a total of 50 realizations of grade, blast movement distance and direction were generated. Figure 5.7 presents a 3D perspective of the bench section with realizations of grade, blast movement distance and directions.

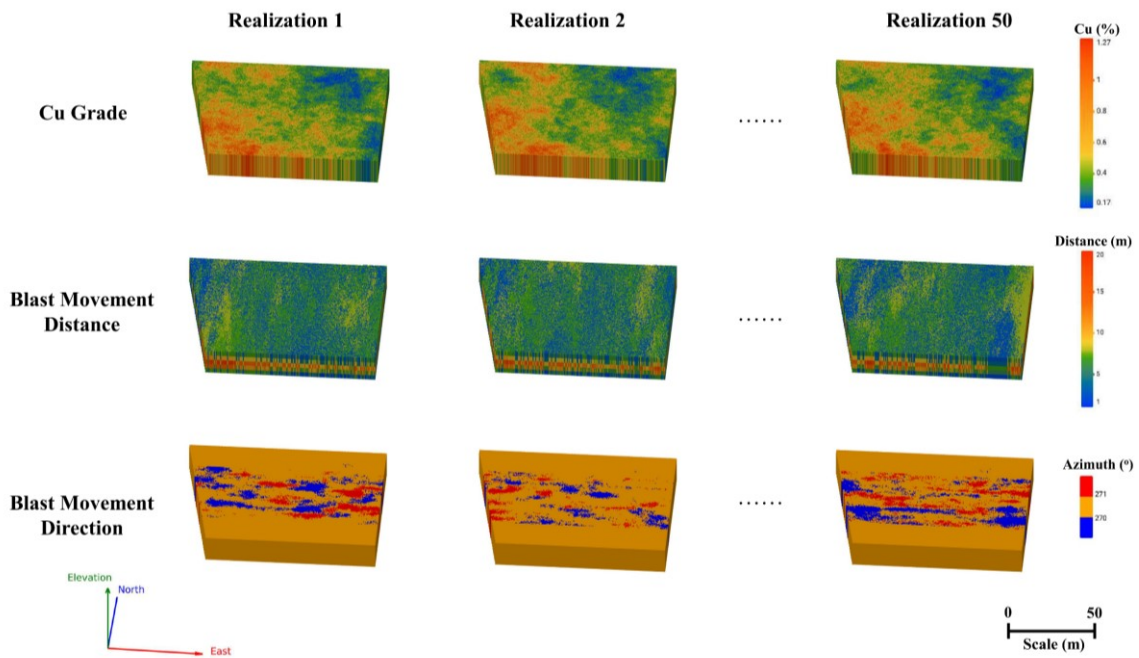


Figure 5.7: 3D perspective of the bench section showing Copper, blast movement distance, and direction realizations.

For each realization, the simulated grade values at every block in the grade control block model are relocated based on a corresponding realization of blast movement distance and direction to generate a post-blast grade realization. This process was applied to all fifty grade realizations, resulting in fifty post-blast grade realizations. Ore loss and dilution were calculated for each realization, and the range of possible ore loss and dilution values is shown in Figure 5.8.

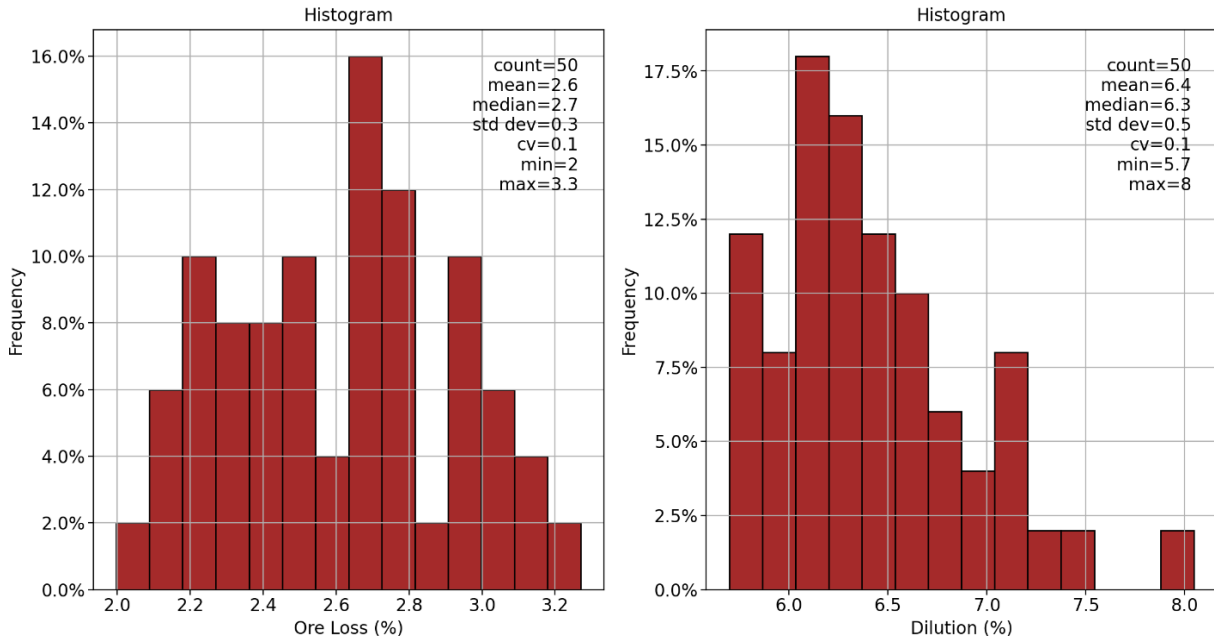


Figure 5.8: Ore loss and dilution calculated on post-blast realizations.

Based on the ore loss and dilution realizations, ore loss due to blast movement is expected to be 2.6%, with a minimum value of 2% and a maximum of 3.3%. Dilution is expected to be 6.4%, with a minimum value of 5.7% and a maximum of 8%.

When dealing with multiple destinations, comparing the pre-blast tonnage of each destination to the post-blast tonnage provides more detailed insights than summarizing results solely in terms of ore loss and dilution. To facilitate this comparison, a material-change classification matrix is proposed to summarize the impact of blast movement on the tonnage of different material types or classes and their respective destinations. This matrix compares the percentage difference between post-blast tonnage and pre-blast tonnage for each material type and destination. For post-blast realizations, a material-change classification matrix is computed for each realization. The average change in tonnage for post-blast material relative to pre-blast tonnage is calculated and shown in Figure 5.9.

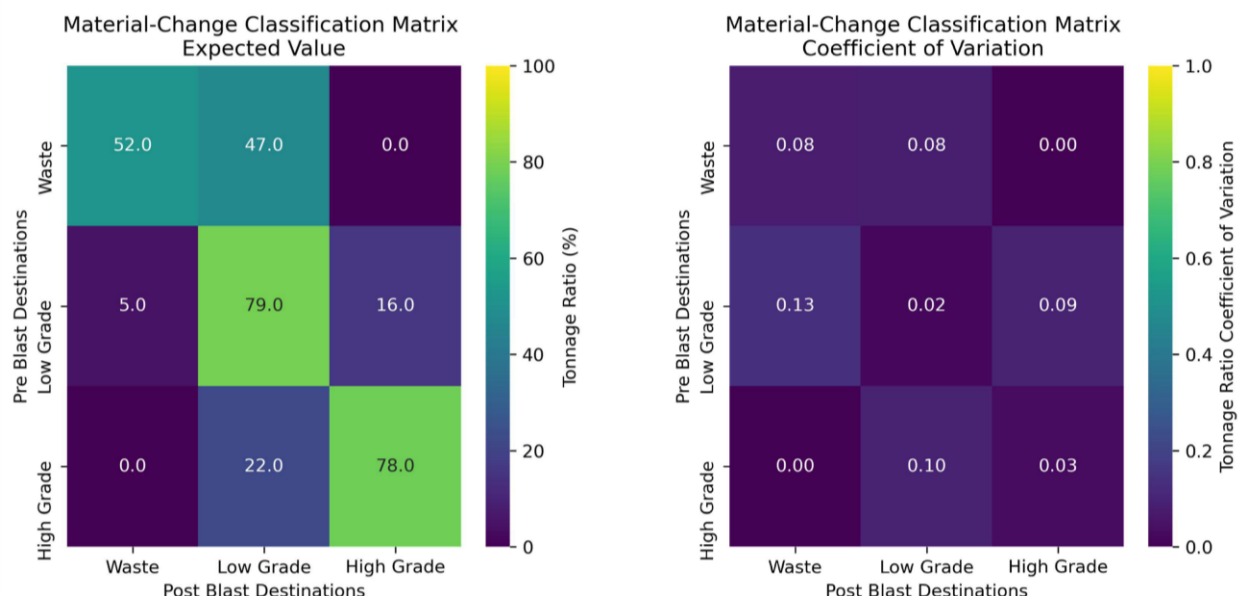


Figure 5.9: Material-change classification matrices showing expected values and coefficient of variation.

The results show that 47.4% of the waste is expected to be moved to the low-grade class and 0.5% to the high-grade class due to dilution caused by blasting. The low-grade class is expected to lose 4.9% of ore to the waste dump while converting 15.8% of low-grade ore to high-grade ore through dilution. No loss of high-grade ore to waste is expected; however, 22.2% of high-grade ore is expected to be processed as low-grade. The coefficient of variation (CV) is used as a measure of uncertainty for the expected material-change values, and the uncertainty in material-change classification is considered low in this case.

When translating material-change classification into dollar value to assess its impact on profit reduction under uncertainty, it is observed that the expected profit pre-blast was reduced from 3.8 million USD to 3.6 million USD, reflecting an expected profit reduction percentage of 4.3%. The distributions of pre-blast profits, post-blast profits, and profit reduction are presented in Figure 5.10.

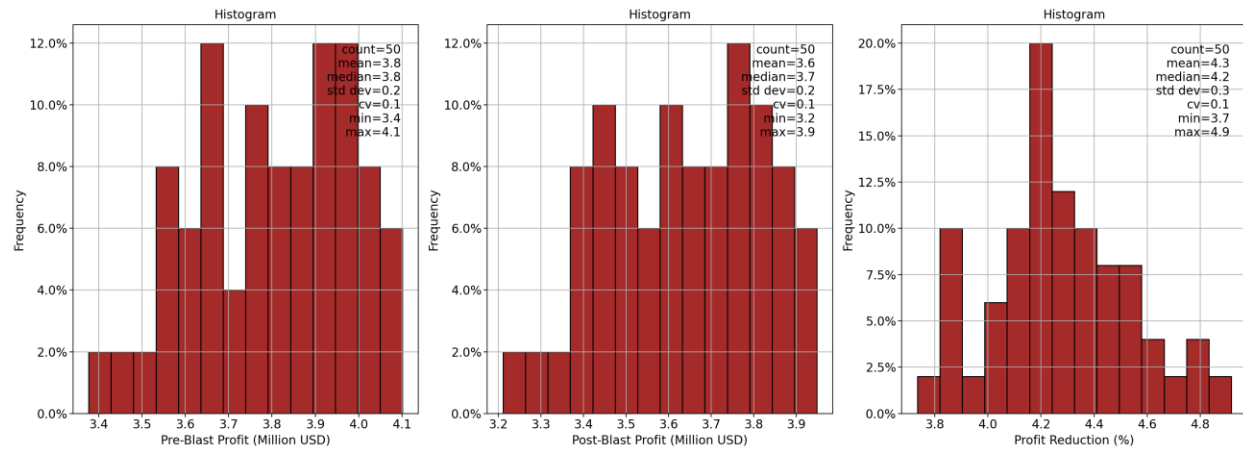


Figure 5.10: Histograms showing pre-blast profits, post-blast profits, and profit reduction percentages.

After generating post-blast realizations and assessing uncertainty in ore loss and dilution caused by grade and blast movement uncertainties, the 3D block model is converted into a 2D model by summing tonnage and contained metal vertically at each XY location in the post-blast grade control block model. To reduce computational costs, the 2D block model was regridded to 5m x 4m in the X and Y directions, respectively, to align with the east-west mining direction.

Destinations based on MEP were determined. Table 5.4 summarized the economical and metallurgical parameters used to calculate the profit for each destination.

Table 5.4: Cutoff grade and block economics parameters.

Item	Unit	Value
Copper price	\$/lb	3.5
Mining cost	\$/tonne	3
Low grade processing cost	\$/tonne	14
High grade processing cost	\$/tonne	25.5
Low grade recovery	%	60
High grade recovery	%	90
Rock bulk density	tonne/m ³	2.7

After determining blasted material destinations based on the MEP method, dig-limits optimization was executed to ensure that materials are sent to their optimal destinations while accounting for equipment selectivity. In this case study, equipment selectivity for this mine is 8m, requiring the dig-limits optimization to apply a 3-block by 2-block constraint in the X- and Y-directions, respectively, to ensure that mining selectivity and direction are considered. No coordinate rotation is applied since the mining direction is aligned with the principal directions. The pre-processing step, which aims to identify confined blocks and free face blocks that can be mined more selectively, did not identify any blocks that may cause issues during the dig-limits optimization step. The economic and metallurgical parameters used in the dig-limits optimization model are the same as those used to determine material destinations based on the MEP method. The outcome destinations from the MEP method and the risk-based dig-limits optimization are shown in Figure 5.11.

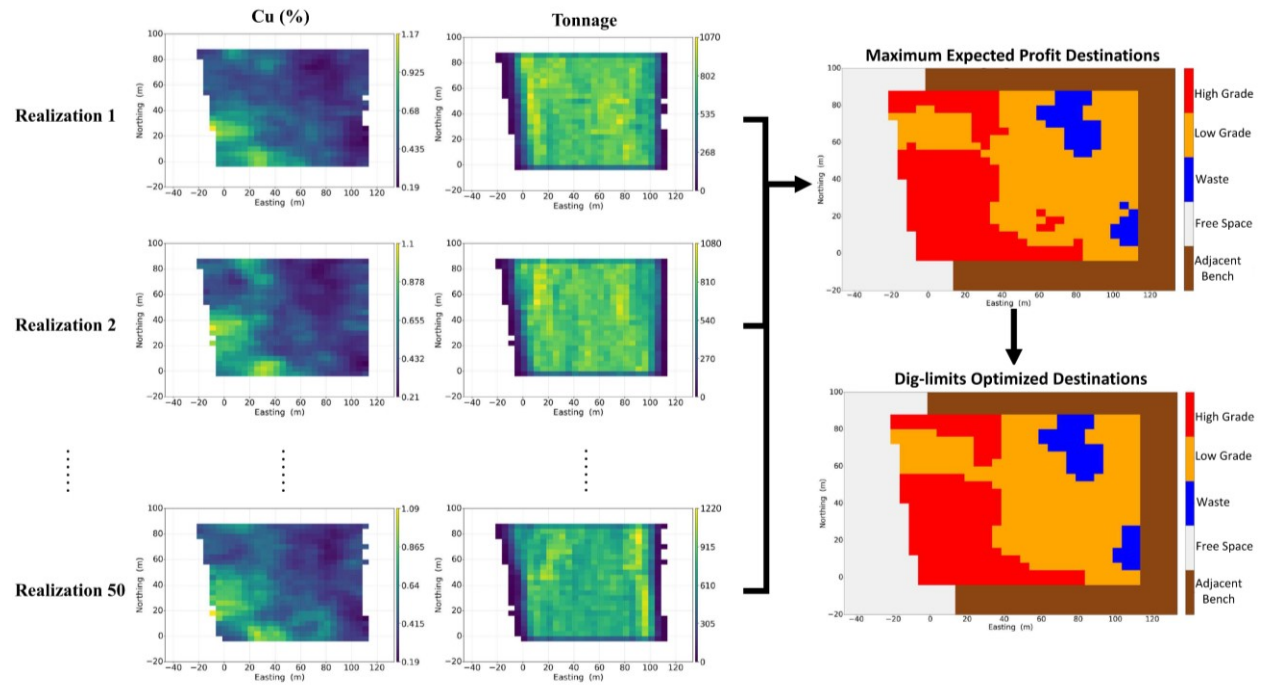


Figure 5.11: Maximum expected profit 2D destinations and risk-based dig-limits optimization destinations.

The results from comparing material destinations due to equipment selectivity relative to MEP destinations are summarized as material-change classification matrix in Figure 5.12.

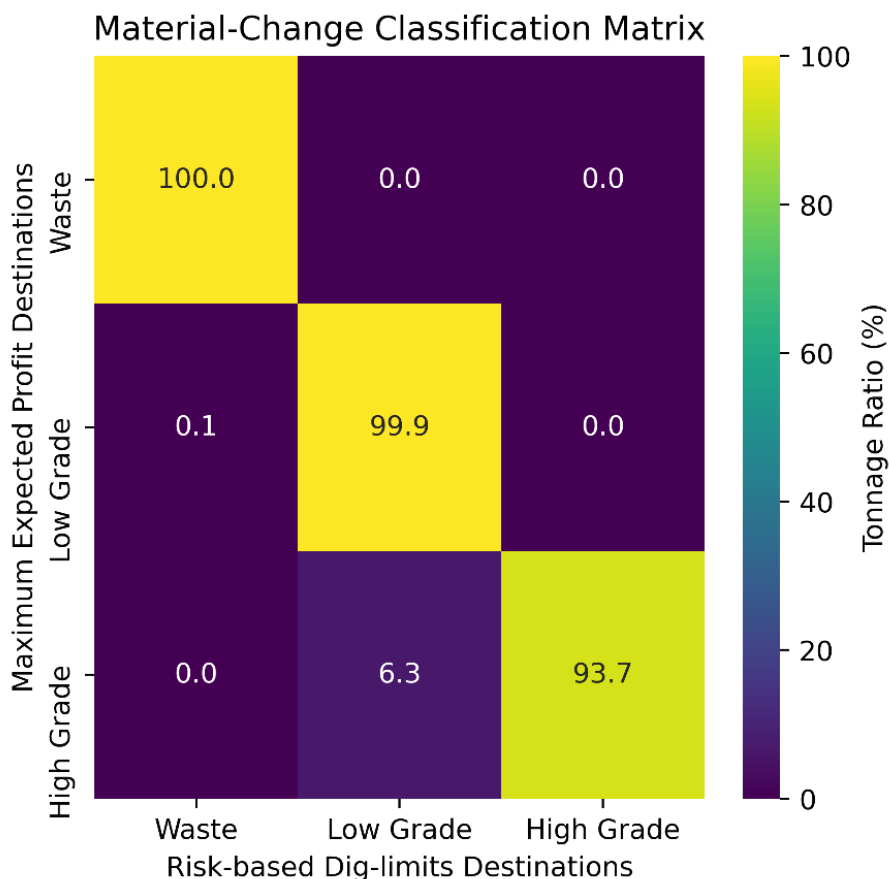


Figure 5.12: Material-change classification matrix between Maximum Expected Profit classification against Risk-based dig-limits classification due to equipment selectivity.

Destinations based on dig-limits optimization resulted in converting 0.1% of low-grade material to waste and 6.3% of high-grade material to low-grade. Other than these changes, materials retained their original destinations as determined by the MEP method. The expected profit from mining and processing this bench section decreased from USD 3,548,300 based on MEP destinations to USD 3,544,800 after applying dig-limits optimization, representing a profit reduction of 0.1%.

In this case study, it is noted that the impact of blast movement on profit reduction is much more significant compared to equipment selectivity. This is expected in the presence of a porphyry

copper deposit, which exhibits more spatial continuity compared to nuggety gold deposits. Care should be taken before blasting bench sections to minimize blast movement, thereby reducing ore loss, dilution, and changes in material classes due to material mixing during the blast.

It is also worth mentioning that the impact of dig-limits optimization on profit reduction can be more pronounced in heterogeneous orebodies (Hmoud and Kumral, 2023). Therefore, capturing both sources of ore loss and dilution as part of the standard grade control procedure is essential. Each deposit is unique, and this assessment should be conducted at every mine and within different zones of the deposit to better understand the main factors controlling ore loss and dilution, as it is difficult to generalize rules for this.

Moreover, the importance of adopting this workflow becomes more significant in the presence of a nonlinear profit function. For example, when the grade-recovery relationship is not linear, it is necessary to use a stochastic method such as the MEP method to determine material destinations under uncertainty without bias.

5.6 DISCUSSION

Implementing the proposed integrated grade control workflow offers mining projects a more efficient, accurate, and responsive approach to short-term mine planning. This approach can be deployed on a cloud-based platform, providing users with easy access to generate optimal risk-based dig-limits while accounting for grade and blast movement uncertainties. The platform facilitates the rapid integration of assay data from blast holes or grade control samples, along with BMM ball measurement data, to update a dynamic grade control model that predicts the location and grade of displaced material post-blasting. This will allow grade control geologists and short-

term mine planning engineers to make more informed decisions and improve operational efficiency on a daily basis.

The benefits of this integrated approach extend beyond operational efficiency. By reducing ore loss and dilution through more precise material classification, mining projects can recover more valuable material, improving overall profitability. The streamlined process minimizes errors in destination planning, allowing for quicker, more informed decisions. As the workflow adapts to real-time mining conditions, it ensures that geological models align closely with what is happening on the ground, reducing the risks associated with grade and blast movement uncertainty. This ultimately leads to more consistent production outcomes, lower waste handling costs, and better financial performance for mining projects.

In addition to operational and economic gains, this workflow contributes to sustainability in mining. By recovering more resources and reducing waste, it helps lower the environmental impact of mining operations. With better material classification, less waste ends up in processing, which means less water and energy are needed for downstream tasks like milling and refining. This also leads to lower energy use for transporting and disposing of materials, cutting down on fuel consumption and emissions. Additionally, by extending the life of the mine and reducing the need to re-handle materials, the proposed workflow helps save energy, reduce wear on equipment, and moves us one step closer to more sustainable mining.

5.7 CONCLUSIONS AND FUTURE WORK

This paper presents an integrated workflow for optimizing dig-limits under grade and blast movement uncertainties in open-pit mining operations. The methodology incorporates these uncertainties into the grade control process to enhance material-type classification and destination

optimization with a goal of minimizing ore loss and dilution. Key findings include the effective use of SGS associated with PPMT to capture spatial uncertainties in blast movement distances and directions. This approach ensures accurate representation of inherent data variability and correlations, providing a robust foundation for subsequent analyses and ensures that the D-like shape resulting from blasting is produced without using physical simulation that is computationally expensive and do not capture uncertainty. The MEP method is demonstrated as an effective approach for determining optimal material-type classification under uncertainty, thereby improving overall mining profitability. The importance of using the MEP method becomes more pronounced when dealing with non-linear profit functions. The risk-based dig-limits optimization model successfully incorporates equipment selectivity, irregular bench shapes, and varying mining directions that may reflect different orebody orientations, resulting in practical and economically viable dig-limits. A case study on a porphyry copper mine is presented to highlight the significant impact of blast movement on ore loss and dilution, emphasizing the need for capturing blast movement uncertainty and integrate this uncertainty into dig-limits optimization workflow and grade control procedures. By accounting for differential blast movement and its uncertainties, the proposed workflow ensures reliable post-blast material classifications, reducing the risk of suboptimal decisions. The practical viability and effectiveness of the workflow in real-world scenarios are demonstrated, showing that incorporating blast movement uncertainty leads to more accurate assessments of ore loss and dilution, providing valuable insights for mine planning and operations.

In conclusion, integrating grade and blast movement uncertainties into the grade control process enhances material classification accuracy and dig-limits optimization, resulting in improved economic outcomes and operational efficiency. Future research should focus on refining blast

movement models to work faster with larger moving frames, exploring their applicability to various ore deposits, and developing automated tools to streamline the implementation of this integrated approach. Incorporating variogram and histogram uncertainties of blast movement distance and direction into the workflow, given the limited BMM ball measurements, will help better capture parameter uncertainty and the overall uncertainty in blast movement. Additionally, linking stochastic dig-limits results to short-term mine planning, especially when multiple benches are mined simultaneously, and considering blending, would further enhance operational planning.

Furthermore, advancements in BMM ball technology, particularly in reducing their cost, would enable the installation of more sensors in each blast, providing more accurate real-time blast movement assessments and reducing modeling uncertainty. Developing accurate and cost-effective BMM balls that can be installed in multiple holes and screened through sorters without needing extraction before mining the muck-pile would significantly enhance this workflow and save time. Additionally, developing automated tools to streamline this integrated approach would further benefit the mining industry by reducing manual interventions and improving decision-making processes.

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Chapter 6 Conclusions and future work

6.1 CONCLUSIONS

This research has contributed to the field of dig-limit optimization by quantifying and integrating grade and blast movements uncertainties in determining optimal dig-limits, with a focus on spatial heterogeneity. A number of conclusions can be drawn from this research.

First, a copula-based simulation approach was effectively used to quantify blast movement uncertainty on a flitch-by-flitch basis, helping to capture its impact on dig-limit optimization outcomes and identify areas with a high risk of ore loss and dilution. Incorporating blast movement realizations significantly improved the estimation of ore loss and dilution, providing a more accurate profit assessment that closely reflects reality. Identifying high-risk material misclassification areas in advance is crucial to ensure that careful grade control procedures are applied in these areas, reducing ore loss and dilution and ultimately increasing project profitability.

Second, Shannon's entropy principle was successfully used to quantify spatial heterogeneity between ore and waste on a bench-by-bench basis at the scale of selective mining units. Two indices measuring spatial entropy globally and locally were introduced and tested through case studies, proving useful in quantifying spatial heterogeneity linked to ore loss and dilution. Understanding the relationship between spatial heterogeneity, blast movement, ore loss, and dilution is essential for predicting the level of ore loss and dilution due to ore spatial heterogeneity and blast movement. By identifying bench sections with increased spatial heterogeneity post-blast, controlled blasting can be applied to these benches to reduce blast movement, thereby reducing dilution and ore losses, and enhancing project profitability.

Third, incorporating grade and blast movement uncertainties into the grade control process enhances material-type classification and dig-limit optimization outcomes. This integration results in a better assessment of ore loss and dilution, thus providing a more accurate profit assessment.

The methodologies developed in this thesis were applied to a range of deposit types, illustrating their adaptability across different geological settings. This demonstrates that the workflow is not restricted to a specific deposit type and can be applied broadly without significant modification.

Finally, the proposed methodologies aim to optimize material destinations, contributing to more sustainable mining practices by improving resource efficiency and reducing waste. The workflows developed in this thesis support more precise material classification and routing, minimizing the processing of low-value material, reducing energy consumption, and lowering the overall environmental impact. These advancements demonstrate how this research aligns with the mining industry's broader goals of responsible and sustainable mining.

6.2 FUTURE WORK

Building upon the findings and methodologies developed in this thesis, several avenues for future research and development can be identified. Improving the performance of dig-limit optimization models could be achieved by exploring advanced computational methods such as parallel computing, aggregation/disaggregation, and nested Bender decompositions, particularly for larger frames and more complex deposits.

The decision to use exact methods, specifically mixed-integer linear programming, was based on the manageable problem size, which allowed for efficient computation within a short timeframe for scenarios involving frames of up to 3 blocks by 3 blocks. However, scaling this approach to larger frames presents significant computational challenges. Overcoming these limitations will

require successful implementation of the aforementioned advanced techniques. An alternative approach involves leveraging metaheuristic, heuristic, or Bayesian optimization methods. While these methods may not guarantee optimality, they offer the potential to generate high-quality solutions in a shorter timeframe. Given the inherent uncertainties in input parameters, the trade-off between computational efficiency and guaranteed optimality may be acceptable, as it aligns more closely with realistic decision-making scenarios.

Additionally, accounting for the angle of repose, which impacts the degree of dilution when extracting blasted material in certain mining directions, could be added to the model to generate more accurate dig-limits. Extending the application of spatial entropy to more than two categories, such as multiple ore types and waste, would allow for a more comprehensive assessment of spatial heterogeneity.

Refining blast movement models to work faster with larger moving frames and exploring their applicability to various ore deposits would also be beneficial, while incorporating variogram and histogram uncertainties of blast movement distance and direction into the workflow would help capture parameter uncertainty more effectively.

Technological advancements in blast movement monitoring, particularly in reducing the cost and improving the accuracy of blast movement monitoring balls, would enable the installation of more sensors in each blast, providing more accurate real-time blast movement assessments and reducing modeling uncertainty. Developing automated tools to streamline the integrated approach for dig-limit optimization would reduce manual interventions and improve decision-making processes, ultimately benefiting the mining industry by enhancing operational efficiency and economic outcomes.

The recent advancement of parallel computing may facilitate the multiphysics modeling of blast movement in reasonable times, and investigating the applicability of using a stochastic approach in modeling blast movement using multiphysics simulation models is an area worth exploring. By using these multiphysics simulation models, the need for blast movement monitoring balls may reduce over time and eventually become unnecessary. Future work should focus on investigating the incorporation of non-Newtonian fluid models, such as the Herschel-Bulkley model, into multiphysics simulations of blasting phenomena. This might provide a better representation of the complex interactions between detonation gases and fractured rock, particularly in predicting gas expansion and blast movement.

Additionally, linking geometallurgy to blast movement can provide valuable information that can be used to characterize the physical characteristics of the rock, enabling better modeling of blast movement. Attributes such as unconfined compressional strength can be predicted on a block-by-block basis using machine learning techniques, increasing the accuracy of physical blast movement simulation models.

Geometallurgical models can also be used to identify different material types that require specific processing methods. For example, they can help distinguish between oxides and sulfides, hard and soft rocks, or ores that contain harmful elements such as arsenic and heavy metals or have the potential to cause acid-rock drainage. By integrating this into the dig-limit optimization models, the objective function can be adjusted to direct these materials to the right destinations. This can contribute to building more sustainable mining operations.

Finally, linking stochastic dig-limit results to short-term mine planning, especially when multiple benches are mined simultaneously and blending is considered, would further enhance operational planning. Investigating holistic optimization approaches that consider various operational

scenarios, including blast movement, fragmentation size, and the cost of loading, hauling, and crushing, could further improve decision-making. Addressing these future research directions will refine and expand the methodologies and findings from this thesis, contributing to more efficient and profitable mining operations in the face of inherent uncertainties.

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