Unified Modelling and Simultaneous Optimization of Open Pit Mining Complexes with Supply Uncertainty

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Doctor of Philosophy

Mining and Materials Engineering

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Montreal, Quebec

2014-08-14

A thesis submitted to McGill University in partial fulfillment of the requirements of the degree of Doctor of Philosophy

ACKNOWLEDGEMENTS

I would like to express my deepest gratitude to my supervisor, Dr. Roussos Dimitrakopoulos, for his endless advice, insight and encourangement. The past few years haven't always been easy, but he has always reminded me to lighten up and stop stressing (admittedly, however, sometimes having the opposite effect — *sorry*). I would also like to thank the past and current students at COSMO Laboratory who I have had the amazing opportunity to learn from, including Francisco, Victor, Luis, Putra, Conor, Martha, Mario, Murilo, Renaud and Fernanda, and the past and current research staff who have helped along the way, including Hussein, Snehamoy, Waqar, David, Amina and Jian.

I have had amazing opportunities to travel around the world to teach, work with and learn from industry practitioners who are genuinely interested in this area of research; this could not have happened without the support of the COSMO Laboratory of McGill University, and its industrial sponsors. The work in this thesis was funded from the National Science and Engineering Research Council of Canada, Collaborative R&D Grant CRDPJ 411270-10 with AngloGold Ashanti, Barrick Gold, BHP Billiton, De Beers, Newmont Mining and Vale.

Most of all, I would like to acknowledge the endless support from my lovely partner, Veronica, who has put up with the long trips, the long hours and my high levels of stress. Thankfully, we have an (over-) abundance of pets that have kept me grounded. I'd also like to thank my mother, Janet, for nudging me on to do a Ph.D.,

and the rest of my family, Alex, Devon, Kerry and Dylan. I'm eternally grateful for their support.

CONTRIBUTION OF AUTHORS

The author of this thesis is the primary author for all manuscripts contained within. Professor Roussos G. Dimitrakopoulos is the supervisor of the author's Ph. D., and is included as a co-author for each of these works.

Chapter 3 - Goodfellow, R. and Dimitrakopoulos, R. (2013) *Algorithmic integration of geological uncertainty in pushback designs for complex multiprocess open pit mines*, Mining Technology, Vol. 122(2), pp. 67-77.

Chapter 4 - Goodfellow, R. and Dimitrakopoulos, R. (submitted) *Mining supply chain optimization under geological uncertainty*, International Journal of Production Economics.

Chapter 5 - Goodfellow, R. and Dimitrakopoulos, R. (submitted) *Global optimization of open pit mining complexes with uncertainty*, Applied Soft Computing.

Chapter 6 - Goodfellow, R. and Dimitrakopoulos, R. (in preparation) *Stochastic global optimization of open pit mining complexes with capital expenditures.* European Journal of Operational Research.

ABSTRACT

A mining complex is an integrated business that extracts materials from open pit or underground mines, treats extracted materials via a series of processing facilities that are interconnected by various material handling methods, and generates a set of products that are sold and delivered to customers and/or the spot market. The primary objective when optimizing a mining complex is to maximize its value for the business and its stakeholders while obeying the technical constraints that limit production. This optimization is traditionally performed by treating the various components independently, leading to the suboptimal use of the natural resources and financial capital, and the underperformance of the mining complex. The global optimization of mining complexes aims to simultaneously optimize the multi-mine production schedules, which define the distribution of materials over time, the destination policies, which define where extracted materials are sent, and the use of the various processing streams for processing, distribution and product marketing. As the size of the mining complex grows, there is a compounded effect that uncertainty has on the various components, and new stochastic optimization methods are needed to manage this risk. This thesis aims to generate a unified modelling and global optimization methodology that integrates geological (supply) uncertainty and manages risk in the design and operation of mining complexes, and can be adapted to suit the needs and objectives of individual operations.

First, Chapter 1 provides an overview of the problem and outlines the goals, objectives and major contributions of this work. Following this, Chapter 2 provides

a literature review of the related work, which highlights the need for new, risk-based optimization approaches. Chapter 3 proposes a method that integrates geological, or supply, uncertainty into pushback designs for mining complexes with multiple processing paths. Pushbacks are used to guide the sequence of extraction and help to ensure that the sequence generates a high net present value and provides consistent quantities of valuable material to the processing streams. An application at the Escondida Norte mine, Chile, indicates that the stochastic pushback design method generates a 61% reduction in risk for tonnages sent to the various processing streams over a deterministic pushback design that is generated with BHP Billiton's Blasor software. This outcome is a result of being able to re-distribute the risk in the pushback design. This method is easily integrated into an industry-standard sequential optimization mine design framework.

The remainder of this thesis is in the context of a more recent paradigm shift in the design of mining complexes, where many intricate and interrelated components, such as mining, processing, distribution and marketing, are simultaneously optimized. Chapter 4 frames a mining complex as a mineral resource supply chain that is owned by a mining company, whose objective is to optimize its destination policies for the mined material and the use of various processing streams in order to satisfy customer demands or maximize the value of the operation. Depending on the type of products, such as iron ore, coal, nickel or copper, in addition to the geology and geographical location, the complexity of mineral resource supply chains (i.e. blending, processing and distribution), can vary substantially. Existing research has focused on models designed for a single application that isn't necessarily useful

for other mining complexes. As a result, a unified methodology is proposed that enables the detailed modelling and optimization of these intricate systems. A case study for the Onça Puma nickel laterite blending chain is used to highlight the need to integrate uncertainty into the definition of destination policies, particularly for multi-element deposits. The results show that the stochastic optimizer is able to generate a complex destination policy that satisfies and reduces risk related to the stringent blending constraints and production targets at the processing plant without compromising on economic value.

Chapter 5 expands on this unified modelling and optimization methodology by including the ability to simultaneously optimize multi-mine production schedules, in addition to the destination policies and processing streams. Through a case study for a copper-gold mine, it is demonstrated that the deterministic-equivalent of the proposed optimizer is able to generate a solution that has 4% higher net present value than an industry-standard deterministic production scheduler. Moreover, the stochastic optimizer is able to generate a design that not only has a 6% higher net present value than the deterministic-equivalent of the stochastic optimizer, based on the P-50 risk profiles, but also is able to meet production targets and reduce the risk of the materials processed, thus helping to ensure that financial forecasts are met in practice.

Existing stochastic optimization models for production scheduling have focused on reducing the risk related to not meeting pre-defined production targets. Chapter 6 addresses the challenge of simultaneously optimizing the production and processing of materials with the ability to increase or decrease capacity constraints via capital

expenditures. In a case study for a copper mining complex, the optimizer is permitted to design the mine's production rates by purchasing or replacing shovels and trucks simultaneously with the mine production schedule, destination policies and the processing streams. Existing commercial schedulers are unable to perform this simultaneous optimization, nor accommodate a highly-detailed model, thus a fair comparison to commercial solutions cannot be made. In this example, the stochastic optimizer defines a practical production rate that consistently feeds the mill processing stream with a minimal amount of risk, which is of critical importance for low-grade mining operations. Additionally, by being able to understand the grade variability of the mined materials and the associated uncertainty, the stochastic design also generates a 5.4% increase in net present value over the same (deterministic-equivalent) method, based on the P-50 risk profiles, that is optimized only using a single, estimated orebody model.

This work advances the related field of knowledge through the development of new models and methods for optimizing mining complexes with uncertainty. This is primarily achieved through five major contributions. First, a stochastic global optimization method is developed to simultaneously optimize multi-mine life-of-mine production schedules, destination policies, processing streams and capital expenditures for capacity design; while existing state-of-the-art methods may address some of these aspects, they have not been previously integrated in a simultaneous optimization model that does not rely on divvying up the global model into sub-problems. Second, a new, unified modelling approach is developed that permits the proposed optimizer to be tested on many different types of mining complexes with a high degree

of modelling detail; as a result of this generalized and unified approach, non-linear relationships can easily be integrated in the optimization models — a limitation of existing deterministic and stochastic methods. Third, and a result of the previous development, a new approach is developed to model the economic value of the products sold, rather than the materials mined. Existing models and methods are limited by the assumption that each block has an economic value, hence the optimal processing stream is known a priori, and the block is treated and sold in isolation from other blocks; in some cases, this may lead to substantially undervaluing the resource. Using the new modelling approach, it is possible to evaluate the economic potential of products at the point of sale, rather than making these unrealistic assumptions at the block-level. Fourth, computationally efficient solvers are adapted and applied using metaheuristics. A combination of particle swarm optimization and a modified simulated annealing algorithm are developed to optimize various aspects of the global optimization problem; these methods have not been previously combined for mine optimization, and requires devising new methods to change designs and ensure that the optimizers do not get trapped in local optima. Finally, the performance, advantages and limitations of the models and methods are analyzed through full-field testing on real-world and large-scale examples. The results consistently reinforce the concept that it is possible to not only reduce the risk of not meeting production targets, thus guaranteeing financial forecasts are met, but also increase the net present value of the operation.

RÉSUMÉ

Un complexe minier est une entreprise intégrée qui extrait du minerai de mines à ciel ouvert ou sous-terraines, traite le minerai extrait via une série d'usines de traitement de métaux, qui sont interconnectées par une variété de méthodes de traitement, et génère un ensemble de produits qui sont vendus et livrés à des clients ou sur le libre marché. L'objectif premier lors de l'optimisation d'un complexe minier est de maximiser sa valeur pour l'entreprise et les parties prenantes tout en respectant les contraintes techniques qui limitent la production. Cette optimisation est faite de façon traditionnelle en considérant chaque composante indépendamment, conduisant à une utilisation sous-optimale des ressources naturelles et capitaux financiers et la sous performance du complexe minier. L'optimisation globale des complexes miniers cherche à optimiser simultanément les calendriers de production de multiples mines, définissant la distribution du minerai dans le temps et les politiques de destination. Celles-ci définissent où le minerai extrait est envoyé, l'utilisation des différents courants de traitement et produits pour le marché. Avec le grossissement de la taille des complexes miniers, l'incertitude produit un effet d'agrégation sur les différentes composantes et donc de nouvelles méthodes d'optimisation stochastiques doivent être développées pour gérer ce risque. Cette thèse a pour objectif de générer un model unifié et une méthodologie d'optimisation globale qui intègre l'incertitude géologique et gère le risque du design et des opérations de complexes miniers et peuvent être adapté pour répondre aux besoins et objectifs pour différentes opérations.

Premièrement, le Chapitre 1 fourni un résumé du problème et introduit les buts,

objectifs et majeures contributions de ce travail. Ensuite, le Chapitre 2 fourni une revue littérature de travails connexes, qui surligne le besoin de nouvelles approches fondées sur le risque. Le Chapitre 3 propose une méthode qui intègre l'incertitude géologique au design de phases minières pour les complexes miniers avec multiples courants de traitement. Les phases minières sont utilisées pour guider le calendrier de production, assurer une grande la valeur nette actualisée pour celui-ci et fournir des quantités consistantes de matériaux précieux au courant de traitement. Une application à la mine Escondida Norte, Chile, indique que la méthode stochastique de design de phases minières génère une réduction de 61% du risque sur le tonnage envoyé aux courants de traitement comparé aux phases minières générées de façon déterministe avec le logiciel Blasor de BHP Billiton. Ce résultat est produit grâce à la redistribution du risque dans le design des phases minières. Cette méthode est facilement intégrable dans un système commercial d'optimisation séquentiel du design de la mine.

Le reste de cette thèse considère un contexte plus général pour le design de complexe minier où plusieurs composantes compliquées et inter reliées, comme l'abattage, le traitement, la distribution et la commercialisation, sont optimisées simultanément. Le Chapitre 4 décrit le complexe minier comme une chaine de valeur de ressources minérales qui est détenue par une compagnie minière qui a pour objectifs d'optimiser les politiques de destinations pour le minerai extrait ainsi que l'utilisation des différents courant de traitement pour satisfaire les demandes des clients ou maximiser la chaine de valeur. Selon le type de produits comme le minerai de fer, le charbon, le nickel ou le cuivre et selon la géographie ou la géologie, la complexité de la chaine de valeur

de ressources minérales (i.e. homogénéisation, traitement, distribution) peut varier substantiellement. Des recherches existantes ont mis l'accent sur le développement de modèles spécifiques à chaque application ce qui n'est pas nécessairement utile pour le cas d'autres complexes miniers. Par conséquent, une méthodologie unifiée est proposée qui permet de modéliser et d'optimiser ces systèmes complexes. Une étude de cas pour la chaine de valeur de nickel latérite Onça Puma est utilisée pour surligner le besoin d'intégrer l'incertitude quant aux politiques de destinations, particulièrement dans des gisements de minerai complexe. Les résultats démontrent que l'optimisation stochastique permet de générer une politique de destinations complexe qui satisfait et réduit le risque relié aux contraintes serrées d'homogénéisation et aux cibles de production des usines de traitement sans compromettre la valeur économique.

Le Chapitre 5 généralise ce modèle unifié et cette méthodologie d'optimisation en incluant la capacité d'optimiser simultanément les politiques de destinations et de courants de traitement ainsi que le calendrier de production de plusieurs mines. Une étude de cas faite appliquée à une mine de cuivre et d'or démontre que l'équivalent déterministe de l'optimiseur est capable de générer une solution possède une valeur nette actualisée améliorée de 4% par rapport à un planificateur de production déterministe utilisé en l'industrie. De plus, l'optimiseur stochastique est capable de générer un design, qui en plus d'augmenter de 6% la valeur nette actualisée par rapport son équivalent déterministe, est aussi capable d'atteindre les cibles de production et de réduire le risque sur les minerai traité, basé sur un profil de risque P-50. Tout ceci aide à assurer que les prévisions financières sont atteintes en pratique.

Les modèles d'optimisation stochastiques existants pour la planification de production minière ont mis l'accent sur la réduction du risque relié à l'atteinte des cibles de production. Le chapitre 6 adresse le défi d'optimiser simultanément la production et le traitement de minerai avec la possibilité d'augmenter ou de décroître les contraintes de capacités via les dépenses en capital. Dans une étude de cas pour un complexe minier de cuivre, l'optimiseur est permis d'ajuster les taux de production par l'achat et le remplacement de pelles et de camions en optimisant simultanément le calendrier de production, les politiques de destinations ainsi que les courants de traitement. Les planificateurs de production commerciaux existants ne peuvent performer cette optimisation simultanée ainsi que de permettre le design de ces modèles détaillés, donc aucune comparaison ne peut être fait avec des solutions commerciales. Dans cet exemple, l'optimiseur stochastique défini un taux de production réalisable en pratique qui alimente constamment le courant de traitement associé au broyeur avec un minimum de risque, ce qui est d'une importance critique pour les mines à minerai de faible qualité. En plus de comprendre la variabilité de la qualité du minerai extrait et son incertitude, le design stochastique génère une augmentation de 5.4% de la valeur nette actualisée par rapport à la même méthode utilisant seulement un modèle de gisement estimé.

Ce travail fait avancer le domaine connexe par le développement de nouveaux modèles et méthodes pour l'optimisation globale de complexes miniers avec incertitude. Ceci est principalement réalisé par cinq contributions majeures. Premièrement,

une méthode d'optimisation stochastique globale est développée pour optimiser simultanément le calendrier de production de multiples mines, les politiques de destinations, les courants de traitements et les dépenses en capital pour le design des capacités, par opposé à l'état de l'art existant qui peut adresser quelqu'un de ces aspects sans considérer l'optimisation simultanée et sans diviser le modèle globale en sous-problèmes. Deuxièmement, une nouvelle approche unifiée est développée qui permet de modéliser et d'être appliqué à différents types de complexes miniers. Un résultat de cette approche généralisée est la capacité d'intégrer facilement des relations non-linéaires dans les modèles d'optimisation, une limite des méthodes déterministes et stochastiques existantes. Troisièmement, une nouvelle approche est développée pour modéliser la valeur économique des produits vendus plutôt que celui du minerai extrait. Les méthodes et modèles existants sont limités par l'hypothèse que chaque bloc d'exploitation possède une valeur économique et donc le courant de traitement optimal est connu a priori. Le bloc d'exploitation est donc traité et vendu séparément des autres blocs et, dans certains cas, ceci peut produire une sous-évaluation substantielle des ressources. En utilisant le modèle unifié, il est possible d'évaluer le potentiel économique des produits aux points de vente plutôt que de faire une hypothèse irréaliste au niveau des blocs. Quatrièmement, les solveurs développés sont efficients et basés sur des méthodes métaheuristiques. Une combinaison d'algorithmes d'optimisation par essaims particulaires et de recuit simulé modifié est développée pour optimiser les divers aspects de ce problème doptimisation globale. Ces méthodes n'ont jamais été combinées pour l'application au domaine minier et donc ceci requiert le design de nouvelles méthodes afin de

modifier le design et s'assurer que l'optimiseur n'est pas coincé dans un optimum local. Finalement, les performances, avantages et limites des modèles et des méthodes présentés sont analysés grâce à des applications sur des exemples de grandes tailles de la vie courante. Les résultats renforcent le concept qu'il est possible de non pas juste réduire le risque d'écart entre les cibles de production attendues et réelles, garantissant l'atteinte des prévisions financières, mais aussi augmente la valeur nette actualisée des opérations.

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CHAPTER 1
Introduction

1.1 Overview

A *mining complex* is a term used to describe a business operation that generates materials from open pit or underground mines, which are subsequently transformed from a bulk material into a group of saleable products via a set of processing streams (Fig. 1–1), in addition to the logistics of transportation and marketing of the products produced. The primary goal for optimizing a mining complex is to maximize the net present value (NPV) of the cash flows generated from mining, processing and selling the metals, while obeying operational and environmental constraints that are unique to each operation. Traditionally, the optimization of a mining complex is treated as independent operations. Mining engineers design and optimize the extraction sequence of the material from the available mines; this optimization is generally a step-wise procedure that is performed for each mine independently. Metallurgical engineers optimize the treatment or processing of the incoming materials from the mines in order to maximize its value. The marketing and sales team sell the products generated to a set of customers, generally in the form of contracts, or the open market, such as a metal exchange. The interaction between these optimization steps often comes from an agreement between the mining and metallurgical engineers and the financial team on what available materials can and should be sent for processing and sold, which is often defined by capacity and blending constraints at the various

locations in the mining complex and customer demand. These optimization steps are strongly interrelated, and optimizing the mines separately from the processing and marketing leads to undervaluing the resources and under-utilizing the mining complex's capabilities.

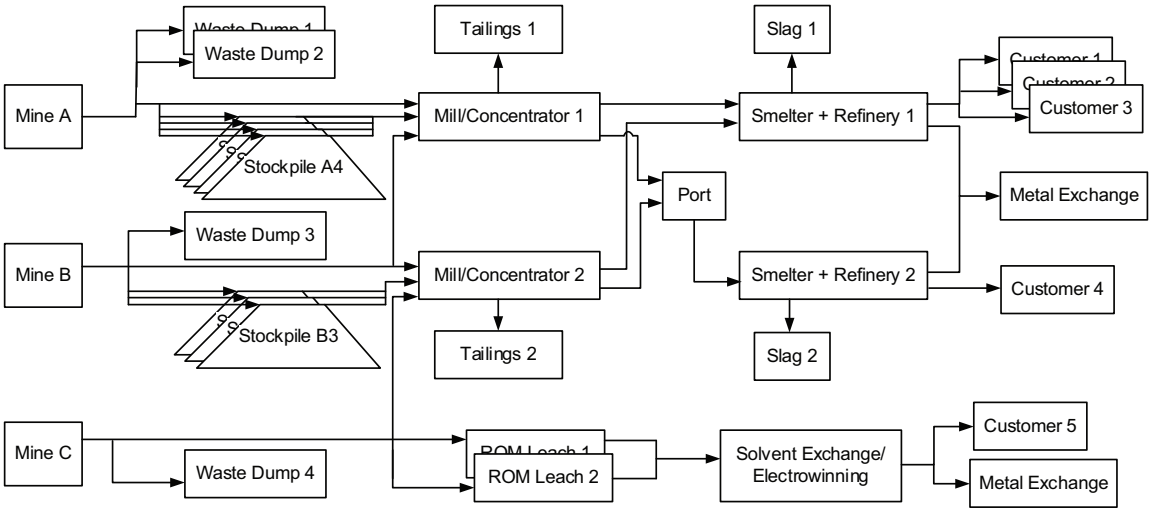

Figure 1–1: A hypothetical mining complex with multiple mines, processing streams and sales options.

Global optimization for mining complexes aims to holistically optimize the extraction sequence of materials from the mines and defines the optimal use of the processing streams to maximize the utility of the extracted materials while meeting contractual obligations. One of the primary challenges related to globally optimizing mining complexes is related to the stockpiling, blending and non-linear transformations (often a result of chemical interactions) of the materials at the various locations in the processing streams. Given that these challenges require complex, non-linear optimization formulations that are computationally difficult to solve, engineers often

resort to severe, simplifying assumptions that result in a linear optimization model that can be solved more efficiently. Often, these simplifying assumptions are implicit in the optimization models by defining the economic value of a unit of mined material (e.g. block), which ignores the impact of blending multiple units together, the optimal processing streams used, and the impact that the unit of material has on capacity and quality constraints at the various locations in the mining complex. In order to overcome these simplifying assumptions, it is necessary to accept that these non-linearities are often required to accurately model the mining complex and employ or devise new optimization methods that can efficiently generate useful solutions in the presence of these non-linearities.

Recently, several stochastic optimization models have been proposed to integrate and manage various forms of risk, such as the geological uncertainty of materials and grades, into the optimization of mining operations. In these models, material type uncertainty has a direct impact on the quantities of materials available for the various processing streams, and multi-element grade uncertainty is used to quantify the risk related to valuable metals, deleterious elements or minerals that are treated in the various processing streams. These methods aim to optimize the mine design and production schedules in order to not only maximize the operation's NPV, but also reduce the risk associated with not being able to meet pre-defined production targets or blending targets over time. These models, however, often neglect the optimization of other critical aspects, such as where to send the mined materials, the optimization of the down-stream processes that are used to treat the materials, and the use of capital expenditures to design the critical bottlenecks in the system. By ignoring

these key aspects, the result is a locally optimum design for the mining complex. In order to adequately quantify and manage risk in mining complexes, it is not only necessary to expand the existing stochastic optimization formulations to model mining complexes, but also explore efficient optimization methods that consider all components of the mining complex simultaneously.

1.2 Goals and Objectives

The goal of this thesis is to develop a unified model and computationally efficient stochastic global optimization methodology that addresses the holistic optimization of open pit mining complexes, including the multi-mine production schedules that define the extraction sequence and available materials over time, the destination policies that define where mined material is sent, the use of the various processing streams through to the final products that are sold and the use of capital expenditures to design the critical bottlenecks in the system. This requires developing a unified methodology that permits modelling and dynamically evaluating the non-linear transformations of blended products in the mining complex at various scales, without relying on severe, simplifying assumptions to maintain a linear model. By integrating both geological and material type uncertainties directly in the model, it is therefore possible to manage the risk related to the various products and elements across the entire mining complex. In order to achieve this goal, the related objectives are outlined as follows:

i Review the traditional deterministic and stochastic mine optimization frameworks, and outline their limitations when adapting these concepts to optimizing mining complexes.

ii Explore new methods to directly integrate both material type and multi-element uncertainty into aspects of mine design optimization, such as pushback design.

iii Develop a new, flexible modelling and stochastic optimization framework for the downstream aspects of mining complexes, including the definition of destination policies for extracted materials that are more appropriate for optimizing mining complexes or mineral resource supply chains in the presence of stockpiling, blending and non-linear transformations of materials.

iv Develop a computationally efficient method for stochastic global optimization, that addresses multi-mine production scheduling, destination policy decisions and the optimal use of processing streams in order to reduce technical risk in the mining complex and unlock higher value.

v Incorporate capital expenditure decisions directly in the global optimization model, which may, for example, be used to optimize the number of trucks and shovels at the various mines, or decide when it is best to invest in new processing streams to improve the financial and operational performance of the mining complex.

vi Conduct full-field tests with real-world and large-scale mining complexes in order to analyze the results and to document the benefits and limitations of the proposed methods.

vii Outline the limitations of the developed methods and suggest opportunities for future research.

1.3 Thesis Outline

This thesis is organized into the following chapters:

- *Chapter 1* provides an introduction to the work presented herein and briefly discusses the topics addressed in this thesis.
- *Chapter 2* provides a literature review of traditional optimization frameworks for mine design and life-of-mine production scheduling with uncertainty, downstream optimization and work related to the global optimization of mining complexes.
- *Chapter 3* describes a method to integrate material and multi-element uncertainty into pushback design using a stochastic optimization model that is optimized using simulated annealing. The benefits of this method are documented through a case study at the Escondida Norte copper mining complex, Chile.
- *Chapter 4* describes a flexible modelling procedure for mining complexes or supply chains with multiple mines, materials and processing streams. A stochastic optimization model aims to generate robust destination policies for materials that are extracted from a fixed multi-mine production schedule. The benefits of this method are demonstrated through an application at the Onça Puma nickel laterite blending complex, Brazil.
- *Chapter 5* expands on the methods developed in the previous chapter by developing a stochastic optimization framework that simultaneously optimizes multi-mine production schedules, destination policies and processing streams. The benefits of this global optimization method are demonstrated through a full-field test for a real-world copper-gold mining complex, whose identity is withheld for confidentiality.

- *Chapter 6* improves on the previous method by permitting the optimizer to make capital expenditure decisions that increase or decrease capacities in the mining complex. A full-field test for a major copper mining complex, whose identity is withheld for confidentiality, demonstrates the ability of the optimizer to not only generate a production schedule, destination policy and the use of stockpiles, but also design the appropriate levels of mine production by simultaneously optimizing truck and shovel purchases.
- *Chapter 7* concludes this thesis by revisiting its major contributions to the stochastic global optimization of mining complexes, and recommends related future work.

1.4 Original Contributions

Stochastic mine design and production scheduling is a recent paradigm shift that has consistently shown the ability to manage the distribution of risk over time, thus ensuring that early and critical cash flows are attained, and increase the net present value of the operation by considering the true variability (low- and high-grades) of the materials. The work contained in this manuscript-based thesis provides five key contributions to this new paradigm:

I. A new, risk-based approach to long-term planning for mining complexes that simultaneously optimizes life-of-mine production schedules, destination policies, processing streams and capital expenditures for capacity design.

All existing industry-standard and state-of-the-art methods are unable to holistically

optimize these critical aspects without first dividing the problem into smaller sub-problems (i.e. ultimate pit limits, pushbacks, or aggregates of blocks), and assume pre-defined mining and processing capacities. This leads to a sub-optimal use of the non-renewable resources, the individual components of the mining complex, and substantial financial resources. The work contained in this thesis not only overcomes these limitations by simultaneously optimizing all aspects of the mining complex without divvying-up the global optimization problem, but also addresses the core issue of how to manage uncertainty, such as geological (supply) uncertainty, directly in the optimization models.

II. Creating a unified approach for generating intricate models of material flow and transformation in mining complexes, regardless of the commodity produced or processing and distribution methods employed.

Industry-standard methods and practices do not provide enough flexibility to accurately model the intricacies, needs and objectives that are specific to a mining operation; they tend to focus on the economic value of blocks, capacity constraints in tonnages and have limited integration of blending constraints. Existing state-of-the-art methods are tailored to a specific commodity, or geological and geographical conditions. This leads to specific models of the processing streams and methods of distribution that are unique to the given operation, and limits the applicability of the models to other mining complexes. The work in this thesis develops a new, two-tiered approach to modelling mining complexes. First, primary attributes are defined, such as metal quantities and tonnages. The flow of these attributes through

a mining complex, from the mines to the customers, can be modelled using three fundamental sets of decision variables, regardless of the complexity of the processing streams or distribution methods: production schedules, destination policies, and processing stream decisions. It is then possible to create detailed representations that define the transformations and non-linear interactions that occur as material is transformed from an input to an output product in the processing streams, based on these primary attributes. This permits highly complex models that can integrate practical, but extremely important, aspects that are unique to each operation, such as truck cycle times and variable shovel rates for selectivity at the mine level, and variable throughput rates, mill energy consumption, grade-recovery curves in the processing streams.

III. A new approach that assesses the economic value of the *products sold*, rather than the economic value of the *materials mined*.

The existing approaches to mine optimization assumes that each block of material in the orebody model has its own economic value. This results in evaluating the value of a discrete volume of material in isolation from others, and also assumes that explicit knowledge of its optimal processing stream is known a priori. This block-based valuation can lead to a severe misrepresentation of the economic viability of a mineral deposit, particularly for mining complexes that use blending to ensure a consistent quality of products sold, such as iron ore. In reality, multiple mining faces and short-term stockpiling are strategies that are frequently employed to blend and homogenize blocks to provide a feed of consistent quality to the various processing

streams, and is therefore erroneous to assume that a block is treated and sold in isolation. Through the development of the unified modelling approach, it is possible to properly model the flow and (potentially non-linear) transformation of metals through the mining complex, and assess the economic values of the products that are sold where the transaction occurs. For example, in an iron ore mining complex, the value may be calculated according to the quantity and quality of products sent from the port (a function of contractual agreements); in this case, it is advantageous to evaluate the blended material sent from the port, rather than the blocks in isolation.

IV. Unique adaptations and applications of metaheuristic-based solvers for the stochastic global optimization of open pit mining complexes.

In order to provide solutions for large-scale, industrial applications, the optimization models developed in this thesis are solved using a unique adaptation of metaheuristic solvers. Previous efforts have focused on a single metaheuristic to solve the open pit mine production scheduling problem. This thesis proposes a novel combination of the simulated annealing and particle swarm optimization algorithms to optimize the critical decision variables that govern the design and flow of materials through a mining complex. This requires developing several new methods to perturb the key decision variables with minimal computational overhead — most of which have not been previously optimized using metaheuristics (destination policies, processing streams and capital expenditures). The simultaneous optimization of these decision variables using metaheuristics poses new challenges that have not been previously documented, such as the relative impact that each set of decision variables has on

the objective function value, which leads to converging on a local optima. Given the strong interrelationship between the key decision variables, a modified simulated annealing algorithm is developed in this thesis to ensure that the optimizer maintains these relationships when exploring the solution space, and helps to ensure that the solution is not trapped in a local optimum.

V. Full-field testing on real-world mining operations to evaluate the actual performance of the proposed methods.

When possible, a comparison is made to an existing design or a design that is generated using industry-standard methods. Comparisons between a deterministic-equivalent and stochastic optimizers are made using the methods developed in this thesis, and highlight the necessity to properly assess the geological variability and spatial uncertainty in the orebodies when optimizing mining complexes. The results consistently reinforce the practical benefits of the stochastic optimization paradigm, including explicit risk management, which helps to guarantee production and financial forecasts, and the economic value added by adopting a stochastic optimization framework.

CHAPTER 2
Literature Review

2.1 Outline

This chapter provides a review of the literature pertinent to the global optimization of open pit mining complexes, and comprises four sections. Section 2.2 provides a review of existing work in the stochastic optimization of mine design and long-term production scheduling; as will be seen, the existing work has primarily focused on the mining decisions, and do not integrate the downstream processes of a mining complex. Section 2.3 reviews some of the work related to downstream optimization with stockpiles and blending; this work is generally decoupled from the mine design and production scheduling problem, and the integration of uncertainty in these models for mining applications has only recently garnered attention. Section 2.4 discusses some of the more advanced deterministic and stochastic models that attempt to simultaneously optimize aspects of mine design and production scheduling with downstream optimization, which leads to the discussion of a need for a framework that tightly couples these two aspects in a stochastic global optimization framework. It is noted that in order to accurately quantify the impact that geological risk has on a mining complex, it is necessary to have accurate models of the underlying geological conditions and uncertainty. Section 2.5 provides a brief review of some of the recent work pertaining to the geostatistical simulation of mineral deposits, with

a focus on computational improvements and newer multiple-point and high-order simulation frameworks.

2.2 A Traditional Open Pit Mine Design Optimization Framework

2.2.1 Sequential optimization of open pit mines

The primary objective when optimizing the use of a mineral resource is to create a mine design and production schedule that maximizes the value of the metal or products produced, while obeying technical constraints, such as mine production and processing capacities, blending constraints and geotechnical stability or access constraints. A step-wise optimization framework is typically used in practice [38, 167] to partition the global optimization problem into computationally manageable steps by successively reducing the number of decision variables involved as the complexity of the optimization model increases (Fig. 2–1). In the traditional framework, the ultimate pit limit is first defined, which aims to define a pit contour that maximizes the economic value of the material contained within, and is often generated using the Lerchs-Grossmann algorithm [113, 162] or a network flow algorithm [75, 134]. By parameterizing the economic value of the ore blocks within the deposit, it is possible to obtain smaller volumes within the ultimate pit limit, called nested pits, which can be grouped to obtain a pushback design that guides the sequence of extraction from the ultimate pit [82]. Pushbacks are large volumes of material that can be mined independently and may be mined over several years (i.e. non-contiguous time periods). They are used to guide the life-of-mine or long-term production schedule, which defines the material that is extracted on an annual basis, hence the discounted cash flows over time [37, 39, 61, 88, 160]. The annual production schedule, and the

resulting cash flows, may then be used to optimize the cut-off grade policy and the use of available stockpiles [109, 145]. An optimal cut-off grade policy, in its most simplistic form, is a policy that decides whether or not to send a block of material for further processing, and accounts for the opportunity cost that is incurred by deferring the treatment of other valuable material. Given the strong relationships between these components, this process is repeated iteratively until a final design is obtained. It is noted that this sequential optimization methodology is frequently used in the mining industry because it is commonly implemented in commercial software; as will be seen in this section, much of the work in stochastic optimization has focused on optimizing a few of these components simultaneously. A modern, integrated optimization approach [154] for open pit mine design and production scheduling that is more adept to global optimization is discussed in Sect. 2.4

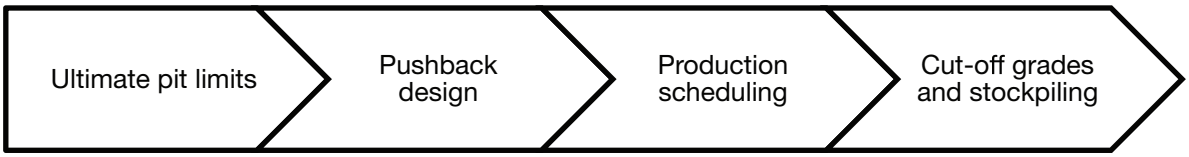

Figure 2–1: A traditional open pit framework for mine design and production scheduling.

2.2.2 From deterministic to stochastic optimization

Historically, mine design and production scheduling optimization frameworks use only a single geological model as input [12, 58, 61, 88, 96, 113, 160]. This model is often generated using geostatistical estimation methods [41, 84], which

defines the expected value of the grade for a block of material in the mine. The economic valuation of a block of mined material is a non-linear transfer function that is traditionally calculated prior to optimization in order to define ore and waste blocks. While the vast majority of the existing optimization models for open pit mine production scheduling are linear formulations, an optimized schedule generated using an average input model will not necessarily perform well on average because of this non-linear valuation function.

Stochastic geostatistical simulation methods [34, 42, 69, 89, 90, 92] have been developed over the past several decades to overcome the shortcomings of traditional estimation methods. These methods aim to generate a set of equally probable simulations that better represent the univariate distribution and spatial correlations for the attributes of interest (e.g., metal grade), and may be used as a group to quantify the geological uncertainty in a mineral deposit. With the emergence of the simulation methods, several studies began to investigate the impact of uncertainty on ore reserve calculations, ultimate pit definition and production scheduling. Ravenscroft [143] concludes that the existing deterministic optimizers are unable to accommodate quantified risk, and that new optimization methods need to be developed. Dimitrakopoulos et al. [45] discuss an application at a gold mine, where an optimized mine design is generated using an estimated orebody model, and is subsequently tested with a set of geological simulations. The authors note that the NPV of the estimated orebody is misleading; the simulations indicate that there is only a 5% chance of actualizing the indicated value, and the median NPV of the simulations for the deterministic design is 25% lower. Additionally, the authors observe a 12.5%

shortfall of ore production from what the estimated orebody model indicates, and that production should cease one year earlier. As a result of the non-linear transfer functions present when optimizing mine designs, the solutions generated from deterministic models and methods can be severely misleading and result in unrealistic and undesirable production rates and cash flows. These studies demonstrate a paradigm shift from a deterministic optimization framework, whereby the input is assumed to be 100% correct, to a framework that attempts to consider uncertainty in order to improve decision making.

Dimitrakopoulos et al. [48] perform a study that attempts to integrate uncertainty into decision making for choosing a mine design based on a maximum upside, minimum downside approach. The authors use traditional, deterministic optimization methods to create a mine design for each geological simulation independently, and compare their performance when tested with the other geological simulations. The quality of a design can be assessed based on the probability of being above or below a set of key performance indicators, such as minimum annual return, ore tonnage or metal production. The authors then rank the designs based on maximizing the upside potential (the potential of performing better than expected) and minimizing the downside loss (the risk of performing worse than expected); the highest ranking design is then selected as the preferred design. The authors demonstrate their proposed method through an application for an epithermal gold deposit, where four different mine designs are analyzed and ranked according to the mine owner's desired minimum annual return while simultaneously meeting a minimum of 70% chance of having more than one million ore tonnes. The authors observe that there

is ambiguity in the selection process; for a given mine design that has a high upside potential on return with minimal downside risk at the beginning of the mine life, the same design may have substantially lower upside potential and downside risk at the end of the mine's life. While this method is conceptually easy to apply, it is essentially generating a set of risk analyses for a group of mine designs that are only optimal for a single geological simulation, and choosing which design better satisfies the decision-maker's goals. Unfortunately, this form of posterior analysis does not attempt to directly manage uncertainty in the optimization step, and therefore guarantees a sub-optimal solution. This highlights the need to shift from deterministic optimizers, which blatantly ignore risk, to stochastic optimization methods, which explicitly manage risk in the mathematical formulations [44].

2.2.3 Probability-driven mine design and production scheduling

More recent optimization formulations attempt to directly integrate geological uncertainty directly into mine design and production scheduling through the use of probabilistic optimization models. Ramazan and Dimitrakopoulos [140] propose a methodology and model that first requires optimizing the schedules individually to obtain a probability distribution for a block being mined in a given period. The authors attempt to maximize the expected net present value, which is defined by the probability of a block being mined in a given period and its expected net present value, and additionally create a spatially smooth schedule for equipment access. This method, however, requires solving a mixed integer program (MIP) for each geological simulation, which can be computationally infeasible for large deposits.

Dimitrakopoulos and Ramazan [49] propose a similar formulation that attempts to minimize the deviations from not having a probability of 100% for a desirable quality (e.g., metal or deleterious element content), while maintaining a spatially smooth schedule; this method is particularly useful for multi-element mines that require blending constraints, and does not require a MIP optimization for each geological simulation. The authors introduce the concept of *risk discounting*, whereby the penalty cost associated with not having the desirable qualities is reduced for each period of time using a discount factor that is similar one used in the calculation of the net present value. The risk discount rate is an input parameter to the optimization model and relates to the heterogeneity of the orebody and the willingness of the mining company to accept risk in the short-, medium- and long-term. Using an extremely high geological risk discount rate effectively forces the optimizer to be short-sighted about the level of risk in the design; the optimizer will seek to only extract blocks with a high level of certainty in the first few periods, but ignores the desire to also meet production targets in the medium- and long-term. A low risk discount rate (i.e. 0) treats the risk in early periods the same as the periods at the end of the mine life, which may introduce a higher level of risk in the short-term than the company is willing to accept, particularly when there is a need to pay back creditors. As a result, this parameter should be calibrated for each operation and should account for the company's willingness to accept and distribute risk over time. It is noted that this formulation does not directly attempt to maximize the net present value of the schedule, but attempts to have a high probability of meeting ore production targets at the beginning of the mine life, and defer riskier material

(i.e., lower probability of having a desirable quality) to later periods of the mine life. The authors test their formulation on a nickel laterite deposit, where the goal is to achieve a target nickel grade of the ore sent to the processing plant. The results indicate that the net present value, when compared to the deterministic model, remains relatively unchanged (2% difference), however the probabilities of meeting the ore production targets are much more appealing. Whereas the deterministic schedule randomly distributes these probabilities through time, the probabilistic schedule shows descending probabilities from the beginning to the end of the mine's life, meaning that the schedule is not controlling the risk at the expense of NPV. This method, however, focuses on the probabilistic representation of each block independently of other blocks (i.e. the probability of each block having a desirable quality). This is a strict, binary (good-or-bad) representation of uncertainty for each block, and ignores the localized values and uncertainty of nearby blocks (referred to herein as *joint local uncertainty*) that could be better represented or controlled using a set of geological simulations. The inclusion of joint local uncertainty in stochastic optimization models, represented via a series of simulations, is preferential because it empowers the optimizer with the ability to blend the risk associated with nearby blocks.

Grieco and Dimitrakopoulos [70] propose a probabilistic MIP model that aims to generate stope designs for underground mines, including the size, location and number of stopes. Grade uncertainty is integrated into the optimization model by requiring that the probability of the stopes being above a specified cut-off grade is also above a specified minimum threshold. By increasing the minimum probability threshold and re-optimizing the model, the optimizer seeks a stope design with an

increased probability of being above a given cut-off grade, often at the expense of a decrease in tonnage for the stopes. Similar to the previously mentioned work, this method does not directly integrate joint local uncertainty, represented by a set of geological simulations, into the optimization model. Additionally, this method does not attempt to generate a production schedule for the stopes that are generated.

2.2.4 Managing ore production risk with simulated annealing

Godoy [65] and Godoy and Dimitrakopoulos [66] propose a method to manage risk in meeting production targets for open pit production schedules that incorporates joint local uncertainty, thus better accounts for any spatial correlation in the geological variables that are represented in the simulations. The objective is to create a production schedule that minimizes the expected deviations from ore and waste production targets, where the deviations are measured using each equally probable geological simulation, rather than a probability for each block. The authors first generate a mine production schedule for each of the geological simulations independently using traditional deterministic methods; this is used to generate a probability distribution for each block belonging to a period. A stochastic production schedule is then generated using the simulated annealing metaheuristic [60, 98], which is based on the Metropolis algorithm [127]. It commences with a starting (input) schedule, and randomly selects blocks that may change production periods without violating slope stability constraints; the transition probability for a block's new extraction period is based on the distribution defined by the schedules for the simulations. The algorithm is permitted to explore the solution space by accepting sub-optimal swaps at the beginning of the algorithm, and the chances of accepting sub-optimal changes

are gradually reduced as the algorithm iterates. The efficacy of the method is demonstrated through a case study at a gold deposit, where the authors demonstrate that the risk of not being able to meet ore production targets is reduced from 13% for the traditional deterministic design to 3% for the stochastic design. Moreover, the authors note a 28% increase in the stochastic design's NPV over the deterministic design, which reinforces the concept that one does not need to compromise value in order to manage risk.

Leite and Dimitrakopoulos [111] apply this simulated annealing-based method to a copper mine, which results in a 15% increase in NPV over the deterministic design (when testing the simulations). Additionally, the authors note that the stochastic schedule has a mine life that is one year shorter than the deterministic schedule, which is a result from the estimated model having more ore material than the simulations in this case study. It is interesting to note that the optimization formulations for both experiments do not explicitly attempt to maximize the NPV of the schedule, yet both experiments have a higher NPV. This is a result of the fact that the transition probability for moving a block between periods is based on the probability distribution, which is in turn defined by each simulation's optimal production schedule that tries to maximize the NPV. Rather than explicitly incorporating NPV in the model's objective function, the potential increase in NPV is accounted for implicitly when changing a block's extraction period.

One of the challenges of the simulated annealing-based models is the lack of clear method to calibrate the algorithm's parameters or to define which starting schedule to use as input for the algorithm. Albor and Dimitrakopoulos [1] investigate these

challenges by performing a sensitivity analysis using the same copper deposit as Leite and Dimitrakopoulos [111]. Given the long pre-processing time required to generate a mine design for each geological simulation, the authors suggest that simulations and their respective production schedule should be added sequentially to the simulated annealing-based optimizer. Through a case study, the authors demonstrate that the quality of the solution, in terms of meeting ore and waste production targets, stabilizes after using 10 geological simulations; this is confirmed by a stabilized coefficient of variation for the block's grades and the number of blocks with 100% chance of being extracted in a single period. The authors also state that the quality of the final stochastic production schedule becomes increasingly dependent on the starting schedule selected when using fewer geological simulations and respective schedules. This study helps to highlight an important concept related to the stochastic optimization of mine production schedules: for increasingly large volumes of materials extracted and treated, particularly up to the scale of interest in long-term production scheduling, the variability of the material decreases or plateaus – the impact of the potentially high variability that arises from each block is mitigated by blending risk from other that are blocks mined in the same period. As a result of this volume-variance relationship [93], it is generally not necessary to consider hundreds of simulations in the optimization model, because a small group of simulations is often sufficient to accurately describe the variability for an appropriate scale of a mining complex.

The development of the simulated annealing-based optimization framework is a substantial improvement over the probabilistic methods because it considers the

joint local uncertainty that is represented through geological simulations, and also does not require a commercial mathematical optimizer (however, it does require commercial mine optimization software); once the schedules are generated for each simulation, the method is computationally efficient and has been used to optimize large deposits in a reasonable amount of time. The method, however, suffers from four major limitations: i) it does not integrate geological risk discounting to defer riskier material to later periods, which would contradict the use of the block transition probabilities; ii) it does not explicitly maximize the net present value of the production schedule (this is implicit when generating the schedules for each simulation); iii) the method focuses on ore and waste production targets, and does not accommodate grade blending constraints, which is a critical aspect of many mining operations; and iv) the algorithm does not guarantee mathematical optimality.

2.2.5 Stochastic mine production scheduling with stochastic integer programming

Ramazan and Dimitrakopoulos [139, 141, 142] propose a two-stage stochastic integer program (SIP) with fixed recourse [16] to address many of the limitations of the simulated annealing-based framework proposed by Godoy [65]. A two-stage SIP is comprised of two sets of variables: the first-stage variables, which are designed to be robust to random events, and the recourse variables, which are used to minimize adverse affects that are result of the first-stage decision variables coupled with the outcomes of the random events. The authors propose a SIP formulation that attempts to maximize the expected net present value of the production schedule, which is defined by the first-stage decision variables, and simultaneously minimize

the deviations from ore processing and mine production targets, grade blending constraints and metal production capacities, which are the recourse variables that are calculated for each geological simulation. The mixed integer linear model is solved using a commercial optimizer, thus provides a better idea of how far the design is from optimality using a duality gap. The authors incorporate geological risk discounting as a means to control the deviation (recourse) variables to force the optimizer to defer risk to later periods in the life of the mine, when more geological information is available.

Leite and Dimitrakopoulos [112] test the SIP formulation on the copper deposit used in [111], where the authors note a 29% increase of expected NPV of the stochastic schedule over the performance of the deterministic schedule when testing with simulations. The authors demonstrate the capability of the SIP formulation to accurately control the risk profiles for the ore and total mine production over the life of the mine, particularly when compared to the deterministic schedule, where there is less than 5% chance of actually producing the desired ore tonnages. Moreover, the authors present a sensitivity analysis to document the impact that the geological discount rate has on the resulting production schedule; they note that after increasing the geological risk discount rate above 10%, there is no longer a substantial impact on the ability of the optimizer to meet production targets. This study highlights the ability for the optimizer to strike a balance between extracting blocks of high grade (and potentially more risk) with blocks of lower economic value but more certainty in grade.

Given the success of the SIP formulation at integrating risk into production scheduling, later research aims to expand on the basic formulation and integrate more realistic complexities. Benndorf and Dimitrakopoulos [14] adapt the model for an application at an iron ore mine. In this application, the authors are interested in investigating uncertainty in multiple elements in order to satisfy target metal grades and blending constraints and guarantee that the ore produced is of satisfactory quality. Additionally, the authors introduce smoothness constraints, similar to Ramazan and Dimitrakopoulos [140], to produce a feasible schedule for equipment mobility. The authors state that the SIP model only has 5% and 20% deviations from the SiO_2 targets in the first two years of production, respectively, whereas the deterministic schedule shows an average deviation of 30%; this is a substantial improvement that aids in quality assurance for the customers that receive the materials from the mine. The authors also provide a discussion regarding the penalty costs that are used to control the deviations from production targets, and demonstrate the effects of an increasing penalty cost, both on the risk profiles and on the schedule itself. Naturally, an increase in penalty cost forces the optimizer to better satisfy the constraints. It is noted, however, that there is a limit to the effect that these penalty costs may have on the optimization model: the optimizer is not capable of completely eliminating risk and, after a certain point, increasing the penalty cost will not drastically change the design.

Jewbali [85] integrates short- and long-term uncertainty through the use of simulated short-scale future grade control drilling in the SIP model. The reason for integrating these two scales of information is because the short-term production

schedules often deviate from long-term production schedules, which is a result of changes in ore and waste classification when performing grade control drilling; this method aims to help reconcile the short- and long-term scales of information in a single SIP model. The authors demonstrate the method through an application at a gold deposit, where the stochastic schedule indicates an increase of 3.6 million tonnes of ore, 2.6 million tonnes of metal and 7.7 million dollars in NPV, when compared to the mine's actual long-term production schedule. These increases are a result of the fact that the dense grade control information indicates a higher average grade above the cut-off and a higher ore tonnage, than the exploration drilling that has been traditionally used.

Results from these studies consistently demonstrate that a stochastic optimizer is able to generate a production schedule with a higher net present value and less risk than a deterministic design. This is an extremely counter-intuitive concept for many, as a popular belief is that one must sacrifice economic value in order to manage or reduce risk. It is necessary to clearly understand what is being compared when claiming there is a risk-reward relationship. Birge and Louveaux [**?**] define the value of the stochastic solution as the difference between the objective function value for a stochastic solution optimized with a set of scenarios (called the "Recourse Problem", RP) and the expected value from a deterministic solution when tested with the same set of scenarios (called the "Expected result of using the Expected Value solution", EEV). The authors note that the objective function value for the RP is always better than the EEV; if it were not, then the solution for the RP would, by definition, not be optimal. In the context of mine production scheduling,

where the objective function evaluates both the net present value and the design's ability to meet production targets, this concept directly translates to having a higher net present value and (or) less risk. It is noted that a deterministic optimizer does not understand the concept of risk, and, as a result, when a deterministic design is introduced to risk, it generally doesn't perform as indicated. It is natural that a stochastic optimizer, which inherently understands risk, is able to generate a more realistic and higher-valued solution.

One of the major challenges associated with SIP formulations is the time required to solve it using commercial mathematical optimizers; for large-scale optimization problems with thousands to millions of blocks, it is often infeasible to even solve the linear relaxation of the integer formulation. More recent work has explored new methods to solve large-scale mining SIP formulations in a reasonable amount of time. Marinho [119] proposes a SIP model that focuses on modelling the first-stage decision variables as surface extraction decisions (i.e. the progression of the exposed surface), rather than block-based extraction decisions. This is an an extension of previous work done by Goodwin et al. [68] for deterministic production scheduling. The model is solved sequentially for each time period to find an approximation to the full-scale SIP model previously discussed. The sequential nature of the method, however, does not guarantee an optimal solution, particularly for cases where a large amount of pre-stripping is required at the beginning of the mine's life to access ore. Chatterjee and Dimitrakopoulos [32] propose a model that is solved using Lagrangian relaxation and the subgradient method sequentially for each production period. In the event that the solution is infeasible, a smaller MIP formulation is employed to

correct the violated constraints. The authors show for one example that the method is able to generate an approximation within 2% of the full SIP's linear relaxation value 37 times faster than a commercial optimizer. The sequential nature of the method, however, is similar to the previously discussed method and therefore does not necessarily perform well in general.

Lamghari and Dimitrakopoulos [106] propose a computationally efficient solution to the SIP model using the Tabu Search metaheuristic [64], and compare two diversification strategies to help the optimizer not get stuck in a local optimum. The authors note that one of the diversification strategies is able to, in one example, obtain a gap of 2.4% in 104 minutes, versus the 15 230.5 minutes it takes to solve the linear relaxation using a commercial optimizer. Lamghari and Dimitrakopoulos [107] propose a variable neighbourhood descent metaheuristic for solving a production scheduling SIP, and investigate the impact that the quality of a starting solution has on the quality of an SIP production schedule model that is optimized using metaheuristics. The authors test two sequential methods to create an initial production schedule prior to optimizing with the algorithm: the first is a method using a mathematical optimizer for the time-separated sub-problem and the second method is a greedy algorithm. The authors demonstrate that the initialization method impacts the quality of the final production schedule; for three different deposits, the final schedule that is initialized with a commercial optimizer has a slightly smaller gap (measured from the linear relaxation of the SIP model) than that of the schedule that is initialized with a greedy heuristic, however, the greedy heuristic outperforms the commercial optimizer-based method in terms of computing time.

Both methods, however, are orders of magnitude faster than only solving the linear relaxation of the SIP model. Lamghari and Dimitrakopoulos [108] benchmark a similar method that uses linear programming and variable neighbourhood descent for deterministic models, and note that the method finds new, tighter bounds for several benchmark problems [53] within a shorter timeframe than other recent methods, thus demonstrating that the use of metaheuristics as an optimization method can perform equally well as mathematical programming-based methods, and generally in substantially less time.

2.2.6 Integrating uncertainty in pushback design and ultimate pits

In addition to the sensitivity of the input parameters related to the simulated annealing algorithm, Albor and Dimitrakopoulos [1] discuss the impact of geological uncertainty on the ultimate pit limits. The majority of the previously mentioned work defines the ultimate pit to be scheduled using a single, estimated orebody model; this ultimate pit is often selected based on the nested pit shell that provides the highest (approximated) NPV using conventional, deterministic optimization methods. In practice, there is often a wide range of pit shells that may be selected as the ultimate pit with similar NPVs but different stripping ratios. Using the simulated annealing formulation, the authors extend the ultimate pit limit for the copper deposit case study by introducing an additional pushback, which is defined by additional pit shells. The stochastically optimized pit limits are shown to be 17% larger in total tonnage, increase the NPV of the schedule by 9% and increase the life of the mine by an additional year.

Early work with SIP models for mine production scheduling were limited by the ability of commercial solvers to optimize mines of realistic size. In order to reduce the size of the problem, a traditional optimization framework was often used, which requires generating the ultimate pit and pushback design to reduce the number of variables in the optimization model. The SIP scheduling model is then solved similar to a sliding time window heuristic, whereby only first few pushbacks and production periods (e.g. 2 and 5, respectively) are considered. After obtaining an optimal solution, the scheduled blocks in the first periods (e.g. 1 to 3) are removed from the model, and the process is repeated. Albor and Dimitrakopoulos [2] use a case study for a copper deposit to analyze the impact that pushback design has on the risk and value of a SIP production schedule. The authors generate mine designs that have 3, 5, 6, 7, 9 and 10 pushbacks, solve the SIP models [142] for each design and compare the results. The differences in the production schedules could be quite drastic; for example, a design with 5 pushbacks has a higher NPV and a lower maximum deviation from the ore production target than the design with 6 pushbacks. More importantly, the authors document a 30% increase in NPV over deterministic methods, which is attributable to better risk management, increased metal extraction and a larger pit limit. This study highlights the impact of the traditional step-wise optimization framework has on the value and risk of the final production schedule. The method, however, is computationally demanding because a SIP model needs to be solved for each pushback design, and fails to address the question of how to directly optimize pushback designs in the presence uncertainty.

Gholamnejad and Osanloo [62] investigate the question of how to directly generate pushback designs that integrate geological uncertainty (as opposed to the previously mentioned methods that select a design), with the goal of generating pushback designs that have material with high grade, low uncertainty and a low stripping ratio. The authors use a parameterized Lerchs-Grossmann [113] algorithm to generate a series of nested pits. Unlike the implementation used by Whittle [167], the authors attempt to integrate uncertainty by parameterizing the block's estimation variance (which is only affected by drilling density and the model of spatial continuity) rather than the block's economic value. This method, therefore, does not necessarily produce nested pits that only target high-valued ore blocks; it is conceivable that an early pushback may contain only low-grade blocks with small estimation variances. The authors compare their pushback designs to Whittle's deterministic design, and state that their method produces pushbacks with substantially lower risk. The method, however, makes the incorrect assumption during the development of their formulation that the block grades are normally distributed, and also incorrectly assumes that a block's kriging variance [84] from estimation is the same as the variance of the grade of the block. Not only does normally distributed random variables for geological phenomena not occur naturally, by definition, the kriging variance defines the error of estimation according to the model of spatial continuity and the locations of surrounding drillhole data. As a result, the penalty applied to a high-grade block or a low-grade (marginally profitable) block with the same local drillhole configurations is exactly the same; the optimizer, therefore, does not understand that the upside

potential of the high-grade block is much more valuable than that of the low-grade block.

Meagher et al. [122] propose a parametric minimum cut approach that aims to integrate geological and metal price uncertainty into ultimate pit and pushback design. This method is an extension of the parametric maximum flow, minimum cut algorithm proposed by Picard [134]. Geological and metal price simulations are integrated into the design of the directed graph through bi-directional, infinite capacity arcs, which ensure that if the optimizer chooses to mine a block, it must do so for all scenarios. Rather than having potentially thousands of geological and metal price scenarios, the graph model can be simplified by adding up the economic value of an ore block across all scenarios, which is treated separately from the sum of the economic values of the waste blocks. The model can be solved in polynomial time using a maximum flow/minimum cut algorithm. The authors note that the method may be used to generate pushbacks by parameterizing the economic value of the ore blocks and re-running the optimization model. This results in a formulation that not only targets high value blocks, but also blocks with a lower risk.

Chatterjee and Dimitrakopoulos [31] test the parametric maximum flow approach on a copper deposit to quantify the differences between a deterministic design and one that includes uncertainty in the ultimate pit and pushback design. Rather than solving an SIP model for the pushback designs, which is computationally demanding, the performance of the designs are compared based on a bench-wise production schedule. The authors note that the stochastic design has a 10% higher

NPV than the deterministic design, which is largely attributable to the method being able to understand not only high-valued material, but also material with low risk. Asad and Dimitrakopoulos [8] extend the previous developments to incorporate time-dependent block economic values and ore reserve constraints [158] using Lagrangian relaxation and the sub-gradient method. The authors propose modifying the Lagrangian parameters in a manner that ensures that the pushbacks do not suffer from the gap problem, which is defined by large discrepancies in size between adjacent pushbacks. More recently, Asad et al. [9] adapt the previous model to incorporate the option for multiple block destinations. In this case, for a large copper mine, the authors note a 8.7% larger pit limit, 10% increase in metal content and 14% increase in discounted cash flows. The stochastic network flow methods have consistently demonstrated that a stochastic ultimate pit limit is larger than a deterministic pit; however, the designs have not been tested with stochastic production scheduling to quantify the impact that the pit designs have on the ability to control risk and increase value on an annual basis. Additionally, with the exception of the work by Asad et al. [9], by simplifying the graph model, all information with respect to the joint local uncertainty that is represented through geological simulations is condensed into a single ore and waste value for each block, which is more similar to the probabilistic approaches previously discussed.

2.2.7 Uncertainty and cut-off grade optimization

Other work has investigated the impact of uncertainty on cut-off grade optimization for a static mine design. Dowd [51] proposes a dynamic programming approach for optimizing the cut-off grade and production levels with metal price uncertainty.

An increase in the cut-off grade results in a faster depletion of the resource, hence the method attempts to find a single cut-off grade policy that balances this depletion rate with uncertain metal prices. Johnson et al. [87] develops a partial differential equation model that aims to generate a cut-off grade policy with metal price uncertainty. The authors test their method using a real-world case study, and conclude that their method is able to increase the NPV by 10%. The method, however, is limited by several severe assumptions, such as ignoring geological uncertainty, assuming that the processor is always the bottleneck in the operation, and that the mine's production can vary with changing commodity prices. The method also assumes a specific, ordered block extraction sequence; if the sequence is changed, the cut-off grade policy would need to be re-optimized. Barr [13] proposes an alternative partial differential equation formulation for optimizing the dynamic cut-off grade with commodity price uncertainty, which includes the option to temporarily shut down the mining operation. Similar to the previous method, however, it is limited in its ability to accommodate geological uncertainty. Asad and Dimitrakopoulos [7] propose a heuristic to optimize a cut-off grade policy that considers geological uncertainty for mining operations with multiple processing streams. This work has the added advantage that it can consider operations that have mining, processing and refining or marketing constraints, rather than only a single processing capacity. The method, however, ignores economic uncertainty and is not capable of optimizing cut-off grade policies for multi-mine and multi-commodity deposits.

2.2.8 Extended formulations for production scheduling with uncertainty

Despite the advances in stochastic mine planning, it is evident that the aforementioned work tends to gravitate towards a traditional, step-wise optimization framework (Fig. 2–1). By optimizing these aspects independently, the mine design is certain to provide a sub-optimal use of the resource. For example, the previously mentioned SIP models for production scheduling assume that the ore and waste classification (i.e. the cut-off grade policy) is defined a priori; conversely, the mentioned work in cut-off grade optimization assumes a fixed production schedule. In both areas of research, authors consistently document an increased value when using stochastic optimization; naturally, these problems are interrelated, and combining these two challenges into a single optimization model would certainly add value to the mining operation. Recent work has investigated simultaneously optimizing aspects of the mine design and production scheduling problem.

Boland et al. [22] propose a multistage stochastic optimization formulation that optimizes the long-term production schedule, cut-off grades and ultimate pit limit. Rather than generating a single schedule that is unaffected by geological fluctuations, whereby the ability to meet production targets, blending constraints and the NPV are unaffected by testing a design with another set of geological simulations, the authors propose an interesting mathematical model that adapts to the uncertainty as it is revealed during extraction. Rather than scheduling blocks, the method requires scheduling aggregates of blocks, which may, for example, be generated using the fundamental tree algorithm [138]. In order to obtain the *scenario-dependent* cut-off grade decision, the model is decomposed into a time-separable processing

sub-problem; the authors demonstrate that when a single processing capacity is considered, choosing the optimal destinations is a matter of solving a common knapsack problem [169]. While being an interesting concept, the model has several limitations. First, the use of aggregates of blocks, which are generated as a pre-processing step, does not guarantee optimality. Second, the method cannot accommodate complex mining operations, such as multiple processors, stockpiles, and constraints that define minimum processing capacities and blending requirements. Third, multistage methods have limited practicality for mine design, because engineers need to modify the design for medium- and short-term scheduling, which cannot be done easily with an adaptive schedule. Finally, if a geological simulation is generated that is quite different from the ones used during optimization, the optimal schedule won't provide a policy for how to react in that case. Boland et al. [24] proposes a novel method for generating the minimal set non-anticipativity constraints required in multistage optimization formulations, which may be used to substantially improve the computational efficiency of their previously proposed method.

Kumral [100] proposes a formulation that integrates geometallurgical information, in the form of simulated mining and processing costs, and recoveries, into a stochastic optimization model. This formulation aims to generate a single mine production schedule and scenario-dependent block ore/waste classifications. Similar to the previously mentioned model, the scenario-dependent ore/waste classifications assume perfect geological knowledge at the beginning of each period, and are not particularly useful if another geological simulation is tested with the "optimal"

schedule. This may require re-optimizing a sub-problem to decide the block classifications, which may itself require a large computational effort, particularly in the presence of blending constraints. Given that the capacity and blending requirements are specified as hard constraints, rather than soft recourse constraints that are penalized in the objective function, this sub-problem is also not guaranteed to have a feasible solution. Moreover, this formulation is currently limited to solving only small-scale problems, given the use of commercial mathematical programming solvers and the number of decision variables required in the model. Kumral [102] proposes an optimization model that attempts to minimize deviations from mining and processing targets and is solved using simulated annealing. This formulation attempts to not only generate a single production schedule, but, unlike the previous formulation, generates a single ore/waste classification for each block. The rationale for optimizing ore or waste classification is that market prices, recoveries and processing costs are uncertain, thus a cut-off grade is not suitable. The author proposes fixing blocks that have more than 80% chance of being ore to being ore, and the blocks with 80% chance of being waste as fixed waste; anything in between is considered to be intermediate, and the classification may be changed by the optimizer. While this may initially sound appealing, this interpretation is somewhat limited because a block that is certain ($\geq$80%) and is slightly above the marginal cut-off grade may be classified as ore; if this block is mined at the beginning of the mine life, the ore classification does not take into the account the opportunity cost of not processing higher-valued material.

Menabde at al. [124] proposes a mixed integer programming (MIP) model that considers geological uncertainty and simultaneously optimizes the ultimate pit, production schedule and cut-off grade policy. Unlike the previous formulations that define the destination of the block for each scenario or across all scenarios, this formulation proposes a compromise between the two methods. The cut-off grade policy is transformed into a binary decision, whereby any blocks for any simulation that are above the optimized cut-off grade will be considered as ore. The issue of misclassification is non-existent; if a block is above the cut-off in one simulation, it will be sent for processing, however, if it is below the cut-off in another simulation, it is sent to the waste dump. By generating a robust cut-off grade policy, the decision is interpreted as a strategic policy that is applied without knowing exactly what is in the ground at the beginning of the production period, rather than the scenario-dependent block destination methods, which assume that the material in the ground is known with absolute certainty on the first day of each production period. The use of a policy also substantially reduces the number of decision variables in the model when compared to block-based decisions. The authors compare the same method for both the deterministic and stochastic models. A variable cut-off grade for deterministic optimization results in a 20% increase in NPV over the deterministic design that only considers the marginal cut-off grade; the stochastic model with dynamic cut-off grades increases the NPV by an additional 4.1%. While this may seem low, the authors do concede that their deposit has very low geological variability, and state that substantial benefits may be realized for highly variable deposits.

2.3 Ore Processing and Downstream Optimization

The previous section discussed some of the major developments in stochastic optimization for open pit mines. While many of these developments are substantial and have shown the ability to generate mine designs and production schedules with less risk and higher value, they are currently limited in their ability to incorporate the downstream processes of a mining complex. This section focuses on some related research in mining and related industries (e.g. oil and gas) that address the issues of stockpiling, blending and complex processing streams or supply chains, often in isolation from the mine design and production scheduling problem.

2.3.1 Optimizing with stockpiles

Stockpiles are locations in the mining complex, downstream from the mines, that may retain material over time for future processing. They often serve two primary functions: first, they may be used to blend materials from various sources (mines) together to attain a more homogenous product; and second, they may be used for strategic purposes to store material above a marginal cut-off grade but below the optimum cut-off for processing in a later period when the optimum cut-off grade is lower. One of the challenges with modelling stockpiles is the fact that the decision to send or retain material in each period produces bilinear terms (a product of two variables) in the constraints that leads to a non-linear formulation; for any given period, the stockpile grade is not only a function of the grade of the material that is coming from the mine, but also needs to consider the grade of the material retained from the previous period.

Ramazan and Dimitrakopoulos [142] modify their original SIP model to incorporate a stockpile. The authors model the quantities of material pulled from the stockpile and sent to the processing plant as a recourse (scenario-dependent) variable; this is useful in order to model the recourse decision at the processing plant whereby a shortfall in ore production directly from the mine may be mitigated by using stockpiled material. In order to circumvent the bilinear stockpile grade term, the stockpile grade (potentially multivariate) is fixed for each period prior to optimization. Unfortunately, this simplification is somewhat unrealistic, and may permit low-grade mined material to be upgraded to a higher-grade material by being processed via the stockpile.

Given the importance in the role of the stockpile for many mining operations, some work has been done in deterministic cases to investigate other methods of approximating the stockpile grade. Caccetta and Hill [29] propose a simplistic long-term optimization model and mention the incorporation of stockpiles, however the specifics of the implementation and solution methods are not given. Sarker and Gunn [151] propose a successive linear programming (SLP) iterative approach to obtaining the correct stockpile grade without considering production scheduling; the problem is first optimized using an estimate of the stockpile grade, then the grade is calculated using the optimal decisions, and the optimization model is updated and re-optimized. This method, however, isn't ideal for large-scale problems such as mine production scheduling and does not guarantee a global optimum. Bley et al. [18, 19] propose a method to integrate a stockpile into a deterministic MIP formulation. The authors propose first solving a branch-and-bound node while ignoring the non-linear

constraint that enforces that the grade of material sent to the processing plant from the stockpile must be equal to the grade of the stockpile. In the event that one of the nodes in the branch-and-bound tree for this relaxed MIP shows a violation in the non-linear constraint, the optimizer is pulling metal from the stockpile and leaving the remaining waste material behind (i.e. the grade of material taken from the stockpile is not the grade of the stockpile itself). In this case, the authors use a spatial branch-and-bound procedure to cut-off this infeasible solution, which can be applied until the stockpile is guaranteed to satisfy the non-linear constraint. This method, therefore, is an effective method for optimizing the mine's production schedule while incorporating non-linear stockpiles with near-optimality. The performance of the method for large-scale tests or stochastic optimization models remains to be seen.

2.3.2 Downstream optimization in the petroleum industry

There may be many downstream storage and processing options for large mining complexes for material after it is extracted; these may include, for example, stockpiles, waste dumps, mills, concentrators, tailings ponds, leach pads, smelters and refineries, among others. In an abstract sense, these are locations in the mining complex that receive materials from sources, store them over time or transform them, and produce a set of products that are retained, sold, or given to another location. Each location may only accept and produce certain products, and may be constrained by chemistry to only receive or produce products within a certain specification. This definition of the downstream options and product requirements resembles the pooling problem commonly seen in the oil and gas industries [11]. The goal of the pooling problem is to decide, given the availability from a selection of feeds (e.g. mines),

what quantities from each feed should be used at intermediate pools (e.g. homogenization piles) to achieve a desirable blended chemistry for the final product at a minimum cost [10]. The generalized pooling problem closely resembles mining complexes, whereby intermediate pools (stockpiles, concentrators, smelters, etc.) may be connected together in series or parallel. Similar to the stockpiling problem, one of the challenges for solving the pooling problem is related to the bilinear terms in the formulation used to model the mixing of materials together.

Audet et al. [10] propose three formulations for the pooling problem; the first formulation relies on flow variables, which may be amenable to network flow models, the second formulation uses proportions of products entering a pool and the third is a combination of the two methods. The authors assume that all attributes are linearly additive when blended together in order to simplify the non-linear chemical reactions that may happen when products are blended. The authors compare the computational results from a globally optimal branch-and-cut algorithm, similar in concept to that of Bley et al. [18], to heuristics and a variable neighbourhood search metaheuristic using a set of benchmark examples. The authors note that they are able to successfully apply the heuristics to obtain a near-optimum solution substantially faster than exact optimization methods. Méndez et al. [126] investigates the optimization of blending and short-term scheduling problems. An iterative algorithm is used to approximate the non-linear blending of attributes, which, when related to the mining context, may be used to iteratively optimize the downstream processes when non-linear throughputs and recoveries are required. Kolodziej et al. [99] note

that the vast amount of research related to the pooling problem assumes a steady-state inventory, which is not realistic for mining operations that use stockpiles and where the quality and grades of the mined materials are time-dependent. A multi-period blend scheduling problem is developed to optimize the profits generated from the flow and blending of materials, which is more aligned with the objectives for optimizing the downstream processes of a mining operation.

2.3.3 Downstream optimization for mining complexes and mineral supply chains

While research related to the generalized pooling problem is quite extensive over the past several decades, only recently have similar concepts and methodologies been applied for optimizing the processing streams in mining complexes. The vast majority of the downstream optimization research in the mining industry has focused on iron ore operations, which may involve several mines and stockpiles, transportation using railways and blending at the port to yield a homogenous product that meets contractual obligations to a set of customers. Everett [54] defines 'stress' as a useful metric of product quality, which is defined by the root-sum of the squared deviations from the target quality for iron, silica, alumina and phosphorus, divided by a tolerance level. This is a somewhat different approach from blending constraints used in production scheduling models, where deviations from target quality are penalized linearly, because the optimizer that seeks to find an optimal blended feed severely penalizes large deviations from target quality, and slightly penalizes smaller deviations. The author proposes a simulation-based approach to decision support for coordinating quality of the products produced from the mine through to the product loaded on ships to be given to customers. Everett [55] and Howard and Everett [79]

improve on the previous decision-support system by more accurately modelling the splitting of fine and lump products and better accounting for stockpile dilution; this is a result of changing the stockpile models to assume continuous changes over time rather than batches.

The previous methods attempt to simulate and provide decision-support in order to coordinate stockpile management and blending from the mines through to the port, however, they do not provide an integrated optimization of the various steps to unlock additional value. Singh et al. [153] propose an integrated, deterministic downstream model that simultaneously optimizes the allocation of trains to mines and material sent from each port, with the goal of maximizing the medium-term revenues for Rio Tinto's iron ore operations in Western Australia. The authors propose an iterative MIP formulation, where the non-linear stockpiling terms are approximated successively, similar to Méndez et al. [126]. The optimizer reduces the solution time to 15 minutes, substantially lower than the five hours of manual computing time previously needed, and results in increasing the amount of material sent to the ports by one million tonnes in a typical planning horizon, which leads to an additional $100 million in sales.

The previously mentioned downstream optimization methods are all deterministic; ignoring uncertainty in downstream optimization can have serious ramifications on the performance of the processing streams because of the variability of each of the attributes considered (e.g. metals), particularly on blending constraints. It is apparent that non-linear models play a significant role in modelling the downstream

processes, thus considering a single, often estimated, input does not necessarily optimize the true performance of the mining complex. Pimentel et al. [136] propose a multistage stochastic capacity planning model that is tested using an iron ore supply chain in Brazil. The formulation models and the flow of material from mines through to final customers, including capacities on transportation methods between the various destinations in the supply chain. Moreover, the multistage model integrates demand (price) uncertainty, which permits important strategic supply chain decisions, such as opening up new facilities (stockpiles, concentrators, etc.) or transportation modes (e.g. trucks, rail, port), investing capital to expand the capacities for existing facilities and transportation modes, and permanently or temporarily shutting down the facilities or transportation modes. The objective of the formulation is to minimize structural, capital and operational costs for the supply chain. Despite the model's complexity, however, it does not integrate the impact of geological uncertainty into the multistage formulation. Moreover, the model does not explicitly incorporate blending constraints, which is often of critical importance for iron ore operations and will have a substantial impact on the strategic decisions made for the supply chain.

Chanda [30] proposes a simplistic capacitated network flow model that aims to optimize a mining complex that includes production from five underground mines, an open pit mine, five concentrators, three smelters and two refineries. The model optimizes the flow from the mines through to the final products at a minimum cost; it is noted, however, that the materials produced from the mines is given as a parameter, and the model does not consider uncertainty. Pimentel [135] creates

a large deterministic model that attempts to better integrate mine-level production with the supply chain, while including mine production capacities from the various mines and blending requirements and the ability to procure mined material from outside sources, in addition to many of the aspects of the previously mentioned work [136]. Using a simplistic example that does not consider metal production or quality from the mines, and the supply chain as a whole, the authors benchmark their relax-and-fit heuristic when compared to a commercial optimizer. However, by not integrating the mined product quality into the example, the authors do not need to consider non-linear constraints that are a result of stockpiling and blending, which would drastically complicate the model. Topal and Ramazan [161] propose a network linear programming model for strategic mine planning that determines the amount of materials to extract from a set of pits, which are subsequently stockpiled or treated through a set of processing streams; in order to avoid a non-linear optimization model, the authors employ grade bins at each location in the mining complex, which may be used to approximate the grade of the material leaving from a destination.

2.4 Global Optimization of Mining Complexes

2.4.1 Overview of global optimization

A substantial amount of research has been done to optimize mine designs, production schedules and the use of mined materials in the downstream processes. There is, however, a clear disconnect in the optimization objectives between the mining and downstream formulations; mining engineers often seek to maximize the NPV of the mine design, however the downstream optimization models tend to minimize costs.

More importantly, the previously discussed methods do not coordinate mine production, processing streams, logistics and marketing in a single optimizer. Global optimization for mining complexes addresses these shortcomings by simultaneously optimizing multiple pits and mining faces, underground mines, multiple elements and materials, cut-off grade decisions, stockpiling opportunities, blending, alternative processing methods and product options [166].

2.4.2 Integrated mine planning and downstream optimization

With the increase in computing capabilities, optimization models in the mining industry have become increasingly detailed and aim to integrate both mine planning and downstream optimization. Urbaez and Dagdelen [163] outline a mathematical model that aims to optimize multi-mine production schedules with multiple stockpile and processing stream options. In their formulation, a decision variable is used to define whether or not a parcel of material from a mine and sequence is sent to a specific destination; the grade of material sent from a stockpile to a process is assumed to be defined a priori. While this formulation does not model the detailed processing paths downstream of the mills, it is an improvement over the research in the previous section (downstream optimization) because it attempts to model mine sequence, stockpiling and processing options. As a result, cut-off grade optimization is implicit in the formulation. Hoerger et al. [76, 77] give an overview of Newmont's in-house MIP optimizer for their Nevada operations, which simultaneously sequences the pushbacks and underground stopes from 50 mines, 20 material types, 60 destinations (including waste dumps) and 8 stockpile areas. Exact details of the model and implementation are omitted from their discussion, however, they do note that their

model is based off that of Urbaez and Dagdelen [163] with further modifications in order to facilitate scaling-up to their large mining complex.

Bodon et al. [21] and Sandeman et al. [150] discuss their discrete event simulation (DES) methodology that is a coupled with an MIP optimizer, which is used to simulate the decision making for mining supply chains from pit to port. DES is a powerful tool that permits modelling a system and the logic that occurs within to identify bottlenecks and potential problems; it is also able to integrate various forms of uncertainty, such as machine breakdowns. The purpose of the optimizer is to plan the movements of ore from the pits through to the ports, and can include intermediate destinations such as processors, transportation systems and stockpiles. The DES is used to take the decisions made from the optimizer and identify issues and bottlenecks that may occur given the set of decisions. The results from this step may be given as feedback to the optimizer to improve decision making, or to plan the next time horizon. One of the primary benefits of this method is that very complex systems may be modelled with a high degree of detail. There are several challenges that are identified by the authors, including non-linear stockpiles and blending (simplified by defining a priori), and the time required to solve each MIP model. This model also does not attempt to optimize the mine design or production schedule directly, but does have some leeway in sequencing the pre-defined material to be mined.

Pimentel [135] proposes an integrated formulation to optimize short-term mining, stockpiling, processing and shipping operations that is designed for iron ore

mining operations. The optimizer has control over which work benches to mine material from (although precedence constraints are not considered), which stockpile the bench goes to, when to form and consume stockpiles, which products to produce at the processing plant (fines or super-fine products). The objective function tries to minimize production and processing costs, deviations between the short- and long-term production schedules, in addition to deviations from stockpile and processing plant material qualities and quantities. The short-term stockpiles are assumed to be consumed a single period, hence there is no linkage between time periods, thus non-linearities. The author analyzes the value of the integrated solution for a simple example with a pit with three working benches, four homogenization stockyards, a processing plant and a product stockyard. The integrated optimal solution is compared to a solution with shipping priority, where shipping operations always get the exact product needed, and mining and processing priority, where the mine, homogenization piles and processing plant all operate at optimum levels. As expected, the integrated solution performs far better than the other approaches, generating a solution that never fails to meet shipping quality and a balance between long-term plan deviations and mined product quality.

Blom et al. [20] propose a multi-mine short-term production scheduling model that integrates mining, transportation via railways and multiple blended products at the port. Their model attempts to maximize revenues and minimize deviations from qualities for a set of products blended at the port. The authors propose an iterative decomposition heuristic to solve this "global problem" to avoid the complexities of the bilinear terms required in the model. First, the a set of candidate short-term

production schedules is generated for each mine. In this step, the objective is to maximize the productivity from each mine for a set of lump or fine products, which is defined by the ability to meet production targets and produce materials of desired qualities. A set of schedules is generated by varying the upper- and lower-bounds of the quality constraints. The sets of schedules from the mines are then given to the port-side optimization problem, which chooses the best multi-mine schedule from the available options, and determines the number of trainloads of ore for fine and lump materials that are railed to a port to form a specific product. The solution from this port-side optimization problem is then used as feedback to the mine-side optimization problem by changing the bounds of the product quality targets. While this is an interesting approach to integrate mine-to-port optimization, it is presently limited by several factors: first, the model assumes that high-grade, low-grade and waste classifications are defined a priori; second, the model assumes unlimited railing capacity; third, the iterative decomposition method is likely to generate sub-optimal solutions for the global problem; and, finally, the method will not likely scale to long-term production scheduling easily, given the substantial increase in block extraction decisions and number of schedules that would need to be generated.

Epstein et al. [52] propose an advanced MIP formulation to optimize Codelco's North Division copper complex. This is an interesting development because the authors permit to optimizer to control the extraction sequence of the panels from the open pit mines in addition to the extraction from the underground stopes for a block caving operation. The model of the mining complex is essentially a capacitated, multi-commodity network flow model. In order to formulate a linear model

that avoids issues with blending and stockpiling, the authors assume averaged grades defined a priori at the various destinations. A heuristic algorithm is used, whereby the linear relaxation is iteratively solved and the solution is then rounded to an integer solution. The authors show results for the optimization model when optimizing the Chuquicamata and Radomiro Tomic mines, and benchmark the integrated optimization model against the existing design and the MIP model solved, both solved independently for each mine. The integrated optimized solution shows an increase of 5% for using an optimizer and an additional 3% for using an integrated approach.

Groeneveld and Topal [71] develop an MIP formulation that attempts to integrate operational flexibility into mining, stockpiling, processing and port capacities. Similar to the previous method, the model is able to choose the extraction sequence of materials from available panels. This model uses a series of stockpiles where the grades are defined by bins (groups); in order to maintain linearity, the optimizer only sends qualified material to the appropriate binned stockpile. This approach analyzes the effects of price, recovery, cost and utilization uncertainty have on the solution; the MIP model is solved for 200 scenarios independently, the the impacts of expending capacities is analyzed. Groeneveld et al. [72] expand on the previously mentioned model by introducing the concept of an operational schedule, which effectively attempts to find a robust schedule for the first few periods of operation, then permits the optimizer to have "flexible" solutions that adapt to each scenario independently for the remaining periods. One of the limitations with the author's definition of "flexibility" is that it is essentially a wait-and-see solution [16] that assumes all stochastic parameters are known with certainty for the remainder of the

life of the operation; by knowing these parameters far in advance, the optimizer is certain to generate an overly optimistic design and will depict inflated NPV. Additionally, given that the authors do not consider geological uncertainty, this model focuses solely on scheduling parcels of materials (e.g. benches) within a prescribed pushback design generated from deterministic optimization.

Whittle [168] provides a general overview of the mechanics of their proprietary global optimization algorithm, ProberC. First, nested pit shells are generated for each mine independently, based on the nested pit implementation of the Lerchs-Grossmann algorithm [113]. The algorithm then randomly schedules a feasible panel production schedule, where a panel is defined as the intersection of phases or pit shells with a mining bench. With the extraction sequence defined, it is known what materials are extracted; the algorithm proceeds to optimize the processing streams using linear programming. It then locally improves the best solutions (for the global problem) iteratively by changing bench start and end dates and re-optimizing. While the mechanics related to the optimization of the downstream processes have not been discussed, the authors outline their approach to modelling the mining complex in a general manner. Similar to the modelling method outlined by Audet et al. [10], the authors model the flow of materials and related attributes through the mining complex. Constraints may be applied to any of these attributes, and any of the attributes may be used to calculate costs, throughput and revenues, for example. The various destinations (e.g. concentrators, smelters and refineries) in the mining complex's processing streams are viewed in an abstract way, and are referred to as "procedures" that act on the materials and their attributes. This results in being

able to model the mining complex with flexibility. The specifics of the algorithms employed, however, are proprietary and are not disclosed.

2.4.3 An integrated approach for the stochastic global optimization of open pit mining complexes

The work discussed in the previous sub-section shows an increased effort for simultaneously optimizing the mine sequencing and mining complex. There are two major limitations to these methods that need to be addressed. First, there needs to be a better coupling between mine production scheduling and downstream decisions. Existing work makes assumptions on pre-defining the pit limit, nested pit shells, phases or material to be mined in the production period. The challenge is that, in the traditional, sequential optimization framework for open pit mine design, each of these are based on the economic value of a block, which is not known with certainty, and is not known prior to optimizing the extraction sequence, because of the potential to blend with other materials. A global optimization framework will require simultaneously optimizing the mine design and production schedule with the destination decisions and down-stream operations.

In order to achieve this goal, it is necessary to re-visit the method in which open pit mines are sequentially optimized. Stone et al. [154] outlines the methodology used by BHP Billiton's in-house optimization suite, Blasor, and is summarized in Fig. 2–2. Rather than using a sequential framework (Fig. 2–1), which assumes a substantial amount of information is known a priori and generates sub-optimal solutions [97], this framework solves the mine production schedule first. The advantage of this methodology is that production scheduling permits integrating multiple mines, each with multiple materials and elements, time value of money, blending, cut-off

grade [123, 124] or destination policy optimization, stockpiling [125] and complex processing streams. The globally optimal production schedule defines the ultimate pit limit and overcomes the limitations of the Lerchs-Grossmann algorithm [113]. A set of pushback designs, which use the optimal production schedule as a guide, may be generated, assessed and compared so the engineer can create practical mining shapes while simultaneously maximizing the value of the design [57]. The panels, which are the intersection of the bench and pushback, may then be scheduled similar to the initial globally optimal production schedule.

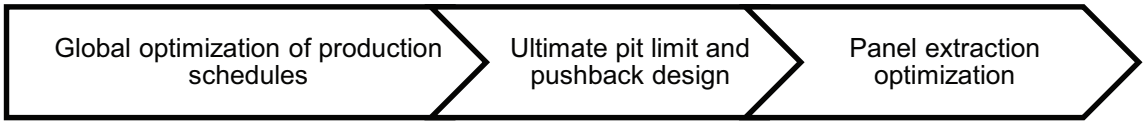

Figure 2–2: A new open pit mine design and production scheduling framework for global optimization.

The work in mine production scheduling optimization with uncertainty demonstrates progress towards a new open pit mine design framework for global optimization (Fig. 2–2). These methods are currently limited to simplistic mining complexes that consist of a single mine and ore process. In order to transition from the previous developments to a global optimization framework, it is necessary to have new formulations that consider all aspects of the mining complex, including multiple mines, blending, destination policies and detailed processing streams. In order to achieve this, it is necessary to shed preconceived notions that have been used for decades, such as "block value", and evaluate these aspects dynamically where the transformations or transactions happen in the mining complex. While these concepts have

helped to simplify the models in the past, they undervalue the value of the mining complex as a whole. The vast majority of efforts in global optimization have focused on deterministic optimization; developments in stochastic optimization with geological uncertainty have demonstrated not only the ability to increase the NPV of the mine, but also to control fundamental aspects of risk, such as the ability to meet production targets and maintain product quality. Increasingly complex operations require increasing complex constraints; relying on deterministic models not only misleads the value of the mining operation, but can also result in serious consequences because the variability is not properly quantified and managed.

2.5 Models of Geological Uncertainty

2.5.1 An introduction to stochastic simulation

All methods of stochastic optimization for mining operations require a set of input models that are used to represent the uncertainty and variability in the mineral deposit. From a global optimization perspective, proper assessment of both material type and grade uncertainty is critical. Material categories may be used to not only define candidate destinations in a mining complex, but may also be used to differentiate qualities of the material that have an impact on the system, such as non-additive geometallurgical attributes like hardness. Grade uncertainty may be used to define the uncertainty related to a metal, deleterious element of mineral of interest, which may be critical to the performance of downstream processes (e.g. iron and silica content in a smelter). Geostatistical simulation methods are tools used to generate equally probable scenarios of a mineral deposit, where each simulation accurately reproduces the spatial statistics of the original drillhole data [42, 89, 92].

This section provides a brief overview of some of the new developments in geostatistical simulation methods. First, computational improvements to the sequential Gaussian simulation framework will be discussed. Following this, a discussion on modern multiple-point and high-order simulation methods is provided.

2.5.2 Computational improvements in traditional geostatistical simulation

Sequential geostatistical simulation algorithms, a variant of Monte Carlo simulation [74], rely on the decomposition of the multivariate cumulative distribution function (*cdf*), which is used to describe the mineral deposit as a whole and can be expressed recursively as product of one-point conditional distribution functions [86, 147, 149]. A simulated value for an unknown point may be generated by randomly sampling the conditional cumulative distribution function (*ccdf*), which is then retained as future conditioning data when simulating other points. One of the primary challenges of sequential simulation methods has been to find ways to obtain a *ccdf*. Isaaks [83] proposes the Sequential Gaussian Simulation (SGS) method for continuous variables, which relies on a Gaussian transformation of the *cdf* that provides the computation of the *ccdf* using the simple kriging method commonly used in geostatistics [40, 41, 84, 89, 120]. SGS has been the predominant method to generate conditional simulations in the mining industry, however suffers from severe computational limitations. For each unknown point to be simulated, the method requires inverting a covariance matrix, a computationally demanding task ($\mathcal{O}(N^3)$), and is therefore limited in practicality when simulating for very large deposits.

To overcome these computational challenges, Dimitrakopoulos and Luo [47] propose generalized sequential Gaussian simulation (GSGS) on group size ν, whereby

groups of nodes that share a similar neighbourhood of conditioning points, such as points that discretize a mining block, are simultaneously simulated using the LU method [42], which only requires a single covariance matrix inversion. The authors demonstrate the mathematical equivalence of the two methods, and the substantial computational improvements that are attained when using GSGS over SGS or LU. Godoy [65] improves on this method by developing the direct block simulation method (DBSIM). This method first uses the GSGS method to simulate a group of nodes that discretize a block, then proceeds to back-transform them from the Gaussian space to the data space. Following this, in the Gaussian space, the method then averages out the simulated nodes in the block, which is then directly used during the simulation of other nodes. Because of the reduction in the size of the covariance matrix when using previously simulated blocks (rather than points), the authors note drastic improvements in computational efficiency when simulating large mineral deposits.

Often, a mineral deposit contains multiple elements of interest, which may be required in the optimization models. The previous methods have focused on simulating a single variable, however, multivariate simulation methods [3, 164] may be used to simulate the variables and maintain any spatial cross-correlations that are present. Desbarats and Dimitrakopoulos [43] adapt the min/max autocorrelation factors (MAF) [157] for applications in geosciences, which is used to spatially decorrelate the variables of interest. Spatially un-correlated variables, which are linear combinations of the variables of interest, are then independently simulated using any available simulation method, and the results are subsequently back-transformed

to the correlated space [148]. Boucher and Dimitrakopoulos [27] propose the direct block MAF simulation method (DBMAFSIM), which is a practical tool for generating multi-variate simulations of large deposits. In this method, DBSIM is used to generate simulations for the uncorrelated MAF factors, thus taking advantage of previous computational improvements. One alternative method used to spatially decorrelate variables is the uniformly weighted exhaustive diagonalization with Gauss iterations (U-WEDGE) [130] method.

2.5.3 Modern simulation algorithms for geostatistics

The traditional geostatistical framework relies on two-point statistics in the form of a covariance or a variogram model. These methods are limited in their ability to describe connectivity and incorporate complex shapes or geometries that are often seen in geology. Journel [91] gives an example (Fig. 2–3) of three distinct images that have similar two-point statistics, despite having different levels of connectivity [92]. This concept gives rise to modern geostatistics algorithms that rely on multiple-point [144] and high-order statistics and shed the prior belief that two-point statistics are sufficient to characterize complex deposits.

Journel and Alabert [92] propose an extended indicator kriging system for use with sequential indicator simulation (SIS) that incorporates high-order spatial moments. This method has rarely been applied in practice because of its practical limitations, including the inability to accurately infer high-order statistics using sparse datasets, along with the computational limitation of kriging with very large matrices. Guardiano and Srivastava [73] propose a multiple point sequential simulation algorithm called ENESIM. In order to alleviate the challenge of inferring multiple

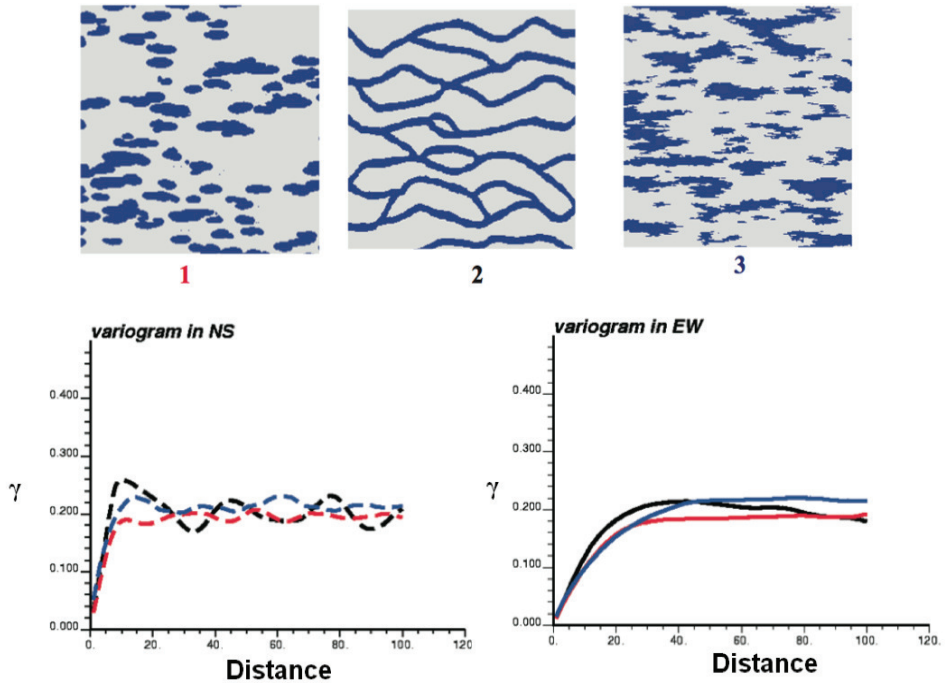

Figure 2–3: Limitations of the ability for two-point statistics to describe complex geometries [91].

point statistics on sparse data sets, the authors introduce the concept of a training image, or a geological analogue, that acts as a supplemental pattern or statistics database for the drillhole data. Given a set of conditioning data, generally from drillhole information or previously simulated nodes, the algorithm relies on scanning a training image for similar "data events" to obtain a conditional probability distribution function, which may be used to simulate a value. This proposed method, however, suffers from computational issues because the training image is searched for each point to be simulated.

Strebelle [156] improves on the method by introducing the single normal equation simulation (SNESIM) algorithm for simulating categorical data (i.e. material

types). Rather than scanning the training image for replicates of a data events to build the conditional distribution function for each point to be simulated, the algorithm stores the frequencies of the data events in a search tree; this helps to drastically reduce the computational effort, however does have limitations in terms of memory required. The method, however, can cause problems when replicates for a data event are not found in the search tree; the algorithm will iteratively remove conditioning points from the data event until a replicate can be found, thus deliberately ignoring existing information. Some enhancements of the SNESIM algorithm to improve the quality of the simulations generated include the use of multiple grids and target histogram matching [116], and the ability to incorporate various local rotations and scales [173]. Initially, one of the practical limitations of the algorithm is the amount of memory that is required to store information in the search tree; as the size of a template (a local window used to scan the training image for data events) increases, the amount of memory required grows exponentially. Straubhaar et al. [155] propose the IMPALA algorithm, which is differentiated by the fact that it uses lists to store the multiple point statistics rather than a search tree; the primary advantage to this implementation is the need for less memory (i.e. a linear increase in memory with increasing template size), and the calculation of the conditional probability density function can be parallelized. Huang et al. [81] propose a GPU-based implementation of the "SNESIM" algorithm. GPU computing is a particularly interesting tool for modern geostatistics algorithms that require training images because of their ability to process the training image or dense data sets very quickly, which is a step that often takes a substantial amount of time. The authors' implementation is more similar

to the ENESIM algorithm, given that the training image is re-scanned for each node to be simulated; the primary difference, however, is that by taking advantage of the GPU's massive parallelism, there is not a massive computational hit when searching for replicates. The authors note a 15x speedup over the conventional SNESIM algorithm. The implementation, however, is currently limited to only two-dimensional simulations and training images.

Pattern-based simulations have recently emerged as a tool to simulate both continuous and categorical variables. These methods are based loosely off image reconstruction algorithms, and attempt to find and paste patterns (rather than a single node value in the SNESIM algorithm) onto the simulation grid that closely match the data event. The manner in which these algorithms differ is generally related to how the pattern database is generated and retrieved. Arpat [4], and Arpat and Caers [5] propose the SIMPAT algorithm. Rather than attempting to reproduce univariate, bi-variate or multiple point statistics, this algorithm attempts to find patterns that are similar to the data events by calculating the pixel-wise distance between the data event and the patterns found in a training image. The algorithm pastes an entire pattern over a group of points, which may be updated at a later iteration. One of the limitations of this method is that it requires a lot of computing time to calculate the pixel-wise distance between the conditioning data and all of the patterns in the training image.

Zhang [174] and Zhang et al. [175] propose the FILTERSIM algorithm, which attempts to condense the patterns found in the training image into a single number using a set of filters that reduce the dimension of the pattern. The filters may be used

to give a score to each pattern's curvature, gradients and mean levels of the variable of interest. The patterns are separated into bins of similarity using a k-means clustering algorithm [117] and a prototype is formed by averaging the patterns within. The conditioning data for a point to be simulated is then compared to the prototype; a pattern within the nearest (pixel-wise) prototype's class is then randomly selected and pasted on the simulation grid. The authors attempt to make the algorithm more data-driven by assigning varying weights when calculating the distance between a data event and a prototype, giving more weight to hard data (drillholes), followed by previously simulated central node of a pasted pattern, and finally, the previously simulated exterior points from a pasted pattern. Wu et al. [170] extend the FILTERSIM method to alleviate some of the computational challenges. The authors note that during the simulation process, evaluating the pixel-wise comparison between the data event and the prototypes is computationally demanding; the authors propose replacing the pixel-wise comparison with a filter-score comparison.

Alternative methods have been proposed to classify patterns into groups represented by a prototype. Honarkhah and Caers [78] propose multidimensional scaling (MDS) to reduce the dimension of a set of patterns. In this method, each pattern is mapped to a point in a new lower-dimensional space where the distances between points represent the dissimilarity between the patterns. The authors then use a kernel clustering algorithm to classify the points in the lower-dimensional space; the authors note that kernel clustering is more appropriate than traditional k-means because of its ability to capture non-linear trends in the data points (patterns). The method, however, may be limited in some cases because of the amount of memory

required to define the dissimilarity matrix, which grows with the size of the training image. Chatterjee et al. [33] propose the *wavesim*, an alternative pattern-based simulation method. The method uses wavelet decomposition to reduce the dimension of the pattern database, thus making it easier and more computationally efficient to classify the patterns and create prototypes. For categorical data sets (such as those used to define material types), the authors also include the ability to draw the value of a central node according to a *ccdf*, rather than simply selecting a pattern to paste from the group of patterns within a prototype. Mustapha et al. [131] introduce *cdfsim*, a pattern-based simulation method, which differs from others in the method that the pattern prototypes are generated. The authors propose mapping each pattern in the database to a one-dimensional real number, which can then be ordered into a cumulative distribution function (*cdf*). The prototypes that define similar patterns may then be generated by splitting the *cdf* into percentiles (or clusters), and selecting the appropriate pattern that maps to each percentile (or cluster). The patterns are then classified to the prototypes using a pixel-wise distance function, and the process of finding the appropriate prototype for a given set of hard data during simulation is similar to the FILTERSIM algoirthm. Rather than visiting all nodes on the simulation grid, the method pastes an inner patch (nodes within a template) that are not modified in later steps. The authors note that the method generates simulations that more accurately depict spatial connectivity than the FILTERSIM method, and requires less emphasis on accurately defining the number of clusters and template size.

Other methods exist that are similar to the SNESIM and pattern-based simulation approaches. Boucher [25] proposes a hybrid between the SNESIM and FILTERSIM algorithms to simulate categorical data. In the proposed method, the training image is first scanned and partitioned using the FILTERSIM algorithm. A search tree is then formed for each of the partitions; this ultimately leads to search trees that capture more relevant information for complex training images. During the simulation process, the algorithm first decides which partition is most accurate for the node to be simulated, then uses the relevant search tree. While this method doesn't attempt to address the computational limitations of either of the methods, it does attempt to provide more accurate, and therefore more useful, simulations. Boucher et al. [26] propose the *contactsim* algorithm, which is more suitable for mining applications where the contacts between two geological domains is of interest. Rather than using all patterns from a training image, which can disproportionately represent many patterns that mostly consist of a single geological unit, the method focuses on the boundaries between different geological units to search for patterns. The search tree method [25] is used to partition these patterns into various search trees, which is used to provide more accurate contact patterns. Mariethoz et al. [118] propose a direct sampling method for multiple point simulation for both continuous, categorical and multivariate datasets. The authors propose randomly scanning the training image and directly sampling if the pattern is close to the data event (within a threshold). The central node is then pasted directly on the simulation grid, and the method continues on to the remaining nodes. This method has the added benefit

that it is one of the only multiple point simulation methods that are able to simulate multivariate data, which is of critical importance for the global optimization of mining complexes. Rezaee et al. [146] extend the method for bunch-pasting, similar to the inner patch used by Mustapha et al [131], and note orders of magnitude speedup. Huang et al [80] propose a GPU implementation of the direct sampling method, which results in 10-100x speedups.

One of the challenges related to the practical use of the previous methods is their lack of mathematical formality and their inability to guarantee that drillhole lower-order statistics are reproduced; these multiple point methods often tend to treat any point within a data event with equal importance. Given that these methods rely on searching the training images for related patterns, if there is a conflict between the drillhole information and the patterns in the training image, these algorithms tend to reproduce the patterns and statistics of the training image [67]. This is particularly problematic for mining applications, which tends to have substantially more drillhole information than the oil and gas industry. To address these challenges, Mustapha and Dimitrakopoulos [132, 133] propose the High-Order Spatial Simulation (HOSIM) algorithm to simulate continuous variables. This method relies on high-order spatial statistics [132] and Legendre polynomials [110] to construct a conditional probability density function for each point to be simulated. By using high-order spatial statistics, rather than multiple point statistics, this framework is mathematically consistent framework and is also leads to a data-driven algorithm because it maximizes the use of the drilling information, and is not heavily influenced by the quality of the training image. Currently, however, the method is under continued development and

has not yet been adapted for use with categorical simulation, block-simulation and multi-variate simulation. Moreover, the method is computationally demanding. Li et al. [115] propose a GPU implementation for the calculation of spatial cumulants, where the authors note 17-50x speedups for two datasets; these methods, however, have not been directly implemented in the HOSIM algorithm.

Multiple-point and high-order stochastic simulation methods have shown to be useful tools to simulate continuous and categorical variables, which are crucial for the proper quantification and management of risk in stochastic optimization. One of the current challenges with existing multiple-point simulation algorithms is their reliance on training images. Often, the performance of the algorithm and quality of the simulation may be over-ridden by the quality of the training image and the sparsity of the conditioning data. It is therefore crucial that great care be taken when generating the simulations to ensure that the simulations accurately represent the deposit of interest. Additionally, there is a need for new, computationally efficient methods to simulate multivariate deposits, with more emphasis on statistical reproduction (e.g. spatial cumulants), rather than pattern reproduction. Research in this field is an ongoing effort, and will ultimately lead to simulation models that better represent the underlying geological conditions, and, therefore, produce better and more accurate results when coupled with a stochastic global optimization method.

CHAPTER 3
Algorithmic Integration of Geological Uncertainty in Pushback Designs for Complex Multi-Process Open Pit Mines

Albor and Dimitrakopoulos [2] investigate the impact of pushback design on life-of-mine production scheduling, and note that the pushback can have a drastic impact on the net present value and the ability to meet ore processing and mine production targets over time. While this method permits *selecting* a pushback design, it does not attempt to directly integrate uncertainty into the optimization and design process. This Chapter presents a new method for integrating geological uncertainty into pushback design. The objective is to use an existing starting design, and generate a *similar* design that mimics the average tonnages for each processing stream contained in each pushback as the original design, while simultaneously reducing the risk to ensure that those quantities are actually in the pushbacks. This method integrates well into a sequential optimization methodology, and may be adapted for use with ultimate pit limits, nested pit shells and production schedules. A full-field study for the Escondida Norte mine, a large deposit that contains over 176 000 blocks, demonstrates the ability to modify an existing pushback design, generated with BHP Billiton's Blasor software, and substantially reduce the risk of materials that can be sent to the various processing streams.

3.1 Overview

The objective of pushback (or cutback or phase) design is to provide a long-term guide for the sequence of extraction of material from an open pit mine over time such that the net present value (NPV) of the mine production schedule is maximized [38, 82]. Conventional mine design and production scheduling [75, 134, 158, 167] is a step-wise procedure that only considers a single input model consisting of the expected metal content for a block of material. In the case of complex deposits, the input models may consist of various material types, which define the set processing destinations that the rock can be sent to, along with zones that require different slope angles to ensure wall stability.

As the complexity of the deposit increases in terms of number of elements (metals, deleterious elements), materials and candidate destinations, the conventional framework for mine design fails in the sense that it only sees an expected value for each element and material type and does not consider the interactions of the uncertainties between the grades and material types and the compounded effect that it may have on the various processing paths and the final economic value of the mine design. The assumption of constant inputs for each block in the orebody model may therefore result in an unrealistic mine design in terms of practicality and ability to meet annual production targets [45]. Frameworks for optimization under geological and economic uncertainty (referred herein as stochastic optimization methods) have been developed over the past decade to address many of the shortcomings that are inherent in conventional optimization methods.

Albor and Dimitrakopoulos [2] integrate pushback design into the stochastic optimization framework by considering the influence of the number of pushbacks and pushback sizes has on the risk profiles for production schedules. The authors propose grouping a series of nested pits into a specified number of pushbacks by evaluating the combinations in terms of approximated discounted economic value. The authors generate life-of-mine production schedules for a mine design based on a varying number of pushbacks using a SIP formulation [50, 142], and document the effects that the step-wise procedure has on the risk profiles of the production schedule. Selecting the best pushback design requires comparing the schedules for the different designs in terms of value, maximum deviations and stripping ratios over the life of the mine. By choosing an optimal pushback design, the authors are able to increase the net present value of the production schedule for a sample copper deposit by approximately 30% over conventional methods; this increase is directly related to the pushback design that enables better risk management and extending the ultimate pit limits. While adding substantial value to the mine design, this framework for pushback design under uncertainty is computationally intensive, given that a SIP needs to be solved for each pushback design. The methodology is also limited by the fact that uncertainty is not directly incorporated in the pushback design but rather incorporated in the life-of-mine production scheduling process.

Meagher et al. [122] extend the parametric maximum flow/minimum cut approach [75, 134] to account for multiple orebody representations and multiple metal price simulations. The push-relabel algorithm generates a minimum cut that minimizes the sum of the ore left outside of the pit plus the waste mined inside. To

account for multiple geological simulations, the authors introduce infinite capacity bi-directional arcs between the blocks in each of the simulations to guarantee that if a block is extracted in one simulation, it must be extracted in all simulations. By incorporating multiple simulations and applying a parameterization, the algorithm is able to see blocks that not only have high value, but also blocks that have a high probability of being ore.

Asad and Dimitrakopoulos [8] extend the parametric maximum flow algorithm for pushback design under uncertainty and attempt to control the relative differences in sizes between pushbacks. The authors use the subgradient method to accommodate knapsack constraints for ore quantities in the pushback [158] and simultaneously minimize the differences in sizes between pushbacks. The authors apply the method on a copper deposit and compare the same method when solved without attempting to control the "gap" between pushbacks; when the gap control is used, the algorithm generates pushbacks that are much more practical (i.e. relatively equally sized pushbacks) than when the gap control is not used.

While the results thus far from the minimum cut/maximum flow framework are promising, the method is limited by its inability to simultaneously accommodate joint local uncertainty for both grades and material types that is available through the geological simulations; in the network flow framework, for any given block in a geological model, all simulations that have a positive economic value are grouped together, and all simulations with a negative economic value are grouped together, disregarding the local combinations of positive and negative blocks that any given geological simulation contains.

One alternative to using the network flow methods for pushback design is to use metaheuristics, which do not guarantee a truly optimal solution, however can be used to find a high-quality solution in a reasonable amount of time. One successful application of metaheuristics in mining is the use of the simulated annealing algorithm [60, 98] for life-of-mine (LOM) production scheduling under uncertainty. Godoy [65] proposes using an initial production schedule design and iteratively shifting blocks between production periods, with the ultimate goal of minimizing deviations from ore and waste production targets. While the concept of an ore or waste production target is not directly transferable from production scheduling to pushback design (given that pushbacks can be defined over a longer and variable timescale), the methodology with simulated annealing used is general and suitable for many types of mining problems. For designing pushbacks in particular, the underlying formulation for minimizing deviations and optimization methodology using simulated annealing is directly transferrable and easily generalized for multiple processes, where the target for each destination (waste or processors) can be described by the average over all simulations.

This chapter contributes a generalized stochastic pushback design algorithm that modifies an initial design to minimize the variability of the material that is sent to each destination over the geological simulations using the simulated annealing algorithm. This method is capable of optimizing real-world deposits, while simultaneously considering multiple (jointly simulated) elements [27, 148], simulated material types [156] that define multiple processing destinations and multiple zones with variable slope angles. The goal of the algorithm is to modify an existing pushback design

to better account for the joint local uncertainty in metal grades and material types simultaneously, while remaining similar to the original design in terms of pushback tonnages and the tonnages sent to the various destinations, which indirectly leads to a reduced level of risk in the economic value of the design. The method is readily and easily incorporated into any conventional or stochastic optimization framework, such as those previously mentioned. In the following sections, two formulations for modifying existing pushbacks that incorporate joint local grade and material type uncertainty are described, along with the implementation using the simulated annealing algorithm. The methods are then applied in a case study for BHP Billiton's Escondida Norte mine, Chile. Finally, conclusions and recommendations for future work are discussed.

3.2 Modifying Pushback Designs to Manage Geological Uncertainty

The contribution of this work is a computationally efficient and general method for modifying an existing pushback design to accommodate joint local uncertainty in terms of grade and material types simultaneously for complex mineral deposits and mining chains, where the material type defines the set of destinations that the block can be sent. Two formulations are proposed to achieve this, and both are similar in nature to the formulations used for mine production scheduling using the simulated annealing algorithm [1, 65, 111]. Unlike the methods used for life-of-mine (LOM) production scheduling, which have specified production targets for mining equipment and destinations, pushback design is much more general and does not necessarily have consistent or constant targets for each phase. For this reason, an initial pushback design is required to specify initial targets for each phase, which can be in the form of

a conventional pushback design, pit shells generated by a parametric algorithm or a randomly generated design with specified pushback size targets. The ultimate result after applying the proposed methodology is a mine design that mimics the same average tonnages in each pushback for each destination as the starting design (hence total size of pushback) and simultaneously minimize the variability of the tonnages sent to the various destinations over the set of geological simulations. Given that the variability in tonnages is strongly linked to the variability in the economic value of the material within the pushbacks, the proposed algorithm also indirectly reduces risk associated with the economic value of the design. The proposed methods can be easily integrated into existing frameworks for mine design and production scheduling (deterministic or stochastic), and does not drastically change the original design in terms of tonnages sent to the various destinations.

3.2.1 Formulations for modifying a starting design

Formulation #1

The first formulation aims to minimize the average absolute deviation (difference) from a target tonnage over all geological simulations [27, 156], where the target tonnage for each destination and pushback is defined as the average tonnage over the available simulations and the starting pushback design. This formulation is similar to the objective function proposed by Godoy [65] for production scheduling, where Godoy's ore and waste production targets are replaced by the average tonnages for

each destination from the starting pushback design. The formulation for the objective function is as follows:

$$\min \quad f(x) = \sum_{d=1}^{D} \sum_{p=1}^{P} E\left\{ \left| T^*_{d,p}(s) - T_{d,p} \right| \right\} \tag{3.1}$$

where:

- $d \in \{1, ..., D\}$ represents the indices for the destinations or processes that mined material can be sent to.
- $p \in \{1, ..., P\}$ represents the indices for the pushbacks.
- $s \in \{1, ..., S\}$ represents the indices for the input orebody simulations.
- $T^*_{d,p}(s)$ represents the quantity of material (i.e. tonnes) sent to process d in pushback p in scenario s for the current (modified) pushback design.
- $T_{d,p}$ represents the average quantity (tonnage) of material sent to process d in pushback p over all geological simulations for the *initial* pushback design, i.e. $T_{d,p} = \frac{1}{S} \sum_{s=1}^{S} T^0_{d,p}(s)$, and remains static throughout the optimization process.
- $E\left\{ \left| T^*_{d,p}(s) - T_{d,p} \right| \right\} = \frac{1}{S} \sum_{s=1}^{S} \left| T^*_{d,p}(s) - T_{d,p} \right|$ represents the expected value of the deviations from the targets over all geological simulations.

Formulation #2

One of the potential issues that may arise with using Formulation #1 is that it only evaluates changes based on a linear change in tonnage's absolute deviation. If there is a drastic difference in target quantities between the destinations, the optimizer may reduce the variability of material sent to one pushback but drastically increase the variability of material sent to another process because it is smaller in size; this is directly related to the well-known volume-variance subject that is studied in

geostatistics [93]. Another proposed formulation is to evaluate the square deviations from the targets and standardize them by their target size. This helps to avoid the issues with large discrepancies in quantities of materials sent to the processes, and provides a better measure of the variability of material going to each process. The formulation of the objective function is as follows, and uses the same variable nomenclature as Formulation #1:

$$\min \quad f(x) = \sum_{d=1}^{D} \sum_{p=1}^{P} \frac{E\left\{\left(T^*_{d,p}(s) - T_{d,p}\right)^2\right\}}{T_{d,p}} \tag{3.2}$$

3.2.2 Implementation using simulated annealing

Given that the proposed objective functions are mathematically non-linear, metaheuristics are well-suited for performing the optimization; rather than having to define an extremely large number of precedence constraints for each block in the model that is required for traditional optimization models, metaheuristics are able to simplify the formulation by implicitly obeying slopes and various other constraints when performing modifications to the pushback design, thus slope constraints are always respected at every iteration of the algorithm. The algorithms use the simulated annealing framework to perform the optimization based on the proposed objective functions [60, 98].

Simulated annealing algorithm

Algorithm 1 outlines the method for pushback design with simulated annealing. The underlying principle of simulated annealing is to start with an initial solution,

and gradually make perturbations, or shifts in the solution vector, where the probability of accepting sub-optimal perturbations (in terms of the variability of the design) decreases as the algorithm proceeds. Initially, the algorithm may accept sub-optimal solution shifts in order to attain a better solution in a future iteration (referred to as solution-space exploration). The starting pushback design is represented as a N-dimensional vector, x^0, where each component $j \in \{1, ..., N\}$ of the vector x^0, $x_j^0 = p$, represents the pushback (p) that block j is initially assigned to. The optimal destination for each block j in each simulation is determined by the undiscounted cash value, and stored in a vector, *dest* (see Sect. 3.2.2). The algorithm commences with a high annealing temperature, T, and is gradually reduced using a cooling factor, k, where $k < 1$. This temperature reduction is performed every $niter_{cool}$ iterations. At each iteration of the simulated annealing algorithm, a block j is randomly selected to be shifted from pushback p to p' (see later for details). The updated solution vector is represented by $x_{new} := x \oplus x(\rho(j), p')$, where $x(\rho(j), p')$ is the original solution with the modifications required to shift the set of blocks $\rho(j)$ to pushback p'. The annealing algorithm then randomly accepts or rejects the shift in solution based on a probability distribution, $P(f(x_{new}), f(x), T) < rand()$:

$$P(f(x_{new}), f(x), T) := \begin{cases} 1 & \text{if } f(x_{new}) \leq f(x) \\ \exp(-|f(x_{new}) - f(x)|/T) & \text{otherwise} \end{cases} \tag{3.3}$$

where $f(x_{new})$ is the value of the objective function (Eq. (3.1) or (3.2)) for the candidate (shifted) pushback design, $f(x)$ is the value of the objective function for the pushback design from the previous solution, and $rand() \in [0, 1]$ is a uniform random

number. As the annealing temperature T decreases, the probability of accepting a sub-optimal shift in solution, in terms of the objective function value, decreases. If the shift is accepted, the current solution x_{new} overwrites the previous solution x. If the current pushback design has the best objective function value discovered up to that point in the algorithm, x_{new} overwrites the global best solution, $x*$. Similarly, if the optimizer is exploring a solution that is too far away from the global best solution by a factor of $divergence_{criteria} > 1$, the global best solution, $x*$, replaces the current and previous solutions, x_{new} and x, respectively.

Block destinations

The proposed implementation requires assumptions based on the destinations for a given block for each geological simulation. Conventional mine design frameworks assume that the pushback design is performed prior to production scheduling, thus the truly optimal destination for each block and each geological simulation is not known a priori. For this reason, a greedy approach is used based on the destination that gives the highest recovered economic value. This assumption is common in pushback design for the vast majority of the currently used mine design optimizers as a means of simplification. The algorithm accepts multiple metals or deleterious elements and performs the calculations for the undiscounted economic block values accordingly. Material codes can be used to define where given block can be sent; for a set of destinations $d \in \{1, ..., D\}$, let $Ex\,(j, s) \subset \{1, ..., D\}$ denote the destinations that block j cannot be sent to in simulation s, given its material code. The value of a block $value_{j,d,s}$ for block j when sent to destination on d in simulation s can be

Algorithm 1 Modifying pushbacks with simulated annealing

x^0 ▷ The starting pushback is read
$dest$ ▷ Vector of destinations for each block & simulation based on $ value
$f(x^0)$ ▷ Objective function value (Eq. (3.1) or Eq. (3.2))
T^0 ▷ Initial temperature
k ▷ Cooling factor
$niter_{cool}$ ▷ The number of iterations to perform before applying k to T
$divergence_{criteria}$ ▷ Decides whether or not to revert to the global best solution
procedure SIMULATEDANNEALING(x^0, $dest$, $f(x^0)$, T^0, k, $niter_{cool}$, $divergence_{criteria}$, end)
    $x \leftarrow x^0$ ▷ Setting the current solution to the starting pushback
    $x_{new} \leftarrow x^0$ ▷ The candidate solution that x can be shifted to
    $x^* \leftarrow x^0$ ▷ The current best solution
    $f(x) \leftarrow f(x_{new}) \leftarrow f(x^*) \leftarrow f(x^0)$
    $T \leftarrow T^0$ ▷ Current temperature
    $niter_{total} \leftarrow 0$ ▷ Number of iterations performed in the algorithm
    $end \leftarrow false$ ▷ Algorithm will terminate when true
    while $end = false$ **do**
        $niter_{total} \leftarrow niter_{total} + 1$
        if $niter_{total} \mod niter_{cool} = 0$ **then** ▷ Reduce the temperature
            $T \leftarrow T \cdot k$
        end if
        $candidate_{found} \leftarrow false$ ▷ Finding a new pushback design
        while $candidate_{found} = false$ **do**
            $j \leftarrow$ RANDOMLYSELECTBLOCK(x) ▷ Select a block to change
            $p = x_j^0$
            $p' \leftarrow$ RANDOMLYSELECTANOTHERPUSHBACK(x, x_j, p)
            if ISACCEPTABLECHANGE(x, j, p, p') **then** ▷ See Sect. 2.2.3
                $\rho(j) \leftarrow$ RETURNCANDIDATEBLOCKS(x, j, p, p') ▷ See Algorithm 4
                $candidate_{found} = true$
                $x_{new} \leftarrow x \oplus x(\rho(j), p')$
            end if
        end while

Algorithm 2 Modifying pushbacks with simulated annealing - Continued

```
        if P(f(x_new), f(x), T) ≥ random() then   ▷ Accept or reject new design
            x ← x_new
            f(x) ← f(x_new)
        end if
        if f(x) > divergence_criteria · f(x*)
            x ← x* then                           ▷ Reset solution to global best
            x_new ← x*
            f(x) ← f(x_new) ← f(x*)
        end if
        if x < x* then                            ▷ Update the global best solution
            x* = x
            f(x*) ← f(x)
        end if
        if niter_total ≥ niter_max then           ▷ Termination condition
            end ← true
        end if
    end while
end procedure
```

calculated as follows:

$$value_{j,d,s} = \sum_{m=1}^{M} g_{j,m,s} r_{j,d,m,s} \left(V_m - SC_m\right) - \left(M_{j,s} + P_{d,j,s}\right) \tag{3.4}$$

where j denotes the block index, $d \in D \setminus Ex\left(j,s\right)$ denotes the potential destinations for block j based on its (possibly simulated) material code, $s \in \{1,...,S\}$ denotes the index of the geological simulation, $m \in \{1,...,M\}$ denotes the metal type (or deleterious element), $g_{j,m,s}$ represents the metal content of block j for metal m in simulation s, $r_{j,d,m,s}$ represents the recovery of metal m in block j when sent to destination d in simulation s (note that sending a block to waste will yield a recovery of 0), SC_m denotes the selling cost for metal m, V_m denotes the value of metal m (which can take on positive or negative values, depending on whether or not it is a deleterious metal), $M_{j,s}$ is the mining cost for block j in simulation s (where a mining cost adjustment factor, MCAF, can be applied) and finally $P_{d,j,s}$ is the cost to treat block j at destination d, which can be dependent on the tonnage of the block in simulation s. It is noted that in practice, Eq. (3.4) may need to be more complex for a given mine, however if it is possible to model the value mathematically, replacing Eq. (3.4) is trivial. The optimal destination d for block j in simulation s can then be determined as the destination that generates the maximum profit or minimum loss:

$$value_{j,s} = \underset{d}{\operatorname{argmax}}\, value_{j,d,s} \tag{3.5}$$

While the previously outlined method for generating block values, hence destinations, is common in most mine design software for generating pushback designs,

there are more advanced methods that should be considered in the future. Generally, the conventional framework for pushback design that the majority of mine planning software currently used in the industry needs to be reversed: the pushback designs should be generated using a production schedule, given that the production schedule better represents the time dimension, which impacts the net present value of the mine design and considers the annual production capacity constraints. One example of software that operates in this manner is BHP Billiton's Blasor [154], which is not commercially available. However, in the proposed framework, this modern methodology is computationally intensive given that the pushback design and production schedule would need to be solved simultaneously. As a result, the proposed methodology is best suited for a conventional framework that generates pushbacks prior to production scheduling.

Solution shifting

A shift of the solution vector is a perturbation of the pushback design; a central block j is randomly selected, and, if it is a candidate for shifting the block from pushback p to p', the block is then permitted to be tested according to the accept or reject criterion of the simulated annealing algorithm (Algorithm 1).

The ability for a candidate block to be shifted to a different pushback is based on the directly adjacent blocks and the number of predecessors or successors (overlying or underlying blocks, respectively) that need to be corrected to ensure that the slope constraints are respected. Consider a randomly selected block j from Algorithm 1 that needs to be checked if it is a suitable candidate for shifting. Let (i, j, k) denote the coordinates (IJK coordinate system) of block j, and let $\rho(j)$ denote the list of

blocks that need to be corrected to guarantee slope stability. Additionally, consider the minimal sets $\mathcal{P}(j)$ and $\mathcal{S}(j)$ of predecessors or successors, respectively, for any block j, which can be calculated with variable slope angles; see Khalokakaie et al. [95] for a description of the algorithm to generate variable predecessor sets. Algorithm 4 is used to verify whether or not the block is a suitable candidate for shifting. The algorithm uses a breadth-first-search (BFS) algorithm [35] to explore the predecessors or successors that may need fixing up; if there are too many corrections that need to be made (greater than or equal to $totFixups$), the algorithm stops exploring j and attempts to find another candidate block. The reason for quitting after $totFixups$ corrections is that Algorithm 2 is the bottleneck for the simulated annealing algorithm's performance, and allowing the algorithm to continue the BFS substantially increases the algorithm's running time. It has been noted experimentally that it is sometimes useful to allow the user to set $totFixups = \infty$, which is referred to as an aggressive shifting strategy, as it can often help the algorithm get out of local optima faster. It is noted that the BFS algorithm uses a queue data structure, where the predecessors or successors are explored in a first-in-first-out (FIFO) manner using generic Enqueue and Dequeue functions [35]. The algorithm either terminates prematurely and states that the randomly selected block j is not a candidate for shifting, or approves the candidate block and returns a set of blocks that need to be shifted to pushback p' to guarantee slope stability.

Algorithm 3 Check if block is a candidate for a solution shift

function IsAcceptableChange(x, j, p, p')
▷ Let $x_{(i,j,k)} = p^*$ denote that the block located at coordinates (i, j, k) is in pushback p'
$isCandidate = true$
▷ Check to see if overlying and underlying block are in the same pushback
if $x_j\,(i, j, k-1) = x_j\,(i, j, k+1)$ **then**
$isCandidate = false$
end if
▷ Check to see if the adjacent blocks are in the same pushback
if $x_j\,(i+1, j, k) = x_j$ and $x_j\,(i-1, j, k) = x_j$ and $x_j\,(i+1, j, k) = x_j$ and $x_j\,(i, j+1, k) = x_j$ and $x_j\,(i, j-1, k) = x_j$ **then**
$isCandidate = false$
end if
return $isCandidate$
end function

3.3 Application at Escondida Norte Mine, Chile

The two proposed formulations for modifying pushback designs to simultaneously account for grade and material type uncertainty are tested at BHP Billiton's Escondida Norte mine, Chile.

3.3.1 Overview of the Escondida Norte Mine

The Escondida mining operation is currently the world's largest open pit copper producer, located 170 km southeast of Antofagasta in northern Chile. For this case study, only a portion of the Escondida mining complex is being considered, called Escondida Norte, which contains approximately 176,000 blocks that are 25·25·15 cubic meters in size. The mine has provided fifty conditional simulations that contain simulated copper grades, recoveries, tonnage and five simulated material types; the material types are classified as waste, sulphides (low or high recovery), mixed and

Algorithm 4 Check if block is a candidate for a solution shift

```
function ReturnCandidateBlocks(x, j, p, p′)
    ρ(j) ← {j}                                   ▷ Initialize the candidate set
    searchUp ← true    ▷ Defines if blocks above/below may violate slope angles
    if p′ > p then
        searchUp ← false
    end if
    for all u ∈ {1, ..., N} do        ▷ Initialized vector of explored blocks in BFS
        explored[u] ← false
    end for
    Enqueue(Q, j)
    while Q ≠ ∅ do
        u = Dequeue(Q)
        fixupCount ← fixupCount + 1
        if fixupCount ≥ totFixups then
            return ∅                              ▷ Too many corrections required.
        end if
        Adj(u) ← ∅
        if searchUp = true then
            Adj(u) ← P(u)                        ▷ Search predecessors (above) of u
        else
            Adj(u) ← S(u)                        ▷ Search successors (below) of u
        end if
        for all v ∈ Adj(u) do
            if explored[v] = false then
                explored[v]← true
                if searchUp = true and x_v > p′ then
                    ρ(j) ← ρ(j) ∪ {v}            ▷ Needs to be corrected to p′
                    Enqueue(Q, v)
                end if
                if searchUp = false and x_v < p′ then
                    ρ(j) ← ρ(j) ∪ {v}            ▷ Needs to be corrected to p′
                    Enqueue(Q, v)
                end if
            end if
        end for
    end while
    return ρ(j)
end function
```

oxides. Escondida Norte has already mined or is in the process of mining the first two pushbacks (out of a total of 10); this study tests the proposed formulations on the remaining pushbacks (#3 to #10), and will herein refer to these pushbacks as #1 to #8. The geological simulations, which contain simulated copper grades and material types, also specify four zones that represent areas that have separate slope angles. In particular, the slopes are set as 33°, 35°, 41° and 35° for the first, second, third and fourth zones, respectively.

Escondida Norte is capable of extracting a total of 500,000 tonnes per day (500 ktpd); of which the waste goes directly to the waste dump (unlimited capacity), the sulphides have the option to go to one of two mills (both having a capacity of 60 ktpd) or the bio-leach pad (unlimited capacity), the mixed ore can go to the bio-leach pad, and finally the oxide material can go to the acid leach plant (60 ktpd). Figure 3–1 shows a flow chart for the potential destinations that each material can be sent to. The copper recoveries for each of the processes are specified for each block in each simulation, and were provided by the mine. The material types (which affect candidate destinations) are also simulated variables, which are specified in the geological simulations. Given that both the Escondida and Escondida Norte mines feed the two mills, the geological simulations show similar recoveries for both mills and the material codes do not distinguish between the two mills, the case study is simplified such that the Escondida Norte material only goes to one of the mills; it is not expected that this decision will have a significant impact on the case study given that the recoveries are similar, and provides a simpler analysis of the algorithm's optimization performance.

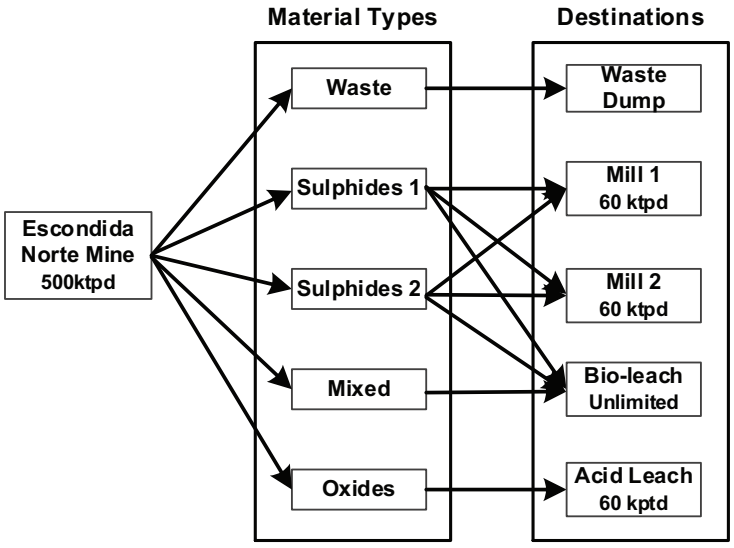

Figure 3–1: Flow chart of materials to various destinations at the Escondida Norte mining operation.

It is assumed that the copper selling price is \$1.90/lb, with a selling cost of \$0.2/lb. The base mining cost is \$1.40/t, with a depth-based mining cost adjustment factor applied and is specified in the geological models. The milling, bioleach and oxide leach costs are assumed to be \$5.50/t, \$1.75/t and \$4.25/t, respectively. Table 3–1 shows the target tonnages for each pushback and process from the original design that the annealing algorithm tries to maintain (while simultaneously reducing the variability). It is noted that the target tonnages have all been scaled proportionately.

3.3.2 Numerical results and analysis

Both proposed formulations are tested on a set of 20 geological simulations. In terms of change in objective function values (evaluating the objective functions proposed in Eqs. (3.1) and (3.2) before and after modifying the pushback designs), Formulation #1 indicates a decrease of 35% and Formulation #2 indicates a decrease

Table 3–1: Target tonnages for each process defined by the average tonnages for each destination over all orebody simulations in the starting design.

Pushback	Mill Target (Mt)	Waste Target (Mt)	Oxide Leach Target (Mt)	Bioleach Target (Mt)
1	100	4.68	0.64	1.41
2	1.13	24.75	20.58	2.71
3	79.54	94.10	9.17	7.91
4	139.89	80.20	23.08	19.67
5	176.22	170.52	0.03	4.61
6	323.77	435.85	0.42	11.47
7	128.40	287.08	10.16	29.52
8	161.84	363.79	2.83	5.83

of 61%. In both cases, it takes approximately 1.5 hours to obtain a solution on a Intel i7 2.66 GHz MacBook Pro with 8GB RAM; it is noted, however, that it takes less than 30 minutes to converge on a pushback design within 10% of the final solution. Table 3–2 shows how the three designs (original and two designs from the proposed formulations) perform when evaluating using the proposed objective functions (Eqs. (3.1) and (3.2)). When considering Formulation #1, described by Eq. (3.2), the annealed design that was optimised using Formulation #1 (Design 1) reduces the original objective function value by 35%. Interestingly enough, when Design 2 (generated by annealing with Formulation #2) is tested using Eq. (3.2), the resulting decrease in objective function value is higher than Design 1, indicating that Formulation #2 is outperforming Formulation #1; this is likely caused by its ability to better handle variability with its square term. When the designs are evaluated using Eq. (3.2), as previously mentioned, Design 2 (generated by annealing with

Table 3–2: Objective function values when the three designs are tested with the proposed objective function formulations (and respective percent reduction).

Phase Design	Formulation #1 (Eq. (3.1))	Formulation #2 (Eq. (3.2))
Original	19,425,023	1,594,155
Design 1	12,624,898 (-35%)	1,099,249 (-31%)
Design 2	11,962,492 (-38%)	629,568 (-61%)

Formulation #2) reduces the initial objective function value by approximately 61%. Design 1 does not perform nearly as well under Eq. (3.2), and only reduces the objective function value in Eq. (3.2) by 31%. This is largely caused by the fact that Formulation #1 does not consider the differences in sizes between the pushbacks. This indicates that the design generated by Formulation #2 outperforms Formulation #1 for either objective function equation, and thus provides better designs, in terms of risk reduction.

Given that the tonnages going to each of the destinations can vary significantly between pushbacks (e.g. mill tonnage in pushback #2), the following analysis has been standardized in terms of the target quantities of materials for each destination and each pushback to provide a clear view of the changes in the risk profile, i.e. for any destination p, pushback t and simulation s, the standardized risk for simulation s is defined as:

$$StdRisk_{d,p,s} = \frac{T^*_{d,p}(s)}{T_{d,p}} \tag{3.6}$$

For this analysis, the risk profiles are defined by the minimum and maximum standardized risk (Eq. (3.6)) over the simulations for each destination and each

pushback; while separating the risk profiles into three graphs for each design and showing the risk profiles for all simulations may lead to a better understanding of the differences in performance between the two algorithms, comparing the minimum and maximum standardized risk values on a single graph is much more clear. It is noted that in the standardized graph, the target value is 1.0 (on the vertical axis); a standardized risk value of 1.6, for example, indicates that one simulation has the potential to have 60% more tonnage than the target indicated in Table 3–1. Similarly, a value of 0.4 indicates the potential to have 40% of the target indicated in Table 3–1.

Figure 3–2 shows the standardized mill tonnage risk profiles for the original pushback design, and the designs after using Formulations #1 and #2. Both proposed formulations are able to substantially decrease the minimum and maximum risk in the second pushback when compared to the original pushback design, however it is noted that the mill tonnage in pushback #2 is substantially smaller than the other pushbacks, hence would not likely have a significant impact on the mine's operations. Both proposed formulations are able to reduce the downside risk (tonnage below the target) in pushbacks #3 through #7 from the original design, which all have large mill tonnages and thus the reduction in risk is likely to have operational significance. In general, both formulations perform very similar in terms of reducing the minimum and maximum risk of the pushback designs, with some improvements from Formulation #2 over Formulation #1 in pushback #2.

Figure 3–3 shows the risk profiles for the waste tonnage for each pushback. It can be seen that the original design has a substantial amount of risk in waste tonnage

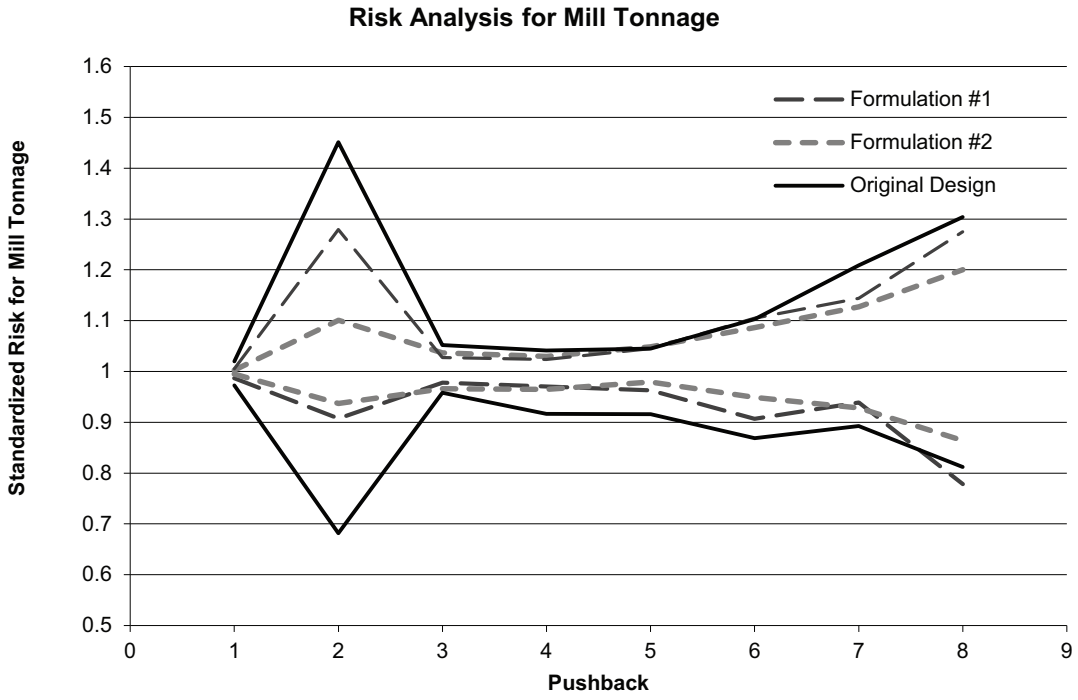

Figure 3–2: Standardized risk analysis for mill tonnage in the original, Formulation #1 and Formulation #2 designs.

for the first pushback; both Formulations #1 and #2 substantially reduce this standardized risk from approximately 50% to (+29%/-15%) and ±8%, respectively. This risk, however, is associated with a small target waste tonnage, hence the reduction in variability is not expected to have any operational significance. Figure 3–3 also indicates a reduction in the downside risk (standardized risk for waste tonnages greater than 1) from both formulations by approximately 10% for pushbacks #4 through 7, which confirms the result of reducing the downside risk in mill tonnage in Fig. 3–2. It is clear that once again, the difference in the performance between the two

proposed objective function formulations is almost negligible for the minimum and maximum standardized risk values. There is a slight improvement for the minimum and maximum standardized risk profiles by Formulation #2 in the final 3 pushbacks, however it is not significant.

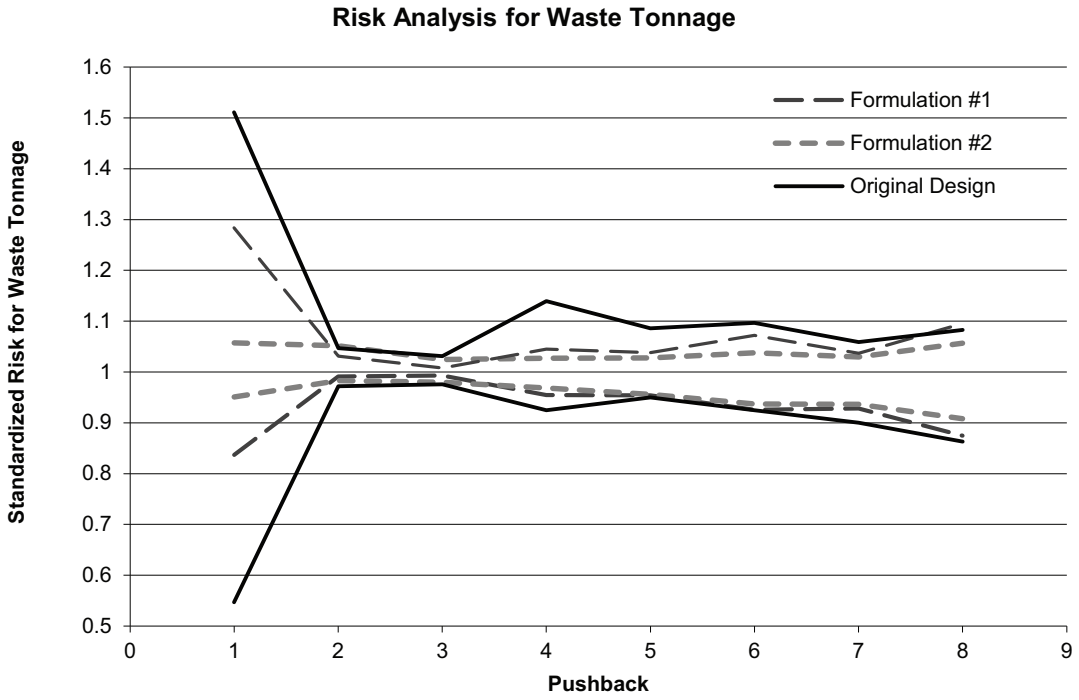

Figure 3–3: Risk analysis for waste tonnage in the original, Formulation #1 and Formulation #2 designs. Note that the waste tonnages have been standardized to the original design's target size.

Figure 3–4 shows the standardized risk profiles for the tonnage sent to the oxide (acid) leach process. All three designs show similar standardized risk profiles, with the exception of pushback #5, where the original design shows a drastic amount of risk in tonnage. Formulation #1's standardized risk profile for pushback #5 shows

an increase in risk, whereas Formulation #2 shows a drastic reduction in risk. It is noted, however, that the total tonnage sent to the acid leach process in pushback #5 amounts to approximately 2 days of the mine's production, and this variability is therefore not expected to have a drastic effect on the mine's overall value.

Figure 3–5 shows the standardized risk profiles for bioleach tonnage. In this case, there is a clear difference between the two methods; Formulation #1 does not appear to obey the target tonnages, which is likely a result of the fact that the target sizes for the bioleach process is smaller than the mill and the optimizer is reducing the overall risk of the design by shifting it to the bioleach process; this is a perfect example for why Formulation #2 was proposed. Formulation #2 generates entirely different risk profiles that lead to higher quantities of mixed (bioleach) material mined in the first and third pushbacks, and reduced quantities mined in the fifth and eighth pushbacks. Overall, it appears that Formulation #2 was able to reduce the risk in the bioleach tonnage over the original design; while the profiles may appear to fluctuate around different means, the difference between the minimum and maximum curves is smaller for this design. This is not a significant change given that the quantities of mixed material in these pushbacks is low compared to the other pushbacks.

Table 3–3 shows the total undiscounted cash value for each of the pushbacks for the three designs; it is noted that the cash values have been scaled arbitrarily and do not reflect reality. Both designs that result from the two proposed formulations have 7-10% higher undiscounted value in the first pushback than the original design. This difference is recovered by the original design in pushback #3, however both

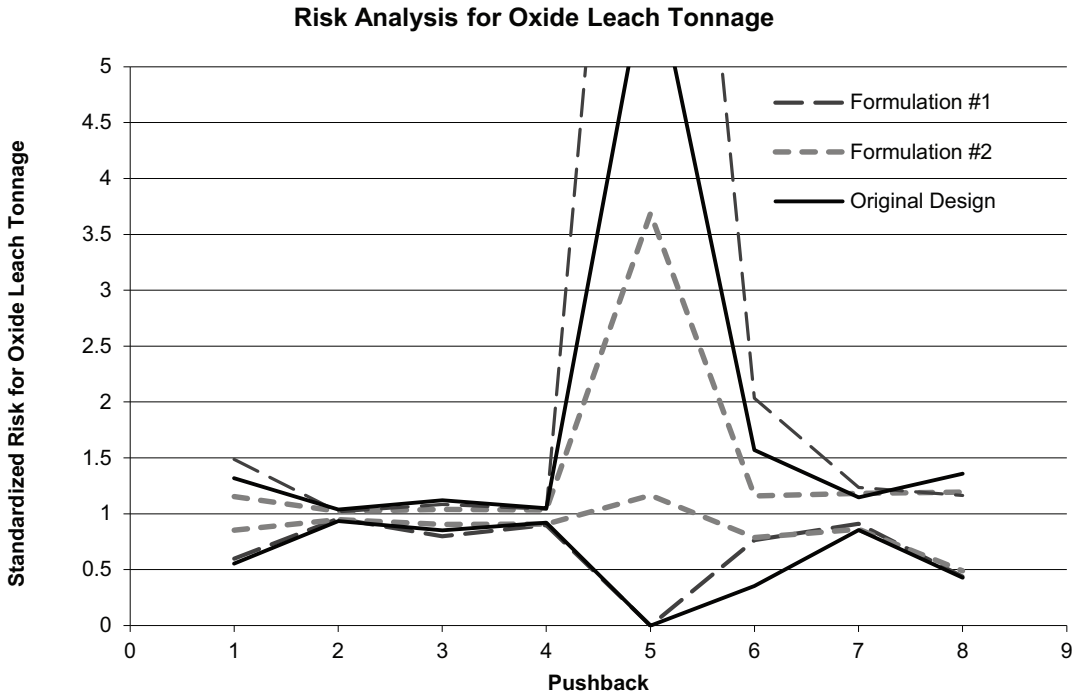

Figure 3–4: Standardized risk analysis for oxide tonnage in the original, Formulation #1 and Formulation #2 designs.

formulations show a 10-12% increase in value for the fourth pushback over the original design. By moving the undiscounted cash values forward through the life of the mine, the designs generated by the proposed formulations may lead to an increased net present value after production scheduling is performed because discounting will favour the higher-valued pushbacks mined early in the mine's life; however, these assumptions will need to be confirmed with stochastic LOM production scheduling. Given that a fair comparison between the deterministic and stochastic mine

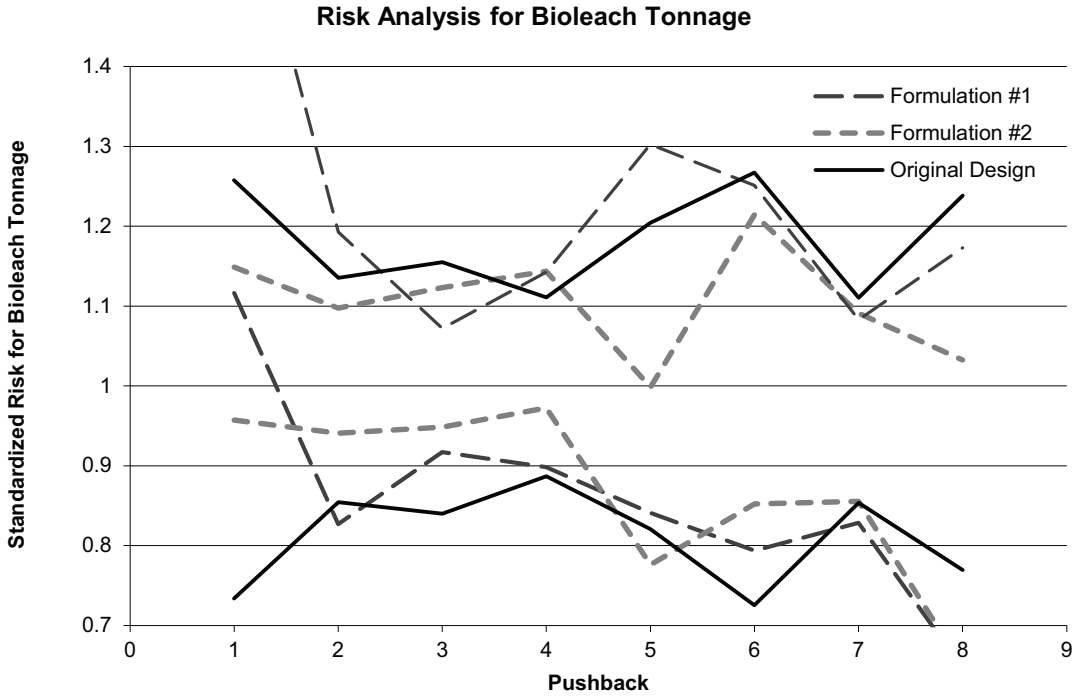

Figure 3–5: Standardized risk analysis for bioleach tonnage in the original, Formulation #1 and Formulation #2 designs.

design frameworks would require a substantial amount of detail regarding long-term production scheduling [2], this comparison is will be a topic of future research.

Figure 3–6 shows the risk analysis of undiscounted values, where the values are standardized in terms of the design's own average undiscounted value (rather than in terms of the original design used for standardized risk analysis of tonnages). For pushbacks #1 to #3, the differences between risk profiles for all three designs are negligible, however the differences are more pronounced for the later pushbacks, where the proposed formulations show substantially less risk than the original design,

Table 3–3: Average undiscounted cash values for each pushback for the starting designs and the designs after applying the proposed formulations (values have been scaled).

Pushback	Average Undiscounted Value		
	Starting design ($M)	Formulation #1 design ($M)	Formulation #2 design ($M)
1	100	106.6	109.82
2	16.37	17.26	17.91
3	66.72	65.19	58.74
4	98.42	107.78	110.28
5	78.65	77.74	80.27
6	131.32	119.00	116.44
7	45.64	46.63	49.12
8	39.12	36.04	33.66

in the order of 10 to 20%. This is a result of reducing the risk for the mill and waste tonnages for pushbacks #4 through #7 discussed previously.

The undiscounted value risk profile for the original design performs extremely well, in specific, there is very little risk in the first pushbacks, and grows gradually for the remaining pushbacks; this result is generally not expected, given that conventional mine design frameworks do not understand uncertainty that is represented through a set of geological simulations, and thus do not understand the effects of deferring it to later pushbacks (or production periods). Using a conventional framework, it is expected that the risk profiles are substantially more erratic than what is shown in Fig. 3–6 because there is no control over the uncertainty. In order to incorporate this risk-deferral of undiscounted cash flows in any of the proposed formulations, one can simply add a penalty cost to the variance of undiscounted cash

flows, where the penalty cost is reduced for each subsequent pushback. Similar concepts are used for production scheduling under uncertainty with stochastic integer programming [50, 142]).

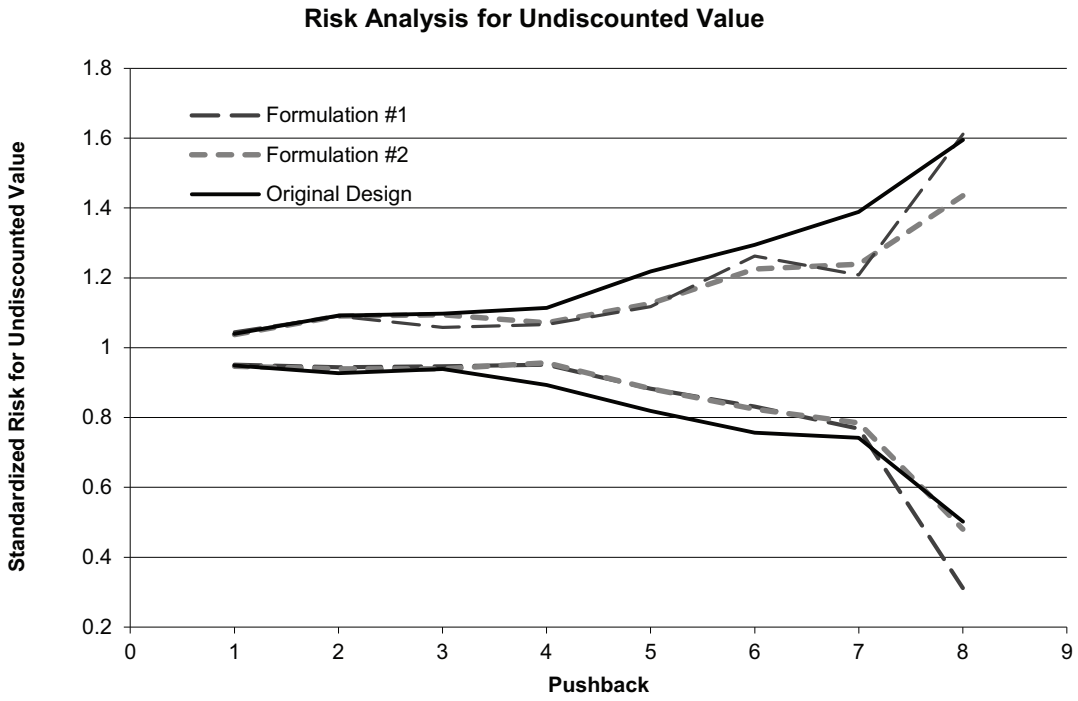

Figure 3–6: Standardized risk analysis for the undiscounted cash values for the original pushback design and Formulation #1 and Formulation #2.

Figure 3–7 shows a sample cross-section from the original pushback design, and the sections from the resulting pushback designs from Formulations #1 and #2. While there are some consistencies in terms of the locations of the pushbacks, the algorithm seems to have chosen pushback designs that are similar to the type of pit shells commonly seen with conventional mathematical optimization tools. It can

be seen that the first four remain relatively unchanged, which is indicative of lower uncertainty near the surface. It is apparent that the algorithm is still respecting the various slope angles, and has changed the initial design considerably to be able to attain the reductions in risk that were previously discussed.

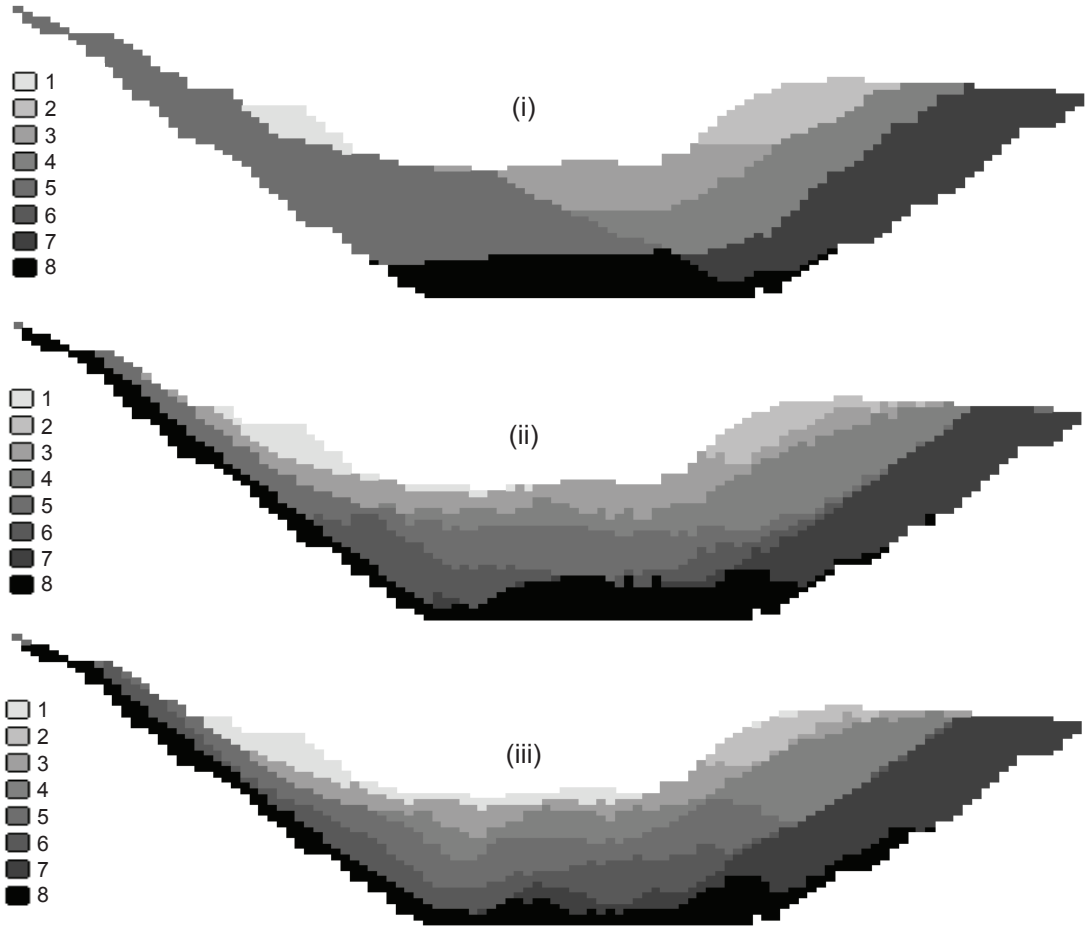

Figure 3–7: Cross sections of the pushback designs. (i) Original design. (ii) Formulation #1 design. (iii) Formulation #2 design.

3.4 Conclusions

This chapter proposes two general formulations to modify an existing pushback design to incorporate joint local geological uncertainty and is implemented using a simulated annealing algorithm. The proposed formulations and algorithm contributes and improves on existing stochastic mine design frameworks because it: (i) can accommodate joint uncertainty between multiple metals and material types and multiple material destinations; and (ii) directly reduces the geological risk associated with pushback designs for each material destination. The proposed formulations and implementation can be easily integrated into any existing framework used by a mine to aid in minimizing the variability of materials, be it a deterministic or stochastic framework. The proposed algorithm using simulated annealing is computationally efficient, and suitable for real-world applications because it is capable of considering multiple metals, materials, destinations and slope zones.

The proposed formulations are tested on BHP Billiton's Escondida Norte mine, Chile. The two proposed formulations often show significant reduction in variability for pushbacks that are particularly problematic, without displacing the risk to other material destinations or pushbacks. The second proposed objective function formulation outperformed the first formulation in this case study, with an overall reduction in objective function by 61% over the original design; additionally, the second design also outperforms the first design when evaluated with the first objective function by an additional 3%. This is a result of its ability to better penalize variability using a squared deviation from target, along with the fact that it treats all processes equally,

regardless of the tonnages going to the various processes. While the proposed designs appear to have higher values and reduced risk in the earlier pushbacks, the true effects on the net present value of the proposed pushback design methodology would need to be tested with long-term production scheduling.

Future work will seek to integrate the proposed pushback design algorithms with stochastic life-of-mine production scheduling, and quantify the impact that stochastic pushback design has on the annual production schedules. Additionally, new methods for pushback design need to be developed for industrial use. These production schedules should, unlike the conventional approaches, integrate uncertainty and consider the pushback design's direct influence on the net present value and annual capacity constraints. However, this type of a development will require substantial enhancement of computational efficiency of the related scheduling methods.

CHAPTER 4
Mining Supply Chain Optimization under Geological Uncertainty

The previous Chapter focuses on integrating geological uncertainty into pushback design for a sequential optimization framework. This method, however, is limited by the fact that critical aspects such as blending, capacity constraints and the time value of money are not considered while optimizing the pushback design; this is reflected by the definition of the "optimal" block destination, which is defined solely by the destination that provides the maximum value or minimum loss. The remainder of this thesis focuses on a new, global optimization framework that first starts with a simultaneous optimization of the production schedule (considering blending, capacities and time value of money), which may then be used to drive the definition of ultimate pit limits and pushback design [57, 123, 125, 154]. This Chapter begins to address this simultaneous optimization by investigating new models and methods for optimizing the downstream (post-mining) aspects of a mining complex, referred to herein as a supply chain. There are several original contributions of this work that form the foundation for the remainder of this thesis. First, a unified modelling approach is created that isolates the flow of primary attributes (metal content, tonnages, etc.) through a mining complex from the secondary information that is of interest to a modeller (e.g. revenues, costs, product specifications, nonlinear transformations). A modeller can then choose various attributes of interest to construct an two-stage stochastic optimization model that suits their own needs

and objectives. As a result, the method is general and may be applied to many different mining complexes with minimal effort. Second, this chapter develops the concept of destination policies, which define where extracted material is sent under geological (supply) uncertainty. This is an important contribution because it is more adept for optimizing mining complexes, and overcomes many of the widely-known limitations of cut-off grade policies. The third contribution is a result of the first: by adopting the proposed modelling methodology, it is possible to shift the focus of the optimizer to evaluating the economic value of the products *sold*, rather than the economic value of the materials *mined*. This overcomes the limitations of the widely-used "block value" because the destination of mined material is no longer assumed to be known a priori, and the economic value is based on the blend of materials that are sold, rather than a block's economic value, which is calculated in isolation from other blocks. The final contribution of this Chapter is the development of an efficient metaheuristic solver that uses a unique combination of two existing algorithms (particle swarm optimization and simulated annealing) to optimize the destination policy and processing stream "flow" variables. A full-field test is performed for the Onça Puma nickel laterite mining complex, Brazil. Results highlight the need to consider geological uncertainty when optimizing the processing streams, particularly for highly variable operations that have stringent feed quality specifications. The stochastic optimizer is successful at generating a robust destination policy that is able to satisfy these stringent quality specifications while simultaneously reducing the risk of not meeting production targets.

4.1 Introduction

As the world's mineral resources are continuously being depleted, mining companies are seeking to extract and process (treat or transform) increasingly complex material profitably. Through tremendous technological and operational improvements, mining companies can often profitably process material that was once considered waste, which is often the result of two interrelated aspects. The first aspect relates to blending material from various sources of different mineralogical quality in order to obtain a homogenous and predictable material that meets a set of specifications prior to processing or delivery to customers. The other aspect is related to mines that contain several commodities (e.g. metals) or materials of distinct mineralogy that can be transformed into multiple products; these cases often require multiple processing streams to separate and treat the various commodities. As the number of options for blending or processing increases, along with the methods for distribution, a supply chain is formed (e.g., Fig. 4–1), which defines the flow of material from the set of mines through the processing streams to final products that are sold to markets or customers. A thorough review of global mining supply chains in the context of iron ore operations is given by Pimentel et al. [137]. The complexity of mineral resource supply chains can vary drastically, depending on the products produced (e.g. iron, coal, nickel, copper) and the geological and geographical conditions. For example, iron ore supply chains in Western Australia often stockpile materials near the mines. Material is transported from the mine stockpiles via a railway to a set of stockpiles at the port. The stockpiled material at the port is then shipped to customers with very stringent product quality constraints. Alternatively, an integrated nickel value chain

may mine, grind, concentrate, smelt and refine the metal that is either delivered to a contracted customer or is sold on the volatile spot market. As the complexity of the supply chains grows, there is a need for tools that optimize the flow and management of materials through the supply chain while simultaneously respecting the practical constraints that may be imposed at each location.

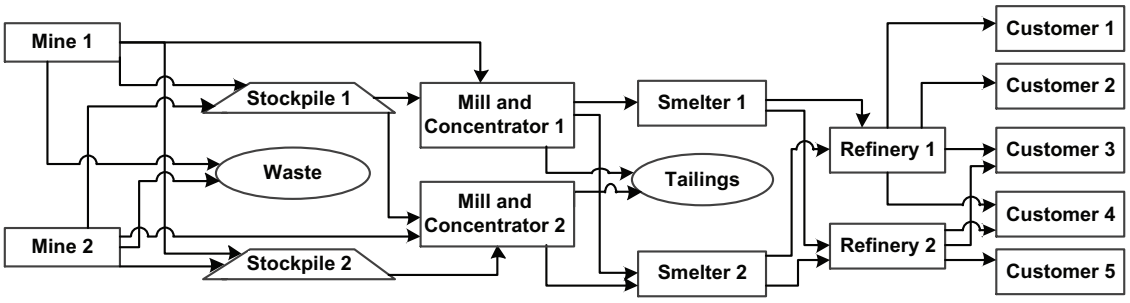

Figure 4–1: Hypothetical example of a mining supply chain.

Without loss of generality, the fundamental concepts of this work are demonstrated through an application for a real-world nickel laterite blending chain (Fig. 4–2) called Onça Puma, which is a useful example to highlight aspects of blending and non-linear constraints in a supply chain. While the example may be simplistic, the general modelling and optimization methodology can be extended to larger instances, such as Fig. 4–1, which may require multiple processing streams, transportation options and customers. Laterites are challenging deposits to mine because of their variability related to the depth, the thickness and the amount of nickel metal contained in a layer of material called saprolite. In addition to this challenge, the pyrometallurgical plant that refines the saprolite into nickel metal imposes quality specifications for the by-products contained in the saprolite, specifically the silica-to-magnesia ratio and iron content; a deviation from the specifications can have severe

economic and operational consequences. To overcome these challenges, blending stockpiles are used as a buffer to not only reduce the impact of the geologic variability, but also to mix the material together to achieve a desirable specification prior to processing. Material is pulled from the buffer stockpiles over time and homogenized prior to being sent and treated at the processing plant (Fig. 4–2). The act of accumulating and partially removing material from the stockpiles over time leads to a non-linear model that is challenging to optimize using existing methods [18]. The nickel metal obtained after processing is then sold on the market. The key decisions and issues that need to be addressed in practice are:

i *Destination policies*: discrete decisions that determine where the mined material initially goes (e.g., waste dump, or stockpiles).

ii *Processing stream decisions*: the quantity of an output product that is sent to the subsequent destinations (e.g., amount of material reclaimed from a stockpile and sent to a homogenization pile).

iii Uncertainty should be integrated into the decision-making process.

iv The mining supply chain needs to be optimized holistically, from the mine through to the final products sold, while obeying practical constraints over the life of the value chain.

Existing work related to iron ore supply chains [20, 136, 153] assumes that all mined material has already been defined as ore or waste a priori; in general, this is a decision that should be directly controlled by an optimizer. This work aims to develop a framework that addresses some limitations of past research through the development of a stochastic supply chain optimization model, and is organized into

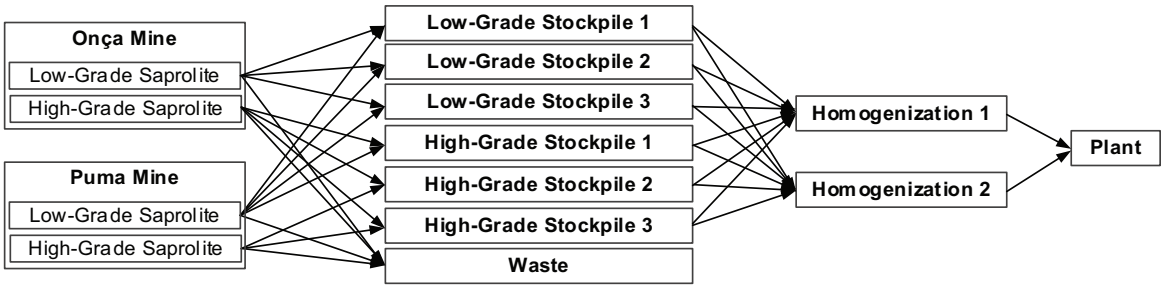

Figure 4–2: Flow of materials at the Onça Puma nickel laterite blending chain.

six sections. Figure 4–3 provides a graphical overview of the proposed methodology. Section 4.2 provides a review of related literature to this work. In Section 4.3, a flexible modelling methodology is developed that can be tailored to suit many supply chain configurations, including Onça Puma. A core aspect of this methodology is related to *destination policies*, which permits the optimizer to define materials as ore or waste, and provides a basis for optimizing the mineral resource supply chain with uncertainty. Section 4.4 provides an overview of the generalized formulations and metaheuristics used to optimize the supply chain model with uncertainty. In Section 4.5, aspects of the deterministic and stochastic methods are demonstrated through an application at the Onça Puma nickel laterite blending chain. Finally, Section 4.6 provides conclusions and outlines future research.

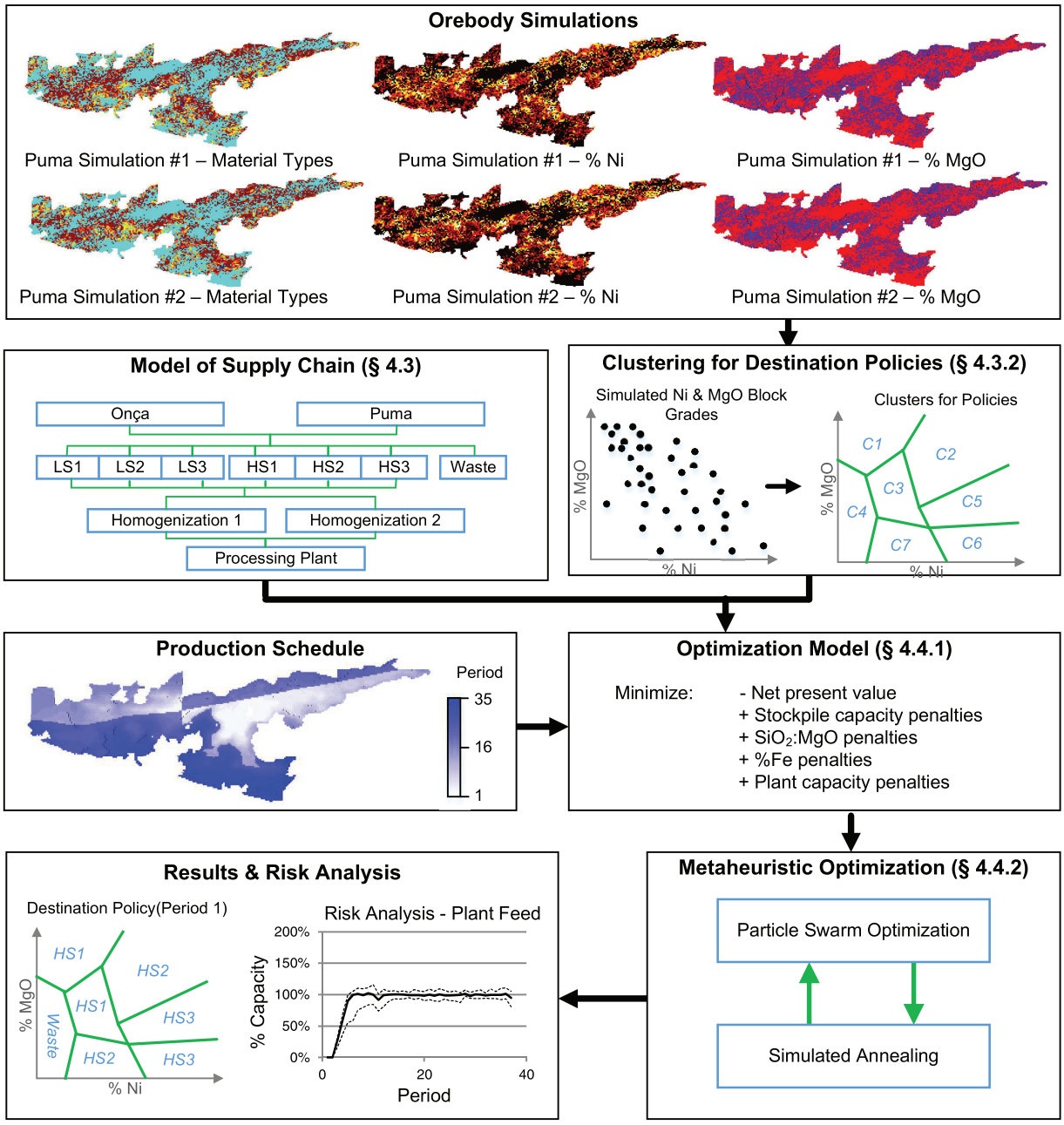

Figure 4–3: Overview of the proposed supply chain optimization methodology.

4.2 Literature Review

The present work relates to the literature in mine planning, material processing optimization and stochastic optimization. The primary objective of long-term optimization of mineral resource supply chains is to maximize the net present value (NPV) of the cash flows; this requires optimizing the long-term mine production schedules and the use of the supply chain's processing streams and distribution methods simultaneously [165, 166]. Long-term production scheduling defines the extraction sequence of materials from the ground (i.e., what to mine and when), thus defines the supply of materials available in a given period of time. Material processing optimization, which will herein be referred to as *supply chain optimization*, outlines how to maximize the utility of the available materials (supply) by defining destination policies, and the use of processing and transportation methods to produce and deliver the products to customers or the open market. The vast majority of related research has focused on long-term production scheduling optimization, where models often require thousands to millions of decision variables. In order to reduce the size of these models, the destination of the extracted material is often defined a priori; these decisions are almost exclusively based on the economic value of a discrete volume of material, referred to herein as a *block* [29, 61, 113, 142]. When the destinations are specified a priori, the models ignore the impact that the aggregate of blocks will have on capacity and material quality constraints that are imposed in the supply chain. More recent production scheduling formulations attempt to integrate material destination decisions directly into the optimization models [15, 23, 63, 100, 105, 124], but are incapable of optimizing non-linear supply chain configurations that are linked

through time, such as the stockpiles that are used at the Onça Puma blending chain (Fig. 4–2). In order to improve these production scheduling models, it is necessary to revisit and develop new models that better-manage the down-stream aspects of the supply chain.

Certain aspects of the material processing optimization of mining supply chains have been introduced in order to address many limitations of related production scheduling research. The majority of these research efforts has focused on supply chain configurations that blend or strategically stockpile material in order to produce a homogenous material tailored to required specifications [55, 76, 77, 151, 154]. Chanda [30] and Epstein et al. [52] propose multi-echelon blending and supply chain models that are solved using linear optimization. Everett [54] discusses simulation algorithms used for decision support to obtain a homogenous product for multi-echelon iron ore supply chains. Singh et al. [153] develop a sophisticated model for an iron ore supply chains and demonstrate substantial improvements in the quantity of iron ore shipped while meeting customer's product specifications. Sandeman et al. [150] present two mining supply chain case studies that are solved using discrete event simulation (DES) and a linear optimization model. While the mentioned methods contribute to an improved understanding of mining supply chain optimization and more accurate models, they are limited by either being tailored for a specific supply chain configuration or require linear optimization assumptions.

The majority of the previously mentioned work assumes that the supply of materials generated from a production schedule is known with certainty. Conventional optimization methods use a single, deterministic representation of the geology, where

the block attributes (i.e., metal content) are expected values that are estimated using geostatistics [84]. The assumption of a single input is fundamentally flawed for mining optimization models for two reasons. First, the geology is estimated using sparse drilling information, hence is not known with certainty prior to extraction. Second, having the option of multiple processing streams leads to non-linear transfer functions (e.g., economic value of material processed or wasted); using an expected value of a random variable as input to the optimization model does not necessarily provide decisions that perform well on average.

A recent trend of research has addressed the integration of uncertainty into mining optimization models; of critical importance to the mining industry is what is termed geological uncertainty, or the uncertainty that arises from not knowing exactly what is in the ground prior to extraction and processing. This supply uncertainty can have serious ramifications in the supply chain as the bulk material from the mines is transformed to salable products [137], and has been shown to be a key contributor to deviations from production targets [45]. Rather than using an optimizer that accepts only a single representation of the geology as input, it is desirable to have an optimizer that relies on a set of equally probable geological simulations of materials and attributes that are generated using Monte Carlo simulation methods [69]. Unlike estimation methods, which tend to smooth out low and high values, geostatistical simulation methods better reproduce the spatial variability, univariate distributions and cross-correlations of the attributes of interest. Integrating geological uncertainty into supply chain optimization models is challenging for two reasons: first, the joint combination of uncertainties that arises when considering multiple mines leads to

an exponential increase in computing time and, more importantly, there does not presently exist a method to transition from deterministic to stochastic models. All related work neglects the opportunity to manage uncertainty and risk to improve the performance and reliability of the supply chain; while some methods permit what-if scenarios, uncertainty has not been integrated into decision-making and could be missing out on value.

4.3 Modelling Mining Supply Chains with Uncertainty

Given the wide diversity, complexity and individual needs or objectives for mineral resource supply chains, this section aims to develop a flexible modelling procedure that integrates uncertainty. First, definitions for materials and related attributes in the context of mining supply chains with uncertainty is provided. Following this, the decision variables that govern the flow of the materials and attributes from the mines and the processing streams will be discussed. These variables are presented as a directed graph in order to depict the flow and transformations of materials from the mines through to the final products. It is important to note that this does not imply that the optimization models (see Sect. 4.4.1) are optimized using network flow methods. A summary of nomenclature used for sets, variables and parameters can be found in Tables 4–1, 4–2 and 4–3, respectively.

4.3.1 Material flow and attributes

Let the directed graph $\mathcal{G}(\mathcal{N}, \mathcal{A})$ represent the flow of materials through the supply chain, where the nodes, $\mathcal{N}$, is comprised of three subsets:

i $\mathcal{C}$: *clusters* of similar types of materials that are extracted from the mines (see Sect. 4.3.2).

Table 4–1: Notations for sets and indices for supply chain optimization.

Sets and Indices	
Set	Description
$\mathbb{P}$	Primary attributes that are being tracked in the supply chain (e.g., metal content, tonnages).
$\mathbb{H}$	Hereditary attributes (derived from primary attributes) that are being tracked in the supply chain (e.g., grades, recoveries, economic values).
$\mathbb{T}$	Time periods that the supply chain operates in (e.g. months, years).
$\mathbb{S}$	Global set of scenarios that are used to represent uncertainty.
$\mathcal{C}$	Clusters of materials at the mines with similar attributes (see Sect. 4.3.2).
$\mathcal{S}$	Destinations in the supply chain that are able to stockpile material.
$\mathcal{P}$	Destinations in the supply chain that process material and must send out all of the recovered output product to subsequent destinations, if available.
$\mathcal{N}$	Nodes in the supply chain graph, i.e., $\mathcal{N} = \mathcal{C} \cup \mathcal{S} \cup \mathcal{P}$.
$\mathcal{A}$	Set of arcs. An arc exists if $i \in \mathcal{N}$ can send material to $j \in \mathcal{N}$.
$\mathcal{I}(i)$	Set of nodes that are connected via incoming arcs to node $i \in \mathcal{N}$, defined by the arcs in $\mathcal{A}$.
$\mathcal{O}(i)$	Set of nodes that are connected via outgoing arcs from node $i \in \mathcal{N}$, defined by the arcs in $\mathcal{A}$.
$\mathcal{G}(\mathcal{N}, \mathcal{A})$	Graph that models the flow of materials in the supply chain.

Table 4–2: Notations for variables for supply chain optimization.

Variables	
$z_{c,j,t} \in \{0,1\}$	Decision of whether or not to send cluster c to destination j in period t.
$y_{i,j,t,s} \in [0,1]$	Proportion of material produced at i sent to j in period t and scenario s.
$v_{p,i,t,s} \in \mathbb{R}$	Value of primary attribute p at node i in period t and scenario s.
$v_{h,i,t,s} \in \mathbb{R}$	Value of hereditary attribute h at node i in period t and scenario s.
$r_{p,i,t,s} \in [0,1]$	Recovery of primary attribute p after processing at node i in period t and scenario s.
$d^+_{h,i,t,s}, d^-_{h,i,t,s} \in \mathbb{R}$	Surplus and shortage, respectively, of hereditary attribute h at node i in period t and scenario s.

Table 4–3: Notations for parameters used for supply chain optimization.

Parameters	
$\gamma_{p,c,t,s}$	Value of attribute p for cluster c in period t and scenario s.
$f_{h,i}(p)$	Function that transforms primary attributes p to hereditary attribute h at location i.
$U_{h,i,t}$	Upper bound for hereditary attribute h at i in period t.
$L_{h,i,t}$	Lower bound for hereditary attribute h at i in period t.
$p_{h,i,t}$	Unit price of hereditary attribute h at i in period t.
$c^+_{h,i,t}$	Unit surplus penalty cost for hereditary attribute h at i in period t.
$c^-_{h,i,t}$	Unit shortage penalty cost for hereditary attribute h at i in period t.

ii $\mathcal{S}$: destinations in the supply chain that are able to *stockpile* material over time. No transformation from input materials to outputs occur at these nodes.

iii $\mathcal{P}$: destinations in the supply chain that are forced to *process* and send all output material to the subsequent nodes.

The graph's arcs, $\mathcal{A}$, are used to describe the possibility to send material from $i \in \mathcal{N}$ to a subsequent destination $j \in \mathcal{N}$. For the purpose of simplification, let $\mathcal{I}(i)$ represent the set of nodes that send materials to $i \in \mathcal{S} \cup \mathcal{P}$, which is defined by the incoming arcs to node i in $\mathcal{A}$. Additionally, let $\mathcal{O}(i)$ represent the set of subsequent destinations that are defined by the outgoing arcs in $\mathcal{A}$ from node i. Let $t : t \in \mathbb{T} = \{1, ..., T\}$ be used to describe the periods of time in which the supply chain operates (e.g., months, years), where T is the maximum number of periods considered. The general case where destinations $i \in \mathcal{S} \cup \mathcal{P}$ are able to receive or produce multiple distinct materials, or nodes $i \in \mathcal{N}$ operating in non-contiguous periods is not discussed in order to simplify the fundamental concepts and notations; however, the method may be generalized to include these complexities.

Attributes are used to quantify characteristics of materials that are of interest, such as metal content, tonnages and economic values, and are categorized into two classes: primary and hereditary. Primary attributes ($p : p \in \mathbb{P}$) are characteristics that are sent from node $i \in \mathcal{N}$ to a node $j \in \mathcal{O}(i)$ and are assumed to be linearly additive; this requirement is necessary for accurately blending the incoming materials from $\mathcal{I}(j)$ at node j. Generally, primary attributes are assumed to be the fundamental variables of interest, such as metal and total tonnages. Hereditary

attributes ($h : h \in \mathbb{H}$) are used to quantify information that is relevant to the optimization model, but are not necessarily passed from node i to j; these attributes are expressed as (non-) linear functions, $f_{h,i}(p)$, of the primary attributes $p \in \mathbb{P}$. For Onça Puma, the primary attributes in the model that are passed from the mined blocks through to the processing plant are are the total, nickel, iron, silica and magnesia tonnages. The primary attributes are transformed into a set of hereditary attributes at the processing plant in order to define the cash flows (processing cost and the market value of the recovered nickel metal) and treated material's chemistry (silica-to-magnesia ratio and iron content). As an example, consider two hereditary functions, Eq. (4.1) and Eq. (4.2), that may be used to model the nickel recovery and profit, respectively, after processing at Onça Puma's pyrometallurgical plant. Recoveries are commonly used in the mining industry to describe the proportion of an attribute p that is liberated from the incoming bulk material at a processor. Let NiT and T represent the primary attributes for nickel and total tonnages received at the pyrometallurgical plant, respectively, and let $NiPrice$ and $ProcCost$ represent the price of nickel metal and processing costs per tonne, which are specified as input parameters. The implementation of the proposed method takes these functions (defined by the modeller) and parses them into expression tree data structures that are dynamically evaluated during optimization (see Sect. 4.4).

$$Rec = f_{rec,plant} = \begin{cases} 0.3 + \left(\frac{NiT}{T}\right) - 0.48 * \left(\frac{NiT}{T}\right)^2 & \text{if } \left(\frac{NiT}{T}\right) \leq 1 \\ 0.82 & \text{otherwise} \end{cases} \tag{4.1}$$

$$Profit = f_{\$,plant} = NiT * Rec * NiPrice - T * ProcCost \tag{4.2}$$

Uncertainty in the supply chain may be described using a set of joint scenarios $\mathbb{S} = \{1, ..., S\}$, where a *scenario* is defined herein to be an equally probable realization (sampling) of all sources of uncertainty. The primary source of uncertainty considered in this work is related to geological uncertainty, however, integrating other sources of uncertainty (e.g., metal prices) in the form of simulated functions $f_{h,i}(p)$ is a natural extension, but, for simplicity, is not considered in this work. As more independent sources of uncertainty are given to the models as input (e.g., simulations for multiple mines), the number of scenarios, S, increases exponentially. For example, if the Onça and Puma deposits are independently simulated and are each represented by 20 geological simulations, there would be 400 joint scenarios to consider for optimization. One of the challenges related to the use of discrete scenarios to represent uncertainty is the determination of the number of scenarios that are required to accurately quantify the uncertainty. Albor and Dimitrakopoulos [1] discuss this subject in-depth by demonstrating the impact that the number of geological simulations has on the quality of the optimized design. It is noted that the scale of interest to the modeller (e.g. mine production tonnages, ore processing tonnages) is much larger than a single block that is extracted; while the simulations may show a large amount of variability for individual blocks, the amount of variability for the scale of interest is substantially less because the risk is blended with other blocks.

This phenomenon is commonly referred to as the volume-variance effect in geostatistics [93]. As a result, there is a point where optimizing with more scenarios does not drastically alter the quality of the resulting solution. Given the substantial computational overhead associated with optimizing with many simulations at once, it is often useful to incrementally add scenarios to the optimization model to analyze how many scenarios are needed. An example of this is discussed in Sect. 4.5.4.

4.3.2 Mine supply and destination policies

The flow of material in a mining supply chain commences at the mines. A mineral deposit is represented by a set of blocks, which often varies from thousands to billions in size. A production schedule specifies the period of extraction for each block. Without classifying the blocks as ore or waste prior to optimization, there are two conceivable options related to where to send the blocks after extraction. The first option is to use a decision variable to decide where each block is sent in the supply chain ($\mathcal{S} \cup \mathcal{P}$). This option is often considered undesirable because the number of decision variables grows linearly with the number of blocks and mines considered in the supply chain. The alternative is to decide on where *similar* types (clusters) of materials go (rather than each block individually), which can be used to reduce the number of decision variables in the model substantially. This work focuses on the latter, however a comparison of the two methods for the deterministic case is provided in Sect. 4.5.3. One of the key advantages for using clusters, other than for computational reasons, is that the method can be extended to consider uncertainty in the quantities of primary attributes, i.e. geological uncertainty.

A pre-processing step is required in order to group materials with similar attributes into clusters, $\mathcal{C} \subset \mathcal{N}$, which form the initial nodes on the graph $\mathcal{G}(\mathcal{N}, \mathcal{A})$. The k-means clustering algorithm [117] is a useful method for classifying data with similar attributes. The algorithm starts by randomly selecting cluster centroids for the attributes of interest and iteratively updates the centroid positions according to the nearest data points; the data points are then assigned a single membership to a single cluster according to the nearest cluster centroid. Consider a set of geological simulations (Fig. 4–4), where each block has a simulated material type (e.g., low- or high-grade saprolite) and primary attributes (e.g., nickel content, tonnage). For each mine and material type, the simulated blocks may be classified or clustered into a pre-defined number of distinct groups based on similar (multi-variate) attributes using the k-means clustering algorithm. For any given scenario, a block's cluster membership may change, given that the material type and attributes may differ between simulations. Let the pre-processed input parameter $\gamma_{p,c,t,s}$ represent the quantity of attribute $p \in \mathbb{P}$ available for cluster $c \in \mathcal{C}$ in time $t \in \mathbb{T}$ and scenario $s \in \mathbb{S}$; this quantity is derived from the quantities of the attributes from the simulated blocks that are members of the cluster for a given scenario and the availability of each block, which is defined by the mine's production schedule. The set of candidate destinations, $\mathcal{O}(c)$, for cluster $c \in \mathcal{C}$ is defined by the material type of the cluster. For example, at Onça Puma (Fig. 4–2), any cluster that belongs to the low-grade saprolite materials has the option of going to one of three low-grade stockpiles or the waste dump.

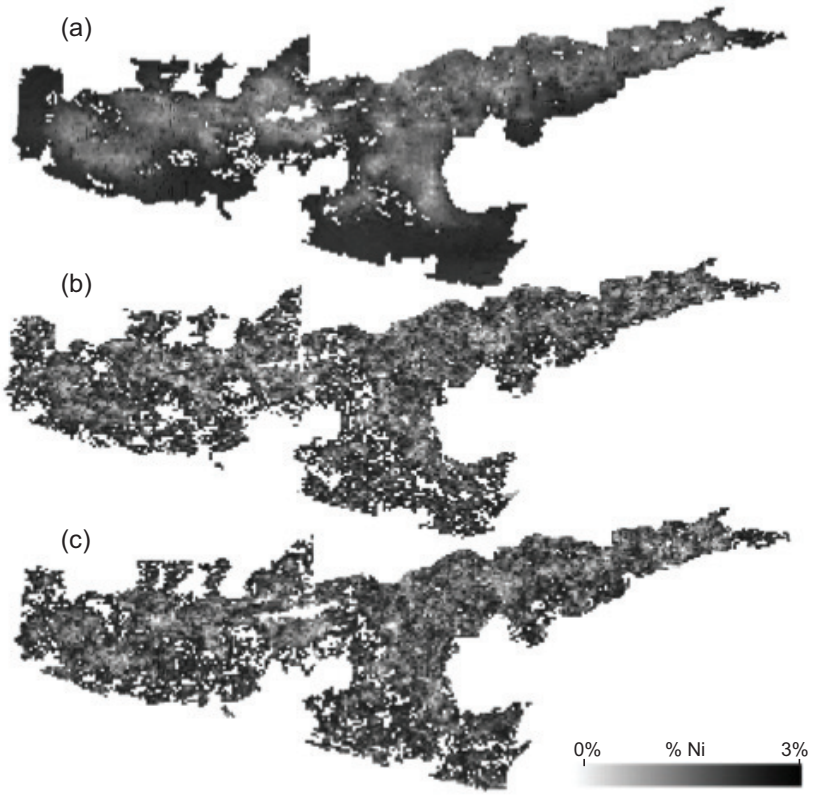

Figure 4–4: Plan view comparison of Puma's nickel content in the high- and low-grade saprolite layers between the (a) estimated model and (b)-(c) simulated models.

Let the decision variable $z_{c,j,t} \in \{0,1\}$ represent the decision of whether (1) or not (0) cluster $c \in \mathcal{C}$ is sent to destination $j \in \mathcal{O}(c)$ in period $t \in \mathbb{T}$. This variable is used to define a *destination policy* that is applied across all scenarios $s \in \mathbb{S}$, regardless of the quantities of the attributes, $\gamma_{p,i,t,s}$, available. For stochastic optimization models, these decisions form the first-stage decision variables [16] that are optimized to be robust to uncertainty. One of the most common types of destination policies used in the mining industry is a cut-off grade policy, whereby the destination of blocks

is decided on based on the metal content (grade) being above or below a threshold. In simplistic cases where only a single attribute (i.e. metal content) and ore or waste definitions are used, the proposed definition of destination policies reduces to that of Menabde et al. [124], whereby the optimizer will seek to determine a robust time-dependent cut-off grade destination policy. The proposed methodology is, however, more general because it can consider the full supply chain and the impacts of multiple metals, blending and deleterious elements — a common issue that a simple cut-off grade policy is unable to address.

Reserve constraints are used to state that a block (or cluster of blocks) must only be sent to a single destination; violating this constraint implies that the mining equipment has a higher degree of selectivity than can be defined using the geological models. The following equation is used to enforce the reserve constraints during the optimization process:

$$\sum_{i \in \mathcal{O}(c)} z_{c,i,t} = 1 \quad \forall c \in \mathcal{C}, t \in \mathbb{T} \tag{4.3}$$

4.3.3 Destinations and processing stream decisions

Destinations in the supply chain are categorized based on their primary functions: stockpiles ($\mathcal{S}$) and processors ($\mathcal{P}$). Let the decision variable $y_{i,j,t,s} \in [0,1]$ define the proportion of material sent from destination $i \in \mathcal{S} \cup \mathcal{P}$ to destination $j \in \mathcal{O}(i)$ in period $t \in \mathbb{T}$ and scenario $s \in \mathbb{S}$. In stochastic optimization, these

are commonly referred to as recourse (adaptive) decisions [16], which are scenario-dependent variables used to maximize the utility of the materials available at the various destinations, which is a result of the first-stage destination policies.

Stockpiles $i \in \mathcal{S}$ require mass flow conservation constraints to ensure that only materials that are available can be sent out to the subsequent destinations $j \in \mathcal{O}(i)$:

$$\sum_{j \in \mathcal{O}(i)} y_{i,j,t,s} \leq 1 \quad \forall i \in \mathcal{S}, t \in \mathbb{T}, s \in \mathbb{S} \tag{4.4}$$

In the case of processors $i \in \mathcal{P}$, mass flow constraints are used to ensure that all material that is available is forwarded to the subsequent destinations:

$$\sum_{j \in \mathcal{O}(i)} y_{i,j,t,s} = 1 \quad \forall i \in \mathcal{P}, t \in \mathbb{T}, s \in \mathbb{S} \tag{4.5}$$

Let $v_{p,i,t,s} \in \mathbb{R}$ represent the value of the primary attribute $p \in \mathbb{P}$ at destination $i \in \mathcal{S} \cup \mathcal{P}$ in period $t \in \mathbb{T}$ and scenario $s \in \mathbb{S}$. The primary attributes may be transformed into hereditary attributes $h \in \mathbb{H}$ using (non-) linear transformation functions, $f_{h,i}(p)$, in order to track meaningful quantities in the optimization model. Let $v_{h,i,t,s}$ represent the value of the attribute h at node i at time t and scenario s, which may be evaluated using $f_{h,i}(p)$ and the values of the related primary attributes, $v_{p,i,t,s}$. Using the previous example, the values of the recovery and profits, $v_{Rec,plant,t,s}$ and $v_{\$,plant,t,s}$, respectively, can evaluated by plugging in the values of $v_{NiT,plant,t,s}$ and $v_{T,plant,t,s}$ into Eqs. (4.1) and (4.2) in lieu of the NiT and T, respectively. As a result, the nickel recovery and profits may vary, depending on the scenario and time period. Let the state variable $r_{p,i,t,s} \in [0,1]$ represent the recovery of the primary

attribute $p \in \mathbb{P}$ at node $i \in \mathcal{S} \cup \mathcal{P}$ at time $t \in \mathbb{T}$ and scenario $s \in \mathbb{S}$. Often, the recoveries are given as a static input parameter, such as 0.8 to represent 80% of the incoming metal is recovered after processing. In general, however, recovery is often defined as a non-linear function based on the metal content; in order to accommodate these concepts in the supply chain optimization models, it is possible to model a hereditary function $f_{h,i}(p)$ based on the metal content of the primary attribute(s) p and set $r_{p,i,t,s} = v_{h,i,t,s}$. An example of this hereditary function is given in Eq. (4.1). In the case of stockpiling destinations, which do not process the incoming materials, it is necessary to set $r_{p,i,t,s} = 1 \; \forall p \in \mathbb{P}, i \in \mathcal{S}, t \in \mathbb{T}, s \in \mathbb{S}$. Using the previously defined state variables, the quantities of the primary attributes that are available at a location of the supply chain are calculated as follows:

$$v_{p,j,(t+1),s} = \underbrace{v_{p,j,t,s} \cdot \left(1 - \sum_{k \in \mathcal{O}(j)} y_{i,k,t,s}\right)}_{\text{Retained attributes from previous period}} + \underbrace{\sum_{i \in (\mathcal{I}(j) \setminus \mathcal{C})} r_{p,i,t,s} \cdot v_{p,i,t,s} \cdot y_{i,j,t,s} + \sum_{c \in (\mathcal{I}(j) \cap \mathcal{C})} \gamma_{p,c,(t+1),s} \cdot z_{c,i,(t+1)}}_{\text{Incoming attributes in current period}}$$
$$\forall p \in \mathbb{P}, j \in \mathcal{S} \cup \mathcal{P}, t \in \mathbb{T}, s \in \mathbb{S} \tag{4.6}$$

It is often useful to track and constrain the quantities of materials that are carried over from period-to-period, such as the tonnages in a stockpile. A hereditary attribute $h \in \mathbb{H}$ may be used to calculate the end-of-period quantities based on a primary attribute $p \in \mathbb{P}$, i.e.:

$$v_{h,i,t,s} = v_{p,i,t,s} \cdot \left(1 - \sum_{j \in \mathcal{O}(i)} y_{i,j,t,s}\right) \quad \forall i \in \mathcal{S}, t \in \mathbb{T}, s \in \mathbb{S} \tag{4.7}$$

4.4 Optimization of Mining Supply Chains

4.4.1 A generalized two-stage stochastic optimization model

Using the set of decision variables introduced in Sect. 4.3, a two-stage mixed integer stochastic optimization model can be created where the first-stage decision variables ($z_{c,j,t,s}$) decide the optimal destination policy for each cluster and the second-stage recourse variables ($y_{i,j,t,s}$) decide the optimal use of the processing streams once the random variables (the attributes of the blocks) are revealed at the first set of destinations. These models will produce a destination policy for mined materials that is robust to uncertainty in the sense that it attempts to improve the supply chain's resilience to disruptions caused by geology, which is often measured through the performance of a set of key project indicators. For a review of two-stage stochastic programming, see Birge and Louveaux [16]. The general mining supply chain optimization model is defined as follows:

Objective:

$$\min \quad \frac{1}{|\mathbb{S}|} \sum_{i \in \mathcal{S} \cup \mathcal{P}} \sum_{t \in \mathbb{T}} \sum_{h \in \mathbb{H}} \sum_{s \in \mathbb{S}} p_{h,i,t} \cdot v_{h,i,t,s}$$
$$+ \frac{1}{|\mathbb{S}|} \sum_{i \in \mathcal{S} \cup \mathcal{P}} \sum_{t \in \mathbb{T}} \sum_{h \in \mathbb{H}} \sum_{s \in \mathbb{S}} \left(c^{+}_{h,i,t} \cdot d^{+}_{h,i,t,s} + c^{-}_{h,i,t} \cdot d^{-}_{h,i,t,s} \right) \tag{4.8}$$

Subject to:

$$v_{h,i,t,s} - d^{+}_{h,i,t,s} \leq U_{h,i,t} \qquad \forall h \in \mathbb{H}, i \in \mathcal{S} \cup \mathcal{P}, t \in \mathbb{T}, s \in \mathbb{S} \tag{4.9}$$

$$v_{h,i,t,s} + d^{-}_{h,i,t,s} \geq L_{h,i,t} \qquad \forall h \in \mathbb{H}, i \in \mathcal{S} \cup \mathcal{P}, t \in \mathbb{T}, s \in \mathbb{S} \tag{4.10}$$

$$d^{+}_{h,i,t,s}, d^{-}_{h,i,t,s} \geq 0 \qquad \forall h \in \mathbb{H}, i \in \mathcal{S} \cup \mathcal{P}, t \in \mathbb{T}, s \in \mathbb{S} \tag{4.11}$$

$$v_{h,i,t,s} = f_{h,i}\left(p\right) \qquad \forall h \in \mathbb{H}, i \in \mathcal{S} \cup \mathcal{P}, t \in \mathbb{T}, s \in \mathbb{S} \tag{4.12}$$

Eqs. (4.3)-(4.7)

$$z_{c,j,t} \in \{0,1\} \qquad c \in \mathcal{C}, j \in \mathcal{O}\left(c\right), t \in \mathbb{T} \tag{4.13}$$

$$y_{i,j,t,s} \in [0,1] \qquad i \in \mathcal{C}, j \in \mathcal{O}\left(i\right), t \in \mathbb{T}, s \in \mathbb{S} \tag{4.14}$$

The objective function (Eq. (4.8)) is comprised of two components. The first component is used to directly incorporate hereditary attributes, $v_{h,i,t,s}$, into the objectives and is multiplied by the unit price $p_{h,i,t}$; this component may represent, for example, time-discounted revenues and costs. The second component is used to penalize deviations from targets or bounds, $U_{h,i,t}$ and $L_{h,i,t}$, whereby the amount of deviation is represented using the variables $d^{+}_{h,i,t,s}$ and $d^{-}_{h,i,t,s}$, which are calculated using Eqs. (4.9) and (4.10), respectively. The deviation variables are penalized using time-discounted penalty cost parameters, $c^{+}_{j,i,t}$ and $c^{-}_{j,i,t}$. These are useful for stochastic optimization because they permit *geological risk discounting*, which attempts to minimize the risky decisions at the beginning of the mine life and defers riskier decisions to later in the mine life when more geological information is available [142].

This geological risk-discounted penalty may be calculated as follows:

$$c^+_{h,i,t} = \frac{c^+_{h,i,0}}{\left(1 + grd^+_{h,i}\right)^t} \tag{4.15}$$

where $grd^+_{h,i}$ is the geological risk discount rate (as a decimal) for attribute h at location i associated with an upper-bound constraint (Eq. (4.9)). In practice, this is a desirable feature in order to meet short-term production targets or demand with a high level of certainty, rather than having the optimizer distribute the risk randomly over time. This risk discount rate is similar to the economic discount rate that is used to calculate the discounted cash flows, and is an input parameter that depends on the modeller's desire to balance short-, medium- and long-term risk. If $grd = 0$, the optimizer is willing to treat the risk in the early periods the same as the periods at the end of the mine's life, which may result in raising the level of risk in the first period and can impact the mine's ability to prepay creditors. A high grd (e.g. 0.5) may result in an overly short-sighted solution that only considers the immediate desire to satisfy production targets. It is noted that the penalty cost is analogous to the economic value that the modeller is willing to pay per unit of deviation for a given constraint. In the absence of a known penalty cost, they may be derived experimentally by trying different orders of magnitude and analyzing the quality of the solution, in terms of ability to satisfy these constraints. For an example, the reader is referred to Benndorf and Dimitrakopoulos [14]. A useful extension of the objective function is to use an exponent term with the deviation variables, $d^+_{h,i,t,s}$ and $d^-_{h,i,t,s}$, which may be used to penalize large deviations more heavily than smaller

deviations [55] and to have greater control of the risk distributed over time (see Chapter 3).

4.4.2 Optimization using metaheuristics

One of the challenges associated with optimizing the optimization model outlined in Sect. 4.4.2 is the high likelihood of having a non-linear optimization model. The goal of this unified modelling approach is for flexibility when testing on different supply chain configurations. As a result of this flexibility, if an exact optimization method were used as a point of comparison to assess the performance of the proposed method, it could vary depending on the supply chain model itself, and the results would not be generalizable. Non-linear, global mathematical optimizers are not currently as computationally efficient as their linear counterpart. Network flow-based optimizers require a specific knowledge of the structure of the model, particularly the constraints, which is generally not compatible with the generalized optimization model proposed. Metaheuristics are powerful optimization algorithms that can be used to obtain high-quality solutions for challenging problems in a reasonable amount of time. It is noted, however, that they do not guarantee mathematical optimality. These methods are useful for simulation-optimization frameworks for supply chains [104, 172] because of their iterative nature: first, simulate the supply chain using a set of randomly generated decision variables, then modify the variables based on the results of the simulation. There are several metaheuristics available that may be used (e.g., simulated annealing, tabu search, genetic algorithms, ant colony optimization, etc.), however, the vast majority are designed for combinatorial

optimization problems and do not easily translate to continuous optimization problems. Particle swarm optimization (PSO) is a population-based metaheuristic that uses cognitive and social behaviour to achieve a high quality, but not necessarily optimal, solutions [94]. It is a particularly appealing algorithm because of its inherent ability to optimize discrete and continuous variables simultaneously. Additionally, unlike continuous variants of combinatorial optimization algorithms, such as simulated annealing, particle swarm optimization inherently simultaneously optimizes all decision variables rather than a single variable at a time; this is a particularly useful feature because the computational cost of evaluating the objective function for the supply chain models discussed herein, relative to the impact that a single decision variable will have. For this reason, the proposed framework uses a combination of PSO with local heuristics, including simulated annealing [60, 98], to optimize the model defined in Sect. 4.4.1. Future work may seek to compare the performance of various metaheuristics.

Consider a group of Q particles (solution vectors), $\mathbf{x}_q \ \forall q = \{1, ..., Q\}$, where each vector is a distinct representation of all destination policy and processing stream variables and may be randomly generated at the beginning of the algorithm. The first $|\mathcal{C}| \cdot T$ elements of $\mathbf{x}_q$ represent encoded destination policy decisions, whereby, for a single cluster $c \in \mathcal{C}$ and period $t \in \mathbb{T}$, the decision variables $z_{c,j,t} \ \forall j \in \mathcal{O}(c)$ map to a single element, $x_k \in \mathbf{x}_q$, $k \in \{1, ..., |\mathcal{C}| \cdot T\}$. An encoding scheme is used to map the candidate destinations $j \in \mathcal{O}(c)$ to the element x_k in the range $[1, |\mathcal{O}(c)|]$, which automatically guarantees that the reserve constraints defined in Eq. (4.3) are satisfied. Note that the range is defined to be continuous for compatibility with

PSO; the continuous value is rounded to yield a discrete destination decision when required. All remaining elements in $\mathbf{x}_q$ have a one-to-one mapping to the processing stream variables $y_{i,j,t,s}$; in the event that Eqs. (4.4) or (4.5) are violated during optimization, the elements are re-normalized.

Additionally, consider three vectors that are of the same size as $\mathbf{x}_q$: $\mathbf{v}_q$, a velocity vector associated with the q^{th} solution vector; $\mathbf{x}_q^b$, which stores particle q's best solution found to date; and $\mathbf{x}^g$, a global best solution vector that stores the positions of the best solution found throughout the algorithm. At iteration $\alpha + 1$, particle q's velocity vector is updated using:

$$\mathbf{v}_q(\alpha+1) = c_1 \cdot \mathbf{v}_q(\alpha) + c_2 \cdot r_1 \cdot \left(\mathbf{x}_q^b - \mathbf{x}_q(\alpha)\right) + c_3 \cdot r_2 \cdot \left(\mathbf{x}^g - \mathbf{x}_q(\alpha)\right) \tag{4.16}$$

where $\mathbf{v}_q(\alpha+1)$ is the new velocity for the q^{th} particle at iteration $\alpha+1$, c_1, c_2 and c_3 are weights for the particle's current inertia, its personal best solution and the global best solution, respectively, and r_1 and $r_2 \in [0,1]$ are random uniform numbers. The particle's velocity is, therefore, a combination of it's own inertia at the previous iteration and an attraction towards its best solution and the global best solution. The particle's position is then updated at iteration $\alpha+1$ using:

$$\mathbf{x}_q(\alpha+1) = \mathbf{x}_q(\alpha) + \mathbf{v}_q(\alpha+1) \tag{4.17}$$

If any element of the particle's solution vector $\mathbf{x}_q$ exceeds its bounds, it is set to the closest (minimum or maximum) bound and the processing streams are re-normalized.

After the velocity and position updates are performed, the supply chain is simulated using the new decision variables at iteration $\alpha+1$. Let $g(\mathbf{x}_q)$, $g\left(\mathbf{x}_q^b\right)$ and $g(\mathbf{x}^g)$

denote the current, best and global best objective function values, respectively, from Eq. (4.8) at iteration $\alpha + 1$. If $g(\mathbf{x}_q) \leq g\left(\mathbf{x}_q^b\right)$, update the particle's best solution vector to $\mathbf{x}_q^b = \mathbf{x}_q$. If $g(\mathbf{x}_q) \leq g(\mathbf{x}^g)$, update the global best solution vector to $\mathbf{x}^g = \mathbf{x}_q$. The iterative update of position and velocity vectors continues until the swarm of particles converges on an optimum solution. PSO has a tendency to get trapped in local optima as the number of discrete decisions increases [171, 114]. Several types of heuristics can be employed on the global best solution vector, $\mathbf{x}^g$, to improve the quality of the solution quickly and ensure that the solution is not a local minimum. Experimentally, we have seen that it is often useful to employ one of the four heuristics:

1 Freeze the cluster destinations, $z_{c,j,t}$, and perform the next set of iterations solely changing the processing stream decisions, $y_{i,j,t,s}$.

2 Restart each of the particles from the global best solution, $\mathbf{x}^g$, with new random velocity vectors.

3 Randomly select a destination policy variable, $z_{c,j,t}$, and change it to a different candidate destination. Accept or reject based on the simulated annealing probability distribution in Eq. (4.18).

4 Randomly select a processing stream decision $y_{i,j,t,s}$ from the global best particle solution vector and multiply it by a random number between 0 and 2 and renormalize (if needed) to satisfy Eqs. (4.4) or (4.5). Accept or reject based on the simulated annealing probability distribution in Eq.(4.18).

Experimentally, we have found that that coupling the PSO algorithm with simulated annealing [98, 60] every few iterations (e.g., 25) proves to be the most effective local

improvement and can often substantially improve the objective function value, thus avoiding being caught in a local optimum. The simulated annealing algorithm uses a probability distribution, $P\left(g\left(\mathbf{x}^{g}\right), g\left(\mathbf{x}^{g'}\right), \delta\right)$, to define whether or not to accept a random change (perturbation) to the solution vector, $\mathbf{x}^{g'}$. The probability distribution is governed primarily by an annealing temperature, δ, which is initially provided as an input parameter and is gradually cooled (reduced) as the annealing algorithm progresses. The initial temperature is often large in order to permit accepting suboptimal changes, which helps to avoid getting trapped in a locally optimal solution. As the annealing temperature decreases, the probability of accepting a suboptimal perturbation decreases and the algorithm converges on the optimal solution. The acceptance distribution is defined as follows:

$$P\left(g\left(\mathbf{x}^{g}\right), g\left(\mathbf{x}^{g'}\right), \delta\right) = \begin{cases} 1 & \text{if } g\left(\mathbf{x}^{g'}\right) \leq g\left(\mathbf{x}^{g}\right) \\ \exp\left(-\left|g\left(\mathbf{x}^{g'}\right) - g\left(\mathbf{x}^{g}\right)\right| / \delta\right) & \text{otherwise} \end{cases} \tag{4.18}$$

It is important to note that, because the optimizer relies on metaheuristics, there is a substantial amount of flexibility for the user to decide when to terminate the optimization process. All parameters related to the metaheuristic can be modified dynamically, scenarios can be introduced slowly as the optimizer progresses, and the optimization may continue without the need to start an entirely new optimization instance.

4.5 Application at a Nickel Mining Complex

4.5.1 Overview of blending supply chain

The Onça Puma nickel laterite mining complex is located 20 km north of Ourilândia do Norte, in Parà State, Brazil, which consists of two deposits (Onça and Puma) that are situated approximately 16 km apart. The primary chemical constituents of interest within the laterite profile are nickel, silica, magnesia and iron. Within the two deposits, five material types are considered in the model and are classified as bedrock, limonite, saprolite waste, low-grade saprolite and high-grade saprolite. Figure 4–2 shows a diagram for material flow at the mine site used for this study; it is noted that the bedrock, limonite and waste saprolite materials only have the option of being sent to the waste dump and are, therefore, excluded from the figure. The low- and high-grade stockpiles ($\sigma\left(i\right)\ \forall i = \{1, ..., 6\}$) are short-term stockpiles with a maximum capacity of U_S dry tonnes, where the capacities are equal in size for all stockpiles, but the tonnage is withheld for confidentiality purposes. Each of the six intermediate stockpiles then feeds two homogenization piles, each having a capacity of U_{HP} dry tonnes per period. The homogenization piles are modelled as processors with a recovery of 1 and alternate between feeding the pyrometallurgy processing plant (P) and being filled by the intermediate stockpiles every 36 days (considered as one production period in this study). During the filling period, material cannot flow from the homogenization pile to the processing plant. The processing plant has a U_{HP} dry tonne capacity that is matched with the homogenization piles and has strict requirements on the incoming feed material. The silica-to-magnesia ratio, $SiO_2 : MgO$, is of primary importance and should lie

between 1.5 and 1.8; however, it is possible to operate between 1.4 and 1.9 for short periods of time. Additionally, the plant has a requirement that the iron grade, %Fe, lies between 12% and 16%. For this case study, it is assumed that there are not any constraints on distribution (e.g., ports), and that any nickel metal produced is sold directly to the spot market (i.e. no contractual obligations); as a result, is not necessary to include these aspects in the model of the nickel value chain.

Estimated resource models of the deposits were given by the mine staff and are used for deterministic optimization. For the stochastic optimization, a set of 20 orebody simulations of the laterite profile for each of the mines was simulated using the direct block Min/Max Autocorrelation Factor simulation method [28, 67]. This results in a total of 400 scenarios that may be considered during optimization. In specific, the limonite and saprolite layer thicknesses were jointly simulated to quantify the high variability and volumetric uncertainty for the deposit. For each lithological simulation, the valuable saprolite layer is retained for further joint simulation of the primary attributes in the model: nickel (Ni), magnesia (MgO), silica (SiO_2), iron (Fe) and the dry tonnages ($\mathcal{T}$) using the same method. A comparison of the nickel grade in the saprolite between the estimated and two of the simulated models is shown in Fig. 4–4.

The production schedule is generated from the mine's long-term (annual) production schedule by sequentially mining each bench until the short-term (36-day) total mining capacity is met. The Onça deposit is first mined for 10 periods, then the Puma deposit is mined from periods 10 through 35. The production schedule,

therefore, spans over 35 consecutive periods and contains a total of 55,622 mining blocks.

4.5.2 Optimization model

Table 4–4 gives an overview of the key hereditary attributes that are included in the objective function and constraints, along with the bounds and associated unit price or deviation costs. For confidentiality purposes, tonnage capacities U_S and U_{HP}, along with the nickel metal recovery (r), metal selling price (p) and processing costs (c^{proc}) are withheld. The penalty costs have been determined experimentally by re-running the optimization with different values, to balance the objective function's priorities. It is noted that the silica-to-magnesia ratio has a large penalty term, because the unit deviations are small, hence a large penalty is required to maintain priority over other aspects, such as stockpile tonnages. Given the fact that this optimization only spans over four years, a geological risk discount rate has not been applied to the penalty costs. It is noted that, for this example, mining costs account for sending material to both the waste dump and stockpiles and are not included in the optimization model because they are not dependent on any decision variables.

4.5.3 Deterministic optimization and risk analysis

As previously stated, using cluster decision variables rather than blocks may substantially reduce the number of variables in the model; this may, however, compromise the quality of the solution by not being able to make fine-grained adjustments on the block-scale. It is useful to provide a comparison of the computational efficiency and quality between the solutions when using cluster or block decision variables for the deterministic case, as there is not a direct way to compare the methods

Table 4–4: Hereditary attributes of interest in the optimization model for a nickel laterite mining complex.

Description of attribute	Transformation equation from primary attributes	Lower, Upper Bounds	Discounted unit price/cost for deviation*
Revenues	$v_{Ni,P,t,s} \cdot r \cdot p$	-, -	$\frac{-1}{1.008^t}$
Processing costs	$v_{\mathcal{T},P,t,s} \cdot c^{proc}$	-, -	$\frac{1}{1.008^t}$
End-of-period stockpile tonnages	$v_{\mathcal{E}(\mathcal{T}),\sigma(i),t,s}$	-, U_S	400, 400, 450, 600, 600, 800**
Processing tonnage	$v_{\mathcal{T},P,t,s}$	U_{HP}, U_{HP}	2 500
Silica-to-magnesia ratio	$v_{SiO_2,P,t,s}/v_{MgO,P,t,s}$	1.5, 1.8	800 000 000
Iron grade	$v_{Fe,P,t,s}/v_{\mathcal{T},P,t,s} \cdot 100$	12, 16	1 200 000

* Note that the objective function defined as a minimization problem, hence revenues have negative values, whereas costs have positive values.
** Stockpile deviation costs are listed in order for the low- and high-grade stockpiles.

for the stochastic case. In order to optimize the block destinations, the same methods and formulations are used as presented in Sect. 4.4; however, the solution encoding scheme is modified to accommodate blocks rather than clusters. The clustering problem uses 15 and 25 clusters for the Onça and Puma mines, respectively, for both the high-grade and low-grade saprolite materials. This results in reducing the block destination problem from 55,622 mining block destination decisions to 3,268 variables (waste, limonite and saprolite waste are each assigned to a different cluster). Through trial and error, the number of particle solution vectors, Q, is set to 15. The inertia (c_1) is set to 0.6, the particle's best inertia (c_2) is set to 0.25, and the global best inertia (c_3) is set to 0.8 (Eq. (4.16)). Initial solutions are randomly generated for each particle. The simulated annealing algorithm commences with a temperature of 800 000 and uses a cooling factor of 0.999 that is applied every 200 iterations. It is noted, however, that these parameters have been calibrated specifically for this

model and would need to be calibrated for other supply chain models. The block destination problem ran for 108 hours and performed 29,400 iterations, whereas the cluster problem ran for 24 minutes and performed 630 iterations. All tests are run on a Windows-based machine with two Intel Xeon 5650 6-core 2.67 GHz processors with 24 GB of RAM.

Figure 4–5 compares the block and cluster decision solutions for a set of key performance indicators. Both the block and cluster solution's $SiO_2 : MgO$ ratio often lie in the target range of 1.5 and 1.8, with a slight dip below the minimum of 1.4; this is likely a result of the lack of availability of material with a high $SiO_2 : MgO$ ratio in Onça's estimated model. The plant feed capacities for the cluster destination solution display a consistent feed to the processing plant, without any disruptions. This solution is substantially better than the block solution, which often drastically exceeds the plant feed's capacity. In both cases, the minimum and maximum bounds on the plant feed's $\%Fe$ are generally satisfied. Figure 4–5 also shows the cumulative NPV of the processed material of the solutions, expressed as a percentage of the NPV of for the deterministic cluster solution for confidentiality purposes. While the NPV for the block destination solution may indicate an 18% increase over the cluster solution, this is a result of the excess plant feed tonnage and should not be used as a bound on the value for the cluster destination solution. It is noted that the problems would ideally generate similar solutions, however, the amount of time required to solve the block destination problem is impractical. The solutions are similar up to the point where the block destination solution sends an excessive amount of material to the processing plant in period 14. In particular, the differences

between the $SiO_2 : MgO$ and the cumulative NPV are minor, indicating that the use of clustering does not substantially impact the quality of the solution.

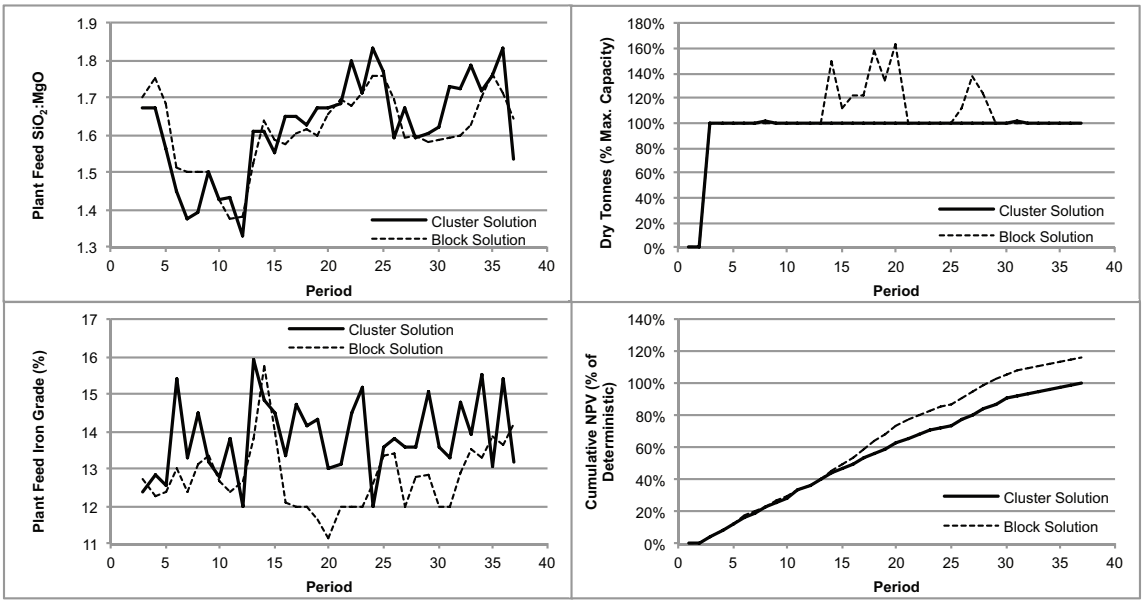

Figure 4–5: Graphs comparing the performance of the algorithm when using block destinations or cluster destinations.

Table 4–5 shows an analysis of the number of clusters used, where the problem is optimized 5 times each time. For confidentiality purposes, the values of the objective functions are expressed relative to the minimum objective function value obtained when using 15 and 25 clusters for Onça and Puma, respectively, for both the high- and low-grade saprolite. One would expect that as the ability to make fine-grain decisions decreases by optimizing with fewer clusters, the quality of the resultant solution would rapidly deteriorate. While this is true, creating too many clusters is computationally excessive and, after a certain threshold, does not generate substantially better solutions. The use of high-dimension solution vectors also comes

at a cost of slightly longer solution times. Table 4–5 also demonstrates some of the instabilities that arise from the randomly selected starting points for the k-means clustering heuristic [59]; in particular, it is noted that the objective function values are often more variable when using fewer clusters. Disaggregating and re-aggregating the clusters coupled with some heuristics during the algorithm may improve the stability of the clusters and reduce the impact that the initial point selection for the k-means algorithm has on the final objective function value.

Table 4–5: Influence of the number of clusters on the objective function. All values are expressed relative to the base case with 15 and 25 clusters for the Onça and Puma deposits, respectively.

Relative Objective Function Values	Number of Clusters for Low- and High-Grade Saprolite (Onça Puma)			
	(5, 10)	(10, 15)	(15, 25)	(20, 30)
Maximum	1.46	1.20	1.08	1.13
Average	1.30	1.15	1.05	1.06
Minimum	1.17	1.09	1.00	0.98
Average run time (min)	17	19	24	28

It is possible to use the destination policies generated from the deterministic optimization to test its performance with a set of geological simulations. In order to perform this risk analysis, the simulated blocks are first classified into the deterministic clusters according to the similarity of the primary attributes. The destination policies ($z_{c,j,t}$) are then applied to the set of simulations, and the recourse variables ($y_{i,j,t,s}$) are re-optimized for each scenario. Figure 4–6 shows a risk analysis for the key parameters from the deterministic destination policy, which is defined using the values for the exceedance probabilities for 10%, 50% and 90% of the simulations'

responses to the deterministic policies, which are commonly referred to as the P-90, P-50 and P-10 values, respectively. It can be seen that when using the destination policies from the estimated models, there is an extremely high chance that the maximum $SiO_2 : MgO$ ratio (1.9) will be exceeded, which would likely cause severe disruptions at the processing plant. The tonnage sent to the plant is often undesirable, given that it is not fed with enough material when mining the Onça deposit and the plant is fed with too much material when mining the Puma deposit. These problems are a result in the difference between the distributions between estimated and simulated geological models. It is known that the distributions for simulations can be quite different from the estimated model [69] hence the cluster locations and destination policies from the estimated model do not appear to be adequate for the simulations. It must be noted that these issues do not indicate that the proposed optimization methodology is ineffective — they give a cautionary tale of what can happen when generating destination policies from estimated models that have very little and unrepresentative variability.

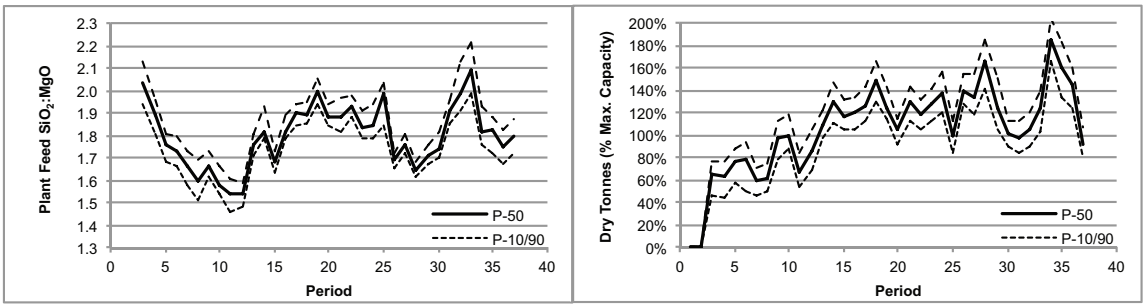

Figure 4–6: Risk analysis of the destination policies made using an estimated orebody model.

4.5.4 Stochastic optimization

Using the proposed stochastic optimization method, it is possible to link the geological simulations together in the optimization process through the cluster destination decision variables to generate a robust destination policy. For this case, the number of clusters is reduced to 5 and 15 clusters for Onça and Puma, respectively; it was found through experimentation that using more clusters results in several empty clusters for certain simulations, which leads to a solution that is not as stable or robust to geological uncertainty. The solution time for optimizing with this method is substantially longer for the stochastic case than the deterministic case; to solve the formulation, it takes an average of 12.6 hours to perform 1400 iterations, where there is no substantial change in objective function value. The reason for this drastic increase is that the optimizer needs to evaluate a supply chain for each scenario of the algorithm. In order to alleviate the computational burden associated with multiple mines, whereby 400 scenarios are evaluated, the optimizer commences with a small number of scenarios (e.g., 4) and gradually increases the number of scenarios considered after the solution has stabilized, and no substantial change in objective function value is attained. It was found experimentally for this case study that the solution becomes stable when using 50 of the 400 scenarios and adding more does not change the quality of the solution.

Figure 4–7 shows the risk profiles when using stochastic optimization. It can be seen that it is possible to make robust destination policies that meet both the plant feed's silica-to-magnesia ratio requirements and the plant's iron grade requirements

with minor probabilities of deviation. The plant feed's median tonnage is often centered on the plant capacity (U_{HP}) and the P-10 and P-90 risk profiles around this target are evenly distributed. Finally, the final cumulative NPV of the material processed is 3% higher than that of the deterministic solution. The stochastic optimizer, therefore, generates robust destination policies that are feasible in practice (unlike the policies from the deterministic model), but it also generates a similar economic value as indicated by the deterministic policy that doesn't consider the risk or variability.

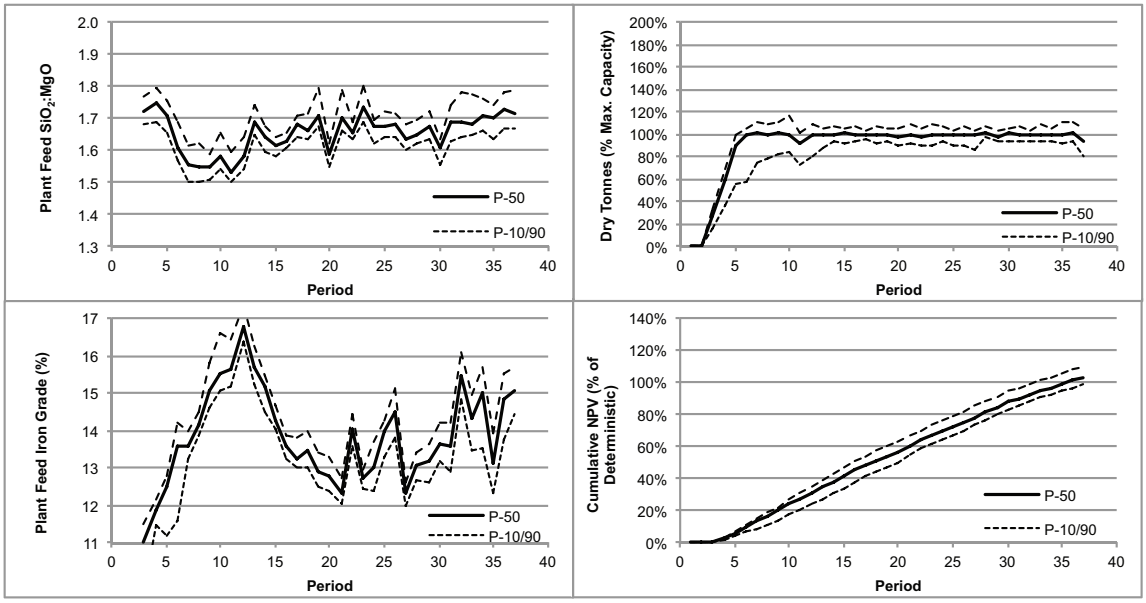

Figure 4–7: Risk analysis for the stochastic destination optimization solution where a common destination policy is applied to all simulations.

4.6 Conclusions

This work proposes a general mining supply chain modelling and stochastic optimization framework that can be adapted to many instances; the method is particularly useful for optimizing complex blending supply chains that have multiple processing streams and purity or material chemistry constraints. Unlike existing methods, the proposed work does not assume an a priori classification of ore and waste materials. Uncertainty is integrated into the decision-making through the use of policy variables that define the destination for mined materials that have similar attributes (e.g., metal content and tonnages). The optimized solution outlines destination policies that are designed to be robust to fluctuations that arise from uncertainty, along with adaptive processing stream decisions that state how to utilize the supply chain's resources to maximize the utility of the materials given the destination policies that have been implemented.

The proposed method is tested at a nickel laterite blending chain. Experimental results indicate that the proposed method works well for optimizing with complex blending operations with non-linear constraints. It was found, however, that caution should be taken when generating destination policies from estimated orebody models because the associated risk is substantial. The stochastic optimization for the supply chain generates a robust destination policy that adheres to the strict blending requirements and tonnages at the processing plant, while maintaining a similar economic value when compared to the deterministic case. Future work will seek to improve cluster stability by disaggregating and re-aggregating the clusters as the algorithm progresses, test the proposed methodology on other mining supply

chains and integrate production scheduling with the supply chain optimization under uncertainty.

CHAPTER 5
Global Optimization of Open Pit Mining Complexes with Uncertainty

The previous Chapter provides a foundation for the simultaneous optimization of mining complexes, and focuses on the creation of a unified modelling methodology and the development of efficient solvers based on metaheuristics in order to optimize the downstream aspects of a mining complex (contextualized as a mineral supply chain). Specifically, the previous Chapter focuses using these metaheuristic solvers to optimize critical variables that affect the flow of materials through a mining complex, such as destination policies and the use of the various processing streams, in order to optimize the objectives defined by the modeller in the form of a two-stage stochastic optimization model with fixed recourse. It is noted, however, that the production schedule that defines the initial quantities available for the various processing streams is assumed to be defined a priori. This Chapter contributes to the previous developments by allowing the optimizer to simultaneously control (multi) mine production schedules, destination policies and processing stream variables, which results in greater control over the geological risk over the life of the mining complex. While the modelling methodology, optimization formulation and metaheuristic solvers are similar to those of Chapter 4, some key additions and modifications are noted. First, the optimization models now consider production scheduling aspects, such as slope and reserve constraints, in addition to mine-level constraints (e.g. mine production capacities), which are not necessary with a fixed

production schedule. More importantly, the metaheuristic solvers are adapted to account for production scheduling changes; this requires combining aspects developed in Chapter 3 to change the production schedule while obeying slope constraints with aspects developed in Chapter 4 to optimize the destination policies and processing stream variables. In order to simultaneously optimize the destination policies and scheduling decisions, a modified simulated annealing algorithm is proposed, which better manages the strong relationship between the two variables that helps to avoid getting trapped in local optima. A full-field test is performed for a copper-gold mining complex (identity withheld for confidentiality) that has 30 098 blocks and six processing streams, including a stockpile. Results indicate that the deterministic equivalent of the proposed method is capable of generating a design with a 4% higher than an industry-standard optimizer. The stochastic optimizer further increases the net present value by an additional 6% over the deterministic-equivalent design, and simultaneously improves the mine's ability to meet production targets with less risk.

5.1 Introduction

Global optimization for mining complexes addresses the issue of integrated mining and processing operations with multiple pits or underground mines, multiple metals or minerals, stockpiles, blending options and alternative processing streams to yield distinct products [166, 168]. The primary objective of a mining enterprise is to maximize the net present value (NPV) of the cash flows; this requires optimizing the long-term mine production schedules and the use of the materials that have been extracted (i.e. the supply). Production schedules define the sequence of extraction for the materials to be mined, thus defines the supply of materials available over

time. Material processing optimization, referred to as supply chain optimization in Chapter 4, defines how to optimally use the mining complex's processing streams to maximize the utility of the available materials, and addresses both the destination policies (where to send material from the mines) and processing stream decisions (where to send stockpiled or processed material). Historically, these two components have been optimized independently, leading to sub-optimal solutions for the mining complex as a whole [61]. Many of the existing attempts at global optimization ignore the compounded effect that uncertainty (i.e. geological or economic) has on the value and operational feasibility of the supply chain [128, 137]. As the complexity of the supply chain increases in terms of number of mines, processing stream options and methods of distribution, there is an increasing importance in integrating all elements simultaneously while considering the uncertainty that arises within the mining complex's various components.

Recent work has focused on integrating geological, or supply, uncertainty into open pit mine production scheduling optimization models. Ramazan and Dimitrakopoulos [142] propose a two-stage stochastic integer programming (SIP) formulation [16] that seeks to maximize the NPV of a production schedule while minimizing the risk of not meeting production targets. Through the use of a geological risk discounting parameter, the optimizer aims to strike a balance between extracting high-value and low-risk material at the beginning of a mine's life, and defer riskier material to later periods when more information is available. The basic SIP model has been tested and improved over time [2, 14, 46, 106, 111], and results consistently

demonstrate that the NPV of the production schedule that considers geological uncertainty can be substantially higher than that of a conventional schedule and less risk in deviating from production targets.

Despite these advances in integrating geological uncertainty into mine production scheduling models, all of the previous formulations assume an a priori decision of what is ore (valuable) and waste material, commonly referred to as a cut-off grade policy [109, 145]. By specifying the destination of material a priori, the models ignore the impact that the aggregate of blocks will have on capacity and material quality constraints that are imposed in the various processing streams of the mining complex. Other work has aimed to integrate dynamic destination decisions with long-term deterministic production scheduling by exploiting the structure of the linear optimization model [15, 23, 63, 105]. Boland et al. [22] propose a multi-stage stochastic optimization model that decides the destinations for each scenario; scenario-dependent destination decisions are overly optimistic because they assume perfect knowledge of the material that will be extracted at the beginning of each time period and do not provide a long-term guide for operations. Extending the aforementioned methods to globally optimizing mining complexes is challenging because of the linear assumptions used to generate optimize the destinations. Alternatively, other work has investigated integrating robust cut-off grade destination policies [124] and robust block destinations [100, 102], which both integrate uncertainty into the policies, however have also not been extended to globally optimizing mining complexes.

The underlying challenge for globally optimizing mining complexes is the extensive amount of non-linearity that is required to accurately model the blending and stockpiling of materials [19] and the complex transformations that occur when refining the bulk input material into a set of output products. There have been several attempts to holistically optimize mining complexes [30, 52, 76, 77, 150, 153, 154, 165, 166, 168], however all models ignore geological uncertainty, and are often limited in the degree of flexibility in modelling the non-linear transformations in the supply chain. Chapter 4 proposes a supply chain ore processing optimization method that not only permits a high degree of flexibility for modelling the non-linear aspects of the supply chain, but does not require simplifying assumptions to generate high-quality optimization solutions. Moreover, the proposed model can create destination policies that are robust to geological uncertainty, while addressing many of the shortcomings of cut-off grade optimization, such as blending, stockpiling and multiple processing streams.

This work expands on the mineral resource supply chain optimization framework for open pit mining complexes, developed in Chapter 4, by enabling the optimizer to make production scheduling decisions in addition to destination policies and processing stream decisions. First, an overview of the modelling approach is given. Following this, a two-stage SIP formulation is proposed, where the first stage decisions are used to optimize multi-mine long-term production schedule and generate robust destination policies, and the second-stage recourse decisions are used to optimize the various processing streams of the supply chain. Following this, the solution method is discussed, which is a hybrid of particle swarm optimization and an adapted version

of simulated annealing. The method is then tested at on a copper-gold deposit data set. Finally, conclusions and future work are presented.

5.2 Modelling Mining Complexes with Uncertainty

This section discusses a flexible modelling procedure for mining complexes with uncertainty. First, definitions for materials and related attributes in the context of mining complexes with uncertainty is provided. Following this, the decision variables that govern the production schedule, the flow of the materials and attributes from the mines, and the processing streams will be discussed. Tables 5–1, 5–2, and 5–3, summarize the required sets, variables and parameters used for modelling.

5.2.1 Material flows and attribute transformations in mining complexes

In a mining complex, *materials* are products that are mined or are generated through blending or processing. Materials are considered to have unique mineralogy or metallurgical attributes and, as a result, may only be sent to a set of certain locations in a mining complex for further treatment. In order to define the flow of materials from the sources (mines) to the final products (refined metals), it is useful to describe a mining complex as a directed graph. Let the graph $\mathcal{G}(\mathcal{N}, \mathcal{A})$ represent the flow of materials through the mining complex. The set of nodes, $\mathcal{N}$, is comprised of three subsets:

i $\mathcal{C}$: *clusters* of mined materials that have similar attributes (e.g., metal content). See Sect. 5.2.3 for a detailed description.

ii $\mathcal{S}$: destinations that are able to *stockpile* material over time. The input material is not treated or transformed at these nodes.

Table 5–1: Notations for sets and indices used for the stochastic global optimization of open pit mining complexes.

Sets and Indices	
Set	Description
$\mathbb{P}$	Primary attributes (e.g. metal tonnages) of interest.
$\mathbb{H}$	Hereditary attributes of interest (e.g. recoveries, profits).
$\mathbb{T}$	Time periods.
$\mathbb{S}$	Global set of scenarios that represent uncertainty.
$\mathbb{M}$	Set of mines in the complex.
$\mathbb{B}_m$	Set of blocks from the mine $m \in \mathbb{M}$.
$\mathbb{O}_b$	Set of overlying blocks that need to be mined prior to extracting block b.
$\mathcal{C}$	Clusters of materials at the mines with similar primary attributes.
$\mathcal{S}$	Destinations in the complex that stockpile material.
$\mathcal{P}$	Destinations in the complex that process material and must send out all of the recovered output product to subsequent destinations, if available.
$\mathcal{N}$	Nodes in the mining complex, i.e. $\mathcal{N} = \mathcal{C} \cup \mathcal{S} \cup \mathcal{P}$.
$\mathcal{A}$	Set of arcs. An arc exists if $i \in \mathcal{N}$ can send material to $j \in \mathcal{N}$.
$\mathcal{I}(i)$	Set of nodes that are connected via incoming arcs to node $i \in \mathcal{N}$, defined by the arcs in $\mathcal{A}$.
$\mathcal{O}(i)$	Set of nodes that are connected via outgoing arcs from node $i \in \mathcal{N}$, defined by the arcs in $\mathcal{A}$.
$\mathcal{G}(\mathcal{N}, \mathcal{A})$	Graph that models the flow of materials in the mining complex.

Table 5–2: Notations for variables used for the stochastic global optimization of open pit mining complexes.

Variables	
$x_{b,t} \in \{0,1\}$	Extraction decision for block b in period t.
$z_{c,j,t} \in \{0,1\}$	Decision of whether or not to send cluster c to destination j in period t.
$y_{i,j,t,s} \in [0,1]$	Proportion of material produced at i sent to j in period t and scenario s.
$v_{p,i,t,s} \in \mathbb{R}$	Value of primary attribute p at node i in period t and scenario s.
$v_{h,i,t,s} \in \mathbb{R}$	Value of hereditary attribute h at node i in period t and scenario s.
$\gamma_{p,c,t,s}$	Value of attribute p for cluster c in period t and scenario s.
$r_{p,i,t,s} \in [0,1]$	Recovery of primary attribute p after processing at node i in period t and scenario s.
$d^+_{h,i,t,s}, d^-_{h,i,t,s} \in \mathbb{R}$	Surplus and shortage, respectively, of hereditary attribute h at node i in period t and scenario s.

Table 5–3: Notations for parameters used for the stochastic global optimization of open pit mining complexes.

Parameters	
$\theta_{b,c,s} \in \{0,1\}$	1 if block b belongs to cluster c in scenario s, or 0 otherwise.
$\beta_{p,b,s}$	Simulated value of attribute p for block b in scenario s.
$f_{h,i}(p)$	Function that transforms primary attributes p to hereditary attribute h at location i.
$U_{h,i,t}$	Upper bound for hereditary attribute h at i in period t.
$L_{h,i,t}$	Lower bound for hereditary attribute h at i in period t.
$p_{h,i,t}$	Unit price of hereditary attribute h at i in period t.
$c^+_{h,i,t}$	Unit surplus cost for hereditary attribute h at i in period t.*
$c^-_{h,i,t}$	Unit shortage cost for hereditary attribute h at i in period t.*

* A geological risk discount rate helps to defer riskier material to later periods, i.e., $c^+_{h,i,t} > c^+_{h,i,(t+1)}$.

iii $\mathcal{P}$: destinations that must *process* and send all output material to the subsequent nodes, if available.

The set of directed arcs, $\mathcal{A}$, defines the possibility to send material from $i \in \mathcal{N}$ to a subsequent destination $j \in \mathcal{N}$. Let $\mathcal{I}(i)$ represent the set of nodes that send materials to $i \in \mathcal{S} \cup \mathcal{P}$, which is defined by the incoming arcs to node i in $\mathcal{A}$. Additionally, let $\mathcal{O}(i)$ represent the set of nodes that receive material from node i, which is defined by the outgoing arcs in $\mathcal{A}$ from node i. Let $\mathbb{T} = \{1, ..., T\}$ describe the set of periods of time in which the mining complex operates (e.g. months, years), where T represents the end of the life for the mining complex. In order to simplify future notations, the general case where destinations $i \in \mathcal{S} \cup \mathcal{P}$ are able to receive or produce multiple distinct materials (e.g. multiple concentrates to be sent to various smelters), or nodes $i \in \mathcal{N}$ operating in non-contiguous periods is not discussed, however the method may be generalized to include these complexities.

Attributes are used to quantify information that is of interest in the optimization model, such as metal quantities and costs. Uncertainty in the attributes may be quantified using a set of joint scenarios $\mathbb{S} = \{1, ..., S\}$, where a *scenario* defines a realization, or sampling, from all sources of uncertainty. Attributes are categorized into two classes:

i Primary attributes ($p : p \in \mathbb{P}$): fundamental variables of interest to the entire model (e.g. metal and total tonnages) that are sent from node $i \in \mathcal{N}$ to a node $j \in \mathcal{O}(i)$. The quantity of the attribute available is denoted by $v_{p,i,t,s}$ $\forall i \in \mathcal{S} \cup \mathcal{P}, t \in \mathbb{T}, s \in \mathbb{S}$. These often originate at the mines, and may flow

through the mining complex to the final products. The amount of attribute recovered after treatment is denoted by $r_{p,i,t,s}$.

ii Hereditary attributes ($h : h \in \mathbb{H}$): information that is relevant to the optimization model, but is not necessarily passed from node i to j (e.g., feed material chemistry, treatment costs, revenues from sales); these attributes may be expressed as (non-) linear functions, $f_{h,i}(p)$, of the primary attributes $p \in \mathbb{P}$. The quantity of the hereditary attribute available is denoted by $v_{h,i,t,s}$ $\forall i \in \mathcal{S} \cup \mathcal{P}, t \in \mathbb{T}, s \in \mathbb{S}$.

5.2.2 Long-term mine production schedules

Mines are the suppliers of bulk materials to the mining complex, and are represented by the set $\mathbb{M}$. Each mine $m \in \mathbb{M}$ is discretized into volumes of material called *blocks*, $\mathbb{B}_m$. In order to quantify the geological uncertainty for both the quantities of materials and attributes, it is assumed that each block $b \in \mathbb{B}_m$ has a simulated material classification and simulated attributes, $\beta_{p,b,s}$ $\forall p \in \mathbb{P}, s \in \mathbb{S}$. Figure 5–1 shows an example of the differences that may appear for both simulated material classifications and attributes in a copper-gold mine.

The long-term production schedule is determined by the decision variables $x_{b,t} \in \{0, 1\}$, which define whether (1) or not (0) block $b \in \mathbb{B}_m$ is extracted in period $t \in \mathbb{T}$. In order to safely extract a block $b \in \mathbb{B}_m$, it is necessary to uncover b by extracting a set of overlying blocks, $\mathbb{O}_b$, in their entirety. The overlying blocks $\mathbb{O}_b$ may be identified for each block b in a pre-processing step by creating an inverted cone from the centre of b and verifying which blocks are within the cone. In more complex cases, it may be necessary to define variable slope angles for the North, East, South

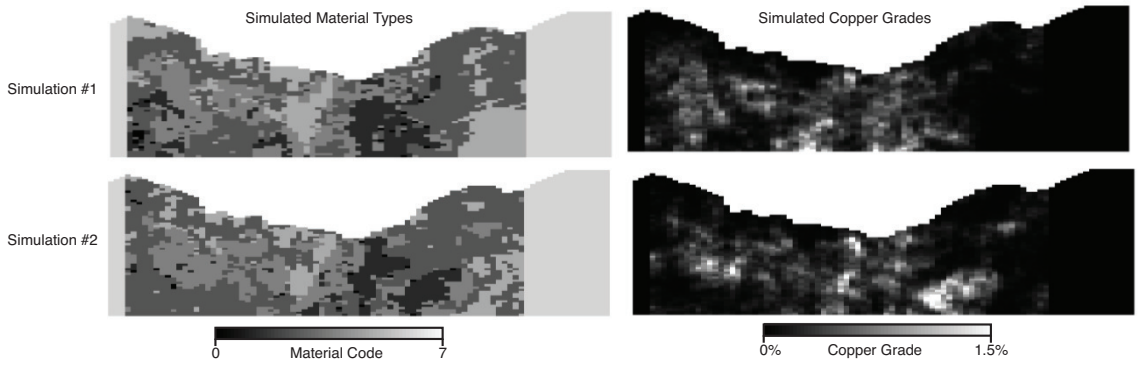

Figure 5–1: Example of a cross-section showing a comparison of simulated material types and copper grades for a copper-gold mine.

and West walls for each block. Figure 5–2 gives a 2D example of how the overlying blocks $\mathbb{O}_b$ are defined with variable slope angles. For a more detailed description of the 3D pre-processing algorithm, the reader is referred to Khalokakaie et al. [95].

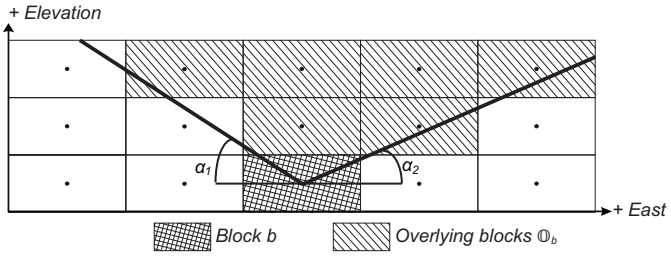

Figure 5–2: 2D example of blocks that must be uncovered ($\mathbb{O}_b$) prior to extracting block b. Note that variable slope angles are defined for the West (α_1) and East (α_2) walls.

5.2.3 Destination policies

Destination policies define where the mined blocks blocks are initially sent in the mining complex. In this work, each material from the mines is decomposed into sub-groups based on attributes with similar quantities (e.g. valuable or deleterious

metal content), and the destination policy outlines for all scenarios where each sub-group of material is sent. A similar concept is introduced by Menabde et al. [124], who separate the univariate distribution of the metal content (grades) into "bins" (i.e., categories based on ranges of metal content), and create a time-varied cut-off grade policy based on the bins (Fig. 5–3A); rather than looking at the individual blocks in the mine [22, 100, 102], the optimizer requires substantially fewer decision variables because it focuses on the distribution of grades. In the more general case, proposed in Chapter 4, the sub-groupings of materials, called clusters, may be created on multivariate distributions, which permits for a higher degree of flexibility when defining the policies (Fig. 5–3B). In both cases, the destination of a single block may change between simulations, depending on the simulated attributes. The general method, however, addresses many of the limitations of cut-off grade destination policies because it is able to consider the impacts of deleterious elements, blending or homogenization, stockpiling and complex processing streams.

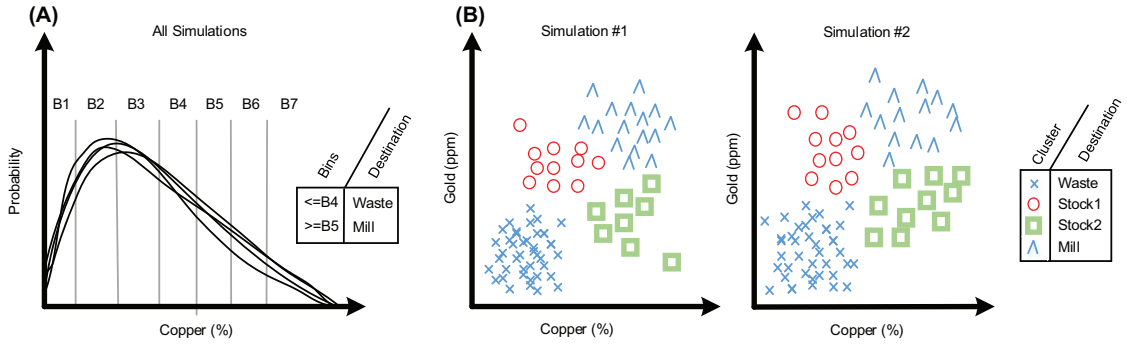

Figure 5–3: (A) Simulated univariate copper distributions with a cut-off grade policy defined using bins. (B) Destination policies using clusters, where the points represent a block's simulated copper and gold attributes.

In order to define destination policies, it is first necessary to classify the simulated blocks into clusters, $\mathcal{C} \subset \mathcal{N}$. The k-means++ clustering algorithm [6, 117] is a useful method for grouping information with similar attributes. The algorithm first creates a pre-determined number of cluster centroids for each mine and material based on the simulated attributes. For a given scenario, a block's cluster membership is determined by the closest centroid, measured using a Euclidean distance from the block's simulated attributes to the centroid's multivariate attributes; this cluster membership is subject to change for a block, given that the material classification and attributes may vary between simulations. Let $\theta_{b,c,s}$ represent the pre-processed parameter that defines whether (1) or not (0) block $b \in \mathbb{B}_m$ is a member of cluster $c \in \mathcal{C}$ in scenario $s \in \mathbb{S}$. The destination policies are determined using the variable variable $z_{c,j,t} \in \{0, 1\}$, which represents the decision of whether (1) or not (0) cluster $c \in \mathcal{C}$ is sent to destination $j \in \mathcal{O}(c)$ in period t. It is noted that the set of candidate destinations, $\mathcal{O}(c)$, is determined by the type of material that the cluster belongs to.

5.2.4 Processing and stockpiling decisions

The destination policy variables described in Sect. 5.2.3 define where to send material after it is mined. Depending on the configuration of the processing streams for a given mining complex, it may be necessary to model the transfer of materials between two locations. Processing decision variables, $y_{i,j,t,s} \in [0, 1]$, define the proportion of an output material sent from $i \in \mathcal{S} \cup \mathcal{P}$ to destination $j \in \mathcal{O}(i)$ in period $t \in \mathbb{T}$ and scenario $s \in \mathbb{S}$. It is noted that, unlike the production schedule and destination policies, these decisions are designed to be adaptive to uncertainty; after the material is received at the initial destination and the uncertainty is revealed, it

is assumed that the mining complex can adapt appropriately. Given that the primary attributes of interest are assumed to be linear and additive, the quantity of the primary attribute sent from destination $i \in \mathcal{S} \cup \mathcal{P}$ to $j \in \mathcal{O}(i)$ is calculated with $y_{i,j,t,s} \cdot v_{p,i,t,s}$.

5.3 Optimizing Mining Complexes

5.3.1 Flexible design of optimization objectives with a two-stage stochastic optimization model

Using the set of decision variables introduced in Sect. 5.2, a two-stage stochastic optimization model [16] can be expressed to meet the objectives and constraints of the mining complex. In this formulation, the first-stage decisions are the long-term production schedule and destination policies, which are designed to be robust to the fluctuations that arise from the uncertainty in the geological attributes. Recourse variables are used to adapt to the first-stage decisions via the processing stream variables and penalties for excessive risk or inability to meet specified targets. The deviation variables may be penalized using their related penalty costs, $c^+_{h,i,t}$ and $c^-_{h,i,t}$. Similar to a discount rate used to calculate the net present value (NPV), these penalty costs my be defined to be monotonically decreasing with respect to time. This phenomenon referred to as *geological risk discounting*, which attempts to defer riskier material to later periods in the mine life [142]. The geological risk discount rate is a parameter that may be used to describe the modeller's desire to balance the ability to meet production targets in the short- and long-terms. These penalty costs relate to the willingness to pay for a unit of deviation from a capacity constraint, or may be determined experimentally by running the optimization model multiple times to obtain a desirable risk profile for all of the constraints of interest [14]. The

general global optimization formulation for open pit mining complexes is defined as follows:

Objective:

$$\max \quad \underbrace{\frac{1}{|\mathbb{S}|} \sum_{i \in \mathcal{S} \cup \mathcal{P} \cup \mathbb{M}} \sum_{t \in \mathbb{T}} \sum_{h \in \mathbb{H}} \sum_{s \in \mathbb{S}} p_{h,i,t} \cdot v_{h,i,t,s}}_{\text{Discounted revenues and costs}}$$
$$\underbrace{- \frac{1}{|\mathbb{S}|} \sum_{i \in \mathcal{S} \cup \mathcal{P} \cup \mathbb{M}} \sum_{t \in \mathbb{T}} \sum_{h \in \mathbb{H}} \sum_{s \in \mathbb{S}} \left(c^{+}_{h,i,t} \cdot d^{+}_{h,i,t,s} + c^{-}_{h,i,t} \cdot d^{-}_{h,i,t,s} \right)}_{\text{Risk-discounted penalties for deviations}} \tag{5.1}$$

Subject to:

I. *Capacity constraints* are used to calculate the value of the hereditary attributes and evaluate the surplus or shortage from a specified target at the various locations in the mining complex. Examples of typical constraints may include, but are not limited to, mine production capacity, stockpile capacity, processing plant feed capacity and grade blending constraints.

$$v_{h,i,t,s} = f_{h,i}(p) \qquad \forall h \in \mathbb{H}, i \in \mathcal{S} \cup \mathcal{P} \cup \mathbb{M}, t \in \mathbb{T}, s \in \mathbb{S} \tag{5.2}$$

$$v_{h,i,t,s} - d^{+}_{h,i,t,s} \leq U_{h,i,t} \qquad \forall h \in \mathbb{H}, i \in \mathcal{S} \cup \mathcal{P} \cup \mathbb{M}, t \in \mathbb{T}, s \in \mathbb{S} \tag{5.3}$$

$$v_{h,i,t,s} + d^{-}_{h,i,t,s} \geq L_{h,i,t} \qquad \forall h \in \mathbb{H}, i \in \mathcal{S} \cup \mathcal{P} \cup \mathbb{M}, t \in \mathbb{T}, s \in \mathbb{S} \tag{5.4}$$

II. *Reserve and block access constraints* ensure a block is extracted at most once, and that the overlying blocks have also been extracted.

$$\sum_{t \in \mathbb{T}} x_{b,t} \leq 1 \quad \forall b \in \mathbb{B}_m \tag{5.5}$$

$$x_{b,t} \leq \sum_{t'=1}^{t} x_{u,t'} \quad \forall b \in \mathbb{B}_m, u \in \mathbb{O}_b, t \in \mathbb{T} \tag{5.6}$$

III. *Destination policy constraints*, which calculate the quantities of the primary attributes for each cluster from the blocks, and ensure that a cluster is sent to a single destination in the mining complex.

$$\gamma_{p,c,t,s} = \sum_{b \in \mathbb{B}_m} \theta_{b,c,s} \cdot \beta_{p,b,s} \cdot x_{b,t} \quad \forall m \in \mathbb{M}, p \in \mathbb{P}, c \in \mathcal{C}, s \in \mathbb{S} \tag{5.7}$$

$$\sum_{i \in \mathcal{O}(c)} z_{c,i,t} = 1 \quad \forall c \in \mathcal{C}, t \in \mathbb{T} \tag{5.8}$$

IV. *Mine extraction constraints* are used to determine the quantities of the primary attributes that are extracted from each mine.

$$v_{p,m,t,s} = \sum_{b \in \mathbb{B}_m} \beta_{p,b,s} \cdot x_{b,t} \quad \forall m \in \mathbb{M}, p \in \mathbb{P}, t \in \mathbb{T}, s \in \mathbb{S} \tag{5.9}$$

V. *Processing stream flow constraints*, which calculate the quantity of the primary attributes that are retained or received at each location, and ensure mass balancing for materials sent from the stockpiles and processors to subsequent destinations in the mining complex.

$$v_{p,j,(t+1),s} = \sum_{i \in (\mathcal{I}(j) \setminus \mathcal{C})} r_{p,i,t,s} \cdot v_{p,i,t,s} \cdot y_{i,j,t,s} + \sum_{c \in (\mathcal{I}(j) \cap \mathcal{C})} \gamma_{p,c,(t+1),s} \cdot z_{c,i,(t+1)} + v_{p,j,t,s} \cdot \left(1 - \sum_{k \in \mathcal{O}(j)} y_{i,k,t,s}\right) \forall p \in \mathbb{P}, j \in \mathcal{S} \cup \mathcal{P}, t \in \mathbb{T}, s \in \mathbb{S} \quad (5.10)$$

$$r_{p,i,t,s} = 1 \quad \forall p \in \mathbb{P}, i \in \mathcal{S}, t \in \mathbb{T}, s \in \mathbb{S} \quad (5.11)$$

$$\sum_{j \in \mathcal{O}(i)} y_{i,j,t,s} \leq 1 \quad \forall i \in \mathcal{S}, t \in \mathbb{T}, s \in \mathbb{S} \quad (5.12)$$

$$\sum_{j \in \mathcal{O}(i)} y_{i,j,t,s} = 1 \quad \forall i \in \mathcal{P}, t \in \mathbb{T}, s \in \mathbb{S} \quad (5.13)$$

VI. End-of-year stockpile quantities (*optional*) are used to calculate the quantities of materials that remain in the stockpile at the end of the production period.

$$v_{h,i,t,s} = v_{p,i,t,s} \cdot \left(1 - \sum_{j \in \mathcal{O}(i)} y_{i,j,t,s}\right) \quad \forall i \in \mathcal{S}, t \in \mathbb{T}, s \in \mathbb{S} \quad (5.14)$$

VII. Binary constraints

$$x_{b,t} \in \{0,1\} \qquad \forall b \in \mathbb{B}_m, t \in \mathbb{T} \tag{5.15}$$

$$z_{c,j,t} \in \{0,1\} \qquad \forall c \in \mathcal{C}, j \in \mathcal{O}(c), t \in \mathbb{T} \tag{5.16}$$

VIII. Continuous variable definitions

$$\gamma_{p,c,t,s} \geq 0 \qquad \forall p \in \mathbb{P}, c \in \mathcal{C}, t \in \mathbb{T}, s \in \mathbb{S} \tag{5.17}$$

$$y_{i,j,t,s} \in [0,1] \qquad \forall i \in \mathcal{S} \cup \mathcal{P}, j \in \mathcal{O}(i), t \in \mathbb{T}, s \in \mathbb{S} \tag{5.18}$$

$$r_{p,i,t,s} \in [0,1] \qquad \forall p \in \mathbb{B}_m, i \in \mathcal{S} \cup \mathcal{P}, t \in \mathbb{T}, s \in \mathbb{S} \tag{5.19}$$

$$v_{p,i,t,s} \geq 0 \qquad \forall p \in \mathbb{P}, i \in \mathcal{S} \cup \mathcal{P} \cup \mathbb{M}, t \in \mathbb{T}, s \in \mathbb{S} \tag{5.20}$$

$$v_{h,i,t,s} \in \mathbb{R} \qquad \forall h \in \mathbb{H}, i \in \mathcal{S} \cup \mathcal{P} \cup \mathbb{M}, t \in \mathbb{T}, s \in \mathbb{S} \tag{5.21}$$

$$d^{+}_{h,i,t,s}, d^{-}_{h,i,t,s} \geq 0 \qquad \forall h \in \mathbb{H}, i \in \mathcal{S} \cup \mathcal{P}, t \in \mathbb{T}, s \in \mathbb{S} \tag{5.22}$$

5.3.2 Optimization with metaheuristics

The generalized global optimization formulation can be challenging to solve using conventional mathematical programming and network flow methods, particularly when the models require non-linear functions. Metaheuristics are generalized optimization algorithms that are useful for such cases because they do not require linear formulations or a special structure in the optimization problem. Metaheuristics do not guarantee a mathematically optimal solution, however have been shown in the past to give useful solutions for mining-related problems [66, 102, 152]. The proposed optimization framework uses a hybrid of particle swarm optimization (PSO) and a modified simulated annealing (SA) algorithm. The reason for combining two metaheuristics is to overcome some of the limitations inherent in the individual method. PSO cannot be easily adapted to make production scheduling decisions without specific assumptions on block destinations and economic values [56]. From experimental testing in Chapter 4, simulated annealing did not prove to be nearly as effective or efficient at optimizing the continuous processing stream variables as PSO, which is a result of the computational overhead required to evaluate the small changes that are incurred by changing a single continuous variable at each iteration. Notably, using two complementary metaheuristics helps to ensure that solutions do not get trapped in a local optimum. For the proposed framework, PSO is used to optimize the destination policies and processing stream decisions, and SA is used to optimize the destination policies and production schedule simultaneously. The two methods are used interchangeably during the algorithm to improve the solution and to move a

solution out of a local optimum. It is noted that there is a wide variety of metaheuristic solvers available; future work may investigate and benchmark the performance of these other methods.

It is first necessary to describe the solution vector, $\mathbf{\Phi} = \{\mathbf{x}, \mathbf{z}, \mathbf{y}\}$, which is used to store all decision variables and comprises three components. The production schedule vector ($\mathbf{x}$) stores all $x_{b,t}$ variables. Each discrete-valued element $x_b \in \mathbf{x}$ represents the extraction period of block $b \in \mathbb{B}_m$, and may take on any value in $\mathbb{T}$. An initial (starting) production schedule may be obtained using industry-standard methods, or by constructing a feasible schedule randomly (this design, however, must obey slope constraints). The destination policy vector ($\mathbf{z}$) stores all $z_{c,j,t}$ variables. Each element $z_{c,t} \in \mathbf{z}$ represents the encoded destination for cluster $c \in \mathcal{C}$ in period $t \in \mathbb{T}$, and may in the range $[1, ..., |\mathcal{O}(c)|]$. A decoding scheme is used to convert the value to the appropriate destination for the cluster. Finally, the processing stream vector ($\mathbf{y}$) stores all $y_{i,j,t,s}$ variables. Each element $y \in \mathbf{y}$ maps directly to a $y_{i,j,t,s}$ variable, and may lie in the range $[0, 1]$.

PSO [94] is a population-based metaheuristic that is capable of optimizing both discrete and continuous variables, making it particularly suitable for optimizing the destination policy and processing decisions. A *particle* is comprised of 3 equally sized vectors: its solution vector, $\mathbf{\Phi}_q$, its best solution vector, $\mathbf{\Phi}_q^b$, and its velocity vector, $\mathbf{v}_q$. Initially, the vectors $\mathbf{\Phi}_q$ and $\mathbf{v}_q$ may be randomly generated to obtain an initial solution for the destination policies and processing stream decisions. A *swarm* is a group of Q particles, and the best solution vector in the swarm is denoted by $\mathbf{\Phi}^g$. At each iteration of the algorithm, $\alpha + 1$, the velocity and solution vectors are updated

using the following equations:

$$\mathbf{v}_q(\alpha+1) = c_1 \cdot \mathbf{v}_q(\alpha) + c_2 \cdot r_1 \cdot \left(\mathbf{\Phi}_q^b - \mathbf{\Phi}_q(\alpha)\right) + c_3 \cdot r_2 \cdot \left(\mathbf{\Phi}^g - \mathbf{\Phi}_q(\alpha)\right) \tag{5.23}$$

$$\mathbf{\Phi}_q(\alpha+1) = \mathbf{\Phi}_q(\alpha) + \mathbf{v}_q(\alpha+1) \tag{5.24}$$

where c_1, c_2 and c_3 are the weights for the particle's inertia, its best solution and the global best solution, respectively, and r_1 and r_2 are uniformly distributed random numbers in the range $[0, 1]$. It is noted that the part of the velocity vector $\mathbf{v}_q$ that relates to the velocity of the production schedule, $\mathbf{x}$, is always set to 0 to ensure the production schedule remains unchanged. If any element of the solution vector $\mathbf{\Phi}_q$ exceeds its bounds, the value is set to the nearest bound. After each solution vector $\mathbf{\Phi}_q(\alpha+1)$ is updated, the objective function value, $g(\mathbf{\Phi}_q(\alpha+1))$, is evaluated. If $g(\mathbf{\Phi}_q(\alpha+1)) \geq g\left(\mathbf{\Phi}_q^b\right)$, the best vector is updated by $\mathbf{\Phi}_q^b = \mathbf{\Phi}_q(\alpha+1)$. Additionally, if $\mathbf{\Phi}_q^b \geq \mathbf{\Phi}^g$, the global best solution is updated by $\mathbf{\Phi}^g = \mathbf{\Phi}_q^b$.

A modified SA algorithm is employed to improve the global best solution vector, $\mathbf{\Phi}^g$, after optimizing with PSO. Two classes of *perturbations* (solution changes) are considered when attempting to improve an existing solution.

i Production scheduling perturbations ($\mathbf{x} \in \mathbf{\Phi}^g$): a block is randomly selected, and its mining period is changed (possibly to not being mined at all). Any blocks that would violate the slope constraints (Eq. 5.6) are also considered as candidates to change extraction periods (see Chapter 3).

ii Destination policy perturbations ($\mathbf{z} \in \mathbf{\Phi}^g$): a cluster destination decision variable is randomly selected and sent to a different destination, if possible.

In the classic simulated annealing algorithm, the acceptance probability for a perturbation of the solution vector for a maximization problem is based on the following acceptance probability distribution:

$$P\left(g\left(\mathbf{\Phi}^{g}\right), g\left(\mathbf{\Phi}^{g'}\right), \delta\right) = \begin{cases} 1 & \text{if } g\left(\mathbf{\Phi}^{g'}\right) \geq g\left(\mathbf{\Phi}^{g}\right) \\ \exp\left(-\left|g\left(\mathbf{\Phi}^{g'}\right) - g\left(\mathbf{\Phi}^{g}\right)\right| / \delta\right) & \text{otherwise} \end{cases} \tag{5.25}$$

where $g\left(\mathbf{\Phi}^{g}\right)$ and $g\left(\mathbf{\Phi}^{g'}\right)$ are the objective function values before and after the perturbation, respectively, and δ is called the annealing temperature. As the algorithm progresses, the temperature is gradually reduced until only minor changes in the objective function are accepted; this is often controlled by the initial temperature at the start of the algorithm, $\delta\left(0\right)$, and the cooling schedule, which is defined by a reduction factor, $k \in [0, 1)$, and a number of iterations before the reduction factor is applied, n^{iter}. One of the primary difficulties of the simulated annealing algorithm is the calibration of the initial temperature; naturally, this parameter is dependent on the magnitude of change in objective function that any given perturbation will have. This parameter is particularly problematic for simulated annealing with multiple neighbourhoods or variables (e.g. $\mathbf{x}$ and $\mathbf{z}$) because the different neighbourhoods for a set of variables may have different effects on the change in objective function value.

Figure 5–4 shows a comparison of the cumulative distributions between the changes in objective function values $\left(g\left(\mathbf{\Phi}^{g}\right) - g\left(\mathbf{\Phi}^{g'}\right)\right)$ for sub-optimal candidate perturbations for the production scheduling and destination policy neighbourhoods.

When using the two neighbourhoods with the classic SA, for a very large temperature, the optimizer will may limit the number of sub-optimal changes in production schedule, but will likely accept all destination policy changes. As the temperature decreases, it becomes more likely that only sub-optimal destination policy changes are accepted, and all sub-optimal production schedule changes are rejected. The classical simulated annealing method with a single temperature ignores the relationship between the two types of decision variables.

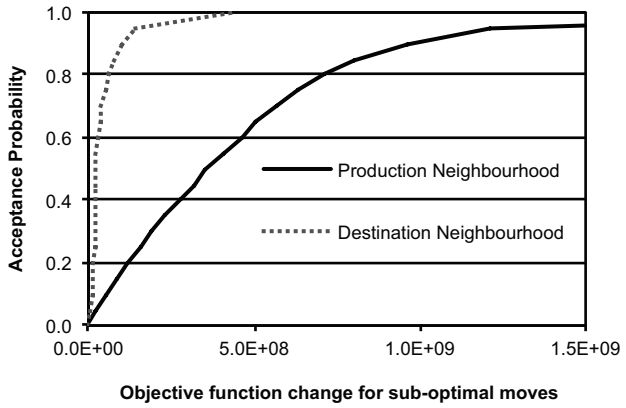

Figure 5–4: Example of cumulative probability distributions of objective function changes for sub-optimal perturbations of two neighbourhoods.

In the proposed modified SA algorithm, the cumulative probability distributions, shown in Fig. 5–4, are constructed for each neighbourhood by proposing random perturbations to $\mathbf{\Phi}^g$. Rather than using a single temperature, δ, for both neighbourhoods in Eq. 5.25, the optimizer uses two independent temperature variables, $\delta_{\mathbf{x}}$ and $\delta_{\mathbf{z}}$, for the production schedule and destination policy neighbourhoods, respectively, which are in turn controlled by a single parameter, $\rho \in [0, 1]$, which represents a cumulative probability. For a fixed ρ, the respective temperature variables ($\delta_{\mathbf{x}}$ and

$\delta_{\mathbf{z}}$) are computed from the cumulative probability plots (Fig. 5–4). The cooling schedule (k, n^{iter}) is then applied to ρ, rather than δ. As the algorithm progresses, the information garnered from any new proposed sub-optimal perturbations is used as feedback to update the cumulative distributions; this better reflects the current search space, rather than the search space when the SA algorithm commenced.

5.4 Case Study – Application at a Copper-Gold Mine

The proposed integrated mine planning optimization framework is demonstrated on a copper-gold mining complex, which is similar to a real-world deposit, but has been modified for the sake of both confidentiality and discussion.

5.4.1 Overview of the mining complex

In the given case study, a single mine supplies materials to a mining complex that produces gold and copper. Figure 5–5 summarizes the definition of the mine's materials and the processing options. The mine contains three main material groups: sulphides, transition and oxides. In order to model the material flows in the processing paths, the sulphide and transition material groups are both separated into two different material types based on being above or below 0.2% copper in order to be compatible with the heap leach chemistry requirements. The oxide materials are automatically classified based on potential ore and waste; the waste group is material below the marginal cut-off grade of the process or is material that is not simulated, thus automatically considered as waste. The deposit's uncertainty is represented by a set of 50 equally probable geological simulations with variable copper, gold, tonnages and material types; 35 of the simulations are used for optimization and the remaining 15 are used to verify the robustness of the stochastic solution (see Sect.

5.4.3). The number of simulations (scenarios) has been determined experimentally by running separate stochastic optimization experiments with a varying number of simulations and comparing the quality of the risk profiles.

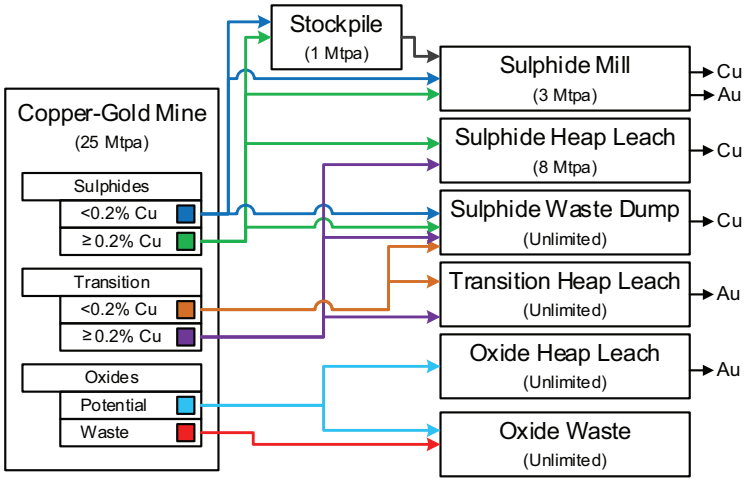

Figure 5–5: Definition of material types at the copper-gold mine, along with the various destinations.

With the exception of the oxide waste dump, all destinations have variable grade-recovery curves that are based on the average grade of the incoming material at a process in a given period (Fig 5–6). The non-linear grade-recoveries have interesting implications when considering the transition materials: for a given block or cluster that has (hypothetically) similar economic values for two processing options, the selected destination would be the one that profits the most from an increase in recovery. As a result, one cannot assume that the destinations can be specified a priori in a greedy manner because it is the recovery of the aggregated material sent to a given process that determines the potential value.

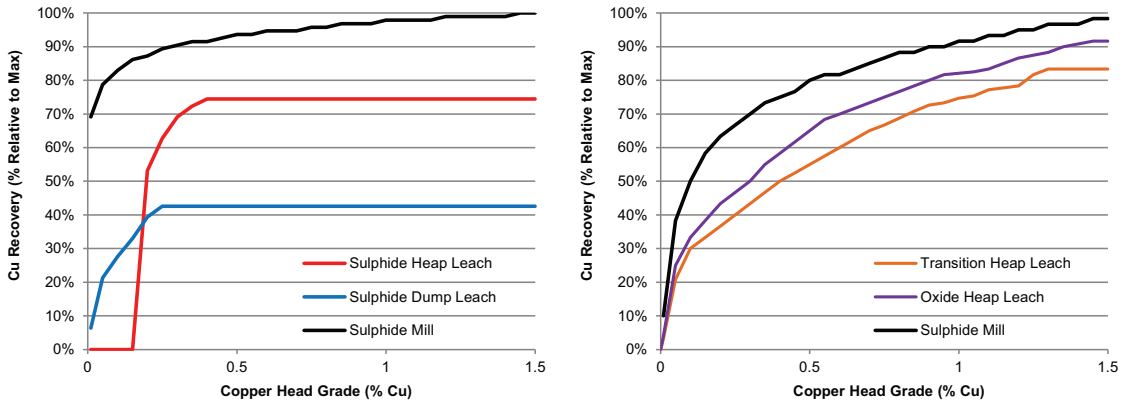

Figure 5–6: Grade-recovery curves for copper (left) and gold (right) at each of the processes.

Table 5–4 provides an overview of the orebody model size and the number of decision variables in the optimization model. The goal for the optimizer is to maximize the net present value of the mining complex, which considers the sale of copper and gold from each of the destinations, along with the processing and mining costs. All cost-related parameters in Table 5–5 are expressed relative to a base cost for confidentiality purposes. Table 5–6 summarizes the constraints in the stochastic optimization model; the capacities used in the deterministic model are shown in Fig. 5–5. A *geological risk discount* rate [142] of 5% is used to penalize the deviations from the production capacities and are used to ensure that riskier material is deferred to later periods when more geological information is available. This discount rate is determined through testing and analyzing the risk profiles, and is used to provide a suitable balance between meeting short- and long-term production targets. The penalty costs have been determined experimentally by incrementally

Table 5–4: Description of orebody and optimization model sizes.

Optimization model	Value
Number of blocks	30 098
Number of periods	22
Mine slope angle	45°
Destination policy variables	2 332
Stockpile variables (deterministic)*	20
Stockpile variables (stochastic, 35 scenarios)*	700

* The stockpile is empty in period 1 and ceases to send material to the mill in period 21.

Table 5–5: Economic parameters used in the copper-gold mining complex model.

Parameter	Value
Mining cost	\$1.00/t
Sulphide mill	\$11.30/t
Sulphide heap leach	\$2.98/t
Sulphide dump leach	\$1.87/t
Transition heap leach	\$2.15/t
Oxide heap leach	\$2.06/t
Copper price	\$2.88/lb
Gold price	\$1480/oz
Economic discount rate	7%
Geological risk discount rate	5%

adding constraints to the model and checking the impact that the cost has on the risk profiles and net present value. For an in-depth discussion on the impacts of penalty costs with the final solutions obtained, the reader is referred to Benndorf and Dimitrakopoulos [14].

All tests are performed on a 64-bit Windows 7-based machine with two 6-core Intel Xeon 5650 processors and 24 GB of RAM. For the PSO algorithm, 15 particles are used, and the inertia coefficients c_1, c_2 and c_3 are set to 0.6, 0.25 and 0.8, respectively. The SA algorithm is applied after 15 iterations of PSO, where $\rho = 0.8$,

Table 5–6: Constraints used in the copper-gold mining complex optimization model.

Description of constraint	Lower, Upper Bounds ($*10^6$)	Penalty ($/tonne)
Mine capacity	-, 25.0	-, 10
Stockpile capacity	-, 1.0	-, 20
Sulphide mill capacity	2.8, 3.2	50, 50
Sulphide heap leach capacity	7.8*, 8.0	10, 25

* Minimum sulphide heap leach bound is removed from periods 11-22 to increase NPV.

$k = 0.999$ and $n^{iter} = 500$. The initial production schedule is discussed in the subsequent section, however the destination policies and processing stream (i.e. stockpile) variables are randomly generated for each particle at the beginning of the algorithm. Given that the SA and PSO algorithms are algorithmic optimizers, the termination condition is defined by the user; this permits the ability to change the PSO and SA parameters and continue optimizing without restarting the entire optimization problem. It is noted that while this approach is not ideal for benchmarking the computational performance of the method, its purpose is to provide a high-quality solution for the end-user that is used for subsequent mine design.

5.4.2 Deterministic optimization and risk analysis

An E-type [69] deterministic orebody model is created by averaging the grades in the given simulations and re-classifying the material types in the same manner that the simulations are classified. An initial production schedule is first obtained using the E-type model and Gemcom's Whittle 4.3.1 commercial mine planning software [166, 168]. Unlike Table 5–6, the deterministic models use a sulphide mill capacity of 3 Mtpa and a sulphide heap leach capacity of 8 Mtpa. The Whittle schedule serves as an initial solution for the proposed optimizer when only a single scenario is considered (i.e. the E-type orebody model). This re-optimized design,

referred to herein as the deterministic-equivalent design, is used to provide a fair comparison between the deterministic and stochastic designs (i.e. obtained using the same optimization methods). For future reference, the schedule that is optimized using the proposed method and a single orebody model will be referred to as the *deterministic-equivalent* design; this provides a consistent comparison between traditional methods that use a single input and a risk-based design. For the deterministic-equivalent optimization, each material is clustered into 15 groups, with the exception of the oxide waste material, which only considers 1; this results in 76 cluster destination policies per period (1,672 total). Figure 5–7 shows a comparison for the sulphide mill, the sulphide heap leach and the cumulative NPV for both the Whittle and deterministic-equivalent designs. Notably, the deterministic-equivalent schedule generated using the proposed method continues to feed the sulphide mill at capacity for an additional three years, and is able to immediately fill the sulphide heap leach up to capacity in the first two periods. This results in a 4.7% higher NPV than the schedule generated by Whittle, and the life of the mine is extended by a year.

A risk analysis is performed by taking the schedule and destination policies generated from the deterministic-equivalent design and testing how the 50 geological simulations perform. This will provide a consistent basis of comparison for the stochastic designs discussed in the following section. Figure 5–7 shows the risk profiles, which is defined by the values of the exceedance probabilities for 10%, 50% and 90% (P-90, P-50, P-10, respectively) of the simulations' responses to the deterministic-equivalent design. The risk profiles for sulphide mill and sulphide heap leach tonnages indicate

that the deterministic-equivalent design performs noticeably worse than the E-type orebody model, which is attributed to the differences in the univariate distributions between the estimated model and the simulations. The E-type model smooths out variability (high- and low-values), whereas the simulations attempt to replicate the variability from the original drilling information. The spike in sulphide mill tonnage in period 10 is similar in concept to when a cut-off grade is applied to the simulations, where the simulations contain excessive quantities of material above the cut-off. This tonnage is unrealistic and inflates the NPV of the risk analysis, which can be seen by a 3.1% increase in NPV for the P-50 value over the E-type model in the deterministic-equivalent design. Moreover, the risk analysis indicates that the deterministic-equivalent design would consistently be unable to meet the production targets at the sulphide heap leach. Interestingly, the NPV of the risk analysis is unaffected by its inability to fill the sulphide heap leach to capacity, which is a result of sending higher valued materials to the sulphide mill. This helps to increase recovery and profits from the sulphide mill, however comes at the expense of underutilizing the sulphide heap leach.

5.4.3 Stochastic optimization

The stochastic optimizer aims to optimize the long-term production schedule, destination policies and the use of the stockpile while considering the geological uncertainty during optimization. The deterministic-equivalent design is used as a starting design for the stochastic optimizer. The final stochastic design is obtained after running for 25 hours using the same machine as the deterministic-equivalent design. Table 5–6 gives the relevant parameters that are used to guide the risk

profiles; similar to the deterministic-equivalent optimization model, 15 clusters are used for all material types, with the exception of the oxide waste material. Figure 5–8 compares the deterministic-equivalent design with risk profiles of the stochastic solution for the sulphide mill tonnages, sulphide heap leach tonnages and the cumulative NPV. It is apparent that the stochastic design is capable of meeting the target capacity of 3 Mtpa at the sulphide mill on average, with substantially less risk than the deterministic-equivalent design (Fig. 5–7). Unlike the deterministic-equivalent solution, the stochastic solution begins ramping down the tonnages of material sent to the sulphide mill in period 10. The stochastic design also meets the target sulphide heap leach tonnage over the first 12 periods, after which the quantities being to decline. It is also possible to see the impact of geological risk discounting for the sulphide heap leach tonnages, where the risk profiles are tight until period 11 (i.e. small difference between P-10 and P-90 profiles) and expand thereafter. This implies that the riskier sulphide heap leach material is being deferred until the end of the mine life. Finally, it is noted that the P-50 NPV of the stochastic design is 6% higher than the P-50 value of the risk analysis for the deterministic-equivalent design.

Figure 5–9 shows the risk profiles of the stochastic design when tested for robustness using the remaining 15 geological simulations. The P-50 tonnages sent to the sulphide mill and sulphide heap leach are similar to those shown in Fig. 5–8, however the variability (i.e. the difference between the P-10 and P-90 profiles) is substantially larger. This implies that the stochastic design successfully extracts sufficient quantities to feed the mill and sulphide heap leach up to capacity, however,

the scheduler is unable to tightly control the geological variability related to the spatial location of the sulphide materials. This variability is noticeable when comparing the material types for two simulations in Fig. 5–1. The variability may be improved by using more simulations during the stochastic optimization process, however there is a limit on the effect that using more simulations has [1]. If the spatial variability of the materials is high for a deposit, the optimizer tends to focus on local details that affect the available sample population of simulations, rather than the global trends that would be depicted using a larger sample of simulations. Admittedly, it is possible that more simulations are required than were provided by the mine. In this case study, it is more important that the sulphide mill and sulphide heap leach targets are met on average over the long-term and that high-valued zones are targeted to maximize the NPV; the local details that are impacted by the spatial variability of the materials can be managed operationally.

5.5 Conclusions

This chapter presents a modelling methodology and global optimization formulation for mining complexes under uncertainty, whereby the solutions give robust long-term open-pit mine production schedules and destination policies. The proposed framework permits a high-degree of flexibility and detail in modelling the mining complex, including the opportunity to integrate non-linear relationships that are generally ignored in existing models because of the challenges associated with non-linear optimization. The mathematical formulations can be generalized as a mixed integer non-linear stochastic programming problem, where the first-stage decisions are the production schedules for the mines along with the destination policies,

and the recourse decisions decide how to best use the processing streams and destinations in order to maximize the value of the material that has been extracted. The optimizer uses a hybrid metaheuristic comprised of particle swarm optimization and a modified simulated annealing optimizer, whereby the particle swam optimizes the destination policies and processing streams and the simulated annealing optimizes the long-term production schedules and destination policies.

The method is tested on a copper-gold mining complex. Experimental results indicate that the stochastic design is able to satisfy the target tonnage capacities, thus ensuring that the mine is able treat profitable material over the life of mine. Additionally, the stochastic solution indicates a 6% higher net present value than the design generated from deterministic-equivalent of the proposed stochastic optimizer (both measured using the P-50 values from the risk profiles), thus making better use of the non-renewable natural resource. Given that the proposed method seeks to generate a single, robust set of destination policies, future research will investigate the use of multistage stochastic optimization in order to permit adaptive policies under both supply (geological) and demand (metal price) uncertainty, which will likely lead to higher economic value.

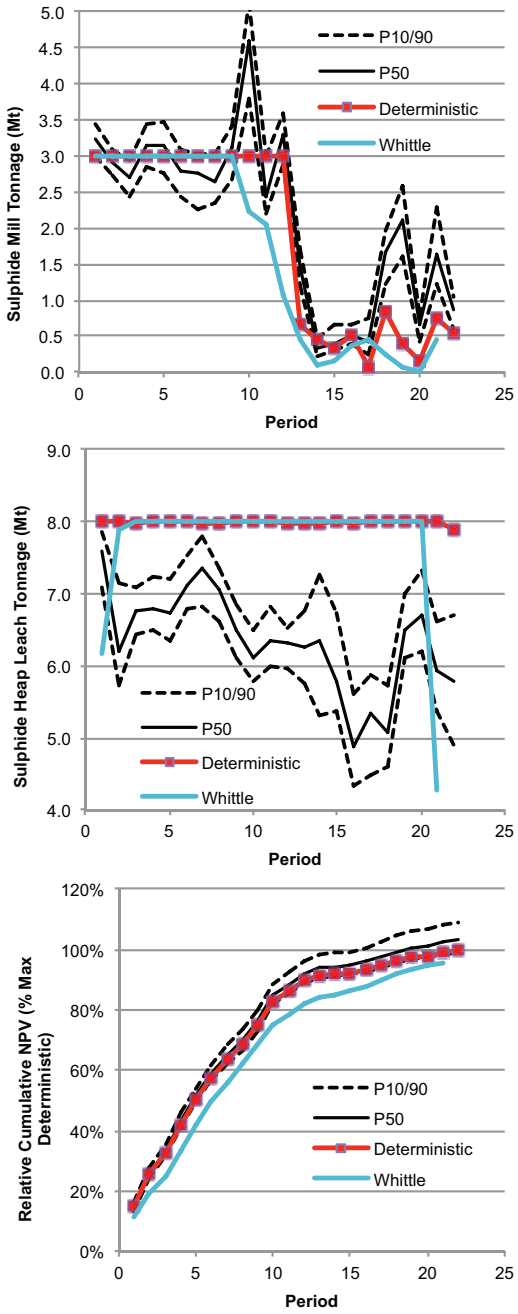

Figure 5–7: Comparison of the deterministic-equivalent design, a design created using Whittle software, and the risk analysis of the deterministic-equivalent design.

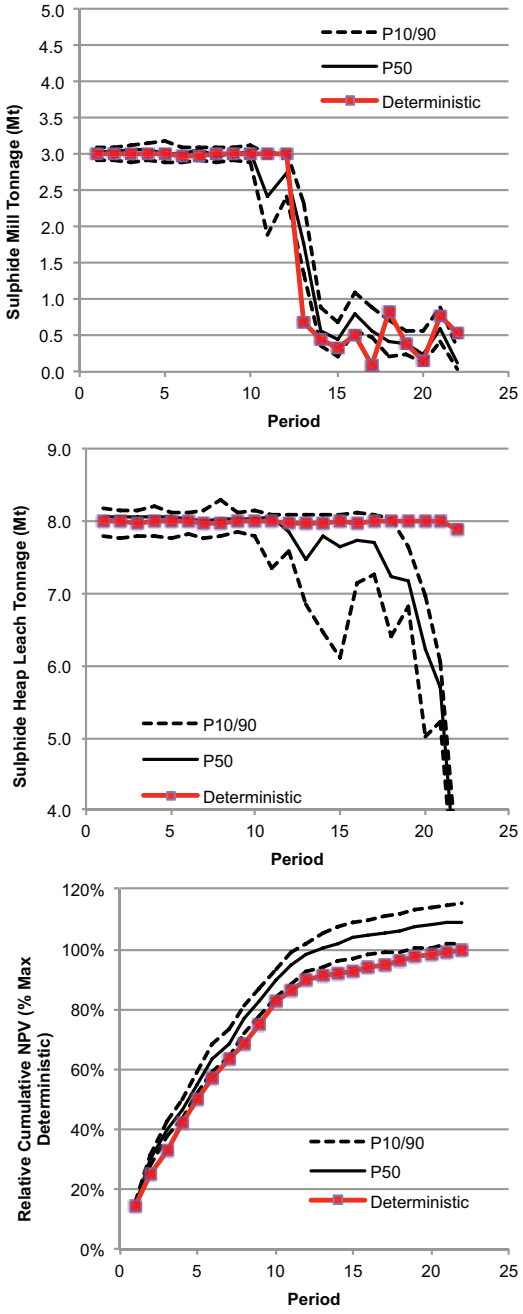

Figure 5–8: Risk profiles of the stochastic solution using 35 geological simulations.

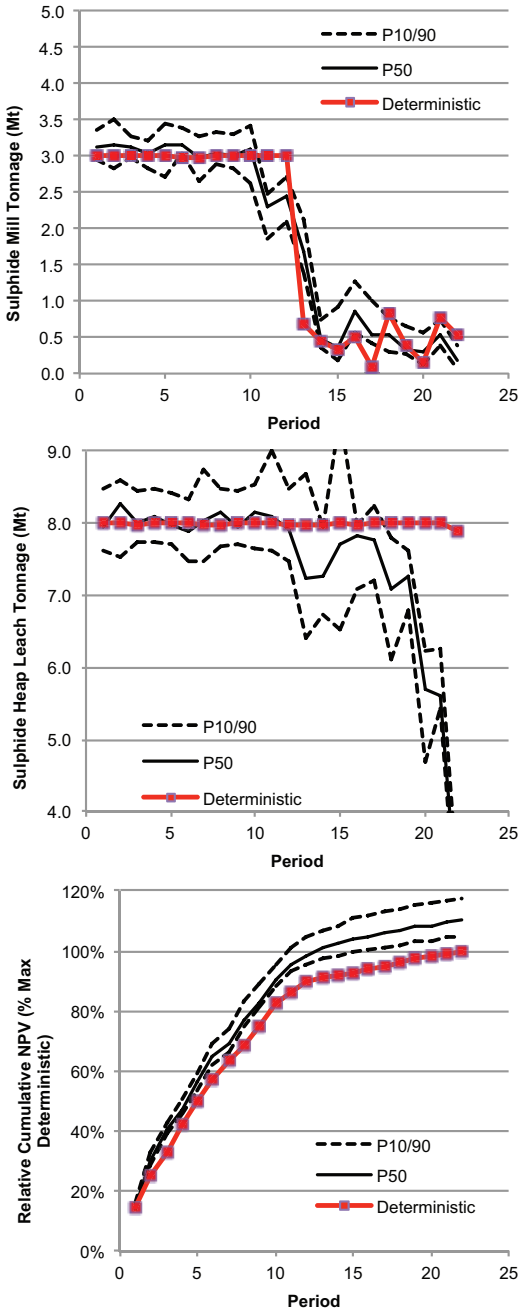

Figure 5–9: Risk profiles to verify the robustness of the stochastic solution using the remaining 15 geological simulations.

CHAPTER 6
Stochastic Global Optimization of Open Pit Mining Complexes with Capital Expenditures

The previous Chapter focuses on the global optimization of open pit mining complexes with geological uncertainty by simultaneously optimizing mine production schedules, destination policies and processing stream variables. Similar to all previous work in stochastic production scheduling, these models assume that the target capacities (e.g. mine production, ore processing, stockpile size) are defined a priori. The solution is likely to be a local optimum because the optimizer does not consider options to increase or decrease these capacities or targets. This Chapter expands on the developments outlined in Chapters 4 and 5 by permitting the optimizer to invest money, in the form of a capital expenditure, in order to increase or decrease capacity constraints. As a result of this contribution, it is possible to simultaneously consider design options, such as expanding mill capacities, opening and closing processing streams, or optimizing the mine production rates by purchasing trucks and shovels. In order to achieve this, minor modifications to the modelling methodology proposed in Chapters 4 and 5 are made in order to give more flexibility when defining variables of interest, and the optimization model is modified to account for capital costs in order to increase or decrease capacity constraints. Capital expenditures pose a challenge to metaheuristic (algorithmic) optimizers: by constantly increasing or decreasing the constraints, it is easy for an algorithmic optimizer to get trapped

in a local optimum because it cannot quickly change the other design variables to truly explore the potential of the new capacity. In order to alleviate this challenge, several new methods are developed to change production schedules during simulated annealing to make large changes to the design to quickly evaluate the advantages of an increased or decreased capacity. The method is evaluated using a full-field test for a copper mining complex (identity withheld for confidentiality), which is comprised of a mine with 128 946 blocks, contains 3 major processing streams, and has 8 stockpiles. In this example, the optimizer has the ability to design the mine production capacity by purchasing and replacing shovels and trucks. Due to the complexity and non-linearity of this model, industry-standard and state-of-the-art mine production schedulers are unable to provide sufficient flexibility to create an accurate optimization model. As a result, the proposed stochastic optimizer is compared with its deterministic-equivalent, and results indicate that the stochastic optimizer is better-able to utilize the mill's available capacity and reduce the risk of not being able to keep the mill fed with materials. Additionally, the stochastic optimizer provides a 5.7% increase in net present value over the deterministic-equivalent.

6.1 Introduction

The primary objective of a mining enterprise is to maximize the value of its assets for its stakeholders. This requires optimizing many strongly interrelated components, such as the amount and timing of capital expenditures (CAPEXs) that are required to develop, maintain or expand an operation, the sequencing of extraction from the mines, and the use of the various processing streams to maximize the utility of the products mined and treated. Naturally, the amount of capital expenditure is

strongly related to the rate of which a mining complex can produce, treat and sell materials. The global optimization of mining complexes addresses the challenge of integrating all relevant aspects of optimizing a mining enterprise. Existing methods have predominantly focused on aspects of mine production scheduling, the use of the processing streams that stockpile, blend, treat and transform the bulk mined material into refined products, and distribution networks that are used to deliver the products to customers [19, 20, 21, 29, 30, 52, 76, 77, 153, 154, 161, 163, 166, 168], while leaving strategic capital expenditure decisions outside of the optimization model in the form of what-if scenarios. Given the strong relationship between capital expenditures, capacities, operating costs, production scheduling and the use of processing streams, this scenario-wise design methodology leads to a sub-optimal use of capital and the non-renewable resource. Moreover, many of the existing attempts at global optimization for mining complexes ignore the compounded effects that uncertainty has on the performance of the mining complex, particularly the ability to fully utilize the capacities that are purchased with a significant capital cost. In order to truly maximize the value of the mining operation, it is necessary to optimize all aspects of the mining complex, including capital expenditures, and simultaneously manage the opportunities and risk that arise in the mining complex's various components.

Recent research as focused on integrating geological uncertainty into mine design and production scheduling optimization. Godoy and Dimitrakopoulos [66] propose a sequential optimization methodology that first uses a modified linear programming model, based off work by Tan and Ramani [159], to determine the optimal production rates (i.e. shovel and truck purchases) from an orebody while considering uncertainty

in metal quantities. A risk-based production scheduling algorithm is then used to find a single production schedule that minimizes the risk of not meeting ore and waste production targets over the life of the mine, which are governed by the previously determined mine production capacities. The authors demonstrate that the method is capable of not only substantially reducing the risk of the stochastic production schedule, but also generates a higher net present value (NPV).

Ramazan and Dimitrakopoulos [142] propose a two-stage stochastic integer programming model (SIP) [16] that aims to generate a production schedule that maximizes the NPV of the design and reduces the risk of not meeting production targets (e.g. ore production capacity, total material movement capacity), metal quantities produced and blending targets. The authors introduce the concept of geological risk discounting, which is a time-dependent discount factor used to ensure that production targets are met at the beginning of the mine life, thus guaranteeing early cash flows, and defer riskier material to later periods when more information is available. This model has been expanded upon and tested [2, 14, 46, 112], and results consistently demonstrate the ability to not only generate a substantially higher NPV, but also minimize the risk of not meeting production targets, metal quantities and blending targets.

The previously mentioned methods for production scheduling with uncertainty, however, are limited by several assumptions. The formulations assume that ore and waste materials are classified a priori, hence are unable to simultaneously optimize cut-off grade decisions [109, 145] or mining complexes with multiple processing

options. Despite the fact that the optimizer will seek to extract blocks with high economic value in early periods, a fixed ore-waste classification can result in low-grade ore that is sent for processing and deferring the processing of higher-grade material that may be readily available. Menabde et al. [124] propose a production scheduling model that simultaneously generates a robust cut-off grade policy, however, it does not explicitly manage the upside or downside risk of not meeting production targets. Boland et al. [22] propose a multistage model that simultaneously generates an adaptive production schedule and scenario-dependent cut-off grade decisions. This, however, leads to overly optimistic destination decisions, as it assumes that the grades of the mined materials are known at the beginning of each period. Kumral [102] proposes a model that attempts to simultaneously optimize the production schedule and define an ore-waste classification for each block. Scenario-independent block classifications have limited applicability for mining complexes that consider multiple material types because certain materials often cannot treated with certain processing streams due to incompatible chemical reactions.

The aforementioned work in stochastic optimization for mine production scheduling attempt to meet production targets over the life of the mine, and reduce the risk associated with not being able to satisfy the targets. These models, however, fail to consider the timing and quantity of capital expenditures that that permit the option to increase or decrease the target capacities. Recent work has sought to incorporate this additional level of decision-making directly in the optimizer. Groeneveld et al. [71] propose a mixed integer program (MIP) model that schedules the mining of benches (a production schedule within pre-defined phases), optimizes destination

decisions, and the timing and quantities of capital expenditures used to increase or decrease target capacities. By solving the optimization model for a set of metal price, cost and utilization simulations independently, which the authors refer to as a "flexible" design, it is possible to obtain a probability distribution for a capital expenditure that can be used to to approximate the timing of the decision. This method, however, does not integrate uncertainty into the optimizer's decision-making, and generates an overly optimistic solution that assumes perfect knowledge of uncertain events (i.e. a wait-and-see solution [16]). Groeneveld et al. [72] improve this model by forcing the optimizer to choose the same decisions at the beginning of the mine life across all scenarios. The authors note that geological uncertainty is not integrated in the models, and that a phase design is required prior to running their proposed model. Geological uncertainty can play a critical role when designing capacities because the uncertainty relates directly to the quantities that are available and sent. Giving the optimizer the ability to do detailed production scheduling can help manage the distribution of risk over time, thus providing a consistent quantity and quality of material at the appropriate capacity with controlled variability.

Chapter 4 proposes a generalized methodology for modelling and optimizing mining supply chains with geological uncertainty, including the ability to model non-linear transformations that occur in the processing streams. This method aims to generate robust destination policies, similar to cut-off grades, which define where materials are sent from the mine, and how to utilize the processing streams to maximize the utility of the materials extracted. The destination policies improve on

cut-off grade policies because they can consider the blending of materials and complex, non-linear processing streams. Chapter 5 improves on the method to consider the simultaneous optimization of multi-mine production schedules, destination policies and processing streams with uncertainty. This chapter expands the previously mentioned developments to include capital expenditure options, which permit the optimizer to change the target capacities in the mining complex (e.g. mine production and ore processing capacities). In the following section, a brief overview of the generalized modelling procedure is outlined. Following this, a mathematical model is given that may be tailored to suit the individual needs of each mining complex. A description of the proposed metaheuristics that are used to perform the optimization is provided. An application at an industry partner's copper mining complex is then discussed. Finally, conclusions and future work are presented.

6.2 Flexible Modelling of Mining Complexes with Uncertainty

6.2.1 Models of material and attribute uncertainty

In a mining complex, a *material* is a term used to define a physical product that is extracted from a mine (e.g., sulphide or oxide) or generated from blending and processing (e.g., tailings, concentrate, slag or refined metal). Materials often have unique mineralogical or geometallurgical properties that have varying impacts at the locations in a mining complex, which limit the choice of where they can be sent for further blending or processing. An *attribute* is a generic term used to describe the property of a material that is of interest to the optimization model, such as metal mass or percent by weight (commonly referred to as grade), total mass, economic values from sale, costs, recoveries or mill residence time (among

many other possibilities). It is useful to categorize the various attributes into one of two groups. Primary attributes are the fundamental variables of interest (e.g. metal content and mass) that are sent from one location in the mining complex to another, and are used to define the total quantity and quality of a single material (in its entirety) in a given period. Hereditary attributes are variables that are of interest for optimization models, and are derived using linear or non-linear expressions from primary attributes. In practice, these may be used to track information such as processing costs, revenues, throughput or energy consumption, among others.

Traditional mine production scheduling optimization frameworks consider only a single representation of the spatial distribution of materials and their attributes, such as metal content. Often these models are generated by kriging [40, 93, 120], a geostatistical method used to estimate the values of the attributes at points or volumes of interest. These estimation methods are known to over-smooth the distributions of the attributes, resulting to less high- and low-grade materials, which ultimately leads to inaccurate financial and production forecasts [45, 143]. Geostatistical simulation methods [34, 69, 89] are ale to overcome the limitations of conventional estimation techniques. They offer the possibility to generate an infinite number of equally probable realizations of the geological conditions, which may be used as a group the quantify the geological uncertainty in each mineral deposit of the mining complex, and also better represent the geological variability (high- and low-values) of the attributes of interest. Several geostatistical simulation techniques exist, which are capable of generating simulations for both material types and multiple attributes [5, 25, 26, 33, 43, 66, 78, 92, 118, 130, 132, 133, 156, 175]. Figure 6–1

shows an example of a cross section for both material type and attribute (copper) simulations at the copper mine used in the case study; it is noted that the simulations for material types provides discrete geological units, whereas the copper grade attribute is a continuous variable.

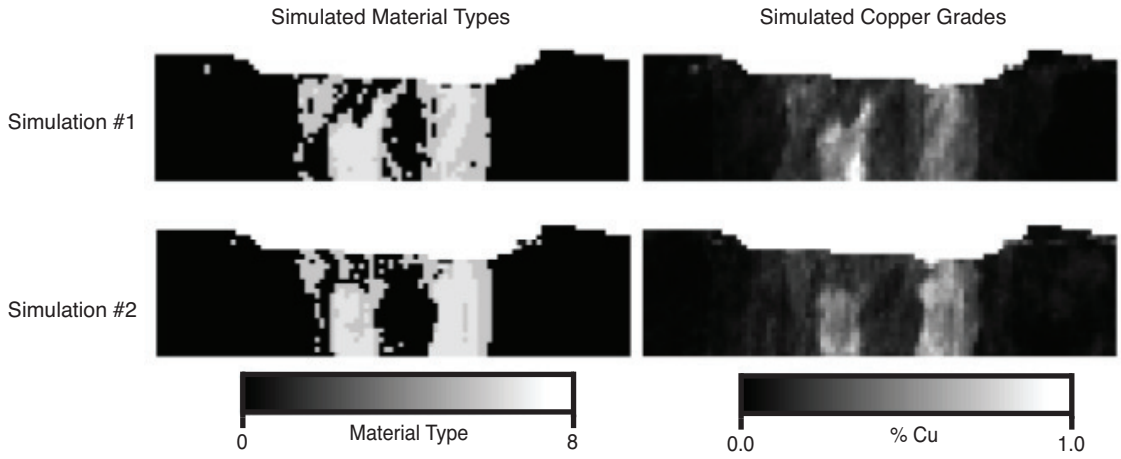

Figure 6–1: Example of simulated material types and copper grades at the copper mine used in the case study.

Let $\mathbb{S}$ represent a set of equally probable *scenarios*, whereby a scenario is a joint sampling from all sources of uncertainty considered in the optimization model. In the case of multi-mine operations, each scenario is indexed, and the number of scenarios is the product of the number of simulations for each orebody model. For example, if two mines are considered, each having 20 geological simulations, $\mathbb{S} = \{1, ..., 400\}$. Naturally, as the number of independent sources of uncertainty (geology, prices, recoveries, costs, etc.), the size of the optimization model grows exponentially.

6.2.2 Material and attribute flow through a mining complex

In order to develop a model for the global optimization of open pit mining complexes, it is first necessary to establish some fundamental terminology. Tables

6–1, 6–2 and 6–3 provide the relevant sets, variables and parameters used in the optimization models. A mining complex is comprised of a set of mines ($\mathbb{M}$), stockpiles ($\mathcal{S}$) and processors ($\mathcal{P}$). For simplicity, this work will consider mining complexes where all mines and locations within operate during a fixed set of periods $t \in \mathbb{T}$; the more general case where the operating periods of mines and processing streams varies with time [136, 161] is omitted. Mines are assumed to be the only sources of materials for the mining complex. Each mine $m \in \mathbb{M}$ is comprised of a set of discrete volumes ($\mathbb{B}_m$), referred to as *blocks*. Each block $b \in \mathbb{B}_m$ has simulated attributes $\beta_{p,b,s}$ for the primary attributes of interest ($p \in \mathbb{P}$), which are assumed to be inputs to the optimization model. Stockpiles are locations in the mining complex that are capable of storing incoming materials (and their attributes) over time and distributing them to subsequent locations when desired. Stockpiles are useful in practice because they can be used to blend materials together, thus creating a more homogenous product, and may also be used to store marginally valuable material that is treated at a later time when the opportunity cost of deferring more valuable is lower (i.e. the cut-off grade is lower). A processor is a generic term used to describe all other locations in the mining complex, which may, but not necessarily be used to transform an incoming bulk product into a purer form, for example, concentrators, smelters, refineries, leach pads. Additionally, in this definition, the set of processors may also contain other elements, such as modes of transport (rail, trucks, ports), which are useful for the optimization model. One of the primary distinctions between a stockpile and a processor, in the generic modelling sense, is that a processor does not store material

Table 6–1: Sets used for optimization with capital expenditures.

Sets and Indices	
Set	Description
$\mathbb{P}$	Primary attributes.
$\mathbb{H}$	Hereditary attributes.
$\mathbb{T}$	Time periods.
$\mathbb{S}$	Joint scenarios for all sources of uncertainty.
$\mathbb{M}$	Mines.
$\mathbb{B}_m$	Blocks at mine $m \in \mathbb{M}$.
$\mathbb{O}_b$	Blocks that overly $b \in \mathbb{B}_m$ that must be extracted prior to b.
$\mathbb{K}$	All capital expenditure options.
$\mathbb{K}^1$	One-time capital expenditures ($\mathbb{K}^1 \subseteq \mathbb{K}$).
$\mathcal{C}$	Sub-groupings (clusters) of blocks with similar attributes.
$\mathcal{S}$	Stockpile destinations.
$\mathcal{P}$	Processors in the mining complex that must forward all products generated to the subsequent destinations, if available.
$\mathcal{N}$	Nodes that describe the clusters and destinations in the mining complex, i.e. $\mathcal{N} = \mathcal{C} \cup \mathcal{S} \cup \mathcal{P}$.
$\mathcal{I}(i) \subseteq \mathcal{N}$	A set of nodes that destination $i \in \mathcal{S} \cup \mathcal{P}$ receives materials from (incoming).
$\mathcal{O}(i) \subseteq \mathcal{S} \cup \mathcal{P}$	A set of nodes that destination $i \in \mathcal{S} \cup \mathcal{P}$ can send material to (outgoing).

over time; all material that is produced is sent out to subsequent destinations, if possible.

The primary and hereditary attributes at the stockpiles and processors are tracked in the optimization models using state variables. Let $v_{p,i,t,s}$ represent the value of attribute $p \in \mathbb{P}$ at location $i \in \mathcal{S} \cup \mathcal{P}$. Additionally, let the state variable $v_{h,t,s}$ represent the value of hereditary attribute $h \in \mathbb{H}$, which is calculated using a (non-) linear function, $f_h(p, i, k)$, of the primary attributes $p \in \mathbb{P}$ at location i and capital expenditure option $k \in \mathbb{K}$. These functions may be used, for example, to calculate non-linear recoveries (see Chapter 5), mill throughputs, profits and costs,

Table 6–2: Variables used for optimizing with capital expenditures.

Decision variables	
$x_{b,t} \in \{0,1\}$	Defines whether or not block $b \in \mathbb{B}_m$ is extracted in period $t \in \mathbb{T}$.
$z_{c,j,t} \in \{0,1\}$	Defines whether or not cluster $c \in \mathcal{C}$ is sent to destination $j \in \mathcal{O}(c)$ in period $t \in \mathbb{T}$.
$y_{i,j,t,s} \in [0,1]$	Defines the proportion of output material sent from $i \in \mathcal{S} \cup \mathcal{P}$ to $j \in \mathcal{S} \cup \mathcal{P}$ in period $t \in \mathbb{T}$ and scenario $s \in \mathbb{S}$.
$w_{k,t} \in \{L_{k,t}, U_{k,t}\}$	Defines how many capital expenditure options $k \in \mathbb{K}$ are exercised in period $t \in \mathbb{T}$.
State variables	
$v_{p,i,t,s} \in \mathbb{R}$	State variable for attribute $p \in \mathbb{P}$ at location or cluster $i \in \mathcal{N} \cup \mathbb{M}$ in period $t \in \mathbb{T}$ and scenario $s \in \mathbb{S}$.
$v_{h,t,s} \in \mathbb{R}$	State variable for attribute $h \in \mathbb{H}$ in period $t \in \mathbb{T}$ and scenario $s \in \mathbb{S}$.
$r_{p,i,t,s} \in [0,1]$	Proportion of attribute $p \in \mathbb{P}$ recovered after processing at node $i \in \mathcal{P}$ in period $t \in \mathbb{T}$ and scenario $s \in \mathbb{S}$.
$d^+_{h,t,s}, d^-_{h,t,s} \in \mathbb{R}$	Surplus and shortage variables, respectively, from a deviation target for attribute $h \in \mathbb{H}$ in period $t \in \mathbb{T}$ and scenario $s \in \mathbb{S}$.

Table 6–3: Parameters used for optimization models with capital expenditures.

Material flow parameters and attribute transformation functions	
$\beta_{p,b,s}$	Simulated value of attribute $p \in \mathbb{P}$ (e.g., metal content) for block $b \in \mathbb{B}_m$ and scenario $s \in \mathbb{S}$.
$\theta_{b,c,s} \in \{0,1\}$	Defines whether or not block $b \in \mathbb{B}_m$ belongs to cluster $c \in \mathcal{C}$ in scenario $s \in \mathbb{S}$.
$f_h(p,i,k)$	A (non)-linear function that is used to calculate the value of attribute $h \in \mathbb{H}$ using attributes $p \in \mathbb{B}$ from locations $i \in \mathcal{S} \cup \mathcal{P} \cup \mathbb{M}$ and capital expenditure options $k \in \mathbb{K}$.
Optimization model parameters	
$U_{h,t}$,$L_{h,t}$	Upper- and lower- bounds, respectively for hereditary attribute $h \in \mathbb{H}$ in period t.
$p_{h,t}$	Price (or cost) of attribute $h \in \mathbb{H}$ in period $t \in \mathbb{T}$.
$c^+_{h,t}$, $c^-_{h,t}$	Unit surplus and shortage costs associated with deviations from bounds for attribute $h \in \mathbb{H}$ in period $t \in \mathbb{T}$.
Capital expenditure parameters	
$p_{k,t}$	Discounted purchase price for capital expenditure $k \in \mathbb{K}$ in period $t \in \mathbb{T}$.
$\kappa_{k,h}$	The per-unit increment for a constraint that capital expenditure $k \in \mathbb{K}$ has on attribute $h \in \mathbb{H}$.
λ_k	The life of capital expenditure $k \in \mathbb{K}$ (e.g. truck life before replacement).
τ_k	Lead time before capital expenditure $k \in \mathbb{K}$ is built or delivered.
$L_{k,t}, U_{k,t}$	Minimum and maximum purchase requirements, respectively for option $k \in \mathbb{K}$ in period $t \in \mathbb{T}$.

among others. It is noted that this definition, unlike that of Chapters 4 and 5, defines hereditary attributes as global functions that may be calculated using the primary attributes p from multiple locations $i \in \mathcal{S} \cup \mathcal{P} \cup \mathbb{M}$ in a single equation. These equations are defined by the modeller, and may be a function of the level of capital expenditures, which is useful when modelling variable operating costs as a function of equipment purchases (i.e. economies of scale from expanding the mill capacity or mine production). Additionally, a recovery variable, $r_{p,i,t,s} \in [0,1]$, may be used to define the quantity of attribute p recovered at a location $i \in \mathcal{S} \cup \mathcal{P}$ for each time period and scenario.

The flow of materials and their respective attributes through a mining complex is defined by three sets of decision variables (Fig. 6–2), namely the mine production schedule decisions, destination policies and processing stream decisions. Production scheduling decision variables, $x_{b,t} \in \{0,1\}$, determine whether (1) or not (0) a block b is extracted in period t; these decision variables define the initial quantities of attributes for each material that is available in each time period. For open pit mines, a set of overlying blocks, $\mathbb{O}_b$ is defined for each block $b \in \mathbb{B}_m$, which are the blocks that must be extracted prior to b in order to ensure slope stability and safety. These sets are generated via a pre-processing step that looks at the overlying blocks within an inverted cone [95], and are of minimum cardinality to avoid excessive memory usage.

After extraction, it is necessary to decide where to send the extracted materials. One method commonly used in the mining industry is a cut-off grade policy, which is

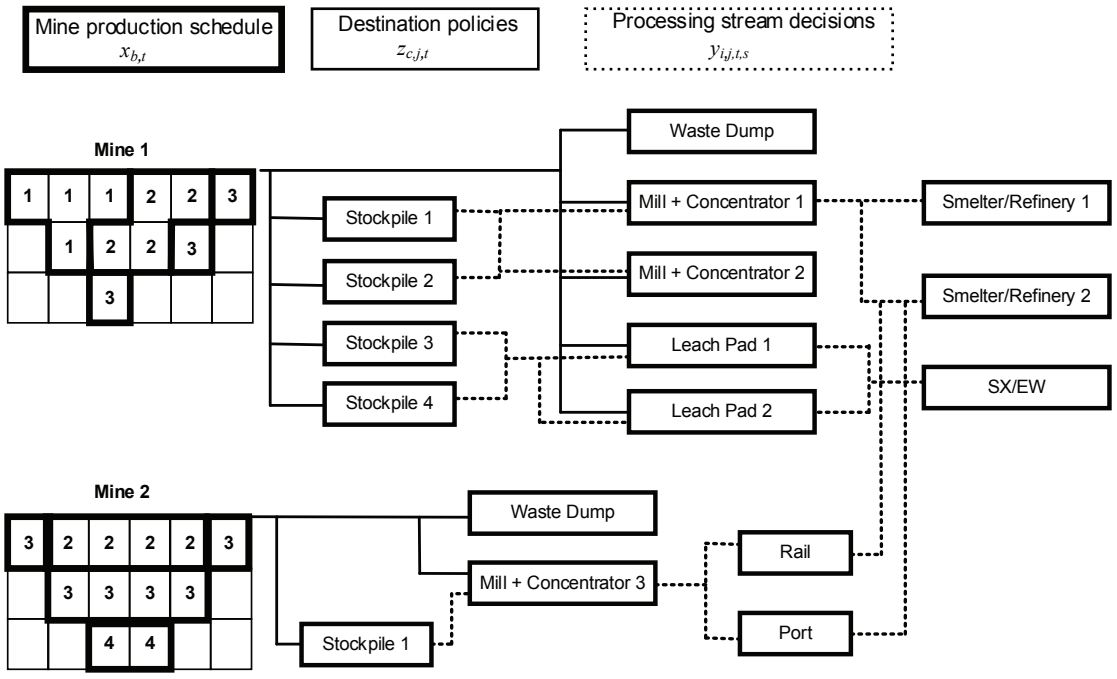

Figure 6–2: Decision variables related to material flowing through a mining complex.

a threshold value that defines where material above or below the prescribed threshold is sent. Simulated block attributes, $\beta_{p,b,s}$, are often sampled from continuous distributions, which complicates the decision of where to send extracted materials because it leads to non-linear formulations. In order to avoid these complex models, some research has instead focused on methods that use binary decision variables to define where material is sent. There are several ways to model the destination decisions, such as scenario-dependent block destinations [22, 100] and robust block destinations [102, 129]. The former, however, leads to overly optimistic solutions that

don't integrate uncertainty into the decisions, whereas, the latter may lead to sending blocks to destinations that are incompatible with a block's simulated material type. Rather than attempting to make destination decisions on the block-scale, Menabde et al. [124] propose a discretization of the continuous attribute into *bins* to define cut-off grade policies. Cut-off grade policies, however, are often not useful for global optimization models that require complex blending constraints and multiple attributes (e.g. multiple metals or deleterious elements). Chapter 4 proposes a generalization of this concept that is useful for mining complexes that consider the impacts of multiple attributes on the entire system. In this method, a set of multivariate bins ($\mathcal{C}$), referred herein as *clusters*, are created in a pre-processing step by clustering [6, 117] the primary block attributes $\beta_{p',b,s}\ \forall p' \subseteq \mathbb{P}, b \in \mathbb{B}_m, m \in \mathbb{M}, s \in \mathbb{S}$ for each material type and for each the mine. Let $\theta_{b,c,s} \in \{0,1\}$ represent a pre-processed parameter that defines whether (1) or not (0) block b belongs to a cluster $c \in \mathcal{C}$ in scenario $s \in \mathbb{S}$. Additionally, let the decision variable $z_{c,j,t} \in \{0,1\}$ decide whether (1) or not (0) cluster $c \in \mathcal{C}$ is sent to destination $j \in \mathcal{O}(c)$ in period $t \in \mathbb{T}$. These variables effectively form a robust destination policy that decides where to send all blocks with similar attributes (e.g. high iron content, medium silica, medium phosphorus) and material types, rather than deciding on the destination of individual blocks. Given that a block's simulated material type and grades may vary between simulations, the membership to a given cluster c for a scenario s may also vary accordingly. As a result, the destination of a block may vary between scenarios, according to its membership c distribution.

Finally, after material is received at the first set of destinations directly from the mines, the material flow through the remainder of the mining complex is governed by a set of processing stream decision variables. Let $y_{i,j,t,s} \in [0,1]$ define the proportion of a material (product) sent from destination $i \in \mathcal{S} \cup \mathcal{P}$ to destination $j \in \mathcal{O}(i) \subseteq (\mathcal{S} \cup \mathcal{P}) \setminus i$ in period $t \in \mathbb{T}$ and scenario $s \in \mathbb{S}$. It is noted that these scenario-dependent decision variables are designed to let the optimizer take recourse decisions [16] after the uncertainty has been revealed at the first set of destinations. For future reference, the set of locations in the mining complex that send material to $i \in \mathcal{S} \cup \mathcal{P}$ is denoted by $\mathcal{I}(i)$.

6.3 Optimization of Mining Complexes with Capital Expenditures

Given the flexibility required to accurately model an individual mining complex, a generalized optimization model is proposed, which can be configured to satisfy the needs of the decision-maker. Using the sets, variables and parameters outlined in Tables 6–1 to 6–3, respectively, it is possible to define a generalized two-stage stochastic integer program [16] that is used to optimize mining complexes with capital expenditures. In this model, the first-stage decisions, which must be made before the uncertainty is revealed, are the mine production schedule(s), destination policies and capital expenditures. The recourse variables, which adapt the optimization model to information garnered after uncertainty is revealed, include the processing stream decisions ($y_{i,j,t,s}$) and penalties related to deviations from production targets. Notably, these penalties are used to manage the upside and downside risk, and may be penalized using time-discounted, monotonically decreasing factors ($c^+_{h,t}$ and $c^-_{h,t}$, respectively) that forces riskier materials to be mined in later periods (*geological risk*

discounting) [142]. These penalty costs, and the associated geological risk discount rate, may be determined experimentally by testing different values, re-optimizing and analyzing the resulting risk profiles. It is noted that it is often necessary to balance the orders of magnitudes for these penalty costs to force the optimizer to consider the differences in order of magnitudes between the constraints (e.g. millions of tonnages compared to a grade measured as a percentage). The optimization formulation is as follows:

Objective function:

$$\max \quad \underbrace{\frac{1}{|\mathbb{S}|}\sum_{s\in\mathbb{S}}\sum_{t\in\mathbb{T}}\sum_{h\in\mathbb{H}} p_{h,t}\cdot v_{h,t,s}}_{\text{Discounted revenues and costs}} - \underbrace{\sum_{t\in\mathbb{T}}\sum_{k\in\mathbb{K}} p_{k,t}\cdot w_{k,t}}_{\text{Capital expenditure costs}} - \underbrace{\frac{1}{|\mathbb{S}|}\sum_{s\in\mathbb{S}}\sum_{t\in\mathbb{T}}\sum_{h\subseteq\mathbb{H}} c^{+}_{h,t}\cdot d^{+}_{h,t,s} + c^{-}_{h,t}\cdot d^{-}_{h,t,s}}_{\text{Risk-discounted penalties for deviations}} \tag{6.1}$$

Subject to:

I. *Mine reserve and slope constraints*, which guarantee that a block is only mined once, if at all, and obeys slope stability requirements.

$$\sum_{t\in\mathbb{T}} x_{b,t} \leq 1 \qquad \forall b \in \mathbb{B}_m \tag{6.2}$$

$$x_{b,t} \leq \sum_{t'=1}^{t} x_{u,t'} \qquad \forall b \in \mathbb{B}_m, u \in \mathbb{O}_b, t \in \mathbb{T} \tag{6.3}$$

II. *Destination policy constraints*, which ensure that the clusters of materials $c \in \mathcal{C}$ are each only sent to a single destination.

$$\sum_{j \in \mathcal{O}(c)} z_{c,j,t} = 1 \qquad \forall c \in \mathcal{C}, t \in \mathbb{T} \tag{6.4}$$

III. *Processing stream constraints*, which calculate the quantities of the primary attributes for each period and ensure mass balance in the mining complex.

$$v_{p,j,(t+1),s} = \underbrace{v_{p,j,t,s} \cdot \left(1 - \sum_{k \in \mathcal{O}(j)} y_{i,k,t,s}\right)}_{\text{Leftovers from previous period}} + \underbrace{\sum_{i \in \mathcal{I}(j) \setminus \mathcal{C}} r_{p,i,t,s} \cdot v_{p,i,t,s} \cdot y_{i,j,t,s}}_{\text{Incoming from other locations}} + \underbrace{\sum_{c \in \mathcal{I}(j) \cap \mathcal{C}} \left(\sum_{b \in \mathbb{B}_m} \theta_{b,c,s} \cdot \beta_{p,b,s} \cdot x_{b,(t+1)}\right) \cdot z_{c,i,(t+1)}}_{\text{Incoming from mines}} \qquad \forall p \in \mathbb{P}, j \in \mathcal{S} \cup \mathcal{P}, t \in \mathbb{T}, s \in \mathbb{S} \tag{6.5}$$

$$\sum_{j \in \mathcal{O}(i)} y_{i,j,t,s} = 1 \qquad \forall i \in \mathcal{P}, t \in \mathbb{T}, s \in \mathbb{S} \tag{6.6}$$

$$\sum_{j \in \mathcal{O}(i)} y_{i,j,t,s} \leq 1 \qquad \forall i \in \mathcal{S}, t \in \mathbb{T}, s \in \mathbb{S} \tag{6.7}$$

IV. *Attribute calculation constraints*, which are used to calculate the value of the state hereditary attributes and quantities of interest at the mine level (e.g. per-period tonnages). Recall that the function $f_h\left(p, i, k\right)$ is defined by the modeller, and

is not necessarily linear.

$$v_{h,t,s} = f_h\left(p, i, k\right) \qquad \forall h \in \mathbb{H}, t \in \mathbb{T}, s \in \mathbb{S} \tag{6.8}$$

$$v_{p,m,t,s} = \sum_{b \in \mathbb{B}_m} \beta_{p,b,s} \cdot x_{b,t} \qquad \forall m \in \mathbb{M}, p \in \mathbb{P}, t \in \mathbb{T}, s \in \mathbb{S} \tag{6.9}$$

V. *Hereditary attribute constraints*, which may be used to track the deviations for variables $h \in \mathbb{H}$ from a upper- and lower- bound capacities (e.g. mining, stockpiling, processing, grade blending capacities). It is noted that these capacities may be increased or decreased by $\kappa_{k,h}$ by investing in the capital expenditure option $k \in \mathbb{K}$, and consider the lifespan of the capacity increment (λ_k) and the lead time to delivery or construction (τ_k).

$$v_{h,t,s} - d^{+}_{h,t,s} \leq U_{h,t} + \sum_{t'=t-\lambda_k+\tau_k}^{t} \kappa_{k,h} \cdot w_{k,t'} \qquad \forall h \in \mathbb{H}, t \in \mathbb{T}, s \in \mathbb{S} \tag{6.10}$$

$$v_{h,t,s} + d^{-}_{h,t,s} \geq L_{h,t} + \sum_{t'=t-\lambda_k+\tau_k}^{t} \kappa_{k,h} \cdot w_{k,t'} \qquad \forall h \in \mathbb{H}, t \in \mathbb{T}, s \in \mathbb{S} \tag{6.11}$$

VI. *Recoveries*, which are constant for stockpiles and may be equal to the value of hereditary attributes for processors; which may, in turn, be a static recovery or a value from a grade-recovery curve.

$$r_{p,i,t,s} = 1 \qquad \forall p \in \mathbb{P}, i \in \mathcal{S}, t \in \mathbb{T}, s \in \mathbb{S} \tag{6.12}$$

$$r_{p,i,t,s} = v_{h,t,s} \qquad \forall p \in \mathbb{P}, i \in \mathcal{P}, t \in \mathbb{T}, s \in \mathbb{S} \tag{6.13}$$

VII. *End-of-year stockpile attribute constraints*, which may be used to calculate and track the quantity of an attribute that remains in a stockpile at the end of the production period.

$$v_{h,t,s} = v_{p,i,t,s} \cdot \left(1 - \sum_{j \in \mathcal{O}(i)} y_{i,j,t,s}\right) \qquad \forall i \in \mathcal{S}, t \in \mathbb{T}, s \in \mathbb{S} \tag{6.14}$$

VIII. *Capital expenditure constraints* for one-time investments (e.g. expanding mill capacity), which ensure that the option is exercised once, if at all.

$$\sum_{t \in \mathbb{T}} w_{k,t} \leq 1 \qquad \forall k \in \mathbb{K}^1 \subseteq \mathbb{K} \tag{6.15}$$

IX. *Variable definitions*

$$L_{k,t} \leq w_{k,t} \leq U_{k,t} \qquad \forall k \in \mathbb{K}, t \in \mathbb{T} \tag{6.16}$$

$$x_{b,t} \in \{0,1\} \qquad \forall b \in \mathbb{B}_m, t \in \mathbb{T} \tag{6.17}$$

$$z_{c,j,t} \in \{0,1\} \qquad \forall c \in \mathcal{C}, j \in \mathcal{O}(c), t \in \mathbb{T} \tag{6.18}$$

$$\gamma_{p,c,t,s} \geq 0 \qquad \forall p \in \mathbb{P}, c \in \mathcal{C}, t \in \mathbb{T}, s \in \mathbb{S} \tag{6.19}$$

$$y_{i,j,t,s} \in [0,1] \qquad \forall i \in \mathcal{S} \cup \mathcal{P}, j \in \mathcal{O}(i), t \in \mathbb{T}, s \in \mathbb{S} \tag{6.20}$$

$$r_{p,i,t,s} \in [0,1] \qquad \forall p \in \mathbb{P}, i \in \mathcal{S} \cup \mathcal{P}, t \in \mathbb{T}, s \in \mathbb{S} \tag{6.21}$$

$$v_{p,i,t,s} \geq 0 \qquad \forall p \in \mathbb{P}, i \in \mathcal{S} \cup \mathcal{P} \cup \mathbb{M}, t \in \mathbb{T}, s \in \mathbb{S} \tag{6.22}$$

$$v_{h,t,s} \in \mathbb{R} \qquad \forall h \in \mathbb{H}, t \in \mathbb{T}, s \in \mathbb{S} \tag{6.23}$$

$$d^{+}_{h,t,s}, d^{-}_{h,t,s} \geq 0 \qquad \forall h \in \mathbb{H}, t \in \mathbb{T}, s \in \mathbb{S} \tag{6.24}$$

Chapter 3 discusses the impact of the scale of the target capacity, and the effects of an optimizer forcing uncertainty onto smaller-scale processes in order to reduce the uncertainty for large-capacity processing streams. This scale effect is generally a result of solely considering the linear deviation from the target capacity, which is defined in Eqs, (6.10) and (6.11). In the case where the optimizer has the ability to change target capacities dynamically, it is possible that these scale issues may be present. In order to avoid these issues, it may be necessary to standardize the deviation variables with respect to the (dynamic) target capacity, as is done

in Chapter 3. These details, however, are omitted in order to provide a simpler optimization model.

6.4 Algorithmic Optimization with Metaheuristics

6.4.1 Hybrid optimization with simulated annealing and particle swarm optimization

The stochastic optimization model presented in Sect. 6.3 is computationally challenging to solve using conventional optimization methods; single orebody models often consist of hundreds of thousands of blocks, which can result in millions of binary decision variables. Moreover, given the flexibility that a modeller has to design these optimization formulations, these models may be non-linear, making exact optimization methods infeasible for realistic-sized mining complexes. While many authors have proposed mine production scheduling models and heuristics that are tailored to be solved using mathematical optimizers [15, 17, 19, 23, 29, 36, 105], the scale of the formulations poses a formidable challenge. Moreover, these methods often are limited by various simplifying assumptions, such as the use of aggregates (reduces scale of decision-making), not being able to accommodate lower bounds or blending constraints (generally required for mining complexes), linearity (to garner information using duality theory) and time-separability properties (precludes the use of stockpiles and complex processing streams). Metaheuristics are a class of optimization algorithms that do not necessarily give mathematical optimality, however, have been used as a tool to generate high-quality optimization solutions (when compared to commercial mine design software and optimization solvers) within a reasonable amount of time [1, 56, 65, 102, 103, 106, 107, 108, 129].

The generalized formulation for optimizing mining complexes with uncertainty (Sect. 6.3) is optimized using a combination of the simulated annealing [60, 98, 127] and particle swarm optimization [94] algorithms. The simulated annealing algorithm has been demonstrated in the past to be capable of optimizing large-scale mine production scheduling models with good resultsc̃itealbor2009, godoy2003, kumral2013. This method, however, is somewhat limited in its ability to handle a large number of continuous variables (i.e. processing stream decision variables); evaluating the objective function for the optimization of mining complexes is generally computationally demanding, and existing simulated annealing algorithms for continuous variables generally work with only a single variable at a time. Through experimental testing in Chapter 4, it was noted that using the simulated annealing algorithm is not particularly effective for optimizing processing stream decisions. As a result of this limitation, the particle swarm optimization algorithm is used because of its inherent ability to modify all continuous variables at each iteration, leading to more changes in the variables per objective function evaluation. Let $\mathbf{\Phi} = [\mathbf{x}, \mathbf{z}, \mathbf{w}, \mathbf{y}]$ represent a solution vector that is used to store the production schedules ($\mathbf{x} = [x_{b,t}]$), destination policies ($\mathbf{z} = [z_{c,j,t}]$), capital expenditure options ($\mathbf{w} = [w_{k,t}]$) and processing stream variables ($\mathbf{y} = [y_{i,j,t,s}]$). The simulated annealing (SA) algorithm is used to optimize the discrete variables ($\mathbf{x}$, $\mathbf{w}$, $\mathbf{z}$), and, after a specified number of iterations (e.g. 10), the particle swam optimization (PSO) algorithm optimizes the continuous variables ($\mathbf{y}$). The methods are used interchangeably to avoid getting trapped in local optima. An initial schedule can be generated using either industry-standard

planning software as input, or start from nothing by setting all blocks to being unmined. The remaining variables for destination policies, processing stream values and capital expenditures can be initialized by randomly generating values that obey their constraints.

In the classic SA algorithm, a *perturbation* to the current solution vector is proposed (see Sect. 6.4.2). Let $g(\mathbf{\Phi})$ represent the objective function value (Eq. (6.1)) for the current solution vector, $\mathbf{\Phi}$, and let $g(\mathbf{\Phi}')$ represent the objective function value for a perturbed solution vector, $\mathbf{\Phi}'$. For an objective function that is being maximized, a proposed perturbation is accepted or rejected according to the following probability distribution:

$$P(g(\mathbf{\Phi}), g(\mathbf{\Phi}'), \delta) = \begin{cases} 1 & \text{if } g(\mathbf{\Phi}') \geq g(\mathbf{\Phi}) \\ \exp\left(-\left|g(\mathbf{\Phi}') - g(\mathbf{\Phi})\right| / \delta\right) & \text{otherwise} \end{cases} \tag{6.25}$$

where δ is commonly referred to as an *annealing temperature*, which is initially defined as an input parameter for the first iteration, $\delta(0)$, and is gradually cooled as the algorithm progresses using a cooling factor, $cf \in [0, 1]$, by applying $\delta = \delta * cf$ every n iterations. If a perturbation is accepted, $\mathbf{\Phi} = \mathbf{\Phi}'$. If the perturbation is the best found by that iteration, the global best solution vector, $\mathbf{\Phi}^g$ is updated, i.e. $\mathbf{\Phi}^g = \mathbf{\Phi}$. One of the difficulties of the classic SA algorithm is related to the use of multiple *neighbourhoods* of perturbations, where a neighbourhood refers to a class of perturbation in solution vector ($\mathbf{x}$, $\mathbf{w}$ or $\mathbf{z}$). A perturbation for each neighbourhood may have a drastically different impact on the objective function value. Figure 6–3 shows an example of a cumulative probability distribution with respect to rejected neighbourhood perturbations ($g(\mathbf{\Phi}') - g(\mathbf{\Phi}) < 0$). In the classic SA algorithm, for a

given δ, Eq. (6.25) will likely reject all capital expenditure perturbations (CAPEX), however accept most production schedule and destination policy perturbations. Naturally, there is a strong relationship amongst these variables, and this phenomenon may result in SA tunneling into a local optimum.

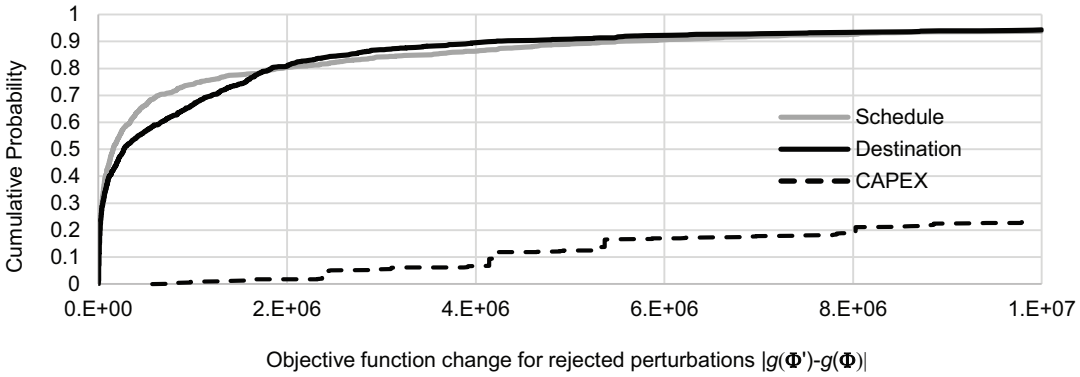

Figure 6–3: Cumulative distribution of change in objective function values for rejected perturbations for production scheduling, destination policy and capital expenditures.

Alternatively, it is possible to modify the simulated annealing algorithm to deal with these multiple neighbourhoods independently. Rather than a fixed δ that applies to all neighbourhoods, the proposed method uses variable annealing temperatures, $\delta_{\mathbf{x}}$, $\delta_{\mathbf{w}}$, $\delta_{\mathbf{z}}$, which are calculated using an *annealing probability temperature* and a distribution function similar to Fig. 6–3. This distribution is first constructed by sampling 1000 perturbations for each neighbourhood prior to the full-scale annealing using Eq. (6.25). An initial annealing probability temperature, $\rho(0)$ is specified as an input parameter (e.g. $\rho\,(0) = 0.8$), and the appropriate annealing temperatures ($\delta_{\mathbf{x}}$, $\delta_{\mathbf{w}}$, $\delta_{\mathbf{z}}$) are derived. Equation (6.25) then uses the appropriate annealing temperature according to the neighbourhood that the proposed perturbation belongs.

This method has the added advantage that the modeller does not need to spend excessive time calibrating the initial temperature $\delta(0)$ for the classic SA algorithm. Similar to the annealing temperatures, the annealing probability temperature is updated by $\rho = \rho * cf$ every n_i iterations As the algorithm progresses and sub-optimal perturbations are discovered ($g(\mathbf{\Phi}') - g(\mathbf{\Phi}) < 0$), the cumulative distributions that define $\delta_{\mathbf{x}}$, $\delta_{\mathbf{w}}$ and $\delta_{\mathbf{z}}$ are updated accordingly. This helps to ensure that the appropriate neighbourhood annealing temperatures are always updated to the local solution space, defined by $\mathbf{\Phi}$.

It is noted that the proposed SA algorithm does not perturb the processing stream variables $\mathbf{y} \in \mathbf{\Phi}$. PSO [94] is a population-based metaheuristic that can optimize both discrete and continuous variables. In the proposed method, PSO is used to optimize the processing streams after a number of defined iterations of the SA algorithm. Unlike Chapters 4 and 5, which uses PSO in conjunction with the global best solution vector, $\mathbf{\Phi}^g$, this method uses the working solution vector, $\mathbf{\Phi}$, and focuses solely on the processing stream decisions $\mathbf{y} \in \mathbf{\Phi}$. A *particle* is a data structure that stores a temporary processing stream solution vector, $\mathbf{y}_i \in \mathbf{\Phi}$, a particle best solution vector, $\mathbf{y}_i^b$ and a velocity vector, $\mathbf{v}_i$ for all particles $i \in \{1, ..., N^P\}$, where N^p is a parameter that defines the total number of particles. Additionally, a vector $\mathbf{y}^g$ is used to store the processing stream solution vector. At each iteration $(\alpha + 1)$ of PSO, the particles (solution vectors) are updated as follows:

$$\mathbf{v}_i(\alpha + 1) = c_1 \cdot \mathbf{v}_i(\alpha) + c_2 \cdot r_1 \cdot \left(\mathbf{y}_i^b - \mathbf{y}_i\right) + c_3 \cdot r_2 \cdot (\mathbf{y}^g - \mathbf{y}_i) \tag{6.26}$$

$$\mathbf{y}_i(\alpha+1) = \mathbf{y}_i(\alpha) + \mathbf{v}_i(\alpha+1) \tag{6.27}$$

$$\mathbf{y}_i^b = \mathbf{y}_i(\alpha+1) \text{ if } g([\mathbf{x},\mathbf{w},\mathbf{z},\mathbf{y}_i(\alpha+1)]) \geq g\left(\left[\mathbf{x},\mathbf{w},\mathbf{z},\mathbf{y}_i^b\right]\right) \tag{6.28}$$

$$\mathbf{y}^g = \mathbf{y}_i^b \text{ if } g\left(\left[\mathbf{x},\mathbf{w},\mathbf{z},\mathbf{y}_i^b\right]\right) \geq \max\left\{g([\mathbf{x},\mathbf{w},\mathbf{z},\mathbf{y}^g]), g\left(\left[\mathbf{x},\mathbf{w},\mathbf{z},\mathbf{y}_j^b\right]\right)\right\} \forall j \in \{1,...,N^p\} \tag{6.29}$$

where c_1, c_2 and c_3 are inertia coefficients (parameters), and r_1 and r_2 are random uniform numbers between 0 and 1. In the previous equations, $g([\mathbf{x},\mathbf{w},\mathbf{z},\mathbf{y}_i(\alpha+1)])$ is used to denote the objective function value (Eq. (6.1)) for a solution vector $[\mathbf{x},\mathbf{w},\mathbf{z},\mathbf{y}_i(\alpha+1)]$, where $\mathbf{x},\mathbf{w},\mathbf{z} \in \mathbf{\Phi}$ from simulated annealing. It is noted that in the event that Eqs. (6.6) and (6.7) are violated when updating Eq. (6.27), the processing stream variables are need to be re-normalized prior to evaluating Eq. (6.28). The PSO algorithm is iterated until all particles converge on an optimum (approximately 0.1%) or after a specified number of iterations (e.g. 100). The processing stream portion of the solution vector is updated upon termination of PSO, i.e. $\{\mathbf{y} \in \mathbf{\Phi}\} = \mathbf{y}^g$.

6.4.2 Neighbourhood perturbations for simulated annealing

As the SA algorithm progresses, it is necessary to find valid perturbations to the solution vector, $\mathbf{\Phi}$. Given the inclusion of capital expenditure options, where a perturbation can drastically increase or decrease a capacity constraint, the optimizer is likely to cycle or converge on a local optimum as it attempts to find a large number of production schedule perturbations that satisfy large changes in the constraints. To

avoid the chances of this occurring, a combination of both small and large production scheduling changes is used. The following neighbourhood perturbation mechanisms are used to modify an existing solution, $\mathbf{\Phi}$ during the simulated annealing algorithm:

i *Destination policy perturbations* ($z \in \mathbf{z} \in \mathbf{\Phi}$) are generated by randomly selecting a cluster $c \in \mathcal{C}$ for a period $t \in \mathbb{T}$, which is currently sent to destination $j \in \mathcal{O}(c)$ and sending it to $j' \in \mathcal{O}(c)$.

ii *Random capital expenditure perturabations* ($\mathbf{w} \in \mathbf{\Phi}$) are generated by randomly selecting a capital expenditure option $w_{k,t} = n$ and choosing a new value, i.e. $w_{k,t} = n' \in [L_{k,t}, U_{k,t}]$. If the option is a one-time decision ($k \in \mathbb{K}^1$), all other variables $w_{k,t'}\ \forall t' \in \mathbb{T} \setminus t$ are set to 0.

iii *Delayed capital expenditure perturabations* ($\mathbf{w} \in \mathbf{\Phi}$) are generated by randomly selecting a capital expenditure option $w_{k,t} = n$ for any $k \in \mathbb{K} \setminus \mathbb{K}^1$ and deferring the purchase of one unit by setting $w_{k,t} = w_{k,t} - 1$ and setting $w_{k,(t+1)} = w_{k,(t+1)} + 1$.

iv *Small production schedule perturbations* ($\mathbf{x} \in \mathbf{\Phi}$) are generated by randomly selecting a block $b \in \mathbb{B}_m$ on the boundary between two periods and advancing or delaying its extraction to a period t' of a randomly selected, directly adjacent block (i.e. above, below or the four adjacent blocks on the same elevation). When advancing the extraction period of block b, slope constraint violations are corrected by searching for blocks $b' \in \mathbb{O}_b$ where $x_{b',t''} = 1$ and $t' < t''$, which are also moved to period t'. A similar procedure is used when delaying a block b's extraction period by searching for slope constraint violations below b.

v *Conical production schedule perturbations* ($\mathbf{x} \in \mathbf{\Phi}$) are similar to small production schedule perturbations, however do not require the condition that the block $b \in \mathbb{B}_m$ lie on the boundary between two periods. The extraction period of the block is advanced or delayed by randomly selecting a new $t \in \mathbb{T}$ (i.e. not considering the directly adjacent blocks). Any slope constraint violations are corrected in a similar manner, however, rather than moving all violating blocks into a single period, which may incur drastic penalty costs for deviations from targets, the blocks are moved into several periods. For example, if the mine can sink two benches (levels in elevation) per period, and a block b's extraction period is advanced, any blocks that lie two or three levels above will be extracted in period $t' - 1$. Similarly, any blocks that lie four or five levels above are extracted in period $t' - 2$. This helps to split a large schedule change into several periods, which is likely to have less of an impact on the penalties in the objective function.

vi *Bench-wise production schedule perturbations* ($\mathbf{x} \in \mathbf{\Phi}$) are generated by first labelling the blocks in each bench (vertical level) of the schedule $\mathbf{x} \in \mathbf{\Phi}$ using a connected component labelling algorithm [35]. This algorithm assigns a unique label to all blocks that are mined within the same period and are spatially connected (Fig. 6–4). Rather than randomly selecting a single block that forms an apex of a cone, a component of a bench is then randomly selected (e.g. Fig. 6–4B, component "K") to delay or advance the extraction period.

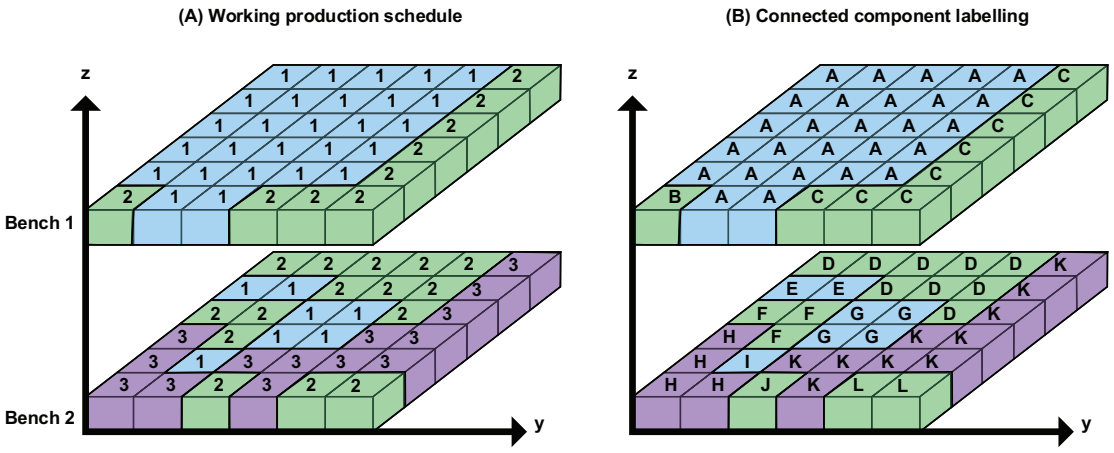

Figure 6–4: Example of labelling a production schedule $\mathbf{x} \in \mathbf{\Phi}$ as connected components for bench-wise production schedule perturbations. Spatially connected blocks in (B) are selected for advancing or delaying their period of extraction.

6.5 Application at a Copper Mining Complex

The proposed method for the global optimization of an open pit mining complex with capital expenditures is applied to a copper mining complex that is supplied by an industrial partner. The name of this mining operation and some of the relevant modelling parameters are withheld for confidentiality purposes.

6.5.1 Overview

The mining complex under study consists of a single mine that primarily produces copper, a group of stockpiles, a mill and concentrator processing stream, a leach pad and a waste dump. The mine produces five main material classes, specifically waste, supergene sulphide, transitional, oxide and hypogene materials. With the exception of the waste material, all material classes are divided into two materials types that are used for modelling the mining complex based on being above

ore below a pre-defined threshold. It is noted that this threshold is defined only to better differentiate the materials that are stockpiled to improve selectivity. The low- and high-grade sulphide, transitional and oxide materials have the option of being sent to a unique stockpile (according to material type), a mill, a leach pad or a waste dump. The stockpiles for these materials, in turn, each sends material to either the mill or the leach pad. The low- and high-grade oxide materials may not be treated at the mill, thus have the option of being stockpiled, sent to the leach pad or the waste dump. The low- and high-grade oxide stockpiles feed only the leach pad. The waste materials are automatically sent to the waste dump.

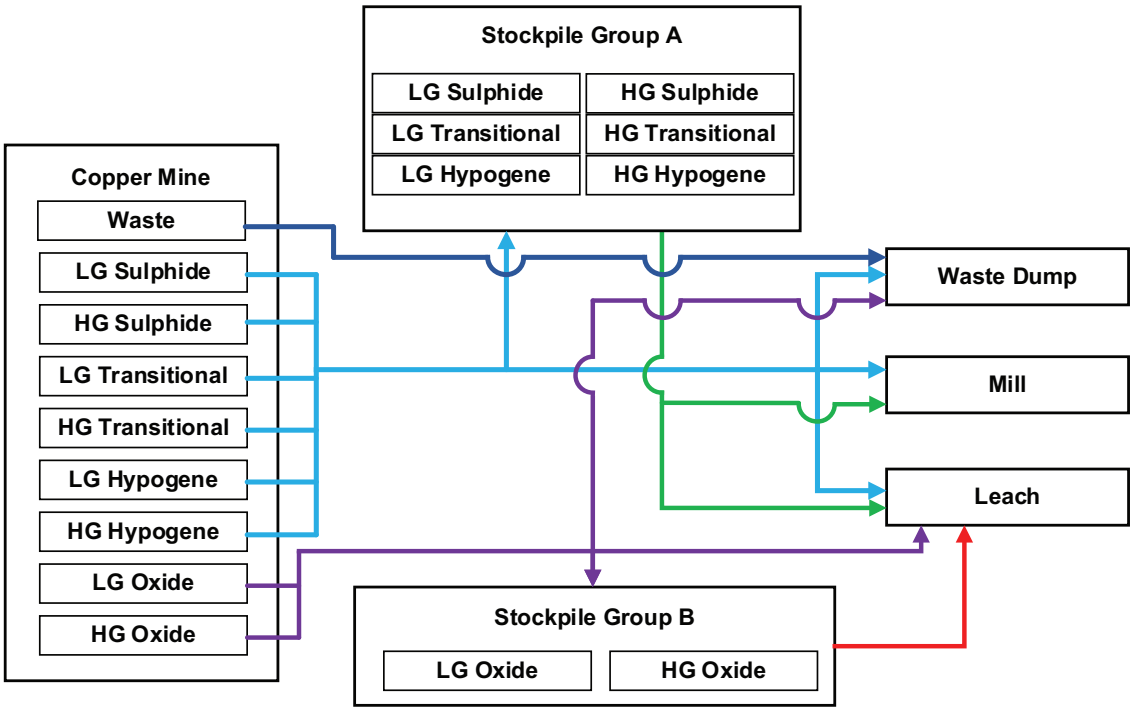

Figure 6–5: Material flow diagram for the copper mining complex.

Table 6–4: Capital expenditure options for the copper mining complex.

Parameters	Shovels	Trucks
Undiscounted cost ($USD)	30,000,000	5,000,000
Life (years)	10	5
Purchase or replacement decision frequency	5 years	4 years
Minimum, maximum purchased per period	0, 4	0, 40
Lead time to delivery (years)	1	1
Capacity increment ($\kappa_{k,t}$)	8670 h	8670 h
Combined productivity and utilization factor	0.58	0.65
Production capacity*	4200 tph (waste) 3100 tph (ore)	230 t

* Shovel production rate (tons per hour) depends on destination policies that classify material as ore or waste.

This case study considers two types of capital expenditures: (i) shovels, which are used to extract material from the ground; and (ii) trucks, which are loaded by shovels and haul material from the mine to the various processing streams. The optimizer, therefore, has full control over the production rates from the mine. Table 6–4 provides an overview of the relevant parameters for the shovel and truck capital expenditures. To provide more consistent production rates, the decision to purchase or replace shovels or trucks only occurs every 5 and 4 years, respectively. While it may be interesting to allow the optimizer to make these decisions annually, the result is a series of fluctuations in fleet size, which, in turn, would result in cycles with excessive amounts of hiring or laying off employees. It is interesting to note that the shovel production rates depends on the block's classification of ore and waste (destination policy variables $z_{c,j,t}$). For selectivity reasons, the shovel is able to load at only 3100 tons per hour for material that is sent to the stockpiles, leach pad or mill, rather than the 4200 tons per hour for material that is sent to the waste dump.

Table 6–5 provides an overview of the key parameters required to model of the mining complex. The mine has provided an estimated orebody model and a set of 50 geological simulations. During as the metaheuristic optimization progresses, simulations are added gradually until a stable solution is obtained (i.e. the objective function value does not change by adding more scenarios). In this case study, this is achieved by starting with 5 simulations, optimizing, then incrementally increasing the number of simulations by 5 and continuing the optimization process. In this case, it was found that the objective function and risk profiles remain stable when using 30 simulations, and adding more does not drastically alter the net present value or risk profiles. Gradually adding simulations (scenarios) to the model has two advantages, when compared to starting with all scenarios: i) it is possible to see how many scenarios are required to obtain a stable design — by starting with all at once, it is not possible to see whether or not more should be added; and ii) the computational time is reduced because fewer simulations are used to converge on a relatively good solution before increasing the computational load by adding more simulations and continuing the optimization process. For an in-depth discussion related to the number of simulations required, the reader is referred to Albor and Dimitrakopoulos [1]. Both the estimated and simulated orebody models contain information for each block related to the copper grade and tonnage (Fig. 6–1), recovery if treated in the mill processing stream, in-pit travel time (i.e. the round-trip time required by a truck to access the block from surface), a mining cost adjustment factor (used to increase the operating costs with depth), block tonnages and material types (Fig. 6–4). With the exception of the in-pit travel time and the mining cost

adjustment factor, which only relate to the spatial location of blocks, all variables have been simulated. Table 6–6 summarizes the relevant information related to clustering the simulations to generated the destination policy variables. It is noted that the quantities of the oxide and transitional materials is small, relative to the quantity of hypogene material, hence fewer clusters are used for these materials to form destination policies. This assumption is made for the sole purposes of reducing the size of the optimization model.

The objective of the optimization model is to maximize the net present value of the cash flows from mining, processing and selling copper, while considering the capital expenditures required to produce at an optimal production rate. Table 6–5 shows the relevant economic parameters used to calculate the revenues associated with the sale of copper concentrate for the mill and the copper from the leaching processing streams. Table 6–7 shows the constraints used to penalize deviations from truck, shovel, mill and leach capacities. It is noted that the capacity constraints are expressed in hours of operation, rather than tonnages, which is typically used in production scheduling models. Given that certain materials are harder than others, the residence time in the mill can vary according to how long it takes to grind material down to a finer size for the concentrator. The shovel and truck constraints are also expressed in hours to more accurately model the shovel's adaptive production rates and the dynamic truck cycle times. The stockpiles are assumed to have unlimited capacity. It is noted that, during the first two years of production, the cash flows are substantially higher than the rest. As a result, the optimizer naturally seeks to extract an infeasible amount of material, and an additional set of constraints

Table 6–5: Modelling parameters for the copper mining complex.

Orebody model	
Number of blocks	128 946
Life (years)	33
Simulations used during optimization	30
Discount rate	6%
Geological risk discount rate	5%
Pit slope angle	39°
Economic parameters	
Copper price	\$3.00/lb
Selling cost (mill)	\$0.40/lb recovered Cu
SX/EW cost (leaching)	\$0.30/lb recovered Cu
Mining cost*	\$1.60/ton
Stockpile rehandling cost	\$0.50/ton
Leach cost (sulphides)	\$8.80/ton
Leach cost (oxides, transitional, hypogene)	\$10.00/ton
Processing cost (mill processing stream)	\$10.50/ton
Copper recovery parameters	
Mill processing stream recovery (variable)	82% - 90%
Leach recovery (sulphides)	78%
Leach recovery (oxides)	70%
Leach recovery (transitional and hypogene)	58%
Truck cycle times (return trip)**	
In-pit travel time (variable with block depth)	3 - 42 mins.
Mine to waste dump	42 mins.
Mine to stockpile	29 mins.
Mine to leach pad	25 mins.
Mine to mill	46 mins.
Stockpile to mill	17 mins.
Stockpile to leach pad	9 mins.

* Before applying a mining cost adjustment factor (costs increase with pit depth).
** Truck cycle times have been adjusted to account for the combined productivity and utilization of the equipment.

Table 6–6: Clustering parameters used to form destination policies

Material type	Destination policy parameter	Number of clusters per period
Waste	Copper tonnage	All: 1
Sulphide	Recoverable copper per mill hour	LG: 20, HG: 20
Transitional	Recoverable copper per mill hour	LG: 5, HG: 5
Hypogene	Recoverable copper per mill hour	LG: 10, HG: 10
Oxide	Copper tonnage	LG: 20, HG: 20
Total number of destination policy variables (all periods):		3 663

for shovel production is used with a high penalty cost to force the optimizer to generate a feasible solution. Additionally, the upper bounds differ for the mill in the deterministic model from the stochastic model (to be discussed in Sect. 6.5.2). The upper bound for the deterministic model is the bound used in practice, however, in the stochastic case, the optimizer avoids having an average production at this limit because of the excessive penalties that are incurred for the simulations that produce above this capacity. This would result in an overly conservative design that will rarely fill the mill up to its capacity. As a result, the stochastic optimization model uses a relaxed upper bound to permit the optimizer to create a design that is able to fully utilize the mill (on average). Finally, given that the optimizer is able to decide how many trucks and shovels to purchase, the lower- and upper-bounds for trucks and shovels, outlined in Table 6–7, are starting points. As the optimizer purchases more equipment, the bounds are changed dynamically using the increments shown in Table 6–4.

6.5.2 Comparison of deterministic-equivalent and stochastic designs

Using the parameters defined in Sect. 6.5.1, a comparison can be made between the deterministic and stochastic designs. First, a deterministic-equivalent design is

Table 6–7: Optimization model constraints.

Constraint Description	Lower, Upper Bounds	Lower, Upper Penalties (Undiscounted)[a]
Truck hours[b]	-, 0[c]	-, 600
Shovel hours (all periods)[b]	-2 190[c], 0[c]	3 000, 7 000
Shovel hours (periods 1 and 2)	-, 2000[c]	-, 2 000 000
Mill hours (deterministic)[d]	8 000, 8 345	10 000, 40 000
Mill hours (stochastic)[d]	8 000, 8 450	10 000, 40 000
Leach capacity (tons)	-, 21 900 000	10

[a] A geological discount rate of 5% is applied to the penalties to manage the distribution of risk over time.
[b] Truck and shovel hour calculations are adjusted for producivity and utilization (Table 6–4).
[c] Constraint bounds are adjusted by the number of trucks or shovels purchased (Table 6–4).
[d] Mill requires a ramp-up to full production in the first four years of production (Fig. 6–6). The same penalty costs are used for these periods. The lower-bound constraints are removed after period 28.

generated using a single, estimated orebody model that is generated using kriging. Recall that estimated models do not depict the spatial and volumetric uncertainty of material types, and estimated methods tend to smooth out the distributions for the attributes of interest (e.g. copper grades). It is worthwhile to note that existing commercial mine design and production scheduling software is not able to incorporate many of the key details that are required in this model, such as variable throughputs, shovel production rates and truck cycle times that depend on ore/waste classification, and targeting production in hours rather than tonnages. It is therefore not possible to provide a benchmark against other methods, as was done in Chapter 5. The deterministic design discussed herein is referred to as the *deterministic-equivalent* design, because it uses the proposed modelling and optimization methodology with a single scenario (i.e. $\mathbb{S} = \{1\}$). In this sense, it is possible to highlight the differences between deterministic and stochastic models, with all other details being the same. For this study, the primary focus of the optimization model is to maximize the net present value (NPV), provide a consistent feed of materials to the mill processing

stream and obey mine production targets, which the optimizer decides by purchasing or replacing shovels and trucks.

All testing is performed using Amazon's EC2 cloud computing platform with Windows 2012-based virtual machines that use Intel Xeon E5-2670 v2 processors (32 virtual CPUs) and have 244 GB RAM. Unlike Chapter 5, both the deterministic and stochastic solutions are generated without the use of an initial schedule. The deterministic design is generated in 20 hours, whereas the stochastic solution requires 49 hours. It is noted that for the stochastic design, four orebody simulations are used at the beginning of the algorithm, and are gradually added; this aids to reduce the computational load at the beginning of the algorithm when trying to find an adequate solution. In this example, the SA algorithm parameters are $\rho = 0.8$, $cf = 0.999$ and $n_i = 500$; the algorithm is run for 600,000 iterations and the parameters are reset and the optimization is re-run. This aids in diversifying the solution to ensure that the solution is not trapped in a local optimum. The PSO algorithm is run every 100 iterations of the SA algorithm, with 15 particles. The PSO inertia parameters, c_1, c_2 and c_3 are set to 0.8, 0.4 and 1.2, respectively. The PSO algorithm terminates after the objective function values for all particles lie within 0.1% of the best-found value. The SA+PSO algorithm is terminated by the user; while this is not ideal for comparing the computational performance, the objective is to obtain a high-quality solution that satisfies the modeller. Admittedly, the computational performance of the method is hindered by the generalized modelling methodology, which requires the use of maps and expression tree data structures to dynamically evaluate the current solution. If one is concerned with the computational performance, it is possible

to adapt the optimization methods proposed with specially tailored models of the mining complex that avoid these data structures.

Figure 6–6 (left) shows the results from the deterministic-equivalent design. With the exception of the first year, the deterministic-equivalent design (red) indicates that the optimizer is able to consistently feed materials to the mill up to its capacity for the first 15 years (Fig. 6–6A). Afterwards, the design is capable of meeting the minimum bounds that are imposed on the mill's operating hours. In a pure NPV maximization approach, where the lower bound on mill hours does not exist, the optimizer would choose to send fewer quantities to the mill. This result can be seen after year 28, where the number of operating hours is drastically reduced. This result, however, is not ideal for long periods of time because of indirect costs that are incurred when not fully utilizing the mill [101]. A minimum bound penalty is used to approximate these indirect costs when production dips below a specified threshold. As a result, the optimizer obeys the lower bound constraint by blending low- and high-grade materials. It is possible to test the sensitivity and risk associated with the deterministic-equivalent design using a set of geological simulations by taking the deterministic-equivalent decision variables (production schedule, destination policies and capital expenditures) and testing how the simulations react to the design. Figure 6–6A also shows the exceedance probabilities (P-10, P-50, P-90), which are used to quantify the risk for the design. Unlike what the estimated model indicates with the deterministic design, there is a large amount of risk associated with the use of the mill. The risk analysis indicates that there are large fluctuations in the utilization of the mill during the first 15 years of production (where the estimated model indicates

it is filled to capacity). After year 15, there aren't sufficient quantities of material to feed the mill using the deterministic-equivalent destination policy (defined in terms of recoverable copper per mill hour). This indicates that the deterministic-equivalent design is a knife's edge solution, where it perform very well for the estimated model, but does not perform well when the uncertainty related to spatial locations, volumes and metal quantities are considered. Figure 6–6B shows the number of hours used by the shovels. In this example, the optimizer has chosen to use three shovels, and replaces the shovels every 10 years. It is clear that the optimizer is capable of staying within the bounds of their production capabilities for the life of the mine. Similarly, Fig. 6–6C shows the number of truck hours for the deterministic design. It is noted that as the mine extracts increasingly deep material, the number of trucks increases accordingly to compensate for the increase in cycle times to bring material to surface.

A stochastic optimizer considers all geological simulations simultaneously to generate a single production schedule, destination policy and capital expenditure strategy that manages the risk associated with uncertainty. Figure 6–6D shows the risk profiles for the stochastic design for mill utilization. The optimizer is capable of fully utilizing the mill during the first 13 years, and the utilization begins to decline thereafter. Unlike the deterministic-equivalent solution (Fig. 6–6A), the stochastic design is able to provide enough material to the mill to satisfy the minimum bound, which is a much more practical solution. Additionally, it is noted that the stochastic optimizer successfully manages the distribution of risk over time; at the beginning of the mine life, the distance between the P-10 and P-90 profiles is relatively small, with an average of 186 hours over the first 10 years, which implies that the stochastic

design has a high probability of consistently feeding the mill and helps to guarantee early cash flows. Later in the mine life, the distance between the P-10 and P-90 profiles widens up to an average of 372 hours for the last 10 years, which is a result of the optimizer deferring the extraction of riskier material through time using the geological risk discount rate. This is an improvement over the risk analysis from the deterministic-equivalent design that randomly distributes risk over time, which is indicated by an average of 283 and 391 hours between the P-10 and P-90 values for the first and last 10 years, respectively. Not only is the stochastic design providing a more consistent feed to the mill, but the risk associated with the materials sent is also reduced. The shovel and truck production rates (Fig. 6–6E and F, respectively) for the stochastic design are similar to those of the deterministic-equivalent solution (Fig. 6–6B and C), however there are some minor differences. Notably, the stochastic optimizer chooses to extract more material between periods 12 and 16, which is indicated by the slight increase in shovel and truck hours. This is a result of the optimizer needing to uncover more material during these periods in order to provide enough material to remain above the mill's minimum bound in the later years of the mine's life. In order to achieve this, the optimizer decides to purchase an additional truck in year 12.

Given the inability to consistently feed the mill up to the desired capacity using the deterministic-equivalent design, and the high risk associated with the quantities sent, a risk analysis of the deterministic solution indicates a 1.7% lower NPV than the deterministic-equivalent design originally indicated (based on the P-50 value).

This minor impact on the NPV, given the inconsistent feed, is a result of the simulations having a higher metal content above the cut-off grade than what the estimated (smoothed) model indicates. The stochastic design is not only able to provide a consistent feed of material to the mill and reduce the risk associated with the quantities sent, but is also able to attain a 5.7% increase in NPV over the deterministic design. While this is not a drastic increase, it is also necessary to also consider the fact that the mill is consistently fed with materials, thus does not incur the large overhead and opportunity costs that are incurred by under-utilizing the mill.

6.6 Conclusions

This chapter proposes a global optimization modelling and optimization methodology for open pit mining complexes that aims to mange risk in the production and processing of mined materials, and, additionally, the capital expenditures required to maximize the value of the operation. The generalized and flexible modelling procedure that is outlined permits the ability to model very large mining complexes with a high-degree of detail, including non-linearities that are typically ignored in practice due to computational and theoretical limitations of conventional mathematical optimizers. Previous formulations for mine production scheduling with uncertainty have attempted to manage risk around a fixed target, such as mine production and mill capacities; this a priori definition of capacities or bottlenecks leads to sub-optimal use of both the depletable natural resource and the large investments required to produce and process material. By incorporating capital expenditure decisions in the model, the optimizer is able to control aspects such as optimal mine production rates and the timing of opening new processing streams that has previously been ignored

in mine production scheduling models. The proposed method uses a combination of simulated annealing and particle swarm optimization to generate multi-mine production schedules, destination policies, capital expenditure strategies, and the use of the available processing streams in order to maximize the performance of the mining complex.

The method is tested on a large-scale, real-world copper mining complex, provided by an industrial partner. Experimental results indicate that the optimizer is able to successfully create a production schedule, destination policy and capital expenditure strategy that manages risk associated with fully utilizing a mill, and also simultaneously decides the timing of purchases or replacement of shovels and trucks. The result is a risk-based design with a 5.7% higher NPV than a deterministic-equivalent design that does not consider risk. Moreover, the stochastic design ensures the smooth operation of the mill. It is noted that, in this example, the complexity of the optimization formulation surpasses the capabilities of commercially available production scheduling methods, thus a comparison cannot be provided at the time of this study. Future work will seek to test the method in cases where both multi-mine production rates and sizing of the mill are considered, and to develop new methods and models that can be used as a basis of comparison for this method.

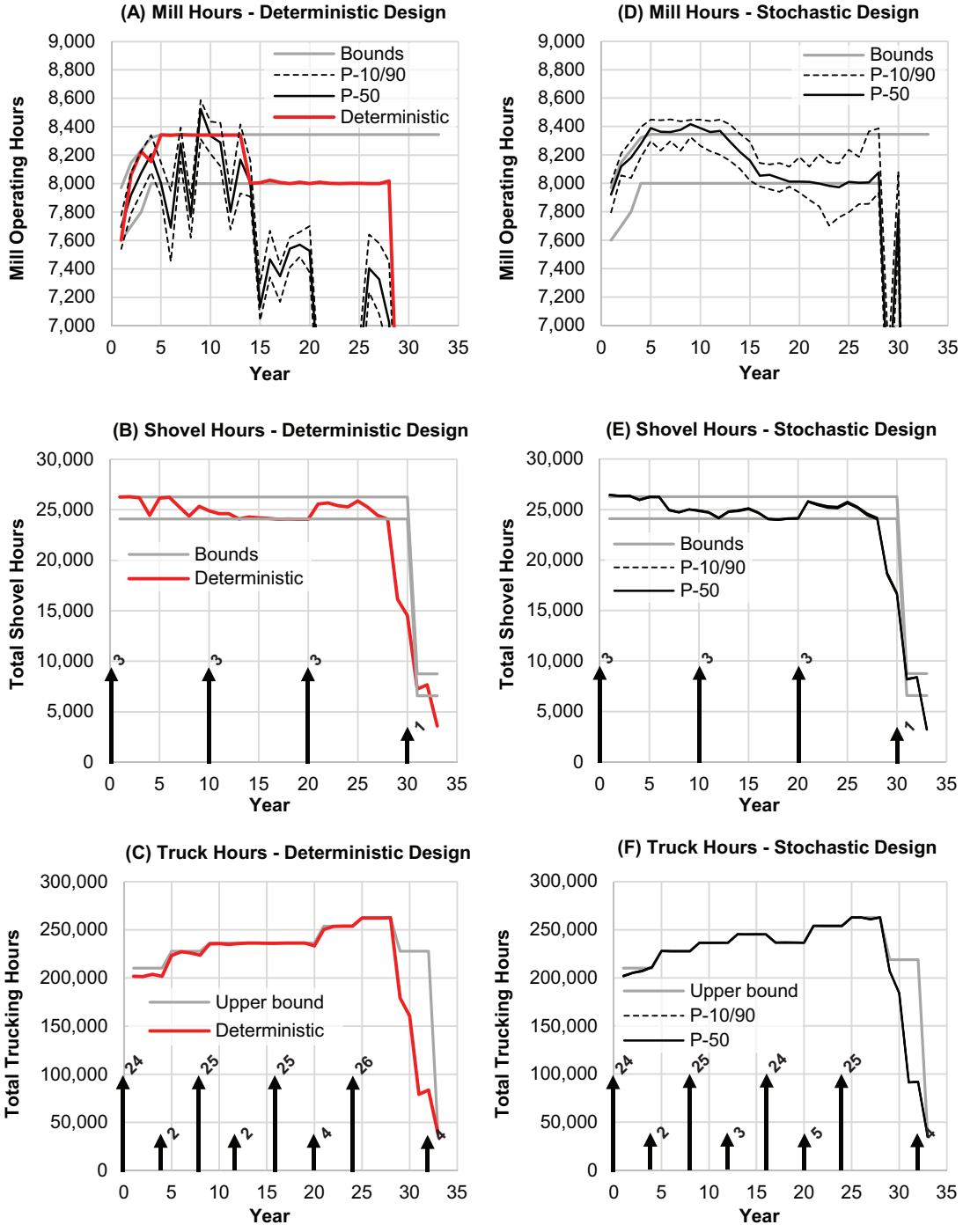

Figure 6–6: Comparison of risk profiles for the deterministic-equivalent and stochastic copper mining complex designs.

CHAPTER 7
Conclusions and Future Work

Integrating uncertainty into mine design and production scheduling is a recent paradigm shift that has consistently demonstrated the ability to not only drastically reduce the levels of risk of not meeting production targets, thus providing reliable financial forecasts, but also increases the economic value of the mining operation. This counter-intuitive result is related to the ability of a stochastic optimizer to understand the variability in the high- and low-grades of mined materials, and exploit it to treat the upside potential of the resource separately from its downside risk. Traditional, deterministic optimization is unable to understand and exploit this variability. This thesis provides five major contributions to this new stochastic optimization paradigm for mine planning. First, a new, simultaneous stochastic global optimization method is developed that holistically optimizes production schedules, destination policies, processing streams and capital expenditures. Existing methods require decomposing this global optimization model into sub-problems, such ultimately lead to sub-optimal designs. Second, a unified modelling approach is created that permits the design of intricate models to represent the flow of materials from the mines through to the final customers, in addition to the non-linear stockpiling and transformations that occur in the various processing streams. This can be used to apply the concepts developed in this thesis to a wide variety of mining complexes, regardless of the commodities produced or the geographical and geological conditions.

Third, and as a result of the second contribution, a new approach is used that enables the economic valuation of the products sold, rather than the materials mined. Existing models focus on the economic values of blocks, which is calculated in isolation from other blocks that may be mined in the same period, and assume prior knowledge of the optimal processing stream. Fourth, computationally efficient solvers are developed using metaheuristics. Previous work in stochastic optimization for mine production scheduling has focused on a single metaheuristic. The solvers developed in this thesis uses a unique combination of two metaheuristics, which require developing new and computationally efficient strategies to change a design in a holistic optimizer. Finally, full-field tests are used to assess the performance of the proposed methods. The results and methods from this work are compared to an industry-standard method, when possible, and compared to the deterministic-equivalent of the proposed method. The results consistently indicate that the risk-based designs can substantially reduce the risk in terms of product specifications or capacity targets, and, when simultaneously optimizing the production schedule, also achieve a higher net present value.

The impact of pushback design on stochastic production scheduling is discussed in detail by Albor and Dimitrakopoulos [2], where the authors note that the performance of the production schedule is heavily reliant on the pushback design. Chapter 3 proposes two formulations that integrate geological uncertainty in pushback design for mining complexes with multiple processing streams. In this work, material types are used to define candidate processing streams; by using a set of geological

simulations that have simulated material types, the uncertainty related to the spatial locations and volumetric quantities can play a critical role in pushback design, hence the performance of a production schedule. This method focuses on modifying existing pushback designs to have similar tonnages of materials going to the various destinations, but with less risk than the original designs. Through an application at the Escondida Norte mine, the stochastic pushback design is found to have an objective function value (measured in risk of having target tonnages for each processing stream) that is 61% lower than the original pushback design that is generated with BHP Billiton's Blasor software. This drastic reduction is a direct result of the optimizer being able to understand and redistribute risk in the design. There are, however, several limitations of this method. First, the method is assumed to be used in a traditional, sequential mine design framework that optimizes pushbacks prior to the production schedule, resulting in a locally optimal pushback design. Moreover, the destinations of blocks is defined a priori, based on a marginal cut-off grade. While this is typical assumption for a traditional framework, it does not accurately accommodate the production capacity and blending constraints that are considered in production scheduling models.

The ability to satisfy capacity and blending constraints is of critical critical importance for many mining operations. A sequential optimization framework ignores these constraints during the definition of the ultimate pit limit and phase design. In extreme cases, such as operations that are controlled by the blending quality, this sequential methodology is leads to infeasible and impractical mine designs that undervalue the resource. Stone et al. [154] propose an integrated approach that starts

with production scheduling, and is thus able to consider capacity and blending constraints. The schedule is then used to dictate the design of the ultimate pit and pushbacks. This approach is much more suitable for the global optimization of mining complexes, where many interrelated aspects are optimized simultaneously, and forms the basis of the remaining chapters of this thesis.

Chapter 4 addresses the optimization of mineral resource supply chains with uncertainty. In this context, a mining complex is a form of a supply chain that generates, treats and distributes products to a set of customers via a set of distribution methods. At each location, there may be technical or contractual obligations on the quality of the products received or produced. This chapter investigates questions related to how to optimize the destination policies that define where mined materials are sent in the presence of supply uncertainty, and how maximize the performance of the supply chain using the various processing streams. A destination policy is a generalization of a cut-off grade policy that is more suitable for mining complexes, and may be used to define, on an operational level, where to send materials to achieve a target blend, without assuming perfect knowledge of the material that will be mined in the future. It is noted that there is a wide diversity in types of mineral resource supply chains. For example, the iron ore value chains in Western Australia consider mining, stockpiling, transporting by rail, blending at a port and shipping to customers. Alternatively, a nickel value chain may only need to consider mining, milling, concentrating, smelting and refining their products to sell to a contracted customer. As a result of this wide diversity, a unified modelling methodology is developed to model the many different mining complexes or supply chains, which

can incorporate a high degree of detail in the optimization models. The method is tested at the Onça Puma nickel laterite blending operation, Brazil. Nickel laterites are known to have a high degree of variability, not only in the quantities of nickel and by-products, but also in the volumes and spatial locations of the valuable saprolite material. A deterministic solution is first generated using the proposed method and an estimated (kriged) orebody model. A risk analysis of the deterministic destination policies indicates that the solution results in infeasible blending ratios at the processing plant. This highlights the need for a new, risk-based framework that is able to consider the impacts that variability and uncertainty have on the performance of the value chain. The stochastic optimizer is able to generate a robust destination policy that consistently meets the stringent blending requirements and production targets at the processing plant with a minimal amount of risk.

Chapter 5 expands on the unified modelling and optimization methodology developed in Chapter 4 to incorporate multi-mine production scheduling. As a result, the optimizer has control over the production, or supply, of materials, the destination policies that define where material is sent, and the use of the processing streams. It is noted that in this integrated approach, there is no need to first generate a ultimate pit limit or a phase design, although they may be used as a starting design for the optimizer to modify. The method is demonstrated through an application for a copper-gold mine. In this case study, the mill and various leach pads use nonlinear recovery curves that are functions of the head grades of the incoming materials (i.e. recoveries are not assumed to be known for individual blocks). These recovery curves are interesting in the sense that they provides an opportunity to blend

low-recovery material, which would have a unprofitable if treated in isolation, with higher grade material, which would have a high recovery in isolation. By blending these blocks together, it is possible to simultaneously increase the size of the resource and its value — an aspect that block-based, linear optimizers that are commercially available neglect to consider. A design is first generated using Whittle, an industry-standard production scheduling tool, using an E-type orebody model. An alternative design is generated using the same orebody model and the deterministic equivalent of the proposed stochastic optimizer to provide a consistent basis of comparison. The deterministic-equivalent design that has a 4.1% higher NPV than the Whittle-based design, despite using the same orebody model as input. The deterministic design, however, ignores material type and grade uncertainty, thus is unable to satisfy production targets when tested with a set of geological simulations. A stochastic design is not only able to generate a schedule that has a 6% higher NPV than its deterministic equivalent (compared using the P-50 risk profiles for both), but also is able to meet production targets and drastically reduce the risk associated with meeting those targets over the life of the mine. These improvements are a direct result of the fact that a stochastic optimizer understands the spatial and volumetric variability of materials, and is also able to separately control the upside potential of metal content variability from the downside risk.

Previous work in stochastic optimization for mine production schedules has focused on meeting pre-defined production targets and reducing the risk associated with the material processed. By defining these capacities a priori, the stochastic solution may not be optimal because it fails to consider alternative targets. Chapter

6 continues to develop on the previous global optimization methodology for open pit mining complexes by including the ability to increase or decrease capacities (i.e. production targets) through capital expenditures. By simultaneously optimizing the capital expenditures in addition to production schedules, destination policies and processing streams, it is possible to consider the impact that timing and quantity of investment may have on the system. This capability is challenging for algorithmic optimizers (i.e. metaheuristics) because of the drastic changes to bottlenecks that can be made with a single capital injection, which ultimately leads to a solution that cycles around a local optimum and doesn't necessarily converge. In order to address this challenge, new classes of perturbations are proposed to create large changes to the production schedule that can quickly meet the fluctuating capacities. The method is tested using a case study for a copper mining complex from an industrial partner. In this example, the optimizer has full control of when to purchase or replace shovels and trucks, thus the optimizer is essentially optimizing mine production rates simultaneously with the production schedule, destination policies and the stockpiles. This model is interesting because of the fact that shovel production rates and truck cycle times vary with ore and waste classification and spatial location in the deposit. Existing deterministic, commercially available production scheduling software are unable to model these complexities. The results indicate the the stochastic optimizer outperforms the deterministic optimizer in its ability to utilize the mill and reduces the risk of not meeting mill production targets — a critical factor for a low-grade deposit. Additionally, the optimizer generates a 5.7% higher

NPV than the deterministic design, when comparing the P-50 values from both risk profiles.

Like any method, there are strengths and weaknesses that need to be weighed when choosing this approach. While the proposed method is extremely flexible and permits a high degree of detail in modelling, this comes with a computational overhead that is a result of the data structures required to store information (e.g. maps and expression trees). Additionally, by using a generic modelling approach, there is little opportunity to target aspects of the algorithm to help it converge on an optimum faster, because it is assumed that the models contain very little structure (aside from the scheduling and material flow constraints). Most importantly, despite the ability to optimize highly complex models that even commercial, deterministic optimizers cannot accommodate, it is often not possible to obtain a reliable reference to measure the quality of the solutions generated from the proposed method. Unfortunately, without other competing optimizers that can accommodate the same level of complexity, it is difficult to assess how far a solution is from a global optimum. Developing these new methods, which may be in the form of new solution perturbation methods or entirely new algorithms, may lead to obtain a better understanding of the convergence properties of the methods proposed in this thesis.

Another limitation of the proposed methods in this thesis is related to the destination policies that are generated using clusters. The purpose of using clusters is to convert a continuous, potentially multi-variate, distribution into discrete decision units that define where materials with similar attributes (e.g. grades) are sent. One must be aware of the impacts that the number of clusters can have on the quality of

the solutions that are generated. If too few clusters are used, the distribution may not be adequately discretized to provide enough granularity to define a destination policy. Conversely, in the case of stochastic optimizers, too many clusters may result in the optimizer being able to over-fit the destination policies for each input scenario, leading to a policy that is not robust to uncertainty. It may be possible to use alternative probabilistic classification methods, such as fuzzy c-means clustering, to overcome these challenges.

There are many opportunities for future research that are a result of this work. The concept of destination policies can be easily used in a traditional, mathematical programming-based framework; these policies not only address a core issue of deciding where to send material under uncertainty, but also provide an added benefit that they drastically reduce the number of decision variables, when compared to block-based destination decisions. It is noted that the proposed methods generate a single destination policy that is static through time; while they can accommodate other forms of uncertainty, such as metal price, throughput and costs, it would be useful to consider a multistage stochastic optimization formulation that allows the policies to vary over time as more information is revealed. This, however, might be computationally demanding for algorithmic optimizers, particularly when simultaneously optimizing mine production schedules. It may be possible to overcome these computational challenges through the use of distributed and GPU computing.

It is also noted that there are many scales from which to consider when optimizing mining complexes. This thesis primarily focuses on the long-term optimization, and thus does not consider operational factors such as drilling, blasting, grinding,

crushing and classification. Some of these operations can influence other downstream aspects of a mining complex, particularly related to processing costs. As computing capabilities increase, it may be possible to simultaneously optimize these short-term aspects in a long-term global optimizer, which will help to better account for the discrepancies between these two levels of planning that is seen in practice.

Incorporating capital expenditures to aid in the design of capacity constraints is also an interesting topic of future research. The case study presented in this thesis considers pre-defined options for shovels and trucks; naturally, it may be useful to consider many different types of equipment for purchase. For linear optimization models, it may be possible that duality theory and shadow prices may provide insight to how much the mining operation is willing to pay to provide a certain level of production, which would in turn, help to reduce the number of fleet purchasing options. Given the relationship between volume and variance for processed materials (e.g. Chapter 3), it is of interest to see the impact that increasing processing capacities has on both the NPV and the risk profiles for a given location. It is conceivable that in some cases, an optimizer might time a mill expansion in order to alter the risk profiles distributed through time. It may also be possible to combine these results with new, computationally efficient cutting plane methods [121] to obtain an optimal solution quickly.

Finally, as the environmental impact of mining operations becomes an increasingly important factor that decides whether or not a mine will open, it will be necessary to explicitly integrate aspects related to waste management (e.g. waste rock, tailings, slag) and rehabilitation into global optimization models. It is known that

environmental aspects, such as acid mine drainage, increase at exponential rates. Stochastic optimizers provide unique insight into these problems because they consider the true variability and uncertainty of the materials that are extracted, and provide an opportunity to reduce many negative environmental impacts that are a result of mining. Additionally, given social and political pressure to adopt carbon taxes and cap-and-trade systems, there will be a need to also integrate emissions and the complex taxation policies directly into the mine design optimization models.

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