Friedmann Equation in Codimension-two Braneworlds

Chen Fang

Master of Science

Department of Physics

McGill University

Montréal, Québec, Canada

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Abstract

We study the expansion of the universe on a deSitter 3-brane in a warped codimension-two brane model. We use analytical and numerical methods to solve the jump conditions and quantify the deviations of the Friedmann equation from its standard form. The radion must be stabilized in this model, and it is shown that the magnitude of the deviations is controlled by the radion mass, though in a quantitatively different way from codimension-one brane models. We also examine the effect of the modified expansion rate on inflation driven by fields on the brane.

Résumé

Nous étudions l'expansion de l'univers sur un desitter 3-brane dans un modèle déformé de brane de dimension 2. Nous employons des méthodes analytiques et numériques pour résoudre les conditions de saut et pour mesurer les déviations de l'équation de Friedmann a partir de son format standard. Le radion doit être stabilisé dans ce modèle, et on démontre que l'importance des déviations est commandée par la masse de radion, cependant de faéon quantitativement différente a partir de modèles de brane de dimension 1. Nous examinons également l'effet du taux modifié d'expansion sur l'inflation conduite par des champs sur le brane.

Originality

This project is done with the collaboration of Jim Cline, Sugumi Kanno. J.C proposed the project and the analytical part of Chapter 2 and checked part of the results in numerical part of chapter 2; S.K checked some of the results in chapter 2 and all of the results in chapter 3 and 4; and I did the calculations in the numerical part and most of those in the analytical part of chapters $2-4$. This work is intended for submission for publication.

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Contents

List of Tables

List of Figures

Chapter 1

Introduction

Recently braneworld models have been widely studied because it is possible that our universe is confined on a 3-brane. In this thesis we consider a codimension-two braneworld model with a 3-brane representing our universe and a 4-brane truncating the extra dimensions. The Friedmann equation on the 3-brane and the stability of the model are studied.

The thesis is organized as follows: in chapter one we present some basic ideas related to our model such as extra dimensions, braneworld and inflation; in chapter two the model is set up and Friedmann equations on the 3-brane are found with and without a Goldberger-Wise scalar field; in chapter three we study the stability of the model in tensor, vector and scalar modes respectively; in chapter four we apply the Friedmann equation to study the inflation on the 3-brane; chapter five is the summary and outlook for future work.

1.1 Why six dimensions or codimension-two

There is no evidence for extra dimensions yet, so why are we interested in such "fictitious" dimensions? First, because if they are finite they can help to explain some quantized fundamental parameters like electric charge [1], [2]. Second, extra dimensions help to resolve the hierarchy problem [3], [4], [5], cosmological constant problem and the mystery of dark energy [6]. Third, string theory and M-theory select a certain number of the space-time dimensions greater than four.

If extra dimensions really exist they must be invisible due to some mechanism. There are only three possibilities to do this. One is that the extra dimensions are very small, *i.e.* compactified, and cannot be detected by current accelerators. The second is that all the standard model particles are confined in some low dimensional space-time and cannot "see" those flat extra dimensions except the gravity. The third is that in addition to the assumption of the confinement of standard model particles we can also have non-flat extra dimensions, *i.e.* warped bulk. The sensitivity of gravity to the extra dimensions in second and third possibilities constrains the scale of the extra dimensions.

Below we first discuss the general ideas of extra dimensions in KK type and string theory respectively and next discuss why six-dimensional or codimension-two models deserve special attention.

1.1.1 Extra dimensions

The first attempt to study extra dimensions can be traced back to almost one century ago to Nordström [7] and Kaluza [8]. The basic idea was to realize Einstein's dream of finding a geometrical origin for all the interactions. Since at that time the only known interactions were gravitation and electromagnetism, Kaluza introduced a fifth spatial dimension to incorporate the electromagnetic $U(1)$ gauge group into the five-dimensional general coordinate transformations. The metric was assumed to be

$$g_{MN} = \begin{bmatrix} g_{\mu\nu} & A_\mu\phi \\ A_\nu\phi & \phi \end{bmatrix} .$$

All of the metric components are independent of the fifth dimension which Kaluza called "cylinder condition". By choosing ϕ to be a constant Kaluza found exactly the four-dimensional Einstein-Maxwell action from the pure five-dimensional Einstein action. Some years later, Klein [1], [2] considered a similar theory but with the extra dimension compactified as a small circle. He thus explained why the fifth dimension is invisible. Klein also expanded all the fields as the Fourier modes of the fifth dimension and found Kaluza's results with the zero mode. In addition, he found that the electric charge is quantized when non-zero modes are considered.

After the standard model of particle physics was constructed people tried to generalize the Kaluza-Klein (KK) theory to higher dimensions to include the larger gauge group $SU(3) \times SU(2) \times U(1)$. Witten [9] found the smallest number of extra dimensions to embed such a group is seven after Nahm [10] proved eleven is the largest number of dimensions to have a consistent supergravity theory. Thus combining higher dimensional KK theory with supergravity uniquely determines the dimension of the theory. Freund *et al.* [11] further showed that there are only two ways to compactify the $d = 11$ supergravity theory which naturally introduces an antisymmetric tensor field with rank 4: flat four dimensions and compactified seven dimensions or flat seven dimensions and compactified four dimensions depending on which manifold the time dimension is in. Moreover, Cremmer *et al.* [12] showed that there is only one choice of extra matter fields that can make the eleven-dimensional supergravity theory consistent. Thus it seems possible to find a unique theory in this framework which describes the real universe.

However, there are several things to be noticed. First, if we insist that the extra dimensions are spontaneously compactified then additional matter fields, at least those corresponding to fermion fields in the standard model, are needed. Second, however, it is impossible to get chiral fermions, as argued by Witten [9]. Third, though the theory seems have unified all the interactions, it does not provide a way to quantize gravity. Last, the compactification scale is generally taken to be the Planck scale which makes it impossible to probe the extra dimensions.

The first of the above problems violates the original idea of unifying all the matter fields in a purely geometrical scenario. One way to naturally add matter fields is through supergravity; see [13], [14], [15]. There are alternatives which can keep the purely geometrical idea but the price to pay is to alter the higher dimensional Einstein equation [16]. To solve the second problem of chirality, people reduced the theory to ten dimensions. However, in ten dimensions the beauty of the KK theory is totally ruined because the standard model gauge group can no longer be interpreted as the symmetry group of the extra dimensions. Meanwhile, the uniqueness of the theory is destroyed: there are two self-consistent ten-dimensional supergravity theories, one is based on $SO(32)$ and the other is based on $E_8 \times E_8$ respectively.

As supergravity was developing string theory was also advancing; for reviews see [17]. It turns out that supergravity and string theory are deeply related. There are five different superstring theories in ten dimensions based on groups $SO(32)$ and $E_8 \times E_8$ respectively. During the 90's, Witten *et al.* [18]-[23] proved that the five superstring theories are related by dualities and are different perturbation expansions of a single theory living in eleven dimensions, called M-theory. The low energy limit of M-theory is just eleven-dimensional supergravity. Furthermore, Salam *et al.* [24] proved the two ten-dimensional supergravity theories are low energy limits of the corresponding superstring theories of the same group.

Much literature is devoted to compactifying the extra dimensions of M-theory or string theory but these papers seldom explain why four of them remain infinite and how the extra dimensions are stabilized. In fact compactification is not the only possibility. One can also imagine that all the dimensions are compactified at first due to the high energy density of the early universe and later four of them unfold through some mechanism during the decrease of energy density. This idea is realized in the [25], [26], [27]). Besides, there exist non-compactified approaches which relax the cylindrical condition and no longer interpret the extra dimensions as space-like. In such ways pure higher dimensional geometrical effects can produce rich enough matter content without adding extra matter fields [28],[29]. Extra time-like dimensions are also studied [30], [31]. However, these cannot be interpreted as real dimensions because time is a measurement of relative movement and one measurement cannot generate two results. Moreover, extra time-like dimensions generally lead to tachyons and violate causality. Most importantly recent progress in braneworld models shows that compactification is not always necessary and we will discuss this in the next section.

1.1.2 Why six dimensions or codimension-two

Six-dimensional models and codimension-two braneworld models received a lot of attention because researchers thought they could help to solve the cosmological constant problem (CC problem). For reviews of the CC problem see [32], [33]; for reviews of attempts to solve it in these models see [34]-[40].

The basic idea of solving the CC problem in six-dimensional models with codimension-two braneworlds is to separate the observed expansion rate from the vacuum energy predicted by the fundamental particle models. In the early work [34], the authors showed that in a warped six-dimensional model the expansion rate is a constant independent of the vacuum energy. Later, it was found that in codimension-two braneworld models the only effect of the 3-brane with tension T is to introduce a deficit angle $\delta\theta = \frac{T}{M_6^4}$. This makes it possible that the vacuum energy only curves the extra dimensions but leaves the 3-brane flat. Codimension-two braneworld models are special mainly because only in such models the compactification scale can be comparable to the observed vacuum energy scale. Above the compactification scale supersymmetry ensures a vanishing cosmological constant. Below it, although supersymmetry is totally broken on the 3-brane, it can be broken only by a small amount in the bulk and this results in a small cosmological constant [41], [42], [43], [44]. So if the compactification scale is comparable to the observed vacuum energy scale the CC problem can be solved. However, those works all considered a thin 3-brane with only pure tension; when matter is added the singularity becomes nonconical. There are two ways to regularize it: one is to add a Gauss-Bonnet term [45], [46] in the original Lagrangian; the other is to assume a thick brane [38]. It was proved [38] that in non-supersymmetric scenarios, changes in the brane tension lead to deSitter or anti-deSitter braneworlds; thus the cosmological constant cannot be relaxed dynamically. Also in supersymmetric scenarios with thick 3-branes [40] no-self-tuning mechanism was found.

In fact, even in the thin brane limit the apparent solution to the CC problem is model dependent. Only in models where orbifold conditions or fine-tuning between two 3-brane tensions are imposed the effect of the curved extra dimensions will cancel the effect of the brane tension. Without these requirements, the change of the brane tension will lead to the standard Friedmann equation at least at low energy; for example see [55] and our work herein. However, static solutions with arbitrary 3-brane tensions always exist and any deviation from the static value of the tension leads to an expanding universe.

1.2 Braneworlds

Braneworlds are now a fruitful subject which suggests new approaches to old problems such as the hierarchy problem [3], [4], [5] and CC problem, and provides new insights into black holes [47]-[50], and holography [51], [52] and gives new perspectives on cosmology [53], [54], [56].

In braneworlds the standard matter fields and gauge fields are all confined on a brane while gravity can propagate freely in the bulk. It changes the concept of extra dimensions relative to KK type theories where the extra dimensions are responsible for the gauge group. There are three different ways to deal with the extra dimensions in the braneworld scenario. First, the space-time is a product of $M_4 \times R_n$ [3], [59] where $n \geq 1$, and M_4 is the usual four-dimensional space-time and R_n is the extra n-dimensional compact manifold with volume V_n. In this framework the 4-dimensional Planck scale M_p is related to the fundamental $(4+n)$-dimensional Planck scale M_n by $M_p^2 = M_n^{2n} \times V_n$. For example if there are only two extra dimensions the fundamental compactification scale is $\sim$mm, thus the only fundamental energy scale in this theory is M_{ew}. In this way the strength of gravity becomes comparable with that of the gauge interactions at the scale M_{ew} and quantum gravity can therefore be probed at this scale. Second, the space-time is warped while the extra dimensions are infinite, though with finite volume [5], [57]. In this framework gravity is localized in the extra dimensions not because they are compactified but because of the curved bulk. Third, the whole space-time is uncompact and flat [58]. This is achieved by adding an additional four-dimensional Ricci scalar term coming from the one loop quantum corrections of the interactions between matter on the brane and gravity. Intuitively the additional term can be understood as follows: since $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ does not vanish identically on manifolds with dimension greater than two, matter on the brane should also affect the intrinsic curvature of the brane as well as that of the whole space-time.

The braneworld scenario has its seeds in string theory. In string theory open strings describe non-gravitational fields and must attach their ends onto surfaces called D-branes, while closed strings can describe gravitational fields and move freely in the bulk. Thus it is possible that matter fields are confined on a d-dimensional D-brane while gravitational fields propagate freely in the bulk [59], [60]. Also this might be the realistic way that strings can describe our universe [61] since it is easier in this way to get unification of different couplings as well as supersymmetry breaking.

1.3 Inflation

The standard model of cosmology is based on general relativity and the cosmological principle that the universe is homogeneous and isotropic at large scales. Despite its great success in explaining the abundance of light elements and cosmic microwave background (CMB) radiation, the standard model of cosmology also generates several problems, including the horizon problem, flatness problem and monopole problem. Inflation [62]-[67] was proposed to solve these three problems.

It is assumed that the universe was in a causally connected region before inflation; while during inflation since the universe was increasing exponentially different regions had no time to change to different states before they became causally unconnected. So distant but similar regions which appear to be causally unconnected now were much closer in the past. This explains the isotropy of the CMB. Also from the Friedmann equation

$$H^2 = \frac{8\pi G}{3}\rho - K\frac{c^2}{a^2} \tag{1.1}$$

where H is the Hubble rate, ρ is the energy density and K is the curvature of the space, we know that the expansion rate was eventually determined by the energy density as the scale factor increased, because the energy density was almost constant during inflation. In other words, the curvature of space was driven to zero during inflation and thus the flatness problem is solved. In fact, here inflation only says that there is no need to fine tune the curvature of the space; it does not give a definite sign of the curvature let alone its exact value in the early universe. The monopoles which were possible after grand unified theory symmetry breaking were diluted during inflation, explaining why they are not observed.

However, inflation is widely accepted not only because it resolved the above problems but because it successfully explained the origin of the density inhomogeneities by quantum perturbations and is able to make predictions about temperature anisotropy in the CMB.

To have inflation, it is assumed that the early universe was dominated by vacuum energy whose equation of state is $\rho = -P$. This is realized by a scalar slowly rolling down a flat potential satisfying the slow roll conditions: $|M_p V'| \ll |V|$ and $|M_p^2 V''| \ll |V|$. Inflation ends when the slow roll conditions are violated and the scalar field starts to oscillate around the minimum of the potential and transform its energy to create ordinary matter fields. There are many scalar fields in fundamental particle physics theories which may act as the inflaton.

However, inflation itself has some problems. First, tracing back inflation in time we still encounter the singularity appearing in the standard model of cosmology [68]. Second, since during inflation there was a large vacuum energy from the scalar potential, how does it evolve to the cosmological constant today? Third, why does the inflaton start away from the minimum of its potential?

To address these problems and to describe the very early universe a fundamental particle theory is needed which includes quantum gravity. Only then can we understand the meaning of the vacuum, the property of singularity, the origin of the quantum fluctuation and space and time.

Chapter 2

Friedmann equation in codimension-two braneworld

Modified Friedmann equations have been found in many extra dimensional models. Some of them are 5D models and included a Gauss-Bonnet term in the action so that matters can be put in consistently [46], [69], [70], [71], [72], [73], [78] while some are 6D models and used flux compactification [55]. Modified Friedmann equations have applied to study dark energy [58], [74] and inflation [75]. In this chapter we study how the Friedmann equation is modified in a codimension-two (6D) braneworld model with only pure tension on the 3-brane and a negative cosmological constant in the bulk. The extra dimensions are truncated by a 4-brane.

2.1 The codimension-two braneworld model

We will consider a codimension-two braneworld with a negative cosmological constant in the six-dimensional bulk space-time. The geometry describes a warped conical throat, which is bounded at large r (specifically at $r = P$) by a 4-brane around which orbifold boundary conditions are imposed, analogous to the Planck brane of the Randall-Sundrum (RS) model. In addition there is a 3-brane at the infrared end of the throat, denoted by $r = \rho$, which can be thought of as the Standard Model brane. The action is

$$S = \frac{1}{2k_6^2}\int d^6x\sqrt{-\mathcal{G}}\,[\,\mathcal{R}-2\Lambda_6\,] - \int d^6x\sqrt{-\mathcal{G}}\left[\frac{1}{2}(\nabla\phi)^2+V(\phi)\right] + \int d^5x\sqrt{-\tilde{g}}\,\mathcal{L}_{4-\mathrm{brane}} - \int d^4x\sqrt{-g}\,\tau_3\,, \tag{2.1}$$

where

$$k_6^2 = \frac{1}{M_6^4} \tag{2.2}$$

is the 6-dimensional gravitational constant and $\mathcal{G}_{AB}$ is the 6-dimensional bulk metric. We denote the induced metrics on the 3-brane and the 4-brane by $g_{\mu\nu}$ and $\tilde{g}_{ab}$, respectively. τ_3 is the 3-brane tension. Here, $\mathcal{L}_{4-\mathrm{brane}}$ is the Lagrangian density of the matter on the 4-brane that includes the 4-brane tension T_4, as well as some additional component which is necessary for satisfying the extra jump (Israel matching) condition which exists in 6D relative to 5D models.

The 6-dimensional Einstein equation derived by varying the above action with respect to $\mathcal{G}^{AB}$ takes the form

$$\sqrt{-\mathcal{G}}\left[G_{AB}+\Lambda_6\mathcal{G}_{AB}-k_6^2T_{AB}\right] = k_6^2\sqrt{-\tilde{g}}\,S_{ab}\,\delta_A^a\,\delta_B^b\,\delta(r-P) - k_6^2\tau_3\sqrt{-g}\,g_{\mu\nu}\,\delta_A^\mu\,\delta_B^\nu\,\delta^{(2)}(r-\rho)\,, \tag{2.3}$$

where $\delta^{(2)}$ denotes the 2-dimensional delta function with support at the position of the 3-brane, and S_{ab} is the stress-energy tensor defined below in eq. (2.12) and the bulk stress tensor is given by

$$T_{AB} = \partial_A\phi\partial_B\phi - \frac{1}{2}\mathcal{G}_{AB}\partial^D\phi\partial_D\phi - V(\phi)\mathcal{G}_{AB}\,. \tag{2.4}$$

The variation with respect to ϕ gives

$$\Box\,\phi - \frac{dV}{d\phi} = 0\,. \tag{2.5}$$

For simplicity we impose azimuthal symmetry on the extra two dimensions and therefore require that the metric depends only on the radial coordinate. The line element has the form

$$ds^2 = a(r)\,[\,-dt^2+e^{2\tilde{H}t}\delta_{ij}dx^idx^j\,] + f(r)K^2\,d\theta^2 + \frac{1}{f(r)}dr^2, \tag{2.6}$$

where $\tilde{H}$ is the Hubble parameter describing de Sitter expansion of the brane, and K is a parameter which determines the deficit angle at the 3-brane, where there is generically a conical singularity. To insure that the position $r = \rho$ of the 3-brane represents a single point in the extra dimensions, we require that $f(\rho) = 0$. The 4-brane at $r = P$ is needed in order to have a compact bulk and localized gravity. The 6D cosmological constant is negative, $\Lambda_6 = -10/\ell^2$, and gives rise to an approximately AdS_6 bulk geometry with curvature length scale ℓ. The pure gravity model has a modulus, the radion, which must be stabilized to make a realistic model. For this reason we have included a bulk scalar field, which can give the radion a mass by the Goldberger-Wise mechanism [114], provided that ϕ couples to the 4-brane in $\mathcal{L}_{4-\mathrm{brane}}$.

The Einstein equations for this model are

$$
\begin{aligned}
\mu\mu: &\quad \frac{6\tilde{H}^2 - 3a'f' - 3a''f - f''a}{2} = a\Lambda_6 + ak_6^2\left(\frac{f}{2}(\partial_r\phi)^2 + V(\phi)\right) \\
\theta\theta: &\quad \frac{K^2 f(12a\tilde{H}^2 - 2a'f'a - fa'^2 - 4a''af)}{2a^2} = K^2 f\Lambda_6 + K^2k_6^2\left(\frac{f^2}{2}(\partial_r\phi)^2 + V(\phi)f\right) \\
rr: &\quad \frac{12a\tilde{H}^2 - 3fa'^2 - 2a'f'a}{2a^2 f} = \frac{1}{f}\Lambda_6 - k_6^2\left(\frac{1}{2}(\partial_r\phi)^2 - \frac{1}{f}V(\phi)\right)
\end{aligned}
\tag{2.7}
$$

where $'$ denotes the derivative with respect to r.

2.1.1 The jump conditions

To completely specify the geometry, we must consider the jump conditions which relate the metric and bulk scalar to the sources of stress-energy at the boundaries.

First, let us consider the jump condition for the 3-brane. Recalling that the position of the 3-brane satisfies

$$f(\rho) = 0\ , \tag{2.8}$$

the deficit angle is given by

$$\Delta\theta = 2\pi\left[1 - \frac{K}{2}f'(r)\right]\Bigg|_{r=\rho} \tag{2.9}$$

A non-vanishing deficit angle indicates the presence of a conical singularity at the position of the 3-brane, related to the brane tension τ_3 by $\Delta\theta = k_6^2\tau_3$. The jump condition at $r=\rho$ can thus be written as

$$K = \frac{2}{f'(\rho)}\left(1 - k_6^2 \frac{\tau_3}{2\pi}\right) . \tag{2.10}$$

In what follows, it will not be necessary to include any coupling of the scalar field to the 3-brane, so the boundary condition of ϕ at $r=\rho$ is simply

$$\phi'(\rho) = 0 \tag{2.11}$$

Next, for the 4-brane, we define the stress-energy tensor of the 4-brane S_{ab} as:

$$\begin{aligned} S_{\mu\nu} &= -\left(T_4 + \frac{c_0}{L_\theta^\alpha}\right)\tilde{g}_{\mu\nu} \, , \qquad L_\theta^2 = fK^2 \\ S_{\theta\theta} &= -\left(T_4 + (1-\alpha)\frac{c_0}{L_\theta^\alpha}\right)\tilde{g}_{\theta\theta} \, . \end{aligned} \tag{2.12}$$

The parameter α, which allows for a difference between the 4D and the $\theta\theta$ components of the 4-brane stress tensor, must be nonzero to accommodate a static solution with $P<\infty$. The physics which could account for the c_0 contribution to the stress tensor can arise in several ways [117, 101]. Conservation of S_{ab} implies that the energy density of the source c_0 must scale like $1/L^\alpha(P)$ [104, 118] where L is the circumference of the 4-brane. If c_0 is due to the Casimir effect of massless fields living on the 4-brane, then $\alpha=5$, whereas if it is due to smearing a 3-brane around the 4-brane then $\alpha=1$.

Thus the jump conditions for the 4-brane at $r=P$ are given by

$$f\left[\frac{3}{2}\left(\frac{a'}{a}\right)' + \frac{1}{2}\left(\frac{f'}{f}\right)'\right]\tilde{g}_{\mu\nu} = k_6^2\sqrt{f}\, S_{\mu\nu}\,\delta(r-P)\, , \tag{2.13}$$

$$2\, f^2K^2\left(\frac{a'}{a}\right)' = k_6^2\sqrt{f}\, S_{\theta\theta}\,\delta(r-P)\, , \tag{2.14}$$

where the term $\sqrt{f}$ in the r.h.s. of each equation comes from the ratio of the determinant of the 4-brane induced metric to the bulk metric, $\sqrt{-\tilde{g}}/\sqrt{-\mathcal{G}}$.

By integrating the above equations across $r=P$, assuming Z_2 orbifold symmetry, we get

$$\begin{aligned} \sqrt{f}\left(3\frac{a'}{a} + \frac{f'}{f}\right) &= k_6^2\left(T_4 + \frac{c_0}{L^\alpha}\right) , \\ 4\sqrt{f}\,\frac{a'}{a} &= k_6^2\left(T_4 + (1-\alpha)\frac{c_0}{L^\alpha}\right) , \end{aligned} \tag{2.15}$$

To stabilize the model with the bulk scalar field, we will allow for the possibility that ϕ couples to the 4-brane through a potential $V_P(\phi)$ contained in $\mathcal{L}_{4-\mathrm{brane}}$. This leads to the boundary condition

$$\phi' = -\frac{1}{2f(P)}\frac{dV_P}{d\phi} \tag{2.16}$$

at $r = P$, using the assumed Z_2 orbifold symmetry.

2.1.2 Background solution

The general solution to Einstein's equations for the metric (2.6) is

$$a(r) = \frac{r^2}{r_0^2}, \qquad f(r) = \left(\frac{r^2}{r_0^2} - \frac{r_1^3}{r^3} + \tilde{H}^2\ell^2\right)\frac{r_0^2}{l^2} \tag{2.17}$$

The constants of integration r_0, r_1 can be set to convenient values by rescaling coordinates,

$$x^\mu \to Ax^\mu, \qquad r \to Br \tag{2.18}$$

Under this transformation, the metric functions change as

$$a \to aA^2B^2, \qquad \tilde{H} \to \tilde{H}/A, \qquad f \to f/B^2, \qquad K \to BK \tag{2.19}$$

By choosing $A = (\frac{r_0}{r_1})^{3/5}$ and $B = (r_0^2 r_1^3)^{1/5}$, we can thus set $r_0 = r_1 = 1$, for convenience.

Notice that in the limit of $\tilde{H} \to 0$, eq. (2.6) reduces to the AdS soliton solution.

Since we have chosen to normalize $a(\rho) = \rho^2$ at the 3-brane instead of the usual value, $a = 1$ the observed Hubble rate on the 3-brane is given by $H = \tilde{H}/\rho$, and the position of the 3-brane, defined by eq. (2.8), satisfies

$$1 + H^2\ell^2 = \frac{1}{\rho^5}\ , \tag{2.20}$$

The jump condition for the 3-brane, eq. (2.10), is then

$$K = \frac{1}{\rho + \frac{3}{2\rho^4}}\left(1 - k_6^2\frac{\tau_3}{2\pi}\right)\ , \tag{2.21}$$

and the jump conditions for the 4-brane, eqs. (2.15), can be expressed as

$$
\begin{aligned}
F &\equiv \frac{P + \frac{3}{2}P^{-4}}{\sqrt{P^2 - P^{-3} + H^2\rho^2\ell^2}} + \frac{3\alpha+1}{\alpha-1}\frac{\sqrt{P^2 - P^{-3} + H^2\rho^2\ell^2}}{P} \\
&= \frac{k_6^2\alpha}{2(\alpha-1)}\ell T_4 \qquad (2.22) \\
G &\equiv \frac{P + \frac{3}{2}P^{-4}}{\sqrt{P^2 - P^{-3} + H^2\rho^2\ell^2}} - \frac{\sqrt{P^2 - P^{-3} + H^2\rho^2\ell^2}}{P} \\
&= \frac{k_6^2\alpha c_0}{2K^\alpha\ell^{\alpha-1}(P^2 - P^{-3} + H^2\rho^2\ell^2)^{\alpha/2}} \qquad (2.23)
\end{aligned}
$$

These conditions cannot be solved analytically; in the next section we will use approximate and numerical methods to solve them and thereby deduce the form of the Friedmann equation in the 6D model.

Before solving the jump conditions (2.23, 2.23) it is important to understand which quantities should be considered as inputs and which are derived. Obviously we can freely specify all sources of stress energy, including Λ_6, T_4, τ_3, c_0 and α; these determine the Hubble parameter H. However our de Sitter brane solutions only exist for special values of P, the position of the 4-brane, which in the absence of the bulk scalar field is an unstabilized modulus, except in the case where $\alpha > 5$ [118]. Therefore eqs. (2.23, 2.23) should be seen as determining H and P given arbitrary sources of stress-energy.

2.2 Friedmann equation without Goldberger-Wise scalar field

In this section we study the Friedmann equation on the 3-brane without the Goldberger-Wise scalar field in the bulk. We first do analytical calculations and find the first several terms of the Friedmann equation and then we calculate the time variation of Newton's constant and finally we do numerical calculations and compare them to analytical results.

2.2.1 Analytical results

By solving eq. (2.23), we can obtain the position of the 4-brane in terms of the physical Hubble rate $P = P(H)$. Substituting P into eq. (2.23), we then find the relation between H and τ_3 since the deficit parameter in eq. (2.23) is given by $K = K(H, \tau_3)$ from eq. (2.21). In this subsection we will carry out this procedure treating the excess 3-brane tension $\delta\tau_3$, or equivalently the Hubble rate H, as a perturbation. Thus we start by finding P as a Taylor series in H using eq. (2.23); then we will eliminate P from eq. (2.23) to obtain H as a Taylor series in $\delta\tau_3$.

The first step in the perturbative approach still requires a numerical procedure to find the value of the 4-brane position $\overline{P}$ and the critical 3-brane tension $\bar{\tau}_3$ which satisfy the jump conditions for a static geometry, with $H = 0$:

$$\frac{\overline{P} + \frac{3}{2}\overline{P}^{-4}}{\sqrt{\overline{P}^2 - \overline{P}^{-3}}} + \frac{3\alpha + 1}{\alpha - 1}\frac{\sqrt{\overline{P}^2 - \overline{P}^{-3}}}{\overline{P}} = \frac{k_6^2 \alpha}{2(\alpha - 1)}\ell T_4\,, \tag{2.24}$$

$$\frac{\overline{P} + \frac{3}{2}\overline{P}^{-4}}{\sqrt{\overline{P}^2 - \overline{P}^{-3}}} - \frac{\sqrt{\overline{P}^2 - \overline{P}^{-3}}}{\overline{P}} = \frac{k_6^2 \alpha c_0}{2K_0^\alpha(\tau)\ell^{\alpha-1}(\overline{P}^2 - \overline{P}^{-3})^{\alpha/2}}\,. \tag{2.25}$$

$\overline{P}$ can then be thought of as a function of α, c_0 and T_4, which is especially sensitive to T_4. In fig. 2.1 we plot the typical dependence of $\overline{P}$ on T_4, showing that it diverges for a critical value $T_{4,0}$ by power of ~ -0.25, and approaches 1 for large values of T_4 by power of ~ -2. Recall that the position of the 3-brane is $\rho = 1$ when $H = 0$ (eq. (2.20)), so the latter behavior corresponds to the shrinking of the radial length to zero, while the former is needed to have a strongly warped solution with a large hierarchy between the 4-brane and the 3-brane.

To perturb around the above Minkowski space background, we will let $\tau_3 = \bar{\tau}_3 + \delta\tau_3$, which gives rise to a small Hubble parameter H, and also changes the position of the 4-brane. We make the expansion

$$P = \overline{P} + \frac{d\overline{P}}{d\mathcal{H}}\mathcal{H} + \frac{1}{2}\frac{d^2\overline{P}}{d\mathcal{H}^2}\mathcal{H}^2 + ... \tag{2.26}$$

where we have defined the dimensionless quantity

$$\mathcal{H} \equiv H^2\ell^2\,. \tag{2.27}$$

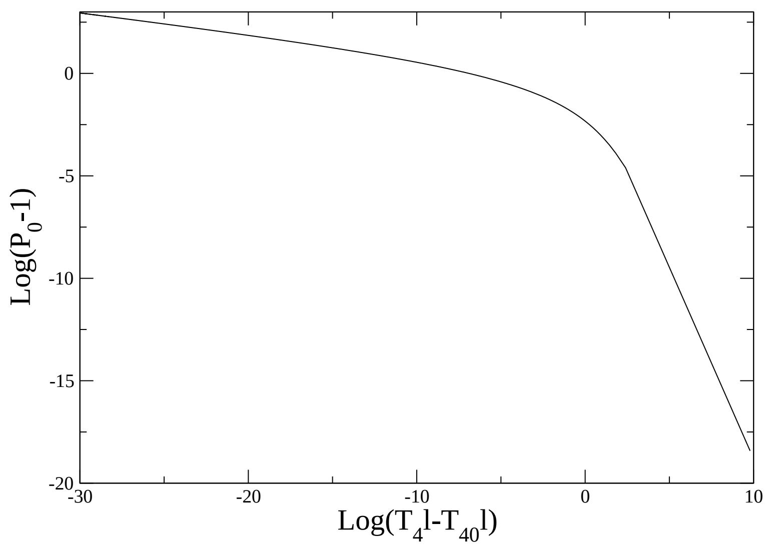

Figure 2.1: $\log(\overline{P}-1)$ as a function of $\log(T_4 l - T_{4,0} l)$, where $T_{4,0}$ is the value for which $\overline{P} \to \infty$

The coefficient $\frac{d\overline{P}}{d\mathcal{H}}$ in this expansion can be found by differentiating eq. (2.23) to get

$$\frac{d\overline{P}}{d\mathcal{H}} = \frac{4\,(\alpha+1)\overline{P}^5 - 9\,\alpha + 1}{10\,(\alpha-5)\overline{P}^5 + 5\,(3\,\alpha+5)}\overline{P}^4 \tag{2.28}$$

To relate H to the 3-brane tension, we use (2.21) and (2.23) to find that

$$1 - k_6^2\frac{\tau_3}{2\pi} = \left(\rho + \frac{3}{2\rho^4}\right)\left(\frac{k_6^2\alpha c_0}{2G(P)\,\ell^{\alpha-1}\,f^{\alpha/2}(P)}\right)^{1/\alpha} \tag{2.29}$$

where we recall that ρ depends on H via eq. (2.21). Recall that $\bar{\tau}_3$ is defined to be the value of τ_3 corresponding to $H=0$ and $P=\overline{P}$. To find the Friedmann equation at leading order in $\delta\tau_3$, we take $\tau_3 = \bar{\tau}_3 + \delta\tau_3$ and substitute $P = \overline{P} + \frac{d\overline{P}}{d\mathcal{H}}\mathcal{H}$ into the r.h.s. of (2.29).

This gives

$$-\frac{k_6^2}{2\pi}\,\delta\tau_3 = -\frac{2}{5}\mathcal{H}(\overline{P}^3 - 1)\left(1 - \frac{k_6^2}{2\pi}\bar{\tau}_3\right) + O(\mathcal{H}^2). \tag{2.30}$$

which after solving for H^2 becomes

$$H^2 = \frac{5}{2}(\overline{P}^3 - 1)^{-1}\left(1 - \frac{k_6^2}{2\pi}\tau_3\right)^{-1}\frac{k_6^2}{2\pi\ell^2}\,\delta\tau_3 + O(\delta\tau^2) \tag{2.31}$$

Ignoring the terms $O(\delta\tau^2)$, eq. (2.31) has the form of the standard Friedmann equation, $H^2 = (8\pi G/3)\rho$, since $\delta\tau_3$ is the excess energy density driving the cosmological expansion. However we need to check that the coefficient of $\delta\tau_3$ in eq. (2.31) is indeed $8\pi G/3$. We can compute Newton's constant, or equivalently the 4D Planck mass $M_4^2 = (8\pi G)^{-1}$, by dimensionally reducing the 6D Einstein-Hilbert action. Ignoring Kaluza-Klein excitations, the 6D and 4D Ricci scalars are related by $R_6 = R_4/a$; thus

$$\begin{aligned} M_4^2 = (8\pi G)^{-1} &= M_6^4 \int \sqrt{-g_6}\, a^{-1} dr\, d\theta = M_6^4 \int aK\ell\sqrt{f}\frac{\ell}{\sqrt{f}}\, dr\, d\theta \\ &= 2\pi M_6^4 K\ell^2 \int_\rho^P r^2\, dr = \frac{2\pi}{3} M_6^4 \ell^2 \frac{P^3-\rho^3}{\rho+\frac{3}{2\rho^4}}\left(1-k_6^2\frac{\tau_3}{2\pi}\right) \end{aligned} \quad (2.32)$$

Notice that in the above expression, the 4D Planck mass depends upon H through the integration limits ρ and P; thus Newton's constant becomes time-dependent when the rate of expansion of the universe is not constant. We will need to check whether this effect can be small enough to be consistent with experimental constraints on the time variation of G (see next subsection). But for the immediate purpose of comparing our obtained Friedmann equation with the standard one, we define a static Planck mass which corresponds to its value when $H = 0$:

$$\overline{M}_4^2 = (8\pi\overline{G})^{-1} = 2\pi M_6^4 K\ell^2 \int_1^{\overline{P}} r^2\, dr = \frac{4\pi}{15}\ell^2 M_6^4(\overline{P}^3 - 1)\left(1 - \frac{k_6^2}{2\pi}\tau_3\right) \quad (2.33)$$

Since $M_6^4 = \kappa_6^{-2}$, we see that indeed eq. (2.31) is consistent with the 4D Friedmann equation, $H^2 = (8\pi\bar{G}/3)\delta\tau_3$. This is in contrast to ref. [104], which mistakenly found deviations from the normal Friedmann equation at this order. The mistake made there was that corrections to the expansion rate due to changes in the 3-brane and 4-brane positions were argued to be negligible. However, we have just shown that it is essential to take account of the changes in ρ and P in order to obtain the correct result.

Having established the Friedmann equation is recovered at leading order in $\delta\tau_3$, we now turn to the corrections at higher order. We need to determine the higher order terms in the expansion of $\overline{P}$ in powers of $\mathcal{H}$. Again differentiating eq. (2.23),

$$\frac{d^2\overline{P}}{d\mathcal{H}^2} =$$

$$2\overline{P}^4\Big[\left(-160+32\,\alpha^3-288\,\alpha-96\,\alpha^2\right)\overline{P}^{18}+\left(-800+288\,\alpha^2-480\,\alpha-32\,\alpha^3\right)\overline{P}^{15}$$
$$+\left(1224\,\alpha+48\,\alpha^2+264\,\alpha^3\right)\overline{P}^{13}+\left(-504\,\alpha^2-24\,\alpha^3+600+3000\,\alpha\right)\overline{P}^{10}$$
$$+\left(-633\,\alpha^2-549\,\alpha^3-159\,\alpha-195\right)\overline{P}^{8}+\left(-2160\,\alpha-1056\,\alpha^2+144\,\alpha^3\right)\overline{P}^{5}$$
$$+\left(-402\,\alpha+230+306\,\alpha^2+378\,\alpha^3\right)\overline{P}^{3}-50+522\,\alpha^2+162\,\alpha^3+390\,\alpha\Big]$$
$$\Big/\left[25\,\left(5+2\,\overline{P}^5\alpha-10\,\overline{P}^5+3\,\alpha\right)^3\right] \tag{2.34}$$

and substituting $P=\overline{P}+\frac{d\overline{P}}{d\mathcal{H}}\mathcal{H}+\frac{1}{2}\frac{d^2\overline{P}}{d\mathcal{H}^2}\mathcal{H}^2$ into the r.h.s. of (2.29) we get the surprising result

$$H^2=\frac{8\pi\overline{G}}{3}\delta\tau_3+O(\delta\tau_3^3) \tag{2.35}$$

so the second order correction vanishes. To find the leading correction, we need to go one order higher. Continuing the procedure, computing $d^3\overline{P}/d\mathcal{H}^3$ in (2.26), using eq. (2.23), we find an unwieldy result which is given in the appendix, eq. (A.1). Substituting $P=\overline{P}+\frac{d\overline{P}}{d\mathcal{H}}\mathcal{H}+\frac{1}{2}\frac{d^2\overline{P}}{d\mathcal{H}^2}\mathcal{H}^2+\frac{1}{6}\frac{d^3\overline{P}}{d\mathcal{H}^3}\mathcal{H}^3$ into the r.h.s. of (2.29) we get

$$\frac{k_6^2}{2\pi}\delta\tau_3=\frac{2}{5}\left(1-\frac{k_6^2}{2\pi}\bar{\tau}_3\right)(\overline{P}^3-1)\left(\mathcal{H}-\mathcal{H}^3J(\alpha,\mathcal{H})+O(\mathcal{H}^4)\right) \tag{2.36}$$

where

$$J(\alpha,\mathcal{H})=$$
$$\Big[(-1120-1056\,\alpha+96\,\alpha^2+32\,\alpha^3)\overline{P}^{24}+(5088\,\alpha+600+144\,\alpha^3+312\alpha^2)\overline{P}^{19}$$
$$+(3000-24\,\alpha^3+360\,\alpha^2-1800\,\alpha)\overline{P}^{18}+(-8\,\alpha^3+120\,\alpha^2-600\,\alpha+1000)\overline{P}^{15}$$
$$+(-459\,\alpha^3-165-3087\,\alpha^2-2433\,\alpha)\overline{P}^{14}+(-108\,\alpha^3-4500+900\,\alpha^2-900\,\alpha)\overline{P}^{13}$$
$$+(-36\,\alpha^3-1500+300\,\alpha^2-300\,\alpha)\overline{P}^{10}+(185+1179\,\alpha^2+783\,\alpha^3-99\,\alpha)\overline{P}^{9}$$
$$+(2250\,\alpha-162\,\alpha^3+270\,\alpha^2+2250)\overline{P}^{8}+(90\,\alpha^2+750-54\,\alpha^3+750\,\alpha)\overline{P}^{5}$$
$$+(-405\,\alpha^2-81\,\alpha^3-375-675\,\alpha)\overline{P}^{3}-225\,\alpha-125-27\,\alpha^3-135\,\alpha^2\Big]$$
$$\Big/\left[25\left((2\,\alpha-10)\overline{P}^5+5+3\,\alpha\right)^3(\overline{P}^3-1)\right] \tag{2.37}$$

Using eq. (2.33) and solving for H^2, we obtain

$$H^2=\frac{8\pi\overline{G}}{3}\delta\tau_3\left(1+\left(\frac{8\pi\overline{G}}{3}\right)^2J\ell^4\delta\tau_3^2\right)+O(\delta\tau_3^4) \tag{2.38}$$

At large $\overline{P}$ the function J increases like $\overline{P}^6$ for generic values of α and like $\overline{P}^{21}$ for $\alpha = 5$ due to the behavior of the denominator in eq. (2.37).

The dependence of J on $\overline{P}$ is illustrated in figure 2.2 for several values of α. For intermediate values of $\overline{P}$, and $1 < \alpha < 5$, J can become negative, as seen between the cusps of $\ln|J|$ in the figure.

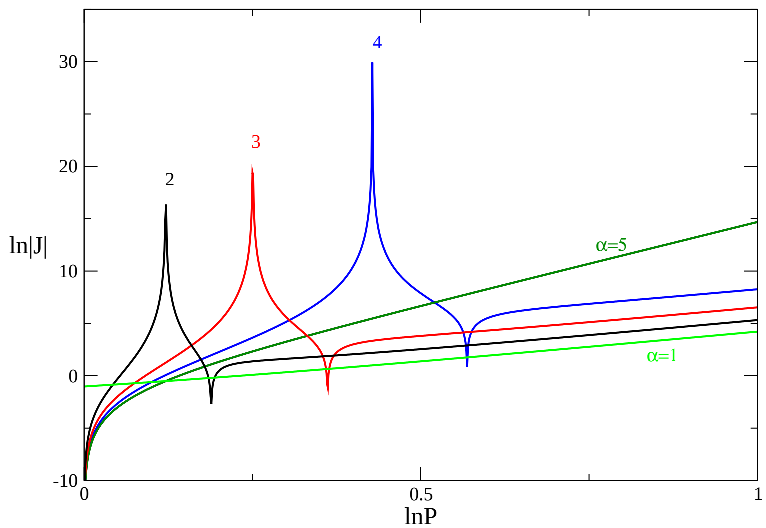

Figure 2.2: $\log|J|$ versus $\log\overline{P}$ for several values of α. J is positive except for the regions between cusps.

2.2.2 Time variation of Newton's constant

From eq. (2.32) we see that the 4D Planck mass depends on H and τ_3, the 3-brane tension. We can therefore anticipate that it will depend on time in a situation where the pure tension τ_3 is replaced by matter or radiation which gets diluted by the expansion of the universe. To quantify this dependence, we will consider the variation of M_p^2 (or equivalently Newton's constant G) with $\delta\tau_3$. We do this both analytically, in the region of small $\delta\tau_3$, and numerically for larger values of $\delta\tau_3$.

For small $\delta\tau_3$ we can differentiate eq. (2.32) with respect to τ_3, keeping in mind that P and ρ depend on $\mathcal{H}$, which in turns depends on τ_3. At leading order, we find that

$$\frac{d\ln G}{d\delta\tau_3} = -\frac{k}{\left(\overline{P}^3-1\right)^2(1-k\bar{\tau}_3)}\left(\frac{3}{2}\,\frac{\overline{P}^6\left(4\;(\alpha+1)\,\overline{P}^5-9\,\alpha+1\right)}{2\;(\alpha-5)\,\overline{P}^5+3\,\alpha+5}+\frac{3}{2}-\overline{P}^3(\overline{P}^3-1)\right) \tag{2.39}$$

at $\delta\tau_3 = 0$, where $k = k_6^2/(2\pi)$. Since the deficit angle should lie in a certain range, $i.e.$ $[0, 2\pi)$, $k\bar{\tau}_3$ must be in $[0, 1)$ which means the energy scale determining the 3-brane tension must be approximately equal to or below that of the 6D Plank mass. In the limit of large warping, $\overline{P} \gg 1$, this simplifies to

$$\frac{d\ln G}{d\delta\tau_3} \cong 2\,\frac{4+\alpha}{5-\alpha}\,\frac{k}{(1-k\bar{\tau}_3)} \tag{2.40}$$

assuming that $\alpha \neq 5$.

Let us compare this to experimental constraints on the time-variation of G, [125]

$$|d\ln G/dt| < 10^{-12}/y. \tag{2.41}$$

Since $d\rho/dt = -2\rho_0 t_0^2/t^3$, we take ρ_0 and t_0 to be the present values and $t \cong t_0 \cong 10^{10}$y, we see that the constraint implies that

$$\frac{d\ln G}{d\delta\tau_3} \lesssim 1.2\times 10^{44}/\ \mathrm{GeV}^4 \tag{2.42}$$

for $\alpha \sim 1$. Substituting into (2.40) we see that this constraint is very loose, requiring only that $M_6 \geq 10^{-11}$ GeV.

we can put another constraint. Since there is no observational changes in G around a dense object like a neutron star, if we treat the 3-brane as a dense object then G should not change very much spatially. From this we find $M_6 \geq$ 1Gev which is a much stronger constraint.

For the special case $\alpha = 5$, the above approximations are not valid, and the time variation of G is relatively large if the radial size of the extra dimension P is much greater than unity. We get

$$\frac{d\ln G}{d\delta\tau_3} \cong -\frac{9}{5}\,\overline{P}^5\,\frac{k}{(1-k\bar{\tau}_3)} \tag{2.43}$$

The constraint on $\bar{\tau}_3$ becomes more stringent in that case, by a factor of $\overline{P}^5$. For the interesting value of $P \sim 10^{16}$ which is required to tackle the hierarchy problem,

$\frac{k}{(1-k\bar{\tau}_3)} \lesssim 10^{-36}/\ (\mathrm{GeV}^4)$ gives $M_6 \geq 10^9$ GeV. From the spatial constraint we get $M_6 \geq M_p$. Alternatively one can have $P \sim 1$ and keep the constraint (2.42). In no case is there any constraint on $\bar{\tau}_3$, except that it cannot be too close to $1/k_6$.

2.2.3 Numerical results

Now we proceed to check and extend the above analytical results with numerical ones. For clarity of presentation we transform the units from $M_6 = 1$ to $\overline{M}_4 = 1$ given by (2.33).

We first check the Friedmann equations for the special case $\alpha = 5$, which displayed the largest derivation from the studied case in the perturbative treatment. Numerically we solve P from eq. (2.23) for each H and then substitute it into (2.23) to find τ_3, thus obtaining the relation of H and $\delta\tau_3 = \tau_3 - \overline{\tau}_3$. The parameters we choose are $T_4 = 1.12 \times 10^{-8}$, $c_0 = 8.17 \times 10^{-7}$ and $\Lambda = -1.56 \times 10^{-9}$. These are not special choices as well as other parameters below though they may seem strange, this is because they have been transformed from $k_6 = 1$ unit to $k_4 = 1$ unit. The main concern is that they should be below the 4D Planck scale. Figure 2.3 shows $\ln H^2$ vs $\ln\delta\tau_3$ at small $\delta\tau_3$ from numerical and analytical calculations. The discrepancy is small for $\ln\delta\tau_3 < -26.5$ which means (2.38) is a good approximation at low energy.

We also present results from numerical calculations for $\alpha = 5$ and $\alpha = 1$ (choosing the parameters $\Lambda = -3.34 \times 10^{-11}, c_0 = 6.01 \times 10^{-14}$ and $T_4 = 2.39 \times 10^{-10}$ because in units of $M_6 = 1$, $\overline{M}_4$ is different for different values of α so it is difficult to get the same parameters in $\overline{M}_4 = 1$ units for all values of α) since $\alpha = 1$ is a special case which has totally different features from others; see fig. 2.4 and fig. 2.5 respectively.

Fig. 2.3 has many different features compared to the Friedmann equation in 5D models. First, the Hubble rate is double valued. This means that there are two sets of points (H_1^2, P_1) and (H_2^2, P_2) corresponding to the same 3-brane tension τ_3; in other words, the 4-brane also affects the Hubble rate on the 3-brane. Second, figure 2.4 shows that there is a pole in H and the 3-brane tension is not continuous but has a gap–$\delta\tau_3$ cannot have values between $\sim 7 \times 10^{-12}$ and $\sim 13 \times 10^{-11}$. The pole occures when the left hand side of (2.23) vanishes. However, above the pole the

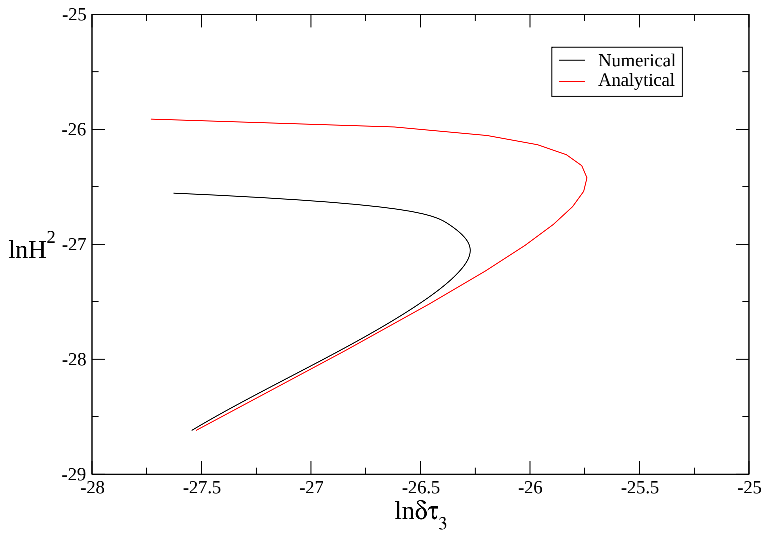

Figure 2.3: Friedmann relation from numerical and analytical methods

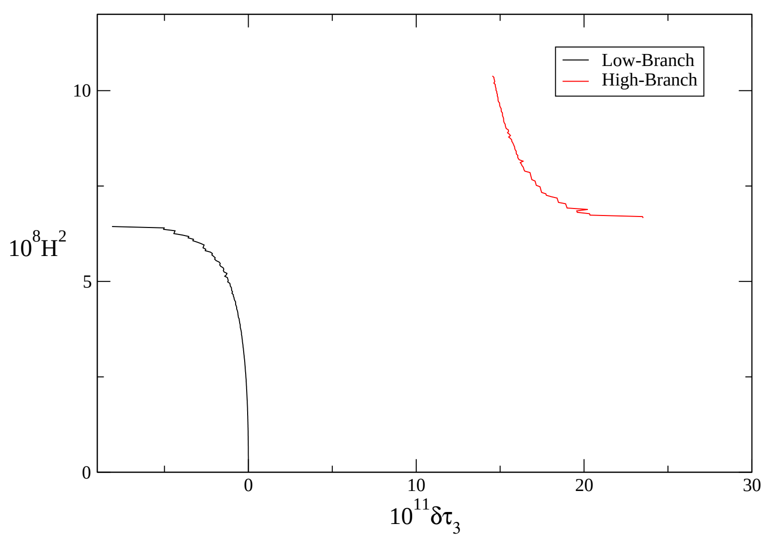

Figure 2.4: Friedmann equation for $\alpha = 5$

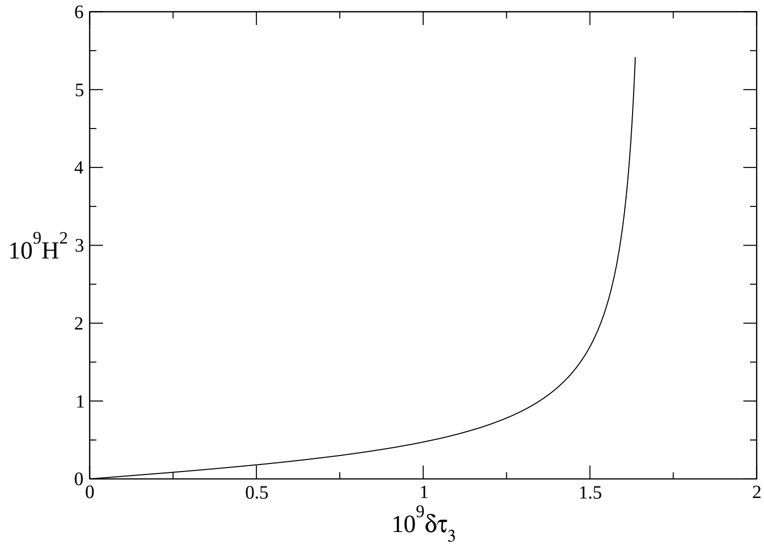

Figure 2.5: Friedmann equation for $\alpha = 1$

deficit angle exceeds 2π which makes the high-H branch unphysical. Also in the low-branch $\delta\tau_3$ cannot be too negative, in order that the 3-brane tension and the deficit angle remaining positive, thus the conical angle remains $< 2\pi$. These are common features shared by all odd values of $\alpha > 1$ cases. For even values of α greater than two two, all the features are the same except that there are no high-energy branches. This is because when the right hand of (2.23) is negative, which corresponds to the high-energy branch, we cannot extract a negative root of it by $1/\alpha$. For $\alpha = 2$, the situation is more complicated. The features of the Friedmann equation for $\alpha = 2$ depends on the parameters since the derivative of $\frac{dh}{dP}$ where $h = (H\rho\ell)^2$ can be zero for some values of parameters; see fig. 2.6. For those values of parameters the position of the 4-brane P is double valued and the Friedmann equation is like fig 2.7; for other values of parameters the Friedmann equation is like that of $\alpha = 5$. In any case in this model for $\alpha > 1$ the Hubble rate as well as the 3-brane tension has a maximum value. High energy corrections from the matter on the 3-brane are needed if large H regions are to be probed. Hereafter we are only interested in the low-energy branch.

For $\alpha = 1$, fig. 2.5 shows that the Hubble rate blows up at finite $\delta\tau_3$ which is very different from $\alpha > 1$ cases. This is because for $\alpha = 1$ there always exists a real

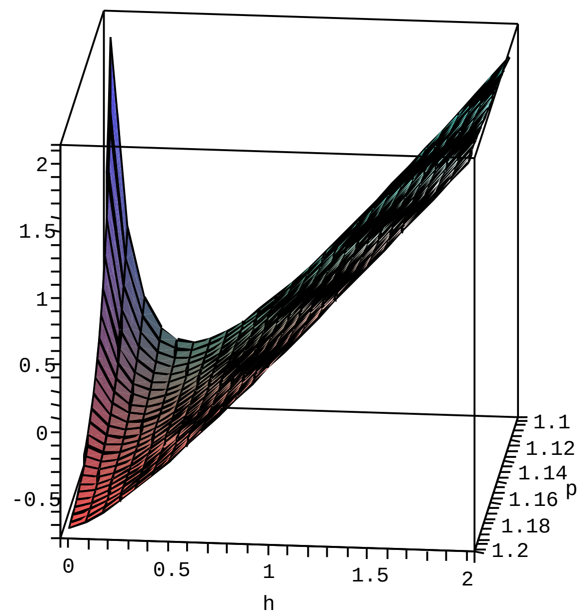

Figure 2.6: $\frac{dh}{dP}$

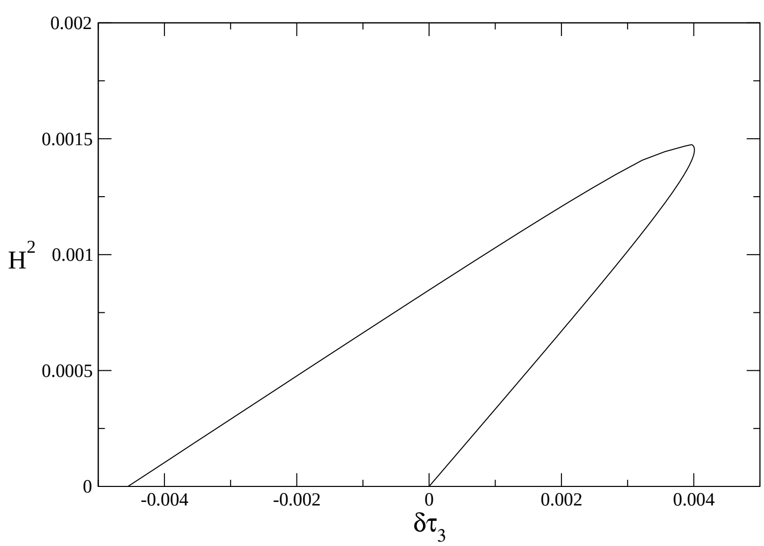

Figure 2.7: Friedmann equation for $\alpha = 2$

value of P which decreases as H increases and the physical distance between the two branes gets increasingly smaller. For $\alpha > 1$, whether P gets smaller or larger depends on $\overline{P}$, but the physical distance between the two branes always gets larger until the solution leaves the physical region. Fig. 2.8 shows the dependence of the physical distance on H for several values of α where for clarity we choose to start at the same $\bar{P}$ for all values of α.

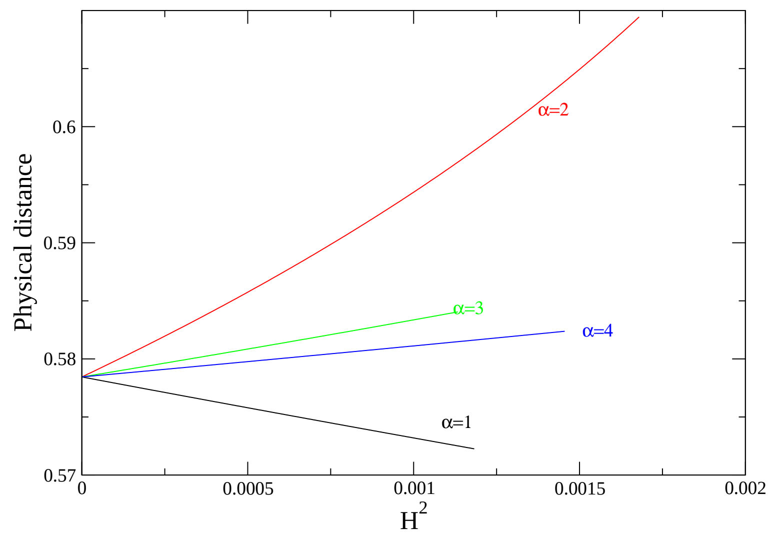

Figure 2.8: The physical distance in $k_6 = 1$ unit between two branes for different values of α

To make sure that the normal Friedmann equation is recovered at low energy which is essential for successful Big Bang Nucleosynthesis (BBN), we also calculate the ratio $\frac{H^2 - \frac{8\pi G^{(1)}}{3}\delta\tau_3}{H^2}$, which is the fractional deviation of H^2 in this model from the standard one. Fig. 2.9 shows that the deviation is 8% around $\delta\tau_3 \sim 3 \times 10^{-12}$ which corresponds to the energy of $(3.2 \times 10^{17}\mathrm{Me}V)^4$, and the deviation is much smaller at lower energies. For BBN to be successful it is required that the deviation of the Hubble rate is small ($< 10\%$) at scales $\sim$ MeV. Thus we confirm that BBN is not spoiled.

At high energy the deviation of H^2 from the standard Friedmann equation becomes large. Fig. 2.10 shows that phenomenon.

We also calculate the deviation for different values of α, see fig. 2.11. The devia-

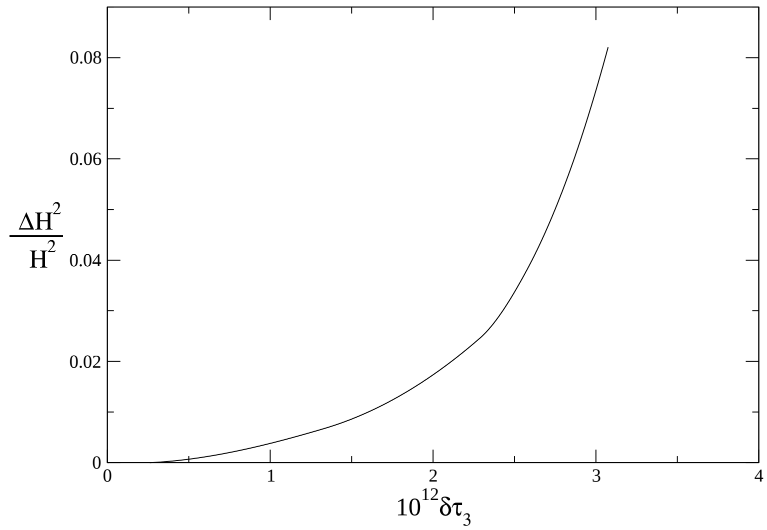

Figure 2.9: Fractional deviation in H^2 $\left(\frac{H^2-\frac{8\pi G^{(1)}}{3}\delta\tau_3}{H^2}\right)$, versus $\delta\tau_3$ at low energy

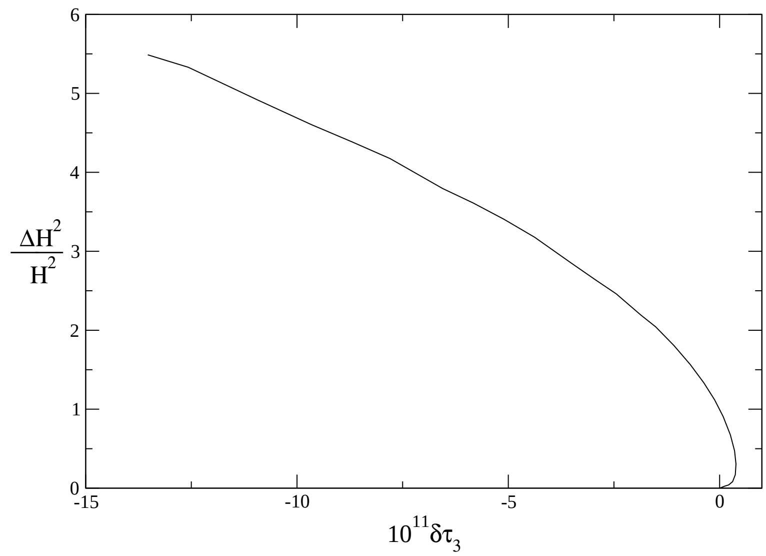

Figure 2.10: Fractional deviation in H^2 $\left(\frac{H^2-\frac{8\pi G^{(1)}}{3}\delta\tau_3}{H^2}\right)$, versus $\delta\tau_3$ for high H^2 region

tion is larger for smaller α and negative for $\alpha = 1$. The radion is stable for the dotted part of $\alpha = 6$ case while unstable for the solid part and other values of α which will be explained in section 3.3.

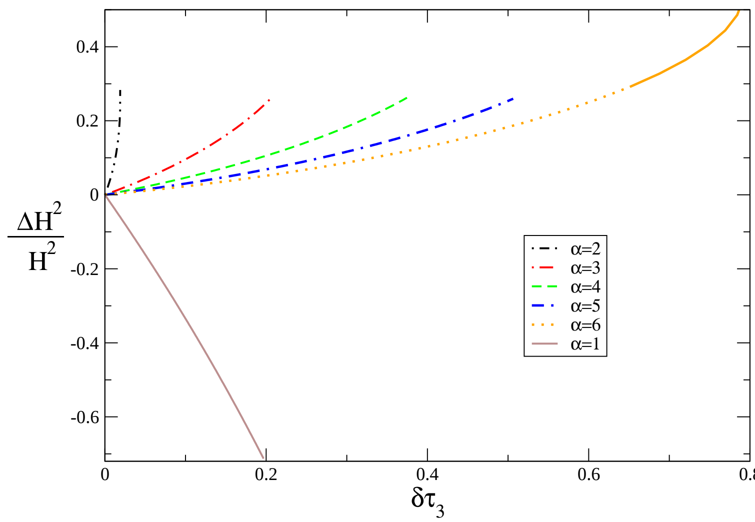

Figure 2.11: Deviation from normal FE for different values of α

Next, we check the variation of G with time. For each value of H we can calculate P, ρ and τ_3 as we did before. Using (2.32) we can find $\ln G$, and then the relation of $\ln G$ to τ_3. Numerically we find $\frac{dlnG}{d\delta\tau_3} < 10^{13}/(\mathrm{GeV}^4)$ at $\delta\tau_3 = 0$. In fact, for our set of parameters the experimental constraint (2.41) is satisfied for all $\delta\tau_3$ except those very close to the turning point; see fig. 2.12.

To summarize, the analytical results agree with the numerical results at low energy, so the analytical ones are good approximations; the Friedmann equation does not deviate from the standard one very much at low energy, so is consistent with BBN; the time variation of Newton's constant is also consistent with the constraint from lunar laser ranging experiments.

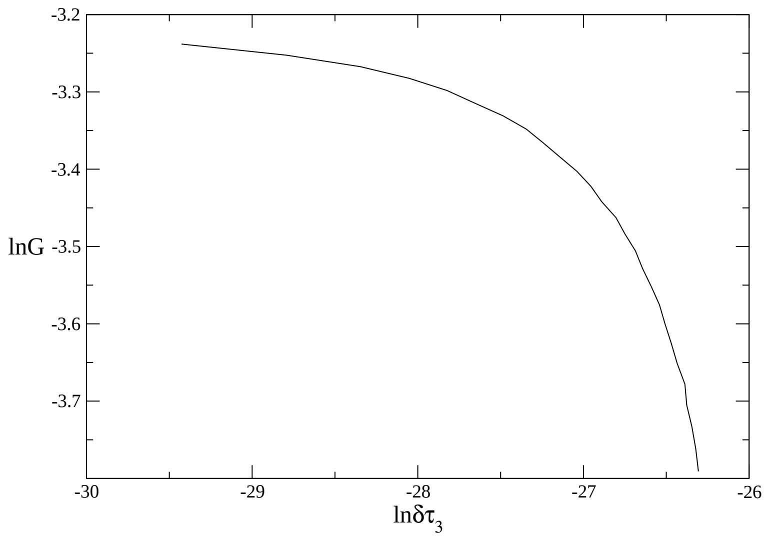

Figure 2.12: Variation of Newton's constant

2.3 Friedmann equation with Goldberger-Wise scalar field

Goldberger and Wise (GW) introduced a bulk scalar field [126], [133], [134] in 5D models to stabilize the bulk; this is important for getting the standard Friedmann equation in those models and for yielding the desired size of the extra dimension to solve the hierarchy problem.

We first briefly review the GW mechanism. In Randall and Sundrum model I (RS I) [5], the fifth dimension has a S_1/Z_2 orbifold symmetry, and two 3-branes with opposite tensions sit at the orbifold fixed points. There is also a fine-tuned bulk cosmological constant serving as a source for 5D gravity. The line element is

$$ds^2 = e^{-2kr_c|\phi|}\eta_{\mu\nu}dx^\mu dx^\nu - r_c^2 d\phi^2 \tag{2.44}$$

where ϕ is the fifth dimension $-\pi \le \phi \le \pi$. The two 3-branes are located at $\phi = 0$ and $\phi = \pi$ respectively. Randall and Sundrum showed [5] that the 4-dimensional Planck mass M_{pl} is related to a fundamental scale M_5 like:

$$M_4^2 = \frac{M_5^3}{k}\left[1 - e^{-2kr_c\pi}\right] \tag{2.45}$$

Because of the exponential factor in the metric, a field on the 3-brane at $\phi = \pi$ with mass m will have a physical mass $me^{-kr_c\pi}$ on the 3-brane at $\phi = 0$, thus if kr_c has a value of 12 the weak scale can be generated from a fundamental scale M_5 which is the same order as M_{pl}, thus solving the hierarchy problem. However, r_c which determines the size of the extra dimension has zero potential and is not fixed. Goldberger and Wise introduced a bulk scalar with interactions on the two 3-branes to generate a potential stabilizing the value of r_c. The bulk action is

$$S_b = \frac{1}{2}\int dx^4 \int_{-\pi}^{\pi} d\phi\sqrt{G}(G^{AB}\partial_A\Phi\partial_B\Phi - m^2\Phi^2) \tag{2.46}$$

and the interaction terms on the hidden ($\phi = 0$) and visible branes ($\phi = \pi$) are

$$\begin{aligned} S_h &= -\int dx^4\sqrt{-g_h}\lambda_h(\Phi^2 - v_h^2)^2 \\ S_v &= -\int dx^4\sqrt{-g_v}\lambda_v(\Phi^2 - v_v^2)^2 \end{aligned} \tag{2.47}$$

respectively. After solving the equation of motion for Φ and substituting it into the action and then integrating out the extra dimension, an effective 4-dimensional potential for r_c is found:

$$V_\Phi = k\epsilon v_h^2 + 4ke^{-4kr_c\pi}(v_v - v_h e^{-\epsilon kr_c\pi})^2\left(1+\frac{\epsilon}{4}\right) - k\epsilon v_h e^{-(4+\epsilon)kr_c\pi}(2v_v - v_h e^{-\epsilon kr_c\pi}) \tag{2.48}$$

where $\epsilon \simeq m^2/4k^2$ is a small quantity. Ignoring terms proportional to ϵ, the potential has a minimum at

$$kr_c = \left(\frac{4}{\pi}\right)\frac{k^2}{m^2}\ln\left[\frac{v_h}{v_v}\right] \tag{2.49}$$

Thus r_c is stabilized and there is no need to fine tune anything to get $kr_c \sim 12$.

Now we include the GW scalar field to see how it stabilizes the bulk in our model and changes the Friedmann equation. In this section we adopt a different coordinate system, whose line element is:

$$ds^2 = M^2(\tilde{r})[-dt^2 + e^{2Ht}dx^2] + d\tilde{r}^2 + L^2(\tilde{r})\, d\tilde{\theta}^2, \tag{2.50}$$

In this coordinate, the 3-brane always sits at the origin and we can rescale the coordinate so that $M(0) = 1$. This is more convenient for numerical calculations. Defining

$u = M'/M$, $v = L'/L$ [135], the Einstein and scalar field equations are

$$
\begin{aligned}
\mu\mu: &\qquad v' + 3u' + v^2 + 6u^2 + 3lu = \frac{\Lambda_4}{M^2} - k_6^2\left(\frac{1}{2}\phi' + V\right)\\
\theta\theta: &\qquad 4u' + 10u^2 = \frac{\Lambda_4}{M^2} - k_6^2\left(\frac{1}{2}\phi' + V\right)\\
\rho\rho: &\qquad 4uv + 6u^2 = \frac{\Lambda_4}{M^2} + k_6^2\left(\frac{1}{2}\phi' - V\right)\\
\phi: &\qquad \phi'' + (4u+v)\phi' = \frac{dV}{d\phi}
\end{aligned}
\tag{2.51}
$$

Here $V = (\frac{b^2}{2} + \frac{5b}{2l})\phi^2 - \frac{5}{32}b^2\phi^4 - \frac{10}{l^2}$, thus $\Lambda_6 = -\frac{10}{l^2}$ is absorbed into V. This potential is special because when $u = v$, *i.e.* in the limit of $\alpha = 0$ or $S_{\mu\nu} = S_{\theta\theta}$, there exists an analytical solution for the Einstein equation. The boundary conditions at the two branes are respectively

$$
\begin{aligned}
L'(0) &= 1 - \frac{\tau_3}{2\pi}\\
V_P + T_4 &= \left(6 + \frac{2}{\alpha}\right)u(P) + \left(2 - \frac{2}{\alpha}\right)v(P)\\
\frac{\alpha c_0}{2L_\theta^\alpha} &= v(P) - u(P)\\
\phi'(P) &= -\frac{1}{2}\frac{dV_P}{d\phi}
\end{aligned}
\tag{2.52}
$$

where V_P is the scalar potential at the 4-brane, which prevents $\phi'(P)$ from vanishing, and for simplicity we take $V_P = -\lambda\phi$.

To solve these equations, we randomly choose $b = 1$ and $\lambda = 4.212$. Note that only $v = \frac{L'}{L}$ is involved in the equations so we can assume $L'(0) = 1$ and then from the first and the third of eq. (2.52) we find

$$
\left(1 - \frac{k_6\tau_3}{2\pi}\right)^\alpha = \frac{\alpha c_0}{2(v(P) - u(P))L_\theta^\alpha}. \tag{2.53}
$$

Given a value of λ, for each H we first randomly choose $\phi(0)$ and integrate the equation system from zero to find a P where the second boundary conditions of eq. (2.52) is satisfied, and then use the fourth boundary condition to adjust $\phi(0)$ and iterate until both the second and the fourth conditions are satisfied. In this way we find $\delta\tau_3$ for each H.

With the same parameters as those in the previous subsection we found a similar unconventional Friedmann Equation for $\alpha = 5$; see fig. 2.13. The features of the Friedmann equation for each α do not change much from those without GW scalar field. However, the deviation at a given energy is much smaller than that in unstabi-

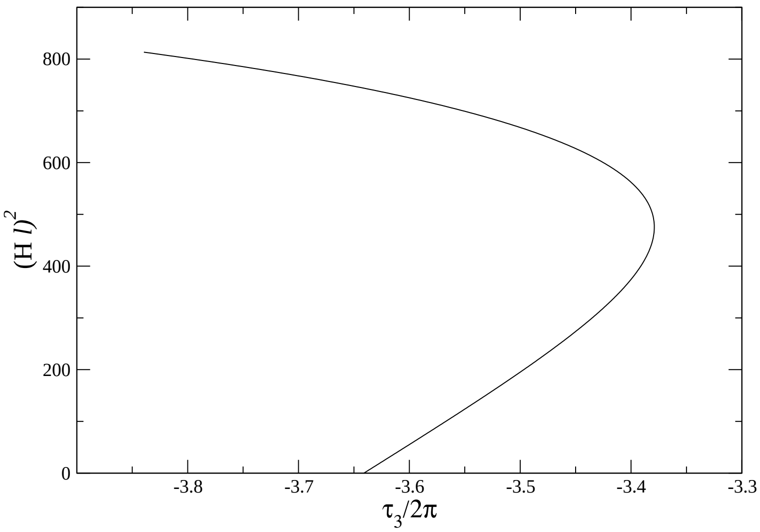

Figure 2.13: Friedmann equation with GW scalar field for $\alpha = 5$

lized cases; see fig. 2.14 for $\alpha = 5$. This suggests that the model is more stable with the scalar field.

This can be understood as follows: using the same procedure as GW mechanism an effective 4D potential of the 4-brane position P would be found, which is a function of λ, b and H^2. The bulk is stabilized if this potential has a minimum, otherwise it is unstable. For given λ and b, the potential only has a minimum for $H < H_c$, notice that $\lambda = b = 0$ represents the special case without the GW scalar field. We will discuss H_c and the stability in greater detail in the next chapter.

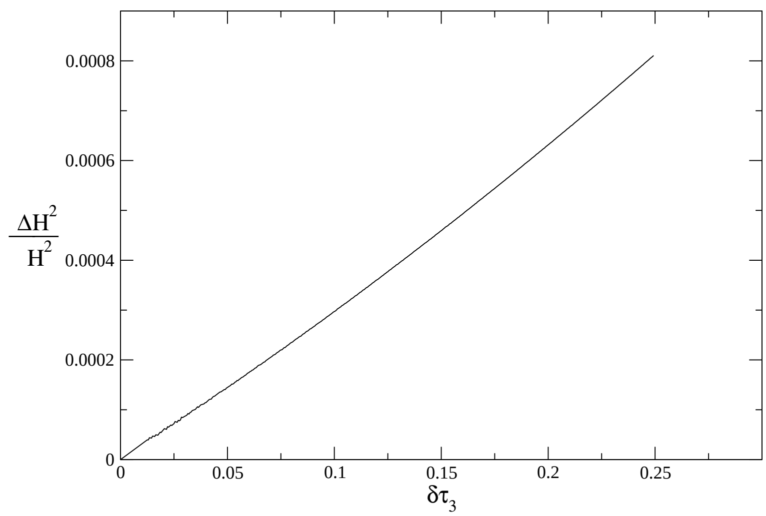

Figure 2.14: $\frac{H^2 - \frac{8\pi G^{(1)}}{3}\delta\tau_3}{H^2}$ versus τ_3 in the case with GW scalar field for $\alpha = 5$

Chapter 3

Stability

Based on experience with 5D models, one may suspect that the unusual Friedmann equation is due to the instability of the model. In the following we study the stability of the model with and without the GW scalar field.

Our stratagy is to perturb around the background solutions. The perturbation can be written as h_{AB}, where

$$ds^2 = (\,\mathcal{G}_{AB} + h_{AB}\,)\, dx^A dx^B \tag{3.1}$$

Without GW scalar field we have already found the background metric:

$$\begin{aligned}
\mathcal{G}_{tt} &= r^2 g_{tt} = -r^2 = -a(r) \\
\mathcal{G}_{ij} &= r^2 g_{ij} = r^2 \delta_{ij} e^{2\tilde{H}t} \\
\mathcal{G}_{\theta\theta} &= \frac{K^2}{\ell^2}\left(r^2 - \frac{1}{r^3} + \tilde{H}^2 l^2\right) = K^2 f(r) \\
\mathcal{G}_{rr} &= \frac{\ell^2}{r^2 - \frac{1}{r^3} + \tilde{H}^2 \ell^2}
\end{aligned} \tag{3.2}$$

Since with the scalar field there is no exact solution, we assume it to be small and treat it as a perturbation. Hence we use the same background to solve the field equation for the GW scalar field.

The general perturbation equation is [119]:

$$\frac{1}{2}\nabla_A \nabla_B h^D{}_D + \frac{1}{2}\nabla^2 h_{AB} - \nabla^D \nabla_{(A} h_{B)D} - \frac{5}{\ell^2} h_{AB} = 0. \tag{3.3}$$

If tachyons are found in any modes, it means that the background is unstable, otherwise it is stable.

The stability is also studied in with the Hubble rate $H = 0$, here we study a general case for each Hubble rate $H \neq 0$. Notice that stable or unstable only apply to a certain value of H not to two different values of H, *i.e.* for different values of H the position of the 4-brane is different but this does not imply instability. In the following, for simplicity, we assume that there is no dependence of h_{AB} on the extra angular dimension θ, *i.e.* we assume azimuthal symmetry.

3.1 Tensor modes

For tensor modes, $h_{\mu\nu}$ satisfies the transverse-traceless condition,

$$h_\mu{}^\mu = h_{\mu\nu}{}^{;\nu} = 0\,, \tag{3.4}$$

where ; denotes the covariant derivatives with respect to $g_{\mu\nu}$. Then eq. (3.3) becomes

$$h''_{\mu\nu} + \frac{f'}{f}h'_{\mu\nu} + \frac{\ell^2}{af}\left(\Box^{(4)}h_{\mu\nu} - h_{\mu\alpha;\nu}{}^{;\alpha} - h_{\nu\alpha;\mu}{}^{;\alpha}\right) + \left[\left(\frac{a'}{a}\right)^2 - \frac{10}{f}\right]h_{\mu\nu} = 0\,. \tag{3.5}$$

Using the condition (3.4), we can rewrite parenthesis in the third term of (3.5) as:

$$\Box^{(4)}h_{\mu\nu} - h^{;\alpha}_{\mu\alpha;\nu} - h^{;\alpha}_{\nu\alpha;\mu} = \left(\Box^{(4)} - 8\tilde{H}^2\right)h_{\mu\nu}\,. \tag{3.6}$$

Also rewriting $h_{\mu\nu}(r, x^\mu)$ as $a(r)\tilde{h}_{\mu\nu}(r, x^\mu)$, we have

$$\begin{aligned}\tilde{h}''_{\mu\nu} &+ \left(2\frac{a'}{a} + \frac{f'}{f}\right)\tilde{h}'_{\mu\nu} + \frac{\ell^2}{af}\left(\Box^{(4)} - 8\tilde{H}^2\right)\tilde{h}_{\mu\nu} \\ &+ \left[\frac{a''}{a} + \left(\frac{a'}{a}\right)^2 + \frac{a'f'}{af} - \frac{10}{f}\right]\tilde{h}_{\mu\nu} = 0\,.\end{aligned} \tag{3.7}$$

Now using the background Einstein equation,

$$\frac{a''}{a} + \left(\frac{a'}{a}\right)^2 + \frac{a'f'}{af} = \frac{10}{f} + \frac{6\tilde{H}^2\ell^2}{af}\,, \tag{3.8}$$

eq. (3.7) can be rewritten as

$$\tilde{h}''_{\mu\nu} + \left(2\frac{a'}{a} + \frac{f'}{f}\right)\tilde{h}'_{\mu\nu} + \frac{\ell^2}{af}\left(\Box^{(4)} - 2\tilde{H}^2\right)\tilde{h}_{\mu\nu} = 0\,. \tag{3.9}$$

We can decompose $h_{\mu\nu}$ as product of $\varepsilon_{\mu\nu}$ and $Z(r)$ depending on x^μ and r respectively,

$$\tilde{h}_{\mu\nu}(r, x^\mu) = \varepsilon_{\mu\nu}(x^\mu) Z(r) \tag{3.10}$$

where $\varepsilon_{\mu\nu}$ satisfies the transverse-traceless condition: $\varepsilon_\mu{}^\mu = \varepsilon_{\mu\nu}{}^{;\mu} = 0$. We finally obtain the perturbation equation in the bulk:

$$Z'' + \left(2\frac{a'}{a} + \frac{f'}{f}\right) Z' + \frac{\ell^2 M^2}{af} Z = 0. \tag{3.11}$$

$$\left(\Box^{(4)} - 2\tilde{H}^2 - M^2\right) \varepsilon_{\mu\nu} = 0 \tag{3.12}$$

where M^2 is a separation constant.

To find the equation at the 4-brane notice that the jump condition for the background Einstein equation is

$$f\left[\frac{3}{2}\left(\frac{a'}{a}\right)' + \frac{1}{2}\left(\frac{f'}{f}\right)'\right] \tilde{g}_{\mu\nu} = k_6^2 \sqrt{f}\, S_{\mu\nu}\, \delta(r - P)\,, \tag{3.13}$$

where $S_{\mu\nu}$ is defined by eq. (2.12). Thus the contribution from the fluctuation of the metric is:

$$-\frac{af}{2\ell^2}\left[Z'' + \left(2\frac{a'}{a} + \frac{f'}{f}\right) Z' + \frac{\ell^2 M^2}{af} Z\right] \varepsilon_{\mu\nu} - k_6^2 \sqrt{f}\, S_{\mu\nu}\, \delta(r - P)\,. \tag{3.14}$$

And the contribution from the fluctuation of matter on the 4-brane is:

$$-k_6^2 \sqrt{f}\, S_{\mu\nu}\, \delta(r - P)\,. \tag{3.15}$$

Thus the effect of the 4-brane matter gets cancelled out by the last term in (3.14) and we obtain the same equations as eq. (3.11).

Next we consider the boundary conditions for eq. (3.11). At $r = \rho$, where $f(\rho) = 0$, for $Z(r)$ to be regular at this point we impose the regularity condition:

$$Z'(\rho) \;\; = \;\; -\frac{M^2 \ell^2 \rho^2}{5 - 2\tilde{H}^2 \ell^2 \rho^3} Z(\rho)\,. \tag{3.16}$$

At $r = P$, since there is no delta-functional matter the junction condition becomes

$$Z'(P) \;\; = \;\; 0\,. \tag{3.17}$$

Note that the physical Hubble rate is related to $\tilde{H}$ by $\tilde{H} = \frac{H^2}{(1+H^2\ell^2)^{2/5}}$.

Table 3.1: Graviton masses for different values of Hubble rate

Mode Number	$H = 0.0497$	$H = 0.0995$
0	0	0
1	0.2056	0.3067
2	0.5301	0.7238
3	1.0250	1.3699
4	1.6933	2.2463

We use the shooting method to solve this differential equation and find the mass spectrum as in Table 3.1. We still work in the units $\overline{M}_4 = 1$ and choose $\Lambda_6 = -0.0995$ which is also used in the vector modes.

Zero modes which have solution $Z(r) = 1$ correspond to the usual 4D gravitational fluctuations. We see that there is no tachyonic excitation in the spectrum of tensor modes.

3.2 Vector modes

For the vector modes, $h_{\theta\mu}$ satisfies the transversality condition:

$$h_{\theta\mu}{}^{;\mu} = 0\,. \tag{3.18}$$

Then eq. (3.3) becomes

$$h''_{\theta\mu} + \frac{a'}{a}h'_{\theta\mu} + \frac{\ell^2}{af}\left(\Box^{(4)}h_{\theta\mu} - h_{\theta\alpha;\mu}{}^{;\alpha}\right) + \left(\frac{a'f'}{af} - \frac{10}{f}\right)h_{\mu\nu} = 0\,. \tag{3.19}$$

Replacing $h_{\theta\mu} = a(r)\tilde{h}_{\theta\mu}$, the above equation becomes

$$\tilde{h}''_{\theta\mu} + 3\frac{a'}{a}\tilde{h}'_{\theta\mu} + \frac{\ell^2}{af}\left(\Box^{(4)}\tilde{h}_{\theta\mu} - \tilde{h}_{\theta\alpha;\mu}{}^{;\alpha}\right) + \left[\frac{a''}{a} + \left(\frac{a'}{a}\right)^2 + \frac{a'f'}{af} - \frac{10}{f}\right]\tilde{h}_{\mu\nu} = 0\,. \tag{3.20}$$

Using the background Einstein eq. (3.8), we get

$$\tilde{h}''_{\theta\mu} + 3\frac{a'}{a}\tilde{h}'_{\theta\mu} + \frac{\ell^2}{af}\left(\Box^{(4)} + 3\tilde{H}^2\right)\tilde{h}_{\theta\mu} = 0 \tag{3.21}$$

Table 3.2: Vector masses for different values of Hubble rate

Mode Number	$H = 0.0497$	$H = 0.09950$
0	0.1458	0.3533
1	0.3583	0.5894
2	0.7674	1.1145
3	1.3483	1.8732
4	2.1040	2.8642

where we have used the relation: $\tilde{h}_{\theta\alpha;\mu}{}^{;\alpha} = 3\tilde{H}^2\tilde{h}_{\theta\mu}$ under the condition eq. (3.18). Thus, introducing the separation constant M^2 and decomposing $\tilde{h}_{\theta\mu}$ as

$$\tilde{h}_{\theta\mu} = Y(r)\, v_\mu(x)\,, \tag{3.22}$$

where $v_\mu{}^{;\mu} = 0$, we finally obtain

$$Y'' + 3\frac{a'}{a}Y' + \frac{M^2\ell^2}{af}Y = -2\left(\frac{f'}{f} - \frac{a'}{a}\right) Y\,\delta(r-P)\,, \tag{3.23}$$

$$\left[\Box^{(4)} + 3\tilde{H}^2 - M^2\right] v_\mu = 0\,. \tag{3.24}$$

Notice that we have included the contribution from the 4-brane:

$$\left[\frac{f}{\ell^2}\left(\frac{f'}{f} - \frac{a'}{a}\right) h_{\theta\mu}\right]\delta(r-P)\,. \tag{3.25}$$

The boundary condition at $r = \rho$ is:

$$Y(\rho) \;\; = \;\; 0\,, \tag{3.26}$$

and at $r = P$ is:

$$Y'(P) = \left(\frac{f'}{f} - \frac{a'}{a}\right)\Bigg|_{r=P} Y(P) = \frac{5 - 2\tilde{H}^2\ell^2P^3}{P^6 - P + \tilde{H}^2\ell^2P^4}\,Y(P)\,. \tag{3.27}$$

Using the same shooting method as before we find the first several excitations shown in Table 3.2. There are no zero modes here for $H \neq 0$, because they do not satisfy the boundary conditions.

3.3 Scalar modes

The metric we use which contains all possible scalar perturbations, is

$$ds^2 = a(r,t)\left[-dt^2 + e^{2\tilde{H}t}\delta_{ij}dx^i dx^j\right] + b(r,t)\,d\theta^2 + c(r,t)\,dr^2\,, \qquad (3.28)$$

We first write down the Einstein equations. The $(tt)+(ii)$, (rr), $(\theta\theta)$ and (tr) components are

$$\begin{aligned}
&2\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} - 3\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{2}\left(\frac{\dot{b}}{b}\right)^2 - \frac{1}{2}\left(\frac{\dot{c}}{c}\right)^2 - \frac{\dot{a}}{a}\left(\frac{\dot{b}}{b} + \frac{\dot{c}}{c}\right) - \tilde{H}\left(2\frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{c}}{c}\right) = 0\\
&\frac{c}{2a}\left[\frac{\ddot{b}}{b} + 3\frac{\ddot{a}}{a} - \frac{1}{2}\left(\frac{\dot{b}}{b}\right)^2 - \frac{3}{2}\left(\frac{\dot{a}}{a}\right)^2 + \frac{\dot{a}}{a}\frac{\dot{b}}{b} + 9\frac{\dot{a}}{a}\tilde{H} + 3\frac{\dot{b}}{b}\tilde{H} + 12\tilde{H}^2\right] - \frac{3}{2}\left(\frac{a'}{a}\right)^2 - \frac{a'}{a}\frac{b'}{b}\\
&\qquad -k_6^2 c\,\Lambda_6 = 0\\
&\frac{c}{2a}\left[\frac{\ddot{c}}{c} + 3\frac{\ddot{a}}{a} - \frac{1}{2}\left(\frac{\dot{c}}{c}\right)^2 - \frac{3}{2}\left(\frac{\dot{a}}{a}\right)^2 + \frac{\dot{a}}{a}\frac{\dot{c}}{c} + 9\frac{\dot{a}}{a}\tilde{H} + 3\frac{\dot{c}}{a}\tilde{H} + 12\tilde{H}^2\right] - 2\frac{a''}{a} - \frac{1}{2}\left(\frac{a'}{a}\right)^2\\
&\qquad +\frac{a'}{a}\frac{c'}{c} - k_6^2 c\,\Lambda_6 + V_\theta\sqrt{c}\delta\left(\tilde{r} - P\right) = 0\\
&6\frac{\dot{a}'}{a} - 6\frac{\dot{a}}{a}\frac{a'}{a} - \frac{\dot{b}}{b}\frac{a'}{a} - 3\frac{\dot{c}}{c}\frac{a'}{a} + 2\frac{\dot{b}'}{b} - \frac{\dot{b}}{b}\frac{b'}{b} - \frac{\dot{c}}{c}\frac{b'}{b} = 0 \qquad (3.29)
\end{aligned}$$

where for simplicity we denoted $S_{\mu\nu} \equiv -V_0\,\tilde{g}_{\mu\nu}$ and $S_{\theta\theta} \equiv -V_\theta\,\tilde{g}_{\theta\theta}$ in eq. (2.12). The small perturbation around the static background corresponds to

$$\begin{aligned}
a(r,t) &= a_0(r)(1 - a_1(r,t))\,, \quad a_0 = r^2\\
b(r,t) &= b_0(r)(1 - b_1(r,t))\,, \quad b_0 = f(r)K^2\,, \quad f(r) = \left(r^2 - \frac{1}{r^3} + \tilde{H}^2\ell^2\right)/\ell^2\\
c(r,t) &= c_0(r)(1 + c_1(r,t))\,, \quad c_0 = f^{-1}(r) \qquad (3.30)
\end{aligned}$$

Now use the ansatz $\ddot{a}_1 + 3\tilde{H}\dot{a}_1 = -m_r^2 a_1$ and similarly for b_1 and c_1 to expand the Einstein equations $(tt)+(ii)$, (rr) and (tr) to first order:

$$2a_1 + b_1 - c_1 = 0 \qquad (3.31)$$

$$\left(3\frac{a_0'}{a_0} + \frac{b_0'}{b_0}\right)a_1' - \frac{a_0'}{a_0}b_1' + \frac{a_0'}{a_0}\left[\frac{3}{2}\frac{a_0'}{a_0} + \frac{b_0'}{b_0}\right]c_1 + \frac{c_0\,m_r^2}{2a_0}(3a_1 + b_1) + 6\tilde{H}^2\frac{c_0}{a_0}a_1 = 0 \qquad (3.32)$$

$$b_1' + 3\,a_1' + \frac{1}{2}\left(\frac{b_0'}{b_0} - \frac{a_0'}{a_0}\right)b_1 + \frac{1}{2}\left(\frac{b_0'}{b_0} + 3\frac{a_0'}{a_0}\right)c_1 = 0 \qquad (3.33)$$

Imposing Z_2 symmetry across the 4-brane for the (tt) and $(\theta\theta)$ components of the Einstein equations, we find

$$\left[\frac{1}{\sqrt{c_0}}\left(3\frac{a'}{a}+\frac{b'}{b}\right)\right]\Big|_{r=P} = k_6^2 V_0\sqrt{c}\,\big|_{r=P}\,, \tag{3.34}$$

$$\frac{4\,a'}{\sqrt{c_0}\,a}\Big|_{r=P} = k_6^2 V_\theta\sqrt{c}\,\big|_{r=P}\,. \tag{3.35}$$

The first order perturbations of these junction conditions are

$$\left[\frac{3a_1'+b_1'}{\sqrt{c_0}}\right]\Big|_{r=P} = \left[-\frac{k_6^2}{2}V_0 c_1 - k_6^2\delta V_0\right]\Big|_{r=P}, \tag{3.36}$$

$$\frac{4a_1'}{\sqrt{c_0}}\Big|_{r=P} = \left[-\frac{k_6^2}{2}V_\theta\, c_1 - k_6^2\delta V_\theta\right]\Big|_{r=P} \tag{3.37}$$

where we have used the background Einstein equations. Furthermore, since $L_\theta = \int\theta\sqrt{b} = 2\pi\sqrt{b}$ we can expand V_0, V_θ (assuming T_4 is constant) as follows:

$$V_0 = T_4 + c_0 L_\theta^\alpha, \qquad \delta V_0 = \alpha\, c_0\, L_\theta^{-\alpha-1}\pi b_1\sqrt{b_0}\,, \tag{3.38}$$

$$V_\theta = T_4 + (1-\alpha)L_\theta^{-\alpha}, \qquad \delta V_\theta = -\alpha(\alpha-1)\, c_0\, L_\theta^{-\alpha-1}\pi b_1\sqrt{b_0} \tag{3.39}$$

Also using the background equations, we find

$$L_\theta^{-\alpha-1} = \frac{1}{k_6^2 2\pi\alpha\sqrt{b_0\, c_0^3}}\left(-\frac{a_0'}{a_0}+\frac{b_0'}{b_0}\right) \tag{3.40}$$

Thus Eqs. (3.36) and (3.37) lead to

$$[\,3a_1'+b_1'\,]\,\big|_{r=P} = \left[-\left(\frac{3a_0'}{2a_0}+\frac{b_0\prime}{2b_0}\right)c_1 - \frac{1}{2}\left(-\frac{a_0'}{a_0}+\frac{b_0'}{b_0}\right)b_1\right]\Big|_{r=P}, \tag{3.41}$$

$$a_1'\,\big|_{r=P} = \left[-\frac{a_0'}{2a_0}c_1 + \frac{\alpha-1}{8}\left(-\frac{a_0'}{a_0}+\frac{b_0'}{b_0}\right)b_1\right]\Big|_{r=P} \tag{3.42}$$

Combining Eqs. (3.41) and (3.42), we finally obtain the junction condition at the 4-brane as

$$[\,b_1' - a_1'\,]_{r=P} = -\frac{1}{2}\left(\frac{b_0'}{b_0}-\frac{a_0'}{a_0}\right)(c_1+\alpha b_1)\,\big|_{r=P} \tag{3.43}$$

Note that there are not really two boundary conditions at the 4-brane. This is because by imposing the momentum constraint eq. (3.33) which implies the conservation of fluctuational momentum on the junction condition eq. (3.42) we will recover eq. (3.43) exactly.

At the 3-brane, since we do not perturb the 3-brane tension τ_3 we assume that the deficit angle is unchanged by perturbations around the static solution. Considering a circle with radius $r = \rho + \epsilon$ from the 3-brane, the circumference L over the physical radius D should be invariant in the limit $\epsilon \to 0$:

$$\lim_{\epsilon \to 0} \delta \left(\frac{L}{D} \right) = \lim_{\epsilon \to 0} \delta \left(\frac{\int d\theta \sqrt{b}}{\int_{\rho}^{\rho+\epsilon} dr \sqrt{c}} \right) = -\frac{L_0}{2D_0} \left(b_1 + c_1 \right) \Big|_{r=\rho} = 0 \tag{3.44}$$

where

$$L_0 = \int d\theta \sqrt{b_0} \quad , D_0 = \int dr \sqrt{c_0} \tag{3.45}$$

In this way, we find the boundary condition at 3-brane to be

$$[\, b_1 + c_1] \,\Big|_{r=\rho} = 0 \,. \tag{3.46}$$

Now define the variable X as

$$X = 3a_1 + b_1 \;. \tag{3.47}$$

By using the linear combinations $(t,t) + e^{2\tilde{H}t}(i,i)$ and (t,r) of the components of the Einstein equation, the variables a_1, b_1 and c_1 can be expressed in terms of X as

$$\begin{aligned} a_1 &= \frac{1}{2B_0'}[(B_0' + A_0')X - X'] \\ b_1 &= \frac{3}{2B_0'}\left[\left(-\frac{B_0'}{3} - A_0'\right) X + X'\right] \\ c_1 &= \frac{1}{2B_0'}[X' + (B_0' - A_0')X] \end{aligned} \tag{3.48}$$

where $\prime$ denotes the derivative with respect to r and $A_0' = -\frac{a_0'}{a_0}$ and similarly for B_0' and C_0'.

3.3.1 Radion Mass Analysis

In this subsection we study the radion mass without GW scalar field stabilization. Combining Eqs. (3.31) , (3.32) and (3.33), we get an

$$X'' + \left[\frac{4}{r} + 2\frac{f'}{f} - \frac{f''}{f'} - \frac{6f}{r^2 f'} + \frac{6\tilde{H}^2 \ell^2}{r^2 f'} \right] X'$$

$$+\left[\frac{m^2\ell^2}{r^2 f}-\frac{2f''}{rf'}+\frac{4f'}{rf}-\frac{12f}{r^3 f'}+\frac{6\tilde{H}^2\ell^2}{r^2 f}+\frac{12\tilde{H}^2\ell^2}{r^3 f'}\right]X=0 \qquad (3.49)$$

The boundary conditions Eqs. (3.43) and (3.46) become

$$\left[X'+\frac{2}{r}X\right]\Bigg|_{r=\rho}=0 \qquad (3.50)$$

$$\left[X''+\left\{\frac{3\alpha+13}{8}\frac{f'}{f}-\frac{f''}{f'}-\frac{3\alpha-7}{4r}\right\}X' \right.$$
$$\left.+\left\{\frac{\alpha+5}{2r}\frac{f'}{f}+\frac{\alpha-1}{8}\left(\frac{f'}{f}\right)^2-\frac{3\alpha+5}{2r^2}-\frac{2f''}{rf'}\right\}X\right]\Bigg|_{r=P}=0 \quad (3.51)$$

Eliminating X'' from eq. (3.51) using eq. (3.49), we find

$$\left[\left\{\frac{3(\alpha-1)}{8}\frac{f'}{f}-\frac{3(\alpha+3)}{4r}+\frac{6f}{r^2 f'}-\frac{6\tilde{H}^2\ell^2}{r^2 f'}\right\}X'+\left\{-\frac{m^2\ell^2}{r^2 f}+\frac{\alpha-3}{2r}\frac{f'}{f}\right.\right.$$
$$\left.\left.+\frac{\alpha-1}{8}\left(\frac{f'}{f}\right)^2-\frac{3\alpha+5}{2r^2}+\frac{12f}{r^3 f'}-\frac{6\tilde{H}^2\ell^2}{r^2 f}-\frac{12\tilde{H}^2\ell^2}{r^3 f'}\right\}X\right]\Bigg|_{r=P}=0 \quad (3.52)$$

Thus if we solve eq. (3.49) under the boundary condition (3.50) and (3.52), we can obtain the radion mass spectrum. In the asymptotic region $r\to\infty$ we have

$$X'\left(\frac{15(5-\alpha)}{2r^6}-\frac{3\tilde{H}^2l^2}{r^3}(1-\alpha)\right)$$
$$+X\left(-\frac{20(\alpha-5)}{r^7}+\frac{4m^2l^2}{r^4}+\frac{8\tilde{H}^2l^2}{r^4}(\alpha+1)\right)\Bigg|_{r=P}=0 \qquad (3.53)$$

Thus the radion mass can be expressed as:

$$m^2=\frac{5(\alpha-5)}{l^2r^3}-2(\alpha+1)\tilde{H}^2+\frac{15(\alpha-5)}{8l^2r^2}\frac{X'}{X}-\frac{3}{4}(\alpha-1)\tilde{H}^2\left(\frac{rX'}{X}\right)\Bigg|_{r=P} \qquad (3.54)$$

which is consistent with the result in [118] when $\tilde{H}=\frac{H^2}{(1+H^2\ell^2)^{2/5}}=0$. For $\alpha=5$ the radion mass is almost zero.

In the region $r\to\infty$ the order of X'/X is r^{-4}, so we ignore the last two terms and get

$$m^2=\frac{5(\alpha-5)}{l^2P^3}-2(\alpha+1)\tilde{H}^2 \qquad (3.55)$$

This expression is accurate as long as the 4-brane is far from the 3-brane, *i.e.* $P\gg 1$.

The radion mass is always negative for $\alpha < 5$, but can be positive, below a critical Hubble rate, for $\alpha > 5$, which has been shown for $\alpha = 6$ in figure. 2.11 as an example. As for $\alpha = 5$, where the anisotropy of the 4-brane stress tensor is provided by the Casimir effect [136], it is still possible to have a positive squared mass which is proportional to $\overline{H}^2$ when the two branes are close enough.

3.3.2 Stability with Goldberger-Wise field

In this section we use the GW mechanism to stabilize the model.

We assume that the bulk scalar is small enough that it does not change the background metric. The perturbed equations become

$$
\begin{aligned}
& X'' + \left(\frac{3A_0'^2 - 4A_0'B_0' - 2B_0'^2 - 2B_0'' - k^2\phi'^2}{2B_0'} - \frac{6c_0\tilde{H}}{a_0 B_0'} \right) X' + \left(\frac{2A_0'B_0'' - 2A_0''B_0'}{2B_0'} \right. \\
& \quad \left. + \frac{(A_0' - B_0')(k^2\phi_0'^2 - 3A_0'^2 - 2A_0'B_0')}{2B_0'} + \frac{c_0}{a_0} m^2 + \frac{6c_0}{a_0}\frac{\tilde{H}^2}{B_0'}(A_0' + B_0') \right) X \\
& \quad + \left(\frac{2B_0'' - 2B_0'\phi_0'' - (3A_0'^2 + 2A_0'B_0') + k^2\phi_0'^3}{2B_0'}\phi_0' + \frac{6c_0}{a_0}\frac{\tilde{H}^2}{B_0'}\phi_0' - c_0\frac{dV}{d\phi} \right) 2k^2\phi_1 = 0 \\
& \phi'' - \frac{1}{2}(4A_0' + B_0' + C_0')\phi_1' + \left(-c_0\frac{d^2V}{d\phi_0^2} + \frac{c_0}{B_0}\frac{dV}{d\phi_0}k^2\phi_0' + \frac{c_0}{a_0}m^2 \right)\phi_1 - \left(\phi_0' + \frac{c_0}{2B_0'}\frac{dV}{d\phi} \right) X' \\
& \quad + \frac{c_0}{2B_0'}(A_0' - B_0')\frac{dV}{d\phi_0} X = 0
\end{aligned}
\tag{3.56}
$$

and the boundary conditions are

$$
\begin{aligned}
& X' - A_0'X = 0|_\rho \\
& \left(\frac{3(\alpha+3)A_0' - 3(\alpha-1)B_0' - 12A_0'^2 + 4k^2\phi_0'^2}{8B_0'} + \frac{6c_0}{a_0}\frac{\tilde{H}^2}{B_0'} \right) X' + \left[-\frac{c_0}{a_0}m^2 \right. \\
& \quad \left. - \frac{6c_0}{a_0}\frac{\tilde{H}^2}{B_0'}(A_0' + B_0') + \frac{(B_0' - A_0')((-7+3\alpha)A_0'B_0' + (\alpha-1)B_0'^2) - 12A_0'^2 + 4k^2\phi_0'^2}{8B_0'} \right] X \\
& \quad + \left(\frac{\phi_0'(1+3\alpha)B_0'^2 + (7-3\alpha)A_0'B_0' + 12A_0'^2 - 4k^2\phi_0'^2}{8B_0'} - \frac{6c_0}{a_0}\frac{\tilde{H}^2}{B_0'}\phi_0' + c_0\frac{dV}{d\phi} \right) 2k^2\phi_1 \\
& \quad - 2k^2\phi_0'\phi_1' = 0|_P
\end{aligned}
\tag{3.57}
$$

In the region $r \to \infty$, the mass spectrum is

$$
\begin{aligned}
m^2 = \quad & \frac{5(\alpha-5)}{l^2r^3} - 2(\alpha+1)\tilde{H}^2 + \frac{15(\alpha-5)}{8l^2r^2}\left(\frac{X'}{X}\right) - \frac{3}{4}(\alpha-1)\tilde{H}^2\left(\frac{rX'}{X}\right) \\
& - \frac{k^2\phi_0'^2}{4l^2}\left(\frac{rX'}{X}\right) - \frac{k^2\phi_0'^2\tilde{H}^2}{4r}\left(\frac{X'}{X}\right)\Big|_{r=P}.
\end{aligned} \tag{3.58}
$$

Notice that X'/X is negative, see fig. 3.1, so the radion mass is indeed larger when including the GW scalar field. However, from fig. 3.1 we see that $e^{\int \frac{x'}{x}} \to C$ when $r \to \infty$ where C is a constant, so the order of X'/X is always smaller than r^{-1}, thus the second term in (3.58) dominates when $\tilde{H}$ or H grows large and eventually the radion mass squared becomes negative.

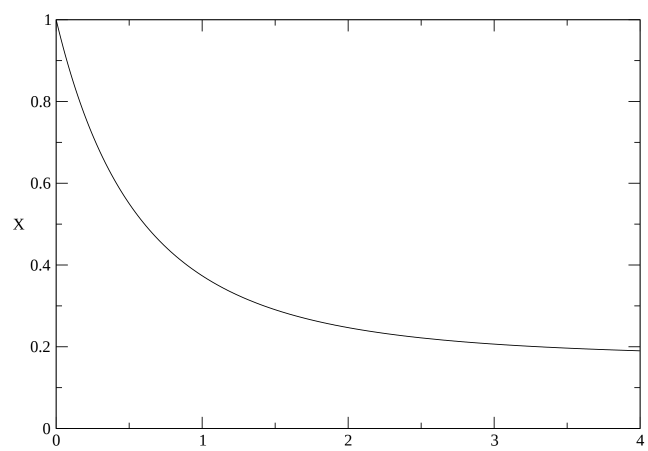

Figure 3.1: Radion wave function X

In conclusion, in the absence of the scalar field the stability of the compactification depends on the value of α and H; in particular, for $\alpha < 5$ the model is unstable for all H, while for $\alpha \geq 5$ it is stable for $H < H_c$ but unstable for $H > H_c$, where H_c is a critical value such that

$$
\frac{H_c^2}{(1+H_c^2\ell^2)^{2/5}} = \frac{5(\alpha-5)}{2(\alpha+1)\ell^2P^3}. \tag{3.59}
$$

When the scalar field is included, we can stabilize all cases up to some H_c', where

$H_c' > H_c$ for each $\alpha \geq 5$. H_c' can be obtained by setting the radion mass to be zero in eq. (3.58). In all cases the model becomes unstable above the critical Hubble rate.

Chapter 4

Application of the Friedmann equation to inflation

Inflation in braneworld models has been investigated extensively [75], [128]-[132]. In this chapter we use the Friedmann equation obtained in section (2.2.1) to study a simple model of chaotic inflation on the 3-brane and find that inflation can occur at the inflaton field values below the 4D Planck scale. We assume that the inflaton field on the 3-brane is varying slowly and thus its relation to H can be approximated as that of 3-brane tension and H. We focus on the low energy part of the results of section 2.2.1 and the special case of $\alpha = 5$.

The Friedmann equation is approximated by

$$H^2 = \frac{\delta\tau_3}{3\bar{M}_4^2} + \frac{J\ell^4\delta\tau_3^3}{27\bar{M}_4^6} \tag{4.1}$$

For $\alpha = 5$, $J = \frac{162\overline{P}^{19} - 459\overline{P}^{14} + 397\overline{P}^{9} - 75\overline{P}^{3} - 25}{625(\overline{P}^{3} - 1)}$.

The slow-roll conditions are

$$V \gg \frac{1}{2}\dot{\varphi}^2, \qquad 3H|\dot{\varphi}| \gg |\ddot{\varphi}| \tag{4.2}$$

which expressed in terms of V are

$$\epsilon = \frac{\bar{M}_4^2}{6}\frac{V'^2}{V^2}\frac{1}{1+\frac{J\ell^4V^2}{9\bar{M}_4^4}} \ll 1, \qquad \eta = \frac{\bar{M}_4^2}{3}\frac{V''}{V}\frac{1}{1+\frac{J\ell^4V^2}{9\bar{M}_4^4}} \ll 1 \tag{4.3}$$

where we have already used the slow roll approximation:

$$3H\dot{\varphi} \simeq -V'(\varphi), \qquad \rho \simeq V(\varphi) \tag{4.4}$$

In the limit $J \to 0$ the usual relation is obtained.

The condition for inflation is [75], [124]

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 > 0 \tag{4.5}$$

This is obviously satisfied if $\dot{H}$ is positive, ingeneral we need

$$\frac{\dot{H}}{H^2} > -1 \tag{4.6}$$

Then the condition for inflation expressed in terms of V becomes

$$\begin{aligned} \zeta &\equiv \frac{\bar{M}_4^2}{6}\frac{V'^2}{V^2}\frac{1+\frac{3J\ell^4V^2}{9\bar{M}_4^4}}{(1+\frac{J\ell^4V^2}{9\bar{M}_4^4})^2} = \epsilon + 2\epsilon\frac{\frac{J\ell^4V^2}{9\bar{M}_4^4}}{1+\frac{J\ell^4V^2}{9\bar{M}_4^4}} \\ &< 3\epsilon \ll 1 \end{aligned} \tag{4.7}$$

So this condition is always satisfied if slow roll condition is satisfied.

The number of e-foldings is given by:

$$\begin{aligned} N &= \int_{t_i}^{t_f} H dt \simeq \int_{\varphi_f}^{\varphi_i} \frac{\frac{V}{\bar{M}_4^2} + \frac{J\ell^4V^3}{9\bar{M}_4^6}}{V'} d\varphi \\ &= \frac{1}{\bar{M}_4^2}\left[\left(\frac{1}{4}\varphi_i^2 - \frac{1}{4}\varphi_f^2\right) + \left(\frac{J\ell^4m^4\varphi_i^6}{432\bar{M}_4^4} - \frac{J\ell^4m^4\varphi_f^6}{432\bar{M}_4^4}\right)\right] \end{aligned} \tag{4.8}$$

Here we used a simple potential $V = \frac{1}{2}m^2\varphi^2$.

The key test of an inflation model is the spectrum of perturbations produced due to quantum fluctuations. The curvature perturbation on uniform density hyper-surfaces is related to the inflation perturbation by

$$\mathcal{R} = -\frac{H}{\dot{\varphi}}\delta\varphi \tag{4.9}$$

If the inflaton mass is negligible, the field fluctuations at Hubble crossing ($k = aH$) in the slow-roll limit are given by $\langle\delta\varphi^2\rangle \simeq (H/2\pi)^2$. For a single inflaton field, the curvature perturbation $\mathcal{R}$ is related to the density perturbation by $P = 4\langle\mathcal{R}^2\rangle/25$. Using the slow-roll equations we find

$$P \simeq \frac{1}{600\pi^2}\frac{m^2\varphi^4}{\bar{M}_4^6}\left(1 + \frac{J\ell^4m^4\varphi^4}{36\bar{M}_4^4}\right)^3 \tag{4.10}$$

The scale-independence of the perturbation is described by the spectral index as

$$n - 1 \equiv \frac{d \ln P}{d \ln k} = -\frac{8\bar{M}_4^2}{\varphi^2} \frac{1 + \frac{J\ell^4 m^4 \varphi^4}{9\bar{M}_4^4}}{\left(1 + \frac{J\ell^4 m^4 \varphi^4}{36\bar{M}_4^4}\right)^2} \tag{4.11}$$

Assuming that inflation ends at $\zeta = 1$, then

$$\frac{2\bar{M}_4^2}{\varphi_f^2} \frac{1 + \frac{J\ell^4 m^4 \varphi_f^4}{12\bar{M}_4^4}}{\left(1 + \frac{J\ell^4 m^4 \varphi_f^4}{36\bar{M}_4^4}\right)^2} = 1 \tag{4.12}$$

and impose the constraint of cobe normalization on the density perturbation of equation (4.10) $P = 4 \times 10^{-10}$ [124], which gives

$$\frac{m^2 \varphi_c^4}{\bar{M}_4^6} \left(1 + \frac{J\ell^4 m^4 \varphi_c^4}{36\bar{M}_4^4}\right)^3 \simeq 2.4 \times 10^{-6} \tag{4.13}$$

we find that if J is very small, *i.e.* $\overline{P}$ is close to 1, then all the terms in N, n, ζ, P containing J can be ignored and the third order correction does not give a different result from ordinary chaotic inflation. However, if J is so large that those terms containing J dominate then we find

$$\begin{aligned} n - 1 &= -\frac{8\bar{M}_4^2}{\varphi^2} \frac{144\bar{M}_4^4}{J\ell^4 m^4 \varphi^4} \\ \frac{J\ell^4 m^4 \varphi_c^6}{432\bar{M}_4^6} - \frac{J\ell^4 m^4 \varphi_f^6}{432\bar{M}_4^6} &= 60 \\ \frac{J\ell^4 m^4 \varphi_f^6}{216\bar{M}_4^6} &= 1 \\ \frac{J^3 \ell^1 2 m^{14} \varphi_c^{16}}{46656\bar{M}_4^{18}} &\simeq 2.4 \times 10^{-6} \end{aligned} \tag{4.14}$$

assuming that $N = 60$. From the above equations we find

$$\begin{aligned} J\ell^4 \varphi_c^{10} &\simeq 2.11 \times 10^6 \bar{M}_4^6 \\ J\ell^4 m^{10} &\simeq 36.1 \bar{M}_4^6 \end{aligned} \tag{4.15}$$

Thus as long as $J\ell^4$ is large enough, for example 2.11×10^6 times of $1/\bar{M}_4^4$ or more, chaotic inflation can occur when the field is below 4D Planck mass. This is possible if $\overline{P}$ is large enough, in other words, if the warp factor is large enough. For our parameters $\ell = \sqrt{80}$ and $\bar{M}_4 = 94.6$ in $k_6 = 1$ unit, to satisfy the above condition for

J, $\overline{P}$ only need to be larger than 4.1×10^{-6} while actually $\overline{P} = 5.1$. However large $\overline{P}$ is the perturbative approach is still valid because notice that when perturbing around the static solution we do not impose any constraint on the value of $\overline{P}$.

Chapter 5

Conclusions and future work

In this thesis we studied a codimension-two braneworld model with a 3-brane sitting at the origin and a 4-brane sitting at some distance away, truncating the extra dimensions. The 3-brane tension is used to model the effects of matter on the brane, and we find that the Hubble rate on the 3-brane changes according to the standard Friedmann equation at low energy but deviates from it at high energy. When the GW scalar field is absent the stability of the model depends on the parameter α which quantifies different mechanisms for generating the difference between the $\mu\nu$ and $\theta\theta$ components of the energy-momentum tensor of the 4-brane. For $\alpha < 5$ the model is unstable whatever value the Hubble rate takes; for any $\alpha \geq 5$ the model is stable for H below a critical value of the Hubble rate H_c but unstable for H above H_c. When the GW scalar field is included the model becomes stable at small Hubble rate for $\alpha < 4$ and the critical values H_c for $\alpha \geq 5$ increase respectively. Using a larger GW scalar field results in larger H_c. We also applied the unusual Friedmann equation to study the inflation on the 3-brane. It shows that if the two branes are far away enough then the chaotic inflation can occur at the value of the inflaton smaller than the 4D Planck mass.

It would be interesting to study in more detail the physics of the stability. As shown by Mukohyama *et al.* [87] the stability may have profound relation with the entropy of the model. To study this we first have to define the entropy. Though the whole space-time is anti-deSitter, it can be considered that there is a deSitter 3-brane at each point in the extra dimensions and thus it is possible to extend the entropy definition of the four dimensional deSitter space-time to this model, *i.e.* define the entropy as one quarter of the surface area of the boundary.

In addition, the unusual Friedmann equation itself might be interesting for studies of dark energy and inflation on the brane. There are other ways to drive inflation on the 3-brane, such as changing the 4-brane tension and the six dimensional cosmological constant. However, if we want to have a more realistic model we should study more complicated cases with matter fields on the brane instead of the pure tension and see what happens for the Friedmann equation and its consequences.

Appendix A

Result of $\frac{d^3\overline{P}}{d\mathcal{H}^3}$

We give the result of $\frac{d^3\overline{P}}{d\mathcal{H}^3}$ used in section 2.2.1 to find the Friedmann equation.

$$
\begin{aligned}
\frac{d^3\overline{P}}{d\mathcal{H}^3} \;=\; & -6\overline{P}^4\Big[\left(512\,\alpha^5 - 6656\,\alpha^4 + 23552\,\alpha^3 - 89600\,\alpha + 5120\,\alpha^2 - 64000\right)\overline{P}^{28} \\
& + \left(-58240\,\alpha^3 + 156800\,\alpha^2 - 280000 + 8512\,\alpha^4 - 56000\,\alpha - 448\,\alpha^5\right)\overline{P}^{25} \\
& + \left(-99840\,\alpha^2 + 106240\,\alpha^3 + 5760\,\alpha^5 + 630400\,\alpha + 64000 - 51200\,\alpha^4\right)\overline{P}^{23} \\
& + \left(54000\,\alpha + 43200\,\alpha^4 + 64800\,\alpha^3 - 32400\,\alpha^5 - 129600\,\alpha^2\right)\overline{P}^{21} \\
& + \left(12880\,\alpha^4 - 655200\,\alpha^2 + 54880\,\alpha^3 + 490000 - 1680\,\alpha^5 + 1246000\,\alpha\right)\overline{P}^{20} \\
& + \left(-194080\,\alpha^3 + 38160\,\alpha^4 - 94000 + 5040\,\alpha^5 - 570000\,\alpha - 495840\,\alpha^2\right)\overline{P}^{18} \\
& + \left(-82350\,\alpha^4 - 47250\,\alpha - 180900\,\alpha^3 - 33750 + 76950\,\alpha^5 + 267300\,\alpha^2\right)\overline{P}^{16} \\
& + \left(-47040\,\alpha^4 + 385280\,\alpha^3 - 280000 - 2016000\,\alpha - 336000\,\alpha^2\right)\overline{P}^{15} \\
& + \left(35280\,\alpha^4 + 170000 + 611360\,\alpha^2 + 19600\,\alpha - 10800\,\alpha^5 + 485280\,\alpha^3\right)\overline{P}^{13} \\
& + \left(-145800\,\alpha^2 + 35100\,\alpha^4 - 67500\,\alpha + 67500 + 167400\,\alpha^3 - 56700\,\alpha^5\right)\overline{P}^{11} \\
& + \left(1225000\,\alpha + 7560\,\alpha^5 + 35000 + 35280\,\alpha^3 + 1089200\,\alpha^2 - 98280\,\alpha^4\right)\overline{P}^{10} \\
& + \left(84700\,\alpha - 1620\,\alpha^5 - 111500 - 134460\,\alpha^4 - 136440\,\alpha^2 - 356040\,\alpha^3\right)\overline{P}^{8} \\
& + \left(4050\,\alpha^4 - 33750 + 60750\,\alpha + 8100\,\alpha^2 + 12150\,\alpha^5 - 51300\,\alpha^3\right)\overline{P}^{6} \\
& + \left(-18900\,\alpha^4 - 318500\,\alpha - 279720\,\alpha^3 + 17500 + 11340\,\alpha^5 - 558600\,\alpha^2\right)\overline{P}^{5} \\
& + \left(-9360\,\alpha^2 + 60048\,\alpha^3 + 13608\,\alpha^5 + 23000 + 56376\,\alpha^4\overline{P}^3 - 12600\,\alpha\right)\overline{P}^{3}
\end{aligned}
$$

$$
+33453\,\alpha^4+5103\,\alpha^5+28875\,\alpha+81270\,\alpha^3+85050\,\alpha^2-4375\Big]
\Big/\Big[125(-10\overline{P}^5+2\,\alpha\overline{P}^5+5+3\,\alpha)^5\Big]
\tag{A.1}
$$

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