Integrating Stope Design and Long-term Production Scheduling Under Uncertainty: Methods and Applications for an Operating Copper Mine

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Contribution of Authors

All the work presented herein has the author of this thesis is the primary author. The work was completed with the supervision and advice of her advisor Prof. Roussos Dimitrakopoulos, who is also the co-author on the two papers. Another co-author of the first paper presented on chapter 2 is Phil Conway, BHP's Principal Strategic Mine Design, and Innovation

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Abstract

Underground long-term mine planning is based on three main components: stope layout, access network design and production scheduling. Due to the complexity of developing a method that is generalized for the various underground mining methods and the decisions related to underground mine operations, these components are traditionally optimized separately. In addition, these traditional frameworks are deterministic, hence geological uncertainty and variability of grades and material types are not considered. Once the interactions among the mine planning components and the risk related to inherent sources of uncertainty is ignored, the optimization process deviates from its main objective, which is to maximize the project's net present value. Recent studies have improved the sequential framework by integrating the stope design and production scheduling in one optimization approach, while considering cumulative development costs and managing the risk of not meeting production targets, in terms of ore quality and quantity. These developments, however, are site-specific and are unable to adapt to different mining methods and variants. Additionally, these stochastic approaches use geostatistical simulations of the orebody that rely on two-point statistics and Gaussian distribution assumptions. Thus, they are not able to properly characterize complex spatial geometries, high-grade connectivity or multiple point statistics of non-Gaussian and non-linear natural phenomena. This thesis presents a model that follows the integrated stochastic framework, by proposing new mathematical formulations and applications for the sublevel longhole open stoping mining method, parametrized as per an operating mine. The impacts of using high-order sequential simulations, that can reproduce geological patterns and infer high-order spatial statistics from data — in the proposed optimization approaches are explored.

The first chapter of the thesis presents a literature review on deterministic and stochastic developments for the optimization of stope design and long-term mine production scheduling. It also reviews relevant stochastic optimization frameworks for open-pit mine production scheduling which are used as state-of-art references for the development of an integrated strategic optimization framework. In addition, methods developed for generating geostatistical simulations of mineral deposits are presented.

The second chapter of the thesis presents a new two-stage stochastic integer programming formulation for the integrated optimization of stope design and long-term mine production

scheduling, considering operational parameters of an operating underground copper mine. The mathematical model contains an objective function that aims to maximize the net present value of the scheduled stopes, by considering horizontal development and haulage costs for different possible haulage systems, while managing geological risk. Backfilling and adjacency criteria are included to suit the method to operational and geotechnical requirements according to the mining zones. A case study is presented for an operating underground copper mine where orebody uncertainty is quantified with geostatistical simulations of copper and related secondary elements. A comparison to a sequential stochastic long approach in which stope boundaries limit the possible locations and shapes of stopes, shows that physically different stope layouts, horizontal networks and extraction sequences are produced. The integrated approach shows a substantial reduction in horizontal development costs and a 6% higher net present value compared to the sequential approach.

The third chapter of this thesis enhances the previously mathematical model to incorporate stockpiling decisions. In addition, the impacts of using a sequential simulation method that infers high-order statistics from available geological data as input for the underground mine production schedule is investigated. High-order sequential simulations of copper grades are generated for a given mining zone of an operating copper mine. These simulations as well as provided second-order sequential Gaussian simulations are used as input to the proposed integrated method. The produced extraction sequences and related final stope layouts are shown to be physically different. It is seen that the optimization process takes advantage of the better representation of high-grade connectivity when high-order sequential simulations are used, allowing a 4% higher metal production and a consequent 6% higher net present value.

Future research may consider expanding these studies for other underground mining methods and for mining complexes with multiple processing streams, stockpiles and mines. In addition, the impact of using high-order simulations with multiple correlated elements on underground productions schedules is a topic for further investigation.

Resumé

La planification minière à long terme en sous-terrain repose sur trois composantes principales: la disposition des chantiers, la conception du réseau d'accès et la planification de la production. En raison de la complexité de développement d’une méthode généralisée pour les différentes méthodes d'exploitation minière souterraine et des décisions liées aux opérations minières souterraines, ces composantes sont traditionnellement optimisées séparément. De plus, ces cadres traditionnels sont déterministes, c'est pourquoi l'incertitude géologique et la variabilité des teneurs et des types de matériaux ne sont pas prises en compte. Une fois ignorés les interactions entre les parties composantes de la planification minière et les risques liés aux sources d’incertitude, le processus d’optimisation dévie de son objectif principal, qui est de maximiser la valeur actuelle nette du projet. Des études récentes ont amélioré ce cadre séquentiel en intégrant la conception des chantiers et la planification de la production dans une approche d'optimisation unique, tout en tenant compte des coûts de développement cumulatifs et la gestion du risque de ne pas atteindre les objectifs de production, en termes de qualité et de quantité du minerai. Ces développements sont cependant spécifiques à chaque opération et ne s'adaptent pas aux différentes méthodes et variantes d'exploitation minière. En plus, ces études qui prennent en compte l’incertitude géologique utilisent des simulations géostatistiques du gisement minéral qui s'appuient sur des statistiques à deux points et des hypothèses de distribution gaussienne. Ainsi, ils ne sont pas capables de caractériser correctement des géométries spatiales complexes, une connectivité des teneurs élevées ou des statistiques à points multiples de phénomènes naturels non gaussiens et non linéaires. Cette thèse présente un modèle qui suit cadre intégré, en proposant de nouvelles formulations mathématiques et applications pour la méthode d'exploitation par sous-niveaux, telle qu'appliquée à une mine en activité. Les impacts de l’utilisation de simulations séquentielles d’ordre supérieur, capables de reproduire des modèles géologiques et de déduire des statistiques spatiales d’ordre supérieur à partir de données, dans les approches d’optimisation proposées sont aussi explorées.

Le premier chapitre de la thèse présente une revue de la littérature sur les développements déterministes et stochastiques pour l'optimisation de la conception des chantiers et de la programmation à long terme de la production minière. Il examine également les optimisations stochastiques les plus pertinentes de la planification de la production minière en carrière à ciel

ouvert, qui sont utilisées comme références pour les développements d'un cadre d'optimisation stratégique intégré. De plus, les méthodes développées pour générer des simulations géostatistiques des gisements minéraux sont présentées.

Le deuxième chapitre de la thèse présente une nouvelle formulation de programmation stochastique en nombres entiers en deux étapes pour l'optimisation intégrée de la conception des chantiers et de la planification de la production minière à long terme, en considérant les paramètres opérationnels d'une mine de cuivre souterraine en exploitation. Le modèle mathématique intègre une fonction objectif visant à maximiser la valeur actuelle nette des chantiers planifiés, en considérant les coûts de développement horizontal et de transport pour différents systèmes de transport possibles, tout en gérant les risques géologiques. De plus, il contient des critères de remblayage et d’adjacence afin de s’aligner sur les exigences opérationnelles et géotechniques spécifiques aux zones minières. Les contraintes opérationnelles telles que la proximité et les capacités d'extraction et de traitement sont prises en compte. Pour illustrer cela, une étude de cas d’une mine de cuivre souterraine en activité est présentée, où l'incertitude du corps minéralisé est quantifiée à l'aide de simulations géostatistiques des teneurs en cuivre et d’éléments secondaires associés. Une comparaison avec une approche séquentielle stochastique de planification de la production minière à long terme, dans laquelle les limites des chantiers restreignent les emplacements et les formes possibles des chantiers, montre que la disposition des chantiers, la conception du réseau horizontal et les séquences d'extraction sont physiquement différentes. L'approche intégrée montre une réduction substantielle des coûts de développement horizontaux et une valeur actuelle nette supérieure de 6 % par rapport à l'approche séquentielle.

Le troisième chapitre de cette thèse améliore la formulation de programmation stochastique en deux étapes proposées précédemment en intégrant les décisions de stockage. De plus, l’impact de l'utilisation d'une méthode de simulation séquentielle reposant sur l’inférence de statistiques d'ordre supérieur est étudié ; l’inférence étant effectuée à partir des données géologiques disponibles dans le calendrier de production de la mine souterraine. Des réalisations d’un modèle séquentiel d'ordre élevé des teneurs en cuivre sont simulées pour une zone minière donnée d'une mine de cuivre en exploitation. Ces simulations ainsi que les simulations gaussiennes séquentielles du second ordre sont utilisées comme arguments dans la méthode intégrée proposée. Les séquences d'extraction produites et la disposition des chambres finales révèlent des différences physiques. On constate que le processus d’optimisation tire parti de la meilleure représentation de la connectivité de haute teneur lorsque des simulations séquentielles d’ordre supérieur sont utilisées, permettant une production de métal 4 % plus élevée et, par conséquent, une valeur actuelle nette plus élevée de 6 %.

Les futures recherches possibles incluent l’extension de ces études à d’autres méthodes d’exploitation minière souterraine et à des complexes miniers, avec de multiples flux de traitement, des stocks et des mines. De plus, l’utilisation de simulations d’ordre supérieur pour plusieurs éléments corrélés et ses impacts sur les calendriers de production souterraine est un sujet à approfondir.

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Chapter 1 – Introduction and Literature Review

1.1 Introduction

Underground mining is described as the extraction of metals and minerals selectively and their transportation to the surface where they are processed and transformed into commercially viable products (Hartman and Mutmansky 2002). The present work focuses on the sublevel longhole open stoping (SLOS) mining method which is based on the definition of mineable shapes that are sequentially drilled, blasted, hauled and usually backfilled (Hamrin 2001; Hartman and Mutmansky 2002; Pakalnis and Hughes 2011). The associated long-term mine planning process generally comprises in the definition of the spatial configuration of stopes with their respective accesses and the schedule of development and extraction of these mineable volumes, given the primary objective of maximizing the project's net present value (NPV) (Alford et al. 2007; Fava et al. 8-10 June 2011; Hauta et al. 2017; Nehring and Topal 2007; Topal 2003; Topal 2008). Conventionally, these components are addressed separately due to the complexity of tailoring the various decisions to suit the distinct mining methods and their variations (Alford et al. 2007; Bootsma et al. 2018; Morin 2001; O'Sullivan et al. 2015; Trout 1995). This sequential approach; however, is not capable of integrating complementary objectives to generate truly optimal schedules (Kumral and Sari 2020; Little et al. 2011; Morin 2001). Therefore, joint optimization of the underground mine design and schedule have been recent studied, showing the impacts of capturing the synergies between these components on the stope layout, on the development decisions and costs and on the NPV (Copland and Nehring 2016; Foroughi et al. 2019; Furtado e Faria et al. 2022a; Hou et al. 2019; Little et al. 2013).

Different sources of uncertainty affect the feasibility of a mining project. The orebody material variability and uncertainty in pertinent properties have an inevitable impact on the quality, quantity, and value of the final products, thus proving to be a critical source of technical risk (Dimitrakopoulos et al. 2002; Dowd 1994; Ravenscroft 1992; Vallée 2000). Equiprobable geostatistical simulations (Dimitrakopoulos and Yao 2020; Goovaerts 1997; Remy et al. 2009; Strebelle 2002) of the orebody are the main inputs to a mine planning framework in which the production schedule is given by the maximization of the expected NPV and the simultaneous minimization of the risk of not meeting the production targets. Stochastic optimization of mine production schedules has been an extensive topic of study for open-pit mine plans (Goodfellow and Dimitrakopoulos 2016, 2017; Leite and Dimitrakopoulos 2007; Montiel and Dimitrakopoulos 2017, 2015, 2018; Ramazan and Dimitrakopoulos 2013, 2005). However, limited research on stochastic optimization addresses the particularities of underground mine designs, production schedules, and their interaction (Carpentier et al. 2016; Dirkx et al. 2018; Furtado e Faria et al. 2022a, 2022b; Huang et al. 2020; Montiel et al. 2016; Noriega et al. 2022; Villalba Matamoros and Kumral 2018). In addition, the typically used simulation methods rely on assumptions on the data distribution, preventing them from capturing high-grade connectivity or reproducing the high-order statistics of the sample data (Chilès and Delfiner 1999; David 1977, 1988; Goovaerts 1997; Journel 2005; Mariethoz and Caers 2015; Rossi and Deutsch 2014). Therefore, recent research has expanded on sequential simulation methods that infer the conditional probability distribution functions (cpdf) from the available data, showing a better reproduction of complex geological patterns. (de Carvalho et al. 2019; Dimitrakopoulos et al. 2010; Dimitrakopoulos and Yao 2020; Minniakhmetov and Dimitrakopoulos 2017b; Minniakhmetov et al. 2018; Mustapha and Dimitrakopoulos 2011). Although the effect of different types of simulations as an input to non-linear transfer functions has been assessed (de Carvalho and Dimitrakopoulos 2019; Goodfellow et al. 2012; Qureshi and Dimitrakopoulos 2005), their impact on the stochastic optimization underground mine productions schedules is a topic to be further explored.

This chapter covers the technical literature in underground mine planning and orebody modelling through geostatistical simulation methods. Section 1.2 reviews the sublevel stoping mining method, including its main definitions and systems and the most relevant variants. Section 1.3 outlines the deterministic frameworks for the stope design and the mine production schedule, including the review of integrated optimization approaches. Section 1.4 presents traditional simulation methods and advances on orebody modelling, using multi-point and high-order statistics. Section 1.5 covers stochastic mine planning optimization for open-pit mining and state-of-the-art simultaneous optimization of mining complexes. Lastly, technical literature on stochastic underground mine planning optimizations methods is reviewed. Section 1.6 presents the goals and objectives of this thesis, and Section 1.7 outlines the content of this thesis.

1.2 Sublevel Stoping Mining Method

The underground mining process consists of the extraction of metals and minerals without contact with the surface. This material is hauled to the surface where it undergoes different milling and concentration processes, transforming them into sellable products (Hamrin 2001; Hartman and Mutmansky 2002). The feasibility of using underground mining techniques over surface mining methods is based on the evaluation of the geology of the deposit, physical and economic requirements, and environmental aspects (Nelson 2011). Among the diversity of the underground mining methods, the selection of the appropriate method must be done based on the orebody characteristics such as its orientation, topology, as well as qualitative and quantitative features regarding the mechanics of the in situ rock material (Alpay and Yavuz 2009; Bullock 2011; Carter 2011; Laubscher 1981). The present work focuses on the sublevel longhole open stoping underground mining method. Thus, this section is dedicated to a review of relevant technical terms and respective definitions constantly used throughout this thesis, followed by an overview of the operational aspects of the sublevel stoping mining method and variants.

1.2.1 Underground Mining Terms and Definitions

Underground mining methods are generally divided into caving and stoping methods. Caving methods rely on gravity, to intentionally induce the rock material collapse. On the other hand, in stoping excavations, the exploitation of ore requires drilling and blasting sequentially selected volumes of the rock mass, called stopes, and retrieving this material from previously developed drawpoints or drawbells, that control the load and haulage of the fragmented rock. Within the stoping methods, there are supported methods and unsupported (or self-supported) methods. The supported methods primarily use artificial supports to guarantee the stability of the openings. In the unsupported class the rock material supports the load of the superincumbent load (Hamrin 2001; Hartman and Mutmansky 2002). For these methods, after the material extraction, light structural supports such as rock bolts or posterior filling are commonly applied (Bullock 2011).

Primary developments directly connect the surface to the orebody. Haulage systems such as slopes, shafts, ramps or declines are examples of primary accesses. Levels define horizontal excavations that connect the orebody to the primary accesses. Between levels, in certain mining methods, sublevels are developed. Those, however, connect the mining area to secondary accesses and define a working horizon. Within levels and sublevels, drifts (horizontal openings usually parallel to the strike) and crosscuts (horizontal or nearly horizontal excavations perpendicular to the strike) are developed to guarantee access to stopes and enable drilling, and blasting, and haulage operations (Hamrin 2001; Hartman and Mutmansky 2002).

1.2.2 Sublevel Stoping Mining Method and Variants

Sublevel stoping is an underground mining method in which large vertical stopes are created within the orebody (Atlas Copco 2007; Hamrin 2001; Hartman and Mutmansky 2002; Pakalnis and Hughes 2011; Soma 2001) as presented in Figure 1-1. The method is applied in lenticular or tabular deposits, preferably with regular boundaries. The dip of the deposit must be steep, such that it exceeds the angle of repose. The ore and the host rock must be competent as a stable footwall and hanging wall are required. Drifts and crosscuts are developed horizontally within each sublevel to enable drilling and blasting. It requires minimal labour underneath blasted volumes, thus little exposure to hazardous conditions is observed. The unitary operations can be developed simultaneously from different levels, which makes this mining method highly efficient in terms of productivity (Hartman and Mutmansky 2002). In addition, usually vertical and horizontal pillar are left between the stopes to guarantee the stability of the openings. After the complete extraction of a stope, rock bolts are usually installed to improve the rock stability. Alternatively, backfilling the stopes adds stability and enables the extraction of larger stopes following an adjacency rule of primary, secondary, and tertiary stopes (Villaescusa 2014).

Several variants of the sublevel stoping mining method are presented in the technical literature. Their naming and application are variable and are usually adapted to the orebody conditions, available equipment and new techniques developed throughout the years. The first variant is sublevel open stoping (Bullock 2011), also called sublevel longhole open stoping (SLOS) (Atlas Copco 2007; Hamrin 2001; Pakalnis and Hughes 2011; Soma 2001) or the blasthole method (Hartman and Mutmansky 2002). On this variation a vertical slot is created at one end of the stope and sublevels are excavated horizontally within levels for drilling fan patterns and blasting sequentially each section of the stope (i.e., ring). The material is mucked from the draw points developed below the stope. Bighole stoping is a large-scale variant of SLOS, in which longer and wider blastholes are drilled following a radial pattern (Atlas Copco 2007; Hamrin 2001). Although this variant allows larger stopes and the available drilling equipment is more accurate than traditional tophammer drilling, it has a higher risk of damaging the rock structures. The second variant of the sublevel stoping method is referred to as the open-ending method by Hartman and Mutmansky (2002), also as blasthole stoping or end slicing (Bullock 2011). This method consists of developing a drilling level at the top of the stope and drilling the blastholes vertically downwards. Subsequently one or several slices are blasted sequentially towards the direction of a previously developed open slot at one end of the stope.

In the following subsections, a general description of the sequence of development with sublevel stoping mining as well as the ordering of the unitary operations defining a production cycle and alternatives of the backfilling procedure are presented.

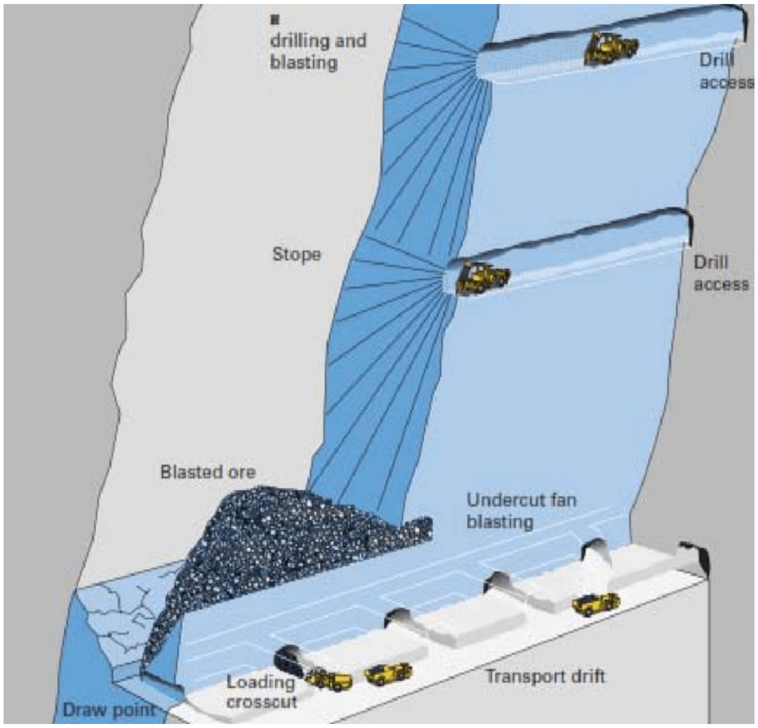

Figure 1-1 – Sublevel stoping mining method (Atlas Copco 2007)

1.2.2.1 Sequence of development

In order to extract the ore material using the sublevel open stoping it is necessary to develop all the preparation with drifts, crosscuts and drawpoint below the targeted stope to allow haulage. In addition, raises and wises can be developed for ventilation. For VRC stoping, an undercut is developed to allow the initial blasted horizontal slice fall with gravity. For the other variants, sublevel crosscuts are developed to create a raise that is enlarged to form a slot. This slot works as a free face allowing blasting of the vertical rings. The blasthole drilling for all methods is done through the sublevels' horizonal developments (Bullock 2011; Hartman and Mutmansky 2002).

1.2.2.2 Mining cycle

The mining cycle consists of the preparation of initial vertical and horizontal tertiary developments. Reinforcements of the stope walls can be conduced by installing cablebolts. Then, drilling according to the pattern related to the mining method variant is performed, followed by blasting operations. Loading occurs in the drawpoints below the stopes through gravity flow with front-end loaders, LHDs, shovels, slushers or belt conveyors. Finally, the material is hauled through the haulage drifts by LHDs, trucks or conveyor belts to the orepass (Hartman and Mutmansky 2002). The way this sequence of operations is performed within the stope or the orebody depends on the stope geometry. Villaescusa (2014) describes two types of internal developments inside a stope that impact its mining cycle: multi-lift and single-lift open stoping. On the multi-lift method, several intermediate sublevels are developed within the stope and entire mining cycle must occur sequentially for each sublevel as shown in Figure 1-2. On the single-lift method, the stope is bounded by the sublevels; thus, the mining cycle is performed once to allow the stope extraction, as shown in Figure 1-3.

In certain applications, once the stope is completely excavated, the backfilling operations are performed. Backfilling allows more recovery of ore pillars, while increasing the host rock wall support and potentially improving the return of the mine. If backfilling is used for pillar recovery, a sequence of extraction of primary, secondary and often tertiary stopes is employed (Atlas Copco 2007; Hamrin 2001; Villaescusa 2014). The choice of this type of adjacency that will drive the final extraction sequence depends on grade requirements, locations of existing developments and induced stress considerations given the maximum void size and filling type (Villaescusa 2004). Figure 1-4 shows an example of a sequence of extraction of stopes to allow backfilling and Figure 1-5 shows the filling sequence considering primary, secondary, and tertiary stopes, difference sequences and precedence rules can be used to guarantee rock-mass stability while stopes are progressively mined and backfilled.

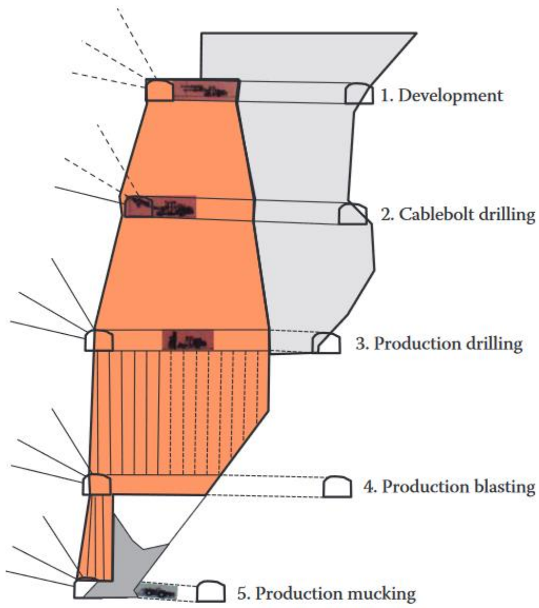

Figure 1-2 – Multiple-lift open stoping mining cycle (Villaescusa 2014)

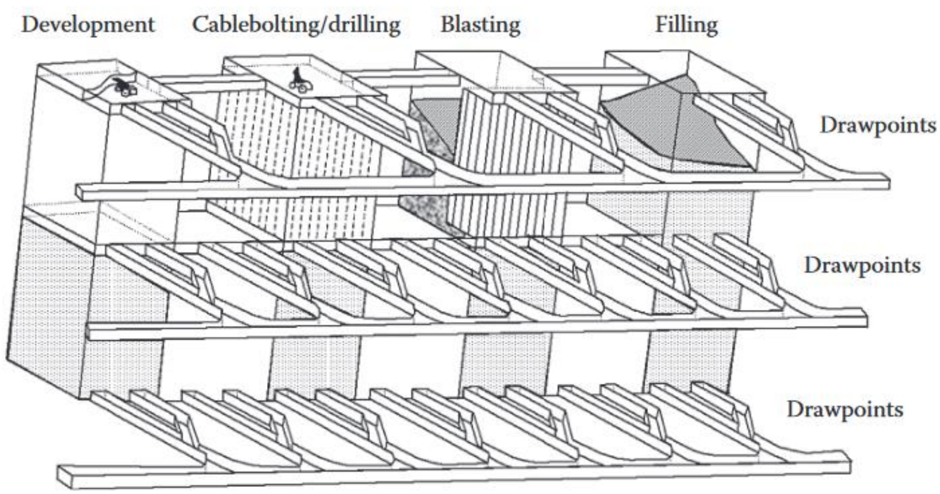

Figure 1-3 – Single-lift open stoping mining cycle (Villaescusa 2014)

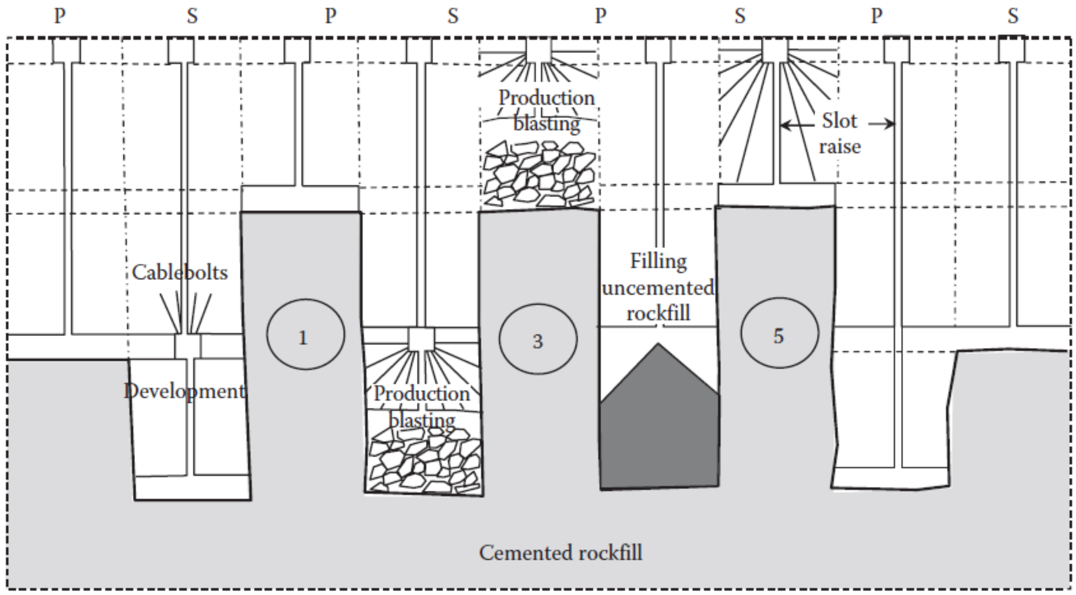

Figure 1-4 – Conceptual longitudinal view of an extraction and filling sequence using primary and secondary stoping geometries (Villaescusa 2004).

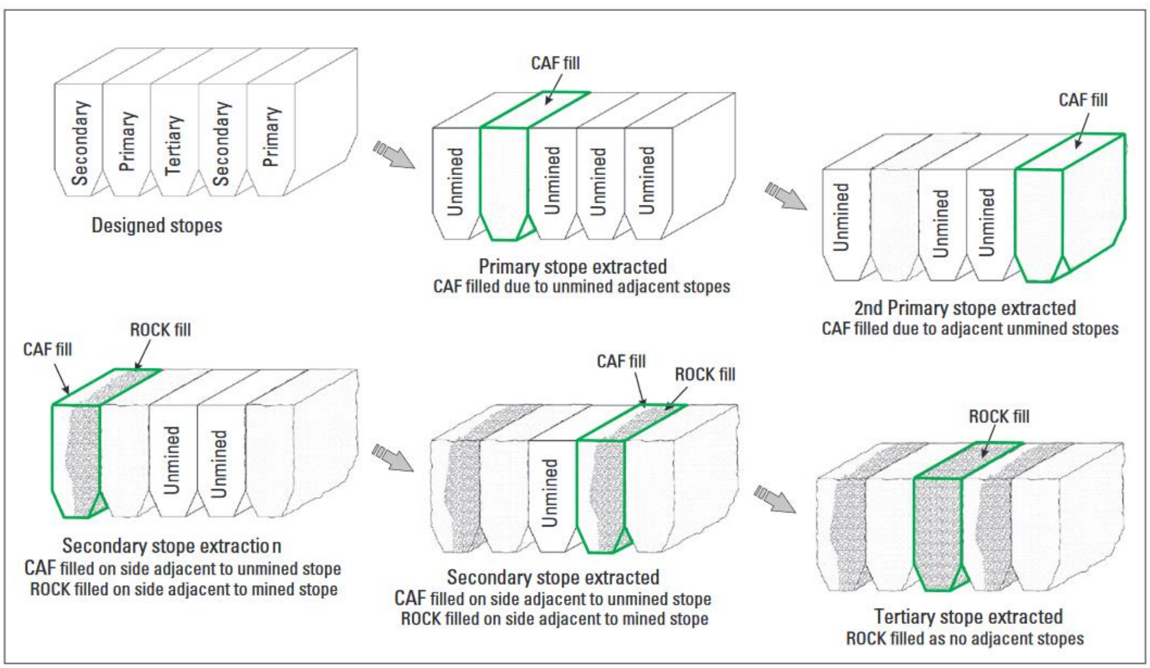

Figure 1-5 – Filling sequence of primary, secondary and tertiary stopes using cemented artificial filling (CAF) (Atlas Copco 2007)

1.3 Deterministic stope design and underground mine production schedule

This chapter reviews traditional deterministic approaches for underground mine planning. It starts with a description of stope design methods, then long-term mine production scheduling is presented, and recent deterministic integrated approaches are discussed. In order to have complete overview of these developments, this literature review comprehensively describes developments that do not apply to a stoping mechanism.

1.3.1 Stope Layout Optimization

Stope design optimization consists of defining mineable volumes and their acceptable shapes, orientations, and positions within the orebody, as well as the necessary infrastructure to guarantee accessibility and stability of the excavation. This optimization proceeds given a definition of the mining method, physical constraints, and the objective of maximizing undiscounted cashflow (Alford et al. 2007). Although this process can be compared to the definition of the ultimate pit limit for an open pit mining problem, few algorithms were developed for the definition of stope boundaries, because it is difficult to define a framework that applies to all existing mining methods and their variants (Appianing et al. 2018; O'Sullivan et al. 2015).

Riddle (1977) presents the first algorithm for the block caving underground mine layout based on dynamic programming. The method is performed in 2D cross-sections by defining rows and columns based on the drawpoint locations and the vertical allowable boundaries. Initially, the profit of each block is calculated individually. Then, moving through the defined columns and dropping the profit values of the ones already evaluated, the cumulative profit for mining the block is calculated. Although the method is optimal for the 2D problem, the process of combining the cross-sections' results to generate a 3D layout makes it non-optimal and infeasible once it is not capable of accommodating the necessary geometrical constraints. In addition, a heuristic approach is applied to define the footwall design (Ataee-Pour 2000). Similarly, Deraisme et al. (1984) presented the downstream geostatistical approach or mathematical morphology approach. The method constructs two algorithms to generate the stope layout for the cut-and-fill and sublevel stoping mining methods given a 2D cross-section of the block model. The proposed approach transforms the original block model into minable volumes subjected to a cut-off grade policy for the mining blocks and stope geometry constraints. In addition to the limitations described for the work presented by Riddle (1977), the mathematical morphology approach does not take into account the profit related to the designed stopes (Erdogan et al. 2017).

The Octree Division Approach (Cheimanoff et al. 1989) is a rule-based heuristic approach to generate mineable stope shapes. The first step of the method relies on gathering the available geological data into a 3D geometric model. Convex mineable volumes are defined such that each volume justifies its separate extraction. Each mineable volume is then divided into octants which are further divided into new sub-volumes until it reaches the smallest allowable mining volume. Throughout this process, the sub-volumes are included or removed from the stope layout according to the geometric and geotechnical constraints, and to the profitability considering the cut-off grade as well as mining and access development costs. This method, however, is not able to analyze the stopes' profitability jointly. In addition, the approach cannot be considered optimal since it is a heuristic (Appianing et al. 2018).

The Floating Stope Algorithm is a practical heuristic based method developed by Alford (1995). This tool has the objective of defining the stope boundaries by maximizing metal grade and content, and the undiscounted accumulated value, while minimizing waste content, given the orebody model, the stopes' geometric parameters, a cut-off grade value, and a minimum head grade. The algorithm assigns each block above the defined cut-off grade to the stope that takes the highest head grade. Thus, two envelopes are generated. The 'outer' envelope includes all overlapping stopes that contain the blocks above the cut-off grade. The 'inner' envelope contains all the blocks above the cut-off grade and respective stopes with the highest grades. The final stope layout is a trade-off of having the highest grades within the 'inner' envelope and the operational shapes within the 'outer' envelope. The search for non-overlapping stopes, although, simplistic, is not able to take into consideration the interaction between stope values and grades, once they are evaluated separately and sequentially. Cawrse (2001) expands this work with the Multiple Pass Floating Stope algorithm that generates a set of stope envelopes for the different input parameters given by the user. This method provides more functionality to the Datamine software. The generated three-dimensional boundaries, however, rely on manual manipulation and decisions made by the user, not guaranteeing optimality in the process.

Alford and Hall (2009) extend the original Floating Stope Algorithm by developing a tool for automated stope design that better adapts to complex geological boundaries. Mineable zones are defined by dividing the orebody model into a regular grid of levels and sections that can represent stopes or mining rings. A seed solution is found by identifying the best combinations of sections and levels. Thus, the stope shape annealing process searches for orthogonal slices to adjust the shape of the stopes based on geometric requirements, and the economic value based on a fixed cut-off grade. The third step adapts the stope layout to satisfy access criteria. The process is automated to allow running it for a range of different possible cut-off grades — that generate nested stope layouts, from which the best layout can be chosen. The geometric and grade outputs, however, do not allow the manual choice to be made in terms of the real stope design profit or considering the effect of the time value of money that can effectively impact the actual cut-off grade throughout the life of the mine (Little et al. 2011).

The Maximum Value Neighborhood (MNV) algorithm (Ataee-Pour 2000) is based on finding the maximum economic value of a stope, subjected to the minimum allowable stope geometry, that defines a block neighborhood. The algorithm evaluates the neighbourhood of each positive valued block and selects the one with the highest economic value. The process is repeated sequentially for all positive blocks that were not included on the best economic value neighborhoods of other blocks, and consequently are not included yet on the stope layout. The algorithm is further extended to the multiple-pass MNV (Ataee-Pour 2006), in which the boundaries generated by the MVN algorithm are further evaluated in terms of ore blocks excluded from the original stope layout, followed by the evaluation of the waste blocks included in the previously generated boundary. The final stope boundaries generated by this algorithm are affected by the order of evaluation of blocks and respective neighbourhoods. In addition, a fixed stope dimension is evaluated, requiring further manual post-processing to generate the actual stope shapes.

Topal and Sens (2010) propose a three-step algorithm to define the stope shapes and locations. The first step consists of plotting within the block model all the possible stopes shapes and locations. Then, the average value of the stope, given a pre-defined envelope of overlapping stopes is calculated and stored with the respective stope. On the third step from a list of all possible stopes, the algorithm greedily chooses the stopes based on the profit, either profit per square meter or the profit per estimated mining time. Once a stope is selected to be included in the final layout, the overlapping stopes are eliminated from the list. This process lacks mathematical background and fails to evaluate all the possible combinations of stopes. Thus, a set of non-overlapping stopes might be neglected if they belong to an envelope in which a third overlapping stope was selected first.

A network flow optimization approach is proposed by Bai et al. (2013) tailored to the sublevel open stoping mining method. The block model is converted from the cartesian grid to a cylindrical grid centered at the initial development raise location. The algorithm is based on graph theory in which the vertical arcs are defined from the center of the block to the footwall and hanging wall considering stope wall constraints while horizontal arcs respect stope width constraints. The maximum flow problem is then solved considering the undiscounted economic values of the blocks.

In contrast to the heuristic method presented by Topal and Sens (2010) and the MVN algorithm (Ataee-Pour 2006) also presented by Sandanayake et al. (2015) is less sensitive to the order in which the stopes are evaluated. The method is based on four main steps: combining stopes based on their allowable size, assigning attributes such as grade and density, accordingly, discarding stopes with negative economic value and generating families of non-overlapping positive stopes. The family with overall highest economic value is selected as the best solution. This last step allows the interaction of stope values and quality as the stopes are not evaluated individually. However, for large-scale applications the method might be inefficient since it requires significant computational power (Erdogan et al. 2017).

The Mineable Shape Optimizer (MSO) (Alford Mining Systems 2022) is a software for stope design that is broadly accepted and used by the industry. The software is based on the algorithm presented by (Alford and Hall 2009) with its functionalities improved throughout the years. The algorithm requires a set of input parameters such as a cut-off grade and respective estimated capacities that are not available at the time of the mine design. In addition, only geometric parameters are considered, neglecting the developments necessary to allow the feasibility of a mine design.

Lastly, Villalba Matamoros and Kumral (2017) propose a heuristic method that aims to maximize the undiscounted profit from selected stopes while minimizing the cost of internal dilution, subjected to maximum acceptable dilution, geometric, and precedence constraints. The mathematical formulation is based on the selection of slices defined as group of blocks. In addition, grades of the slices and their combination into stopes must be above a defined cut-off grade. The heuristic algorithm will evaluate each possible stope configuration starting at any block location. This process, however, might get stuck in a local optimal solution. Therefore, Villalba Matamoros and Kumral (2018) propose a genetic algorithm that takes this output as an initial solution to improve it. In addition, the authors explore the presence of grade uncertainty and variability by proposing a stochastic programing formulation to be further discussed on Section 1.5.2.

1.3.2 Underground Mine Production Schedule

The final step of the conventional production scheduling framework is to define an extraction sequence of the mining units based on the defined design while considering geotechnical, production, and operational constraints. Similarl to open pit long-term mine production scheduling, the general main objective of this process is to optimize the project's NPV. On the other hand, according to the selected mining method, the extraction, preparation, and support activities directly affect the yearly production schedule. Therefore, other than just scheduling the stope sequence, some approaches take into account the sequence of activities (i.e., developments, drilling, blasting, mucking, and backfilling) (Nehring et al. 2010; Sotoudeh et al. 2020). In addition, most of the developments are tailored for specific mining methods as they are applied in real operations. Thus, they are not directly adaptable to other underground mining method variants and applications (Brickey 2015).

Williams et al. (1973) first explored the optimization of underground production scheduling through the development of a linear programing (LP) formulation for the sublevel stoping mining method. The method aims to minimize the fluctuations in terms of ore tonnage production. The assumption in terms of material homogeneity for each level mischaracterizes the selectivity related to the mining method. Early developments presented by Gershon (1983) and Barbaro and Ramani (1986) address mine production scheduling through mixed integer programming (MIP), generalizable to surface and underground mining by considering discrete decisions in terms of the selective mining units. These approaches, however, were not tested for underground mining.

Starting from Trout (1995), a MIP model is proposed to model underground mine production scheduling. The mathematical formulation includes four integer decision variables to describe the selection of time periods for preparation, extraction, backfilling, and an intermediate time period when there is a production phase scheduled for a stope and it remains void. The objective function maximizes the NPV under the production timeframe, capacity, activity precedence, adjacency, and production target constraints. The method is applied to Mt. Isa and Cannington mines in a two-year timeframe and on a monthly scheduling basis. (Nehring and Topal 2007) extend this approach by adding constraints to limit multiple fill mass exposures, such that when a stope is being backfilled, a limit of two adjacent stopes are allowed to be under the other three production phases.

Development activities are added to the MIP formulations by Carlyle and Eaves (2001). Binary decision variables control the drilling and development activities on a quarterly basis considering precedence constraints in order to guarantee the expected operational safety for the cut-and-fill plan. The proposed objective is to maximize the discounted metal ounces produced and consequently the discounted profit. Although these methods incorporate the main phases of the mine production schedule, their application is restricted to limited timeframes since the number of decision variables and constraint necessary to the describe the life-of-mine (LOM) production schedule increases the problem’s complexity. Besides the work presented by Little et al. (2011) and Topal (2008), in order to reduce the number of decision variables of these models, Smith et al. (2003) use aggregation of stopes into mining areas or blocks that are scheduled independently in a yearly basis. Continuous variables describe the quantities and area mined, while integer decision variables control mining and development precedence relationships between blocks and potential capital expansion. Although fewer decision variables are required, a set of initial assumptions such as a fixed predefined cut-off grade is necessary and heuristics are necessary to get a solution in a feasible amount of time.

Kuchta et al. (2004) propose a model that is based on load-haul-dump-fleet (LHD) allocation for long-term sublevel caving production scheduling. The main objective of the method is to minimize deviations from ore production targets, subjected to production capacity, vertical and horizontal precedence, and physical capacity constraints. Newman and Kuchta (2007) propose an aggregation technique in terms of production periods to solve the problems in less of time. A decomposition heuristic is proposed by Martinez and Newman (2011) for solving the scheduling problem for the same mining methods. In this study the equipment allocation is scheduled in long term, while the blocks' production is scheduled in the short term.

McIsaac (2008) proposed an MIP with the objective of maximizing the cash flow and minimizing fixed costs related to development. The author defines a set of independent economic sectors or zones which are scheduled such that mill feed requirements are met. Constraints for maximum development rate are defined for each zone, while ore production and grade blending requirements are applied to all zones simultaneously. The method is applied to a narrow vein deposit, and the effect of each objective is evaluated separately and compared to the developed mathematical formulation.

O'Sullivan et al. (2015) proposes an MIP for the production schedule of Lisheen Mine that combines room-and-pillar, longhole stoping, and drift-and-fill mining methods. The objective function is modeled in terms of maximizing discounted metal production. In fact, no economic information is included as the schedule considers the activity sequencing monthly. The mine design, the mining method assigned to each mining zone, and the cut-off grade are fixed.

Similarly to O'Sullivan et al. (2015) and Smith et al. (2003), Brickey (2015) aggregates the stope layout into different ventilation domains according to the development design. The domains are scheduled in terms of mining activities and resource constraints limit the required airflow for each of these activities, while the mine plan accomplishes the objective of maximizing the NPV.

The Schedule Optimiztion Tool (SOT) developed by Fava et al. (2013) is a software that uses an evolutionary algorithm that searches for the sequence of development networks and extraction of stopes to maximize the discounted cash flows. Hauta et al. (2017) expands the approach to incorporate backfilling requirements and implements the GeoSequencing module that evaluates the best stope-to-stope links through an iterative procedure.

A review of long-term underground mine production schedules is done by Sotoudeh et al. (2020). It is seen that most traditional stepwise production scheduling approaches are based on activity scheduling given the duration of activities in a monthly or quarterly basis. Therefore, the complexity of the mathematical models does not allow an application for the LOM other then when aggregation techniques or heuristics are applied. In fact, recent studies (Campeau et al. 2022) treat these activity scheduling methods as short-term or medium-term approaches. In contrast, integrated approaches (Copland and Nehring 2016; Foroughi et al. 2019; Hou et al. 2019; Little et al. 2013; Little et al. 2011) that do not assumed a predefined stope layout are able to generate a yearly LOM production schedule, and will be reviewed on Section 1.3.3.

1.3.3 Integrated Optimization of Stope Design and Long-term Mine Production Scheduling

The steps concerning underground mine planning are conventionally addressed in a stepwise manner. The previous sections present studies, algorithms and tools that singularly treat the stope layout and the underground production schedule. In order to have an operational design and extraction sequence, another step is required to generate a network layout that links the production areas to the surface. The approaches presented for the network layout optimization minimize the undiscounted development and haulage costs (Brazil et al. 2008; Brazil et al. 2003; Brazil and Thomas 2007). Therefore, the time value of money and the sequence of development corresponding to the mining sequence are neglected when generating this infrastructure layout. In order to consider the interaction among these three main components, an iterative procedure is commonly used. Starting from a predefined cut-off grade, the stope layout is generated, the respective network infrastructure is designed, and the LOM production schedule is optimized (Bootsma et al. 2018; Poniewierski et al. 2003). This process is repeated for different cut-off grades and usually requires manual intervention as the original stope layout might contain stopes with negative economic values or have difficult access when the accesses are considered. Although this procedure presents a convenient way of unifying the underground mine planning components by using commercially available software, they are repetitive and are unable to truly integrate and optimize these interconnected elements (Little et al. 2011). In addition, the requirement of a fixed cut-off grade as the first input deviates the mine planning outputs from the main objective to maximize the NPV.

Little et al. (2011) and Little et al. (2013) propose an integrated approach for the stope layout and production schedule. The process to generate the necessary inputs follows the same traditional stope layout idea in which a set of overlapping stopes is generated. In this approach, the potential stopes have different shapes and locations such that all allowable geometries are explored. In order to reduce the number of decisions, a preprocessing step eliminates the potential stopes with negative economic values. As for the underground scheduling, the proposed mathematical formulation is constructed as an MIP, where a decision variable selects the starting period of a stope production, while the NPV is maximized. Non-overlapping, adjacency, geotechnical offset, and leveled drawpoint basis constraints to allow an operational design and schedule. An application at a gold deposit shows a higher NPV compared to the traditional stepwise approach. This approach, however, does not integrate the related discounted development costs and the practical cumulative development distances to access the stopes.

Copland and Nehring (2016) expand the previous proposed MIP by adding a decision variable that controls the sequence of each level development. These binary variables define if a level is opened, and in which period the level is developed. Each stope is flagged according to its extraction level. Therefore, linking constraints ensure that the stope extraction occurs only after level access is constructed. Besides the time discounted profit of a stope, the objective function accounts for the discounted value of developing a level. Thus, the mathematical formulation aims to maximize a better estimate of the actual NPV. The final output accounts for the stope layout, stope extraction sequence, and level development sequence. However, the method assumes that once the level is developed, all the stopes within that level are accessible, ignoring horizontal cumulative development costs of drifts and crosscuts.

Hou et al. (2019) proposed a joint optimization of the stope layout and scheduling with vertical and horizontal access constraints. The proposed MIP takes a set of overlapping stopes that are grouped into level subsets, a vertical shaft position, and a piecewise horizontal development for each possible level. The decisions variables select when to mine each possible stope, when to develop the shaft segments according to each level, and when to develop the horizontal access segments according to each level and stope position. A set of constraints ensure that only accessible levels can have horizontal access developed and only accessible stopes can be extracted. The objective function maximizes the discounted cash flow. Similarly, Foroughi et al. (2019) develop a MIP that maximizes two weighted objectives: the NPV and metal recovery, while jointly considering the production schedule and stope layout. This work differs for the latter, once instead of having fragmented drifts, it is assumed that a single drift is developed at each level, connecting the most centralized stope entry to the shaft. In addition, the method is applied to a three-dimensional iron ore deposit and is solved using a genetic algorithm. Although these methods jointly cover the three main components of the mine planning optimization, they are still not able to account for the cumulative time-valued horizontal development of drifts and crosscuts. Also, the assumption of piecewise shaft development does not correspond with real mining practice.

1.4 Mineral Deposit Modeling

The previous sections present a review of deterministic approaches for underground mine planning. A single estimated orebody model is the considered as the main input for these methods, which contains the necessary information of spatially distributed geological attributes of the mineral deposit. Consequently, these deterministic approaches fail to incorporate or to manage the geological uncertainty and spatial variability of the mineral deposit, which are shown to be critical sources of technical risk in a mining project (Baker and Giacomo 1998; Vallée 2000). This material supply uncertainty is due to the limited amount of sampled data from exploration and further grade control sampling that do not provide a full precise knowledge of the orebody material characteristics (Goovaerts 1997; Rossi and Deutsch 2014).

Traditionally used estimation approaches such as kriging provide the minimum-error-variance linear unbiased estimate (David 1977, 1988; Goovaerts 1997; Isaaks and Srivastava 1989; Journel and Huijbregts 1978). This commonly used method, however, generates a smooth representation of mineral deposit. Specifically, the proportions of high and low grades are misrepresented. Therefore, this smoothing effect does not allow the reproduction of spatial statistics of sample data and underlying geological patterns observed in the ground, generating an average-type representation of the orebody (Chilès and Delfiner 1999; David 1977, 1988; Goovaerts 1997; Journel and Huijbregts 1978; Rossi and Deutsch 2014). An average input to a non-linear transfer function does not generate an average output. Thus, an average-type orebody model does not guarantees an average assessment of the long-term production schedule (Dimitrakopoulos et al. 2002; Dowd 1994; Qureshi and Dimitrakopoulos 2005; Ravenscroft 1992).

Stochastic simulations can be used to generate equally probable representations of the orebody to model the spatial uncertainty and variability of material quality and quantity, while reproducing spatial statistics (e.g., mean, variance, variograms) of available sample data (David 1977, 1988; Deutsch and Journel 1997; Goovaerts 1997; Journel 1994; Mariethoz and Caers 2015; Mustapha and Dimitrakopoulos 2010b). The impact of supply uncertainty on deterministic mine production schedules has been evaluated by various authors through risk assessment. It has been shown that misleading forecasts are produced in terms of grades, tonnages and cashflows when conventional approaches are used (Dimitrakopoulos et al. 2002; Dowd 1994; Ravenscroft 1992). Recent studies show similar outcomes for underground mine designs. Dimitrakopoulos and Grieco (2009) show how conventional methods for underground mine design are unable to capture the upside potential and/or downside risk of meeting forecasts, which are tied to the presence of uncertainty and variability of grades and material types. Furtado e Faria et al. (2022b) compare the propose stochastic optimization of stope design to a bechmarked deterministic stope layout optimization software. Besides the flexibility and the more realistic assumptions of cumulative horizontal devlopment costs, a risk analysis shows that there were pontentialy high-grade stopes to be included in the laylout that were not included by the determistic approach. It is concluded that the somoothing effect underestimates high-grade blocks, therefore this upside potential is masked when an average-type block model is given as the input.

In the following sections, traditional and more recent developments on stochastic simulations of mineral deposits are discussed.

1.4.1 Sequential Simulation Framework

Conceptually, a random field or a random function (RF) is a set of a random variables over a domain and can be fully described by a joint probability density function (jpdf). Geological phenomena can be modeled as a stationary and ergodic RF in which the attributes of interest, such as grades, densities and material types are described by spatially distributed random variables (Chilès and Delfiner 1999; Goovaerts 1997; Journel 1994; Journel and Huijbregts 1978; Rossi and Deutsch 2014).

Considering $Z(\boldsymbol{u}_i)$ a stationary ergodic RF indexed in R^n, where $\boldsymbol{u}_i$ represents the location of the points $i = 1 \dots N$ of the grid to be simulated in a domain $D \subseteq R^n$. The set $\boldsymbol{d}_n = \{z(\boldsymbol{u}_\alpha), \alpha = 1 \dots n\}$ denotes the original sample data conventionally obtained by exploration data. A set Λ_i represents the conditioning data for each node indexed by *i*. Therefore, $\Lambda_0 = \{\boldsymbol{d_n}\}$ is the conditioning data when the first point is simulated and only sample data is available and $\Lambda_i = \{\Lambda_{i-1} \cup Z(\boldsymbol{u}_i)\}$ is the conditioning data for the subsequent points being simulated that includes the original sample data and previously simulated points. Accordingly, the sequential simulation approach defines that the joint probability density function of the random field $Z(\boldsymbol{u}_i)$ can be decomposed into the product of conditional multivariate distributions as shown in Eq.1.1 (Dimitrakopoulos and Luo 2004; Goovaerts 1997; Journel 1994)

$$f_{\boldsymbol{Z}}(u_1, \dots, u_N; z_1, \dots, z_N | \boldsymbol{d_n}) = f_{Z_1}(\boldsymbol{u}_1; z_1 | \Lambda_0) \times \dots \times f_{Z_N}(\boldsymbol{u}_{\mathrm{N}}; z_N | \Lambda_{N-1})$$
$$= \prod_{i=1}^{N} f_{Z_i}(\boldsymbol{u}_i, z_i | \Lambda_{i-1}) \tag{1.1}$$

Therefore, the sequential simulation process starts by defining a simulated path to visit the non-simulated nodes. At each node location, a random value is drawn via Monte Carlo sampling from the conditional pdf initially generated based on the original sample data. This new value is added to the conditioning data set (Λ_i). The process is repeated for all the nodes of the simulation grid by always considering the original samples and previously simulated nodes as conditioning data to generate a simulation of the orebody. In order to obtain additional realizations, the same process is repeated following a different random path. For any simulation method applied, each generated scenario must reproduce the spatial statistics of the original data, including histograms, variograms (covariance), and higher-order statistics (Chilès and Delfiner 1999; Dimitrakopoulos et al. 2010; Goovaerts 1997; Journel 1994; Mariethoz and Caers 2015; Remy et al. 2009; Rossi and Deutsch 2014).

1.4.2 Sequential Gaussian Simulation Methods

The sequential Gaussian simulation (SGS) method is a conventionally used approach based on the sequential simulation paradigm. This method assumes a multi-Gaussian RF model; thus, each univariate distribution is Gaussian. Although natural phenomena might not follow a normal (or Gaussian) distribution, this assumption is made because a Gaussian distribution can be fully characterized by its mean and variance (first and second order cumulants), since all its higher order cumulants equal zero. Therefore, it is called a parametric approach that uses the Kriging system to obtain the conditional mean and variance to generate the conditional cumulative distribution function (ccdf) from which the simulated values will be randomly sampled. In order to use the SGS method, the data must be transformed to normal space prior to following the sequential simulation steps. The experimental variogram must be calculated from this transformed data and the variogram model must be inferred (David 1977, 1988; Deutsch and

Journel 1997; Goovaerts 1997; Isaaks and Srivastava 1989; Journel and Huijbregts 1978; Remy et al. 2009). Once the simulated values are available, they are back-transformed to the original data space.

The lower-upper (LU) triangular decomposition technique is proposed to be used for conditional simulation (Davis 1987). The method is used to simulate a set of values simultaneously which has been shown to be faster than SGS using screen effect approximation (SEA) (Luo 1998), which limits the amount of conditioning data to a searching neighbourhood. However, the LU simulation technique has a higher memory requirement in order to manage sparce covariance matrix. Luo (1998) proves the equivalence of the triangular decomposition method to SGS, allowing the development of the generalized sequential Gaussian simulation (GSGS) method (Dimitrakopoulos and Luo 2004). The GSGS method takes advantage of overlapping neighbourhoods of simulation nodes, by simulating each group of values simultaneously via LU decomposition while using SGS assumptions and the sequential simulation process. The equivalence of GSGS to the other two described methods is explored by Dimitrakopoulos and Luo (2004) by examining the number of nodes to be simulated (N) and the number of nodes in the neighbourhood (v). It is shown that GSGS is computationally equivalent to SGS when $v = 1$ and equivalent to LU simulation if $v = N$. Thus, the balance between the computational efficiency and the effectiveness of the GSGS method related to the group size v can be assessed through the SEA loss (Benndorf and Dimitrakopoulos 2018).

Computational improvement in terms of memory handling using the direct block simulation algorithm (DBSIM) is proposed by Godoy (2003). The method simulates all the nodes within a block simultaneously as in GSGS and only retains the averaged value of these points that corresponds to the final simulated block value (Benndorf and Dimitrakopoulos 2018). In contrast to the previously discussed methods where the simulations are performed in point support, this process eliminates the need of reblocking the simulated values in terms of the size of the selective mining unit. All the simulation methods discussed require input data transformation to the normal space. It is known, however, that a Gaussian RF has maximum entropy. Consequently, the simulated values misrepresent the connectivity of extreme grades. Godoy (2003) show improvements in terms of high-grade connectivity when DBSIM is used.

Multiple attributes of interest are frequently necessary to describe the important mineral deposit characteristics. These attributes' values are usually correlated and must be simulated jointly to improve predictions regarding the mineral deposit. An SGS co-simulation approach is proposed by Verly (1993) that integrates the simulation of different types of variables. The necessity of modeling the cross-covariances in addition to the covariances and the size increase of the kriging systems in terms of the number of variables being simulated make the application of this method computationally expensive and impractical. A decorrelation technique using principal component analysis (PCA) is proposed prior to simulating each decorrelated factor separately (David 1988). This method, however, is based on the decorrelation of the variance-covariance matrix at lag zero, which means that the correlation of the spatially distributed data is ignored. Desbarats and Dimitrakopoulos (2000) study the use of minimum/maximum autocorrelation factors (MAF) to simulate multiple attributes by performing additional PCA decorrelation assuming a small lag distance and generating the corresponding number of MAF factors which can be simulated individually and then back transformed to the correlation space using the inverse mathematical process. In addition, Boucher and Dimitrakopoulos (2009, 2012) propose the DBMAFSIM algorithm that uses MAF to generate decorrelated factors that are individually simulated directly on block support.

1.4.3 Geostatistical Simulation Methods that Account for Multiple-point and High-order Statistics

The previously reviewed traditional simulation techniques are based on second-order statistics that are able to fully characterize Gaussian random functions. It is known, however, that natural phenomena do not follow a normal distribution. Therefore, this assumption limits a proper characterisation of complex geological patterns in the presence of non-Gaussianity and non-linearity (Journel and Alabert 1989). In addition, Gaussian RFs maximize the entropy, or spatial disorder, especially in terms of high-grade connectivity (Dimitrakopoulos et al. 2010; Journel 2005; Journel and Deutsch 1993). Thus, methods that can infer the natural spatial connectivity and reproduce higher-order statistics are proposed and reviewed in this section, as well as their impact on mine production schedules and forecasts.

1.4.3.1 Multiple-point simulation methods

Methods based on multiple-point statistics (MPS) were initially applied to oil and gas reservoir modeling of curvilinear structures (Guardiano and Srivasta 1993; Journel 2005; Remy et al. 2009; Strebelle 2002; Zhang et al. 2006). In contrast to traditional random field simulation frameworks, each conditional probability distribution function (cpdf) is inferred from a training image (TI), without making any assumptions about it (Guardiano and Srivasta 1993; Journel 2005; Mariethoz and Caers 2015). TIs represent the complex geological structures but do not necessarily contain a local description of the geological attribute of interest. They contain densely sampled information acquired from exploration, or grade-control data. Alternatively, they can be artificially generated (Osterholt and Dimitrakopoulos 2018; Strebelle 2002). A searching neighborhood configuration or spatial template, is used to search for replicates in the TI and build the cpdf for the data-event of the center node being simulated. This type of algorithm is called 'pixel-based' and it follows the sequential approach described herein (Guardiano and Srivasta 1993; Journel and Deutsch 1993). ENESIM (Guardiano and Srivasta 1993), SNESIM (Strebelle 2002), Direct Sampling (Mariethoz et al. 2010) and IMPALA (Peredo and Ortiz 2011) are exemples of 'pixel-based' algorithms. In contrast, 'pattern-based' algorithms store different patterns found in the TI and compare them with the configuration of the conditioning data using a similarity metric to determine the most similar pattern, which is directly pasted to the simulation grid. 'Pattern-based' methods include SIMPAT (Arpat and Caers 2007), FILTERSIM (Zhang et al. 2006), WAVESIM (Chatterjee et al. 2012) and CCSIM (Tahmasebi et al. 2012). While 'pixel-based' methods are computationally expensive, 'pattern-based' algorithms have a larger memory requirement. These MPS approaches tend to be limiting in terms of the reproduction of sample data statistics. The direct inference of patterns from a TI lead to simulations that reproduce the spatial statistics of this TI. A consistent mathematical modeling approach should be data-driven (Goodfellow et al. 2012; Osterholt and Dimitrakopoulos 2018; Yao et al. 2018).

1.4.3.2 High-order simulation methods

Overcoming the limitations of MPS algorithms coming from the lack of a mathematical formalism, Dimitrakopoulos et al. (2010) introduce the use of high-order cumulants to explicitly infer high-order statistics from data. Cumulants and moments describe the behavior of a distribution. The knowledge of all infinite moments allows the full description of a RF.

Therefore, assessing the cumulant values beyond the second-order enables the modelling of non-linear and non-Gaussian random functions. Studies investigate the suitability of using moments and cumulants to describe geological patterns (Dimitrakopoulos et al. 2010; Mustapha and Dimitrakopoulos 2010c). The high-order simulation (HOSIM) algorithm uses the concept of cumulants and the sequential simulation method to generate stochastic simulations of the mineral deposit that show the natural connectivity of high grades and reproduce complex geometries (de Carvalho et al. 2019; Dimitrakopoulos and Yao 2020; Minniakhmetov and Dimitrakopoulos 2017b; Minniakhmetov et al. 2018; Mustapha and Dimitrakopoulos 2010b, 2011; Yao et al. 2018). HOSIM uses TIs as complementary information to the original exploration data to calculate the cpdf. Since no assumptions are made in terms of the data distribution, studies propose the use of a series of orthogonal polynomials to approximate the cpdf. Legendre polynomial (Mustapha and Dimitrakopoulos 2010b, 2011), Laguerre polynomials (Mustapha and Dimitrakopoulos 2010a) and Legendre-like orthogonal splines (Minniakhmetov and Dimitrakopoulos 2021; Minniakhmetov et al. 2018) are some of the orthonormal functions explored for the mentioned use. Eq. 1.2 shows the approximation of the cpdf using Legendre polynomials where $\bar{P}_{m_n}$ are the normalized Legendre polynomials of order *m* and $\bar{L}_{m_0,m_1 \dots m_n}$ are the Legendre coefficients that are inferred by first defining a spatial template based on neighbourhood nodes formed by the conditioning data and then searching for replicates in the exploration data and in the TI. This approach is shown to be data-driven, reproducing the geological patterns as well as the low and high-order statistics of the conditioning data (Goodfellow et al. 2012).

$$f(u_{\mathrm{i}}; z_i | \Lambda_0, \Lambda_{i-1}) = \frac{\sum_{m_0=0}^{\omega_0} \sum_{m_1=0}^{\omega_1} \dots \sum_{m_n=0}^{\omega_n} \bar{L}_{m_0,m_1 \dots m_n} \bar{P}_{m_0}(z_0) \dots \bar{P}_{m_n}(z_n)}{\int f(u_{\mathrm{i}}; \lambda_0, \lambda_{i-1}; z_0, \Lambda_0, \Lambda_{i-1}) du_i} \quad (1.2)$$

Minniakhmetov and Dimitrakopoulos (2017b) expand the proposed method to allow the simulation of multiple spatially correlated variables. As MAF relies on multi-Gaussian distribution assumption that does not coexist with the proposed simulation method, a diagonal domination of high-order cumulants condition is used to decorrelate the attributes, which can then be simulated independently. Minniakhmetov and Dimitrakopoulos (2017a) Minniakhmetov and Dimitrakopoulos (2021) propose a data-driven high-order simulation method using high-order indicator moments. The method uses a recursive B-spline approximation to calculate high-order spatial cumulants from hard data, showing better computational efficiency than the previous implementations. Yao et al. (2018) propose a numerical approximation of the cpdf using a multivariate Legendre polynomial series through a recursive approach. This method does not need the explicit calculation of cumulants, allowing the parallel computation of replicates. A natural extension of these approaches for mining applications through direct block HOSIM is proposed by (de Carvalho et al. 2019). The method requires a TI or geological analog both in point and block support, as well as the corresponding spatial temples.

The high-order simulations reproduce low- and high-order spatial statistics of the sample data and show better spatial connectivity of high-grade values in comparison to the traditionally used SGS method (Mustapha and Dimitrakopoulos 2010b). In addition, the impact of using different simulation algorithms to generate the geological inputs for non-linear transfer functions has been studied by Qureshi and Dimitrakopoulos (2005). de Carvalho and Dimitrakopoulos (2019) compare the open-pit mine production schedules and forecasts when SGS and HOSIM are used to generate the simulated orebody models that are input to a simultaneous stochastic optimization framework. This application has shown that the sequence of extraction of mining blocks favors the high-grade continuity area when HOSIMs are used. Also, different pit limits are seen and more gold is produced at the end of the life-of-mine (LOM) causes a higher NPV when HOSIMs are the given inputs.

1.5 Long-term Stochastic Mine Planning

As previously discussed, conventional mine planning frameworks are deterministic. Thus, they use a single estimated, or average-type, orebody model as an input to produce mine production schedules. It has been shown, however, that these deterministic approaches produce unrealistic forecasts in terms of ore production, grades and cashflows (Baker and Giacomo 1998; Dowd 1994; Ravenscroft 1992; Vallée 2000). The limitations of using a smooth representation of the orebody and ignoring the geological uncertainty motivate the development of stochastic frameworks. This section starts with a review of open-pit stochastic production scheduling studies that lead to state-of-the-art simultaneous stochastic optimization of mining complexes. Then, stochastic stope design and underground production scheduling methods are presented.

1.5.1 Simultaneous Stochastic Optimization of Open Pit Mine Planning

The first approach that incorporates geological risk in long-term mine planning is proposed by Dimitrakopoulos et al. (2007). The approach generates multiple schedules according to each realization of the orebody using a conventional open-pit mine planning method. Subsequently, a risk analysis is carried out for each LOM plan in order to assess the maximum-upside potential and minimum downside risk, and then select the schedule that shows the best performance based on key performance indicators (KPI), such as a minimum acceptable return (MAR) on investment. Although this method is simple by allowing the use of an existent deterministic scheduling software, it relies on the subjective choice of a KPI that well describes the projects performance. In addition, it is not capable of directly incorporating risk management during the scheduling decision making process.

Geological uncertainty is directly incorporated into an MIP by Dimitrakopoulos and Ramazan (2004). The method takes a probabilistic orebody model that is built up based on multiple simulations and the respective probability of each block showing a certain attribute value, such as being within a grade range. The formulated objective function maximizes the probability of meeting ore tonnage and grade requirements, while deferring the extraction of areas with lower probabilities of having the target properties, introducing the idea of risk discounting. Dimitrakopoulos and Grieco (2009) also uses a similar probabilistic framework to generate a stope design for a copper mine. This approach is reviewed with more details in the next section. Probabilistic methods, however, uses limited information regarding geological variability in the deposit. In addition, the assessment of uncertainty is restricted to individual blocks, stopes, or panels instead of capturing the joint uncertainty of the combination of these mining units that are extracted simultaneously.

Godoy and Dimitrakopoulos (2004) propose a multi-step mine production scheduling framework under geological uncertainty. Initially, the cumulative graph of ore production and waste removal is calculated considering the best case (pit-shell-by-pit-shell) and the worst case (bench-by-bench) scenarios of conventional open-pit mine planning for each orebody simulation, given a pit-limit and set of cutbacks. Therefore, a stable solution domain (SSD) can be derived considering the inner part of all the cumulative ore and waste graphs. A linear programing (LP) formulation is used to optimize the schedule of ore production and waste removal, obtaining the optimal mining rates by maximizing the discounted cashflow. Subsequently, a conventional scheduler is used to generate one mining sequence for each orebody simulation, using the previously determined optimal mining rates. These extractions sequences are combined using a simulated annealing algorithm (Kirkpatrick et al. 1983) that minimizes the deviations from ore and waste production targets and produces a single production schedule. An application at a gold mine shows an expected improvement of 28% in the NPV compared to the conventional deterministic schedule and lower deviations from ore and waste production targets. Leite and Dimitrakopoulos (2007) present a case study using this method at a copper deposit in which improvements on NPV and ore and waste production are also seen. A study is conducted by Albor Consuegra and Dimitrakopoulos (2009) to analyze the impact of the number of simulations in the final schedule. It is concluded that 10 to 15 simulations produce stable final schedules, justified by the support-scale effects. The authors also investigate the potential of having different pit limits than those obtained with conventional ultimate pit limit (UPL) optimizers (Lerchs and Grossman 1965), observing larger pit limits and higher NPV when the simulated-annealing based method is applied.

Stochastic integer programming (SIP) has been presented in the technical literature as a mathematical background to address strategic mine planning under uncertainty (Birge and Louveaux 2011). A two-stage SIP can be defined as follows:

$$\min z = c^T x + E_\xi[\min q(\omega)^T y(\omega)] \tag{1.3}$$

$$s.t. Ax = b \tag{1.4}$$

$$T(\omega)x + Wy(\omega) = h(\omega) \tag{1.5}$$

$$x \geq 0, y(\omega) \geq 0 \tag{1.6}$$

Where x represents a vector of the first-stage decision variables and $y(\omega)$ are the second-stage decision variables. The minimization of the objective value z is presented in Eq. 1.3, with two components. The first component $c^T x$ corresponds to a vector of objective coefficients associated with the first-stage decisions. The second component consists in the second-stage vectors coefficients $q(\omega)^T$ associated with the second stage decision variables $y(\omega)$, which are function of a random event ω. Eq. 1.4 shows the constraints associated only with the first-stage decisions that are taken before the uncertainty is revealed, with the matrices of coefficients A and

b. Eq. 1.5 shows the stochastic constraints which allow the second-stage decisions to adapt to the first-stage decisions once the random scenarios are observed. Thus, the matrix $T(\omega)$ and W are the matrices of coefficients related to x and $y(\omega)$, constrained by the random vector $h(\omega)$. $\xi = [q(\omega)^T, T(\omega), h(\omega)0\ ]$ is, thus, the vector of second-stage coefficients. Lastly, Eq. 1.6 presents the non-negativity constraints. In addition, it is mathematically proven that the value of a stochastic program (VSP) or value of a stochastic solution (VSS), that is, the difference between the expected stochastic programming value solution (ESS) and the deterministic expected value solution (EVS) is always non-negative (Birge and Louveaux 2011).

The first formulation of the stochastic open-pit mine production scheduling using a two-stage SIP is presented by Ramazan and Dimitrakopoulos (2005). The objective function considers the maximization of the NPV, while managing the risk of not meeting production targets. The geological risk management term is multiplied by a geologic risk discounting coefficient (GRD), that attributes different penalty costs at different time periods encouraging the schedule to postpone the extraction of more uncertain areas. The first stage decision variables are binary and define the production period that a block is extracted. The second stage decision variables correspond to the deviations from ore tonnage, metal, and grade production targets. The model is applied on a two-dimensional gold deposit. Using this mathematical framework, Dimitrakopoulos and Ramazan (2008) show through a case study that the VSP for a gold deposit corresponds to $64M (9% increase in the original expected NPV) and $60M at a copper deposit (25% increase in the original expected NPV). An extension of the mathematical model to include stockpiling decisions and reclamation of blocks is presented in Ramazan and Dimitrakopoulos (2013). In order to maintain the linearity of the problem, a fixed yearly grade for the stockpile is assumed. Benndorf and Dimitrakopoulos (2013) extend the SIP to incorporate grade blending requirements in a multi-element deposit and also implement smoothing operational constraints similarly to Dimitrakopoulos and Ramazan (2004). Several applications of this framework show generally higher expected NPV, higher recovered metal, and lower deviations from production targets compared to deterministic schedules (Benndorf and Dimitrakopoulos 2013; Leite and Dimitrakopoulos 2014; Ramazan and Dimitrakopoulos 2013). In addition, the effect on the order of magnitude of the per unit penalty costs for each type of deviation is investigated. Besides the assumption of perfect homogenization at the stockpiles, this approach is based on economic values of blocks. Therefore, a predefined cut-off grade policy (Lane 1964, 1988; Rendu 2014) defines the processing stream destinations of individual blocks, instead of capitalizing on the values of products sold.

Menabde et al. (2007) propose a non-anticipative stochastic model to optimize the cut-off grade and the mine production schedule jointly by maximizing the NPV of the project. The proposed model does not explicitly integrate risk management. Capacity constraints are thus applied on the average outcome of all simulated scenarios. In addition, this approach can only deal with a single element. Boland et al. (2008) propose a muti-stage SIP that relies exclusively on scenario dependent decision variables where mining and processing decisions can change according to the uncertainty. Non-anticaptivity constraints restrict mining decisions to be changed in a one-year lag, while processing constraints can change immediately. This approach, however, assumes that each geological scenario individually represents reality, which is not true. Thus, it generates a set of production schedules which is not possible to be operationally implemented.

In order to produce a single risk resilient mine production schedule that simultaneously accounts for the different processes and the values of products sold, the simultaneous stochastic optimization of a mining complex is explored (Goodfellow and Dimitrakopoulos 2016, 2017; Montiel and Dimitrakopoulos 2017, 2015, 2018). A mining complex is defined as a mineral value chain where various interacting activities allow the extraction and transformation of material into sellable products. A 'global optimization' of complex operations is proposed by Whittle (2007); Whittle (2010) as a commercial tool known as Prober C. This tool allows an optimization of the extraction sequences of multiple deposits followed by the definition of the processing stream decisions. Although Prober C incorporate various parts of a mining complex into its optimization framework, it still relies on a stepwise process that does not optimize all components of interest simultaneously. In the past decade, several approaches were presented to overcome the limitations of stepwise methods by jointly optimizing the interconnected components of mining complexes using MIP (Hoerger et al. 1999; Stone et al. 2007). Nevertheless, these approches are deterministic and rely on simplifications, such as block aggregation that missrepresents the materials' selectivity, unceratinty, and the variability. Therefore, two-stage SIPs are proposed for the simultaneous stochastic optimization of mining complexes. The objective functions follow the general idea presented in Ramazan and

Dimitrakopoulos (2005), which aims to maximize the NPV while mininimizing the deviations from production tragets, while simultaneously considering all scenarious that describe supply uncertainty as well as other sources of uncertainty. In contrast to the previous propositions based on cut-off grade destination policy, block-based (Montiel and Dimitrakopoulos 2017, 2015, 2018) and clustered-based (Goodfellow and Dimitrakopoulos 2016) destination policies are explored. Capital investment (Capex) decisions (Goodfellow and Dimitrakopoulos 2015, Del Castillo and Dimitrakopoulos 2019, Farmer 2016), tailing managements (Saliba and Dimitrakopoulos 2019), transportantion options and process operating modes (Montiel and Dimitrakopoulos 2015) are other components and decisions incorporated in the optimization. This stochastic framework is extended to incoporate different sources of uncertainty beyond the typical grade uncertainty. Kumar and Dimitrakopoulos (2017) incorporate geo-metallurgical uncertainty, commodity price uncertainty is added by Saliba and Dimitrakopoulos (2019) and Both and Dimitrakopoulos (2020) include simulated productivity and availability of shovels and trucks, respectively. Most of these approaches were not tested or do not suit directly to mining complexes with underground operations.

1.5.2 Stochastic Stope Design and Underground Mine Production Scheduling

The advances on stochastic optimization for underground mine production scheduling are limited compared to those addressing open-pit scheduling. Besides the difficulty of developing a mathematical formulation that adapts to all underground mining methods and their variants, a large number of operational components must be taken into account that lead to highly complex problems (O'Sullivan et al. 2015). In addition, most of the existing stochastic approaches address the stope layout or mine production schedule individually. The literature review that follows goes beyond the sublevel stoping methods in order to provide an overview on how uncertainty is incorporated to underground applications.

Grieco and Dimitrakopoulos (2007) propose a probabilistic method to address stope design optimization under uncertainty for the sublevel open stoping mining method. First, the block model is regularized into mineable rings, which are aligned transversely in terms of panels and form horizontal layers bounded by haulage levels. An MIP is developed for the selection of rings within a panel, by maximizing the metal content while assuming an average grade of the ring above a specified cut-off grade and its respective probability of occurrence. These two values are calculated based on a set of geostatistical simulations of the mineral deposit. Geometric constraints are imposed such that a minimum and maximum number of sequential stopes within a panel respect the allowable stope size and proportional pillar dimensions. In addition, a constraint controls the acceptable level of risk of a ring being above the defined cut-off grade. The method is applied to a copper mine considering different levels of acceptable risk. Although the design generated with 100% acceptable risk shows better downside potential, it is not able to take advantage of blending low and high grades and meeting production demands. The method can be used to assist the mine planner to manage project parameters while assessing its upside potentials and downside risks. This probabilistic approach, however, is not able to explicitly manage the geological risk, or capture the spatial uncertainty and variability.

Villalba Matamoros and Kumral (2018) propose a stope design framework that incorporates geological uncertainty. The method uses the same mathematical background as in Villalba Matamoros and Kumral (2017), through an MIP that maximizes the stope profit and minimizes the internal sector dilution while a stope layout is obtained through a heuristic approach for each geological scenario. Subsequently, a clustering process based on the Euclidian distances, or dissimilarities, among the set of stope designs generated on the previous step defines an average stope layout. This new layout, however, might violate geotechnical and operational requirements regarding stope shapes and adjacencies. Therefore, two additional steps to fix potential constraint violations and reduce domain size are performed. Based on the average stope layout, a pool of initial feasible solutions is generated and combined using a genetic algorithm (GA) to produce a single final stope layout. An application at a gold deposit shows that the GA method is able to generate a near-optimal solution in reasonable amount of time. In addition, a comparison of the approach with a corresponding deterministic approach shows an improvement of 12% in terms of the undiscounted profit and a considerable difference on the internal stope dilution. This approach, however, does not directly manage the geological uncertainty. The clustering step, instead, smoothes out the uncertainty and variability of grades within stopes that are given as a feasible pooling parameter to generate the initial solution candidates to the GA.

A deterministic two-stage heuristic algorithm is proposed by Nikbin et al. (2020) that combines a dynamic programming technique to find the best stopes within different strips and a greedy search to combine each strip layout into the final stope boundaries, for the sublevel stoping method. This approach is modified by Wilson (2020) to incorporate grade uncertainty, by first defining a stochastic objective function that maximizes the expected economic value of individual blocks or potential stopes while minimizing the respective standard deviation according to a set of realizations of the orebody and a penalty parameter. Although a set of realizations of the orebody is a direct input to this method, the mathematical formulation does not allow a true stochastic optimization that is able to manage the uncertainty. The second component of the objective function that accounts for the variance minimization does not reflect a minimization of risk. Instead, it might lead to an avoidance of high-grade zones and miss upside potential in contrast to the downside risk and low tolerance for grade blending. In addition, the penalty parameter plays and important role in the heuristic procedure, not necessarily representing a financial representation of risk.

Furtado e Faria et al. (2022b) presents a new SIP for stope design with the sublevel open stoping mining method. The proposed method starts with preprocessing steps in which a set of overlapping levels and overlapping stopes are generated according to the block model geometry. A level and stope adjacency search is also performed in order to define sets in which geomechanical constraints will be further imposed. Three main binary decision variables are defined to select the type of vertical access, the level positions, and the corresponding stopes. The proposed objective function maximizes the undiscounted profit of selected stopes by considering the overall vertical and horizontal development costs while minimizing the total deviations from the project's capacities and the difference between the metal content of all blocks in a selected level and the recoverable metal within a stope. The minimization terms account for an economic penalty applied to each second-stage decision variable associated with a geological scenario. Linking, adjacency, and capacity constraints are defined to generate an operational stope design. The stochastic method is compared to an industry-standard stope design tool, showing the advantages of incorporating grade uncertainty and variability in the optimization. In addition, this new method better manages dilution, as it is able to define the position of production levels and pillars, which are an input to the benchmark tool. Finally, the stochastic optimization produced a layout with 21% higher recoverable metal and 4% higher undiscounted profit.

Montiel et al. (2016) propose a two-stage SIP to globally optimize mining complexes with open-pits, underground mines, stockpiles, and processing stream destinations under geological uncertainty. Although the method comprises the previously reviewed state-of-the-art optimization approaches for mining complexes (Goodfellow and Dimitrakopoulos 2016, 2017; Montiel and Dimitrakopoulos 2017, 2015, 2018), it requires a predefined stope design. In addition, the general mathematical formulation does not allow a specific application that will generate an operational extraction sequence for specific underground mining methods. The lack of specific physical and operational constraints allows only a general application in which the underground mine within a mining complex is scheduled in terms of production zones.

Carpentier et al. (2016) propose a two-stage SIP that takes as input mineralized lenses, considered as mining units that can be accessed independently through ramps and horizontal drifts. A set of grade-tonnage curves represents each lens in considering the information contained in the orebody simulations. Three binary decision variables control the time-period selection of development and extraction activities, the cut-off grade of each lens, and when the lens is mined. Continuous decision variables control the proportion of each activity developed at each time-period, the amount of waste moved to and from each mine, and the deviations form production targets. It is assumed that each lens belongs to a mine, with a single associated cut-off grade. The objective function maximizes the NPV from mining the lenses while also considering the minimization of the selected development costs and operational costs. In addition, the last component of the objective function minimizes the penalty costs associated with deviations of productions targets, thus, managing the geological risk. An application at a nickel mining complex that uses the cut-and-fill and sublevel longhole open stoping mining methods shows that a deterministic equivalent schedule overestimates the NPV by approximately 47%. The risk analysis of both deterministic and stochastic approaches shows that the latter produces a 22% higher expected NPV. These results show the advantages of incorporating and managing risk through the optimization. This method, however, is only applicable to lenticular deposits with predefined boundaries.

A stochastic optimization framework is proposed for the open-pit to underground mining transition by MacNeil and Dimitrakopoulos (2017). The authors propose a sequential optimization of the open-pit production schedule followed by the underground optimization for a set of crown pillar candidates. The mathematical formulation model follows the same two-stage SIP first proposed by Ramazan and Dimitrakopoulos (2005) with the difference that the first stage decisions are made in terms of stopes. The objective function maximizes the NPV while maximizing the risk of not meeting production targets, subject to physical, capacity, and production target constraints. The stope precedence constraints follow the same mathematical implementation of the analog slope constraints. Therefore, a predefined stope design and stope precedence rule must be given as input to the model. An application at a gold mine is evaluated, in which the selected mining method is cut-and-fill. A comparison with a deterministic equivalent for the underground schedule and a commercial software for the open-pit production schedule shows 9% higher NPV for the stochastic approach. In addition, a risk analysis shows that the NPV of the deterministic approach has 90% probability of being lower than forecasted, confirming the inability of deterministic methods to manage geological uncertainty and variability, and that an average-type input does not generate average outputs. Besides the limited capability of this method to simultaneously evaluate the open-pit and underground production schedules and related layouts, the method is incapable of incorporating development costs, especially vertical development costs, that are directly associated with the transition depth.

Dirkx et al. (2018) present a two-stage SIP for the stochastic optimization of underground production scheduling assuming a block caving operation. In addition to grade uncertainty, delays from hang-ups in draw points represent another source of uncertainty that is modeled though a discrete evet simulation. The decision variables select the activities at the draw points, the opening of a draw point, the activity in terms of a mining slice, and the percentage of the slice extraction at a timeframe. Therefore, the availability and production of a draw point and the uncertain obstruction of it directly impact the mine production schedule. The case study at a copper mine shows that by incorporating the second source of uncertainty, the optimizer can better manage the extraction of slices, avoiding a reduction in the production due to delays related to draw point obstruction.

Another block caving scheduling method is proposed by Sepúlveda et al. (2018), that incorporates geological and geometallurgical uncertainties. An SIP is proposed to maximize the expected discounted net smelter return (NSR) while minimizing or maximizing a risk measure. The defined possible risk measurement is the standard deviation of the NSR, the value at risk of

NSR, or the deviations from ore production targets. An integer decision variable controls the number of blocks extracted by the available draw points. Capacity constraints control the number of blocks extracted at the drawpoints according to upper and lower bounds. Because the ore recovery is a non-linear geometallurgical parameter, a genetic algorithm is (GA) is proposed to solve the mathematical model. A case study at a multi-element underground mine tests the various secondary objectives to assess the different effects on risk descriptions. This method, however, requires a predefined layout of drawpoints and a deterministic sequence that defines a possible order of extraction of blocks.

Huang et al. (2020) propose a two-stage SIP tailored to underground production scheduling for an operation that uses cut-and-fill. The mathematical programing formulation has three binary decision variables that schedule the periods for developing advancements (i.e., crosscuts, transport drifts, and ventilation raises) and the periods for extracting stopes. It is assumed that a stope can be scheduled in multiple periods, and internal portions of it can be scheduled independently. The model aims to maximize the NPV while minimizing deviations from grade blending requirements, subject to precedence constraints in terms of advancements and extraction of stopes, adjacency and capacity constraints in terms of advancement rates, processing and backfilling constraints, as well as stochastic constraints that deal with grade blending requirements. An application at a gold mine makes the assumptions of an input stope layout and network design. The outputs of this stochastic approach are compared to the deterministic equivalent forecast using a single estimated block model. A consistently higher NPV is shown for the stochastic case. In addition, the stochastic schedule forecasts higher ore production and better grade quality in terms of the production targets.

Nesbitt et al. (2021) propose a multistage stochastic integer programming (MSIP) model for underground mine production scheduling under grade and activity duration uncertainty. Although binary, scenario dependent decision variables are proposed to model the scheduling of mining activities, they have the ability to adapt to first-stage decision variables that define a time-interval baseline for each activity. Therefore, a single baseline schedule of activities is produced, generating a practical sequence that can be used during operation. The objective function aims to maximize the expected NPV given the adaptative policies. The proposed MSIP, however, does not explicitly incorporate risk management. In addition, the method deals with a general activity scheduling under precedence constraints, not considering any specific production demands. Due to the computational complexity of scheduling activities, the application of this method is restricted to short-term schedules with a predefined mine layout. Although a case study is presented for a gold mine, the results focus on the efficiency of the proposed heuristic solution approach, not showing forecasts and the capability of the method to adapt to geological risk.

The approaches presented above for the underground mine production scheduling rely on a common assumption of an existing underground mine design. Furtado e Faria et al. (2022a) takes advantage of the integrated approaches previously reviewed in Section 1.3.3 and truly stochastic mathematical formulations for open-pit mine production scheduling to develop a two-stage SIP that aims to maximize the expected NPV while minimizing the geological risk (Ramazan and Dimitrakopoulos 2013, 2005). The authors use the same inputs and preprocessing steps as in (Furtado e Faria et al. 2022b). Also, the mathematical programing formulation is expanded to incorporate time-period decisions in terms of scheduled stopes, incremental vertical and horizontal development costs, and second stage decision variables related to deviations from production targets. This approach, however, assumes that the orebody is accessed through a vertical shaft in a predefined position. An application at a gold mine that uses the sublevel open stoping mining method with two geotechnical zones and multiple allowable stope shapes shows an 11% higher NPV when compared to a stepwise stochastic approach. In addition, physically different schedules and layouts are observed. It can be noted that this method is tailored for the aforementioned mining method, not presenting a general mathematical formulation that can be adapted to other mining methods or variants. Also, the final schedule is mainly guided by the relative positions of the selected stopes to the main access and to the footwall. Although sill and rib pillar requirements are included in the non-overlapping requirements, no adjacency constraints are imposed because the model assumes rock mass stability requirements. Therefore, the method is not applicable to mining methods that use pillar recovery or backfilling operations.

1.6 Goal and Objectives

The goal of the research presented in this thesis is to explore the integrated stochastic optimization of stope design and long-term mine production scheduling for the sublevel longhole open stoping mining method with backfilling, as applied in an operating underground copper mine. The following objectives are set to meet this goal:

- Review the technical literature related to the sublevel longhole open stoping mining method, deterministic and stochastic approaches for stope design and mine production scheduling, and methods used for simulating mineral deposits.
- Develop an integrated stochastic optimization model that accounts for geological uncertainty and adjacency constraints to allow backfilling and different haulage costs according to mining zone characteristics, and implement it with a copper mine.
- Expand the proposed integrated stochastic optimization model to incorporate stockpiling decisions and evaluate the impact of using high-order sequential simulations of the mineral deposit as input to the developed mathematical approach.
- Summarize the contributions and conclusions of this thesis and provide suggestions for future research.

1.7 Thesis Outline

This thesis is organized into the following chapters:

Chapter 1 presents a review of the available literature related to the sublevel stoping mining method and variants, deterministic and stochastic advances for the optimization of stope design and underground mine production scheduling, and methods related to geostatistical simulation of mineral deposits.

Chapter 2 presents a stochastic optimization mathematical model that integrates the stope design and production schedule for an operating copper mine with secondary elements that uses the sublevel longhole open stoping mining method. An application for an underground cooper mine is compared to a stochastic stepwise approach, showing the benefits of integrating all planning components in one optimization framework.

Chapter 3 expands the proposed stochastic mathematical model to incorporate stockpiling decisions. In addition, the same copper mineral deposit is simulated using a high-order sequential simulation framework and the schedule and forecast are compared to those generated using sequential Gaussian simulation.

Chapter 4 summarizes the contributions presented in previous chapters and overall conclusions and presents suggestions for future work.

Chapter 2 - Integrated Stochastic Optimization of Stope Design and Long-term Mine Production Scheduling at an Operating Underground Copper Mine

2.1 Introduction

Long-term underground mine production scheduling conventionally consists of the definition of the stope layout, the access design, the ventilation system requirements, and the extraction sequence (Alford et al. 2007; Appianing et al. 2018; Bootsma et al. 2018; Poniewierski et al. 2003). Initially, potential mineable volumes are defined based on geomechanical and geological properties. This step requires fixed input values for operational parameters, such as mining capacity, as well as a cut-off grade in order to maximize the undiscounted profit of the stopes (Alford 1995; Alford and Hall 2009; Alford Mining Systems 2022; Cawrse 2001; Erdogan et al. 2017; Topal and Sens 2010). Subsequently, accesses such as declines, ramps, and shafts connecting production areas to main haulage and ventilation systems are defined (Brazil et al. 2008; Brazil et al. 2003; Brazil and Thomas 2007). The stope layout and network designs are used as inputs to the life-of-mine (LOM) production schedule optimization, which maximizes the net present value (NPV) of the mining asset under a unit operation timeframe, while considering economic and capacity constraints (Brickey 2015; Fava et al. 8-10 June 2011; Fava et al. 2013; Hauta et al. 2017; Little et al. 2011; Nehring et al. 2010; Newman et al. 2010; Topal 2003; Trout 1995). An iterative procedure ensures that these steps are repeated for different cut-off grade values until the one that produces a schedule with the highest NPV is found and selected (Alford and Hall 2009; Bootsma et al. 2018; Poniewierski et al. 2003). However, by considering separately the stope design and extraction sequence, the synergies between these two components are not captured in the optimization process. In addition, this traditional approach is deterministic in which a single estimated (average type) representation of the orebody is considered, which does not quantify geological uncertainty and variability, representing a critical source of technical risk for a mining project (Vallée 2000).

Little et al. (2011) show that stope boundaries should be an outcome of the production schedule so that interdependencies among stope grades, development costs, and the time value of money are incorporated into an integrated process. Accordingly, Little et al. (2011, 2013) propose an integrated underground mine design and production scheduling optimization approach, based on a mixed integer programming (MIP) formulation, aiming to maximize the discounted cashflow of stopes mined in an extraction sequence accounting for their size and location, while constrained by ore production and backfilling capacities. This approach, however, does not account for critical costs and decisions associated with the development of main and secondary accesses. Copland and Nehring (2016) address the stope boundaries and scheduling with a binary integer programming formulation that considers the discounted profit from stopes and incorporates level access development decisions. The development of drifts and shafts as one-time decisions is integrated into the MIP by Hou et al. (2019). Nevertheless, it assumes a predefined network design in terms of the position of the longitudinal ore drives and the shaft. In addition, the application of the proposed method is restricted to a stratiform two-dimensional deposit. Foroughi et al. (2019) develop a MIP that maximizes the NPV and metal recovery, while jointly considering the production scheduling and stope layout. The above-mentioned method is applied to an iron ore deposit and is solved using a genetic algorithm. The development and application of the methods delineated thus far are suited to the sublevel open-stoping (SOS) mining method. The development of ventilation systems and the internal accessibility of stopes are not thoroughly investigated in the studies previously presented, not allowing the generalization to other mining methods or variants of the SOS that require specific backfilling practices given different stope types (Villaescusa 2014). In addition, these integrated methods are deterministic and are, thus, unable to account for uncertainty.

Deterministic LOM approaches lead to forecasts and production schedules that devaite from key production targets (Dimitrakopoulos et al. 2002; Dowd 1994; Leite and Dimitrakopoulos 2007; Ravenscroft 1992). Dimitrakopoulos and Grieco (2009) analyze how conventional methods for underground mine design are unable to capture the upside potential and/or downside risk of meeting forecasts, which are tied to the presence of uncertainty and variability of grades and material types. In order to account for geological uncertainty, stochastic simulations of the orebody are used as the main input to probabilistic and stochastic frameworks to generate a stope layout (Dimitrakopoulos and Grieco 2009; Furtado e Faria et al. 2022b; Grieco and Dimitrakopoulos 2007; Villalba Matamoros and Kumral 2018). The stochastic optimization of the underground mine production schedule, given the stope boundaries, is modeled for cut-and-fill, block-caving, and long-hole stoping mining methods (Carpentier et al. 2016; Dirkx et al.

2018; Huang et al. 2020; Noriega et al. 2022). Starting from Brickey (2015) a resource-constrained project scheduling problem is proposed to adress the selection of undergorund mine activities following their related precedeces and duration. Nesbitt et al. (2021) improve this previous work by incorporating uncertainty in the activities duration and grades, while Hill et al. (2022) address computational techniques to reduce the problem size and algorithms to solve the optimization. This activity scheduling approach, however, is suitable for shorter-term planning since the duration of each undergorund mining activity cannot be described on a yearly basis. Recently, a two-stage stochastic integer programming (SIP) (Birge and Louveaux 2011) formulation that jointly optimizes the stope design and production schedule for the SOS mining method is developed by (Furtado e Faria et al. 2022a). The mathematical model presented selects the stopes' shapes and locations, and levels of extraction in order to maximize NPV while considering effective vertical and horizontal development costs. It also manages the geological risk by minimizing deviations from production targets. The related case study shows an improvement in the NPV when compared to the sequential framework, where a stope layout is given as input to the optimization of the production schedule. Nevertheless, this method is tailored for a specific variant of SOS that does not account explicitly for adjacency constraints when backfilling is used.

A new integrated stochastic optimization of stope design and long-term production scheduling is proposed herein to go beyond the mining method specificities of previous approaches, in order to be applied to the sublevel longhole open stoping (SLOS) mining method with backfilling. A variant of the SLOS is considered with assumptions and parameters derived from an existing operational mine is considered. Horizontal extraction levels define the vertical boundaries of stopes, and sublevel drifts and crosscuts are developed to enable longhole drilling. Primary, secondary, and tertiary stopes, which are aligned with extraction and backfilling procedures, and are combined with a bottom-up extraction approach to create precedence rules among stopes (Hamrin 2001; Hartman and Mutmansky 2002; Pakalnis and Hughes 2011; Soma 2001). In addition, geometric parameters for shapes and sizes of stopes are defined for different mining zones according to geotechnical characteristics and requirements.

The proposed integrated optimization is formulated as a two-stage SIP (Birge and Louveaux 2011). The method considers the selection of the stopes' period of extraction and mining zone configuration that relates to the definition of stope shape parameters and types, as well as to sublevel positions. The maximization of the NPV as per the objective function considers cumulative horizontal development costs and different haulage costs for different possible network systems. Geological uncertainty is modeled through a set of equiprobable geostatistical simulations of the mineral deposit (Goovaerts 1997; Journel and Huijbregts 1978; Rossi and Deutsch 2014). The risk of not meeting production targets, which is inherent to geological uncertainty, is managed in the objective function through the incorporation of geological risk discounting (GRD) (Ramazan and Dimitrakopoulos 2005). This discounting factor is applied to the objective function component minimizing deviations from production targets, leading to the extraction of more uncertain materials in later periods on the LOM, when more information about the mineral deposit becomes available (Ramazan and Dimitrakopoulos 2005, 2013; Montiel and Dimitrakopoulos 2013, 2015; Montiel et al. 2016; Goodfellow and Dimitrakopoulos 2016, 2017). Physical and capacity constraints are included. Precedence rules of stopes defined based on geotechnical requirements enable the use of backfill after a stope is extracted.

The subsequent sections of this chapter progress as follows. First, the main inputs to the integrated stochastic optimization for the underground mine production scheduling and the correspondent mathematical programming formulation are outlined. Then, a case study at an operating underground mine is presented, including a comparison with a sequential stochastic framework. Conclusions and future work follow.

2.2 Methodology

A method for the integrated stochastic optimization of stope design and mine production scheduling for the sublevel longhole open stoping (SLOS) mining method with backfilling is considered and presented below. The approach considers that a mining cycle, defined by all unit operations, such as drilling, blasting, loading, hauling, and backfilling, is completed for each stope on its mining period. A pattern of extraction according to the type of stopes (e.g., primary, secondary, and tertiary) ensures that geotechnical constraints are met. Backfilling guarantees the stability of stope walls, removing the need for pillars. The sublevels are all aligned with the stopes’ lower bounds and can be used as extraction levels.

A set of geostatistical simulations $s \in S$ of the orebody describes the geological uncertainty. Initially, the orebody is represented in terms of blocks $i \in I$ that are, then, grouped into stopes $j \in J$. The bottom of the stopes that share the same sublevel $l \in L$ must be vertically aligned in order to define the location of developments, such as drifts $d \in D_l$. The requirement for the creation of possible cross-cuts $c \in C_l$ is that the stopes must also be laterally aligned throughout a mining direction. A set of possible configurations $b \in B$ of stopes that overlap each other is generated, allowing the stopes to have different allowable shapes, locations, and type options $a \in A$ that define the ordering of primary, secondary and tertiary stopes.

The indices, sets, technical, economical, and geometrical parameters as listed in Table 2-1 to Table 2-4, respectively. The decision variables of the proposed mathematical formulation are shown in

Table 2-5 and Table 2-6.

Table 2-1 – List of indices

Index	Definition
i	Block index
j	Stope index
b	Mining zone configuration index
d	Drift direction index
c	Crosscut index
l	Production level/sublevel index
a	Stope type option
k	Primary, secondary, and tertiary stope index
s	Index of a scenario quantifying the considered sources of uncertainty
h	Primary access system (shaft or ramp) index
ε	Element (metal) index
t	Production period index

Table 2-2 – List of sets

Index	Definition
H	Set of primary access systems (shaft, ramp)
H_{ramp}	Sub-set of primary access systems *h* which are ramps
H_{shaft}	Sub-set of primary access systems *h* which are shafts
B	Set of all possible configurations *b* for the mining zone
B_h	Set of possible mining zones *b* using the primary access system *h*

Index	Definition
A_b	Set of stope type option *a* for a mining zone *b*
L_b	Set of sublevels *l* for mining zone configuration *b*
D_l	Set of drift directions *d* in sublevel *l*
C_l	Set of potential crosscuts *c* in sublevel *l*
J_b	Set of potential stopes j in a mining zone configuration *b*
J_{ba}	Set of potential stopes j in a mining zone configuration *b*, for stope type option a
J_{bl}	Set of potential stopes j in a mining zone configuration *b* and using sublevel *l*
J_{bdl}	Set of possible stopes in level *l* and drift direction *d*, and in mining zone configuration *b*
J_{bcl}	Set of possible stopes in level *l* and cross-cut *c* and in mining zone configuration *b*
Φ_{ja}	Set of predecessors of stope *j* in stope sequencing option *a*
I_b	Set of blocks in a possible mining zone configuration *b*
I_{jba}	Set of blocks i in stope j in mining zone configuration *b*, for stope type option a
S	Set of scenarios s describing the considered sources of uncertainty
T	Set of mining periods t
E	Set of elements ε
K	Set of stope types (primary, secondary, tertiary)

Table 2-3 – List of technical and economic parameters

Index	Definition
v_i	Volume of block i
v_{jba}	Volume of stope *j* in mining zone configuration *b* for stope type option *a* $$v_{jba} = \sum_{i \in I_{jba}} v_i$$
w_{is}	Tonnage of block i in scenario *s*
w_{jbas}	Tonnage of stope *j*, in mining zone configuration *b*, for stope type option a and in geological scenario *s* $$w_{jbas} = \sum_{i \in I_{jb}} w_{is}$$
$g_{i\varepsilon s}$	Grade of element ε in block i, in scenario *s*
$g_{jba\varepsilon s}$	Average grade of element ε within stope j in mining zone b, for stope type option a and in scenario *s* $$g_{jba\varepsilon s} = \sum_{i \in I_{jb}} w_{is} g_{i\varepsilon s} / w_{jbas}$$
π_{kjba}	Extraction sequence type indicator (primary, secondary or tertiary) $\pi_{kjba}=1$ if stope j is of type k in stope sequencing option a, in mining zone configuration b, and 0 otherwise

Index	Definition
ρ_{jba}	Backfilling density ton/m^3 in stope *j*, in mining zone configuration *b*, in stope sequencing option *a*
R_{ε}	Processing recovery in percent of element ε
P_{ε}	Metal selling price \$/t of element ε
v_{jbas}	Economic value of stope j in mining zone b, stope sequencing option a and in scenario s
C_l^{hor}	Unit horizontal development cost in sublevel *l* in \$/km
$C_k{}^{mine}$	Mining cost for type *k* stopes in \$/t
C^{proc}	Processing cost in \$/t
C_{hb}^{haul}	Haulage cost from mining zone *b* to primary access if $h \in H_{ramp}$ in $\$/(\text{tons} \cdot \text{km})$ and if $\in H_{shaft}$ in \$/t
F_b	Fixed cost for keeping the mining zone configuration *b*
f_t^{EDR}	Economic discount factor for period t given an economic discount rate
f_t^{GRD}	Geologic discount factor for period t given a geologic discount rate
U_{ht}^{haul}	Hoisting capacity of primary access system *h* in period t (tons/year).
U_t^{proc}	Processing capacity in period t (tons/year).
$U_{\varepsilon t}$	Maximum target of element ε in period t (% element/year).
$L_{\varepsilon t}$	Minimum target of element ε in period t (% element/year).
U_{kt}^{bf}	Backfilling capacity for backfill type associated to *k* in period *t* (tons/year)
U_t^{dev}	Development capacity in period *t* (m/year)
c_h^{haul}	Penalty costs associated with the surplus deviations from the haulage tonnage capacity.

Table 2-4 – List of geometric parameters

Index	Definition
$\gamma_{b,j}^{x}, \gamma_{b,j}^{y}$	Stope $j \in J_b$ sizes along direction x and y in the horizontal plane, in terms number of blocks for mining zone configuration $b \in B$.
$\gamma_{b,j}^{z,min}, \gamma_{b,j}^{z,max}$	Stope $j \in J_b$ minimum and maximum sizes along direction z (vertical plane) and in number of blocks for mining zone configuration $b \in B$.
$\alpha_{b,l}^{z}$	Sublevel spacing in *z* direction, for a mining zone configuration *b* and sublevel *l*
$\delta_{h,b,l}$	Length in km from the surface to the sublevel *l* of mining zone configuration *b* for primary access if $h \in H_{ramp}$ and 1 if $h \in H_{shaft}$
$\delta_{j,d,l,b}^{drift}$	Distance in a drift from a stope *j* to the h access point/ ore pass along drift direction d in level *l*, in mining block design *b*
$\delta_{j,c,l,b}^{crosscut}$	Horizontal distance from stope *j* to the drift position in the footwall in sublevel *l* under mining zone configuration *b*
$\eta_b^{sublevels}$	Number of sublevels in mining zone configuration *b*
η_b^{stopes}	Number of stopes in mining zone configuration *b*

Table 2-5 – Binary decision variables

Index	Definition
$z_{b,a}$	Mining zone selection decision variable, equal to 1 if mining zone configuration b using stope type option a is selected, and 0 otherwise
$y_{j,b,a,t}$	Stope selection decision variable, equal to 1 if stope j in mining zone configuration b is selected, for stope sequencing option a, in period t, and 0 otherwise

Table 2-6 – Continuous decision variables

Index	Definition
$\psi_{d,l,b,t}^{\text{drift}}$	Drift's development distance in sublevel l, in mining zone configuration b, along with drift direction $d \in D_l$ in period t
$\psi_{d,l,b,t}^{\text{drift}^*}$	Effective drift's development distance in sublevel l, in mining zone configuration b, along with mining directions $d \in D_l$ in period t
$\psi_{c,l,b,t}^{crosscut}$	Crosscut's development distance in sublevel l in mining zone configuration b, for crosscut $c \in C_l$ in period t
$\psi_{c,l,b,t}^{crosscut^*}$	Effective crosscut's development distance in sublevel l in mining zone configuration b, for crosscut $c \in C_l$ in period t
d_{hts}^{haul}	Surplus deviation of total hoisting/haulage capacity for the system h, in period t and scenario s
d_{ts}^{proc}	Surplus deviation of total processing capacity, in period t, and scenario s
${d_{\varepsilon ts}}^{+}$	Surplus deviation from the maximum grade target of element ε, in period t, and scenario s
${d_{\varepsilon ts}}^{-}$	Shortage deviation from the maximum grade target of element ε, in period t, and scenario s

2.2.1 Input Data Processing

Figure 2-1 shows the three main steps to generate potential stopes from an orebody model by considering a given mining zone; that is, a portion of the deposit that has the same geomechanical characteristics. It is assumed that, for a given mining zone, stopes share the same geometrical parameters that define their shapes. The first step consists of dividing the orebody model into different sublevels and mining fronts. This process is done repeatedly for all allowable sublevel spacing and stope widths, producing a set of possible mining zone configurations B. Subsequently, stope-type options A_b are mapped for each configuration $b \in B$. Each option $a \in A_b$ assign a different type (primary, secondary, or tertiary) to sets of stopes J_{ba}. These stopes undergo an additional stage of processing that determines the stope height that yields a probabilistic highest profitability. Steps 1 and 2 are invariant to the geostatistical simulations of the orebody, while step 3 considers all orebody simulations jointly. The output of this process defines possible geometries and positions of the stopes and sublevels, which, combined with the grade and material type simulations of the orebody, are finally used as inputs to the proposed two-stage SIP.

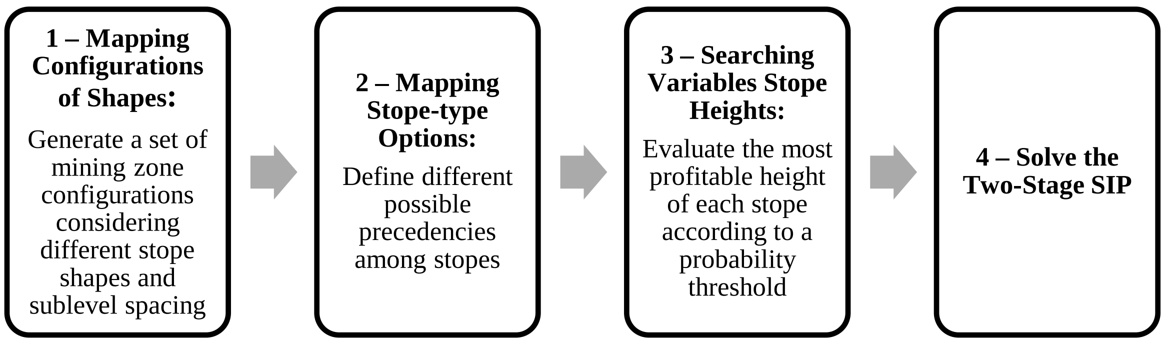

Figure 2-1 – Steps of the stope design and scheduling optimization.

2.2.1.1 Mapping Configurations of Shapes

First, starting from a block $i \in I$ in a mining zone, blocks are grouped according to stope dimensions, $\gamma^x_{b,j}, \gamma^y_{b,j}$ and the sublevel spacing $\alpha^z_{b,l}$ defined for a given mining zone configuration $b \in B$, possible stopes J_b and possible sublevels $l \in L_b$ within that mining zone configuration. This process is repeated for all possible configurations. Figure 2-2 shows two different mining zone configurations $b, b' \in B$, where configuration b assumes three equally spaced sublevels and configuration b' has five sublevels. It is worth noting that two different indexed mining zone configurations can have an identical structure in terms of number of mining fronts and sublevels, as they differ in the assigned haulage system option h in terms of type (i.e., shaft of ramp) and position.

It is important to note that the stope dimensions in the horizontal plane can be variable within the mining zone configuration and are directly related to the final possible stope shapes. These dimensions follow mainly geotechnical requirements related to minimum and maximum dimensions of stopes. The option of having variable shapes considering the horizontal plane is critical when there are different mining or backfilling costs for different stope types, as it enables the management of these costs according to the stope volume and type. On the other hand, for the vertical direction, the distance between sublevels, which are also potential extraction levels, is considered. The spacing $\alpha^{z}_{b,l}$ only defines the sublevels l bounding the stopes and does not determine the final height of the potential stopes. A specific step to determine the actual stope height, which considers not only geometric parameters, but the economic profitability of the given mineable volume is presented in Section 2.2.1.3. In addition, the total number of blocks in a given direction might not be divisible by the stope dimension in that direction. Thus, configurations in which the grouping of blocks into stopes starts from different blocks in the mining zone can be generated.

As seen in Figure 2-3, cross-cuts c are developed parallel to a defined mining direction, in order to meet ventilation and backfilling requirements. Drifts d are developed perpendicularly to cross-cuts. The approximated dimensions of crosscuts ($\delta^{crosscut}_{j,c,l,b}$) and drifts ($\delta^{drift-h}_{j,d,l,b}$) are calculated considering the position of the sublevels l. The drift length is always associated with the access point of a haulage system h and its respective mining zone configurations $b \in B_h$.

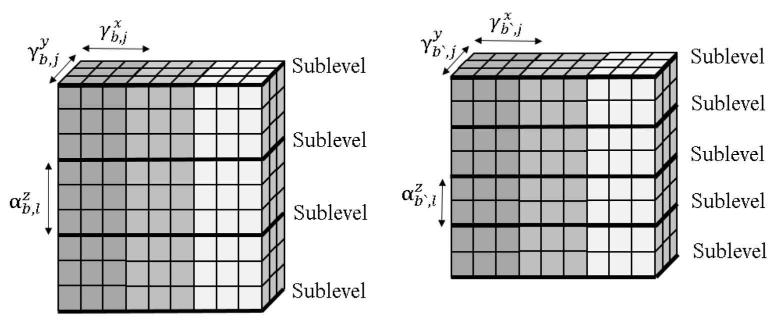

Figure 2-2 – Two mining zone configurations $b, b` \in B$ generated in the mapping of shapes step.

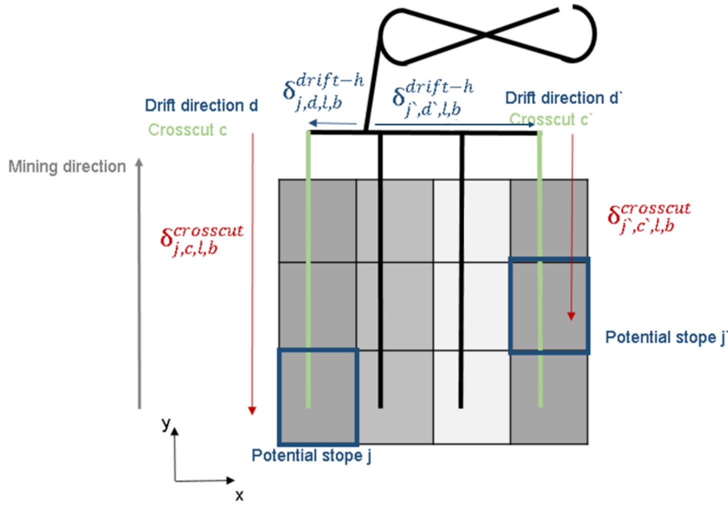

Figure 2-3 – Drifts and cross-cuts distances for a haulage system h, a potential sublevel l (in a plan view) for two potential stopes j and $j`$.

2.2.1.2 Mapping Stope Type Options

Once the possible mining zone configurations are defined, for each configuration b, stope type options $a \in A_b$ are generated. A pattern of extraction is defined following geotechnical constraints. Figure 2-4 shows a stope-to-stope dependency that is adapted from different patterns presented by Villaescusa (2014), in which the numbers define the predecessors and the coloring defines the stope type. A stope with number 1 has no predecessors, a stope tagged with number 2 has stopes with number 1 as a predecessor, and those with number 3 have all the previous ones as predecessors; this rule follows for all other stopes. Therefore, the adjacencies are mapped, such that stopes with lower numbers in Figure 2-4 are predecessors ($\varphi \in \Phi_{j,a}$) of stopes with higher numbers ($j \in J_b$). It is important to note that this pattern does not predefine the extraction sequence and it is used only to define adjacencies for rock mass stability purposes.

The generated stopes, that are outputs from the previous step, are flagged with an indicator π_{kjba} type, where $k \in K$ defines the type of a stope j (i.e. primary, secondary, or tertiary), in a mining zone configuration b and considering a type option a that follows a dependency pattern. Thus, each type option a shows a different combination of types k and stopes j. Figure 2-5 represents the process of mapping the stope type options. It is observed that different combinations of mining zone configurations and stope type options are generated.

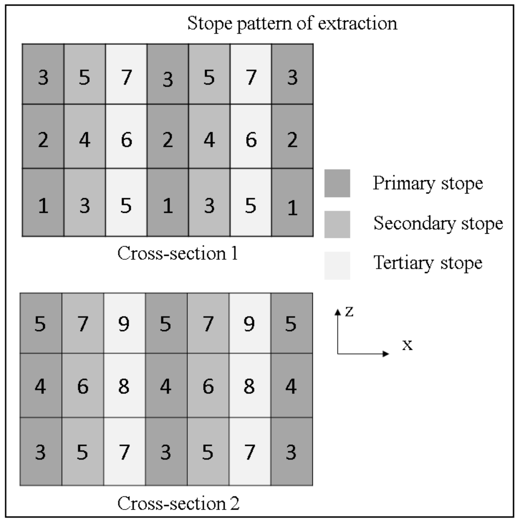

Figure 2-4 – Two consecutive cross-sections of the stope patter of extraction.

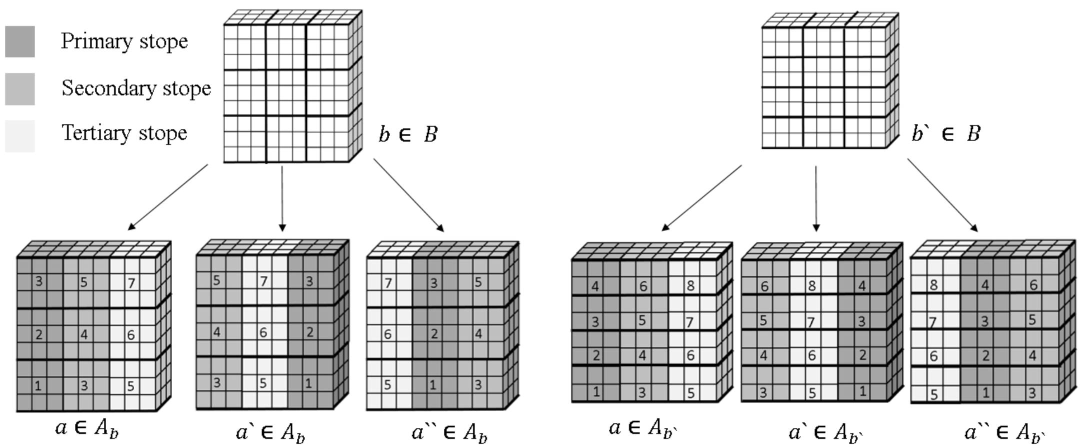

Figure 2-5 – Stope type option for different mining zone configurations.

2.2.1.3 Searching Variable Stope Heights

In the previous steps, the stopes generated come from the grouping of blocks by considering that they fully occupy the space between sublevels. In order to manage dilution, the profitability of having stopes with heights that are smaller than the distance between sublevels ($\alpha^{z}_{b,l}$) and greater than the minimum stope height ($\gamma^{z,min}_{b,j}$) is evaluated. This means that the alignment of the bottom of the stopes according to the sublevel that defines their lower bound is kept. However, the height might be variable within the same sublevel, and there may be waste material above certain stopes, being left as pillars. For this variant of SLOS, a sublevel can serve as an extraction level. Therefore, it is assumed that stopes above the waste portion can still be mined subsequently.

For a given mining zone configuration b, a stope type option a and a stope j, the economic value of each stope is calculated, as shown in Eq. 2.1, for all its possible vertical dimensions and for each simulated orebody scenario s of grades of different elements $\varepsilon \in E$ and stope tonnages w_{jbs}. For each vertical dimension, given a probability threshold, the economic value that defines this quantile is calculated ($v_{height}^{\%}$). For instance, if the P90 defines the probability threshold, there will be an economic value associated with a scenario s where 90% of all scenarios will either be equal to or will not exceed this value. Then, the economic value associated with different vertical dimensions ($v_{height}^{\%}$) are compared, the dimension associated with the highest economic value is retained ($\gamma_{j,b,a}^{z,best}$), defining the final possible stope shape, for that configuration b and the associated stope type option a. Figure 2-6 illustrates how the best stope height is chosen. It should be noted that the stope type option a definition described on the previous section (i.e., step two) does not consider dimensional parameters. However, the mining cost associated to the different stope types have an impact while searching for the best stope height. Consequently, on this third step, the stope height, volume, weight, and grade will be corelated to its possible type and indexed accordingly.

$$v_{j,b,a,s} = w_{jbs}\left(\sum_{\varepsilon\in E} g_{j,b,\varepsilon,s} R_\varepsilon P_\varepsilon - \left(C^{proc} + \sum_{k\in K} \pi_{kjba} C_k{}^{mine}\right)\right), \forall\, j \in J_b, b \in B, a \in A, s \in S \quad (2.1)$$

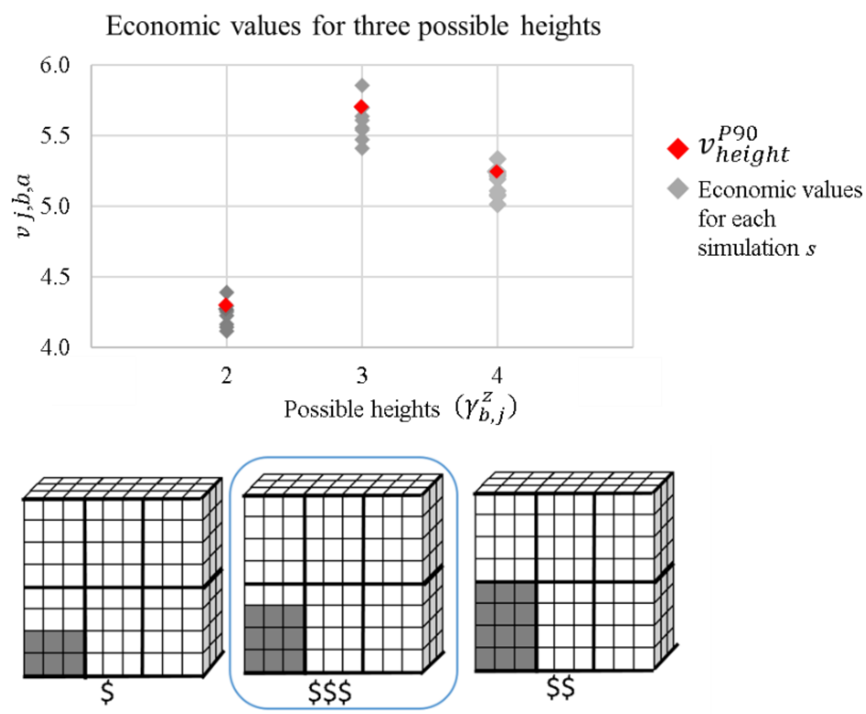

Figure 2-6 – Example of how to define the stope height considering $\gamma^{z,min}_{b,j} = 2$ and $\gamma^{z,max}_{b,j} = 4$. The $ represents the economic value of a stope considering the v^{P90}_{height}. The circled stope is the best stope height $\gamma^{z,best}_{j,b,a}$ selected.

2.3 Stochastic Integer Programming Formulation

This section presents a mathematical programming formulation model, which is developed to optimize the underground mine production schedule and stope boundaries jointly, while considering uncertainty in material supply. Two binary decision variables are shown in

Table 2-5. The mining zone configuration decision variables $z_{b,a} \in \{0,1\}$ control which mining zone configuration $b \in B$ and respective stope type option is selected $a \in A_b$. These decision variables impact directly the selections of stopes shapes and types. A mining zone configuration is always associated with a single haulage system $h \in H$. This means that identical mining zone configuration options (b and $b`$) in terms of stope shapes and sublevels can exist, but they will be associated with different available haulage systems ($b \in B_h$, and $b` \in B_{h`}$). It is assumed that vertical accesses compatible with the haulage systems are already developed, which enables the variable $z_{b,a}$ to be time-independent. The stope selection decision variables $y_{j,b,a,t} \in \{0,1\}$ determine if a stope $j \in J_b$ in a mining zone configuration $b \in B$, using stope type option $a \in A_b$ is mined in period *t*. It is assumed that all the unitary operations; that is, development of secondary accesses, drilling, blasting, hauling, and backfilling, are ready or done at the period scheduled for mining a given stope.

Two continuous decision variables $\psi^{drift}_{d,l,b,t}$ and $\psi^{crosscut}_{c,l,b,t}$ correspond to the developed distance of a drift *d* or a cross-cut *c*, for a sublevel *l*, in a mining zone configurations *b*, in period *t*. To take into account the available structures developed in previous years, effective development distance $\psi^{drift^*}_{d,l,b,t}$ and $\psi^{crosscut^*}_{c,l,b,t}$ are used in practice, they correspond to cumulative horizontal development distances. The remaining decision variables presented in Table 2-6 refer to surplus deviations from haulage capacities for different haulage systems h (d^{haul}_{hts}) and processing capacity (d^{proc}_{ts}), and deviations from lower and upper bounds for different elements $\varepsilon \in E$ requirements, $d_{\varepsilon ts}{}^{-}$ and $d_{\varepsilon ts}{}^{+}$ respectively.

2.3.1 Objective Function

This section introduces the objective function of the proposed SIP and describes its main components as follows.

$$\max \underbrace{\frac{1}{|S|}\sum_{s \in S}\sum_{t \in T}\sum_{b \in B}\sum_{a \in A_b}\sum_{j \in J_b} f^{EDR}_t v_{j,b,a,s}\, y_{j,b,a,t}}_{\textbf{Part I}:Discounted\ Revenue\ from\ Scheduled\ Stopes}$$

$$-\underbrace{\frac{1}{|S|}\sum_{s \in S}\sum_{t \in T} f^{EDR}_t \left(\sum_{h \in H}\sum_{b \in B_h}\sum_{l \in L_b}\sum_{a \in A_b}\sum_{j \in J_{bl}} y_{j,b,a,t}\, w_{j,b,s}\, \delta_{h,b,l} C^{haul}_{h,b}\right)}_{\textbf{Part II}:Haulage\ Costs}$$

$$-\underbrace{\sum_{t \in T}\sum_{b \in B}\sum_{l \in L_b} f^{EDR}_t\, C^{hor}_l \left(\sum_{d \in D_l} \psi^{drift^*}_{d,l,b,t} + \sum_{c \in C_l} \psi^{crosscut^*}_{c,l,b,t}\right)}_{\textbf{Part III}:Development\ Costs}$$

$$-\underbrace{\sum_{b \in B}\sum_{a \in A_b} f^{EDR}_t F_b\, z_{b,a}}_{\textbf{Part IV}:Fixed\ Cost}$$

$$-\underbrace{\frac{1}{|S|}\sum_{s\in S}\sum_{t\in T} f_t^{GRD}\left(c_h^{haul} d_{h,t,s}^{haul} + c^{proc} d_{t,s}^{proc} + \sum_{\varepsilon\in E} c_\varepsilon^{+}\, d_{\varepsilon,t,s}{}^{+} + c_\varepsilon^{-} d_{\varepsilon,t,s}{}^{-}\right)}_{\boldsymbol{Part\ V}:Geological\ Risk\ Management} \tag{2.2}$$

The objective function shown in Eq.2.2 has five parts. Part I aims to maximize the discounted revenue from scheduled stopes. Part II of the objective function minimizes the haulage cost of mined stopes. When a mining zone configuration is associated with a ramp system, the haulage cost C_{hb}^{haul} is expressed in \$/(t*km). Thus, the length $\delta_{h,b,l}$ of the ramp until a sublevel l must be taken into consideration, while for the shaft these values will always equal one. Part III of Eq.2.2 minimizes the horizontal development costs. Part IV minimizes particularly fixed costs associated with different mining zone configurations and patterns of extraction. Part V manages the geological risk, by minimizing deviations from production targets, related to mining and processing capacities and grade blending requirements. For that purpose, penalty costs c_h^{haul}, c^{proc}, c_ε^{+} and c_ε^{-} are applied correspondently to the production requirements and targets, as they are discounted by a geological risk discounting factor f_t^{GRD}. Therefore, riskier stopes will be scheduled in later periods when more information regarding grades and material quality is available (2013, 2004).

2.3.2 Constraints

The objective function is subjected to the following constraints.

$$\sum_{b\in B}\sum_{a\in A_b} z_{b,a} \le 1 \tag{2.3}$$

Eq. 2.3 imposes that a single mining zone configuration with a correspondent stope option is selected.

$$\sum_{t\in T} y_{jbat} \le z_{ba}, \qquad \forall\, b\in B, a\in A_b, j\in J_{ba} \tag{2.4}$$

Liking constraints (Eq. 2.4) ensure that a scheduled stope belongs to the chosen mining zone configuration.

$$\sum_{t\in T} y_{jbat} \le 1, \qquad \forall b\in B, a\in A_b, j\in J_{ba} \tag{2.5}$$

Eq.2.5 defines reserve constraints, where the stope can be mined only once.

$$\left|\Phi_{ja}\right| y_{jbat} \leq \left|\Phi_{ja}\right| - \sum_{a \in A_b} \sum_{\varphi \in \Phi_{ja}} \sum_{t'=t+1}^{|T|} y_{\varphi ba,t'}, \qquad \forall\, j \in J_b, b \in B, t \in T \tag{2.6}$$

Eq. 2.6 ensures that adjacency constraints are met. The sublevel open stope mining method allows that stopes can be mine if their predecessor are mined before or are left unmined. Differently from open-pit mine scheduling problems, the set of predecessors $\Phi_{j,a}$ of a stope *j* must contain all direct and indirect predecessors. Thus, due to this constraint, the complexity of the model is highly impacted by the size of the orebody.

$$\psi_{dlbt}^{drift} \geq \max_{j \in J_{bdl}} \left(\delta_{jdlb}^{drift} \sum_{a \in A} y_{jbat} \right), \qquad \forall b \in B, l \in L_b, d \in D_l, t \in T \tag{2.7}$$

$$\psi_{dlb1}^{drift^*} = \psi_{dlb1}^{drift}, \qquad \forall b \in B, l \in L_b, d \in D_l, t = 1 \tag{2.8}$$

$$\psi_{dlbt}^{drift^*} \geq \left\{ \left[\psi_{dlbt}^{drift} - \sum_{t'=1}^{t-1} \psi_{dlbt'}^{drift^*} \right] \right\}, \qquad \forall b \in B, l \in L_b, d \in D_l, t > 1 \tag{2.9}$$

Eq. 2.7 to 2.9 show how the drift development costs are calculated. As shown in Figure 2-7 the drift development distance $\psi_{d,l,b,t}^{drift}$ corresponds to the distance from the furthest stope mined in a year *t* to the access point in the sublevel *l*. This value is used to calculate the effective development distance $\psi_{d,l,b,t}^{drift^*}$, that considers only the remaining length to be developed in a given period *t* considering the developments done in the previous years.

$$\psi_{clbt}^{crosscut} \geq \max_{j \in J_{bcl}} \left(\delta_{jclb}^{crosscut} \sum_{a \in A} y_{jbat} \right), \qquad \forall \quad b \in B, l \in L_b, c \in C_l, t \in T \tag{2.10}$$

$$\psi_{clb1}^{crosscut^*} = \psi_{clb1}^{crosscut}, \qquad \forall b \in B, l \in L_b, c \in C_l, t = 1 \tag{2.11}$$

$$\psi_{clbt}^{crosscut^*} \geq \left\{ \left[\psi_{clbt}^{crosscut} - \sum_{t'=1}^{t-1} \psi_{clbt'}^{crosscut^*} \right] \right\}, \forall b \in B, l \in L_b, c \in C_l, t > 1 \tag{2.12}$$

Eq. 2.10 to 2.11 show how the crosscut development distances are calculated. The same method is used for the crosscuts when compared to the drifts. However, by considering a mining direction that should be strictly followed, the process of deferring the crosscut development costs will also define how far from the access is worth mining.

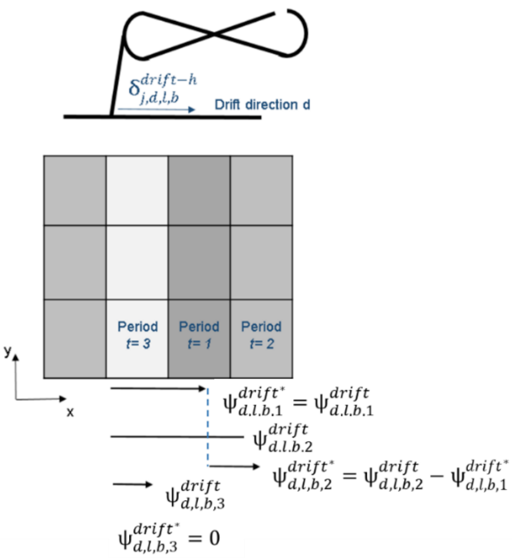

Figure 2-7 – Example of drift development costs $\psi^{drift}_{d,l,b,t}$ and effective development costs $\psi^{drift^*}_{d,l,b,t}$ for the extraction of three stopes in three periods.

$$\sum_{b \in B_h} \sum_{a \in A_b} \sum_{j \in J_{ba}} \left(y_{jbat}\, w_{jbas}\right) - d_{ts}{}^{h} \leq U_t^h, \qquad \forall h \in H, t \in T, s \in S \tag{2.13}$$

$$\sum_{b \in B} \sum_{a \in A_b} \sum_{j \in J_{ba}} \left(y_{jbat}\, w_{jbas}\right) - d_{ts}{}^{p} \leq U_t^p, \qquad \forall\, t \in T, s \in S \tag{2.14}$$

Eq. 2.13 defines an upper bound for the extracted material considering the hoisting or haulage capacity of the haulage system associated to the mining zone configuration. Also, a constraint that limits the production of ore mined is considered (Eq. 2.14). These two constraints are modelled as soft constraints by allowing deviations from the defined boundaries considering a general case where block tonnages, consequently, stope tonnages, are variable for different geological scenarios $s \in S$.

$$\sum_{b \in B} \sum_{a \in A_b} \sum_{j \in J_{ba}} \left(y_{j,b,a,t}\, \rho_{jba} \nu_{jba}\right) \leq U_{kt}^{bf}, \qquad \forall k \in K, t \in T \tag{2.15}$$

Backfilling capacity constraints (Eq. 2.15) are considered in a way that different upper bounds U_{kt}^{bf} can be chosen for different types of backfilling $k \in K$. In this case, deviations are not allowed since no uncertainty is associated with the stope volume ν_{jb}.

$$\sum_{b \in B} \left(\sum_{l \in L_b} \sum_{d \in D_l} \psi^{drift^*}_{d,l,b,t} + \sum_{l \in L_b} \sum_{C \in C_l} \psi^{crosscut^*}_{c,l,b,t} \right) \leq U_t^{develop}, \qquad \forall\, t \in T \tag{2.16}$$

Horizontal development capacities are added in terms of the maximum length of the total drifts and crosscuts (Eq.2.16).

$$\sum_{b \in B} \sum_{a \in A_b} \sum_{j \in J_b} (y_{jbat}\, w_{jbsa})(g_{jba\varepsilon s} - U_{\varepsilon t}) - d_{\varepsilon ts}{}^{+} \leq 0, \qquad \forall \varepsilon \in E, t \in T, s \in S \qquad (2.17)$$

$$\sum_{b \in B} \sum_{a \in A_b} \sum_{j \in J_b} (y_{jbat}\, w_{jbsa})(g_{jba\varepsilon s} - L_{\varepsilon t}) + d_{\varepsilon ts}{}^{-} \geq 0, \qquad \forall \varepsilon \in E, t \in T, s \in S \qquad (2.18)$$

In order to guarantee that grade requirements for different elements $\varepsilon \in E$ are achieved, grade blending constraints are added to the formulation (Eq. 2.17 and 2.18). These constraints allow deviations $d_{\varepsilon ts}{}^{+}$and $d_{\varepsilon ts}{}^{-}$ from upper and lower bounds respectively for each scenario$s \in S$.

$$y_{jbat} \in \{0,1\}, \qquad \forall\, b \in B, a \in A_b, j \in J_{ba}, t \in T \qquad (2.19)$$

$$z_{ba} \in \{0,1\}, \qquad \forall\, b \in B, a \in A_b \qquad (2.20)$$

$$\psi_{dlbt}^{drift}, \psi_{dlbt}^{drift^*} \geq 0 \;, \qquad \forall\, b \in B, l \in L_b, d \in D_l, t \in T \qquad (2.21)$$

$$\psi_{clbt}^{crosscut}, \psi_{clbt}^{crosscut^*} \geq 0 \;, \qquad \forall\; c \in C_l\; b \in B, l \in L_b, c \in C_l, t \in T \qquad (2.22)$$

$$d_{hts}{}^{haul} \geq 0 \;, \qquad \forall\, h \in H, t \in T, s \in S \qquad (2.23)$$

$$d_{ts}{}^{proc} \geq 0 \;, \qquad \forall\, t \in T, s \in S \qquad (2.24)$$

$$d_{\varepsilon ts}{}^{+}, d_{\varepsilon ts}{}^{-} \geq 0 \;, \qquad \forall \varepsilon \in E, t \in T, \in S \qquad (2.25)$$

Eq. 2.19 to Eq. 2.25 refer to integrality and non-negativity constraints.

2.4 Case Study – Application at an Operating Underground Copper Mine

In this section, an application of the proposed method at an operating underground copper mine is presented, where gold and uranium are secondary elements. First, the results for the underlined integrated stochastic framework are analyzed. Then, these results are compared to a stochastic sequential framework in which a stope design is given as an input to the method, fixing the available stopes to be scheduled, as well as its types.

As the main input, 10 geostatistical simulations of Cu, Au, and U_3O_8 in a grid of size 5mx 5m x 5m, for a mining zone of the considered mineral deposit are used. Previous studies show through a sensitivity analysis of the in the stochastic optimization of long-term mine planning, that 10-15 simulations are sufficient to produce stable results. This conclusion is attributed to the support-scale effect, once a large number of blocks are grouped to generate the schedule of a given production period (Albor Consuegra and Dimitrakopoulos 2009; Dimitrakopoulos and Lamghari 2022; Montiel and Dimitrakopoulos 2017). Figure 2-8 shows two realizations for copper, gold, and uranium grades. Mine accesses (i.e. ramp and decline) and ventilation systems are also fixed inputs. Figure 2-9 shows the available infrastructure in the mine and the defined mining direction. In addition, geometrical parameters for the shapes of the stopes were provided by the mining company operating the mine. The maximum and minimum dimensions of stopes are considered given geotechnical and drilling equipment requirements. Two possible mining zone configurations associated with these allowable shapes are considered (Table 2-7). These configurations account for the position of the access point at each sublevel according to the available ramp design. Three option types are used as an input, meaning that all stopes will have the possibility of being primary, secondary, or tertiary.

For this case study the stopes are considered to be blasted and extracted bottom-up, with the pattern of extraction represented in Figure 2-5, and are subsequently backfilled with cemented aggregate fill (CAF), which is not considered a limiting feature in mine production. Thus, the same mining cost is used for all stope types. A horizontal development capacity is considered in terms of the maximum length that can be developed. A single haulage system is available with its maximum capacity constraining ore production. Once the sources of uncertainty considered do not directly affect the ore tonnage production, the mining capacity constraints are modeled as hard constraints. On the other hand, uncertainty in terms of copper, gold, and uranium grades is taken into consideration, thus, penalty costs for deviations from the minimum and maximum grades for these three elements' requirements are applied and are discounted throughout the years in order to manage the geologic risk. Table 2-8 displays the technical and economic parameters used in the optimization of the copper mine. It is worth noting that a fixed unit mining cost of 50 $/t is considered. This cost incorporates drilling, blasting, mucking and other fixed yearly costs such as ventilation costs and backfilling that were scaled in terms of the yearly production rate.

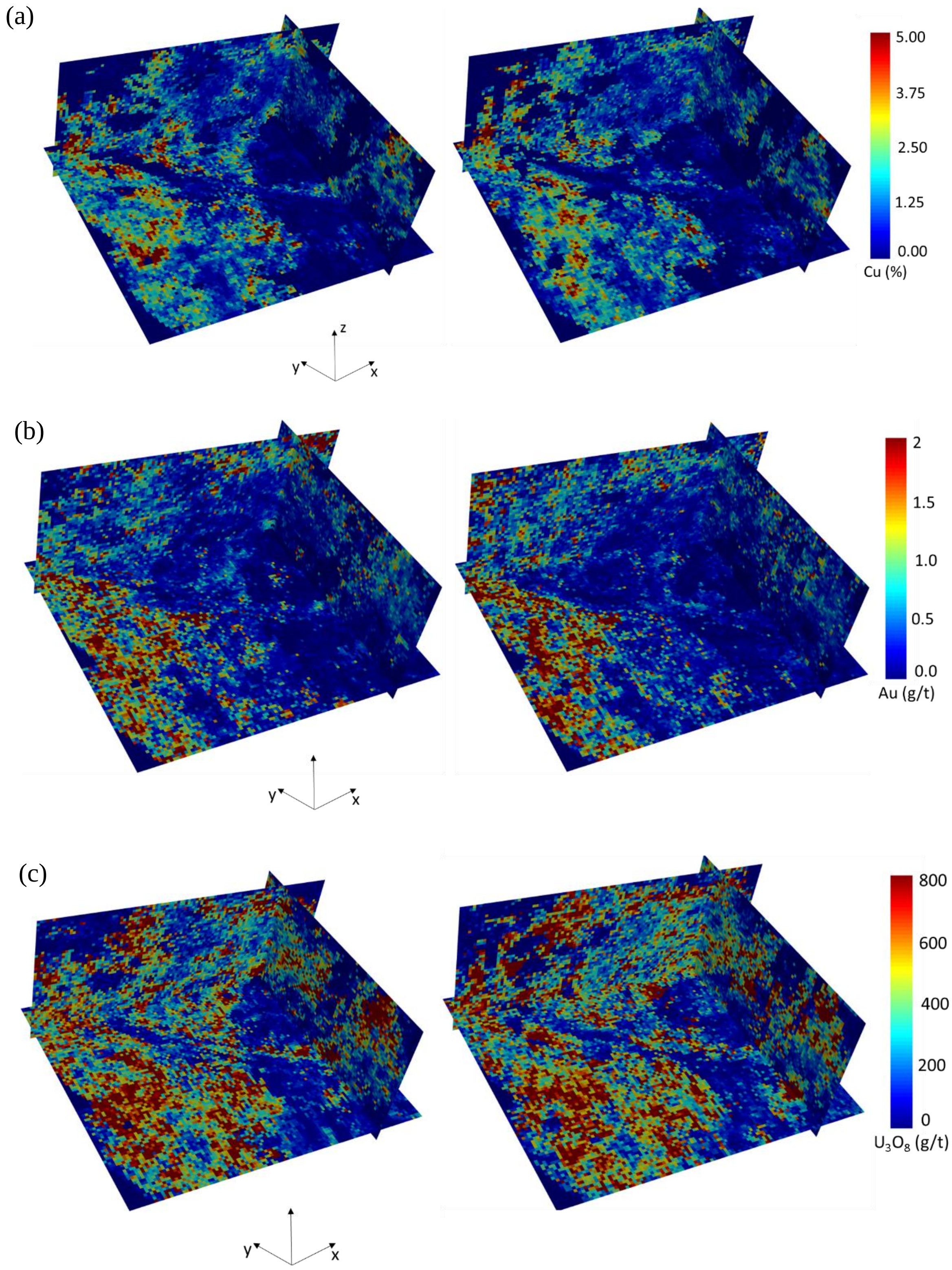

Figure 2-8 – Realizations of (a) copper, (b) gold, and (c) uranium grades in a grid of 5m x 5m x 5m.

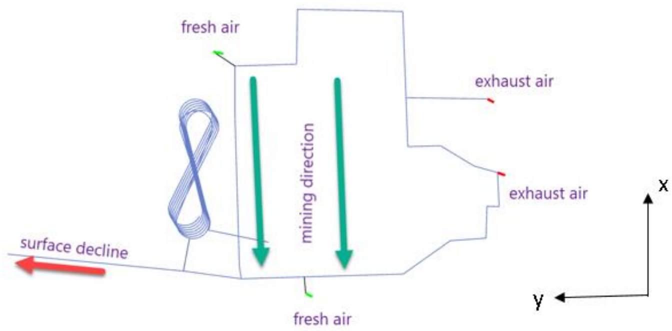

Figure 2-9 - Plan view of the developed infrastructure.

Table 2-7 – Stope geometrical parameters

Parameter	Value/Description
Minimum dimensions	10m x 30m x 30m
Maximum dimensions	30m x 30m x 150m
Configuration 1	15m x 30m x 40m (3,240 potential stopes)
Configuration 2	30m x 30m x 80m (810 potential stopes)

Table 2-8 – Technical and economic parameters used as input in the optimization

Parameter	Value
Cu price ($/t)	8,500
Au price ($/ozt)	1,200
U_3O_8 price ($/t)	71,904
Economic discount rate	10%
Geologic discount rate	10%
Processing recovery Cu (%)	94%
Processing recovery Au (%)	70%
Processing recovery U_3O_8 (%)	70%
Mining cost ($/t)	50
Processing cost ($/t)	13.5
Haulage cost ($/t*km)	5
Drifts development cost ($/m)	12,000
Density (t/m3)	3.2
Block tonnage (t)	400
Mining capacity (Mt/y)	3
Drift development capacity (m/y)	5,000
Minimum copper mill head grade (%)	1.8

Parameter	Value
Minimum gold mill head grade (g/t)	1.0
Maximum gold mill head grade (g/t)	2.0
Minimum uranium mill head grade (g/t)	420
Maximum uranium mill head grade (g/t)	600
Penalty cost for deviations below minimum Cu mill head grade ($/unit)	100
Penalty cost for deviations below minimum and above maximum Au mill head grade ($/unit)	100
Penalty cost for deviations below minimum and above maximum U_3O_8 mill head grade ($/unit)	10

2.4.1 Results of the Integrated Stochastic Optimization

The outputs of the proposed integrated stochastic approach are presented next. Figure 2-10 shows the optimal stope shapes, types, and sequence of extraction throughout a 12-year life-of-mine. The selection of the mining zone configuration represents the trade-off between selecting grades and development costs. The optimal configuration 2 (Table 2-7) guarantees a lower development cost, while deviations from grade production targets are well managed. The green curves in Figure 2-11 and Figure 2-12 show the risk profiles of the integrated stochastic stope design and schedule in terms 10%, 50%, and 90% probabilities (i.e. P10, P50 and P90). An NPV of 3.53 B$ (Figure 2-11a) considering the P50 and a cumulative development cost of 157 M$ (Figure 2-11b) are observed. Also, the ore production follows the maximum mining capacity (Figure 2-11c) and the Cu, Au, and U_3O_8 grades have small deviations from the defined bounds and tend to decrease through the years showing the effect of geological risk management (Figure 2-12). These results are compared to a stochastic sequential framework in the next subsection.

The formulation was implemented in C++ on Visual Studio 15 and solved with CPLEX v.12.8.0. The present application is compounded by 117,684 binary decision variables and 2,172,580 constraints. Using a standard personal computer with six cores and 32 GB RAM, the preprocessing and optimization steps took approximately 24 hours and were constrained by memory allocation limitations, with a 15% optimality gap.

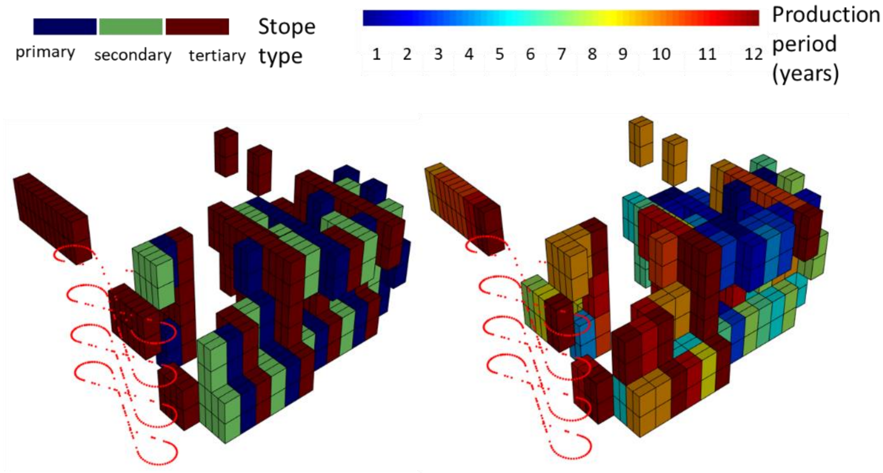

Figure 2-10 – Integrated stochastic optimization outputs from left to right: the stope types option selected and the extraction sequence

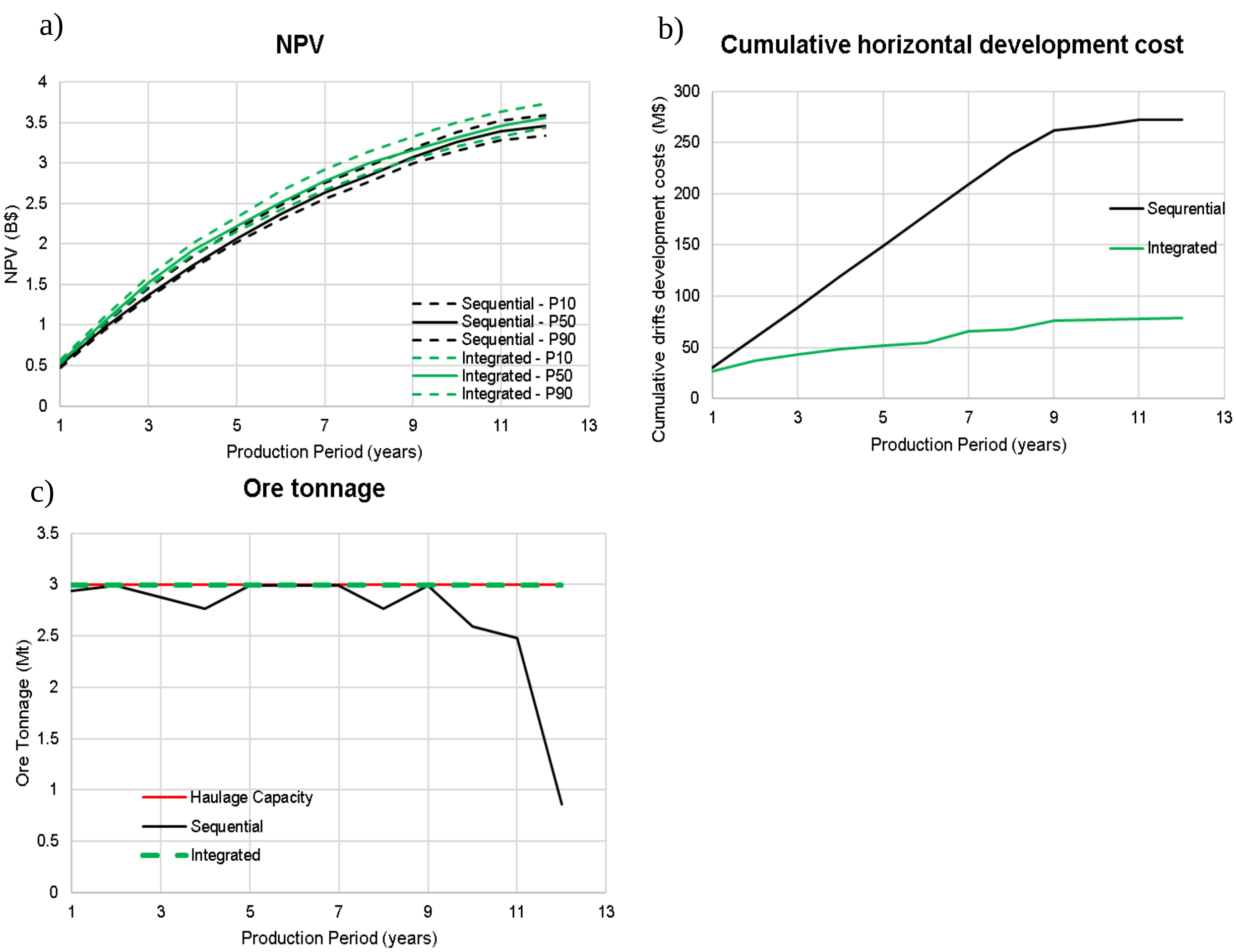

Figure 2-11 – Risk profiles of the integrated (green curves) and sequential (black curves) stochastic frameworks: a) NPV; b) cumulative horizontal development cost; c) ore tonnage.

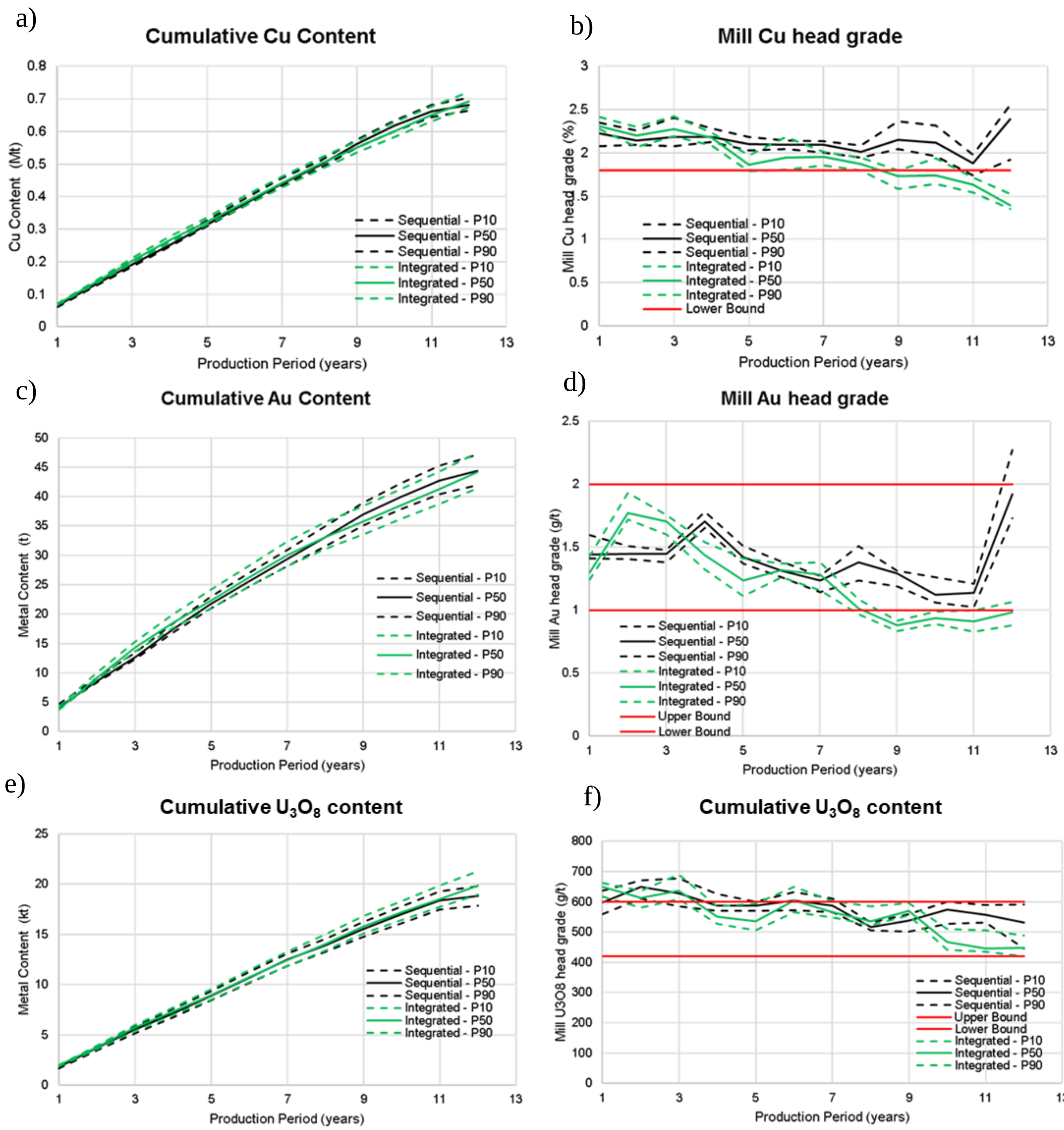

Figure 2-12 – Risk profiles of the integrated (green curves) and sequential (black curves) stochastic frameworks: a) Cumulative Cu content; b) mill Cu head grade; c) cumulative Au content; d) mill Au head grade; e) cumulative U_3O_8 content; f) mill U_3O_8 grade.

2.4.2 Comparison between the Integrated and the Sequential Stochastic approaches

To show the importance of jointly optimizing the stope design and extraction sequence, the proposed method is compared to the stochastic sequential approach. In this sequential approach, the preprocessing steps generate the possible mining zone configurations with the respective allowable stope shapes. From these configurations and possible stopes, a stope design that maximizes the undiscounted cashflow, regardless of adjacency constraints and the horizontal development costs, is first generated. The selected stope locations and shapes are used as the inputs to generate the extraction sequence using the SIP formulation presented in Section 2.3.

The black curves in Figure 2-11and Figure 2-12 show the results of this sequential approach. The same technical, geometric, and economic parameters presented in Table 2-7 and Table 2-8 are used. It is seen in Figure 2-11a, that the NPV of the integrated approach is 3.54 B$ while in the sequential approach it is 3.35 B$, considering the P50 in both cases. Therefore, the integrated approach is expected to produce a NPV 190 M$ higher than the sequential approach. Although less stopes are mined when the sequential approach is used, the development cost for the sequential approach is almost three times higher than that in the integrated approach (Figure 2-11b). Therefore, many stopes that have a positive impact economic value when the cumulative development costs are not considered become uneconomical and inaccessible when this information is actually taken into account. Figure 2-13 shows the comparison of the extraction sequence of the integrated and sequential approaches. In Figure 2-13b, the black wireframe shows the initial stope design generated with step-wise approach, that is noted to be physically different to the one produced by the integrated approach. In addition, it is seen that several stopes included in the initial design in Figure 2-13b are not included in the schedule, showing that, in fact, the effective development costs are critical in the economic value of stopes. It can be also noted that the stopes chosen in the sequential approach are smaller (i.e. configuration 1 in Table 2-7), generating more selectivity in terms of grades, but producing comparable metal contents for the three elements, Cu, Au, and U_3O_8 and meeting grade blending requirements (Figure 2-12). The integrated approach, however, shows a higher metal production in early periods, which has a greater positive impact on the NPV, due to the time value of money.

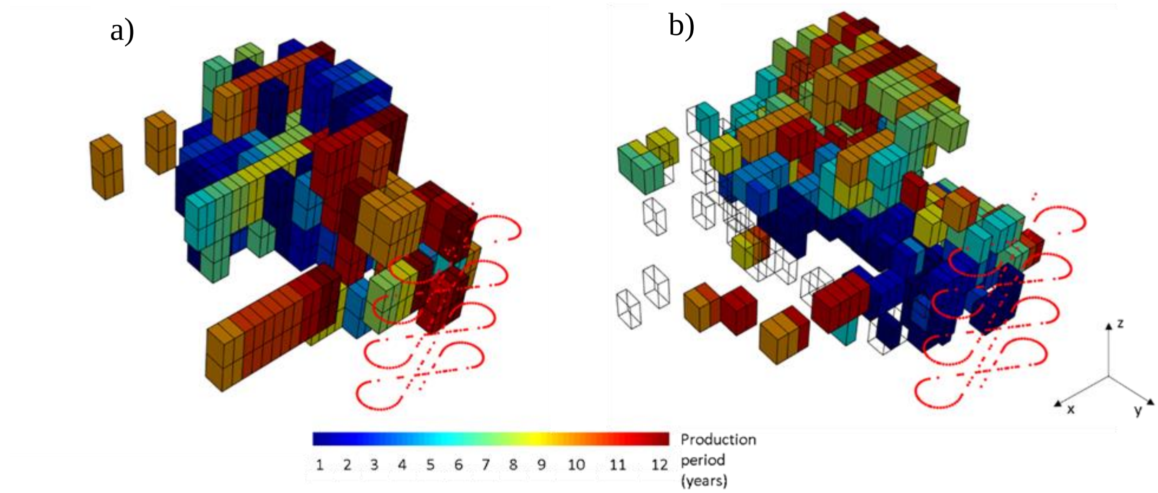

Figure 2-13 - Comparison of the extraction sequence of the a) integrated approach and b) sequential approach (the wireframe corresponds to the stopes selected in the stope design).

2.5 Conclusions

A new mathematical programming formulation for the integrated stochastic optimization of stope design and long-term mine production scheduling is presented, along with its application at an operating underground copper mine. The method is developed based on the sublevel longhole open stoping (SLOS) with backfilling underground mining method and overcome the limitations of previously proposed approaches that are tailored to specificities of other mining methods. The SLOS variation considered in the present study follows the assumptions and parameters derived from an existing operational mine. The proposed method generates jointly the stope boundaries and the extraction sequence assuming that all stopes follow the same geometrical parameters. In addition, a pattern of extraction and a mining direction define adjacencies among stopes and a mining cycle can be completed in one year, which defines the period of extraction.

The proposed two-stage stochastic integer programming (SIP) maximizes the NPV, as well as considers metal prices for different elements, mining costs for different types of stopes, horizontal development costs and haulage costs for the different systems available, while also minimizing the risk of not meeting production targets. The output of the optimization is an operational selection of mining zone configuration that defines the stope shapes and respective types, as well as the extraction sequence of stopes that respects the optimal adjacencies and defined mining direction, generating a risk resilient production schedule.

The proposed method is applied to an underground mine that has copper as the main element, as well as gold and uranium as the secondary elements. The mining zone studied has an available ramp and a ventilation system that defines a mining direction. It follows a pattern of extraction of primary, secondary, and tertiary stopes that defines the adjacencies. The integrated framework is compared to a sequential stochastic framework, in which a design that defines the mining zone configuration and stopes that maximize the undiscounted cashflow is used as a fixed input to the proposed SIP. Firstly, it is seen that physically different stope designs and, consequently, different extraction sequences are produced for these two different frameworks. Furthermore, the integrated approach shows an NPV that is 6% higher than the step-wise approach. The horizontal development costs of the sequential approach are shown to be clearly higher than in the integrated approach. This difference can be attributed mainly to the fact that the actual development costs cannot be considered when generating the initial stope design, which, thereby, limits the decisions in the extraction sequence for the stepwise framework. Therefore, the importance of having an integrated method that is capable of exploiting the relationships between the optimization components is validated.

A case study that accounts for multiple sources of uncertainty simultaneously is a topic for future research, once the presented SIP can directly accommodate commodity price uncertainty. The application and the several aspects of the method are built based on the assumption of a large underground mine divided into mining zones. In the presented case study, only one mining zone schedule was optimized due to the complexity associated with the orebody size and computational limitations. Considering that multiple mining zones can be mined simultaneously and can contribute to the production of ore that feeds the processing plant, an application that optimizes multiple mining zones simultaneously could be considered. In addition, the simultaneous optimization for a mining complex that assumes the existence of multiple mines, stockpiles, and processing streams, with critical considerations in terms of vertical development costs is an extension for future work. Furthermore, computational efficiency limitations are seen when a commercial solver is used, which restricts the model's application to larger problems. Thus, a metaheuristic solver should be implemented in future developments.

Chapter 3 - Integrated Stochastic Underground Mine Planning with Long-term Stockpiling: Method and Impacts of Using High-order Sequential Simulations

3.1 Introduction

Sublevel longhole open stoping (SLOS) is an underground mining method in which the orebody is divided into vertically oriented open stopes that are self-supported by rock pillars and are posteriorly backfilled. Horizontal extraction levels define the vertical boundaries of the stopes, and sublevel drifts and crosscuts are developed to enable longhole drilling (Hamrin 2001; Hartman and Mutmansky 2002; Pakalnis and Hughes 2011). The long-term mine planning process for this mining method relies on three main components. The stope layout defines the spatial design of mineable volumes according to geomechanical and geological properties (Alford 1995; Alford and Hall 2009; Alford Mining Systems 2022; Cawrse 2001). A network design of ramps, shafts, raises, wises, and other developments is done in order to define accesses and ventilation systems (Brazil et al. 2008; Brazil et al. 2003; Brazil and Thomas 2007). The last component defines the production schedule of stopes by maximizing the net present value (NPV) of the related life-of-mine (LOM) (Brickey 2015; Fava et al. 8-10 June 2011; Fava et al. 2013; Hauta et al. 2017; Little et al. 2011; Newman et al. 2010; Topal 2003; Trout 1995). Little et al. (2011) show that these three components should be optimized simultaneously, so that the interdependencies among stope grades, development costs, and the time value of money are captured in the mine planning optimization. In addition, geological uncertainty in grades and material types is known as a critical source of risk for mining projects and its management is essential for meeting production targets and generating realistic forecasts (Dimitrakopoulos 2011; Dimitrakopoulos et al. 2002; Dowd 1994; Grieco and Dimitrakopoulos 2007; Ravenscroft 1992).

A mixed integer programming (MIP) model that integrates underground mine design and production scheduling was first proposed by Little et al. (2011, 2013)Little et al. (2013); Little et al. (2011). The MIP maximizes the discounted cash flow of the mined stopes and considers the stope size and location, while under the constraints of the ore production and backfilling capacities. Therefore, the stope boundaries are an outcome of the production schedule. However, the costs associated with the development of access are not covered in this approach. Copland and Nehring (2016) incorporate level access development decisions in an integer program (IP) that maximizes the discounted revenue from mined stopes while minimizing the level`s development costs. Foroughi et al. (2019) optimize the underground mine production scheduling and stope layout through an IP that aims to jointly maximize two weighted objectives, the NPV and the overall metal recovery. Hou et al. (2019) integrate the mathematical formulation into the development of longitudinal drives and shaft level segments as unitary decisions linked to the stope`s extraction decisions. Although an analysis of the forecasts given different simulations of the orebody is done, the risk associated to the geological uncertainty is not managed or assessed through this approach. Furtado e Faria et al. (2022a) propose an integrated stochastic framework for the stope design and long-term mine production scheduling, tailored for the sublevel open stopping (SLOS) mining method. The authors develop a two-stage stochastic integer programming (SIP) (Birge and Louveaux 2011) formulation that jointly optimizes the stope design and production scheduling, while considering the cumulative development costs and managing the geological risk. Carelos Andrade et al. (2024) also explores the integrated stochastic approach to the SLOS mining method variant that uses backfilling practices and adjacency patterns of primary, secondary and tertiary stopes (Villaescusa 2014). This optimization framework aims to maximize the NPV while managing the geological risk, by minimizing deviations from production targets, which are related to mining and processing capacities, as well as the grade blending requirements. An application at an operating copper mine with secondary elements shows significant improvement in terms of the NPV when compared to a stochastic sequential framework (Carelos Andrade et al. 2024). Nonetheless, an important aspect that has not been addressed in these models is the presence of stockpiles, that are typically used in mining operations. It has been shown that, for long-term open pit mine planning and production scheduling, the consideration of all components of a mining complex in the optimization process leads to more realistic assumptions and forecasts (Dimitrakopoulos 2018; Dimitrakopoulos and Lamghari 2022; Goodfellow and Dimitrakopoulos 2016, 2017; Montiel and Dimitrakopoulos 2015).

In order to assess and manage the spatial uncertainty and variability of grades and material types, a set of geostatistical simulations of the orebody is used as the main input to the optimization of the stope design and production scheduling (Carpentier et al. 2016; Dimitrakopoulos and Grieco 2009; Dirkx et al. 2018; Furtado e Faria et al. 2022a, 2022b; Grieco and Dimitrakopoulos 2007; Hou et al. 2019; Villalba Matamoros and Kumral 2018). Geological attributes, such as metal grades and material types, can be modeled through geostatistical simulation methods that build upon the concept of spatial random fields (Chilès and Delfiner 1999; David 1988; Goovaerts 1997; Journel and Huijbregts 1978; Mariethoz and Caers 2015; Rossi and Deutsch 2014). The sequential simulation paradigm allows for the assessment of an attribute at an unsampled location, by its conditioning value to sample data and previously simulated values, via Monte Carlo sampling of a probability distribution function (Goovaerts 1997). As an example, the sequential Gaussian simulation (SGS) (Goovaerts 1997; Journel 1994) can be mentioned as a simulation method that is conventionally employed. The SGS method, however, does not reproduce the connectivity of high grades given that a Gaussian random function model has the character of maximum entropy (Journel and Deutsch 1993). In addition, this traditional method relies on two-point spatial statistics. Although second-order statistics can fully characterize Gaussian random functions, they do not describe complex geological patterns in the presence of non-Gaussianity and non-linearity (Dimitrakopoulos et al. 2010; Guardiano and Srivasta 1993; Journel 2005; Remy et al. 2009).

Methods based on multiple-point statistics (MPS) are introduced to overcome the limitations of the aforementioned traditional simulation methods (Arpat and Caers 2007; Chatterjee et al. 2012; Guardiano and Srivasta 1993; Journel 2005; Mariethoz and Caers 2015; Mariethoz et al. 2010; Remy et al. 2009; Strebelle 2002; Zhang et al. 2006). These methods infer the conditional probability distribution function (cpdf) by extracting multiple point patterns from a training image (TI) or geological analogue, without making any assumptions about it. These MPS-based simulation approaches tend to reproduce the spatial statistics of the TI, while a consistent mathematical modeling approach should be driven by the sample data (Goodfellow et al. 2012; Osterholt and Dimitrakopoulos 2018; Yao et al. 2018). Dimitrakopoulos et al. (2010) introduce the use of high-order cumulants to explicitly infer high-order statistics from the spatial data. Thus, the high-order simulation (HOSIM) algorithm follows the sequential simulation framework and uses spatial cumulants to derive the cpdf from available data, generating realizations that show the natural connectivity of high grades and reproduce complex geometries (de Carvalho et al. 2019; Dimitrakopoulos and Yao 2020; Minniakhmetov and Dimitrakopoulos 2017b; Minniakhmetov et al. 2018; Mustapha and Dimitrakopoulos 2010b, 2011; Yao et al. 2018).

The impact of using different simulation algorithms to generate mineral deposit models used as inputs for stochastic mine planning in open pit mines has been studied by de Carvalho and Dimitrakopoulos (2019). The related work compares the long-term open-pit mine production schedules and forecasts when SGS and HOSIM (de Carvalho et al. 2019) are used to generate the simulated orebody models serving as inputs to a simultaneous stochastic optimization framework (Goodfellow and Dimitrakopoulos 2016, 2017; Montiel and Dimitrakopoulos 2017, 2015, 2018). The application shows that the long-term sequence of extraction of mining blocks favors the high-grade continuity areas when simulations generated using the HOSIM method are used. The comparison also shows different final pit limits. In addition, more gold is produced at the end of the life-of-mine (LOM), leading to a higher expected NPV, when the optimization uses simulations generated with the HOSIM method. Thus, it is of interest to investigate how a HOSIM approach impacts the stochastic stope design and mine production schedule, particularly given that underground mining methods make assumptions in terms of the ore selectivity and spatial configuration of mineable volumes.

The current work presents the extension of the integrated stochastic optimization of stope design and mine production scheduling proposed by Carelos Andrade et al. (2024) adding long-term stockpiling decisions to the previously developed SIP and related material destination decisions to the previously proposed SIP formulation. In addition, the sensitivity of the proposed scheduling model to different methods used for the geostatistical simulations of the mineral deposit involved is investigated in a case study at an operating underground copper mine. The case study presents the practical aspects of the proposed mathematical programming model based on simulated realizations of the copper deposit generated using a high-order sequential simulation approach (Minniakhmetov et al. 2018). In addition, the extraction sequence and forecasts are compared to those obtained when the deposit realizations are generated using sequential Gaussian simulation. The following section presents a description of the underground mine planning approach with the integration of a linear stockpile. A brief review of the sequential simulation methods, as relevant to the present study, is presented. Subsequently, a case study at an operating copper mine is presented, followed by conclusions and suggested future work.

3.2 Methodology

The following subsection presents an extended stochastic integer programming (SIP) formulation, which incorporates long-term stockpiling decisions into the integrated stochastic optimization of stope design and mine production scheduling. The method builds upon the work on the optimization framework considering the sublevel longhole open stoping (SLOS) mining method with backfilling (Carelos Andrade et al. 2024). The two stochastic simulation approaches (Goovaerts 1997; Minniakhmetov et al. 2018) used in the case study to generate orebody models quantifying geological uncertainty, that are inputs to the optimization process are also summarized.

3.2.1 Mathematical formulation of the stochastic long-term underground mine production scheduling with stockpiling

An extended stochastic integer program (SIP) (Birge and Louveaux 2011) that incorporates long-term stockpiling decisions into the integrated stochastic optimization of stope design and mine production scheduling is proposed herein. The proposed method follows the optimization framework presented by Carelos Andrade et al. (2024) for the sublevel longhole open stoping (SLOS) mining method with backfilling, as per an operating copper mine. Therefore, only the new aspects of this methodology are detail herein. The method aims to optimize jointly the extraction sequence of stopes $j \in J$ and horizontal development costs of drifts $d \in D_l$ and crosscuts $c \in C_l$ that will lead to stope boundaries that respect the stopes' geometric parameters. The approach assumes the optimization of a mining zone of a large orebody that defines a volume with unique geotechnical requirements. A set of geostatistical simulations $s \in S$ of the orebody describes the geological uncertainty. Initially, the orebody is represented in terms of blocks $i \in I$ that are, subsequently, grouped into stopes $j \in J$.

Three data processing steps are needed to generate the inputs necessary to the proposed two-stage SIP as shown in Figure 3-1. The first preprocessing step generates different mining zone configurations $b \in B$ by dividing the mining zone into different mining fronts and sublevels $l \in$

L according allowable stope and sublevel dimensions**.** Crosscuts c are developed within each mining front, parallel to a defined mining direction. Drifts *d* are developed perpendicularly to the cross-cuts. The approximated dimensions of crosscuts δ^{C}_{jclb} and drifts δ^{D}_{jdlb}, as well as the length δ^{V}_{hlb} from the surface to the access point of a haulage system *h* and its respective mining zone configurations $b \in B_h$, for each sublevel $l \in L_b$. In the second preprocessing step, shown in Figure 3-1, for each configuration *b*, stope type options $a \in A_b$ are generated. Each stope type option a defines a possible ordering of primary, secondary, and tertiary stope types $k \in K$, as exemplified Figure 3-2. Each stope is identified by an indicator parameter π_{kjba}, according to its type. Therefore, the set of predecessors $\varphi \in \Phi_{ja}$ of a stope $j \in J_{j,a}$ can be defined following geotechnical constraints. Along these steps, the stopes are assumed to fully occupy the space between the sublevels. To manage dilution, in the third step, the profitability of having stopes with heights γ^{z} that are smaller than the distance between sublevels and greater than the minimum stope height is evaluated. Thus, for a given mining zone configuration *b*, a stope type option *a*, a stope *j* and elements $\varepsilon \in E$, the economic value of each stope $v_{jbas}(\gamma^{z})$ is calculated, as shown in Eq. 3.1, for all its possible vertical dimensions γ^{z} and for each simulated orebody scenario *s*. A probability of non-exceedance threshold (e.g., P50) is predefined and only the economic value that corresponds to that threshold $(\text{e.g.}, v^{\text{P50}}_{jba}(\gamma^{z}))$ is analyzed. The final stope heights are those that maximize the probabilistic economic value of a stope $(v^{\mathcal{P}}_{j,b,a}(\gamma^{z}))$, as in Eq. 3.2, as follows

$$v_{jbas}(\gamma^{z}) = w_{jbas}(\gamma^{z})\left(\sum_{\varepsilon\in E} g_{jb\varepsilon s}(\gamma^{z}) R_{\varepsilon} P_{\varepsilon} - \left(C^{P} + \sum_{k\in K} \pi_{kjba} {C_k}^{M}\right)\right), \quad \forall\, j \in J_b, b \in B, a \in A, s \in S \tag{3.1}$$

$$\underset{\gamma^{z}}{\operatorname{argmax}}\, v^{P}_{jba}(\gamma^{z}), \forall\, j \in J_b, b \in B, a \in A \tag{3.2}$$

where $w_{jbs}(\gamma^{z})$ and $g_{jb\varepsilon s}(\gamma^{z})$ are respectively the tonnage and grade of element ε within stope *j* in mining zone *b*, in scenario s, as function of the stope height γ^{z}, R_{ε} is the metal recovery of element ε and P_{ε} is the related metal price, C^{P} is the unitary processing cost and ${C_k}^{M}$ is the unitary mining cost for each stope type. The function argmax, in Eq. 3.2, returns the value of a stope height (γ^z) that maximizes the probabilistic economic value of a stope $\left(v_{jba}^{P}(\gamma^z)\right)$.

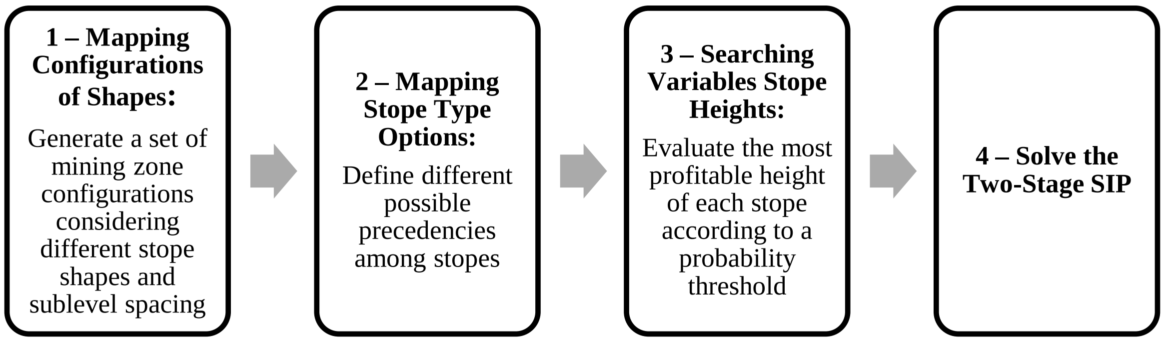

Figure 3-1 - Steps of the stope design and scheduling optimization (Source: Carelos Andrade et al. 2024).

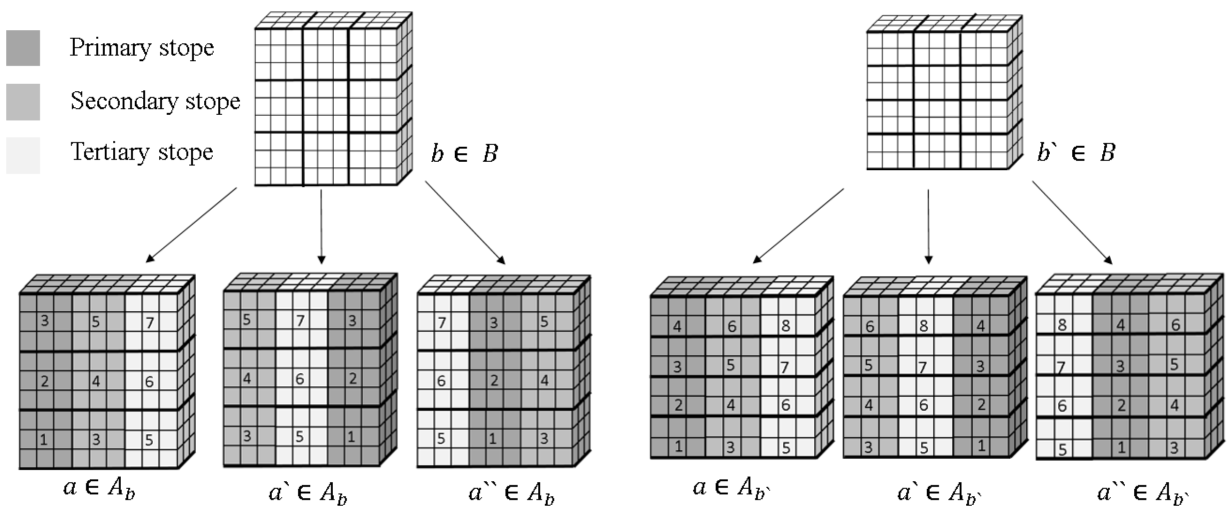

Figure 3-2 - Stope type option for different mining zone configurations (Source: Carelos Andrade et al. 2024).

Finally, the information generated in the three steps described above are used as input to the proposed extended two-stage SIP, that defines the fourth step of the presented method. The new SIP addresses the stope design and long-term production scheduling with the complement of stockpiling decisions. The related decisions variables, objective function and main constraints are described next. The mining zone configuration decision variables $z_{ba} \in \{0,1\}$ control which mining zone configuration $b \in B$ and respective stope type option is selected $a \in A_b$. These decision variables directly impact the selections of stope shapes and types. A mining zone is associated with one or more haulage systems $h \in H$. Therefore, identical mining zone configuration options (b and b'), in terms of stope shapes and sublevels, can exist but they will be associated with different available haulage systems ($b \in B_h$, and $b' \in B_{h`}$). It is assumed that vertical accesses compatible with the haulage systems are already developed, which enables the variable z_{ba} to be time-independent. The stope selection decision variables $y_{jbat} \in \{0,1\}$ determine if a stope $j \in J_b$ in a mining zone configuration $b \in B$, using stope type option $a \in A_b$ is mined in period $t \in T$ and sent directly to the processor. A variable $x_{jbatt'} \in \{0,1\}$ controls the extraction sequence and posterior reclamation of stockpiled stopes by defining if a stope $j \in J_b$ in a mining zone configuration $b \in B$, using stope type option $a \in A_b$ is mined and sent to a stockpile in period t and rehandled at period $t' > t$. Thus, it is assumed that a stockpile for each time period t' can exist, or that the selection of the material of stopes within a stockpile is possible, in order to have a linear formulation (Brika 2019).

Two continuous decision variables ψ_{dlbt}^{drift} and $\psi_{clbt}^{crosscut}$ correspond to the developed distance of a drift d or a cross-cut c, for a sublevel l, in a mining zone configurations b, in period t. To account for the available structures developed in previous years, effective development distances $\psi_{dlbt}^{drift^*}$ and $\psi_{clbt}^{crosscut^*}$ are used in practice and they correspond to the cumulative horizontal development distances. The remaining decision variables refer to surplus deviations from haulage capacities for different haulage systems $h\left(d_{hts}^{haul}\right)$, processing capacity ($d_{ts}^{proc}$) and stockpiling capacities (d_{ts}^{sp}), and deviations from lower and upper bounds for different elements $\varepsilon \in E$ requirements, ${d_{\varepsilon ts}}^{-}$ and ${d_{\varepsilon ts}}^{+}$ respectively. Additional technical and economical parameters are presented in Table 3-1.

Table 3-1 – List of technical and economic parameters

Index	Definition
w_{jbas}	Tonnage of stope j, in mining zone configuration *b*, stope sequencing option *a* and in geological scenario *s*
$g_{jb\varepsilon s}$	Grade of element ε within stope j in mining zone *b*, in scenario *s*
f_t^{EDR}	Economic discount factor for period t given an economic discount rate
C_{lt}^{H}	Discounted horizontal development discounted cost in sublevel *l*, at period *t* in $/km

Index	Definition
${C_{kt}}^{M}$	Discounted mining cost for type *k* stopes at period *t* in $/t
C^{P}	Unitary processing cost $/t
C_{t}^{rehan}	Discounted rehandling cost at period t in $/t
C_{hbt}^{haul}	Discounted haulage cost at period *t* if $h \in H_{ramp}$ in $/(tons*km) and if $\in H_{shaft}$ in $/t
F_{bt}	Fixed discounted cost for keeping the mining zone configuration b
U_{t}^{sp}	Stockpiling capacity at period *t*(tons/year).

Once stockpiling decisions are considered, mining, rehandling, and processing costs can be incurred in different periods for a given stope. Therefore, Eq. 3.3 describes the general profit of a stope at the period during which this stope is processed.

$$p_{jbast} = f_{t}^{EDR} w_{jbas} \left(\sum_{\varepsilon \in E} g_{jb\varepsilon s} R_{\varepsilon} P_{\varepsilon} - C^{P} \right), \forall\, j \in J_{b}, b \in B, a \in A, s \in S \qquad (3.3)$$

Eq. 3.4 presents the objective function in five parts. Part I aims to maximize the discounted revenue from the stopes that are mined and processed at the same period. Part II maximizes the revenue from the scheduled stopes that are stockpiled by applying the discounted mining and rehandling costs according to the year in which they are incurred in the production schedule. Haulage costs are managed in different ways depending on the transportation systems available and chosen by the optimizer. If material is hauled through a ramp, the distance from the sublevel to the surface (δ_{hbl}) must be considered; otherwise, if the material is hauled through a skip, the parameter δ_{hbl} is set as one. Part III of the objective function minimizes the effective development costs and part IV minimizes a fixed cost for keeping the mining zone in operation. Finally, part V manages the geological risk by minimizing the deviations from the production targets related to mining, stockpiling and processing capacities, and grade blending requirements. For that purpose, penalty costs c_{h}^{haul}, c^{P}, c^{sp} c_{ε}^{+} and c_{ε}^{-} are applied to correspond to the production requirements and targets, as they are discounted by a geological risk discounting factor f_{t}^{GRD} (Ramazan and Dimitrakopoulos 2013, 2005).

$$max\frac{1}{|S|}\underbrace{\sum_{s\in S}\sum_{t\in T}\sum_{b\in B}\sum_{a\in A_b}\sum_{j\in J_b}\left(p_{jbast}-w_{jbs}\left(\sum_{k\in K}\pi_{kjba}C_{kt}^{P}-\sum_{h\in H}\sum_{l\in L_b}\delta_{hbl}C_{hbt}^{haul}\right)\right)y_{jbat}}_{Part\ I}$$

$$\underbrace{+\frac{1}{|S|}\sum_{s\in S}\sum_{t}^{|T-1|}\sum_{t'=t+1}^{|T|}\sum_{b\in B}\sum_{a\in A_b}\sum_{j\in J_b}\left(p_{jbast'}-w_{jbs}\left(\sum_{k\in K}\pi_{kjba}\,C_{kt}^{M}-\sum_{h\in H}\sum_{l\in L_b}\delta_{hbl}C_{hbt}^{haul}-C_{t'}^{rehan}\right)\right)x_{jbatt'}}_{Part\ II}$$

$$\underbrace{-\sum_{t\in T}\sum_{b\in B}\sum_{l\in L_b}C_{lt}^{hor}\left(\sum_{d\in D_l}\psi_{dlbt}^{drift^*}+\sum_{c\in C_l}\psi_{clbt}^{crosscut^*}\right)}_{Part\ III}$$

$$\underbrace{-\sum_{t\in T}\sum_{b\in B}\sum_{a\in A_b}F_{bt}\,z_{ba}}_{Part\ IV}$$

$$\underbrace{-\frac{1}{|S|}\sum_{s\in S}\sum_{t\in T}f_t^{GRD}\left(c_h^{haul}d_{hts}^{haul}+c^{P}d_{ts}^{P}+c^{sp}d_{ts}^{sp}\sum_{\varepsilon\in E}c_\varepsilon^{+}\,d_{\varepsilon ts}{}^{+}+c_\varepsilon^{-}d_{\varepsilon ts}{}^{-}\right)}_{Part\ V}\quad(3.4)$$

The objective function is subjected to reserve, adjacency, non-overlapping, and capacity constraints. The addition of decision variables that control both the extraction sequence and the stockpiling decisions requires a simple adaptation of the reserve, adjacency and capacity constraints proposed by Carelos Andrade et al. (2024). New constraints are included to control the stockpiling capacity.

$$\sigma_{t,s}=\sum_{b\in B}\sum_{a\in A_b}\sum_{j\in J_b}\left(w_{jbs}\sum_{t`=2}^{|T|}x_{j,b,a,t,t'}\right)\ \forall t=1,s\in S \qquad (3.5)$$

$$\sigma_{t,s}=\sum_{b\in B}\sum_{a\in A_b}\sum_{j\in J_b}\left(w_{jbs}\left(\sum_{t`>t}^{|T|}x_{j,b,a,t,t'}-\sum_{t'=1}^{|T|}x_{j,b,a,t',t}\right)\right)+\sigma_{t-1,s},\ \ \forall t>1,s\in S \qquad (3.6)$$

$$\sigma_{t,s}-d_{t,s}^{sp}\leq U_t^{sp},\ \ \forall t\in T,\ s\in S \qquad (3.7)$$

Eq. 3.5 and 3.6 calculate the value of an auxiliar variable $\sigma_{t,s}$ that defines the tonnage left at the stockpiles at the end of period *t* for scenario *s.* This tonnage is constrained by a maximum yearly capacity U_t^{sp} that can be left stockpiled at the end of each period *t*, as shown in Eq. 3.7.

3.2.2 Mineral deposit modeling using sequential simulations

Consider $Z(\boldsymbol{u}_i)$ a stationary ergodic random field indexed in R^n, where $\boldsymbol{u}_i$, represents the location of the points $i = 1 \dots N$ of the grid to be simulated in a domain $D \subseteq R^n$. The set $\boldsymbol{d}_n = \{z(\boldsymbol{u}_\alpha), \alpha = 1 \dots n\}$ denotes the original sample data conventionally obtained by the exploration data. A set Λ_i represents the conditioning data for each node index by *i*. Therefore, $\Lambda_0 = \{\boldsymbol{d_n}\}$ is the conditioning data when the first point is simulated and only sample data is available and $\Lambda_i = \{\Lambda_{i-1} \cup Z(\boldsymbol{u}_i)\}$ is the conditioning data for the subsequent points being simulated that includes the original sample data and previously simulated points. Accordingly, the sequential simulation paradigm defines that the joint probability density function (pdf) of the random field $Z(\boldsymbol{u}_i)$ can be decomposed into the product of conditional univariate distributions (Goovaerts 1997; Journel 1994)

$$f(u_1, \dots, u_N; z_1, \dots, z_N|\boldsymbol{d_n}) = f(u_1, z_1|\boldsymbol{d_n}) \prod_{i=2}^{N} f(\boldsymbol{u}_i, z_i|\Lambda_{i-1}) \,. \tag{3.8}$$

The conditional probability distribution function (cpdf) for any node u_i can be written according to the Bayes' rule as

$$f(u_\mathrm{i}; z_i|\Lambda_0, \Lambda_{i-1}) = \frac{f(u_\mathrm{i}; \lambda_0, \lambda_{i-1}; z_0, \Lambda_0, \Lambda_{i-1})}{\int f(u_\mathrm{i}; \lambda_0, \lambda_{i-1}; z_0, \Lambda_0, \Lambda_{i-1}) du_i} \,, \tag{3.9}$$

where λ_0 and $\lambda_{i-1}$are the locations of the points in the conditioning data sets Λ_0 and Λ_{i-1}, respectively and $f(u_\mathrm{I}; \lambda_0, \lambda_{i-1}; z_0, \Lambda_0, \Lambda_{i-1})$ is the joint pdf.

3.2.3 High-order simulation using Legendre-like orthogonal splines

To generate geostatistical simulations that account for high-order spatial statistics (Dimitrakopoulos et al. 2010; Dimitrakopoulos and Yao 2020; Minniakhmetov et al. 2018), the method proposed by Minniakhmetov et al. (2018), where the joint cpdf is approximated using high-dimensional polynomials combined with high-order spatial cumulants is used herein and summarized bellow.

$$f(u_{\mathrm{i}};\lambda_0,\lambda_{i-1};z_0,\Lambda_0,\Lambda_{i-1}) = \sum_{m_0=0}^{\omega_0}\sum_{m_1=0}^{\omega_1}\dots\sum_{m_n=0}^{\omega_n} L_{m_0,m_1\dots m_n}\varphi_{m_0}(z_0)\varphi_{m_1}(z_1)\dots\varphi_{m_n}(z_n)\ , \tag{3.10}$$

where $L_{k_0,k_1,\dots k_n}$ are coefficients of approximation and $\varphi_{m_0}(z_0)\varphi_{m_1}(z_1)\dots\varphi_{m_n}(z_n)$ follows the orthogonality property as

$$\int_a^b \varphi_m\varphi_k(z)dz = \delta_{mk}\ , \tag{3.11}$$

where $\delta_{m_n k_n} = \begin{cases}1, m = k\\ 0, m \neq k\end{cases},\ \forall k = 0 \dots \omega$ is the Kronecker delta.

The orthogonal functions considered herein are Legendre-like orthogonal splines (Minniakhmetov et al. 2018) and the Legendre coefficients $L_{k_0,k_1,\dots k_n}$ can be approximated experimentally by calculating

$$L_{k_0,k_1,\dots k_n} \approx E\left[\varphi_{k_0}(z_0)\varphi_{k_1}(z_1)\dots\varphi_{k_n}(z_n)\right] \approx \frac{1}{N_{h_1,h_2\dots h_n}}\sum_{k=1}^{N_{h_1,h_2\dots h_n}} \varphi_{k_0}\left(z_0^k\right)\varphi_{k_1}\left(z_1^k\right)\dots\varphi_{k_n}\left(z_n^k\right), \tag{3.12}$$

where $z_i^k, i = 0 \dots n$ are values taken from a training image (TI), or geologic analog, that contains densely sampled geological information and represents complex geological structures.

The method relies on the definition of a spatial template formed by the central node being simulated and neighbouring values separated by lag vectors $\boldsymbol{h}_i = \boldsymbol{u}_i - \boldsymbol{u}_0,\ i = 1 \dots n$, which is used to scan the TI, to calculate the Legendre coefficients. The high-order sequential simulation algorithm follows:

1. Define a random path for visiting all unsampled nodes on the simulation grid.
2. For each node $\boldsymbol{u}_0$ in the path:
    a. Find the closest neighbor nodes $\boldsymbol{u}_{1,}\boldsymbol{u}_2, \dots \boldsymbol{u}_n$.
    b. Obtain the spatial template configuration by calculating the lag vectors $\boldsymbol{h}_i$.
    c. Scan the TI and find values $z_i^k, i = 0 \dots n$ given the spatial template configuration.
    d. Calculate the spatial Legendre coefficients $L_{k_0,k_1,\dots k_n}$ using Eq. 3.12.

e. Build the cpdf $f(u_I; z_i|\Lambda_0, \Lambda_{i-1})$ by calculating the joint probability density function as in Eq. 3.10 and normalizing it as shown in Eq. 3.9.
f. Draw a uniform random value in [0,1] to sample z_0 from the cumulative cpdf derived on the previous step.
g. Add z_0 to the set of conditioning data Λ_i and move to the next node

3. Repeat steps 1 and 2 to generate different realizations.

3.2.4 Sequential Gaussian simulation

The case study presented in Section 3.3 also uses sequential Gaussian simulation (SGS) (Goovaerts 1997; Isaaks 1990; Journel 1994) as an input, so that the related schedules and the forecast are compared to the ones obtained when the HOSIM is used. The method follows the sequential simulation paradigm, while assuming a Gaussian conditional probability distribution function (cpdf) $f(u_I; z_i|\Lambda_0, \Lambda_{i-1})$ that can be parametrized by its mean and variance. Initially, the original sample data is transformed to the Gaussian space, the experimental variogram is calculated from the transformed data, and the variogram model is inferred. Then, at each node, the Kriging system is used to obtain the conditional mean and variance, allowing the definition of a normal cpdf from which the simulated values will be sampled.

3.3 Case study at an operating copper mine

The case study presented herein shows first an analysis of the simulations produced by the high-order sequential simulation (HOSIM) method described in Section 3.2.3 and a comparison to sequential Gaussian simulations (SGS) of a copper deposit related to an operating underground copper mine. These simulations obtained through the HOSIM method are used as an input to the proposed extended integrated stochastic optimization of the underground mine design and production schedule with stockpiling, the extraction sequence, and forecasts presented. A comparison between these outputs to the ones obtained with the same optimization framework and technical parameters, but with simulations generated with the SGS method, is subsequently shown.

3.3.1 High-order sequential simulations of the mineral deposit, results and comparisons to sequential Gaussian simulations

The realizations of the copper deposit using the high-order sequential simulation (HOSIM) method are done in a grid size of 5m x 5m x 5m x 5m with 466,560 nodes. In order to generate the simulations, 1,510 exploration drillholes with 5 m composites are spatially distributed as fans, with centers at approximately every 30 m, which are used as the sample data, as shown in Figure 3-3. In addition, a training image (TI) generated from densely sampled blast hole data was used. A set of 20 high-order sequential simulations is generated in point support and then go through the previously described preprocessing steps to generate the inputs to the proposed optimization method. From this set, 10 simulations are used directly as an input to the optimization and the remaining 10 simulations are further used for the risk analysis of the related forecasts. The number of simulations follows previous studies that show that 10-12 simulations are sufficient to produce stable results for the mine planning optimization (Albor Consuegra and Dimitrakopoulos 2009; Dimitrakopoulos and Lamghari 2022; Montiel and Dimitrakopoulos 2017).

Ten simulations of the copper deposit, using the same exploration data, are generated based on the sequential Gaussian simulation (SGS) method are generated, to be used as means of comparison to the ones obtained using HOSIM. Figure 3-4 shows the grade-tonnage curves for the two sequential simulation frameworks being compared, considering the minimum stope dimensions (i.e., 15m x 30m x 40m). The blue and green curves in the graph overlap each other for some of the simulated scenarios indicating that, for both methods, the grade and tonnage proportions are similar. Thus, the simulation method does not directly impact the metal quantities. Figure 3-5 shows cross sections of simulations using high-order and Gaussian sequential simulations. A visual inspection indicates that both realizations reproduce the spatial distribution of copper grades of the exploration data. However, the realization generated with the SGS shows a more dispersed behavior, representing the effect of maximum entropy when the data is transformed into Gaussian space. The highlighted high-grade areas show better connectivity for the realization generated with HOSIM, as expected.

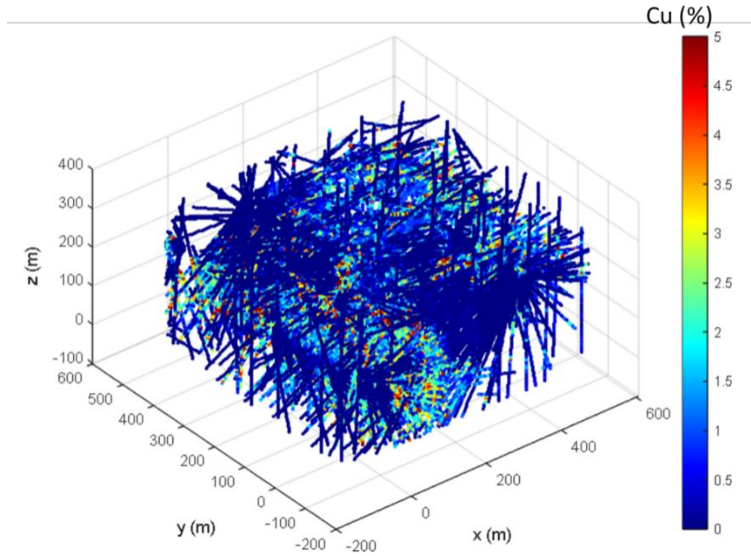

Figure 3-3 – Exploration data with underground drilling fans

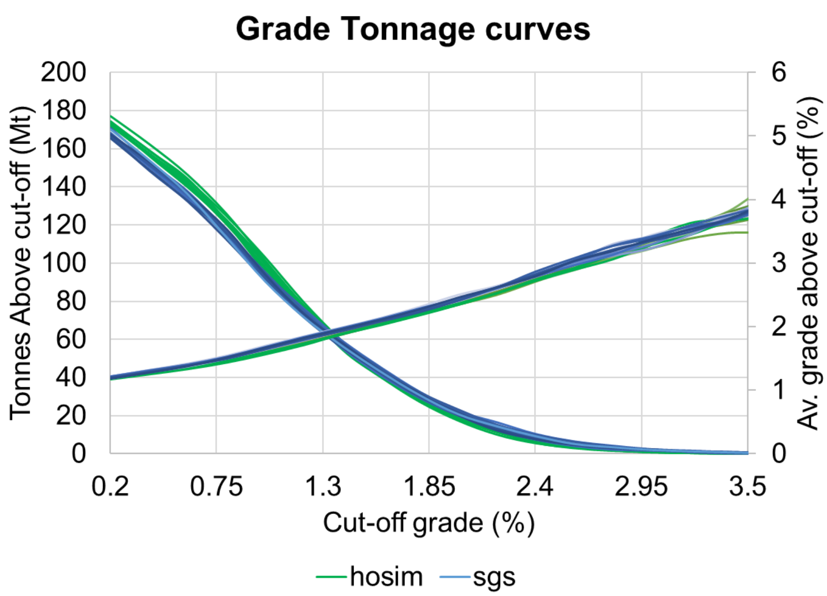

Figure 3-4 – Grade-tonnage curves for simulated copper deposit using SGS and HOSIM, for stopes $15x30x40m^3$

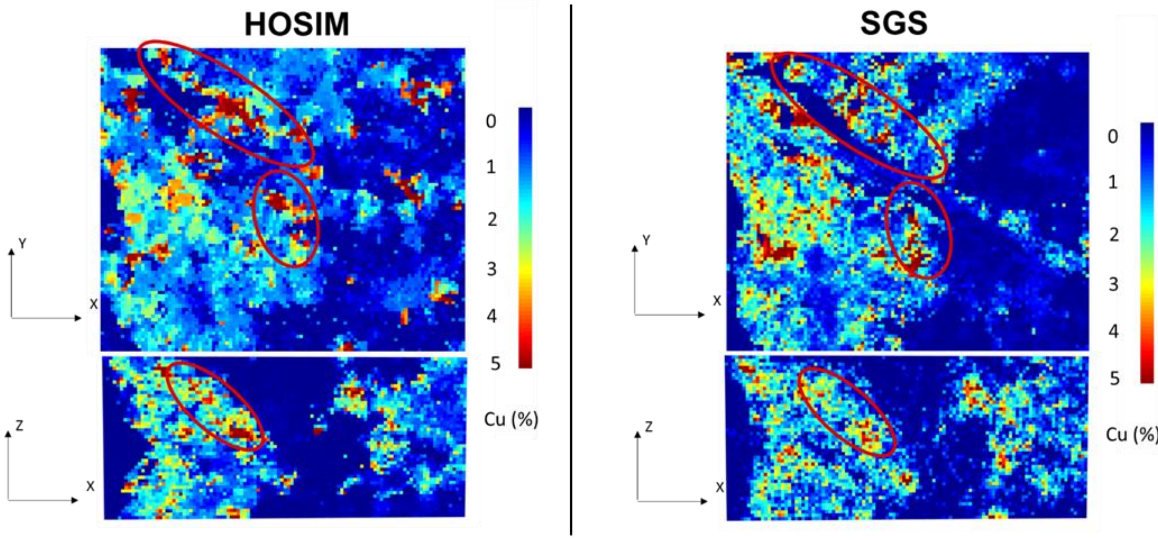

Figure 3-5 – Cross-sections of the simulations high-grade areas highlighted in red.

The reproduction of spatial statistics of the simulations in terms of the exploration data is evaluated. Figure 3-6 shows the histograms of the sample data, TI, and high-order sequential simulation realizations. Similarly, Figure 3-7 to 10 show the variograms in x and y directions, 3rd and 4th order cumulant maps form the sample data, TI, and of a realization produced by HOSIM and SGS methods. The areas with red circles highlight the main differences among the cumulant maps. It is seen that both methods can reasonably reproduce the histograms and variograms of the exploration data. For 3rd and 4th order cumulants, however, the realization obtained with the HOSIM method shows a closer reproduction to the sample data, compared to what is shown for the realization obtained with the SGS method. In addition, although the described HOSIM method uses a TI to infer the conditional probability distribution function (cpdf), the simulated values reproduce the low and high-order statistics of the exploration (i.e., sample) data. In fact, the TI assumes an auxiliary part in the simulation procedure, while the initial sample data serves as conditioning data and is also used to calculate the cpdf.

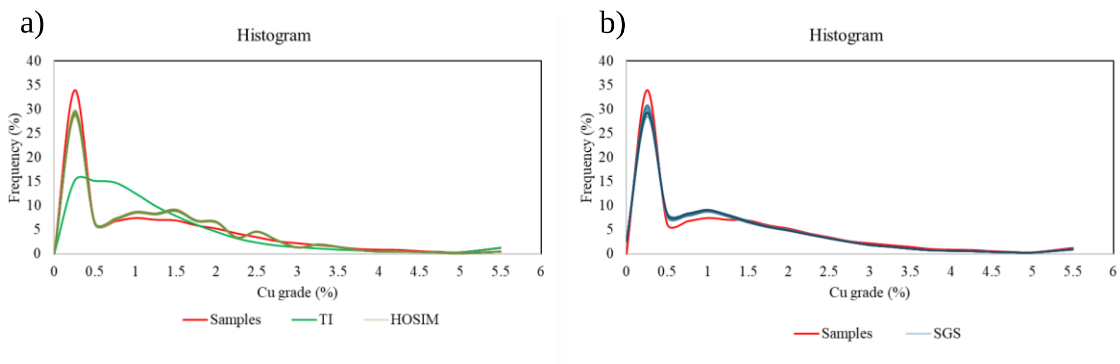

Figure 3-6 – Histograms of samples (red), TI (green), a) HOSIM realizations (grey) and b) SGS realizations (blue)

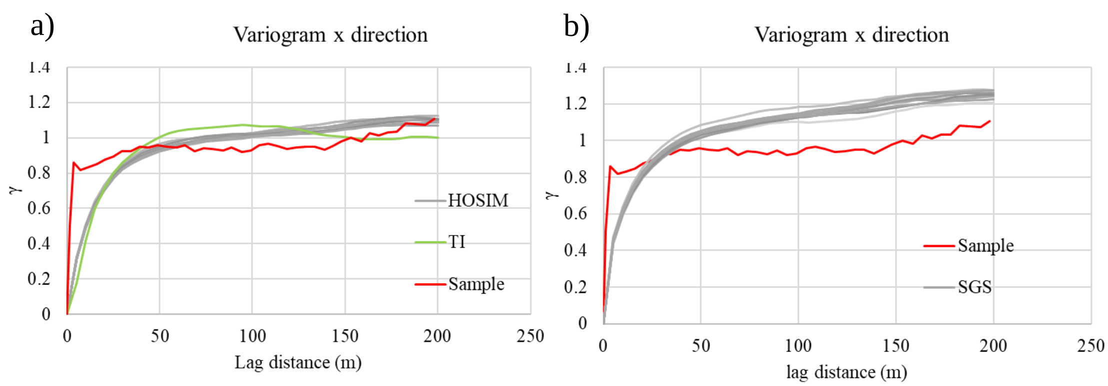

Figure 3-7 – Variograms in x direction of samples (red), TI (green), a HOSIM realizations (grey) and b) SGS realizations (grey)

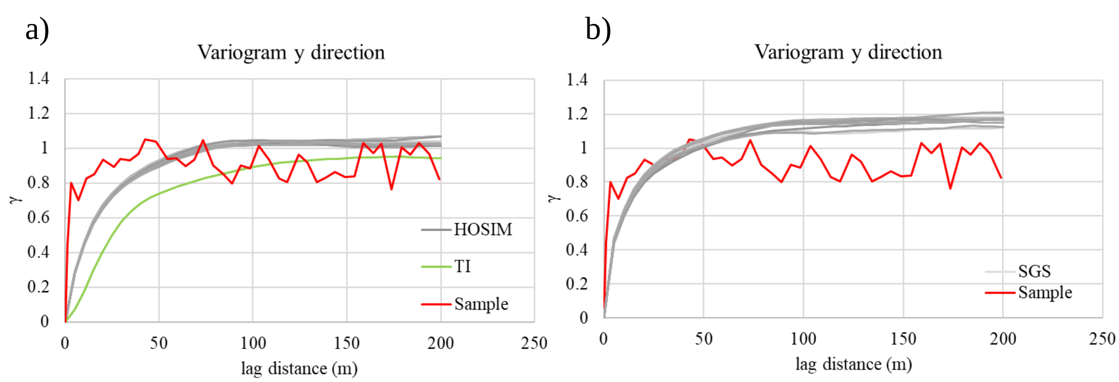

Figure 3-8 - Variograms in y direction of samples (red), TI (green), a) HOSIM realizations (grey) and b) SGS realizations (grey)

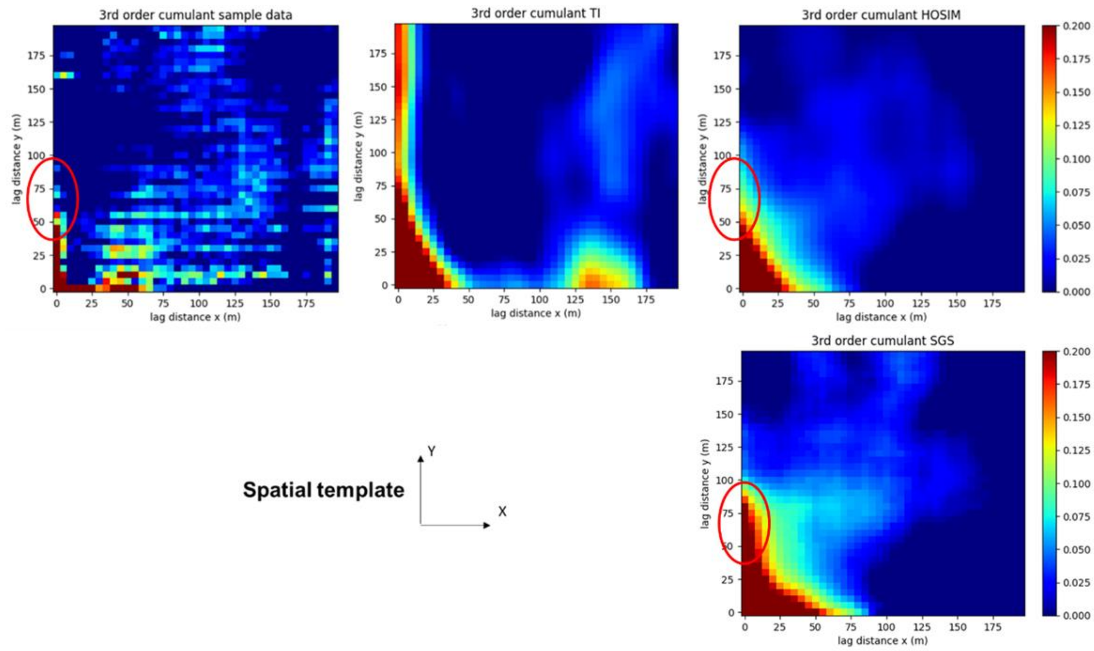

Figure 3-9 – 3rd order cumulant maps of sample data, the used TI, HOSIM simulated realization and SGS simulated realization, where areas highlighted in red show differences in cumulative maps.

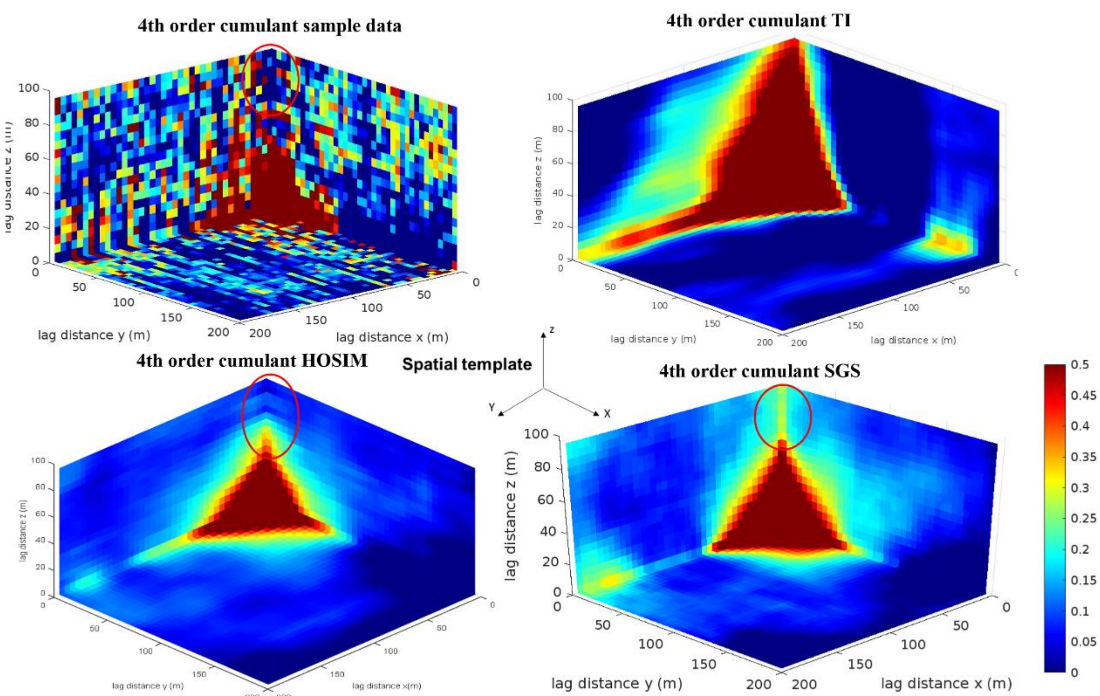

Figure 3-10 – 4th order cumulant maps of sample data, the used TI, HOSIM simulated realization and SGS simulated realization, where areas highlighted in red show differences in cumulative maps.

3.3.2 Integrated stope design and scheduling optimization and forecasting

The application of the proposed integrated stochastic stope design and production scheduling with long-term stockpiling at a copper deposit is presented in this section. The comparison of the stope design, extraction sequence, and related forecasts using high-order and Gaussian sequential simulations presented in the previous section, are used as inputs to the optimization approach, the result of which is then analyzed.

The present case study is developed for an operating underground copper mine. Therefore, mine accesses (i.e. ramp and decline) and ventilation systems are given as inputs. Figure 3-11 shows the available infrastructure in the mine and the defined mining direction. In addition, geometrical parameters for the shapes of the stopes were provided by the mining company operating the mine. The maximum and minimum dimensions of the stopes are considered, given geotechnical and drilling equipment requirements, as shown in , where the green arrow indicates the mining direction, and the red arrow indicates the direction to the surface decline.

Table 3-2. These possible configurations account for the position of the access point at each sublevel according to the available ramp design. Three option types are used as an input, meaning that all stopes will have the possibility of being primary, secondary, or tertiary. In addition, the stopes are blasted and extracted bottom-up and are subsequently backfilled with cemented aggregate fill (CAF), which is not considered a limiting feature in the mine production. Thus, a single mining cost is used for all stope types. A horizontal development capacity is considered in terms of the maximum length that can be developed. A single haulage system is available with its maximum capacity constraining the mining capacity, and a processor with a smaller capacity that controls the copper concentrate product production. An annual stockpile capacity is considered in order to manage the grade blending, while uncertainty in terms of copper grades is taken into consideration. Thus, penalty costs for deviations from the minimum and maximum grades for this element's requirements are applied and are discounted throughout the years to manage the geologic risk (Ramazan and Dimitrakopoulos 2013, 2005). Table 3-3 displays the technical and economic parameters used in the optimization of the copper mine. The proposed SIP model is programmed in the C++ language and solved using the CPLEX v.12.8.0 software's solver engine (IBM ILOG 2017).

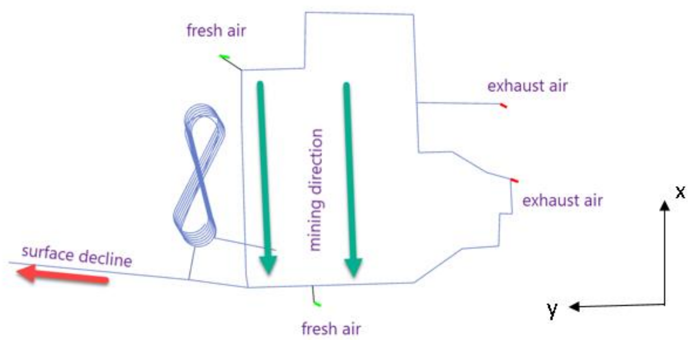

Figure 3-11 - Plan view of the developed infrastructure, where the green arrow indicates the mining direction, and the red arrow indicates the direction to the surface decline.

Table 3-2- Stope geometrical parameters

Parameter	Value/Description
Minimum dimensions	15m x 30m x 30m
Maximum dimensions	30m x 30m x 150m

Table 3-3 - Technical and economic parameters used as input in the optimization

Parameter	Value
Cu price	8,500 $/t
Economic discount rate	10%
Geologic discount rate	10%
Processing recovery Cu	94%
Mining cost	50 $/t
Processing cost	13.5 $/t
Haulage cost	5 $/t·km
Rehandling cost	0.5 $/t
Drifts development cost	12,000 $/m
Density	3.2 t/m3
Haulage capacity	3 Mt/year
Processing capacity	2.5 Mt/year
Stockpiling capacity	400 kt/year
Drift development capacity	5,000 m/year
Minimum copper mill head grade	1.8 %
Penalty cost for deviations below minimum Cu mill head grade	100 $/unit

Figure 3-12 displays the extraction sequence and the optimum stope types when the simulations generated with the HOSIM method are used. An operational extraction sequence that follows the mining direction and the bottom-up extraction approach, while respecting the adjacencies given by the optimized stope types is produced. The output results obtained with the high-order sequential simulations are compared to those obtained when the simulation generated using the SGS method are the inputs. Similar to Figure 3-12, Figure 3-13 displays the extraction sequence and the optimum stope types when the simulations generated with the SGS method are used. It is observed that, although for both cases the same parameters are considered, the extraction sequence displayed in Figures 12 and 13 are different, and relevant differences can be noticed on the final stope layout. Once the simulated grades are averaged into possible stope volumes, the more connected the high grades are, the higher the stope grades will be. When the simulations generated by the HOSIM method are used as inputs, areas further to the access point can be mined, once the profit generated from the metal content of this stope prevails over the cumulative horizontal development costs. On the other hand, when sequential Gaussian simulations are used, it is observed that the stopes closer to the access points are preferred once they incur lower development costs, as highlighted with the red circles Additionally, when the realizations generated by HOSIM are used, smaller stope sizes are chosen, allowing more selectivity in terms of high-grade stopes and less dilution.

Figure 3-14 show the risk profiles considering the decisions optimized using the different inputs with 10 additional high-order sequential simulations. The results are presented in terms of P10, P50 and P90, representing the 10^{th}, 50^{th} and 90^{th} percentiles of the related performance indicators, respectively. Figure 3-14a shows that the produced copper content is 4% higher when the realization generated with the HOSIM method are the related inputs. This can be explained by the maximum entropy that the Gaussian-based approaches generate with respect to the high grades. Thus, after the optimization process, it is observed that the high-grade areas are a better target when a better representation of extreme grade continuity is given as an input. The copper production impacts directly on the NPV, which is 6% higher for the HOSIM case compared to the SGS case, as shown in Figure 3-14b. Although the mined tonnage is similar for both cases (Figure 3-14c), the cumulative stockpiled tonnage is approximately 5% higher for the SGS case (Figure 3-14e), which means that the use of the stockpile is necessary to achieve the grade blending requirements, incurring higher rehandling costs. Figure 3-14d shows that grade blending requirements are overall achieved for both cases, with probable deviations from the lower bound for the SGS case. As previously inferred from the extraction sequences (Figure 3-12 and Figure 3-13), in the SGS case, the areas closer to the access points are privileged. In the graph displayed in Figure 3-15, the horizontal development costs for the SGS case are, in fact, lower for early periods. However, the total horizontal development cost is similar for both cases. Considering the ability of the optimization process to decide whether to stockpile mined material or not and when over the life of this mine, the analysis demonstrates that, in both scenarios, it is profitable to direct material to stockpiles, given the associated rehandling costs. This strategic choice results in an optimal NPV and minimum deviations from the required lower bound of copper grade Figure 3-16 shows the number of active stockpiles and tonnage left at the stockpiles for each period and for each case, assuming that multiple stockpiles can be used to assist the selection of the stockpiled material to be processed. The maximum yearly stockpiling capacity of 400,000 tons is respected and a maximum of three active stockpiles are needed for both cases. This finding underscores the significant role of stockpiling decisions in long-term mine planning, as they can conform to operational considerations and requirements.

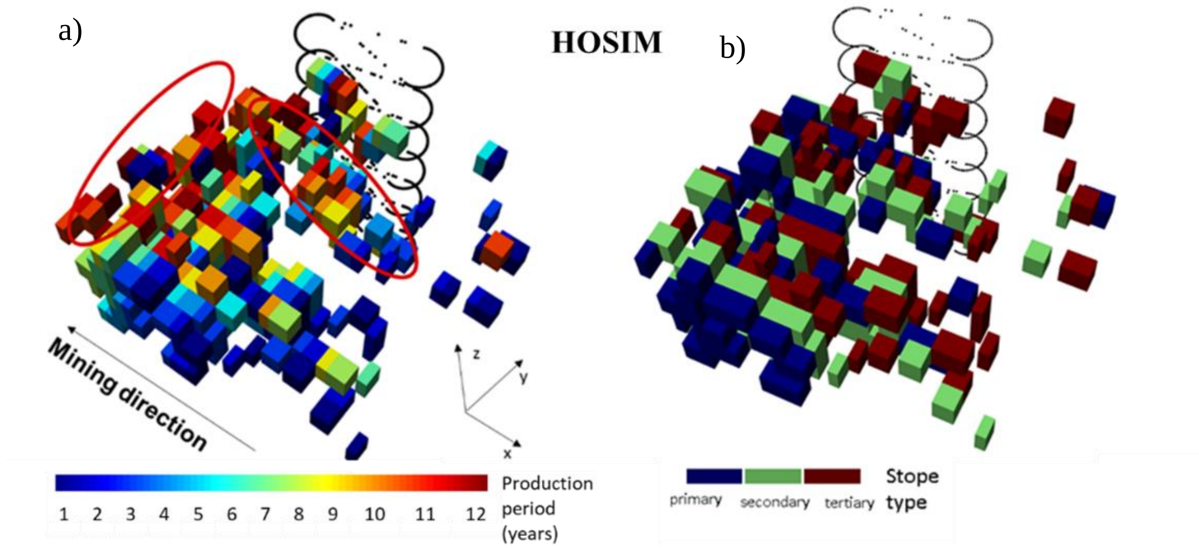

Figure 3-12 – a) Extraction sequence and b) final stope types using input realization from HOSIM, red circles highlight the correspondent high-grade areas.

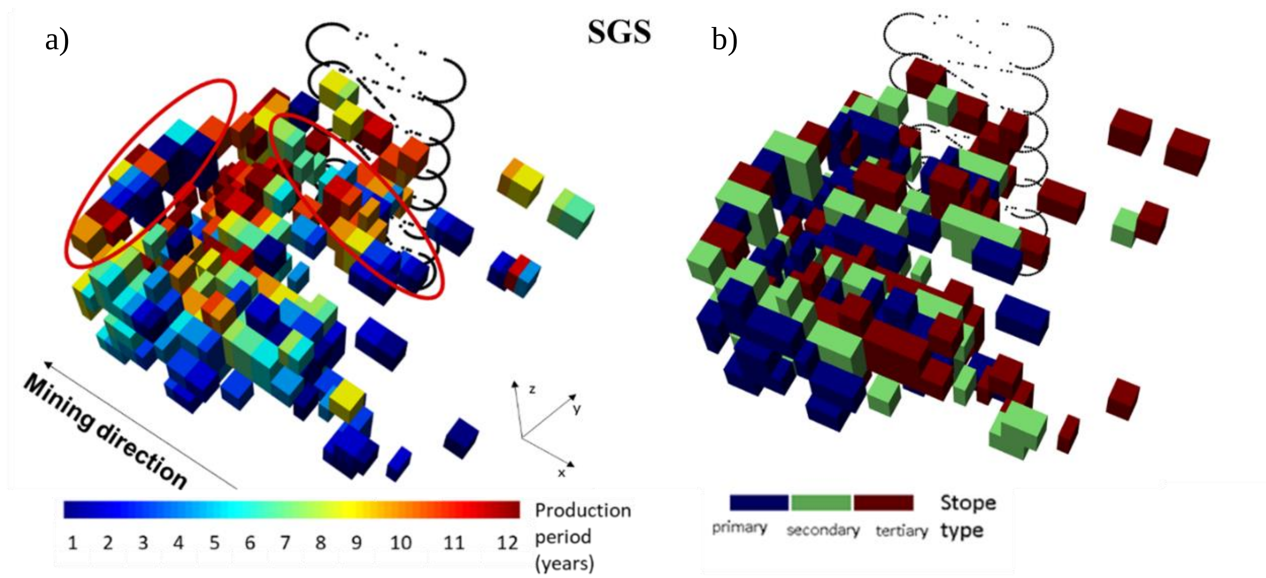

Figure 3-13 – a) Extraction sequence and b) final stope types using input realization from SGS, red circles highlight the correspondent high-grade areas.

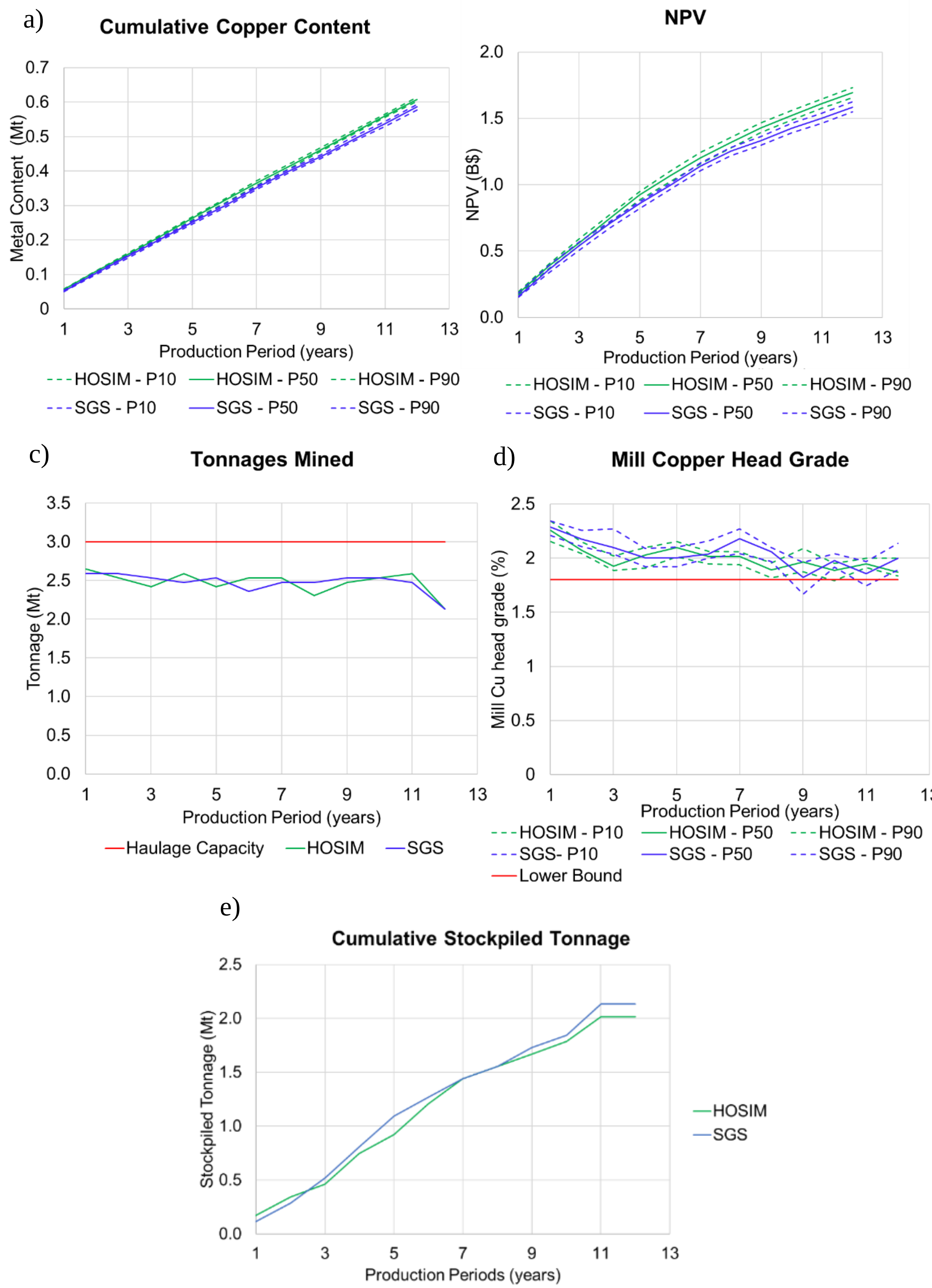

Figure 3-14 – Risk profiles for a) cumulative recovered copper, b) NPV, c) total tonnages mined, d) mill copper head grade, and e) cumulative stockpiled tonnage

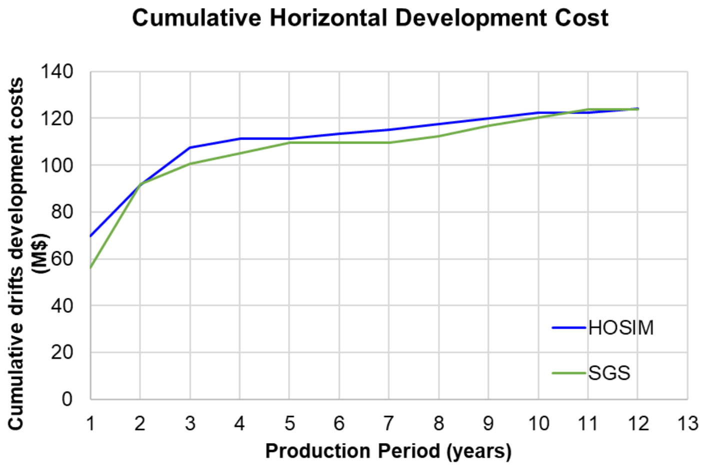

Figure 3-15 - Cumulative development costs for drifts and crosscuts

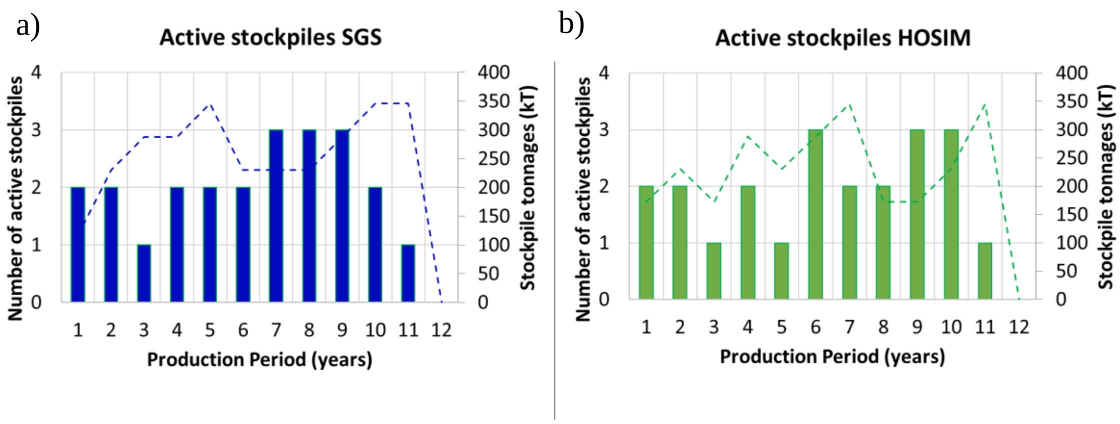

Figure 3-16 – Number of active stockpiles and tonnage left at the stockpiled for each periods

3.4 Conclusions

This chapter proposes an extension of previous work on the integrated stochastic optimization of stope design and mine production scheduling, through a two-stage stochastic integer programming (SIP) linear formulation that accounts for long-term stockpiling decisions for the sublevel open stoping (SLOS) mining method. The operational aspects and impacts on the mine production scheduling with this additional component is evaluated. The objective function of the proposed SIP aims to maximize the net present value (NPV) of the project while managing the geological uncertainty, by minimizing deviations from production targets, subjected to operational constraints are presented. Additionally, the effects of using the high-order sequential simulation (HOSIM) method to generate the realizations of a copper deposit to be used as inputs for the proposed stochastic optimization formulation are also presented. This simulation method infers high-order spatial statistics from available data, enabling the reproduction of complex geological patterns of natural phenomena. The optimized stope layout and production schedule along with the related forecasts are compared against a case in where the conventional sequential Gaussian simulations are the main inputs for the optimization formulation. It is observed that these simulation methods can be fairly compared against each other once they produce similar grade-tonnage pro-portions and reproduce related statistics, that is, histograms and variograms of the available sample data. It is seen, however, that, due to the maximum entropy property of the Gaussian-based methods, the extreme high grades are more spatially dispersed, showing a misrepresentation of the spatial connectivity of the high grades. It is also verified that the realizations obtained with the HOSIM method reproduce the sample data high-order spatial statistics despite utilization of a training image (TI).

The application of the proposed method shows that the long-term stockpiles can be operationally implemented. Under the assumption of having multiple active stockpiles, to facilitate operational mining aspects, it is seen that a maximum of three active stockpiles are needed and the stockpile tonnage after each production year does not reach the maximum capacity of 400,000 tones, for both HOSIM and SGS input cases. In addition, it is observed that the optimization process takes advantage of the more connected high-grade representations of the copper deposit to generate the stope designs and production schedules. Notable differences are observed on the final stope boundaries and extraction sequences comparing the two cases. The HOSIM case tends to target the high-grade continuity areas to produce a 6% higher NPV, while the SGS case initially mines areas that incur a smaller horizontal development cost. This outcome is observed once the simulated values are averaged into large stope volumes; thereafter, the realizations with better connected high grades generate higher-grade possible stopes. As a result, the higher the stope grade, the lower the impact of the horizontal development cost on its profit. The HOSIM case produces a 4% higher copper content at the end of 12 years of production, which directly impacts on the cumulative cashflow. In addition, the HOSIM case is able to produce ore material that follows the grade-blending requirement of the mill by sending 5% less material to the stockpile.

A case study that accounts for high-order sequential simulations of multiple elements is a topic for future research, once secondary and deleterious elements also play an important role in decision-making for mine planning activities. In addition, the simultaneous optimization of a mining complex that assumes the existence of multiple mines, stockpiles, and processing streams is an extension for future work. Also, this simultaneous optimization approach should generalize to different types of underground mining methods, while further facilitating the interaction between underground and open-pit mining operations. Furthermore, as more components are included in the mathematical programming formulation, the development of alternative solvers, rather than the ones commercially available, is a proposed for future contribution. The proposed method addresses long-term underground stope layout, mine planning and production scheduling. However, it is recognized that intricate operational considerations, such as the scheduling of individual activities, prediction of technical parameters, and updates on geotechnical parameters, are important for short-term planning. Future developments related to short-term planning could consider integrating these aspects, as well as enabling the interaction between short and long-term planning.

Chapter 4 - Conclusions and Future Research

4.1 General Conclusions

Underground long-term mine planning is conventionally addressed with a stepwise optimization framework that starts with the definition of a stope layout, followed by the design of haulage and ventilation network systems that connect the mining areas to the surface. Finally, the optimization of the life-of-mine production schedule is performed, aiming to maximize the net present value (NPV). This conventional stepwise framework is unable to exploit the synergies between the mine planning components, leading to suboptimal solutions. In addition, the traditional mine planning frameworks are deterministic, hence geological uncertainty and variability of grades and material types are not considered. Recent stochastic approaches have improved the sequential framework by jointly optimizing stope design and production scheduling. These few developments present specific mathematical formulations that are not generalizable to various mining methods and its variants. Additionally, several components and practices currently used in the mining industry, such as stockpiling and backfilling, are not incorporated in the developed methods. These observations motivate the development of stochastic optimization frameworks that are able to integrate the stope design and production schedule using operational considerations of an existing underground mine. The first model proposed in this thesis presents a joint stochastic optimization of stope design and extraction sequence for a sublevel longhole open stoping (SLOS) mining method with backfilling and its application at an operating copper mine. The second method expands the first stochastic integer programming (SIP) formulation to incorporate stockpiling decisions and investigate the impacts of using high-order sequential simulations as inputs.

The first method presented in Chapter 2 refers to the joint optimization of stope design and mine production scheduling for the SLOS mining method with backfilling. The proposed two-stage SIP maximizes the NPV, while managing the geological risk by minimizing the deviations from production targets. The outputs of the optimization include the operational selection of mining zone configuration that defines the stope shapes and respective types, as well as the stope design and extraction sequence of stopes that respects the optimal backfilling adjacencies and have optimal horizontal development costs. An application of the proposed method to an underground copper mine with gold and uranium as secondary elements is presented. As an operating mine, a ramp and a ventilation system that defines a mining direction are available infrastructure given as inputs. It follows a pattern of extraction of primary, secondary, and tertiary stopes. The integrated framework is compared to a sequential stochastic framework, in which a stope layout is used as a fixed input to the proposed SIP. It is seen that physically different stope designs and, consequently, different extraction sequences are produced for these two different frameworks. The proposed integrated approach outperforms the stepwise counterpart in terms of the NPV which is 6% higher. Similarly, the horizontal cumulative development costs are shown to be substantially higher when the sequential approach is used. These results show that the limited knowledge of time discounting and effective development distances when first obtaining the stope layout has a critical impact on the final production schedules. Thus, by jointly optimizing the underground mine planning components, it is possible to capitalize on their synergies to generate a truly optimal schedule.

Chapter 3 presents the extension of the previously proposed method to incorporate long-term stockpiling decisions into the SIP. Additionally, the effect of using high-order sequential simulations (HOSIM) of the mineral deposit is evaluated. This investigation is motivated by the fact that conventionally used sequential simulation methods such as sequential Gaussian simulation (SGS) rely on two-point spatial statistics that are not able to properly characterize complex spatial geometries or high-grade connectivity of natural phenomena. Also, the assumption of dealing with Gaussian random functions incurs high entropy in terms of extreme grades. An application of the proposed method for an operating mine compares the cases where the geological simulated orebody models of copper grades are generated with both HOSIM and SGS. The output stope designs and related production schedules are shown to be physically different. When the simulations generated with HOSIM are used, it is seen that fewer stopes are sent to the stockpiles. In addition, a higher copper metal production and a consequent 6% higher NPV are observed when high-order sequential simulations are used. These results are explained by the fact that the realizations generated with HOSIM show better high-grade connectivity allowing the availability of higher-grade stopes compared to the realizations generated with SGS.

4.2 Recommendations for Future Research

Future research on the proposed topic can be conducted with an application that considers multiple sources of uncertainty simultaneously as the presented SIPs can directly accommodate commodity price uncertainty. The presented applications consider the optimization of a single mining zone with common geotechnical and geometrical parameters. Therefore, an application that optimizes multiple mining zones simultaneously could be considered. In addition, the simultaneous optimization for a mining complex that assumes the existence of multiple mines, stockpiles, and processing streams is an extension for future work. A case study that accounts for high-order sequential simulations of multiple elements is a topic for future research, as secondary and deleterious elements also play an important role in decision-making for mine planning activities. As more components are included in the mathematical model, higher computational efficiency is needed. The use of commercial solvers restricts the model's application to larger problems. Thus, alternative solvers that use metaheuristic can be implemented in future developments.

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