THESIS ON
SECONDARY STRESSES
IN BRIDGE TRUSSES INDUCED BY
RIVETED JOINTS

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THESIS RESPECTFULLY SUBMITTED
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Munich, Bavaria, in a paper in which he drew the distinction between these stresses and Direct or Primary Stresses.

To exemplify the nature and occurence of Secondary Stresses, consider a column of area A, moment of inertia I, supporting a disc of weight W. (Plate I) If the centre of gravity of the disc and column be in the same straight line as in Fig.1, the unit compressive stress in the column equals $\frac{W}{A} = F$ = direct or primary stress.. This unit stress is uniform across the section of the column as designated by the stress diagram a b c d.

Next consider the disc to be placed eccentrically on the column, with its C of G a distance "X" from the C of G of the column, which is still supporting the disc W, giving a uniform stress F across the section as in a b c d. Now the eccentricity "X" of the weight W sets up a moment $M = W \cdot X$ giving rise to an additional or secondary stress $F_1 = \frac{M\,Y}{I}$, which gives a stress diagram of the form a b c_1 d_1.

Apply this analysis in the case of a riveted truss member, of area A, and Moment of inertia I, as shown in Fig III, Plate I. Suppose the member to be in tension under the action of a force P. The direct stress is therefore $\frac{P}{A} = F$, as represented by a b c d. In addition consider the member to be acted upon by a moment M at its extremity, due to the riveting of the member and the distortion of the truss. Then $M = \frac{I F_1}{Y}$ AND F_1 represents the secondary stress set up by the rigidity of the joint, and varies across the member from F_1 tension to F_1 compression, giving the new stress diagram a b $c_1 d_1$. Thus the maximum stress in the member which was a-d under the action of the direct force P has been increased to a-d_1 by the deformation of the truss.

The moment and therefore the secondary stress varies throughout the length of the member but is a maximum at the joints. (there is one exception to this in the case of a compression member of single curvature, which we do not consider) Therefore, if the moments at the joint can be determined, it would be possible to calculate the maximum secondary stress in the member. It will be seen that, in the case of a member in which the direct stress is small, the secondary stress may be conceived to greatly exceed
reverse
or even reserve the sign of the direct stress.

Thus the Primary or Direct stresses are those whose resultant passes through the C of G of the member and act uniform along its axis and across the section producing elongation or contraction of the member. The Secondary or Additional Stresses are those stresses produced by bending, shearing, or tortion of the members and are not uniform in their action along the member or across its section.

In determining the primary stresses in a bridge truss, it is assumed that the ends of the truss members are unrestrained or are free to take any position of revolution, about the panel points, required by the elastic deformation of the truss. This assumption indicates an ideal truss with friction-less pins, in which the axes of the truss members remain straight during the deformation and the stresses induced in a member are uniform throughout its length and section.

This condition is never realized in practice, as even in pin connected trusses there is a greater or less restraint of the joints due to frictional resistance. This frictional resistance varies considerably depending upon the size of the pin, or the surface in contact, and the stress on the member. If the frictional resistance of the surface of the pin exceeds the torsional moment caused by the member, the joints must be considered as rigid. The profession is beginning to realize that possibly too great value may be attached to the pin joint as a means of reducing the secondary stresses, and that, in the case of a poorly designed pin, a riveted joint might be used to greater advantage.

Now, since the truss is distorted under the action of the load, the several members tend to assume different angles relative to one another around their common panel points. This condition, occurring in the idea truss with friction less pins, is impossible in the riveted truss, as the members are retained in their initial relative positions by the rigidity of the joints. Therefore, due to this restraint, each member of the truss is bound to be deformed in assuming its new position, its axis will be distorted and bending or secondary stresses will be set up in addition to the direct or primary stress.

The problem which therefore presents itself is the calculation of the moments induced at the extremities of each member by the deflection of the truss, from which we can straight way obtain the secondary stresses by the single formula $M = \frac{I F}{Y}$.

The efforts of German scientists are largely responsible for the introduction and present development of the theory of secondary stresses. Considering the subject one of importance, the Munich polytechnic school offered a prize in 1877 for the solution of this problem. A "highly scientific and mathematical paper" entitled "The Calculation of Secondary Stresses which occur in simple trusses as a consequence of rigid joints" by Manderla, was awarded this prize, and was the first great advance towards its solution. A graphic solution was advanced by Professor Landsberg in 1885, in which he assumed only the chords to be restrained. Next Professor Muller Breslau developed an analytic solution and Professor Ritter introduced a graphic method.

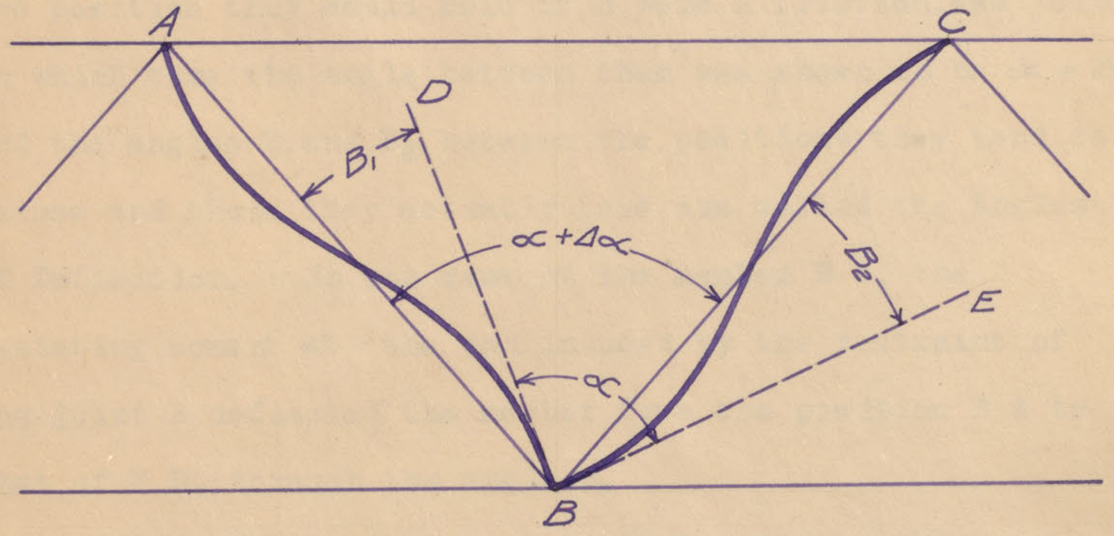
The first work published in English dealing with secondary stresses due to riveted joints was written by Isami Horoi of Tokio Imperial University, under the title of "Statically Indeterminate Stresses" in which he deals with the solution of the problem by the method of Least Work.

The next publication on the subject was by G.R. Grimm in 1908. In this work he introduces the methods of Manderla, Muller-Breslau, Ritter and Mohr, with examples of the calculated secondary stresses for different types of bridge trusses. In presenting the subject he remarks "The nature of the problem is such that it offered great obstacles to a solution; in fact this problem is one of the most difficult in technical mechanics, and although Manderla's solution is a very great step forward, the problem in all its aspects has not yet been completely solved."

In his report to the Royal Commission on the design of the Quebec Bridge, C.C. Schneider drew the attention of the Commission to the importance of secondary stresses in a structure of such magnitude. He further gave a brief outline of the theory, with the calculation of these stresses for the bottom chord members of the cantilever and anchor arms, assuming only the chords themselves to be restrained. This details the only available information on secondary stresses published in English.

The calculations entailed in the solution of secondary stresses are very extensive and laborious and in this Thesis Manderla's method will be indicated and solution executed by the integration of the equation of the elastic line.

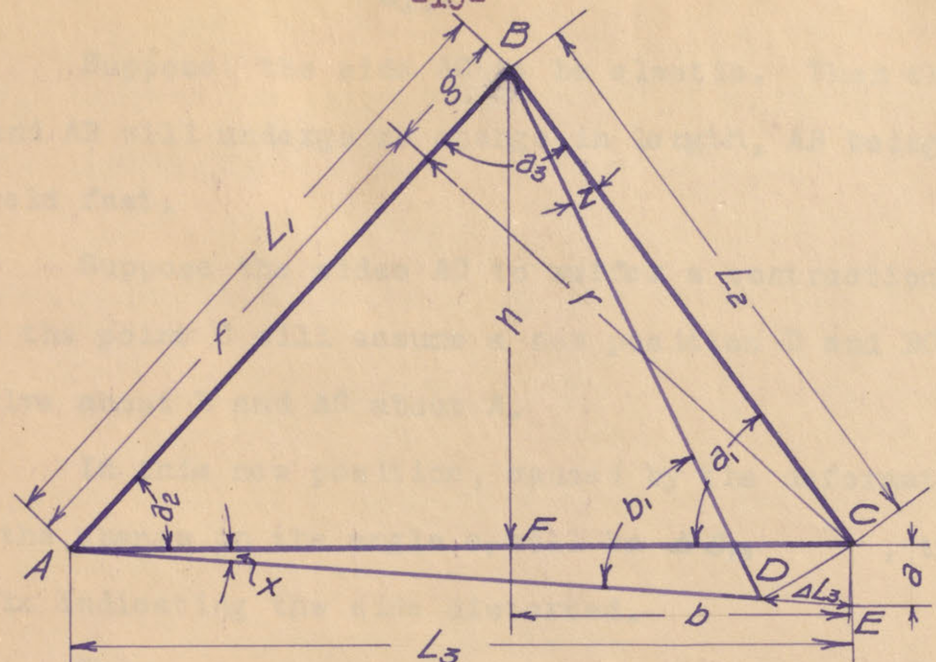
In the determination of secondary stresses, in the members of any truss, under the action of exterior forces, certain preliminary assumptions must be made. The exterior forces acting upon the truss, together with the deflections and deformations accompanying the same, must occur wholly within the plane of the truss. No torsion should exist. The load is considered as applied at the panel points, and each member is assumed to be of uniform section throughout. This final assumption does not take into account the sudden increase in section at the panel points due to the gusset plates. Manderla further assumes that in a riveted truss, the positions of the panel points are the same, under the influence of the exterior forces, as in a truss with frictionless joints.



Now considering two members of a truss A B and B C. Let the joint at B be considered a frictionless pin offering no resistance to the relative movement of the members under the deformation of the truss. Suppose the original angle between A B and B C to have been $\propto$. Under the influence of the deflection, and with complete freedom of rotation about B, the angle is changed to $\propto + \Delta\propto$.

Reversing the argument, consider the members A B and B C to be in complete restraint at B. Then in the deformation of the truss, the extremities of these members will retain their original angle $\propto$ to one another, but will be distorted throughout their length, their elastic lines being represented by the curved lines A B and B C, the tangents B D and B E to the elastic lines at B containing the angle α . The tendency of these members is to assume the position they would hold if B were a frictionless joint, in which case the angle between them was shown to be $\alpha + \Delta\alpha$ and the angles B_1 and B_2 between the positions they tend to assume and those they actually take are called the Angles of Deflection. In the case of the member B A, the resisting moment at the end induced by the restraint of the joint B deflected the member from the position B A to that of B D, through the angle B_1.

The problem then is: first to express the angle deflections in terms of the exterior forces, second to establish a relation between the angle deflections and the moments of restraint at the extremities of the members. This gives us the moments in terms of the exterior forces, from which the secondary stress can be readily deduced.



Consider a truss element ABC. Under the influence of the external forces the sides L_1, L_2 and L_3 of the element will be deformed, and altered in length due to their being strained. Now if the joints of the truss offer no resistance to the rotation of the members, their change in length will cause a corresponding chain in their contained angles.

The elastic deformation of each member will have an influence on the deformation of the angle a_1, and in this analysis, it is assumed that two of the members are non-elastic, the effect of the deforming of one being considered at a time. The effect of the straining of each member with regard to the deformation of the angle a_1, is thus separately determined and the summation of these for the three sides gives the resulting total deflection of the angle.

Suppose the side AC to be elastic. Then the sides BC and AB will undergo no change in length, AB being imagined as held fast.

Suppose the sides AC to suffer a contraction Then the point C will assume a new position D and BC must revolve about B and AC about A.

In this new position, caused by the deformation of AC, the change in the angle a_1 will be Δa_{13} , the suffix indicating the side distorted.

Then

$$\Delta a_{13} = a_1 - b_1$$

where b_1 is the new angle between the sides BC and AC.

Now $a_1 = 180^\circ - a_2 - a_3$

and $b_1 = 180^\circ - a_2 - a_3 - X + Z$

$\therefore \quad \Delta a_{13} = X - Z$

In DCE and BCF we have two similar triangles.

Now $Z = \frac{DC}{BC}$ for small angles

Again $a_1 = DCE$

Therefore $DC = \frac{a_1}{Cos. a_1}$

or $BC \times Z \times Cos. a_1 = a$

or $Z = \frac{a}{BC\ Cos} = \frac{a}{b} = \frac{\Delta L_3}{h}$

Again $\frac{a}{b} \times \frac{b}{L_3} = \frac{\Delta L_3 b}{h L_3}$. as $X = \frac{a}{L_3}$

$\therefore X = \frac{\Delta L_3 b}{h L_3}$

Therefore knowing Z and X

$$\Delta a_{13} = -\frac{\Delta L_3}{h} + \frac{\Delta L_3 b}{h L_3}.$$

If S_3 denotes the total stress in AC when the cross section is A_3 then $\frac{S_3}{A_3} = s_3 =$ unit stress.

Therefore the deformation of a_1 due to the alteration in AC

$$= \Delta a_{13} = -\frac{S_3}{EA_3} \times \frac{L_3 - b}{h} = -\frac{S_3}{EA_3} \text{Cot.}\, a_2 = -\frac{s_3}{E} \text{Cot.}\, a_2$$

Similarly if BC is considered elastic we get the change in the angle. a_1.

$$\Delta a_{12} = -\frac{s_2}{E} \text{Cot.}\, a_3$$

If AB only is elastic

$$\Delta a_{11} = \frac{\Delta L_1}{r} = \frac{S_1 L_1}{EA_1 r} = \frac{S_1}{EA_1}\left(\frac{f}{r} + \frac{g}{r}\right) = \frac{s_1}{E}(\text{Cot}\, a_2 + \text{Cot}\, a_3).$$

The total result of these deformations of the angle a_1 is found by the sum of these three results,

$$\Delta a_1 = \Delta a_{11} + \Delta a_{12} + \Delta a_{13}.$$

By proceeding in a similar manner with the angles a_2 and a_3 the other three equations for the deformation of the angles of the truss increment are derived.

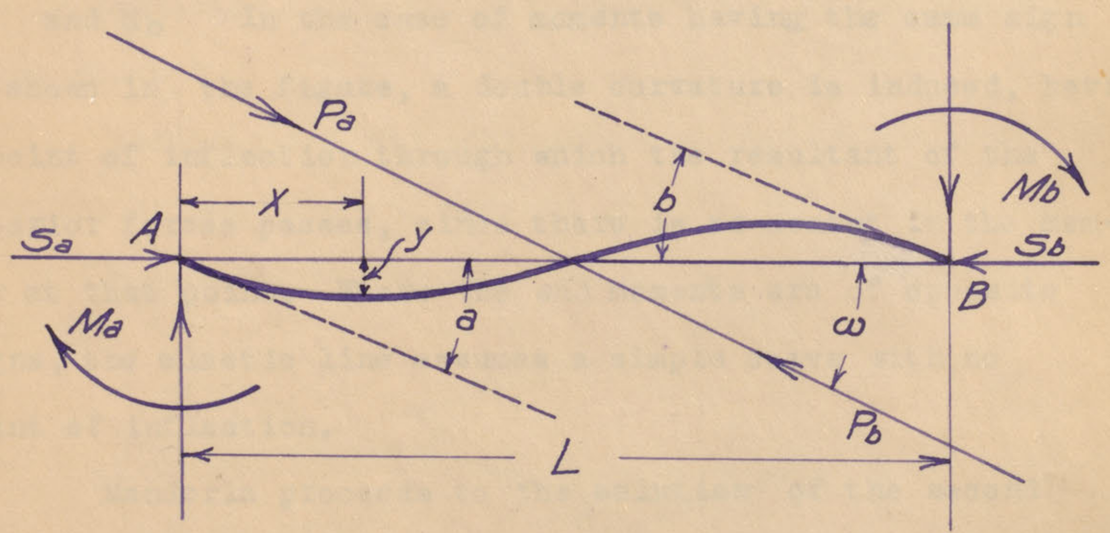
$$\Delta a_1 = \frac{s_1}{E}(\text{Cot}\, a_2 + \text{Cot}\, a_3) - \frac{s_2}{E}\text{Cot}\, a_3 - \frac{s_3}{E}\text{Cot}\, a_2$$

$$\Delta a_2 = \frac{s_2}{E}(\text{Cot}\, a_1 + \text{Cot}\, a_3) - \frac{s_1}{E}\text{Cot}\, a_3 - \frac{s_3}{E}\text{Cot}\, a_1$$

$$\Delta a_3 = \frac{s_3}{E}(\text{Cot}\, a_2 + \text{Cot}\, a_1) - \frac{s_2}{E}\text{Cot}\, a_1 - \frac{s_1}{E}\text{Cot}\, a_2$$

These are the fundamental equations in the solution of secondary stresses and are the basis of both Manderla's and the Least Work methods.

Having considered the behaviour of the members relatively to one another from the point of their combined action as a structure, the next step is the investigation of a single member, its deformation and the analysis of the forces acting upon it.



Take the member AB. Considering it entirely independant of the truss, let the resultant of all the external forces imposed upon it be represented by P_a, acting at an angle with its chord. P_a can be resolved into three forces. A direct force S_a, a transverse force Q_a and a moment M_a. The combined action of these forces causes the member to assume the double curvature as shown by its elastic line. Due to the equilibrium of the member in the truss,

$$S_a = S_b, \quad Q_a = Q_b \text{ and } \frac{M_a + M_b}{L} = Q_b$$

The transverse force Q is uniform throughout the length of the member, as there is no external force acting between its extremities.

The form assumed by the elastic line of the member depends upon the magnitude and direction of the end moments M_a and M_b In the case of moments having the same sign as shown in the figure, a double curvature is induced, having a point of inflection through which the resultant of the exterior forces passes, since there is no moment in the member at that point. Where the end moments are of opposite signs, the elastic line assumes a simple curve with no point of inflection.

Manderla proceeds to the solution of the second step in the problem, namely, the establishing of a relation between the moments at the extremities of the member and the deflection angles, from the equation of the elastic line,

$$\frac{d^2y}{dx^2} = -\frac{M_x}{EI}.$$

Where M_x,= a moment at any point of the axis of the member.

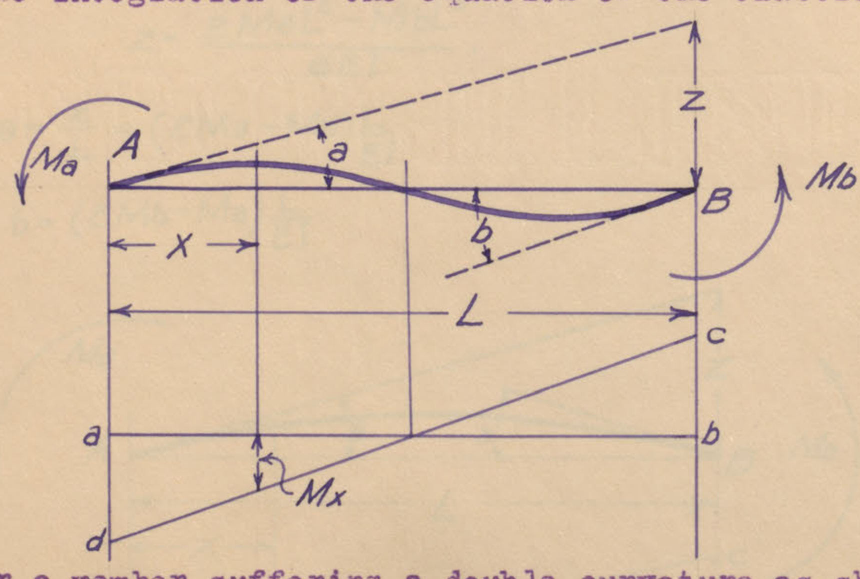
Considering all the forces acting on the member,

$$M_x = +Sy - Q_a x + M_a.$$

From this he obtained an expression of the bending moments in terms of the deflection angles using hyperbolic functions for the intergrations. Expressing the deformation angles in terms of the original angles, he found the result by trial.

Generally it is assumed that in well designed compression members, the tension members not being so important, where ample allowance has been made against buckling, the value of the lever arm Y is so small that the moment S Y can be neglected, together with the effect of the transverse force Q

The simplest and most direct method of obtaining the relation between the end moments and the deflection lines is by the integration of the equation of the elastic line.



Consider a member suffering a double curvature as shown, neglecting all other forces but the end moments M_a and M_b. Representing these moments graphically $M_a = ad$ and $M_b = cb$.

At any point X on the axis of the member

$$M_x = M_a - (M_a + M_b)\frac{x}{L}.$$

$$\text{Now } \frac{d^2y}{dx^2} = \frac{M_x}{EI} = \frac{1}{EI}\left[M_a - (M_a + M_b)\frac{x}{L}\right].$$

$$\frac{dy}{dx} = \frac{1}{EI}\left[M_a x - (M_a + M_b)\frac{x^2}{2L}\right] + C$$

when X = 0 $\quad \frac{dy}{dx} = 0 \quad \therefore C = 0.$

$$y = \frac{1}{EI}\left[\frac{M_a X^2}{2} - (M_a + M_b)\frac{X^3}{6L}\right] + C_1$$

when X = 0 Y = 0 $\quad \therefore C_1 = 0$

X = L Y = Z

Where Z = total displacement of the point B due to resisting moment.

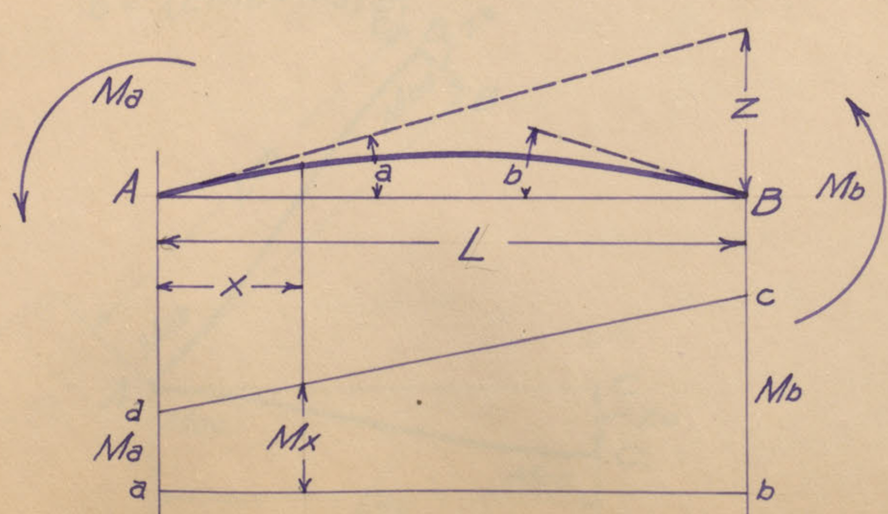
$$z = \frac{1}{EI}\left(\frac{M_a L^2}{2} - \frac{M_a L^3}{6L} - \frac{M_b L^3}{6L}\right).$$

$$z = \frac{2M_a L^2 - M_b L^2}{6EI}.$$

Now

$$a = \frac{z}{L} = (2M_a - M_b)\frac{L}{EI}$$

$$b = (2M_b - M_a)\frac{L}{EI}.$$



Applying this method to the case of a member having end moments of an opposite sign, the curve assumed by the elastic line is a single one with no point of inflection.

The bending moment on the axis at X = Mx = $M_a + (M_b - M_a)\frac{x}{L}$.

$$\frac{d^2y}{dx^2} = \left[M_a + (M_b - M_a)\frac{x}{L}\right]\frac{1}{EI}.$$

$$\frac{dy}{dx} = \frac{1}{EI}\left[M_a x + (M_b - M_a)\frac{x^2}{2L}\right] + C.$$

When X = 0 $\quad \frac{dy}{dx} = 0. \quad \therefore C = 0.$

$$y = \frac{1}{EI}\left[\frac{M_a x^2}{2} + (M_b - M_a)\frac{x^3}{6L}\right] + C_1$$

When X = 0, y = 0 $\qquad C_1 = 0$

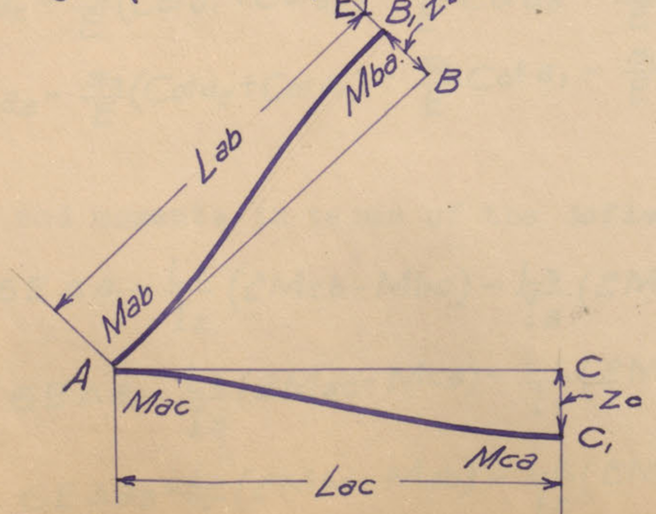
X = L y = Z

$$z = \frac{1}{EI}\left(\frac{M_a L^2}{2} + \frac{M_b L^3}{6L} - \frac{M_a L^3}{6L}\right)$$

$$z = (2M_a + M_b)\frac{L^2}{EI}.$$

Now $\qquad a = \frac{z}{L} = (2M_a + M_b)\frac{L}{EI}$

and $\qquad b = (2M_b + M_a)\frac{L}{EI}$



In finding the deflections in terms of the bending moments, the formulae for the case of moments with the same sign is ~~equally~~ generally used. Keeping to this formula and maintaining the same order round the various truss elements, the signs will take care of themselves.

Applying the equation to the members AB and AC at the joint A

$$\Delta BAC = \frac{Zc}{Lac} - \frac{Zb}{Lab}.$$

Therefore $6E\Delta BAC = \frac{Lac}{Iac}(2Mac - Mca) - \frac{Lab}{Iab}(2Mab - Mba).$

Summing up the results of the theoretical investigation of the secondary stresses of the truss elements ABC due to rigidity of its joints, the relations are:-

For the deflection angles in terms of the exterior forces.

$$\Delta a_1 = \frac{S_1}{E}(Cot\, a_2 + Cot\, a_3) - \frac{S_2}{E} Cot\, a_3 - \frac{S_3}{E} Cot.\, a_2$$

$$\Delta a_2 = \frac{S_2}{E}(Cot\, a_1 + Cot\, a_3) - \frac{S_1}{E} Cot\, a_3 - \frac{S_3}{E} Cot\, a_1$$

$$\Delta a_3 = \frac{S_3}{E}(Cot\, a_2 + Cot\, a_1) - \frac{S_2}{E} Cot\, a_1 - \frac{S_1}{E} Cot\, a_2$$

For the end moments in terms of the deflection angles.

1. $6E\Delta a_1 = \frac{L_2}{I_2}(2Mcb - Mbc) - \frac{L_3}{I_3}(2Mca - Mac).$

2. $6E\Delta a_2 = \frac{L_3}{I_3}(2Mac - Mca) - \frac{L_1}{I_1}(2Mab - Mba).$

3. $6E\Delta a_3 = \frac{L_1}{I_1}(2Mba - Mab) - \frac{L_2}{I_2}(2Mbc - Mcb).$

Further by the equilibrium of each joint $\Sigma M = 0$, therefore

$$4.\quad M_{cb} + M_{ca} = 0$$

$$5.\quad M_{ac} + M_{ab} = 0$$

$$6.\quad M_{ba} + M_{bc} = 0$$

Thus we have as many equations as unknown moments in the truss element, and by the solutions of these six equations the required end moments are obtained and the theoretical determination of the secondary stresses is accomplished.

This method was applied in the determination of the theoretical secondary stresses occurring in the Warren Type Truss shown in Plates II and IX, Fig.I for a load of 13,050 lbs. at the panel points D and F as shown. In designating the moments, strain, or other observation on a member, the first letter indicates the end of the member at which they occur.

- Compression + Tension.

The following table gives the direct stresses and general information of the action of the truss under this load.

Member	Length	M.of I.	Area	Tot.Stress	Unit Str.	Elong'n
AB	31.88"	1.28	2.795	-17,339.8	-6203.8	-.006,592
BD	42.0 "	1.28	2.795	-22,837.5	-8170.8	-.011,439
DF	42.0 "	1.28	2.795	-34,256.2	-12256.2	-.017,158
CE	42.0 "	1.28	2.795	34,256.2	12256.2	.017,158
AC	42.0 "	.77	2.21	11,418.7	5166.8	.007,233
BC	31.88"	.77	2.21	+17,399.8	+ 7810.7	+.008,300
DC	31.88"	.77	2.21	-17,339.8	- 7810.7	-.008,300
DE	31.88"	.77	2.21	0	0	0

Cot. BAC = .87492

Cot. ACB = .87492

Cot. ABC = .13402

The results of this table were found by considering the truss to have pin connected and frictionless joints.

To determine the angular changes which would occur with free and unrestrained joints, use the formulae previously determined.

Δ BAC = .0002603 (.13402 + .87492) + .0002067 (.13402)
- .0001722 (.87492) = .0001397

Δ ACB = -.0002067 (.13402 +.87492) - .0002603 (.13402)
- .0001722 (.87492) = -.0003939

Δ CBA = .0001722 (.87492 +.87492) + .0002067 (.87492)
- .0002603 (.87492) = .0002544

Δ DBC = - .0002603 (.87492 + .13402) - .0002603 (.13402)
+.0002723 (.87492) = - .0000592

Δ BCD = - .0002723 (.87492 + .87492) + .0002603 (.87492)
- .0002603 (.87492) = - .00047648

Δ CDB = .0002603 (.87492 + .13402) + .0002723 (.87492)
+.0002603 (.13402) = .0005356

Δ DCE = 0 - .0004085 (.87492) + .0002603 (.13402) = - .0003226

Δ CED = - .0002603 (.13402 +.87492) - 0 - .0004085 (.87492)
= - .0006200

Δ EDC = .0004085 (.87492 + .87492) - 0 + .0002603 (.87492)
= .0009425

Δ FDE = 0 - 0 + .0004085 (.87492) = .0003574

Δ DEF = - .0004085 (.87492 + .87492) = - .0007148

Applying the previous equations to all the angles successively, reading found each truss increment, in a clockwise direction, the signs taking care of themselves.

1. $$6\,E\;.0001397 = \frac{42}{.77}(2\,Mac - Mca) - \frac{31.88}{1.28}(2\,Mab - Mba)$$

2. $$6\,E\;-.0003939 = \frac{31.88}{.77}(2\,Mcb - Mbc) - \frac{42}{.77}(2\,Mca - Mac)$$

3. $$6\,E\;.0002544 = \frac{31.88}{1.28}(2\,Mba - Mab) - \frac{31.88}{.77}(2\,Mbc - Mcb)$$

4. $$6\,E\;-.0000592 = \frac{31.88}{.77}(2\,Mbc - Mcb) - \frac{42}{1.28}(2\,Mbd - Mdb)$$

5. $$6\,E\;-.0004764 = \frac{31.88}{.77}(2\,Mcd - Mdc) - \frac{31.88}{.77}(2\,Mcb - Mbc)$$

6. $$6\,E\;.0005356 = \frac{42}{1.28}(2\,Mdb - Mbd) - \frac{31.88}{.77}(2\,Mdc - Mcd)$$

7. $$6\,E\;-.0003226 = \frac{42}{1.28}(2\,Mce - Mec) - \frac{31.88}{.77}(2\,Mcd - Mdc)$$

8. $$6\,E\;-.0006200 = \frac{31.88}{.77}(2\,Med - Mde) - \frac{42}{1.28}(2\,Mec - Mce)$$

9. $$6\,E\;.0009425 = \frac{31.88}{.77}(2\,Mdc - Mcd) - \frac{31.88}{.77}(2\,Mde - Med)$$

10. $$6\,E\ .0003574 = \frac{31.88}{.77}\,(2\,Mde - Med) - \frac{42}{1.28}\,(2\,Mdf - Mfd)$$

11. $$6\,E\ -.00071480 = -\,\frac{63.76}{.77}\,(2\,Med - Mde)$$

12. $$Mac + Mab = 0$$

13. $$Mba + Mbc + Mbd = 0$$

14. $$Mca + Mcb + Mcd + Mce = 0$$

15. $$Mdb + Mdc + Mde + Mdf = 0$$

The solution of these fifteen equations gives the moments at the extremities of the members and from these the secondary stresses by $M = \frac{I\,F}{Y}$. when $Y = 1.5$"and 1.34".

Mab = -735.95	in/lbs.	Stress per sq. in.	= 862.4
Mba = -754.18	"	"	= 883.8
Mbd = 1518.98	"	"	=1780.
Mdb = 4696.13	"	"	=5503.2
Mdf = -2309.93	"	"	=2705.8
Mce = - 623.91	"	"	= 731.1
Mec = 2078.04	"	"	=2435.5
Mac = 735.95	"	"	=1280.5
Mca = 1339.66	"	"	=2330.8
Mbc = -764.80	"	"	=1330.7
Mcb = 41.85	"	"	= 72.82
Mdc = - 282.5	"	"	= 491.5

Mcd = -757.6 in/lbs. Stress per sq.in = 1318.2

Mde = -2103.7 " " " 3660.5

Med = -274.90 " " " 478.3

As this solution does not take into account the size of the gusset plates and their undoubted effect on the moment at the end of the members, another solution was made in which only the free length of the member was considered. That is, the distance from the out~~t~~ermost rivet of the gusset plates at the ends of the member was taken as the length of the member.

$$1 \quad \frac{31.25}{.77}(2\,Mac - Mcd) - \frac{18.13}{1.28}(2\,Mab - Mba) = 25{,}146.$$

$$2. \quad \frac{17.68}{.77}(2\,Mcb - Mbc) - \frac{40.59}{.77}(2\,Mca - Mac) = -70{,}902.$$

$$3. \quad \frac{18.13}{1.28}(2\,Mba - Mab) - \frac{17.68}{.77}(2\,Mbc - Mcb) = 45{,}792.$$

$$4. \quad \frac{17.68}{.77}(2\,Mbc - Mcb) - \frac{29.75}{1.28}(2\,Mbd - Mdb) = -10{,}656.$$

$$5. \quad \frac{17.68}{.77}(2\,Mcd - Mdc) - \frac{17.68}{.77}(2\,Mcb - Mbc) = -85{,}762.$$

$$6. \quad \frac{29.75}{1.28}(2\,Mdb - Mbd) - \frac{17.68}{.77}(2\,Mdc - Mcd) = 96{,}408.$$

$$7. \quad \frac{31.75}{1.28}(2\,Mce - Mec) - \frac{17.68}{.77}(2\,Mcd - Mdc) = -58{,}068$$

$$8. \quad \frac{20.68}{.77}(2\,Med - Mde) - \frac{31.75}{1.28}(2\,Mec - Mce) = -111{,}600.$$

9. $\frac{17.68}{.77}$ (2 Mdc - Mcd) - $\frac{20.68}{.77}$ (2 Mde - Med) = 169,650

10. $\frac{20.68}{.77}$ (2 Mde - Med) - $\frac{33}{1.28}$ (2 Mdf - Mfd) = 64,332

11. $\frac{41.36}{.77}$ (2 Med - Mde) = 128,664

12. Mac + Mab = 0

13. Mba + Mbc + Mbd = 0

14. Mca + Mcb + Mcd + Mce = 0

15. Mdb + Mdc + Mde + Mdf = 0

This gives the end moments and the secondary stresses to be :-

Mab =	- 879.30 in/lbs.	Stress per sq.in.	= 1030.4
Mba =	- 342.48 "	"	= 401.3
Mbd =	1214.3 "	"	= 1423.0
Mdb =	3822.8 "	"	= 4478.1
Mdf =	-2363.8 "	"	= 2770.
Mce =	- 801.3 "	"	= 939.
Mec =	3145.8 "	"	= 3683
Mac =	879.3 "	"	= 1530
Mca =	1633.7 "	"	= 2843.3
Mbc =	- 871.8 "	"	= 1517.4
Mcb =	131.44 "	"	= 228.7
Mdc =	673.6 "	"	= 1172.2
Mcd =	- 963.8 "	"	= 1681.9

Mde = -2132.6 in/lbs . Stress per sq.in. = 3711.5

Med = 130.9 " " = 227.9

These stresses if added to the calculated direct stress, will give the maximum stress occurring in the member. Their signs are plus for one side of the member and minus for the opposite side. A comparison of these calculated secondary stresses with the observed secondary stresses, together with their percentage of the calculated direct stresses is given in Table III.

THE EXPERIMENTAL DETERMINATION.

The experimental method of detemining the secondary stresses induced in truss members by the actual observation of their distortion,when subjected to definite exterior loads, is entirely new and hitherto undeveloped.

If a truss member (as shown in Fig.III, Plate I) be subjected to a direct stress and a bending moment M at its extremity, and if the elongation or contraction on the opposite sides of the member be observed by means of extensometers, then the unit stresses ad_1 and bc_1 could be determined, and hence the direct stress ad and the secondary stress dd_1 or cc_1.

The truss used in these experiments was of the Warren type, with 4 - 3'6" panels, length 14' c - c of bearings, depth 2'. It was constructed of 4"-7.5# and 4"-9.5# I beams as shown (Plates II and III) the end panel of the bottom chord and the web members being of the lighter metal. I beams were used to avoid eccentric connections and on account of their comparatively high moment of inertia at right angles to the plane of the truss providing against possible deflection in this direction. Their small moment of inertia in the plane of the truss gave ample opportunity to observe the secondary stresses. They were also more convenient in this instance than built up sections, and apply equally well for these experiments.

$\frac{1}{2}$" rivets and $\frac{1}{4}$" gusset plates were used throughout, with 7/16" splice plates at the splices at C and G in the bottom chord. 2 - $3\frac{1}{2}$" x $2\frac{1}{2}$" x $\frac{1}{4}$" x $9\frac{1}{2}$" angles with a $9\frac{1}{2}$" x 5/16" x 0' - $9\frac{1}{2}$" bearing plate composing the end bearings.

In preparing the truss, prior to its being placed in the testing machine, the neutral axes of the members were accurately layed off and the panel points determined. It should be stated that the panel points obtained by the intersection of the neutral axes of the members, check with remarkable accuracy. This is a testimony to the precise methods of construction of the Dominion Bridge Co., to whose kindness and generosity we are indebted for this truss and the possibility of carrying out these experiments.

Having determined the neutral axes, the positions of the ~~extremities~~ extensometers were marked out. These were placed as close to the panel points as the gusset plates would allow, it being considered that, though there undoubtedly was bending within the plate, the maximum bending moment in the member would occur just beyond the restraint of the last rivet in the gusset plate.

For experiments of this nature the Testing Laboratory of McGill University is probably the best and most completely equipped on the continent, and includes one of the three Wicksteed or Buck~~st~~toin machines in use in America. This machine, whose front and rear elevations are shown in Plates IV and V respectively, is particularly adapted for the loading required on the truss described.

The beam of the machine A, rests upon a knife edge B, as shown in Plates IV, V, VIII and X. The length of beam on one side of the knife edge greatly exceeds that on the opposite side, and the balance is maintained by the moving weight C. From the knife edge D, which is fixed to the beam is suspended the loading system E, F, G. The Plates VIII and X show the arrangement of the machine for two systems of loading in which H and H are two hydraulic rams exerting a force upon the end bearings of the truss, which in turn transfers this force to the plate G, thence it passes through the suspension bars F F to the head E and so to the knife edge D.

By advancing the moving weight C along the beam, we maintain the equilibrium of the beam and measure the load applied by the rams. Actually, the moving weight C is placed in a position requiring a certain force on the hydraulic rams to balance the beam. Fluid under pressure is then admitted to the rams from the accumulator I (Plate V) till the beam is balanced.

In order to accommodate the depth of the truss and the different arrangements used for the several systems of loading, it was found necessary to raise the rams some 18" by means of hardwood blocks as shown in accompanying plates.

Plates VIII and IX show the arrangement for a symetrical loading at the panel point F and D. A loading block was placed on the plate G, and upon this 2-7"-15# I beams, side by side. Upon these and directly under the predetermined panel points D and F, two steel loading blocks were placed. The white arrows indicate the pointsof application of the loads. The arrangement of the loading system was made with the greatest possible care, the whole being symetrically placed with regard to the centre of the truss and of the machine.

In the observation of strain due to secondary stress, the quantities observed are very small, requiring high accuracy in the method of determination. With the importance of this in mind, a simple but exceedingly efficient extensometre, made in the McGill labatory, was used. It was a modification of the Martens type and consisted of a cast steel extensometer bar 3/8" x 3/32" x 4-1/8" over all (Plate VII). The bar was made of comparatively large section to overcome the effects of temperature changes. One end was turned at right angles 5/16" and brought to a tempered knife edge as shown. From this knife edge 4" was accurately guaged to a precisely cut V shaped notch which was at right angles to the axis of the bar and parallel to the knife edge.

A steel diamond, having two highly tempered, and accurately ground knife edges was attached to one end of a steel rod 1/16" diameter, to whose other extremity was fastened a small mirror ½" square. One knife edge of this diamond rested in the notch of the bar, while the other edge, and that at the extremity of the bar were placed on the member 4" apart and held firmly in place by a spring S as shown in Plates VI and X. Any deformation of the member, elongation or contraction, altered the distance between the knife edges on the member, causing the diamond to rock and the mirror at the end of bar to revolve.

A telescope with a scale as shown in Plate VII recorded the deflection of the mirror, Each extensometer was calibrated, that is the distance of the scale from the mirror was determined at which a certain elongation of a member would give a definite reading on the scale: in this case the calibration was made so that one inch on the scale indicated .002" elongation of the member. This gave a very sensitive and satisfactory extensometer.

Eight extensometeres were numbered and calibrated in this way, wooden rods being cut the calibrated distance for each instrument.

In placing the extensometers on the members it was first thought that it would give the required results if they were placed on the front face of the member, near the edge. But as the secondary stress varies from zero at the axis to a maximum at the top fibre of the member, and as the knife edges of the diamonds were 7/16" long, the elongation of the speciman would vary over the 7/16". Now as the knife edge of the diamond and that of the bar are held parallel and as the elongation of the member varies over their width, therefore one or other knife edge must slip on the member, thus making a reliable or consistent result impossible. Comparative tests prove this to be true and the observations taken on the side were from 25% to 50% less than those taken at the top fibre.

Extensometers were therefore placed on the edge or extreme fibre of the member where there could be no possible variation of strain across the knife edges. By this arrangement, as shown in E and E_1 Plate VI , it was necessary to read the instruments by placing the telescopes parallel to the plane of the truss. This necessitated longer bars (8") supporting the mirrors to bring them clear of obstruction by the machine so they could be read. (See M, Plates VIII and X).

As stated previously, the elongation or contraction must be measured on opposite sides of the member, therefore the extensometers E and E_1 were arranged as shown in Plate VI and were placed accurately over one another, the mirror rods being parallel.

These two extensometers did not have exactly the same calibrated distance, and as they were but 3" apart it was very inconvenient to read them separately with two telescopes. It was therefore necessary to observe the two instruments with one telescope, having them reading on the same scale. To overcome the difference in calibration, the rod A (Plate VI) supporting the scale B and adjusted by the thumb screw C was provided with two aluminium collars D and D_1 , with adjusting screws. These collars were so arranged that, when observing the mirror of the greatest calibrated distance, the scale and rod A were moved back till the collar D which was set for the calibrated distance, came in contact with the rod support. Reading the other instrument, the scale was advanced, till the collar D_1 came in contact with the support. This allowed of the two instruments being read on the same scale.

In order to admit of both instruments being read by the telescope, the telescope was first sighted on the top mirror, then an aluminum disc F, machined so that its faces were absolutely parallel, was placed between the vertical adjusting screw G and its bearing, and given a few turns to insure its true setting. This disc was so guaged as to be just deep enough to bring the other mirror squarely into the field of view.

The height of the telescope was such as to bring its level half way between the mirrors of the instruments. The telescope stand was then set at approximately the calibrated distance from the mirrors and the aluminum collars on the scale rod used to make the final adjustment. The mirrors being trained on the scale, was set with their zero readings on a level with their centres. This was done by revolving them about their supports, to which they are held by friction by small aluminum clamps. Since their travel on the scale was so limited, this setting eliminates any possible error due to the obliquity of the line of sight. This combined arrangement of telescope and extensometers required much care in the setting, but when accurately done the results obtained warranted the trouble taken. Having considered the elimination of errors due to the construction and arrangement of the extensometers and telescope, the next possible source of error is that arising from the deflection of the truss under the loading. The telescope and scale, on which the readings are taken, are separate from the truss and extensometers. Due to the deflection of the truss, the extensometers nearest the end of the truss will move slightly relative to the scale. At the extreme end of the truss, the deflection under the first system of loading by $\delta = \Sigma \frac{pul}{E}$, taking account of the deflection of the 7" I beams is .16". Now if the plane of the mirror remained exactly at the same angle to the plane of the scale throughout this vertical movement, there would be no error in the readings due to such deflection, but the mirror might pass from the field of view. As it is the angle of the mirror to the plane of the scale changes due to the mirrors having revolved about the centre of the truss, the amount that the angle has changed being equal to the angle subtented by .16" at the centre of the truss. As the result is a comparative one, and the two mirrors would be moved the same amount, this error need not be recognized in determining the secondary stresses.

Another source of error which must be considered is that due to temperature changes. Although no special observations were made along these lines, it was discovered early in the experiment, that, to obtain a consistent result, the temperature must be maintained as nearly uniform as practical. To prevent local temperature effects, the truss was wholly protected from the direct rays of the sun. The first series of experiments was made with a symetrical loading of 13,050 lbs. at the top panel points F and D (Plates VIII and IX). 2 - 7" - 15lb. I beams were used, as previously described with loading blocks as shown to distribute the load uniformly to F and D. The hydraulic rams were carefully set, so that their centres coincided with the centres of the end bearings.

In order to avoid error in the results due to the depression of one end of the truss by unequal loading of the hydraulic rams, a very sensitive 18" level was placed directly in the centre, and over the panel point E. In loading the truss particular care was taken to keep the level bubble strictly in the centre. Further a plate guage K (Plate X) was used to balance the beam in exactly the same position for successive loadings.

Trial runs were made to check the stresses in corresponding members on opposite ends of the truss. These proved entirely symetrical, checking to within 800 lbs. in a members whose calculated total stress was 34,000 lbs. Tests were further made as to the stresses on opposite sides of the same members. These were not as satisfactory as the previous ones, but were within reasonable limits and were constant. This small variation in reading of the front from the back of the truss would not effect the secondary stresses, which are the comparative results of the distortion of the top and bottom of the member on the same side. It would to a slight extent effect the co-direct stress, or the direct stress measured at the same time as the secondary stress. The front of the truss was in all cases taken as the standard.

After setting the instruments on the members of the truss, it was found advisable to load the structure to 50% or 75% of the intended loading in order that the extensometers might work into position or limber up.

In setting the zero of reading, the weight of the truss was not considered. An initial load of 100 lbs. was taken as the zero reading in all cases. The load was applied in 2000 lb. increments to a maximum load of 13,050 lbs. at F and D. The load was then removed and the extensometers checked back to the zero reading, which they did in every case to within .00001 or .00002 of an inch.

The method of hydraulic loading of the Wicksteed machine, proved the ideal one for this work and with care, the load could be applied very steadily without shock, thus giving most uniform results. See Table I for the method of recording observations.

It was found inconvenient to observe more than two members at a time, but this did not influence the final results as exactly the same conditons were maintained for each run.

In addition to the determination of the secondary stresses, observations were made for the direct stresses.

The extensometers were placed upon the neutral axes of the members at their centres, as shown in Plate X on the member EF.

In order to determine the effect of secondary stresses in a cantilever, an unsymetrical loading of 26,100 lbs was concentrated at the panel point B by the arrangement as shown in Plate IX, Fig.2 and Plate X. The same methods and precautions were used as in the case of the symetrical loading. Observations were made for secondary and direct stresses in the same manner as before. More care was required in loading the truss as the pressure exerted by the two rams was not equal. In the case of the maximum load of 26,100 lbs, the ram at A caused a reaction of 16,312 lbs. and at K, 9,787½ lbs.

Tables II and IV give the total observed elongation or compression of the members under the maximum load in the two cases, together with the distance of the nearest knife edge of the extensometer from the panel point. By the formula $F = \frac{1}{L}$ E. (E = 30,000,000), these observations give F, the stress in the extreme fibre of the member due to its distortion. In the case of F being of the same sign on each side of the member, $\frac{Ft - Fb}{2}$ is the secondary stress induced, and $\frac{Ft - Fb}{2} + Fb$ = the co-direct stress, or the direct stress observed at the same time as the secondary stresses. (See Plate I, Fig. II and III).

It will be noticed, that in the case of the member DE (Case I) we have an elongation on one side and a compression on the opposite side of the member. This shows that the direct stress is very small, and that the secondary stresses have reversed the sign of the maximum stress.

By observing whether the greatest distortion occurs at the top or the bottom fibre and noting its sign, the form assumed by the elastic line of the member can be determined. In Plate IX, Fig. II and III, the heavy lines indicate the various positions into which the elastic lines are deflected by the loading shown, the dotted lines represent the positions given by the calculated moments.

In Tables III and IV, the comparisons of the results of observation and calculation are given for the symetrical and unsymetrical loading, for both the secondary and direct stresses.

Before entering upon a discussion of the comparison of the secondary stresses, it might be well to first consider the relation of the calculated to the observed direct stresses, and its significance. It is very evident from the figures of Tables III and IV, especially in the case of the cantilever, that the observed direct stresses and those determined by calculation are not the same. The explanation of this is in the form and style of the truss on which the experiments were made. A rivetted truss, with rigid joints, differs from a pin connected truss, with frictionless joints, not only in the fact that the rigidity of the joints set up secondary or bending stresses at the extremities of the member, but the fixture of the joints also alters the distribution of the direct stresses among the various members.

For example, in the case of the first system of loading, for exterior forces within certain limits, the chord member AC could be cut, and the truss would not collapse, as the resistance offered by the joint B would maintain the equilibrium. This shows that, for varying conditions, depending on the loading, the design of its end connections and its position in the truss, the actual direct stress in any member may be greater or less than that determined by calculation.

The joints having the heaviest connections and offering the greatest resistance are those at A,B C, and the corresponding points at the opposite end of the truss. It is therefore at these points that the greatest discrepancies between the theory and the practice might be expected to occur, Considering the members AC and CD in the first case (Plate IX, Fig.I). Due to the fact that the rigidity of the joint B absorbed part of the direct stress that, in a pin connected truss, would go to AC and CD, the observed direct stresses in these members were 3,900 lbs. per square inch, and 6,975 lbs. per square inch as compared with the calculated stress of 5,166.8 and 7,810.7 lbs. per square inch.

Again in the member DE, the calculated stress is zero, but the observed stress is 375 lbs. per square inch, induced by the restraint of the joints, and the distribution of the stresses thus introduced.

In the unsymetrical loading, the members KG and GF show this point very clearly, while the web members in general show the effect of the redistribution of stress.

It is therefore evident that proper allowance must be made for these conditions in determining the secondary stresses. In the calculation of these stresses, it will be remembered that the stresses considered as causing the elongation of the members, and hence the angle distortions, were the direct stresses obtained by calculation. Now since the actual direct stresses differ to a greater or less degree from the calculated, the secondary stresses obtained from the latter and those given by observation will vary correspondingly. It will be seen in Table III that 75% of the calculated secondary stresses are greater than the observed, and that the members in which the reverse is ~~through~~ true have a direct relation to the aforementioned joints. (R in Table III indicates a difference in the sign of the moment obtained by observation and calculation).

In comparing the two results, it must be further remembered that the moments obtained in the case of observation are farther along the member than those given by calculation. As the moment at the axis at the bar varies throughout its length, therefore there will be a proportionable difference between the moment at the end of the member and that at the extensometer, depending on the distance of the instrument from the panel point. If the nature of the curvature of the elastic line was taken from the solution, a correction could be applied to the calculated value for the position of the extensometer.

For these reasons it is very apparent that the comparison of the theoretical and observed results will not mean a check in the cases where these discrepancies are large.

In this experimental truss, these effects greatly exceed those encountered in practice. The general structure, the size of the plates, and the weight of the web members compared with the other dimensions of the truss all tend to enhance these, and in actual practice, the effect of the restraint of the joints on the general distribution of the direct stresses might very properly be neglected.

In case I, the secondary stresses, which it is customary to express as percentage of the direct stresses, are a maximum in the chord, and increase towards the end. The web members show a tendency to increase towards the centre. This is a fortunate coincidence, as the excess of section of the chord and web members generally occur at these respective points. In case II, the maximum percentage of secondary stress occurs in the chord members towards the end of the cantilever.

Though this Thesis has been confined to the calculation and observation of secondary stresses due to riveted joints, there are other important sources of these stresses occurring in bridge construction. Among these may be mentioned the secondary stresses induced in the posts of the truss due to the connection and deformation of the floor beams, the portal, and the lateral bracing. These stresses are in a plane at right angles to the plane of the truss, and in the case of shallow floor beams or heavy wind loads are of considerable magnitude. In erection or in the case of inaccurately constructed or misfit members, important, though somewhat indeterminate secondary stresses are set up. The designer must take into account all possible maximum conditions and make ample allowance for cases of this kind.

In conclusion, this investigation clearly demonstrates the importance of considering the secondary stresses in all bridge construction in which the riveted or rigid joint is used. It further shows the nature of the subject to be such as to forbid of a quick determination of these stresses, and the impossibility of deducing definite rules or formulae. In view of the increasing popularity of the riveted truss for long spans, in American bridge practice, further and more exhaustive investigations and comparisons of these stresses would be timely and well directed effort.

Gordon McLeod Pitts
B. Sc

Montreal, April 1st, 1909.

TABLE I.

Tabulated Form of Notes.

Run No. 12 March 11th, P.M.

Member G H at G. Dist. to P.P. 8.00

	Top Extens 2		Bottom Extens.6	
Load	Reading		Reading.	
100	.01850		.01265	
		.00002		.00008
2100	.01852		.01257	
		.00002		.00009
4100	.01854		.01248	
		.00003		.00008
6100	.01857		.01240	
		.00003		.00009
8100	.01860		.01231	
		.00002		.00010
10100	.01862		.01221	
		.00003		.00009
12100	.01865		.01212	
		.00003		.00010
14100	.01868		.01202	
		.00003		.00010
16100	.01871		.01192	
		.00003		.00010
18100	.01874		.01182	
		.00003		.00010
20100	.01877		.01172	
		.00003		.00009
22100	.01880		.01163	
		.00002		.00008
24100	.01882		.01155	
		.00003		.00008
26100	.01885		.01147	
Returning				
100	.01851		.01263	

Total Difference +.00035 +.00118"
(Indicates that both sides of the member have elongated)

TABLE II

Total Observed Strain.

Symetrical Loading.

Chord Member	Top Extens.	Bottem Extens.	Dist. to P.P.
AB	.00003 C	.00072 C	8.00"
BA	.00130 C	.00042 C	6.50"
BD	.00153 C	.00064 C	8.00"
DB	.00083 C	.00117	6.50"
DF	.00137 C	.00169 C	5.5 "
CE	.00195 T.	.00128 T	7.75"
EC	.00199 T	.00118 T	5.25"
AC	.00062 T	.00055 T	6.75"
CA	.00057 T	.00073 T	6.00"
Web Members			
BC	.00168 T	.00035 T	8.00"
CB	.00048 T	.00152 T	8.00"
DC	.00112 C	.00107 C	7.50"
CD	.00096 C	.00087 C	7.50"
DE	.00023 T	.00030 C	6.50"
ED	.00013 T	.00011 C	6.50"

TABLE III.

Comparison of Observed and Calculated

Secondary and Direct Stresses.

Members	Cal. 2nd.St.	Obser. 2nd St.	Co.Dir. St.	Obser. Dir.St.	Calcul. Dir.St.	% obser. 2nd of Dir.	% Cal. 2nd of Dir.
Chords.							
AB	862.4R	412.5	-5812.5	-6225.	-6203.8		14%
BA	883.8	3300.	-6450.	-6225.	-6203.8	54%	
BD	1780.	3337.	-8137.	-8325.	-8170.8	40%	
DB	5503.2	1275.0	-7500.	-8325.	-8170.8		67%
DF	2705.8	1200.	-11475.	-11325.0	-12265.2	10%	22%
CE	731.1	2512.5	12112.5	11325.	12256.2		
EC	2435.5	3037.5	11887.5	11325.	12256.2	27%	20%
AC	1280.R	262.5	4387.0	3900.	5166.8		
CA	2330.8R	600.	4875.	3900.	5166.8	15%	45%
Web.							
BC	1330.7	4987.5	7612.5	7875.	7810.7	63%	17%
CB	72.8R	3900.	7500.	7875.	7810.7		
DC	491.5R	187.5	-8212.5	-6975.	-7810.7		
CD	1318.2	462.5	-6987.5	-6975.	-7810.7	6%	17%
DE	3660.5	1987.5	262.5	375.	0	500%	3660.5 lbs
ED	478.3	900.	75.0	375.	0		

TABLE IV.

Total Observed Strain.
Unsymetrical Loading.

Chord Members.	Top Extens.	Bottom Extens.	Dist. to P.P.
AB	.00102 C	.00105 C	8"
BA	.00149 C	.00067 C	6.50"
BD	.00170 C	.00088 C	8.00"
DB	.00105 C	.00174 C	6.50"
DF	.00142 C	.00194 C	5.50"
FD	.00119 C	.00180 C	5.50"
FH	.00020 C	.00126 C	6.50"
HF	.00048 C	.00117 C	8.00"
HK	.00116 C	.00017 C	6.50"
KH	.00027 C	.00100 C	8.00"
KG	.00083 T	.00005 T	6.75"
GK	.00022 T	.00090 T	6.00"
GE	.00150 T	.00092 T	7.75"
EG	.00108 T	.00111 T	5.25"
EC	.00239 T	.00184 T	5.25"
CE	.00231 T	.00153 T	7.75"
CA	.00093 T	.00087 T	6.00"
AC	.00081 T	.00070 T	6.75"
Web Members			
BC	.00178 T	.00070 T	8.00"
CB	.00119 T	.00140 T	8.00"
CD	.00110 C	.00106 C	7.50"
DC	.00153 C	.00106 C	7.50"
DE	.00058 C	.00114 C	6.50"
ED	.00048 C	.00102 C	6.50"
EF	.00048 T	.00108 T	6.50"
FE	.00059 T	.00088 T	6.50"
FG	.00059 C	.00088 C	7.50"
GF	.00029 C	.00120 C	7.50"
GH	.00035 T	.00118 T	8.00"
HG	.00039 T	.00106 T	8.00"

TABLE V

Comparison of Observed and Calculated
Secondary and Direct Stresses.

Members	Cal 2nd Str.	Obser. 2nd St.	Co. Dir. St.	Obser. Dir. St.	Calcul. Dir. St.	% Obser. 2nd of Dir.	% Cal. 2nd. of Dir.
Chords							
AB		112.5	-7762.5	-10575.	-7754.8		
BA		3075.	-8100.	-10575.	-7754.8	29%	
BD		3075.	-9675.	-10275.	-10214.	29%	
DB		2587.5	-10462.5	-10275.	-10214.		
DF		1950.	-12600.	-13050.	-12256.0		
FD		2287.5	-11212.5	-13050.	-12256.	17%	
FH		3225.	-4725.	-7275.	-6128.6	44%	
HF		2587.5	-6187.5	-7275.	-6128.6		
HK		3712.5	-4987.5	-4475.	-4652.4	83%	
KH		2737.5	-4762.5	-4475.	-4652.4		
KG		2925.	+3300.	2250.	3875.	130%	
GK		2550.	4200.	2250.	3875.		
GE		2175.	9075.	8250.	9192.6	26%	
EG		112.5	8212.5	8250.	9192.6		
EC		2062.5	15862.5	14925.	15320.5		
CE		2925.	14400.	14925.	15320.5	19%	
CA		225.	6750.	6825.	6458.7		
AC		400.	5650.	6825.	6458.7	5%	
Web Members.							
BC		4050.	9300.	11100.	9807.4	36%	
CB		787.5	9712.5	11100.	9807.4		
CD		1050.	-9300.	-7800.	-9807.4		
DC		1762.5	-9712.5	-7800.	-9807.4	22%	
DE		2100.	-6450.	-4125.	-5884.0	50%	
ED		2025.	-5625.	-4125.	-5884.0		
EF		2250.	5850.	6900.	5884.0	32%	
FE		1087.5	5512.5	6900.	5884.0		
FG		1087.5	-5512.5	-4125.	-5884.0		
GF		3412.5	-5587.5	-4125.	-5884.0	82%	
GH		3112.5	5737.5	7200.	5884.0	43%	
HG		2512.5	5437.5	7200.	5884.0		

PLATE I.

THESIS ON

SECONDARY STRESSES

IN BRIDGE TRUSSES INDUCED BY
RIVETED JOINTS

FIG. I.

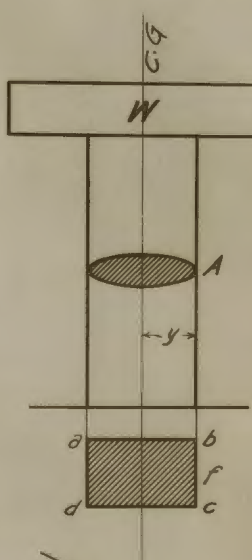
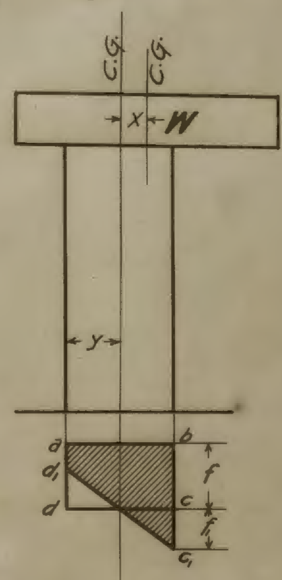


FIG. II.



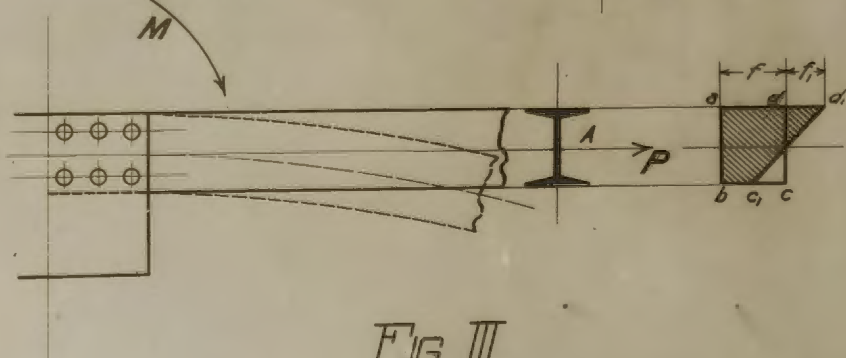


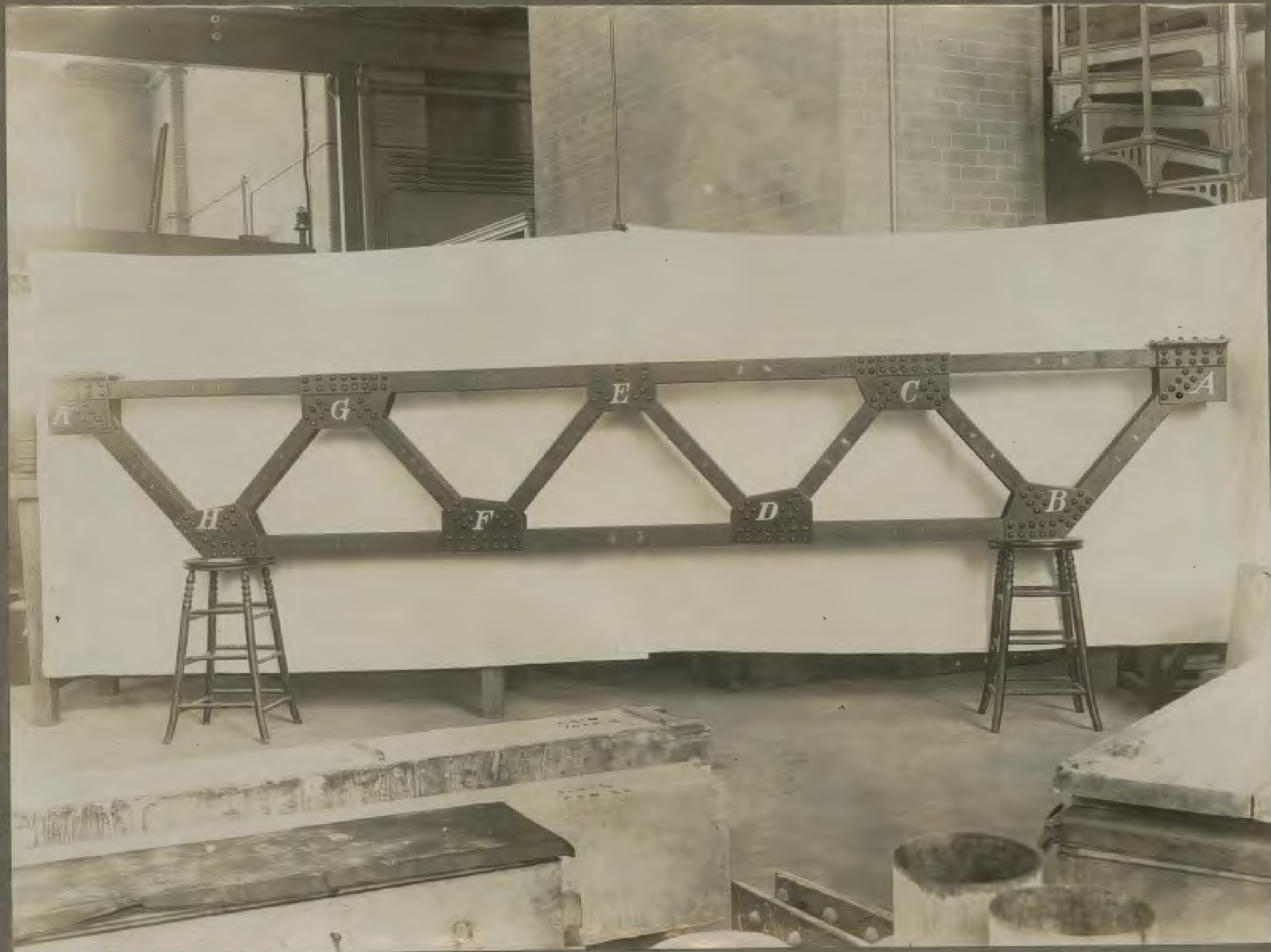
FIG. III.

PLATE II.

THESIS ON

SECONDARY STRESSES

IN BRIDGE TRUSSES INDUCED BY
RIVETED JOINTS



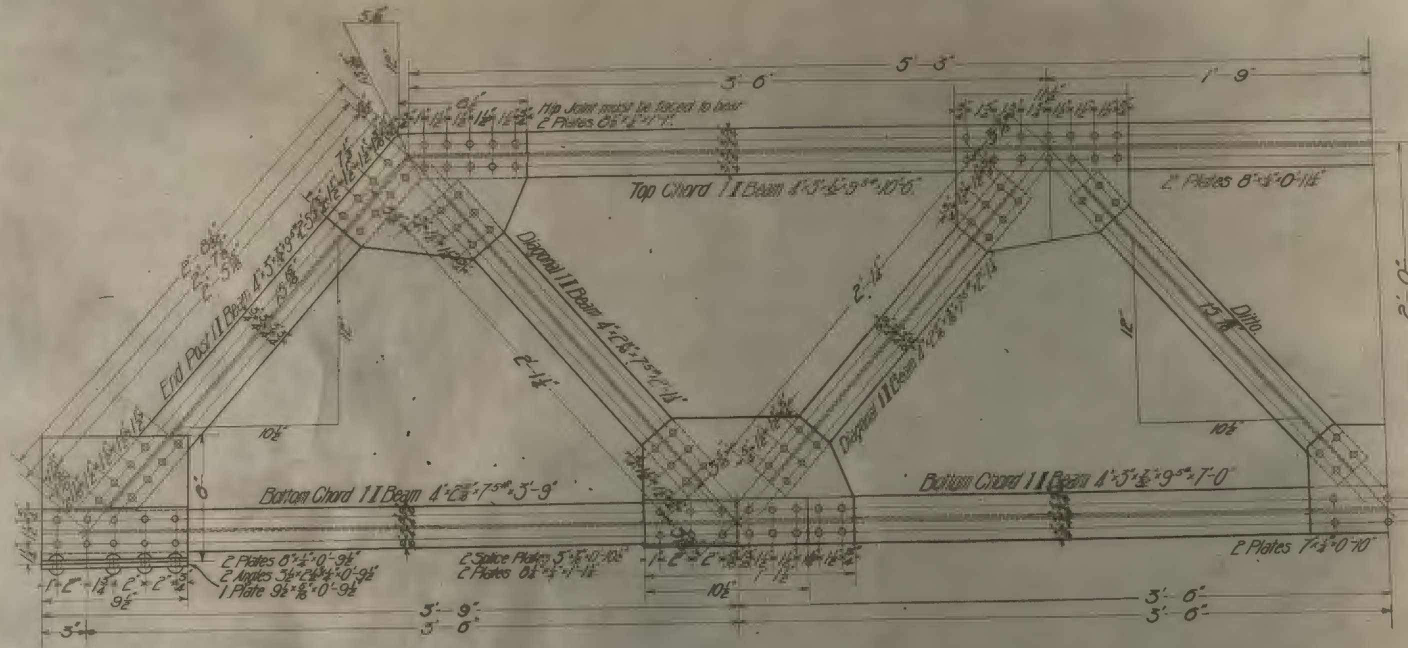
A
B
C
D
E
F
G
H

PLATE III.

THESIS ON

SECONDARY STRESSES

IN BRIDGE TRUSSES INDUCED BY
RIVETED JOINTS



Number.	Description.	Weight per lin. ft.	Total Weight.
1	I Beam 4"×3"×¼"×9.5#×10'-6"	9.5	99.7
1	" 4"×3"×¼"×9.5#×7'-0"	9.5	66.5
2	" 4"×3"×¼"×9.5#×2'-5⅜"	9.5	47.6
2	" 4"×2⅝"×⅛"×7.5#×3'-9"	7.5	56.2
6	" 4"×2⅝"×⅛"×7.5#×2'-1¼"	7.5	94.2
4	Gusset Plates 8"×¼"×0'-11½"	Clipped	25.0
4	" " 8½"×¼"×1'-1"	-	40.0
4	" " 8"×¼"×1'-1½"	-	30.[illegible]
4	" " 8"×¼"×0'-9½"	6.8	21.5
2	" " 7"×¼"×0'-10"	5.95	9.9
4	Splice Plates 3"×7/16"×0'-10½"	4.46	15.5
2	Plates 9½"×5/16"×0'-9½"	10.09	16.0
4	Angles 3½"×2½"×¼"×0'-9½"	4.9	15.5
848	⅜" Rivet Heads		40.8
	Total Weight of Truss		587.[illegible] lbs

14 FT. WARREN TRUSS
USED IN RESEARCH ON SECONDARY STRESSES DUE TO RIVETING

Scale: 1"=4"

Plate IV.

Thesis on

SECONDARY STRESSES

IN BRIDGE TRUSSES INDUCED BY
RIVETED JOINTS

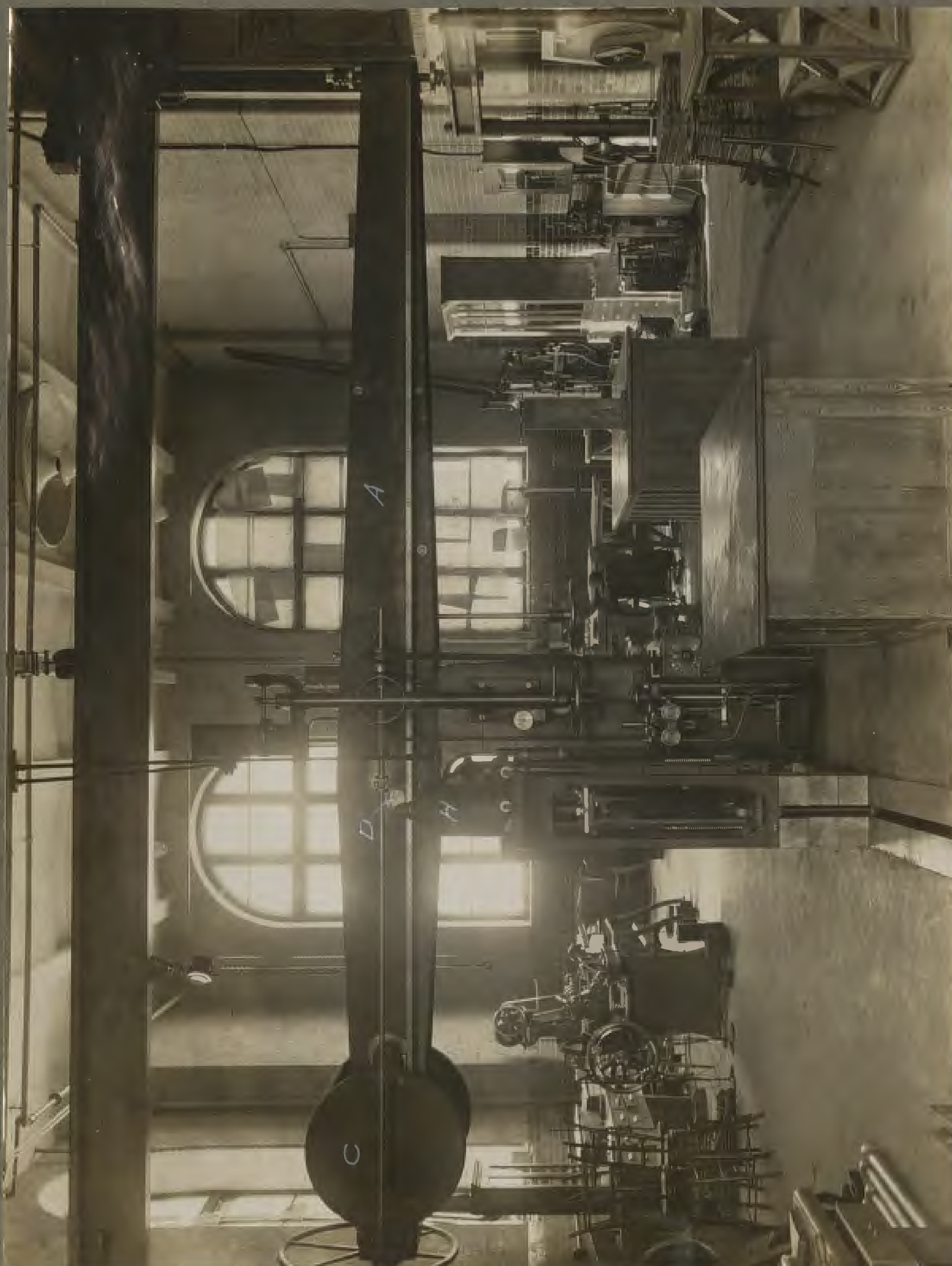

A
D
H
C

PLATE V.

THESIS ON

SECONDARY STRESSES

IN BRIDGE TRUSSES INDUCED BY
RIVETED JOINTS

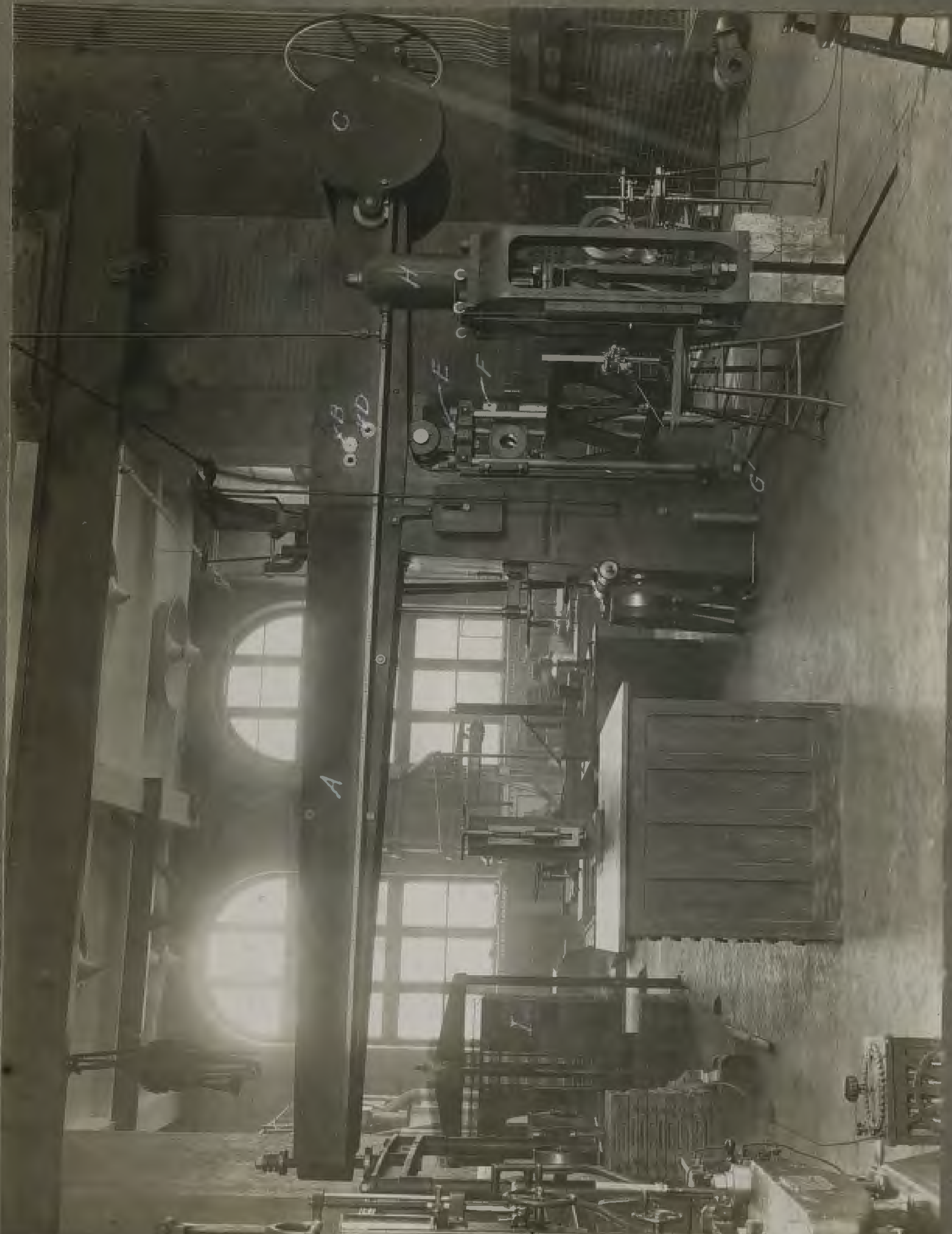

A
B
C
D
E
F
G
H
I

Plate VI.

Thesis on

SECONDARY STRESSES

IN BRIDGE TRUSSES INDUCED BY
RIVETED JOINTS

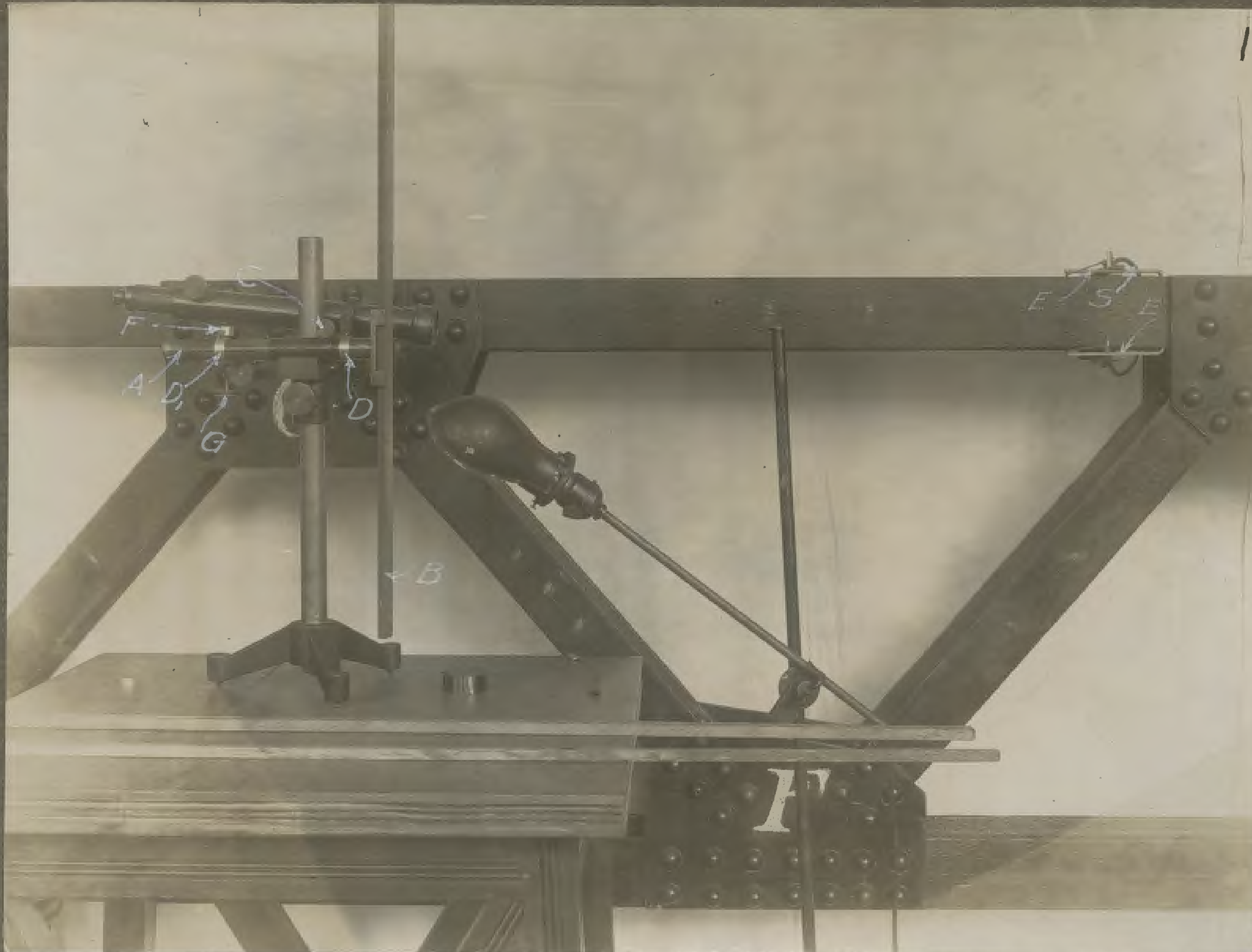
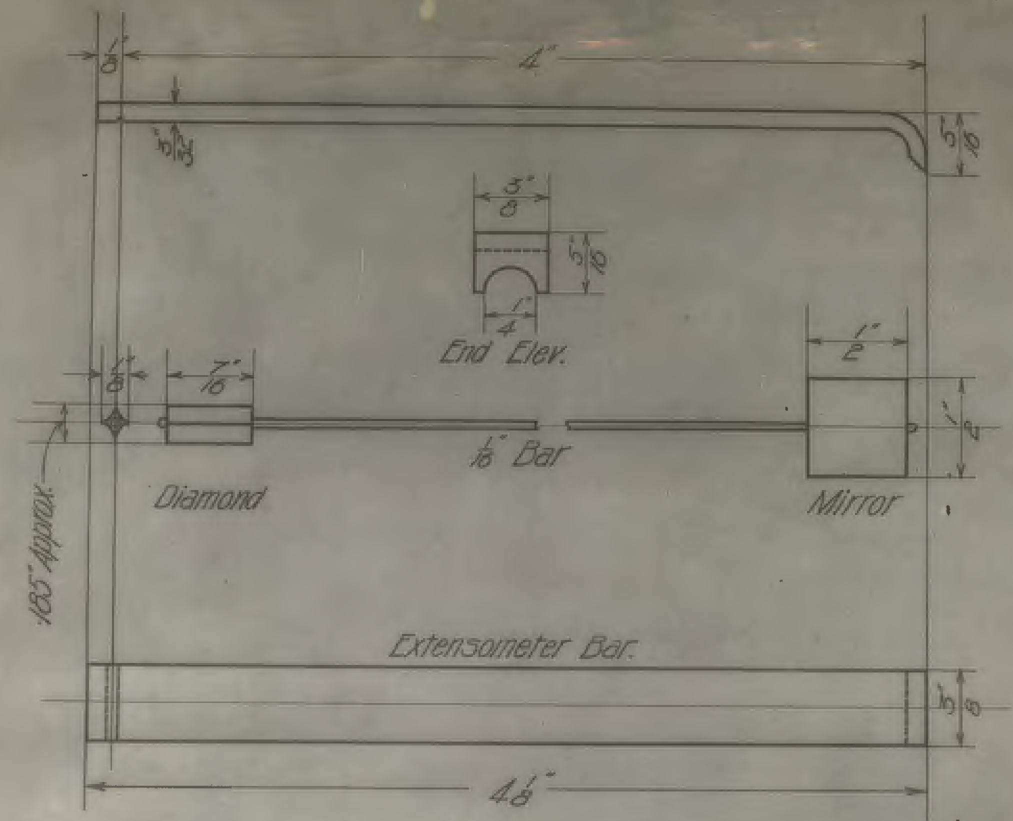

C
F
A
D,
G
D
B
E
S
E
F

Plate VII.

Thesis on

SECONDARY STRESSES

IN BRIDGE TRUSSES INDUCED BY
RIVETED JOINTS



THE MARTENS EXTENSOMETER.

PLATE VIII.

THESIS ON

SECONDARY STRESSES

IN BRIDGE TRUSSES INDUCED BY
RIVETED JOINTS

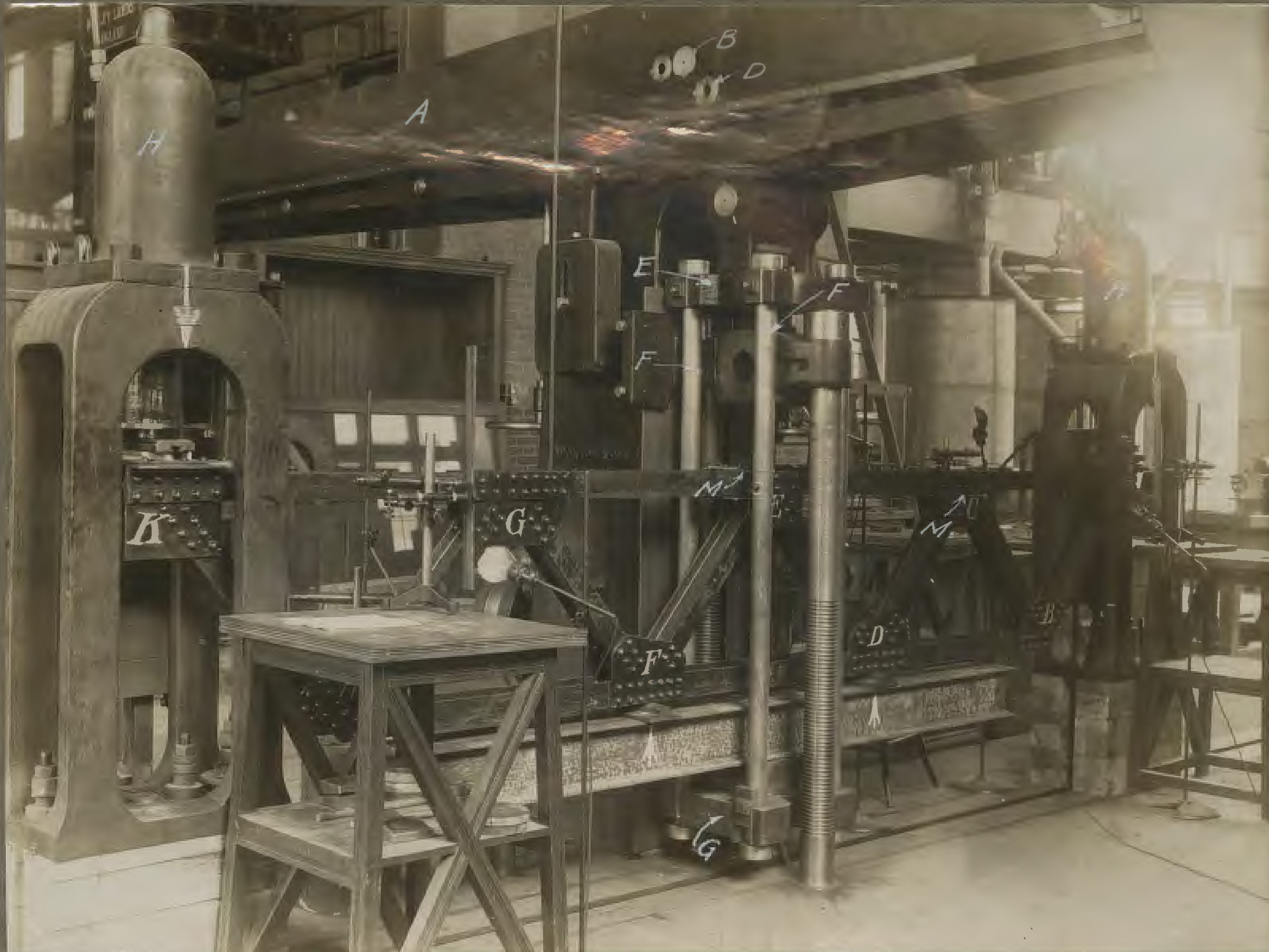

A
B
D
H
E
F
F
K
G
M
M
F
D
B
G

PLATE IX.

THESIS ON

SECONDARY STRESSES

IN BRIDGE TRUSSES INDUCED BY
RIVETED JOINTS

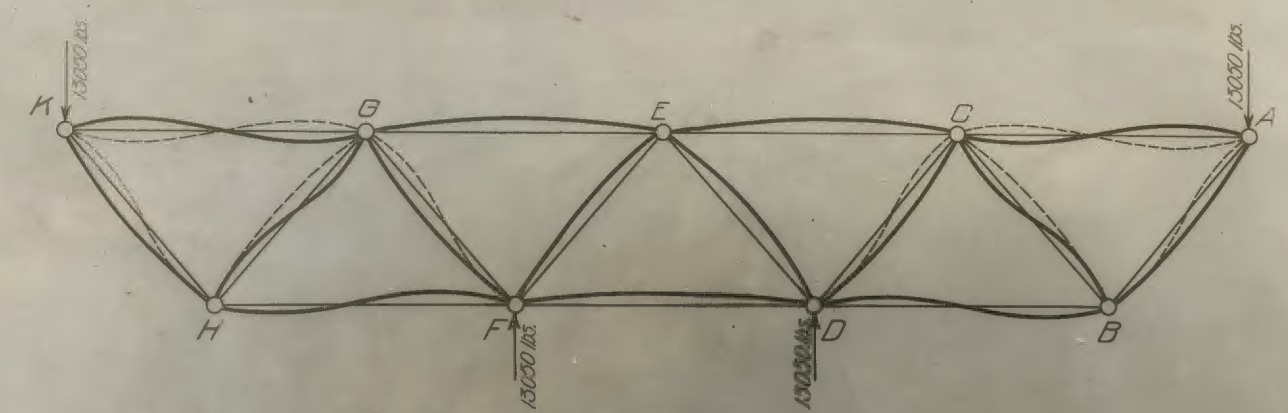

13050 lbs.
K
G
E
C
13050 lbs.
A
H
F
13050 lbs.
D
13050 lbs.
B

Fig. I.

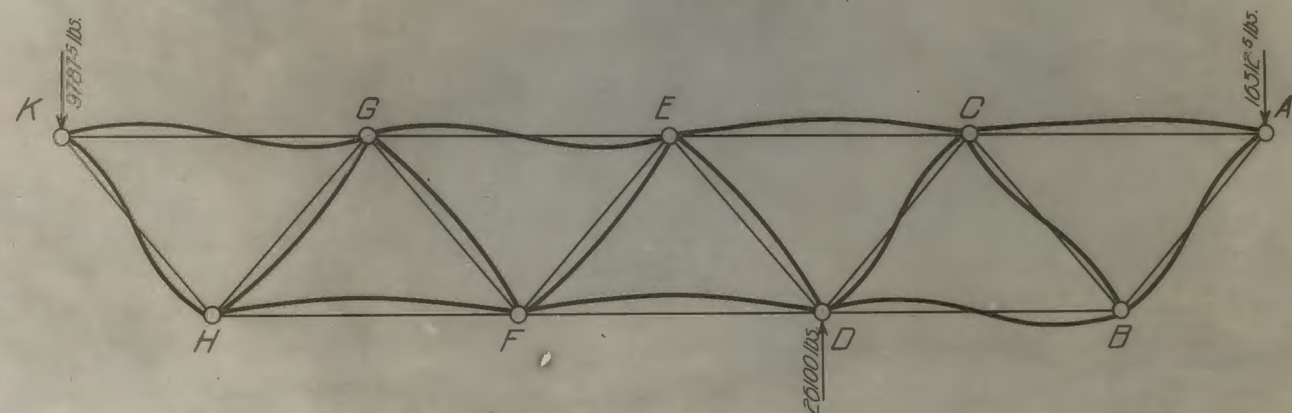

9787.5 lbs.
K
G
E
C
16312.5 lbs.
A
H
F
26100 lbs.
D
B

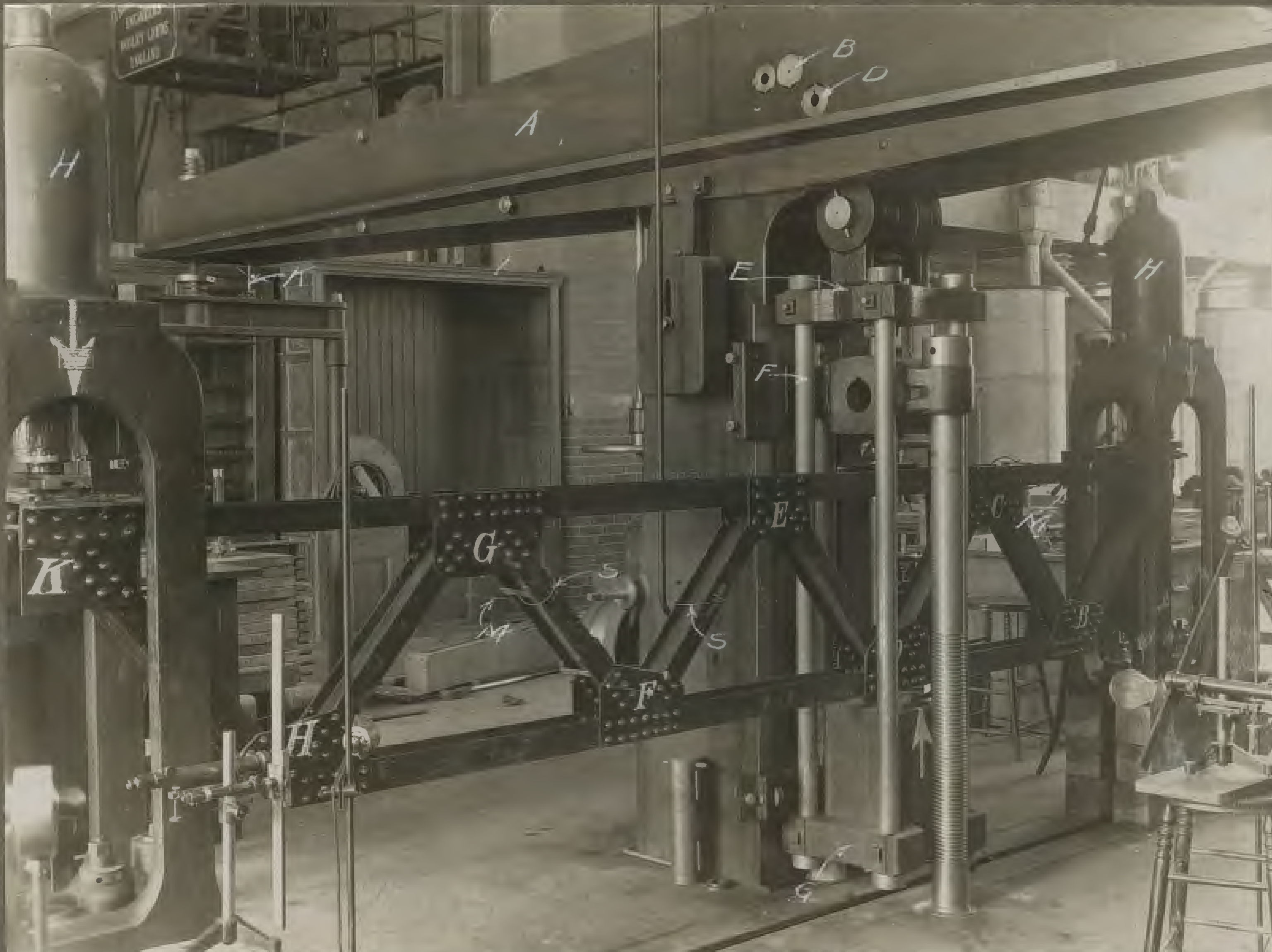
Fig. II.

Plate X.

Thesis on

SECONDARY STRESSES

IN BRIDGE TRUSSES INDUCED BY
RIVETED JOINTS


A
B
D
H
H
E
F
G
E
F
H
K
S
S
M
C