The Development of a Reflectometer Type of Standing Wave Indicator

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M. Eng. Thesis Elect. Eng.

Submitted May 3rd, 1947.

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I - Introduction

In the study of the propagation of radio frequency power along a transmission line, one of the most important factors is the manne in which the voltage and current vary with distance along the line. The theory of voltage and current variation along a radio frequency transmission line has been treated in detail in many texts, (1, 2, and some of the conclusions of particular interest to the subject o this paper will be repeated here.

In the installation and use of radio frequency transmission lines the engineer must have means of making accurate measurements of the voltage and current conditions along the line. It is the purpose of this paper to discuss various methods of measurement in use and to present the theory and experimental data on a new type of instrument having distinct advantages for many applications.

Three different types of measurements are employed for determining the conditions existing along a transmission line.

(a) Impedance measurement by means of a radio frequency bridge or an impedance measuring set.

(b) Standing wave measurements in which the magnitude and position of standing voltage or current waves are determined by voltage or current measurements along the line.

(c) Reflection measurements in which the absolute or relative value of power going from the generator to the load is compared with the amount being reflected back from the load to the generator.

Impedance and standing wave measurements are well known and until recently have been the only methods in use. For many applications, however, they suffer from serious disadvantages and a simpler and more rapid method of obtaining the information is most desirable.

The first information on a method of making reflection measurements appeared in 1943 (9). The possibilities of this method of making transmission line measurements seemed most attractive and the work described in this paper was undertaken by the author assisted by some of his associates, with the purpose of developing a practical instrument which would be dependable and accurate.

No further references to reflection measurements appeared in the literature until the lifting of wartime secrecy resulted in the publication of several articles on the directional coupler used in making reflection measurements on wave guides. Within the past few months descriptions of several methods of making reflection measurements on coaxial lines have appeared. The present interest and activity in this phase of the problem is prompted by the large number of Frequency Modulation transmitters being installed. Reflection measurements on the transmission lines of these installations provide an excellent means of checking the behaviour of the antenna and transmission line systems.

The fundamental principles presented in this paper are applicable to any type of transmission line balanced or unbalanced, open wire or coaxial. However, since all the development work was done on balanced open wire line, the practical details mentioned refer to this type of line in particular. Application of the principles to unbalanced open wire or coaxial lines involves different equations and constructional details which will not be considered here.

II - Relevent Basic Transmission Line Data

The fundamental principles involved in the propagation of radi frequency energy along transmission lines are well established. Certain of these principles which are of particular interest to the problem under study are summarized below.

1- The losses in a properly designed transmission line are quite small and usually may be neglected. In the following analysis it is quite permissable to assume the losses are negligible and all equations are based on this assumption.

2- When losses are negligible the characteristic impedance Z_o of the transmission line is a pure resistance.

3- If the line is terminated in a load of impedance Z_L and energy fed down the line from a generator to the load, there is in general a wave reflected back from the load towards the generator. The vector ratio of the voltage of the reflected wave to the voltage of the forward wave is called the reflection coefficient and its value is given by

$$K = \frac{V_f}{V_r} = \frac{Z_L - Z_o}{Z_L + Z_o}$$

In the particular case where $Z_L = Z_o$ the reflection coefficient becomes zero, there is no reflected wave, and the forward wave is completely absorbed in the load.

4- When both forward and reflected waves exist on the line, the total voltage at any point on the line is the sum of the forward voltage plus the reflected voltage at that point. Since the two waves are moving in opposite directions their relative phase changes with distance along the line. At some particular point on the line the voltages of the two waves will be in phase and the resultant voltage will be a maximum. One quarter wave length towards the load from this point the forward wave will be retarded by 90° and the reflected wave will be advanced by 90°. Hence the two are in phase opposition and the resultant voltage is a minimum.

The value of the voltage on the line may be found from

$$E = E_L \ (\cos \theta + j\frac{Z_o}{Z_L} \sin \theta)$$

where E_L is the voltage at the load and θ is the distance from the load to the point at which the voltage E is desired. θ is expressed in electrical degrees. This equation describes a voltage whose value varies along the line in a wave-like manner, as shown in fig. 1.

The ratio of the voltage at a maximum point (E max) to the voltage at a minimum point (E min) is called the voltage standing wave ratio and is here given the symbol $Q = \frac{E\ max}{E\ min}$.

Although we are interested primarily in voltages in this discussion, it should be noted that there are current standing waves coexistent with the voltage standing waves. The I max occurs at the same point as the E min and the I min occurs at the same point as the E max.

5- The standing wave ratio may be obtained from the reflection coefficient by the relation

$$Q = \frac{1+(K)}{1-(K)} = \frac{V_f + V_r}{V_f - V_r}$$

where (K) is the magnitude of the reflection coefficient.

6- The amount of power going towards the load is $P_f = \frac{{V_f}^2}{Z_o}$

The amount of power reflected from the load is

$$P_r = \frac{{V_r}^2}{Z_o}$$

The Power delivered to the load is

$$P_L = \frac{V_f^2 - V_r^2}{Z_o}$$

Expressed as a fraction the ratio of reflected power to forward power is given by

$$\frac{P_r}{P_f} = \left(\frac{V_r}{V_f}\right)^2 = \left(\frac{Q-1}{Q+1}\right)^2$$

Fig. 2 shows the relationship between the standing wave ratio and the percentage reflected power.

III - Summary of Present Measurement Methods

(a) Radio Frequency Bridge Measurements:

There does not seem to be any commercially available measuring equipment particularly designed for measurements on balanced lines. Equipment developed by the British Broadcasting Corporation for this purpose is described in their report on the subject (10). This report gives details of the oscillator and receiver used with the bridge, but gives very little information on the bridge itself.

The General Radio Company's 516C or 916A radio frequency bridges have been used for some time for impedance measurements on unbalanced lines. Good accuracy is obtainable but the bridge with its associated signal generator and detector - usually a radio receiver - is rather bulky.

A recently developed and very simple bridge is described in (18). It consists essentially of a unity ratio resistance bridge of which the transmission line to be measured forms one arm. The instrument is built in the shape of a short section of coaxial line and is available in different sizes, designed to plug into standard 1-5/8 inch or 3-1/8 inch diameter coaxial line. The detector is a crystal rectifier and DC meter. When the load side of the bridge is open circuited, the input power from the transmitter is adjusted to about 2 watts to give full scale meter indication. Then the bridge is plugged into the line being tested and the meter reading drops to some low value depending on the standing wave ratio on the line. A calibration curve is provided with each instrument to relate standing wave ratio to meter reading. The detector voltage is shown to be directly proportional to the magnitude of the reflection coefficient.

Another simple bridge circuit is embodied in a recently developed instrument called a Micromatch. As described in reference (21) the instrument is intended for use on either balanced or unbalanced lines. It differs from the previously described bridge in that two of the arms are capacitors and the bridge is not of the unity ratio type. The bridge resistor in series with the transmission line is made quite small (1 ohm) so that the power loss in the bridge is low. Hence the instrument may be used under actual operating conditions for power levels up to about 1000 watts. The indicating meter is calibrated in standing wave ratio and in relative power. Once the power calibration has been determined for a given line, the calibration is independent of frequency over a wide range. A disadvantage of this instrument is that it is affected by any unbalanced currents existing on a balanced line.

(b) Voltage or Current Measurements:

Although these measurements are simple in theory, they are rather difficult in practice and considerable care must be taken if accuracy is to be obtained. One difficulty is that the measuring instrument must respond to either current or voltage but not to both simultaneously. A thermocouple ammeter with a pick-up loop for instance, although intended to indicate current may also give some indication due to the voltage on the line. It is also essential that the measuring instrument should produce no appreciable reflection on the line to which it is coupled.

Voltage measurements are preferable to current measurements under most circumstances. For coaxial lines, a probe type vacuum tube voltmeter measures the voltage between the inner and outer conductors through a slot or series of holes in the outer conductor (12, 13). Balanced open wire lines may be checked by a vacuum tube or crystal rectifier type radio frequency voltmeter, capacity coupled to the line. It is fairly simple to design a radio frequency voltmeter which is insensitive to current in the line. The most important precaution is to keep the closed loops, formed by the wiring of the instrument, of as small physical dimensions as possible, and in addition, to keep the plane of these loops at right angles to the transmission line (11).

Fig. 3 shows a radio frequency voltmeter for use on a balanced line carrying 20 watts at 40 megacycles and having two inch spacing between conductors. A 1N34 germanium crystal is used as the rectifier. The two resistors isolate the radio frequency voltage from the meter and at the same time provide the resistance necessary in the meter circuit for the desired sensitivity and linearity. The machine screws, which support the 1N34 and the resistors, together with the wiring to these components, provide the necessary capacity coupling to the line conductors.

Current measurements may be made by clipping an RF ammeter across a short section of one conductor of the line or by connecting an ammeter in a single turn loop and coupling the loop to one or both line conductors. The clip-on type is simple, but not very convenient to use because of the nuisance of clipping the meter on and off the wire at a great many places for each measurement and because of variable contact resistance. In addition, it introduces a slight unbalance on the line, the seriousness of which depends on meter size, line spacing, and frequency. The coupled type eliminates contact difficulties but requires some type of insulating guide to maintain constant spacing between the line and the pick-up loop. A satisfactory method of construction uses a long insulating handle with a trolley wheel at the top and has the meter and loop mounted on the handle in such a way that the spacing between the loop and the trolley wheel is adjustable. This type, shown in fig. 4a, couples to one side of the line. Fig. 4b illustrates a type for coupling to both sides.

If absolute voltage or current values are desired the instruments described above must be calibrated on the line and at the frequency on which they are to be used. For standing wave measurements only relative values are required and an absolute calibration is not necessary.

(c) Measurement of Reflected Power:

This method has found wide application in microwave installations where directional couplers are used to determine the amount of forward and reflected power flowing in a wave guide. In its simple form a directional coupler is essentially a single frequency device since the coupling slots in the coupler must be spaced one quarter wave length apart (14). Recent improvements have produced broad band couplers (16, 17) which will operate reasonably well over a 2 to 1 frequency range.

A method for the measurement of forward and reflected power on transmission lines will be discussed in detail. The instrument has been called a reflectometer, and although it is similar to a directional coupler in some respects, it does not suffer from the frequency limitations of that instrument.

IV - The Reflectometer

The basic principles of a reflectometer type of standing wave indicator were outlined in a discussion by G.W.O. Howe (9) in comments on an article by Pistolkors and Neumann in the Russian journal Elektrosvyaz. The analysis presented below includes part of the work in reference (9) and extends the theory to enable calculation of the absolute sensitivity of the device. In addition, several working models are described together with some notes regarding precautions required to obtain the desired accuracy.

Principle of Operation

A short section of transmission line terminated at each end in a resistance equal to its characteristic impedance is suspended in the field of, and parallel to, a transmission line carrying RF power from a generator to a load. See fig. 5. As the forward wave moves along the main line it induces a voltage E_1 at the end A of the coupled section and a short time later induces an equal voltage E_2 at end B. If the velocity of propagation is the same for the line section as for the main line, the voltage induced at A will have travelled forward in the line section and arrive at B at the same instant that E_2 is induced. The net voltage existing at end B is the difference between these two, and since they are equal and in phase, the difference is zero. Hence no current will flow through the load resistor, and there will be no voltage across it.

As soon as the voltage E_2 is induced at B it travels backwards on the line section and arrives at A delayed in phase with respect to E_1 by twice the time taken to travel the length of the line section. There is therefore a net voltage of $E_1 - E_2$ at end A causing current to flow through the terminating resistor at A and a voltage drop to appear across it.

Referring to the vector diagram in fig. 5b the voltage induced at end A is taken as $E_1 = E\angle 0°$

If the line section is θ° in length, the voltage induced at B is $E_2 = E\angle -\theta$

By the time this voltage has travelled back along the section to A it is retarded θ° more and becomes $E_2' = E\angle -2\theta$

The net voltage at A is

$$E_A = E_1 - E_2' = E(1\angle 0 - 1\angle -2\theta)$$

$$E_A = 2E \sin\theta\angle 90-\theta$$

Since E_A is in series with the terminating resistor and the line impedance, one half of the voltage appears across the terminating resistor.

Hence the voltage across the terminating resistor at end A, due to the forward wave, is

$$E_R = E \sin \theta\angle 90 - \theta \qquad \ldots\ldots(1)$$

Since the phase angle of this voltage is of no interest it may be dropped from the expression giving $E_R = E \sin \theta$ ......(2)

It has already been shown that the voltage at B due to the forward wave is zero.

If a reflected wave exists on the main line, the same reasoning shows that it will produce a voltage at B and none at A. Consequently the terminating resistor at A has a voltage across it proportional to the forward wave on the main line, and the resistor at B has a voltage across it proportional to the reflected wave.

Equation (2) shows that the voltage appearing across the resistor terminating the line section is dependent only on the sine of the line length and the voltage in space existing across the terminals of the line section. If the line section is very short (10° or less in length) the voltage across the terminating resistor becomes almost directly proportional to the line length. For a given physical length, therefore, the voltage becomes proportional to frequency. If it is desired to operate this reflectometer over a wide frequency range a simple resistance capacity network, having an inverse voltage-frequency characteristic may be placed between the load resistor and the radio frequency voltmeter. Over the range in which the compensation is effective, the indication from the instrument will be independent of frequency.

The value of the induced voltage E is shown in ref. (6) to be the difference in potential between the ends of the line section conductors due to the field from the main line. Using the theory of logarithmic potentials (8), the potential E_p of a point in space due to a potential $+\frac{E_L}{2}$ on one conductor of the main line and $-\frac{E_L}{2}$ on the other conductor of the main line, is

$$E_P = \frac{E_L}{2}\left(\frac{276}{Z_o}\log\frac{r_2}{r_1}\right)$$

When the coupled section is symmetrically placed with respect to the main line so that their zero potential planes are coincident, we have from the geometry of fig. (6)

$$r_1 = \sqrt{D^2 + \left(\frac{a-b}{2}\right)^2} \quad \text{and} \quad r_2 = \sqrt{D^2 + \left(\frac{a+b}{2}\right)^2}$$

$$\text{Hence } E_P = \frac{E_L}{2}\,\frac{276}{Z_o}\log\sqrt{\frac{D^2 + \left(\frac{a+b}{2}\right)^2}{D^2 + \left(\frac{a-b}{2}\right)^2}}$$

$$= \frac{E_L}{2}\,\frac{138}{Z_o}\log\frac{D^2 + \left(\frac{a+b}{2}\right)^2}{D^2 + \left(\frac{a-b}{2}\right)^2}$$

The total voltage between the ends of conductors of the coupled section is $E = E_P - (-E_P) = 2\,E_P$

$$\text{Therefore } E = E_L\,\frac{138}{Z_o}\log\frac{D^2 + \left(\frac{a+b}{2}\right)^2}{D^2 + \left(\frac{a-b}{2}\right)^2}$$

This may be combined with equation (2) to obtain the complete expression for the sensitivity of the reflectometer

$$E_R = E_L \frac{138}{Z_o} \left(\log \frac{D^2 + \left|\frac{a+b}{2}\right|^2}{D^2 + \left|\frac{a-b}{2}\right|^2} \right) \sin \theta \qquad \ldots\ldots(3)$$

In this expression

E_R = the voltage across the resistors terminating the coupled section.

E_L = the voltage across the main line.

Z_o = the characteristic impedance of the main line.

θ = the length of the coupled section in electrical degrees.

D = the distance between the plane containing the main line and that containing the coupled section.

a = the spacing between conductors of the main line.

b = the spacing between conductors of the coupled section.

All distances are in the same units.

V - Practical Applications

There are a number of different methods of obtaining meter readings from the voltages developed across the line section terminating resistors. One method is to insert a thermocouple in series with each terminating resistor. This will give deflections on the indicating meter proportional to the square of the direct and reflected voltage waves and hence directly proportional to direct and reflected power. Fig. 7 shows a simple metering system which will permit measuring direct, reflected and delivered power. A practical difficulty with this circuit is that the thermocouples are susceptible to the presence of unbalanced line currents. In addition, the thermocouples are easily burned out during any adjustments on the line or antenna. Reference to fig. 2 shows that a standing wave ratio of 2 corresponds to a reflected power of only 11%. Hence a meter which reads reflected power linearly is a very insensitive indicator of low values of standing wave ratio.

A more desirable form of indicating circuit is a radio frequency voltmeter which will indicate the voltage existing across the terminating resistor at the ends of the line section. The DC outputs from the voltmeter rectifiers may be used in several different ways.

1- They may be fed directly to DC meters which will show the absolute or relative values of forward and reflected voltage waves.
2- They may be fed into a ratio meter in order to obtain an indication proportional to the ratio of the reflected to forward voltage waves and independent of their amplitudes. One type of ratio meter has two stationary field coils at

2- (cont'd) right angles to each other. The meter needle is connected to a soft iron vane which is pivoted in the center of the coils and is free to assume a position parallel to the resultant magnetic field inside the coils. The direction of this field is dependent only on the ratio of the currents in the two coils and not on their magnitudes.

3- A circuit which combines the features of 1 and 2 is shown in fig. 8. A single meter is switched to one position for indication of the forward wave and the variable resistor adjusted for full scale indication of the meter. The meter is then switched to the other position where it reads the reflected wave. If the meter scale is calibrated linearly from 0-100 it will read the reflected wave as a percentage of the forward wave.

Since $Q = \frac{V_f + V_r}{V_f - V_r}$ the meter scale may be calibrated to read standing wave ratio directly from 1.0 at the minimum of the scale to infinity at the maximum of the scale. The value of 3.0 occurs at mid scale so that it is possible to read moderate standing wave ratios quite accurately. If it is desired to read low standing wave ratios more accurately it is a simple matter to use a more sensitive range on the DC meter when it is indicating on the reflected wave. This expands the lower end of the scale but eliminates the upper end. A convenient combination provides one scale reading standing wave ratios from 1.0 to infinity and another from 1.0 to 2.0.

A working model of the instrument described in the preceding paragraph is illustrated in fig. 9 and the diagram shown in fig. 10. This model was designed to work on a line carrying 50 kilowatts over a range from 6 to 20 megacycles. It may be operated on either

110 volts AC or from internal batteries for portable use.

The transmission line section is the two outer lengths of 3/8 inch copper tubing. The central piece of tubing contains all the DC voltages as well as the heater circuit for the diode rectifiers. The small boxes at each end contain the frequency compensating networks, diode load resistors, etc. The 6H6 diode rectifiers and the line section terminating resistors are on the outside of the boxes.

Since the cathode to heater capacity is greater than the cathode to plate capacity in the diode rectifiers, an external capacity is added to balance the circuit. The second diode section of the 6H6 is used to provide a bucking voltage to balance the initial meter current caused by the contact potential of the measuring diode.

The terminating resistors were developed specially for this particular instrument. They are capable of dissipating about 30 watts and have very low reactance up to 20 mc. The resistance wire is wound around a mica insulated piece of sheet copper. A second layer of mica is placed over the wire and then a pair of brass plates over the mica. Machine screws at the four corners of the outside plates clamp the whole assembly securely. The proximity of the conducting plates to the resistance wire causes the series inductance to be quite low since eddy currents are induced in the plates which oppose the flux producing them and hence reduce the self inductance of the wire. There is some distributed capacity between the resistance wire and the plates and over a limited frequency range this capacity tends to compensate for the residual series inductance. By adjusting the compression on the plates by means of the corner bolts it has been found possible to vary the capacity and inductance in such a way that the characteristics of the resistor are good up to about 20 mc.

The mica insulation used is a fabricated type composed of mica scales and a binder. After the compression has been adjusted to the desired value as indicated by measurements of the resistor characteristics on a radio frequency bridge, the resistor is heated by applying about 30 watts of 60 cycle power to it. The binder in the mica softens with the heat and after heating for a short time the assembly becomes well bonded together and quite stable.

Performance

In spite of the precautions taken in the construction of this model, its performance in the field was unsatisfactory. Although it gave accurate indications of standing wave ratio in the laboratory, it did not maintain this accuracy when installed. The loss of accuracy was due to the difficulty of obtaining an exact balance of the condenser compensating for heater to cathode capacity of the diodes. The presence of two additional 50 kilowatt transmitters, one on the broadcast and one on the shortwave band, caused fairly serious extraneous voltages on the line where the reflectometer was expected to operate. Any slight unbalance in the diode circuit made the instrument responsive to such voltage and gave false readings.

Improved Model

The recently developed germanium crystal rectifiers (Sylvania type 1N34) permitted a considerable improvement and simplification of the original model of the reflectometer. These crystals have a peak voltage rating of 50 volts and consequently can be used over a reasonable voltage range without danger of burnout.

Advantages to be gained by using the 1N34 instead of a vacuum tube diode are:

1- No heater current is required thus eliminating the power supply and simplifying construction.

2- The 1N34 has no inherent unbalanced capacity to ground such as the heater-cathode capacity of a vacuum tube rectifier. Stray capacities to ground are very much smaller than for a vacuum tube.

3- The crystal rectifier has no contact potential thus eliminating the bucking circuit and its initial adjustment.

A laboratory model of the reflectometer using 1N34 rectifiers is shown in fig. 11. This instrument operates at 40 mc. coupled to a line carrying about 20 watts. By making the line section small and keeping the coupling to the main line fairly loose, the power dissipated in the line section terminating resistors becomes quite small so that ordinary half watt resistors of suitable radio frequency characteristics may be used. Because of the physical and electrical symmetry of this model its discrimination against unbalanced line voltages is excellent. When compared with the original model, the reduction in size and complexity achieved in this model is very apparent.

The operation is as follows:

1- Set the switch to the "Calibrate" position and adjust the sensitivity control to give full scale indication on the meter.

2- Set the switch to the "Read" position and read the standing wave ratio directly on the meter.

VI - Experimental Check of Theory.

The model shown in fig. 11 was operated under the conditions given below. The main line was terminated with 25 watt tungsten lamp as load, which was a sufficiently good match for the main line that the reflectometer indicated a standing wave ratio of only 1.14 or a reflection coefficient of 0.064. This corresponds to a reflected power of only $(0.064)^2 = 0.41\%$ which is negligible.

The radio frequency power was measured by using a photo-electric exposure meter to check the brilliancy of the lamp and substituting direct current to get the same brilliancy. A voltmeter-ammeter method was used to check the DC power.

The RF voltage across the line is then $E = \sqrt{PZ_o}$

The main line had 2 inch spacing between conductors and a $Z_o = 476^{\omega}$. At a power of 5.65 watts used for the test the line voltage was $E_L = \sqrt{5.65 \times 476} = 51.8$ volts.

Data for the coupled section was

Length = 19-7/8 inches = 24.2° at 40 mc.

Conductor spacing = 2 inches

Spacing below main line = 1 inch

Z_o of coupled section = 466^{ω}

Terminating resistors = 461^{ω} (DC measurement)

The voltage across the terminating resistor is given by equation (3)

$$E_R = \frac{138E_L}{Z_o}\left(\log \frac{D^2 + \left(\frac{a+b}{2}\right)^2}{D^2 + \left(\frac{a-b}{2}\right)^2}\right)\sin\theta$$

$$= \frac{138E_L}{476}\left(\log \frac{1^2 + 2^2}{1}\right)\sin 24.2°$$

$$= .0835\ E_L$$

Hence for $E_L = 51.8$ volts

$E_R = 4.32$ volts

The voltage indicated on the RF voltmeter across the terminating resistor was actually 10% below this value. 5% of this was caused by the input admittance of the voltmeter shunting the terminating resistor (see appendix I). The remaining 5% error may be due to the general difficulty of making measurements of this sort to a higher degree of accuracy.

Other Designs

One difficulty in the construction of this type of reflectometer particularly in the case of small, high frequency models, is that of keeping the terminating resistor, rectifier, and other associated components in a sufficiently compact form that there will be negligible effect on the main line. This difficulty may be easily overcome by bending the line section down from the main line and thus removing the terminating components from the field of the main line. Fig. 12 illustrates application of this principle to a transmission line composed of Amphenol 300 ohm Twin-Lead, a line having a polyethylene web between conductors.

The voltage is induced into that part of the coupled section which bends away from the main line. Since no reflections come back from the properly terminated ends of the coupled section, it does not matter how far the terminations are from the main line.

Fig. 13 shows a version in which resistors at the end of the horizontal part of the section form part of the termination, the remainder of the termination being a hanging section of properly terminated line of lower impedance. Fig. 14 illustrates the use of sections of line having different physical dimensions but the same characteristic impedance. The one restriction on using different types of lines is that the velocity of propagation along the portion of the coupled line running parallel to the main line should be the same as that of the main line itself.

VII - Design Considerations

Considerable flexibility is possible in the design of the coupled line section for the reflectometer but the following general principles should be followed.

1- The amount of power absorbed by the reflectometer should be negligible with respect to the power on the main line. This means that the coupling between the main line and the coupled section should be small. The factors controlling the coupling are the length and spacing between conductors of the coupled section as well as the spacing between the coupled section and the main line.

2- The coupled section should be mounted symmetrically with respect to the main line. The spacing between the coupled section and the main line must be maintained accurately but at the same time, the number and size of supporting insulators must be kept at a minimum in order to avoid undesired reflections.

3- If more than one transmission line is in use, care must be taken that the amount of energy coupled into the reflectometer from adjacent lines is quite negligible.

If the reflectometer is used without the frequency compensating network, consideration must be given to the fact that an adjacent line may be operating on a higher frequency which may make the ratio of desired coupling less favourable. Keeping the coupled section close to its own line will help to obtain the best ratio of desired to undesired coupling.

The voltage induced into the reflectometer due to the presence of an adjacent line may be calculated using the equation

$$E_P = E_L \frac{138}{Z_o} \log \frac{r_2}{r_1}$$

The potential E_P is calculated for the position in space of each conductor of the coupled section and the difference between these two values gives the voltage appearing across the line section terminating resistor.

4- The coupled section must be accurately terminated in a pure resistance. This is not too difficult below 50 megacycles, but at higher frequencies stray capacities and inductances present a serious problem and considerable care must be taken to minimize their effects. The errors introduced by resistance mismatch are treated in detail in Appendix I (c).

VIII - Conclusion

A complete analysis of the operation of a reflectometer type of standing wave indicator has been presented. The equations necessary for the design of the instrument for use with a balanced transmission line have been developed and the magnitude of certain possible errors calculated. The simplicity of construction and ease of operation found in this instrument make it valuable for both field and laboratory measurements. It is particularly useful during the installation and adjustment of matching stubs on a transmission line feeding an antenna. In addition the reflectometer provides a convenient means of continuously monitoring the standing wave ratio on the transmission line of the completed installation.

Acknowledgement

The writer wishes to express his appreciation to Dr. F.S. Howes, of McGill University for suggestions in the preparation of this paper, as well as to his associates in the Canadian Broadcasting Corporation, who cooperated in the construction of the equipment. Mr. W.G. Roxburgh deserves particular recognition for his work in the construction of the original model of the reflectometer.

APPENDIX I

Equivalent Circuit Analysis

The action of a reflectometer may be analysed in terms of an equivalent circuit and although the procedure is somewhat more complicated than the vector representation already given, the circuit analysis is useful for evaluating the effects of imperfections in the line section terminations or other departures from the assumed conditions.

Section (a) below is chiefly from reference (9) and sections (b) and (c) have been derived by the author.

(a) Reflectometer Null Action

The coupled section of transmission line may be represented as an equivalent T section (1 - p. 172) and since a lossless line is assumed, the hyperbolic functions are replaced by circular functions in obtaining the reactances for the arms of the T section (see fig. 15).

The values of the arms of the T are

$$X_1 = X_2 = \omega L = R_o \tan \frac{\theta}{2}$$

$$X_3 = \frac{1}{\omega c} = \frac{R_o}{\operatorname{Sin} \theta}$$

θ = the line section length in electrical degrees.

R_o = the characteristic impedance of the section.

R_o is used instead of Z_o to stress the fact that it is a pure resistance.

Let $R_1 = R_2$

Then $Z_1 = Z_2 = R_1 + j\omega L$

$$Z_3 = \frac{1}{j\omega C}$$

E_1 will produce a current I_1' at A and I_2' at B

$$I_1' = E_1 \frac{Z_2 + Z_3}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1} \text{ and } I_2' = E_1 \frac{Z_3}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}$$

Similarly E_2 produces a current I_2'' at B and I_1'' at A.

$$I_2'' = E_2 \frac{Z_1 + Z_3}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1} \text{ and } I_1'' = E_2 \frac{Z_3}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}$$

The resultant currents at the ends are

$$I_1 = I_1' - I_1'' = \frac{E_1(Z_2 + Z_3) - E_2Z_3}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1} \text{ and } I_2 = I_2'' - I_2' = \frac{E_2(Z_1 + Z_3) - E_1Z_3}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}$$

I_1 or I_2 will be zero if the numerators are zero.

Since the line section is assumed to be terminated in a pure resistance equal to its characteristic impedance

$$R_1 = R_2 = R_o$$

Therefore $Z_1 = Z_2 = R_o + j\omega L = R_o(1 + j \tan \frac{\theta}{2})$

$$Z_3 = \frac{R_o}{j \sin \theta}$$

The numerator of $I_2 = E_2R_o(1+j \tan \frac{\theta}{2} - j \frac{1}{\sin \theta}) + E_1R_oj \frac{1}{\sin \theta}$ ....(4)

For a forward wave moving from A to B, E_2 lags E_1 by $\theta°$

Therefore $E_1 = E_2e^{j\theta} = E_2 (\cos \theta + j \sin \theta)$

Substituting these in equation (4)

Numerator $I_2 = E_2R_o (1 + j \tan \frac{\theta}{2} - j \frac{1}{\sin \theta} + e^{j\theta} j \frac{1}{\sin \theta})$

Let $\frac{\theta}{2} = x$

$$\text{Numerator } I_2 = E_2R_o(1 + j \tan x - j \frac{1}{\sin 2x} + j \frac{\cos 2x + j \sin 2x}{\sin 2x})$$

$$= E_2R_oj(\tan x - \frac{1}{\sin 2x} + \frac{\cos 2x}{\sin 2x})$$

$$= E_2R_oj(\tan x - \frac{1 - \cos 2x}{2 \sin x \cos x})$$

$$= E_2R_oj(\tan x - \frac{2 \sin^2 x}{2 \sin x \cos x})$$

$$= 0$$

Therefore $I_2 = 0$ at end B for a wave travelling from A to B.

Similarly a wave from B to a produces no current at A.

(b) Reflectometer Sensitivity

In the case of a reflected wave travelling along the main line from B to A, the only change in the above initial conditions is that E_2 now leads E_1 by $\theta°$ whereas in (a) E_2 lagged E_1 by $\theta°$.

Substituting $E_1 = E_2 e^{-j\theta}$

in the equation $I_2 = \frac{E_2(Z_1 + Z_3) - E_1 Z_3}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}$

gives $$I_2 = \frac{E_2(Z_1 + Z_3 - Z_3 e^{-j\theta})}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}$$

Consider the numerator only and let $\frac{\theta}{2} = x$

Then numerator I_2

$$= E_2 \left| R_o (1 + j \tan x) - j \frac{R_o}{\sin 2x} + j \frac{R_o}{\sin 2x} e^{-j2x} \right|$$

$$= E_2 R_o \left| 1 + j \tan x - j \frac{1}{\sin 2x} + j \frac{1}{\sin 2x} (\cos 2x - j \sin 2x) \right|$$

$$= E_2 R_o \left| 2 + j \tan x - j \frac{1}{\sin 2x} + j \frac{\cos 2x}{\sin 2x} \right|$$

$$= E_2 R_o \left| 2 + j \left(\frac{\sin x}{\cos x} + \frac{\cos 2x - 1}{\sin 2x}\right) \right|$$

$$= E_2 R_o \left| 2 + j \left(\frac{\sin x}{\cos x} + \frac{-2 \sin^2 x}{2 \sin x \cos x}\right) \right|$$

$$= E_2 R_o \left| 2 + j \left(\frac{\sin x}{\cos x} - \frac{\sin x}{\cos x}\right) \right|$$

$$= 2 E_2 R_o$$

The denominator of $I_2 = Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1$

$$= R_o^2 (1 + j \tan x)^2 + 2 R_o (1 + j \tan x) \frac{R_o}{j \sin 2x}$$

$$= R_o^2 \left| 1 + 2 j \tan x - \tan^2 x + \frac{2}{j \sin 2x} + \frac{2 j \tan x}{j \sin 2x} \right|$$

$$= R_o^2 \left| 1 - \frac{\sin 2x}{\cos^2 x} + \frac{1}{\cos^2 x} + 2 j \left(\frac{\sin x}{\cos x} - \frac{1}{2 \sin x \cos x}\right) \right|$$

$$= R_o^2 \left| 1 + \frac{1 - \sin^2 x}{\cos^2 x} + 2j \left(\frac{2 \sin^2 x - 1}{2 \sin x \cos x}\right) \right|$$

$$= R_o^2 \left| 2 + 2 j \left(\frac{-\cos 2x}{\sin 2x}\right) \right|$$

$$= 2 R_o^2 \left| 1 - j \cot 2x \right|$$

Hence $$I_2 = \frac{2 E_2 R_o}{2 R_o^2 (1 - j \cot 2x)} = \frac{E_2}{R_o} \left(\frac{1}{1 - j \cot 2x}\right)$$

Replacing θ for 2 x

$$I_2 = \frac{E_2}{R_o} \frac{1}{1 - j \cot \theta} = \frac{E_2}{R_o} \frac{1}{1 - j \frac{\cos \theta}{\sin \theta}}$$

$$= \frac{E_2}{R_o} \sin \theta \frac{1}{\sin \theta - j \cos \theta}$$

$$= \frac{E_2}{R_o} \sin \theta (\sin \theta + j \cos \theta)$$

$$= \frac{E_2}{R_o} \sin \theta \left| \cos (90 - \theta) + j \sin (90 - \theta) \right|$$

$$I_2 = \frac{E_2}{R_o} \sin \theta \underline{/90 - \theta}$$

Since the voltage drop across the terminating resistor is

$$E_R = I_2 R_o$$

Hence $E_R = E_2 \sin \theta \underline{/90 - \theta}$

This is the same expression as equation (1) which was obtained by the vector solution of the problem.

(c) Errors due to a Resistance Mismatch of the Coupled Section.

Suppose the line section is terminated at each end in resistances such that $R_1 = R_2 = nR_o$

then for the forward wave when

$E_1 = E_2 e^{j\theta}$ and letting $2x = \theta$

the numerator of

$$I_2 = R_o E_2 \left| n + j \tan x - j \frac{1}{\sin 2x} + j \frac{\cos 2x + j \sin 2x}{\sin 2x} \right|$$

$$= R_o E_2 \left| n - 1 + j \left(\tan x - \frac{1}{\sin 2x} + \cot 2x\right) \right|$$

$$= R_o E_2 \; (n - 1)$$

The denominator of $I_2 = Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1$

$$= R_o^{\,2} \; (n + j \tan x^2)^2 + 2 \frac{R_o^{\,2}}{j \sin 2x} (n + j \tan x)$$

$$= R_o^{\,2} \left| n^2 + 2\,n\,j \tan x - \tan^2 x + \frac{2n}{j \sin 2x} + \frac{2 \tan x}{\sin 2x} \right|$$

$$= R_o^{\,2} \left| n^2 - \frac{\sin^2 x}{\cos^2 x} + \frac{2 \sin x}{\cos x \; (2 \cos x \sin x)} + 2j\,n \left(\tan x - \frac{1}{\sin 2x}\right) \right|$$

$$= R_o^{\,2} \left| n^2 - \frac{\sin^2 x}{\cos^2 x} + \frac{1}{\cos^2 x} + 2j\,n(-\cot 2x) \right|$$

$$= R_o^{\,2} \left| n^2 + 1 - j\,2\,n \cot 2x \right|$$

$$\text{Therefore } I_2 = \frac{R_o E_2 \; (n-1)}{R_o^{\,2} (n^2 + 1 - j\,2n \cot \theta)}$$

$$= \frac{E_2 \; (n-1)}{R_o \; (n^2 + 1 - j\,2n \cot \theta)}$$

This shows that there is not a complete cancellation at end B as there was when the coupled section was terminated properly so that $R_1 = R_2 = R_o$.

For the reflected wave when $E_1 = E_2 e^{-j\theta}$

The numerator of I_2

$$= E_2 \left| R_o(n + j \tan x) - j \frac{R_o}{\sin 2x} + j \frac{R_o}{\sin 2x} (\cos 2x - j \sin 2x) \right|$$

$$= E_2 R_o \left| 1 + n + j \tan x - j \frac{1}{\sin 2x} + j \frac{\cos 2x}{\sin 2x} \right|$$

$$= E_2 R_o (1 + n) \text{ since other terms} = 0 \text{ as before.}$$

$$\text{Therefore } I_2 = \frac{E_2 R_o \ (1+n)}{R_o^2 (n^2 + 1 - j\ 2n \cot \theta)}$$

$$= \frac{E_2}{R_o} \frac{1 + n}{(n^2 + 1 - j\ 2n \cot \theta)}$$

The magnitude of this effect may be found for the reflectometer shown in fig. 11.

Its length was 24.2° so that $\cot \theta = 2.225$

Suppose the terminating resistors R_1 and R_2 were 10% higher than the R_o of the section so that $n = 1.1$

Then for the forward wave

$$I_2 = \frac{E_2}{R_o} \frac{n - 1}{n^2 + 1 - j\ 2n \cot \theta} = \frac{E_2}{R_o} \frac{0.1}{5.37\angle -65.7}$$

$$= \frac{E_2}{R_o} 0.0186\angle 65.7$$

For the reflected wave

$$I_2 = \frac{E_2}{R_o} \frac{1 + n}{n^2 + 1 - j\ 2n \cot \theta} = \frac{E_2}{R_o} \frac{2.1}{5.37\angle -65.7}$$

$$= \frac{E_2}{R_o} 0.391\angle 65.7$$

For a perfect match, the forward wave gives $I_2 = 0$ and the reflected wave gives

$$I_2 = \frac{E_2}{R_o} \sin \theta \angle 90 - \theta = \frac{E_2}{R_o} 0.4099\angle 65.8$$

The same relative errors would exist at the other end of the reflectometer in the magnitude of I_1.

In measuring the magnitude of the forward wave on a perfectly matched main line, the voltage ecross the R_1 of the reflectometer would be

$$R_1 I_1 = \frac{1.1\ R_o E_2\ 0.391}{R_o} = 0.430\ E_2$$

The true value = $.4099\ E_2$

Hence the error is $\frac{.430}{.4099} = 1.049 = 4.9\%$ high.

In addition it gives a false indication of a reflected wave whose indicated value is $\frac{.0186}{.391} = .0475$ of the forward wave. This corresponds to a standing wave ratio of $\frac{1 + .0475}{1 - .0475} = 1.1$ instead of the true value of 1.0.

If the main line has a reflected wave on it, the errors will depend on the relative phases of the forward and reflected waves. Since the phases vary with position along the line, the errors will also vary with the location of the reflectometer along the line.

This variation in error may be readily understood if it is considered that the coupled section, when terminated in resistors which are greater than the R_o of the section, tends to act as a line voltage indicator. Conversely, if the terminating resistors are less than R_o, the section tends to act as a line current indicator. This sensitivity to voltage or current is superimposed on the desired reflectometer action resulting in the errors described above.

18- H.R. Whaley, New High Frequency Bridge, Western Electric Oscillator, 1947.

19- J.F. Morrison, E.L. Younker, A Method of Determining and Monitoring Power and Impedance at High Frequencies, Western Electric Company, 1947.

20- B. Parzen, A. Yalow, Theory and Design of the Reflectometer, Electrical Communication, Vol. 24, No. 1, P. 94, 1947.

21- M.C. Jones, C. Sontheimer, The Micromatch, Q S T, Vol. 31, No. 4, P. 15, 1947.

22- N.I. Korman, The Theory and Design of Several Types of Wave Selectors, Proceedings of the National Electronics Conference 1946, P. 404, published in Feb. 1947.

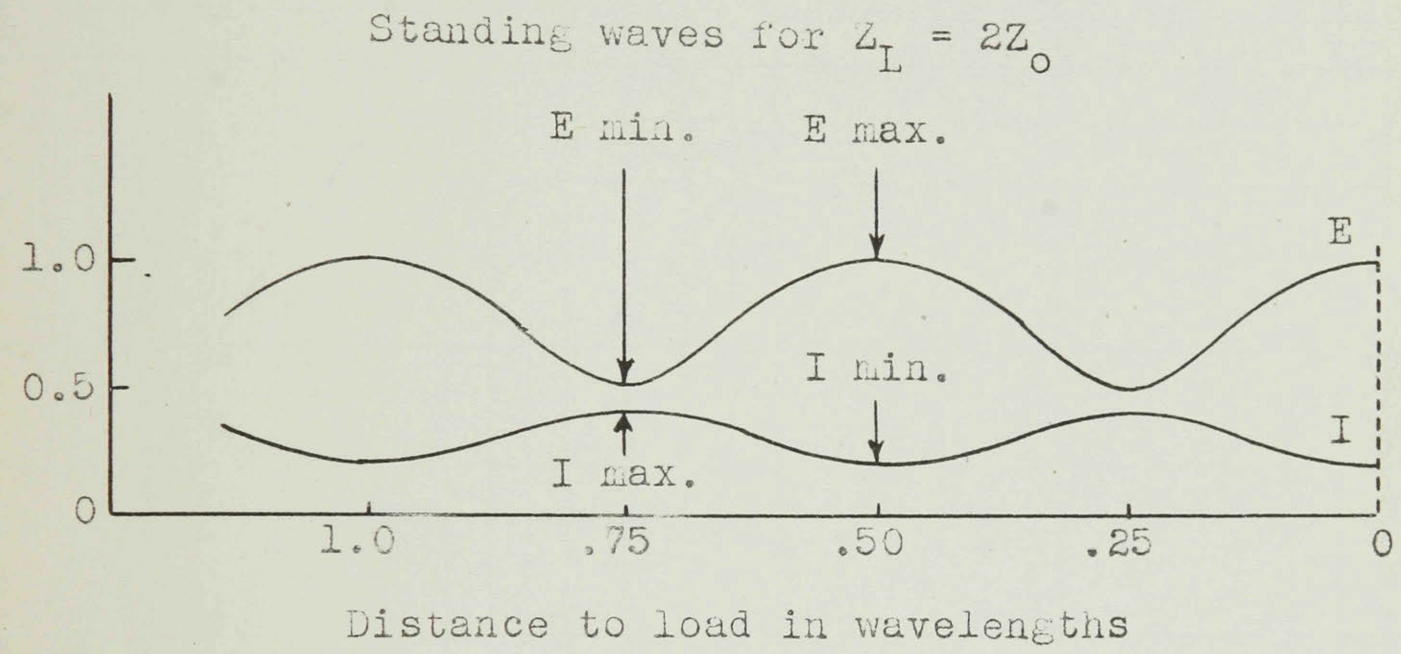


Fig. 1 - Standing Waves on a Transmission Line.

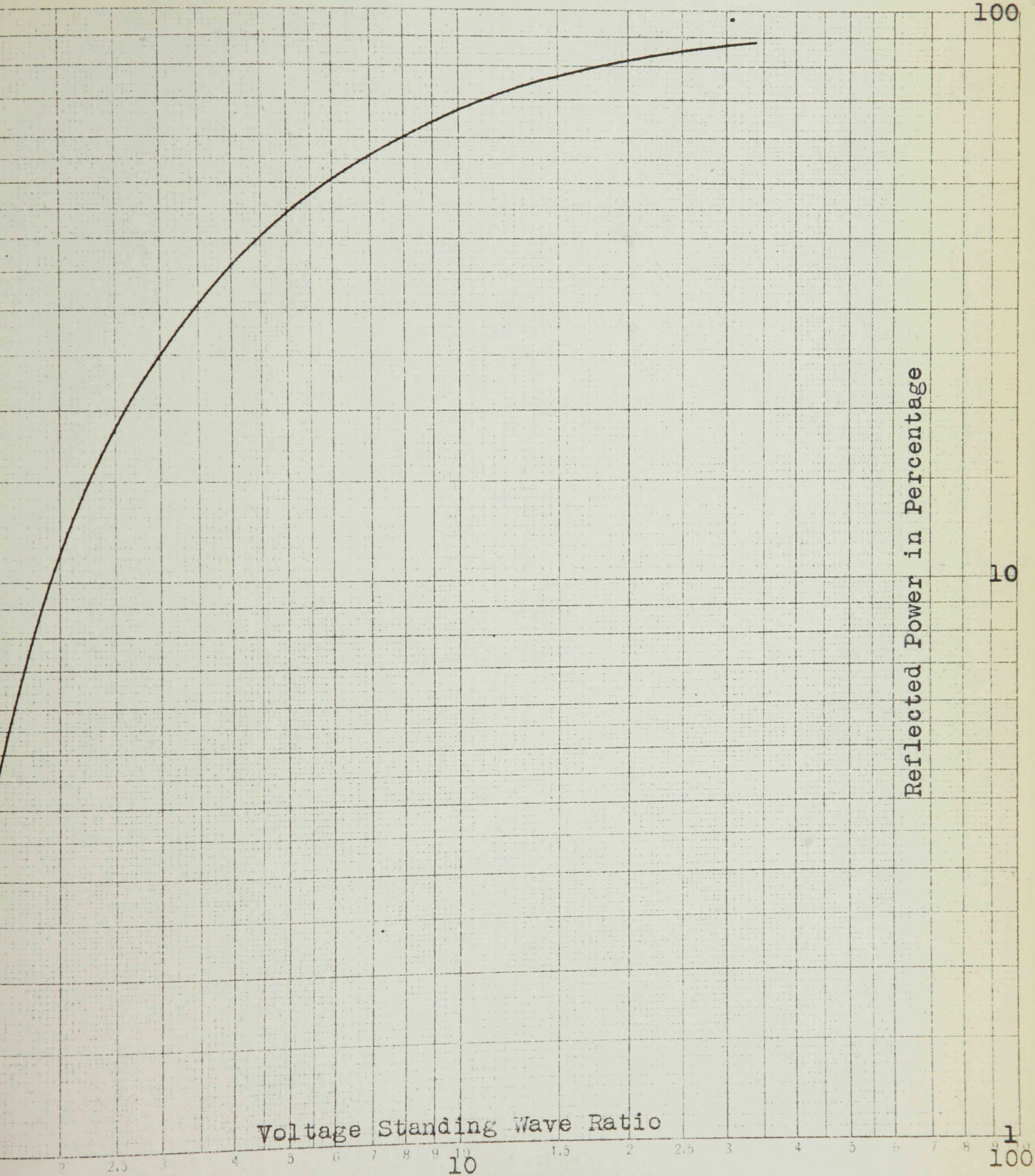


Fig. 2 - Reflected Power vs. Standing Wave Ratio.

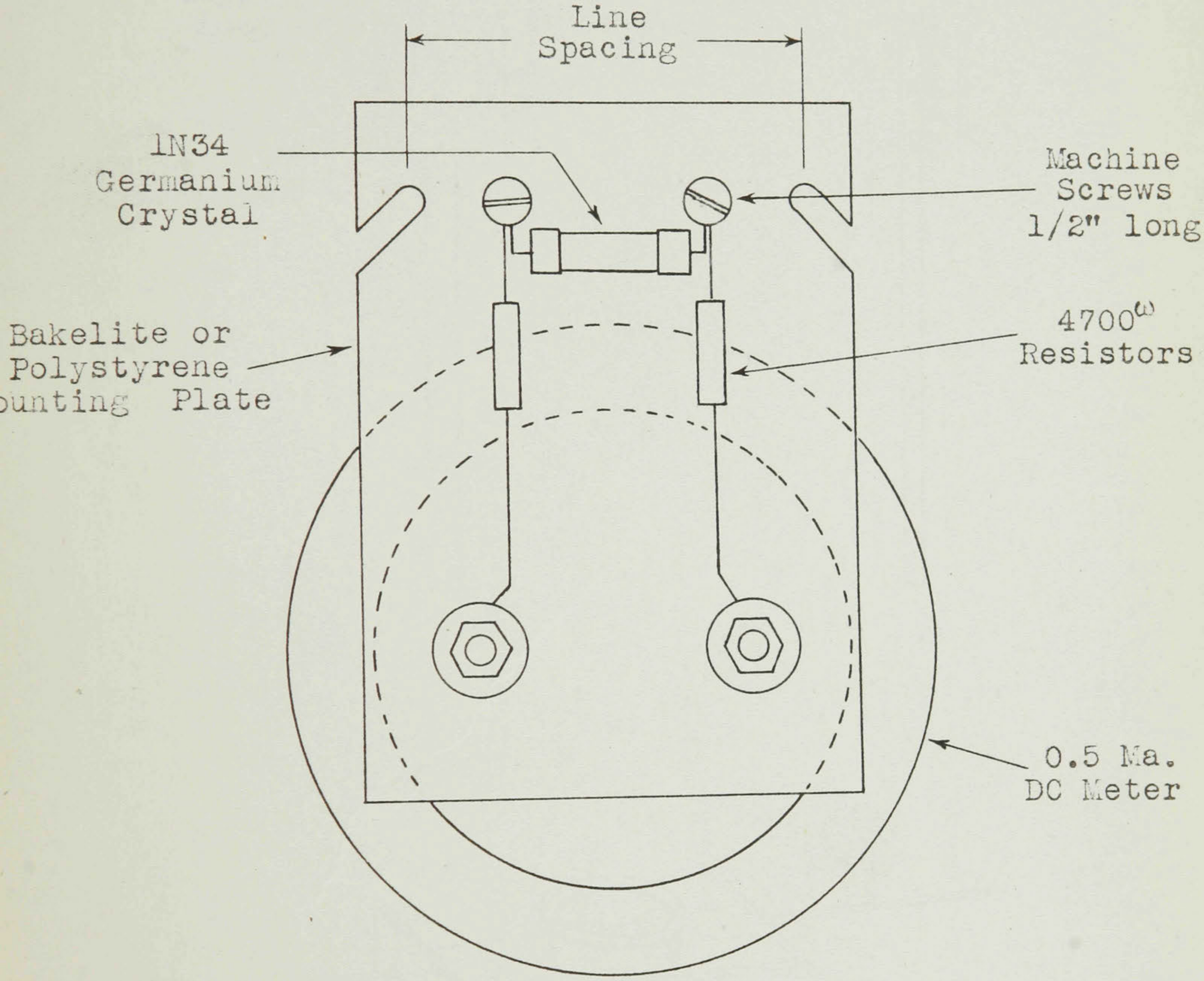


Fig. 3 - Voltage Standing Wave Indicator.

(a)

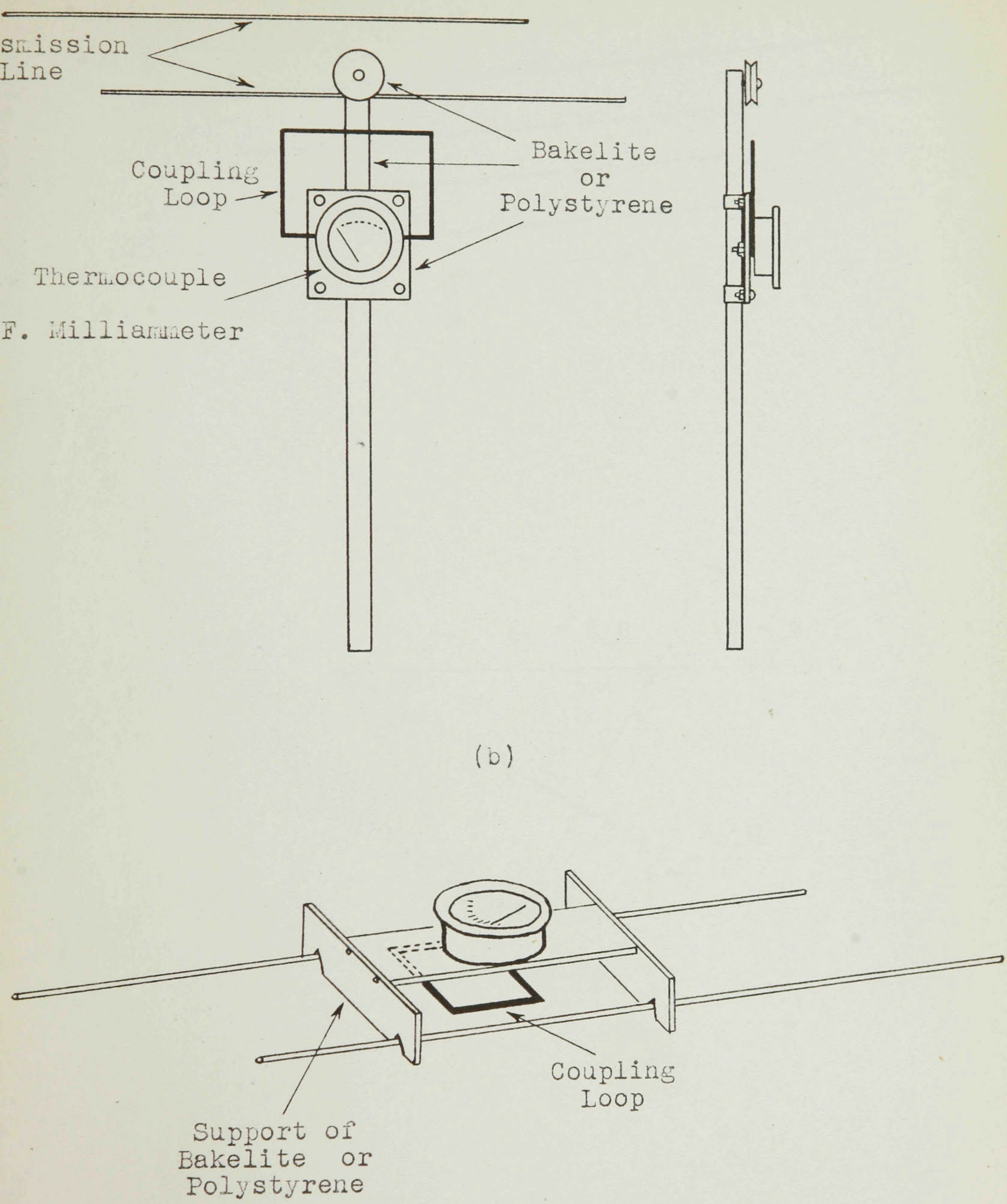


Fig. 4 - Current Standing Wave Indicators.

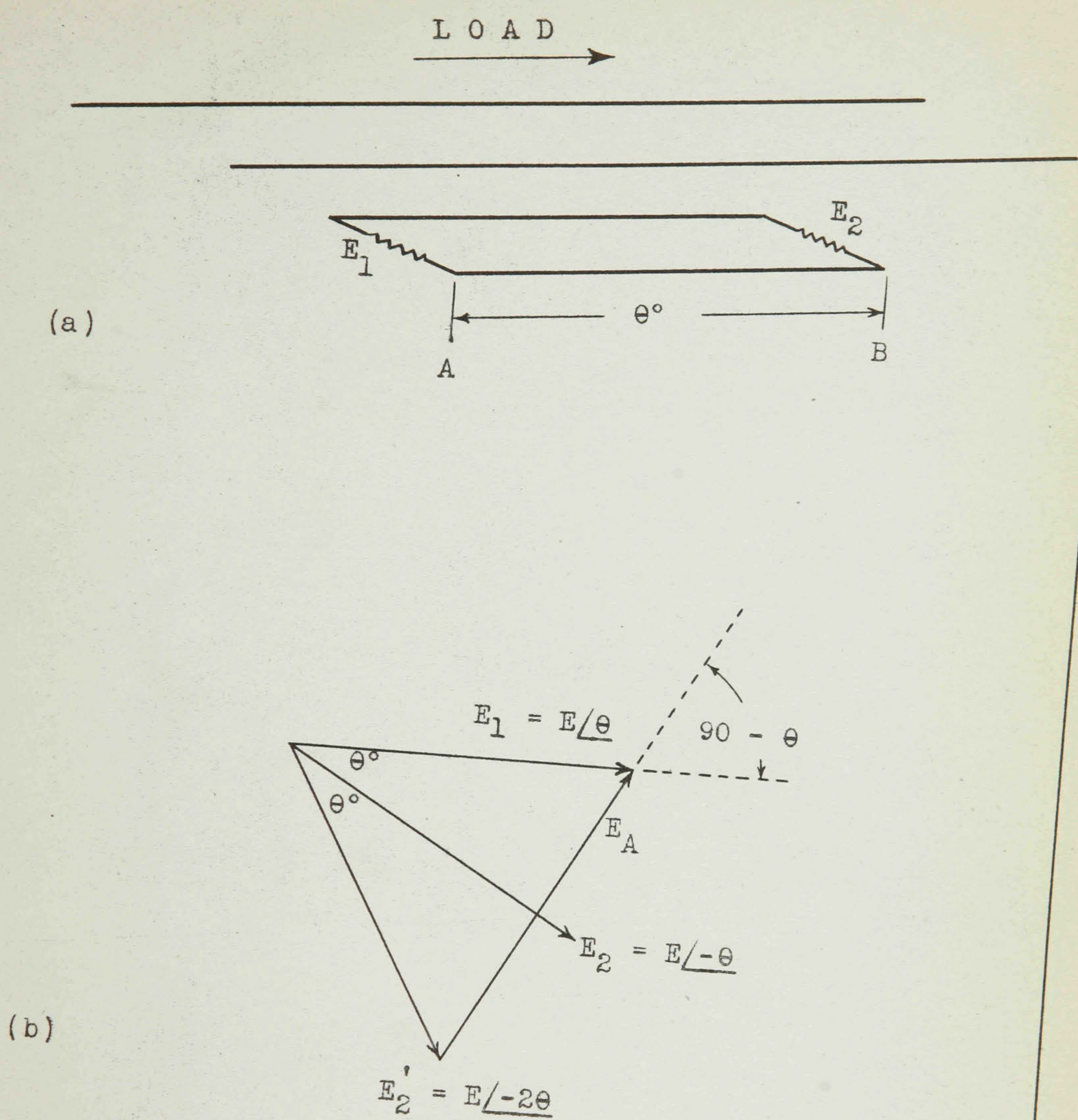


Fig. 5 - The Reflectometer.

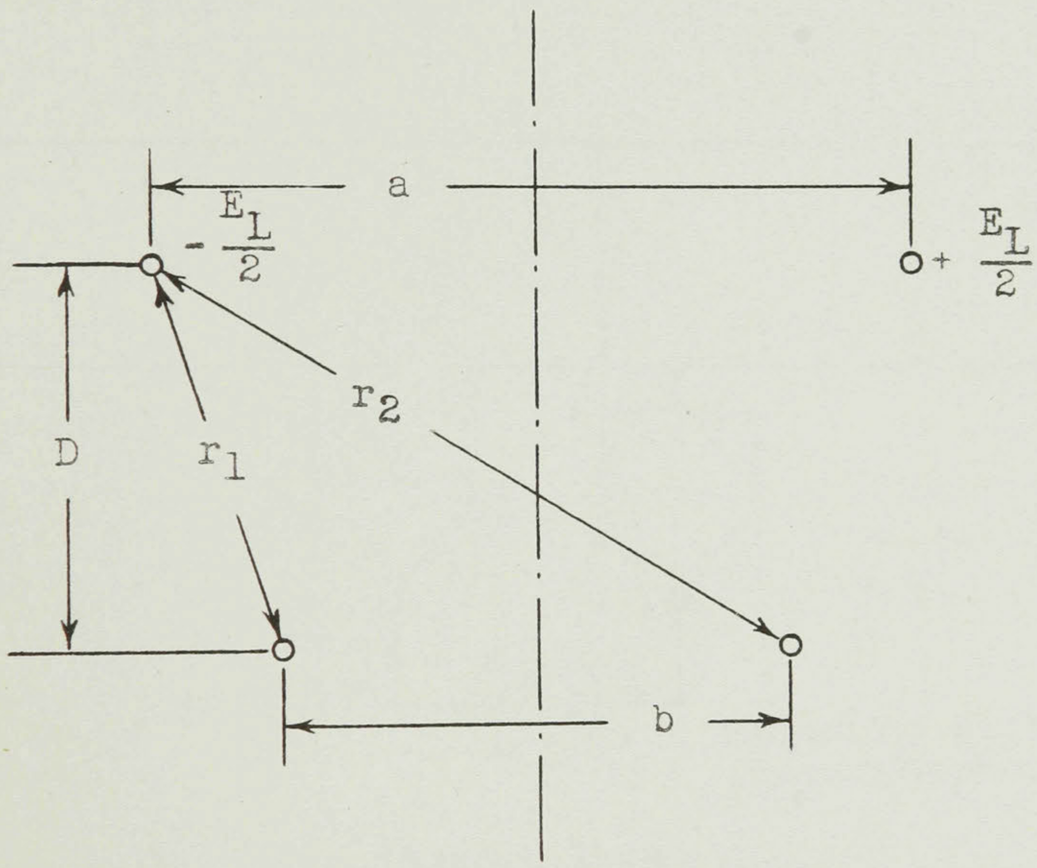
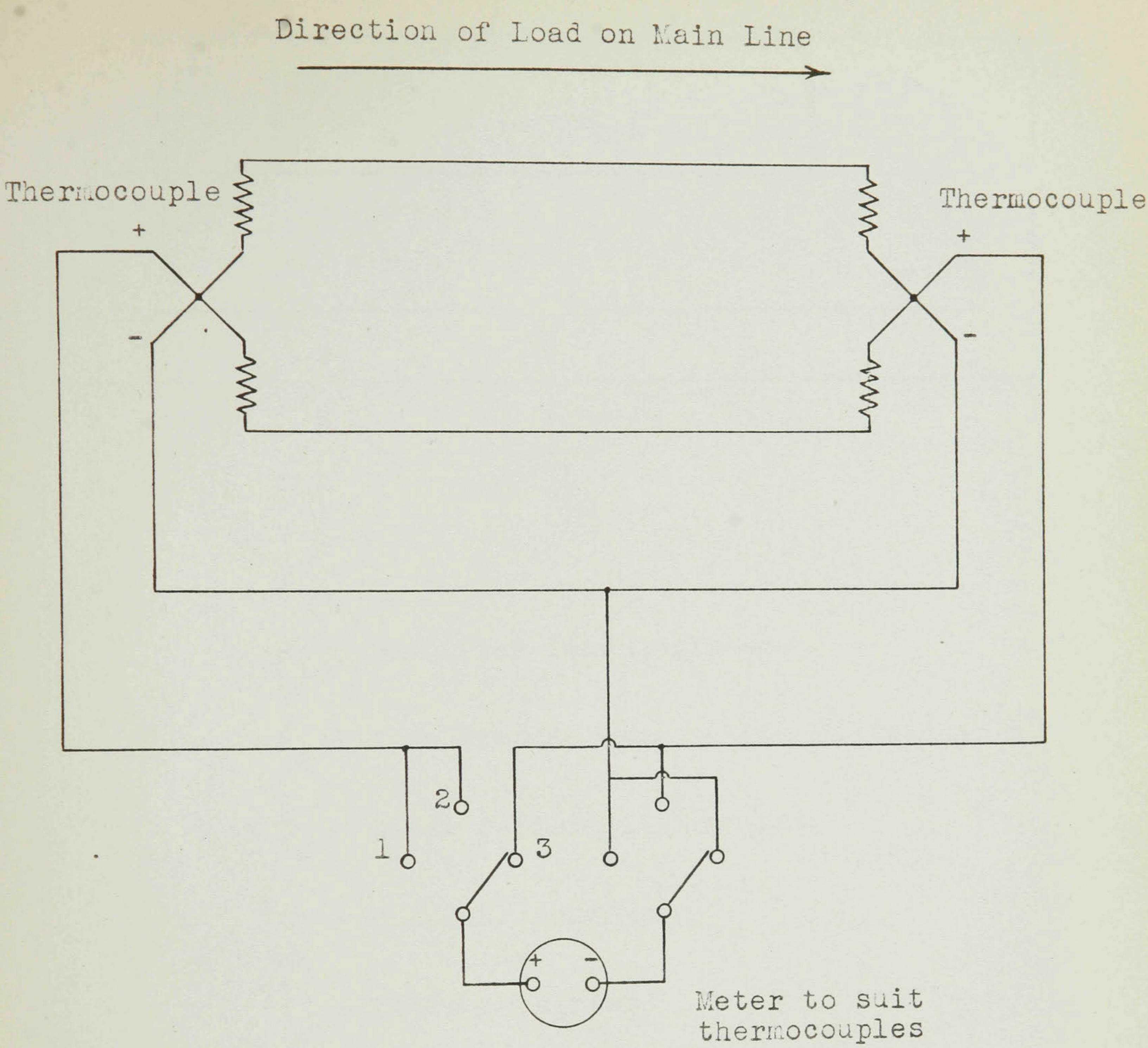


Fig. 6 - Cross Section of Line and Reflectometer.



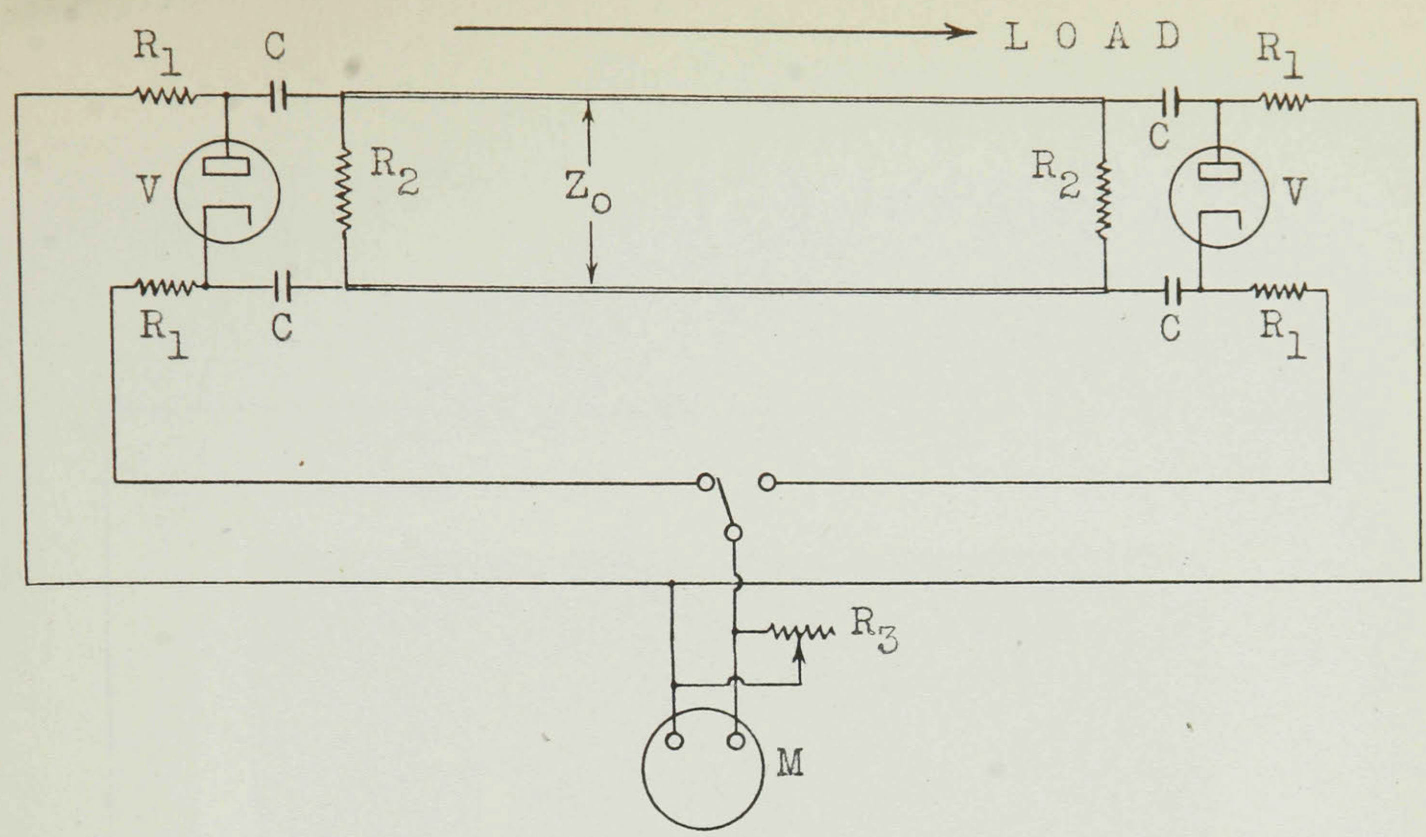
1 - Forward Power

2 - Delivered Power

3 - Reflected Power

Note - Thermocouples must be identical.

Fig. 7 - Power Measurement Circuit.



R_1 - Rectifier load resistors

R_2 - Line Section terminating resistors, $R_2 = Z_o$

R_3 - Meter calibration resistor

C - Coupling condenser

V - Diode rectifier

M - DC meter

Fig. 8 - Circuit for Reading Reflection coefficient or Standing Wave Ratio.

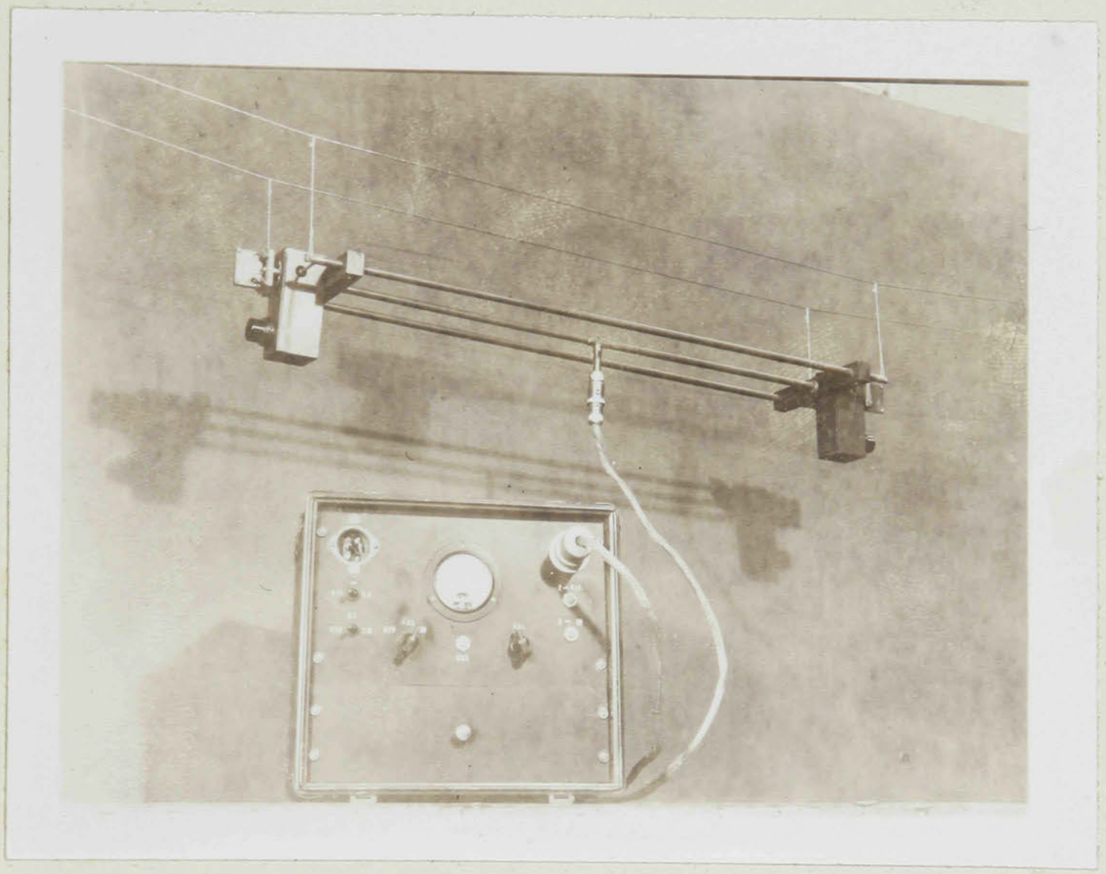
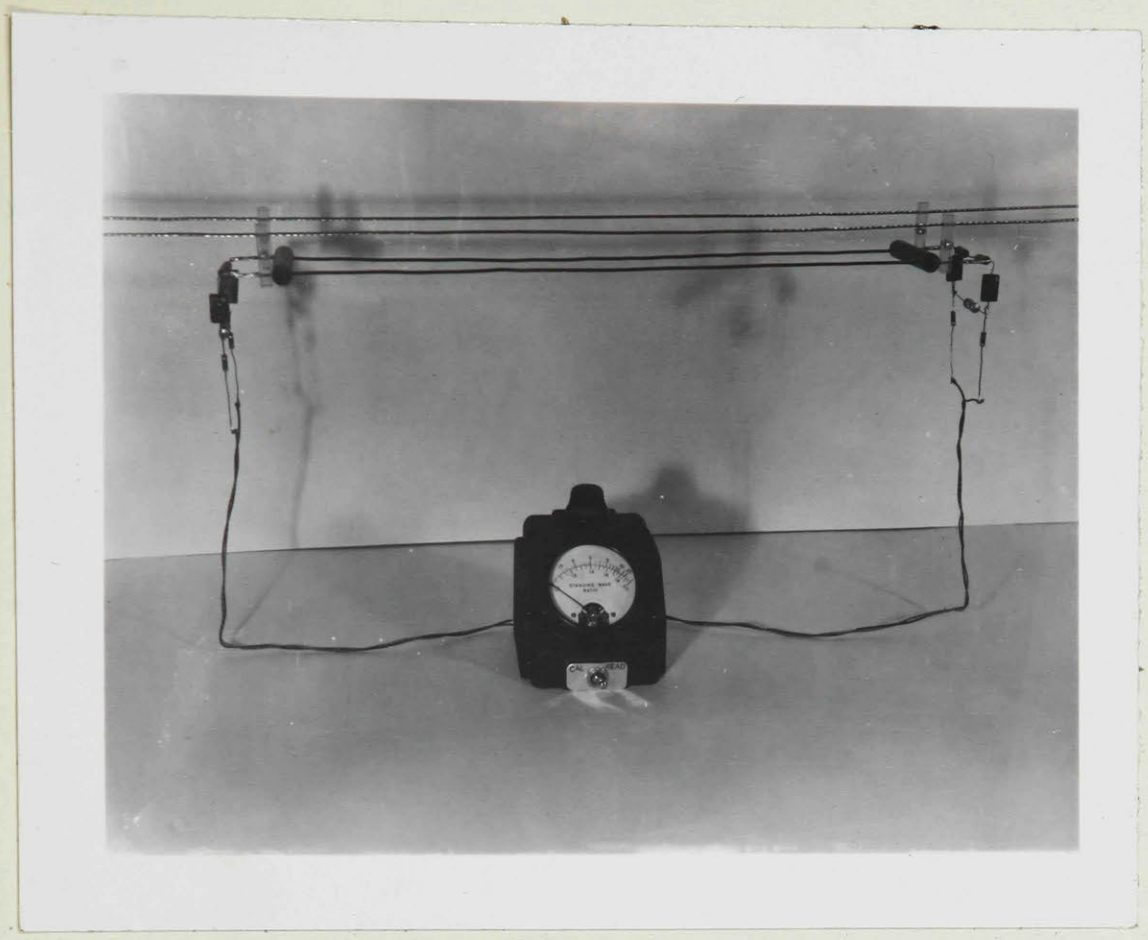


Fig. 9 - Photograph of Original Model.

(a)



(b)

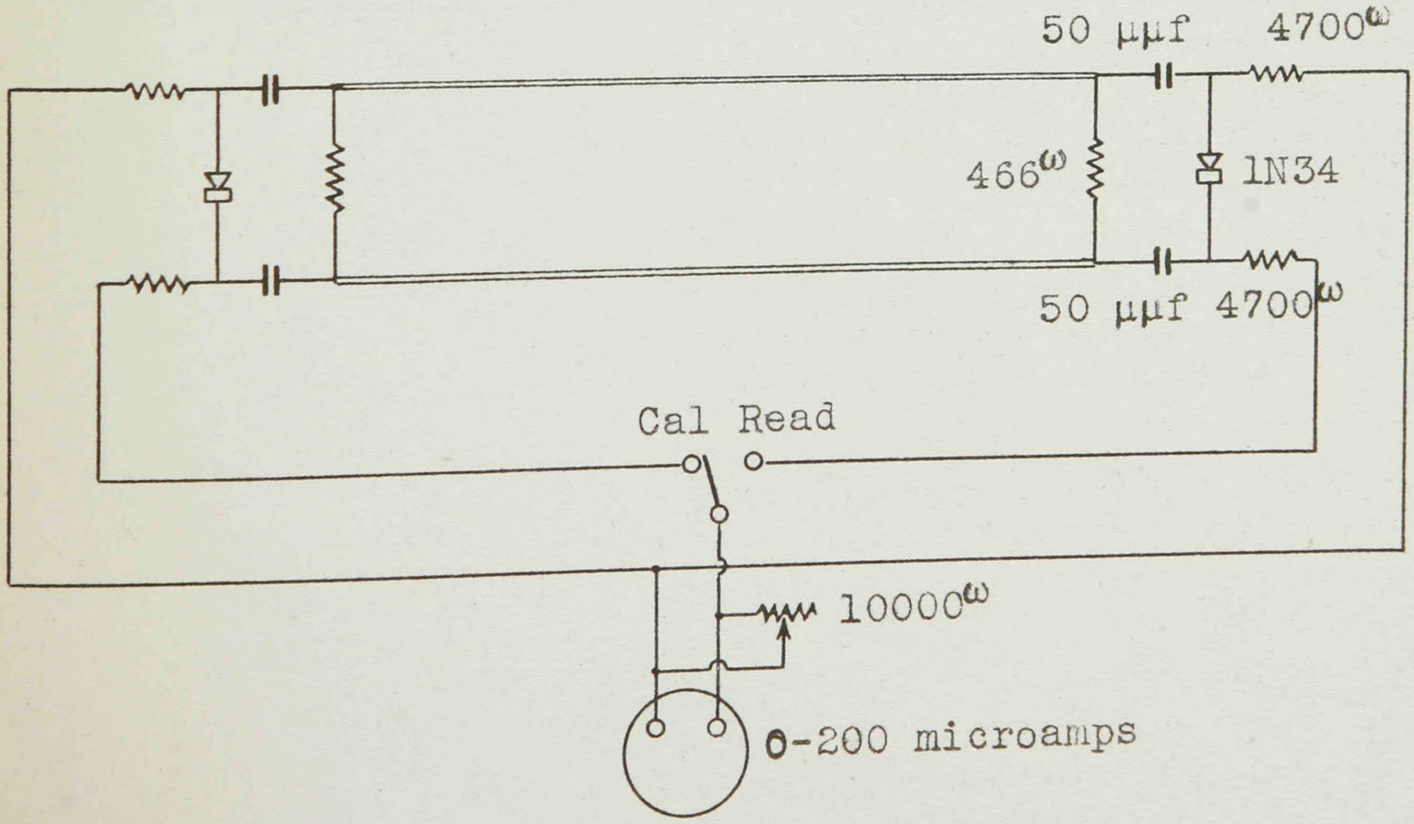


Fig. 11 - Laboratory Model of the Improved Reflectometer.

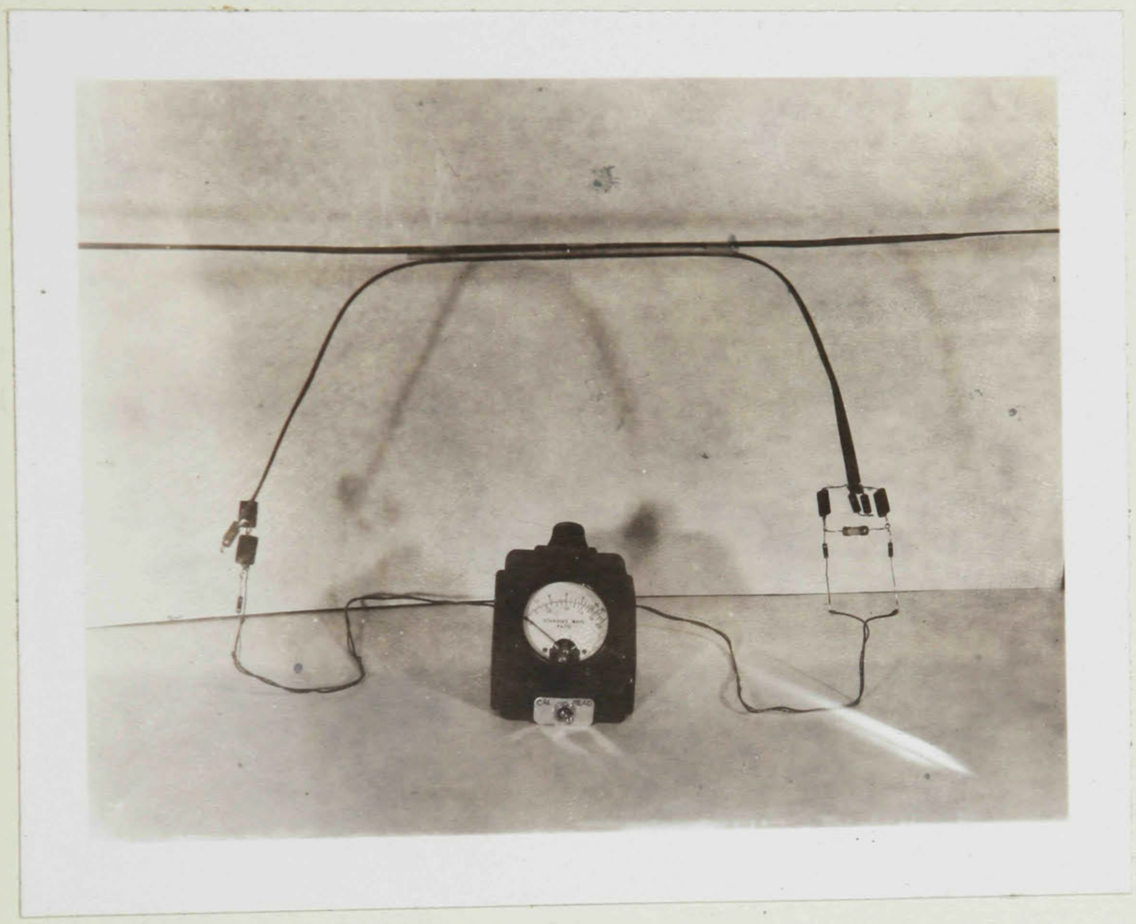


Fig. 12 - Reflectometer for use with "Twinlead" transmission line.

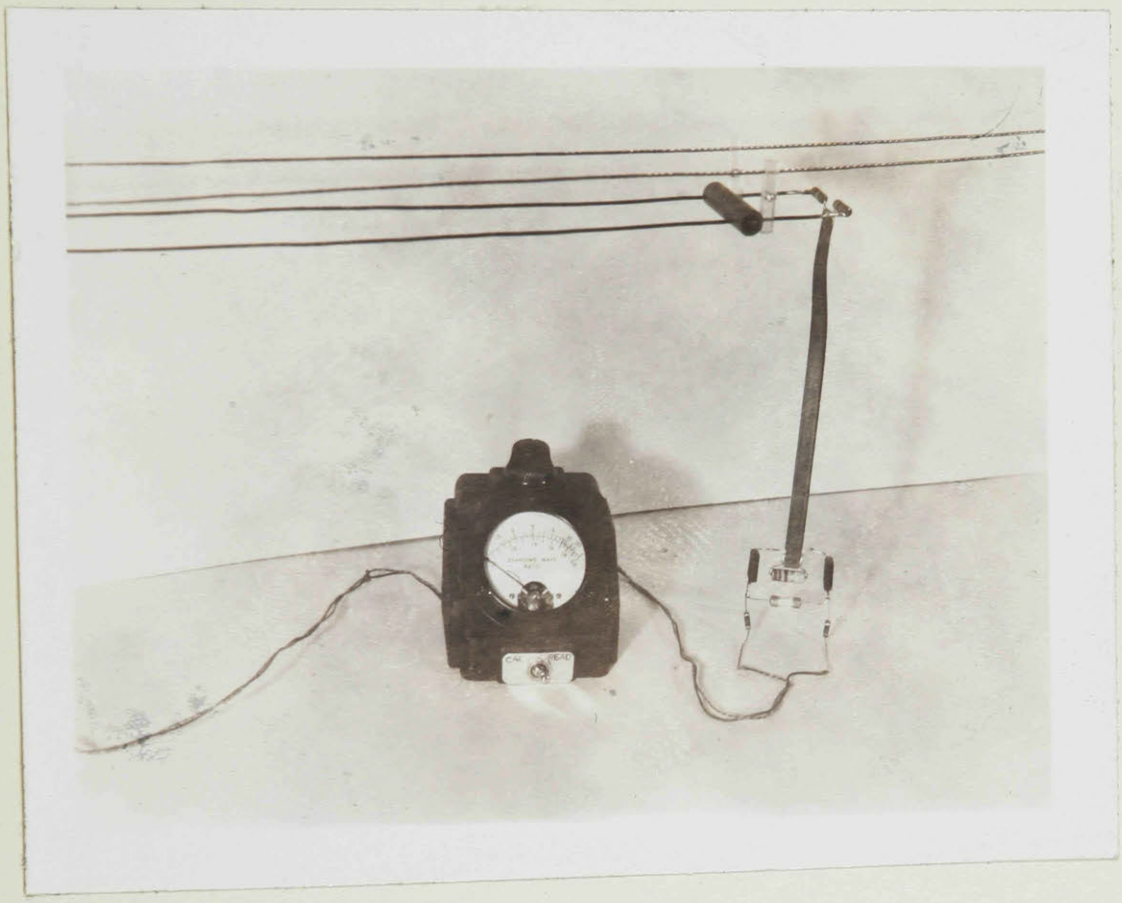


Fig. 13 - Reflectometer with split terminating resistors.

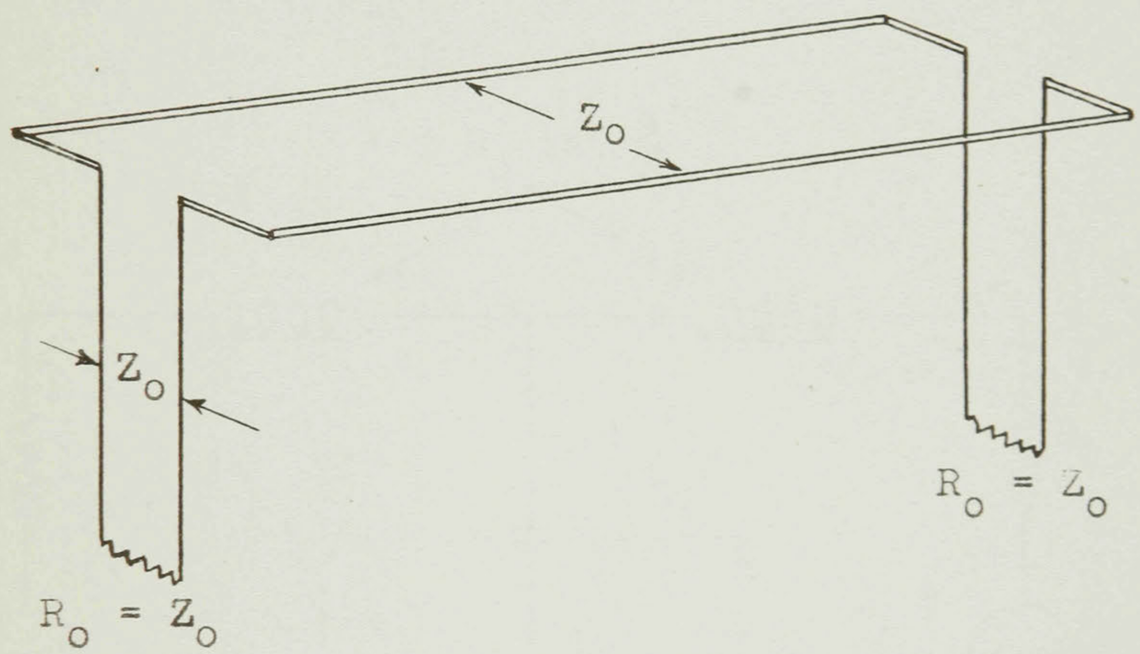


Fig. 14 - Reflectometer with Remote Terminating Resistors.

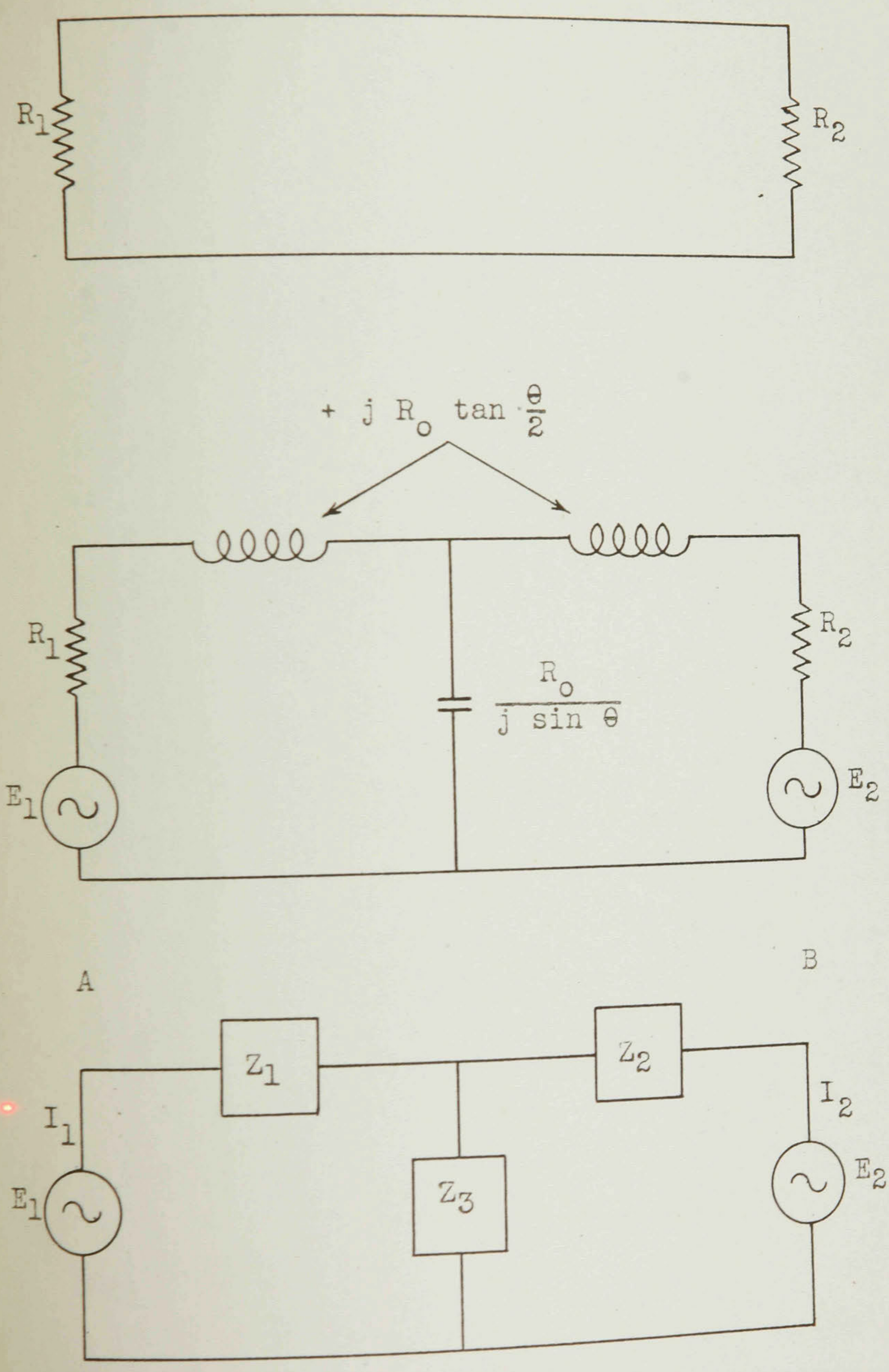


Fig. 15 - Equivalent T section of a Reflectometer.