GEOTECHNICAL RESEARCH CENTRE

PREDICTION OF MOBILITY AND DRAWBAR-PULL IN TOW-BIN TESTS USING CONE, VANE-CONE, AND BEVAMETER TYPE TOOLS

by

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SOIL MECHANICS SERIES NO. 37

OCTOBER 1977

McGill University
Montreal, Que Canada

Report to Defence Research Establishment Ottawa

Department of Supply and Services
Contract No. 08SU CD 3237007
Serial No. 2SU77-00087

ISSN 0541-6329

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submitted to

Defence Research Establishment Ottawa
Department of National Defence
Ottawa, Ontario.

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EXECUTIVE SUMMARY

This report deals with an examination of three kinds of field testing instruments presently advocated for use as measuring tools in the field to provide input for prediction of mobility. There are undoubtedly many more kinds of field measuring devices. However, these were chosen because of their immediate availability and greater common usage. The tools considered are the cone penetrometer, similar to the one used at WES; a laboratory adaptation of the bevameter-type of test device; a vane-cone test instrument presently used at McGill University.

The results of the study indicate that it is most important to obtain proper simulation from these test devices of the mechanics of vehicle-terrain interaction if a feasible prediction for mobility is to be made. We note that whilst measurements can be obtained from these instruments which may or may not be pertinent to the actual mechanics of vehicle-terrain interaction, the other important consideration for mobility prediction is the actual analytical framework within which these measurements must participate. Thus, it is not only the instruments that are used that are important for prediction of mobility, but also the <u>method of application</u> of the measurements obtained from these tools for analysis and prediction of mobility. Where the proper appreciation of the actual system problem is obtained, both from a measurement and an analytical point of view, a viable prediction for mobility

is obtained. These points are examined in this report, and the results indicate that the tool designed to account for the necessary mechanical properties participating in the mechanics of vehicle-terrain interaction, does indeed produce the better correlation with respect to prediction of mobility as determined in laboratory tow-bin tests.

Note that regardless of the tool used, *if the measurements are rationally obtained, and if these measurements are utilized in analyses which properly reflect the physical interaction phenomenon to be predicted, sensible results will be obtained.*

The above statement is best highlighted by quoting an example of misuse of measurements for computations leading to prediction of mobility.

> * *Measurements of rolling resistance of a family of rigid wheels in sand and clay have recently been made using the apparatus shown in Fig. 3 (i.e., Fig. 3 in paper by Reece). These show that the basic assumption of all rolling resistance theories, that work put into rolling a free-to-turn wheel equals that involved in making the rut by means of vertical pressure, is wrong.*

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* Reece, A.R. (1965) *Principles of Soil-Vehicle Mechanics.* Proc. of the Institution of Mechanical Engineers (Automobile Division), Vol.180, Part 2, p.45.

PREDICTION OF MOBILITY AND DRAWBAR-PULL IN TOW-BIN TESTS USING CONE,VANE-CONE, AND BEVAMETER-TYPE TOOLS

1.0 PROBLEM DEVELOPMENT

1.1 PRELIMINARY STATEMENTS

In order to properly demonstrate the problem at hand, it is necessary to review, very briefly, the essential elements which participate in the production of mobility for a vehicle, together with the basic mechanics of the vehicle-terrain interaction. The reasons for these review observations are found in the fact that if rational correlation is to be desired for tools that are required to predict mobility from measurements of terrain properties, the sensing tools need to provide a proper reflection of the kinds of mechanisms developed in the vehicle-soil interaction problem.

These mechanisms are examined in this section from the vehicle-soil interaction point of view for the purpose of development of the the requirements for field sampling tools. Following this, three kinds of tools available are listed and their respective performance mechanisms developed, to show how they might relate to the problem at hand.

1.2 DEVICES FOR TERRAIN SENSING FOR MOBILITY

Figure 1-1 shows a brief summary capsule of the three principal tools presently used for the prediction of vehicle mobility. A fuller discussion of these tools will be given in section 2.0. In this section, we will restrict the discussion to statements providing a brief appreciation

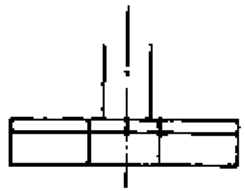
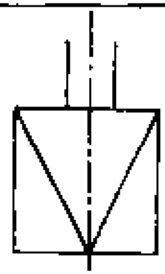
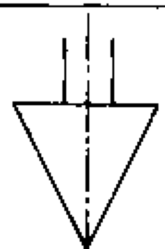
DEVICE	DATA OBTAINED	APPROACHES USED
(a)  Bevameter	(1) Bevameter values at the surface, C, ϕ, K_c, $K\phi$, n. (2) Tangential shear stress at the surface.	Drawbar Pull obtained by computations involving determination of Rolling and Bulldozing resistances.
(b)  Vane-cone	(1) Vane-cone values at different penetration depths, CI, C, ϕ. (2) Tangential stress at different depths. (3) Soil density at different depths (in case of sand). (4) Complete soil profile.	(1) Energy approach. (2) Clay number approach. (3) Power number approach. (4) Mobility index approach. (5) Simplified energy approach.
(c)  Cone	(1) Cone index at different depths, CI. (2) Soil density at different depths (in case of sand). (3) Some data about soil profile.	(1) Energy approach. (2) Clay number approach. (3) Power number approach. (4) Mobility index approach.

FIG. 1-1 Some simple devices and method of utilization.

of the kinds of actions and reactions provoked by the use of the tool in the terrain. Figures 1-2, 1-3 and 1-4 show the *cause and effect* problem as demonstrated by the tools together with the descriptions of the kinds of activities generated from the use of the respective tools. The direct application of all these tools, together with their corresponding test results will be presented in section 3.0 of this report.

1.2.1 Vehicle-Soil Interaction Mechanics

Interaction between vehicle and terrain is achieved through the running gear system which is either a wheel system or a track/device, or some other equivalent system. The mechanics of interaction between wheel-soil and track-soil have been developed previously in many published reports by Yong.* The essential items for the interaction between these running gears and the terrain are summarized in Figs. 1-5 and 1-6.

Note that for mobility purposes, it is desirable that the vehicle be able to move from one point to another specified position. To achieve this, the vehicle must be supported by the terrain. In addition, the terrain must provide sufficient resistance (capability) wherein thrust can be developed between the running gear and the terrain itself. Immobilization therefore can arise as a result of lack of ground support or lack of strength in the terrain to provide the wherewithal for development of thrust. These are demonstrated as sinkage immobilization, or slip immobilization. There are obviously combinations between the two which may provide for immobilization through interaction between these two primary mechanizations.

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* See References at the end of this Report.

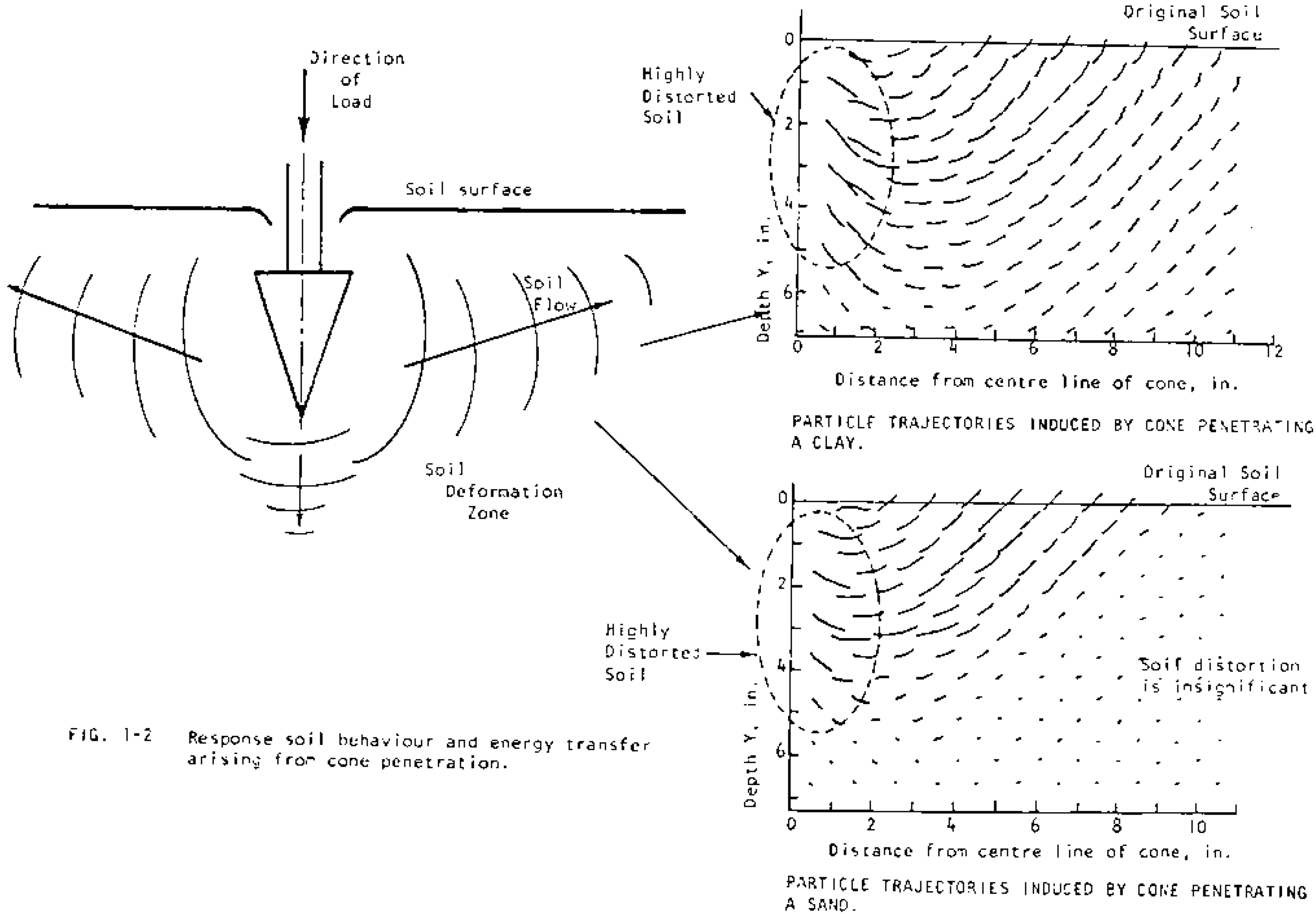

FIG. 1-2 Response soil behaviour and energy transfer arising from cone penetration.

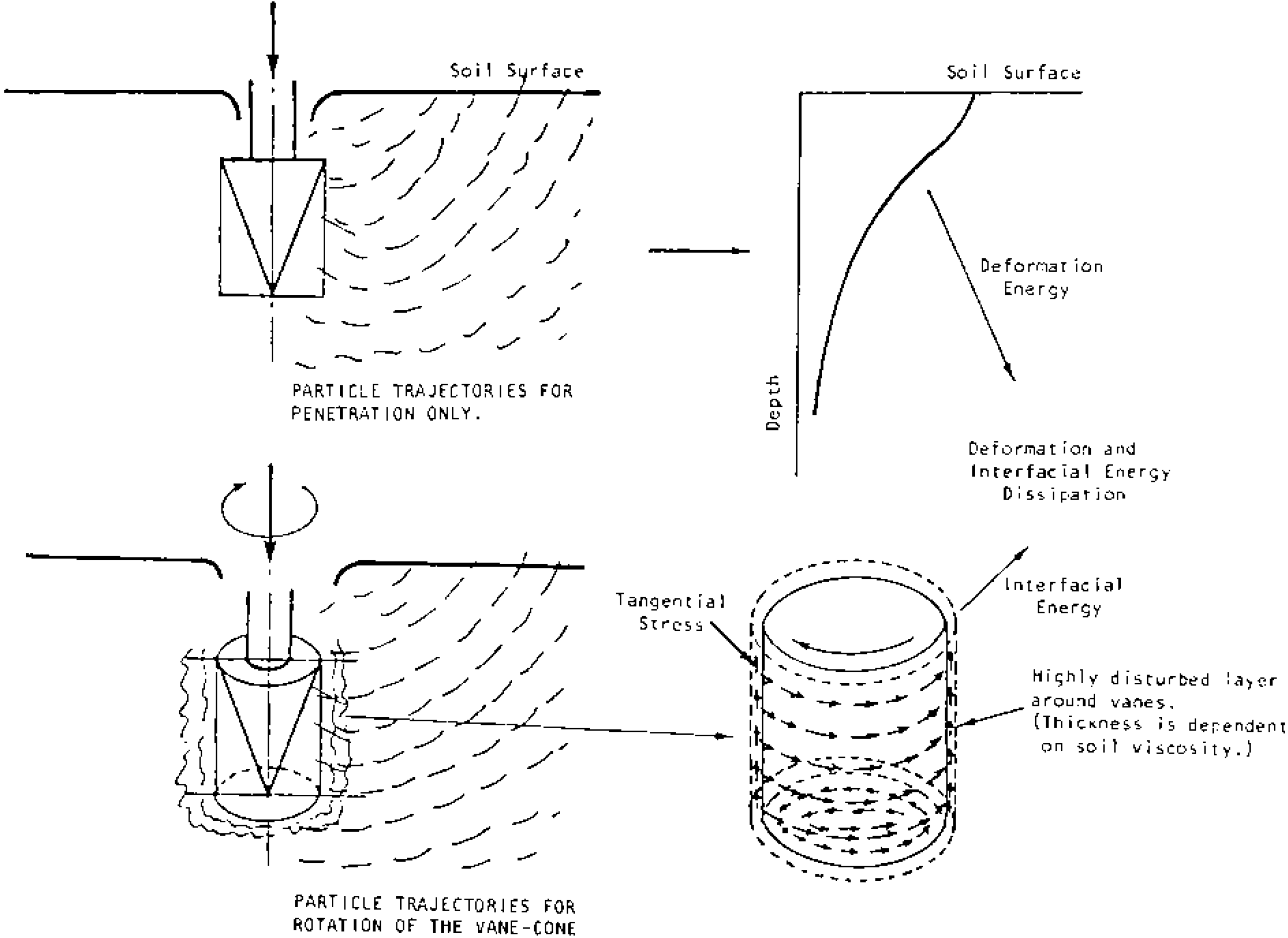

FIG. 1-3 Response soil behaviour and energy transfer due to utilization of a vane-cone penetrometer.

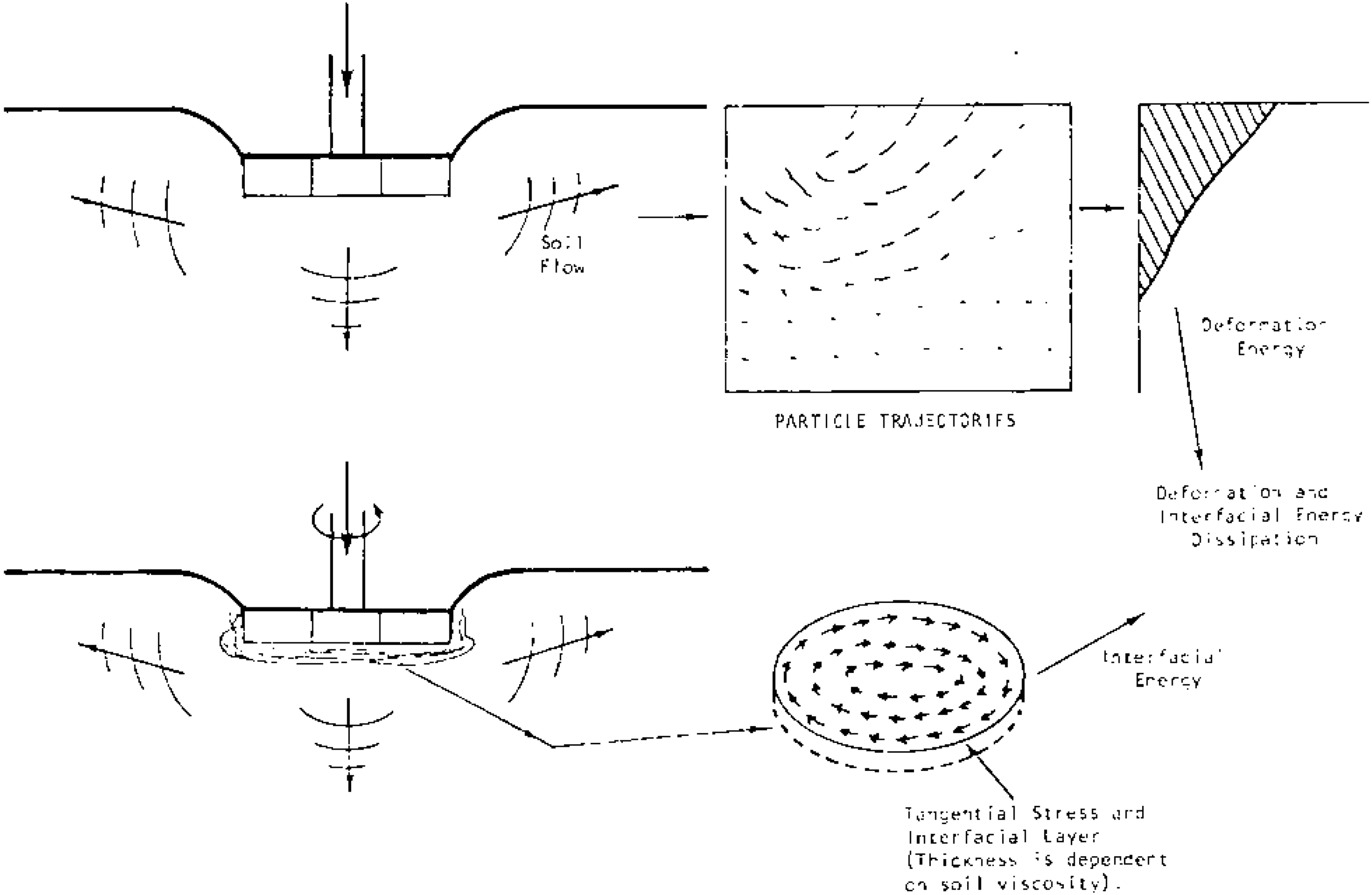

FIG. 1-4 Response soil behaviour and energy transfer due to application of an annular shear plate.

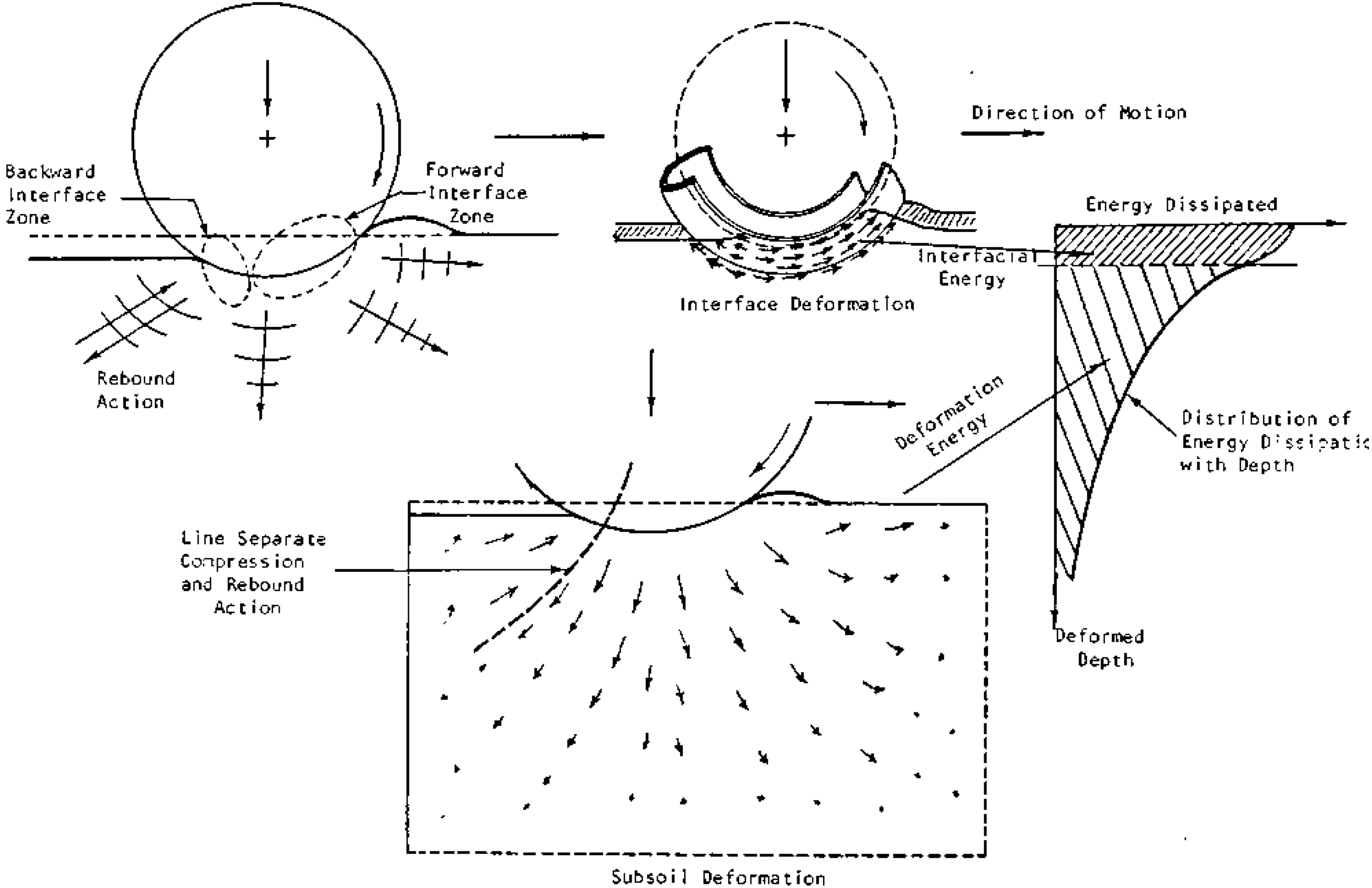

FIG. 1-5 Response soil behaviour and energy transfer beneath a moving rigid wheel.

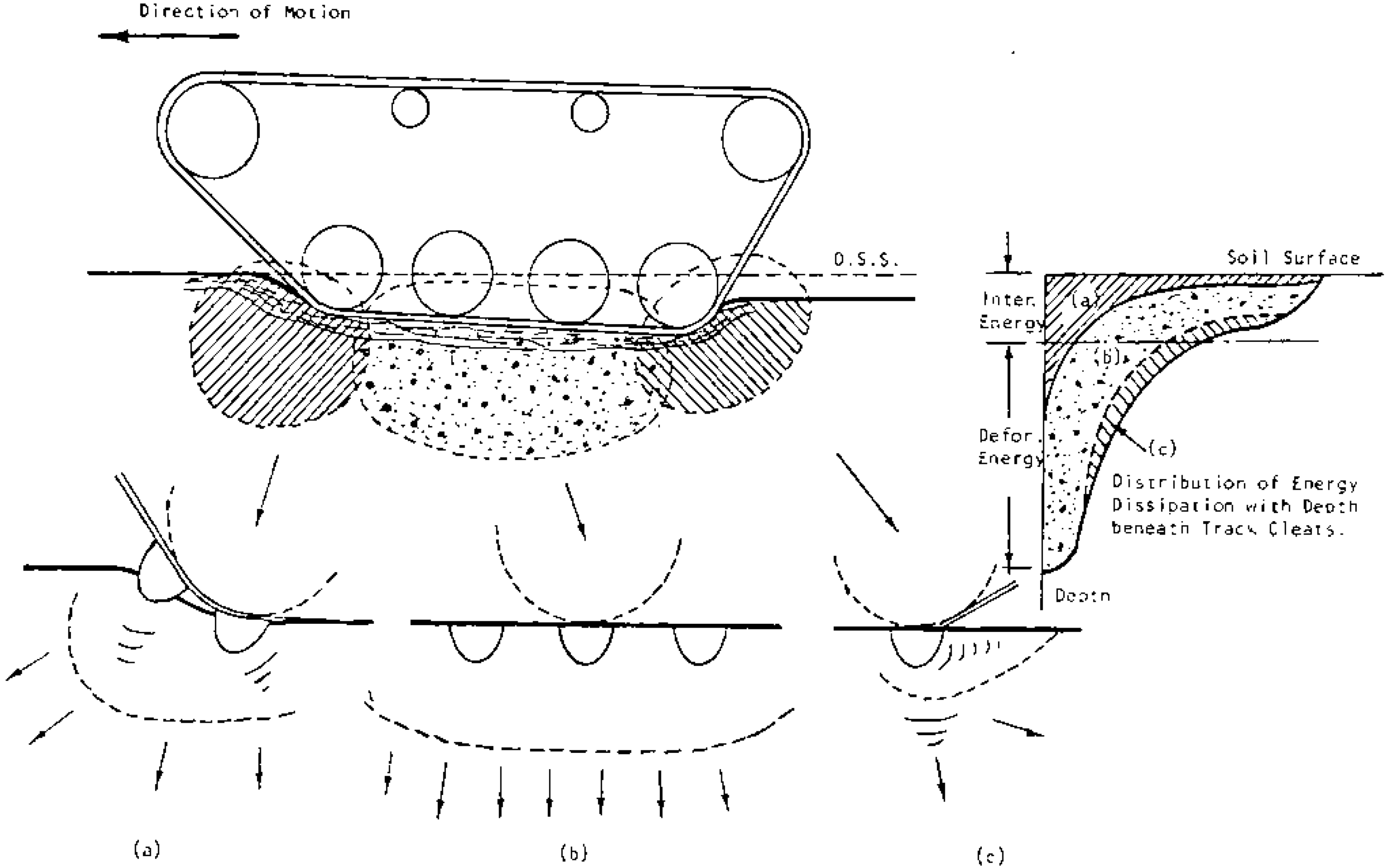

FIG. 1-6 Response soil behaviour and energy transfer beneath a moving track.

1.3 SIMILARITY REQUIREMENTS FOR PROPER PREDICTION

In order that one might predict the behaviour of a system, it is essential that a proper knowledge of the kinds of mechanisms involved be understood. If a prediction is to be made, based upon a sampling of the individual components or of the interactive components, it is necessary for the kinds of mechanisms developed and sensed by the sampling procedures to be similar in essence to those identified as responsible for the actual system behaviour needed to be predicted. Figure 1-7 illustrates the general requirement.

If the sampling or predictive techniques do not mirror or generate mechanisms similar to that provoked by the actual or real system to be predicted, it becomes immediately obvious that the ability to predict the actual or real system becomes severely hampered. The further one departs from the *similar mechanism* requirement, the less is one's ability to successfully predict the behaviour of the real system. In the case of a wheel-soil interaction problem, it is obvious that the ideal test system should use a wheel moving on the soil. In this case the test wheel would be properly instrumented to provide for the various kinds of measurements which can be translated directly into a large wheel performance and subsequently into a total vehicle performance.

Correspondingly, for a track-soil interaction problem, the ideal tool would be a small track acting on soil with the proper associated instrumentation. Since these evidently would become cumbersome and since the requirement for the model wheels or model tracks seeks to duplicate both geometric and other kinds of scaling essentials, the problem becomes horrendously complex.

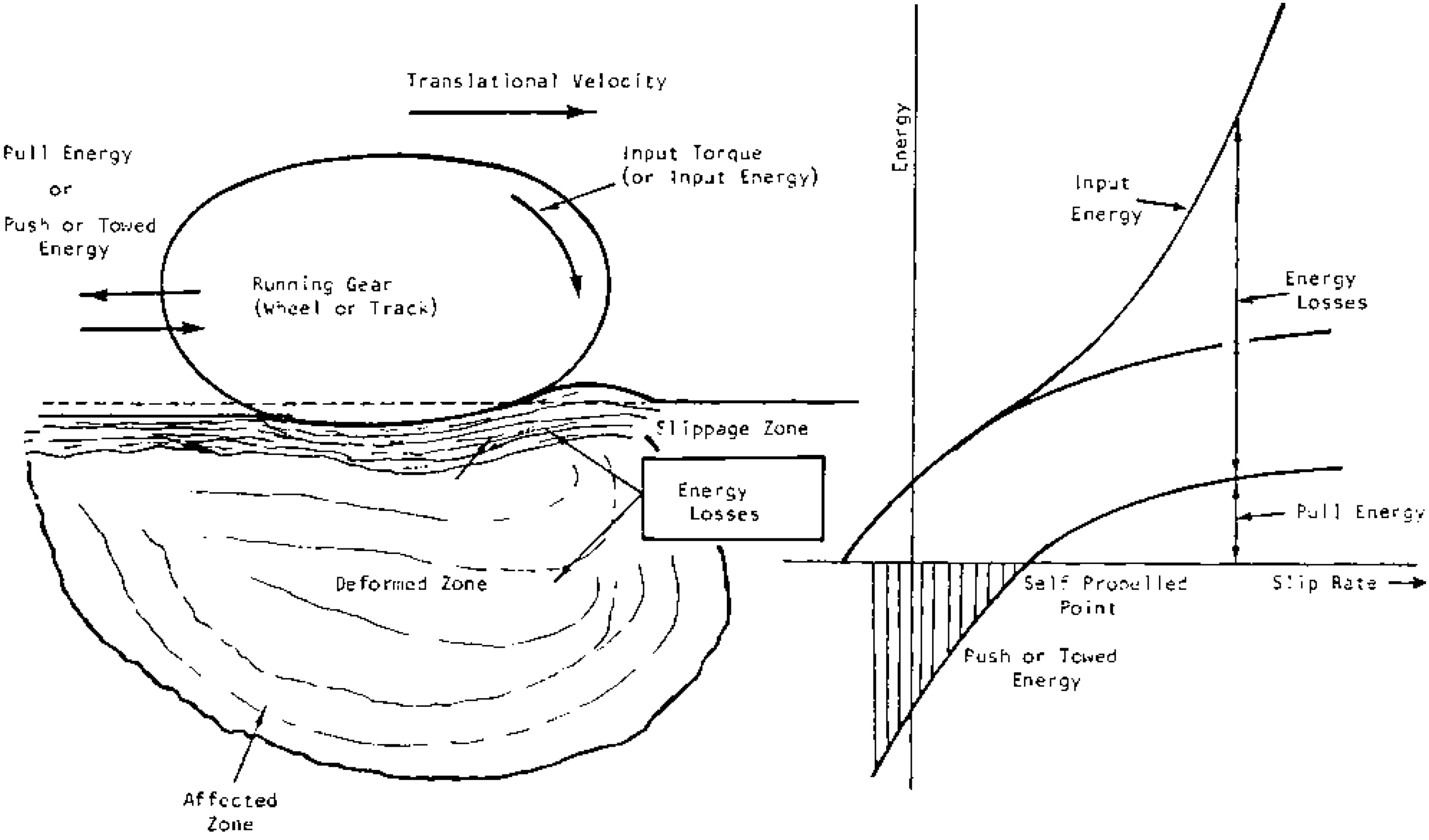

FIG. 1-7 Energy input and dissipation in production of useful work [pull energy]

If one were to depart from the ideal prediction problem requiring modelling of the actual field situation to small test tools, the three kinds of tools given in Figs. 1-1 through 1-4 offer themselves as prime candidates. The question that needs to be asked is how relevant are these tools with respect to their appreciation of the actual mechanics of the real system itself.

To reduce the problem to its most simple form, and to allow for the greatest ease in development of similarity modelling, the concept of energy conservation and its application provides for the most attractive form of comparison performance. The details given in Fig. 1-8 portray the kinds of energy fields or quantities demanded by the wheel and track soil interaction problem, together with the essential items produced by the sensing devices. Note that because of the fact that energy is a scalar quantity, there is no particular requirement for a duplication of the strain or strain rate fields, the stress or the deformation fields associated between the real system and the test devices.

1.3.1 Deformation and Slip

As noted in Fig. 1-8, all the three tools produce deformation fields, for which the energy associated with the deformation of the soil can be obtained. How these relate as direct quantities to the wheel-soil or track-soil problem is one which requires analysis of the test results using a modelling procedure which reflects the actual mechanics of the running gear-soil interaction,* i.e. there must be a supportive analytical

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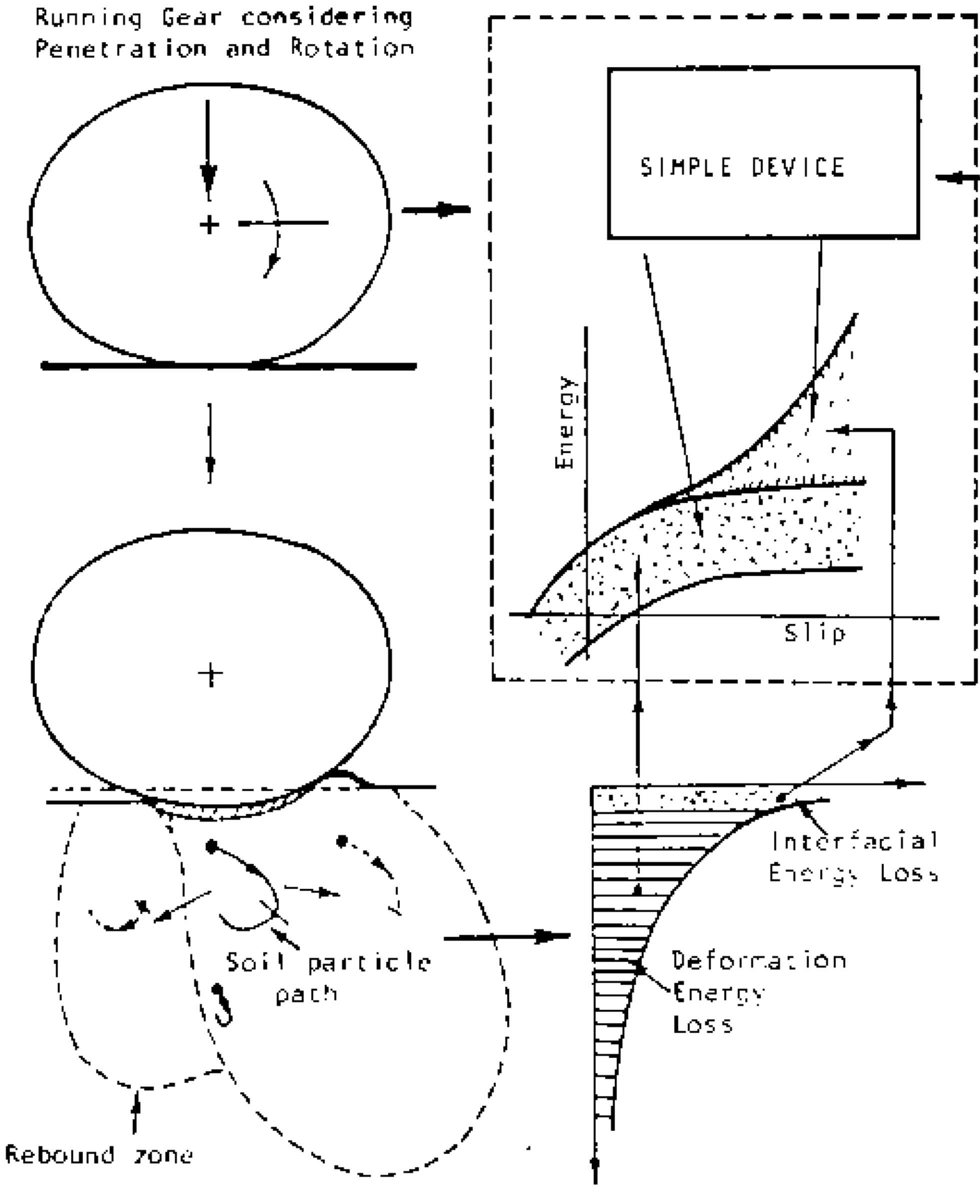
* Note that the same principles apply to the problem of snow mobility and trafficability.



Device	Data used in terms of Energetics*
CONE	1. Data available for deformation energy prediction at different depths of subsoil. 2. No data available for interfacial energy prediction.
VANE-CONE	1. Data available and used for deformation energy prediction at different depths of subsoil. 2. Data available and used for interfacial energy prediction at different strain rates and different penetrations.
BEVAMETER	1. Data available for deformation energy prediction in subsoil. 2. Data available for interfacial energy prediction.

* *Note that whilst the data available (for the various tools) for utilization in energy prediction are shown as indicated, proponents of some of these tools may not view their use in these terms.*

FIG. 1-8 Assessment of the three devices as prediction tool for analyses using the mechanics of energy transfer (Energetics).

theory which utilizes energetics in the formulation of the mobility equation.

Note that the energy terms per se do not provide one with an actual direct prediction of mobility. The terms provide an appreciation of the amount of energy expended in shear distortion and in deformation of the soil. With reference to the sensing tools, soil deformation energy evaluation is obtained either through the penetration of the cone or vane-cone devices, or direct bearing of the plate from the bevameter test. One might equate the subsoil deformation aspects of the phenomenon material for a static loading situation.

Recognizing that the running gear-soil interaction problem requires both deformation and slip for a complete appreciation of the mechanics of the vehicle-terrain interaction problem, we observe that the slip requirement is not generally directly met by any of the tools. However, the generative mechanisms needed to provide for computational success in determination of slip energy can be obtained from a knowledge of the shear strength of the material. The use of measured shear stresses for computation of slip energy loss requires the establishment of a proper analytical framework.

Suffice it to say that the applicability of any sensing tool insofar as mobility prediction is concerned, depends not only on the measurements made by the tool, but also on the manner of utilization of the measurements consistent with the analytical framework describing the vehicle-soil interaction problem. This aspect of the problem cannot be overstated.

1.4 MEASUREMENT AND ANALYSIS

As stated previously, the measurement made by a sampling tool is one aspect of the problem. The other is the need for the transfer of that measurement into a mobility prediction. What is frequently overlooked in prediction requirements is that whilst the tool itself is an important aspect of the problem, the other significant contribution for prediction is the manner in which the obtained measurements are applied. Thus whilst the tool itself might not specifically provoke the kinds of mechanisms associated with the real system, perhaps some similarity or correlative relationships might be utilized which could serve as a transfer function, providing thereby a degree of respectability (or acceptability) for the tool insofar as mobility prediction is concerned. Thus, whilst the various tools might produce subsoil or terrain deformation in one part of the test procedure, and show shear resistance in the secondary part of the procedure, it is not necessarily true nor does it follow that these items will be rationally used in the production of prediction for mobility. How one interprets and uses one measurement now becomes the crux of the problem.

The above point can be amply demonstrated by considering the example of a small wheel as a tester where measurements are made with the small model wheel as a predictive tool. If the measurements made from such a model wheel tester are used to predict total wheel performance, it becomes important to apply the proper kinds of scaling since the measurements essentially are a direct one to one correspondence between model and prototype performance so far as mechanisms are concerned. However, if one were to use a plate as a bearing tool and to utilize the bearing capability of the plate to predict wheel performance, several questions arise. These are (a) the relationship between a static plate bearing performance and a dynamic wheel phenomenon, and, (b) the translation of a bearing capacity term to a mobility or drawbar pull quantity.

Figure 1-9 illustrates this quandary. In the discussion provided in Fig. 1-9, it is obvious that application of a static bearing pressure prediction of mobility becomes difficult as a direct one to one prediction. However, if one chooses to perform a correlative testing program where a body of actual test data or wheel performance becomes available, it is not unlikely that some degree of correlation might be achieved. However, under such circumstances, the correlation function becomes sensitive to several parameters and factors associated not only with the terrain, but particularly with the forcing function - i.e. the vehicle or the wheel or the track etc. as a real system.

1.5 THE DILEMMA AND THE PROBLEM

From the previous discussions and diagrams, it becomes obvious that for one to be able to predict mobility and drawbar pull production with any sense of realism and confidence, the kinds of sensing tools needed would become cumbersome and complex, and the measurements needed would perhaps be too comprehensive in scope. Since the usefulness of any kind of predictive tool is its simplicity, and for this particular situation, its portability, one is then forced into the situation where certain sacrifices and trade-off must be made. The value of the sensing tool becomes immeasurably increased if it were to be simple and portable allowing, therefore for greater usage over a wider scope of terrain situations. Therein lies the dilemma. Since portability and simplicity will require a certain amount of sacrifice in the detailed requirements for the tool, the question that is asked is *how well can this kind of system predict a complex behaviour pattern?*

The problem therefore is how can one devise a test system that would be simple, portable and accurate in predicting mobility through -

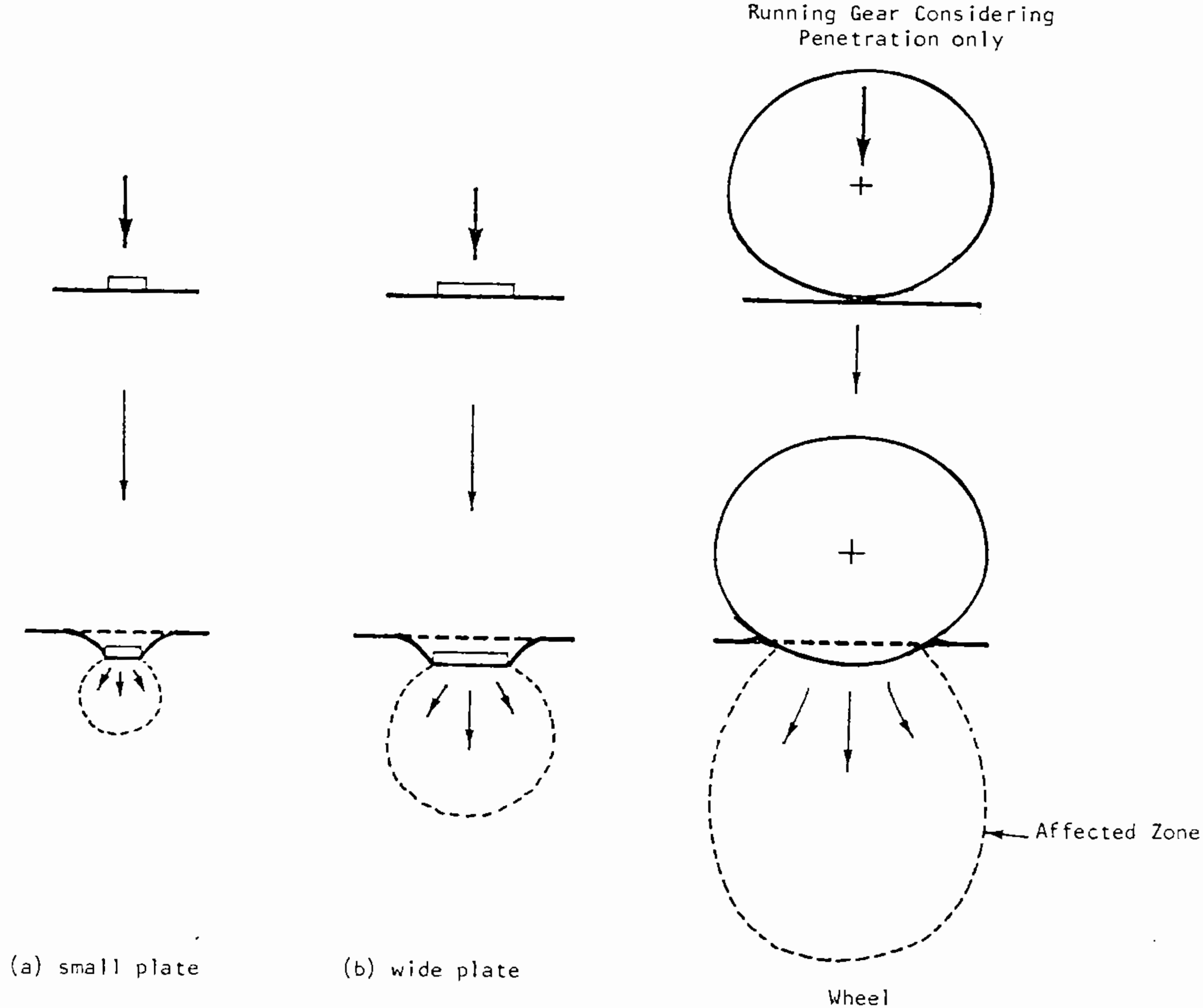


FIG. 1-9 Penetration characteristics of plate test and running gear element under a static vertical load only.

(a) a proper measurement of the kinds of parameters participating in the problem of mobility, and

(b) a use of these measurements through a proper body of analysis germane to and relevant to the actual mechanics of the vehicle-soil interaction problem.

2.0 PENETROMETERS AND TESTING DEVICES

2.1 INTRODUCTION

The three types of field testing tools studied as identified in section 1.0 are:

(1) The W.E.S. type cone penetrometer,

(2) The bevameter-type of tester adapted for laboratory use,

(3) The vane-cone device developed in the mobility laboratory of the Geotechnical Research Centre.

In this section, the mechanisms developed with the use of these devices are compared with those thought to be responsible for the vehicle-soil interaction problem.

From Figs. 1-5 and 1-6 the problem of vehicle-soil interaction can be reduced to one which examines the situation in energetics, i.e. the mechanics of energy transfer where the two principal parasitic energy components are identified as interfacial (slip) energy loss, and compression energy loss.* Note that the principles illustrated in Figs. 1-5 and 1-6 can be equally applied to vehicle-snow interaction, except that the relationships developed for these parasitic energy components need to be more fully developed.

For simplicity in presentation of the figures in this section, the running gear will be specified in some general form. It is understood that this running gear can either be a wheel system or track device.

2.2 THE CONE PENETROMETER

As noted in Fig. 1-2 in the previous section, the application of a penetrating force by the cone into the soil produces a compression zone in

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* Note that proof for proper prediction of mobility using the mechanics of energy transfer (energetics) has been shown in various publications, e.g. References 8, 9, 10, 11, and 12.

the soil which results in material being displaced in the direction generally normal to the face of the cone as it penetrates. The exact direction of flow depends on the properties of the soil and the surface properties of the cone. For a frictionless soil, and for the condition of zero friction at the cone surface, the direction of soil flow away from the cone surface can be taken to be normal to its surface. If the soil is frictional, with friction angle ϕ, and if the soil-cone friction angle is δ, the direction of soil flow from the cone surface is a function of ϕ and δ. Yong and Chen (1976) have discussed this in detail.

At some distance away from the face of the cone, following the discussion given above, the material moves along prescribed paths and directions (slip lines) dictated by the properties of the material being penetrated and the boundary conditions of the problem. In the diagram given as Fig. 1-2 in section 1.0, note that the particle paths for typical points within the soil being penetrated describe trajectories which in actual fact conform to those that would be predicted if one applied limit equilibrium theory. The corresponding energy losses that can be computed, with a knowledge of these deformations or particle paths, would provide us with the details as shown in Fig. 2-1.

From detailed experimentation and previous studies by Yong, Chen and Sylvestre-Williams (1972), and Yong and Chen (1976), it is observed that the development of the deformation zone is also sensitive to the rate of penetration and the size of cone (in addition to soil properties). However, these can be normalized and the results obtained thereby can be analyzed accordingly as shown by Yong et al. (1972).

Hence if the objections to the use of the cone as a penetrating device for assessment of mobility hinge around the fact that cone penetrometers are:

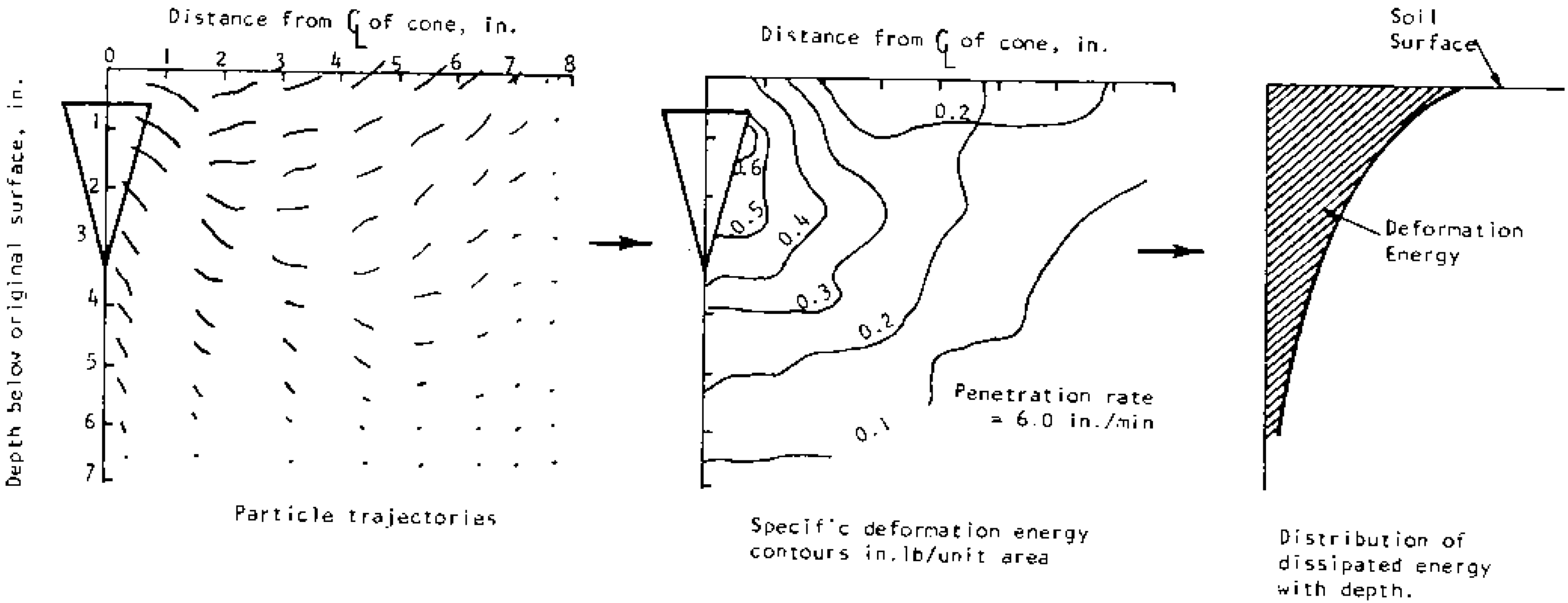

FIG. 2-1 Soil particle trajectories and corresponding dissipative energy field due to penetration of a 30° cone penetrometer.

(1) sensitive to size and geometry,

(2) sensitive to rate of application of penetrating force,

previous studies have shown that these can be accounted for in the reduction of data, and the results used with due attention to these factors - Yong et al. (1972), Yong and Chen (1976), Sylvestre-Williams (1973), Chen (1972). The point that should be made in actual fact is not that the device shows the sensitivities, (*since these can be properly accounted for*), but that these devices will only produce response mechanisms in the soil which are primarily compressive in nature. Thus the dissipative energy component of prime importance is the compression energy term. Because of the action of the cone in penetrating the material, if the cone surface is smooth, very little slip energy is lost (with respect to friction) at the interface between the cone and the material. Thus in effect the interface or slip energy loss obtained at the wheel-soil interface during forward motion of a wheel is not modelled or sensed.

2.2.1 Requirement for Comparison and Prediction

It is apparent from Fig. 2-1 that only one kind of trafficability consideration is sensed by the cone. In effect, if soil compression characteristics leading to stability characterization is the sole criterion for mobility, then the tool provides us with a very useful means for describing (vehicle) flotation mobility. We observe from this technique that if low slip is encountered insofar as vehicle movement is concerned, the tool should provide one with a reasonable means for correlation so long as the proper transfer functions (i.e. sensing tool to vehicle) are found. The situation becomes less clear and indeed more tenuous if mobility at high slips is to be assessed or evaluated. This is because the sensing tool does not provide a means for determination of slip energy losses. Its main and prime capability is with respect to a sensing of the material property (compression resistance) which provides for stability in static bearing. We observe that the cone penetration tool is a common tool in most geotechnical engineering practices where sounding measurements are used to provide for input into bearing stability considerations.

It is because of the above considerations that it is not improper to use the cone for evaluation of go, no-go situations through field correlations and testing. There is very little pretense in the application of the cone measurements for situations beyond the go, no-go cases, Where extension of the cone penetrometer readings is necessary, several adaptations of these readings have been made, such as those given in section 3.3. These by and large depend upon correlative techniques and dimensional analyses which have been developed into empirical functions. The direct input for these are then seen in terms of computer modelling such as the AMC-74.

2.3 BEVAMETER-TYPE LABORATORY TOOL

The bevameter-type laboratory tool used is as described in section 3.0 (following). The general principle of the tool does not differ from that presented by Bekker, except that the circular plate (for compression of the soil), and the annulus used to provide the surficial shear resistance test, are adapted for laboratory purposes. The technique followed is identical to that recommended by Bekker - i.e. using at least two differing sizes of circular plates for compression followed by shear with the annulus vane.

Figures 1-4 and 2-2 show the kinds of mechanisms for compression and shear developed with the bevameter tool technique. As indicated in the figures, the circular plates will provide compression in the soil, developing thereby the deformation characteristics similar to that identified with a simple load bearing problem. Note that this part of the procedure is identical to the plate-load tests used in geotechnical engineering. The results obtained from such tests in geotechnical engineering are used directly to obtain static foundation design requirements.

The energy consumed by loading the circular bearing plate can be expressed in the form of soil deformation energy. Note again that this is a direct load-bearing problem, and that it does not differ from a standard geotechnical soil mechanics problem of stability of foundations. However, from detailed experiences from general geotechnical engineering practice, the problems of plate sizes, loading technique, utilization of test data, etc. have been throughly investigated - for at least 50 years!

The application of the annulus shear device at the surface produces a shear resistance phenomenon not unlike that given as a direct shear test.* The value of the shear strength obtained is obviously therefore a function of the rate of rotation of the annulus. By constructing the analytical framework accordingly, one can obtain a shear-slip energy relationship which is sensitive to the rate of rotation. It is pertinent to note that a similar type of surface vane tool is used in geotechnical engineering investigations. The tool measures about two inches in diameter, and is considered and used as a hand-held instrument. The results obtained are directly translated in terms of shear resistance of the soil.

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* Note that a small hand-held annulus shear device is available on the market as a commercial "rough and ready" tester used in Geotechnical Engineering soil exploration. The tester is small enough to be held in the palm of the hand, and the procedure calls for rotation of the tester in soil whilst "palming" the tester. The results obtained are "interpreted" in terms of friction angle ϕ and cohesion c of the soil material tested.

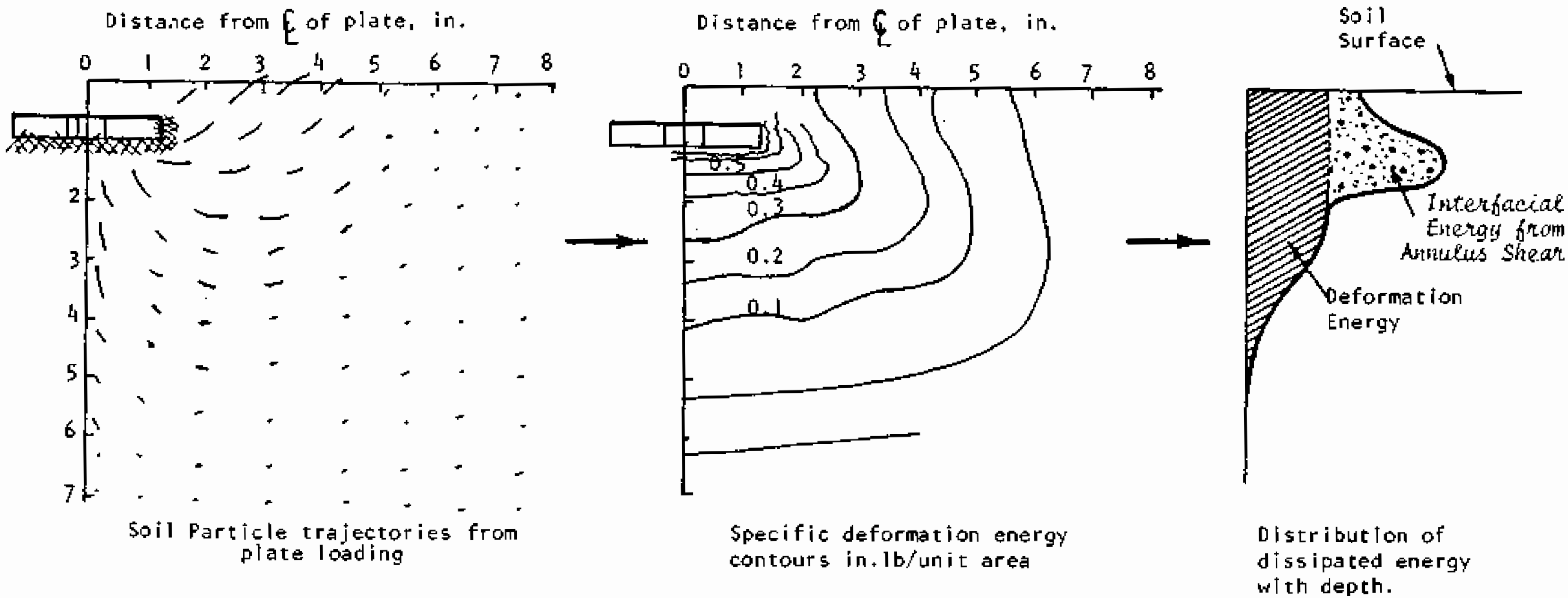


FIG. 2-2 Soil particle trajectories and corresponding dissipative energy field due to plate loading

2.3.1 Application for Mobility Prediction

It is obvious from Figs. 1-4 and 2-2 that the two prime components for energy dissipation in the mobility problem can be sensed with the bevameter-type technique where both the plate and the annulus shear devices are used. It now becomes necessary to translate these measurements into application to the running gear-soil interaction problem.

Note that the size of plates used for the bearing compression problem has been thoroughly investigated and that corrections can be made to account for size effects. In the present method of application of the test results, the constants derived from reduction of the bearing plate data depends upon the validity of the limit equilibrium principle which is sufficiently well constrained and conditioned.

2.4 THE VANE-CONE DEVICE

The vane-cone device shown in section 1.0 was developed with the knowledge that the two prime parasitic energy components are compression and interfacial energy loss. In addition, the requirement for simplicity and portability dictated that the instrument be sufficiently simple.

Figure 2-3 shows the kinds of mechanisms developed with application of the vane-cone device. As in the cone aspect of the device, the kinds of mechanisms developed due to penetration of the cone are not unlike those shown in section 2.1. Thus only the deformation energy component is sensed. As remarked previously, the cone, in essence, does not develop a significant shear stress at the interface if the surface of the cone is smooth and if the soil is frictionless. Thus one is left with devising a technique which would provide for slip energy loss characterization.

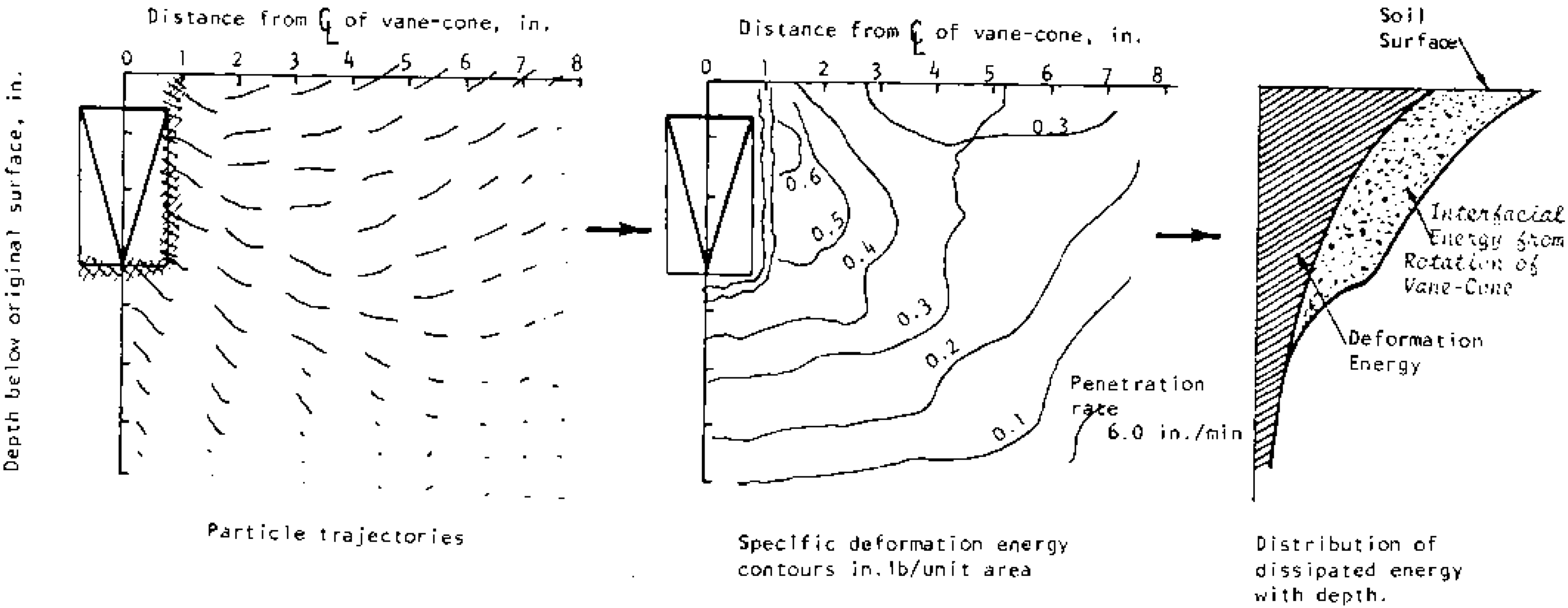


FIG. 2-3 Soil particle trajectories and corresponding dissipative energy field developed by penetration of a 30° vane-cone penetrometer.

The vane portion of the vane-cone device will provide a shear strength evaluation of the soil if the vane-cone is rotated. The technique requires that the cone be penetrated to the required depth and maintained at that depth for rotation of the vane. In actual fact one would note that the mechanisms developed as the vane-cone is rotated are not unlike that for the annulus vane in the bevameter-type device.* The resistance values or shear strength measurements obtained thereby can be converted into slip energy loss relationships not unlike that obtained for the annulus device if, and only if, the proper energy transfer mechanics framework is provided with the annulus device. In actual fact, we note that the bevameter annulus device does not utilize this kind of a framework, and therefore the direct translation of annulus information to slip energy loss has not been obtained.

2.4.1 Application for Mobility Prediction

It is clear from Fig. 2-3 that the shear and resistance mechanisms developed with the vane-cone device are not dissimilar to that produced with the bevameter-type device. Whilst both deformation and interfacial energy losses can be sensed, the energy transfer analytical framework which utilizes the kinds of sensing device measurements made has only been developed in conjunction with the vane-cone device. Thus, insofar as mobility prediction can be obtained through energetics consideration, it is clear that the vane-cone measurements can be suitably used.

If only the cone measurements are used, the Power Method described in section 3.3.2 can be used to arrive at a prediction of mobility. Its success and credibility can be seen in the results presented in section 4.0.

* Note that the vane portion of the vane-cone device is in actual fact not dissimilar to the commercially available vane shear devices used in the Geotechnical Engineering practice. It is common practice in Geotechnical Engineering to use vane shear values for evaluation of soil properties and strength.

With the addition of the vane shear measurements obtained by rotating the vane-cone device, the added feature of slip-energy loss can be factored into the general energy conservation equation to permit evaluation of drawbar pull. This application is graphically illustrated in Figs. 2-3, 1-7 and 1-8. Section 4.0 shows the kinds of predictions obtained with the vane-cone in comparison to the cone and bevameter predictions and actual tow-bin test results.

3.0 EXPERIMENTATION AND ANALYTICAL TECHNIQUES

3.1 INTRODUCTION

In this section, the types of experiments performed are given together with the methods used for reduction of the various kinds of data for application to the vehicle-soil interaction problem. Thus for instance, if the cone is used, the measurements obtained from the experimental tests are reduced according to the procedures used by the agencies applying this particular technique as a predictive device. Extensions of the approaches used have been consistent with the philosophy of the technique itself, especially with regard to the cone. This should be evident in the presentation of the method of analysis since, by and large, common usage of the cone penetrometer for prediction of mobility has been with respect to go or no-go situations.

Recent advancement in the use of the cone penetrometer, especially for application with the AMC-71 and the AMC-74 has seen the use of the so-called mobility numbers associated with the cone penetration readings. These have by and large taken the cone penetration device one step beyond the go no-go situation.

3.2 EXPERIMENTATION

The kinds of experimentation performed in the study included three kinds of predictive tools, and wheel-soil tests in a tow bin to provide data for comparison with the predictions obtained using the various sensing devices and their measurements.

3.2.1 Soil Preparation for Predictive Tool Tests and Tow Bin Tests

The soil used was a kaolinitic soil with moisture content of about 47 to 51 percent. This represented a saturation of about 95 percent. The soil was compacted in layers in the tow bin and also in the test box used for application of the predictive tool tests. The density of the soil varied between 99 and 103 pounds per cubic foot and the corresponding shear strengths of the soil determined through plane strain triaxial tests provided values for cohesion and friction as follows:

Cohesion = 0.8 psi
Friction angle = 5 to 10°

3.2.2 Tow Bin Tests and Apparatus

The tow bin facility and associated instrumentation have been described in previous reports and papers as follows:

Yong, R.N., Boyd, C.W. and Windisch, S.J. (1965) *Soil Vehicle Interaction Study*, 5th Meeting Quadrupartite Group, Report No. D.Phys.R(G) Misc. 21, Directorate of Physical Research, DRB, Canada.

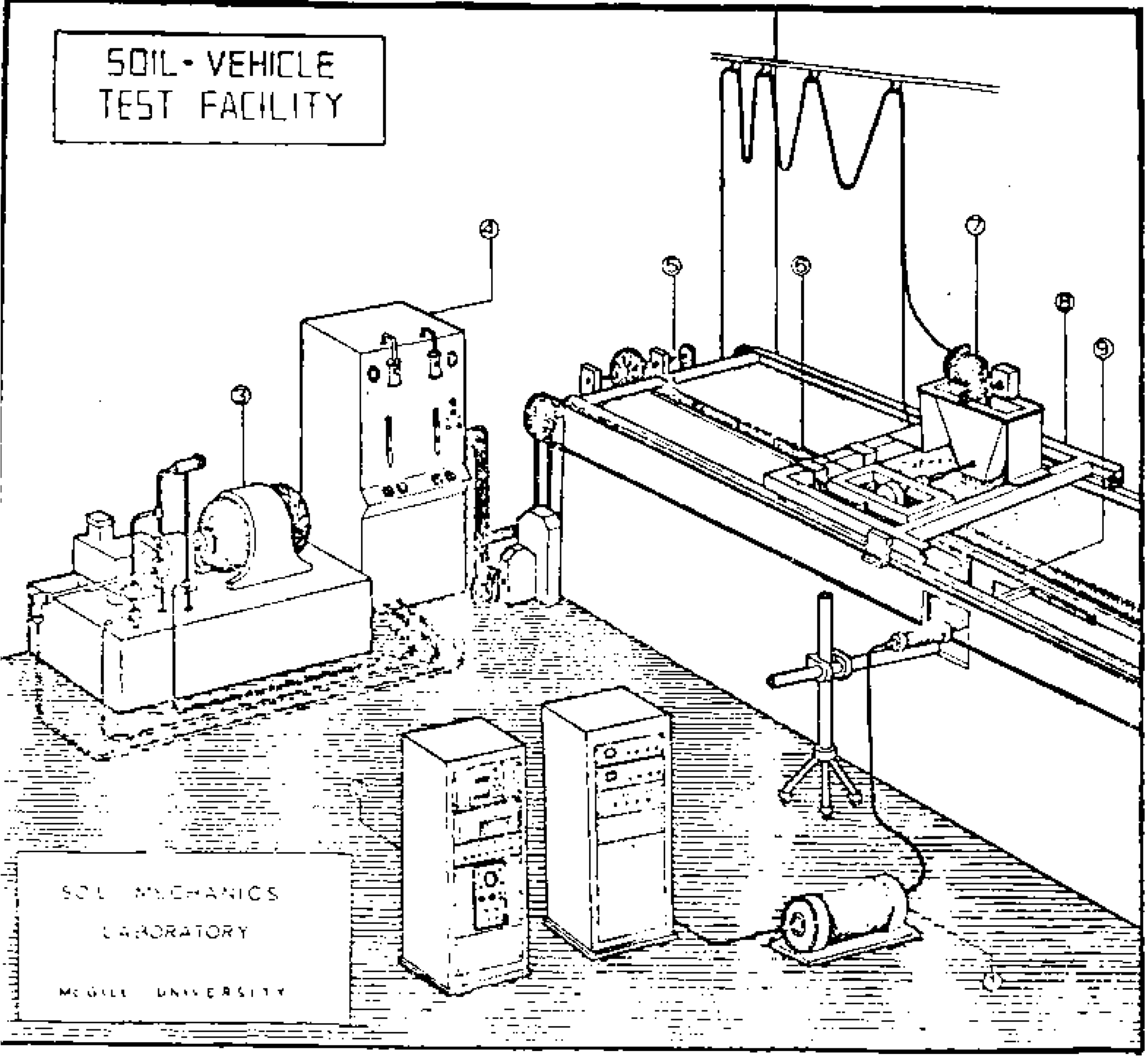
Yong, R.N. and Webb, G.L. (1969) *Energy Dissipation and Drawbar Pull Prediction in Soil-Wheel Interaction*, Proc., 3rd Int. Conf. ISTVS, Vol.1:93-142.

Yong, R.N. and Windisch, S.J. (1970) *Determination of Wheel Contact Stresses from Measured Instantaneous Soil Deformations*, J. Terramechanics, Vol.7, Nos. 3 & 4, pp.57-67.

The soil vehicle test referred to in the above is shown in Figure 3-1.

3.2.3 Test Facility for Predictive Tools

The test facility used for all three predictive tools is shown in Fig. 3-2. The physical facility allows for interchangeability of the testing tool and with the cone, bevameter annulus device, plate, or vane-cone. Recording of



① X - RAY UNIT

② RECORDER, POWER SUPPLY, AMPLIFIERS

③ HYDRAULIC PRESSURE SUPPLY

④ HYDRAULIC CONTROL PANEL

⑤ CHAIN DRIVE FOR MOBILE X-RAY CASSETTES

⑥ FLEXURE FRAME & TEST WHEEL OR SECTION TRACK

⑦ D.C MOTOR, TRANSMISSION & TACHOMETER

⑧ DYNAMOMETER CARRIAGE

⑨ SOIL SAMPLE CONTAINING BURIED MARKERS

FIGURE (3-1) SOIL - VEHICLE TEST FACILITY

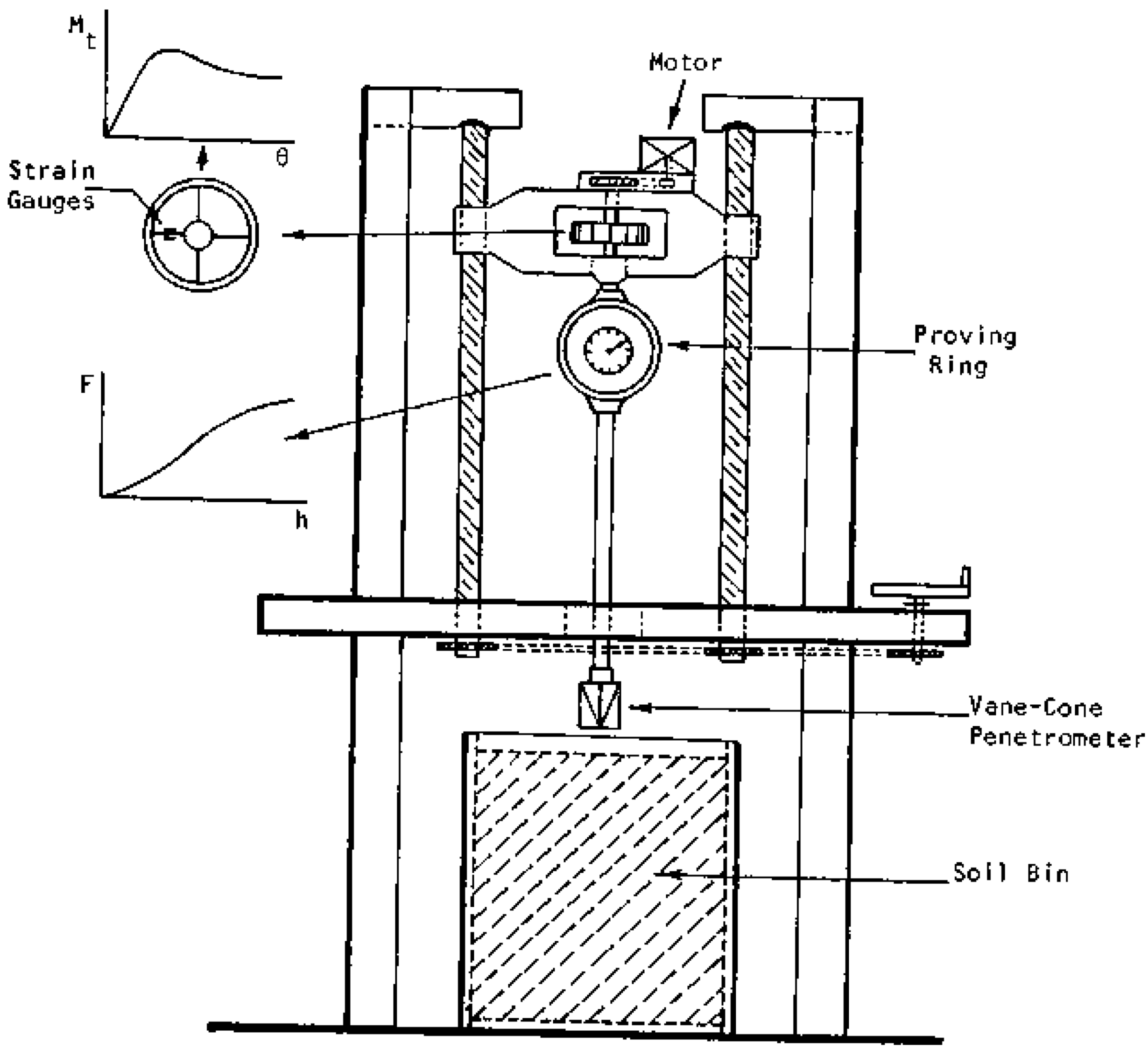


FIG. 3-2 Test facility used for Cone, Vane-Cone, and Bevameter-type tools.

the vertical thrust or rotational torque values was obtained through the associated instrumentation as shown in Fig. 3-2. Rotation of the shear unit of the vane-cone or the bevameter annulus was obtained through a motorized unit.

Following preparation of the test soil in the test facility, the proper test tool was placed in the facility and the tool advanced into the soil material. Depending upon the kind of tool used, the measurements taken provided a record of either the penetration force together with the penetration depth, or the other measurements associated with the rotational shear aspect of the test.

3.2.4 Cone Penetration Tests

In addition to the size of the cone used which was similar to that as given by the specifications from Waterways Experiment Station, other variable cone sizes were also examined. Only one soil type was used and cone penetration readings were recorded as load-penetration relationships.

The technique for the penetration tests was identical to that used by Waterways Experiment Station. Variable rates of penetration ranging from 0.1 to 6 in./min penetration rate up to a depth of 6.0 in. were also investigated. The influence of cone size and rates of penetration has been fully examined and documented previously by Yong and Chen (1976) and Yong, Chen and Sylvestre-Williams (1972). Typical load penetration curves are given in Fig. 3-3.

3.2.5 Bevameter-type Tests

Three sizes of pressure plates were used in the tests. They were 1.5 x 1.5, 1.5 x 3.0, and 1.5 x 4.5 in. plates.* These were individually mounted on to the device shown in Fig. 3-2 and separate plate penetration tests conducted.

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* Note that plate bearing theory allows one to substitute rectangular plates for circular plates so long as the boundary conditions are properly specified.

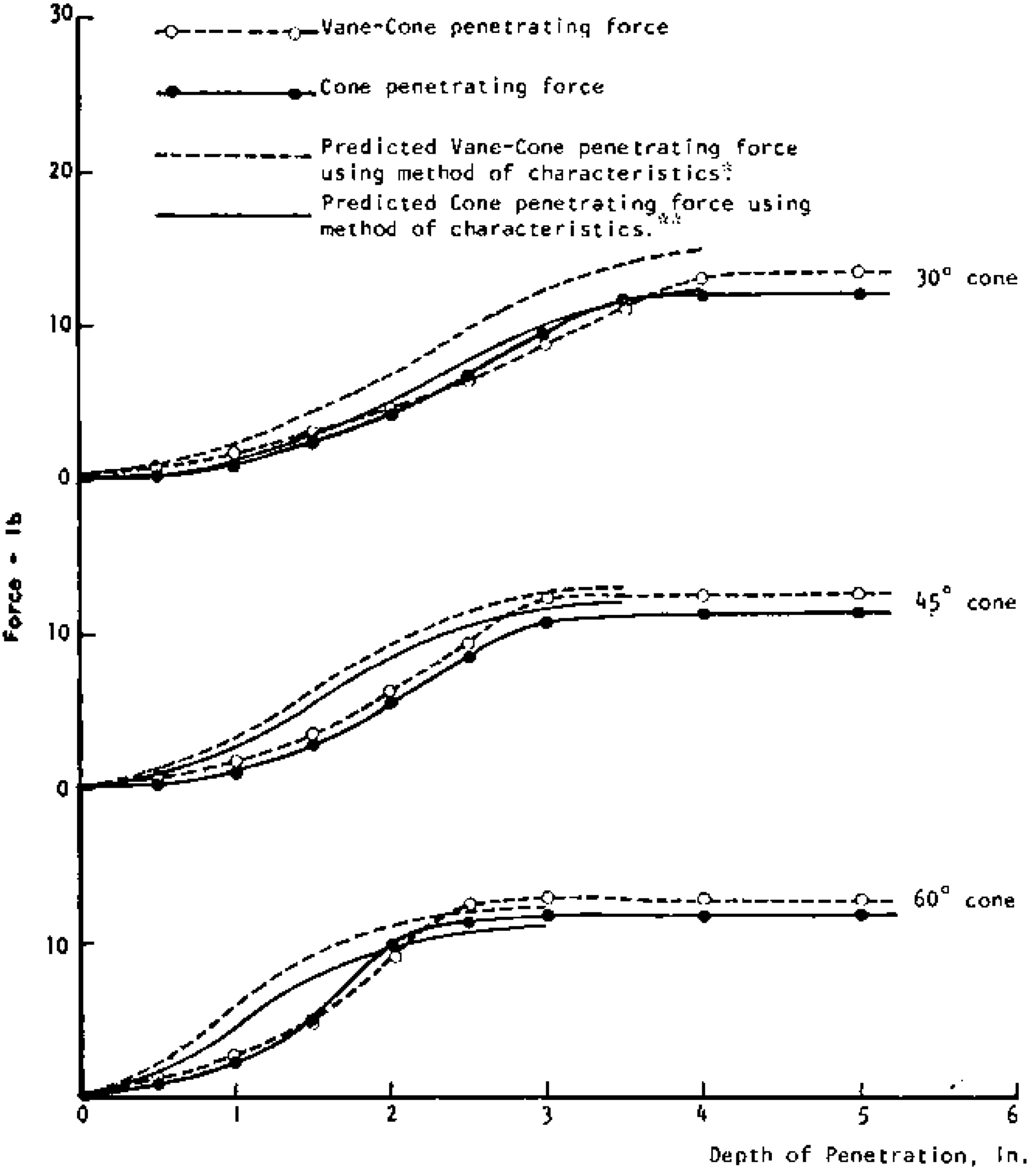

FIG. 3-3 Penetration force vs depth of penetration for Cone and Vane-Cone penetrometers at 6.0 in./min penetration speed.

* From Youssef, (1977)

** From Yong et al. (1972)

The size of the bevameter-type annulus shear device used was 1.625 in. I.D. and 3.125 in. O.D. with shear vanes protruding 0.375 in. The number of shear vanes on the annulus was four.

The rate of penetration of the pressure plates into the test soil was 3.0 in./min, and the rate of angular rotation for the annulus was 72 degrees/min. Figures 3-4 and 3-5 show the results for the plate penetration tests and the annulus shear tests. Note that a schematic of the annulus shear device is shown in Fig. 3-5 together with typical test results.

3.2.6 Vane-Cone Tests

The size of the vane-cone used is shown in schematic form together with the typical set of results for cone penetration in Fig. 3-6. The corresponding shear tests for the vane portion of the vane-cone device are shown in Fig. 3-7.

The procedure for application of the vane-cone device requires that the cone be penetrated into the soil at the same rate as that used in the WES cone - i.e. less than 72 in./min.* At the required specified depth of penetration, the cone is maintained at that depth of penetration whilst being rotated to produce the vane shear sensing aspect of the vane-cone device. Note the importance for the requirement for the vane-cone to be maintained at that position with no penetration whilst the vane-cone is being rotated, since the analyses performed must relate soil strength to the depth of soil stratum sampled. The rate of rotation of the vane-cone is conditioned by the shear resistance of the soil. If desired, the vane-cone may be further penetrated into the soil and the procedure for vane shear repeated at a lower depth whilst maintaining the cone at the new penetrated depth.

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* Note from Fig. 3-3 that the penetrating force for the vane-cone is almost identical to that of the cone (by itself). To all intents and purposes, one would consider the penetration of the vane-cone equal to the cone.

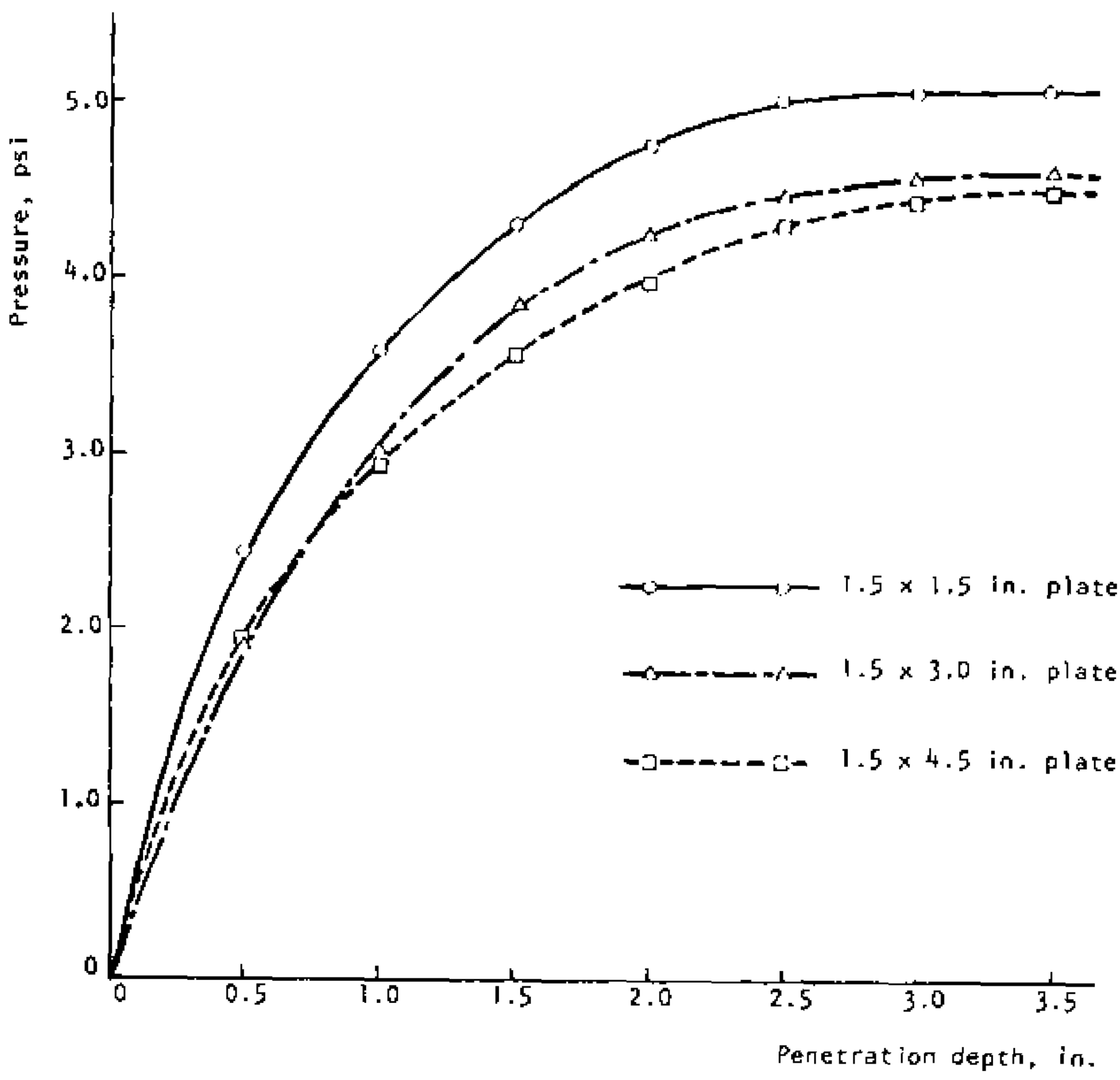

FIG.3.4 Pressure penetration relation for rectangular plates with different width to length ratios.

Note that plate bearing theory allows for rectangular plates to be substituted for circular plates. Experiments conducted in soil mechanics studies confirm the validity of the plate theory cited.

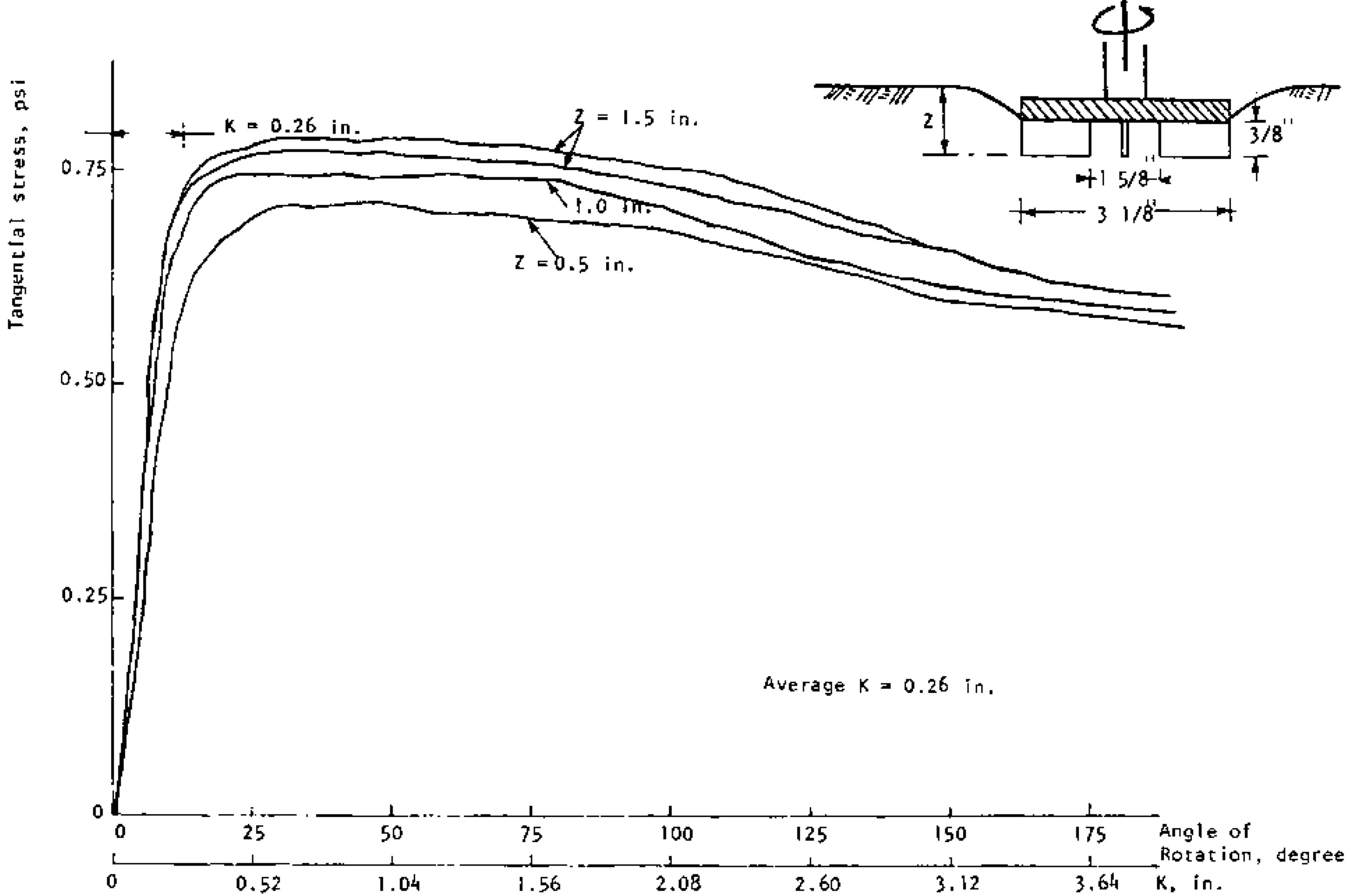

FIG. 3-5 Tangential stress vs angle of rotation for shear ring.

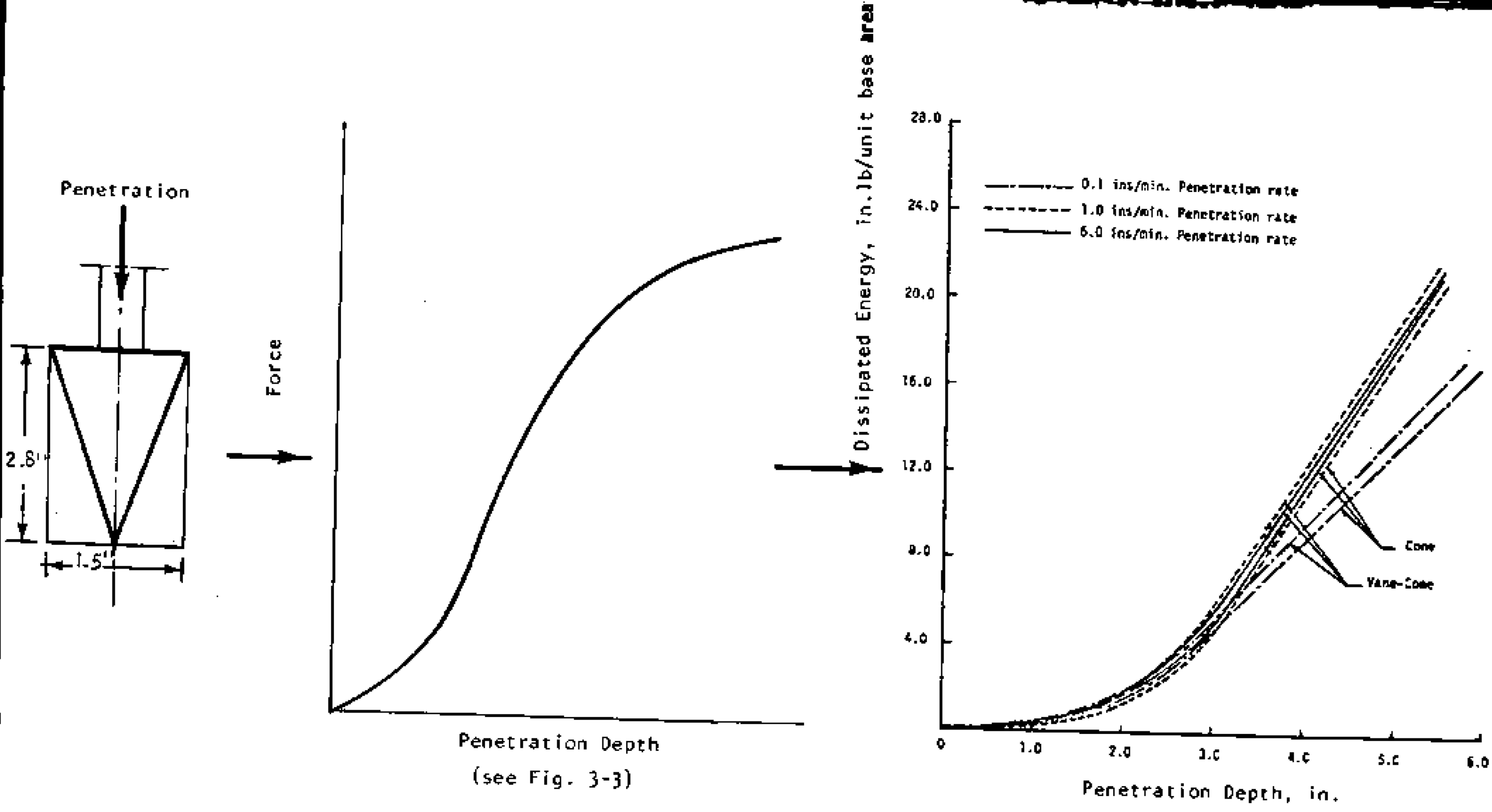

FIG. 3-6 Vane-cone device and results obtained using the cone component.

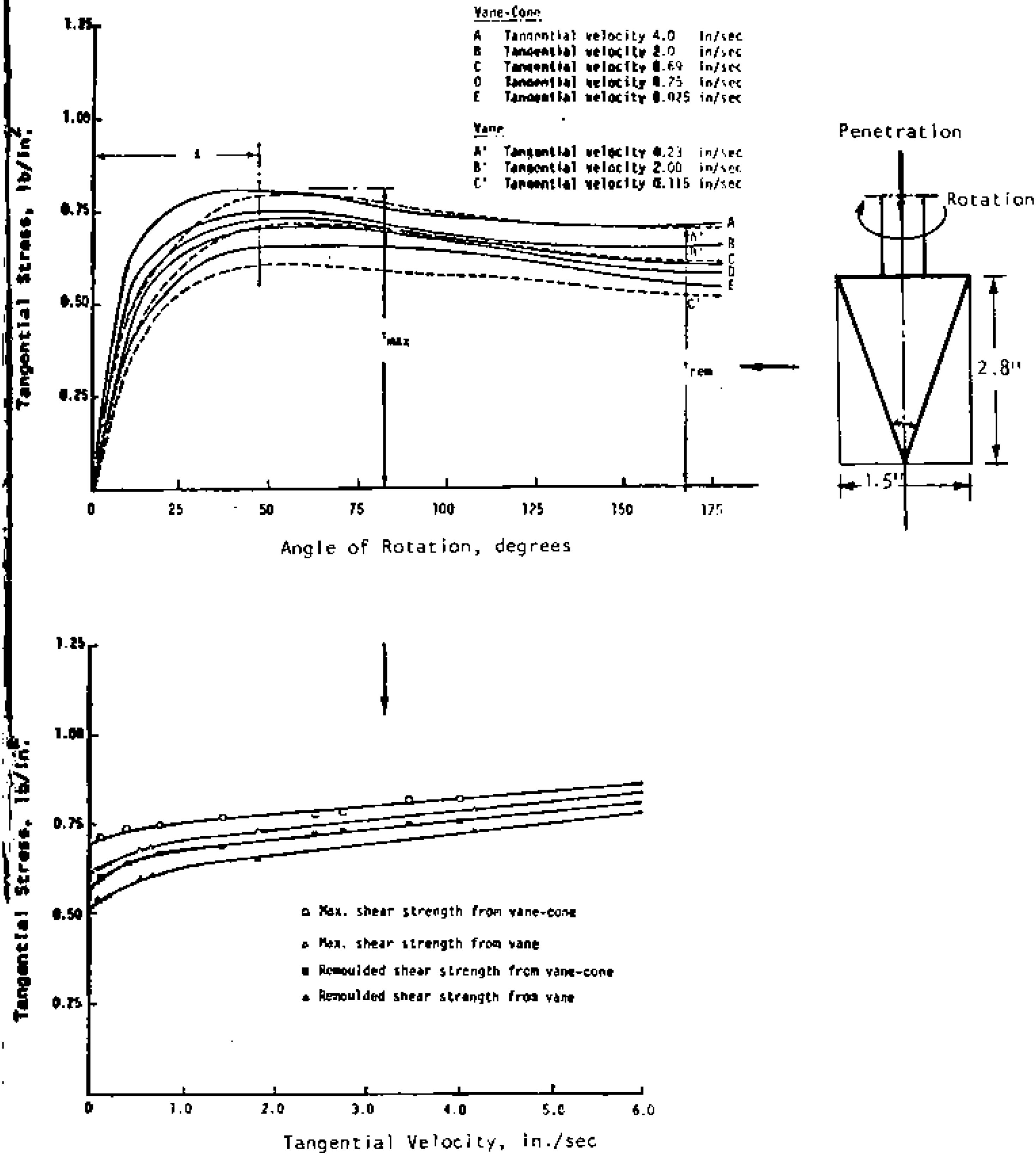

FIG. 3-7 Vane-cone device and results obtained using vane position.

3.3 PROCEDURE FOR UTILIZATION OF CONE PENETRATION READINGS FROM WES PENETROMETER

Initial use of the cone readings has been to assess mobility through empirical correlations where cone penetration readings given as cone indices were correlated with actual performance of typical vehicles. Through empirical correlations with such vehicles, judgments and evaluations were made as to go or no-go criteria where direct one to one correspondence with the cone indices can be stated. By and large, this has been the traditional visualization of the application of cone penetration readings for prediction of mobility.

Since the development of the AMC-71 and AMC-74 require more than descriptive go no-go predictions, the cone indices have been extended in application to provide for a means for prediction of mobility performance with respect to slip. In all, together with the previous go no-go situation producing thereby a mobility number approach, there are two other applications which may be considered as extensions and improvements of the cone penetration test application. All these are basically dimensionless techniques which include wheel or track parameters and soil data obtained by the penetrometer.

3.3.1 The Clay and Sand Number Approach (WES)

This approach depends on the production of a graphical relationship developed between dimensionless numbers for clay or sand and a pull or tow coefficient.

$\frac{P}{W} = \frac{\text{Pull}}{\text{Weight}}$ and $\frac{P_T}{W} = \frac{\text{Towed Force}}{\text{Weight}}$ = pull and tow coefficients.

The clay number can be written as:

$$N_c = \frac{Cbd}{W} \left(\frac{\delta}{h}\right)^{\frac{1}{2}} \frac{1}{1 + b/2d} \qquad (3.1)$$

where

C = average cone penetration resistance,

b, d = unloaded tire width and diameter,

W = vertical load or vehicle weight,

δ = tire deflection,

h = tire section height.

3.3.2 Power Number Approach (WES)

This approach also depends on the production of a graphical relation between power number (PN) and pull coefficient ($\frac{P}{W} = \frac{Pull}{Weight}$) as shown in Fig.3-8 where:

$$PN = \frac{M\omega}{WV_a} \qquad (3.2)$$

M = torque,

ω = rotational velocity,

W = wheel load,

V_a = test carriage or vehicle speed.

3.3.3 Extension of Power Number Approach

With the results given in Fig. 3-8, the approach can be extended using information obtained with the McGill vane-cone device. To implement this procedure, the dynamic sinkage, and tangential stresses can be obtained using vane-cone measurements. The input torque can be obtained or calculated from a knowledge of the dynamic sinkage and tangential stresses. With a knowledge of the slip and the corresponding angular velocity, the power number can be calculated. With this kind of information, Fig. 3-8 can now provide the corresponding pull ratio at different slips.

It is evident that the augmentation of the power number approach using information from the McGill vane-cone can increase the capability of the cone for

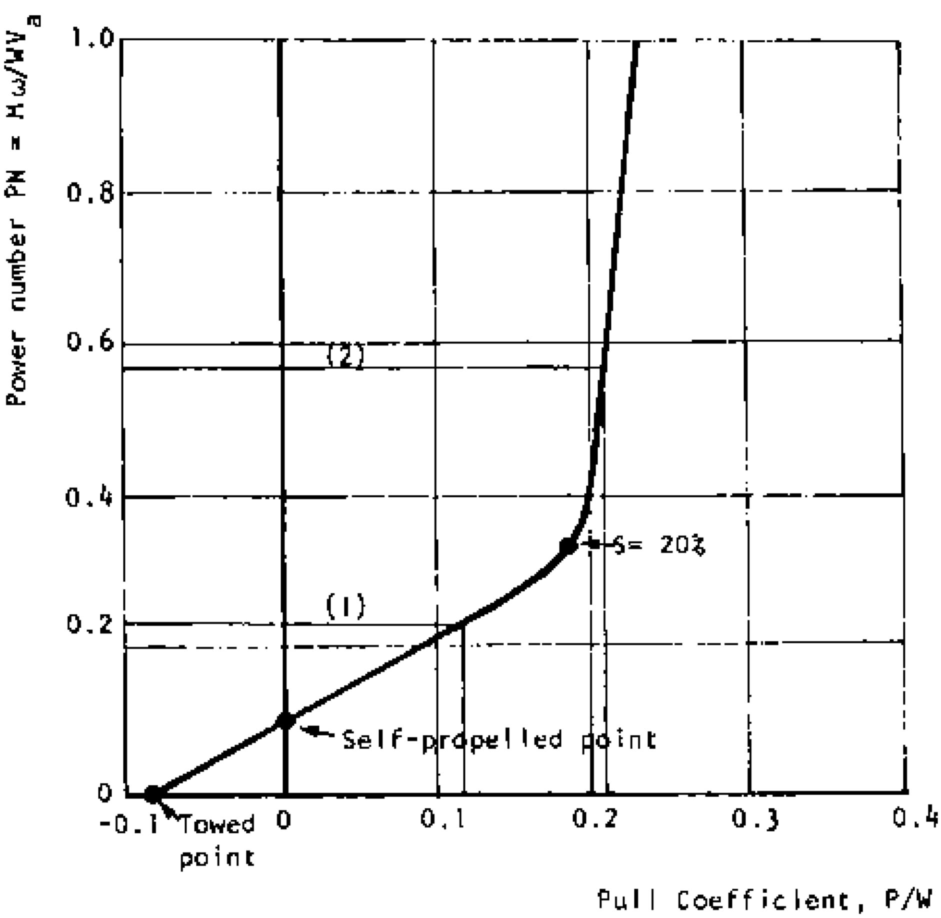

FIG.3-8 Representative relation of power number to pull coefficient (Melzer, 1972)

prediction of drawbar pull with slip. We note in passing that in effect, cone penetration can actually be obtained with the cone portion of the vane-cone device.

3.4 UTILIZATION OF BEVAMETER-TYPE READINGS USING BEKKER APPROACH*

The use of bevameter readings for prediction is as follows:

(a) Thrust or traction prediction.

(b) Motion resistance prediction (which includes rolling and bulldozing resistances).

(c) Drawbar pull prediction which can be obtained by subtracting (a) from (b) as follows:

$$DP = H - R \tag{3.3}$$

where

DP = drawbar pull,

H = thrust or traction,

R = resistance.

i.e. R = [Rolling (or towing) Resistance] + [Bulldozing resistance if the wheel moves on hard surface covered with loose soil or mud or snow]

3.4.1 Thrust Prediction

$$H = \iint \tau \, dA \qquad \text{(Bekker, 1956)} \tag{3.4}$$

where

τ = tangential shear stress obtained as discussed previously from annulus shear test and shown in Fig.3-9.

$$= (C + p\tan\phi)\,(1 - e^{-j/K}) \tag{3.5}$$

A = contact area.

To calculate the contact area, a sinkage formula based also on the plate equation has been used by Bekker,* as:

$$Z = \left[\frac{3W}{(3-n)\,(K_c + bK_\phi)\,\sqrt{D}}\right]^{\frac{2}{2n+1}} \tag{3.6}$$

---

* Note that a recent modification of the Bekker formulation for sinkage can be used. The results obtained do not deviate measurably from the data given in this section. The recent modification of the Bekker formulation is given in the Appendix.

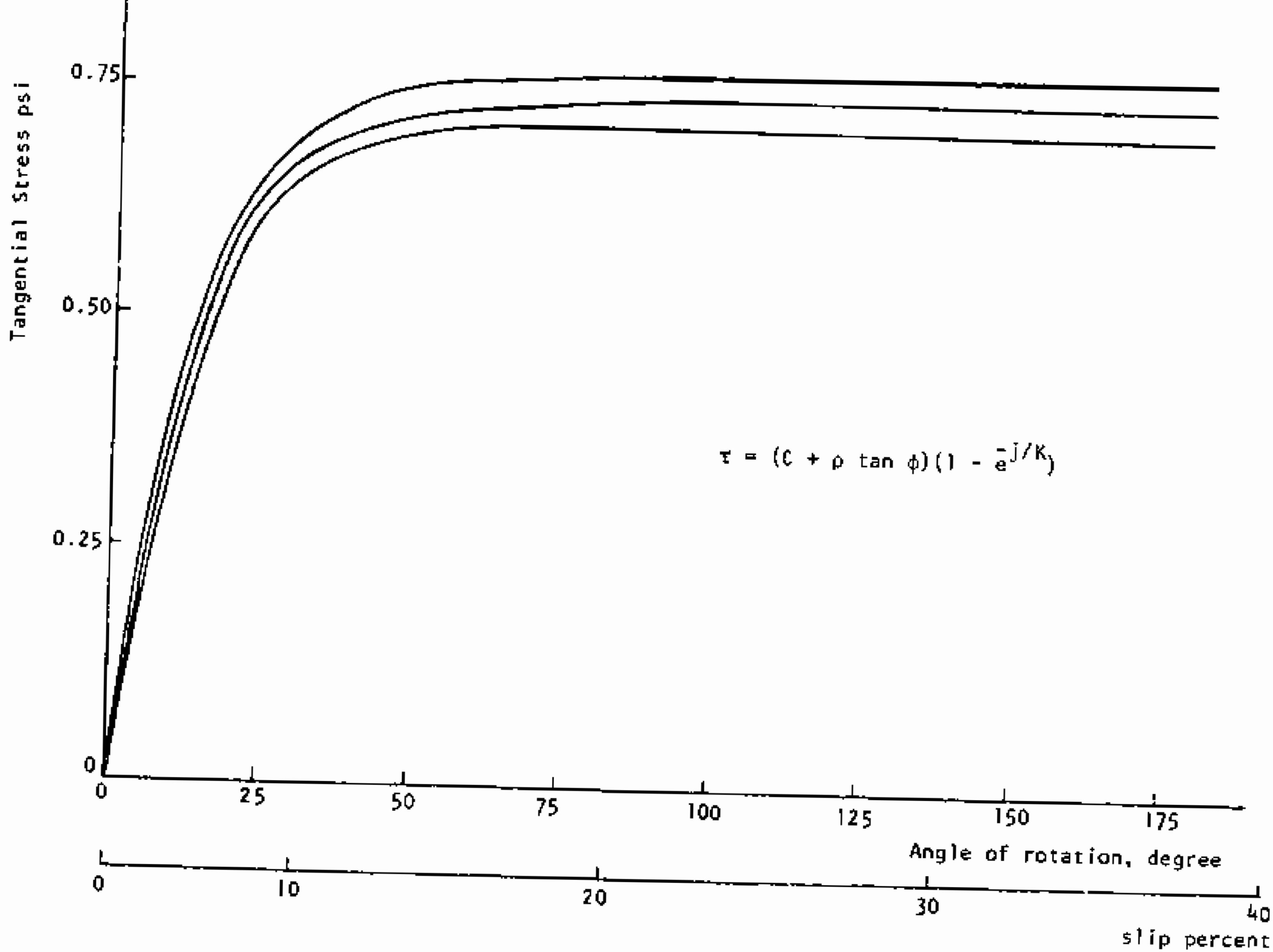

FIG. 3-9 Tangential stress vs slip rotation

where K_c, K_ϕ and n are soil values,

W = wheel load,

D = wheel diameter,

b = wheel width.

Note that the relationship is essentially a static bearing stability formula. The soil values are obtained from three separate plate load tests as shown in Fig. 3-9. The pressure sinkage relations shown in this figure are replotted again on log P-log Z graph as shown in Fig. 3-10.

Considering the idealized straight lines shown in Fig. 3-10, the following equation can be applied:

$$\text{Log } P_j = \log (K_c/b_i + K_\phi) + n \log Z_j \tag{3.7}$$

where i and j are specified quantities obtained from the tests. Picking values of Z_1 = 2 inches, and Z_2 = 1 inch and the corresponding values for P from Fig. 3-10, the following equations are obtained:

$$\log 4.6 = \log [(K_c/0.83) + K_\phi] + n \log 2.0 \tag{3.8a}$$

$$\log 4.1 = \log [(K_c/1.16) + K_\phi] + n \log 2.0 \tag{3.8b}$$

$$\log 3.9 = \log [(K_c/1.45) + K_\phi] + n \log 2.0 \tag{3.8c}$$

$$\log 3.5 = \log [(K_c/0.83) + K_\phi] \tag{3.9a}$$

$$\log 3.0 = \log [(K_c/1.16) + K_\phi] \tag{3.9b}$$

$$\log 2.9 = \log [(K_c/1.45) + K_\phi] \tag{3.9c}$$

Note that 0.83, 1.16 and 1.45 are the effective radii of the plates used in this study.

Solution of the above equations provides the average soil values as:

K_c = 1.0

K_ϕ = 2.25

n = 0.45

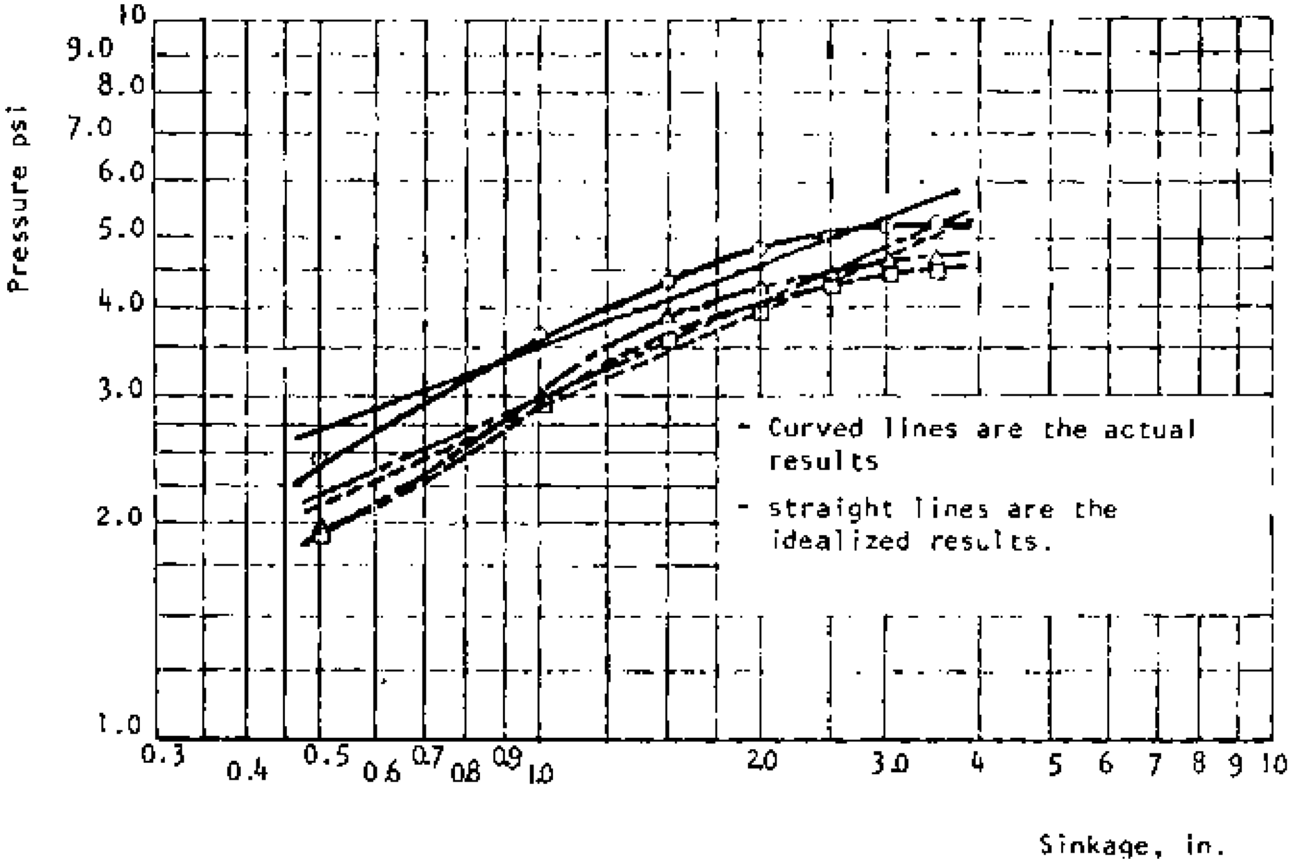

FIG. 3-10 Log-log plot of load-penetration curves

Substituting these soil values in Eq. (3.6) the sinkage obtained for 13.5 in. diameter test wheel used and 3.75 in. width loaded with 34, 54, and 74 lb as in the actual laboratory test condition (for the tow-bin tests) provides:

Z_{34} = 1.16 in. (for 34 lb test wheel),

Z_{54} = 1.88 in. (for 54 lb test wheel),

Z_{74} = 2.60 in. (for 74 lb test wheel).

The wheel contact areas corresponding to these sinkages are:

A_{34} = 14.88 $in.^2$

A_{54} = 18.88 $in.^2$

A_{74} = 22.05 $in.^2$

Thus, with knowledge of the contact areas and the tangential stresses the total thrust or traction can be calculated from Eq. (3.4). The calculated tractions are given in Table 3-1. These values and results have been used to provide for the comparison of predictions and measured wheel performances in section 4.0.

3.4.2 Resistances

(1) Rolling (or towing) Resistance - The generalized equation for rolling or towing resistances given by Bekker (1969) is:

$$R = \frac{(3W)^{\frac{2n+2}{2n+1}}}{(3-n)^{\frac{2n+2}{2n+1}}\,(n+1)\,(K_c+bK_\phi)^{\frac{1}{2n+1}}\,D^{\frac{n+1}{2n+1}}} \qquad (3.7)$$

where

W = wheel load,

b and D = wheel width and wheel diameter,

K_c, K_ϕ and n = soil constants as obtained previously.

TABLE 3-1

Predicted Thrust and Drawbar Pull using Bevameter Approach

Slip %	Average Shear Stress τ psi	Wheel Load 34 lb Traction lb	34 lb Pull lb	54 lb Traction lb	54 lb Pull lb	74 lb Traction lb	74 lb Pull lb
0	0	0	-8.64	0	-17.5	0	-28.4
5	0.5	7.44	-1.20	9.44	- 8.06	11.03	-17.37
10	0.675	9.67	1.03	12.74	- 4.76	14.88	-13.52
15	0.72	10.71	2.07	13.59	- 3.91	15.88	-12.52
20	0.73	10.86	2.22	13.78	- 3.72	16.10	-12.30
30	0.74	11.01	2.37	13.97	- 3.53	16.32	-12.08
40	0.74	11.01	2.37	13.97	- 3.53	16.32	-12.08
50	0.74	11.01	2.37	13.97	- 3.53	16.32	-12.08
60	0.74	11.01	2.37	13.97	- 3.53	16.32	-12.08
70	0.74	11.01	2.37	13.97	- 3.53	16.32	-12.08

Applying the wheel and soil data into the above relationship the following results are obtained:

R_{34} = 8.64 lb (for 34 lb wheel load),

R_{54} = 17.50 lb (for 54 lb wheel load),

R_{74} = 28.4 lb (for 74 lb wheel load).

(ii) Bulldozing Resistance - The bulldozing resistance equation as given by Bekker (1969) is:

$$R_b = \frac{b \sin (\alpha+\phi)}{2 \sin \alpha \cos \phi}[2Z\, C\, K_c + \gamma Z^2\, K_\gamma] + \frac{\pi L_0^2\, \gamma(90-\phi)}{540}$$

$$+ \frac{C\pi L_0^2}{180} + C\, L_0^2 \tan (45) + \frac{\gamma}{2} \qquad (3.8)$$

where

K_c, K_γ = soil constants,

L_0 = dimension, function of sinkage and angle of internal friction,

α = angle of approach.

In the present case, there is no need for this resistance as defined in Eq. (3.3), since the soil used in the bin is a cohesive soil.

3.4.3 Drawbar Pull

The drawbar pull can be obtained at each degree of slip by considering the wheel thrust and wheel resistances as discussed in the preceding subsections and in Table 3-1. The same results are restated in Fig. 3-11.

The results can be represented in terms of specific energy values for the purpose of comparison with the energy analysis as identified with vane-cone predictions. The values given in Table 3-2 are obtained as follows from Table 3-1:

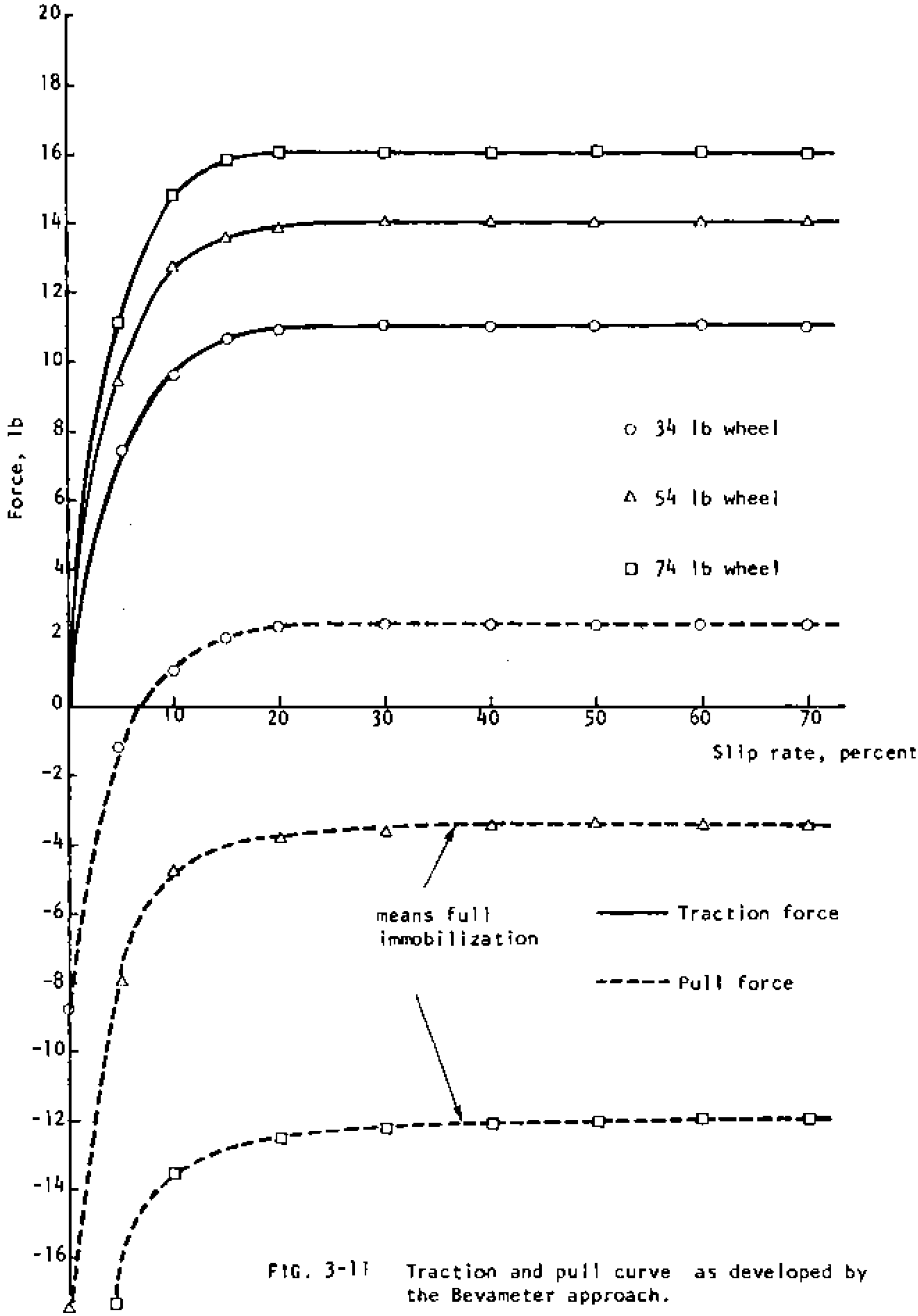

FIG. 3-11 Traction and pull curve as developed by the Bevameter approach.

TABLE 3-2

Predicted Specific Energy Components using Bevameter Approach
(Input Energy and Pull Energy)

Slip %	Average Shear Stress τ psi	Wheel Load 34 lb Specific Input Energy	34 lb Specific Pull Energy	54 lb Specific Input Energy	54 lb Specific Pull Energy	74 lb Specific Input Energy	74 lb Specific Pull Energy
0	0	0	-2.3	0	-4.67	0	-7.57
5	0.5	1.86	-0.32	2.37	-2.15	2.76	-4.63
10	0.675	2.55	0.27	3.36	-1.27	3.93	-3.61
15	0.72	3.00	0.55	3.81	-1.04	4.45	-3.34
20	0.73	3.22	0.59	4.08	-0.99	4.77	-3.28
30	0.74	3.73	0.63	4.73	-0.94	5.53	-3.22
40	0.74	4.35	0.63	5.51	-0.94	6.44	-3.22
50	0.74	5.23	0.63	6.64	-0.94	7.75	-3.22
60	0.74	6.52	0.63	8.28	-0.94	9.66	-3.22
70	0.74	8.90	0.63	11.04	-0.94	12.88	-3.22

$$\text{Specific pull energy} = \frac{\text{Pull force x translational speed}}{\text{Wheel width x translational speed}}$$

Specific interfacial energy =

$$\frac{\text{Traction force x wheel radius x angular velocity}}{\text{Wheel width x translational speed}}$$

Specific energy has units of lb in./unit width/unit of travel.

Note that the results comparing the predictions of drawbar pull etc. between the various test devices and actual tow bin test wheel measurements are given in section 4.0.

3.5 VANE-CONE PREDICTION METHOD

In this approach it is recognized that the actual wheel or track performance, in regard to subsoil deformation, is influenced by many factors such as wheel or track dimensions, loads, contact surface characteristics and angular and translational velocities. This performance may be discussed in terms of energy transfer mechanisms at the wheel or track interface.

The energy transfer component associated with the subsoil deformation must obviously depend on the mechanics of transfer at, and beneath, the interface and is thus related directly to the interface input energy component. The procedure requires an analysis of the overall problem in terms of the rationale for partitioning the components of parasitic energy.

It is desirable to separate the vane-cone method of prediction into two approaches:

(a) Rigorous approach which considers most of the variables and parameters affecting the real problem.

This might require a more detailed examination of experimental measurements coupled with theoretical analyses. This approach is not developed in this report. The detailed procedures, requirements etc. of this rigorous approach are discussed separately in another report.

(b) "Simple" approach using approximate simplifying equations developed as a result of both theoretical and experimental analyses. The first order simplification of the working equation is shown here for wheels moving over soft soil. Note that these can be used directly with the simple vane-cone device. Note also that continued development and generalization of such type of equations for all vehicles and all types of soils is continuing.

The application of these predictive equations for drawbar pull evaluation of the actual tow-bin tests is given in section 4.0.

3.5.1 Vane-Cone Prediction - Simple Approach

The simple approach for vane-cone prediction for wheel performance moving over soft soil can be achieved using the following equations:

(a) Input Energy Prediction, I.E.

$$I.E. = \frac{mbd}{2}\left[\frac{W^2(1+2S)^2}{N_c\, p^2\, b^2} + \delta d\right]^{\frac{1}{2}} \cdot \tau \cdot \omega \qquad (3.9)$$

(b) Slippage (or Interfacial) Energy Prediction, S.E.

$$S.E. = \frac{mbd}{2}\left[\frac{W^2(1+2S)^2}{N_c\, p^2\, b^2} + \delta d\right]^{\frac{1}{2}} \cdot \tau\left(\omega - \frac{2V_c}{0.935d}\right) \qquad (3.10)$$

(c) Deformation Energy Prediction, D.E.

$$D.E. = b.V_c.[A]_o^{\,y} \quad \text{(See Fig. 3-12).} \qquad (3.11)$$

where

$$y = K\left[\frac{W^2}{N_c\, p^2\, b^2 d}\right] + \alpha\, e^{\beta\left(\frac{h}{2r}\right)}$$

(d) Pull Energy Prediction, P.E.

$$P.E. = I.E. - S.E. - D.E. \qquad (3.12)$$

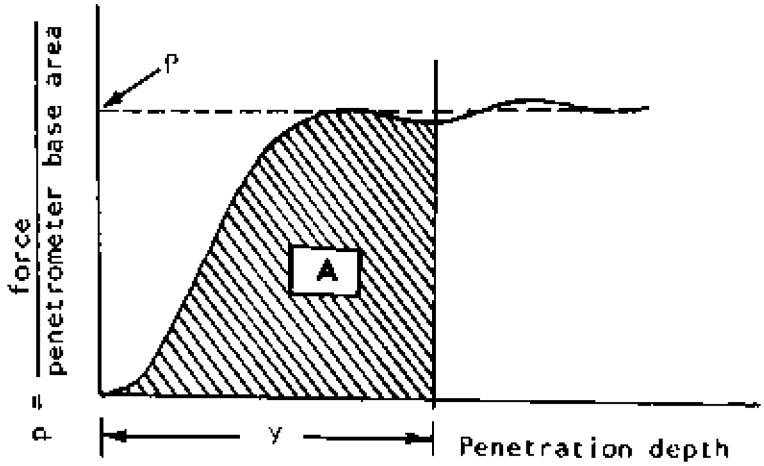

FIG. 3-12 Pressure-penetration diagram for vane-cone penetrometer.

where*

W = wheel load,

p = pressure beneath the penetrometer,

b, d = wheel width and diameter,

h, r = penetrometer height and radius,

δ = tread height,

S = slip,

V_c = translational velocity,

ω = angular velocity,

τ = tangential shear stress obtained at the outer edge of the vane component,

m = factor takes the effect of backward contact area equal to 1.31,

N_c = bearing stability factor which is dimensionless and equal to 0.225 for $\phi = 0$,

K, α, β = dimensional factors obtained from visioplasticity analyses (semi-analytical analyses) for the purpose of deformation energy prediction. For cohesive material, $\phi = 0$ and in the case of lb in. units, $K = 1.21$, $\alpha = 0.97$ and $\beta = 0.230$.

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* Note that the items listed are readily obtained in measurements.

4.0 COMPARISON OF DRAWBAR PULL PREDICTION RESULTS USING THE THREE SENSING DEVICES

4.1 INTRODUCTION

In this section, we compare the predictive capability of the three sensing devices with actual wheel test results from laboratory tow bin tests. The test instruments used were discussed in section 2.0, and the methods of utilization of test data for application to prediction of mobility have been given in section 3.0. As noted in section 2.0, the tests conducted by the three devices produce results either in terms of compression alone or in terms of compression penetration and shear.

As observed in section 3.0, the actual raw data available from each of the test devices should be used with the analytical theories associated with the sensing devices to compute the drawbar pull. It is noted that the drawbar-pull can be obtained directly using the vane-cone device or the bevameter-type tool, and indirectly with extension into the power number concept with the Vicksburg penetrometer tool.

4.2 TOW BIN TESTS CONDUCTED

Three types of wheels were used in the tow bin tests. These were:

(a) a polished aluminum wheel,

(b) a rigid wheel with its contact surface coated with a 1/8 inch thick rubber,

(c) the same rubber wheel coated with a tread surface. (tread height = ¼ inch)

Three loads were used for all the tests with these three types of wheels. The loads were 34 lb, representing the weight of the wheel itself, 54 lb, and 74 lb. The size of the wheel was 13.5 inches in diameter, and 3.75 inches wide.

4.3 COMPARISON OF MEASURED AND PREDICTED VALUES

Figures 4-1 through 4-3 show a comparison of the test results with those predicted based upon measurements made with the three sensing devices and the methods for data reduction given in section 3.0*. As evident from all three figures, the kinds of correlation obtained between predicted and measured values for the various test devices varies between adequate to poor. This is especially true for correlations with experiments where wheel surface characteristics are varied.

The inability for some of the tools to completely predict the slip drawbar pull performance does not necessarily lie in the manner in which the sensing devices work, but more so in the manner of utilisation of the measured results obtained from the sensing tools themselves. It is apparent from the previous discussions given in sections 2.0 and 3.0 that accountability for slip performance is not necessarily provided even if the shear strength of the soil is given. It must be emphasized that if slip accountability is not provided, this is not totally a fault of the measuring technique itself but the method of utilization of the data - i.e. the analytical framework used in association with the sensing tool results.

The exception to the above is the cone penetrometer which does not provide a direct shear strength result and thus cannot easily lead to slip loss prediction. Whilst the power number application for the Vicksburg penetrometer results has extended the capability of the penetrometer for computation of slip-drawbar pull results beyond a go no-go situation to over-

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* Note that the comparison for the vane-cone predictions in Figs. 4-1 through 4-3 utilizes the simple approximate approach. The comparison of results using the rigorous approach for prediction of vane-cone results is given in section 4.4.

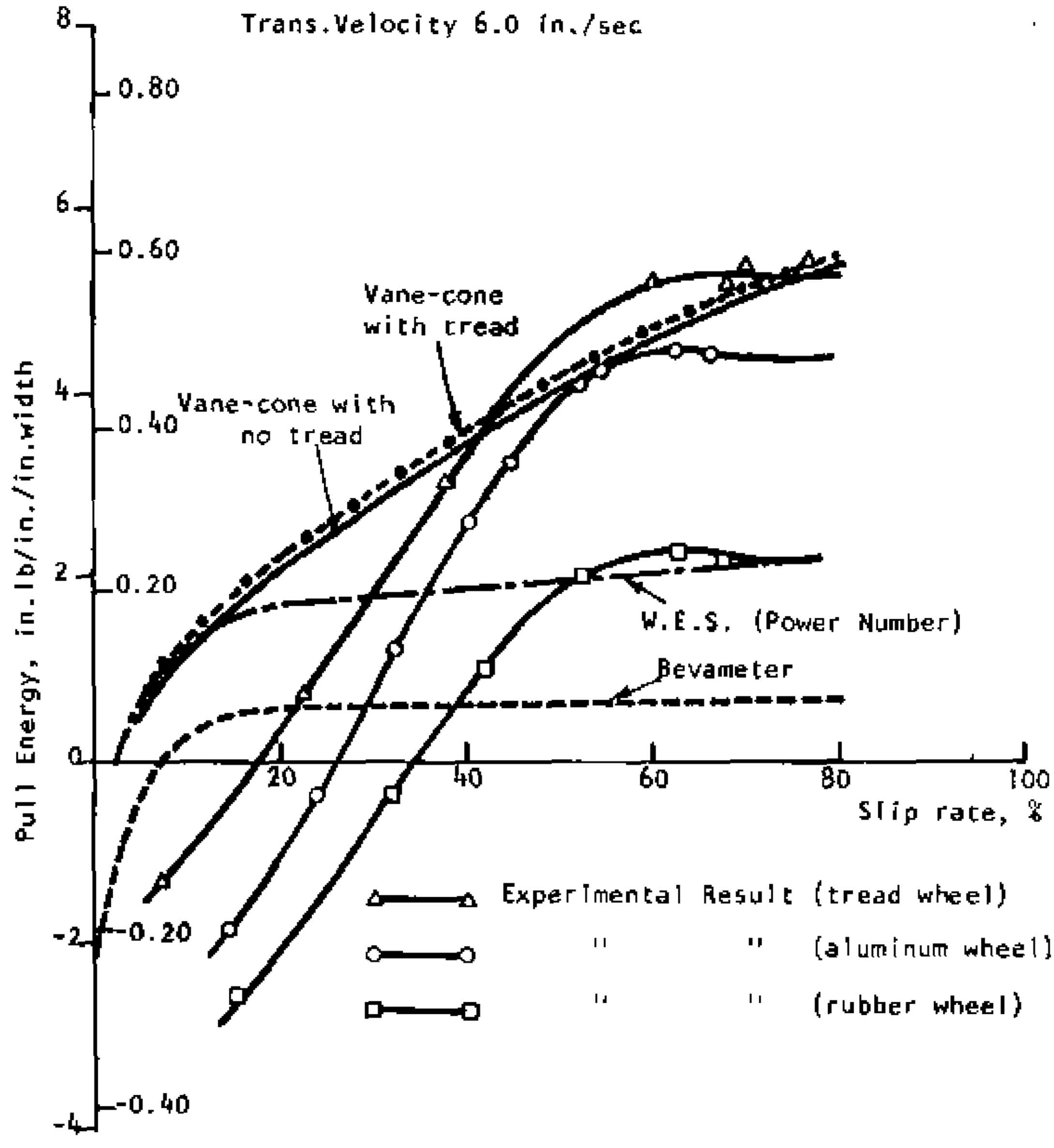

FIG. 4-1 Measured and predicted pull energy using Bevameter, W.E.S. Power Number, and approximate Vane-Cone equations.

(for 34 lb wheel load)

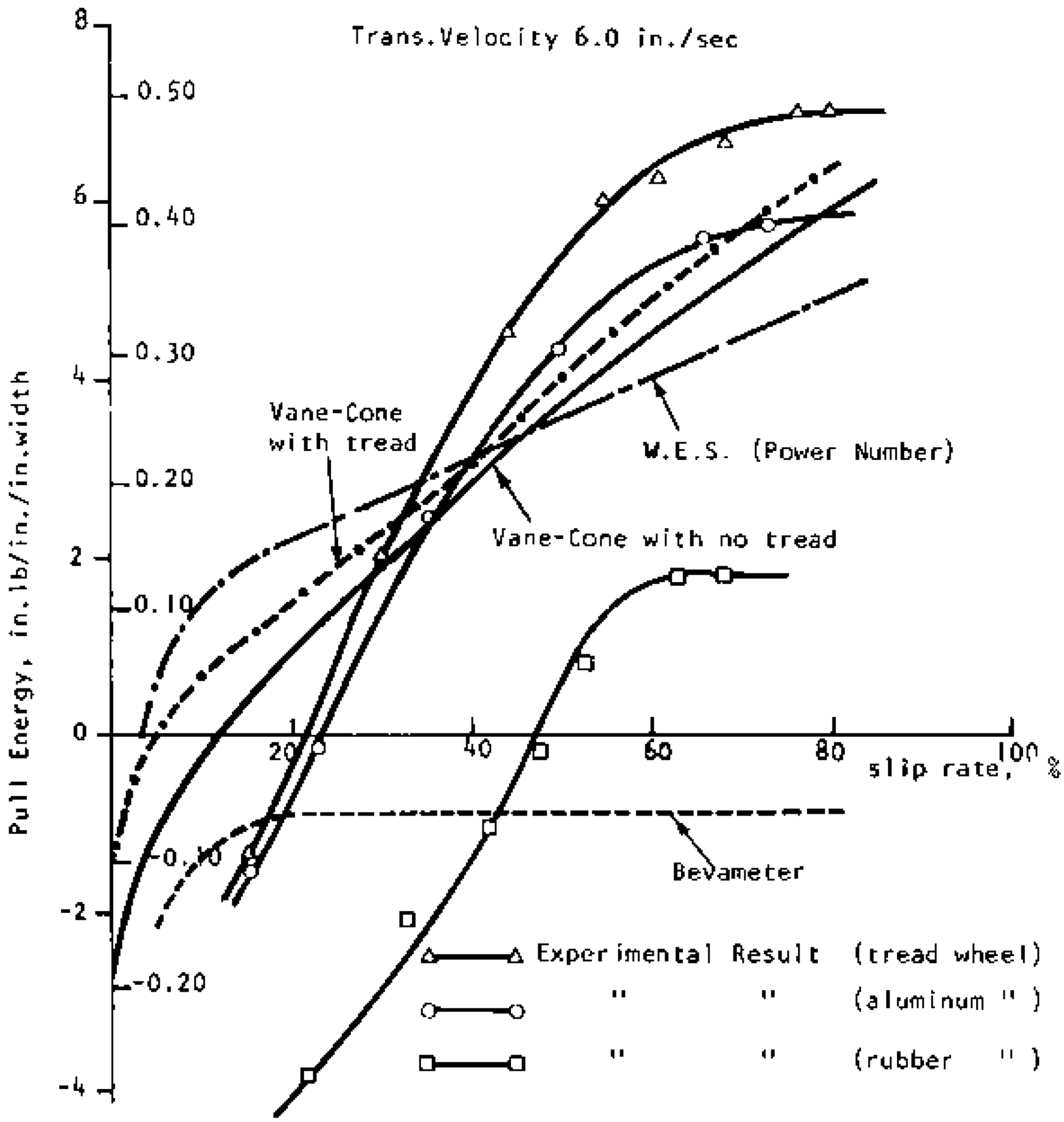

FIG. 4-2 Measured and predicted pull energy using Bevameter, W.E.S. Power Number, and approximate Vane-Cone equations.

(for 54 lb wheel load)

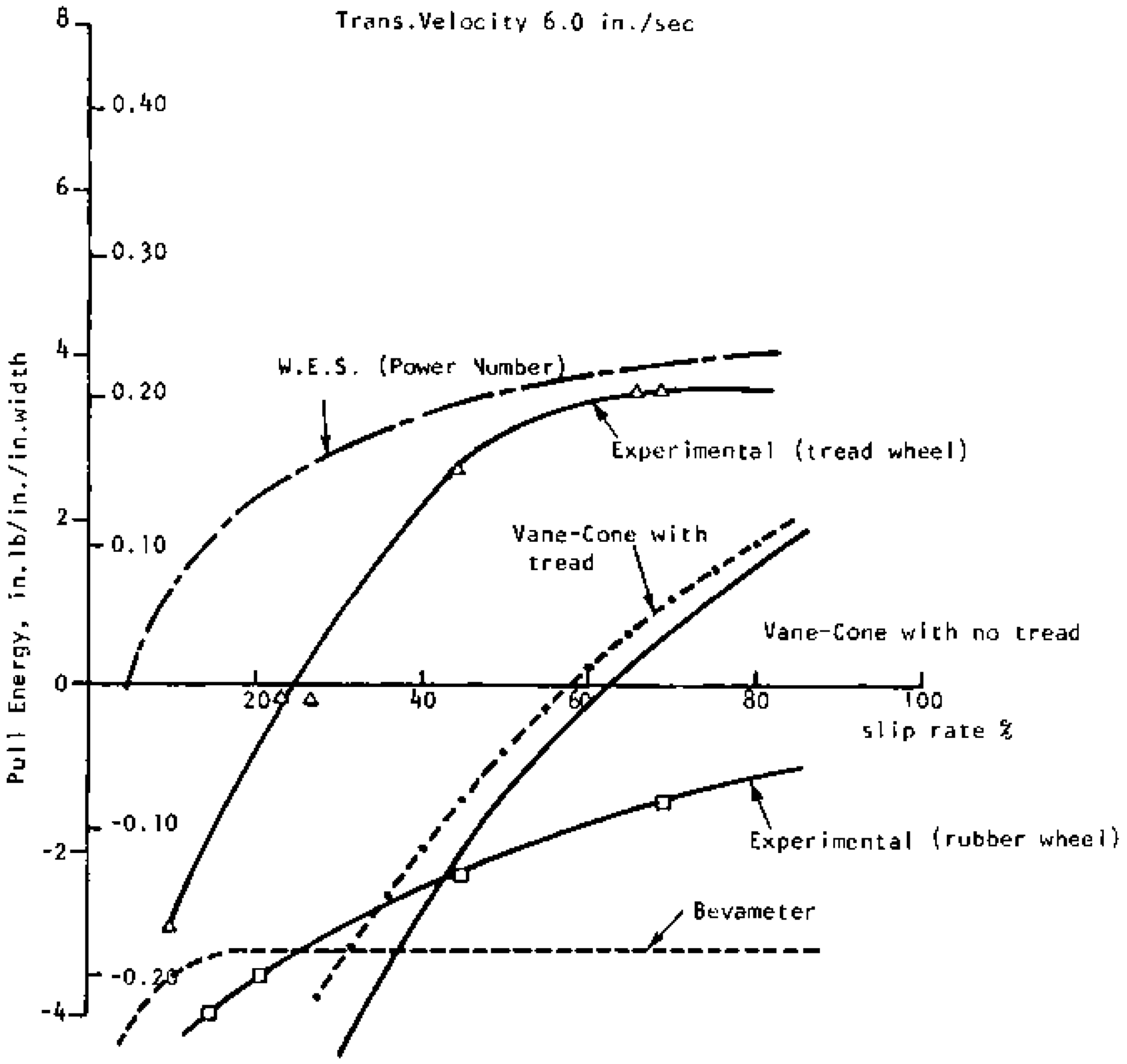

FIG. 4-3 Measured and predicted pull energy using Bevameter, W.E.S. Power Number, and approximate Vane-Cone equations.

(for 76 lb wheel load)

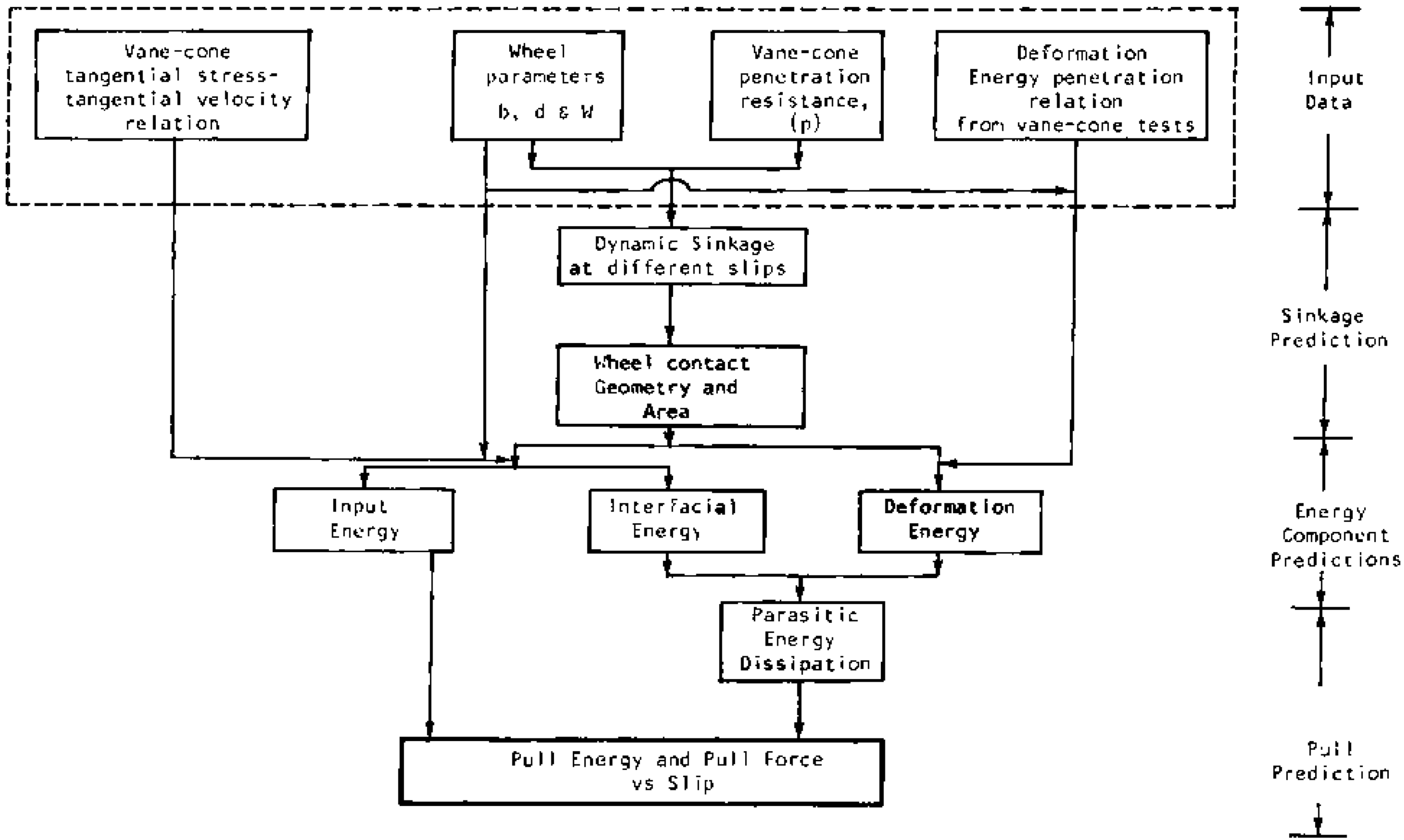

FIG. 4-4 Flow Chart for Method of Data Analysis for Vane-Cone Device for "exact" and "rigorous" methods of Mobility Prediction

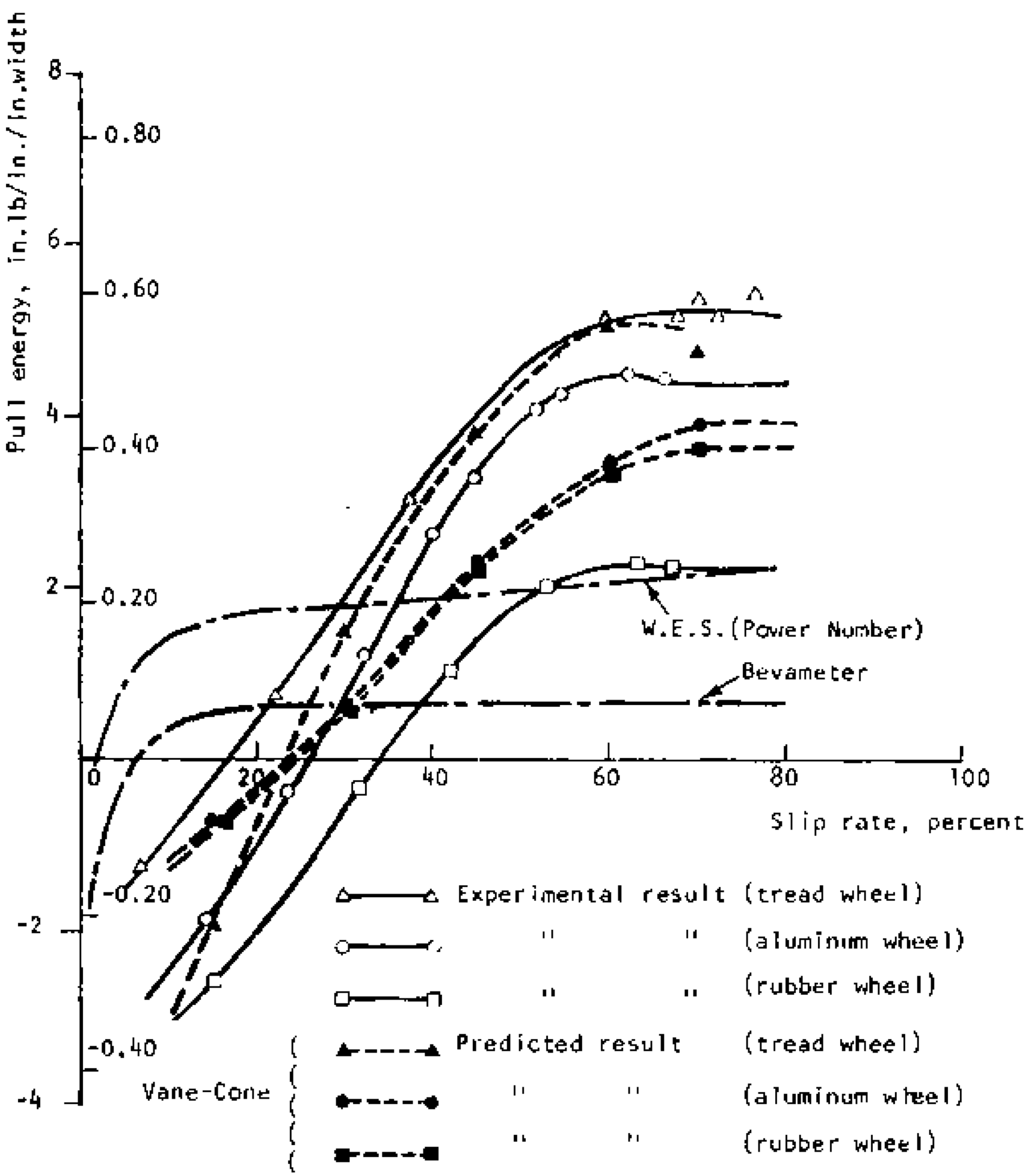

FIG. 4-5 Measured and predicted pull energy and pull to weight ratio vs slip rate for 34 lb wheel.

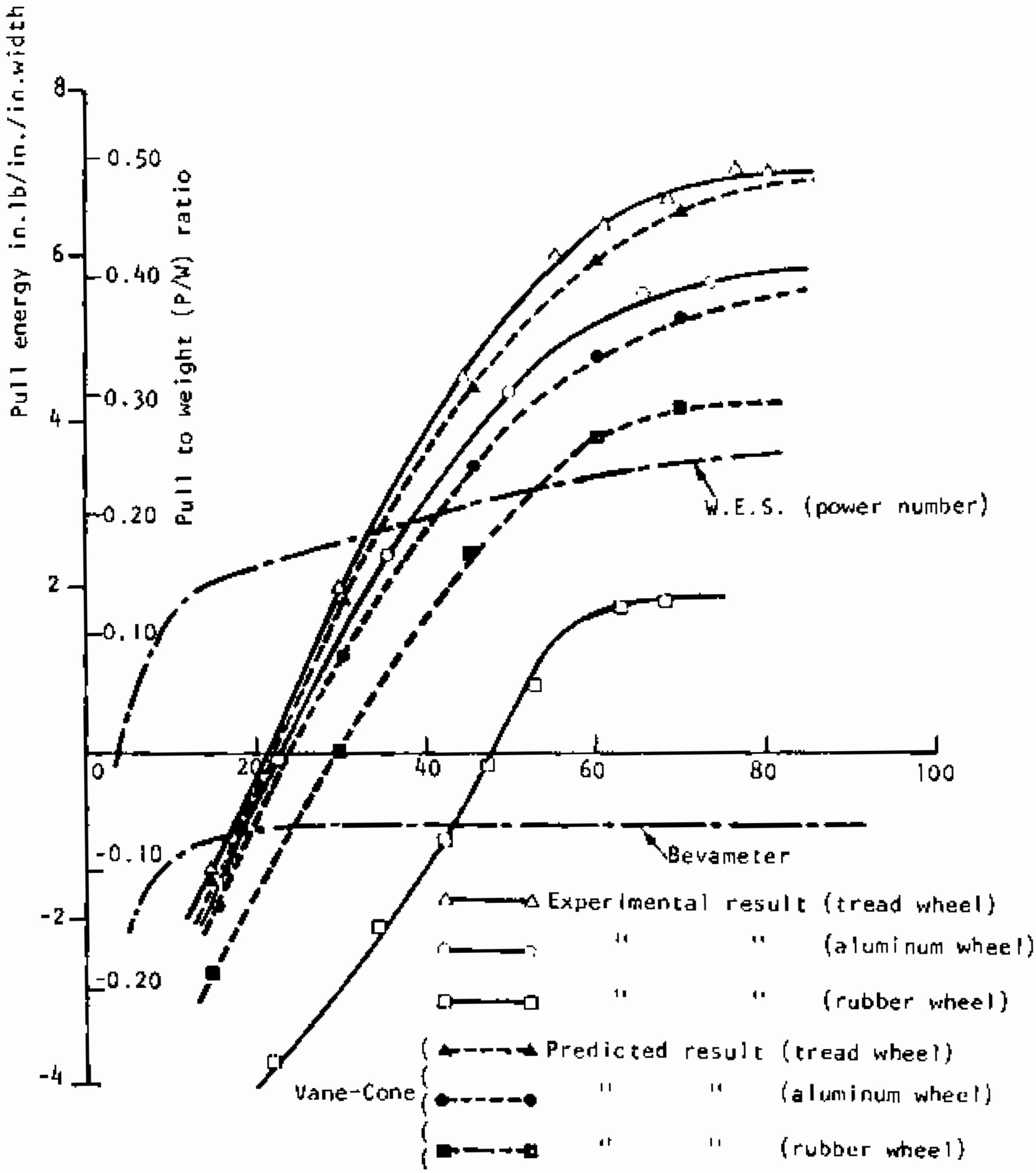

FIG.4-6 Measured and predicted pull energy and pull to weight ratio vs slip rate for 54 lb wheel.

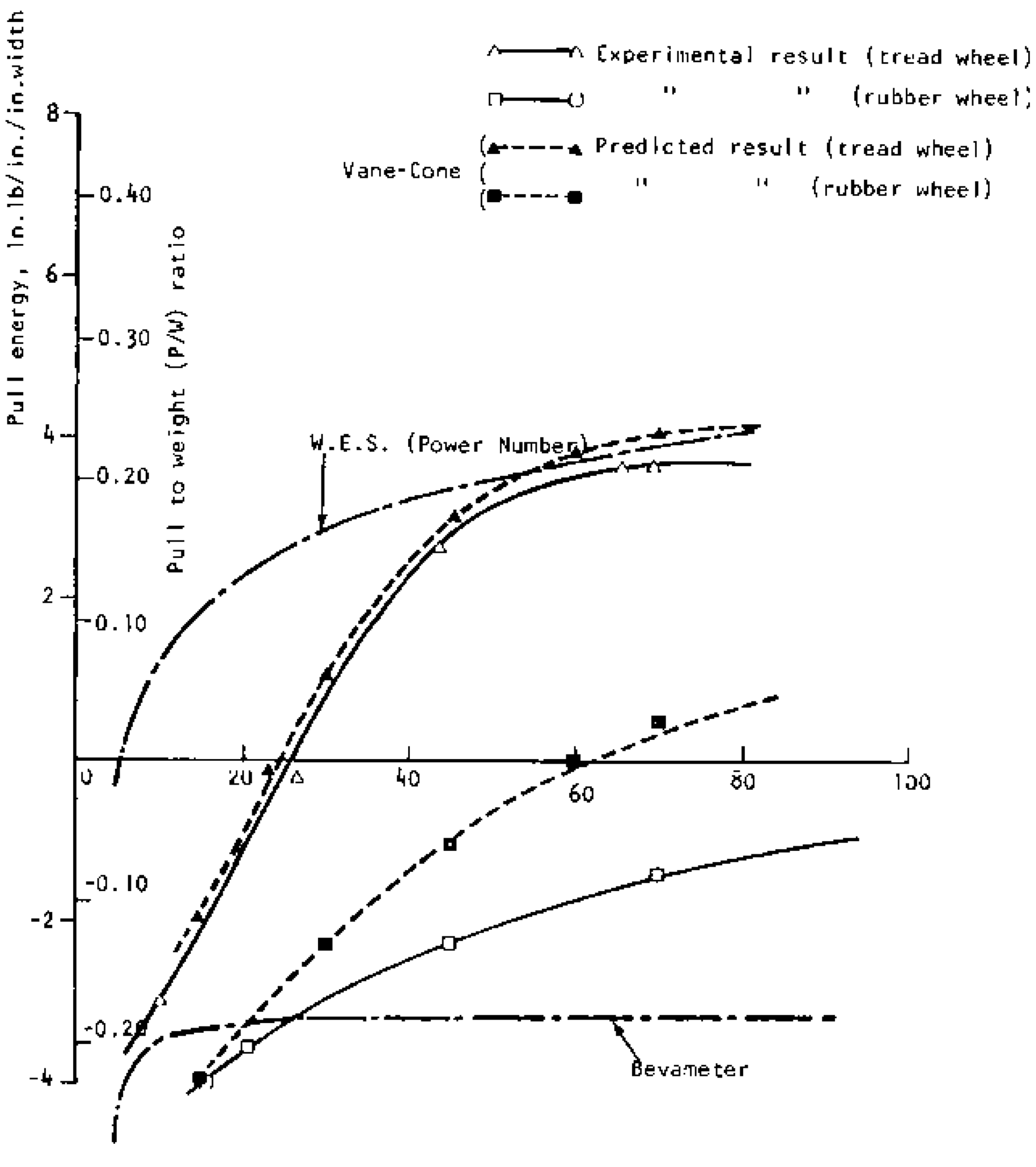

FIG. 4-7 Measured and predicted pull energy and pull to weight ratio vs slip rate for 74 lb wheel.

come the lack of direct shear strength measurements, it is seen that this technique is deficient and cannot adequately predict the performance in the tow bin for the three types of wheels used.

It should be noted that whilst the vane-cone predictions are perhaps the better correlated sets of predictions [with the test data], the "simple" approximate method adopted has indeed restricted the accuracy in correlation. Nevertheless, the results shown in Figs. 4-1 through 4-3 show that if the proper framework for analysis of data is provided, a degree or measure of correlative fit can be obtained.

4.4 RIGOROUS ANALYSIS FOR VANE-CONE MEASUREMENTS

In the report *The Utilization of Vane-Cone Device for Prediction of Mobility*, Report No. SMS-39 by Yong and Youssef, three levels of sophistication in application of the vane-cone measurements have been presented. It will be seen that the order of rigor or sophistication can be expressed as:

(1) Exact method - representing the highest level of sophistication in analysis of data.

(2) Rigorous method - denoting an intermediate level.

(3) Approximate method - representing a simple approach and is the lowest level of sophistication.

Without repeating the Report SMS-39, we show the summary DBP results using the "Rigorous Approach" to provide the reader with an appreciation of the kind of predictive correlation that can be obtained with the vane-cone results.

Figure 4-4 shows the flow chart for the method of data analysis, whilst Figs. 4-5 through 4-7 show the results obtained with the Rigorous Method in comparison to the test measurements.

Whilst it might be stated that there is a certain degree of deja vu with the application of the vane-cone results, it must be stated and emphasized that the design, development and comstruction of the vane-cone specifically addressed the problem of energy transfer mechanics. Thus, not only are the measurements designed to provide the basic information for use, the framework for analysis of data has been directly and specifically developed in terms of the mechanics of energy transfer. It is thus not surprising that the predictive capability with the rigorous method is good.

4.5 CONCLUDING REMARKS

As can be seen from the figures shown, the ability to predict the measured drawbar pull in the tow bin tests, with the kinds of measurements made with the vane-cone device are not too dissimilar from that of the bevameter-type tool, the associated analytical framework for utilisation of the vane-cone results - i.e. utilisation of the data in terms of energetics provides for a method of analysis and prediction which responds more closely to the actual problem at hand - especially if the higher level of sophistication of analysis is used. Thus it is not surprising that the data obtained with the vane-cone device, together with the utilisation of energetics as a method of analysis can indeed provide for a closer predictive correlation with the results obtained in actual tow bin tests.

In the report dealing with the vane-cone device, *The Utilization of Vane-Cone Device for Prediction of Mobility*, by Yong and Youssef, SMS-39*, the more exact methods for application of vane-cone test results given together with the simple method of analysis as described in this present report show the effect of simplification in the analysis and prediction capability.

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* February 1978

APPENDIX A

APPENDIX A

I. MODIFIED BEKKER APPROACH

The same Bekker approach discussed in section 3.4 is used again in this Appendix where these modified equations are obtained - and have been used as the Bekker equations recently [1977] by Thompson (Hovey, Ottawa).

(a) Sinkage

$$Z_r = \left[\frac{3W}{(3-n)\left[\frac{K_c}{\sqrt{DZ_r - Z_r^2}} + K_\phi\right] 2\sqrt{2rZ_r - Z_r^2}\ \sqrt{D}} \right]^{\frac{2}{(2n+1)}} \tag{A-1}$$

(By trial and error substitution of Z_r to obtain equality to the nearest tenth of an inch.)

(b) Rolling Resistance

$$R_c = \frac{b\ K\ (Z)^{(n+1)}}{(n+1)} \tag{A-2}$$

$$K = \frac{K_c}{b} + K_\phi$$

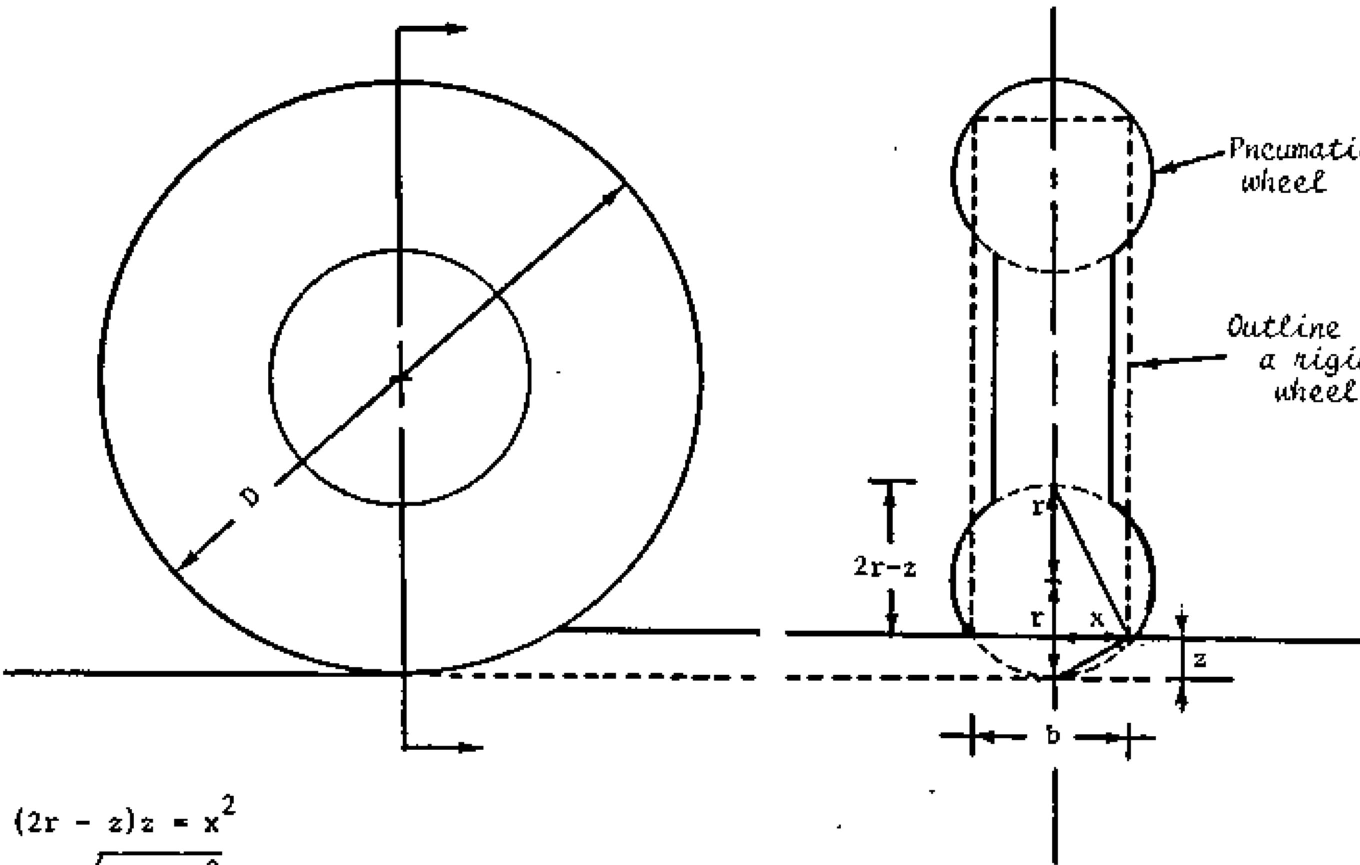
(c) Traction Force

$$H = (b\ L\ C + W \tan \phi)\ (1 - e^{-j/K}) \tag{A-3}$$

(d) Using Eq.(3-3) the drawbar pull can be calculated:

$$DP = H - R \tag{3-3}$$

All the above variables are given before in section 3.4. Note that Eq. (A-1) is from the model shown in Fig. A-1. This is contrasted with the initial simplification given in Eq. (3-6).



$$(2r - z)z = x^2$$

$$x = \sqrt{2rz - z^2}$$

$$b = 2x$$

$$b = 2x = 2\sqrt{2rz - z^2}$$

FIG A-1 Geometrical model for Eq. A-1.

2. COMPARISON BETWEEN THE BEKKER RESULTS

Comparisons between the results obtained using the above equations and the equation given before in section 3.4 are shown in Table A-1 and Fig. A-2. These comparisons demonstrate that there is essentially no difference between the results obtained in using the two types of equations. Thus the comparisons shown in section 4.0 are valid for all cases.

TABLE A-1 Comparison between Bekker (1969) and Modified Bekker

Slip %	Wheel Load 34 lb DP	34 lb DP_{med}	54 lb DP	54 lb DP_{med}	74 lb DP	74 lb DP_{med}
0	-8.64		-17.5		-28.4	
10	1.03	0.68	- 4.76	-4.44	-13.52	-12.23
20	2.22	2.58	-3.72	-2.70	-12.30	-10.73
30	2.37	2.99	-3.53	-2.45	-12.08	-10.58
40	2.37	3.09	-3.53	-2.42	-12.08	-10.57
50	2.37	3.11	-3.53	-2.41	-12.08	-10.57
60	2.37	3.11	-3.53	-2.41	-12.08	-10.57
70	2.37	3.11	-3.53	-2.41	-12.08	-10.57

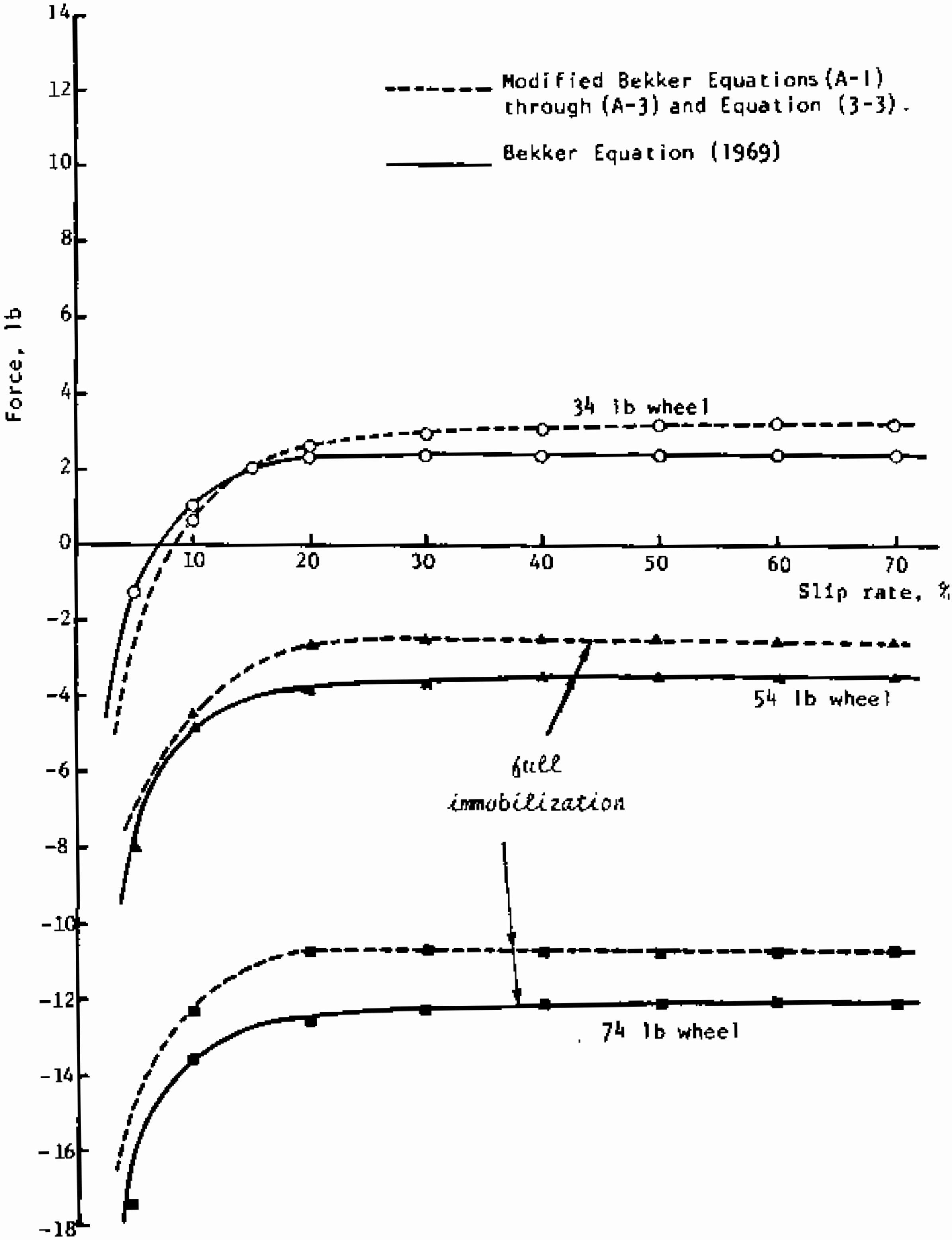


FIG.A-2 Traction and pull curve as developed by the Bevameter approach

3. COMPUTER SOLUTION FOR EQUATIONS A-1 THROUGH A-3 AND EQUATION 3-3

```
      IMPLICIT REAL (A-Z)
      INTEGER   IVEHIC,IJ,IK,IL
      READ(5,20) GAMA,KPHI,KC,PHIDEG,N,C
      DO 2000 IVEHIC=1,3
      READ(5,20) W,NO,B,D
   20 FORMAT(8F10.0)
      WRITE(6,31)
   31 FORMAT('1',15X,'  W   ',14X,'  Z  ',14X,'  RC  ',14X,'  HM  ',14X,
     *'D. P.',14X,'% SLOPE',//)
      PHI=PHIDEG*0.017453
   60 Z=0.1
      DO 63 IJ=1,30
      A=KC/SQRT(D*Z-Z**2.) + KPHI
   61 E=B*SQRT(D)
   62 TEMP=(3.*W)/((3.-N)*A*E)
      TEMP=TEMP**(2./(2.*N+1.))
      IF(ABS(TEMP-Z).LE.0.1) GO TO 65
      Z=TEMP
   63 CONTINUE
      WRITE(6,101)
  101 FORMAT(' ',' Z NOT CONVERGED AFTER 30 ITERATIONS',/)
   65 K=KC/B + KPHI
   69 RC=(NO*B*K*Z**(N+1.))/(N+1.)
      X=SQRT(D*Z-Z*Z)
      L=SQRT(Z*Z + X*X)
      HM=NO*B*L*C + W*TAN(PHI)
      DP=HM-RC
      IF(DP.LE.0.) GO TO 86
      ALPHA=0.017453
      DO 83 IK=1,30
      F=W*SIN(ALPHA)+RC-NO*B*L*C-W*COS(ALPHA)*TAN(PHI)
      FPRIME=W*COS(ALPHA)+W*SIN(ALPHA)*TAN(PHI)
      TEMP=ALPHA-F/FPRIME
      IF(ABS(TEMP-ALPHA).LE.0.017453) GO TO 85
      ALPHA=TEMP
   83 CONTINUE
      WRITE(6,103)
  103 FORMAT(' ',' SLOPE CALCULATION NOT CONVERGED AFTER 30 ITERATIONS
     *',/)
      GO TO 87
   85 IF(TEMP.GT.1.50) GO TO 87
      SLOPE=TAN(ALPHA)*100.
      IF(SLOPE.LT.0.) GO TO 87
      GO TO 88
   86 SLOPE=0.
      GO TO 88
   87 SLOPE=9999999.99
   88 WRITE(6,89) W,Z,RC,HM,DP,SLOPE
   89 FORMAT(' ', 8X,6(F12.2, 8X),////)
      WRITE(6,32)
   32 FORMAT(' ',15X,'  W   ',14X,'  I  ',14X,'  RC  ',14X,'  T  ',14X,
     *'D. P.',//)
      DO 99 IL=1,7
      VARA=FLOAT(IL)*0.1*L
      COMPNT=-VARA/0.26
      T=(B*L*C+W*TAN(PHI))*(1.-EXP(COMPNT))
      DP=T-RC
      WRITE(6,90) W,VARA,RC,T,DP
   90 FORMAT(' ',8X,5(F12.2,8X),/)
   99 CONTINUE
 2000 CONTINUE
      STOP
      END

$DATA
```

4. RESULTS

Wheel Load lb	Slip %	Deformation in.	Rolling Resistance lb	Traction Force lb	Drawbar Pull lb
W	I	J	RC	T	D. P.
34.00	10	0.39	7.97	8.65	0.68
34.00	20	0.79	7.97	10.55	2.58
34.00	30	1.18	7.97	10.96	2.99
34.00	40	1.58	7.97	11.05	3.09
34.00	50	1.97	7.97	11.07	3.11
34.00	60	2.36	7.97	11.08	3.11
34.00	70	2.76	7.97	11.08	3.11

W	I	J	RC	T	D. P.
54.00	10	0.51	16.72	12.29	-4.44
54.00	20	1.02	16.72	14.02	-2.70
54.00	30	1.53	16.72	14.27	-2.45
54.00	40	2.03	16.72	14.30	-2.42
54.00	50	2.54	16.72	14.31	-2.41
54.00	60	3.05	16.72	14.31	-2.41
54.00	70	3.56	16.72	14.31	-2.41

W	I	J	RC	T	D. P.
74.00	10	0.60	27.57	15.34	-12.23
74.00	20	1.21	27.57	16.84	-10.73
74.00	30	1.81	27.57	16.98	-10.58
74.00	40	2.42	27.57	17.00	-10.57
74.00	50	3.02	27.57	17.00	-10.57
74.00	60	3.63	27.57	17.00	-10.57
74.00	70	4.23	27.57	17.00	-10.57

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