

Three essays on credit risk, fixed income and derivatives

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ABSTRACT

This dissertation comprises three essays. In the first essay, we provide results for the valuation of European style contingent claims for a large class of specifications of the underlying asset returns. Our valuation results obtain in a discrete time, an infinite state-space setup using the no-arbitrage principle. Our approach allows for general forms of heteroskedasticity in returns. It also allows for conditional non-normal return innovations, which is critically important because heteroskedasticity alone does not suffice to capture the option smirk. The resulting risk-neutral return dynamics are from the same family of distributions as the physical return dynamics. Our framework nests the valuation results obtained by Duan (1995), and Heston and Nandi (2000) by allowing for a time-varying price of risk and non-normal innovations.

In the second essay, we develop a methodology to study the linkages between equity and corporate bond risk premia and apply it to a large panel of corporate bond transaction data. We find that a significant part of the time variation in bond default risk premia can be explained by equity-implied bond risk premium estimates. We compute these estimates using a recent structural credit risk model. In addition, we show by means of linear regressions that augmenting the set of variables predicted by typical structural models with equity-implied bond default risk premia significantly increases explanatory power. This, in turn, suggests that time-varying risk premia are a desirable feature for future structural models.

In the third essay, we first document empirically that embedded put option values are related to proxies for term structure risk, default risk and illiquidity. In a second step, we develop a valuation model that simultaneously captures default and interest rate risk. We use this model to disentangle the reduction in yield spread enjoyed by puttable bonds that can be attributed to each risk. Perhaps surprisingly, the most important reduction is due to mitigated default or spread risk, followed by term structure risk. The reduction in the non-default component is present but rather small.

RÉSUMÉ

Cette thèse comprend trois essais. Dans le premier essai nous avons développé des résultats pour l'évaluation des actifs contingents de type Européen pour une vaste classe de spécification du rendement de l'actif sous-adjacent. Notre méthode est obtenue dans une économie à temps discret et espace infini en utilisant seulement la condition de non arbitrage dans le marché. Notre approche permet une forme générale d'hétéroskedasticité pour les rendements. Les résultats pour les cas d'homoskedasticité sont retrouvés comme des cas spéciaux. Notre approche permet d'accommoder les cas où l'innovation dans la dynamique du rendement est conditionnellement non normale. Cette flexibilité est extrêmement importante car l'hétéroskedasticité seulement n'est pas suffisant pour capturer le phénomène du "smirk" dans les prix des options. Nos résultats emboîtent ceux obtenue dans Duan (1995) et Heston et Nandi (2000).

Dans le deuxième essai nous avons développé une méthodologie pour étudier le lien entre la prime de risque dans les obligations corporatives et celle de l'actif risqué de la firme. Nous avons appliqué notre méthode sur une large base de données des transactions des obligations corporatives. Nous avons trouvé qu'une importante partie de la variation temporelle du risque de défaut dans ces obligations peut être expliquée par des estimées de la prime de risque du défaut reconstruite à partir de l'actif risqué de la firme seulement. En plus, nous avons démontré à l'aide des régressions linéaires qu'augmentant la série des variables prédites par le modèle structurel par notre estimé de la prime du risque de défaut ajoute une explication significative.

Dans le troisième essai nous avons montré empiriquement que la valeur des obligations corporatives du type "puttable" est reliée aux risques de défaut, de liquidité et celui dû aux taux d'intérêts. Dans la deuxième étape de ce projet nous avons développé un modèle d'évaluation qui capture simultanément ces risques. Nous avons documenté que la plus grande réduction est due à l'assurance que procure ce type d'instrument contre le défaut, suivi par la structure des taux. La réduction due aux autres types de risques dont la liquidité est présente mais négligeable.

DEDICATION

To my parents.

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CONTRIBUTION OF AUTHORS

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The second chapter is a joint collaboration between Redouane Elkamhi and Jan Ericsson, associate professor of finance, Desautels Faculty of Management, McGill University. Redouane Elkamhi performed the data management and analyses that make up the chapter. Redouane Elkamhi and Jan Ericsson developed the main research ideas behind the chapter and discussed revisions of the chapter.

The third chapter is a joint collaboration between Redouane Elkamhi, Jan Ericsson and Hao Wang, assistant professor of Finance at Tsinghua University. Redouane Elkamhi together with Hao Wang performed the data management, estimation and data analysis that comprise the chapter. Redouane Elkamhi, Hao Wang and Jan Ericsson developed the main research ideas and discussed revisions of the chapter.

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Chapter 1

Introduction

The contingent claims literature starts with the seminal work of Black & Scholes (1973) and Merton (1974). Their original work has been extended in several ways later in both the option pricing and the credit risk literatures.

In the option pricing area, most of the literature on contingent claims and most of the applications of the Risk Neutral Valuation Relationship (RNVR) have been cast in continuous time. While the continuous-time approach offers many advantages, the valuation of contingent claims in discrete time is also of substantial interest. For example, when hedging option positions, rebalancing decisions must be made in discrete time. In the case of American and exotic options, early exercise decisions must be made in discrete time as well. Moreover, as only discrete observations are available for empirical study, discrete time models are often more econometrically tractable.

In the discrete-time option valuation, characterizing conditions on preferences are needed to obtain risk-neutral valuation. For example, Brennan (1979) characterizes the bivariate distribution of returns on aggregate wealth and the underlying asset under which a risk-neutral valuation relationship obtains in the homoskedastic case. Duan (1995) extends this framework to the case of heteroskedasticity of the underlying asset return. Amin and Ng (1993) also study the heteroskedastic case. Although they formulate the problem in terms of the economy's stochastic discount factor, they begin by making an assumption on the bivariate distribution of the stochastic discount factor and the underlying return process. There is also a growing literature that values options for discrete-time return dynamics with non-normal innovations. A number of other papers obtain risk-neutral valuation relationships either under the maintained assumption of non-normal innovations, or under the maintained assumption of heteroskedasticity, or both. Madan and Seneta (1990) use the symmetric and i.i.d. variance gamma distribution, Heston (1993b) presents results for the gamma distribution and Heston (2004) analyzes a number of infinitely divisible distributions. Camara (2003) uses a transformed normal innovation and Duan (1999) uses a heteroskedastic model with a transformed normal innovation. Christoffersen, Heston and Jacobs (2006) analyze a heteroskedastic

return process with inverse Gaussian innovations.

In the credit risk literature, tremendous work has been done at extending the underlying assumptions of Merton (1974). Among others, Black & Cox (1976) allow for default before debt maturity. Leland (1994) endogenizes default barrier and derives an analytical solution with perpetual debt. Leland & Toft (1996) extends the Leland model to allow for nonperpetual debt. Anderson & Sundaresan (1996), Mella-Barral & Perraudin (1997) and Fan & Sundaresan (2000) investigate the implications of strategic default. François & Morellec (2004) and Broadie et al (2006) examine the impact of default procedures, and Morellec (2004) gauges the manager-shareholders conflicts through managers' incentives for over-investment.

This thesis spans two closely interrelated areas: credit risk and derivatives modeling. In the first theme, we derive valuation results for contingent claims in a discrete-time infinite state setup. The valuation argument applies to a large class of conditionally normal and non-normal stock returns with flexible time-varying mean and volatility, as well as a potentially time-varying price of risk. Within my second research theme I develop methodologies to disentangle the default risk premium contained in the credit spread, analyze its time series and cross sectional dimensions, and shed light on the degree of integration between the equity and corporate bond markets. Bringing these ideas to the analysis of the puttable bonds constitutes the third essay of this thesis.

In the first essay we focus on providing valuation results for contingent claims in discrete-time infinite state space setup (ex GARCH). The valuation argument applies to a large class of conditionally normal and nonnormal stock returns with flexible time-varying mean and volatility, as well as to a potentially time-varying price of risk. The setup generalizes the result in Duan(1995) to the extent that we do not restrict the returns to be conditionally normal, nor do we restrict the price of risk to be constant. The results apply to some of the most widely used discrete-time processes in finance, such as GARCH processes. We are able to provide more general valuation results than the existing literature. In our opinion, the analysis in Brennan (1979) and Duan (1995)

addresses two important questions simultaneously: First, a mostly technical question that characterizes the risk-neutral dynamic and the valuation of options; second, one of a more economic nature that characterizes the equilibrium underlying the valuation procedure. The existing discrete-time literature for the most part has viewed these two questions as inextricably linked, and has therefore largely limited itself to (log)normal return processes as well as a few special non-normal cases. We argue that it is possible and desirable to treat these questions one at a time, and we provide some general results on the valuation of options under conditionally non-normal asset returns without resorting to equilibrium techniques. The same separation of questions occurs in the literature on option valuation using continuous time stochastic volatility models, such as, for instance, in Heston's (1993a) model. For any assumption on the price of volatility risk in Heston, we can find the risk neutral dynamic and the price of contingent claims. Although the question of which utility function supports this price of risk is an interesting one in its own right, it can be treated separately. We also show how the normal model and the available conditional nonnormal models are special cases of our setup.

To demonstrate the empirical relevance of this approach, we provide an empirical analysis of a heteroskedastic return dynamic with a standardized skewed variance gamma distribution, which is constructed as the mixture of two gamma variables. In the resulting dynamic, conditional skewness and kurtosis are directly governed by two distinct parameters. We estimate the model on return data using quasi-maximum likelihood, and compare its performance with that of the heteroskedastic conditional normal model which is standard in the literature. Diagnostics clearly indicate that the conditionally nonnormal model outperforms the conditionally normal model, and an analysis of the option smirk demonstrates that the former provides substantially more flexibility to value options.

The second essay focuses on the default risk premia in corporate bond markets. Investors in credit markets need a framework to assess whether a given defaultable security is fairly priced. The spread itself may not be an adequate metric to respond to this

question. The investor needs to know if the spread contains (i) acceptable compensation for expected default losses, and (ii) a sufficient risk premium to induce participation. We develop two methodologies capable of disentangling risk premia and expected losses, and we measure default risk premia in a large panel of US corporate bond data spanning a 10-year period. Like previous work, we find that the risk premium is highly time varying. We also document similar time-series behavior to that in Berndt et al (2005), Berndt et al (2006), and Saita(2006). We find that the expected loss and default premium components behave differently over time. The risk premium is at its most important for high-grade debt, whereas the expected loss component increases monotonically with the default probability. We show that the time-series variation of the risk premium is closely related to the overall market volatility, whereas the expected loss component appears more closely related to the average total volatility across firms. This result is contrasted with the finding in Campbell et al (2002). The risk premia which we measured are translations of risk premia as measured in equity markets. As such, they do not capture risk premia that may be specific to fixed income markets. Given the burgeoning debate over the choice of risk-free curve, we test the sensitivities of our results with respect to different proxies for the risk-free rate. Also, in an attempt to understand whether time-varying risk premia may partly account for the documented failure of structural models to explain credit spreads behavior, we carry out a regression analysis on credit spreads in line with recent studies by, among others, Collin-Duffresne et al (2001), Campbell & Taksler (2002), and Cremers et al (2004). We take as a benchmark a regression motivated by the key drivers implied by structural models. Our results suggest that augmenting the regressions by our measures of equity-implied risk premia improves explanatory power for both levels, and changes in bond credit spreads considerably.

In the third essay, we investigate the puttable bonds. We shed light on which risks are insured against by embedded puts, and to what extent. The most important drivers of corporate bond prices are likely to be interest rate risk, default risk, and illiquidity. Intuitively, the option to return the bond to the issuers would provide insurance against

all three. Using a sample of puttable bond and comparable regular bond transactions, we find that the put option feature does significantly hedge against all three risks. The reduction in corporate bond yield spread due to the presence of a put represents, on average, just over 40% of the yield spread. By means of regression analysis, we show that the put option value (as measured by the spread reduction) is related to proxies for default, interest rate, and marketability risk. To further understand the composition of the put option feature, we develop a bivariate lattice model that simultaneously captures correlated credit and term structure risks. The model is then applied to price regular and puttable bonds to decompose the risk components contained in the put options. We find that the dominant source of spread reduction is attributable to default risk - an average of 60% of the reduction. But, we find that when default is imminent and the firm may not be able to honor the option, the put option value is significantly reduced. Perhaps surprisingly, only a small fraction (7%) of the spread reduction by put option is due to other nondefault factors including illiquidity. Put options are less valuable for bonds issued by larger firms which enjoy better marketability. The values of put options increase as market liquidity drops. Finally, we find that put options are more likely to be exercised when interest rates are high, which, in turn, increases their values.

Chapter 2

Option Valuation with Conditional Heteroskedasticity and Non-Normality

2.1 Introduction

A contingent claim is a security whose payoff depends upon the value of another underlying security. A valuation relationship is an expression that relates the value of the contingent claim to the value of the underlying security and other variables. The most popular approach for valuing contingent claims is the use of a Risk Neutral Valuation Relationship (RNVR).

Most of the literature on contingent claims and most of the applications of the RNVR have been cast in continuous time. While the continuous-time approach offers many advantages, the valuation of contingent claims in discrete-time is also of substantial interest. For example, when hedging option positions, rebalancing decisions must be made in discrete time. In the case of American and exotic options, early exercise decisions must be made in discrete-time as well. Moreover, as only discrete observations are available for empirical study, discrete-time models are often more econometrically tractable.

As a result, most of the stylized facts characterizing the underlying securities have been studied in discrete time models. One very important feature of returns is conditional heteroskedasticity, which can be addressed in the GARCH framework of Engle (1982) and Bollerslev (1986).¹ Presumably, because of this evidence, most of the recent empirical work on discrete time option valuation has also focused on GARCH processes.² The GARCH model amounts to an infinite state space setup, with the innovations for underlying asset returns described by continuous distributions. In this case, the market is incomplete, and it is generally not possible to construct a portfolio containing the contingent claim and the underlying asset in some proportions so that the resulting portfolio becomes riskless.³

¹See, for example, French, Schwert and Stambaugh (1987) and Schwert (1989) for early studies on stock returns. The literature is far too voluminous to cite all relevant papers here. See Bollerslev, Chou and Kroner (1992) and Diebold and Lopez (1995) for reviews on GARCH modeling.

²See Bollerslev and Mikkelsen (1996), Satchell and Timmermann (1996), Garcia and Renault (1998), Heston and Nandi (2000), Christoffersen and Jacobs (2004), and Christoffersen, Heston and Jacobs (2006) for applications to option valuation.

³In a discrete-time finite state space setting, Harrison and Pliska (1981) provide the mathematical

To obtain a RNVR, the GARCH option valuation literature builds on the approach of Rubinstein (1976) and Brennan (1979), who demonstrate how to obtain RNVRs for lognormal and normal returns in the case of constant mean return and volatility, by specifying a representative agent economy.⁴ The resulting first-order condition yields an Euler equation that can be used to price any asset. For a given dynamic of the underlying security, specific assumptions have to be made on preferences in order to obtain a risk-neutralization result. For lognormal stock returns and a conditionally heteroskedastic (GARCH) volatility dynamic, the standard result is the one in Duan (1995). Duan's result relies on the existence of a representative agent with constant relative risk aversion or constant absolute risk aversion.

However, because it is difficult to characterize the general equilibrium setup underlying a RNVR, very few valuation results are currently available for heteroskedastic processes with non-normal innovations.⁵ In this paper, we argue that it is possible to investigate option valuation for a large class of conditionally non-normal heteroskedastic processes, provided that the conditional moment generating function exists. It is also possible to accommodate a large class of time-varying risk premia. Our framework differs from the approach in Brennan (1979) and Duan (1995), and is more intimately related to the approach adopted in continuous-time option valuation: we only use the no-arbitrage assumption and some technical conditions on the investment strategies to show the existence of an RNVR. We demonstrate the existence of an EMM and characterize it, without first making an explicit assumption on the utility function of a representative agent. We then show that the price of the contingent claim defined as the expected value of the discounted payoff at maturity is a no-arbitrage price and characterize the risk-neutral

framework to obtain the existence of the risk neutral probability measure, to demonstrate uniqueness in the case of complete markets, and to get a RNVR for any contingent claim.

⁴Camara (2003) uses this approach to obtain valuation results for transformed normal dynamics of returns and state variables. See also Schroder (2004). Perrakis (1986), Perrakis and Ryan (1984) and Ritchken and Kuo (1988) provide alternative discrete time approaches.

⁵Duan, Ritchken and Sun (2005) analyze a heteroskedastic model with Poisson-normal innovations and Duan (1999) analyzes a conditionally fat-tailed heteroskedastic model. Christoffersen, Heston and Jacobs (2006) use a heteroskedastic return dynamic with inverse Gaussian innovations.

dynamic.

Why are we able to provide more general valuation results than the existing literature. In our opinion, the analysis in Brennan (1979) and Duan (1995) addresses two important questions simultaneously: First, a mostly technical question that characterizes the risk-neutral dynamic and the valuation of options; second, a more economic one that characterizes the equilibrium underlying the valuation procedure. The existing discrete-time literature for the most part has viewed these two questions as inextricably linked, and has therefore largely limited itself to (log)normal return processes as well as a few special non-normal cases. We argue that it is possible and desirable to treat these questions one at a time, and we provide some general results on the valuation of options under conditionally non-normal asset returns without resorting to equilibrium techniques. We also show how the normal model and the available conditional non-normal models are special cases of our setup.

The same separation of questions occurs in the literature on option valuation using continuous-time stochastic volatility models, such as, for instance, in Heston's (1993a) model.⁶ For any assumption on the price of volatility risk in Heston, we can find the risk-neutral dynamic and the price of contingent claims. The question of which utility function supports this price of risk is an interesting one in its own right, but it can be treated separately. See, for instance, Heston (1993a) and Bates (1996, 2000) for a discussion.

The paper proceeds as follows. In Section 2, we define the class of conditional stock return processes we can accommodate, and derive an appropriate class of EMMs which in turn is used to derive a no-arbitrage option price. Section 3 characterizes the risk-neutral dynamics, and Section 4 discusses several return dynamics that can be analyzed using our approach. In Section 5, an empirical illustration demonstrates the importance of

⁶Interestingly, GARCH models can sometimes be viewed as discrete-time approximations to underlying continuous time diffusions. See, for example, Nelson and Foster (1994), Foster and Nelson (1996), Nelson (1996) and Ritchken and Trevor (1999). Corradi (2000) points out that such limiting results must be interpreted with caution.

allowing for volatility dynamics as well as conditional non-normality in option valuation models. Section 6 discusses related research, and Section 7 concludes.

2.2 Theoretical results

We define the probability space (Ω, \mathbb{F}, P) to describe the physical distribution of the states of nature. The financial market consists of a zero-coupon, risk-free bond index and a stock. The dynamics of the bond are described by the process $\{B_t\}_{t=1}^T$ normalized to $B_0 = 1$ and the dynamics of the stock price by $\{S_t\}_{t=1}^T$. The information structure is given by the filtration $\mathbb{F} = \{F_t | t = 1, \dots, T\}$ generated by the stock and the bond process.

2.2.1 The stock price process

The underlying stock price process is assumed to follow the conditional distribution D under the physical measure P . We write

$$R_t \equiv \ln \left(\frac{S_t}{S_{t-1}} \right) = \mu_t - \gamma_t + \varepsilon_t \quad \varepsilon_t | F_{t-1} \sim D(0, \sigma_t^2), \quad (2.1)$$

where S_t is the stock price at time t , and σ_t^2 is the conditional variance of the log return in period t . The mean correction factor, γ_t , is defined from

$$\exp(\gamma_t) \equiv E_{t-1} [\exp(\varepsilon_t)]$$

and it serves to ensure that the conditional expected gross rate of return, $E_{t-1} [S_t/S_{t-1}]$, is equal to $\exp(\mu_t)$. More explicitly,

$$\begin{aligned} E_{t-1} [S_t/S_{t-1}] &= E_{t-1} [\exp(\mu_t - \gamma_t + \varepsilon_t)] = \exp(\mu_t) \\ \iff \exp(\gamma_t) &= E_{t-1} [\exp(\varepsilon_t)] \end{aligned}$$

Note that our specification (2.1) does not restrict the risk premium in any way nor

does it assume conditional normality.

We follow most of the existing discrete-time empirical finance literature by focusing on conditional means μ_t and conditional variances σ_t^2 that are F_{t-1} measurable. We do not constrain the interest rate r_t to be constant. It is instead assumed to be an element of F_{t-1} as well. Our framework is able to accommodate the class of ARCH and GARCH processes proposed by Engle (1982) and Bollerslev (1986) and used for option valuation by Amin and Ng (1993), Duan (1995, 1999), and Heston and Nandi (2000).⁷

In the following, we show that we can find an EMM by defining a probability measure that makes the discounted security process a martingale. We derive more general results on option valuation for heteroskedastic processes compared to the available literature, because we focus on the narrow question of option valuation while ignoring the economic question regarding the preferences of the representative agent that support this valuation argument in equilibrium.

We use a no-arbitrage argument that is similar to the one used in the continuous-time literature. We first prove the existence of an EMM. Subsequently, we demonstrate the existence of a RNVR by demonstrating that the price of the contingent claim, defined as the expected value of the discounted payoff at maturity, is a no-arbitrage price under this EMM.⁸ The proof uses an argument similar to the one used in the continuous-time literature, but is arguably more straightforward as it avoids the technical issues involved in the analysis of local and super martingales.

⁷Our results will also hold for different types of GARCH specifications, such as the EGARCH model of Nelson (1991) or the specification of Glosten, Jagannathan and Runkle (1993). For our results to apply, all that matters is that the volatility process is predetermined. While our framework can accommodate a wide range of interesting processes, it must be noted that it is not able to handle potentially interesting processes such as discrete-time stochastic volatility models. See Ghysels, Harvey and Renault (1995) for a review of these models.

⁸Duan (1995) refers to RNVR as Local RNVR in the case of GARCH. The reason for the distinction is that the conditional volatility is identical under the two measures only one period ahead. In the remainder of the paper we will drop this distinction for ease of exposition. We emphasize that the result that the conditional volatility differs between the two measures for more than one period ahead is to be expected as volatility is random in this case. This feature is very similar to the continuous time case, which has random volatility for any horizon.

2.2.2 Specifying an equivalent martingale measure

The objective in this section is to find a measure equivalent to the physical measure P that makes the price of the stock discounted by the riskless asset a martingale. An EMM is defined as long as the Radon-Nikodym derivative is defined. We start by specifying a candidate Radon-Nikodym derivative of a probability measure. We then show that this Radon-Nikodym derivative defines an EMM that makes the discounted stock price process a martingale. This result in turn allows us to obtain the distribution of the stock return under this EMM.

For a given sequence of a random variable, ν_t , we define the following candidate Radon-Nikodym derivative

$$\frac{dQ}{dP} \Big| F_t = \exp \left(- \sum_{i=1}^t (\nu_i \varepsilon_i + \Psi_i(\nu_i)) \right) \quad (2.2)$$

where $\Psi_t(u)$ is defined as the natural logarithm of the moment generating function

$$E_{t-1} [\exp(-u\varepsilon_t)] \equiv \exp(\Psi_t(u))$$

Note that we can think of the mean correction factor in (2.1) as $\gamma_t = \Psi_t(-1)$. Note also that in the normal case we have $\Psi_t(u) = \frac{1}{2}\sigma_t^2 u^2$.

We can now show the following lemma

Lemma 1 $\frac{dQ}{dP} \Big| F_t$ is a Radon-Nikodym derivative

Proof. We need to show that $\frac{dQ}{dP} \Big| F_t > 0$ which is immediate. We also need to show that $E_0^P \left[\frac{dQ}{dP} \Big| F_t \right] = 1$. We have

$$E_0^P \left[\frac{dQ}{dP} \Big| F_t \right] = E_0^P \left[\exp \left(- \sum_{i=1}^t (\nu_i \varepsilon_i + \Psi_i(\nu_i)) \right) \right].$$

Using the law of iterative expectations we can write

$$\begin{aligned}
E_0^P \left[\frac{dQ}{dP} \middle| F_t \right] &= E_0^P \left[E_1^P \dots E_{t-1}^P \exp \left(- \sum_{i=1}^t (\nu_i \varepsilon_i + \Psi_i(\nu_i)) \right) \right] \\
&= E_0^P \left[E_1^P \dots E_{t-2}^P \exp \left(- \sum_{i=1}^{t-1} \nu_i \varepsilon_i - \sum_{i=1}^t \Psi_i(\nu_i) \right) E_{t-1}^P \exp(-\nu_t \varepsilon_t) \right] \\
&= E_0^P \left[E_1^P \dots E_{t-2}^P \exp \left(- \sum_{i=1}^{t-1} \nu_i \varepsilon_{i+1} - \sum_{i=1}^t \Psi_i(\nu_i) \right) \exp(\Psi_t(\nu_t)) \right] \\
&= E_0^P \left[E_1^P \dots E_{t-2}^P \exp \left(- \sum_{i=1}^{t-1} \nu_i \varepsilon_i - \sum_{i=1}^{t-1} \Psi_i(\nu_i) \right) \right]
\end{aligned}$$

Iteratively, using this result we get

$$\begin{aligned}
E_0^P \left[\frac{dQ}{dP} \middle| F_t \right] &= E_0^P [\exp(-\nu_1 \varepsilon_1 - \Psi_1(\nu_1))] \\
&= \exp(-\Psi_1(\nu_1)) \exp(\Psi_1(\nu_1)) \\
&= 1
\end{aligned}$$

and the lemma obtains. ■

We are now ready to show that we can specify an EMM using this Radon-Nikodym derivative.

Proposition 2 *The probability measure Q defined by the Radon-Nikodym derivative in (2.2) is an EMM if and only if*

$$\Psi_t(\nu_t - 1) - \Psi_t(\nu_t) - \gamma_t + \alpha_t \sigma_t^2 = 0$$

where $\alpha_t = \frac{\mu_t - r_t}{\sigma_t^2}$.

Proof. We need $E^Q \left[\frac{S_t}{B_t} \middle| F_{t-1} \right] = \frac{S_{t-1}}{B_{t-1}}$ or equivalently $E^Q \left[\frac{S_t}{S_{t-1}} / \frac{B_t}{B_{t-1}} \middle| F_{t-1} \right] = 1$.

We have

$$\begin{aligned}
E^Q \left[\frac{S_t}{S_{t-1}} / \frac{B_t}{B_{t-1}} \middle| F_{t-1} \right] &= E^P \left[\left(\frac{\frac{dQ}{dP} \middle| F_t}{\frac{dQ}{dP} \middle| F_{t-1}} \right) \frac{S_t}{S_{t-1}} / \frac{B_t}{B_{t-1}} \middle| F_{t-1} \right] \\
&= E^P \left[\left(\frac{\frac{dQ}{dP} \middle| F_t}{\frac{dQ}{dP} \middle| F_{t-1}} \right) \frac{S_t}{S_{t-1}} \exp(-r_t) \middle| F_{t-1} \right] \\
&= E^P [\exp(-\nu_t \varepsilon_t - \Psi_t(\nu_t)) \exp(\mu_t - \gamma_t + \varepsilon_t) \exp(-r_t) \middle| F_{t-1}] \\
&= \exp(-\Psi_t(\nu_t) + \mu_t - r_t - \gamma_t) E^P [\exp((1 - \nu_t) \varepsilon_t) \middle| F_{t-1}] \\
&= \exp(-\Psi_t(\nu_t) + \mu_t - r_t - \gamma_t + \Psi_t(\nu_t - 1)) \\
&= \exp(\Psi_t(\nu_t - 1) - \Psi_t(\nu_t) - \Psi_t(-1) + \alpha_t \sigma_t^2)
\end{aligned}$$

Thus Q is a probability measure that makes the stock discounted by a riskless asset a martingale if and only if

$$\Psi_t(\nu_t - 1) - \Psi_t(\nu_t) - \Psi_t(-1) + \alpha_t \sigma_t^2 = 0 \quad (2.3)$$

This result implies that we can construct an EMM by choosing the sequence, ν_t , to make (2.3) hold. ■

2.2.3 The valuation of European style contingent claims

We have demonstrated that in a general return model with time-varying conditional mean and volatility and non-normal shocks, there exists an EMM Q that makes the stock discounted by the riskless asset a martingale.

We now turn our attention to the pricing of European style contingent claims. Existing papers on the pricing of contingent claims in a discrete-time infinite state space setup, such as the literature on GARCH option pricing in Duan (1995), Amin and Ng (1993) and Heston and Nandi (2000) value such contingent claims by making an assumption on the bivariate distribution of the stock return and the endowment, or an equivalent assumption. While this approach, which most often amounts to the characterization of

the equilibrium that supports the pricing, is an elegant way to deal with the incompleteness that characterizes these markets, we argue that it is not strictly necessary to characterize the equilibrium. Instead, we adopt an approach which is more prevalent in the continuous-time literature, and proceed to pricing derivatives using a no-arbitrage argument alone.

To understand our approach, the analogy with option valuation for the stochastic volatility model of Heston (1993a) is particularly helpful. In this incomplete markets setting, an infinity of no-arbitrage contingent claims prices exist, one for every different specification of the price of risk. When one fixes the price of volatility risk, however, there is a unique no-arbitrage price. For the purpose of option valuation, one can simply pick a price of volatility risk, and the resulting valuation exercise is purely mechanical.

The question whether a particular price of risk is reasonable is of substantial interest in its own right, and an analysis of the representative agent utility function that support a particular price of risk is very valuable. However, this question can be analyzed separately from the option valuation problem. For the heteroskedastic discrete-time models we consider, a similar remark applies. We can value options provided we specify the price of risk. The link between our approach and the utility-based approach in Brennan (1979), Rubinstein (1976) and Duan (1995) is that assumptions on the utility function are implicit in the specification of the risk premium in the return dynamic in our case.⁹ The representative agent preferences underlying this assumption are of interest, but it is not necessary to analyze them in order to value options. Of course, we note that the main difference with the continuous-time stochastic volatility models is that GARCH models are one-shock models, and that therefore there is only one price of risk.

We have already found an EMM Q . We therefore want to demonstrate that the price at time t is defined as

$$C_t = E^Q \left[\frac{C_T(S_T)}{B_T} B_t \middle| F_t \right].$$

⁹See Bick (1990) and He and Leland (1993) for a discussion of assumptions on the utility function implicit in the specification of the return dynamic for the market portfolio. We proceed along the lines of Jacob and Shiryaev (1998), and Shiryaev (1999).

The proof proceeds in a number of steps and requires defining a number of concepts that are well-known in the literature. Fortunately, even though our methodology closely follows the continuous-time case, we economize on the number of technical conditions in the continuous-time setup, such as admissibility, and avoid the concepts of local martingale and super martingale. The reason is that the integration over an infinite number of trading times in the continuous-time case is replaced by a finite sum over the trading days in discrete time.

Definitions

1. We denote by η_t , δ_t and ψ_t the units of the stock, the contingent claim and the bond held at date t . We refer to the F_t predictable processes η_t , δ_t and ψ_t as investment strategies.
2. The value process

$$V_t = \eta_t S_t + \delta_t C_t + \psi_t B_t$$

describes the total dollar amount available for investments at date t .

3. The gain process

$$G_t = \sum_{i=0}^{t-1} \eta_i (S_{i+1} - S_i) + \sum_{i=0}^{t-1} \delta_i (C_{i+1} - C_i) + \sum_{i=0}^{t-1} \psi_i (B_{i+1} - B_i).$$

captures the total financial gains between dates 0 and t .

4. We call the process $\{\eta_t, \delta_t, \psi_t\}_{t=0}^{T-1}$ a self financing strategy if and only if $V_t = G_t$ $\forall t = 1, \dots, T$.
5. The definition of an arbitrage opportunity is standard: we have an arbitrage opportunity if a self financing strategy exists with either $V_0 < 0$, $V_T \geq 0$ a.s. or $V_0 \leq 0$, $V_T > 0$ a.s.

6. We denote the discounted stock price at time t as $S_t^B = \frac{S_t}{B_t}$ and the discounted contingent claim as $C_t^B = \frac{C_t}{B_t}$. Similarly, the discounted value process is denoted $V_t^B = \frac{V_t}{B_t}$ and the discounted gain process $G_t^B = \frac{G_t}{B_t}$.

Note that for a self financing strategy, we have $V_t^B = G_t^B$ because $V_t = G_t$ and $B_t > 0$. Furthermore, we can show the following.

Lemma 3 *For a self financing strategy we have*

$$G_t^B = \sum_{i=0}^{t-1} \eta_i (S_{i+1}^B - S_i^B) + \sum_{i=0}^{t-1} \delta_i (C_{i+1}^B - C_i^B) \quad \forall t = 1, \dots, T$$

Proof. The proof involves straightforward but somewhat cumbersome algebraic manipulations of the above definitions. See the Appendix for the details. ■

We know that under the EMM we defined, the stock discounted by the risk free asset is a martingale. We now need to show that the contingent claims prices obtained by computing the expected value of the final payoff discounted by the risk free asset also constitute a martingale under this EMM.

Lemma 4 *The stochastic process defined by the discounted values of the candidate contingent claims prices is an F_t martingale under the EMM.*

Proof. We defined our candidate process for the contingent claims price under the EMM as $C_t = E^Q \left[\frac{C_T(S_T)}{B_T} B_t \middle| F_t \right]$. The process for the discounted values of the contingent claims prices is then defined as

$$C_t^B \equiv \frac{C_t}{B_t} = E^Q \left[\frac{C_T(S_T)}{B_T} \middle| F_t \right]$$

We use the fact that the conditional expectation itself is a Q martingale. This in turn follows from the law of iterated expectations and the European style payoff function.

Taking conditional expectations with respect to F_s on both sides of the above equation yields

$$E^Q \left[\frac{C_t}{B_t} \middle| F_s \right] = E^Q \left[E^Q \left[\frac{C_T(S_T)}{B_T} \middle| F_t \right] \middle| F_s \right] \quad \forall t > s$$

Now using the law of iterated expectations we get

$$E^Q \left[\frac{C_t}{B_t} \middle| F_s \right] = E^Q \left[\frac{C_T(S_T)}{B_T} \middle| F_s \right] = \frac{C_s}{B_s} = C_s^B \quad \forall t > s$$

which gives the desired result. ■

Lemma 5 *Under the EMM defined by (2.2), the discounted gain process is a martingale.*

Proof. Under the EMM Q , the process $\{S_t^B\}_{t=1}^T$ is a Q martingale. Using a standard property of martingales the process defined as $SS_t^B = \sum_{i=0}^{t-1} \eta_i (S_{i+1}^B - S_i^B)$ then is a Q martingale, since the investment strategy η_t is included in the information set.¹⁰ Furthermore, from Lemma 3 we get that $\{C_t^B\}_{t=1}^T$ is also a Q martingale. Then using the fact that δ_t is an F_t predetermined process and using the same martingale property as above we get that the process $CC_t^B = \sum_{i=0}^{t-1} \delta_i (C_{i+1}^B - C_i^B)$ is a Q martingale. Then since from Lemma 2 the discounted gain process $\{G_t^B\}_{t=1}^T$ is the sum of two Q martingales, SS_t^B and CC_t^B , it is itself a Q martingale. ■

At this stage, we have all the ingredients to show the following main result.

Proposition 6 *If we have an EMM that makes the discounted price of the stock a martingale, then defining the price of any contingent claim as the expected value of its payoff, taken under this EMM and discounted at the riskless interest rate constitutes a no-arbitrage price.*

Proof. From Lemma 4 G_t^B is a Q martingale. Because we are considering self financing strategies we get that V_t^B is a martingale. We prove the absence of arbitrage

¹⁰Note that because we are working in discrete time there is no need to investigate the integrability of SS_t^B .

by contradiction. If we assume the existence of an arbitrage opportunity, then there exists a self financing strategy with type 1 arbitrage ($V_0 < 0, V_T \geq 0$ a.s.) or type 2 arbitrage ($V_0 \leq 0, V_T > 0$ a.s.). Both cases lead to a clear contradiction. Consider type 1 arbitrage: we start from the existence of a self financing strategy with $V_0 < 0$ that ends up with a positive final value. $V_0 < 0$ implies that $V_0^B < 0$ since the numeraire is always positive by definition. Also since $V_T \geq 0$ we have $V_T^B \geq 0$. Taking expectations and using the fact that V_t^B is a Q martingale yields $V_0^B = E_0^Q[V_T^B] \geq 0$. This is a contradiction because we assumed that we start with a negative value $V_0 < 0$. A similar argument works for type 2 arbitrage. Thus, the C_t from the EMM Q must be a no-arbitrage price.

■

In summary, we have demonstrated that in a discrete-time infinite state space setting, if we have an EMM that makes the underlying asset price a martingale, then the expected value of the payoff of the contingent claim taken under this EMM, discounted at the riskless asset, is a no-arbitrage price. In Section 2.2, we derived such an EMM. Altogether, we have therefore demonstrated that for any contingent claim paying a final payoff $C_T(S_T)$ the current price C_t can be computed as

$$C_t = E^Q \left[\frac{C_T(S_T)}{B_T} B_t \middle| F_t \right].$$

2.3 Characterizing the risk-neutral distribution

The previous section demonstrates how options can be priced using the EMM directly. However, when pricing options using Monte Carlo simulation, knowing the risk neutral distribution is valuable. In this section, we derive an important result that shows that for the class of models we investigate, the risk neutral distribution is from the same family as the original physical distribution.

We first need the following lemma, where we recall that $\Psi_t(u)$, denotes the one-day log conditional moment generating function of the innovation ε_t

Lemma 7

$$E_{t-1}^Q [\exp(-u\varepsilon_t)] = \exp(\Psi_t(\nu_t + u) - \Psi_t(\nu_t))$$

Proof.

$$\begin{aligned} E_{t-1}^Q [\exp(-u\varepsilon_t)] &= E^P \left[\left(\frac{\frac{dQ}{dP}|F_t}{\frac{dQ}{dP}|F_{t-1}} \right) \exp(-u\varepsilon_t) | F_{t-1} \right] \\ &= E^P [\exp(-\nu_t \varepsilon_t - \Psi_t(\nu_t)) \exp(-u\varepsilon_t) | F_{t-1}] \\ &= \exp(\Psi_t(\nu_t + u) - \Psi_t(\nu_t)) \end{aligned}$$

■

From this lemma, if we define $\Psi_t^Q(u)$ to be the log conditional moment generating function of ε_t under the risk neutral probability measure, then we have

$$\Psi_t^Q(u) = \Psi_t(\nu_t + u) - \Psi_t(\nu_t)$$

From this we can derive

$$E_{t-1}^Q [\varepsilon_t] = \left. \frac{\partial \exp(\Psi_t^Q(-u))}{\partial u} \right|_{u=0} = -\Psi_t'(\nu_t)$$

which represents the risk premium. Define the risk neutral innovation

$$\varepsilon_t^* = \varepsilon_t - E_{t-1}^Q [\varepsilon_t]$$

The risk-neutral log-conditional moment generating function of ε_t^* , labeled $\Psi_{t-1}^{Q*}(u)$, is then

$$\Psi_t^{Q*}(u) = -u\Psi_t'(\nu_t) + \Psi_t^Q(u) \tag{2.4}$$

We are now ready to show the following

Proposition 8 *If the physical conditional distribution of ε_t is an infinitely divisible distribution with finite second moment, then the risk-neutral conditional distribution of ε_t^**

is also an infinitely divisible distribution with finite second moment.

Proof. See the appendix. ■

Because of the one-to-one mapping between moment generating functions and distribution functions, this result can be used to derive specific parametric risk-neutral distributions corresponding to the parametric physical distributions assumed by the researcher.

2.4 Parametric examples

In this section we demonstrate how a number of existing models are nested in our setup. We also give an example of a model that has not yet been discussed in the literature but can be handled by our setup.

2.4.1 Conditionally normal returns

In the conditional normal case we have the return dynamics

$$R_t = \mu_t - \gamma_t + \varepsilon_t \quad \varepsilon_t | F_{t-1} \sim N(0, \sigma_t^2)$$

where the conditional variance, σ_t^2 , can take on any GARCH-type specification.

The normal log MGF is $\Psi_t(u) = \frac{1}{2}\sigma_t^2 u^2$ so that $\gamma_t = \Psi_t(-1) = \frac{1}{2}\sigma_t^2$ and our EMM condition

$$\Psi_t(\nu_t - 1) - \Psi_t(\nu_t) - \Psi_t(-1) + \alpha_t \sigma_t^2 = 0$$

from (2.3) is solved by choosing

$$\nu_t = \alpha_t = \frac{\mu_t - r_t}{\sigma_t^2}$$

In this normal case, the probability measure Q defined by the Radon-Nikodym derivative

$$\frac{dQ}{dP} \Big|_{F_t} = \exp \left(- \sum_{i=1}^t (\nu_i \varepsilon_i + \Psi_i(\nu_i)) \right) = \exp \left(- \sum_{i=1}^t \left(\alpha_i \varepsilon_i + \frac{1}{2} \alpha_i^2 \sigma_i^2 \right) \right)$$

is therefore an EMM.

From Section 3 we have the risk neutral conditional log MGF in the general case

$$\Psi_t^{Q*}(u) = -u \Psi'_t(\nu_t) + \Psi_t(\nu_t + u) - \Psi_t(\nu_t)$$

Using $\Psi_t(u) = \frac{1}{2} \sigma_t^2 u^2$, we get

$$\Psi_t^{Q*}(u) = \frac{1}{2} \sigma_t^2 u^2.$$

so that in the normal case the risk neutral distribution is also normal. The results in Section 3 also imply that the risk neutral innovation generally can be written

$$\varepsilon_t^* = \varepsilon_t + \Psi'_t(\nu_t)$$

so that in the normal case we have

$$\varepsilon_t^* = \varepsilon_t + v_t \sigma_t^2 = \varepsilon_t + \alpha_t \sigma_t^2 = \varepsilon_t + \mu_t - r_t$$

2.4.2 Flexible risk premium specifications

One of the advantages of our approach is that we can allow for time-varying risk premia. Here we discuss some potentially interesting ways to specify the risk premium in the return process for the underlying asset. In order to demonstrate the link with the available literature and for computational simplicity, we assume conditional normal returns, although this assumption is by no means necessary.

The conditional normal models in the Duan (1995) and Heston and Nandi (2000)

models are special cases of our set-up. In our notation, Duan (1995) assumes

$$r_t = r, \text{ and } \mu_t = r + \lambda \sigma_t$$

which in our framework corresponds to a Radon-Nikodym derivative of

$$\frac{dQ}{dP} \Big| F_t = \exp \left(- \sum_{i=1}^t \left(\frac{\varepsilon_i}{\sigma_i} \lambda - \frac{1}{2} \lambda^2 \right) \right)$$

and risk neutral innovations of the form

$$\varepsilon_t^* = \varepsilon_t + \lambda \sigma_t.$$

Heston and Nandi (2000) instead assume

$$r_t = r, \text{ and } \mu_t = r + \lambda \sigma_t^2 + \frac{1}{2} \sigma_t^2$$

which in our framework corresponds to a Radon-Nikodym derivative of

$$\frac{dQ}{dP} \Big| F_t = \exp \left(- \sum_{i=1}^t \left(\left(\lambda + \frac{1}{2} \right) \varepsilon_i - \frac{1}{2} \left(\lambda + \frac{1}{2} \right)^2 \sigma_i^2 \right) \right)$$

and risk neutral innovations of the form

$$\varepsilon_t^* = \varepsilon_t + \lambda \sigma_t^2 + \frac{1}{2} \sigma_t^2.$$

However, many empirically relevant cases are not covered by existing theoretical results. For example, in the original ARCH-M paper, Engle, Lilien and Robins (1987) find the strongest empirical support for a risk premium specification of the form

$$\mu_t = r_t + \lambda \ln(\sigma_t) + \frac{1}{2} \sigma_t^2$$

which cannot be used for option valuation using the available theory. In our framework it simply corresponds to a Radon-Nikodym derivative of

$$\frac{dQ}{dP} \Big| F_t = \exp \left(- \sum_{i=1}^t \left(\frac{\lambda \ln(\sigma_i) + \frac{1}{2}\sigma_i^2}{\sigma_i^2} \varepsilon_i - \frac{1}{2} \left(\frac{\lambda \ln(\sigma_i) + \frac{1}{2}\sigma_i^2}{\sigma_i^2} \right)^2 \sigma_i^2 \right) \right)$$

and risk neutral innovations

$$\varepsilon_t^* = \varepsilon_t + \lambda \ln(\sigma_t) + \frac{1}{2}\sigma_t^2$$

Our approach allows for option valuation under such specifications whereas the existing literature does not.

2.4.3 Conditionally inverse Gaussian returns

Christoffersen, Heston and Jacobs (2006) analyze a GARCH model with an inverse Gaussian innovation, $y_t \sim IG(\sigma_t^2/\eta^2)$. We can write their return dynamic as

$$R_t = r + (\lambda + \eta^{-1}) \sigma_t^2 + \varepsilon_t$$

where ε_t is a zero-mean innovation defined by

$$\varepsilon_t = \eta y_t - \eta^{-1} \sigma_t^2$$

and where the conditional return variance, σ_t^2 , is of the GARCH form.

From the MGF of an inverse Gaussian variable, we can derive the conditional log MGF of ε_t as

$$\Psi_t(u) = \left(u + \frac{1 - \sqrt{1 + 2u\eta}}{\eta} \right) \frac{\sigma_t^2}{\eta}$$

The EMM condition

$$\Psi_t(\nu_t - 1) - \Psi_t(\nu_t) - \Psi_t(-1) + \alpha_t \sigma_t^2 = 0$$

is now solved by the constant

$$\nu_t = \nu = \frac{1}{2\eta} \left[\frac{(2 + \lambda\eta^3)^2}{4\lambda^2\eta^2} - 1 \right], \forall t$$

which in turn implies that the EMM is given by

$$\begin{aligned} \frac{dQ}{dP} \Big|_{F_t} &= \exp \left(- \sum_{i=1}^t \left(\nu \varepsilon_i + \left(\nu + \frac{1 - \sqrt{1 + 2\nu\eta}}{\eta} \right) \frac{\sigma_i^2}{\eta} \right) \right) \\ &= \exp \left(-\nu t \bar{\varepsilon}_t - \delta t \bar{\sigma}_t^2 \right) \end{aligned}$$

where $\bar{\varepsilon}_t = \frac{1}{t} \sum_{i=1}^t \varepsilon_i$, $\bar{\sigma}_t^2 = \frac{1}{t} \sum_{i=1}^t \sigma_i^2$, and $\delta = \frac{\nu}{\eta} + \frac{1 - \sqrt{1 + 2\nu\eta}}{\eta^2}$.

These expressions can be used to obtain the risk-neutral distribution from Christoffersen, Heston and Jacobs (2006) using the results in Section 3. Recall that in general the risk neutral log MGF is

$$\Psi_t^{Q*}(u) = -u\Psi_t'(\nu) + \Psi_t(\nu + u) - \Psi_t(\nu)$$

In the GARCH-IG case we can write

$$\Psi_t^{Q*}(u) = \left(u + \frac{1 - \sqrt{1 + 2u\eta^*}}{\eta^*} \right) \frac{\sigma_t^{*2}}{\eta^*}$$

where

$$\eta^* = \frac{\eta}{1 + 2\nu\eta} \text{ and } \sigma_t^{*2} = \frac{\sigma_t^2}{(1 + 2\nu\eta)^{3/2}}$$

This implies that the risk neutral model can be written as

$$R_t \equiv \ln \left(\frac{S_t}{S_{t-1}} \right) = r - \Psi_t^{Q*}(-1) + \varepsilon_t^* = r + (\lambda^* + \eta^{*-1}) h_t^* + \varepsilon_t^*$$

where

$$\lambda^* = \frac{1 - 2\eta^* - \sqrt{1 - 2\eta^*}}{\eta^{*2}} \text{ and } \varepsilon_t^* = \eta^* y_t^* - \eta^{*-1} \sigma_t^{*2}$$

The risk neutral process thus takes the same form as the physical process which is exactly what our Proposition 3 in Section 3 shows.

2.4.4 Conditionally Poisson-normal jumps

Another interesting model that can be easily nested in our framework is the heteroskedastic model with Poisson-normal innovations in Duan, Ritchken and Sun (2005).¹¹ For expositional simplicity, we consider the simplest version of the model. More complex models, for instance with time-varying Poisson intensities, can also be accommodated. We can write the underlying asset return as

$$R_t = \kappa_t + \varepsilon_t$$

The zero-mean innovation ε_t equals

$$\varepsilon_t = \sigma_t (J_t - \lambda \bar{\mu})$$

where J_t is a Poisson jump process with N_t jumps each with distribution $N(\bar{\mu}, \bar{\gamma}^2)$ and jump intensity λ . The conditional return variance equals $(1 + \lambda(\bar{\mu}^2 + \bar{\gamma}^2))\sigma_t^2$, where σ_t^2 is of the GARCH form. The log return mean κ_t is a function of σ_t^2 as well as the jump and risk premium parameters.

We can derive the conditional log MGF of ε_t as

$$\begin{aligned} \Psi_t(u) &= \ln(E_{t-1}[\exp(-u\sigma_t(J_t - \lambda\bar{\mu}))]) \\ &= u\lambda\bar{\mu}\sigma_t + \frac{1}{2}u^2\sigma_t^2 + \lambda \left[e^{(-\bar{\mu}u\sigma_t + \frac{1}{2}\bar{\gamma}^2u^2\sigma_t^2)} - 1 \right] \end{aligned}$$

The approach taken in Duan et al (2005) corresponds to fixing $\nu_t = \nu$ and setting

$$\kappa_t = r + \Psi_t(\nu) - \Psi_t(\nu - 1)$$

¹¹Maheu and McCurdy (2004) consider a different discrete-time jump model.

which in turn implies that the EMM is given by

$$\frac{dQ}{dP} \Big| F_t = \exp \left(-\nu t \bar{\varepsilon}_t - \nu \lambda \bar{\mu} t \bar{\sigma}_t - \frac{1}{2} t \nu^2 \bar{\sigma}_t^2 + \lambda t - \lambda \sum_{i=1}^t e^{\left(-\bar{\mu} \nu_i \sigma_i + \frac{1}{2} \bar{\gamma}^2 \nu_i^2 \sigma_i^2 \right)} \right)$$

where $\bar{\varepsilon}_t$ and $\bar{\sigma}_t^2$ are the historical averages as above.

We can again show that the risk-neutral distribution of the risk neutral innovation is from the same family as the physical

$$\begin{aligned} \Psi_t^{Q*}(u) &= \ln E_{t-1}^Q [\exp(-u \varepsilon_t^*)] \\ &= u \lambda_t^* \bar{\mu}_t^* \sigma_t + \frac{1}{2} u^2 \sigma_t^2 + \lambda_t^* \left[e^{\left(-\bar{\mu}_t^* u \sigma_t + \frac{1}{2} \bar{\gamma}^2 u^2 \sigma_t^2 \right)} - 1 \right] \end{aligned}$$

where

$$\lambda_t^* = \lambda \exp \left(-\bar{\mu} \nu \sigma_t + \frac{1}{2} \bar{\gamma}^2 \nu^2 \sigma_t^2 \right) \text{ and } \bar{\mu}_t^* = \bar{\mu} - \bar{\gamma}^2 \sigma_t \nu$$

2.4.5 Conditionally skewed variance gamma returns

We now introduce a new model where the conditional skewness, s , and excess kurtosis, k , are given directly by two parameters in the model.¹² Consider the return of the underlying asset specified as follows

$$\begin{aligned} R_t &= \mu_t - \gamma_t + \varepsilon_t \\ &= \mu_t - \gamma_t + \sigma_t z_t, \end{aligned} \quad z_t \stackrel{i.i.d}{\sim} SVG(0, 1, s, k)$$

The distribution of the shocks, $SVG(0, 1, s, k)$, is a standardized skewed variance gamma distribution which is constructed as a mixture of two gamma variables.¹³ The conditional variance, σ_t^2 , can take on any GARCH specification. We will provide an empirical

¹²In Christoffersen, Heston and Jacobs (2006), conditional skewness and kurtosis are driven by functions of the same parameter.

¹³See Madan and Seneta (1990) for an early application of the symmetric and i.i.d. variance gamma distribution in finance.

illustration in the next section using a leading GARCH dynamic.

Let z_1 and z_2 be independent draws from two gamma distributions

$$z_{i,t} \stackrel{i.i.d}{\sim} \Gamma(4/\tau_i^2), \quad i = 1, 2$$

parameterized as

$$\tau_1 = \sqrt{2} \left(s - \sqrt{\frac{2}{3}k - s^2} \right) \text{ and } \tau_2 = \sqrt{2} \left(s + \sqrt{\frac{2}{3}k - s^2} \right)$$

If we construct the SVG random variable from the two gamma variables as

$$z_t = \frac{1}{2\sqrt{2}} (\tau_1 z_{1,t} + \tau_2 z_{2,t}) - \sqrt{2} \left(\frac{1}{\tau_1} + \frac{1}{\tau_2} \right)$$

then z_t will have a mean of zero, a variance of one, a skewness of s , and an excess kurtosis of k , thus allowing for conditional skewness and kurtosis in the GARCH model as intended.¹⁴

The log moment generating function of ε_t can be derived from the gamma distribution MGF as

$$\Psi_t(u) = \sqrt{2} (\tau_1^{-1} + \tau_2^{-1}) u \sigma_t - 4\tau_1^{-2} \ln \left(1 + \frac{1}{2\sqrt{2}} \tau_1 u \sigma_t \right) - 4\tau_2^{-2} \ln \left(1 + \frac{1}{2\sqrt{2}} \tau_2 u \sigma_t \right)$$

so that the mean correction variable, γ_t , for the return can be found as $\gamma_t = \Psi_t(-1)$.

Using the formula for the risk neutral conditional log MGF

$$\Psi_t^{Q*}(u) = -u\Psi'_t(\nu_t) + \Psi_t(\nu_t + u) - \Psi_t(\nu_t)$$

¹⁴The special cases where τ_1 or τ_2 are zero can be handled easily by drawing from the normal distribution for the relevant mixing variable $z_{1,t}$ or $z_{2,t}$. When both τ_1 and τ_2 are zero then the normal distribution obtains for z_t .

we can show that the risk neutral model is

$$R_t = r_f - \gamma_t^* + \varepsilon_t^* \quad (2.5)$$

where

$$\Psi_t^{Q^*}(u) = \sqrt{2} \left(\tau_1^{-1} \sigma_{1,t}^* + \tau_2^{-1} \sigma_{2,t}^* \right) u - 4\tau_1^{-2} \ln \left(1 + \frac{1}{2\sqrt{2}} \tau_1 \sigma_{1,t}^* u \right) - 4\tau_2^{-2} \ln \left(1 + \frac{1}{2\sqrt{2}} \tau_2 \sigma_{2,t}^* u \right)$$

with

$$\sigma_{i,t}^* = \frac{\sigma_t}{\sqrt{2} + \frac{1}{2} \tau_i \sigma_t \nu_t}, \quad \text{for } i = 1, 2. \quad (2.6)$$

We see that $\Psi_t^{Q^*}(u)$ is exactly of the same form as $\Psi_t(u)$, and therefore that $\gamma_t^* = \Psi_t^{Q^*}(-1)$. This model will be investigated empirically in the next section.

2.5 Empirical illustration

In this section we demonstrate how the greater flexibility and generality allowed for by our approach can lead to more realistic option valuation models. To do so, we analyze the GARCH-SVG model in Section 4.5, which allows for conditional skewness and kurtosis, and which has not yet been analyzed in the literature. We compare its empirical implications with the more standard conditional normal model of Section 4.1. We compute option prices from both models using parameters estimated from return data, and subsequently construct option implied volatility smiles. We also compare the two heteroskedastic models to two benchmark models with independent returns.

2.5.1 Parameter estimates from index returns and stylized facts

We start by illustrating some key stylized facts of daily equity index returns using the S&P500 as a running example.

Figure 1 shows a normal quantile-quantile plot (QQ plot) of daily S&P500 returns,

using data from January 2, 1980 through December 30, 2005 for a total of 6,564 observations. The returns are standardized by the sample mean and standard deviation. The data quantiles on the vertical axis are plotted against the normal distribution quantiles on the horizontal axis. The plot reveals the well-known stark deviations from normality in daily asset returns: actual returns include much more extreme observations than the normal distribution allows for in a sample of this size. The largest negative return is the famous 20 standard deviation crash in October 1987, but the normal distribution has trouble fitting a large number of extremes in both tails of the return distribution. The actual returns range from -20 to +9 standard deviations but the normal distribution only ranges from -4 to +4 standard deviations in a sample of this size.

Figure 2 shows the sample autocorrelation function of the squared daily returns for the sample. The significantly positive correlations at short lags suggest the need for a dynamic volatility model allowing for clustering in volatility.

Figures 1 and 2 clearly suggest the need for a GARCH model which can capture potentially both the volatility clustering in Figure 2 and the non-normality in Figure 1.

As a benchmark, we use the conditional normal NGARCH model of Engle and Ng (1993)

$$R_t = \mu_t - \gamma_t + \sigma_t z_t, \quad z_t \stackrel{i.i.d}{\sim} N(0, 1) \quad (2.7)$$

where

$$\begin{aligned} \mu_t &= r_t + \lambda \sigma_t \\ \gamma_t &= \frac{1}{2} \sigma_t^2 \\ \sigma_t^2 &= \beta_0 + \beta_1 \sigma_{t-1}^2 + \beta_2 \sigma_{t-1}^2 (z_{t-1} - \beta_3)^2 \end{aligned}$$

Notice that the β_3 parameter in the GARCH variance specification allows for an asymmetric variance response to positive versus negative shocks, z_{t-1} . This captures the so-called leverage effect, which is another important empirical regularity in daily equity index returns.

Table 1 reports the maximum likelihood estimates of the GARCH parameters. We also report parameter estimates for a version of the model where the GARCH dynamics have been shut down, that is, where $\beta_1 = \beta_2 = \beta_3 = 0$. Notice the large increase in the Log-likelihood function from including the GARCH dynamics.

Figure 3 shows the autocorrelation function for the observed squared GARCH shocks, z_t^2 . If the GARCH model has adequately captured the volatility clustering then the shocks should be independent and in particular the squared shocks should be uncorrelated. Figure 3 suggests that the GARCH model does a good job of capturing the volatility dynamics in the daily index returns.

Figure 4 assesses the conditional normality assumption by plotting a QQ plot of z_t against the normal distribution. It is clear from Figure 4 that much of the non-normality in the raw returns has been removed by the GARCH model. This is particularly true for the right tail, where the non-normality was least pronounced to begin with. Unfortunately, the left tail of the shock distribution still exhibits strong evidence of non-normality with negative shocks as large as -10 standard deviations compared with the normal distribution's -4.

From Figures 3 and 4, we conclude that while the normal GARCH model appears to provide adequate dynamics for capturing volatility clustering, the conditional normality assumption is violated in the data and must be modified in the model.

For the implementation of the GARCH-SVG model, μ_t and σ_t^2 are the same as in the conditional normal model in (2.7). We can calibrate the s and k parameters in the GARCH-SVG model from Section 4.5 by simply equating them to the sample moments from the z_t sequence from the QMLE estimation of the GARCH model. These sample moments are reported in Table 1.

Figure 5 shows the QQ plot of the GARCH shocks against the SVG distribution. Compared with the normal QQ plot in Figure 4, we see that the SVG captures the left tail of the shock distribution much better than the normal does. Impressively, the SVG model only has trouble fitting the two most extreme negative shocks, whereas the normal

distribution misses a whole string of large negative shocks.

2.5.2 Option prices and implied volatilities

Armed with estimated return processes we are ready to assess the option pricing implications of the different models. From Section 2.3 we have the general option price relationship which for a European call option with strike price K is

$$C_t(T, K) = E_t^Q \left[\text{Max}(S_T - K, 0) \frac{B_t}{B_T} \right]$$

Using the estimated physical process from Section 4 we can now simulate future paths for S_T from the current S_t and compute the option price as the simulated sample analogue to this discounted expectation.

We present evidence on the option pricing properties of the various models in Figures 6 and 7. Figure 6 considers an i.i.d. normal and an i.i.d. SVG model where the GARCH dynamics have been shut down ($\beta_1 = \beta_2 = \beta_3 = 0$), and s and k have been set to the sample skewness and kurtosis from the raw returns which are reported in Table 1. Figure 7 considers the normal GARCH-Normal and GARCH-SVG models. The parameter estimates used are again from Table 1.

We first compute option prices for various moneyness and maturities and we then compute implied Black and Scholes (1973) volatilities from the model option prices. Implied volatilities are plotted against moneyness on the horizontal axis. The three panels correspond to maturities of 1 day, 1 week, and 1 month respectively.

The i.i.d. SVG model in Figure 6 (solid lines) shows a strong implied volatility “smile” for the 1-day maturity driven by the large excess kurtosis of 27.33 from Table 1. Interestingly, as the maturity increases the smile becomes an asymmetric “smirk” driven by the skewness parameter of -1.21 in Table 1. The i.i.d. normal model in Figure 6 (dashed line) results in a flat implied volatility curve.

The GARCH-SVG model in Figure 7 shows a smirk at the 1 day maturity compared

with the flat implied volatility for the GARCH-Normal model where the conditional 1 day distribution is normal. The GARCH-Normal model generates a non-trivial volatility smirk for horizons beyond 1 day where the conditional distribution is no longer normal. However, the GARCH-SVG model is capable of capturing much more non-normality than the GARCH-Normal model at all horizons. This is important because the empirical option valuation literature often finds that existing models are unable to fit short term option prices where the implied degree of non-normality is large.¹⁵

From this empirical illustration we conclude that it is possible to build relatively simple models capturing the conditional volatility and non-normality found in index returns data, and more importantly that such models provide the flexibility needed to price options.

2.6 Relationship with the existing literature

Our results are intimately related to the theoretical and empirical literature on GARCH option valuation, which in turn builds on the discrete-time option valuation results of Brennan (1979) and Rubinstein (1976). The aim of this literature is to obtain a risk-neutral valuation relationship, but these papers typically obtain such relationship by characterizing conditions on preferences needed to obtain risk-neutral valuation. For example, Brennan (1979) characterizes the bivariate distribution of returns on aggregate wealth and the underlying asset under which a risk-neutral valuation relationship obtains in the homoskedastic case. Duan (1995) extends this framework to the case of heteroskedasticity of the underlying asset return. Amin and Ng (1993) also study the heteroskedastic case. Although they formulate the problem in terms of the economy's stochastic discount factor, they start by making an assumption on the bivariate distribution of the stochastic discount factor and the underlying return process.

There is also a growing literature that values options for discrete-time return dynamics

¹⁵See Bates (2003) for an excellent discussion of this and other stylized facts in option markets.

with non-normal innovations. A number of other papers obtain risk-neutral valuation relationships either under the maintained assumption of non-normal innovations, or under the maintained assumption of heteroskedasticity, or both. Madan and Seneta (1990) use the symmetric and i.i.d. variance gamma distribution. Heston (1993b) presents results for the gamma distribution and Heston (2004) analyzes a number of infinitely divisible distributions. Camara (2003) uses a transformed normal innovation and Duan (1999) uses a heteroskedastic model with a transformed normal innovation. Christoffersen, Heston and Jacobs (2006) analyze a heteroskedastic return process with inverse Gaussian innovations.

Our paper differs in a subtle but important way from most of the studies that use heteroskedastic processes, in the sense that we do not attempt to characterize the bivariate distribution of preferences and returns that gives rise to the risk-neutral valuation relationship. Strictly speaking, the only assumption we make is on the return dynamic. Establishing the equivalent martingale measure that makes the discounted stock price process a martingale does not amount to an additional assumption. It is simply a mathematical manipulation required to obtain the benefits of risk-neutral valuation. All assumptions needed for risk-neutral valuation are given by the specification of the return dynamic, or, in other words, the assumptions on the equilibrium supporting the valuation problems are implicitly incorporated in the risk premium assumption for the return dynamic. The specification of the price of risk may be—but does not need to be—explicitly motivated by a utility-based argument.

To motivate our approach, consider the available literature on option valuation in continuous time, and in particular option valuation with continuous-time stochastic volatility models, such as the one in Heston (1993a). It is well-known (see e.g. Karatzas and Shreve (1998)) that in this case there are different equivalent martingale measures for different specifications of the volatility risk premium. However, for a given specification of the volatility risk premium, we can find an EMM and characterize the risk-neutral dynamic using Girsanov's theorem. To perform this manipulation, and to value options, there is

no need to characterize the utility function underlying the volatility risk premium. Characterizing the utility function that generates a particular volatility risk premium is a very interesting question in its own right, but differs from characterizing the risk-neutral dynamic and the option value for a given physical return dynamic.¹⁶ The latter is a purely mathematical exercise. The former provides the economic background behind a particular choice of volatility premium, and therefore helps us understand whether a particular choice of functional form for the risk premium, which is often made for convenience, is also reasonable from an economic perspective.

In the same sense, our paper should be interpreted as providing a set of tools that can be used to value options for a large class of discrete-time return dynamics that are characterized by heteroskedasticity and non-normal innovations. Whether this valuation exercise makes sense from an economic perspective depends on the nature of the assumed risk premium, and the general equilibrium setup that gives rise to such risk premium. There are two questions: a mostly technical one that characterizes the risk-neutral dynamic and the valuation of options, and a more economic one that characterizes the equilibrium underlying this valuation procedure. In our opinion, the existing discrete-time literature for the most part has viewed these two questions as inextricably linked, and has therefore largely limited itself to (log)normal return processes. We argue that it is possible and desirable to treat these questions one at a time, and we provide new results on the question of option valuation with conditionally non-normal returns.

There are many other papers that are in some way related to our contribution. First and foremost, we emphasize that we do not claim to be the first to analyze no-arbitrage pricing in discrete-time models. There is a rich tradition of discrete-time finite state space modeling in discrete time, going back to Harrison and Kreps (1979), Cox, Ross and Rubinstein (1979) and Cox and Ross (1976). However, the infinite state space, conditionally non-normal return dynamics we analyze are arguably the most empirically

¹⁶See Bollerslev, Gibbons and Zhou (2005) for a recent treatment.

relevant descriptions of return data available,¹⁷ and the option valuation literature that uses GARCH processes has hitherto focused on equilibrium arguments. Because of this, the available valuation results in this literature are quite limited, and our paper shows that we can obtain additional results by using a simple no-arbitrage approach. Second, it is likely that our risk neutralizations can equivalently be derived using the specification of a candidate stochastic discount factor, rather than through our approach which starts with the specification of a Radon-Nikodym derivative and derives the EMM.¹⁸ However, in most applications that we are aware of, existing work actually starts out by assuming a bivariate distribution for the stochastic discount factor and the stock return (see for example Amin and Ng (1993)).¹⁹ This assumption clearly goes beyond the existence of no-arbitrage and is closer in spirit to the general equilibrium setup of Duan (1995) and Brennan (1979). See Garcia, Ghysels and Renault (2006) for a discussion on how some of these assumed joint distributions effectively amount to degenerate distributions. Our approach is also related to the risk-neutral valuation argument used in Heston (1993b, 2004) and Christoffersen, Heston and Jacobs (2006), but in our opinion our approach is more transparent. Duan, Ritchken and Sun (2005) use a risk neutralization for a Poisson-normal heteroskedastic model that has some similarities with our approach. However, they do not apply their principle to the investigation of more general return dynamics.

Finally, at an empirical level, combining non-normality with heteroskedasticity attempts to correct the biases associated with the conditionally normal GARCH model. These biases are similar to those displayed by the Heston (1993) model, which the continuous-time literature has sought to remedy by adding (potentially correlated) jumps in returns and volatility. This paper is therefore also related to empirical studies of jump models. See for example Bakshi, Cao and Chen (1997), Bates (2000), Broadie, Chernov and Johannes (2006), Carr and Wu (2004), Eraker, Johannes and Polson (2003), Eraker

¹⁷The empirical evidence suggesting GARCH type processes is strong. See Bollerslev, Chou and Kroner (1992) and Diebold and Lopez (1995) for overviews.

¹⁸See for example Hansen and Richard (1987) for a characterization of risk neutralization using the stochastic discount factor.

¹⁹See Gouriéroux and Montfort (2006) for a notable exception.

(2004), Huang and Wu (2004) and Pan (2002).

2.7 Conclusion

This paper provides valuation results for contingent claims in a discrete time infinite state space setup. Our valuation argument applies to a large class of conditionally normal and non-normal stock returns with flexible time-varying mean and volatility, as well as a potentially time-varying price of risk, provided that these moments are predetermined one period ahead. Our setup generalizes the result in Duan (1995) in the sense that we do not restrict the returns to be conditionally normal, nor do we restrict the price of risk to be constant. Our results apply to some of the most widely used discrete time processes in finance, such as GARCH processes. For the class of processes we analyze in this paper, the risk neutral return dynamic is the same as the physical dynamic, but with a different parameterization which we characterize.

To demonstrate the empirical relevance of our approach, we provide an empirical analysis of a heteroskedastic return dynamic with a standardized skewed variance gamma distribution, which is constructed as the mixture of two gamma variables. In the resulting dynamic, conditional skewness and kurtosis are directly governed by two distinct parameters. We estimated the model on return data using quasi maximum likelihood and compare its performance with the heteroskedastic conditional normal model which is standard in the literature. Diagnostics clearly indicate that the conditionally nonnormal model outperforms the conditionally normal model, and an analysis of the option smirk demonstrates that this model provides substantially more flexibility to value options.

We leave a couple of important issues unaddressed. First, while we obtain a unique EMM given the choice of Radon-Nikodym derivative, we cannot exclude that even for a given specification of the risk premium, there exist other EMMs corresponding to different functional forms of the Radon-Nikodym derivative. Second, while we advocate separating the valuation issue and the general equilibrium setup that supports it, the

general equilibrium foundations of our results are of course very important. It may prove possible to characterize the equilibrium setup that gives rise to the risk neutralization proposed for some of the processes considered in this paper, such as the empirically interesting dynamics considered in Section 5. However, this is by no means a trivial problem, and it is left for future work.

2.8 Appendix

Proof of Lemma 2. For a self financing strategy we have

$$\begin{aligned} G_{t+1} &= V_{t+1} = \eta_t S_{t+1} + \delta_t C_{t+1} + \psi_t B_{t+1} \\ &= \eta_{t+1} S_{t+1} + \delta_{t+1} C_{t+1} + \psi_{t+1} B_{t+1} \end{aligned}$$

We also have

$$G_t = \sum_{i=0}^{t-1} \eta_i (S_{i+1} - S_i) + \sum_{i=0}^{t-1} \delta_i (C_{i+1} - C_i) + \sum_{i=0}^{t-1} \psi_i (B_{i+1} - B_i).$$

It follows that

$$G_{t+1} - G_t = \eta_t (S_{t+1} - S_t) + \delta_t (C_{t+1} - C_t) + \psi_t (B_{t+1} - B_t)$$

We can trivially also write

$$G_{t+1}^B - G_t^B = G_{t+1}^B - G_t^B + \underbrace{\left(\frac{G_{t+1}}{B_t} - \frac{G_{t+1}}{B_t} \right)}_{=0}$$

This implies that

$$\begin{aligned} G_{t+1}^B - G_t^B &= (\eta_t S_{t+1} + \delta_t C_{t+1} + \psi_t B_{t+1}) \left(\frac{1}{B_{t+1}} - \frac{1}{B_t} \right) \\ &\quad + \frac{1}{B_t} (\eta_t (S_{t+1} - S_t) + \delta_t (C_{t+1} - C_t) + \psi_t (B_{t+1} - B_t)). \end{aligned}$$

$$\begin{aligned}
&= \eta_t \left[S_{t+1} \left(\frac{1}{B_{t+1}} - \frac{1}{B_t} \right) + \frac{1}{B_t} (S_{t+1} - S_t) \right] \\
&\quad + \delta_t \left[C_{t+1} \left(\frac{1}{B_{t+1}} - \frac{1}{B_t} \right) + \frac{1}{B_t} (C_{t+1} - C_t) \right] \\
&\quad + \underbrace{\psi_t B_{t+1} \left(\frac{1}{B_{t+1}} - \frac{1}{B_t} \right) + \frac{1}{B_t} \psi_t (B_{t+1} - B_t)}_{=0}
\end{aligned}$$

Then

$$\begin{aligned}
G_{t+1}^B - G_t^B &= \eta_t \left[S_{t+1} \left(\frac{1}{B_{t+1}} - \frac{1}{B_t} \right) + \frac{1}{B_t} (S_{t+1} - S_t) \right] + \delta_t \left[C_{t+1} \left(\frac{1}{B_{t+1}} - \frac{1}{B_t} \right) + \frac{1}{B_t} (C_{t+1} - C_t) \right] \\
&= \eta_t (S_{t+1}^B - S_t^B) + \delta_t (C_{t+1}^B - C_t^B) + \left(\eta_t \frac{S_{t+1}}{B_t} - \eta_t \frac{S_{t+1}}{B_t} \right) + \left(\delta_t \frac{C_{t+1}}{B_t} - \delta_t \frac{C_{t+1}}{B_t} \right)
\end{aligned}$$

and therefore

$$G_{t+1}^B - G_t^B = \eta_t (S_{t+1}^B - S_t^B) + \delta_t (C_{t+1}^B - C_t^B), \quad \forall t = 1, \dots, T-1$$

Because $G_0 = G_0^B = 0$ the discounted gain can be written as the sum of past changes

$$G_t^B = \sum_{i=0}^{t-1} (G_{i+1}^B - G_i^B) \quad \forall t = 1, \dots, T.$$

Therefore the discounted gain can be written

$$G_t^B = \sum_{i=0}^{t-1} \eta_i (S_{i+1}^B - S_i^B) + \sum_{i=0}^{t-1} \delta_i (C_{i+1}^B - C_i^B)$$

and the proof is complete.

Proof of Proposition 3. From Lukacs (1970), page 119, we have the Kolmogorov canonical representation of the log-moment generating function of an infinitely divisible distribution function. This result stipulates that a function Ψ is the log-moment generating function of an infinitely divisible distribution with finite second moment if, and

only if, it can be written in the form

$$\Psi(u) = -uc + \int_{-\infty}^{+\infty} (e^{-ux} - 1 + ux) \frac{dK(x)}{x^2}$$

where c is a real constant while $K(u)$ is a nondecreasing and bounded function such that $K(-\infty) = 0$. Applying this theorem gives the following form for $\Psi_t(u)$,

$$\Psi_t(u) = -uc_{t-1} + \int_{-\infty}^{+\infty} (e^{-ux} - 1 + ux) \frac{dK_{t-1}(x)}{x^2} \quad (2.8)$$

where c_{t-1} is a random variable known at $t-1$, and $K_{t-1}(x)$ is a function known at $t-1$, which is nondecreasing and bounded so that $K_{t-1}(-\infty) = 0$. Using relation (2.4) and the characterisation (2.8) we can write $\Psi_t^{Q*}(u)$ as

$$\Psi_t^{Q*}(u) = \int_{-\infty}^{+\infty} (e^{-ux} - 1 + ux) \frac{dK_{t-1}^*(x)}{x^2}$$

where

$$K_{t-1}^*(x) = \int_{-\infty}^x e^{-\nu_{t-1}y} dK_{t-1}(y)$$

This implies that

$$K_{t-1}^*(-\infty) = 0$$

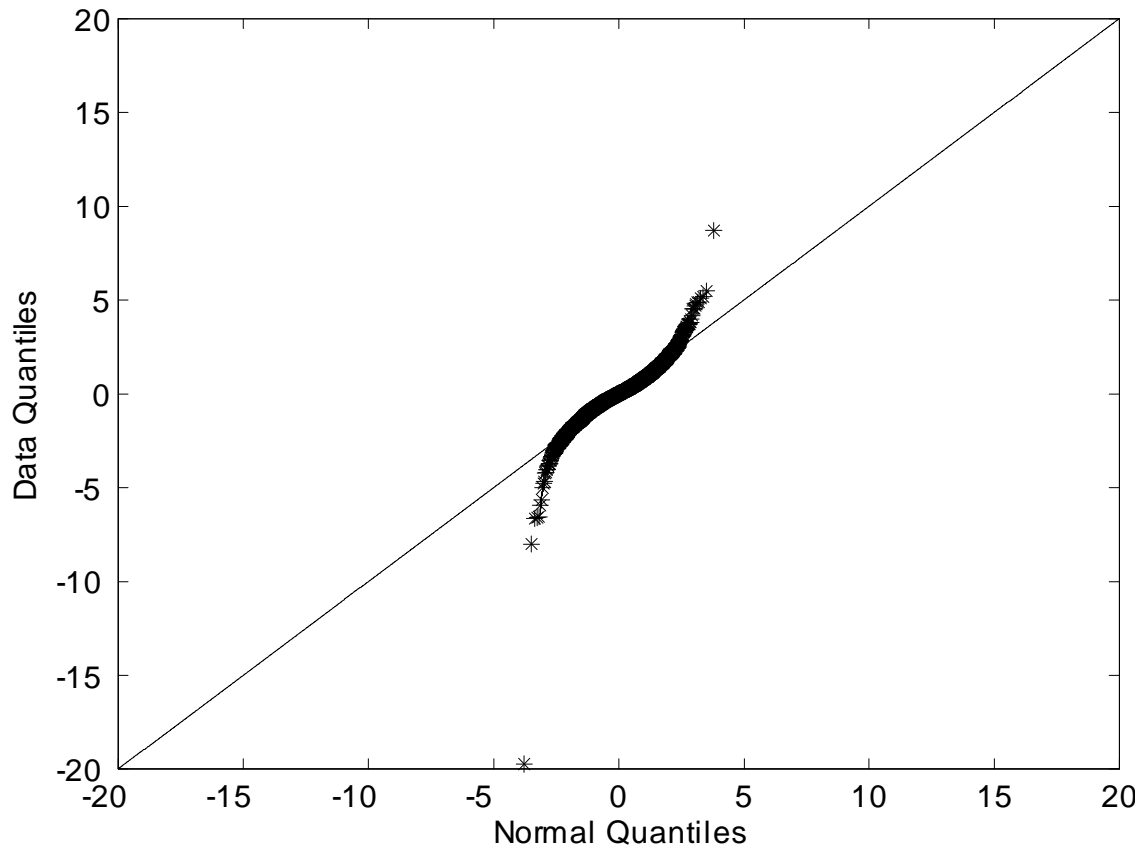
$K_{t-1}^*(x)$ is obviously non-decreasing since $K_{t-1}(x)$ is non-decreasing, $K_{t-1}^*(\infty) < \infty$, because $K_{t-1}(\infty) < \infty$, and $e^{-\nu_{t-1}y}$ is a decreasing function of y which converge to 0. Recall that ν_t is the price of risk, which is positive and known at time $t-1$.

In conclusion we have constructed a constant c_{t-1}^* ($= 0$) and a non-decreasing bounded function $K_{t-1}^*(x)$, with $K_{t-1}^*(-\infty) = 0$, such that

$$\Psi_t^{Q*}(u) = -uc_{t-1}^* + \int_{-\infty}^{+\infty} (e^{-ux} - 1 + ux) \frac{dK_{t-1}^*(x)}{x^2}.$$

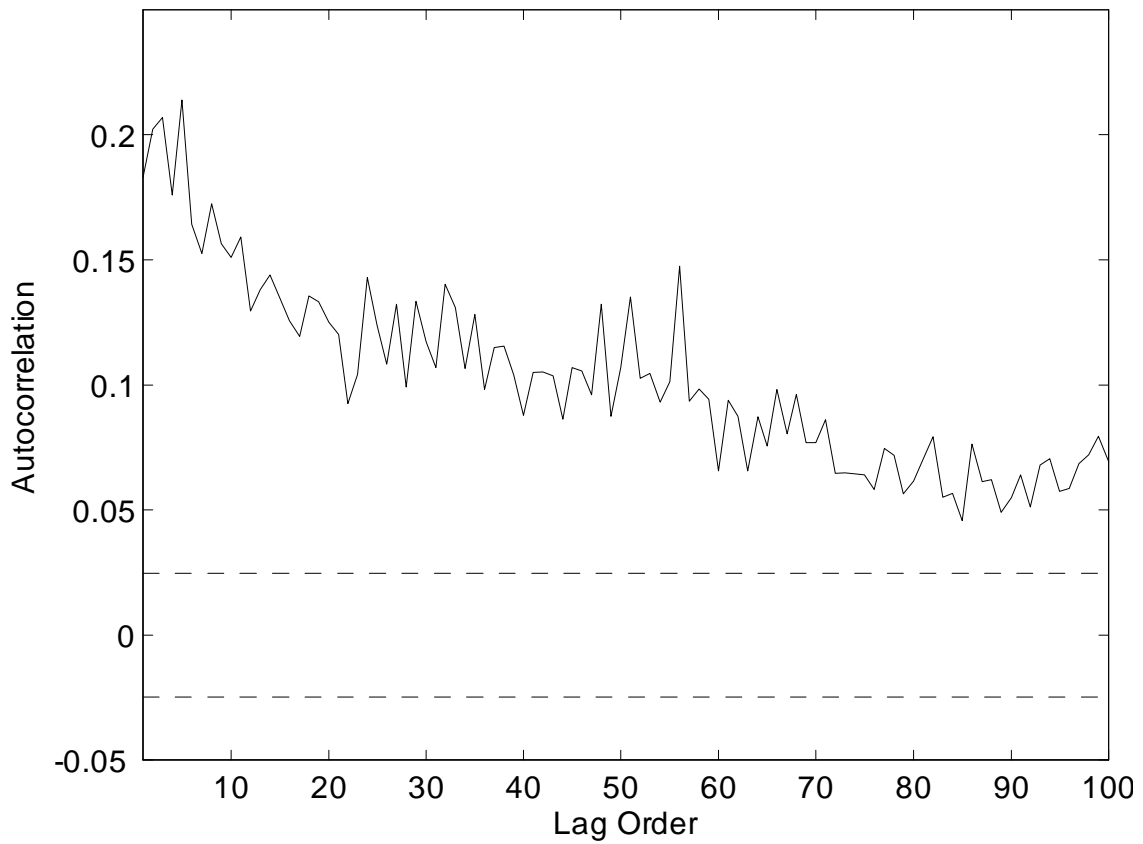
Hence, according to the Kolmogorov canonical representation, the conditional distribution of ε_t^* is infinitely divisible.

Figure 1: Quantile-Quantile Plot of S&P500 Returns Against the Normal Distribution



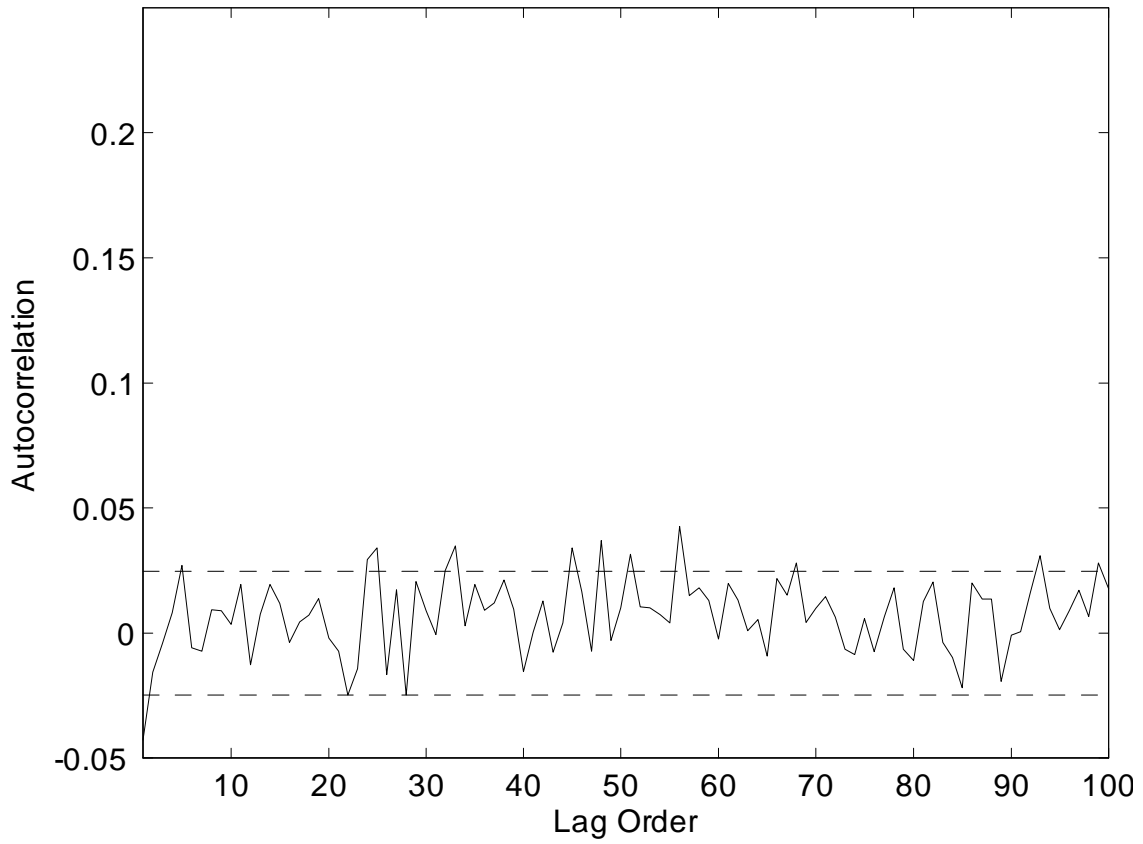
Notes to Figure: We take daily returns on the S&P500 from January 2, 1980 to December 30, 2005 and standardize them by the sample mean and sample standard deviation. The quantiles of the standardized returns are plotted against the quantiles from the standard normal distribution.

Figure 2: Autocorrelation Function of Absolute S&P500 Returns



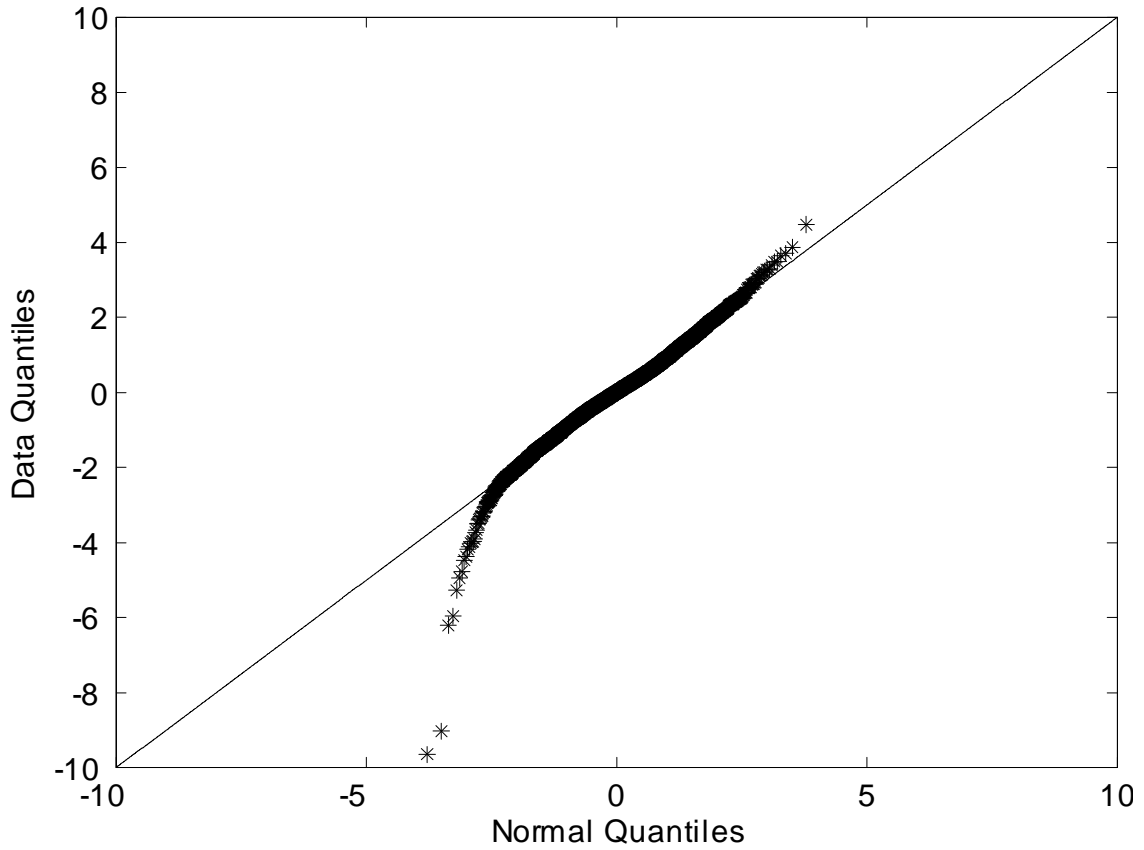
Notes to Figure: From daily absolute returns on the S&P500 from January 2, 1980 to December 30, 2005 we compute and plot the sample autocorrelations for lags one through 100 days. The horizontal dashed lines denote 95% Bartlett confidence intervals around zero.

Figure 3: Autocorrelation Function of Absolute GARCH Innovations



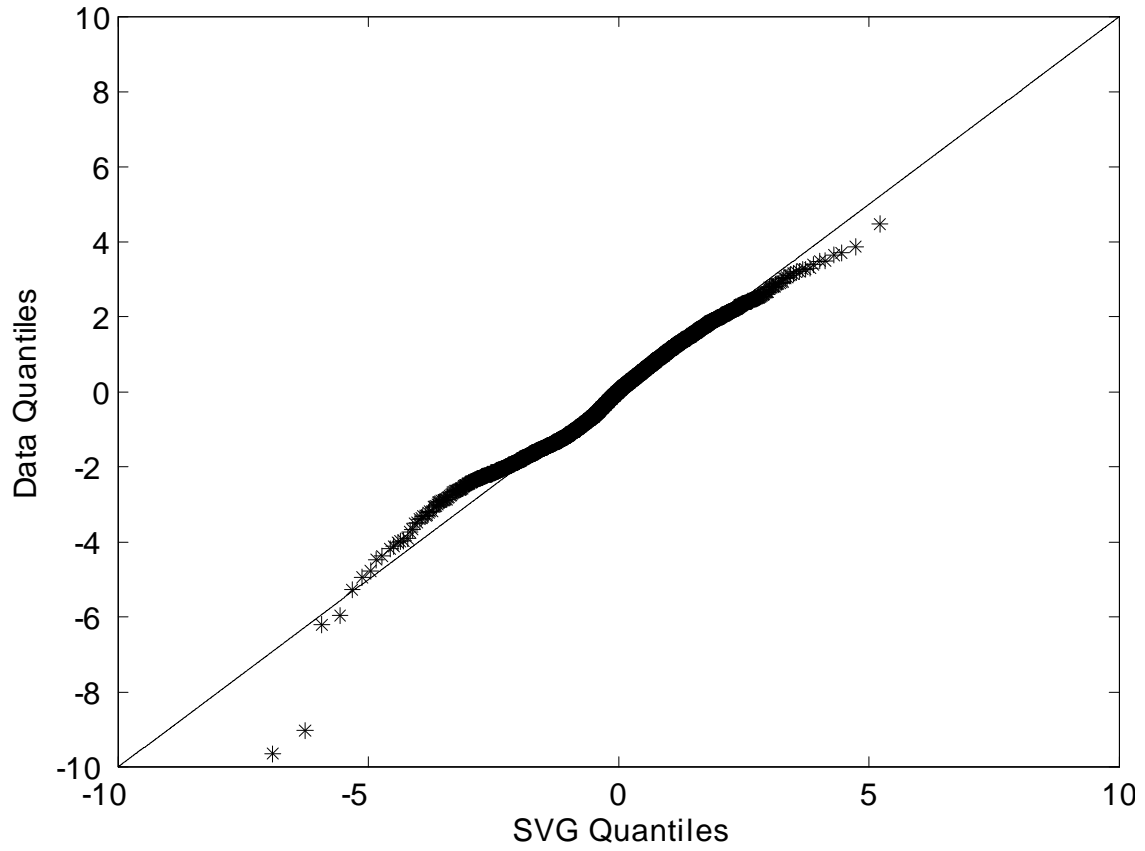
Notes to Figure: From the estimated GARCH model in Table 1 we construct the absolute standardized sequence of shocks and plot the sample autocorrelations for lags one through 100 days. The horizontal dashed lines denote 95% Bartlett confidence intervals around zero.

Figure 4: Quantile-Quantile Plots of GARCH Innovations Against the Normal Distribution



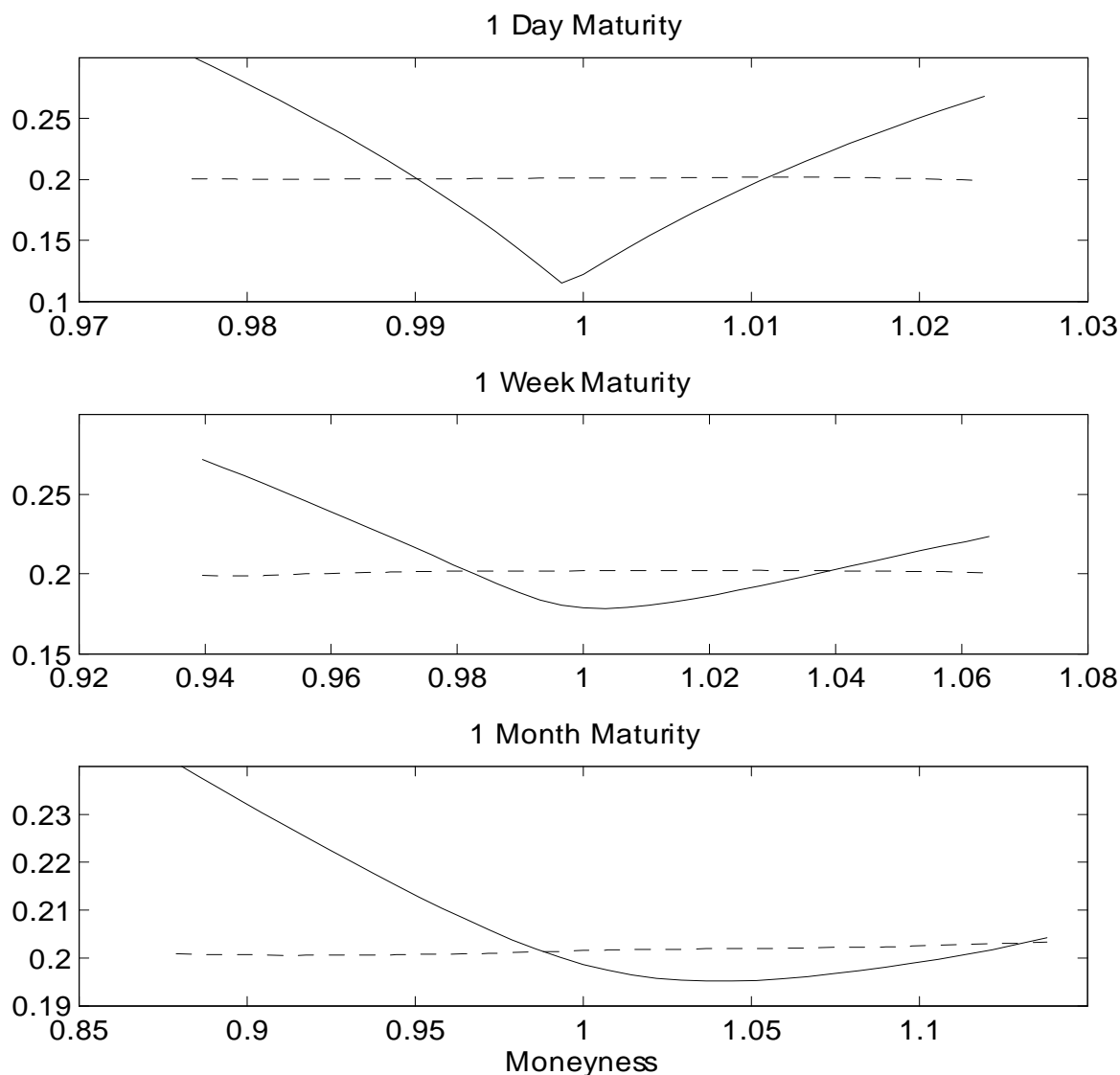
Notes to Figure: From the estimated GARCH models in Table 1 we compute the time series of dynamically standardized S&P500 returns. The quantiles of these GARCH innovations are plotted against the quantiles from the standard normal distribution.

Figure 5: Quantile-Quantile Plots of GARCH Innovations Against the SVG Distribution



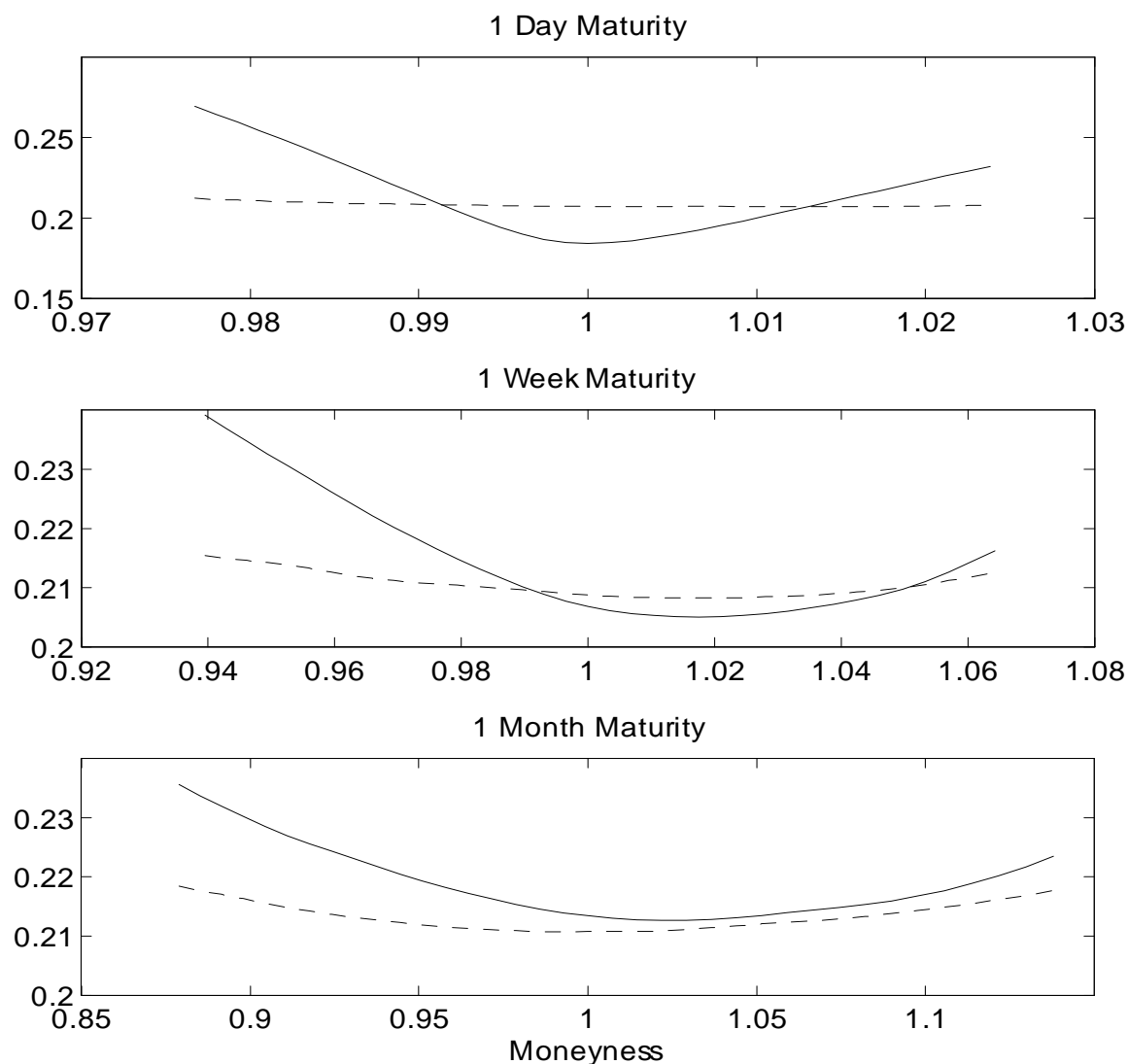
Notes to Figure: From the estimated GARCH models in Table 1 we compute the time series of dynamically standardized S&P500 returns. The quantiles of these GARCH innovations are plotted against the quantiles from the skewed variance gamma (SVG) distribution.

Figure 6: Implied Volatility Functions for Normal and SVG Independent Return Models



Notes to Figure: From the estimated independent return model in Table 1 we compute call option prices for various moneyness and maturities and we then compute implied Black-Scholes volatilities from the model option prices. Implied volatility is plotted against moneyness on the horizontal axis. The three panels correspond to maturities of 1 day, 1 week, and 1 month respectively. The solid lines show the i.i.d. SVG model and the dashed lines the i.i.d. Normal models.

Figure 7: Implied Volatility Functions for Normal and SVG GARCH Models



Notes to Figure: From the estimated GARCH model in Table 1 we compute call option prices for various moneyness and maturities and then we compute implied Black-Scholes volatilities from the model option prices. The implied volatilities are plotted with moneyness on the horizontal axis. The three panels correspond to maturities of 1 day, 1 week, and 1 month respectively. The solid lines show the SVG GARCH model and the dashed lines the Normal GARCH model.

Table 1: Parameter Estimates and Model Properties
Daily S&P500 Returns. January 2, 1980 - December 30, 2005.

<u>Parameters</u>	Independent Returns		GARCH Returns	
	<u>Estimate</u>	<u>Standard Error</u>	<u>Estimate</u>	<u>Standard Error</u>
r	1.370E-04		1.370E-04	
λ	0.0313	0.0129	0.0312	0.0121
β_0	1.111E-04	9.9781E-06	1.516E-06	6.050E-07
β_1			0.8916	0.0274
β_2			0.0617	0.0154
β_3			0.7422	0.0808

<u>Properties</u>	Independent Returns	GARCH Returns
Log-Likelihood	20,615.00	21,586.28
Volatility Persistence	0	0.9873
Annual Volatility	0.1673	0.1734
Conditional Skewness	-1.2105	-0.4127
Conditional Kurtosis	27.3304	3.4935

Notes: We use quasi maximum likelihood to estimate an independent return and a GARCH return model on daily S&P500 returns from January 2, 1980 to December 30, 2005 for a total of 6,564 observations. We report various properties of the two models including conditional skewness and excess kurtosis which are later used as parameter estimates in the SVG models.

Chapter 3

Time Varying Risk Premia in Corporate Bond Markets

3.1 Introduction

Yield spreads are an imperfect measure of the expected return on a corporate bond. For asset allocation or performance evaluation purposes, a key input is the risk premium earned by holding a particular security. However, bond spreads also contain significant compensation for expected losses - even in the absence of a risk premium. Two firms with the same default probability can earn very different spreads depending on their systematic risk. Two firms with the same spread can earn significantly different risk premia. This paper studies risk premia extracted from yield spreads.

The equity risk premium has received intense attention in the finance literature. In stark contrast, the literature on the risk return characteristics of corporate bonds is only now emerging.¹ One reason is likely the scarcity of corporate bond return data. Another is that disentangling the risk premium from spreads requires estimates of objective default probabilities, the measurement of which is in itself a complex task.² Using a large sample of US corporate bond transactions over a 10 year period, we measure corporate bond risk premia. Using a structural model, we first estimate actual default probabilities to compute the expected loss component of spreads. Second, we use this estimate to disentangle the risk premium component.³

Theoretically, a given firm's equity and bond returns should be closely related. Nevertheless, many papers have documented difficulties in relating equity factors and bond returns. Fama and French (1993) find that factors that explain the time series and the cross section of equity returns well are not that successful in explaining corporate bond returns. Recent work has produced mixed results on the impact of financial distress on bond and stock returns. Some studies have documented that firms with higher default

¹Notable exceptions, discussed further below, include Elton, Gruber, Agrawal & Mann (2001), Driessen (2005), Huang & Huang (2002), Berndt, Douglas, Duffie, Ferguson & Schranz (2004), Berndt, Lookman & Obreja (2006), Saita (2006) and Chen, Collin-Dufresne & Goldstein (2006).

²Recent work on estimating default probabilities includes Shumway (2001), Barath & Shumway (2004), Leland (2004) and Duffie et al. (2006).

³There is currently a debate in the literature regarding the choice of appropriate risk free rate. To address this, we carry out our analysis both using US Treasury and interest rate swap benchmark curves.

risk have low average stock returns.⁴ Others have found that firms with a high likelihood of default experience high average stock returns.⁵

Our main contribution is to revisit the link between equity and bond risk premia. A visual inspection of time series of average equity risk premia and bond risk premia would likely suggest that they move largely independently of each other. In particular, between 1997 and 2002 equity premia trended slowly downwards, whereas the much more volatile bond premia increased irregularly. Our theoretical framework predicts a highly nonlinear relationship between these premia. This relationship depends on financial leverage, operating risk as well as bond specific characteristics. Empirically we find that equity returns are in fact useful in explaining bond risk premia when these are taken into account. Interestingly, we show that the risk premium, as a function of the likelihood of distress is non-monotonic. For healthy firms risk premia will increase in risk, but for firms approaching distress, they can in fact decrease. As distress becomes more likely, uncertainty about the arrival of default decreases and then so do risk premia.

Previous studies have documented surprisingly volatile risk premia in default swap markets.⁶ We find a very similar time series behavior and degree of time variation, which is interesting given that our study is based on different data, a different financial instrument and a different methodology.

Another contribution of our work is to document the characteristics of the expected loss components across firms and time. Previous work has shown that expected losses explain about a quarter of corporate bond spread levels.⁷ Our overall average is of the same magnitude, however, the relative importance of expected loss component is (i) highly time varying and (ii) tends to be higher when spreads are high. For example, we find that across our sample of about 400 firms, it reaches a high of about 70% of the spread over government bonds in 2000, up from an average near a third the five

⁴See Dichev (1998), Campbell et al. (2008), and Garlappi et al. (2008).

⁵See Vassalou & Xing (2004).

⁶See Berndt et al. (2004), Berndt et al. (2006).

⁷See Elton et al. (2001).

preceeding years. Ignoring this time variation may lead to significant biases in estimating risk premium levels in spreads.

Risk premium and expected loss components of bond spreads behave quite differently from one another. As a proportion of the total spread, risk premia tend to be high in times of low spreads, whereas expected losses dominate during periods of high defaults. We are also able to shed light on recent events in credit markets. For example it appears that the spike in spreads after the LTCM episode in the late summer of 1998 is driven by increased risk premia rather than expected losses. On the other hand, the expected loss component is the dominant spread component during 2001, a period of unprecedented default losses in the US corporate bond markets.

Both spread components are intimately related to measures of volatility. The risk premium component appears to be closely tied to systematic volatility. The expected loss component on the other hand is closely tied to total risk. This is intuitive as idiosyncratic risk matters for default probabilities, while it should not influence risk premia. This allows us to comment on the results of Campbell & Taksler (2003). They document a period where corporate bond spreads increase in tandem with stock prices, an apparent contradiction. They attribute this to an increase in idiosyncratic equity volatility. We find support for this conclusion in that in our sample it is the expected loss component of spreads which increases, driven by an increase in total asset risk. In this period, market volatility exhibited no clear trend.

Implicitly, our study shows that structural credit risk models are useful tools in translating equity risk premia into the corporate bond specific counterparts. Although this casts the models in a favorable light, it begs the question why they have been relatively unsuccessful at explaining changes in corporate bond yield spreads, by relying on the variables implied by their specification. A candidate explanation that arises from our work is the strong degree of time variation that our measures of risk premia exhibit. Although structural models provide powerful cross-sectional predictions on the no-arbitrage relationship between debt and equity, they tend to be silent on both the level and the

time variation of the risk premium.

In an attempt to understand whether time varying risk premia can be part of the reason for the documented failure of structural models to explain credit spreads behavior, we carry out a regression analysis on credit spreads in the spirit of what has recently been done in the literature.⁸ We take as a benchmark a regression motivated by the key drivers implied by structural models. We find that augmenting the regressions by our measures of equity-implied risk premia improves explanatory power considerably, in particular for high grade bonds.

The paper is organized as follows. The following section reviews the literature and relates our paper to existing work. Section 3.3 describes our methodology for measuring bond risk premia in corporate bond markets and compares it to alternative approaches used in the literature. Section 3.4 explains how we translate equity risk premia into bond risk premia. In sections 3.5 and 3.5.3 we present our estimation methods and data. In section 3.6 we discuss our main findings, while section 3.6.3 presents our regression results. In section 3.7 we examine the implications of a structural model for empirical work on risk premia. Section 3.8 closes our study.

3.2 Related literature

In what follows we will review work related to corporate bond risk premia. The first papers we discuss deal with the link between equity and corporate bond returns for varying levels of financial distress risk. Next, we consider papers that deal more directly with the risk premium in credit markets.

As mentioned above, Fama and French (1993) find that equity return factors have difficulties in explaining corporate bond returns. In theory, however, bond and stock returns should be closely related. For example, if financial distress is imminent, bond and stock expected returns should both be high. Among others, Dichev (1998) and

⁸See among others Collin-Dufresne, Goldstein & Martin (2001), Campbell & Taksler (2003), Cremers, Driessen, Maenhout & Weinbaum (2004).

Campbell et al. (2008) have documented that stock returns for firms with high degrees of distress risk are surprisingly low. Garlappi et al. (2008) provide a justification for this finding based on the relative bargaining strengths of the parties in financial distress. On the other hand some studies have shown a common variation in the time-series of returns to portfolios of stocks and corporate bonds (see for example Keim & Stambaugh (1986) and Ferson & Harvey (1991)).

Elton, Gruber, Agrawal & Mann (2001) show that in addition to compensation for expected losses, corporate bond yield spreads appear to contain compensation for tax effects and that there is a non-trivial residual component, related to the Fama-French factors and thus interpreted as a risk premium. Our study differs from theirs in that we consider firm specific data, study the time series of both expected losses and risk premia; and most importantly we rely on a model which provides an exact non-linear relationship between risk premia in equity markets and those in bond specific yield spreads.

An interesting related paper by Huang & Huang (2002) measures how much of observed credit spreads over the Treasury curve can be explained by structural models. Their analytical approach allows for time varying risk premia but their study does not focus on measuring risk premium components in bond spreads. They find that it is difficult to reconcile observed and model spreads. Interestingly, Leland (2004) finds that a selection of structural models, faced with difficulty in explaining corporate bond prices, are in fact quite successful at predicting default probabilities consistent with historical levels.

Chen, Collin-Dufresne & Goldstein (2005) consider whether existing asset pricing models that have proven successful in explaining equity returns can, if reasonably calibrated, explain the levels and volatilities of credit spreads. They have some success with models that exhibit time varying risk premia, in particular if the default boundaries are permitted to be countercyclical. Perhaps the main conclusion of their paper is the necessity of time varying risk premia to explain credit spreads. Our study clearly illustrates the dramatic time variation of these premia in the marketplace and documents the

explanatory power of equity-implied risk premium estimates for bond market spreads.

Using a reduced form credit risk model, Driessen (2005) decomposes corporate bond yield spreads into tax, liquidity, interest rate risk and risk premium components. He finds that the ratio of risk neutral to objective default intensities is greater than one, suggesting that default event risk is priced. He obtains cross-sectional estimates of spread components and risk premia but does not explore the time series variation of risk premia nor does he explore the link with equity markets.

In a closely related paper, Berndt, Douglas, Duffie, Ferguson & Schranz (2004) (BDDFS) use expected default frequencies from Moody's KMV together with default swap prices to extract historical and risk neutral default intensities respectively. The ratio of these is interpreted as a measure of the risk premium observed in the marketplace. They document substantial time series variation in premia with a peak in the third quarter of 2002 and a subsequent dramatic drop. They show that their measure of the risk premium is strongly dependent on general stock market volatility after controlling for idiosyncratic equity volatility. They also find that their measure is increasing in credit quality. We document a similar behavior of the ratios of our risk neutral to objective default probabilities in our longer corporate bond sample, and show that this is in fact a prediction of the Leland & Toft (1996) structural credit risk model.

Berndt et al. (2006) (BLO) extract a factor representing the part of default swap returns, implied by a reduced form credit risk model, that does not relate to interest rate risk, expected default losses and the Fama-French factors. They find that this factor is priced in the corporate bond market but that they cannot establish with the same confidence that it is a factor for equity returns.

Saita (2006) studies the risk and return profiles of corporate bond portfolios using estimates of objective default probabilities obtained using a novel methodology.⁹ He finds strikingly high levels of expected excess returns that appear difficult to explain given the measured risks. For example, bond portfolio Sharpe ratios can be multiple times higher

⁹See Duffie et al. (2006).

than the corresponding measure for the S&P 500 index.

In summary, although many studies have attempted to relate pricing in corporate credit and equity markets, the precise link between risk premia in the two markets is not yet well understood.

3.3 Measuring risk premia with corporate bond data

In equity markets, expected returns are most often proxied by average historical returns. Measuring expected returns and risk premia in corporate bond markets is a more daunting task due to the absence of long historical time series of regularly spaced data. Perhaps as a result, researchers tend to focus on bond yield spreads instead. This leaves us with another complication which is that the yield spread is an imperfect measure of the risk premium - it requires an adjustment as we shall explain in detail below.

Some recent related empirical work on risk premia in credit markets have relied on reduced form models. To permit a comparison of those results with ours, we briefly outline, in an appendix, the key theoretical results on risk premia in that literature. Our objective is also to estimate risk premia, but we rely on a contingent claims approach. In a first step, we seek to understand if this approach, based on using information on firm specific equities and bond specific contractual details, is able to explain the behavior of the risk premium reported by Berndt et al. (2004), Saita (2006) and Berndt et al. (2006).

To study the corporate bond risk premium, two different approaches avail themselves. First, one can measure the expected excess return directly and second, one can compute the part of the bond spread that represents a risk premium. The second approach has the advantage of also allowing us to learn about the expected loss component in the promised yield which has been central in many recent studies.¹⁰

¹⁰See for example Elton et al. (2001).

3.3.1 Excess returns

In order to compute the return on a corporate discount bond that matures at time T , we would need to solve

$$R_B(t, T) - r(t, T) = \frac{E_t^P \left[\frac{B(v_T, T)}{B(v_t, T)} - 1 \right]}{T - t} - r(t, T)$$

where $R_B(t, T)$ is the expected return and $r(t, T)$ is the risk free rate with maturity $T - t$.

However, when computing risk premia for holding coupon bearing bonds until maturity, we need to account for coupons as well as the rate at which these coupons are reinvested. There is no received solution to this problem, perhaps because most work on corporate bonds has focused on yield spreads instead of returns.

We follow the methodology of Driessen & DeJong (2005) to estimate bond market implied risk premia. They show that

$$R_B(t, T) = [P_t(\tau < T)(1 - l) + (1 - P_t(\tau < T))](1 + y(t, T))^T - 1 \quad (3.1)$$

where $R_B(t, T)$ is the expected return on a corporate discount bond that matures at time T , τ is the default time, l is the proportional loss given default, $r(t, T)$ the relevant benchmark risk free rate and $y(t, T)$ the corporate bond yield-to-maturity. The objective probability of a default prior to maturity is denoted $P_t(\tau < T)$, where τ represents the default time.

This expression is valid for discount bonds for which default losses are incurred at maturity. Maintaining the default timing assumption, this expression still holds for coupon bonds if we are willing to assume that coupons are reinvested at the initial yield.¹¹ The approach is also valid if we assume that coupons are reinvested at today's

¹¹An analogous assumption is made when using yield-to-maturity as a measure of promised return in the government bond market.

prevailing forward rates. Similar procedures for calculating expected corporate bond returns have been applied by Elton et al. (2001) and Campello et al. (2006).¹²

To estimate $P_t(\tau < T)$, we rely on the Leland & Toft (1996) (LT) model together with historical estimates of firm specific asset value risk premia.¹³ Using aggregate data, Leland (2004) studies the ability of the LT model to predict default probabilities. He finds that the model is able to fit historical default experience for A, Baa and B rated debt reasonably well for horizons of 5 years and longer. The model underestimates shorter term default probabilities.

Recent work by Berndt et al. (2004) and Berndt et al. (2006) on risk premia relies on Moody's KMV expected one-year default frequencies.¹⁴ These are based on using a structural credit risk model to compute a firm's distance to default. This metric is then mapped into historical probabilities using an extensive database of default experience. Conceptually, this approach is very similar to our method of using the LT model to predict default probabilities.¹⁵ The key difference is that KMV only use the model to rank companies according to default riskiness, whereas we combine the model with firm risk premium estimates to arrive at default probabilities.

3.3.2 Yield spread components

An example

There is an important distinction between a bond risk premium and yield spread. Consider, for simplicity, a unit zero discount bond with zero recovery in default issued by a firm that can only default at time T . The value of that bond is $B(t) = e^{-r(t,T)T} E_t^Q [I_{\tau > T}]$,

¹²Campello et al. (2006) use an Ito's lemma approximation to compute the expected return from the duration and the convexities of bonds.

¹³Details of their estimation are provided below. The closed form solution for the default probability is provided in the appendix.

¹⁴Saita (2006) uses a different approach, based on Duffie et al. (2006), where default probabilities are allowed to depend on both firm specific and macro-economic variables. One of the firm specific variables is distance-to-default.

¹⁵See Leland (2004) for an interesting discussion of the two approaches.

or $e^{-r(t,T)T}Q_t(\tau > T)$, the present value of the risk adjusted survival probability.

Assume for the time being that default is not a priced risk. Then there will be no distinction between historical and risk neutral survival probabilities. Suppose $P_t(\tau > T) = Q_t(\tau > T) = 80\%$, where P_t denotes the objective survival probability. Assume further that $r(t, T) = 10\%$, $T = 10$. Then the price of the bond is $B = e^{-0.10 \times 10} 0.8 = 0.2943$ and its continuously compounded yield is 12.23%. Thus the bond pays a spread of 223 basis points, while there is no risk premium.

In other words, the presence of a positive yield spread by no means implies that there is a risk premium for default, merely an actuarially fair compensation for expected losses – in this case the present value of expected default losses is $EL = e^{-0.1 \times 10} 0.2 = 0.073576$.

Now consider an economy where default is a priced risk, implying that $P_t(\tau > T) > Q_t(\tau > T)$. Assume the same parameters as above except that $Q_t(\tau > T) = 70\% < P_t(\tau > T) = 80\%$. Now the bond price is lower at $B = e^{-0.10 \times 10} 0.7 = 0.25752$ and accordingly, the yield spread has increased by 134 to 357 basis points. This increase (denoted π) reflects the risk premium for bearing default risk.

Another way of expressing this is that the value of a bond can be written as either (i) the present value (at the risk adjusted rate) of the expected payment at maturity or (ii) the present value of the full face value discounted at the risk free rate augmented by a spread s . This spread contains both a component γ which adjusts for expected losses (in this example 223 basis points) and a risk premium part π (134 basis points):

$$e^{-(r(t,T)+\pi)T} E[B(v_T)] = e^{-(r(t,T)+s)T} 100$$

$$\text{with } s = \gamma + \pi$$

Note that the π component corresponds to the excess return $R_B(t, T) - r(t, T)$ discussed above.

Disentangling yield components in practice

In what follows we describe our methodology for measuring risk premia in yield spreads, π . The most common application of a structural model such as the Leland & Toft model is to use balance sheet information, together with estimates of asset value and volatility to infer term structures of risk neutral default probabilities, $\{Q_t(\tau < s); s \in (t, \infty)\}$. However, like Leland (2004), we use the model together with estimates of asset value risk premia to provide term structures of objective default probabilities, $\{P_t(\tau < s); s \in (t, \infty)\}$.

In order to disentangle the risk premium component from market bond spreads, we use these objective probabilities. Given knowledge of $\{P_t(\tau < s); s \in (t, \infty)\}$, we obtain an estimate of the price that would prevail in a market without risk premia:

$$\begin{aligned} B_{t,T} = & \sum_{i=1}^N d_i \cdot c_i \cdot (1 - P_t(\tau < s_i)) + d_N \cdot p \cdot (1 - P_t(\tau < T)) \\ & + R \cdot p \cdot \int_t^T d_s \cdot dP_t(s), \end{aligned} \quad (3.2)$$

where d_i are risk free discount factors, p the face value, c_i denote promised coupon payments and R represents the recovery rate in default.

In such a market the yield spread would only compensate for average losses. We call this spread the expected loss spread (c.f. the γ component above). The difference between the actual yield spread and the lower spread obtained using the price in (3.2) is our estimate of the risk premium components of corporate bond spreads.

In the next section we discuss our method for estimating bond risk premia without bond price data, using only equity and balance sheet data.

3.4 Estimating corporate bond risk premia with equity data

Since stocks and bonds are contingent claims on a firm's assets, it is natural to expect a large proportion of risk premia observed in the bond market to be explained by premia inferred from the equity market.

As noted above, common factors shown to explain equity returns have met with limited success in explaining corporate bond returns.¹⁶ However bond returns cannot be viewed as linear functions of stock returns. They will depend on the issuing firm's characteristics such as leverage and business risk while also incorporating information about bond specific contractual features. In the remainder of this section we will delineate our methodology for extracting equity-implied bond specific risk premia, while providing a brief explanation of the model we use in the process.

To estimate excess returns and bond spread components, we need a model that give us the price of the equity and the bond as well as the sensitivity of the equity and the bond with respect to the value of the asset at any time t . We will base our discussion on the LT model, although this is not crucial to the implications. Leland & Toft (1996) assume that the value of a firm's assets evolves as a geometric Brownian motion:

$$dv_t = (\mu_v - \beta) v_t dt + \sigma v_t dW_t$$

where β is the payout ratio, σ is the volatility of the asset value return and W_t is a Brownian motion.

Default is triggered by the shareholders' endogenous decision to stop servicing debt. The value of the firm differs from the value of the assets by the values of the tax shield and the expected bankruptcy costs. Coupon payments are tax deductible at a rate τ and

¹⁶For example, Fama & French (1993) show that common factors in the equity market have some explanatory power for the bond market but mainly when augmenting the set of equity factors with bond market factors (term structure and default premium).

the realized costs of financial distress amount to a fraction α of the value of the assets in default (i.e. L). The firm continuously issues debt of maturity M , while retiring older vintages. Hence, at any given time, the firm has many overlapping debt contracts outstanding. The LT framework allows closed form solutions for the value of the firm's equity and liabilities.¹⁷ In addition, it allows us to derive straightforward closed form solutions for a bond's price as well as its sensitivity to changes in the asset value. These will prove useful in the next step as we turn to computing excess returns.

3.4.1 Equity-implied bond excess returns

Following Campello et al. (2006), we use the Euler equation together with Ito's lemma and explicitly link the risk premia for stocks and bonds. The key to this approach is that it allows estimates of instantaneous expected equity return $R_S(t)$ to be translated into bond specific instantaneous expected return $R_B(t)$. Note that this relation requires only the existence of a state price density and that the mean rate and the volatility of the asset return are functions of time and the value of the asset itself only. More precisely

$$(R_B(t) - r) = \Delta_{B/S} \cdot (R_S(t) - r), \quad (3.3)$$

$$\text{where } \Delta_{B/S} = \left(\frac{\frac{\partial B(v_t, t)}{\partial v_t} S(v_t, t)}{\frac{\partial S(v_t, t)}{\partial v_t} B(v_t, t)} \right),$$

$$R_S(t) dt = E_t \left[\frac{dS(v_t, t)}{S(v_t, t)} \right] \text{ and}$$

$$R_B(t) dt = E_t \left[\frac{dB(v_t, t)}{B(v_t, t)} \right]$$

where S and B denote stock and bond prices respectively. We use the Leland & Toft (1996) model for the sensitivities $\frac{\partial B(v_t, t)}{\partial v_t}$ and $\frac{\partial S(v_t, t)}{\partial v_t}$ and the term structure of risk-adjusted

¹⁷The relevant expressions are reproduced in the appendix.

default probabilities used in pricing the bond.

The key determinants of the bond risk premium in (3.3) are (i) the premium in the equity market, (ii) the characteristics of the firm and (iii) the contractual features of the bond.

For example, two bonds issued by the same firm may have different expected excess returns simply due to differences in maturity and cash flow structure. Identical bonds issued by two different firms with the same objective default probability (as measured by the credit rating) may be different depending on the degree of systematic risk at the firm level (e.g as measured by beta). To date most empirical work has ignored bond characteristics and intra rating category differences in systematic risk.

We now turn to our second risk premium metric: the part of a bond's yield spread that compensates for systematic risk.

3.4.2 Equity-implied yield spread components

We have already discussed how to compute the bond price that would result in a market without systematic risk. In a market with risk premia on the other hand the bond price is given by

$$\begin{aligned}
B_t = & \sum_{i=1}^N d_i \cdot c_i \cdot (1 - Q_t(\tau < s_i)) + d_N \cdot p \cdot (1 - Q_t(\tau < T)) \\
& + R \cdot p \cdot \int_t^T d_s \cdot dQ_t(s),
\end{aligned} \tag{3.4}$$

where d_i are risk free discount factors, p the face value, c_i promised coupon payments and R the recovery rate, respectively. Risk adjusted probabilities are denoted $Q(\cdot)$.

The difference between the model yield spread obtained using the price in (3.4) and the lower spread obtained using the price in (3.2) defines our equity implied measure of the risk premium component of a corporate bond spread.

We now turn to the empirical implementation of our framework.

3.5 Empirical implementation

In this section we will describe our estimation methodology for equity-implied and bond market measured risk premia.

Since we do not observe government bond yields or swap rates for all relevant maturities, we estimate the term structure of default free zero coupon interest rates using the extended Nelson & Siegel form due to Svensson (1995):

$$r(t, T) = \delta_{1,t} + \delta_{2,t} \frac{1 - e^{-\delta_{3,t}(T-t)}}{\delta_{3,t}(T-t)} + \delta_{4,t} e^{-\delta_{3,t}(T-t)} + \delta_{5,t} \frac{1 - e^{-\delta_{6,t}(T-t)}}{\delta_{6,t}(T-t)}$$

Each day from 1990 to 2004 we estimate the parameters $\delta_{1,t}, \delta_{2,t}, \delta_{3,t}, \delta_{4,t}, \delta_{5,t}, \delta_{6,t}$ by minimizing the sum of squared bond pricing errors for constant maturity treasury yields and interest rate swap yields.¹⁸

3.5.1 Estimating structural credit risk models

From equation (3.3), it is clear that to estimate equity implied bond risk premia, we require estimates of equity risk premia as well as the price of the bonds and the sensitivities of the bond and the equity with respect to the asset value. This is equally valid when disentangling risk premia from bond yield spreads.

Bond prices and sensitivities

In addition to benchmark term structures, the following inputs are needed to price bonds, and to compute the sensitivity of the stock and the bond with respect to the value of the assets

- the bond's principal amount, p , the coupons c , maturity T and the coupon dates, t_i ;

¹⁸For robustness, alternative term structure specifications have been used - including cubic splines with smoothness conditions ruling out negative forward rates. The specification has negligible results for our study.

- the recovery rate of the bond, ψ ;
- the total nominal amount of debt, N , coupon C and maturity M
- the costs of financial distress, α
- the tax rate, τ
- the rate, β , at which earnings are generated by the assets, and finally
- the current value, v , and volatility of assets, σ

The bond's principal amount, p , the coupons c , maturity T and the coupon dates are readily observable. The recovery rate of the bond in financial distress is not. We set it equal to 40%, roughly consistent with average defaulted debt recovery rate estimates for US industrial entities between 1985-2003.

The nominal amount of debt is measured by the total liabilities as reported in COMPUSTAT. Since book values are only available at the quarterly level, we linearly interpolate in order to obtain daily figures. For simplicity, we assume that the average coupon paid out to all the firm's debt holders equals the risk-free rate: $c = r \cdot N$.¹⁹ We set the maturity of newly issued debt equal to 6.76 years, consistent with empirical evidence reported in Stohs & Mauer (1994).

Finally, we assume that 15% of the firm's assets are lost in financial distress before being paid out to debtholders and fix the tax rate at 20%. The choice of 15% distress costs lies within the range estimated by Andrade & Kaplan (1998). The choice of 20% for the effective tax rate is consistent with the previous literature (see e.g. Leland (1998)) and is intentionally lower than the corporate tax rate to reflect personal tax benefits to equity returns, thus reducing the tax advantage of debt.

The payout rate β is an important parameter. We compute β as the weighted average of net of tax interest expenses (relative to total liabilities (TL)) and the equity dividend

¹⁹This assumption is made for convenience. We checked this assumption by considering randomly selected firms' actual interest expense ratios. We found that our approximation performs well.

yield (DY):

$$\beta = \frac{IE}{TL} \times lev \times (1 - TR) + DY \times (1 - lev) \quad (3.5)$$

where

$$lev = \frac{TL}{TL + MC}$$

where MC denotes the firm's equity market capitalization and TR is the effective tax rate. The average net debt payout rate in our sample is 2.9%.²⁰

We then require estimates of asset value and volatility. The methodology utilized was first proposed by Duan (1994). The maximum likelihood estimation relies on a time series of stock prices, $E^{obs} = \{\mathcal{E}_i^{obs} : i = 1 \dots n\}$. If we let $w(\mathcal{E}_i^{obs}, t_i; \sigma) \equiv E^{-1}(\mathcal{E}_i^{obs}, t_i; \sigma)$ be the inverse of the equity function, the likelihood function for equity can be expressed as

$$\begin{aligned} L_{\mathcal{E}}(\mathcal{E}^{obs}; \sigma, \mu) &= L_{\ln v}(\ln w(\mathcal{E}_i^{obs}, t_i; \sigma, \mu) : i = 2 \dots n; \sigma) \\ &\quad - \sum_{i=2}^n \ln v_i \left. \frac{\partial \mathcal{E}(v_i, t_i; \sigma)}{\partial v_i} \right|_{v_i = w(\mathcal{E}_i^{obs}, t_i; \sigma)} \end{aligned} \quad (3.6)$$

$L_{\ln v}$ is the standard likelihood function for a normally distributed variable, the log of the asset value, and $\frac{\partial \mathcal{E}_i}{\partial v_i}$ is the “delta” of the equity formula. An estimate of the asset values is computed using the inverse equity function: $v_t = w(\mathcal{E}_n^{obs}, t_n; \hat{\sigma})$. Once we have obtained the pair $(\hat{v}_t, \hat{\sigma})$ it is straightforward to compute equity and debt values as well as sensitivities. See appendix.

²⁰ An alternative method for estimating the cash flow rate is to use bond coupons as a proxy for the firm's proportional interest expenses. Our estimates of the cash flow rate will be lower than if we had used this approach. First, coupons are pre-tax and second corporate bonds are long term instruments. While the bond coupon may proxy well for the interest expense on long term liabilities, we find that in our sample it overestimates the interest expenses paid on short term debt. Our average net of tax interest expense ratio is about 3% which is just less than half the average bond coupon of 7.2% in our sample.

3.5.2 Estimating default probabilities

Above, we describe our methodology for extracting observed risk premia. To apply equation (3.1), we need to estimate bond spreads, loss rates and have a proxy for the default probabilities for different horizons. This is also necessary for disaggregating spreads into risk premia and expected losses.

We set the loss rate l equals to 60%, roughly consistent with average defaulted debt recovery rate estimates for US entities between 1985-2003. Like much of previous work, our paper is limited by not considering stochastic recovery rates.

Previous studies on the default risk premium Berndt et al. (2004), Saita (2006) and Berndt et al. (2006) used Expected Default frequencies (EDFs) provided by Moody's KMV as their estimate of the historical default probabilities. In this paper, we estimate company specific default probabilities using the Leland & Toft (1996) model. This methodology yields estimates conceptually similar to EDFs.

The default probabilities $P_t(\tau > T_i)$ are provided in closed form in the appendix. The only parameter that still needs to be estimated at this point is the expected return of the asset value under the objective measure denoted μ_v .

Previous studies such as Leland (2004) and Huang & Huang (2002) have used the CAPM beta of the firm multiplied with an average market risk premium figure to provide an estimate of the expected asset return. In contrast, we use the same methodology that we applied above to link the bond risk premium and the equity risk premium. Equity is a contingent claim on the asset value and we can write

$$\begin{aligned} \mu_v - r &= (R_v(t) - r) = \Delta_E \cdot (R_E(t) - r) \\ \text{where } \Delta_E &= \left(\frac{\frac{\partial E(v_t, t)}{\partial v_t} v}{E(v_t, t)} \right)^{-1} \end{aligned}$$

where $\frac{\partial E(v_t, t)}{\partial v_t}$ is computed using the LT model and $(R_E(t) - r)$ is the estimated equity risk premium. Equity risk premia are constructed using average realized returns. For every transaction in our bond data, we check if 1200 daily returns prior to the transaction

date are available. Otherwise a bond is dropped.

3.5.3 Data

We use the following data for our estimation: firm market equity values, balance sheet information, and term structures of swap rates. Daily equity values are obtained from CRSP. Quarterly firm balance sheet data are taken from COMPUSTAT. Since balance sheet information is only available at quarterly level, we transform it into daily data through linear interpolation. Swap rates are acquired from DataStream. We take the US constant maturity Treasury rates from the Federal Reserve Board data archive.

Our bond transaction data are sourced from the National Association of Insurance Commissioners (NAIC). Bond issue- and issuer-related descriptive data are obtained from the Fixed Investment Securities Database (FISD). The majority of transactions in the NAIC database take place between 1994 and 2004. Cleaning of the raw NAIC database was carried out in four steps.

1. Bond transactions with counterpart names other than clearly recognizable financial institutions were removed. Transactions without a clearly defined counterparty were deemed unreliable.
2. We restricted our sample to fixed coupon rate USD denominated bonds with issuers in the industrial sector. Furthermore, we eliminated bond issues with option features, such as callables, putables, and convertibles. Asset-backed issues, bonds with sinking funds or credit enhancements were also removed to ensure bond prices in the sample truly reflect the underlying credit quality of issuers.
3. We eliminated bonds issued by Municipal, provincial, and any special agencies to ensure that the bond prices in the sample are cleaned from any special tax treatment inherent the issuer type.
4. The last step involves selecting those bonds for which we have their issuers' complete and reliable market capitalizations as well as accounting information about

liabilities. Then we eliminated those bonds for which there is less than 1200 previous daily equity returns available, for the purpose of computing the equity risk premia at that transaction date.

3.6 Empirical results

We begin, in Table 1, by describing, on an aggregate basis, the inputs to our estimation (panel A) and the intermediate firm specific outputs (panel B). The data covers a wide variety of firms and bonds. Firm sizes vary between just over 130 million dollars to just less than half a trillion. Leverage ratios range from naught to almost 100%. The bonds vary widely in maturity (between a few months and 100 years), in yield spreads (1 to 1050 basis points relative to the Treasury curve) and credit rating (AAA to defaulted). Our MLE estimation yields estimates of firm asset values of on average 49 billion dollars, asset volatilities of 20% on average. Our estimates of asset volatility are consistent with previous work by Schaefer & Strebulaev (2004). Figure 1 plots their estimates and ours by rating categories. Our across sample average is somewhat lower but the pattern across rating categories is strikingly similar with the exception of the lowest category.²¹

²¹We only have about 130 transactions, most of which relate to one firm in our CCC category, whereas they have more than 1600. This suggests that the noticeable difference between our estimates in this particular category may be outlier driven.

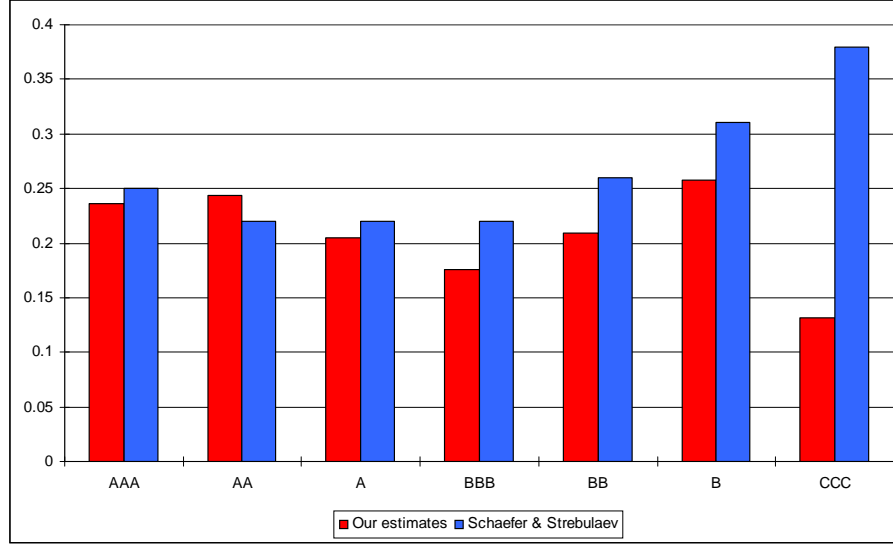


Figure 1

Table 1: Summary statistics

Panel A. Summary statistics of firm specific variables.					
	<i>N</i>	Mean	Std. Dev.	Min	Max
Firm size (USD billion)	34,414	51,610	78.88	0.13	471.35
Leverage (%)	34,414	45.48	20.28	1.31	96.83
# of transactions / firm	34,414	216	285	1	1947
Bond maturity (<i>T</i>)	34,414	11.80	9.77	0.01	100
Bond yield spread (bps)	34,414	146	131	1	1055
S&P rating	34,414	8.3	4.4	1	27
Historical equity volatility (%)	34,414	22.26	6.31	9.94	45.74
Firm equity β	34,414	0.86	0.24	-0.12	1.92

Panel B. Summary statistics of estimated firm specific characteristics

	N	Mean	Std. Dev.	Min	Max
Model yield spread		97	151	0	1077
asset value (v_t) (USD million)		48,815	74,552	119.72	442,031
Asset volatility (σ) (%)		20.24	10.45	2.19	89.93
Default threshold (L)		16,163	36,209	0.69	391,350

Before moving to our estimates of risk premia in corporate bond markets, we present intermediate results on our estimation of objective default probabilities. For our risk premium estimates to be quantitatively reasonable, the employed default probabilities need to be as well. Note that most work prior to Berndt et al. (2004) rely on historical rating based default probabilities. For example Elton et al. (2001) use a constant rating transition matrix for a ten year period and across all firms within a rating category. Recent work by Campello et al. (2006) relies on time varying rating based default probabilities. Our approach permits us to capture simultaneously the variation in default probabilities across time and across firms within rating categories.

To begin, we benchmark our estimates of default probabilities across horizons and rating categories. Figure 2 plots our estimates together with historical averages provided by Moody's cumulative default rates for the period 1920-2004. Given that the sample periods are very different, it is not clear what to expect. It is interesting to note that for the largest rating category in our sample, BBB, our average estimates across the 20 different years are quite similar to historical averages.

Table 2 provides a more detailed overview of the relationship between our estimated default probabilities and historical averages per rating category and horizon.

	Mod	Actual	Mod	Actual	Mod	Actual	Mod	Actual	Mod	Actual
	AA	AA	A	A	BBB	BBB	BB	BB	B	B
1	0.00%	0.06%	0.12%	0.08%	0.42%	0.31%	0.32%	1.39%	1.08%	4.56%
2	0.00%	0.19%	0.34%	0.25%	1.29%	0.93%	1.63%	3.36%	3.96%	9.97%
3	0.02%	0.32%	0.71%	0.54%	2.32%	1.69%	3.34%	5.48%	7.06%	15.24%
4	0.07%	0.49%	1.21%	0.87%	3.41%	2.55%	5.11%	7.71%	9.93%	19.85%
5	0.14%	0.78%	1.76%	1.22%	4.50%	3.40%	6.80%	9.93%	12.48%	23.80%
6	0.23%	1.11%	2.34%	1.58%	5.54%	4.28%	8.38%	12.01%	14.74%	27.13%
7	0.34%	1.48%	2.93%	1.98%	6.53%	5.12%	9.83%	13.84%	16.74%	30.16%
8	0.45%	1.85%	3.51%	2.34%	7.45%	5.95%	11.16%	15.65%	18.51%	32.62%
9	0.57%	2.20%	4.07%	2.76%	8.32%	6.83%	12.37%	17.25%	20.08%	34.74%
10	0.70%	2.57%	4.61%	3.22%	9.12%	7.63%	13.48%	19.00%	21.49%	36.51%
11	0.83%	3.01%	5.13%	3.71%	9.87%	8.42%	14.50%	20.60%	22.77%	38.24%
12	0.95%	3.50%	5.63%	4.21%	10.56%	9.22%	15.43%	22.16%	23.92%	39.80%
13	1.07%	3.98%	6.10%	4.65%	11.21%	10.00%	16.29%	23.72%	24.96%	41.23%
14	1.19%	4.48%	6.55%	5.09%	11.82%	10.70%	17.08%	25.10%	25.92%	42.67%
15	1.31%	4.87%	6.97%	5.56%	12.38%	11.32%	17.82%	26.31%	26.80%	43.92%
16	1.42%	5.13%	7.38%	6.02%	12.92%	11.91%	18.50%	27.44%	27.60%	45.21%
17	1.53%	5.35%	7.76%	6.30%	13.41%	12.51%	19.14%	28.59%	28.35%	46.15%
18	1.64%	5.57%	8.13%	6.60%	13.88%	13.04%	19.73%	29.70%	29.04%	46.89%
19	1.74%	5.87%	8.47%	6.89%	14.32%	13.49%	20.28%	30.58%	29.69%	47.52%
20	1.84%	6.09%	8.80%	7.19%	14.74%	13.95%	20.80%	31.48%	30.29%	47.79%

Table 2. Historical and model predicted default probabilities by rating categories and horizon. The historical probabilities represent Moody's cumulative default rates for 1920- 2004.

Some previous work has relied on KMV expected default frequencies (EDFs) as measures of objective default probabilities. As noted above, KMV use a structural model to estimate firm specific default metrics which are mapped into default probabilities using historical default experience. Given the relative similarity of historical default experience as reported by Moody’s and our estimates of default probabilities we don’t expect a systematic bias to be induced by our methodology relative to using EDFs. Hence we expect that our results can be related to those reported in BLO and BDDFS. In what follows, we turn to a discussion of our estimated risk premium metrics.

3.6.1 Risk premia measured in bond markets

In table 3 we report the two measures of bond risk premia discussed above: the expected excess return and the risk premium component of bond yield spreads.

Consider first our measure of risk premia based on equation (3.1). Figure 3 plots the time series of average excess return imputed from bond market spreads using the methodology in section 3.3.

Table 3: Market measured bond risk premium metrics

	N	Mean	Std. Dev.	Min	Max
Bond Excess return $E[R_B(t) - r]$ measured in bond markets (bps)	34414	77	110	-821	950
Spread RP component measured in bond markets	34414	91	123	-822	1219

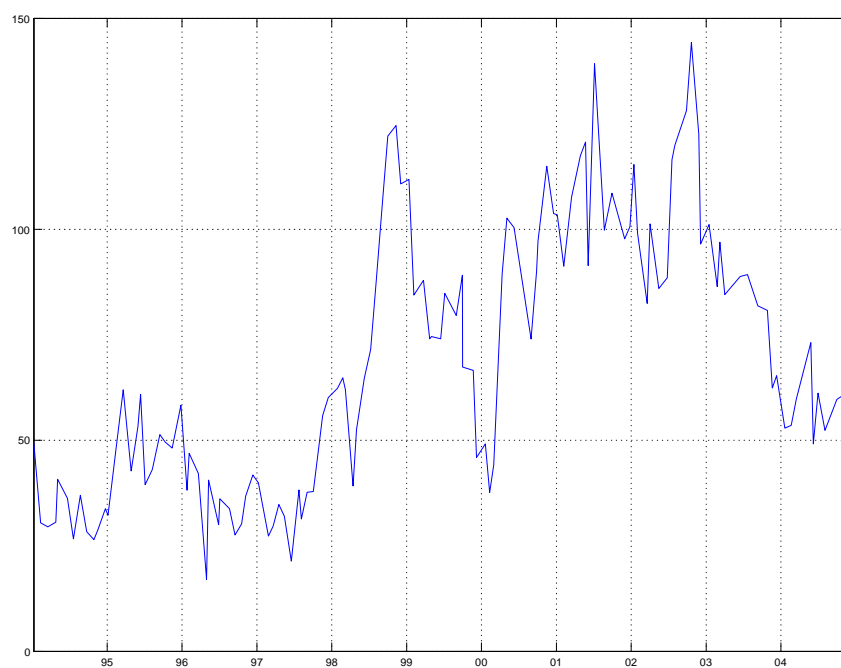


Figure 3: Monthly average bond market measured excess returns 1994-2004. For each bond the excess return is computed using equation (3.1).

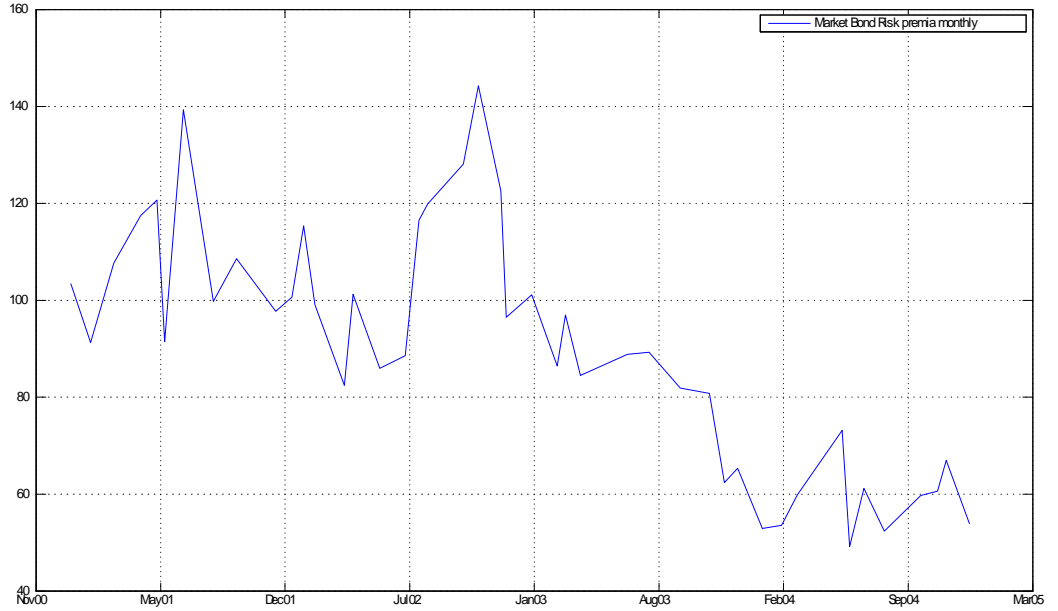


Figure 4: Monthly average excess returns for the sample period in Berndt et al. (2004).

Clearly this measure of the bond risk premium is highly time varying. There is a peak after the LTCM crisis in late 1998 followed by a sharp drop until early 2000. After late 2000, the overall level seems to have shifted up to a higher level of about 100 basis points, then decreasing until the end of the sample.

BDDFS study the period 2001-2004. For ease of comparison, Figure 4 shows the same risk premium metric as Figure 3 for the same sample period. They document a peak in the third quarter of 2002 with a steady drop until the end of 2003, in particular for the broadcasting industry. We find the same pattern, although for the whole of our sample: the risk premium peaks at 140 basis points decreasing to levels of about 50 basis points by the end of that year. The observed similarity in patterns is all the more striking when one considers that we are using a dataset with a different cross section of firms, a different financial instrument (bonds rather than credit swaps) and employ a different methodology. We interpret this as an implicit robustness test of our default probability estimates, which we argued above should be similar to the EDFs used by BDDFS.

We are also able to identify two earlier peaks in risk premia: one after the LTCM period in 1998 and another lesser in mid 2001. The post LTCM peak is followed by a drop in premia of the same magnitude as the 2002 episode. All three peaks appear to be short-lived, lasting no longer than 2-3 months.

Next we turn to a decomposition of bond yield spreads. We separate the risk premium component from the total spread. This will provide a robustness check on our results and express them in an easily interpretable unit. Another important motivation is that this allows us to analyze the impact of expected losses on bond yield spreads. As we shall see the two components behave quite differently.

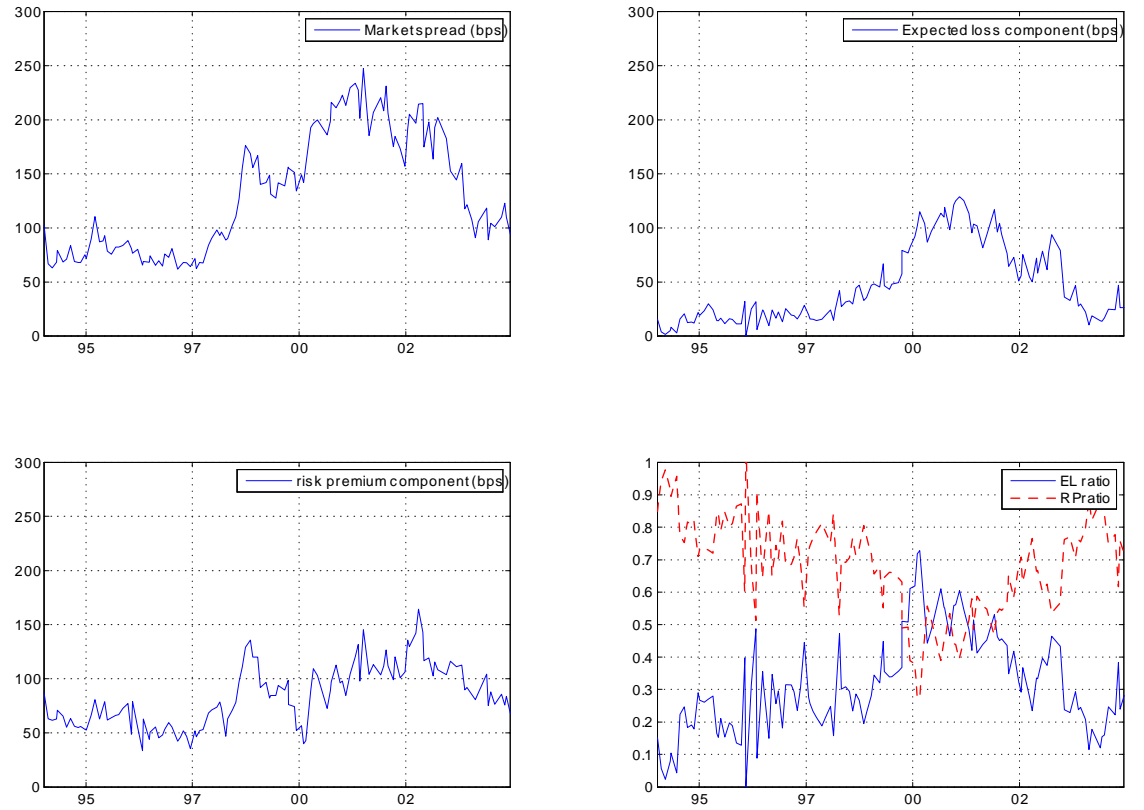


Figure 5. Overview of estimated market spread components. Panel A plots the monthly

average market spread relative to the CMT curve for our full sample. Panels B and C plots the expected loss and risk premium components respectively. Panel D plots the ratio of the two components to the total spread.

Panel A of figure 5 plots the average monthly market bond spread in our sample. Spreads were stable during the first part of our sample, whereas the post LTCM period is marked by a dramatic increase in spread levels and volatility. Considering the expected loss spreads in panel B, we find that these peak quite a bit later - towards the beginning of 2000. Panel C reports the risk premium component of the spreads in panel A. The risk premium component does not simply mirror the behavior of the spread. For example it appears that the spike in spreads at the end of 1998 is driven by increased risk premia rather than expected losses. Post LTCM, risk premia decline to reach a low of just less than 50 basis points in early 2000, whereas the expected loss component on the other hand is reaching a high plateau which persists during 2001.

It is interesting and reassuring to note that the spread risk premium component in panel C behaves similarly to the measured excess return in figure 3. A striking way of depicting the relative importance of risk premia and expected losses is provided in panel D. It plots the respective percentage of the total spread explained by risk premia and expected losses. In the earlier part of the sample, the dominant component of the spread is the risk premium. Its importance trends downwards until the beginning of 2000 when it begins to recover and eventually reach a level of about 75% towards the end of the sample. The period when the expected loss component is the most important coincides with a period of unprecedented default losses in the US corporate bond markets.

In summary, we have measured risk premia in corporate bond markets. Our results are consistent with previous findings based on reduced form models in credit derivative markets for those subperiods when our samples overlap. So far we have used a structural model to estimate default probabilities. We now wish to see if a structural model taken together with estimates of risk premia in equity markets can explain the observed risk premia in credit markets.

3.6.2 Risk premia inferred from equity markets

We now discuss the results for our bond risk premia estimated from equity risk premia. In brief, bond risk premia can be thought of as a non-linear translation of equity risk premia. In a general contingent claims model of a firm's security prices, we can derive a relationship between risk premia on bonds, stocks and firm values (see equation (3.3) above). To operationalize this relationship we rely on the Leland & Toft (1996) model as a link between equity prices, asset value and volatility.

Most work in asset pricing measures excess returns relative to Treasury rates. In fixed income markets, the choice of benchmark rate is an important and more subtle issue. In fixed income derivatives markets, practitioners typically rely on interest rate swap rates to construct a reference curve. In the cash market, corporate bond spreads are also often measured against this curve. One reason for this is the arbitrage relationship between credit default swaps and corporate bonds. The argument links the basis point price of default protection with the spread of a corporate bond over a floating rate benchmark, in practice the swap curve.²² Recent work suggests that structural models are able to predict the level of default protection prices and that of bond spreads over the swap curve, while underestimating spreads relative to the government curve.²³ For our purposes in this paper, we need an unbiased model of the bond spread in order to avoid a bias in the decomposition of spreads into risk premia and expected loss components respectively. This leads us to rely on the swap curve as the key benchmark, but initially we also report results for the government benchmark curve for completeness.

As an input, we require an estimate of the risk premium in equity markets, which we obtain as described above. Table 4 summarizes the inputs as well as key outputs of this exercise. The average equity risk premium during our sample is 10.3%. This translates into an implied bond excess return of 80 basis points for the government curve and 63

²²For details see e.g. Rajan et al. (2007).

²³See Ericsson et al. (2006). Choudhry (2006) provides an interesting discussion of the default swap basis - the differential pricing of credit risk in bond and credit derivative markets.

basis points for the swap curve. When yield spreads are measured against the Treasury curve, 41 basis points of that spread represents compensation for default risk as implied by the premium in equity markets. When the swap curve is used, the risk premium spread is 34 basis points. Table 5 provides a summary of our bond market measured risk premia based on the swap curve.

The equity implied excess returns appear able to capture the excess returns in the bond market on average, regardless of the benchmark employed. For the spread components, the choice of benchmark will be a key determinant of the average levels.

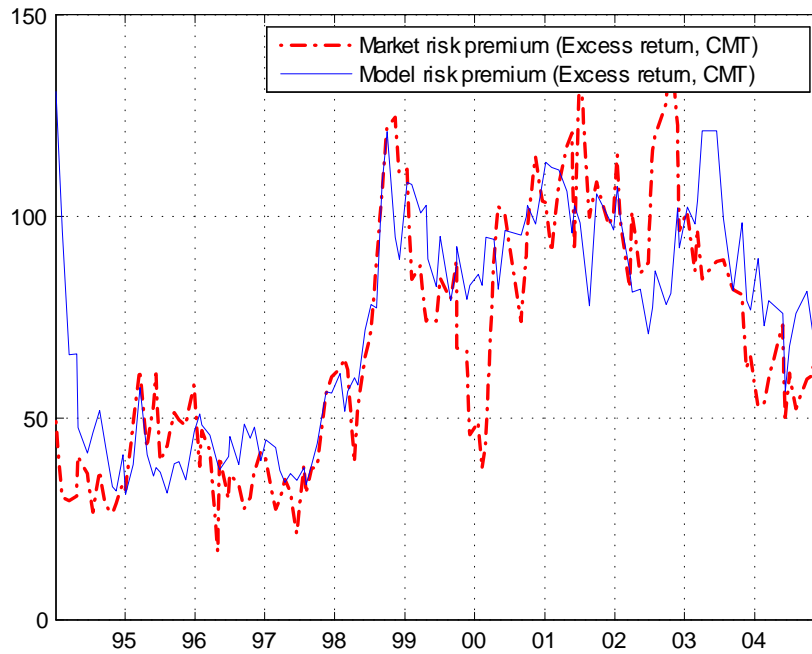
Table 4: Equity implied bond risk premium metrics

	N	Mean	Std. Dev.	Min	Max
$\Delta_{B/S}$		0.078	0.071	0	0.283
Equity risk premium (%)	34414	10.30	2.93	-1.47	23.08
$(R_S(t) - r)$					
Bond Excess return $E[R_B(t) - r]$	34414	80	78	-6	486
estimated from equity data (CMT)					
Spread RP component	34414	41	49	-89	1158
estimated from equity data (CMT)					
Bond Excess return $E[R_B(t) - r]$	34414	66	70	-6	473
estimated from equity data (Swap curve)					
Spread RP component	34414	34	43	-86	342
estimated from equity data (Swap curve)					

Table 5: Market measured bond risk premium metrics (Swap curve)

	N	Mean	Std. Dev.	Min	Max
Bond Excess return $E[R_B(t) - r]$ measured in bond markets (bps)	34414	63	115	-788	985
Spread RP component measured in bond markets	34414	51	121	-760	1031

Figure 6 plots our measured excess returns in bond markets and our equity implied bond excess returns. Clearly, on average the fit is rather good.²⁴ There are period of divergence, such as the period 1999-2000 where our model overpredicts the risk premium and in late 2002 when it underestimates the premium. The latter time period coincides with the period during which BDDFS document, like us, a sharp increase in the risk premium. This figure suggests that across firms on average, there is a clear relationship between equity and corporate bond risk premia.



²⁴Although not reported, a similar pattern is found when the swap curve is used.

Figure 6: bond market measured and equity implied bond excess returns. Based on using the CMT curve as risk free benchmark.

Figure 7 plots the average raw equity risk premia that are used as inputs in equation (3.3) together with the average bond risk premia. It is clear that bond risk premia are highly non-linear translations of risk premia for the corresponding stocks. Note for example the period 1998-2003. During this period equity risk premia trended slowly downwards, while bond risk premia in fact did the opposite in a less regular fashion. In addition the volatility of the bond risk premium seems more variable than that of the equity premia. In a structural model, keeping leverage and volatilities constant the relationship between the two should be positive, just like for spreads - when stock prices increase bond prices should increase as well. Thus, the explanation must lie in time variation in either leverage or volatility. We return to a more detailed discussion of this below, but note that this result is related to findings in Campbell & Taksler (2003). They find that during the late nineties, bond spreads increased as stock markets were performing well, an apparent contradiction.

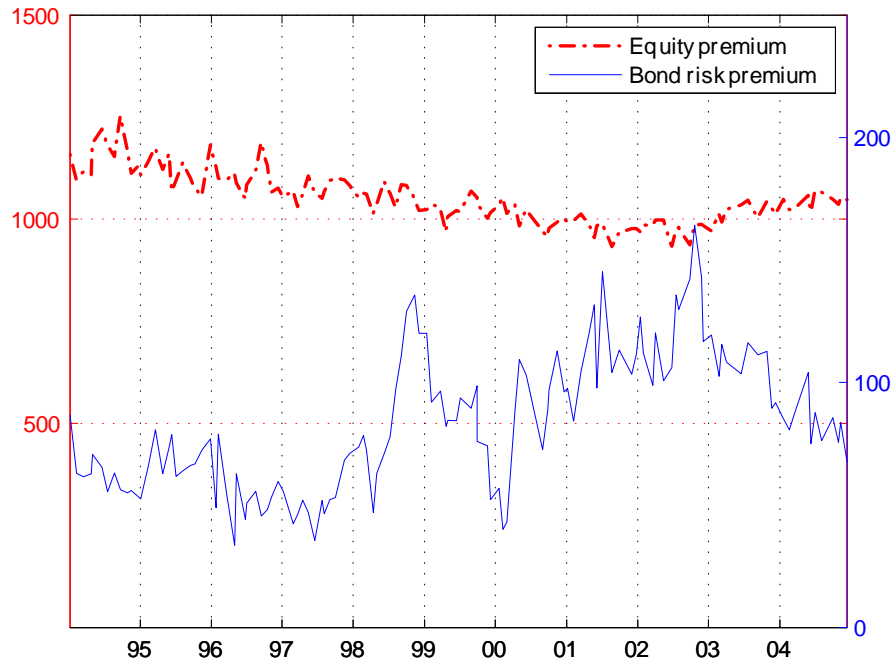


Figure 7: Equity and corporate bond risk premia. Equity risk premia are measured using the Fama & MacBeth (1973) methodology and bond risk premia using equation (3.1).

In order to provide a more disaggregated view of risk premia, Figure 8 reports a breakdown of risk market and model premia across 8 industries: manufacturing, media, oil and gas, rail, retail, services, transportation and telephone. The equity implied bond excess returns track their bond market measured counterparts quite well for those industries that represent a large fraction of our dataset, in particular manufacturing.



Figure 8: market and model risk excess return across industries. Monthly averages. The CMT curve is used as benchmark. Results based on excess returns using the swap curve as benchmark are, although not reported, very similar.

As discussed above, an alternative way of measuring the risk premium is achieved by decomposing bond yield spreads into an expected loss component and a risk premium component. Much of the academic work on bond spreads has relied on Treasury securities as risk free assets. In contrast, most practitioners will argue that a more informative measure of yield spreads is obtained when using the swap curve as a benchmark. In

addition it has recently been shown in the literature on structural models that the oft documented underprediction appears to be related to the choice of benchmark curve.²⁵

We find the same result - when the Leland & Toft model is used to predict bond yield spreads, it fits well on average when the swap curve is used and underestimates when the Treasury curve is used. Figures 9 & 10 clearly illustrate this result. There is no systematic gap between average market and model spreads relative to the swap curve. That is not to say that the model cannot at times under or overpredict as it does interchangeably as of 2000. The spread underestimation as of mid 2001 can at least partially be explained by recent findings in Feldhütter & Lando (2007). They decompose swap spreads - the difference between fixed rates on interest rate swaps and corresponding Treasury yields - into three components: a convenience yield for holding Treasuries, a credit risk component and a swap market specific factor. The period of extremes in spread underprediction in our findings coincide with a period when they document an unusually negative swap factor which yields low swap rates. They provide an explanation based on hedging activity in the mortgage backed securities market. Thus the observed spreads in our sample appear higher than they should be, had the swap rate been a better proxy for the risk free rate.

²⁵See Ericsson et al. (2006).

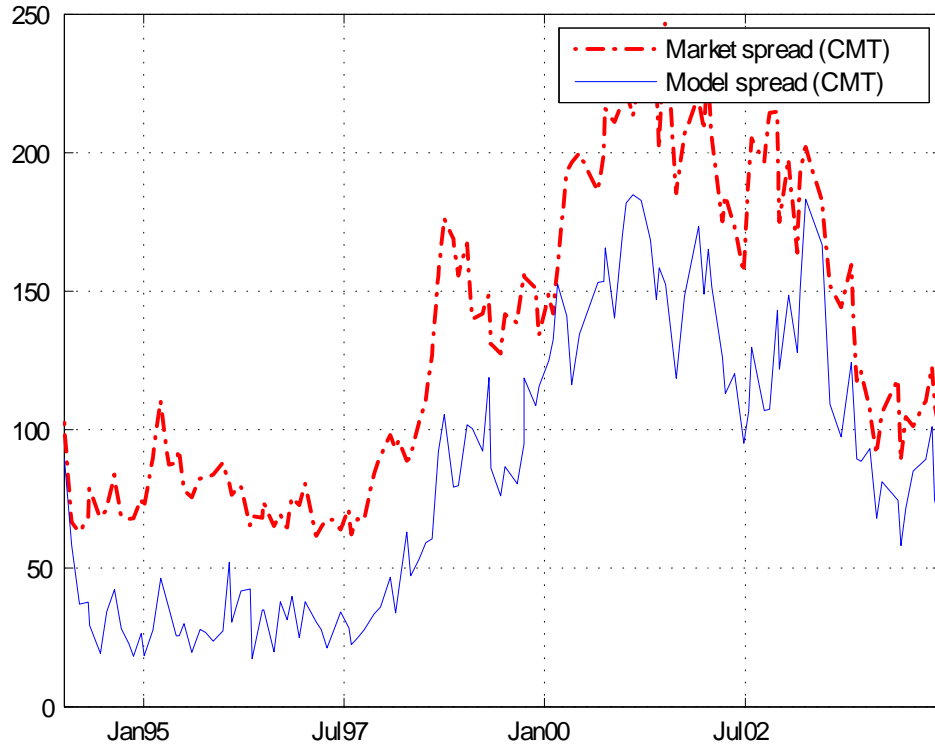


Figure 9: Monthly averages of model and market bond spreads relative to the CMT curve.

Although imperfect, the swap curve appears to be the better choice for our purposes. In addition to mitigating the influence of Treasury market liquidity effects it removes the need to correct for differential taxation between corporate bonds and treasury bonds.²⁶ We will from now on report only results based on this curve.

²⁶See Elton et al. (2001).

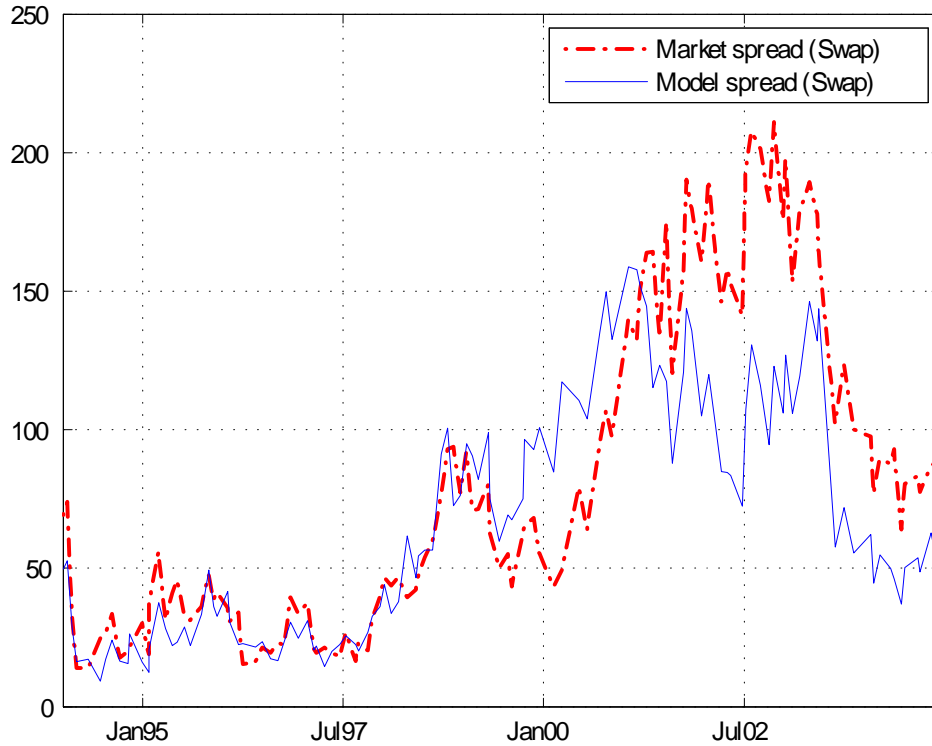


Figure 11: Monthly averages of model and market bond spreads relative to the swap curve.

Figure 12 plots the equity implied risk premium in spreads vs. the premium inferred from bond spreads. The model predicted risk premium component tracks the market measured quite well to begin with, but under and overestimates in the same way as the spread in the second part of the sample. For example, if hedging demand in MBS biases spreads over the swap curve, then this bias will be inherited by the risk premium component. Note that the effects are mitigated when the excess return measure is used (see Figure 6). At this stage, we conclude that our equity implied risk premia are partially successful in explaining the time series variation of average risk premia as measured in corporate bond markets. Nevertheless there appears to be an important unexplained component to market measured risk premia, unrelated to equity risk premia. This direction is pursued in interesting work by Berndt et al. (2006) and Saita (2006).

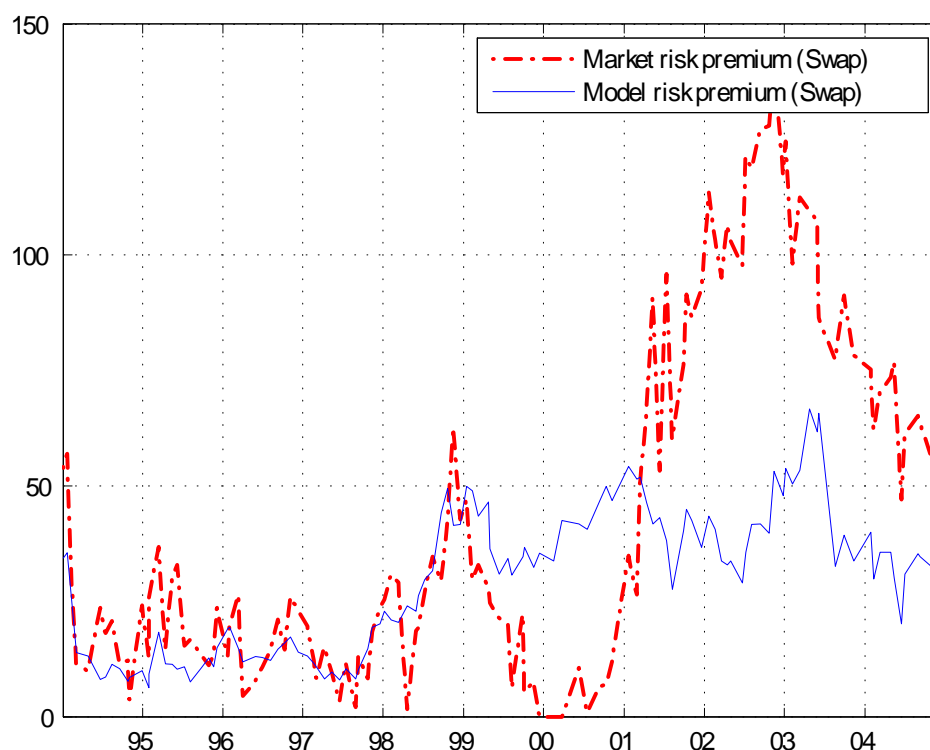


Figure 12: equity implied and bond market measured risk premium components in basis points.

It is interesting to observe the time series behavior of spread components over time. A number of interesting observations can be made from Figure 13, which plots the time series of the average model risk premium and expected loss component. First the expected loss component of spreads is more volatile than the risk premium. Second, although most of the time they appear to move together there are notable exceptions. For example during 2002, risk premia increased steadily while expected losses moved around without clear trend. Between 1999 and mid 2002 on the other hand there was no clear trend in risk premia while expected losses first increased to a peak in early 2001 and then decreased, although irregularly until the end of the period. The expected loss component is relatively larger in periods of high spreads and defaults (for example in 2001 - a year with spectacular defaults) and is similar or lower than the risk premium in lower spread

periods.

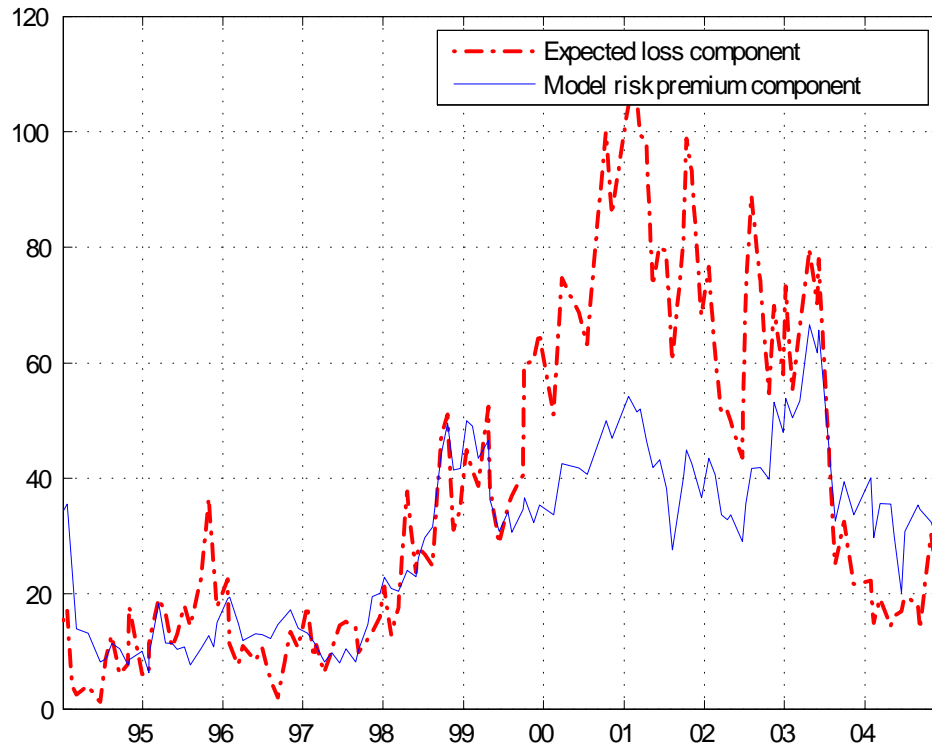


Figure 13: expected losses vs. the risk premium component in bond spreads.

Campbell & Taksler (2003) point out that during the late nineties, bond yield spreads increased while stock prices were rising. They argue that this puzzling pattern can be explained by an increase in idiosyncratic volatility over the same period. Our analysis allows us to make a related observation. Note that during 1999-2001, the increase in spreads was largely driven by an increase in the average expected loss component. The risk premium component should depend critically on systematic risk, whereas the expected loss should derive from a firm's total risk. Figure 14 plots the average trailing 250 day S&P500 return volatility and the average asset volatility as proxies for these two risk sources respectively.

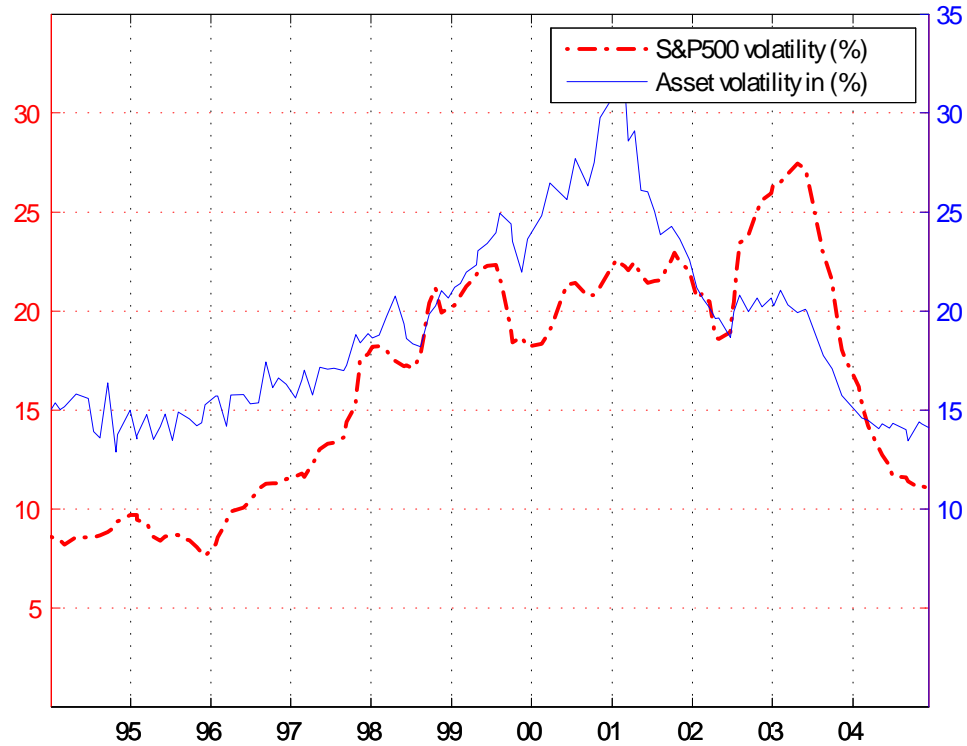


Figure 14: volatility measures. The solid line represents the average asset volatility across firms in our sample. The dashed line draws the 250 day historical volatility on the S&P 500 index. The latter is intended as a proxy for market volatility, whereas the asset volatility measures total risk, including idiosyncratic volatility.

Although these two metrics are not directly comparable, one being an average of asset volatilities and the other an equity volatility, the pattern is suggestive. Between 1999 and 2001, there is no clear trend in the market equity volatility, while average asset volatilities increased steadily. Absent a trend in firms' average leverage, this suggests that idiosyncratic risk increased during this period whereas systematic risk did not.²⁷ This is consistent with our observed pattern in the expected loss and risk premium spread components respectively.

²⁷There is no clear trend in leverage during our sample. The average leverage oscillates around 45% between 1994 and 2004.

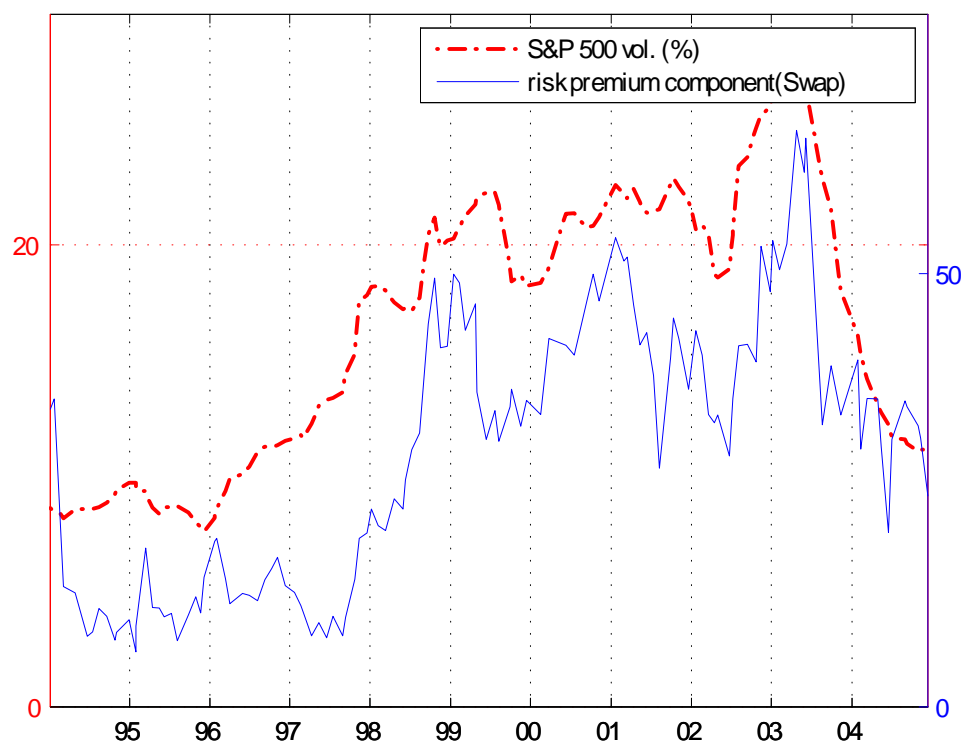


Figure 15. The risk premium in bond spreads vs. S&P 500 volatility as a proxy for systematic risk.

In fact, plotting risk premia together with the market equity volatility and the expected loss component with the average asset volatility, it becomes quite clear that risk premia move closely with systematic volatility whereas the expected loss component is aligned with a measure of total volatility. In summary, we find that the idiosyncratic equity volatility increase in the late nineties documented by Campbell & Taksler (2003) is in fact due to an increase in firm specific asset volatility leading to a higher spread as a result of higher expected losses.

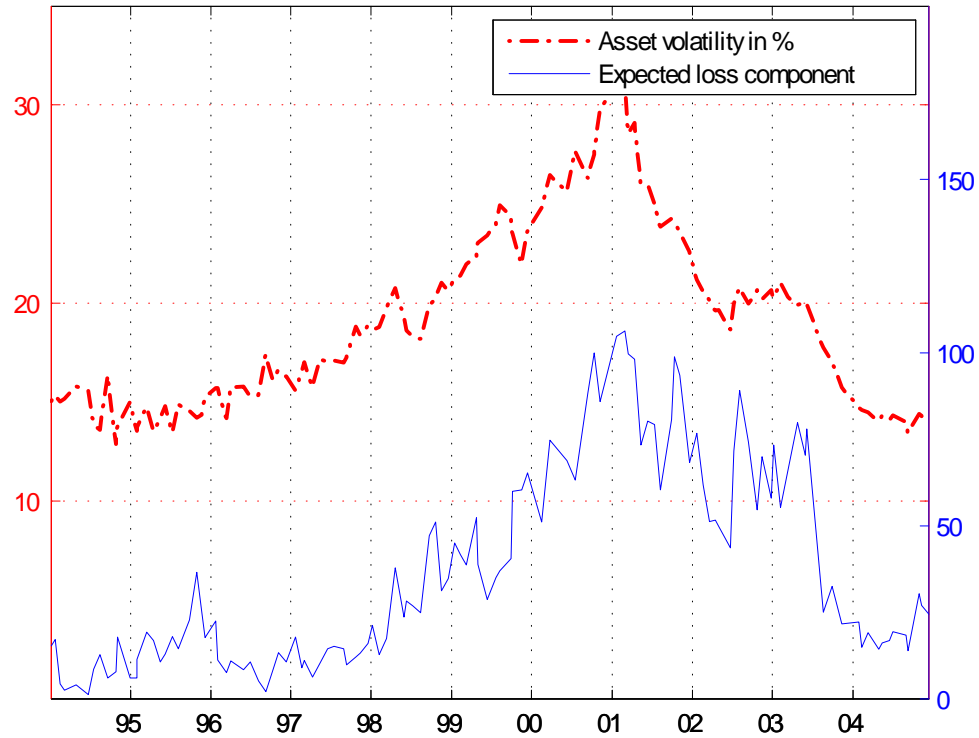


Figure 16: the expected loss component in bond spreads vs. asset volatility which measures total risk.

We now turn to a discussion of the determinants of credit spreads, more precisely we will discuss to what extent our metric for bonds' risk premia can help to explain spread dynamics empirically.

3.6.3 Explaining credit spreads

We have made extensive use of the contingent claims approach to valuing corporate securities in our analysis. Although the evidence may be mixed on the ability of structural models to correctly predict levels of the risk component of bond spreads, there is ample evidence that they cannot fully explain their time series variation – see for example

Collin-Dufresne et al. (2001).²⁸

The relative success of our use of structural models in translating equity market risk premia into bond specific risk premia suggests that these models do have merit. However, the typical such model predicts that levels of bond spreads and equity prices depend mainly on two unknowns – asset value and volatility. To date however, the models are almost always silent on risk premia. Studies that have considered risk premia in the context of structural models do not consider their dynamics (see e.g. Huang & Huang (2002) and Leland (2004)). With this in mind, we now consider the possibility that using information about the time series of risk premia can help structural models to explain the variation in credit spreads.

Developing and implementing a structural model with time varying risk premia lies well beyond the scope of this paper.²⁹ Instead we follow Collin-Dufresne et al. (2001) in working with linear regression analysis with a choice of variables motivated by the theoretical underpinnings of structural models.³⁰

We first run the following panel regression

$$\begin{aligned} s_{it} = & \alpha_i + \beta_1 LEV_{i,t} + \beta_2 EVOL_{i,t} + \beta_3 ERET_{i,t} \\ & + \beta_4 SLOPE_t + \beta_5 r_{i,t} + \beta_6 RPI_{it} + \varepsilon_{it} \\ \text{where } \varepsilon_{it} = & \rho \varepsilon_{i,t-1} + \eta_{it} \end{aligned}$$

where *LEV* denotes a firm’s leverage, *EVOL* and *ERET* its historical equity volatility

²⁸Many early studies of corporate bond pricing using structural models resulted in yield spread estimates substantially below their market counterparts - see for example Jones et al. (1984), Jones et al. (1985), Ogden (1987) and Lyden & Saranati (2000). Recent work has produced more mixed evidence (see e.g. Eom et al. (2004) and Ericsson et al. (2006)) and it has been suggested that the reason for the underestimation may be the presence of important non-default components (see e.g. Longstaff et al. (2004) and Ericsson & Renault (2000)).

²⁹Interesting work in this direction has been done by Chen et al. (2005).

³⁰Due to the irregular spacing of our data, we chose for simplicity to work with levels of credit spreads. Clearly a case can be made for working with changes in spreads. However, as we are mainly interested in the marginal importance of an additional variable rather than the absolute level of explanatory power we feel that our choice is adequate. Papers in the field have varied in their approach. See e.g. Campbell & Taksler (2003) and Cremers et al. (2004).

and return respectively, $SLOPE$ is the difference between the 10- and 2-year swap rates, r_{it} , the swap rate corresponding to the maturity of the particular bond, and RPI is our equity-implied measure of the spread's risk premium component. We run the regression with and without the risk premium metric in order to gauge the marginal gain in explanatory power by including this variable.

Table 3 reports the results. The first two columns relate to the full sample, whereas the remaining columns provide disaggregated evidence based on firm leverage quartiles. The reported R-squares in the benchmark regressions are comparable to those reported for credit spread level regressions by Campbell & Taksler (2003).³¹ The pattern across rating quartiles for the regressions is not dissimilar to what is found by Collin-Dufresne et al. (2001), although they work with credit spread changes rather than levels.

For the full sample the R-square increases by 5% to about 36% when including our risk premium variable, a non-trivial increase. The R-square increases for all leverage quartiles, although by varying degrees. The risk premium variable is more important for the lowest leverage firms (an R-square increase by 12%) than for the highest leverage firms (and increase by 2%). This suggests that spreads for firms with lower default risk have higher proportional risk premia. This is consistent with our findings above as well as with the results of BDDFS. As we shall see below, it is also consistent with the predictions of a structural model.

3.7 Structural models and risk premia

Our regression results suggest that risk premia are a relatively more important determinant of high grade debt spreads than for lesser quality bonds. Similarly, we have seen that on average risk premia appear to be more important during periods of low default rates. In high default periods, the expected loss component is more important. As noted

³¹It is important to note that the R-squares measure explained variation over and above the fixed effects. For example, the reported R-square in Table IIa for the first regression is about 31%. In contrast the total R-square for a standard pooled regression is just less than 60%.

above, Berndt et al. (2004) also find that the default premium is higher for high-quality firms. Next, we consider comparative statics of the Leland & Toft (1996) model to determine whether this finding can be explained. The four panels of Figure 16 plot the following quantities against leverage: (i) total spread, expected loss and risk premia, (ii) the ratio of risk premium to total spreads, (iii) ratio of risk neutral to objective 5 year default probabilities and (iv) the difference between risk neutral and objective 5 year default probabilities.

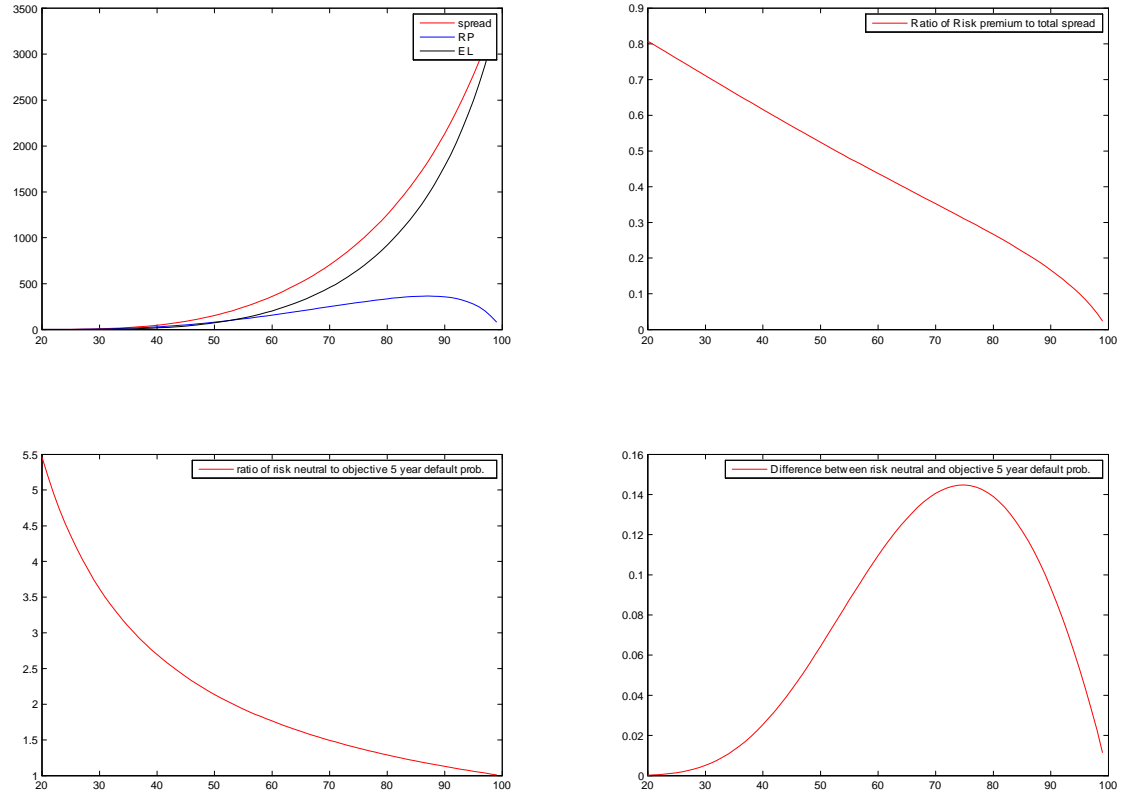


Figure 16

The first two are relevant to our results on bond spread components whereas the last two are intended for comparison with the results of the BDDFS study. The first panel shows,

not surprisingly, that the expected loss spread and total spread increase monotonically in leverage. The risk premium component on the other hand does not. For low levels of leverage it increases in tandem with the expected loss spread. But for some (high) level of leverage, it decreases and eventually disappears as the distress becomes more and more certain. This is intuitive, as when default is certain, debt needs to compensate for the imminent losses, but there is in fact little risk. The second panel shows that the fraction of the risk premium in the total spread decreases monotonically as leverage increases. This is consistent with our regression coefficients on the risk premium being highest in the lowest leverage quartile and monotonically decreasing as leverage increases.

The third panel illustrates the same effect using a proxy for the risk premium similar to what BDDFS use in their empirical study. The ratio of risk neutral to objective default probabilities behaves much like the ratio of the risk premium spread component to the total yield spread - it decreases monotonically with leverage. This suggests that the empirical finding of BDDFS, also found in our study is consistent with the prediction of a structural model.³² The final panel plots the absolute difference between risk neutral and objective default probabilities - the resulting pattern echoes what can be seen in the first panel: for extremely high and low default probabilities, the difference between the risk neutral and objective probabilities disappears.³³ Again - this is intuitive. Both probabilities have to converge to zero or one at the extremes.

3.8 Concluding discussion

Investors in credit markets need a framework to assess whether a given defaultable security is fairly priced. The spread itself may not be an adequate metric to respond to this question. The investor needs to know if the spread contains (i) acceptable compensation

³²For robustness, we have also established that this result can be generated within the much simpler Merton (1974) model. It would thus seem that it is not very sensitive to the choice of a particular model for the computation of the risk premia.

³³Berndt et al. (2006) work with the difference in intensities rather than ratios.

for expected default losses and (ii) a sufficient risk premium to induce participation.

With a methodology capable of disentangling risk premia and expected losses, we measure risk premia in a large panel of US corporate bond data spanning a ten year period. We find, like previous work, that the risk premium is highly time varying. We also document similar time series patterns as previous work. We find that the expected loss and default components behave differently over time. The risk premium is at its most important for high grade debt, whereas the expected loss component increases monotonically with the default probability. We show that the time series variation of the risk premium is closely related to the overall market volatility whereas the expected loss component appears more closely related to the average total volatility across firms.

Perhaps our two most important findings are that (i) the time series variation observed in the risk premium in bond markets can be replicated using equity market measured risk premia translated to corporate bond risk premia and (ii) that including our risk premium metric in a linear regression of bond spreads on theoretical determinants of corporate bond risk premia increases explanatory power, suggesting that time varying risk premia is a desirable feature of future structural credit risk models.

The risk premium we have measured is a translation of risk premia measured in equity markets. As such it does not capture risk premia that may be specific to fixed income markets. We have already discussed the sensitivity of our results to a swap market specific factor. Another example of market specific risks that influence prices has been documented by Newman & Rierson (2004), who show that issuance activity may play a role in the pricing for seasoned securities in particular market segments. Recent work by Berndt et al. (2006) suggests the presence of a credit market specific risk factor. In addition, recent work documents the commonality of illiquidity risk within and across markets. The unexplained part of our market risk premia may well contain information about some or all of these additional risk premia.

3.9 Appendices

3.9.1 Default risk premia in reduced form models

The default intensity λ^P of a firm is the instantaneous mean arrival rate of the first event of a non explosive counting process N . Conditional on survival to time t and all additional available information, the probability of default in a short time between t and $t + \Delta$ is approximately $\lambda^P \Delta$. In a Cox process framework, the probability of survival of an obligor for some time h , conditional on survival up to time t is

$$P_t(\tau > t + h) = E_t^P \left[\exp \left\{ - \int_t^{t+h} \lambda_s^p ds \right\} \right], \quad (3.7)$$

with E_t^P denoting the expectation operator conditional on the information available up to time t .

In the absence of arbitrage and market frictions, there exists a risk neutral probability measure.³⁴ In reduced form models, it is the change from historical to risk-neutral default intensities that defines the risk premium. Information about risk-neutral default intensities can be extracted from market prices of corporate bonds or credit derivatives. The historical default probabilities need to be inferred from a different source such as historical default frequencies conditional on rating categories or alternative measures such as Moody's KMV expected default frequencies (EDFs).³⁵

In order to describe the concept of risk premia in reduced form credit risk models, we will borrow from a discussion in Lando (2003), who provides an illustrative setting where under the historical probability measure P , we can write the dynamics of the default intensity as follows (assuming deterministic interest rates):

$$d\lambda_t = \gamma(\bar{\lambda} - \lambda_t)dt + \sigma_\lambda \sqrt{\lambda_t} dW_t, \quad (3.8)$$

³⁴See Harrison & Kreps (1979) and Delbaen & Schachermayer (1994) for technical conditions.

³⁵See e.g. Berndt et al. (2004) and Driessen (2005).

where W is a Brownian motion under P , γ is the speed of mean reversion, $\bar{\lambda}$ the long run mean and σ_λ is the volatility of the intensity process. Under an equivalent measure Q :

$$d\lambda_t = (\gamma\bar{\lambda} - (\kappa - \psi_\lambda)\lambda_t)dt + \sigma_\lambda\sqrt{\lambda_t}d\tilde{W}_t, \quad (3.9)$$

where \tilde{W} is a Brownian motion under Q .

The parameter ψ_λ is the price of risk for a unit change in the intensity. Note that the stopping time of default that has an intensity process λ with a process defined in (3.8) under the probability measure P is no longer a stopping time under Q with intensity dynamics defined in (3.9). It is however a stopping time with intensity process $\lambda_t^Q = \mu_t\lambda_t$, where μ_t is assumed constant for simplicity.³⁶

Now, consider a h maturity zero coupon bond with zero recovery. Its price at time t is

$$B_{t,h} = E_t^Q \left[\exp \left\{ - \int_t^{t+h} (r_s + \lambda_s^Q) ds \right\} \right], \quad (3.10)$$

where r_s is the short term risk free rate. An application of Ito's lemma permits us to write the instantaneous expected excess return for the bond as

$$R_B(t) - r_t = F_\lambda \sigma_\lambda \psi_\lambda \lambda_t + (\mu - 1) \lambda_t 1_{\{N_t=0\}}, \quad (3.11)$$

where $R_B(t)$ is the drift rate of the corporate bond under P and F_λ is the loading for intensity risk. The excess return on the bond thus consists of two distinct components. First, there is a positive contribution from the risk of a jump to default itself – if $\mu \neq 1$. Second, the bond is subject to price volatility due to fluctuations in the default intensity, contributing to the risk premium whenever ψ_λ is nonzero. The first type of risk is commonly referred to as default event risk, whereas the second can intuitively be thought of as spread risk.

³⁶See Lando (2003) for details.

Jarrow et al. (2005) show that under certain conditions, asymptotically, $\mu \rightarrow 1$, implying that default event risk should not be priced. Driessen (2005) shows empirically that μ exceeds 1. This is supported by Berndt et al. (2004) who estimate the risk premium as the ratio between historical and risk neutral default intensities, which corresponds to μ in equation (3.11). They obtain λ^p by calibrating Moody's KMV Expected default frequencies to $\{P_t(\tau < s); s \in (t, \infty)\}$ in equation (3.7), and use market prices for default swaps to recover λ^Q from equation (3.10). Berndt et al. (2006) investigate the source for common variation in the portion of returns on default swaps that is not explained by changes in risk-free rates or expected default losses. Their estimate for risk premia corresponds to $(\mu - 1)\lambda_t 1_{\{N_t=0\}}$ in equation (3.11).

Appendix: The Leland & Toft Model

The value of the firm is the same as in Leland (1994). The value of debt is given by

$$\mathcal{D}(v_t) = \frac{C}{r} + \left(N - \frac{C}{r}\right) \left(\frac{1 - e^{-rM}}{rM} - I(v_t)\right) + \left((1 - \alpha)L - \frac{C}{r}\right) J(v_t)$$

The bankruptcy barrier

$$L = \frac{\frac{C}{r} \left(\frac{A}{rM} - B\right) - \frac{AP}{rM} - \frac{\tau Cx}{r}}{1 + \alpha x - (1 - \alpha)B}$$

where

$$A = 2ye^{-rM}\phi\left[y\sigma\sqrt{M}\right] - 2z\phi\left[z\sigma\sqrt{M}\right] - \frac{2}{\sigma\sqrt{M}}n\left[z\sigma\sqrt{M}\right] + \frac{2e^{-r\Upsilon}}{\sigma\sqrt{M}}n\left[y\sigma\sqrt{M}\right] + (z - y)$$

$$B = -\left(2z + \frac{2}{z\sigma^2 M}\right)\phi\left[z\sigma\sqrt{M}\right] - \frac{2}{\sigma\sqrt{M}}n\left[z\sigma\sqrt{M}\right] + (z - y) + \frac{1}{z\sigma^2 M}$$

and $n[\cdot]$ denotes the standard normal density function.

The components of the debt formulae are

$$I(v) = \frac{1}{r\Upsilon} (i(v) - e^{-r\Upsilon} j(v))$$

$$i(v) = \phi[h_1] + \left(\frac{v}{L}\right)^{-2a} \phi[h_2]$$

$$j(v) = \left(\frac{v}{L}\right)^{-y+z} \phi[q_1] + \left(\frac{v}{L}\right)^{-y-z} \phi[q_2]$$

and

$$J(v) = \frac{1}{z\sigma\sqrt{M}} \begin{pmatrix} -\left(\frac{v}{L}\right)^{-a+z} \phi[q_1] q_1 \\ +\left(\frac{v}{L}\right)^{-a-z} \phi[q_2] q_2 \end{pmatrix}$$

Finally

$$q_1 = \frac{-b - z\sigma^2 M}{\sigma\sqrt{M}}$$

$$q_2 = \frac{-b + z\sigma^2 M}{\sigma\sqrt{M}}$$

$$h_1 = \frac{-b - y\sigma^2 M}{\sigma\sqrt{M}}$$

$$h_2 = \frac{-b + y\sigma^2 M}{\sigma\sqrt{M}}$$

and

$$y = \frac{r - \beta - 0.5\sigma^2}{\sigma^2}$$

$$z = \frac{\sqrt{y^2\sigma^4 + 2r\sigma^2}}{\sigma^2}$$

$$x = y + z$$

$$b = \ln\left(\frac{v}{L}\right)$$

3.9.2 Bond pricing

Next we need a pricing formula for the corporate bond obligation . To this end, we apply a bond pricing model that takes discrete coupons, nominal repayment and default

recovery into account.³⁷ To express the value of the bond we make use of two building blocks, a binary option $H(v_t, t; S)$ and a dollar-in-default claim $G(v_t, t; S)$. The former pays off \$1 at maturity S if the firm has not defaulted before that, the latter pays off \$1 upon default should this occur before S ; the value of both depend upon the firms asset value v_t and current time t . The formulae for the binary option and the dollar-in-default claim are, for a given default barrier L .

Proposition 9 *A straight coupon bond. The value of a coupon bond with M coupons c paid out at times $\{t_i : i = 1..M\}$ is*

$$\begin{aligned} \mathcal{B}(v_t, t) = & \sum_{i=1}^{M-1} c \cdot H(v_t, t; t_i) \\ & + (c + p) \cdot H(v_t, t; T) \\ & + R \cdot p \cdot G(v_t, t; T) \end{aligned}$$

The formulae for H and G are given in the appendix.

The value of the bond is equal to the value of the coupons (c), the value of the nominal repayment (P) plus the value of the recovery in a default (Rp). Each payment is weighted with a claim capturing the value of receiving \$1 at the respective date.

Note that the above formula for the reference bond is not directly related to the debt structure of the firm. Specifically, coupon payments to the bond are unaffected by the debt redemption schedule elaborated in the Leland & Toft model. The choice of model affects the bond formula solely via the default barrier L .

³⁷This bond pricing model was used in Ericsson & Reneby (2004) and was shown to compare well to reduced form bond pricing models.

3.9.3 Building blocks for bond valuation

First, define default as the time (τ) the asset value hits the default boundary from above, $\ln \frac{v\tau}{V_{B,\tau}} \equiv 0$. Then define $G(v, t)$ as the value of a claim paying off \$1 in default:

$$G(v_t, t) \equiv E^B [e^{-r(\tau-t)} \cdot 1]$$

We let E^B denote expectations under the standard pricing measure. The value of G is given by

$$G(v_t, t) = \left(\frac{v_t}{V_{B,t}} \right)^{-\theta}$$

with the constant given by

$$\theta = \frac{\sqrt{(h^B)^2 + 2r + h^B}}{\sigma}$$

and

$$h^B = \frac{r - \beta - 0.5\sigma^2}{\sigma}$$

Define the dollar-in-default with maturity $G(v_t, t; T)$ as the value of a claim paying off \$1 in default *if* it occurs before T

$$G(v_t, t; T) \equiv E^B [e^{-r(\tau-t)} \cdot 1 \cdot (1 - I_{\tau \leq T})]$$

and define the binary option $H(v_t, t; T)$ as the value of a claim paying off \$1 at T if default has *not* occurred before that date

$$H(v_t, t; T) \equiv E^B [e^{-r(T-t)} \cdot 1 \cdot I_{\tau > T}]$$

$I_{\tau > T}$ is the indicator function for the *survival event*, i.e. the event that the asset value (v_T) has not hit the barrier prior to maturity ($\tau \not\leq T$). The price formulae for the last two building blocks are given below. They contain a term that expresses the probabilities (under different measures) of the survival event – or, the *survival probability*. To clarify

this common structure, we first state those probabilities in the following lemma.³⁸

Lemma 10 *The probabilities of the event $(\tau \not\leq T)$ (the “survival event”) at t under the probability measures $Q^m : m = \{B, G, obj\}$ are*

$$Q^m(\tau \not\leq T) = \phi\left(k^m\left(\frac{v_t}{V_{B,t}}\right)\right) - \left(\frac{v_t}{V_{B,t}}\right)^{-\frac{2}{\sigma}h^m} \phi\left(k^m\left(\frac{V_{B,t}}{v_t}\right)\right)$$

where

$$\begin{aligned} k^m(x) &= \frac{\ln x}{\sigma\sqrt{T-t}} + h^m\sqrt{T-t} \\ h^G &= h^B - \theta \cdot \sigma = -\sqrt{(h^B)^2 + 2r} \\ h^{obj} &= \end{aligned}$$

$\phi(k)$ denotes the cumulative standard normal distribution function with ϵ -ration limit k .

The probability measure Q^G is the measure having $G(v_t, t)$ as numeraire (the Girsanov kernel for going to this measure from the standard pricing measure is $\theta \cdot \sigma$). Using this lemma we obtain the pricing formulae for the building blocks in a convenient form. The price of a down-and-out binary option is

$$H(v_t, t; T) = e^{-r(T-t)} \cdot Q^B(\tau \not\leq T)$$

The price of a dollar-in-default claim with maturity T is

$$G(v_t, t; T) = G(v_t, t) \cdot (1 - Q^G(\tau \not\leq T))$$

To understand this second formula, note that the value of receiving a dollar if default occurs prior to T must be equal to receiving a dollar-in-default claim with infinite ma-

³⁸The probabilities are previously known, as is the formula the down-and-out binary option in Lemma 3.9.3 (see for example Björk (1998)).

turity, less a claim where you receive a dollar in default conditional on it *not* occurring prior to T :

$$G(v_t, t; T) = G(v_t, t) - e^{-r(T-t)} E^B [G(v_T, T) \cdot I_{\tau \not\leq T}]$$

Using a change of probability measure, we can separate the variables within the expectation brackets (see e.g. Geman et al. (1995)).

$$\begin{aligned} G(v_t, t; T) &= G(v_t, t) - e^{-r(T-t)} E^B [G(v_T, T)] \cdot E^G [I_{\tau \not\leq T}] \\ &= G(v_t, t) \cdot (1 - Q^G(\tau \leq T)) \end{aligned}$$

3.9.4 Objective default probabilities

$$P_t(\tau > T_i) = N(d_{T_i}^P(\frac{v_t}{L})) - (\frac{v_t}{L})^{-2\frac{\mu_v - 0.5\sigma^2}{\sigma^2}} N(d_{T_i}^P(\frac{L}{v_t}))$$

$$\begin{aligned} \text{with } d_{T_i}^P(\frac{v_t}{L}) &= \frac{\ln(\frac{v_t}{L}) + (\mu_v - 0.5\sigma^2)(T_i - t)}{\sigma\sqrt{T_i - t}} \\ \text{and } d_{T_i}^B(\frac{L}{v_t}) &= \frac{\ln(\frac{L}{v_t}) + (\mu_v - 0.5\sigma^2)(T_i - t)}{\sigma\sqrt{T_i - t}} \\ \mu_v &= \text{the realized mean of the time series of } v(t) \end{aligned}$$

Don't forget to insert the table!!!

Chapter 4

What Risks Do Corporate Bond Put Features Insure Against?

4.1 Introduction

Corporate bond prices are influenced by a number of risk factors, the most important of which are likely interest rate risk, default risk, and illiquidity.¹ Puttable bonds give their owners the right to sell, or put, their bond to the issuer prior to the bond's maturity date. This embedded option appears designed to provide insurance against any or all of these risks. Understanding the relative contribution of the distinct risks to put values may help shed further light on the components of corporate bond yields. Furthermore, while callable and convertible bonds have well-understood embedded option features,² this paper constitutes, to the best of our knowledge, the first empirical study of puttable bond valuation.³

We study a matched sample of more than a thousand pairs of puttable and non-puttable bond transactions. We first perform a linear regression analysis on the relationship between puttable and regular bond yield spreads and default, non-default and interest rate proxies suggested by theory and previous empirical work.⁴ Across all bonds, we find that the estimated coefficients of the three classes of proxies are not only consistent with our expectations, but also statistically and economically significant. In addition, we find that puttable bond spreads are economically less sensitive to those proxies. This confirms that put options embedded in corporate bonds help to reduce bondholders' exposures to those risks. In a second step, we split the yield reduction due to the put option into separate components for each of the risk reductions. To do this we develop and implement a novel valuation methodology, which can be used for all types of corporate bonds with embedded options. We find that the most important reduction in yield is

¹See Huang & Huang (2002), Elton et al. (2001), Driessen & DeJong (2005) and Longstaff et al. (2004).

²A non-exhaustive list of work on callable and convertible bonds would include Brennan (1979), Brennan & Schwartz (1977), Brennan & Schwartz (1980), Ingersoll (1977), Longstaff (1992) and McConnell & Schwartz (1986).

³One recent paper, David (2001), deals with the strategic values of poison put bonds. Poison put bonds are excluded from our sample.

⁴See Collin-Dufresne et al. (2001), Campbell & Taksler (2003), and Ericsson et al. (2004).

due to a decreased exposure to default risk, followed by term structure risk and last by non-default risks such as illiquidity.

In the first part of the paper, we attempt to identify the risks that embedded put options insure against. Intuitively, a puttable bond is simply a regular bond with a put option attached. The price of a puttable bond can therefore be split into the price of an otherwise identical regular bond and the price of the put. We measure the market value of the put option with the difference in yield spreads between the regular and puttable bonds for the same issuer. The reduction in corporate bond yield spread due to the presence of a put is, on average, just over 40% of the yield spread. We show that the value of a put option is positively and significantly correlated to credit proxies, including firm leverage, equity volatility, and Moody's default premium. This suggests that their value increases as default become more likely. In addition, analysis on illiquidity proxies shows that a put option is less valuable for the bonds issued by relatively large firms. A larger firm is likely to enjoy the attention of a larger number of investors and to enjoy better marketability of its securities. Furthermore, the value of a put option increases when market liquidity, as measured by the Pastor-Stambaugh Index, deteriorates. The risk-free rate shows a strong and positive correlation. This confirms the intuition that the puts are more likely to be exercised when interest rates are high, which, in turn, increases their value.

The market value of the put is also significantly influenced by contractual features. Our results indicate that the value of a put increases as the time to the earliest exercise date decreases. Moreover, its value increases with the remaining life of the put. Not surprisingly, a put option is more valuable the lower the bond price. A puttable bond with more frequent put dates is more valuable.

In the second part of the paper, given that we have established that put values are related to the conjectured risks, we proceed to measure the proportion of the put option value that can be attributed to insurance against the different risks. To do this, we require a model that prices puttable bonds. We develop a bivariate lattice model

that simultaneously captures correlated default and interest rate risk. Our model is closely related to Das & Sundaram (2007). Theirs is also an integrated model for pricing securities subject to equity, default and interest rate risk. Their approach is based on observed equity prices, term structure and credit default swap prices. The latter are necessary for extracting the default intensity. In our model, we follow a different approach where the key sources of uncertainty are the value of a firm's assets and term structure fluctuations. We draw on recent developments in the literature on structural credit risk models. Implementing the model involves, in a first step, the estimation of asset values, volatilities, and the historical correlation between asset values and interest rates. In a second step, we construct a recombining lattice Heath et al. (1992) (HJM) term structure model. Our method offers a fast and accurate approach for valuing corporate bonds with embedded options. The model is flexible enough to be useful also for bonds with most other types of embedded options. Perhaps the key difference between our model and that proposed by Das & Sundaram (2007) is that ours does not require a market observed credit spread yield curve for the issuer, as implied by e.g. default swaps as in Das & Sundaram (2007). Instead the term structure of risk-adjusted default probabilities is inferred from equity and balance sheet data.

Applying the model to price regular and puttable bonds, we illustrate that most of the reduction in the puttable bond spread (about 60%) is due to a decrease in the default component of the spread. A third of the reduction is due to mitigated term structure risk. The smallest fraction (7%) represents a reduction in the illiquidity component of the bond spread.

The remainder of this paper is structured as follows. Section 2 introduces the theoretical frameworks, and data and regression analyses on put options. Section 3 describes the bivariate lattice model. In Section 4, the value of put options as insurance against various risk factors is decomposed. Finally, Section 5 concludes.

4.2 Analytical Framework

In this section, we study the relationship between putable bond yields and variables predicted to be the main determinants by theory. This literature starts with the seminal work of Black & Scholes (1973*b*) and Merton (1974). Although their basic model has since been extended in various ways,⁵ structural models share a number of common determinants of default risk. Leverage is a key factor - all else equal, a firm with higher leverage has a higher likelihood of default. The underlying asset return volatility is another critical determinant of the default probability. Moreover, structural models predict that risk-free interest rates negatively influence the yield spread.⁶ Under the risk-neutral measure, high interest rates lead the firm's underlying asset value to grow at a higher rate, reducing the probability of financial distress.

The value of a putable bond should be subject to the same risks, unless the option were to provide full insurance against them. Accordingly, the yield spread of the putable bond as well as the market value of the put option should be determined by the leverage ratio of the underlying firm, the volatility of the firm asset return, the riskless spot rates, and the maturity of the individual bond. We denote the leverage of the firm i at time t as $LEV_{i,t}$, the equity volatility as $\sigma_{i,t}$, and the bond maturity as $MAT_{i,t}$. We define the risk-free rate variable to be the 5-year swap yield, denoted by r_t^5 .

Empirical research shows that in practice, corporate bond yield spreads contain compensation for non-default risks as well as risk premia which may be difficult to identify without aggregate macro variables.⁷ For this reason, we will not limit our analysis to the

⁵Among others, Black & Cox (1976) allows for default before debt maturity. Leland (1994) endogenizes default barrier and derives an analytical solution with perpetual debt. Leland & Toft (1996) extends the Leland model to allow for nonperpetual debt. Anderson & Sundaresan (1996), Mella-Barral & Perraudin (1997), and Fan & Sundaresan (2000) investigate the implications of strategic default. Leland (1998) and Ericsson (2000) study the effects of asset substitution and hedging policies. François & Morellec (2004) and Broadie et al. (2006) examine the impact of default procedures, and Morellec (2004) gauges the influence of manager-shareholders conflicts through investment.

⁶Studies that explicitly model stochastic riskfree interest rate show the same result. Those includes Longstaff & Schwartz (1995) and Collin-Dufresne & Goldstein (2001).

⁷See e.g. Newman & Rierson (2004) and Longstaff et al. (2004) on non-default risks and Berndt et al. (2004) and Berndt et al. (2006) on the topic of default risk premia.

traditional theoretically motivated regressors. We augment our set of variables by the return of the S&P 500, denoted by $S\&P_t$, used in Collin-Dufresne et al. (2001) to proxy for the overall state of the economy. Moody's default premium is denoted by MDP_t ; and is meant to capture the default risk premium in the corporate bond market. The S&P ratings of the underling firms are denoted by $SPR_{i,t}$; we expect they may contain incremental accounting information and proxy for clientele effects.⁸ Following Collin-Dufresne et al. (2001), we use the option-implied volatility based on the S&P 100 Index options denoted by VIX_t .

With respect to the empirical liquidity proxies, we include the bond age - denoted by $AGE_{i,t}$. Older Bonds are expected to be less liquid - much like seasoned issues in Treasury bond markets. Another liquidity factor we use is the firm size denoted by $SIZE_{i,t}$. We expect larger firms to attract a greater number of investors and to enjoy better marketability of their securities. Furthermore, we use the Pastor & Stambaugh Index, denoted by PSI_t , as a general measure of market liquidity, mindful that this measure captures liquidity in the equity market and may not be an ideal proxy for the corporate bond market. Nevertheless Driessen & DeJong (2005) find evidence that bond spreads increase when equity market liquidity decreases. We include coupon rates, CPN_i , which may proxy for potential tax, duration or investor preference effects (see e.g. Longstaff et al. (2004)).

The above discussed theory and empirical studies therefore suggest the following regression equations (where PS and RS respectively denote putable and regular bond spreads)

$$\begin{aligned}
 PS_{i,t} = & \alpha_i + \beta_{i,1}^{ps} LEV_{i,t} + \beta_{i,2}^{ps} \sigma_{i,t} + \beta_{i,3}^{ps} MAT_{i,t} + \beta_{i,4}^{ps} r_t^5 + \beta_{i,5}^{ps} S\&P_t + \beta_{i,6}^{ps} MDP_t \\
 & + \beta_{i,7}^{ps} SPR_{i,t} + \beta_{i,8}^{ps} VIX_t + \beta_{i,9}^{ps} AGE_{i,t} + \beta_{i,10}^{ps} SIZE_{i,t} + \beta_{i,11}^{ps} PSI_t + \beta_{i,12}^{ps} CPN_i + \varepsilon_{i,t},
 \end{aligned} \tag{4.1}$$

⁸Many investors are constrained to hold only bonds above certain rating categories.

$$\begin{aligned}
RS_{i,t} = & \alpha_i + \beta_{i,1}^{RS} LEV_{i,t} + \beta_{i,2}^{RS} \sigma_{i,t} + \beta_{i,3}^{RS} MAT_{i,t} + \beta_{i,4}^{RS} r_t^5 + \beta_{i,5}^{RS} S\&P_t + \beta_{i,6}^{RS} MDP_t \\
& + \beta_{i,7}^{RS} SPR_{i,t} + \beta_{i,8}^{RS} VIX_t + \beta_{i,9}^{RS} AGE_{i,t} + \beta_{i,10}^{RS} SIZE_{i,t} + \beta_{i,11}^{RS} PSI_t + \beta_{i,12}^{RS} CPN_i + \varepsilon_{i,t}.
\end{aligned} \tag{4.2}$$

Regarding the analysis of the put option value as measured by the spread reduction in the putable spreads, we regress the put option value, $PV_{i,t}$, on some variables described above. We discard the bond-specific factors such as maturity, age and coupon rates because the putable and the regular bonds do not share the same features in general.⁹ The following regression ensues:

$$\begin{aligned}
PV_{i,t} = & \alpha_i + \beta_{i,1}^{pv} LEV_{i,t} + \beta_{i,2}^{pv} \sigma_{i,t} + \beta_{i,3}^{pv} r_t^5 + \beta_{i,4}^{pv} S\&P_t + \beta_{i,5}^{pv} MDP_t \\
& + \beta_{i,6}^{pv} VIX_t + \beta_{i,7}^{pv} SIZE_{i,t} + \beta_{i,8}^{pv} PSI_t + \varepsilon_{i,t}.
\end{aligned} \tag{4.3}$$

Table 1 summarizes our expectations regarding the relationship between the regressors and the three dependent variables. The first four regressors are simply key variables implied by most structural models of credit risk. As default risk increases via the leverage and volatility channels, bond spreads should increase. If the embedded put mitigates default risk, the effect should be less pronounced for putable bonds.¹⁰ The bond maturity is ambiguous. Depending on the degree of distress risk, structural models may predict either increasing, decreasing or hump shaped term structures of credit spreads.

The interest rate plays a more subtle role. Merton (1974) would predict that an

⁹We, however, include the bond characteristic variables for both regulars and putables as independent variables in unreported robustness check regressions.

¹⁰From the theoretical point of view, massive put exercise might accelerate an issuer's default, especially when the issuer is of low credit quality. See David (2001) and Cremers et al. (1993).

increased interest rate yields lower spreads as the risk adjusted drift of the asset value increases in the rate. In other words the risk adjusted default probability is decreasing in the risk free rate.

When we introduce embedded puts, the impact of the interest rate becomes more complex. Consider a floating rate corporate puttable bond. For such a bond the interest rate hedge provided by the put is relatively unimportant. If the interest rate increases, the straight bond spread should decrease; while for the puttable, the default insurance value decreases, offsetting some or all of the spread reduction.¹¹ So for a floating rate corporate, we would expect the interest rate coefficient to be negative for a puttable and non-puttable alike, while larger in absolute value for the latter. However, the bonds in our sample are fixed rate bonds. For such bonds, both the interest rate and default hedge components will matter. To understand the impact of the term structure insurance in isolation, consider fixed rate non-defaultable puttable and non-puttable bonds respectively. When the interest rate increases, the put becomes more valuable and renders the spread of the puttable over the benchmark non-puttable more negative. We thus have two offsetting effects. Overall, we expect the sign of the interest rate to be negative, but cannot predict in which way the embedded put influences the overall relationship.

The rating should increase the spreads and the value of the put, either if it measures default risk or is interpreted as a proxy for participation. If fewer investors are able to hold speculative grade bonds then the price of a bond may decrease in value due to decreased demand as the credit quality deteriorates. The predictions regarding MDP , SPR and VIX are similar to that for LEV and σ .

¹¹Consider the stylized case of a floating rate corporate puttable bond where the benchmark floating rate is reset continuously. If this bond were also continuously puttable at par, then an increase in the interest rate would have no effect. In the case of less frequent reset and put dates this neutrality would of course break down.

Table 1: Expected Signs in Linear Regressions

	Mnemonic	Putable bond spread (4.1)	Regular bond spread (4.2)	Put option value (4.3)
Leverage ratio	<i>LEV</i>	> 0	$>> 0$	$>> 0$
Equity volatility	σ	> 0	$>> 0$	$>> 0$
Bond maturity	<i>MAT</i>	≤ 0	≤ 0	≤ 0
5-year swap rate	r^5	< 0	$<< 0$	≤ 0
S&P numerical rating	<i>S&P</i>	> 0	$>> 0$	$>> 0$
Moody's Baa index spread - Aaa index	<i>MDP</i>	> 0	$>> 0$	$>> 0$
S&P500 return	<i>SPR</i>	< 0	< 0	< 0
CBOE implied volatility index	<i>VIX</i>	> 0	$>> 0$	$>> 0$
Bond age in years	<i>AGE</i>	> 0	$>> 0$	$>> 0$
Firm size	<i>SIZE</i>	< 0	$<< 0$	$<< 0$
Pastor Stambaugh equity market liquidity index	<i>PSI</i>	< 0	$<< 0$	$<< 0$
Bond coupon	<i>CPN</i>	≤ 0	≤ 0	≤ 0

We think of a bond's age as a proxy for its liquidity in a way akin to the on- and off-the-run labels used in Treasury bond markets. An older bond will be less liquid and thus promise a higher yield spread. If an embedded put insures against illiquidity, we would expect the relationship to be positive and economically less significant for puttable bonds. Firm size should influence bond spread analogously - a larger firm will garner more

investor attention, have more liquid bonds with lower spreads, all else equal. Aggregate market liquidity, as measured by PSI , should decrease spreads and be more influential for non-putable bonds.

The coupon rate has sometimes been argued to proxy for tax effects in corporate bond markets. Corporate bonds are more heavily taxed at the investor level than Treasury bonds. As a result, a higher coupon corporate would have to compensate investors with a higher spread. If that is the case the coupon should carry a positive sign when included as a regressor. However, other interpretations are quite possible and the empirical results to date are mixed. As a result, we do not have any strong expectations regarding regular bonds. For putable bonds there is a more direct effect at play. All else equal, the higher the coupon of a bond, the less likely it is that a bond can be profitably put back to the issuer. Thus a higher coupon should correspond to a higher spread for putables. Although the overall effect remains ambiguous given a lack of clear expectation for non-putables, we expect a more positive (less negative) relationship between coupons and spreads on putable bonds relative to their non-putable counterparts.

4.3 Data

Swap rates are acquired from DataStream. Term structures of swap rates are constructed using polynomial splines, and then bootstrapped to obtain the term structures of zero swap rates. We use the 5-year maturity for regression purposes, and check the robustness of our maturity choice using longer maturities. For robustness, we also consider the US constant maturity Treasury rates as alternative regressors and as benchmark rates.

Our bond transaction data are sourced from the National Association of Insurance Commissioners (NAIC). Bond issue- and issuer-related descriptive data are obtained from the Fixed Investment Securities Database (FISD). The majority of transactions in the NAIC database take place between 1995 and 2004.

We implement rigorous filters to improve the reliability of the bond transaction data.

Bond transactions without recognizable counterparty names are removed. We restrict our sample to fixed coupon USD denominated bonds with issuers in the industrial sector. Furthermore, we eliminate bond issues with option features other than putables, such as callables and convertibles. Asset-backed issues, and bonds with sinking funds or credit enhancements were also removed to ensure bond prices in the sample truly reflect the underlying credit quality of issuers. The third step involves selecting bonds for which we have issuers' complete and reliable equity data as well as accounting information.

In total, 57 firms have transactions for both putable and regular bonds in the NAIC data. We then take an intersection of the putable and regular bond data. Requiring the time between the putable and regular bond transactions to be at most three days in either direction, we are left with 1,039 pairs from 45 distinct entities.

Table 2 reports the descriptive statistics of issuing companies, putable bonds, and regular bonds. We see that firm sizes vary from 1.6 to 469 billion with an average of 52 billion dollars. Bond issuers' S&P credit ratings range between AA and CCC, with a majority between BBB+ and BBB. The average regular bond issue size is 324 million dollars. The average regular bond transaction size is approximately 2.65 million dollars. In comparison, putable bonds have a smaller average transaction size of 0.33 million dollars, and a smaller average issue size of 270 million dollars. There is no significant difference in age between the two bond types but putables tend to have a longer maturity.

Table 2, Descriptive Statistics - Firm & Bond Characteristics

Variable	Obs	Mean	Std. Dev.	Min	Max
Firm size (billion \$'s)	1039	52.49	75.27	1.60	469.067
Equity (billion \$'s)	1039	22.58	27.63	0.17	183.40
Nom. debt (billion \$'s)	1039	29.91	65.77	0.88	446.61
Leverage	1039	52%	20%	6%	96%
S&P rating	1039	8.63	3.92	1.00	27.00
Regular bond					
Trans size (million \$'s)	1039	2.65	3.79	0.01	31.60
Offering amount (million \$'s)	1039	324.33	203.72	1.50	1000.00
Age	1039	4.30	3.38	0.00	16.75
Time to maturity	1039	15.05	14.36	0.52	99.71
Coupon	1039	7.25	1.11	2.25	11.13
Putable bond					
Trans size (million \$'s)	1039	0.33	0.51	0.00	4.80
Offering amount (million \$'s)	1039	269.89	132.88	10.00	600.00
Age	1039	5.00	3.57	0.00	17.73
Time to maturity	1039	26.26	10.75	2.28	99.55
Coupon	1039	7.33	0.99	5.55	10.20

To check whether there exists any selection bias for our matched sample of regular and putable bonds, we compute and compare the statistics of all industry regular bonds and putable bonds in our sample. With 74,990 and 3,463 transactions for regular and putable bonds respectively, we find: the average transaction sizes for regular (putable) bonds are 2.71 (3.41) million dollars; the average offering amounts are 393.4 (241.8) million dollars; the average ages are 3.92 (5.06) years; the average times to maturity are 11.18 (26.87)

years; the average coupon rates are 7.24 (7.33); the average S&P ratings are 8.61 (8.73) respectively. Our matched sample provides a close representation of the entire sample of industry regular and putable bonds.

Table 3 provides descriptive statistics for the regressors used in the analysis below. The leverage of a firm is defined as

$$Lev = \frac{Book\ Value\ Of\ Debt}{Market\ Value\ Total\ Equity + Book\ Value\ Of\ Debt}$$

Equity volatility for each firm (for every transaction) is computed using a moving window of 250 daily returns obtained from CRSP.

Table 3: Descriptive Statistics of Regressors in Regression Analysis

Variable	Obs	Mean	Std. Dev.	Min	Max
Dependent Variables					
Moody's Default Premium	1039	0.86	0.24	0.50	1.48
VIX	1039	23.01%	6.14%	10.66%	44.92%
S&P500 EWRETD ¹²	1039	0.11%	1.18%	-3.66%	5.88%
Pastor & Stambaugh index	1039	-0.033	0.068	-0.250	0.162
5-year swap rate	1039	4.93%	1.27%	2.12%	7.82%

The default premium measures the aggregate price of default risk. It is computed as Moody's Baa index spreads minus Aaa index spreads, which has a mean of 86 basis points for the transaction days in our sample. Another default proxy, VIX, an option-implied volatility index acquired from CBOE, averages 23% and ranges between 11% and 50%. Daily equally weighted S&P 500 returns average 0.11%. We use the Pastor & Stambaugh index level from CRSP as a proxy for market liquidity, which has a mean of 0.033 and volatility of 0.068. The average 5-year swap rate in our sample is 4.93%.¹³

¹²S&P500 Equally Weighted Rate of Return Daily.

¹³Paster & Stambaugh level provides a measure of equity market liquidity. Given the partial integration

4.4 Regression Analysis

We fit a panel data regression model run as random effects GLS with an AR(1) error structure. We rely on a methodology developed by Baltagi & Wu (1999) to adjust for the unbalanced nature of the data panel.

4.4.1 Analysis of Putable and Regular Bond Yield Spreads

Tables 4 and 5 report the results of the regressions for market putable and regular bond spreads. The independent variables include a set of proxies for credit risk, liquidity risk, a tax component, and interest rate risk. To proxy credit risk, we use financial leverage, equity return volatility, the S&P rating, Moody's default premium, and S&P500 returns. Firm size, bond age, maturity, and the Pastor-Stambaugh index level are used as liquidity proxies.

between the equity and fixed income markets, the measure should also be able to capture the liquidity in corporate bond markets. Therefore, as a proxy for market liquidity measures, we choose the closest (monthly) Pastor & Stambaugh levels to our transaction dates. For details of the index, please see the original paper of Pastor (2003).

Table 4, Regression Analysis for Market Putable Bond Spreads

Regressions are run as random effects GLS with an AR(1) error structure, and use a method developed by Baltagi & Wu (1999) to adjust for the unbalanced nature of the data panel.

Regressor		Coef.	Std. Err.	z	P>z	[95% Conf.	Interval]
Leverage ratio	<i>LEV</i>	234.5074	16.1833	14.49	0.0000	202.789	266.226
Equity volatility	σ	140.7576	23.6786	5.94	0.0000	94.348	187.167
S&P numerical rating	<i>SPR</i>	0.5331	1.0863	0.49	0.6240	-1.596	2.662
Moody's Baa index spread - Aaa index	<i>MDP</i>	2.7253	8.4681	0.32	0.7480	-13.872	19.322
CBOE implied volatility index	<i>VIX</i>	0.7677	0.2633	2.92	0.0040	0.252	1.284
S&P500 EWRETD	<i>SPR</i>	194.3806	112.7540	1.72	0.0850	-26.613	415.374
Bond maturity	<i>MAT</i>	0.0000	0.0006	-0.07	0.9430	-0.001	0.001
Bond age in years	<i>AGE</i>	0.0004	0.0026	0.13	0.8930	-0.005	0.006
Firm size	<i>SIZE</i>	-0.0001	0.0001	1.02	0.3060	0.000	0.000
Pastor Stambaugh equity market liquidity index	<i>PSI</i>	-42.8082	21.0816	-2.03	0.0420	-84.127	-1.489
Bond coupon	<i>CPN</i>	35.2034	3.7038	9.5	0.0000	27.944	42.463
5-year swap rate	r^5	-28.6989	2.0799	-13.8	0.0000	-32.775	-24.622

Adjusted $R^2 = 0.5533$

For putable bond spreads, as reported in Table 4, leverage, equity volatility and VIX are significant at the 5% level. The credit rating, default premium have the expected signs but are not statistically significant. The S&P500 return is significant at the 10% level. The positive sign may at first appear puzzling at first glance. However note that our sample period covers the same period studied in Campbell & Taksler (2003). They document that although stocks performed well in the late nineties, bond spreads increased. They attribute this to an increase in idiosyncratic volatility. Thus the positive sign need not be inconsistent with a theoretical model such as Merton (1974).

Amongst the illiquidity variables, *PSI* is significant at the 5% level and carries the expected sign. Putable bond spreads are positively related to age and negatively to firm size and *PSI*. A higher coupon tends to increase bond spreads, while spreads increase in maturity on average. In addition, bond spreads are strongly and negatively related to interest rates, consistent with Duffee (1998).

Table 5, Regression Analysis on Market Regular Bond Spreads

	Coef.	Std. Err.	z	P>z	[95% Conf.	Interval]
Leverage	646.5838	39.0379	16.56	0.000	570.071	723.097
Equity volatility	285.3367	59.1994	4.82	0.000	169.308	401.365
S&P rating	1.5620	2.1283	0.73	0.463	-2.609	5.734
Moody's def. prem.	50.9624	22.6497	2.25	0.024	6.570	95.355
VIX	0.0176	0.6957	0.03	0.980	-1.346	1.381
S&P500 EWRETD	245.9434	302.5429	0.81	0.416	-347.030	838.917
Bond maturity	0.0028	0.0008	3.34	0.001	0.001	0.004
Bond age	0.0126	0.0044	2.9	0.004	0.004	0.021
Firm size	-0.0003	0.0001	-2.89	0.004	-0.001	0.000
P-S index	-188.7807	56.5153	-3.34	0.001	-299.549	-78.013
Coupon	-1.0154	4.8268	-0.21	0.833	-10.476	8.445
5-year swap rate	-7.5403	4.6811	-1.61	0.107	-16.715	1.635
Adjusted $R^2 = 0.2149$						

Qualitatively similar results are observed for regular bond spreads. It seems that both putable and regular corporate bond spreads contain compensation to credit, liquidity and interest risks. However, the economic significance of the regressors is quite different for the two classes of bonds. For example, non-putable bonds are much more sensitive to changes in leverage and equity volatility. The difference can be thought of as a measure of the effectiveness of the embedded put against a deterioration in credit quality. The same pattern can be gleaned from the liquidity proxies. Age increases spreads for regular bonds more than for putables. As a regressor it is only significant for regular bonds. Firm size significantly decreases spreads for regular bonds - the same cannot be statistically verified for putables. The *PSI* variable decreases spreads and is statistically significant for both bond types but the coefficient is four times larger for regular bonds. These

findings are consistent with our expectations outlined above. The put option appears to absorb much of the sensitivity to changes in credit quality as well as deteriorations in bond-specific or marketwide illiquidity.

Consistent with our discussion above, the coefficient on the coupon rate is larger for puttable bonds than for regular bonds. In fact it is negative and insignificant for the latter while it is positive and significant for the puttables. Thus in our sample, there appears to be no support for the tax effect discussed in e.g. Elton et al. (2001). The sign of the coefficient for the puttables likely arises from the effect described above - a high coupon bond will likely trade further away from the strike price of the put and is thus less effectively insured than a low coupon bond.

Like previous work, we document a negative relationship between bonds spreads and the level of the risk free term structure, in our case proxied by the 5 year swap rate. This is consistent with an increased interest rate decreasing the risk-adjusted default probability and thus the spread. In addition, we find that the effect is four times stronger for puttable bonds and statistically more significant. Recall that when term structure risk is of second order importance, e.g. for a floating rate bond, the effect of the embedded put would be to weaken the negative relationship between the rate and the spread. Thus our finding of a stronger negative relationship suggests that a dominant effect at play here is the increased value of the put as a hedge against interest rate increases.

The regression of puttable spreads displays stronger statistical significance ($R^2 = 55.3\%$) compared to the regression of regular spreads does ($R^2 = 21.5\%$). Interestingly, puttable spreads show less sensitivity to the risk proxies - the coefficients of puttable regression are generally lower than those of regular regression. Given that a puttable bond can be seen as a regular bond plus put options, this suggests that put options reduce puttable bonds' risk exposure.

In summary, we find that both regular and puttable bond spreads relate broadly as expected to all our regressors. We conclude from this that our proxies for the core risk sources are reasonable and, more importantly, that puttable bonds appear to at least

partially protect their holders against default risk, term structure risk and non-default risks such as illiquidity.

We now turn to an analysis of a market proxy for the put option value directly. Further below, we will use a valuation model to quantify the relative importance of each risk source.

4.4.2 Analysis of Put Option Values

Since there is no direct way of observing the put value, we rely on the difference between market spreads for the matched regular and puttable bonds as a proxy. It is clear that this measure is contaminated by differences in puttable and regular bond characteristics such as maturity and coupon rates.¹⁴

The average difference between regular and puttable bond spreads is 49 basis points. Table 6 reports the results of the regression analysis. Put values are positively and significantly correlated to leverage, equity volatility and the default premium proxy. The S&P500 return and the VIX are not significant.

Table 6, Regression Analysis on Put Options (Market)

¹⁴For robustness check, we perform regression analysis with the bond characteristic variables included as independent variables as well. The inclusion of the variables in regression does not change the results qualitatively. We therefore do not report the results.

Regressors	Coef.	Std. Err.	z	P>z	[95% Conf.	Interval]
leverage	429.1973	34.3088	12.51	0.000	361.9534	496.4413
equity sigma	135.7978	51.6791	2.63	0.009	34.5085	237.0870
Moody's def. prem.	42.5883	19.8582	2.14	0.032	3.6670	81.5097
VIX	-0.6711	0.6100	-1.1	0.271	-1.8666	0.5245
S&P return	131.5280	265.7805	0.49	0.621	-389.3923	652.4482
firm size	-0.0004	0.0001	-3.98	0.000	-0.0006	-0.0002
P-S index	-131.4099	49.5915	-2.65	0.008	-228.6074	-34.2124
5-year swap rate	16.6322	3.8086	4.37	0.000	9.1675	24.0969

Adjusted $R^2 = 0.2340$

The values of put options are negatively correlated to the illiquidity proxies - firm size and pslevel. This implies that the put options are less valuable for holders of bonds issued by relatively larger firms. Such a firm is able to attract a larger number of investors and enjoys better marketability of its securities. Therefore, the put options are less valuable as insurance against illiquidity. In addition, the values of embedded puts increase as market liquidity drops as reflected by the negative and significant impact of the Pastor & Stambaugh Index. The risk-free rate shows a strong and positive correlation. This confirms our intuition that put options are more likely to be exercised when interest rates are high, which, in turn, increases their values.

It is also interesting to consider tables 4-6 in combination. For example, the default premium variable is highly significant for regular bonds and insignificant for putables. It is an important determinant of put value. This suggests that the risk associated with this variable is quite effectively hedged by the option. Consider next the PSI variable: it is economically much more significant for the regular bond, although it retains explanatory power for putables (and the option in isolation). Thus, we see that the put is an imperfect hedge against marketwide liquidity, likely due to limitations in their contractual design,

which we study next.

We regress put option values on various contractual characteristics of the puttable bonds. We use the following variables: the number of dates to the last scheduled put date (DLP), the number of dates to the first put date (DFP), ratio of the number of outstanding put dates to maturity of the bond (NPM), ratio of the number of the dates to the first put date to maturity (FTM), and a dummy variable for in-the-money (ITM). Due to the limited time variation in the characteristics variables, we also include controls from the previous regression.

$$\begin{aligned}
PV_{i,t} = & \alpha_i + \beta_{i,1}^{pv} DLP_{i,t} + \beta_{i,2}^{pv} DFP_{i,t} + \beta_{i,3}^{pv} NPM_{i,t} + \beta_{i,4}^{pv} FTM_{i,t} + \beta_{i,5}^{pv} ITM_{i,t} \\
& + \beta_{i,6}^{pv} SPR_{i,t} + \beta_{i,7}^{pv} MDP_t + \beta_{i,8}^{pv} VIX_t + \beta_{i,9}^{pv} SIZE_{i,t} + \beta_{i,10}^{pv} PSI_t + \varepsilon_{i,t}.
\end{aligned} \tag{4.4}$$

Table 7 reports the results of the regression analysis. First note that the explanatory power of the regression is higher by about a third, suggesting that the characteristics are important determinants of option values. Dates to first put, dates to last put, and in-the-money variables are statistically significant. The value increases the longer the time between the transaction date the last put date, consistent with the intuition that a put option value increases in time to expiration. The value of a put option increases as the earliest put date approaches. Intuitively, the closer an investor is to an exercise date, the more effective and thus valuable his option. The ratio of numbers of outstanding put dates to maturity carries a positive coefficient, implying that, by controlling for maturity, a puttable bond with a more frequent put schedule enjoys a relatively higher value. The result is marginally statistically significant. The deeper in-the-money the put is, the more valuable it is. The lower the credit rating of the issuer, the more valuable the insurance provided by the put.

Table 7, Impact of Put Option Features on the Value of the Option

Regressors		Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
Dates to last put	<i>DLP</i>	0.029137	0.004935	5.9	0	0.0194638	0.03881
Dates to first put	<i>DFP</i>	-0.03145	0.008033	-3.91	0	-0.0471929	-0.0157
Number of put/maturity	<i>NPM</i>	107001.7	66847.04	1.6	0.109	-24016.11	238019.5
Dates to first put/maturity	<i>FTM</i>	-50.4856	55.10592	-0.92	0.36	-158.4912	57.52003
In-the-money	<i>ITM</i>	103.0036	9.635411	10.69	0	84.11859	121.8887
S&P rating	<i>SPR</i>	3.95331	0.97625	4.05	0	2.039896	5.866724
Moody's def. prem.	<i>MDP</i>	47.82498	17.57578	2.72	0.007	13.37708	82.27287
VIX	<i>VIX</i>	-0.33385	0.704944	-0.47	0.636	-1.715518	1.04781
Firm size	<i>SIZE</i>	-0.00019	6.23E-05	-3.1	0.002	-0.0003151	-7.1E-05
P-S index	<i>PSI</i>	-116.393	57.73799	-2.02	0.044	-229.557	-3.22829

Adjusted $R^2 = 0.3022$

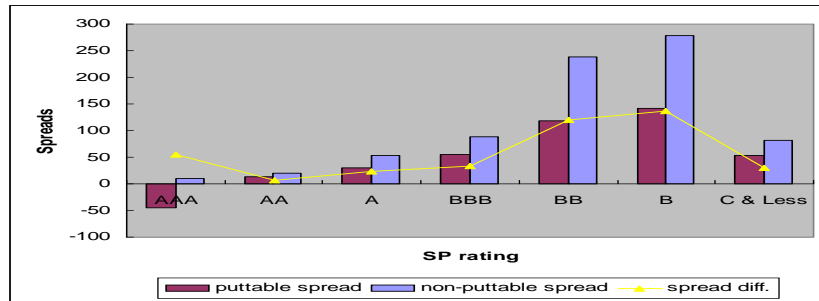
Figure 1 provides additional evidence that the values of the put options are affected by issuing firms' credit quality and market liquidity. We subgroup our market put option values by issuing firms' credit ratings and market liquidity conditions. It is noticeable that put values, proxied as before by the difference in spreads for putable and regular bonds, increase as firms' credit ratings deteriorate - to a point. Below a rating of *B*, they decrease as default becomes increasingly imminent. This observation again confirms that embedded puts do hedge against changes in default risk, but that the provided insurance is limited given that it is written by the very firm whose default is insured¹⁵. When an

¹⁵This stands in contrast to the norm in credit derivative markets where buyer and seller of default insurance are distinct from the reference entity. In the absence of any other significant counterparty risk, insurance premia in these markets should be monotonically decreasing in credit quality.

issuing firm is near bankruptcy it is unlikely to be able to honour the put.¹⁶ In addition, we observe that the values of put options generally increase as market liquidity, signaled by the Pastor-Stambaugh Index, decreases. This is consistent with the regression analysis that put options become more valuable when liquidity is relatively low.

Figure 1, Market Put Option Values by Credit Rating and Liquidity Index

Figure 1(A) shows regular and puttable bond yield spreads, and the values of put options at different credit ratings. The purple columns represent regular bond spreads; the blue columns represent puttable bond spreads; the yellow line represents the values of put options, proxied by the difference in regular spreads and puttable spreads¹⁷.



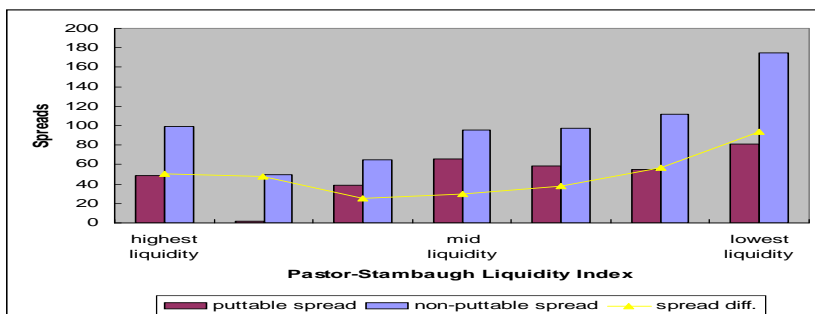
(A)

Figure 1(B) shows regular and puttable bond yield spreads, and the values of put options at different market liquidity levels represented by the Pastor-Stambaugh Index. The purple

¹⁶Puts could also derive strategic value if they can be used to trigger default by the issuing firm. See David (2001).

¹⁷The spread for puttable bonds at AAA is negative. This could be due to that swap rates are used as riskfree benchmark. In an earlier version of the paper, we use Treasury rates as riskfree rates and find a positive spread for AAA puttables. Sample bias might explain that the spreads for puttables and regulars at C&Less are lower than those at B. We have few observations in C&Less rating category.

columns represent regular bond spreads; the blue columns represent putable bond spreads; the yellow line represents the values of put options, proxied by the difference in regular spreads and putable spreads.



(B)

In summary, we find that put options embedded in corporate bonds contain insurance against default risk, illiquidity risk, and interest rate risk. The next logical step of interest is to quantify the proportion of the insurance against each individual risk. Clearly, understanding not only which risks one is exposed to but also to what extent is important. We now proceed to develop a methodology for the valuation of corporate bonds with embedded features.

4.5 A Valuation Model

In this section, we develop a recombining bivariate lattice model for the joint risk-neutral process of the asset value and the forward interest rate curve. To price corporate bonds, we require our model to have two important features: first, given the scarcity of corporate bond data and the inexistence of the market for default swap data during a large part of our sample period, the implementation should rely only on equity and accounting data

in the estimation; Secondly, suggested by recent empirical evidence (e.g. Duffee (1998)) and the regression analysis results on the relationship between corporate bond spreads and riskfree rates, the model needs to capture the correlation between interest rates and default rates - driven by asset values. The lattice is constructed in the following four steps:

1. Build the interest rate lattice in a HJM setting;
2. Estimate, using maximum likelihood, initial asset values V_0 , asset return volatilities σ_V , and default barriers V_b using the Leland & Toft (1996) model.¹⁸ Estimation of the correlation between the asset value and the risk-free rate is embedded in the maximum likelihood estimation;
3. Construct the asset value (default process) lattice using the inputs from the estimation in (2);
4. Combine the two lattices while imposing the recombining conditions.

4.5.1 The Term Structure Model

Our construction of a risk-free rate tree is based on the discrete-time form of the HJM model (see Heath et al. (1990) and Heath et al. (1992)). The HJM framework uses forward rates and the term structure of forward rate volatilities as inputs to determine the evolution of short rates. For any given pair of time-points (t, T) with $0 \leq t \leq T$, let $f(t, T)$ denote the forward rate on default-free bonds applicable to the period $(T, T + h)$. In other words, $f(t, T)$ is the rate as viewed at time t for a default-free loan transaction over the interval $(T, T + h)$. All interest rates in the model are expressed in continuously compounded terms. The forward rate curve is assumed to follow the stochastic process below:

$$f(t + h, T) = f(t, T) + \alpha(t, T)h + \sigma(t, T)X_f\sqrt{h}, \quad (4.5)$$

¹⁸We have found in previous work that the Leland & Toft model performs well in pricing corporate bond spreads over the swap curve. See Ericsson et al. (2006) and Elkamhi & Ericsson (2007).

where X_f is a standard normal variate, $\alpha(t, T)$ and $\sigma(t, T)$ are F_t -adapted processes for all $T > t$.¹⁹ The variable h represents the length of a single period. A recursion relates the risk-neutral drifts α to the volatilities σ . Given the risk-neutral dynamics of forward rates, the no-arbitrage value of $\alpha(t, T)$ can be written as:

$$\alpha(t, s) = \sigma(t, s)^T \sum_{i=1}^{s-t} \sigma(t, t + ih), \quad (4.6)$$

where $\sigma(t, t + ih)$ is the volatility of the forward rate between time t and $t + ih$. In the same spirit of Hull & White (1993), our modeling of the interest rate tree takes into account the whole term structure of forward rate volatilities.

4.5.2 The Asset Value Model

We assume the risk-neutral discrete-time asset value follows:

$$\ln \left[\frac{V(t+h)}{V(t)} \right] = (r(t) - \beta)h + \sigma_V X_V(t) \sqrt{h}, \quad (4.7)$$

where σ_V is the asset return volatility. The random variable $X_V(t)$ is standard normal.²⁰

Under this specification, a probability measure is chosen such that the one-period expected return of asset value is set to equal $r(t)h$, and the variance of return is $\sigma_V^2 h$. Since the same numeraire (the money market account) is also used in the valuation of bonds, we generate a lattice that is arbitrage free in both the bond and asset value markets.

As asset value evolves in time, default occurs when the asset value hits the default barrier for the first time. The latter is endogenously determined as the *ex post* optimal level for the shareholders to relinquish ownership of the firm. Upon default, corporate bondholders receive the face value of bonds that they own multiplied by the recovery rate.

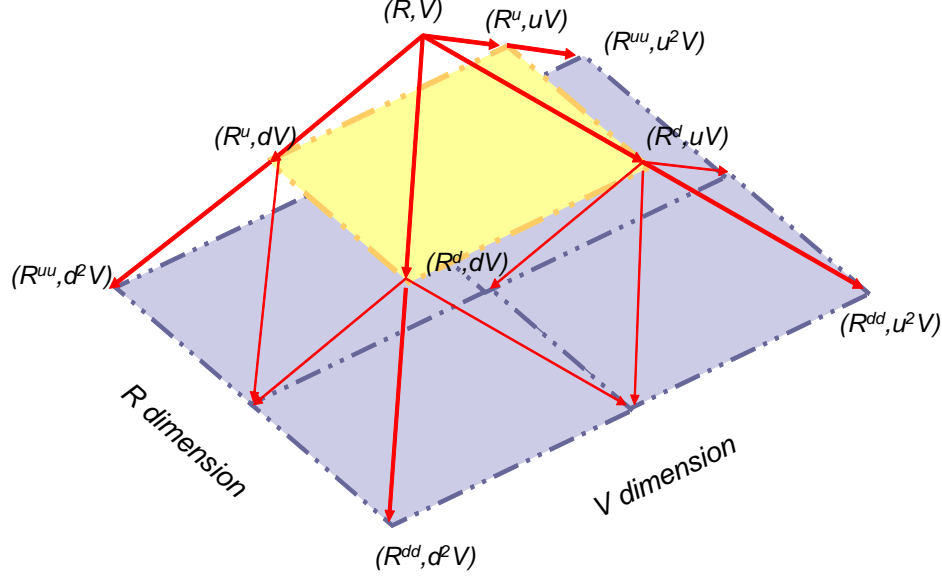
¹⁹In the lattice implementation, we let X_f be a Bernoulli random variable with outcomes $\{-1, +1\}$.

²⁰In the lattice implementation, we treat X_V like X_f , and let it take on values of $\{-1, +1\}$.

4.5.3 The Joint Process

In this final step, we combine the two processes for the term structure and the asset value introduced above in a bivariate lattice. In Figure 2, R and V denote the short interest rate and asset value (implied from the Leland & Toft (1996) model) at time t respectively. The superscripts u and d on R , the interest rate dimension, stand for upper and lower nodes respectively. But the u and d for the asset value, V , dimension represent the increments.

Figure 2, A Bivariate Lattice



In the model section above we take X_f and X_V to be normally distributed. The normality assumption for the forward rates dynamic is a necessary condition to satisfy the no-arbitrage restriction on the forward rate drift in the continuous time version of the HJM model. The normality assumption for the asset value is to conserve the same discrete time infinite state space version of the Leland & Toft model that we use to infer the initial asset values and volatilities. In the implementation of the bivariate model, we set the shocks to be captured by a Bernoulli random variable that takes values in $\{-1, 1\}$. This discrete time finite state space specification of the asset value and the forward rate dynamics converges to the usual normal distribution as the time step converges to zero.

We impose two conditions for the construction of the bivariate lattice. The first is that the tree recombines. This condition is imposed for practicality. Second, we set the conditional probabilities in the joint process in such way that the model implied correlation between the asset return and the term structure of interest rates equals the empirical correlation denoted by ρ . The model contains two correlated sources of risks

on each node - the term structure risk and the firm asset value risks. From each node on the lattice, we observe four departing branches. The corresponding probabilities depend on the realization of X_f and X_V - the shocks to the forward rate and the firm value respectively. Please see the appendix for detailed mathematical treatment for constructing the bivariate lattice.

We construct the bivariate lattice of 500 time steps. This means, for a bond of 10-year maturity, the length of one time step h is about 1 week ($7.3 \text{ days} = 365 \times 10/500$). For a bond of shorter maturity, the length of one time step in the valuation bivariate lattice shortens accordingly. The reverse holds for bonds of longer maturities.

4.5.4 Estimating and implementing the model

We rely on the Leland & Toft (1996) model (see appendix) to estimate initial asset value V_0 , asset return volatility σ_V , and default barrier V_b . The fundamental variable in the models is the value of the firm's assets, which is assumed to evolve as a geometric Brownian motion under the risk-adjusted measure:

$$d\omega_t = (r - \beta)\omega_t dt + \sigma\omega_t dW_t.$$

The constant risk-free interest rate is denoted r , β is the payout ratio, σ is the volatility of the asset value, and W_t is a standard Wiener process under the risk-adjusted measure.

Default is triggered by the shareholders' endogenous decision to stop servicing debt. Although the exact asset value at which this occurs is determined by several parameters as well as by the characteristics of the respective models, it is always a constant which we denote by V_b . In Leland & Toft (1996), the firm continuously issues debt of maturity Υ ; therefore, the firm also continuously redeems debt issued many years previously. Hence, at any given time, the firm has many overlapping debt contracts outstanding, each serviced by a continuous coupon. Coupons to individual debt contracts are designed such that the total cash flow to debt holders (the sum of coupons to all debt contracts

plus nominal repayment) is constant.

We use Maximum Likelihood estimation together with Leland & Toft (1996) to compute the initial asset value V_0 , default barrier V_b , and the volatility of the asset returns σ_V .

The methodology utilized, first proposed by Duan (1994) in the context of deposit insurance, uses price data from one or several derivatives written on the assets to infer the characteristics of the underlying, unobserved, process. In principle, the "derivative" can be any of the firm's securities but, in practice, only equity is likely to offer a precise and undisrupted price series.

The maximum likelihood estimation relies on a time series of stock prices, $E^{obs} = \{\mathcal{E}_i^{obs} : i = 1 \dots n\}$. A general formulation of the likelihood function using a change of variables is documented in Duan (1994). If we let $w(\mathcal{E}_i^{obs}, t_i; \sigma) \equiv E^{-1}(\mathcal{E}_i^{obs}, t_i; \sigma)$ be the inverse of the equity function, the likelihood function for equity can be expressed as

$$L_{\mathcal{E}}(\mathcal{E}^{obs}; \sigma) = L_{\ln \omega}(\ln w(\mathcal{E}_i^{obs}, t_i; \sigma) : i = 2 \dots n; \sigma) - \sum_{i=2}^n \ln \omega_i \left. \frac{\partial \mathcal{E}(\omega_i, t_i; \sigma)}{\partial \omega_i} \right|_{\omega_i = w(\mathcal{E}_i^{obs}, t_i; \sigma)}. \quad (4.8)$$

$L_{\ln \omega}$ is the standard likelihood function for a normally distributed variable, the log of the asset value, and $\frac{\partial \mathcal{E}_i}{\partial \omega_i}$ is the "delta" of the equity formula.

The value of the correlation between firm asset values and the term structure of interest rate denoted ρ is estimated using a window of 250-day of implied asset returns and risk-free rates observed prior to the transaction date t .

On the bivariate lattice, default occurs when asset value V falls below the barrier V_b , which is estimated using the Leland & Toft (1996) model. The value of V_b is determined at the transaction time t , and remains the same along the tree. Backward induction is used to solve for the bond price at t . On every node, if default does not occur, the value of the bond equals the one-period coupon plus the continuation value of the bond. Should default occur, the bond is recovered with ψ of its face value.

We use the following parameter values to price regular bonds and putable bonds:

- the bond's principal amount, p , the coupons c , maturity T and the coupon dates, t_i ;
- the recovery rate of the bond, ψ ;
- the total nominal amount of debt, N , coupon C and maturity Υ
- the costs of financial distress, α
- the tax rate, τ
- the rate, β , at which earnings are generated by the assets, and finally
- the current value, v , and volatility of assets, σ

The bond's principal amount, p , the coupons c , maturity T and the coupon dates are readily observable. The recovery rate of the bond in financial distress is not. We set it equal to 40%, roughly consistent with average defaulted debt recovery rate estimates for US industrial entities between 1985-2003.

The nominal amount of debt is measured by the total liabilities as reported in COMPUSTAT. Since book values are only available at the quarterly level, we linearly interpolate in order to obtain daily figures. For simplicity, we assume that the average coupon paid out to all the firm's debt holders equals the risk-free rate: $c = r \cdot N$.²¹ We set the maturity of newly issued debt equal to 6.76 years, consistent with empirical evidence reported in Stohs & Mauer (1994).

Finally, we assume that 15% of the firm's assets are lost in financial distress before being paid out to debtholders and fix the tax rate at 20%. The choice of 15% distress costs lies within the range estimated by Andrade & Kaplan (1998). The choice of 20% for the effective tax rate is consistent with the previous literature (see e.g. Leland (1998))

²¹This assumption is made for convenience. We checked this assumption by considering randomly selected firms' actual interest expense ratios. We found that our approximation performs well.

and is intentionally lower than the corporate tax rate to reflect personal tax benefits to equity returns, thus reducing the tax advantage of debt.

The payout rate β is an important parameter. We compute β as the weighted average of net of tax interest expenses (relative to total liabilities (TL)) and the equity dividend yield (DY):

$$\beta = \frac{IE}{TL} \times lev \times (1 - TR) + DY \times (1 - lev) \quad (4.9)$$

where

$$lev = \frac{TL}{TL + MC}$$

where MC denotes the firm's equity market capitalization and TR is the effective tax rate. The average net debt payout rate in our sample is 2.72%.²²

We use the following data for our estimation: firm market equity values, balance sheet information, and term structures of swap rates. Daily equity values are obtained from CRSP. Quarterly firm balance sheet data are taken from COMPUSTAT. Since the balance sheet information is only available at the quarterly level, we transform them into daily data through linear interpolation.

4.6 Decomposing Put Option Values

We first examine the statistic properties of market puttable and regular bond yield spreads over swap rates (reference risk-free benchmark) with matched maturities. As reported in Table 8, the average of observed puttable bond spreads over swap rates is approximately 50 basis points. The average of observed regular bond spreads over swap rates is approximately 99 basis points. Our model estimates an average puttable bond spread of 44 basis

²²An alternative method for estimating the cash flow rate is to use bond coupons as a proxy for the firm's proportional interest expenses. Our estimates of the cash flow rate will be lower than if we had used this approach. Corporate bonds are long term instruments. While the bond coupon may proxy well for the interest expense on long term liabilities, we find that in our sample it overestimates the interest expenses paid on short term debt. Our average net of tax interest expense ratio is about 2.72% which is just less than half the average bond coupon of 7.3% in our sample.

points, while the average of model regular bond spreads is 90 basis points. The model underestimates putable and regular bond spreads by 10 and 6 basis points on average. The discrepancies could be attributed to the liquidity premia that are not captured by the model.

Table 8, Bond Spread Summary

This table reports the market regular and putable bond spreads and their model counterparts. The hypothetical regular bond spreads are computed by applying the bivariate lattice model to price putable bonds while shutting down put options. Therefore, the hypothetical regular bonds share the same features, except for put options, as the putable bonds in the sample.

Variable	Obs	Mean	Std. Dev.	Min	Max
Putable market spread (bps)	1039	49.9	78.0	-161.9	476.6
Putable model spread (bps)	1039	44.0	97.9	-151.9	719.3
Regular market spread (bps)	1039	99.1	153.4	-120.1	2318.2
Regular model spread (bps)	1039	89.8	135.5	0.0	1044.6
Hypothetical regular spread (bps)	1039	83.5	106.7	0.0	719.3

Table 9 displays a statistic summary of the parameters estimated by implementing the LT model, and some independent variables used in our regression analysis. The average asset value is proximately 49.7 billion dollars, which is 2.8 billion lower compared to the firm value reported in Table 1. Asset value can be viewed as unlevered firm value, while firm value reflects tax benefits of debt and potential bankruptcy loss. The average default barrier is 26.4 billion dollars, and equals 98% of the average nominal debt amount. The average asset return volatility is 18%. The correlation between firm

assets and risk-free interest rates is -2.33% with negative and positive extremes of -37% and 26% , respectively. The payout rate of assets is 2.7% on average, similar to 2.64% in Ericsson et al. (2006).

Table 9, Leland & Toft (1996) Estimates

Variable	Obs	Mean	Std. Dev.	Min	Max
Leland & Toft Estimates					
Asset values V (billion \$'s)	1039	49.737	71.742	1.537	466.585
Barrier (billion \$'s)	1039	26.412	61.108	0.660	421.247
Asset return volatility	1039	17.78%	10.45%	1.00%	79.24%
Correlation between V and r	1039	-2.33%	7.04%	-37.25%	25.66%
Payout ratio β	1039	2.72%	1.25%	0.00%	17.03%

We rely on our above model to decompose the values of put options. We compute the model puttable spreads, then we calculate the model spreads of the comparable non puttable bonds. We obtain the model values of the put options by subtracting the model puttable spreads from the model spreads of comparable regular bonds that have the same features as the puttable bonds with no put options attached.

The average difference between regular and puttable bond spreads is 49 basis points. This is our proxy for the value of the market put options; we fully realize that a proportion of this difference is due to the difference in coupons and maturities. Later in this section, we will discuss and correct this aspect. Figure 3(I) illustrates the market values of the puttable and the non-puttable bonds spreads, and their model counterparties.

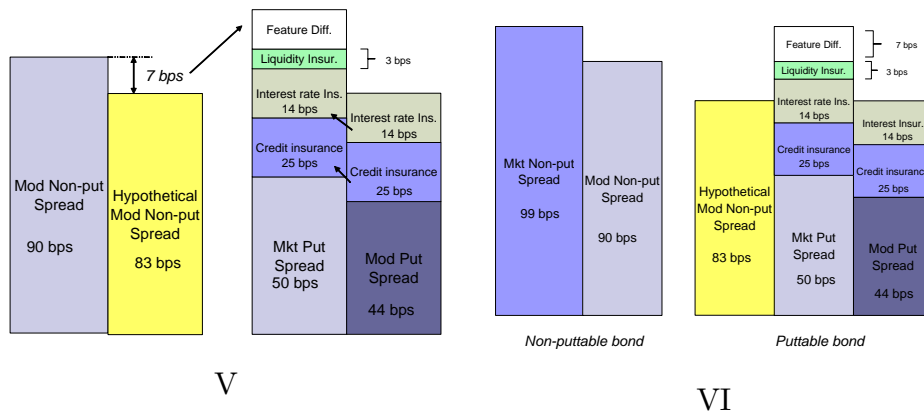
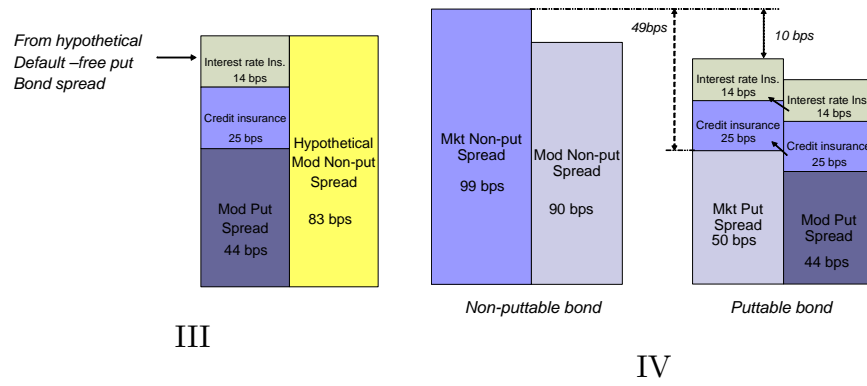
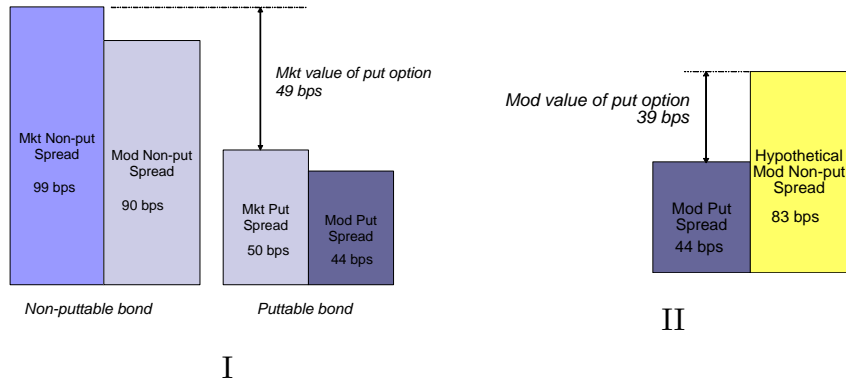
Using our model, we compute the value of the put option embedded in a puttable and an otherwise identical non-puttable bond. As illustrated in Figure 3(II), the model spreads of comparable regular bonds and puttable bonds are 83 and 44 basis points on

average. The 39 basis point difference represents the average of the model values of the put options. Given that our model captures interest rate risk and credit risk, these 39 bps comprise insurance against default risk and interest risk.

We turn our attention to investigating the model value of the put option. In essence, we wish to quantify what proportion of the 39 bps is due to credit risk insurance, and what proportion is attributed to interest rate risk. Hence, we compute the spreads of comparable default-free puttable bonds -we reestimate puttable bonds while shutting down the default dimension of the bivariate lattice. The average comparable default-free puttable bond model spread is approximately -14 basis points. To clarify the negative sign that may seem counter-intuitive, the comparable puttable default-free bond yield should be lower than the benchmark swap rate because of the embedded put. This implies that, amongst the 39 basis points, 14 basis points are insurance against interest rate fluctuations, and the remaining 25 basis points comprise insurance against default risk. This is illustrated in figure 3(III) .

Figure 3, Decomposition of Put Option Values

This figure illustrates the steps to decompose the values of put options. (I) reports the market value of the put options; (II),(III) and (IV) decompose the portions attributable to insurance against default risk and term structure risk; (V) and (VI) show how to compute the put value attributable to insurance against non-default and non-term structure risks.



After decomposing the credit risk and the term structure risk in the model price of the put option, we direct our attention to the core task of decomposing the market implied price of the put option. Figure 3(IV) shows the proportions of interest rate risks and credit risks contained in the average put values of 49 basis points.

There are 10 basis points left unexplained in the market spreads of the put options ($99bps - 50bps - 39bps$). To understand the remaining 10 basis points, we first measure the proportion due to the property difference between regular bonds and puttable bonds. In order to do so, we compare the spreads between the model regular bonds and the model comparable regular bonds that have the same coupon rate and time to maturity as the regular bonds. The average difference is 7 basis points, which captures the difference in credit risk and interest risk premia due to feature difference. This suggests that the remaining 3 basis points are attributed to other factors including liquidity enhancement provided by the put options²³. The result is consistent with the difference between the residual (market – model) spreads of regular bonds, 10 basis points, and puttable bonds, 6 basis points.

We find that the average value of insurance against risk factors provided by the put options attached to corporate bonds in term of spread is approximately 42 basis points. 60% of the spreads is insurance against default risk. 33% is insurance against interest rate risk. 7% is due to other factors including liquidity enhancement.

4.7 Conclusion

The most important drivers of corporate bond prices are likely to be interest rate risk, default risk, and illiquidity. The option to put back the bond to the issuers provides insurance against all three. In this article, we shed light on which risks are insured against by embedded puts, and to what extent.

²³The 3 basis points are unlikely due to measurement error, given that it is computed based on the average value of put options. In addition, in an unreported regression analysis, we relate the residual values of put options to liquidity proxies and find a significant relationship.

Using a sample of puttable bond and comparable regular bond transactions, we find that the put option feature does significantly hedge against all three risks. The reduction in corporate bond yield spread due to the presence of a put represents, on average, just over 40% of the yield spread. By means of regression analysis we show that the put option value (as measured by the spread reduction) is related to proxies for default, interest rate, and marketability risk.

To further understand the composition of the put option feature, we develop a bivariate lattice model that simultaneously captures correlated credit and term structure risks. The model is then applied to price regular and puttable bonds to decompose the risk components contained in the put options.

We find that the dominant source of spread reduction is attributable to default risk - an average of 60% of the reduction. But, we find that when default is imminent and the firm may not be able to honor the option, the put option value is significantly reduced.

Perhaps surprisingly, only a small fraction (7%) of the spread reduction due to the put option attributed to other non-default factors including illiquidity. Given swap spreads are used, this measure is clean of the liquidity premia contained in Treasury yields. Put options are less valuable for bonds issued by larger firms which enjoy better marketability. The values of put options increase as market liquidity drops. Put options are more likely to be exercised when interest rates are high, which, in turn, increases their values.

4.8 Appendix

4.8.1 The Leland & Toft Model

We provide a brief introduction of the Leland & Toft model used in our estimation.

The value of debt is given by

$$\mathcal{D}(\omega_t) = \frac{C}{r} + \left(N - \frac{C}{r}\right) \left(\frac{1 - e^{-r\Upsilon}}{r\Upsilon} - I(\omega_t)\right) + \left((1 - \alpha)L - \frac{C}{r}\right) J(\omega_t).$$

The bankruptcy barrier

$$L = \frac{\frac{C}{r} \left(\frac{A}{r\Upsilon} - B \right) - \frac{AP}{r\Upsilon} - \frac{\tau Cx}{r}}{1 + \alpha x - (1 - \alpha)B},$$

where

$$A = 2ye^{-r\Upsilon} \phi \left[y\sigma\sqrt{\Upsilon} \right] - 2z\phi \left[z\sigma\sqrt{\Upsilon} \right] - \frac{2}{\sigma\sqrt{\Upsilon}} n \left[z\sigma\sqrt{\Upsilon} \right] + \frac{2e^{-r\Upsilon}}{\sigma\sqrt{\Upsilon}} n \left[y\sigma\sqrt{\Upsilon} \right] + (z - y)$$

$$B = - \left(2z + \frac{2}{z\sigma^2\Upsilon} \right) \phi \left[z\sigma\sqrt{\Upsilon} \right] - \frac{2}{\sigma\sqrt{\Upsilon}} n \left[z\sigma\sqrt{\Upsilon} \right] + (z - y) + \frac{1}{z\sigma^2\Upsilon},$$

and $n[\cdot]$ denotes the standard normal density function.

The components of the debt formulae are

$$I(\omega) = \frac{1}{r\Upsilon} \left(i(\omega) - e^{-r\Upsilon} j(\omega) \right)$$

$$i(\omega) = \phi[h_1] + \left(\frac{\omega}{L} \right)^{-2a} \phi[h_2]$$

$$j(\omega) = \left(\frac{\omega}{L} \right)^{-y+z} \phi[q_1] + \left(\frac{\omega}{L} \right)^{-y-z} \phi[q_2],$$

and

$$J(\omega) = \frac{1}{z\sigma\sqrt{\Upsilon}} \begin{pmatrix} - \left(\frac{\omega}{L} \right)^{-a+z} \phi[q_1] q_1 \\ + \left(\frac{\omega}{L} \right)^{-a-z} \phi[q_2] q_2 \end{pmatrix}.$$

Finally,

$$q_1 = \frac{-b - z\sigma^2\Upsilon}{\sigma\sqrt{\Upsilon}}$$

$$q_2 = \frac{-b + z\sigma^2\Upsilon}{\sigma\sqrt{\Upsilon}}$$

$$h_1 = \frac{-b - y\sigma^2\Upsilon}{\sigma\sqrt{\Upsilon}}$$

$$h_2 = \frac{-b + y\sigma^2\Upsilon}{\sigma\sqrt{\Upsilon}},$$

and

$$y = \frac{r - \beta - 0.5\sigma^2}{\sigma^2}$$

$$z = \frac{\sqrt{y^2\sigma^4 + 2r\sigma^2}}{\sigma^2}$$

$$x = y + z$$

$$b = \ln\left(\frac{\omega}{L}\right).$$

4.8.2 Constructing the Bivariate Lattice

We propose the following parametrization of the conditional risk neutral probabilities on each node:

X_f	X_V	$RN \text{ probability}$	
1	1	$\frac{1}{4}(1 + m_1)$	
1	-1	$\frac{1}{4}(1 - m_1)$	(4.10)
-1	1	$\frac{1}{4}(1 + m_2)$	
-1	-1	$\frac{1}{4}(1 - m_2)$	

where m_1 and m_2 are two distinct parameters, whose values are determined by satisfying three conditions: a recombining tree, no-arbitrage and a matching of the correlations between V and R . We first look at the conditions for the term structure of the interest rate tree to be recombining. Then we analyze the no-arbitrage and recombining conditions for the asset value tree. Lastly, we set the necessary and sufficient conditions so that combining the univariate trees produces an correlation between V and R that matches the empirical one.

For the Hull and White version of the HJM model to be recombining, two conditions

need to be satisfied:

$$\begin{aligned} E[X_f] &= \frac{1}{4} [(1 + m_1) + (1 - m_1) - (1 + m_2) - (1 - m_2)] \\ &= 0, \end{aligned} \tag{4.11}$$

and

$$var(X_f) = E[(X_f - E[X_f])^2] = E[(X_f)^2] = 1. \tag{4.12}$$

It is important to note that the proposed parametrization of the conditional probabilities satisfies the recombining conditions for the forward rate tree for any values of m_1 and m_2 . The necessary and sufficient condition for the other dimension of the bivariate lattice, the V process, to be recombining is that the drift of the asset value process is normalized to be zero. Then the stochastic process for asset value in equation (4.7) becomes

$$\ln \left[\frac{V(t+h)}{V(t)} \right] = \sigma_V X_V(t) \sqrt{h}. \tag{4.13}$$

This discrete time finite state process specification converges to a geometric Brownian motion specification of the asset value. It is in the same spirit as the Cox et al. (1979) binomial tree model for option pricing. The mean and variance of the random variable X_V in the asset value process are

$$\begin{aligned} E[X_V] &= \frac{1}{4} [(1 + m_1) + (1 + m_2) - (1 - m_1) - (1 - m_2)] \\ &= \frac{m_1 + m_2}{2}, \end{aligned} \tag{4.14}$$

and

$$\begin{aligned}
var(X_V) &= E[(X_V - E[X_V])^2] \\
&= \left[\frac{1}{4}(1 + m_1) + \frac{1}{4}(1 + m_2) \right] \left(1 - \frac{m_1 + m_2}{2} \right)^2 \\
&\quad + \left[\frac{1}{4}(1 - m_1) + \frac{1}{4}(1 - m_2) \right] \left(-1 - \frac{m_1 + m_2}{2} \right)^2 \\
&= 1 - \left(\frac{m_1 + m_2}{2} \right)^2.
\end{aligned} \tag{4.15}$$

At this stage, the above parametrization for the risk neutral probabilities produces a perfectly recombining interest rate tree. But the firm asset value shock has non-unit variance. Therefore, the asset value tree is not recombining unless $\frac{m_1 + m_2}{2}$ is set to be zero or at least converges to zero. The values of m_1 and m_2 at this stage still provide two degrees of freedom. One is to insure that the discounted asset value, using the money account as numeraire, is a martingale under the risk-neutral measure Q . The second is to fit the correlations between asset value and term structure.

The first condition to infer m_1 and m_2 is

$$E \left[\frac{V(t+h)}{V(t)} \right] = E \left[\exp \left(\sigma_V X_V(t) \sqrt{h} \right) \right] = \exp(rh), \tag{4.16}$$

where $X_V(t)$ is a random variable whose mean and variance are defined in equations (4.14) and (4.15), respectively.

The other condition that needs to be satisfied to infer m_1 and m_2 stems from matching the correlation between the random variables $X_f(t)$ and $X_V(t)$ to the empirical correlation between the asset value and the riskfree rate, ρ :

$$Cov[X_f(t), X_V(t)] = \frac{m_1 - m_2}{2} = \rho. \tag{4.17}$$

Solving the two equations leads to:

$$\begin{aligned} m_1 &= \frac{A+B}{2} \\ m_2 &= \frac{A-B}{2}, \end{aligned} \tag{4.18}$$

where

$$\begin{aligned} A &= \frac{4 \exp(r(t)h) - 2(a+b)}{a-b} \\ B &= 2\rho, \end{aligned}$$

and where $a = \exp(\sigma_V \sqrt{h})$ and $b = \exp(-\sigma_V \sqrt{h})$.

Note that $\frac{m_1+m_2}{2}$ is different from zero, which implies that the asset value tree is non recombining. Given any parametrization of the conditional probabilities on each node, the asset value shock, X_V , would have a variance that is node dependent, when the shock to the forward rate, X_f , is set to have mean of 0 and variance of 1. As shown in equation (4.15), the variance of the asset return is node dependent. Using equation (4.16), it implies

$$\begin{aligned} \text{var}(\ln(\frac{V(t+h)}{V(t)})) &= \text{var}(\sigma_V X_V(t) \sqrt{h}) \\ &= \sigma_V^2 h \times \text{var}(X_V(t)) \\ &= \sigma_V^2 h \left(1 - \left(\frac{m_1 + m_2}{2} \right)^2 \right). \end{aligned} \tag{4.19}$$

Given that σ_V is assumed to be constant in the model, for the asset value tree to be recombining in the limit, the necessary and sufficient condition is that $\left(\frac{m_1+m_2}{2} \right)^2$ tends to zero. From equation (4.18), we have

$$\left(\frac{m_1 + m_2}{2} \right)^2 = \frac{2 \exp(f(t|node)h) - (\exp(\sigma_s \sqrt{h}) + \exp(-\sigma_s \sqrt{h}))}{\exp(\sigma_s \sqrt{h}) - \exp(-\sigma_s \sqrt{h})}.$$

By structure, no bivariate lattice is perfectly recombining in a framework that interest rate risk and other sources of risks are modeled simultaneously. However, as the length of one time step h in the lattice goes to zero, the variance of asset returns tends to converge

to $\sigma_V^2 h$, which is node independent.

$$\begin{aligned}
\lim_{h \rightarrow 0} \left(\frac{m_1 + m_2}{2} \right)^2 &= \lim_{h \rightarrow 0} \frac{2f(t|node)h - (\sigma_s \sqrt{h} - \sigma_s \sqrt{h})}{\sigma_s \sqrt{h} + \sigma_s \sqrt{h}} \\
&= \lim_{h \rightarrow 0} \frac{2f(t|node)h}{2\sigma_s \sqrt{h}} = \lim_{h \rightarrow 0} f(t|node) \sqrt{h} \\
&= 0.
\end{aligned} \tag{4.20}$$

4.8.3 Table of Firms

Firm Name	N	Size	Lev	Payout	Asset &Rf
		\$billion		Rate	Correlation
ALBERTSON'S INC	4	16706	39%	0.036	0.031
ALLEGIANCE CORP	9	3634	56%	0.017	-0.039
AMERCO	4	3208	80%	0.033	-0.015
ANADARKO PETROLEUM CORP	10	12012	36%	0.012	0.011
BAXTER INTERNATIONAL INC	5	22024	30%	0.057	0.005
BELLSOUTH TELECOMMUNICATIONS	36	81388	20%	0.026	-0.020
BOEING CO	27	65084	44%	0.017	0.001
BURLINGTON NORTHERN SANTA FE	16	29815	57%	0.020	-0.025
CSX CORP	31	22001	67%	0.029	-0.048
CHAMPION INTERNATIONAL CORP	19	10175	57%	0.026	-0.018
COCA-COLA ENTERPRISES INC	107	29173	66%	0.025	-0.048
CONAGRA FOODS INC	64	23633	47%	0.032	-0.004
CORNING INC	7	24548	28%	0.013	0.006
DOW CHEMICAL	33	50901	48%	0.034	-0.012
EASTMAN CHEMICAL CO	31	7972	52%	0.032	-0.041
EATON CORP	27	11033	47%	0.033	-0.009
FEDERATED DEPT STORES	19	17600	56%	0.022	-0.032
FORD MOTOR CO	8	284411	84%	0.044	-0.044
GENERAL MOTORS CORP	55	314864	88%	0.033	-0.109
GRAND MET PLC -ADR	5	12685	96%	0.045	0.076
HALLIBURTON CO	8	20306	36%	0.019	0.056
HARRIS CORP	3	2950	32%	0.015	-0.024
HERTZ CORP	32	9644	82%	0.038	-0.001
INGERSOLL-RAND CO LTD	14	13215	47%	0.024	-0.039
JOHNSON CONTROLS INC	14	14326	52%	0.018	-0.074
LOWE'S COMPANIES INC	29	38267	20%	0.007	-0.007

(Continued)

Firm Name	N	Size	Lev	Payout	Asset & Rf
		\$billion		Rate	Correlation
MCI COMMUNICATIONS	7	30983	41%	0.016	-0.065
MCCORMICK COMPANY INC	1	2919	30%	0.026	-0.013
MEAD CORP	6	6056	49%	0.027	-0.032
MERCK CO	11	161317	17%	0.023	-0.005
MOTOROLA INC	77	58797	33%	0.015	-0.012
NOVA CHEMICALS CORP	2	2815	93%	0.035	-0.063
OCCIDENTAL PETROLEUM CORP	29	20648	60%	0.041	-0.004
PENNEY (J C) CO	98	22460	64%	0.033	-0.015
PEP BOYS-MANNY MOE JACK	10	2146	63%	0.032	-0.028
PROCTER GAMBLE CO	49	127167	20%	0.022	-0.010
SYSCO CORP	3	21410	16%	0.012	0.008
TRW INC	3	13504	54%	0.026	-0.032
TENNECO INC	20	12673	45%	0.030	0.051
TEXACO INC	8	39887	42%	0.038	-0.009
UNION CARBIDE CORP	12	10145	46%	0.022	-0.019
WASTE MANAGEMENT INC-OLD	31	27408	48%	0.029	-0.030
WILLAMETTE INDUSTRIES	12	6450	42%	0.032	-0.019
XEROX CORP	43	42132	61%	0.030	-0.027

Chapter 5

Conclusion and Summary

This thesis contains three essays that explore various topics in fixed income and derivative Markets. These essays have in common their focus on different empirical and theoretical features of contingent claims.

In the first essay we provide valuation results for contingent claims in a discrete time infinite state space setup. Our valuation argument applies to a large class of conditionally normal and non-normal stock returns with flexible time-varying mean and volatility, as well as a potentially time-varying price of risk. Our setup generalizes the result in Duan (1995). For the class of processes we analyze in this paper, the risk neutral return dynamic is the same as the physical dynamic, but with a different parameterization which we characterize. We postulate that conditional non normality is important for index return. The heteroskedacity alone is not enough to capture the “smirk” in index options.

To demonstrate the empirical relevance of our approach, we provide an empirical analysis of a heteroskedastic return dynamic with a standardized skewed variance gamma distribution, which is constructed as the mixture of two gamma variables. Diagnostics clearly indicate that the conditionally nonnormal model outperforms the conditionally normal model, and an analysis of the option smirk demonstrates that this model provides substantially more flexibility to value options.

Unlike the first essay that looks exclusively at European style derivatives, the two other essays focus on corporate bonds. In the first one of them we develop a methodology capable of disentangling risk premia and expected losses components of the yield spread. We measure default risk premia in a large panel of US corporate bond data spanning a ten year period. We find that the risk premium is highly time varying. We show that the expected loss and default components behave differently over time. The risk premium is at its most important for high grade debt, whereas the expected loss component increases monotonically with the default probability. We show that the time series variation of the risk premium is closely related to the overall market volatility whereas the expected loss component appears more closely related to the average total volatility across firms. Perhaps our two most important findings are that (i) the time series variation observed in the risk premium in bond markets can be replicated using equity market measured risk premia translated to corporate bond risk premia and (ii) that including our risk premium metric in a linear regression of bond spreads on theoretical determinants of corporate bond risk premia increases explanatory power, suggesting that time varying risk premia is a desirable feature of future structural credit risk models. The risk premium we have measured is a translation of risk premia measured in equity markets. As such it does not capture risk premia that may be specific to fixed income markets. We conjecture that the unexplained part of our market risk premia may well contain information about illiquidity and other specific fixed income factors.

As documented in the second essays, the most important drivers of corporate bond prices are likely to be interest rate risk, default risk, and illiquidity. Thus, the option to put back the bond to the issuers should provide insurance against all three. In this third essay, we shed light on which risks are insured against by embedded puts and to what extent. Using a sample of puttable bond and comparable regular bond transactions, we find that the put option feature does significantly hedge against all three risks. The reduction in corporate bond yield spread due to the presence of a put represents, on average, just over 40% of the yield spread. By means of regression analysis we show

that the put option value (as measured by the spread reduction) is related to proxies for default, interest rate, and marketability risk. To further understand the composition of the put option feature, we develop a bivariate lattice model that simultaneously captures correlated credit and term structure risks. The model is then applied to price regular and puttable bonds to decompose the risk components contained in the put options. We find that the dominant source of spread reduction is attributable to default risk – an average of 60% of the reduction. However, we find that when default is imminent and the firm may not be able to honor the option, the put option value is significantly reduced. Perhaps surprisingly, only a small fraction (7%) of the spread reduction by put option is due to other nondefault factors including illiquidity. This finding confirms relatively our finding in the second essay concerning the size of the non default component in the credit spread of corporate bonds. More specifically we find that put options are less valuable for bonds issued by larger firms which enjoy better marketability. The values of put options increase as market liquidity drops. Finally, we show that put options are more likely to be exercised when interest rates are high. This explains why the put option increases in value when interest rates increase.

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