FRACTAL CHAOTIC SYSTEMS: INVESTIGATION OF THE GEOLOGICAL SYSTEM AND ITS SEDIMENTATION BEHAVIOUR

by

Costas Karavas

A thesis submitted to the Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of
Master of Science.

Geophysics Laboratory
Department of Geological Sciences
McGill University
Montreal, Quebec
Canada
December, 1990

First of all Chaos came into being, and next broad-bosomed Earth, for all things a seat unshaken for ever,...

–Hesiodos

Abstract

Chaos theory has only recently been related to various phenomena in the earth sciences. Here, using systems theory in a description of geological processes, we study the chaotic development of sedimentary sequences.

The geosystem is treated as a partially specified system in order to apply qualitative stability analysis in the investigation of sedimentation behaviour and interactions among geological processes. The analysis suggests that the sedimentary system is unstable. This instability in conjunction with the system's sensitive dependence to internal fluctuations (i.e., those generated within the system) provide supporting evidence to suggest a chaotic behaviour for the sedimentation system.

We suggest that chaos could act as the common underlying mechanism which is manifest as the fractal-flicker noise character observed in reflectivity well logs. Acoustic impedance variations – the geophysical measures of lithologic variability – represent the internal organization of the interacting geological processes. This organization under a chaotic regime is responsible for the common statistical character found in various sedimentary basins.

Résumé

La théorie du chaos a vu récemment ses applications s'étendre à plusieurs phénomènes des sciences de la terre. Cette étude se sert de la théorie des systèmes, dans un contexte géologique, pour étudier le développement chaotique de séquences sédimentaires.

Le géosystème est définit comme étant un système partiellement spécifié afin d'exécuter une analyse qualitative de stabilité du comportement de la sédimentation et de l'interaction de divers processus géologiques. L'analyse du système semble indiquer que le système sédimentaire est instable. Cette instabilité, combinée à une dépendance de la sensibilité du système aux fluctuations internes (c.a.d., celles générées par le système), contribuent à appuyer l'idée d'un comportement chaotique du système sédimentaire.

Nous pensons que le chaos pourrait être le mécanisme de base expliquant la présence de bruits de type fractal-scintillation généralement observés dans les diagraphies de forage de réflectivité. Les variations d'impédance acoustique – la mesure géophysique des variations lithologiques–représentent l'organisation interne de l'interaction des processus géologiques. Une telle organisation, sous un régime chaotique, contrôle le caractère statistique communément déterminé dans divers basins sédimentaires.

Acknowledgements

I am greatly indebted to my thesis supervisor, Dr. O. Jensen, for introducing me to the "new" science of fractals and chaos theory, for many helpful comments, and critical review of this thesis. I also would like to thank her for helping to arrange funding through the Department of Geological Sciences for the purchase of my personal computer. Imperial Oil Limited which supported this research through a University Research Grant to Dr. O. Jensen, is acknowledged.

I would like to express my gratitude to the geophysics community of McGill University and to thank Dr. D. Crossley, Dr. J. Arkani-Hamed, and Dr. P. Toft for their support.

I would like to thank the Department of Geological Sciences at McGill University for a Reinhardt Scholarship. Support provided by the Canadian Society of Exploration Geophysics through a Scholarship (sponsored by BP Canada) is also gratefully acknowledged.

I would like to thank my fellow students Michael Manga and especially George Agapeew and Mark Gregotski for many enlightening and interesting discussions, for their unfailing friendship, and for making my time at McGill a memorable one. I also appreciate Nathalie Guillemette's effort in translating the abstract into French.

Finally, I wish to thank my family for their love, moral support, and encouragement.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Describing the sedimentary sequence

Chaotic phenomena in the earth sciences have only recently been recognized. Earthquakes (Huang and Turcotte, 1990; Scholz, 1989), and thermal mantle convection (Stewart and Turcotte, 1989) are some particular phenomena showing a fundamentally chaotic behaviour.

Previously, Thornes (1983) had suggested that multiple stable states exist for geomorphological systems. He describes the braiding-meandering river system, showing how shifts in the control variables of this sytem (in this particular example the control variables are slope and discharge) can shift the system to different equilibrium stable states.

Sedimentation development, a process which reflects the conspiracy of many geological processes, such as sea level fluctuations, tectonism, biological activity, climatic variability, etc., has long been considered in geological modelling. Quantitative modelling of sedimentary sequence development has been based on Markov chain theory (Swarzacher, 1975; Velezeboer, 1981; Pan, 1987; Strauss and Sadler, 1989). These models however, do not fully describe complete sedimentary sequence development. Their failure in representing depositional processes derives from the assumption that sequence development is governed by Brownian statistics. This implies that: i) sedimentation is a non-stationary [1] process and ii) present sedimentation depends essentially on the immediately preceding sedimentation state and not on long past influences. Such a condition can be described by a first-order Markov process.

A first-order Markov process describes a system at time t_n dependent only on the state of the system at time t_{n-1}, and not directly on the history previous to time t_{n-1} (Carr, 1982). It is inadequate in describing sedimentary sequence development in that it conflicts with our knowledge of basin-fill processes. Deposition of a sedimentary sequence is a result of conspiracy of many geological processes which have acted during different temporal scales. The deposition of a particular lithology in a sedimentary basin depends not only on the existing conditions during that time, but also on the broad regional conditions that persisted during earlier times (Keshner, 1982). These conditions directly affect the sedimentology and structural style of sedimentary basins (Montgomery, 1983).

Acoustic impedance (the product of density and seismic velocity) is an important measure of geological sequences. In addition to sedimentation processes, the acoustic impedance is also affected by post-depositional processes; these are diagenetic processes. Rock velocity and density depend on mineral composition, matrix structure, cementation, interstitial fluids in the solid rock matrix, porosity, depth of burial and geological age. The average seismic velocity of igneous rocks is higher and with a narrower range of variation than sedimentary or metamor-

[1] Stationarity is the condition where the probabilistic parameters of a random process are time invariant (Gardner, 1986).

phic rocks (Dobrin, 1988). Laboratory experiments on calcite and quartz have shown that increase in porosity decreases P-wave velocity (Gardner *et al*, 1974). Porosity normally decreases with burial depth which has a consequence of velocity increasing with depth (overburden pressure). At shallow depths of burial, the velocity of sedimentary rocks increases rapidly with increasing pressure. Beyond the depth of consolidation, the influence of variation in pressure on velocity has a smaller effect. At these depths, porosity and mineral composition of the grains become dominant in governing seismic velocity (Dobrin, 1988).

The presence of interstitial fluids also affect the velocity of rocks. Laboratory experiments by Domenico (1974) show that the presence of gas in sands reduces the seismic velocity. Also, presence of water in the pore spaces has a larger effect in increasing velocity than the presence of oil.

In general, many empirical relationships have been obtained from laboratorty experiments on rock samples in order to determine the various responses of rock properties to seismic velocity.

It is noted however, that laboratory experiments can only imitate real processes of the earth to a certain extent. The interval of time, over which these processes act, is usually not taken into account in the laboratory.

However, in this thesis, we shall not deal with post-depositional diagenetic processes, but rather relate sedimentary processes to a chaotic geosystem.

In attempting to describe a sedimentary sequence (or an equivalent sedimentation process) as a Brownian process, we assume implicitly that the seismic velocities of the sedimentary layers would wander very far from the initial values

(Todoeschuck and Jensen, 1988). This challenges a fundamental tenet of geology: the Principle of Uniformitarianism, which implicitly suggests that acoustic impedance changes would be stationary throughout the Earth's history. The inconsistentcy of Brownian processes with Uniformitarianism would commonly suggest an unsatisfactory description of sedimentary processes.

Acoustic impedance is a geophysical measure of formation lithologies. Changes in acoustic impedance correspond to sedimentation variability and diagenetic processes. Todoeschuck and Jensen (1988) show how relative changes in acoustic impedance correspond to seismic reflection coefficients or seismic reflectivity (Appendix A). Such changes in acoustic impedance further correspond in variations in the sedimentation processes and hence relate directly to processes in geology. An examination of the statistics of reflectivity sequences derived from acoustic impedance well logs from a wide distribution of sedimentary basins, have shown that reflection coefficients are not spectrally white but have a fall-off of power at low frequencies (Hosken, 1980; Walden and Hosken, 1985; Todoeschuck and Jensen, 1989; Todoeschuck *et al.*, 1990) typically with a f^{β} spectrum (Fig. 1.1). The equivalent acoustic impedance functions show a "Joseph" power spectrum, proportional to $1/f^{\beta}$, f is frequency, and where β is approximately equal to 1 (i.e., a flicker noise process). "*...for stationary geology the power spectral density of the impedance function must have β less than or equal to unity.*" (Todoeschuck and Jensen, 1988). Such a spectrum characterizes a flicker noise process (Hosken, 1980).

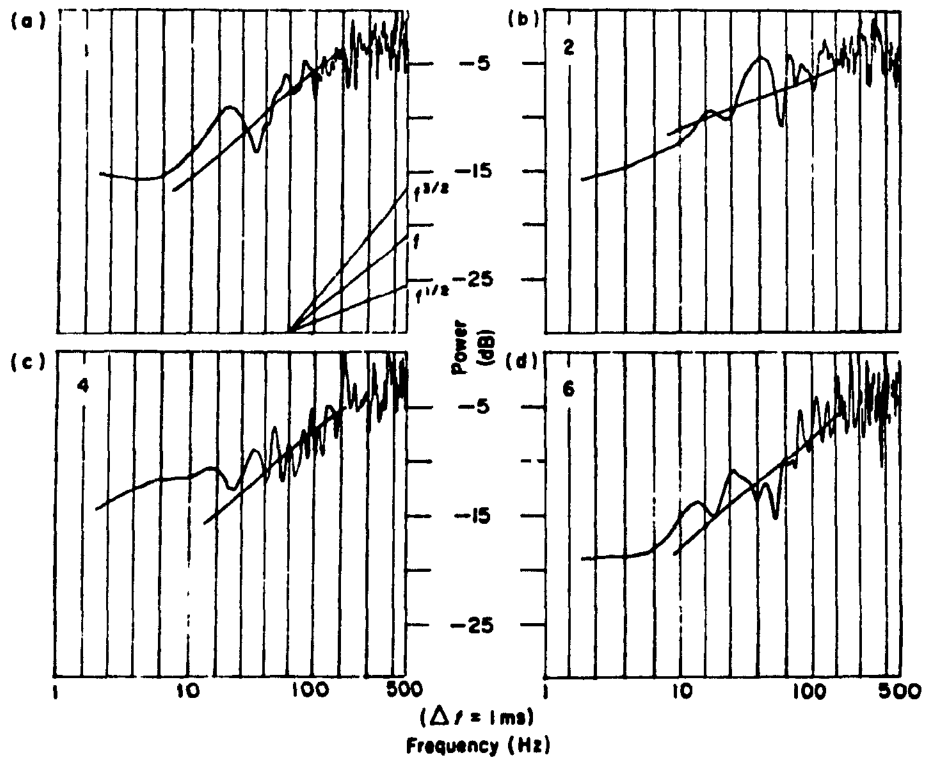

Figure 1.1: Power spectra of reflection sequences from various sonic logs. The reflectivity sequence is proportional to f^{β} (f is frequency) where $1/2 < \beta < 3/2$ (After Walden and Hosken, 1985).

Spectra of these type belong to a family of spectrally scaling noises (Fig. 1.2). With β equal to zero, we have uncorrelated noise (white noise). This is an unreasonable model for geology:

> *"The earth's stratification is the result of natural laws,...these provide some predictable constraints, and consequently...the outcome is not completely random"* (O'Doherty and Anstey, 1971).

With $\beta = 2$ we have a Brownian process. Flicker noise is *"intermediate between white and brownian noises and exhibits a balance between randomness and correlation on all time scales"* (Voss, 1983).

These particular statistical characteristics of the acoustic impedance function, and equivalently of the reflectivity sequences, reflect the variability in sedimentation and thus, the organizational structure of geological processes, as well as post-depositional processes.

1.2 Fractal character

The adjective *"fractal"* was coined by Benoit B. Mandelbrot (1977, 1983) to describe the geometry of objects that look similar at different length scales. These geometrical concepts are thoroughly discussed in his book "The Fractal Geometry of Nature" (1983). Scale-invariance is demonstrated (Mandelbrot, 1983) from the power-law (Pareto) distribution:

$$\frac{dN}{dr} = A\, r^{-p}, \tag{1.1}$$

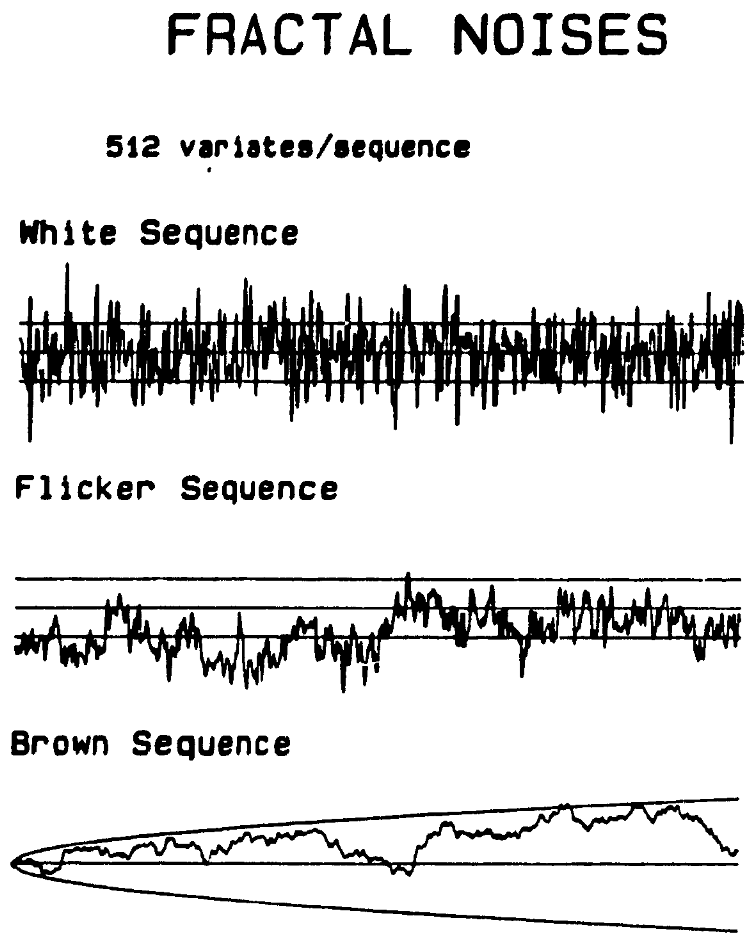

Figure 1.2: Spectrally self-scaling noises. A white Gausiian noise has a $1/f^0$ spectrum. Flicker noise, with $1/f$ spectrum lies at the boundary of the stationary scaling noises. Brown noise or "random walk" is non-stationary and has a $1/f^2$ spectrum (After Jensen and Mansinha, 1987).

where dN is the number of features with size between r and $r + dr$. When the length scale is changed from r to kr, the functional form of the equation (1.1) remains the same. Only the constant A is changed to

$$A' = A\, k^{-p} \tag{1.2}$$

This implies that the object looks the same upon scale change; it is scale-invariant. Fractals have been ubiquitously observed in many physical systems. The Chandler wobble of the Earth's rotation axis is modelled by a fractal-flicker noise excitation (Jensen and Mansinha, 1987). Crustal velocity variability in seismic refraction experiments have been modelled with self-scaling noises (Crossley and Jensen, 1989). The statistics of near-surface magnetic sources have been described as fractal (Gregotski *et al.*, 1990).

In geological cases, Plotnick (1986) demonstrates that the pattern of stratigraphic hiatuses show evidence of fractal organization based upon the "Cantor bar" model. A fractal power-law relationship has been demonstrated between tonnage and grade (concentration) relations for economic ore deposits (Turcotte, 1986). In the study of sinuous stream channels, Snow (1989) has treated meandering river traces as fractal curves. Other applications of fractals in geophysics and geology are described by Turcotte (1989). A more general usage of fractals in the sciences and arts is discussed by Schroeder (1989).

Acoustic impedance variations in the geological record exhibit statistical self-similarity - they are fractal (Todoeschuck and Jensen, 1988; Agapeew, 1989). Traces characterized as $1/f$ noise are statistical self-similar (Mandelbrot, 1983). This means that in a vertical sedimentary sequence *"each small section looks* *like its neighbours and like scaled down versions of the whole, while retaining its individual identity*", over a wide range of scales (Walden and Hosken, 1985). That is, they are scale-invariant.

1.3 Problématique

Fractal-flicker noise statistics adequately represent the response of acoustic impedance variations in the sedimentary record. However, there is no general theory that explains this observation.

Previous work dealing with the geological justification of flicker acoustic impedance suggests that a hierarchy of independant random geological processes of discrete frequency bandwidth superimpose to produce the natural state which determines the flicker noise model (Agapeew, 1989). Agapeew's work focused on a detailed and complex description of geological processes on various temporal and spatial scales.

Further examination of this problem is the main focus of this research. The objective of this thesis is to relate the statistical properties of acoustic impedance variations to the development of sedimentary sequences. This will be accomplished by introducing a new mechanism which will describe sedimentary sequence development, and thus acoustic impedance changes.

The problem will be approached from a different viewpoint than Agapeew took. Methodologies of system science will be used to examine the geological system and the behaviour of its processes. The interaction among the processes is investigated rather than each individual process.

If we were to examine what *"rule"* governs changes in lithologies and thus in acoustic impedance, we recognize that the "*cause*" or explanation of this phenomenon is related to the internal organization of the geological processes which interact to produce the self-scaling and $1/f$ character of sedimentary sequences. Thus, emphasis will be given to the organizational structure of the geological processes through the study of process interaction. This organization follows a chaotic regime which is suggested by analogies between fundamental properties of chaos and geological process behaviour:

> "*Chaos describes the origins of variability in our processes. All variability arises from interactions, either in space or in time*" (McGill, 1989).

In this thesis, the concept of chaos as a mechanism for the development of stratigraphic sequences is investigated. The basic fundamental properties (conditions) of chaos in systems are examined and their effect on stratigraphy is discussed.

The problem at this stage will be dealt with on a theoretical basis as no strict mathematical description can yet be obtained. The absence of theorems and rules in geology forces us to accept documented geological cases as "*guides*" to our analysis. As Rogers (1989) states:

> "*Here we ask questions about what the new interrelations tell us about nature and how geologic processes operate. As we develop explanations for different aspects of a geologic process, observational patterns*

that integrate a large amount of data act as high-level rules in our reasoning schemes."

Our analysis will not be restricted to a particular site; rather, we will endeavour to describe fundamental characteristics of all sedimentary basins, world-wide.

Chapter 2

Systems Science Theory

2.1 Methodology

Sedimentary sequences are deposited under a conspiracy of various geological processes. These processes act on different spatial and temporal scales which are difficult to separate. In examining geological systems, one should not focus on each individual process, but rather should examine their behaviour relative to one another. Only a study of the geological system as a whole, will provide information about its structure and internal operations. One methodology which allows for a comprehensive view of the sedimentation process is based on systems science theory. We abandon the classical viewpoint through which the system is analysed in terms of its separable components. For a real dynamic and complex geological system, this is not appropriate. In this thesis, we shall attempt to study geological processes holistically. Through this strategy we shall explore the interactive nature of the different processes and their behaviour.

2.2 Principles of system science

System science is defined as *"the total collection of knowledge, methods and skills available for the identification, abstraction, modelling, quantification, analysis, synthesis, evaluation and control of rational systems and their behaviour"* (Sandquist, 1985, p.1).

The principle of causality [1] that *"a measurable cause produces a measurable effect"* is an essential and implicit assumption in system science theory. In many systems, it is not easily possible to distinguish cause from effect. Geological processes which represent components of the complex geological system, are often difficult to identify and isolate. The geological system may possess many inputs interacting through feedback and system coupling of the outputs (Legget, 1985).

Using methodologies of system science we can begin to understand the importance of interactions between fundamental components of the geological system.

2.3 Defining the system

A physical or conceptual boundary is created which isolates the system from its external environment except for the inputs and outputs which can act across it. The choice of boundary depends on what aspects of the system we seek to investigate as well as on spatial and temporal scales. For the purpose of this study, the essential boundary conditions are dependent on factors controlling sedimentation such as tectonic activity, climate, sea level fluctuations, sediment supply, biological activity, etc.. Orders of 10^6 to 10^7 years will typically specify

[1] For every effect or output response exhibited by any rational system, there exists a definite set of causes or input stimuli that influenced and produced that effect (Sandquist, 1985).

temporal boundaries, while spatial boundaries will be of sedimentary basin scale.

After the system is defined with boundaries, one must identify the inputs and outputs of the system. This identification is not always easy. There may be multiple inputs to and outputs from a specific system. The greater the number of inputs and outputs in a system the greater the number of possible interactions occur between them. The exact relationships between system inputs and outputs are not always known for the geological system. This, in conjunction with not always knowing the time interval over which an input has acted (Gardner *et al.*, 1987; Milenkovic, 1989) complicates the study of the geological system.

2.4 Classifying the geological system

Classification of the geological system in terms of sedimentation is based upon the number of independent inputs and outputs associated with it. The geological system here is considered a multiple input-single output system. For the purpose of this study various geological processes comprise the multiple inputs and the resulting sedimentary sequence, the single output of the "geosystem". When trying to classify the complex geosystem, some simplifying assumptions are necessary in order to render our study manageable. We assume that the inputs to the geological system are independent. Also, we assume the geosystem is continuous in the sense that the system responds with continuously varying output to continuously varying inputs.

The main goal of system analysis as applied to the geosystem is to determine and evaluate the relationships between the inputs imposed on the system and the output produced; that is, the organizational relationship between the geological processes.

Chapter 3

Sedimentary Sequence Development

3.1 Factors affecting sedimentation

Interacting processes can be examined through the study of their physical relationships (Allison, 1981). This idea is adopted for the case of the geosystem. It is essential to consider all factors which contribute to sediment deposition. Absence of some parameters is a limiting factor in both stochastic and deterministic models of sediment prediction in sedimentary reservoirs as a function of time (Soare *et al.*, 1982). So many factors contribute to sedimentation that for reasons of manageability only those with high influence on sediment deposition will be considered in this study of the geosystem.

It is always necessary in basin sequence analysis to distinguish the *controlling factors* for the occurrence of transgressive or regressive sedimentary sequences in sedimentary basins. Most important of these are the rate of sediment influx and the rate and direction of sea-level change. (Watts, 1982).

The major controls of stratigraphic sequences has been widely debated. Sloss and Speed (1974) favoured tectonic movements as the prevailing control on stratigraphic sequences, introducing three episodes or modes during stratigraphic sequence development: "oscillatory", "emergent" and "submergent". They argued a synchronous tectonic control over broad sedimentary basins. Vail *et al.*, (1977), favour global sea level as the dominant factor controlling sedimentary sequences in continental interiors and margins of sedimentary basins. Donovan and Jones (1979) come to the same conclusion. However, Bally (1980) agrees that tectonic events correlated over wide regions and associated with major plate reorganizations could account as the major control on stratigraphic sequences. Sea-level changes in his model play only a secondary role. Construction of thermal and mechanical models of tectonic evolution of passive margins based upon the assumption of no sea-level change through time were used to estimate the contribution of tectonics to stratigraphic sequence development by noting the amount of modelled "onlap" (Watts, 1982). Watts mentions that coastal onlap is a characteristic feature of the tectonic evolution of a cooling passive margin and occuring sea-level change is not required to produce it. Alternately, studies by Pittman (1978) on transgressive sequences during the Eocene on margins of sedimentary basins in North America and Africa show that they have been deposited from the interaction of both changes in the sea-level and the steady subsidence of these margins.

The evolution of sedimentary basins depends upon basement movement, sediment accumulation and compaction, and variations of eustasy with time. The stratigraphy of the sediments that fill a basin is dependent on the source and depositional setting of the sediments, the role tectonism plays in controlling basin fill, and the influence of eustasy. The difficulty of separating these three influences has plagued sedimentary stratigraphic interpretation (Burton *et al.*, 1987). A genetic stratigraphic sequence is the product of the ongoing interplay among sediment supply, basin subsidence and uplift, and eustatic sea-level change (Galloway, 1989). Variations in the rates of these variables will yield a full range of depositional architectures.

It is now accepted that there is not one dominant control of sedimentary sequence deposition, but rather there exists an interplay of many processes. Stratigraphic sequences of deep water deposition and their facies patterns are primarily controlled by the rates and type of sedimentation, local crustal movements and eustatic sea-level. These three controls act together; that is, they "interplay". The facies anatomy of carbonate shelves and platforms reflects the combined effect of these three processess.

> *"Often it is difficult to separate the relative importance of these three primary controls whatever the sediments. This is because the same geometry and facies distribution may be the result of an infinite number of combinations of these controls"* (Kendal and Schlager, 1981).

They recognize a complex interplay of sea level change, crustal movement and reef growth (sediment supply). Thus, sedimentary sequences cannot be distinguished in terms of the precursors which led to their deposition. They exist "within" a ternary system but not as pure "end members" (Fig. 3.1).

Next we examine the relationships between the geological processes.

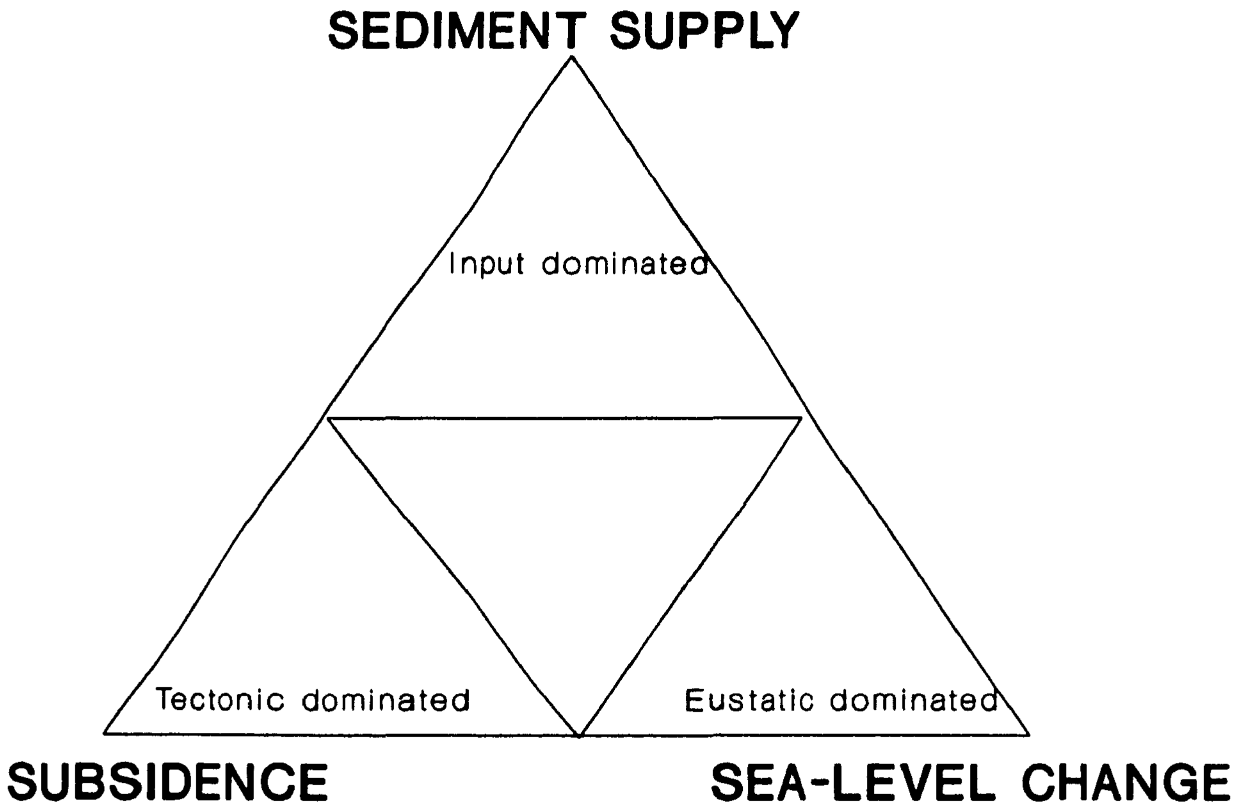

Figure 3.1: Ternary diagram with main factors influencing sedimentary sequence deposition. Most sequences are plotted within the inner triangle (Modified from Galloway, 1989).

3.2 The interaction of geological processes

Sea-level changes are one of the most important factors affecting sedimentation (Schlager, 1981). World-wide changes in relative sea-level have occurred repeatedly throughout geologic time, producing characteristic responses in carbonates (e.g. drowned reefs, rim differentiation, etc.). They are not constant with time. Their rate is about 1cm/1000 years. Relative sea-level changes are caused by many factors (Miall, 1984): endogenic, epigenic and exogenic processes. These processes act on the Earth's outer part and are responsible for the vertical movements of sedimentary blocks and the basic configuration of sedimentary basins.

Endogenic processes have their origin in the Earth's "heat engine" in the upper mantle. These processes are responsible for large surface displacements. Average vertical rates could reach as high as several metres per 1000 years (e.g. in orogenic areas).

Epigenic processes have their origin in the hydrological cycle and in atmospheric circulation and meteorological phenomena. These processes cause sea-level fluctuations, glaciations and erosion. The resultant processes act on different temporal scales and modify the geological setting of basins.

Chappell (1981) describes an interaction between endogenic and epigenic forces through upper mantle mass transfer and by altering crustal stress fields. However, these forces differ in rate and duration. Sea-level studies show that typical tectonic uplift rates are about 5 m per 1000 years and persist for 10^5 to 10^7 years. Epigenic glacio-isostatic displacement rates, can exceed 50 m per 1000 years with relaxation time constants of about 10^3 years. Thus, typical epigenic movements are faster than tectonic displacements but their durations are much shorter.

Exogenic processes are those with origin outside the Earth. Some of these are manifest in *"Milankovitch cycles"* (Berger, 1988). Barron *et al.*, (1985) mention that sedimenatry cycles in the stratigraphic record are attributed to specific orbital frequencies. They argue that they relate orbital variations to the climatic induced sedimentation of Cretaceous bedding sequences. However, sedimentation cycles can also be produced by mechanisms independent of orbitally modulated climatic change (Algeo and Williamson, 1987).

Except for climatic induced sedimentation, sea-level change also plays an important role in sedimentation. Relative rise in sea-level will produce three different responses in carbonate platforms: drowned reefs, rimmed reefs, and reefs with flat surfaces at sea-level. Falls of relative sea level are associated with karsts, deposition of subtidal evaporites in restricted basins, and clastics in open marine basins (Kendal and Schlager, 1981).

Interaction between geological processes is argued by Schlager (1981) in order to explain the characteristic sedimentation behaviour of *"drowned"* reefs. He states that it is more likely to have superposition of a steady process with pulses of another, rather than two processes creating simultaneously a strong pulse. This is demonstrated from sea-level and subsidence rate curves for the Tertiary (Fig. 3.2). Changes in the rate of subsidence (a steady process) change the likelihood of carbonate platform *"drowning"* by eustasy (a pulse).

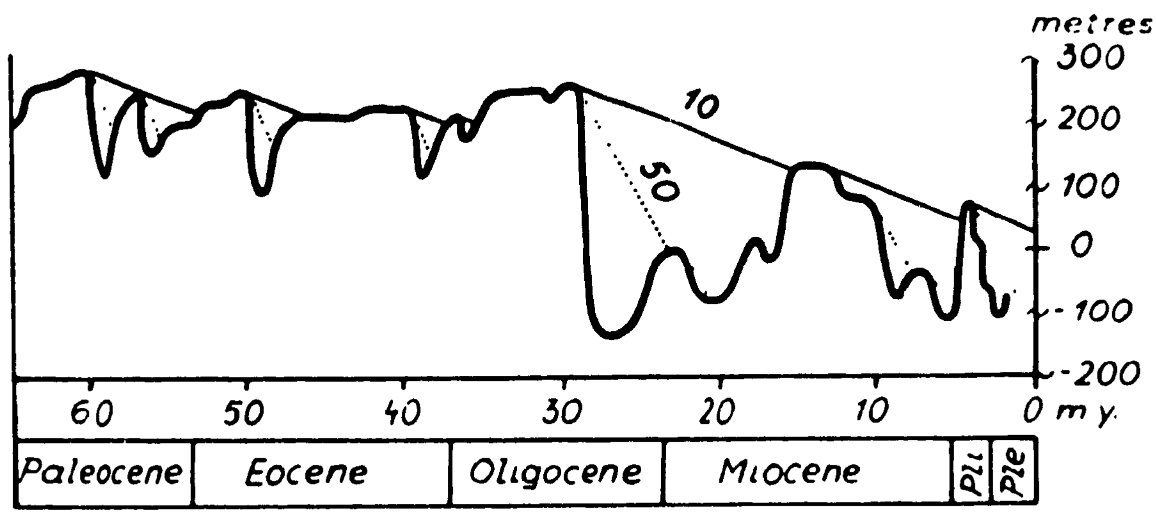

Figure 3.2: Sea level curve for the Tertiary. The superposition of two effects is demonstrated. The straight and dotted lines represent steady subsidence rates. At rates of 50 μ m/yr subsidence, sea-level rise in the Miocene has the potential to initiate platform "drowning" (After Schlager, 1981).

Pacific ocean sedimentation rates for the past 50 million years have been correlated with global sea-level fluctuations. Worsley and Davies (1979) show that high sedimentation rates coincide with low sea-level stands while high sea-levels allow high rates of biogenous precipitation on the shelves, thus starving the ocean basins of terrigenous sediments (Fig. 3.3). During low sea-level stands, detritus sediments bypass the continental shelves and reach the deep sea floor, increasing accumulation rates. However, they note that it is not possible to determine to what extent input of land-derived material to the deep sea basins is a function of the land/ocean ratio. This is due primarily to the existence of other processes such as climate which is especially important in basin sedimentation.

Lisitsyn *et al.*, (1982) have determined the distribution of terrigenous and biogenic pelagic sediments in the world's oceans (Table 3.1).

Their data set covers 90 % of the ocean floors. In all oceans, biogenic accumulations (i.e. carbonate and siliceous) are slightly lower than terrigenous ones. However, study of 334 sites, worldwide, in the North, Central Atlantic and Pacific oceans have demonstrated an increase in carbonate accumulation rates during the Tertiary (Davies *et al.*, 1977). On the contrary, sites from the Indian ocean show variable results; they offer no explanation. According to Worsley and Davies (1979) a complete sea-level cycle (i.e. fall-rise-fall) causes also a cycle in pelagic sedimentation rates. However, Donnelly (1982) doesn't see this varying effect in his data suggesting that sea-level change does not affect pelagic sedimentation. He contradicts Worsley and Davies' work by attributing the discrepancy to cli-

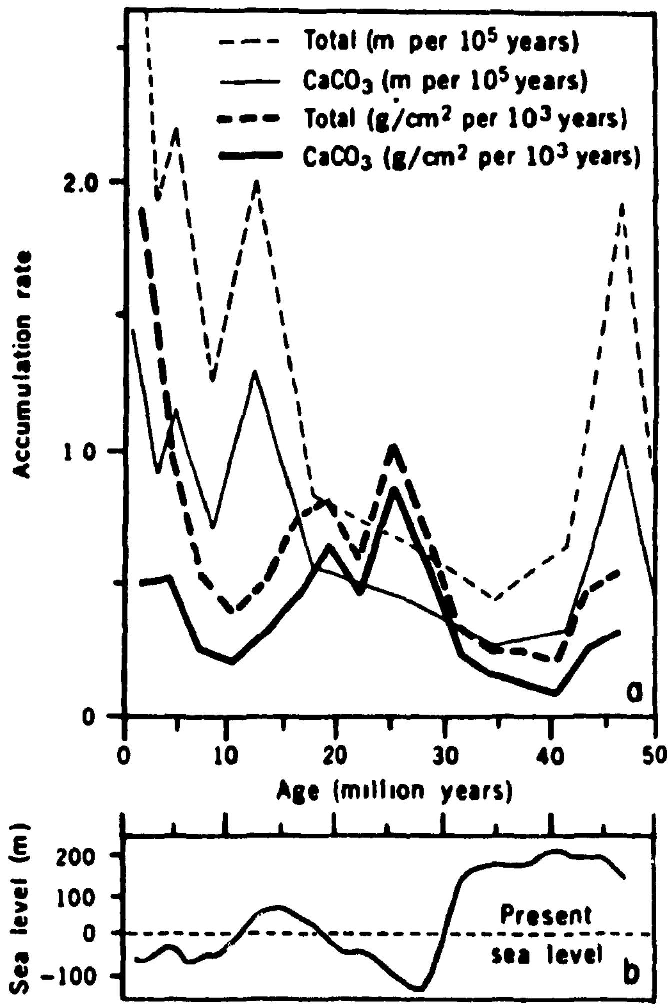

Figure 3.3: Sedimentation rates and eustatic sea-level changes from the Pacific (After Worsley and Davies, 1979).

Table 3.1
World-wide oceanic sedimentation distribution %
Pelagic sediments

	Atlantic	Pacific	Indian	Total
Terrigenous	52.2	67.9	50.9	58.0
Biogenous	47.8	32.1	49.1	42.0
Carbonate	44.1	26.5	38.7	36.2
Siliceous	3.7	5.6	10.4	5.8
Total	100.0	100.0	100.0	100.0

(From Lisitsyn *et al.*, 1982).

matic variability. This shows once more, what substantial impact climate has on sea-level and consequently on sedimentation rates.

There have been wide fluctuations in the rate of sediment deposition in the oceans with time. Sedimentation rates in deep basins are higher today than during Cretaceous times (100 Ma ago). Increase in sedimentation rate during the past 5 Ma represents a net unloading of the continents and a deposition onto the sea floor. If no other factors are contributing to sedimentation, this increase is equivalent to a sediment layer of 40 m average thickness worldwide. Harrison *et al.* (1981) took account of the isostatic adjustment of the ocean floor in response to the sediment load, and computed an effective rise in sea-level of 7.3 m. However, in the past 10,000 years, sea-level rise has been so rapid that sedimentation has not kept up (Pittman, 1978).

The processes which affect sedimentation in deep oceans are different from those of sedimentary basins. Basin sedimentation, in general, displays a larger variability of sediment type than in oceans. Sedimentation in oceans are mainly comprised of fine-grained material such as clays, calcareous and siliceous oozes, and also volcanics from submarine volcanoes. Basin sediments on land and in near-bounded continental margins show a wide range of depositional environments. Geological processes of sea-level change, glaciation, tectonism and sediment supply have a different affect on sedimentary basin deposition as opposed to deep ocean deposition. A sea-level change directly effects the character of sediment deposition. For example, a large sea-level drop could produce an unconformity in the rock record of a sedimentary basin, thus affecting the acoustic impedance response. On the contrary, in ocean basins, due to the larger volume of water, the same sea-level chnage does not have such an effect.

Except for sea-level change, glaciers also affect sediment deposition. Glaciers are responsible for huge transport and deposition of sediments on land and in near near-shore environments creating supraglacial sequences in sedimentary basins (Edwards, 1975). The availability of reworked material from erosion on land, is a large source of sediment supply to sedimentary basins. Ocean basins, however, receive only a very small portion of this eroded material, mainly by transport through submarine canyons.

These are just a few examples which illustrate the relative importance of the geological processes on sedimentation in basins and in ocean floors. Sediment type and deposition, in sedimentary basins compared to oceanic, varies according to the different structural types of sedimenatry basins. Also, the various environmental settings near the continental margin, account for more variable sediments as opposed to ocean floor sediments, which are fine-grained and "monotonous" (Reineck and Singh, 1980).

The geological processes are here viewed as general processes in the sense that they occur worldwide. This does not assume that their affect on sedimentation is the same everywhere. These processes are more pronounced in sedimentary basins. The description of oceanic sedimentation has been dealt in order to give broader information on geological process-activity in other regimes.

In summary, we have described the relative magnitudes of influences of the processes on sedimentation as well as their interrelationships. These processes are sea-level fluctuations, sediment supply, tectonic activity, and climate. Sea-level fluctuations are interrelated with other geological processes in a complex manner. Sea-level changes cannot be directly related to sedimentation rates. Other processes come into play (i.e. climate) constituting the relationship complex. The link between climate and sea-level change, and climate and sediment accumulation is not well defined.

The study of geological process behaviour through *qualitative stability analysis* comes from the necessity of the indirect and complex manner by which geological processes relate.

Chapter 4

Qualitative Stability Analysis

4.1 Partially specified systems

The geosystem is comprised of many subprocesses related in a complex manner. In principle, their interrelationships can be mathematically represented as non-linear differential equations. This, now, mathematical system model comprises a large number of parameters, inputs and outputs, an elaborate set of initial and boundary conditions all of which relate through complex functional inter-dependencies. While possible, in principle, quantitative solutions are intractable (Slingerland, 1981).

The mathematical simplification which includes that which is basic to the geosystem is described as a ***partially specified system.*** Such systems form a class for which *"the types of feedback among variables (positive or negative) are known but the exact functional relationships are not"* (Slingerland, 1981).

Qualitative stability analysis - a technique used for determining the response of partially specified systems - will be used here for the description of the sedimentary system (i.e. the gesosystem). The results of this analysis will judge the system's response to changes, and determine whether or not it is stable or unstable.

Qualitative stability analysis has successively been applied to individual sedimentary basins (Slingerland, 1981; Philips and Steila, 1984; Philips, 1985, 1987). Here, this methodology is used with broader spatial and temporal scales. It does not try to explain the sedimentation character of individual basins; instead, we describe a geosystem which is appropiate to the study of fundamental characteristics of "*any*" particular sedimentary basin "*anywhere*" in the world. Among the numerous state-variables which model the implicated geological processes, only those that have the largest influence in sedimentary sequence deposition will be considered. These are: sediment supply, tectonic activity, sea-level fluctuation and glaciation.

Stability is defined as the ability of the geosystem to recover from a disturbance or change of state. A stable system will return to a previous state. An unstable system will tend to evolve to a different state (mode) after a disturbance or perturbation. In the case of the geosystem, a different state will represent a new set of conditions responsible for the deposition of new lithologies. The change of lithology reflects geophysically, changes in seismic acoustic impedance.

4.2 Theory

Qualitative stability analysis was first applied in population ecology (May, 1973; Levins, 1974), and has been more recently introduced into the earth sciences by Slingerland in 1981. If we consider a geological system of n variables, $\vec{X}$, whose levels vary over time as functions $\vec{F}$ of each other, we have:

$$\frac{d\vec{X}}{dt} = \vec{F}(\vec{X}). \tag{4.1}$$

If $\vec{C}$ is an equilibrium point for the system, a deviation $\vec{x}$ of $\vec{X}$ from equilibrium is given by

$$\vec{x} = \vec{X} - \vec{C}. \tag{4.2}$$

From equation (4.1) we obtain

$$\frac{d\vec{X}}{dt} = \vec{F}(\vec{x} + \vec{C}). \tag{4.3}$$

A Taylor expansion of the right side of equation (4.3) gives

$$\frac{d\vec{X}}{dt} = \vec{F}(\vec{C}) + \mathbf{A}\vec{x} + \vec{g}(\vec{x}) \tag{4.4}$$

where $\vec{g}(\vec{x})$ is a vector of polynomials with terms of two or higher order, each small compared to $\vec{x}$. They vanish at $\vec{x} = 0$. $\mathbf{A}$ is an n x n interaction matrix whose elements are of the form $\partial\, F_i(\vec{C})\ /\ \partial X_j$.

$$\mathbf{A} = \begin{pmatrix} \frac{\partial \mathrm{F}_1\,(\vec{\mathrm{C}})}{\partial \mathrm{X}_1} & \cdots & \frac{\partial F_1\,(\vec{C})}{\partial X_n} \\ \vdots & & \vdots \\ \frac{\partial F_n(\vec{C})}{\partial X_1} & \cdots & \frac{\partial F_n(\vec{C})}{\partial X_n} \end{pmatrix}$$

The matrix entries reflect interactions among the geological processes (geo-variables). When $\vec{x}$ is very small, $\vec{g}$ is also very small with respect to $\mathbf{A}\vec{x}$, and $\vec{F}(\vec{C}) = 0$ (equilibrium). Thus, from equation (4.4) we obtain the linearized system

$$\frac{d\vec{X}}{dt} = \mathbf{A}\vec{x} \tag{4.5}$$

Solutions of equation (4.5) have the form

$$\vec{x}(t) = \vec{v}\,(\vec{C})\; exp\,(\lambda t), \tag{4.6}$$

where $\vec{v}$ are eigenvectors of $\vec{A}$ corresponding to eigenvalues λ.

The system's response to small changes depends on the eigenvalues. The stability of equation (4.5) is determined by whether or not the real parts of the eigenvalues, λ of $\mathbf{A}$ are each greater or less than zero. To analyse the behaviour of the system, we investigate the roots (i.e. the eigenvalues) of the characteristic equation of the matrix $\mathbf{A}$.

If all eigenvalues are negative, the system is stable. This can be seen from equation (4.6); if all λ are < 0 then all $\vec{x}$ approach zero as t approaches infinity. Then from equation (4.2), all $\vec{X}$ re-approach $\vec{C}$. Thus, the system re-approaches the stable equilibrium condition $\vec{C}$. If any (at least one) eigenvalue of $\mathbf{A}$ has a positive real part, the system is unstable (Braun, 1983, p.354) and the values of the variables increasingly deviate from their equilibrium values with time.

The signs of the eigenvalues can be determined indirectly without having to actually solve the characteristic equation. This is accomplished using the Routh-Hurwitz criterion (May, 1973). The signs of λ are determined by the a_{ij} components of the system matrix. The criterion states that the roots of a real characteristic equation have negative parts if and only if all the coefficients a_{ij} of the equation are positive (Slingerland, 1981).

Applying qualitative stability analysis to the geosystem involves determining the cause-effect relationship (a_{ij}) between the geological processes. Each a_{ij} represents a particular interaction within the geosystem and thus evaluation of the geosystem's stability is accomplished in terms of the relationships between the geological processes.

a_{ij} represents an interaction between two variables. The first subscript i denotes the end point of the link and the second, j, the point of origin. Therefore, a_{ij}, signifies a link to i from j. The sign of a_{ij} depends on whether or not the relationship is positive or negative, as it reflects the effect on i from j.

It is noted that not all interactions between the geological processes need be considered for qualitative stability analysis. The geological processes which play the role of geo-variables were chosen on the basis of their importance to sedimentation, and their generality of participation to geological environments. These are sediment supply, sea-level change, tectonic uplift and glaciation, and comprise the components of the geoprocess model.

4.3 The geoprocess model and geosystem interaction matrix

Sediment supply in sedimentary basins is a continuous process though varying with time. The most important types of sediments are clastics (mainly sandstone and shale) and carbonates (mainly limestone and dolomite). Evaporites, volcanics and organic-carbon-rich sediments are also significant. Sediment supply varies depending on the depositional environment. For example, major rivers provide a larger and faster sediment supply than does pelagic settling (Stow, 1985). Stratigraphic analysis of the Western Canada Basin has shown that during the

Jurassic and Cretaceous, sedimentation varied from 3 m to 300 m/1000 years (Stott, 1984).

Eustatic sea-level change can be either glacial, tectonic or geoidal. Glacial eustatic changes result from the changing volume of ice caps, and have rates of up to 10 m/1000 years (Pitman, 1978). Pitman and Golovchenco (1983) argue that a sea-level rise of about 60 m would result, if all present glaciers and ice caps would melt (-a_{23}).[1] A sea-level rise would also tend to melt glaciers. Ruddiman and Wright (1987) state that rising sea-level increases calving of icebergs (-a_{32}). Eustatic changes due to tectonism are mainly due to large-scale lithospheric plate interactions. In the case of small sedimentary basins, Jordan (1982) documents that tectonic uplift in the Cretaceous western interior of North America has caused seaward advance of the shoreline ($-a_{21}$).

Worsley and Davies (1979) argue that high sea-levels produce a decrease in the land/sea ratio, and hence a slower chemical erosion of the continents, thus providing less sediment supply (-a_{42}). Also, Stow (1986) states that during periods of high sea-level, sediment sources such as rivers do not have direct access to the basin slopes of continental margins, hence decreasing sediment supply (-a_{42}).

Glaciers are a large mass of ice formed by the compaction and recrystallization of snow, creeping downslope or outward due to the stress of its own weight. About 10 % of the earth's continental surface is now covered by glacial ice. In the Quaternary, the coverage was 30 % (Edwards, 1986). Glaciers act as erosional agents and thus supply glacial sediments (a_{43}). Glacial erosion takes place by

[1]This and each subsequent bracketed coefficient indicates the corresponding interaction in the "geoprocess" model (Figure 4.1).

"plucking" and *abrasion*. In the first case, blocks of rock are loosened, detached from the bedrock, by the movement of the glacier. The process of abrasion acts as a mechanical wearing or grinding of rock surfaces on valley walls by friction from the moving glacier. This process mainly produces fine-grained glacial sediments: silt and clay. Glacial deposits are observed in both marine and terrestrial settings and in a variety of tectonic and climatic situations. Pressure exerted from glaciers cause tectonic loading and lowering of the crust ($-a_{13}$). This effect of glaciation is well demonstrated in the classical Scandinavian example, where glacio-isostatic rebound adjustment has been occuring since the end of the last glacial period. The Gulf of Bothnia, located between Swedan and Finland, must rise a further 200 m to achieve isostatic equilibrium (Heiskanen and Vening Meinesz, 1958). Appart from effects of glaciers, sediment supply could also initiate tectonic activity. Sediment loading caused by sediment supply in basins could act as a force which contributes to lithospheric subsidence (Turcotte *et al.*, 1977; Cloetingh *et al.*, 1984), (a movement opposite to tectonic uplift) ($-a_{14}$).

The interacting nature of the geological processes is simplified above and represents common sedimentary environments, excluding unique situations such as desert environments, and localized settings such as attols. From the above described interrelationships, we construct a signed, directed graph of the geological variables of the geosystem (Fig. 4.1), and then translate this graph into a geosystem interaction matrix (Table 4.2). Each link between the geological processes in Fig. 4.1, corresponds to the matrix elements a_{ij}.

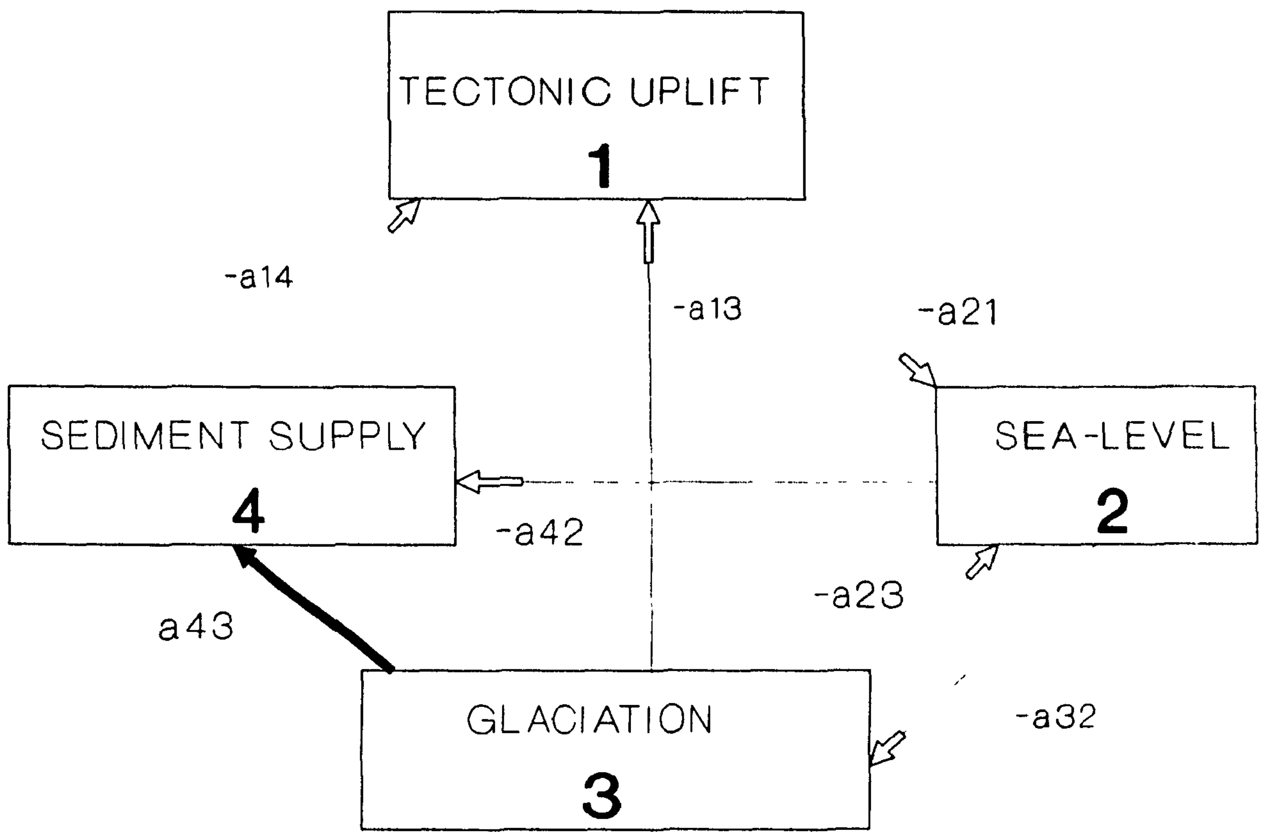

Figure 4.1: Interaction "geo-process" model. Bold arrow represents enhancing effect and dotted arrows signify an opposite effect. Each coefficient a_{ij} is positive; negative relationships are indicated by sign.

Table 4.2
Geosystem interaction matrix

	tectonic uplift	sea-level	glaciation	sediment supply
tectonic uplift	0	0	-a_{13}	-a_{14}
sea-level	-a_{21}	0	-a_{23}	0
glaciation	0	-a_{32}	0	0
sediment supply	0	-a_{42}	a_{43}	0

Rewriting Table 4.2 in the form of a matrix we have:

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & a_{13} & a_{14} \\ -a_{21} & 0 & -a_{23} & 0 \\ 0 & a_{32} & 0 & 0 \\ 0 & -a_{42} & a_{43} & 0 \end{pmatrix},$$

where each a_{ij} represents a particular interaction within the geosystem.

4.4 Results

The characteristic equation of the interaction matrix **A** is calculated (Appendix B) and is:

$$\lambda^4 - (a_{23}a_{32})\,\lambda^2 + (a_{13}a_{32}a_{21} + a_{21}a_{14}a_{42})\,\lambda + det\ \mathbf{A} = 0 \qquad (4.7)$$

The outcome of the analysis is dependent on the sign of the coefficients of the characteristic equation. Examination of equation (4.7) reveals that the coefficient of the second term, is generally, negative (refer to p.34). Thus, since all the coefficients of the equation are not all positive, according to the Routh-Hurwitz criterion, the roots of the equation do not all have negative parts. So, at least one eigenvalue of **A** has a positive real part, and thus, the system is unstable.

Qualitative stability analysis of the partially specified geosystem has shown that it is normally unstable. The sedimentation sequence records the evolution of the unstable geosystem for millions of years. The geosystem has many quasiequilibrium states which persist for long periods of time. These periods coincide with geological conditions of formation lithologies. Changes in lithologies in the sedimentary record are the result of the evolving geosystem. The evolution of the geosystem is a manifestation of a chaotic attractor.

Chapter 5

Chaos Theory

5.1 Deterministic chaos

The study of dynamical systems has been, to a large extent, the origin of chaos theory. Dynamical systems are most completely modelled by nonlinear systems of equations. Nonlinear phenomena often exhibit sensitive behaviour. Small changes in initial values can cause very large changes at later time, and thus, the behaviour of systems is unpredictable. A dynamical system may be described in terms of its: i) state, which is the essential information about the system, and ii) a rule that describes how its state evolves with time. For example, in the case of the pendulum, where velocity and position are all that are required to determine its motion, the state is a point in a plane, whose coordinates are position and velocity. A mathematical differential equation, is the rule that describes how the state evolves (Crutchfield *et al.*, 1986).

A visualized representation of an evolving dynamical system can be illustrated in phase- or state-space. A phase-space is an abstract space whose coordinates are components of the state. The phase-space gives the ability to represent behaviour in geometric form.

A point in the state–space specifies the state of a system at a given time. An orbit through that space specifies the motion of a particular system as time passes. A rough definition of an "attractor" is what state of the system "settles down to", or is "attracted to". "Basins" are the set of points that evolve towards an attractor. Systems may cycle periodically through a sequence of states. A state-space diagram in this case corresponds to a cycle, or periodic orbit. These attractors are called limit cycles. An even more complicated form of an attractor is a torus of N-1 dimensions, where N is the number of degrees of freedom in the system. For a system with three degrees of freedom, the orbits are made up of two independent quasi-periodic oscillations.

All the attractors described above are, in principle, predictable; the evolution of systems towards these attractors is non-chaotic.

Traditionally it has been accepted that simple systems with a few degrees of freedom (or equivalently those sufficiently described by few parameters) are associated with "order" and complex systems with "chaos". Campbell and Rose (1983) provide three fundamental reasons why this association is not totally correct. They state that: i) complex systems (with many degrees of freedom) can undergo very orderly motion (e.g. a fluid in a laminar flow moves in a totally predictable manner), ii) simple physical systems can exhibit chaotic behaviour (a rigid plane pendulum could become chaotic), and iii) chaos which has been observed in very complicated systems can be understood quantitatively using simple physical models which have only a few degrees of freedom (Swinney, 1983; Huang and Turcotte, 1990).

Chaos can arise from the behaviour of systems which are governed by simple sets of deterministic equations (Ruelle, 1989). Simple nonlinear interaction of only a few components within a system can lead to chaotic behaviour (Crutchfield *et al.*, 1986). A chaotic system is deterministic and its evolution with time is characterized by sensitive dependence to initial conditions (Ford, 1989). Small changes in initial values can cause very large changes at later time, and thus, the behaviour of systems is unpredictable. "*...chaos ensures that the uncertainties will quickly overwhelm the ability to make predictions* (Crutchfield *et al*, 1986). Ford (1983) states that *"Almost all dynamical systems are now known to exhibit chaotic orbits"*.

The evolution of the geosystem can be represented in a state-space. However, the behaviour of the geosystem is manifestly chaotic and, consequently, its evolution cannot be described in terms of predictable attractors. The complex nature of its evolution requires the more elaborate *"chaotic"* or *"strange"* attractor. Chaotic attractors have a complicated geometric form.

5.2 The "strange" attractor

Fluctuations of one variable of a system can easily be displayed by a time series (Fig. 5.1). However, to show the changing relationships for example of more than one variable, variables, the evolving system is more adequately represented in phase-space. At any given moment, the variables fix the location of a point in space. As the system changes state, the locus of this point moves. The system

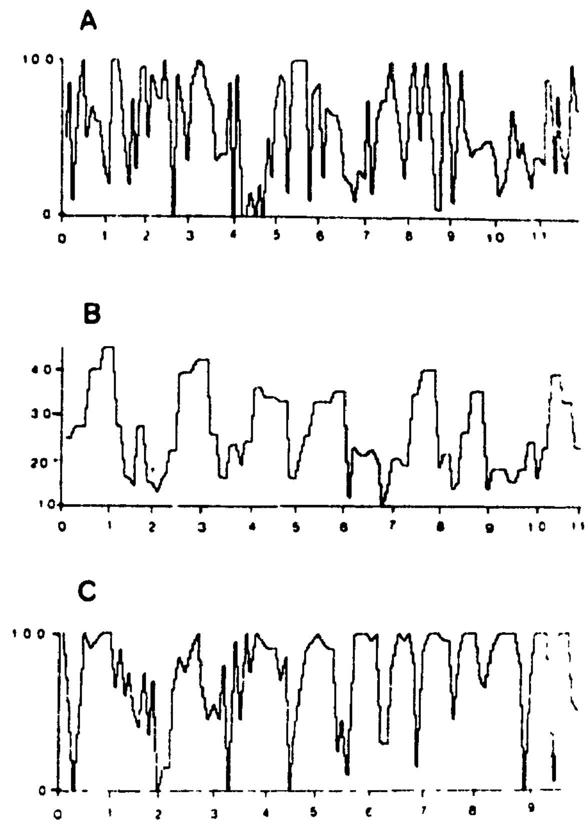

Figure 5.1: Examples of stratigraphic time series. The vertical scale in (a) and (c) represent limestone percentages, while in (b) it shows thickness of limestone beds. The horizontal scale in all examples represent the stratigraphic position in meters (After Schwarzacher, 1985).

may never repeat itself exactly (i.e. the trajectories never intersect with each other). Chaotic or strange attractors are unpredictable as well as non-repeating. A dynamical system may evolve towards more than one attractor. Different initial conditions may select from among different attractors.

The description of Edward N. Lorenz' motions of a fluid flow (his description of a turbulent cell in the atmosphere) with only 3 degrees of freedom behaved in a random manner which could not be characterized by the predictable attractors then known. His attractor was an example of a chaotic (strange) attractor (Fig. 5.2). The Lorenz system is a simple three-dimensional system with a strange attractor. Lorenz found that microscopic perturbations are amplified and that two orbits with nearby initial conditions diverge geometrically. This observation is a basic condition (property) found for all strange attractors. *Chaotic systems are extremely sensitive to initial conditions* (Fig. 5.3).

Strange or chaotic attractors, represented by the systems of lines in phase-space, show fractal geometry; an infinite number of points on the attractor show self-similar detail on various spatial scales (Middleton, 1990).

5.3 Bifurcations

Study of the equilibrium points and their stability of nonlinear systems can reveal information about their behaviour. The number and stability of equilibrium points of a set of differential equations could change, when the coefficients of the equation change (Middleton, 1990). Such changes could cause abrupt changes in the behaviour of the system; these are called *bifurcations*.

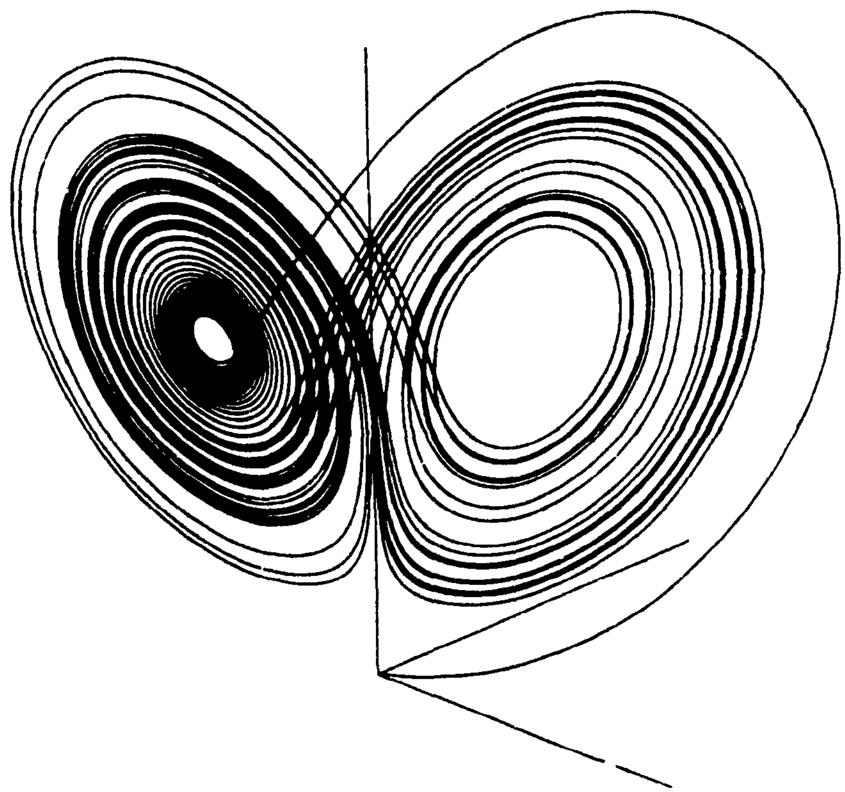

Figure 5.2: The Lorenz attractor. Evolution of three variables in phase space are presented by a chaotic attractor. A point on the attractor signifies the variables in three-dimensional space at that given time. (After Moon, 1987).

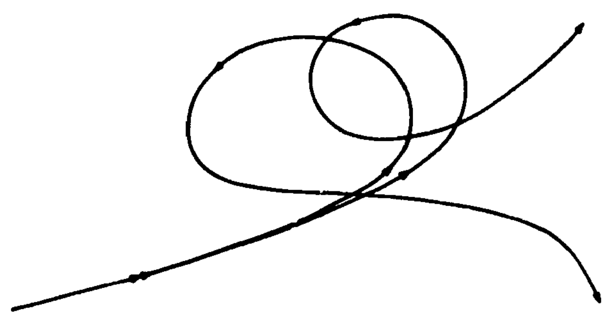

Figure 5.3: Sensitivity to initial conditions. On a strange attractor, two neighbouring phase trajectories always diverge, regardless of their initial proximity. The trajectory actually followed by the system depends crucially on its starting point. (After Bergé *et al.*, 1986).

At a bifurcation point a system "switches" from one stable operating mode to another (Nicolis, 1989). Bifurcations cause a system to either: i) fragment itself (period-doubling), or ii) stabilize to a new behaviour through feedback loops that couple the new change to its environment. A system that has passed through a bifurcation point and has stabilized by its feedback, will remain in the new state until its next bifurcation. Beyond some threshold value which depends on the particular system, the state of the system can branch out to different paths. The path which the system will follow is not predictable: *"Only chance will decide, through the dynamics of the fluctuations"* (Nicolis, 1989, p.334). Once the path is chosen, the system will evolve towards a new stable operating state. In chaotic systems, the internal dynamics (i.e. the interactions among the state-variables) drive the branchings to new states.

In summary, the basic properties of chaos are: sensitive dependence on initial conditions, abrupt shifts in stable operating states (modes) through bifurcation points, and long-term unpredictability. We justify the presence of the › properties and show their inherence in the sedimentation system.

5.4 A chaotic geosystem

We now show that stratigraphic sequence development in sedimentary basins is consistent with chaos. Predictable attractors described by Thornes (1983) for various geological systems (Fig. 5.4) are inadequate to represent the complex nature of the geosystem. A chaotic geosystem would have to show the definitive properties of sensitivity to initial conditions, abrupt shifts in operating mode,

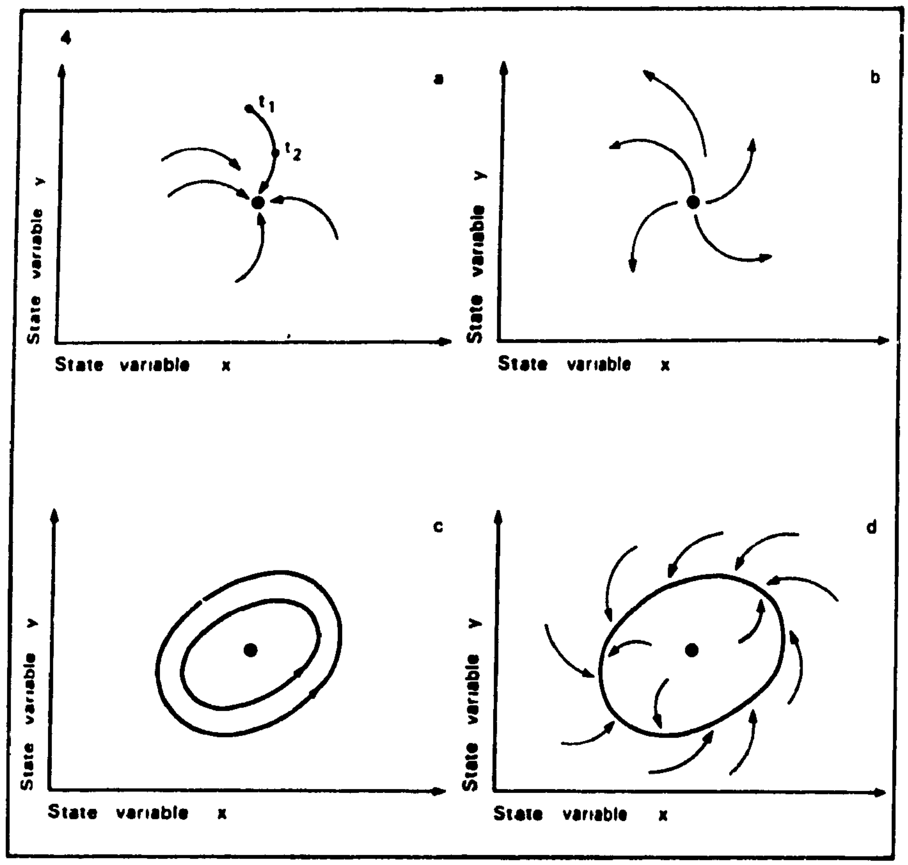

Figure 5.4: Common attractors in geological systems. The attractors correspond to: (a),(b) gully growth, alluvial fans and caliche development (c) meanders and (d) pool and riffle sequences, beach cusps. (After Thornes, 1983).

and unpredictability.

Climate is an important process in large-scale sedimentation. Mathematical modelling of climate shows that only a small alteration of parameters is required to cause a sudden potential for glaciation over large areas of the Earth's surface (North and Crowley, 1985). They state that change of atmospheric carbon dioxide over geological time, can induce radical changes in continental ice cover. Also, small changes in the mean climate result in dramatic changes in the frequency of extreme events such as temperature and meteoric precipitation (di Castri and Malcolm, 1988). When some stable climatic variables fall below some critical threshold value, we have the appearance of extreme "nonlinear" climatic events (Wigley, 1985). Abrupt climatic change in geological records from the Quaternary are documented (Woillard and Mook, 1982; Broecker *et al.*, 1985), which show evidence for rapid climatic "swings" in the interstadial preceding the last glacial maximum.

This sensitive climatic behaviour affects sedimentary processes of the Earth. Changes in deep water circulation alters heat transport and effects carbon storage and oxygen levels in waters and in the atmosphere (Crowley and North, 1988). Thus, abrupt shifts in climate have an effect on changing carbonate sedimentation resulting in creation of different lithotypes. CO_2 measurements from ice cores have shown that the Earth has two modes of ocean-atmosphere operation on a timescale of 10^5 years (Broecker *et al.*, 1985). Oscillations observed in ice core data involve a "jump" from one stable operating mode to another. One interpretation of this phenomenon, proposed by Oeschger *et al.*, (1984), is that the sediment production rate *"kick"* the ocean-atmosphere system from one quasi-stable mode to another. Broecker *et al.*, (1985) find this idea tempting, however they argue that it is difficult to hold.

Abrupt shifts of the climatic subsystem which affects sedimentation are also indicated by worldwide sedimentation rates which are not equal and constant with time (Davies *et al.*, 1977) (Fig. 5.5). During Oligocene and Paleocene times, sediment rates were low, while in the Miocene and Eocene rates were high. Different modes of weathering during Paleocene-early Eocene and late Eocene-early Miocene have been suggested for this discrimination. This implies that there was climatic variability during those periods in which oscillations of the climate proceeded through different states or modes of stability. Long term climatic change has been suggested to exhibit deterministic, unstable dynamics of the chaotic attractor (Nicolis, C., and Nicolis, G., 1984). These changes are reflected as sedimentation variability in sedimentary basins; they do not exhibit periodicity. However, some periodicities are still observed in the sedimentation record. Lutz (1987) claims that there is no statistical motivation for favouring periodicity to explain various geological events. He refers to the time series distribution of different geological events as "noisy periodicity models". In addition, Shaw (1987) stresses that causitive periodic forcing is not required to develop apparent periodic cycles in the sedimentation record. He offers an explanation suggesting that the observed periodicities can arise from *"nonlinear coupling of interacting resonances"*. Here we use use the term *"nonlinear chaotic behaviour"* in describing such phenomena. Baksi (1990) more rigorously states that the search for periodicities in the rock record have, so far, been subjective and that such studies

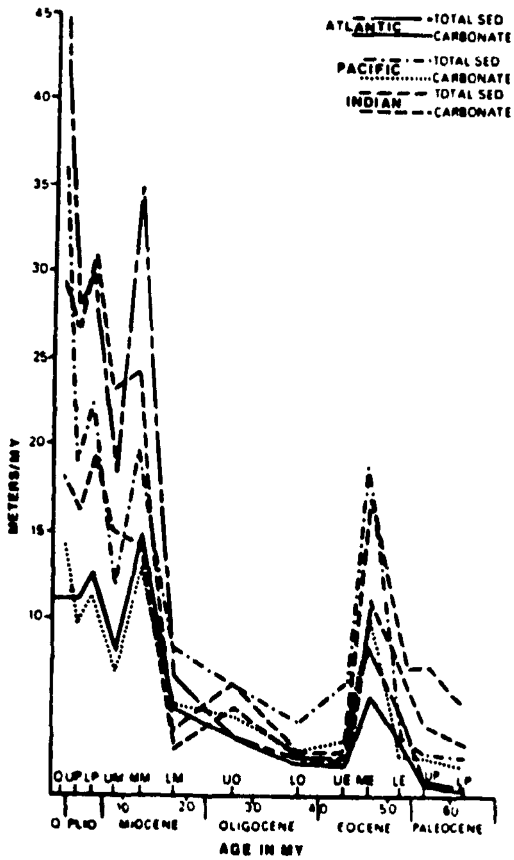

Figure 5.5: Total rates of sediment accumulation. (After Davies *et al.*, 1977).

should be postponed until high-quality radiometric age data are available for the Phanerozoic.

Other geological/geomorphological phenomena are known to be vulnerable to climatic variation. The geometrical structure of individual bends in river meanders show great sensitivity to earlier meander-train geometry and climatic variability. Patterns of North American river channels show adjustment (structural and sedimentological) to previous climatic conditions in the Pleistocene (Montgomery, 1983). In attempting to describe the river meandering process, Furbish (1988) states that the *"sensitive dependence, and the property of recurrent self-organized structure, are the hallmarks of nonlinear chaotic behavior"*.

The abrupt shifts from one mode (stable state) to another arises through bifurcation. Bifurcations, in the case of the geosystem, are represented as internal thresholds. Transition between two states is particularly complex in geological systems due to multiple feedback relationships among the interacting processes (Chappell, 1983). Representation of processes in a magnitude–frequency distribution is adequate in presenting the effect of threshold on operating domains (Fig. 5.6). When process rate $g(x)$ increases with magnitude of event x, the product of the process function (sedimentary bed) is described by a smooth curve with a single peak (R) in the magnitude-frequency distribution. There is no crossing of an *"internal threshold"*. In the case of an existing threshold, the process-resultant curve has two modes of operation (R_1 and R_2). In crossing this internal threshold, the system arrives at a new state.

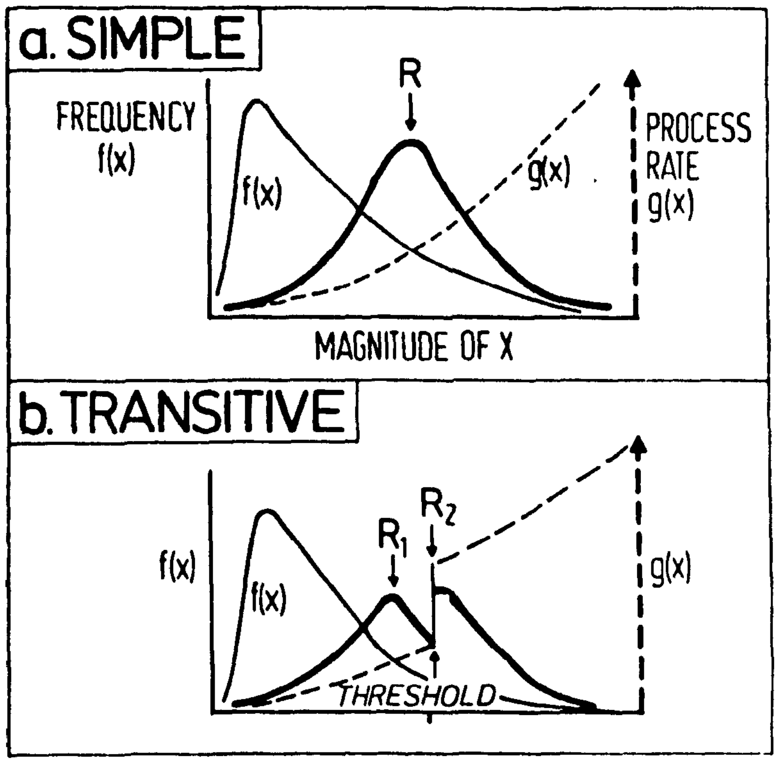

Figure 5.6: Magnitude-frequency distribution of processes: (a) the process resultant curve (sedimentary bed in the case of the geosystem) is denoted with R, (b) in the case of internal threshold the process curve has two modes of operation R_1 and R_2. (After Chappell, 1983).

A geological environment which has adjusted to the environmental conditions is in a state of equilibrium (a stable state). Such stable states could still be punctuated by *"episodic"* events which will abruptly shift the environmental equilibria to other stable states. These abrupt changes or *"punctuated events"*, are more frequent than we have previously realized (Phillips, 1986). Geologists have referred to such events as *"punctuated aggradational cycles"* (PACs) in explaining episodic stratigraphic accumulations (Goodwin and Anderson, 1985). These episodes or *"crises"*, which superpose on the normal sedimentation trends, originate within the geosystem itself (Berger *et al.*, 1984). *"Abrupt changes are inherent in normal geological processes"* (Parker, 1985). Oscillations or disturbances caused by changes in the endogenetic system control the sediment output in time and space (Montgomery, 1983).

We view such a behaviour of the sedimentary system as being indicative of chaos. Sedimentation exhibits long periods of relative stability separated by abrupt shifts. During times of dynamic equilibrium, the geosystem is relatively stable. Under that quasi-stable state, geological conditions persist which are responsible for the deposition of a particular lithology. Due to the instability of the sedimentation system, it shifts to other operating states. Under these new geological, temporarily stable conditions, a new lithology deposits. The geosystem is *"attracted"* towards a stationary state. Following some unpredictable interval of time, the system evolves through the next bifurcation point, to another new state. The duration the system spends in each state determines the thickness of the formation lithologies.

These successive, abrupt shifts in the operating mode of the sedimentation system, and its known sensitive dependence on initial conditions- initial conditions of climate cause sensitive sedimentological and structural effects (Montgomerey, 1983; see also Furbish, 1988)–cause an accumulation of a series of different lithologies which form the sedimentary sequence. Thus, sedimentological processes demonstrate qualities of chaotic behaviour. Lithological changes (and equivalently, their geophysical measures; i.e. acoustic impedance) may be represented in phase-space by a strange attractor. The chaotic attractor represents evolution of the sedimentary system through different lithological states. A sedimentary sequence describes a high order strange attractor (Fig. 5.7). Thus, the strange attractor acts as the common underlying mechanism of variation of acoustic impedance.

The relationship between $1/f$ spectra and strange attractor solutions have been addressed for electronic systems (Shaw, 1980). He provides arguments in support of the idea that strange attractors or ensembles of strange attractors commonly show the $1/f$ power spectrum of scaling flicker noise. In addition, he states that any chaotic mechanism which would purport to explain the ubiquitous occurence of $1/f$ noise must itself occur ubiquitously. Mandelbrot (1983) notes that *"Many scaling noises have remarkable implications in their fields and their ubiquitous nature is a remarkable generic fact."* Mandelbrot and Voss (1983) also directly state, that *"...$1/f$ noise may be related to turbulence or chaotic behaviour and nonlinearities"*.

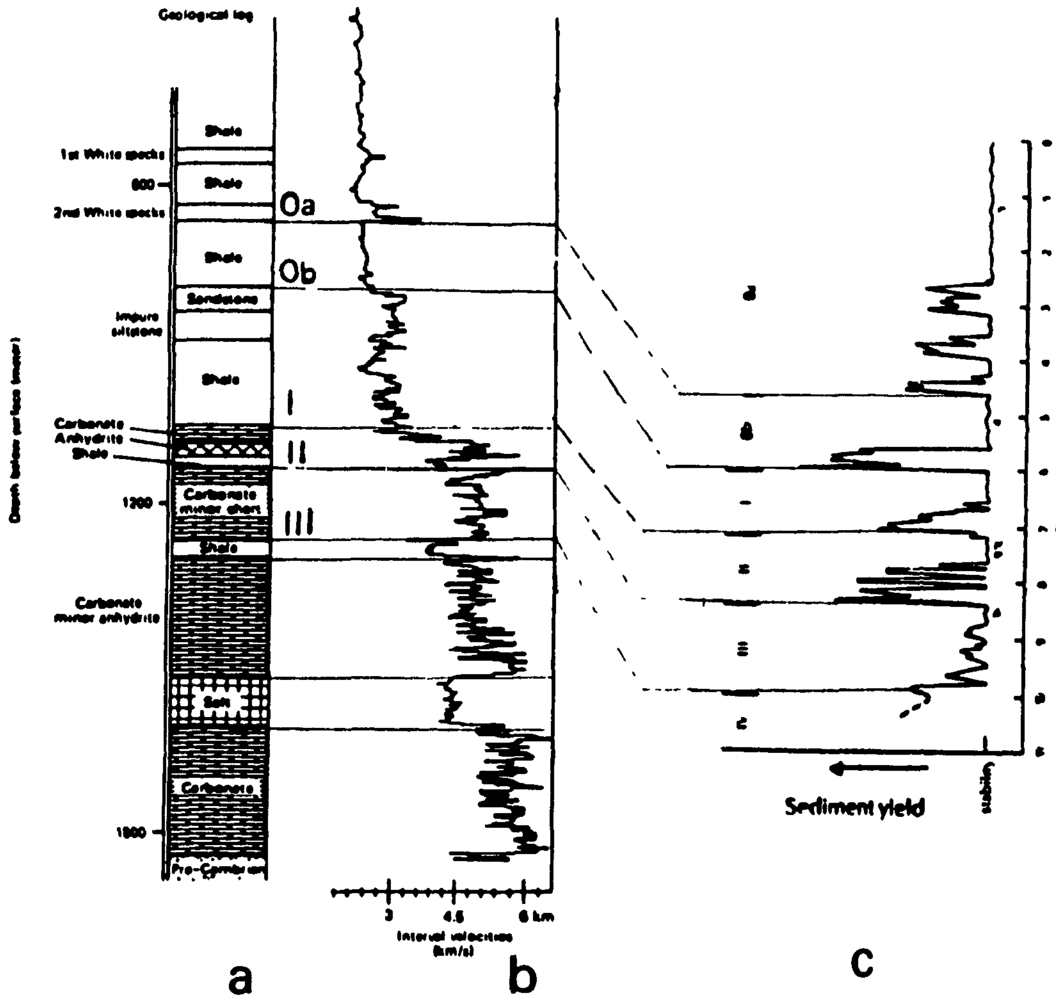

Figure 5.7: Chaotic reflectivity model of sedimentary sequence development. Schematic diagram showing the relationship of change in acoustic impedance and resultant lithological states; (a) geological section, (b) corresponding sonic well log, (c) relative stability fields. The chaotic sedimentary geosystem evolves to different states. These states represent conditions for the deposition of a particular lithology. Abrupt shifts to other states, create new conditions responsible for deposition of new lithologies, and so on. The created lithologies correspond to layers of the sedimenatry sequence. (Modified from Sharma, 1986; and Johnson and Thorn, 1982).

In summary, accoustic impedance variations – the geophysical measures of lithological variability – result from the chaotic internal organization of the interacting geological processes. This organization is responsible for the common statistical characteristics observed in reflectivity well logs from all sedimentary basins. The statistical properties of the fractal-flicker noise character of the acoustic impedance record, are consistent with a geosystem governed by chaos.

Chapter 6

Summary and conclusions

6.1 The chaotic sedimentation system

The sedimentation system has, here, been treated as partially specified system in order to employ qualitative stability analysis. For this purpose, the interrelationships among the geological processes were examined to aid in the construction of the interaction matrix used in this analysis. This allows for abrupt shifts from one stable mode reflecting one lithotype to another new state, reflecting a different lithology. We have argued that changes arising from the internal structure of the geosystem (i.e. within the system) cause the geosystem to evolve among its various stable operating states.

Evidence from geological process interaction over long time periods reveals a sensitive dependence on initial conditions (Montgomerey, 1983; Furbish, 1988). A small deviation of one geological process can have a tremendous effect on the geosystem's output response – sediment deposition. The exact lithology however, cannot be predicted due to the essentially chaotic character of the geosystem.

This research supports the hypothesis that a sedimentary sequence is the result of a chaotic geosystem. Sedimentary sequence development exhibits all the fundamental characteristics of chaos. The strange attractor of the chaotic geosystem is manifest as stable lithological formations. The chaotic behaviour is inherent in the internal interactions of the geosystem.

The acoustic impedance function of depth is a geophysical measure of the sedimentary sequence. Its well known fractal, self-scaling ($1/f$) noise character (Todoeschuck *et al.*, 1990) is consistent with a chaotic generation process. This further supports Agapeew's (1989) arguments, justifying a fractal flicker noise model for the acoustic impedance function.

6.2 Recommendations for future work

Here we have set a basis for considering sedimentary accumulations and evolution from a unique quasi–mathematical viewpoint. The next step would be to obtain a strict mathematical description of the concept of the phase- space representation for specific geological environments: active, passive continental margins and cratons. The dimensionality of the attractor solutions should be determined. This could be accomplished through the determination of the Lyapunov dimension (Tabor, 1989) and the correlation exponent which measure the number of dimensions minimally necessary to specify the region of an attractor in phase space. The dimension of a chaotic attractor is often a non-integer number which specifies a fractal character. Thus, by determining this number, we should be able to determine whether any specific geosystem is chaotic or not.

Chaotic behaviour can also result from time delays in system interactions (McGill, 1989). In the case of the geosystem, there are several feedback loops between processes which may exhibit time–delayed interactions (Legget, 1985). These must be identified and examined to determined whether or not they enhance the chaotic behaviour or perhaps suppress it.

Finally, recognizing that the geosystem is much more complex than the simple *four– component geoprocess model* studied in this thesis, and that it must comprise other dynamic internal physical, chemical and biological processes, the geosystem model is open to arbitrary elaboration and extension.

6.3 Claims of originality

This research has obtained a modelling of sedimentary sequence development under a whole new perspective. The importance of chaos has been recognized describing lithological accumulations in sedimentary basins. Also, geology has been treated as a dynamical system with geological processes studied holistically from a systems – science viewpoint.

Benefits accrue from treating geology in the above manner:

1. It is especially convenient to deal with the complex interactive nature of geological processes.
2. We may see past "raw" detail in the geological processes and allow for a view of essential geosystem behaviour.

Traditionally, geology advances through the description of many individual examples. Our (geo)physical approach here is perhaps, unique in the study of stratigraphic sequences in that we didn't concentrate on specific sedimentary basins, but instead attempted to describe a generally applicable, chaotic geosystem.

Furthermore, we have introduced an original way of studying sedimentation systems. We have described measures of geological environments in a phase-space. We have argued that their evolution with time can be described by a single point of a trajectory of strange attractor in a multidimensional space. We have taken one innovative step towards quantitative stratigraphic modelling.

Appendix A

Power spectral density relationships

This appendix gives the relationship between the power spectra of the reflectivity sequence and that of the acoustic impedance depth function.

We show that when the power spectal density of the acoustic impedance behaves as $1/f^{\beta}$, the reflection coefficients or the reflectivity sequence has a power spectrum proportional to $f^{\beta-2}$.

If $R(t)$ denotes the sequence of small reflection coefficients as a function of time, and $V(t)$ is the acoustic impedance as a function of time, we have:

$$R(t) = 1/2 \; d[ln \; V(t)]/dt. \qquad (A.1)$$

Integration of equation (A.1) gives

$$ln \; [V(t)/V(0)] = 2 \int_0^t R(t') \, dt'. \qquad (A.2)$$

If the reflection coefficients are very small then $V(t) \simeq V(0)$ and we can approximate:

$$\frac{V(t)}{V(0)} \simeq 1 + \frac{V(t)}{V(0)}, \qquad (A.3)$$

where $V(t)/V(0)$ is a small quantity. The relationship $ln(1 + x) \simeq x$ and equation (A.3) are used to rewrite equation (A.2) as

$$\frac{V(t)}{V(0)} \simeq 2\int_0^{t} R(t')\, dt'. \qquad (A.4)$$

Fourier transformation of equation (A.4) gives:

$$\tilde{V}(f) \simeq \frac{V(0)}{\pi i f}\, \tilde{R}(f), \qquad (A.5)$$

where $\tilde{V}(f)$ and $\tilde{R}(f)$ are the Fourier transforms of $V(t)$ and $R(t)$ respectively. Thus, from equation (A.5) if we take the second power we get:

$$\mid \tilde{U}(f) \mid^2 \simeq (\frac{V(0)}{\pi f})^2 \mid \tilde{r}(f) \mid^2. \qquad (A.6)$$

Equation (A.6) shows, that the power spectrum of the acoustic impedance function is approximately proportional to the power spectrum of the reflection coefficients divided by the square of the frequency.

Therefore, if the acoustic impedance spectrum is flicker ($1/f^{\beta}$, where $\beta = 1$), the power spectrum of the reflections is proportional to $(1/f^{\beta-2}) = f$.

Appendix B

Calculation of the characteristic equation

The characteristic equation of Matrix **A** is calculated from the determinant which is set to zero:

$$\begin{Vmatrix} -\lambda & 0 & -a_{13} & -a_{14} \\ -a_{21} & -\lambda & -a_{23} & 0 \\ 0 & -a_{32} & -\lambda & 0 \\ 0 & -a_{42} & a_{43} & -\lambda \end{Vmatrix} = 0$$

Evaluating the determinant using elements of the first column we get:

$$-\lambda \begin{vmatrix} -\lambda & -a_{23} & 0 \\ -a_{32} & -\lambda & 0 \\ -a_{42} & a_{43} & -\lambda \end{vmatrix} + a_{21} \begin{vmatrix} 0 & -a_{13} & -a_{14} \\ -a_{32} & -\lambda & 0 \\ -a_{42} & a_{43} & -\lambda \end{vmatrix} = 0$$

We solve and obtain:

$$-\lambda(-\lambda) \begin{vmatrix} -\lambda & -a_{23} \\ -a_{32} & -\lambda \end{vmatrix} + a_{21} \left[a_{13} \begin{vmatrix} -a_{32} & 0 \\ -a_{42} & -\lambda \end{vmatrix} \quad -a_{14} \begin{vmatrix} -a_{32} & -\lambda \\ -a_{42} & a_{43} \end{vmatrix} \right] = 0$$

Modifying the above we have:

$$\lambda^2(\lambda^2 - a_{23}a_{32}) + a_{21}[a_{13}(\lambda a_{32}) - a_{14}(-a_{32}a_{43} - a_{42}\lambda)] = 0$$

Finally, we get:

$$\lambda^4 - a_{23}a_{32}\,\lambda^2 + (a_{13}a_{32}a_{21} + a_{21}a_{14}a_{42})\,\lambda + a_{21}a_{14}a_{32}a_{43} = 0$$

References

Agapeew, G., 1989. *Geological justification of fractal acoustic impedance models*, M.Sc. thesis, McGill University.

Algeo, T.J., and Wilkinson, B.H., 1987. Periodicity of mesoscale phanerozoic sedimentary cysles and the role of Milankovitch orbital modulation, *Jour. Geol.*, **96**, 313-322.

Allison, I., 1981. *Antarctic sea ice growth and oceanic heat flux*, in: Sea level ice and climatic change. International Association of Hydrological Sciences, no.131, 471 p.

Bally, A.W., Basins and subsidence-summary, in *Dynamics of plate interiors*, Bally, A.W., Bender, P.l., McGetchin, T.R., Walcott, R.I., eds., American Geophysical Union Geodynamics Series, Vol.1, 5-20, 1980.

Baksi, A.K., 1990. Search for periodicity in global events in the geologic record: Quo vadimus?, *Geology*, **18**, 983-986.

Barron, E.J., Arthur, M.A., and Kauffman, E.G., 1985. Cretaceous rhythmic bedding sequences: a plausible link between orbital variations and climate, *Earth Plan. Sci. Lett.*, **72**, 327-340.

Bergé, P., Pomeau, Y., Vidal, C., 1986. *Order within Chaos*, John Wiley and Sons Publ. 329 p.

Berger, A., Milankovitch theory and climate. 1988, *Reviews of Geophysics*, **26**, 624-657.

Berger, W. H., Short-term changes affecting atmosphere, oceans, and sediments during the Phanerozoic: Group report, in *Holland, H. D. and Trendall, A. F., eds., Patterns of change in earth evolution*, 171-205, Springer-Verlag, Heidelberg, 1984.

Braun, M., 1983. *Differential equations and their applications*, N.Y. Springer-Verlag.341 p.

Broecker, W.S., Peteet, D.M., and Rind, D., 1985. Does the ocean-atmosphere system have more than one stable mode of operation?, *Nature*, **315**, 21-26.

Burton, R., Kendall, Ch.G.St.C., and Lerche, I., 1987. Out of our depth: on the impossibility of fathoming eustacy from the startigraphic record, *Earth Sci. Rev.*, **24**, 237-277.

Campbell, D., and Rose, H., Nonlinear phenomena-Order in Chaos, in *Proceedings of the International Conference on Order in Chaos, Center for Nonlinear Studies, Los Alamos, New Mexico, May 24-28, 1982, Physica 7D*, Flaschka, H., Ford, J., Newell, A.C., Scott, A. Swinney, H.L., eds., 1-46, 1983.

Carr, T.R., 1982. Log-linear models, Markov chains and cyclic sedimentation, *Jour. Sed. Petrology*, **52**, 905-912.

Chappell, J., Relative and average sea level changes, and endo-, epi, and exogenic processes on the Earth, in *Proceedings of the General Assembly of the International Union of Geodesy and Geophysics, December 7-9, 1979, Canberra*, International Association of Hydrological Sciences No. 131, 411-430, 1981.

Chappell, J., 1983. Thresholds and lags in geomorphologic changes, *Australian Geographer*, **15**, 357-366.

Cloetingh, S.A.P.L., Wortel, M.J.R., and Vlaar, N.J., 1984. Passive margin evolution, initiation of subduction and the Wilson cycle, *Tectonophysics*, **109**, 147-163.

Crossley, D.J. and Jensen. O.G., 1989. Fractal velocity in refraction seismology, *Pageoph.*, **131**, 61-76.

Crowley, T.J., and North, G.R., 1988. Abrupt climate change and extinction events in earth history, *Science*, **240**, 996-1002.

Crutchfield, J.P., Farmer, J.D., Packard, N.H., and Shaw, R.S., 1986. Chaos, *Scientific American*, **255**, 46-57.

Davies, T.A., Hay, W.W., Southam, J.R., and Worsley, T.R., 1977. Estimates of Cenozoic oceanic sedimentation rates, *Science*, **197**, 53-55.

di Castri, F., and Malcolm, H., 1988. Enhancing the credibility of ecology: Interacting along and across hierarchical scales, *GeoJournal*, **17**, 5-35.

Dobrin, M.B., and Savit, C.H., 1988. *Introduction to geophysical prospecting*, 4th edition, McGraw Hill Co. New York.867 p.

Domenico, S.N., 1974 Effect of water saturation on seismic reflectivity of sand reservoirs encased in shale, *Geophysics*, **39**, 759-769.

Donnelly, T.W., 1982. Worldwide continental denudation and climatic deterioration during the late Tertiary: Evidence from deep-sea sediments, *Geology*, **10**, 451-454.

Donovan, D.T., and Jones, E.J.W., 1979. Causes of world-wide changes in sea level, *J. geol. Soc. Lond.*, **136**, 187-192.

Edwards, M.B., 1975. Glacial retreat sedimentation in the Smalfjord Formation, Late Pre-Cambrian, North Norway, *Sedimentology*, **22**, 75-94.

Edwards, M., Glacial environments, in *Sedimentary environments and facies*, Reading, H.G. ed., Blackwell Scien. Publ., 445-470, 1986.

Ford, J., 1983. How random is a coin toss?, *Physics Today*, **36**, 40-47.

Ford, J., What is chaos, that we should be mindful of it ?, in *The new physics*, Davies, P., ed., Cambridge University Press, 348-372, 1989.

Furbish, D.J., 1988. *The river meandering process: Evidence for nonlinear chaos*, Geological Society of America. Abstract-1104

Galloway, W.E., 1989. Genetic stratigraphic sequences in basin analysis I: Architecture and genesis of flooding-surface bounded depositional units, *Am. Ass. Pet. Geol. Bull.*, **73**, 125-142.

Gardner, G.H.F., Gardner, L.W., and Gregory, A.R., 1974. Formation velocity and density: the diagnostic basis for stratigraphic traps, *Geophysics*, **39**, 770-780.

Gardner, T.W., Jorgensen, D.W., Shuman, C., and Lemieux, C.R., 1987. Geomorphic and tectonic process rates: Effects of measured time interval, *Geology*, **15**, 259-261.

Gardner, W.A., 1986. *Introduction to random processes: With applications to signals and systems*, Macmillan Publishing Company, New York, 434 p.

Goodwin, P.W., and Anderson, E.J., 1984. Punctuated aggradational cycles: A general hypothesis of episodic stratigraphic accumulation, *Jour. Geology*, **93**, 515-533.

Gregotski, M.E., Jensen, O.G., and Arkani-Hamed, J., 1990, *Fractal stochastic modeling of aeromagnetic data*, Geophysics (submitted).

Harrison, C.G.A., Brass, G.W., Saltzman, E., Sloan, J., Southam, J., Whitman, J.M., 1981. Sea level variations, global sedimentation rates and the hypsographic curve, *Earth Plan. Sci. Lett.*, **54**, 1-16.

Heiskanen, W.A., and Veining Meinesz, F.A., 1958. *The earth and its gravity field*, McGraw-Hill, N.Y..470 p.

Hosken, J.W.J., 1980. *A stochastic model of seismic reflections*, 50th Ann. Mtg. Soc. Explor. Geophys. Abstract G-69.

Huang, J., and Turcotte, D.L., 1990. Are earthquakes an example of deterministic chaos ?, *Geophys. Res. Lett.*, **17**, 223-230.

Jensen, O.G., and Mansinha, L., Excitation of geophysical systems with fractal, flicker noise, in *Time series and econometric modelling*, 165-188, MacNeill, I.B. and Umphrey, G.J., eds., D. Reidel Publ. Co., Dordrecht, 1987.

Johnson, W.H., and Thorn, C.E., Interrelationships among geomorphic interpretations of the stratigraphic record, process geomorphology and geomorphic models, in *Proceedings of the Binghamton symposia in Geomorphology, No.12, Binghamton, NY., 1982*, Space and Time in Geomorphology, Thorn, C.E., editor, 219-241, 1982.

Kendal, Ch.G.St.C., Wolfgang, S., 1981. Carbonates and relative changes in sea level, *Mar. Geol.*, **44**, 181-212.

Keshner, M.S., 1982. $1/f$ noise, *Proc. IEEE*, **70**, 212-218.

Legget, J.K., Deep-sea pelagic sediments and palaeo-oceanography: a review of recent progress, in *Sedimentology: recent developments and applied aspects. Geological Society Special Publications 18*, Brenchley, P.J., Williams, B.P.J., eds., Blackwell Scientific Publications, 95-121, 1985.

Levins, R., 1974. The qualitative analysis of partially specified systems, *N. Y. Acad. Sci. Ann.*, **231**, 123-138.

Lisitsyn, A.P., Lukashin, V.N., Gurvich, Y.G., Gordeyev, V.V., and Demina, L.L., 1982. The relation between element influx from rivers and accumulation in ocean sediments, *Geochem. Intern.*, **19**, 102-110.

Lutz, T.M., 1987. Limitations to the statistical analysis of episodic and periodic models of geologic time series, *Geology*, **15**, 1115-1117.

Mandelbrot, B.B., 1977. *Fractals form, chance, and dimension*, W.H. Freeman and Co., San Francisco, 365 p.

Mandelbrot, B. B., and Voss, R. F., Why is nature fractal and when should noises be scaling?, in *Noise in physical systems and 1/f noise*, Elsevier Science Publishers, Amsterdam, 31-39, 1983.

Mandelbrot, B.B., 1983. *The Fractal Geometry of Nature*, W.H. Freeman and Co., San Francisco, 468 p.

May, R.M., 1973. *Stability and complexity in model ecosystems*, Princeton, N.J., Princeton Univ. Press. 235 p.

McGill, B.K., Return to chaos, in *43rd Annual Quality Congress Transactions vol.43, May 8-10, 1989, Toronto, Ontario*, ASQC Milwaukee, 781-792, 1989.

Miall, A.D.W., 1984. *Principles of sedimentary basin analysis*, Springer-Verlag New York Inc. 490 p.

Middleton, G.V., 1990. Nonlinear dynamics and chaos: Potential applications in the earth sciences, *Geoscience Canada*, **17**, 3-11.

Milenkovic, B.S., 1989. Verification of the hypothesis regarding the random nature of fluctuation of geological properties, *Jour. Geology*, **97**, 109-116.

Moon, F.C., 1987. *Chaotic vibrations; an introduction for applied scientists and engineers*, John Wiley and Sons, New York.309 p.

Montgomery, K., 1983. Concepts of equilibrium and evolution in geomorphology: the model of branch systems, *Progress in Phys. Geography*, **13**, 47-66.

Nicolis, C., and Nicolis, G., 1984. Is there a climate attractor, *Nature*, **311**, 529-532.

Nicolis, C., and Nicolis, G., 1987. *Irreversible phenomena and dynamical systems analysis in geosciences*, D. Reidel Publishing Co. 578 p.

Nicolis, G., Physics of far-from-equilibrium systems and self-organization, in *The New Physics*, 316-347, Davies, P., ed., Cambridge University Press, 1989.

North, G.R., and Crowley, T.J., 1985. Application of a seasonal climate model to Cenozoic glaciation, *J. geol. Soc. Lond.*, **142**, 475-482.

O'Doherty, R.F., and Anstey, N.A., 1971. Reflections on amplitudes, *Geophys. Prosp.*, **19**, 430-458.

Oeschger, H., Beer, J., Siegenthaler, U., Stauffer, B., Dansgaard, W., Langway, C.C., Late glacial climate history from ice cores, in *Am. Geophys. Un.*

Monogr. Ser. 29, Climate processes and climate sensitivity, M. Ewing vol. 5, Hansen, J.E., Takahashi, T., eds., 299-306, 1984.

Pan, G., 1987. A stochastic approach to optimum decomposition of cyclic patterns in sedimentary processes, *Math. Geol.*, **19**, 503-521.

Parker, R.B., 1985. Buffers, energy storage, and the mode and tempo of geologic events, *Geology*, **13**, 440-442.

Phillips, J.D., and Steila, D., 1984. Hydrologic equilibrium status of a disturbed watershed in eastern North Carolina, *GeoJour.*, **9**, 351-357.

Phillips, J.D., 1985. Stability of artificially-drained lowlands: A theoretical assessment, *Ecol. Modelling*, **27**, 69-79.

Phillips, J.D., 1986. Sediment storage, sediment yield, and time scales in landscape denudation studies, *Geographical Analysis*. **18**, 161-167.

Phillips, J.D., 1987. Sediment budget stability in the Tar river basin, North Carolina, *Amer. Jour. Science*, **287**, 780-794.

Pitman, W.C., 1978. Relationship between eustacy and stratigraphic sequences of passive margins, *Geol. Soc. Amer. Bull.*, **89**, 1389-1403.

Pitman, W.C., and Golovchencho, The effect of sea level change on the shelf edge and slope of passive margins, in *The shelfbreak*, Stanley, D.J., and Moore, G.T., ed., Spec. Publ. Soc. econ. Paleont. Miner. **33**, 41-58, 1983.

Plotnick, R.E., 1986. A fractal model for the distribution of stratigraphic hiatuses, *Jour. Geology*, **94**, 885-890.

Reineck, H.E., and Singh, I.B., 1980. *Depositional sedimentary environments*, Springer-Verlag, New York.549 p.

Rogers, R.D., 1989. Use of observational patterns in geology, *Geology*, 17, 131-134.

Ruddiman, W. F. and Wright, H. E. Jr., Introduction, in *Ruddiman, W. F. and Wright, H. E. Jr., eds., The geology of North America v. K-3, North America and adjacent oceans during the last deglaciation*, 1-12, The Geological Society of America, Boulder Colorado, 1987.

Ruelle, D., 1989. *Chaotic evolution and strange attractors*, Cambridge University Press, New York, 93 p.

Sandquist, G., 1985. *Introduction to system science*, Prentice-Hall, Inc. 567 p.

Schlager, W., 1981. The paradox of drowned reefs and carbonate platforms, *Geol. Soc. Amer. Bull.*, **92**, 197-211.

Scholz, C.H., 1989. Global perspectives of chaos, *Nature*, **338**, 459-460.

Schwarzacher, W., 1975. *Sedimentation models and quantitative stratigraphy*, Developments in Sedimentology 19, Amsterdam Elsevier, 382 p.

Schwarzacher, W., Principles of quantitative lithostratigraphy, in *Quantitative stratigraphy*, 361-386, Gradstein, F.M., Agterberg, F.P., Brower, J.C., and Schwarzacher, W.S., eds., D. Reidel Publ. Co., Dordrecht, 1985.

Schroeder, M.R., 1989. Self-similarity and fractals in science and art, *J. Audio Eng. Soc.*, **37**, 795-808.

Sharma, P.V., 1986. *Geophysical methods in geology*, Elsevier Science Publ. Co., Inc., 442 p.

Shaw, R.H., 1980. Strange attractors, chaotic behavior, and information flow, *Z. Naturrforsch.*, **36a**, 80-112.

Shaw, R.H., 1987. The periodic structure of the natural record and nonlinear dynamics, *EOS, American Geophysical Union Transactions*, **68**, 1651-1665.

Slingerland, R., 1981. Qualitative stability analysis of geologic systems, with an example from river hydraulic geometry, *Geology*, **9**, 491-493.

Slingerland, R., Predictability and chaos in quantitative dynamic stratigraphy, in *Quantitative dynamic stratigraphy*, 45-53, Cross, T.A., ed., Prentice Hall Inc, 1989.

Sloss, L.L., and Speed, R.C., Relationships of cratonic and continental margin tectonic episodes, in *Special Publ. Soc. Econ. Palaeont. Miner. 22*, Tectonics and sedimentation, Dickinson, W.R., ed, 98-119, 1974.

Snow, R.S., 1989. Fractal sinuosity of stream channels, *Pageoph*, **131**, 99-109.

Soares, E.F., Unny, T.E., and Lennox, W.C., 1982. Conjuctive use of deterministic and stochastic models for predicting dediment storage in large reservoirs, *Jour. Hydrol.*, **59**, 49-82.

Stewart, C.A., and Turcotte, D.L., 1989. The route to chaos in thermal convection at infinite Prandtle number: Some trajectories and bifurcations, *J.*

Geophys. Res., **94**, 13707-13717.

Stott, D.F., Cretaceous sequences of the foothills of the Canadian Rocky Mountains, in *The Mesozoic of Middle North America*, Stott, D.F., and Glass, D.J. ed., CSPG Memoir 9, 85-107, 1984.

Stow, D.A.V., Deep-sea clastics: where are we and where are we going ?, in *Sedimentology: recent developments and applied aspects. Geological Society Special Publications 18*, Brenchley, P.J., Williams, B.P.J., eds., Blackwell Scientific Publications, 67-93, 1985.

Stow, D.A.V., Deep clastic seas, in *Sedimentary environments and facies*, Reading, H.G. ed., Blackwell Scien. Publ., 399-444, 1986.

Strauss, D., and Sadler, P.M., 1989. Stochastic models for the completeness of stratigraphic sections, *Math. Geol.*, **21**, 37-59.

Swinney, H.L., 1983. Observatios of order and chaos in nonlinear systems, *Physica*, **7D**, 3-15.

Tabor, M¿, 1989. *Chaos and integrability in nonlinear dynamics: an introduction*, New York, Wiley 1989.364 p.

Thornes, J.B., 1983. Evolutionary geomorphology, *Geog.*, **68**, 222-235.

Todoeschuck, J.P., and Jensen, O.G., 1988. Joseph Geology and seismic deconvolution, *Geophys.*, **53**, 1410-1414.

Todoeschuck, J.P. and Jensen, O.G., 1989. Scaling geology and seismic deconvolution, *Pageoph.*, **131**, 273-287.

Todoeschuck, J.P., Jensen, O.G., and Labonte, S., 1990 Gaussian scaling noise model of seismic reflection sequences: Evidence from well logs, *Geophys.*, **55**, 480-484.

Turcotte, D.L., Ahren, J.L., and Bird, J.M., 1977. The state of stress at continental margins, *Tectonophysics*, **42**, 1-28.

Turcotte, D.L., 1986. A fractal approach to the relationship between ore grade and tonnage, *Econ. Geology*, **81**, 1528-1532.

Turcotte, D.L., 1989. Fractals in geology and geophysics, *Pageoph*, **131**, 171-196.

Vail, P.R., Mitchum, R.M.Jr., Todd, R.G., Widmier, J.M., Thompson, S.,III, Sangree, J.B. Bubb, J.N., and Hatleilid, W.G., Seismic stratigraphy and global changes of sea level, in *Seismic stratigraphy -Applications to hydrocarbon exploration, Amer. Ass. Petr. Geol. Mem. 26*, Playton, C.E., ed., 49-212, 1977.

Velzeboer, C.J., 1981. The theoretical seismic reflection response of sedimentary sequences, *Geophysics*, **46**, 843-853.

Voss, R.F., $1/f$ Flicker noise: A brief review, in *Proceedings of the 33rd Annual Symposium on Frequency Control*, Atlantic City, 40-46, 1983.

Walden, A.T., and Hosken, J.W.J., 1985. An investigation of the spectral properties of primary reflection coefficients, *Geophys. Prosp.*, **33**, 400-435.

Watts, A.B., 1982. Tectonic subsidence, flexure and global changes of sea level, *Nature*, **297**, 469-474.

Wigley, T.M.L., 1985. Impact of extreme events, *Nature*, **316**, 106-107.

Woillard, G.M., and Mook, W.M., 1982. Carbon 14 dates at Granda Pile: correlation of land and sea chronologies, *Science*, **215**, 159-161.

Worsley, T.R., and Davies, T.A., 1979. Sea-level fluctuations and deep-sea sedimentation rates, *Science*, **203**, 455-456.